SECONDARY SCHOOL MATHEMATICS TEACHERS' PEDAGOGIC REASONING AND ACTION IN RELATION TO LEARNER ERRORS AND MISCONCEPTIONS IN QUADRATIC INEQUALITIES

BY

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DECLARATION

I, Edgar Marange do hereby declare that this dissertation entitled secondary school mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities is being submitted for the Master of Education at the Wits School of education and has not been submitted by me or anyone else for a degree at any another university before, and I further declare that this is my own work and all the materials consulted have been properly acknowledged.

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DEDICATION

Firstly, I dedicate this study to my lovely wife, Alice Nakai, our three children, Tadiwanashe, Tinovimbanashe and Anotidaishe who were always there for me in good times as well as when my enthusiasm in the study was ebbing away. You provided me with the strength and courage throughout this study. I am further indebted to you for your prayers and words of encouragement and the fact that you had to forego family time for the sake of my study and also further accepted to sacrifice the meagre financial resources just for me to study. I also further dedicate this study to my mother, Gladys and my late father Kilford who showed me the door to education and planted in me an attitude of wanting to learn and achieve more.

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ABSTRACT

Learner achievement in school mathematics has been of great concern not only to South Africa but to the whole world at large. Many and varied reasons have been postulated as the underlying factors for the unconvincing learner achievement. Of particular importance is the mathematical pedagogical content knowledge or mathematical knowledge for teaching, a monopoly of knowledge by the mathematics teachers. However, on the flip side there is the subject of errors and misconceptions learners make in their endeavour to actively construct knowledge on the social plane. It is for this reason that this study sought to explore secondary school Mathematics teachers' Pedagogic Reasoning and Action (PRA) in relation to learner errors and misconceptions in quadratic inequalities. The study was framed within Vygotsky's socio-cultural theory of learning for which teachers are regarded as key in mediating learning in their learners' Zones of Proximal Development (ZPD). Located in the interpretivist paradigm and adopting a qualitative approach, which foregrounds multiple and subjective forms of reality, the study utilised an exploratory case study as the design genre or strategy of inquiry. In this regard, the research design was a case study of three secondary school mathematics teachers. Data were generated using interviews, observations and learners' written work. The analysis of data followed a qualitative content analysis strategy to unpack and lay bare the teachers' PRA in relation to the problem of learner errors and misconceptions in quadratic inequalities. Among the findings of the study was the ability of secondary school Mathematics teachers to identify the procedural errors, linear extrapolation and conceptual errors which learners make in quadratic inequalities. The secondary school Mathematics were also able to interpret the underlying factors for such errors. The study thus concluded that the ability of secondary school Mathematics teachers to evaluate learner performance with regard to the errors they make in quadratic inequalities should be the starting point for devising instructional strategies to address such errors. Therefore, the key recommendation is that the teachers' PRA should be focussed on eliciting powerful forms of instructional strategies, which are adaptive and tailored to the variations in learner ability and backgrounds.

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CAPS	Curriculum and Assessment Policy Statement
CCK	Common Content Knowledge
DBE	Department of Basic Education
FET	Further Education and Training
HCK	Horizon Content Knowledge
KCS	Knowledge of Content and Students
KCT	Knowledge of Content and Teaching
MKT	Mathematical content Knowledge for Teaching
NSC	National Senior Certificate
PCK	Pedagogic Content Knowledge
PISA	Programme for International Student Assessment
PRA	Pedagogic Reasoning and Action
RME	Realist Mathematics Education
SCK	Specialized Content Knowledge
SMK	Subject Matter Knowledge
TIMSS	Trends in International Mathematics and Science Study
ZPD	Zone of Proximal Development

CHAPTER 1
BACKGROUND AND CONTEXT OF THE STUDY

1.1 Introduction

The focus of this study was to explore secondary school Mathematics teachers' pedagogic reasoning and action (Shulman, 1987), in relation to learner errors and misconceptions in quadratic inequalities. The theoretical framework adopted for the overall study was Vygotsky's socio-cultural theoretical framework. The study was also further premised on Peng and Luo's (2009) analytical framework for interrogating the mathematics teacher knowledge in error analysis. The implications of the teachers' mathematics teaching philosophy are also dealt with in this study. The study followed an interpretivist paradigm and a qualitative approach utilising an exploratory case study of three secondary school mathematics teachers as the design genre or strategy of inquiry. The research site was Johannesburg North District in Gauteng chosen on account of its ability to generate robust research data due to its richness of diversity in the research participants' backgrounds. Further to this, the topic of the study is located within the South African school curriculum. In the next section of this chapter the discussion focuses on the contextual background of the study, the research problem, aims and significance of the study. The chapter then concludes with a brief outline of how the rest of the study is structured.

1.2 Contextual background of the study

Pedagogic Reasoning and Action (PRA) is a construct that deals with how teachers think of what to teach, how best to teach what has to be taught (pedagogical reasoning) and the operationalisation (action) of such thinking for effective learner understanding of the subject matter (Shulman,1987). It involves the application of the teachers' knowledge base in order to provide grounds for the justification of prospective, enactive and retrospective choices and actions in teaching. This means that PRA draws from the teacher's mathematical knowledge for teaching (MKT). MKT is basically a unique body of knowledge required for the effective teaching of mathematics (Ball, Thames & Phelps, 2008). In light of the above, the PRA of school mathematics teachers can be implicated in learner achievement. This is because they are involved in the contextualization of the school mathematics curriculum. In view of this, it is perhaps important to begin by looking at the state of learner achievement on the international front.

Many and varied international studies on learner achievement in school mathematics have been conducted over the years. Of note are the Trends in International Mathematics and Science Study (TIMSS) and the Programme for International Student Assessment (PISA) (Stephens et al.,2016) .To this end, some countries have been found to be top achievers yet others are a distance behind in terms of learner achievement in mathematics (Spaull, 2013; Stephens et al., 2016). The following East Asian countries have been dominating in mathematics achievement internationally: Singapore, Republic of Korea, Hong Kong and Japan (Reddy et al., 2016). In Europe, Finland and Netherlands have posted remarkable achievements over the years whilst African countries have disappointingly been occupying the lower end of the rankings (Reddy et al., 2016). One African country ranked in the lower end of the international ranks is South Africa, which was second from last in the 2015 TIMSS results (Reddy et al., 2016). What can be discerned from the PRA is reflected in the pedagogic practices of school mathematics in these countries. A brief focus of the pedagogic practices in some of the high performing countries is highlighted in the subsequent section.

Singapore's education has generated considerable interest on the international stage, over the years, as a result of its remarkable achievement on the international stage (Hogan et al., 2013). Its educational practices have become the subject of envy and emulation in the world (Hogan et al., 2013). One might say that the success of Singapore mirrors the PRA of Singapore teachers, which is presumably focused on the procedural and conceptual understanding of learners (Ho, 2009). From Ho (2009) one can put forward the notion that mathematics as a process is given primacy. Pedagogic practices in East-Asian countries are reportedly characterised by traditional forms of practice (Hogan et al., 2013). However, Singapore does not consider itself as traditional or constructivist in orientation. Its stance is reportedly pragmatic (Hogan et al., 2013). Manifested in the teachers' pedagogic reasoning is the notion of non-routine, open-ended and authentic real-world problems (Toh et al., 2019).

Though emphasis is on problem solving, there are divergent teacher pedagogic practices for implementing the intended curriculum (Toh et al., 2019). One can argue that the successful operationalisation of Singapore's mathematical problem-solving curriculum rests on teacher quality. This implies that in the teaching and learning of mathematics the mathematical knowledge base of the teachers is viewed through the lens of their PRA. In the context of other top performing East-Asian countries, this can be learnt in terms of the PRA of teachers in these countries as it is in the Singapore scenario. The PRA of the teachers in East-Asian countries is rooted in traditional approaches to teaching (Pang, 2016) and this implies that the teachers make use of the direct instructional methods of teaching. Teachers engage in lesson study (Pang, 2016), which describes an approach to teaching originating from Japan (Doig & Groves, 2011) where teachers with a common focus meet and plan lessons aimed at building learner skills or understanding of the subject or content (Doig & Groves, 2011). These lessons are referred to as research lessons and typically a lesson study provides guidance to teachers regarding what they are supposed to do in a lesson. Through lesson studies, teachers are thought to be bestowed with opportunities to improve their views and knowledge of the subject, its pedagogy and curriculum (Lewis, Perry, & Hurd, 2009, Meyer, & Wilkerson, 2011). The focus is mainly on what the teacher will and will not do amidst what learners will or will not do. This again impacts on the PRA of the teachers and thus makes a case of their mathematical knowledge of teaching and it might also be of interest to explore the nature of pedagogic practices in some of the successful countries in Europe.

Related literature on Finland's success in mathematics on the international stage reveals many and varied reasons (OFSTED, 2010). In essence the Finnish educators' pedagogic practices are characterised by careful exposition and questioning (OFSTED, 2010). This implies that the teachers' PRA is focussed on the explicit exposition of the subject matter so as to build the bridge between what the teacher knows and that desired of the learners. Emphasis is thus given to a clear and concise presentation and representations of mathematics (Hemmi & Ryve, 2014). Drawing from the foregoing, it can be deduced that in Finnish schools the teacher's PRA is projected towards eliciting learner thinking. It can also be discerned that the teachers' prospective thinking processes are influenced by best forms of presentations of what has to be taught, a situation that is in contrast with the pedagogic practices in Netherlands, which is yet another successful country in mathematics on the international stage (Reddy et al., 2015).

Netherlands' terrain of school mathematics is illuminated by Realistic Mathematics Education (RME) (van den Heuvel-Panhuizen & Wijers, 2005, Doorman et al., 2007). RME describes a theoretical view towards the teaching and learning of mathematics premised on the notion of mathematics as a human activity (Moffet & Corcoran, 2007, Makonye, 2014). The theory is grounded on the *"what" and "how*" of mathematics as well as *"how"* learners are perceived towards their learning of the subject (Moffet & Corcoran, 2007). In this regard the teaching of mathematics has to be linked to reality whether physical or imagined (Doorman et al, 2007). It is in light of this that the teaching and learning of mathematics should not be divorced from the learners' world of experiences and the dictates of society. Thus, the teachers' PRA are rooted in the mathematization of the subject. According to Doorman, et al. (2007), mathematization describes an approach to mathematics learning centred on two key aspects namely; the activity and the process of learning mathematics. It follows ,therefore that little consideration is given to mathematics as a product, which implies that value is given to relational understanding of mathematics as opposed to an instrumental understanding (Skemp, 1976)

In view of the successful pedagogic practices which directly or indirectly impact the teachers' PRA, what can be reported about African countries and South Africa in particular? African countries and South Africa in particular have perennially been occupying the lower end in terms of learner achievement in international assessments (Spaull, 2013; Arends, Winnar & Mosemege, 2017). This means that South Africa's performance has not been pleasing. Further to this, the learners' performance in local mathematics examinations has been a far cry from what is expected (Spaull, 2013).Spaull (2013) reports that South Africa's National Senior Certificate (NSC) achievement in mathematics is not convincing. This is because of the fewer number of learners achieving a pass rate of 50% or above in mathematics. This probably reflects deficiencies in the teachers' PRA in their pedagogic practices. This draws attention to South Africa's envisaged teacher's pedagogic practices as reflected in its curriculum, which according to (DBE, 2011) also clearly imply that Mathematics must be viewed in two folds: as a language and a human activity. Consequentially, the teacher pedagogical reasoning and action entails, instructional methods focussed on promoting learner access to the mathematical language as well as developing problem solving and cognitive abilities of the learners (DBE, 2011). In a nutshell, one might argue that learner achievement in mathematics in a particular country partially reflects the collective and individual PRA of teachers in the country. In the ensuing discussion the problem statement is examined.

1.3 Problem statement

Mathematics is typically problematic to many learners in South African public schools (Pournara, Hodgen, Saunders & Adler, 2016). This has resulted in general discontentment by stakeholders regarding learner achievement in Mathematics. Spaull (2013), for example, reports that in addition to the convincing evidence of fewer learners pursuing Mathematics in the Further Education and Training Band, grades 10-12 levels, achievement in the subject has been dismal. It is further reported that, some researchers in mathematics education have expressed concern over teacher pedagogic practices (Spaull, 2013). However, there has been research into different aspects of school mathematics and its teaching. For instance, Brodie's (2014) research is on teacher pedagogic practices while other scholars, such as Luneta and Makonye (2010) have researched on learner errors and misconceptions in calculus.

Of particular interest to this study is the large and growing body of literature focused on learner errors and misconceptions in algebra (Pournara et al., 2016), which is possibly attributed to the generational, transformational and meta-level aspects of algebra (Kieran, 2007). According to Kieran (2007), the generational aspects involve the different dimensions of variation of algebra such as algebraic expressions, quadratic equations and inequalities. The transformational aspects of algebra are concerned with the procedural aspects of dealing with the different dimensions of variation of algebra (Kieran, 2007). An example of that is how to solve equations and inequalities. Lastly, Kieran (2007) postulated that the meta-level aspects of algebra involve the use of algebra as a tool in other topics of mathematics. An example of that involves applying one's knowledge of quadratic inequalities to solve a trigonometric question. Reasoning from the above, it is evident that in-depth studies have been conducted on learner errors and misconceptions in algebra. To that end, available research seem to show that many of the learner errors and misconceptions made are attributable to two main aspects, which are the learner's prior knowledge as well as the teacher's pedagogic practices (Brodie & Berger, 2010; Brodie, 2014).

A key aspect of the teachers' pedagogic practice is whether their PRA is a function of instrumental understanding or relational understanding (Skemp, 1976). For instance, a teacher whose PRA is focused on instrumental understanding may be concerned with correct answers only and this implies that if a learner gives a wrong answer, the teacher may lack the interest to pursue the learner's ways of thinking. The teacher will not be interested in gaining insight into the learner's reasoning (Moru & Qhobela, 2013). Such limited pedagogic thinking on the part of the teacher does not help learners much. Therefore, understanding the way some mathematics teachers deal with learner errors and misconceptions in their PRA is vital. No wonder Brodie (2014) argues that this has the potential of assisting or denying learner access to real mathematical knowledge. Therefore, it is critical for researchers to focus their research on what use, if any, mathematics teachers make of the learner errors and misconceptions in the subject and this certainly dovetails with the aim and objectives of the current study.

1.4 Rationale

Despite the large and growing body of research focussed on learner errors and misconceptions in algebra there seems to be paucity study, particularly in the South African context, regarding learner errors and misconceptions in inequalities in general and quadratic inequalities in particular. This is despite the integration of inequalities in other mathematical topics such as functions, series and trigonometry. It is for this reason that this study sought to investigate the teachers' PRA in relation to learner errors and misconceptions in quadratic inequalities at the grade 11 level. Further to the above, it is also of paramount importance to note that the need to develop an understanding of the way mathematics teachers ought to deal with learner errors and misconceptions in their PRA is vital. This rationale is also shared by Brodie (2014) in her argument that this has the potential of assisting learners' access to real mathematical knowledge. It is therefore fundamental for educational researchers, particularly mathematics educators to focus their research on what use, if any, mathematics teachers make of their diagnosis of the learner errors and misconceptions their learners make in the subject. This will culminate in best practices in their mathematical classroom activities.

1.5 Aim of the study

The study seeks to investigate secondary Mathematics teachers' pedagogical reasoning and action (PRA) in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level.

1.6 Research questions

The following main and subsidiary research questions guided the study;

1.6.1 Main research question

Using Peng and Luo (2009)'s analytic framework, what is the pedagogical reasoning and action of secondary school mathematics teachers with respect to learner errors and misconceptions in quadratic inequalities at grade 11 level? (cf. section 2.2 on Peng and Luo (2009)'s framework).

1.6.2 Subsidiary questions

1.6.2.1 What types of learner errors and misconceptions in quadratic inequalities do secondary mathematics teachers know?

1.6.2.2 What interpretation, do secondary Mathematics teachers make of learner errors and misconceptions in quadratic inequalities at grade 11 level?

1.6.5.3 How do secondary Mathematics teachers evaluate learners' written and verbalised responses in relation to errors and misconceptions in quadratic inequalities at grade 11 level?

1.6.5.4 What remedial actions, if any, do secondary Mathematics teachers employ when addressing learner errors and misconceptions in quadratic inequalities at grade 11 level?

1.7 Significance of study

Quadratic inequalities are an algebraic topic taught in grade 11 only, but is constantly examined in paper one of the grade 12 National Senior Certificate (NSC) examinations. This makes it to be an important topic in the grade 11 mathematics curriculum. The failure of secondary school mathematics teachers to address learner errors and misconceptions in quadratic inequalities at grade 11 level might have a significant bearing on learner achievement in the NSC examinations. Basing on the researcher's experience of teaching secondary school mathematics, in the Further Education and Training (FET) band, he notes that some questions in topics such as functions, trigonometry and sequences and series require learners' prior knowledge in quadratic inequalities. This means that learners will have epistemological obstacles in the topic if their prior knowledge in quadratic inequalities is contaminated with errors and misconceptions. It follows that the learners' prior knowledge in quadratic inequalities may position them favourably or unfavourably to respond, to questions in other topics where such prior knowledge is needed. Thus, if a single misconception is not addressed it may lead to a cluster of errors because mathematical ideas are thought to be hierarchical (Siyepu, 2012). This makes the practical contribution imperative for secondary mathematics teachers to address learner errors and misconceptions in quadratic inequalities. In this regard, it is envisaged that the study will offer some practical and theoretical contributions to teachers of secondary school mathematics particularly in terms of unmasking the nature of learner errors and misconceptions in quadratic inequalities, offering practical and theoretical strategies for remedial actions that may be used to address such errors and misconceptions. Further to this, it is also anticipated that the study will theoretically add to the existing body of literature regarding the teachers' PRA in the context of learner errors and misconceptions in quadratic inequalities. In terms of contribution to policy, it is assumed that the study will also influence the teaching and learning policy of mathematics particularly in respect of the teaching and learning of quadratic inequalities at the secondary school level. Thus, the significance of the study is three-fold given that it is envisaged to offer practical, theoretical and policy contributions. The next section presents a summary of the content of this chapter.

1.8 Summary

In this chapter the study presented an introduction and background to the problem. Further to this, the chapter has also highlighted research problem, aims, research questions and the significance of the study. The next chapter's focus is on the literature guiding the study. The chapter discusses the theoretical framework, analytical framework and literature review.

CHAPTER 2
THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 Introduction

An outline of the theoretical and conceptual framework guiding this study is carried out in this chapter. This is substantiated by a review of the related literature influencing the study. The sub-headings guiding the literature review herein are thus as follows; Pedagogical Reasoning and Action (PRA), theoretical framework, conceptual frameworks and literature review with respect to the teachers' philosophy of mathematics teaching, the teacher's mathematical knowledge for teaching, the nature of learner errors and misconceptions in secondary school mathematics, learner errors and misconceptions in quadratic inequalities and summary of the chapter. Detailed discussions under each of these sub-headings are given in the subsequent sections of the chapter.

2.2 Pedagogical Reasoning and Action (PRA)

PRA is a construct that was coined by Shulman (1987). It is based on the notion that teachers' pedagogic practices are a product of some thinking processes that teachers engage in before, during and after enacting their teaching practices. One starts by thinking of "what to teach" and "how to teach"; this is then followed by the operationalisation of one's thoughts and the subsequent evaluation and reflection upon the whole process. Thus, Shulman (1987) argues that PRA is characterised by the activities of comprehension, transformation, instruction, evaluation and reflection. Comprehension implies in this case, understanding what has to be taught in breadth and depth (Shulman, 1987). For instance, in teaching learners to overcome errors and misconceptions in quadratic inequalities, one has to have a deep comprehension of the topic, the possible sources of the errors and misconceptions and how such errors and misconceptions can be addressed. Transformation is characterised by aspects such as the choice of examples, demonstrations and explanations which makes the subject palatable to learners (Shulman, 1987). Instruction is about the teacher's "presentation, questioning and other observable acts of teaching" (Shulman, 1987, p.15). This may be characterised by the use of a learner centred approach in which the teacher uses a discussion method for teaching. It might also imply adopting a teacher-centred approach in which the teacher employs behaviourist instructional methods (Marshman & Goos, 2018). However, I argue for instructional methods which are learner centred in which the teacher and learners are co-constructors of knowledge in the classroom (Ahn & Class, 2011).

Evaluation, involves the mechanism that the teacher uses to assess learners' conceptions or misconceptions of what they are taught (Shulman, 1987). For example, evaluating the learners' responses by adapting a behaviourist approach in which the teacher just put a cross against the learners' incorrect responses (Moru & Qhobela, 2013). Lastly, Shulman (1987) notes that reflection is the ability of the teacher to look back at own teaching and how learning occurred. The teacher re-constructs and re-enacts the lesson for purposes of improvement and learning from experience. In this case, the focus is on teacher's ability to think about the intervention strategies used to remediate learner errors and misconceptions in quadratic inequalities. This also includes, but is not limited to, a reflection on errors and misconceptions remediated as well as emerging errors that may still need to be dealt with in future lessons.

2.3 Theoretical framework

There are many and varied theoretical frameworks. Examples include, but are not restricted to the following, Piaget's cognitive theory of learning, Vygotsky's sociocultural theory and Sfard's commocognition theory. This research is guided by Vygotsky's socio-cultural theoretical framework. A discussion of the framework is carried out below. The framework is based on the assumption that human activities occur in cultural contexts and are mediated by semiotic mechanisms (including psychological tools) and other systems (Vygotsky, 1978; John-Steiner & Mahn, 2011). The following key two aspects can be drawn from this assumption: cultural context and mediation. Vygotsky (1978) argues that learning occurs in a socio-cultural context through mediation. The socio-cultural context drives learning (Vygotsky, 1978; Makonye, 2012). This means that what a child has to learn and how the child has to learn is influenced by the culture of the society to which the child is exposed. Thus, the nature of the knowledge that a child has to learn is drawn from the child's socio-cultural context. In this regard, learning entails being enculturated into the society's form of knowledge accrued over time. It follows that social interaction is indispensable in the child's learning process

In light of the above Vygotsky (1978) argues that every function in the cultural development of the child occurs twice: first on the social plane and second on the child's individual plane. The social plane is referred to as the inter-psychological plane and the child's individual plane is referred to as the intra-psychological plane (Vygotsky, 1978, John-Steiner & Mahn, 2011). On the inter-psychological plane, a child learns through interaction with other people more capable than the child. On the intra-psychological plane learning occurs inside the child (Vygotsky, 1978). Therefore, there is interdependence between the child's inter-psychological plane and the intra-psychological plane. In the context of learning mathematics, this implies that a child learns the subject first through interaction with others and second inside the child. Therefore, Vygotsky (1978) makes a case for the dynamic interdependence of the social and the child's individual processes. On the social plane the interaction may be between the child and the teacher or with other peer collaborators more capable than the child. An example of the interaction could be a class discussion on how to solve a given mathematical problem. As mentioned elsewhere in this section, learning on the intra-psychological plane occurs inside the child through the child's reconceptualization of newly acquired knowledge from the social plane. Knowledge acquisition at this stage is characterised by the integration of newly acquired knowledge with the already existing knowledge. This process is akin to Piaget's theory of accommodation and assimilation. Thus, errors and misconceptions occur as the child attempts to integrate newly acquired knowledge with the already existing knowledge. Focus is now turned to mediation as a key aspect of the socio-cultural perspective.

Daniels (2015) defines mediation as a process through which the social and the individual mutually configure each other. This means that human action on both the inter-psychological plane and intra-psychological plane is mediated (John-Steiner & Mahn, 2011; Vygotsky, 1978). There are different forms of mediation which are material, psychological semiotic and other human beings (Vygotsky, 1978). In view of what has been mentioned above, one may say that learning on the social and individual plane occurs through mediation. On the social plane the teacher may mediate the child's learning while on the individual plane psychological tools mediate the child's learning. Forms of psychological tools consist of, but are not limited to: language; diagrams; symbols; conventional signs and works of art. What then is the role of mediation? Its role is to transform the child's mental skills from lower to higher cognitive functions as the child progresses from existing knowledge to new knowledge forms (Daniels, 2015). Thus, mediation scaffolds learning.

Vygotsky's socio-cultural perspective also characterises how the child's mental development occurs. According to Vygotsky (1978), there are two levels of mental development. These are the actual development level of the child and the zone of proximal development (ZPD). The actual development level of a child is characterised by the mental functions of the child that have reached maturation as a result of previous teaching or experience. It consists of authentic knowledge that is free from misconceptions. The level is shown by what the child is able to do independently without the collaboration of others. In mathematics, this can be shown when a child is able to solve a particular mathematical problem without assistance. On the other hand, the ZPD is characterized by “the level of potential development as determined through problem solving under adult guidance or in collaboration with more capable peers” (Vygotsky, 1978; p.86). The ZPD delineates mental functions that have not yet matured but are likely to be manifested at a future date. An example of this would be when a child is able to solve a particular mathematical problem through the guidance or help of the teacher or more capable learner. The mental functions of the adult guider (teacher in this case) or the more capable others should be at the actual development level in order to successfully mediate in the child’s ZPD. Otherwise, the mediated efforts of the teacher or the more capable peers will give rise to error and misconceptions in the child’s re-construction of the mediated knowledge. Thus, at the heart of this study is the learners’ ZPD, their errors and misconceptions manifesting in the process through which they receive guidance from the teacher or a more capable peer collaborator (John-Steiner & Mahn, 2011). Therefore, the researcher would like to explore the teachers’ mediated efforts on the learner’s social plane in their efforts to facilitate the learner’s re-construction of knowledge in the intra-psychological plane. Hence, the focus is on examining the teachers’ PRA as reflected in their mediated efforts meant to scaffold learning in the learners’ ZPD.

Makonye (2012) points out that there is need for teachers to accurately determine the learner’s ZPD. This involves the accurate determination of the learners’ existing knowledge through the teacher’s error analysis of the learner’s written work or verbal talk. This should then lead to a form of teaching directed at ameliorating such errors and misconceptions (Makonye& Stepwell, 2016). Drawing from the teacher’s mathematical knowledge for teaching and pedagogical content knowledge (Ball, Thames & Phelps, 2008), appropriate mediated strategies on the learner’s social plane may then be devised. However, as argued by Riccomini (2005), teachers may accurately describe the error patterns in learners’ responses but base their instructional focus on re-teaching the content of the topic instead of directing their instructional strategies on the error patterns identified. Such an instructional strategy may not necessarily address the error patterns identified (Riccomini, 2005).

In conclusion, this study is grounded within the socio-cultural theoretical framework. The framework affords a valuable lens through which children’s learning can be examined as well as how that learning can be facilitated, in a social context, under the guidance of teachers or more capable peers. The next section is about the conceptual framework guiding this study.

2.4 Conceptual framework: Peng and Luo framework (2009)

A conceptual framework is an argument for the concepts chosen in terms of their appropriateness and usefulness in the context of the research problem under investigation. It offers a conceptual structure for the justification of one's research as well as mapping the researcher's territory for investigation (Marshall & Rossman, 2006). In this study, Peng and Luo (2009)'s framework for mathematical knowledge as used in error analysis is adopted. Peng and Luo (2009)'s framework is essentially built on literature based on students and teacher perspectives on errors in mathematics in general. On the other hand, the student's perspective is based on the nature of mathematical error and the teacher's perspective on type of error analysis. According to Peng and Luo (2009) the students' nature of mathematical errors is based on four categories which are mathematical, logical, strategic and psychological. The teacher perspective, which is the focus of this study, is based on the type of error analysis (Peng & Luo, 2009). It is also categorised into four main components which are identify, interpret, evaluate and remediate. A discussion of the components of the teacher perspective is carried out below.

2.4.1 Identify

This involves the teacher's awareness about the existence of the error, for instance, knowing that learners view quadratic inequalities as quadratic equations and hence use procedures for solving quadratic equations to solve quadratic inequalities. For example, a quadratic inequality such as $(x-3)(x+5)>0$ is solved in the same way as the equation $(x-3)(x+5)=0$ is solved. So, when solving the quadratic inequality, the learner will solve it using the same procedures as those applied in quadratic equations. In this case the learner will solve the quadratic inequality in the following way:

$$(x-3)(x+5)>0$$
$$x-3>0 \; or \; x+5>0$$
$$x>3 \; or \; x>-5$$

Peng and Luo (2009) argue that teachers should know about the existence of such errors and be able to identify them. The identification of learner errors and misconceptions should lead to the teacher's interpretation of such errors and misconceptions (Brodie, 2014).

2.4.2 Interpret

This involves the teacher's ability to interpret the underlying reasons of the mathematical error. School mathematics teachers should draw from their mathematical knowledge for teaching and be able to interpret the underlying rationality of the error made. Learners make such errors as a result of overgeneralizing their previously acquired correct knowledge of quadratic equations to quadratic inequalities (Bicer, Capraro & Capraro, 2014). Using the example in section 2.2.1; the learner overgeneralized his or her knowledge of the product rule for solving quadratic equations to quadratic inequalities. The product rule $a \times b = 0$ as used in solving quadratic equations requires that at least either $a = 0 \; or \; b = 0$. This however, is not the case with quadratic inequalities. For quadratic inequalities $if \; a \times b > 0$, it requires that both, a and b , must be of the same sign. That means that a and b can all be less than zero or a and b should all be greater than zero. Learners who, for instance, solve a quadratic inequality of type $a \times b > 0$ in the same way as they do when solving quadratic equations, fail to recognise that the multiplication of two negative numbers yields a positive number. Therefore, such learners will give $x - 3 > 0$ and $x + 5 > 0$ as their response to the quadratic inequality $(x - 3)(x + 5) > 0$ .The learners fail to realise that $x - 3 < 0$ and $x + 5 < 0$ also make the inequality true.

2.4.3 Evaluate

This means that the teacher should be able to evaluate the learner's performance according to the error. School mathematics teachers should be able to evaluate the learner's level of performance according to the error .The level of performance of the learner is realised through a formative assessment of the learner's work .For example, the teacher may acknowledge the correct responses given by the learner by way of ticking the successful steps completed by the learner and underlining or circling incorrect parts of the learner's responses .The teacher may also write comments informing the learner about what he or she has or has not achieved in the task given.

2.4.4 Remediate

This involves the teacher's ability to present a teaching strategy to get rid of the error. After identifying, interpreting and evaluating the error, the teacher should be able to present a mediating strategy to eliminate the error. One such strategy is to use a diagnostic teaching strategy directed at the errors identified in the pre-intervention task (Makonye & Stepwell, 2016). In conducting the remedial strategy, the teacher plays a mediation role on the learner's social plane, through the use of appropriate mediation tools. For example, the teacher may decide to use the graphical approach as a method to solve quadratic inequalities (Ndlovu, 2019). The mediation tool in this case will be, but not limited to, the appropriate language, both mathematical and non-mathematical, and the sketch of the graph that the teacher will use to demonstrate the use of the method. Thus, classroom practices used by teachers as they interact with learners play a significant role in the learner's comprehension of mathematical concepts and overall performance in the subject (Arends, Winnaar & Mosimege, 2017).

Drawing from the above insights, I locate my study within Peng and Luo (2009)'s conceptual framework. Table 1 below provides a summary of the constructs guiding my study in exploring the teacher's pedagogic reasoning in relation to learner errors and misconceptions in quadratic inequalities at the identified grade 11.

Dimension	Analytical category	Description
Phrase (type) of error misconception analysis	Identify	Knowing the existence of the error
	Interpret	Interpreting the underlying rationality of mathematical error
	Evaluate	Evaluating student's levels of performance according to mathematical error
	Remediate	Presenting teaching strategy to eliminate mathematical error

Table1: Adapted from Peng & Luo (2009) framework for examining mathematics teacher knowledge as used in error analysis

To summarize, it may be argued that Peng & Luo (2009)'s teacher perspective for error analysis makes a case for the mathematical knowledge for teaching. There is converging evidence that it draws upon the appropriate forms of the teacher's mathematical knowledge for teaching for one to successfully operationalise the components of this framework. The next section is about a review of the literature guiding my study.

2.5 Literature review

This section provides an account of the reviewed literature based on the following sub-headings: the teacher's philosophy of teaching school mathematics; the teacher's mathematical knowledge for teaching; the teacher's pedagogic reasoning and action; learner errors and misconceptions in school mathematics and lastly learner errors and misconceptions in inequalities.

2.5.1 Teachers' philosophy of mathematics teaching

What school mathematics teachers do in their classroom practices provides a lens from which the philosophical position of a country as well as that of the teacher regarding the teaching and learning of mathematics can be viewed and evaluated (Sriraman & English, 2010; Ernest, 2012). Broadly stated and for brevity, there are two main divergent and competing views on mathematics based on the philosophy of mathematics: the behaviourist and constructivist perspective (Handal, 2009). These views impact on the way mathematics is taught in schools (Handal, 2009).

In the face of the two broad standpoints in mathematics, one might need to know the basis of mathematics teachers' assumptions of the "what of mathematics" and how it is taught and learnt. With regard to the context of this study the "what" refers to mathematical errors and misconceptions in quadratic inequalities and the "how" implies the way in which these errors and misconceptions are dealt with in the context of the teacher's pedagogical reasoning and action. Thus, "understanding the nature of mathematics and its philosophical underpinnings" is imperative for one to teach the subject thoughtfully (Ernest, 2012, p.9). Drawing from Ernest (2012)'s insights it brings to light the notion that each mathematics teacher has a personal philosophy of mathematics which underlies the way mathematics is taught and learnt in the classroom.

For example, one may teach instrumentally yet somebody may focus on relational understanding (Skemp, 1976). The teacher who teaches the subject instrumentally focuses on teaching procedures, facts and skills (Marshman & Goos, 2018). Thus, the subject is taught in a behaviourist way. In view of that, little attention is given to why and how the procedures work. The interest of the teacher is on the final product. Very little attention, if at all, is given to the learners' forms or reasoning. It is argued that many teachers have an instrumental view of mathematics (Makonye, 2012). It means such teachers do not give due consideration to the underlying structure of the subject. Hence, the teachers may not engage in diagnosing learner errors and misconceptions in the subject with a view of making the errors and misconceptions a teaching resource. I argue that the instrumentalist mathematics teacher may be conscious of the learner errors and misconceptions but teaches the subject traditionally. The teaching strategies employed, in this case, entail conventional strategies characterised by direct verbal exposition of the content of the subject matter (Confrey, 1990;

Riccomini, 2005). I further argue that the pedagogic reasoning of such teachers is directed on mathematics as a product and attention is given to conventional processes giving rise to the product. Thus, the emphasis of the instrumentalist teacher is on procedural competencies (Makonye, 2014)

On the other hand, teachers with a relational view of mathematics focus on the understanding of relationships and connections between mathematical concepts and ideas (Makonye, 2012). In this regard, learners are taught what to do and why. The learning of the subject is centred on making use of the learner’s prior knowledge.

