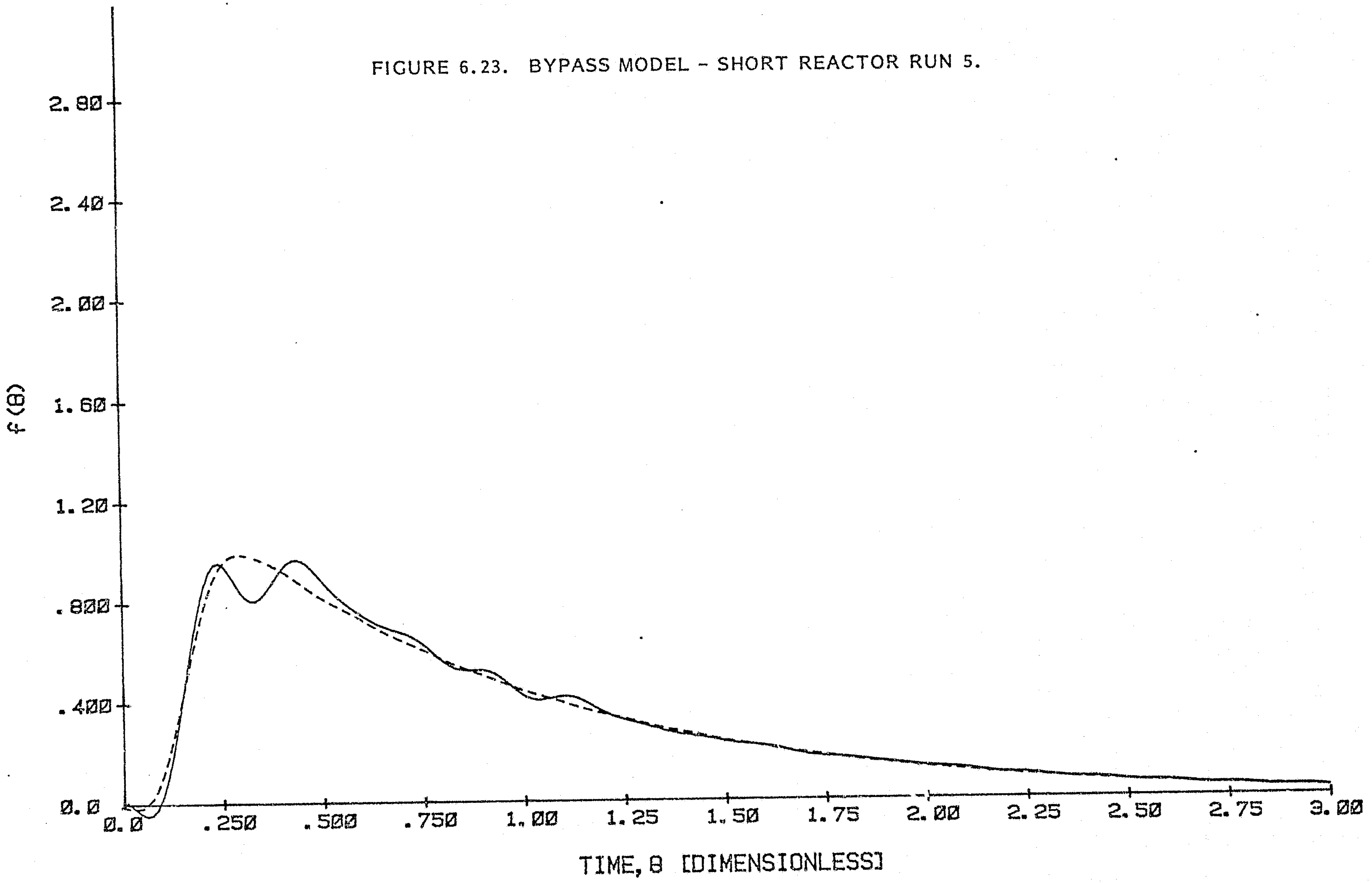
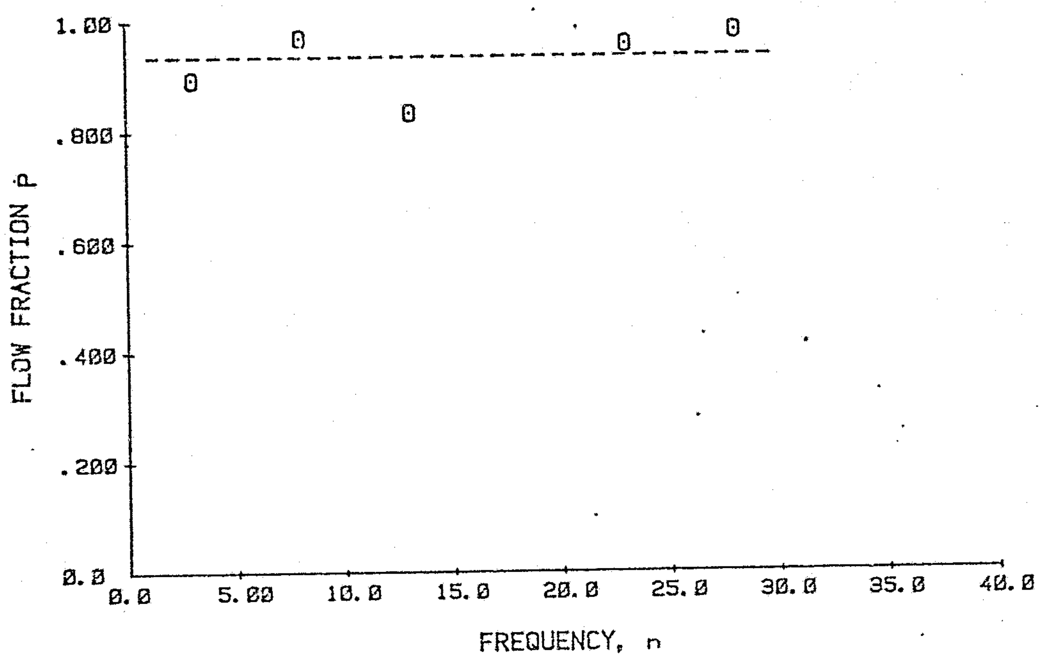


FIGURE 6.23. BYPASS MODEL - SHORT REACTOR RUN 5.

FIGURE 6.24. PARAMETERS OF THE BYPASS MODEL - SHORT REACTOR RUN 2.



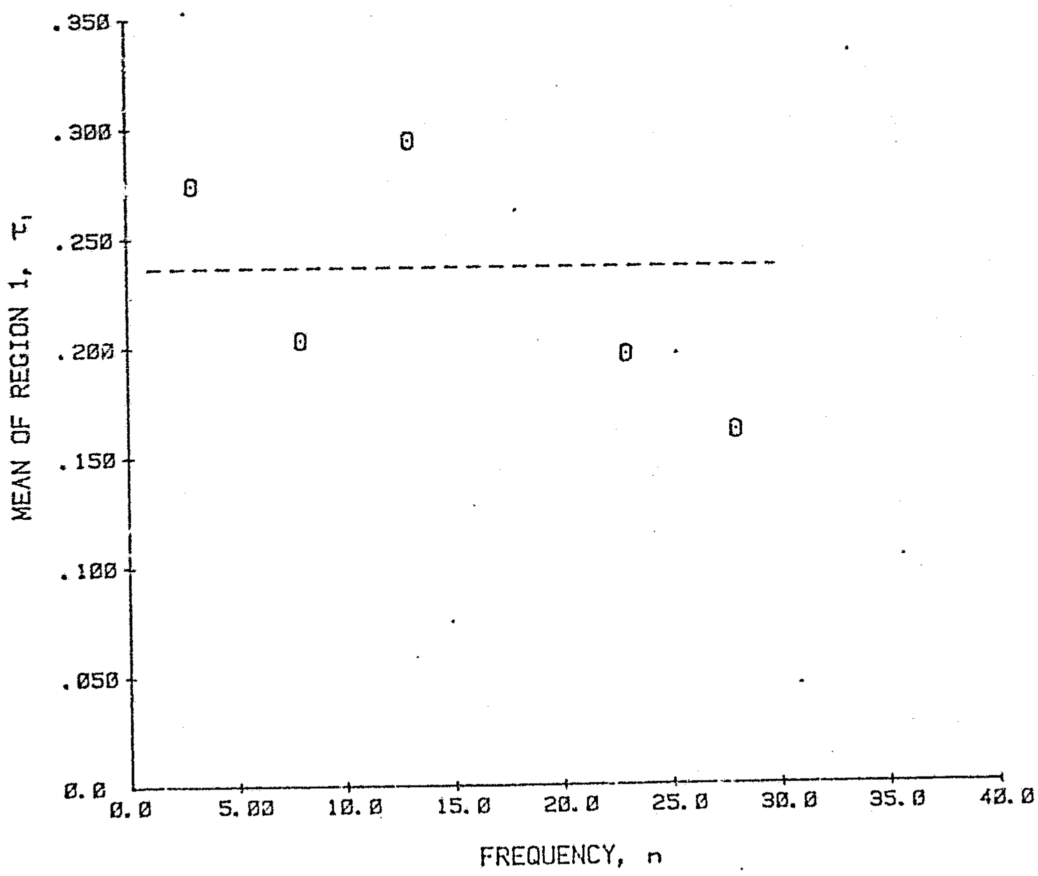
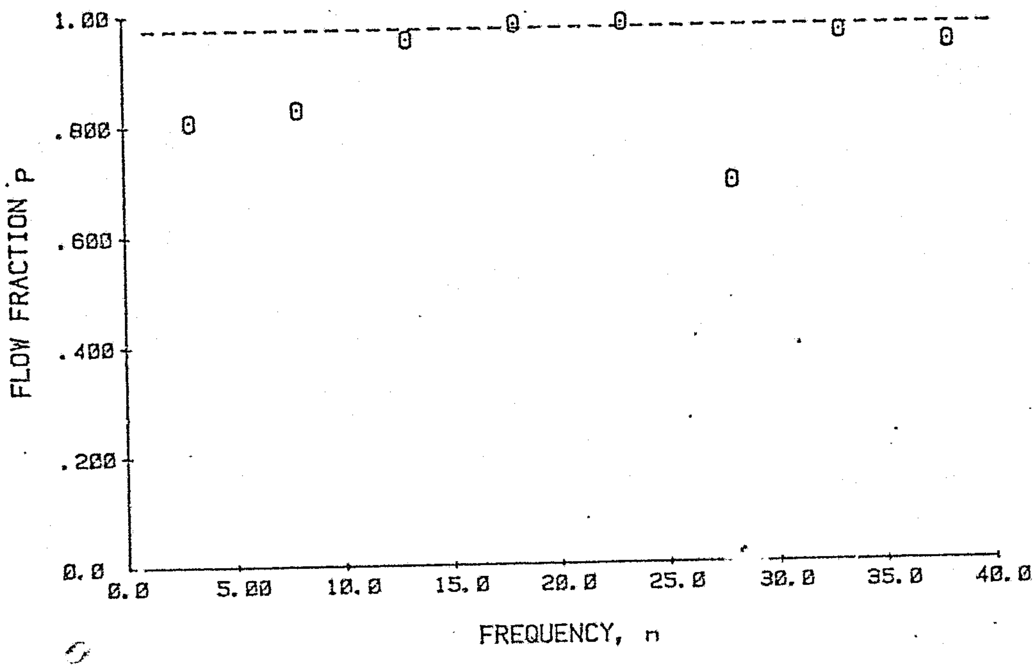
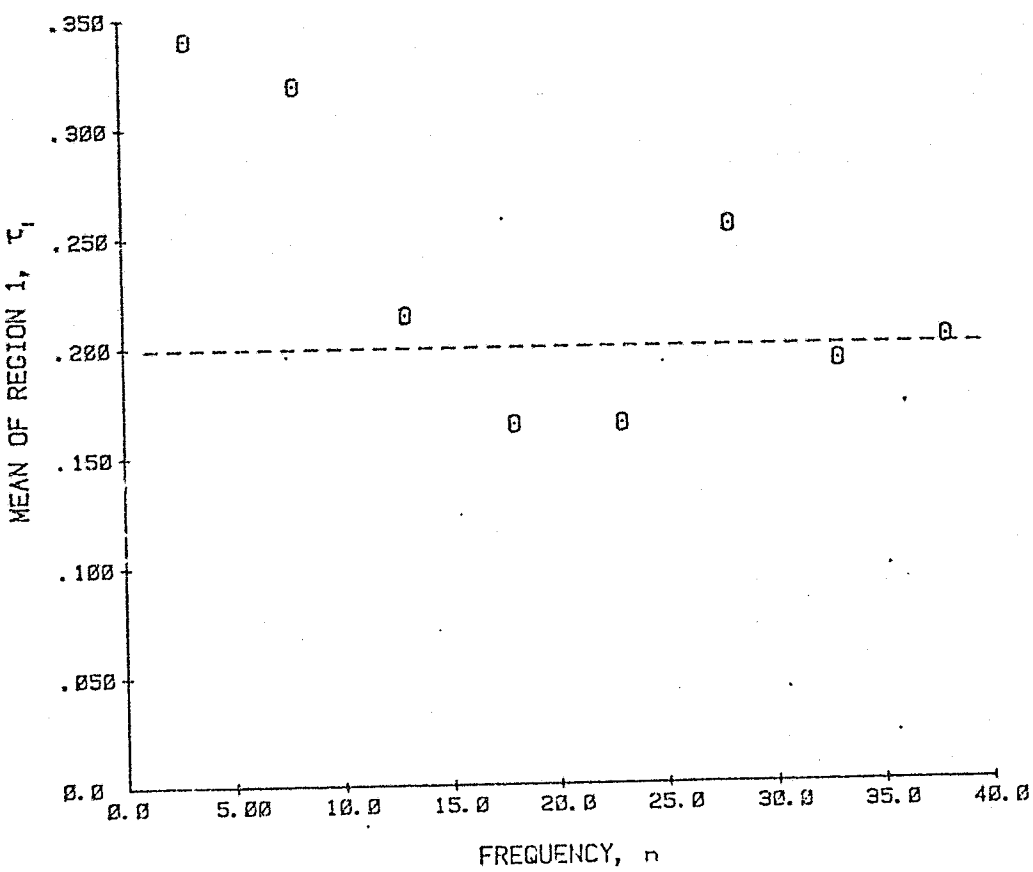


FIGURE 6.25. PARAMETERS OF THE BYPASS MODEL - SHORT REACTOR RUN 5.





7. CONCLUSIONS

Digital radiation counting techniqes are found to be suitable for measuring very short residence time distributions (down to a mean residence time of 0,4 sec). By using high count rates and carefully selected counting periods, the statistical radiation error and instrument error can be kept small.

Flow patterns in the vessels are shown to fluctuate with time, and experiments have to be repeated and averaged to obtain a representative curve.

The flow patterns are independent of flowrate over the range investigated (turbulent conditions). This is in agreement with the findings of Clegg (11) for a cylindrical vessel also exhibiting internal recycle.

In comparing the RTD curves of the two lengths of reactor, it can be seen that the effect of reducing the vessel length is to increase the "mixedness" in the vessel. The short reactor RTD curve does not differ significantly from that of a CSTR, while the long reactor curve shows the peaks characteristic of recycling. For both vessels the tails of the curves decay exponentially. The large first peak in the long reactor RTD curve would result in lower conversions per unit volume for this reactor shape than for the short reactor for most reaction schemes. If one were to increase the reactor length further, the first peak would become more emphasised but its position would shift towards $\theta = 1$ and the recycle fraction would decrease, approaching plug flow for very large l/d ratio. It thus appears that the l/d ratio given by the long reactor is less attractive than either a small l/d (well mixed) or a much larger l/d (plug flow). One cannot however extrapolate the results of only two reactor lengths to much larger l/d ratios, and further research would be required. There are naturally other factors which must be considered in comparing reactor types, e.g. the effect of recirculation in heating up the reactants in the case of autothermal reactions.

Flow patterns in the longer vessel are well modelled by a simple recycle model. The forward flow region shows a small amount of backmixing and is adequately represented by a gamma distribution (tanks-in-series model) or an axial dispersion model. The recycle region is well mixed and can be represented by a CSTR. The mean residence time in the forward region is about 0,2 of the overall mean residence time and this region comprises about 70% of the total reactor volume. The recycle fraction is approximately 0,7.

Less information could be extracted from the smoother RTD curves of the short vessel. This reactor can adequately be represented by a gamma distribution with little backmixing, followed by a CSTR, with a small bypass (4%) around the CSTR. The first region makes up about 20% of the vessel volume. For approximate engineering calculations however, the short vessel can be assumed to be a CSTR.

Scale-up of the results to a full sized reactor is simplified by the fact that the flow patterns are independent of flowrate. Dynamic similarity is therefore not required, so long as the Reynolds number is high enough so that the flow is turbulent. The only two scaling criteria as far as flow patterns are concerned are therefore geometric similarity and allowances for density changes in a reacting system. Allowance for density changes is usually made by assuming that all gas expansion takes place immediately upon entering the reactor. The inlet nozzles are therefore scaled by an additional factor equal to the square root of the ratio of expansion of the gases in the reactor (23,24). Expansion due to chemical reaction (change in number of moles) as well as temperature change is treated in this way.

The type of reactor investigated in this work can be used in many applications where intense mixing of gases is required. Its greatest application is where internal recirculation is required e.g. for autothermal reactions such as combustion systems. It can also be used for gas-particle reactions where a high level of turbulence is required e.g. pulverised coal gasification. Besides their application to represent flow inside vessels, the recycle flow models developed here can be applied to numerous other processes based on recirculation, e.g. catalytic cracking, particle coating, granulation and crystallisation (15).

NOMENCLATURE

a_n	Fourier coefficient (sine) at frequency $n\pi/T$
AR	Amplitude ratio of a frequency response
b_o	Fourier coefficient at zero frequency
b_n	Fourier coefficient (cosine) at frequency $n\pi/T$
c	Concentration of tracer
CSTR	Constant flow stirred tank reactor
d	Diameter
f(t)	Statistical probability density function used to describe input and output concentrations and RTD
F(s)	Laplace transform of f(t)
i	$\sqrt{-1}$
Im[ ]	Imaginary part of a complex number
l,L	Length
M	Number of points in an experimental RTD curve
n	Fourier coefficient number (frequency)
n_1, n_2	Number of CSTR's in a tanks-in-series model
N	Last Fourier coefficient number before truncation of the Fourier series
p	Fraction of the flow going to region 2 of the model
P	Pressure
r, R	Radiation count rate (counts/second)
R	Recycle ratio of recycle model $(p/(1-p))$
Re	Reynolds number
Re[ ]	Real part of a complex number
RTD	Residence time distribution
s	Laplace transform variable $(\alpha + iw)$
S^2	Sum of squared errors
t	Time
T	Half of the time duration of a Fourier Series representation of a RTD curve
T	Temperature
Δt	Time interval between readings in an experimental RTD curve
u	Velocity

V	Volume
$\dot{V}$	Volumetric flowrate

Greek letters

α_K	k-th moment of $f(t)$
θ	Dimensionless time, t/τ
μ	Dynamic viscosity
ρ	Density
σ^2	Variance of $f(t)$
τ	Mean residence time: mean of $f(t)$
τ_D	Instrument dead time (section 2.3.4)
ϕ	Phase angle of frequency response
w	Frequency, $n\pi/T$

Subscripts

in, i	Input curve
out, o	Output curve
n	Fourier coefficient number
1	Flow region 1 of flow model
2	Flow region 2 of flow model

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APPENDIX I. CALIBRATION CURVES

FIGURE I.1: Calibration curve for the rotameter for measuring gas sampling flowrate.

The calibration curve is for air at 15°C and 101,32 kPa abs. Correction for other temperatures and pressures is:

$$\dot{V}\ [l/min]\ (actual) = \dot{V}\ [l/min]\ (graph)\ \sqrt{\frac{101,3}{P}\ \frac{T}{288,15}}$$

where P = upstream pressure (kPa)
T = air temperature (K)

FIGURE I.2: Calibration curve for the orifice plate flowmeters used for measuring the air flowrate to the reactor.

This curve has been calculated using the British Standard BS1042 The orifice plate was constructed to the specifications of the Standard.

Orifice type: Square edged with corner tappings
Pipe diameter: 80 mm
Orifice diameter: 56 mm

The curve is for air at 20°C and upstream pressure of 88 kPa abs. Correction for other temperatures and pressures is:

$$\dot{V}\ [Nm^3/h]\ (actual) = \dot{V}\ [Nm^3/h]\ (graph)\ \sqrt{\frac{P}{88}\ \frac{293,15}{T}}$$

where P = upstream pressure (kPa)
T = air temperature (K)

FIGURE I.1 SAMPLING ROTAMETER CALIBRATION CURVE

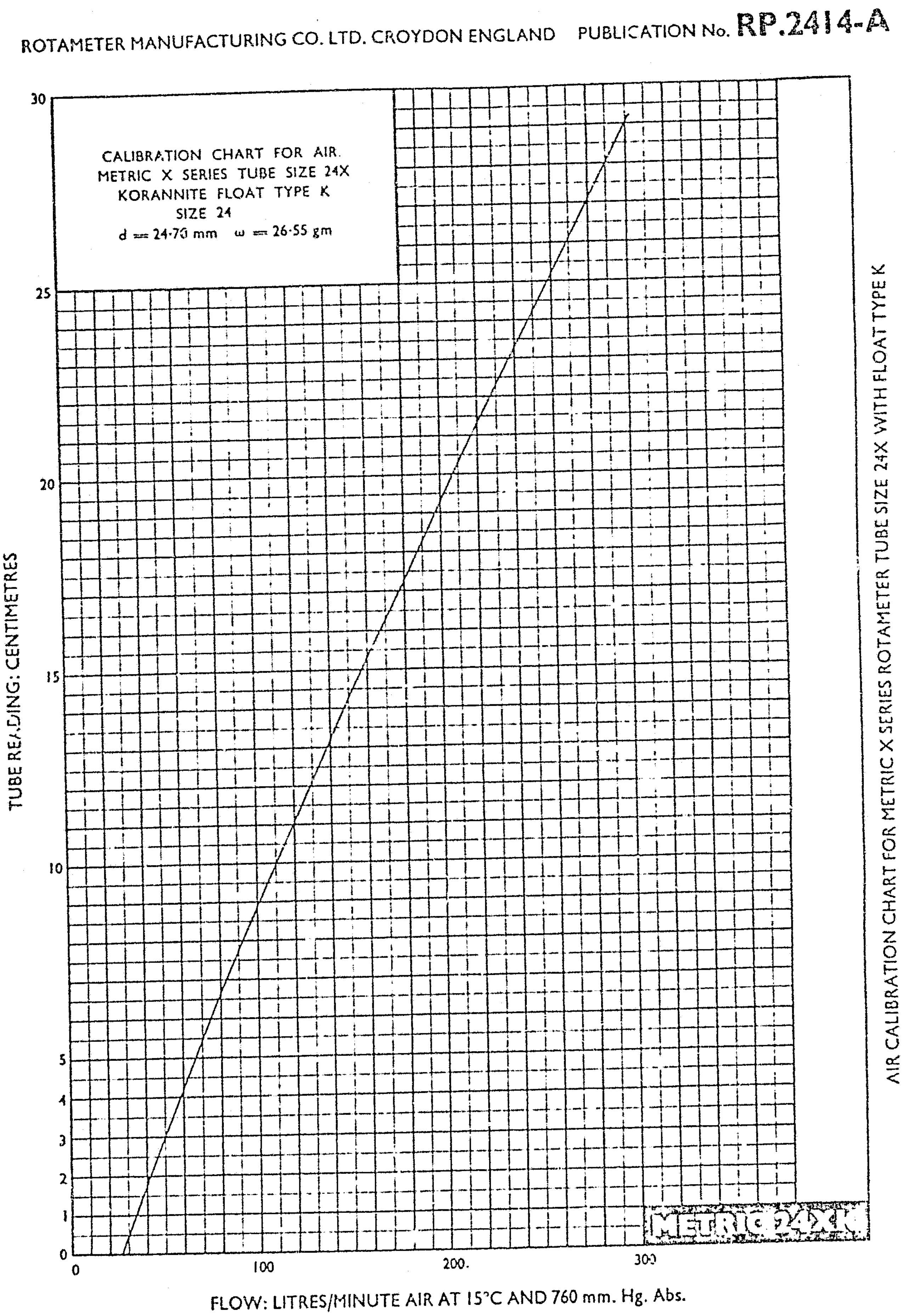
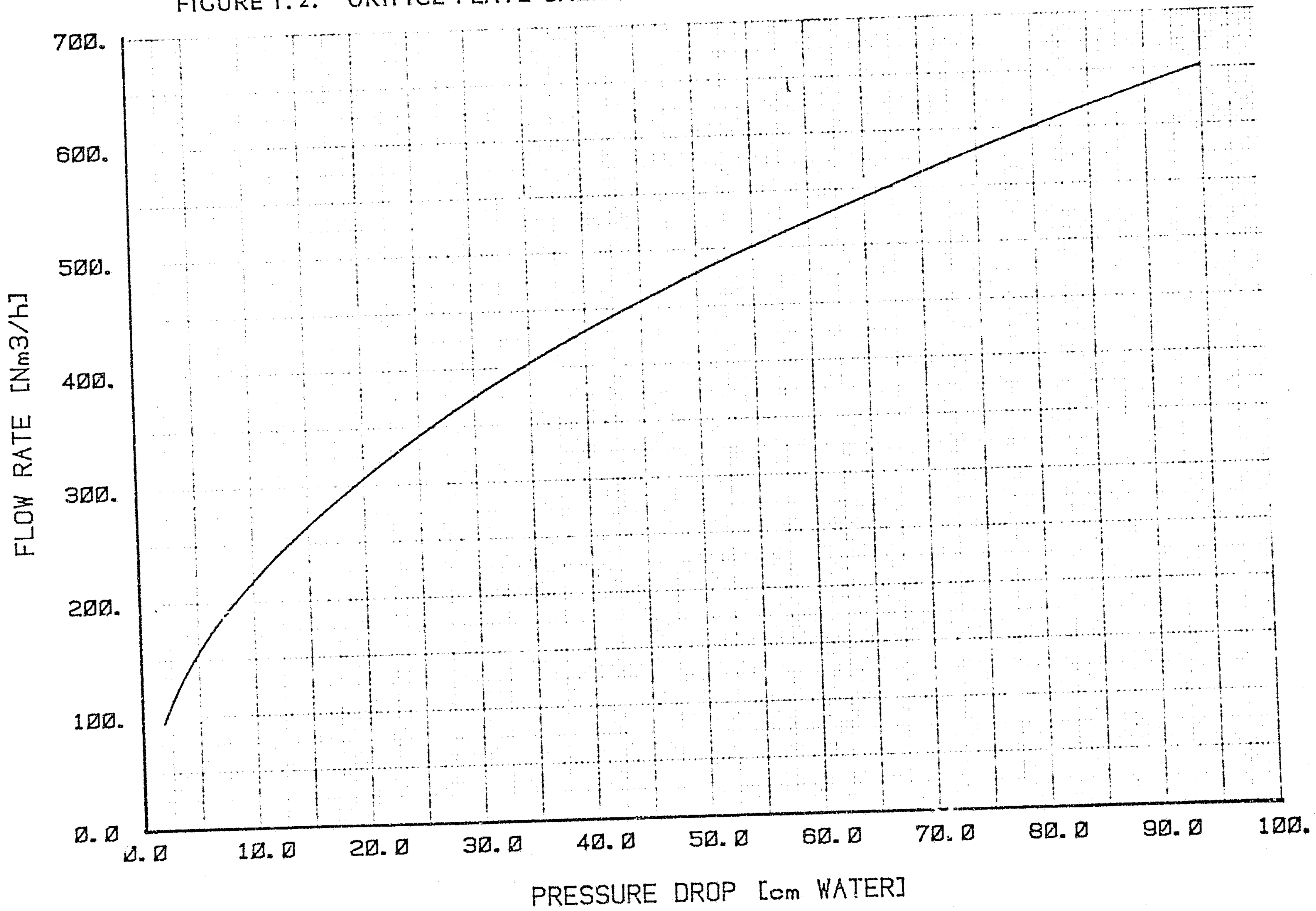


FIGURE I.2. ORIFICE PLATE CALIBRATION CURVE - AIR AT 20°C AND 88 kPa abs.



APPENDIX II. GRAPHS OF THE OUTPUT CURVES FOR THE LONG REACTOR

FIGURES II.1 to II.5 : Output curves for runs 1-5 (each "run" is for a different flowrate - see table 3.1)

FIGURES II.6 to II.10 : The mean and standard deviation of the output curves of each run.

