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A computer and experimental simulation of Stirling cycle machines

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T_i (T(I)) temperature of gas in cell i.

Tb local working gas bulk temperature.

Tc (TC) time dependent compression space gas temperature.

Te (TE) time dependent expansion space gas temperature.

To (ETO)[K] temperature at which the dynamic viscosity of the working gas is defined.

TC constant gas temperature in compression space (Schmidt).

TE constant gas temperature in expansion space (Schmidt).

TH (TH) constant gas temperature in heater (ideal pseudo-Stirling).

TK (TK) constant gas temperature in cooler (ideal pseudo-Stirling).

Tn_i (TN(I)) temperature of gas at node i.

Tw_i (TW(I)) temperature of wall in cell i.

Tsu (ETSU)[K] Sutherland constant.

Twh (TWH) constant temperature of heater wall.

Twk (TWK) constant temperature of cooler wall.

TNC, TNE conditional temperatures (ideal pseudo-Stirling).

TSU normalised Sutherland constant.

Twr_i (TWR(I)) temperature of the regenerator wall in cell i.

$Tfilm$ (TFILM) film temperature.

u specific internal energy.

U internal energy.

v specific volume of gas.

V control volume; volume of cell void space.

v_i (VS(I)) specific volume of gas in cell i.

V_i (V(I)) void volume of cell i.

vc (VSC) specific volume of gas in compression space.

ve (VSE) specific volume of gas in expansion space.

Vc (VC) compression space volume.

Vd (VD) dead volume.

Ve (VE) expansion space volume.

Vr (VR) regenerator void volume.

Vs (EVS)[m] total power stroke volume.

VC compression space stroke volume.

VE expansion space stroke volume.

VH (VH) heater void volume.

VK (VK) cooler void volume.

VT total working gas volume.

vn_i (VSN(I)) specific volume of gas convected across node i.

Vn_i elemental volume associated with the momentum equation.

Vcl (VCL) clearance volume in working spaces.

w mass flow rate.

W (W) indicated work; reversible work.

wc mass flow rate out of compression space.

we mass flow rate out of expansion space.

wh mass flow rate from regenerator to heater.

wk mass flow rate from cooler to regenerator.

Wc (WC) indicated or reversible work at compression space.

We (WE) indicated or reversible work at expansion space.

Wt total reversible work (Schmidt).

W_v work per unit volume.

wco_2 mass flow rate of CO_2 gas.

x (X) length; heat exchanger axial position variable.

xp (XP) length of heat exchanger pipe bundle.

xho position in heater at which the gas flow is at rest.

xko position in cooler at which the gas flow is at rest.

xro position in regenerator at which the gas flow is at rest.

y dependent variable.

Y(I) array in integration routine.

z height of protrusions in flow channels in order to define the relative roughness Ro.

Greek

α (ALPHA) phase angle advance of the expansion space to the compression space volume variation.

β combination of terms (Schmidt analysis).

γ (GAMMA) ratio of specific heats of working gas (Cp/Cv).

δ phase displacement between the compression and expansion pressure profiles.

Δ(DEL, DL) difference operator; small increment; DEL denotes a spacial increment, DL a time increment.

Δpc contraction pressure drop.

Δpe expansion pressure rise.

Δx cell length.

Δx_i (DELX(I)) length of the i'th cell.

ε error.

εb error in overall energy balance.

ε_i error at any point i.

εm fractional error in measurement.

ε_R highest expected fractional error.

εa_i absolute fractional error.

Σ summation.

η (EFF) thermal efficiency.

θ crank angle.

μ dynamic viscosity.

μo (EMUO)[kg/m s] actual dynamic viscosity of the working gas at temperature To.

μk (RMUK) dynamic viscosity of the working gas at temperature $Tfilm$

ν velocity of the gas; kinetic viscosity.

ξ compound angle.

ρ density.

σ viscous stress tensor.

ϕ combination of terms (Schmidt); dissipation function; combination of terms (crank slider formula).

ψ combination of terms (Schmidt); area ratio.

ω (OMEGA) oscillation frequency; angular frequency.

Superscripts

' conditional parameter.

* normalised parameter (Fourier series only).

· flow rate.

~ vector quantity.

¯ arithmetic average or mean quantity.

Subscripts

e outflow conditions.

i inflow conditions.

cyl defined at working space cylinder.

R highest expected error.

reg defined at regenerator.

1 INTRODUCTION

1.1 General

The rising costs of energy supplies and the increasing awareness of pollution and noise has made it important to investigate alternative prime movers. Major requirements for these devices are that they be efficient, non-polluting, reliable, economical and socially acceptable.

The development and utilisation of such systems are important not only for first world countries, but possibly more so for third world and developing countries. Widespread usage of efficient prime movers can have a considerable effect on the balance of payments situation for countries that import energy supplies [Fin77] . While the high cost of fuel is particularly stifling to countries which are on the verge of industrialisation.

The Stirling engine has aroused considerable interest in recent times because of its many favourable characteristics. These include extremely clean and cool exhaust gases, silent and practically vibrationless operation, low fuel consumption and the advantage of using energy from many different sources.

Stirling engines can be used either for stationary power generation or for automotive engines. The use of nuclear energy as a heating medium has been suggested by Kolin [Ko68] for stationary Stirling engines driving electric generators. Other stationary applications include the conversion of solar energy to electric or mechanical energy [Be76]. This could be especially attractive in remote and unattended areas [Ta67, CF74]. The automotive field is presently being investigated by the Philips Gloilampenfabrieken (Holland), Ford Motor Company (U S A), United Stirling (Sweden) and M A N (The German Federal Republic). However, most of these

firms are working under licence to Philips Gloilampenfabrieken.

1.2 The Stirling Cycle

The Carnot theorem (1824) states that the efficiency of all reversible engines operating between the same two temperatures is the same, and no irreversible engine working between the same two temperatures can have a greater efficiency than this. This statement was later shown by Clausius and Kelvin to be a necessary consequence of the Second Law of Thermodynamics. The process during which heat is transferred must be done isothermally. Thus the two temperatures referred to by Carnot are the higher temperature at which heat is added reversibly, and the lower temperature at which heat is extracted reversibly. The processes connecting the heat addition to the heat extraction must then be externally adiabatic when taken together, and must obviously also be reversible to achieve a cycle which is completely reversible.

Such a cycle suggested by Sadi Carnot in 1824, and subsequently known as the Carnot cycle is shown in Figure 1.1 as process 1-2-3-4. Here the two processes which connect the isothermals (2-3 and 4-1) are each independently adiabatic.

Another reversible cycle devised by either James or Robert Stirling (there appears to be some contention here [Ko72]) in 1816, some eight years before Carnot's classic paper (Reflection on the Motive Power of Fire - 1824) is shown for comparison in Figure 1.1 as process 1-2'-3-4'.

