CHAPTER 5

VERTICAL SHEAR

5.1. Introduction

This chapter discusses the vertical shear in composite slabs using lipped channel members. The vertical shear strength of the member is based on the strength at which a flexure-shear crack forms. In the case of composite members using lipped channel beams without shear reinforcement the shear force is re-distributed across the inclined crack by the dowel action.

A well known formula for shear strength of a beam without shear reinforcement is used to quantify the shear strengths of experimental beams tested at the University of the Witwatersrand (Komakec,2002), California State University (Nguyen,1989) and University of Windsor (Addel-Sayed,1982). It was discovered from the comparative study that the equation used and discussed in the balance of the chapter gives accurate results for beams of high slenderness ratios (as based on the shear length of the beam), and seem to give very conservative values for beams of low to intermediate slenderness ratios. Nguyen(1989) concluded that vertical shear failure values as derived from the flexural-shear failure equation if far too inaccurate and that the shear-bond rupture equation as discussed in chapter 3 is more relevant.

5.2. Background Theory

The bending stress and shear stress distribution over the depth of a beam consisting of a homogeneous, isotropic and linearly elastic material is shown on Fig. 5.1 (a) and Fig. 5.1 (b). From classic theory of mechanics, the normal stress (f) and the shear stress (υ) of the elements A_1 and A_2 at a distance y from the neutral axis are :

$$f = \frac{My}{I}$$

and

$$\upsilon = \frac{VA\bar{y}}{Ib}$$

where :

	M =	bending moment at cross section a-a
	V =	shear force at cross section a-a
	A =	cross sectional area of the section at a plane passing through the centroid of the elements $a_1 - a_1$ and $a_2 - a_2$
	y =	distance from the element to the neutral axis
	$\bar{y}$ =	distance from the centroid of A to the neutral axis
	I =	moment of inertia of the cross section
	b =	width of beam

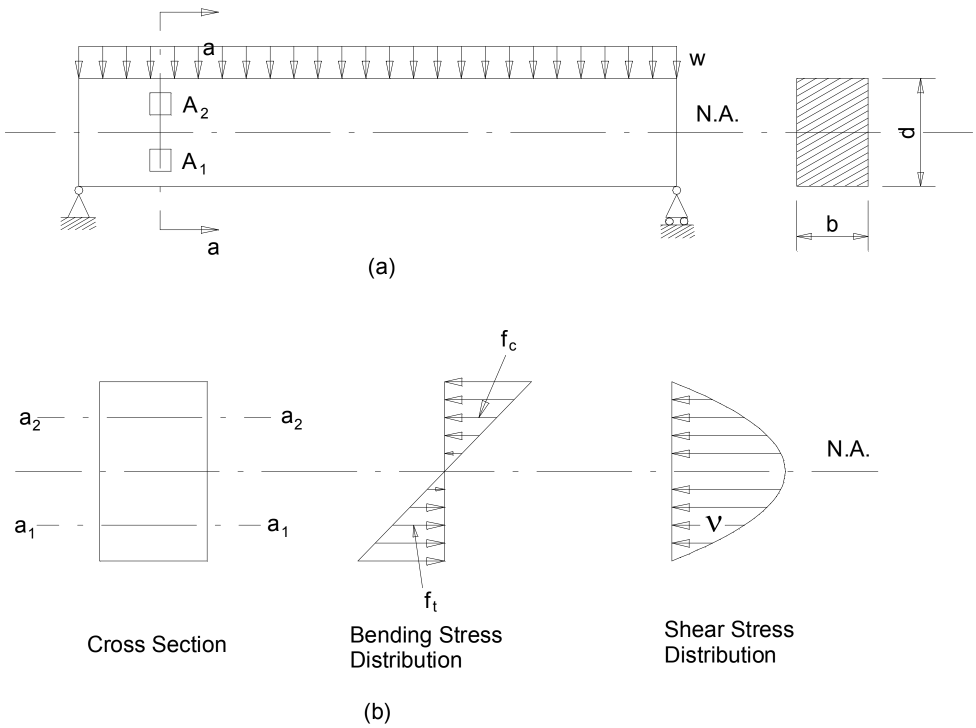


Fig 5.1 (a) & (b) Stress distribution for a typical homogeneous rectangular beam

The internal stresses acting on infinitesimal elements A_1 and A_2 are shown in Fig 5.2 as well as Mohr's circle representation. The principle stresses for element A_1 in the tensile zone below the neutral axis are :

$$f_{t(\max)} = \frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + \upsilon^2} \tag{5.1}$$

$$f_{c(\max)} = \frac{f_t}{2} - \sqrt{\left(\frac{f_t}{2}\right)^2 + \upsilon^2} \tag{5.2}$$

where $f_{t(\max)}$ = principal tension

$f_{c(\max)}$ = principal compression

and

$$\tan 2\theta_{\max} = \frac{\upsilon}{f_t / 2} \tag{5.3}$$

In Fig 5.3, we see a plot of trajectories of principal stresses in a homogeneous isotropic beam. In regions close to the supports the bending moment and hence f_t approaches zero, while the shear stress approaches its maximum. At the point of zero moment the element A_1 reaches a state of pure shear whereby the principal tensile stress is equal to the shear stress (υ) and acts at a plane of 45^0 to the normal of the section. In concrete sections, due to the low tensile strength of the material, diagonal cracking develops in the tension zone along planes perpendicular to the direction of the principal stresses, which is referred to as diagonal tension cracking.

