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VOLUME I

(Main sections)

AN ETHNOMATHEMATICAL STUDY OF TCHADJI – ABOUT A MANCALA TYPE
BOARDGAME PLAYED IN MOZAMBIQUE AND POSSIBILITIES FOR ITS USE IN
MATHEMATICS EDUCATION

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*A thesis submitted to the Faculty of Science, University of the Witwatersrand, in fulfillment of the
requirements for the degree of Doctor of Philosophy*

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DECLARATION

I declare that this thesis is my own, unaided work. It is being submitted for the Degree of Doctor of Philosophy in the University of Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

A handwritten signature in black ink, appearing to read 'Ismael', with a large, stylized initial 'I'.

(Abdulcarimo Ismael)

Johannesburg, May 30th, 2002

DEDICATION

To my wife

Rita (Reyana) and my daughters Livra and Laysha

for their love and understanding.

Abstract

AN ETHNOMATHEMATICAL STUDY OF *TCHADJI* – ABOUT A MANCALA TYPE BOARDGAME PLAYED IN MOZAMBIQUE AND POSSIBILITIES FOR ITS USE IN MATHEMATICS EDUCATION

Abdulcarimo Ismael

This research is in the field of ethnomathematics. The study was carried out in three phases with the *Tchadji*-game being its principal focus. *Tchadji* is a traditional game played at Ilha de Moçambique, a small Island situated on the northeast coast of Mozambique. *Tchadji* belongs to a class of board games known as Mancala, which are thousands of years old and are played throughout the African continent as well as in other parts of the world. Mancala games have only recently received due attention as a topic for research.

The first phase of the research was carried out in the school mathematics classroom. The outcomes of this phase indicated that games like *Tchadji* and the three stones (a variant of the *Muravarava* game) as a part of Mozambican culture are also rich in providing opportunities for activities in the mathematics classroom related to the development of key probabilistic concepts. Quasi-experimental research involving the researcher, 4 mathematics teachers and 162 students showed statistically significant positive effects on attitude towards the learning of, and performance in probability. These outcomes were corroborated by qualitative research.

The second phase of this study consisted of ethnographic research amongst master *Tchadji* players, which explored the mathematical ideas embedded in the *Tchadji*-game and in the procedures for playing the game. The results of this phase of the research revealed that, the *Tchadji*-players had mathematical knowledge, skills and ways of thinking such as counting, logical thinking, calculation, visualisation, recognition of different numerical patterns and infinity. These results are described in terms of examples taken from different critical moments of the recorded matches of *Tchadji*. This way of

presenting the results gives insight into the complexity of the methodology used. It also uncovers different aspects of playing *Tchadji*, like rules, strategies and tactics. A need for further investigations of this nature for uncovering mathematical ideas in traditional culture is also indicated by this research.

As advocated in ethnomathematical research an intervention with (pre and in-service) student teachers formed the third phase of the research. The results of this phase indicated that the 24 teachers, who participated in the research, showed enthusiasm, satisfaction and excitement in experiencing the mathematical richness of *Tchadji* and in appreciating possibilities for the use of *Tchadji* in the mathematics classroom. They were able to analyse the game independently and to identify embedded mathematical ideas in the game, like logical thinking, counting and empirical and mental calculation.

The research makes contributions to the field of ethnomathematics itself, to ethnographic research methodology and to the pedagogy of mathematics.

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LIST OF ABBREVIATIONS

SNE.....	National System of Education
MERP	Mozambique's Ethnomathematics Research Project
SAREC.....	Swedish Agency for Research Cooperations with Developing Countries
SIDA.....	Swedish Authority for International Development
UP.....	Mozambique's Pedagogical University
UEM.....	Mozambique's Eduardo Mondlane University
MINED.....	Mozambique's Ministry of Education
ISP.....	Mozambique's Higher Pedagogical Institute
ISRI.....	Mozambique's Higher Institute for International Relations
ISPU.....	Mozambique's Higher Institute Politechnic and University
ISCTEM.....	Higher Institute of Science and Technology of Mozambique
ICTM.....	Higher Institute of Transport of Mozambique
UCM.....	Catholic University of Mozambique
UMBB.....	Mozambique's University Mussa Bin Bique
SPSS.....	Statistical Package for Social Sciences
ANCOVA...	Analysis of Covariance
SD.....	Standard Deviation

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CHAPTER ONE: INTRODUCTION

1.1 The research context and Motivation

1.1.1 Education in Africa: a general concern

In most formerly colonized countries in Africa, education did not succeed in appeasing the hunger for knowledge of the masses, in spite of efforts taken after independence and years later. This process was accompanied with a dramatic explosion in numbers of students in schools. In terms of efforts in this respect, Mozambique, for example, has adapted the existing educational system to respond to the immediate needs of the society and has established teacher training centres (See section 1.1.4).

The mean illiteracy rate for Africa, for example, was still 66% in 1980. In most of these African countries there are certainly other factors, like overcrowded classrooms as a result of the explosion in student numbers, a shortage of qualified teachers, and a lack of teaching resources. These factors have certainly also contributed to low levels of attainment (Cf. Gerdes, 1988).

Concerned with this situation in Africa some organizations, amongst which UNESCO, based in Paris (France), organised commissioned studies, which resulted in some important publications appearing in the early 1990s.

"The challenge of the South", The Report of the South Commission, led by the former President of Tanzania, Julius Nyerere, criticizes development strategies that minimize cultural factors. Such strategies only provoke indifference, alienation and social discord. The development strategies followed until today *"have often failed to utilize the enormous reserves of traditional wisdom and creativity and enterprise in the countries of the Third World. Instead, the cultural wellsprings of the South should feed the process of development"* (p.46).

In "**African Thoughts on the Prospects of Education for All**", a selection from papers commissioned for the Regional Consultation on 'Education for All', held in Dakar, in November 1989, the crisis of African cultures is emphasised as the issue of African cultural identity (p.10). Africa needs culture-oriented education that would ensure the survival of African cultures. Originality of thought and creativity are encouraged (p.15). Scientific appreciation of African cultural elements and experience is considered to be *"one sure way of getting Africans to see science as means of understanding their cultures and as tool to serve and advance their cultures"*(p.23).

"Educate or Perish: Africa's Impasse and Prospects", a study directed by the great African historian Joseph Ki-Zerbo, shows that today's African educational systems favour foreign consumption without generating a culture that is both compatible with the original civilization and truly promising. Education for all, as discussed by Ki-Zerbo, should be an attempt to encourage the development of initiative, curiosity, critical awareness, individual responsibility, respect for collective rules, and a taste for manual work. Africa needs a *"new educational system, properly rooted in both society and environment, and therefore apt to generate the self-confidence from which imagination springs"* (p. 104).

1.1.2 The Mozambican Educational System

Mozambique is situated on the south-eastern coast of the African Continent. It has an area of 801 590 square kilometres. According to the 1997 Population Census, today the country has a population of about 15.278.334 people¹, mostly Bantu, with small minorities of Arab, European, Indian, and Chinese origin, and a considerable population of mixed origins (INE, 1999).

Mozambique gained independence from Portugal in 1975 after ten years of liberation struggle. The newly independent country inherited a little-developed school network and an illiteracy rate of about 95%. Besides the Portuguese language, which is the mother

¹ <http://www.ine.gov.mz/>

tongue of only 6,5% of Inhabitants, most children have a Bantu language as mother tongue (See figure 1.1). Portuguese was declared as the language of instruction in the country and it was considered to be the "symbol of national unity." English and French are also taught in secondary schools in Mozambique.

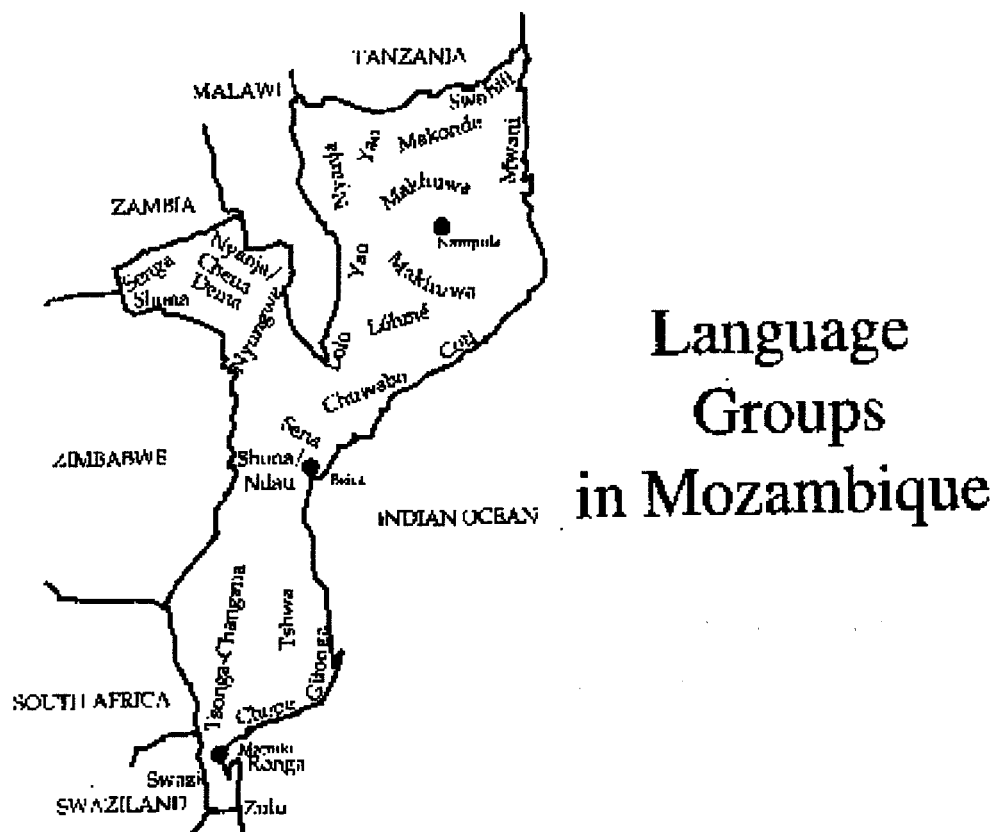


Figure 1.1: Geographical distribution of Bantu languages in Mozambique (Gerdes, 1993)

Following the immediate efforts taken towards the situation of education in the country after independence and as a result of much debate, a new National Educational System (SNE) was established in 1983. General education in the SNE includes the primary cycle of 7 compulsory years (Grades 1-7) of schooling and the secondary cycle with a total of 5 years (Grades 8-10 as the lower secondary and grades 11-12 as the upper secondary or pre-university). The vocational and technical training includes three levels of general education. All schools in Mozambique are co-educational.

Besides the technical and vocational training Institutes in the area of Agriculture, Commerce and Industry, with an entry level equivalent to completion of lower secondary school (grade 10) of SNE, there are three other public institutions for further training. The first public institution is the Eduardo Mondlane University (UEM), former 'Universidade de Lourenço Marques', which was established in 1962 during the colonial period. The second is the Pedagogical University (UP), former 'Instituto Superior Pedagógico', the first higher education institution founded after independence in 1986. The UP is a vocational institution specialised in the training of teachers for all levels of instruction in Mozambique. The third institution is the 'Instituto Superior de Relações Internacionais' (ISRI) created in 1986, where training takes place in the area of international relations and diplomacy mainly for officials of the Ministry of Foreign Affairs and Cooperation.

In early 1994, a legislation permitting the constitution of private educational institution at all level of instruction was approved by Parliament. Since that period five other tertiary institutions (the 'Universidade Católica de Moçambique' (UCM), the 'Instituto Superior Politécnico e Universitário' (ISPU), the 'Instituto Superior de Ciências e Tecnologia de Moçambique' (ISCTEM), the 'Universidade Mussa Bin Bique' (UMBB) and 'Instituto Superior dos Transportes de Moçambique' (ISTM)) were created as well as many private schools around the country.

However, the contribution of private institutions to post-secondary training, in terms of the number of students enrolled in these institutions, is still very small. One of the reasons for

this situation is the fact that academic fees are very high and the large majority of the people cannot afford them.

1.1.3 Learning mathematics in school: issues of concern

In Mozambique, but also in many other countries, 10-15% of the pupils' time is devoted to mathematics². Mathematics is also included as a compulsory subject in practically every curriculum, at any level of instruction, in all countries of the world (Cf. D'Ambrosio, 1990). For example, *"a strong mathematical background is a pre-requisite to many career and job opportunities in the technological world today"* (El Tom, 1984:3). As Bishop (1988a: 1) stated, *"Mathematics is felt to be one of the most important subjects in the school curriculum by educators, parents and society at large - second only to the national language"*.

Particularly, *"African countries face the problem of low 'levels' of attainment in mathematics. Math anxiety is widespread"* (Gerdes, 1995: 6). This is specially so for the majority of young people, who do not succeed in mathematics. They feel it is important, but it is also difficult - impossibly so for many - mysterious, meaningless and boring. Mathematics creates a feeling of fear, feelings of a lack of confidence and, indeed, feelings of hatred. For some it even creates feelings of oppression and of being dominated by someone, they know not whom (Bishop, 1988a).

These aspects are indeed not so surprising, because of the misuse of mathematics. *"Mathematics is universally recognised as the most effective educational filter"* (El Tom, 1984:3) and *"No other subject in school serves so well the purpose of reinforcement of power structure as does mathematics. And the main tool for this negative aspect of mathematics education is evaluation"* (D'Ambrósio, 1983:363).

²In Mozambique about 200 hours are spent per year in secondary school on mathematics.

Why does mathematics provoke such feelings in students? Is the problem with the students/learning or with the teachers/teaching? Is it perhaps in the teaching approach adopted by the teachers or in the selection of material used during mathematics lessons? Is it a combination of more than one factor including some not mentioned here?

Over the past 14 years, as a lecturer at the Mathematics Department of the UP in Maputo (Mozambique), I have been involved in a variety of activities related to mathematics education. My previous work in secondary schools as a mathematics teacher, and later as coordinator of student teaching practices, has led me to observe a variety of practical teaching and learning situations, which might relate to the low 'levels' of attainment in mathematics.

For example, as I observed, in the teaching of mathematics the use of the approach of one-way transmission of knowledge from teacher to the students has been dominant. In most cases I observed, mathematics teachers have been teaching without taking into consideration the prior knowledge, which the students have from other experiences in- and outside school.

Also, concerning the students, their participation in the process of learning mathematics has been very poor, their interest in and motivation for the subject and their attitudes towards mathematics have not been given the necessary attention. I believe that, as a consequence, achievement in mathematics has been very poor, the failure rates in mathematics have increased and the performance of Mozambican pupils in mathematics has been generally very weak.

Resnick (1989) argued that generally, schools place too much emphasis on the transmission of syntax (procedures) rather than on the teaching of semantics (meaning) and this "*discourages children from bringing their intuitions to bear on school learning tasks*" (p.166). In fact, in most schools knowledge acquired all too often grows out of the transmission paradigm of instruction and is largely devoid of meaning, i.e., there is a lack

of context, relevance and specific goals. This could be the case in the Mozambican educational system.

How to improve this situation? In Mozambique, there is a need for efforts to improve the quality of education in general and of mathematics education in particular. There is a need for renovation in all areas of the education system. Studies have to be conducted to understand the real problems and give more justified recommendations for solutions in terms of actions. As Ki-Zerbo stressed "*all educational renovation in Africa has to be based on research. In fact, in Africa there is generally a surprising lack of research to back up proposals for educational reforms*" (quoted in Gerdes, 1995: 8). Mozambique is not an exception.

With respect to mathematics education, researchers of different areas need to conduct studies with the main concern of increasing the quality of mathematics education. It was also with the problem of the quality of mathematics education in Africa in mind that, during the 1970s and 1980s, emerged as a general concern among mathematics educators in Africa. In order to improve the quality of mathematics education, to make it more interesting, to enhance the social and cultural confidence of the pupils in their mathematical abilities, to improve the pupil's motivation, several studies point to the importance of integrating mathematical traditions and practices into the school curriculum (Cf. for example: Ale, 1989; Doumbia, 1989; Eshiwani, 1979; Jacobsen, 1984; D'Ambrosio, 1985a; Bishop, 1988b; Zaslavsky, 1989a; Gerdes, 1985b, 1988, 1995).

1.1.4 Mathematics teacher education in Mozambique

Before independence, most secondary and higher education teachers in school were Portuguese. Most of them returned to Portugal after independence. As a result of increased school access, an educational explosion took place in the country. So, there was an urgent need to rapidly train teachers.

It was in the enthusiastic context of the first years after independence that mathematics teacher education started in 1977, at UEM, the then only university in the country. At that time, there were only five trained Mozambican secondary mathematics teachers, so intensive and accelerated teacher training programmes became necessary. The first teacher education programme for secondary school was of ten months duration and with an entrance requirement of only grade 9 (Gerdes, 1980, 1981; see also Gerdes, 1998b).

In spite of a liking for mathematics, the few students who entered the Mathematics Teacher Education Programme, spoke frequently about mathematics as something from another world, as 'the beast with seven heads', apparently having no utility to society and no roots in the Mozambican and African cultures. The course on "Applications of Mathematics in the Daily Life of the Population" introduced into the programme, included examples, which after were relevant for the development of the country. These examples were also part of the experience of the majority of the population, which means in the lives of the peasants (Cf. Gerdes, 1982, 1990, 1998).

For example, one student in the mathematics teacher training course explained how in his home region the rectangular bases for traditional houses were constructed on the ground by beginning with the sides: *"The house builders start by laying down on the floor two long bamboo sticks of equal length. Then these first two sticks are combined with two other sticks also of equal length, but normally shorter than the first ones. Now the sticks are moved to form a closure of a quadrilateral. The figure is further adjusted until the diagonals, measured with a rope, become equally long. Then lines are drawn where the sticks are now lying on the floor and the building of the house can begin."* (Gerdes, 1998b: 37).

Another student from another region explained how the rectangular bases for houses were constructed on the ground by beginning with the diagonals: *"The house builders start with two ropes of equal length that are tied together at their midpoints. A bamboo stick, whose length is equal to that of the desired breadth of the house, is laid down on the floor and its endpoints pins are hit into the ground. An endpoint of each of the ropes is tied to one of*

the pins. Then the ropes are stretched and at the remaining two endpoints of the ropes, new pins are hit into the ground. These four pins determine the four vertices of the house to be built" (Gerdes, 1998b: 38).

