Valuation of Structured Bonds in Illiquid Markets

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DECLARATION

I, Benard Gora declare that the research work reported in this dissertation is my own, except where otherwise indicated and acknowledged. It is submitted for the degree of Master of Management in Finance and Investment at the University of the Witwatersrand, Johannesburg. This thesis has not, either in whole or in part, been submitted for a degree or diploma to any other universities.

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Abstract

Corporations often find it difficult to raise capital in illiquid markets such as most African markets and if they do they pay a premium which is not only costly to them but also propagates illiquidity in these markets. Convertible bonds provide a cheaper source of financing for issuers with the optionality of maintaining targeted capital structures. However, these instruments are not popular in these markets as they are less understood compared to their conventional counterparts and if used are often mispriced. The main objective of this research is to provide a valuation framework for structured bonds, specifically convertible bonds where market imperfections such as illiquidity are prevalent. This will entail customising the standard valuation framework so that these market imperfections are incorporated in the model. The valuation framework of the convertible bond is then applied to an illiquid market where several deviations from the perfect-market benchmarks exist and then observe what effect these deviations have by comparing the theoretical value to that of a convertible bond assuming the market is liquid.

Acknowledgements

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1 Introduction

Corporations could not have grown to their present size without being able to find innovative ways to raise capital to finance expansion. Corporations raise capital through the sale of equities and bonds. They also raise capital through a hybrid security of stocks and bonds. Raising capital by issuing bonds is a popular alternative to selling shares, as it allows a company to avoid relinquishing ownership of part of the business. However it is susceptible to interest rate volatilities. A hybrid security provides characteristics that limit the downside risk while providing better or similar potential of both equity and debt.

1.1 Problem Statement

Essentially, a convertible bond is a bond that can be converted into equity. Often, a convertible bond is described as a bond warrant, long-rated call option that is issued by a corporation on its own stock, attached. Raising capital through equity or straight debt presents challenges for corporations especially in illiquid markets due to risk exposures faced by investors while their capital is tied up in those assets as there's lack of immediacy in that market. Convertible bonds are hybrid instruments with both debt and equity traits which offers flexibility especially for external investors in that they can change their portfolio holdings from debt to equity depending on which one creates better value. Also, they offer investors better exit strategies in illiquid markets. Illiquid markets present challenges in terms of capturing the effects of the deviations from traits observed in liquid or near-perfect markets to both issuers and investors. Valuation of assets under such conditions becomes complicated and requires more art than science to fully capture the complex behaviour of assets. Investors are willing to invest in such markets if they understand the underlying risk factors and if those risk factors are considered to be fairly priced. While issues arising from equity and bonds are extensively studied by researchers, convertible bonds research still trails a short way behind and that may be the reason why they are still not as popular as other forms of debt which have a more active market presence as issuers and investors do not understand them. The African market which currently has a lot of potential is being eyed by investors for capital injection. These structured bonds need to be understood by both issuers and investors so that they can be fairly valued so as to attract more capital inflow. On the same breath, issuers should not be called upon to pay a premium for the risk that has been mispriced.

1.2 Purpose of Study

The purpose of this research is to study the valuation framework of structured bonds, specifically convertible bonds, in illiquid markets. This valuation framework will then be used to analyse the characteristics of a convertible to find out how the traits of these assets deviate from those that have been valued using standard valuation methods and are currently trading in the secondary markets. The research aims to show the effects of embedded options on bonds in illiquid markets which is characterised by lack of information and not much trading activities of the underlying assets.

1.3 Questions of Study

The theoretical and empirical literature of this research will address four basic questions:

(i) What is the effect of embedding an option on a bond?

(ii) How does market illiquidity affect the value of the bond?

(iii) What is the valuation framework of a convertible bond in an illiquid market?

(iv) How does this valuation framework differ from that of a convertible bond in a liquid market?

1.4 Significance of Study

Convertible bonds are innovative financial instruments which despite their name, have greater similarities with exotic options than with conventional bonds. The popularity of convertible bonds with issuers has been due to several reasons. By issuing convertible bonds they can lower their cost of debt funding compared to straight debt alone. Lower-credit rated companies which may not be able to access the straight debt market can often still issue convertible bonds. Companies which anticipate equity appreciation can use convertible bonds to defer equity financing to a time when growth has been achieved. Investors find several features of convertible bonds appealing. Convertible bonds offer greater stability of income than common stock thus offering protection against losses, enabling investors to invest in riskier firms without exceeding their risk exposure limits. They provide a yield that is often higher than dividend yield of common stock. Finally, because they are often under-priced they may provide a cheap source of common stock volatility that offset the firm's credit risk. At higher volatilities the convertible value increases because the value of the option to exchange the straight bond for stock is greater. Understanding the characteristics of convertible bonds in the illiquid markets will give corporations better tools for pricing convertible bonds and allow investors to better balance and hedge their portfolios. This gives corporations an opportunity to attract both internal and external investors to the markets where they are raising capital and also giving them an optionality of converting debt to equity as they grow. The optionality allows corporations to maintain or not deviate much from their targeted capital structures.

By the same token, it would be a fallacy to conclude that convertibles therefore provide better risk-return traits compared to equity and straight bonds as equity will perform much better in rising equity prices and straight bonds will fair much better in falling share prices.

1.5 Background Literature

The first convertible bond was issued in 1881, with J.J Hill (Calamos (2003))by a railroad magnate who needed to raise capital for the railroad project but needed some form of cheaper financing other than ordinary debt issues. Since then the convertible bond market has grown, with well-developed markets in the U.S., Europe, and Japan, the U.S market being the most liquid of them and the Japan market being extensively controlled. The convertible bond markets in the U.S and Japan are of primary global importance as they are the two largest domestic markets in terms of capitalization. A growing European convertible bond market has become an important source of financing even though it experienced a slump in the 2007-2009 period. Over time, the convertible securities market has matured, surpassing $230 billion in market capitalization as of December 31, 2010. The first convertible bond in South Africa was publicly issued in 2009 by Aquarius Platinum, a mining company. The most recent was the Shoprite, a retailer, offering which took place in May 2012 mainly to raise capital for their expansion purposes into the African market.

Convertible bonds cannot be classified as either equity or bonds or simply a combination of both as they present unique characteristics and challenges. It is therefore a hybrid security with characteristics of both debt and equity. When the bondholder decides to convert, they exchange the bond for a fixed number of shares, thus giving up the remaining bond cash flows for the equity cash flows. The convertible bond begins as a plain vanilla bond and then becomes a hybrid with characteristics of both a bond and equity then transforms into an equity asset. This transition is dependent on the conditions of conversion and the price of the underlying stock. Because convertible bonds are hybrids of bonds and stocks, they must inherit all the complexities of the underlying instruments and behave in an often complicated fashion as a mix of two securities. While convertible securities may offer many opportunities for investing and formulating portfolio strategies they are also relatively complicated to analyse. The convertible bond price movement is mainly influenced by the underlying stock price movement and thus the valuation of convertibles is based on equity valuation analysis. However the equity valuation analysis tools like the Black-Scholes Merton model do not provide valuations where deviations from the underlying assumptions are observed. This thesis offers an analytic framework that will enable bond issuers and investors to deal with some of the difficulties especially in illiquid markets.

1.6 Methodology

Although structured bonds are available with a wide variety of underlying assets, this thesis will focus on convertible bonds. This thesis intends to provide a valuation framework for structured bonds, specifically convertible bonds, in illiquid markets. The valuation framework will then be tested to observe how changes in parameters will have an effect on the value of the convertible bond. An example of a convertible bond that has already been issued in the market will be used for the analysis and conclusions will be drawn based on the behaviour of that convertible bond. The terms of the BancABC convertible bond that was issued to the IFC in Botswana will be used to analyse the features of the convertible bond using a valuation framework that has been derived specifically for illiquid markets. These features will then be compared to the convertible bond behaviour in liquid markets that have been valued using the standard valuation framework so that the effect of deviating from the underlying assumptions can be quantified.

1.7 Outline of Study

In this thesis, standard valuation methods of conventional bonds and some bonds with embedded options in liquid markets will be introduced in Chapter 2. In this chapter basic terms and effects of embedding options on straight bonds will be analysed and the effects will be incorporated on the price of the straight bond. Chapter 3 will introduce the issue of illiquidity in markets which will create deviations from the assumptions made on the standard valuation methods introduced in Chapter 2. This chapter will provide a valuation framework for exotic options, since they have similarities to convertibles, in illiquid markets. The main contribution of this thesis is in Chapter 4, where the valuation framework introduced in Chapter 3 will be used to value a convertible bond in an illiquid market, namely the (BancABC) convertible bond that was issued in Botswana. Comparisons will be made with the prices of the convertible bond to observe how the issue of market illiquidity has an effect on the value of the convertible bond. The valuation framework will then be compared to the framework used in a market such as Bloomberg to quantify the extent of deviation and mispricing of convertibles in illiquid markets. The conclusion will focus on the improvements that need to be done on the current models to address the issue of pricing in illiquid markets.

2 Structured Bonds and Valuation Methodologies

Structured bonds are debt securities that feature customised terms. They are hybrid securities that combine fixed income or stock and derivative characteristics and as a result are extremely popular among all investors' classes. Examples of structured bonds are callable bonds, convertible bonds, corridor bonds etc. The addition of a derivative changes the asset's risk-return profile to make it more suited to the potential investors. This would make it possible to invest in an asset class that would otherwise be considered too risky especially for insurance companies and pension funds who have regulatory restrictions attached. The main characteristics of such bonds consists of the general terms of issue that define maturity, repayment and interest rates (coupons). Structured bonds are attractive alternatives to conventional debt securities because their terms can be defined flexibly.

Although now finding favour with private investors, structured bonds were originally designed to protect corporate investors against adverse movements in financial costs. If a corporation needed to protect a loan against a possible rise in interest rates, for example, a bank or finance company would be called upon to structure a bond whose income grew when interest rates rose. Structured bonds attract investors for a number of reasons. The most significant of these are the attractive earnings opportunities, the ability to solve individual problems and the chance of gaining access to restricted or complex markets. Another major attraction is the ease with which these bonds can be used. By buying these bonds, investors access markets traditionally reserved for investment banks and sophisticated funds and are exposed to risk and returns normally prohibitive to them for various reasons, such as lack of financial expertise, size, regulatory constraints, etc. Attractions to issuers are that they allow issuers to obtain funds more cheaply than is the case with conventional products. The standard procedure is to issue a structured bond, which is swapped simultaneously with the bank that arranged the issue and thus less administrative issues in the issuing process. However, these bonds are complex and if not clearly understood by both parties they can be a source of huge losses. They have been blamed for the turmoils in Greece, Italy and the subprime crisis in the U.S.

Bond portfolio managers and other investors are faced with challenges when they consider pricing and purchasing structured bonds. Practical issues that need to be addressed with structured bonds are those related to the building of cash flows and discounting factors. This in turn is related to the ability to manage calender dates and schedules. Among questions posed about structured bonds are usually whether they are 'fairly priced', whether investors are adequately presented for the associated risks this type of investment entails, whether the expected returns reflect the risks assumed by investors. Many times heavy criticisms lie on the funding cost of the issuer which is hidden behind the structuring process and cannot be easily calculated as it has allegedly happened in the Lehman case. On the other hand complexity is usually equated to excessive risk which is not necessarily true. Extensive empirical studies have dealt with the problem of whether structured products are 'fairly priced' or not. Usually fairness is measured as a percentage price difference between the issue (or transacted) price and a theoretical model price or as a difference between the volatility used in the embedded option of the structured product and the volatility appearing in listed options of similar type of maturity. Although results of price differences are usually reported with respect to specific products and markets, sometimes they are reported on a relative basis compared to other products.

2.1 Structure of structured bonds

Structured bonds provide partial or 100% capital protection depending upon the investor's needs. When an investor buys a structured bond, he/she actually has bought a package which consists of a bond and an option and the fee on top of it. The payoff of a structured bond is equal to the par amount of the bond plus a commodity/equity linked coupon. The payoff is either

i. zero if the underlying has depreciated from the initial agreed upon strike level.

ii. or the participation rate times the percentage change in the underlying commodity/equity times the par amount of the bond.

2.1.1 The bond component

The bond component of a structured bond is the most important part of it. It is also the major part of any structured bond. The bond component ensures that the investor will receive the agreed amount of his/her investment at maturity. The agreed amount can be 100% of the invested capital or it can be a partial protection depending on the product. Structured bonds can have the characteristics of a zero coupon bond or it can have coupon payments (annual or semi-annual)

2.1.2 The option component

The option component is the second part of a structured bond. It provides the chances of payoff. Options are of two types that is a call and a put. A call option provides the chances to its holders the right to buy an asset at a certain date on a pre-specified price while a put option gives its holders the right to sell an asset at a pre-specified price on a certain date. Call options are normally embedded in structured bonds because it is easy to earn something whose price is increasing rather than decreasing as in case of a put option. The option component is also the risky part of any structured bond because the payoff depends on the performance of the underlying. If the option embedded in a structured bond expires out of the money (that is the strike price of the call option is higher than the corresponding price of the underlying) then it will not be exercised and the holder will get no profit and will instead lose money used to buy that option but will still receive the invested capital. If on the other hand, an option expires in the money then it will be exercised and the holder will earn profit along with guaranteed capital.