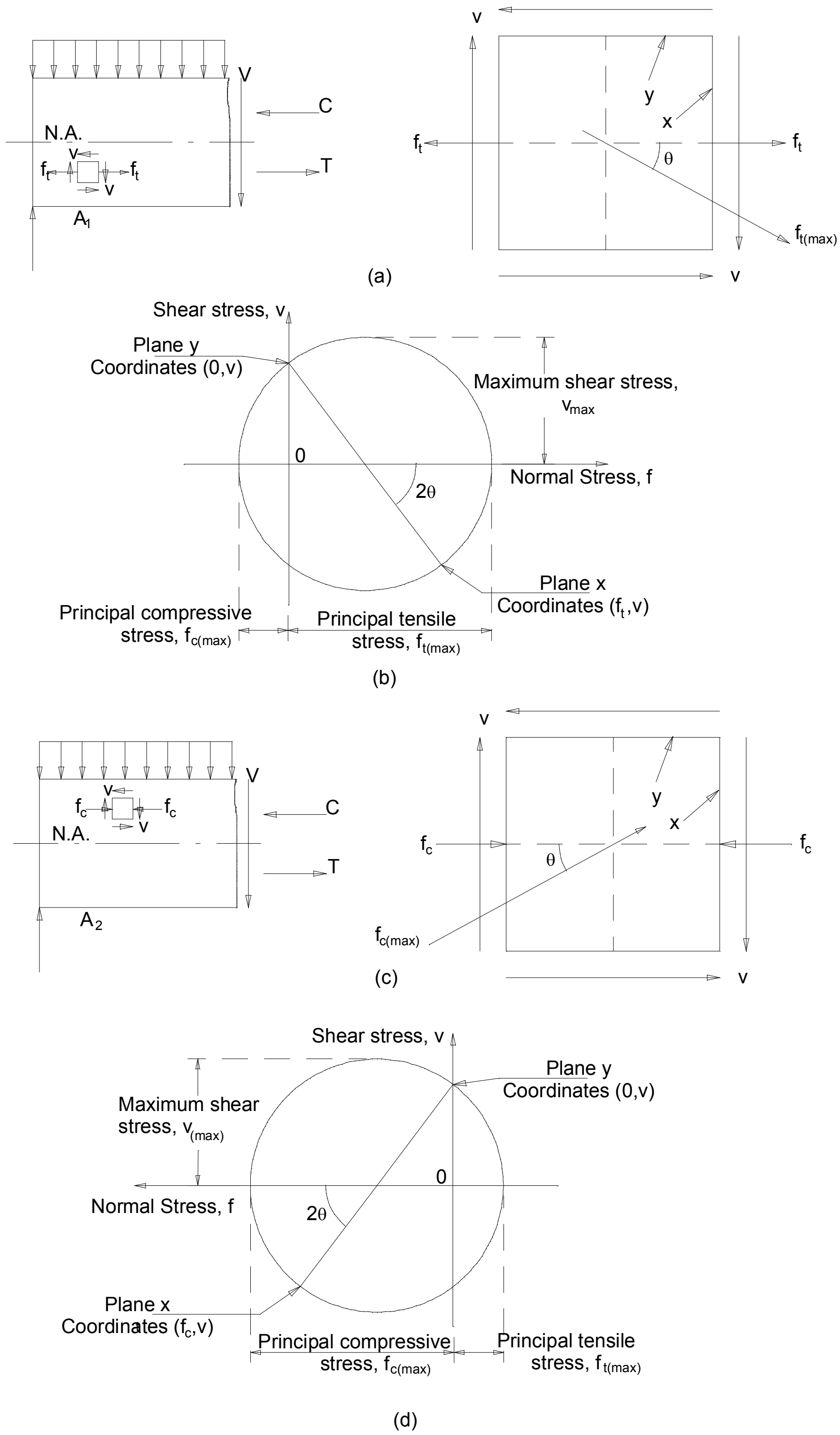


Fig 5.2 (a) & (b) State of Stress for element A_1 ; (c) & (d) State of Stress for Element A_2

