

https://dspace.org/

Supergravity solutions with AdS(4) from non-Abelian T-dualities

Zayas, L.A.P.; Rodgers, V.G.J.; Whiting, C.A.

2016-02
Springer
http://hdl.handle.net/10539/22027
Zayas, L.A.P., Rodgers, V.G.J. and Whiting, C.A. 2016. Journal of High Energy Physics (1), Article number 061.

Downloaded from a DSpace repository. https://dspace.org/

JHEP

Published for SISSA by Springer

Received: December 5, 2015
Accepted: January 27, 2016
Published: February 9, 2016

Supergravity solutions with AdS_4 from non-Abelian T-dualities

Leopoldo A. Pando Zayas,[a,b] **Vincent G.J. Rodgers**[c] **and Catherine A. Whiting**[c,d]

[a] *The Abdus Salam International Centre for Theoretical Physics, Strada Costiera 11, 34014 Trieste, Italy*

[b] *Michigan Center for Theoretical Physics, Department of Physics, University of Michigan, Ann Arbor, MI 48109, U.S.A.*

[c] *Department of Physics and Astronomy, The University of Iowa, Iowa City, IA 52242, U.S.A.*

[d] *National Institute for Theoretical Physics, School of Physics and Mandelstam Institute for Theoretical Physics, University of the Witwatersrand, Johannesburg, WITS 2050, South Africa*

E-mail: lpandoz@umich.edu, vincent-rodgers@uiowa.edu, catherine.whiting@wits.ac.za

Abstract: We present a large class of new backgrounds that are solutions of type II supergravity with a warped AdS_4 factor, non-trivial axion-dilaton, B-field, and three- and five-form Ramond-Ramond fluxes. We obtain these solutions by applying non-Abelian T-dualities with respect to SU(2) or SU(2)/U(1) isometries to reductions to 10d IIA of 11d sugra solutions of the form $AdS_4 \times Y^7$, with $Y^7 = S^7/\mathbb{Z}_k, S^7, M^{1,1,1}, Q^{1,1,1}$ and $N(1,1)$. The main class of reductions to IIA is along the Hopf fiber and leads to solutions of the form $AdS_4 \times K_6$, where K_6 is Kähler Einstein with $K_6 = \mathbb{CP}^3, S^2 \times \mathbb{CP}^2, S^2 \times S^2 \times S^2$; the first member of this class is dual to the ABJM field theory in the 't Hooft limit. We also consider other less symmetric but susy preserving reductions along circles that are not the Hopf fiber. In the case of $N(1,1)$ we find an additional breaking of isometries in the NAT-dual background. To initiate the study of some properties of the field theory dual, we explicitly compute the central charge holographically.

Keywords: Supergravity Models, AdS-CFT Correspondence

ArXiv ePrint: 1511.05991

Open Access, © The Authors.
Article funded by SCOAP[3].

doi:10.1007/JHEP02(2016)061

JHEP02(2016)061

Contents

1 Introduction **2**

2 Freund-Rubin seeds for solution generating techniques using non-Abelian T-duality **3**

3 Background from $S^7/\mathbb{Z}_k$ **5**
3.1 NATD($\Downarrow_\tau$ ($AdS_4 \times S^7/\mathbb{Z}_k$)) 5

4 Background from $AdS_4 \times S^7$ **7**
4.1 NATD($\Downarrow_{\psi_2}$ ($AdS_4 \times S^7$)) 8

5 Backgrounds from $AdS_4 \times M^{1,1,1}$ **9**
5.1 NATD($\Downarrow_\tau$ ($AdS_4 \times (M^{1,1,1})$)) 10
5.2 NATD of SU(2)/U(1) isometry in $\Downarrow_\tau$ ($AdS_4 \times (M^{1,1,1})$) 11
5.3 Alternate reduction: $\Downarrow_{\phi_1}$ ($AdS_4 \times M^{1,1,1}$) 11
5.3.1 NATD($\Downarrow_{\phi_1}$ ($AdS_4 \times M^{1,1,1}$)) 12

6 Backgrounds from $AdS_4 \times Q^{1,1,1}$ **13**
6.1 Alternate reduction: $\Downarrow_{\phi_i}$ ($AdS_4 \times Q^{1,1,1}$) 15
6.1.1 NATD($\Downarrow_{\phi_i}$ ($AdS_4 \times Q^{1,1,1}$)) 15

7 Backgrounds from $AdS_4 \times N(1,1)$ with less supersymmetry **16**
7.1 NATD($\Downarrow_\beta$ ($AdS_4 \times N(1,1)$)) 17

8 Dual CFT central charge **19**
8.1 Dual CFT central charge for coset NATD 22

9 Discussion and conclusions **25**

A Review of NATD **26**
A.1 SU(2) isometries 26
A.2 Coset spaces: SU(2)/U(1) isometry 28

B Review of supergravity EOM **28**

JHEP02(2016)061

1 Introduction

The AdS/CFT correspondence [1] conjectures a complete equivalence between a field theory and a string theory. This correspondence provides an explicit recipe to connect observable quantities on both sides of the correspondence and it is particularly potent when the field theory is conformal [2–4]. A central conceptual and intuitive role is played by the geometerization of conformal invariance which on the gravity side is realized as the presence of an AdS factor whose isometries coincide with the conformal group of the theory living on the boundary.

More precisely, the conformal group of a field theory in d dimensions is $\mathrm{SO}(d,2)$ for $d > 2$; this group is precisely the isometry group for AdS_{d+1}. This relation has prompted the search for solutions in type II supergravities and M-theory that contain AdS_{d+1} as a space-time factor. Most of the effort has, naturally, been concentrated on searching for solutions with an AdS_5 factor (see, for example, [5, 6] and [7]). There are, however, strong efforts in other dimensions. Recent efforts include AdS_7 [8]. In the context of theories connected to massive IIA, this search can be extended to AdS_6 [9, 10] and more recently to AdS_5 [11] and AdS_4 [12]. Another recent approach to the construction of gravity duals to conformal backgrounds has been presented in [13–15]. There has also been recent progress in the direct search for solutions with AdS_{d+1} factors; interesting examples are the geometries interpolating between AdS_7 and AdS_5 presented in [16, 17] and further studied and generalized in [18, 19].

Recently, there has been a revival of NATD [20–22] including its systematic extension to the Ramond-Ramond sector [23, 24]. This resurrected symmetry has already been used to generate solutions from various seed backgrounds in the context of the AdS/CFT correspondence [25–47]. NATD is a powerful tool in the search for solutions because it can be considered as a solution generating technique. Even more importantly, as shown in for example [39], it leads to supersymmetric solutions with very few apparent isometries other than those of AdS.[1] This makes NATD a unique technique which is able to probe the space of solutions in directions that cannot be accessed by various general classification approaches which are based on explicit symmetries.

The main goal of this paper is to further use T-duality and NATD to construct supergravity backgrounds that contain an AdS_4 factor in the metric. In a very direct sense, this manuscript is an extension of [39] which was devoted to solutions with an AdS_5 factor. There are various interesting applications for conformal field theories in $d = 3$. For example, they have historically played an important role in our understanding of critical phenomena through the ϵ expansion of the Wilson-Fisher fixed point. More formally and recently, 3d conformal theories have helped clarify aspects of the M2 brane theory. We hope that the backgrounds presented in this manuscript will add some interesting new examples and motivate studies of the corresponding field theory duals which are arguably conformal or superconformal field theories in 3d. The quintessential example in this class is the ABJM field theory [48]; the NATD to the gravity background dual to the ABJM theory has been discussed explicitly in [37].

[1] In this paper we present an example where the NATD destroys even more isometries than the previously known examples.

JHEP02(2016)061

$$\mathrm{AdS}_4 \times Y^7 \xrightarrow{Reduction} \Downarrow (\mathrm{AdS}_4 \times Y^7) \xrightarrow{NATD} \mathrm{NATD}(\Downarrow (\mathrm{AdS}_4 \times Y^7))$$

$$F_4 \xrightarrow{Reduction} (F_2, F_4) \xrightarrow{NATD} (F_1, F_3, F_5), H_3$$

$$Q_{D2} = N_{M2} \xrightarrow{Reduction} Q_{D2} = N_{D2} \xrightarrow{NATD} \begin{pmatrix} Q_{D5} = N_{D5} \\ \Delta Q_{D3} = n N_{D5} \end{pmatrix}$$

Figure 1. A schematic description of some properties of the supergravity solutions discussed in this manuscript and their field theory duals.

We summarize our strategy and main results in figure 1. Namely, we generate various families of 10d Type IIB solutions containing an AdS_4 factor by first starting with a Freund-Rubin type solution in 11d supergravity of the form $\mathrm{AdS}_4 \times Y^7$ and subsequently reduce the solution to IIA, which is followed by applying a non-Abelian T-duality to either an SU(2) or an SU(2)/U(1) isometry. Generically, the final Type IIB backgrounds have all of the RR fluxes turned on, with the exception of the very symmetric case of S^7. Finally, we compute the Page charges of these backgrounds and the central charge of the corresponding dual field theories. We also investigate the change in the Page charges under large gauge transformations in B_2.

The paper is organized as follows. In section 2 we sketch the procedure that we apply to generate the various explicit solutions in the rest of the paper. Sections 3, 4, 5, 6 and 7 correspond respectively to the application of our solution generating techniques to: $S^7/\mathbb{Z}_k$, S^7, $M^{1,1,1}$, $Q^{1,1,1}$ and $N(1,1)$. In section 8, we discuss our recipe for computing the central charge of the dual field theory holographically and present the results for one representative background from each section. We also propose a method for determining the ranges of the dual coordinates in backgrounds generated from NATD with coset space isometries, where B_2 is absent. We discuss some arguments for the preservation of supersymmetry in some of the new backgrounds and conclude in section 9. We relegate to appendix A the rules for non-Abelian T-duality and appendix B the form of the 11d and 10d equations of motion as we used them to verify that the new backgrounds obtained in the manuscript are solutions.

2 Freund-Rubin seeds for solution generating techniques using non-Abelian T-duality

In this section we review a construction of solutions to 11-dimensional supergravity based on the Freund-Rubin Ansatz [49]. Essentially every Sasaki-Einstein 7d manifold provides a supersymmetric solution to 11d supergravity. A fairly complete description of solutions of seven dimensional manifolds, providing Freund-Rubin solutions to 11d supegravity, was cataloged in Duff-Nilsson-Pope [50]. The list includes further specification about those

JHEP02(2016)061

which are supersymmetric and states what fraction of the supersymmetry is preserved. An exhaustive list of Sasaki-Einstein seven-dimensional manifolds is presented in [51]. Here we focus on a particular but wide set of solutions, hoping to elucidate some of the most generic aspects for other AdS_4 solutions.

We start by considering Freund-Rubin type solutions to 11d supergravity,

$$\begin{aligned} ds^2 &= ds^2(\mathrm{AdS}_4) + ds^2(Y^7), \\ F &= 3d\Omega_4, \end{aligned} \tag{2.1}$$

where ds_7^2 is an Einstein metric on Y^7. In the case when Y^7 is also Sasaki, the corresponding solution is supersymmetric. In this manuscript we are particularly interested in solutions that are supersymmetric. The metric ds_4^2 could also contain an asymptotically AdS_4 black hole but we do not consider explicitly that case as it breaks supersymmetry. If the metric on Y^7 admits a U(1) isometry, which is always the case in the backgrounds we consider in this paper, it can be written as

$$ds_7^2 = a(y_n)(d\tau + \mathcal{A})^2 + ds^2(K_6), \tag{2.2}$$

where $\mathcal{A}$ is a connection on K_6 and the y_n represent coordinates on K_6. Then we can reduce the corresponding 11d solution to 10d Type IIA via the following rules:

$$\begin{aligned} ds_{11}^2 &= e^{-\frac{2}{3}\Phi} ds_{10}^2 + e^{\frac{4}{3}\Phi}(d\tau + C_{(1)})^2, \\ C_{(3)}^M &= C_{(3)}^{IIA} + B_{(2)} \wedge d\tau. \end{aligned} \tag{2.3}$$

If one is concerned with the correct dimensional scalings while performing the reduction, it is important to note that the 11d backgrounds have l_p as the natural length scale. Therefore the dilaton must pick up this scale to compensate for the factors of the 11d radius, R, inherited from the $a(y_n)$. This can be seen in the examples provided below.

Once we arrive at a Type IIA supergravity background using the prescription in eq. (2.3), we can further apply a Non-Abelian T-duality (NATD) to the corresponding background, provided that the appropriate symmetries are present. We will find that there are generically two cases where we can perform an NATD. In the first case there is a structure resembling an S^3 and in this case we apply the rules of NATD directly as in [23, 25]. In the second case we merely have one or more S^2 subspaces on which we may implement NATD by exploiting the coset structure of S^2 and following the prescriptions and results of [24].

Before we begin the presentation of the backgrounds, let us first define some notational conventions used, and clarify a few points about the NATD gauge fixing. In all of the backgrounds presented in this paper, we choose a gauge fixing for NATD such that all three of the Lagrange multipliers are kept as coordinates in the dual background, i.e. $g = 1$. We will introduce them in terms of spherical polar coordinates, $(\theta_i, \phi_i, \psi) \to (\rho, \chi, \xi)$. We have also left arbitrary the gauge fixing constant that we label m_2, which arises from $v_i \to m_2 v_i$. We direct the reader to appendix A or [25] for more on gauge fixing in NATD. In section 8 we will use the fact that m_2 may be absorbed into the definition of ρ.

JHEP02(2016)061

Note that we will adopt the following conventions for AdS_4, $d\Omega_{\mathrm{AdS}_4}$ and 2-spheres throughout this document:

$$\begin{aligned} ds^2(\mathrm{AdS}_4) &= \frac{r^2}{\mathrm{R}^2}(-dx_0^2+dx_1^2+dx_2^2)+\frac{\mathrm{R}^2}{r^2}dr^2, \\ d\Omega_{\mathrm{AdS}_4} &= -\frac{r^2}{\mathrm{R}^2}dr\wedge dx_0\wedge dx_1\wedge dx_2, \\ ds^2(\Omega_2^{(i)}) &= d\theta_i^2+\sin^2\theta_i d\phi_i^2, \qquad & ds^2(\hat{\Omega}_2) &= d\chi^2+\sin^2\chi d\xi^2, \\ d\Omega_2^{(i)} &= \sin\theta_i d\theta_i\wedge d\phi_i, & d\hat{\Omega}_2 &= \sin\chi d\chi\wedge d\xi. \end{aligned} \tag{2.4}$$

3 Background from $S^7/\mathbb{Z}_k$

We begin our presentation with the background $\mathrm{AdS}_4\times S^7/\mathbb{Z}_k$, which is given by,

$$ds^2=\frac{1}{4}ds^2(\mathrm{AdS}_4)+ds^2(S^7/\mathbb{Z}_k). \tag{3.1}$$

The metric on $S^7/\mathbb{Z}_k$ is:

$$\begin{aligned} ds^2(S^7/\mathbb{Z}_k) &= \lambda_2^2\Big[d\alpha^2+\cos^2\frac{\alpha}{2}ds^2(\Omega_2^{(1)})+\sin^2\frac{\alpha}{2}ds^2(\Omega_2^{(2)}) \\ &\quad +\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}(d\psi+\cos\theta_1\,d\phi_1-\cos\theta_2\,d\phi_2)^2\Big]+\frac{\lambda_3^2}{k^2}(d\tau+k^2\mathcal{A})^2, \\ \mathcal{A} &= \cos\alpha\,d\chi+\lambda_1\cos^2\frac{\alpha}{2}\cos\theta_1\,d\phi_1+\lambda_1\sin^2\frac{\alpha}{2}\cos\theta_2\,d\phi_2. \end{aligned} \tag{3.2}$$

with $\lambda_1=2$, $\lambda_2=\frac{1}{2}$, $\lambda_3=\frac{1}{4}$. The angle τ parameterizes the U(1) fiber over $\mathbb{CP}^3$, defined in the brackets of eq. (3.2) above. The ranges of the angles are $0\le\alpha,\theta_1,\theta_2\le\pi$, $0\le\phi_1,\phi_2\le 2\pi$, $0\le\psi\le 4\pi$ and $0\le\tau\le\frac{2\pi}{k}$. This geometry is supported by an $F_4=-\frac{3}{8\mathrm{R}}\,d\Omega_{\mathrm{AdS}_4}$.

After a reduction to Type IIA via τ, we recover the familiar $\mathrm{AdS}_4\times\mathbb{CP}^3$ background, which we label $\Downarrow_\tau(\mathrm{AdS}_4\times S^7/\mathbb{Z}_k)$ to match our labeling conventions. In addition to the metric, the supergravity background has a dilaton, and 4-form and 2-form field strengths from the Ramond-Ramond (RR) sector,

$$e^{2\Phi}=\frac{\mathrm{R}^3\lambda_3^3}{l_p^3k^3}\,,\qquad F_4=-\frac{3}{8\mathrm{R}}\,d\Omega_{\mathrm{AdS}_4}\,,\qquad F_2=l_pk\,d\mathcal{A}\,. \tag{3.3}$$

Here $d\Omega_{\mathrm{AdS}_4}$ is the volume form on AdS_4 and F_2 is proportional to the Kähler form on $\mathbb{CP}^3$. This IIA background is dual to a supersymmetric field theory with $\mathcal{N}=6$ susy in 3D; therefore the IIA background preserves the same number, 24, supercharges [48].

