AN EQUIVALENT CIRCUIT MODEL FOR A THREE PHASE HARMONIC MITIGATING TRANSFORMER

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DECLARATION

I declare that this dissertation is my own unaided work. It is being submitted for the Degree of Master of Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

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ABSTRACT

A harmonic mitigating transformer is capable of preventing the propagation of a triplen harmonic current from the secondary side of the transformer to the primary side. The transformer is able to achieve this via its zig-zag connected secondary windings. This investigation developed an equivalent circuit model for a harmonic mitigating transformer. The development of the model was based on the premise that a transformer can be modelled as a network of mutually coupled inductors. The equivalent circuit model was used to evaluate the transformer under ideal and non-ideal coupling conditions. The evaluation revealed that the non-ideal model is capable of mitigating more that 99% of the third harmonic current. This result showed that the non-ideal couplings within a harmonic mitigating transformer do not significantly affect its ability to mitigate the third harmonic. Consequently when seeking to optimise the harmonic mitigating transformer the area of focus should be on the couplings between windings on the same core limb rather than on the couplings between windings on adjacent core limbs.

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CHAPTER 1: INTRODUCTION

1.1 Introduction

One of the main causes for the proliferation of harmonics in a power system is the numerous loads connected to the system that draw a non-linear current waveform from the supply. Excessive harmonics circulating within the power system not only affect the supply but also other loads connected to the system. The effect of harmonics on a load is dependent on the type of load connected to the system. For example many electronic devices such as personal computers and programmable logic controllers depend on a constant sinusoidal input at the fundamental frequency in order to ensure the integrity of the data processing and handling [1]. The proliferation of harmonics in such loads causes the equipment to malfunction resulting in erroneous outputs which could have serious consequences. The effect of harmonics on motors causes oscillations within the motor that result in excessive heating which leads to an overall decrease in the motor efficiency. The point of the matter is that the proliferation of excessive harmonics in a power system is undesirable. The problem though is that it is not often a simple task to remove the harmonics once they proliferate within the system. One can argue that the loads generating the harmonics must be disconnected from the system in order to remove the harmonics. This however is unlikely to happen because the majority of the loads generating the harmonics are required in everyday life. For example personal computers, mobile telephone battery re-charges and network equipment all use switched mode power supplies to provide them with Direct Current (DC). Such power supplies draw non-sinusoidal current waveforms that invariably cause harmonics to proliferate. This leads to the statement once again that it is not a simple task to remove the harmonics.

One method of dealing with the harmonics is to filter them out of the power system. The term filter in this case means that the non-linear load will be prevented from drawing a non-sinusoidal current from the supply. This means that the filter should only allow the fundamental frequency to be drawn by the load. This can be done since a harmonic current is simply a sinusoidal current at a frequency that is an integer multiple of the fundamental. This means that passive or active filters can be designed to remove specific harmonics. The design and operation of such filters will therefore be determined by the nature of the harmonic present in the power system. One novel method of filtering out the harmonics is to use a harmonic mitigating transformer. The use of a transformer in this manner may seem foreign however consider the fact that a transformer is capable of providing electrical isolation and phase shifting between the supply and load. Naturally the harmonic mitigating transformer would have to be a three phase transformer in order to achieve the phase shifting between the currents supplying the transformer and the currents supplying the load. By exploiting the fact that harmonic currents are sinusoidal the transformer, via phase shifting, can force the currents to cancel out hence preventing them from propagating from the load side to the supply side. A transformer capable of mitigating harmonics would be a desirable solution to the problem of harmonics since the transformer will not only remove harmonics but it will still perform as a typical transformer. This means that in a new installation where there are expected to be numerous non-linear loads, a harmonic mitigating transformer can perform the task of two devices, namely a transformer and a harmonic filter.

The first step to understanding how a harmonic mitigating transformer operates would be to consult its equivalent circuit model. Upon performing this task it was discovered that there is minimal literature available on such a model. Generally speaking there are equivalent circuit models available for single phase and general three phase transformers. By using such models as a starting point, a more relevant circuit model for a harmonic mitigating transformer can be developed. The reason for requiring an equivalent circuit model of a harmonic mitigating transformer is twofold. Firstly if such a transformer were to be installed in a typical power system, an accurate model of it is required in order to understand how it will perform in relation to the other equipment connected to the system. Secondly if the performance of the transformer were to be improved, a model of it is required in order to determine which parameters affect its performance. The aim of this investigation is to use existing methodologies for transformer modelling and an understanding of the nature of harmonics to develop a concise equivalent circuit model of a harmonic mitigating transformer. The model can then be used to predict the transformers behaviour it terms of its ability to mitigate harmonics and provide a means for improving the transformer's performance.

1.2 Problem statement

An equivalent circuit model for a three phase harmonic mitigating transformer will be developed using coupled inductor circuit theory. This investigation involves the development of the equivalent circuit model and the implementation of the model from a theoretical and practical point of view. This is done in order to perform a comparison between the ideal model and practical model. From this comparison, the effects of the coupling on the transformer's ability to mitigate harmonics are established. The reason for developing such a model is because it was found that there is limited information on modelling this particular type of transformer. The development of such a model will prove to be beneficial in terms of understanding the transformer and its operation. This will allow for the transformer to potentially be used in a wide range of applications.

1.3 Formal definition of harmonics

In order to understand the problem statement, it is first necessary to understand harmonics and their influence on a power system. The sections that follow within Chapter 1 explain in detail the nature of harmonics and how they are generated within a system. The chapter covers much theory on harmonics and presents methods for identifying, quantifying and mitigating harmonics. The subsequent sections of Chapter 1 then focus on harmonic mitigating transformers and their applications. Finally a discussion on the need for a three phase equivalent circuit model for a harmonic mitigating transformer is presented. This is done in order to relate the content of Chapter 1 back to the problem statement.

In modern power systems, power is transmitted via sinusoidal waveforms at frequencies of 50 Hz. Such a frequency is therefore referred to as the *fundamental frequency*. A sinusoidal waveform that transpires within the power system at a frequency higher than the fundamental frequency will be referred to as a harmonic. The definition of a harmonic is therefore a sinusoidal component of a periodic current or voltage waveform that occurs at a frequency that is an integer multiple of the fundamental frequency [1]. A power system can therefore contain several harmonics each at an integer multiple of the fundamental. Such harmonics are typically classified as either odd or even harmonics meaning that the integer multiple is either odd or even. Furthermore harmonics can also be classified as being triplen. Triplen harmonics occur at integer multiples of 3 for example 3, 6, 9 etc. If the fundamental harmonic is 50 Hz the first triplen harmonic or the third harmonic would occur at a frequency of 150 Hz. Harmonics can also be classified as being *characteristic* or *non-*

characteristic. Characteristic harmonics are typically generated by semiconductor converter equipment operating under normal conditions whereas non-characteristic harmonics can emerge as a result of imbalances within the power system [1].

A common technique used in analysing the behaviour of harmonics is Fourier analysis. The method of analysis involves decomposing a distorted periodic waveform into a series of sinusoidal waveforms each at an integer multiple of the fundamental i.e. a harmonic. Furthermore the analysis allows a relationship to be formed whereby a time domain waveform can be represented in the frequency domain [2]. This method reveals the frequencies that the harmonics occur at within the system along with their magnitudes' and phases'. Initially only the magnitude of the harmonics is considered because the magnitude provides an indication as to whether the harmonics are of concern or not. If the harmonic presence is high then the phase of the harmonics is considered in order to determine the nature of the harmonics and how they propagate through a system. The coefficients of the Fourier Series provide information on the nature of the waveform. For example the Fourier coefficients will reveal whether a waveform exhibits odd or even symmetry. The coefficients will also ascertain as to whether the waveform exhibits half-wave symmetry or not. The symmetry of a waveform plays an important role in determining the type of harmonics that will be present within a power system. It follows that waveforms that exhibit half-wave symmetry only contain odd harmonics [2]. For this study only systems that contain waveforms exhibiting odd or even half-wave symmetry will be considered as in the case of a square wave. Therefore this study is only concerned with harmonics occurring at odd integer multiples including the odd triplen harmonics. In the case where an even harmonic transpires, it has been noted and discussed within context.

1.4 The proliferation of harmonics

As mentioned in the previous section, a distorted periodic waveform can be decomposed using Fourier analysis into a series of sinusoidal waveforms at various frequencies. Therefore it can be said that any distorted signal or waveform within a power system can be represented as a sum of sinusoidal waveforms at varying frequencies [2]. The question arises as to what causes a signal or waveform to become distorted. Alternatively, what causes the generation of the different sinusoidal waveforms? It is a well-documented fact that the proliferation of harmonics within a power system is a result of the introduction of non-linear loads into the system [2] [3] [4] [5] [6]. A non-linear load is a load that draws a current that is not proportional to the instantaneous voltage [4]. Non-linear loads are typically used in the conversion of AC power into DC power. Examples of such loads include Switched Mode Power Supplies (SMPS) and Adjustable Speed Drives (ASDs). SMPS and ASDs contain semiconductor devices that perform the rectification of an AC waveform into DC. Such devices perform the rectification by drawing a current from the source in short pulses [4]. The current pulses are then combined and with the aid of a smoothing capacitor, constant DC is produced. The effectiveness of the rectification is determined by the percentage of ripple current found in the DC component. Ideally the better the rectification of the rectifier, the less ripple current there will be.

The means of power transmission nowadays is primarily by three phase transmission. A typical power system would comprise of three conductors each transmitting the power for one phase and in some cases a neutral conductor is present. The neutral is used to provide a return path for the current in each phase. There are two basic load profiles that can be present in a three phase power system. The first profile is one whereby the loads are phase-to-neutral loads and the second profile is one whereby the loads are phase-to-phase loads or three-phase loads [7]. When considering non-linear devices attached to a system, it is important to establish which load profile they fall under. The reason is because the proliferation of certain harmonics is dependent on the type of connection method of the load.

1.4.1 Non-linear phase to neutral loads

An example of a non-linear phase to neutral load would be a single-phase full-wave bridge rectifier. A single-phase rectifier is made up of four semiconductor diodes and is used to convert alternating current into direct current. Such a rectifier typically forms the primary conversion stage in a SMPS used in modern electronics such as personal computers [8]. For this discussion, three-single phase full wave-bridge rectifiers along with a smoothing capacitor and arbitrary resistor load have been used to represent a typical phase-to-neutral non-linear load. For simulation purposes it is assumed that the rectifiers are ideal meaning that there is zero ripple in the output current [8]. The circuit diagram in Figure 1.1 shows the three full-wave bridge rectifiers connected between a phase and the neutral of the supply line (the sources in this case are balanced so the neutral will be at zero potential hence it is connected to ground). The supply to each rectifier is an AC voltage source each with a magnitude of 230 V and a frequency of 50 Hz. The sources are phase shifted by 120° from each other.

Figure 1.2 shows a graphical representation of the harmonics and their magnitudes present in the circuit. The plot in Figure 1.2 was obtained by taking the Fast Fourier Transform of the line current in each of the phases of the circuit as well as the neutral. From Figure 1.2 it can be seen that the line current for each phase has a fundamental harmonic that has a magnitude of approximately 4.2 A whereas the third harmonic (150 Hz) has a magnitude of approximately 3.8 A. There is also a noticeable presence of the fifth (250 Hz) and seventh harmonic (350 Hz). From Figure 1.2 it can be seen that the current in the neutral has a significant third harmonic component. In fact the third harmonic magnitude is approximately three times that of the line current in each phase. The ninth harmonic is also quite prominent. The plot in Figure 1.3 presents the phase angle of the line currents in each phase. From the plot it can be seen that the third harmonic currents are in phase. The phase angle however is not important at this stage because it is the absolute value of the third harmonic that is of concern. The ninth harmonic in each line are also in phase. It can also be seen that the fundamental, fifth and seventh harmonic currents are 120° out of phase.

The circuit in Figure 1.1 represents a basic example of the layout of a distribution system for single phase load as may be found in a commercial or residential building [5]. From Figure 1.2 it can clearly be seen that even in the simplified ideal case, there is a significant harmonic presence particularly the third harmonic for this type of non-linear load. It can also be seen from Figure 1.3 that the triplen harmonics in each line are in phase whereas the non-triplen harmonics are 120° out of phase. This explains the significant presence of the triplen harmonics in the neutral conductor. In other words because the triplen harmonics are in phase, they add up in the neutral as oppose to cancelling like the non-triplen harmonics. Therefore the presence of the neutral conductor provides a path for the triplen harmonics to flow. If more single phase non-linear loads are added to the system there will be an increase in the harmonics circulating within the system. In particular there will be an increase in the triplen harmonics within the neutral conductor of the system. This is one of the main concerns when introducing phase-to-neutral non-linear loads to a power system.

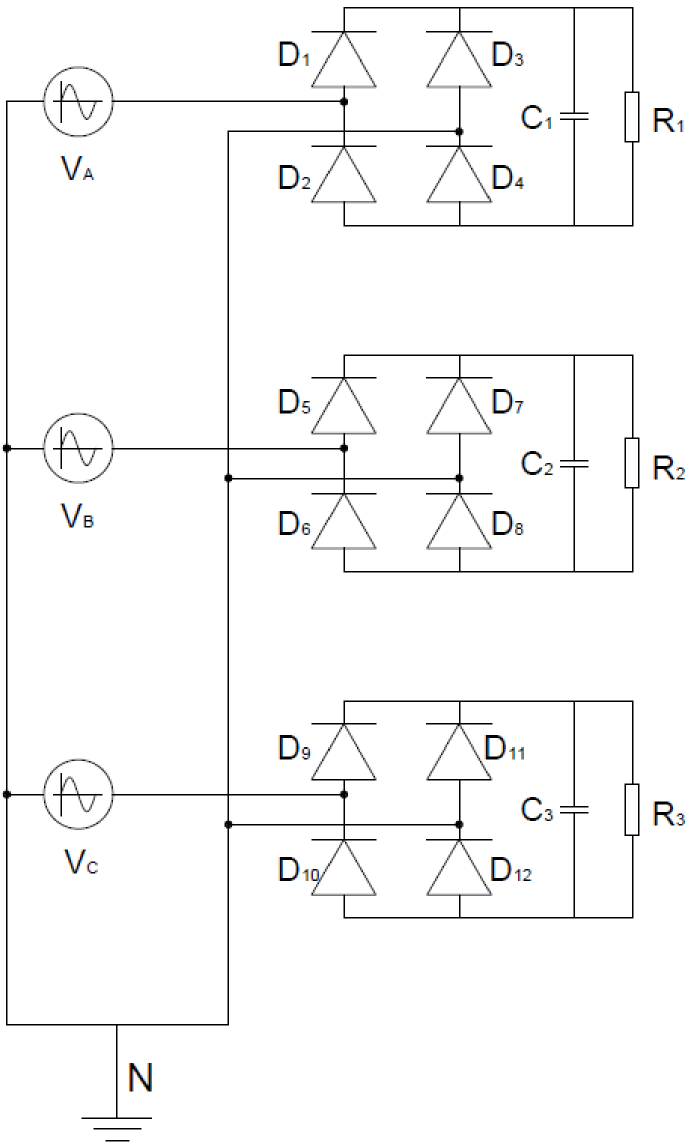


Figure 1.1: Example of a three phase system with a single phase rectifier load on each phase. Note that $R_1 = R_2 = R_3$.

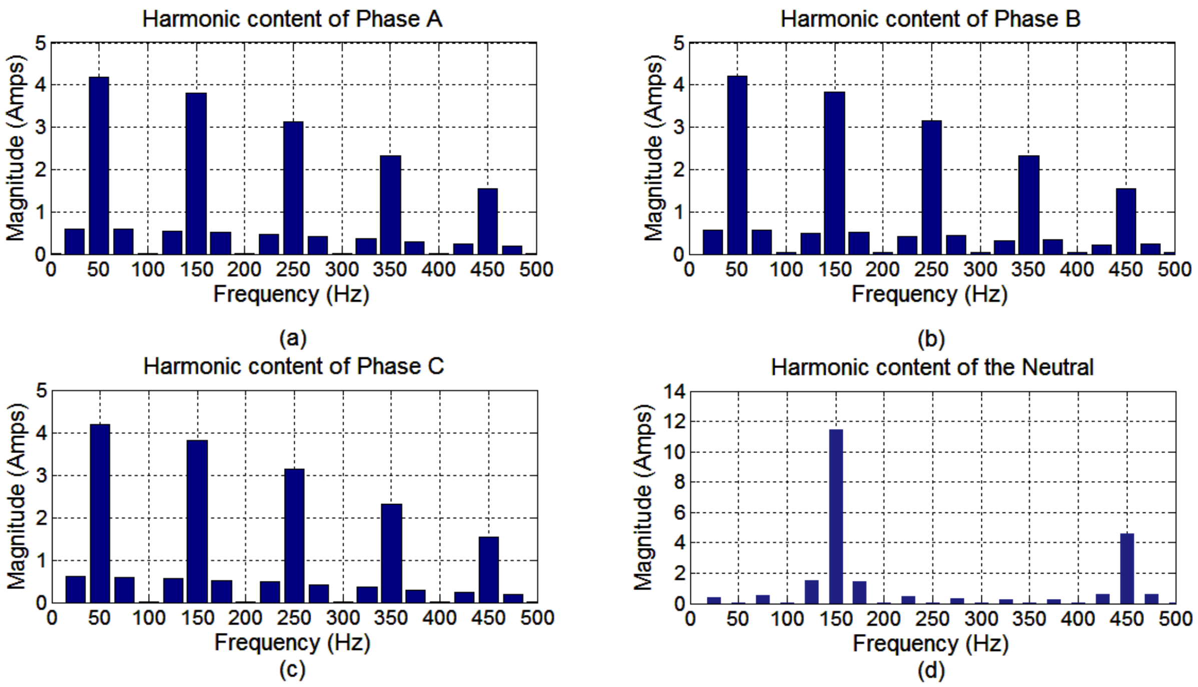


Figure 1.2: (a) (b) (c) Harmonic content for the line current in each of the phases and (d) the neutral of the circuit in Figure 1.1. It can be seen that there is a broad spectrum of harmonics in each phase. The third harmonic at 150 Hz can be seen to have a substantially high magnitude in the neutral conductor of the circuit.

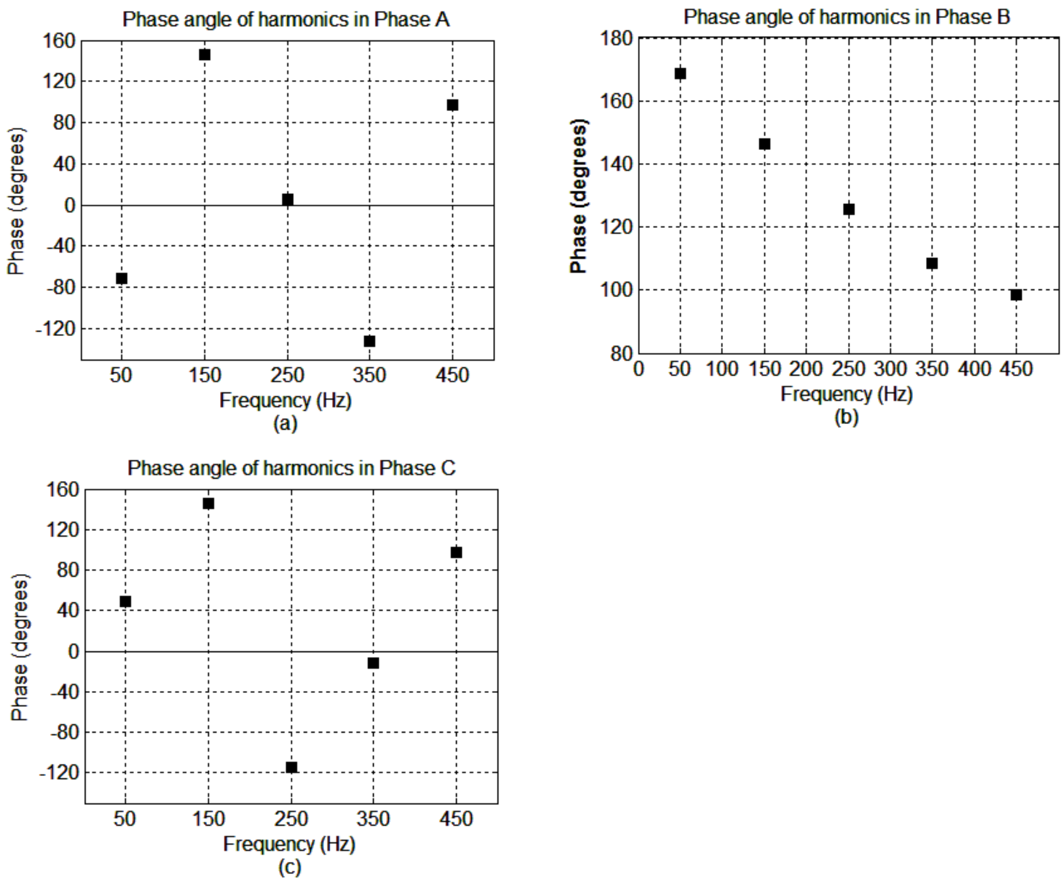


Figure 1.3: (a) (b) (c) Phase angles of the currents for the phase-to-neutral loads at the frequencies 50, 150, 250, 350 and 450 Hz. It can be seen that the triplen harmonics are in phase whereas the other harmonics are 120° out of phase.

1.4.2 Non-linear three phase and phase to phase loads

An example of a three-phase non-linear load would be a six-pulse full-wave bridge rectifier. Such a rectifier consists of six semiconductor diodes connected so that the three phase alternating current is converted into a direct current. The rectifier is typically the initial power conversion stage in AC and DC motor ASDs. A basic three-phase six-pulse rectifier is used for the discussion in this section. It will be assumed that the source is balanced so this means that the rectifier will, under normal operating conditions, generate the characteristic harmonics i.e. harmonics caused by the switching of the semiconductor diodes [1]. Also as with the single phase rectifiers, for simulation purposes, it is assumed that the rectifier is ideal. Figure 1.4 shows the connection of the six-pulse rectifier to the three phase system. The supply to the rectifier comes from three 50 Hz AC voltage sources each with a magnitude of 230 V and phase shifted by 120° from each other. Figure 1.5 shows a graphical representation of the harmonics present in each of the phases of the circuit. The plot in Figure 1.5 was obtained by taking the Fast Fourier Transform of the line currents in each of the phases of the circuit. From Figure 1.5 it can be seen that there is no third harmonic. There is however a noticeable presence of the fifth and seventh harmonic. The plot in Figure 1.6 presents the phase angles of the line currents for each phase. From the plots it can be seen that each harmonic is 120° out of phase.

A comparison of the plots in Figure 1.2 and Figure 1.5 reveals that a non-linear phase-to-neutral load is capable of generating triplen harmonics whereas a non-linear three-phase load is not. The reason for this can be attributed to the fact that in the former case there is a neutral conductor present which provides a path forth triplen harmonics. It can be seen however that both loads generate the non-triplen odd harmonics particularly the fifth and seventh.

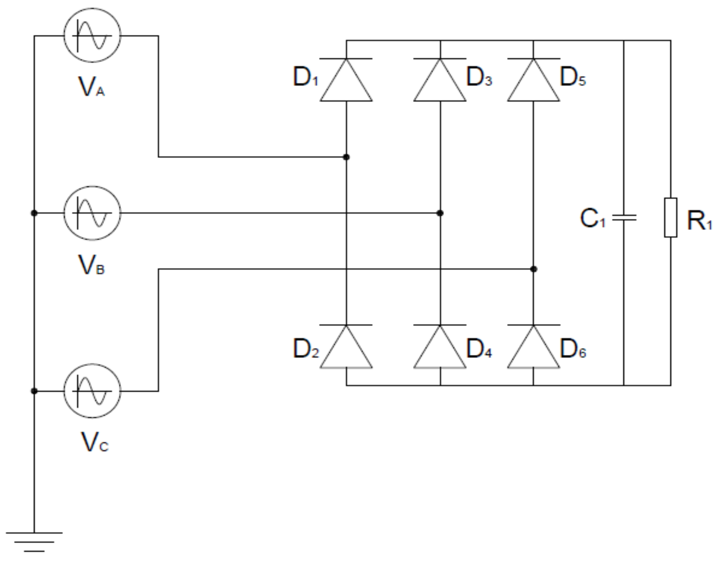


Figure 1.4: Example of a three phase system with a three phase full wave six pulse bridge rectifier with a resistive load R_1.

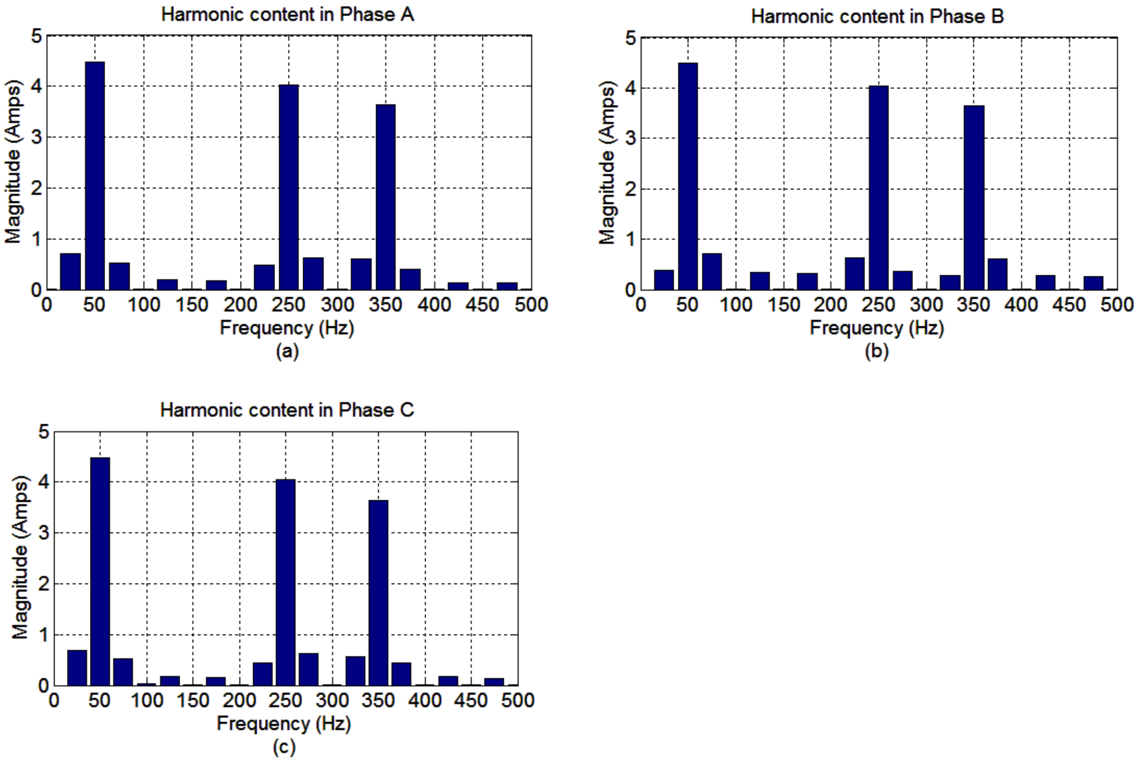


Figure 1.5: (a) (b) (c) The harmonic content for the line current in each of the phases for the circuit in Figure 1.4. The third harmonic is not present however the fifth (250 Hz) and seventh (350 Hz) harmonics are prominent

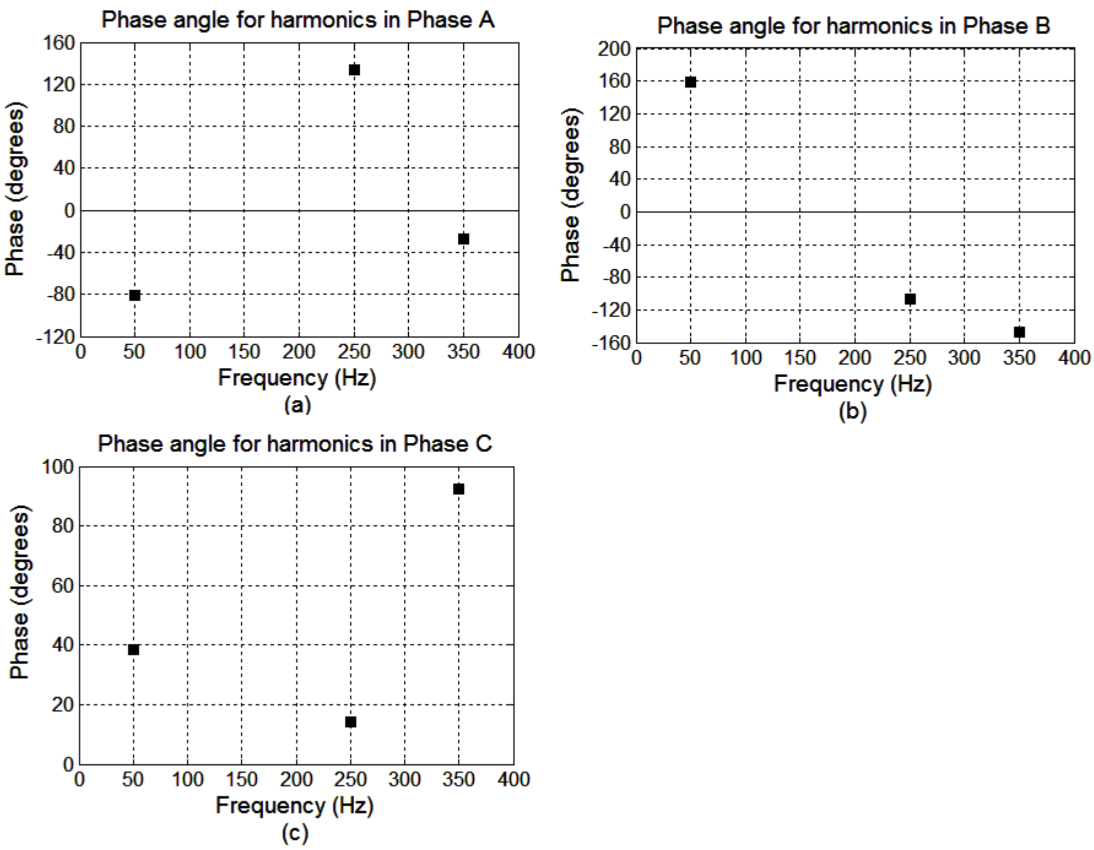


Figure 1.6: (a) (b) (c) Phase angles of the currents for the three phase load at the frequencies 50, 250, and 350 Hz. It can be seen the harmonics are 120° out of phase

From the discussions and simulations above it is clear that non-linear loads, whether phase-to-neutral, three-phase or phase-to-phase, generate harmonics, in particular harmonic currents. This is because, as mentioned, a non-linear load draws a current that is not proportional to the instantaneous voltage [4]. The proliferation of harmonic voltages within a system occurs when harmonic currents flow through the impedance of the various components connected within a system such as transformers. The harmonic currents cause voltage drops across the impedances within the system. Such voltage drops in turn distort the source voltage waveform. The distorted voltage waveform, much like the distorted current waveform, will be made up of the fundamental frequency waveform and waveforms at the different harmonic frequencies. When considering the effects of harmonics, the focus is typically on the harmonic currents because if one can mitigate the harmonic currents, the harmonic voltages will in turn also be mitigated.

1.4.3 Other methods of harmonic generation

Proliferation of harmonics due to unbalanced supply

If one considers the circuit in Figure 1.4 of the non-linear three phase load, from the plots in Figure 1.5 it can be seen that it produces the odd non-triplen harmonics. These harmonics are therefore the characteristic harmonics. If a triplen harmonic were to proliferate within the system, the triplen harmonic in this case would be considered a non-characteristic harmonic. Non-characteristic harmonics transpire within a system as a result of variations in the supply voltage. The slightest imbalance will cause the ASD rectifier to generate non-characteristic harmonics [9]. Non-characteristic harmonics in this case occur not because of the non-linearity of the rectifier but rather because of the interaction of the unbalanced source waveforms with the rectifier [1]. Figure 1.7 presents a plot of the harmonic content for the line currents in the circuit in Figure 1.4. The supply voltages for each phase were intentionally unbalanced to simulate an unbalanced supply. Phase A remained at 230 Volts whereas Phase B was increased by 5% and Phase C was decreased by 5%. From the plots it can be seen that there is a significant presence of the triplen harmonics in Phase A and Phase C. It can therefore be seen that the type of harmonic generated within a power system is typically dependent on the type of load connected to the system and the stability of the voltage source of the system.

Proliferation of harmonics due to the excitation current waveform of a transformer

Consider the fact that in an iron core transformer, the actual core has to be magnetised in order to establish the flux within the core. The current required to do this is known as the magnetising current. Furthermore due to the reluctance of the core, a small current known as the core loss current is produced. The excitation current therefore is the sum of the magnetising current and core loss current. The excitation current arises when a voltage is applied across the terminals on the primary side of the transformer. The excitation current is dependent on the reluctance of the iron core which in turn is dependent on the magnetisation properties of the iron core. The magnetisation characteristics of the core are determined by the B-H magnetisation curve of the core material which is non-linear in nature [10]. The non-linear characteristics of the core therefore cause the excitation current to be non-sinusoidal especially when the core is driven into saturation and as discussed in the preceding sections, a non-sinusoidal current contains harmonics. In a three phase transformer the non-sinusoidal excitation current may become significant particularly when the transformer is connected in either star or delta [11] because as mentioned, certain harmonics can sum together resulting in large neutral currents or certain harmonics can circulate within the windings.