FIGURE II.1. OUTPUT CURVES - LONG REACTOR RUN 1

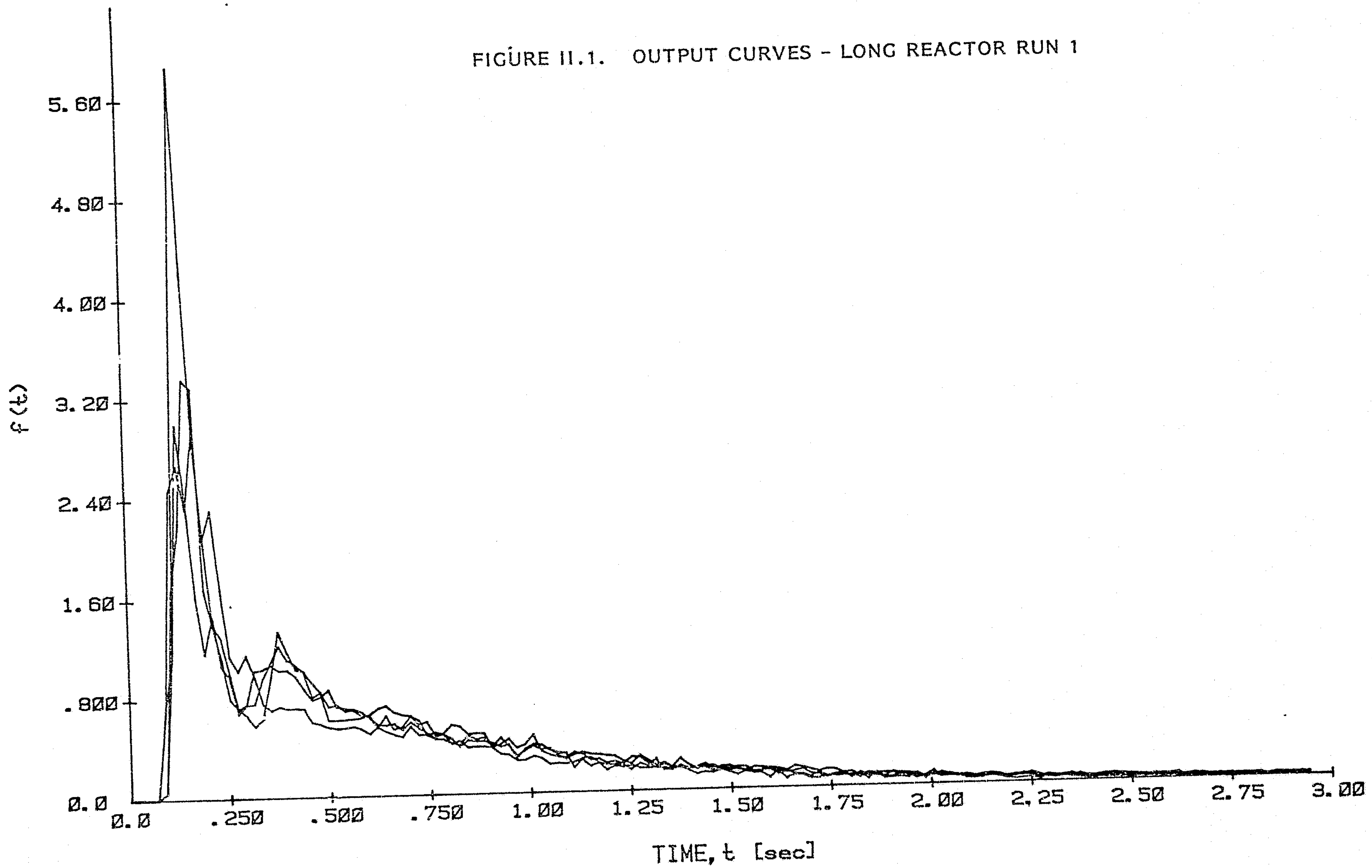


FIGURE II.2. OUTPUT CURVES - LONG REACTOR RUN 2

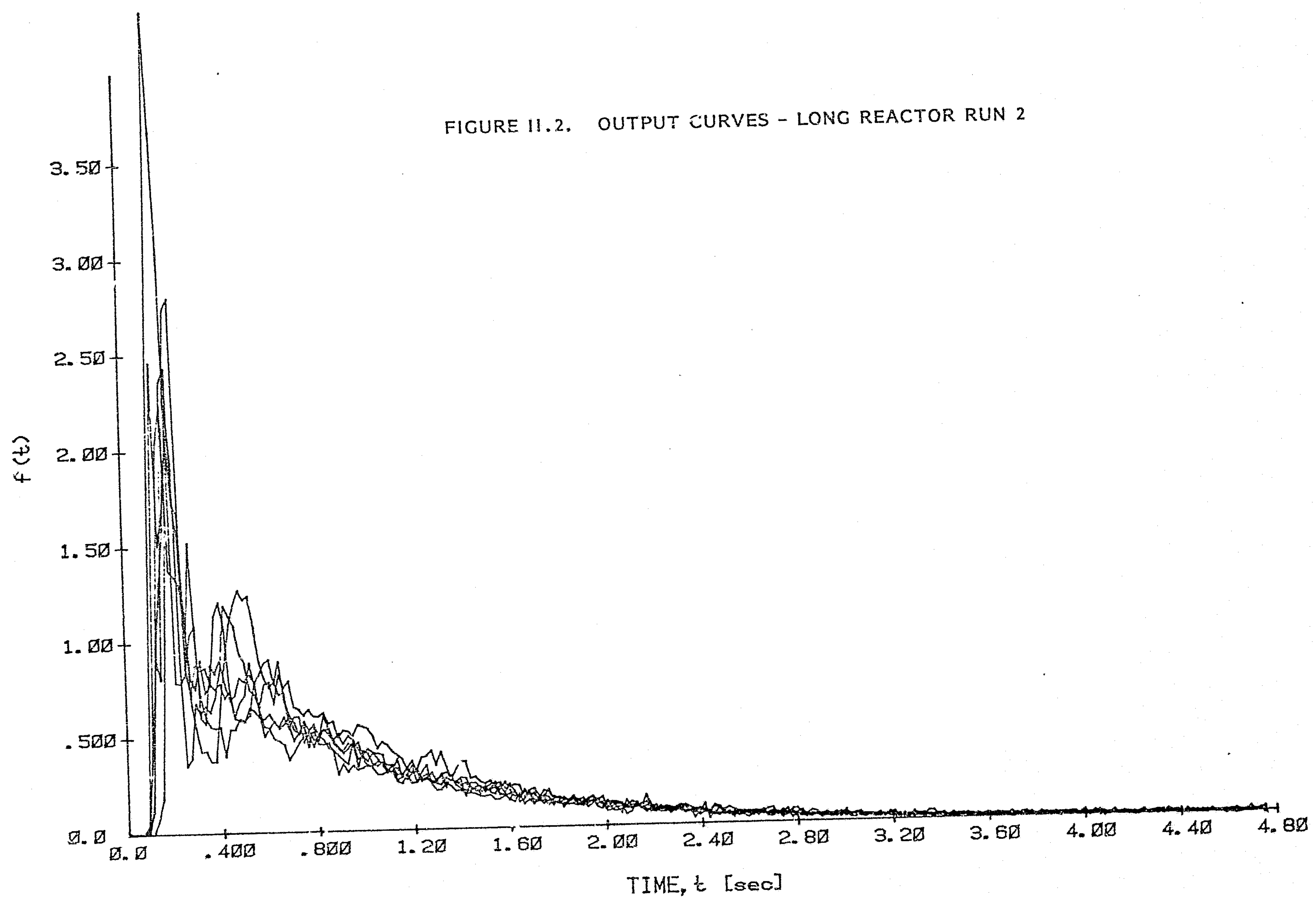


FIGURE II.3. OUTPUT CURVES - LONG REACTOR RUN 3

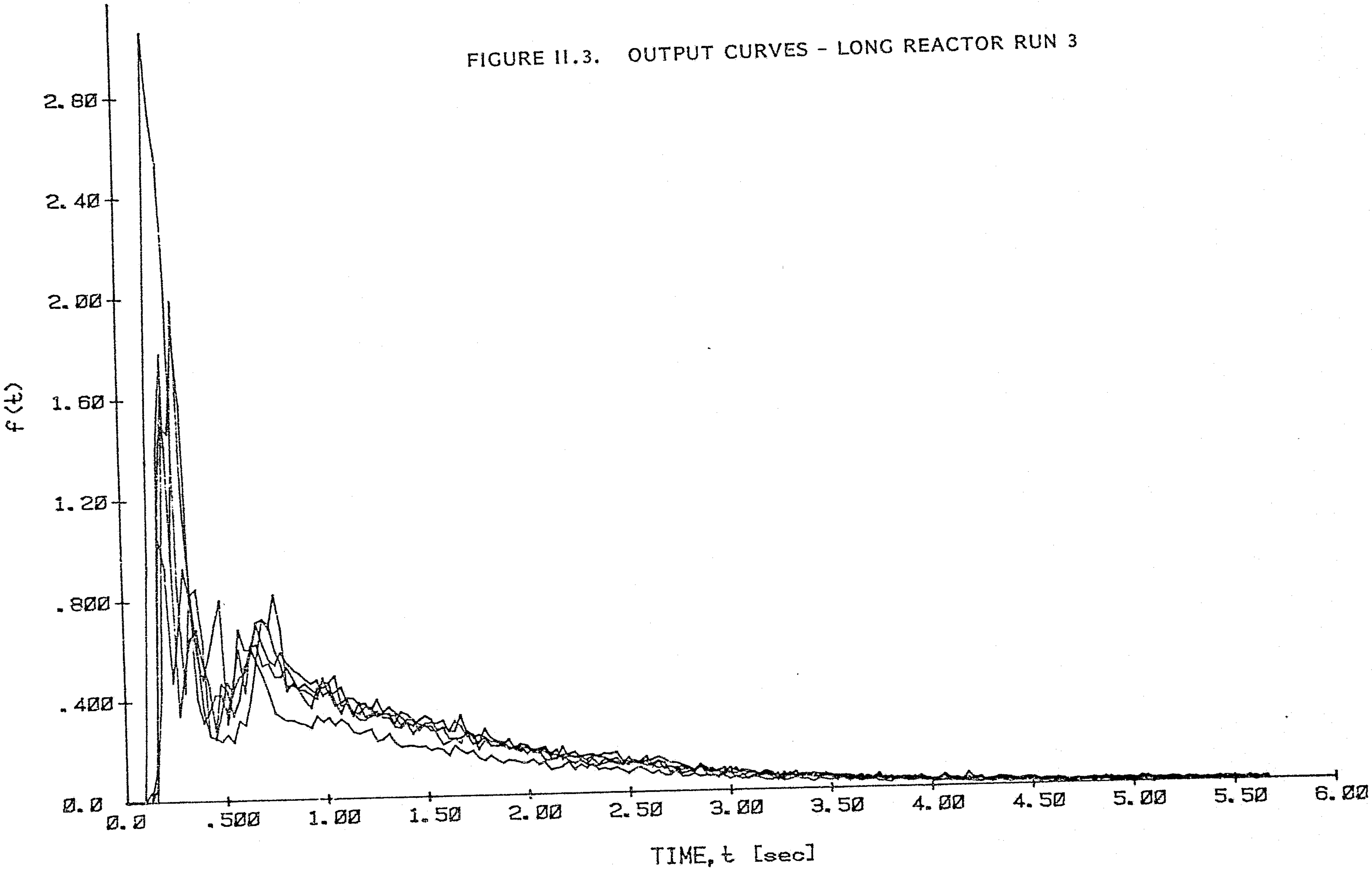


FIGURE II.3. OUTPUT CURVES - LONG REACTOR RUN 3

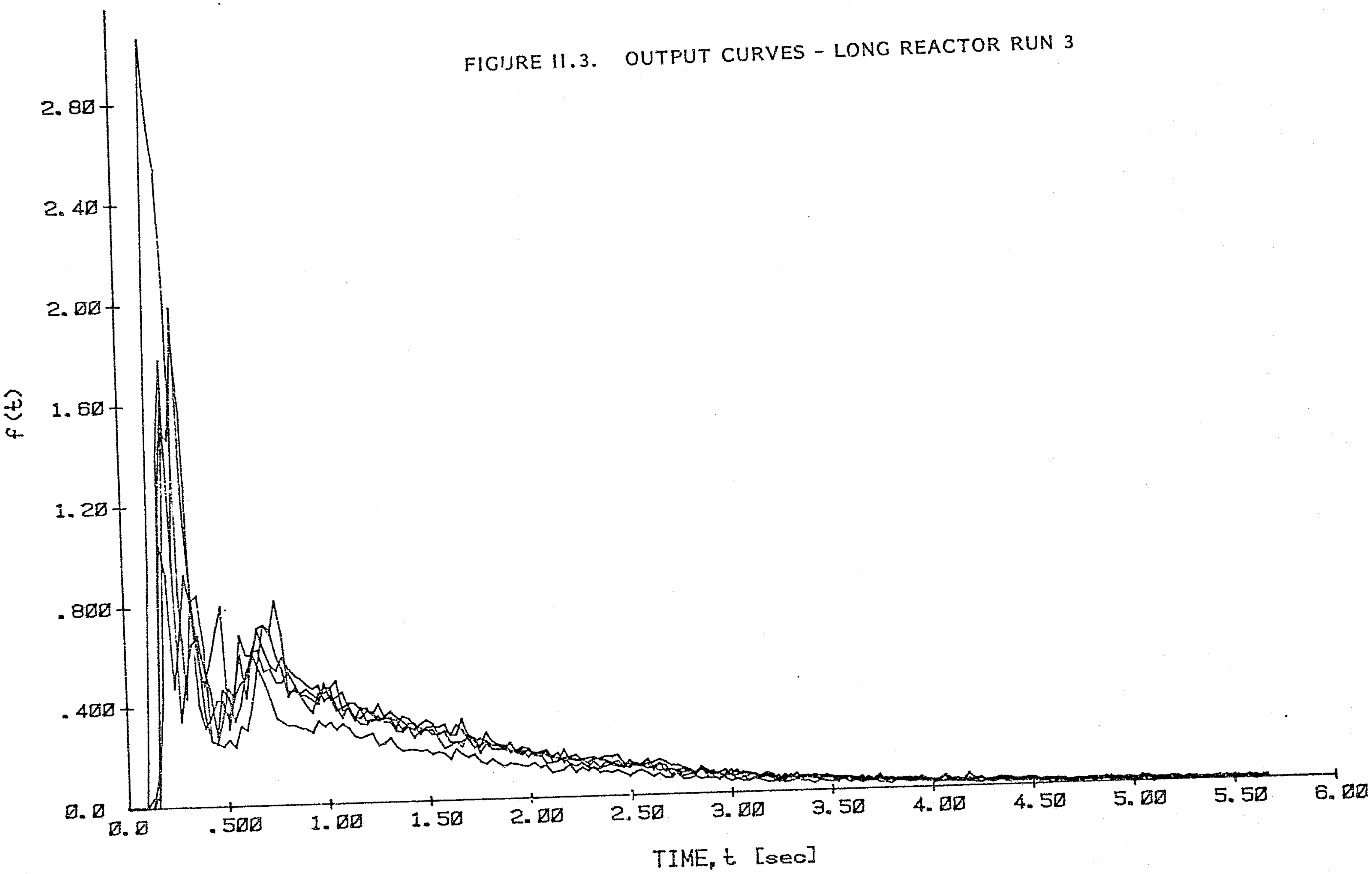


FIGURE II.4. OUTPUT CURVES - LONG REACTOR RUN 4

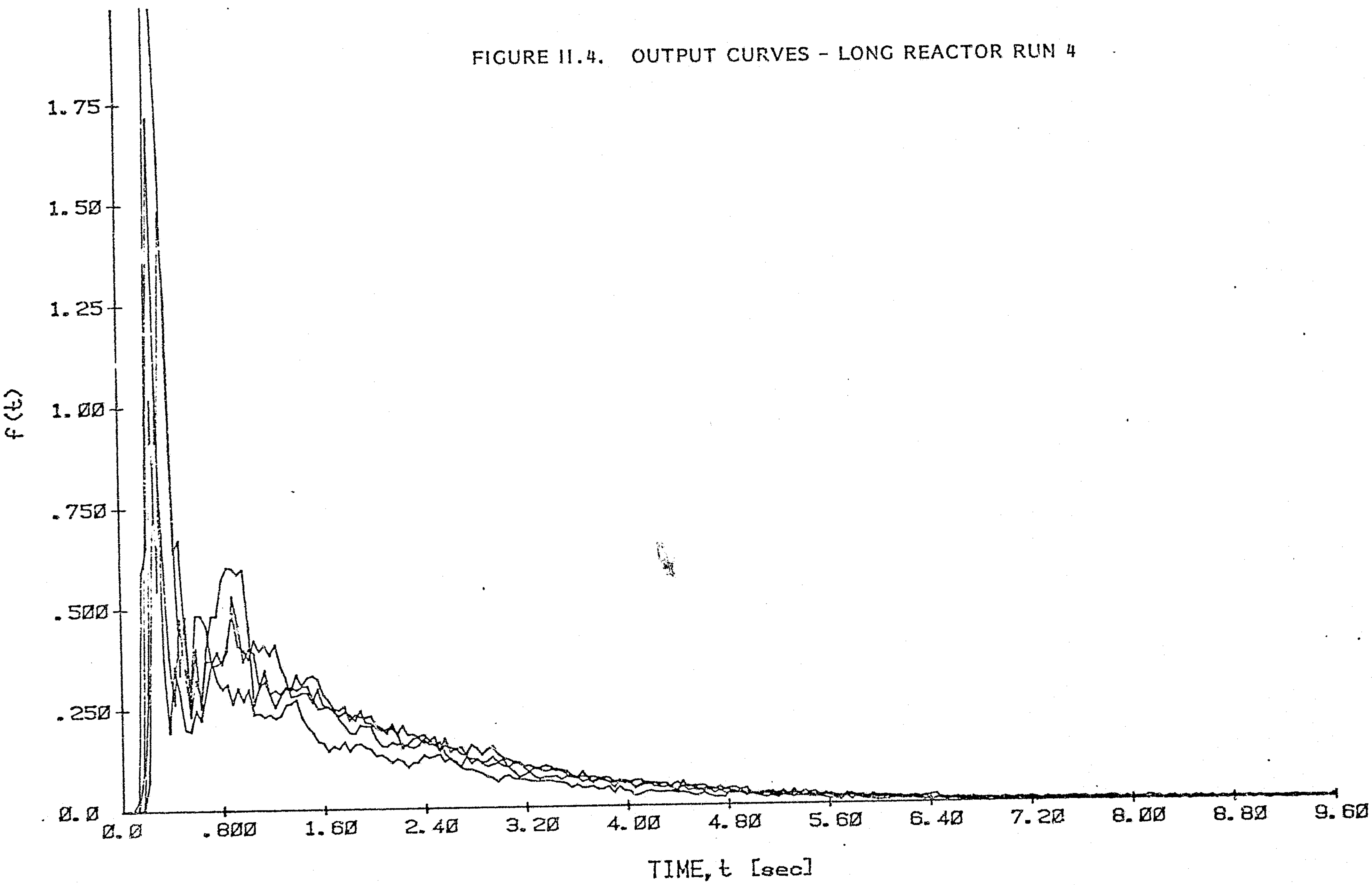


FIGURE II.5. OUTPUT CURVES - LONG REACTOR RUN 5

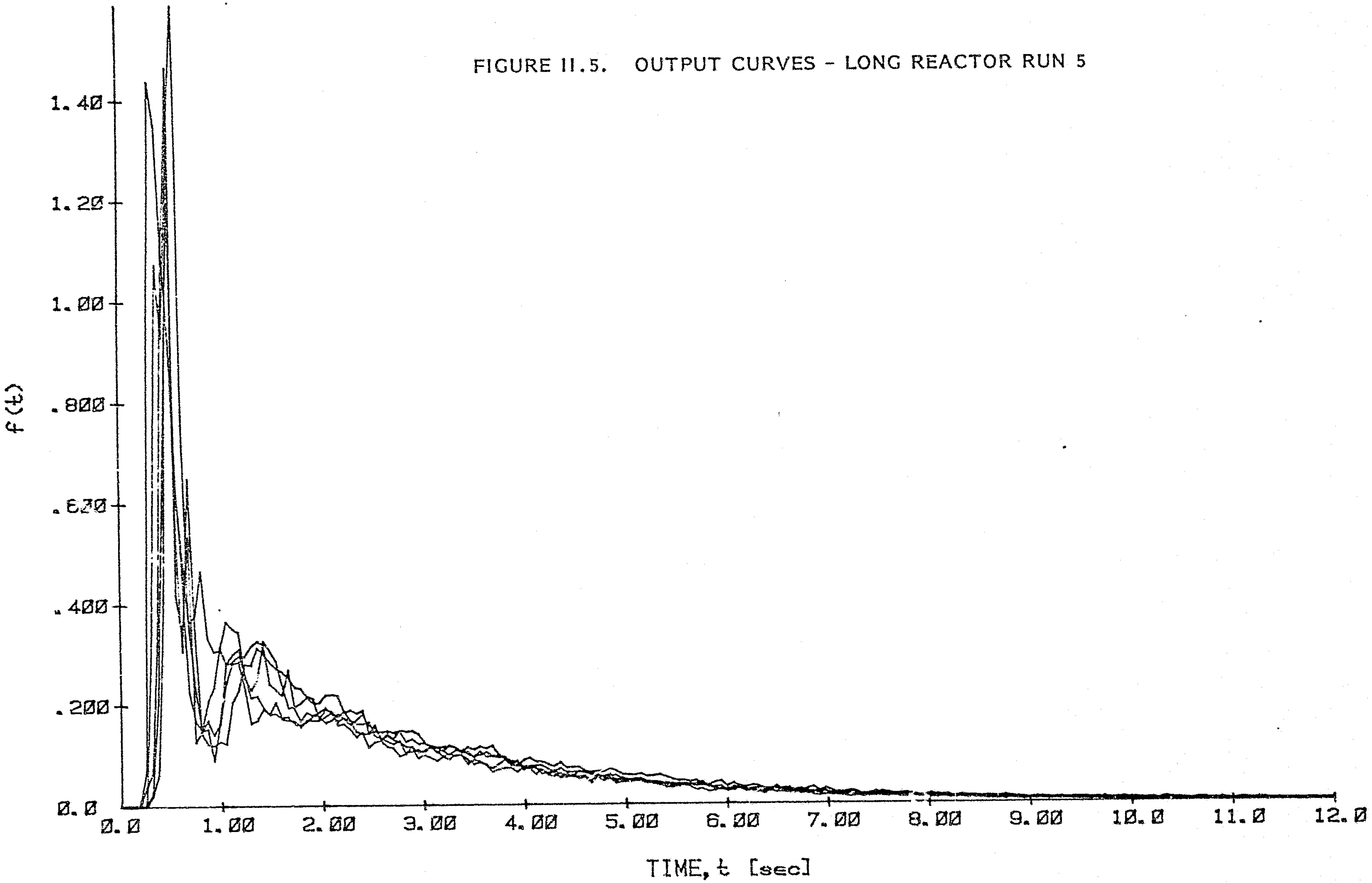


FIGURE II.6. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - LONG REACTOR RUN 1.

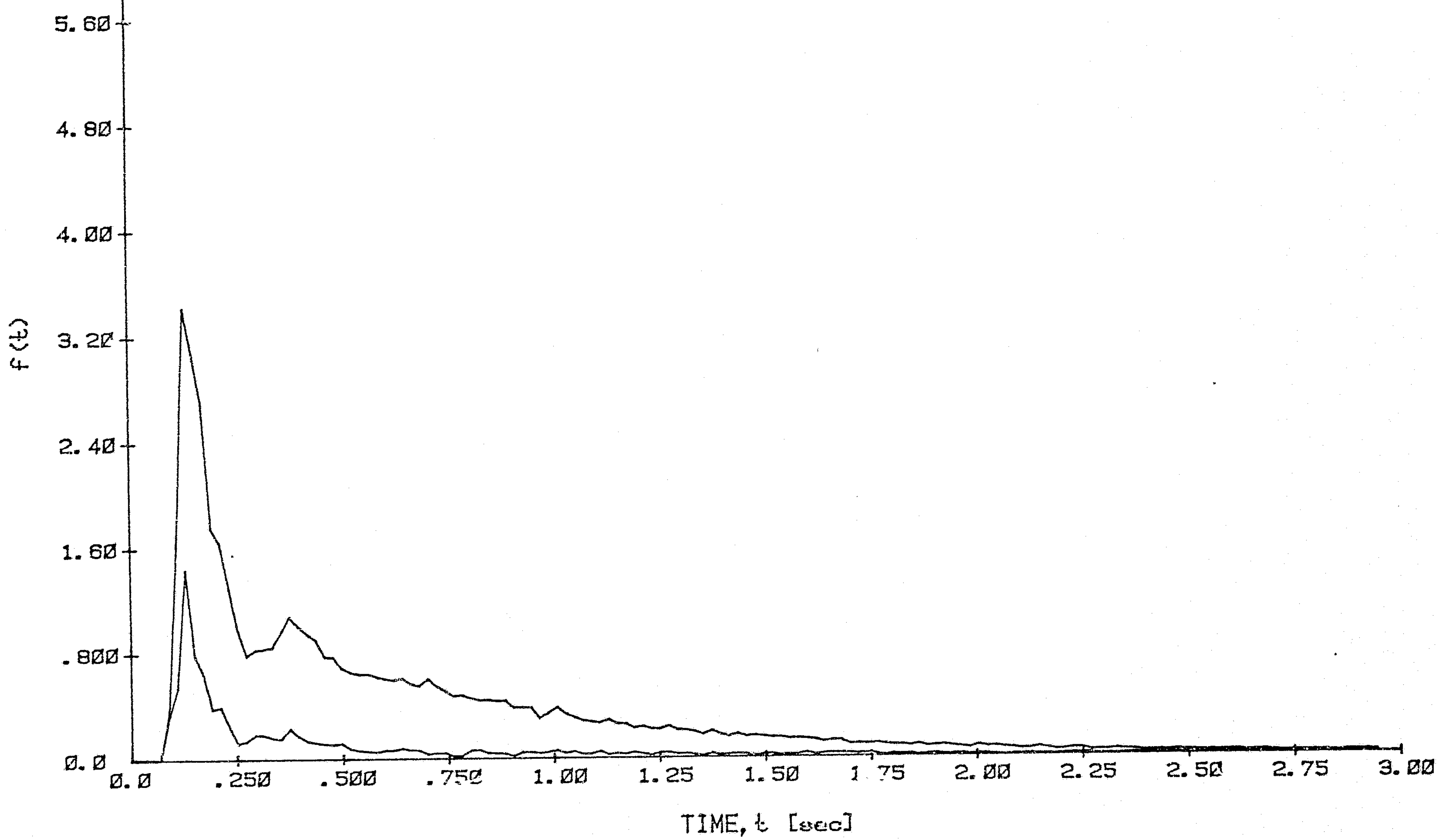
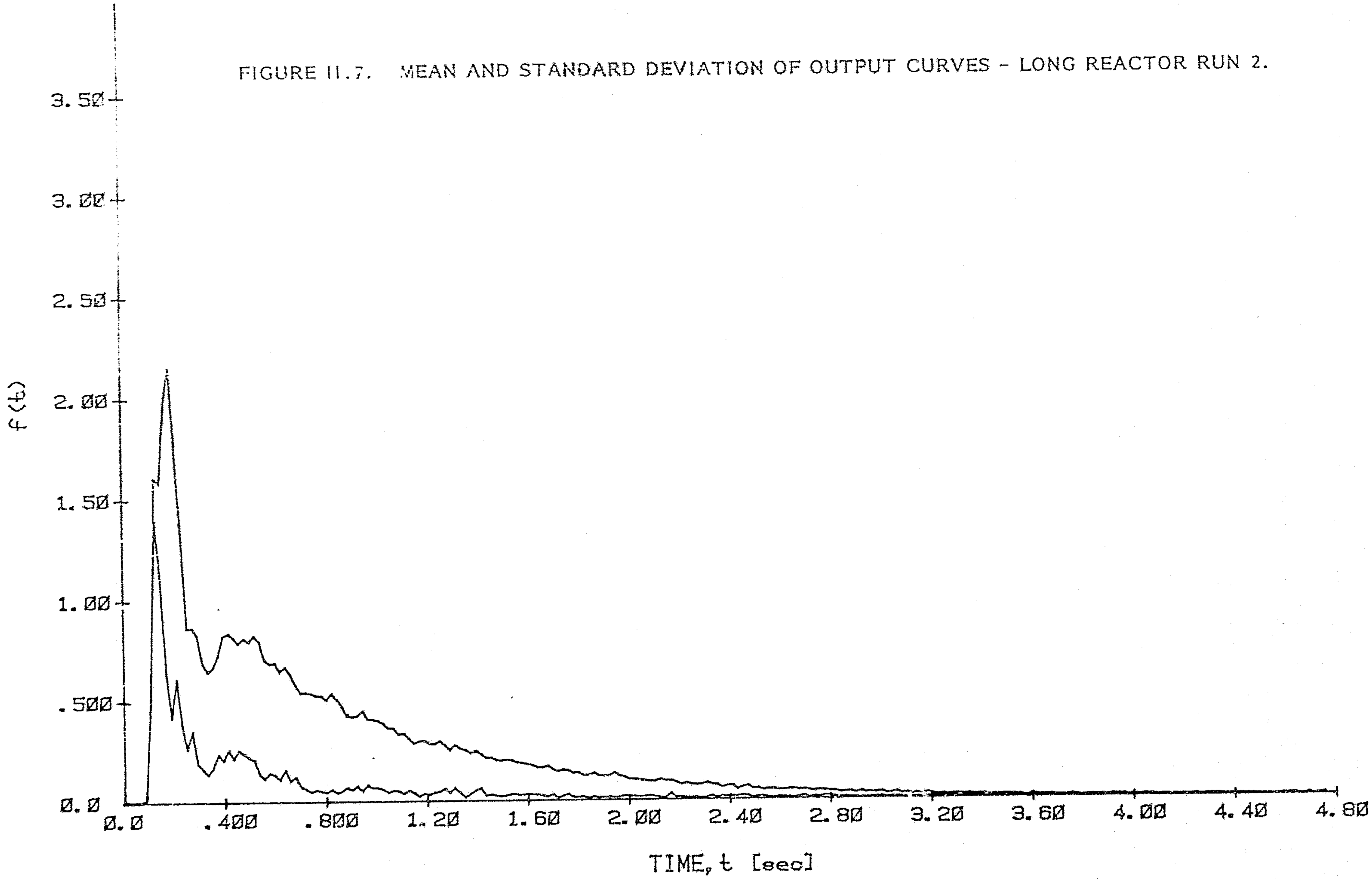


FIGURE II.7. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - LONG REACTOR RUN 2.



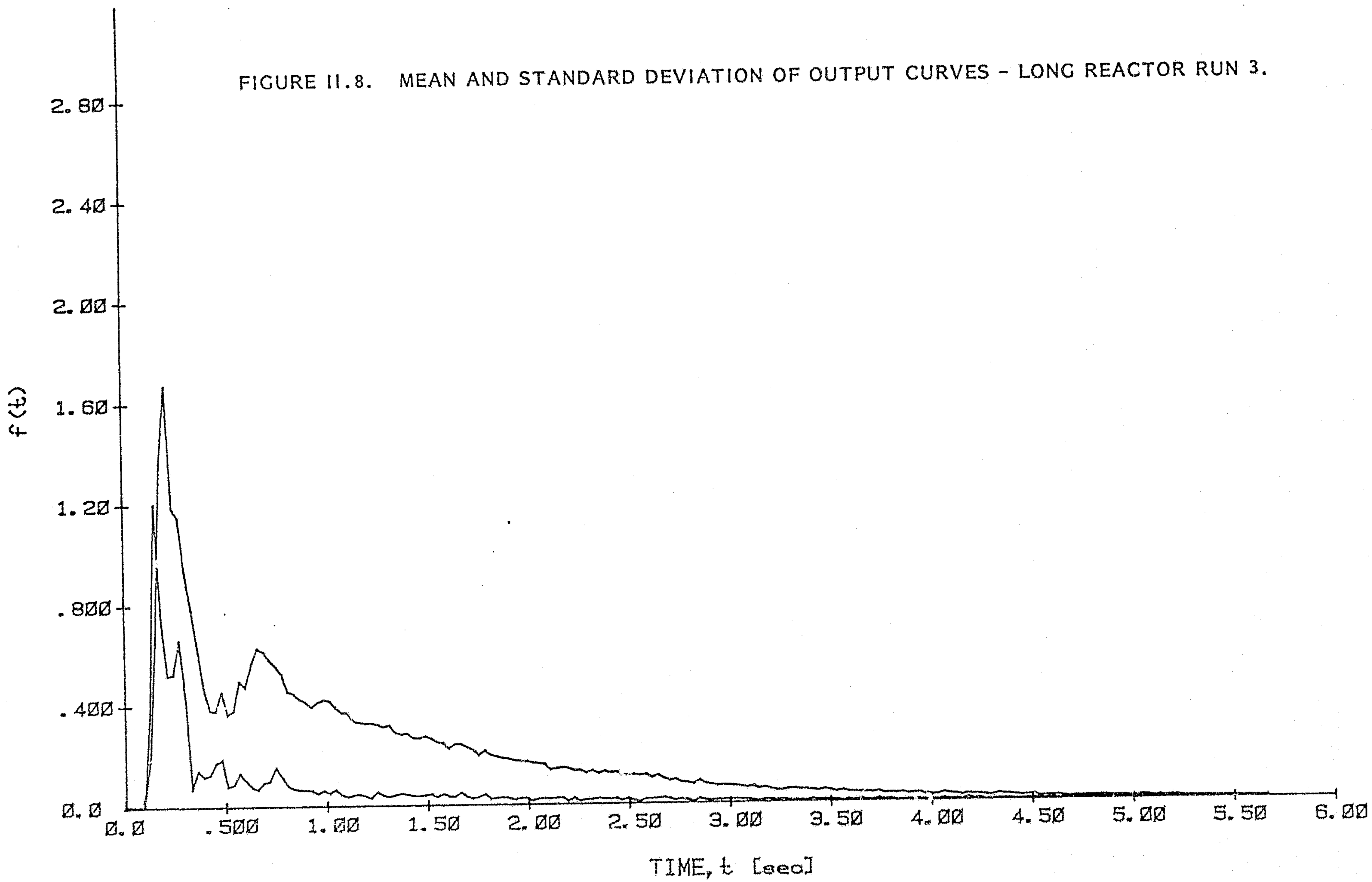


FIGURE II.8. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - LONG REACTOR RUN 3.

FIGURE II.9. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - LONG REACTOR RUN 4.

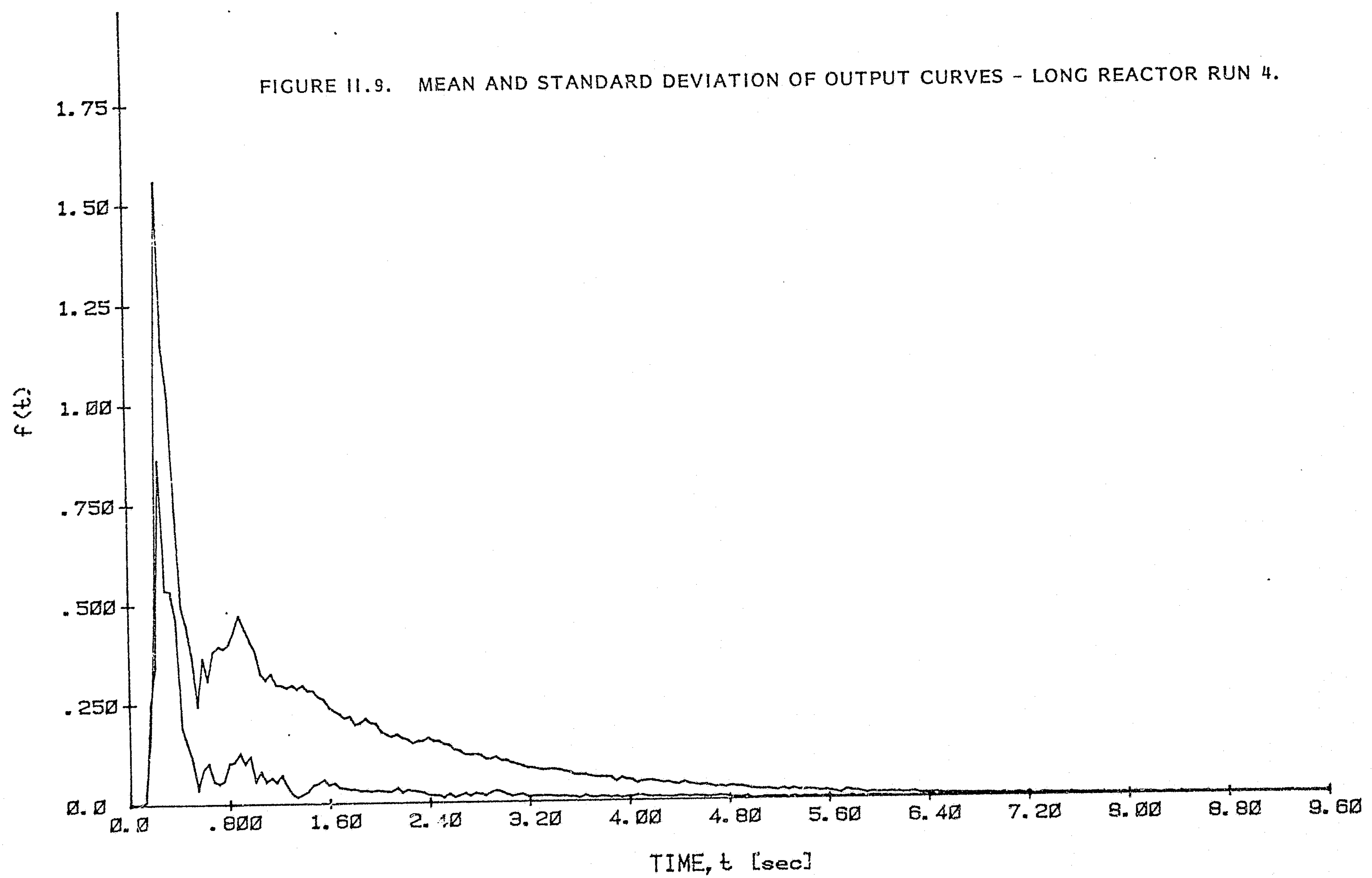
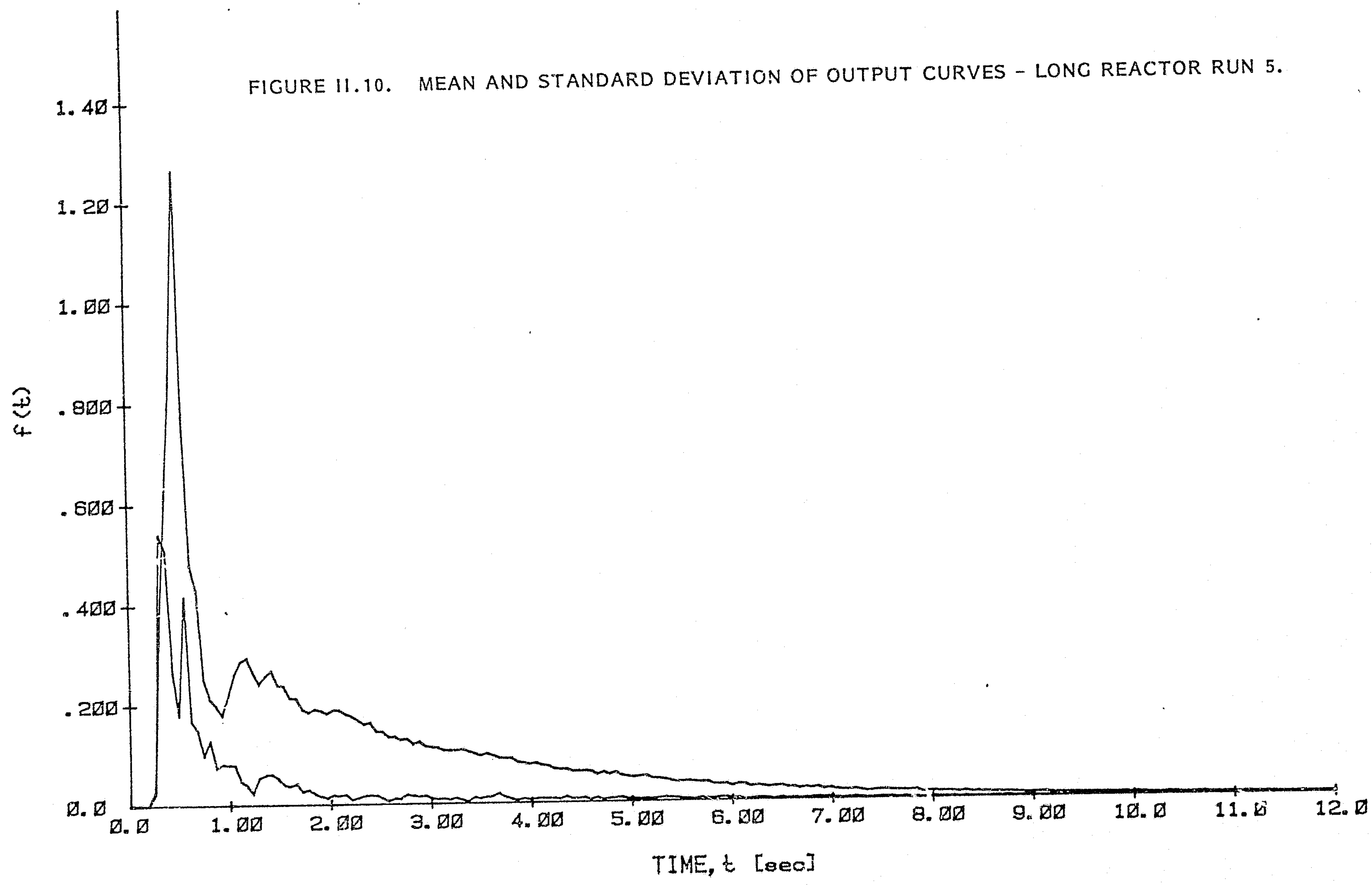


FIGURE II.10. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - LONG REACTOR RUN 5.



APPENDIX III. GRAPHS OF THE OUTPUT CURVES FOR THE SHORT REACTOR

FIGURES III.1 to III.5 : Output curves for runs 1–5 (each "run" is for a different flowrate – see table 3.1).

FIGURES III.6 to III.10 : The mean and stadard deviation of the output curves of each run.

FIGURE III. 1. OUTPUT CURVES - SHORT REACTOR RUN 1.

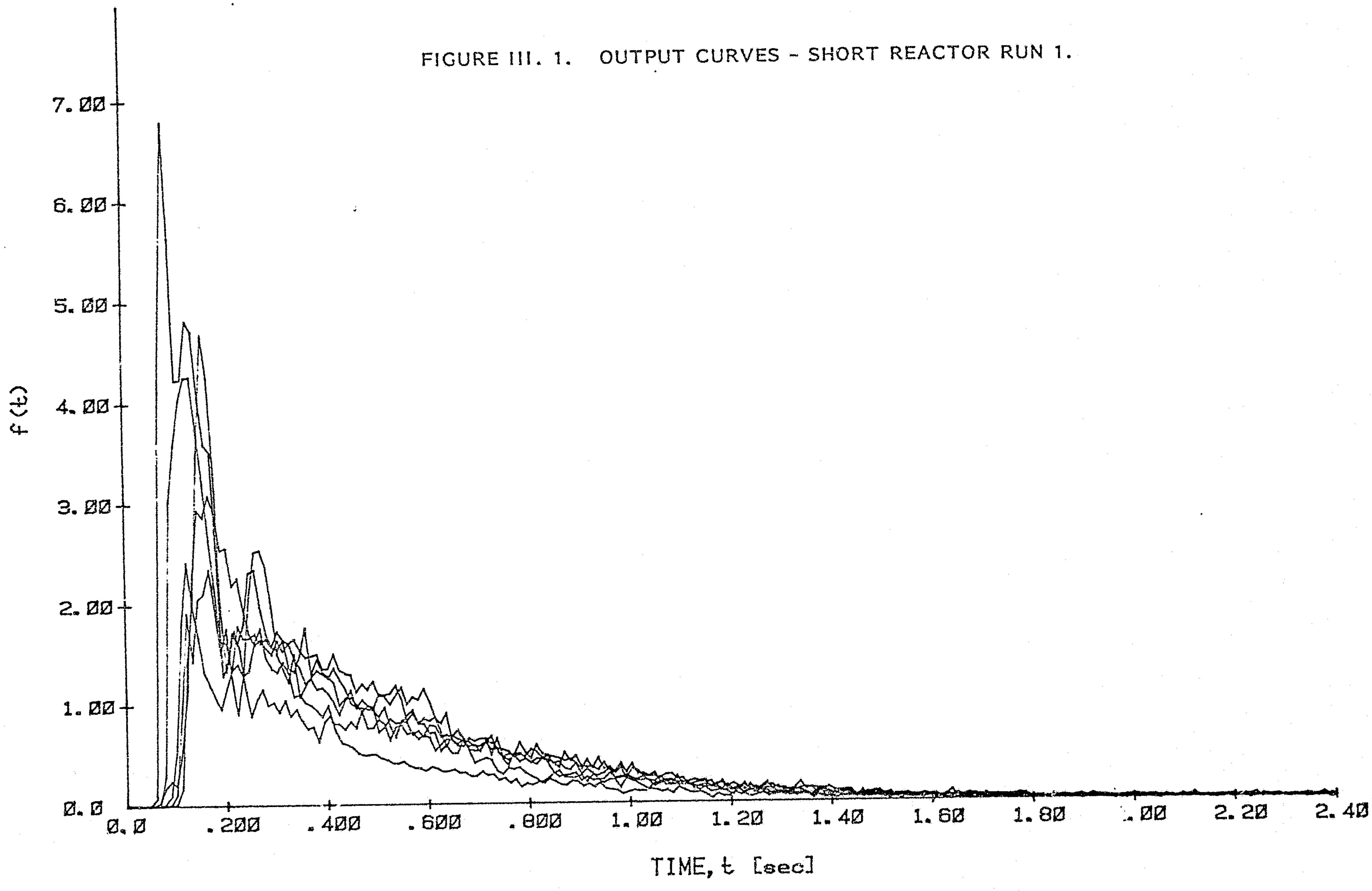


FIGURE III.2. OUTPUT CURVES - SHORT REACTOR RUN 2.

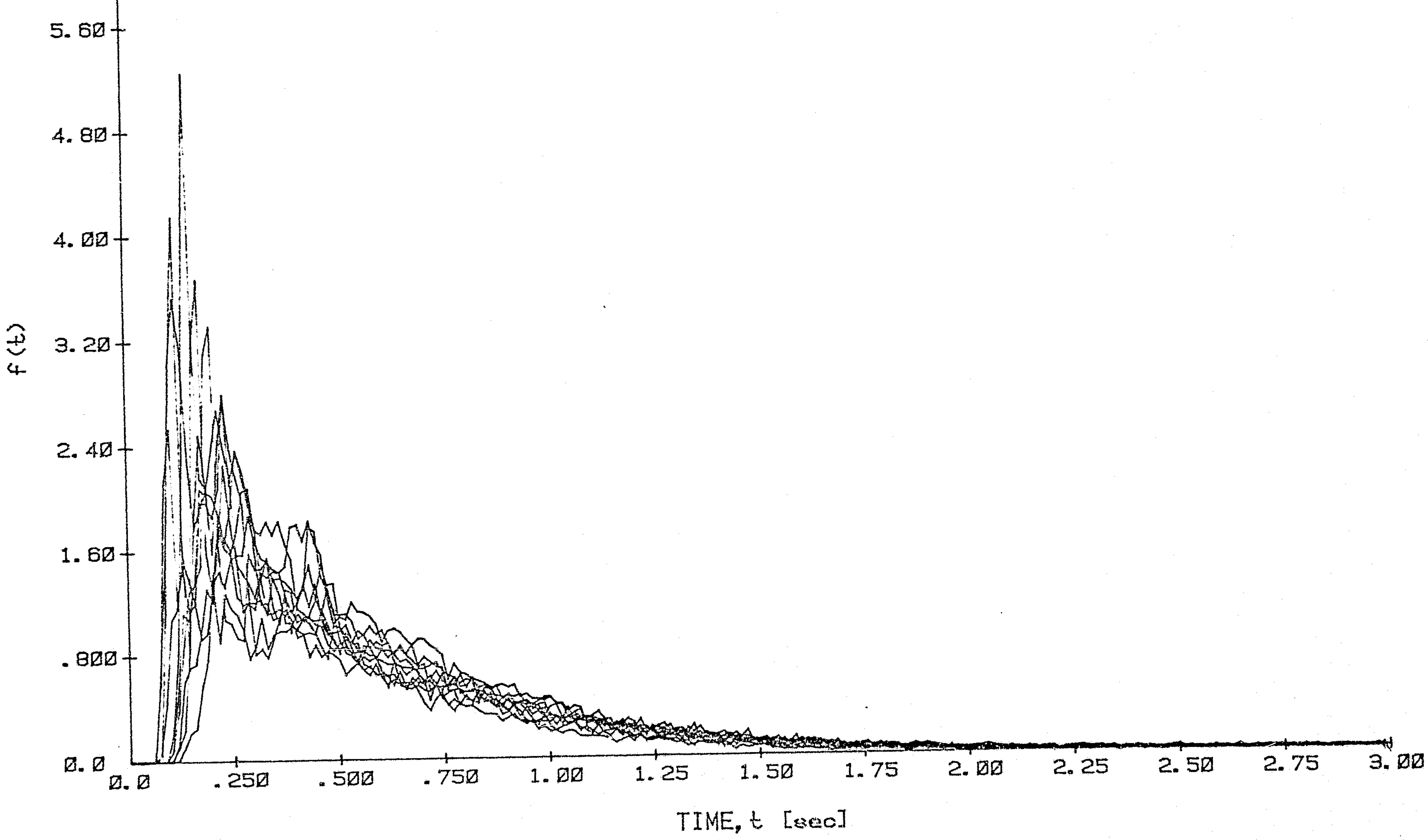


FIGURE III.3. OUTPUT CURVES - SHORT REACTOR RUN 3.

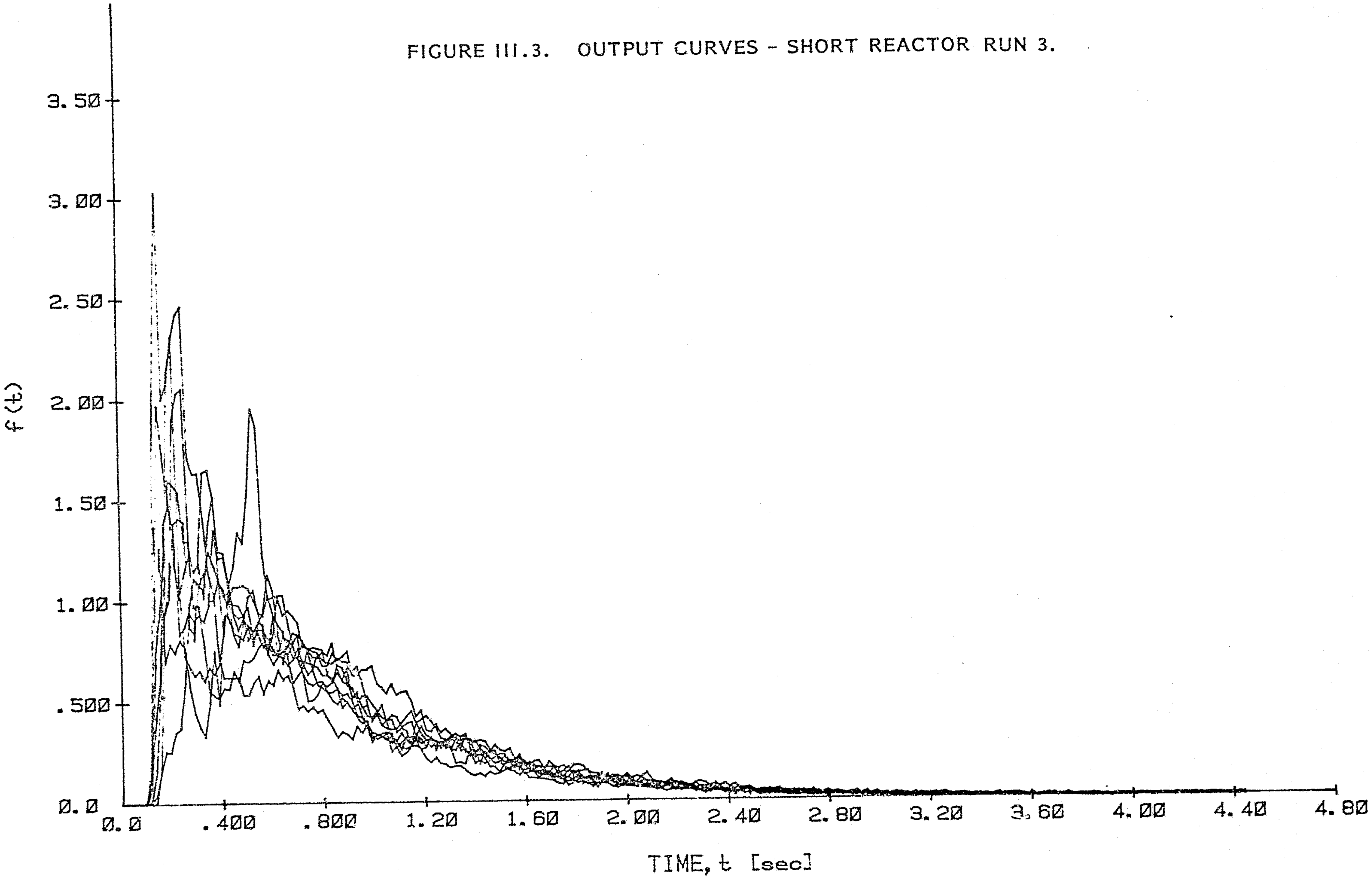


FIGURE III.4. OUTPUT CURVES - SHORT REACTOR RUN 4.

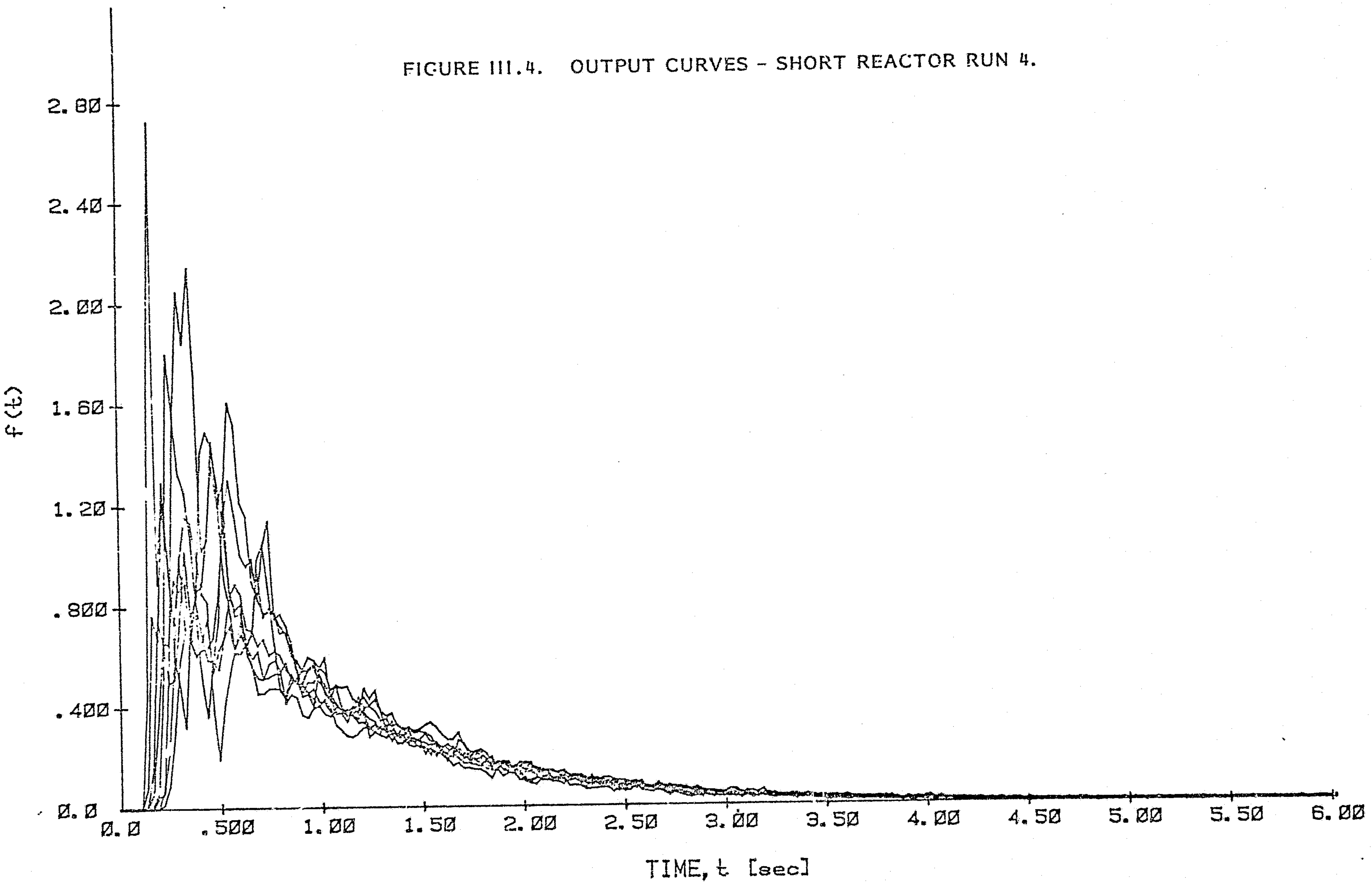
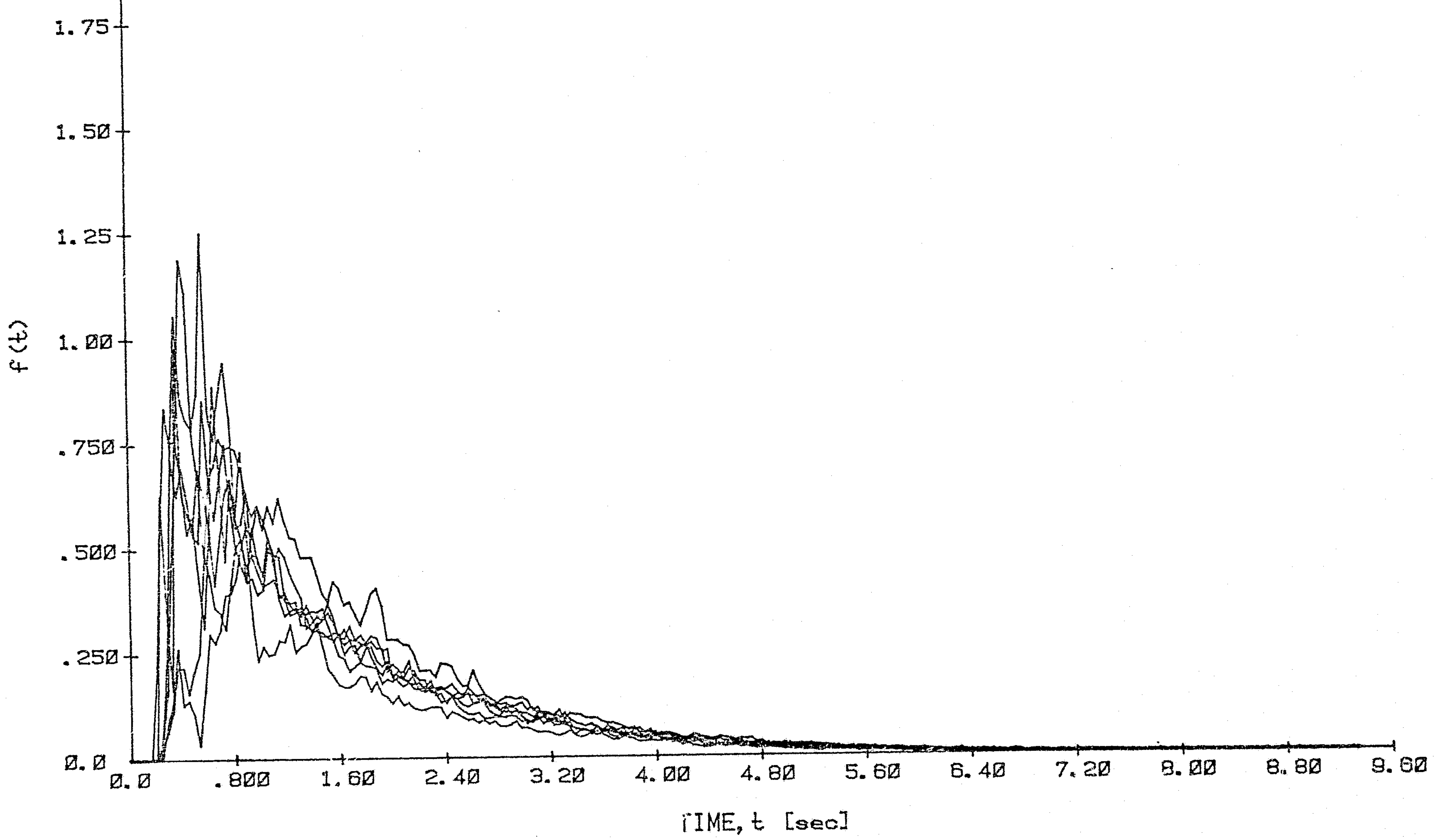


FIGURE III.5. OUTPUT CURVES - SHORT REACTOR RUN 5.