The relational teacher develops interest in the learner’s ways of thinking and errors and misconceptions essentially provide the spring board for effective pedagogic practices (Confrey, 1990). I therefore, argue that the pedagogic reasoning of teachers who engage the subject and learners in a relational way are propelled towards constructivist learning aimed at building the conceptual understanding of the process of doing mathematics and the product thereof. Hence, in the event of learner errors and misconceptions, the teacher engages in the underlying reasons giving rise to the errors and misconceptions (Peng & Luo, 2009).In doing so, appropriate intervention strategies to deal with the errors and misconceptions are devised as the learner errors and misconceptions are viewed and exploited as teaching resources (Makonye & Stepwell, 2016). Thus, I argue that the central focus in this case, is on building connections amongst mathematical concepts with shifts and gains in conceptual change as the barometer for proficiency in the subject. Therefore, relational teachers employ cognitive conflict strategies of teaching (Makonye & Stepwell, 2016). Thus, the teacher is seen as a facilitator, an explainer, whose focus is on helping learners towards the conceptual understanding of the subject (Marshman & Goos, 2018).

However, despite what has been discussed above, school mathematics should possess the appropriate knowledge required to teach the subject. A discussion of what this knowledge entails is carried out in the next section.

2.5.2 The teachers’ mathematical knowledge for teaching

Notwithstanding the teachers’ personal philosophy in mathematics, teachers need to be in possession of a certain body of knowledge for teaching (Shulman, 1986). Various models of the nature of knowledge that school mathematics should possess have been developed and reported, but there seems to be no universally agreed model for describing the mathematical knowledge for teaching (Tirosh & Even, 2007). However, in this study, the mathematical knowledge for teaching (MKT) school mathematics is premised on Ball, Thames and Phelps (2008)'s conceptualisation of the knowledge base that school mathematics teachers possess and apply in the work of teaching. A brief discussion of this knowledge is given below**.** The model splits the teacher's knowledge base into two main components; subject matter knowledge (SMK) and pedagogical content knowledge (PCK) (Ball, et al. 2008). These components are further categorised into subparts. SMK is divided into specialised content knowledge (SCK), common content knowledge (CCK), and horizon content knowledge (HCK) (Ball, et al., 2008). On the other hand, PCK is made up of knowledge of content and students (KCS), knowledge of content and teaching (KCT) and knowledge of content and curriculum (KCC) (Ball, et al., (2008). Focus is given to each of the subparts identified above.

CCK refers to the mathematical knowledge and skills which can be used in any other context besides that of teaching (Ball, et al., 2008; Petrou & Goulding, 2011). According to Ball, et al. (2008), SCK is the mathematical knowledge that is uniquely the county of mathematics teachers. For example, in quadratic inequalities, dealing with emerging learner errors and misconceptions requires the teacher's SCK. This form of knowledge is solely employable in classroom mathematics teaching and is neither unusable nor needed in other settings other than teaching (Ball, et al., 2008; Petrou & Goulding, 2011). KCS is characterised by the interconnectedness of the teacher's conceptual understanding of mathematics, consciousness and familiarity of students and their associated thinking in mathematics (Ball, et al., 2008). For example, the ability of the teacher to proactively anticipates learner errors. In the topic under study, this can be exemplified by the teacher's knowledge of learner errors and misconceptions in quadratic inequalities and the ability to deal with such errors and misconceptions.in a proactive way. KCT entails knowledge that blends "knowledge about mathematics and knowledge about teaching" (Ball, et al., 2008; Petrou & Goulding, 2011). Petrou & Goulding (2011) note that, amongst other decisions, KCT influence the teacher's decisions concerning the sequencing of particular content, for example, the sequencing of a section on quadratic inequalities. Consequently, this has a strong bearing on the nature of the teacher's exemplifications as they engage learners in deep conceptual and procedural understanding of the content (Petrou & Goulding, 2011). In the context of errors and misconceptions in quadratic inequalities, KCT implies the teacher's pedagogical reasoning in crafting instructional methods that will assist learners to deal with errors and misconceptions. Figure 1 below is a summary of MKT as constructed by Ball et al., (2008, p.403).

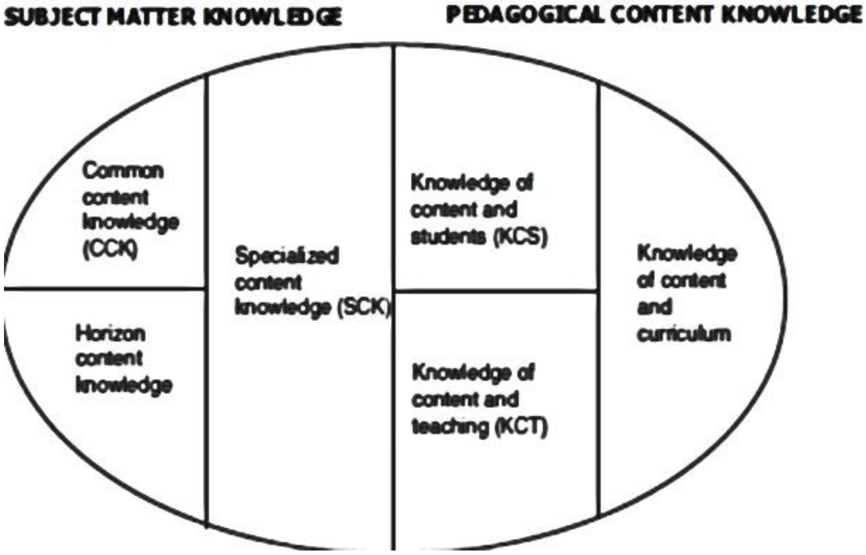


Figure 1: Domains of Mathematical knowledge for teaching (Ball, et al., 2008, p.403)

In view of the above, I argue that the secondary mathematics teacher's pedagogical reasoning and action draws from the teacher's MKT. Thus, I further argue that MKT provides the bed rock upon which to engage learner errors and misconceptions in the subject. The next section examines the nature of errors and misconceptions learners make in mathematics.

2.5.3 Nature of learner errors and misconceptions in school mathematics

An error is an incorrect verbal or written response which is a result of a mistake, slip, or inaccurate deviation from factual information (Luneta &Makonye, 2010). According to Nesher (1987), errors do not occur randomly but have a historical connotation to prior knowledge which might not have been properly acquired or new knowledge interfering with previously acquired correct knowledge. A misconception is a pseudo idea originating from a particular way of thinking which culminates in a sequence of errors (Nesher, 1987). One might argue that misconceptions are grounded in incorrect underlying assumptions about a particular subject matter or content. Olivier (1989) argues that misconceptions occur in the process of the active construction of one's knowledge. What gives rise to the misconceptions in mathematics? Misconceptions are rooted in one's attempt to overgeneralise previously acquired correct or incomplete knowledge to an extended domain where it is inapplicable (Olivier, 1989; Makonye, 2012). An example of this is applying the multiplicative inverse rule for solving linear equations to linear inequalities (Kroll, 1987; Tsamir & Bazzini, 2004). When solving linear inequalities, this rule collapses because dividing or multiplying both sides of an inequality reverses the inequality sign.

Researchers in learner errors in school mathematics have conceptualised the nature of learner errors in different ways. The following are typical cases to support my claim. Donaldson (1963) categorised learner errors into structural, arbitrary and executive errors; Kiat (2005) conceptualised learner errors into conceptual, procedural and technical errors; Siyepu (2012) classified them into interpretation, arbitrary, linear extrapolation and conceptual errors; Herholdt & Sapire (2014) categorised learner errors into procedural and conceptual errors. From the few cases stated above, what is evident is that researchers into learner errors in school mathematics seem to agree that learners are prone to certain errors but the nature of these errors has been conceptualised differently. On the basis of my observation, errors in this study should be conceptualised in terms of conceptual, procedural and linear extrapolation errors. A discussion of the nature of each of the three categories of errors identified is as presented below.

Procedural errors manifest when a learner incorrectly applies a procedure or rule to solve a particular problem (Kiat, 2005; Herholdt &Sapire, 2014). Radatz (1980) described such errors a arising from the wrong execution or operation of a procedure. In the topic under study such errors include, but are not limited to the following: failing to factorise the quadratic expression of the inequality question in order to determine the critical values of the quadratic inequality; failing to reverse the inequality sign when dividing or multiplying both sides of an inequality by a negative number (Bicer, Capraro & Capraro; 2014), and failing to sketch the correct graph of the quadratic function when solving quadratic inequalities using the graphical approach. For a procedural error, the learner may be aware of the concept or concepts required to solve the problem but fail to execute the procedures (Siyepu, 2012).

Drawing from the above discussion, it might be plausible to argue that a learner may be aware that he or she is supposed to apply the concept of factorisation in order to determine the critical values of a quadratic inequality but proceeds to execute the factorisation process incorrectly. Conceptual errors are grounded in the learner's inability to correctly comprehend the underlying ideas and connections among ideas associated with a given mathematical problem (Siyepu, 2012; Brown & Kim 2016). In quadratic inequalities this can be exemplified, in the following ways: failing to establish the relationship between the two sides of a quadratic inequality; inability to interpret the inequality signs (Vaiyavutjamai & Clements, 2006); inability to interpret solutions of the quadratic inequality from a graph or number line and lack of conceptual understanding of logical connectors (Almog & Ilany,2012)

Linear extrapolation errors occur as a result of learners applying already existing successfully acquired knowledge from previous learning to an extended domain where it is not applicable (Matz, 1980; Siyepu, 2012). In quadratic inequalities, we may think of such errors as occurring when learners overgeneralise the product rule for solving quadratic equations (Bazzin & Tsamir, 2004; Bicer, et al., 2014). The rule $a \times b = 0$ is used to solve quadratic equations. Using the rule, if $a \times b = 0$, it means that at least $a = 0 \; or \; b = 0$. Learners may apply the same rule to solve quadratic inequalities. An example of that would be when required so solve the quadratic inequality such as $(x-2)(x+4) > 0$. The leaner exhibiting linear extrapolation errors will proceed to solve the quadratic inequality in the following way:

$$(x-2)(x+4) > 0$$

$$(x-2) > 0 \; or \; (x+4) > 0$$

$$x > 2 \; or \; x > -4$$

The learner may have full conceptual understanding and procedures to solve quadratic equations of the form $a \times b = 0$ but fails to recognise that the same rule cannot be used when solving quadratic inequalities. What this learner may fail to conceptualise is that if $(x-2)(x+4) > 0$ then $x - 2 < 0$ and $x + 4 < 0$ will also be true for $(x-2)(x+4)$ to be greater than zero. This is because of the fact that if the product of the two factors is greater than zero or is positive, then the two factors must be of the same sign. In this case the two factors can either be positive or negative. This section has focussed on the nature of errors which learners make in school mathematics. It has also categorised these errors as procedural, conceptual and linear extrapolation errors. In the next section a review of the learner errors and misconceptions in inequalities based on researched evidence is carried out.

2.5.4 Learner errors and misconceptions when solving inequalities

There is a relatively small body of literature based on learner errors and misconceptions in inequalities (Ndlovu, 2019). Basing on the few studies conducted and the researcher's observation on these studies, considerable focus has been on errors and misconceptions in linear inequalities. From the studies conducted there is converging evidence that learners are prone to procedural and conceptual errors in inequalities in general and quadratic inequalities in particular. The following is an account of some of the available literature on learner errors and misconceptions in inequalities. The literature also highlights the methods that were used to arrive at the findings. Available literature from international studies is given first followed by studies in the South African context.

Verikios and Farmaki (2010) conducted a qualitative study concerning the teaching and learning of school algebra using a function-based approach to a class in grade 8 in Athens. 26 students participated in the study. Part of the findings was that: learners divided both sides of the inequality without reversing the inequality sign and that inequality signs were treated as equal signs. Though Verikios and Farmaki (2010) did not arrive at the conclusion that learners were prone to procedural and conceptual errors, their findings show that the learners were victims of these errors. The inability of the learners to reverse the inequality sign when dividing both sides of an inequality by a negative number signifies the existence of a procedural error. The error is associated with the overgeneralisation of the multiplicative inverse rule for solving linear equations applied to linear inequalities (Kroll, 1987; Bicer et al.; 2014). On the other hand, treating inequality signs as equal signs shows the existence of the absence of the semantic meaning of the inequality signs (Kroll, 1987; Blanco &Grote, 2007). Therefore, I would like to argue that this is an indication of a conceptual error arising from the learners' inability to differentiate an inequality sign from an equal sign. The conceptual error arises from the learners' prior knowledge about the equal sign that was correctly acquired applied but extended to a domain involving inequalities where it is inapplicable.

Vaiyavutjamai and Clements (2006) conducted a study involving 231 grade 9 learners in Thailand with respect to linear algebraic inequalities. 13 linear algebraic lessons on inequalities were conducted in which there were pre- and post-interviews with the learners. The findings were that linear inequalities were solved in the same way as linear equations except that the inequalities signs were maintained. However, the study did not indicate how the lessons were conducted. The instructional methods used to teach the lessons were also not highlighted. It would be of interest to know whether the learners were taught in an instrumental or relational way. Nonetheless, it can be argued and concluded that these learners displayed conceptual errors with respect to the interpretation of the inequality signs. The learners also lacked a conceptual understanding of the difference between a linear equation and a linear inequality. Naseer (2015) conducted a study based on the analysis of 2411 examination papers for students enrolled in a pre-university mathematics course in Maldives. The findings revealed that the students committed procedural and conceptual errors. The following is a typical question that the students were required to solve .Solve (x+5) (x-3) > 0. It is reported that a number of the students wrote (x+5) > 0 or (x-3) > 0 as their first step. This shows that the students overgeneralised the product rule for solving quadratic equations to quadratic inequalities (Almog & Ilany, 2012).

The aforementioned might be a result of the interference of prior knowledge with newly acquired knowledge or vice versa. Bicer et al. (2014) conducted a quantitative study with pre-service mathematics teachers regarding their understanding of linear and quadratic inequalities. It is reported that the pre-service mathematics teachers treated the solution of inequalities in the same way as the solution of equations. It was also reported that the pre-service teachers failed to interpret inequality solutions. From the findings it is clear that the pre-service teachers displayed conceptual errors in inequalities. One might argue that the misconceptions giving rise to these errors were of a historical background (Nesher, 1987). Thus, it might be that the pre-service teachers acquired these misconceptions at high school and the misconception were not diagnosed and remediated. In the South African context, it has been observed that there is very little published literature reported about learner errors and misconceptions in quadratic inequalities. However, (Godden, 2012; Makonye, 2014; Makonye and Mhonda, 2014), have reported on learner errors and misconceptions in quadratic inequalities in the South African context. Godden (2012) conducted a qualitative research study in which 1959 scripts of question one of paper1 of the grade 12 examinations were analysed. The findings were that a large number of the candidates were unable to solve quadratic inequalities. Part of the examination required candidates to solve the following inequality $4 + 5x > 6x^2$. An excerpt of how one of the candidates responded to this question is given below.

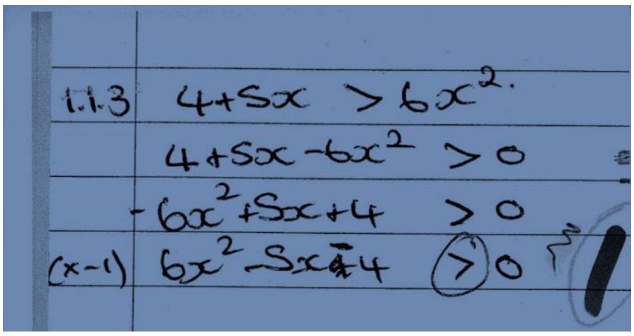


Figure 2: Learners' response to the question: Solve $4 + 5x > 6x^2$ adapted from Godden (2012, p.45)

Godden (2012 reported that learners made procedural errors when solving the above inequality. In this case the learner could not reverse the inequality sign when multiplying both sides of the inequality by a negative number. This shows the existence of a conceptual error in the candidate's execution of the procedures for solving quadratic inequalities.

Makonye and Mhonda (2014) conducted a qualitative research study with 27 grade 11 learners based on how learners respond to a task involving quadratic inequalities. Part of the findings were that: learners could not factorise the quadratic part of the inequality in order to determine the critical values; the inequality signs were treated in the same way as equal signs; the inequality was treated as an equation and incomplete interpretation of the solution set from the quadratic factors. Thus, learners were found to be prone to procedural and conceptual errors connected to algebraic processes (Makonye & Mhonda, 2014). Overall, the available literature on learner errors and misconceptions in quadratic inequalities provide compelling evidence that secondary school learners are prone to learner errors and misconceptions in quadratic inequalities. Learners commit procedural, conceptual and linear extrapolation errors. The interpretation secondary school mathematics teachers make of these errors, how, if at all, these errors are evaluated by secondary school teachers and what intervention strategies, if any, are used by secondary school teachers to address the errors and misconceptions remain the focus of this research.

2.6 Summary

This chapter has contextualised the study by focussing on the review of related literature in terms of the following aspects; the teacher's pedagogic reasoning and action, theoretical framework; conceptual framework; the teacher's philosophy of teaching mathematics; the nature of leaner errors and misconceptions in general and some specific errors that learners make inequalities. In the next chapter, the focus of the research is on the methodology guiding this study.

CHAPTER 3

RESEARCH METHODOLOGY

3.1 Introduction

This chapter discusses the research methodology under the following headings; the research paradigm, research approach and design, population and sampling, research setting, data collection and analysis methods as well as ethical considerations and measures to ensure trustworthiness. A detailed discussion of each of these aspects is presented in the subsequent sections of the chapter.

3.2 Research paradigm

All research activities are located within a paradigm (Creswell, 2014). Creswell (2014, p.6), further defines a paradigm as "a general philosophical orientation about the world and the nature of the research that a researcher undertakes". There are a variety of paradigms, which are but not limited to, positivism, post-positivism, interpretivism, critical theory and pragmatism. This study adopts an interpretivist paradigm. This is on account of its assumption about reality or truth which is believed to be relative (Maree, 2011). This essentially implies that what is reality is subjectively determined through multiple forms of reality (Yilmaz, 2013). In this study, it meant the interpretation of what constitute reality needed to be understood from the perspective of the secondary school mathematics teachers' pedagogic reasoning within the social context of their pedagogic practices.

The interpretivist paradigm also assumes a naturalistic view by valuing the study of participants in their natural settings (Lincoln & Guba, 1985). By studying people in their natural settings opportunities to understand their thoughts and behaviour are strengthened (Yilmaz, 2013). This justified the choice of my paradigm for this study. I intended to gain an in-depth understanding of the phenomenon: secondary school mathematics teacher's pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities. In the next section the research approach is examined.

3.3 The research approach

Creswell (2014) and Leedy, Ormrod and Johnson (2014) identify the main research approaches as basically quantitative, qualitative and the mixed methods. The approach followed in this study was qualitative. It implies an emergent, inductive, interpretive and naturalistic approach to the study of people, phenomena, social situations and processes in their natural settings (Nieuwenhuis, 2016). The justification for adopting this research approach was the researcher's keen interest in generating an in-depth understanding of the phenomenon: secondary school mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities. Secondary school mathematics teachers were the participants used to reveal in descriptive terms the meanings they attach to their experiences of the classroom world in relation to learner errors and misconceptions in quadratic inequalities. Reasoning from the aforementioned view, it is clear that the approach is holistic (Leedy, Ormrod, & Johnson, 2014). The researcher also enters the research field with an open mind and a readiness to be immersed in the complexity of the phenomenon through interacting with the participants (Leedy & Ormrod, 2010). The in-depth understanding and interpretation of the participants' experiences were centred on their subjective experiences as secondary school mathematics teachers. From a qualitative approach reality is socially and psychologically constructed through a framework that is value laden, flexible, holistic and context sensitive (Yilmaz, 2013). This further justified the choice of the approach in the study. It enabled the researcher to gain a deeper insight into the teachers' pedagogic reasoning and action as they engaged learner errors and misconceptions in the social context of their pedagogic practices. Within the context of the approach described above, the next section focuses on the research design adopted for this study.

3.3 Exploratory case study research design

A research design is a plan that one adopts in conducting research (Yilmaz, 2013; Creswell; 2014). As viewed by Stake (1995: xi), an exploratory case study is "one focused on the particularity and complexity of a single situation or case, coming to understand its activities within important circumstances". It seeks an in-depth exploration of the phenomenon or specific phenomena based on an extensive data collection process (Creswell, 2014). The exploratory case study involves cases described and compared in order to gain insight into the phenomenon under study (Creswell, 2014). In the context of my study the case entailed three secondary school mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities. I therefore adopted a multi-perspective analysis of the teachers' pedagogical reasoning and action in relation to learner errors and misconceptions in quadratic inequalities. This became the central focus of the investigation and so the researcher took into consideration the voices and perspectives of the participants regarding the phenomenon and context under examination (Maree, 2011).

3.4 Population and sampling

The population for this study was secondary school mathematics teachers in Gauteng. It was from this population that a non-probability sample of three mathematics teachers was chosen. A non-probability sample is a sample where the researcher has no way of forecasting or guaranteeing that each element of the population will be represented (Leedy & Ormrod, 2010). There are four types of non-probability sampling (Maree, 2011): purposive, convenient, quota and snowball sampling. In this study a purposive and convenient sample was chosen. Purposive sampling involves the deliberate selection of the participants based on specific qualities that they possess (Pacho, 2015). According to Maree 2011, p.178), a purposive sample is used “in special situations where the sampling is done with a specific purpose in mind”.

The three mathematics teachers were chosen because they were deemed to have in-depth knowledge of the phenomenon under study by virtue of their professional role or experience. The three teachers had more than ten years of teaching secondary school mathematics in the South African context. However, the sample was not representative of secondary school mathematics as it was selected on the basis of its relevance to the researcher. The sample chosen was also a convenient sample because the participants were easily and conveniently available to the researcher (Maree, 2011; Leedy & Ormrod, 2010). Thus, the findings from this study are not generalised but transferable (Lincoln & Guba, 1985). This means that findings from this study may be applied in other contexts similar to the context in which this study is based.

3.5. Empirical site

The study was located within three research sites, which were three secondary township schools in Johannesburg North, a district in Gauteng province and three teachers, one from each school, participated in the study. The three teachers were of the black nationality, each with more than ten years of teaching secondary school mathematics. About the schools they teach, the following can be said: one teacher taught at a predominantly coloured school, 29 of the grade 11 learners that he taught participated in the study. The other two teachers taught at predominantly black schools. Of the two teachers one had 15 learners who participated in the study and the other worked with 32 learners.

The schools from which the three teachers hailed were government owned and fall in the non-fee-paying category. Additionally, the three schools are located within a radius of ten kilometres from each other. At the time of conducting research, the researcher was located in the same district but teaching at an independent secondary school.

3.6 Data collection methods

For this study, data were collected using the following methods: interviews, observations, audio-recordings and analysis of the learners' written task. Utilising these methods, data were collected in four stages namely, through semi-structured interviews that were used to generate the data, secondly, learners' written task, which also served as the source for data generation, followed by observation to generate the data from a remedial lesson conducted by each of the three participants and then finally, the semi-structured interviews were used again as a means for data generation. The interviews and lesson observations were audio recorded and transcribed to facilitate the data analysis process. Focus was thus given to each of the four stages. It is however important to note that the four stages were carried out outside the normal teaching time of the participants. Therefore, the data were collected during study periods in the three teachers' respective schools.

3.6.1 Interviews

An interview is a process in which two or more people engage in a dialogue or conversation for a particular purpose known to the parties involved (Lichtman, 2010). The purpose of the interviews conducted were explained and understood by each of the three teachers.

There are different forms of interviews: structured, semi-structured and unstructured interviews. In this study, semi-structured interviews were used. A semi-structured interview involves the asking of questions which allow further probing in order to obtain more information (Castellan, 2010). They allow for the probing and clarification of responses (Maree, 2011). In this study two semi-structured interviews were conducted. The purpose of the first interview was to gather the teacher's knowledge about learner errors and misconceptions in quadratic inequalities. The second interview conducted on the fourth stage of the data collection process was an evaluative interview. Its purpose was to gather the participants' thoughts about the remedial lessons conducted. The intention was to gain insight into the strategies used in remediating learner errors and misconceptions in quadratic inequalities.

The level of researcher involvement when conducting semi-structured interviews has been put on the spotlight and widely acknowledged (Pezalla, Pettigrew & Miller-Day, 2012). This is based on certain attributes that the researcher possesses, which had the potential to influence data collection using semi-structured interviews. For example, in my case, my personal experiences of teaching secondary mathematics at the grade level identified in this research may have influenced the quality and direction in which the interviewing processes took place. However, as argued by Ellis and Berger (2003), interviewer reflexivity is encouraged. Therefore, throughout the two sets of interviews conducted, the role of the researcher was that of the researcher as the primary instrument for data collection. In this regard I had to bracket out my own personal experiences in the subject and focussed on the interviewees' perspectives and experiences about the phenomenon under study.

3.6.2 The learners' diagnostic task

A learners' diagnostic task prepared by the researcher was administered by each teacher to his class. The task was written in the presence of the researcher. Each teacher was given four days to mark his learner's scripts. The marked scripts were then collected by the researcher for further analysis. The purpose of this stage, in the data collection process was to determine the teachers' transformation of their comprehended ideas (Shulman, 1987), as gathered in first interview. Part of that involved the teacher's ability to detect errors and misconceptions in the learners' written work. Therefore, this enabled further answering of the first research question as well as the second and third research questions. In addition, the errors and misconceptions identified by the teachers formed the basis of the teacher's planning and execution of a remedial lesson. Attention is now given to the data collection through observation.

3.6.3 Observations

According to Maree (2011), observation is a process of recording the behaviour patterns at a research site of subjects, objects and occurrences without necessarily questioning or communicating with them. One may argue that observation enables the collection, from a research site, of open-ended first-hand information about a phenomenon under study. According to Leedy and Ormrod (2010), observation as a tool for data collection facilitates a deeper insight and comprehension of the phenomenon under investigation. There are four types of observations: complete observer; observer as participant; participant as observer;

and complete participant (Maree, 2011). With respect to the study, the researcher was a complete observer, a non-participant observer and had to focus on his role as a complete observer. My role was to observe the teachers' pedagogic reasoning and action as they dealt with learner errors and misconceptions in quadratic inequalities. The thrust was on the teacher as a mediation tool as well as the teacher as a mediator in the learners' ZPD. I had to observe the mediation tools that the teacher employed as well as the instructional methods at work. The purpose of this stage in the research process was to enable the answering of research question 4 of the study.

3.7 Analysis of data

Qualitative data analysis is a process that brings order, structure and meaning of mass data collected (Marshal & Rossman, 2014). According to Luneta (2013), the process involves observing and extracting patterns by way of conducting a methodical and critical examination of the data collected. Therefore, the data analysis process reduces the data, without losing its meaning, so as to transform the data into findings. Furthermore, in the process of analysing the data, categories emerge from the data resulting in contextual information, patterns or theories that aid in explaining the data (Yilmaz, 2013). Lastly, the qualitative analysis of data is directed at textual loosely structured or nonstandard observations and interviews (Leedy & Ormrod, 2010).

These are several methods for data analysis in qualitative data, some of which are grounded theory, content analysis and thematic analysis. In this study the data was analysed using the qualitative content analysis method. The method involves the subjective interpretation of the content of the text data through "the systematic classification process of coding and identifying, categories, themes and patterns (Hsieh & Shannon, 2005, p.1278). The researcher chose this method for its advantages. The method has the following advantages: it is flexible as it allows the use of the inductive and deductive analysis of the data; it enables the researcher to code the visible and surface meaning of the text as well as the underlying meaning of the text (Cho & Lee, 2014). Thus, the data was analysed deductively and inductively and reported at the semantic and latent meaning (Zhang &Wildemuth, 2005).

The analysis of the data was guided by the identification of codes, categories and themes arising within and across the three cases (Miles & Huberman, 1994). In order to facilitate the coding process, descriptive codes, evaluate codes, process codes and in-vivo codes were used. Therefore, emerging codes were classified into categories according to their relationships. The categories were synthesized into themes. Lastly, Peng and Luo (2009)'s analytical framework (see section 2.2) was then used to analyse the whole data in the context of the phenomenon under study. Table 2 below gives a summary of the analysis of each research question in the study

Question	Data analysis method
1. What type of learner errors and misconceptions in quadratic inequalities do Secondary Mathematics teachers know?	Content analysis Peng and Lou (2009)'s analytical framework
2. What interpretation, do Secondary Mathematics teachers make of learner errors and misconceptions in quadratic inequalities at grade 11 level?	Content analysis Peng and Lou (2009)'s analytical framework
3. How do Secondary Mathematics teachers evaluate learners written and verbalized responses in relation to errors and misconceptions in quadratic inequalities at grade 11 level?	Content analysis Peng and Lou (2009)'s analytical framework
4. What remedial actions, if any, do Secondary Mathematics teachers employ when addressing learner errors and misconceptions in quadratic inequalities at grade 11 level?	Content analysis and Peng And Luo (2009)'s analytical framework

Table 2: Summary of the analysis of each research question

3.8 Measures to ensure trustworthiness

The following measures were adopted to ensure the trustworthiness of the data collected: triangulation; pilot study and member checking. A description of each of these measures is outlined in the sections below.

3.8.1 Triangulation

Triangulation involves the use of multiple data sources in which their use converges on a consistent conclusion (Leedy & Ormrod, 2010). The multiple data collection techniques and sources used by the researcher helped to minimize researcher bias (Yilmaz, 2013). In view of the argument of the three scholars cited above the credibility of this study was achieved through the use of multiple data sources. Thus, a combination of interviews, observations and document analysis (in this case the analysis of the learners' written task) were used in this study. This contributed to a rigorous research study. The method of data collection enabled the capturing of and respected a multi-perspective of the phenomenon (Yilmaz, 2013). This was done in order to improve the trustworthiness and credibility of the study.

3.8.2 Pilot study

The instruments for data collection (semi-structured interviews, task instrument for learners and observation protocol or schedule) were piloted. The pilot study thus involved using a mathematics teacher at a school in the same district with the three teachers participating in this study. This helped guaranteeing the instruments' transferability (Lincoln & Guba, 1985)

3.8.3 Member checking

According to Harper and Cole (2012), member checking is an important quality control process in qualitative research. In view of this, the three teachers who participated in the study were given opportunities to verify transcripts of the semi-structured interviews and observation data. The field notes of the lesson observations were also presented to them. To enhance the credibility of the findings, participants were given the opportunity to check and evaluate the final report to determine if its descriptions and themes accurately reflected their standpoints (Yilmaz, 2013).

3.9 Ethical considerations

According to Punch and Oancea (2014), ethics refer to the rules of behaviour which state what is acceptable in a profession. The researcher applied for an ethical clearance from the University of Witwatersrand Ethics Committee and the Department of Education, Gauteng Province to access its public schools. The ethical clearance was duly issued after satisfying all the necessary conditions required. Permission to access the research sites used for this research was sought from principals of the respective schools. Permission was also sought from the Head of the Mathematics department of each school, the teachers, parents and learners who participated in this study. The right to voluntary participation and withdrawal was observed at all times. Thus, the participants were informed that they could voluntarily withdraw from participating in the study should they feel so. Furthermore, the identity of the participants and schools where the research was conducted were kept anonymous (Babbie &Mouton, 2010). Thus, the names of the teachers and learners used in this study are not their real names. Pseudo names were used.

3.10 Summary

This chapter has examined and brought to fore the research paradigm, methodology, and design, population and sampling process as adopted for the study. In pursuit of that, the following key ideas about the chapter were identified and discussed: the research paradigm, research approach and design; population and sample; research setting, data generation and analysis; measures adopted to ensure the trustworthiness of the study as well as ethical considerations. In the next chapter a chronicle of the data analysis of the study is presented.

CHAPTER 4

DATA ANALYSIS, INTERPRETATION AND DISCUSSION

4.1 Introduction

This chapter zooms on data analysis and discussion of the research results. The data analysis is guided by Peng and Luo's (2009) analytical framework. Data analysis in qualitative research involves observing and extracting patterns by way of carrying out a methodical and critical examination of the data collected (Luneta, 2013). According to Hatch (2002), data analysis is characterised by the organisation and interrogation of the data with the aim of establishing patterns, relationships, explanations and the identification of themes to guide in the interpretation of the data. It calls for critical thinking in terms of analysis and discussions of collected data. However, there are many methods for analysing qualitative data. Some of these methods are; the grounded theory, narrative, thematic and content analysis methods. Each method provides a unique interpretive approach of the qualitative data (Floersch, Longhofer, Kranke & Townsend, 2010; Cho &Lee, 2014). Among the various approaches employed under each method, the deductive and inductive approaches are prominent. A brief description of each approach is done.

The deductive approach to analysing qualitative data is theory driven (Luneta, 2013; Makhubele; 2014; Cho & Lee, 2014). It involves the use of pre-conceived codes, categories or a coding frame derived from apriori theory, research or literature guiding the study (Luneta, 2013; Cho & Lee, 2014). What is critical in this approach is the pre-identification and generation of constructs for use in data analysis from theory or previous studies. Thus, the approach is typological and theory driven and theory verifying (Glaser & Strauss, 1967). In view of this the coding process of the data is mapped onto a pre-existing coding frame or specific research question. On the other hand, the inductive approach is data driven (Braun and Clarke, 2006).