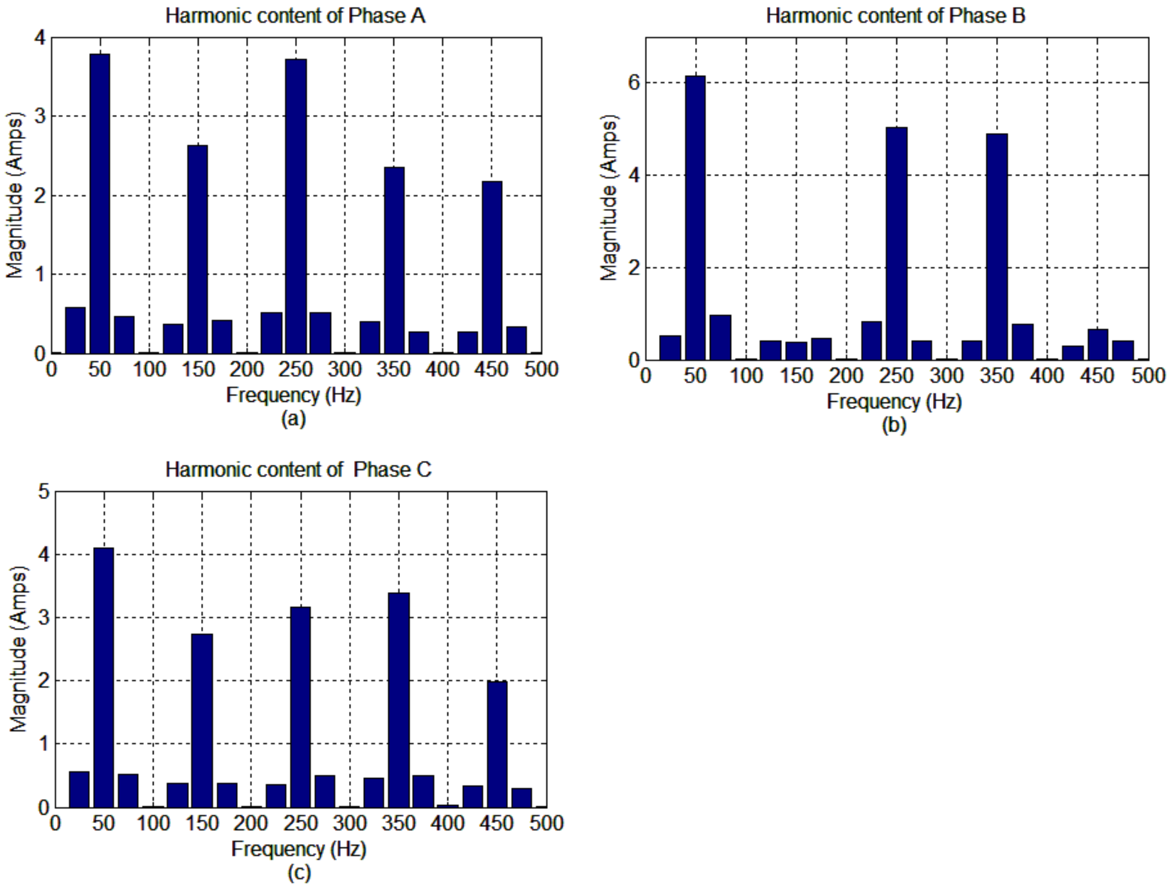


Figure 1.7: The harmonic content for the line current in each of the phases of the circuit in Figure 1.4 with the supply voltage unbalanced. It can be seen that in (a) and (c) the triplen harmonic at 150 Hz and 450 Hz has proliferated whereas in (b) the triplen harmonics are negligible.

1.5 Harmonics: Positive, negative and zero sequence

In a three phase system with a balanced linear three phase load, the three voltage phasors will be equal in magnitude and positioned 120 electrical degrees apart. This will apply to the three current phasors as well. The sum of the three voltage phasors will be zero as will the sum of the three current phasors. This means that the voltage and current phasors are balanced. In this situation the system is relatively simple to analyse because the system can be represented by an equivalent single-phase system. The problem arises when the three phase load is not balanced. An unbalanced three phase load will draw a current such that three current phasors will become unbalanced. If the current phasors are unbalanced, the system cannot be represented by an equivalent single-phase system. The analysis of the unbalanced system is therefore complex and difficult to solve. In order to deal with unbalanced systems, a technique of Symmetrical Components as discussed in [12] was developed. The theorem states that three unbalanced phasors of a three phase system can be resolved into three balanced systems of phasors [12]. The balanced sets of phasors are known as the symmetrical components. The symmetrical components are either positive sequence, negative sequence or zero sequence. Positive sequence components comprise three balanced phasors spaced 120 electrical degrees apart and having the same rotation as the original unbalanced phasors. Negative sequence components also comprise three balanced phasors spaced 120 electrical degrees apart however the rotation is opposite to the original phasors. Zero sequence which consists of three phasors equal in magnitude with zero rotation between them. It therefore stands to reason that in a balanced system, there will be no negative or zero sequence components implying that all voltages and currents will be positive sequence [12]. The method of symmetrical components therefore provides a means for identifying the nature of the unbalance in terms of its phase sequence. Knowledge of the sequence will provide insight into the effect of the imbalance on the power system. For example, if the current in an unbalanced system contains negative sequence symmetrical components then it will cause a counter rotating field in the stator of a motor attached to the system therefore causing the motor to experience a braking force.

The reason for mentioning the method of symmetrical components is because the technique can be used to determine the sequence of a harmonic. Recall that a non-linear load present in a three phase power system will cause a non-sinusoidal current to be drawn from the supply. The non-sinusoidal waveform drawn essentially causes the system to become unbalanced at certain instances in time. By using Fourier analysis, the individual sinusoidal harmonic waveforms can be obtained and then by using the method of symmetrical components the sequence of each waveform can be determined. The first step in achieving this is to obtain the phasor form of each harmonic signal. It was shown in the previous section that by taking the Fourier Transform of a non-linear waveform, the magnitude and phase angle of each harmonic making up the waveform can be obtained. Recall that the magnitude of the harmonics making up the non-linear waveform in each phase is presented in Figure 1.2 and Figure 1.5 and the phase angle of each harmonic is presented in Figure 1.3 and Figure 1.6. Using this information, the phasor form of each harmonic making up a waveform can be obtained and an individual harmonic can then be studied in isolation. Using this method, the magnitude and phase angle of the third harmonic current present in each phase of the circuit in Figure 1.1 was determined. Once the information about the third harmonic current in each phase was acquired, the symmetrical components were obtained.

Figure 1.8 presents a plot of the symmetrical components of the third harmonic. It can be seen that the magnitude of the zero sequence current is substantially higher than the magnitude of the positive and negative sequence current. Furthermore the phase angle of each third harmonic is the same, this may not be apparent from the plot since the phasors are overlapping. By considering the magnitude of the neutral current one can see that the third harmonic current is almost three times the magnitude of the fundamental current in each phase. This is because the zero sequence currents in each conductor are in phase therefore they tend to add up in the neutral rather than cancel out as is the case with the positive and negative sequence currents. If one considers the rest of the triplen harmonics, they too will exhibit similar characteristics leading to the conclusion that the triplen harmonics have a dominant zero sequence. A similar analysis was performed on the fifth and seventh harmonic currents present in the circuit of Figure 1.1. The plot in Figure 1.9 reveals that the fifth harmonic current has a dominant negative sequence whereas the plot in Figure 1.10 reveals that the seventh harmonic current has a dominant positive sequence.

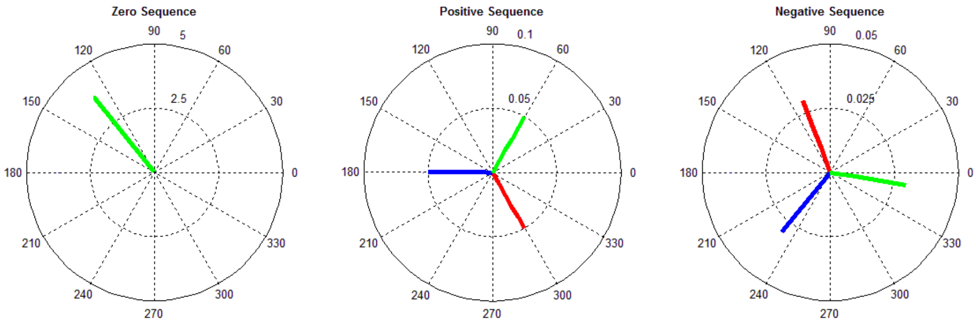


Figure 1.8: Symmetrical components of the third harmonic in a system with non-linear phase to neutral loads. It can be seen that the zero sequence current magnitude is substantially higher than the positive and negative sequence current magnitude. This indicates that third harmonic is predominantly zero sequence.

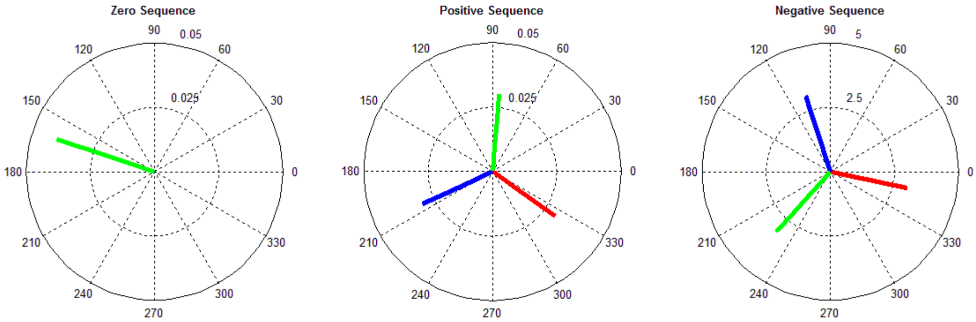


Figure 1.9: Symmetrical components of the fifth harmonic in a system with non-linear phase to neutral loads. It can be seen that the fifth harmonic is associated predominantly with the negative sequence.

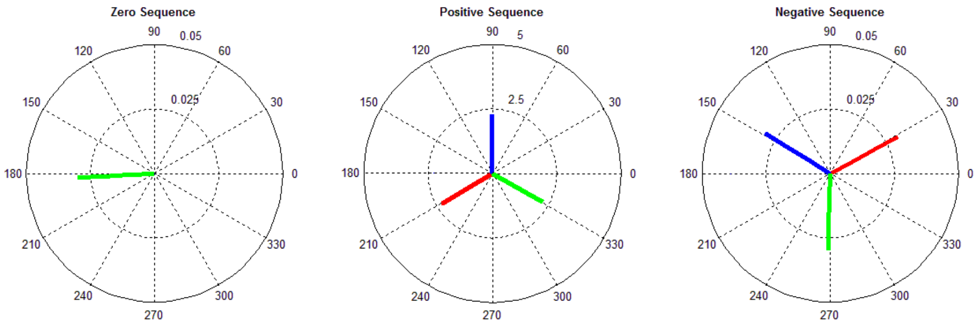


Figure 1.10: Symmetrical components of the seventh harmonic in a system with non-linear phase to neutral loads. It can be seen that the seventh harmonic is associated predominantly with the positive sequence.

Table 1.1: Harmonic number and associated phase sequence, where n = 1, 3, 5…

Harmonic Number	Sequence
1 (Fundamental)	+
3	*0*
5	-
7	+
9	*0*
11	-
13	+
k = 3n	*0*
k = 6n-1	-
k = 6n+1	+

1.6 The adverse effects of harmonics

The sequence of a harmonic provides an understanding as to how the harmonic propagates through a system. Positive sequence harmonics have the same phase rotation as the fundamental frequency however, they affect the power factor of the system resulting in large currents being drawn from the source in order to compensate for the reduction in real power [13]. Negative sequence harmonics have a phase rotation opposite to the fundamental frequency. In rotating machines such as electric motors, the negative sequence current will oppose the rotation of the rotor resulting in the motor overheating and increasing the vibrations within the motor. Zero sequence harmonics have no phase associated with them however because of this, the zero sequence harmonics from each phase will sum up in the neutral line of a power system. This will result in the neutral being subjected to currents almost three times that of the individual line and phase currents [7]. This was seen in the plot of the neutral current in Figure 1.2. The presence of the zero sequence currents invariably overloads the neutral conductor which eventually results in its failure. Zero sequence harmonics are also known to circulate in the delta windings of a transformer because of the absence of a neutral path for them to flow in. This inevitably causes additional heating within the transformer which leads to an increase in the transformer losses.

Transformers play a vital role in power transmission networks therefore the adverse effects of harmonics on transformers are of particular concern. It is a well-documented fact that harmonics propagating through a transformer increases the transformer losses [1] [2] [4] [6] [14]. Transformer losses can be classified as being either load losses or no-load losses [14]. Load losses are further divided into the I^2R losses due to the winding resistance and the stray losses. According to the IEEE Std. 519-1992, stray losses are caused by stray electromagnetic flux in the windings, core clamps and other structural parts of the transformer [1]. No-load losses are the core losses or iron losses which comprise hysteresis losses and eddy current losses within the magnetic core. Harmonics present within a system affect the load losses and no-load losses of a transformer. In particular, current harmonics cause an increase in the load losses and voltage harmonics cause an increase in the no-load losses [1]. According to IEEE Std. 519-1992 the losses within a transformer caused by the current and voltage harmonics are frequency dependent implying that the higher the harmonic frequency, the higher the losses [1]. Essentially an increase in the transformer losses results in additional heating within the transformer. In cases where the heating exceeds predetermined design levels, the effect could result in insulation failure which in turn could lead to overall transformer failure. In essence the lower the transformer losses, the better the transformer will operate and therefore it is necessary to understand how harmonics affect a transformer. Standards such as the IEEE Recommended Practise for Establishing Liquid-Filled and Dry-Type Power and Distribution Transformer Capability When Supplying Non-Sinusoidal Load Currents and the IEEE Recommended Practises and Requirements for Harmonic Control in Electrical Power Systems have been developed in order to provide standardised methods of quantifying harmonics in order to allow designers to compensate for the harmonics that may proliferate in a system. The standards also present guidelines for the permissible levels for the presence of harmonic currents and voltages in a power system.

The majority of power systems nowadays contain numerous loads of varying capacity, age and construction. In such systems there is also a variety of protection and transmission medium. The presence of harmonics in such a power system will have an effect on each of the components within the system. The reason that most systems are not designed to handle the harmonics. For example the designers of a building wiring system that was installed several years ago would not have been able to predict the amount of SMPS that would be connected to the system. SMPS in this case are used in personal computers, printers, cellular telephone battery chargers, fax machines etc. Such a system would therefore be subjected to large triplen harmonics which in turn could result in the failure of the distribution transformer or burn out of the neutral conductor. The point is that the majority of power systems are not able to cope with the presence of harmonics. The harmonics generated by the non-linear loads propagate through the system affecting the components attached to the system. In some situations, the power system cannot keep up with the demand for power and the system inevitably becomes overloaded. The proliferation of harmonics in a system that is already operating at its maximum capacity will have dire consequences due to the reasons stated in the two preceding paragraphs. It is therefore necessary to prevent the generation of harmonics by designing devices that meet IEEE standards. This may not be possible so it may therefore be necessary to install devices that mitigate the harmonics.

1.7 Harmonic mitigation

Two common methods used to mitigate harmonics include the use of passive filters and the use of active filters [3]. When considering the use of active or passive filters, the filters can be placed within the device generating the harmonics i.e. the within the non-linear load or at the supply transformer supplying the power system [5]. Passive filters can be very effective at reducing high frequency harmonics but the performance is dependent on the source impedance which is not always accurately determinable and varies as the system changes [5]. An active filter monitors and tracks the changes in the harmonic current and adjusts the filtering accordingly, therefore providing stable operation against system variations [5]. Active filters however are costly and complex to install especially in established systems. Another method of dealing with harmonic components in terms of transformers is the K-rating method [4]. This method involves designing a transformer that can withstand the effects of the harmonics rather than reduce the effects [4]. In other words, the transformer is rated higher so as to compensate for the losses associated with harmonics. This means that the transformer will be under-utilised and the problem of harmonics will still persist. A not so common method of mitigating harmonics is the use of a harmonic mitigating transformer.

1.8 Harmonic mitigating transformer

A harmonic mitigating transformer is designed to have a low impedance so as to reduce the magnitude of the voltage distortion caused by the harmonic currents. The main aim is to prevent the transfer of harmonic currents from the secondary windings to the primary windings of the transformer. In other words the transformer itself acts like a filter preventing the harmonics generated by a non-linear load from transferring to the supply side. The design of the transformer involves connecting the secondary windings in such a way so as to obtain zero sequence flux cancellation and phase shifting within the secondary windings [1]. Typically a transformer with a zig-zag connected secondary winding would be classed as a harmonic mitigating transformer. In this configuration, the transformer will have one winding per primary phase and two windings per secondary phase (please refer to Figure 2.2 for a diagram of the connection scheme). Typical connection schemes would be star zig-zag or delta-zig-zag. Autotransformers have also been known to be connected in the zig-zag manner. In this case the autotransformer is used as a grounding transformer or is used to create a neutral point in a three wire system. The autotransformer prevents the harmonics generated by a load from entering into the supply. In essence the zig-zag connection is what performs the harmonic mitigation. The transformer can either be connected in parallel with the distribution transformer as in the case with an autotransformer or it can be connected directly between the load and distribution transformer or between the load and the supply as in the case of a star or delta-zig-zag transformer. In this case the transformer is analogous to an isolation transformer.

As with many mitigation techniques however, each has its own pros and cons as does a harmonic mitigating transformer. For example it must be emphasised that the zig-zag connection is only able to remove the zero sequence currents. In other words the zig-zag connection can only remove or mitigate the triplen harmonics. This might not be ideal for situations whereby there is a high presence of fifth and seventh harmonics. In other words, a transformer with a zig-zag connection is best suited for applications where there are phase to neutral loads. It follows then that this study focuses only on systems whereby a harmonic mitigating transformer can be applied. Therefore for simulations and experimental purposes, only phase to neutral loads will be used. Continuing with the notion that a harmonic mitigating transformer may not be the best mitigation method, in the case of a zig-zag autotransformer, it may be necessary to add an additional inductor or filter to aid in situations where the utility voltages are unbalanced [15]. In the case of a star or delta-zig-zag transformer, its mitigation effectiveness is also compromised in situations where there is severe system instability [16]. The main point though is that although the harmonic mitigating transformer may not be ideal for certain applications, the benefit of using a mitigating transformer over conventional methods is sometimes more cost effective and practical [15].

The question must be asked though as to why a model must be developed when harmonic mitigating transformers are already available from numerous transformer manufacturers around the world. The answer is that although many harmonic mitigating transformers are available, there is not much literature on how they are constructed, how they actually perform the mitigation and whether or not the transformer itself suffers from losses due to the harmonics. There is much theory on harmonics, harmonic mitigating techniques and transformers however when it comes to harmonic mitigating transformers, there are many voids in the theory. Several authors present studies on how harmonic mitigating transformers perform and how to model them for specific applications [15] [16] [17] [18], however none of them focus on how the model relates to the physical aspects of the transformer and in particular the coupling between the various windings. In other words although the authors discuss and analyse the harmonic mitigating transformers, the fundamentals of the harmonic mitigating transformer are not clearly presented. In their defence they are merely presenting their findings under the premise that one understands how the harmonic mitigating transformer works. However upon researching harmonic mitigating transformers, one cannot find a clear description from an equivalent circuit model perspective. Several textbooks provide detailed steps for modelling a transformer and designing a transformer [10] [12] [19] [20], however it is agreed that such steps are necessary for the design of any type of transformer but not specifically a harmonic mitigating transformer. Using the knowledge of transformer design and the knowledge of harmonics, this investigation develops a three phase equivalent circuit model for a harmonic mitigating transformer that is complete in a sense that it can be used as a basis for any harmonic mitigating transformer design. The model provides a clear understanding of the electromagnetic nature of a harmonic mitigating transformer from a coupled circuit perspective and from this understanding a practical transformer can be designed. The development of a complete three phase equivalent circuit model for a harmonic mitigating transformer therefore provides information on how best to optimise the transformer for use in a wider range of applications.

1.9 Organisation of the dissertation

This dissertation is divided into six main chapters followed by a list of references and six appendices. Chapter 1 of this dissertation is the introductory chapter and it presents the topic being investigated in this dissertation. The chapter provides information regarding the background to the investigation and the motivation for performing the investigation namely the reason why an equivalent circuit model for a harmonic mitigating transformer is being developed. Chapter 2 presents the transformer modelling techniques that will be used in this investigation. The chapter shows that a mathematical model of a harmonic mitigating transformer can be developed by applying the concepts used to model single phase and conventional three phase transformers. Chapter 3 then presents the procedure for obtaining an ideal coupling model of the harmonic mitigating transformer. Such a model is necessary in order to provide a platform upon which to compare the practical model. Chapter 4 presents a non-ideal coupling model for a physical harmonic mitigating transformer. The model is based on a physical harmonic mitigating transformer that was constructed specifically for this investigation. Chapter 5 then presents a comparison between the ideal coupling model and the non-ideal coupling model of the harmonic mitigating transformer. The comparison is based on each model’s ability to mitigate the third harmonic. Such a comparison is required in order to establish whether or not the transformer couplings in a harmonic mitigating transformer play any role in the mitigation of the third harmonic. Finally chapter 6 presents a conclusion and a discussion on future work. Six appendices containing supplementary information relevant to this investigation are also included.

1.10 Assumptions

For this study numerous assumptions have been made in order to simplify the modelling process. These assumptions are used throughout this study unless otherwise stated. The following assumptions apply:

1. Steady state conditions apply throughout this study implying that any transient affects particularly those associated with the loads are neglected,
2. Only systems that contain waveforms that exhibit odd or even half-wave symmetry are considered,
3. The transformer excitation current is sinusoidal implying that there will be no harmonics present in the excitation current,
4. The losses associated with the transformer core can be neglected implying that the excitation current only has a magnetising component i.e. the core loss current will be zero,
5. Triplen harmonics are all assumed to be zero sequence,
6. All sources are balanced implying that only characteristic harmonics are considered,
7. Only transformers that operate in the low frequency range i.e. 50 Hz to say 1 kHz are considered. This means that capacitive effects can be neglected,
8. Transformers are dry type and not in any enclosures at this stage.

1.11 Conclusion

The focus of this chapter is to introduce the main aim of this investigation and that is to develop an equivalent circuit model for a harmonic mitigating transformer. The first part of the chapter provides a concise discussion on harmonics and their effects on a power system. The discussion highlights the fact that the third harmonic is particularly troublesome in systems with phase to neutral non-linear loads. It would therefore be beneficial to remove or filter out this harmonic in order to prevent it from causing undesirable effects such as over loading the neutral of the power system. One method of doing this would be to connect a harmonic mitigating transformer between the load generating the third harmonic and the supply. The harmonic mitigating transformer will then be able to filter out the third harmonic via phase cancellation among its secondary windings. Using a transformer in this manner is not common place and therefore there is minimal literature on an equivalent circuit model for this transformer. The aim of this investigation is then to develop such a model for use in numerous applications. In order to develop this model several assumptions have been made. Two such assumptions include the assumption that steady state conditions apply and that the third harmonic has a zero sequence associated with it. The sections that follow in this dissertation present the modelling procedure used for developing an equivalent circuit model.

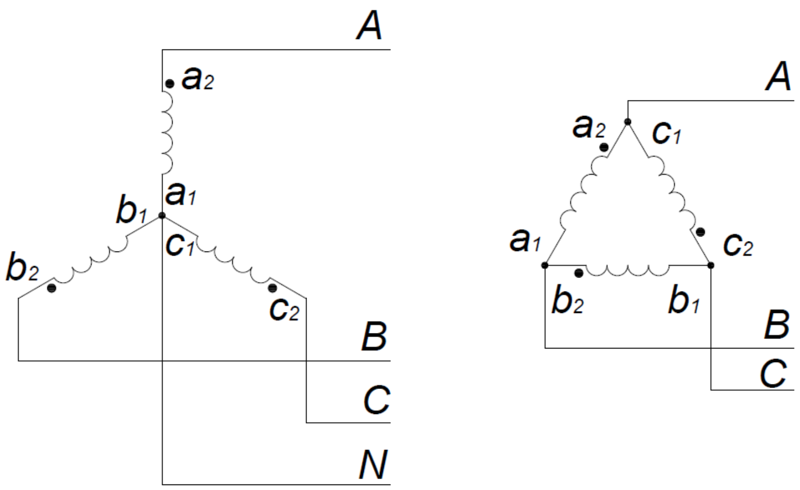
CHAPTER 2: TRANSFORMER MODELLING

2.1 Introduction

The following chapter provides a detailed discussion on the modelling of a three phase transformer. In particular a three phase equivalent circuit model for a harmonic mitigating transformer is developed using coupled circuit theory. At this stage it is important to note that this investigation will specifically focus on the star or delta-zig-zag transformer as opposed to a zig-zag autotransformer. The reason why is because the star or delta-zig-zag transformer can be used to mitigate harmonics and can also be used as a distribution transformer that provides both energy conversion and electrical isolation. The use of such a transformer in a typical building wiring system would be beneficial because one piece of equipment will be able to perform multiple functions. The idea of a transformer being used in such a manner is not uncommon. A delta-star connected transformer can also be used as a means of mitigating the triplen harmonics. In essence, because the delta connection lacks a neutral, the delta connection would 'trap' the third harmonics thereby preventing them from propagating through to the supply side. The transformer would still perform the energy conversion while at the same time stopping the triplen harmonics from entering the supply. The circulating triplen harmonics however would cause a significant increase in transformer losses [16], therefore causing the transformer to heat up which as discussed in Chapter 1 could cause the transformer to fail. The need for a transformer that can eliminate harmonics rather than trap them would therefore be more beneficial, hence this study focuses on developing a model for delta-zig-zag connected transformers. A transformer connected in a delta-star configuration will be used for simple design comparative purposes. The delta-star connected transformer also provides a starting point for modelling the harmonic mitigating transformer which can now referred to as a delta-zig-zag connected transformer

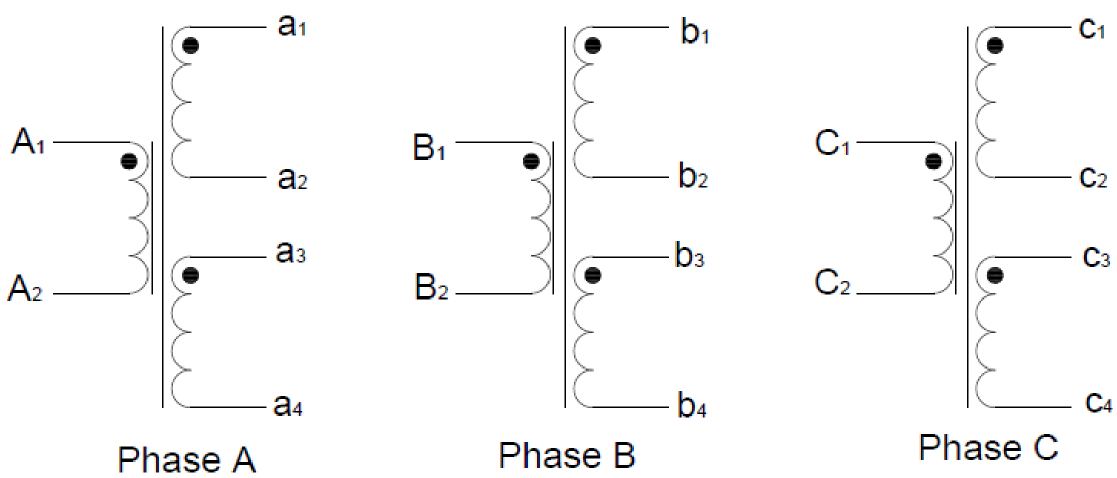
To begin with, Section 2.2 of this chapter looks at modelling a transformer using controlled sources. An ideal model of a delta-star transformer and a delta-zig-zag transformer is developed. From these models, the nature of the triplen harmonics will be demonstrated using basic circuit theory. Section 2.3 focuses on the transformer parameters necessary for developing a practical transformer model. In this section coupled circuit theory is used to show that a single phase two winding transformer can be modelled as two mutually coupled inductors. Using this notion, the single-phase two-winding transformer can be expressed as a matrix equation whereby the voltages are related to the currents via an impedance matrix. Section 2.4 shows the development of the three-phase equivalent circuit model for the delta-star transformer and the delta-zig-zag transformer using the methodology set out in Section 2.3. Firstly though, it is important to understand exactly what is meant when the term zig-zag is used in reference to a transformer connection.

A harmonic mitigating transformer can be referred to as a zig-zag transformer. The term zig-zag comes from the way the secondary windings of the transformer are connected. In practise, the two most common connections that are used are the delta and star or wye connections shown in Figure 2.1 (a) and (b) respectively. These connections show that there is one primary winding per phase and one secondary winding per phase. In a zig-zag connection (also known as an interconnected star), there is one winding per primary phase and two windings per secondary phase. Figure 2.2 (a) shows the connection. From the labelled connection points shown in Figure 2.2 (a) it can be seen that although there are two secondary windings per core limb the two windings are not connected together rather each secondary winding on the same core limb is connected to an adjacent winding on a separate core limb. Figure 2.2 (a) shows the vector diagram for the zig-zag transformer. Using the law of cosines, it can be shown that the voltage across each individual winding is the resultant voltage, V, divided by the square root of three. This demonstrates that there is a phase associated with the voltage across each individual winding. This means that the voltage across two connected windings is the vector sum of the individual voltages. It is also important to note though that the connections for the delta or star are external whereas for the zig-zag connection, the windings have to be physically split and then interconnected i.e. the connections are internal as well as external. This means that the zig-zag connection has to be done during the transformer construction, it cannot be done after. So for example a delta-star transformer cannot simply be reconnected to form a delta-zig-zag transformer.

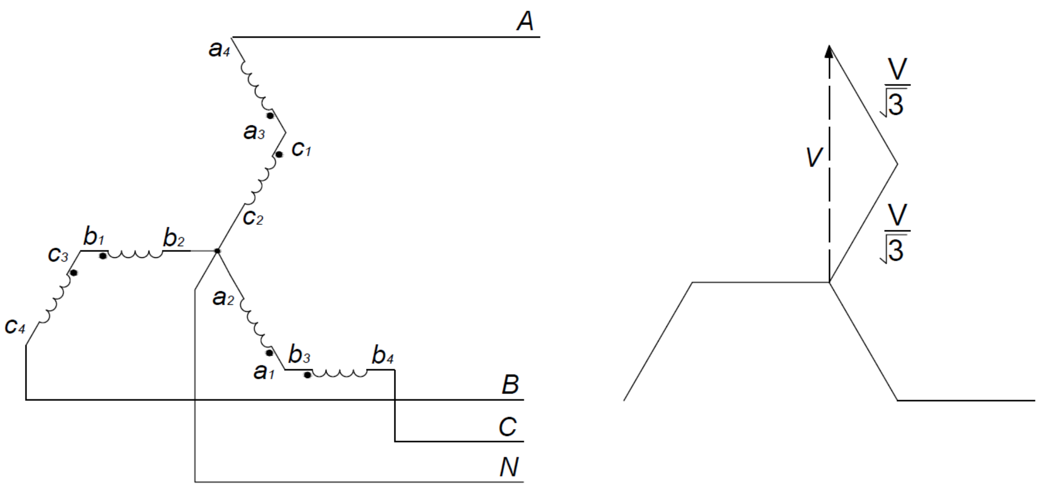


(a) (b)

Figure 2.1: (a) Star connection on one side of a three phase transformer, (b) Delta connection on one side of a three phase transformer



(a)



(b) (c)

Figure 2.2: (a) Un-connected zig-zag transformer showing two windings per secondary (b) Zig-zag connection scheme on the secondary side and its corresponding (c) vector diagram

The diagram in Figure 2.3 shows the connection scheme for a three phase delta-zig-zag transformer. According to the diagram, depending on where a voltage or current is measured, a factor of root three has to be considered. Furthermore the turns ratio ‘a’ between the primary windings and secondary windings has to be taken into account when considering the voltages and currents on the secondary side. This once again is shown in Figure 2.3. The value ‘b’ shown in the diagram is used to represent the fact that the voltage across one of the secondary windings is not simply half of the voltage across two of the windings. Therefore the value of ‘b’ has to √3 as shown in Figure 2.2 (b) in order to ensure the correct resultant voltage is achieved across the connection.

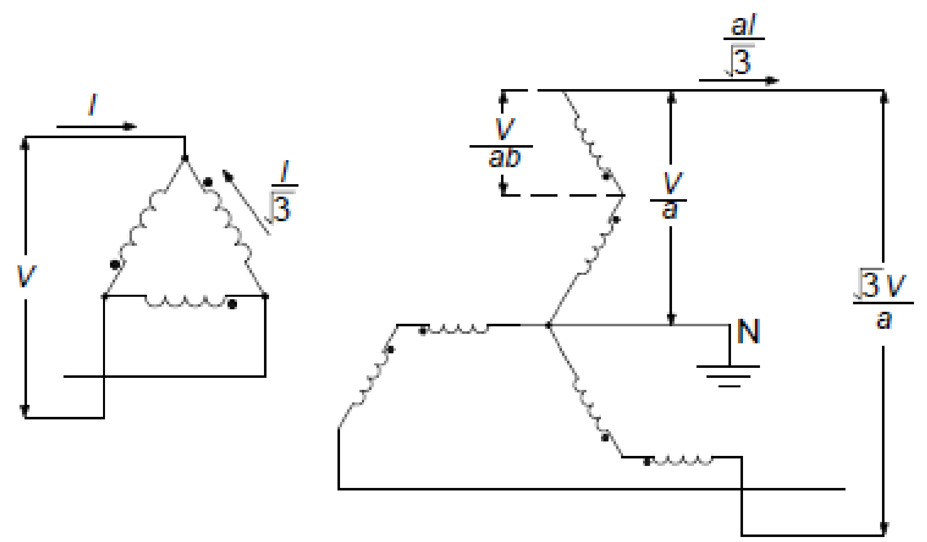


Figure 2.3: Transformer connection for a three phase delta-zig-zag transformer. Note the inclusion of the turns ratios a and b.

2.2 Transformer modelling using controlled sources

For this section only the ideal transformer has been considered. For the ideal case, the transformer winding resistances are negligible, there are no leakage fluxes and the permeability of the core is infinite [10]. The reason for only considering the ideal case is because the main aim is to show how the zig-zag connection mitigates the triplen harmonics. By representing a transformer and its connection scheme using controlled sources, one can then use circuit theory for the analysis. By using circuit analysis, the fundamental concept of the zig-zag connection can be shown without the complexity of electromagnetic effects. In subsequent sections of the investigation a practical model of the transformer and its connection schemes will be developed. A typical model of the ideal transformer is shown in Figure 2.4. The relationship between the windings and the voltage is given in Equation (2.1). The primary and secondary voltages are v_1 and v_2 respectively and the number of turns for the primary and secondary is given by N_1 and N_2 respectively. The value a is hence the ratio of N_2 to N_1. Similarly the relationship for the current and the windings is given in Equation (2.2) where i_1 and i_2 are the primary and secondary currents respectively. Furthermore it can be shown that the power rating P of the transformer is constant regardless of which side the transformer is referred to. This is shown in Equation (2.3). Using these relationships, a transformer can be modelled using a voltage controlled voltage source and a current controlled current source. The circuit model is shown in Figure 2.5.

$$\frac{N_2}{N_1} = a$$

$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = \frac{1}{a}$$

$$\boldsymbol{v_2 = a v_1} \tag{2.1}$$

$$\frac{i_1}{-i_2} = \frac{N_2}{N_1} = a$$

$$i_1 = -a i_2 \tag{2.2}$$

$$P = v_1 i_1 = -\frac{v_2}{a} a i_2 = -v_2 i_2 \tag{2.3}$$

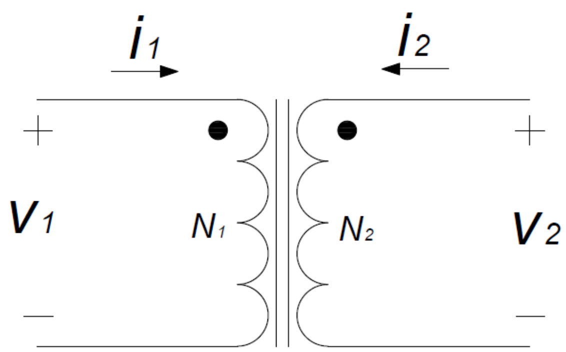


Figure 2.4: Model of a transformer using inductors on a common core to represent the windings

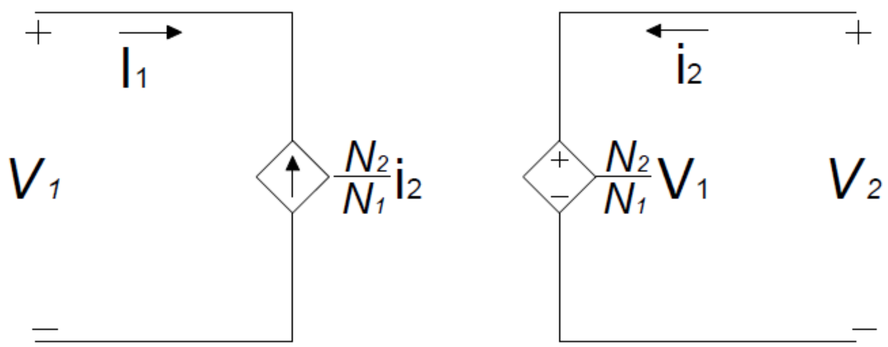


Figure 2.5: Model of a transformer using a voltage controlled voltage source and a current controlled current source.