Gerdes (1998) commented that other students from the same course who had grown up in the towns became surprised. They had never imagined that rectangles could be constructed without starting with constructing the right angles one by one, and using some instrument for their measurement. The examples of students' cultural environments were afterwards used by Gerdes (1988 & 1990) in looking for possible interesting didactic alternatives to the axiomatic constructions of Euclidean geometry in mathematics teacher education (See more in Gerdes, 1988, 1990, 1998b).

In the mid-eighties, the country needed many more teachers, but it could not immediately absorb many more teachers, as a consequence of the destructive and destabilising war it was suffering. A new opportunity was created, namely to raise the entrance level for the secondary school teacher education programme to grade 12, and to extend their duration from (at that time) 2 years to the 5 years of a university 'licenciatura' degree, so that more highly qualified teachers could be trained. This important task was taken over by the newly created institution, 'Instituto Superior Pedagógico, and later renamed 'Universidade Pedagógica' (See section 1.1.2).

The long duration of the programme and the recruitment of both experienced lecturers and young lecturers (most of them recently returned from their studies abroad) and other graduates from the institution itself, interested in the social-cultural aspects of mathematics education gave rise to the possibility of generating new experiences in raising social- and cultural-mathematical awareness among future mathematics teachers. Moreover, this provided the possibility of expanding research on culture, mathematics, and education to be organised as a research project on ethnomathematics.

One of the objectives of the required course "Mathematics in History" for the second year students enrolled in the 'licenciatura' in the teaching of mathematics and physics, is to

contribute to a broader historical, social and cultural understanding of mathematics. The first theme of the course, "Counting and Numeration Systems," gives a good start, because the students can begin to analyse and compare the various ways of counting and numeration they learned in their own spoken and concrete counting systems as part of the reality of their daily life. After they have discovered the rich variety at national level, they then are brought into contact with systems from other parts of Africa and the world, and from other historical periods (Cf. Gerdes, 1993, 1994, 1998b; Ismael & Soares, 1993).

This approach is used to make future teachers aware of mathematical elements in their own culture(s) and in African cultures and has proved to be effective. Also, the fact that the mathematics teacher education classroom in this process became a place of reflection, of experimenting with the raising of mutual cultural awareness, where the social-cultural tensions relate to the differences across rural versus city, manual work versus intellectual work, and models coming from abroad versus models with roots in the cultural environment, has shown very encouraging results (Cf. Gerdes, 1985, 1986, 1998b).

In fourth year, optional courses are offered in physics or mathematics related areas. Two of those courses offered by the ethnomathematics research group have been "Symmetry and Geometry" and "Ethnomathematics and the Teaching of Mathematics in Secondary School." Both courses have a strong cultural component. All participants in these courses are encouraged to do fieldwork on a specific cultural aspect and also to develop specific (sub)themes for experimentation in secondary schools.

The main concern in all the processes described above has been to ponder on the strategies that might motivate all future mathematics teachers, to stimulate teachers to multiculturalise mathematics teaching (Cf., e.g., Gerdes, 1994). How might one enhance their self-confidence? In the case of mathematics, it seemed also important for all mathematics teachers to become aware of mathematical elements in their own culture(s) and in other African cultures. So, this approach helped to transform the mathematics teacher education classroom into a place of reflection, of experimenting with the raising of mutual cultural awareness. The future mathematics teachers here became aware of the existence of

mathematical ideas and practices in their own and their future students' cultural environment and on how useful and necessary it was to take into consideration the children's environment in their teaching (Gerdes, 1981, 1982, 1998b).

This has been a process described by Gerdes (1998b) as *"the process of developing among future mathematics teachers an awareness of the social and cultural bases of mathematics, which contributes both to enhance their self-confidence and their capacity, and readiness and openness to work in a multicultural environment"* (p.33).

Gerdes (1998b) also wrote on possible dimensions of developing this awareness. According to him:

(1) *"it is important for teachers to develop an awareness of mathematics as an universal activity in order to never underestimate the capacities, know-how, and wisdom of their students and the students' communities. It is important for teacher educators to help pre-service and in-service teachers to see how they can give each other examples of mathematics in their life and history, and it is important for (teacher) students to see people like themselves doing mathematics and to reflect on their future in relation to mathematics"*.

(2) Teachers need to be aware that *"generally the development of mathematics is not unilinear, and that the learning of mathematics ideas, even in an apparently homogeneous cultural context, does not need always to follow the same path"* (p.47).

(3) Teachers need to be aware of *"influences of socio-cultural factors on the teaching and learning of mathematics"* (p.48). (4) Teachers need to develop an awareness of the mathematical potential of their students independent of their race, gender, ethnic group, linguistic group, religious group, and so on.

This process of developing an awareness of the social and cultural bases of mathematics in the process of mathematics teacher education in Mozambique still continues. It has to be a continuous process as long as teachers are trained and educated both in-service and pre-service. The use of cultural instruments and activities such as cultural games is a

continuation of this process of developing this awareness among mathematics teachers in Mozambique.

1.1.5 Ethnomathematics: an Alternative?

Having analysed developments in mathematics and particularly in mathematics education (with a focus on ethnomathematics) in post independence Mozambique, I will now locate my work within those developments.

In late 1980s, I became interested on the field of ethnomathematics (See chapter 2) and started working on it. I participated in Mozambique's Ethnomathematics Research Project (MERP), created and headed by Paulus Gerdes (See section 1.1.5.1).

My main interest in the project has been on the educational side, i.e., how ethnomathematics could be useful for the teaching and learning of mathematics. However, during an early period I worked through the "*problematizing reality approach*" suggested by Gerdes (1985b, 1988). The main concern there was to gain greater understanding of the 'hidden' mathematical histories of different cultures, the 'hidden' mathematics of traditional societies, the mathematical activities of different groups in the society (Cf. Ismael: 1989, 1993, 1994; Gerdes [ed.]: 1993, 1994). Gerdes expressed similar concerns in some of his previous work (Cf. Gerdes: 1985a, 1988, 1990, 1991, 1994a, 1994b, 1994c).

On the other hand, Gerdes always stressed the need for didactical uses of ethnomathematical research results in the mathematics classrooms of Mozambique. In this context, I participated in studies involving either teacher students at UP or children, focusing on the possibilities of using ethnomathematical activities in classrooms (See Gerdes: 1988, 1990, 1994a, 1995; Gerdes [ed.]: 1993, 1994c; Ismael, 1996). For example, student teachers have been able to mathematise many experiences and to reflect on the

possibilities of using those experiences in their classrooms (See more in Gerdes [ed.]: 1993, 1994; 1997).

I understand that in this respect much more has to be done since "*Ethnomathematical-educational research, including the study of possible educational implications of ethnomathematical research, is still in its infancy*" (Gerdes, 1996: 927). So, I formed the impression that there is a need to follow specific educational related research questions. I wanted to pursue Gerdes' concerns about ethnomathematics and education in a more specific setting.

1.1.5.1 Mozambique's Ethnomathematical Research Project

Ethnomathematical research in Mozambique has a long tradition and is widely recognized. Founded by Paulus Gerdes, a Mozambican mathematician, mathematics educator, teacher educator and anthropologist, MERP³ has made important contributions to the development of ethnomathematics as a research area.

In the words of Mary Harris (1987), "*Gerdes' work, and the work of others in the field of ethnomathematics offer a rather threatening confrontation to the traditional standard setters. Gerdes is up against a number of factors that until recently have tried to determine the education, or previous lack of it, in his country. The freshness of his work and his illustration of the mathematics that already exists in Mozambican culture and how he is setting about 'defrosting' it (...)*"(p.26).

In the opinion of Alan Bishop (1989), "*Gerdes, in Mozambique, (...) has done a great deal of work in this area [developing a greater awareness of one's own culture]. He seeks not only to demonstrate interesting mathematical aspects of Mozambican life but also to*

³This project has as general objectives: the reconstruction and analysis of mathematics in African cultures in general and in Mozambican cultures in particular, the elaboration of a methodology for this reconstruction, the analysis and experimentation of the possibilities of using cultural elements of African societies in mathematics education and, on the basis of the reconstructed mathematics, reflection on didactical alternatives, on (early) history of mathematics and on epistemological and philosophical problems of mathematics.

develop the process of 'defreezing' the 'frozen' mathematics, which he uncovers. For example, with the plaiting methods used by fishermen to make their fishtraps, he demonstrates significant geometric ideas which could easily be assimilated into the mathematics curriculum" (p.13).

Bill Barton (1999) also notes support as follows: *"Paulus Gerdes's magnificent research project associated with teacher education in Mozambique is also a paradigm of what can be achieved when a good idea is accompanied by total commitment and dedication"* (p.32)

1.1.5.2 Looking for local resources: cultural games

Learning does not occur in a vacuum of abstractness. The learner is living in an environment and his/her learning is affected by the context provided by that environment. The claim is that, people learn very much better from lived contexts and can remember the good things that they have experienced. In this case, there is less abstractness, the learner has some degree of familiarity with the things he/she is experiencing from his/her environment (See also section 2.2.1.3.2).

Teaching by considering the local environment was a general concern among researchers and mathematics educators in Africa, during the 1970s. In 1978, at an International Colloquium on mathematics and Development, held in France, it was proposed to give consideration to the 'local environment' in mathematics education⁴.

El Tom (1979) at the International Conference on Developing Mathematics in 'Third World' Countries, held in Sudan, advocated that *"Pupils should learn mathematics as far as possible through active practical experience and with learning aids drawn from the environment. Generalizations and structures should come out of pupils' experiences, rather than from formal assertions from the teachers"* (p.182).

⁴see more in 'rapport final du Colloque International 'Mathématiques et Développement"', 1978: 2

Ginsburg (1978) also argued that, *"teaching of basic skills could be more effective if the curricula were oriented to the particular styles of each culture. For African children, the answers seem obvious: to be effective, curricula must be responsive to local culture"* (p.42). He continued by arguing that, *"the same is likely to be true for subgroups of the American poor"* (Ginsburg, 1978:43). So, the issue of learning from practical experiences and from the local environment seems not to be exclusively an issue relevant to third world countries.

Games, in general, are part of human culture and are part of the local environment. Zaslavsky (1989b, AMUCHMA 3) suggested that *"it could be important in the reconstruction of the history of mathematical thinking in Africa to investigate further mathematical aspects of traditional games"*(p.6).

In the early 1990s, the research team of MERP identified as one of its tasks, the exploration of the educational potential of traditional games (Cf. Gerdes, 1992) with mathematical *"ingredients"* (Zaslavsky, 1973a), in particular games involving string figures, games of the three-in-a-row type (in Mozambique called *Muravarava*) and Mancala-type games (Cf. Zaslavsky, 1982).

Also, the editorial group of the first edition of the International Study Group of Ethnomathematics (ISGEm) stressed that *"(...) examples of Ethno-mathematics derived from culturally identifiable groups, and related inferences about patterns of reasoning and models of thought can be led to curriculum development projects that build on the intuitive understandings and practiced methods students bring with them to school. Perhaps the most sticking need for such curriculum development may be in Third World countries, yet there is mounting evidence that schools in general do not take advantage of their students' intuitive mathematical and scientific grasp of the world"* (ISGEm-Newsletter, 1985, Vol.1)

With regard to *N'tchuva*, another Mancala type game of four rows played in Mozambique, a newspaper article ('Notícias', 26.10.1976), entitled "*N'tchuva in our schools?*"⁵, proposed the introduction of this game in schools in Mozambique instead of Chess, claiming that "*N'tchuva, this game of holes and stones, is a game which can not only develop deductive thinking but also quick thinking ability. So, why not consider introducing this game in our schools as part of recreational activity through championships and tournaments?*" (My translation).

Much later in another short article in a Sports newspaper ('Desafio', 10.04.1995)⁶, which included a photograph of a group of young people playing *N'tchuva*, the need to disseminate this cultural practice among young people was expressed, since they really felt interested in these games.

1.2 The research problem

My recent research interest in the area of Ethnomathematics has been to analyse so-called traditional games, in particular, the game called *Tchadji* (sometime also called *M'palé*), which is a Mancala type game played at Ilha de Moçambique (see more in chapter 5). Since I was born at this area in the North of Mozambique, I was quite familiar with the game. Indeed, whenever I went to the Island on vacation, I used to spend some time watching people playing *Tchadji*.

In the context of MERP, I organized extra-curricular activities with student teachers. The focus of these activities was to analyse the game from a mathematical point of view, in particular from a probabilistic point of view. Results of this preliminary work and related reflections were included in short papers (Ismael, 1996 & 1997). The present study represents also a continuation of this work (See section 6.2.1).

⁵ 'Noticias' is published in Portuguese.

⁶ 'Desafio' is also published in Portuguese.

Muravarava is another game with many variations, which is practised in Mozambique. For example, there is a simple variant of this game called three stones (See more in chapter 6). Card games and games using coins are other games with mathematical ingredients known and played in Mozambique. Doumbia (1992) suggested the use of cowry shells for teaching probability. Within MERP, my general concern has always been on how to integrate cultural practices in the teaching of mathematics in Mozambique, i.e., to teach mathematics by adopting an increased level of cultural orientation, and then to determine the impact this has on learning. So, the questions in my mind have been **how I could develop an approach for teaching the unit of probability in school by using those games and what the impact of implementing it would be.**

Thus, the **first phase** of study within MERP was a **mathematical investigation of four types of cultural games** (*Tchadji*, the three stones game as variant of *Muravarava*, Card games, and Coin and Cowry shells games) in order to make apparent any underlying probability concepts in playing these games, and to relate the potential of these games to the teaching of probability at secondary school level and subsequently develop appropriate learning and teaching materials. The main issue was to develop an appropriate probability teaching approach and to conduct experimental research in secondary school mathematics classrooms, in order to determine the effects of teaching probability by using the four types of games (game-approach) on attitudes towards the learning of mathematics, on motivation for the study of probability, on performance in a test on probability, and on the understanding of critical probability concepts.

The use of cultural games (traditional games) for the purpose of teaching probabilistic ideas has not been much researched so far. The reactions of the pupils, their engagement and performance when confronted with cultural games, in particular, this combination of four games, in the mathematics classroom have, to my knowledge, not been published yet. How do the students in the mathematics classroom cope with games? Do they enjoy learning mathematics through games or do they react negatively?

On the other hand, cultural games and their playing processes, specially the Mancala type games are widely recognized by mathematicians, educators and even anthropologists as involving various mathematical aspects such as logical thinking, mathematical thinking, counting, calculation skills, geometrical reasoning and problem solving. It seems that the process of playing Mancala games, for example, is empirically related to calculations, counting and recognition of geometrical patterns in the game. A good player necessarily possesses these skills. However, the literature fails in most cases to describe these skills (Cf. Zaslavsky, 1973a, 1989; Bell & Cornelius, 1988; Russ, 1984; Barta & Schaelling, 1998; Bishop, 1991; Doumbia, 1989; Doumbia & Pil, 1992; Silva, 1994, 1995; Lumpkin & Strong, 1995; Cheska, 1987; Odeleye, 1977).

Some researchers have argued that the level of mathematics used in Mancala does not extend beyond counting and simple addition, as to casual observers this appears to be all that is required. My understanding is that the Mancala type games involve a considerable amount of thinking and reflection, which might relate to mathematics. The playing of *Tchadji* in Mozambique has a long tradition. Regarding it, I could not find any book or article specific to this topic. It seems that the game and the mathematical ideas embedded in it are not documented at all, besides the apparent recognition of its mathematical potential.

Probabilistic ideas in the *Tchadji*-game were previously researched in the first phase of this study. The methodology employed has been the possible in that phase. A need emerged to investigate other possible mathematical ideas embedded in the *Tchadji*-game. I felt that a contact with the game and its players in their playing environment would be beneficial for this kind of research. For this reason, I felt it to be important to undertake ethnographic research.

Thus, the aim of the **second phase** of the study was **to investigate the mathematics embedded in the *Tchadji*-game amongst its players and to document it**. This phase of the study is in the context of exploring mathematical aspects of cultural activities, in this

case the *Tchadji*-game, with the purpose of their possible use in the mathematics classroom.

Some of the interventions of MERP in the educational area, have been in teacher education through the introduction of optional courses, "circles of interest" or clubs centred on "Ethnomathematics and Education" with a strong cultural component (See more in Gerdes, 1998a). Bishop (1991) points to teacher education as being the key focus for research in ethnomathematics to be used so that, correct values go back to the classroom. Once again in theory this is fine but work needs to be done to ensure methods of practical implementation and to create the necessary resources for teacher educators, teachers, students and pupils. Also, it has long been suggested that ethnomathematics provides teachers with useful resources and helps motivate students to study mathematics (Cf. Zaslavsky, 1991; Gerdes, 1989).

Taking into consideration the concerns of MERP, i.e., the incorporation of cultural traditions in the curriculum, and considering the role that the teacher plays in this process as stressed above, the **third phase** of this study sought **to investigate, by means of a workshop, whether or not with the *Tchadji*-game, at least, a higher degree of awareness of one's own culture can be developed among student teachers.**

According to Gerdes (1998b) *"It is particularly important for teachers to develop an awareness of mathematics as a universal activity in order to never underestimate the capacities, know-how, and wisdom of their students and the students' communities. It is important for educators to help pre-service and in-service teachers to see how they can give each other examples of mathematics in their life and history, and it is important for (teacher) students to see people like themselves doing mathematics and to reflect on their future in relation to mathematics"* (p.47).

In fact, my involvement in teacher education has led me to observe how important it is for teachers to generate their own resources for teaching. Involving prospective teachers in the

"Process of developing an awareness of the social and cultural bases of mathematics, which contributes both to enhance their self confidence and their capacity, and their readiness and openness to work in a multicultural environment" (Gerdes, 1998b: 33) is one way of encouraging the teachers to use, in the future, the resources they developed in their mathematics classrooms.

I feel that research like this embodying a way of creating teaching materials, if successful, could be used as an example of what student teachers and practicing teachers could do in their own teaching situations. Since the focus on cultural games is much more recent, the student teachers' reactions, when confronted with the study and exploration of cultural games for the purpose of developing appropriate teaching resources, have not been sufficiently researched. How do student teachers cope with the *Tchadji*-game as a cultural instrument from their environment, in terms of whether they are able to explore the game for teaching purposes and how do their attitudes towards mathematics change in this process? This study aims to contribute to this debate.

