APPENDIX A THE MPUMALANGA TO WESTERN CAPE POWER ANGLE

This appendix computes the power angle, δ, that develops between the voltage, $\boldsymbol{E}'_{MPU}$, behind transient reactance of the Mpumalanga generation pool and the voltage, $\boldsymbol{E}'_{KBG}$, behind transient reactance of the Western Cape generation pool.

δ is computed for the case where the generators are modelled using the PSS/E generator model GENROU (Chapter 4, section 4.1.1).

When the generators are modelled using the constant voltage behind reactance model the voltage behind transient reactance is situated along the generator q-axis – i.e. angle δ_i shown in figure A.2 is zero.

A.1 EQUATION FOR δ

δ is the power angle by which $\boldsymbol{E}'_{MPU}$ leads $\boldsymbol{E}'_{KBG}$. Therefore, δ can be computed using (figure A.1):

$$\delta = \delta_{MPU} - \delta_{KBG} \tag{A.1}$$

Where δ_{MPU} is the angle by which the voltage, $\boldsymbol{E}'_{MPU}$, behind transient reactance of the Mpumalanga generation pool leads the synchronous rotating reference and δ_{KBG} is the angle by which the voltage, $\boldsymbol{E}'_{KBG}$, behind transient reactance of the Western Cape generation pool leads the synchronous rotating reference.

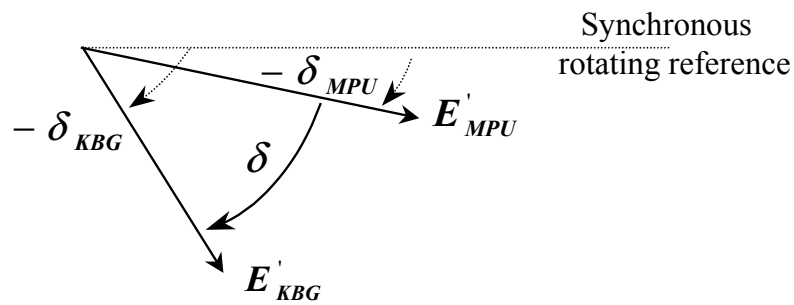

FIGURE A.1 Defining the angles δ, δ_{MPU} and δ_{KBG}

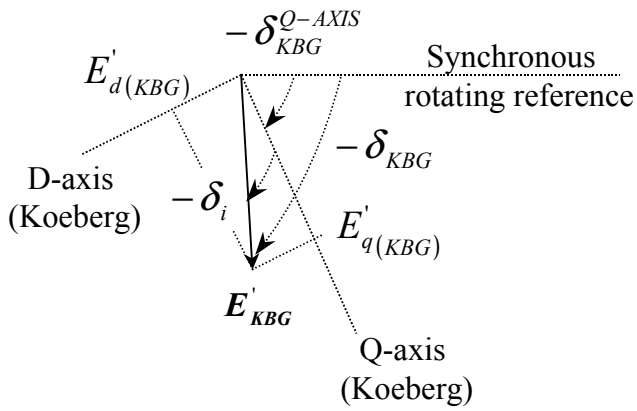

FIGURE A.2 Defining δ_{KBG}, δ_{KBG}^{Q-AXIS}, δ_i, the d-axis, $E'_{d(KBG)}$, and q-axis, $E'_{q(KBG)}$, components of the voltage behind transient reactance of the Koeberg generator

δ_{KBG} is made up (figure A.2) of the angle, δ_i, by which E'_{KBG} leads the generator q-axis and the angle, δ_{KBG}^{Q-AXIS}, by which the generator q-axis leads the synchronous rotating reference. Therefore, δ_{KBG} can be computed using:

$$\delta_{KBG} = \delta_{KBG}^{Q-AXIS} + \delta_i \tag{A.2}$$

PSS/E computes δ_{KBG}^{Q-AXIS}, $E'_{d(KBG)}$ and $E'_{q(KBG)}$ at each time step. Therefore, δ_{KBG} can be computed using (figure A.2):

$$\delta_{KBG} = \delta_{KBG}^{Q-AXIS} \pm \tan^{-1}\left(\frac{E'_{d(KBG)}}{E'_{q(KBG)}}\right) \tag{A.3}$$

The sign of $\tan^{-1}\left(E'_{d(KBG)} / E'_{q(KBG)}\right)$ is determined by whether $\boldsymbol{E'_{KBG}}$ lags the q-axis, negative sign, or leads the q-axis, positive sign.

Figure A.3 shows that $E'_{q(KBG)} > 0$ and $E'_{d(KBG)} > 0$ for at least the first slip cycle - first slip cycle is completed when the simulated time is 1.474 seconds (section A.3).