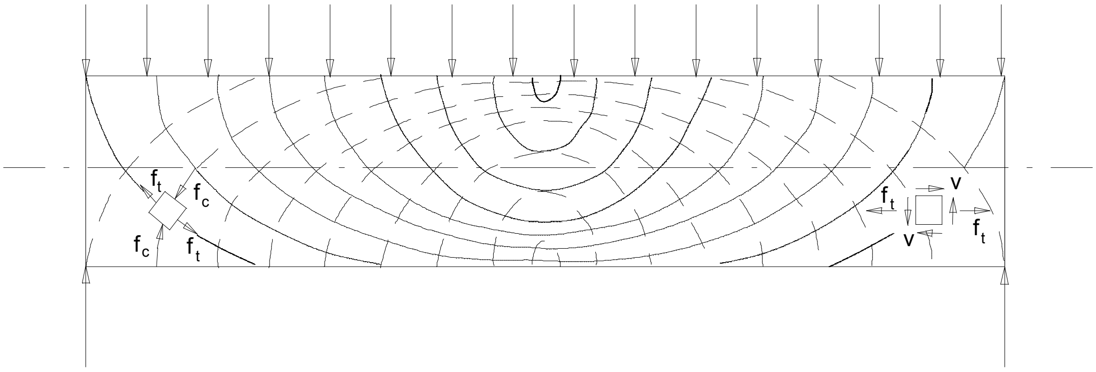


Fig 5.3 Trajectories of Principal Stresses in Homogeneous Isotropic Beams. Solid Lines : Tensile ; Dashed Lines : Compression

Composite concrete beams using cold formed lipped channel sections behave similar to reinforced concrete beams without shear reinforcement. In regions of large bending moments and relatively low shear, flexural cracks develop almost perpendicular to the axis of the beam. In regions of large shear forces and small bending moment, minor flexural cracking develops and extends to inclined diagonal tension cracks, which is termed flexural-shear cracking. In regions of negligible bending moment diagonal tension stresses are inclined at 45^0 and are equal to the shear stresses. The cracking that develops is termed web-shear cracking. All failure patterns are schematically shown in Fig 5.4.

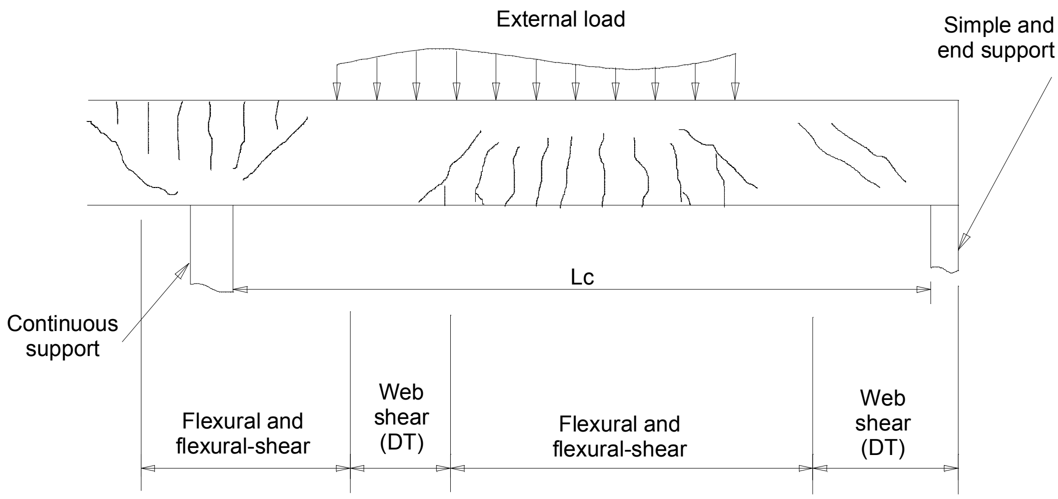


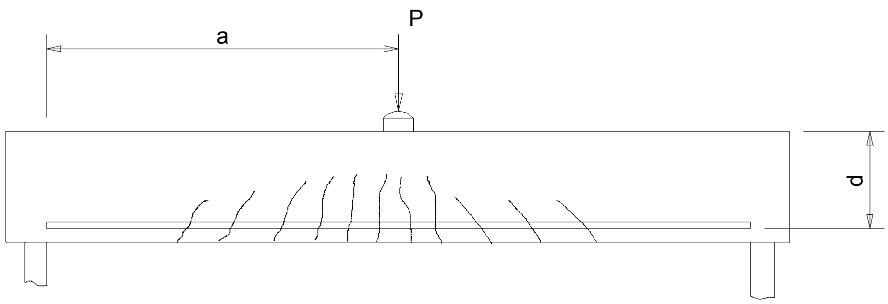
Fig 5.4. Web Shear; Flexural Shear & Flexural Cracking

Fundamentally, three modes of failure occur in composite beams without shear reinforcement. They are :

- Flexural failure shown in Fig.5.5 (a) in which mainly vertical cracks middle third of the beam and are caused by a dominant flexural stress f. Very little contribution to the cracking is caused by the shear stress υ.
- Diagonal tension failure shown in Fig 5.5 (b) which occurs in beams of intermediate slenderness (shear span/depth ratio of 2.5 to 5.5 for beams with concentrated loading and of 11 to 16 for beams with distributed loading). In this mode of failure diagonal cracking is initiated by the destruction of bond between the tension steel and adjacent concrete. Two or three diagonal tension cracks develop at approximately 1.5d to 2.0d from the face of the support. One of the diagonal tension cracks develops into a principal diagonal tension crack and extends to the top compression fibers indication total failure.