Here the constant volume processes 2'-3 and 4'-1 absorb and reject heats Q_3 and Q_4 respectively. Since the working medium of a Stirling machine is assumed to be an ideal fluid, Q_3 is identically equal to Q_4 but opposite in sign. Stirling thus realised that by storing heat Q_4 in a thermal 'sponge' (the

regenerator) which could later be extracted as Q_3, the processes joining the isothermals could be made to be externally adiabatic with a consequent increase in efficiency.

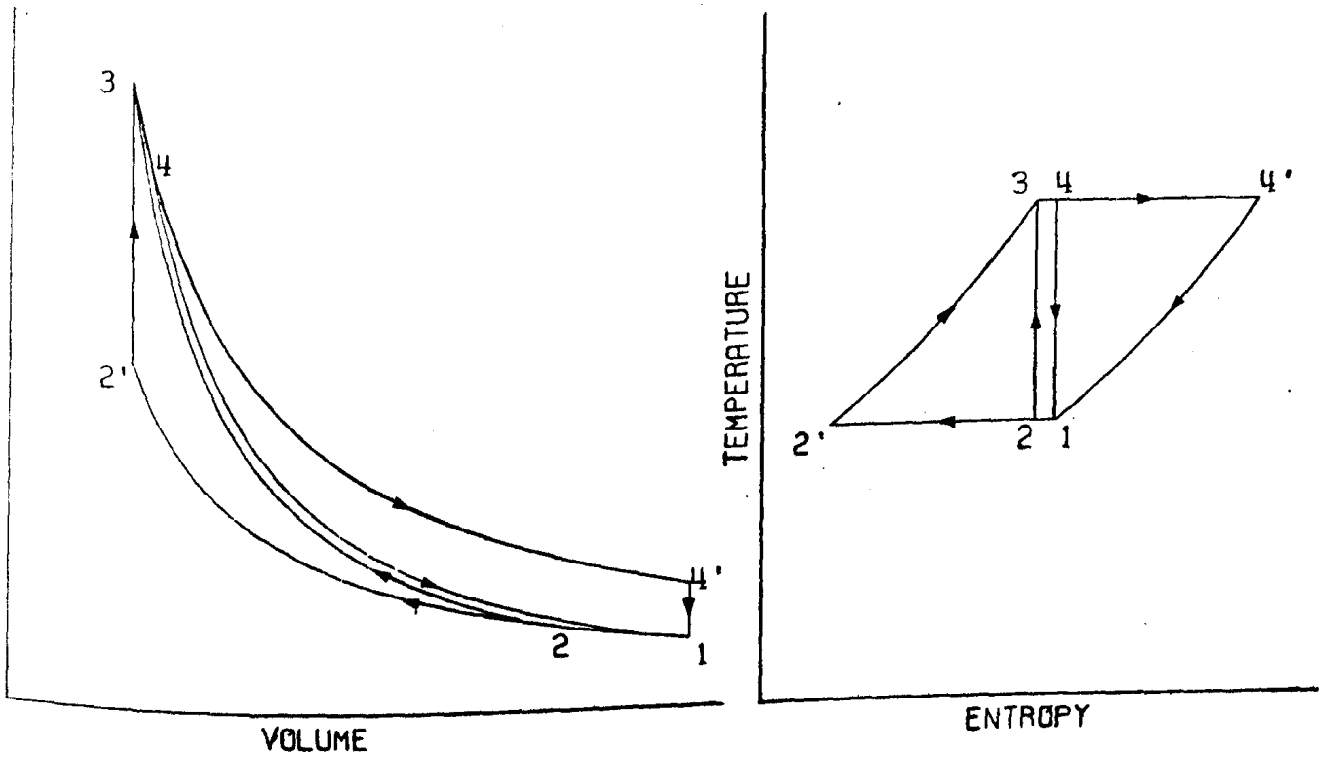


Figure 1.1 *p V* and *T S* Diagrams

A fact which he could not have guessed is that this cycle fulfils the condition for reversibility, and thus the ideal Stirling engine has the Carnot efficiency which is the maximum theoretical efficiency obtainable for any heat engine. Further, as can be seen from Figure 1.1, the areas enclosed by the *p V* and *T S* diagrams for the Stirling cycle are very much larger than those enclosed for the Carnot cycle. Thus the work and heat transferred by the Stirling cycle is very

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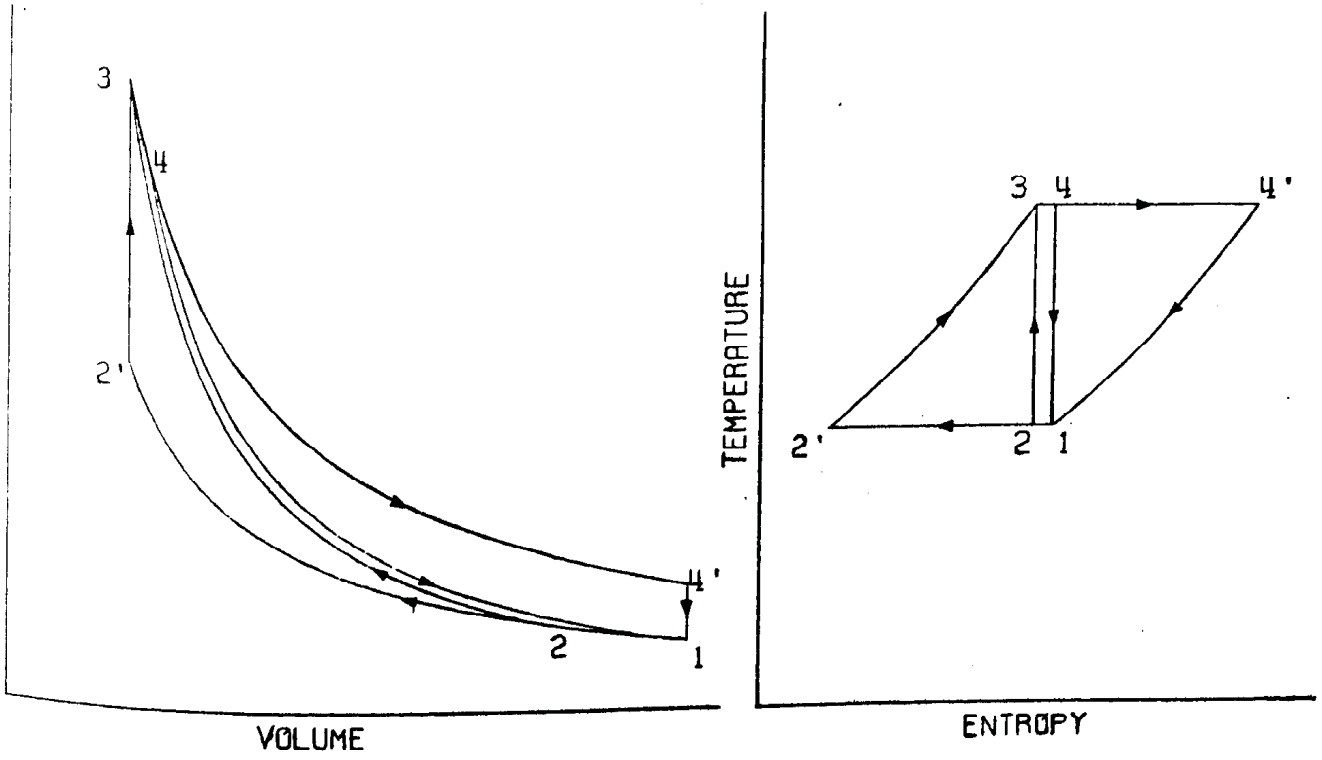


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much larger than an equivalently sized Carnot cycle. In fact the Stirling cycle gives the highest specific work of all known reversible cycles under the constraints of pressure, volume and temperature limits.