1.3 Aims of the research

1.3.1 In the first phase

The first phase of the study incorporates:

1.1. A mathematical investigation of the following games:

- * *Tchadji* (Ismael, 1996 & Cf. Russ, 1984),
- * Three stones (simple variant of *Muravarava*), know as *Three-in-a-row* games (Zaslavsky, 1973b),
- * *Burro (donkey)* card game
- * Games of coins and cowry shells (Zaslavsky, 1973a; see also Doumbia & Pil, 1992);

in order to:

- a) Make apparent any underlying probability concepts in the playing of these games.
- b) Relate the potential of these games to the teaching of probability at secondary school level and thus develop appropriate learning and teaching materials.

1.2. Investigation of the effect of using these games in secondary school mathematics classrooms on

- a) Attitudes towards the learning of mathematics,
- b) Motivation for the study of probability,
- c) Performance in a test on probability, and
- d) The understanding of critical probability concepts.

1.3.2 In the second and third phases

The initial broad intent was to develop a better understanding of mathematical ideas related to cultural games. As a consequence of positive indicators during the first phase, I

focused my attention on one particular game - the *Tchadji*-game. More specifically, the second and third phases of the study have the following aims:

2. The second phase aims to explore any mathematical ideas involved in the *Tchadji*-game and in the procedures for playing it.

In order to achieve this aim it was necessary to:

- i. understand 'who plays *Tchadji*'?
- ii. identify the characteristics of a Master player of *Tchadji* (the "masters" are the best players);
- iii. explore 'how *Tchadji* is played' or 'what the rules of the *Tchadji*-game are' and to document this;
- iv. explore and document the strategies and the tactics involved in the playing of *Tchadji*;
- v. understand the geographical and historical context within which the game is played;
- vi. understand 'How people learn to play *Tchadji*';

3. The third phase aims to research the impact, in terms of attitudes towards mathematics and mathematical ideas developed, of interaction with the *Tchadji*-game in teacher education in Mozambique.

1.4 Research Questions

1.4.1 For the first phase

For the first phase of the study the following research questions were considered:

1.1. What aspects of probability concepts are embedded in the playing of *Tchadji*, the three stones version of the *Muravarava* game, the donkey card game, and in coin and cowry shell games?

1.2. What is the potential of *Tchadji*, the three stones version of the *Muravarava* game, the donkey card game, and of the coin and cowry shell games for teaching probability concepts at secondary school level in Mozambique?

1.3. Does teaching probability concepts by using *Tchadji*, the three stones version of the *Muravarava* game, the donkey card game, and the coin and cowry shell games at secondary school level

- a) develop positive students' attitudes towards the learning of mathematics,
- b) enhance students' motivation for the study of probability,
- c) improve the performance in a test on probability?

1.4.2 For the second and third phases

The second and third phases of the study aimed at exploring the *Tchadji*-game among its players in their playing environment and in mathematics teacher education in Mozambique. The following research questions thus apply to those phases:

2.1 What mathematical ideas can be found in the *Tchadji*-game and in the procedures for playing it?

In order to explore this question it was necessary to examine:

2.1.1 Who plays *Tchadji*?

2.1.2 What are the characteristics of a master player of *Tchadji*?

2.1.3 How is *Tchadji* played?

2.1.4 What are the rules of the *Tchadji*-game?

2.1.5 What are the strategies and the tactics involved in the playing of *Tchadji*?

2.1.6 Where is *Tchadji* played?

2.1.7 How do people learn to play *Tchadji*?

3.1 What are the attitudes of a group of student teachers towards the use of the *Tchadji*-game as starting point for doing mathematics in the classroom?

3.2 What changes in attitudes take place during the workshop with the *Tchadji*-game? How able are the student teachers in coping with embedded mathematics in the *Tchadji*-game and in seeing its potential for teaching mathematics in school?

3.3 How able are the student teachers in analysing the mathematical ideas embedded in the *Tchadji*-game?

1.5 Overview of the thesis

The remaining chapters of this thesis show how I met the aims of this study.

Chapter two: *Background and Literature Review* discusses the literature related to this study. There is a body of literature on ethnomathematics which provides an insight into the area and also gives an overview of the research on ‘hidden’ mathematics, on the use of games in mathematics education and in mathematics teacher education, on issues related to students’ attitudes towards and motivation to mathematics and also on performance in mathematics.

Chapter three: *Epistemological Grounding of the Research* discusses the theoretical framework of the study, i.e., the theoretical and philosophical points of view which have direct or even indirect links to the field of ethnomathematics as the field of the present study

Chapter four: *The Research Methodology* includes a description of the approaches and procedures employed for investigating, first, the use of the game in the teaching approach in the classroom for the probability syllabus as stated in aim 1.2; second, the mathematical ideas among the players of *Tchadji* as in aim 2, and third, the intervention with *Tchadji* among student teachers in Mozambique as in aims 3 (See sections 1.3). The instruments used and data analysis procedures in all phases of the study are also described in this chapter.

Chapter five: *The Tchadji-game and Its Players* gives historical and background knowledge related to Mancala type games, in particular, the *Tchadji*-game. It also gives the background of the players of *Tchadji*. Some informations regarding the tradition of playing *Tchadji* at Ilha de Moçambique are also presented in this chapter.

Chapter six: *Probabilistic Ideas in Tchadji and other Cultural Games and their Use in the Classroom (Phase 1)* gives the results of the reflections and investigations for the construction of the probability teaching intervention (using the four types of games). Also included in this chapter, is background knowledge related to the three other types of games. The chapter presents the results of the first phase of the study, both qualitative and quantitative, and discussion thereof in relation to the research questions.

Chapter seven: *Tchadji and Mathematical Ideas from the Players' and Researcher's Perspective: Analysis and Discussion (Phase 2)* discusses the results of the exploration of the *Tchadji*-game among its players. In this chapter, the mathematical ideas found in the game are explicitly described.

Chapter eight: *Tchadji and Mathematical Ideas from Perspective of Student Teachers: Analysis and Discussion (Phase 3)* discusses the results of the workshop on the *Tchadji-game* with student teachers. In this chapter, the mathematical ideas resulting from the independent exploration of the *Tchadji-game* by the student teachers and their ideas for possibilities of use in the mathematics classroom are described. A discussion of student teachers' attitudes towards mathematics and thoughts when confronted with the game are also included in this chapter.

Chapter nine: *Conclusion and Implications*: reviews the study and reflects: on what can be concluded from the study and on possible implications for activities in the professional development of teachers, and further research on cultural issues in mathematics education. A discussion of the possible research contribution of the study is presented in this chapter.

CHAPTER TWO: BACKGROUND AND LITERATURE REVIEW

2.1 Introduction

The present chapter discusses the literature related to this study. There is a body of literature on ethnomathematics which provides an insight into this area and also gives an overview of the research on 'hidden' mathematics, on the use of games in mathematics education and in mathematics teacher education, on issues related to students' attitudes and motivation to mathematics and also on performance. In addition to it, a review of current literature on cultural games, in particular, Mancala type games, is included.

2.2 Cultural issues and mathematics

The view of mathematics as being culture-free and universal has been dominant for a long time and maybe still is. Millroy (1992) expressed this in the following terms: *"Mathematical Knowledge has commonly been viewed as consisting of universal truths which are independent of people and which are discovered by mathematicians through a process of formal reasoning. Mathematics reasoning, unlike any other cognitive activity, is believed to be a descontextualised activity, tied to formal system, which relies upon a specifically defined set of symbols. These ideas have led to a view of mathematics as divorced from ordinary human activity and devoid of social, cultural and political considerations"* (p.1).

In studying different cultures, Bishop (1988a) has argued that there are six fundamental mathematical activities, i.e., counting, locating, measuring, designing, playing and explaining (See Appendix H), that *"are both universal, in that they appear to be carried out by every cultural group ever studied, and also necessary and sufficient for the development of mathematical knowledge"* (p.182). Mathematics, as cultural knowledge, *"derives from humans engaging in these six universal activities in a sustained, and conscious manner"* (Bishop, 1988a: 183).

2.2.1 Ethnomathematics

Ethnomathematics is a relatively new field of interest. D'Ambrósio (1985) describes it as *"The evolution of the concepts of mathematics in a cultural and anthropological framework"* (p.44), which emerged later than other ethnosciences. It emerged as a consequence of recent anthropological and cross-cultural studies. These studies show that not only does mathematics have a cultural history, but different types of mathematics have come from different cultural histories (Bishop, 1988; D'Ambrósio, 1985; Gerdes, 1985). So, Ethnomathematics derives from the social, cultural, anthropological and historical milieu of the society and consequently of the learners.

From an ethnomathematical point of view, mathematics education is grounded in the social and cultural milieu of the learners. So, ethnomathematics relies on the theory of social constructivism. The ethnomathematically aware teacher aims at constructing mathematics the pupils have to learn in the curriculum - from their daily environment.

A mathematics educator concerned with ethnomathematics should ask the question of *"What kind of knowledge do children bring with them into school, and what are the socio-cultural constraints which allow, or prevent, the use of outside knowledge in school?"* (Bishop, 1988:1). So, in the educational context, ethnomathematics is seen as bridging the gap between home culture and modern science (Gerdes, 1995), between street and school mathematics (Carraher, 1985).

Asked to refer to the issue why the area is called ethnomathematics and not a different name at the International Congress of Mathematics Education in 1992 (ICME7), in Quebec (Canada), Ubiratan D'Ambrosio, the president of ISGEm, said that *"it doesn't matter which name other people use to call it, we call it ethnomathematics"*. D'Ambrosio is known as the intellectual father of ethnomathematics. However, the mathematicians, ethnographers, psychologists, and educationalists: Wilder, White, Fettweis, Luquet, and Raum; may be considered as forerunners of ethnomathematics (Cf. Gerdes, 1997).

In fact, in the process of the development of ethnomathematics as a research domain there have been many attempts to clarify it. So, many explanations of ethnomathematics appear in the literature. Among them I shall distinguish the following: 'sociomathematics' (Zaslavsky, 1973), 'mathematics in the (African) socio-cultural environment' (Touré & Doumbia, 1980), 'informal mathematics' (Posner, 1982), 'oral mathematics' (Carraher et al, 1982), 'non-standard mathematics' (Carraher, 1982; Gerdes, 1985; Harris, 1987), 'folk mathematics' (Mellin-Olsen, 1986), 'spontaneous mathematics' (D'Ambrosio, 1982, 1985), 'hidden or repressed mathematics' (Gerdes, 1982, 1986), 'frozen mathematics' (Gerdes, 1985), 'mathematics codified in know-how' (Ferreira, 1987), 'implicit and non-professional mathematics' (Ascher & Ascher, 1981; Zaslavsky, 1984).

The prefix *ethno* refers to " *identifiable cultural groups*". In this regard, D'Ambrosio (1985a) gave the following definition: "*Ethnomathematics is the mathematics practised among identifiable cultural groups, such as a national-tribal societies, labor groups, children in certain age brackets, professional classes, and so on.*" (p. 45).

Or, as Powell & Frankenstein (1997) have argued, " *ethnomathematics includes the mathematical ideas of peoples, manifested in written or non written, oral or non-oral forms, many of which have been ignored or otherwise distorted by conventional histories of mathematics*"(p.9)

Scott (1985) incorporated the meaning of *ethno*, together with mathematics content, in his definition of ethnomathematics. He argued that Ethnomathematics lies at the confluence of mathematics and cultural anthropology. At one level, it is what might be called "*math in the (socio-cultural) environment*" (see also Doumbia, 1994) or "*math in the community*". It may also be defined as "*the cultural Anthropology of mathematics and mathematics education*" (Cf. Gerdes, 1985b). At another, related level, ethnomathematics is the particular (perhaps the peculiar) way of classifying, ordering, counting and measuring (Cf. Gerdes, 1985b).

Coming back to the early eighties, the general concern of mathematics educators has always been to increase the quality of mathematics education and thus many mathematics educators around the world have been looking for ways of doing it. It was in this period that the area of ethnomathematics emerged and it has been seen as an approach improving the quality of mathematics education. So, the initial inspiration of ethnomathematics has been of pedagogical nature (See more in sections 2.2.1.2 & 2.2.1.3).

D'Ambrosio himself relates the concept of ethnomathematics to the influence that socio-cultural factors have on the learning and teaching of mathematics (Scott, 1985). Also Gerdes (1985b, 1991, 1995) stated that ethnomathematics was conceived in light of the struggle that developing countries undergo in devising ways for children to learn mathematics meaningfully within their own culture.

Barton (1996) defined ethnomathematics as the field of study, which examines the way people of different cultures, understand, articulate and use concepts and practices which are from their culture and which the researcher describes as mathematical. He separates the researcher from the research field. For him the cultural practices may be or may not be mathematical practices. According to him, it is the researcher, who describes it as mathematics or as 'hidden' mathematics. For him the existence of different forms of mathematics is not in question.

Patrick Scott, the former secretary of the ISGEm, explained during the hearing 'What can we expect from Ethnomathematics?', at ICME6 in Budapest, 1988, that there are "*two major models in ethnomathematics: the D'Ambrosio-Gerdes Model (Ethnomathematics in Third World countries to bridge the gap between home culture and 'modern' science) and the Zaslavsky-Ascher model (ethnomathematics in First World countries to bring the world into the classroom and to have an appreciation of the mathematics of other cultures)*"(see Keitel at al, 1989).

According to Gerdes (1995), both models may be considered as complementary. Indeed, both industrialized and Third World countries see themselves nowadays confronted with the need to *multiculturalise* their mathematics curricula.

In general, ethnomathematics can be seen as having two related branches of research involved in it. The first branch is the unfreezing of mathematics inherent to specific cultures, which can be considered as anthropological or historical in nature (Cf. Gerdes: 1985, 1989, 1995; Ascher, 1991). The second is the pedagogic branch, which uses the unfreezed mathematics from the cultural artifacts as teaching and learning aids in order to improve the quality of mathematics education, to augment the cultural and social self-confidence of all pupils (Cf. D'Ambrosio, 1985a & b; Bishop, 1988a & 1988b; Mellin Olsen, 1986; Gerdes 1995; Zaslavsky, 1989).

In fact, only when the relationship between mathematical ideas and culture is adequately studied, can the question of incorporating these ideas into education be addressed. For this reason, I incorporated in this study an ethnographic research, where I extensively investigated mathematical ideas related to a cultural game. The idea in my mind has been always to look for ways of using these ideas in classroom (See results in Chapters 6, 7 & 8).

So, Ethnomathematics can be considered as the study of the relationship between the mathematics and the culture of a given people or population group (D'Ambrosio, 1985a; Barton, 1998). Mathematics is seen as a cultural factor and so it may have cultural influences. Mathematics cannot be seen as culture-free. Mathematics has to be seen as an element of culture, as a culture clue or as a cultural system. Mathematics as a creation of humans is a part of human culture (Bishop, 1988a).

With regard to the present thesis, the term ethnomathematics has to be understood in two ways, which are in some ways related to each other. Ethnomathematics is in the first instance used to represent a field of mathematics which studies the different types of mathematics arising from different cultures, i.e., the way Gerdes (1991,1992,1993, 1995)

and Ascher (1991) use the term. However, ethnomathematics has its implications for mathematics education, and this is my primary concern here. As Stieg Mellin-Olsen (1986) referred to it, *"If knowledge is related to culture by the processes which constitute knowledge - as Paulo Freire expresses it - this must have some implications for how we treat knowledge in the didactical processes of (mathematical) education"*(p.103). So, my second use of the word ethnomathematics is in a pedagogical sense: how to use the mathematics in the culture to best teach mathematics in school (D'Ambrosio, 1985; Bishop, 1988).

In the first edition of the newsletter of ISGEm, an editorial comment appears, entitled 'Ethnomathematics: What might it be?'. According to this comment, Ethnomathematics lies at the confluence of mathematics and of cultural anthropology. Ethnomathematics may be called, at first level, 'mathematics-in-the-(cultural)-environment' or mathematics-in-the-community' and, in the second level, related to the first, ethnomathematics is *"the particular (and perhaps peculiar) way that specific cultural groups go about the tasks of classifying, ordering, counting and measuring"* (ISGEm-Newsletter, Vol.1, 1985).

Some authors express their views of ethnomathematics in a different manner, uniting both levels. For example, Hunting (1985) considers ethnomathematics as *"Mathematics used by a defined cultural group in the course of dealing with environmental problems and activities"* (quoted in ISGEm-Newsletter, Vol.2, 1986). Ferreira & Imenes (1986) consider ethnomathematics as *"mathematics incorporated in the popular culture"* (p.4).

Borba (1988) presents a definition in the line of D'Ambrosio's view of ethnomathematics. He defines it as *"mathematics practiced by cultural groups, as tribal societies, workgroups or neighborhoods"* (p.20). D'Ambrosio (1987) defines ethnomathematics as *"different forms of mathematics, which are proper to cultural groups"* (p. 5). Ascher & Ascher (1986) defined ethnomathematics as the study of mathematics of non-literate people.

All these views have certain points in common. The common ideas here can be summarised in the following sentence: ethnomathematics includes mathematics practiced, used or simply incorporated in the cultural practices or activities of different groups in society. The pedagogical implication of ethnomathematics is not stressed in these views.

2.2.1.1 The ethnomathematical movement

By comparing different views of ethnomathematics, Bishop (1989) concluded that the concept of ethnomathematics did not yet constitute a well defined term and appealed that *"in view of the ideas and data we now have, perhaps it would be better not to use that term but rather to be more precise about which, and whose, mathematics one is referring to in any context"* (p.13). Gerdes (1995) agreeing with Bishop stated that *"Maybe it is better to speak provisionally about an ethnomathematical accent in research and mathematics education, or of an ethnomathematical movement"*(p.17, 18)

So, the term '*ethnomathematical movement*', which refers to the commitment of many scholars working in the area of ethnomathematics, can be traced to Gerdes (1989). Gerdes (1996) define the concept in the first instance, as the adoption of a broad concept of mathematics, which includes, in particular, the Bishop's six fundamental mathematical activities, i.e., counting, locating, measuring, designing, playing and explaining (See appendix H). Ferreira (1991, 1993) relates to this concept as an *ethnomathematical paradigm*. The researchers involved in this movement may be called *ethnomathematicians* (See also Gerdes: 1991, 1994, 1995, 1996).

