Understanding pedagogic shifts from concrete to abstract conceptions of number

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A research report submitted in fulfilment of the requirements for the degree of Masters in Education

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September 2013

DECLARATION

I declare that this research report is my own work, except as indicated in the acknowledgements, the text and the references. It is being submitted in fulfilment of the requirements for the degree of Masters in Education at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other institution.

Signed Date: September 2013

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Abstract

My research study aimed to explore the pedagogic shifts between working with concrete to more abstract conceptions of number. By using a case study approach focused on a grade 2 (G2) Foundation Phase (FP) teacher who retained her class into grade 3 (G3), I gathered data on her teaching over two years (2012-13) in the context of the 'Lesson Starters Project' (LSP). In addition, the teacher also participated in another project within the Wits Maths Connect Primary project (WMC-P) which was focused on developing content knowledge related to primary mathematics during 2013. Whilst content knowledge course assessment indicated gains through this year, the teacher's results indicated gaps in mathematical content knowledge - a feature that literature has highlighted as quite common amongst primary teachers in South Africa and internationally. My focus in this study is on the extent to which this teacher in the LSP professional development project specialised content and modes of representation and showed connections between these aspects.

The findings showed that there were varying degrees of specialisation of content and specialisation of representations. In other words, the teacher is seen to make the mathematics more sophisticated in conjunction with the use of a variety of representations or strategies. There was evidence that the degree of shifts towards more abstract strategies depended at least partially on the teacher's beliefs about the abilities of different learners in her class.

Key words: number sense, abstract calculations, concrete calculations, connections, representations, specialisation of content, specialisation of representations, Foundation Phase, South Africa

Acknowledgements

Firstly, I would like to thank God for giving me the opportunity to complete this Masters studies and the research report. It is by His grace that this research report has been completed. I wish to thank most deeply my professors, lecturers and research report supervisor, Professor Hamsa Venkatakrishnan, who have provided, without hesitation, and with unflagging support, constructive comments, suggestions and criticism, all of which provided a strong backing for this research work. I would like to thank her for her support and the intellectual energy that she gave to this report and for her ability to listen with care and critique with rigour. I also wish to thank those who, from the start, provided constant constructive comments and set out a clear direction for me.

My sincerest appreciation goes to my sponsor, the DHET who funded my studies through my supervisor Professor Hamsa Venkatakrishnan, I give many thanks. I also thank our project leader Mr Corin Mathews for his unwavering support, dedication and much needed advice in order to stay on the academic path. The many tears, laughter and sheer irrationality were part of the memorable stepping stones on this erratic journey. To my employers, firstly the Gauteng Department of Education (GDE) for consenting to my leave and for allowing me to be away on study leave as well as the Chairperson of Horizon View Primary School, Mr Advocate Segone, for allowing me the opportunity to be developed professionally. Please accept my deepest gratitude.

To you, members of the case study and other men and women involved in this research, what can one say? Many thanks for taking the time to share your most valuable experience, knowledge and in-depth perception on teaching mathematics.

As with all success in life, this would not have happened without the concern and encouragement of those closest to my heart. The most significant accolade goes to my family, friends and colleagues whose loving guidance provided me with moral support; for allowing me to be away so many times and for being there for me when I needed them the most. For having made the journey less wobbly, I am most grateful and very proud of knowing and having you all in my life.

Dedications

This research report is dedicated to my siblings Brigitte, Edmund and Kurt and close friends Abigail Pophaim, Ms Sam Mitha, Ms Astrid Meng, Stuart Akers and Corin Mathews to name but a few, who believed in me when I faltered and whose support and prayers sustained me over the long period of study. Also, to my dearly loved mother, Mrs Sheila A. Alexander, who emphasised the value of education, hard work and earnest prayer. For these reasons and amongst others she has been and continues to be a great influence on my life.

Glossary of terms

‘Count-all’ involves the counting of all the items to obtain the solution, and usually involves three separate counts from 1 - for the first number, the second number and the total number

‘Count-on’ entails holding one number mentally (initially the first in an addition sum) and counting on from it to obtain the answer.

‘Activity’ is a task that sets out what needs to be done or what is required of the learners to do in order to demonstrate their understanding or lack thereof in a particular theme or concept.

‘Specialisation of content’ is linked to the progression of tasks or activities over a period of time that require shifts from concrete counting (count-all with unit counting) to calculations by counting (count-on, count-from larger, group counting) to calculating without counting (recalled number facts like bonds, multiplication tables, derived facts by looking for patterns, compensation and base 5 or base 10 strategies).

‘Specialisation of representation’ refers to the shift of use over time from visible, concrete apparatus to the gradual use of more abstract apparatus as the representations become more symbolic and present as appropriate internalised mental representations.

‘Internalised’ refers here to demonstrating comprehension of concepts and strategies without physical representations in the calculation process is the ultimate goal for mental calculation strategies.

‘Specialised’ refers to the use of more advanced strategies or mathematical terminology, representations and talk.

‘Content’ refers to what is being taught

‘Representations’ refers to different types of apparatus used in a concrete to semi-concrete to abstract way.

‘Mathematical language’ refers to language that is subject specific namely mathematical vocabulary whether it is symbols, representations or words.

Abbreviations

G1 - Grade 1

G2 - Grade 2

G3 - Grade 3

G4 - Grade 4

G6 - Grade 6

G9 - Grade 9

LSP - Lesson Starters Project

LSA - Lesson Starter Activities

FP - Foundation Phase

DBE - Department of Basic Education

DoE - Department of Education

GDE – Gauteng Department of Education

ANA - Report on the Annual National Assessments

FNWS – Forward Number Word Sequence

BNWS – Backward Number word Sequence

OTLM - Opportunity to Learn Mathematics

CAPS - Curriculum and Assessment Policy Statements

SEAL - Stages of Early Arithmetic Learning

WMC-P - Wits Maths Connect - Primary

LFIN - Learning Framework in Number

WC - Whole Class

IW - Individual Work

Contents

Chapter 1

Introduction to the Study

1.1 Overview

This study aims to explore and understand developments in the teaching of number concepts of a Foundation Phase (FP) teacher who showed evidence of significant gaps in her mathematical content knowledge in the context of a developmental project. The study is located in a broader context of evidence of widespread gaps in primary teacher mathematics content knowledge (Carnoy, Chisholm, & Chilisa, 2012) and evidence of weaknesses in learner performance in number (DBE, 2011). Within this context, I am involved in a teacher development project - the Lesson Starters Project (LSP), within the Wits Maths Connect-Primary (WMC-P) project focused on developing the teaching of number. The LSP is an attempt to help teachers in their work with number focused mental activities through using various strategies at the beginning of the lesson for fifteen to twenty minutes. The LSP is set within the WMC-P research and development project, which seeks to improve primary mathematics teaching and learning in ten partner schools.

In this research, I want to understand and analyse how a FP teacher develops her teaching of number across a two year period. My purposive selection of this teacher is based on an interest in understanding the kinds of shifts that might be possible for teachers with significant gaps in mathematical content knowledge, with specific emphasis on teaching that supports moves towards more abstract conceptions of number.

1.2 Problem statement

International and national test data have shown that the majority of South African learners are performing below the minimum requirement level of performance stipulated in the curriculum documents (Department for Basic Education, 2011; Ensor, Hoadley, Jacklin, Kuhne, Schmitt, Lombard & van den Heuvel-Panhuizen, 2009). This is evident in the Schollar and Associates (2004) summary of findings from a range of national, regional and international tests:

> ‘NSE: Grade 6 (2005) - 81% of learners below minimum competence level; SACMEQ: Grade 6 (2000) - 84% of learners below minimum competence

> level; TIMMS: Grade 8 (2003) - 82% of learners below minimum competence level. The SACMEQ study found that 52% of grade 6 learners were achieving scores in mathematics at the Grade 3 level or lower' (p. 2).

Furthermore, a key overall finding of the 2011 ANA report (Department for Basic Education (DBE), 2011b) was that learners, especially disadvantaged learners, continued to perform well below the expected grade level competencies. In the FP, the ANA results have indicated that learners are operating far below the required level of performance, for example, 'in the national G3 numeracy data, 34% of the learners attained a level of performance that represented at least a partial achievement - above 35%' (DBE, 2011, p. 30). However, the 2012 ANA report for the FP indicated that the mathematics at the Foundation Phase was largely of satisfactory quality and level, with mean scores showing that: 'G1 learners improved from 63% in 2011 to 68% in 2012, G2 learners from 55% in 2011 to 57% in 2012 and G3 from 28% in 2011 to 41% in 2012' (Department for Basic Education (DBE), 2012, p. 68). Learners' achievement at the Intermediate Phase level showed a wide range of deficiencies in basic knowledge and competencies in G4 (from obtaining 28% in 2011 it increased to 37% in 2012, in G6 (from obtaining 30% in 2011 it decreased to 27% in 2012) and especially in G9, where the mean score was 13% in 2012. These results concurred with the TIMSS study that ranked South Africa at the bottom. This trend can be observed by looking at the regressing mathematics average percentage results from G1 to G9 '68% in G1 to 13% in G9' (DBE, 2012, p. 23). It is evident that whilst the ANA 2012 report acknowledges increase and improvement in the mathematics results in the FP, overall performance is still low.

Schollar and Associates (2004), in a study focused on 7 028 diagnostic test scores of learners in the Intermediate Phase from 154 schools in all 9 provinces found that G3 learners still used counting-based strategies to solve number problems. Counting-based strategies in addition situations rely on 'count-all' which involves the counting of all the items to obtain the solution, and usually involves three separate counts from 1 for the first number, the second number and the total number. For example, 4+6 = 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, after counting out the 4 and 6 each in ones.

This strategy contrasts with 'count-on' which entails remembering one number mentally (initially the first in an addition sum) and counting on from it to obtain the answer. For example, 9+2=□ starts with 9 and counts on to 10, 11. Schollar and Associates found 'no evidence of calculation by structuring, for example grouping or breaking up numbers'. From this, they concluded that the learners whom they tested were effectively operating at G1 or at best early G2 levels. Aligned with these findings, the WMC-P project's baseline data analysis indicated that three-quarters of their G2 learners' sample (n=231, based on six learners drawn from across the attainment range in each G2 class across their ten project schools) were assessed as operating at levels on an international research-based oral interview test that indicated 'count-all' strategies on addition and subtraction problems in the 1-20 number range (Venkat, 2011). Literature on early number strategies, detailed in chapter 2, describes 'count-all' as a highly concrete strategy.

A range of issues have been highlighted in relation to primary mathematics teaching in South Africa. Problems identified in the teaching of number specifically are: the failure to extend learners from concrete counting strategies (which involve working with objects or counting on fingers with unit counting) towards more abstract calculation-based strategies (involving diagram representations or more symbolic number line-based representations and eventually mental calculations and the grouping and decomposing of number mentioned above). Ensor et al. (2009) criticise numeracy teaching in FP for the lack of sufficient shift towards more 'specialised' number content (working with abstract number concepts rather than counting-based number concepts) and the more 'specialised' representations of number (concrete to iconic to indexical to symbolic to calculating without counting representations) that support progression to more 'specialised' content. In addition, they, and others, highlight slow pace and lack of learners doing individual work as crucial aspects that need to be addressed in order to help learners to work with number more efficiently (Carnoy et al., 2012; Ensor et al., 2009). Ensor et al. (2009) indicate that teachers hold learners back in concrete counting and argue that learners' lack of ability to abstract from concrete representations as well as the less frequent use of written mathematical elements in class work inhibits the development of number concepts. Ensor et al. (2009) also state that factors that inhibited the development of number were that teachers tended to use concrete methods, representations and strategies in their teaching. In so doing, they limited the occasions for learners to access symbolic representations. An implication of their findings is that teachers should focus on supporting the gradual move to more abstract representations of number within their teaching, alongside the presentation of tasks that require more specialised number content.

Venkat and Naidoo (2012) identified problems with poor connections within teacher talk and classroom based activities in one lesson. The examples provided by the teacher were not connected in teacher talk by strategies where learners could apply known answers to derive new answers by using established number facts. Poor connections were also seen where the teacher's talk did not explicitly make known the links between the different strategies used in solving the number problems. For example, when subtracting, the teacher explained by relying only on unit counting with the use of concrete apparatus, as noted by Ensor et al. (2009) and Venkat & Askew (forthcoming). In addition, learners who were able to give the answer as a recalled fact were not asked to explain their method; instead they were pushed back towards concrete counting strategies. Furthermore, they noted disruptions to the connected learning of number concepts as a result of random selection and sequencing of mathematics activities (Venkat & Naidoo, 2012).

Problems with learner performance in mathematics have also been linked with the gaps in primary teachers' mathematical content knowledge. The gaps are seen in teachers lacking the necessary skills and tools to help the learners to acquire fundamental mathematical concepts through the use of a string of well-sequenced activities (Carnoy et al., 2012). What is of particular interest is that research points to widespread gaps in primary teachers' mathematical content knowledge in South Africa. Research conducted in sub-Saharan Africa on mathematics teaching shows that weak content knowledge makes it very difficult for these teachers to work constructively and progressively with learners on mathematics (Carnoy et al., 2012).

The CAPS curriculum supports the development of number sense in its prescription with respects to specialisation of content over specified time frames. The CAPS curriculum document contains more specialised related subject matter which is progressively developed and it also identifies relevant prior knowledge needed before teaching a new concept. It also makes the practice of mental activities mandatory for approximately twenty minutes a day which is an integral part for developing number sense. It is in this mental activity slot focused on number sense that the bulk of the empirical data in this study is focused. Another policy intervention required teaching with prescribed lesson scripts in the main activity. Hence my decision (aligned with the broader project) to focus attention on number sense in the mental starter assigned time. The focus on number sense further encourages the teacher to use benchmarks and various bases: 5, 10, 20, 50 and 100 facts to facilitate mental calculations (Anghileri, 2006). It continues to make mention of a range of strategies and representations such as 'subitising, temporal sequencing, empty number lines, arrays, decomposition and partitioning which aid the development of number sense, thus making it more specialised if applied correctly. The promotional benchmarks have been raised in the CAPS document (rating of 3 for mathematics) which infers that there is a need to specialise the mathematical content, representations and mathematical talk if learners are to meet these promotional requirements (Department for Basic Education, 2011). With this in mind, I will be observing the teacher based on the enacted curriculum which is founded on her selection choices of tasks which are context based or not, representations and teacher talk. Specialisation will also be observed if the number range is in accordance with the relevant grade requirement alongside a more abstract representation and/or strategy. I also note if there is a shift from class to more individualised work. The challenge that I perceive the teacher to have is whether knowing what to teach is sufficient as neither the CAPS document nor the training elaborate or provide insight into new concepts or illustrate how to use various representations in a differentiated way, namely from concrete to semi-concrete to abstract. It further fails to enlighten teachers on the different strategies included in the curriculum document such as 'subitising, temporal sequencing, decomposition and the use of arrays' which are subject specific terms which the teacher may not encounter in her everyday life (Department for Basic Education (DBE), 2011a). With this said, the way the teacher interprets the curriculum document and models the above is of great importance for my study.

In addition, the central problem identified here within the teaching of number is related to the lack of shift from concrete to abstract number concepts and strategies. In the LSP, attention was given to progression from more concrete to more abstract number concepts using the Stages of Early Arithmetical Learning (SEAL) within the Learning Framework in Number (LFIN) model devised by Wright, Martland and Stafford (2006). The strategies associated with gradual shifts by learners from more concrete to more abstract number concepts in Wright et al.'s (2006) work are summarised below: Their framework highlights how children work with number and includes different aspects of early number learning. Furthermore, they provide the stages for early arithmetical learning (SEAL) which provides descriptive and progressive counting strategies that are relevant to FP number learning. The authors note that this framework can aid teachers in diagnosing learners' understanding of number sense. They differentiate between five stages of counting as follows:

> Stage 0 - emergent counting which implies that learners are unable to count visible items or they do not know the corresponding number name;
> Stage 1 - perceptual counting means that the learners can count those items that are visible but not those items concealed from them;
> Stage 2 - figurative counting is when the learner chooses to 'count-all' the items as opposed to 'counting on';
> Stage 3 - initial number sequence is when the learners use 'count-on strategies';
> Stage 4 - intermediate number sequence is when the learners are able to 'count-down-to' to solve problems and
> Stage 5 - facile number sequence is the ability to count using more advanced strategies which do not involve counting in ones, i.e. range of non-count-by-ones strategies (abridged version, of Wright et al.'s SEAL framework, 2006, p. 22).

A shift towards more abstract number conceptions can be seen within the more sophisticated counting strategies that occur within the higher stages. My study's focus is not on the LSP intervention. Rather, I focus on shifts (if any) in the teaching of number of one FP teacher who participated in the LSP project. I selected this teacher for two reasons. Firstly, she worked in a school where the G2 teachers retained their class into G3, allowing the LSP project (which began work with the 2011 G2 cohort and followed this cohort into G3 in 2012) to gather data on her teaching over two years. Secondly, her participation in another project within the

WMC-P project focused on developing content knowledge. This indicated that her content knowledge related to primary mathematics (whilst improved over the course of 2012) contained gaps, a feature that literature has highlighted as quite common amongst primary teachers in South Africa and internationally. Following my summary of problems related to number teaching, my focus in this study is on the extent to which this teacher in the LSP professional development project specialises content and modes of representation and shows connections between these aspects. These issues are central to my research questions, as stated below.

1.3 Specific research questions

My interest is in investigating any shifts made towards more abstract ways of working with number of a teacher within the WMC-P teacher development project. My research questions are focused on teacher talk and the representations used within the classroom to communicate early number ideas. I am interested in the ways these two aspects promote or constrain possibilities for learners to shift towards more abstract ways of working with number.

Following Ensor et al.'s (2009) development of the idea of 'specialisation', I investigated how the teacher's talk and use of representations in the classroom specialise and connect between:

a. Content (where specialising entails shifts over time from 'count-all' strategies to 'count-on' to 'count-from larger' to mental calculations using recalled, known and derived facts).
b. Modes of representations (where shifts over time entail the use of concrete representations to iconic apparatus to indexical apparatus to symbolic number-based apparatus to symbolic syntactical apparatus to calculating without the use of representations).

1.4 Rationale

As noted above, numerous authors have argued that many South African children do not have a grasp of number sense and number concepts as they are too reliant on concrete operations (Ensor et al., 2009). Therefore, there is a need for a shift over time from more concrete to more abstract ways of calculating (Gray, 2008; Anghileri, 2006). Therefore my focus in this study is on a teacher in the context of a project that aims to help teachers to help learners to shift to more abstract number concepts.

The Lesson Starter Project (LSP) has discussed and shared activities and representations to develop number concepts and strategies to address this gap. My study involves analysis of teaching of starter activities focused on number by one teacher across 2011-12, within the ‘mental activity’ lesson slot that is advocated within Numeracy lessons in FP. This data consists of six videotaped observations of her teaching; two of these observations are drawn from whole class lessons that were videotaped at the start of both years as part of the WMC-P project’s ‘baseline’ data, and the remaining videos are based on teaching of a short, early number-related starter activity. Additional data on the teacher’s perceptions of number teaching and learning were gathered through field notes taken in the LSP workshops and two follow-up interviews. Whilst the focus of this study is on teaching, an assumption that I make in a South African context where authoritative teaching styles have been described as dominant (Hoadley, 2010) is that as a teacher expands her number concept repertoires, this in turn will increase learners’ access to more abstract concepts and representations, and thus increase opportunities for learners to work with abstract calculations which are efficient and flexibly applied (Gray, 2008).

My research attempts to explore the developments of the teaching of number concepts by doing an in-depth study of a teacher’s pedagogic practice relating to number concepts in the context of lesson starter activities. In order to explore this teaching, I focus on the role of teacher’s talk while working with early number, which was not dealt with centrally by Ensor et al. (2009), who paid more attention to tasks and representations (which I also focus on). I aim to understand pedagogic shifts from concrete to abstract conceptions of number in order to explore whether it is possible to move a teacher with significant gaps in her mathematical content knowledge towards more abstract conceptions of number, as well as moving teachers on from a reliance on concrete counting strategies to more abstract mental calculations. The findings of my research may be used primarily to inform teachers about the relevance and impact of their choices and selections with regard to early number learning. More broadly, my study explores developments in teaching of number concepts in the Foundation Phase.

1.5 Why the case study approach

A case study allows me to do an in-depth study of a FP teacher's pedagogic practices in number teaching as opposed to working for breadth (Creswell, 2012, p. 465). Case study allows me to purposively sample. As stated earlier, I intentionally selected the FP teacher at the centre of this study for her experience, the presence of significant gaps in her mathematical content knowledge and her willingness to participate in two teacher development projects; namely the content knowledge course and the LSP project over 2 years as well as her willingness to be part of the study. Taylor (2008) notes that significant gaps in experienced teachers' mathematical content knowledge are widespread in South African schools. Thus, in this respect, this teacher is a representative case of a much broader problem. The reason for choosing an experienced teacher was based on the view that experience was likely to be linked to more willingness to speak, and to explain herself and share her ideas and rationales; thus the participant is 'information rich' (Patton, 1990, p. 169). Purposive qualitative sampling helped me to develop an in-depth understanding of the teacher's classroom practice which in turn provided me with useful information to answer my research questions (Creswell, 2012).