A transformer with a delta-zig-zag connection will have three windings per phase i.e. one winding for the primary and two windings per secondary. A typical model of a three winding transformer is presented in Figure 2.6. In this case N_2 and N_3 are the number of turns for the two secondary windings. The voltages across the two secondary windings are given by v_2 and v_3 and the currents in the secondary windings are i_2 and i_3. Figure 2.7 represents the three winding transformer using two voltage controlled voltage sources and two current controlled current sources. Essentially this is the method that is used to represent the connections of a zig-zag transformer. Using the voltage controlled voltage source and a current controlled current source model of the transformer, a three phase zig-zag transformer can easily be developed and analysed using circuit theory and simulations.

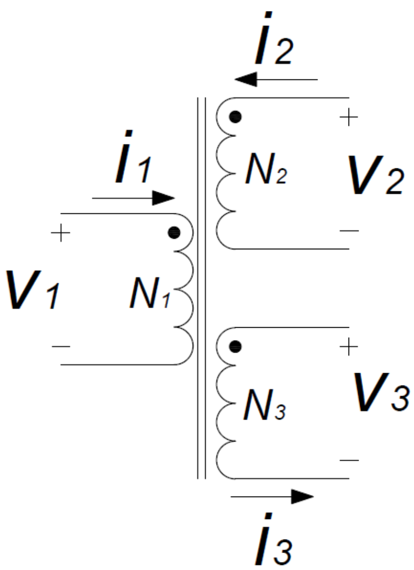


Figure 2.6: Model of a three winding transformer using inductors to represent the windings

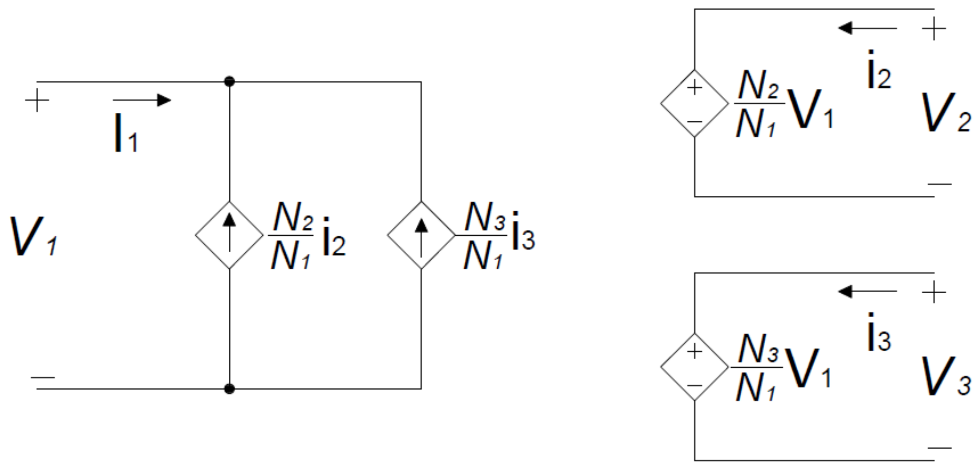


Figure 2.7: Model of a three winding transformer using two voltage controlled voltage sources and two current controlled current sources

2.2.1 Controlled source model of a delta-star transformer

The controlled source model of a three phase two winding transformer is presented in Figure 2.8. The figure shows that no particular transformer connection scheme has been applied. Figure 2.9 presents the delta-star connected controlled source model of a three phase two winding transformer. From Chapter 1 it is known that a phase to neutral non-linear load generates the triplen harmonics, particularly the third harmonic. The two current sources connected to each secondary phase shown in Figure 2.9 are in parallel. One of the current sources on a phase produces a current waveform (i_f) at the fundamental frequency whereas the second current source produces a current waveform (i_h) at the third harmonic frequency. Using Kirchhoff's Current Law the controlled source circuit can be analysed. The equations that follow focus mainly on one phase of the transformer as it is assumed that the transformer is operating under balanced steady state conditions. The variable a used in the model is the ratio of the primary windings to the secondary windings. In order to ensure a 1 to 1 ratio, this value may not necessarily be 1. The reason being is because of the nature of the delta connection however the value is not critical at this stage. From Figure 2.9 using Kirchhoff's Current Law it can be seen that,

$$i_2 = i_{f1} + i_{h1}$$

Where i_2 is the phase or line current on the secondary side of the transformer and i_{f1} is the current at the fundamental frequency and i_{h1} is the current at the third harmonic frequency.

It can also be seen that,

$$I_1 = -ai_2$$

Therefore,

$$I_1 = -a(i_{f1} + i_{h1}) \quad \textbf{(2.4)}$$

Similarly it can be shown that,

$$I_3 = -a(i_{f2} + i_{h2}) \quad \textbf{(2.5)}$$

$$I_5 = -a(i_{f3} + i_{h3}) \quad \textbf{(2.6)}$$

Where I_1, I_3 and I_5 are the phase currents on the primary side of the transformer.

Equation (2.4), Equation (2.5) and Equation (2.6) represent the phase currents in the delta connections. From the equations it can be seen that the third harmonic (i_{h1}, i_{h2} and i_{h3}) is present in the phase currents implying that the third harmonic propagated from the secondary side to the primary side. If the analysis is continued it can be shown that the line current in phase A is,

$$I_A = I_1 - I_5 \quad \textbf{(2.7)}$$

Substituting Equation (2.4) and (2.5) into Equation (2.7) results in,

$$I_A = -a\left(i_{f1} - i_{f3}\right) - a(i_{h1} - i_{h3}) \quad \textbf{(2.8)}$$

Similarly it can be shown that,

$$I_B = -a\left(i_{f2} - i_{f1}\right) - a(i_{h2} - i_{h1}) \quad \textbf{(2.9)}$$

$$I_C = -a\left(i_{f3} - i_{f2}\right) - a(i_{h3} - i_{h2}) \quad \textbf{(2.10)}$$

Noting that from Chapter 1 it is known that the third harmonic currents are in phase with each other and if the system is balanced, they are also equal in magnitude. Therefore from Equation (2.8), Equation (2.9) and Equation (2.10) it can be seen that the third harmonic from each phase does not enter into the line currents instead they cancel out. This is the main reason why in the past a delta connection was used to remove harmonics within a system. However from Equation (2.4), Equation (2.5) and Equation (2.6) it can be seen that the third harmonics are still present in the phase currents leading to the observation that the third harmonics circulate within a delta winding. As mentioned already, the circulating third harmonics increase the overall transformer losses.

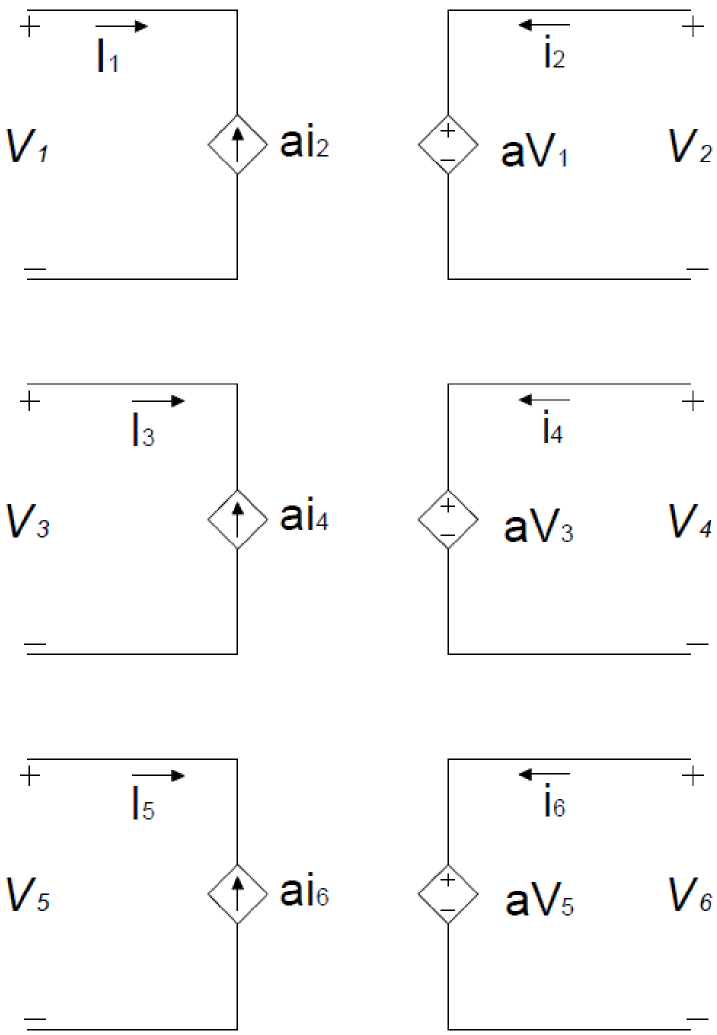


Figure 2.8: Controlled source model of a three phase two winding transformer. No particular transformer scheme has been applied.

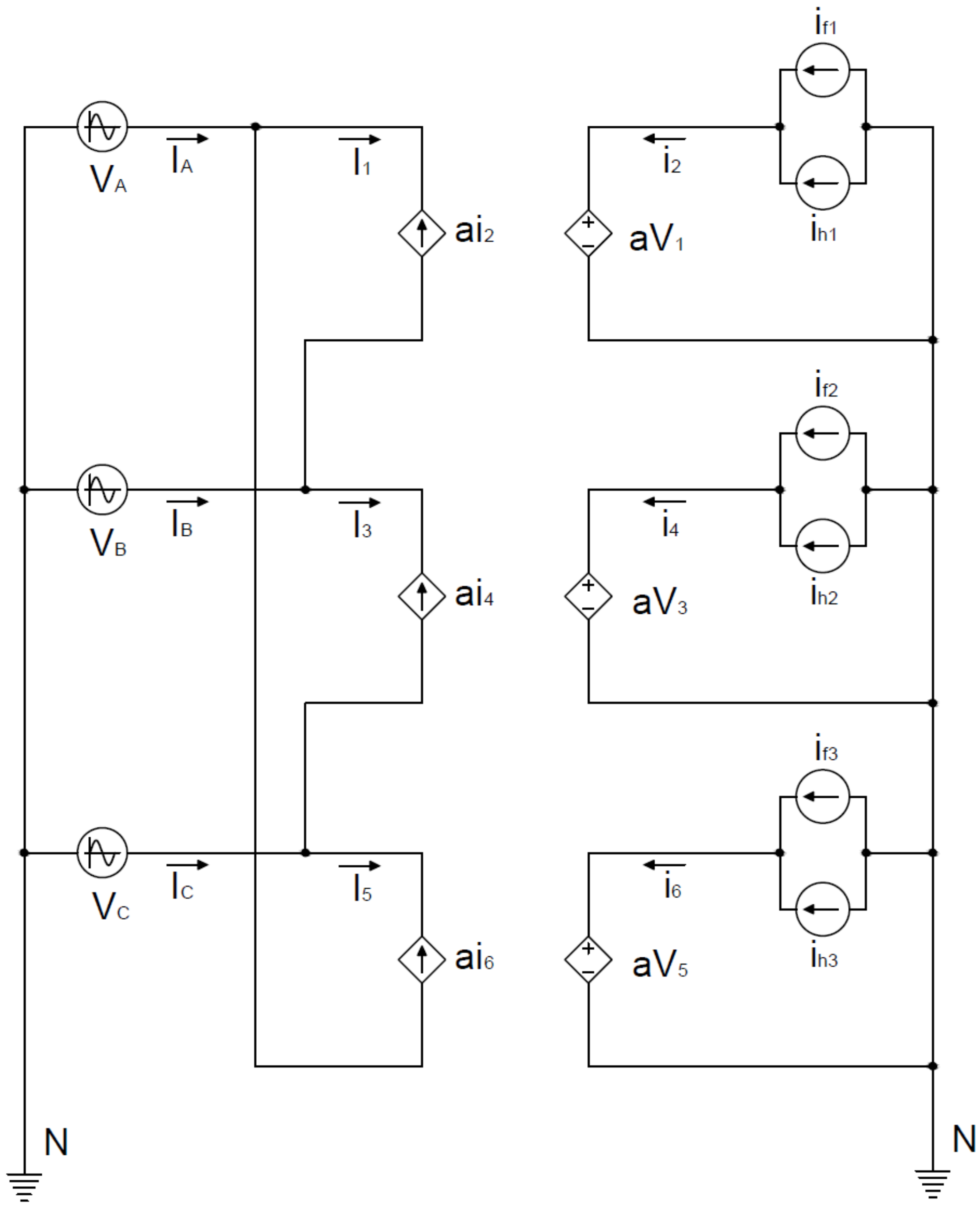


Figure 2.9: Controlled source model of a delta-star transformer. Two current sources have been used to represent the non-linear load i.e. one current source produces a current waveform at the fundamental frequency and the other current source produces a current waveform at the third harmonic frequency.

2.2.2 Controlled source model of a delta-zig-zag transformer

The controlled source model of a three phase three winding transformer is presented in Figure 2.10. The figure shows that no particular transformer connection scheme has been applied. Figure 2.11 presents the delta-zig-zag connected controlled source model. The connections have been done in accordance with Figure 2.2 and Figure 2.3. Once again from Chapter 1 it is known that a phase to neutral non-linear load generates the triplen harmonics, particularly the third harmonic. As in the delta-star circuit, using Kirchhoff's current law the circuit can be analysed. As with the previous section, the equations that follow focus mainly on one phase of the transformer. This is because it is assumed that the transformer is operating under steady state balanced conditions. The variable b used in the model is the ratio of a primary winding to its corresponding secondary windings. The variable a is as per the delta-star model. Figure 2.3 presents a clear description of the use of variable a and variable b. From Figure 2.11 using Kirchhoff's Current Law it can be shown that,

$$i_3 = -i_8 = i_{f1} + i_{h1}$$

$$i_6 = -i_2 = i_{f2} + i_{h2}$$

$$i_9 = -i_5 = i_{f3} + i_{h3}$$

Where i_2, i_3, i_5, i_6, i_8 and i_9 are the phase currents on the secondary side of the transformer.

It can be shown that,

$$I_1 = -ab(i_2 + i_3)$$

$$I_4 = -ab(i_5 + i_6)$$

$$I_7 = -ab(i_8 + i_9)$$

Where I_1, I_4 and I_7 are the phase currents on the primary side of the transformer.

By performing the appropriate substitutions it can be shown that,

$$I_1 = -\left[-ab\left(i_{f2} + i_{h2}\right) + ab\left(i_{f1} + i_{h1}\right)\right]$$

$$I_1 = -\left[ab(i_{f1} - i_{f2}) + ab(i_{h1} - i_{h2})\right] \quad \textbf{(2.11)}$$

$$I_4 = -\left[-ab\left(i_{f3} + i_{h3}\right) + ab(i_{f2} + i_{h2})\right]$$

$$I_2 = -\left[ab(i_{f2} - i_{f3}) + ab(i_{h2} - i_{h3})\right] \quad \textbf{(2.12)}$$

$$I_7 = -\left[-ab\left(i_{f1} + i_{h1}\right) + ab(i_{f3} + i_{h3})\right]$$

$$I_7 = -\left[ab(i_{f3} - i_{f1}) + ab(i_{h3} - i_{h1})\right] \quad \textbf{(2.13)}$$

Noting once again that from Chapter 1 it is known that the third harmonic currents are in phase with each other and if the system is balanced, they are also equal in magnitude. Therefore from Equation (2.11), Equation (2.12) and Equation (2.13) it can be seen that the third harmonics cancel out. In other words there is no third harmonic component in the phase currents of the primary side. This is the fundamental reason for using the zig-zag connection.

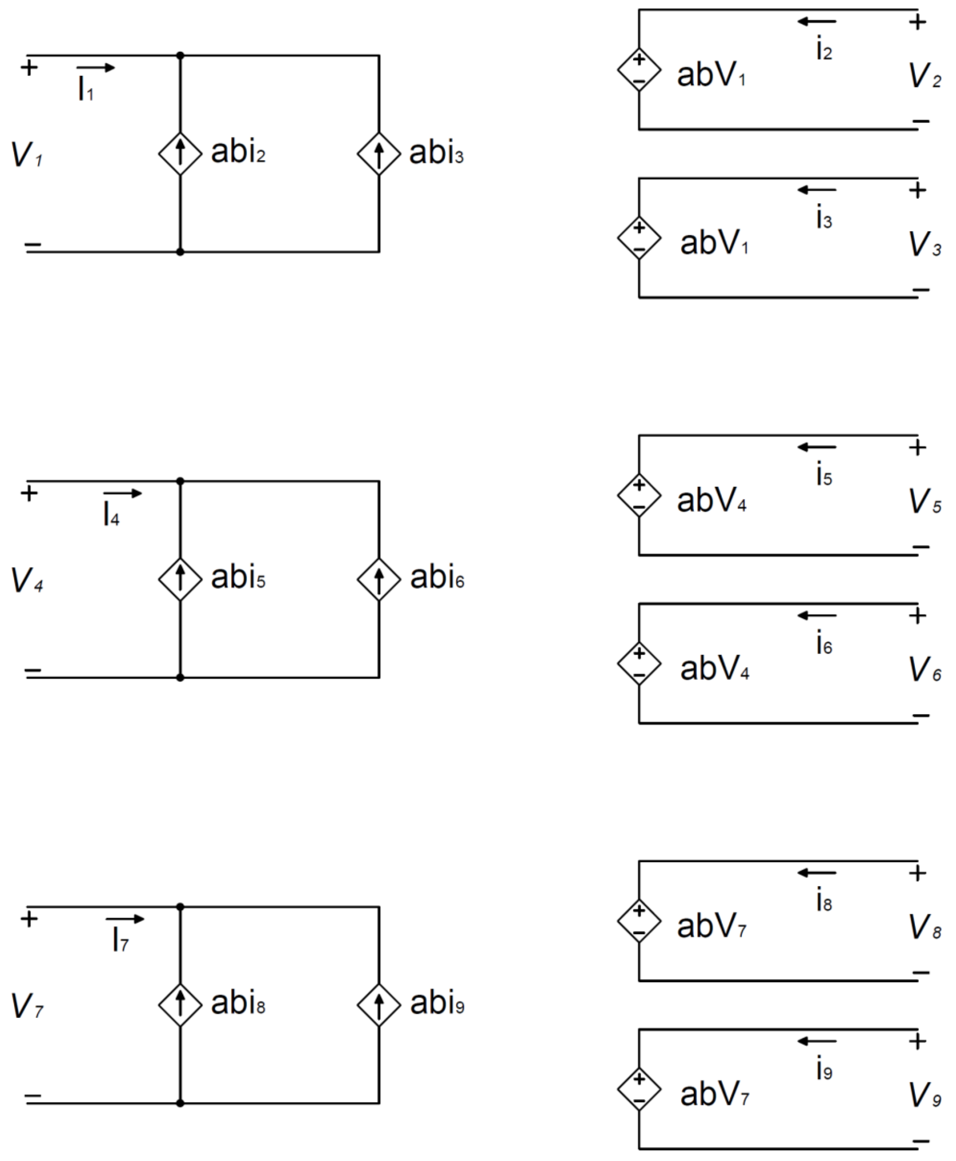


Figure 2.10: Controlled source model of a three phase three winding transformer. No particular transformer scheme has been applied.

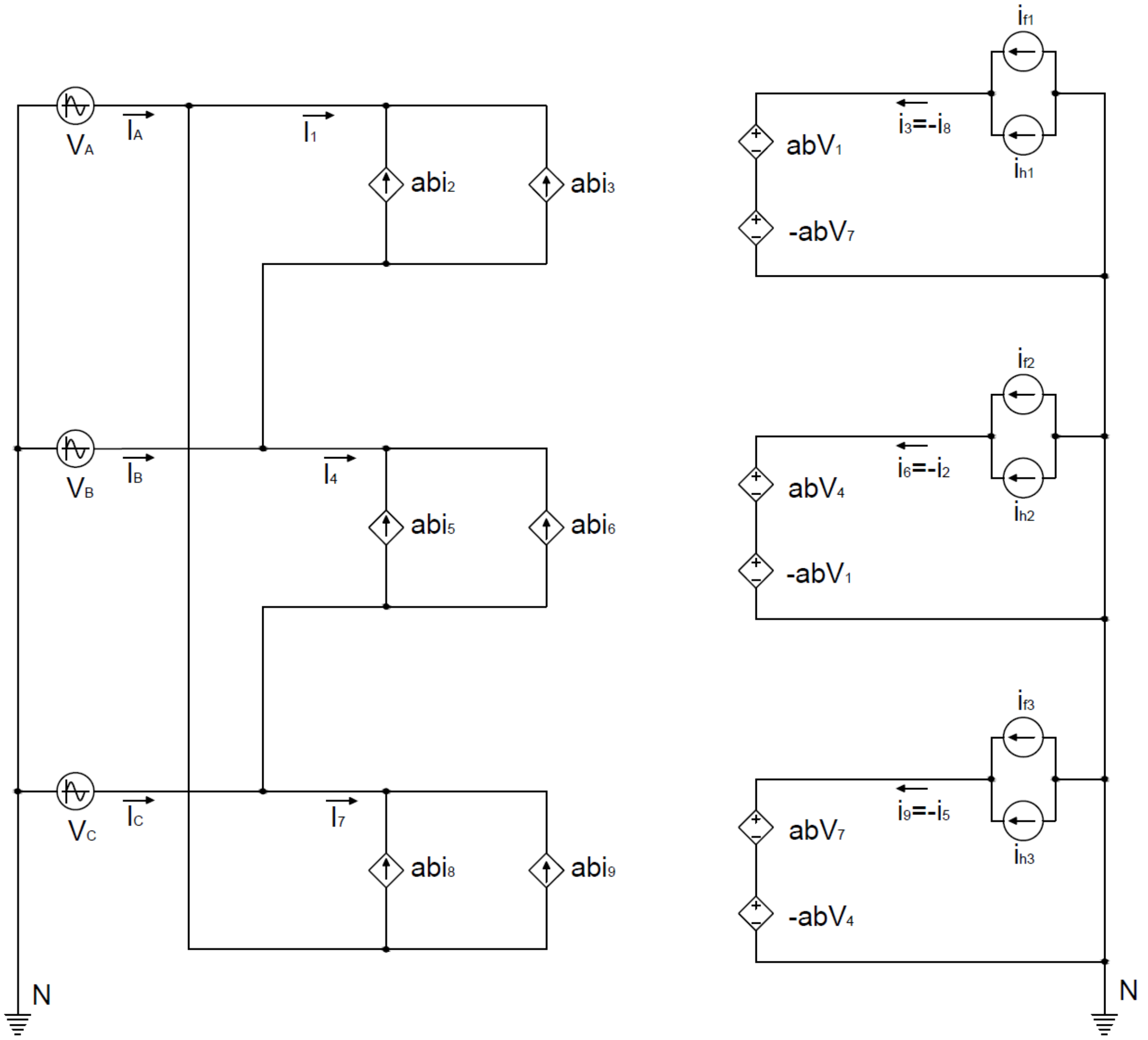


Figure 2.11: Controlled source model of a delta-zig-zag transformer. Two current sources have been used to represent the non-linear load i.e. one current source produces a current waveform at the fundamental frequency and the other current source produces a current waveform at the third harmonic frequency.

2.2.3 Discussion on controlled source models

The circuit models in Figure 2.9 and Figure 2.11 reveal how the third harmonic behaves within a transformer. In particular the delta-star connection allowed the third harmonic current to propagate from the secondary winding into the primary. The third harmonic current was therefore seen to manifest in the phase currents of the delta connected transformer windings. The delta-zig-zag connection however prevented the third harmonic from propagating to the primary winding. The zig-zag connection exploits the fact that the third harmonic components are in phase which therefore forces them to cancel out within the secondary windings. By cancelling the third harmonic components in the secondary windings the third harmonics will not circulate within the delta connection. The controlled source models also provide an indication as to where to measure or observe the harmonics. For example, one will not detect any triplen harmonics in the line currents of the delta connection as they will have cancelled out however by consulting the controlled source model, it can be seen that the triplen harmonic currents can be measured or observed in the phase currents of the delta connection of the delta-star transformer. Similarly if any triplen harmonics are detected in the phase currents of the delta connection of the delta-zig-zag transformer then the effectiveness of the mitigation can be questioned. The circuit models in Figure 2.9 and Figure 2.11 only focus on the ideal transformer. The use of such models will only reveal how the harmonic currents propagate within a transformer. In order to fully understand how the transformer couplings affect the harmonic currents, a practical circuit model has to be considered.

2.3 Single phase two winding transformer equivalent circuit model

As mentioned, harmonics within a power system have an adverse effect on transformers, in particular the transformer losses. One method of mitigating harmonics in a power system is to use a harmonic mitigating transformer. The question arises though as to what role do the transformer losses play in a harmonic mitigating transformer. In other words, will the transformer losses be less than or greater than a normal transformer's losses and how will such losses affect the transformer's ability to mitigate harmonics? In order to answer the question at hand, one needs to first understand the differences between an ideal transformer and a practical transformer. A practical transformer differs from an ideal transformer in that the practical transformer windings have resistances, not all windings link the same flux and the permeability of the core material is not infinite [10]. In essence the main difference between an ideal transformer and a practical transformer is that a practical transformer has transformer losses. The practical transformer equivalent circuit model must therefore include parameters that represent such losses.

2.3.1 Modelling a single phase transformer under no-load conditions

As discussed in Chapter 1, transformer losses can be classified as being either load losses or no-load losses [14]. Load losses are further divided into the I^2R losses and stray losses due to stray electromagnetic flux in the windings, core clamps and other structural parts of the transformer [1]. No-load losses are the core losses or iron losses which comprise hysteresis losses and eddy current losses. For this investigation only the load losses will be considered this is because in order to use coupled circuit theory to model a transformer, two conditions have to be met namely core losses must be neglected and the magnetic non-linearity of the core must also be neglected [11]. The condition regarding the omission of core losses will be explained in this section and the condition regarding the non-linearity of the magnetic core is explained in Appendix A.

If one considers the load losses of a transformer, one will see that the I^2R losses are due to the resistance of the transformer winding conductors. Figure 2.12 shows a basic diagram of a single phase transformer [12]. The diagram shows a common core with a primary winding on one leg and a secondary winding on the other leg. The resistance of the primary winding is given by r_1 and the secondary winding resistance is given by r_2. When a current i_1 flows through the primary winding, it encounters the resistance r_1 hence resulting in the I^2R losses. The same can be said when current i_2 flows through r_2. If current i_1 flows through the primary winding, it produces a flux Φ_{11} that links only winding 1 as well as a flux that links the primary winding to the secondary winding shown in Figure 2.12 as Φ_{21} [12]. Similarly if only current i_2 flows in the secondary winding it produces a flux Φ_{22} that links only winding 2 as well as a flux that links the secondary winding to the primary winding shown in Figure 2.12 as Φ_{12}. The two flux values Φ_{21} and Φ_{12} are equal and are referred to as the mutual flux. According to coupled circuit theory, the resultant flux produced as a result of winding 1 and the resultant flux produced as a result of winding 2 can be expressed by Equation (2.14) and Equation (2.15) respectively [11].

$$\Phi_1 = \Phi_{11} + \Phi_{12} \tag{2.14}$$

$$\Phi_2 = \Phi_{21} + \Phi_{22} \tag{2.15}$$

By considering Kirchhoff's Voltage Law, the time varying voltage on the primary side can be expressed as:

$$v_1 = r_1 i_1 + N_1 \frac{d\Phi_1}{dt}$$

$$v_1 = r_1 i_1 + N_1 \frac{d\Phi_{11}}{dt} + N_1 \frac{d\Phi_{12}}{dt} \tag{2.16}$$

Similarly, the induced secondary voltage can be expressed as:

$$v_2 = r_2 i_2 + N_2 \frac{d\Phi_2}{dt}$$

$$v_2 = r_2 i_2 + N_2 \frac{d\Phi_{22}}{dt} + N_2 \frac{d\Phi_{21}}{dt} \tag{2.17}$$

Using Faraday's Law of Induction [12], the self-inductance L_{11} and the self-inductance L_{22} can be determined using the relationships in Equation (2.18) and Equation (2.19) respectively. The same relationships shown in Equation (2.20) and Equation (2.21) are used to determine the mutual inductance L_{12} or L_{21}. By substituting Equation (2.18) and Equation (2.21) into Equation (2.16) an expression for the primary voltage can be obtained in terms of the self-inductance and coupled inductance of the transformer. This is shown in Equation (2.22). Similarly if Equation (2.19) and Equation (2.20) are substituted into Equation (2.17) then a revised expression for the secondary voltage can be obtained as shown in Equation (2.23) [11]. Equation (2.22) and Equation (2.23) form the basis for the classical theory of coupled inductor circuits [11].

$$N_1\Phi_{11} = L_{11} i_1 \tag{2.18}$$

$$N_2\Phi_{22} = L_{22} i_2 \tag{2.19}$$

$$N_1\Phi_{12} = L_{12} i_2 \tag{2.20}$$

$$N_2\Phi_{21} = L_{21} i_1 \tag{2.21}$$

$$v_1 = r_1 i_1 + L_{11} \frac{di_1}{dt} + L_{12} \frac{di_2}{dt} \tag{2.22}$$

$$v_2 = r_2 i_2 + L_{22}\frac{di_2}{dt} + L_{21}\frac{di_1}{dt} \tag{2.23}$$

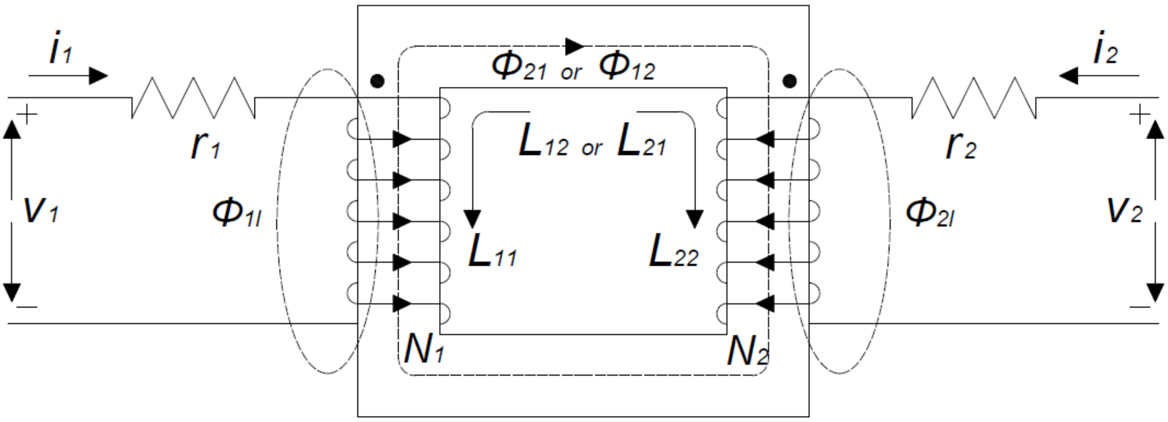


Figure 2.12: Single phase transformer model illustrating the leakage flux paths for the flux of each winding and the mutual flux between the primary and secondary windings

Up to this point Equation (2.22) and Equation (2.23) can be seen to explicitly represent the I^2R losses of a practical transformer. This is seen by the inclusion of the resistance r_1 and r_2 in the equations. What are not explicitly represented are the stray losses due to stray electromagnetic flux in the windings. Such a flux is commonly known as leakage flux. Leakage flux occurs on both sides of the transformer and arises due to the fact that not all the flux produced by one winding links the flux of the second winding. The leakage flux in Equation (2.22) and Equation (2.23) is not explicitly shown however it is incorporated into the flux of each winding. Consider the diagram in Figure 2.12 which illustrates the path of the leakage flux [12]. The leakage flux of winding 1 is expressed in Equation (2.24) and the leakage flux of winding 2 is expressed in Equation (2.25) [11]. The resultant flux for winding 1 and winding 2 via simple substitution can be expressed in Equation (2.26) and Equation (2.27) respectively [11]. This means that the leakage flux associated with each winding is implicitly included in the coupled circuit equations in Equation (2.22) and Equation (2.23). Therefore two of the main aspects causing a transformer to deviate from ideal conditions are suitably captured and coupled inductor circuit theory can be used as a basis for modelling a practical transformer.

$$\Phi_{1l} = \Phi_{11} - \Phi_{21} \tag{2.24}$$

$$\Phi_{2l} = \Phi_{22} - \Phi_{12} \tag{2.25}$$

$$\Phi_1 = \Phi_{l1} + \Phi_{21} + \Phi_{12} \tag{2.26}$$

$$\Phi_2 = \Phi_{l2} + \Phi_{12} + \Phi_{21} \tag{2.27}$$

2.3.2 Open circuit impedance matrix model of a single phase transformer

A transformer works under the premise that alternating currents and voltages are applied to the windings. Using this notion and the assumption that steady state conditions apply, Equation (2.22) and Equation (2.23) can be transformed into the phasor form shown in Equation (2.28) and Equation (2.29) [11] [12]. Using the variable assignments in Equation (2.28) and Equation (2.29) for the impedances the matrix equation in Equation (2.30) can be formed [12].

$$V_1 = (r_1 + j\omega L_{11})I_1 + (j\omega L_{12})I_2 \tag{2.28}$$

Let: $z_{11} = (r_1 + j\omega L_{11})$

and let: $z_{12} = (j\omega L_{12})$

$$V_2 = (r_2 + j\omega L_{22})I_2 + (j\omega L_{21})I_1 \tag{2.29}$$

Let: $z_{22} = (r_2 + j\omega L_{22})$

and let: $z_{21} = (j\omega L_{21})$

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$
$$[\mathbf{V}] = [\mathbf{Z}][\mathbf{I}] \tag{2.30}$$

The equivalent circuit model represented in Equation (2.30) successfully captures the resistances of the windings, the self-inductance of each winding and the mutual inductance between windings [12]. The model however does not capture the core losses. For a complete description of a single phase transformer, the Steinmetz exact transformer model can be used whereby the core losses are modelled as a resistor in parallel with the inductor representing the mutual inductance [21]. The current flowing to the parallel branch would be known as the excitation current which comprises a magnetising current flowing through the inductor and a much smaller current, known as the core loss current, flowing through the resistor. In reality the core losses are dependent on the material used and because of this, predefined values for the core losses are often stated on core material data sheets. The omission of the core losses would result in a small error in the efficiency calculations. The error in the efficiency will not be considered at this stage as the main aim is to develop an equivalent circuit model that can be successfully extrapolated to the three phase case. The self and mutual impedance values in Equation (2.30) can be determined by performing an open circuit test as shown in Appendix A. Thus the impedance matrix in Equation (2.30) is referred to as the open circuit impedance matrix. The discussion in Appendix A also presents an alternative method for modelling the single phase transformer using the short circuit admittance values. Such a method is best suited for load flow studies and is not required for this investigation.