3.1 NATD($\Downarrow_\tau(\mathrm{AdS}_4\times S^7/\mathbb{Z}_k)$)

In this section, we perform an NATD along the (θ_1,ϕ_1,ψ) directions defining an SU(2) isometry. This NATD was performed in [37], however, the NATD there acted along the (θ_2,ϕ_2,χ) SU(2) isometry. It is true that the two NATD backgrounds are related by a simple map (in the notation of [37] it is $\zeta\to\zeta+\frac{\pi}{2}$). Our results differ slightly, particularly in the factors of k, due to the presence of the warp factor $e^{-\frac{2}{3}\Phi}$ left over from the reduction to

JHEP02(2016)061

IIA, absent from [37]. Our choice here leads to a slightly different structure of singularities in the resulting background.

$$
\begin{aligned}
\hat{ds}^2 &= \frac{\mathrm{R}\lambda_3^2}{4kl_p} ds^2(\mathrm{AdS}_4) + \frac{\lambda_2^2\lambda_3\mathrm{R}^3}{kl_p}\left(\sin^2\frac{\alpha}{2} ds^2(\Omega_2^{(2)}) + d\alpha^2\right) \\
&\quad + \frac{1}{4\Delta k^2\alpha' l_p^2}\Big[4k^2\alpha'^2 l_p^2 m_2^4\rho^2 d\rho^2 + 4\lambda_2^4\lambda_3^2 m_2^2\mathrm{R}^6\cos^4\frac{\alpha}{2}(\cos\chi d\rho - \rho\sin\chi d\chi)^2 \\
&\quad + \lambda_2^4\lambda_3^2 m_2^2\mathrm{R}^6\sin^2\alpha\cos^2\frac{\alpha}{2}\left(\sin^2\chi\left(\rho^2\left(d\xi - \cos\theta_2 d\phi_2\right)^2 + d\rho^2\right)\right. \\
&\quad \left. + \rho^2\cos^2\chi d\chi^2 + \rho\sin 2\chi d\rho d\chi\right)\Big], \\
\hat{B}_2 &= \frac{\lambda_2^2\lambda_3 m_2\mathrm{R}^3}{4\Delta kl_p}\left[\frac{\sin^2\alpha\cos\theta_2}{k^2\alpha'^2 l_p^2}\left(\cos\chi\left(k^2 m_2^2\rho^2\alpha'^2 l_p^2 + \lambda_2^4\lambda_3^2\mathrm{R}^6\cos^4\frac{\alpha}{2}\right)d\rho\right)\wedge d\phi_2\right. \\
&\quad \left. - \lambda_2^4\lambda_3^2\rho\mathrm{R}^6\cos^4\frac{\alpha}{2}\sin\chi d\chi - m_2^2\rho^2\sin\chi\left(\rho\Theta d\chi + 4\cos^4\frac{\alpha}{2}\cos\chi d\rho\right)\wedge d\xi\right], \\
e^{-2\hat{\Phi}} &= \left(\frac{kl_p}{\mathrm{R}\lambda_3}\right)^3\Delta,
\end{aligned}
\tag{3.4}
$$

where we have additionally defined,

$$
\begin{aligned}
\Delta &= \frac{\lambda_2^2\lambda_3\mathrm{R}^3}{4k^3 l_p^3\alpha'^3}\left(k^2 l_p^2 m_2^2\rho^2\alpha'^2\Theta + \lambda_2^4\lambda_3^2\mathrm{R}^6\sin^2\alpha\cos^4\frac{\alpha}{2}\right), \quad \text{and} \\
\Theta &= \left(\sin^2\alpha\cos^2\chi + 4\cos^2\frac{\alpha}{2}\sin^2\chi\right),\ \Gamma = \left(4k^2 m_2^2\rho^2\alpha'^2 l_p^2 + \lambda_2^4\lambda_3^2\mathrm{R}^6\sin^2\alpha\cos^2\frac{\alpha}{2}\right).
\end{aligned}
$$

The RR sector contains the following field strengths:

$$
\begin{aligned}
\hat{F}_1 &= -\frac{km_2 l_p\left(\lambda_1\cos^2\frac{\alpha}{2}(\rho\sin\chi d\chi - \cos\chi d\rho) + \rho\sin\alpha\cos\chi d\alpha\right)}{\sqrt{\alpha'}}, \\
\hat{F}_3 &= \frac{\lambda_2^6\lambda_3^3 m_2^2\rho^2\mathrm{R}^9\sin^3\alpha\cos^4\frac{\alpha}{2}\cos\chi}{4\Delta k^2\alpha'^{5/2} l_p^2} d\alpha\wedge\left(\cos\theta_2\sin\chi d\chi\wedge d\phi_2 - d\hat{\Omega}_2\right) \\
&\quad + \frac{\lambda_1\lambda_2^2\lambda_3 m_2^4\rho^3\mathrm{R}^3\sin^2\alpha\cos^2\frac{\alpha}{2}\cos\chi}{4\Delta\sqrt{\alpha'}} d\rho\wedge\left(\cos\theta_2\sin\chi d\chi\wedge d\phi_2 - d\hat{\Omega}_2\right) \\
&\quad + \frac{\lambda_2^2\lambda_3 m_2^2\rho\mathrm{R}^3\sin\alpha\cos^2\frac{\alpha}{2}\sin^2\chi\Gamma}{4\Delta k^2\alpha'^{5/2} l_p^2} d\alpha\wedge d\rho\wedge\left(\cos\theta_2 d\phi_2 - d\xi\right) \\
&\quad + \left(k\lambda_1 m_2^2\rho\sqrt{\alpha'}\sin^2\frac{\alpha}{2} l_p d\rho + \frac{3\lambda_2^6\lambda_3\mathrm{R}^6\sin^3\alpha d\alpha}{4k\alpha'^{3/2} l_p}\right)\wedge d\Omega_2^{(2)}, \\
\hat{F}_5 &= \frac{\lambda_2^6\lambda_3^2 m_2^3\mathrm{R}^9\rho^2\sin^2\frac{\alpha}{2}\cos^2\frac{\alpha}{2}\sin\alpha}{4\Delta k^2\alpha'^{3/2} l_p^2}\Big[(\lambda_1\lambda_3\sin\alpha d\rho - 3\lambda_2^2\rho\Theta d\alpha)\wedge d\hat{\Omega}_2 \\
&\quad - 12\lambda_2^2\cos^4\frac{\alpha}{2}\sin^2\chi\cos\chi d\alpha\wedge d\xi\wedge d\rho\Big]\wedge d\Omega_2^{(2)} \\
&\quad - d\Omega_{\mathrm{AdS}_4}\wedge\left(\frac{\lambda_1\lambda_2^2\lambda_3^3\mathrm{R}^5\sin\alpha\cos^2\frac{\alpha}{2} d\alpha}{32k^2\alpha'^{3/2} l_p^2} + \frac{3m_2^2\rho\sqrt{\alpha'} d\rho}{8\mathrm{R}}\right).
\end{aligned}
\tag{3.5}
$$

JHEP02(2016)061

The Einstein frame Ricci scalar for this background is (after setting $\mathrm{R}=1, l_p=1, \alpha'=1, m_2=1, k=1$ for simplicity),

$$\begin{aligned}\mathcal{R}_E = & \frac{1}{\left(4096\rho^2\cos^2\frac{\alpha}{2}\cos 2\chi+(2048\rho^2-1)\cos\alpha+2\cos 2\alpha+\cos 3\alpha-6144\rho^2-2\right)^3}\Big[\\ & 4096\ 2^{3/4}\sec^4\frac{\alpha}{2}\Big(-\cos^2\frac{\alpha}{2}\Big(4096\rho^2\cos^2\frac{\alpha}{2}\cos 2\chi+\left(2048\rho^2-1\right)\cos\alpha+2\cos 2\alpha\\ & +\cos 3\alpha-6144\rho^2-2\Big)\Big)^{3/4}\left(65536\rho^4(-\cos\alpha+\cos 2\alpha-8)\right.\\ & -128\rho^2(4\cos\alpha+\cos 2\alpha-21)\cos^6\frac{\alpha}{2}\cos 2\chi\\ & -32\rho^2\cos^4\frac{\alpha}{2}\left(-209\cos\alpha+6\cos 2\alpha+\cos 3\alpha-16384\rho^2\cos 2\chi+266\right)\\ & \left.+\sin^2\frac{\alpha}{2}(16\cos\alpha+\cos 2\alpha-39)\cos^8\frac{\alpha}{2}\right)\Big],\end{aligned} \tag{3.6}$$

which has singularities when $\alpha=\pi$, or when $\alpha=0$ simultaneous with $\rho=0$, or $\alpha=0$ simultaneous with $\chi=0$. This can also be recognized, as in many examples of NATD, in the vanishing of Δ, which is given by the $\det M$ (see eq. (A.8)). These singularities are generated by the NATD, since the $\mathrm{AdS}_4\times S^7/\mathbb{Z}_k$ background and its reduction to Type IIA are smooth. NATD generates singularities whenever the duality is performed along a collapsing cycle, as in Abelian T-duality.

In [37] the Killing spinor of $\mathbb{CP}^3$ was computed and shown to be dependent only on α (ζ in their coordinates) and two constants. More precisely, [37] showed that there are only 2 linearly independent Killing spinors on $\mathbb{CP}^3$ that are independent of the SU(2) angles (θ_1,ϕ_1,ψ) in the required frame. Based on the demonstration given in [38], which showed that supersymmetry is preserved when the Killing spinor of the original background is independent of the isometry direction, we can conclude that this NATD background preserves supersymmetry (at least $\mathcal{N}=2$ in 3d).

4 Background from $\mathbf{AdS_4\times S^7}$

In this section we consider a reduction to IIA along the ψ_2 direction of the $\mathrm{AdS}_4\times S^7$ background, which we define as:

$$\begin{aligned} ds_{11}^2 &= ds^2_{\mathrm{AdS}_4}+ds^2_{S^7},\\ ds_7^2 &= d\mu^2+\frac{1}{4}\sin^2\mu\omega_i^2+\frac{1}{4}\lambda^2(\nu_i+\cos\mu\omega_i)^2,\\ \nu_i &= \sigma_i+\Sigma_i, \qquad \omega_i=\sigma_i-\Sigma_i,\\ F_4 &= -\frac{3}{\mathrm{R}}d\mathrm{Vol}(\mathrm{AdS}_4),\end{aligned} \tag{4.1}$$

where σ_i and Σ_i are left-invariant SU(2) Maurer Cartan 1-forms given by,

$$\begin{aligned}\sigma_1 &= -\sin\psi_1 d\theta_1+\cos\psi\sin\theta_1 d\phi_1,\\ \sigma_2 &= \cos\psi_1 d\theta_1+\sin\psi\sin\theta_1 d\phi_1,\\ \sigma_3 &= \cos\theta_1 d\phi_1+d\psi_1,\end{aligned} \tag{4.2}$$

JHEP02(2016)061

and similarly for the Σ_i, but with coordinates $(\theta_2, \phi_2, \psi_2)$. The range of μ is $0 \leq \mu \leq \pi$, as given in [50]. This background is supersymmetric for $\lambda = 1$, corresponding to the round S^7, and $\lambda = \frac{1}{\sqrt{5}}$ corresponding to the squashed S^7. We will focus on the round S^7. Here we perform the reduction along the U(1) angle ψ_2 defined in the σ's above.[2] The 10D background NS and RR sectors take the form:

$$\begin{aligned}
ds_{10}^2 &= \frac{\mathrm{R}}{l_p}\cos\frac{\mu}{2}\left(ds^2(\mathrm{AdS}_4) + \mathrm{R}^2\left(\sin^2\frac{\mu}{2}\Sigma_i^2 + \cos^2\frac{\mu}{2}ds^2(\Omega_2^{(2)}) + d\mu^2\right)\right), \\
B_2 &= 0, \qquad\qquad e^{2\Phi} = \left(\frac{\mathrm{R}}{l_p}\right)^3\cos^3\frac{\mu}{2}, \\
C_1 &= l_p\cos\theta_2 d\phi_2, \quad F_2 = -l_p d\Omega_2^{(2)}, \\
F_4 &= \frac{3}{\mathrm{R}}d\mathrm{VolAdS}_4.
\end{aligned} \tag{4.3}$$

This Type IIA background has Einstein frame Ricci scalar, $\mathcal{R}_E = -\frac{3l_p^{1/4}(1+15\cos\mu)}{64\mathrm{R}^{9/4}\cos^{9/4}\frac{\mu}{2}}$. The singularity at $\mu = \pi$, however, can be understood by the presence of D6 branes. Indeed, the presence of the F_2 flux suggests a dual C_7 potential along the directions of AdS_4 and Σ_i. More importantly, one can introduce near $\mu = \pi$ the following coordinate: $(\pi - \mu)d\mu^2/2 = dr^2/\sqrt{r}$. In this new coordinate, near the singular point, the metric takes the form

$$ds^2 = \sqrt{r}\left(ds^2(\mathrm{AdS}_4) + \Sigma_i^2\right) + \frac{1}{\sqrt{r}}\left(dr^2 + r^2 ds^2(\Omega_2^{(2))})\right), \tag{4.4}$$

which coincides, precisely, with the metric near a D6 brane source. Under the subsequent NATD transformations this singularity will persist, but we know its origin.

4.1 NATD($\Downarrow_{\psi_2}$ ($\mathbf{AdS_4 \times S^7}$))

We will now perform a NATD along the SU(2) isometry defined by the Σ_i. The NATD metric, B_2, and dilaton have the following form,

$$\begin{aligned}
\hat{ds}^2 &= \frac{\mathrm{R}}{l_p}\cos\frac{\mu}{2}\Big(ds^2(\mathrm{AdS}_4) + \mathrm{R}^2(\cos^2\frac{\mu}{2}ds^2(\Omega_2^{(2)}) + d\mu^2)\Big) \\
&\quad + \frac{m_2^2\mathrm{R}^6\rho^2\sin^2\mu\sin^2\frac{\mu}{2}}{4l_p^2\alpha'\Delta}ds^2(\tilde{\Omega}) + \frac{2l_p m_2^2\alpha'^2}{\mathrm{R}^3\sin\mu\sin\frac{\mu}{2}}d\rho^2, \\
\hat{B}_2 &= \frac{\mathrm{R}^3 m_2^3\rho^3\sin\mu\sin\frac{\mu}{2}}{2l_p\Delta}\sin\chi d\xi\wedge d\chi, \quad e^{-2\hat{\Phi}} = \frac{l_p^3\Delta}{\mathrm{R}^3\cos^3\frac{\mu}{2}}, \\
\Delta &= \frac{\mathrm{R}^3\sin\mu\sin\frac{\mu}{2}}{2l_p^3\alpha'^3}(l_p^2 m_2^2\alpha'^2\rho^2 + \frac{1}{4}\mathrm{R}^6\sin^2\mu\sin^2\frac{\mu}{2}).
\end{aligned} \tag{4.5}$$

The non-trivial dual RR Fluxes are,

$$\hat{F}_3 = \left(3\frac{\mathrm{R}^6}{8l_p\alpha'^{3/2}}\sin^3\mu d\mu + l_p m_2^2\sqrt{\alpha'}\rho d\rho\right)\wedge d\Omega_2^{(2)},$$

[2] Note that it is only U(1) for the round S^7. For the squashed S^7 only ϕ_1 and ϕ_2 are U(1) angles.

JHEP02(2016)061

$$\hat{F}_5 = d\text{Vol}(\text{AdS}_4) \wedge \left[\frac{\text{R}^5 \sin\mu \sin^2\frac{\mu}{2}}{2l_p^2\alpha'^{3/2}}d\mu + \frac{3m_2^2\sqrt{\alpha'}\rho}{\text{R}}d\rho\right] \tag{4.6}$$
$$- \frac{\text{R}^9 m_2^2\rho^2 \sin^3\mu \sin\frac{\mu}{2}}{16l_p^2\alpha'^{3/2}\Delta}\left(3\rho\sin\mu d\mu - 2\sin^2\frac{\mu}{2}d\rho\right) \wedge d\Omega_2^{(2)} \wedge d\tilde{\Omega}_2.$$

After this NATD, the Einstein frame Ricci scalar is,

$$\mathcal{R}_E = -\frac{l_p\alpha'^{3/4}(29 + 164\cos\mu + 63\cos 2\mu)}{32\text{R}^3\sin^{5/2}\mu(4l_p^2m_2^2\alpha'^2\rho^2 + \text{R}^6\sin^2\frac{\mu}{2}\sin^2\mu)^{1/4}},$$

where here we can again see that the singularities that appear correspond to the zeros of Δ ($\mu = 0, \pi$), plus the singularity at $\mu = \pi$, inherited from the Type IIA background.