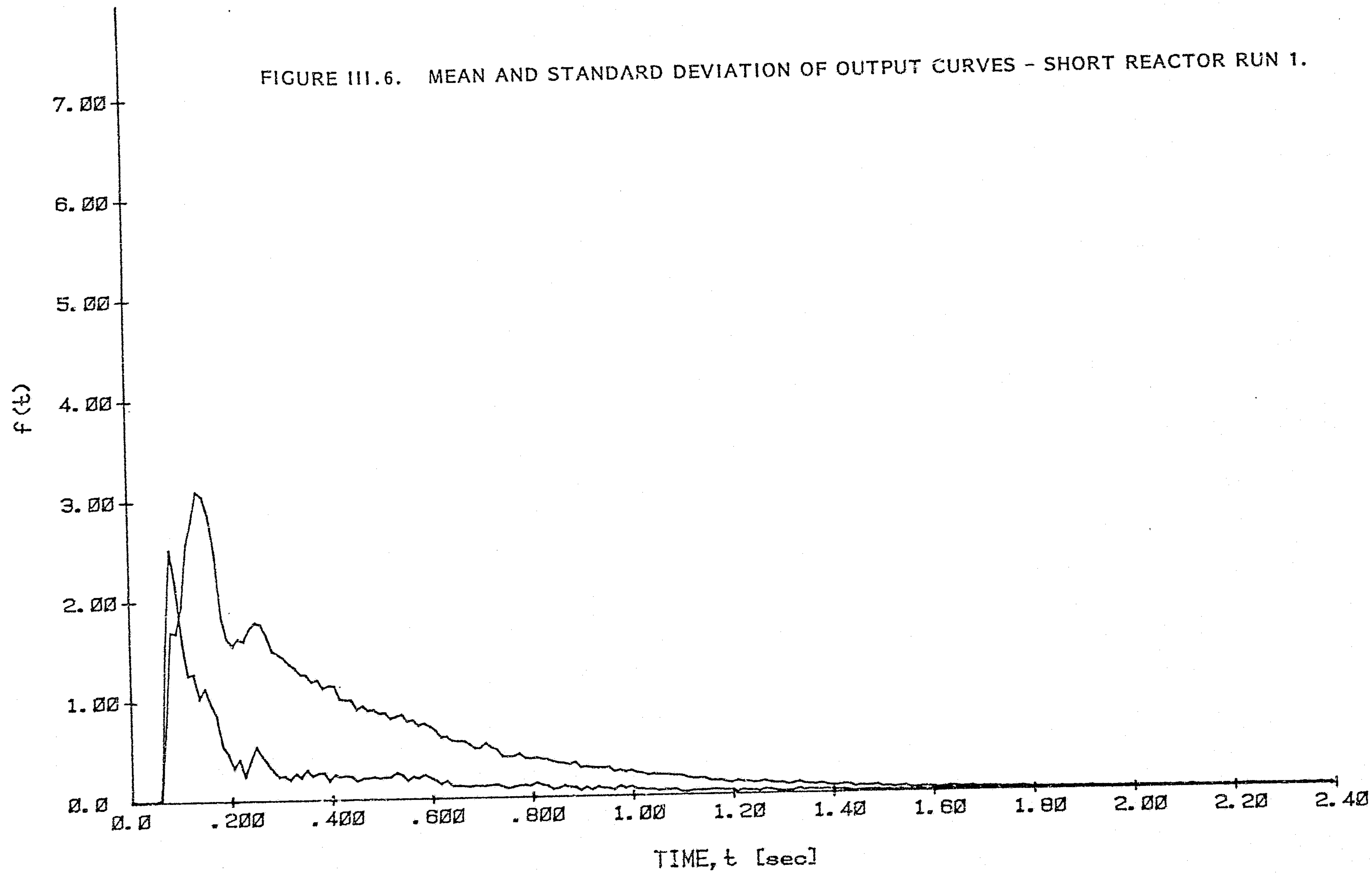


FIGURE III.6. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - SHORT REACTOR RUN 1.

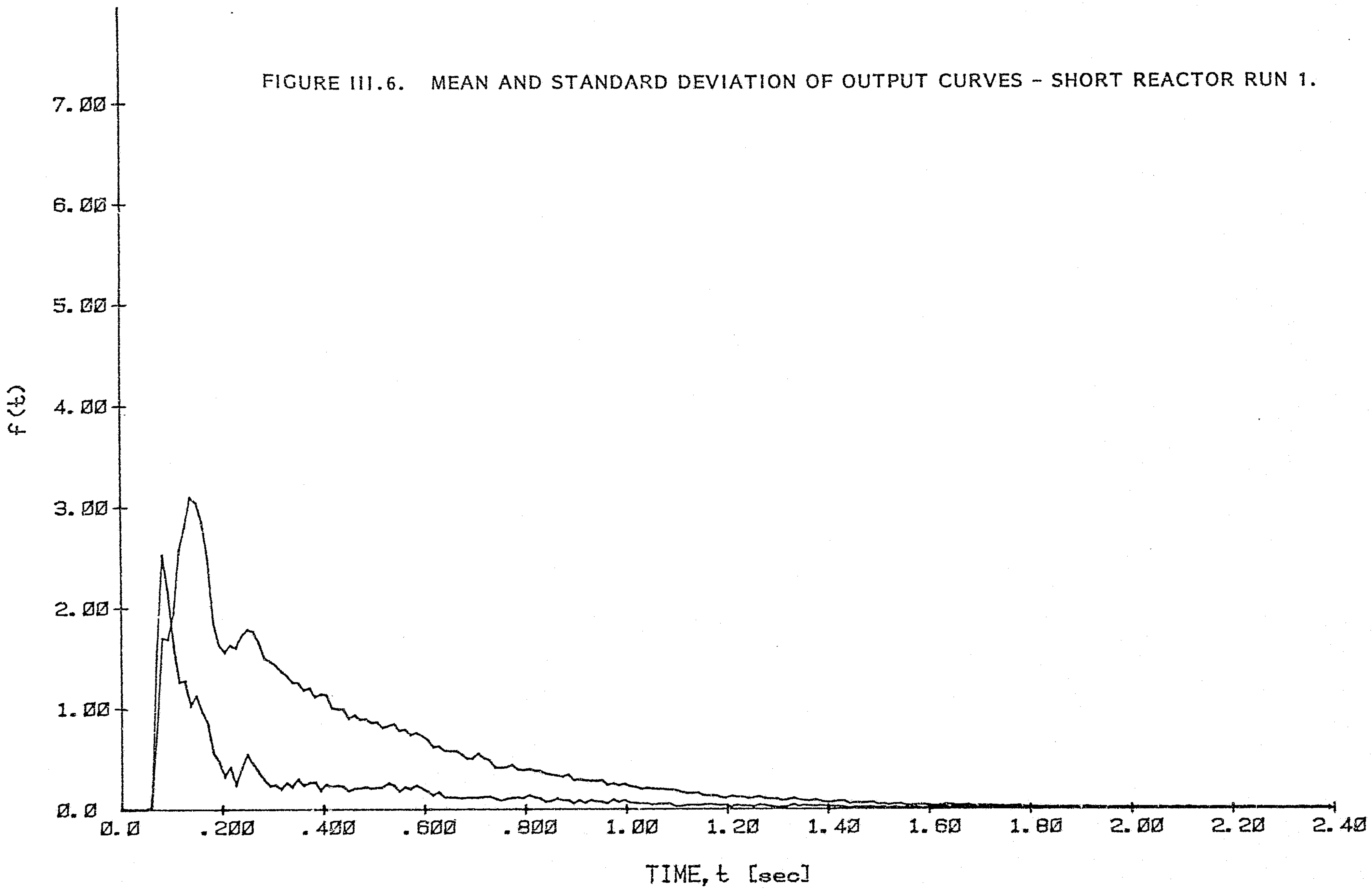


FIGURE III.6. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - SHORT REACTOR RUN 1.

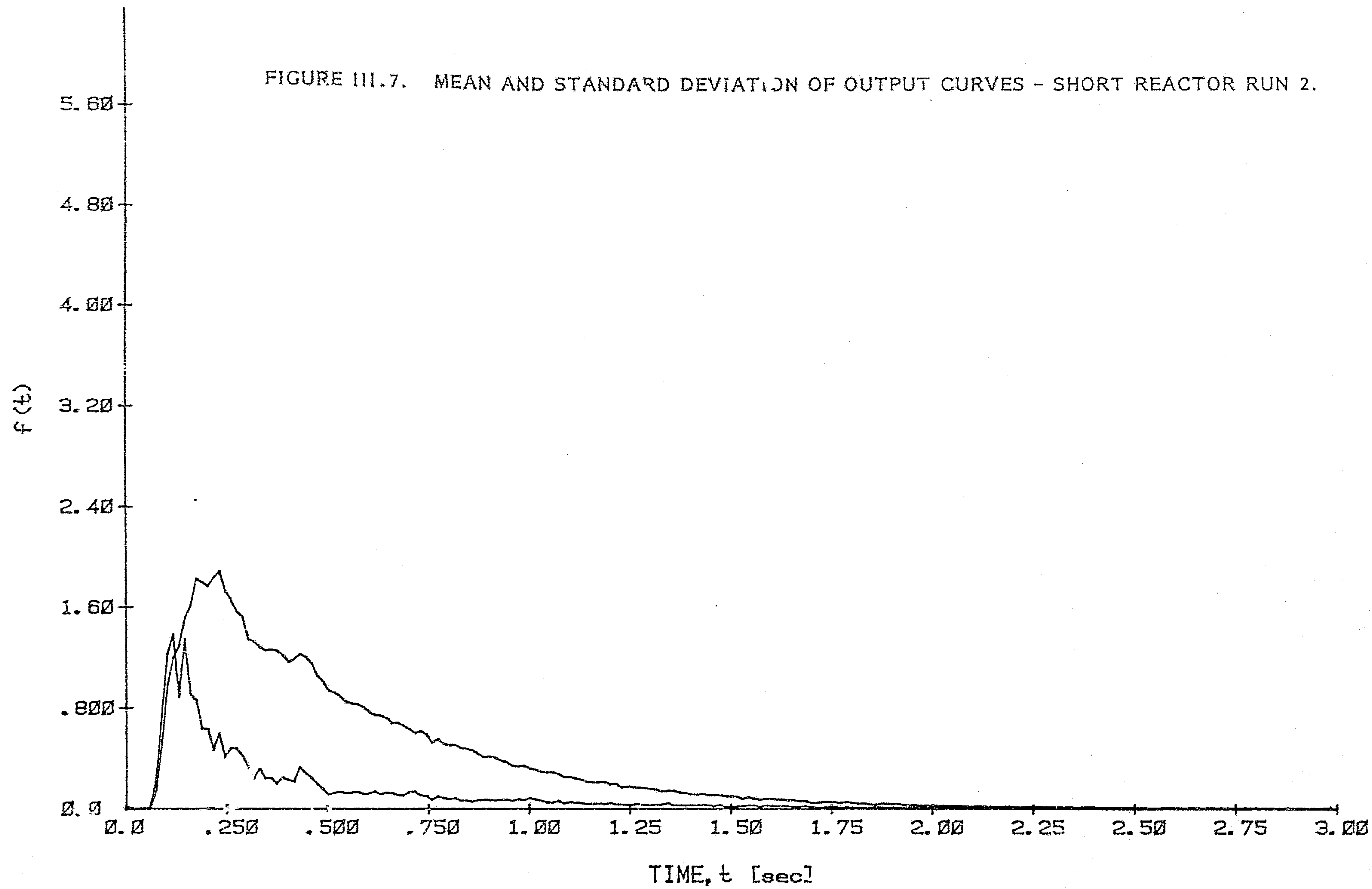


FIGURE III.7. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - SHORT REACTOR RUN 2.

FIGURE III.8. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - SHORT REACTOR RUN 3.

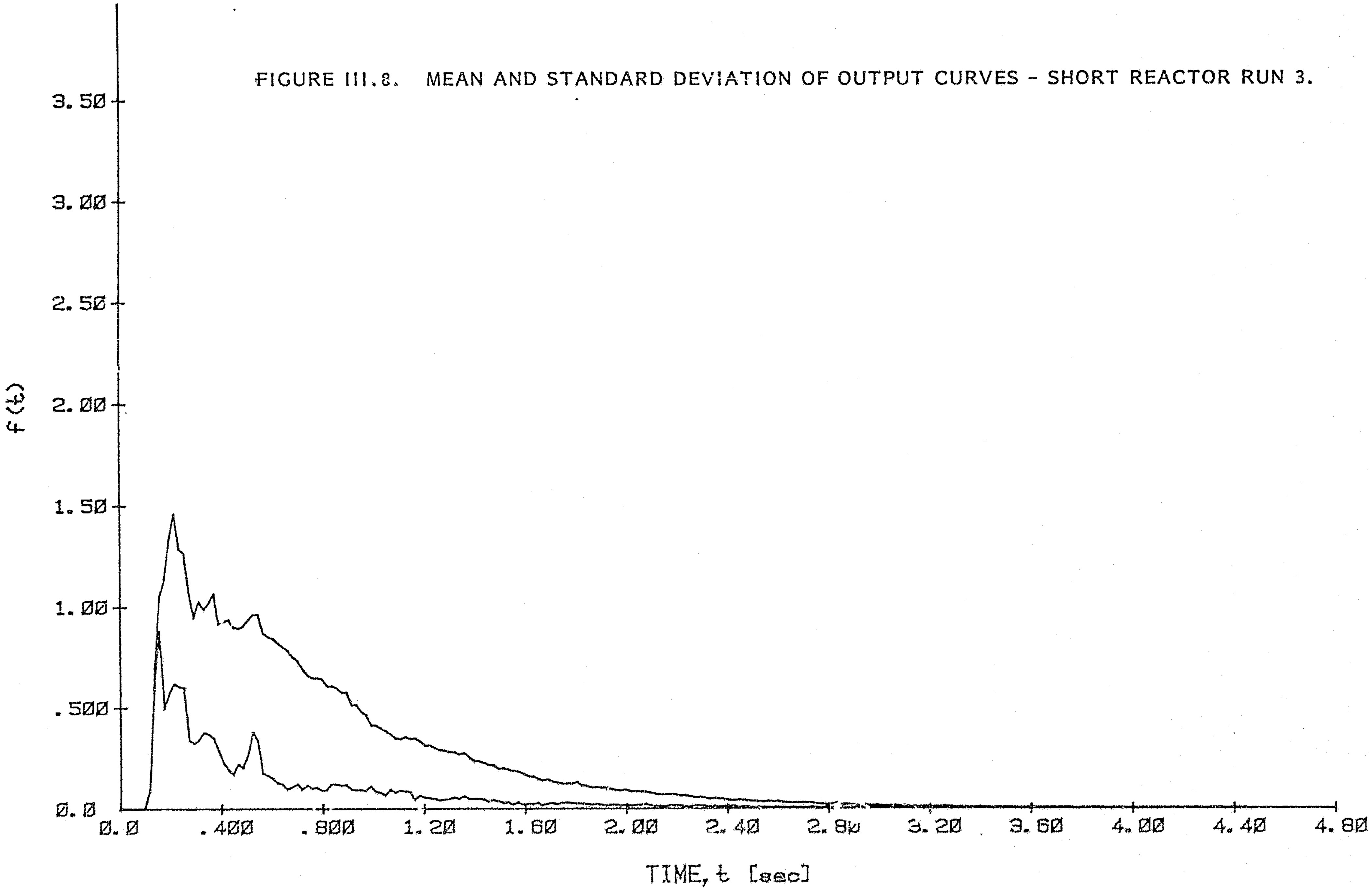
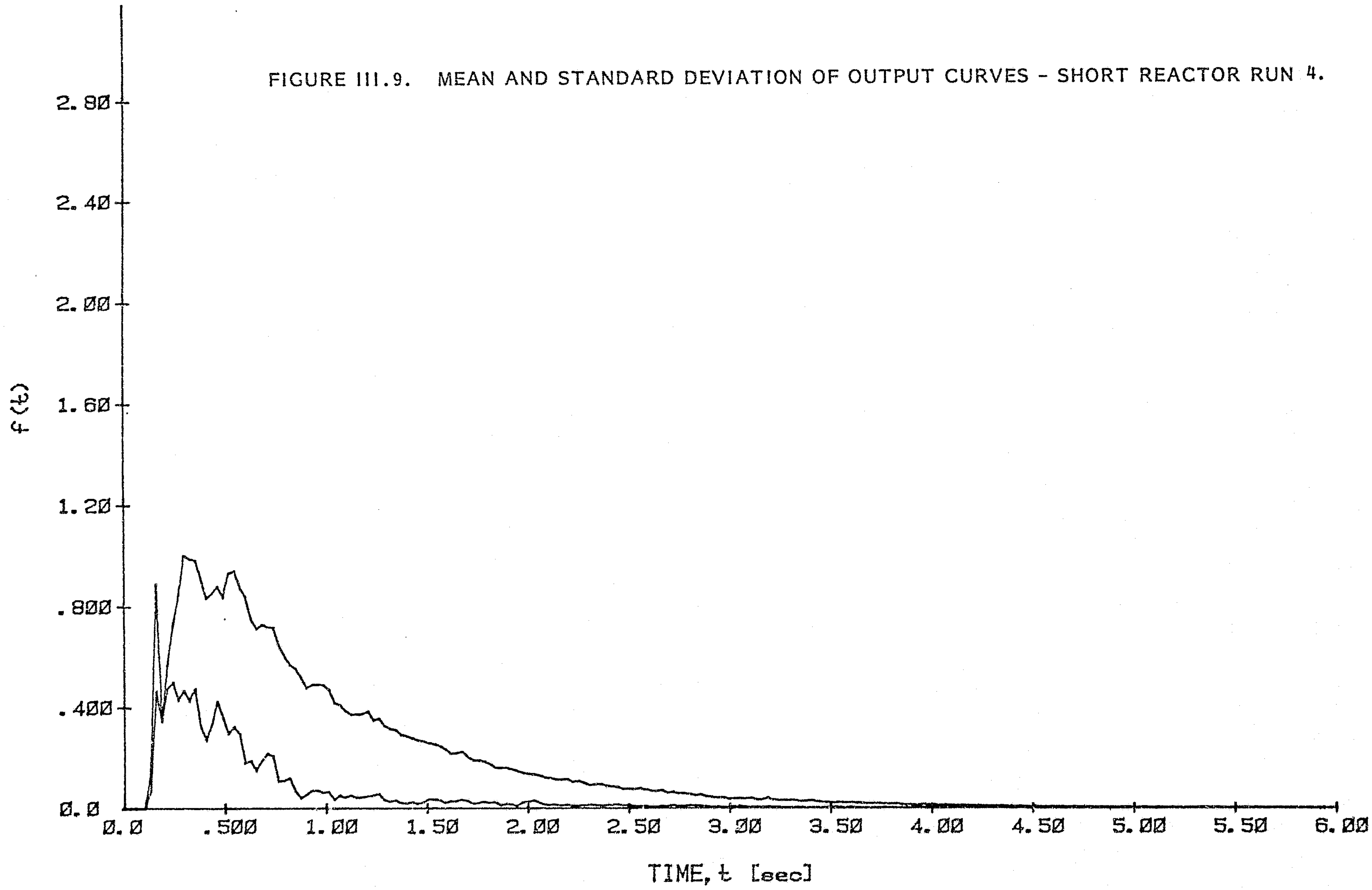


FIGURE III.9. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - SHORT REACTOR RUN 4.



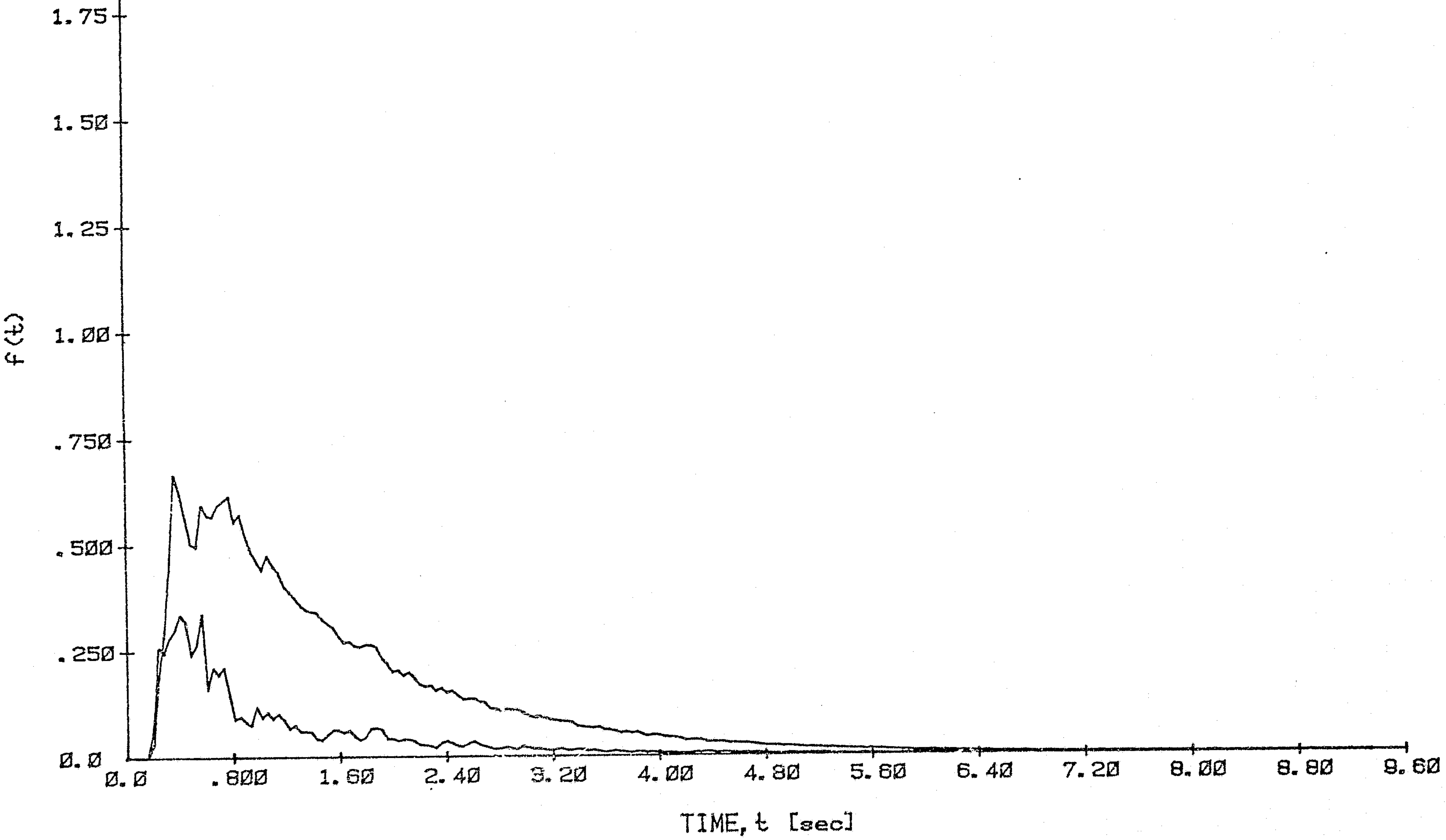


FIGURE III.10. MEAN AND STANDARD DEVIATION OF OUTPUT CURVES - SHORT REACTOR RUN 5.

APPENDIX IV. GRAPHS OF THE FOURIER COEFFICIENTS, AMPLITUDE RATIO AND PHASE ANGLE FOR THE OUTPUT CURVES OF THE LONG REACTOR

FIGURE IV.1. FOURIER COEFFICIENTS - LONG REACTOR RUN 1.

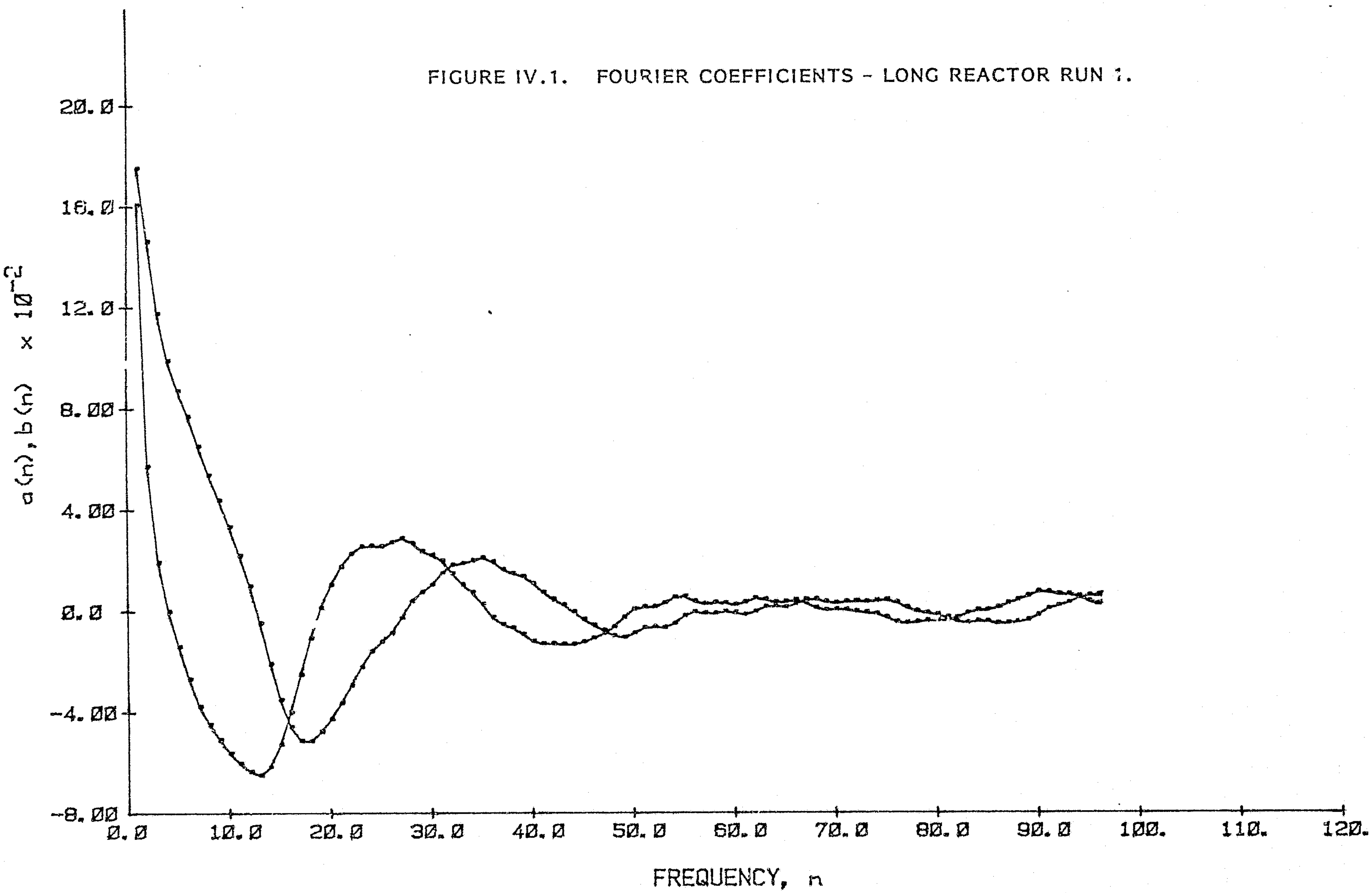


FIGURE IV.2. AMPLITUDE RATIO - LONG REACTOR RUN 1.

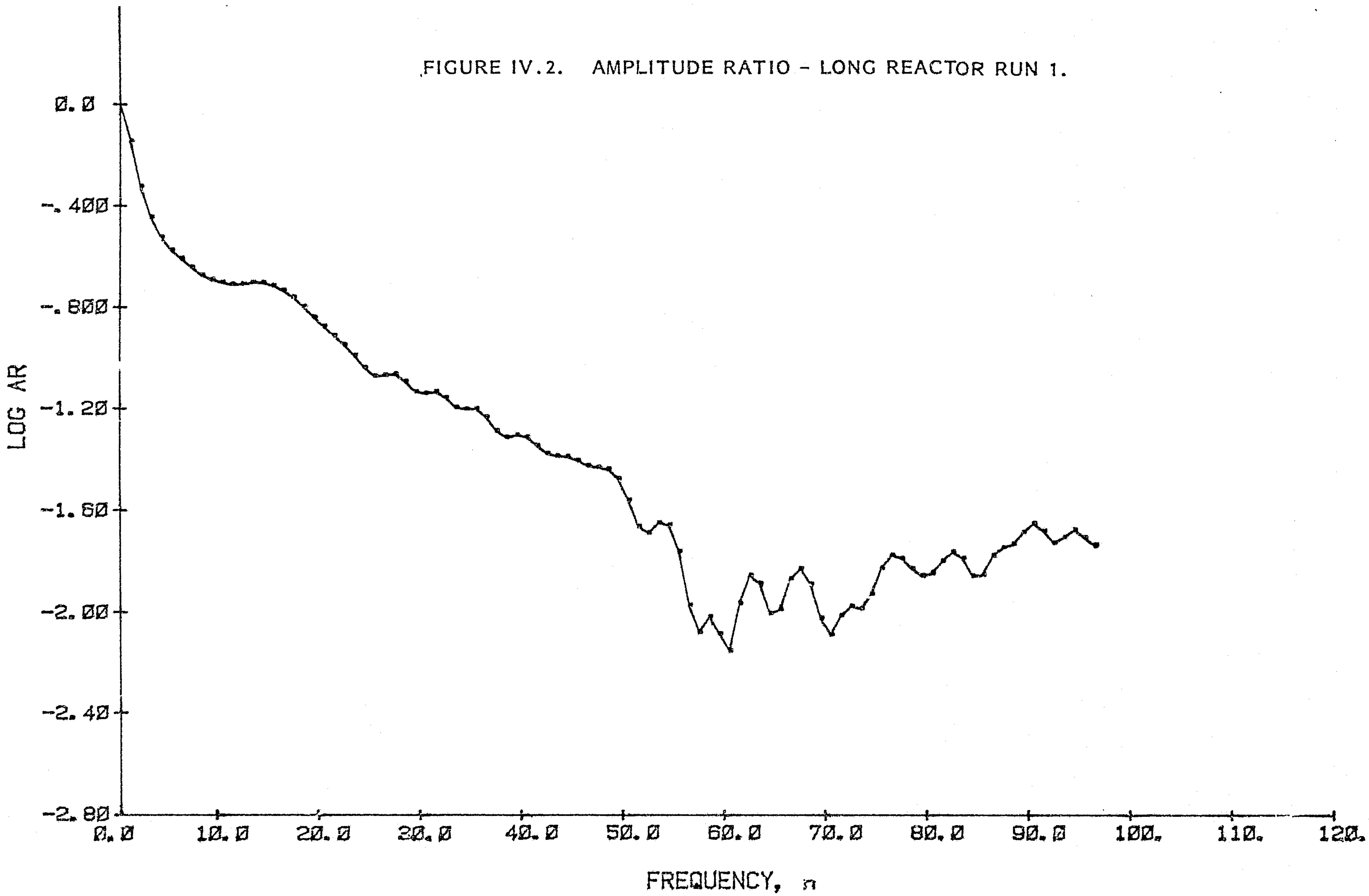


FIGURE IV.3. PHASE ANGLE - LONG REACTOR RUN 1.

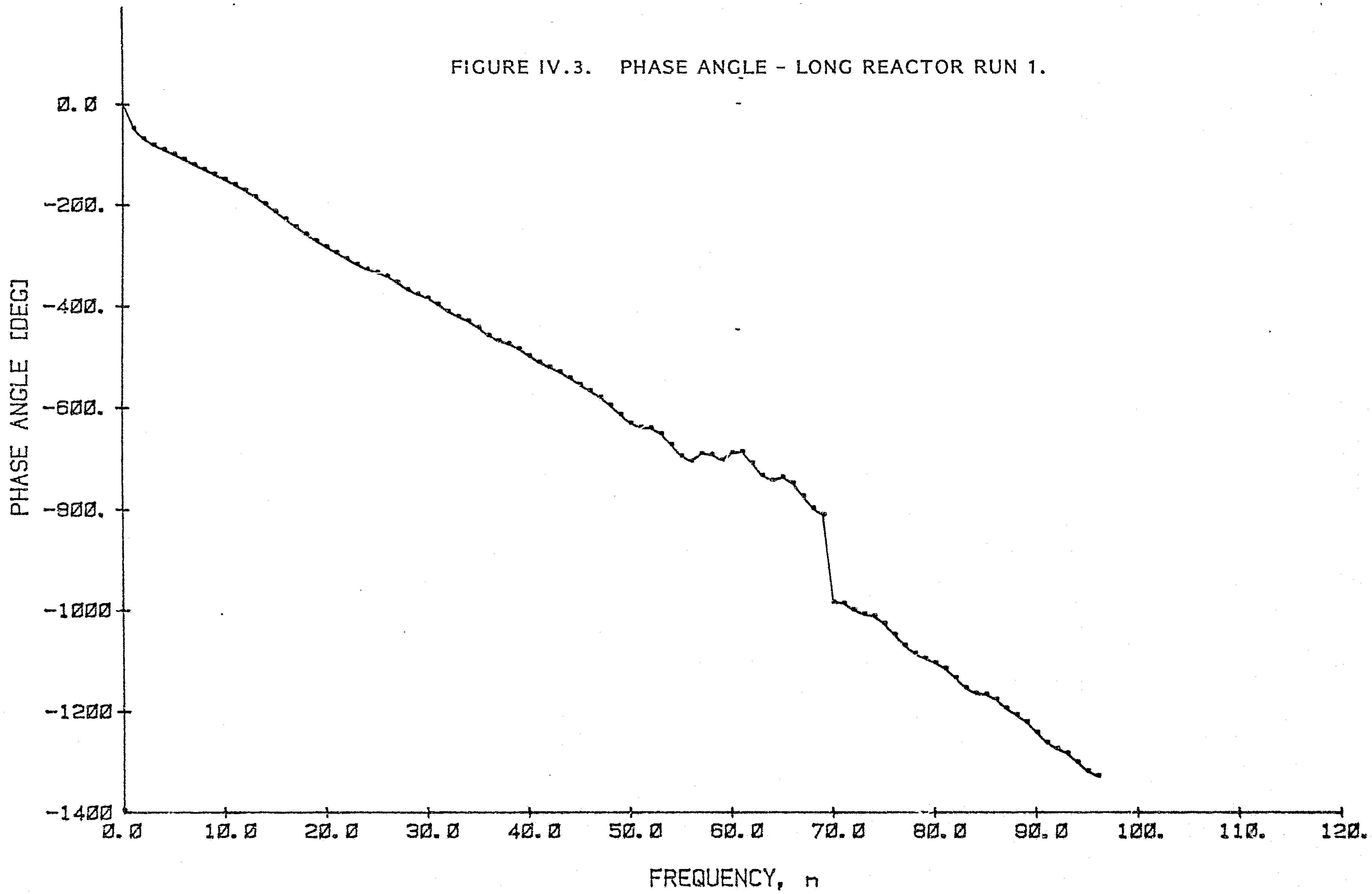


FIGURE IV.4. FOURIER COEFFICIENTS - LONG REACTOR RUN 2.

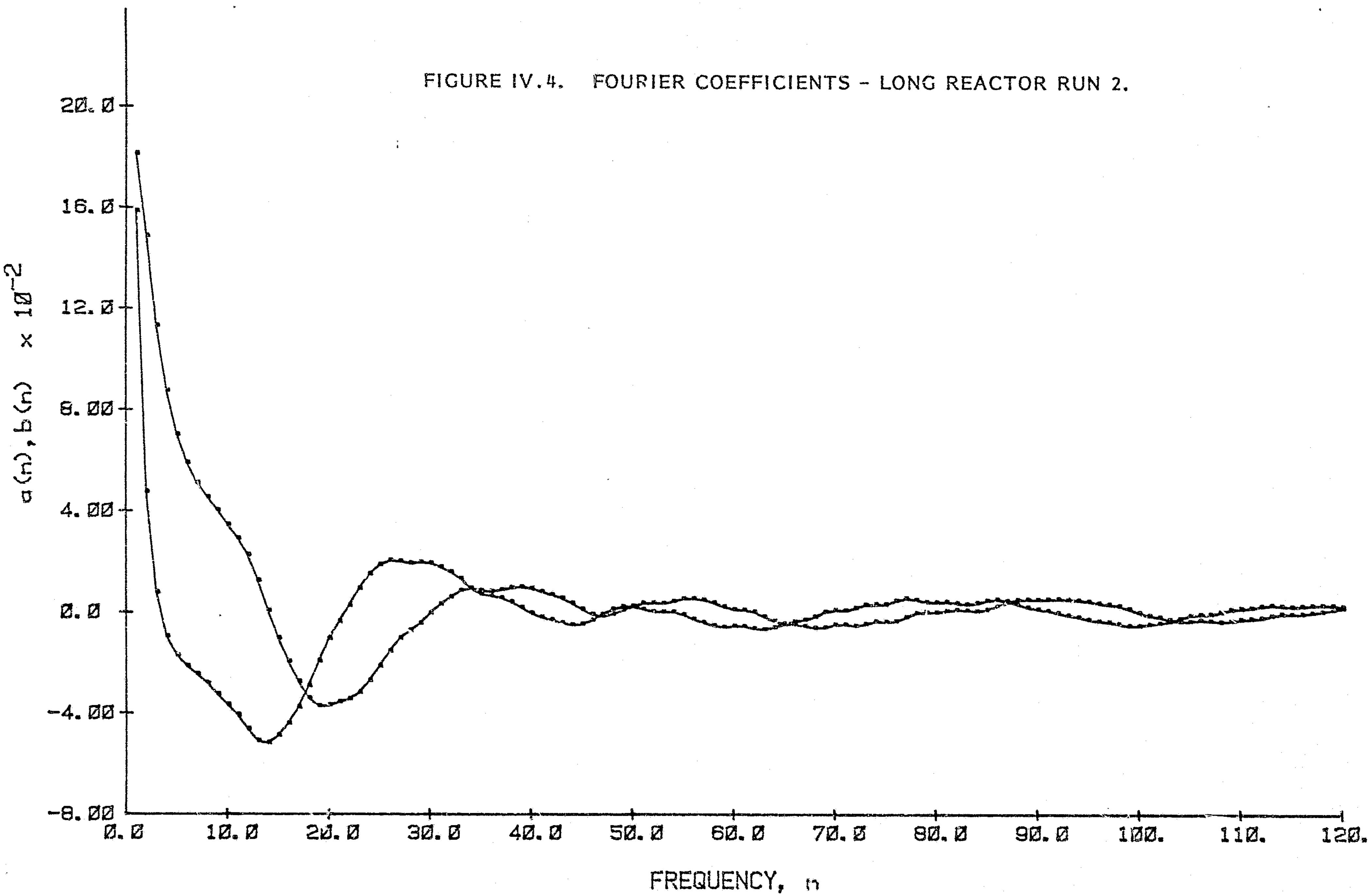


FIGURE IV.5. AMPLITUDE RATIO - LONG REACTOR RUN 2.

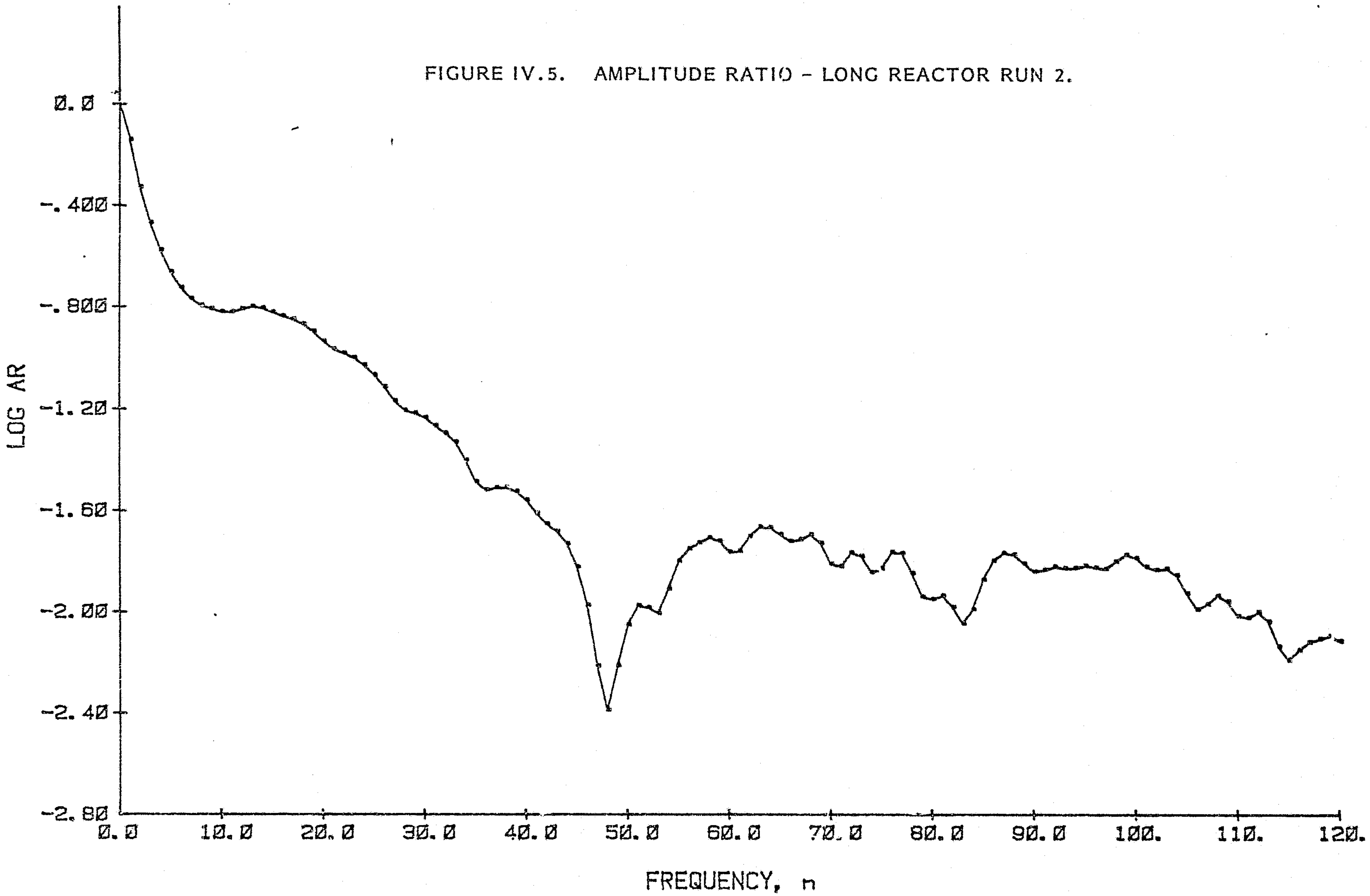


FIGURE IV.6. PHASE ANGLE - LONG REACTOR RUN 2.