1.6 Outline of the study

For this reason, my research aimed to understand the shifts in teaching with specific emphasis on teaching that supports moves towards more abstract conceptions of number in terms of the specialisation of content and the use of more specialised representations over time.

In the following chapter, I look at a range of literature related to my research question in order to develop a better understanding of number in the FP. The subsequent chapters of the report are structured as follows: in Chapter 2, I provide a review of the relevant literature and I describe the conceptual framework that was used as the analytical framework, namely Ensor et al.'s (2009) forms of specialisation. Chapter 3 describes the research design and methodology used in this study. In Chapter 4, the findings are presented and discussed using excerpts from the lesson observations and interview transcripts to support claims and interpretations of the data. Finally, in Chapter 5, the implications of the research findings and the overall significance of the study are discussed.

Chapter 2

Conceptual Framework and Literature Review

2.1 Introduction

In this study, the research questions are focused primarily on the ways in which teacher talk and use of representations promote or constrain possibilities for learners to shift towards more abstract ways of working with number. Following Ensor et al.'s (2009) development of the idea of 'specialisation', I want to explore and understand how the teacher's talk and use of representations in the classroom specialises and connects between:

a. Content (where specialising entails shifts over time from 'count-all' strategies to 'count-on' to 'count-from larger' to mental calculations).
b. Modes of representations (where shifts over time entail the use of concrete representations to iconic apparatus to indexical apparatus to symbolic number-based to recalled facts without counting and to known or derived facts).

Therefore in this chapter, I briefly recap on the motivation for these foci in this study by detailing findings in relation to problems with specialisation and connections in South Africa. Thereafter, I use international mathematics education literature on early number to describe what counts as number sense.

Then I expound on how the key tenets of specialisation of content [based on Ensor et al.'s (2009) descriptions] connect with the mathematics education literature base on number sense. In conjunction with the former, I expand on what would count as specialisation of representations according to the international literature base. Alongside this, the notions of what count as 'good' connections in early number with reasons for why this is viewed as important in the international literature are discussed. I then move to looking at literature that discusses how pedagogy can promote specialisation of content and representations as well as connections in mathematics teaching. Finally, I conclude with a summary of the tenets of my conceptual framework that is based on Ensor et al.'s (2009) notion of 'specialisation'.

2.2 Contextual findings in relation to problems with specialisation and connections in South Africa.

Firstly, widespread problems have been identified with teachers moving to more sophisticated strategies in the following studies done in South Africa. Ensor et al., (2009) carried out their longitudinal study in the Western Cape amongst G1 to G3 learners. They found that the G3 learners remained highly dependent on concrete strategies to solve problems. The authors argue that this was a result of classroom practices that favoured the use of concrete modes of representation, consequently limiting the access to more abstract ways of working with number and using the representations in a more abstract way. This was exacerbated by the inefficient use of class time as well. Even though there was evidence of some degree of specialisation of representations (numbers were represented as numerals in symbolic form) across the grades and there was some evidence of written mathematical statements used, albeit infrequently in G3), the authors argued that the nature and extent of this specialisation was not in accordance with the standard required of that grade by the curriculum.

Schollar and Associates' (2004) empirical study highlighted that G5 and G7 learners relied mainly on unit counting and repeated operations, namely the use of repeated addition and subtraction to solve multiplication and division problems. When the number range was extended, learners demonstrated an inability to calculate effectively and efficiently as some resorted to tally counting or using repeated operations. Their study further revealed that the complexity of the content that learners were exposed to was entirely dependent on the teacher. The need for teachers to be provided with suitable material, if the content was to be more specialised was highlighted by their study (Schollar & Associates, 2004).

Secondly, problems with connections between task and teachers' modelling of more sophisticated strategies through the use of appropriate representations were also identified in these studies. This is apparent in the Venkat and Naidoo (2012) research, which analysed number teaching in one G2 lesson exploring the ways in which the language used by the teacher communicated meaning. They used systemic functional linguistics (Halliday & Hasan, 1985) and variation theory (Marton, Runesson, & Tsui, 2004) to argue that meaning is constructed through strong connections between the teacher talk within and across episodes in the lesson and activities and materials used to develop conceptual understanding. It should be noted that neither of these notions of connections take learners' conceptual understanding nor their actual ability to answer the questions themselves into account which is a feature that would involve learners making connections between tasks, and the representations and procedures needed to answer the problem. These foci on connections and Ensor et al.'s (2009) focus on specialisation both deal with teachers' handling of number ideas, without taking learner responses or their working centrally into account.

The study done by Carnoy et al. (2012) revealed that teachers with an above average mathematical content knowledge taught lessons of better quality and more of the curriculum was also covered as they tended to provide more mathematical lessons to their learners. This is of interest to my study, as my teacher has significant gaps in her content knowledge and it thus becomes interesting to explore the nature and extent of connectedness and specialisation within the number work that the teacher exposes the learners to and the way the teacher engaged with the representations chosen to develop mathematical understanding by using the correct language (mathematical terminology and instructional talk) to communicate and connect number ideas unambiguously (Carnoy et al., 2012).

My study is similar to Ensor et al.'s (2009) in that I also look at the specialisation of content and representations over time but differs in that I do not look at the specialisation of text and time within lessons. Eight years on from Schollar and Associates' study, my research wants to understand the possibilities for pedagogic shifts from concrete to more abstract ways of working with number across lessons over time. From the above mentioned literature, my study aims to determine the extent to which the teacher models more efficient calculation strategies in the numeracy lessons as well as the teacher's ability to select appropriate tasks and to use a variety of representations in more flexible ways. As well as selecting and moving over time towards more specialised modes of representations of early number, I also focus on how the teacher's talk provides connections between more specialised content and representations in the classroom motivated by some of the problems identified in Venkat and Naidoo's (2012) study. These foci lead to the conceptual framework for this study being focused on Ensor et al.'s specialisation, but prior to this I link the issues identified with teaching early number to the international literature on teaching early number.

2.3 International literature on teaching early number

Numerous authors have argued that many South African children do not have a grasp of number sense or number concepts as they are too reliant on concrete operations. Therefore it is important to understand what number sense is and why its development is important for learners to be able to calculate more effectively and efficiently by using more specialised strategies and representations. I view number sense as an overarching goal in the teaching of mathematics as it provides the building blocks and it is crucial for the learning of more complex mathematics in order to calculate more efficiently and effectively (Anghileri, 2006; Askew, 2002; Mcintosh, Reys, & Reys, 1992).

Mcintosh et al. (1992) defined number sense as one's general understanding of number and operations together with the ability to use one's understanding in flexible ways to make mathematical judgements and to generate useful strategies when working with number and operations. In other words, learners need to be comfortable working with numbers. Mcintosh et al. (1992, p. 4) asserted that number sense is having a conceptual understanding and knowledge of the structure of numbers and the size of numbers (Markovitz & Sowder, 1994), having an understanding of the effect and meaning of different operations, understanding of equivalent representations (graphic or symbolic) of number as well as having the ability to apply them to 'computational settings'.

Furthermore, learners need to be able to use numeral benchmarks in order to make calculating more easily (Yang & Hsu, 2009; Anghileri, 2006; Berch, 2005; Markovitz & Sowder, 1994; Mcintosh et al., 1992). This notion of benchmarks assists in the understanding of more difficult calculations, namely base five aids the understanding of working with base ten and later twenty, twenty-five and one hundred respectively.

Mcintosh et al., (1992) further emphasised that contexts (a rich and varied range of contexts) can support children's conceptual understanding of number. To consolidate, Anghileri (2006) further suggested that teachers should expose learners to a variety of contexts in which they can enjoy doing mathematics by practising, talking and recording their mathematical ideas in multiple modes. In so doing, important foundations are laid for learning more complex mathematics later on.

For these reasons, my study intends to explore whether the mathematics over time was made more sophisticated with respect to the content taught and the representations used as expressed in the teacher's talk.

2.3.1 The importance of developing number sense

Number sense in different parts of the world is considered to be an important component and objective of primary schools' mathematics curriculum (Department of Basic Education, 2011; Australian Educational Council, 1990; National Council of Teachers of Mathematics, 1989). It is considered useful to help learners to realise that mathematics can make sense and is not merely the memorisation of a series of facts but the ability to judge the reasonableness of solutions and knowing that there are different ways to finding a solution (Anghileri, 2006; Mcintosh et al., 1992; Howden, 1989). A good sense of number builds learner confidence which in turn impacts their views on mathematics which enables them to apply mathematics and cope better with more advanced mathematics. This kind of access and success can open up career opportunities and it is also useful in other fields for learners who specialise in mathematics in higher grades (Howden, 1989; Committee of Inquiry into the Teaching of Mathematics in the Schools, 1982).

Previously, there was a strong reliance on computational procedures and algorithms which did not translate into learners having a conceptual understanding of number. This resulted in learners not being able to solve problems encountered in a variety of contexts, especially in a society that is technologically advanced (Anghileri, 2006; Mcintosh et al., 1992). For this reason, it is important for learners to develop their conceptual understanding of number sense and for this to be developed through teaching practice where content and representations are specialised in the teaching of mathematics over time.

Therefore, in my research I am interested in Ensor et al.'s (2009) conceptual framework of specialisation of content and specialisation of representations which highlight the notion that teachers need to shift learners from concrete strategies to abstract calculating in order to support learners' development of number sense and vice versa. In this literature review, I therefore concentrate primarily on writing focused on classroom practices that support the development of number sense. Anghileri (2006) and Wright et al. (2010) argue emphatically for the efficacy of number sense in facilitating the learning of more complex mathematics, suggesting that evidence of the encouragement of number sense would be useful to look for within my dataset.

2.3.2 Literature that expands on the notion of 'specialisation of content'

Specialisation of content can be seen in the tasks (examples) selected by the teacher, by the modelling of more advanced strategies by the teacher as the number range extends, in the teacher's talk and by her allowing more specialised strategies to be used across different learners when they need to solve a variety of problems. Ensor et al.'s (2009, p. 8) focus is on the building blocks needed for mastering numeracy: 'progression in acquiring the number concept, the shift from concrete to abstract reasoning and the related move from counting to calculating.' In addition, the language being made more mathematical is an aspect of specialising content that emerged within the data. This will be commented on in the data analysis and findings chapter.

For Ensor et al. (2009), specialisation of content can occur within episodes within a lesson, across episodes as well as across multiple lessons over time. Lesson activities were divided into different episodes which consisted of a single theme or mathematical concepts which were demarcated according to their level of 'specialisation' or change of mathematical foci or format. A single theme usually comprised of a number of tasks and activities that related to the unitary theme or mathematical concept. Therefore, an activity was driven by a task that set out what needed to be done or what was required of the learners to do in order for them to demonstrate their understanding or lack thereof in a particular theme or concept.

Wright et al.'s (2006) framework, introduced in the last chapter, provides detail on the specialisation of counting strategies within early addition, subtraction, multiplication and division. The progression in counting strategies can be used as a tool to note the shifts from concrete to more abstract working with number by the teacher. Their analytical framework (summarised earlier) is referred to as the Learning Framework in Number (LFIN). According to Wright et al. (2006), this knowledge of where the learners fit into the LFIN can be combined with instructional frameworks that support shifts to more sophisticated strategies.

For learners to progress across the stages, they would need to demonstrate that they can count in more sophisticated ways to answer counting, additive and subtractive tasks. Wright et al.'s (2006), framework provides steps in the shifts of moving from concrete to abstract number concepts (specialising content), which may be useful as a tool for looking at the teaching of number in my study. As I look at the data, I want to ascertain whether there is progression across the five stages, namely the emergent counting stage which implies that learners were unable to count visible items or they did not know the corresponding number name for the objects; the perceptual counting stage entailed that the learners could count those items that were visible but not those items concealed from them; the figurative counting stage is when the learners' chose to 'count-all' the items as opposed to 'counting-on'; the initial number sequence is when the learners used 'count-on' strategies, the intermediate number sequence is when the learners were able to 'count-down-to' (the numbers are close to each other making it easier for the learners to find the difference between the numbers) to solve problems and the facile number sequence stage is where the learners demonstrated the ability to count using more advanced strategies which do not involve counting in ones (non-count-by-ones strategies).

Wright, Ellemor-Collins, and Tabor (2012, p. 17) argue that shifts to more sophisticated strategies and reasoning, in the context of early number, are often associated with extending the number range and what they call the 'progressive distancing of the setting of materials'. 'Progressive distancing' refers to learners who are initially given concrete materials like counters, match sticks, fingers, coins and so on to make sense of numbers. Thereafter, the teacher gradually and strategically removes the reliance on concrete apparatus, allowing learners to understand and solve verbal or written number work without additional contexts by using recalled number facts (Wright et al., 2012). The setting of materials can also be distanced through a process wherein the teacher initially makes the counters visible all the time, then shows the counters briefly and shields the items, next the task could be asked verbally with the items all shielded from the learners' view and lastly the teacher could ask the task verbally without any reference to the concrete items (Wright et al., 2012). As Ensor et al. (2009) have noted, this notion of 'distancing' reflects the close interplay between the possibilities for the specialisation of content and the availability and nature of use of representations within and across the solving of problems.

According to Gray (2008), the compression of numbers involves the shift from unit counting of real objects to seeing number as an instant recognition of a recognisable pattern or as a whole number. This makes calculating procedures more efficient and more effective. Gray also states that teachers need to give learners a sufficient range of activities to explore flexibility with respect to number work. Thus the requirement is for teaching to present number concepts flexibly, with this being essential for learners to cope with the demands of the curriculum in higher grades. For example, the use of the empty number line has been seen to support the flexible use of numbers to solve problems (Wright et al., 2012; Anghileri, 2006; Askew, 1998).

In our number system, the numbers are organised into 1s, 10s, 100s and 1000s and so forth. Therefore it is important that teachers introduce the learners into 'base-ten thinking' and in this way the mathematical content would become more specialised (Wright et al., 2012, p. 16). This is first developed through 'structured' representations that can be worked with mentally and then becomes more formalised, as written place value notations or conventions are developed. For example, 25+57 = ? The learner may add 10 to 57 = 67 and then an additional 10 is added to make 77 and then add 5 = 82. This aids the learner to calculate more efficiently and effectively which is the end goal of specialising both content and representations.

Wright et al. (2012, p. 15) further asserted that 'progressive mathematisation[1]' is the development of mathematical sophistication over time which is similar to the specialisation of content and representations as they are interrelated or

[1] The term mathematisation can be accredited to numerous authors like Treffers (1987) who identified and described several characteristics of Realistic Mathematics Education (RME) which involved the use of contexts, the use of models, the use of learners' own constructions and inventions and the interactive nature of the teaching process.

interconnected. In this way, learners continuously develop their mental organisation and visualisation until they are able to solve the task without the need for a supporting setting (Wright et al., 2012). The focus on making mathematical thinking more sophisticated is seen as the main function of mathematics instruction (Wright et al., 2012).

With this in mind, I intend to analyse to what extent, if any, the modelling of strategies within the teacher's talk and representations across the enacted mathematics activities were made more sophisticated either through extending the number range and/or through the use of a more sophisticated strategy.

Wright, Stanger, Stafford & Martland (2006) stated that learners normally count forward/produce the forward number word sequence (FNWS) more easily than counting backwards/backward number word sequence (BNWS). In specialisation terms, tasks which involve dealing with the starting number in abstract terms, rather than in concrete terms. For example, count-on from 7 in ones, can represent specialisation of content, in comparison to always starting from 1, and counting backwards is a small further specialisation. They further stated that forward counting is not the mere naming of number names, instead it entails the co-ordination of knowing the number names with the corresponding item (numerosity) whether it is visualised or concrete. Both forward and backward counting do not only involve counting in ones. For example, counting forward in tens from 34 or counting backwards by twenty from 145 can be seen as specialising content (Wright et al., 2006). This implies that the teacher needs to be aware of the types of problems that arise when counting backwards in order to provide sufficient practice or intervention to scaffold learners to the next stage or to be alert to difficulties within subtraction problems. For example, 17 minus 4 as sixteen, fifteen, fourteen, thirteen! (Wright, Stanger et al., 2006, p. 35).

In addition, learners could confuse teens with decades or they could have difficulty progressing to the next lowest decade or omit sequences in backward counting that they would not necessarily omit in counting forward. It is important that teachers take cognisance of the examples in the assessment tasks that they set for the learners as these form the basis for eliciting a range of strategies from learners which depend on their level of early number learning.

Furthermore, the identification and recognition of numerals is essential for numerical development as the gradual increase of the number range (numerals to 10, 20, 100 and 1000) can show the extent to which the content is being specialised. For example, were learners able to identify by stating the name of the numeral revealed and recognise a numeral by choosing the specified numeral from a randomly grouped arrangement of selected numerals of one, two or three digit numbers? (Wright, Martland, & Stafford, 2006).

Teaching that is indicative of a more abstract form of calculating is evident when the teacher explicitly uses a range of developmental strategies such as group counting rather than by using visible items and by counting in ones to obtain the answers. Next, learners may use visible items but count now in groups using skip counting strategies. For example, learners given 20 counters can arrange the counters in four groups of five. In this example, the learners are able to count perceptually in multiples as the items are visible. With time, the learners ought to be able to count in multiples without any reliance on visible or concrete items. For this to happen, the teacher needed to have developed the learners' conceptual understanding of the structure of the number. This means that the learners are aware of both the specified number's unitary and composite parts (Wright et al., 2006). In the case of the teacher providing significant experiences with addition, subtraction, multiplication and division, the learners would be able to use repeated addition to solve multiplication problems and repeated subtraction to solve division problems. Following on from this, learners would be able to use known facts to solve unknown facts. For example, the learner could use $5 \times 6 = 30$ to work out $30 \div 6$. This example illustrates that known facts are used to derive the answer as well as showing the connection between inverse operations which is a form of specialising content.

2.3.3 Literature that expands on the notion of ‘specialisation of modes of representations’

Ensor et al. (2009), provide a trajectory that illustrates shifts from concrete to abstract representations of number, namely concrete to iconic to indexical to symbolic and to calculating without representations.

In my study, I am interested in the modes of representation that the teacher selects for a given mathematical task or activity and the manner in which these are utilised by and spoken about by the teacher (teacher’s talk concerning the structure of the apparatus, if any). Following this, I took note of how the teacher engaged and worked with the apparatus and the strategies she employed as they are of concern to me for my research work. The foci on connections and specialisations both deal with the teacher’s way of handling number ideas, without taking learner responses or working centrally, into account. This rather exclusive focus on pedagogy follows the identification of problems within this feature in the South African literature, and reflects a focus on what teachers make available to learn, rather than on learning itself.

Representations, actions, and artefacts that support concrete to abstract shifts of number work are highlighted below. In the teaching of early number, the literature suggests that learners should be encouraged to use materials to solve early number problems for as long as they need the resources, but with the introduction and connection to written numerals alongside (Anghileri, 2006; Department of Basic Education, 2011; Wright, Stanger, et al., 2006). Wright et al. (2006) recommended that teachers should use instructional strategies that assist learners to progress to the next stage where learners were not reliant on visible or concrete materials. It is therefore crucial that teachers are made aware of the different ways to shift learners from working concretely with visible items to working more abstractly with visualised representations. This is also referred to as ‘distancing the setting’ by Wright et al. (2012). This means in the instructional design, initial tasks often involve an instructional setting involving ten-frame or base-ten materials. Learners can be progressively distanced from the setting through steps such as:

[a) manipulating the materials;

b) seeing the materials but not manipulating them;

c) seeing the materials only momentarily; and

d) solving tasks posed in verbal or written form without materials (p. 17).]

From the onset, Ensor et al. (2009) acknowledges that the use of multiple representations is essential in the teaching of number concepts and that learners need to shift from the use of concrete strategies to more abstract ways of calculating. Carruthers and Worthington (2008) claim that children's representations (number charts, number lines, arrays) reveal their thinking. They also demonstrated that children's representations helped them to move 'from mathematics with more concrete materials into more abstract mathematical thoughts and symbols' (Carruthers & Worthington, 2008, p. 130). They also suggest that teachers need to provide an environment that allows learners to choose their own representations or strategies that is supportive of their own mathematical thinking and processes. This means that it is important for the teacher to allow the learners to engage with activities so that they can derive alternative ways of working with number since they have a better understanding of number and their relationships to one another. Therefore, learners would be more able to select appropriate strategies for what is needed to calculate efficiently and accurately. In more directed ways, Harries, Barmby, and Suggate (2008) note that different representations emphasise different aspects of a concept, and thus advocated having a range of representations with teaching focused on including the understanding of their structures as this helps with the understanding of the concepts. This use of representations further serves as a 'tool to think with and can be a focus for discussions' (Harries et al., 2008, p. 172).

Gelman and Gallistel (1986) are emphatic about the need for teaching to provide learners with ample opportunities to work initially with counters (on one-to-one correspondence). Thereafter the process should progress from the counting of perceived items, to screened items, to temporal units (clapping of hands or stepping) and finally to verbal and abstract counting of numbers. In so doing, learners ought to, through working with a variety of representations, develop the ability to partition, recognise numbers and patterns, thereby helping them to understand the numerosity of a number by realising that the cardinal number of a set of items is the same, irrespective of the order they are counted in (Gelman & Gallistel, 1986).