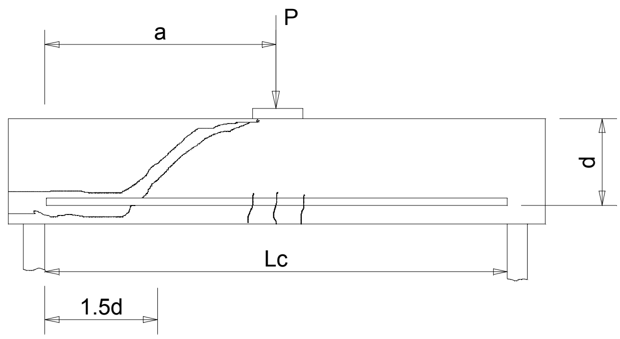
- Shear compression failure shown in Fig 5.5 (c), which occurs in deep beams of small shear span/depth ratio (in the region of 1.0 to 2.5 for beams with concentrated loading and 1.0 to 5.0 for beams with distributed loading). In this mode of failure diagonal tension failure is preceded by the crushing of the concrete in the top compression fiber. Thereafter, the rate of failure is quick as the diagonal tension crack joins the crushed concrete zone.

We have seen that the mode of failure very much depends on the slenderness of the beam (shear span /depth ratio ; a/d for concentrated loading and L_c/d for distributed loading). For a concentrated loading the shear span is the distance from the face of the support to the point of application of the load (a) and for a distributed load the shear span is the clear span of the beam (L_c). Table 5.1 summarizes the effect of slenderness values.

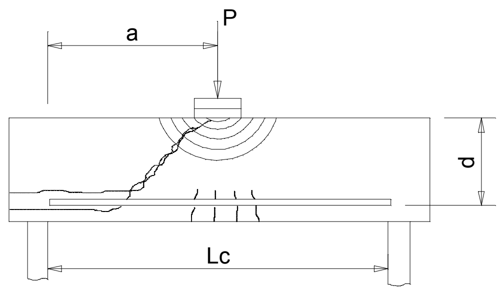


(a)

Fig 5.5 (a) Flexure Failure



(b)



(c)

Fig 5.5 (b) Diagonal Tension Failure ; (c) Shear Compression Failure

Beam Category	Failure Mode	Shear span/depth ratio as a measure of slenderness[a]	
		Concentrated load, a/d	Distributed load, l_c/d
Slender	Flexure (F)	Exceeds 5.5	Exceeds 16
Intermediate	Diagonal Tension (DT)	2.5 - 5.5	11 - 16[b]
Deep	Shear Compression (SC)	1 - 2.5	1 - 5[b]

[a]a = shear span for concentrated loads

l_c = shear span for distributed loads

d = effective depth of beam

[b]For a uniformly distributed load, a transition develops from deep beam to intermediate beam effect

Table 5.1 Slenderness Effect on Mode of Failure

5.3. Diagonal Tension Analysis of Lipped Channel Composite Members

The shear strength of a composite beam without shear reinforcement is determined by the development of the first inclined crack. This in turn depends on the controlling principal stress in the concrete which is as a result of the shearing stresses (υ) due to the factored shear force (V_u) and the horizontal flexural stresses (f_t) due to the factored bending moment (M_u). The following expression is derived based on a large number of tests (ACI-ASCE Committee 326) on beams without shear reinforcement to failure.

From equation 5.1 :

$$f_{t(\max)} = f_t^{'} = \frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + \upsilon^2} = K\sqrt{f_c^{'}} \tag{5.4}$$

where $f_t^{'} =$ tensile splitting strength of plain concrete

and K = constant

Re-arranging we have :

$$\sqrt{f_c^{'}} = K_1\left[\frac{f_t}{2} + \sqrt{\left(\frac{f_t}{2}\right)^2 + \upsilon^2}\right] \tag{5.5}$$