According to Zhang and Wildemuth (2005), the application of the inductive approach involves the active generation of categories and themes emerging from the data. This is realised through the researcher's careful examination of the data and the constant comparison (Glasser & Strauss, 1967). As argued by Cho and Lee (2014), the inductive approach is centred on formulating codes, categories or themes embedded in the data. This implies that the researcher immerses him or herself in the data and actively generates emerging themes from the data. Further to this, the researcher may, but not exclusively, bracket out his or her theoretical interest regarding the topic under investigation (Braun & Clarke, 2006). Clarke and Braun (2006) point out that the generated themes are strongly attached to the data. Therefore, the coding process is carried in ways which exclude attempts to fit the data into pre-existing constructs from previous theory or studies (Zhang & Wildemuth, 2005; Hsieh & Shannon, 2005). In a nutshell, the choice of an approach within a particular method of the qualitative analysis of data is either theory or data driven. Hatch (2002) argues that qualitative data should be analysed using the deductive and inductive approach. In light of Hatch (2002)'s argument, the researcher employed an analysis that incorporates both approaches forwards and backwards.

A focus on the data analysis method for this study is made. The qualitative content analysis of data was used. The method involves the subjective interpretation of the content of text data through a methodological classification process involving coding and the identification of categories or themes (Hsieh& Shannon, 2005). The content of the text data as referred to in the foregoing statement was in three parts. It consisted of transcribed interview data, observation data and the learners' scripts on a task based on quadratic inequalities.

The qualitative content analysis method was chosen for its advantages. Part of its advantages is that it enables data to be analysed deductively and inductively (Zhang & Wildemuth, 2005). Qualitative data should be analysed inductively and deductively (Hatch, 2002; Elliot, 2018).As argued by Creswell (2014), the use of the deductive approach only restrict the analysis through the application of pre-determined codes without opening up the analysis to reflect the views of participants in a traditional qualitative way. In view of this the qualitative content analysis method is a flexible approach not limited in its scope of analysis. A brief focus is given to the data of this study.

The data for this study was about a case of three teachers' PRA concerning learner errors and misconceptions in quadratic inequalities at grade 11 level. The following research questions provided the frame of reference throughout the data analysis process:

1. What types of learner errors and misconceptions in quadratic inequalities do secondary Mathematics teachers know?

2. What interpretation, do secondary Mathematics teachers make of learner errors and misconceptions in quadratic inequalities at grade 11 level?

3. How do secondary Mathematics teachers evaluate learners' written and verbalised responses about learner errors and misconceptions in quadratic inequalities at grade 11 level?

4. What remedial actions, if any, do secondary Mathematics teachers employ when addressing learner errors and misconceptions in quadratic inequalities at grade 11 level?

4.2 A brief outline of how the data was collected

The initial exploration of the teachers' PRA concerning learner errors and misconceptions analysis in quadratic inequalities involved an examination of the teachers' knowledge about learner errors and misconceptions in quadratic inequalities. Interview data from each of the three key participants participating in this study was gathered. Each interview was conducted in each teacher's (participant's) respective school. The teachers were interrogated about learner errors and misconceptions in quadratic inequalities. The data collected at this stage was used to examine the participants' ability to theorize, based on experience, specific errors and misconceptions committed by learners in quadratic inequalities. In addition, this data was also meant to explore the participants' theoretical interpretation, evaluation and remediation of the learners' errors and misconceptions in quadratic inequalities based on grade11 content. This part was followed by the collection of data from learners' scripts.

The thrust of the data from the learners' scripts was the teachers' practical assessment of the errors and misconceptions arising from the learners' written task on quadratic inequalities. This was followed by observation data from a remedial lesson conducted by each teacher. Each teacher conducted a remedial lesson structured on the basis of the errors and misconceptions arising from their learners' written task. Lastly, the researcher collected interview data centred on each participant's remedial lesson. Drawing from the above and in response to the research questions highlighted in this section, the data collection process was interactive and therefore, justified the methodology of study- a qualitative research study (Makonye, 2011). The next section chronicles the sequence in which the analysis of data for this study was carried out.

4.3 The data analysis sequence

The analysis of the data was done in a sequence similar to how the data was collected. I started by analysing the first interview data from each of the three participants. Secondly, I analysed the learners' scripts marked by the three key participants influencing this study.

Thirdly, the lesson observation data of the participants was analysed and fourthly the analysis of the second interview data of the participants was done. Lastly, Peng and Luo (2009)'s framework was then applied to the whole analysed data. Attention is now given to how the analysis of each of the two sets of the interview and observation data gathered.

4.4 The analysis of the interview data and observation data

The interview data were analysed through a systematic coding process. The coding was deductively and inductively done. The decision to code deductively or inductively was not independent of other decisions such as the research question, conceptual framework and structuring of the data (Punch, 2013). The deductive coding process was centred on aspects of the teachers' pedagogic reasoning and action (PRA) as identified by Shulman (1987).Thus a priori coding framework was used and the coding process was done at the level of meaning (Decuir-Gunby, Mashal & McCulloh, 2011).Descriptive and process codes were used for this part. After the deductive coding, an inductive coding of the same data was done. The coding was data driven and a line-by-line, at sentence or phrase level, coding process was carried out. In-vivo codes, descriptive and process codes were used in this part. Thus, codes emerged through the researcher's active engagement with the data. The data driven codes were then examined in relation to the theory driven codes. This was done in order to get rid of any overlaps (Decuir-Gunby, et al, 2011). Focus is now turned to the analysis of the first interview data.

4.5 Analysis of the first interview data

The interview data were analysed deductively. In line with Hsien and Shannon (2005)'s view, existing theory from related literature was used to develop a coding frame before analysing the data. The frame was adapted to fit the purpose of the interview data. The interview data were meant to unpack the participants' pedagogic reasoning prospectively. Hence the participants' thoughts under-pinning what they knew, how and why with respect to learner errors and misconceptions in quadratic inequalities were explored. This exploration was to guide further exploration of their PRA into the topic of study. Based on the aforementioned I developed a coding frame from Shulman (1987)'s model of pedagogic reasoning and action. The coding frame was centred on the first two aspects of the model. These are comprehension and transformation. The two aspects were chosen because they focus on the form of rehearsals and the action plan that guide performances for teaching a specific subject matter (Shulman, 1987). The subject matter for the study was errors and misconceptions in quadratic inequalities and therefore in recognition of the foregoing, table 3 below gives the frame that was formulated for the analysis of the first interview data.

Category	Description of the category	Example	Code
1.Comprehension	The ability of the teacher to distinctively show a clear understanding of what they teach in several ways	Understanding the different types of errors and misconceptions which learners make in quadratic inequalities. For instance, the inequality sign interpreted as an equal sign	C_1
2.Transformation	Transforming the comprehended ideas		
2.1. Preparation	2.1 Coming up with the text-material (errors and misconceptions in quadratic inequalities) and putting them into forms adapted to the teacher's understanding and in prospect suitable for teaching	2.1 The teacher's ability to understand the instructional materials- learner errors and misconceptions in quadratic inequalities and making them part of the subject matter.	TP
2.2. Representation	2.2 Ability to think about the central ideas in the text-material and identifying possible ways of representing them to learners	2.2. The analogies, metaphors, examples and demonstrations used by the teacher in order to assist learners to clear their fault lines of reasoning	TR
2.3. Instructional selection	2.3 The various approaches of strategies of teaching that the teacher employs in addressing learner errors and misconceptions in quadratic inequalities	2.3 Use of teaching strategies such as the lecture method, guided method, demonstration, and reciprocal teaching to name but a few.	TI
2.4 Adaptation	2.4 Modifying and fitting the represented material to suit the characteristics of the learners	2.4 -Prior knowledge contributing to learner errors and misconceptions.	TA
2.5 Tailoring	2.5 The fitting of the text material in terms of the specific errors and misconceptions committed by participant's learners.	2.5 Ability of the participant to fit representations to the group of learners that one teaches.	TT

Table 3: Coding frame for the analysis of the data

The typology framed above was used to analyse the interview data pertaining to the teachers' thoughts about learner errors and misconceptions in quadratic inequalities. However, before making use of the coding frame I had to read and re-read, several times, the transcribed audio recordings of the interview data .This was inter-twinned with memoing (Maree, 2011). The purpose for that was to gain "an intimate, interpretative familiarity with every datum in the corpus" (Miles, Huberman &Saldana, 2014; 73).This was followed by the use of the coding frame given above. In using the frame, I had to label using the codes identified in the frame a statement, sentence or sentences from each of the participant's interview data relating directly or indirectly to a particular aspect concerning learner errors and misconceptions in quadratic inequalities. For example, the code C_1 was assigned to each statement, sentence or sentences relating to the teacher's understanding of the different types of learner errors and misconceptions which learners make in quadratic inequalities. The same approach was used regarding the application of the other codes identified in table .Thus ,the coding of the data using the coding frame in table 3 was done at the level of meaning .This coding strategy was followed by sorting and categorising the codes based on the relationship that existed among them (Huberman &Saldana, 2014). Finally the underlying meaning within and across the codes was established leading to the formulation of the two themes; teacher comprehension and transformation of learner errors and misconceptions in quadratic inequalities.

Drawing from the above description, the coding frame was therefore used as a yardstick to determine the themes emerging from the interview data. The identified themes were later used for further analysis of the data and discussion of the findings. Focus is now given to each of the two themes.

4.5.1 Participants' comprehension of learner errors and misconceptions in quadratic inequalities

The three participants were given the following pseudonyms in order to protect their identity and confidentiality of their contributions: Mr Dlomo, Mr Zama and Mr Mthembu. In order to demonstrate their knowledge of learner errors and misconceptions in quadratic inequalities, I identified the different aspects which they emphasised and the comparison thereof. Focus is made to Mr Dlomo's comprehension of learner errors and misconceptions in quadratic inequalities.

4.5.1.1 Mr Dlomo's comprehension of learner errors and misconceptions in quadratic inequalities

Mr Dlomo identified the following aspects as central in learner errors and misconceptions in quadratic inequalities: quadratic inequalities are solved in the same way as quadratic equations, symbols are misinterpreted, incorrect interpretation and application of critical values, interpretation of solutions of quadratic inequalities graphically or using the number line method. Mr Dlomo explained that learners view the solving of quadratic inequalities in the same way as solving quadratic equations. He explained that this possibly stems from the sequencing of the topics. In the curriculum quadratic equations are taught before quadratic inequalities. As such, Mr Dlomo reckoned that learners struggle to realise the difference, resulting in the generalisation of methods of solving quadratic equations to quadratic inequalities. He articulated that this was mainly because of the learners' insufficient conceptual understanding of the inequality signs.

The following transcript between the researcher (interviewer) and Mr Dlomo provides an insight into Mr Dlomo's pedagogical reasoning with respect to learner errors and misconception in quadratic inequalities.

Researcher: What are your thoughts regarding learner thinking in inequalities in general and quadratic inequalities in particular.

Mr Dlomo: I can say the topic is a big challenge to learners in the sense that they basically struggle to understand algebra on its own. They mix up a lot of issues. Talk about equations, talk about directed numbers, talk about interpreting graphs of functions. When it comes to inequalities the symbols are misinterpreted as to when to use greater than, greater than or equal to, less than and less than or equal to? To the extent that along the way you find kids changing the inequalities of any nature to equations because they don't just understand the concept of inequalities. When it comes to the inequality signs, they are misinterpreted. You find kids sometimes think that the inequality signs have the same meaning as the equal sign. You can see this in the manner, they solve quadratic inequalities

Researcher: What are the possible sources of learners' incorrect solutions in quadratic inequalities?

Mr Dlomo**:** To them quadratic inequalities are just too difficult to comprehend, especially when it comes to the issue of critical values. To say what exactly do we mean by critical values? Because they have already done quadratic equations, but the moment you introduce a quadratic inequality and say your answer goes beyond this answer. This is when you ask yourself: do they really understand the concept of inequality to begin with? Factorisation is not a problem with the learners but incorporating the inequality part of it makes it difficulty, especially when interpreting the solution from the graph or number line. These problems stem from lower grades where possibly the concepts are misrepresented in the child's mind and they carry that through. Trying to unlearn obviously you know from education it is difficult.

From Mr Dlomo thoughts, as articulated in the transcript above, the concept of critical values, the learners' interpretation of the inequality signs, the learners' background knowledge about quadratic equations and numbers in their generalised form seem to be the main epistemological hindrance to learners' mastering of quadratic inequalities.

According to Mr Dlomo, learner errors and misconceptions have a historical background. It starts with the learner's comprehension of algebra. However, Mr Dlomo's view was that learners basically struggle to understand algebra as they "mix up a lot of issues". This suggests that learners' knowledge in algebra in fragmented. Mr Dlomo also makes a case of directed numbers. His thinking signals that, learners' struggle with negative and positive numbers have a bearing on their conceptualisation of quadratic inequalities. Further to this, he reasoned that the learners' obstacles with quadratic inequalities are a function of their misinterpretation of inequality signs of any kind. Mr Dlomo thinks learners view quadratic inequalities as being congruent to quadratic equations. The next part of this section focusses on Mr Zama's compression of leaner errors and misconceptions in quadratic inequalities.

4.5.1.2 Mr Zama's compression of leaner errors and misconceptions in quadratic inequalities

Regarding Mr Zama's comprehension of the errors and misconceptions in quadratic inequalities, reference is made to the following transcript. The transcript lays bare Mr Zama's comprehension of learner errors and misconceptions in quadratic inequalities at the identified grade level.

Researcher: What are your thoughts regarding learner thinking in inequalities in general and quadratic inequalities in particular?

Mr Zama: Learners generally regard inequalities as equations. So, to them inequalities would actually be synonymous with equations to the extent that they simply replace the inequality sign with an equal sign, then they get solutions. So, for a quadratic inequality they will simply come up with two solutions in the form of equations and not in the form of inequalities. Thereby, coming up with an impartially complete solution as it was, because what is supposed to be critical values, to them they are solutions of the quadratic inequality. Coming to the solutions of quadratic inequalities, it seems like learners do not link sketches of quadratic function to the quadratic inequality. When it comes to interpreting the solutions, with most learners it is a mess. It seems like most learners use trial and error, they do it in any manner not applying concepts of curve sketching and inequalities. Then there is also the number line method. Learners usually make mistakes there. To be able to substitute values correctly into the original inequality and when that substitution leads to an inconsistent statement. For example, when we say 10<0, that is an inconsistent statement. It means the region where the value that was substituted is coming from is not a solution. But some learners would actually say `10< 0, but still go on to select that same region where that value substituted to give that particular inconsistent statement is coming from. So, learners do not seem to know when to accept the solution and when to reject using the number line method.

From the above transcript Mr Zama's comprehension of learner errors and misconceptions in quadratic inequalities reveals the following: Learners fail to differentiate between quadratic inequalities and quadratic equations. It follows that they overgeneralize their knowledge of equations to inequalities (Kroll, 1987; Almog & Ilany, 2012). Mr Zama reasoned that when solving quadratic inequalities learners simply replace the inequality sign with an equal sign. Therefore, he reasoned that for some learners the solutions of quadratic inequalities are in the form of equations. Additionally, learners fail to establish the connection between the graphical representation of the quadratic function and the inequality statement. The same thing is also observed when it comes to solving quadratic inequalities using the number line method. According to Mr Zama, learners exhibit some epistemological obstacles when it comes to discriminating the regions apportioned by the critical values on a number line. Mr Zama's reasoning seems to indicate that learners execute procedures for solving quadratic inequalities without any conceptual connections. This implies that learners view the inequality sign as a nexus with the same meaning and application as the equal sign (Kroll, 1987; Blanco & Garrote, 2007). He also articulated that learners make errors and misconceptions about the critical values. The critical values of quadratic equations are considered as the solutions of the quadratic inequalities. Having discussed Mr Zama's reasoning regarding learner errors and misconceptions in quadratic inequalities, focus is given to Mr Mthembu's reasoning.

4.5.1.3 Mr Mthembu's comprehension of leaner errors and misconceptions in quadratic inequalities

The following transcript taps into Mr Mthembu's comprehension of learner errors and misconceptions in quadratic inequalities.

Researcher: What are your thoughts regarding learner thinking in inequalities in general and quadratic inequalities in particular?

Mr Mthembu: Before quadratic inequalities, learners are introduced to factorisation, linear equations, quadratic equations, linear inequalities and functions. Now when it comes to quadratic inequalities, it's another thing. It appears what they are taught, their prior knowledge, especially of quadratic equations interferes with what they are supposed to know about quadratic inequalities. To some learners there is no difference between quadratic inequalities and quadratic equations, these are the same. When it comes to quadratic inequalities it's, a matter of the interpretation of the inequality signs, the calculation of critical values and using these critical values to find the solutions of the quadratic inequality. What I have seen is that critical values are not understood properly. Learners take critical values as solutions of quadratic inequalities. In some cases, they are written in the form of inequalities in other cases they are written as equations. I have also realised that most of my learner's face problems when it comes to the methods of calculating the critical values. I encourage them to use the quadratic formula because it is easy to use. It's a question of substituting into the formula. Now when it comes to interpreting the solutions of the quadratic inequality, it's a problem. Some learners struggle to differentiate the inequality signs, for instance, differentiating the greater than sign from the less than sign. Again, when interpreting solutions of quadratic inequalities from a graph they tend to combine their solutions even in cases where it's not applicable. Also, in some cases when dealing with negative numbers, they don't change the direction of the inequality sign when dividing or multiplying by a negative number. Another thing is that, when interpreting solution of the quadratic inequality using the graphical approach, learners do not know how to conclude in terms of structuring their answer. They struggle to know when to include or exclude the critical values.

Based on the reference made in the transcript above, the analysis of the data unmasks the following key aspects about Mr Mthembu's pedagogic reasoning:

The learners' prior knowledge about quadratic equations gives rise to learner errors and misconceptions in quadratic inequalities. There is a tendency by some learners to conceptualise quadratic inequalities as quadratic equations. Because of that quadratic inequalities are solved in the same way as quadratic equations.

The concept of critical values is also weakly conceptualised. Learners tend to think that the critical values are the solutions of quadratic inequalities. In addition, there are also misconceptions in methods used to determine the critical values. The interpretation of inequality signs was also the origin and root of the learner's inability to establish the solutions of quadratic inequalities graphically. It appears there are epistemological barriers experienced by learner when interpreting solutions of quadratic inequalities graphically. Thus, Mr Mthembu reasoned that learners do not know how to structure their final answers in relation to the sketches of the quadratic functions. This, he said, result in learners making improper conclusions from the sketches drawn. The next section focusses on the three teachers' transformation of the learner errors and misconceptions in quadratic inequalities.

4.5.2 Participants' transformation of the learner errors and misconceptions in quadratic inequalities.

Transformation is characterised by the teacher's thinking process which culminates in a strategy to execute a lesson (Shulman, 1987). According to Shulman (1987), transformation like comprehension is a rehearsal process for the performance of a lesson which has not yet occurred. In view of this, the researcher had to focus on how the participants transformed their comprehended ideas about learner errors and misconceptions in quadratic inequalities. Below is each of the participant's pedagogic reasoning with respect to the topic under study.

4.5.2.1 Mr Dlomo's transformation of learner errors and misconceptions in quadratic inequalities

In the interview with Mr Dlomo, the following key ideas about quadratic inequalities were identified. Defining terms, critical values as a key concept, inequality signs and their interpretation, factorisation, dividing both sides of an inequality by a negative number, sketching of parabolic functions, the regions demarcated by the critical values, methods of solving quadratic inequalities: the number line method and the graphical approach. In his bid to transform his comprehended ideas about quadratic inequalities, Mr Dlomo highlighted the examples, demonstrations and analogies that he uses in the act of teaching quadratic inequalities. For instance, in dealing with the concept of critical values, Mr Dlomo emphasised the need of explaining the concept using real life situations where there are boundaries or demarcations. With regard to the use of the number line method, Mr Dlomo stressed the need to test each region defined by the inequality and identifying the nature of the inequality statement in each region. However, of the methods suggested for solving quadratic inequalities, Mr Dlomo viewed the graphical approach as critical in the teaching of quadratic inequalities. Thus, during the interview, Mr Dlomo indicated that graphs should be sketched "whenever we are solving quadratic inequalities".

What then is the instructional selection that Mr Dlomo would use when dealing with learner errors and misconceptions in quadratic inequalities? With respect to this question, the teacher-learner centred approach and the lecture method were put on the spotlight. For example and alluding to a teacher-learner centred approach, Mr Dlomo articulated that he involves learners by inviting them to present their solutions to the board. Thus, he said:

> As learners present their solutions on the board you will always see something, you can even read their mind. If a learner has a problem of interpreting… that inequality symbol you will see by the manner in which they will choose their answers.

Further to this, Mr Dlomo also indicated the need for the use of physical objects in illustrating regions defined by a particular quadratic inequality. In light of this he explained that "what I normally do in my case I use a ruler. I place it across or super-impose it on the x- axis and say what this ruler has done is cut our curve into the region above and the region below". He went on to demonstrate how he uses a ruler as an aid to assist learners in visualising the process of solving quadratic inequalities. The foregoing highlights the instructional selection with regard to Mr Dlomo's pedagogic reasoning in transforming his comprehended ideas based on the topic of study.

Finally, Mr Dlomo also theorized about adaptation. Adaptation involves fitting the represented material to the characteristics of the learners (Shulman, 1987). In view of this Mr Dlomo said:

> You need to know your learners. What kind of learners are you dealing with? … And use a method that is most appropriate to their needs.

The next section focuses on Mr Zama's transformation of his comprehended ideas regarding learner errors and misconceptions in quadratic inequalities.

4.5.2.2 Mr Zama's transformation of learner errors and misconceptions in quadratic inequalities

Outlined below is an account of how Mr Zama transformed his comprehended ideas about learner errors and misconceptions in quadratic inequalities. With respect to that he indicated the need to make learner errors and misconceptions a part of the material content to be dealt with in class. However, in thinking about the key ideas about quadratic inequalities, the following aspects were central to his pedagogic reasoning; sketching graphs of quadratic functions, interpretation of inequality signs, finding critical values and coming up with solutions of quadratic inequalities. In addition to the above, Mr Zama spoke about the number line method and substitution as important concepts in solving quadratic inequalities. With regard to the number line method, Mr Zama mentioned the importance of correctly determining the three regions defined by the critical values. He also stressed the substitution of specific values picked from the regions defined by the critical values on the number line. Thus, he noted: "after coming up with the three regions learners have to choose a value that falls in any of the three regions [and] substitute into the inequality". Emphasis was placed on correct substitution and being able to detect when the substitution of a particular value would lead to an inconsistent statement. According to Mr Zama an inconsistent statement is a statement such as $10<0$.Mr Zama noted that upon arriving at an inconsistent statement, the region where the value substituted was picked will not form part of the solution of the quadratic inequality.

When discussing the embodiment of the representation of his comprehended ideas about quadratic inequalities, Mr Zama highlighted the lecture method and the learner- centred approach to teaching. In exemplifying the application of the lecture method, Mr Zama said:

> If an error has been made, then I would use that error and its source and try to come up with another strategy to go around the error. That might mean changing the method, if I have seen that the method is challenging to learners, I will re-explain the concept a little bit more if the errors are coming from misunderstanding of the underlying concept on the particular topic.

On the other hand, and alluding to the learner-centred approach this is what Mr Zama said:

> It is also interesting when a learner makes a mistake or an error to involve other learners to try to correct such errors. So, if you actually involve other learners, you can also find it interesting to realise that you can even learn better methods or better ways of countering those errors and misconceptions.

In addition to highlighting his instructional material with respect to the topic of study, Mr Zama gave an account of his adaptation to the topic. In view of that, Mr Zama explained that learners face great challenges in coming up with solutions of quadratic inequalities. He noted that in some cases learners are able to determine the correct regions defined by a given quadratic inequality but still use trial and error in selecting their solutions

Additionally, Mr Zama spoke about using the relevant language when dealing with the key ideas about quadratic inequalities. In line with that, he explained that learners need to know how to find the critical values and what these critical values are. He also explained that learners need to know the concept of a number line as well as substituting into a given expression. Mr Zama added that the prior knowledge of learners involving the number line was critical. Thus, he noted that, some learners place a negative number to the right of a positive number on a number line. According to him, such learners focus on the numerical size of the number ignoring the meaning of the signs attached to the numbers.

Lastly, Mr Zama spoke about adaptation with regard to language wherein emphasis was placed on the correct use of language in dealing with the key concepts involving quadratic inequalities. Thus, the following key concepts were identified by Mr Zama: finding critical values; the number line; substitution and the rejection criteria in coming up with the solution of a quadratic inequality.

Attention is now given to Mr Mthembu's pedagogic reasoning with respect to the transformation of his comprehended ideas about learner errors and misconceptions in quadratic inequalities.

4.5.2.3 Mr Mthembu's transformation of learner errors and misconceptions in quadratic inequalities

Presented below is an extract of the transcript of the interview data representing Mr Mthembu's transformation of his comprehended ideas about learner errors and misconceptions in quadratic inequalities.

Researcher: How do you address learner errors and misconceptions in quadratic inequalities?

Mr Mthembu: First, I need to think about the learners' knowledge of quadratic equations and the methods of solving such equations. So, I always start by going over the learners' knowledge about solving quadratic equations and the different methods of solving such equations. When it comes to quadratic inequalities, it's about the correct interpretation of inequality signs and reaching the correct conclusions. So, I make sure that learners have sufficient knowledge content knowledge of inequalities taught in lower grades.by revisiting this content with the learners. However, I have noticed that learners have insufficient content knowledge of inequalities from lower grades. In most cases learners fail to differentiate inequality signs. They struggle to address the meaning of the different types of inequality signs. Because of that, it makes it important for me to ensure that learners have an adequate interpretation of the different inequality signs. The correct interpretation of the inequality signs assists learners in the correct determination of the solutions of quadratic inequalities.

Researcher: What then will be your key ideas in your teaching of quadratic inequalities with respect to learner errors and misconceptions in quadratic inequalities?

Mr Mthembu: I always emphasize the correct interpretation of inequality signs; the correct determination of critical values using methods of solving quadratic equations and their presentation; the graphical method of solving quadratic inequalities and the interpretation of solutions of quadratic inequalities from the graph. In most cases I use the learner errors and misconceptions to re-explain the critical aspects required to solve quadratic inequalities. For example, some learners, don't know how to deal with dividing both sides of a quadratic inequality by a negative number; in most cases such learners don't reverse the inequality sign, it's something which I also consider important when dealing with quadratic inequalities

From the above presentation, Mr Mthembu showed bias towards the lecture method when solving quadratic inequalities. He highlighted that he makes use of the errors and misconceptions in quadratic inequalities learners make to re-explain the relevant aspects of the topic. In adapting his represented material with respect to his own experience, Mr Mthembu highlighted that some learner errors and misconceptions in quadratic inequalities are a result of the knowledge gap in content from lower grades He also mentioned that some learners do not change the inequality sign when dividing both sides of a quadratic inequality by a negative number.

4.5.3 Summary of the teachers' transformation of their comprehended ideas.

Drawing from the three participants' transformation of their comprehended ideas, the following aspects were identified as critical in one's rehearsal to remediate errors and misconceptions in quadratic inequalities: the correct interpretation of inequality signs; use of the proper terminology of the topic; the concept critical values and how to calculate these values; the graphical and number line method for solving quadratic inequalities. The next section is about an analysis of the data collected from a task based on quadratic inequalities. The task was written by learners taught by the three participants influencing this study.

4.6 Analysis of the task written by learners

The purpose for the analysis of the data from the learners' scripts was two-fold. Firstly, the analysis was meant to determine, if any, the specific errors and misconceptions in quadratic inequalities committed by learners taught by each of the three teachers influencing the research topic. Secondly, the analysis also sought to determine the teachers' PRA with respect to the transformation of their comprehended ideas as captured in the first interview.

In order to facilitate the analysis of the learners' task as outlined above, the researcher started by categorising the learners' misconceptions in quadratic inequalities. The categorisation was deductively and inductively carried out. This led to the formulation of codes which were then used to code the learners' scripts.

4.6.1 Categories of learner misconceptions and codes used in analysing learner errors in this study

To gain insight into the nature of the learners' errors and misconceptions in quadratic inequalities, nine categories were used. Of the nine, six were derived from literature and three emerged from the data collected. However, the researcher had to design his own coding. The coding was based on the following two main thinking processes; generalisation and application. The decision to design the researcher's own coding system was influenced by the absence of coding of such errors in related literature. Table 4 below shows the coding frame that was formulated in order to code the learners' written responses.

Thought process	Misconception	Reason for misconception	Code
1.Generalisation Product rule: If $a \times b = 0$ then $a = 0 \; or \; b = 0$	1.1 (a) Overgeneralisation of previous correct learning to an extended domain (Olivier, 1989). For example: $If \; (x-3)(x+4) < 0$, then $x < 3 \; or \; x < -4$ 1.1(b) Overgeneralisation of previous incorrect learning to an extended domain (Olivier, 1989). For example: $If \; (x+3)(x+4) > 2$, then $x+3 > 2 \; or \; x+4 > 2$	Overgeneralisation of the product rule for solving quadratic equations to quadratic inequalities (Bazzin &Tsamir ,2004; Almog & Ilany, 2012)	OP_A OP_B
1.2 Multiplying or dividing both sides of an inequality by a negative quantity, the inequality sign is reversed.	1.2 Overgeneralisation of previous correct learning to an extended domain (Olivier, 1989). For example: $If -5x > 10 \;\; then \; x > -2$	Overgeneralisation of the multiplicative inverse rule in linear equations to inequalities (Kroll,1987)	OMR
2. Application 2.1 Methods of solving quadratic equations as tools to solve quadratic inequalities	2.1 Insufficient conceptual understanding of methods of solving quadratic equations (Hodes, 1998). For example: $If \;\; x^2 - x - 6 > 0 \,, then$ $(x+6)(x-1) > 0$	2.1 Concept of factorisation known, but cannot be applied correctly in the specific situation or question (Hodes, 1998)	ICF-MSQ
2.2 Inequality sign does not behave like an equal sign	2.2 Misapplication of previous correct learning to an extended domain. For example: If $x^2 - 3x - 4 < 0$ then $x = 4, x = -1$	Inequality sign a nexus with the same meaning and application as the equal sign (Kroll, 1987, Blanco & Garrote,2007)	IN
2.3Use of the logical connectors "and" and "or"	2.3 Absence of semantic meaning of logical connectors used in inequalities. For example: $If \;\; x^2 - 4 \geq 0$ then $x \leq -2 \;\; x \geq 2$	Absence of semantic meaning of logical connectors and/or (Almog & Ilany, 2012).	ULC

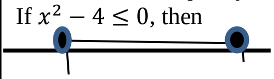
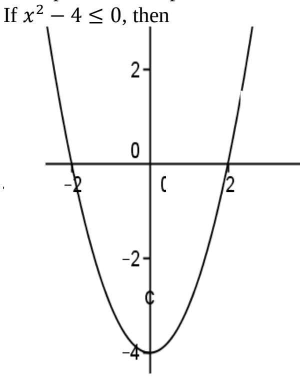
2.4 Critical values	2.4 Absence of semantic meaning of critical values as applied in inequalities. For example, if $x^2 - 9 \geq 0 \quad CV: x = 3 \ and \ x = -3$	The incomplete application of the procedures of solving quadratic inequalities	CV
2.5 Use of a visual representation to interpret and communicate solution sets of inequalities	2.5.1 Use of the number line without conceptual connection to inequality concepts. For example: If $x^2 - 4 \leq 0$, then  $-2 < x < 2$	Procedures without conceptual connection to inequality concepts	RN
2.5 Use of a visual representation to interpret and communicate solution sets of inequalities	2.5.2 Use of a graph without conceptual connection to inequality concepts. For example: If $x^2 - 4 \leq 0$, then  $-2 < x < 2$	Procedures without conceptual connection to inequality concepts	RG

Table 4: Categories of learner misconceptions and codes

Having tabulated the coding frame for the analysis of the learners' written responses, the next section presents how the analysis of the learners' scripts was carried out. The section details the analysis of the learners' scripts as marked by the three key participating teachers who partook in the study. The learners' scripts were in response to a written task that was prepared by the researcher and aligned to the South African mathematics Curriculum and Assessment Policy Statement (CAPS) content as covered at grade 11. The task was written by a total of 76 learners who volunteered to participate in the study upon seeking consent from their parents. Of the 76 learners, 29 were from Mr Dlomo's school, 15 were from Mr Zama's school and 32 were from Mr Mthembu's school.

4.6.2 How the analysis was done

Selected vignettes of the learners' marked scripts from each of the three teachers participating in the study were made use of. The vignettes identified displayed the nature of the misconception or misconceptions committed by the learner or learners. An analysis of the nature of the misconception or misconceptions was made. This was followed by analysing how each of the three teachers evaluated the learners' script. The reason for doing that was two -fold. Firstly, it was to seek consistence in the teachers' pedagogic reasoning as theorised in section 4.5. Secondly it was meant to examine how the teacher reasoned in the process of detecting and correcting errors in the learners' written responses. The reasons identified assisted the researcher to zoom into each of the three teachers' transformation of their comprehended ideas about learner errors and misconceptions in quadratic inequalities at the identified grade level. However, it must be noted that the researcher presented the analysis of five out of the eight learner's written responses. Question 3, 7 and 8 were left out .The decision to leave out the responses to these questions was based on the fact that the nature of the learner errors and misconceptions for question 2, 3 and 7 were identical. Those of question 4 and 8 were also identical. As explained in the opening part of this paragraph, attention is now focussed on selected vignettes of the learners' marked scripts.

The vignettes chosen capture the misconceptions exhibited by the learners in selected questions and how these were evaluated by the respective teachers participating in the study. The three teachers who participated in this study were given the following pseudo names: Dlomo, Zama and Mthembu. Similarly, their learners were also given pseudo names. Learners in Mr Dlomo's class all start with the letters DML followed with a number. For example DML1.Those in Mr Zama's class start with the letters ZML, followed with a letter, for example ZML1.Lastly, those in Mr Mthembu's class start with the letters MTL followed with a number, for example MTL1. Throughout the next section the responses of learners in Mr Dlomo's class would be analysed first, followed by Mr Zama's and lastly Mr Mthembu's.

4.6.3 Analysis of the learners' responses and how they were evaluated by the three teachers

Consideration is given to the first question of the task that was written by the learners. The first question was: *Solve the inequality* $-5x^2 \geq 0$.

Mr Dlomo's learners' responses to the question; solve *the inequality* $-5x^2 \geq 0$

The following vignettes are typical responses from learners in Mr Dlomo's class.

