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Adoption of Homotopy Perturbation Method (HPM) for non-Darcy Flow in Porous Media

Amirhossein Arvin[a], **Mohammad Hadi Fattahi**[a], **Mohammad Sedghi-Asl**[b], **and Seyyed Abbas Mohammadi**[c]

[a]*Dept. of Civil Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran*
[b]*Dept. of Soil Science, College of Agriculture, Yasouj University, Yasouj, Iran*
[c]*School of Computer Science and Applied Mathematics, University of The Witwatersrand, Braamfontein 2000, Johannesburg, South Africa*

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ABSTRACT

For non-Darcy flow in coarse porous media, an analytical solution based on the Homotopy Perturbation Method (HPM) is proposed. Subsurface water profile data of a laboratory model are used for six different inlet discharges in both rounded and crushed coarse porous media types. The model slope is S = 0.00001 close to the horizon. Different upstream and downstream water level boundary conditions are considered. The results of the analytical solution of non-Darcy flow by the HPM method are compared with experimental data. The normal objective function (NOF) is used for better comparison between the results of analytical solutions and experimental data. Results depict that the HPM method provides acceptable solutions in both rounded and crushed media types. The analytical results of flow rates q = 26.25 lit/s with NOF of 0.000294586 percent in rounded porous media and q = 30 lit/s with NOF of 0.00028660 percent in crushed one are the most consistent with the experimental data. The proposed HPM solution performs very well, particularly at higher flow rates, in both rounded and crushed porous media.

1. Introduction

Darcy equation has many limitations such as linearity between flow velocity and hydraulic gradient, etc. If the above-mentioned conditions are not satisfied, the flow regime will be non-Darcy and the flow enters a new phase called the non-Darcy flow. Today the rounded and crushed materials are used in hydraulic structures for filtration, gabion fabrication, canal coating, stilling basins, and rock fill dams due to their special properties (Sedghi-Asl, 2010).

The flow behavior in coarse porous media is very complex because of the large particle size, pores, and the occurrence of high velocities, and high turbulence (Bear, 1972). Gill (1977) used the perturbation method to solve the spatially variable flow equation and converted the spatially variable differential equation from nonlinear to linear, obtained an algebraic solution for it, and showed that the results of the perturbation solution for a range of subcritical flow in the frictionless state give satisfactory results compared to Li (1955). Stephenson (1979) solved the ordinary differential equation of constant steady-state variable flow using analytical integration in rectangular channels. So, there was a good correlation, comparing the results with the observations of the experimental flume results. In this way, it became clear that Stephenson's analytical equation could only be applied if the turbulent flow was fully developed. Moutsopoulos and Tsihrintzis (2005) presented an analytical solution for fully-developed turbulent flows in the finite porous media under short-term and long-term, inertial, and non-Darcy flow conditions. Hosseini and Joy (2006) developed a non-linear non-stable model for flow inside gravel dams and compared the results of their model with experimental data and observed a good correlation. Sedghi-Asl and Rahimi (2011) presented a second-order relationship based on the friction factor and Manning formula for coarse porous media by studying six diameters of coarse materials with a particle range between 2.83 to 8.56 mm. HPM method was first proposed by He (1999, 2003, 2004 and 2005). Ghotbi et al. (2011) studied infiltration of the water flow in unsaturated soils using an HPM method. Khan and Wu (2011) used the Homotopy Perturbation Transform

CORRESPONDENCE Mohammad Hadi Fattai fattahi.mh@miau.ac.ir Dept. of Civil Engineering, Marvdasht Branch, Islamic Azad University, Marvdasht, Iran