2.1.3 Participation Rate

Participation rate determines how much the product will participate in the performance of the underlying. It can be defined as the exposure of a product to movements in the price of its underlying. A participation rate of 100% means that the investor would receive the return that will be exactly equal to the rise in the price of the underlying. For example if the underlying has increased by 25% at maturity, then the investor will also receive 25% return. Participation rate depends on the value of the option embedded in the structured product, the administration and other issuing costs and the present value of the bond component of the structured bond. Participation rate is generally not set prior to the expiry of the issuance period and it appears as an estimate in the prospectus. The participation rate can be calculated by the following relation

$$PR = \frac{P_i - PV_B}{OP} \times 100 \tag{1}$$

where PR is the participation rate, P_i is the issue price, PV_B is the present value of the bond and OP is the price of the option component.

Participation rate is also known as Gearing. The above relation also shows that there are other factors which determine the participation rate. For example, the discount rate, the term of the structured bond and volatility of the underlying asset.

2.2 Structured Bond Valuation

The primary objective is to show that investors and issuers value structured bonds in a different way that depends on the level of sophistication, on the ability to access the derivatives market, on the different hedging strategies they employ and on the level of risk they want to bear. Although in theory the pricing of a structured bond does not present any perculiarity, in practice, different risk assessments, restrictions to access particular markets related to coupon performance, inability to dynamically replicate the pay-off and asymmetry of information making pricing more complicated and not unique for all the various parties participating in the transaction. Small-sized, private complex structures without active secondary markets aggravate the situation. From a theoretical standpoint, the price of a structured bond equals the expected discounted (adjusted for issuer's credit rating) cash flows. In practice, the notion of 'fair price' becomes very subjective and represents different things to different people.

There are theoretically three approaches to the valuation of financial instruments. The first valuation approach would be to find the closed form solution that is estimating the value by a combination of functions. The second approach values the instruments numerically such as numerical partial differential equations (PDEs) approach. Binomial tree valuation is used in this approach as a generalised numerical method to solve PDEs. This is one of the common approaches used in practice and in this thesis most of the valuation models are based on this approach. The third valuation approach is based on monte carlo simulation whereby a number of possible outcomes are sampled in the algorithm that calculates the value of the financial instrument.

2.3 Valuation of Plain Vanilla Bonds

The theoretical price of a bond can be calculated as the present value of all the cash flows that will be received by the holders of the bond. These cash flows are the bond's principal which is received at maturity. The other cash flows are coupon payments, which are received at fixed time intervals (semi-annually, in most cases). For bonds with coupon payments, c, yield to maturity, y, maturity in years, T, principal, P and assuming the coupon payments are made semi annually then the price is given by

$$B_t = \frac{c}{2}e^{-y_1 t_1} + \frac{c}{2}e^{-y_2 t_2} + \frac{c}{2}e^{-y_3 t_3} + ... + (P + \frac{c}{2})e^{-y_{2T} t_{2T}} \tag{2}$$

this can also be written as

$$B_t = \sum_{i=1}^{2T} \frac{c}{2}e^{-y_i t_i} + Pe^{-y_{2T} t_{2T}} \tag{3}$$

2.3.1 Example 1: Vanilla bond value calculation

We provide a hypothetical bond which will be used throughout this research to demonstrate the effects of changing the bond or deviating from the underlying assumptions that are used in the valuation of standard models.

Bond characteristics:

Principal: R100

Coupon: 10% per year

Maturity: 5years

Risk free rate: 5% per year

Risk premium Credit Spread: 500 basis points

Given these characteristics the bond price is given by,

$$\begin{aligned} B_t &= 10e^{-0.1\times 1} + 10e^{-0.1\times 2} + 10e^{-0.1\times 3} + 10e^{-0.1\times 4} + 110e^{-0.1\times 5} \\ &= 98.07 \end{aligned}$$

2.4 Valuation of a Callable Bond

A callable bond is a bond that contains a provision allowing the issuing firm to buy back the bond at a predetermined price at uncertain times in the future. Callable bonds cannot usually be called for the first few years of their life. After that the call price is usually a decreasing function of time. Bonds with call features generally offer higher yields than bonds with no call features due to call uncertainties and therefore investors would like to increase returns before call date. Often callable bonds cannot be called until 5 or 10 years after they have been issued. When this is the case the bonds are said to be call protected. The date when the bonds are called is called the call date. Bonds may be called for managerial reasons. The firm may wish to retire some debt in order to lower the debt ratio and consequently increase the corporate bond rating, or it may wish to have a simpler balance sheet to ready itself for merger and acquisition activities. In such cases investors may not suffer financially from the call because the call price may be significantly higher than an equivalent bond. There are two call provisions:

(i) **Traditional Fixed-Price Call Provision**

These bonds are called at a fixed price. By calling existing bonds, the issuer forces the bondholders to surrender their bonds in exchange for the fixed call price. This is a disadvantage to investors since they can only reinvest funds at lower interest rates. Consequently these callable bonds are less attractive to investors than non callable bonds and hence sell at a lower price.

(ii) **Make-Whole Call Provision**

If a callable bond has a make-whole call provision, bondholders receive approximately what the bond is worth if the bond is called. This call provision gets its name because the bondholder does not suffer a loss in the event of a call; that is, the bondholder is made whole when the bond is called. Unlike a fixed-price call provision, a make-whole call provision requires the borrower to make a lump-sum payment representing the value of all payments that will not be made because of the call. The discount rate used to calculate the present value is usually equal to the yield on a comparable risk free rate instrument plus a fixed, pre-specified make-whole premium.

The holder of a callable bond has sold a call option to the issuer. The strike price or call price in the option is the predetermined price that must be paid by the issuer to the holder. The plain vanilla bond has no option embedded in it, thus the closed form solution will be the same as the discounted cash flow valuation of the bond while callable bonds have an option embedded in a plain vanilla bond and thus the closed form solution will be different. The presence of an embedded call option on a callable bond introduces the issue of randomness as the issuer can call the bond at uncertain times. This randomness can be represented by a geometric Brownian motion in a continuous case where a closed form solution can then be used to provide a valuation framework of the bond. In a discreet case a binomial tree valuation framework is used to capture the randomness by incorporating possible movements of the bond prices with uncertain possibilities of the bond being called back by the issuer. The binomial tree valuation is expected to be different from the closed form solution, but will tend to be close with a decrease in the time periods between the steps. It is extremely important for the investor to realise that the presence of the embedded call option in a bond affects the value of the bond. The two valuation frameworks are presented and compared below.

2.4.1 The Closed Form Solution Valuation Approach

The fair value of a vanilla bond with principal P, continuously compounded risk-free rate from time t to t_i, $r(t,t_i)$, coupon rate c and a credit spread $\alpha(t)$ is given by

$$B_t = cP\sum_{i=1}^{T} e^{(r(t,t_i)+\alpha(t_i)(T-t))} \tag{4}$$

where T is the maturity in years. The credit spread $\alpha(t)$ indicates the level of credit risk. Credit risk is the possibility that a bond issuer will default, by failing to repay the principal and the interest rate at the expected time. Bonds issued by the government bear no risk of default, for the most part and therefore the cost to the government is the risk free rate while bonds issued by corporations are more likely to be defaulted on, since corporations can go bankrupt. Thus in the pricing of corporate bonds the probability of default is incorporated by adding a credit spread on the risk free interest rate to quantify credit risk. The credit spread influences the present value of the bond. So the higher the credit spread the larger the discount rate that will be used to calculate the present value of the bond indicating a higher probability of default. Widening credit spreads indicate growing concern about the ability of corporate borrowers to service their debt. If a large value of the discount rate is used then the present value of the bond will be lower compared to a borrower with a lower credit spread.

Consider the dynamics of the bond price to be as follows

$$dB_t = rB_t dt + \sigma B_t dW_t \tag{5}$$

The price of the European call option, C_t, on a bond with strike price K and time to maturity T is given by the Black-Scholes pricing formula

$$C = B_t N(d_1) - e^{-r(T-t)} K N(d_2) \tag{6}$$

Here $N(\cdot)$ is the cumulative distribution function for the standard normal, $N(0,1)$, distribution and

$$d_1 = \frac{1}{\sigma\sqrt{T-t}}\left(\ln\left(\frac{B_t}{K}\right) + \left(r + \frac{1}{2}\sigma^2\right)(T-t)\right), \tag{7}$$

$$d_2 = d_1 - \sigma\sqrt{T-t} \tag{8}$$

The fair value of a callable bond is given by

$$B^c = B_t - C_t \tag{9}$$

This equation shows that holding a callable bond, B^c, is the same as holding a non-callable bond, B_t and shorting a call option on that bond, C_t. Holding a callable bond is the same as holding a vanilla bond and also having a call option to sell the same bond. The holder of the bond has the optionality to redeem the bond before maturity as can be seen from the Black Scholes formula for a call option where the N(.) represents the choice for the holder of the option to exercise or not to exercise at a particular time. The issuer determines the periods when the bond can be redeemed by the holder and thus the holder can be said to be short a call option. From an investors view a fall in interest rates will lead to a higher price of the option free bond, but part of that price increase is mitigated by the loss of the call option position. From an issuers perspective, in issuing a callable bond the firm has issued an option free bond but simultaneously has bought some call options of the bond. In doing so the firm has the right to retire the bond at a fixed price but at the cost of the option premium. Therefore the callable bond price should be lower than that of the corresponding non-callable bond. In practice callable bonds can be priced using binomial trees and therefore a look at how this approach is used is important for valuing these bonds.

2.4.2 The Binomial Tree Valuation Approach

The general approach adopted in the binomial tree is similar to that in an important paper adopted by Cox, Ross and Rubinstein (1979). Binomial trees provide a generalised numerical method to solve PDEs. It is essentially a discreet version of the Black Scholes Merton (BSM) European option formula under the risk-neutral valuation framework, and gives a good approximation value for assets without dividends/coupons to the BSM model. The binomial tree approach involves dividing the life of an asset into a large number of small time intervals of length, $\triangle t$. It assumes that the price of the underlying asset moves from its initial value of S to one of two new values S_u which is an upward movement with probability p and S_d which is a downward movement with probability $1-p$.

Suppose the asset provides a yield of q. The expected return in the form of capital gains must be $r-q$, where r is the risk free rate of return. This means that the expected value of the asset price at the end of the time interval of length $\triangle t$ must be $Se^{(r-q)\triangle t}$, where S is the asset price at the beginning of the time interval. It follows that

$$Se^{(r-q)\triangle t} = pSu + (1-p)Sd \tag{10}$$

The variance of the percentage change in the asset price in a small time interval of length $\triangle t$ is $\sigma^2 \triangle t$. There is probability p that the percentage change is u and $1-p$ that it is

d. The expected percentage change is $e^{(r-q)\triangle t}$. It follows that

$$pu^2 + (1-p)d^2 - e^{2(r-q)\triangle t} = \sigma^2 \triangle t \tag{11}$$

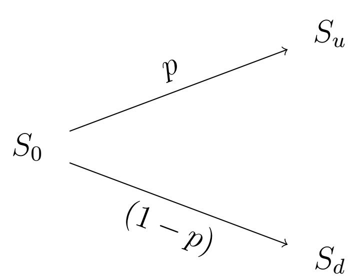
Solving (10) for p we find

$$p = \frac{e^{(r-q)\triangle t} - d}{u - d} \tag{12}$$

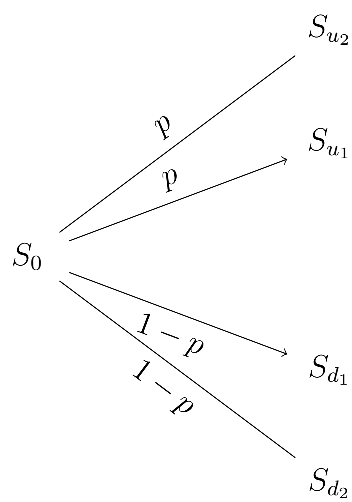
p can be interpreted to be the subjective probability of an up movement in the stock price, in this case. The variable $1 - p$ is then the subjective probability of a downward movement. Substituting p into (11) and ignoring terms in $\triangle t^2$ and higher powers of $\triangle t$ from the Taylor series we get

$$u = e^{\sigma\sqrt{\triangle t}} \quad \text{and} \quad d = e^{-\sigma\sqrt{\triangle t}} \tag{13}$$

From the above it can be seen that the variables are inverses of each other and they are also functions of volatility.



An increase in volatility implies the value of u will be large and the value of d will be small. The difference between S_u and S_d will be wider as a result. The above one period model will have much more acute slopes from period zero to period one implying that with the same probabilities the stock price will increase more or decrease more from period zero to period one as compared to when the volatility is not high. The diagram below shows what happens as volatility increases.



Let σ_1 represent the volatility leading to the movement from S_0 to either S_{u_1} or S_{d_1} and σ_2 represent the volatility leading to the movement to either S_{u_2} or S_{d_2}. Given that $\sigma_2 > \sigma_1$ then the distance between the upward node and the downward node will be wider. Thus it becomes much riskier to hold an asset at high volatilities but the returns are also likely to be higher.