2.3.4 Importance of establishing connections amongst the teacher's talk, tasks and representation

Connections can be related to both the specialisation of content within teaching and the teacher's scaffolding of learners' input. This means that the teacher needs to be knowledgeable of how the learners learn mathematics and the specific needs of the learners in her classroom. In addition, the teacher also needs to understand how to explain and select effective activities to teach conceptual understanding (Askew, 2002). As a 'connectionist' orientated teacher, the teacher has a sense of herself being a learner too and depicts mathematics as a network of connections (Askew, 2002). For example, the teacher will show the connections between different topics in her lesson (addition and subtraction or multiplication and division were taught together). Mathematical discourse or teacher talk is also looked at, as shared discussions are used to develop meaning of the mathematics. This research base directs my attention towards whether the learners are given the opportunity to share their methods and to whether the teacher also provides more advanced, alternative strategies. It is pertinent for my study to look at how the teacher engages with the apparatus and to analyse both her actions and her explanations for showing learners the relationships between numbers and structure of the apparatus.

Anghileri (2006, p. 1), states that children who have an 'awareness of relationships that enable them to interpret new problems in terms of results and known number facts, have a 'feel' for numbers or number sense'. Furthermore, she embraces Vygotsky's principles when she advises that good teaching will take into account learners' prior knowledge, and argues that working with number should be done as part of a social activity and in a context. Teachers should help children to focus on the diversity of the language associated with learning about the four operations and how such language may be related to counting patterns and the links among them should be addressed.

Whilst Askew (2002) and Anghileri (2006) look at both connecting mathematical ideas and learner thinking, Venkat and Naidoo (2012) deal primarily with how teachers connect mathematical ideas. According to Venkat and Naidoo (2012), the nature of teaching should encompass the following key aspects: content of lessons should contain meaningful connections within and across teacher discourse, tasks, representations used, examples employed and materials used to support learning activity as well as having clear goals for each lesson. Furthermore, they advocate that the afore-mentioned aspects need to be related to learners' prior knowledge if they are to make any meaningful sense of their mathematical learning (Venkat & Naidoo, 2012). From the empirical analysis, they claim that there is a lack of connection of mathematical ideas within and across episodes which they refer to as 'extreme localisation' (Venkat and Naidoo, 2012).

In addition, the teacher may use an integrated teaching approach involving working across key representations, since early number learning has been described as involving logical connections modelled consistently amongst the oral or written words/language, actions on objects and the mathematical symbols (Haylock & Cockburn, 2008).

Furthermore, Venkat and Naidoo (2012), analysing number teaching in one G2 lesson, found incoherence within teacher explanation, and a lack of connection between examples in terms of building towards more sophisticated strategies. Neither of these notions of connections though takes into account any indication of the learners' conceptual understanding nor their actual ability to answer the questions themselves, a feature that involves learners making connections between tasks, and the representations and procedures needed to answer the problem. With this connection at the fore, Brodie (2007) argues that the connections that a teacher should encourage are based on interactions that maintain the degree of task difficulty, respond genuinely to learners' questions and those which support conversations amongst learners. She further advises that genuine dialogue is supported by different kinds of questioning to either elicit from learners what they already know or to stimulate their thinking processes or to help them to think differently and more deeply about a problem. In this way, learners are given an opportunity to articulate their understanding, explain their strategies and to question one another to increase conceptual understanding. Askew (2002, p. 12) speaks of a connectionist-orientated teacher as one who 'valued the use of a range of methods by learners and teachers as well as discussing the merits of the different methods including ones suggested by the teacher'.

The above mentioned authors have stated that for learners to be able to calculate flexibly and innovatively, make reasonable choices as well as for learners to be in the position to choose mathematics as a subject in order to open up future career paths, teachers will have to apply the key points discussed. Teaching should focus on individual work for learners to develop different strategies of working with number (Ensor et al., 2009; Mcintosh, 1992; Carruthers & Worthington, 2008; Gelman & Gallistel, 1986). For this reason my study will also note if the teacher makes provision for the shift from primarily focussing on group work to more individual work.

2.3.5 Pedagogies that support specialisation of content

For the teacher to be competent with specialising, both the content and the representations used during teaching would require insight and self-awareness on the teacher's side about the following: how students learn mathematics, understanding how different learners think about number (multi-digit addition or base 10), designing and using assessments, being alert to the learners' new ways of learning mathematics and their use of number relationships (Wright et al., 2012), for example, that 47 is close to 50. Additionally, having an awareness of one's own teaching by being cognisant of how to scaffold a learner who derived a solution quickly through knowing how to choose a task and the objectives for the choice of activity are markers of competence with specialisation. In other words, this involves knowing what mathematical aim she (the teacher) wanted to achieve and knowing when to keep the learners struggling through the problem and when to adjust the task (Wright et al., 2012). In my study, I used two interviews to help me ascertain whether the teacher had an inkling of what was needed to move learners from concrete based work to more abstract forms of calculating.

Numerous authors have studied number development and have described key components to move learners on from concrete to abstract ways of counting. These authors have argued that shifts from concrete to abstract ways of working with number are important for early number learning. In order to develop number, teachers need to allow learners to work with concrete apparatus and gradually shift them towards abstract ways of calculating (using specialising representations).

According to Stein, Grover & Henningsen (1996), classrooms are complex environments and activities (tasks, teaching and learning aids and talk) used therein can be used to develop learners' capacity for mathematical thinking and reasoning. Of special importance to the teaching of progressive number concepts in learners are two broad factors that teachers need to make provision for in their teaching of number: using a range of problem contexts, and supporting number-based discourse and the use of a range of representational forms.

Firstly, Anghileri (2006) advised that teachers should provide opportunities to learn number sense within a context so that it can make sense and allow the doing of mathematics to become more meaningful to learners. Secondly, teachers should encourage and allow learners to bring real-life problems to the class.

Additionally, teachers should involve learners in a number of conversations since number sense is further developed when teachers allow learners to work on graded tasks with one another in different ways; by using different types of games, searching for patterns and by having mathematical discourse which the teacher should model (Anghileri, 2006, p.128).

Wright et al. (2010) further suggests that teachers should expose learners to a variety of contexts in which they can enjoy doing mathematics by practising, talking and recording their mathematics ideas in multiple modes. In so doing, important foundations are laid for learning more complex mathematics later on.

Thus, Anghileri (2006) argues that teaching should give learners the opportunity to practise estimating and justifying their solutions, identifying recognisable patterns, seeing numbers as being part of other numbers and constructing mental pictures of the size and value of numbers. This in turn will provide learners with the much needed mathematical proficiency demanded in the work place and to manage more complex concepts of mathematics. Ultimately, flexible and efficient calculations based on number sense are the desired outcome of number teaching and holistic development and critical thinking (Gray, 2008). In so doing, learners will rely on their creative and innovative interpretations of real-life problems and will be able to select an appropriate mental strategy to solve problems accurately and efficiently. In my study, I want to look at whether, and if so, how the teacher provides the learner with the opportunity to record and connect their ideas in a variety of ways. This can be seen in the lessons recorded if the teacher modelled a range of ways to represent and connect ideas. The ability to show one's reasoning in different ways is fundamental for learners to learn more complex mathematics in the Intermediate Phase.

Likewise, good teaching will expose learners to multiple representations and the reasons for their use as well as ensuring that the use of a representation shifts from concrete to more abstract ways of calculating in order to develop learners' ability to work with number more efficiently and effectively (Harries et al., 2008; Anghileri, 2006; Mcintosh, 1992; Gelman & Galistel, 2008). More generally, good teaching is viewed as helping learners make meaning of their strategies (Askew, Brown, Rhodes, William, & Jonson, 1997).

Ensor et al. (2009), provides a trajectory that illustrates shifts from concrete to abstract representations of number, namely concrete to iconic to indexical to symbolic and to calculating without representations. Furthermore, they also elaborate on the specialising strategies of content (count-all to count-on to count-from larger to mental calculations) for the acquisition of early number. As these specialisations are crucial for my study as part of the conceptual framework, they have been detailed below.

2.4 Conceptual framework - Ensor et al.'s (2009) notion of 'specialisation'

My research is underpinned by the notion of specialisation and the notion of connecting. The conceptual framework of specialising within my research is based on Ensor et al.'s (2009) specialising strategies for content for the acquisition of early number and the specialisation of representations within classroom based activities and classroom talk to analyse the data collected. I then look at the ways in which connections are made within pedagogy between content and representations.

Ensor et al.'s (2009) notion of specialisation provides a trajectory for the teaching of number. The categories used in showing the trajectory for 'specialising content' was taken from the Ensor et al. (2009) study which emerged from their empirical data analysis as well as from a range of literature on the acquisition of early number. In my study, I have made use of the same categories to analyse my data but have also supplemented the categories with additional literature that specifically supports and relates to the specialisation of content and representations. Ensor et al. (2009) adapted 'the specialisation of representations' from Dowling's (1998) sociological perspective on mathematics education in which he analysed modes of representations in a mathematics textbook scheme which propagated that mathematical content and mathematical expressions [language] should shift from the public domain (content of tasks are non-mathematical and language is related to an everyday context) to the esoteric domain (where both the language as part of the representational repertoire and the content are specialised).

Therefore, I will be looking for evidence of differentiated strategies being promoted that aid the shift from counting to calculating, as central to the specialisation of content. The diagram on page 31 below explains that the specialisation of content in the teaching of number involves a shift from 'counting, to calculating-by-counting, to calculating without counting' (Ensor et al., 2009, p. 15). Each of these is described below in turn.

2.4.1 Specialisation of pedagogic content

Based on the framework used by Ensor et al. (2009, p. 15 - 16), specialisation of content in the teaching of number entails the shift from counting, to calculating-by-counting, to calculating without counting. This 'specialisation of content' is linked to the progression of tasks or activities over a period of time that require shifts from concrete counting (counting-all, unit counting) to calculations by counting (count-on, count-from larger, group counting) to calculating without counting (recalled number facts like bonds, multiplication tables, derived facts by looking for patterns, compensation and base 5 or base 10 strategies). The categories presented below emerged from my analysis of the classroom data collected for this study, as well as from aspects gleaned from literature on early number acquisition (Ensor et al., 2009). In addition, I have also included the examples evident in my data collected and have indicated whether the language used by the teacher shifted from an everyday context to being more mathematical as well. As for this study 'specialisation of content' would focus on the type of strategies used, examples or tasks with respect to the progression of number range and the types of strategies that can be used to solve them efficiently, the language used in the teacher's talk to show explicit links between the strategies used and relationships between numbers.

Ensor et al. (2009) define specialisation of content in terms of shifts over time:

- from counting,
- to calculating-by-counting,
- to calculating without counting.

By counting I refer to the presentation of a range of tasks such as:

- Acoustic or oral counting, which encourages learners to memorise number sequences and number patterns. Wright, Stanger, et al. (2006) & Anghileri (2006) argued that counting forwards and backwards, counting in 1s, 2s 3s, 5s and so on develops an understanding of patterns that would assist in early addition and subtraction. This kind of acoustic counting was present in all the lessons observed. Examples included counting forwards in 1s, 2s, 3s, 5s, 10s, 20s (seen in only one lesson) and backwards in 1s, 20s (seen only in 1 lesson).
- Counting out objects entails mapping a number sequence onto a set of objects, which includes counting animals on a poster, counting socks on a number line, counting out a set of counters, matchsticks, unifix cubes, beads, dots and tallies. Included in this category is the teacher's division of a sweet in order to introduce the notion of a fraction as a result of sharing.

  Further sub-aspects of the features identified above are detailed here:

  Producing and recognising a written number sequence. Acoustic counting enables learners to memorise number sequences, which they need to be able to recognise and to reproduce in written form. Important foundations in terms of moving from concrete to symbolic representations need to be laid for learning more complex mathematics later on.

  Locating numbers on a number chart or number line and learning number facts, which includes identifying numbers on a number chart, finding numbers closest to a given number, bigger or smaller than a given number, or between two given numbers. It also entails the use of expanding number cards to represent numbers.
- Calculating-by-counting tasks use counting for the purpose of calculation. For example, learners used a variety of counting strategies to find the solution to problems. Therefore, tasks that are modelled through count-all, count-on and to count-on-from the larger number to solve the problems can all fall within this category.

- Calculating without counting tasks entail solving adding, subtracting, multiplying and dividing tasks without relying on counting; instead they rely on memorised number facts such as number bonds and multiplication tables. This can involve the addition of two-digit numbers using expanding cards (partitioning of numbers). Both Anghileri (2006) and Wright, Ellemor-Collins & Lewis (2007) state that learning about numbers occurs together with calculating and organising numbers.

Mathematics education literature provides a range of aspects that figure within the presentation of calculating without counting as detailed here. Benchmark numbers are numbers that simplify calculations, for example, by working through: 5, 10, 20, 25, 50, 100, 50%, and ½ (Anghileri, 2006; Markovitz & Sowder, 1994; Mcintosh et al., 1992). The teacher therefore needs to provide sufficient experience with base 5 before learners can master base 10 and so on. Eventually these strategies will assist learners with larger benchmarks (CAPS document, 2011).

Place value plays a major role in learners being able to calculate effectively, mentally and in written computations. Through sufficient experience of counting, learners understand the connections between quantity and position of a digit, for example

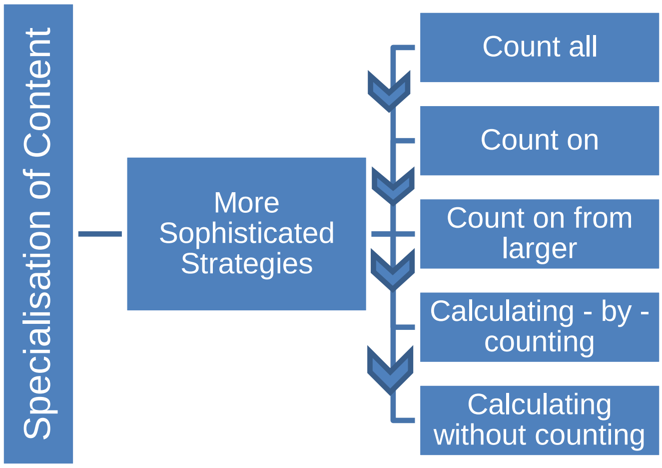
5 463 - each digit in a multiple digit number represents a quantity (CAPS, 2011, p. 188, 216, 253; Askew, Bibby, & Brown, 2001).

Number relationships and number patterns are necessary for the learners to understand how topics and operations are connected through language, tasks and representations used (Anghileri, 2006; Berch, 2005; Markovitz & Sowder, 1994; Greeno, 1991). Learners need to know that numbers relate to one another (for example 8 is one less than 9, but 3 more than 5) and related vocabulary (more, less, equal to, how many more or how many less, sequencing of numbers, composition and decomposition of numbers) (CAPS, 2011, p. 93, 144). Number lines can be used as a tool to help visualise the relationships between numbers and operations. Activities are provided for learners to 'copy, describe, extend' and develop number sequences and to think about what they are doing, which in turn will help them to notice number patterns - 2.2 (CAPS, 2011, p. 48). Furthermore, learners should be able to represent the same number in various ways depending on the context and reason for the representation, according to Berch (2005). In the CAPS document

(2011, p. 143, 144), the tasks and activities provided use visual and symbolic representations and are related to real-life experiences. For example, the number 6 is represented visually using two different coloured counters, then the writing is related to addition and subtraction number sentences in order to illustrate the relationship between the operations as well as the commutative property of addition and the reinforcement of number bonds.

Furthermore, this category includes the use of recalled facts and derived facts. They also expound on the progressive development of more complicated and advanced mental strategies needed for learners to calculate without using counting strategies but rather through the use of more specialised representations and content which can involve the use of an empty number line and more formal written calculations which entail the use of mathematical conventions and language. The progression trajectory that Ensor et al. (2009) outlined above illustrates the increasing specialisation of pedagogic content from counting, to calculating-by-counting to calculating without counting. The term counting is indicative of a range of tasks ranging from oral counting to counting out objects, to tasks that require number identification and recognition in order to produce a written number sequence to learners using number facts to solve problems to calculating by counting (progression from count-all to count-on to count-from the larger number) and finally by calculating without counting as the learner is reliant on previously memorised facts such as bonds, multiplication facts and mental strategies involving partitioning and compensation (Ensor et al., 2009). In concise terms, I will look for the progression of strategies in relation to the task selection and the increase of the number range to support the use of more advanced strategies. In addition to Ensor et al.'s (2009) categories, I look to see how the teacher connects the mathematical ideas and whether there are shifts from context-based problems to context-free problems. Alongside this, evidence of the language being made more mathematical in order for learners to cope with more complex mathematics which is a component of specialising content, is looked for in my data.

The diagram below (Figure 1) summarises the increasing specialisation of pedagogic content visually:



Next I look at the specialisation of forms of representations.

2.4.2 Specialisation of representations

Ensor et al. (2009) define specialisation of representations in terms of shifts over time from:

- concrete use of apparatus (counters, fingers, cards);
- to iconic apparatus which involves images from everyday life (cartoons, drawings, photographs);
- to indexical apparatus which entails the use of apparatus depicting reality (tallies, shapes, sticks, dots);
- to symbolic number-based apparatus that makes use of numerals to represent numbers (number charts, number cards and number lines);
- to symbolic syntactical apparatus which makes use of mathematical notations to derive logical mathematical statements;
- to calculating without the use of representations as the learner is able to calculate mentally in an efficient and effective way (Ensor et al., 2009, p. 17 - 18).

It is necessary for teaching purposes to expose learners to multiple representations and for teachers to understand the different ways a piece of apparatus may be used in order to shift learners from concrete to abstract conceptions of number. In concise terms, 'specialisation of representation' refers to the shift of use over time from visible, concrete apparatus to the gradual use of more abstract apparatus as the representations become more symbolic and present as appropriated mental representations.

The following forms of representations of number were used in the lessons observed in this study:

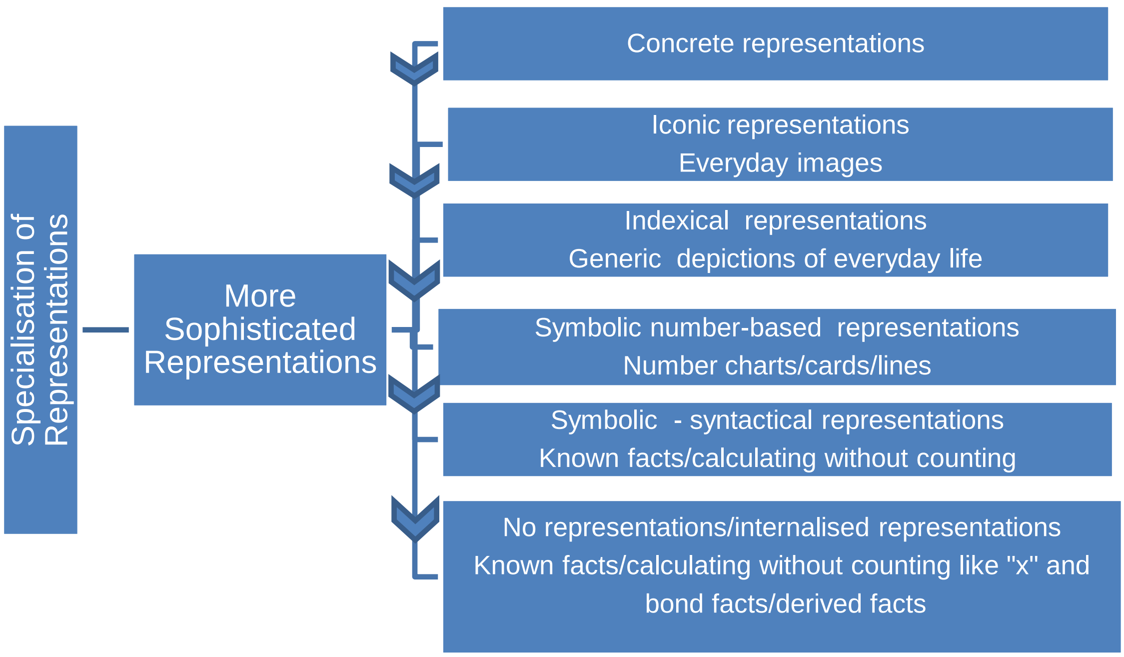
Concrete apparatus that entailed the manipulation of objects such as fingers, money (real or plastic), matches, unifix cubes, counters, single beads or string beads. This apparatus was used for counting and for calculation-by-counting. Therefore, I will be looking for evidence of differentiated strategies being promoted that aid the shift from counting to calculating - as central to the specialisation of content. The diagram below explains that the specialisation of representations in the teaching of number involves shifts over time which entail the use of ‘concrete apparatus to iconic apparatus to indexical apparatus to symbolic number-based apparatus to symbolic syntactical apparatus to calculating without the use of apparatus ’ (Ensor et al., 2009, p. 15).