Method (HPTM) for nonlinear equations. They showed that HPTM solves nonlinear problems without the use of Adomian polynomials. Khan et al. (2011) studied the effects of Reynolds numbers using various analytical methods, and they solved the momentum equation using an HPM method and showed the variations in results of the HPM method are the resultant of an unknown parameter *a.* This causes the HPM convergence to be poor and utmost care must be taken in selecting these unknown parameters. Khan (2017) introduced a series of solution for governing Navier-Stokes equations of MHD flow of linear visco-elastic fluid over a shrinking/stretching sheet by using an HPM method. Faraz (2011) and Faraz et al. (2019a) studied the effect of Reynolds (R) numbers on long, circular porous sliders. Madani et al. (2012) investigated the circular porous slider by using an HPM method, and also analyzed its lift and drag. Eck et al. (2012) provided an analytical solution for non-Darcy flow in sloping porous layers. This solution includes the complete Forchheimer equation and can be used for an extensive range of flows. Khan (2013) used the HPTM to solve the time-dependent nonlinear drainage. Khan et al. (2013) developed an auxiliary Laplace parameter method (ALPM) using Adomian and polynomials. The proposed method solves nonlinear problems without any discretization or limiting assumption. Sedghi-Asl et al. (2014a) presented an analytical solution to calculate the subsurface water profile inside a rock drain layer and they indicated that their solution had a better correlation than the others, but it was only valid for the horizontal slopes. Sedghi-Asl et al. (2014b) performed a laboratory packed column test to study high-velocity flow through granular materials. They employed six different sizes of rounded aggregates and applied different hydraulic gradients to assess the non-Darcy flow behavior. After that, Sedghi-Asl et al. (2014c) and Sedghi-Asl and Ansari (2016) presented two different analytical solutions for sloping coarse porous media. Khan et al. (2014) proposed a modified Homotopy Perturbation Transform Method (MHPTM) for nonlinear boundaries, in which he demonstrated the convergence of the MHPTM method. Khan (2014) studied the two-dimensional stable magnetic field of hydrodynamic chemical reaction (MHD) on a shrinking sheet by suction injection, in which the flow of the MHD boundary layer was analyzed analytically and numerically. Similarity conversion was used to convert partial differential equations, an analytical solution was obtained, and the validity of the obtained results was finally confirmed using the HPM method. Faraz et al. (2019b) studied three-dimensional hydro-magnetic flow in a long porous slider and a circular porous slider with velocity sliding using analytical methods and showed that the homotopy analysis method (HAM) yields a convergent series solution. Sedghi-Asl and Ansari (2016) presented an analytical solution for the fully-developed turbulent flow based on the extended Dupuit-Forchheimer theory. To the best knowledge of the authors, no analytical solution has been presented by HPM for non-Darcy flows in porous media. Moreover, the HPM method capability regarding the porous media grading has not been investigated yet. In this study, a new HPM solution for non-Darcy flows in rounded and crushed porous media will be proposed and evaluated using the experimental data of Sedghi-Asl (2010). The proposed HPM solution performs very well, particularly at higher flow rates, in both rounded and crushed porous media.

2. Materials and Methods

2.1 Laboratory Model

In this study, the laboratory data of Sedghi Asl (2010) from a laboratory model at the University of Stuttgart, Germany, has been used. A rectangular flume with length, width, and height specifications of 6.4, 0.8, and 1 meter, respectively was selected to investigate the subsurface water profiles in coarse (rounded and crushed) porous media.

Fig. 1. General View of the Laboratory Flume of Sedghi-Asl (2010)

This laboratory study was performed on a slope close to the horizon S = 0.00001 for both rounded and crushed materials. Subsurface water profile data were obtained in the laboratory by a calibrated plate attached to the Plexiglas flume wall. The computed subsurface water profiles of the HPM were compared with the experimental data, which is presented in other parts of this study.

2.2 Theory

He (1999) has developed the HPM method to solve equations and nonlinear problems. This method has been considered by researchers in mathematics, physics and engineering all over the world due to its simplicity and high accuracy (He, 1999; Xu, 2007; He, 2012; He, 2014). The differential equation which governs the non-Darcy flows in the coarse porous medium is nonlinear and employing an HPM method we obtain the Maclaurin expansion of its solution. Indeed, an analytical solution for the nonlinear differential equation is calculated. In this paper, a non-Darcy flow equation is considered as follows (Sedghi-Asl et al., 2014c):