The flexural stress (f_t) in the concrete is proportional to the steel stress in the longitudinal reinforcement or the moment of resistance of the section :

$$f_t \propto \frac{E_c}{E_s} f_s \propto \frac{E_c M_n}{E_s A_s d} \tag{5.6}$$

If we substitute the reinforcement ratio $\rho_w = A_s / bd$ and regard the modular ratio E_c/E_s as a constant the we have :

$$f_t = F_1 \frac{M_n}{\rho_w bd^2} \tag{5.7}$$

The shear stress at a specific cross section of size bd due to the vertical external shear force is :

$$\upsilon = F_2 \frac{V_n}{bd} \tag{5.8}$$

From equations (5.5) ; (5.7) and (5.8) as well as evaluating the constants K_1; F_1 and F_2 from the experimental models we obtain the following regression expression (Imperial Units) :

$$\frac{V_n}{bd\sqrt{f_c^{'}}} = 1.9 + 2500\rho_w \frac{V_n d}{M_n \sqrt{f_c^{'}}} \leq 3.5 \tag{5.9}$$

A graphic representation of equation (5.9), as well as the test results are shown in Fig (5.6). If we note that $\upsilon_n = V_n / bd$ and $M_u / V_u d = a / d$, which is the shear span to depth ratio (slenderness ratio) then the dashed line shown if Fig 5.6 is a representation of the allowable average shear stress as a function of slenderness ratio of the beam.

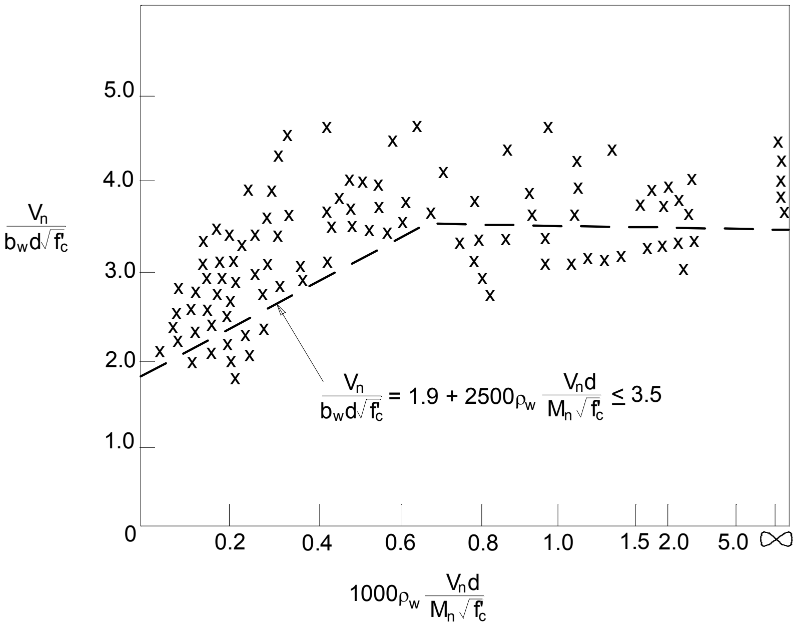


Fig 5.6 Shear Resistance of Reinforced Concrete Beam Webs (ACI-ASCE Committee 326 – 1962)

If we convert equation (5.9) into S.I.Units and substitute the term, V_n, on the left hand side of the equation with V_c (the nominal shear resistance of a composite beam having no diagonal shear tension steel), we get the following equation :

$$\upsilon_c = \frac{V_c}{b_w d} = 0.16\sqrt{f_c^{'}} + 17.2\rho_w \frac{V_u d}{M_u} \leq 0.29\sqrt{f_c^{'}} \qquad (5.10)$$

Equation (5.10) indicates that in regions of large shear forces and small moments the diagonal tension cracks form at a nominal shear stress of about $0.16\sqrt{f_c^{'}}$ and when large bending moments occur the nominal shear stress at which diagonal cracks appear is $0.29\sqrt{f_c^{'}}$, which is roughly half the value. Equation (5.10) will be used to further evaluate the shear resistances as obtained from experimental results.

5.4. Quantitive Assessment : Experimental Beams

The following section compares the results from three different experimental test programs, viz. experimental beams tested at the University of the Witwatersrand (Komacec, 2002), beams tested at the California State University (Nguyen, 1989) and beams tested at the University of Windsor (Addel-Sayed, 1982). A brief overview of the test programs are given in Chapter 3.

5.4.1. Comparative Study

A comparative study between the test programs is shown below. Tables 5.2, 5.3 and 5.4 are the computed values for each of the test series of Komacec, Nguyen and Addel-Sayed. Equation (5.10) is used to predict the nominal shear force, V_{cr} at which diagonal flexural-shear cracking develops. The tables also give the ratio between the actual shear failure value and the predicted shear force, V_u/V_{cr}.