- Concrete apparatus refers to the manipulation of objects such as fingers, money (real or plastic), matches, unifix cubes, counters, single beads or string beads.
- Iconic refers to images of everyday context or realistic depictions. Apparatus included in this category are drawings like washing lines, photographs and cartoons. This apparatus is also used for counting and for calculation-by-counting strategies.
- Indexical refers more to generic depictions than realistic depictions of everyday contexts. This includes drawings of sticks, tallies, dots, circles and other shapes or pictures to represent everyday objects.
- Symbolic number-based representations involve the use of numerals/symbols to represent numbers. Apparatus includes number lines (structured and semi-structured), number charts and number cards. Askew (1998) defines unstructured apparatus as materials or items that relate to normal, commonplace materials that are used for counting and measuring, like bottle tops, string, matchsticks, unifix cubes, stones and any other form of counters. Structured items are defined as objects or materials that support a particular

mathematical idea or pattern that aids calculating more efficiently, like dominoes which illustrates the concept of subitising (regular patterns), number charts and the abaci are arranged in multiples of 10, drawings illustrating part-part-whole, arrays and different dice. This mode of representation supports calculation without counting but can also be used for calculation-by-counting tasks.

- Symbolic syntactical refers to the use of mathematical notation to produce mathematical statements. This mode of representation is abstract, and entails the deciphering and production of mathematical statements. It relies on known number facts and facts which can be derived without counting by working with the number patterns identified.
- Working without physical representations in the calculation process is the ultimate goal for mental calculation strategies. In lessons this is visible when tasks which learners are given are completed without the use of physical or iconic modes of representations. This suggests that the representations are internalised.

Figure 2 summarises this specialisation of representations visually:



These representations of number illustrate the shifts from concrete counting using objects and fingers, to the use of tallies to the use of mathematical notation without any reliance on any form of concrete representations. It is expected that learners progress through the different modes of representations as they develop a good sense of working with number.

The relationship between the teacher's talk, her use of multiple and gradually more specialised representations to explain more specialised strategies is important for my research to ascertain if rich connections are made during the lesson activities. The purpose of the mediated activity as described by Sherin, Reiser, and Edelson (2004) is for the teacher to maintain direction of mathematical instruction, provide support to learner responses, adherence to mathematical task content, control learners' frustrations, extend the range of learners' tasks and finally to control freedom during learning interaction by allowing alternative (and especially more sophisticated) methods to be shared. This implies that the teacher needs to have a solid grasp and knowledge of the specialisation potential within and across mathematical tasks, namely, mathematical content knowledge and pedagogic knowledge (Kilpatrick, Swafford, & Findell, 2001; Stein, Grover, & Henningsen, 1996).

The teacher modelling the use of various forms of representations in combination with various counting strategies provides information on whether the content and/or the representation are being specialised. According to Haylock and Cockburn (2008), representations in early number learning should be modelled by the teacher in a connected way over a sustained period of time. In other words, connections between the word or language, pictures or drawings (subitising into recognisable patterns), symbol and the action imply that the activity should initially be modelled by the teacher. Thus tasks mediated by representations form the connections that are made between the content and apparatus used by the teacher within my study.

I intend to analyse the empirical data using Ensor et al.'s (2009) categories of content and representations primarily. Reference is made within my analysis to instances of providing learners with a range of contexts and the use of more mathematical language as aspects of specialising content rather than as a separate theme as in Bernstein (2000) terms when he speaks of 'specialisation of expression'. This follows Ensor et al.'s (2009) foci on content and representations, with the addition of my focus on the nature of connections between them, within and across episodes. Teacher talk and specifically, shifts towards more formal and symbolic mathematical language, provided important means for the specialisation of content, and thus, in my analysis, I incorporate excerpts of teacher talk from both lessons and interviews.

In the chapter that follows, the research design and methodology are discussed. The context in which the study was undertaken is described, the methodological approach used is explained and the rationale for using a case study research approach is provided. Data collection methods are mentioned and explained.

Chapter 3

Research Design and Methodology

3.1 Introduction

The aim of this chapter is to describe the context in which the research was completed, the methodological approach chosen for this research and the specific research techniques or procedures used in collecting and analysing the data. The reasons for choosing a case study research approach and the procedures used are justified in this chapter. My data sources for investigating the nature of specialisation of content, modes of representation and forms of expression in this study consist of: 6 videotaped lessons with field notes, a post-lesson sequence interview that was video and audiotaped, and a further audiotaped, reflective interview in which I returned to seek clarification on the points made following some initial analysis. Background information on the teacher's mathematical content knowledge was gathered through looking at her pre and post-test scripts drawn from her work in the WMC-P content knowledge course.

The chapter further discusses all the data sources, including the interview process and how it unfolded after the six lessons were observed and videotaped. Thereafter, I conclude this chapter with comments on issues related to validity, reliability and research ethics.

3.2 Research design

This case study research is set in a qualitative research paradigm since 'profound understanding can be obtained through observations and conversations in the natural setting rather than through experimental manipulations under artificial conditions' (Anderson & Arsenault, 2002, p. 119). Denzin and Lincoln (2000) define qualitative research as:

> 'Qualitative research is multi-method in focus, involving an interpretive, naturalistic approach to its subject matter. This means that qualitative researchers study things in their natural setting, attempting to make sense of, phenomena in terms of the meanings people bring to them. Qualitative research involves the studied use and collection of a variety of empirical materials (case study, personal experience, introspection, life story, interview,

observational, historical, interactional, and visual texts) that describe routine and problematic moments and meanings in individuals' lives' (Denzin & Lincoln, 2000, p. 3).

3.3 Rationale for qualitative approach

A case study allowed me to do an in-depth study of a FP teacher's pedagogic practices in number teaching as opposed to breadth (Creswell, 2012, p. 465). Case study methodology is situated within a qualitative paradigm. I have chosen a qualitative approach because it produces descriptive data that is created socially and lends itself to interpretation. An exploratory case study allowed me to collect extensive data within the classroom and the inclusion of interviews allowed me the opportunity to ascertain what informed the participants' actions and choices on a first hand basis. Whilst the study is limited by its focus on one teacher, this afforded me the opportunity to gain a more in-depth perspective, with detail that will allow me to relate my findings to similar contexts (Bassey, 1999).

3.4 The research site

The research was conducted in a primary school in an urban district in Gauteng Province, over 18 months in the broader project (2011/2012) and 6 months for additional information for my research. The school was selected in liaison with the Foundation Phase facilitator who is part of the broader project. This school is surrounded by an informal settlement and is graded as a non-paying fee school. The intake of learners is largely from the surrounding areas as well as foreign learners from Mozambique and Zimbabwe. Consequently, the language of instruction and learning is complex and South African learners' different mother tongues are used up to G3 but from G4, English is utilised extensively with code switching between the predominant languages such as Northern Sotho, Zulu and Tsonga. On average, the multilingual class sizes are approximately 45 learners per class, but this varies according to language groups. Learners in the G2 and subsequent G3 class in focus in this study were of varying academic abilities, ethnic groups and gender, with the majority in each class speaking the language of learning and teaching (Tsonga) as part of their home language repertoire.

3.5 Research sample

Case study allows me to purposively sample. According to Merriam (1998, p. 60), sampling is the selection of the research site, time, people and events in the field.

The sample in the research can have a significant impact on the trustworthiness of the findings and so the process of deciding on the sample was one of the pivotal stages of my research process. Purposive sampling according to Patton (1990) provides a researcher with the power to select the participant who is relevant to the purpose of the research. The selected participant is referred to as an 'information rich case' that is 'a person from whom one can learn a great deal about issues of central importance to the purpose of the research '(Patton, 1990, p. 169). The former guiding principles were instrumental in the selection of the participant in this study.

As stated already, I intentionally selected the FP teacher for her experience, the significant gaps in her mathematical content knowledge and her willingness to participate in two teacher development projects, namely the Opportunity to Learn Mathematics (OTLM) and LSP project over 2 years, as well as her willingness to be part of the study. Taylor (2008) notes that significant gaps in experienced teachers' mathematical content knowledge is widespread in South African schools. The reason for choosing an experienced teacher was an expectation of lack of hesitation to speak and explain herself, as well as a willingness to share her ideas. Thus the participant is 'information rich' (Patton, 1990, p. 169). Purposive qualitative sampling helped me to develop an in-depth understanding of the teacher's classroom practice which in turn provided me with useful information to answer my research questions (Creswell, 2012). This wealth of information was made further possible by the teacher having taught the class in G2 and in G3, enabling her to speak about any changes in her teaching practice.

3.6 Data sources

My two key data sources are lesson observations and post-lesson observation interviews as these are compatible with the qualitative case study approach. The data collected from these instruments allowed me to answer my research questions. As with any form of inquiry, the respondent may become self-conscious or intimidated by the sight of electronic devices; therefore I made sure that I placed and used them in an unobtrusive way (Opie, 2004). In addition, before I commenced with the interviews, I first obtained permission from the participant to use both the videotaping and audio recording devices to record the interviews (Opie, 2004).

3.6.1 Lesson Observations

The data was collected across a snapshot of 6 lesson starter activities that focused primarily on number development. The first of these lessons each year, formed part of the baseline data collected where single lessons across all the G2 classes of the project were videotaped and viewed. Within my research, I have 2 whole lessons and the remaining four lessons were videotaped in the context of the LSP. In these lessons, the duration varied between 15-20 minutes. The first lesson of 2011 and 2012 were whole lessons of 50 minutes each. The remaining 4 lessons were approximately 20 minutes each. Therefore, in total, the duration across all 6 lessons totalled 180 minutes.

The lessons observed and videotaped were initially three G2 and thereafter three G3 lessons as the same teacher followed the G2 cohort to G3. The transcripts derived from these videotaped lessons were translated verbatim into English with all aspects related to mathematics teaching, namely the entire teacher's talk, and teacher-learner interactions were captured. Descriptions of the teacher's representations or actions were recorded as well. Learner-learner talk was not captured. Furthermore, teacher's talk which was not relevant to my research questions, such as 'interruptions' or 'classroom organisation' focused talk was not recorded. The only footage captured on video was where the teacher was engaged with the learner in order to support or scaffold the learners' number understanding. In addition, the tasks that the teacher gave out or wrote on the board were also captured on video.

The transcripts of the lesson activities were divided into different episodes. An episode was classified according to a single theme or concept. A single theme entailed a unitary focus which in many cases consisted of a number of activities that related to this theme or concept. However, if the strategy or representation used illustrated a more advanced mathematical form of reasoning or thinking then I coded it as a new episode or if the form of the activity shifted from whole class to individual work, I changed the episode.

Also, if the number range increased with the use of different representations and forms of calculating then it marked a new episode as well. For example, when the teacher switched from acoustic counting to exploration of group counting or the use of a number line, this was taken to mark the end of one episode and the start of a new episode.

My research work is similar to Ensor et al. (2009) in that they also divided their transcripts into sets of pedagogic tasks where the different segments of the lesson was focused around a single theme or goal. Their tasks were divided into segments when the teacher changed the focus by moving from one topic to another or if the teacher changed the classroom organisation within the same topic. This included a shift to a different representation used by the teacher or learner and if there was a shift from whole class activity to individual work. The latter does not apply to my study as there was very little individual work evident across the six lessons. Their study does not focus on the specialisation of language or modes of expression which my study is interested in as an aspect of specialising content with respect to the teacher talk as the language used by the teacher communicates meaning as stated by Venkat and Naidoo (2012). By this they imply that classroom discussions should be comprised of strong connections among the classroom talk (teacher/learner explanations and reasoning) and activities or materials used to develop conceptual understanding (Venkat & Naidoo, 2012) which is one of the foci of my research study. However my study does not look at analysing for coherence, which Venkat and Naidoo's (2012) paper focused on primarily.

Therefore, lesson observations allowed me to gain insight into the classroom as I observed this teacher implementing lesson starter activities (LSA). Furthermore ,the videotaped classroom observations (±6) provided validity as I was able to follow up on ideas and probe for responses in the initial and follow-up, reflective interview (Anderson & Arsenault, 2002).

3.6.2 Pre and post-test

The data obtained from the pre and post-test of my participant provided empirical evidence which illustrated whether any shifts from concrete to abstract calculating were evident in the teacher's work. From the post-test it was apparent that the teacher found the following mathematical concepts challenging: estimating, work on decimals, percentages and fractions, being able to choose the correct operations in problem-based calculations and basic algebraic expressions. In addition, I also had field notes taken from workshops, but these were used for background information only.

3.6.3 Post-lesson observation interviews

The objective of interviews and lesson observations is to ‘collect concrete insights, understandings, meanings and perspectives’ of the interviewee’s own experiences or knowledge on various aspects of the study (Denzin & Lincoln, 2005, p. 645).

In this study, I used post-lesson observation interviews to explore pertinent points made by the respondents in-depth, and to elicit reasons for her choices of specific mathematical representations, as well as to gather her reflective input based on artefacts shown from her actual lesson (Opie, 2004).

Before conducting the post-lesson observation interviews, I drafted the interview schedule that guided me on the sequencing of the questions during the interviews. In other words, it was the order in which I intended to ask the questions. The interview questions were then piloted on a different mathematics teacher who was not involved in this research. The teacher of the pilot study was selected according to the same criteria as the case study participant and was also based in the same school, so that the contextual factors were the same, namely, language policy for LoLT, teaching and learning interventions and the socio-economic milieu of the school. The aim of this pilot was to determine the appropriateness of the interview questions as well as to refine the questions to be answered. In other words, was the teacher able to access the language used in the phrasing of the questions since English was her additional language? It was also intended to assess the structure and clarity of the interview questions as well as other issues such as the layout and duration of the interview. This led to some amendments in the interview questions.

The interviews were semi-structured as these offered more depth and flexibility in the answers than written ones (Osborne & Gilbert, 1980). The use of ‘face-to-face’ interviews allowed me the opportunity to seek further clarification on crucial points as they came up during the interview (Posner & Gertzog, 1982). It also gave me the leeway to explore the teacher’s motives and reasons for any of her given responses. In this way, it enabled me to accumulate rich data from which the required explanation was constructed. Finally, the interviews allowed me the opportunity to observe non-verbal cues, facial expression, voice tone as well as cues in the interviewee's natural context and surroundings in which the 'natural language was being preserved' (Opie, 2004, p. 121).

It needs to be noted that even though the mathematics teacher was given the option of speaking during the interview in her mother tongue (Tsonga), which both the translator and the mathematics teacher could understand, she opted for the interview to be conducted and to respond in English. These post-lesson observation interviews were conducted in November 2012.

The post-lesson observation interviews focused on gathering information in the following key areas:

- To explore the teacher's understanding of her classroom practice/pedagogy. This is important in order to understand the teacher's rationale for her selections of tasks, strategies and representations.
- To explore the teacher's understanding of the need to use different modes of representations in specialised ways to move learners from concrete working with the resources to more abstract ways of calculating.
- To explore the teacher's understanding of specialising the mathematical content over time through her talk and selection of activities/tasks.

3.6.4 Conducting the post-lesson observation interviews

The process of conducting the post-lesson observation interviews included initial steps of obtaining relevant permissions for access and initial meetings with the principal, HOD and the mathematics teacher of the institution in order to brief them on the nature of the study. I provided information regarding the purpose of the interviews as well as the intended duration thereof. Following Anderson & Arsenault's (2002), advice, no interview exceeded 40 minutes.

The pilot and the first follow-up interview were conducted in the staff room within the normal programme of the school to avoid disrupting the normal running of the school, whereas the reflective interview occurred on the university campus. In this way, I ensured minimal disruptions, which facilitated the effortless running of the interview process (Bell, 1993). Audio and videotaping allowed for the capture of non-verbal cues that could reveal misunderstandings or boredom.

All the questions posed were semi-structured as explained in section 4.6.3, with the interview process providing opportunities for further probing by the interviewer if necessary. Both the pilot and case study mathematics teachers were asked a similar set of key questions, though sometimes in a different order, depending on how they answered a particular question or what was recorded in their particular lesson observations. The flow of information the mathematics teacher gave shaped the interview as I avoided using specialised language or unusual words (Fraenkel & Wallen, 2000, p. 337). Finally, the interviewer's logic was not forced on the interviewee and I endeavoured to use the respondent's own language when responses were rephrased or clarity was needed about an issue (Novale & Govan, 1984).

Validity and trustworthiness were ensured as these video and audiotaped interviews allowed me to revisit or reanalyse information repeatedly when needed, including for verification purposes by persons who checked the raw data for capturing inaccuracies, to check against bias or misinterpretation of data; by supervisors and other future researchers (Opie, 2004, Merriam, 2001). By using recorded interviews, I was able to engage with the respondents and not miss nuances and/or data while making field notes or trying to record the respondent's views without processing them at the same time.

Therefore, the use of post-lesson observation interviews were instrumental in gathering rich information from the mathematics teacher about substantive meanings that she gave to the way number sense was taught and her understanding of the shifts that needed to take place in her teaching practice for learners to work more abstractly, efficiently and effectively with number work.

3.6.5 The reflective interview

The reflective interview was conducted with the mathematics teacher after all the lesson observations and the post-lesson observation interview were concluded. This interview depended on the lessons observed and were facilitated by showing the mathematics teacher extracts from the initial interview which contained examples used or the explanations given for a particular question. This interview was important because it was a follow-up to some issues that were seen during the initial interview, for example, questions to explain which strategies she observed her stronger and weaker learners used when they solved number-related problems. This provided information on her views on what she regarded as important strategies for learners to use according to their abilities when they worked on classroom activities. The reflective interview was audio recorded as it sought to obtain clarity on inconsistencies or explanations of examples used in the initial interview.

3.7 Data analysis approach

As mentioned above, I used Ensor et al.'s (2009) categories related to specialisation of content and representations to analyse my data. This implied a typological analysis of the data which involved a systematic, continuous, cyclical process that is integrated into all stages of qualitative research. Hatch (2002, p. 152) defines a typology as a process of analysing data 'by dividing the overall data into categories based on predetermined typologies'. Hatch's (2002) identification of steps to aid the analysis of data was followed: As I read the data, I made entries related to the identified typologies in the margins. Thereafter, I only reread the data within the typology of interest and I recorded the main ideas on a summary sheet. Next, I looked for patterns, relationships and themes within typologies. Furthermore, I kept a record of the codes used to identify the different patterns. In addition, I verified whether my patterns were supported by the data and I also took note of examples that did not relate to the existing patterns. During my data analysis, I sought permission to return to the site, if needed, to obtain additional data or to verify emerging patterns. According to McMillan & Schumacher (2010, p. 36), 'making sense of the data depends entirely on the researcher's intellectual rigour and tolerance for tentativeness of interpretations until the entire analysis is completed'. Finally, I selected excerpts that would support my identification of key patterns in the findings. Qualitative analysis of nature of connections between the two aspects of specialisation followed.

3.8 Ethical Issues (Access and permission)

Information letters describing the aims of my study and the data collected were given to the principal of the school, the teacher involved in the study and the learners in the class (Creswell, 2012). Following the ethical guidelines of the university, separate information letters were distributed for classroom observations, audiotaping and videotaping. An example of one of these information letters to the teacher about the project is included in appendix 1. The participant was respected and I explained that participation was on a voluntary basis with the right to withdraw at any time with no consequences. All stakeholders were kept anonymous with pseudonyms used in all transcriptions of interviews and videotaped lessons [school's name, participants' names].

Within the broader project, learner and teacher assessment data had prior clearance. I used this data as contextual background detail on learner and teacher performance as this is drawn from data collected within the broader WMC-P project. The data collected for this study received its own ethical clearance from my institution.

3.9 Validity and reliability

Rigour should be maintained throughout the process to ensure the quality and trustworthiness of the data and findings. To combat the limitations of the various instruments, I used triangulation to ensure findings were authentic by consulting a range of data sources and data collection strategies that provide confirming evidence. Creswell (2012, p. 259) defines 'triangulation as the process of corroborating evidence from different individuals, types of data, or methods of data collection in descriptions and themes in qualitative research'.

Validity refers to the degree to which the findings described by the researcher are the real representation of the data collected (Creswell, 2012). I have included excerpts from transcripts in the report which provide ample opportunities for others to assess my interpretations and follow the reasoning process. In this way validity and trustworthiness of the claims would be ensured.

I used 'member checking' by consulting the participant to verify the accuracy of the report and, if need be, make adjustments. Another activity that was undertaken to ensure credibility was collecting data from various sources (Creswell, 2012). He argues that such an approach would help to test the reliability and credibility of the findings through cross-referencing and triangulation of accounts.

'Face Validity' refers to the extent to which an instrument appears to measure the concepts which it purports to measure (Creswell, 2012). In this instance, I worked with my supervisor, who is an expert, to check for both 'content and construct validity', in other words for factors that might affect the results so that these could be eliminated. Internal validity was pursued by ensuring that claims made were justified by the research data (Cohen & Manion, 1994).

Thus, the purpose of this study is not to make generalizations about the claims but to fill a gap in the literature by presenting the findings to make a contribution to the existing literature and to shed light on an under-researched area of mathematics teaching in primary mathematics education.

3.10 Summary and conclusion

This chapter outlines the research design. I have elaborated further on the methodology, namely the case study approach and the methods of data collection employed and detailed the research process used in this study. Also since the procedure was qualitative in nature, the methods of data collection used were consistent with the tenets of qualitative research. Finally, issues pertaining to the reliability and validity of this research, as well as ethical considerations that guided me in the process were also discussed in relation to the research site, the nature of participation, data collection methods and what will happen to the collected and analysed data. The limitations and relatability of this research were also explained.

In the next chapter, a detailed description of the findings and data analysis is presented.