$$\frac{dx}{dy} = \frac{(\cos\theta)^3}{aq^2}(y - x\tan\theta)^2 . \tag{1}$$

In Eq. (1), q is regarded as discharge per unit width, x is the longitudinal distance, y is the flow depth, θ is the bed slope, and a is the non-Darcy flow coefficient. Suppose x is a function in terms of y. It is important to point out that angel theta (θ) is constant and independent of x, and y. Let $y - x\tan\theta = z$ then:

$$\frac{dz}{1-\frac{\sin\theta(\cos\theta)^2}{aq^2}z^2} = dy . \tag{2}$$

It is straightforward to check that the solution of this equation is as follows:

$$y = \frac{2\tanh\left(\frac{\cos\theta\sqrt{a\sin\theta}(c_1+y)}{qa}q\sqrt{a\sin\theta}\right)}{\sin(2\theta)} + x, \tag{3}$$

where c_1 is the constant of integration. Calculating y using this implicit nonlinear equation is difficult and computationally time-consuming.

2.2.1 HPM Method for Non-Darcy Flow in Porous Media

Equation (1) is a nonlinear equation in terms of the variables x and y. It is simply not possible to work with. Therefore, to find y as a function in terms of x, in this paper, the HPM method is selected. To the best of our knowledge, this is the first time this method has been used to solve Eq. (1).

To show a simpler form of the equation, the following change of variables is used. Let $z = y - x\tan\theta$.

Now using the change of variables, we will have

$$\frac{(dy-dz)\cot\theta}{dy} = \frac{(\cos\theta)^3}{aq^2}z^2 \tag{4}$$

$$\Rightarrow 1-\frac{dz}{dy} = \frac{\cos^2\theta\sin\theta}{aq}z^2 \tag{5}$$

$$1-\frac{\cos^2\theta\sin\theta}{aq^2}z^2 = \frac{dz}{dy} . \tag{6}$$

Setting

$$\omega^2 = \frac{\cos^2\theta\sin\theta}{aq^2},$$

We will arrive at the following equation

$$1-\omega^2 z^2 = \frac{dz}{dy} . \tag{7}$$

Consider z as a function of y, then indeed we have

$$z = z(y) \text{ and } y \in \Omega , \tag{8}$$

and Ω is a set in which y is defined. So Eq. (7) can be rewritten as follows:

$$z' + \omega^2 z^2 = 1 . \tag{9}$$

The resulting equation is a typical nonlinear differential equation. To solve this equation, the HPM method for $y \in \Omega$ is used. Let us define the operator $A(z)$ as follows:

$$A(z) = z' + \omega^2 z^2 . \tag{10}$$

According to the HPM Method, here A is decomposed into two linear operators L and nonlinear operator N. Indeed, we define

$$L(z) = z' , \tag{11}$$

$$N(z) = \omega^2 z^2 . \tag{12}$$

The right-hand side function $f(y)$ which usually is considered in the HPM method, is defined:

$$f(y) = 1 . \tag{13}$$

Now, the Homotopy corresponding to Eq. (9) is:

$$\begin{aligned} Z(u,P) = L(u(y,P)) - L(z_0(y)) + PL(z_0(y)) + \\ P[N(u(y,P)) - f(y)] = 0, \end{aligned} \tag{14}$$

where is defined as a Homotopy which maps the set $\Omega \times [0,1]$ to real numbers. In this formula, u is a function of y and P belongs to the closed interval [0,1]. The function $z_0(y)$ is the initial approximation of the solution of Eq. (9). In differential equations, we usually see $y = y(x)$ in the literature. For simplicity we use the familiar notations, actually, we write y instead of z and x instead of y and so we have:

$$1-\omega^2 y^2 = \frac{dy}{dx} . \tag{15}$$

Now, we employ the HPM method for Eq. (15) with the initial guess for its solution $y_0(x) = \alpha$, where α is an unknown numerical value that is obtained in the following in a way that the solution derived by HPM satisfies the upstream and downstream boundary conditions of the governing equation. Then, we have

$$A(y) = y' + \omega^2 y^2 = 1, \tag{16}$$

$$L(y) = y', \tag{17}$$

$$N(y) = \omega^2 y^2, \tag{18}$$

$$Y(u,P) = L(u(x,P)) - L(y_0(x)) + PL(y_0(x)) + P[N(u(x,P)) - f(x)]$$
$$= u'(x) - y_0'(x) + Py_0'(x) + P(\omega^2 u^2(x) - 1) = 0. \tag{19}$$