Furthermore, by taking into consideration the pedagogical aspect of ethnomathematics, Gerdes (1985, 1989, 1995, 1996) argued that the ethnomathematical movement may be characterised as follows:

* Ethnomathematicians, in their research, emphasise and analyse the influences of socio-cultural factors on the teaching, learning and development of mathematics. They also

emphasise that, school mathematics of the transplanted, imported 'curriculum' type is apparently alien to the cultural traditions of Africa, Asia, and South America.

* Ethnomathematicians argue that the techniques and truths of mathematics are a cultural product and have to be seen as universal and a panhuman activity. They stress that all people- every culture and subculture - develop their particular forms of mathematics, i.e., the development of mathematics should not be seen as uni-linear. In 'Third world' countries, they look for mathematical traditions, which survived colonization, especially for mathematical activities in people's daily life, and try to develop ways of incorporating them into the curriculum.

* Ethnomathematicians try to contribute to and affirm the knowledge of the mathematical realizations of the formerly colonized peoples. They try to reconstruct the original mathematical modes of thinking, concepts, and principles. They also look for cultural elements and activities that may serve as a starting point for doing and elaborating mathematics in the classroom.

* In educational context, ethnomathematicians generally favour a socio-critical view, an interpretation of mathematics education which enables students to reflect on the realities in which they live, and empowers them to develop and use mathematics in an emancipatory way (Cf. Bishop, 1989, 1988a and b; Ascher & Ascher, 1986; Doumbia, 1989; Borba, 1987; Ferreira & Imenes, 1986; Shirley, 1988; Zaslavsky 1989a; Vithal & Skovsmose, 1997).

In general, the ethnomathematical movement, in its concern with increasing the quality of mathematics education through the incorporation of daily life cultural activities in the classroom, requires more for acting through a realistic instruction, i.e., *"The learner is given tasks that proceed from reality, that is, from within the learner's ever expanding living world, which in the first instance require horizontal mathematising"* and for

empiricist instruction, i.e., "...*The world is a reality, where man can acquire useful experiences (...). Provided with material from the living world, learners get the opportunity to acquire useful experiences, but they are not prompted to systematise and rationalise these experiences in order to break the barriers of the environment and to expand the reality they are familiar with*" (Freudenthal, 1991: 135).

The ethnomathematical movement does not claim that we should give up school mathematics and remain with ethnomathematics. According to D'Ambrosio (1985a), mathematics curricula in schools should incorporate ethnomathematics in such a way that they facilitate the acquisition of knowledge, understanding, and the compatibility of known and current popular practices with mathematics. Furthermore, D'Ambrosio (1985b) points out that in order to be able to incorporate ethnomathematics into the curriculum it will be necessary to "*identify within ethnomathematics a structured body of knowledge*" (p.47)

2.2.1.2 Ethnomathematics and Education

"Up to ten years ago, Mathematics was generally assumed to be culture-free and value-free knowledge, explanations of 'failure' and 'difficulty' in relation to school mathematics were sought either in terms of the learners' cognitive attributes or in terms of the quality of the teaching they received; there were several attempts to make mathematics teaching more effectively satisfactory to the learners, with few long term benefits, and 'social' and 'cultural' issues in mathematics education were rarely considered" (Bishop, 1993: 202)

The ideas of the Brazilian pedagogue Paulo Freire, in particular, in his books "*Pedagogy of the Oppressed*" (1970) and "*Education for Critical consciousness*" (New York, 1973) influenced deeply the reflections of ethnomathematicians such as Paulus Gerdes, Marilyn Frankenstein, Stieg Mellin-Olson, Marcelo Borba, Arthur Powell and Eduardo Ferreira.

A part of every culture is the everyday happenings of the people belonging to that culture. As such, mathematical thinking and learning occurs in this everyday practice. As Lave (1988) explained, 'everyday' "*is not a time of day, a social role, nor a set of activities, particular social occasions, or settings for activity. Instead the everyday world is just that: what people do in daily, weekly, monthly, ordinary cycles of activity*"(p.15)

Mathematics educators, particularly in Africa, pointed out in their studies that there is a need for integrating mathematical traditions and practices of Africa into the school curriculum in order to improve the quality of mathematics education (Ale, 1989; Doumbia, 1989; Eshiwani, 1979; Jacobsen, 1984; D'Ambrosio, 1985; Bishop, 1988a; Zaslavsky, 1989a, Gerdes, 1995). Also, it has long been suggested that ethnomathematics provides teachers with useful resources (Zaslavsky, 1989b; Gerdes, 1990).

For most of these mathematics educators the important issue was to bring the learner's social and cultural contexts into the curriculum. This should also include critical reflection and awareness of that situation. As Pompeu (1992) pointed out mathematical knowledge should, as far as possible, be developed from students' own situations. He argued that students should be familiar with those situations, identify with them and feel self-confident. This has also been stressed by D'Ambrosio for many years as in his address at ICME5 (1984), when he related that *"the big challenge in education, in rapidly changing societies, is how to bring their cultural diversity into curriculum design"* and that *"the key issue in curriculum design for the years to come seems to be in meeting this challenge"* (p.5).

Students need in-school mathematical experiences to build on and formalise the mathematical knowledge gained outside the school. An important part of mathematical experience in school is the guidance and structure that can be provided by a teacher to help students make connections among mathematical ideas. When children are faced with learning mathematics apart from their everyday experiences, they have difficulty linking the in-school mathematics with their out-of school mathematics. The end result is that the students memorise information in school without understanding how it fits with the realities of their lives (Masingila, 1992).

Masingila et al (1996: 194) pointed out the necessity of connecting both school and out-of school experiences. They argued that, *"knowing and using students' out-of-school mathematics practices is important in school situations because it provides contexts in which students can make connections. Connections making is essential in constructing*

mathematical knowledge but at present is often absent in classrooms" (Cf. Bishop & De Abreu, 1991).

Mathematics learning should not be "*limited to acquisition of the formal algorithmic procedures passed down by mathematicians to individuals via school.*" Mathematics learning should "*occur as well during participation in cultural practices as children and adults attempt to accomplish pragmatic goals*"(Saxe, 1988:14). In this context, learning and doing mathematics has to be seen as an act of sense-making and comprises both cultural and cognitive phenomena which cannot be separated (Masingila, 1992; See more about this in section 3.3).

2.2.1.3 Ethnomathematical research

The original inspiration for the field of ethnomathematics was of a pedagogical nature, i.e., it arose from the desire to enhance the quality of mathematics education. In spite of this, most early ethnomathematical research has been directed towards identifying and studying various cultural forms of mathematics (the hidden mathematics), which differ from standard 'academic' and 'school' mathematics (Cf. Gerdes, 1985, 1989, 1995 & 1996).

The basic idea in research on ethnomathematics is that "*Ethnomathematics is a synthesis of mathematics and cultural anthropology, and has as its basic tenet that, as Marcia Ascher writes, mathematical ideas are cultural expressions embedded within cultural context*" (Zaslavsky, 1999:281).

In fact, all societies throughout the world and all eras of history have developed mathematical ideas relevant to their needs and interests. As Alan Bishop (1988) explains, all societies count objects, measure various quantities, and invent calendars and other ways to describe the passage of time. They design works of art, plan buildings, and play games that involve mathematical concepts. Furthermore, they invent terminology and symbols that enable them to communicate about these activities. How each society and subgroup

within the society carries out these practices, as well as the factors that have led to their particular evolution, is the province of ethnomathematical research.

According to Bishop (1994; see also Vithal & Skovsmose, 1997) in ethnomathematical research three major approaches have been used:

(a) Investigations of mathematical knowledge in traditional cultures, which have been informed by the *anthropological approach*. In this research, the mathematics of traditional cultures and of indigenous peoples, who may have been colonized but continued with their practices, is analysed. These cultures have been explored in relation to: number systems, gestures and symbolism, games and puzzles, geometry, space, shape, pattern, symmetry, art and architecture, time, money, networks, graphs or sand drawings, kin relations and artifacts (see for example, Ascher, 1991; Zaslavsky, 1973; Gerdes, 1985, 1987, 1989, 1993, 1994; Ismael & Soares, 1993; Ismael, 1994);

(b) Investigations of mathematical knowledge in non-Western societies with more historical direction. This research is related to the fact that the contributions of cultures outside Europe to the body of mathematical knowledge that is paradoxically referred to as 'Western' mathematics is frequently ignored, devalued, distorted or marginalized. Also, the fact that only marginal attention, if any at all, is paid to the history of mathematics in cultures that have not directly contributed to 'Western' mathematics such as sub-Saharan Africans and American Indians (See for example, Zaslavsky, 1973; Joseph, 1991; Gerdes, 1991, 1995; Ismael, 1994).

(c) Investigations of mathematical knowledge of different groups within the same society, whose emphasis has been socio-psychological, i.e., social construction of mathematical knowledge associated with the practices of the groups. Much of this research has been generated through focusing on the connections between cognition, culture and context. The everyday practices of different groups are explored such as dairy workers, construction foremen, farmers, child street vendors, carpenters, candy sellers, shoppers

and fishermen (see for example, Abreu & Carraher, 1988; Saxe, 1988; Carraher, Carraher & Schliemann, 1982 & 1987; Millroy, 1992; Abreu, 1993).

On the other hand, as initially stressed, ethnomathematics emerged among mathematics educators and teachers of mathematics. So, there is a general conception of the need of integrating the mathematical ideas embedded in the cultural activities into mathematical curricula, i.e., the focus is on the relationship between ethnomathematics and mathematics education. Bishop (1994) has argued that in order to experiment with the incorporation of ethnomathematical research findings in education, a basic and radical assumption needs to be recognised: *"all formal mathematics education is a process of cultural interaction, and that every child (and teacher) experiences some degree of cultural conflict in this process"* (p.16).

In spite of this relationship being evident in studies related to all three ethnomathematical research directions mentioned above, the idea of producing multicultural curriculum materials illustrating how an ethnomathematics perspective may be used in mathematics education, there are relatively few detailed descriptions of the actual implementation and outcomes of the use of such an approach within the formal school system. This is still an under-researched area. This research is aimed to contribute to this debate.

In Mozambique, Gerdes and his team, have carried out much work (see Gerdes: 1985, 1989, 1990, 1994, 1995, 1997) in describing the unfreezing of mathematics from cultures in the context of MERP (See section 1.1.5.1). This research is more a continuation of the research initiated by Gerdes. In this research, an attempt to idealise curriculum material by student teachers has been worked out. Also, the use of developed material in the classroom was implemented in early stage.

According to Gerdes (1995), ethnomathematics, in general, is the study of the interrelationship between (the) mathematics and (the) culture(s) of a given people or population group. The ultimate goals of ethomathematical studies are to improve the quality of mathematics education and to accelerate the access of the people of third world

countries to 'World Mathematics', the common heritage of mankind, as a (possibly) useful instrument to improve the quality of life. This view of ethnomathematics is clearly reflected in the work of Gerdes and his team (Gerdes: 1995; 1993).

The research in ethnomathematics as it is described in the literature and according to what is understood of what ethnomathematics is, has taken different focuses. Some researchers have focused their work on *mathematical practices or activities of distinct cultures* and ways of incorporating them in the school curriculum and other have studied the mathematical practices in *everyday situations within cultures* and its contrast with the school mathematics (Cf. Bishop, 1984). Again, the ultimate goal of these researchers has been almost always to find ways of incorporating these cultural practices and these everyday situations into the school curricula. In following sub-section, I will present ideas beyond these two focuses of research on ethnomathematics.

2.2.1.3.1 Research on 'hidden' or 'frozen' mathematics

Among the various concepts proposed within the context of ethnomathematics to provide a contrast between ethnomathematics and the academic 'school mathematics,' such as '*indigenous mathematics*', '*socio-mathematics*', '*informal mathematics*', '*folk mathematics*'.

Gerdes (1985) developed the concept of *hidden or frozen mathematics*. As he explained it, "*Although, probably, most of the mathematical knowledge of formerly colonized peoples has been lost, one may try to identify, reconstruct, thereby 'unfreeze' the mathematical thinking which is 'hidden' or 'frozen' in old techniques (like, for example, basket making)*" (e.g. Gerdes, 1995:15; 1996:914). These concepts have not only contributed a great deal to the development of an understanding of what ethnomathematics might be, but also they may be understood as a research orientation in this field.

The 'uncovering' of 'hidden mathematics' or the 'unfreezing' of 'frozen mathematics', has been the first issue for consideration in late 1970's, when ethnomathematical research started in Mozambique. As Gerdes (1996) describes it, "*As most 'mathematical' traditions*

which survived colonisation and most 'mathematical' activities in the daily life of the Mozambican people are not explicitly mathematical, in the sense that the mathematics is partially 'hidden', the first aim of this research was to 'uncover' the 'hidden' mathematics" (p.925).

For example, in his prize-winning 1998 book, *Women, Art and Geometry in Southern Africa*, Gerdes recognized and analysed the mathematical thinking inherent in various artistic activities, like weaving of handbags and mats, pottery decoration, tattooing and body painting, string figures and mural decoration, in which women engage. This ethnographic research has led to the development of new curriculum materials: lessons that empower students and teachers to respect and value their cultural heritage, and that also may serve as a model for curriculum development in other parts of the world (See more in Gerdes, 1998).

The research on 'uncovering' of 'hidden mathematics' or the 'unfreezing' of 'frozen mathematics' is a particular example of research on mathematical ideas embedded in practices or activities of distinct cultures (c.f. Gerdes, 1985, 1986). Studying the mathematical ideas, concepts and processes embedded in the *Tchadji*-game, as it is the main concern of this study, is an example of examining or unfreezing the frozen mathematics.

2.2.1.3.2 Research on in and out-of-school mathematics

For many decades conventional wisdom has assumed that mathematics learning takes place only in school. In fact, recent research evidence justifies that, mathematical learning *"is not limited to acquisition of formal algorithmic procedures passed down by mathematicians to individuals via school. Mathematics learning occurs as well during participation in cultural practices as children and adults attempt to accomplish pragmatic goals"* (Saxe, 1988:14).

Analogous to literacy in language usage, D'Ambrosio (1985b) has used the term 'matheracy' to refer to the competence in using numbers, and further distinguished between *spontaneous matheracy* (arising out of one's cultural experiences) and *learned matheracy* (gained through formal schooling - usually western schooling). D'Ambrosio (1985b) has argued in this respect that learned matheracy eliminates spontaneous matheracy: an individual who is matherate in his or her own culture, "*creates a psychological blockage*" when facing "*a completely new and formal approach to the same facts and needs*" (p.45). The main reason for this barrier was the gap between the mathematics practice in school and that in out-of-school situations: the ways in which, for example, Bishop's six mathematical activities (see appendix H) are traditionally taught in schools are often vastly different from the way they are carried out by various cultural groups (D'Ambrosio, 1985b).

It was the increasing body of research on everyday mathematics that challenged the view that school is the central source of everyday mathematics practice. This research demonstrates that mathematical problems in out-of-school situations are often approached efficiently and creatively without using school-taught procedures in any observable manner (see for example, Carraher, Carraher & Schliemann, 1982 & 1987; Saxe, 1988; Abreu, 1992).

Regarding this issue of 'Mathematics learned outside the school', Subgroup 4 of Working Group 21: 'Teaching mathematics in different cultures' at ICME8 concluded that, "*Regarding the experience that is concerned with the mathematics that the students use outside the school and teachers' perspectives of those mathematical activities in students' lives, the discussion revealed that (i) mathematics exists outside of school (ii); it is useful to introduce or discuss out-of-school mathematics in the classroom to improve skill(s), and then consciously use again outside of school; (iii) there is a need to exercise restraint in emphasizing students' reflections on mathematical aspects of their lives, just in case they might stop doing it altogether*" (Becker, 1998: 204).

2.2.2 Cultural games and mathematics

Playing is one of Bishop's six fundamental mathematical activities (See Appendix H). As Bishop (1988a) explain it: *“playing [is] devising, and engaging in games and pastimes, with more or less formalised rules that all players must abide by Puzzles, [which include] paradoxes; models; games; rules; procedures; strategies; prediction; guessing; chance; hypothetical reasoning; games analysis”* (p.182).

The activity of playing is of primary interest to me in this study, since it is also part of study the exploration of mathematical ideas, concepts and processes embedded in a particular game, the *Tchadji*-game. On the other hand, playing is a mathematical activity that, if well worked out, seems to fit well in classroom and this correspond to another intention of this research.

2.2.2.1 Views on mathematics and cultural games

Games are part of the reality of life in the world, i.e., human culture. It cannot be considered a separate aspect. For example, *“During recent centuries in the West, one of the most common forms of race game has been based on the idea of reaching a goal in life, overcoming difficulties and hindrances on the way”* (Bell, 1979: 12).

As defined by Cheska (1989:14), *“A game is organized play. Essential elements of game include: 1) participants, opponents or sides; 2) organized procedure, a fixed sequence of actions or plot; and 3) uncertain outcome, resolution or end result. Without these elements a game cannot exist. Games fluctuate between two activity poles: play and sports. At one end, play stresses more the participant's expressiveness while doing the activity, whereas sport stresses the participant's instrumentality or the achievement of some goal. Games contain both to varying degrees - one might say, 'I enjoyed doing it' (expressiveness) or 'It was great to win' (instrumentality).”*

Bell & Cornelius (1995) stated that, "*There can be few people who have not, at some time, been excited or stimulated by a game. We enjoy leading an army to victory after outwitting an opponent, winning a race after devising a cunning and careful strategy, or defeating an adversary by skilful tactical play in a game of position*" (p.1). In fact, since many thousands of years games have been a recreational activity for adults and children in all parts of the world.

Zaslavsky, in her famous work on 'Africa Counts' (1973a) underlined important aspects on traditional games played in Africa. She suggested that it could be important in the reconstruction of the history of mathematical thinking to investigate further mathematical aspects of traditional games (Zaslavsky, 1989b: 6).

Among the games with mathematical 'ingredients', she distinguishes the Mancala type games (see section 3.2.1) and three-in-a-row games (see more in Zaslavsky, 1973a). Zaslavsky (1979) stated that the mathematics of the games requires skills of counting and remembering numbers as well as higher-level strategic thinking and problem solving. Referring to Mancala, she (1996) argued that "*It is based entirely on mathematical principles, and some experts rate it among the ten best games in the world*" (p.189). She (1993, 1994 & 1996) set out concrete activities, which could be performed in the mathematics classroom in dealing with concepts such as Numbers, Measurements, concepts of Geometry and even probability.