VERTICAL SHEAR : KOMACEC

Beam Specimen No.	h	b	d	ρ	A_s	Shear Length L	Slenderness L/d	f_c'	ρVd/M	V_{cr}	V_u	V_u/V_{cr}
	(mm)	(mm)	(mm)		(mm^2)	(mm)		(MPa)	(mm4)	(kN)	(kN)	
T1/1	200.0	150.0	179.0	3.326%	892.95	800.00	4.47	22.080	5.95E-02	47.679	101	2.12
T1/2	200.0	150.0	179.0	3.326%	892.95	800.00	4.47	22.080	5.95E-02	47.679	130	2.73
T1/3	200.0	150.0	179.0	3.326%	892.95	800.00	4.47	22.080	5.95E-02	47.679	142.5	2.99
T1/4	200.0	150.0	179.0	3.326%	892.95	800.00	4.47	22.080	5.95E-02	47.679	112.5	2.36
T2/1	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	22.080	2.65E-02	32.405	112.5	3.47
T2/2	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	22.080	2.65E-02	32.405	55	1.70
T2/3	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	22.080	2.65E-02	32.405	57	1.76
B2/1	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	26.720	2.65E-02	34.425	78.5	2.28
B2/2	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	26.720	2.65E-02	34.425	77.5	2.25
B2/3	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	26.720	2.65E-02	34.425	80	2.32
B2/4	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	29.280	2.65E-02	35.465	108	3.05
B2/5	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	29.280	2.65E-02	35.465	105	2.96
B2/6	200.0	150.0	179.0	3.326%	892.95	1800.00	10.06	29.280	2.65E-02	35.465	105	2.96
B4/1	200.0	150.0	179.0	3.326%	892.95	3700.00	20.67	26.720	1.29E-02	28.151	34	1.21
B4/2	200.0	150.0	179.0	3.326%	892.95	3700.00	20.67	26.720	1.29E-02	28.151	40	1.42
B4/3	200.0	150.0	179.0	3.326%	892.95	3700.00	20.67	26.720	1.29E-02	28.151	40	1.42
B4/4	200.0	150.0	179.0	3.326%	892.95	3700.00	20.67	26.720	1.29E-02	28.151	40	1.42

Table 5.2 Computed vs Actual Shear Values : Komacec

VERTICAL SHEAR : NGUYEN

Beam Specimen No.	h (mm)	b (mm)	d (mm)	ρ	A_s (mm^2)	Shear Length L (mm)	Slenderness L/d	$f_c^{'}$ (MPa)	$\rho Vd/M$ (mm4)	V_{cr} (kN)	V_u (kN)	V_u/V_{cr}
S-1	157.18	100.03	145.44	1.776%	258.32	431.80	2.97	20.106	5.98E-03	11.934	18.95	1.59
S-2	154.00	99.34	141.94	1.901%	268.06	431.80	3.04	20.106	6.25E-03	11.632	18.95	1.63
S-3	159.79	103.07	146.63	1.721%	260.13	431.80	2.94	20.106	5.84E-03	12.362	19.88	1.61
S-4	158.34	103.51	146.69	1.633%	247.94	431.80	2.94	20.106	5.55E-03	12.342	19.97	1.62
S-5	153.19	98.98	141.25	1.767%	247.04	431.80	3.06	35.558	5.78E-03	14.729	22.60	1.53
S-6	155.58	100.91	143.76	1.697%	246.20	431.80	3.00	35.558	5.65E-03	15.251	22.62	1.48
S-7	205.87	100.97	194.16	1.240%	243.04	457.20	2.35	20.106	5.26E-03	15.840	29.60	1.87
S-8	206.38	100.84	194.79	1.269%	249.29	457.20	2.35	20.106	5.41E-03	15.919	26.64	1.67
S-9	203.99	125.02	187.68	1.752%	411.06	457.20	2.44	20.106	7.19E-03	19.736	36.49	1.85
S-10	202.41	125.1	186.13	1.768%	411.70	457.20	2.46	20.106	7.20E-03	19.588	34.75	1.77
S-11	200.03	100.51	188.32	1.341%	253.74	457.20	2.43	20.106	5.52E-03	15.377	29.54	1.92
S-12	201.63	99.87	189.97	1.333%	252.84	381.00	2.01	20.106	6.64E-03	15.780	35.72	2.26
S-13	254.64	155.58	234.75	1.620%	591.79	571.50	2.43	20.106	6.66E-03	30.384	47.87	1.58
S-14	250.83	155.07	230.96	1.607%	575.66	571.50	2.47	20.106	6.50E-03	29.696	47.73	1.61
S-15	153.19	127.79	137.13	2.397%	420.02	304.80	2.22	20.106	1.08E-02	15.823	28.47	1.80
S-16	155.58	131.78	139.65	2.297%	422.73	304.80	2.18	20.106	1.05E-02	16.534	27.67	1.67