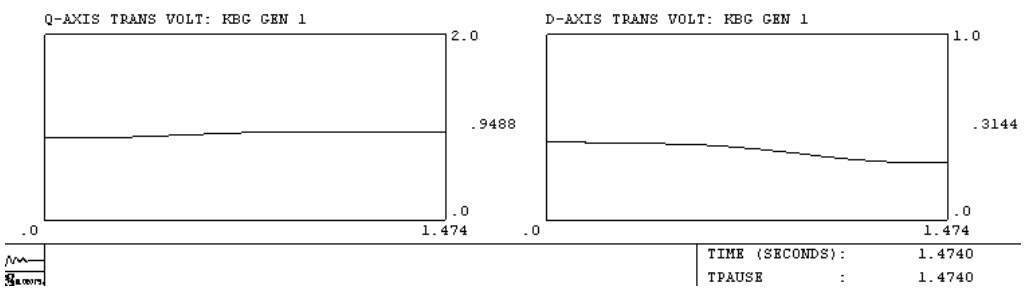

FIGURE A.3 Values the d-axis and q-axis components of the voltage behind transient reactance obtain at Koeberg during the first slip cycle

The above shows $\boldsymbol{E'_{KBG}}$ lags the q-axis. Consequently, δ_{KBG} can be computed using:

$$\delta_{KBG} = \delta_{KBG}^{Q-AXIS} - \tan^{-1}\left(\frac{E'_{d(KBG)}}{E'_{q(KBG)}}\right) \tag{A.4}$$

The Mpumalanga generation pool is made up of the Primary Pool and the Secondary Pool (Chapter 3, section 3.8.1). Therefore, δ_{MPU} can be computed using:

$$\delta_{MPU} = \frac{\delta_{PP} + \delta_{SP}}{2} \tag{A.5}$$

Where δ_{PP} is the angle by which the voltage, $\boldsymbol{E}'_{PP}$, behind transient reactance of the Primary Pool leads the rotating reference and δ_{SP} is the angle by which the voltage, $\boldsymbol{E}'_{SP}$, behind transient reactance of the Secondary Pool leads the rotating reference.

Matla power station represents the northern border and Arnot power station represents the southern border of the Primary Pool (Chapter 3, section 3.8.1). Therefore, the angle δ_{PP} can be computed using:

$$\delta_{PP} = \frac{\delta_{MAT} + \delta_{ARN}}{2} \tag{A.6}$$

Where δ_{MAT} is the angle by which the voltage, $\boldsymbol{E}'_{MAT}$, behind transient reactance of the Matla generator leads the rotating reference and δ_{ARN} is the angle by which the voltage, $\boldsymbol{E}'_{ARN}$, behind transient reactance of the Arnot generator leads the rotating reference.

The Secondary Pool is made up of the Tutuka and Kendal power stations (Chapter 3, section 3.8.1). Therefore, the angle δ_{SP} can be computed using:

$$\delta_{SP} = \frac{\delta_{TUT} + \delta_{KEN}}{2} \tag{A.7}$$

Where δ_{TUT} is the angle by which the voltage, $\boldsymbol{E}'_{TUT}$, behind transient reactance of the Tutuka generator leads the rotating reference and δ_{KEN} is the angle by

which the voltage, $\boldsymbol{E}'_{KEN}$, behind transient reactance of the Kendal generator leads the rotating reference.

The factors considered when deriving the equations for δ_{MAT} and δ_{ARN} are similar to the factors considered when deriving the equation for δ_{KBG}. This implies, δ_{PP} can be computed using:

$$\delta_{PP} = \frac{\left[\delta_{MAT}^{Q-AXIS} \pm \tan^{-1}\left(\frac{E'_{d(MAT)}}{E'_{q(MAT)}}\right)\right] + \left[\delta_{ARN}^{Q-AXIS} \pm \tan^{-1}\left(\frac{E'_{d(ARN)}}{E'_{q(ARN)}}\right)\right]}{2} \qquad \text{(A.8)}$$

Where δ^{Q-AXIS} is the angle by which the generator q-axis leads the rotating reference - the sub script, e.g. MAT, indicates the generator referred to; E'_d is the d-axis component of the transient voltage and E'_q is the q-axis component of the transient voltage.

The $\pm$ operator present in equation A.8 becomes the – operator when $\boldsymbol{E}'$ lags the generator q-axis. When $\boldsymbol{E}'$ leads the generator q-axis the + operator is used. In our application both $\boldsymbol{E}'_{MAT}$ and $\boldsymbol{E}'_{ARN}$ lag the generator q-axis.