It seems that, "*In trying to decide how best to play a particular game a player is forced into the need for logical thinking and thence, normally, to some mathematical thinking*" (Bell & Cornelius, 1995: 2)

Shirley (1995) seems to recognize the mathematical potential of games for use in classrooms when he argued that "*Recreational mathematics is informal, often coming from games and puzzles, and it is pure, usually without a direct practical application beyond enjoyment and intellectual beauty, (but) it is rich in opportunities for classroom*

activities. One that is appropriate for all levels of school is the game generically called *Mancala*" (p. 39).

Crump (1990) seems to recognize the connection between games and mathematical skills when he argued that, "*Games foreshadow the acquisition of elementary mathematical skills in the intellectual development of the child. More generally, play - which, in the absence of rules fixed by outside convention, may be seen as the precursor of games - is the main source of the symbols used by the child to create its own cognitive universe. At an early stage the child can then accept rules, which define a true game, even though their formal mathematical character is beyond its understanding*" (p.117).

Barta & Shaelling (1998: 388) argued that, "*Games allow us to practice and enhance our skills, challenge our intellect, and improve our ability to solve problems, all while we are having fun. [They] also involve a great deal of mathematics, so much so that play is considered one of the six universal mathematical activities of all cultures*" (Cf. Bishop, 1991). In general, there is a recognition that games might involve mathematics, but there is little research to uncover the mathematics, which is hidden in the games. This research attempted to close this gap with regard to Mancala type games.

Doumbia's work (1989; with Pil, 1992; 1994) has focused on mathematical analysis of traditional African games and exploration of the possibilities of using them in the mathematics classroom. She and her colleagues, were involved in a project called 'Mathematics and the African socio-cultural environment', created by Saliou Touré, former general secretary of African Mathematical Union. The aim of the project was to mathematise different games of West Africa.

Doumbia (1994) classified games as "*les jeux verbaux, les jeux pour deviner, les jeux de position, les jeux d'exhibition, les jeux de stratégie et les jeux de hasard*" (p.5). Also Cheska (1987) has given a tentative classification of games. Oldfield (1991a) gives a classification in terms of the educational reason for using a particular game.

Doumbia (1994) distinguishes games like the "Yé gonan", which in Yacouba language spoken in Ivory Coast, means "nous sommes en train de jouer", a game of memory practiced by children in the Ivory Coast; "L'Awalé", a 'Mancala' type game, also known as Ayo, Bao, Wari, Mweso, N'tchuva (Cf. Townshend, 1979); "Les jeux de Cauris", a Cowry Shell game, as an alternative *"from the classical case of heads or tails with a coin"* (Doumbia, 1989: 175).

Silva (1994: 8) referring to "Ouri", a 'Mancala' type game, argues that it is, apparently, very easy to play the "Ouri". It is not the case of just learning the rules in order to play it. It requires reflection, calculation and practical experience. Here, Silva recognizes that the playing of the "Ouri" game involves some mathematics. He did not go so far as to describe what kind of mathematics is involved in the game. This makes the present research necessary.

Regarding the "Ouri" game, Sawyer (1949) refer to the following: *"It is, to the best of my knowledge, the most arithmetical game with a mass following anywhere in the world. African children must benefit from the familiarity with numbers that this game gives, and it may have classroom use in other countries"* (quoted in Mucavele, 1998: 63). Also, other scholars, mathematicians or not, recognize as well that the 'Mancala' type games in general involve skills of counting and calculations (Strong & Lumpkin, 1995: 51; Cheska; 1987: 45).

Mucavele (1998) working with children aged 8-14 years, explored mathematical aspects of two games played in Mozambique, the 'Mathakuzana' and the 'N'tchuva', a Mancala type game. He found out that some children use many mathematical ideas in playing these games, e.g., cardinal numbers, mental and oral addition and subtraction, logical thinking.

Odeleye (1977), referring to 'Ayo', a type of 'Mancala' game, describes it as *"A serious game involving a considerable skill and mental exertion. As an intellectual exercise it is on a level with chess. Experts can play it at great speed and it is not generally realized that it is a game requiring a considerable amount of skill at calculation"* (p.12).

So, games in general are part of the human culture. As stated above, games and also their playing processes are widely recognized by mathematicians, educators and anthropologists as involving various 'mathematical' aspects such as logical thinking, mathematical thinking, counting, calculation skills, geometrical reasoning, problem solving and probability reasoning (Zaslavsky, 1973a, 1989, Bell & Cornelius, 1995; Barta & Schaelling, 1998; Bishop, 1991; Doumbia, 1989, Doumbia & Pil, 1992; Silva, 1994, 1995; Strong & Lumpkin, 1995, Cheska, 1987; Odeleye, 1977; Ismael, 1996). The literature, however, fails to describe such mathematical aspects. This study is also aimed to contribute for this context.

2.2.2.2. Research on cultural games and mathematics education

Zaslavsky (1973) presented a series of cultural games with mathematical 'ingredients'. Furthermore, she (AMUCHMA 3, 1989:6) suggested that it could be important in the reconstruction of the history of mathematical thinking in Africa to investigate further mathematical aspects of traditional games.

Some scholars have focused part of their attention in studying various mathematical aspects of cultural games (see, for example: Russ, 1984; Crowe, 1987; Silva, 1994, 1995; Zaslavsky, 1979, 1982, 1989b; De Voogt, 1997, 1998; Doumbia & Phil, 1992, Doumbia, 1994, 1995; Mosimege, 1997, 1998). So far, they have at least recognised that games, in particular, cultural or traditional games, involve mathematical ideas.

Recreational activities such as games and puzzles have become quite popular activities in the classroom. They engage students' attention by challenging them either to win against an opponent or to complete an interesting task. Games should thus be more suitable in creating motivation and interest in learning mathematics. As Bell & Cornelius (1988) in this respect commented "*it is interesting to observe that pupils would carry out some quite complex analyses when given tasks which were based on a game but that they would be less willing and less interested to carry out the same tasks in isolation*" (p.96). As far as I

am concerned, there has not much research to challenge the affective aspect of pupils and teachers through the use of traditional games. This aspect has been the motivational point to advance with this research.

Some games are already known as potential for use in the classroom. Regarding other types of games, like the traditional games, e.g., Mancala type games, their uses in the mathematics classroom have not been researched so far. Do they also challenge the students' attention? Are they also interesting for the students? These games also involve complex movements and strategies. Is there any mathematical thinking involved?

2.2.3 Research Perspectives in Ethnomathematics

El Tom (1995), former chair of the African Mathematical Union Commission on Mathematics Education, sees ethnomathematical education research and experimentation as a necessary activity in a context in which most African countries attempt to imitate major curriculum reforms in the West.

Ethnomathematics is a relatively new area of research. As Gerdes (1998a) argued, *"notwithstanding the research already realized, ethnomathematical-educational experimentation and, generally, the study of possible educational implications of ethnomathematical research, both internationally and in Africa, are relatively still in the beginning. In order to experiment a basic and radical assumption, namely that - in the words of Alan Bishop - 'all formal mathematics education is a process of cultural interaction, and that every child experiences some degree of cultural conflict in that process', - has to be taken into account. Established theoretical constructs of mathematics education are not based on this assumption"* (p. 231)

There are, as referred earlier, some important issues in this area. In fact, *"generally ethnomathematical research findings oblige to reflect about fundamental mathematical-educational questions: Why teach mathematics?; What and whose mathematics should be taught, by whom and for whom?; Who participates in curriculum development?; How to*

organize the school practices in order to minimize the effects of the possible disruptive relationship between home and school culture and mathematics?" (Gerdes, 1998a: 231)

In the case of Mozambique, and maybe more generally in Africa, a fundamental objective of ethnomathematical research consists in looking for possibilities to improve the teaching and learning of mathematics by embedding it in the cultural context of pupils and teachers. This type of mathematics education is intended to succeed in dignifying and valuing the scientific knowledge inherent in the culture. This is done by using this knowledge to lay the foundations in providing quicker and better access to the scientific heritage of all humanity (Cf. Gerdes, 1998a). This thesis aims to contribute to these needed aspects of research in ethnomathematics.

2.3 The Use of Games in Mathematics Education

There are many and varied reasons for using games in the teaching and learning of mathematics. However, the use of games should be implemented with care, i.e., the choice of the game should be done with specific purposes in mind (Mosimege, 1997). As Oldfield (1991a) suggested, the researcher needs to ask himself: what types of games are useful in the mathematics classroom. In fact, crucial to this aspect, is the purpose of using a particular game.

As Fisico (1994) argued, *"to have a more conducive atmosphere for learning mathematics, a creative teacher who is also diligent and resourceful should always consider giving mathematics activities which not only help in the learning processes but also captivate the attention, the curiosity, and the sense of wonder of students"* (p.95). Games can serve as a teaching resource for a variety of mathematical topics.

"Many different games have been used for various purposes in the teaching of a variety of topics in mathematics, and their effects have been studied by a large number of researchers" (Bright et al., 1985, quoted in Schroeder, 1989, 40). Games have been used for the purpose of learning the language and vocabulary of mathematics; for developing

and practicing mathematical skills; developing abilities in mathematics; devising strategies on problem solving; generating a variety of mathematical activities, e.g., investigational work in mathematics (Kirkby, 1992).

In the recent past the use of games in the teaching and learning of mathematics has increased (Vithal, 1992). Particularly games of chance have often been used to introduce and explore concepts of probability, since they provide familiar and practical instances of these ideas (Schroeder, 1989). Traditional games like Mancala type games, however, have not been used so far. There is no information on how these games could be used. Their mathematical richness is recognised but the manner of their use for the purpose of teaching mathematics have not yet been researched. This thesis aims to contribute to this aspect.

Crane (1982) referring to the context of the United States, stated that *"More and more frequently, African games are being used in the classroom. The work of Claudia Zaslavsky, Donald W. Crowe, and other American specialists in mathematics who consulted with their counterparts in Africa has helped American teachers discover that African board games and the related geometric patterns and designs are invaluable resources for studying mathematics, art, architecture, and other subjects requiring mathematical concepts"* (p. 2)

Games, in general, can serve many other purposes, e.g., for reinforcing mathematical concepts and for stimulating mathematical discussion (Oldfield, 1991a, 1991b, 1991c, 1991d, 1992). Powell (1998) argued that *"Games of numbers and games of strategy stimulate children's imagination and thinking. When children feel appropriately challenged by a game, they seek to discover the secret of winning (or avoiding loss). The sheer pleasure of playing games enables children to learn the mathematical ideas of the games as a natural by-product. While playing games, children construct intellectual frames that will enable them to comprehend complex mathematical ideas, strategies, and theories"*. These aspects have so far not been researched with regard of traditional games.

As it is suggested therefore, *"It seems only natural that educators should increase their use of games in the classroom, since playing them is an important human activity that affords substantial opportunities to experience and explore mathematics within the context of culture"* (Barta & Shaelling, 1998: 388). In other words, there is a need for concrete actions with regard to the use of games in the mathematics classroom. This thesis constitute a contribution in terms of action.

2.3.1 Teaching Probability Using Games

As Borovcnik & Peard (1996: 240) note *"There are two main motives for legitimating probability for curricula at any level. One is the formation of a specific type of thinking, a probabilistic thinking. This should be comparable to other types of mathematical reasoning such as geometric thinking and can be improved by mathematical treatment. There should emerge a component extending one's senses, some sort of intuitive insight, which is more than a superficial knowledge of mathematical terms and procedures. Second is the need for applications"* (cf. Munisamy & Doraisamy, 1996; Peard, 1995)

One of the question that should drive the research in teaching probability, as identified by Shaughnessy (1992), is the following: *"Are there optimum teaching and learning techniques in probability?"* (p.488). Also, Ahlgren & Garfield (1991) argue that there is very little persistent research on how best to teach probability.

As Zaslavsky (1973a) note, *"it is incredible that African games were actually discouraged by the colonial education authorities in favour of ludo, snakes-and-ladders, and similar games of European origin"* (p.131). Also Cheska (1987) referring to issues of concern in research on Africa games, notes that one problem *"is the apparent loss of many traditional African ethnic activities in part by the substitution of "colonial" originated activities taught in school and encouraged by clubs; e.g., the sports of soccer and cricket"* (p. 13).

As Bishop (1988a) notes, developing ideas about chance and prediction are important mathematical activities, and that gambling games are part of modern western society at present. Preliminary investigations in our mathematical culture, e.g. traditional games, indicate a connection with experimental probability (Ismael, 1996). The issue of accessibility of teaching resources in developing countries in Africa is a big problem. Traditional games, however, are part of the culture of these countries. Would it not be much more effective to explore these local resources with regard to the possibility of their use in education? This is one of the aims behind this research.

According to Falk & Levin (1980), one method to help develop young children's potential for the understanding of probability is to let them play games that assist in conceptualising aspects of probability. Playing games gives the child some experience with the operation of the laws of probability, but the child's role is usually passive. It would be preferable to offer a game of chance where the child would be required to make decisions and to cope with the probabilities that determine the course of the game. The way that this research was planned offered a range of activities in the classroom with, for example, the *Tchadji*-game. It is interesting to know what happened then, and this is an aim of this research.

Steinbring (1991) suggests that the experiments with the games that students play should not be taken simply as a motivational aid or as point of departure for a course in which the intended goal will thereafter be reached in the step-by-step transmission fashion. This concrete context of their experiences with the game is a fundamental source for students, one which has to be maintained throughout the whole process of developing the concept of chance.

Fischbein & Gazit (1984), referring to improvement of probabilistic intuitions in the classroom, argued that, *"The new intuitive attitudes can be developed only through the personal involvement of the learner in the practical activity. Intuitions (cognitive beliefs) cannot be modified by verbal explanations only. Therefore, a teaching program, which intends to develop and improve an efficient intuitive background for probability concepts and strategies, along with the corresponding formal knowledge, must provide the learner*

with frequent opportunities to experience actively, even emotionally, Stochastic situations. In these situations, the learner will confront his plausible expectations with empirically obtained outcomes" (p. 2). Playing appropriate games provides such opportunities.

Shaughnessy (1997) reflecting on research in stochastic (as some European researchers refer to the area of probability and statistics) argued that: *"In reflecting on the current state of affairs of our research efforts in Probability and Statistics, I find that we have made a good deal of progress in the first arena - uncovering students' conceptions and beliefs about chance and data. However, we have made very little progress in the second arena, the documentation of student growth and change as they interact with chance and data tasks or curriculum materials. I believe this is a missed opportunity in our research"* (p. 6). My research is intended to contribute to the issue on what changes in students, both cognitive and affective, are observed in the process of dealing with *Tchadji*-game.

Munisamy & Doraisamy (1996) found, in their study in Malaysian Schools, that probabilistic reasoning is not an easily acquired skill for most pupils. They reported that even after instruction, many pupils had difficulties developing an intuition about certain probabilistic concepts. They further noted that, *"the results suggest that probability concepts are unlikely to develop either incidentally or through maturation, which means that appropriately planned experience must be provided. Teachers should introduce probability concepts through activities and simulations, not abstractions; use visual illustration and emphasize exploratory data methods; create situations requiring probabilistic reasoning that corresponds to the pupils' view of the world"*(p. 43). In fact, as they concluded, *"Much work remains and much help is needed to complete the task of arming tomorrow's citizens with basic concepts of probability"* (p. 45).

2.3.2 Attitudes Towards and Motivation for Mathematics

Attitude is a psychological construct. Oskamp (1977) pointed out some aspects, which have been stressed in various definitions of attitudes. He stated that 'attitude is a mental set or disposition' or 'attitude is a readiness to respond.' *"Attitudes are evaluated beliefs which*

predispose the individual to respond in a preferential way. That is, attitudes are predispositions to react positively or negatively to some social object" (Burns, 1995: 332).

Still according to Burns (1995), most definitions of attitudes suggest that there are three major components: the cognitive, affective and behavioural components. The cognitive component involves what a person believes about the object, whether true or not; the affective component concerns the feelings about the object, which influences its evaluation; the behavioural component reflects the actual behaviour of the individual, though this is rather an unreliable indication of an attitude.

Olliges (1988) points out three facets of mathematical attitudes: (1) enjoyment of mathematics, (2) security and confidence with mathematics, and (3) appreciation of the usefulness and value of mathematics. Aiken (1979), however, has extended the view of students' attitudes towards mathematics by considering four aspects: the enjoyment of mathematics, the importance of mathematics, their motivation in studying mathematics and freedom from fear of mathematics (Cf. Coulson, 1992)

According to Oldfield (1991a), teachers using games in their mathematics lessons frequently report on powerful motivation, excitement, involvement, and positive attitudes. Using cultural issues, e.g., traditional games, in the teaching of mathematics (as suggested by Barton (1996)) and the use of history of mathematics, can result in increasing motivation for learning, making mathematics less frightening, giving mathematics a human face as well as changing students' perception of mathematics (Cf. Fauvel, 1991).

Other researchers have reported that the use of games increases students' interest, satisfaction and continuing motivation (Sleet, 1985; Strauss, 1986). On the other hand, some researchers have reported that playing a game does not influence student satisfaction (De-Vries & Slavin, 1978).

Ernest (1994) notes that it is widely remarked in the mathematics education literature that students' attitudes and perceptions of mathematics are important factors in learning. Some theorists have argued that instructional games are motivational because they generate enthusiasm, excitement, and enjoyment, and because they require students to be actively involved in learning (Ernest, 1986; Wesson, Wilson, & Mandelbaum, 1988; as quoted in Klein & Freitag, 1991). Doumbia (1989) stated that "*the best teaching must create motivation, and games have this advantage*" (p. 174).

However, other educators theorised that instructional games can even decrease student motivation, particularly for those students who do not regularly win (Klein & Freitag, 1991). However, the need for good motivation, involvement, and the development of positive attitudes by pupils has long been recognized (Bell, 1978; Ernest, 1986). There is some kind of inconsistency with regard to the theory on instructional gaming, which supports the need for further research, particularly when a research study also aims to contribute to related theory.

On the other hand, the literature on instructional design theories fails to address instructions and recommendations on how to design practice that is motivational. Instructional games can have a motivational effect on the student. It depends on the way in which they are used (Cf. Oldfield, 1991a; Mosimege, 1997).