Chapter 4

Data Analysis and Findings

4.1 Introduction

This chapter presents an analysis of the data collected. In the presentation of data analysis, the research questions are dealt with separately with respect to specialisation of content and specialisation of representations but connections in teacher talk are looked at holistically, since the specialisation of the mathematical task and its corresponding representations could take on various forms. For example, the teacher may de-specialise content through modelling more basic strategies even whilst using more specialised representations (Venkat & Askew, forthcoming). Schollar and Associates (2004) provide evidence of this at learner level. There could also be no specialisation of either content or representations used or extensive specialisation of both.

As discussed in chapter 3, data collected in this study was analysed and interpreted using categories and sub-categories that were derived from Ensor et al.'s (2009) specialisation of content and specialisation of representations. Whilst I used Ensor et al.'s (2009) literature base as a source, I also expanded on this by looking more broadly into the literature on progression within number. In addition, the teacher's talk was analysed for connections between the different components as noted by Haylock and Cockburn (2008).

As discussed in the previous chapter, data were derived from six videotaped lessons, which were transcribed and then analysed. Discussions with the teacher during both interviews were transcribed. The data collected focused primarily on number development. The lessons observed and videotaped were initially three G2 and thereafter three G3 lessons as the same teacher followed the G2 cohort to G3. The videotaped lessons were translated verbatim into English with all aspects related to mathematical teaching captured - namely the teacher's talk, teacher-learner interactions, and descriptions of the teacher's representations and actions were recorded as well. Thus, all the footage where the teacher was demonstrating or modelling or explaining mathematical ideas or was engaged with the learner in order to support or scaffold the learners' understanding was captured verbatim. In addition, the tasks that the teacher gave out or wrote on the board, and any representations drawn on the board were also captured on video.

Furthermore, the transcripts of the lesson activities were divided into different episodes. An episode was classified according to a single theme or concept. A single theme entailed a unitary focus which in many cases consisted of a number of activities that related to this theme or concept. However, in line with the conceptual focus of this study, if the strategy or representation used illustrated a more advanced mathematical form of reasoning or thinking, then I coded it as a new episode.

Similarly, if the number range increased with the use of different representations and form of activity of calculating then it marked a new episode as well. This can be seen, for example, when the teacher switched from acoustic counting to exploration of group counting or when there was a switch to the use of a number line and/or when there was a switch in activity format from whole class work to individual work.

My study focused on the extent to which the teacher offered more specialised content in relation to the type of representation and strategy used (specialisation of representations). In addition, I also focused on whether the language was made more mathematical as an aspect of specialising content, following Venkat and Naidoo's (2012) argument that classroom discussions should be comprised of strong connections among the classroom talk (teacher/ learner explanations and reasoning) and activities/materials used to develop conceptual understanding.

4.2 Introducing the analysis

In my discussion I explore the ways in which, and the extent to which 'specialising content' - count-all, count-on, count-on from larger to calculating without counting (which entails the use of benchmarks, regrouping, composition and decomposition of number) occurred. From the literature, two further aspects that allow for this specialisation of content are in focus: expansion of number range and the move to more mathematical language in discussions across the 6 lessons. Thus, in my analysis, I add in a focus on the number range covered. Teacher talk also figures as one route towards the specialisation of content. I also used excerpts of teacher talk and specifically shift toward more formal and symbolic mathematical language to understand the specialisation of content.

With respects to ‘specialising representations’, the following aspects were in focus: the language used to describe the structure of the representation (where applicable), how it was modelled and what types of representation were used in the lesson (from concrete counters to more abstract symbolic forms to the reliance of recalled facts without the use of apparatus). ‘Specialising representations’ is also referred to as ‘distancing the setting’ by Wright et al. (2012). Therefore, in my analysis I incorporated excerpts of teacher talk from both lessons and interviews, relating to the use of representations and learner interactions in the overview tables.

In order to understand the nature and extent of connections, I looked at the sequencing of the tasks and examples within and across episodes, and whether the teacher modelled the relationship between the word, picture, symbolic notation and action in an integrated way or as separate entities. Additionally, I noted whether the teacher talk explicitly showed the relationship between operations and number facts.

My findings and analysis are presented as follows: I begin by providing an overview of each lesson. In most instances, in the analytical discussion that follows the overview table, I refer to the episodes that are presented in the table, and add in data from the video transcript to add texture and substance to the profile I want to make. In occasional instances though, a point of interest occurred in another episode and I drew on that. I deal with the interviews in more detail towards the end of this chapter. However, where aspects of the interview data provided explanatory details on the teacher’s selection of tasks and use of examples or justifying her rationale for doing a specific task (specialising of content), I have drawn these into the discussions of the lessons.

If very similar patterns of teacher talk, use of representations and teacher/learner interactions ensued across all examples in an episode, I provided excerpts from the most illustrative case/s. In many instances, a first example was associated with such talk and explanations with limited further explanations for the remaining examples.

4.3 Teaching context of the school

In this class the learners are taught primarily in their mother tongue which is Tsonga. With this said, the teacher consistently taught the learners the English equivalent of the Mathematics terms and operations, namely addition (+), subtraction (-), multiplication (x) and division (÷). The teacher is required to translate the education department’s documents into their respective mother tongue and there is emerging anecdotal evidence in the broader WMC-P project that teachers prefer to teach some sections in English as the language is easier for them and the learners to understand and use. This pattern was seen across the six lessons in this study.

I begin with an overview of the counting strategies that occurred within each lesson before summarising the detail across each individual lesson.

4.4 Overview and analysis of counting strategies across episodes in lesson 1

Overview of lesson 1				
Episode Number (E1)/Format and Time duration	**Themes/ Concepts**	**Examples**	**Representations**	**Summary of teacher instructions, explanations and responses.**
E1 [00:00] Whole class (WC)	Forward Counting	‘Let's count-from 1 to 100’.	1 - 100 number chart	Teacher monitors as learners follow counting on individual 1 - 100 number chart.
E2 [02:05] WC	Backward Counting	‘Let’s count backwards from100 - 1.	1 - 100 number chart	Teacher orchestrated the counting.
E3 [04:47] WC	Forward Grouped Counting	Count in 2s	1 - 100 number chart	Teacher orchestrated the counting. Class counting in 2s from 2 to 100.
E4 [05:46] WC	Forward Grouped Counting	Count in 5s	1 - 100 number chart	Teacher orchestrated the counting. Class counting in 5s from 5 until 100.

E5[06:10] WC	Naming months of the year, days of the week and discussed the weather of the day and the time	No counting activities within this episode/theme		
E6 [07:27] WC	Subtraction [using one abacus row]	6.1 5-3 6.2 4-0 6.3 6 - 2 6.4 8 - 5 6.5 10 - 5	Abacus	Count-all strategy modelled involving three separate counts. In other words, the teacher demonstrated unit count out of 5 beads, unit taking away of 3 beads and unit counting of the remaining 2. Other examples (6.2/6.3/6.4/6.5) demonstrated in the same way.
E7 [18:12]	Subtraction [using two abacus rows]	7.1 12-4 7.2 15-5 7.3 13-9	Abacus	Teacher demonstrated as above with a triple process of unit counting. This usually involves three separate counts from 12 for the first number, the second number and the total number.
E8 [33:37] Individual work (IW)	These subtraction examples are happening simultaneously		iconic representations: fingers	Learners attempt 8.1a/b and 8.2a/b individually.

	For stronger group [using two abacus rows] For weaker group [using one abacus row]	8.1a 20 - 15 8.2a 20 - 10 8.1b 3 - 1 8.2b 4 - 2	Indexical representations : drawing of apples and the abacus Symbolic representations	The stronger learners are capable of doing the work without further input from the teacher and she simply checks that the solution is correct. Teacher supports weaker group where 8.2a and 8.2b is demonstrated as above with a triple process of unit counting.

Analysis of lesson 1

4.4.1 Specialisation of content

A recurring pattern across the counting related episodes in lesson 1 is the demonstration of count-all strategies involving a triple unit counting process (seen in episodes 6, 7 and 8). Interestingly, this unit counting occurs in the context of the use of a structured resource, the abacus, but there is almost no reference to the visibility of the base 10 structure. Disjunction in that grouped counting seen in early part of the lesson, is not used in the context of subtraction (grouped counting which is seen in episodes 3 and 4, is working without any visible unit counting procedures). A shift from forward to backward counting is seen, which is viewed by Wright, Stanger, et al. (2006) as one form of specialisation of content.

Additionally, in the shifts from unit counting (E1 and E2) to grouped counting (E3 and E4), specialisation of content in Ensor et al.'s (2009) terms is observed, however, in the context of subtraction there is a reversion to unit counting strategies. This reversion to unit counting by the teacher can be seen as an act of despecialisation. Furthermore, of the 10 examples modelled by the teacher, 5 are below the range of 10 and the other 5 examples are below the range of 20. In relation to the curriculum advocated for G3, this range is below the range specified for subtraction (solve problems in context with answers up to 999).

4.4.2 Specialising of representations

In this lesson a number chart and an abacus were used to demonstrate the mathematical ideas of various forms of counting and subtraction. A number chart is used in the context of the skip counting episodes. It can be seen that some learners were following the counting orally without using the number chart, yet others who were able to follow the counting orally were indicating the wrong numbers on their number chart (E1). According to literature, children can know their number names [can recite the list of number names] but in ways that are not connected to quantity or ordinality of numbers (Gelman & Gallistel, 1986).

The teacher elected to use a range of counters (fingers), iconic apparatus (drawing of apples) and an abacus (indexical apparatus) to support the learners understanding of subtraction. An abacus used in the context of subtraction, is seen as a structured resource (Askew, 1998). Structured resources make visible important aspects of the structure of number. It aims to make some of the structuring of number into tens and units very distinct. However, the teacher's enactment in the use of the resource in this lesson ignores the structure of number and uses the representation in a highly despecialised way (at a very low level of specialisation) because she promotes unit counting strategies.

In the discussion that follows, I discuss both aspects of specialisation together, noting the ways in which each is intertwined with the other.

In lesson 1, episode 7, the teacher demonstrates unit counting to obtain 5 then proceeds to remove 3 by counting the objects individually to get to the answer of 2. There is no evidence of the teacher taking away 3 as a group as it is possible to visualise 3 objects as a whole. This would have shown a more abstract way of working with the abacus. The teacher also repeatedly reminds the learners to use the abacus to help with the solution and there is no modelling showing learners how to use number facts like partitions of 5 or 10 to assist with the solution. Instead, the modelling by the teacher showed a reliance on using the abacus in a concrete way by unit counting or 'counting-all'.

With this said, there is an attempt to explain to the learners the structure of the abaci in episodes 7 and 8 when the teacher increases the number range of the examples the learners need to work with in class. The teacher first demonstrates that the first row of the abacus has 10 balls by counting them out individually then later refers to the first row as having ten and the learners need to 'count-on' using the second row for two-digit numbers (Lesson 1, episode 7). Thus there is implicit reference to their being 10 beads in the first row, but no explicit mention of this being part of the general structure of the abacus. Further on in the lesson in episode 7 the teacher is seen to use the structure of the abacus to help the learner to 'count-on':

> T: Huh? 4? Let's start again [Teacher continues to hold the 10 beads then resets the abacus]. You use two lines when you want to count 13.
>
> [The learner begins counting again but stops at 10 again.]
>
> T: Start from 10, **continue counting and go to the second row**.
>
> [Learner starting at 1 again]
>
> T: [Teacher resets the second row] **When you get to 10, you don't go back to 1**

you go forward. We say 11; 12; 13 [The teacher moves the beads in the second row, counting the remaining 3 for the learner in illustration], from 10 we say 11; 12; 13, right?'

She also specialises the content and the representation according to the learners' abilities in her class based on her perceived notions of their mathematics capabilities. This can be seen when the teacher reverted to the use of a range of counters (fingers), iconic apparatus (drawing of apples) in conjunction with the abacus. For example, in episode 8 while the more able learners worked on subtraction involving two-digit numbers, the weaker learners were given subtraction of one-digit numbers (3 - 1 = 2) to work on. Of interest, in relation to the somewhat patchy picture of specialisation of content/representations seen in this lesson are the comments made by the teacher in the two final interviews. I note here that the teacher following participation in the LSP and OTLM projects seemed aware of many of the limitations of unit counting as a strategy, and showed awareness of more sophisticated strategies and related representations.

4.4.3 Connections between content and representations

Interestingly there are no errors in her connections within episodes in this lesson, but there is a lack of connection between episodes. For example, the teacher did grouped counting in episodes E3 and E4 in ways which could be useful in the context of the abacus subtraction tasks as the number chart is an appropriate resource for grouped counting and the abacus is an appropriate resource for subtraction tasks. The lack of connection across episodes means that there is a lack of use of more specialised strategies used with skip counting, and therefore, a lack of specialising content.

There are also disconnections between episodes as the learners are seen to know how to count in 2s (2, 4, 6...) and yet the grouped counting that came before the subtraction lesson was not used by the teacher when she demonstrated the examples in E6 (6 - 2; 8 - 5 and 10 - 5). The teacher is seen to unit count out 6 beads as opposed to counting in 2s which was practised in the previous episodes. Therefore, I note that the grouped counting strategies that the learners were familiar with in the previous episodes were not brought into the teaching, which is more specialised than unit counting of the beads.

4.5 Overview and analysis of counting strategies across episodes in lesson 2

Overview of lesson 2				
Episode Number (E1)/Format and Time duration	**Themes/ Concepts**	**Examples**	**Representations**	**Summary of teacher instructions, explanations and responses.**
E1 [00:00] WC	Money Identification	1.1 5c 1.2 10c 1.3 50c 1.4 20c 1.5 R1	Concrete representations: coins Iconic representations: money chart Symbolic representations	Coin identification activity The teacher looked at different coins with respect to colour, size and the picture embossed on them to support coin identification. No additional detailed explanation provided for 1.2 to 1.5
E2 [04:00] WC	Money equivalences	2.1 50c 2.2 20c		The teacher makes use of grouped counting strategies, expanded notation and recalled facts when working with equivalences between coins. No additional detailed explanation provided for 2.2
E3 [33:37] WC	Money equivalences using recalled facts	3.1 10c 3.2 Units of measurement : c, R		No additional detailed explanation. Recalled facts

Analysis of lesson 2

4.5.1 Specialising of content

In lesson 2 the main focus is on identifying different types of coins (episode 1) and knowing what makes the coins different from one another. The teacher deals with equivalences using the context of money in episodes 2 and 3. For example, she uses the different coins namely 5c, 10c, 20c, 50c and R1 coins and highlights the unit of measurement involved when calculating with money equivalences of different coins and what each symbol stood for, (c for cents and R for rands). In episode 2, the teacher links working out equivalences to using the expanded notation (50c = 10c + 10c + 10c + 10c + 10c) as well as counting in groups of 10. In this way, the teacher has specialised the content in relation to the grouped counting strategy used which is a more advanced strategy than sharing by unit counting, to work out the expanded notation calculation.

> **Lesson 2, episode 2**
>
> T: Now we want to know on 50c, how many 10c do we have? How many 10c do we have on 50c? Let's see, let's count in 10s and see how many 10c do we have? Let's count in 10s'
>
> [Class counts in 10s from 10 to 50 [using fingers]
>
> T: 50, so I ask you, on 50c how many 10c do we have? How many 10c do we have on 50c?
>
> Class: 5
>
> T: 50c is equals to 10c + 10c+ 10c+ 10c+ 10c [Writing the sum on the board] right

In episode 3, the learners were able to rely on recalled facts to work out the equivalence, which is a more advanced strategy than sharing by unit counting to work out the equivalences. The number range that the teacher is working with is relatively low in relation to the curriculum, and thus the content used was not specialised by the teacher according to curriculum as learners in G2 should be able to solve money problems involving totals and change up to R99 and in cents up to 90c. In G3 they should be able to convert between rands and cents (Department of Basic Education, 2011).

4.5.2 Specialising of representations

In this lesson the teacher used both concrete apparatus (a variety of coins) and iconic apparatus (a money chart) for the learners to feel and see the differences in the coins. Here the coin representations are used to support counting on in tens, a grouped counting, rather than a unit counting strategy.

4.5.3 Connections between content and representations

There are no errors in connections within the episodes as the teacher used a range of more sophisticated strategies that she drew from previous work, namely grouped counting strategies, expanded notation and recalled facts which is seen in Ensor et al.'s (2009) terms as specialising content as it is more efficient than unit counting. However, there is a disconnection between episodes as the examples selected did not become progressively more challenging. Instead the teacher reverted to easier examples (learners had to find the equivalences of 50c, 20c and 10c). For example, the teacher did equivalences on 50c initially, and could have used this to find the equivalences for R1, R2 and R5 respectively, thus providing the learners with the opportunity to apply what they are working with to consecutive examples.

Thus, whilst there is some specialising of content seen in the use of grouped counting strategies, the sequencing of examples tends to work against a specialising of content.

4.6 Overview and analysis of counting strategies across episodes in lesson 3

Overview of lesson 3				
Episode Number (E1)/Format and Time duration	**Themes/ Concepts**	**Examples**	**Representations**	**Summary of teacher instructions, explanations and responses.**
E1[00:00] WC	Division without a remainder	1.1 12÷2 1.2 10÷2 1.3 14 ÷2	Concrete apparatus: counters Indexical apparatus: drawings of circles Symbolic apparatus: calculation written down numerically	The teacher communicated the concept of dividing by 2 with related terminology such as 'sharing', 'equal', 'evenly distributed' and 'don't forget they must get the same amount of sweets'. From the onset, the teacher records the symbolic notation on the board, for example 12 ÷ 2 = 6. Thereafter, the teacher modelled the division process using a triple process of unit counting. First she demonstrated unit counting to count out the dividend of 12, then she drew two circles for the learners to share the 12 counters 1 - by 1, and unit counted to share the items in each circle. No additional detailed explanation provided for 1.2 and 1.3.
E2 [09:27] WC	Division with a remainder	2.1 15÷2		In this example, the teacher modelled division with a remainder using the same concrete approach as discussed above but in this instance she deals with an incorrect answer by helping the children to see that if each child only received 7 sweets then one sweet remained. A learner was able to see that the remaining sweet would 'have to be cut in half'. Additionally, the learners used recalled facts to check their answer by knowing that 8 + 8 = 16. She further demonstrates what a half means by actually

				breaking a piece of chalk in the class before writing the word 'half' and then the symbol '½' on the board. In this example, the teacher used everyday language and context to aid the learners' understanding of the concept before introducing the mathematical symbolic notation for half. Additionally, extension of concept through including the remainder example is a form of specialising content. [Makes chopping movements to indicate sharing in pieces]

Analysis of lesson 3

4.6.1 Specialising of content

In lesson 3, the focus is division without a remainder first and then division with a remainder, which comes to be viewed in relation to fractional answers. According to the CAPS document (2012), learners are expected to group and share in G1 up to 20, G2 up to 50 and in G3 up to 100. In the 20 minute lesson activity on division, the teacher did not extend the concept in grade three which requires learners to have a firm foundation of grouping and sharing up to a 100 which was not achieved as the minimum requirements were not met in grade two (should be able to share and group up to 50). The teacher's examples cover two-digit numbers below 20. This selection is far below the minimum requirement for grade 2; thus the content taught was not specialised and progression is impeded.

The teacher provided a range of terms to explain the concept of dividing by 2 with related terminology such as 'sharing', 'equal', 'evenly distributed' and 'don't forget them must get the same amount of sweets'. The teacher shifts the use of everyday language to more mathematical language, which is important for specialising content. Thereafter, the teacher modelled the division process using a triple process of unit counting in the excerpts from lesson 3, episode 1. It is observed that the teacher first counts out the 12 objects to represent the dividend and then used a diagram when she drew two groups for her to share the 12 objects one-by-one until there were no objects left. Thereafter she asks learners to count one-by-one how many items were left in each of the circles drawn in order to write down the quotient of the division calculation. She also has the learners repeat the answer numerous times before writing it down next to the number sentence. The teacher instructs the learners to 'give one sweet at a time' (counters) and by doing this favours a concrete way of working based on unit counting with number which does not specialise the content or the representations used. This can be seen in the example $12 \div 2 = 6$. The learners are expected to work through the remaining three examples by counting out the objects needed to divide, then to share them equally between two people one-by-one. This is both time consuming and the strategy selected is grounded in working tangibly with the objects (see the excerpt on the next page).

Lesson 3, episode 1

T: Its 12 ÷ 2 [Writing the sum 12 ÷ 2 on the board and drawing 2 big circles] Here are our 12 counters and we must divide for 2 people [Pointing to the counters on the board] How many people must we divide for?

Class: 2

T: Let's take the counters and divide for them [Dividing the counters one by one putting them in the 2 circles]

[Class counts the counters as the teacher divides them. 1 for the 1st circle and then 1 for the 2nd circle, 2 for the 1st circle and then 2 for the 2nd circle and so on]

T: You must give 1 person 1 sweet at a time [The teacher points one by one in opposite directions to demonstrate sharing] Until all of the sweets are evenly distributed and they both have the same amount of sweets that's when you will get the answer of how many sweets will each person have. How many sweets will each person get?