Then, the homotopic solution will be a finite series in the following form:

$$u(x,P) = \sum_{i=0}^{\infty} P^i u_i(x). \tag{20}$$

The derivative in Eq. (20) is obtained as follows:

$$u'(x,P) = \sum_{i=0}^{\infty} P^i u_i'(x). \tag{21}$$

By substituting this into Eq. (19) we obtain:

$$\sum_{i=0}^{\infty} P^i u_i^i(x) - y_0'(x) - Py_0'(x) + P[\omega^2 \sum_{i=0}^{\infty} P^i u_i(x)^2 - 1] = 0. \tag{22}$$

By expanding Eq. (22) one has:

$$P^0 u_0'(x) + P^1 u_1'(x) + P^2 u_2'(x) + \ldots - y_0'(x) + Py_0'(x) + \ldots$$
$$+ P(\omega^2 (P^0 u_0(x)) + P^1 u_1(x) + P^2 u_2(x) + P^3 u_3(x) + \ldots)^2 - 1) = 0. \tag{23}$$

By setting the coefficients of common powers P in this equation to zero, we will finally arrive at the final HPM equation, which yields a formula for Y:

$$\lim_{p-1} Y(u,P) = u_0(x) + u_1(x)P + u_2(x)P^2 + u_3(x)P^3 + \ldots$$

$$\lim_{p\to 1} \mathrm{Y}(u,P) = \alpha - (\alpha\omega+1)(\alpha\omega-1)(x-\alpha)P + \omega^2(\alpha\omega+1) +$$
$$\alpha(\alpha\omega-1)(x-\alpha)^2 P^2 - \left(\alpha^2\omega^2 - \frac{1}{3}\right)(\alpha\omega+1)\omega^2(\alpha\omega-1)$$
$$(x-\alpha)^3 P^3 + \alpha(\alpha\omega+1)\omega^4\left(\alpha^2\omega^2 - \frac{2}{3}\right)(\alpha\omega-1)(x-\alpha)^4 P^4 -$$
$$\left(\alpha^4\omega^4 - \alpha^2\omega^2 + \frac{2}{15}\right)(\alpha\omega+1)\omega^4(\alpha\omega-1)(x-\alpha)^5 P^5. \tag{24}$$

$$Y(u,P) = \alpha - (\alpha\omega+1)(\alpha\omega-1)(x-\alpha) + \omega^2(\alpha\omega+1)$$
$$\alpha(\alpha\omega-1)(x-\alpha)^2 - \left(\alpha^2\omega^2 - \frac{1}{3}\right)(\alpha\omega+1)\omega^2(\alpha\omega-1)$$
$$(x-\alpha)^3 + \alpha(\alpha\omega+1)\omega^4\left(\alpha^2\omega^2 - \frac{2}{3}\right)(\alpha\omega-1)(x-\alpha)^4 - \tag{25}$$
$$\left(\alpha^4\omega^4 - \alpha^2\omega^2 + \frac{2}{15}\right)(\alpha\omega+1)\omega^4(\alpha\omega-1)(x-\alpha)^5.$$

In order to obtain the solution in the form of the original variables, we set y instead of x and z instead of Y in Eq. (25) and then we set $y - x \tan\theta$ instead of z. The final formula which is calculated by the HPM method for Eq. (1) is:

$$y - x\tan\theta = \alpha - (\alpha\omega+1)(\alpha\omega-1)(y-\alpha) + \omega^2(\alpha\omega+1)$$
$$\alpha(\alpha\omega-1)(y-\alpha)^2 - \left(\alpha^2\omega^2 - \frac{1}{3}\right)(\alpha\omega+1)\omega^2(\alpha\omega-1)(y-\alpha)^3$$
$$+\alpha(\alpha\omega+1)\omega^4\left(\alpha^2\omega^2 - \frac{2}{3}\right)(\alpha\omega-1)(y-\alpha)^4 - \left(\alpha^4\omega^4 - \alpha^2\omega^2 + \frac{2}{15}\right)$$
$$(\alpha\omega+1)\omega^4(\alpha\omega-1)(y-\alpha)^5, \tag{26}$$