2.3.3 Operation of a single phase transformer under load conditions

Consider the diagram in Figure 2.13 of a single phase transformer connected to a supply and a load. If the coupled circuit equations used in Equation (2.22) and Equation (2.23) are applied around loop 1 and loop 2 then according to Kirchhoff's Voltage Law:

$$v_s = (r_s + r_1)i_1 + L_{11}\frac{di_1}{dt} - L_{12}\frac{di_L}{dt} \tag{2.31}$$

$$0 = L_{21}\frac{di_1}{dt} - (r_2 + r_L)i_L - L_{22}\frac{di_L}{dt} \tag{2.32}$$

Where r_S is the internal resistance of the supply, r_L is the load resistance and i_L the current drawn by the load. If steady state conditions apply then Equation (2.32) and Equation (2.33) can be represented as:

$$V_1 = (r_s + r_1 + j\omega L_{11})I_1 - (j\omega L_{12})I_L \tag{2.33}$$

Let: $z_{11} = (r_s + r_1 + j\omega L_{11})$

and let: $z_{12} = -(j\omega L_{12})$

$$0 = (r_L + r_2 + j\omega L_{22})I_L - (j\omega L_{21})I_1 \tag{2.34}$$

Let: $z_{22} = (r_L + r_2 + j\omega L_{22})$

and let: $z_{21} = -(j\omega L_{21})$

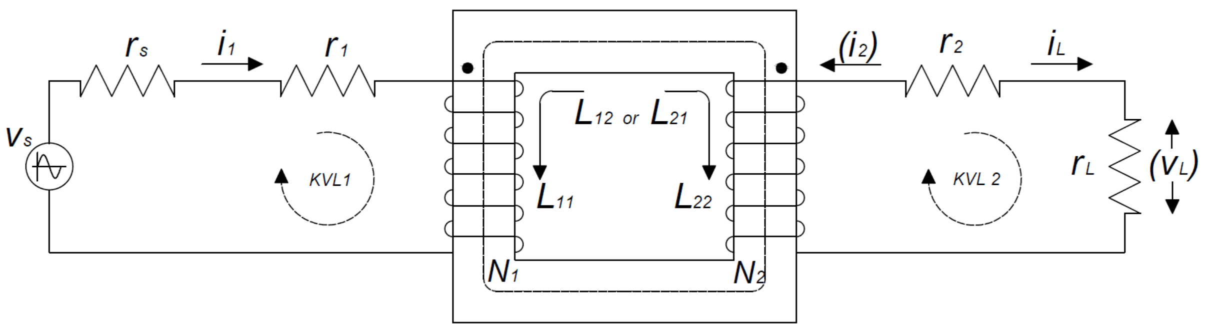


Figure 2.13: Single phase transformer model with a load connected. Note the direction of the load current is opposite in direction to the secondary current (i_L = -i_2).

Equation (2.33) and Equation (2.34) represent the voltage and current relationships in the single phase transformer operating under normal load conditions whereby r_L is a linear load. The current i_2 is shown in brackets just to indicate the conventional direction for the current on the secondary side therefore as seen in the equations i_L=-i_2. It is clear that under no-load conditions r_L is equal to zero and the un-loaded transformer equations apply.

2.4 Three phase transformer equivalent circuit model

The preceding section presents a method for modelling a single phase two winding transformer. The method relies on the fact that the transformer windings can be modelled as two mutually coupled coils. This is in accordance with electromagnetic coupled circuit theory. Using this notion, a model of a three phase transformer can be developed. It must be noted that the model developed for this investigation is specifically for a three phase transformer whereby the windings are on a common core. This is important to note because in many cases a three phase transformer is modelled as three single phase transformers connected to form a three phase transformer bank. In this situation the coupling characteristics between the windings will not be adequately accounted for in the model. The first part of this section focuses on developing a model for a three phase two winding transformer in particular a delta-star connected transformer. The second part focuses on developing a model for a three phase three winding transformer and in this case a delta-zig-zag connected transformer.

2.4.1 Three phase two winding transformer equivalent circuit model

Three phase two winding transformer open circuit impedance matrix

The diagram in Figure 2.14 presents a basic illustration of a three limbed core with six windings i.e. two windings on each limb [22]. The diagram does not show any flux paths or resistors as it is merely a representation of a physical core with six windings. It can be observed that according to Faraday's Law of Induction if a current were to flow in winding 1, 3 and 5 (considered the primary windings) a voltage will be induced in winding 2, 4 and 6 (considered the secondary windings) respectively. It follows that if current flows in the primary windings, Winding 1 will be mutually coupled to Winding 2, Winding 3 will be mutually coupled to Winding 4 and Winding 5 will be mutually coupled to Winding 6. Through the mutual coupling, a mutual inductance is established. It must be noted however that because the windings share the same core, there will also be a mutual inductance between adjacent windings. For example there will be a mutual inductance between winding 1 and winding 6. Therefore in order to represent the three phase two winding transformer, the mutual inductance between each winding as well as the self-inductance and leakage inductance of each winding must be represented within the model. In order to do this, the method used for the single phase transformer can be used whereby the six windings of the transformer can be modelled as a network of mutually coupled inductors. An impedance matrix will be developed, as in the single phase case, whereby the three primary voltages and three secondary voltages can be expressed in terms of the primary and secondary currents and the open circuit impedances.

From the single phase matrix of Equation (2.30) and by using basic algebraic principals, the matrix in Equation (2.35) can be deduced**.** From the matrix it can be seen that the diagonal impedance values (from top left to bottom right) are in fact the self-impedance values for each winding. It can also be seen that the impedance values on the top right side of the diagonal are symmetrical with the impedance values on the bottom left of the diagonal. The matrix of Equation (2.35) effectively captures the open circuit impedances between windings on the same core or limb and the open circuit impedances of windings on adjacent cores. In order to determine the values in the impedance matrix, 36 open circuit measurements will have to be taken. If one assumes flux symmetry within the core, then only 21 measurements will be required [22]. For this investigation, 36 open circuit measurements are taken in order to illustrate the symmetry. The impedance matrix in Equation (2.35) is effectively called the winding impedance matrix because it shows the relationship between the voltages and currents without the inclusion of any transformer connection.

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} \\ z_{51} & z_{52} & z_{53} & z_{54} & y_{55} & z_{56} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} \quad \textbf{(2.35)}$$

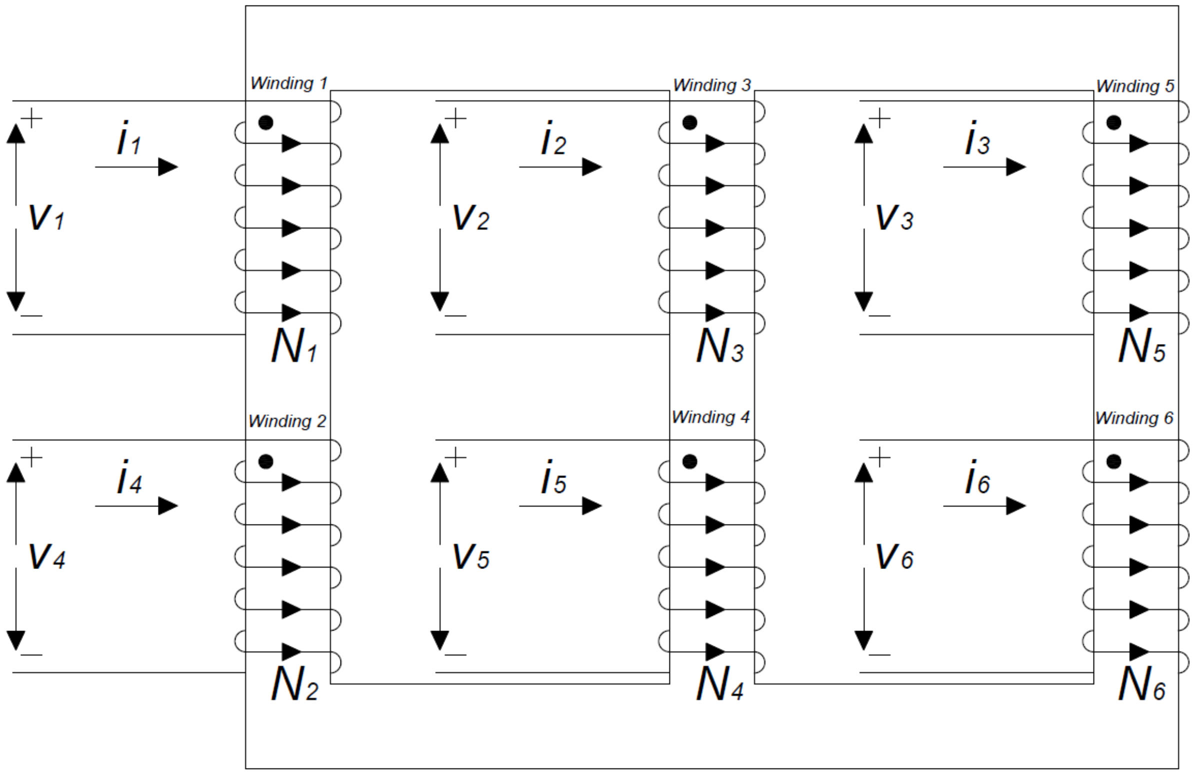


Figure 2.14: Simplified model of a two winding three phase transformer on a common core. Note that no particular connection scheme has been assigned to the transformer and the values are not referred to any particular side.

Application of the delta-star connection

As mentioned, the matrix equation in Equation (2.35) does not demonstrate any type of transformer connection. In other words the equation does not show whether the transformer is connected in a delta-star or not. In order to represent the connection, a connection matrix is required [22]. Consider the diagram in Figure 2.15 which shows the connection of the six windings on a common core as in Figure 2.14. The connection is such that windings 1, 3 and 5 are delta connected and windings 2, 4 and 6 are star connected. The voltages V_A, V_B and V_C are the primary node or supply voltages and voltages V_a, V_b and V_c represent the secondary line or phase voltages. The voltages of V_1 to V_6 represent the winding voltages. Using basic circuit analysis on Figure 2.15 it can be seen that:

$$V_1 = V_A - V_B$$
$$V_2 = V_a$$
$$V_3 = V_B - V_C$$

$$V_4 = V_b$$
$$V_5 = V_C - V_A$$
$$V_6 = V_c$$

By rewriting the voltage relationships in matrix form, the matrix equation in Equation (2.36) can be developed. The matrix that converts the phase voltages on the supply and load side to the connection specific winding voltages is therefore called the connection matrix. Such a matrix can be developed for any type of transformer connection.

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \\ V_a \\ V_b \\ V_c \end{pmatrix} \quad \textbf{(2.36)}$$

$$[\mathbf{V_{winding}}] = [\mathbf{C}][\mathbf{V_{phase-supply-load}}]$$

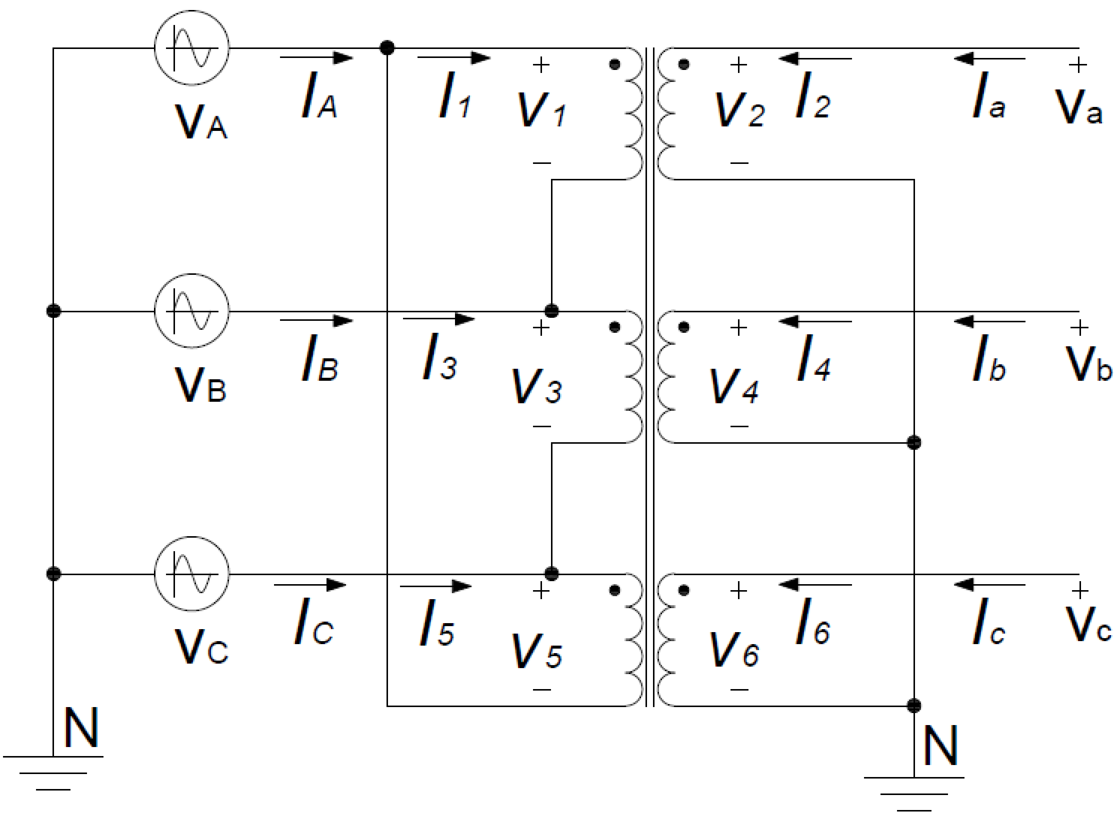


Figure 2.15: Simplified diagram illustrating the delta-star connection between the six windings on a three limbed core shown in Figure 2.13.

The development of the connection matrix also provides a means for determining the correct current values for the delta-star transformer. Consider Equation (2.37) below which uses the connection matrix to obtain the correct delta connection currents where **[C]** is the connection matrix and **[C ^{T}]** is the transpose of the connection matrix [23]. The connection therefore ensures that the correct values for the phase and line currents and phase and line voltages are obtained when applying a particular connection scheme to the transformer. The use of the connection matrix may appear to be trivial at this stage however the use of it will become more apparent when considering the delta-zig-zag connection scheme.

$$\begin{pmatrix} I_A \\ I_B \\ I_C \\ I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{pmatrix} \tag{2.37}$$

$$[\mathbf{I_{phase-supply-load}}] = [\mathbf{C^T}][\mathbf{I_{winding}}]$$

2.4.2 Three phase three winding transformer equivalent circuit model

Three phase three winding open circuit impedance matrix

The discussion in Section 2.4.1 presents a modelling procedure for a three phase two winding delta-star connected transformer. The main focus of the procedure is to develop the three phase two winding open circuit impedance matrix as shown in Equation (2.35). Using this same procedure, a model for a three phase, three winding transformer can be developed. In order to establish the open circuit impedance matrix the first step would be to consider the transformer as a three limbed core this time with nine windings i.e. three windings on each limb. The diagram in Figure 2.16 presents an illustration of this and once again it must be noted that the diagram does not show any flux paths or resistors as it is merely a representation of a physical core with nine windings. As with the six winding case, using Faraday's Law of Induction if a current were to flow in winding 1, 4 and 7 (considered the primary windings) a voltage will be induced in winding 2, 3, 5, 6, 8 and 9 (considered the secondary windings) respectively.

It follows that if current flows in the primary windings, Winding 1 will be mutually coupled to Winding 2 and 3, Winding 4 will be mutually coupled to Winding 5 and 6 and Winding 7 will be mutually coupled to Winding 8 and 9. Through the mutual coupling, a mutual inductance is established. It must be noted however that because the windings share the same core, there will also be a mutual inductance between adjacent windings. For example there will be a mutual inductance between winding 1 and winding 9. Therefore in order to represent the three phase three winding transformer, the mutual inductance between each winding as well as the self-inductance of each winding must be represented within the model. Using the procedures of Section 2.4.1, the open circuit impedance matrix in Equation (2.38) can be developed which expresses the primary and secondary voltages as functions of the impedance values and primary and secondary current values.

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} & z_{18} & z_{19} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} & z_{28} & z_{29} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} & z_{38} & z_{39} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} & z_{48} & z_{49} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} & z_{58} & z_{59} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} & z_{68} & z_{69} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} & z_{78} & z_{79} \\ z_{81} & z_{82} & z_{83} & z_{84} & z_{85} & z_{86} & z_{87} & z_{88} & z_{89} \\ z_{91} & z_{92} & z_{93} & z_{94} & z_{95} & z_{96} & z_{97} & z_{98} & z_{99} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} \qquad \textbf{(2.38)}$$

$$[\mathbf{V_{winding}}] = [\mathbf{Z_{winding}}][\mathbf{I_{winding}}]$$

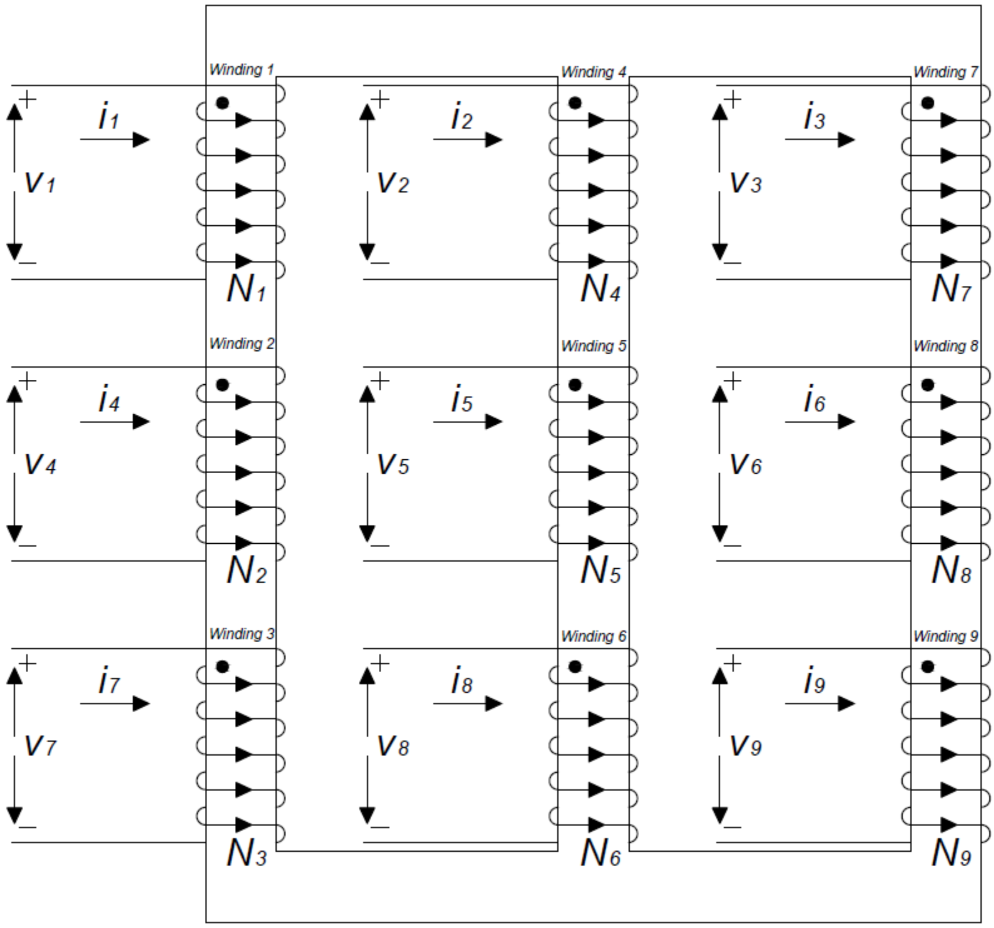


Figure 2.16: Simplified model of a three winding three phase transformer on a common core. Note that at this stage no connection scheme has been assigned to the transformer.

Application of the delta-zig-zag connection

As already mentioned with regard to the delta-star model, the matrix equation in Equation (2.38) does not represent the delta-zig-zag connection, instead a connection matrix is required. One thing to note is that the interconnections between the secondary windings will have to be taken into account first before considering the connection matrix. What this means is that certain windings will be connected such that some of the secondary winding currents in Equation (2.38) will be forced to be equal. Consider the diagram in Figure 2.17 which represents a zig-zag connection between the secondary windings based on the schematic in Figure 2.2. The dotted line is used to illustrate the fact that the zig-zag connection is an internal connection and that once the connection has been done the transformer essentially resembles a three phase two winding transformer. From the diagram it can be seen that the terminal voltages can be written in terms of the winding voltages. The lower case letter *w* is used to represent the terminal voltages and currents.

$$V_{w1} = V_1$$
$$V_{w3} = V_4$$
$$V_{w5} = V_7$$
$$V_{w2} = V_3 - V_8$$
$$V_{w4} = V_6 - V_2$$
$$V_{w6} = V_9 - V_5$$

The expressions above can be re-written in matrix form to yield the matrix in equation in Equation (2.39) where the upper case letter *W* is used to represent the internal connection matrix.

$$\begin{pmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \\ V_{w4} \\ V_{w5} \\ V_{w6} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{pmatrix} \qquad \textbf{(2.39)}$$

$$[\mathbf{V_{terminal}}] = [\mathbf{W}][\mathbf{V_{winding}}]$$

The terminal currents can therefore be expressed in terms of the winding currents as:

$$I_1 = I_{w1}$$
$$I_2 = -I_{w4}$$
$$I_3 = I_{w2}$$
$$I_4 = I_{w3}$$
$$I_5 = -I_{w6}$$
$$I_6 = I_{w4}$$
$$I_7 = I_{w5}$$
$$I_8 = -I_{w2}$$
$$I_9 = I_{w6}$$

The currents above can also be expressed in matrix form to yield Equation (2.40) where **[W ^T^]** is the transpose of the internal connection matrix **[W]**.

$$\begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \qquad \textbf{(2.40)}$$

$$[\mathbf{I_{winding}}] = [\mathbf{W^T}][\mathbf{I_{terminal}}]$$

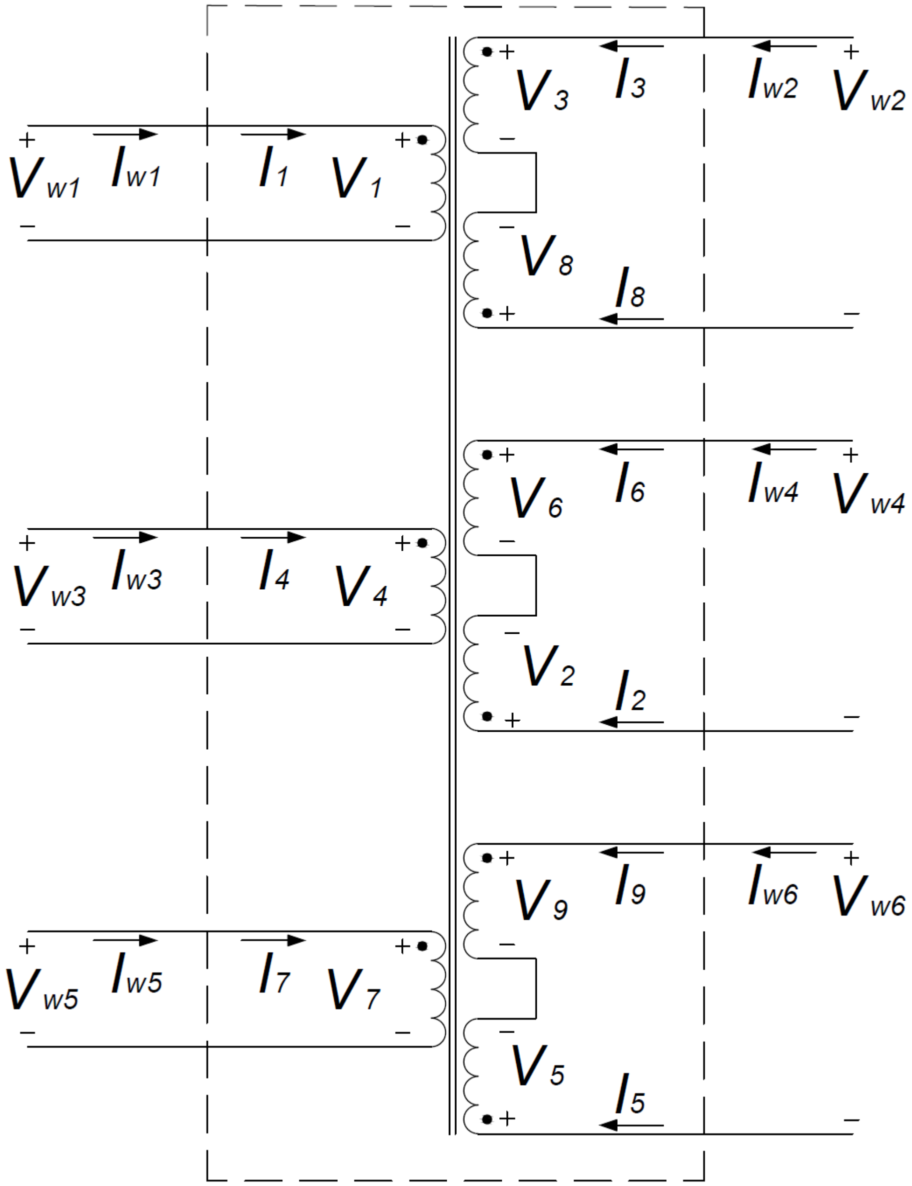


Figure 2.17: Simplified depiction of the windings in a three phase three winding transformer with the secondary side connected using the zig-zag connection scheme

If the terminals are to be connected to represent the complete model then the circuit diagram in Figure 2.18 would apply. From the circuit diagram, the relationship between the terminal voltages and the phase supply and load voltages are as follows:

$$V_{w1} = V_A - V_B$$
$$V_{w2} = V_a$$
$$V_{w3} = V_B - V_C$$
$$V_{w4} = V_b$$
$$V_{w5} = V_C - V_A$$
$$V_{w6} = V_c$$

By rewriting the voltage relationships in matrix form, the matrix equation in Equation (2.41) can be developed. The matrix that converts the phase voltages on the supply and load side to the connection specific winding voltages of the delta- zig-zag transformer is therefore the external connection matrix **[C]**.

$$\begin{pmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \\ V_{w4} \\ V_{w5} \\ V_{w6} \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} V_A \\ V_B \\ V_C \\ V_a \\ V_b \\ V_c \end{pmatrix} \qquad \textbf{(2.41)}$$

$$[\mathbf{V_{terminal}}] = [\mathbf{C}][\mathbf{V_{phase-supply-load}}]$$

The terminal currents can also be expressed in terms of the phase currents. This is shown by the matrix equation in Equation (2.42)

$$\begin{pmatrix} I_A \\ I_B \\ I_C \\ I_a \\ I_b \\ I_c \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & -1 & 0 \\ -1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \qquad \textbf{(2.42)}$$

$$[\mathbf{I_{phase-supply-load}}] = [\mathbf{C^T}][\mathbf{I_{terminal}}]$$

At this stage, there are three important aspects concerning the harmonic mitigating transformer or delta-zig-zag transformer that have been presented. Firstly, a nine-by-nine impedance matrix is used to represent the self-impedances and mutual impedances of a three limbed core with three windings on each limb. This matrix is called an open circuit impedance matrix or winding impedance matrix. The values within the matrix are based on the fact that when a current flows in one winding, a current will be induced in the surrounding windings due to the flux linkage within the common core (Faraday's Law of Induction). By considering Ohm's Law, the voltages across each winding can be represented by the impedance matrix values multiplied by the current circulating in each winding. This relationship is shown in Equation (2.38). Secondly if the core with the nine windings is used to represent the delta-zig-zag transformer, it must be connected accordingly. The connection however involves two parts, first an internal connection and then an external connection. The internal connection is necessary in order to form the zig-zag connection. The external connection is then based on a typical delta-star connection which can be seen by comparing Equation (2.36) and Equation (2.41). Finally concerning the triplen harmonics, it was shown in Section 2.2.1 that for a delta-star connection, the triplen harmonics will only be detectable in the primary and secondary winding currents $[\mathbf{I_{winding}}]$ and not in the phase supply and load currents $[\mathbf{I_{phase\text{-}supply\text{-}load}}]$. For the harmonic mitigating transformer however it was shown in Section 2.2.2 that the third harmonic will only be detectable in the secondary side terminal currents $[\mathbf{I_{terminal}}]_{\mathbf{Secondary}}$ and not in the primary side terminal currents $[\mathbf{I_{terminal}}]_{\mathbf{Primary}}$ nor will the third harmonic be detectable in the phase supply and load currents $[\mathbf{I_{phase\text{-}supply\text{-}load}}]$.

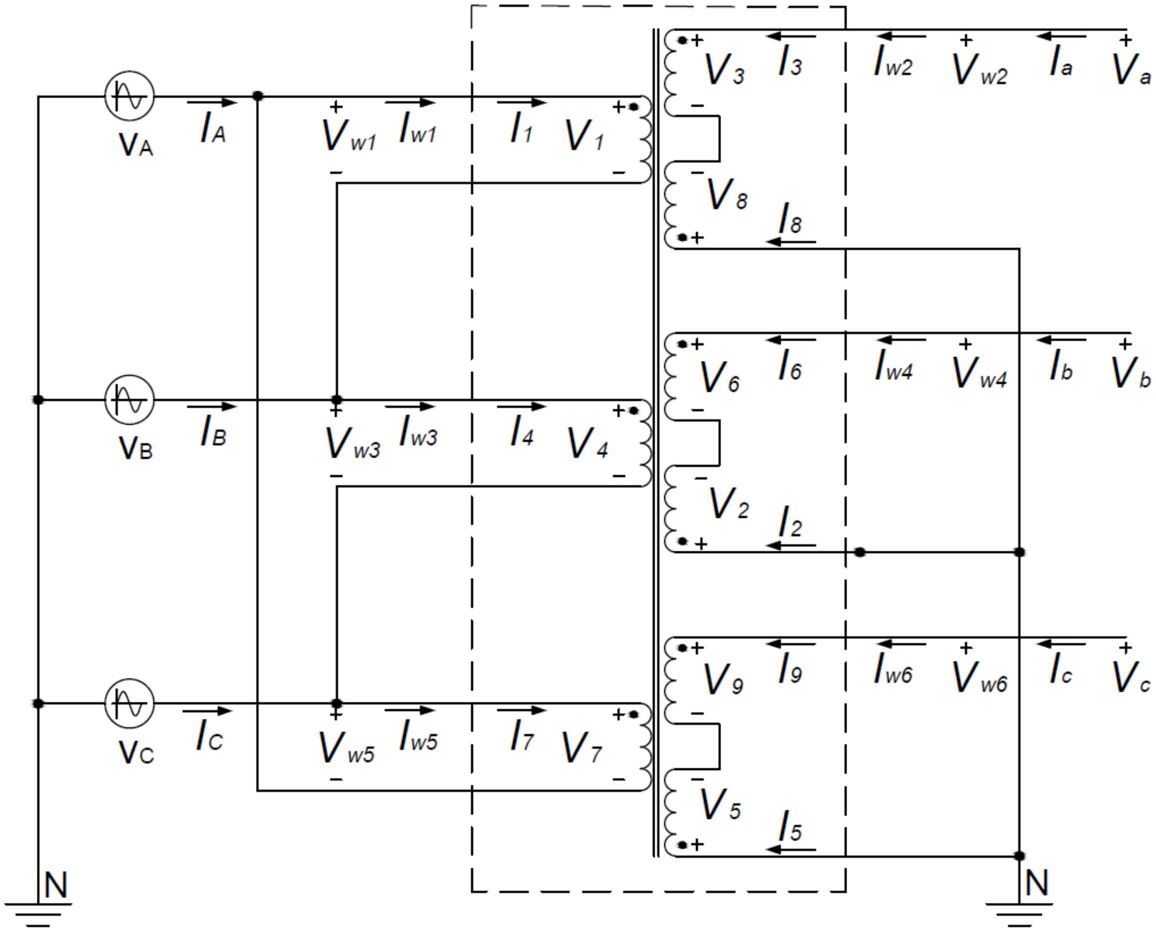


Figure 2.18: Simplified depiction of the windings in a three phase three winding transformer with the primary side connected in the delta connection scheme and the secondary side connected using the zig-zag connection scheme. A three phase source represented by V_A, V_B and V_C is also shown connected to the primary side of the transformer.

2.5 Harmonic mitigating transformer equivalent circuit model

2.5.1 Modelling the harmonic mitigating transformer under no-load conditions

The delta-zig-zag transformer model presented in the previous section is considered as being a harmonic mitigating transformer particularly because of the zig-zag connected secondary side. This section seeks to develop the model further so that it can be used in the more general case whereby the primary can assume any type of transformer connection. Recall that for a three phase three winding transformer it was shown that the currents circulating within each winding, when a voltage is applied to the primary windings can be expressed in terms of the terminal phase currents by applying the internal connection matrix, as shown in Equation (2.40). Now according to the controlled source models in Section 2.2 the triplen harmonic currents would be associated with terminal currents I_{w2}, I_{w4} and I_{w6} and if the transformer is a harmonic mitigating transformer then there should be no triplen harmonic currents in I_{w1}, I_{w3} and I_{w5}. In order to confirm that the open circuit impedance model of a harmonic mitigating transformer can indeed achieve this, the equivalent circuit model must suitably represent these currents.

Consider the equation in Equation (2.43) which represents the three phase three winding transformer open circuit impedance model. If the secondary side of the transformer is zig-zag connected then the terminal voltages can be determined via the use of an internal connection matrix, shown in Equation (2.44) as [W]. By substituting Equation (2.43) into Equation (2.44), the terminal voltages can be expressed in terms of the winding impedances and winding currents as shown in Equation (2.45). If Equation (2.40) expressing the winding currents in terms of the terminal phase currents is substituted into Equation (2.45) then the terminal voltages can be expressed in terms of the terminal currents as shown in Equation (2.46) which consequently leads to an expression for the terminal voltages in terms of the terminal impedances and terminal currents as shown in Equation (2.47). Equation (2.47) shows that the nine-by-nine winding matrix can be reduced to a six-by-six matrix which in a general sense allows the model to be used as an unconnected three phase two winding transformer which simply requires the connection matrix [C] to make it a delta-star transformer. For this part of the investigation, the external connection matrix is not explicitly required henceforth the equivalent circuit model for a harmonic mitigating transformer will be represented by Equation (2.47).