Class: 6

When the teacher modelled division with a remainder, she used the same concrete approach as discussed above, but in this instance she deals with an incorrect answer by making them think about the logic of their answer that was given, which helps them to get to the correct answer. The learners used recalled facts to check their answer through stating that 8 + 8 = 16. The checking strategy was more specialised than the strategy of unit counting used to solve 2.1. Given the presence of grouped counting in Lesson 1, and its use here within verification of the answer, the insistence on unit counting for the sharing process can be considered as pulling backwards against specialisation of content, as noted by Ensor et al. (2009).

The teacher demonstrates what a half means by actually breaking a piece of chalk in the class before writing the word 'half' and then the symbol '½' on the board as observed in the next excerpt from lesson 3, episode 2:

Lesson 3, episode 2

T: But how do you say the answer, there were 15 sweets and they shared 7, 7 and 1 remains and you said they must cut it in half and I want to know how you say the answer? [The teacher then breaks her chalk in half to illustrate what the learner said]

L13: 8

T: 8?

Class: No

T: Which means 1 person will get 8 and the other 1 will get 7? [The teacher's hands wave in half gesture as she explains]

L14: They will get 8, 8.

T: What is 8 + 8? [Points randomly to the board] Class: 16 T: That means it is not correct because we want 15 divide by 2 [The teacher points to the sum] L15: 7 and ½

T: Say it louder.

L16: 7 and 1/2 T: 7 and?

Class: half

T: 7 and?

Class: half

T: How many sweets will each person get?

Class: 7 and ½

T: When we divide 1 sweets in the middle we say it's half [The teacher shows her chalk and symbolises breaking it in half once more] Half and half [Shows both halves] Equals to 1 whole sweet [Teacher puts the two chalk halves together to show the class a whole] Which is not cut in half. This will give us 15. [Points to the sum] It will give us what?

Class: 15

T: 1 Person will get 7[Writes 7 after the = of 15 divide by 2], how do I write half? [Writing 15 divided by 2 = 7 and ½]

T: I'll get one…over…two [As she writes 1 over 2 next to the 7] [Class says one over two as the teacher writes]

T: How many sweets will 1 person get?

Class: 7 ½

T: How many sweets will 1 person get?

Class: 7 ½

T: You must teach yourself even if things are not equal must share equally in half and quarters.

[Makes chopping movements to indicate sharing in pieces]

In this example, the teacher used everyday language and context to aid the learners' understanding of the concept before introducing the mathematical symbolic notation for half, thereby making the language more mathematical, which is an aspect of specialising content. Additionally, the extension of a concept through including the remainder example is a form of specialising content.

4.6.2 Specialising of representations

In this lesson the teacher made use of a variety of representations such as concrete apparatus, (counters, chalk), indexical apparatus (drawing of circles) and symbolic apparatus (calculation written down numerically). By including the symbolic notation symbol '½' there is some specialising of representations. This indicates that the teacher attempts to specialise the representation but in a disconnected way in that the teacher's enactment in the use of the resource ignores the grouped counting of 2s which was modelled in lesson 1, episode 3. Thus, she used the concrete representation in a highly despecialised way (at a very low level of specialisation) because she promotes unit counting strategies.

4.6.3 Connections between content and representations

In the structure of the lesson the teacher moved from division without a remainder to division with a remainder, back to division without a remainder. The teacher only did one example with a remainder and then she reverted back to $14 \div 2 = 7$ which was done concretely. The learners were not moved on to more challenging tasks and thus the content was not specialised. This showed a lack of connectedness between one example to the next example used. Furthermore, learners were seen to count competently in 2s in lesson 1 but this was not applied to or integrated with work on division, and hence the strategies and the representations were used in a despecialised way by the teacher.

4.7 Overview and analysis of counting strategies across episodes in lesson 4

Overview of lesson 4					
Episode Number (E1)/Format and Time duration	**Themes/ Concepts**	**Examples**	**Representations**	**Summary of teacher instructions, explanations and responses.**	**Correlated learner actions.**
E1 [00:00] WC	Forward Counting	Let's count-from 1 to 100.	1 – 200 number chart	The teacher assists certain learners who are struggling with the tracking of their counting, by following the numbers with the learner helping him/her to track the numbers for a period.	Each learner is following and tracking their counting on a personal number chart stored in their flip files.
E2 [02:06] WC	Backward Counting	Okay, let's start from 100 going backwards.	1 – 200 number chart	And put your finger where you ended, backwards.	Class counting from 100 to 1. The class continues to use the number chart to help them track their counting. The learners again seem to be tracking correctly.

E3 [05:12] WC	Forward Group Counting	Count-from 10 to 100	No representation: Internal	Let's count in 10s.	Class counting in 10 up to 100 and clapping their hands at each 10.
E4 [02:05] WC	Repeated Addition/ Adding repeatedly	4.1 5 +5+5=15/5x3=15 4.2 4+4+4=12/3x4=12 4.3 5x4=20 4.4 4x4=16	Pictorial representations Concrete apparatus: counters Indexical apparatus: drawings of circles Symbolic apparatus: calculation written down numerically	Episode 4 dealt with repeated addition primarily. All four examples used were below the range of 20 with answers less than 25. The teacher starts the lesson by stating that the focus of the lesson is 'repeated addition' but immediately afterwards she writes '5 x 3' on the board, suggesting a focus on connecting repeated addition to multiplication. She continues by stating that the learners are going to add by using groups. For example, 5 groups of 3. The teacher explicitly states that she wants to see 'repeated addition of plus and not multiplication'. She then proceeds to draw three circles on the board to represent the three groups in which she places 5 counters	

				by unit counting to share them out. Thereafter, she links the repeated addition to grouped counting in 5s by allowing the learners to use their fingers to represent the groups, followed by them counting in 5s until they reach 15. O + O + O 5 + 5 + 5 = 15 3 x 5 = 15 **4.4 modelled in a similar way to 4.3**	

Analysis of lesson 4

4.7.1 Specialising of content

The first three episodes of this lesson dealt with counting, in units and then in groups, as in Lesson 4. Excerpt 1, episode 4 dealt with repeated addition (adding repeatedly) primarily:

> **Excerpt 1, episode 4**
>
> T: Let's look at the board. We have 3 X 5.
>
> **[The teacher writes the word (Hlanganisa) addition on the board before writing 3 X 5 on the board]**
>
> T: Do we know this? [Writing a multiplication sign next to 3 X 5 on the board]
>
> Class: Yes
>
> T: We are going to add here. We are going to learn with groups. We are talking about groups.
>
> Class: About groups
>
> T: Let's look at the board. [Teacher writing "mintlawa (group) on the board ya 3 ya vo 5" on the board.] I'm going to show you with the counters, when we say repeated what do we mean. How do we do repeated addition? We are going to do 5 groups of 3. How many groups are we making?
>
> Class: We are making 5 groups of 3.
>
> T: We are adding, doing repeated addition, here I make the first group, here another one and another one. We are making 3 groups of 5. What are we doing? [Teacher drew 3 circles on the board and gradually put counters into the circles on the board, eventually forming 3 groups of 5 as she explains.]
>
> Class: We are making 3 groups of 5.

Of the four examples used, all were below the number range of 20 which is below the requirement range for G3 (solution to problems using multiplication with answers up to 100) according to the CAPS document (2012). It was observed that the teacher in excerpt 2, episode 4 showed the learners how to count in a more efficient way by not relying on unit counting methods. She encouraged the learners to use their fingers to represent the groups and then to count in multiples of 5 until they reach 15. The shift from unit counting of all items to grouped counting in 5s is a form of specialising content as it is a more efficient strategy.

Excerpt 2, episode 4

T: What we have done here, it takes us back to multiplication. **Here we are just making it short. Here are our groups** [3 × 5, pointing to the 3 on the board]. We are multiplying, we are just writing it in a short way. Let's count them.

T: Show me 15 with your fingers, your pinkie finger. Repeat is 15, raise your 3 fingers, and raise them so that I can see them. [She asks the learners to hold up **3 fingers** in the air, **representing the number of groups**. She explains that one represents a group of 5] Let's count in 5s [The learners then count in fives upon the three fingers they hold up.]

T: What number must we count in?

Class: 5

T: We know how to count in 5s right?

Class: Yes

T: Let's count in 5s and stop on the third finger.

In this way, the teacher made use of the grouped counting in 5s which was covered in lesson 1, episode 4. In addition, she continues to show the connection or link between repeated addition and multiplication. In this way, she is attempting to specialise the content as acoustic counting allowed learners to memorise number sequences, for example, 5, 10, 15 and so forth, a skill which is used here to ascertain quantity in grouped arrangements (Ensor et al., 2009).

In excerpt 3, episode 4, we see the teacher holding the learners back even though they have shown that they are capable of working with the more efficient strategy:

Excerpt 3, episode 4

> T: I want to see everyone doing it, do it. I want to see your sum; I want to see your sum done. Groups of four, right. Let's write. Are you writing? **I want to see repeated addition of plus, I don't want to see multiplication.** I'm not in multiplication yet, I want to see plus the many plus. I want to see repeated addition of plus. No, I don't want you to do groups of 5. You were doing groups of 4. [Many of the learners are seen to be making 3 circles with four blocks in each, they then write out the problem as 4 + 4 + 4 =12 underneath the diagrams]
>
> T: Alright Good. Who is going to write it for me? [Teacher writing on the board 4+4+4] what is the answer?

She did not provide the learners with a more challenging task; rather she chose to let them revert to the repeated addition method – 'I want to see repeated addition of plus, I don't want to see multiplication'. This phenomenon has been noted by Ensor et al. (2009) and can represent a pulling back of the learner who can produce a more specialised recalled fact strategy. It is also a method noted as widespread amongst older learners by Schollar and Associates (2004).

4.7.2 Specialising of representations

In lesson 4 the teacher used a range of apparatus, namely concrete apparatus (unifix cubes for counting), indexical apparatus (circles to represent groups) and symbolic number-based apparatus (5 + 5 + 5 = 15 and 5 X 3 = 15) and a 1 - 200 number chart. The teacher modelled step-by-step how the learners needed to do repeated addition in a concrete way most of the time. However, in episode 4 the teacher then linked the repeated addition (5 + 5 + 5 = 15) to multiplication (3 X 5 = 15) which is a shorter and a more efficient method to use, thereby specialising the representations (this is a form of calculating without counting by using known facts) as well.

4.7.3 Connections between content and representations

In excerpt 4, episode 4, the question asked by the teacher could be interpreted in an ambiguous way:

Excerpt 4, episode 4

T: Who is going to tell us what the difference between these 2? What is the difference between these 2 on the board? 5 + 5 + 5 = 15 and 5 X 3 =15

L: The answers are the same.

T: Who can tell me what is different [The class keeps quiet.]

T: Grade 2 work. What's the difference?

L: Nothing

T: There is no difference? The difference is this one is multiplication, this one is short we multiply and this 1 is long we add, right.

For example, difference could be seen as what is the difference between the two solutions which are the same so there is no difference. However, if the question was asked as, 'What is the difference between the two methods used to solve the problem?' This could have provided openings for seeing that one method is shorter and therefore quicker than the other method. Providing learners with the opportunity to discuss the value of using a specific method is advocated as good mathematical practice by a number of authors (Anghileri, 2009 & Askew, 2002).

In excerpt 5 from lesson 4, episode 4, the teacher does not show the connection between 2 X 4 = 8 or 4 groups of 2 = 8 and 4 X 2 = 8 or 2 groups of 4 = 8:

Excerpt 5, episode 4

[Teacher draws 8 circles in sets of two on the board] I wanted 4 of 2, so let's count. I said we are doing 4 groups of 2. You are making noise. Let's count then write [Learner counting] 1, 2 [Teacher writes 2 underneath a set of two circles.] 1, 2 [Teacher writes 2 underneath a set of two circles.] 1, 2 [Teacher writes 2 underneath a set of two circles.] 1, 2 [Teacher writes 2 underneath a set of two circles] and inbetween we put plus [Teacher writes a plus sign in-between a set of 2] in-between we have?

Class: Plus

[Teacher writes plus between the next set of 2]

T: In-between we have?

Class: Plus [Teacher writes plus between the next set of 2]

T: In-between we have?

Class: Plus [Teacher writes plus between the next set of 2]

T: In-between we have?

Class: Plus [Teacher writes plus between the next set of 2]

T: See like this we can count 2, 4, 6 [Pointing to each 2] and we don't have to count by 1. So now let's count. [Class counts as the teacher points to each circle]

In this example, the number base would have been developed by the teacher emphasising the relevance of the alternative representations for given numbers like 2 X 4 = 8, but also 4 X 2 = 8 (Gray, 2008). Thus, the teacher missed an opportunity to allow the learners to discuss their different calculations which yielded the same product. In response to the learner's different approach, the teacher chose to repeat her instructions of what and how she wanted the calculation to be done. The lack of connections between the teacher's talk and representations, and the content not being made more sophisticated, appeared to relate to the learners working out the examples provided in a concrete way.

There is evidence that some learners are held back even though they are capable of doing the calculation without counting using the more efficient and quicker method (excerpt 3 from lesson 4, episode 4). Instead the learner calculated using the multiplication method and checked her answer by counting. In addition, the learners who want to write down the symbolic number sentence were told to show their groupings first, which is a less abstract and more time consuming way to calculate the answer.

Whilst there is evidence of a move to higher numbers in this episode, which might be seen as specialisation of content, the availability and use of concrete representations can be seen as holding the learners back:

Excerpt 6, episode 4

L: Mam are we writing on paper?

T: No I am still checking you guys. Where are the other groups, we must have 4 groups. Where are your groups I want to see four groups? You must not mix this counters and remove the ones that you are not using. Let me see, where are your 4 groups?

T: Now let's write, I want to see repeated addition start. This group not that, don't mix it will confuse you. 4 of 2. Write; write on your paper, I gave you a paper write. I want see it. Here are your papers Write. What you have done on the table also do it on paper. Where are your groups of 4? Write on your paper I want to see it, **I want to see repeated addition.**

L: Is that correct?

T: I want to see 4 groups but what you did is 4 + 4 there is no 4 + 4. I said how many groups are we doing?

Excerpt 7, episode 4

L: Are we writing yet?

T: No work with your counters only, I want to see the counters 1s

[Learner working out the problem that the professor gave her]

L: [Learner writes down 5 X 4 = 20, and then uses the counters to check]

[Learner writes the 5 backwards however]

4.8 Overview and analysis of counting strategies across episodes in lesson 5

Overview of lesson 5					
Episode Number (E1)/Format and Time duration	**Themes/ Concepts**	**Examples**	**Representations**	**Summary of teacher instructions, explanations and responses.**	**Correlated learner actions**
E1 [00:00] WC	Forward Group Counting	Count in 3s	1 – 200 number chart	Let's count in 3s. Range: 3 to 180.	Learners count in 3s.
E2 [03:01] WC	Division 2 – digit from one-digit	2.1 48 ÷ 3 2.2 20 ÷ 5	Pictorial representations Concrete representations: matches	The teacher starts the lesson by writing the operation on the board and ascertains from the learners if they understand what the division sign means. She also introduces the mathematical language and the symbolic representation (48 ÷ 3) from the onset. Thereafter the teacher	

			Indexical apparatus: drawings of children/apples Symbolic apparatus: calculation written down numerically	draws three children on the board and unit shares the matches amongst them. 'T: So when we share we must take one cake at a time and give them'. She proceeds by drawing the matchsticks/1 under each sketch. The teacher constantly checks how many sticks were shared out, and in this way she keeps track of what still needs to be shared out. The teacher specialised the checking strategies by using doubling as opposed to unit counting in ones. No additional detailed explanation provided for 2.2	
E3 [17:52] WC	Number bonds of 150	3.1 40+110=150 3.2 120+30=150 3.3 100+50=150		The teacher explains what number bonds are: '2 numbers that we can add and it gives us 150'.	The learners provided the following bonds for 150 without any visible counting seen]

		3.4 111+30=141 3.5 90+60=150 3.6 30+120+150 3.7 125+25=150 3.8 80+70=150		The teacher demonstrated a ‘grouped count-on strategy’ to verify whether the answers provided by the different learners were correct (count in 5s if the unit digit ended in a 5 or they counted on in tens if the unit digit was 0). ‘T: Let’s count the big number in your head and count in 5s and see if it will get us there [Pointing at 150 on the board] Class: Count in 5s from 125 up to 150 T: If we were counting in 10s. It was going to pass it was not going to allow us to get to 150, right?’ No additional detailed explanation for 3.2 to 3.8	L: 40+110;120+30; 100+50; 30+111; 90+60; 30+120; 125+25;

Analysis of lesson 5

4.8.1 Specialising of content

Unit counting has been dispensed with in this lesson, with Episode 1 involving counting in 3s. In the excerpt from Lesson 5, episode 2, the teacher focused on division of two-digit numbers by a one-digit number and both the examples that she modelled are an improvement from the range she worked with in Lesson 3.

The teacher's talk infers that the learners should do unit counting when working out the sum: 'T: So when we share we must take one cake at the time and give them'. The learners are seen counting as the teacher is sharing the cakes between the children by drawing matchsticks each time below the child. She made the children check at different intervals how many cakes were shared. During this process she also checks the learners' number recognition abilities for them to track how much they have shared already: 'T: When we look at 36 does it look the same as the number on the board?' which is more abstract than acoustic counting.

She further used recalled number facts of doubling to aid the learners' tracking of what still needs to be shared: 'T: When you add three 16s here are our 16s [The teacher writes three 16s] 16 + 16 is equal to what? Double numbers 16 + 16, two 16s. What does it give us? Class: 32. T: We take 32 and add the other 16, what will it give us? Class: 48'

She also showed the learners how to make use of group counting of 5s to determine the answer without having to share the objects one by one (20 ÷ 5 = 4) but reverts to drawing of apples to verify the answer. It is therefore apparent that the teacher moved the learners from the initial unit counting of sharing the match sticks to more efficient strategies of keeping track of what was shared.

In excerpt 2 of Lesson 5, episode 3, the main theme is number bonds of 150. The teacher proceeds by explaining what it is and gives different learners the opportunity to provide a set of bonds that gives 150. Then the class checks the correctness of the answer before proceeding to the next example on offer. It is noted that the teacher demonstrated with the learners 'count-on' strategies and 'count-on from the larger number' which they seem to be familiar with. It is also interesting that when a learner gave an incorrect answer, the teacher gave the class the chance to evaluate whether the answer was correct or not. This is viewed as important in the development of the learners' number sense (Kilpatrick et al., 2001). Further on, the teacher also justifies the reason for choosing a particular strategy which learners need to know so that they can also think about their choice of strategy when calculating. In this lesson activity, the learners have used more advanced strategies than concrete acoustic or unit counting. The number range has also been extended which makes the content more specialised, although no explicit connections are made between, for example bonds for 15 and bonds for 150.

4.8.2 Specialising of representations

In this episode, she made use of indexical apparatus (matchsticks), symbolic-number-based apparatus ($48 \div 3 = 16$) and iconic apparatus (drew children to represent the groups) and has contextualised the problem to help the learners understand that they need to share the cakes between the 3 children, a concept which they can relate to.

4.8.3 Connections between content and representations

I noticed that there are strong connections and strong rationales for these selection choices of tasks across lessons of similar topics like division. However, the sequencing of the examples used were disconnected in that she proceeded by extending the number range in lesson 5 in comparison to lesson 3 ($48 \div 3 = 16$) then reverts to an easier example ($20 \div 4 = 5$). This would have a direct impact on specialising content and the notion of pursuing efficiency as advocated by Venkat and Askew (forthcoming) as sufficient modelling with more advanced tasks is limited. Counting in 5s is present across both episodes here, presenting some continuities in skills. But bonds of 150 are focused on without making any connections to bonds of 15, and there is no push to work systematically either; just get some answers. Both of these aspects represent important connections in mathematics education but are absent here.

4.9 Overview and analysis of counting strategies across episodes in lesson 6

Overview of lesson 6					
Episode Number (E1)/Format and Time duration	**Themes/ Concepts**	**Examples**	**Representations**	**Summary of teacher instructions, explanations and responses.**	**Correlated learner actions.**
E1 [00:00] WC	Forward Group Counting	Let's count in 20s from 20 to 1000	No representations	Forward grouped counting in 20s which was aligned to the curriculum as the learners counted up to 1000. Teacher monitors as learners count without using any representations.	Several of the weaker learners are seen to follow on a number chart which the teacher allowed for differentiation purposes.
E2 [03:18] WC	Backward Group Counting	Start counting in 20 from 200 counting backwards up to 0.	No representations	Backward grouped counting in 20s is seen as a form of specialising content since it is perceived to be more difficult to cross over to the lower decade.	
E3 [03:05] WC	Forward Number Patterns	Larger ' + '/plus 700, 720, 740 1. From 700 in 20s to 1000.	No representations used	From the onset, the teacher explicitly established the rule and the pattern. For example, the rule and the operation needed to increase the number by 20 namely counting forward in 20s.	

		2. From 621 in 20s 621, 641…			
E4 [03:29] WC	Backward Number Patterns	' - '/minus Smaller 1. From 900 in 20s 900, 880, 860		Next, the teacher explicitly reinforced the rule and the pattern. For example, the rule and the operation needed to decrease the number by 20 namely counting backwards in 20s. Strong connections occurred within and across episodes as the teacher made use of the grouped counting used in episode 2.	