5 Backgrounds from $\text{AdS}_4 \times M^{1,1,1}$

The space we are concerned with in this section is a U(1) bundle over $\mathbb{CP}^2 \times S^2$ with characteristic numbers n_1 and n_2 and metric given by,

$$\begin{aligned} ds^2 &= c_1^2 ds^2(\text{AdS}_4) + \text{R}^2 ds^2(M^{1,1,1}), \\ ds^2(M^{1,1,1}) &= c_2^2\big(d\tau - n_1\sin^2\mu\sigma_3 + n_2\cos\theta_1 d\phi_1\big)^2 + c_3^3(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2) \\ &\quad + c_4^2\big(d\mu^2 + c_5^2\sin^2\mu(\sigma_1^2 + \sigma_2^2 + \cos^2\mu\sigma_3^2)\big), \end{aligned} \tag{5.1}$$

where $c_1^2 = \frac{3}{2\Lambda}$, $c_2^2 = \frac{3}{32\Lambda}$, $c_3^2 = \frac{3}{4\Lambda}$, $c_4^2 = \frac{9}{2\Lambda}$, $c_5^2 = \frac{1}{4}$. The supersymmetric case corresponds to $n_1 = 3, n_2 = 2$; in the notation used by Duff-Nilson-Pope [50] this space is naturally denoted by $M(-3, 2)$. Here, however, we follow a slightly more modern notation widely used in the literature, $M^{1,1,1}$. The corresponding 11D geometry has Ricci scalar $\mathcal{R} = -\frac{\Lambda}{\text{R}^2}$ and admits a 4-form Flux,

$$C_3 = c_1^3\frac{r^3}{\text{R}^3}dx_0 \wedge dx_1 \wedge dx_2, \qquad F_4 = \frac{3c_1^3}{\text{R}}d\text{Vol}(\text{AdS}_4), \tag{5.2}$$

and $0 \leq \mu \leq \frac{\pi}{2}$. In the IIA reduction along τ, we obtain,

$$\begin{aligned} ds_{IIA}^2 &= \frac{c_2\text{R}^3}{l_p}\Big(c_1^2 ds^2(\text{AdS}_4) + c_4^2(d\mu^2 + c_5^2\sin^2\mu(\sigma_1^2 + \sigma_2^2 + \cos^2\mu\sigma_3^2)) \\ &\quad + c_3^2(d\theta_1^2 + \sin^2\theta_1 d\phi_1^2)\Big), \\ B_2 &= 0, \qquad e^{2\Phi} = \frac{c_2^3\text{R}^3}{l_p^3}. \end{aligned} \tag{5.3}$$

This implies that, $C_3^{IIA} = C_3$, and $C_{(1)} = l_p(-n_1\sin^2\mu\sigma_3 + n_2\cos\theta_1 d\phi_1)$. This reduced metric has Einstein frame Ricci Scalar, $\mathcal{R}_E = 0$. Note that the metric on the six dimensional subspace, which we denote $\Downarrow_\tau (M^{1,1,1})$, is simply the product of $S^2 \times \mathbb{CP}^2$. The solution is supported by the Kähler forms which appear as $dC_{(1)}$. The field theory of this background was examined in [52] where it was shown that this τ reduction of $\text{AdS}_4 \times M^{1,1,1}$ is $\mathcal{N} = 2$ supersymmetric.

JHEP02(2016)061

5.1 NATD($\Downarrow_\tau$ ($\mathbf{AdS_4} \times (M^{1,1,1})$))

We are now poised to perform a Non-Abelian T-duality along the SU(2) isometry made explicit by the SU(2) invariant one-forms σ.

$$
\begin{aligned}
\hat{ds}^2 &= \frac{c_2 \mathrm{R}^3}{l_p}(c_1^2 ds^2(\mathrm{AdS}_4) + c_3^2 ds^2\Omega_2^{(1)} + c_4^2 d\mu^2) \\
&+ \frac{m_2^2}{\alpha' l_p^2 \Delta}\left(c_2^2 c_4^4 c_5^4 \mathrm{R}^6 \sin^4\mu\left(\rho^2 d\xi^2 \cos^2\mu \sin^2\chi + \rho^2 d\chi^2\left(1-\sin^2\mu\cos^2\chi\right)\right.\right. \\
&\left.\left. + d\rho^2\left(1-\sin^2\mu\sin^2\chi\right) - 2\rho d\rho d\chi \sin^2\mu \sin\chi\cos\chi\right) + m_2^2\rho^2\alpha'^2 d\rho^2 l_p^2\right), \\
\hat{B} &= \frac{c_2 c_4^2 c_5^2 m_2^3 \rho^2 \mathrm{R}^3 \sin^2\mu\sin\chi}{4 l_p \Delta} d\xi \wedge \left(\rho\left(-2\sin^2\mu\cos 2\chi + \cos 2\mu + 3\right) d\chi\right. \\
&\left. -4\sin^2\mu\sin\chi\cos\chi d\rho\right), \\
e^{-2\hat{\Phi}} &= \frac{l_p^3 \Delta}{c_2^3 \mathrm{R}^3}, \\
\Delta &= \frac{c_2 c_4^2 c_5^2 \mathrm{R}^3 \sin^2\mu}{\alpha'^3 l_p^3}\Big(\cos^2\mu\left(c_2^2 c_4^4 c_5^4 \mathrm{R}^6 \sin^4\mu + m_2^2\rho^2\alpha'^2 l_p^2\cos^2\chi\right) \\
&+ m_2^2\rho^2\alpha'^2 l_p^2 \sin^2\chi\Big).
\end{aligned}
\tag{5.4}
$$

The RR sector contains,

$$
\begin{aligned}
\hat{F}_1 &= \frac{m_2 n_1 l_p \sin^2\mu(\rho d\chi \sin\chi - \cos\chi(2\rho d\mu\cot\mu + d\rho))}{\sqrt{\alpha'}}, \\
\hat{F}_3 &= \frac{1}{\alpha'^{3/2} l_p}\left(-\frac{3c_2 c_3^2 c_4^4 c_5^3 \mathrm{R}^6 \sin^3\mu\cos\mu}{c_1} d\mu - m_2^2 n_2 \rho \alpha'^2 l_p^2 d\rho\right) \wedge d\Omega_2^{(1)} \\
&+ \frac{c_2 c_4^2 c_5^2 m_2^2 n_1 \rho^2 \mathrm{R}^3 \sin^4\mu\cos^2\mu\sin\chi\cos\chi}{\alpha'^{5/2} l_p^2 \Delta}\left(2c_2^2 c_4^4 c_5^4 \mathrm{R}^6 \sin^3\mu\cos\mu d\mu\right. \\
&\left. - m_2^2\rho\alpha'^2 l_p^2 d\rho\right) \wedge d\xi \wedge d\chi \\
&+ \frac{2 m_2^2 n_1 \rho \sin\mu\cos\mu\sin^2\chi\left(\alpha'^3 l_p^3\Delta - c_2^3 c_4^6 c_5^6 \mathrm{R}^9 \sin^8\mu\cos^2\mu\cos^2\chi\right)}{\alpha'^{5/2} l_p^2\left(\cos^2\mu\cos^2\chi + \sin^2\chi\right)\Delta} d\mu \wedge d\xi \wedge d\rho, \\
\hat{F}_5 &= \frac{c_1^3}{c_3^2 \mathrm{R}\alpha'^{3/2} l_p^2} d\mathrm{Vol}(\mathrm{AdS}_4) \wedge \left(3c_3^2 m_2^2 \rho \alpha'^2 l_p^2 d\rho + c_1 c_2^3 c_4^4 c_5^3 n_2 \mathrm{R}^6 \sin^3\mu\cos\mu d\mu\right) \\
&+ \frac{c_2^2 c_4^6 c_5^5 m_2^3 \rho^2 R^9 \sin^5\mu\cos\mu\sin\chi}{\alpha'^{3/2} l_p^2 \Delta}\left(\frac{3c_3^3\left(\sin^2\mu\sin^2\chi + \cos^2\mu\right)}{c_1} d\mu\right. \\
&\left. - c_2 c_5 n_2 \sin\mu\cos\mu d\rho\right) d\xi \wedge d\chi \wedge d\Omega_2^{(1)} \\
&+ \frac{3 c_2^4 c_3^2 c_4^{10} c_5^9 m_2^3 \rho^2 \mathrm{R}^{15} \sin^{11}\mu\cos\mu\cos\chi}{64 c_1 \alpha'^{15/2} l_p^8 \Delta^3}\left(8 c_2^2 c_4^4 c_5^4 \mathrm{R}^6 \sin^4\mu\cos^2\mu\sin\chi\right. \\
&\left. + m_2^2\rho^2\alpha'^2 l_p^2\Big((\cos 2\mu + 7)\sin\chi - 2\sin^2\mu\sin 3\chi\Big)\right)^2 d\xi \wedge d\mu \wedge d\rho \wedge d\Omega_2^{(1)}.
\end{aligned}
\tag{5.5}
$$

This background has singularities at $\mu = 0$ and $\mu = \frac{\pi}{2}$, simultaneous with $\rho = 0$ or $\chi = 0$, all generated by the NATD.

JHEP02(2016)061

5.2 NATD of SU(2)/U(1) isometry in $\Downarrow_\tau$ ($\mathbf{AdS_4 \times (M^{1,1,1})}$)

After the reduction to IIA of $\mathrm{AdS}_4 \times M^{1,1,1}$ we have a six-dimensional Einstein-Kähler manifold $\Downarrow_\tau (M^{1,1,1}) = \mathbb{CP}^2 \times S^2$. In the previous section we performed an NATD along the SU(2) isometry present in $\mathbb{CP}^2$ (see its explicit form in eq. (5.3)). Since S^2 is a coset manifold and there is a prescription for applying NATD on coset manifolds [24], we now proceed to apply this coset NATD along the S^2 factor. We review this method in appendix A.2. The dual NS fields are given by

$$\begin{aligned}
\hat{ds}^2 &= \frac{c_2 \mathrm{R}^3}{l_p}\left(c_1^2 ds^2(\mathrm{AdS}_4) + c_4^2(d\mu^2 + c_5^2 \sin^2\mu(\sigma_1^2+\sigma_2^2+\sigma_3^2))\right) \\
&\quad + \frac{m_2^2 l_p \alpha'^2}{c_2 c_3^2 \mathrm{R}^3}\left(\frac{z}{\rho}dz + d\rho\right)^2 + \frac{c_2 c_3^2 \mathrm{R}^3}{l_p \rho^2} dz^2, \\
\hat{B} &= 0, \quad e^{-\hat{\Phi}} = \frac{m_2 c_3 l_p \rho}{c_2\sqrt{\alpha'}},
\end{aligned} \tag{5.6}$$

with the RR sector given by,

$$\begin{aligned}
\hat{F}_1 &= -\frac{m_2 n_2 l_p}{\sqrt{\alpha'}} dz, \\
\hat{F}_3 &= m_2^2 n_1 l_p \sqrt{\alpha'} z dz \wedge \left(\sin^2\mu d\Omega_2^{(2)} - 2\sin\mu\cos\mu d\mu \wedge \sigma_3\right), \\
\hat{F}_5 &= \frac{3m_2}{c_1 l_p \sqrt{\alpha'} \mathrm{R}^3}\Big(- c_1^4 m_2 l_p \mathrm{R}^2 \alpha' d\mathrm{Vol}(\mathrm{AdS}_4) \wedge (z dz + \rho d\rho) \\
&\quad + c_2 c_3^2 c_4^4 c_5^3 \mathrm{R}^9 \cos\mu \sin^3\mu dz \wedge d\mu \wedge d\psi \wedge d\Omega_2^{(2)}\Big).
\end{aligned} \tag{5.7}$$

This background has a singularity at $\rho = 0$, as can be seen in the behavior of the dilaton. This is generated by the collapsing cycle in eq. (5.3) given by $\theta_1 \to 0$.

5.3 Alternate reduction: $\Downarrow_{\phi_1}$ ($\mathbf{AdS_4 \times M^{1,1,1}}$)

In this section we present an alternate, supersymmetric reduction to Type IIA of $\mathrm{AdS}_4 \times M^{1,1,1}$ presented originally in [53]. We then perform an NATD on this background, thus providing a new supersymmetric Type IIB solution with an AdS_4 factor.

In [53] it was shown that reducing along the U(1) ϕ_1 angle yields a supersymmetric Type IIA background of the following form,[3]

$$\begin{aligned}
ds^2 &= \sqrt{A} c_1^2 ds^2(\mathrm{AdS}_4) + ds_6^2, \\
B &= 0, \qquad e^{2\Phi} = \frac{A^{3/2}\mathrm{R}^3}{l_p^3}
\end{aligned} \tag{5.8}$$

where

$$ds_6^2 = \frac{\mathrm{R}^3}{l_p}\sqrt{A}\left[c_4^2 d\mu^2 + c_3^2 d\theta_1^2 + c_4^2 c_5^2 \sin^2\mu(\sigma_1^2+\sigma_2^2+\cos^2\mu\sigma_3^2) + c_2^2 c_3^2 \sin^2\theta_1 (d\tau - n_1 \sin^2\mu\sigma_3)^2\right], \tag{5.9}$$

and $A = c_2^2 n_2^2 \cos^2\theta_1 + c_3^2 \sin^2\theta_1$.

[3] using the notation in eq. (5.1) above.

JHEP02(2016)061

The RR sector contains the $F_4 = -\frac{3c_1^3}{\mathrm{R}} d\mathrm{Vol}(\mathrm{AdS}_4)$ flux proportional to the differential volume on AdS_4, while the reduction produces an additional C_1 and F_2:

$$
\begin{aligned}
C_1 &= c_2^2 n_2 l_p \frac{\cos\theta_1}{A}(d\tau - n_1 \sin^2\mu\sigma_3),\\
F_2 &= \frac{c_2^2 n_2 l_p}{A^2}\Big[n_1 \sin\mu\Big(A\cos\theta_1(\sin\mu d\Omega_2^{(2)} - 2\cos\mu d\mu\wedge(\cos\theta_2 d\phi_2 + d\psi))\\
&\quad + W\sin\mu(d\Omega_2^{(2)} - d\psi\wedge d\theta_1)\Big) + W\, d\tau\wedge d\theta_1\Big].
\end{aligned}
\tag{5.10}
$$

5.3.1 NATD($\Downarrow_{\phi_1}$ ($\mathbf{AdS_4}\times \boldsymbol{M^{1,1,1}}$))

Now we move on to present the Non-Abelian T-dual with respect to an SU(2) isometry given in the σ's of eq. (5.9) above.

$$
\begin{aligned}
\hat{ds}^2 &= \sqrt{A}c_1^2 ds^2(\mathrm{AdS}_4) + \frac{\mathrm{R}^3}{l_p}\sqrt{A}\big[c_4^2 d\mu^2 + c_3^2 d\theta_1^2 + \frac{c_2^2 c_3^2 c_4^4 c_5^2 \sin^2\theta_1}{Q} d\tau^2\big] + \hat{ds}_3^2,\\
\hat{ds}_3^2 &= \frac{m_2^2\sin^4\mu}{\alpha' l_p^2}\Bigg(\frac{c_4^2 c_5^2 \mathrm{R}^3\alpha'^3\sqrt{A}l_p^3\csc^6\mu\sec^2\mu(\cos\chi d\rho - \rho\sin\chi d\chi)^2}{V}\\
&\quad + \frac{\left(c_4^2 c_5^2 Q\mathrm{R}^6\cos\mu(\sin\chi d\rho + \rho\cos\chi d\chi) + m_2^2\rho^2\alpha'^2 l_p^2\csc^4\mu\sec\mu\sin\chi d\rho\right)^2}{\Delta V}\\
&\quad + \frac{c_4^2 c_5^2\rho^2\mathrm{R}^6\sec^2\mu\sin^2\chi\left(Q\cos^2\mu d\xi - c_2^2 c_3^2 n_1\sin^2\theta_1 d\tau\right)^2}{\Delta Q}\Bigg),\\
B &= \frac{\sqrt{A}c_2^2 c_3^2 c_4^4 c_5^4 m_2 n_1\rho\mathrm{R}^9\sin^2\theta_1\sin^6\mu\sin\chi d\tau\wedge d\chi}{\Delta\alpha'^2 l_p^3}\\
&\quad + \frac{c_2^2 c_3^2 m_2 n_1\mathrm{R}^3\sin^2\theta_1\sin^2\mu\cos\chi\left(Ac_4^4 c_5^4\mathrm{R}^6\sin^4\mu + m_2^2\rho^2\alpha'^2 l_p^2\right)d\rho\wedge d\tau}{\sqrt{A}\Delta\alpha'^2 l_p^3}\\
&\quad + \frac{m_2^3\rho^3\mathrm{R}^3\sin^2\mu\sin^3\chi\left(Ac_4^2 c_5^2 + Q\cos^2\mu\cot^2\chi\right)d\xi\wedge d\chi}{\sqrt{A}\Delta l_p}\\
&\quad + \frac{m_2^3\rho^2\mathrm{R}^3\sin^2\mu\sin^2\chi\cos\chi\left(Q\cos^2\mu - Ac_4^2 c_5^2\right)d\xi\wedge d\rho}{\sqrt{A}\Delta l_p},\\
e^{-2\Phi} &= \frac{l_p^3}{\mathrm{R}^3 A^{3/2}}\tilde{\Delta},
\end{aligned}
\tag{5.11}
$$

with $\Delta = \frac{\mathrm{R}^3\sin^2\mu}{\alpha'^3\sqrt{A}l_p^3}\left(c_4^2 c_5^2 m_2^2\rho^2\alpha'^2 A l_p^2\sin^2\chi + Q\cos^2\mu\left(c_4^4 c_5^4\mathrm{R}^6 A\sin^4\mu + m_2^2\rho^2\alpha'^2 l_p^2\cos^2\chi\right)\right)$, $Q = c_4^2 c_5^2 A + c_2^2 c_3^2 n_1^2\sin^2\theta_1\tan^2\mu$, $V = c_4^2 c_5^2 Q\mathrm{R}^6 + m_2^2\rho^2\alpha'^2 l_p^2\csc^4\mu\sec^2\mu\sin^2\chi$, and $W = A\sin\theta_1 + A'\cos\theta_1$. This NATD generates singularities at $\mu = 0$ and $\mu = \frac{\pi}{2}$, with $\rho = 0$, or $\chi = 0$.