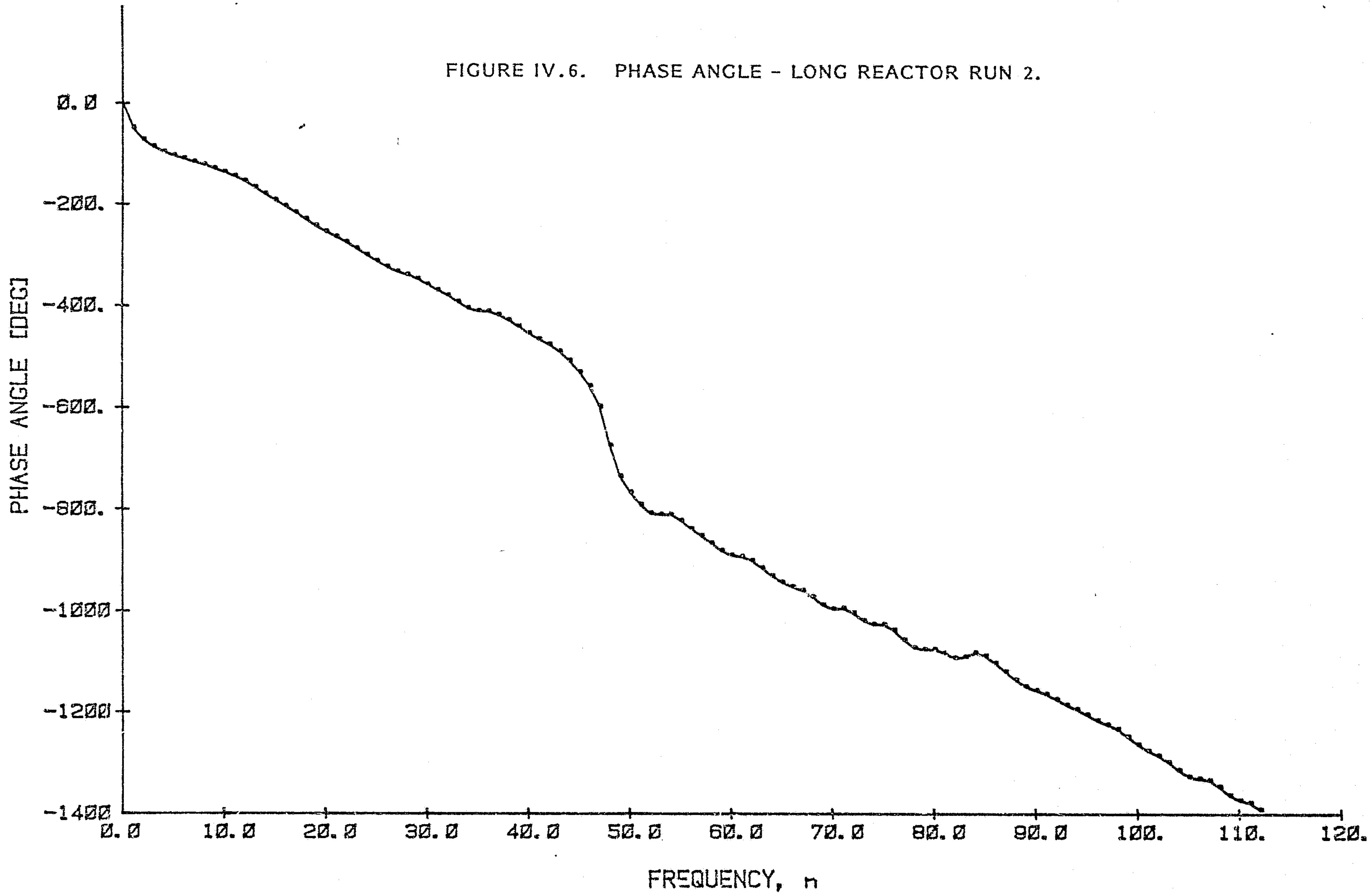
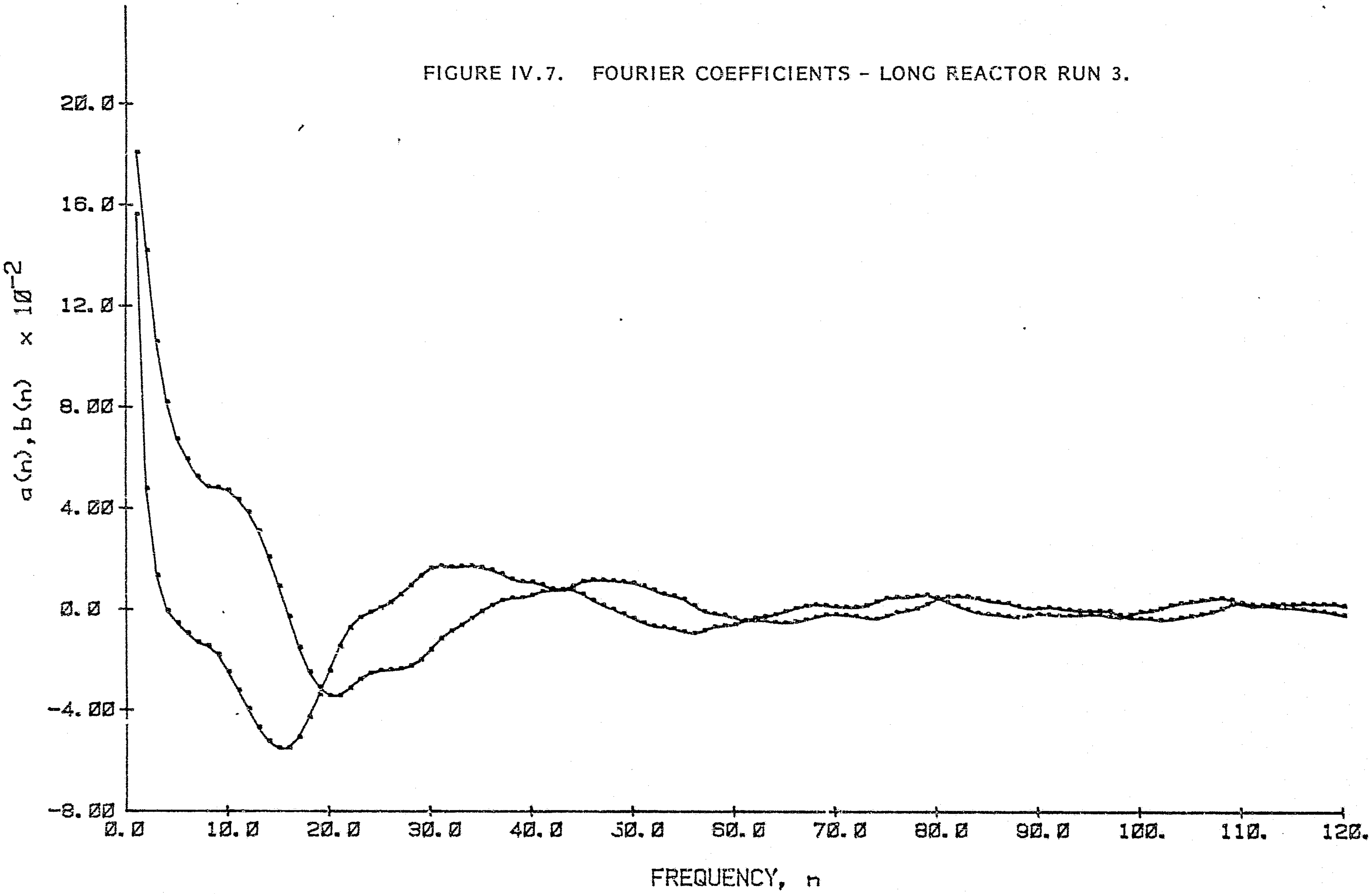


FIGURE IV.7. FOURIER COEFFICIENTS - LONG REACTOR RUN 3.



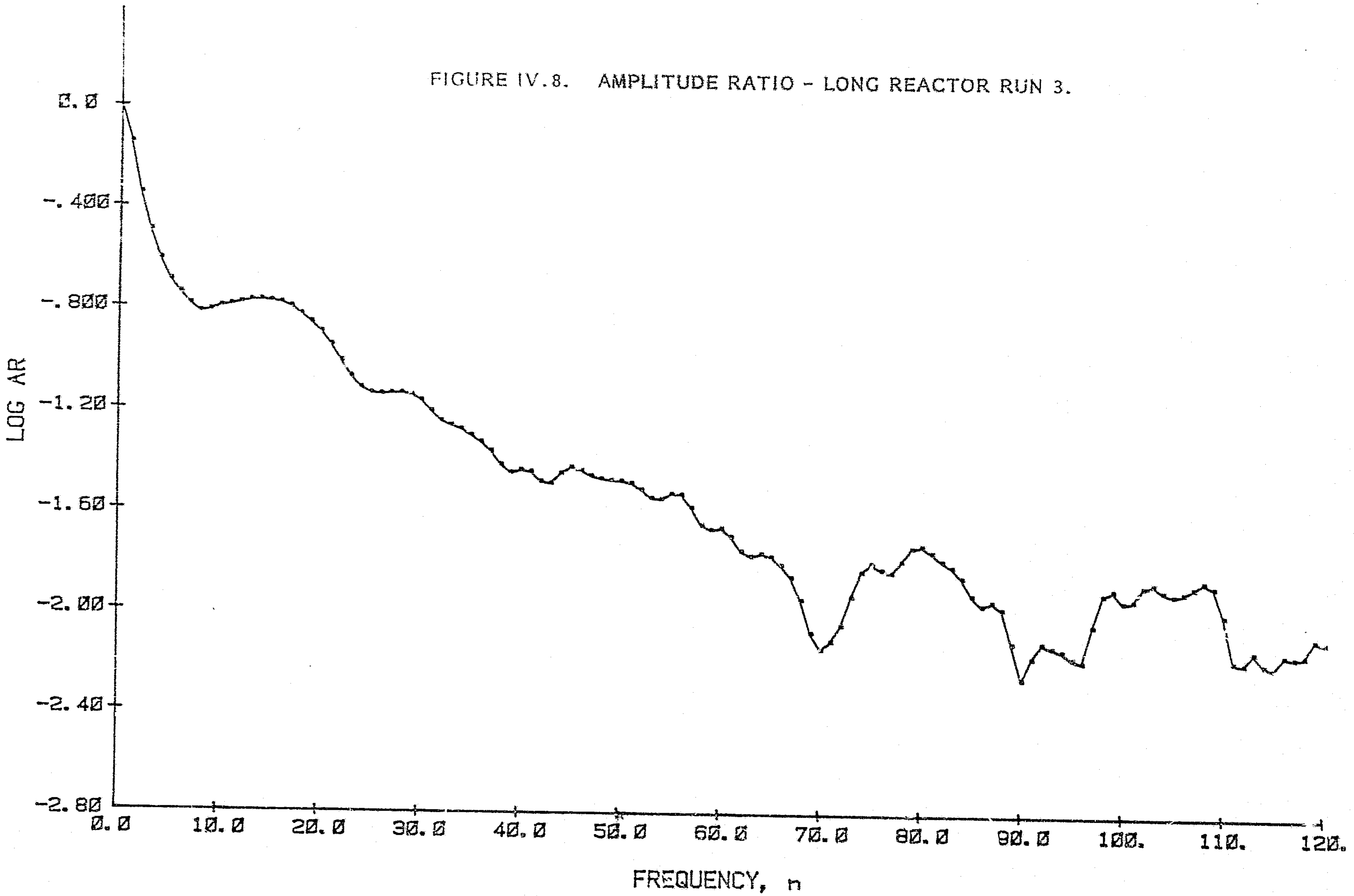


FIGURE IV.8. AMPLITUDE RATIO - LONG REACTOR RUN 3.

FIGURE IV.9. PHASE ANGLE - LONG REACTOR RUN 3.

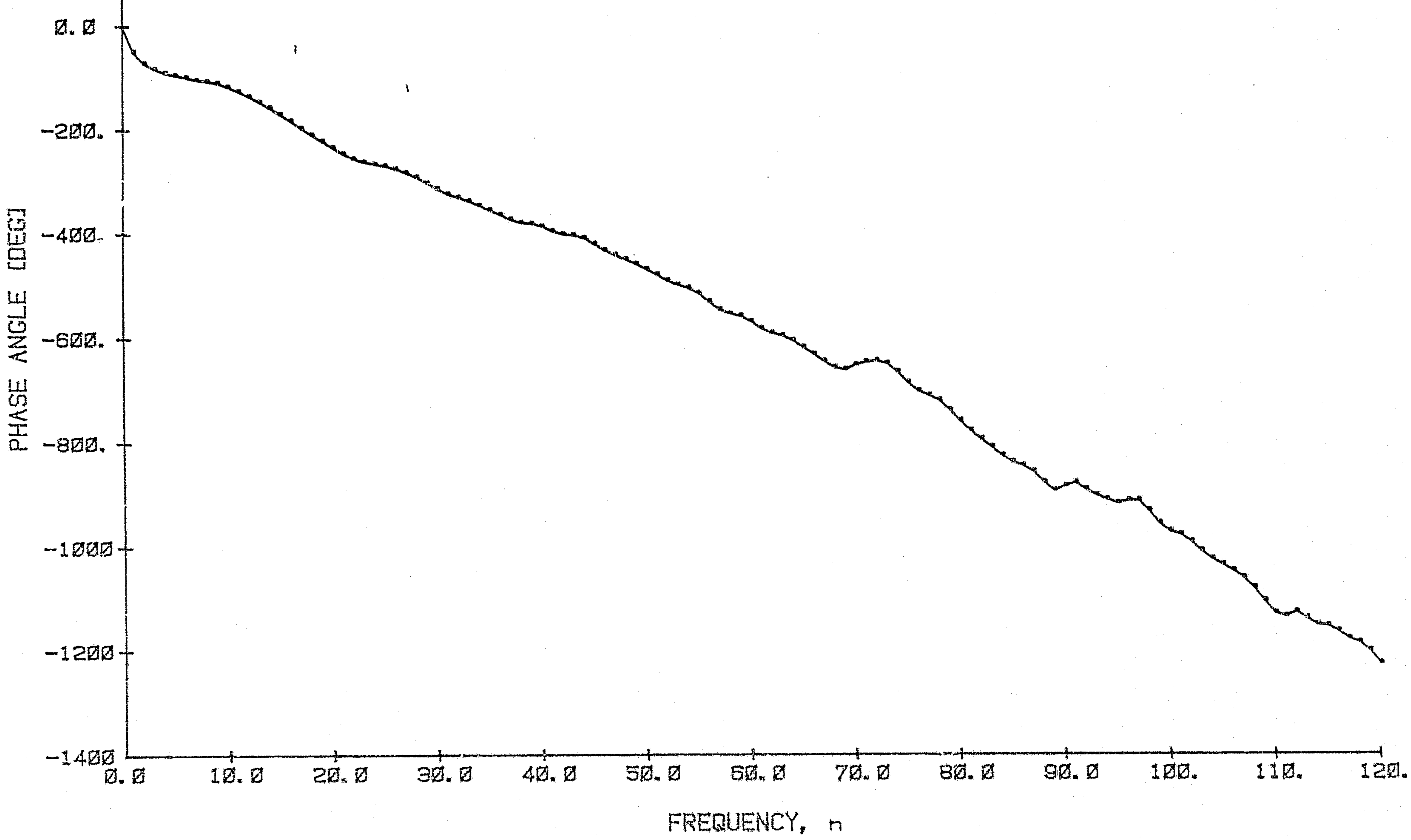


FIGURE IV.10. FOURIER COEFFICIENTS - LONG REACTOR RUN 4.

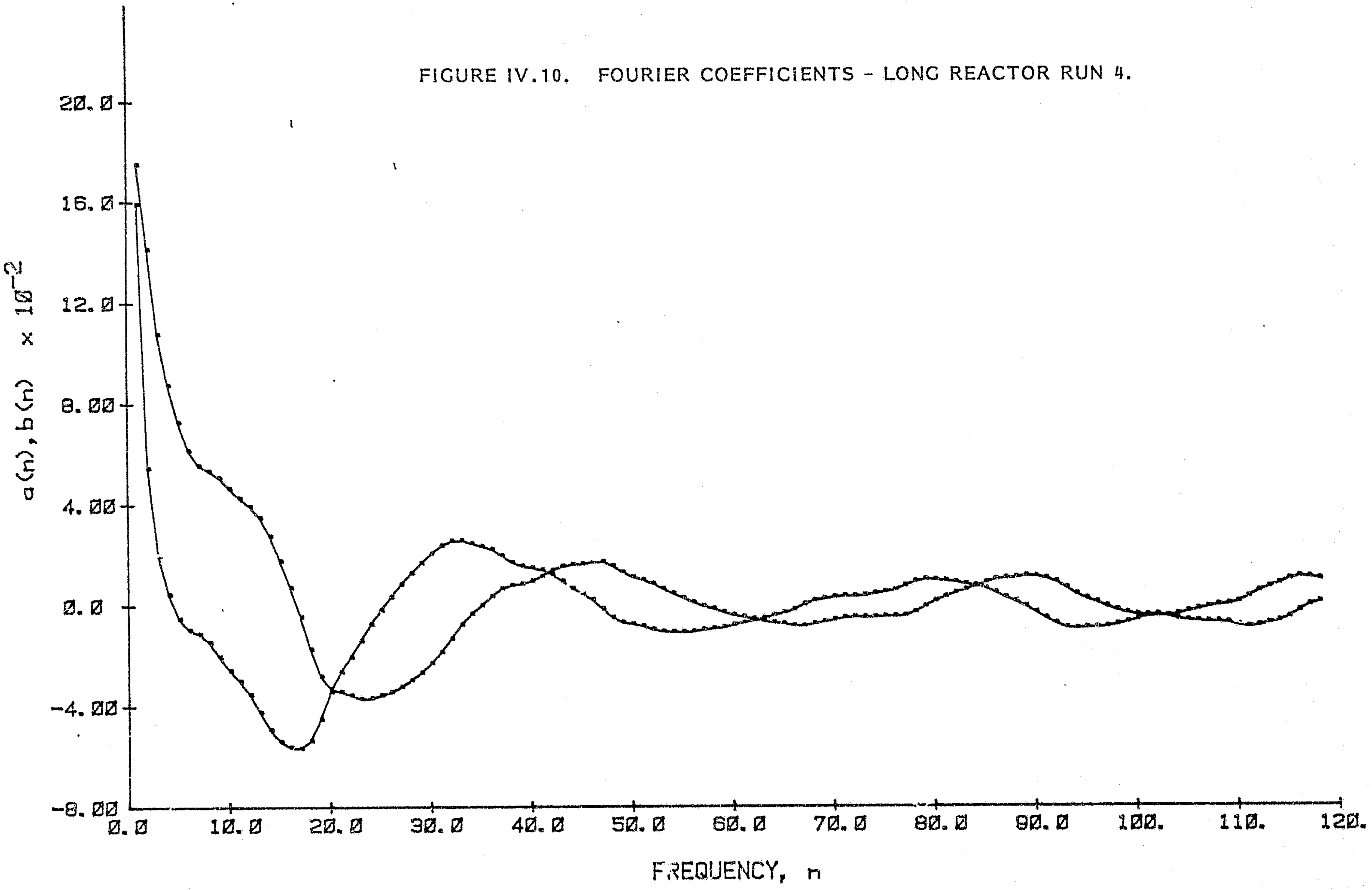
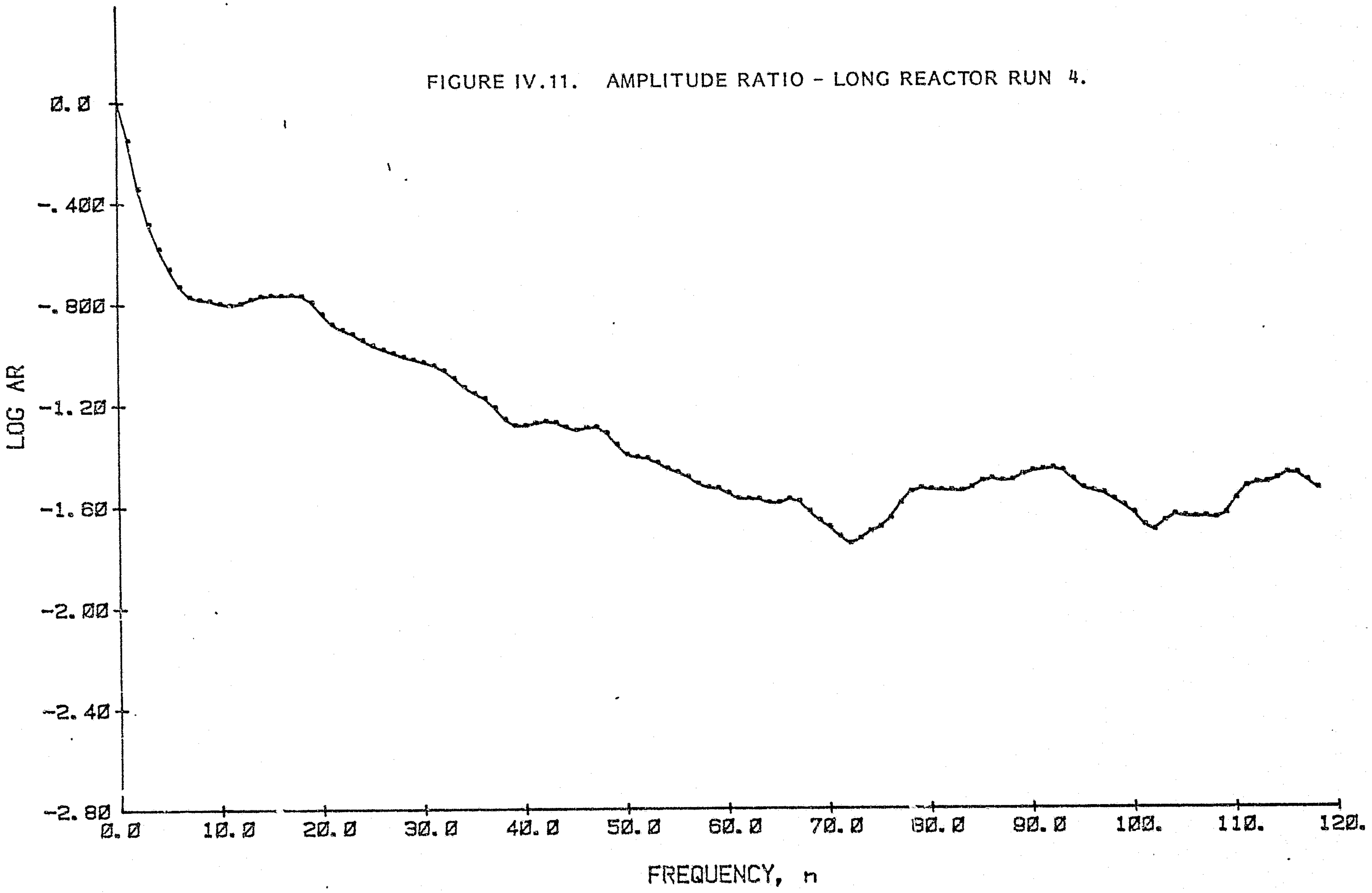


FIGURE IV.11. AMPLITUDE RATIO - LONG REACTOR RUN 4.



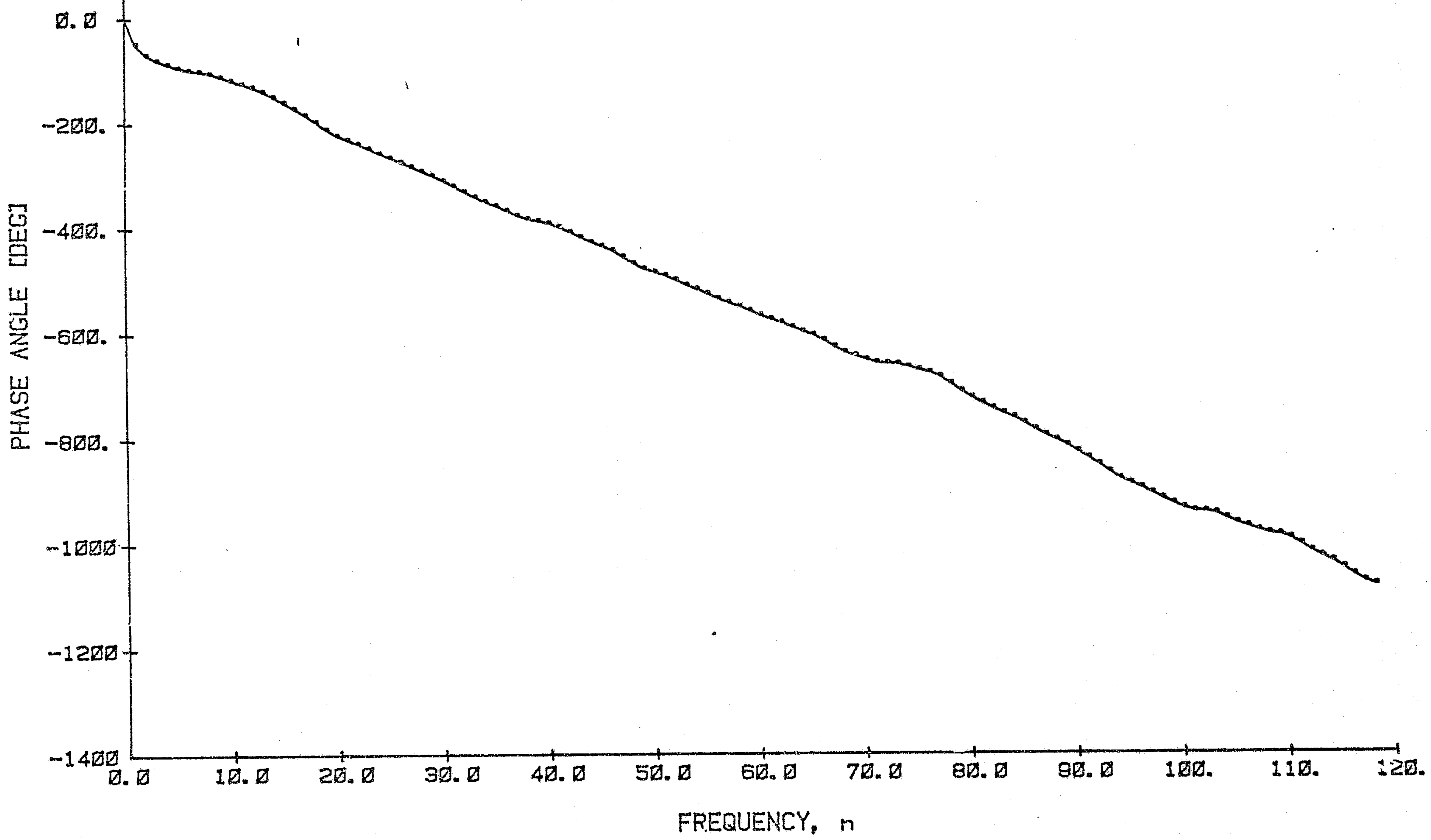
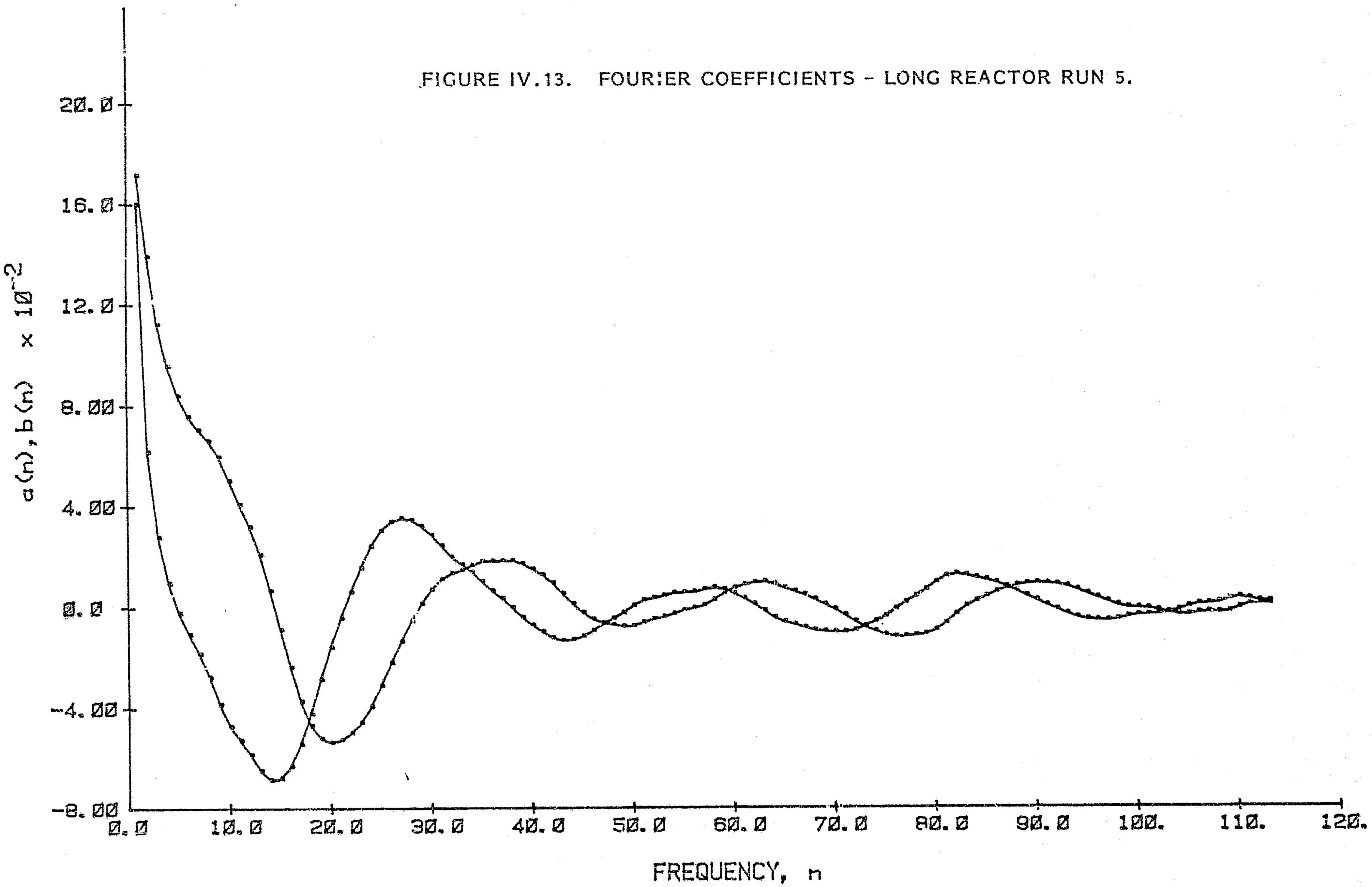


FIGURE IV.12. PHASE ANGLE - LONG REACTOR RUN 4.

FIGURE IV.13. FOURIER COEFFICIENTS - LONG REACTOR RUN 5.



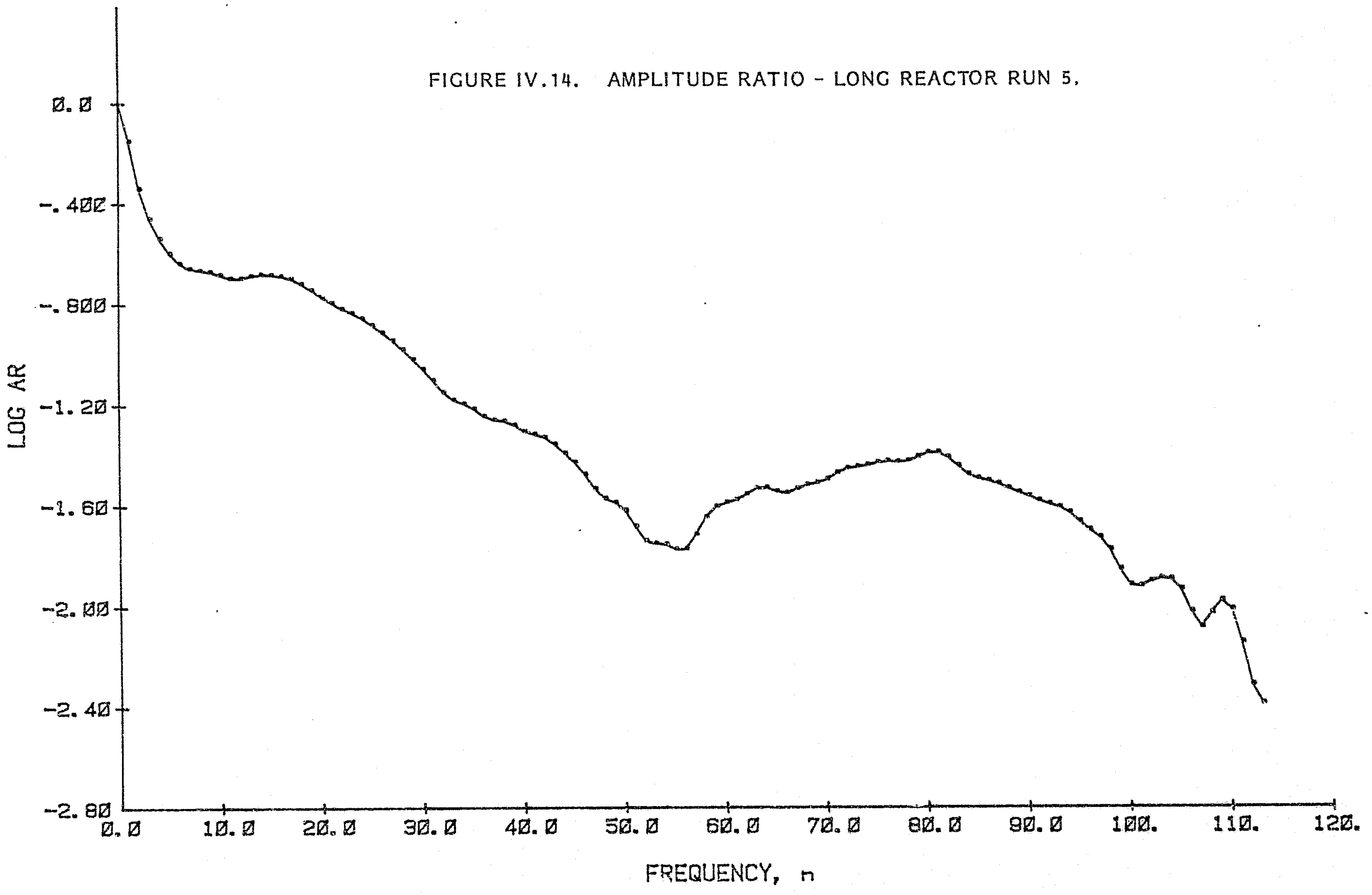
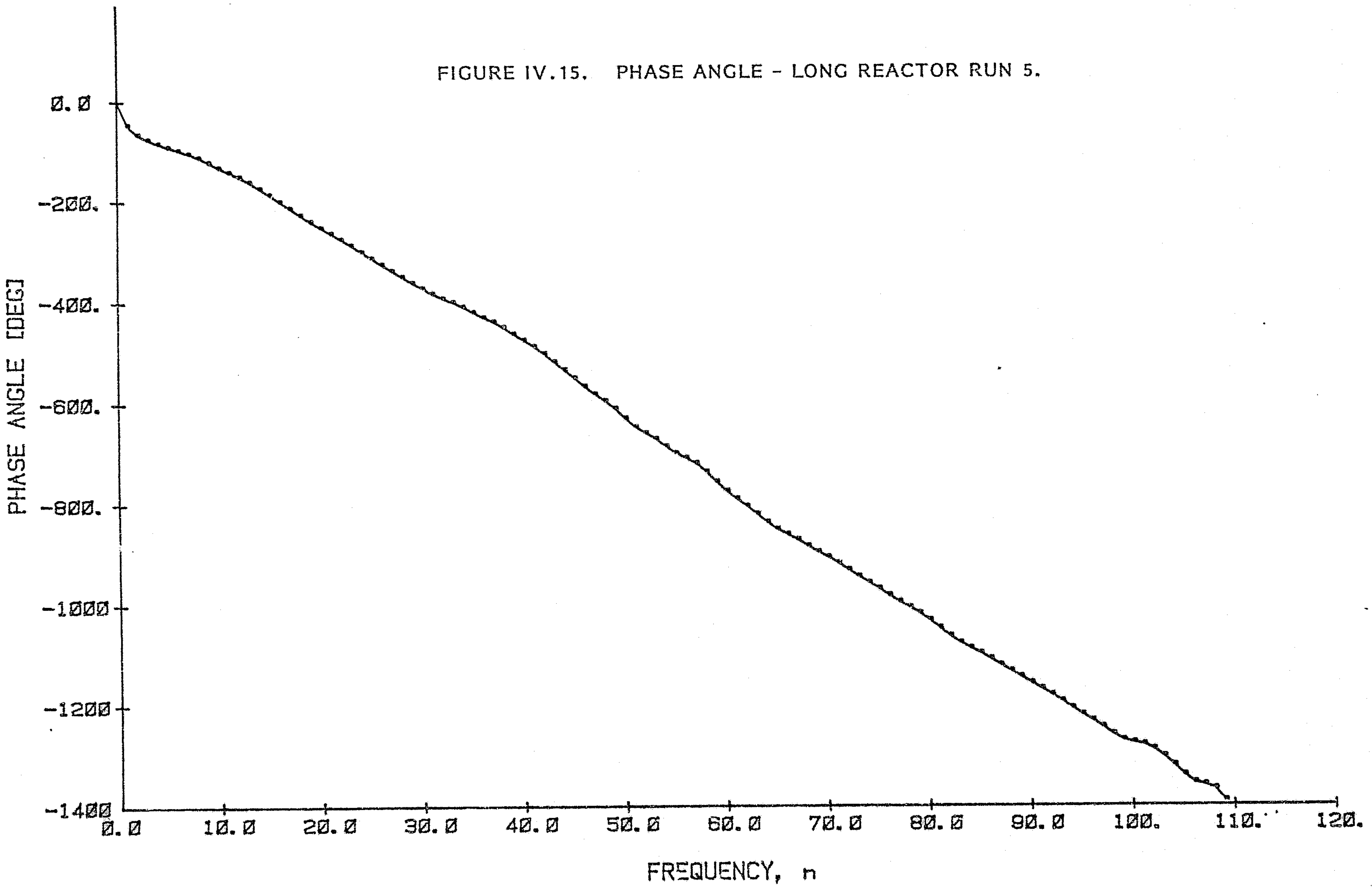


FIGURE IV.14. AMPLITUDE RATIO - LONG REACTOR RUN 5.

FIGURE IV.15. PHASE ANGLE - LONG REACTOR RUN 5.



APPENDIX V. GRAPHS OF THE FOURIER COEFFICIENTS, AMPLITUDE RATIO AND PHASE ANGLE FOR THE OUTPUT CURVES OF THE SHORT REACTOR

FIGURE V.1. FOURIER COEFFICIENTS - SHORT REACTOR RUN 1.

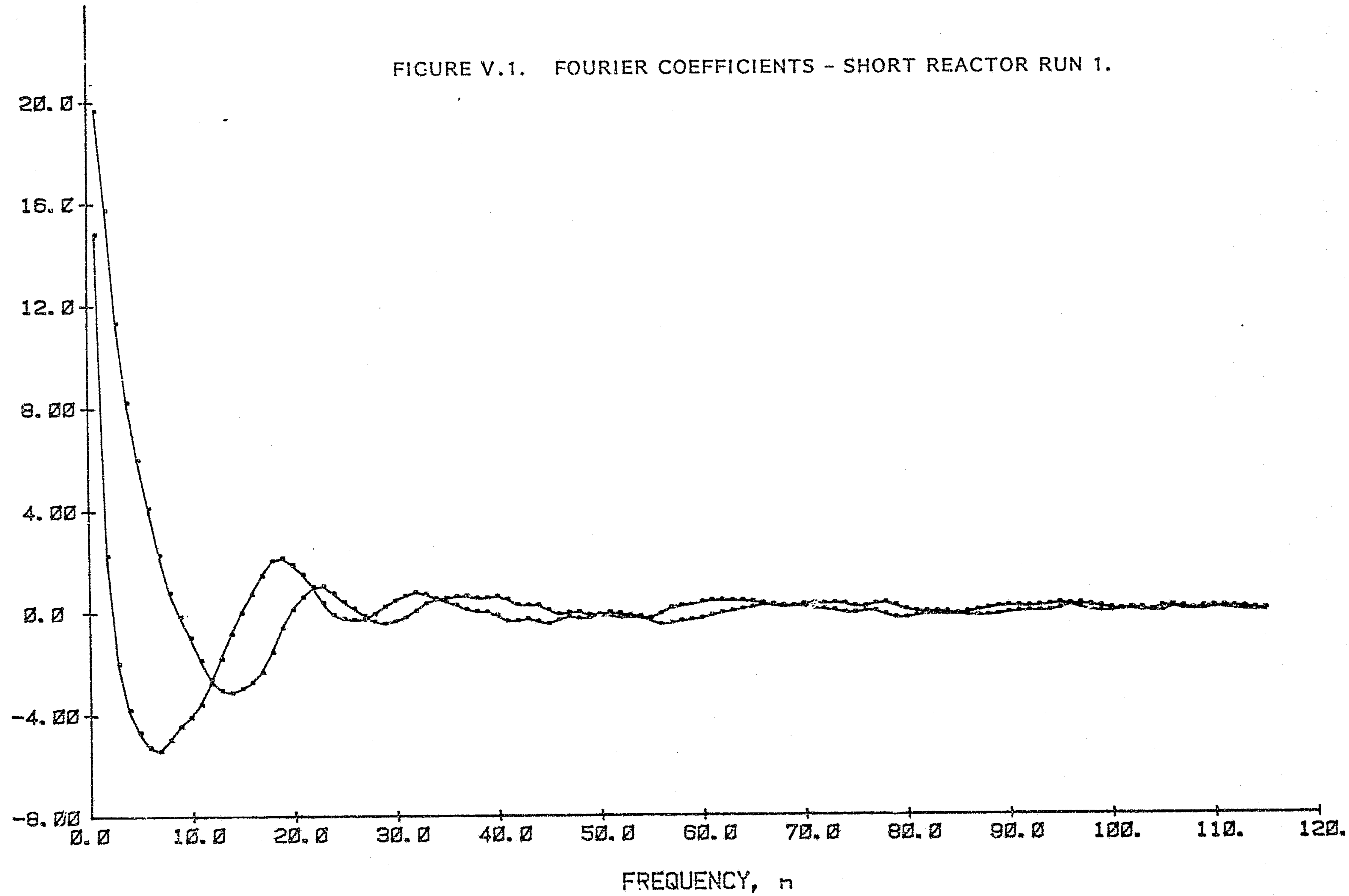
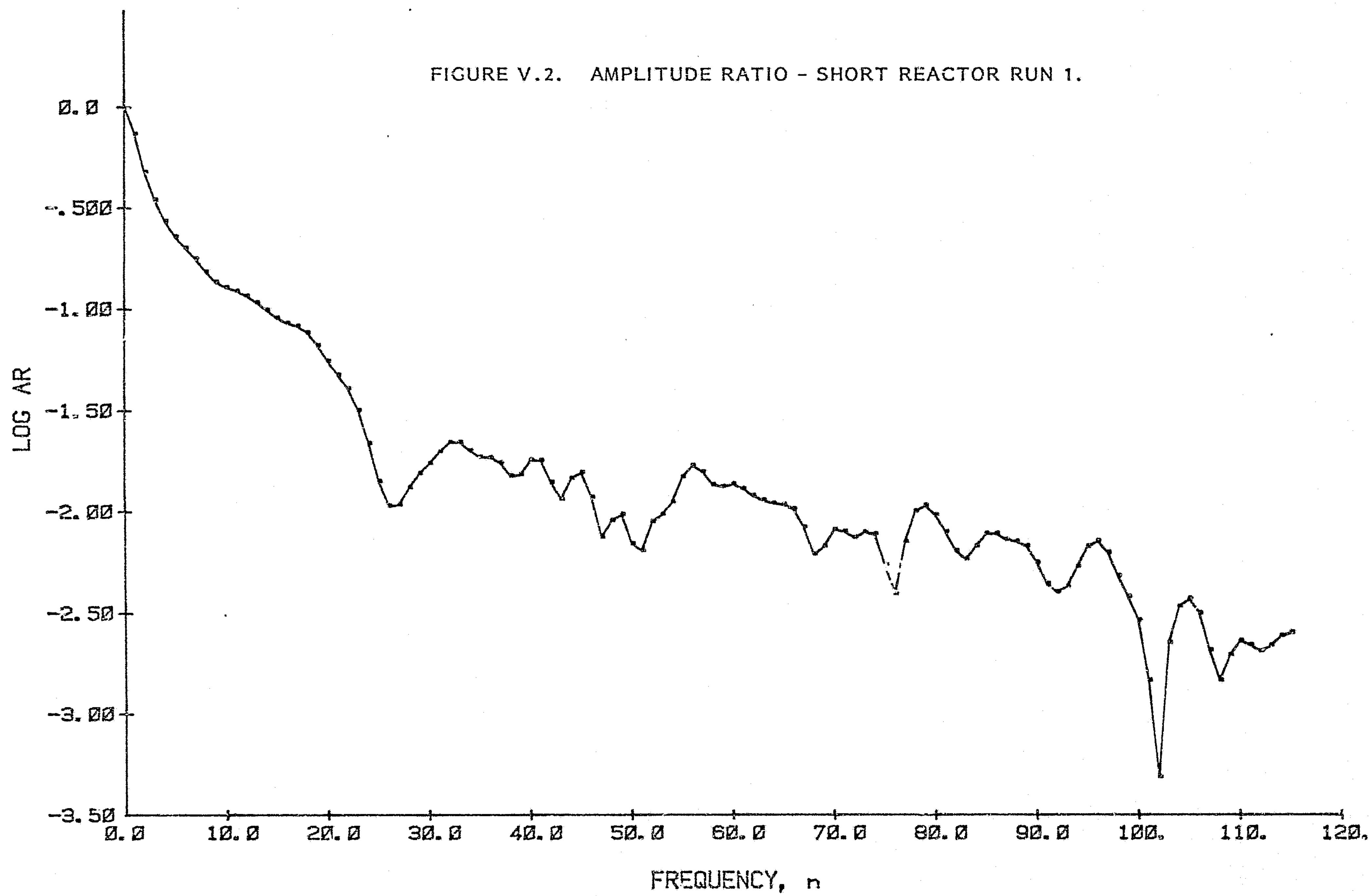


FIGURE V.2. AMPLITUDE RATIO - SHORT REACTOR RUN 1.