Analysis of lesson 6

4.9.1 Specialising of content

In lesson 6, episode 3, the main theme was forward number sequence or patterns and in episode 4, it was backward number sequence or patterns. The teacher extended the number range up to a 1000 which is in line with the CAPS document (2012). In the excerpts below the teacher demonstrates what forward and backward number patterns entail. She focuses on the learners establishing the direction and rule of the sequence and uses various vocabularies to describe the action of the numbers, like: small, shrinking, bigger, and growing. She also lets learners take note of the differences between the patterns of forward number patterns and backward number patterns. In this lesson the teacher is making the content more sophisticated as she does backward number patterns as well. According to Wright et al. (2006), counting involves knowing the problems the learners would make when crossing over to the next decade. The teacher provides the learners time to identify their mistakes and to correct them. She also specifies when learners should count in 10s or 5s when checking their solutions. Within this pedagogic support, she provides what Askew et al. (1997) view as good teaching in terms of helping learners make meaning of their strategies. An example of this kind of support is given below:

Excerpt 1 from Lesson 6, episode 3

T: Which means on our 700 to get 720 we must add 20, this is what we call a rule when we are counting forward we must add a plus (writing +20 on the board) this is our rule when we are doing our pattern on 20 when we are counting forward, we must add 20. What must we add?

Class: 20

L8: Plus 20

T: Let’s see here we have 720, we have 740 and 700. Let’s look at the difference between the two patterns. Let’s read the first pattern what does it says?

Class: 700, 720, 740

T: Let’s look at the second pattern

Class: 740, 720, 700

T: I want you to tell me what is the difference between this pattern and this pattern? (Teacher pointing on the board) raise your hand up I don't want a choir. On this on we start with 700 and on the other one we start with 740 what is the difference between these patterns we started with 700 and on the other side we started with 740 what is the difference? Before you tell me the difference, what does this pattern do? Does it grow or shrink? (Pointing at the 700, 720, 740 pattern)

L9: It grows

T: What is the reason to make it grow? Is because we say plus 20.

4.9.2 Specialising of representations

In lesson 5, no diagrammatic or symbolic representations were used as the learners relied on recalled facts to provide the bonds that equal 150. In Ensor et al.'s (2009) terms, this is a highly specialised form of representations as it implies that the learner has internalised the representation and does not rely on any form of counting.

4.9.3 Connections between content and representations

Strong connections occurred within and across episodes as the teacher made use of the grouped counting used in episode 1. Additionally, there are strong connections within and across the episodes as the teacher explicitly demonstrated the difference between the two episodes' activities (forward counting in 20s and backward counting in 20s). In addition the teacher gave the learners the opportunity to work out the rule for the increase or decrease depending on the example provided. Both the content and the representation was specialised in that no representations were used (learners relied on recalled facts) and the number range was extended (counting up to 1000) which is in line with the government's curriculum for grade 3.

4.10 Overview and analysis of the counting strategies employed across the 6 lessons.

It is observed that acoustic counting occurred in 4 of the 6 lessons. In the overview of the counting strategies it is noted that the teacher at the onset would instruct the learners to use a number chart in G2 to track their counting which always started from the digit the teacher called out. For example, learners would count-from 1s or 2s or 3s or 5s or 10s or 20s respectively. Towards the end, the learners showed more fluency in their counting and did not rely on any representations suggesting that the number sequence had at least been internalised as Ensor et al. (2009) advocated. The teacher accredited this to the learners being more familiar with their numbers (initial interview) which made them more able to count quickly. The teacher also moved from unit counting in the earlier lessons to acoustic grouped counting in the last two lessons which implies that there was some specialisation across lessons.

From the excerpt below, I could not ascertain whether the learners have an understanding of the numerosity of the numbers or the structure of the number by the regular practising of acoustic counting. But the teacher seemed to show an understanding that rote or habitual learning is meaningless unless they understand what they are doing. The practice of the different counting strategies is important as I pointed out earlier, as it aids the development of seeing patterns that would help with early addition and subtraction since learners are given the opportunity to memorise number sequences and number patterns (Anghileri, 2006).

The teacher also uses group counting in different lesson activities to model checking or showing a faster strategy to get to the answer (For example, in lesson 5 and lesson 6). The use of symbolic-number based representations which ranged from number charts from 1 to 100 in the first term to 1 to 200 and later on to 1000 which is more abstract according to Ensor et al. (2009) than concrete apparatus, iconic and indexical materials. The teacher, according to the initial interview also used the number chart in less concrete ways towards the end as she allowed the learner to count-from different numbers so that they can be more 'flexible' with numbers. The teacher provided ample opportunities for counting forwards, backwards and in groups and she also increased the number range from 100 in G2 to 1000 in G3, in accordance with the CAPS document. It was also observed that several learners encountered problems when counting backwards as they struggled to progress to the next lowest decade which resulted in them using a number chart or following the more abled learners (Wright, Stanger, et al., p. 35). Wright et al. (2006) stated that the teacher increasing the number range with the appropriate use of a corresponding representation is a form of the teacher specialising the content. Furthermore, the teacher has introduced the learners into 'base-ten thinking' since our number system is organised into 1s, 10s, 100s, 1000s and so forth (Wright et al., 2012, p. 16). This is evident in the interview when she chooses to count in 50s, 100s to solve the sum and the reverting to group counting (5s, 10s and 20s) to facilitate the solving of the calculation in several lessons. So previous learning is being made use of in some of the later lessons, which points to connections across lessons.

There is also evidence across all six lessons that learners have progressed across the five stages devised by Wright et al. (2006). Several learners seen in the 2011 videos were functioning at the emergent counting stage which implies that they were unable to count visible items or they did not know the corresponding number name for the objects. The weaker learners functioned at the 'figurative counting stage' when they solved subtraction sums of numbers below 10, by choosing to 'count-all' the items as opposed to 'counting-on'. This was also the method demonstrated by the teacher in Lesson 1. In some instances, the learners used and the teacher modelled in the subtraction lesson 'count-on' strategies (initial number sequence). The teacher demonstrated the ability to count using more advanced strategies which do not involve counting in ones (non-count-by-ones strategies) to solve problems (the facile number sequence stage) during the interviews. In this way the content has to some extent been made more specialised. There appears to be some awareness of the possibility of 'fading' the representation as learners came to internalise its features, as was apparent in the initial interview:

> I: [15:22] Sometimes you used to say use your number chart but of late you are not telling the learners to use their number chart.
>
> T: [15:25] They are no longer using their number charts, they know their numbers. I have to tell them to count in two – it does not mean you have to start from two; you can start from any number as long as you know you have to skip two numbers. But some are still struggling but they are getting there.
>
> I: [15:51] **So you allow them to practise counting from different numbers.**
>
> T: [15:55] I want them to be flexible, so not to cram 2, 4, 6. It becomes a habit for them. Oh, Mam said we must count in two: 2, 4, and 6. I can start from any number - count-from 52 in two's or count-from 55 in two's. They have to know from 55 it is 57, 59, 61 until…

4.11 Analysis of the interview data

As noted earlier, the interview data provided commentary and rationales that went beyond the six lessons that had been videotaped for this study. In lesson 1, the teacher encouraged learners to use unit counting as the weaker learners especially are seen to have struggled when they worked with the apparatus. This concurs with the teacher's interview comments when she intimated that she preferred the learners to work with the string beads as each colour represents 10 and the learners are able to manipulate the string beads more easily than the abacus. The following excerpt shows a greater awareness by the teacher for learners to use apparatus that they can manipulate easily and accurately.

Excerpt from the initial interview

> T: [9:32] Because this one - the abacus is too small and they usually make a mistake that is why I prefer these [referring to the colour coded string beads] so that they can know that here is ten [each colour on the string beads represents 10] so that they can use them so here referring now to the abacus when we say count - 1, 2, 3 but they don't see one [of the objects] is in the corner. They don't care what ever, so I have to use the string beads or the empty number line. The empty number line and they enjoy it because my learners they know that or if it is minus or whatever I have to make **friendly numbers**. They can start by moving two spaces can know how to count in tens by putting two aside or they can start by jumping 2 spaces then they can count in tens or in twenties so that I **can be quicker.**

The teacher also suggested that she does not use the abacus in G3 as it encourages the learners to unit count which is time consuming. She rather prefers them to use the string beads or part-part-whole. This indicated that, at least in retrospect, the teacher had an awareness and understanding that learners need to calculate more quickly by using more efficient strategies. This is crucial if learners are to cope with the more challenging mathematics in the Intermediate Phase which is the pivotal reason for specialising both content and the representations used.

In the example below, the number concept was first developed mentally and then formalised, written place value notation or conventions were developed. The teacher was further able to provide examples that substantiated her view of working more efficiently:

A calculation to show how learners used friendly numbers to work with numbers in a flexible way

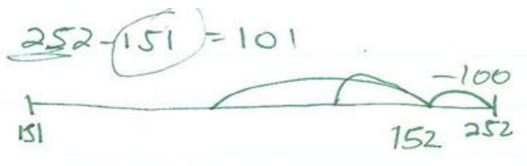
$$199 + 101 =$$

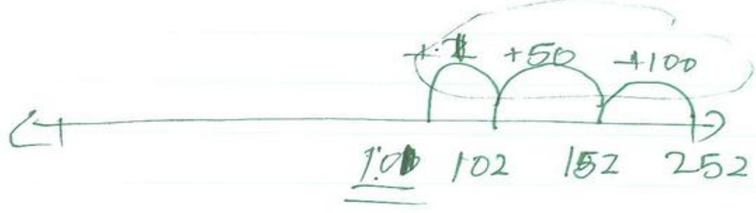
$$(199 + 1) + 100$$

$$200 + 100$$

$$= 300$$

In the excerpt below, the teacher explains how different strategies are used by different learners depending on their abilities. According to Markovitz and Sowder (1994), it is important that a teacher shows an understanding of equivalent representations and is able to apply them to different computational contexts. In the initial interview, whilst the teacher did show different strategies that learners were observed using, she was hesitant and made errors moving from one representation to the next. In spite of this though, she was able to correct herself when asked questions which indirectly highlighted the error. In the interviews, there is evidence of her awareness of more specialised content and use of representations as the number range shifted to three-digit numbers above 100 in comparison to the lesson examples which were less than 20. In the interview she noted that the learners were able to do the work themselves and draw their own number lines by using their own intervals. In the example provided, according to the teacher the learners would jump in 50s or 100s and so on.





This is therefore broader reported evidence of the content becoming more specialised as the learners are making use of calculating-by-counting strategies rather than working with concrete objects, as the excerpt below shows:

Reflective interview follow-up explanation given in post-lessons interview

T: The example done in the interview: 252-151=□.

I: Draw your own number line.

T: They can subtract 100 here and they will be left with 152 and they can subtract another 50 which is going to be 102 then they subtract 2 which is going to be 100. Am I correct?

I: Why are they subtracting two? They have subtracted 100 and they have subtracted 50

T: No, they are going to subtract 1, which is going to be 101. So when they add this it is going to get this [151: pointing to the subtrahend]

I: [11: 31] And those who don't use an empty number line, what do they use?

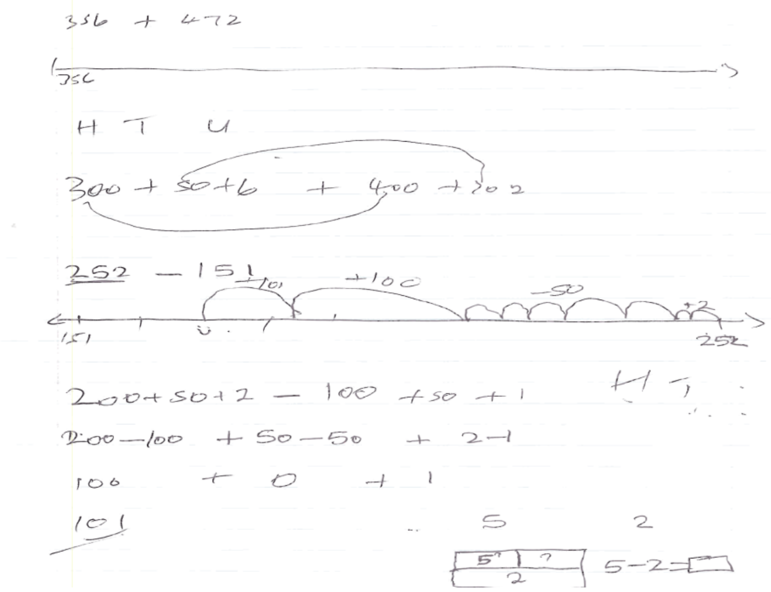
T: [11:33] They like breaking down numbers. Like in addition but here they need to use the minus sign. They will start breaking it by H, T and U (200 + 50 + 2) but in the middle they must use minus 100 + 50 + 1. Then when they come here, they have to group: 200 - 100 + 50 – 50 + 2 - 1. From here they know oh 100 + 0 + 1. They know that the answer is 101. That is how they do it.

I: [12:26] And your weaker learners, what do they do?

T: [12:29] It is a disaster with them. I try to do this method [referring to the breaking down method]. It is easier to do than the H, T and U method because this method needs you to borrow. So when it comes to borrowing it confuses [the weaker learners]. So this method [breaking down] I find it easier for them to do it. Number line ha-ha [not an option] it is another story for them, they can't do it. So they do try [breaking down method], they can break down the numbers from here and when it comes here they confuse plus and minus, they confusing the operation. You find them writing plus, plus, plus. I explain to them that it is not plus but now you are subtracting, taking away the answer. But they are still doing the same thing.

In the above examples, the teacher illustrated how learners partitioned the numbers into hundreds, tens, and units. Thereafter, they would regroup the hundreds together, then the tens and the units before adding.

Scanned example of how learners used decomposition to solve an addition and subtraction calculation



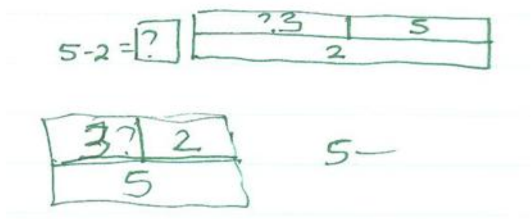
She commented that with subtraction, the able learners coped well with this method but the weaker learners found working with decomposed number operation and the subtraction operation confusing. The teacher opted to revert to a simpler method like part-part-whole or using the bead strings for the weaker learners, which is explained as follows:

I:[13:11] So if they still doing the same thing.

T:[13:16] So I go back to small numbers and I use part-part-whole.

I: [13:20] Show me part-part-whole.

T: I can use the small numbers and I can tell **them a story like** Mary has 5 oranges and her mother. Oh I want to go back to addition. Let me come back, Mary has 5 oranges and he took two oranges and gave two oranges to her friends. I can draw a diagram like this



Mary had 5 oranges and gave two to his friend. Where must I put two? Woo! I want to know where it will be written 5 – 2 = □ I want to know the answer. So it is where he can go back to the string beads, use the abacus or they use bottle tops for them to count out five objects and put them on the table. They count, okay now, they take away two bottle tops - throw them. How many bottle tops are left on your table? They won't tell you it is three or what. They will say another number. So that is how we do it with the slow learners.

Representations and strategies used by the weaker learners as illustrated by the teacher.

The interview data, thus pointed to purposive use of more concrete strategies and representations in some instances, geared towards the needs of lower attainers. There was evidence in Lesson 1 of this kind of differentiation at the level of tasks backing up the teacher comment. I come back to this point in the final chapter. Whilst there is a push in Ensor et al.'s (2009) and Schollar & Associates' (2004) work for specialisation, it is important that specialisation carries learners along, not useful if learners are left behind.

Further evidence of the teacher's willingness to allow differentiation is seen in the excerpt below where the teacher claimed that the learners choose which strategy they prefer to use when solving problems. Both the content and the representations have been made more advanced as the learners are calculating without counting and the number range has been extended to three-digit numbers, which is in line with the CAPS curriculum document (2012). The more sophisticated strategies that the learners have used to solve the problem were base ten blocks, partitioning, using an empty number line flexibly and column addition (H, T, and U). In the case were the teacher believed the learners were not coping, she provided them with strategies that they could cope with, such as part-part-whole or the string beads. She states categorically in the interview that she does not promote the use of the abacus in G3 since learners tend to unit count-on it, which she sees as time consuming. For this reason, she prefers the string beads as learners are able to manipulate it better.

Excerpt from the initial interview

> T: They can draw the number line. If it is 356 + 472 = □. They know where to start. They can draw a number line. And they start at They start where? At 356 and they count in tens until they reached 346. So they know what the answer is that is needed. Or they can use the ten base block. Some learners prefer to use Hundreds (H), Tens (T) and Units (U) and some prefer breaking down. For example: 300 + 50 + 6 + 400 + 70 + 2. Then they group the hundreds together, they group the tens then they group the units. This way is easy for them, that's how they do it
>
> I: And you said your weaker learners normally count using the abacus.
>
> T: [8:26] Not always, I have this - I use the string beads and they know that they are colour coded and that these are 10 referring to the orange and 10 green bottle tops. Sometimes I lead with a number line and I ask them to show me what is the number starting from the first interval and I told them that there are also numbers between the intervals - if it is zero and we are counting, if we are using 5, 10, 15, 20 then they must know that on the number line they must use 5, 10, 15, 20. But in the mind they must keep it that there are numbers between the numbers also even if there are no sticks. They have to know there are numbers there and to use them to help them with the units.

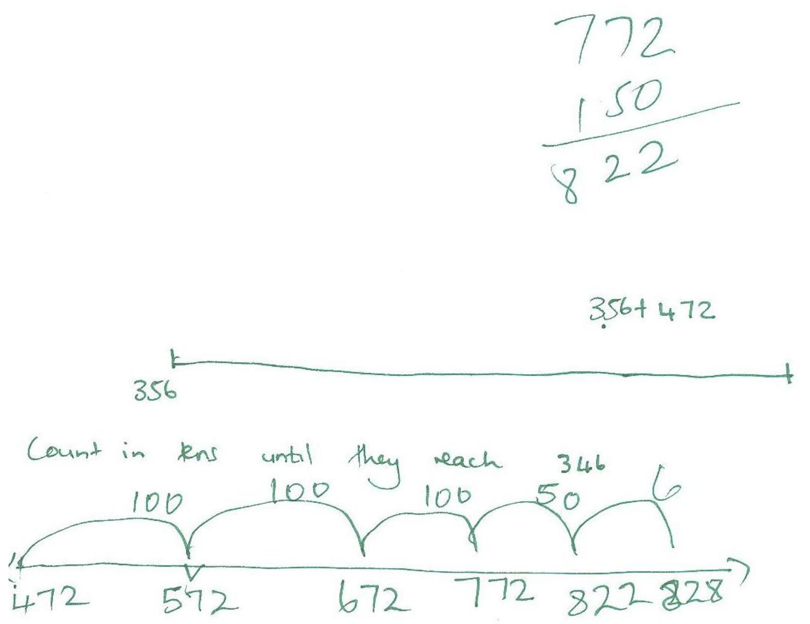
Excerpt from the reflective interview

From the explanation below of how the more able learners will use an empty number line, it is apparent that there is awareness of how the content and the representations can be specialised.

> T: Let me see. They can start from 356 and the can add until Let say 800 whatever I don't know. They must add 472 to 356. So which means I should have started with 472 then they add 356 to get the answer.
>
> I: So redo it at the bottom for me.

T: They are adding so let me start with 472. And then they can jump in 100s. Let me break it again [356] to 572 then it is 672 then break it again with 100 then it is going to be 772 then they add 50 and from here it is going to be 822 [does unit addition to get the total] then they can add 6 and it is going to be 828.

Scanned example of teacher's representation



Of interest in relation to the videotaped lessons is the teacher's awareness of strategies that are more efficient than unit counting, which is made clear in the interview. Additionally, the interview data allowed me to note an awareness that I did not always see in her practice. In some ways I heard more in her interview than I saw in the classroom about the differentiated specialising strategies and representations used by the teacher for her able and weaker learners.

4.12 Overview of the specialisation of forms of representations across the 6 lessons

Forms of representations and number of tasks used by the teacher for each lesson activity or episode were recorded in the table below. Multiple forms of representations can be found within each episode as an activity lends itself to different forms of representations depending on the learners' abilities according to the teacher's belief system.

	Counting		Calculating by counting		Calculating		Total
	G2	G3	G2	G3	G2	G3	
Concrete	L1,E1 L1,E2 L1,E3 L2,E1 L2,E2 L3,E1 L3,E2	L4,E1 L4,E2 L4,E3 L4,E4 L5,E3 L6,E2					13
Iconic	L1,E5 L1,E8 L2,E2	L5,E2					4
Indexical	L1,E6 L1,E7 L1,E8 L3,E1 L3,E2	L4,E4 L5,E2		L5,E2			8
Symbolic	L1,E1 L1,E2 L1,E3 L1,E6 L1,E7 L1,E8 L2,E1 L2,E2 L2,E3 L3,E1	L4,E1 L4,E2 L4,E3 L4,E4 L5,E2 L5,E3					16
Syntactical							
None		L6, E2					1
Total	25	16		1			42

This table shows that concrete apparatus is present in most of the episodes for counting in both G2 and G3. The teacher also made extensive use of the symbolic number based form of representation which is a more abstract form of representation used when calculating. In addition, the table shows that the teacher utilised a variety of representations to model the mathematical ideas in her six lessons. The table further shows that there is greater prevalence of concrete and symbolic than the other representations, suggesting that there may be a JUMP between these, rather than a scaffolding and connecting via the iconic and indexical. In the interview there was strong mention of calculation without counting being demonstrated by the teacher of what she observed in her class: column addition and subtraction, partitioning, using base 10 blocks and part-part-whole. The need for learners to be able to use a range of representations to express their mathematical understanding as this would show a more strategic way of working with number more flexibly. In addition, the learner needs to work with the representations provided by the teacher and gradually to use apparatus of their own choosing and design (be able to draw own number line and add calibration themselves).