The binomial tree concept can be extended to interest rates so that its derivatives can also be valued using binomial trees. Interest rates have an impact on the prices of bonds. There is an inverse relationship between the price of a bond and interest rates, that is, when the interest rate increases the price of the bond decreases. A look at the interest rate models using binomial trees will give an insight on the bond price movements. Binomial interest rate trees are a bit more complex to model than share prices, as they cannot take on arbitrary values, since this would be inconsistent with the arbitrage valuation framework. The study of interest rate models was pioneered by Merton (1974). Some of the popular models are the ones created by Vasicek (1977) and Cox, Ingersoll and Ross (1985), Ho-Lee (1986) and Hull-White (1990). The interest rate r, under the risk neutral dynamics is given by

$$dr_t = \mu(t, r_t)dt + \sigma(t, r_t)dW_t \tag{14}$$

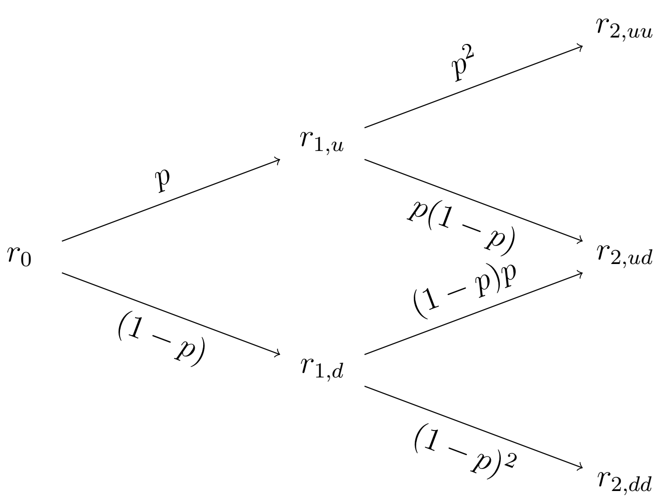
where $\mu(t, r_t)$ and $\sigma(t, r_t)$ are drift and volatility functions, respectively, W_t is a Brownian motion. The higher the volatility the more the interest rate becomes more unpredictable as there will be too much noise around the drift. The above model can be expressed in discrete time as

$$\triangle \ln r_t = \ln r_{t+\triangle t} - \ln r_t = \mu \triangle t + \sigma\sqrt{\triangle t} Z_t \tag{15}$$

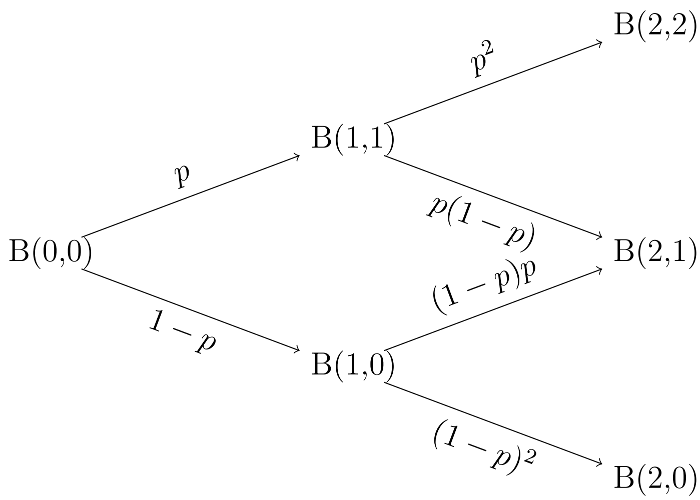
where Z_t is a random variable that takes on the value +1 and -1 with risk neutral probability p and $1-p$ respectively. The variable Z_t can be seen as a random walk. Thus there are two movements in each node

$$\ln r_{t+\triangle t} = \begin{cases} \ln r_t + \mu \triangle t + \sigma\sqrt{\triangle t} & \text{with} \quad p = 0.5 \\ \ln r_t + \mu \triangle t - \sigma\sqrt{\triangle t} & \text{with} \quad 1-p = 0.5 \end{cases}$$

Using the above discreet interest rate model, we can calculate the price of a callable bond starting from the end and working backwards to get the price at the beginning of the period. The binomial tree for the interest rates is shown below



Another binomial tree for the bond prices should be constructed and the above interest rates in the binomial tree can be used as discount rates. These bonds are like zero-coupon bonds with unit principal. $B(n, i)$, n is the period and i is the state.



Suppose the firm announces a call price a month before the call date. The valuation will start from the end of period 2, where all the nodes will be having the same price at maturity and then work backwards to period 1 using the interest rates in the first binomial tree as discount rates. That is discount $B(2,1)$ with the interest rate $r_{1,d}$ and multiply it by the upward probability which is 0.5 and then discount $B(2,0)$ with $r_{1,d}$ interest rate and multiply it by the downward probability. If you add these two calculated values you will get the price at $B(1,0)$. The bond price at each node will be given by

$$B(n,i) = \min[B^c, B_t] + c \tag{16}$$

where $B(n,i)$ is the bond price at period n and state i, B^c is the call price, B_t is the calculated price and c is the coupon payment. If the same process is continued then the value of the callable bond at node 0 will be derived which will be the price.

2.4.3 Example 2: Callable bond binomial tree calculation

Bond characteristics:

Principal: R100

Coupon: 10% per year

Maturity: 5years

Risk free rate: 5% per year

Risk Premium Credit Spread: 500 basis points

Volatility: 10%

Interest rates: up and down probability 0.5

The pricing formula in each node: $[p \times P_u + (1 - p) \times P_d]/(1 + i)$

Call Price: R115 after 2 years decreasing by R5 each year up to maturity

where p is the upward probability, $1 - p$ is the downward probability, P_u is the up price, P_d is the down price and $i$is the interest rate used for discounting.

Given these parameters we get,

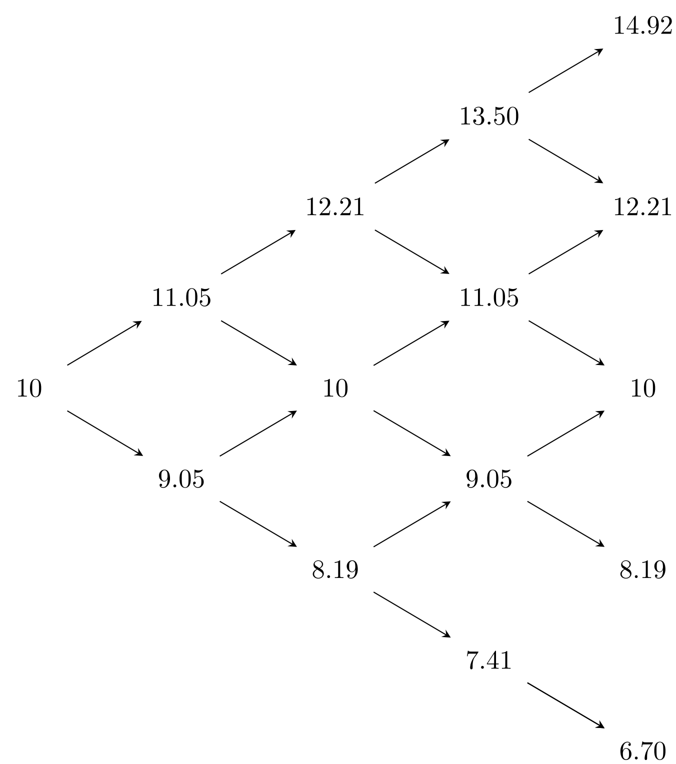


Figure 1: Rates Binomial Tree

To value both the plain vanilla bond (non callable bond) and the callable bond one must consider the volatility of interest rates as their volatility will affect the possibility of the call option being exercised for a callable bond. The interest rate binomial tree shown in Figure 1 models the random evolution of future interest rates. The volatility dependent one-period forward rates produced by this tree can then be used to discount the cash flows of the bond in order to arrive at the present value of the bond. Corporates are risky and therefore cannot be discounted by risk free rates and investors do not know exactly how risky these bonds are and how much higher the return should be. The interest rate binomial tree presents a consistent averaging model for calculating fair price of bonds. Each node in the tree is labeled by the interest rate for the period of a year that begins in that node. A look at some nodes to clarify how the interest rates evolved from one period to the adjacent periods. The evolution of the interest rate through the binomial tree is based on the short rate model (15). The interest rate is 10% at period 0 which is at the beginning. Assuming the drift function is 0, then the upward rate at period 1 is given by $\ln r_{11} = \ln(0.1) + 0.1\sqrt{1} = -2.2035$ and the downward rate at period 1 is given by $\ln r_{10} = \ln(0.1) - 0.1\sqrt{1} = -2.403$. Now taking exponentials of both the upward and downward calculated figures we get the interest rates shown in period 1. This process is continued taking each node to be the initial node and calculating the upward and downward rate up to period 4 and these are the rates that will be used to discount the calculated values in the bond binomial tree.

The bond prices can be calculated using the interest rate binomial tree. To value the bond in any node of the tree, each cash flow in the future is discounted backwards, the node corresponding to the specific road of the interest rate and averaged on all possible roads. This is equivalent to sequentially averaging and discounting the prices of the bonds to each adjacent pair of nodes to determine the price of the bond in the prior node.

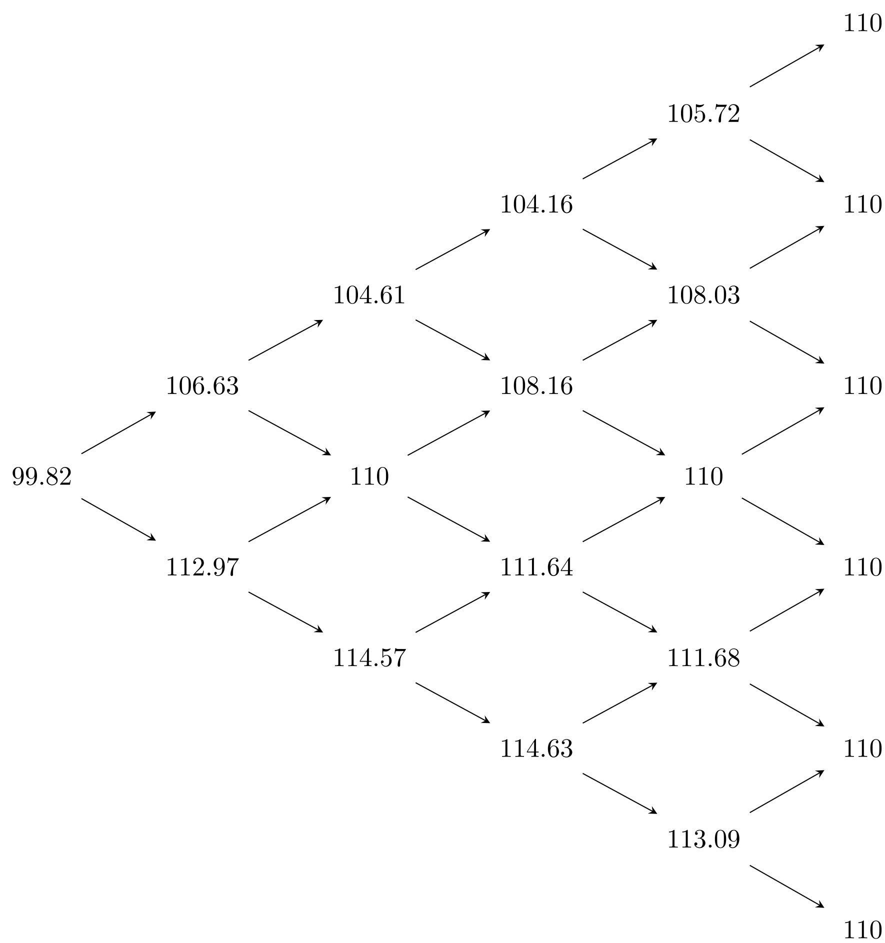


Figure 2: Non Callable Bond Binomial Tree

Figure 2 shows bond prices that are below the call prices at each node and there the bond will not be called and the price of the callable bond will be the same as the price of the plain vanilla bond in Figure 2. The call option will not be exercised. If we change the call price at year 2 to R103 and then decrease it by R1 for each year up to maturity, then the callable bond price will be different. A look at the calculation of the bond price at some of the nodes. At maturity in year 5 the bond pays out 110 (principal plus accrued interest) and the call price is also the same amount as the principal therefore at each node in year 5 the bond is worth 110. The interest rates in period 4 will be used to discount the bond values at node 5 to get the present values at period 4. A look at the calculation of the bond value in period 2, $(0.5 \times 110 + 0.5 \times 113)/(1 + 0.0905) = 102.25$.

Applying (16), we compare the call price which is 103 with the calculated price and then take the minimum of the two which is 102.25 and then adding the accrued interest for that period will give the value of the bond at that node which is 112.25. A look at the node in period 2 where the price is 113, we observe that the calculated value in Figure 2, is $114,57$, excluding the accrued interest, 10, the calculated value is 104.57. Comparing this calculated value to the call price which is 103 we see that the minimum of the two values is 103 and then adding the coupon will give 113. The call periods are year 2,3 and 4 and the calculated values in these periods will be compared to the call prices for those periods to get the bond prices at those particular nodes. Elsewhere the bonds will be discounted from one node to the prior nodes as calculated above.