It must be noted though that the matrix equation for the transformer in Equation (2.47) essentially represents Ohm's Law. This implies that if all the terminal phase currents and impedances are known then the terminal phase voltages can easily be determined. This is true, however one has to take into account the fact that the model is for a transformer. What this means is that technically only the primary voltages have to be known and using the winding ratios (which are incorporated in the impedance values), the model should be able to predict or determine the primary currents, secondary voltages and secondary currents.

$$[\mathbf{V_{winding}}] = [\mathbf{Z_{winding}}]\,[\mathbf{I_{winding}}] \tag{2.43}$$

$$[\mathbf{V_{terminal}}] = [\mathbf{W}]\,[\mathbf{V_{winding}}] \tag{2.44}$$

$$[\mathbf{V_{terminal}}] = [\mathbf{W}]\,[\mathbf{Z_{winding}}]\,[\mathbf{I_{winding}}] \tag{2.45}$$

$$\begin{pmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \\ V_{w4} \\ V_{w5} \\ V_{w6} \end{pmatrix} = [\mathbf{W}] \begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} & z_{18} & z_{19} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} & z_{28} & z_{29} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} & z_{38} & z_{39} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} & z_{48} & z_{49} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} & z_{58} & z_{59} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} & z_{68} & z_{69} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} & z_{78} & z_{79} \\ z_{81} & z_{82} & z_{83} & z_{84} & z_{85} & z_{86} & z_{87} & z_{88} & z_{89} \\ z_{91} & z_{92} & z_{93} & z_{94} & z_{95} & z_{96} & z_{97} & z_{98} & z_{99} \end{pmatrix} [\mathbf{W^T}] \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \tag{2.46}$$

$$[\mathbf{V_{terminal}}] = [\mathbf{W}]\,[\mathbf{Z_{winding}}]\,[\mathbf{W^T}][\mathbf{I_{terminal}}]$$

$$\begin{pmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \\ V_{w4} \\ V_{w5} \\ V_{w6} \end{pmatrix} = \begin{pmatrix} z_{11} & z_{13} - z_{18} & z_{14} & z_{16} - z_{12} & z_{17} & z_{19} - z_{15} \\ z_{31} - z_{81} & z_{33} - z_{83} - z_{38} + z_{88} & z_{34} - z_{84} & z_{82} - z_{32} - z_{86} + z_{36} & z_{37} - z_{87} & z_{85} - z_{35} - z_{89} + z_{39} \\ z_{41} & z_{43} - z_{48} & z_{44} & z_{46} - z_{42} & z_{47} & z_{49} - z_{45} \\ z_{61} - z_{21} & z_{63} - z_{23} - z_{68} + z_{28} & z_{64} - z_{24} & z_{22} - z_{62} - z_{26} + z_{66} & z_{67} - z_{27} & z_{25} - z_{65} - z_{29} + z_{69} \\ z_{71} & z_{73} - z_{78} & z_{74} & z_{76} - z_{72} & z_{77} & z_{79} - z_{75} \\ z_{91} - z_{51} & z_{93} - z_{53} - z_{98} + z_{58} & z_{94} - z_{54} & z_{52} - z_{92} - z_{56} + z_{96} & z_{97} - z_{57} & z_{55} - z_{95} - z_{59} + z_{99} \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \tag{2.47}$$

$$[\mathbf{V_{terminal}} = [\mathbf{Z_{terminal}}]\,[\mathbf{I_{terminal}}]$$

2.5.2 Operation of the harmonic mitigating transformer under load conditions

The matrix equation for the transformer in Equation (2.47) essentially represents the harmonic mitigating transformer with no-load attached. If a load were to be connected to each phase on the secondary side of the transformer then Equation (2.33) and Equation (2.34) for a single phase transformer can be applied. Equation (2.33) shows that the impedance of the supply can be lumped together with the self-impedance of the primary winding and Equation (2.34) shows that the impedance of the load can be lumped with the self-impedance of the secondary winding. In terms of the harmonic mitigating transformer, if a load were to be attached to the secondary side, then the current flowing through the load would cause a voltage to develop. The voltage will be the secondary voltage and the relationship for the current and voltage for each phase will be:

$$|V_{w2}| = r_L |I_{w2}|$$

$$|V_{w4}| = r_L |I_{w4}|$$

$$|V_{w6}| = r_L |I_{w6}|$$

This leads to Equation (2.48) which is a representation of the harmonic mitigating transformer under load conditions. The secondary terminal voltages are shown to be zero on the left hand side of the equation since they are implicitly included via the inclusion of the load resistance with the secondary self-impedances. The equation shows that only the primary terminal phase voltages and load resistance values are required in order to solve for the primary terminal phase currents and secondary terminal phase currents. Equation (2.48) can also be used under no-load conditions by setting the value of each load resistance to equal a very large value so as to represent an open circuit. The model can also be used to represent the transformer under short circuit conditions by setting each load resistance value to zero. The model in Equation (2.48) reveals that there are six unknown variables that must be solved for, namely the terminal phase currents. The model also shows that there are six equations that can be used to solve for the six unknowns. This means that the six unknowns can be solved for using simultaneous equations. The solution although linear is somewhat difficult to solve for by hand therefore a numerical computing program can be used to implement the model in Equation (2.48). The numerical program used in this investigation is Matlab and to solve for the terminal phase currents a function was written in Matlab to implement the model in Equation (2.48). The input to the function is the winding impedance values, the primary terminal phase voltages (or secondary terminal phase voltages as discussed in Chapter 5) and the value of the resistive load and source resistance.

$$\begin{pmatrix} V_{w1} \\ 0 \\ V_{w3} \\ 0 \\ V_{w5} \\ 0 \end{pmatrix} = \begin{pmatrix} z_{11} + r_s & z_{13} - z_{18} & z_{14} & z_{16} - z_{12} & z_{17} & z_{19} - z_{15} \\ z_{31} - z_{81} & z_{33} - z_{83} - z_{38} + z_{88} + r_L & z_{34} - z_{84} & z_{82} - z_{32} - z_{86} + z_{36} & z_{37} - z_{87} & z_{85} - z_{35} - z_{89} + z_{39} \\ z_{41} & z_{43} - z_{48} & z_{44} + r_s & z_{46} - z_{42} & z_{47} & z_{49} - z_{45} \\ z_{61} - z_{21} & z_{63} - z_{23} - z_{68} + z_{28} & z_{64} - z_{24} & z_{22} - z_{62} - z_{26} + z_{66} + r_L & z_{67} - z_{27} & z_{25} - z_{65} - z_{29} + z_{69} \\ z_{71} & z_{73} - z_{78} & z_{74} & z_{76} - z_{72} & z_{77} + r_s & z_{79} - z_{75} \\ z_{91} - z_{51} & z_{93} - z_{53} - z_{98} + z_{58} & z_{94} - z_{54} & z_{52} - z_{92} - z_{56} + z_{96} & z_{97} - z_{57} & z_{55} - z_{95} - z_{59} + z_{99} + r_L \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \quad \textbf{(2.48)}$$

2.6 Application of the equivalent circuit model

The matrix model in Equation (2.47) is the equivalent circuit model of a harmonic mitigating transformer. In order to understand the nature of the harmonic mitigating transformer and how best to optimise it, the model was used to demonstrate the performance of an ideal and non-ideal harmonic mitigating transformer. For the demonstration, two particular cases were investigated the details of which are presented in Chapter 3 and Chapter 4. For now though a brief description of each case is presented.

2.6.1 Case A – Transformer with ideal couplings

For this situation the equivalent circuit model is used to represent a transformer with ideal couplings. In order to understand what is meant by ideal couplings, the characteristics that allow a transformer to be ideal are examined. The results from the examination present a procedure for calculating the values of the individual impedances of the open circuit impedance matrix. The values in the matrix will adhere to the ideal conditions which will allow for the relationship between the transformer voltages and currents under ideal conditions to be determined. Testing the model under no-load and load conditions allowed for the calculation of the voltage regulation for the ideal transformer. A more detailed discussion is presented in Chapter 3.

2.6.2 Case B – Transformer with non-ideal couplings

In order to use the equivalent circuit model to represent a transformer with non-ideal couplings, a physical harmonic mitigating transformer is required. Such a transformer was acquired and using the transformer, two sets of measurements were obtained. The first set of measurements obtained was the voltages and currents of the physical transformer under no-load and load conditions. From these measurements, the voltage regulation of the physical transformer is calculated. The second set of measurements taken was the actual self and mutual impedance values of the transformer. These values were used to populate the open circuit impedance matrix so that an equivalent circuit model with non-ideal couplings could be used to represent the physical transformer. The model is then evaluated under no-load and load conditions and from these results the voltage regulation of the model is calculated. A more detailed discussion is presented in Chapter 4.

2.7 Conclusion

This chapter presented two key concepts. The first is the idea that an ideal harmonic mitigating transformer can remove all the triplen harmonics that proliferate in a power system. The transformer is able to achieve this via the zig-zag connection scheme which prevents the flow of the triplen harmonics to the primary side of the transformer. This concept was demonstrated using a controlled source model of a harmonic mitigating transformer supplied with a third harmonic current. The second key concept demonstrated that a non-ideal harmonic mitigating transformer can be suitably modelled as a network of nine coupled inductors or windings on a common core. This concept is based on the classical theory of coupled inductor circuits and by using this concept, an open circuit impedance matrix of a harmonic mitigating transformer was obtained.

The use of an open circuit impedance matrix allows one to model the relationships between the transformer's terminal voltages and currents. The matrix is able to achieve this because the impedances within the matrix encompass the winding ratios of the transformer and the coupling relationships between the windings. The impedance matrix also implicitly represents the leakages occurring within the transformer therefore allowing a true representation of the transformer from an equivalent circuit model perspective. Based on this, the focus of the next two Chapters is to use the equivalent circuit model to develop an ideal and a practical model of a harmonic mitigating transformer. The aim will be to determine the nature of the parameters in the practical model that cause it to deviate from the ideal model and in particular the parameters that reduce its ability to mitigate the triplen harmonics.

CHAPTER 3: MODEL OF A HARMONIC MITIGATING TRANSFORMER WITH IDEAL COUPLINGS

3.1 Introduction

As discussed in Section 2.6 of Chapter 2, one of the main aspects of this dissertation is to compare an ideal harmonic mitigating transformer to a physical one using the equivalent circuit model developed in Chapter 2. The eventual aim of this comparison is to establish whether the non-ideal characteristics of a physical harmonic mitigating transformer affect its ability to mitigate the third harmonic. The answer to this question is discussed in Chapter 5. This chapter presents an ideal model of a harmonic mitigating transformer based on the modelling techniques presented in Chapter 2. The term ideal in this case means that the variables within the model will be chosen such that the performance of the model will resemble the ideal case. The variables of particular concern in this situation will be the individual impedance values within the open circuit impedance matrix. In order to select values for the individual impedances that will allow the model to resemble the ideal case, one has to determine the characteristics of the transformer that prevent it from resembling the ideal case. Naturally at this stage one will agree that it is the transformer leakages. This chapter seeks to establish how the transformer leakages can be removed or at least minimised so that they no longer affect the transformer subsequently allowing a true ideal model of a harmonic mitigating transformer to be developed.

3.2 Conditions pertaining to an ideal harmonic mitigating transformer

When considering an ideal transformer it is understood that the following conditions apply [11]:

1. The resistances of the windings are negligibly small;
2. The core of the transformer has infinite permeability which means that the exciting current required to establish the flux in the core is negligible [10];
3. The core losses are negligibly small,
4. Symmetrical and even flux distribution which implies that:
    - windings on the same side of the transformer have to be identical i.e. all properties of the primary windings are the same and all properties of the secondary windings are the same,
    - all points in the core material are homogenous,
    - all flux paths between windings are equal (i.e. for a three limbed core, no limb is in the middle rather the limbs are arranged in a triangular shape);
5. There is no leakage flux between the primary and secondary windings and
6. Capacitive effects of the windings can be ignored

3.2.1 Condition 1: Negligible winding resistance

With these conditions in mind the first step to setting up the ideal transformer model would be to consider the diagram is shown in Figure 2.12 [12]. Considering condition 1, it is clear that the resistances r_1 and r_2 in the diagram must be zero. This implies that for the steady state case the self-impedances in the matrix equivalent circuit model shown in Equation (2.30) must not contain a real part i.e. they must be purely inductive. What is not explicitly stated in Equation (2.30) is that the mutual impedances must also be purely inductive. This means that any real part of a mutual impedance is not ideal and must therefore be considered to be negligibly small.

3.2.2 Condition 2: Infinite core permeability

In order to satisfy condition 2 the inductances of an ideal transformer have to be infinitely large which ensures that the magnetising current is negligible [11]. Condition 2 can be satisfied theoretically and can be suitably applied when using the impedance matrix; however a problem arises if the admittance matrix were to be used. As mentioned in Appendix A, the admittance matrix is determined by the inversion of the impedance matrix. If the coupling between the two windings approximates unity then the inversion process becomes numerically unstable. If the admittance matrix were to be used care must be taken when performing the inversion. The self-impedance values for the ideal model will have to be selected based on practical values noting that according to condition 1, the values will not have real components. Once the self-impedance values are selected, the mutual impedance values can be determined based on the coupling coefficient between each winding. This is further explained under condition 4.

3.2.3 Condition 3: Omission of core losses

With regard to condition 3 it has already been discussed that core losses will be omitted from both the ideal and practical models for reasons already discussed in Chapter 2.

3.2.4 Condition 4: Symmetrical and even flux distribution

When considering the flux circulating in an ideal transformer, if the flux paths are symmetrically distributed and the windings are identical then the impedance values in Equation (3.1) below for the steady state case can be grouped accordingly in order to represent similar self and mutual impedance values as one value that will represent the collective as shown in Equation (3.2). If condition 1 were to apply then all the impedance values in Equation (3.2) would have to be imaginary i.e. the impedances would have no real part.

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} & z_{18} & z_{19} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} & z_{28} & z_{29} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} & z_{38} & z_{39} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} & z_{48} & z_{49} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} & z_{58} & z_{59} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} & z_{68} & z_{69} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} & z_{78} & z_{79} \\ z_{81} & z_{82} & z_{83} & z_{84} & z_{85} & z_{86} & z_{87} & z_{88} & z_{89} \\ z_{91} & z_{92} & z_{93} & z_{94} & z_{95} & z_{96} & z_{97} & z_{98} & z_{99} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} \quad \textbf{(3.1)}$$

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{pmatrix} = \begin{pmatrix} z_{pp} & z_{ps1} & z_{ps1} & z_{pp1} & z_{ps2} & z_{ps2} & z_{pp1} & z_{ps2} & z_{ps2} \\ z_{ps1} & z_{ss} & z_{ss1} & z_{ps2} & z_{ss2} & z_{ss2} & z_{ps2} & z_{ss2} & z_{ss2} \\ z_{ps1} & z_{ss1} & z_{ss} & z_{ps2} & z_{ss2} & z_{ss2} & z_{ps2} & z_{ss2} & z_{ss2} \\ z_{pp1} & z_{ps2} & z_{ps2} & z_{pp} & z_{ps1} & z_{ps1} & z_{pp1} & z_{ps2} & z_{ps2} \\ z_{ps2} & z_{ss2} & z_{ss2} & z_{ps1} & z_{ss} & z_{ss1} & z_{ps2} & z_{ss2} & z_{ss2} \\ z_{ps2} & z_{ss2} & z_{ss2} & z_{ps1} & z_{ss1} & z_{ss} & z_{ps2} & z_{ss2} & z_{ss2} \\ z_{pp1} & z_{ps2} & z_{ps2} & z_{pp1} & z_{ps2} & z_{ps2} & z_{pp} & z_{ps1} & z_{ps1} \\ z_{ps2} & z_{ss2} & z_{ps2} & z_{ps2} & z_{ss2} & z_{ss2} & z_{ps1} & z_{ss} & z_{ss1} \\ z_{ps2} & z_{ss2} & z_{ps2} & z_{ps2} & z_{ss2} & z_{ss2} & z_{ps1} & z_{ss1} & z_{ss} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} \quad \textbf{(3.2)}$$

Self-impedance values of the primary windings:

$$z_{pp} \rightarrow z_{11}, z_{44}, z_{77}$$

Self-impedance values of the secondary windings (Note that the number of turns on the secondary windings is equal):

$$z_{ss} \rightarrow z_{22}, z_{33}, z_{55}, z_{66}, z_{88}, z_{99}$$

Mutual impedance values between the primary winding secondary windings on the same core limb:

$$z_{ps1} \rightarrow z_{12}, z_{21}, z_{45}, z_{54}, z_{78}, z_{87}, z_{13}, z_{31}, z_{46}, z_{64}, z_{79}, z_{97}$$

Mutual impedance values between the secondary windings on the same core limb:

$$z_{ss1} \rightarrow z_{23}, z_{32}, z_{56}, z_{65}, z_{89}, z_{98}$$

Mutual impedance values between the primary winding on one core limb and the primary winding on the adjacent core limbs:

$$z_{pp1} \rightarrow z_{14}, z_{41}, z_{17}, z_{71}, z_{47}, z_{74}$$

Mutual impedance values of the primary winding on one core limb and the secondary windings on the adjacent core limbs:

$$z_{ps2} \rightarrow z_{15}, z_{51}, z_{18}, z_{81}, z_{42}, z_{24}, z_{48}, z_{84}, z_{72}, z_{27}, z_{75}, z_{57}, z_{16}, z_{61}, z_{19}, z_{91}, z_{43}, z_{34}, z_{49}, z_{94}, z_{73}, z_{37}, z_{76}, z_{67}$$

Mutual impedance values of the secondary windings on one core limb and the secondary windings on the adjacent core limbs:

$$z_{ss2} \rightarrow z_{25}, z_{52}, z_{26}, z_{62}, z_{28}, z_{82}, z_{29}, z_{92}, z_{35}, z_{53}, z_{36}, z_{63}, z_{38}, z_{83}, z_{58}, z_{85}, z_{59}, z_{95}$$

3.2.5 Condition 5: No leakage flux between windings

Single phase two winding transformer

As already discussed in Section 2.3.1, a practical transformer has a certain amount of leakage flux associated with it. The leakage flux path per winding in a single phase transformer is shown in Figure 2.12 and is expressed as Φ_{1l} for winding 1 and Φ_{2l} for winding 2. The leakage flux per winding can be represented by the relationship in Equation (3.3) and Equation (3.4) respectively. It should be apparent that from Equation (3.3) in order to ensure that there is no leakage flux then the flux produced in winding 1, Φ_{11}, must be equal to the mutual flux, Φ_{12}. The same can be said for the leakage flux of winding 2. The flux for winding 1 and winding 2 is expressed in Equation (3.5) and Equation (3.6) respectively and the mutual flux between each winding is expressed in Equation (3.7) and Equation (3.8). By substituting Equation (3.5) and Equation (3.8) into Equation (3.3) the leakage flux Φ_{1l} can be expressed in terms of the self and mutual inductance of winding 1 as shown in Equation (3.9) noting that the turns ratio a has to be taken into account. The same can be applied to winding 2 this time by substituting Equation (3.6) and Equation (3.7) into Equation (3.4) the leakage flux Φ_{2l} can be expressed in terms of the self and mutual inductance of winding 2 as shown in Equation (3.10).

$$\Phi_{1l} = \Phi_{11} - \Phi_{21} \tag{3.3}$$

$$\Phi_{2l} = \Phi_{22} - \Phi_{12} \tag{3.4}$$

$$\Phi_{11} = \frac{L_{11} i_1}{N_1} \tag{3.5}$$

$$\Phi_{22} = \frac{L_{22} i_2}{N_2} \tag{3.6}$$

$$\Phi_{12} = \frac{L_{12} i_2}{N_1} \tag{3.7}$$

$$\Phi_{21} = \frac{L_{21} i_1}{N_2} \tag{3.8}$$

$$\begin{gathered} \Phi_{1l} = \Phi_{11} - \Phi_{21} = \frac{L_{11} i_1}{N_1} - \frac{L_{21} i_1}{N_2} = \frac{i_1}{N_1 N_2} \left(N_2 L_{11} - N_1 L_{21}\right) = \frac{i_1}{N_1} \left(L_{11} - aL_{21}\right) \\ \Phi_{1l} = \frac{i_1}{N_1} \left(L_{11} - aL_{21}\right) \end{gathered} \tag{3.9}$$

$$\begin{gathered} \Phi_{2l} = \Phi_{22} - \Phi_{12} = \frac{L_{22} i_2}{N_2} - \frac{L_{12} i_2}{N_1} = \frac{i_2}{N_1 N_2} \left(N_1 L_{22} - N_2 L_{12}\right) = \frac{i_2}{N_2} \left(L_{22} - \frac{L_{12}}{a}\right) \\ \Phi_{2l} = \frac{i_2}{N_2} \left(L_{22} - \frac{L_{12}}{a}\right) \end{gathered} \tag{3.10}$$

Equation (3.11) shows the relationship between the mutual inductance and self-inductance. The variable k is the coefficient of coupling which will assume a value between zero and unity depending on the arrangement of the transformer windings. Equation (3.12) expresses the self-inductances in terms of the number of turns per winding i.e. the turns ratio. Substituting Equation (3.12) into Equation (3.11) and simplifying yields the equations in Equation (3.13) and Equation (3.14) depending on whether L_{11} or L_{22} was solved in Equation (3.12). By substituting Equation (3.13) into Equation (3.9) it can be seen from Equation (3.15) that in order for there to be zero leakage flux for winding 1, the coefficient of coupling between the primary winding and secondary winding must be unity. Similarly if Equation (3.14) is substituted into Equation (3.10) it can be seen from Equation (3.16) that in order for there to be zero leakage flux for winding 2, the coefficient of coupling between the secondary winding and primary winding must be unity. It can therefore be seen that the coefficient of coupling plays a vital role in establishing whether or not a transformer behaves ideally or not.

$$L_{12} = L_{21} = k\sqrt{L_{11}L_{22}} \tag{3.11}$$

$$\frac{L_{11}}{L_{22}} = \left(\frac{N_1}{N_2}\right)^2 \tag{3.12}$$

Let:

$$L_{11} = L_{22}(\frac{N_1}{N_2})^2$$

$$L_{12} = k\sqrt{L_{22}L_{22}(\frac{N_1}{N_2})^2} = kL_{22}(\frac{N_1}{N_2}) = akL_{22}$$

$$L_{12} = akL_{22} \tag{3.13}$$

Let:

$$L_{22} = \frac{L_{11}}{(\frac{N_1}{N_2})^2}$$

$$L_{21} = k\sqrt{L_{11}\frac{L_{11}}{(\frac{N_1}{N_2})^2}} = kL_{11}(\frac{N_2}{N_1}) = \frac{k}{a}L_{11}$$

$$L_{21} = \frac{k}{a}L_{11} \tag{3.14}$$

$$\Phi_{1l} = \frac{i_1}{N_1}\ (L_{11} - aL_{21}) = \frac{i_1}{N_1}\ (L_{11} - a(\frac{k}{a}L_{11})) = \frac{i_1}{N_1}\ (L_{11} - kL_{11})$$

$$\Phi_{1l} = \frac{i_1}{N_1}\ (1 - k)L_{11} \tag{3.15}$$

$$if\ k = 1\ then\ \Phi_{1l} = 0$$

$$\Phi_{2l} = \frac{i_2}{N_2}\ (L_{22} - \frac{L_{12}}{a}) = \frac{i_2}{N_2}\ (L_{22} - \frac{akL_{22}}{a}) = \frac{i_2}{N_2}\ (L_{22} - kL_{22})$$

$$\Phi_{2l} = \frac{i_2}{N_2}\ (1 - k)L_{22} \tag{3.16}$$

$$if\ k = 1\ then\ \Phi_{2l} = 0$$

An important point that must be highlighted is that from the calculations and derivations above, it can be clearly seen that the leakage flux is dependent on the coupling coefficient *k*. Furthermore in terms of the coupled circuit theory, if the leakage fluxes for each winding were to be zero shown as shown Equation (3.17) and Equation (3.18) respectively, it will imply that resultant flux produced in each winding is consequently the super position of the mutual flux generated when a current flows in each winding at the same time. The resultant fluxes are shown in Equation (3.19) and Equation (3.20) respectively.

$$\Phi_{1l} = \Phi_{11} - \Phi_{21} = 0 \quad \textbf{(3.17)}$$

$$\Phi_{2l} = \Phi_{22} - \Phi_{12} = 0 \quad \textbf{(3.18)}$$

$$\Phi_{1} = \Phi_{21} + \Phi_{12} \quad \textbf{(3.19)}$$

$$\Phi_{2} = \Phi_{12} + \Phi_{21} \quad \textbf{(3.20)}$$

Three phase three winding transformer

By taking Equation (3.3) and applying it to three phase three winding case, the leakage flux for winding 1 can be expressed as:

$$\Phi_{1l} = \Phi_{11} - \Phi_{21} - \Phi_{31} - \Phi_{41} - \Phi_{51} - \Phi_{61} - \Phi_{71} - \Phi_{81} - \Phi_{91}$$

Applying the steps above for the two winding case, it can be shown that the leakage flux for winding 1 in the three phase three winding case is dependent on the coefficient of coupling between each of the windings. The same can be said for the leakage fluxes for the other windings that is:

$$\Phi_{1l} = \frac{i_1}{N_1}(1 - k_{21} - k_{31} - k_{41} - k_{51} - k_{61} - k_{71} - k_{81} - k_{91})L_{11}$$

$$\Phi_{2l} = \frac{i_2}{N_2}(1 - k_{12} - k_{32} - k_{42} - k_{52} - k_{62} - k_{72} - k_{82} - k_{92})L_{22}$$

$$\Phi_{3l} = \frac{i_3}{N_3}(1 - k_{13} - k_{23} - k_{43} - k_{53} - k_{63} - k_{73} - k_{83} - k_{93})L_{33}$$

$$\Phi_{4l} = \frac{i_4}{N_4}(1 - k_{14} - k_{24} - k_{34} - k_{54} - k_{64} - k_{74} - k_{84} - k_{94})L_{44}$$

$$\Phi_{5l} = \frac{i_5}{N_5}(1 - k_{15} - k_{25} - k_{35} - k_{45} - k_{65} - k_{75} - k_{85} - k_{95})L_{55}$$

$$\Phi_{6l} = \frac{i_6}{N_6}(1 - k_{16} - k_{26} - k_{36} - k_{46} - k_{56} - k_{76} - k_{86} - k_{96})L_{66}$$

$$\Phi_{7l} = \frac{i_7}{N_7}(1 - k_{17} - k_{27} - k_{37} - k_{47} - k_{57} - k_{67} - k_{87} - k_{97})L_{77}$$

$$\Phi_{8l} = \frac{i_8}{N_8} \left(1 - k_{18} - k_{28} - k_{38} - k_{48} - k_{58} - k_{68} - k_{78} - k_{98}\right) L_{88}$$

$$\Phi_{9l} = \frac{i_9}{N_9} \left(1 - k_{19} - k_{29} - k_{39} - k_{49} - k_{59} - k_{69} - k_{79} - k_{89}\right) L_{99}$$

If one assumes that all the primary windings are identical and all the secondary windings are identical and that in an ideal transformer there must be perfectly symmetrical flux distribution it can be shown that:

$$k_{12} = k_{21} = k_{13} = k_{31} = k_{45} = k_{54} = k_{46} = k_{64} = k_{78} = k_{87} = k_{79} = k_{97} = k_{ps1},$$

which is the coupling coefficient between a primary winding and a secondary winding on the same core leg and

$$k_{14} = k_{41} = k_{17} = k_{71} = k_{47} = k_{74} = k_{pp1},$$

which is the coupling coefficient between a primary winding and a primary winding and

$$k_{23} = k_{32} = k_{56} = k_{65} = k_{89} = k_{98} = k_{ss1},$$

which is the coupling coefficient between a secondary winding and a secondary winding on the same core leg and

$$k_{25} = k_{52} = k_{35} = k_{53} = k_{26} = k_{62} = k_{36} = k_{63} = k_{28} = k_{82} = k_{38} = k_{83} = k_{29} = k_{92} = k_{39} = k_{93} = \\ k_{58} = k_{85} = k_{68} = k_{86} = k_{59} = k_{95} = k_{69} = k_{96} = k_{ss2},$$

which is the coupling coefficient between a secondary winding and a secondary winding on a different core leg and

$$k_{15} = k_{51} = k_{16} = k_{61} = k_{18} = k_{81} = k_{19} = k_{91} = k_{42} = k_{24} = k_{43} = k_{34}$$

$$= k_{48} = k_{84} = k_{49} = k_{94} = k_{72} = k_{27} = k_{73} = k_{37} = k_{75} = k_{57} = k_{76} = k_{67} = k_{ps2},$$

which is the coupling coefficient between a primary winding and a secondary winding on a different core leg.

Substituting in the renamed coupling coefficients leads to:

$$\Phi_{1l} = \frac{i_1}{N_1} \left(1 - k_{ps1} - k_{ps1} - k_{pp1} - k_{ps2} - k_{ps2} - k_{pp1} - k_{ps2} - k_{ps2}\right) L_{11}$$

$$\Phi_{2l} = \frac{i_2}{N_2} \left(1 - k_{ps1} - k_{ss1} - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps2} - k_{ss2} - k_{ss2}\right) L_{22}$$

$$\Phi_{3l} = \frac{i_3}{N_3} \left(1 - k_{ps1} - k_{ss1} - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps2} - k_{ss2} - k_{ss2}\right) L_{33}$$

$$\Phi_{4l} = \frac{i_4}{N_4} \left(1 - k_{pp1} - k_{ps2} - k_{ps2} - k_{ps1} - k_{ps1} - k_{pp1} - k_{ps2} - k_{ps2}\right) L_{44}$$

$$\Phi_{5l} = \frac{i_5}{N_5} \left(1 - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps1} - k_{ss1} - k_{ps2} - k_{ss2} - k_{ss2}\right) L_{55}$$

$$\Phi_{6l} = \frac{i_6}{N_6}\ (1 - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps1} - k_{ss1} - k_{ps2} - k_{ss2} - k_{ss2})L_{66}$$

$$\Phi_{7l} = \frac{i_7}{N_7}\ (1 - k_{pp1} - k_{ps2} - k_{ps2} - k_{pp1} - k_{ps2} - k_{ps2} - k_{ps1} - k_{ps1})L_{77}$$

$$\Phi_{8l} = \frac{i_8}{N_8}\ (1 - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps1} - k_{ss2})L_{88}$$

$$\Phi_{9l} = \frac{i_9}{N_9}\ (1 - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps2} - k_{ss2} - k_{ss2} - k_{ps2} - k_{ss2})L_{99}$$

Simplifying:

$$\Phi_{1l} = \frac{i_1}{N_1}\ (1 - 2k_{ps1} - 2k_{pp1} - 4k_{ps2})L_{11} \qquad \textbf{(3.21)}$$

$$\Phi_{2l} = \frac{i_2}{N_2}\ (1 - k_{ps1} - k_{ss1} - 2k_{ps2} - 4k_{ss2})L_{22} \qquad \textbf{(3.22)}$$

$$\Phi_{3l} = \frac{i_3}{N_3}\ (1 - k_{ps1} - k_{ss1} - 2k_{ps2} - 4k_{ss2})L_{33}$$

$$\Phi_{4l} = \frac{i_4}{N_4}\ (1 - 2k_{ps1} - 2k_{pp1} - 4k_{ps2})L_{44}$$

$$\Phi_{5l} = \frac{i_5}{N_5}\ (1 - k_{ps1} - k_{ss1} - 2k_{ps2} - 4k_{ss2})L_{55}$$

$$\Phi_{6l} = \frac{i_6}{N_6}\ (1 - k_{ps1} - k_{ss2} - 2k_{ps2} - 4k_{ss2})L_{66}$$

$$\Phi_{7l} = \frac{i_7}{N_7}\ (1 - 2k_{ps1} - 2k_{pp1} - 4k_{ps2})L_{77}$$

$$\Phi_{8l} = \frac{i_8}{N_8}\ (1 - k_{ps1} - k_{ss1} - 2k_{ps2} - 4k_{ss2})L_{88}$$

$$\Phi_{9l} = \frac{i_9}{N_9}\ (1 - k_{ps1} - k_{ss1} - 2k_{ps2} - 4k_{ss2})L_{99}$$

Continuing with the notion that for the ideal case, there must be no leakage flux, it can be seen that the coefficients of coupling will have to assume particular values in order to ensure that in each case above, the result of the sum in brackets is zero. Now the question arises as to how to choose values for the different coupling coefficients. The first step is to look at windings on the same core. It can be said that in the ideal case, windings on the same core limb will have a coupling coefficient of unity that is k_{ps1} = 1 and k_{ss1} = 1. The reason being is because the mutual flux produced only circulates within the common core limb therefore all the flux produced by one winding, will link the windings on the same core leg.

To understand how the coupling coefficient comes into play when considering windings on different core limbs, consider the flux circulating within the core. Figure 3.1 shows a three limbed core with a winding on each limb. If a current were to be applied to the central winding, Winding 1, a flux φ_{12} will be produced that links Winding 1 and Winding 2 and a flux φ_{13} will be produced that links winding 1 and winding 3. In the ideal case, the two fluxes produced are equal i.e. $\varphi_{12} = \varphi_{13}$. Now because the core is a three limbed core, there are two paths for the flux to flow therefore each flux must be divided by two as shown in the figure. This is similar to Kirchhoff's current Law for electric circuits. It must be noted however that if the true ideal case were to be assumed then there would be no middle core leg so to speak, rather the core would be arranged in a triangular manner allowing the flux to split in half for *each* leg not just the middle leg i.e. the flux produced will be evenly distributed between the core limbs – Condition 4. If this were not the case then the flux would split unevenly and different values for the coupling coefficients would be obtained. Therefore the values for the coupling coefficients between windings not on the same core limb can be said to be 0.5 that is k_{ps2} = 0.5, k_{ss2} = 0.5 and k_{pp1} = 0.5. The fact that the coupling is not unity does not mean that there is leakage flux. The non-unity value in this case merely shows that only half the flux links one winding and the other half links the other winding. This means that all flux is accounted for and confined to the core.