There may be many different ways of using various games. Their effect on student motivation will vary depending also on other factors, e.g., personality, previous knowledge of the game and the context in which the game is used. Also, I believe that there are games, which are more instructional than others. There is a need to know how instructional a game can be, i.e., what cognitive and affective outcomes does it reach. Studies that aim to use the games in a way that promotes active participation and cooperation, and which provides immediate feedback to participants based on specific educational objectives are strongly recommended.

Much of the research concerning the effects of instructional games on motivation and learning has been conducted using flawed experimental designs and methods, and many studies have not investigated the integration of games in an instructional system. These are some explanations for the inconsistency of their findings (Klein & Freitag, 1991). As Wolfe (1985) indicates, "*No rigorous research has examined a game's motivational power, (or) what types of students are motivated by games*" (p.279). I believe that games can be powerful with regard to the motivational aspect. This research was included the practice of traditional games, in the classroom context. Students were playing games and experimenting their view within games (See sections 4.2.3 & 4.2.4).

Research on attitudes towards mathematics has a relatively long history. There have been a large number of studies of attitudes towards mathematics over the years. The research on attitudes should be linked more closely to the study of cognitive factors in learning (McLeod, 1992). This is the intention in this study.

2.3.3 Performance in Mathematics

Concerning performance, some educators have theorised that games are effective for improving student performance, because they make practice more effective and students become active in the learning process (Ernest, 1986; Wesson at al., 1988). Others have suggested that games foster incorrect responding and inefficient use of instructional time (Allington & Strange, 1977; Andrew & Thorpe, 1977).

Researchers have reported that instructional games are effective for assisting students to acquire, practice, and transfer mathematical concepts (Bright, 1980; Bright, Harvey, & Wheeler, 1979; Rogers and & Miller, 1984). In Nigeria, for example, recent evidence from groups of pupils taught through making use of simple ideas from the world they recognize as real or meaningful showed improved performance over groups taught more traditionally (Ochepa, 1997).

Other researchers have found out that games are effective for assisting slow learners to practice mathematical skills, but not the more able students (Friedlander, 1977). There are researchers who have argued that research on use of games has produced inconclusive or non-significant findings (Boseman & Schellenberger, 1974; Greenlaw & Wyman, 1973). Research findings concerning the effect of the use of games on performance are inconsistent.

Barta & Shaelling (1998) argued that, *"students who create and play games construct personally meaningful understandings for the concepts they are applying. Mathematics games and culture become synonymous with fun and learning"* (p. 393).

2.4 Mathematics teacher education

2.4.1 The role of teacher education in the ethnomathematical programme

Bishop (1988b) argues that a mathematical education can only be called education if it transmits values, including cultural values. Otherwise it is not an education at all but merely training. Mathematical education should make values explicit and develop the learner's capacity for choosing critically. Teacher education is fundamental here because teachers must be made fully aware of their task in this respect. Much value learning occurs implicitly in the mathematics classroom, and hence if contexts are excluded, positive attitudes are not encouraged often enough. Bishop suggests that, in order for these values to contribute positively to the pupils' mathematical education, the values need to be made more explicit and meaningful.

Regarding the issue of 'Preparing Teachers to Teach to Diversity', the Subgroup of Working Group 21: 'Teaching mathematics in different cultures' at ICME8, gave the following recommendations:

- a) Teachers should adopt a child-centred method of teaching in which they facilitates the student's construction of mathematical knowledge;
- b) Teaching to diversity means recognition of the fact that students bring different (cultural) background to the classroom, taking care not to destroy these cultural backgrounds while empowering them to modify or extend their background in ways that will give them greater access to mathematical understanding and competence;
- c) Regarding the importance of ethnomathematics, the mathematics education of children should begin, when appropriate, with the mathematics of their "cultural practice", i.e., their ethnomathematics. Ethnomathematics should be part of teacher education curricula in order to enable the teachers to see it as "authentic mathematics" (Becker, 1998)

Mathematics learning does not only take place in school but also outside the school. In addition, it is necessary to take into consideration those experiences that the pupils bring

with them from their own out of school environment while teaching mathematics. Obviously, the role of teacher is therefore crucial in all this process.

In fact, *"Every people, every culture and every subculture, including every social group and every individual, constructs and develops its own, in a certain way particular, mathematics"* (Gerdes, 1998b: 47). So, a teacher should be aware of these cultural differences and the fact that they can lead to different developments in mathematics, not forgetting that *"one of the most crucial aspects of the teacher's role is to motivate young people's interest in mathematics. One way to achieve this is by appealing to the pupils' own experience"* (Sellinger, 1994:113).

The teachers themselves have to be prepared for this task. For teachers to become aware of cultural differences and different cultural experiences of the pupils is not something that happens of itself. Something has to be done to achieve it. An important moment for doing it, or at least for beginning to do it, is during teacher education courses both pre-service and in-service. During teacher education courses there is a need to involve teachers in activities that can help them develop their own awareness of cultural differences.

As Gerdes (1998b) stated, *"developing, in the process of mathematics teacher education, an awareness of the social and cultural bases of mathematics education involves developing an awareness of the influence of socio-cultural factors on the teaching and learning of mathematics. Trying to discover and analyse together, in the teacher educational environment, those factors that influenced one's own learning and that of one's colleagues may contribute to this awareness raising"* (p. 48).

Regarding this issue of developing an awareness of cultural differences in teacher education, Gerdes (1998b) explains, *"People may be doing mathematics, may be engaged in thinking that involves mathematical through processes, without themselves calling their activity "mathematical"; they may even say that they do not know mathematics, or that they are not able to do mathematics. Teachers becoming aware of these phenomena, and the reasons for them will not accept feelings such as "fear of mathematics" and "I lack*

mathematical ability" by their students or students' parents as natural, normal, or insurmountable. On the contrary, the teacher will try to place these feelings in their social, cultural, and historical context, being motivated to challenge them through her/his work as a teacher. So, the development of this capacity may be a focal point in pre-service and in-service mathematics teacher education"(p. 48).

It is important to act in order to bring changes in teachers, as according to Abreu (1993), "*(1) teachers need to be able to cope with different mathematical tools and to take on an analytical role which contrasts with the traditional prescriptive role; (2) teachers need to structure immediate social interactions in a pattern which differs from traditional teaching; (3) teachers need to develop awareness of which social representations inform their positions in order to be able to understand the children's perspective. If however these three basic changes in the role of the teacher are to be put into effect, more research is needed"* (p. 271). Clearly, meeting the implied challenge is a long process. There have been some initiatives in this direction (For example, Pompeu, 1992) and the results have been encouraging. This research is an attempt to contribute to these efforts.

2.4.2 Student teachers' attitudes towards mathematics

Research on the affective issue of teachers is very recent (McLeod, 1992; Thompson, 1992). Most professional development efforts in mathematics education focus on teachers' knowledge, understandings, or behaviour, rarely addressing the emotional aspects of their lives (Weissglass, 1994). It is generally agreed that teachers' emotional issues, such as feelings, beliefs and general attitudes, play a critical role in determining how teachers teach (Pajares, 1992). For example, "*beliefs about the nature of mathematics and how mathematics is done are important not only because they influence how one thinks about, approaches, and follows through on mathematical tasks but also because they influence how one attends to mathematics instruction"* (Garofalo, 1989: 502).

It is not that natural that, teachers will feel motivated and will be interested to act for changes in their way of teaching and working with students. Aiken (1976) gives two

reasons why research on the attitudes of pre-service elementary school teachers is of interest and fairly abundant. First, these teachers have the potential of greatly influencing their future pupils' attitudes. Secondly, prospective teachers are a readily available population. Although Aiken (1970) has also pointed out that it is unfair to indict teachers too strongly as creators of negative students attitudes towards mathematics, the results of research have suggested that the teacher, perhaps even more than parents, is an important determiner of students' attitudes.

Aiken (1970), in reviewing dozens of journal articles, doctoral theses and research reports concerned with attitudes towards mathematics, concluded that teachers' attitudes were being viewed as being of particular importance. Attitudes towards mathematics are highly related to student's statements about previous mathematics teachers (Aiken and Dreger, 1961). It is the pupil's attitude towards mathematics and his/her self-concept of his/her ability to succeed in mathematics that are factors that affect the progress and the future of the pupil (Olliges, 1988:21).

Evans (1965) argues that *"teacher who are confident, secure and interested in children are more likely to be successful, whilst those who are insecure, critical of the system, and the work conditions, tend to be sarcastic, overtly controlling and authoritarian towards pupils"* In addition, he says, *"this kind of teachers will tend to impose a barrier between himself and the children, and neither he nor they will be happy in the classroom"* (p. 136).

The quality of mathematics instruction at all school levels depends on the preparation of pre-service primary teachers to teach mathematics. *"Many teacher educators believe that developing positive attitudes towards mathematics should be a major goal of both mathematics and mathematics methods courses for pre-service primary teachers"* (Battista, 1986:10). Thus, student teachers developing a positive attitude toward mathematics could be a potential positive influence on their pupils, and hence, on the attitudes that those pupils might develop. Attitude is however a difficult variable to measure. Ringness (1975) states that attitudes are learnt, they are impermanent and can be changed, and there is hope that if things are corrected, the negative attitudes that teachers

may have towards their pupils may be obliterated. If these are obliterated, teachers themselves will, to some extent, be able to foster positive attitudes among their pupils.

Acknowledging the importance of teacher's attitudes toward mathematics, numerous educators have attempted to influence those attitudes by specific methods or courses. A general result of these efforts is that the attitudes of prospective primary teachers, relative to their pre-course attitudes or those of groups of students completing no course, improve as a result of a method-content course (Aiken, 1976).

A sound basic knowledge of mathematics is fundamental to life in modern society. Mathematical ideas are employed to organize, describe, regulate and foster our human activities. We all are consumers of mathematics and need to be aware of it to become interactive citizens in modern society. Mathematics is used to solve useful problem. It should not be seen to create a division between the mathematical world and the real world. If teachers are convinced and believe that mathematics is so useful or will be useful to the student in the future, then there is a reason to assume that they will work harder to ensure that their students learn mathematics (Fennema & Leder, 1990). Teachers do talk and advise their students about the need for and usefulness of doing mathematics. There is therefore little doubt that teachers can influence students' attitudes towards mathematics.

There is always a positive correlation between attitudes and behaviour. So, teachers' attitudes towards mathematics can be expected to influence their practice in teaching mathematics. Promoting positive attitudes towards mathematics is a goal of many teachers training courses and varying degrees of success have been reported in attaining this goal (Cf. Coulson, 1992).

This study is also aimed at incorporating the *Tchadji*-game as a cultural activity in mathematics teacher education. As Gerdes (1996) argues the "*Incorporation into the curriculum of elements belonging to the socio-cultural environment of the pupils and teachers, as starting point for mathematical activities in the classroom, is likely to increase the motivation of both pupils and teachers*" (p.928)

2.5 Summary

Through the time devoted to it in school and its compulsory character in many school curricula, mathematics is considered one of the most important subjects in the school curriculum (D'Ambrosio, 1990) and, at the same time, as it is considered by many learners, mathematics is a difficult, impossible, mysterious, meaningless and boring subject (Bishop, 1988a, Gerdes, 1995).

In many countries in Africa, educators and researcher, recognizing the generally lack of research as stressed by Joseph Ki-Zerbo (Cf. Gerdes, 1995), have been asking themselves why the process of teaching and learning mathematics is connected with so many problems.

In Mozambique, for example, the problems preoccupy even more, when certain situations are frequently observable, like the methods most frequently used for teaching being the teachers exposition, when the teachers are not aware of neither the previous learners' experiences from in- and out of school situation, the learners' social and cultural environment (Cf. Masingila et al., 1996), nor the learners' affective site, their attitudes, their beliefs, interests and motivation to mathematics.

Ethnomathematics claims that mathematics is neither culture-free nor value-free (Bishop, 1988a, 1988b). In the last fifteen years many researcher investigated the cultural roots of mathematics and the results are widely recognized. The initial inspiration of ethnomathematics has been pedagogical. It appears always the question, whether or not there are any implications of the cultural aspect of mathematics in the teaching and learning. In fact, they have been some efforts to materialize this aspect (See Gerdes, 1995), but I have the impression that much more has to be done. As argued by Gerdes (1996) this aspect is still in its infancy (Cf. Bishop, 1993).

Games are part of the human culture. The games itself and also their playing process are widely recognized by mathematicians, educators and Anthropologists as involving various mathematical aspects such as logical thinking, mathematical thinking, counting, calculation skills, geometrical reasoning, problem solving and probability reasoning (Zaslavsky, 1973a, 1989, Bell & Cornelius, 1995; Barta & Schaelling, 1998; Bishop, 1991; Doumbia, 1989, Doumbia & Pil, 1992; Silva, 1994, 1995; Strong & Lumpkin, 1995, Cheska, 1987; Odeleye, 1977; Ismael, 1996).

Many different games have been used for various purposes in the teaching of a variety of topics in mathematics. Their effects have been studied by a large number of researchers and in recent years the use of games in the teaching and learning of mathematics has increased (Bright et al., 1985, quoted in Schroeder, 1989; Vithal, 1992, Kirkby, 1992).

Regarding teaching of probability in school, researchers are still asking whether there are any optimum teaching and learning techniques in probability (Shaughnessy, 1992, 1997). In fact, there is very little persistent research on how best to teach probability (Ahlgren & Garfield, 1991).

For example, research on the effects of using games has produced inconclusive and not significant findings (Cf. Boseman & Schellenberger, 1974; Greenlaw & Wyman, 1973, Oldfield, 1991a; Klein & Freitag, 1991; Ernest, 1986; Sleet, 1985; Strauss, 1986; DeVries & Slavin, 1978; Wolfe, 1985). This suggests a need for further research and the aims of this study are directly related to this aspect. The study is furthermore indirectly aimed to explore the influences of the social and cultural environment of the students in the learning of probability as it is recommended by Borovcnik & Peard (1996).

On the other hand, many researchers have suggested that in order to develop young children's potential for understanding of probability, it is necessary to provide them with opportunities to experience actively and emotionally stochastic situations. Also probability should be introduced through activities and simulations and not abstractions (Falk &

Levin, 1980; Steinbring, 1991; Fischbein & Gazit, 1984; Munisamy & Doraisamy, 1996). Games, in general, generate such kind of activities (Bishop, 1988a, Falk & Levin, 1980).

The area of traditional games is an extensive area of study. For example, the Mancala type games are played in many parts of the world and they involve many variants (See section 5.2). Most of these games have not been investigated. These games involve also complex movements and strategies. Is there any mathematical thinking involved? Do they challenge the students' attention? Are they also interesting for the students? The mathematical potential of Mancala type games and the potential for their use in the classroom are not known at all. Here, there is a role for ethnomathematical research.

What is the role of teachers in all process of implementing ethnomathematical activities in the classroom? Teachers need to become aware of cultural differences and of the different cultural experiences of their pupils and this is not something that happens naturally. Something has to be done to achieve it. An important opportunity for realising this it is provided by teacher education courses both pre-service and in-service.

“Teachers need to be able to cope with different mathematical tools and to take on an analytical role which contrasts with the traditional prescriptive role” (Abreu, 1993: 271). This can be realised during teacher education courses. Once they are able to cope with it, how do they feel? Are they interested in, at least, trying to do it? What are their motivations? How does their attitudes towards teaching mathematics develop? Here, more research is certainly needed.

CHAPTER THREE: EPISTEMOLOGICAL GROUNDING OF THE RESEARCH

3.1 Introduction

Over the last two decades mathematics education research witnessed a diversification of research thrusts and theoretical models coming from sociology, anthropology, and linguistics (Cf. Atweh et al, 2000). Since Ubiratan D'Ambrosio (1985a), in his plenary address at the ICME-congress in Adelaide (Australia), formally established the field of ethnomathematics, this word has been used and misused in curriculum development in many countries.

For instance, Barton (1999) emphasises that, *"as a community of teachers, mathematics educators, academics, mathematicians and others interested in ethnomathematics, we create and are responsible for this field. We do have a stake in the way the term is used, in the way the notion is understood by others who are not so involved and in the way the undertaking has a political and educational effect in society"* (p.32)

Barton (1999) raises also concerns about the necessity of articulating a theory, which could drive the work on ethnomathematics. He argued that, *"In fact, much of the theory which can be found in ethnomathematical writings is based on such self-reflection: hermeneutics, semiotics, post-modernism, critical theory and fields of sociology and anthropology in general contain reflexive characteristics"* (p. 32). I understand that, it is as a result of profound reflection of scholars working in the area of ethnomathematics, which is included in their writings that a consistent basic theory in this area will come out. The present chapter is aimed to contribute for this debate.

The theoretical framework for inquiring into the mathematical ideas embedded in cultural elements and activities can be considered as part of a new disciplinary paradigm: ethnomathematics. Gerdes, based on the work of D'Ambrosio (1990), has made major theoretical and empirical contribution to this field (See, e.g., Gerdes: 1985, 1989 & 1995),

whose theoretical origins are owed to the Brazilian mathematician, Ubiratan D'Ambrosio (See, for instance, D'Ambrosio, 1990).

Theoretically, D'Ambrosio (1997¹) recognizes that "*The complexity of every society, so different one from another, is responsible for the generation of codes, norms, rules and values in the direction of organizing, classifying, comparing, and ordering the action of its individuals. Instances of these codes, norms, rules and values are instruments of analyses, of explanations, and of actions, such as more or less, small and big, few or many, near or far, and in and out. These codes, norms, rules and values - for instance, cardinality and ordinality, counting and measuring, and sorting and comparing - take different forms according to the cultures in which they were generated, organized, and accepted. To recover these forms and behaviours in different environments has been the main thrust of ethnomathematics(...)*"(p.xvi)

One of the current directions of ethnomathematics is the academic debate concerning its philosophy, legitimacy and relationship with other disciplines and theories. This debate has been going on in a number of articles (e.g., issue 14(2) of *For the Learning of Mathematics* and in *Educational Studies in Mathematics*, e.g. Chevallard, 1990; Skovsmose, 1990; Barton, 1996; Vithal & Skovsmose, 1997). These, and other examples, have been shown to be useful for their motivation, contributions and practice (Barton, 1999).