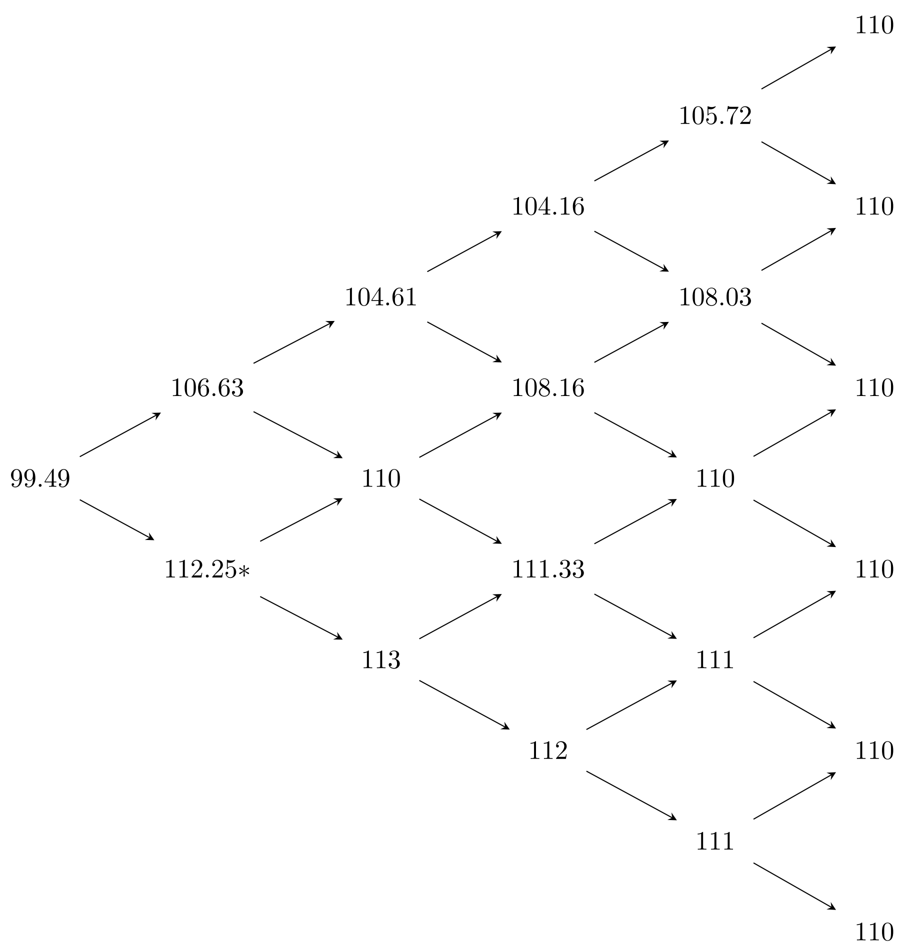


Figure 3: Callable Bond Binomial Tree

The price of the bonds shown in Figure 2 and Figure 3 are different, with the callable bond price being lower than that of the plain vanilla bond. This shows that calling the bond lowers the price of the bond.

2.5 Convertible Bonds

In its plain vanilla form a convertible bond is a bond that pays a frequent coupon like any other bond but also entitles the investor the right to purchase a certain number of shares (convert the bond) of the issuing firm during the life of the bond. It can be viewed as a plain vanilla bond with a relatively low coupon and an option on the equity of the firm. So depending on the performance of the company, if the stock is more volatile the bond part is more valuable and as such the convertible bond begins to mimic a fixed income security.

Some convertible bond basics

Conversion:

(i) The number of common stock shares in exchange for each converted bond is called the conversion ratio:

$$\textit{Conversion Ratio} = \textit{Number of stock shares acquired by conversion}$$

(ii) The par value of a convertible bond divided by its conversion ratio is called the bonds conversion price:

$$CP = \frac{B_{pv}}{CR} \tag{17}$$

where CP is the conversion price, B_{pv} is the bond par value and CR is the conversion ratio.

(iii) The market price per share of common stock acquired by conversion times the bonds conversion ratio is called the bonds conversion value:

$$CV = S \times CR \tag{18}$$

where CV is the conversion value, S is the price per share of stock.

When convertible bonds are originally issued, their conversion ratio is automatically set to yield a conversion value of 10% to 20% less than par value. Thereafter the conversion ratio is fixed, but each bond's conversion value becomes linked to the firm's stock price, which may rise or fall in value. The price of a convertible bond reflects the conversion value of the bond. The higher the conversion value the higher the bond price, and vice versa. The rational decision of convertible bondholders to postpone conversion as long as possible, to continue receiving coupons, is limited since convertible bonds are almost always callable. Firms automatically call outstanding convertible bonds when their conversion value has risen by 10% to 15% above bond par value. Calling the bonds forces holders to make an immediate decision whether to convert to common stock or accept cash payment of the call price. A convertible bond is said to be *in-the-money* bond when its conversion value is greater than its call price. A bondholder will convert when the bond is called. When the conversion value is less than the call price, a convertible bond is said to be *out-of-the-money*. Bondholders will accept the call price.

Lower Bounds of a Convertible Bond:

First, a convertible bond's price can never fall below its *intrinsic bond value*, also commonly called its investment value or vanilla bond value. This value is what the bond will be worth if it were not convertible. Second, a convertible bond can never sell for less than its conversion value because, if it did, investors could simply buy the bond and convert, thereby realising an immediate risk-free profit. Thus the floor value of a convertible bond is its intrinsic bond value or its conversion value, whichever is large.

Call and Conversion Strategies:

Most convertible bonds are issued with a call option that the issuer can exercise during a specified interval or throughout the life of the bond. The call allows the issuer to buy back the security at a predetermined price, and consequently limit the investor's returns when interest rates fall or stock prices rise. However conversion is given priority, that is, even if the issuer calls the bondholder can still convert the bond if he so wishes. Calling the bond enables the issuer to control the price of the convertible bond and if necessary refinance the debt with a new cheaper issuance. As both parties now have options this introduces an optimal trading strategy for each of the two parties concerned. The issuer's objective is to maximise shareholder equity by forcing the bondholder to either convert or redeem the convertible bond, thus the issuer cuts back on coupon payments, and this equates to minimising the price of the convertible bond, whilst the bondholder's objective is to maximise the price of the convertible bond or minimise the shareholder's wealth.

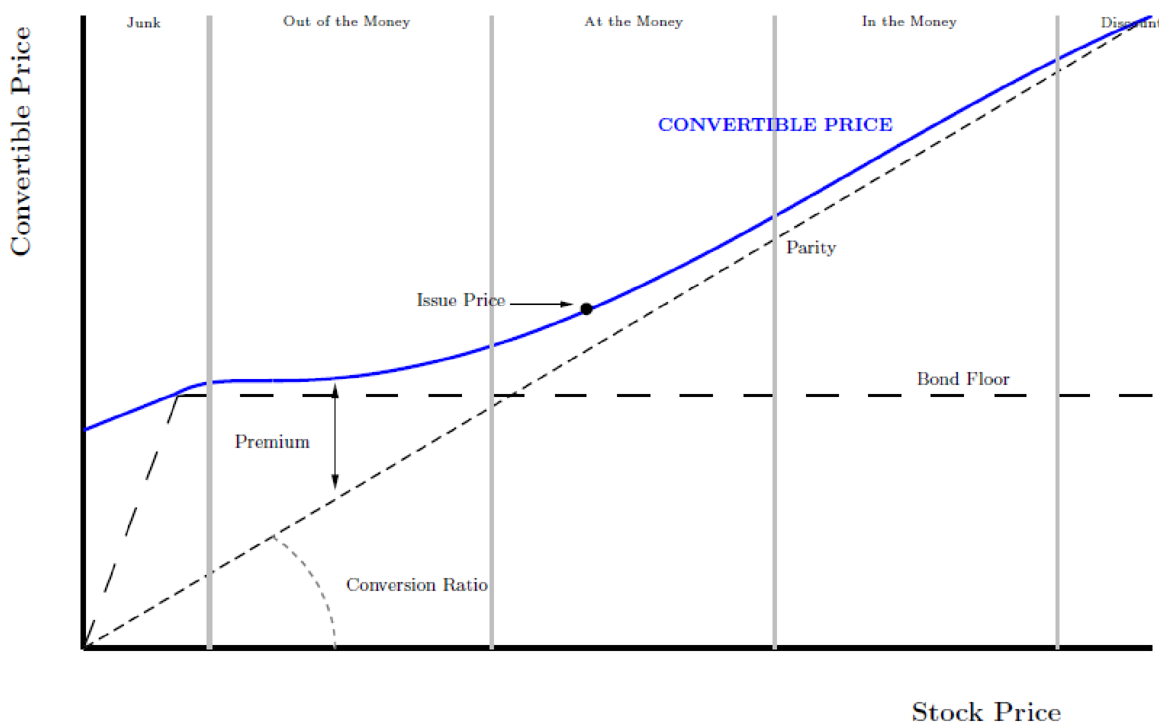


Figure 4: Convertible Bond Performance in Secondary Markets

Phases of a Convertible bonds:

Distressed Debt or Junk Status. In this region the convertible bond is close to a default event. If a default event occurs, a sum proportional to the recovery rate R is paid out to the holder of a convertible bond. The value of the convertible bond is highly sensitive to the credit spread in this region. Due to the subordination of the convertible bond, if a share price drops to very low levels, credit risk increases and so the convertible bond value falls below its vanilla bond value, and actually possesses some negative gamma, vega and increasing delta with respect to the share price.

Busted Convertible Bond. A term often used to describe a convertible bond that is out of the money but above the distressed zone, and shows characteristics similar to a pure bond. The reason for them being classified as out of the money is attributed to the under performing share price since the issue of the convertible and share price still being below the conversion price. Despite their name, out of the money convertibles will provide profitable returns under four different scenarios

(i) If the underlying share price rises.

(ii) If interest rates fall. This would cause bond prices to rise thus raising the bond floor and hence raising the price of convertible bonds.

(iii) If the market's perception of the credit quality of the company improves. This reduces the credit spread of the bond and raising the bond floor, increasing the price of the convertible bond.

(iv) if the convertible bond's premium to its bond floor increases as a result of a general richening of convertible valuation.

Hybrid Zone. The convertible bond displays behaviour similar to that of a stock and a straight bond. The convertible bond is at the money. At the money convertible bonds are often regarded as 'balanced' convertible bonds as they tend to have asymmetric risk-return traits (upside participation with downside protection)

Equity Zone. The convertible bond price is more equity-like than debt. Credit risk factors become insignificant since the firms credit rating is high due to the high stock value. The convertible bond is in the money. The stock price begins to rise above the conversion price and there is a high possibility of conversion.

2.6 Developments Made in the Valuation of Convertible Bonds

Generally there are two approaches to valuing convertible bonds, using firm value or structural approach where the asset value of the firm is taken as the underlying asset and equity value approach where the firms share price is the underlying asset.
Starting from Ingersoll (1977) and Brennan and Schwartz (1977), the early literature on convertible bonds followed a one-factor structural firm-value approach along the lines pioneered by Merton (1974). In this approach, one starts with a stochastic process for the firm value (typically geometric Brownian motion), and all the corporate securities issued by the firm are treated as contingent claims on the firm value. Brennan and Schwartz (1980) introduced a second stochastic factor into this structural framework, by modelling stochastic default-free interest rates (see also Nyborg (1996)).

While appealing from the corporate finance theory standpoint, the structural approach has not been widely adopted by the convertible bond practitioner community for at least two reasons. First, the structural approach is necessarily highly stylized. A detailed model would require modelling of all the corporate securities issued by the firm simultaneously. This might involve modelling dozens of different straight and convertible bond issues, taking into account seniority. Secondly, the firm value process is not directly observable. The observable data in the market include the stock price, credit spreads (corporate bond yields and, more recently, credit default swaps (CDS) spreads in the credit derivatives market), and prices (and implied volatilities) of stock options.

In practice, a practitioner would have a difficult time ascertaining any precise numerical value for the value of the firm, not to mention the difficulties in estimating its process parameters, such as volatility. In contrast, the stock price, corporate credit spreads, and implied volatilities of equity options are continuously observed in the market. From the practical standpoint, it makes sense to develop convertible bond models that can be calibrated to the prices of liquid benchmark securities, such as stock options and CDS, and then used to value convertible bonds. Moreover, since the firms common stock, plain vanilla stock options, CDS, and straight corporate debt are all available for trading, convertible bond arbitrageurs need models that price convertible bonds consistently with these more liquid instruments.