Similarly, δ_{SP} can be computed using:

$$\delta_{SP} = \frac{\left[\delta_{TUT}^{Q-AXIS} \pm \tan^{-1}\left(\frac{E'_{d(TUT)}}{E'_{q(TUT)}}\right)\right] + \left[\delta_{KEN}^{Q-AXIS} \pm \tan^{-1}\left(\frac{E'_{d(KEN)}}{E'_{q(KEN)}}\right)\right]}{2} \qquad \text{(A.9)}$$

Where δ^{Q-AXIS} is the angle by which the generator q-axis leads the rotating reference - the sub-script, e.g. TUT, indicates the generator referred to; E'_d is the

d-axis component of the transient voltage and $E_q^{'}$ is the q-axis component of the transient voltage.

The $\pm$ operator present in equation A.9 becomes the – operator when $\boldsymbol{E}^{'}$ lags the generator q-axis. When $\boldsymbol{E}^{'}$ leads the generator q-axis the + operator is used. In our application, both $\boldsymbol{E}_{\boldsymbol{TUT}}^{'}$ and $\boldsymbol{E}_{\boldsymbol{KEN}}^{'}$ lag the generator q-axis.

Consequently, the power angle δ can be computed using:

$$\delta = \frac{\left[\frac{\delta_{MAT} + \delta_{ARN}}{2} + \frac{\delta_{TUT} + \delta_{KEN}}{2}\right]}{2} - \delta_{KBG} \qquad \text{(A.10)}$$

A.2 COMPUTING δ_{KBG}

To illustrate how δ is computed it is sufficient to show how one of the angles that make up δ is computed. We chose to compute the angle δ_{KBG}.

The PSS/E routine that computes δ, sets δ_{KBG} equal to -232.940° at the end of the first slip cycle (figure A.4) - first slip cycle is completed when the simulated time is 1.474 seconds (section A.3).

During the first slip cycle $\boldsymbol{E}_{\boldsymbol{d}(\boldsymbol{KBG})}^{'} > 0$ and $\boldsymbol{E}_{\boldsymbol{q}(\boldsymbol{KBG})}^{'} > 0$ (Figure A.4). Hence, δ_{KBG} can be computed using:

$$\delta_{KBG} = \delta_{KBG}^{Q-AXIS} - \tan^{-1}\left(\frac{E_{d(KBG)}^{'}}{E_{q(KBG)}^{'}}\right) \qquad \text{(A.11)}$$

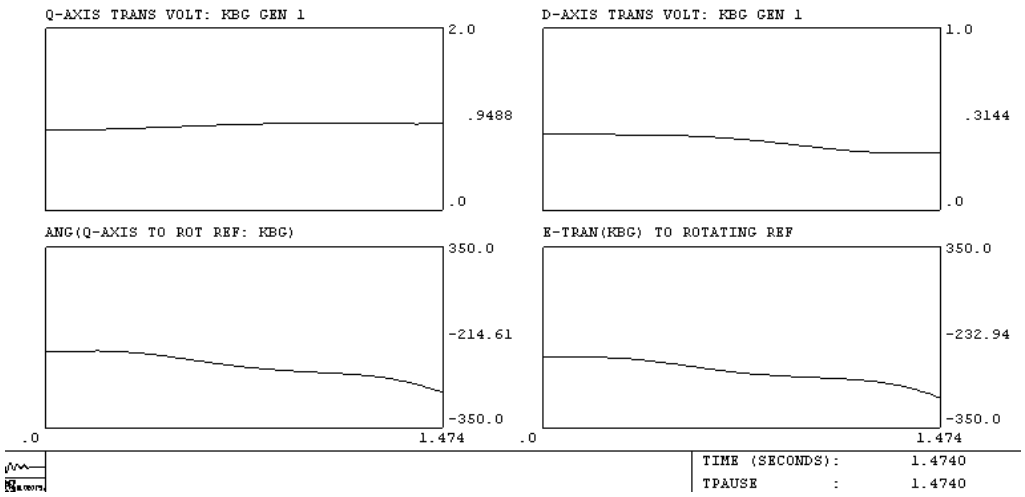

FIGURE A.4 The values of the d- and q-axis components of the voltage, $\boldsymbol{E}'_{\boldsymbol{KBG}}$, behind transient reactance at Koeberg; the rotor angle at Koeberg and the angle that develops between the rotating reference and $\boldsymbol{E}'_{\boldsymbol{KBG}}$ at the end of the first slip cycle

According to figure A.4, $E'_{d(KBG)} = 0.3144$ per unit, $E'_{q(KBG)} = 0.9488$ per unit and $\delta_{KBG}^{Q-AXIS} = -214.61^{\circ}$ at the end of the first slip cycle.

Consequently:

$$\delta_{KBG} = \delta_{KBG}^{Q-AXIS} - \tan^{-1}\left(\frac{E'_{d(KBG)}}{E'_{q(KBG)}}\right)$$

$$= -214.610^{\circ} - \tan^{-1}\left(\frac{0.3144}{0.9488}\right)$$

$$= -232.943^{\circ} \qquad \text{(A.12)}$$

A.3 COMPUTING δ

The PSS/E routine used to compute δ sets δ equal to 95.179° when the network is in steady state. Consequently, when $\delta = 455.179^{\circ} = (95.179^{\circ} + 360.0^{\circ})$ one slip cycle is completed (figure A.5).