There has not been consensus on the theoretical framework to support the work on ethnomathematics as a research field. For example, some scholars have argued that ethnomathematics is more an anthropological work (Ascher, 1986 & 1991; Barton, 1996). Others supported the idea that ethnomathematics is also an educational activity (Gerdes: 1985, 1989 & 1995; D'Ambrosio, 1985, 1990; Zaslavsky, 1989a). Others, like Powell & Frankenstein (1997), suggest that, ethnomathematics is an interplay between mathematics, education, culture and politics. The purpose of present chapter is to discuss theoretical and

¹ Foreword to Powell & Frankenstein (1997)

philosophical points of view, which have direct or even indirect links to the field of ethnomathematics as the field of the present study.

3.2 Cross-cultural approach to mathematical knowledge

For many years mathematics educators and researchers in mathematics education have focused on the classroom as the primary setting in which mathematics learning takes place. Analyses of children's knowledge of mathematics (for example, Carraher, Carraher & Schliemann, 1985, 1987; Ginsburg, 1977; Resnick, 1984; Draisma, 1992, De Abreu, 1992) have shown that much mathematical knowledge is acquired outside the school. This brings new variables into the analyses of mathematics learning and teaching. Is the mathematics learned outside the school the same as that taught in school? How can teachers identify and capitalize on mathematics learned outside school? (See also section 2.2.1.3.2).

D'Ambrosio (e.g. 1990) has used the expression *ethnomathematics* to refer to forms of mathematics knowledge that vary as a consequence of being embedded in cultural activities whose purpose is other than *doing mathematics*. Everyday activities such as building houses, exchanging money, weighing products, calculating proportions for recipes, involve numbers, calculations, and precise geometrical patterns. These applications of mathematics often look different to those used in school. Everyday mathematics also varies across countries, because of differences in numeration systems used, for example, or the devices used for calculating. These differences may be perceived as deep- or surface differences, depending on what views one holds of mathematical knowledge (See also section 2.2.1).

Two distinct approaches to the study of cultural influences on mathematical knowledge can be identified in the literature. One view, espoused by Stigler and Baranes (1988), holds that "*Mathematics is not a universal, formal domain of knowledge ... but rather an assemblage of culturally constructed symbolic representations. As children develop, they*

incorporate representations and procedures into their cognitive system, a process that occurs in the context of socially constructed activities"(p.258).

Here, the definition of mathematical knowledge is implicit and appears to be based on the content of knowledge. Activities that involve number, geometrical patterns, calculations, and so forth are treated as applications of mathematical knowledge. This view stresses differences rather than similarities across cultures. Within this approach, for example, variations across cultures of reciting strings of numbers are treated as reflections of the differences in language and numeration systems.

A second perspective, illustrated in the works by Gal'perin and Georgiev (1969) and D'Ambrósio (1986), suggests that the analysis of cultural influences on mathematical knowledge can demonstrate both differences and invariance in mathematical knowledge across cultures. In this view, 'mathematising' reality is representing reality in such a way that (a) more knowledge about the represented reality can be generated through inferences using mental representations, and (b) there is no need to manipulate reality further in order to verify this new knowledge. Invariant logical structures are embedded in mathematical knowledge, regardless of whether mathematical knowledge is developed in or out of school. It is the ability to make inferences using these structures - not the content of knowledge - that distinguishes mathematical knowledge.

Understanding cultural influences on mathematical learning must involve both the analysis of the differences among particular approaches to mathematising reality and the recognition of the logical invariants underlying these differences. Cultural differences exist also across cultures. Within a culture, practices of mathematics vary depending on their purpose. Research on mathematics education can benefit from the analysis of various solutions to the same problem that co-exists in a single culture. In cultures, where there are both oral and written representations of numbers, it is likely that two practices of arithmetic co-exist (Reed & Lave, 1981). For example, Draisma and his team at Beira Branch of UP investigated oral counting practices of pupils in primary schools and the

results showed differences from the current written numbers' practices in schools in Mozambique (See, e.g., Draisma, 1998, 2000)

For example, when a teacher faces a class with the aim of teaching a particular concept, the teacher must ponder whether the pupils are likely to have used the concept in everyday life. Until more research is available on this subject, teachers will have the task of looking for everyday uses of the concept and attempt to analyse the logical invariants that underlie everyday concepts and procedures. Everyday procedures will often differ from school-taught procedures, and teachers will need to look beneath the surface to recognize which invariants are being respected in everyday practices. After identifying an everyday situation in which the concept is used, the teacher will need to know how to use the situation to promote an awareness and understanding of the concept.

3.3 Piaget versus Vygotsky

The Piagetian paradigm can be described as follows: the learner constructs new knowledge on the basis of his/her experience. It was this aspect of Piaget's work that met the need for a theory, which could bring new insight into problems concerning learning and teaching. Many educators were very worried about the prevailing use of drill methods in mathematics education as promoted by behaviourism.

Piaget's epistemological base is that knowledge, including intelligence as general knowledge, exists in the context of actions, such as physical actions and mental actions. Knowledge is thus constructed as the result of the individual's actions on his environment (world). In the literature, we find the word constructivism being related to Piaget's ideas of knowledge acquisition (See, for example, Wertsch, 1979; Von Glasersfeld, 1995). Piaget's theory is based on actions, or as it is referred in the literature, is an "activity theory" (C.f. Mellin-Olsen, 1987). Development and learning arise from the activities of the individual, and how he or she constructs knowledge on the basis of these activities, i.e., experience is the basis for action adopting cognitive structures.

Didactical theory based on Piaget's general epistemology is not complete. Aspects that are wanting include the pupil's evaluation of the kind of learning situation she/he is confronted with; the recognition of the fact that the pupil is in a position to reject the kind of activity he/she is invited to participate in; the role that her/his own judgment of the educational situation she/he is part of, plays in affecting her/his learning behaviour, and how all this relates to the design of the educational situation; the conception of communication or dialogue which is object-oriented and in which both teacher and learners participate (Mellin-Olsen, 1987). These are important issues when it comes to consider, for example, the pupils' environment in the teaching.

Vygotsky, on the other hand, emphasises the role of language in expressing emotions, needs, claims, experiences, everything which was important in the child's life, i.e., the socio-cultural milieu, rather than as a tool for doing ready-made exercises from some textbooks. His theory is clearly embedding cognition in socio-cultural experiences. He is recognized as one of the great founders of activity theory. It disregards major social structures which we today experience as important in order to understand learning processes within an educational system. The emphasis on Vygotsky's socio-cultural approach arose out of recognition of the limitations of Piagetian cross-cultural research (Cf. Cobb & Bauersfeld, 1995; Abreu, 2000).

Psychologists studying learning in socio-cultural contexts over the last decades, both in and out-of-school, have been moving away from a cross-cultural experimental psychology, passing through a Vygotskian-based socio-cultural psychology, to the most recent calls for a cultural psychology. They have realised that the diversity of the socio-cultural characters of the human higher functions, such as linguistic habits, moral ideas or ideological convictions, could no longer be studied effectively by traditional experimental methods. For this, they could be explored only by the non-experimental methods of anthropology, sociology and social psychology. They realise that the cultural psychology perspective could be useful in explaining the uses and learning of mathematics in and out-of-school (Cf. Abreu, 1998).

The studies of Gay and Cole (1967) with the Kpelle in Liberia seem to fit into the Vygotskian framework. First, the manner in which Kpelle drew on local cultural resources, such as local forms of measuring, to articulate their thinking, parallels Vygotsky's view of cognition as mediated by cultural tools. Second, the insight that their own relationship with the people they were studying changed the nature of their observations was in line with Vygotsky's view of cognition as mediated by social interactions (Cf. Abreu, 2000).

Gay and Cole (1967) did not take school knowledge as the reference and elaborate tasks from this perspective to see whether individuals transfer it to other settings. They used ethnographic observations outside the school contexts. They organised research tasks based on activities within the community studied. Similarly, this study looks at a cultural resource, the *Tchadji*-game, and uncovers the 'hidden' mathematical ideas embedded in the game.

According to Vygotskian research and simple observations of the children's play, we realise that children possess important thinking-tools in the form of sign-systems. These tools are functional for children's play. Where Piaget tends to say: "child has not", Vygotsky tends to say: "the child has, look how she uses her thinking-tools". Where the followers of Piaget construct tasks which direct children away from their daily activities, Vygotsky would ask us to start there, and connect school activities to thinking tools, having regard to the historical and social dimensions of activity (Mellin-Olsen, 1987).

As an alternative to Piaget's theory, which focused on the progressive maturation of cognitive structures inside people's minds, some researchers looked for explanations for their conflicting results in cultural contexts. Differences did not appear to be a question of rate of development as suggested by Piaget's theory, but a question of different kinds of cognitive development. These researchers found support for their findings in an alternative theory of cognitive development, which originated in Vygotsky's work and emphasised cognitive development as a socio-cultural process.

For example, the work of Gay and Cole (1967) is a pioneering effort in cross-cultural research. This work is consistent with Vygotsky's theoretical constructs in the sense that the *"emphasis has been primarily on a contextual analysis - an analysis of the way the conceptual abilities of individuals are influenced by and adapted to particular problem-solving Contexts"* (Saxe & Posner, 1983: 301)

Ethnomathematical research is related to socio-cultural mathematical experiences. Anthropological work has to be undertaken, when it comes to know better these experiences. It was also this direction that made the emergence of ethnomathematics to be more concerned with the historical and anthropological studies of mathematics of different social and cultural groups. Ethnomathematics shares with the cultural psychology and cognitive approach the view that each culture develops its own mathematical knowledge.

As Abreu (1993) concluded, *"cross-cultural studies concerned with the development of mathematical cognition have provided a basis for the acknowledgment of mathematics as a cultural form of knowledge. The Challenge to the Piagetian assumption of the universality of cognitive development has led to the need to account for culture not as an external variable that could accelerate or delay the rate of cognitive development, but as a constitutive part of mathematical cognition. The scarcity of descriptions of non-Western mathematics has led Western psychologists to adopt what has been traditionally the role of anthropologists, undertaking ethnographic descriptions of the mathematics of the cultures in which they aim to carry out their studies"* (p.16).

3.4 The philosophical point of view

Traditionally, mathematical knowledge has been understood as universal and absolute knowledge, whose epistemological status sets it above all other forms of knowledge. The traditional foundationalist schools of formalism, logicism and intuitionism sought to establish the absolute validity of mathematical knowledge. Although modern philosophy

of mathematics has in part moved away from this dogma, it is still very influential and needs to be critiqued (Ernest, 1991).

In the last two decades, due to the growing acceptance of the weakness of the universalistic and absolutist accounts of mathematical knowledge and of the impossibility of establishing knowledge claims absolutely, a need for reconceptualisation of philosophy of mathematics has arisen (Ernest, 1997).

Imre Lakatos, with his historical approach to mathematical invention through proofs and refutations, has been a great contributor to the rejection of absolutism in establishing mathematical knowledge. His work has shown the need for the reconceptualisation of the philosophy of mathematics to include the history and methodology of mathematics (Ernest, 1998).

Ernest (1991) proposes social constructivism as a basis for a philosophy of mathematics. Social constructivism agrees with Lakatos' approach to mathematical invention through negotiation and acceptance of mathematical knowledge, concepts and proofs. The second basic assumption of social constructivism is the fact that mathematical knowledge is taken to rest on socially situated linguistic practices, including shared rules, meanings and conventions, i.e., on both tacit and explicit knowledge and symbolic practices. In fact, the novelty of social constructivism is to realise, both that mathematical knowledge is necessary, stable and autonomous, but also that this co-exists with its contingent, fallibilist, and historically shifting character (Ernest, 1998).

In addition, Ernest (1998) concludes that *"social constructivism links both the learning of mathematics and research in mathematics in an overall scheme in which knowledge travels either embodied in a person or in a text, and the processes of formation and warranting in two contexts are parallel. Social constructivism provides an explanation of how mathematics and logic seems irrefutably certain, yet are contingent, historical creations"* (p.168).

Fallibilism is the position of social constructivism, which claims that mathematics is a relative, contingent, historical construct, and the concepts, definitions and rules of mathematics were invented and evolved over millenia, including rules of truth and proof. Thus, mathematical knowledge is based on contingency, due to its historical development and the resourcing and direction of mathematics. Mathematics consists of language games with deeply entrenched rules and patterns that are very stable and enduring, but which always remain open to the possibility of change. So, the range of the possible to invent within mathematical systems, remain open (Ernest, 1998).

Social constructivism and fallibilism reject absolutism. The latter involves the following three sub-theses. First of all, there is the thesis of universalism, which asserts that all knowing beings at all times and in all cultures would agree on truth and on mathematical knowledge. This thesis is rejected by social constructivism. So, mathematical knowledge can not be regarded as universal. Ethnomathematics also rejects this position.

Secondly, there is the thesis of objectivism, which asserts that truth depends on objective reality, not on the views of persons or groups. This thesis is unsatisfactory because it is not what mathematicians intuit which is taken as objective truth, but rather what they prove that is regarded as true. Thus how mathematical truth 'depends' on mathematical reality that plays no part in its justification is unclear. Third, there is the thesis of foundationalism, which asserts that there is a unique permanent foundation for knowledge. This foundation has not been identified historically. Foundationalist philosophies of mathematics (Logicism, Formalism, Intuitionism) have all failed. Furthermore, the idea that a basic foundation for mathematical knowledge exists leads to a vicious cycle, because no basic set of assumptions can ever ultimately be dispensed with. So, this is also rejected by social constructivism.

Ethnomathematics, as a field of study, originated from the issues concerned with the universality of mathematical knowledge. The claims that mathematics cannot be seen as a culture free discipline and that it is indeed a pan-human activity, has been dominant in this

field. Thus, ethnomathematics links more directly with the fallibilist, quasi-empirical epistemology of socio-cultural constructivism (Cf. Laridon, 1995).

3.5 Theoretical approach of ethnomathematics

Mathematics is no longer seen as disconnected from culture. The ideas of D'Ambrosio (1985), Mellin-Olsen (1987), Zaslavsky (1987), Gerdes (1988) and Bishop (1988) have certainly largely contributed to this view of mathematics as a cultural phenomenon. With this consideration a new theoretical approach has emerged, the so-called ethnomathematical approach or theory.

A theory of knowledge can be built on five strands: (1) Recognition and description of ad hoc knowledge, (2) How do methods give rise to theories?, (3) How do theories lay the ground for invention?, (4) What are the socio-political frameworks of knowledge?, and (5) How is knowledge generated? So, the theory of ethnomathematics should also focus in these five strands. Much of the current research in ethnomathematics has been directed more to descriptions of knowledge and legitimising of methods (See Gerdes, 1996). There is indeed a need for more research to enlarge and support the emerging ethnomathematical theory. For example, as ethnomathematics is considered a interplay between mathematics, culture, education and politics, research should challenge all these interconnection in ethnomathematics. Much of research in ethnomathematics has been directed to the issue on mathematics and culture and there is much evidence in this issue in terms of findings, which supports theoretical constructs. There are indeed still some controversies with regard to the methodology. For example, research on the educational part of ethnomathematics need to give more evidence.

3.6 Concluding remarks

There is no doubt that the research on socio-cultural aspects in mathematics and mathematics education, as part of ethnomathematical research, has been influenced by Vygotsky's socio-cultural approach and that socio-cultural constructivism, as an approach

to the philosophy of mathematics, constitutes a support philosophy for the work of ethnomathematicians.

This study is guided by a socio-cultural constructivist approach to the acquisition of knowledge, i.e., all knowledge is constructed by the individual and it is connected with its socio-cultural contexts. Mathematical knowledge is constructed, at least in part, through a process of reflective abstraction. The mind of the individuals plays an important role in knowledge construction.

Vygotskian activity theory, which is a "*theoretical framework which affords the prospect of an integrated account of mind-in-action*" (Masingila, 1992: 9), stresses that the individual "*acts within social structures, and thus both creates these and is created by them*" (Mellin-Olsen, 1987:40).

Vygotskian activity theory also constitutes a guiding framework for this study, specially, in the investigation of the mathematics embedded in the *Tchadji* - game, as a daily life activity of a certain cultural group. It also constitutes the framework for research on the implication of these activities when it comes to incorporate them into school mathematics, i.e., the study of the relationship between cultural context (cultural tools) and learning and uses of mathematical knowledge (cognition).

CHAPTER FOUR: RESEARCH METHODOLOGY

4.1 Introduction

The present chapter includes a description of the methodology and procedures employed in investigating, first, the use of the games approach for teaching the probability syllabus as stated in aim 1.2; second, the mathematical ideas of the players of *Tchadji* as in aim 2 and, third, the intervention with *Tchadji* among student teachers in Mozambique (See details in chapter 1). The instruments used and data analysis procedures in all phases of the study are also described in this chapter.

4.2 The experimental study (phase 1)

From the ideas presented in my paper (Ismael, 1996), that included a reflection on the possible connection of the games of type Mancala, its procedures of playing and its players' thinking, with probabilistic ideas, a exploration of the *Tchadji*, a Mancala type game, was undertaken and the results of this investigation of any underlying probability ideas embedded in the *Tchadji* and other three types of games and the development of a teaching approach to probability are described in the Chapter 6. The present section describes the experimental approach and the instruments employed for studying the use of these probabilistic ideas in the classroom.

The idea of this part of the study was to compare the outcomes of two groups of secondary school students taught elementary probability using two different approaches. Generally, the literature suggested the use of experimental research methods (i.e., "*the focus of which is the identification of causes or what leads to what*" (Rosnow & Rosenthal, 1996: 16) for a study whose objective is to investigate the effect of any kind of intervention (e.g., Cohen & Manion, 1994; Wiersma, 1995).

In experimental research we can distinguish the "true" experimental from the quasi-experimental methods. The principal characteristic that distinguishes a research design, as a true experiment is the random assignment of treatment to the sampling units, also called randomisation. In this case, equivalence of the groups is achieved within the limits of

random fluctuation due to random assignment (Rosnow & Rosenthal, 1991; Wiersma, 1995).

Random assignment to treatments is not always possible in research involving human subjects. In educational research, for example, there are many naturally formed intact groups of subjects, such as the students in a classroom. For this reason, researchers have employed a number of alternative approaches that could serve as an approximation of true experimental design. Quasi-experimental methods provide alternative approaches that use intact groups of subjects. Quasi-experimental approaches resemble true experiments in that both types have treatments, outcome measures, and sampling units (Rosnow & Rosenthal, 1991).