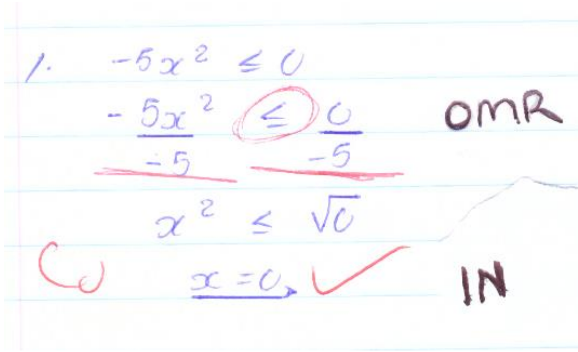


Figure 3: For DML1

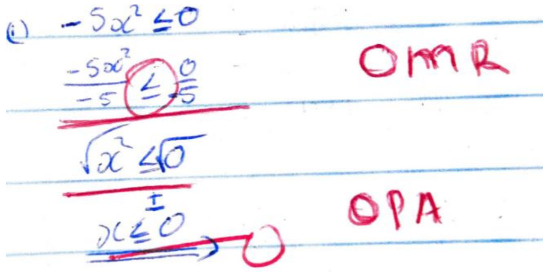


Figure 4: For DML2

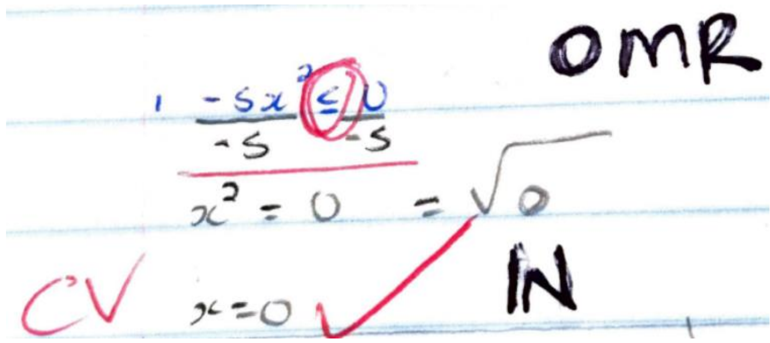


Figure 5: For DML3

The three vignettes highlight the most common misconceptions exhibited by learners in Mr Dlomo's class. An analysis of the above vignettes revealed some underlying pseudo concepts associated with the learner's construction of knowledge about inequalities.

What is common in DML1 to DML3's responses is the misconception associated with the overgeneralisation of previous correct knowledge in linear equations. The learners extended their previous correctly acquired knowledge of solving linear equations to quadratic inequalities. In this case the learners overgeneralised the multiplicative inverse rule in linear equations to quadratic inequalities (OMR). They divided both sides of the inequality and did not reverse the inequality sign. DML2 also overgeneralized procedures for solving quadratic equations to quadratic inequalities (OPA) and arrived at the solution$x \leq 0$. However, DML1 and DML3 treated the critical values as the solution of the quadratic inequality (CV). DML1 and DML3 also displayed a misconception associated with the interpretation of the inequality sign. They interpreted the inequality sign as a nexus carrying the same meaning as the equal sign. Thus, they wrote $x = 0$ as the solution of the inequality, yet $x = 0$ is a repeated critical value of the inequality and not a solution.

Having discussed how the learners responded to this question, focus was then turned to how Mr Dlomo evaluated the learner's written response.

Mr Dlomo's evaluation of the learners' responses

Mr Dlomo evaluated the learners' response by way of underlining and circling where the learners displayed forms of misconceptions. This is evident in DML1 to DML3's responses as displayed in the vignettes above. Further to that, Mr Dlomo commented that the solution which DML1 and DML3 found were critical values. That was done by writing CV next to the learner's solution and placing a tick in front of the learner's solution. The ticks implied the teacher's acknowledgement that what the learners wrote and thought to be the solution of the quadratic inequality were in fact the critical values. Attention is turned to the analysis of the responses to the same question by learners in Mr Zama's class.

Mr Zama's learners' responses to the question; solve *the inequality* $-5x^2 \geq 0$

The vignettes below are typical responses from learners in this class:

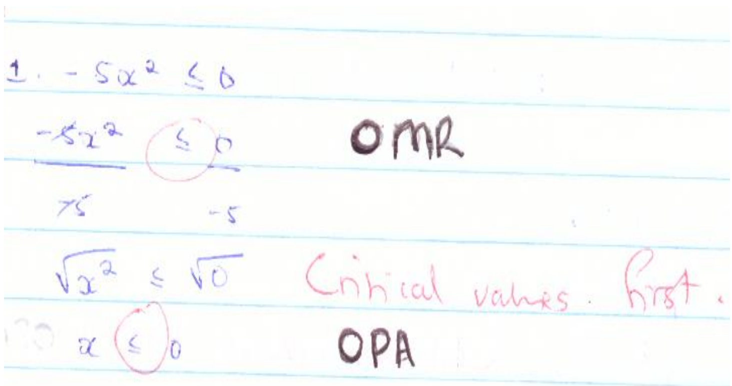


Figure 6: For ZML1

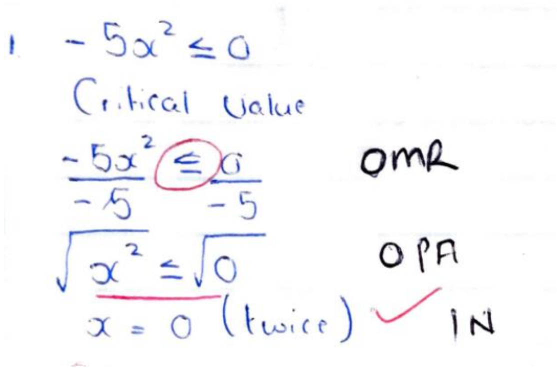


Figure 7: For ZML2

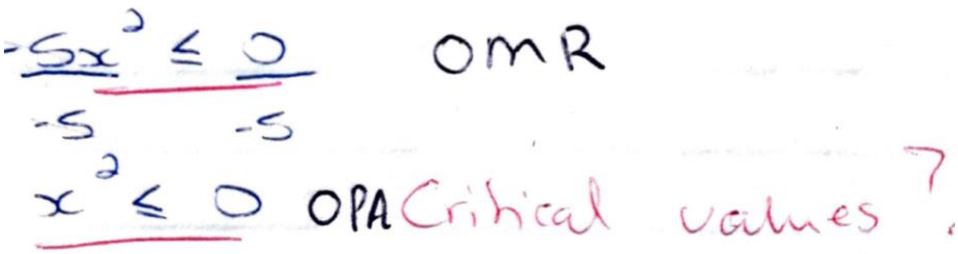


Figure 8: For ZML3

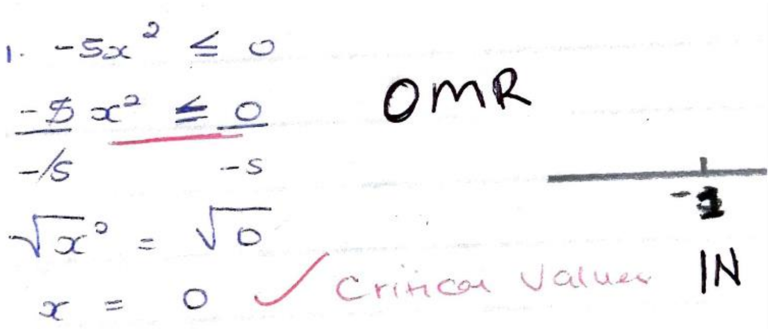


Figure 9: For ZML4

The learners in this class, like DML1 to DML2 in Mr Dlomo's class, divided both sides of the quadratic inequality by negative five but did not reverse the inequality sign. This is evident in ZML1 to ZML4's responses. The learners displayed the same misconceptions as those observed in Mr Dlomo's class. However, ZML1 and ZML2 treated the critical values as the solution of the quadratic inequality. For these learners, the inequality sign was attached the same meaning as the equal sign (IN). However, ZML3 arrived at the solution $x \leq 0$, an indication that the learner also overgeneralised his or her previously acquired correct knowledge about the procedures for solving quadratic equations to quadratic inequalities (OPA).

Consideration is now given to how Mr Zama evaluated the learners' responses.

Mr Zama's evaluation of the learners' responses

Mr Zama circled and underlined where learners displayed some misconceptions. This is evident in ZML2 to ZML3's responses. In some cases, he also circled the final answers as was done to ZML1's answer. Comments were also written to some learners' responses. For example, Mr Zama indicated that ZML4 found critical values of the inequality. For ZML3; a comment in the form of a question about critical values was paused. Mr Zama also placed some ticks on certain aspects of the learners' work. For example, in ZML2 and ZML4, ticks were placed to indicate to these learners that they determined critical values. Turning to how learners in Mr Mthembu's class responded to question one.

Mr Mthembu's learners' responses to the question; solve *the inequality* $-5x^2 \geq 0$

The vignettes below are typical responses by learners in this class:

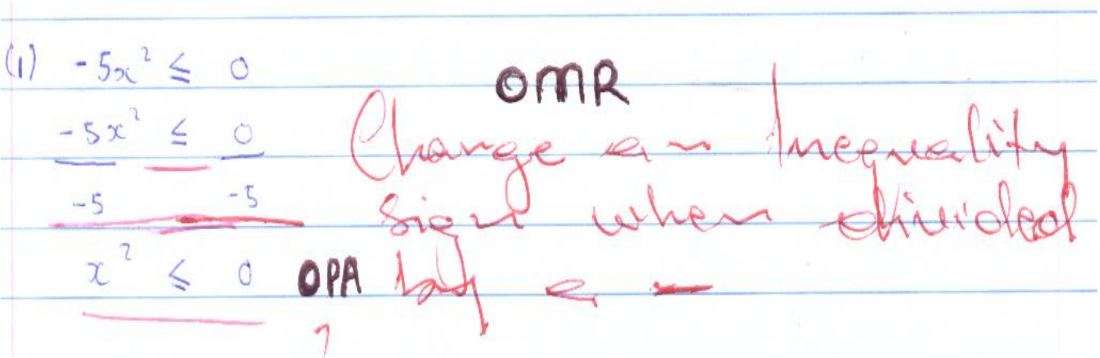


Figure 10: For MTL1

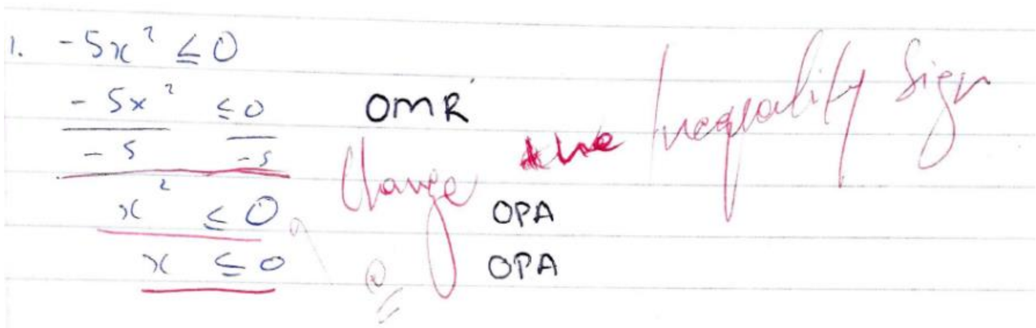


Figure 11: For MTL2

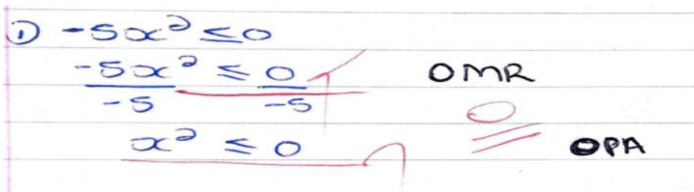


Figure 12: For MTL3

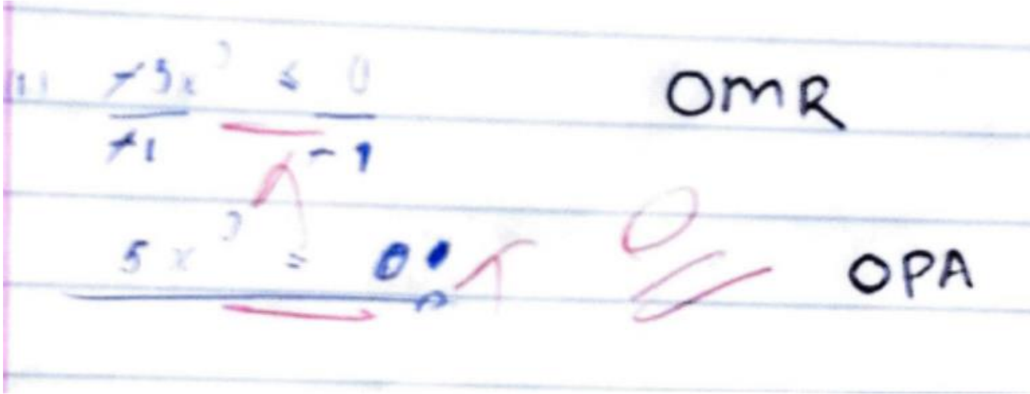


Figure 13: For MTL4

An analysis of how MTL1 to MTL4 responded to question one revealed the following: the learners overgeneralised the multiplicative inverse rule for solving linear equations to quadratic inequalities (OMR). Furthermore, they also overgeneralised their previous correct knowledge about solving quadratic equations to quadratic inequalities (OPA). Hence, they arrived at the solution $x \leq 0$. The misconceptions were identical to those displayed by some learners in Mr Dlomo and Zama's class as has been analysed in the foregoing paragraphs. Attention is now given to how Mr Mthembu evaluated the learners' responses to question one.

Mr Mthembu's evaluation of the learners' responses

Mr Mthembu evaluated the learner's response by underlining stages where misconceptions were evident in the learners' written responses. This is evident in MTL1 to MTL4's responses. In some cases, written comments were made. For instance, on MTL1 and MTL2's responses, he commented that the learners were supposed to change the inequality sign when dividing by a negative number. Consideration is now given to the second question of the task that was written by the learners. The second question was: *Solve the inequality* $x^2 - 9 \leq 0$ .The researcher started by analysing the responses to the question by Mr Learners in Mr Dlomo's class.

Mr Dlomo's learners' responses to the question; solve *the inequality* $x^2 - 9 \leq 0$.

Given below are typical responses by learners in Mr Dlomo's class:

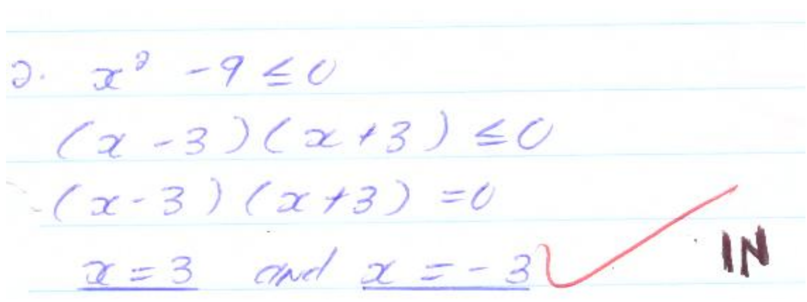


Figure 14: For DML5

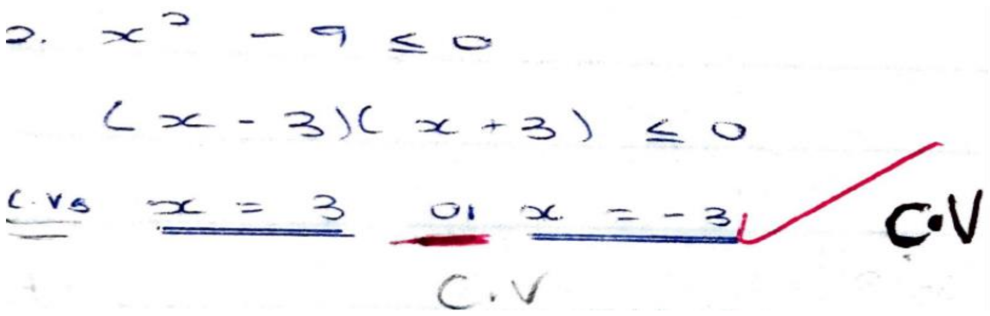


Figure 15: For DML6

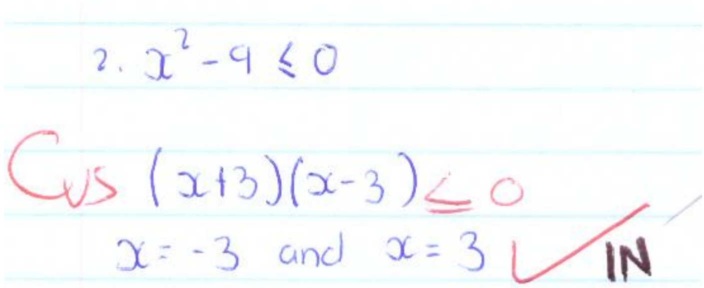


Figure 16: For DML7

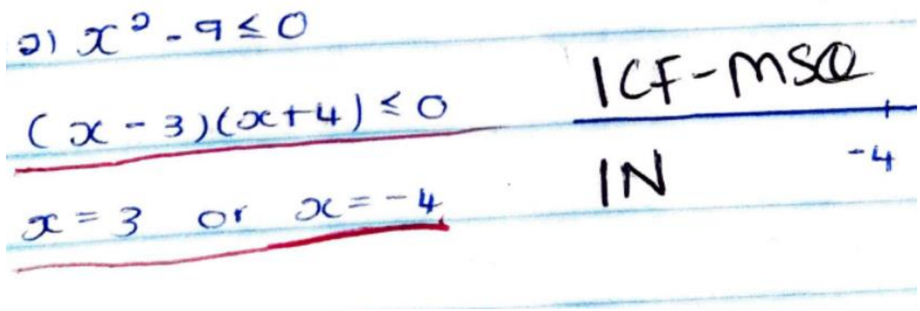


Figure 17: For DML8

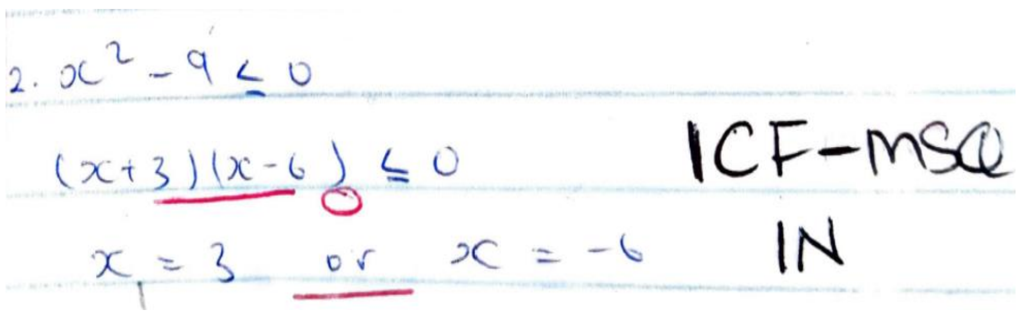


Figure 18: For DML9

The learners' responses, except that of DML6 (figure 15), show that the learners had insufficient conceptual knowledge of the inequality sign. The inequality sign was attached the same meaning as the equal sign (IN).These learners also had misconceptions regarding the solutions of quadratic inequalities. They conceptualised the solutions of quadratic inequalities as equalities. However, DML8 (figure17) and DML9 (figure18) also had an insufficient conceptual understanding of the factorisation of quadratic expressions. These two learners applied previous incorrect learning about factorisation (ICF-MQS). This led to the incorrect determination of the factors of $x^2 - 9$.

With respect to DML6's response (figure 15), the learner had an absence of the semantic meaning of critical values and logical connectors used in quadratic inequalities. In solving the given quadratic inequality DML 6 ended prematurely and the critical values of the quadratic inequality were conceptualised as the solution of the quadratic inequality (CV).

Therefore the learners simply carried out procedures associated with solving quadratic equations without connecting to the relevant concepts about quadratic inequalities. As a result, the solutions of quadratic inequalities mirrored solutions of the equivalent quadratic equations. How Mr Dlomo did evaluate these learners' responses? Shown below is a description of the learners' responses were evaluated by Mr Dlomo.

Mr Dlomo's evaluation of the learners' responses to the question; solve *the inequality* $x^2 - 9 \leq 0$

The teacher's evaluation of the learners' responses reveals that Mr Dlomo acknowledged the learners' calculation of critical values. This was done by way of ticking the final answers of the learners' responses. However, the teacher went on to underline the usage of the logical connector "or" in DML6's response. The same format of evaluation was made to the responses of other learners who approached the question in a similar way. Attention is now given to Mr Zama's learners' responses to question two.

Mr Zama's learners' responses to the question; solve *the inequality* $x^2 - 9 \leq 0$.

The vignettes below are typical responses by some learners in this class**.**

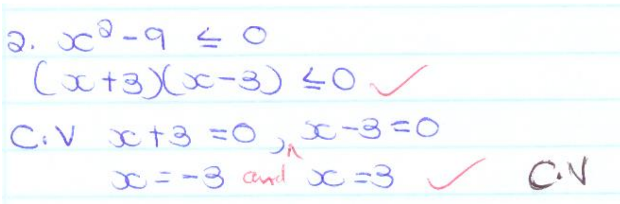


Figure 19: For ZML6

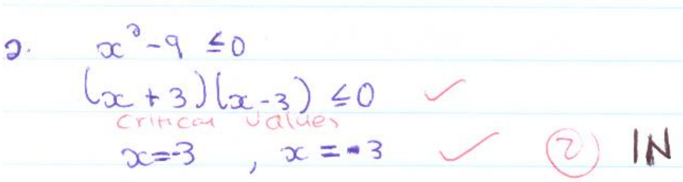


Figure 20: For ZML7

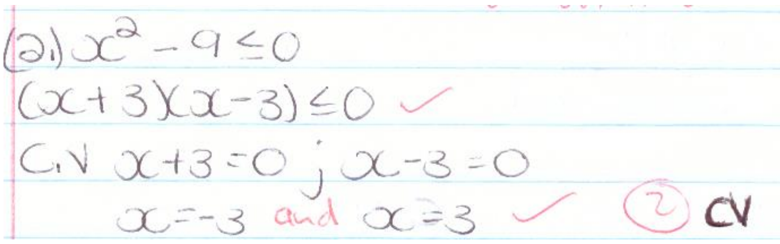


Figure 21: For ZML8

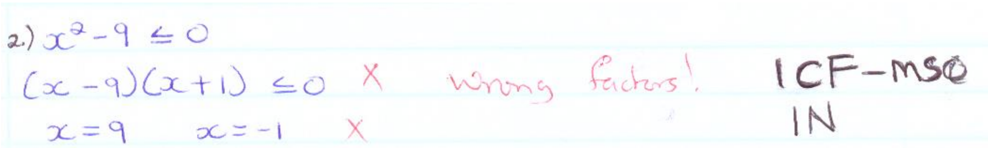


Figure 22: For ZML9

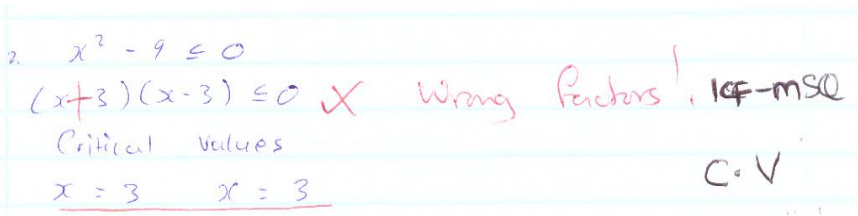


Figure 23: For ZML10

The analysis of ZML6 to ZML10's responses, with the exception of ZML7 (figure 20) and ZML9 (figure 22), showed that these learners treated critical values as solutions of quadratic inequalities (CV). The learners did not complete the procedures for solving quadratic inequalities. They were supposed to go beyond the determination of the critical values. These learners exhibited an absence of the semantic meaning of critical values and logical connectors (Almog &Illany, 2012). ZML9 and ZML10 (figure 23) applied their previous incorrectly acquired knowledge about the factorisation of quadratic expressions (ICF-MSQ) in the process of solving the quadratic inequality. Because of that the two learners incorrectly factorised the expression $x^2 - 9$. With regard to ZML 7(figure20) and ZML9(figure22),the two learners interpreted the inequality sign as having the same meaning as the equal sign(IN).Hence ,they wrote $x = -3$ and $x = 3$ and $x = 9$ and $x = -1$ respectively as the solution of the quadratic inequality. The two learners regard the solution of quadratic inequalities as equalities.

Drawing from the analysis of the learners' responses in the foregoing paragraph, it is clear that the learners had an insufficient conceptual understanding of solving quadratic inequalities. The next part of this analysis is about Mr Zama's evaluation of the learners' responses.

Mr Zama's evaluation of the learners' responses to the question; solve *the inequality* $x^2 - 9 \leq 0$

Mr Zama evaluated these learners' responses by acknowledging the correct thinking processes completed by the learners by way of ticks. These are reflected in ZML6 to ZML8's responses. However, in some cases, he indicated that learners found critical values. This evident in the comment inserted in ZML7's response. He also inserted the correct logical connector in learners' responses. This is shown, in ZML 6, and ZML8's responses. Mr Zama indicated that the learners were supposed to make use of the logical connector "and".

Mr Zama also evaluated the learners' responses by putting a cross($\times$) in front of the first step where the learner factorised the left-hand side of the given inequality. This was done in ZML9's response. A comment "wrong factors" was written in front of the cross. Further to that, a cross in front of the final answer was made. Focus is now given to the analysis of the responses to question two by learners in Mr Mthembu's class.

Mr Mthembu's learners' responses to the question; solve *the inequality* $x^2 - 9 \leq 0$

The vignettes below highlight typical responses by some learners in this class:

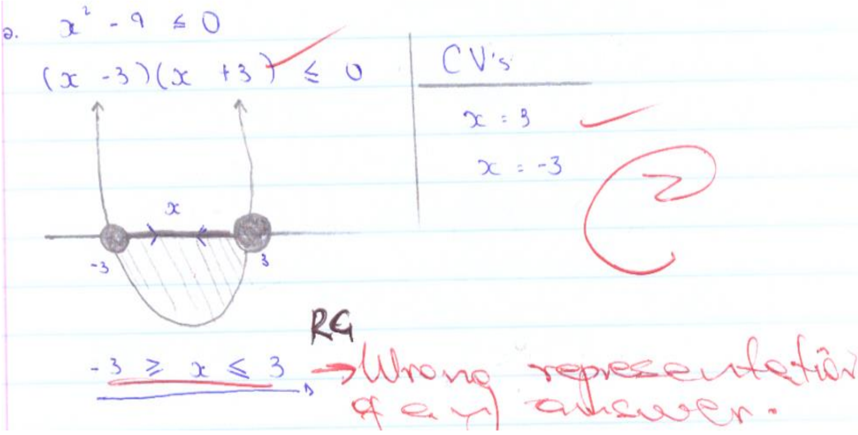


Figure 24: MTL5

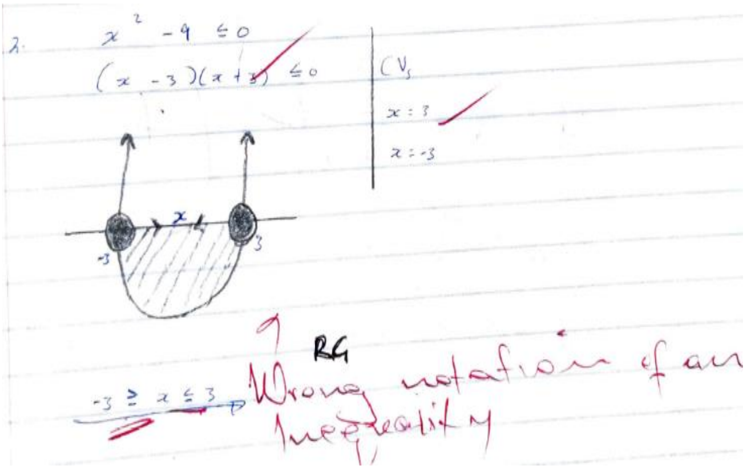


Figure 25: MTL6

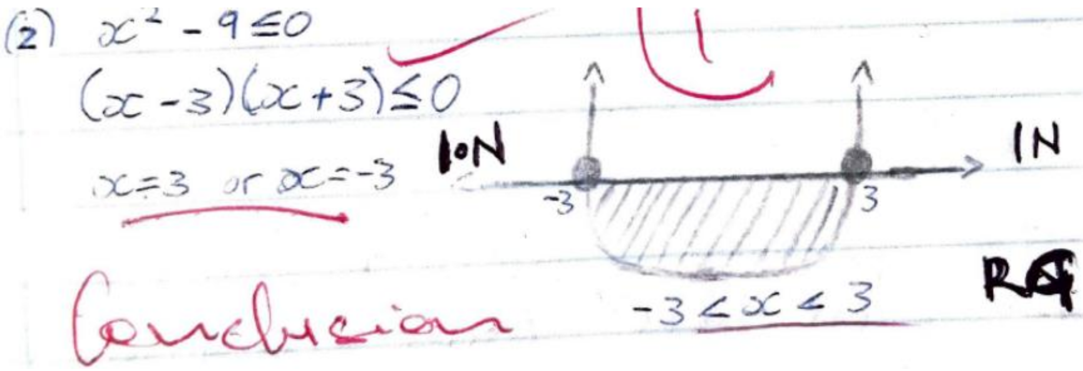


Figure 26: MTL7

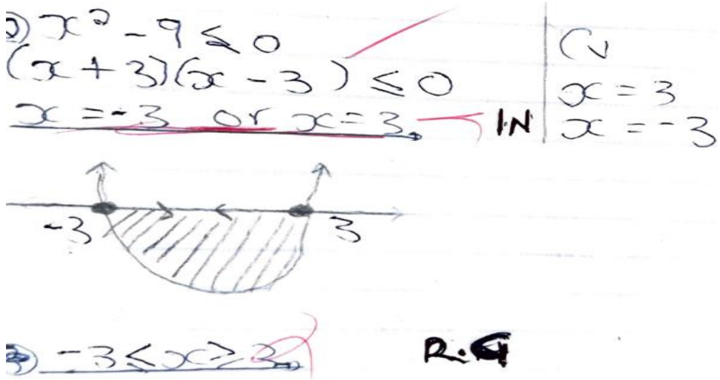


Figure 27: MTL9

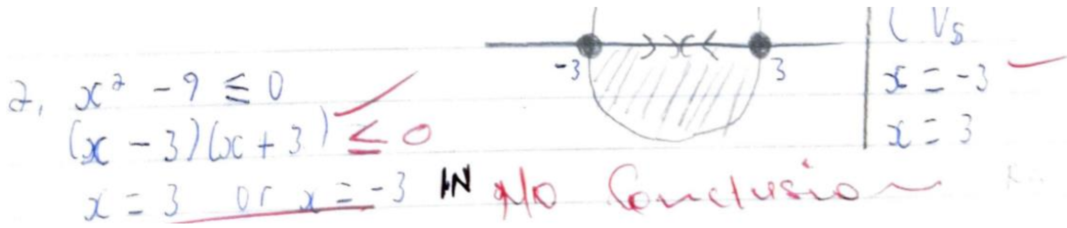


Figure 28: MTL10

An analysis of the learners' responses, from ZMTL5 (figure24) to MTL10 (figure 28), reveal that the learners were able to factorise the left-hand side of the inequality and proceeded to state the correct critical values. However, the learners showed misconceptions in the interpretation of solutions of the quadratic inequalities from the graph (RG). With respect to the use of the graphical method, the learners used the method without the conceptual connection to the inequality concepts. For example, MTL5 (figure 24), wrote:$-3 \geq x \leq 3$. The learner did not make use of the appropriate logical connector neither did the learner show a conceptual understanding of the inequality signs. This showed epistemological obstacles in the learners' interpretation of inequality signs and the reading of solutions of quadratic inequalities from a graph. Thus, the learners' visual interpretation of the solutions of the quadratic inequalities showed an insufficient conceptual connection between the graph and the inequality problem. This is evident in the solutions presented in the vignettes above. A further analysis of the learners' responses show that learner MTL7 (figure 26), MTL9 (figure 27) and MTL 10(figure 28) treated the inequality sign as a nexus with the same meaning and application as the equal sign (IN). Focus is now turned to how Mr Mthembu evaluated the learner's responses.

Mr Mthembu's evaluation of the learners' responses to the question; solve *the inequality* $x^2 - 9 \leq 0$

In terms of Mr Mthembu's evaluation of these learners' responses, the teacher acknowledged each successfully completed thinking stage attained by way of ticks. This is evident in MTL5 to MTL10's responses. Some of the final answers are underlined and some relevant comments made. For example, Mr Mthembu underlined MTL6's final answer and made the comment; "[w]rong representation of an answer". The same thing was done to MTL7 and MTL9's responses, though the nature of the comment differed to that of MTL6.For MTL6's response, Mr Mthembu commented "wrong notation of an inequality sign". For MTL7, he indicated that the learner did not conclude. However, for MTL9 and MTL10, Mr Mthembu acknowledged the correct thinking steps achieved by the learners by way of placing ticks but underlined the final incorrect solutions presented by the learners. The next part of this section focusses on the analysis of question four. Learners were required to solve the inequality $x^2 + x - 12 > 0$ .I now present the learners' responses to this question by learners in Mr Dlomo's class.

Mr Dlomo's learners' responses to the question; *solve the inequality* $x^2 + x - 12 > 0$

Below are vignettes which are representative of how some learners in Mr Dlomo's responded to this question.