The dissatisfaction with the structural models led practitioners to propose a number of low-factor (typically one- or two-factor) models where the issues of credit risk were handled in a somewhat ad-hoc manner by either splitting the convertible bond into its fixed income and equity components and discounting the components at different rates (e.g., Tsiveriotis and Fernandes (1998)) or adjusting discount rates according to somewhat ad-hoc rules depending on the stock price level (e.g., Bardhan et al. (1994)). However, these models lack consistent theoretical underpinnings. On the other hand, the intensity-based reduced-form credit risk modelling literature has enjoyed remarkable development over the past decade (see the recent monographs by Bielecki and Rutkowski (2002), Duffie and Singleton (2003), Lando (2004), and Schonbucher (2003)for state-of-the-art surveys). Modelling of convertible bonds in this modern intensity based framework was initiated by Davis and Lischka (2002), who proposed a convertible bond model that incorporated a Black-Scholes stock price (equity risk), a stochastic short rate (interest rate risk), and a default intensity (hazard rate of default) dependent on the stock price (credit risk linked with the stock price level). Such models with stochastic stock price, stochastic interest rate, and default intensity taken to be a deterministic function of the stock price became known in the industry as two-and-a-half-factor models (see Andersen and Buffum (2004), Takahashi et al. (2001), and Ayache et al. (2003) for detailed studies of one-and-a-half-factor models with the default intensity taken to be a deterministic function of the stock price, and deterministic interest rates). Several typical specifications of the default intensity as a function of the underlying stock price were later solved in closed form in the case of European-style securities by Linetsky (2006) and Carr and Linetsky (2006) (unfortunately these solutions do not extend to convertible bonds, which are American-style securities). However, all of these models still fail to account for the empirical fact that while the default intensity and credit spreads are strongly influenced by the stock price, they are not perfectly correlated. Moreover, these models fail to take into account stochastic volatility of the stock price, an essential empirical feature in the realm of stock options modelling.

Carr and Wu (2005) introduced an interesting three-factor reduced-form affine model of default. In this model, the stock price drops to zero at default. Prior to default, the stock price follows a continuous process with stochastic volatility. The default intensity and the instantaneous stock variance follow a bivariate diffusion process intricately specified to capture empirical evidence on stock option prices and corporate debt and CDS spreads. Carr and Wu (2005) showed in an extensive empirical study that their model was well suited for joint modelling and valuation of stock options and credit default swaps. This model was also well suited to serve as a basis for a convertible bond model. However, while Carr and Wus main interest is in relatively shorter maturity stock options and CDS spreads, they took the short rate as deterministic. In contrast, convertible bonds have maturities up to thirty years. For example, an Eastman Kodak convertible issue has thirty year maturity (on October 15, 2003 Eastman Kodak & Co. issued $575 million of 3.375% convertible senior notes due October 15, 2033). While it may be a reasonable approximation to assume constant interest rates for the valuation of shorter maturity securities, valuing long-term bonds assuming constant interest rates is highly unrealistic and prone to serious errors.

2.7 Valuation Methodology

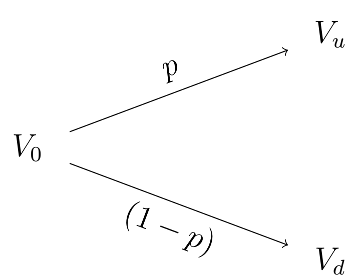
When determining the value of a convertible using the binomial tree approach the following assumptions are made:

(i) The distribution of future stock prices is lognormal with known volatility (the same model used for the calculation of callable bonds).

(ii) All the future interest rates the risk-free rate, the stock loan rate and the issuers credit spread are known with certainty.

(iii) All the information we need about the default risk is contained in the credit spread for the issuer's vanilla bonds.

The Cox, Ross and Rubinstein method of formulating the Black-Scholes equation is used for the valuation. We start by building a binomial tree of stock prices in a risk neutral world, where any security has an expected total return equal to the risk-free rate less any rebates for borrowing securities. The tree of future stock prices is used to build a corresponding tree of future convertible bond prices. The value of the convertible bond can be calculated at each convertible tree node by starting at maturity, where its value is known with certainty and then moving backwards in time down the tree, period by period, to calculate the value at earlier nodes. Let V_0 be the value of the convertible bond at the start of the period. We can find V_0 by comparing choices available to the issuer and the investor, assuming that each behaves in a rational manner, knowing that only possible convertible values one period in the future are V_u and V_d.



The holding value, H, of the convertible bond at the start of the period in a binomial model is the expected present value of V_u and V_d , plus the present value of any convertible coupons paid during this period. H is the value the investor can realise by waiting for one further time period without converting, assuming no provisions are applicable during that time. The calculation of the value, V, of the convertible bond at the current node for all combinations of provisions that maybe in effect is given by:

(i) *No active call provision.* The investor can either hold the bond for one more period or convert it to stock. Therefore the investor will choose to make V the maximum of the holding value H and the corresponding stock price.

(ii) *The convertible bond is callable at price B^c.* The issuer will call the bond when the call value (defined as B^c plus the accrued interest) is less than the holding value H. If the bond is called, the investor can still choose to convert it to stock, or accept the issuers call. Note that in this case V is the maximum of the two values.

The call provision allows the investor to receive accrued interest. An investor who converts will forfeit the accrued interest. Convertible bonds pay coupons and return principal, which are both subject to default, and therefore the risk-free rate will not be entirely appropriate especially if the stock price is below the conversion price. If at the next node, the stock prices are high, the discount rate is the risk-free rate since there is no risk of default. If the stock is below the conversion price, then the investor will definitely not convert and thus holds a plain vanilla bond, the discount rate will be the risky rate obtained by adding the issuers credit spread to the risk-free rate. At the intermediate stock prices, the credit adjusted discount rate is used. This credit adjusted discount rate is a weighted average of risk-free rate and the risky rate depending on the probability of conversion at the next node.

2.7.1 Example 3: Convertible bond valuation

We now illustrate how to use the model by valuing a hypothetical convertible bond. We choose a large credit spread of 500 basis points to make its effect on the credit adjusted discount rate easily observable. (The current risk free interest rate (JIBAR- Johannesburg Inter Bank Agreed Rate) is 5.13%. A credit spread of 500 basis points implies that the discount rate will be at 10.13%.)

Bond characteristics:

Principal: R100

Coupon: 10% per year

Maturity: 5years

Conversion Ratio: 1

Calls: R115 in year 2 declining by R5 every year to maturity

Stock characteristics:

Current stock price: R100

Stock dividend rate: 0%

Volatility: 30% per year

Credit Spread: 500 basis points

Figures 5 and 6 below show the stock and convertible binomial trees with one-year time steps and this model is based on the model proposed in Goldman Sachs (1994). At each node on the stock tree we show the stock price. At each node on the convertible tree we show the theoretical convertible value, together with a letter code to indicate the action that has been taken at that node. The letter codes are explained below:

Letter codes used in the convertible bond binomial tree

X: Investor converts to stock

C: Issuer exercises call

H: Investor holds convertible bond from one period to another

R: Issuer redeems the convertible bond at maturity

The evolution of the stock binomial given these parameters is shown below.

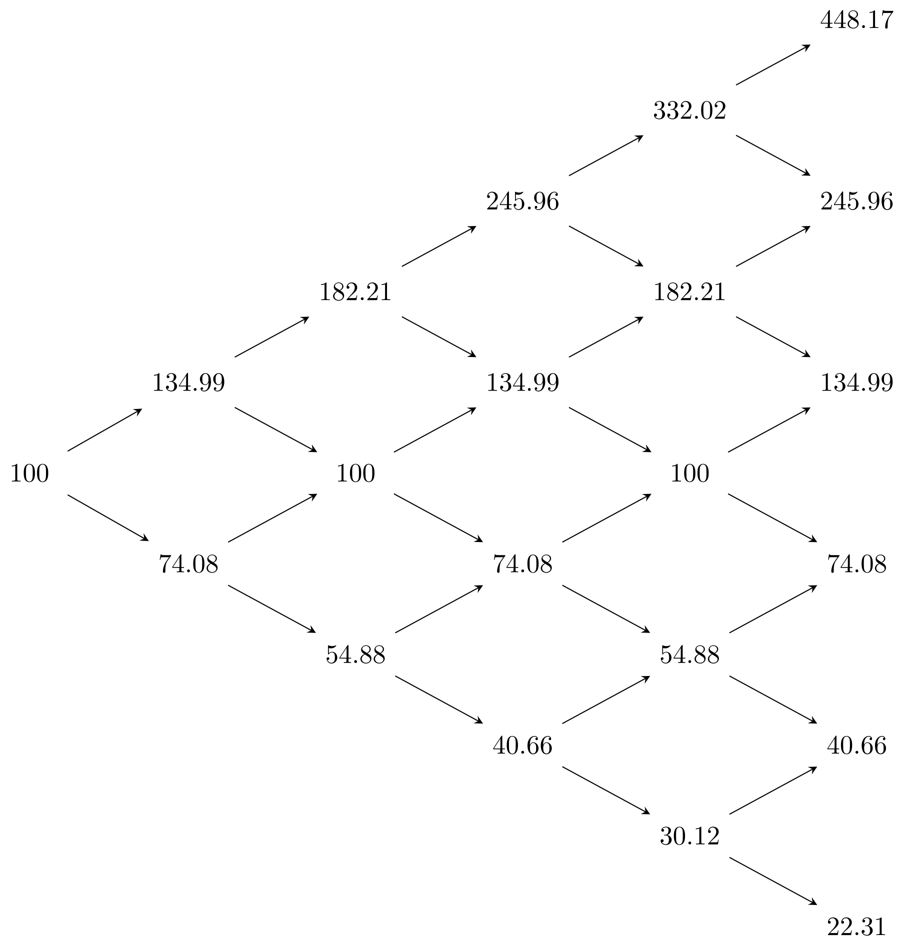


Figure 5: Stock Binomial Tree

A look at some nodes to see how the values are calculated on the trees:

At the start, the stock has a value of 100. After one year, it moves up to 134.99 or down to 74.08. These are the moves that correspond to a one-year return volatility of 30%. Applying (13), we calculate the values of u and d and the stock price at each node is multiplied by u to get the upward movement of the stock price and d to get the downward movement of the stock price. For example, the movement from 100 to 138.99 and 74.08

is calculated by multiplying the calculated value of u, which is 1.34986 by 100 for the upward movement and multiplying d which is 0.7408 by 100. Repeating the process will lead to the evolution of the stock binomial tree shown in Figure 5.

Now looking at the convertible bond binomial tree, at maturity in 5 years, the convertible bond pays out 110 (Principal plus accrued interest) if the investor does not convert into stock. Therefore at each node, the bond is worth the maximum of stock price at the corresponding node on the stock tree and 110. At these nodes where the maximum is the stock price, the conversion probability is 1.0 the nodes letter code is X to indicate conversion, and the credit adjusted discount rate is the risk-free rate (5%). At these nodes where the maximum is 110 the conversion probability is 0.0, the letter code is R, for redemption and the credit adjusted discount rate is the risk-free rate plus the credit spread (500 basis points), or 10%. Now look at the stock node with the price of 115 in year 4. The corresponding convertible node in year 4 can evolve in up and down-nodes at maturity that are each worth 110 and carry code R with a conversion probability of 0.0 and 134.99 and carry code X with conversion probability 1. The credit adjusted discount rate at each of these nodes is $(0.5)(110/1.1)+(0.5)(134.99/1.05) = 114.28$. The stock price in the stock binomial tree at the same node gives a value of 100 and the call price in that period is 105. The criteria use to get the bond price at that node is given by

$$\text{Price} = \max[S_t, \min(B^c, B_t) + c] \tag{19}$$

where S_t is the calculated stock price in the stock binomial tree at the same node, B^c is the call price, and B_t is the calculated bond price. Comparing the call price (105) and the calculated bond price(114.28) we see that the call price is the lower of the two values and adding the coupon will give 115. Now comparing the stock price and the call price plus coupon we see that the stock price is lower and thus the call price plus the coupon is the price of the bond at that node.