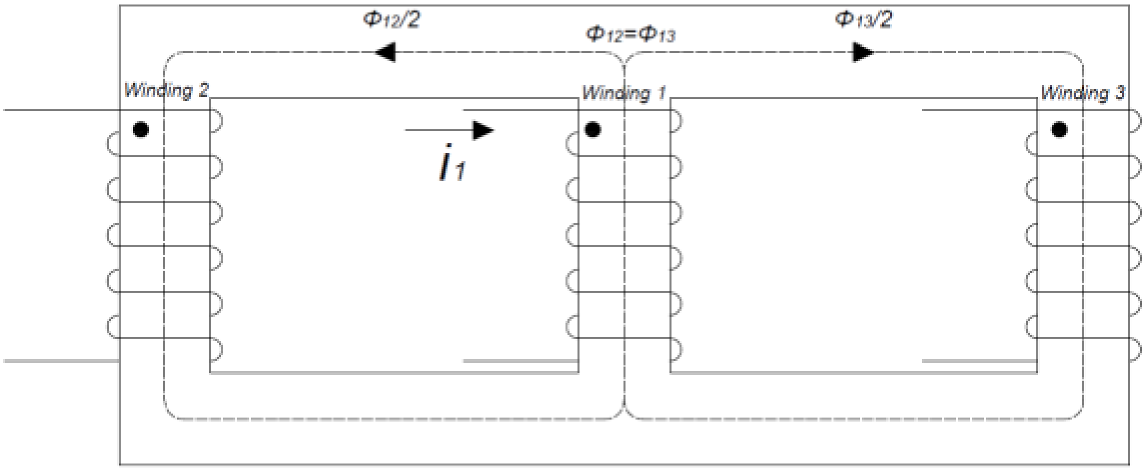


Figure 3.1: Flux distribution in a three limbed core with a single winding on each core limb

It was stated that the variable *k* is the coefficient of coupling which will assume a value between zero and unity depending on the arrangement of the transformer windings. This is true however the coupling coefficient is also dependent on the direction of the current flow. Consider the bracketed term in Equation (3.21) and Equation (3.22) expressed in Equation (3.23) and Equation (3.24) respectively. If the coupling coefficients chosen in the preceding text were applied directly then the equations would not equal zero. In order for the equations to equal zero, k_{ps1} = -1 and k_{ss1} = -1. The fact that the coupling coefficient is negative merely indicates that the direction of the one of the currents is reversed; alternatively the polarity of the windings has to be taken into account.

$$\left(1 - 2k_{ps1} - 2k_{pp1} - 4k_{ps2}\right) = 0 \tag{3.23}$$

$$(1 - 2(-1) - 2(0.5) - 4(0.5)) = 0$$

$$(1 + 2 - 1 - 2) = 0$$

$$\left(1 - k_{ps1} - k_{ss1} - 2k_{ps2} - 4k_{ss2}\right) = 0 \tag{3.24}$$

$$(1 - (-1) - (-1) - 2(0.5) - 4(0.5)) = 0$$

$$(1 + 1 + 1 - 1 - 2) = 0$$

Upon inspecting the individual elements in the open circuit impedance matrix in Equation (3.2), it should be apparent that for the ideal case the coupling coefficients can be expressed in Equation (3.25) as:

$$\begin{aligned} k_{ps1} &= \frac{|z_{ps1}|}{\left|\sqrt{z_{pp}z_{ss}}\right|} = 1 \\ k_{ps2} &= \frac{|z_{ps2}|}{\left|\sqrt{z_{pp}z_{ss}}\right|} = 0.5 \\ k_{pp1} &= \frac{|z_{pp1}|}{\left|\sqrt{z_{pp}z_{pp}}\right|} = 0.5 \\ k_{ss1} &= \frac{|z_{ss1}|}{\left|\sqrt{z_{ss}z_{ss}}\right|} = 1 \\ k_{ss2} &= \frac{|z_{ss2}|}{\left|\sqrt{z_{ss}z_{ss}}\right|} = 0.5 \end{aligned} \tag{3.25}$$

3.2.6 Condition 5: No capacitive effects

Condition 5 only really comes into effect under high frequency conditions which is not the case for this investigation.

3.3 Equivalent circuit model of a harmonic mitigating transformer with ideal couplings

3.3.1 Open circuit impedance matrix

The purpose of developing the harmonic mitigating transformer model with ideal couplings is to provide a benchmark that can be used to rate the performance of a physical harmonic mitigating transformer. The goal will be to identify any major discrepancies between the two transformers and use the areas of discrepancy as the starting point for optimising the physical transformer. The chief foreseeable discrepancy will pertain to the assumption that there is no leakage flux and that the core is symmetrical and evenly distributed regardless of the position of the core limbs. What this means is that the mutual coupling between the windings may not be as predicted in Equation (3.25). The values in the equation however can still be used to determine theoretical values for the open circuit impedance matrix in Equation (3.2) noting that the theoretical values will have to be determined in accordance with condition 1 and condition 2. Once a theoretical winding impedance matrix is established it can be used along with the internal connection matrix to set up the ideal coupling model of the transformer.

In order to develop a theoretical or an ideal winding impedance matrix, values for the self-inductances had to be selected. According to condition 2 these self-inductances would have to be extremely large however it was shown in Appendix A and under condition 4 that this need not necessarily be the case. In other words so long as the couplings are ideal, the inductance values will not have to be infinitely large. So for this investigation an ideal value for the self-inductances was initially selected. The values were calculated based on the parameters of the physical harmonic mitigating transformer presented in Chapter 4. The self-inductance values for a transformer with ideal couplings between the windings are presented in Table 3.1. Using the self-inductance values and the ideal coupling coefficient values presented in Equation (3.25), the mutual inductances were calculated and the values also presented in Table 3.1. The ideal complex impedances were calculated and presented alongside the corresponding inductances in Table 3.1. For more information on how the actual values in Table 3.1 were calculated, please refer to Appendix B.

In Table 3.1, it can be seen that there are two values for each ideal impedance value. The first value was calculated at a frequency of 50 Hz i.e. the fundamental frequency and the second value was calculated at 150 Hz i.e. the third harmonic frequency. The impedances calculated at the third harmonic are used in Chapter 5 however they are presented here to show that the impedance increases when a higher frequency is applied. This indicates that the magnitude of the corresponding current waveform will consequently decrease accordingly [1]. This can be seen in the plots in Figure 1.2 and Figure 1.5. This suggests that one method of stopping the harmonic would be to ensure that the corresponding impedance is substantially high. This however may not be simple to do since the impedances for the fundamental will also be affected. The zig-zag connection however is able to reduce the magnitude of the third harmonic by means of altering the path of the associated flux. This is explicitly shown in Chapter 5.

Table 3.1: Ideal open circuit impedance values for a harmonic mitigating transformer. Values have been presented for a frequency of 50 Hz and 150 Hz.

Ideal coupling coefficient name	Ideal coupling coefficient value	Inductor name	Inductor value (H)	Impedance ($\omega = 2\pi f$)	Impedance value (Ω) f = 50 Hz	Impedance value (Ω) f = 150 Hz
-	-	L_{pp}	4.13	$Z_{pp} = jX_{pp} = j\omega L_{pp}$	1297.48i	3892.43i
-	-	L_{ss}	0.46	$Z_{ss} = jX_{ss} = j\omega L_{ss}$	144.20i	432.60i
k_{ps1}	1.00	L_{ps1}	1.38	$Z_{ps1} = jX_{ps1} = j\omega L_{ps1}$	432.54i	1297.63i
k_{ps2}	0.50	L_{ps2}	0.69	$Z_{ps1} = jX_{ps1} = j\omega L_{ps2}$	216.27i	648.82i
k_{pp1}	0.50	L_{pp1}	2.07	$Z_{pp1} = jX_{pp1} = j\omega L_{pp1}$	648.82i	1946.22i
k_{ss1}	1.00	L_{ss1}	0.46	$Z_{ss1} = jX_{ss1} = j\omega L_{ss1}$	144.20i	432.60i
k_{ss2}	0.50	L_{ss2}	0.23	$Z_{ss1} = jX_{ss1} = j\omega L_{ss2}$	72.10i	216.27i

3.3.2 Ideal coupling model under no-load conditions

Equation (3.26) presents the equivalent circuit model for a harmonic mitigating transformer with ideal couplings. The impedance values in Table 3.1 are used to populate the terminal impedance matrix in this equation. The results for the open circuit test are presented in table form in Appendix F. The source resistance is set to zero in order to emphasise that ideal conditions apply. The primary voltages have equal magnitudes spaced 120 electrical degrees apart therefore representing positive sequence voltage. The primary terminal phase currents are essentially the magnetising currents and are seen to lag the corresponding voltages by 90° which is consistent with the fact that the ideal windings are purely inductive. The secondary terminal currents are zero which is consistent with the open circuit condition.

Since the practical transformer used in this investigation has a 1:1 ratio between the primary and secondary line voltages, the secondary terminal voltages are divided by a factor of √3. The angle of the secondary terminal phase voltages also lags the primary terminal phase voltages by 30°. This is to ensure that when the secondary line voltage is taken it will have the same phase angle as the primary terminal voltage or primary line voltage. This is essentially the reason why the associated delta-zig-zag connection is Dzn0. The phase angle of the secondary terminal phase voltages are expressed as θ since the value is determined from the type of connection. This means that for the Dzn0 connection the phase angle between the line voltages on the primary and secondary side will always be 0° regardless of the load connected.

$$\begin{pmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \\ V_{w4} \\ V_{w5} \\ V_{w6} \end{pmatrix} = \begin{pmatrix} z_{pp} & z_{ps1}-z_{ps2} & z_{pp1} & z_{ps2}-z_{ps1} & z_{pp1} & 0 \\ z_{ps1}-z_{ps2} & 2(z_{ss}-z_{ss2}) & 0 & z_{ss2}-z_{ss1} & z_{ps2}-z_{ps1} & z_{ss2}-z_{ss1} \\ z_{pp1} & 0 & z_{pp} & z_{ps1}-z_{ps2} & z_{pp1} & z_{ps2}-z_{ps1} \\ z_{ps2}-z_{ps1} & z_{ss2}-z_{ss1} & z_{ps1}-z_{ps2} & 2(z_{ss}-z_{ss2}) & 0 & z_{ss2}-z_{ss1} \\ z_{pp1} & z_{ps2}-z_{ps1} & z_{pp1} & 0 & z_{pp} & z_{ps1}-z_{ps2} \\ 0 & z_{ss2}-z_{ss1} & z_{ps2}-z_{ps1} & z_{ss2}-z_{ss1} & z_{ps1}-z_{ps2} & 2(z_{ss}-z_{ss2}) \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \quad \textbf{(3.26)}$$

3.3.3 Ideal coupling model under load conditions

Equation (3.27) presents the ideal equivalent circuit model for a harmonic mitigating transformer under load conditions. The impedance values in Table 3.1 are used to populate the terminal impedance matrix. The results for the load test are presented in table form in Appendix F. For the test, a resistive load of 1000 Ohms is used. The source resistance is set to zero Ohms. The primary voltages have equal magnitudes spaced 120 electrical degrees apart therefore representing positive sequence voltage. The primary terminal phase currents still lag the corresponding voltages this time by less than 90°. This indicates that the inductance of the windings still has a significant effect on the current phase angles. The magnitude of the secondary terminal currents is consistent in terms with the secondary terminal phase voltage across the resistive load. The angle of the secondary terminal phase currents are in phase with their corresponding secondary terminal phase voltages. This is consistent with the fact that the voltage and current phase angle relationship through a resistive load is the same. The angle of the secondary terminal phase voltages still lags the primary terminal phase voltages by 30°.

$$\begin{pmatrix} V_{w1} \\ 0 \\ V_{w3} \\ 0 \\ V_{w5} \\ 0 \end{pmatrix} = \begin{pmatrix} z_{pp} + r_s & z_{ps1} - z_{ps2} & z_{pp1} & z_{ps2} - z_{ps1} & z_{pp1} & 0 \\ z_{ps1} - z_{ps2} & 2(z_{ss} - z_{ss2}) + r_L & 0 & z_{ss2} - z_{ss1} & z_{ps2} - z_{ps1} & z_{ss2} - z_{ss1} \\ z_{pp1} & 0 & z_{pp} + r_s & z_{ps1} - z_{ps2} & z_{pp1} & z_{ps2} - z_{ps1} \\ z_{ps2} - z_{ps1} & z_{ss2} - z_{ss1} & z_{ps1} - z_{ps2} & 2(z_{ss} - z_{ss2}) + r_L & 0 & z_{ss2} - z_{ss1} \\ z_{pp1} & z_{ps2} - z_{ps1} & z_{pp1} & 0 & z_{pp} + r_s & z_{ps1} - z_{ps2} \\ 0 & z_{ss2} - z_{ss1} & z_{ps2} - z_{ps1} & z_{ss2} - z_{ss1} & z_{ps1} - z_{ps2} & 2(z_{ss} - z_{ss2}) + r_L \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \quad \textbf{(3.27)}$$

3.3.4 Voltage regulation for the ideal coupling model

A good measure of a transformer's performance is to determine the voltage regulation of the transformer. The voltage regulation is essentially a measure of the difference between the magnitude of the secondary side open circuit voltage and the secondary voltage under load conditions. The higher the value the greater the volt drop across the internal impedance of the transformer when the transformer is loaded [10]. According to reference [10] voltage regulation is defined as the change in magnitude of the secondary voltage as the load current changes from the no-load condition to the loaded condition. This implies that a high value for the regulation is undesirable. The only way to ensure the value remains low under loaded conditions is to design a transformer with low internal impedance. The calculation for the voltage regulation is presented in Equation (3.28).

$$Voltage\ regulation\ (VR)\ (\%) = \frac{|V_{No\ load}| - |V_{load}|}{|V_{No\ load}|} \times 100\% \quad \textbf{(3.28)}$$

Equation (3.28) is therefore used to determine the voltage regulation of the ideal harmonic mitigating transformer. The calculation is presented in Equation (3.29) and shows that the voltage regulation for an ideal harmonic mitigating transformer is zero. This makes sense since the transformer has very low internal impedance due to the fact that it has no leakages. In a practical transformer, the way to achieve maximum voltage regulation is to ensure the power factor angle of the load matches the angle of the internal impedance and the power factor angle must be lagging [10].

$$VR\ (\%) = \frac{230.968 - 230.968}{230.968} \times 100\% \ = 0.000\% \quad \textbf{(3.29)}$$

3.4 Conclusion

This chapter presents a step by step procedure for establishing a model of a harmonic mitigating transformer with ideal couplings. Certain conditions have to be met in order for such a model to be established. Such conditions include ensuring that there is negligible winding resistance and zero leakage flux. An important condition when considering the couplings between the windings is the distribution of the flux. For the ideal case, the flux was assumed to be symmetrically and evenly distributed, a condition that may be difficult to obtain in the practical case. If all the conditions are met a purely theoretical model can be developed. The model was based on using the ideal coupling coefficients between the various windings to calculate the actual theoretical open circuit impedance values. The model developed was evaluated under no-load and load conditions. This was achieved by applying a primary voltage to the transformer model and hence using the model to predict or calculate the corresponding primary and secondary currents and secondary voltages. The model was shown to perform satisfactorily and this was shown by the zero value obtained for the voltage regulation.

CHAPTER 4: MODEL OF A HARMONIC MITIGATING TRANSFORMER WITH NON-IDEAL COUPLINGS

4.1 Introduction

In this chapter, a physical harmonic mitigating transformer is assessed in order to determine the practical parameters necessary for developing a model based on the non-ideal couplings of the physical transformer. As pointed out in Section 2.6 of Chapter 2, a physical transformer was acquired in order to perform this task. The first part of this chapter presents a description of the physical transformer used. Information regarding its construction is presented and a brief comparison with an equivalent kVA rated delta-star transformer is presented. The transformer is then tested under no-load and load conditions in order to obtain the measurements necessary for calculating its voltage regulation. The next part of the investigation presents a measurement procedure to obtain the open circuit impedances of the physical transformer. This is done in order to establish the non-ideal coupling model of the physical transformer. The model of the physical transformer is then tested under no-load and load conditions and the voltage regulation obtained. The aim then is to compare the voltage regulation for the physical transformer and that obtained for the non-ideal model. This comparison will reveal whether or not the non-ideal model provides a suitable approximation of the physical transformer.

4.2 Physical harmonic mitigating transformer used in this investigation

4.2.1 Transformer construction

The physical harmonic mitigating transformer used for this investigation is a delta-zig-zag connected transformer rated at 1 kVA. The specifications for the transformer are presented in Appendix C. A transformer with a 1:1 voltage ratio was chosen in order to make it easier to perform the required measurements. In other words the same voltage source could be used when performing measurements on either side of the transformer. The ratio in this case refers to the ratio between the primary line voltage and the secondary line voltage. The ratio does not take into account the actual transformer connection therefore implying that the connection scheme is arbitrary so long as the voltage between the lines is the same or scaled by some factor. In reference to the delta-zig-zag connected transformer this means that if, for example, the line voltage on the primary side is 400 V then the phase to neutral voltage must be 230 V. This means that the voltage across one of the secondary windings has to be $230/\sqrt{3}$ which is equal to 133.33 V. This highlights the fact that the voltage across each winding has a phase associated with it implying that the turns ratio is not necessarily the same as the desired voltage ratio. With regard to the zig-zag connection, typically one would not be able to measure the voltage across each winding because as discussed, the zig-zag connection is an internal one. The transformer however can be constructed to allow the connection to be done externally as is the case with the transformer used for this investigation. Simply put the start and finish of each secondary winding has been 'brought out' to the top of the transformer in order to allow for the interconnections to be done manually. This means that there will be 12 connection points for the secondary side.

In terms of connections between the primary windings, the start and end of each winding has also been 'brought out', this time it was done in order to allow for the phase currents of the delta connection to be measured.

Typically in a normal delta construction, the connection is internal so one cannot measure the phase current in a winding. As mentioned it will be necessary to measure the phase current because as mentioned, it would contain the third harmonic if it is present. So in total there will be 18 connection points and 19 if an earth is included. Figure 4.1 presents a schematic of the top of the transformer where the connections are done. Technically this transformer can be connected in any type of connection i.e. star-star or star zig-zag however a 1 to 1 voltage ratio may not be achieved because the number of turns per winding may not allow for it. Therefore the transformer will only be connected as a delta-zig-zag transformer as shown in the connection scheme in Figure 4.2.

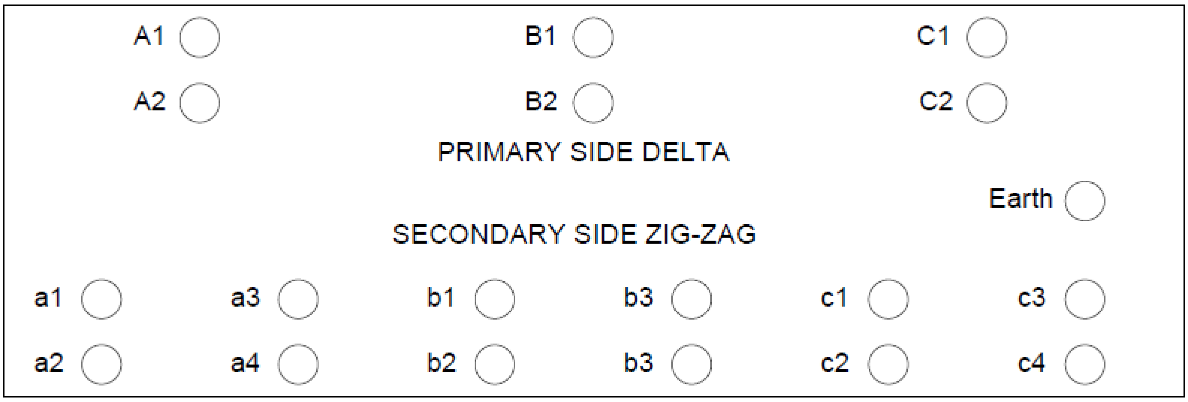


Figure 4.1: Diagram of the connection points for the practical delta-zig-zag transformer.

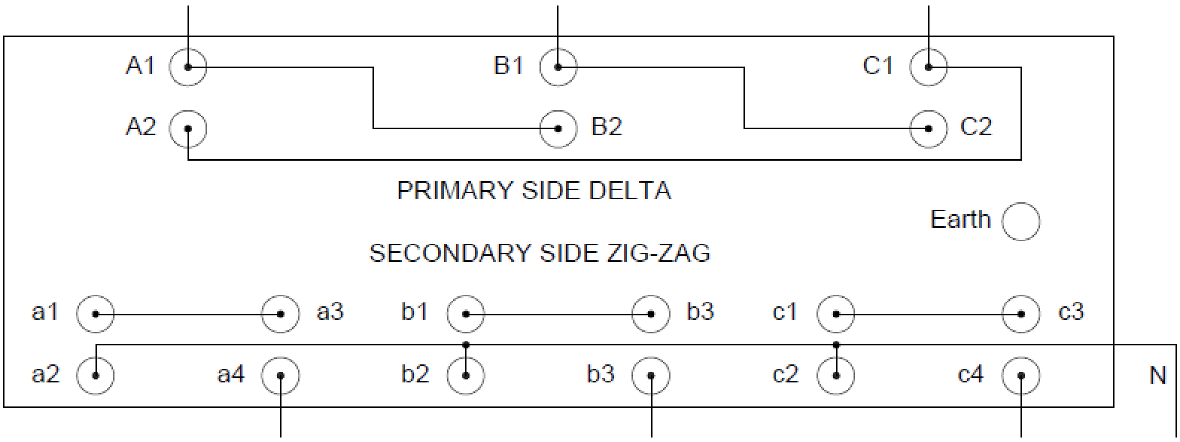


Figure 4.2: Diagram showing the connection scheme for the delta-zig-zag transformer.

4.2.2 Comparison with a delta-star

Although a delta-star transformer cannot mitigate harmonics like a delta-zig-zag transformer, it can still trap them therefore suggesting that it could be used to prevent harmonics from entering into the supply. It is therefore a worthwhile exercise to perform a basic comparison between the two transformers in terms of cost and construction. The specifications for a 1 kVA delta-star transformer are also presented in Appendix C. Upon comparing the two transformers, several discrepancies between certain values were noted and are presented in Table 4.1. From the values in the table, it can be seen that the increase in the number of secondary turns in order to accommodate the required voltage, has a direct influence on the wire mass and I^2R wire loss. The largest increase in terms of percentage however was observed in the cost. A 33.67 % increase in the cost to make a delta-zig-zag transformer was calculated. Since the differences in material used were marginal, the cost increase is predominantly due to labour. This is because the zig-zag connection is more complex than a standard star connection therefore it requires more time to construct. The increase in price is significant and this could possibly be the main reason why delta-zig-zag transformers are not more common place in power systems where third harmonics are rife. By modelling the delta-zig-zag transformer, a good understanding of its construction will be obtained which could help future designers to minimise costs when constructing delta-zig-zag transformers.

Table 4.1. A basic comparison between a delta-star and a delta-zig-zag transformer.

Parameter	Delta-star	Delta-zig-zag	Percentage difference
Number of turns per winding (Secondary)	646	715	9.65 %
Wire mass (Secondary)	3.29 kg	3.65 kg	9.76 %
Total I^2R wire loss	12.43 Watts	14.55 Watts	14.59%
Total core loss	20.12 Watts	22.54 Watts	10.74 %
Cost of transformer (ZAR)	R2620.00	R3950.00	33.67 %

4.3 Measurement procedure to determine the voltage regulation of the physical harmonic mitigating transformer

4.3.1 Setup procedure

The procedure used to measure the physical harmonic mitigating transformer involves connecting a resistive load to each phase of the transformer and measuring the respective terminal phase voltage and current values. The layout for the verification setup is presented in Figure 4.3. A three phase 50 Hz sinusoidal voltage source expressed as V_A, V_B and V_C is applied to the primary side of the transformer. Each corresponding terminal phase current (which is the delta connection phase current, a value measurable only because the delta connection was done externally) is measured using current transformer CT1, CT2 and CT3 respectively. Note that the voltage V_{CT1}, V_{CT2} and V_{CT3} across resistor R_{CT1}, R_{CT2} and R_{CT3} respectively is the actual value that is measured and the relationship between each voltage and resistor is the current I_{CT1}, I_{CT2} and I_{CT3} respectively. Current I_{CT1}, I_{CT2} and I_{CT3} are then multiplied by some scaling factor to represent the true primary terminal phase currents I_{w1}, I_{w3} and I_{w5} respectively. Figure 4.3 shows that a resistive load R_{load1}, R_{load2} and R_{load3} is connected to the secondary side of the transformer. Just as with the primary currents, the secondary terminal phase currents are measured using current transformers CT4, CT5 and CT6 respectively. The same method as for the primary side is employed to obtain the values for the secondary currents $I_{w2\ load}$, $I_{w4\ load}$ and $I_{w5\ load}$. It must be noted that the direction of the secondary currents has been reversed such that $I_{w2} = -I_{w2\ load}$, $I_{w4} = -I_{w4\ load}$ and $I_{w6} = -I_{w6\ load}$ [19]. The reason for this is to show the fact that a current flows 'into' the load rather than out of the load.

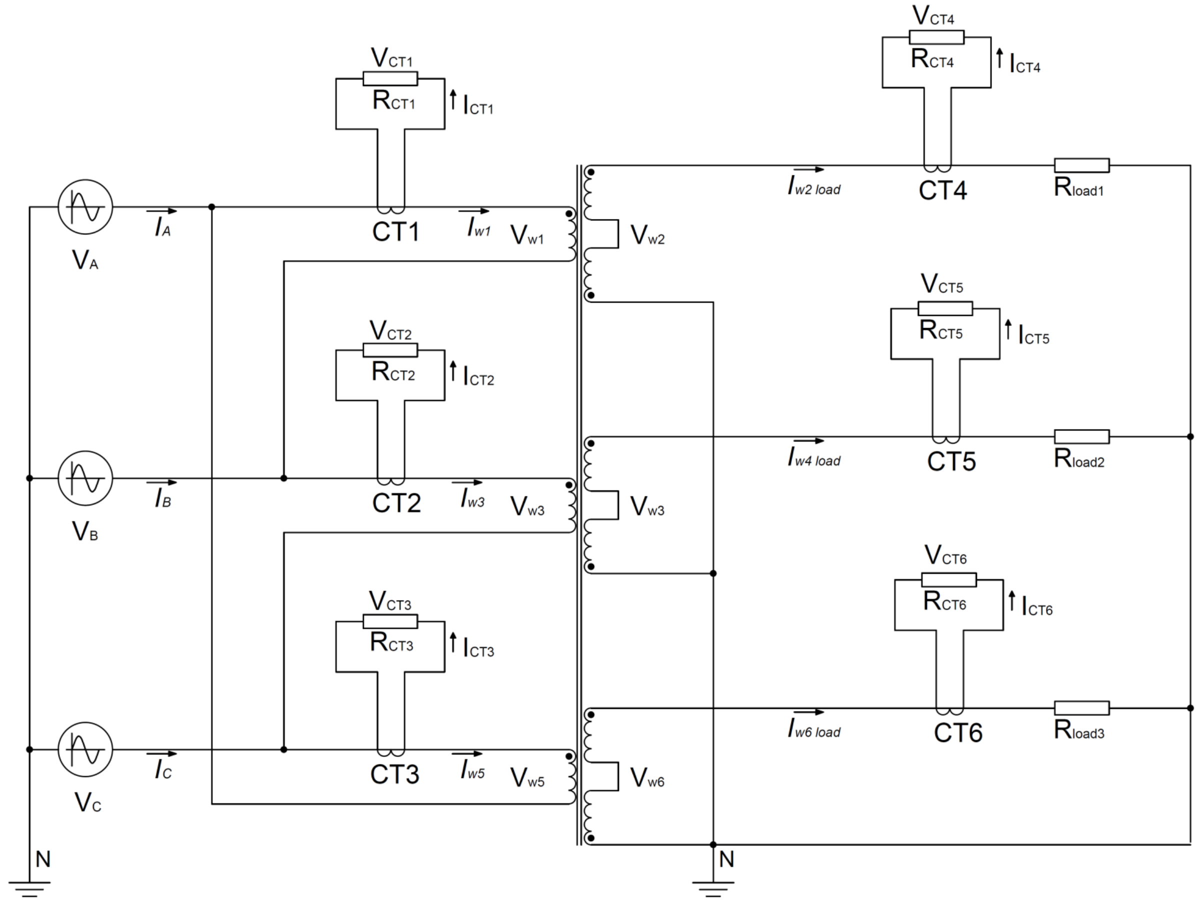


Figure 4.3: Experimental setup to verify the performance of the physical delta-zig-zag transformer. A linear resistive load is connected to each phase.

4.3.2 Physical transformer under no-load conditions

The experimental setup shown in Figure 4.3 is used to obtain the results for the open circuit test performed on the physical harmonic mitigating transformer. The one adjustment required for the setup is that instead of a load resistor, the circuit was open circuited. The input primary voltages have equal magnitudes spaced 120 electrical degrees apart therefore representing positive sequence voltage. The results are presented in table form in Appendix F. All values show reasonable agreement with the ideal coupling model values also presented in Appendix F, except for the magnitude of the primary terminal phase currents. The discrepancy could be accounted for in the fact that the physical transformer has a certain amount of leakage flux and a magnetising current associated with it. The values in table are based on time sampled measurements hence when they were converted to phasor form; the values were rounded to two decimal places to simplify the calculation process.

4.3.3 Physical transformer under load conditions

The experimental setup shown in Figure 4.3 is used to obtain the results for the load test performed on a physical harmonic mitigating transformer. The load resistor was set at 1000 Ohms for each load. The test results are presented in table form in Appendix F. The values show slight discrepancies when compared to the ideal coupling model results also shown in the table yet once again this could be accounted for in the fact that there are leakage fluxes present in the physical transformer.

4.3.4 Voltage regulation for the physical transformer

The results for the no-load and load test performed on the physical harmonic mitigating transformer are used to calculate the voltage regulation of the transformer. The result for the calculation is presented in Equation (4.1) and it shows that the physical transformer experiences a slight drop in the terminal voltage when it is loaded.

$$VR\ (\%) = \frac{226.133 - 220.617}{226.133} \times 100\% \ = 2.439\% \qquad \textbf{(4.1)}$$

4.4 Non-ideal coupling model of a physical harmonic mitigating transformer

4.4.1 Open circuit impedance matrix of a physical harmonic mitigating transformer

Consider the open circuit impedance matrix in Equation (4.2) which is used to represent the model of an unconnected delta-zig-zag transformer. In order to solve for the impedance values, the open circuit test will have to be done on each of the windings. As discussed in Appendix D further measurements will be required in order to determine the mutual impedance values. Although there are nine windings, the same method as shown in Appendix D can be used. In order to demonstrate how this will work, the first two voltage values in the impedance matrix will be used namely V_1 and V_2. The expanded equations for V_1 and V_2 are presented in Equation (4.3) and Equation (4.4) respectively. It can be seen that in order to determine the open circuit value for z_{11}, all currents except I_1 must be zero that is I_2 to I_9 must equal zero. Similarly this is done for winding 2 this time letting I_1 equal to zero and I_3 to I_9 equal to zero. Therefore the solutions for the values of z_{11} and z_{22} are shown in Equation (4.5) and Equation (4.6) respectively.

To solve for the mutual impedances, one will have to open circuit all windings except winding 1 where the measurement will be taken and winding 2 which will essentially be short circuited. This process is presented in more detail in Appendix D. The main aspect to note is that in each connection instance the voltages V_3 to V_9 are equal to zero i.e. all windings except winding 1 and winding 2 are open circuited. The method is based on the idea that if the all the currents and voltages are known or at least can be measured then the resistances, or in this case impedances, can be determined. This is essentially Ohm's Law. The first part of the measurement process will have to be performed nine times in order to determine the self-impedance values. The part whereby the mutual impedance values are determined will have to be performed 72 times however due to the reciprocity theorem, only 45 measurements can be taken in order to populate the impedance matrix in Equation (4.2). Once the impedance values have been determined then the internal connection matrix can be applied in order to establish the true equivalent circuit model for a harmonic mitigating transformer.

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \\ V_9 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} & z_{13} & z_{14} & z_{15} & z_{16} & z_{17} & z_{18} & z_{19} \\ z_{21} & z_{22} & z_{23} & z_{24} & z_{25} & z_{26} & z_{27} & z_{28} & z_{29} \\ z_{31} & z_{32} & z_{33} & z_{34} & z_{35} & z_{36} & z_{37} & z_{38} & z_{39} \\ z_{41} & z_{42} & z_{43} & z_{44} & z_{45} & z_{46} & z_{47} & z_{48} & z_{49} \\ z_{51} & z_{52} & z_{53} & z_{54} & z_{55} & z_{56} & z_{57} & z_{58} & z_{59} \\ z_{61} & z_{62} & z_{63} & z_{64} & z_{65} & z_{66} & z_{67} & z_{68} & z_{69} \\ z_{71} & z_{72} & z_{73} & z_{74} & z_{75} & z_{76} & z_{77} & z_{78} & z_{79} \\ z_{81} & z_{82} & z_{83} & z_{84} & z_{85} & z_{86} & z_{87} & z_{88} & z_{89} \\ z_{91} & z_{92} & z_{93} & z_{94} & z_{95} & z_{96} & z_{97} & z_{98} & z_{99} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \\ I_7 \\ I_8 \\ I_9 \end{pmatrix} \qquad \textbf{(4.2)}$$

$$V_1 = z_{11}I_1 + z_{12}I_2 + z_{13}I_3 + z_{14}I_4 + z_{15}I_5 + z_{16}I_6 + z_{17}I_7 + z_{18}I_8 + z_{19}I_9 \quad \textbf{(4.3)}$$

$$V_2 = z_{21}I_1 + z_{22}I_2 + z_{23}I_3 + z_{24}I_4 + z_{25}I_5 + z_{26}I_6 + z_{27}I_7 + z_{28}I_8 + z_{29}I_9 \quad \textbf{(4.4)}$$

Let I_2 to $I_9 = 0$ therefore:

$$V_1 = z_{11}I_1$$

$$z_{11} = \frac{V_1}{I_1} \quad \textbf{(4.5)}$$

Let $I_1 = 0$ and let I_3 to $I_9 = 0$ therefore:

$$V_2 = z_{22}I_2$$

$$z_{22} = \frac{V_2}{I_2} \quad \textbf{(4.6)}$$

As per the measurement procedure discussed, the individual impedance values for the open circuit impedance matrix in Equation (4.2) were determined and the values are presented in table form in Appendix E. The most important observation at this stage is the fact that the mutual impedances measured have a real component meaning there is a mutual resistance associated with each of the mutual impedances. Recall that the winding impedance matrix is a nine-by-nine matrix and by applying the internal connection matrix, it reduces the nine-by-nine matrix to a six-by-six matrix. This six-by-six matrix is the terminal phase impedance matrix and is essentially the matrix that will be used to establish the non-ideal coupling model of the physical harmonic mitigating transformer. The terminal phase impedance matrix is presented in table form in Table 4.2. As mentioned, the mutual impedances have mutual resistances; this can also be seen in the table.

Table 4.2: Terminal phase impedance matrix for a physical harmonic mitigating transformer. The values were determined using the equation $[Z_{terminal}] = [W][Z_{winding}][W^T]$. Note that all values must be multiplied by 1 x 10^3.