An additional consequence of a reversible cycle is that it can be used as a refrigerator or a heat pump, and the Stirling cycle has found popular use in these fields [KJ54, Kö60].

Though there are many configurations for realising this cycle mechanically, the two most common, shown in Figure 1.2, are known respectively as the dual piston and piston-displacer arrangements.

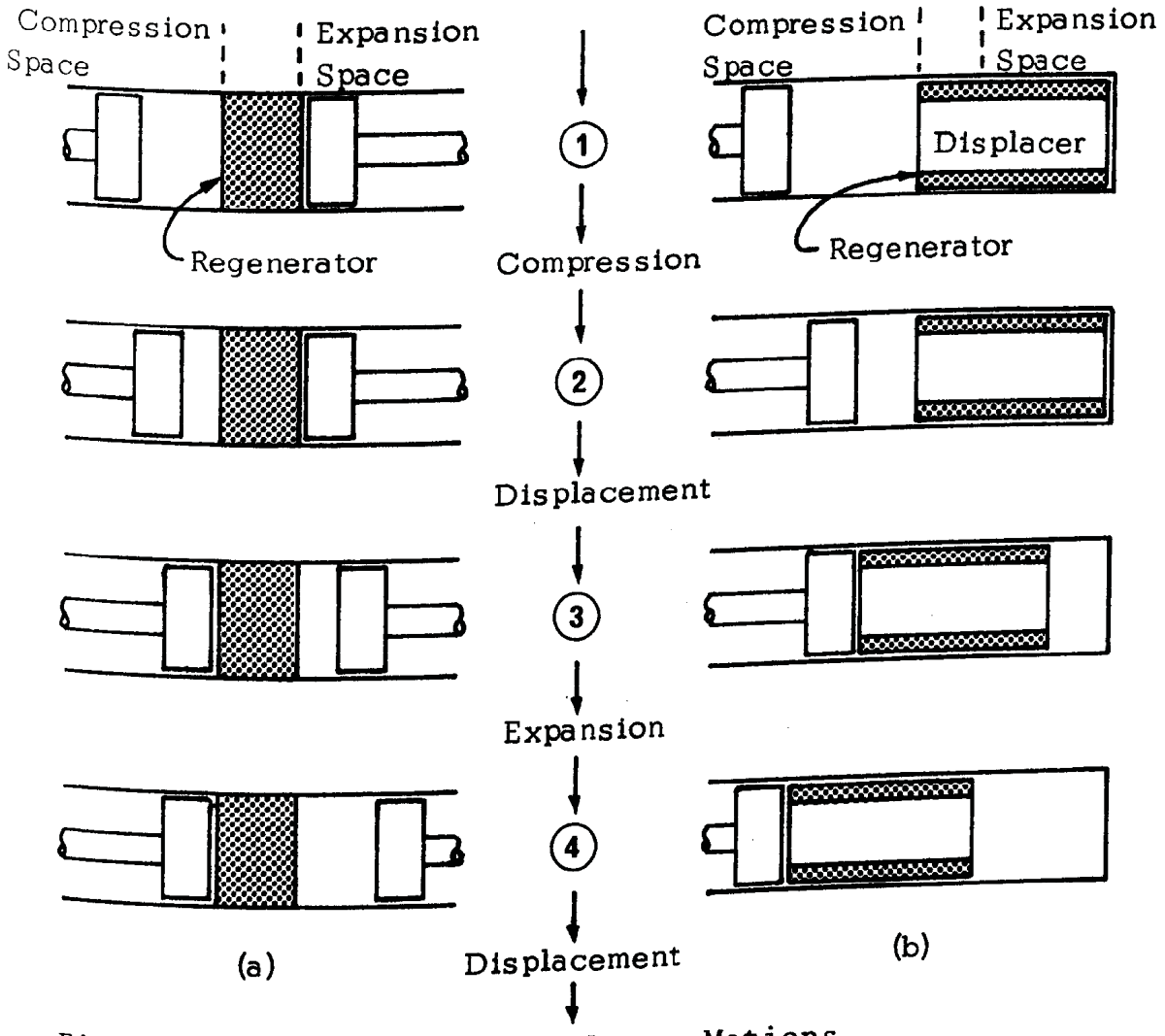


Figure 1.2 Piston and Displacer Motions

From Figure 1.2 it can be seen that the regenerator is never entirely purged in a practical cycle. Also the motions of the displacers and pistons are continuous resulting in a certain overlap of the processes. It can be shown however (Appendix A) that these two effects do not change the ideal theoretical efficiency at all, but that they have considerable effect on the heat transferred and the work done. Factors which affect the efficiency as well as the energy transferred are the irreversibilities which exist in a real cycle. These are the real gas flow effects such as friction, time lag effects and the efficiency of the heat exchangers. It is often impractical to effect heat transfer in the expansion and compression spaces, thus separate heaters and coolers are occasionally introduced, as shown in Figure 1.2. These heaters and coolers can introduce a considerable amount of dead space (unswept volume) and must thus be designed to be extremely efficient so as to be as compact as possible. The Stirling cycle which has separate heaters and coolers has been referred to as the Pseudo-Stirling cycle.