Table 5.3 Computed vs Actual Shear Values : Nguyen

VERTICAL SHEAR : ADDEL-SAYED

Beam Specimen No.	h (mm)	b (mm)	d (mm)	ρ	A_s (mm^2)	Shear Length L (mm)	Slenderness L/d	$f_c^{'}$ (MPa)	$\rho Vd/M$ (mm4)	V_{cr} (kN)	V_u (kN)	V_u/V_{cr}
A-1	304.80	101.60	296.93	1.07%	323.15	1092.20	3.68	33.303	2.91E-03	28.986	20.69	0.71
A-2	304.80	101.60	283.24	1.29%	371.52	1092.20	3.86	32.062	3.35E-03	27.373	24.03	0.88
A-3	304.80	101.60	293.12	0.98%	292.19	1092.20	3.73	31.028	2.63E-03	27.528	16.47	0.60
A-4	304.80	101.60	288.04	1.13%	330.89	1092.20	3.79	32.407	2.98E-03	27.792	16.47	0.59
A-5	304.80	101.60	282.7	1.29%	369.59	1092.20	3.86	27.028	3.33E-03	25.212	28.48	1.13
A-6	304.80	101.60	283.46	1.03%	297.35	1092.20	3.85	39.095	2.68E-03	29.746	9.79	0.33
A-7	304.80	101.60	278.13	1.16%	328.31	1092.20	3.93	29.649	2.96E-03	25.721	24.25	0.94
A-8	304.80	101.60	272.29	1.30%	359.27	1092.20	4.01	37.302	3.24E-03	28.206	19.80	0.70
B-1	304.80	101.60	266.7	1.45%	393.45	863.60	3.24	32.200	4.48E-03	26.358	27.06	1.03
B-3	304.80	101.60	293.12	0.98%	292.19	863.60	2.95	35.027	3.33E-03	29.522	25.14	0.85
B-4	304.80	101.60	288.04	1.13%	330.89	863.60	3.00	32.131	3.77E-03	28.079	28.39	1.01
B-6	304.80	101.60	283.46	1.03%	297.35	863.60	3.05	32.407	3.39E-03	27.553	29.50	1.07
B-7	304.80	101.60	278.13	1.16%	328.31	863.60	3.11	31.993	3.74E-03	27.044	33.06	1.22
B-8	304.80	101.60	272.29	1.39%	385.07	863.60	3.17	34.268	4.39E-03	27.648	33.86	1.22
C-3	304.80	101.60	293.12	0.98%	292.19	863.60	2.95	31.028	3.33E-03	27.886	25.10	0.90
C-4	304.80	101.60	262.64	1.24%	330.89	863.60	3.29	31.234	3.77E-03	25.267	30.84	1.22
C-6	304.80	101.60	283.46	1.03%	297.35	863.60	3.05	31.579	3.39E-03	27.221	29.19	1.07
C-7	304.80	101.60	278.13	1.16%	328.31	863.60	3.11	30.890	3.74E-03	26.606	31.95	1.20
C-8	304.80	101.60	272.29	1.30%	359.27	863.60	3.17	32.131	4.09E-03	26.698	32.13	1.20
D-1	355.60	127.00	339.85	1.11%	477.30	965.20	2.84	29.235	3.89E-03	39.723	42.10	1.06
D-2	355.60	127.00	339.85	1.11%	477.30	965.20	2.84	26.477	3.89E-03	37.942	27.68	0.73
D-3	406.40	152.40	389.89	0.88%	522.45	965.20	2.48	27.166	3.55E-03	52.509	57.05	1.09
D-4	406.40	152.40	389.89	0.88%	522.45	965.20	2.48	26.201	3.55E-03	51.632	68.09	1.32
D-5	406.40	152.40	389.89	0.88%	522.45	965.20	2.48	29.028	3.55E-03	54.155	47.62	0.88
D-6	406.40	152.40	387.35	0.89%	522.45	965.20	2.49	28.338	3.55E-03	53.203	48.73	0.92