The RR sector fields take the following form,

$$
\begin{aligned}
F_1 = \frac{c_2^2 m_2 n_1 n_2}{A\sqrt{\alpha'}}\Bigg(&\frac{1}{A}\rho l_p\sin^2\mu\cos\chi W d\theta_1\\
&- \cos\theta_1 l_p\sin^2\mu\cos\chi d\rho + \rho\cos\theta_1 l_p\sin^2\mu\sin\chi d\chi - \rho\cos\theta_1 l_p\sin\mu\cos\mu\cos\chi d\mu\Bigg),
\end{aligned}
$$

JHEP02(2016)061

$$
\begin{aligned}
F_3 = &\Big(\frac{3c_2c_3^2c_4^4c_5^3\mathrm{R}^6}{c_1 l_p \alpha'^{3/2}}\cos\mu\sin^3\mu\sin\theta_1 d\mu + \frac{c_2^2c_4^2c_5^2m_2^2n_2 l_p\sqrt{\alpha'}}{AQ}W\rho d\rho\Big)\wedge d\theta_1\wedge d\tau\\
&+\frac{c_2^2m_2^4n_1n_2\mathrm{R}^3}{\sqrt{\alpha'}A^{3/2}\Delta M}\cos\theta_1\sin^4\mu\cos\chi\sin\chi\big(\cos^2\mu Q d\xi - c_2^2c_3^2n_1\sin^2\theta_1 d\tau\big)\wedge d\rho\wedge d\chi\\
&+\frac{c_2^2c_4^2c_5^2m_2^2n_1n_2\rho\mathrm{R}^3W\sin^4\mu\sin\chi}{A^{3/2}\Delta\alpha'^{5/2}l_p^2}\big(\sin\chi\big(c_4^2c_5^2Q\mathrm{R}^6\sin^4\mu\cos^2\mu + m_2^2\rho^2\alpha'^2l_p^2\big)d\rho\\
&+c_4^2c_5^2\rho Q\mathrm{R}^6\sin^4\mu\cos^2\mu\cos\chi d\chi\big)\wedge d\xi\wedge d\theta_1\\
&-\frac{c_2^4c_3^2c_4^2c_5^2m_2^2n_1^2n_2\rho\mathrm{R}^3W\sin^2\theta_1\sin^2\mu\sin\chi}{A^{3/2}\Delta Q\alpha'^{5/2}l_p^2}\big(m_2^2\rho^2\alpha'^2l_p^2\tan^2\mu\sin\chi d\rho\\
&+c_4^2c_5^2Q\mathrm{R}^6\sin^6\mu\,(\rho\cos\chi d\chi+\sin\chi d\rho)\big)\wedge d\tau\wedge d\theta_1\\
&+\frac{c_2^2c_4^2c_5^2m_2^2n_1n_2\rho\mathrm{R}^3\cos\theta_1\sin^3\mu\cos\mu}{\sqrt{A}\Delta\alpha'^{5/2}l_p^2}\big(c_4^2c_5^2\rho Q\mathrm{R}^6\sin^4\mu\cos^2\mu\sin 2\chi d\chi\\
&+2\sin^2\chi\big(c_4^2c_5^2Q\mathrm{R}^6\sin^4\mu\cos^2\mu+m_2^2\rho^2\alpha'^2l_p^2\big)d\rho\big)\wedge d\mu\wedge d\xi\\
&-\frac{2c_2^4c_3^2m_2^2n_1^2n_2\rho\mathrm{R}^3\sin^2\theta_1\cos\theta_1\sin^3\mu\cos\mu\cos\chi}{A^{3/2}\Delta\alpha'^{5/2}l_p^2}\big(Ac_4^4c_5^4\rho\mathrm{R}^6\sin^4\mu\sin\chi d\chi\\
&+\cos\chi\big(Ac_4^4c_5^4\mathrm{R}^6\sin^4\mu+m_2^2\rho^2\alpha'^2l_p^2\big)d\rho\big)\wedge d\mu\wedge d\tau,\\
F_5 = &\frac{c_1^3}{\mathrm{R}\alpha'^{3/2}}d\mathrm{Vol}(\mathrm{AdS}_4)\wedge\Big[-\frac{\mathrm{R}^6}{l_p^2}c_1c_2^3c_3^2c_5n_1^2n_2\sin 2\theta_2\sin^4\mu d\theta_1 + 3m_2^2\alpha'^2\rho d\rho\\
&+\frac{\mathrm{R}^6}{l_p^2}\frac{c_1c_2c_4^4c_5^3n_2}{c_3^2}\cos\mu\sin^3\mu\csc\theta_1 W d\mu\Big]\\
&+\frac{c_2c_4^4c_5^3m_2^3\mathrm{R}^9\cos\mu\sin^5\mu\sin\chi}{c_1l_p^2\alpha'^{3/2}\sqrt{A}\Delta}\Big(c_4^2c_5^2A\sin\theta_1\Big(3c_3^2\sin\chi d\theta_1\wedge d\tau\wedge d\xi\wedge[d\mu\wedge\\
&(\cos\chi d\rho+\rho\sin\chi d\chi)+c_1c_2c_5n_2\cos\mu\sin\mu d\rho\wedge d\chi]\Big)\\
&+c_3^2\sin\theta_1\Big(3\cos^2\mu\cos\chi Q d\theta_1\wedge d\mu\wedge\xi\wedge d\tau\wedge(\sin\chi d\rho+\rho\cos\chi d\chi)\\
&+c_1c_2^3c_5n_1^2n_2\sin 2\theta_1\sin^2\mu d\mu\wedge d\xi\wedge d\rho\wedge d\tau\wedge d\chi\Big)\\
&+c_1c_2c_4^2c_5^3n_2\cos\theta_1\cos\mu\sin\mu A' d\theta_1\wedge d\xi\wedge d\rho\wedge d\tau\wedge d\chi\Big).
\end{aligned}
\tag{5.12}
$$

6 Backgrounds from $\mathbf{AdS_4\times Q^{1,1,1}}$

The $Q(n_1,n_2,n_3)$ spaces (Equation 9.2.12 in Duff-Nilsson-Pope) [50] are defined as:

$$
ds^2 = c^2\left(d\tau+\sum_{i=1}^{3}n_i\cos\theta_i d\phi_i\right)^2+\sum_{i=1}^{3}\frac{1}{\Lambda_1}(d\theta_i^2+\sin^2\theta_i d\phi_i^2).
\tag{6.1}
$$

The $n_1=n_2=n_3=1$ is $\mathcal{N}=2$ supersymmetric. Aspects of the supersymmetry were first presented in [54]. In [55] the field theory dual to $\mathrm{AdS}_4\times Q^{1,1,1}/\mathbb{Z}_k$ was studied and various gauge invariant chiral operators were matched geometrically.

JHEP02(2016)061

The 11D metric on $\mathrm{AdS}_4 \times Q^{1,1,1}$ is defined such that

$$ds^2 = t_1^2 ds^2(\mathrm{AdS}_4) + \mathrm{R}^2 ds^2(Q^{1,1,1}), \tag{6.2}$$

where we define, $t_1^2 = \frac{3}{2\Lambda}$, $t_2^2 = \frac{3}{4\Lambda}$, $t_3^2 = \frac{3}{8\Lambda}$, and

$$ds^2_{Q^{1,1,1}} = t_3^2(d\tau - \mathcal{A})^2 + t_2^2(ds^2(\Omega_2^{(1)}) + ds^2(\Omega_2^{(2)}) + ds^2(\Omega_2^{(3)})), \tag{6.3}$$

where $\mathcal{A} = (\alpha - \cos\theta_1)d\phi_1 + (\beta - \cos\theta_2)d\phi_2 + (\gamma - \cos\theta_3)d\phi_3$, and the $ds^2(\Omega_2^{(i)})$ are 2-spheres with coordinates θ_i, ϕ_i. The 11D Ricci scalar is $\mathcal{R} = -\frac{\Lambda}{L^2}$, and the F_4 flux is normalized according to $F_4 = \frac{3t_1^3}{\mathrm{R}} d\mathrm{Vol}(\mathrm{AdS}_4)$. The reduction to IIA along the τ angle yields the metric of the form,

$$\begin{aligned} ds^2 &= \frac{\mathrm{R}t_3}{l_p}\left[t_1^2 ds^2(\mathrm{AdS}_4) + t_2^2 \mathrm{R}^2 (ds^2(\Omega_2^{(i)}))\right], \\ e^{2\Phi} &= \frac{\mathrm{R}^3 t_3^3}{l_p^3}, \quad C_1 = l_p \mathcal{A}, \end{aligned} \tag{6.4}$$

along with the same F_4 carried through the reduction. The total Ricci scalar vanishes for this background, $\mathcal{R} = 0$.

The NATD of the background in eq. (6.4) when dualizing along the $(\mathrm{SU}(2)/\mathrm{U}(1))^3$ isometry was found in [24]. Here we present the full results for dualizing along just one of the S^2's, i.e. one $(\mathrm{SU}(2)/\mathrm{U}(1))$ isometry. Furthermore, we pay attention to the factors of R, the 11D radius, l_p and α' throughout the dualization. We review general aspects of NATD with coset spaces in appendix A.2 and refer the reader there for more details. We find that,

$$\begin{aligned} d\hat{s}^2 &= \frac{t_1^2 t_3 \mathrm{R}}{l_p} ds^2(\mathrm{AdS}_4) + \left(\frac{l_p \alpha'^2 m_2^2}{\mathrm{R}^3 t_2^2 t_3 \cos^2\chi} + \frac{\mathrm{R}^3 t_2^2 t_3 \tan\chi^2}{l_p \rho^2}\right) d\rho^2 \\ &\quad + \frac{\mathrm{R}^3 t_2^2 t_3}{l_p}\left[ds^2(\Omega_2^{(1)}) + ds^2(\Omega_2^{(2)}) + d\chi^2 + \frac{2\tan\chi}{\rho} d\rho d\chi\right], \\ \hat{B} &= 0, \quad e^{-\hat{\Phi}} = \frac{l_p m_2 t_2 \rho \cos\chi}{t_3 \sqrt{\alpha'}}. \end{aligned} \tag{6.5}$$

This background has Einstein frame Ricci scalar, $\mathcal{R}_E = \frac{\mathrm{R}^6 t_1^2 t_2^4 t_3^2 + \alpha'^2 l_p^2 m_2^2 \rho^2 (7t_1^2 - 12t_2^2)}{2\alpha'^{7/4} \mathrm{R}^3 l_p^{3/2} m_2^{5/2} \rho^{5/2} t_1^2 t_2^{5/2} \sqrt{t_3} \cos^{\frac{5}{2}}\chi}$. The RR sector has all fields turned on,

$$\begin{aligned} \hat{F}_1 &= -\frac{l_p}{\sqrt{\alpha'}} m_2 (\sin\chi d\rho + \rho\cos\chi d\chi), \\ \hat{F}_3 &= l_p \sqrt{\alpha'} m_2^2 \rho d\rho \wedge (d\Omega_2^{(1)} + d\Omega_2^{(2)}), \\ \hat{F}_5 &= -\frac{3 m_2 t_1^3 \sqrt{\alpha'} \rho}{\mathrm{R}} d\mathrm{Vol}(\mathrm{AdS}_4) \wedge d\rho \\ &\quad - \frac{3 m_2 t_2^6 t_3 \mathrm{R}^6}{t_1 l_p \sqrt{\alpha'}} (\sin\chi d\rho + \rho\cos\chi d\chi) \wedge d\Omega_2^{(1)} \wedge d\Omega_2^{(2)}. \end{aligned}$$

This background has singularities at $\rho = 0$ and $\chi = 0$, both generated by the NATD.

JHEP02(2016)061

6.1 Alternate reduction: $\Downarrow_{\phi_i}$ ($\mathbf{AdS_4 \times Q^{1,1,1}}$)

In this section, we perform a different reduction to Type IIA on the $\mathrm{AdS}_4 \times Q^{1,1,1}$ background, in such a way that two SU(2) isometries are manifest. In order to see these isometries, we must set the parameters appearing in the Kähler form, α, β, γ, to zero. We must also make use of the relations between the t_i's such that $t_1 = 2t_3$, and $t_2 = \sqrt{2}t_3$. (NB: the equations of motion are satisfied for any value of t_3.)

Here we will present explicitly the reduction along ϕ_3, however from the symmetry of $Q^{1,1,1}$ it is evident that reducing along any of the ϕ_i will yield backgrounds of the same general form.

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\big[4t_3 ds^2(\mathrm{AdS}_4) + \mathrm{R}^2 2t_3^2\big(d\Omega_2^{(1)} + d\Omega_2^{(2)} + d\theta_3^2 \\ &\quad + \frac{\sin^2\theta_3}{A}(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + d\psi)^2\big)\big], \\ B_2 &= 0, \quad e^{2\Phi} = \frac{t_3^3 \mathrm{R}^3 A^{3/2}}{2\sqrt{2} l_p^3}, \end{aligned} \tag{6.6}$$

where $A = 3 - \cos 2\theta_3$. We can rewrite the metric above in terms of the σ_i,

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\big[4t_3 ds^2(\mathrm{AdS}_4) + \mathrm{R}^2 2t_3^2\big(d\Omega_2^{(1)} + \sigma_1^2 + \sigma_2^2 + d\theta_3^2 \\ &\quad + \frac{2\sin^2\theta_3}{A}(\cos\theta_1 d\phi_1 + \sigma_3)^2\big)\big]. \end{aligned} \tag{6.7}$$

The RR sector contains,

$$\begin{aligned} C_1 &= \frac{l_p 2\cos\theta_3}{A}(\cos\theta_1 d\phi_1 + \cos\theta_2 d\phi_2 + d\psi), \ F_2 = dC_1 \\ F_4 &= -\frac{24 t_3^3}{\mathrm{R}} d\Omega_{\mathrm{AdS}_4}. \end{aligned} \tag{6.8}$$

6.1.1 NATD($\Downarrow_{\phi_i}$ ($\mathbf{AdS_4 \times Q^{1,1,1}}$))

Now we will present the results for an NATD on the Type IIA background found in the previous section. The NATD background we will present will be along the (θ_2, ϕ_2, ψ) SU(2) isometry, however the results from the (θ_1, ϕ_2, ψ) SU(2) isometry can easily be read off by replacing $(\theta_1 \to \theta_2, \phi_1 \to \phi_2)$. It is given by,

$$\begin{aligned} \hat{ds}^2 &= e^{\frac{2}{3}\Phi}\big[4t_3 ds^2(\mathrm{AdS}_4) + \mathrm{R}^2 2t_3^2\big(d\Omega_2^{(1)} + d\theta_3^2\big)\big] + \hat{ds}_3^2, \\ \hat{ds}_3^2 &= \frac{m_2^2}{l_p^2 \alpha'}\Bigg[\frac{\left(\cos\chi\left(2A\mathrm{R}^6 t_3^6 + m_2^2\rho^2\alpha'^2 l_p^2\right) d\rho - 2A\rho\mathrm{R}^6 t_3^6 \sin\chi d\chi\right)^2}{\Delta\Xi} \\ &\quad + \frac{\sqrt{2}\sqrt{A}\mathrm{R}^3 t_3^3 \alpha'^3 l_p^3 (\sin\chi d\rho + \rho\cos\chi d\chi)^2}{\Xi} \\ &\quad + \frac{4\rho^2 \mathrm{R}^6 t_3^6 \sin^2\theta_3 \sin^2\chi \left(\cos\theta_1 d\phi_1 + d\xi\right)^2}{\Delta}\Bigg], \\ \hat{B}_2 &= \frac{\sqrt{2} t_3^3 m_2 \mathrm{R}^3}{l_p \Delta\sqrt{A}}\Bigg[m_2^2\rho^2\Big(\rho\left(A\sin^2\chi + 2\sin^2\theta_3\cos^2\chi\right) d\hat{\Omega}_2 + 2\sin^2\chi\cos\chi d\rho\wedge d\xi\Big) \\ &\quad + \frac{2\sin^2\theta_3\cos\theta_1}{\alpha'^2 l_p^2}\Big(2A\rho\mathrm{R}^6 t_3^6 \sin\chi d\chi - \cos\chi d\rho\left(2A\mathrm{R}^6 t_3^6 + m_2^2\rho^2\alpha'^2 l_p^2\right)\Big)\wedge d\phi_1\Bigg], \end{aligned}$$