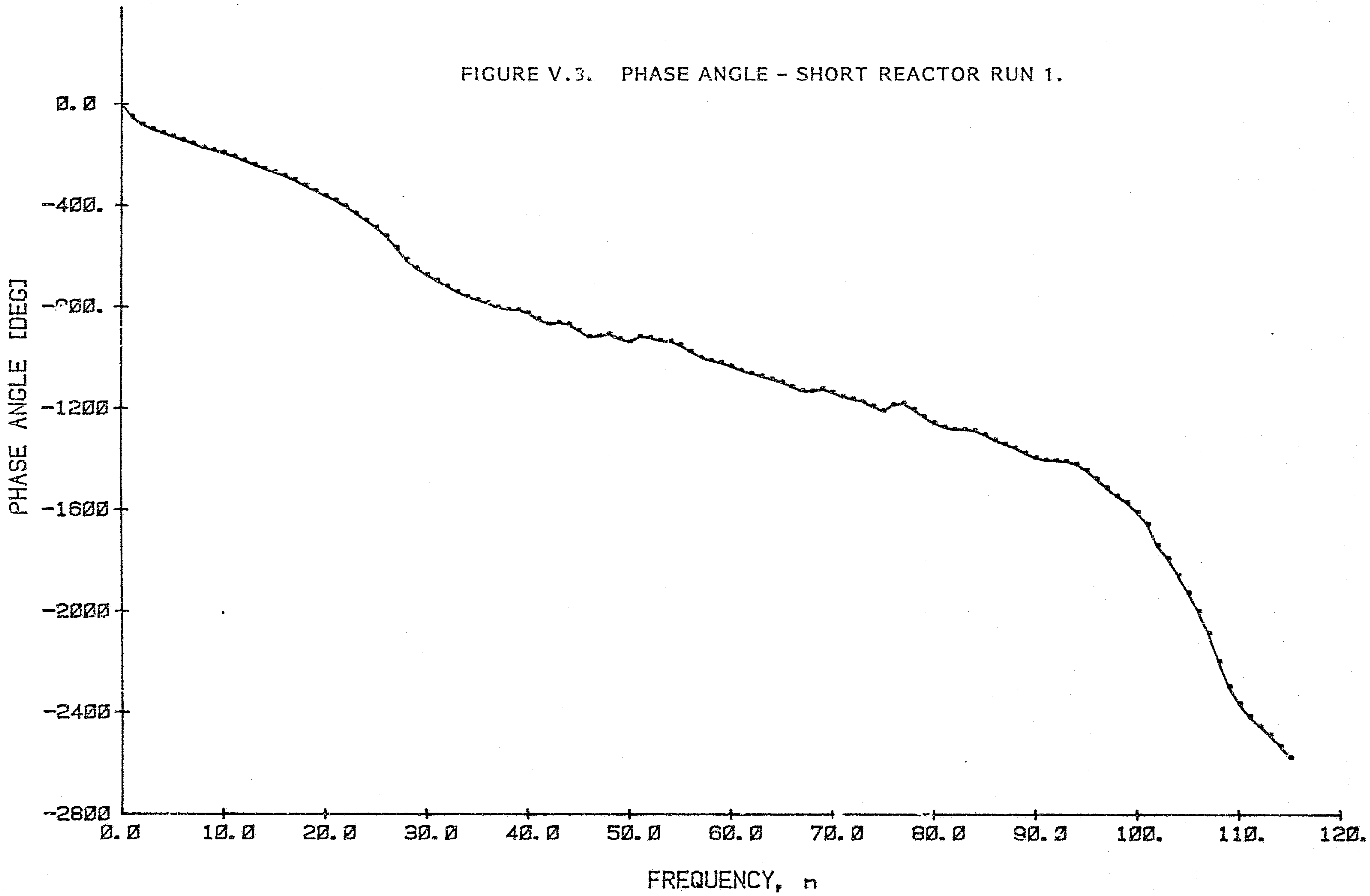


FIGURE V.3. PHASE ANGLE - SHORT REACTOR RUN 1.

FIGURE V.4. FOURIER COEFFICIENTS - SHORT REACTOR RUN 2.

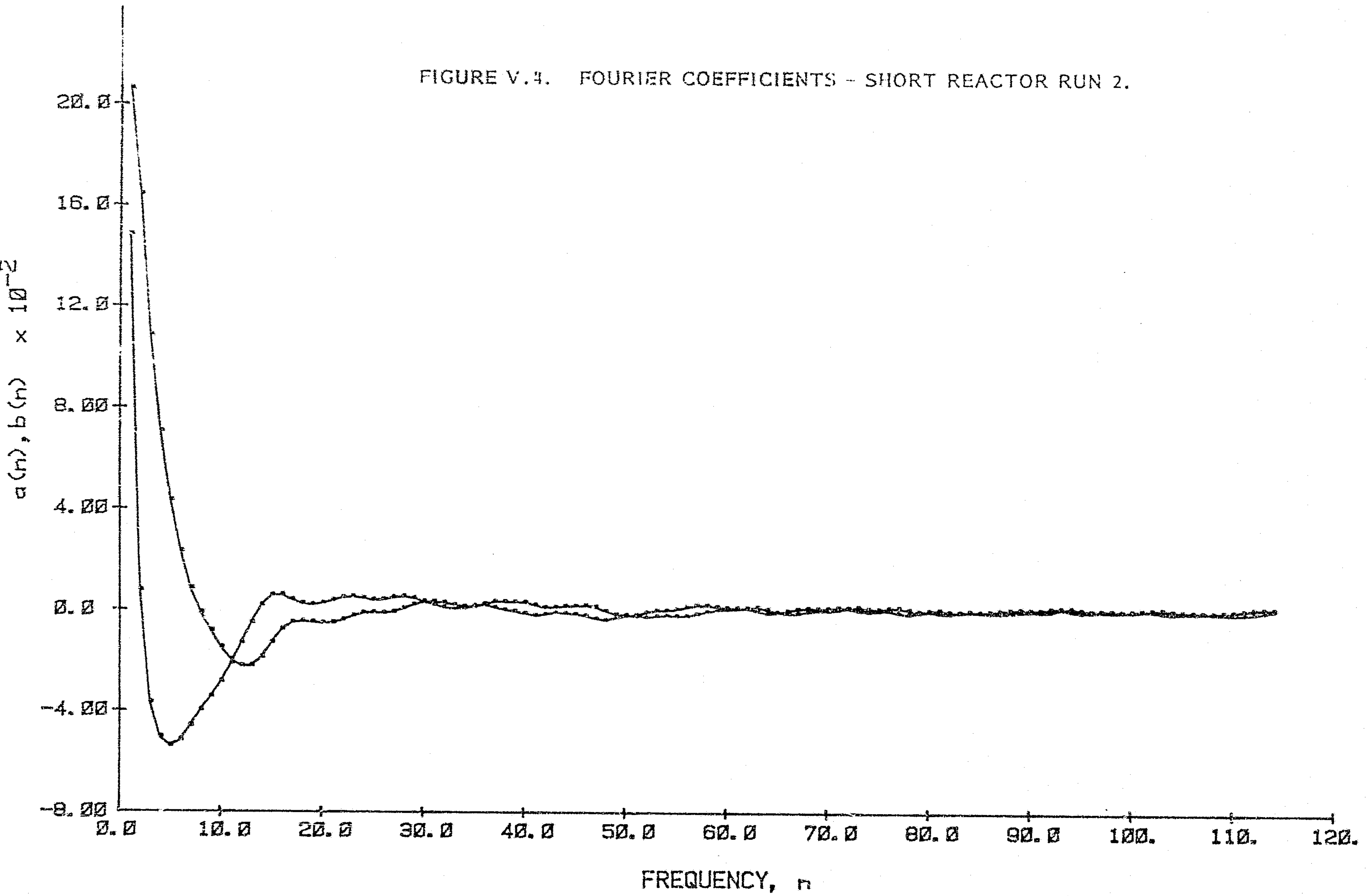


FIGURE V.5. AMPLITUDE RATIO - SHORT REACTOR RUN 2.

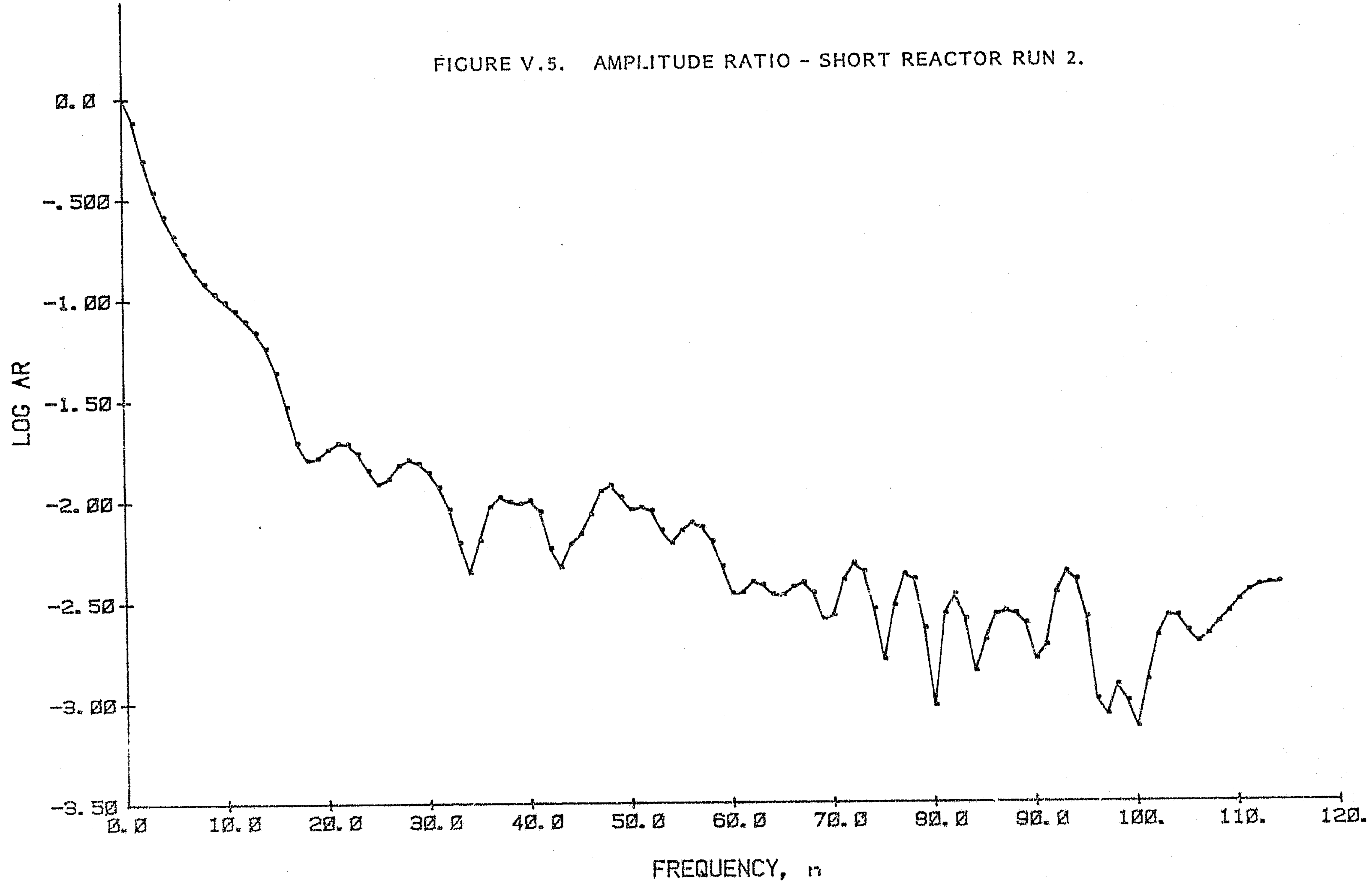
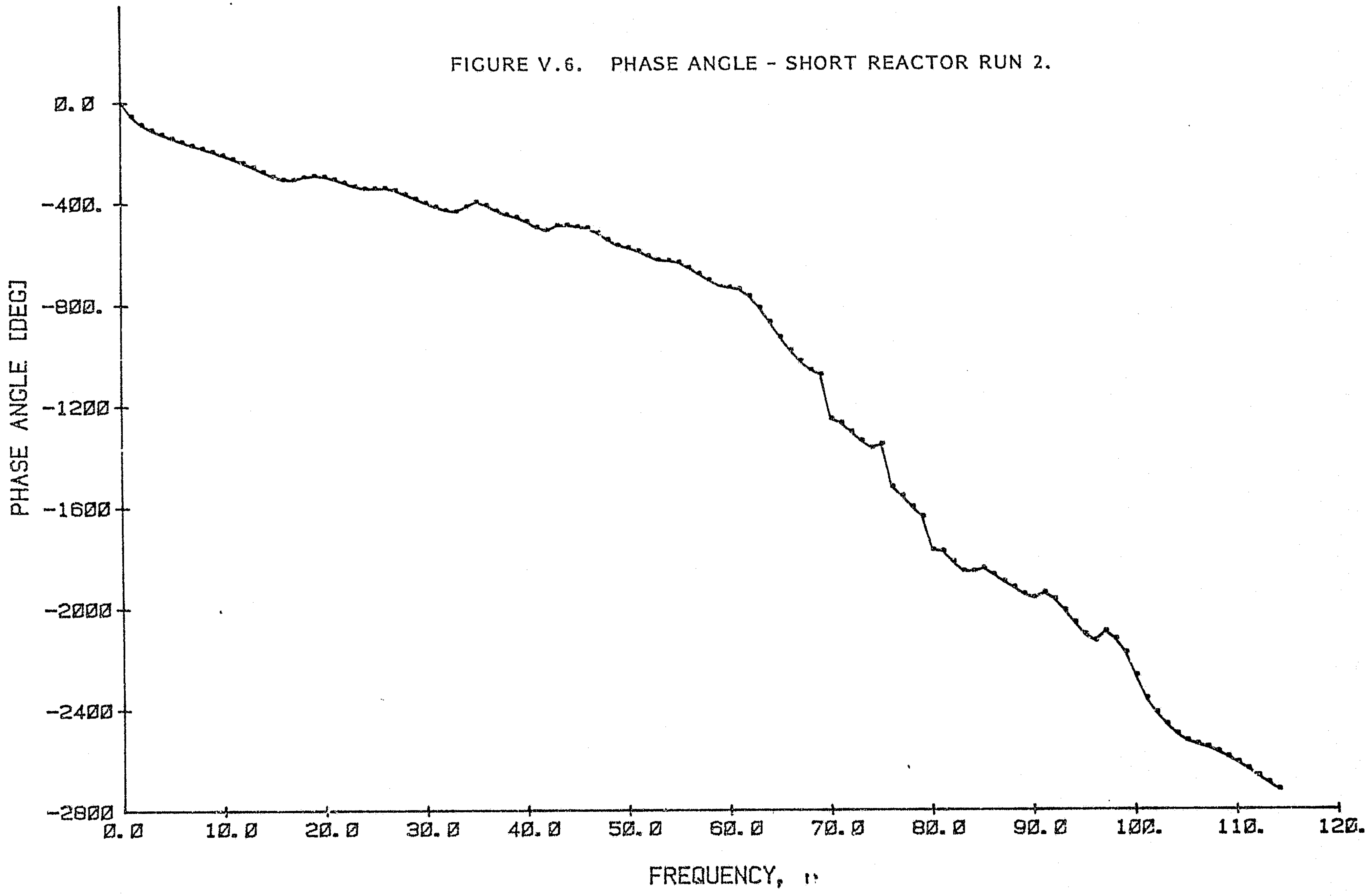


FIGURE V.6. PHASE ANGLE - SHORT REACTOR RUN 2.



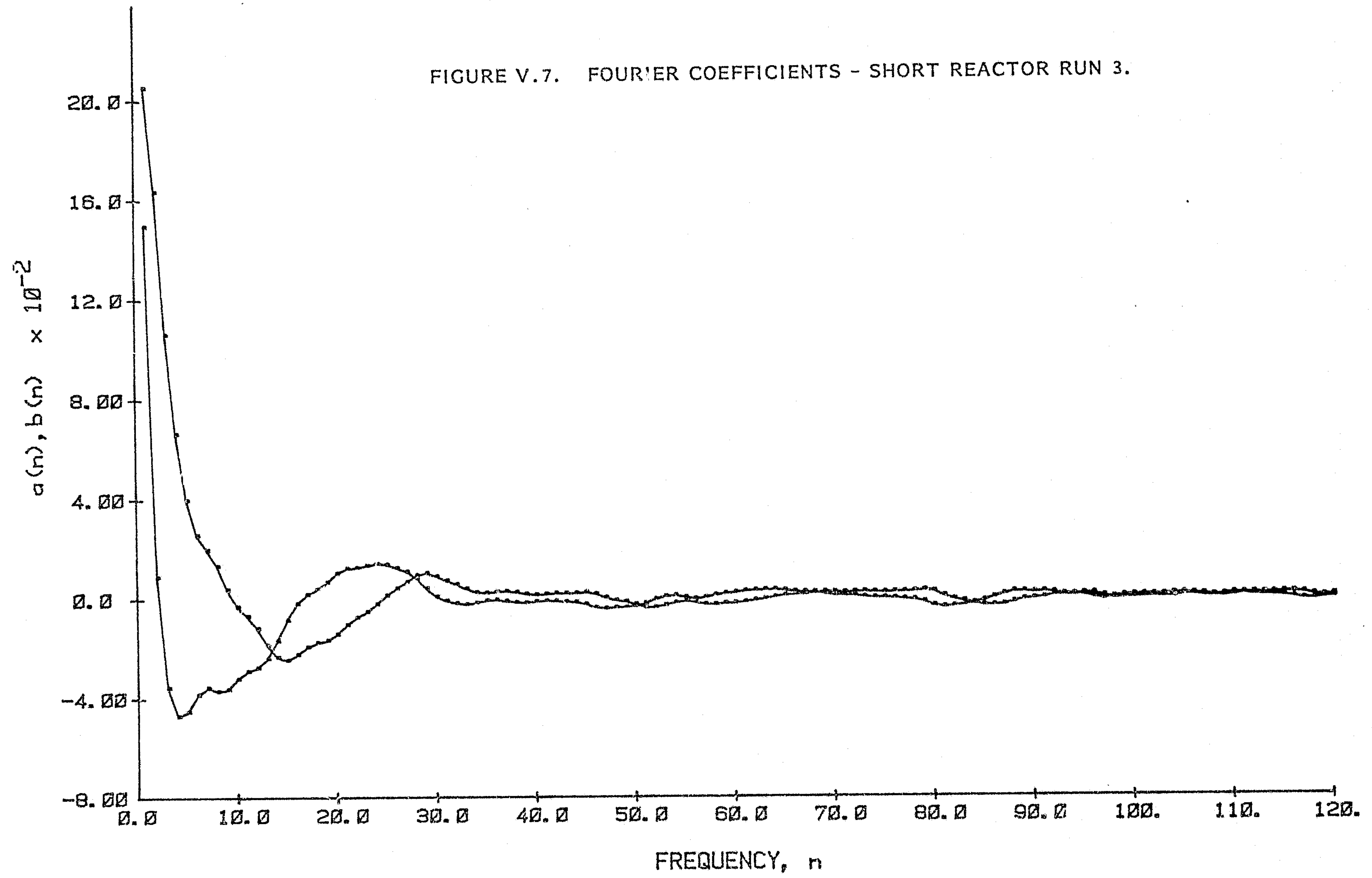


FIGURE V.7. FOUR'ER COEFFICIENTS - SHORT REACTOR RUN 3.

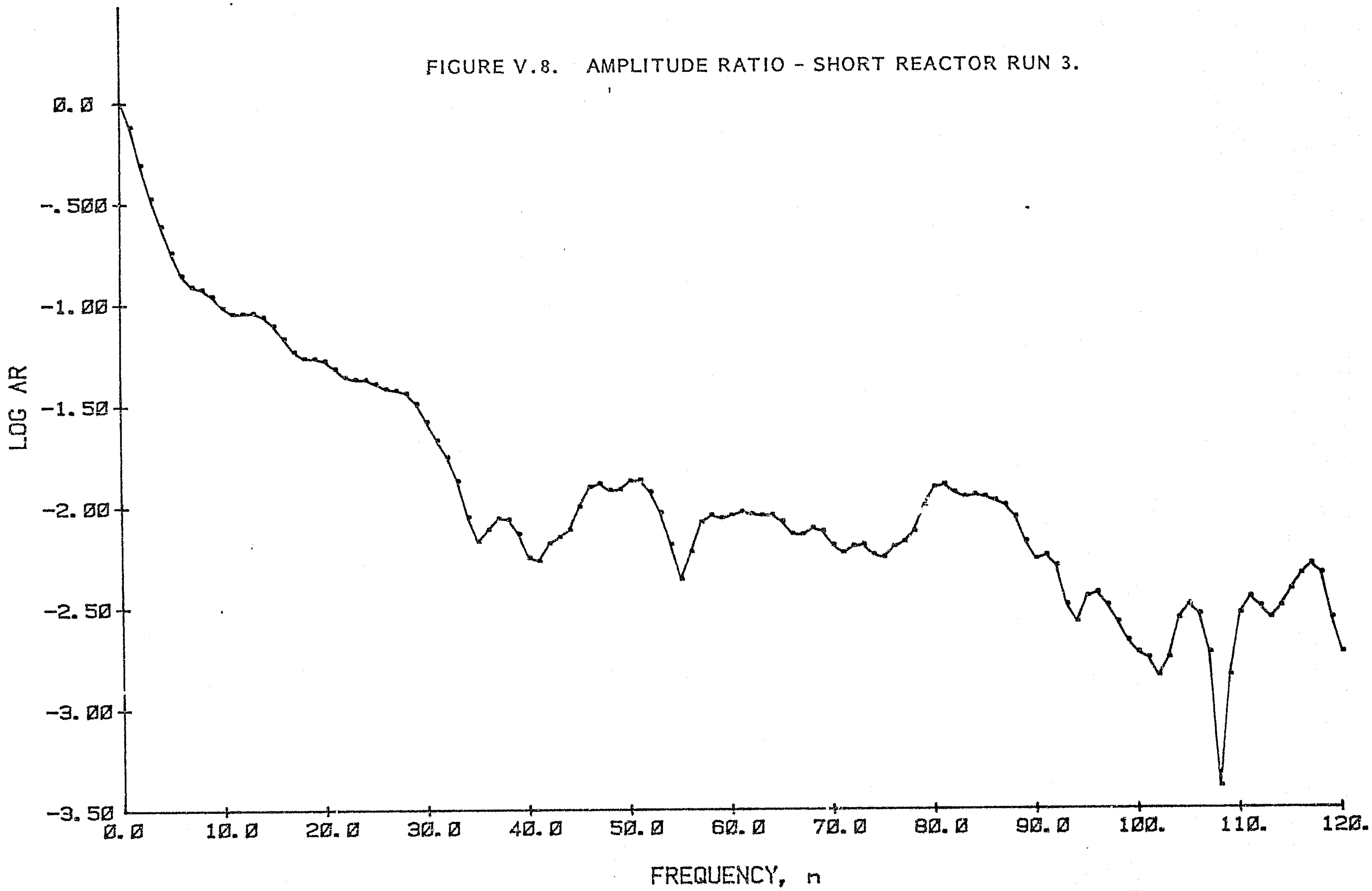


FIGURE V.8. AMPLITUDE RATIO - SHORT REACTOR RUN 3.

FIGURE V.9. PHASE ANGLE - SHORT REACTOR RUN 3.

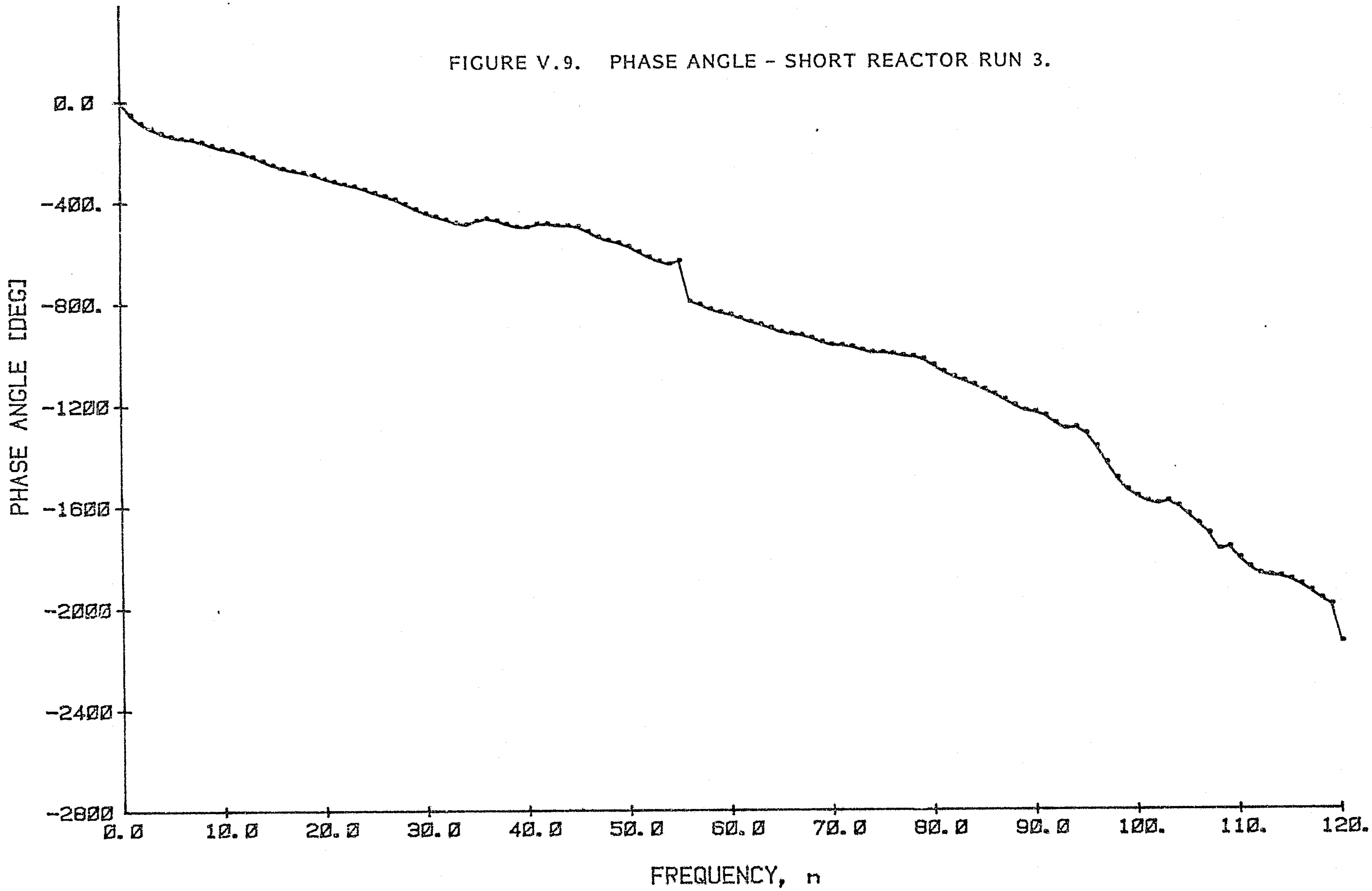


FIGURE V.10. FOURIER COEFFICIENTS - SHORT REACTOR RUN 4.

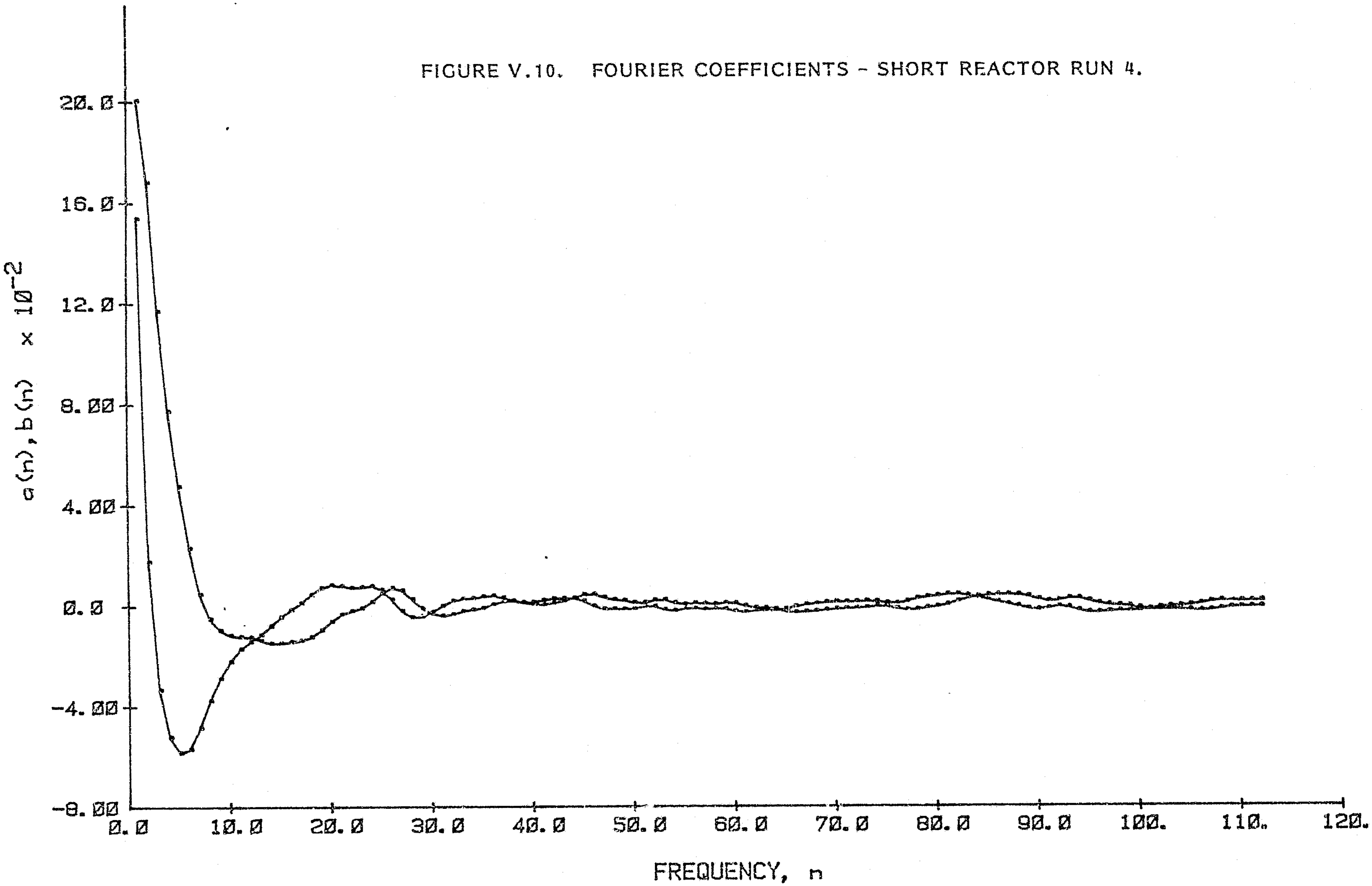


FIGURE V.11. AMPLITUDE RATIO - SHORT REACTOR RUN 4.

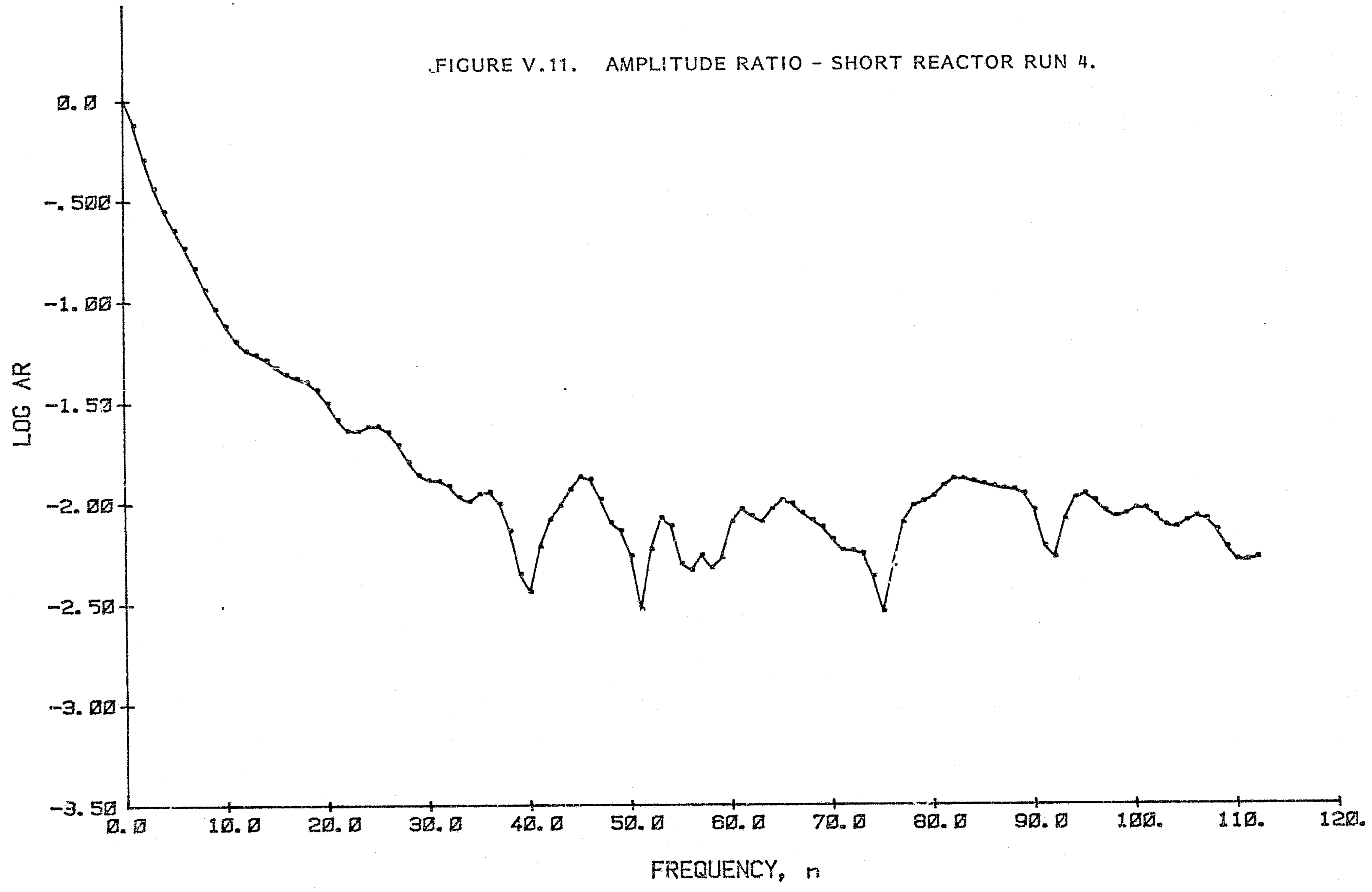
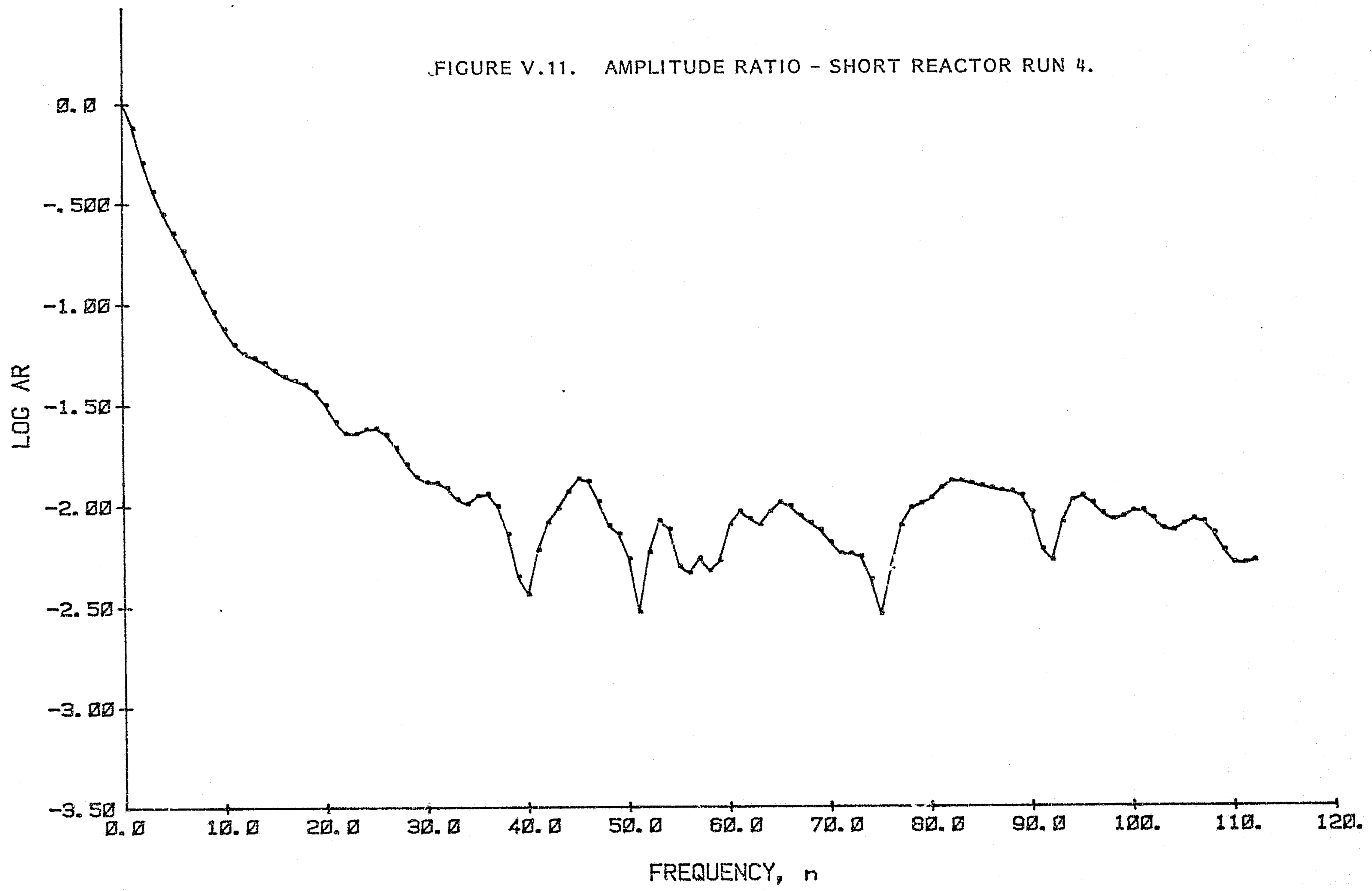


FIGURE V.11. AMPLITUDE RATIO - SHORT REACTOR RUN 4.



FICURE V.12. PHASE ANGLE - SHORT REACTOR RUN 4.

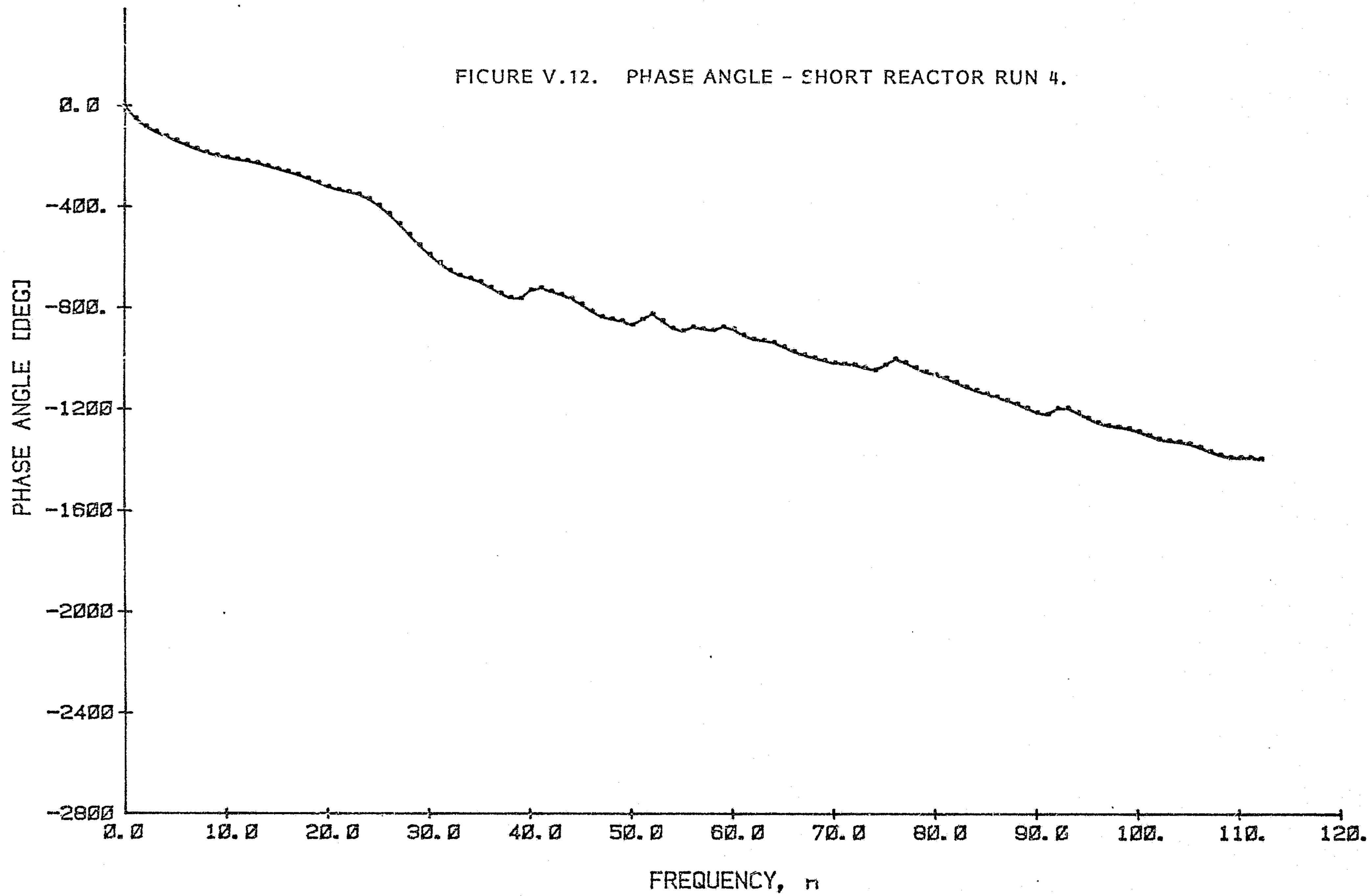


FIGURE V.13. FOURIER COEFFICIENTS - SHORT REACTOR RUN 5.