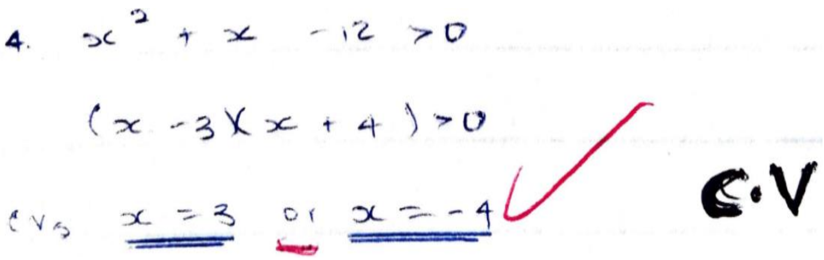


Figure 29: For DML10

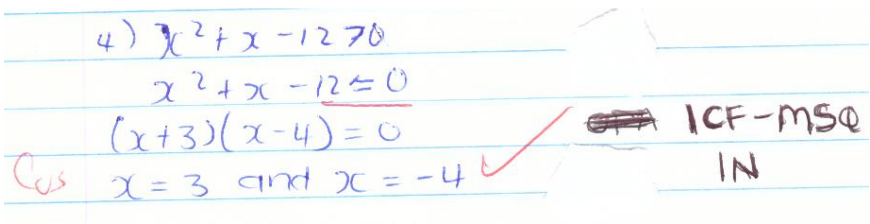


Figure 30: For DML11

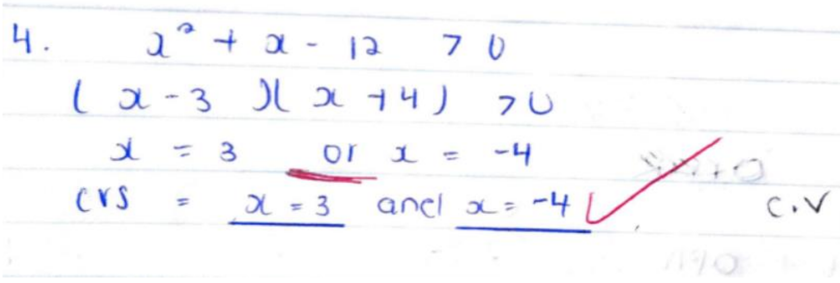


Figure 31: For DML12

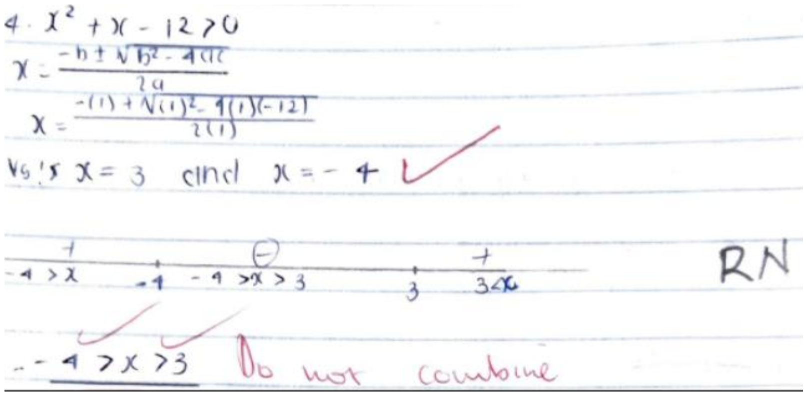


Figure 32: For DML13

An analysis of DML10 (figure 29) and DML12's (figure 31) responses show that the learners conceptualised the critical values as the solution of the quadratic inequality (CV). For these learners, solving a quadratic inequality means determining the critical values using procedures identical to solving quadratic equations. Additionally, the solutions of the quadratic inequalities are understood as equalities. The learners showed an absence of the semantic meaning of critical values as applied in inequalities. The two learners applied incomplete procedures of solving quadratic inequalities.

An analysis of DML11's (figure 30) response shows that the learner incorrectly factorised the expression $x^2 + x - 12$ (ICF-MSQ). This shows that the learner had an insufficient conceptual understanding of factorisation. The learner seems to be aware of the concept of factorisation but cannot execute the factorisation correctly. Further to this, DML11 presented the solution of the quadratic inequality as equality. The learner interpreted the inequality sign as a nexus with the same meaning and application as the equal sign (IN).

A focus on DML13's (figure32) response shows that the learner was able to correctly determine the critical values of the quadratic inequality using the quadratic formula. A correct segment of the number line was also drawn by DML13. However, the learner was unable to use the visual tool to interpret and communicate the solution of the quadratic inequality. Thus, DML13 used the number line method to solve the quadratic inequality without a conceptual connection to the inequality concepts (RN). This is evident in the solution presented by the learner. The learner presented the solution of the inequality as$-4 > x > 3$. This indicates a misconception in the learner's interpretation of the inequality sign as well as the use of logical connectors.

Mr Dlomo's evaluation of the learners' responses to the question; solve *the inequality* $x^2 + x - 12 > 0$

Mr Dlomo's evaluated these learners' scripts by way of putting ticks on the parts which were correctly done by the learner. This is evident in DML10 to DM13's responses in the vignettes presented above. Also, on each first occurrence where the learner reasoned incorrectly, that learner's incorrect step was underlined. This is reflected in DML10 to DML12's responses. For DML13, Mr Dlomo reasoned that the learner was not supposed to combine the solutions. He then underlined the final solution presented by the learner and commented that the learner was not supposed to combine the solutions. The ticks on the critical values (-4 and 3) shows the teacher's acknowledgement that though the solution is incorrect, the critical values stated by DML13 partially constitute the solution of the quadratic inequality though used incorrectly. The teacher's reasoning with respect to the ticks placed on the critical values was however not linked to the evidence on the number line presented by the learner. On the number line it is clear that the learner misinterpreted the solution of the quadratic inequality using the number line. I now present the responses to question 4 by learners in Mr Zama's class.

Mr Zama's learners' responses to the question; *solve the inequality* $x^2 + x - 12 > 0$

Turning to the responses by learners in Mr Zama's class, the excerpts below typify some of the learners' thinking.

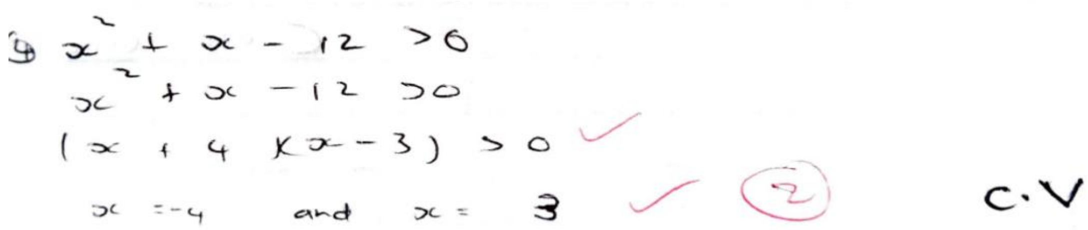


Figure 33: For ZML11

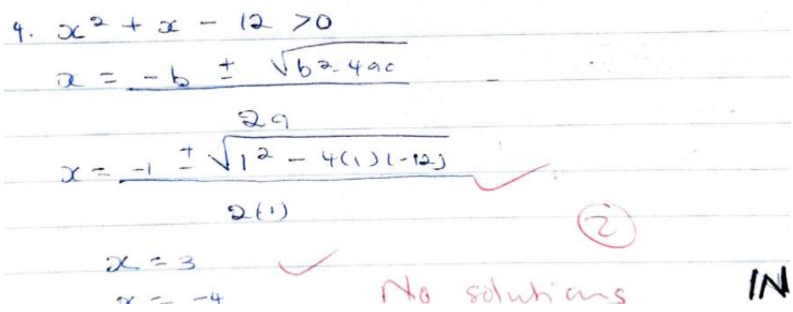


Figure 34: For ZML12

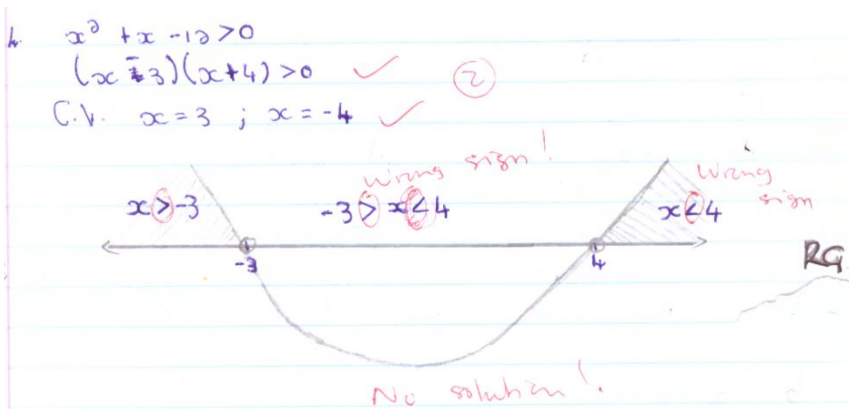


Figure 35: For ZML13

An analysis of ZML11 (figure 33) and ZML12 (figure34) shows that these learners' conceptual understanding of solutions of quadratic inequalities is flawed. Solutions of quadratic inequalities are seen as equalities. The inequality sign is regarded as having the same meaning as the equal sign. Therefore, the critical values of the inequality are considered as the solutions of the quadratic inequality (CV).

However, learner ZML13 (figure 35) displayed a different form of reasoning. The learner had misconceptions regarding interpreting the solution of quadratic inequalities from a graph (RG). ZML13 also had epistemological deficiencies in inequality signs. The learner displayed deficiencies in the semantic meaning of equal signs. Coming to how Mr Zama evaluated the learners' responses to question four, his pedagogic reasoning is explained below.

Mr Zama's evaluation of the learners' responses to the question; solve *the inequality* $x^2 + x - 12 > 0$

Mr Zama evaluated these learners' responses by putting ticks on statements that were successfully and correctly completed by the learners. This was done in ZML11 to ZM13's responses. Mr Zama went on to circle the parts which were incorrectly done by the learner. This is evident in ZML13's response. A comment about the incorrect application of the inequality signs was also written in ZML13's response. Lastly, Mr Zama commented that the learners did not find the solution of the quadratic inequality. This comment is reflected in ZML12 and ZML13's responses. I now focus on the presentation and analysis of the learners' responses to question 4 by learners in Mr Mthembu's class.

Mr Mthembu's learners' responses to the question; solve *the inequality* $x^2 + x - 12 > 0$

The following paragraph discusses the analysis of question four with respect to responses by learners in Mr Mthembu's class. To facilitate that analysis the vignettes reflecting typical misconceptions by learners in this class are presented below.

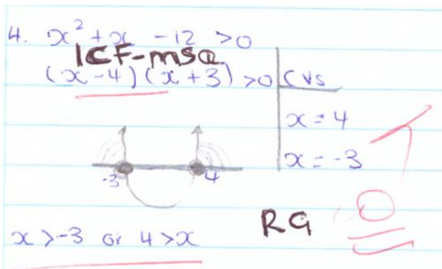


Figure 36: For MTL12

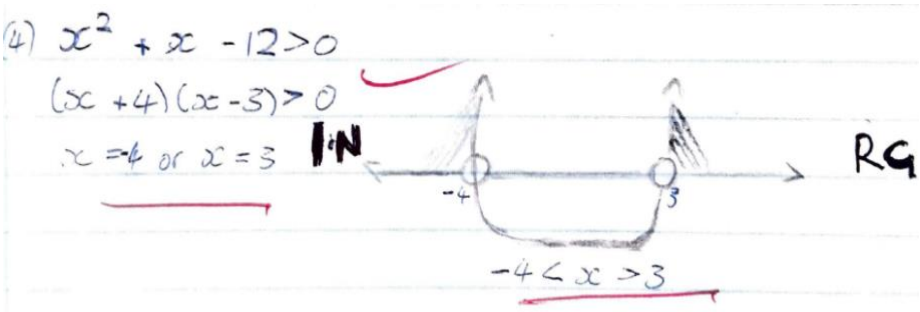


Figure 37: For MTL13

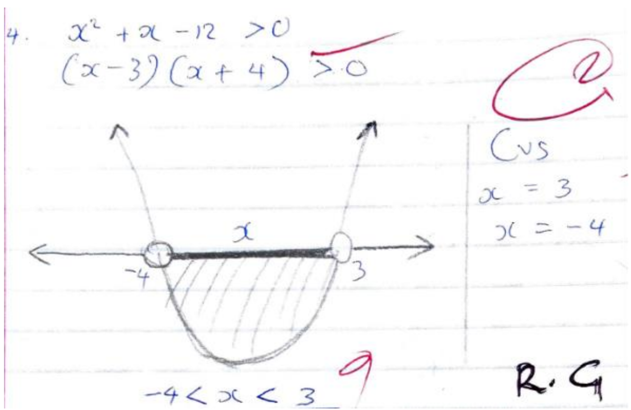


Figure 38: For MTL14

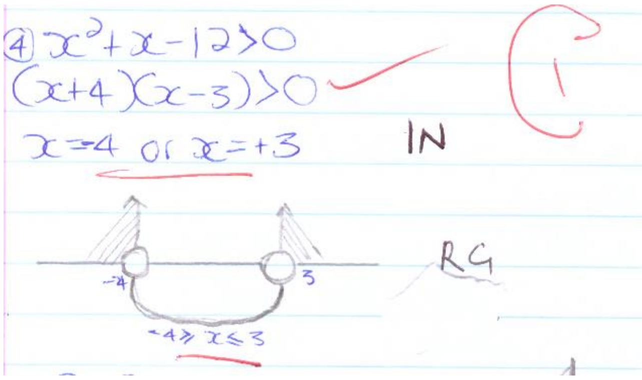


Figure 39: For MTL15

An analysis of MTL12’s (figure 36) response shows that this learner could not factorise the left-hand side of the inequality (ICF-MSQ). Though the learner was aware of the need to factorise, the learner could not correctly execute the process. Thus, the leaner had pseudo knowledge about factorisation. Apart from the incorrect factorisation, an analysis of the graph drawn by MTL12 (using the incorrect critical values from the incorrect factorisation done), MTL13 (figure 37), and MTL15 (figure 39) show that these learners correctly identified by way of shading on the graph the correct regions described by the inequality. However, the three learners were unable to use their graphs to interpret and communicate the solution set of the inequality. An analysis of what the three learners wrote show obstacles in the interpretation of the solution of the inequality from the graph using the inequality notation. Thus, the learners used the graphical method without a conceptual connection to the inequality concepts (RG).For example MTL12 wrote $x > -3$ or $4 > x$ as solution of the inequality read from the graph drawn. This shows the learners’ inability to use graphs to interpret and communicate solutions of quadratic inequalities.

Turning to MTL14 (figure 38), the learner shaded the unwanted region and correctly presented it in inequality form. Hence, the four learners’ responses to the given inequality show that they applied procedures for solving quadratic inequalities using the graphical method without a conceptual connection to the inequality concepts (RG). Apart from this, MTL13 and MTL15 also used the inequality sign as a nexus with the same meaning and application as the equal sign (IN).Attention is now given to how Mr Mthembu evaluated the learners’ responses to question 4.

Mr Mthembu’s evaluation of the learners’ responses to the question; solve *the inequality* $x^2 + x - 12 > 0$

Focussing on how Mr Mthembu evaluated the learners' responses, firstly attention is drawn to MTL12's responses. It can be seen that Mr Mthembu underlined the incorrect factors found by the learner. A cross was also put on the incorrect critical values found by the learner. Mr Mthembu proceeded to underline the final answer found by the learner and a zero (0) was put at the end of his evaluation of this learner's response. Considering Mr Mthembu's pedagogic reasoning in his evaluation of MTL13 (figure37) to MTL15 (figure 39), the teacher reinforced the correct thinking processes successfully completed by the learners. This was done by way of putting ticks on each successfully completed step towards the solving of the quadratic inequality. Nevertheless, the final answers were underlined to show that the answers arrived at were incorrect.

In his evaluation of these learners' responses Mr Mthembu reinforced the correct thinking processes successfully completed by the learners. This was done by way of putting ticks on each successfully completed stage. This is evident in MTL12 to MTL15's responses. Nevertheless, these learners' final answers were underlined to show that the final answers arrived were incorrect.

Attention is now given to the analysis of question five. Learners were required to solve the inequality; $2x^2 - 7x - 4 \geq 0$ .I now presents some of the learners' responses and the analysis of the question. Mr Dlomo's learners' response to this question is presented first.

Mr Dlomo's learners' responses to the question; solve *the inequality* $2x^2 + 7x - 4 \geq 0$

The vignettes below are typical responses from some learners in Mr Dlomo's class:

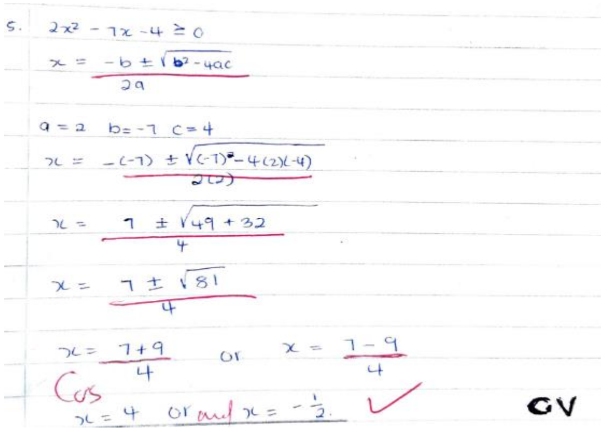


Figure 40: For DML14

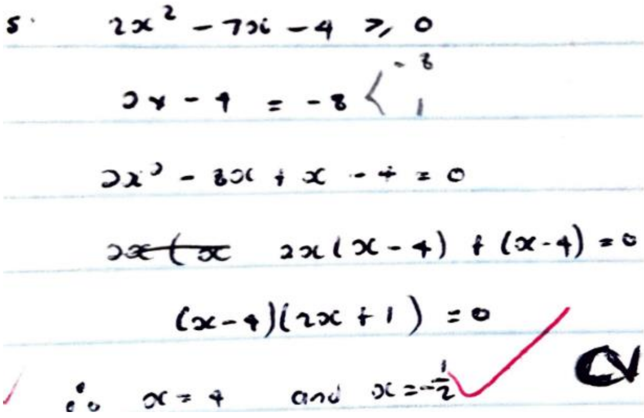


Figure 41: For DML15

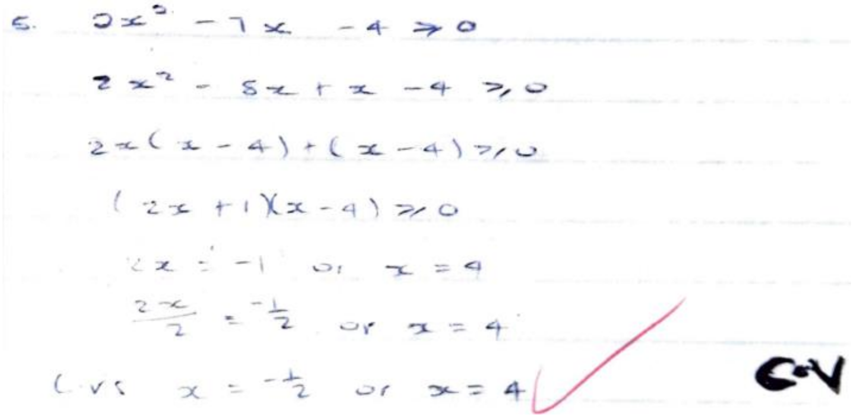


Figure 42: For DML16

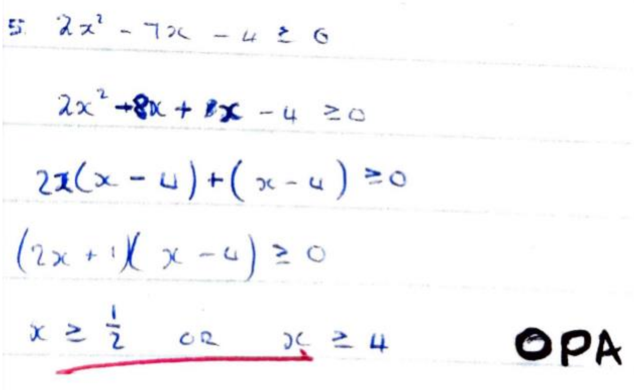


Figure 43: For DML17

An analysis of the data responses of DML14 (figure40) to DML16 (figure42), show that the learners treated the inequality sign as a nexus with the same meaning as the equal sign. The learners applied methods of solving quadratic equations. These learners presented equalities as solutions of the quadratic inequalities (CV). Reference is made to how DML16 (figure 43) responded to the question given. The learner overgeneralised the correct knowledge of the procedures of solving quadratic equations to quadratic inequalities (OPA). Focus is now turned to how the teacher evaluated the learners' responses.

Mr Dlomo's evaluation of the learners' responses to the question; solve *the inequality* $2x^2 + 7x - 4 \geq 0$

In terms of the teacher's evaluation of these learners' responses, the teacher acknowledged each stage towards solving the quadratic inequality that was successfully completed by the learners. This is evident in DML14 (figure 40) to DML16's (figure 42) responses. For these learners, the teacher rewarded the learners by way of ticking the correct thinking attained by the learners in terms of the correct determination of critical values. For DML14's (figure 40) response, the teacher detected some omissions in the learner's response and included the concepts which the learner should have used. Thus, Mr Dlomo judged that what the learner found were critical values and included the appropriate logical connector which the learner was supposed to have used. However, for DML17 (figure 43), Mr Dlomo underlined the learner's final answer; an indication of disapproving the learner's solution to the question. I now make the presentation and analysis of Mr Zama's learners' response to the question; solve the quadratic inequality $2x^2 + 7x - 4 \geq 0$.

Mr Zama's learners' responses and analysis to the question; solve the quadratic inequality $2x^2 + 7x - 4 \geq 0$

With respect to the analysis of question five, the following excerpts display typical incorrect responses in Mr Zama's class as well as how these responses were evaluated by the teacher.

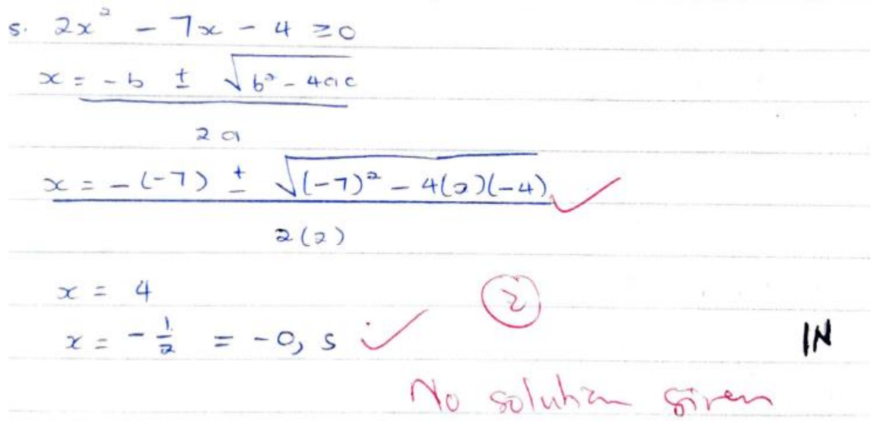


Figure 44: For ZML14

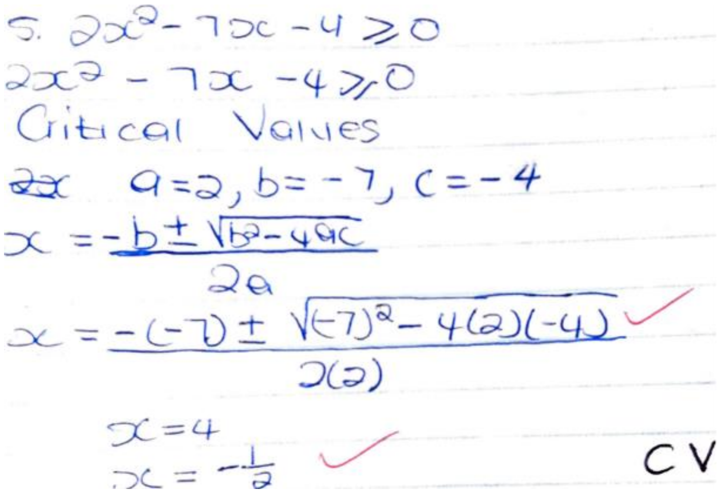


Figure 45: For ZML 15

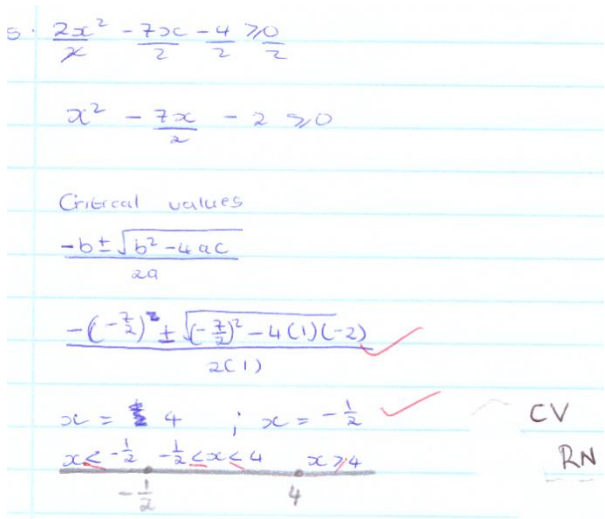


Figure 46: For ZML16

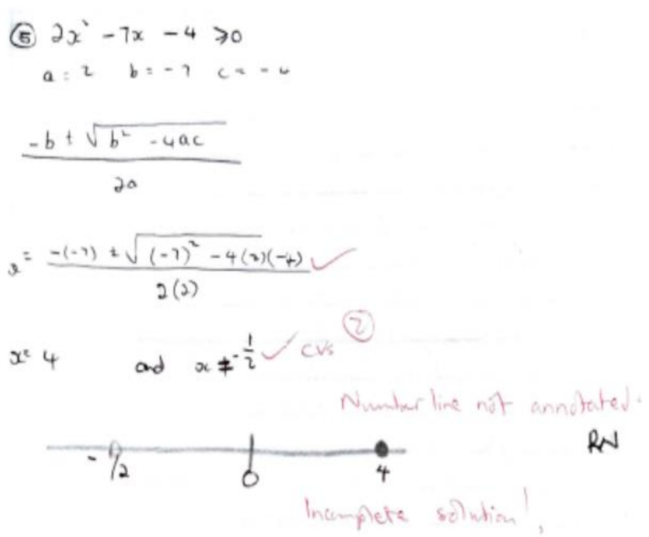


Figure 47: For ZML17

For ZML14 (figure 44), the learner treated the inequality sign as a nexus with the same meaning as the equal sign (IN). Hence, the learner applied procedures for solving quadratic inequalities without conceptual understanding. This led to the critical values to be taken as the solutions of the quadratic inequality.

In terms of ZML15 (figure 45), the learner understood the critical values to be the solution of the quadratic inequality (CV). However, ZML16 (figure 46), used the number line without conceptual connection to the inequality concepts (RN). For ZML 17(figure 47), the learner could not present the solution of the inequality on the sketch of the number line drawn. Hence, the two learners, ZML16 and ZML17, could not interpret the solution of the quadratic inequality from the number lines sketched. Attention is now focussed on Mr Zama's evaluation of these learners' scripts.

Mr Zama's evaluation of the learners' responses to the question; solve *the inequality* $2x^2 + 7x - 4 \geq 0$.

The teacher evaluated these learners' responses by reinforcing the stages which were successfully completed. This is evident in ZML14 (figure 44) to ZML17 (figure 47)'s responses. The teacher rewarded the learners' correct thinking processes by way of ticking each successfully completed step towards solving the quadratic inequality. For ZML14's response (figure44) the teacher reasoned that the learner did not find the solution of the quadratic inequality. For learner ZML16 (figure 46), Mr Zama detected the absence of the semantic meaning of the inequality signs. The teacher had to indicate, on the learner's response the correct usage of the inequality signs in the context of the question. This was done by inserting the correct representation of the inequality signs on the number line drawn by ZML16. Regarding how learners in Mr Mthembu's class responded to question five, a presentation and analysis of the learners' responses is carried out below.

Mr Mthembu's learners' responses and analysis to the question; solve the quadratic inequality $2x^2 + 7x - 4 \geq 0$

The following vignettes are typical learners' responses to the question; solve the inequality $2x^2 + 7x - 4 \geq 0$

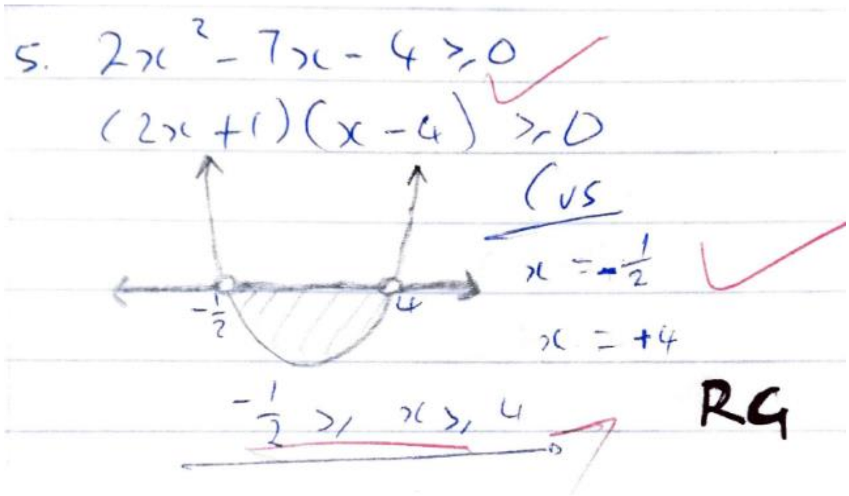


Figure 48: For MTL16

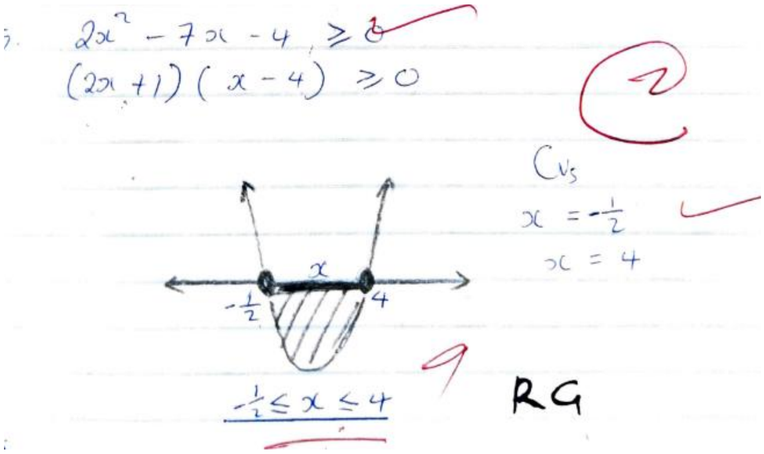


Figure 49: For MTL17

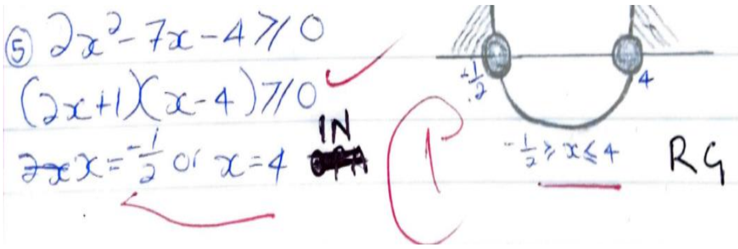


Figure 50: For MTL18

From the above vignettes, it is evident that the learners displayed some underlying misconceptions pertaining to the interpretation of inequality signs in relation to the graphs drawn (RG).The learners were aware of the procedures of solving the quadratic inequality using the graphical method .However the graphs were drawn without a conceptual connection to the inequality concepts. This led to the learners' inability to interpret and communicate their solutions correctly thus signifying a vacuum in knowledge between the learners' conceptualisation of the graphs drawn and the inequality concepts. An analysis of MTL16's response (figure48) shows that the learner used the graph without a conceptual connection to the inequality signs (RG). In the context of the given question the learner presented $-\frac{1}{2} \geq x \geq 4$ as the solution of the inequality. This indicates that the learner had some epistemological obstacles in reading, interpreting and communicating the solution of the inequality from the graph drawn. For MTL17 (figure 49), the learner could not differentiate a greater than or equal to sign from a less than or equal to sign in relation to the graph drawn. Just like MTL16, this learner used the graph without a conceptual connection to the inequality signs (RG). On the other hand, MTL18 (figure 50), could not differentiate an inequality sign from an equality sign (IN). For MTL18, the inequality sign and equality sign had same meaning conceptual meaning. Hence this learner applied procedures for solving quadratic equations and got $x = -\frac{1}{2}$ or $x = 4$.Additionally, the learner could not connect the sketch of the graph correctly drawn to the appropriate inequality concepts (RG).MTL18 was unable to use the graph to interpret and communicate, in inequality notation, the solution of the quadratic inequality. The next part of this presentation is about an analysis Mr Mthembu's evaluation of these learners' responses.

Mr Mthembu's evaluation of the learners' responses to the question; solve *the inequality* $2x^2 + 7x - 4 \geq 0$.

An analysis of Mr Mthembu's evaluation of these learners' responses shows that he reinforced the correct thinking processes completed by the learners. This was achieved by placing ticks and scoring stages successfully attained by the learner. This is reflected in MTL16 (figure 48) to MTL18's (figure 50) responses. However, the learners' incorrect thinking in terms of the presentation of ideas of solving the given inequality was underlined. For learner ML16 and ML17; a cross was put at the end of their final answers.

Having discussed the analysis of the learners' responses to question five as well as how the teachers evaluated the learners' errors and misconceptions, the next part of this section focusses on the analysis of question six. In this question, learners were required to solve the inequality $(y - 3)(y + 4) < 0$. In line with the order of presentation and analysis of the previous questions, Mr Dlomo's learners' responses and analysis to question six is presented first.

Mr Dlomo's learners' responses and analysis to the question; solve the quadratic inequality$(y - 3)(y + 4) < 0$

The following vignettes are typical responses and analysis of the nature of the misconceptions displayed by some learners in Mr Dlomo's class.