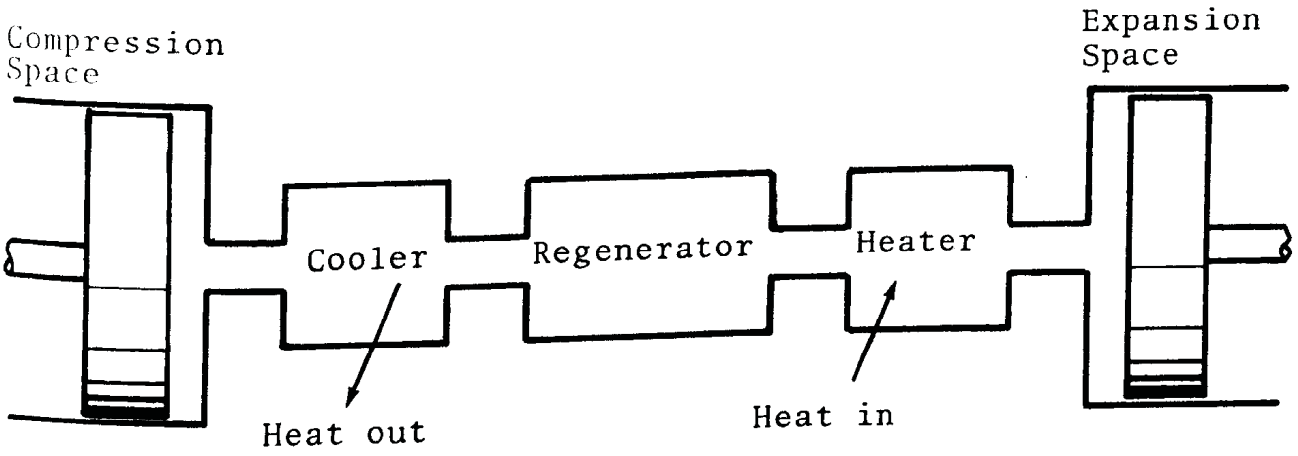


Figure 1.3 The Pseudo-Stirling Cycle Engine

1.3 Problem Areas

As Urieli [Ur77] points out, the problem areas in the

development of Stirling machines can be divided into practical and theoretical branches.

The practical problems are especially important in the development of engines. Stirling engines must be designed to operate at the greatest possible temperature difference between the hot and cold ends. Since power is directly proportional to pressure (in the first instance) it is also necessary to pressurise the working gas to the highest possible value in order to ensure reasonable specific powers. These requirements in turn create unique problems in the reduction of creep and in sealing. Material Science thus becomes an especially important subject in Stirling engine development [TC77, BS77].

The detailed behaviour of the working gas and heat exchangers in Stirling machines is very complex. Since the gas is shuttled at reasonably high velocities, it is important to evaluate momentum effects, and since there is considerable heat transfer, the energy state of the gas and heat exchangers is continuously changing, both with time and with position in the machine.

In order to analytically describe such a machine, all the fundamental equations must be solved: continuity, momentum and energy. In addition the equations cannot be simplified into steady flow or incompressible formats and this complicates analytical efforts even further. Ideal cycle analyses such as the Schmidt type are highly optimistic. They only give a reasonable indication of performance at the point of maximum efficiency since at this point irreversibilities must be minimum. Off-design characteristics, which are also important, are thus difficult to estimate.

In short it can be said that to design and optimise a Stirling

cycle machine with any degree of success a sophisticated technology is required. This technology was not forthcoming to Stirling and his co-workers in their time, nor even for a long time thereafter. It is for this reason that the Stirling engine could not compete with the rising successes of the early internal combustion engines and electric motors.

The work presented here will only be directed towards modern techniques for analysing the gas-dynamic and thermodynamic aspects of the Stirling cycle.

1.4 Modern Analytical Techniques

Here the computer simulation methods are possibly the most important. The describing differential equations are numerically simulated on high speed digital computers and in this way a complete system is developed which will numerically describe any desired machine.

Urieli [Ur77] argues that computational analysis is separate from mathematical and experimental analysis in that it is a developmental technique in its own right. He goes on to say that though computational analysis relies upon fundamental mathematical concepts it is in fact much closer to experimentation. The analyst 'turns on' the computer and waits to see what happens, just as the experimenter does. It is also more convenient to try different configurations and working conditions. In addition there is the advantage of being able to observe the detailed behaviour of the working gas at any point in the system without being concerned with the effects of the measuring instruments.

However, a computer simulation is still a model and thus by definition is an approximation to reality. Therefore it is

not possible for such simulations to ever totally replace experimentation. They do, however, serve to complement experimental and mathematical work.

Mathematical analysis on the other hand is generally used to determine controlling parameters of a particular system. Where closed form mathematical solutions or relations are possible, it is generally easier to mathematically optimise a system in terms of its basic design parameters. Often it is necessary to simplify a system before a closed form solution can be obtained. These solutions can be very useful in determining 'ball court' concepts, bearing in mind that the more a system is simplified the more it departs from reality.

The basic cycle analysis presented by Rallis *et al* [RU77] and ideal analyses such as that of Schmidt (Appendix A) give good indications as to the selection of overall design parameters. However, the idealised nature of these analyses preclude an accurate determination of the performance of a machine, and a computer simulation must be opted for for the final stages of analysis. This process cuts down considerably on the number of computer simulation runs which are generally very expensive.

Physical experimentation has two important functions in engineering: a) to investigate the agreement between the theoretical prediction and reality and b) to generate empirical data for situations where theoretical formulation is either too complex or not possible. Development work, when done empirically, would also fall into the second category.

It is important to note that experimental work is often carried out for systems that can far more easily and economically be

simulated by computer. Ideally mathematical analysis, computer simulation and experimental work should be co-ordinated for the most optimum pay-off from each technique.

1.5 Purpose of this Study

The most sophisticated computer simulation published to date is that developed by Urieli [Ur77, UR77]. This work was not validated against experiment and this as such has become part of the purpose of this study.

A critical appraisal of Urieli's work and a comparison with the equivalent ideal cycle is also included. The problems associated with using steady state friction factors are investigated and an effort is made to include viscous dissipation terms.

For the sake of clarity, this study has been directed towards prime movers. This is not a limitation of the analysis presented here since it is equally valid for refrigerators or heat pumps.

1.6 Conventions Used

1.6.1 Operators: To make the text more readable and more convenient to type, the following hierarchy of operators are adhered to:

Priority	Operator
1 (top)	Exponentiation
2	Multiplication & division
3 (bottom)	Addition and subtraction

The rules associated with these operators are:-

1) They are all *binary* operators, ie, they can only operate between *two* terms.

2) In a subexpression of given parenthesis depth, all operations of a given priority are carried out before any operations of lower priority.

3) Operations of the same priority are carried out in order from left to right.

The solidus (/) has been used to denote division. In some texts the solidus is used to denote a fraction and is thus not a binary operator. To avoid confusion, an example is given of the usage of the solidus in this work.

$$a/b + c = a \div b + c = \frac{a}{b} + c \neq \frac{a}{b + c} \tag{1.1}$$

The above are the standard conventions used in algebra as well as the FORTRAN computer language.

For vectors and tensors, several kinds of multiplication are possible. These operations are indicated by the use of special multiplication signs: the 'dot' (.), the double'dot' (:), and the 'cross' (x). The parentheses enclosing these special multiplication operators indicate the type of quantity produced by the multiplication:

$$\begin{aligned} (\ ) &= \text{scalar} \\ [\ ] &= \text{vector} \\ \{\ \} &= \text{tensor} \end{aligned} \tag{1.2}$$

No special significance is attached to the kind of parentheses if the operation enclosed is addition or subtraction, or if they are used in a context where no vectors are evident.

In this work the usage of the cross product never arises.

1.6.2 References: The Rallis system of reference designation is used. It is considered to provide a better means of information retrieval than the usual numerical sequencing of articles.