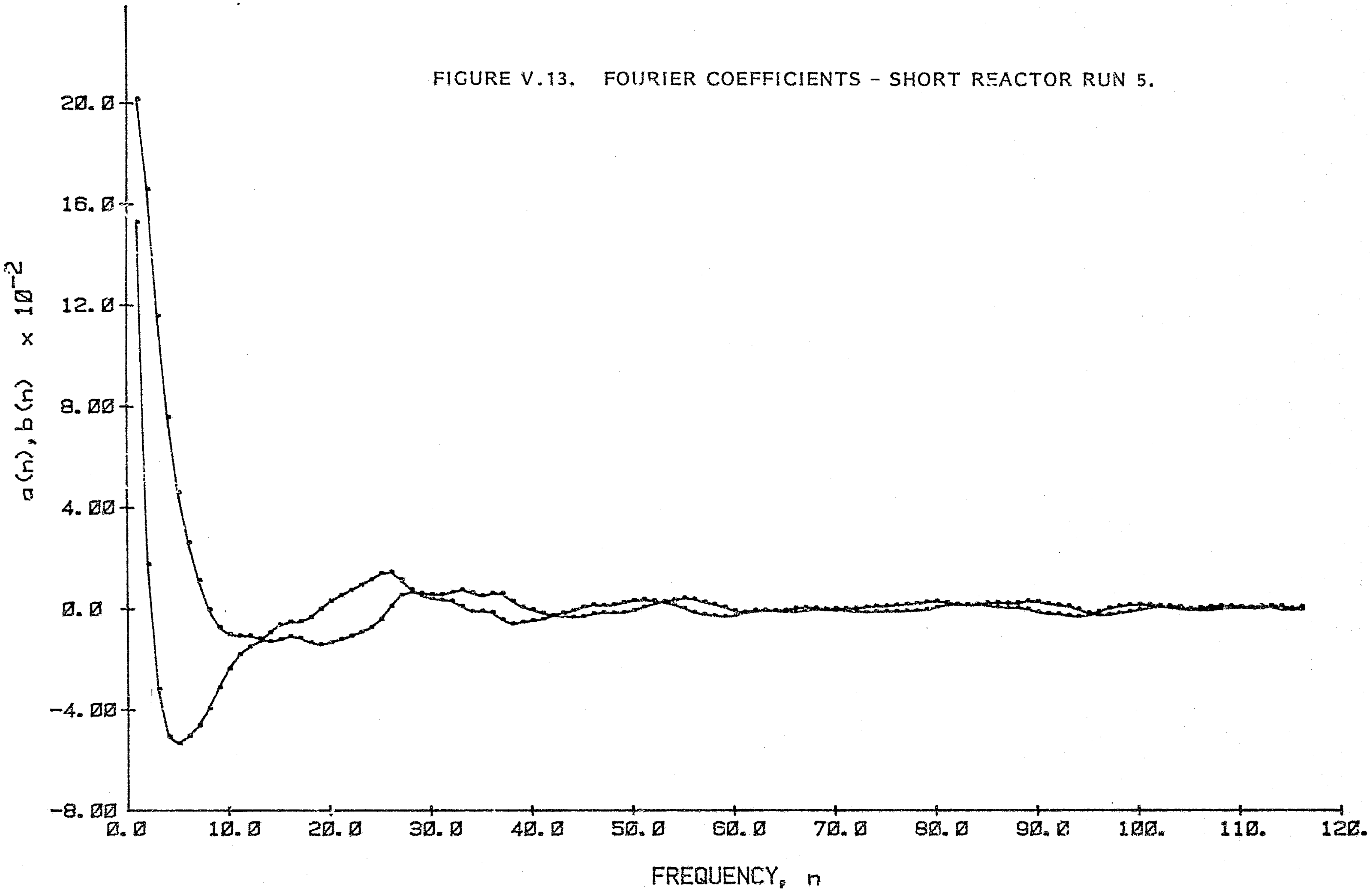
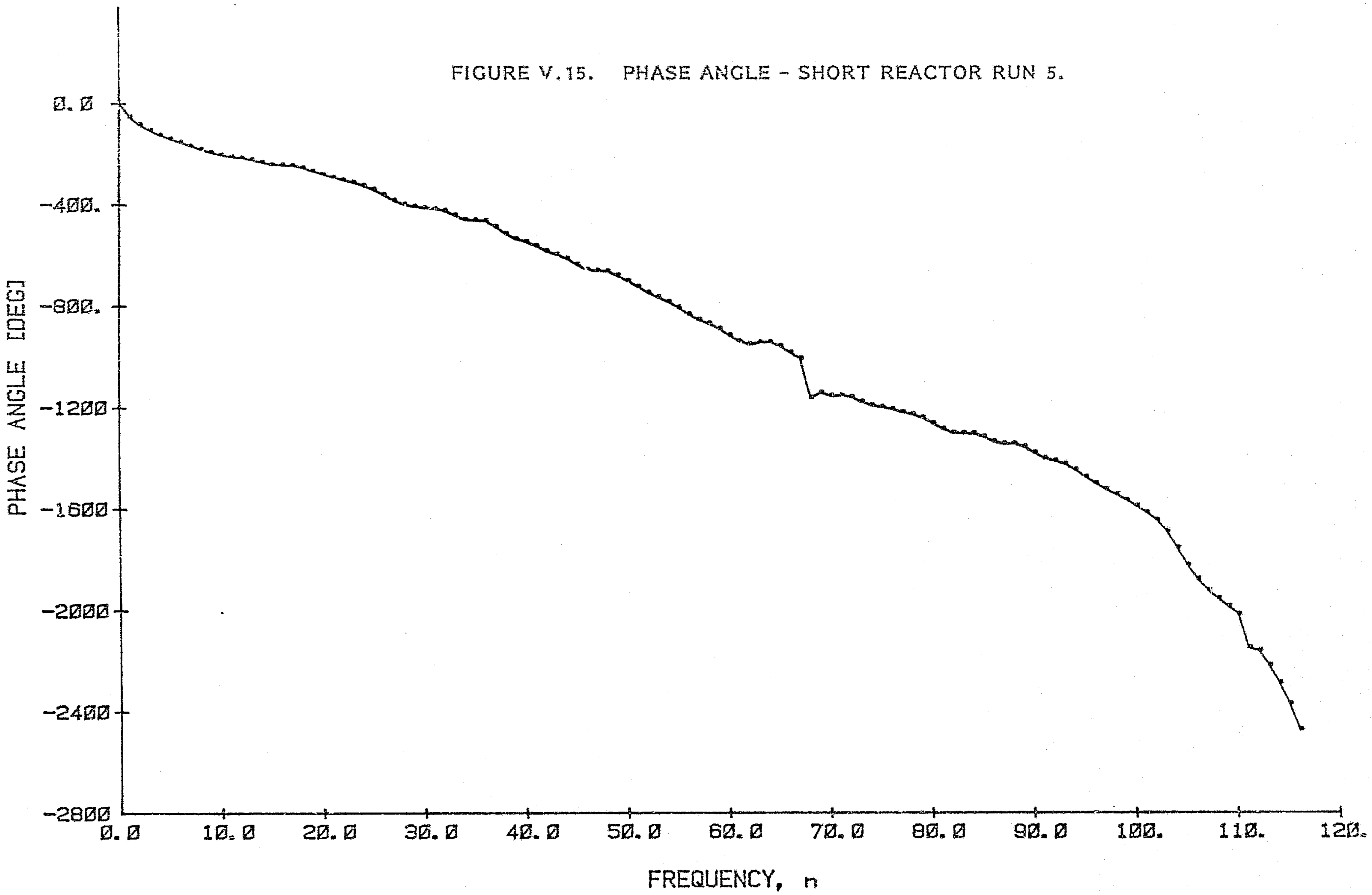


FIGURE V.14. AMPLITUDE RATIO - SHORT REACTOR RUN 5.

FIGURE V.15. PHASE ANGLE - SHORT REACTOR RUN 5.



APPENDIX VI. EQUATIONS ASSOCIATED WITH NORMALISATION AND EXPONENTIAL TAIL FITTING OF EXPERIMENTAL DATA

An exponential curve of the form

$$f(t) = c.\exp(-bt) \qquad (t_1 \le t \le \infty) \tag{VI.1}$$

is fitted to the tail of the response curves from time t_1, where c and b are calculated from the intercept and slope of a ln f(t) vs t plot.

The response curve f(t) therefore now has the form

$$f(t) = \begin{cases} f_1(t), & 0 \le t \le t_1 \quad \text{(not normalised)} \\ c.\exp(-bt), & t > t_1 \end{cases}$$

where $f_1(t)$ is the recorded curve, up to time t_1.

The moments of f(t) are given by

$$\alpha_k = \int_0^\infty t^k f(t)\, dt$$

$$\alpha_0 = \int_0^{t_1} f_1(t)\, dt + \int_{t_1}^\infty c \exp(-bt)\, dt$$

$$= \int_0^{t_1} f_1(t)\, dt + (c/b) \exp(-bt_1) \tag{VI.2}$$

$$\alpha_1 = \int_0^{t_1} t\, f_1(t)\, dt + \int_{t_1}^\infty t\, c \exp(-bt)\, dt$$

Integrating the second integral by parts,

$$\alpha_1 = \int_0^{t_1} t\, f_1(t)\, dt + (c/b) \exp(-bt_1)\, (1/b + t_1) \tag{VI.3}$$

$$\alpha_2 = \int_0^{t_1} t^2 f_1(t)\, dt + \int_{t_1}^\infty t^2 c \exp(-bt)\, dt$$

Integrating the second integral twice by parts,

$$\alpha_2 = \int_0^{t_1} t^2 f_1(t)\,dt + (c/b)\exp(-bt_1)\,[2/b^2 + 2t_1/b + t_1^2] \qquad (VI.4)$$

The mean and variance are now given by

$$\tau = \alpha_1 / \alpha_0$$
$$\sigma^2 = \alpha_2 / \alpha_0 - \tau^2$$

Normalisation: $f(t) = f(t)/\alpha_0$

Tail: $c = c/\alpha_0$

The area under the $f(t)$ curve now equals unity.

Conversion to dimensionless time units:

$$\theta = t/\tau$$
$$f(\theta) = f(t).\tau$$
$$\tau_\theta = 1$$
$$\sigma_\theta^2 = \sigma^2/\tau^2$$

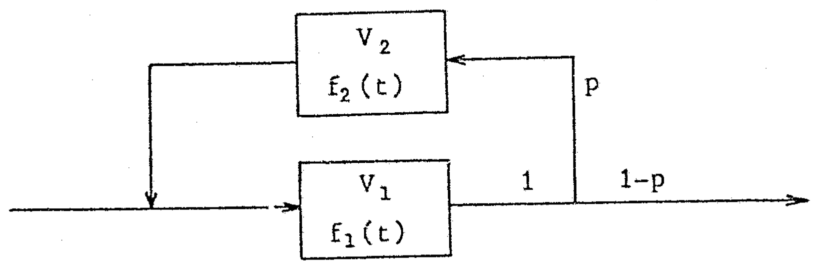
Tail: $c_\theta = c.\tau$

$$b_\theta = b\tau$$

The above equations are included in the computer program "NORM" (see Appendix XI.3)

APPENDIX VII. TIME DOMAIN SOLUTIONS AND MOMENTS FOR THE MATHEMATICAL MODELS

VII.1. RECYCLE MODEL



The transfer function for the recycle model was derived as

$$F(s) = \frac{(1-p)\ F_1(s)}{1 - pF_1(s)F_2(s)} \qquad (5.2)$$

$$= (1-p).F_1(s).[1 - pF_1(s)F_2(s)]^{-1}$$

The above expression may be expanded using the binomial theorem:

$$F(s) = (1-p)\sum_{j=1}^{\infty} F_1(s).[pF_1(s)F_2(s)]^{j-1}$$

Inverting F(s) term by term yields:

$$f(t) = (1-p)f_1(t) + (1-p)\sum_{j=2}^{\infty} p^{j-1}[f_1^{*j} * f_2^{*(j-1)}](t) \qquad (VII.1)$$

where $f_1(t)$ and $f_2(t)$ are the RTD functions of regions 1 and 2 respectively. In equation (VIII.i) the asterisk denotes the convolution operator, and $[f^{*j}](t)$ is the j-fold convolution of f(t) with itself, defined as follows:

$$[f_1 * f_2](t) = \int_0^t f_1(x)f_2(t-x)dx = \int_0^t f_1(t-y)f_2(y)dy$$

$$[f_1^{*1}](t) = f_1(t)$$

$$[f_1^{*2}](t) = [f_1 * f_1](t)$$

$$[f_1^{*j}](t) = [f_1^{*(j-1)} * f_1](t)$$

$$\mathcal{L}\{[f_1 * f_2](t)\} = F_1(s)F_2(s)$$

The terms of equation (VII.1) have a physical significance as they represent the RTD functions of material that has passed j times through the system.

The RTD function f(t) is derived here for the case where regions 1 and 2 are represented by <u>mixed tanks in series</u> (gamma distribution):

$$f_1(t) = \frac{1}{\Gamma(n_1)} \frac{n_1}{\tau_1} \left(\frac{n_1 t}{\tau_1}\right)^{n_1 - 1} e^{-n_1 t/\tau_1}$$

$$f_2(t) = \frac{1}{\Gamma(n_2)} \frac{n_2}{\tau_2} \left(\frac{n_2 t}{\tau_2}\right)^{n_2 - 1} e^{-n_2 t/\tau_2} \qquad \text{(VII.2)}$$

where τ_1 = mean of $f_1(t)$
= (no. of tanks) x (mean residence time in each tank)

n_1= number of CSTR's in region 1

and similarly for τ_2 and n_2. n_1 and n_2 need not be whole numbers.

From equation (VII.1),

$$f(t) = (1-p)\frac{1}{\Gamma(n_1)} \frac{n_1}{\tau_1} \left(\frac{n_1 t}{\tau_1}\right)^{n_1 - 1} e^{-nt/\tau_1}$$

$$+ (1-p) \sum_{j=2}^{\infty} p^{j-1} \left[\left\{ \frac{1}{\Gamma(jn_1)} \frac{n_1}{\tau_1} \left(\frac{n_1 t}{\tau_1}\right)^{jn_1 - 1} e^{-n_1 t/\tau_1} \right\} \right.$$

$$\left. * \left\{ \frac{1}{\Gamma((j-1)n_2)} \frac{n_2}{\tau_2} \left(\frac{n_2 t}{\tau_2}\right)^{(j-1)n_2 - 1} e^{-n_2 t/\tau_2} \right\} \right]$$

$$f(t) = (1-p)\frac{1}{\Gamma(n_1)}\frac{n_1}{\tau_1}\left(\frac{n_1 t}{\tau_1}\right)^{n_1-1} e^{-nt/\tau_1}$$

$$+ (1-p)\sum_{j=2}^{\infty} p^{j-1}\frac{1}{\Gamma(jn_1)}\frac{1}{\Gamma\{(j-1)n_2\}}\frac{n_1}{\tau_1}\frac{n_2}{\tau_2}$$

$$\cdot\int_0^t \left(\frac{n_1 x}{\tau_1}\right)^{jn_1-1} e^{-n_1 x/\tau_1}\left(\frac{n_2(t-x)}{\tau_2}\right)^{(j-1)n_2-1} e^{-n_2(t-x)/\tau_2}dx \qquad \text{(VII.3)}$$

Solution of the above model RTD function would thus require a series of numerical integrations at each point of time required. Gibilaro (12) has solved the convolution integral in eq (VII.3) but the result is complicated, containing a double summation. Thus in order to do a least squares fit of equation (VII.3) in the time domain, f(t) would have to be evaluated at about 240 time points, the sum of squared differences computed, and this whole process repeated for a large number of iterations (about 150). This would required excessive computer time, even using the best minimisation routine.

For the case of instantaneous recycle ($F_2(s)=1$), equation (VII.3) reduces to

$$f(t) = (1-p)\sum_{j=1}^{\infty} p^{j-1} f_1^{*j}(t)$$

$$f(t) = (1-p)\sum_{j=1}^{\infty} p^{j-1}\frac{1}{\Gamma(jn_1)}\frac{n_1}{\tau_1}\left(\frac{n_1 t}{\tau_1}\right)^{jn_1-1} e^{-n_1 t/\tau_1} \qquad \text{(VII.4)}$$

which is the same as the equation derived by Fu (13) (eq.29).

Moments:

The moments of the RTD are defined by (7,20):

$$\alpha_k = \int_0^\infty t^k f(t)\, dt \qquad \text{(VII.5)}$$

and may be obtained from the transfer function as follows :

$$\alpha_k = (-1)^k \frac{d^k}{ds^k} F(s)\Big|_{s=o} \qquad \text{(VII.6)}$$

The mean and variance of f(t) are defined as

$$\tau = \alpha_1 / \alpha_o \qquad \text{(VII.7)}$$

$$\sigma^2 = \alpha_2/\alpha_o - \tau^2 \qquad \text{(VII.8)}$$

($\alpha_o = 1$ for an RTD curve)

and can be calculated as follows:

$$\tau = -\frac{d}{ds} \ln F(s)\Big|_{s=o} \qquad \text{(VII.9)}$$

$$\sigma^2 = \frac{d^2}{ds^2} \ln F(s)\Big|_{s=o} \qquad \text{(VII.10)}$$

For the recycle model (with any half loop RTDs):

$$\ln F(s) = \ln (1-p)F_1(s) - \ln (1 - pF_1(s)F_2(s)$$

$$\frac{d}{ds} \ln F(s) = \frac{F_1'(s)}{F_1(s)} + \frac{p[F_1(s)F_2'(s) + F_1'(s)F_2(s)}{1 - pF_1(s)F_2(s)} \qquad \text{(VII.11)}$$

where the prime indicates differentiation.

$$\frac{d^2}{ds^2} \ln F(s) = \frac{F_1(s)F_1''(s) - F_1'(s)^2}{F_1(s)^2}$$

$$+ \frac{p[1 - pF_1(s)F_2(s)][F_1(s)F_2''(s) + F_1'(s)F_2'(s) + F_1'(s)F_2'(s) + F_1''(s)F_2(s)]}{[1 - pF_1(s)F_2(s)]^2}$$

$$+ \frac{p^2[F_1(s)F_2'(s) + F_1'(s)F_2(s)][F_1(s)F_2'(s) + F_1'(s)F_2(s)]}{[1 - pF_1(s)F_2(s)]^2} \qquad \text{(VII.12)}$$

For any half-loop RTD function $f_1(t)$ (similarly for $f_2(t)$), we have

$$F_1(s)\Big|_{s=0} = \int_0^\infty f_1(t)\,dt = 1$$

$$F_1'(s)\Big|_{s=0} = -\tau_1 \qquad \text{(from III.6)}$$

$$F''(s)\Big|_{s=0} = \sigma_1^2 - \tau_1^2 \qquad \text{(from III.6)} \qquad \text{(VII.13)}$$

Substitution of the above three equations into equations (VII.11) and (VII.12) yields:

$$\frac{d}{ds}\ln F(s)\Big|_{s=0} = \frac{1}{1-p}(\tau_1 + p\tau_2)$$

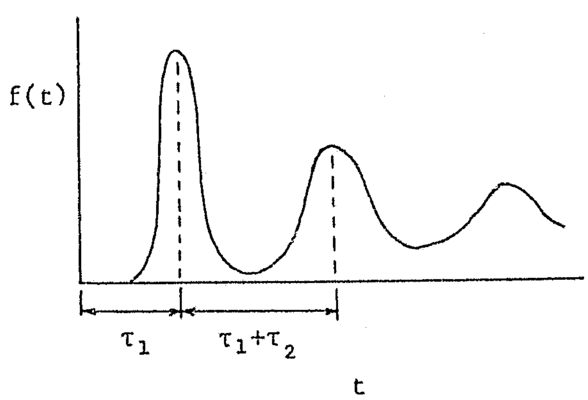
$$\frac{d^2}{ds^2}\ln F(s) = \frac{p}{(1-p)^2}(\tau_1+\tau_2)^2 + \frac{1}{1-p}(\sigma_1^2 + p\sigma_2^2)$$

$$\tau = \frac{1}{1-p}(\tau_1 + p\tau_2) \qquad \text{(VII.14)}$$

$$\sigma^2 = \frac{p}{(1-p)^2}(\tau_1+\tau_2)^2 + \frac{1}{1-p}(\sigma_1^2 + p\sigma_2^2) \qquad \text{(VII.15)}$$

Estimation of Model Parameters:

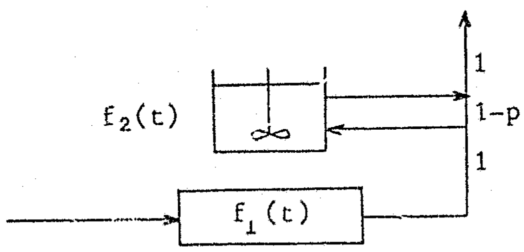
If the variances of the half loop RTD functions are small (i.e. close to plug flow) ($\sigma^2/\tau^2 \leq 0{,}1$), τ_1 and τ_2 can be estimated from the location of the peaks of the RTD curve, as shown below:



The recycle fraction p can then be found from equation (VII.14) (where τ is calculated by numerical integration of the RTD curve; τ equals unity for normalized curves). Also, the area under the first peak should equal 1-p. Furthermore, σ_1^2 can be estimated by examining the shape of the first peak, and σ_2^2 can subsequently be calculated from equation (VII.15).

The method described above eliminates the need to determine higher order moments and is preferable to the method of moments. However, when the half loop variances are not small (as is the case in this work), the successive peaks overlap and are not as symmetrical, and only rough estimates of τ_1 and τ_2 can be obtained by this method.

VII.2. BYPASS MODEL



The transfer function for the bypass model was derived as

$$F(s) = F_1(s).[1-p + pF_2(s)] \tag{5.3}$$

Inverting F(s) yields

$$f(t) = (1-p)f_1(t) + pf_1(t)*f_2(t) \tag{VII.16}$$

For the case where region 1 is represented by mixed tanks in series and region 2 by a single CSTR, we have

$$f(t) = \frac{1}{\Gamma(n_1)} \frac{n_1}{\tau_1} \left(\frac{n_1 t}{\tau_1}\right)^{n_1-1} e^{-n_1 t/\tau_1}$$

$$f_2(t) = \frac{1}{\tau_2} e^{-t/\tau_2} \tag{VII.17}$$

$$\text{Therefore } f(t) = (1-p)\frac{1}{\Gamma(n_1)} \frac{n_1}{\tau_1} \left(\frac{n_1 t}{\tau_1}\right)^{n_1-1} e^{-n_1 t/\tau_1}$$

$$+ p\frac{1}{\Gamma(n_1)} \frac{n_1}{\tau_1} \frac{1}{\tau_2} \int_0^t \left(\frac{n_1 x}{\tau_1}\right)^{n_1-1} e^{-n_1 x/\tau_1} e^{-(t-x)/\tau_2}\, dx \tag{VII.18}$$

The bypass model RTD function is thus simpler than that of the recycle model (eq. VII.3), containing a single convolution integral. A least squares fit in the time domain would however still require excessive computer time.

Moments:

$$\frac{d}{ds} F(s) = (1-p)F_1'(s) + p[F_1(s)F_2'(s) + F_1'(s)F_2(s)]$$

$$\frac{d^2}{ds^2} F(s) = (1-p)F''(s) + p[F_1(s)F_2''(s) + 2F_1'(s)F_2'(s) + F_1''(s)F_2(s)]$$

Using equations (III.13) we obtain:

$$\frac{d}{ds} F(s)\Big|_{s=0} = -\tau_1 - p\tau_2$$

$$\frac{d^2}{ds^2} F(s)\Big|_{s=0} = \sigma_1^2 + \tau_1^2 + p(\sigma_2^2 + \tau_2^2 + 2\tau_1\tau_2)$$

From equations (VII.7) and (VII.8):

$$\tau = \alpha_1 = -\frac{d}{ds} F(s)\Big|_{s=0}$$

$$\sigma^2 = \alpha_2 - \tau^2 = \frac{d^2}{ds^2} F(s)\Big|_{s=0} - \tau^2$$

$$\tau = \tau_1 + p\,\tau_2 \qquad \text{(VII.19)}$$

$$\sigma^2 = \sigma_1^2 + p\sigma_2^2 + p(1-p)\,\tau_2^2 \qquad \text{(VII.20)}$$

APPENDIX VIII. DERIVATION OF THE FOURIER COEFFICIENT EQUATIONS FOR THE RECYCLE MODEL

For the recycle model,

$$F(s) = \frac{(1-p)F_1(s)}{1 - pF_1(s)F_2(s)} \tag{5.2}$$

VIII.1. BOTH REGIONS PLUG FLOW

$$F_1(s) = \exp(-s\tau_1)$$

$$F_2(s) = \exp(-s\tau_2) \qquad \text{where } \tau_2 = \frac{(1-p)\,\tau - \tau_1}{p}$$

The independent variables are therefore p and τ_1.

Substituting in 5.2,

$$F(s) = \frac{(1-p)\exp(-s\tau_1)}{1 - p\exp[-s(\tau_1 + \tau_2)}$$

Substituting (iw) for s,

$$F(iw) = \frac{(1-p)\exp(-iw\tau_1)}{1 - p\exp[-iw(\tau_1 + \tau_2)]}$$

$$= \frac{(1-p)\exp(-iw\tau_1)}{1 - p\cos w(\tau_1 + \tau_2) + ip\sin w(\tau_1 + \tau_2)}$$

$$= \frac{(1-p)[\cos w\tau_1 - i\sin w\tau_1][1 - p\cos w(\tau_1 + \tau\ ) - ip\sin w(\tau_1 + \tau_2)]}{[1 - p\cos w(\tau_1 + \tau_2)]^2 + [p\sin w(\tau_1 + \tau_2)]^2}$$

which after manipulation and regrouping of terms, gives

$$F(iw) = \frac{(1-p)(\cos w\tau_1 - p\cos w\tau_2) - i(1-p)(\sin w\tau_1 + p\sin w\tau_2)}{1 + p^2 - 2p\cos w(\tau_1 + \tau_2)}$$

$$a_n = -(1/T)\ \mathrm{Im}\,[F(iw)] \qquad (5.8)$$

$$= \frac{1}{T}\ \frac{(1-p)(\sin w\tau_1 + p\sin w\tau_2)}{1 + p^2 - 2p\cos w(\tau_1 + \tau_2)}$$

$$b_n = (1/T)\ \mathrm{Re}\,[F(iw)] \qquad (5.9)$$

$$= \frac{1}{T}\ \frac{(1-p)(\cos w\tau_1 - p\cos w\tau_2)}{1 + p^2 - 2p\cos w(\tau_1 + \tau_2)}$$

$$A = \sqrt{Re^2 + Im^2}$$

$$= \frac{1 - p}{\sqrt{1 + p^2 - 2p\cos w(\tau_1 + \tau_2)}}$$

$$\phi = \tan^{-1}(Im/Re)$$

$$= -\tan^{-1}\left[\frac{\sin w\tau_1 + p\sin w\tau_2}{\cos w\tau_1 - p\cos w\tau_2}\right]$$

VIII.2. BOTH REGIONS GAMMA DISTRIBUTIONS (MIXED TANKS IN SERIES)

$$F_1(s) = \frac{1}{(\tau_1 s/n_1 + 1)^{n_1}}$$

$$F_2(s) = \frac{1}{(\tau_2 s/n_2 + 1)^{n_2}}$$

where

$$\tau_2 = \frac{(1-p)\,\tau - \tau_1}{p}$$

$n_1 = \tau_1^2/\sigma_1^2$ = number of mixed tanks in region 1

$n_2 = \tau_2^2/\sigma_2^2$ = number of mixed tanks in region 2

n_1 and n_2 need not be whole numbers.

The independent variables are p, τ_1, σ_1^2 (or n_1), σ_2^2 (or n_2)

Substituting $F_1(s)$ and $F_2(s)$ in equation (3.15),

$$F(s) = \frac{(1-p)(\tau_1 s/n_1 + 1)^{-n_1}}{1 - p(\tau_1 s/n_1 + 1)^{-n_1}(\tau_2 s/n_2 + 1)^{-n_2}}$$

$$F(iw) = \frac{(1-p)(\tau_2\, iw/n_2 + 1)^{n_2}}{(\tau_1\, iw/n_1 + 1)^{n_1}(\tau_2\, iw/n_2 + 1)^{n_2} - p}$$

$$= \frac{(1-p)\,r'\,\exp(in_2\theta_2)}{r_1^{n_1}\exp(in_1\theta_1)\,r_1^{n_2}\exp(in_2\theta_2) - p}$$

where

$$r_1 = (\tau_1^2 w^2/n_1^2 + 1)^{\frac{1}{2}}$$

$$r_2 = (\tau_2^2 w^2/n_2^2 + 1)^{\frac{1}{2}}$$

$$\theta_1 = \tan^{-1}(\tau_1 w/n_1)$$

$$\theta_2 = \tan^{-1}(\tau_2 w/n_2)$$

$$F\ (iw) \quad = \quad \frac{(1-p)r_2^{\,n_2}(\cos n_2\theta_2 + i \sin n_2\theta_2)}{r_1^{n_1}r^{n_2}[\cos(n_1\theta_1 + n_2\theta_2) + i \sin(n_1\theta_1 + n_2\theta_2)] - p}$$

$$= \frac{\begin{array}{c}(1-p)r_2^{n_2}(\cos n_2\theta_2 + i \sin n_2\theta_2)[r_1^{n_1}r_2^{n_2}\cos(n_1\theta_1 + n_2\theta_2) \\ - p - ir_1^{n_1}r_2^{n_2}\sin(n_1\theta_1 + n_2\theta_2)]\end{array}}{\begin{array}{c}r_1^{2n_1}r_2^{2n_2}\cos^2(n_1\theta_1 + n_2\theta_2) - 2pr_1^{n_1} \quad (n_1\theta_1 + n_2\theta_2) \\ + p^2 + r_1^{2n_1}r_2^{2n_2}\sin^2(n_1\theta_1 \quad n_2\theta_2)\end{array}}$$

which after manipulation gives

$$F\ (iw) \quad = \quad \frac{\begin{array}{c}(1-p)r_2^{n_2}(r_1^{n_1}r_2^{n_2}\cos n_1\theta_1 - p \cos n_2\theta_2) + i(1-p)r_2^{n_2} \\ .(-r_1^{n_1}r_2^{n_2}\sin n_1\theta_1 - p \sin n_2\theta_2)\end{array}}{r_1^{2n_1}r_2^{2n_2} - 2pr_1^{n_1}r_2^{n_2}\cos(n_1\theta_1 + n_2\theta_2) + p^2} \qquad (5.10)$$

$$a_n = -(1/T)\ \mathrm{Im}\ [F(iw)]$$

$$= \frac{1}{T}\ \frac{(1-p)r_2^{n_2}(r_1^{n_1}r_2^{n_2}\sin n_1\theta_1 + p \sin n_2\theta_2)}{r_1^{2n_1}r_2^{2n_2} - 2pr_1^{n_1}r_2^{n_2}\cos(n_1\theta_1 + n_2\theta_2) + p^2} \qquad (5.11)$$

$$b_n = (1/T)\ \mathrm{Re}\ [F(iw)]$$

$$= \frac{1}{T}\ \frac{(1-p)r^{n_2}(r_1^{n_1}r_2^{n_2}\cos n_1\theta_1 - p \cos n_2\theta_2)}{r_1^{2n_1}r_2^{2n_2} - 2pr_1^{n_1}r_2^{n_2}\cos(n_1\theta_1 + n_2\theta_2) + p^2} \qquad (5.12)$$

$$AR = \frac{(1-p)r_2^{\,n_2}}{[r_1^{2n_1}r_2^{2n_2} + p^2 - 2pr_1^{n_1}r_2^{n_2}\cos(n_1\theta_2 + n_2\theta_2)]^{\frac{1}{2}}}$$

$$\emptyset = -\tan^{-1}\left[\frac{r_1^{n_1}r_2^{n_2}\sin n_1\theta_1 + p \sin n_2\theta_2}{r_1^{n_1}r_2^{n_2}\cos n_1\theta_1 - p \sin n_2\theta_2}\right]$$

The above equations reduce to those for plug flow in both loops, by taking the limit as n_1 and $n_2 \longrightarrow \infty$.

Then $\quad r_1 = r_2 = 1$

$\qquad\ \ \theta_1 = \theta_2 = 0$

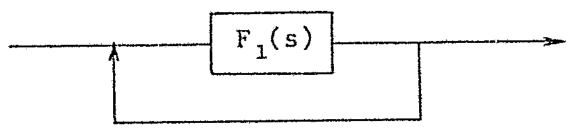
$$\lim_{n_1 \to \infty} n_1\theta_1 = \lim_{n_1 \to \infty} \frac{\tan^{-1}(\tau_1 w/n_1)}{1/n_1}$$

$$= w\tau_1 \quad \text{(by application of L'Hospital's rule)}$$

Similarly, $n_2\theta_2 = w\tau_2$

Further useful special cases can be derived, e.g.

Instantaneous recycle:



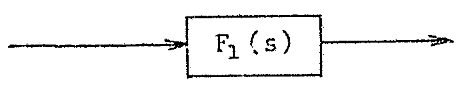
$$\tau_2 = 0$$

Therefore $r_2 = 1$ and $\theta_2 = 0$

$$a_n = \frac{1}{T} \frac{(1-p)r_1^{n_1} \sin n_1\theta_1}{[r_1^{2n_1} - 2pr_1^{n_1}\cos n_1\theta_1 + p^2]}$$

$$b_n = \frac{1}{T} \frac{(1-p)(r^{n_1}\cos n_1\theta_1 - p)}{[r_1^{2n_1} - 2pr_1^{n_1}\cos n_1\theta_1 + p^2]}$$

Simple gamma distribution:



$\tau_2 = 0, \quad p = 0$

$$a_n = (1/T)r_1^{-n_1}\sin n_1\theta_1$$
$$b_n = (1/T)r_1^{-n_1}\cos n_1\theta_1$$

VIII.3. DISPERSION MODEL IN BOTH LOOPS

The equation for axial dispersion with flow (25) is

$$D \frac{\partial^2 c}{\partial x^2} - u \frac{\partial c}{\partial x} = \frac{\partial c}{\partial t}$$

where D = dispersion coefficient
u = velocity
c = concentration
x = distance along flow path
t = time

Turner (7) has transformed this equation to give the transfer function

$$F(s) = \exp \left[\frac{uL}{2D} - \sqrt{\frac{u^2L^2}{4D^2} + \frac{sL^2}{D}} \right]$$

The mean and variance can be found by the usual methods:

$$\text{Mean } \tau = L/u$$

$$\text{Variance } \sigma^2 = \frac{2DL}{u^3} = \frac{2D\tau}{u^2}$$

The half loop transfer functions are expressed in terms of τ and σ^2, to allow easy comparison with the other models:

$$F_1(s) = \exp \left[\frac{\tau_1^2}{\sigma_1^2} - \sqrt{\frac{\tau_1^4}{\sigma_1^4} + \frac{2\tau_1^3 s}{\sigma_1^2}} \right]$$

$$F_2(s) = \exp \left[\frac{\tau_2^2}{\sigma_2^2} - \sqrt{\frac{\tau_2^4}{\sigma_2^4} + \frac{2\tau_2^3 s}{\sigma_2^2}} \right]$$

where $\tau_2 = \dfrac{(1-p)\,\tau - \tau_1}{p}$

The independent variables are p, $\tau_1, \sigma_1^2, \sigma_2^2$

Substituting $F_1(s)$ and $F_2(s)$ in equation (3.15),

$$F(s) = \frac{(1-p)\exp\left[\frac{\tau_1^2}{\sigma_1^2} - \sqrt{\frac{\tau_1^4}{\sigma_1^4} + \frac{2\tau_1^3 s}{\sigma_1^2}}\right]}{1 - p\exp\left[\frac{\tau_1^2}{\sigma_1^2} + \frac{\tau_2^2}{\sigma_2^2} - \sqrt{\frac{\tau_1^4}{\sigma_1^4} + \frac{2\tau_1^3 s}{\sigma_1^2}} - \sqrt{\frac{\tau_2^4}{\sigma_2^4} + \frac{2\tau_2^3 s}{\sigma_2^2}}\right]}$$

$$F(iw) = \frac{(1-p)\exp[A_1 - (A_1^2 + iB_1)^{1/2}]}{1 - p\exp[A_1 + A_2 - (A_1^2 + iB_1)^{1/2} - (A_2^2 + iB_2)^{1/2}]}$$

where $A_1 = \tau_1^2/\sigma_1^2$ $\quad B_1 = 2\tau_1^3 w/\sigma_1^2$

$A_2 = \tau_2^2/\sigma_2^2$ $\quad B_2 = 2\tau_2^3 w/\sigma_2^2$

$$F(iw) = \frac{(1-p)\exp[A_1 - r_1\exp(i\theta_1)]}{1 - p\exp[A_1 + A_2 - r_1\exp(i\theta_1) - r_2\exp(i\theta_2)]}$$

where $r_1 = (A_1^4 + B_1^2)^{1/4}$ $\quad \theta_1 = 0{,}5\tan^{-1}(B_1/A_1^2)$

$r_2 = (A_2^4 + B_2^2)^{1/4}$ $\quad \theta_2 = 0{,}5\tan^{-1}(B_2/A_2^2)$

$$F(iw) = \frac{(1-p)\exp(A_1 - r_1\cos\theta_1 - ir_1\sin\theta_1)}{1 - p\exp[A_1 + A_2 + r_1\cos\theta_1 - r_2\cos\theta_2 - i(r_1\sin\theta_1 + r_2\sin\theta_2)]}$$

$$= \frac{(1-p)\exp(A_1 - r_1\cos\theta_1)[\cos(r_1\sin\theta_1) - i\sin(r_1\sin\theta_1)]}{[1 - p\exp(A_1 + A_2 - r_1\cos\theta_1 - r_2\cos\theta_2)\cos(r_1\sin\theta_1 + r_2\sin\theta_2)] + i[p\exp(A_1 + A_2 - r_1\cos\theta_1 - r_2\cos\theta_2)\sin(r_1\sin\theta_1 + r_2\sin\theta_2)]}$$

Multiplying by the complex conjugate of the denominator, and writing

$\alpha_1 = r_1\cos\theta_1$ $\quad \beta_1 = r_1\sin\theta_1$

$\alpha_2 = r_2\cos\theta_2$ $\quad \beta_2 = r_2\sin\theta_2$

$$F(iw) = \frac{(1-p)\exp(A_1-\alpha_1)\,(\cos\beta_1 - i\sin\beta_1)[1 - p\exp(A_1 + A_2 - \alpha_1 - \alpha_2)\cos(\beta_1+\beta_2) - ip\exp(A_1 + A_2 - \alpha_1 - \alpha_2)\sin(\beta_1+\beta_2)]}{1 - 2p\exp(A_1 + A_2 - \alpha_1 - \alpha_2)\cos(\beta_1+\beta_2) + p^2\exp[2(A_1 + A_2 - \alpha_1 - \alpha_2)]}$$

Now let $a_1 = A_1 - \alpha_1$

$a_2 = A_2 - \alpha_2$

Multiplying out the numerator gives

$$F(iw) = \frac{(1-p)\exp a_1(\cos\beta_1 - p\exp(a_1 + a_2)\cos\beta_2 - i\sin\beta_1 - ip\exp(a_1+a_2)\sin\beta_2)}{1 - 2p\exp(a_1 + a_2)\cos(\beta_1+\beta_2) + p^2\exp(2a_1 + 2a_2)} \tag{5.14}$$

$$a_n = \frac{1}{T}\;\frac{(1-p)\exp a_1\,[\sin\beta_1 + p\exp(a_1 + a_2)\sin\beta_2]}{1 - 2p\exp(a_1 + a_2)\cos(\beta_1 + \beta_2) + p^2\exp(2a_1 + 2a_2)} \tag{5.15}$$

$$b_n = \frac{1}{T}\;\frac{(1-p)\exp a_1\,[\cos\beta_1 - p\exp(a_1 + a_2)\cos\beta_2]}{1 - 2p\exp(a_1 + a_2)\cos(\beta_1 + \beta_2) + p^2\exp(2a_1 + 2a_2)} \tag{5.16}$$

$$AR = \frac{(1-p)\exp a_1}{[1 - 2p\exp(a_1 + a_2)\cos(\beta_1 + \beta_2) + p^2\exp(2a_1 + 2a_2)]}$$

$$\phi = -\tan^{-1}\left[\frac{\sin\beta_1 + p\exp(a_1 + a_2)\sin\beta_2}{\cos\beta_1 - p\exp(a_1 + a_2)\cos\beta_2}\right]$$

APPENDIX IX. DERIVATION OF THE FOURIER COEFFICIENT EQUATIONS FOR THE BYPASS MODEL

For the bypass model

$$F(s) = (1-p)F_1(s) + pF_1(s)F_2(s) \qquad (5.3)$$

For a series of stirred tanks (gamma distribution) in region 1,

$$F_1(s) = \frac{1}{(\tau_1 s/n_1 + 1)^{n_1}}$$

Region 2 is a CSTR:

$$F_2(s) = \frac{1}{(\tau_2 s + 1)}$$

where $\tau_2 = \dfrac{\tau - \tau_1}{p}$

There are 3 independent variables: p, τ_1, σ_1^2

Therefore

$$F(s) = \frac{1-p}{(\tau_1 s/n_1 + 1)^{n_1}} + \frac{p}{(\tau_1 s/n_1 + 1)^{n_1}(\tau_2 s + 1)}$$

$$F(iw) = \frac{(1-p)(\tau_2 wi + 1) + p}{(\tau_1 wi/n_1 + 1)^{n_1}(\tau_2 wi + 1)}$$

$$= \frac{[(1-p)\tau_2 wi + 1](1 - \tau_1 wi/n_1)^{n_1}(1 - \tau_2 wi)}{(1 + \tau_1^2 w^2/n_1^2)^{n_1}(1 + \tau_2^2 w^2)}$$

Let $\quad r_1 = \tau_1^2 w^2/n_1^2 + 1 \qquad \theta_1 = \tan^{-1}(\tau_1 w/n_1)$

$$F(iw) = \frac{[1 + (1-p)\tau_2^2 w^2 - ip\tau_2 w] r_1^{n1/2} (\cos n_1\theta_1 - i \sin n_1\theta_1)}{r_1^{n1}(1 + \tau_2^2 w^2)}$$

$$F(iw) = \frac{(1 + (1-p)\tau_2^2 w^2)\cos n_1\theta_1 - p\tau_2 w \sin n_1\theta_1 - i[(1 + (1-p)\tau_2^2 w^2)\sin n_1\theta_1 + p\tau_2 w \cos n_1\theta_1]}{r_1^{n1/2}(1 + \tau_2^2 w^2)} \qquad (5.18)$$

$$a_n = \frac{1}{T} \frac{[1 + (1-p)\tau_2^2 w^2]\sin n_1\theta_1 + p\tau_2 w \cos n_1\theta_1}{r_1^{n1/2}(1 + \tau_2^2 w^2)} \qquad (5.19)$$

$$b_n = \frac{1}{T} \frac{[1 + (1-p)\tau_2^2 w^2]\cos n_1\theta_1 - p\tau_2 w \sin n_1\theta_1}{r_1^{n1/2}(1 + \tau_2^2 w^2)} \qquad (5.20)$$

$$AR = \frac{\sqrt{p\tau_2^2 w^2 + [1 + (1-p)\tau_2^2 w^2]^2}}{r_1^{n1/2}(1 + \tau_2^2 w^2)}$$

$$\phi = -\tan^{-1} \frac{[1 + (1-p)\tau_2^2 w^2]\sin n_1\theta_1 + p\tau_2 w \cos n_1\theta_1}{[1 + (1-p)\tau_2^2 w^2]\cos n_1\theta_1 - p\tau_2 w \sin n_1\theta_1}$$

APPENDIX X. SIGNIFICANCE OF THE SHAPE OF THE AMPLITUDE RATIO PLOT OF THE LONG REACTOR

The amplitude ratio and phase lag plots of the long vessel (appendix 4) all have a similar characteristic shape, with a hump at about n=15 (the natural frequency). In some cases, further maxima at higher frequencies (harmonics) can be discerned. Typical plots are reproduced below.