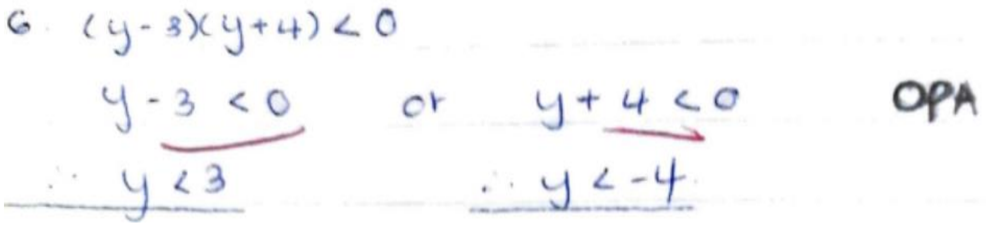


Figure 51: For DML 18

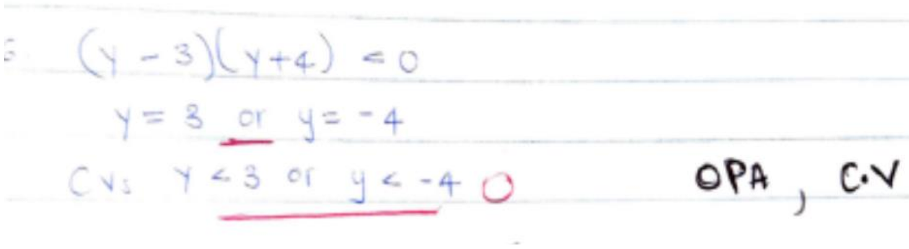


Figure 52: For DML 19

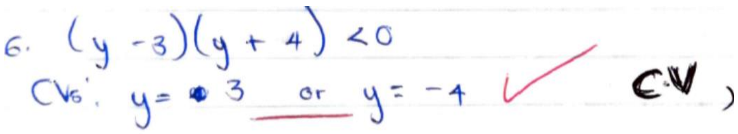


Figure 53: For DML 20

In terms of the analysis of DML18 (figure51) and DML19's (figure52) responses, the learners overgeneralised the procedures of solving quadratic equations to quadratic inequalities (OPA) On the other hand, DML20's (figure 53) response to the question shows an underlying misconception regarding the solutions of quadratic inequalities. For this learner, the critical values were considered to be the solutions of the quadratic inequality (CV). The learner also displayed an absence of the semantic meaning of the logical connectors used in inequalities (ULC). This line of thinking was also evident in DML19's reasoning. Apart from using procedures for solving quadratic equations, DML19 also thought of critical values as solutions of quadratic inequalities. Additionally, the critical values were presented in the form of inequalities. Attention is now focussed on Mr Dlomo's pedagogic reasoning in the evaluation of the learners' responses to question six.

Mr Dlomo's pedagogic reasoning in the evaluation of the learners' responses to the question; solve *the inequality* $(y-3)(y+4) < 0$

Mr Dlomo under lined the faulty stages in the learners' responses to this question. This is shown in DML18 (figure 51) to DML20's (figure53) responses. However, for DML20's response (figure 53), Mr Dlomo acknowledged the determination of the correct critical values; $y = 3$ and $y = -4$ .This was done by way of placing a tick at the end of the learner's presentation of the solution to the given question. Nevertheless, Mr Dlomo underlined the learner's incorrect use of the logical connector "or". Reference is now made to how learners in Mr Zama's class responded to question five, a presentation and analysis of the learners' responses to this question was done below.

Mr Zama's learners' responses to the question; solve *the inequality* $(y-3)(y+4) < 0$

The vignettes below typify the responses by learners in Mr Zama class.

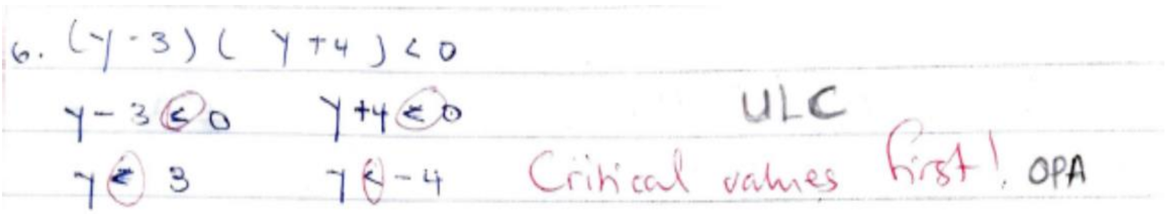


Figure 54: For ZML 18

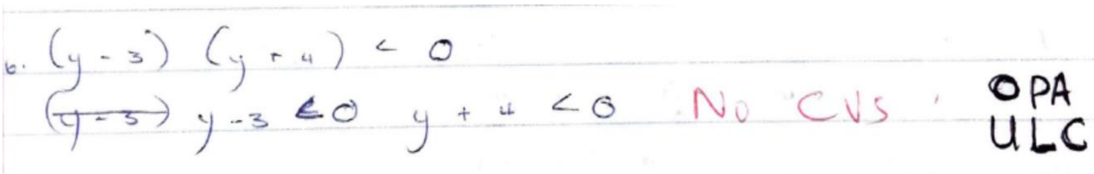


Figure 55: For ZML 19

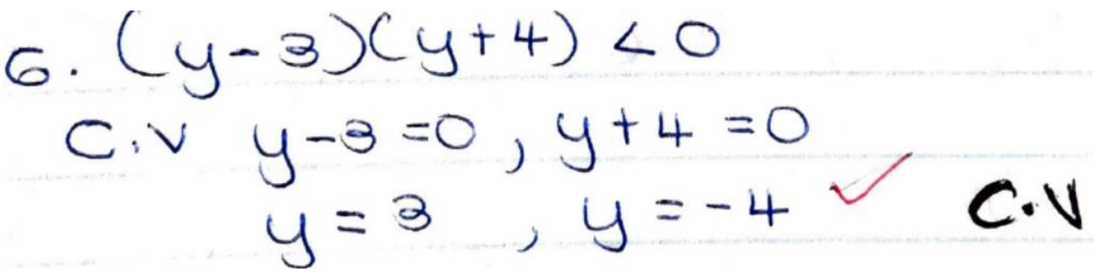


Figure56: For ZML 20

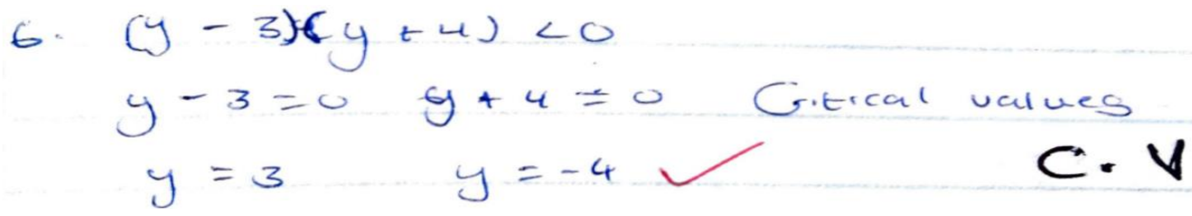


Figure 57: For ZML 21

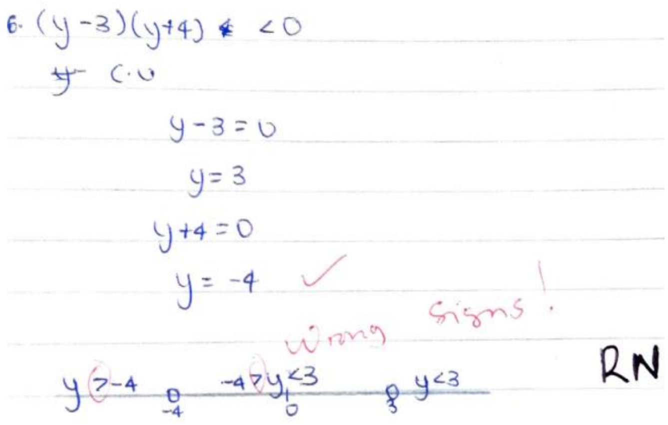


Figure 58: For ZML 22

An analysis of the learners' responses shows that the learners had misconceptions on solving quadratic inequalities. Learner ZML18 (figure 54) and ZML19 (figure 55), overgeneralised procedures for solving quadratic equations to quadratic inequalities (OPA). Hence, they solved the quadratic inequality using the same procedures as are used when solving quadratic equations.

Regarding ZML20 (figure56) and ZML21 (figure 57); these learners viewed the critical values as solutions of the quadratic inequality (CV). Thus, the learners' solutions to the quadratic inequality were presented as equalities. Mr Zama evaluated these learners' responses by putting ticks for the correct determination of critical values. Attention is now given to ZML22's response (figure 58). This learner had misconceptions with respect to the interpretation of the solutions of a quadratic inequality from the number line (RN). Having presented the analysis of the learners' responses to question six, a presentation of Mr Zama's pedagogic reasoning concerning his evaluation of the learners' responses to the same question is presented below.

Mr Zama's pedagogic reasoning in the evaluation of the learners' responses to the question; solve *the inequality* $(y-3)(y+4) < 0$

In evaluating these learners' response, Mr Zama circled the inequality signs for ZML18 (figure 54) and indicated to this learner the need to determine the critical values first. For ZML19 (figure 55), Mr Zama commented that the learner omitted the determination of the critical values. For consistency purposes, perhaps Mr Zama should also have indicated to ZML19 the need to determine the critical values first and possibly circled the incorrect usage of the inequality signs. For ZML20 (figure 56) and ZML21 (figure 57), Mr Zama indicated that the learners determined the critical values. His evaluation of ZML22's response (figure 58) shows that he circled the learner's usage of the inequality signs and commented that the learner used wrong signs. However, what is common in Mr Zama's pedagogic reasoning regarding his evaluation of ZML20 to ZML22 is that he reinforced the learners' determination of the critical values by way of ticks. The next part of this section focusses on typical responses to question six by learners in Mr Mthembu's class.

Mr Mthembu's learners' responses and analysis to the question; solve *the inequality* $(y-3)(y+4)<0$

The vignettes below show typical responses of the above question by learners in Mr Mthembu's class.

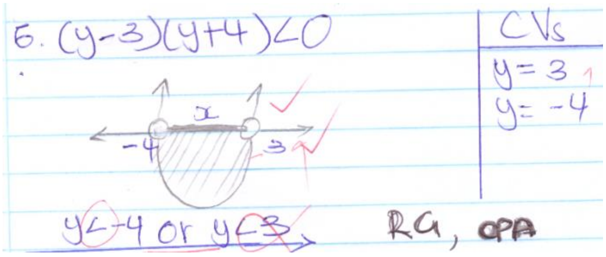


Figure 59: For MTL19

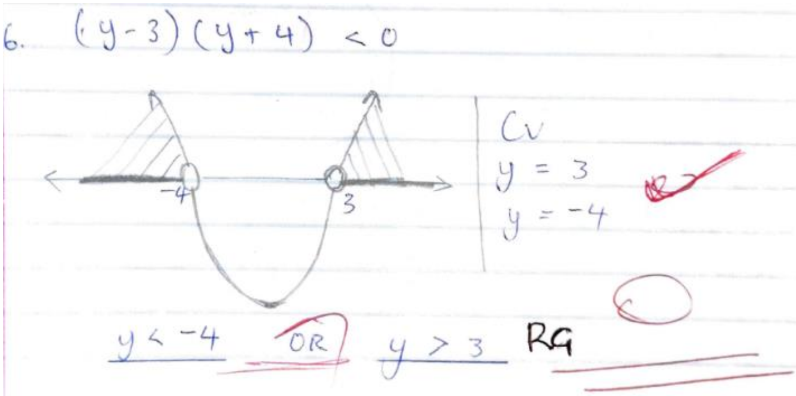


Figure 60: For MTL 20

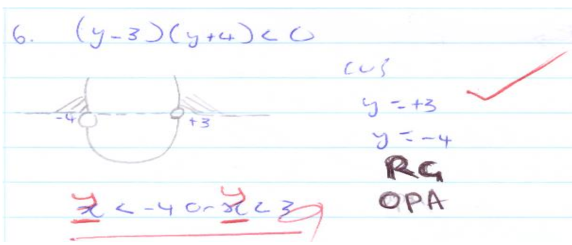


Figure 61: For MTL21

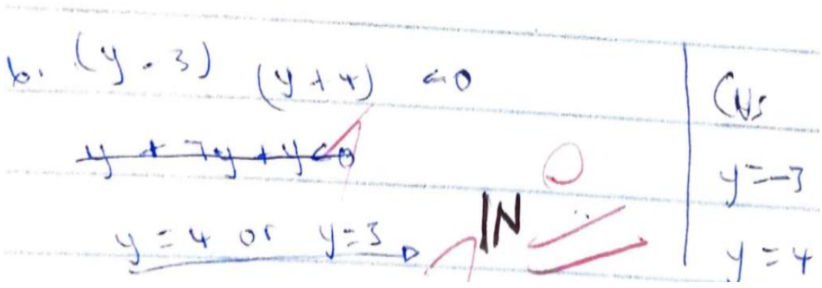


Figure 62: For MTL22

The responses to this question by learners in Mr Mthembu's class showed that the learners had misconceptions regarding interpreting solutions of quadratic inequalities from the graph. This is evident in learner MTL19 (figure 59) to learner MTL22's (figure 62) responses. For MTL19 (figure 59) and MTL21 (figure 61), it is evident that they overgeneralised procedures for solving quadratic equations graphically to quadratic inequalities (OPA).Apart from that MTL19 ,unlike MTL21, correctly identified by way of shading on the graph the correct region described by the quadratic inequality. However, the learner was unable to use the graph to interpret and communicate the solution of the quadratic inequality. For MTL20 (figure 60) and MTL21 (figure 61), the two learners used the sketch of the quadratic inequality without a conceptual connection to the inequality concepts. The interpretation of the inequality sign in conjunction with the quadratic graph drawn was an epistemological hindrance to MTL20 and MTL21. Learner MTL22 (figure 62) displayed a different form of misconception. This learner conceptualised the inequality sign in the same way as an equal sign (IN). Because of that, the solution of the quadratic inequality was presented as $y = 4\ or\ y = 3$.

Mr Mthembu evaluated the learners' responses by way of reinforcing the correct thinking processes attained by the learners. This is reflected in MTL19 to MTL21's responses. Mr Mtembu ticked the stages in the learners' responses which were successfully and correctly completed. However, the final answers of the learners' responses were underlined. For MTL19 (figure 59), a cross was included at the end of the final answer. A slightly different form of evaluation was done in learner MTL20's response (figure20), a question mark was put on the final answer and the learner was scored a zero. MTL21's final answer was also corrected including the inclusion of the variable y. The inclusion of the variable y was misinterpreted by this learner. The learner presented the solution in terms of x instead of y. This was possibly a lack of closure on the learner's thinking concerning the use of the variable x.

For MTL22 (figure 62), Mr Mthembu's evaluation of the learner's response shows that he did not approve any of the steps made by the learner. He awarded the learner a score of zero. On reflecting on this learner's response, Mr Mthembu could have credited the learner for determining the correct critical values of the quadratic inequality. The next section presents a summary of the misconceptions identified from the learners' responses and how they were evaluated by the three teachers.

4.6.4 Summary of the learner errors and misconceptions and how they were evaluated by the three teachers

From the analysis of the learners' responses, a number of misconceptions were identified. The following are the misconceptions which were identified. Learners displayed misconceptions involving the overgeneralisation of the product rule for solving quadratic equations to quadratic inequalities (Kroll, 1987; Almog &Ilany, 2012; Bicer et al., 2014). In most cases, the overgeneralisation was based on previous correct knowledge about solving quadratic equations. Learners also showed misconceptions regarding the overgeneralisation of the multiplicative inverse rule applied in solving linear equations (Kroll, 1987; Almog & Ilany, 2012). They could not reverse the inequality sign when dividing both sides of the inequality by a negative number. In addition, the inequality signs were misinterpreted and given the same meaning as the equal sign. Misconceptions also occurred with regard to interpreting solutions from sketches of the number line and graphs drawn.

Furthermore, there were misconceptions associated with the misapplication of methods of solving quadratic equations to determine the critical values. For instance, some learners were aware of the concept of factorisation but could not execute the factorisation process correctly in order to determine the critical values. Lastly some learners considered the critical values as solutions of quadratic inequalities. The next paragraph gives a summary of how the three teachers evaluated the learners' responses. The three teachers evaluated the learners' responses in ways that were identical in many respects. Common to their evaluation of the learners' responses was the use of ticks to reinforce the correct thinking processes attained by the learners. Underlining and circling of the stages where faulty lines of reasoning were identified in the learners' responses was also characteristic of the three teachers' evaluation of the learners' work. However, unlike Mr Dlomo, Mr Zama and Mr Mthembu made use of a cross (×) in some sections of their evaluation of the learners' responses as a way of indicating some faulty lines of reasoning in the learners' thinking. In addition, written comments were also made based on some responses given by the learners.

However, the types and nature of comments were not the same. The preceding section was centred on the analysis of the learners' responses as well as how these were evaluated by the three key participating teachers in the study. The next section is about an analysis of the observation data based on the teachers' remediation of the errors and misconceptions committed by learners in quadratic inequalities.

4.7 Analysis of the observation data

Firstly, the audio recording of the lesson observation data from the three teachers participating in this study was transcribed. The transcribed data was then read and re-read in conjunction with the field notes that were prepared. The data was then coded deductively and inductively. In coding the data, a coding frame with two categories deductively formulated and three categories that were inductively formulated was constructed. The deductive categories were adapted from Shulman (1987)'s model of pedagogic reasoning and action. The researcher formulated his own codes for coding the data. The reason for doing that was the absence of such coding from related literature. The table 5 below gives the categories that were formulated from the coding process conducted.

Category	Description of the category	Example	Code
Teacher strategy	The practical action exhibited by the teacher in class when addressing learner errors and misconceptions	Teacher demonstration, examples or illustrations used to address learner errors and misconceptions	TS
Teacher method	The way used by the teacher to execute the teaching strategy in the process of addressing the learner errors and misconceptions	Lecture method, question and answer method and discussion method, to name just a few	TM
Explanation and descriptions	The teacher's verbal talk when addressing the learner errors and misconceptions	Clear and concise use of mathematical language and non-mathematical language in explaining and describing mathematical processes	ED
Questions and probes	The nature of the questions and follow up questions that the teacher uses in addressing learner errors and misconceptions identified	Questions asked and the type of follow up questions made by the teacher.	QP
Teacher interaction	The nature of the teacher -learner interaction	Teacher lecturing, learner-initiated discussions or teacher acting as a guider facilitating the development of learner ideas accompanied by appropriate comments.	TI

Table 5: Frame for the analysis of the observation data

The above coding frame was used in an approach similar to the one highlighted in table 3(cf. section 4.5).Hence after reading and re-reading the transcribed data of the audio recording of the lesson observation data , codes were assigned to each teacher's lesson observation data. A label was assigned to a data corpus relating directly or indirectly to a particular aspect concerning the teacher's PRA when remediating learner errors and misconceptions in quadratic inequalities. For instance, the code TS was assigned to the practical action shown by the teacher connected to addressing learner errors and misconceptions in quadratic inequalities. The same approach was employed with respect to the application of the other codes stated in table 5.Finally; the codes were sorted forming five categories which were then synthesised into a single theme. Thus, the theme, the teacher's remedial action when addressing learner errors and misconceptions was formulated. The following is a discussion of the theme with respect to the observation data.

4.7.1 The teachers' remedial actions when addressing learner errors and misconceptions

Outlined below is the analysis of each of the participant's pedagogic reasoning regarding the remedial actions undertaken with respect to the way they remediated the learner errors and misconceptions. Mr Dlomo's observational data was analysed first followed by Mr Zama's and Mr Mthembu's data was analysed last. A summary of the teachers' remedial actions with respect to the similarities and differences in their pedagogic reasoning and actions in the context of the topic under study was carried out at the end of this section. Attention is now given to the analysis of Mr Dlomo's observational data.

Observation of Mr Dlomo's remedial lesson

Mr Dlomo began his remedial lesson by re-visiting the concept of an inequality and discussing the various inequality symbols. He went on to discuss the different types of inequalities. Mr Dlomo then addressed the representation of inequalities on a number line using examples. An analysis of Mr Dlomo pedagogic reasoning and action at this stage shows that his approach towards the remedial lesson was based on re-teaching the topic. His focus was on assisting learners to re-construct their knowledge about inequality concepts. At this stage reference was not made to the learner errors and misconceptions identified in the task they wrote. The following excerpt of the dialogue between Mr Dlomo and learners in his class support the foregoing statement.

Mr Dlomo: What is an inequality?

DML1: A statement which shows that two or more statements are not equal.

Mr Dlomo: Anyone with a different definition of an inequality.

DML2: A statement where one statement has more value or less value than the other.

Mr Dlomo: Wow, wow, come again with that statement. I like that. I never thought of it that way. What did you say? Come again.

DML3: Where a statement has more value or less value than the other.

From the above dialogue, it is evident that Mr Dlomo made no attempt to correct his learners' responses; rather he kept on asking for the different ways of defining an inequality. This prompted other learners to position themselves in the ensuing discussion. In pursuance of the learners' re-construction of their knowledge about the solving of quadratic inequalities, through re-teaching, Mr Dlomo went on to introduce the graphical method of solving quadratic inequalities. He began this part by presenting a quadratic function on the board. Thus, the statement $f(x) = x^2 -3x-4$ was presented. With respect to this function the following dialogue occurred.

Mr Dlomo: Suppose you are given this statement. It reads $f(x) = x^2 -3x-4$. It's a function. We want to sketch, the graph of the function. Then use the graph to determine the values of x for which $f(x) > 0$. Going back to the function, what kind of an expression is this? [Teacher was pointing to the right-hand side of the function $f(x) = x^2 - 3x - 4$].

Chorus response : Quadratic expression

Mr Dlomo: What I want you to do is to sketch the graph of $f(x) = x^2 -3x-4$. As you sketch don't forget to factorise first, get your x-intercept and determine the turning point of your graph. When using the graphical approach to solve quadratic inequalities, by finding your x-intercepts, at the back of your mind, you are finding the critical values of the inequality in question.

After guiding learners to sketch the graph of $f(x) = x^2 -3x-4$, Mr Dlomo proceeded to demonstrate how the sketch could be used to solve the inequality $x^2 -3x-4 >0$

> You place your ruler on the x-axis like this. What happens is that the ruler has cut my curve into two regions, it's like now this is a river. Those who are this side of the river and those that are that side of the river, those that are below it or those are above it. But according to the question, which one do you want? Do you want those who are below according to your inequality or you want those that are above?

From the practical demonstrations and subsequent class discussion Mr Dlomo's learners were able to correctly identify the portion of the graph which satisfied the given quadratic inequality. Emphasis was placed on the x coordinates of the x-intercepts. Mr Dlomo gave the following explanation.

> You can't get your answers without knowing the x-intercepts of your graph. Thus, you need to find the x-intercepts. At the back of your mind, you must know that the x coordinates of your x-intercepts are your critical values.

The class was then directed to find the critical values of the inequality and to present the solution of the inequality. In doing so the following dialogue took place between the teacher and the class. The names of the learners are not their real names.

Mr Dlomo: Who can come and combine the solutions for us if ever they combine?

DML6: Can they be combined? They can't be combined. They can't be combined

Mr Dlomo: Why?

There was a chorus response by the class in which it was mentioned that the solutions of the inequality could not be combined because the region required was above the x-axis. The teacher then concluded by giving a summary of the procedures for solving quadratic inequalities using the graphical approach. After that the class was given the following question to solve using the graphical approach. Solve $x^2 - 4x \geq -x - 2$.

In conclusion, Mr Dlomo's PRA with respect to his remedial lesson was centred on the direct instructional method of method. He led the learners through some examples of questions involving the solving of quadratic inequalities using the conventional teaching approach (Confrey, 1990). The teacher chose re-teaching facts about inequalities and the solving of quadratic inequalities. He did not base his instruction on the errors and misconceptions patterns identified (Riccomini, 2005). The next part of this section focuses on the analysis of the observation data from Mr Zama's remedial lesson.

In the course of the remedial lesson, Mr Zama invited learners to voluntarily present solutions of selected questions from the task that was written. From the presentation made, he would then highlight challenges that were faced by learners in responding to the questions in context. With reference to $-5x^2 \leq 0$ the first question of the written task, ZML29, a learner in Mr Zama's class volunteered to present his ideas about how he had responded to the question. ZML29 solved the inequality $-5x^2 \leq 0$, using the number line method. Shown below is what ZML29 presented to the whole class.

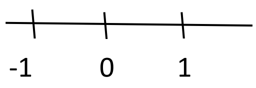
$$-5x^2 \leq 0$$

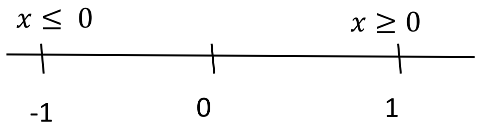
$$-5x^2 = 0$$

$$\sqrt{x^2} = \sqrt{0}$$

Critical values $x = 0, x = 0$

$0 \leq x \leq 0$





Mr Zama started by inviting contributions from other learners in the class regarding ZML29's presentation. Upon realising that no one was ready to add towards ZML29's ideas, Mr Zama began by highlighting that the graphical method was a challenge to some learners who attempted to solve the inequality using that method. To avoid such challenges, he advocated for the use of the number line method. He then proceeded to guide the learners in using the number line method to solve the given inequality. In doing so, he acknowledged that ZML29 had correctly found the critical values of the inequality. He went on to mention that what was unique in this particular question was the existence of a single repeated critical value, zero (0). Mr Zama then demonstrated, using the number line drawn by ZML29, how to find the solution of the quadratic inequality. The following excerpt highlights how Mr Zama reasoned as he guided his leaners. He said:

> We have one critical value, which becomes a challenge with this particular question. So, on the number line, we have zero at this point dividing the number line into two parts. One part is on the left-hand side of zero and another part on the right-hand side of zero. Off-course ZML29 has written $0 \leq x \leq 0$, there, which happens to be the middle part. But actually, the middle part does not exist because it's just a point. Under such cases what you need to do is to choose one value on the left-hand side of zero and test it in the equality. You test $x = -1$; negative one qualifies our inequality so it means this part of the number line to the left of zero is a solution. You don't stop there; you test the other side of zero, the right-hand side of zero. You test $x = 1$. Again, it's consistent with the given inequality. So, this side of the number line is also a solution. That's what ZML29 has shown us there. Coming to what ZML29 wrote here, he wrote his final answer as $x \leq 0$, $x \geq 0$ .That sum up as x is an element of real numbers.

From the above excerpt, Mr Zama used the demonstration method of teaching. The focus was on developing the learner ideas at the same time addressing the cognitive conflict faced by ZML29. Thus, he spoke about the existence of a single critical value unlike in other cases where learners are used to questions with two distinct critical values. He also addressed ZML29's misconception in writing $0 \leq x \leq 0$. He called that a point on the number line. The point divided the number line into two parts, the left-hand side and the right-hand side of zero. Mr Zama then proceeded to illustrate how the number line method is used to solve quadratic inequalities and the interpretation of solutions from the number line. Mr Zama used the same approach throughout the remedial lesson.

The analysis of the data reveals that Mr Zama's approach was learner centred in which the discussion method of teaching was employed. His actions mirrored those of a guider in which the teacher's reasoning reflected that of a facilitator. The teacher facilitated the re-construction of learner ideas in quadratic inequalities. That was accompanied by appropriate comments meant to dissolve forms of cognitive conflicts giving rise to certain learner errors and misconceptions.

In summing up his reasoning, Mr Zama stressed the following ideas when solving quadratic inequalities: using the number line method; the correct determination of critical values; making a sketch of the number line with three segments apportioned by critical values; testing of specific values from the segments in order to determine the solution of the quadratic inequality and interpreting solutions of quadratic inequalities from a number line. He also highlighted the need to "shade" or "un-shade" as a way of showing that a value is included or excluded respectively. The following is an analysis of the observation data from Mr Mthembu's remedial lesson.

Mr Mthembu started his remedial lesson by highlighting errors committed by learners in the written task. He mentioned that errors were made when determining and writing critical values, interpreting solutions graphically and when dividing both sides of an inequality by a negative number. The following excerpt is an example of how Mr Mthembu remediated the misconception about the presentation of critical values.

> Let us say eh… you are given $x^2 - 4 < 0$. We all know that we can re-write the statement as $(x + 2)(x - 2) < 0$. Now to write the critical values, you should write $x = -2$ and $x = 2$ .In the task that you wrote most of you were using or instead of and when writing the critical values. The error is coming from your knowledge of quadratic equations. In quadratic equations you present your solution as x is equal to this or x is equal to this what-so-ever. It's because of the product rule. That way of writing is unacceptable when writing critical values. The product rule is inapplicable in quadratic inequalities and critical values should be connected by the word and.

From the above excerpt, Mr Mthembu identified the type of error committed by his learners with respect to the use of logical connectors. He was also able to identify the source of the error and correctly explained how learners should present the critical values. With respect to the interpretation of inequality solutions using the graphical method, Mr Mthembu guided learners in solving the inequality $7x^2 + 18x - 9 < 0$. The class was requested to find the critical values first and correctly present them using the appropriate logical connector. A question-and-answer session then followed before Mr Mthembu presented the sketch of $7x^2 + 18x - 9$ and used it to illustrate how to solve the inequality $7x^2 + 18x - 9 < 0$. The following excerpt shows the nature of the question-and-answer discussion session between Mr Mthembu and his class.

Mr Mthembu: I want to sketch the graph of $7x^2 + 18x - 9$. Tell me, is the graph of $7x^2 + 18x - 9$ concave up or concave down?

MTL1: Concave up.

Mr Mthembu: Why?

MTL2: Because the value in front of x^2 in the quadratic expression is positive.

Mr Mthembu: Correct. We call that the coefficient of x^2. Now I want know, are my critical values included or excluded?

MTL3: Excluded.

Mr Mthembu: Explain.

MTL3: Because of the inequality. It's a less than sign. Last time you said a less than sign is a strict inequality and if it's a strict inequality the critical values are not included.

Mr Mthembu: Good. Now my focus is on the sketch of $7x^2 + 18x - 9$. The sketch is like this. [The teacher went on to draw the sketch of the function on the board.]

Because our inequality is $7x^2 + 18x - 9 < 0$, our focus should be on the part of the graph which is below the x- axis. This is because we are looking for values of $7x^2 + 18x - 9$ less than zero. On the sketch, $7x^2 + 18x - 9$ is less than zero for all values of x between -3 and $\frac{3}{7}$. Remember -3 and $\frac{3}{7}$ are the critical values. I have put some empty circles on the critical values to show that the critical values are excluded as part of our solution as mentioned by MTL3 earlier on. Is that so?

Chorus response: Yes

Mr Mthembu: I now want to interpret my solution from the graph. Looking at the values of x from the graph, we are interested in values of x to the right of -3 . We know this from our knowledge of the number line. Remember the x axis is a horizontal number line. Now we have so many values greater than -3. As an inequality statement we write that as $x > -3$. But earlier on I said values of x are between -3 and $\frac{3}{7}$. This means that the values of x should not go beyond $\frac{3}{7}$. Also $\frac{3}{7}$ is excluded in the values of x that we are looking for. Remember we said critical values are excluded. So, it means the values of x should be less than $\frac{3}{7}$. Using inequality symbols, we write that as $x < \frac{3}{7}$. No, we need to combine the two statements $x > -3$ and $x < \frac{3}{7}$. We can write that as a single statement as $-3 < x < \frac{3}{7}$.

Besides guiding his learners in interpreting and presenting the solution of the given inequality graphically, the teacher further exposed his class to typical incorrect reasoning in the interpretation and presentation of the solution of the quadratic inequality. The following excerpt shows how he reasoned.

Now let's look at the way some of us interpret the solution of the inequality from the graph. Some of us write their solution as $-3 > x > \frac{3}{7}$. I want us to try to make sense of this. I am going to break it down into $-3 > x$ and $> \frac{3}{7}$. Looking at $-3 > x$, what does that mean? It

> means values of x less than -3. These values are on the left-hand side of -3 on the number line which is our x-axis in this case. Can you see that this is contradicting the information that we have on the graph? We have seen that on the graph we are interested in values of x to the right of -3. Now looking at the other part of the statement that I have, I wrote $x > \frac{3}{7}$. It means we are now looking at values of x greater than $\frac{3}{7}$. This again is a contradiction, because we said we are interested in values of x between our critical values -3 and $\frac{3}{7}$

An analysis of Mr Mthembu's remedial strategy shows that he blended the direct instructional method and the cognitive conflict method in addressing the learner's pseudo concepts in quadratic inequalities. In the first excerpt presented about critical values, the direct instructional method was used. The cognitive conflict method was then used when he dealt with the interpretation of solutions of quadratic inequalities graphically. In this case learners were exposed to typical incorrect interpretation of the solution of quadratic inequalities graphically. This was accompanied by a reasoned illustration indicating the pitfalls in some learners' thinking. Lastly, Mr Mthembu's strategy of remediating the learner's misconceptions involved representation of his ideas using examples and illustrations. He used a variety of teaching methods. Among the methods used was the question-and-answer method which encompassed using probing questions. Having presented the teachers' remedial actions with respect to learner errors and misconceptions, the last part of this section presents the summary of the analysis of the observation data.

4.7.2 Summary of the analysis of the observation data.

There were similarities and differences in the three teacher's remedial actions when addressing the learner errors and misconceptions in quadratic inequalities. Common to their enactive actions was the use of the direct instruction method. Additionally, their strategy in terms of the representation of the topic knowledge involved the use of examples. The method of teaching involved question and answer as well as the discussion method. The teachers' explanations involved the use of the correct mathematical register, though Mr Dlomo and Mr Mthembu used every day language in certain instances to substantiate their mathematical explanations.

However, there were notable differences in the three teachers' remedial actions. For example, Mr Dlomo focussed his remediation on re-teaching the topic using the conventional teaching method. He did not base his remediation on the errors and misconceptions patterns arising from the task written by the learners. On the other hand, Mr Zama and Mr Mthembu centred their remedial actions on the learner errors and misconceptions in the written task. Nonetheless, the two teachers also differed in the way they approached their remedial lesson. Mr Zama's approach was learner-centred. He adopted a guider's role in the process tasking the more capable learners in the class to spearhead the remedial lesson. He was more focussed in dissolving forms of cognitive conflict in the learners' ways of reasoning where such reasoning occurred. On the other hand; Mr Mthembu's approach involved a blend of the conventional and cognitive conflict method. He also made use of questions similar to those encountered by learners in the written task as his conceptual tools. Having discussed the analysis of the observation data, the next section focusses on the analysis of the post- interview data. The data was based on each teacher's reflective thoughts based the remedial lesson conducted.