JHEP02(2016)061

$$e^{-2\hat{\Phi}} = \frac{2\sqrt{2}l_p^3}{\mathrm{R}^3 t_3^3 A^{3/2}}\Delta, \tag{6.9}$$
$$\Delta = \frac{\sqrt{2}t_3^3\mathrm{R}^3}{\sqrt{A}l_p\alpha'^3}\left(4t_3^6\mathrm{R}^6 A\sin^2\theta_3 + l_p^2 m_2^2\alpha'^2\rho^2(2\cos^2\chi\sin^2\theta_3 + A\sin^2\chi)\right),$$

and $\Xi = 2A\mathrm{R}^6 t_3^6 + m_2^2\rho^2\alpha'^2 l_p^2\cos^2\chi$. The RR sector contains the following fluxes,

$$\begin{aligned}
\hat{F}_1 &= \frac{2m_2 l_p\left(A\cos\theta_3\cos\chi d\rho - \rho\left(\cos\chi\left(A\sin\theta_3 + \cos\theta_3 A'\right)d\theta_3 + A\cos\theta_3\sin\chi d\chi\right)\right)}{A^2\sqrt{\alpha'}},\\
\hat{F}_3 &= \frac{2}{\alpha'^{3/2}l_p}\left(\frac{m_2^2\rho\alpha'^2\cos\theta_3 l_p^2 d\rho}{A} - 6\mathrm{R}^6 t_3^6\sin\theta_3 d\theta_3\right)\wedge d\Omega_2^{(1)}\\
&\quad + \frac{2\sqrt{2}m_2^4\rho^3\mathrm{R}^3 t_3^3\sin^2\theta_3\cos\theta_3\sin 2\chi}{A^{3/2}\Delta\sqrt{\alpha'}}\left(d\xi - \cos\theta_1 d\phi_1\right)\wedge d\rho\wedge d\chi\\
&\quad - \frac{2\sqrt{2}m_2^2\rho\mathrm{R}^3 t_3^3\sin\chi\left(A\sin\theta_3 + \cos\theta_3 A'\right)}{A^{3/2}\Delta\alpha'^{5/2}l_p^2}(d\xi + \cos\theta_1 d\phi_1)\wedge d\theta_3\wedge\\
&\quad \left(4\rho\mathrm{R}^6 t_3^6\sin^2\theta_3\cos\chi d\chi + \sin\chi\left(m_2^2\rho^2\alpha'^2 l_p^2 + 4\mathrm{R}^6 t_3^6\sin^2\theta_3\right)d\rho\right),\\
\hat{F}_5 &= 8t_3^3\left[\frac{4\mathrm{R}^5 t_3^6\sin\theta_3\cos\theta_3 d\theta_3}{\alpha'^{3/2}l_p^2} - \frac{3m_2^2\rho\sqrt{\alpha'}d\rho}{\mathrm{R}}\right]\wedge d\Omega_{\mathrm{AdS}_4}\\
&\quad - \frac{4\sqrt{2}m_2^3\rho^2\mathrm{R}^9 t_3^9\sin\theta_3\sin\chi}{\sqrt{A}\Delta\alpha'^{3/2}l_p^2}\Big[3\rho\left(A\sin^2\chi + 2\sin^2\theta_3\cos^2\chi\right)d\chi\wedge d\theta_3\\
&\quad - 3\sin 2\chi d\rho\wedge d\theta_3 - 2\sin\theta_3\cos\theta_3 d\rho\wedge d\chi\Big]\wedge d\xi\wedge d\Omega_2^{(1)}
\end{aligned} \tag{6.10}$$

A simple check of the zero's of Δ for this background reveals that $\theta_3 = 0$ with $\rho = 0$ or $\chi = 0$ are the singular points (generated by the NATD). We have also checked this by examining the Einstein frame Ricci scalar.

7 Backgrounds from $\mathbf{AdS_4}\times N(1,1)$ with less supersymmetry

The $N(k,l)$ spaces are SU(3)/U(1) cosets where the U(1) acts as $diag(\exp(ik\theta),\exp(il\theta),\exp(-i(k+l)\theta))$. The particular case of $N(1,1)$ is supersymmetric and the metric can be viewed as an SO(3) bundle over $\mathbb{CP}^2$,

$$\begin{aligned}
ds^2 &= d\mu^2 + \frac{1}{4}\sin^2\mu(\sigma_1^2 + \sigma_2^2 + \cos^2\mu\sigma_3^2)\\
&\quad + \lambda^2\left[(\Sigma_1 - \cos\mu\sigma_1)^2 + (\Sigma_2 - \cos\mu\sigma_2)^2 + \left(\Sigma_3 - \frac{1}{2}(1+\cos^2\mu)\sigma_3\right)^2\right],
\end{aligned} \tag{7.1}$$

with $\mathcal{R} = -\frac{27}{5L^2}$. There are two values of λ yielding supersymmetric solutions, $\lambda^2 = 1/2$ corresponds to $\mathcal{N} = 3$ and $\lambda^2 = 1/10$ corresponds to $\mathcal{N} = 1$ susy. In this section we will focus on the more supersymmetric version of the background but an analogous treatment can be applied to the less supersymmetric case. We expect a similar situation in the case

JHEP02(2016)061

of the squashed S^7 which also preserves less supersymmetry compared to the round sphere background. We will not pursue this direction here but it is certainly an interesting one.

In this section we consider the metric on $\mathrm{AdS}_4 \times N(1,1)$ with $\lambda^2 = \frac{1}{2}$, which corresponds to the case of $\mathcal{N} = 3$ supersymmetry. There are several U(1) angles with which one can perform the reduction to IIA. We have examined all of the possible reductions. The one that leads to a tractable solution is the U(1) isometry given by the linear combination of $\psi_1 + \psi_2$. We label α to be the coordinate corresponding to $\psi_1 - \psi_2$.

The reduced background is given by,

$$\begin{aligned} ds^2 &= e^{\frac{2}{3}\Phi}\Big[\frac{1}{2}ds^2(\mathrm{AdS}_4) + \mathrm{R}^2\Big(d\mu^2 + \frac{1}{4}\sin^2\mu ds^2(\Omega_2^{(2)}) + \\ &\quad + \frac{1}{2}\left((s_2 - \cos\mu d\theta_2)^2 + (s_1 - \cos\mu\sin\theta_2 d\phi_2)^2\right) + \frac{2}{Z}\cos\mu(s_3 - \cos\theta_2 d\phi_2)^2\Big)\Big], \\ B_2 &= 0, \quad e^{2\Phi} = \frac{\mathrm{R}^3\sin^3\mu Z^{3/2}}{512 l_p^3} \end{aligned} \tag{7.2}$$

in which we have recovered an SU(2) isometry in $(\theta_1, \phi_1, \alpha)$, characterised by the Maurer-Cartan forms s_i. We also define $Z = 3 + \cos 2\mu$. The RR fluxes are given by,

$$\begin{aligned} C_1 &= \frac{l_p}{Z}((-5 + \cos 2\mu)d\alpha - 8\cos\theta_1 d\phi_1 + 4\cos\theta_2\sin^2\mu d\phi_2), \qquad F_2 = dC_1 \\ F_4 &= -\frac{3}{2\sqrt{2}\mathrm{R}}d\Omega_{\mathrm{AdS}_4}. \end{aligned} \tag{7.3}$$

7.1 NATD($\Downarrow_\beta$ ($\mathbf{AdS_4 \times N(1,1)}$))

In this section, we present the results of a Non-Abelian T-duality applied to the SU(2) isometry in eq. (7.2) above. This background is somewhat unique compared to the other backgrounds presented in this paper and other NATD backgrounds presented thus far in the literature. It is unique in the sense that there is mixing between the spectator coordinates (θ_2, ϕ_2) with the Maurer Cartan forms, s_1, s_2. Mixing terms with s_3 have been fairly common (ex. the ABJM, Klebanov Witten backgrounds), however, the non-symmetric mixing with s_1, s_2 leads to a rich Type IIB NATD background. An interesting consequence of this mixing is the breaking of the U(1) isometry normally found in the ξ coordinate after NATD.

$$\begin{aligned} \hat{ds}^2 &= e^{\frac{2}{3}\Phi}\left[\frac{1}{2}ds^2(\mathrm{AdS}_4) + \mathrm{R}^2\left(d\mu^2 + \frac{1}{4}\sin^2\mu ds^2(\Omega_2^{(2)})\right)\right] + \hat{ds}_3^2, \\ \hat{ds}_3^2 &= \frac{m_2^2\mathrm{R}^6\sin^2\mu}{64\Delta\alpha' l_p^2}\Big[\cos^2\mu\rho^2\left(\cos\mu\sin\xi\cos\chi d\theta_2 - \mathrm{G}d\phi_2 + \sin\chi d\xi\right)^2 \\ &\quad + \frac{1}{4K}\left(\rho\mathrm{K}\left(\cos\mu\left(\sin\theta_2\sin\xi d\phi_2 - \cos\xi d\theta_2\right) + d\chi\right) - 2\sin^2\mu\sin\chi\cos\chi d\rho\right)^2\Big] \\ &\quad + \frac{16 m_2^2\sqrt{Z}\alpha'^2 l_p\csc\mu}{\mathrm{R}^3\mathrm{K}}d\rho^2, \end{aligned}$$

JHEP02(2016)061

$$
\begin{aligned}
\hat{B}_2 = \frac{m_2 \mathrm{R}^3 \sin\mu}{1024\Delta\sqrt{Z}\alpha'^2 l_p^3}\Big[&\big(\mathrm{H}\rho\mathrm{R}^6 Z\sin^2\mu\cos^3\mu d\theta_2 - \mathrm{G}\rho\mathrm{R}^6 Z\sin^2\mu\cos^2\mu d\chi\\
&+ \mathrm{J}\mathrm{R}^6 Z\sin^2\mu\cos^2\mu d\rho\big)\wedge d\phi_2 + 64 m_2^2\rho^2\alpha'^2 l_p^2\Big(K d\hat{\Omega}_2\\
&-\big(\mathrm{M}\cos\mu d\phi_2 + \sin^2\mu\sin\chi\sin 2\chi d\xi + Z\cos\mu\sin\xi\sin\chi d\theta_2\big)\wedge d\rho\Big)\\
&+\mathrm{R}^6 Z\cos^3\mu\Big(\rho\sin^2\mu\cos\xi\sin\chi d\xi\wedge d\theta_2 + \sin^2\mu\sin\xi\sin\chi d\rho\wedge d\theta_2\\
&+\rho\sin^2\mu\sin\xi\cos\chi d\chi\wedge d\theta_2 - \rho\sin\theta_2\sin^2\mu\sin\xi\sin\chi d\xi\wedge d\phi_2\Big)\Big],\\
e^{-2\hat{\Phi}} = \frac{512 l_p^3}{\mathrm{R}^3\sin^3\mu Z^{3/2}}\Delta,&\qquad (7.4)\\
\Delta = \frac{\mathrm{R}^3\sin\mu}{1024 l_p^3\sqrt{Z}}\Big(&\mathrm{R}^6\cos^2\mu\sin^2\mu Z + 64 l_p^2\alpha'^2 m_2^2\rho^2(4\cos^2\mu\cos^2\chi+\sin^2\chi Z)\Big).
\end{aligned}
$$

To make the presentation of the background slightly more succinct we have had to define a number of functions,

$$
\begin{aligned}
K &= 4\cos^2\mu\cos^2\chi + Z\sin^2\chi,\\
G &= \cos\theta_2\sin\chi - \sin\theta_2\cos\mu\cos\xi\cos\chi,\\
H &= \cos\theta_2\cos\xi\sin\chi - \sin\theta_2\cos\mu\cos\chi,\\
J &= \sin\theta_2\cos\mu\cos\xi\sin\chi + \cos\theta_2\cos\chi,\\
M &= 4\cos\theta_2\cos\mu\cos\chi + Z\sin\theta_2\cos\xi\sin\chi,\\
N &= \sin^2\mu\sin^2\chi + 2\cos^2\mu,\\
V &= 64 m_2^2\rho^2\alpha'^2 l_p^2\sin^2\chi + \mathrm{R}^6\sin^2\mu\cos^2\mu,\\
X &= \mathrm{R}^9\sin^5\mu\cos^2\mu\cos^2\chi - 512\Delta\sqrt{Z}\alpha'^3 l_p^3.
\end{aligned}
\qquad (7.5)
$$

The reduction to Type IIA along $\psi_1+\psi_2$ generates a singularity at $\mu=0$, while the NATD adds singular points at $\mu=\frac{\pi}{2}$ with $\rho=0$ or $\chi=0$ to the dual background.

The dual RR sector can be written as,

$$
\begin{aligned}
\hat{F}_1 = &\frac{8 m_2 l_p}{K Z^2\sqrt{\alpha'}}\left(K\rho Z\sin\chi d\chi - 2\cos\chi\left(2K\rho\sin\mu\cos\mu d\mu\right.\right.\\
&\left.\left.+Z\left(\sin^2\mu\sin^2\chi + 2\cos^2\mu\right)d\rho\right)\right),\\
\hat{F}_3 = &\frac{\mathrm{R}^9 m_2^2\rho^2\cos^3\mu\sin^4\mu\cos\chi}{32 Z^{3/2}\Delta l_p^2\alpha'^{5/2}}\Big[d\mu\wedge d\chi\wedge\big(\mathrm{G}d\phi_2 - \cos\mu\cos\chi\sin\xi d\theta_2\big)\\
&+\sin\chi d\mu\wedge d\xi\wedge\Big(d\chi - \cos\mu\cos\xi d\theta_2 + \cos\mu\sin\xi\sin\theta_2 d\phi_2\Big)\Big]\\
&+\frac{m_2^2\mathrm{N}\rho\cos^3\mu\sin^2\mu\sin\chi\cos\chi\mathrm{R}^9}{64\sqrt{Z}\Delta\mathrm{K} l_p^2\alpha'^{5/2}}d\xi\wedge d\rho\wedge\Big(\sin\mu\cos\xi d\theta_2 - \sin\xi\sin\theta_2 d\phi_2\Big)\\
&+\frac{m_2^2\mathrm{R}^9\rho^2\cos^3\mu\sin^3\mu\sin^2\chi}{128\sqrt{Z}\Delta l_p^2\alpha'^{5/2}}d\xi\wedge d\chi\wedge\Big(-\cos\xi d\theta_2 + \sin\xi\sin\theta_2 d\phi_2\Big)\\
&+\frac{m_2^2\mathrm{R}^3\rho\cos\mu\sin\mu}{128\sqrt{Z}\Delta l_p^2\alpha'^{5/2}}d\chi\wedge d\theta_2\wedge\Big(\mathrm{R}^6\mathrm{H}\rho\cos^2\mu\sin^2\mu\sin\chi d\phi_2 - \mathrm{V}\sin\xi d\rho\Big)
\end{aligned}
$$

JHEP02(2016)061

$$
\begin{aligned}
&- \frac{\mathrm{R}^3 \cos\mu \sin\mu}{128 Z^{3/2} \Delta l_p^2 \alpha'^{5/2}} \Big[4\mathrm{R}^3 \sin^2\mu \left(\mathrm{H}\mathrm{R}^3 \rho^2 \cos\chi \sin\mu m_2^2 \cos^3\mu \right. \\
&\left. + 3 Z^{3/2} \Delta \sin\theta_2 l_p \alpha' \right) d\mu \wedge d\theta_2 \wedge d\phi_2 \\
&- \rho m_2^2 \left(256 \rho^2 \cos\mu \cos\chi \cos\theta_2 \sin\chi l_p^2 m_2^2 \alpha'^2 + \mathrm{V} Z \cos\xi \sin\theta_2 \right) d\rho \wedge d\chi \wedge d\phi_2 \Big] \\
&+ \frac{2\rho^3 \cos^2\mu \cos\chi \sin\mu \sin\chi m_2^4 d\xi \wedge d\rho \wedge d\chi \mathrm{R}^3}{Z^{3/2} \Delta \sqrt{\alpha'}} \\
&- \frac{\rho m_2^2}{64 Z^{3/2} \Delta \mathrm{K} l_p^2 \alpha'^{5/2}} \left(\frac{1}{4} \mathrm{R}^9 Z \cos^2\mu \sin^3\mu \left(-Z \sin^2\mu \sin^2\chi \sin\theta_2 \right. \right. \\
&\left. 4 \cos\mu \cos\chi \left(\mathrm{H}\mathrm{N} - \cos\mu \cos\chi \sin^2\mu \sin\theta_2 \right) \right) \\
&-16 \mathrm{R}^3 \rho^2 \mathrm{K}^2 \sin^3\mu \sin\theta_2 l_p^2 m_2^2 \alpha'^2 \Big) d\rho \wedge d\theta_2 \wedge d\phi_2 \\
&+ \frac{m_2^2 \mathrm{X} \rho \sin\chi}{16 Z^{3/2} \Delta \mathrm{K} l_p^2 \alpha'^{5/2}} d\mu \wedge d\rho \wedge \Big(\cos^2\mu \cos\chi \sin\mu \sin\xi d\theta_2 - \mathrm{G} d\phi_2 + \sin\chi d\xi \Big), \\
\hat{F}_5 = &\frac{m_2^3 \rho^2 \mathrm{R}^9 \sin^3\mu \cos\mu}{256 \Delta \sqrt{Z} \alpha'^{3/2} l_p^2} \Big[\left(-\frac{3}{2} K \sin\mu d\mu + \cos\mu \left(\sin^2\mu - 2\cos^2\mu \right) d\rho \right) \wedge d\hat{\Omega}_2 \\
&+ 3 \sin^3\mu \sin^2\chi \cos\chi d\mu \wedge d\xi \wedge d\rho \Big] \wedge d\Omega_2^{(2)} \\
&+ \left(\frac{3 m_2^2 \rho \sqrt{\alpha'} d\rho}{2\sqrt{2}\mathrm{R}} - \frac{\mathrm{R}^5 \sin\mu \cos\mu \left(\sin^2\mu - 2\cos^2\mu \right)}{128\sqrt{2} \alpha'^{3/2} l_p^2} d\mu \right) \wedge d\Omega_{\mathrm{AdS}_4}. \qquad (7.6)
\end{aligned}
$$