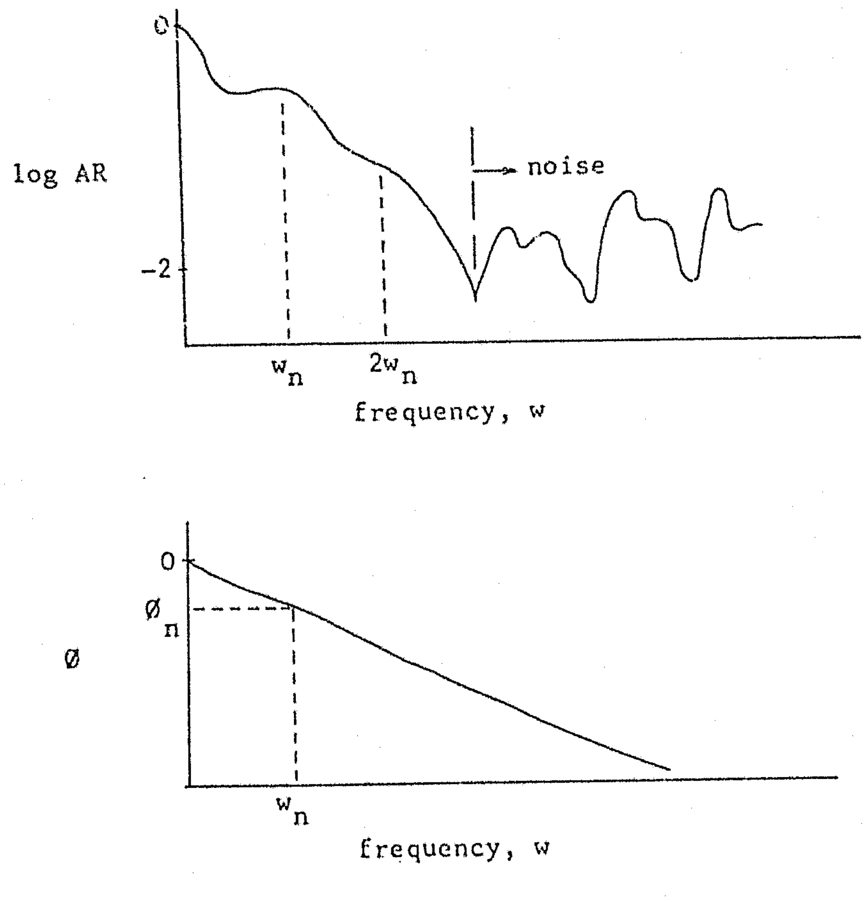


FIGURE X.1. SHAPE OF EXPERIMENTAL AR AND Ø PLOTS OF THE LONG VESSEL

The most satisfactory explanation for the occurence of peaks in the AR plot is recycling of material in the reactor. This also accounts for the oscillatory nature of the RTD curve. In order to investigate the significance of these peaks, we shall study the AR and Ø plots of the "recycle model" developed in section 5.

We first investigate the shape of these curves for the case of plug flow in both loops of the recycle model.

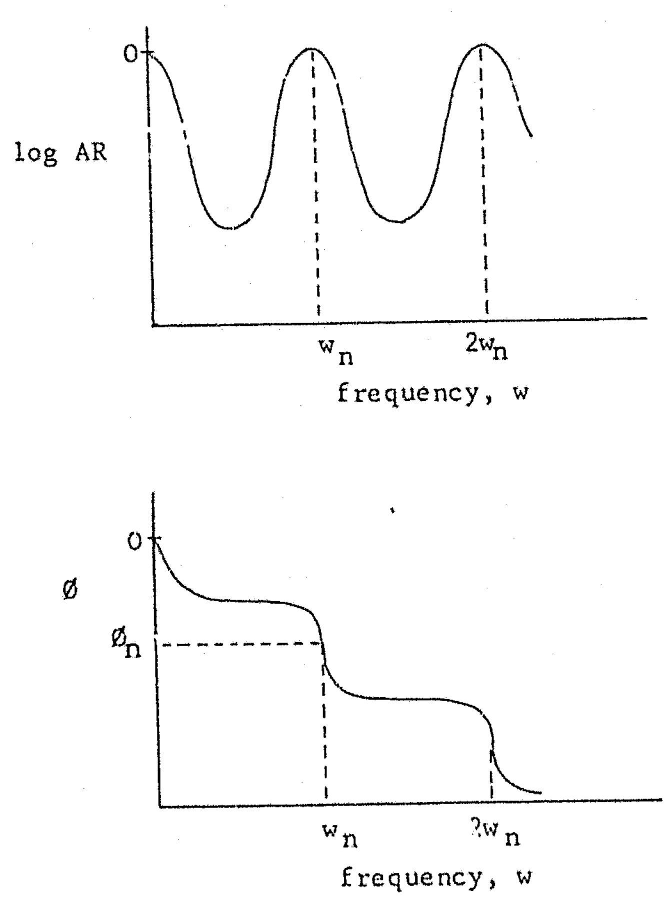


FIGURE X.2. SHAPE OF AR AND Ø PLOTS OF THE RECYCLE MODEL WITH PLUG FLOW IN BOTH LOOPS .

The natural frequency (w_n) can be determined by differentiating the equation for the AR wrt frequency. From appendix VIII.1, with both regions of the model plug flow,

$$AR = (1-p)\left[1 + p^2 - 2p\cos w(\tau_1 + \tau_2)\right]^{-\frac{1}{2}} \quad (X.1)$$

$$\frac{d\,AR}{dw} = -\,p(1-p)(\tau_1+\tau_2)\left[1 + p^2 - 2p\cos w(\tau_1+\tau_2)\right]^{-3/2}\sin w(\tau_1+\tau_2) \quad (X.2)$$

For a maximum (or minimum) $\frac{d\,AR}{dw} = 0$, therefore

either $\quad 1 + p^2 - 2p\cos w(\tau_1 + \tau_2) = 0 \quad$ (a)

or $\quad \sin w(\tau_1 + \tau_2) = 0 \quad$ (b)

Solution (a) is not possible, as

$1 + p^2 > 2p \quad$ for $0 < p < 1$

and $\quad -1 \leq \cos w(\tau_1 + \tau_2) \leq 1$

Therefore $\quad 1 + p^2 > 2p\cos w(\tau_1 + \tau_2)$

$1 + p^2 - 2p\cos w(\tau_1 + \tau_2) \neq 0 \quad$ for $0 < p < 1$

The stationary points are therefore given by (b):

$$w = \frac{n\pi}{\tau_1 + \tau_2} \qquad n = 0,1,2,\ldots$$

The nature of these points is given by the second derivative:

$$\frac{d^2AR}{dw^2} = -\,p(1-p)(\tau_1+\tau_2)^2\Big[\{1 + p^2 - 2p\cos w(\tau_1+\tau_2)\}^{-3/2}\cos w(\tau_1+\tau_2) - 3p\{1 + p^2 - 2p\cos w(\tau_1+\tau_2)\}^{-5/2}\sin^2 w(\tau_1+\tau_2)\Big]$$

Substituting $w = \frac{n\pi}{\tau_1 + \tau_2}$ into the above,

$$\left.\frac{d^2AR}{dw^2}\right|_{w=\frac{n\pi}{\tau_1+\tau_2}} = \begin{cases} -p(1-p)(\tau_1+\tau_2)^2(1+p^2-2p)^{-3/2} & \text{for n even} \\ p(1-p)(\tau_1+\tau_2)^2(1+p^2+2p)^{-3/2} & \text{for n odd} \end{cases}$$

$$= \begin{cases} \dfrac{-p(\tau_1+\tau_2)^2}{(1-p)^2} < 0 & \text{for n even} \\ \dfrac{p(1-p)(\tau_1+\tau_2)^2}{(1+p)^3} > 0 & \text{for n odd} \end{cases}$$

Therefore maxima occur at $w = n\pi/(\tau_1+\tau_2)$ with $n = 0,2,4\ldots$
and minima occur at $w = n\pi/(\tau_1+\tau_2)$ with $n = 1,3,5\ldots$ (X.4)

From equation (X.1): AR(max) = 1, AR(min) = (1-p)/(1+p)

The natural frequency is thus $w_n = 2\pi/(\tau_1+\tau_2)$ (the first peak in the AR plot) and further peaks are harmonics of the natural frequency.

Furthermore, the phase angle at the natural frequency is given by

$$\phi_n = -\tan^{-1}\left[\frac{\sin w_n\tau_1 + p\sin w_n\tau_2}{\cos w_n\tau_1 - p\cos w_n\tau_2}\right]$$

$$= -\tan^{-1}\left[\frac{\sin\dfrac{2\pi\tau_1}{\tau_1+\tau_2} + p\sin\left(2\pi - \dfrac{2\pi\tau_1}{\tau_1+\tau_2}\right)}{\cos\dfrac{2\pi\tau_1}{\tau_1+\tau_2} - p\cos\left(2\pi - \dfrac{2\pi\tau_1}{\tau_1+\tau_2}\right)}\right]$$

$$= -\tan^{-1}\left[\frac{\sin w_n\tau_1 - p\sin w_n\tau_1}{\cos w_n\tau_1 - p\cos w_n\tau_1}\right]$$

$$-\tan^{-1}\left[\frac{\sin w_n\tau_1}{\cos w_n\tau_1}\right]$$

$$-w_n\tau_1 \qquad \text{(X.5)}$$

The above expressions for w_n and $\emptyset_n$ can also easily be deduced by physical reasoning, as follows.

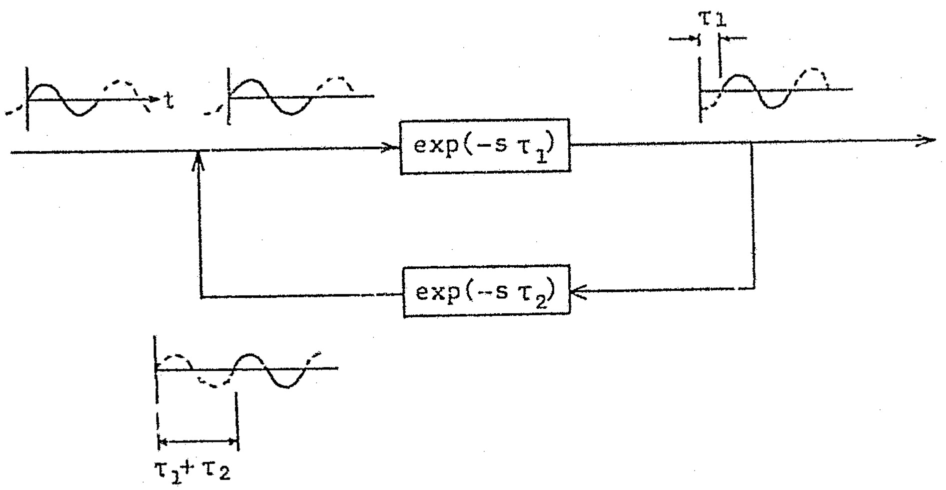


FIGURE X.3. FREQUENCY RESPONSE AT THE NATURAL FREQUENCY

For the frequency response to have maximum amplitude, the peaks of the input and recycled sine waves must coincide, as in figure X.3, i.e. the period must be $\tau_1 + \tau_2$ and frequency $2\pi/(\tau_1+\tau_2)$. It therefore follows that the phase lag at the exit is attributable only to region 1 and is given by $w_n \tau_1$.

Seen differently, $\emptyset_1 + \emptyset_2 = 2\pi$

$$w_n(\tau_1+\tau_2) = 2\pi$$

$$w_n = 2\pi/(\tau_1 + \tau_2)$$

The 2 model parameters, τ_1 and τ_2 $[p = (\tau_1-\tau_2)/(\tau_1+\tau_2)]$ are thus easily estimated from equations (X.4) and (X.5).

Although the above expressions for w_n and $\emptyset_n$ are valid only for plug flow in the loops, they can be used to obtain first estimates of two of the model parameters, τ_1 and τ_2 (or p) for the recycle model with more complex distributions in each loop. For the case of a tanks-in-series model (gamma distribution) in each loop, the phase angle at the natural frequency is also only dependent on region 1 and is given by

$$\phi_n = - n_1 \tan^{-1}(w_n \tau_1/n_1)$$

w_n is found from

$$n_1 \tan^{-1}(w_n \tau_1/n_1) + n_2 \tan^{-1}(w_n \tau_2/n_2) = 2\pi$$

For large n_1,

$$\tan^{-1}(w_n \tau_1/n_1) \cong w_n \tau_1/n_1$$

and similarly for n_2

Then the expressions for ϕ_n and w_n become

$$\phi_n \cong - w_n \tau_1$$
$$w_n \cong 2\pi/(\tau_1 + \tau_2)$$

The more well-mixed are the regions in the two loops of the recycle model, the less well-defined are the peaks in the AR plot and the more the phase lag plot approaches a straight line.

APPENDIX XI. COMPUTER PROGRAMS

XI.1: Program for taking RTD readings.

XI.2: Program for averaging a number of RTD curves.

XI.3: Program for normalisation, exponential tail fitting and plotting of RTD curves.

XI.4: Program for calculating Fourier coefficients, amplitude ratio, phase lag and the Fourier series for the experimental curve.

Programs for least-squares fitting of mathematical models to the experimental Fourier coefficients, producing the model parameters:

XI.5: Recycle model with gamma distributions in both loops.

XI.6: Recycle model with axial dispersion in both loops

XI.7: "Bypass model"

Programs for calculating model parameters as functions of frequency:

XI.8: Recycle model with gamma distributions in both loops.

(Q-MIX (XI.8) is a modification of P-MIX (XI.5). The programs of appendices XI.6 and XI.7 can similarly be modified for calculating parameters as functions of frequency).

APPENDIX XI.1

```
10   !      PROGRAM: READ
20   !
30   !      THIS PROGRAM RECORDS THE RTD CURVE AND STORES IT ON TAPE;
40   !      FLOWRATES,Re NUMBER,THEORETICAL RESIDENCE TIME,DELAY TIMES,ETC ARE CALCULATED     !

50   !      MAIN PROGRAM
60   !      ------------
70   !
80   OPTION BASE 1
90   COM Y(400),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
100  DIM Q$[1]
110  PRINTER IS 16
120  PRINT LIN(1),TAB(30);CHR$(27);"&LD";"PROGRAM: READ";CHR$(27);"&d@"
130  INPUT "ARE A NEW SET OF RTD READINGS TO BE TAKEN? Y/N",Q$
140  IF Q$="N" THEN GOTO Rec
150  CALL Readings
160  INPUT "MUST THE NEW SET OF RTD READINGS BE STORED ON TAPE? Y/N",Q$
170  IF Q$="Y" THEN CALL Rtd_store
180  GOTO P1
190 Rec: INPUT "MUST A PREVIOUS SET OF READINGS BE RECALLED FROM TAPE? Y/N",Q$
200  IF Q$="Y" THEN CALL Rtd_retrieve
210  IF Nor()0 THEN PRINT "ERROR: Nor=";Nor
220  !
230  !      INPUT OF VARIABLES AND PRELIMINARY CALCULATIONS FOLLOW
240 P1: PRINT LIN(1),"PRELIMINARY CALCULATIONS BEGIN NOW"
250  INPUT "MUST )Time1( BE CHANGED BECAUSE OF INACC TRIGGERING? Y/N",Q$
260  IF Q$="Y" THEN INPUT "INPUT:Time1",Time1
270  INPUT "MUST )Interval( BE CORRECTED? Y/N",Q$
280  IF Q$="Y" THEN INPUT "INPUT:Interval",Interval
290  INPUT "ARE PRELIMINARY CALCS REQUD? Y/N",Q$
300  IF Q$="N" THEN Nocalc
310  INPUT "INPUT: MODEL SIZE S/L,NOZZLE DIA [mm],INJEC POINT [mm]",Mod$,Dia_noz,L_noz
320  INPUT "INPUT: BAR.PRESS [mmHg]",P_bar
330  INPUT "INPUT: INDIC ORIFICE FLOW [Nm3/h],TEMP [C],PRESS [cmH2O]",Vn_or,T_or,P_or
340  INPUT "INPUT: INDIC ROTAM FLOW [l/min],TEMP [C],PRESS [mmHg]",V_rot,T_rot,P_rot
350  INPUT "INPUT: LENGTH OF SAMPLE PIPE [mm]",L_samp
360  Dia_samp=.01
370  IF Mod$="S" THEN Dia=.449        ![m]
380  IF Mod$="L" THEN Dia=.471        ![m]
390  IF Mod$="S" THEN Vol=.1043       ![m3]
400  IF Mod$="L" THEN Vol=.2879       ![m3]
410  !
420  !      CONVERSION TO SI UNITS
430  Dia_noz=Dia_noz/1000             ![m]
440  L_noz=L_noz/1000                 ![m]
450  P_bar=P_bar*.1333                ![kPa]
460  Vn_or=Vn_or/3600                 ![m3/s]
470  P_or=P_or*.09807                 ![kPa]
480  V_rot=V_rot*1.667E-5             ![m3/s]
490  P_rot=P_rot*.1333                ![kPa]
500  L_samp=L_samp/1000               ![m]
510  !
520  !      AIR FLOWRATE
530  Pa_or=P_or+P_bar
540  Vn_or=Vn_or*SQR(Pa_or/88*(293.15/(273.15+T_or)))     ![Nm3/s]
550  Pa_mod=P_bar+.18*P_or
560  V_mod=Vn_or*(101.325/83.5)*((273.15+T_rot)/273.15)  ![m3/s] (ONE SIDE)
570  V=2*V_mod                        ![m3/s]
580  !
590  !      REYNOLDS NUMBER AND THEORETICAL MEAN RES TIME
600  Dens_mod=Pa_mod*1000/(287*(273.15+T_rot))
610  Visc_mod=1.075E-6*SQR(T_rot+273.15)
620  Re_ave=4*V_mod*Dens_mod/(PI*Visc_mod*Dia)      !ONE SIDE
630  Mean_th=Vol/V
640  !
650  !      INJECTION DELAY TIME
660  IF Dia_noz=.076 THEN Area_noz=.00867     ![m2]
670  Vel_noz=V_mod/Area_noz           ![m/s]
680  Time_noz=L_noz/Vel_noz           ![s]
690  !
```

```
700  !     SAMPLING DELAY TIME AND FLOWCELL RES TIME
710  Pa_rot=P_bar-P_rot
720  Pa_cell=Pa_mod-.76*(Pa_mod-Pa_rot)
730  Pa_samp=(Pa_mod+Pa_cell)/2
740  Vn_rot=V_rot*SQR(Pa_rot/101.325*(288.15/(273.15+T_rot)))*(273.15/288.15)    ![Nm3/s]
750  V_cell=Vn_rot*(101.325/Pa_cell)*((T_rot+273.15)/273.15)    ![m3/s]
760  V_samp=Vn_rot*(101.325/Pa_samp)*((T_rot+273.15)/273.15)    ![m3/s]
770  Vel_samp=V_samp*4/(PI*Dia_samp^2)    ![m/s]
780  Time_samp=L_samp/Vel_samp       ![s]
790  Time_cell=5.63E-5/V_cell        ![s]
800  !
810  !     TOTAL DELAY TIME CORRECTION
820  Timedelay=Time_noz+Time_samp
830  Timei=Timei-Timedelay
840  !
850  !     PRINTOUT OF PRELIMINARY RESULTS
860  PRINTER IS 7,1,WIDTH(130)
870  FIXED 4
880  PRINT PAGE
890  PRINT TAB(40);CHR$(27);"&dD";"PROGRAM: RTD";CHR$(27);"&d@"
900  PRINT LIN(2),TAB(3),CHR$(27);"&dD";"ANALYSIS OF RESIDENCE TIME DISTRIBUTION IN A HIGH SPEED GA
S REACTION VESSEL";CHR$(27);"&d@"
910  PRINT LIN(3),,,CHR$(27);"&dD";"MODEL GEOMETRY";CHR$(27);"&d@"
920  PRINT LIN(1),"MODEL VOLUME=",Vol;"[M3]",,"INJECTION DISTANCE=",L_noz;"[m]"
930  PRINT "MODEL AVE DIA=",Dia;"[m]",,"SAMP PIPE LENGTH=",L_samp;"[m]"
940  PRINT "NOZZLE DIA=",Dia_noz;"[m]",,"SAMP PIPE DIA=",Dia_samp;"[m]"
950  PRINT LIN(2),,,CHR$(27);"&dD";"PHYSICAL DATA AND FLOWRATES";CHR$(27);"&d@"
960  PRINT LIN(1),"ORIFICE AIR TEMP=",T_or;"[C]",,"ROTAM AIR TEMP=",T_rot;"[C]"
970  PRINT "                PRESS=",Pa_or;"[kPa]",,"           PRESS=",Pa_rot;"[kPa]"
980  PRINT "                FLOW=",Vn_or*3600;"[Nm3/h]",,"             FLOW=",Vn_rot*3600;"[Nm3/h]"
990  PRINT LIN(1),"MODEL AIR TEMP=",T_rot;"[C]",,"SAMP PIPE PRESS=",Pa_samp;"[kPa]"
1000 PRINT "              PRESS=",Pa_mod;"[kPa]",,"           FLOW=",V_samp*3600;"[m3/h]"
1010 PRINT "              DENS=",Dens_mod;"[kg/m3]",,"           VELOCITY=",Vel_samp;"[m/s]"
1020 PRINT "              VISC=",Visc_mod*1E6;"[10^6kg/ms]",,"DELAY TIME IN SAMP PIPE=";Time_samp;"[s]"
1030 PRINT "              FLOW=",V*3600;"[m3/h](total)"
1040 PRINT "NOZZLE VELOCITY=",Vel_noz;"[m/s]",,"FLOW CELL PRESS=",Pa_cell;"[kPa]"
1050 PRINT "NOZZLE DELAY TIME=",Time_noz;"[s]",,"           FLOW=",V_cell*3600;"[m3/h]"
1060 PRINT ,,,"               RES TIME=",Time_cell;"[s]"
1070 PRINT LIN(2),"MODEL REYNOLDS NUMBER,Re=";Re_ave;" (based on one side)"
1080 PRINT "THEORETICAL MEAN RES TIME=";Mean_th;"[s]"
1090 PRINT "SUM OF DELAY TIMES=";Timedelay;"[s]"
1100 PRINT "TIME OF FIRST READING CORRECTED TO";Timei;"[s]"
1110 STANDARD
1120 PRINTER IS 16
1130 !
1140 !
1150 Nocalc:INPUT "MUST THE NEW SET OF RTD READINGS BE STORED ON TAPE? Y/N",Q$
1160 IF Q$="Y" THEN CALL Rtd_store
1170 STOP    !


1180 !      SUBROUTINE: Readings
1190 !      --------------------
1200 !
1210 !      THIS SUBROUTINE TAKES THE EXPERIMENTAL READINGS FOR THE RTD CURVE
1220 !      IN RADIATION COUNTS/SEC, USING THE HP2240A PROCESSOR
1230 SUB Readings
1240 OPTION BASE 1
1250 COM Rcoi(*),INTEGER Number,REAL Interval,Timei,Mode$[1],Nor
1260 DIM Rco(400),R(400)
1270 INTEGER Cond(5),Sec,Msec,Co(400),Dco(400)
1280 PRINTER IS 16
1290 Proc=702
1300 Dt=1.85E-6   !COUNTER DEAD TIME
1310 OUTPUT Proc USING "K";"SN!"
1320 ENTER Proc;Cond(1)
1330 IF Cond(1)<>0 THEN GOSUB Status
1340 INPUT "TOT OR FREQ MODE? T/F",Mode$
1350 IF Mode$="F" THEN GOTO Freq
1360 !
1370 !      TOTALISE MODE READINGS - ONE BATCH
1380 Tot:Number=230        !245 MAX
1390 ! Interval=20 msec APPROX
1400 REDIM Co(Number),Dco(Number),Rco(Number),Rcoi(Number)
1410 !
1420 !      A TRIGGER FOR PRESETTING THE CLOCK IS SET UP USING THE COUNTER/STEPPER CARD
1430 ON INT #7,15 GOTO T
```

```
700  !     SAMPLING DELAY TIME AND FLOWCELL RES TIME
710  Pa_rot=P_bar-P_rot
720  Pa_cell=Pa_mod-.76*(Pa_mod-Pa_rot)
730  Pa_samp=(Pa_mod+Pa_cell)/2
740  Vn_rot=V_rot*SQR(Pa_rot/101.325*(288.15/(273.15+T_rot)))*(273.15/288.15)   ![Nm3/s]
750  V_cell=Vn_rot*(101.325/Pa_cell)*((T_rot+273.15)/273.15)   ![m3/s]
760  V_samp=Vn_rot*(101.325/Pa_samp)*((T_rot+273.15)/273.15)   ![m3/s]
770  Vel_samp=V_samp*4/(PI*Dia_samp^2)    ![m/s]
780  Time_samp=L_samp/Vel_samp       ![s]
790  Time_cell=5.63E-5/V_cell        ![s]
800  !
810  !     TOTAL DELAY TIME CORRECTION
820  Timedelay=Time_noz+Time_samp
830  Time1=Time1-Timedelay
840  !
850  !     PRINTOUT OF PRELIMINARY RESULTS
860  PRINTER IS 7,1,WIDTH(130)
870  FIXED 4
880  PRINT PAGE
890  PRINT TAB(40);CHR$(27);"&dD";"PROGRAM: RTD";CHR$(27);"&d@"
900  PRINT LIN(2),TAB(8),CHR$(27);"&dD";"ANALYSIS OF RESIDENCE TIME DISTRIBUTION IN A HIGH SPEED GA
S REACTION VESSEL";CHR$(27);"&d@"
910  PRINT LIN(3),,,CHR$(27);"&dD";"MODEL GEOMETRY";CHR$(27);"&d@"
920  PRINT LIN(1),"MODEL VOLUME=",Vol;"[M3]",,"INJECTION DISTANCE=",L_noz;"[m]"
930  PRINT "MODEL AVE DIA=",Dia;"[m]",,"SAMP PIPE LENGTH=",L_samp;"[m]"
940  PRINT "NOZZLE DIA=",Dia_noz;"[m]",,"SAMP PIPE DIA=",Dia_samp;"[m]"
950  PRINT LIN(2),,,CHR$(27);"&dD";"PHYSICAL DATA AND FLOWRATES";CHR$(27);"&d@"
960  PRINT LIN(1),"ORIFICE AIR TEMP=",T_or;"[C]",,"ROTAM AIR TEMP=",T_rot;"[C]"
970  PRINT "              PRESS=",Pa_or;"[kPa]",,"             PRESS=",Pa_rot;"[kPa]"
980  PRINT "              FLOW=",Vn_or*3600;"[Nm3/h]",,"             FLOW=",Vn_rot*3600;"[Nm3/h]"
990  PRINT LIN(1),"MODEL AIR TEMP=",T_rot;"[C]",,"SAMP PIPE PRESS=",Pa_samp;"[kPa]"
1000 PRINT "           PRESS=",Pa_mod;"[kPa]",,"           FLOW=",V_samp*3600;"[m3/h]"
1010 PRINT "           DENS=",Dens_mod;"[kg/m3]",,"           VELOCITY=",Vel_samp;"[m/s]"
1020 PRINT "           VISC=",Visc_mod*1E6;"[10^6kg/ms]",,"DELAY TIME IN SAMP PIPE=";Time_samp;"[s]"
1030 PRINT "           FLOW=",V*3600;"[m3/h](total)"
1040 PRINT "NOZZLE VELOCITY=",Vel_noz;"[m/s]",,"FLOW CELL PRESS=",Pa_cell;"[kPa]"
1050 PRINT "NOZZLE DELAY TIME=",Time_noz;"[s]",,"           FLOW=",V_cell*3600;"[m3/h]"
1060 PRINT ,,,"           RES TIME=",Time_cell;"[s]"
1070 PRINT LIN(2),"MODEL REYNOLDS NUMBER,Re=";Re_ave;" (based on one side)"
1080 PRINT "THEORETICAL MEAN RES TIME=";Mean_th;"[s]"
1090 PRINT "SUM OF DELAY TIMES=";Timedelay;"[s]"
1100 PRINT "TIME OF FIRST READING CORRECTED TO";Time1;"[s]"
1110 STANDARD
1120 PRINTER IS 16
1130 !
1140 !
1150 Nocalc:INPUT "MUST THE NEW SET OF RTD READINGS BE STORED ON TAPE? Y/N",Q$
1160 IF Q$="Y" THEN CALL Rtd_store
1170 STOP    !



1180 !      SUBROUTINE: Readings
1190 !      --------------------
1200 !
1210 !      THIS SUBROUTINE TAKES THE EXPERIMENTAL READINGS FOR THE RTD CURVE
1220 !      IN RADIATION COUNTS/SEC, USING THE HP2240A PROCESSOR
1230 SUB Readings
1240 OPTION BASE 1
1250 COM Rco1(*),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor
1260 DIM Rco(400),R(400)
1270 INTEGER Cond(5),Sec,Msec,Co(400),Dco(400)
1280 PRINTER IS 16
1290 Proc=702
1300 Dt=1.85E-6  !COUNTER DEAD TIME
1310 OUTPUT Proc USING "K";"SN!"
1320 ENTER Proc;Cond(1)
1330 IF Cond(1)<>0 THEN GOSUB Status
1340 INPUT "TOT OR FREQ MODE? T/F",Mode$
1350 IF Mode$="F" THEN GOTO Freq
1360 !
1370 !      TOTALISE MODE READINGS - ONE BATCH
1380 Tot:Number=230       !245 MAX
1390 ! Interval=20 msec APPROX
1400 REDIM Co(Number),Dco(Number),Rco(Number),Rco1(Number)
1410 !
1420 !      A TRIGGER FOR PRESETTING THE CLOCK IS SET UP USING THE COUNTER/STEPPER CARD
1430 ON INT #7,15 GOTO T
```

```
1440 CONTROL MASK 7;128
1450 CARD ENABLE 7
1460 !
1470 OUTPUT Proc USING "K";"SN;SM,0,1;ST,1,1,1,1!"
1480 ENTER Proc;Cond(2)
1490 IF Cond(2)<>0 THEN GOSUB Status
1500 PRINT "SYSTEM READY FOR TRIGGER"
1510 CRT OFF
1520 W:GOTO W
1530 T:OFF INT #7
1540 OUTPUT 702 BFHS  NOFORMAT;"TP,0,0;ST,1,2,1,0;RP,230;WN,18;RC,1,2,1;NX;TE!"
1550 ENTER 702;Cond(3),Co(*),Sec,Msec
1560 CRT ON
1570 PRINT "TRIGGER ACTIVATED"
1580 PRINT "READINGS COMPLETE"
1590 IF Cond(3)<>0 THEN GOSUB Status
1600 Elap_time=Sec+Msec/1000     !SECONDS
1610 Interval=Elap_time/Number    !sec
1620 Dco(1)=Co(1)
1630 Rco(1)=Dco(1)/Interval
1640 Rcoi(1)=Rco(1)/(1-Dt*Rco(1))
1650 FOR I=2 TO Number
1660   IF Co(I)<0 THEN Co(I)=Co(I)+32768
1670   Dco(I)=Co(I)-Co(I-1)
1680   IF Dco(I)<0 THEN Dco(I)=Dco(I)+32768
1690   Rco(I)=Dco(I)/Interval
1700   Rcoi(I)=Rco(I)/(1-Dt*Rco(I))    !CORRECTION FOR DEAD TIME
1710 NEXT I
1720 Timei=Interval/2       !ESTIMATE
1730 GOTO Pr
1740 !
1750 !      FREQUENCY MODE READINGS - ONE BATCH
1760 Freq:Number=392
1770 ! GATE TIME =10msec
1780 ! Interval=12msec APPROX
1790 REDIM Dco(Number),Rco(Number),Rcoi(Number)
1800 !
1810 !      A TRIGGER FOR PRESETTING THE CLOCK IS SET UP USING THE COUNTER/STEPPER CARD
1820 ON INT #7,15 GOTO S
1830 CONTROL MASK 7;128
1840 CARD ENABLE 7
1850 !
1860 OUTPUT Proc USING "K";"SN;SM,0,1;ST,1,1,1,1!"
1870 ENTER Proc;Cond(4)
1880 IF Cond(4)<>0 THEN GOSUB Status
1890 PRINT "SYSTEM READY FOR TRIGGER"
1900 CRT OFF
1910 V:GOTO V
1920 S:OFF INT #7
1930 OUTPUT 702 BFHS  NOFORMAT;"TP,0,0;SF,1,4,1;RP,392;WN,11;RC,1,4,1;NX;TE!"
1940 ENTER 702;Cond(5),Dco(*),Sec,Msec
1950 CRT ON
1960 PRINT "TRIGGER ACTIVATED"
1970 PRINT "READINGS COMPLETE"
1980 IF Cond(5)<>0 THEN GOSUB Status
1990 Elap_time=Sec+Msec/1000    !sec
2000 Interval=Elap_time/Number    !sec
2010 FOR I=1 TO Number
2020   Rco(I)=Dco(I)/.01
2030   Rcoi(I)=Rco(I)/(1-Dt*Rco(I))
2040 NEXT I
2050 Timei=Interval-.006      !ESTIMATE
2060 !
2070 Pr:PRINT "ELAPSED TIME =";Elap_time;"[Sec]"
2080 PRINT "NUMBER OF READINGS =";Number,SPA(5),"INTERVAL =";Interval;"[sec]"
2090 Nor=0
2100 INPUT "IS A CRT PRINTOUT OF THE CORRECTED C-RATES REQUIRED? Y/N",Q$
2110 IF Q$="N" THEN SUBEXIT
2120 M=INT(Number/10-.001)+1
2130 FIXED 0
2140 MAT R=Rcoi
2150 REDIM R(10*M)
2160 PRINT LIN(2)
2170 FOR I=1 TO M
2180   PRINT LIN(0)
2190   FOR K=0 TO 9
2200     PRINT USING Im3;R(K*M+I)
2210 Im3:  IMAGE #,MDDDDDD,1X
2220   NEXT K
2230 NEXT I
2240 STANDARD
```

```
2250 SUBEXIT
2260 !
2270 !      PROCESSOR STATUS READ, IN CASE OF ERROR
2280 Status: OUTPUT Proc USING "K";"$T2"
2290 ENTER Proc;Stat$
2300 PRINT LIN(1)
2310 IF Cond(1)<>0 THEN PRINT "ERROR OCCURED AT SYSTEM NORMALISE - Cond1"
2320 IF Cond(2)<>0 THEN PRINT "ERROR OCCURED WHEN TRIG WAS SET - Cond2"
2330 IF Cond(3)<>0 THEN PRINT "ERROR OCCURED WHEN READINGS WERE TAKEN - Cond3"
2340 IF Cond(4)<>0 THEN PRINT "ERROR OCCURED WHEN TRIG WAS SET - Cond4"
2350 IF Cond(5)<>0 THEN PRINT "ERROR OCCURED WHEN READINGS WERE TAKEN - Cond5"
2360 PRINT "EXTENDED STATUS IS:";Stat$
2370 RETURN
2380 SUBEND      !



2390 !      SUBROUTINE: Rtd_store
2400 !      ---------------------
2410 !
2420 !      THIS SUBROUTINE STORES THE RTD CURVE ON TAPE
2430 SUB Rtd_store
2440 OPTION BASE 1
2450 COM Rcount(*),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
2460 DIM File$[6]
2470 INPUT "OPTIONAL TRUNCATION OF NUMBER OF READINGS; INPUT N (or 0)",N
2480 IF N<>0 THEN Number=N
2490 REDIM Rcount(Number)
2500 DISP "INSERT THE DATA STORAGE CASSETTE"
2510 PAUSE
2520 INPUT "MUST EXISTING FILES BE DISPLAYED? Y/N",Q2$
2530 IF Q2$="Y" THEN CAT
2540 INPUT "ENTER THE FILENAME FOR THE CURRENT DATA",File$
2550 Bytes=8*(Number+7)
2560 Rec=INT(Bytes/256)+1
2570 CREATE File$,Rec
2580 ASSIGN #1 TO File$
2590 PRINT #1;Mode$,Number,Interval,Time1,Rcount(*),Nor
2600 IF (Nor=1) OR (Nor=2) THEN PRINT #1;Mean,Var
2610 IF Tail$="Y" THEN PRINT #1;C,B
2620 SUBEND      !



2630 !      SUBROUTINE: Rtd_retrieve
2640 !      ------------------------
2650 !
2660 !      THIS SUBROUTINE RETRIEVES A PREVIOUS RTD CURVE FROM TAPE
2670 SUB Rtd_retrieve
2680 OPTION BASE 1
2690 COM Rcount(*),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
2700 DIM File$[6]
2710 DISP "INSERT THE DATA CASSETTE"
2720 PAUSE
2730 INPUT "MUST EXISTING FILES BE DISPLAYED? Y/N",Q1$
2740 IF Q1$="Y" THEN CAT
2750 INPUT "ENTER THE FILENAME TO BE RETRIEVED",File$
2760 ASSIGN #1 TO File$
2770 READ #1,1
2780 READ #1;Mode$,Number,Interval,Time1
2790 REDIM Rcount(Number)
2800 READ #1;Rcount(*),Nor
2810 IF (Nor=1) OR (Nor=2) THEN READ #1;Mean,Var
2820 INPUT "DOES THIS CURVE HAVE AN EXPONENTIAL TAIL? Y/N",Tail$
2830 IF Tail$="Y" THEN READ #1;C,B
2840 PRINT LIN(1),File$;" HAS BEEN RETRIEVED";SPA(5);"MODE IS ";Mode$
2850 PRINT USING Im1;Time1
2860 PRINT "NUMBER=";Number;SPA(5);
2870 PRINT USING Im2;Interval
2880 IF Tail$="Y" THEN PRINT "EXP TAIL: Y=C.EXP(-Bt)  C=";C;"  B=";B
2890 IF Nor=1 THEN PRINT USING Im3;Mean,Var
2900 IF Nor=2 THEN PRINT USING Im4;Mean,Var
2910 Im1:IMAGE "FIRST READING AT",M.DDDDDD
2920 Im2:IMAGE "INTERVAL=",.DDDDDD
2930 Im3:IMAGE "CURVE IS NORMALISED"/"MEAN=",D.DDD,"[sec] AND VARIANCE=",DD.DDD," [sec^2]"
2940 Im4:IMAGE "CURVE IS NORMALISED"/"MEAN=",D.DDD,"[sec] AND VARIANCE=",D.DDD," [DIMLESS]"
2950 SUBEND      !
```

APPENDIX XI.2

```
10   !      PROGRAM: RTD_AV
20   !
30   !      THIS PROGRAM RETRIEVES AND AVERAGES UP TO 10 RTD CURVES AND STORES THE RESULT ON TAPE !

40   !      MAIN PROGRAM
50   !      ------------
60   !
70   OPTION BASE 1
80   COM Y(245),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var
90   PRINTER IS 16
100  PRINT LIN(1),TAB(30);CHR$(27);"&dD";"PROGRAM: RTD_AV";CHR$(27);"&d@"
110  INPUT "HOW MANY CURVES TO BE AVERAGED?",Nave
120  CALL Average(Nave)
130  INPUT "MUST THE AVERAGED RTD CURVE BE STORED ON TAPE? Y/N",Q$
140  IF Q$="Y" THEN CALL Rtd_store
150  STOP    !

160  !      SUBROUTINE: Rtd_store
170  !      ---------------------
180  !
190  !      THIS SUBROUTINE STORES THE RTD CURVE ON TAPE
200  SUB Rtd_store
210  OPTION BASE
220  COM Rcount(*), 'TEGER Number,REAL Interval,Time1,Mod $[1],Nor,Mean,Var
230  DIM File$[6]
240  REDIM Rcount(Number)
250  DISP "INSERT THE DATA STORAGE CASSETTE"
260  PAUSE
270  INPUT "MUST EXISTING FILES BE DISPLAYED? Y/N",Q2$
280  IF Q2$="Y" THEN CAT
290  INPUT "ENTER THE FILENAME FOR THE CURRENT DATA",File$
300  Bytes=8*(Number+7)
310  Rec=INT(Bytes/256)+1
320  CREATE File$,Rec
330  ASSIGN #1 TO File$
340  PRINT #1;Mode$,Number,Interval,Time1,·count(*),Nor
350  IF (Nor=1) OR (Nor=2) THEN PRINT #1;Mean,Var
360  SUBEND    !

370  !      SUBROUTINE: Average
380  !      -------------------
390  !
400  !      THIS SUBROUTINE FINDS THE AVERAGE AND STD.DEV. OF THE RTD CURVES AT EACH POINT OF TIME
410  !      TIME SCALES OF CURVES MUST BE THE SAME,AND AREA UNDER EACH CURVE MUST EQUAL UNITY
420  !      THE MEAN AND VARIANCE OF THE AVERAGE RTD CUVE ARE ALSO CALCD
430  SUB Average(Nave)
440  OPTION BASE 1
450  COM Y(*),INTEGER N,REAL Interval,Time1,Mode$[1],Nor,Mean,Var
460  DIM Yi(Nave,245),X(245)
470  DISP "INSERT THE DATA CASSETTE"
480  PAUSE
490  INPUT "MUST EXISTING FILES BE DISPLAYED? Y/N",Q$
500  IF Q$="Y" THEN CAT
510  DIM File$(Nave)[6],Meani(Nave),Vari(Nave)
520  INPUT "INPUT NAMES OF RTD DATA FILES",File$(*)
530  !
540  !      RECALLING RTD CURVES AND CHECKING CONSISTENCY
550  ASSIGN #1 TO File$(1)
560  READ #1,1
570  READ #1;Mode$,N,Interval,Time1
580  REDIM Yi(Nave,N),X(N),Y(N)
```

```
590  FOR I=1 TO N
600    READ #1;Y1(1,I)
610  NEXT I
620  READ #1;Nor
630  IF Nor<>1 THEN PRINT "ERROR Nor=";Nor
640  IF Nor=1 THEN READ #1;Mean1(1),Var1(1)
650  FOR J=2 TO Nave
660    ASSIGN #1 TO Files$(J)
670    READ #1,1
680    READ #1;Mode1$,N1,Interval1,Time11
690    IF Mode1$<>Mode$ THEN PRINT "ERROR -MODES DIFFER"
700    IF N1<>N THEN PRINT "ERROR -NUMBER OF READINGS DIFFER"
710    IF Interval1<>Interval THEN PRINT "ERROR -INTERVALS DIFFER"
720    IF Time11<>Time1 THEN PRINT "ERROR -TIME1 DIFFERS"
730    FOR I=1 TO N
740      READ #1;Y1(J,I)
750    NEXT I
760    READ #1;Nor
770    IF Nor<>1 THEN PRINT "ERROR -Nor=";Nor
780    IF Nor=1 THEN READ #1;Mean1(J),Var1(J)
790  NEXT J
800  !
810  !      CALCS AND PRINTOUT FOLLOW
820  PRINTER IS 7,1,WIDTH(210)
830  PRINT PAGE,,,CHR$(27);"&dD";"AVERAGING OF RTD CURVES";CHR$(27);"&d@"
840  PRINT LIN(1),CHR$(27);"&dD";" I ";CHR$(27);"&d@";SPA(2);CHR$(27);"&dD";"t[sec]";CHR$(27);"&d@"
;
850  PRINT SPA(2);CHR$(27);"&dD";"f-ave";CHR$(27);"&d@";SPA(2);CHR$(27);"&dD";"S-DEV";CHR$(27);"&d@
";
860  FOR J=1 TO Nave
870    PRINT SPA(2);CHR$(27);"&dD";"f-";J;CHR$(27);"&d@";
880  NEXT J
890  PRINT USING "/"
900  FOR I=1 TO N
910    X(I)=Time1+(I-1)*Interval     ![sec]
920    Sum11=Sum12=0
930    FOR J=1 TO Nave
940      Sum11=Sum11+Y1(J,I)
950      Sum12=Sum12+Y1(J,I)^2
960    NEXT J
970    Y(I)=Sum11/Nave
980    Std_dev=SQR(Sum12/Nave-Y(I)^2)     ![f(t) UNITS]
990    PRINT USING "#,DDD,1X,MDD.DDD,1X,MD.DDD,2X,D.DDD";I,X(I),Y(I),Std_dev
1000   FOR J=1 TO Nave
1010     PRINT USING "#,1X,MD.DDD";Y1(J,I)
1020   NEXT J
1030   PRINT " "
1040 NEXT I
1050 Sum1=Sum2=Sum3=0
1060 FOR I=1 TO N
1070   Sum1=Sum1+Y(I)
1080   Sum2=Sum2+Y(I)*X(I)
1090   Sum3=Sum3+Y(I)*X(I)^2
1100 NEXT I
1110 Mean=Sum2/Sum1     ![sec]
1120 Var=Sum3/Sum1-Mean^2     ![sec^2]
1130 Area=Interval*Sum1     !CHECK FOR NORMALISATION
1140 PRINT LIN(1)
1150 PRINT USING Ima1;Mean
1160 Ima1:IMAGE #,"MEAN[sec]:",3X,D.DDD,7X
1170 FOR J=1 TO Nave
1180   PRINT USING "#,2X,D.DDD";Mean1(J)
1190 NEXT J
1200 PRINT " "
1210 PRINT USING Ima2;Var
1220 Ima2:IMAGE #,"VAR[sec^2]:",2X,D.DDD,7X
1230 FOR J=1 TO Nave
1240   PRINT USING "#,2X,D.DDD";Var1(J)
1250 NEXT J
1260 PRINT ""
1270 PRINTER IS 16
1280 PRINT LIN(1),"CURVE HAS BEEN AVERAGED"
1290 PRINT "AREA UNDER CURVE=";Area
1300 SUBEND   !
```