Each publication is referred to by two letters followed by two numerals. The latter refer to the year of publication. Where the publication has a single author the first two letters of the surname appear, the second in lower case. Where there is more than one author the initial letters of the surnames of the first two, both in upper case, are used. Exceptions occur when the reference is prior to 1900, then four numerals denoting the year appear. If the name of the author is unknown, three letters denoting the name of the journal are used instead. The references are listed in alphabetical order according to the names of the authors.

2 REVIEW OF RECENT STIRLING CYCLE MACHINE ANALYSIS

2.1 Introduction

Rallis [RU77] has suggested that the analysis of all regenerative closed cycles can be divided into three orders of increasing complexity.

1) The first order analysis. This entails basic cycle analysis based on discrete thermodynamic equilibrium states. This type of analysis provides useful comparative information on different cycles, and can also be used to investigate the effects of non-ideal processes.

2) The second order analysis treats the entire machine as a single entity, and thus recognises the fact that the gas is in numerous bulk thermodynamic states throughout the machine at any instant. Simplifying approximations are generally made so that the analysis becomes mathematically tractable. This results in a simplified picture of the requirements for a particular performance. Analyses which belong to this category include Schmidt-type analyses and also lumped parameter analyses where an effort has been made to include some real effects such as heat exchanger efficiencies. Characteristics of the second order analysis include ease of geometrical optimisation and shorter times to solve the cyclic behaviour of a machine.

3) The third order analysis approaches closest to reality. The describing differential equations for the system are solved numerically by computer and as few as possible simplifying assumptions are made. Such analyses are generally very complex and unwieldy. Extreme care must be exercised when setting up the describing differential equations since an inaccurately modelled effect may easily go unnoticed. The time required to

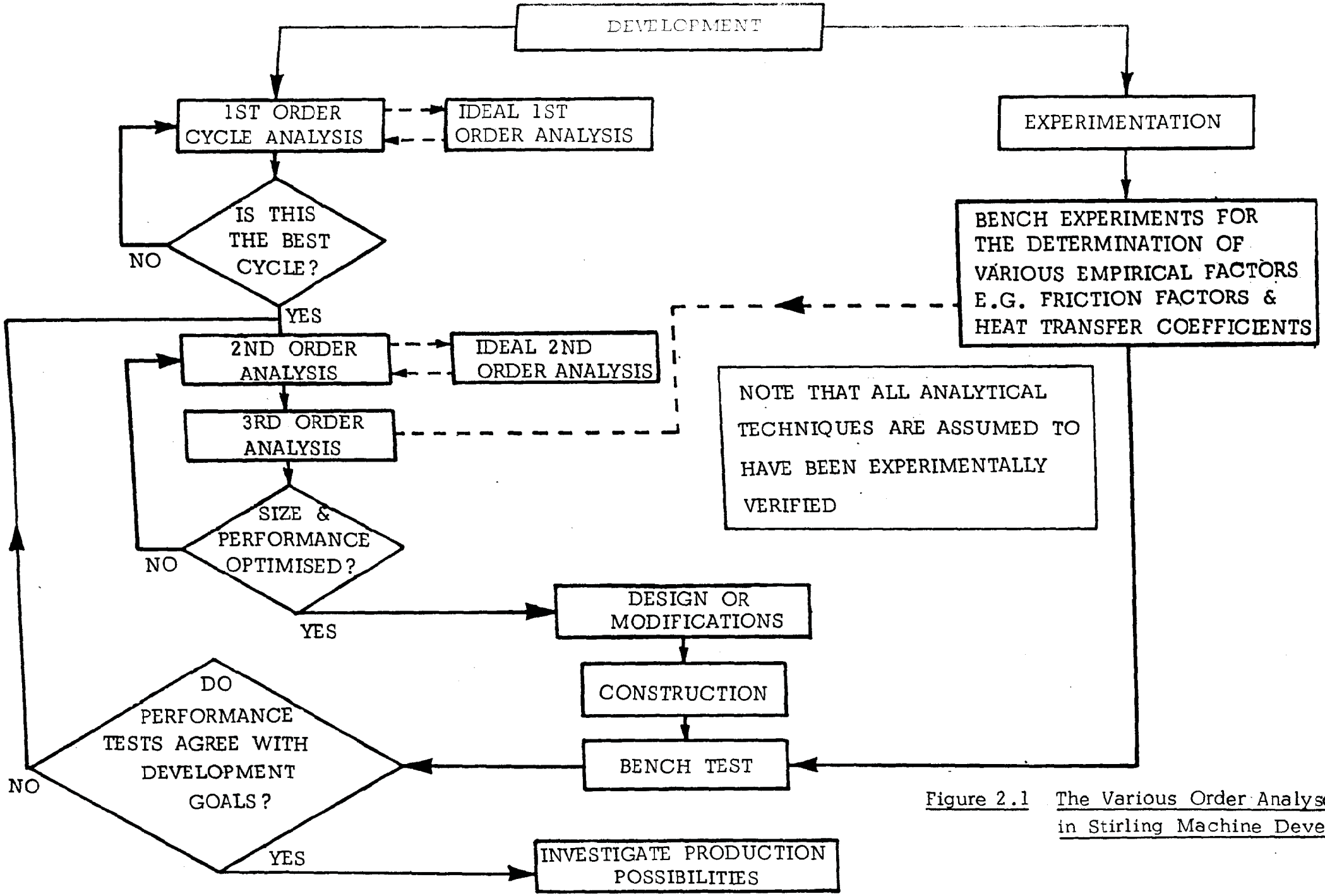


Figure 2.1 The Various Order Analyses in Stirling Machine Developme

solve the cyclic behaviour of a particular machine is much longer than for the second order analysis, and thus such analyses become very expensive when used indiscriminately.

Figure 2.1 shows how the three orders of analysis fit into the development programme of a real machine.

First order analyses are not included in this work and the interested reader is referred to Urieli [Ur77], Rallis *et al* [RU77] and Kovtun *et al* [KN67]. Comprehensive second order analyses which may include such effects as pressure drop, non-isothermal behaviour and non-ideal regeneration [Ki65, Fi60.2, QS68, Qv67, RS70] are also not included in this work. Urieli [Ur77] covers the developments and shortcomings of the second order analyses published thus far.

Ideal second order analyses are presented, ie, the Schmidt analysis and the ideal pseudo-Stirling analysis.

A critical appraisal of Urieli's third order analysis [Ur77, UR77] is presented and an experimental verification is also included.

Note that it is possible to do an ideal analysis within any of the three orders. The definition of an ideal analysis is taken to be any analysis which does not include losses. It would,however,seem that to do an ideal analysis using the third order approach would be an exercise in futility.

2.2 The Schmidt Cycle Analysis [Sc1871]

The Schmidt analysis has evolved into what may rather be known as the Schmidt approach. Recent forms of this analysis bear little resemblance to that presented by Schmidt in 1871 [Ko72]. An important feature of this analysis is that it is the only known Stirling machine analysis which produces closed

form mathematical solutions to the performance of such machines.