0.0963+ 0.8926i	0.0337+0.2568i	0.0510+0.5517i	-0.0150-0.1235i	0.0107+0.2002i	-0.0189-0.1309i
0.0337+ 0.2568i	0.0342+0.1884i	0.0099+0.0277i	-0.0126-0.0870i	-0.0329-0.2539i	-0.0177-0.1015i
0.0510+ 0.5517i	0.0099+0.0277i	0.1191+1.1684i	0.0227+0.2191i	0.0408+0.5198i	-0.0322-0.2427i
-0.0150- 0.1235i	-0.0126-0.0870i	0.0227+0.2191i	0.0210+0.1235i	0.0178+0.1268i	-0.0032-0.0345i
0.0107+ 0.2002i	-0.0329-0.2539i	0.0408+0.5198i	0.0178+0.1268i	0.0993+0.9021i	0.0174+0.1332i
-0.0189- 0.1309i	-0.0177-0.1015i	-0.0322-0.2427i	-0.0032-0.0345i	0.0174+0.1332i	0.0265+0.1380i

4.4.2 Non-ideal coupling model under no-load conditions

Equation (4.7) presents the practical equivalent circuit model for a harmonic mitigating transformer. The impedance values in Table 4.2 are used to populate the terminal impedance matrix in Equation (4.7). The input primary voltages have equal magnitudes spaced 120 electrical degrees apart therefore representing positive sequence voltage. The results for the test are presented in table form in Appendix F. The primary terminal phase currents are the magnetising currents and are seen to lag the corresponding voltages by less than 90°.

This indicates that the internal impedance is not purely inductive as in the ideal-coupling model. Upon comparing the results for the no-load tests for the physical transformer it can be seen that there are some discrepancies between the magnitudes of the primary terminal phase currents and the phase angles of the primary terminal phase currents. The discrepancies however could lie in the fact that the non-ideal coupling model does not take into account the core losses. This means that the primary currents in the physical transformer case will have a core loss component whereas the primary currents in the non-ideal coupling model represent solely the magnetising current. This explains the difference in magnitudes and the difference in phase angles.

The secondary terminal currents are zero which is consistent with the open circuit condition. The secondary terminal phase voltages are slightly larger than the values for the ideal coupling model and physical transformer model as shown in the table in Appendix F. This could be due to the measurement technique used to determine the impedance values. The technique used an alternating current bridge to measure the impedance values while the transformer was disconnected from a supply and load. Furthermore the measurements were taken at a voltage lower than the rated voltage. These factors could result in slight differences between the transformer impedances. In order to confirm this, the conventional open and short circuit test can be performed on the transformer whereby rated voltage and current are applied respectively in order to determine the open circuit and short circuit impedances of the transformer respectively.

$$\begin{pmatrix} V_{w1} \\ V_{w2} \\ V_{w3} \\ V_{w4} \\ V_{w5} \\ V_{w6} \end{pmatrix} = \begin{pmatrix} z_{11} & z_{13}-z_{18} & z_{14} & z_{16}-z_{12} & z_{17} & z_{19}-z_{15} \\ z_{31}-z_{81} & z_{33}-z_{83}-z_{38}+z_{88} & z_{34}-z_{84} & z_{82}-z_{32}-z_{86}+z_{36} & z_{37}-z_{87} & z_{85}-z_{35}-z_{89}+z_{39} \\ z_{41} & z_{43}-z_{48} & z_{44} & z_{46}-z_{42} & z_{47} & z_{49}-z_{45} \\ z_{61}-z_{21} & z_{63}-z_{23}-z_{68}+z_{28} & z_{64}-z_{24} & z_{22}-z_{62}-z_{26}+z_{66} & z_{67}-z_{27} & z_{25}-z_{65}-z_{29}+z_{69} \\ z_{71} & z_{73}-z_{78} & z_{74} & z_{76}-z_{72} & z_{77} & z_{79}-z_{75} \\ z_{91}-z_{51} & z_{93}-z_{53}-z_{98}+z_{58} & z_{94}-z_{54} & z_{52}-z_{92}-z_{56}+z_{96} & z_{97}-z_{57} & z_{55}-z_{95}-z_{59}+z_{99} \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \quad \textbf{(4.7)}$$

4.4.3 Non-ideal coupling model under load conditions

Equation (4.8) presents the practical equivalent circuit model for a harmonic mitigating transformer under load conditions. The impedance values in Table 4.2 are used to populate the terminal impedance matrix. The results for the load test are presented in table form in Appendix F. For the test, a resistive load of 1000 Ohms is used. The results show that the primary terminal phase currents lag the corresponding voltages however the lag is less than the open circuit value for the model shown in the table in Appendix F. This indicates that inclusion of the load causes the impedance of the circuit to become less inductive. The magnitude and phase of the primary currents are seen to be in reasonable agreement with those presented in the table for the physical transformer. The secondary voltages are still larger than those presented in the table for the physical transformer. The reason for this is already presented in the previous section.

$$\begin{pmatrix} V_{w1} \\ 0 \\ V_{w3} \\ 0 \\ V_{w5} \\ 0 \end{pmatrix} = \begin{pmatrix} z_{11}+r_s & z_{13}-z_{18} & z_{14} & z_{16}-z_{12} & z_{17} & z_{19}-z_{15} \\ z_{31}-z_{81} & z_{33}-z_{83}-z_{38}+z_{88}+r_L & z_{34}-z_{84} & z_{82}-z_{32}-z_{86}+z_{36} & z_{37}-z_{87} & z_{85}-z_{35}-z_{89}+z_{39} \\ z_{41} & z_{43}-z_{48} & z_{44}+r_s & z_{46}-z_{42} & z_{47} & z_{49}-z_{45} \\ z_{61}-z_{21} & z_{63}-z_{23}-z_{68}+z_{28} & z_{64}-z_{24} & z_{22}-z_{62}-z_{26}+z_{66}+r_L & z_{67}-z_{27} & z_{25}-z_{65}-z_{29}+z_{69} \\ z_{71} & z_{73}-z_{78} & z_{74} & z_{76}-z_{72} & z_{77}+r_s & z_{79}-z_{75} \\ z_{91}-z_{51} & z_{93}-z_{53}-z_{98}+z_{58} & z_{94}-z_{54} & z_{52}-z_{92}-z_{56}+z_{96} & z_{97}-z_{57} & z_{55}-z_{95}-z_{59}+z_{99}+r_L \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \quad \textbf{(4.8)}$$

4.4.4 Voltage regulation for the non-ideal coupling model

The voltage regulation for the non-ideal coupling model was calculated using the results for the no-load and load test. The regulation for each phase was calculated and the results shown in Equation (4.9), Equation (4.10) and Equation (4.11) respectively. The regulation across all three phases can be seen to demonstrate an acceptable level of consistency. Furthermore the regulation is approximately 1 % higher than that calculated for the ideal coupling model in Equation (3.29). This is because the non-ideal coupling model includes the winding and mutual resistances which increase the overall impedance of the transformer. The regulation value can be seen to be approximately 1.5 % lower than the physical transformer value shown in Equation (4.1). A possible explanation for this could lie in the fact that the regulation of a transformer depends on the power factor of the load. In the physical transformer test, the loads were not ideal suggesting that the power factor of the load was also not ideal. This is in contrast to the loads applied to the non-ideal coupling model which were ideal.

$$VR\ (V_{w2})(\%) = \frac{252.492 - 249.708}{252.492} \times 100\% \ = 1.103\% \qquad \textbf{(4.9)}$$

$$VR(V_{w4})\ (\%) = \frac{241.052 - 238.534}{241.052} \times 100\% \ = 1.045\% \qquad \textbf{(4.10)}$$

$$VR\ (V_{w6})\ (\%) = \frac{254.696 - 252.125}{254.696} \times 100\% \ = 1.010\% \qquad \textbf{(4.11)}$$

4.5 Conclusion

The first part of this chapter presented a description of the physical harmonic mitigating transformer used in the investigation. A comparison between a delta-zig-zag harmonic mitigating transformer and an equivalent kVA rated delta-star transformer was also presented. The comparison revealed that the harmonic mitigating transformer is substantially more expensive to manufacture than the delta-star transformer. Therefore the delta-star transformer provides a less expensive solution for preventing the third harmonic from entering the supply however the trade-off is that the transformer losses will increase significantly. A more effective solution would be to use a harmonic mitigating transformer however the trade-off in this case is the cost.

From the results presented in Appendix F regarding the performance of the transformers, a decision has to be made as to whether or not the non-ideal coupling model of a harmonic mitigating transformer can be used to represent the physical harmonic mitigating transformer. The results presented in Appendix F demonstrate that upon comparing the model to the physical transformer, certain discrepancies between the models become apparent. Some of these discrepancies can be attributed to the fact that the model does not take into account the core losses. Furthermore the measurement procedure for obtaining the impedances for the model do not measure the impedances while the transformer is under load and supply conditions therefore slight variations in the impedance values may lead to discrepancies between the terminal voltages. The deciding factor however for using the non-ideal coupling model to represent the physical transformer is the voltage regulation. This value shows that the model has a consistent regulation among the three phases and the value lies within an acceptable range when compared to physical transformer and the ideal coupling model.

CHAPTER 5: COMPARISON BETWEEN THE IDEAL COUPLING MODEL AND NON-IDEAL COUPLING MODEL IN TERMS OF ABILITY TO MITIGATE THE THIRD HARMONIC

5.1 Introduction

The main aim of this investigation is to develop an equivalent circuit model for a harmonic mitigating transformer. Chapter 2 presented a method for modelling the transformer using coupled circuit theory. Chapter 3 and Chapter 4 evaluated the performance of the model under ideal and practical conditions respectively. Up to this point the ideal coupling model and non-ideal coupling model proved to be sufficiently accurate to be able to predict the behaviour of the harmonic mitigating transformer under normal operating conditions. Normal operating conditions in this case imply that the system is free from harmonics and the load is linear. In a typical power system however this is rarely the case as pointed out in Chapter 1. It is necessary to evaluate the two models in terms of their ability to remove harmonics particularly the third harmonic. The assumption then is that the ideal coupling model presented in Chapter 3 will be able to remove the third harmonic due to the ideal coupling relationships between the windings. The non-ideal coupling model presented in Chapter 4 is expected to not be able to remove the third harmonic completely as the couplings between the windings are not ideal. This chapter presents a comparison between the two models and the outcome is expected to reveal whether the couplings in a harmonic mitigating transformer affect its ability to mitigate the third harmonic. To begin, the first section of this chapter presents a discussion on non-linear load modelling. This is necessary in order to understand how the third harmonic can be applied to the model.

5.2 Non-linear phase to neutral load

In Chapter 1 a definition of a non-linear phase to neutral load was presented. The main focus was on the fact that a non-linear phase to neutral load is capable of generating triplen harmonics even if the load is ideal. The aim of a harmonic mitigating transformer as already discussed is to remove the triplen harmonics generated by such a load. The triplen harmonics would proliferate in the form of currents drawn by the non-linear load. The nature of such currents in relation to two types of transformers was demonstrated using controlled source models of each transformer as shown in Section 2.2 of Chapter 2. The non-linear phase to neutral load was represented by two current sources in parallel supplying the secondary side of each transformer. One of the current sources generated current at the fundamental frequency (50 Hz) and the other generated current at the third triplen harmonic frequency (150 Hz). The path of the third harmonic current was traced through each transformer model and it was shown that the delta-star model allowed the third harmonic current to pass through to the primary side of the transformer whereas the delta-zig-zag model did not. The third harmonic in the delta-star model was detected by measuring the primary phase currents. This implied that no third harmonic would be detected in the phase currents of the delta-zig-zag model.

In reality, a true current source is difficult to implement therefore an alternative method for representing a non-linear phase to neutral load is required in order to successfully test the ideal harmonic mitigating transformer model. With this in mind recall that from Chapter 1 it was stated that a harmonic current cause’s voltage drops across the impedances within a power system and that the voltage drop in turn distorts the source voltage. The distorted waveform in turn will also consist of harmonics. The power system in this case is a transformer and the impedances referred to would therefore be those of the transformer and the supply. In order to represent a third harmonic current, a load is required that draws a third harmonic current. Therefore by evoking Ohm's Law, a voltage source at the third harmonic frequency applied to a *linear* resistive load, would draw a current at the third harmonic frequency thereby representing a non-linear load. The resistive load would therefore form part of the impedances of the system and provided the resistance is large enough, the current drawn by the resistive load would be the dominant current i.e. in this case, the third harmonic current. It is important to note that although the controlled source model of the harmonic mitigating transformer demonstrated the path of two currents, the analysis in this section only focuses on one current, namely the third harmonic current. The reason is because the impedance of the system changes depending on the frequency as shown in Table 3.1. This means that the effect of the third harmonic on the transformer can be observed in isolation.

In order to understand the concept in the preceding text, consider the diagram in Figure 5.1 of a single phase transformer connected to a supply and a load. If the coupled circuit equations used in Equation (2.22) and Equation (2.23) are applied around loop 1 and loop 2 then according to Kirchhoff's Voltage Law:

$$v_s = (r_s + r_1)i_1 + L_{11}\frac{di_1}{dt} - L_{12}\frac{di_L}{dt} \quad \textbf{(5.1)}$$

$$0 = L_{21}\frac{di_1}{dt} - (r_2 + r_L)i_L - L_{22}\frac{di_L}{dt} \quad \textbf{(5.2)}$$

Where r_s is the internal resistance of the supply, r_L is the load resistance and i_L the current drawn by the load. If steady state conditions apply then Equation (5.1) and Equation (5.2) can be represented as:

$$V_1 = (r_s + r_1 + j\omega L_{11})I_1 - (j\omega L_{12})I_L \quad \textbf{(5.3)}$$

Let: $z_{11} = (r_s + r_1 + j\omega L_{11})$

and let: $z_{12} = -(j\omega L_{12})$

$$0 = (r_L + r_2 + j\omega L_{22})I_L - (j\omega L_{21})I_1 \quad \textbf{(5.4)}$$

Let: $z_{22} = (r_L + r_2 + j\omega L_{22})$

and let: $z_{21} = -(j\omega L_{21})$

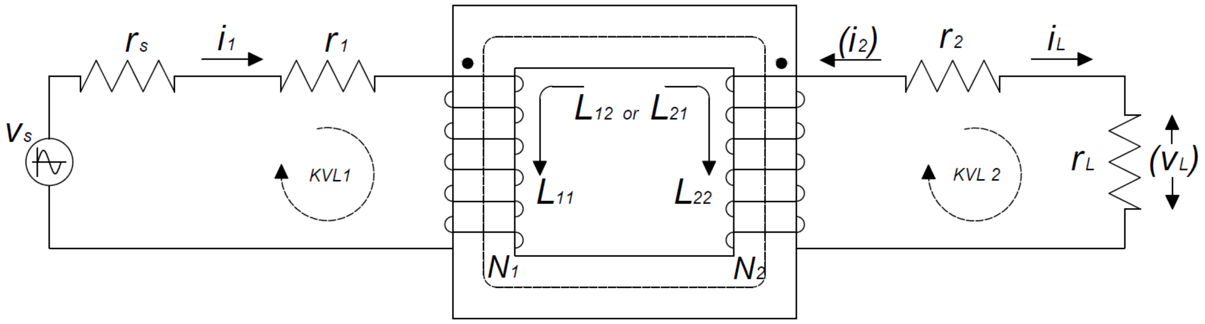


Figure 5.1: Single phase transformer model with a load connected. Note the direction of the load current is opposite in direction to the secondary current (i_L = -i_2).

Equation (5.3) and Equation (5.4) represent the voltage and current relationships in the single phase transformer operating under normal load conditions whereby r_L is a linear load. The current i_2 is shown in brackets just to indicate the conventional direction for the current on the secondary side therefore as seen in the equations i_L=-i_2. Also the term v_L is the induced voltage across the load noting that because of ideal conditions $v_2 = v_L$. Recall that if the third harmonic current were to be applied then a voltage source at the harmonic frequency and a resistor would have to be connected to the secondary side, this is shown in Figure 5.2. From the figure it can be seen that the direction of the current has changed and the primary source is omitted. By considering the new voltage loops, the following steady state equations apply noting that the f in ω will be at the harmonic frequency:

$$0 = (r_s + r_1 + j\omega L_{11})I_{1'} - (j\omega L_{12})I_2 \quad \textbf{(5.5)}$$

Let: $z_{11} = (r_s + r_1 + j\omega L_{11})$

and let: $z_{12} = -(j\omega L_{12})$

$$V_{harmonic} = (r_L + r_2 + j\omega L_{22})I_2 - (j\omega L_{21})I_{1'} \quad \textbf{(5.6)}$$

Let: $z_{22} = (r_L + r_2 + j\omega L_{22})$

and let: $z_{21} = -(j\omega L_{21})$

Where $V_{harmonic}$ represents the voltage source at the harmonic frequency, r_L is still the same resistive load as before and I_2 is the secondary side current at the harmonic frequency. Equation (5.5) and Equation (5.6) suggest that the harmonic current will induce a voltage on the primary side. This is true since the circuit is merely the transformer operating from the secondary side. Therefore V_1, which is not shown, would still be the primary voltage this time measured across r_s. This therefore shows that the primary current I_1 which is at the harmonic frequency causes a voltage drop, V_1, across the source. V_1 in this case will also be at the harmonic frequency. Therefore, in a harmonic mitigating transformer, V_1 and I_1 should be zero. By representing a non-linear load in terms of a voltage source at a harmonic frequency and a resistor ensures consistency with regards to the coupled equations for a transformer. The reason for insisting on this will become apparent in the next section.

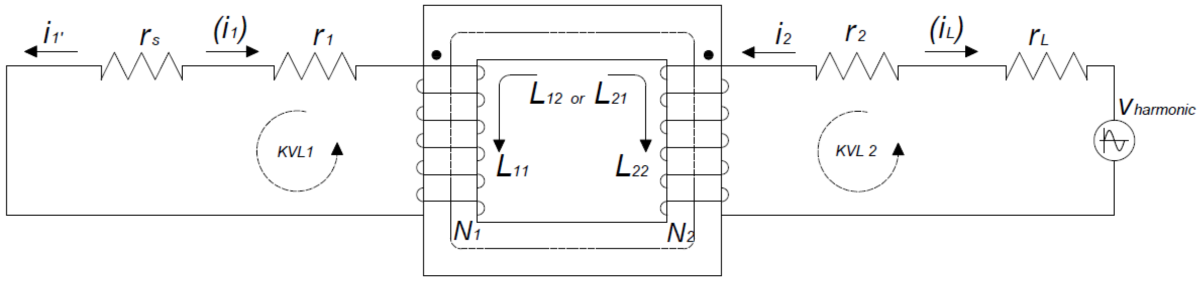


Figure 5.2: Single phase transformer with a voltage source at 150 Hz and a load attached to the secondary which together represent a non-linear load. The primary side is short circuited.

5.3 Ideal coupling model with a non-linear load connected

Equation (5.7) shows the ideal coupling model operating under harmonic condition. The impedance values used for the model are the values calculated at 150 Hz shown in Table 3.1. The input to the model is the secondary terminal phase voltages which have to have the same phase angle in order to be considered zero sequence which in turn is assumed to be the sequence of the third harmonic. The magnitude of the voltages is not significant at this stage so it was chosen to be a round number. The resistive load had to be chosen to be suitably large in order to ensure the current through it is the dominant current. The source resistance was chosen to be 1 Ohm. The results for the application of the third harmonic to the model are presented in Table 5.1. The solution for the terminal phase currents shows that the primary terminal phase currents are indeed zero which is expected. From the results it can be said that the ideal coupling model is capable of mitigating 100 % of the third harmonic meaning that 0 % of the third harmonic current is detectable in the primary terminal phase currents.

$$\begin{pmatrix} 0 \\ V_{w2} \\ 0 \\ V_{w4} \\ 0 \\ V_{w6} \end{pmatrix} = \begin{pmatrix} z_{pp} + r_s & z_{ps1} - z_{ps2} & z_{pp1} & z_{ps2} - z_{ps1} & z_{pp1} & 0 \\ z_{ps1} - z_{ps2} & 2(z_{ss} - z_{ss2}) + r_L & 0 & z_{ss2} - z_{ss1} & z_{ps2} - z_{ps1} & z_{ss2} - z_{ss1} \\ z_{pp1} & 0 & z_{pp} + r_s & z_{ps1} - z_{ps2} & z_{pp1} & z_{ps2} - z_{ps1} \\ z_{ps2} - z_{ps1} & z_{ss2} - z_{ss1} & z_{ps1} - z_{ps2} & 2(z_{ss} - z_{ss2}) + r_L & 0 & z_{ss2} - z_{ss1} \\ z_{pp1} & z_{ps2} - z_{ps1} & z_{pp1} & 0 & z_{pp} + r_s & z_{ps1} - z_{ps2} \\ 0 & z_{ss2} - z_{ss1} & z_{ps2} - z_{ps1} & z_{ss2} - z_{ss1} & z_{ps1} - z_{ps2} & 2(z_{ss} - z_{ss2}) + r_L \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \tag{5.7}$$

Table 5.1: Results for the application of the third harmonic to the ideal coupling model.

Input	Value
V_{2w}	$100.000\angle 0.000°\,V$
V_{4w}	$100.000\angle 0.000°\,V$
V_{6w}	$100.000\angle 0.000°\,V$
r_L	$1000.000\,\Omega$
r_s	$1.000\,\Omega$
Output	
I_{1w}	$0.000\angle 0.000°\,A$
I_{2w}	$0.100\angle 0.000°\,A$
I_{3w}	$0.000\angle 0.000°\,A$
I_{4w}	$0.100\angle 0.000°\,A$
I_{5w}	$0.000\angle 0.000°\,A$

I_{6w}	$0.100\angle 0.000°\,A$
V_{w1}	$0.000\angle 0.000°\,V$
V_{w3}	$0.000\angle 0.000°\,V$
V_{w5}	$0.000\angle 0.000°\,V$

5.4 Non-ideal coupling model with a non-linear load connected

Equation (5.8) shows the non-ideal coupling model under harmonic conditions. The impedance values used for the model are the values presented in Appendix E however the values have been multiplied by a factor of three to ensure that they are at 150 Hz (i.e. 3 x 50 Hz). The input to the model is the secondary terminal phase voltages at zero sequence. The results for the application of the third harmonic to the model are presented in Table 5.2. The solution shows that a current is detectable in the primary terminal phase currents however it is significantly small. From the results it can be said that the non-ideal coupling model is capable of mitigating approximately 100 % of the third harmonic. This is proven by the fact that only 0.3 % of the third harmonic current was detectable in the primary terminal phase currents.

$$\begin{pmatrix} 0 \\ V_{w2} \\ 0 \\ V_{w4} \\ 0 \\ V_{w6} \end{pmatrix} = \begin{pmatrix} z_{11} + r_s & z_{13} - z_{18} & z_{14} & z_{16} - z_{12} & z_{17} & z_{19} - z_{15} \\ z_{31} - z_{81} & z_{33} - z_{83} - z_{38} + z_{88} + r_L & z_{34} - z_{84} & z_{82} - z_{32} - z_{86} + z_{36} & z_{37} - z_{87} & z_{85} - z_{35} - z_{89} + z_{39} \\ z_{41} & z_{43} - z_{48} & z_{44} + r_s & z_{46} - z_{42} & z_{47} & z_{49} - z_{45} \\ z_{61} - z_{21} & z_{63} - z_{23} - z_{68} + z_{28} & z_{64} - z_{24} & z_{22} - z_{62} - z_{26} + z_{66} + r_L & z_{67} - z_{27} & z_{25} - z_{65} - z_{29} + z_{69} \\ z_{71} & z_{73} - z_{78} & z_{74} & z_{76} - z_{72} & z_{77} + r_s & z_{79} - z_{75} \\ z_{91} - z_{51} & z_{93} - z_{53} - z_{98} + z_{58} & z_{94} - z_{54} & z_{52} - z_{92} - z_{56} + z_{96} & z_{97} - z_{57} & z_{55} - z_{95} - z_{59} + z_{99} + r_L \end{pmatrix} \begin{pmatrix} I_{w1} \\ I_{w2} \\ I_{w3} \\ I_{w4} \\ I_{w5} \\ I_{w6} \end{pmatrix} \quad \textbf{(5.8)}$$

Table 5.2: Results for the application of the third harmonic to non-ideal coupling model.

Input	Value
V_{2w}	$100.000\angle 0.000°\,V$
V_{4w}	$100.000\angle 0.000°\,V$
V_{6w}	$100.000\angle 0.000°\,V$
r_L	$1000.000\ \Omega$
r_s	$1.000\ \Omega$
Output	
I_{1w}	$0.0003\angle -170.657°\,A$
I_{2w}	$0.096\angle -0.070°\,A$
I_{3w}	$0.0001\angle 121.015°\,A$
I_{4w}	$0.096\angle -0.073°\,A$
I_{5w}	$0.0004\angle 170.865°\,A$
I_{6w}	$0.096\angle -0.076°\,A$
V_{w1}	$0.0003\angle -170.657°\,V$
V_{w3}	$0.0001\angle 121.015°\,V$
V_{w5}	$0.0004\angle 170.865°\,V$

5.5 Discussion on the effects of coupling on the mitigation of the third harmonic

The percentage coupling between the windings of an ideal three phase three winding transformer are presented in Table 5.3. The areas highlighted in grey show the per cent coupling for windings on the same core and as expected, the couplings are 100 per cent. The non-grey areas show couplings of 50 per cent between windings not on the same core. This is consistent with the idea of symmetrical and even flux distribution for the ideal transformer. The percentage couplings of a physical three phase three winding transformer are presented in Table 5.4. Once again the areas highlighted in grey show the coupling for windings on the same core limb. The couplings can be seen to approximate the ideal case. The couplings in the non-grey areas however deviate from the ideal case and as discussed in Chapter 2, the losses occurring within a transformer can be expressed in terms of the coupling between the transformer windings.

The deviation of the coupling from the ideal values suggests that the flux distribution is not symmetrical or even and that a certain amount of leakage flux exists. According to the results for the harmonic load test presented above however, the deviation of the couplings from the ideal case appears to have a minimal effect of the transformer's ability to mitigate the third harmonic. This means that even though the couplings between the windings on the outer core limbs are not ideal, they have little effect on the transformer's ability to mitigate the third harmonic. Therefore the couplings of concern would appear to be mainly the couplings on the same core limb. This could explain the 0.3 % of the third harmonic still detectable in the primary currents. This means that the area of focus when seeking to optimise a harmonic mitigating transformer should be on the couplings between windings on the same core limb rather than windings on adjacent limbs.

Table 5.3: Percentage coupling values for the open circuit impedance values of an ideal three phase three winding transformer. The areas highlighted in grey show the percentage coupling for windings on the same core limb.

100.00	100.00	100.00	50.00	50.00	50.00	50.00	50.00	50.00
100.00	100.00	100.00	50.00	50.00	50.00	50.00	50.00	50.00
100.00	100.00	100.00	50.00	50.00	50.00	50.00	50.00	50.00
50.00	50.00	50.00	100.00	100.00	100.00	50.00	50.00	50.00
50.00	50.00	50.00	100.00	100.00	100.00	50.00	50.00	50.00
50.00	50.00	50.00	100.00	100.00	100.00	50.00	50.00	50.00
50.00	50.00	50.00	50.00	50.00	50.00	100.00	100.00	100.00
50.00	50.00	50.00	50.00	50.00	50.00	100.00	100.00	100.00
50.00	50.00	50.00	50.00	50.00	50.00	100.00	100.00	100.00

Table 5.4: Percentage coupling values for the open circuit impedance values of a practical three phase three winding transformer. The areas highlighted in grey show the percentage coupling for windings on the same core limb.

100.00	99.80	99.82	54.03	53.88	54.07	22.32	22.34	22.39
99.79	100.00	99.50	57.42	57.26	57.39	22.62	22.48	22.51
99.80	99.51	100.00	57.81	57.68	57.85	23.66	23.51	23.56
50.88	50.77	50.97	100.00	99.88	99.87	50.63	50.49	50.69
52.27	52.18	52.34	99.87	100.00	99.63	52.15	52.08	52.20
52.62	52.54	52.72	99.85	99.64	100.00	52.62	52.48	52.64
23.36	23.39	23.44	54.15	54.13	54.23	100.00	99.81	99.79
23.34	23.26	23.31	57.52	57.44	57.59	99.80	100.00	99.48
23.23	23.14	23.24	57.72	57.65	57.75	99.78	99.48	100.00

5.6 Conclusion

In order to determine whether or not the equivalent circuit model for a harmonic mitigating transformer developed in this investigation is capable of removing a third harmonic current, a suitable load had to be applied to the model. This load was shown to be a zero sequence voltage source connected in series with the load on the secondary side of the transformer. The current therefore drawn by the load would be at the third harmonic. By representing a non-linear load in this manner, the ideal coupling model and a non-ideal coupling model of a harmonic mitigating transformer were able to be tested under harmonic conditions. The ideal model showed that it could successfully prevent the proliferation of the third harmonic in the primary terminal phase currents. The non-ideal model demonstrated that it was capable of removing nearly 100 % of the third harmonic. The performance of the physical model indicated that even though the couplings within the model are non-ideal i.e. even though the transformer has a certain amount of leakage flux, this does play a significant role in the transformer's ability to remove the third harmonic.

CHAPTER 6: CONCLUSION AND FUTURE WORK

This dissertation presented an equivalent circuit model for a three phase harmonic mitigating transformer. The chapters in the dissertation presented a detailed procedure for developing and testing the model under load conditions. A procedure was also presented whereby the performance of the model was rated according to its ability to mitigate the third harmonic. The reason for developing such a model was because at the time of this investigation no concise model of such a transformer could be found. Numerous equivalent circuit models are available for standard three phase transformers. Such models at the time however could not explicitly demonstrate or capture the main aspects that are unique to the harmonic mitigating transformer one of them being its unique zig-zag connected secondary windings. Although such models were inadequate, the techniques used for modelling them provided a platform upon which to develop the harmonic mitigating transformer model. One such technique that proved to be a suitable starting point was to model the transformer using controlled sources. By modelling the transformer in this way, the relationship between the primary and secondary voltages and currents due to the zig-zag connected secondary windings could be demonstrated. This modelling technique was able to demonstrate the defining function of the transformer and that is its ability to prevent the propagation of a third harmonic current from the secondary side of the transformer to the primary side. The model showed that the transformer is able to do this by exploiting the fact that the third harmonic typically has a zero sequence. This means that because of the zig-zag connection, the third harmonic is forced to cancel out before reaching the primary side of the transformer.

The controlled source model of the transformer, although able to demonstrate the basic operation of the transformer, was limited in its application as it was not able to demonstrate the coupling characteristics of the transformer windings. It could therefore not be used as an equivalent circuit model. One modelling technique showed that a transformer can be modelled as a network of mutually coupled inductors. Modelling the transformer in this way meant that the relationships between the transformer voltages and currents could be determined using coupled inductor circuit theory. The theory shows that under steady state conditions, the windings of the transformer can be represented as a series of self and mutual impedance values. By applying Kirchhoff's Voltage Law, the voltage and current relationships of the transformer can be expressed in terms of the self and mutual impedances of the transformer windings. By representing the impedances, voltages and currents in matrix form, a suitable representation of a transformer with multiple windings was developed.

The representation of the impedances of the transformer in a matrix, the size of which is determined by the number of windings, proved to be a fundamental starting point for modelling the harmonic mitigating transformer. The reason is because a three phase harmonic mitigating transformer has nine windings, one primary winding per phase and two secondary windings per phase. A nine-by-nine impedance matrix was therefore used to capture the self and mutual impedances of the windings. At this stage the matrix did not incorporate any specific transformer connection scheme. For a transformer to be considered a harmonic mitigating transformer, the secondary side has to be connected in a zig-zag scheme. This means that each secondary winding has to be connected with an adjacent secondary winding. By doing this however the impedances in the nine-by-nine matrix would have to be grouped accordingly to express the connection. This task was made simple by the inclusion of a connection matrix. The connection matrix is based on the relationships between the currents or voltages associated with the unconnected windings and the currents and voltages once the connection is applied. For example, by applying the zig-zag connection the currents across two connected windings would be equal and a single value can be used to represent it. By applying the connection matrix, the nine-by-nine winding impedance matrix was reduced to a six-by-six terminal phase impedance matrix. This terminal phase impedance matrix was then able to express the transformer in terms of three primary voltages and three secondary voltages; the same can be said for the currents. By using this modelling procedure, an equivalent circuit model for a harmonic mitigating transformer was developed.

One fundamental assumption that applies to the model developed is that the model does not take into account the core losses of the transformer. This is a fundamental point because if the core losses were included then the initial starting point whereby the transformer is modelled as nine coupled inductors would no longer be applicable. With this said core losses were neglected throughout the investigation, an assumption that is not uncommon when first developing a model for a transformer. The equivalent circuit model however can be used to represent the load losses of a transformer namely the winding resistances and leakage fluxes. These values are appropriately included in the impedance values of the nine-by-nine winding impedance matrix. Once the model was developed, certain conditions were applied to it in order to use the model to gain some insight into the nature of the couplings of the transformer and their effect on harmonic mitigation. The first of these conditions was to assume that the transformer has ideal couplings.

By making the assumption that the transformer has ideal couplings, a purely ideal coupling model of the harmonic mitigating transformer was presented. The model was evaluated and it was shown that the model had a 0 % voltage regulation. This indicated that the equivalent circuit model is able to predict the performance of the harmonic mitigating transformer under ideal constraints. The next step was to evaluate the model under practical conditions. For this part a physical harmonic mitigating transformer was used. The voltage regulation of the transformer was calculated and it was shown to be higher than the ideal model. This was expected since the ideal model did not take into account the winding resistances and mutual resistances of the transformer. Once the performance of the physical transformer was established, the open circuit impedances of the transformer were measured and the values obtained were entered into the equivalent circuit model. The model developed in this case was a model demonstrating the non-ideal coupling characteristics of the physical transformer. This model was also evaluated and its voltage regulation was shown to lie between the ideal and physical value. The conclusion from this investigation was that the equivalent circuit mode of a harmonic mitigating transformer can indeed be used to express the nature of the non-ideal couplings in a physical harmonic mitigating transformer.

Up to this point the equivalent circuit model demonstrated its ability to perform under normal operating conditions. If the transformer were a standard three phase transformer then this would be where the investigation ends. However because the transformer is classed as a harmonic mitigating transformer, the model had to be evaluated in terms of its ability to mitigate harmonics. From this evaluation the model would reveal whether or not the non-ideal couplings of a harmonic mitigating transformer affect its ability to mitigate harmonics and if they do how the transformer could be optimised. For this evaluation a model of a harmonic load was required. Such a load had to exclusively present a zero sequence current to the model because it was shown that the harmonic mitigating transformer is only capable of mitigating the triplen harmonics which have a dominant zero sequence associated with them. To provide a benchmark for the evaluation, the ideal coupling model was used. The harmonic current was then applied to both the ideal coupling model and the non-ideal coupling model.

It turns out that the non-ideal coupling model is capable of mitigating almost 100 % of the third harmonic. This result indicated that when considering the couplings between the windings of a harmonic mitigating transformer the couplings between the windings on the outer core limbs play a minor role in the mitigation of the third harmonic. The main point then is that when considering optimising a harmonic mitigating transformer the place to start would be at the couplings between windings on the same core limb since improving these couplings would also increase the overall performance of the transformer.