4.8 Analysis of the second interview data

The analysis of the second-interview data unpacked how the participants evaluated, through reflecting on the remedial lesson conducted, the learner errors and misconceptions in quadratic inequalities. The analysis was done using the conventional content analysis as propounded by Hsien and Shannon (2005). Hence, the data was analysed inductively. In this regard the researcher did not make use of any preconceived categories. Categories used emerged from the data after repeatedly reading the transcripts of the audio-recordings of the data several times. It must also be stated that the categories used in this analysis were formulated after an inductive coding process. The coding was done at different levels in which case descriptive, process and in vivo codes were used. Thus, coding was done at sentence, paragraph or meaning level. Table 6 shows the categories which were formulated after the inductive coding of the data.

Category	Description of category	Example	Code
Comprehension of the teaching meth	The method of teaching used by the teacher	Discussion method	CM
Comprehension of the method of solv quadratic inequalities	Teacher's understanding of the method of solving quadratic inequaliti	Graphical method	MS
Misconceptions which emerged in th remedial lesson	Ability of the teacher to reconstruct the lesson conducted and deliberate on misconceptions identified	Critical values written using inequality signs, for in $x < 2$ and $x < 3$	ME

Table 6: Categories generated from the second interview data

The categories generated were then synthesised into a single theme; the teacher's evaluation of the remedial lesson with respect to learner errors and misconceptions in quadratic inequalities. Focus is given to the generated theme.

4.9 Teacher's evaluation of the remedial lesson with respect to learner errors and misconceptions

The following is the analysis of the interview data based on teachers' evaluation of the remedial lesson with respect to learner errors and misconceptions in quadratic inequalities. As has been the case throughout this analysis, Mr Dlomo's evaluation of his lesson is presented first, flowed by Mr Zama's and Mr Mthembu's evaluation is presented last.

4.9.1 Mr Dlomo's evaluation of his remedial lesson

Mr Dlomo highlighted that some learner errors and misconceptions emerged during the lesson that he conducted. For illustrative purposes, the following is a transcription of the interviewer (Researcher) and Mr Dlomo with respect to his pedagogic reasoning based on the lesson that he conducted.

> Researcher: What errors and misconceptions emerged from your lesson?
>
> Dlomo: They were quite a number of misconceptions, firstly the determination of critical values. Some learners fail to understand why we introduce the equal sign when we are dealing with critical values. Critical values seem to be a challenge for some learners to understand. I picked it up that after factorizing their expression, learners would still want to maintain the critical values in the form of an inequality. That made it difficult for them to locate boundaries leading to a failure to come to the right conclusion. Secondly, the interpretation of regions thereof involved, talking about the open internal intervals, open external interval, and closed interval and on. For learners, it's quite a challenge. To some, if a quadratic inequality contains a less than symbol, they would expect all their answers to contain that symbol. Also, regardless of the region, learners would always want to combine the solutions even if it not applicable or necessary. Their minds are always channelled towards that direction; to say whenever I am solving a quadratic inequality my solution needs to be somehow combined. So, this popped up in today's lesson.

He noted that some learners were still prone to misconceptions regarding the interpretation of solutions of quadratic inequalities. For instance, he mentioned that if a quadratic inequality contains a less than sign, some learners would expect their answer to contain that symbol as well. Further to this, he noted that learners would want to combine solutions of inequalities even in cases where it is not applicable. Mr Dlomo also indicated that the concept of critical values is poorly conceptualised by learners. He identified the conceptualisation of this concept as the major source of the learner misconceptions in quadratic inequalities. During the lesson he observed that some learners presented "critical values in the form of inequalities". This shows that learners find it difficult to conceptualise the use of an equal sign when writing the critical values. Mr Dlomo mentioned that by presenting their critical values in the form of inequalities, it makes it difficult for learners to locate boundaries. He stated that critical values are signifiers of boundaries which help in establishing solutions of quadratic inequalities.

The researcher had to interrogate the strategies that Mr Dlomo employed in dealing with the learner errors and misconceptions in the lesson that he conducted. The transcript below is an extract of the dialogue between the researcher and Mr Dlomo.

> Researcher: What strategies, if any, did you use to address the errors and misconceptions which emerged from your lesson?
>
> Mr. Dlomo: I deliberately started with equations and moved on to linear inequalities, because linear inequalities are solved differently from quadratic inequalities in the sense that you can transpose variables, transpose numerical values but with quadratic inequalities the approach is slightly different. My approach was meant to conscientize learners, to say in as much as linear inequalities can be solved like this; it may necessarily not be the same with quadratic inequalities. This is because with quadratic inequalities you may find that there is more than one region and involved of which not any regions can make the inequality true. So, for that reason you need to identify your critical values and forthwith stop using the inequality signs approach it via critical values

Judging from the above transcript, Mr Dlomo's strategy was centred on re-teaching the topic. His reasoning was for learners to conceptualize the difference between solving equations, and also getting the difference between solving linear inequalities and quadratic inequalities. Mr Dlomo reckoned that he deliberately chose this strategy of teaching in order to facilitate the learners' re-construction of their knowledge about quadratic inequalities. This was based on the observation that his learners' existing knowledge of quadratic inequalities was riddled with many misconceptions. He explained the need for learners to realise that quadratic inequalities are solved in ways that are different from linear inequalities and quadratic equations. He went on to give his reasoning behind the strategy. This is reflected in the extract of the transcript of the interview between the researcher and Mr. Dlomo.

> Researcher: How did the strategy that you used compare with other strategies that you have used in the past?
>
> Mr. Dlomo: What I liked the most is that it cleared so many misconceptions in the way the symbols are misinterpreted. I used learners to getting to know the difference between the different types of inequality symbols. Sticking to my method served me very well because at the end of it all learners could follow what was happening. Although somewhere along the way you could see that some learners were trying to come up with their own old methods. Their methods, you could see where conflicting with what they were being taught.

From the above, transcript, Mr Dlomo, recognized that his strategy was mainly focused on getting learners to comprehend the difference between solving inequalities and solving equations and the difference between solving linear inequalities and solving quadratic inequalities. The strategy yielded some positive results. It cleared so many misconceptions regarding the interpretation of inequality signs. Mr Dlomo observed that learners could connect between the different types of concepts involved in solving quadratic inequalities. However, he also observed that some learners still wanted to use their own old methods of solving quadratic inequalities. These methods were in conflict with what they were being taught. This shows that misconceptions are not easy to eradicate. The next part of this section is on the analysis of Mr Zama's evaluation of the remedial lesson that he conducted.

4.9.2 Mr Zama's evaluation of his remedial lesson

Following the observation of Mr Zama's lesson, the researcher had to gather his thoughts regarding the way he conducted his lesson. An extract of the transcript of the interview data presented below gives an insight into Mr Zama's thoughts.

> Researcher: What are your thoughts regarding the way you conducted your lesson with specific reference to learner errors and misconceptions in quadratic inequalities?
>
> Mr. Zama: My best approach into looking at errors and misconceptions would be to make use of the learners themselves, wherein the learner gives a response, whether right or wrong and the other learners would deliberate on that. Every learner input would be considered, with the aim of maximizing classroom participation. So, when other learners identify and highlight misconceptions in other learners, everyone stand to benefit out of it. I chipped in where I realized that no one in class could offer an appropriate explanation concerning the existence of misconceptions in certain learners' responses.

In addressing the learner errors and misconceptions, Mr Zama made use of the discussion method of teaching. The method of teaching used was learner centred in which learners played a central role in responding to particular questions involving quadratic inequalities. Mr Zama reflected that peer learners deliberated on particular responses presented by other fellow learners in the class. In light of that, he made use of the more capable peer learners to address the errors and misconceptions arising from other fellow learners in the class. He focussed on building a community of learners in which those at the meta- level of the topic took a central role in assisting those whose conceptualisation of the topic was infested with misconceptions. However, on being probed about the errors which emerged in his lesson, the extract of the transcript below typifies Mr Zama's pedagogic reasoning.

> Researcher: What errors emerged out your lesson?
>
> Mr. Zama: The errors which emerged out of my lesson were minimized, because initially with the graphical method, the sketching of the quadratic function created even more errors. Some of the learners could not make correct sketches and relating to the concept above and below the x-axis is even more challenging to some learners. In some cases, they even shade outside the required region. With the use of the number line method, most of the errors arising from the graphical method became minimized. I also noticed that some learners struggled to use factorization to determine the critical values.

Mr Dlomo observed that the use of the graphical method caused more errors as a significant number of learners in the class failed to sketch correct graphs of the quadratic part of the inequality. He further articulated that some learners struggled to comprehend the concept of the graph "being above or below the x-axis" in relation to the inequality sign. He also explained that, the identification of the region to shade or un-shade created problems to a significant of number of the learners. Because of that, he had to switch to the use of the number line method. By using the number line method, he observed that misconceptions arising from the use of the graphical method were minimized. Based on the use of the number line method, the researcher had to further interrogate Mr Zama about the use of this method. An extract of the transcript of the interview data below shows how Mr Zama reasoned.

> Researcher**:** May you highlight your strategy in using the number line method.
>
> Mr. Zama: The learner must first come up with correct critical values. These must then be put on a number line thereby creating three sections of the number line. Learners must then choose a specific value and indicate on the number line the appropriate position where that chosen value is substituted into the inequality; learners would need to assess to find out whether the result will be consistent with the inequality. If it is not, for example, say you come up with an inequality statement such as -5> 0, it means it is not consistent with the inequality since -5 is never greater than zero, so it means that the region where the selected value is coming from is a no solution. If it is the middle part of the number line, it means the solutions are on either side of the number line. So, that way, it appears simpler to most learners to come up with the correct solution.

By using this method to remediate the learner errors he noted that its success is centred on the following aspects:

(a) ability of the learner to correctly determine the critical values;

(b) sketching of the correct segment of the number line with critical values correctly labelled.

Despite, using the number line method, Mr Zama observed that some of learners in his class still could not determine the correct critical values. He reasoned that this was as a result of misconceptions in the method chosen to determine the critical values. His reasoning was that some of his learners still exhibited misconceptions in factorisation or using the quadratic formula to determine the critical values. Additionally, he also highlighted misconceptions associated with the ordering of numbers on a number line. Having analysed Mr Zama's pedagogic reasoning with respect to the evaluation of his remedial lesson, the next section focusses on Mr Mthembu's evaluation of the remedial lesson that he conducted.

4.9.3 Mr Mthembu's evaluation of his remedial lesson

With regard to Mr Mthembu's evaluation of the remedial lesson, his pedagogic reasoning focussed on: content gap, determining of critical values, use of logical connectors, methods of solving quadratic inequalities and the learners' affective domain. The following is an extract of the transcript of the interview data between Mr Mthembu and the researcher with regard to his pedagogic reasoning based on the lesson that he conducted.

Researcher: What are your thoughts regarding the way you conducted your lesson with specific reference to learner error and misconceptions in quadratic inequalities?

Mr. Mthembu: I could realize that there is a content gap from the previous grade. Remember they deal with linear inequalities in grade 10, where they should understand the interpretation of inequalities signs, representation of inequalities on a number line and interpretation of inequalities from a number line. It is in grade 10, where they are introduced to words associated with inequalities such as excluded value and included value, shading and un-shading. That is why you saw I had to spend some time revisiting inequalities covered in grade 10. I wanted to close that gap before engaging the learners with grade 11 quadratic inequalities in the context of learner errors and misconceptions.

Mr Mthembu reasoned that the continual existence of learner errors and misconceptions in quadratic inequalities are a result of the gap in knowledge about inequalities as learners move from grade10 to grade 11. He articulated that the misconceptions in the interpretation of inequality signs in quadratic inequalities have its roots in grade 10 linear inequalities. Mr Mthembu explained that there is need to re-visit the solving of grade 10 linear inequalities before dealing with quadratic inequalities. In light of that he reasoned that emphasis should be placed on the interpretation and presentation of the different inequality symbols before dealing with quadratic inequalities. Thus, he reasoned that learners should have a full conceptual understanding of inequality signs and their interpretation in grade 10 in order to minimise the existence of such errors in quadratic inequalities.

However, in his remedial lesson, he observed that learners exhibited misconceptions in determining the critical values despite the existence of three methods which can be used to determine such values. In this regard, Mr Mthembu stated that some learners still struggle to calculate critical values. His reasoning, about critical values is reflected in the following extract from the transcript of the interview data.

> Researcher: What errors and misconceptions emerged from your lesson?
>
> Mr Mthembu: Critical values are a major problem. This was a big issue in my lesson. Quite a number of learners had problems with coming up with critical values and how to use them. You know, there are mainly three methods for calculating the critical values. Either one can use factorization, the quadratic formula or completing the square. But what I can say is that the quadratic formula seems to work best for the majority of my learners. I also, generally regard it as the simplest method because substituting into the formula is easy to learners. But what I saw is that some learners prefer using factorization but cannot factorize. . I witnessed this again in the lesson.

Thus, Mr Mthembu observed that some of his learners still struggled to determine the critical values. This was mainly because of the learners' inability to use factorisation to determine the critical values. He explained that the use of the quadratic formula led to the correct determination of the critical values by most of his learners. He attributed that to the learners' ability to correctly substitute into the formula. On being asked to reflect on the strategy that he used to deal with learner errors and misconceptions in the lesson that he conducted, Mr Mthembu's thoughts are captured in the extract of the transcript of the interview data presented below.

> Researcher: What strategies, if any, did you use to address the errors and misconceptions which emerged from your lesson?
>
> Mr. Mthembu: Firstly, I had to encourage learners to use the quadratic formula to determine the critical values. I also had to bring to my learners' attention that the quadratic formula works out best to those who are not good at factorization. I did not want to handle so many things. I just wanted to focus on solving quadratic inequalities without going back to the factorization of quadratic expressions. About the methods of solving quadratic inequalities, I had to stick to the use of the graphical method. This is because it is linked to quadratic functions and learners can partly relate to this topic in its application. Learners find the number line method difficult to use and I have seen that it gives rise to a lot of misconceptions.

As regards the method of solving quadratic inequalities, Mr Mthembu advocates for the graphical method over the number line method. According to him, the graphical method has advantages. He mentioned that the method is linked to quadratic functions and learners can partly relate to functions in its application. For him the number line method is too abstract for his learners and it gives rise to many other misconceptions.

Nonetheless, Mr Mthembu observed that quite a number of his learners still displayed misconceptions when interpreting the solutions of quadratic inequalities graphically. Finally, on being asked about his thoughts about whether there is a lasting solution to learner errors and misconceptions in quadratic inequalities, his reasoning was captured in the following extract of the transcript of the interview data.

> Researcher: So, do you think there might be a lasting solution to minimize the errors and misconceptions which learners make is quadratic inequalities?
>
> Mr. Mthembu: I think so; the solution could be different teaching methods and getting to understand the ability levels of your learners to carry out tasks. But we must also not forget the intensity and quality of the learners' motivation towards the subject. One of the greatest problems we have in mathematics is the learner attitude towards the subject. So, it may not necessarily be that they are unable to understand the concepts of quadratic, but it may be everything to do with learner attitude.

Mr Mthembu articulated that the continual existence of learner misconceptions in the topic despite efforts to remediate them could be linked to the learners' affective domain. Thus, he said "the intensity and quality of the learners' motivation" and attitude towards the topic could be the reason for the persistent existence of the misconceptions in the topic despite efforts to remediate them. The last section of this part of the teachers' evaluation of the remedial lesson conducted is a presentation of the summary of the analysis.

4.9.4 Summary of the evaluation of the remedial lessons conducted by the three teachers

An analysis of the three teachers' evaluation of the lesson they conducted, reflects similarities and differences in the teachers' pedagogic reasoning. What is of particular interest and common to the three teachers' pedagogic reasoning is that, apart from their remedial efforts, some learner errors and misconceptions still persisted. This shows that errors and misconceptions in quadratic inequalities are independent of the teachers' methods. However, there were some differences in the methods used to solve quadratic inequalities. For instance, Mr Zama reasoned that the use of the number line method yielded fewer errors and misconceptions in quadratic inequalities. This was contrary to Mr Mthembu's reasoning. Mr Mthembu advocated for the graphical method. Thus, the existence of the different methods for solving quadratic inequalities and the use of these methods is dependent on the teacher's philosophy of teaching the subject as well as the teacher's knowledge of the learners. Focus is now turned to the last section of the data analysis. The last section of the data analysis, as highlighted in section 4.3, focusses on the application of Peng and Luo (2009)'s analytical framework.

4.10 Analysis of the data using Peng and Luo (2009)'s analytic framework

Peng and Luo (2009)'s analytical framework was used to analyse the three teachers' PRA with respect to learner errors and misconceptions in quadratic inequalities. In order to facilitate that, the three teachers' PRA, as reflected in the previous sections, were subjected to Peng and Luo (2009)'s teacher perspective of error analysis. The teacher error analysis as propounded by Peng and Luo (2009) is characterised by four key aspects. These are; identify, interpret, evaluate and remediate. Focus is given to each of these aspects.

4.10.1 Identification: Knowing the existence of the mathematical error

With respect to knowing the existence of the mathematical error, the three teachers identified learner errors in quadratic inequalities as masked in the following misconceptions:

(a) learners solve quadratic inequalities as quadratic equations
(b) learners consider critical values as solutions of quadratic inequalities
(c) learners misinterpret solutions of quadratic inequalities
(d) learners do not reverse the inequality sign when dividing or multiplying both sides of a quadratic inequality by a negative number

With regards to interpreting the underlying rationality of the mathematical error, the teachers' pedagogic reasoning was put on spotlight.

4.10.2 Interpretation: Interpreting the underlying rationality of the mathematical error

The teachers reasoned that learners solve quadratic inequalities using procedures for solving quadratic equations. They attributed this to the teaching of quadratic equations before quadratic inequalities. They reasoned that learners tend to overgeneralise their knowledge of quadratic equations to quadratic inequalities. The teachers further articulated that some learner errors in quadratic inequalities are a result of the learners' insufficient conceptualisation of the inequality signs. The teachers mentioned that learners treat the inequality signs as an equal sign. Thus, learners lack the semantic meaning of the inequality signs. Mr Dlomo and Mr Mthembu further attributed the inadequacy of the semantic meaning of inequality signs to the learners' insufficient knowledge of linear inequalities taught in previous grades.

Turning to critical values, the teachers implicated the methods of solving quadratic inequalities. They mentioned that determining critical values using methods of solving equations make learners view critical values as solutions of quadratic inequalities. Additionally, they also reasoned that learners determine incorrect critical values because of their insufficient mastery of the methods of solving quadratic equations. Mr Mthembu noted that learner errors about critical values were also caused by the learner's inability to use the appropriate logical connectors.

The teachers highlighted that learner errors involving the interpretation of solutions of quadratic inequalities were linked to the learners' inability to interpret the inequality signs. For instance, Mr Dlomo explained that if a quadratic inequality contains a less than sign, "some learners would expect their answer to contain that symbol as well". Lastly, the teachers also reasoned that some learners commit errors in quadratic inequalities because of their inability to reverse the inequality sign when dividing both sides of the quadratic inequality by a negative number (Kroll, 1987; Almog & Ilany, 2012). The teachers attributed this to the learners' previous knowledge about solving linear equations, where the use of the multiplicative inverse rule is permissible. The teachers' ability to identify and interpret the underlying rationality of the mathematical errors meant that they were able to evaluate and provide feedback on the learners' levels of performance according to the mathematical errors.

4.10.3 Evaluation: Evaluating the level of learner performance according to the mathematical error

In evaluating the learners' performance levels in the written task, the correct reasoning in the learners' presentation of ideas about solving quadratic inequalities was credited. This was done using ticks. Where a particular mathematical error existed, the teachers underlined or used a cross to indicate disapproval of the learner's reasoning. However, in some cases the underlining or crossing of the learner's response was accompanied by a written comment describing how the learner should have reasoned. Turning to the learners' verbal responses, in the remedial lesson conducted, their level of performance according to the type of error was evaluated by way of verbal comments. Thus, words such as good, that's it and correct were used by the teachers. The words were used as a sign of approval of the learners' conceptualisation of their knowledge in quadratic inequalities. With regard to the last aspect of the teacher error analysis, the focus was on the ability of the teacher to present a teaching strategy to eliminate the errors.

4.10.4 Remediation: Presenting a teaching strategy to eliminate the mathematical error

Each teacher employed his own teaching strategy targeted at the identified errors. Mr Dlomo's teaching strategy was centred on the direct instruction method. He used the discussion method in which focus was mainly on re-teaching the solving of quadratic inequalities. He made use of his own carefully chosen examples as conceptual tools to facilitate the remedial process. In presenting his teaching strategy, Mr Dlomo settled for the demonstration method. Lastly, the teacher used the graphical method as a cognitive tool for solving quadratic inequalities. Mr Dlomo also made use of everyday objects and examples to mediate the solving of quadratic inequalities using the stated method. For instance, a board ruler was used as an artefact to illustrate how the graphical method of solving quadratic inequalities works. However, in his remedial lesson, Mr Dlomo addressed the following aspects; the concept of inequality and the different types of inequalities, interpretation of inequality signs, determination of critical values and the interpretation of solutions of quadratic inequalities using the graphical method.

With regard to Mr Zama's remediation of the errors identified, his teaching strategy was mainly learner-centred. The discussion method of teaching in which the teacher and the more capable learners were collaborators of the remedial process was used. Learners would present their ideas and the teacher would get involved upon realising areas of cognitive conflict in the learners' thinking and presentation of ideas. The content to address the mathematical errors was drawn from the task written by the learners. Mr Zama used the number line method as a cognitive tool for solving quadratic inequalities. Turning to the errors that were addressed, the teacher addressed errors about: critical values, methods of determining critical values, and the interpretation of solutions of quadratic inequalities using the number line method.

In terms of Mthembu's remedial lesson, the following was central to his PRA. The teacher employed a blend of the direct instruction method and the cognitive conflict method. The teacher settled for the graphical method as a cognitive tool for solving quadratic inequalities. Regarding the errors that were remediated, Mr Mthembu addressed errors involving critical values and methods of determining the critical values, dividing both sides of an inequality by a negative number, the interpretation of inequality signs and the interpretation of inequality solutions using the graphical method.

In summary, it was observed that there were similarities and differences in the strategies used by the three teachers. However, the choice of the teaching strategy used by the teachers was based on the following; the teacher's knowledge of content and students, the teacher's knowledge of content and teaching, the teacher's common content knowledge and the teacher's specialised content knowledge (Ball, et al.;2008). Through analysing the three teachers' pedagogic reasoning regarding their interpretation, evaluation and remedial actions the following conclusion can be made: the errors and misconception which learners make in quadratic inequalities have a historical background. They are linked to previous learning correctly or incorrectly acquired (Olivier, 1989; Nesher, 1987). In addition, it can be noted that misconceptions in quadratic inequalities are stubborn constructs which are difficult to eradicate. Lastly, it can also be noted the teacher's choice of the remedial method of teaching and method of solving quadratic inequalities is a function of the teacher's PRA. However, the teacher's PRA draws from Ball, et al. (2008)'s mathematical knowledge for teaching and the teacher's pedagogical knowledge for teaching.

4.11 Discussions

This section discusses the data analysis conducted from section 4.5 to section 4.8 in relation to the literature review and theoretical framework. The analysis of the data brings to fore the essence of the teacher's pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities. It is important to state that learner errors and misconceptions in quadratic inequalities have not been widely researched in the South African context. However, studies on learner errors and misconceptions have shown that they are related to the learners' past learning experiences (Nesher, 1987).In pursuance of the data analysis, in conjunction with the reviewed literature and theoretical framework, six themes were identified. The first two themes are centred on the nature of learner errors and misconceptions in quadratic inequalities, and the last four are based on the teacher's pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities.

Though the focus of the study was not primarily on learner errors and misconceptions in quadratic inequalities, it was imperative to focus on the nature of learner errors and misconceptions in quadratic inequalities. It provided the researcher with the platform to explore the teachers' pedagogic reasoning and action in the context of the topic of study. In view of that, two themes were identified to highlight the nature of the learner errors and misconceptions in quadratic inequalities. In the discussion of themes below, the first two themes as explained in the preceding sentence are discussed first. This is followed by a discussion of four themes centred on the teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities.

Theme 1: Overgeneralization of; quadratic equations to quadratic inequalities

The analysis of the learners' responses in section 4.6.2.1 revealed that the learners' previous knowledge about quadratic inequalities interferes with the learners' construction of their knowledge of quadratic inequalities. Thus, learners overgeneralize the product rule for solving quadratic equations to quadratic inequalities (Bazzin & Tsamir, 2004; Almog & Ilany, 2012; Bicer et al., 2014). This was evident, for instance, in learner DML 18's response (figure 51). In solving the quadratic inequality$(y-3)(y+4)<0$., the learner wrote $y<3$ and $y<-4$ as the solution of the quadratic inequality. This shows an overgeneralization of the product rule for solving quadratic equations to quadratic inequalities. When solving the quadratic equation, for instance $(x-3)(y+4)=0$, it implies that one of the two factors or both must be equal to zero, for the product of the two factors to equal zero. However, this is inapplicable when solving quadratic inequalities. For quadratic inequalities, if $(y-3)(y+4)<0,$ it means that one of the factors must be negative for the product to be less than zero. This means that a different approach to that of solving quadratic equations is required. Attention is now given to the second theme.

Theme 2: Interpretation of inequality signs

A common and persistent misconception which emerged from the analysis of the learners' responses in section 4.6.2.1 is that learners treated inequalities as equations. In this case, the learners attach the same meaning to the inequality sign as the equal sign (Kroll, 1987; Blanco& Garrote, 2007; Bicer, et al., 2014). Based on this, I put forward the premise that learner errors and misconceptions in quadratic inequalities cannot be divorced from the learners' previous learning particularly the solution of linear equations and quadratic equations. This is evident, for example, in learner DML3 (figure 5). In solving the quadratic inequality$-5x^2 \leq 0$, the learner gave the solution $x=0$, meaning the learner actually solved the quadratic equation$-5x^2=0$. Also inherent in this solution is the misconception of the equal sign. The misconception about the equal sign is linked to the learner's previous experience with mathematics when dealing with the equal sign. For example:

$2+3=$ ___or $to\ solve\ for\ x\ in\ 3x+8=14$, is the reason behind the faulty line of reasoning. Therefore, learners transfer their previous knowledge of the meaning of the equal sign to inequalities and fail to distinguish between the two (Blanco & Garote, 2007; Vaiyavutjamai & Clemence, 2006; Bicer et al., 2014). It was also noted that learners interpret the critical values of the quadratic inequalities to be the solutions of the quadratic inequalities. For instance, ZML15 (figure 45), in solving the quadratic inequality $2x^2 - 7x - 4 \geq 0$ the learner gave the critical values, $x = 4$ and $x = -\frac{1}{2}$ as the solution of the quadratic inequality.

Having discussed the background of the nature of the learner errors and misconceptions in quadratic inequalities as analysed in section 4.6.2.1, focus is now turned to the teachers' pedagogic reasoning and action (PRA) in relation to the learner errors and misconceptions in quadratic inequalities. I put forward the argument that secondary school mathematics teachers are knowledgeable about learner errors and misconceptions in quadratic inequalities.

Theme 3: Secondary school mathematics teachers are knowledgeable about learner errors and misconceptions in quadratic inequalities.

Based on the analysed interview data, secondary school mathematics teachers explained the nature of learner errors in quadratic inequalities encapsulated in certain misconceptions. The secondary school mathematics teachers explained that learners solve quadratic inequalities in the same way as quadratic equations. This was clearly articulated in the interview with Mr Dlomo and Mr Zama (see section 4.5.1).This is consistent with the findings of (Bazzin &Tsamir, 2004; Almog Ilany, 2012; Bicer, et al.; 2014). However, the difference is that these previous studies focused on learners as the subject of study in coming up with this finding whilst in this study the finding was arrived at by focusing on the teachers' pedagogic reasoning in relation to learner errors and misconceptions in quadratic inequalities.

From the analysis of the data in section 4.8.3 Mr Zama and Mthembu explained, that learners are unable to factorize the quadratic part of the inequality in order to determine the critical values. This concurs with the finding in the studies by Makonye and Mhonda (2014). Furthermore, the secondary school mathematics teachers identified that learners lack conceptual understanding of the use and application of logical connectors (Bazzin & Tsamir, 2004). It was also established that learners consider the critical values to be the solutions of quadratic inequalities .Thus; the solutions of quadratic inequalities are treated as equalities (Blanco & Garrote, 2007; Almog & Ilany, 2012). Additionally, the secondary school mathematics teachers identified that learners misinterpret solutions of quadratic inequalities regardless of the method used. Lastly, the teachers reasoned that learners fail to reverse the inequality sign when multiplying or dividing both sides of a quadratic inequality by a negative number (Kroll, 1987; Verikios & Farmaki, 2010; Godden, 2012; Bicer, et al, 2014).

In the above section the teachers' reasoning with respect to the nature of learner errors and misconceptions has been discussed. This means that the three secondary school mathematics teachers were knowledgeable about the type of learner errors and misconceptions which learners make in quadratic inequalities The next theme discusses the teachers' pedagogic reasoning regarding the underlying rationality for the learner errors and misconceptions in quadratic inequalities.

Theme 4: The teachers' pedagogic reasoning regarding the underlying rationality for the learner errors and misconceptions in quadratic inequalities

With respect to that, the teachers reasoned; the learner errors and misconceptions which learners make can be traced to the learners' conceptualization of topics linked to quadratic inequalities. For instance, in the interview with Mr Mthembu (c.f section 4.8.3), he highlighted the existence of a content gap in the learners' knowledge about inequality signs as they progress from grade 10 to grade 11.Therefore, the learners' comprehension of topics taught before quadratic inequalities such as linear equations, linear inequalities, quadratic equations and factorization interferes with the learners' conceptualization of quadratic inequalities. For example, learners over-generalize, the product rule for solving quadratic equations to quadratic inequalities (Almog & Ilany, 2012; Bicer et al.; 2014). It also confirms Olivier (1989)'s viewpoint that learners tend to overgeneralize previously acquired correct or incorrect knowledge to an extended domain where it is inapplicable. Hence, a manifestation of procedural, conceptual or linear extrapolation errors become evident in the learners' written or verbal responses in quadratic inequalities. In view of the above standpoints, I put forward the argument that learner errors and misconceptions in quadratic inequalities are a manifestation of the tension in the learners' thinking. The tension is a function of the prior knowledge and the new knowledge that learners are exposed to. As a result, learner errors and misconceptions are inevitable in the teaching and learning process and teachers need to proactively anticipate learner errors (Ball et al. 2008). This calls for an accurate identification and interpretation of such errors and misconceptions. Peng and Luo (2009), argue that an accurate identification and interpretation of the learner errors and misconceptions lead to the proper evaluation of the learners' performance in terms of the mathematical error. In view of the focus of this study, I can report that the three secondary school mathematics teachers were able to interpret the learner errors and misconceptions in quadratic inequalities. Further to that, the teachers were able to provide the underlying rationality for such errors and misconceptions. Attention is now given to the teacher's evaluation of the learner errors and misconceptions in quadratic inequalities.

Theme 5: The evaluation of the learner errors and misconceptions in quadratic inequalities

Regarding the teachers' evaluation of the learner errors and misconceptions in quadratic inequalities, the analysis of the data reveals that this can be done in multiple ways. However, the teachers' philosophy of teaching the subject is implicated. Concomitantly, the teachers' evaluation of the learner errors and misconceptions in quadratic inequalities reflected aspects of instrumental and relational approaches to teaching (Skemp, 1976). For example, in evaluating learner MTL6's written response (figure 14), Mr Mthembu placed ticks where learner MTL6's correct reasoning was reflected in the process of solving the quadratic inequality$x^2 - 9 \leq 0$. However, the teacher underlined the final incorrect solution $-3 \geq x \leq 3$ and commented about the incorrect notation of the inequality sign. I argue that this showed a relational approach in the evaluation of the learner's work.

Hence, the learner's written responses were evaluated by means of ticks, underlines, circles, crosses or written comments. The nature of responses was a function of the teachers' thinking processes as well as the nature of the responses exhibited by the learners. Furthermore, in the remedial lessons conducted, the learners' verbal responses were evaluated by means of verbal comments. This means that the three secondary school mathematics teachers were able to evaluate the learners' written and verbalised responses in relation to the errors and misconceptions made by the learners in quadratic inequalities.

Finally, the teacher's identification, interpretation and evaluation of the learner errors and misconceptions lead to the appropriate remediation of the learner's faulty lines of the conceptualization of the mathematical ideas (Peng &Luo, 2009). In this study, teaching strategies to remediate the learner errors and misconceptions were revealed.

Theme 6: Teaching strategies to remediate the learner errors and misconceptions in quadratic inequalities

The conventional teaching strategy (Confrey, 1990; Pang, 2016) and the cognitive conflict strategy (Riccomini, 2005) were implicated in the strategies employed by the teachers in remediating learner errors and misconceptions in quadratic inequalities. A typical case of the conventional teaching strategy is reflected in the teaching strategy that was employed by Mr Dlomo (c.f section 4.8.1). In his approach, Mr Dlomo had to re-teach the topic and started by explaining to the learners the difference between equations and inequalities. Much of what the teacher did was based on the direct verbal exposition of solving quadratic inequalities. It follows that each mathematics teacher has a personal philosophy of mathematics which underlies the way the person teaches (Ernest, 2012). Nonetheless, the target for the remediation of the learner errors and misconception in quadratic inequalities is the learner's zone of proximal development (ZPD). Thus, for the learners' mental functions to reach their actual mental developmental level secondary school mathematics teachers play a mediational role in the learner's ZPD (Vygotsky, 1978). This draws on the teacher's PRA; a veil that reflects the quality of the teacher's mathematics knowledge for teaching. A case for the teacher's philosophy of teaching secondary school mathematics was implicated in this study. This implies that the three secondary school mathematics teachers employed remedial actions which were influenced by their philosophical underpinnings of teaching school mathematics.

4.12 Summary

This chapter has examined the data generated with a view to establishing and describing the teacher's pedagogic reasoning and action in respect of the learners' errors and misconceptions in quadratic inequalities. The misconceptions and the errors exhibited by the learners across the three cases were remarkably persistent and to a larger extent similar across contexts, or teaching methods. However, the teacher's PRA underscores the teacher's mathematical knowledge for teaching; a lens through which the teacher's philosophy of teaching mathematics is viewed. Thus, the teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities was revealed. In the next chapter a summary of the major findings, conclusions and recommendations of the study is provided. Further to this, the chapter also discusses the limitations of the study and the recommendations for the teaching and learning of quadratic inequalities.