APPENDIX XI.3

```
10  !     PROGRAM: NORM
20  !
30  !     THIS PROGRAM NORMALISES AND PLOTS THE RTD CURVE   !




40  !     MAIN PROGRAM
50  !     ------------
60  !
70  OPTION BASE 1
80  COM Y(260),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
90  DIM Q$[1]
100 PRINTER IS 16
110 PRINT LIN(1),TAB(30);CHR$(27);"&dD";"PROGRAM: NORM";CHR$(27);"&d@"
120 CALL Rtd_retrieve
130 INPUT "IS AN RTD PLOT OR PRINTOUT REQUIRED? Y/N",Q$
140 IF Q$="Y" THEN CALL Graph
150 INPUT "MUST >Time1< BE CHANGED? Y/N",Q$
160 IF Q$="Y" THEN INPUT "INPUT:Time1",Time1
170 INPUT "MUST >Interval< BE CORRECTED? Y/N",Q$
180 IF Q$="Y" THEN INPUT "INPUT:Interval",Interval
190 INPUT "MUST THE RTD CURVE BE NORMALISED? Y/N",Q$
200 IF Q$="Y" THEN CALL Normalise
210 INPUT "MUST THE RTD CURVE BE STORED ON TAPE? Y/N",Q$
220 IF Q$="Y" THEN CALL Rtd_store
230 INPUT "IS AN RTD PLOT OR PRINTOUT REQD? Y/N",Q$
240 IF Q$="Y" THEN CALL Graph
250 INPUT "MUST THE RTD CURVE BE CONVERTED TO DIMENSIONLESS TIME UNITS? Y/N",Q$
260 IF Q$="Y" THEN CALL Theta
270 INPUT "MUST THE RTD CURVE BE STORED ON TAPE? Y/N",Q$
280 IF Q$="Y" THEN CALL Rtd_store
290 INPUT "IS AN RTD PLOT OR PRINTOUT REQUIRED? Y/N",Q$
300 IF Q$="Y" THEN CALL Graph
310 INPUT "MUST THE AREA UNDER A SECTION OF THE RTD CURVE BE CALCD? Y/N",Q$
320 IF Q$="Y" THEN CALL Part
330 INPUT "IS A PLOT OF Ln F REQD?",Q$
340 IF Q$="N" THEN Nolog
350 FOR I=1 TO Number
360   IF Y(I)<=0 THEN Y(I)=.0005
370 NEXT I
380 MAT Y=LOG(Y)
390 CALL Graph
400 Nolog: STOP   !




410 !     SUBROUTINE: Rtd_store
420 !     ---------------------
430 !
440 !     THIS SUBROUTINE STORES THE RTD CURVE ON TAPE
450 SUB Rtd_store
460 OPTION BASE 1
470 COM Rcount(*),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
480 DIM File$[6]
490 INPUT "OPTIONAL TRUNCATION OF NUMBER OF READINGS; INPUT N (or 0)",N
500 IF N<>0 THEN Number=N
510 REDIM Rcount(Number)
520 DISP "INSERT THE DATA STORAGE CASSETTE"
530 PAUSE
540 INPUT "MUST EXISTING FILES BE DISPLAYED? Y/N",Q2$
550 IF Q2$="Y" THEN CAT
560 INPUT "ENTER THE FILENAME FOR THE CURRENT DATA",File$
570 Bytes=8*(Number+7)
580 Rec=INT(Bytes/256)+1
590 CREATE File$,Rec
600 ASSIGN #1 TO File$
610 PRINT #1;Mode$,Number,Interval,Time1,Rcount(*),Nor
620 IF (Nor=1) OR (Nor=2) THEN PRINT #1;Mean,Var
630 IF Tail$="Y" THEN PRINT #1;C,B
640 SUBEND   !
```

```
650  !      SUBROUTINE: Rtd_retrieve
660  !      ------------------------
670  !
680  !      THIS SUBROUTINE RETRIEVES A PREVIOUS RTD CURVE FROM TAPE
690  SUB Rtd_retrieve
700  OPTION BASE 1
710  COM Rcount(*),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
720  DIM File$[6]
730  DISP "INSERT THE DATA CASSETTE"
740  PAUSE
750  INPUT "MUST EXISTING FILES BE DISPLAYED? Y/N",Q1$
760  IF Q1$="Y" THEN CAT
770  INPUT "ENTER THE FILENAME TO BE RETRIEVED",File$
780  ASSIGN #1 TO File$
790  READ #1,1
800  READ #1;Mode$,Number,Interval,Time1
810  REDIM Rcount(Number)
820  READ #1;Rcount(*),Nor
830  IF (Nor=1) OR (Nor=2) THEN READ #1;Mean,Var
840  INPUT "DOES THIS CURVE HAVE AN EXPONENTIAL TAIL? Y/N",Tail$
850  IF Tail$="Y" THEN READ #1;C,B
860  PRINT LIN(1),File$;" HAS BEEN RETRIEVED";SPA(5);"MODE IS ";Mode$
870  PRINT USING Im1;Time1
880  PRINT "NUMBER=";Number;SPA(5);
890  PRINT USING Im2;Interval
900  IF Tail$="Y" THEN PRINT "EXP TAIL: Y=C.EXP(-Bt)  C=";C;"  B=";B
910  IF Nor=1 THEN PRINT USING Im3;Mean,Var
920  IF Nor=2 THEN PRINT USING Im4;Mean,Var
930 Im1: IMAGE "FIRST READING AT",M.DDDDDD
940 Im2: IMAGE "INTERVAL=",.DDDDDD
950 Im3: IMAGE "CURVE IS NORMALISED"/"MEAN=",D.DDD,"[sec] AND VARIANCE=",DD.DDD," [sec^2]"
960 Im4: IMAGE "CURVE IS NORMALISED"/"MEAN=",D.DDD,"[sec] AND VARIANCE=",D.DDD," [DIMLESS]"
970  SUBEND    !


980  !      SUBROUTINE: Graph
990  !      -----------------
1000 !
1010 !      THE NEXT FOUR SUBROUTINES PLOT THE RTD CURVE ON THE HP7225A PLOTTER
1020 SUB Graph
1030 OPTION BASE 1
1040 COM Y(*),INTEGER N,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
1050 DIM X(260),X1(260),X2(260),Y1(260),Y2(260),Xtit$[30]
1060 Conex:IF Tail$="Y" THEN Nexp=N
1070 IF Tail$="Y" THEN INPUT "EXPON CURVE MUST BE PLOTTED TO TIME t;ENTER t(or 0)",Tf
1080 IF (Tail$="Y") AND (Tf<>0) THEN N=INT((Tf-Time1)/Interval+1)
1090 REDIM Y(N)
1100 REDIM X(N),X1(N),X2(N),Y1(N),Y2(N)
1110 Plt 705
1120 X(1)=Time1
1130 FOR I=2 TO N
1140   X(I)=Time1+(I-1)*Interval
1150 NEXT I
1160 IF (Tail$<>"Y") OR (Tf=0) THEN Tf0
1170 FOR I=Nexp+1 TO N
1180   Y(I)=C*EXP(-B*X(I))
1190 NEXT I
1200 Tf0:INPUT "WHAT IS REQU? PRINTOUT(PR),PLOT(PL),OR BOTH(B)",W$
1210 IF W$="PL" THEN GOTO Noprint
1220 !
1230 !      PRINTOUT OF RTD CURVE VALUES
1240 PRINTER IS 7,1,WIDTH(130)
1250 PRINT LIN(3),,CHR$(27);"&dD";"RTD MEASUREMENT DATA";CHR$(27);"&d@"
1260 PRINT LIN(1),"MODE IS ";Mode$;SPA(5);"NUMBER OF READINGS=";N
1270 IF Nor=1 THEN Nor1
1280 IF Nor=2 THEN Nor2
1290 PRINT USING Im4;Time1,Interval
1300 Im4: IMAGE "FIRST READING AT TIME",M.DDDDDD,"[sec]",5X,"INTERVAL=",.DDDDDD,"[sec]"
1310 PRINT LIN(2),"I","TIME[sec]","Y(T)[cps]"
1320 PRINT "-","---------","---------"
1330 FOR I=1 TO N
1340   PRINT USING "DDD,16X,MDD.DDD,13X,MDDDDDD";I,X(I),Y(I)
1350 NEXT I
1360 PRINTER IS 16
1370 GOTO Noprint
1380 Nor1: PRINT USING Im4;Time1,Interval
1390 IF Tail$="Y" THEN PRINT "EXPON TAIL: f(t)=C.EXP(-B.t)  C=";C;"  B=";B
```

```
1400 IF (Tail$="Y") AND (Tf()0) THEN PRINT "LAST READING BEFORE TAIL IS NO.";Nexp
1410 PRINT USING Im7;Mean,Var
1420 Im7:IMAGE "CURVE IS NORMALISED"/"MEAN=",DD.DDD,"[sec]",5X,"VARIANCE=",D.DDD,"[sec^2]"
1430 PRINT LIN(2),"I","TIME[sec]","f(t)"
1440 PRINT "-","---------","----"
1450 FOR I=1 TO N
1460   PRINT USING "DDD,16X,MDD.DDD,12X,MDD.DDDD";I,X(I),Y(I)
1470 NEXT I
1480 PRINTER IS 16
1490 GOTO Noprint
1500 Nor2: PRINT USING Im5;Time1,Interval
1510 Im5:IMAGE "FIRST READING AT THETA=",M.DDDDDD,5X,"INTERVAL=",.DDDDDD,"[DIMENSIONLESS]"
1520 IF Tail$="Y" THEN PRINT "EXPON TAIL: f(Theta)=C.EXP(-B.Theta)  C=";C;"  B=";B
1530 IF (Tail$="Y") AND (Tf()0) THEN PRINT "LAST READING BEFORE TAIL IS NO.";Nexp
1540 PRINT USING Im6;Mean,Var
1550 Im6:IMAGE "CURVE IS NORMALISED"/"MEAN=",DD.DDD,"[sec]",5X,"VARIANCE=",D.DDD,"[DIMLESS]"
1560 PRINT LIN(2),"I","THETA","f(THETA)"
1570 PRINT "-","-----","--------"
1580 FOR I=1 TO N
1590   PRINT USING "DDD,15X,MDD.DDD,15X,MD.DDDD";I,X(I),Y(I)
1600 NEXT I
1610 PRINTER IS 16
1620 Noprint:IF W$="PR" THEN Graphex
1630 !
1640 !      PLOT OF RTD CURVE(S)
1650 Nice=3          !(0,1,2,3)
1660 IF Nor=2 THEN Ytit$="f(THETA)"
1670 IF Nor=1 THEN Ytit$="f(t)"
1680 IF Nor=0 THEN Ytit$="COUNT RATE [cps]"
1690 IF Nor=2 THEN Xtit$="DIMENSIONLESS TIME, THETA"
1700 IF (Nor=0) OR (Nor=1) THEN Xtit$="TIME,t [sec]"
1710 INPUT "INPUT LINE TYPE (0-7: 7=CONTINUOUS,0=NO LINE)",Lintyp
1720 Lt$=VAL$(INT(Lintyp))&";"
1730 IF Lintyp=7 THEN Lt$=";"
1740 INPUT "INPUT SYMBOL FOR POINTS (N=NO SYMBOL)",Symb$
1750 IF Symb$="N" THEN Symb$=";"
1760 Head$="RTD CURVE"
1770 Nn=N
1780 Xlen=250    !250 MAX
1790 Ylen=170    !170 MAX
1800 Xp=1000+Xlen/.025
1810 Yp=1000+Ylen/.025
1820 INPUT "INPUT NUMBER OF SECTIONS FOR RTD PLOT",Sec
1830 INPUT "MUST ONLY POINTS BE PLOTTED (SAME AXES)? Y/N",P$
1840 IF P$="Y" THEN Points
1850 INPUT "MUST THIS CURVE ONLY BE USED FOR SCALING? Y/N",Sc$
1860 INPUT "WHICH AXES MUST BE RE-SCALED? X/Y/BOTH(B)/NONE(N)",R$
1870 IF R$="B" THEN OUTPUT Pltr USING "K";"IN;IP1000,1000,",Xp,",",Yp
1880 IF R$="B" THEN OUTPUT Pltr USING "K";"SC0,",Xp-1000,",",0,",",Yp-1000
1890 IF (Sec=1) AND ((R$="X") OR (R$="B")) THEN CALL Scale(X(*),Nn,Xlen,Nice,X0,Xdiv)
1900 IF (Sec=1) AND ((R$="X") OR (R$="B")) THEN Xfct=800/Xdiv
1910 IF (R$="Y") OR (R$="B") THEN CALL Scale(Y(*),Nn,Ylen,Nice,Y0,Ydiv)
1920 IF (R$="Y") OR (R$="B") THEN Yfct=800/Ydiv
1930 IF (Sc$="Y") AND (Sec=1) THEN Nex1
1940 IF Sec=1 THEN CALL Axis(0,Xlen,X0,Xdiv,Xtit$)
1950 IF Sec=1 THEN CALL Axis(1,Ylen,Y0,Ydiv,Ytit$)
1960 !
1970 !      CURVE IS PLOTTED IN SECTIONS
1980 Points:IF Sec()1 THEN Sec2
1990 MAT X1=X
2000 MAT Y1=Y
2010 Num1=N
2020 GOTO Sec1
2030 Sec2:Num=INT(N/Sec-.001)+1
2040 REDIM X1(Num),Y1(Num),X2(Num),Y2(Num)
2050 Extra=Num*Sec-N
2060 FOR I=1 TO Sec
2070   IF I=1 THEN INPUT "IS PLOT OF FIRST SECTION REQUD? Y/N",Q$
2080   IF I()1 THEN INPUT "IS PLOT OF NEXT SECTION REQUD? Y/N",Q$
2090   IF Q$="N" THEN Nex
2100   IF I=Sec THEN Num1=Num-Extra
2110   IF I(Sec THEN Num1=Num
2120   FOR J=1 TO Num1
2130     X1(J)=X(J+(I-1)*Num)
2140     Y1(J)=Y(J+(I-1)*Num)
2150   NEXT J
2160   IF I(Sec THEN L11
2170   IF Extra=0 THEN L11
2180   FOR J=Num1+1 TO Num
2190     X1(J)=Time1+(J+(I-1)*Num-1)*Interval
```

```
2200     Y1(J)=0
2210   NEXT J
2220   Num1=Num
2230 L11:  IF P$="Y" THEN Sec1
2240   IF (R$="X") OR (R$="B") THEN CALL Scale(X1(*),Num1,Xlen,Nice,X0,Xdiv)
2250   IF (R$="X") OR (R$="B") THEN Xfct=800/Xdiv
2260   IF Sc$="Y" THEN Nex1
2270   CALL Axis(0,Xlen,X0,Xdiv,Xtit$)
2280   CALL Axis(1,Ylen,Y0,Ydiv,Ytit$)
2290 Sec1:  CALL Trans(X1(*),X2(*),X0,Xfct)
2300   CALL Trans(Y1(*),Y2(*),Y0,Yfct)
2310   !
2320   !      POINTS ARE NOW PLOTTED
2330   OUTPUT Pltr USING "K";"SM",Symb$
2340   OUTPUT Pltr USING "K";"PU;LT",Lt$,"PA",X2(1),",",Y2(1),";PD"
2350   FOR K=2 TO Num1
2360     OUTPUT Pltr USING "K";"PA",X2(K),",",Y2(K)
2370   NEXT K
2380   OUTPUT Pltr USING "K";"SM;"
2390   !
2400   !      TITLE IS PLOTTED
2410   IF (P$="Y") AND (Sec=1) THEN Nex1
2420   IF P$="Y" THEN Nex
2430   Xp1=Xlen*20
2440   Yp1=Ylen*36
2450   OUTPUT Pltr USING "K";"PU;PA",Xp1,",",Yp1
2460   Nb=-.5*LEN(Head$)
2470   OUTPUT Pltr USING "K";"CP",Nb,",",0
2480   OUTPUT Pltr USING "K";"LB",Head$&CHR$(3),";PU"
2490   IF Sec=1 THEN Nex1
2500 Nex:NEXT I
2510 Nex1:INPUT "MUST ANOTHER CURVE BE RETRIEVED AND PLOTTED? Y/N",Q$
2520 IF Q$="N" THEN Nex2
2530 CALL Rtd_retrieve
2540 GOTO Cunex
2550 Nex2:OUTPUT Pltr USING "K";"IN"
2560 Graphex:IF Tail$="Y" THEN INPUT "MUST N BE CHANGED BACK TO EXCLUDE TAIL?",Q$
2570 IF (Tail$<>"Y") OR (Q$="N") THEN SUBEXIT
2580 N=Nexp
2590 REDIM Y(N)
2600 SUBEND    !


2610 SUB Scale(X(*),Nn,Axlen,Nice,X0,Xdiv)
2620 !      THIS SUBROUTINE SCALES THE PLOTTER,RETURNING X0 AND Xdiv
2630 OPTION BASE 1
2640 DIM Acc(8)
2650 Acc(1)=15
2660 Acc(2)=20
2670 Acc(3)=25
2680 Acc(4)=40
2690 Acc(5)=50
2700 Acc(6)=80
2710 Acc(7)=100
2720 Acc(8)=150
2730 Xmax=Xmin=X(1)
2740 FOR I=2 TO Nn
2750   Xmax=MAX(Xmax,X(I))
2760   Xmin=MIN(Xmin,X(I))
2770 NEXT I
2780 Ndiv=Axlen/20
2790 Xdiv=ABS((Xmax-Xmin)/Ndiv)
2800 M=0
2810 Iff1:IF Xdiv>10 THEN GOTO Iff2
2820       M=M+1
2830       Xdiv=Xdiv*10
2840       GOTO Iff1
2850 Iff2:IF Xdiv<=100 THEN Ok
2860       M=M-1
2870       Xdiv=Xdiv/10
2880       GOTO Iff2
2890 Ok:FOR I=1 TO 7
2900   IF Xdiv<Acc(I) THEN Found
2910 NEXT I
2920 STOP
2930 Found:Xdiv=Acc(I)/10^M
2940 X0=INT(Xmin/Xdiv)*Xdiv
2950 IF X0+Ndiv*Xdiv>=Xmax THEN 3140
2960 IF Nice=3 THEN 3120
2970 FOR Jj=2 TO 0 STEP -2
```

```
2980   OUTPUT Xmin$ USING "MDD.DE";Xmin
2990   Xmin$[5-Jj,5-Jj]="0"
3000   Xmin$[5,5]="0"
3010   Nd=3-Jj/2
3020   IF Xmin<0 THEN Xmin$[Nd,Nd]=VAL$(VAL(Xmin$[Nd,Nd])+1)
3030   X0=VAL(Xmin$)
3040   IF X0+Ndiv*Xdiv>=Xmax THEN 3140
3050   IF Nice=2 THEN 3120
3060 NEXT Jj
3070 IF Nice=1 THEN 3120
3080 IF Nice<>0 THEN STOP
3090 Xdiv=(Xmax-X0)/Ndiv
3100 Xdiv=DROUND(Xdiv,3)
3110 SUBEXIT
3120 I=I+1
3130 GOTO Found
3140 SUBEND   !


3150 SUB Axis(Xory,Llen,Mini,Divi,Titi$)
3160 !      THIS SUBROUTINE DRAWS THE AXES
3170 DIM Tit$[35]
3180 Min=Mini
3190 Div=Divi
3200 Tit$=Titi$
3210 Pltr=705
3220 Len=Llen/.025
3230 M=INT(Llen/20)
3240 Len=Llen/.025
3250 IF NOT ((Xory=1) OR (Xory=0)) THEN STOP
3260 OUTPUT Pltr USING "K";"PU;PA0,0;PD"
3270 Xinc=800*(1-Xory)
3280 Yinc=800*Xory
3290 IF Xory=0 THEN T$=";XT"
3300 IF Xory=1 THEN T$=";YT"
3310 FOR I=1 TO M
3320   OUTPUT Pltr USING "K";"PR",Xinc,",",Yinc,T$
3330 NEXT I
3340 Xinc=Len*(1-Xory)
3350 Yinc·   *Xory
3360 OUTF    ltr USING "K";"PA",Xinc,",",Yinc,";PU;PA0,0"
3370 IF Xory=1 THEN GOTO Laby
3380 Labx:Xinc=-2.5
3390      Yinc=-1
3400      Di$="1,0;"
3410      GOTO Lab
3420 Laby:Xinc=-5.7
3430      Yinc=-.25
3440      Di$="0,1;"
3450 Lab:OUTPUT Pltr USING "K";"CP",Xinc,",",Yinc,";PR0,0"
3460 M=M+1
3470 Xinc=800*(1-Xory)
3480 Yinc=800*Xory
3490 L=Min
3500 R=Min+(M-1)*Div
3510 Hi=MAX(ABS(L),ABS(R))
3520 Lo=MIN(ABS(L),ABS(R))
3530 IF L*R<0 THEN Lo=Div
3540 IF Lo=0 THEN Lo=Div
3550 IF LGT(Hi/Lo)>=5 THEN E_notat
3560 E=INT(LGT(Lo))
3570 IF (Lo>=.1) AND (Hi<99999) THEN E=0
3580 FOR I=1 TO M
3590   Label=(Min+(I-1)*Div)/10^E
3600   IF Label<>0 THEN 3630
3610   Fmt$="XD.D"
3620   GOTO 3690
3630   N=INT(LGT(ABS(Label)))+1
3640   IF N>=0 THEN 3670
3650   Fmt$="M.DDD"
3660   GOTO 3690
3670   Fmt$="M"&RPT$("D",N)
3680   IF N<=3 THEN Fmt$=Fmt$&"."&RPT$("D",3-N)
3690   OUTPUT Lab$ USING Fmt$;Label
3700   L1=LEN(Lab$)
3710   Lab$[L1-1,L1]=""
3720   Lab$=Lab$&CHR$(13)&CHR$(3)
3730   OUTPUT Pltr USING "K";"LB",Lab$
3740   OUTPUT Pltr USING "K";"PU;PR",Xinc,",",Yinc
3750 NEXT I
```

```
3760 IF E=0 THEN 3780
3770 Tit$=Tit$&"  x 10"
3780 Tit$=TRIM$(Tit$)
3790 !
3800 L1=LEN(Tit$)
3810 Xp=Len/2*(1-Xory)
3820 Yp=Len/2*Xory
3830 OUTPUT Pltr USING "K";"PA",Xp,",",Yp
3840 Xinc=-7*Xory-L1/2*(1-Xory)
3850 Yinc=-3*(1-Xory)-L1/4*Xory
3860 OUTPUT Pltr USING "K";"CP",Xinc,",",Yinc
3870 Tit$=Tit$&CHR$(3)
3880 OUTPUT Pltr USING "K";"DI",Di$,"LB",Tit$
3890 IF E=0 THEN 3950
3900 OUTPUT Pltr USING "K";"CP0.0,0.35"
3910 FIXED 0
3920 E$=VAL$(E)&CHR$(3)
3930 STANDARD
3940 OUTPUT Pltr USING "K";"LB",E$
3950 OUTPUT Pltr USING "K";"DI 1,0"
3960 OUTPUT Pltr USING "K";"PU;PA0,0"
3970 SUBEXIT
3980 E_notat:FLOAT 2
3990 FOR I=1 TO M
4000   Lab$=VAL$(Min+(I-1)*Div)&CHR$(13)&CHR$(3)
4010   OUTPUT Pltr USING "K";"LB",Lab$
4020   OUTPUT Pltr USING "K";"PU;PR",Xinc,",",Yinc
4030 NEXT I
4040 STANDARD
4050 E=0
4060 GOTO 3780
4070 OUTPUT Pltr USING "K";"PU;PA0,0"
4080 SUBEND   !


4090 SUB Trans(X(*),X2(*),X0,Xfct)
4100 !      THIS SUBROUTINE TRANSFORMS VECTOR X INTO VECTOR Y IN PLOTTER UNITS
4110 OPTION BASE 1
4120 MAT X2=X
4130 MAT X2=X2-(X0)
4140 MAT X2=X2*(Xfct)
4150 MAT X2=X2+(.5)
4160 MAT X2=INT(X2)
4170 SUBEND     !



4180 !      SUBROUTINE:Normalise
4190 !      --------------------
4200 !
4210 !      THIS SUBROUTINE NORMALISES THE  TD CURVE,ie AREA UNDER CURVE=UNITY
4220 !      THE MEAN AND VARIANCE ARE ALSO CALCULATED
4230 !      AN EXPONENTIAL CURVE IS FITTED TO THE TAIL IF REQUD
4240 SUB Normalise
4250 OPTION BASE 1
4260 COM Y(*),INTEGER N,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
4270 REDIM Y(N)
4280 !
4290 !      OPTIONAL CORRECTION FOR BACKGROUND RADIATION
4300 IF (Nor=1) OR (Nor=2) THEN Back2
4310 Nback=0
4320 INPUT "OPTION:FIRST n READINGS ARE BACKGROUND;INPUT n or 0",Nback
4330 IF Nback=0 THEN Back1
4340 Totback=0
4350 FOR I=1 TO Nback
4360   Totback=Totback+Y(I)
4370 NEXT I
4380 Yback=Totback/Nback      ![cps]
4390 MAT Y=Y-(Yback)
4400 FOR I=1 TO Nback
4410   Y(I)=0
4420 NEXT I
4430 GOTO Back2
4440 Back1:INPUT "OPTION: READINGS 1 TO n ARE 0,AND m TO END ARE BACKGROUND;INPUT n,m (or 0,0)",Nba
ck,Mback
4450 IF Mback=0 THEN Back2
4460 Totback=0
4470 FOR I=Mback TO N
```

```
4480    Totback=Totback+Y(I)
4490 NEXT I
4500 Yback=Totback/(N-Mback+1)
4510 MAT Y=Y-(Yback)
4520 FOR I=1 TO Nback
4530    Y(I)=0
4540 NEXT I
4550 FOR I=Mback TO N
4560    Y(I)=0
4570 NEXT I
4580 !
4590 !       OPTIONAL FITTING OF EXPONENTIAL TAIL
4600 Back2:Int1=Int2=Int3=Int4=0
4610 INPUT "MUST AN EXPON TAIL BE FITTED? Y/N",Tail$
4620 IF Tail$="N" THEN Tail1
4630 INPUT "INPUT LAST READING NO.BEFORE START OF EXP TAIL",N
4640 REDIM Y(N)
4650 INPUT "INPUT INTERCEPT AND /SLOPE/ OF ln(f) vs t PLOT",A,B
4660 C=EXP(A)
4670 !
4680 !       CALC OF MEAN AND VARIANCE       !FOR START OF INTEGRATION
4690 Tt=Time1+(N-.5)*interval                               !INT f.dt
4700 Int1=C/B*EXP(-B*Tt)                                    !INT t.f.dt
4710 Int2=C/B*EXP(-B*Tt)*(1/B+Tt)                           !INT t^2.f.dt
4720 Int3=C/B*EXP(-B*Tt)*(2/B^2+2/B*Tt+Tt^2)
4730 Tail1:Sum1=Sum2=Sum3=0
4740 FOR I=1 TO N
4750    Xt=Time1+(I-1)*Interval
4760    Sum1=Sum1+Y(I)*Interval
4770    Sum2=Sum2+Y(I)*Xt*Interval
4780    Sum3=Sum3+Y(I)*Xt^2*Interval
4790 NEXT I                                      ![sec]
4800 Mean=(Sum2+Int2)/(Sum1+Int1)                ![sec^2]
4810 Var=(Sum3+Int3)/(Sum1+Int1)-Mean^2
4820 PRINT LIN(1),"Mean=";Mean;"[sec]","VAR=";Var;"[sec^2]"
4830 !
4840 !       NORMALISATION
4850 INPUT "MUST CURVE BE NORMALISED? Y/N",Q$
4860 IF Q$="N" THEN SUBEXIT
4870 C0=Sum1+Int1                        ![NORMALISED]
4880 MAT Y=Y/(C0)
4890 IF Tail$="Y" THEN C=C/C0
4900 Nor=1
4910 !
4920 ! CHECK OF NORMALISATION
4930 IF Tail$="Y" THEN Int4=C/B*EXP(-B*Tt)
4940 Sum4=0
4950 FOR I=1 TO N
4960    Sum4=Sum4+Y(I)*Interval
4970 NEXT I
4980 Area=Int4+Sum4
4990 PRINT "RTD CURVE IS NORMALISED"
5000 IF Tail$="Y" THEN PRINT "C NOW=";C
5010 PRINT "AREA UNDER CURVE=";Sum4;"+";Int4;"=";Area
5020 SUBEND     :



5030 !       SUBROUTINE: Theta
5040 !       -----------------
5050 !
5060 !       THIS SUBROUTINE CHANGES THE RTD TIME SCALE TO THETA (THETA=t/MEAN RES TIME)
5070 !       THE RTD CURVE MUST BE NORMALISED
5080 SUB Theta
5090 OPTION BASE 1
5100 COM Y(*),INTEGER N,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
5110 REDIM Y(N)
5120 IF Nor<>1 THEN PRINT "ERROR -RTD NOT NORMALISED"
5130 Time1=Time1/Mean                    ![DIMLESS]
5140 Interval=Interval/Mean              ![DIMLESS]
5150 MAT Y=Y*(Mean)
5160 Var=Var/Mean^2                      ![DIMLESS]
5170 IF Tail$="Y" THEN C=C*Mean
5180 IF Tail$="Y" THEN B=B*Mean
5190 !
5200 !       CHECK OF NORMALISATION
5210 Sum1=0
5220 FOR I=1 TO N
```

```
5230   Sumi=Sumi+Y(I)*Interval
5240 NEXT I
5250 IF Tail$="Y" THEN Tt=Timei+(N-.5)*Interval
5260 IF Tail$="Y" THEN Inti=C/B*EXP(-B*Tt)
5270 Area=Sumi+Inti
5280 PRINT LIN(1),"TIME UNITS ARE NOW THETA"
5290 PRINT "Timei=";Timei;SPA(5);"INTERVAL=";Interval
5300 PRINT "VARIANCE=";Var;"[DIMLESS]"
5310 IF Tail$="Y" THEN PRINT "C NOW=";C;SPA(5);"B NOW=";B
5320 PRINT "AREA UNDER CURVE=";Sumi;"+";Inti;"=";Area
5330 Nor=2
5340 SUBEND   !



5350 !      SUBROUTINE  Part
5360 !      ----------------
5370 !
5380 !      THIS SUBRCUTINE CALULATES THE AREA UNDER A SECTION OF THE NORMALISED RTD CURVE,
5390 !      ie.THE FRACTION OF THE TOTAL FLOW WITH RES TIME IN THAT INTERVAL
5400 SUB Part
5410 OPTION BASE 1
5420 COM (*),INTEGER N,REAL Interval,Timei,Mode$[1],Nor
5430 REDIM Y(N)
5440 IF (Nor<>1) AND (Nor<>2) THEN PRINT "ERROR- Nor=";Nor
5450 PRINTER IS 7,1,WIDTH(130)
5460 PRINT LIN(3),,CHR$(27);"&dD";"PARTIAL AREAS";CHR$(27);"&d@",LIN(1)
5470 Partlab:INPUT "INPUT BEGIN AND END READING NUMBERS",N1,N2
5480 Sum=0
5490 FOR I=N1 TO N2
5500   Sum=Sum+Y(I)
5510 NEXT I
5520 Areap=Sum*Interval
5530 PRINT USING Imp;N1,N2,Areap
5540 Imp:IMAGE "AREA UNDER CURVE FROM I=",DDD," TO I=",DDD," IS:",D.DDDD
5550 INPUT "IS ANOTHER PARTIAL AREA REQUIRED? Y/N",Q$
5560 IF Q$="Y" THEN Partlab
5570 PRINTER IS 16
5580 SUBEND   !
```

APPENDIX XI.4

```
10   !       PROGRAM: F_ANAL
20   !
30   !       THIS PROGRAM CALCULATES THE FOURIER COEFFICIENTS AND THE CORRESPONDING F-SERIES
40   !       OF A NORMALISED RTD CURVE.
50   !       THE AMPLITUDE RATIOS AND PHASE LAGS ARE ALSO CALCULATED
60   !       CORRECTION FOR A NON-IDEAL INPUT PULSE IS ALSO MADE    !



70   !       MAIN PROGRAM
80   !       ------------
90   !
100  OPTION BASE 1
110  COM SHORT Y(1400),INTEGER Number,REAL Interval,Time1,Mode$[1],Nor,Mean,Var,Tail$,C,B
120  SHORT X(1400),F(1400),Num(150),Freq(150),A(150),B(150),A1(150),A2(150),B1(150),B2(150),Ar(150)
,Phl(150)
130  DIM Q$[1]
140  PRINTER IS 16
150  PRINT LIN(1),TAB(30);CHR$(27);"&dD";"PROGRAM: F_ANAL";CHR$(27);"&d@"
160  INPUT "MUST F-COEFFS BE RETRIEVED FROM TAPE(T) OR CALCULATED(C)? Y/N",F$
170  IF F$="C" THEN Coeffs
180  CALL F_retrieve(M,Interval,Time1,N,T,B0,A(*),B(*))
190  REDIM X(M),Y(M),Num(N),Freq(N),A(N),B(N),A1(N),A2(N),B1(N),B2(N),Ar(N),Phl(N),F(M)
200  FOR K=1 TO N
210    Freq(K)=K*PI/T
220  NEXT K
230  Nor=2
240  Mode$="T"
250  GOTO Amp
260  !
270  !       COMPUTATION OF FOURIER COEFFICIENTS [A(K),B(K),B0]
280 Coeffs: CALL Rtd_retrieve
290   INPUT "INPUT NUMBER OF READINGS BEFORE TRUNCATION,M",M
300   INPUT "INPUT NUMBER OF PAIRS OF FOURIER COEFFICIENTS,N",N
310   REDIM X(M),Y(M),Num(N),Freq(N),A(N),B(N),A1(N),A2(N),B1(N),B2(N),Ar(N),Phl(N),F(M)
320   IF Tail$<>"Y" THEN Noexp
330   FOR I=Number+1 TO M
340     X(I)=Time1+(I-1)*Interval
350     Y(I)=C*EXP(-B*X(I))
360   NEXT I
370 Noexp: Number=M
380   T=(Time1+(M-.5)*Interval)/2      !FUNCTION Y IS ENCLOSED BY INTERVAL [0,2T]
390   INPUT "IF THIS IS AN INJECTED PULSE THEN INPUT T OF THE CORRESPONDING RTD CURVE (or 0)",T1
400   IF T1<>0 THEN T=T1
410   RAD
420   B0=1/(2*T)                          !ASSUMING AREA UNDER CURVE=1
430   DISP "-----PROGRAM RUNNING-----"
440   L=0
450   FOR K=1 TO N
460     Freq(K)=K*PI/T                    ![rad/sec]
470     S=SIN(Freq(K)*Interval)
480     C=COS(Freq(K)*Interval)
490   ! S1=SIN(Freq(K)*Time1)             ! Y(1) USUALLY =0
500   ! C1=COS(Freq(K)*Time1)             !  "      "      "
510     S2=SIN(Freq(K)*(Time1+Interval))
520     C2=COS(Freq(K)*(Time1+Interval))
530     Suma=Y(2)*S2                      ! +Y(1)*S1
540     Sumb=Y(2)*C2                      ! +Y(1)*C1
550     Si1=S2
560     Ci1=C2
570     FOR I=3 TO M
580       Si=Si1*C+Ci1*S
590       Ci=Ci1*C-Si1*S
600       Suma=Suma+Y(I)*Si
610       Sumb=Sumb+Y(I)*Ci
620       Si1=Si
630       Ci1=Ci
640     NEXT I
650     A(K)=Suma*Interval/T
660     B(K)=Sumb*Interval/T
670   NEXT K
```

```
680  BEEP
690  PRINT LIN(1),"FOURIER COEFFS HAVE BEEN COMPUTED"
700  !
710  !      STORAGE OF F-COEFFS
720  INPUT "MUST THE F-COEFFS BE STORED ON TAPE? Y/N",Q$
730  IF Q$="Y" THEN CALL F_store(M,Interval,Time1,N,T,B0,A(*),B(*))
740  !
750  !      COMPUTATION OF AMPL RATIO [Ar(K)],AND PHASE LAG [Phl(K)]
760 Amp: L=0
770  FOR K=1 TO N
780    Ar(K)=T*SQR(A(K)^2+B(K)^2)
790    DEG
800    Phl(K)=-ATN(A(K)/B(K))
810    IF K=1 THEN Ok
820    IF (Phl(K)>0) AND (Phlk<0) THEN L=L+1
830 Ok:    Phlk=Phl(K)
840    Phl(K)=Phl(K)-L*180                !ATN() GIVES PRINCIPAL VALUE
850    RAD
860  NEXT K
870  Foldf=PI/Interval                    !FOLDING FREQUENCY
880  PRINT "AR AND PH LAG HAVE BEEN COMPUTED"
890  !
900  !      PRINTOUT OF FREQUENCY ANALYSIS DATA
910  INPUT "IS A PRINTOUT OF F-COEFFS,AR,AND PH LAG REQUD? Y/N",Q$
920  IF Q$="N" THEN Plot
930  PRINTER IS 7,1,WIDTH(130)
940  FIXED 3
950  PRINT LIN(3),,,CHR$(27);"&dD";"FREQUENCY ANALYSIS DATA";CHR$(27);"&d@",LIN(1)
960  PRINT "K","FREQ[rad/s]","A(K)","B(K)","AR","LOG AR","P LAG[deg]"
970  PRINT "-","-----------","----","----","--","------","----------"
980  PRINT 0,0,0,B0,1,0,0
990  FOR K=1 TO N
1000   PRINT K,Freq(K),A(K),B(K),Ar(K),LGT(Ar(K)),Phl(K)
1010 NEXT K
1020 PRINT LIN(1),"FOLDING FREQ=";Foldf;"[rad/sec]"
1030 PRINT "NUMBER OF READINGS=";M
1040 PRINT "FOURIER INTERVAL 2T=";2*T
1050 STANDARD
1060 PRINTER IS 16
1070 !
1080 !      PLOT OF FREQUENCY ANALYSIS DATA
1090 Plot:FOR K=1 TO N
1100   Num(K)=K
1110 NEXT K
1120 INPUT "IS A PLOT OF THE F-COEFFS REQUD? Y/N",Q$
1130 IF Q$="Y" THEN CALL Plot(Num(*),A(*),N,"k","a(k),b(k)","FOURIER COEFFICIENTS",X0,Xfct,Y0,Yfct)
1140 IF Q$="Y" THEN CALL Plot(Num(*),B(*),N,"k","a(k),b(k)","FOURIER COEFFICIENTS",X0,Xfct,Y0,Yfct)
1150 INPUT "IS A PLOT OF A.R.vs FREQ REQUD? Y/N",Q$
1160 IF Q$="Y" THEN CALL Plot(Freq(*),Ar(*),N,"FREQUENCY [rad/sec]","A.R.","AMPLITUDE RATIO PLOT",X
0,Xfct,Y0,Yfct)
1170 INPUT "IS A PLOT OF LOG AR vs FREQ REQUD? Y/N",Q$
1180 IF Q$="N" THEN Phase
1190 MAT Ar=LGT(Ar)
1200 CALL Plot(Freq(*),Ar(*),N,"FREQUENCY [rad/sec]","LOG AR","AMPLITUDE RATIO PLOT",X0,Xfct,Y0,Yfc
t)
1210 Phase:INPUT "IS A PLOT OF PH-LAG vs FREQ REQUD? Y/N",Q$
1220 IF Q$="Y" THEN CALL Plot(Freq(*),Phl(*),N,"FREQUENCY [rad/sec]","PHASE LAG [DEG]","PHASE LAG P
LOT",X0,Xfct,Y0,Yfct)
1230 !
1240 !      CORRECTION OF F-COEFFS FOR INJECTION
1250 INPUT "MUST THE F-COEFFS BE CORRECTED FOR INJECTION? Y/N",Q$
1260 IF Q$="N" THEN Ser
1270 MAT A2=A
1280 MAT B2=B
1290 CALL F_retrieve(M1,I1,T1,N1,T1,B10,A1(*),B1(*))
1300 IF N1<>N THEN PRINT "ERROR: NUMBER OF COEFFS DIFFER"
1310 IF T1<>T THEN PRINT "INTERVAL LENGTHS (T) DIFFER"
1320 IF (B10<>1/(2*T)) OR (B0<>1/(2*T)) THEN PRINT "ERROR:B0#1/2T"
1330 FOR K=1 TO N
1340   D=B1(K)^2+A1(K)^2
1350   A(K)=(B1(K)*A2(K)-A1(K)*B2(K))/(T*D)
1360   B(K)=(B1(K)*B2(K)+A1(K)*A2(K))/(T*D)
1370 NEXT K
1380 !
1390 INPUT "MUST THE NEW F-COEFFS BE STORED ON TAPE? Y/N",Q$
1400 IF Q$="Y" THEN CALL F_store(M,Interval,Time1,N,T,B0,A(*),B(*))
1410 !
1420 !      COMPUTATION OF FOURIER SERIES APPROXIMATION F AND ITS MEAN AND VARIANCE
1430 Ser:INPUT "IS A COMPUTATION OF THE FOURIER SERIES APPROXIMATION REQUD? Y/N",Q$
1440 IF Q$="N" THEN St
```