This dissertation focused on developing an equivalent circuit model for a harmonic mitigating transformer from a first principal perspective. The purpose was to demonstrate the nature of the transformer from a coupling point of view. The model developed was predominately looked at in isolation from a typical power system. What this means is that the model was evaluated under a limited range of operating conditions. It is suggested then that any future work regarding the use of the model should start at the list of assumptions stated in Chapter 1. This will provide a good starting point for improving the model and consequently enable the model to be adapted for a wider range of applications. An example of such an assumption was that the supply to the transformer was balanced. It was briefly shown that unbalances in the supply also lead to the proliferation of harmonics. A valuable investigation would be to determine how this affects the model and whether or not the model can accommodate unbalanced power supplies. Another assumption was that the steady state conditions apply. It would be beneficial to determine how transients affect the model and if so by how much. In summary the model developed in this investigation can be used as a starting point and as the assumptions are addressed, the model can be adapted accordingly.

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APPENDIX A: OPEN CIRCUIT TEST ON A HARMONIC MITIGATING TRANSFORMER

A.1 Introduction

In order to establish the relationship between a transformer's primary voltages and currents and secondary voltages and currents, the impedance of the transformer has to be known. One test that can be used to obtain this value is the open circuit test. The test involves applying a pre-determined voltage to either the primary or secondary terminal and then measuring the resulting voltage and current. For the test though, one side of the transformer must be open circuited. Once the relevant voltage and current is measured, the relationship between them can be used to determine the impedance of the transformer. The impedance in this case is called the open circuit impedance and the value obtained provides some information on the transformer core losses. An alternative test that can be performed is the short circuit test. This test involves similar steps to the open circuit test except one of the terminals will be short circuited. This section focuses on using the open circuit test to obtain the internal impedance of the transformer. The values obtained from the test are used to populate the open circuit impedance matrix which is used to characterise the transformer. A brief discussion on the difference between the open circuit test and short circuit test is also presented whereby the focus is on which method is best suited for this investigation.

A.2 Open circuit impedance matrix

The values for the impedance matrix shown in Equation (A.1) are determined by performing the open circuit test on the single phase transformer. The test is done from the primary side of the transformer in order to determine the values for z_{11} and z_{21} in and then the test is performed from the secondary side in order to determine the values for z_{22} and z_{12}.

$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix} \tag{A.1}$$

Multiplying out the variables in the matrix in Equation (A.1) yields Equation (A.2) and Equation (A.3). Using these equations and the results from the open circuit test, the impedance values can be determined. The procedure is presented in the sections that follow.

$$V_1 = z_{11}I_1 + z_{12}I_2 \tag{A.2}$$

$$V_2 = z_{21}I_1 + z_{22}I_2 \tag{A.3}$$

A.1.1 Open circuit test from the primary side to determine the open circuit impedances

For the first open circuit test, the secondary side of the transformer is open circuited. The diagram in Figure A.1 presents an illustration of the procedure. By open circuiting the secondary side, it is clear that the secondary current I_2 will be zero.

If $I_2 = 0$ then Equation (A.2) and Equation (A.3) respectively reduce to:

$$V_1 = z_{11} I_1$$

$$V_2 = z_{21} I_1$$

Solving for the impedances yields:

$$z_{11} = \frac{V_1}{I_1} \quad \textbf{(A.4)}$$

$$z_{21} = \frac{V_2}{I_1} \quad \textbf{(A.5)}$$

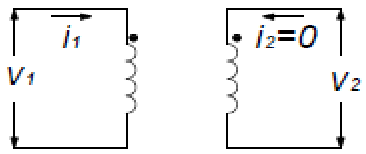

Figure A.1: Circuit diagram representing the open circuit test done from the primary side of the transformer.

A.1.2 Open circuit test from the secondary side to determine the open circuit impedances

For the second open circuit test, the primary side of the transformer is open circuited. The diagram in Figure A.2 presents an illustration of the procedure. By open circuiting the secondary side, it is clear that the primary current I_1 will be zero.

If $I_1 = 0$ the Equation (A.2) and Equation (A.3) respectively reduce to:

$$V_1 = z_{12} I_2$$

$$V_2 = z_{22} I_2$$

Solving for the impedances yields:

$$z_{12} = \frac{V_1}{I_2} \quad \textbf{(A.6)}$$

$$z_{22} = \frac{V_2}{I_2} \quad \textbf{(A.7)}$$

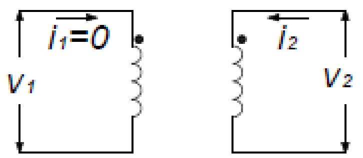

Figure A.2: Circuit diagram representing the open circuit test done from the secondary side of the transformer

A.1.3 Short circuit test to obtain the short circuit impedances

When considering load flow studies it is the short circuit impedance value of a transformer that is important [24] [22]. In terms of the open circuit tests presented above, the method for determining the short circuit impedance is presented in Equation (A.8). The short circuit impedance in Equation (A.8) is the value that would be obtained if the short circuit test were done from the primary side of the transformer i.e. with $V_2 = 0$ as in Figure A.3. Equation (A.9) presents the value for the short circuit impedance if the short circuit test were performed from the secondary side i.e. $V_1 = 0$ as in Figure A.4.

$$\frac{V_1}{I_1} = z_{SC1} = z_{11} - \frac{z_{12}z_{21}}{z_{22}} \quad \textbf{(A.8)}$$

$$\frac{V_2}{I_2} = z_{SC2} = z_{22} - \frac{z_{12}z_{21}}{z_{11}} \quad \textbf{(A.9)}$$

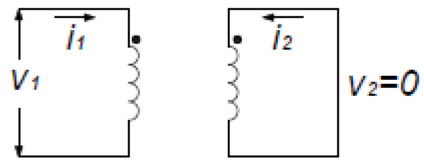

Figure A.3: Circuit diagram representing the short circuit test done from the primary side of the transformer

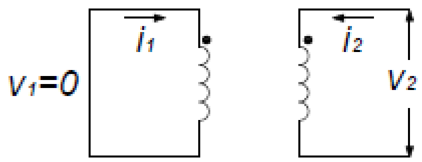

Figure A.4: Circuit diagram representing the short circuit test done from the secondary side of the transformer

A.1.4 Discussion

The current value used in the open circuit test to determine the open circuit impedances is essentially the excitation current. The impedances values determined from the test can be used to model the transformer provided that the magnitude of the excitation current is kept low. The low magnitude of the excitation current ensures that the core does not saturate. If the core saturates, the magnetic properties of the core become non-linear which consequently causes the excitation current to become non-sinusoidal. As mentioned in this study, a non-sinusoidal current waveform will contain harmonics. The harmonics in this case would affect the open circuit impedance values as well because the impedance values are frequency dependent. In essence, the open circuit test results can be suitably used to model a transformer so long as the excitation current is sinusoidal.

As mentioned in the preceding section, it is the short circuit impedance of the transformer that is of interest in load flow studies therefore in this case the open circuit test would not suffice. Consider Equation (A.8) and Equation (A.9). From the equations it can be seen that actual short circuit test need not be performed on the transformer in order to determine the short circuit impedance values. Instead the open circuit test can be done and the open circuit impedances can be used. The open circuit impedances in this case will be significantly larger than the short circuit impedances. This can be observed by the difference between the excitation current magnitude which is significantly smaller than the short circuit current (although the excitation current has to be kept small to prevent harmonics, if allowed to reach saturation levels, it would still be smaller than the short circuit current which is the rated current). What this means is that the short circuit impedances in Equation (A.8) and Equation (A.9) will be determined by the difference between two very large numbers. This may not appear to be problematic however consider the case when the coupling is close to unity. If the transformer impedances are converted to per unit form then:

$$z_{11} \approx z_{22} \approx z_{12} \ or \ z_{21}$$

It should therefore be apparent that the short circuit impedances in Equation (A.8) and Equation (A.9) will approximate zero. In this situation, important information regarding the transformer's performance would be lost. The only way to make certain that the short circuit characteristics do not become negligible would be to ensure that the resolution, and consequently the accuracy, of the measurements is suitably high [24]. In the majority of cases however the measuring equipment used is not be able to achieve the level of accuracy required therefore the short circuit test will have to be performed in order to obtain the short circuit impedance values.

A.2 Short circuit admittance matrix

In some cases the excitation current is so small due to the high impedance of the core that it can effectively be ignored. If the magnetising current were to be ignored however, the matrix equation of Equation (A.1) cannot be used at all since its derivation is based on the magnetising current [24]. Instead the impedance matrix of Equation (A.1) can be inverted to yield the short circuit admittance matrix shown in Equation (A.10) [12].

$$\begin{pmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{pmatrix}^{-1} = \frac{1}{z_{11}z_{22} - z_{12}z_{21}} \begin{pmatrix} z_{22} & z_{12} \\ z_{21} & z_{11} \end{pmatrix} = \begin{pmatrix} y_{11} & -y_{12} \\ -y_{21} & y_{22} \end{pmatrix}$$

$$[\mathbf{Z}]^{-1} = [\mathbf{Y}] \tag{A.10}$$

The equations for the self-admittances are presented in Equation (A.11) and Equation (A.12) respectively. Inverting the self-admittance values in Equation (A.11) and Equation (A.12) lead to Equation (A.13) and Equation (A.14) respectively which are essentially the same equations used for the determining the short circuit impedance values. It follows then that by using the short circuit admittance values an alternative model for the transformer can be formed as shown in Equation (A.15).

$$y_{11} = \frac{z_{22}}{z_{11}z_{22} - z_{12}z_{21}} \tag{A.11}$$

$$y_{22} = \frac{z_{11}}{z_{11}z_{22} - z_{12}z_{21}} \tag{A.12}$$

$$\frac{1}{y_{11}} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{22}}$$

$$z_{SC1} = \frac{1}{y_{11}} = z_{11} - \frac{z_{12}z_{21}}{z_{22}} \quad \textbf{(A.13)}$$

$$\frac{1}{y_{22}} = \frac{z_{11}z_{22} - z_{12}z_{21}}{z_{11}}$$

$$z_{SC2} = \frac{1}{y_{22}} = z_{22} - \frac{z_{12}z_{21}}{z_{11}} \quad \textbf{(A.14)}$$

$$\begin{pmatrix} I_1 \\ I_2 \end{pmatrix} = \begin{pmatrix} y_{11} & -y_{12} \\ -y_{21} & y_{22} \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \end{pmatrix} \quad \textbf{(A.15)}$$

It must be noted that although the short circuit admittance matrix is derived by inverting the open circuit impedance matrix, in reality this does not work. In other words one cannot simply perform the open circuit test and then invert the impedance matrix to obtain the admittance matrix. Consider the inversion process in Equation (A.10). The equations for the self-admittances are presented in Equation (A.11) and Equation (A.12) respectively. From the equations it can be seen that if the coupling is close to unity and if the impedance values are in per unit form, then the denominator in the inversion term in Equation (A.11) and Equation (A.12) would approximate zero which causes the inversion process to become numerically unstable [22]. This leads to the conclusion that in practical situations the open circuit impedance matrix and the short circuit admittance matrix are singular and only one method should be used depending on the application. For this investigation, the open circuit impedance matrix is used.

A.3 Conclusion

Two methods were presented for determining the internal impedance of a transformer. The first method showed that the internal impedance of the transformer can be obtained by performing the open circuit test. The value obtained for the test in this case is known as the open circuit impedance. In order to ensure the values for the test are accurate, the excitation current must be linear. This is to prevent harmonics from entering into the current as a result of the core non-linearity which can be seen in the saturation region of the B-H curve. The short circuit test is used exclusively when the short circuit impedance is required. This value can be obtained from the open circuit test however problems arise when the open circuit impedances are large and the self-impedance values approximate the mutual impedance values. This is overcome by performing the short circuit test and by using the short circuit admittance matrix rather than the open circuit admittance matrix. The short circuit admittance matrix is obtained by inverting the open circuit admittance matrix however in reality this may not work because the matrices are singular and the inversion process becomes numerically unstable. In essence, either the short circuit impedance or the open circuit impedance value of the transformer can be used to model the transformer. For this investigation though the open circuit impedance matrix is used because it leads to a straight forward solution when dealing with the connection matrices used in the modelling of the three phase transformer.

APPENDIX B: CALCULATIONS FOR THE IDEAL SELF INDUCTANCE VALUES OF A HARMONIC MITIGATING TRANSFORMER

B.1 Introduction

In terms of modelling a transformer, it is recommended that an ideal model be developed first in order to provide a benchmark upon which a practical model can be compared. In order to develop the ideal model however, it is sometimes necessary to use the practical transformer as a starting point for determining the scale of the parameters and the relationships between them. This section therefore uses the physical parameter of a practical harmonic mitigating transformer to determine values for the ideal self-inductances required for the formulation of the ideal open circuit impedance matrix.

B.2 Calculations

The physical parameters necessary for the calculation of the ideal self-inductances for a three phase three winding transformer are presented in and Table B.1 and Table B.2 the construction is such that there is one primary winding and two secondary windings per core limb. It can be assumed that the two secondary windings are identical in terms of wire diameter, length and number of turns. The calculations therefore required are the self-inductance value for the primary winding and the self-inductance value for one of the secondary windings. The mutual inductances are determined via the use of the coupling coefficient between two windings. The parameters in Table B.1 present the rating of the practical harmonic mitigating transformer. The reason why a value for the voltage across one of the secondary windings is included is because it is this voltage that will determine the number of turns for one of the secondary windings. In other words the value of this voltage is used to ensure that the overall voltage ratio of 1:1 is maintained between the line to line voltage on the primary and secondary side. The value is set to 133.33 Volts which means that when the zig-zag connection is applied, the voltage between one of the zig-zag connections and neutral will be 230.94 Volts. This means that the line to line voltage will be 400.00 Volts i.e. the voltage ratio of 1:1 is maintained. The steps for calculating the self-inductance of one of the secondary windings are presented below. The steps will essentially are the same for the primary winding.

Table B.1: Harmonic mitigating transformer rating as per datasheet.

Transformer specifications: Transformer rating		
Parameter	**Symbol**	**Value**
Transformer power rating (VA)	S	1000.00
Primary voltage (Volts)	$V_p = V_1$	400.00
Secondary voltage (Volts)	V_s	400.00
Secondary voltage (Volts)	V_2	133.33
Primary amps (A)	$I_p = I_1$	2.50
Secondary amps (A)	I_s	7.50
Frequency (Hz)	f	50.00

Table B.2: Harmonic mitigating transformer core specifications as per design details obtained from the manufacturer. The core is a typical core type core.

Transformer specifications: Transformer core information		
Parameter	**Symbol**	**Value**
Maximum flux density of core (Tesla)	B	1.20
Cross sectional area of core (m^2)	A	1.23E-03
Length of core perimeter (m)	L	0.14
Relative permeability of the core	μ_r	250.00
Permeability of air (H/m)	μ_o	1.26E-06
Permeability of the core (H/m) ($\mu_r\mu_o$)	μ	3.14E-04

B.2.1 Steps to calculate the self-inductance of the primary windings

The steps below assume that the voltage is sinusoidal and at the fundamental frequency [20].

1. Determine the number of windings required,
   - $N_2 = \frac{V_2}{4.44BAf}$
   - $N_2 = \frac{133.33}{(4.44)(1.20)(1.26\times10^{-3})(50.00)}$
   - $N_2 = 408.60\ turns$
2. Determine the reluctance of the core,
   - $R_c = \frac{L}{\mu_r\mu_o A}$
   - $\mathrm{R_c} = \frac{0.14}{(250.00)\left(1.26 \times 10^{-6}\right)(1.23\times10^{-3})}$
   - $R_c = 363782.721\ \mathrm{At.Wb^{-1}}$
3. Determine the magnetic flux required choosing the applied current I_2 to be 1.000 Amp,
   - $\phi_{22} = \frac{I_2N_2}{R_c}$
   - $\phi_{22} = \frac{(1.00)(408.60)}{363782.72}$
   - $\phi_{22} = 1.12 \times 10^{-3}\ \mathrm{Wb}$
4. Determine the flux linkage due to the current chosen in step 2,
   - $\lambda_{22} = \phi_{22}N_2$
   - $\lambda_{22} = (1.12 \times 10^{-3})(408.60)$
   - $\lambda_{22} = 0.46\ \mathrm{Wb.turns}$
5. Determine the self-inductance once again using the current chosen in step 2,
   - $L_{22} = \frac{\lambda_{22}}{I_2}$
   - $L_{22} = \frac{0.459}{1.000}$
   - L_{22}=0.46 H

The value for the self-inductance of the primary winding was calculated using the same steps except the voltage was chosen to be 400.00 Volts. The value for the self-inductance is therefore:

- L_{11}=4.13 H

B.3 Conclusion

The steps necessary for determining the ideal self-inductances of a practical harmonic mitigating transformer are presented in this section. The values form the basis for developing an open circuit impedance matrix for the transformer. When calculating the values, care must be taken to ensure the correct voltage ratio is maintained. The values together with the ideal coupling coefficients can be used to determine the mutual inductances.

APPENDIX C: PHYSICAL SPECIFICATIONS FOR A DELTA-ZIG-ZAG AND A DELTA-STAR TRANSFORMER

Table C.1: Practical delta-zig-zag transformer specifications.

Dzn0 transformer specifications	
Parameter	**Value**
Power rating	1 kVA
Frequency	50 Hz
Connection type	Dzn0
Rating	400 V / 400 V (1:1)
Flux density	1.19 T
Primary side	
Number of primary windings per phase	1
Primary voltage	400 V (line and phase voltage)
Primary current	2.5 A (line current) 1.45 A (phase current)
Number of turns per winding	1104
Wire diameter	0.71 mm
Wire mass	2.26 kg
Secondary side	
Number of windings per phase	2
Secondary voltage	400 V (phase to phase) 230 V (phase to neutral) 133.33 V (phase to neutral of one secondary winding)
Secondary current	1.45 A (line and phase current)
Number of turns per winding	715 split. 357.5 turns per secondary winding
Wire diameter	0.90 mm
Wire mass	3.65 kg
Core information	
Core	Core type
Window length	120 mm
Window width	40 mm
Core mass	8.05 kg
Losses	
Total I^2R wire loss	14.55 Watts
Total core loss	22.54 Watts
Cost of transformer (ZAR)	**R3950.00**

Table C.2: Practical delta-star transformer specifications.

Dyn1 Transformer specifications	
Parameter	**Value**
Power rating	1 kVA
Frequency	50 Hz
Connection type	Dyn1
Rating	400 V / 400 V (1:1)
Flux density	1.19 T
Primary side	
Number of primary windings per phase	1
Primary voltage	400 V (line and phase voltage)
Primary current	2.5 A (line current) 1.45 A (phase current)
Number of turns per winding	1104
Wire diameter	0.71 mm
Wire mass	2.28 kg
Secondary side	
Number of windings per phase	1
Secondary voltage	400 V (phase to phase) 230 V (phase to neutral)
Secondary current	1.45 A (line and phase current)
Number of turns per winding	646
Wire diameter	0.90 mm
Wire mass	3.29 kg
Core information	
Core	Core type
Window length	120 mm
Window width	40 mm
Core mass	8.05 kg
Losses	
Total I^2R wire loss	12.43 Watts
Total core loss	20.12 Watts
Cost of transformer (ZAR)	**R2620.00**

APPENDIX D: MEASUREMENT PROCEDURE TO DETERMINE THE OPEN CIRCUIT IMPEDANCE VALUES OF A PHYSICAL HARMONIC MITIGATING TRANSFORMER

D.1 Introduction

Appendix A presents a discussion on the typical open circuit test and short circuit test that is performed on a transformer in order to obtain the values for the respective matrices. The measurement procedure presented is easily performed on a single phase transformer however as the number of phases increase and consequently the number of windings, the measurement procedure becomes rather cumbersome. In order to circumvent the complexity experienced when considering multiple windings, the terminals of the transformer can be connected in a range of combinations so as to represent inductors in series or parallel. Furthermore some windings can be short circuited or open circuited. This will allow for the impedances to be measured using an alternating current bridge. It is important to note that the alternating current bridge performs the measurements when the transformer is not connected to a load or a supply. Furthermore the alternating current bridge obtains the impedance measurement by applying a low voltage across the terminals and then measures the resulting current. From the applied voltage and measured current, the bridge is able to determine the impedance. The voltage applied across the terminals is significantly lower than the rated voltage which means that the core losses will have minimal effect on the impedance measurement.

D.2 Measurement procedure

The alternating current bridge only has two terminal points where a measurement can be taken from. This implies that the transformer has to be connected in such a way so as to present two terminals to the bridge. In order to achieve this, consider the theoretical three winding system in Figure D. 1 (Note that no core is shown). All three windings are mutually coupled and have a voltage and current associated with them. By considering the modelling techniques discussed in the main text of this investigation, it can be seen that the three windings can be expressed by the matrix equation in Equation (D.1) taking note that the impedance values are used.

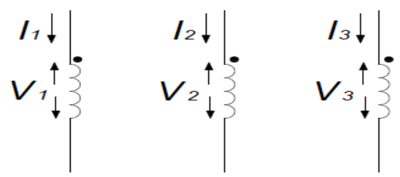

Figure D. 1: Theoretical three winding system to demonstrate the measurement technique used.

$$\begin{pmatrix} V_1 \\ V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} z_{11} & z_{12} & z_{13} \\ z_{21} & z_{22} & z_{23} \\ z_{31} & z_{32} & z_{33} \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \\ I_3 \end{pmatrix} \quad \textbf{(D.1)}$$

The variables in the matrix in Equation (D.1) can be multiplied out to yield Equation (D.2), Equation (D.3) and Equation (D.4). The main aim then is to determine the self-impedance values and the mutual impedance values by only measuring two points on the circuit. The sections that follow present the methods in order to achieve this.

$$V_1 = z_{11}I_1 + z_{12}I_2 + z_{13}I_3 \quad \textbf{(D.2)}$$

$$V_2 = z_{21}I_1 + z_{22}I_2 + z_{23}I_3 \quad \textbf{(D.3)}$$

$$V_3 = z_{31}I_1 + z_{32}I_2 + z_{33}I_3 \quad \textbf{(D.4)}$$

D.3 Determining the self-impedance values

As mentioned in the introduction, the windings have to be connected so that two terminals are available for the bridge. The connection scheme to determine the self-impedance value for z_{11} is presented in Figure D.2. From the figure it can be seen that the measurement is taken across winding 1 with winding 2 and 3 open circuited.

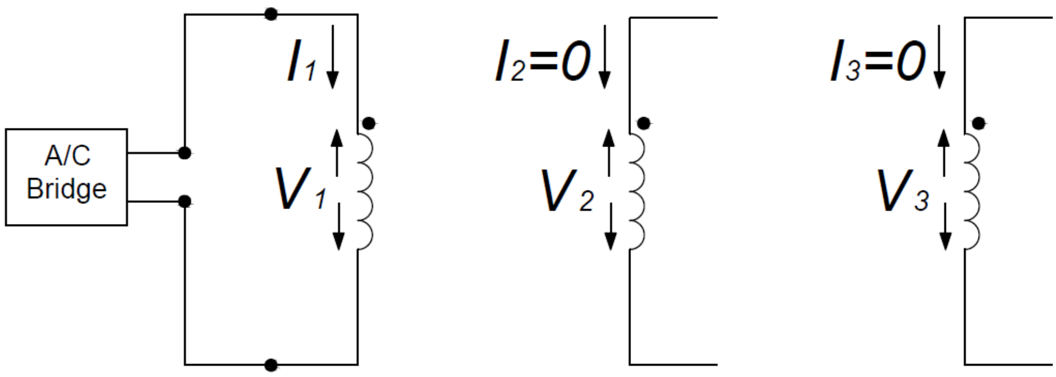

Figure D.2: Connection scheme to determine the self-impedance value for winding 1. The same technique can be applied to the other windings to determine their self-impedances.

If winding 2 and 3 are open-circuited, it is clear that I_2 and I_3 are equal to zero which means that Equation (D.2), Equation (D.3) and Equation (D.4) reduce to:

$$V_1 = z_{11}I_1$$

$$V_2 = Z_{21}I_1$$

$$V_3 = z_{31}I_1$$

Solving for z_{11}:

$$z_{11} = \frac{V_1}{I_1} \quad \textbf{(D.5)}$$

It is important to note that the values for z_{21} and z_{31} cannot be determined because the value for V_2 and V_3 cannot be measured at the same time. The reason is because the addition of another two alternating current bridges would interfere with the measurement for z_{11}. This implies that a different connection has to be implemented in order to measure the mutual impedance values. The remaining self-impedance values, z_{22} and z_{33} however can still be determined one at a time using the same method presented in Figure D.2 on each winding. Equation (D.5) can be re-written in the general form of Equation (D.6) whereby there are n number of windings and i is the winding number in the n by n impedance matrix.

$$z_{ii} = \frac{V_i}{I_i} \quad \text{(D.6)}$$

D.4 Determining the mutual impedance values

In order to determine the mutual impedance values consider the diagram in Figure D.3. From the figure it can be seen that winding 3 is open circuited while winding 2 is short circuited and the two measurement points are across winding 1.

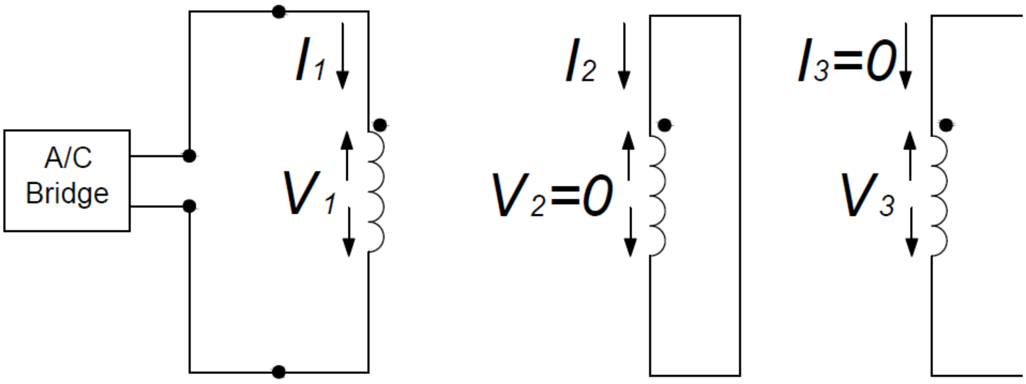

Figure D.3: Connection scheme of the three windings in order to determine the mutual admittance values

If winding 2 is short circuited and winding 3 is open circuited, then Equation (D.2), Equation (D.3) and Equation (D.4) reduce to:

$$V_1 = z_{11}I_1 + z_{12}I_2 \quad \text{(D.7)}$$

$$0 = z_{21}I_1 + z_{22}I_2 \quad \text{(D.8)}$$

$$V_3 = z_{31}I_1 + z_{32}I_2 \quad \text{(D.9)}$$

Up to this point only Equation (D.7) and Equation (D.8) will be considered because it is clear once again that V_3 cannot be measured at the same time using a bridge so Equation (D.9) is not used. So in Equation (D.8) solve for I_2:

$$0 = z_{21}I_1 + z_{22}I_2$$

$$I_2 = -\frac{z_{21}I_1}{z_{22}} \quad \textbf{(D.10)}$$

Substitute Equation (D.10) into Equation (D.7):

$$V_1 = z_{11}I_1 + z_{12}I_2$$

$$V_1 = z_{11}I_1 + z_{12}\left(-\frac{z_{21}I_1}{z_{22}}\right)$$

$$V_1 = I_1\left(z_{11} - \frac{z_{21}z_{12}}{z_{22}}\right)$$

$$\frac{V_1}{I_1} = \left(z_{11} - \frac{z_{21}z_{12}}{z_{22}}\right)$$

Up to this point it must be noted that z_{12} and z_{21} are equal therefore $z_{12} = z_{21} = z_m$ and it can be said that $\frac{V_1}{I_1} = z_{SC}$ which is the impedance measured by the bridge therefore:

$$z_{SC} = \left(z_{11} - \frac{{z_m}^2}{z_{22}}\right)$$

Solving for z_m:

$$z_{SC} - z_{11} = \left(-\frac{{z_m}^2}{z_{22}}\right)$$

$$\frac{{z_m}^2}{z_{22}} = z_{11} - z_{SC}$$

$$z_m = \sqrt{z_{22}(z_{11} - z_{SC})} \quad \textbf{(D. 11)}$$

The value z_{SC} is the value measured across winding 1 in Figure D.3. The connection scheme in Figure D.3 can be used to determine the other mutual impedance values provided the appropriate connection is done. For example to get the value for z_{23} (or z_{32}), winding 1 will be open circuited, winding 2 will be where the measurement is taken and winding 3 will be short circuited. Equation (D. 11) can be expressed in the more general form of Equation (D.12) for the case of *n* windings whereby *i* is the row and *k* is the column in the *n* by *n* impedance matrix [24].

$$z_{ik} = z_{ki} = \sqrt{z_{kk}\left(z_{ii} - z_{SC}^{ik}\right)} \qquad \textbf{(D.12)}$$

D.5 Conclusion

In order to measure the open circuit impedance values of a single phase transformer, the process is relatively straight forward however as the number of windings increase, so the complexity of the measurement procedure increases. This section proposes a convenient method for determining the open circuit impedances for a multiple winding transformer. The measurements involve the use of an alternating current bridge. The bridge is capable of measuring the impedance of each winding by applying a small voltage across the winding and measuring the resulting current. Using this technique the self-impedance of each winding can be measured. A slight variation in the measurement procedure is required in order to measure the mutual impedances between windings. This variation involves either open or short-circuiting the windings such that the mutual impedance can be measured indirectly. It is important to note that the transformer must not be connected to a load or a supply and that the voltage applied across the terminals is considerably lower than the transformer rated voltage.

APPENDIX E: OPEN CIRCUIT IMPEDANCE VALUES FOR A THREE PHASE THREE WINDING TRANSFORMER

Table E.1: Open circuit impedance values for a three phase three winding transformer. The grey areas highlight the impedance values for windings on the same core limb. Note that all values must be scaled by 1 x 10^3. The position of each value in the table corresponds to the position of the value in the matrix for example the first cell in the table corresponds to the matrix position row 1, column 1 in the nine-by-nine matrix.

0.0963 + 0.8926i	0.0387 + 0.3299i	0.0387 + 0.3309i	0.0510 + 0.5517i	0.0237 + 0.2052i	0.0237 + 0.2064i	0.0107 + 0.2002i	0.0051 + 0.0741i	0.0048 + 0.0743i
0.0387 + 0.3299i	0.0195 + 0.1224i	0.0176 + 0.1222i	0.0279 + 0.2172i	0.0123 + 0.0808i	0.0124 + 0.0812i	0.0050 + 0.0752i	0.0022 + 0.0276i	0.0021 + 0.0277i
0.0387 + 0.3309i	0.0176 + 0.1222i	0.0206 + 0.1231i	0.0285 + 0.2193i	0.0126 + 0.0816i	0.0127 + 0.0820i	0.0066 + 0.0789i	0.0030 + 0.0290i	0.0029 + 0.0290i
0.0510 + 0.5517i	0.0279 + 0.2172i	0.0285 + 0.2193i	0.1191 + 1.1684i	0.0506 + 0.4352i	0.0506 + 0.4363i	0.0408 + 0.5198i	0.0185 + 0.1916i	0.0183 + 0.1924i
0.0237 + 0.2052i	0.0123 + 0.0808i	0.0126 + 0.0816i	0.0506 + 0.4352i	0.0252 + 0.1625i	0.0233 + 0.1623i	0.0219 + 0.1996i	0.0096 + 0.0737i	0.0096 + 0.0739i
0.0237 + 0.2064i	0.0124 + 0.0812i	0.0127 + 0.0820i	0.0506 + 0.4363i	0.0233 + 0.1623i	0.0263 + 0.1633i	0.0228 + 0.2020i	0.0099 + 0.0744i	0.0100 + 0.0747i
0.0107 + 0.2002i	0.0050 + 0.0752i	0.0066 + 0.0789i	0.0408 + 0.5198i	0.0219 + 0.1996i	0.0228 + 0.2020i	0.0993 + 0.9021i	0.0395 + 0.3327i	0.0393 + 0.3329i
0.0051 + 0.0741i	0.0022 + 0.0276i	0.0030 + 0.0290i	0.0185 + 0.1916i	0.0096 + 0.0737i	0.0099 + 0.0744i	0.0395 + 0.3327i	0.0196 + 0.1232i	0.0176 + 0.1226i
0.0048 + 0.0743i	0.0021 + 0.0277i	0.0029 + 0.0290i	0.0183 + 0.1924i	0.0096 + 0.0739i	0.0100 + 0.0747i	0.0393 + 0.3329i	0.0176 + 0.1226i	0.0206 + 0.1233i

APPENDIX F: NO-LOAD AND LOAD TEST RESULTS FOR THE HARMONIC MITIGATING TRANSFORMER

Table F. 1: results for the no-load and load tests performed on the ideal coupling model, physical transformer and non-ideal coupling model

	Non-ideal coupling model		Physical transformer		Ideal coupling model	
	No-load	**Load = 1000 Ω**	**No-load**	**Load = 1000 Ω**	**No-load**	**Load = 1000 Ω**
I_{1w}	$0.481\angle-60.926°\,A$	$0.531\angle-53.012°\,A$	$0.18\angle-81.00°\,A$	$0.48\angle-75.00°\,A$	$0.617\angle-90.000°\,A$	$0.630\angle-77.795°\,A$
I_{2w}	-	$0.250\angle-32.769°\,A$	-	$0.23\angle-30.00°\,A$	-	$0.231\angle-30.000°\,A$
I_{3w}	$0.529\angle155.730°\,A$	$0.540\angle165.173°\,A$	$0.17\angle159.00°\,A$	$0.46\angle165.00°\,A$	$0.617\angle150.000°\,A$	$0.630\angle162.205°\,A$
I_{4w}	-	$0.239\angle-151.470°\,A$	-	$0.22\angle-150.00°\,A$	-	$0.231\angle-150.000°\,A$
I_{5w}	$0.449\angle16.252°\,A$	$0.434\angle28.469°\,A$	$0.17\angle39.00°\,A$	$0.47\angle45.00°\,A$	$0.617\angle30.000°\,A$	$0.630\angle42.205°\,A$
I_{6w}	-	$0.252\angle90.046°\,A$	-	$0.23\angle90.00°\,A$	-	$0.231\angle0.000°\,A$
V_{w2}	$252.492\angle-30.000°\,V$	$249.708\angle-32.769°\,V$	$226.10\angle-30.000°\,V$	$220.61\angle-30.00°\,V$	$230.968\angle-30.000°\,V$	$230.968\angle-30.000°\,V$
V_{w4}	$241.052\angle-150.000°\,V$	$238.534\angle-151.470°\,V$	$226.10\angle-150.00°\,V$	$220.61\angle-150.00°\,V$	$230.968\angle-150.000°\,V$	$230.968\angle-150.000°\,V$
V_{w6}	$254.696\angle90.000°\,V$	$252.125\angle90.046°\,V$	$226.10\angle90.00°\,V$	$220.61\angle90.00°\,V$	$230.968\angle90.000°\,V$	$230.968\angle90.000°\,V$