The convertible price will be sold at premium. However there is another feature, the call option, that has an effect on the price of the bond. The effect of the call feature was

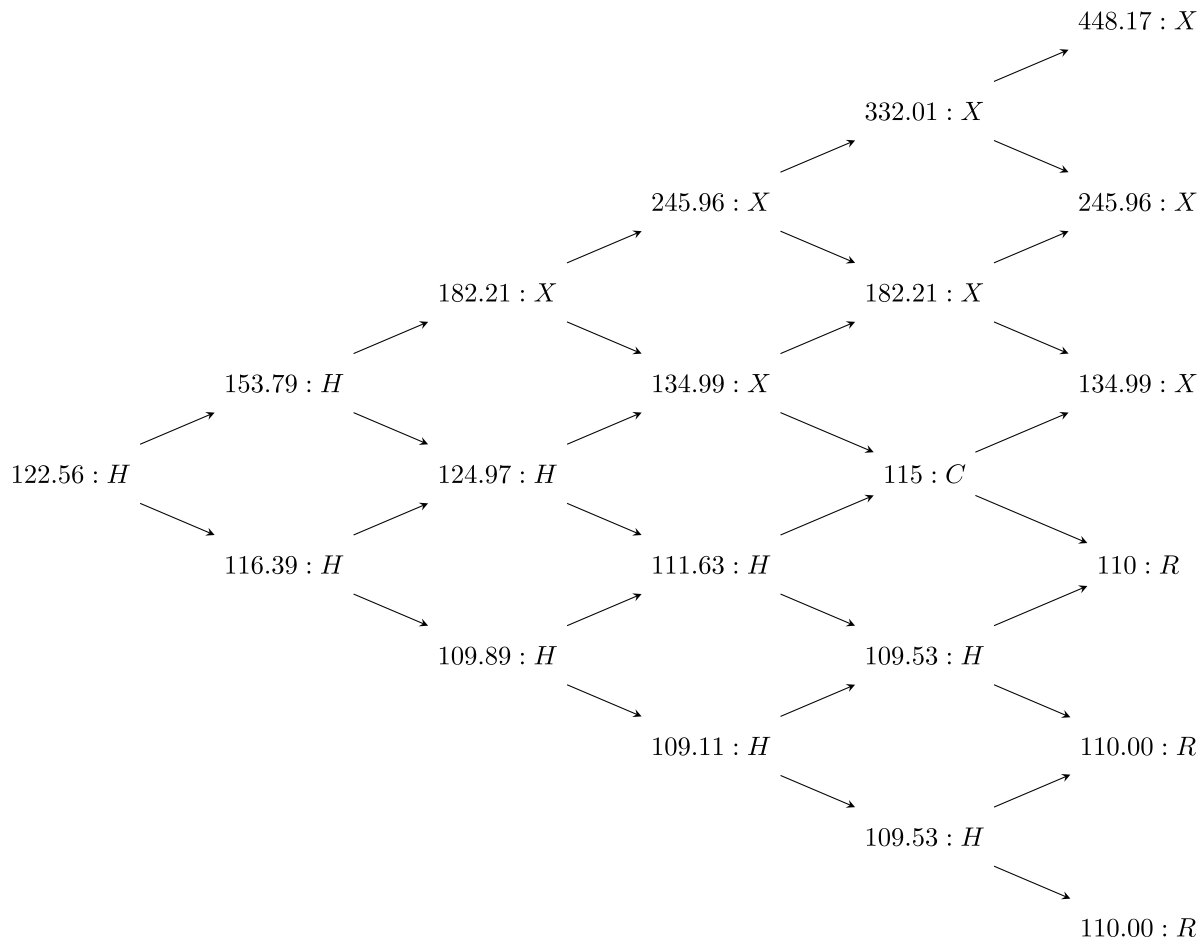


Figure 6: Convertible Bond Binomial Tree

investigated earlier and was shown to lower the price of the bond. Also, the credit spread should have an effect on the value of the convertible since it determines the discount rate that will be used. The lower the credit rating the wider the credit spread and the larger the discount rate. Hull (2009) introduced a default intensity in the model where there is a probability $\lambda \triangle t$ that there will be a default in each short period $\triangle t$, where the variable λ is the risk-neutral default intensity. In the event of default the stock price falls to zero and there is a recovery on the bond. The stock and convertible binomial trees are shown in Figure 7 and Figure 8 respectively. The default intensity can be calculated given the credit spread, and then the up and down variables can then be calculated using the default intensity.

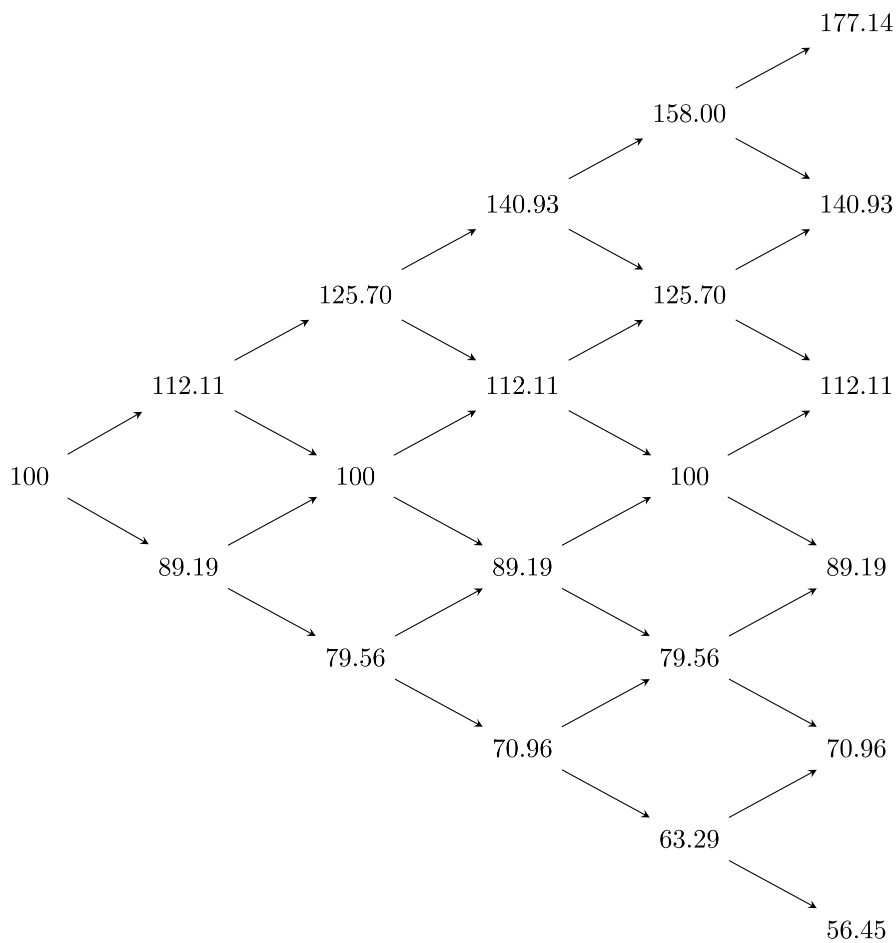


Figure 7: Stock Binomial Tree Using the Hull Model

The calculation of the convertible bond price at each node is similar to the criteria used in Figure 6, where the stock and bond prices will be compared to determine the value maximum value of the bond. If there is a call feature then the call price will be compared to the calculated bond price to determine the minimum value before comparing that minimum value to the stock price. Since the default intensity is already embedded in the calculation of the probabilities the discount rate will be the risk free rate. However, at each node the probability of default is calculated and then multiplied with the recovery amount and this value will be added to the calculated bond price at each node before discounting to present value at each particular node. A drawback with the Hull model is that the probability of default is independent of the stock price.

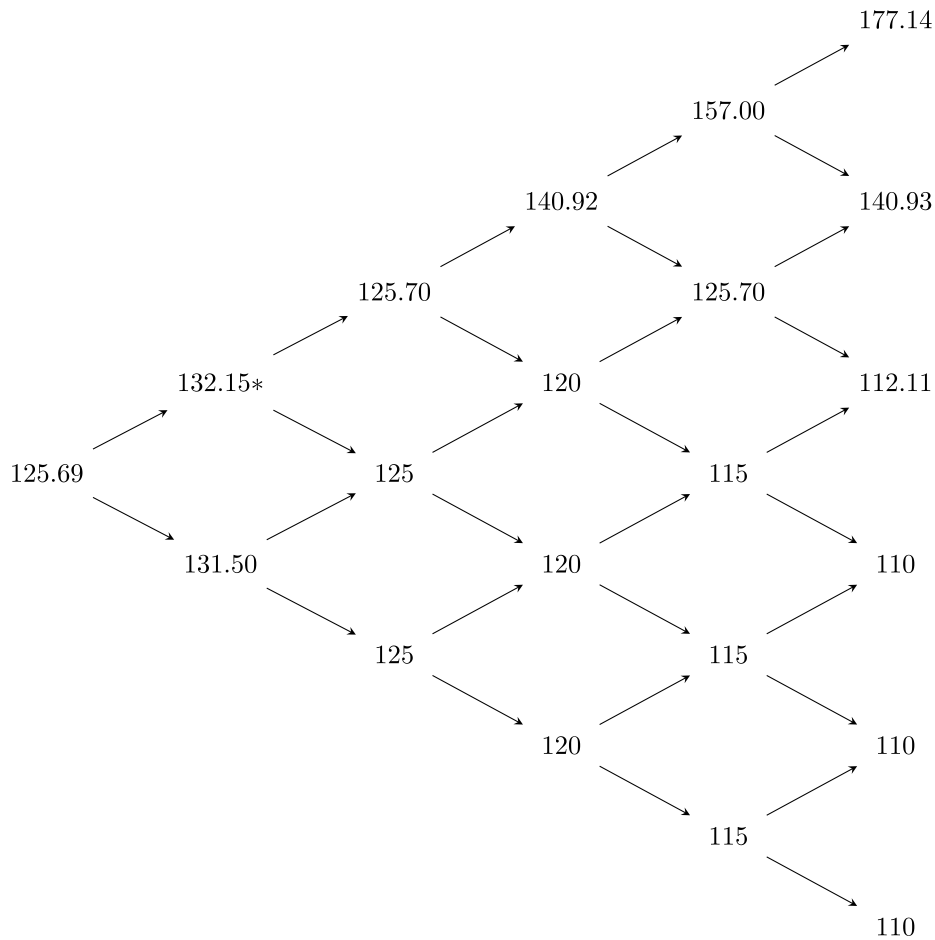


Figure 8: Convertible Bond Binomial Tree using the Hull Model

A look at some nodes to see how the values were calculated. Below are the formulas used for calculating the variables that will be used in the calculation of the prices. A risk free rate of 5% stated in the above examples is used in the calculation.

$$\lambda = \frac{CS}{1-R}, \quad u = e^{\sqrt{(\sigma^2-\lambda)\triangle t}}, \quad d = \frac{1}{u}, \quad p_u = \frac{e^{r\triangle t} - de^{-\lambda\triangle t}}{u-d}, \quad P_d = 1 - e^{-\lambda\triangle t}$$

The default intensity is calculated from the credit spread, CS, given and the recovery rate, R is assumed to be 35% and therefore the recovery amount will be R35. This will give a default intensity of 7.7% . The default intensity will then be used in the calculation of u which will give a value of 1,12 and d a value of 0.89. The probabilities based on these values will be $p_u = 0.98$ and $p_d = 0.02$ and the probability of default, P_d will be 0.074. Looking at the upward node in period 1, the calculated value is given by $(p_u \times 125.70 + p_d \times 125 + P_d \times 35) \times e^{0.05 \times 1} = 122.15$. Since the value calculated is greater than the stock price the bond will be held to the next period, the accumulated coupon amount will be added to get the value in that node. If there is a call feature in that period then criteria (19) will be used to determine the value at that particular node. The convertible bond prices calculated in Figure 6 and Figure 8 are close showing that the model used earlier is consistent with some of the models used in practice.

2.8 Impact of Step Sizes to the Value of Bond Prices

The calculation of bond prices discussed in the previous section has been based on yearly step sizes. The step size chosen has an effect on the price of the asset. The binomial tree is a discreet approximation of the continuous price calculation which requires a stochastic valuation framework. Table 1 shows the prices of the three bonds calculated earlier with varying step sizes, that is the periods from one node to the next node. The step sizes considered are yearly, semiannual, quarterly and monthly periods.

Table 1: Bond Price Sensitivity to Step Sizes

Bond	Yearly	Semiannual	Quarterly	Monthly
Vanilla	99.78	98.79	98.28	98.09
Callable	99.45	98.49	98	97.82
Convertible	122.56	121.34	119.42	119.06

The bond prices in the table are converging. This shows that as the step sizes become smaller the prices are getting closer to the continuous approximation. We can go further in constructing weekly and daily binomial trees of the bonds to make the values more granular but the process is much more cumbersome.

The calculation of the bond price presented in this chapter has been fairly easy because most if not all the underlying assumptions were satisfied. For the underlying assumptions to be satisfied the market where the bonds are being traded have to be liquid. However, not all markets are liquid and if the market is liquid it does not necessarily imply that the asset itself is liquid. The asset may not be attracting investors as other assets in that market and therefore trading of the asset will be less frequent. In the following chapters we present issues related to illiquid markets where parameters that are used for the calculation of the prices are not directly observable. The aim will then be to identify those parameters that are not directly observable and then provide solutions of overcoming those challenges.

It is clear from the above discussion that the risk factors for a convertible bond are interest rate, credit and equity. As long as these risk factors are observable, the valuation of these instruments is straight forward. In chapter 4, we will show that in a liquid market these risk factors are not observable and there is no market consensus on values of these risk factors. Thus, it is difficult to value convertible bonds in an illiquid market

3 Valuation of Structured Securities in Illiquid Markets

Structured financial instruments or derivatives are priced on the assumption that financial markets are frictionless. One can then find an asset-buying-and-selling strategy that only requires an initial investment that ensures that the portfolio generates the same payoff as the derivative. This is called 'replicating the portfolio'. The value of the derivative must be the same as that of replicating the portfolio, otherwise there would be a way to make a risk free profit by buying the portfolio and selling the derivative. Derivatives are priced by constructing a hypothetical replicating portfolio. The main gain from derivatives is therefore to permit individuals and firms to achieve payoffs that they would not be able to achieve without derivatives, or could only achieve at greater costs. Derivatives make it possible to hedge risks that otherwise would not be possible to hedge. When economic market makers can manage risk better, risks are borne by those who are in the best position to bear them and firms can take on riskier but more profitable projects by hedging. A second important benefit is that derivatives can make underlying markets more efficient.

Derivatives that trade in liquid markets can, for the most part be bought or sold at the market price, standard valuation models can be used to characterise them. Valuation is more problematic when trading is illiquid. In these cases, specialised models have to be constructed.