8 Dual CFT central charge

As a first step into the interpretation of the new backgrounds we produce in this manuscript we perform an analysis of the central charge of the dual field theories. In order to compute the dual field theory central charge, we first consider the quantized Page charges [56, 57], defined by

$$
Q_{M2} = \frac{1}{2\kappa_{11}^2 T_{M2}} \int_{\Sigma_7} \star F_4 = M2, \qquad (8.1)
$$

$$
Q_{Dp} = \frac{1}{2\kappa_{10}^2 T_{Dp}} \int_{\Sigma_{8-p}} \left(\sum_i F_i \right) \wedge e^{-B_2} = N_{Dp}, \qquad (8.2)
$$

where $\kappa_{11}^2 = (2\pi)^8 l_p^9$, $\kappa_{10}^2 = (2\pi)^7 \alpha'^4$ and the brane tensions are $T_{M2} = \frac{1}{(2\pi)^2 l_p^3}$, and $T_{Dp} = \frac{1}{(2\pi)^p \alpha'^{\frac{p+1}{2}}}$. We will present the general results of the Page charges and central charge for a few sample backgrounds considered in this manuscript. In particular, we discuss the importance of the factors of the 11d radius R, and their role in the N scaling of the central charge. Our method for computing the central charge is based on [58], as adjusted and generalized in [39]. The main modifications take into consideration a potential dependence on the coordinates of the internal manifold perpendicular to the field theory directions. A generic string frame metric in type II string theory, dual to a QFT in $(d+1)$-dimensions is defined to have the following form,

$$
ds^2 = a dz_{1,d}^2 + abdr^2 + \mathrm{R}^2 g_{ij} d\theta^i d\theta^j. \qquad (8.3)
$$

JHEP02(2016)061

As in [39], we define the modified internal volume to be,

$$\hat{V}_{\text{int}} = \int d\vec{\theta} \sqrt{e^{-4\Phi} \det[g_{\text{int}}] a^d}, \tag{8.4}$$

so that the function $\hat{H}$ is in general given by,

$$\hat{H} = \hat{V}_{\text{int}}^2. \tag{8.5}$$

Then, the central charge for a QFT in $(d+1)$ spacetime dimensions is defined to be [58]:

$$c = d^d \frac{b^{d/2} \hat{H}^{(2d+1)/2}}{G_N (\hat{H}')^d}. \tag{8.6}$$

The G_N factor is needed to cancel the length dimensions in $\hat{H}$. We will use $G_{11} = l_p^9$, and $G_{10} = l_s^8 = \alpha'^4$. In our case of AdS_4, the functions are,

$$a = \frac{r^2}{R^2} f(\theta^i), \quad b = \frac{R^4}{r^4}, \quad d = 2. \tag{8.7}$$

In the case of 11 spacetime dimensions, we obtain $\hat{V}_{\text{int}} \sim \mathrm{R}^9$, $\hat{H} \sim \mathrm{R}^{18} r^3$, therefore, $c \sim \frac{L^9}{l_p^9}$. Due to our choice of Ansatz, eq. (2.1), the relevant objects are always M2 branes, whose normalization yields a scaling relation, $\mathrm{R}^6 \sim l_p^6 N_{M2}$, which means $c \sim N_{M2}^{3/2}$, as known previously in [4], for example. In all of the reductions to Type IIA considered in this paper the F_4 flux comes through the reduction unaffected and an $F_2 = dC_{(1)}$ flux is generated. The normalization of the $D2$ branes leads to a relation, $\mathrm{R}^6 \sim l_p \alpha'^{5/2} N_{D2}$. The 'raw' central charge after the reduction now scales like, $c_{IIA} \sim \frac{\mathrm{R}^9}{l_p \alpha'^4}$, therefore $c_{IIA} \sim \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$. We also note the presence of D6 branes in the Type IIA backgrounds, however they are independent of R and instead are scaled with factors of l_p only, suggesting they are topological charges. In the case of ABJM, the factor of k present in F_2 has the field theory interpretation as the level number of the dual gauge theory.

The relevant objects to the central charge after the NATD are the $D5$ branes, as they always inherit a term with the R^6 scaling factor. As observed previously in [37] for the ABJM background, we see that the effect of NATD is to preserve the $N^{3/2}$ scaling, but to change the numerical coefficient of the central charge.

After the NATD we wish to constrain the range of the dual coordinate ρ, so that we may compute the internal Volume, $\hat{V}_{\text{int}}$. Following the prescription first hinted at in [32], and further discussed in [37, 39], we compute the periodic quantity b_0 defined by,

$$b_0 = \frac{1}{4\pi^2 \alpha'} \oint_{\Sigma_2} \hat{B}_2 \subset [0, 1]. \tag{8.8}$$

The relevant two-cycle is,

$$\Sigma_2 = [\chi, \xi]. \tag{8.9}$$

JHEP02(2016)061

In each case we restrict to a submanifold, which we specify below:

Background	Submanifold
NATD($\Downarrow_\tau$ ($AdS_4 \times S^7/\mathbb{Z}_k$))	$\alpha = 0$, ρ fixed
NATD($\Downarrow_{\psi_2}$ ($AdS_4 \times S^7$))	$\mu = 0$
NATD($\Downarrow_\tau$ ($AdS_4 \times M^{1,1,1}$))	$\mu = 0$
NATD($\Downarrow_{\phi_3}$ ($AdS_4 \times Q^{1,1,1}$))	$\theta_3 = 0$, ρ fixed
NATD($\Downarrow_\beta$ ($AdS_4 \times N(1,1)$)	$\mu = 0$, ρ fixed

We should point out that in all of these cases the cycle is placed on a singularity, which could be problematic. However, the strategy still leads to reasonable results and was also observed in the case of NATD of $AdS_5 \times S^5$ in [39]. Presumably in a full homology theory, it would be possible to show independence of this procedure on position in the manifold, but we leave that question to future investigations. For the remainder of this section we assume the prescription can be trusted, and move on to compute,

$$b_0 = \frac{1}{4\pi^2\alpha'} \oint_{\Sigma_2} (\alpha' \rho \sin\chi) = \frac{\rho}{\pi} \subset [0,1], \tag{8.10}$$

where $0 \leq \chi \leq \pi$, $0 \leq \xi \leq 2\pi$, and we have absorbed the m_2 gauge fixing constant into the definition of ρ. However, as discussed in [46], there is one NS-five brane every time ρ crosses integer multiples of π. Therefore we take the range of ρ from $0 \leq \rho \leq (n+1)\pi$. Next we compute $H_3 = d\hat{B}$,

$$H_3 = \alpha' \sin\chi d\rho \wedge d\xi \wedge d\chi. \tag{8.11}$$

Following [46], we normalize the NS flux to N_{NS5} using $T_{NS5} = \frac{1}{(2\pi)^5}$,

$$Q_{NS5} = \frac{1}{2\kappa_{10}^2 T_{NS5}} \int H_3 = (n+1) = N_{NS5} \tag{8.12}$$

We are now in a position to compute the Page charges and dual field theory central charge after the NATD. The results are summarized in tables 1, 2, 3, 4, and 5 below for the backgrounds with $S^7/\mathbb{Z}_k$,[4] S^7, $M^{1,1,1}$, $Q^{1,1,1}$, and $N(1,1)$, respectively. In the first column we present the flux normalization result, obtained from eq. (8.1) and the corresponding cycle used to integrate. The second column contains the 'Raw' central charge, which is directly computed from eq. (8.6). The third and final column is the substitution of the flux normalization into the 'Raw' central charge. In the cases where we have left some of the numerical constants generalized, we also present the result with the values of these constants replaced.

As noted in [37], NATD maps integer charges onto non-integer charges, due to a violation of the condition, $T_{p-n} = (2\pi)^n T_p$ on the D brane tensions. In fact, we see a generic difference of $\frac{\pi}{2}$ between the D2 and D5 brane normalizations in these cases. In the

[4] A careful reader will notice a factor of 2 difference between our results for the central charge and what was found in [37] for the ABJM background. This is due to the Jacobian factor when changing coordinates from $\alpha \to 2\zeta$.

JHEP02(2016)061

Background	Page Charge Cycle	'Raw' CC	CC
$\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$	$Q_{M2} = \left(\frac{\mathrm{R}}{l_p}\right)^6 \frac{2\lambda_2^6\lambda_3}{k^2\pi^2} = N_{M2}$ $\Sigma_7 = [\alpha, \theta_1, \phi_1, \theta_2, \phi_2, \psi, \tau]$	$\frac{4\lambda_2^6\lambda_3\pi^4}{3k^2}\left(\frac{\mathrm{R}}{l_p}\right)^9$	$\frac{\sqrt{2}\pi^7 k}{3\lambda_2^3\sqrt{\lambda_3}} N_{M2}^{3/2}$ $= \frac{16}{3}\sqrt{2}\pi^7 k N_{M2}^{3/2}$
$\Downarrow_\tau (\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k)$	$Q_{D2} = \frac{\mathrm{R}^6}{l_p\alpha'^{5/2}} \frac{2\lambda_2^6\lambda_3}{k\pi^2} = N_{D2}$ $\Sigma_6 = [\alpha, \theta_1, \phi_1, \theta_2, \phi_2, \psi]$	$\frac{2\lambda_2^6\lambda_3\pi^3}{3k}\frac{\mathrm{R}^9}{l_p\alpha'^4}$	$\frac{\pi^6\sqrt{k}}{3\sqrt{2}\lambda_2^3\sqrt{\lambda_3}}\frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$ $= \frac{8}{3}\sqrt{2}\pi^6\sqrt{k}\frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$
$\mathrm{NATD}(\Downarrow_\tau (\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k))$	$Q_{D5} = \frac{\mathrm{R}^6}{l_p\alpha'^{5/2}} \frac{\lambda_2^6\lambda_3}{k\pi} = N_{D5}$ $\Sigma_3 = [\alpha, \theta_1, \phi_1]$	$\frac{m_2^3\pi^5\lambda_2^6\lambda_3}{18k}\frac{\mathrm{R}^9}{l_p\alpha'^4}$	$\frac{m_2^2\pi^6\sqrt{k_5}}{9\sqrt{2}\lambda_2^3\sqrt{\lambda_3}} N_{NS5}^2 N_{D5}^{3/2}$ $= \frac{8\sqrt{2}m_2^2\pi^6\sqrt{k_5}}{9} N_{NS5}^2 N_{D5}^{3/2}$

Table 1. Results for $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$ and its supergravity duals.

case of $\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$, we note, as in [37], that after the NATD, k is no longer a well-defined level. We define a new level for the dual theory according to,

$$k_5 = \frac{1}{2\kappa_{10}^2 T_{D5}} \int_{\Sigma_3} (\hat{F}_3 - \hat{B}_2 \wedge \hat{F}_1) = \frac{l_p}{\sqrt{\alpha'}} \frac{m_2^2\pi^3}{2} k N_{NS5}^2, \tag{8.13}$$

where the cycle of integration is $\Sigma_3 = [\rho, \theta_1, \phi_1]$, and ρ is integrated over $[0, (n+1)\pi]$. This leads to a N_{NS5}^2 scaling[5] in the central charge, which differs from the other cases. In the cases without k, a factor of N_{NS5}^3 arises from integration over ρ^2 in $\hat{V}_{\mathrm{int}}$.

Finally, we consider changes in the Page charges under large gauge transformations in B_2, particularly $\Delta B_2 = n\pi\alpha' \sin\chi d\chi \wedge d\xi$. The Page charge associated to the $D5$ branes is always either zero or independent of R on the cycle of interest. Therefore, we compute ΔQ_{D3} using

$$\Delta Q_{D3} = -\frac{1}{2\kappa_{10}^2 T_{D3}} \int_{\Sigma_5} \left(-\Delta B_2 \wedge \hat{F}_3 + \frac{1}{2}\Delta B_2 \wedge \Delta B_2 \wedge \hat{F}_1 \right), \tag{8.14}$$

and $\Sigma_5 = \Sigma_3 + [\chi, \xi]$. In all of the cases examined, we find the relation,

$$\Delta Q_{D3} = n Q_{D5}. \tag{8.15}$$

8.1 Dual CFT central charge for coset NATD

In this section, we present a proposal to investigate the effect of large gauge transformations on cases where the NATD is performed on an SU(2)/U(1) coset isometry. The same method used above in eq. (8.8) cannot be directly applied here, as there is no 2-cycle with which to compute b_0 and thus, restrict the range of the dual coordinates. We propose

[5] We are thankful to Niall Machpherson for pointing this out to us.

JHEP02(2016)061

Background	Page Charge Cycle	'Raw' CC	CC
$\mathrm{AdS}_4 \times S^7$	$Q_{M2} = \left(\frac{\mathrm{R}}{l_p}\right)^6 \frac{2}{\pi^2} = N_{M2}$ $\Sigma_7 = [\mu, \theta_1, \phi_1, \psi_1, \theta_2, \phi_2, \psi_2]$	$\frac{32\pi^4 \mathrm{R}^9}{3l_p^9}$	$\frac{8\sqrt{2}}{3}\pi^7 N_{M2}^{3/2}$
$\Downarrow_{\psi_2} (\mathrm{AdS}_4 \times S^7)$	$Q_{D2} = \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} \frac{1}{\pi^2} = N_{D2}$ $\Sigma_6 = [\mu, \theta_1, \phi_1, \psi_1, \theta_2, \phi_2]$	$\frac{8\pi^3}{3} \frac{\mathrm{R}^9}{l_p \alpha'^4}$	$\frac{8\pi^6}{3} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$
$\mathrm{NATD}(\Downarrow_{\psi_2} (\mathrm{AdS}_4 \times S^7))$	$Q_{D5} = \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} \frac{1}{2\pi} = N_{D5}$ $\Sigma_3 = [\mu, \theta_2, \phi_2]$	$\frac{2m_2^3\pi^5}{9} \frac{\mathrm{R}^9}{l_p \alpha'^4} N_{NS5}^3$	$\frac{4\sqrt{2}m_2^3\pi^{13/2}}{9} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{NS5}^3 N_{D5}^{3/2}$

Table 2. Results for $\mathrm{AdS}_4 \times S^7$ and its supergravity duals.

Background	Page Charge Cycle	'Raw' CC	CC
$\mathrm{AdS}_4 \times M^{1,1,1}$	$Q_{M2} = \left(\frac{\mathrm{R}}{l_p}\right)^6 \frac{3c_2c_3^3c_4^4c_5^3}{c_1\pi^6} = N_{M2}$ $\Sigma_7 = [\mu, \theta_1, \phi_1, \theta_2, \phi_2, \psi, \tau]$	$16c_1^2c_2c_3^2c_4^4c_5^3\pi^4 \left(\frac{\mathrm{R}}{l_p}\right)^9$	$\frac{16\pi^7 c_1^{7/2}}{3\sqrt{3}\sqrt{c_2}c_3c_4^2c_5^{3/2}} N_{M2}^{3/2}$ $= \frac{128\pi^7}{9\sqrt{3}} N_{M2}^{3/2}$
$\Downarrow_\tau (\mathrm{AdS}_4 \times M^{1,1,1})$	$Q_{D2} = \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} \frac{3c_2c_3^3c_4^4c_5^3}{2c_1\pi^2} = N_{D2}$ $\Sigma_6 = [\mu, \theta_1, \phi_1, \theta_2, \phi_2, \psi]$	$4c_1^2c_2c_3^2c_4^4c_5^3\pi^3 \frac{\mathrm{R}^9}{l_p \alpha'^4}$	$\sqrt{\frac{2}{3}} \frac{8c_1^{7/2}\pi^6}{3\sqrt{c_2}c_3c_4^2c_5^{3/2}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$ $= \sqrt{\frac{2}{3}} \frac{64\pi^6}{9} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$
$\mathrm{NATD}(\Downarrow_\tau (\mathrm{AdS}_4 \times M^{1,1,1}))$	$Q_{D5} = \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} \frac{4c_1\pi}{3c_2c_3^2c_4^4c_5^3} = N_{D5}$ $\Sigma_3 = [\mu, \theta_1, \phi_1]$	$\frac{c_1^2c_2c_3^3c_4^4c_5^3\pi^5}{3} \frac{R^9}{l_p \alpha'^4} N_{NS5}^3$	$\frac{8c_1^{7/2}\pi^{13/2}}{9\sqrt{3}\sqrt{c_2}c_3c_4^2c_5^{3/2}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{NS5}^3 N_{D5}^{3/2}$ $= \frac{64\pi^{13/2}}{27\sqrt{3}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{NS5}^3 N_{D5}^{3/2}$

Table 3. Results for $\mathrm{AdS}_4 \times M^{1,1,1}$ and its supergravity duals.