The Schmidt analysis provides for the harmonic motion of the moving parts and includes the fact that the gas is in a multiple of thermodynamic states throughout the machine. In particular, expansion does not take place completely in the hot space, nor compression completely in the cold space. With suitably selected phasing of the volume variations, such that the volume variation of the hot space leads that of the cold space by about 80^{o} to 90^{o}, it is possible to arrange that expansion takes place mainly in the hot space and compression mainly in the cold space, with a net positive work output per cycle.

Schmidt assumed sinusoidal motions of the pistons and displacers but noted that this was not the case in practice since an infinitely long connecting rod would be required. He also assumed perfect regeneration and isothermal expansion and compression. No losses are accounted for and the pressure is assumed constant throughout the system at any instant.

The Schmidt analysis is considered to be the classical analysis for Stirling machines and has thus become the ideal standard against which to compare actual machine performance.

A comparison of the Schmidt analysis and the computer simulation will be given in Chapter 7.

It is important to note at this stage that the Schmidt analysis assumes that heat transfer occurs by virtue of the varying working space volumes. This is not the case in most practical machines and it is usual to add separate hot and cold heat exchangers. When applying the Schmidt analysis to include these spaces it is found that no net heat is transferred over a cycle by the separate heat exchangers (Appendix A).

There have been many recent papers written using the Schmidt approach [RD46, Fi53, De53, KJ54, KM58, Fi60.1, Kö60, Ki62,Wa62.1, Ki63, Cr65, Me68, UR76, BR77, Ur77, MW77].

Many of these papers have described methods of optimising the geometry of Stirling machines, a procedure to which the Schmidt analysis is particularly well suited [Fi60.1, Wa62.1, Wa62.2, Ki62, Wa73].

Walker and Agbi [WA73] have extended the Schmidt analysis to include two-phase two-component working fluids and Metwally and Walker [MW77] have similarly described chemically reactive working fluids in Stirling engines.

The main reasons for the reduced efficiencies of real engines are the irreversibilites associated with any real system. These irreversibilities are due to local temperature gradients and viscous dissipation in the working gas. Both these effects combine to produce flow losses within the system. Ideal analyses, by their very nature, are unable to account for these irreversibilities. Thus the Schmidt analysis can only give a good indication of the performance of machines which are operating at their maximum efficiency, since at this point irreversibilities will be at a minimum. Another point of contention arises when deciding on what temperatures to use for the upper and lower temperatures in the analysis. If, as is often done, these temperatures are respectively taken as the heater wall and cooler wall temperatures then this is tantamount to assuming that the heaters and coolers are operating at 100% efficiency. This assumption can thus introduce additional error. Alternatively, these temperatures may be assumed to be the average upper and lower temperatures of the gas in the working spaces. This assumption is more in keeping with the actual analysis and tends to produce better

results (Chapter 7). A further advantage is that the efficiencies of the heat exchangers can to some degree be evaluated separately.

Owing to the large number of papers presented on the Schmidt analysis, it was decided to include in this work a complete and as up to date a form of the analysis as possible. This work is presented in Appendix A and a comparison with the computer simulation is presented in Chapter 7. Optimisation curves and surfaces are not presented since it was felt that this would not be pertinent to the work presented here. The interested reader,however, can refer to the relevant references already cited.

2.3 The Ideal Pseudo-Stirling Analysis

In most practical engines heat is mainly transferred by way of separate heat exchangers for the cooler and heater, the expansion and compression spaces are more or less adiabatic [Mc59]. In such situations then, the isothermal Schmidt analysis is not the ideal analysis for comparison and it would thus seem that an ideal analysis based on adiabatic working spaces would do better. Refrigerators on the other hand can be expected to transfer heat mainly at the working spaces (or at least for the expansion space). This may be part of the reason for their generally better performance.

Finkelstein [Fi60.2] derived an analysis which was based on the Schmidt approach in that the analysis described an ideal cycle except in so far as the working spaces were concerned. Here he defined a non-dimensional heat transfer coefficient for the heat transferred at the working spaces. As a special case he calculated the performance of a machine with adiabatic working spaces, ie, with the heat transfer coefficient equal to zero. A closed form solution was not possible and it was necessary to integrate two simultaneous differential equations

numerically. The most important result of this work was the demonstration that Carnot efficiency is not obtained for any condition other than isothermal working spaces.

Walker and Khan [WK64] expanded Finkelstein's analysis for the adiabatic working spaces case. This work included optimisation curves and surfaces, and the effect of geometry on efficiency was also investigated. This was the first analysis devoted entirely to the behaviour of the ideal pseudo-Stirling cycle.

Both these analyses were based on sinusoidal motion of the reciprocating elements which is valid as a general approach to the problem.

The second order ideal pseudo-Stirling analysis presented in this work (Appendix B) assumes perfect adiabatic working spaces and proper crank-slider formulae are used to determine the motion of the reciprocating elements. In addition the instantaneous heat flow rates in the cooler, heater and regenerator are also presented for comparison with the computer simulation (Chapter 7). Optimisation analysis is not included as it is not pertinent to this work.

2.4 Computer Simulation Analysis

The two most sophisticated Stirling machine computer simulation studies published to date are those of Finkelstein [Fi62, Fi64, Fi67.1, Fi67.2, Fi75] and Urieli [Ur77, UR77].

Other work known to be in progress in this field is that of Schock [Sc77], but this work has not yet been published.

Neither Finkelstein's nor Urieli's analyses have ever been fully verified in the open literature. Finkelstein's work has however been applied to real machines [Fi75] and he has stated that excellent agreement with experimental work has been achieved [Fi64].

2.5 Finkelstein's Model

This work is the result of a continuing development programme to simulate the performance of Stirling cycle machines making as few assumptions as possible. Finkelstein published his first paper on the analysis of non-ideal Stirling cycles in 1960 [Fi60.2]. He has always approached the problem by considering the machine as a complete and continuous entity, and therefore in an attempt to introduce increasingly real effects, his analysis has evolved into a simulation procedure which is solved with the aid of a computer.

Owing to the developmental nature of Finkelstein's work, only his most recent paper need be considered here. A discussion of this work was published in Walker's book [Wa73] before it was presented at the 10th IECEC [Fi75]. Urieli [Ur77] includes a full description of Finkelstein's earlier work.

Finkelstein's work is based on the standard integral forms of the energy and continuity equations. A momentum equation as such has not been used. Instead he assumes that the flow resistance is concentrated at the inlet/exit of the expansion and compression spaces and that the mass flow rate can be calculated from the following formula:

$$dmc/dt = Kc\ pr(pr - pc) \qquad (2.1)$$

where

Kc is an empirical factor
pr is the spatial average pressure in the regenerator

Equation (2.1) is written for the mass flow rate at the inlet/exit of the compression space.

This formula is similar to that used by Kirkley [Ki65] in a comprehensive second order analysis. His result is as follows:

$$(dm/dt)^n = 2p\Delta p/(f\ R\ T) \qquad (2.2)$$

where

f is a friction factor
R is the gas constant
n is an index chosen for best correlation.