3.1 Market Liquidity

If an investor buys a widely traded plain vanilla derivative, it is generally easy to sell. However it can be harder to get out of long maturity contracts and complicated derivatives. The former is liquid and the latter is said to be illiquid. First it is much more likely that there is risk involved in the replicating strategy for such derivatives. Second, a complicated derivative only appeals to a small number of counter-parties who both want that particular set of risk characteristics and are 'confident' that they understand what they are getting into.

We define market liquidity as the difference between the transaction price and the fundamental value. Market illiquidity has many potential implications for asset pricing and raises a number of fundamental issues. How do equilibrium asset prices in an illiquid market differ from those in a liquid market? Who bears the risks created by the market illiquidity? What are the welfare implications of market illiquidity. These are the issues that we address by providing a valuation framework in these markets. Because asset pricing is driven by different factors when agents cannot always trade, prices in an illiquid market can be very different from those in a liquid market.
Financial markets deviate, to varying degrees, from the perfect-market ideal in which there are no impediments to trade. Trade impediments reduce the liquidity that markets offer. A lot of research traces illiquidity to underlying market imperfections such as asymmetric information, different forms of trading costs and funding constraints.

(i) **Participation costs:** In the perfect-market benchmark, all agents are present in the market in all periods. Thus a seller (liquidity demander), for example, can have immediate access to the entire population of buyers (liquidity suppliers). In practice, however, agents face costs of market participation costs, for example buying trading infrastructure or membership of a financial exchange, having capital available on short notice, monitoring market movements, etc. This has an effect on the supply of liquidity as liquidity suppliers might not be willing to participate. Participation cots lower the price of assets because sellers are more willing to participate than buyers. The intuition is that sellers receive a larger risky endowment and hence are more concerned about the risk that additional market shocks will leave them with a larger risk exposure.

(ii) **Transaction costs:** In addition to costs of market participation, agents typically pay costs when executing transactions. Transaction costs drive a wedge between the buying and selling price of an asset. They come in may forms. for example brokerage commissions, transaction taxes and bid-ask spreads. Some types of transaction costs can be viewed as a consequence of other market imperfections, for example costly participation can generate price-impact costs. Transaction costs deter liquidity suppliers from buying and they also deter liquidity demanders from selling.

(iii) **Asymmetric information:** In the perfect-market benchmark, all agents have the same information about the payoff of the risky asset. In practice, agents can have different information because they have access to different sources of information or have different abilities to process information from the same source. There is less or no information transparency to all the agents. Note that because liquidity suppliers are uninformed, supply of liquidity is influenced by suppliers' concern about trading against better informed agents. Cespa and Foucault (2011) show that asymmetric information can generate liquidity spillovers: because asset payoffs are correlated, a drop in liquidity in one asset reduces the information available on other assets, hence reducing the liquidity of those assets. Asymmetric information can raise or lower prices, with the effect being zero when probability distributions are symmetric,- as in the case under normality assumption.

(iv) **Imperfect competition:** In the perfect-market benchmark, agents are competitive and have no effect on prices. In markets, some agents are large relative to others in the sense that they can influence price, either because of their size or because of their information advantage. When liquidity suppliers behave monopolistically, imperfect competition obviously influences the supply of liquidity. More surprisingly, it can influence the liquidity supply when liquidity demanders behave monopolistically and suppliers do not. When information is asymmetric, imperfect competition lowers liquidity, even though liquidity suppliers are competitive. The intuition is that when liquidity demanders take into account their effect on price, they trade less aggressively in response to their information signal and their liquidity shock. This reduces the size of both information and liquidity generated trades. Imperfect competition reduces price informativeness.

(v) **Funding constraints:** Agents' portfolio often involve leverage, that is, borrow cash to establish a long position in a risky asset, or borrow a risky asset to sell it short. In the perfect-market benchmark, agents can borrow freely provided that they have enough resources to repay the loan. But as Corporate Finance literature emphasises, various frictions can limit agents' ability to borrow and fund their positions such as the lack of commitment of liquidity suppliers to supply cash. The literature on funding constraints in financial markets can be viewed as part of a broader literature on the limits of arbitrage. Both literatures emphasise the idea that some traders rely on external capital, which is costlier than internal capital, and this affects liquidity and asset prices. Gromb and Vayanos(2002) link market liquidity to the capital of financial intermediaries and their funding constraints. Investors are subject to liquidity shocks and can realise gains from trade across segmented markets by trading with intermediaries. Intermediaries exploit price discrepancies, and in doing so supply liquidity to investors, they buy low in a market where investors are eager to sell, and sell high in a market where investors are eager to buy, thus supplying liquidity to both sets of investors. Intermediaries fund their position in each market using collateralised debt, and face a funding constraint. Shocks to asset prices that trigger capital losses by intermediaries, tighten the intermediaries funding constraints and force them to reduce their positions. This lowers market liquidity and amplifies shocks.

(vi) **Search:** In the perfect-market benchmark, the market is organised as a centralised exchange. Many markets have a more decentralised form of organisation. For example, in the over-the-counter markets, investors negotiate prices bilaterally with dealers. Locating suitable counter-parties in these markets can take time and involve a lot of search. In a centralised market, when the number of meetings between liquidity demanders and liquidity suppliers decreases, that is less trading activity, the search process becomes less efficient and trading volume decreases reducing the liquidity levels in the market.

3.1.1 Measures of illiquidity

An intuitively and widely used measure of liquidity is the bid-ask spread, where the difference between the bid and ask prices will determine how liquid the market is, with a smaller difference suggesting a very liquid market and a large difference suggesting an illiquid market. Several measures exist. One is the quoted spread, defined as the difference between the quoted ask and bid prices. A drawback of quoted spread is that many trades are executed inside the spread, that is at more favourable prices. A measure remedying this drawback is the effective spread, defined as the difference between transaction price and mid-point of the quoted spread. This difference is taken in absolute value, and is multiplied by two so that it is expressed in the same terms as the quoted spread. Another measure is the realised spread, defined as the reversal between consecutive (or near-consecutive) transactions: the price of the current transaction minus the price of the future transaction if the transaction is above the mid-point, and the opposite if the current transaction is below. This difference is multiplied by two, as with effective spread. The realised spread measures profits earned by liquidity suppliers. These can be a compensation for risks, transaction costs or participation costs, or rents from monopoly power. If instead liquidity suppliers are risk-neutral, competitive and incur no costs, then the realised spread is zero. A detailed discussion of quoted, effective and realised spreads is in Huang and Stoll (1996), who compare liquidity of New York Stock Exchange (NYSE) and the NASDAQ based on these measures.

While the bid-ask spread is an intuitive measure of illiquidity, it has some limitations. First, its estimation requires detailed data on transactions and quotes. Second, because the spread is valid only for transactions up to a certain size, it provides no information on the prices at which larger transactions might take place. By the same token, it provides no information on how the market might respond to a long sequence of transactions in the same direction. In addition to the measures of illiquidity described so far, empirical papers have also used a number of more heuristic measures. Some of these measures relating to trading activity. For example, Bhushan (1994) measures illiquidity by the inverse of trading volume. Lesmond, Ogden and Trzcinka (1999) propose the LOT measure, which is based on the number of non trading days. Fleming (2003) uses trading volume and trade size, alongside bid-ask spread and price impact, to measure the illiquidity of Treasury bonds.

The measures of illiquidity presented so far concern an individual asset. Illiquidity can vary over time in a correlated manner across assets and markets. Hence it is useful to also measure it at a more aggregate level. A number of papers measure the aggregate illiquidity of an asset class by averaging the measures presented so far over individual assets within the class.

3.1.2 Challenges of illiquid markets

In active markets, investors, traders and lenders generally establish security fair values based on primary and secondary market trades and valuation tools including third-party analytics, external pricing sources and proprietary models. In illiquid markets, however, pricing sources and models become less precise and reliable. In extreme scenarios, illiquid markets identified by wide or non-existent bid-ask spreads, companies are left with limited market information, principles based accounting guidance and potentially difficult valuation challenges. Investors have two primary options to determine asset fair value. The first option involves reliance on available market data as a primary valuation input. The second option, involves cash flow modelling (income approach or market to model).

(i) **Market Approach**

Under the market-based valuation approach, investors typically collect prices and inputs from industry recognised sources and apply the prevalent bid-ask spread. The market approach is generally required, as available, as it reflects observed and current transactions and promotes price independence. The market approach may be difficult to apply in markets characterised by wide liquidity risk premiums, as pricing levels are established based on potentially unrepresentative transactions, for example, forced liquidations and other distressed transactions.

(ii) **Income Approach**

Under the income approach (market to model) investors generate market indicative prices using third-party-supported or internally developed models and management-defined assumptions. Compared to the market approach, the income approach may provide participants with increased flexibility in estimating illiquid asset values. Common income-based valuations are developed using (i) observable market inputs: (ii) similarly traded asset class data (spreads, default probabiltiy, recovery rate) adjusted for instrument-specific risk: (iii) unobservable inputs e.g management-applied judgement regarding assumptions other market participants would use to price the asset or liability when observable market inputs are not available. (iv) Despite increased flexibility, the income approach is highly governed by current accounting standards and requires strict adherence to regulatory pronouncements.

Measuring the effect of illiquidity on asset values and expected returns requires separating them from the effects of risk. Tests can then be derived by identifying assets with identical cash flows that differ only in liquidity. Illiquidity can be viewed as a consequence of various forms of market imperfections. These market imperfections cause the behaviour of assets to deviate from the theoretical properties and lead to complex behaviours. Understanding the risk factors and behaviour associated with lack of liquidity can then aid in deriving a reliable framework. The risk factors for valuing convertible bonds are credit spread, volatility and interest rate and these risk factors are observable in a liquid market. In chapter 4 we will be investigating how these variables affect the prices of convertible bonds and then discuss how those that have a huge effect can be approximated in illiquid markets where they are not directly observable.

4 Valuation of Convertible Bonds in Illiquid Markets

In illiquid markets there is no active swap markets and therefore no access to the most liquid interest rate futures. In case the issuer has a liquid credit curve, the investor should use this curve for discounting the projected cash flows. In practice, this is not always the case. In most cases the investor uses a benchmark curve(usually a government one) and adds a uniform spread to this curve for the riskiness of the issuer. The choice of the curve used is subjective and depends on the investor's perception about risk. One distinct feature of the illiquid markets has been the absence of active secondary markets of both the convertible bond and underlying asset and because of this the stock price movement is not as random. Issues of valuing convertible bonds in illiquid markets. We illustrate our methodology using BancABC convertible bond issued as an example.

BancABC is a large financial service provider in sub-Saharan Africa with subsidiaries in Botswana, Mozambique, Tanzania, Zambia and Zimbabwe. The shares of stock of the holding company, ABC Holdings Limited have a dual listing with a primary listing on the Botswana Stock Exchange (BSE) and a secondary listing on the Zimbabwe Stock Exchange (ZSE) where they trade under the symbol:ABCH.

4.1 Botswana Stock Exchange (BSE)

The Botswana Stock Exchange has about 35 market listings and 3 stock indices: the Domestic Company Index (BSE DCI); the Foreign Company Index (BSE FCI), incorporating companies which are are dual listed on the BSE and another stock exchange; and the All Company Index, which is a weighted average of the DCI and FCI. 90% of the market capitalisation is from foreign based mining companies, 10% from private investors. The secondary market is not active and can therefore be regarded as an illiquid market. The figure below shows the price movement of the ABC Holdings share price. As expected there is less movement observed in the share price with less frequent shocks when trading takes place.

Figure 9: BancABC Share Price in the BSE

4.2 Zimbabwe Stock Exchange (ZSE)

The Zimbabwe Stock Exchange is the official stock exchange of Zimbabwe. It has been open to foreign investment since 1993. The exchange has about a dozen members and over 65 listed securities. As of March 2009 trade has been thin, with very few foreign investors willing to risk trading on the market. Most stocks trade in the U.S-cent range, with at least 26 different stocks not trading at all. The figure below shows the ABC Holdings share price movement. The stock is more active compared to that in the BSE, but still less active compared to liquid markets.