JHEP02(2016)061

Background	Page Charge Cycle	'Raw' CC	CC
$\mathrm{AdS}_4 \times Q^{1,1,1}$	$Q_{M2} = \left(\frac{\mathrm{R}}{l_p}\right)^6 \frac{12t_2^6 t_3}{\pi^2 t_1} = N_{M2}$ $\Sigma_7 = [\mu, \theta_1, \phi_1, \theta_2, \phi_2, \psi, \tau]$	$64\pi^4 t_1^2 t_2^6 t_3 \left(\frac{\mathrm{R}}{l_p}\right)^9$	$\frac{8\pi^7 t_1^{7/2}}{3\sqrt{3} t_2^3 \sqrt{t_3}} N_{M2}^{3/2}$ $= \frac{32\pi^7}{3\sqrt{3}} N_{M2}^{3/2}$
$\Downarrow_{\phi_3} (\mathrm{AdS}_4 \times Q^{1,1,1})$	$Q_{D2} = \frac{\mathrm{R}^9}{l_p \alpha'^{5/2}} \frac{6t_2^6 t_3}{\pi^2 t_1} = N_{D2}$ $\Sigma_6 = [\theta_3, \theta_1, \phi_1, \theta_2, \phi_2, \psi]$	$16\pi^3 t_1^2 t_2^6 t_3 \frac{\mathrm{R}^9}{l_p \alpha'^4}$	$\sqrt{\frac{2}{3}} \frac{4\pi^6 t_1^{7/2}}{3t_2^3 \sqrt{t3}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$ $= \sqrt{\frac{2}{3}} \frac{16\pi^6}{3} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$
$\mathrm{NATD}(\Downarrow_{\phi_3} (\mathrm{AdS}_4 \times Q^{1,1,1}))$	$Q_{D5} = \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} \frac{24t_3^6}{\pi} = N_{D5}$ $\Sigma_3 = [\theta_3, \theta_1, \phi_1]$	$\frac{256 m_2^3 t_3^9 \pi^5}{3} \frac{R^9}{l_p \alpha'^4} N_{NS5}^3$	$\sqrt{\frac{2}{3}} \frac{8\pi^{13/2} m_2^3}{9} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{NS5}^3 N_{D5}^{3/2}$

Table 4. Results for $\mathrm{AdS}_4 \times Q^{1,1,1}$ and its supergravity duals.

Background	Page Charge Cycle	'Raw' CC	CC
$\mathrm{AdS}_4 \times N(1,1)$	$Q_{M2} = \left(\frac{\mathrm{R}}{l_p}\right)^6 \frac{3}{16\pi^2} = N_{M2}$ $\Sigma_7 = [\mu, \alpha, \beta, \theta_1, \phi_1, \theta_2, \phi_2]$	$\frac{\pi^4}{2\sqrt{2}} \left(\frac{\mathrm{R}}{l_p}\right)^9$	$\sqrt{\frac{2}{3}} \frac{16\pi^7}{3} N_{M2}^{3/2}$
$\Downarrow_\beta (\mathrm{AdS}_4 \times N(1,1))$	$Q_{D2} = \frac{\mathrm{R}^9}{l_p \alpha'^{5/2}} \frac{3}{64\pi^2} = N_{D2}$ $\Sigma_6 = [\mu, \alpha, \theta_1, \phi_1, \theta_2, \phi_2]$	$\frac{\pi^3}{16\sqrt{2}} \frac{\mathrm{R}^9}{l_p \alpha'^4}$	$\sqrt{\frac{2}{3}} \frac{16\pi^6}{3} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D2}^{3/2}$
$\mathrm{NATD}(\Downarrow_\beta (\mathrm{AdS}_4 \times N(1,1)))$	$Q_{D5} = \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} \frac{3}{128\pi} = N_{D5}$ $\Sigma_3 = [\mu, \theta_2, \phi_2]$	$\frac{m_2^3 \pi^5}{192\sqrt{2}} \frac{R^9}{l_p \alpha'^4} N_{NS5}^3$	$\frac{16\pi^{13/2} m_2^3}{9\sqrt{3}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{NS5}^3 N_{D5}^{3/2}$

Table 5. Results for $\mathrm{AdS}_4 \times N(1,1)$ and its supergravity duals.

JHEP02(2016)061

to introduce a nonzero B_2 which is closed, and therefore does not affect the equations of motion: $\tilde{B}_2 = \alpha' d\rho \wedge d\chi$. This is equivalent to a large gauge transformation on B_2. We compute b_0,

$$b_0 = \frac{1}{4\pi^2\alpha'} \int_{\rho_o \chi_o} \tilde{B}_2 = \frac{\rho_o \chi_o}{2}, \tag{8.16}$$

where ρ and χ are integrated over $[0, \rho_o \pi]$ and $[-\chi_o \pi, \chi_o \pi]$, respectively. Demanding $b_0 = [0, 1]$ restricts the range of ρ and χ. Normally for polar coordinates, we would expect the χ angle to take the range $0 \leq \chi \leq 2\pi$ (or $-\frac{\pi}{2} \leq \chi \leq \frac{\pi}{2}$), however the modified internal volume $\hat{V}_{\rm int}$ computed from eq. (8.4), restricts the range of χ to be $-\frac{\pi}{2} \leq \chi \leq \frac{\pi}{2}$, due to a factor of $\cos\chi$. Then we see that ρ would have to take the range $0 \leq \rho \leq 4\pi$. Note that this condition was different in the cases of SU(2) NATD.

Considering the case of the τ reduction of $\mathrm{AdS}_4 \times Q^{1,1,1}$ as an example, we compute the Page charge for D3 branes,

$$\hat{F}_5 - \tilde{B}_2 \wedge \hat{F}_3 = -\frac{3 m_2 t_2^6 t_3}{t_1} \frac{\mathrm{R}^6}{l_p \sqrt{\alpha'}} (\sin\chi d\rho + \rho \cos\chi d\chi) \wedge d\Omega_2^{(1)} \wedge d\Omega_2^{(2)}. \tag{8.17}$$

Integrating over the cycle containing ρ and the two 2-spheres, we obtain the normalization condition,

$$\frac{1}{2\kappa_{10}^2 T_{D3}} \int \hat{F}_5 - \tilde{B}_2 \wedge \hat{F}_3 = \frac{12 m_2 t_2^6 t_3}{t_1 \pi} \frac{\mathrm{R}^6}{l_p \alpha'^{5/2}} = N_{D3}. \tag{8.18}$$

After the NATD, we find that the central charge is,

$$\begin{aligned} c &= \frac{512 m_2^3 t_1^2 t_2^6 t_3 \pi^5}{3} \frac{\mathrm{R}^9}{l_p \alpha'^4} = \frac{64 m_2^{3/2} t_1^{7/2} \pi^{13/2}}{9\sqrt{3} t_2^3 \sqrt{t_3}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D3}^{3/2} \\ &= \frac{256 m_2^{3/2} \pi^{13/2}}{9\sqrt{3}} \frac{\sqrt{l_p}}{\alpha'^{1/4}} N_{D3}^{3/2}. \end{aligned} \tag{8.19}$$

If we had instead integrated over the χ plus the two 2-spheres cycle, the final central charge would be different by a factor of $\sqrt{2}$.

9 Discussion and conclusions

In this manuscript we have used Non-Abelian T-duality to produce new supergravity backgrounds. We have focused on solutions that preserve an AdS_4 factor. These solutions are particularly relevant in the context of the AdS/CFT correspondence since they describe strongly coupled conformal field theories in three dimensions. One of the prominent members of the class we consider as a seed case is related to the ABJM field theory. Another interesting example was provided by the $N(1,1)$ spaces, where the NATD apparently destroyed all of the isometries, other than the isometries of AdS and a residual U(1) isometry. We expect that these new backgrounds will ultimately enrich the number of entries in the AdS/CFT dictionary.

We have only briefly mentioned the supersymmetry in the case of NATD($\Downarrow_\tau$ ($\mathrm{AdS}_4 \times S^7/\mathbb{Z}_k$)), which we have argued preserves $\mathcal{N} = 2$ supersymmetry. In the other cases, we

JHEP02(2016)061

propose that the reductions along the Hopf fibre coordinate, τ, preserve supersymmetry. In light of the recent work [38] on supersymmetry and SU(2) NATD we argue that the NATD backgrounds also preserved some fraction of supersymmetry. The main result in [38] shows that, at least for backgrounds with Bianchi IX symmetry, supersymmetry is preserved if the Killing spinors of the original background do not depend on the SU(2) isometry directions. A more covariant way to re-state the independence of the Killing spinor on certain coordinates is the Kosmann derivative; this is akin to the Lie derivative being the covariant way of stating that the metric has some invariance. For the reduction along U(1)'s other than the Hopf fiber we do not have a general geometric argument and relied on some explicitly known cases. It would be interesting to systematically and explicitly study the supersymmetry of the resulting backgrounds.

One interesting open problem would be to track, on the field theory side, the effects of various NATD's. In particular, the analysis we have presented of the holographic central charges seems to point to marked differences when one considers reducing along the Hopf fiber or along some other, susy preserving, U(1) direction. The implications for the dual quiver field theories and its potential cascading phases are only hinted at in the gravity side and it would be quite interesting to explore those phases.

Acknowledgments

We are particularly grateful to Carlos Núñez for many discussions and clarifications and, particularly, for sharing [46] before publication with us. We are also thankful to Niall Macpherson, Fernando Quevedo and Dimitrios Tsimpis for comments and encouragement. LAPZ acknowledges Wolfgang Mück and the hospitality of INFN Sezione di Napoli at Università degli Studi di Napoli Federico II during the conclusion of this work. This research was supported in part by the National Science Foundation under Grant Nos. 1067889 (University of Iowa). C.A.W. was partially supported by the National Research Foundation of South Africa NITHP. This research is also supported by funds provided by the Abdus Salam ICTP.

A Review of NATD

In this appendix we will briefly review the procedures for applying Non-abelian T-dualities for backgrounds admitting SU(2) and SU(2)/U(1) isometries. The procedures are described in more depth in [25] for SU(2) and [24] for SU(2)/U(1), as well as other coset isometries.

A.1 SU(2) isometries

We follow [25] in the generalized 3-step Büscher procedure and consider backgrounds with an SU(2) isometry such that the metric can be written in the form of

$$ds^2 = G_{\mu\nu}(x)dx^\mu dx^\nu + 2G_{\mu i}(x)dx^\mu L^i + g_{ij}(x)L^i L^j, \tag{A.1}$$

where $\mu, \nu = 1, \ldots 7$ and $i, j = 1, 2, 3$. The L^i's are the $su(2)$ Maurer-Cartan forms ($L^i_\pm = -i\mathrm{Tr}(t^i g^{-1}\partial_\pm g)$, with g an element of SU(2)). All of the Type IIA backgrounds

JHEP02(2016)061

considered in this paper have $B_2 = 0$, so for simplicity we omit its contribution from the general procedure. The Lagrangian density for the NS sector fields (omitting the dilaton contribution) is given by,

$$\mathcal{L}_0 = Q_{AB}\partial_+ X^A \partial_- X^B, \tag{A.2}$$

where $A, B = 1, \ldots, 10$ and

$$Q_{AB} = \left(\begin{array}{c|c} Q_{\mu\nu} & Q_{\mu i} \\ \hline Q_{i\mu} & E_{ij} \end{array}\right), \quad \text{and} \quad \partial_\pm X^A = \left(\partial_\pm X^\mu,\ L^i_\pm\right), \tag{A.3}$$

with

$$Q_{\mu\nu} = G_{\mu\nu}, \quad Q_{\mu i} = G_{\mu i}, \quad Q_{i\mu} = G_{i\mu}, \quad E_{ij} = g_{ij}. \tag{A.4}$$

We then gauge the SU(2) isometry by changing derivatives to covariant derivatives and introduce gauge fields, $A_\pm$ according to $\partial_\pm g \to D_\pm g = \partial_\pm g - A_\pm g$. The next step is to add a Lagrange multiplier term to eq. (A.2) to ensure that the gauge fields have vanishing field strength. The Lagrange multiplier term is given by,

$$-i\mathrm{Tr}(\alpha' v F_\pm), \quad F_\pm = \partial_+ A_- - \partial_- A_+ - [A_+, A_-]. \tag{A.5}$$

Since the dimension of SU(2) is three, we have introduced three new dynamical variables in the form of the Lagrange multipliers, v_i, so we must eliminate three of the variables by making a gauge fixing choice. A natural choice is $g = \mathbb{I}$, so that all of the Euler angles in the SU(2) are zero and all three of the Lagrange multipliers become dual coordinates. The last step is to integrate out the gauge fields to obtain the dual Lagrangian density,

$$\hat{\mathcal{L}} = \hat{Q}_{AB}\partial_+ \hat{X}^A \partial_- \hat{X}^B, \tag{A.6}$$

where we can read off the dual components of $\hat{Q}_{AB}$ from,

$$\hat{Q}_{AB} = \left(\begin{array}{c|c} Q_{\mu\nu} - Q_{\mu i}M^{-1}_{ij}Q_{j\nu} & Q_{\mu j}M^{-1}_{ji} \\ \hline -M^{-1}_{ij}Q_{j\mu} & M^{-1}_{ij} \end{array}\right), \quad \text{and} \quad \partial_\pm \hat{X}^A = \left(\partial_\pm X^\mu,\ \partial_\pm v^i\right). \tag{A.7}$$

We have additionally defined:

$$M_{ij} = E_{ij} + f_{ij}, \quad \text{with} \quad f_{ij} = m_2 \alpha' \epsilon_{ij}{}^k v_k. \tag{A.8}$$

We can identify the dual metric and the generated $\hat{B}_2$ field as the symmetric and antisymmetric components of $\hat{Q_{AB}}$, respectively. The transformation of the dilaton is given by

$$\hat{\Phi} = \Phi - \frac{1}{2}\ln\left(\frac{\det M}{\alpha'^3}\right). \tag{A.9}$$

JHEP02(2016)061

In order to transform the RR Fluxes, one must construct a bispinor out of the RR forms and their Hodge duals, (in Type IIA):

$$P = \frac{e^{\Phi}}{2} \sum_{n=0}^{5} \not{F}_{2n}, \tag{A.10}$$

where $\not{F}_p = \frac{1}{p!}\Gamma_{\mu_1\dots\mu_p} F_p{}^{\mu_1\dots\mu_p}$. The dual fluxes simply arise from inverting Ω:

$$\hat{P} = P \cdot \Omega^{-1}, \tag{A.11}$$

where

$$\Omega = (A_0 \Gamma^1\Gamma^2\Gamma^3 + A_a\Gamma^a)\Gamma_{11}/\sqrt{\alpha'^3}, \tag{A.12}$$

and

$$A_0 = \frac{1}{\sqrt{1+\zeta^2}}, \qquad A^a = \frac{\zeta^a}{\sqrt{1+\zeta^2}}. \tag{A.13}$$

We also need to define $\zeta^a = \kappa^a{}_i z^i$ with $\kappa^a{}_i \kappa^a{}_j = g_{ij}$ and $z^i = \frac{1}{\det \kappa}(b_i + v_i)$.