For small temperature fluctuations (2.2) can be written as follows:

$$(dm/dt)^n = K\ p\Delta p \qquad (2.3)$$

which is similar to Finkelstein's except that Kirkley found that best results were obtained for $n = 2$.

Both (2.1) and (2.3) neglect the inertia effects of the gas in that for zero pressure drop, the mass flow will also be zero.

In an attempt to investigate the validity of (2.1) and (2.3), the mass flow rate which was numerically calculated from the complete momentum equation (derived in Appendix D) is plotted against the pressure term $p\Delta p$. This is shown in figure 2.2 for a Stirling cycle machine operating at a very low speed. A low speed was chosen so that second order effects would be minimal.

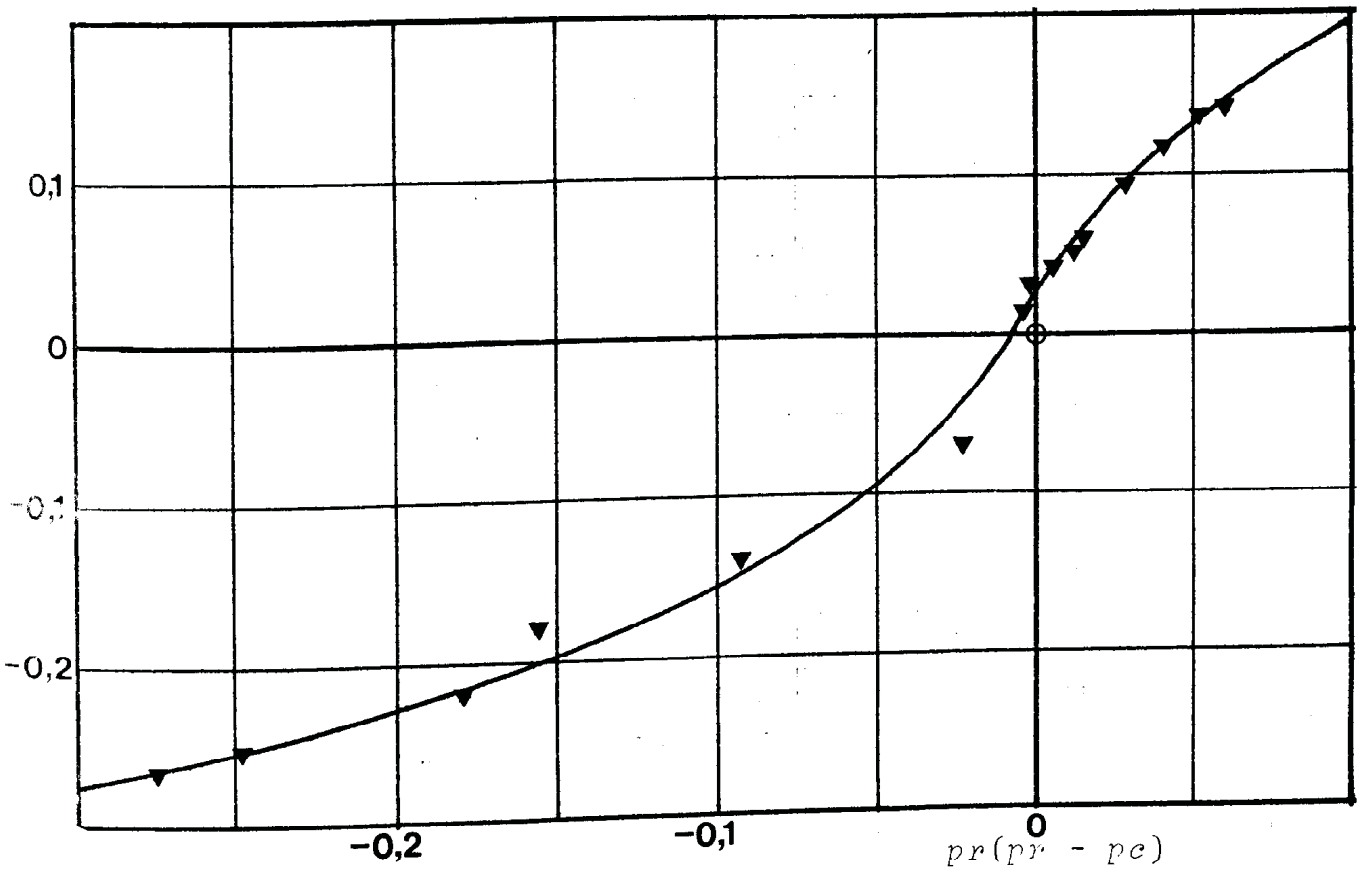


Figure 2.2 g Versus $pr(pr - pc)$ (Normalised)