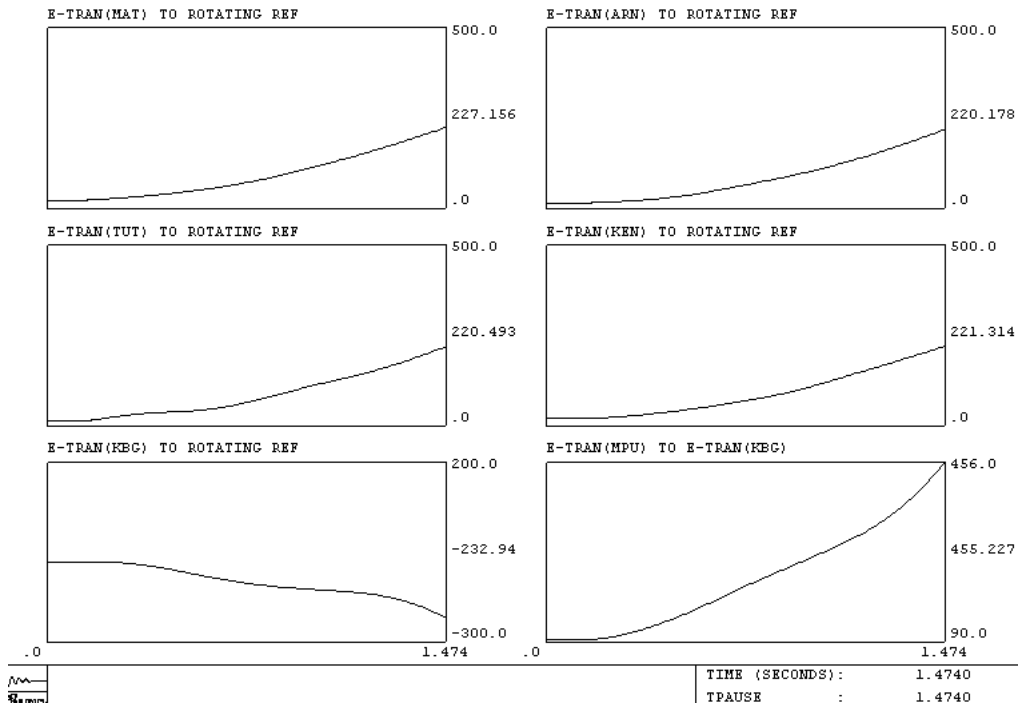

FIGURE A.5 The angles that develop between the synchronous rotating reference and $\boldsymbol{E}'_{KBG}$, $\boldsymbol{E}'_{MAT}$, $\boldsymbol{E}'_{ARN}$, $\boldsymbol{E}'_{TUT}$ and $\boldsymbol{E}'_{KEN}$

When $\delta = 455.179^{\circ}$, the simulated time is 1.474 seconds (Figure A.5) – i.e. first slip cycle is completed when the simulated time is 1.474 seconds. Note, the first 0.1 seconds of the time simulated models steady state - done to see if the steady state network is numerical stable.

The remainder of this section shows how $\delta = 455.179^{\circ}$ is computed.

According to equation A.10, δ is made up of the angles δ_{MAT}, δ_{ARN}, δ_{TUT}, δ_{KEN}, δ_{KBG} and can be computed using:

$$\delta = \frac{\left[\frac{\delta_{MAT} + \delta_{ARN}}{2} + \frac{\delta_{TUT} + \delta_{KEN}}{2}\right]}{2} - \delta_{KBG} \tag{A.13}$$

At the end of the first slip cycle $\delta_{KBG} = -232.94^{\circ}$, $\delta_{MAT} = 227.156^{\circ}$, $\delta_{ARN} = 220.178^{\circ}$, $\delta_{TUT} = 220.493^{\circ}$ and $\delta_{KEN} = 221.314^{\circ}$ (Figure A.5). Therefore, at the end of the first slip cycle δ is equal to:

$$\delta = \frac{\left[\frac{\delta_{MAT} + \delta_{ARN}}{2} + \frac{\delta_{TUT} + \delta_{KEN}}{2}\right]}{2} - \delta_{KBG}$$

$$= \frac{\left[\frac{227.156^{\circ} + 220.178^{\circ}}{2} + \frac{220.493^{\circ} + 221.314^{\circ}}{2}\right]}{2} - \left(-232.94^{\circ}\right)$$

$$= 222.285^{\circ} + 232.94^{\circ}$$

$$= 455.225^{\circ} \tag{A.14}$$