> T: The number line in G2 the teacher was writing all the numbers on the number line and we never gave any children to write on empty number lines and start filling up numbers on their own. But in G3, I have noticed a difference. They know how to draw a number line and if I write a topic there - Do this addition using a number line. They will write a number sentence, they will take a ruler and make the own number line and put the numbers and do the addition or the subtraction on it. That’s a huge difference and I am happy about it because there number sense is developing very fast.

The following statement made by the teacher in her closing interview comment infers that she has observed this practice in her class:

> T: In G2, I think I was doing a lot of work and they were relaxed so in G3 now they have to work on their own. That is the difference that I have seen. Now my learners know how to work on their own-I can write a sum on the board, explain how to do it, sit down and they can do it. They can draw; make pictures when it comes to problem solving they have to know to read the problem first and analyse it. They must know if it is addition or subtraction. They have to make number sentences. In G2 we are not

doing those things. So in G3 I can see it is different for learners have to reason. They have to know 'warr' Mum baked 48 cakes and want to share it equally to her three children. So they have to start drawing children, drawing 48 cakes and start sharing if it is a group. They know and that is the difference that I see and in G2 we are doing everything for them.

The interviews suggest acknowledgement of the need for learners to appropriate representations and use them independently, with the teacher's sense that this is being achieved. It was observed that the teacher used the numbers more flexibly and with differentiation. Literature on mathematics teaching has shown that number relationships and number patterns are necessary for the learners to understand how topics and operations are connected through language, as well as tasks and representations used (Anghileri, 2006; Berch, 2005; Markovitz & Sowder, 1994; Greeno, 1991). Learners need to know that numbers relate to one another (8 is one less than 9, but two more than 6) and related vocabulary (more, less, equal to, how many more/less, sequencing of numbers, composition and decomposition of numbers (CAPS, 2011, p. 93, 144). It was observed in the teacher's lessons that the teacher used a variety of vocabulary (small (shrinking), bigger (growing), forward, backward, less and more) to show learners the relationship between numbers and between numbers and the operations. In other words, the teacher showed that when adding, the number increases or gets bigger and when subtracting the numbers decrease or get smaller and that sharing is linked to dividing. In other words the teacher did focus on the diversity of the language associated with learning about the different operations and how such language may be related to counting patterns and links amongst them.

Number lines have been described as a tool to help visualise the relationships between numbers and operations. In the interview the teacher frequently asserted that the learners enjoyed working with the empty number line as they could use the numbers 'flexibly' (See scanned examples of an addition and subtraction sum). Furthermore, the teacher also stated that the learners in G3 were able to draw their own number line and choose the intervals they wished to work with in comparison to G2 where learners were not given their own number line and had to work from the teacher's number line which had all the demarcations on already. Askew (1998); Anghileri (2006) and Wright et al. (2012) advocate that the use of an empty number line supports the flexible use of numbers to solve problems. Use of the number line was not seen though, in any of the three lessons observed in G3, and the table also shows for the 6 lessons that there is still a strong reliance of concrete apparatus into G3.

The teacher is aware that she has to make the mathematics more challenging (initial interview) and certain about her motives for doing this. She succinctly states that she wants the learners to be ‘more familiar with numbers’ and ‘that their knowledge of mathematics will be more strategic, not to focus on only 2, 4, 6’.

4.13 Results of the pre and post-test of the teacher

The pre-test indicated significant gaps in her mathematical content knowledge. She obtained 34% in the pre-test and progressed to 46% after the 6 contact sessions of the 20 (OTML) day course. The interview data indicated some lack of fluency in moving between representations. Interestingly, she was more fluent with symbolic representations but had difficulty in using some indexical representations (for example, part-part-whole representations). Of interest was her ability to correct these errors once these were brought to her attention. Thus, while representing part-part-whole graphically posed a challenge, the symbolic number base representations were always executed confidently and with ease, perhaps relating to the pattern of representation used as seen in the table in 4.12. The teacher exhibited a very positive attitude and was keenly aware of her abilities and did not hesitate to ask for assistance when she felt the need to do so. She was also able to opt for an alternative method to help the weaker learners who did not understand the method modelled in the class.

4.14 Conclusion

In this chapter an in-depth analysis of the data and research findings are presented. The data analysed was derived from 6 videotaped lessons with field notes, 1 audio and videotaped interview, and 1 reflective interview and the pre and post-test of the teacher. Lesson transcripts were analysed against Ensor’s et al.’s (2009) framework which was based on the specialisation of content and representations respectively. After each lesson excerpt, a concise summary of the findings was presented and discussed in conjunction with the relevant supporting interview extracts. However large portions of the interview excerpts were analysed and discussed separately to the lesson overview.

In the interview the teacher showed an awareness of the need to connect her specialisation of strategies and representations to the learners in her class. Therefore the interview suggested that the teacher was aware and able to adapt her lessons according to the learners' varied abilities which were prevalent in all 6 videotaped lessons.

In most cases, each lesson was overviewed and discussed and in turn in chapter 5 I discuss across all 6 lessons. Therefore, in the following final chapter, an overview of the research findings and analysis, and limitations of the study are presented, discussed, concluded and future research possibilities suggested.

Chapter 5

Conclusions and Research Implications

5.1 Introduction

The purpose of this study was aimed at exploring and understanding the developments in the teaching of number concepts of a Foundation Phase (FP) teacher who showed evidence of significant gaps in her mathematical content knowledge.

The aim was to investigate any shifts made towards more abstract ways of working with number, by a teacher within the WMC-P teacher development project. My research questions are focused on teacher talk and the representations used within the classroom to communicate early number ideas. I was interested in the ways these two aspects promoted or constrained possibilities for learners to shift towards more abstract ways of working with number.

In this chapter, an overview of the research findings, and limitations of the study are presented and discussed and future research possibilities suggested.

5.2 Overall findings

It would appear to me that the teacher showed an increased awareness and understanding of the move from concrete to abstract working with number than was gleaned from the videotaped lessons. As Ensor et al.'s (2009) framework is primarily about the relationship between the teachers and how they engaged with the mathematics, namely how the content is specialised or the representations are specialised. I have noted that much of the specialisation of content depended on a style of teaching which involved largely telling and procedural connections as described by Bernstein (2000).

In the interview the teacher noted that some of her selections were based on her understanding of the learners' abilities which is a feature that Ensor et al.'s (2009) framework did not take into account. The differentiation seen in terms of task setting and learner strategy use provides evidence of this playing through into practice. Interview data notes that lack of specialisation can be intentional and related to learner needs. When tied with differentiation as seen in some of these lessons, this can mean that content and pedagogy are better tailored to learner understanding. This suggests that specialisation on its own is not enough.

It was also observed that the teacher used different contexts in all of her lessons to facilitate the learners' understanding of the mathematical concept or theme. The notion of contextualised teaching is supported by Anghileri (2006), as she asserted that teachers should provide learners with opportunities to learn number sense within different contexts so it can make sense and permit the doing of mathematics to become more meaningful.

I noticed that there are strong connections between the task selections as it was based on the teacher's perception of the learners' abilities, which was frequently the premise for selecting specific content and the representations used. An example that is illustrative of this is in lesson 1, episode 8, where the teacher used pictorial representations (iconic apparatus) for the weaker learners to tangibly derive the answer whereas she provided an abacus for the more able learners who chose to use it to check their answer as they were capable of applying recalled facts to solve the calculation.

5.2.1 Specialisation of content

From the onset, I would like to state that there is a misalignment with the curriculum as the curriculum is urging the teachers to forge ahead yet literature suggests that learners only be moved on from concrete apparatus once they have grasped the numerosity of the number and understand the structure of numbers as well. I noted in my observations that the weaker learners or the 'special needs' learners according to the teacher's assessment, worked with a range below 10 and for the more able learners the teacher extended the number range.

In a number of the lessons, the teacher introduced the problem using a story and then proceeded to act on it by using various forms of counters that were then represented in a symbolic notation for the learners to complete. In this way the teacher has specialised the representations. The teacher also mentioned in the interview a range of strategies that the learners employed to calculate, such as the use of counters to 'count-on' or 'to count-from the larger number', pictorial drawings, part-part-whole, base ten blocks, partitioning (decomposition), empty number line and column addition and subtraction. From the range mentioned, it can be inferred that the teacher does shift from concrete ways of working to more abstract forms of calculating.

The teacher has explicitly specialised the forms of expression in her lessons which is a key component of specialising content. For example, the teacher links the word divided to sharing, shrinking of numbers to minus or smaller than and bigger than to numbers growing and doubling is also used. In this way the teacher used an everyday context to make the mathematics more meaningful for the learners and linked it to the corresponding mathematical language. This is reinforced in the interview when the teacher said that the learners need to be able to read the word problems themselves, determine the operation that is involved and then to select a strategy to work with; whether it is drawings, counters or symbolic notations.

Specialisation with respect to counting is evident as both the number range increased with no reliance on support material nor was it prescribed by the teacher in the 6 videotaped lessons. However, those learners who needed it were permitted to use a number chart whereas in the beginning the teacher told the learners to use their number chart and to track their counting. The reason given by the teacher for the change in her approach was that the learners are now familiar with their numbers and they need to be challenged 'I must make things difficult for them by letting them count-from different numbers and odd numbers as they struggle with it'.

In the examples used to explain the mathematical idea, the necessary progression and use of appropriate range was lacking in most of the videotaped lessons. For example, the range for the division examples was less than 20 and further on the range was extended to numbers less than 50 but for G3 the range ought to be less than a 100 according to the CAPS document (2012). But this was not the case in the examples done in the interview which showed that the learners progressed to working with three-digit numbers and are able to use the numbers flexibly and to change the given numbers to 'friendly' numbers to calculate efficiently and effectively. 'Friendly numbers' as I gleaned from the teacher's example are numbers that are worked to multiples of 10, 20, 50, and 100 and so on (and this was a term used in the OTLM course).

Their framework as it stands does not connect to learning. With this said, the mandatory incorporation of mental mathematics for approximately 20 minutes a day for the development of memorisation skills, consistent drill exercises of much needed number facts and for the regular grounding of learnt content over an extensive period of time has yielded some improvement in the learners' overall results.

It is evident from the interview and the lesson that the teacher made use of a range of more sophisticated strategies to explore a concept. For example, partitioning (decomposition), column addition and subtraction, part-part-whole group counting and the use of recalled facts are some of the strategies used by abled and weaker learners in the class. A point of interest based on data from the broader WMC-P study, was that in learner testing where I observed this teacher's class, there was some prevalence of recalled facts and derived facts amongst the learners, seen in their ability to immediately recall the answer.

5.2.2 Specialisation of representations

There are incidences where the learners are comfortable with the symbolic notation. But there are instances where the learners are not comfortable with the symbolic notation, and thus in those situations, the teacher is seen to revert to pictorial representations and patterns via the use of iconic apparatus. It is seen in all the lessons that the teacher modelled the mathematical idea by firstly using concrete apparatus and gradually shifted them towards abstract ways of calculating (using specialised representations). For example, Wright et al. (2010) suggests that teachers should expose learners to different contexts in which they can enjoy doing mathematics by practising, talking and recording their mathematical ideas in multiple modes. Through doing this, important foundations are laid for the learning of more complex mathematics in the Intermediate Phase. In several of the lessons, the structure of the apparatus was not used explicitly to facilitate efficient calculating strategies; a row of the abacus is representative of 'base ten' and yet it was predominantly used for unit counting which demonstrated that the apparatus was used concretely. Furthermore, the connection of the structure of the abacus was not linked explicitly to the decomposition of the numbers into tens and units; instead the teacher linked it to an everyday context as opposed to on a more specialised level based on the decimal structure of number. For example, for 13 - 9, instead of using unit counting to obtain 13 the teacher could have modelled by shifting the whole row of ten to one side to represent 10 instantly and take three as a group to obtain 13 (10 + 3 = 13). Thereafter, the teacher could immediately take 9 away as it is one less than 10 for the learners to see that the answer is 4 (3 + 1 = 4). The strategy modelled by the teacher made use of counting strategies rather than the use of compressed numbers (instant number recognition) to facilitate calculating efficiently and flexibly (Gray, 2008). As Venkat and Askew (forthcoming) stated, for efficiency to be pursued by both teachers and learners they would have to have prior experiences of moving numbers of beads between 1 to 10 with a single shift. More of this increased efficient and specialised working with the apparatus is needed across all six lessons.

It is evident from the interviews and the lessons that the teacher made use of a range of representations to explore a concept. For example, part-part-whole, the use of the base 10 blocks, colour coded string beads, the empty number line and the abacus. In addition, she also used various forms of counters (unifix cubes, fingers and matchsticks) as well as pictorial depictions such as the drawing of children and apples in some of the tasks. In more directed ways, Harries et al. (2008) noted that different representations emphasised different aspects of a concept, and thus advocated having a range of representations with teaching focused on including the understanding of their structures as this would help with the understanding of the concepts. This use of representations, further serves as a 'tool to think with and can be a focus for discussions' (Harries et al., 2008, p. 172).

Also, sufficient provision of complex tasks and representation over a sustained period of time within and across lessons would be needed if the learners are to progress to more advanced mathematics in the Intermediate Phase. To conclude, whilst my research is focused on teaching, an assumption that I made is that as the teacher expanded her number concept repertoires, she then in turn increased the learners' access to more abstract strategies and representations. Thereby, increasing the opportunities for learners to work with performing more abstract calculations that are efficient and flexibly applied (Gray, 2008). The interviews and the examples provided by the teacher of what strategies and representations her learners used to perform calculations suggest that there are shifts from concrete to more abstract forms of calculation both with respect to specialisation of content and specialisation of representations. In other words there is some degree of specialisation (numbers were represented as numerals in symbolic form) and there was some use of written mathematical statements in G3 ($48 \div 3 = 16$). However, the degree and nature of specialisation is still below the required standard for G3 and the FP in general.

5.2.3 Connections between content and representations

Whilst overall the trajectory across lessons for the focal teacher in the study was a trajectory where specialisation was seen, I have also noted disjuncture occurring. For this teacher, there was broadly a trajectory of specialisation happening. Notwithstanding that, there were elements where potentials for specialisation by either specialising within the episodes in terms of the strategies presented or building connections between episodes and/or lessons. The potential is there in the examples seen but this is not always made use of in pedagogy.

5.3 Limitations encountered in my research framework

Ensor et al.'s (2009) framework did not take much account of what the learners did; only what the teachers did. Thus, Ensor et al.'s (2009) framework did not connect to learning. As well as in several lessons with an instruction where the teacher tells, it does not tell if some specialisation of content occurred. One of the limitations of this reading is that the teacher appears to be able to adapt the specialisation to the learners but the framework does not deal with this aspect.

5.4 Challenges and limitations of the study

5.4.1 Time constraints

A key limitation for my research was that the amount of time I was able to spend in the field with the participant was relatively limited and restricted in the context of the intervention project. My study focused on Ensor et al.'s (2009) framework which focused on the teacher's modelling of mathematics, with less focus on the teacher dealing with learner inputs, which literature describes as important. The interview data allowed me to gain insight into the teacher's rationale for selecting specific tasks and representations in ways that she defended in relation to appropriateness for specific learners.

5.4.2 Teacher used learners' home language as the LoLT

An additional limitation was that I was reliant on a transcriber to transcribe all my video footage, as the teacher I had chosen to focus on did most of her teaching in the learners' Home Language (Tsonga). To ensure that the quality of the transcript was not compromised, I used a transcriber with a mathematical background and who spoke Tsonga as a first language to transcribe all the interactions/episodes that had a mathematical focus with respect to teacher talk, teacher-learner interactions and representations enacted by the teacher. Verbatim transcriptions with respects to all interactions of a mathematical nature except in the following instances like classroom management and interruptions which were noted, but not recorded word for word posed a limitation.

5.4.3 Data collection

Since the data was collected as part of the broader project there were strict time frames about videotaping which was also communicated to the teachers involved in the project. This meant that for the lesson starter activities not more than twenty minutes was videotaped and for the whole lessons not more than an hour was recorded. This does mean that at times one did not see completed activities on the videotaped lesson. The implication of the way the data was collected meant that for the data to be ‘valid’ I needed to look for aspects that were ‘recurring regularities’ across the lessons that would suggest that they were common features of the teacher’s practice. In addition, the data collected was at different times and on different topics because I wanted to observe the teacher’s pedagogic practice as it occurred without her being prescribed to by my research work. This meant I could not make any compelling statements about my findings as I did not track a specific concept or theme over a long period of time for me to do so. Whilst a focus on number concepts provided a way of linking across the lessons, a more rigorous design could involve a more specific focus on a single concept within number, in order to make specialisations of content and representations more visible.

5.5 What future interventions should encompass

After completing my research and through observation on how the teacher engaged and chose to work with the content, representations and her selection choice of tasks or examples to teach the mathematical ideas, the following needs emanated which would improve teaching practice if addressed. Future interventions for teachers should encompass:

- The teaching of the structure of various teaching aids and how to use them more abstractly with the corresponding advanced strategy.
- How to design tasks or select examples that will elicit the use of a more advanced strategy and representation.
- Even though the reflective interview revealed that the teacher had an awareness of what is needed to progress from concrete to more abstract ways of calculating, a more sustained long term intervention is needed in addition to the periodic workshops on offer for teachers. A weekly contact session in which lesson plans, teacher strategies and learner activities could be discussed and improved upon. Tasks or worksheets used need to be screened by experts in the field to ascertain

and approve the cognitive demand of the task, otherwise the projected improvement would be minimised. Interventions should enable and equip teachers to develop quality materials, provided ample practice with more advanced strategies and representations so that they are confident to implement them in the mathematics classroom.

5.6 Future research possibilities

The findings of my research may be used to inform firstly teachers about the relevance and impact of their choices and selections with regard to early number learning. Secondly, it could create more research opportunities in primary mathematics education which can be used by both curriculum developers and future researchers.

Since literature advocates that learners who are taught in their mother tongue would communicate more conceptually, this was not the case in my research work. It would be informative to know if it is more likely secondary learners than FP learners who possess a less developed mathematical knowledge base.

Finally, the interview data suggests a broader awareness of the limitations of less specialised content and less specialised representations. A point of further interest for study would be to see what this teacher is able to put into place going forward in FP. Furthermore, to track a unit of work like addition over a sustained period of time as opposed to a variety of topics over a short period of time will yield more substantial empirical evidence of both specialisations and connections within and across lessons.

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7 Appendix

Appendix 1: Information letter to teacher

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UNIVERSITY OF THE WITWATERSRAND JOHANNESBURG

DATE: 07-11-2012

Dear Teacher

My name is Michele Alexander and I am a student in the School of Education at the University of the Witwatersrand.

I am doing research in Primary Mathematics Education focused on understanding pedagogic shifts from concrete to abstract conceptions of number.

My research involves exploring developments in teaching of number concepts by a Foundation Phase Teacher across the Grade 2/3 cohort.

My research focuses on a teacher's teaching of number work and what informs her selection of teaching material within Grade2/3 mathematics in your school. The specific activities will be: observations of a teacher teaching number work with respect to her Lesson Starter Activities and semi -structured post observation interviews related to the use of selected mathematical representations to obtain clarification on observed data as well as teacher's talk. In these activities, I will be observing and interviewing the teacher in grade 3. In my study, I will be focusing on any shifts in your practices with regard to you using multiple representations/strategies and the use of abstract ways of calculating. My study extends over a 6 months period as learners are taught mental calculations on a daily basis. The teacher will be working on her own in her particular grade. I propose to meet you at pre-arranged times to clarify and reflect on your Lesson Starter Activities taught and to ascertain what you found beneficial in the workshops. The data collecting procedures will include video recorded lesson observations of your Lesson Starter Activities and your interviews will be video recorded as well.

The reason for choosing your school is because the school as well as the Foundation Phase teachers are involved in a broader WMC-Primary development project and are exposed to Lesson Starter Activities which in turn would provide me with valuable information regarding the topic under research. Therefore, I was wondering whether you would mind if I could use your class as a research site and whether you would be willing to be part of my research study.

I am aware of the ethics in human participation research and I have put in place a number of provisions to safeguard both yourself and the learners in this project. For example, I don't intend to disrupt any school activity. Lessons will be observed at the time and in the way they are conducted in the normal school schedule. Teacher interviews will be conducted after school on the day of the observation of the lesson or mutually agreed upon times.

Your name and identity will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

You will not be advantaged or disadvantaged in any way. Your participation is voluntary, so you can withdraw your permission at any time during this project without any penalty. There are no foreseeable risks in participating and you will not be paid for this study.
Please let me know if you require any further information.
Thank you very much for your help.

Yours sincerely,

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SIGNATURE

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