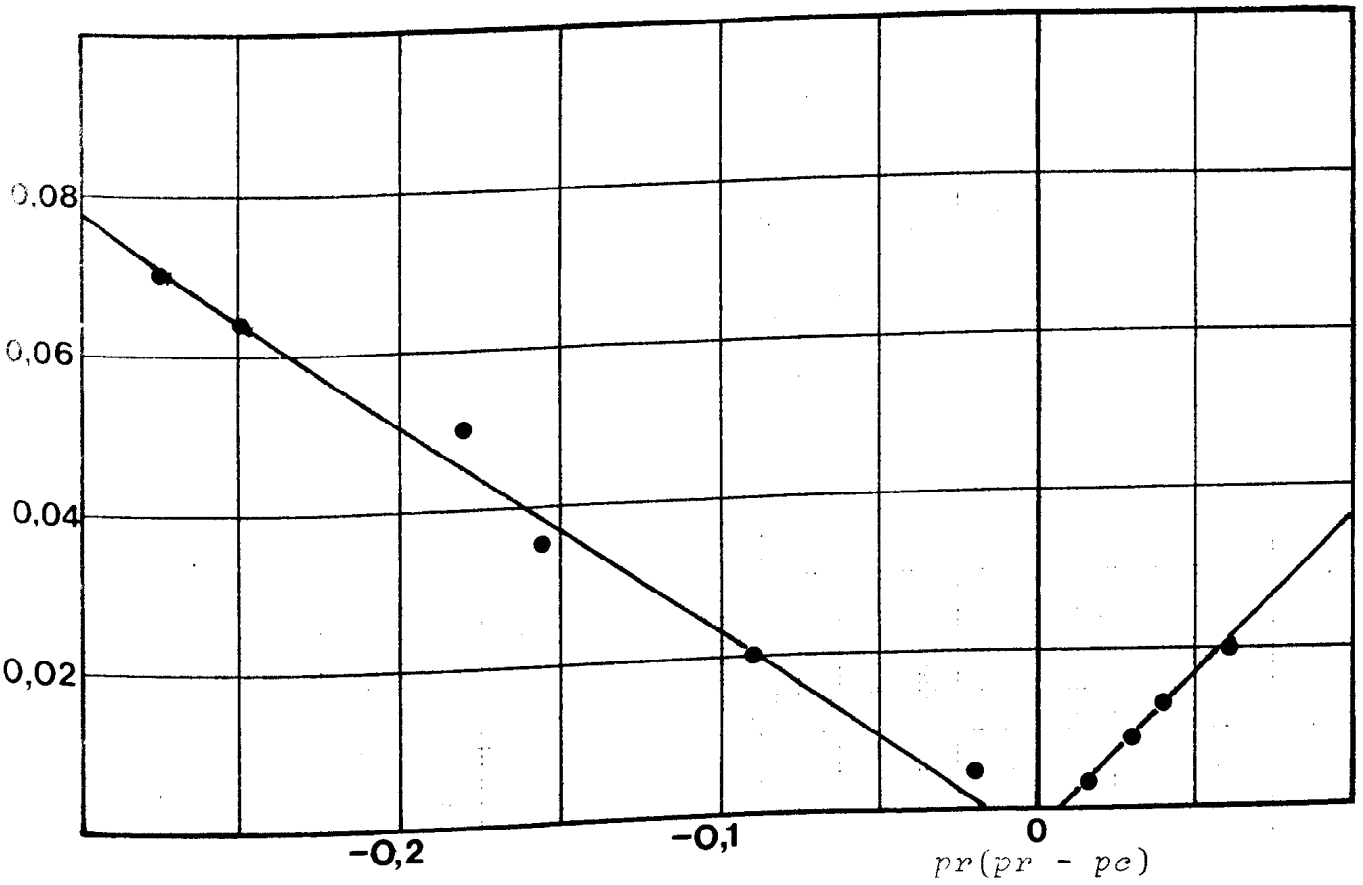


Figure 2.3 g^2 Versus $pr(pr - pc)$ (Normalised)

Two things are apparent from Figure 2.2, *viz*: a) for a zero pressure drop it *is* possible to obtain a finite mass flow and b) the curve is not linear as suggested by Finkelstein's formula. For higher speeds these two effects will become more pronounced.

For the sake of comparison, the square of the mass flow rate is plotted against the pressure term in Figure 2.3. It appears from this plot that Kirkley's formula could be fairly accurate. The zero error is due to the inertia of the gas, however this could be corrected for by adding an empirical constant to Kirkley's formula.

Finkelstein also neglects kinetic and potential energy of the gas. The contribution made by the potential energy to such systems would undoubtedly be small. Neglecting the kinetic energy, though, can present some problems since this assumption also implies that the gas inertia is negligible (Appendix D). In any event, the inertia of the gas was not allowed for and thus no analytical contradiction arises here.

The application example presented by Finkelstein was the analysis of an infrared detector cooler (ie, a refrigerating Stirling cycle). The system was subdivided into three discrete inter-related networks (Figure 2.4) as follows:

1) Machine Temperature Distribution Network
2) Gas Temperature Distribution Network
3) Gas Mass Distribution Network.

All heat transfer or any equivalent energy transport between nodes was accounted for by a 'conductance' and in each case the energy transferred was written in the form

$$C\,dT/dt = \Sigma h(Tw - T) + Q \qquad (2.4)$$

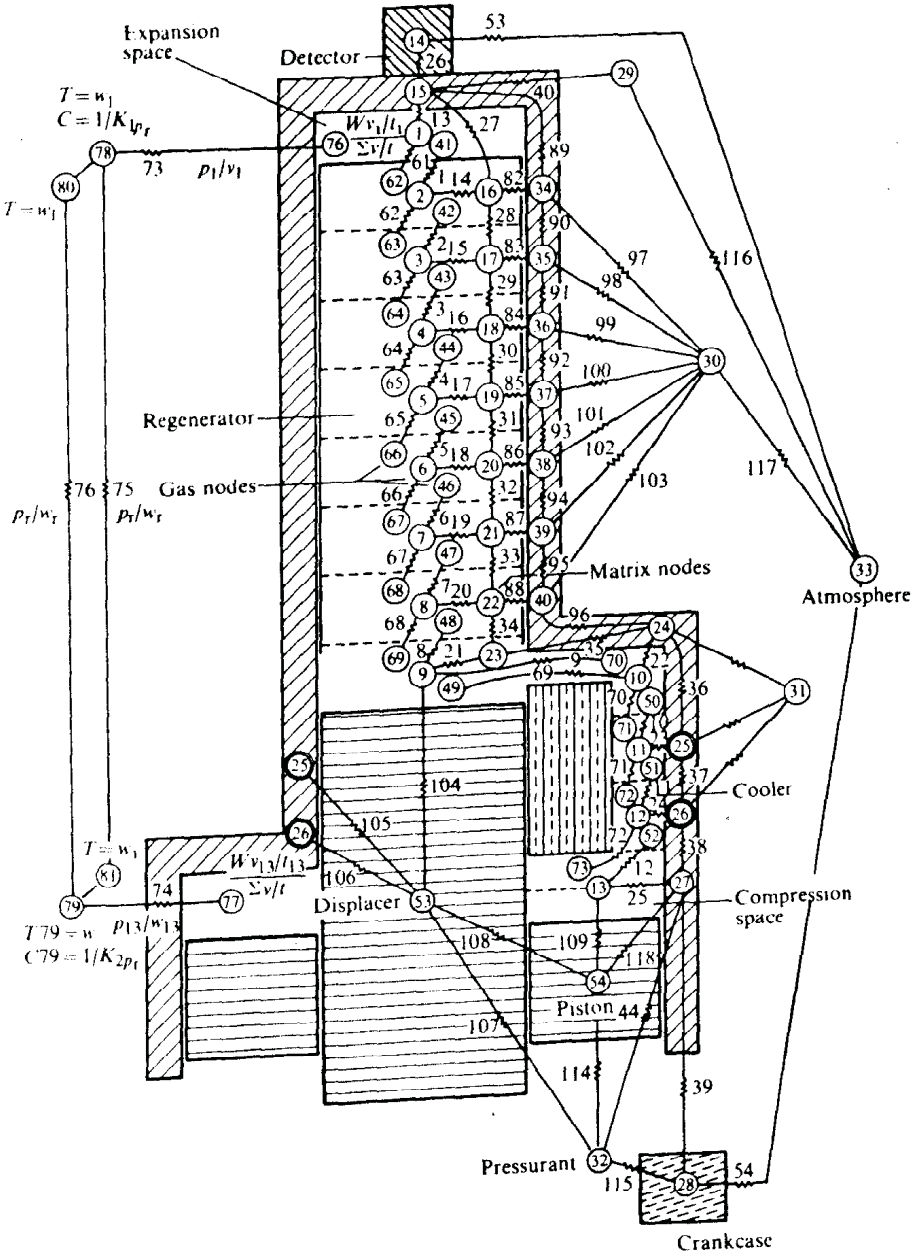


Figure 2.4 Representation of the Nodal Network for Digital Computer Simulation of a Small Stirling-Cycle Cryogenic Cooling Engine (After Finkelstein[Fi75])