$$\alpha - (\alpha\omega+1)(\alpha\omega-1)(y-\alpha) + \omega^2(\alpha\omega+1)\alpha(\alpha\omega-1)(y-\alpha)^2 -$$
$$\left(\alpha^2\omega^2 - \frac{1}{3}\right)(\alpha\omega+1)\omega^2(\alpha\omega-1)(y-\alpha)^3 + \alpha(\alpha\omega+1)\omega^4$$
$$\left(\alpha^2\omega^2 - \frac{2}{3}\right)(\alpha\omega-1)(y-\alpha)^4 - \left(\alpha^4\omega^4 - \alpha^2\omega^2 + \frac{2}{15}\right)$$
$$(\alpha\omega+1)\omega^4(\alpha\omega-1)(y-\alpha)^5 - y + x\tan\theta = 0. \tag{27}$$

To find the subsurface water profile by the HPM method, first we apply one of the upstream or downstream boundary conditions in Eq. (27) and let = 15.57. Comparing the realistic values of the flow depths and the values x from the above or the downstream to the other side, the values of the subsurface water profiles are calculated. The results of the depths obtained by the HPM method will be then compared with the experimental data of Sedghi-Asl (2010).

2.3 Justification of the HPM Solution

In this part of the study, results of the depths obtained by the HPM method are evaluated using Sedghi-Asl (2010) laboratory data. The NOF is the ratio of Root Mean Square Error (RMSE) to the overall mean X of the experimental data, which is expressed as (Moutsopoulos, 2009):

$$NOF = \frac{RMSE}{X}, \tag{28}$$

$$RMSE = \sqrt{\frac{\sum_{i=1}^{N}(y_{obs(i)} - y_{HPM(i)})^2}{N}}, \tag{29}$$

$$X = \frac{1}{N}\sum_{i=1}^{N} Y_{obs(i)}. \tag{30}$$

In which, $y_{obs(i)}$ are the experimental values of subsurface water profiles depth (observed depths); $y_{HPM(i)}$ are the values computed by Eq. (27), and N is the total number of values. In order to find an excellent match, NOF should be less than 1.0. In such a case, the theoretical method is still reliable and can be used with acceptable accuracy (Moutsopoulos et al., 2009; Gikas et al., 2006).

3. Results and Discussion

In this section of the paper, we evaluated the HPM method's generated findings using Sedghi-Asl (2010)'s experimental data (see Figs. 2 to 7). As can be seen in Figs. 2 – 4, the obtained subsurface water profiles from the HPM method are compared to the observed subsurface water profiles (Sedghi-Asl, 2010) for

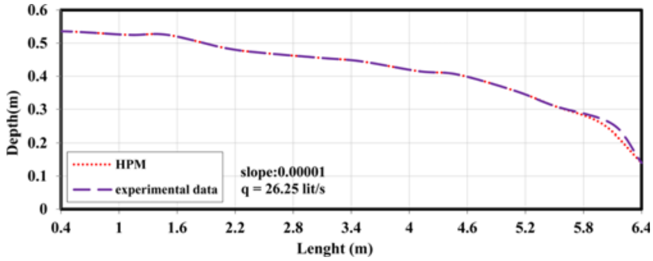

Fig. 2. Comparison of the Experimental Data (subsurface water profiles) of Sadghi-Asl (2010) with the HPM Results on a Slope Close to Horizon S = 0.00001 with Flow Rate q = 26.25 lit/s in Rounded Materials