CHAPTER 5

SUMMARY OF MAJOR FINDINGS, CONCLUSIONS AND RECOMMENDATIONS

5.1 Introduction

The study sought to explore the teachers' pedagogical reasoning and action (PRA) with respect to resourcing learner errors and misconceptions in teaching quadratic inequalities at grade 11 level. PRA relates to the teacher's pedagogic practice, a product of the thinking processes, which the teacher undergoes before, during and after enacting a teaching practice. The study adopted the Vygotskian socio-cultural learning theory. With respect to this theory, it is argued that learners are susceptible to errors and misconceptions in the process of the active construction of new knowledge in the zone of proximal development (ZPD). Mediators of learning, such as teachers, play an active role in dissolving the cognitive dissonance in the learner's ZPD. Hence, the teacher's mediational role in the learner's ZPD was central in exploring the phenomenon under study. To facilitate the exploration, related literature about the teacher's PRA, learner errors and misconceptions in general and particularly in quadratic inequalities was reviewed. What was surprising is the existence of very little literature about learner errors and misconceptions in quadratic inequalities particularly in South Africa. To assist in the exploration of the phenomenon under study, the following research questions guided the study:

1. What types of learner errors and misconceptions in quadratic inequalities do secondary mathematics teachers know?
2. What interpretation, do secondary mathematics teachers make of learner errors and misconceptions in quadratic inequalities at grade 11 levels?
3. How do secondary mathematics teachers evaluate learners' written and verbalised responses about learner errors and misconceptions in quadratic inequalities at grade 11 level?
4. What remedial actions, if any, do secondary mathematics teachers employ when addressing learner errors and misconceptions in quadratic at grade 11 levels?

In order to tap into the teachers' PRA with respect to the topic under study, qualitative data was collected using interviews and observations. A learners' written diagnostic task was also a source of data for this study. The data was analysed deductively and inductively using the content analysis method. To further unmask the phenomenon studied, Peng and Luo

(2009)'s analytical framework was applied as an overarching lens through which the data was interpreted and informed the findings. The next section gives an account of the main findings from the study.

5.2 Summary of major findings

The findings are presented according to the research questions guiding this study. Presented below are the findings.

5.2.1 First research question

In terms of the first research question, the study revealed that:

Secondary mathematics teachers are knowledgeable about conceptual, linear extrapolation and procedural errors which learners make in quadratic inequalities. They were able to identify that the errors are masked in the following misconceptions: learners solve quadratic inequalities in the same way that quadratic equations are solved; critical values are considered as solutions of quadratic inequalities; lack of semantic meaning of logical connectors when writing critical values, misinterpretation of solutions of quadratic inequalities and the inequality sign is not reversed when dividing or multiplying both sides of the quadratic inequality by a negative number (Godden, 2012).

5.2.2 Second research question

With regard to the second research question, it was found that:

Secondary school mathematics teachers interpreted that learners conceptualise quadratic inequalities as quadratic equations resulting in certain errors and misconceptions. In the process, procedures of solving quadratic equations are used in the extended domain of quadratic inequalities leading to a manifestation of procedural errors. Thus, learners overgeneralise their previously acquired correct and incorrect knowledge (Olivier, 1989), about quadratic equations to an extended domain involving quadratic inequalities. An inadequacy in the conceptualisation of the inequality signs was also signalled. This makes a case of conceptual errors in solving quadratic inequalities. Implicated is the lack of differentiation of the equal sign from the inequality sign. The inequality sign is viewed as a nexus without a semantic meaning.

The existence of misconceptions associated with the lack of the semantic meaning of the inequality sign has a historical connotation. Inequality signs are introduced in lower grades;

hence they are assimilated incorrectly and reside as a body of knowledge only to manifest as "bugs" in higher grades.

There are conceptual errors in quadratic inequalities associated with the misinterpretation of critical value. The critical values are considered as solutions of quadratic inequalities. This can be traced back to methods used to determine the critical values. Critical values are determined using factorisation, the quadratic formula or completing the square. The conceptual errors in this case are twofold: Firstly, the determination of the critical values implies solving the quadratic inequality. In this case, the critical values are conceptualised as solutions of the quadratic inequality. Secondly there are procedural errors arising from the incorrect execution of methods used to determine the critical values. For example, the incorrect factorisation of the quadratic expression of the inequality leads to the incorrect determination of critical values (Makonye & Mhonda, 2014).

The use of logical connectors when writing critical values is also weakly conceptualised. Learners seem not to know when to use "and" and when to use "or". This again can be traced back to the knowledge of quadratic equations interfering with that of quadratic inequalities. Learners present critical values using the same logical connector that is used when presenting solutions of quadratic equations.

Conceptual errors occur in the interpretation of solutions of quadratic inequalities regardless of the method used. In this case it was found that the number line method and graphical methods are poorly conceptualised. Learners execute methods for solving quadratic inequalities without the conceptual connection to the inequality concepts. Thus, solutions are misinterpreted regardless of the method. Procedural errors are committed when dividing or multiplying both sides of an inequality by a negative number (Godden, 2012). The error occurs as a result of the overgeneralisation of the multiplicative inverse rule applied in solving linear equations to quadratic inequalities (Kroll, 1987).

5.2.3. Third research question

Regarding the third research question the findings were that: meaningful feedback is essential when evaluating learners' level of performance in quadratic inequalities. Focus should be in the correct diagnosis of the learners' ideas in quadratic inequalities. Therefore, there is need to consciously track the learners' synthesis and presentation of ideas. Correct reasoning in the learners' presentation of ideas should be credited. This may be done by way of approval ticks placed at each successfully completed step in the learners' written work.

The existence of misconceptions leading to particular errors may be indicated by underlining, circling, or inserting a cross whenever a misconception is detected. This may be accompanied by appropriate written comments. On the question of learners' verbal responses whereby particular misconceptions manifest, these should be responded to productively. Correct responses may be positively reinforced. Where misconceptions exist, there is need to dig deeper into the learners' ways of reasoning and unearth the underlying causes of the misconception. This may be done through the meaningful execution of probing questions directed at eliciting learners' reasoning.

5.2.4 Fourth research question

With respect to the fourth research question the study revealed that:

There are different teaching strategies that might be used to remediate learner errors and misconceptions in quadratic inequalities. These are the direct and cognitive conflict instruction method. The focus on the identified methods is on the teacher as a mediation tool for mitigating learner errors and misconception in quadratic inequalities. Therefore, the choice of a strategy must be aimed at assisting in bridging the gap between what the teacher knows, as a more capable other, and what is expected and desired for the learners. The researcher is of the view that, this may be significant in the enculturation of learners into a community of practice. To facilitate that, the teacher's representational methods include, but are limited to, discussions, demonstrations, and illustrational methods. Such methods should be targeted at remediating the errors and misconception identified.

The graphical and number line methods are the main methods used to solve quadratic inequalities. The choice of a method depends on the teachers' knowledge of the subject (in this case knowledge of quadratic inequalities) and the teachers' knowledge of the characteristics of the learners that they teach (Ball et. al., 2008).

5.3. Limitation of the study

A single remedial lesson was conducted by each of the three teachers who were the main participants in the study. In hind sight, with resources and time permitting, more than one remedial lesson per teacher could have been more ideal. Based on the learners' errors and misconceptions that were identified by the teachers prospectively, it was observed that it may have affected the teacher presentation of their remedial lessons. I observed that an attempt was made to remediate all the errors and misconceptions identified in the learners' written task in the single lesson conducted. With the time that was allocated to each teacher there is a possibility that some errors and misconception were not fully remediated.

As part of evaluating the teacher's strategies the researcher could have broadened the data collection process by including a second written task for learners. Nesher (1987) argues that in the real time of instruction (the enacted lesson), committing errors and misconceptions is negatively viewed by other learners. In view of that, learners may have deliberately concealed their thoughts by choosing not to participate for fear of being perceived negatively by peers.

Additionally, conducting interviews with learners after the remedial lessons might have played a dual purpose. Firstly, it was to further substantiate the effectiveness of the intervention strategies enacted by the teachers. Secondly, the leaner interviews, after the remedial process by each teacher might have enhanced the authenticity of the teacher's evaluative thoughts in terms of errors and misconceptions claimed to have been remediated. Lastly, I could also have included female mathematics teachers in order to promote gender parity and equity. It is argued that women participation in mathematics education should be promoted and gender stereotypes with regard to mathematics should be countered (Leder, 2015).

5.4 Recommendations

This section provides the recommendations for this study. The recommendations are classified into three categories. These are: further research; practising teachers and teacher development. Focus is given to each of the categories

5.5 Further research

Further research in learner errors and misconceptions in inequalities with a specific focus on quadratic inequalities is needed because there is paucity research in this area. There is substantial evidence that learners' prior knowledge in quadratic inequalities is integrated into other secondary school mathematics topics such as functions, trigonometry, calculus and sequences and series. If learner errors and misconceptions in quadratic inequalities remain an unchartered territory, learner achievement in secondary school mathematics may continue to be less convincing. It may also be of interest to investigate the possible impact of learner knowledge in inequalities in topics that integrate inequalities such as those mentioned above.

The researcher also recommends the need for more research into the PRA with respect to instructional strategies that may be used in order to minimize learner errors and misconceptions in quadratic inequalities.

Lastly, and more broadly, there is also need to incorporate female mathematics teachers in the exploitation of secondary school mathematics teacher's PRA with respect to learner errors and misconceptions in quadratic inequalities. This may enhance the credibility and transferability of the findings through adopting a more inclusive approach in the selection of participants.

5.6 Practising teachers

The findings of the study have a number of practical implications. Firstly, it is essential for practising mathematics teachers to acknowledge that learner errors and misconceptions are inevitable in the teaching and learning process. Therefore, learners are vulnerable to certain errors and misconceptions in the active construction of their knowledge. Concomitantly, the learner's prior knowledge may negatively interfere with new forms of knowledge or vice versa. Secondly, an accurate identification of the errors and misconceptions which learners make may enable the conceptualization of the underlying reasons to be unearthed. Thirdly, a proper evaluation of learner performance may be established and thus triggering appropriate intervention strategies. This may lead to the adaptation and tailoring of the strategies to the specific characteristics of the learners that one teaches. Thus, the teacher's PRA should be targeted at eliciting powerful forms of instructional strategies that are tailored to accommodate differences in learner abilities and the socio-cultural context to which they are exposed.

5.7 Teacher Development

The teacher's PRA in relation to learner errors and misconceptions in inequalities in general is an unexplored territory. It is recommended, through this study, that in-service and pre-service mathematics teachers be empowered with appropriate evaluative and remedial strategies targeted at minimizing learner errors and misconceptions in inequalities. The unevaluated errors and misconceptions may transform into detrimental errors and misconceptions. This has the potential of cascading into other topics which integrate inequalities. Hence, the need for appropriate evaluative and remedial strategies for teaching the topic initiated and supported at the level of policy.

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APPENDICES

Appendix: 1 Certificate of language editing

CERTIFICATE OF LANGUAGE EDITING

Date: 29 March 2021

Name of Client: Edgar Marange

Institution: Wits School of Education

This is to certify that language editing has been carried out on **the following dissertation:**

Secondary school mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities

Language editing was carried out to appropriate academic standards, including syntax, grammar and style.

Edmore Mutekwe (PhD), (M.Ed), (B.Ed), (BA), (Cert in Ed), (Dip. Pers Man) University of Johannesburg, University of Zimbabwe

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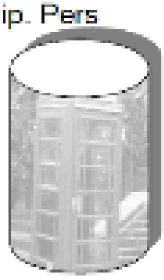
emutekwe@yahoo.com



Signature:

Appendix 2: Semi-structured interview protocol for the first interview

Time 25-30 minutes

Semi-structured interview protocol; First interview before a lesson observation.

The researcher started by explaining to the participant the purpose of the interview. The purpose was to gather their thoughts regarding learner errors and misconceptions in quadratic inequalities based on their classroom experience of teaching the topic.

1.0 What are your thoughts regarding learner thinking in inequalities in general and quadratic inequalities in particular?

 1.1 From your classroom experience, what are your learner's conceptions, if any, of quadratic inequalities?

 1.2 What would you consider to be typical correct or incorrect reasoning by your learners in quadratic inequalities?

2. What are the possible sources of learners' incorrect solutions in quadratic inequalities?

3. How do you make use of learner errors and misconceptions in quadratic inequalities in your teaching? How significant, if at all, are learner errors and misconceptions in quadratic inequalities in your teaching?

4.0 How important, if at all, is the mathematical register in the teaching and learning of quadratic inequalities in your teaching?

4.1 How do you make use of the mathematical register, if at all, in your teaching of quadratic inequalities?

5. How do you address your learner errors and misconceptions in quadratic inequalities?

Appendix 3: Diagnostic task sheet for learners

Task for learners administered by the teachers

Duration: 30 minutes

The researcher started by explaining to the learners that the purpose of the task was to find out how their teacher was to evaluate their scripts. Learners were also, informed that the task had nothing to do with their promotional marks, nor were it for denigrating them nor fault finding.

Do not write your name on the answer sheets provided

Solve the following quadratic inequalities

1. $-5x^2 \geq 0$

2. $x^2 - 9 \geq 0$

3. $x^2 + 7x < 0$

4. $x^2 + x - 12 > 0$

5. $2x^2 - 7x - 4 \geq 0$

6. $(y - 3)(y + 4) \leq 0$

7. $3x - 5x^2 \geq 0$

8. $x^2 > 3(x + 6)$

Appendix 4: An observation protocol schedule

Time 45 minutes

An observation protocol schedule for observing the teacher's (participant) lesson

The researcher started by explaining to the participant the purpose of the lesson observation. The purpose was to gather their thoughts and strategies as they engage learners in addressing the learners' errors and misconceptions in quadratic inequalities

Observation notes; Teacher (participant's) engagement with learner errors and misconceptions in quadratic inequalities

Setting: Classroom of participant:

Observer: Researcher:

Participant: Teacher:

Date:

Time:

Role of Observer; observing the teacher as he engages learners in addressing their errors and misconceptions in quadratic inequalities

Length: 45minutes

Focus of observation	Reflective Notes
1. Teachers' identification of errors and misconceptions in quadratic inequalities	
2. Teachers' interpretation of the underlying rationality of the learner's error and misconceptions in quadratic inequalities	
3. Teachers' evaluation of the learner's verbal of written errors and misconceptions quadratic inequalities	
4. Teachers' strategies of eliminating learner's error and misconceptions in quadra inequalities.	

Appendix 5:Semi-structured interview protocol for the second interview

Semi-structured interview protocol: Second interview after a lesson observation.

The researcher started by explaining to the participant the purpose of the interview. The purpose was to gather their evaluative thoughts based on their engagement with learners in addressing their errors and misconceptions in quadratic inequalities

1 What are your thoughts regarding the manner in which you conducted your lesson with specific reference to learner errors and misconceptions in quadratic inequalities?

2. What errors and misconceptions, if any, emerged from your lesson?

3. How do these errors, if any, compare to the errors and misconceptions that you have experienced before?

4. What strategies, if any, did you use to address the errors and misconceptions which emerged from your lesson?

5. How do the strategy (strategies) which you used, if any, compares (compare) with the previous strategies that you have used?

Appendix 6: University Ethical Clearance

Wits School of Education

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

27 St Andrews Road, Parktown, Johannesburg, 2193 • Private Bag 3, Wits 2050, South Africa
Tel: +27 11 717-3221 • Fax: +27 11 717-3009 • E-mail: enquiries@educ.wits.ac.za • Website: www.wits.ac.za

02 November 2018

Student Number: 1755012

Protocol Number: 2018ECE048M

Dear Edgar Marange

Application for Ethics Clearance: Master of Education

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

Secondary Mathematics teacher's pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities.

The committee recently met and I am pleased to inform you that clearance was granted. Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

M Mosese

Wits School of Education
011 717-3416

cc Supervisor - Dr Judah Makonye

Appendix 7: GDE approval letter

GAUTENG PROVINCE
Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	**13 August 2018**
Validity of Research Approval:	**05 February 2018 – 28 September 2018** **2018/231**
Name of Researcher:	**Marange E.**
Address of Researcher:	**9098 Orleander Street**
	Protea Glen
	Extension 12, 1919
Telephone Number:	**081 459 0065 078 109 6596**
Email address:	**mande.078@gmail.com**
Research Topic:	**Secondary Mathematics teachers; pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities**
Type of qualification	**Masters of Education**
Number and type of schools:	**Three Secondary Schools**
District/s/HO	**Gauteng West, Johannesburg North.**

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

13/08/2018

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management
7th Floor, 17 Simmonds Street, Johannesburg, 2001
Tel: (011) 355 0488
Email: Faith.Tshabalala@gauteng.gov.za
Website: www.education.gpg.gov.za

Appendix 8: Information sheet to principal and SGB chair

University of the Witwatersrand, Wits School of Education, 97 St Andrews Rd, Parkton, Johannesburg,

The Principal/SGB Chair

Date: ------------------------------------

Dear Sir

<u>REF: Permission to conduct research in your school</u>

My name is Edgar Marange. I am studying for a Masters of Education in Mathematics in the School of Education at Witwatersrand University. I am seeking for permission to do research at your school.

I am doing research on: secondary Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level. My focus is on public schools in Johannesburg. I want to explore how, if at all, grade 11 Mathematics teachers make use of learner errors and misconceptions as conceptual tools in their teaching. I have noticed that quite a number of learners struggle with this topic both in grade 11 and grade 12 examinations. I am also motivated by the fact that the topic is taught in grade 11 only but regularly examinable in grade 12 Mathematics paper one. I strongly believe that one of the ways in which learner errors and misconceptions can be dealt with is to examine how Mathematics teachers use the learners' errors and misconceptions as conceptual tools for mediating learning. My research intends to contribute to the South Africa's school Mathematics education through exploring ways of improving the teaching and learning of school mathematics.

I intend carrying out two semi-structured interviews and two lesson observation sessions with one of the grade 11 Mathematics teachers in your school. This shall be done outside the normal teaching time in order to minimize interfering with the normal teaching and learning activities of the teacher and learners.

The first interview, to last at most 35 minutes, is meant to determine the teacher's knowledge of learner errors and misconceptions in inequalities in general and quadratic inequalities in particular. I intend gathering this knowledge from the teacher's experience of teaching inequalities. I also want to find out how, if at all, the teacher makes use of the errors and misconceptions as tools for mediating learning. This interview will be followed by a short task involving quadratic inequalities that the teacher will write. The written task will take at most 10 minutes. The purpose of this task will be to substantiate the teacher's knowledge as gathered in the interview. This will be followed by a short test which the teacher is to administer to the learners that he or she teaches. The test is going to be administered in the presence of the researcher and will last for 30 minutes. The teacher is going to mark the learners' scripts with the aim of noting any errors and misconceptions arising from the learners' written responses. I also plan to observe a 45-minute remedial lesson to be conducted by your teacher. The purpose of this lesson observation will be to find out how the teacher engages learners in addressing the identified errors and misconceptions. I intend observing the teacher's strategies in dealing with the identified errors and misconceptions.

After the afore-mentioned lesson observation, a second semi-structured interview will be conducted with the teacher. This interview will last for, at most 35 minutes. The purpose of this interview will be to gather the teacher's evaluative thoughts based on their engagement with the learners' errors and misconceptions in the remedial lesson conducted.

The research participants will not be disadvantaged in any way. Their personal integrity will be guaranteed and respected. They will be re-assured that they can withdraw from participating in this research at any time without any penalty. There are also no foreseeable risks in participating in this study. The participants will not be paid for participating in this study. Participation in the study is for educational purposes and not to blame or fault finding.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. The individual privacy of the participants will be maintained in all published and written data resulting from the study.

All research data will be destroyed within 3-5 years after completion of the research report.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Edgar Marange

9098 Orleander Street, Protea Glen, Extension 12. 1919

mande.078@gmail.com

Mobile: 0814590065/0781096596

Research Supervisor: Dr Judah Makonye

Phone : 011 717 3206

Mobile: 078 689 4572.

Email : Judah.Makonye@wits.ac.za

Appendix 9: Information sheet to teacher and HOD

University of the Witwatersrand, Wits School of Education, 97 St Andrews Rd, Parkton, Johannesburg,

DATE:

Dear ……………………………………………………………………………………………….

My name is Edgar Marange. I am a Master of Education (in Mathematics) student in the School of Education at the University of the Witwatersrand.

I am doing research on: **Secondary Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level.** My focus is on public schools in Johannesburg. I want to explore how, if at all, grade 11 Mathematics teachers make use of learner errors and misconceptions as conceptual tools in their teaching. My research is centred on quadratic inequalities, a topic taught in the first term of the school Mathematics curriculum. I have noticed that quite a number of learners struggle with this topic both in grade 11 and grade 12 examinations. I am also motivated by the fact that the topic is taught in grade 11 only but regularly examinable in grade 12 Mathematics paper one. Further to this, it is frequently reported in the grade 12 Mathematics examinations diagnostic reports that learners persistently make certain errors in the topic under consideration. I strongly believe that one of the ways in which learner errors and misconceptions can be dealt with is to examine how Mathematics teachers use the learners' errors and misconceptions as conceptual tools for mediating learning. My research intends to contribute to South Africa's school Mathematics education through exploring ways of improving the teaching and learning of school Mathematics.

My research involves grade 11 Mathematics teachers and learners as participants. I intend to carry out two semi-structured interviews and two lessons observation sessions with you as a participant. The interviews and one of the lessons (second lesson) to be observed will be audio-taped. This shall be done outside the normal teaching time in order to minimize interfering with your normal teaching activities.

The first interview is to last at most 35 minutes. It is meant to determine your knowledge of learner errors and misconceptions in inequalities in general and quadratic inequalities in particular. I intend gathering this knowledge from your experience of teaching inequalities. I also want to find out how, if it all, you make use of learner errors and misconceptions as conceptual tools for mediating learning. This interview will be followed by a short-written task in which you will respond to questions involving quadratic inequalities. The written task will take at most 10 minutes of your time. The purpose of this task will be to substantiate your knowledge as gathered in the interview. This will be followed by a short test which you are to administer to the learners that you teach. The test is going to be administered in my presence and will last for 30 minutes.

We are going to mark the learners' scripts together with the aim of noting any difficulties arising from the learners' written responses. Identified errors and misconceptions are going to form the basis of the second lesson observation which I will request you conduct. I intend to be a non-participant observer in this lesson. The duration of the lesson should be 45 minutes. The purpose thereof will be to find out how you plan and engage learners in dealing with the errors and misconceptions in the written test. I also intend observing how you respond to the incorrect verbal responses of your learners as you conduct your remedial lesson. Thus, I would like to observe your ways of dealing with learner errors and misconceptions.

After the aforementioned lesson observation, I intend to conduct a second semi-structured with you. This interview will last at most 35 minutes. The purpose of this interview will be to gather your evaluative thoughts based on your engagement with the learner's errors and misconceptions in the remedial lesson conducted.

The reason I have chosen your school is because of your rich experience in teaching grade 11 and 12 Mathematics as well as your experience as an examiner of the matric examinations. Therefore, I have purposively and conveniently chosen you to participate in this study.

I am wondering whether you would mind my invitation for you to participate in this study.

I need your help in participating in two semi-structured interviews (each interview is to last at most 35 minutes), writing of a short task involving quadratic inequalities (10 minutes) and two lessons (administering a 30 minutes test to your learners and conducting 45 minutes remedial lesson). I am going to audio-tape the two interviews and remedial lesson to be conducted. The purpose for the audio-recording will be for the accurate transcription of the data when doing data analysis of my research report.

You will not be advantaged or disadvantaged in any way by participating in this study. Your participation is voluntary. You can withdraw your participation in this study at any time during this project without any penalty or loss of benefits. You will also not be remunerated for participating in this study. Your withdrawal is re-assured should you feel so. There are no foreseeable risks in participating in this study.

Your personal and professional integrity will be guaranteed and respected at all times. Your participation in this study is for educational purposes and not to blame or fault finding.

Your name and identity as well as that of your school will be kept confidential at all times and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the research report.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Thank you very much for your help.

Yours sincerely,

SIGNATURE…………………………………………………………………….
Date……………………………………………

Edgar Marange

9098 Orleander Street, Protea Glen, Extension 12.

mande.078@gmail.com

Mobile: 0814590065/0781096596

Research Supervisor: Dr Judah Makonye

Phone : 011 717 3206

Mobile : 078 689 4572.

Email : Judah.Makonye@wits.ac.za

Appendix 10: Information sheet to the Parents

University of the Witwatersrand

Wits School of Education

Name of school of learner

23 July 2018

To: ____ s' parent/guardian.

REF: Information Sheet

My name is **Edgar Marange.** I am a Mathematics teacher and studying Master of Education (mathematics) in the School of Education at the University of the Witwatersrand.

I am conducting a study on: secondary Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level. My focus is on public schools in Johannesburg. I want to explore how, if at all, grade 11 Mathematics teachers think and act in response to learner errors and misconceptions in their teaching. My research is centred on quadratic inequalities, a topic taught in the first term of the school Mathematics curriculum. I have noticed that quite a number of learners struggle with this topic both in grade 11 and grade 12 examinations. I am also motivated by the fact that the topic is taught in grade 11 only but regularly examinable in grade 12 Mathematics paper one. Further to this, it is frequently reported in the grade 12 Mathematics examinations diagnostic reports that learners perpetually make certain errors in the topic under consideration. I strongly believe that one of the ways in which learner errors and misconceptions can be dealt with is to examine how Mathematics teachers use these errors and misconceptions as conceptual tools for mediating learning. My research intends to contribute to the South African system through exploring ways of improving the teaching and learning of school Mathematics.

My research involves grade 11 Mathematics teachers and learners as participants. I intend carrying out two semi-structured interviews and two lesson observation sessions with the Mathematics teacher teaching your child. This shall be done outside the normal teaching time in order to minimize interfering with the normal teaching and learning activities of your child. I intend to make use of your child's study time in the afternoon.

The first interview, planned for at most 35 minutes, is meant to determine the teacher's knowledge of learner errors and misconceptions in inequalities in general and quadratic inequalities in particular. I intend gathering this knowledge from the teacher's experience of teaching inequalities. I also want to find out how, if it all, the teacher makes use of the errors and misconceptions in their teaching. This interview will be followed by a short-written task that the teacher will respond to involving quadratic inequalities. The written task will take at most 10 minutes. The purpose of this task will be to substantiate the teacher's knowledge as gathered in the interview. This will be followed by a lesson in which the teacher is to administer a short test to the learners (your child included) that he or she teaches. The test is going to be administered in the presence of the researcher (I will be a non-participant observer) and will last for 30 minutes. The teacher and I are going to mark the learners' scripts with the aim of taking particular note of any difficulties learners encounter when dealing with quadratic inequalities.

Identified errors and misconceptions are going to form the basis of the second lesson observation. I intend observing a 45-minute lesson for which your child will be a participant. The purpose of this lesson observation will be to find out how the teacher engages learners in addressing errors and misconceptions identified in their written task. This will also include any incorrect verbal responses arising from the learners during the lesson. I intend observing the teacher's strategies in dealing with the identified errors and misconceptions.

After the above lesson observation, a second semi-structured interview will be conducted with the teacher. This interview will last at most 35 minutes. The purpose of this interview will be to gather the teacher's evaluative thoughts based on his or her engagement with the learners' difficulties in the remedial lesson conducted. I also plan to audio-tape the remedial lesson that your child will participate in. The purpose for this recording will be for accurate transcription when writing part of my research report. This will ensure that I do not miss out some important information of the lesson observation.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing about the study. Your individual privacy and that of your child will be maintained in all published and written data resulting from this study.

All research data will be destroyed within 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

Edgar Marange

9098 Orleander Street, Protea Glen, Extension 12.1919

mande.078@gmail.com

Mobile: 0814590065/0781096596

Signature ____________________Date ___________________

Research Supervisor: Dr Judah Makonye

Phone : 011 717 3206

Mobile : 078 689 4572

Email : Judah.Makonye@wits.ac.za

Parents' Informed Consent Form for written task (30 minutes test to be written by the parent's child).

University of the Witwatersrand

Wits School of Education

Informed Consent Form for Conducting Research in Mathematics classrooms

Please fill in the reply slip below if you agree to let me use your child's responses to the tasks on quadratic inequalities. I will use your child's written responses for my study called: Secondary Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level.

Parents' Informed Consent

I ____________________________ the parent/guardian of __________________

1. Hereby confirm that I have been informed by ………………………………………. about the nature of the study. Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
3. I am aware that my child's response in the test will be processed without mentioning his/her real name. Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No
5. My child can at any stage, without prejudice, withdraw his/her participation in the study. Yes/No
6. I have had sufficient time to ask questions and (of my free will) give consent to my child to write the research tasks.

Signature of parent: ____________________________________ Date: ________

Details of contact person:

Name: __________________________Phone: ____________________________

Parent audio recording Consent Form (45 minutes lesson observation in which the parent's child is to participate)

University of the Witwatersrand

Wits School of Education

Informed Consent Form for Audio Recording in Mathematics classrooms

Please fill in the reply slip below if you agree to let me audio record an interview in which your child will be a participant. I will use the audio-recording in my studies called: **Secondary Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level.**

The purpose of audio-recording of the lesson that I am going to observe will be for accurate transcription when analysing the data of my research report.

Parents' Informed Consent

I ______________________________ the parent/guardian of ____________________

1. Hereby confirm that I have been informed by ………………………………………… above the nature of the study. Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study. Yes/No
3. I am aware that my child's responses in the audio recording will be processed without mentioning his/her real name. Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No
5. My child can at any stage, without prejudice, withdraw his/her participation in the study. Yes/No
6. I have had sufficient time to ask questions and (of my free will) give consent for my child to write the research tasks. Yes/No

Signature of parent: _____________________________________ Date: ________

Details of contact person:

Name: ____Phone: __________

Appendix 11: Information sheet to the learners

University of the Witwatersrand

Wits School of Education

Address of school

13 September 2018

To: ____

REF: Information Sheet

My name is Edgar Marange. I am studying for a Masters of Education (Mathematics) in the School of Education at the University of the Witwatersrand.

I am doing research on: secondary Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level. My focus is on public schools in Johannesburg. I want to explore how, if at all, grade 11 Mathematics teachers make use of learner errors and misconceptions as part of resources for teaching.

My research is centred on quadratic inequalities, a topic taught in the first term of the school Mathematics curriculum. I have noticed that quite a number of learners struggle with this topic both in grade 11 and grade 12 examinations. I am also motivated by the fact that the topic is taught in grade 11 only but regularly examinable in grade 12 Mathematics paper one

My research involves grade 11 Mathematics teachers and learners as participants. I intend to administer a short task to you. The duration of the task is 30 minutes and your teacher is going to supervise it. This will be followed by a remedial lesson. I intend observing this lesson using an observation protocol. This lesson will be based on errors and misconceptions identified from your written task. The lesson will last for 45 minutes and it will be audio-recorded. The purpose for this will be for the accurate transcription of your responses when doing my data analysis. Both the task and the remedial lesson are going to be done outside your normal conduct time. I am going to make use of your study time. This is in order to minimize interfering with your normal learning activities.

I am wondering whether you would mind my invitation for you to participate in this study.

I need your help in participating in the writing of the short task involving quadratic inequalities as well as your participation in the lesson that your teacher is going to conduct.

Remember, the task is not going to be used for promotional purposes; it is not for marks and it is voluntary, which means that you have the freedom not to do it. The task is also not meant to undermine your knowledge or blame you. In addition, if you decide halfway through that you prefer not to continue, this is completely your choice and will not affect you negatively in any way. Your participation is solely to assist me in doing my project.

I will not be using your name but I will make one up so that no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed within 3-5 years after I have completed my project.

Your parent/guardian is also going to be given an information sheet and consent form, but at the end of the day it is your decision to join me in my study.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Edgar Marange

9098 Orleander Street, Protea Glen, Extension 12.

mande.078@gmail.com Mobile: 0814590065/0781096596

Learner Assent Form

Secondary school Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level

Please fill in the reply slip below if you agree to participate in my study called:

My name is: ________________________

Kindly circle the appropriate response

Permission for a task

I agree to write a test for this study. YES/NO

Permission to observe you in class

I agree to be observed in class. YES/NO

Permission to be audiotaped

I agree to be audiotaped during the observation lesson YES/NO

I know that the audiotapes will be used for this project only YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign_____________________________ Date___________________________

Learners' Informed Assent Form for writing task.

University of the Witwatersrand

Wits School of Education

Informed Consent Form for Conducting Research in Mathematics classrooms

Please fill in the reply slip below if you agree to let me use your responses to the tasks on quadratic inequalities. I will use your responses for my studies called:

Secondary school Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level

Learners' Assent

I ____

1. Hereby confirm that I have been informed by ………………………………………… about the nature of the study Yes/No
2. Have also received, read and understood the Information and Consent sheets regarding the educational study.
   Yes/No
3. I am aware that my responses in the test will be processed without mentioning my identity.
   Yes/No
4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher.

   Yes/No
5. I can at any stage, without prejudice, withdraw my participation in the study.

   Yes/No
6. I have had sufficient time to ask questions and (of my free will) consent to write the research task.
   Yes/No

Name of learner: _______________________________Signature of learner:_____________________ Date: ____________

Learners' audio recording Consent Form

University of the Witwatersrand

Wits School of Education

Informed Consent Form for Audio Recording in Mathematics classrooms

Please fill in the reply slip below if you agree to let me audio tape you during the lesson observation. I would like to audio-tape the lesson that your teacher is going to present. I will use the audio-recording in my study called:

Secondary school Mathematics teachers' pedagogic reasoning and action in relation to learner errors and misconceptions in quadratic inequalities at grade 11 level

The purpose of the audio-recording will be for accurate transcription when writing my research report.

Learners' Informed Consent

I ____

1. Hereby confirm that I have been informed by ………………………………………. above the nature of the study.
Yes/No

2. Have also received, read and understood the Information and Consent sheets regarding the educational study.
Yes/No

3. I am aware that my responses in the test and audio-recording will be processed without mentioning my real name.
Yes/No

4. In view of the requirements of the research, I agree that the data collected during this study can be processed in a computerized system by the researcher. Yes/No

5. I am aware that I can at any stage, without prejudice, withdraw my participation in the study. Yes/No

6. I have had sufficient time to ask questions and of my free will consent to be audio-taped. Yes/No

**Name of learner: __________________________Signature of learner: ---------------------------------
--Date: -------**