A.2 Coset spaces: SU(2)/U(1) isometry

Here we present a quick review of performing the dualization procedure on a background with an SU(2)/U(1) isometry. A more complete description of generating the Non-Abelian T-dual of a background with coset space isometries was first presented in [24]. We start with a metric of the form of eq. (A.1), where $\mu, \nu = 1, \dots 8$ and $i, j = 1, 2$, and the L^i's are replaced with $L_1 = d\theta$, $L_2 = \sin\theta d\phi$. The procedure is essentially the same as in appendix A.1 above, but we modify,

$$E_{ij} = \mathrm{diag}(g_{ij}, \lambda), \tag{A.14}$$

where g_{ij} is 2-dimensional. Due to this, we must additionally gauge fix one of the three Lagrange multipliers. In all of the cases considered, we choose $(v_1 = 0, v_2 = m_2\alpha'\rho\cos\chi, v_3 = m_2\alpha'\rho\sin\chi)$. After inverting M_{ij}, we take $\lambda \to 0$ and construct the dual frames from

$$\hat{e} = \kappa M^{-T} dv. \tag{A.15}$$

B Review of supergravity EOM

In this appendix we will briefly review the 11 and 10D supergravity equations of motion, presented in the convenient form of [25, 59]. The 11D supergravity action is given by [60],

$$S_{11D} = \frac{1}{\kappa_{11}^2} \int_{M_{11}} \sqrt{g} \left[R - \frac{1}{2}\left(\frac{F_4^2}{4!} + F_4 \wedge F_4 \wedge C_3 \right) \right] \tag{B.1}$$

The 11D Einstein's equations and Bianchi identities can be expressed as,

$$R_{AB} = \frac{1}{12}(F_4{}^2)_{AB} - \frac{1}{6} g_{AB} \frac{1}{4!}(F_4{}^2)_{AB}, \tag{B.2}$$

$$d \star F_4 = \frac{1}{2} F_4 \wedge F_4, \qquad dF_4 = 0. \tag{B.3}$$

JHEP02(2016)061

The action of massive Type IIA supergravity in string frame without sources is given by,

$$S_{IIA} = \frac{1}{\kappa^2}\int_{M_{10}}\sqrt{g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{H^2}{12}\right) - \frac{1}{2}\left(F_0^2 + \frac{F_2^2}{2} + \frac{F_4^2}{4!}\right)\right] \\ -\frac{1}{2}\left(dC_3 \wedge dC_3 \wedge B_2 + \frac{F_0}{3}dC_3 \wedge B_2^3 + \frac{F_0^2}{20}B_2^5\right). \tag{B.4}$$

In this case, the Bianchi identities are given by,

$$dF_0 = 0, \quad dF_2 = F_0 H_3, \quad dF_4 = H_3 \wedge F_2, \tag{B.5}$$

which can be deduced from the definitions of the fluxes,

$$F_0 = m, \quad F_2 = dC_1 + F_0 B_2, \quad F_4 = dC_3 - H_3 \wedge C_1 + \frac{F_0}{2}B_2 \wedge B_2. \tag{B.6}$$

The equations of motion that follow from varying (B.4) with respect to the metric are,

$$R_{\mu\nu} + 2D_\mu D_\nu \Phi = \frac{1}{4}H_{\mu\nu}^2 + e^{2\Phi}\left[\frac{1}{2}(F_2^2)_{\mu\nu} + \frac{1}{12}(F_4^2)_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\left(F_0^2 + \frac{1}{2}F_2^2 + \frac{1}{4!}F_4^2\right)\right], \tag{B.7}$$

and the dilaton equation is,

$$R + 4D^2\Phi - 4(\partial\Phi)^2 - \frac{1}{12}H^2 = 0. \tag{B.8}$$

The action of Type IIB supergravity in string frame without sources is given by,

$$S_{IIB} = \frac{1}{\kappa^2}\int_{M_{10}}\sqrt{g}\left[e^{-2\Phi}\left(R + 4(\partial\Phi)^2 - \frac{H^2}{12}\right) - \frac{1}{2}\left(F_1^2 + \frac{F_3^2}{3!} + \frac{1}{2}\frac{F_5^2}{5!}\right)\right] \\ -\frac{1}{2}C_4 \wedge H \wedge dC_2 \tag{B.9}$$

The Bianchi identities are an additional constraint on the fluxes given by,

$$dH = 0, \quad dF_1 = 0, \quad dF_3 = H_3 \wedge F_1, \quad dF_5 = H \wedge F_3. \tag{B.10}$$

Here, the definition of the fluxes take the form,

$$H_3 = dB_2, \quad F_1 = dC_0, \quad F_3 = dC_2 - C_0 H_3, \quad F_5 = dC_4 - H \wedge C_2. \tag{B.11}$$

The equations of motion that follow from varying (B.9) with respect to the metric are,

$$R_{\mu\nu} + 2D_\mu D_\nu \Phi = \frac{1}{4}H_{\mu\nu}^2 + e^{2\Phi}\left[\frac{1}{2}(F_1^2)_{\mu\nu} + \frac{1}{4}(F_3^2)_{\mu\nu} + \frac{1}{96}(F_5^2)_{\mu\nu} - \frac{1}{4}g_{\mu\nu}\left(F_1^2 + \frac{1}{3!}F_3^2\right)\right] \tag{B.12}$$

and the dilaton equation is again given by,

$$R + 4D^2\Phi - 4(\partial\Phi)^2 - \frac{1}{12}H^2 = 0. \tag{B.13}$$

JHEP02(2016)061

Open Access. This article is distributed under the terms of the Creative Commons Attribution License (CC-BY 4.0), which permits any use, distribution and reproduction in any medium, provided the original author(s) and source are credited.

References

[1] J.M. Maldacena, *The large-N limit of superconformal field theories and supergravity*, *Int. J. Theor. Phys.* **38** (1999) 1113 [hep-th/9711200] [INSPIRE].

[2] S.S. Gubser, I.R. Klebanov and A.M. Polyakov, *Gauge theory correlators from noncritical string theory*, *Phys. Lett.* **B 428** (1998) 105 [hep-th/9802109] [INSPIRE].

[3] E. Witten, *Anti-de Sitter space and holography*, *Adv. Theor. Math. Phys.* **2** (1998) 253 [hep-th/9802150] [INSPIRE].

[4] O. Aharony, S.S. Gubser, J.M. Maldacena, H. Ooguri and Y. Oz, *Large-N field theories, string theory and gravity*, *Phys. Rept.* **323** (2000) 183 [hep-th/9905111] [INSPIRE].

[5] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Supersymmetric AdS_5 solutions of M-theory*, *Class. Quant. Grav.* **21** (2004) 4335 [hep-th/0402153] [INSPIRE].

[6] J.P. Gauntlett, D. Martelli, J. Sparks and D. Waldram, *Supersymmetric AdS_5 solutions of type IIB supergravity*, *Class. Quant. Grav.* **23** (2006) 4693 [hep-th/0510125] [INSPIRE].

[7] E. O Colgain and B. Stefanski Jr., *A search for $AdS_5 \times S^2$ IIB supergravity solutions dual to $N = 2$ SCFTs*, *JHEP* **10** (2011) 061 [arXiv:1107.5763] [INSPIRE].

[8] F. Apruzzi, M. Fazzi, D. Rosa and A. Tomasiello, *All AdS_7 solutions of type-II supergravity*, *JHEP* **04** (2014) 064 [arXiv:1309.2949] [INSPIRE].

[9] F. Apruzzi, M. Fazzi, A. Passias, D. Rosa and A. Tomasiello, *AdS_6 solutions of type-II supergravity*, *JHEP* **11** (2014) 099 [*Erratum ibid.* **1505** (2015) 012] [arXiv:1406.0852] [INSPIRE].

[10] H. Kim, N. Kim and M. Suh, *Supersymmetric AdS_6 Solutions of Type IIB Supergravity*, *Eur. Phys. J.* **C 75** (2015) 484 [arXiv:1506.05480] [INSPIRE].

[11] F. Apruzzi, M. Fazzi, A. Passias and A. Tomasiello, *Supersymmetric AdS_5 solutions of massive IIA supergravity*, *JHEP* **06** (2015) 195 [arXiv:1502.06620] [INSPIRE].

[12] A. Rota and A. Tomasiello, *AdS_4 compactifications of AdS_7 solutions in type-II supergravity*, *JHEP* **07** (2015) 076 [arXiv:1502.06622] [INSPIRE].

[13] J.B. Gutowski and G. Papadopoulos, *Supersymmetry of AdS and flat backgrounds in M-theory*, *JHEP* **02** (2015) 145 [arXiv:1407.5652] [INSPIRE].

[14] S.W. Beck, J.B. Gutowski and G. Papadopoulos, *Supersymmetry of AdS and flat IIB backgrounds*, *JHEP* **02** (2015) 020 [arXiv:1410.3431] [INSPIRE].

[15] S. Beck, J.B. Gutowski and G. Papadopoulos, *Supersymmetry of IIA warped flux AdS and flat backgrounds*, *JHEP* **09** (2015) 135 [arXiv:1501.07620] [INSPIRE].

[16] I. Bah, C. Beem, N. Bobev and B. Wecht, *AdS/CFT Dual Pairs from M5-Branes on Riemann Surfaces*, *Phys. Rev.* **D 85** (2012) 121901 [arXiv:1112.5487] [INSPIRE].

[17] I. Bah, C. Beem, N. Bobev and B. Wecht, *Four-Dimensional SCFTs from M5-Branes*, *JHEP* **06** (2012) 005 [arXiv:1203.0303] [INSPIRE].

JHEP02(2016)061

[18] I. Bah, *Quarter-BPS AdS_5 solutions in M-theory with a T^2 bundle over a Riemann surface*, *JHEP* **08** (2013) 137 [arXiv:1304.4954] [INSPIRE].

[19] I. Bah, *AdS5 solutions from M5-branes on Riemann surface and D6-branes sources*, *JHEP* **09** (2015) 163 [arXiv:1501.06072] [INSPIRE].

[20] X.C. de la Ossa and F. Quevedo, *Duality symmetries from nonAbelian isometries in string theory*, *Nucl. Phys.* **B 403** (1993) 377 [hep-th/9210021] [INSPIRE].

[21] B.E. Fridling and A. Jevicki, *Dual Representations and Ultraviolet Divergences in Nonlinear σ Models*, *Phys. Lett.* **B 134** (1984) 70 [INSPIRE].

[22] E.S. Fradkin and A.A. Tseytlin, *Quantum Equivalence of Dual Field Theories*, *Annals Phys.* **162** (1985) 31 [INSPIRE].

[23] K. Sfetsos and D.C. Thompson, *On non-abelian T-dual geometries with Ramond fluxes*, *Nucl. Phys.* **B 846** (2011) 21 [arXiv:1012.1320] [INSPIRE].

[24] Y. Lozano, E. O Colgain, K. Sfetsos and D.C. Thompson, *Non-abelian T-duality, Ramond Fields and Coset Geometries*, *JHEP* **06** (2011) 106 [arXiv:1104.5196] [INSPIRE].

[25] G. Itsios, C. Núñez, K. Sfetsos and D.C. Thompson, *Non-Abelian T-duality and the AdS/CFT correspondence:new $N = 1$ backgrounds*, *Nucl. Phys.* **B 873** (2013) 1 [arXiv:1301.6755] [INSPIRE].

[26] E. Caceres, N.T. Macpherson and C. Núñez, *New Type IIB Backgrounds and Aspects of Their Field Theory Duals*, *JHEP* **08** (2014) 107 [arXiv:1402.3294] [INSPIRE].

[27] Y. Lozano, E. Ó Colgáin, D. Rodríguez-Gómez and K. Sfetsos, *Supersymmetric AdS_6 via T Duality*, *Phys. Rev. Lett.* **110** (2013) 231601 [arXiv:1212.1043] [INSPIRE].

[28] G. Itsios, C. Núñez, K. Sfetsos and D.C. Thompson, *On Non-Abelian T-duality and new $N = 1$ backgrounds*, *Phys. Lett.* **B 721** (2013) 342 [arXiv:1212.4840] [INSPIRE].

[29] N.T. Macpherson, *Non-abelian T-duality, generalised geometry and holography*, *J. Phys. Conf. Ser.* **490** (2014) 012122 [arXiv:1309.1358] [INSPIRE].

[30] E. Gevorgyan and G. Sarkissian, *Defects, Non-abelian T-duality and the Fourier-Mukai transform of the Ramond-Ramond fields*, *JHEP* **03** (2014) 035 [arXiv:1310.1264] [INSPIRE].

[31] N.T. Macpherson, *Non-Abelian T-duality, G_2-structure rotation and holographic duals of $N = 1$ Chern-Simons theories*, *JHEP* **11** (2013) 137 [arXiv:1310.1609] [INSPIRE].

[32] Y. Lozano, E. Ó Colgáin and D. Rodríguez-Gómez, *Hints of 5d Fixed Point Theories from Non-Abelian T-duality*, *JHEP* **05** (2014) 009 [arXiv:1311.4842] [INSPIRE].

[33] J. Gaillard, N.T. Macpherson, C. Núñez and D.C. Thompson, *Dualising the Baryonic Branch: Dynamic* SU(2) *and confining backgrounds in IIA*, *Nucl. Phys.* **B 884** (2014) 696 [arXiv:1312.4945] [INSPIRE].

[34] D. Elander, A.F. Faedo, C. Hoyos, D. Mateos and M. Piai, *Multiscale confining dynamics from holographic RG flows*, *JHEP* **05** (2014) 003 [arXiv:1312.7160] [INSPIRE].

[35] S. Zacarías, *Semiclassical strings and Non-Abelian T-duality*, *Phys. Lett.* **B 737** (2014) 90 [arXiv:1401.7618] [INSPIRE].

[36] P.M. Pradhan, *Oscillating Strings and Non-Abelian T-dual Klebanov-Witten Background*, *Phys. Rev.* **D 90** (2014) 046003 [arXiv:1406.2152] [INSPIRE].

JHEP02(2016)061

[37] Y. Lozano and N.T. Macpherson, *A new AdS_4/CFT_3 dual with extended SUSY and a spectral flow*, *JHEP* **11** (2014) 115 [arXiv:1408.0912] [INSPIRE].

[38] Ö. Kelekci, Y. Lozano, N.T. Macpherson and E. Ó Colgáin, *Supersymmetry and non-Abelian T-duality in type-II supergravity*, *Class. Quant. Grav.* **32** (2015) 035014 [arXiv:1409.7406] [INSPIRE].

[39] N.T. Macpherson, C. Núñez, L.A. Pando Zayas, V.G.J. Rodgers and C.A. Whiting, *Type IIB supergravity solutions with AdS_5 from Abelian and non-Abelian T dualities*, *JHEP* **02** (2015) 040 [arXiv:1410.2650] [INSPIRE].

[40] K.S. Kooner and S. Zacarías, *Non-Abelian T-Dualizing the Resolved Conifold with Regular and Fractional D3-branes*, *JHEP* **08** (2015) 143 [arXiv:1411.7433] [INSPIRE].

[41] T.R. Araujo and H. Nastase, *$\mathcal{N} = 1$ SUSY backgrounds with an AdS factor from non-Abelian T duality*, *Phys. Rev.* **D 91** (2015) 126015 [arXiv:1503.00553] [INSPIRE].

[42] Y. Bea et al., *Compactifications of the Klebanov-Witten CFT and new AdS_3 backgrounds*, *JHEP* **05** (2015) 062 [arXiv:1503.07527] [INSPIRE].

[43] Y. Lozano, N.T. Macpherson, J. Montero and E. Ó Colgáin, *New $AdS_3 \times S^2$ T-duals with $\mathcal{N} = (0, 4)$ supersymmetry*, *JHEP* **08** (2015) 121 [arXiv:1507.02659] [INSPIRE].

[44] Y. Lozano, N.T. Macpherson and J. Montero, *A $\mathcal{N} = 2$ supersymmetric AdS_4 solution in M-theory with purely magnetic flux*, *JHEP* **10** (2015) 004 [arXiv:1507.02660] [INSPIRE].

[45] T.R. Araujo and H. Nastase, *Non-Abelian T-duality for nonrelativistic holographic duals*, *JHEP* **11** (2015) 203 [arXiv:1508.06568] [INSPIRE].

[46] N.T. Macpherson, C. Núñez, D.C. Thompson and S. Zacarías, *Holographic Flows in non-Abelian T-dual Geometries*, *JHEP* **11** (2015) 212 [arXiv:1509.04286] [INSPIRE].

[47] H. Dimov, S. Mladenov, R.C. Rashkov and T. Vetsov, *Non-abelian T-duality of Pilch-Warner background*, arXiv:1511.00269 [INSPIRE].

[48] O. Aharony, O. Bergman, D.L. Jafferis and J. Maldacena, *N=6 superconformal Chern-Simons-matter theories, M2-branes and their gravity duals*, *JHEP* **10** (2008) 091 [arXiv:0806.1218] [INSPIRE].

[49] P.G.O. Freund and M.A. Rubin, *Dynamics of Dimensional Reduction*, *Phys. Lett.* **B 97** (1980) 233 [INSPIRE].

[50] M.J. Duff, B.E.W. Nilsson and C.N. Pope, *Kaluza-Klein Supergravity*, *Phys. Rept.* **130** (1986) 1 [INSPIRE].

[51] T. Friedrich and I. Kath, *Seven-dimensional compact Riemannian manifolds with Killing spinors*, *Commun. Math. Phys.* **133** (1990) 543 [INSPIRE].

[52] D. Martelli and J. Sparks, *Moduli spaces of Chern-Simons quiver gauge theories and AdS_4/CFT_3*, *Phys. Rev.* **D 78** (2008) 126005 [arXiv:0808.0912] [INSPIRE].

[53] M. Petrini and A. Zaffaroni, *N=2 solutions of massive type IIA and their Chern-Simons duals*, *JHEP* **09** (2009) 107 [arXiv:0904.4915] [INSPIRE].

[54] R. D'Auria, P. Fré and P. van Nieuwenhuizen, *$N = 2$ Matter Coupled Supergravity From Compactification on a Coset G/H Possessing an Additional Killing Vector*, *Phys. Lett.* **B 136** (1984) 347 [INSPIRE].

[55] S. Franco, I.R. Klebanov and D. Rodríguez-Gómez, *M2-branes on Orbifolds of the Cone over $Q^{1,1,1}$*, *JHEP* **08** (2009) 033 [arXiv:0903.3231] [INSPIRE].

JHEP02(2016)061

[56] D.N. Page, *Classical Stability of Round and Squashed Seven Spheres in Eleven-dimensional Supergravity*, *Phys. Rev.* **D 28** (1983) 2976 [INSPIRE].

[57] D. Marolf, *Chern-Simons terms and the three notions of charge*, in *Quantization, gauge theory, and strings. Proceedings, International Conference dedicated to the memory of Professor Efim Fradkin*, Moscow, Russia, June 5–10 2000. Vol. 1+2, pp. 312–320, hep-th/0006117 [INSPIRE].

[58] I.R. Klebanov, D. Kutasov and A. Murugan, *Entanglement as a probe of confinement*, *Nucl. Phys.* **B 796** (2008) 274 [arXiv:0709.2140] [INSPIRE].

[59] A. Barranco, J. Gaillard, N.T. Macpherson, C. Núñez and D.C. Thompson, *G-structures and Flavouring non-Abelian T-duality*, *JHEP* **08** (2013) 018 [arXiv:1305.7229] [INSPIRE].

[60] E. Cremmer, B. Julia and J. Scherk, *Supergravity Theory in Eleven-Dimensions*, *Phys. Lett.* **B 76** (1978) 409 [INSPIRE].

JHEP02(2016)061