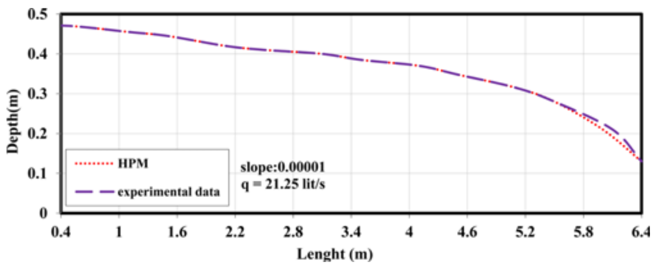

Fig. 3. Comparison of the Experimental Data (subsurface water profiles) of Sadghi-Asl (2010) with the HPM Results on a Slope Close to Horizon S = 0.00001 with Flow Rate q = 21.25 lit/s in Rounded Materials

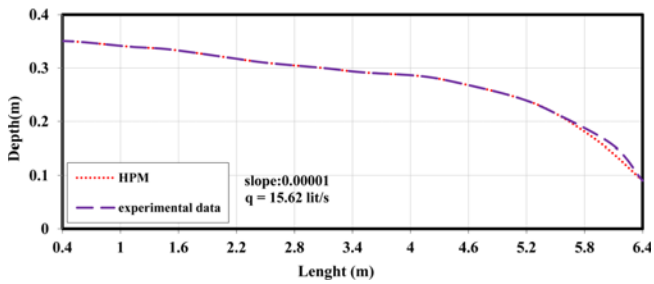

Fig. 4. Comparison of the Experimental Data (subsurface water profiles) of Sadghi-Asl (2010) with the HPM Results on a Slope Close to Horizon S = 0.00001 with Flow Rate q = 15.62 lit/s in Rounded Materials

rounded materials under approximately horizontal slope conditions. The HPM results show acceptable values when compared to the Sedghi-Asl data. We can see better approximations of the subsurface water profiles for higher flow rates (26.25 lit/s) compared to lower flow rates (15.62 lit/s) for round materials.

Figures 5 to 7 show the comparison of the HPM results with the observed water surface profiles (Sedghi-Asl, 2010) for crushed materials under approximately horizontal slope condition. The results of the HPM method show acceptable values, compared

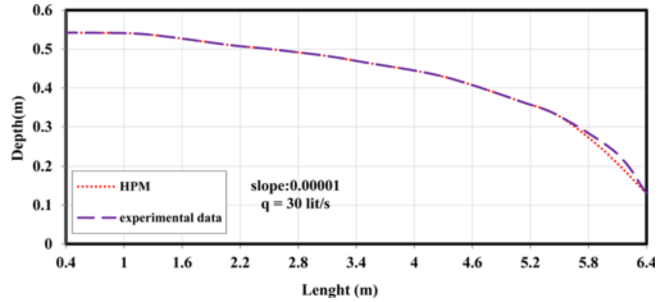

Fig. 5. Comparison of the Experimental Data (subsurface water profiles) of Sadghi-Asl (2010) with the HPM Results on a Slope Close to Horizon S = 0.00001 with Flow Rate q = 30 lit/s in Crushed Materials

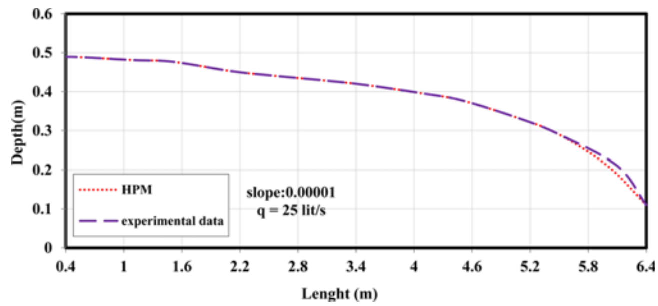

Fig. 6. Comparison of Experimental Data Water Level Profiles of Sadghi-Asl (2010) with the HPM Results on Slope Close to Horizon S = 0.00001 with Flow Rate q = 25 lit/s in Crushed Materials