Table 5.4 : Computed vs Actual Shear Values : Addel Sayed

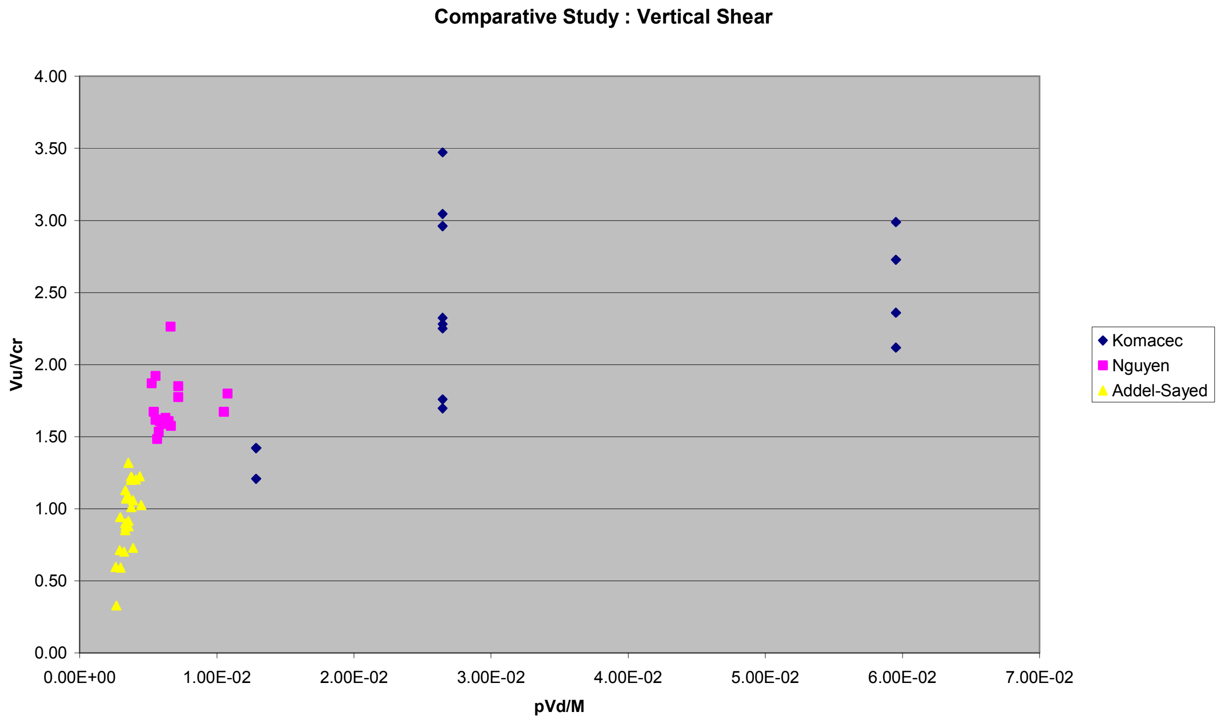


Chart 5.1 V_u/V_{cr} vs ρVd/M

Chart 5.1 gives a plot of the last columns of Tables 5.2 to 5.4 vs the dimensionless quantity ρVd/M. The expression Vd/M can be re-written as d/a, where a is the shear length of the beam. The lower axis of the chart is thus an indication of the inverse of the slenderness ratio (stiffness ratio). Chart 5.1 indicates that for low slenderness ratios equation (5.10) is not very accurate while for intermediate to high slenderness ratios the equation is accurate and also gives a lower bound limit. An explanation why equation is inaccurate for low slenderness ratios is that once a diagonal crack is formed the shear force is still transmitted by the uncracked portion of the concrete, V_c, and the forces across the vertical legs of the channel, V_d. The force, V_d, causes vertical tension stresses (Addel-Sayed : 1982) as shown if Fig 5.7. Final shear failure occurs when these tension stresses cause de-bonding of the concrete from the adjacent concrete or splitting of the concrete along the plane of the lip of the channel.

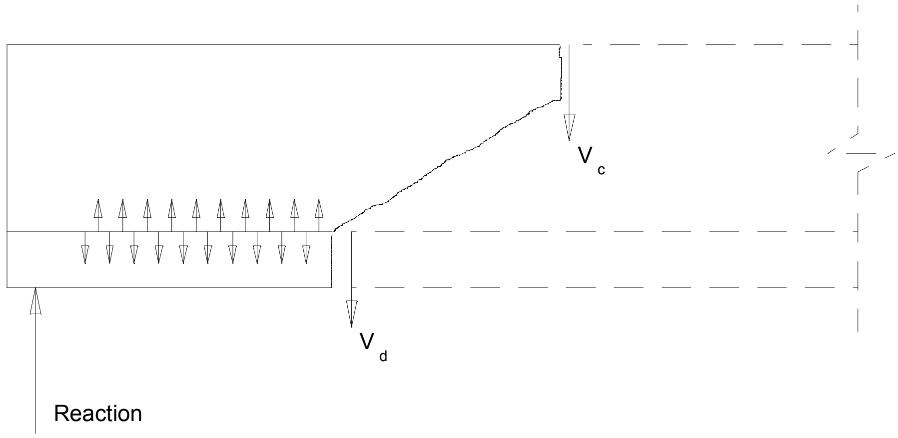


Fig 5.7 Shear Forces after Development of Diagonal Crack

5.5. Conclusion

The following can be summarized with regard to vertical shear in composite lipped channel beams with no shear connectors :

- The A.C.I. homogenous equation as developed for vertical shear prediction in reinforced concrete beams without shear reinforcement.
- The equation is accurate in beams of high shear slenderness ratios.
- In beams of low (deep beams) to intermediate slenderness ratios the equation is far too conservative and the shear-bond equation as developed for horizontal shear seems to be more accurate.