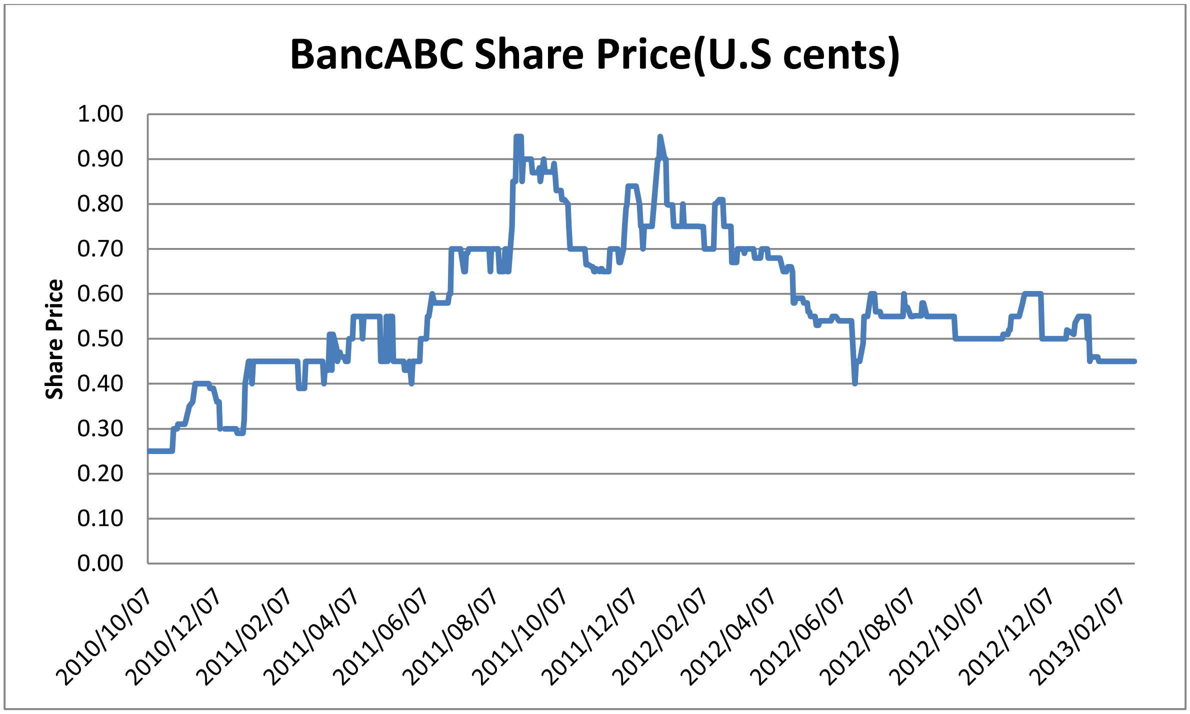


Figure 10: BancABC Share Price in the ZSE

4.3 Sensitivity Analysis

In this chapter several sensitivity analyses on the parameters of the convertible bond will be performed and the effects these changes in the parameters will have on the value of the convertible bond will then be observed. Sensitivity analysis is used to determine how sensitive a model is to changes in the value of the parameters of the model. Parameter sensitivity is performed as a series of tests where different parameter values are set and then observe how the characteristics of the convertible bond is going to change when the parameter values change. The focus will be on two risk parameters, credit spread and volatility. This will help to build a reliable framework. Sensitivity analysis therefore is a useful tool in model building and evaluation since it shows how the characteristics of the model change in relation to changes in parameter values.

In May 2008, BancABC issued a US Dollar denominated convertible loan to International Finance Corporation (IFC) for US \$13.5 million. The loan attracted interest of 6 months LIBOR + 3.75% per annum, payable semi-annually and it is convertible at IFCs option as follows:

(i) BWP3.15 per share at any time during the period from 13th May 2011 to 12th May 2012;

(ii) BWP3.24 per share at any time during the period from 13th May 2012 to 12th May 2013; or if at any time during the conversion period, the Group raises additional capital, a price equal to the price of the shares issued as part of such a capital raising exercise.

The redemption dates for the principal amount are as follows: 15th March 2013 $3,500,000; 15th September 2013 $3,500,000; 15th March 2014 $3,500,000; 15th September 2014 $3,048,969.

4.3.1 Parameter Sensitivity Analysis

Throughout this section representations of the effects of changing values of parameters and the behaviour of the convertible bond will be observed. The analysis will be based on the convertible bond value calculated using monthly step sizes in chapter 2. We start off by observing how the convertible bond price changes as the value of the share changes. We expect the the convertible bond to have a similar behaviour to the one observed in Figure 4.

It can be seen that at low share prices the bond component is performing quite well as the bond floor is way above the share price except for the fact that the bondholder is now exposed to default risk. The convertible bond performs fairly well compared to the share price up to when the convertible bond is at-the-money. If the bondholder converts the bond then s/he forfeits any future appreciation of the bond and the coupon payments attached to the bond.

Table 2: Sensitivity of Convertible Bond Price with Varying Stock Prices

Stock Prices	Convertible Bond Prices
20	100.66
40	101.09
60	103.40
80	109.56
100	119.42
120	131.13
140	146.92
160	162.63
180	180.28
200	200

Figure 11 below shows the effect of introducing the call feature on the convertible bond. A similar outcome that was observed in chapter 2 is seen. The lower the call value the earlier the chance of conversion of the bond as bondholders will be forced to either redeem or convert. The call feature lowers the price of the convertible bond as the time value of the conversion option is reduced. The effect of the call feature is more noticeable at higher share prices.

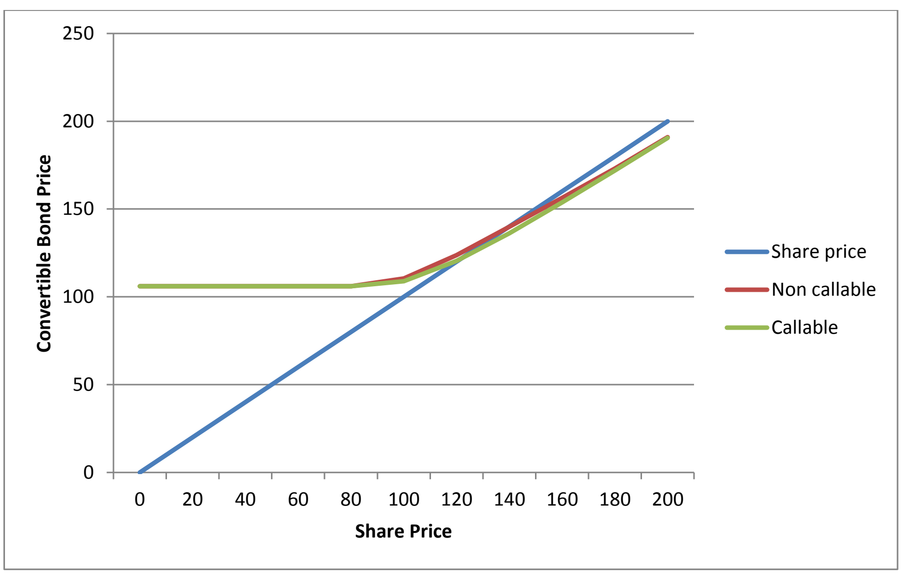


Figure 11: Effect of Call Feature on Convertible Bond with Variation in Share Price

Also having effect is the call date, moving the call dates away from the date of issue does not have the same effect as moving the call date close to the date of issue. As the period between the date of issue of the convertible bond and the call date increases the value of the convertible bond also increases. This can be attributed to the fact that as the call date is pushed away from the issue date the time value of conversion increases.

In an illiquid market the stock volatility is not directly observable due to no or less trading activities taking place. The volatility can be inferred by observing the stock of a company in a liquid market that has similar features to the company under analysis. The volatility will then be adjusted to incorporate the risk associated with lack of liquidity. An increase in volatility increases the expected risk as the share price is likely to fall to very low prices and therefore increase the likelihood of default. By the same token, an increase in volatility increases the expected return. From Table 3 it can be observed that as the stock volatility increases the value of the convertible bond also increases. This result shows that the expected increase in returns dominates the other effect.

Table 3: Sensitivity of Convertible Bond Price with Varying Volatility

Volatilities	Convertible Bond Prices
5%	110.29
10%	108.96
15%	111.23
20%	113.42
25%	116.40
30%	119.42
35%	122.22
40%	124.70
45%	127.13
50%	129.52
55%	131.87
60%	134.18
65%	136.44
70%	138.65
75%	140.82
80%	142.94
85%	145
90%	147.01
95%	148.97
100%	150.88

Perhaps the most important of the risk factors is the credit spread, as it provides an indicator of how the investors perceive the ability of the issuer to repay. A wide credit spread will increase the discount rate and therefore lower the value of the convertible. As expected Table 4 shows that as the credit spread decreases the value of the convertible bond increases. There is an inverse relationship between the credit spread and the value of the convertible bond.

Table 4: Sensitivity of Convertible Bond Price with Varying Credit Spreads

Credit Spread	Convertible Bond Prices
5%	133.11
10%	131.55
15%	130
20%	128.45
25%	126.93
30%	125.43
35%	123.95
40%	122.5
45%	120.94
50%	119.42
55%	117.92
60%	116.44
65%	114.95
70%	113.44
75%	111.97
80%	110.53
85%	109.11
90%	107.78
95%	106.48
100%	105.20

The above sensitivity analyses served as confirmation that the risk factors mentioned earlier have an effect on the price of the convertible bond. The variables are not transparent in an illiquid market. A financial services provider such as BancABC has no clear credit rating and therefore the credit spread cannot be easily inferred. The stock price rarely trades in both markets thus the evolution of the stock price is not as random as in liquid markets. By the same token, the volatility parameter cannot be easily observed or calculated from the market. Another issue is determining the yield curve since there are no active interest rate instruments in a market like the BSE or ZSE. These are the challenges that come with valuing the BancABC convertible bond.

4.4 Valuing the Convertible Bond

In this section we will provide ways of dealing with the risk factors that affect a convertible bond price and are not transparent in an illiquid market, the focus will be mainly on BancABC and how they can value their convertible bond. The valuation framework provided in chapter 2 can then be used once the unobservable risk factors have been approximated. Each risk factor will be dealt with separately.

Interest Rate

In a market like Botswana where there is no active interest rate market, a benchmark or proxy has to be used for constructing a yield curve. The 1 year Treasury Bill will be used as the benchmark risk free interest for that market. The yield curve can then be constructed based on the traded government securities.

Credit Spread

BancABC does not have a credit rating from an established rating agency and therefore the credit spread is not automatically given. There are several ways the credit spread can be extrapolated. First we can note the characteristics of BancABC, that is market share size and the industry it operates in. This can then be used in trying to identify a company that has similar characteristics in a more liquid market, for example a bank in South Africa. This company's credit spread can then be noted and then adjusted for liquidity risk. If the credit spread of the company cannot be determined directly then the credit rating of that company can then be used to approximate the credit spread. The S& P and Moody's rating agencies provide the credit spreads for each credit rating. Once the credit spread has been determined from a liquid market then it is adjusted for liquidity risk. It is therefore expected that the credit spread of a company in an illiquid market will be wider compared to the credit spread of a similar company in a liquid market.

Volatility

The weekly, monthly and yearly volatilities of the stock prices will be calculated separately. Then the average value of these volatilities will then be calculated and this will be the volatility that will be used in the valuation framework.

Therefore issues of valuation of structured bonds in an illiquid market relate to proxies, benchmarking and determination of the unobservable risk. This chapter has proposed a few ways in which how one may determine these risk factors.

5 Conclusion

Structured bonds are complex instruments that have characteristics that are closer to derivatives than conventional bonds. Most of them are exotic as they are issued privately to meet the needs of specific investors. With sophistication comes new characteristics and behaviours that may present new risks which may be embedded in these instruments. Some investors do not understand the cash flows coming from structured bonds and the risks attached to them. This often leads to mispricing and/or inadequate hedging of the risks. However, structured bonds increase liquidity in a market as they attract investors from all spheres including less sophisticated and those that are often hindered by regulatory implications associated with participating in certain markets like pension funds. The lack of regulation around these instruments has been condemned for the turmoils in the recent financial crisis.

In this research we have seen how plain vanilla and structured bonds are valued in liquid markets. Two structured bonds that we focused on are callable bond and convertible bond. These two structured bonds have options embedded in them. A callable bond has the call feature on a plain vanilla bond while a convertible bond has the conversion option and the call option. The call feature has the effect of lowering the value of a plain vanilla bond on any bond. The conversion option has the effect of increasing the value of the bond. The convertible bond protects the investors from downside risk while giving them unlimited upside opportunities. More features, like put option, reset clauses and adjustment to conversion price, can be added to the callable convertible bond to make it more appealing to the potential investors. However the more features added to the bond the more complex the behaviour and the more difficult it will be to understand the expected cash flows and valuation thereof.

Theoretical valuation of convertible bonds provide a nice framework where basic characteristics of convertible bonds can be investigated easily but in practice the market is not as perfect. Market imperfections present their own challenges in the valuation as some of the risk factors are not as transparent as in a liquid market. Imperfections such as in illiquid markets require that the unobservable variables be taken from similar assets in liquid markets and then adjusted for liquidity risk. As discussed in chapter 4, these deviations from perfect market conditions will have the effect of reducing the value of the bond as the credit spread will be wider but investors will be expecting higher returns.

There are many ways of valuing convertible bonds. The model used was compared to the one proposed by Hull (2009) to check for consistency. However the two approaches were different in that in the model presented in Goldman Sachs, the probability of default is incorporated in the credit spread whereas in the model that was presented by Hull the credit risk is embedded in u and d, the upward and downward movement variables. There are other models that are being created as these models are treating the bond component and the option component as two separate assets but Figure 4 shows that the behaviour of the convertible bond is a hybrid of both the stock and bond and should therefore not be treated separately. But these models that try to capture this hybrid nature of convertible bonds are more mathematically involved and may require programming tools. However all these models have underlying assumptions which may not necessarily be true in real life and therefore fair values may not always be as fair.

The South African convertible bond market is developing but it needs to gather more speed as this may provide a relatively better source of capital for the African market. This research has discussion a valuation framework for convertible bonds in a binomial tree model setting. We have also presented the challenges and proposed a way in which one may overcome challenges of valuing convertible bonds in an illiquid markets. The proposed methodology to overcome such valuations can be applied to all structured products in less developed markets.

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