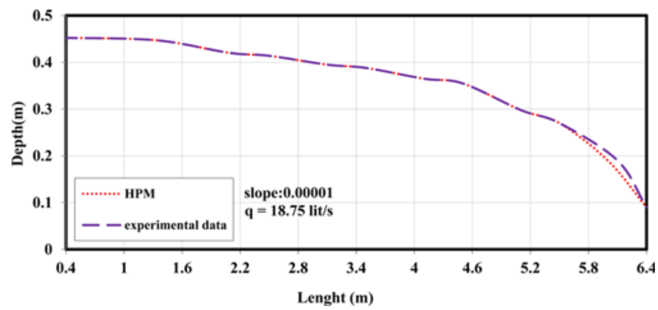

Fig. 7. Comparison of the Experimental Data (subsurface water profiles) of Sadghi-Asl (2010) with the HPM Results on a Slope Close to Horizon S = 0.00001 with Flow Rate q = 18.75 lit/s in Crushed Materials

to the observed data of Sedghi-Asl (2010). It can be seen that the HPM results provide more accurate water level profiles for higher flow rates (30 lit/s) compared to lower flow rates (18.75 lit/s) for crushed materials. The flow rate in the crushed material exceed to that of the round porous medium due to its high porosity. Therefore, the crushed materials have a greater capacity

Table 1. Assessment of HPM Solutions by NOF for Rounded Porous Media

NOF %	Downstream water level (m)	Upstream water level (m)	Information
0.000294586	0.14	0.535	Rounded aggregates, q = 26.25 lit/s
0.00033766	0.13	0.47	Rounded aggregates, q = 21.25 lit/s
0.00044695	0.09	0.35	Rounded aggregates, q = 15.62 lit/s

Table 2. Assessment of HPM Solutions by NOF for Crushed Porous Media

NOF %	Downstream water level (m)	Upstream water level (m)	Information
0.00028660	0.13	0.542	Crushed aggregate, q = 30 lit/s
0.00032011	0.11	0.489	Crushed aggregate, q = 25 lit/s
0.00034620	0.09	0.452	Crushed aggregate, q = 18.75 lit/s

for conveyance of water through the pores.

Table 1 shows the NOF criterion between the HPM solution of subsurface water profile in rounded porous media with various boundary conditions and slope close to the horizon. As it is seen, the NOF in the rounded porous media are 0.000294586, 0.00033766 and 0.00044695 when q = 26.25 lit/s, q = 21.25 lit/s and q = 15.62 lit/s, respectively. The results reveal that NOF in the rounded porous medium increases with the decreasing flow rate.

Table 2 shows the NOF % of the HPM solutions of water surface in crushed porous media with various boundary conditions and a slope close to the horizon. Similar to the rounded materials, NOF increases with the reduction in flow rate.

4. Conclusions

The governing equation of non-Darcy flow in porous aggregate media was solved in this paper by the HPM method for the first time. In addition, results are compared with the results of Sedghi-Asl (2010) under the upstream and downstream conditions for different rates and slope S = 0.00001 close to the horizon. NOF criterion has been employed to find out the accuracy of the presented HPM solutions. According to NOF results, HPM solutions resulted in more precise solutions for higher flow rates. Results depicted quite acceptable accordance between HPM solutions and the experimental data, even in crushed porous media. This result promises the HPM method applicability in porous media of high-velocity flow rates. Best result for crushed porous media was obtained at flow rate q = 30 lit/s with NOF of 0.00028660. The subsurface water profiles calculated using the HPM method, illustrates greater precision in crushed porous media with the experimental data when compared to rounded porous media. Besides, HPM method results depicts quite acceptable results for rounded porous media. Adopting the HPM method for non-Darcy governing equation in this research, presents remarkable accurate solutions for water movement in both, rounded and crushed porous media.

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Nomenclature

a = non-Darcy flow coefficient
c = constant of integration
q = Discharge per unit width
NOF = Normal Objective Function
s = Flume slope
x = longitudinal distance
y = Flow depth
X = Overal mean
θ = Bed slope
$y_{obs(i)}$ = Values of subsurface water profiles depth
$y_{HPM(i)}$ = Values computed by the final HPM solution

ORCID

Mohammad Hadi Fattahi https://orcid.org/0000-0003-3855-3383
Mohammad Sedghi-Asl https://orcid.org/0000-0002-9159-251X
Seyyed Abbas Mohammadi https://orcid.org/0000-0002-3339-4929

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