As Campbell & Stanley (1963) recommend, a quasi-experimental design should be used only "*if better designs are not feasible*" (Pedhazur & Schmelkin, 1991: 277). The situation in Mozambican schools (overcrowded classes and lack of rooms) makes the random selection of subjects practically impossible (See Chapter 1). This phase of the research has been implemented by means of quasi-experimental methods. A pre- and post-test, non-equivalent control group design has been employed. "*The non-equivalent-groups design is the most widely used quasi-experimental method. It refers to non randomised research in which responses of a treatment group and a control group are compared on measures collected at the beginning and end of the research*" (Rosnow & Rosenthal, 1991: 92)

4.2.1 Research design

Probability is first taught in Mozambican schools in Grade 11 (senior secondary school), Unit VI with 9 lessons, which usually occurs between October and November of each academic year. The unit includes the following contents (Ministério de Educação, 1993: 22):

1. Concept of randomness and random phenomena

(Types of events, event spaces, union of events, intersection of events, certain and impossible events, independent and complementary events)

2. Absolute and relative frequency of an event and properties of relative frequency

3. The notion of probability through the notion of relative frequency; axiomatisation of the concept of probability in a finite space
4. Determination of probability when the elementary events are equally likely (Ministério de Educação, 1993: 22).

In Mozambique, there is only one public senior secondary school per province. In order to avoid contact between the teachers and students of the groups being taught through the alternate approaches, which could introduce bias into the study, the research has been conducted in two different schools, in the 'Escola Secundária de Nampula' in the Nampula Province and 'Escola Secundária de Lichinga' in the Niassa Province.

Six intact groups (three experimental and three control) of grade 11 senior secondary school students were used (See Table 4.1). The experimental groups were from the Province of Niassa. These groups were taught by use of the game approach. The control groups were from Nampula, and have been taught in the traditional way. Both places are situated in the Northern part of Mozambique. The choice made, considered the variants of the traditional games, which are proper for the designated area.

	Intact Classes	Pre- test	Pre-attitude- motivational questionnaire	Treatment/ Experimental variable	Post- test	Post-attitude- motivational questionnaire
G1	Class 1	O1	O7	Game approach (X)	O13	O19
G2	Class 2	O2	O8	Game approach (X)	O14	O20
G3	Class 3	O3	O9	Game approach (X)	O15	O21
G4	Class 4	O4	O10	Non-game approach	O16	O22
G5	Class 5	O5	O11	Non-game approach	O17	O23
G6	Class 6	O6	O12	Non-game approach	O18	O24

-----2 weeks' instruction-----

Table 4.1: The research design

4.2.2 Participants

The classes in Mozambican school are usually overcrowded with a minimum of 35 pupils. A total of 194 Grade 11 students (94 from Nampula and 100 from Niassa) participated in the study. However for data analysis only 162 students (87 from Nampula and 75 from Niassa) have been considered (See table 4.2).

	Male	Female	Totals
CLASS 1npl	31		31
2npl	17	11	28
3npl	24	4	28 87
1nias	26	4	30
2nias	20	2	22
3nias	19	4	23 75
Total	137	25	162

Table 4.2: Class-gender cross-tabulation

These students had most likely participated in all sessions. 85,5% of the sample were males and 14,5% females. From the 87 subjects from Nampula, 15 were females, i.e., 17%, and from 75 subjects in Niassa, 10 were females, i.e., 13%. One group in Nampula had only male students.

Four teachers were involved in the study, two in Nampula and two in Niassa each teaching one class. The third class, in both Nampula and in Niassa, were taught by myself. The teaching experience of these teachers varied from 5 to 15 years. Three teachers, one from Nampula (control conditions) and the two from Niassa (experimental conditions) had participated in teacher training courses at university. The two teachers from Niassa also attended a university degree course for teaching mathematics. The second teacher from Nampula attended a university degree course in technology with a strong mathematics component but no pedagogical component.

4.2.3 Research instruments

4.2.3.1 Overview

A teaching approach using four kinds of cultural games had been developed for use in the three classes in Lichinga, the capital of Niassa Province. Here concrete materials have been developed for classrooms use, like worksheets and materials for playing the games (see Chapter 6). However, for the data collection, appropriate instruments were developed as described below.

For measuring attitudes towards mathematics, questionnaires were considered the most appropriate instruments in the study because, according to Cohen & Manion (1995), each respondent is given the same set of questions as everybody else in the sample, phrased in exactly the same way, and the data obtained from questionnaires are more comparable than information obtained by means of interviews. They argue further that a questionnaire is objective because it is less human. People are more likely to express aspects of thoughts, feelings and values in a questionnaire. It is more economical than the interview in terms of money and time. However, the motivation of the respondents cannot be checked using a questionnaire and which makes it difficult to judge the validity of their responses.

For measuring performance on probability a post-test was prepared reflecting the probability syllabus. In addition observations have been carried out by the researcher for the purpose of reporting the teaching style and participants' behaviour, and, as far as was possible, the dialogue-interactions were audio taped for further supportive analysis. It was important here to record any questions asked during the lessons and any comments by the teachers or students.

4.2.3.2 Validity and Reliability

For all the instruments used, a general concern has been about validity and reliability. Validity of a test concerns what the test measures and how well it does so. So, validity is the extent to which an instrument measures what is supposed to be measured (Cf. Rosenthal & Rosnow, 1991; Burns, 1995; Oppenheimer, 1992). Reliability of an

instrument is its degree of consistency in measuring whatever it measures. Reliability measures can take values of 0 to 1.0, inclusive.

We distinguish content validity, which involves the systematic examinations of the test content to determine whether it covers a representative sample of the behaviour domain to be measured, and face validity, which pertains to whether the test "looks valid" to the subjects who take it, the administrative personnel who decide on its use, and other technically untrained observers.

Validity involves two concepts simultaneously, internal validity and external validity. Internal validity is the extent to which results can be interpreted accurately, and external validity is the extent to which results can be generalized to populations, situations and conditions. To a large extent, internal validity is a pre-requisite for external validity because if the results can not be interpreted if it is not likely that they can be generalised (Wiersma, 1995). Validity of research is always a matter of degree. It is practically impossible to attain "perfect" internal and external validity in a study. The researcher attempts to attain a balance so that, results can be interpreted with reasonable certainty and still have some useful generalisability.

The teachers working in the experimental conditions had the opportunity to comment on the items of the post-test. Their opinion was that the test was appropriate for testing the probability unit.

Reliability refers to the consistency of scores obtained by some persons when reexamined with the same test on different occasions, or with different sets of equivalent items, or under other variable test conditions. In other words, it concerns the replicability and consistency of methods, conditions, and results. Reliability is a necessary characteristic for validity. Essentially, reliability and validity establish the credibility of research. Reliability focuses on the replicability of the research and validity on the accuracy and generasability of the findings (Wiersma, 1995).

Since *"all instruments, including measurement of behaviour are subject to fluctuations (also called errors) that can affect reliability and validity"* (Rosenthal & Rosnow, 1991:

46), attempts have been made to maintain an acceptable reliability. So, for the pre- and post-attitude motivational questionnaire and for the pre-and post test a Crombach's alpha reliability coefficient (Table 4.3) has been computed to test their internal consistency (Cf. Cohen & Manion, 1994, Rosenthal & Rosnow, 1991; Wiersma, 1995).

Instruments	No items	Alpha
Pre-test (general mathematics)	20	0.6166
Post-test (Probability)	12	0.3079
Pre-questionnaire (Attitudes & Motivations)	30	0.7474
Post-questionnaire (Attitudes & Motivations)	32	0.7458

Table 4.3: Crombach's alpha reliability coefficients

Thus, the reliability coefficient value for the pre- and post-attitude motivational questionnaire and for the pre-test can be considered satisfactory. According to Rosenthal & Rosnow (1991), for experimental research, instruments with reliability coefficient value much lower than 0.85, as the case with the post-test, may be accepted as satisfactory.

There are several factors that should be considered in interpreting reliability coefficients:

- a) The more heterogeneous a group is on the trait that is measured, the higher the reliability.
- b) The more items there are in the instrument, the higher the reliability.
- c) The greater the range of scores, the higher the reliability.
- d) Achievement test of medium difficult level will result in a higher reliability than either very hard or very easy tests.
- e) Reliability, like validity, is usually based on a morning group and strictly speaking, the reliability is demonstrated only for students whose characteristics are similar to those of the morning group.
- f) The more that items discriminate between high and low achievers, the greater the reliability of the test.

Thus an alternative procedure giving the research an impression of internal reliability can be via good item discrimination and difficulty (Crombach, 1990)

4.2.3.3 Pre-test on general mathematics

A pre-test on previous mathematical knowledge useful for learning and understanding probability, consisting of twenty multiple-choice items was used (See Appendices A10 & A11). The intention of using this test has been to compensate, in the analysis of data, for the initial differences of the intact groups. The test items were mostly written by myself or adapted from other assessment tests. The internal consistency reliability measure from the test (0.6166) was considered satisfactory.

4.2.3.4 Pre Attitude Motivational Questionnaire

In order to have an idea of the students' initial attitudes to mathematics and their motivation for learning mathematics a pre-attitude-motivational questionnaire with thirty Likert type items with five steps in the scale from 'Strongly Agree' to 'Strongly Disagree' (Burns, 1995), was prepared (See Appendices A10 & A12). The items of the questionnaire were single sentences or statements about mathematics and have been chosen and readapted from Coulson (1992), Osei (1995) and Oskamp (1977). According to Oppenheim (1992), an attitude statement is a single sentence that expresses a point of view, a belief, a preference, a judgment, an emotional feeling, a position for or against something.

For Rosenthal & Rosnow (1991), the term "Likert Scaling" (also called the method of summated ratings) is among the most misused terms in behavioral science. Many researchers mistakenly call any attitude scale a Likert scale if it instructs the subject to indicate the degree of agreement or disagreement on a five-step measurement scale. In order to be a true Likert scale, an item analysis has to be performed when constructing the questionnaire. Item analysis is a method developed by Rensis Likert (1932), which results in a self-rating attitude questionnaire which, in appearance, is a type of numerical rating scale in that numbers are associated with different response alternatives (e.g., Strongly agree, agree, undecided, disagree, strongly disagree) to statements that are easily classified as favourable or unfavourable.

Rosenthal & Rosnow (1991) indicated the steps to follow in order to design a Likert questionnaire. The first step in constructing a Likert questionnaire to measure people's attitudes is to gather a large number of statements on the topic of interest. These are given to a sample of subjects from the target population, who indicate their evaluation usually by means of a five-point numerical rating scale. The technique of item analysis is then used to sort out the data in order to select the best statements for the final version. This involves calculating the extent to which the responses to individual statements are correlated with the total score (the sum of all items). Statements that correlate well with the total score are then chosen for the final scale. The rationale is that statements that have low correlations with the total score will not be good discriminatory between those respondents with positive attitudes and those with negative attitudes.

Due to lack of time and the fact that most items were adapted from an existing Likert questionnaire (See Coulson, 1992), these steps were not followed to construct the questionnaire. So, this was not an authentic Likert questionnaire. It was more a questionnaire of Likert type, since it used the procedure of responses scaling employed by Likert. An additional self-appraisal questionnaire consisting of eight items using a scale from 5 = 'Excellent' to 1 = 'weak', designed for triangulation purposes was also used (See Appendix A15).

4.2.3.5 Post-Test on Probability

At the end of the interventions in the two towns (Control and experimental conditions), a post-test on the understanding of probability concepts entirely prepared by myself was applied. The test consisted of twelve multiple-choice items (See Appendices A10 & A13).

The items consisted of the concepts taught in the classroom whether using games or not. The concepts covered were the ones, which constitutes the main objectives of the syllabus in the program. These concepts were: random experience; elementary complementary, impossible, contrary and independent events; probability; estimation of probability. The internal consistency-reliability measure of this test was weak (See table 4.3).

4.2.3.6 Post-Attitude Motivational Questionnaire

A post-questionnaire on motivation for learning probability and on attitudes towards learning mathematics consisting of 32 items (See Appendices A10 & A14) was adopted from the instrument developed by Coulson (1992) with high internal consistency reliability coefficient. The items of this questionnaire were of the same nature as the pre-questionnaire with few exception. For example, compared with the pre-questionnaire, this one had two more items. Most of the items were taken from the pre-questionnaire, but they were reformulated more specifically for the probability topic.

Coulson (1992) conceptualised attitudes to science as a multi-dimensional construct and developed an instrument consisting of four scales, assessing students' confidence in doing science, their enjoyment of science at school, their feelings about the personal usefulness of doing science, and their evaluation of the appropriateness of science for young children. Each of these dimensions has been measured separately with 8-short items of the Likert scale type. As Foddy (1993) recommended, a short questionnaire can avoid boredom by respondents.

The items were divided in four parts following Coulson (1992). However, since Coulson's instrument was for science rather than mathematics education, the items developed by Coulson were readapted (Cf. Osei, 1995; Oskamp, 1977).

In addition to this, in order to support the analysis of what really happened in the mathematics classrooms for both treatment conditions, the students were asked to respond to a 38 item questionnaire on frequent events in the classroom, scaled as follow: 1 = 'it almost never happened', 2 = 'it seldom happened', 3 = 'it sometimes happened', 4 = 'it often happened', 5 = 'it very often happened' (See Appendix A16).

4.2.3.7 Interview Schedule

A randomly selected sample of students was interviewed, for the purpose of triangulation (Cohen and Manion, 1994; Mathison, 1988), in order to establish their understanding of

critical probability concepts like randomness, equally likely events and chance. In the experimental condition 15 students (5 from each group) were interviewed, whereas in the control condition only five students were interviewed.

4.2.4 Procedures

The lessons based on the games approach were implemented with a variety of classroom activities. Each aspect related to probability was taught through active participation of the students in exploring the playing of the games, discussing strategies in groups and solving probability problems within the games. The teachers posed different questions to guide and enrich the discussion. Worksheets were provided to the students with a variety of probability questions based on the use of the games (See Chapter 6).

For the non-game approach the lessons have been taught in the traditional way. The teachers used mainly direct methods, i.e., first defining the concepts verbally and then solving simple exercises. They did not actively involve the students in different activities, neither did they use any kind of games. In some lessons they did mention games like coin tossing and card games, but they did not experiment with such games in their classrooms.

4.2.4.1 Pilot Study

The pilot study provided an opportunity to assess the appropriateness and practicability of the data collection instruments. It also demonstrated the adequacy of the research procedures. In view of the nature of this research it was only possible to pilot some of the research instruments. So, for its complexity of containing sentences, which could be misunderstood, the pre-attitude-motivational questionnaire was piloted in a group of Grade 11 students in Maputo. The results were useful for improving the formulation of the items so as to avoid false interpretations. In general, I had the impression that the items were easy to understand and this could be done in more or less ten minutes.

4.2.4.2 Main experimental study

The main study was implemented over a period of five weeks during October and November 1997. Since all the schools follow the same program and the units are more or less taught in the same period of the school year, arrangements were made to allow me to be present in both places. The Nampula school teachers in the 3 classes had agreed to bring their lessons for the Probability unit forward and so it was possible to finalise the lessons in Nampula, then travel to Niassa, and work with the teachers and observe the lessons there.

The lessons of four classes, two of each treatment condition, were conducted by their own mathematics teachers. The two teachers of the experimental groups were assisted by myself in preparing for their lessons. General teaching aids and criteria, just as a reminder, have been provided to the teachers of the control groups in order to avoid the Hawthorne effect, i.e., the notion that the mere fact of being observed experimentally can influence the behaviour of those being observed (Rosenthal & Rosnow, 1991). This precaution contributed to the validity of the study.

The pre-test and the pre-attitude-motivational questionnaire were administered during the first session of the unit. The post-test and the post-attitude-motivational questionnaire were administered to the groups during the last treatment lesson. The interviews with the students were carried out after the last lesson and were audio taped for further analysis.

4.2.5 Analysis of Data

Since with a quasi-experimental design, the confidence that can be placed in the validity of results depends in large part on the case that can be made for the similarity of the groups (Wiersma, 1995), attempts have been made to minimize the initial differences of the groups in the analysis of the data. Thus, for the analysis of the data on the performance measure, analysis of covariance (ANCOVA) was used to test for differences between the groups. The use of ANCOVA is recommended *"for the purpose of adjusting for initial differences between non-equivalent groups on the pre-test"* (Pedhazur & Schmelkin, 1991: 283). However, according to Pedhazur & Schmelkin (1991), *"much of the*

controversy surrounding the validity of the comparisons among non-equivalent groups resolves around the validity of using ANCOVA for the purpose of adjusting for initial differences among the groups" (p.574). Also, the t-test was used for evaluating the differences in means for the performance measure.

The data generated by the attitude-motivational questionnaires were analysed to produce descriptive statistics, Pearson Correlation and Chi-squared (1900). Chi-squared is a statistic that tells us how unlikely it is that the relationship investigated has occurred by chance. It does not tell us immediately about the strength of the relation between the variables. Chi-squared can be thought of as a comparison of counts. Specifically, it tests the relation between two variables by assessing the discrepancy between a theoretically expected frequency and an obtained or observed frequency. All the statistics were obtained by using a statistical data analysis computer package for windows (SPSS 10.0).

From the students interview data, transcripts were analysed for different categories of responses, according to the frequency distribution of the responses. The analysis of the observations were used to enrich the information about occurrences during the lessons. A questionnaire on occurrences in the lessons completed by the students was used to categorize the main differences between the two treatments.

4.3 Field Research (phase 2)

One of the games used in the first phase of this study, *Tchadji*-game, presents several possibilities for further exploration with regard to its relationship to mathematics. The investigation done in the early phase of this study did not include a direct contact with the people who play the games. These people, for example the *Tchadji*-players, would certainly give much more information concerning this activity. This information could enrich more the ideas of integrating of this kind of activities into the curriculum.

The need for the recognition, incorporation and compatibilisation of ethnomathematics into the curriculum has long been stressed (e.g., D'Ambrosio, 1984). This integration of mathematical traditions "*requires the development of quite difficult anthropological-research methods relating to mathematics*" (D'Ambrosio, 1985a:47).

Traditional counting methods, e.g. by means of knots in strings; counting systems, the conical roves and circular or rectangular bases that the traditional Mozambican houses have; hexagonal holes displayed in traditional fishtraps; the rectangular forms of mats; the circular form of baskets or the various types of symmetries used for decoration in many cultures, are relatively easily recognised as mathematics (Cf. Gerdes, 1995). But, what about mathematical thinking? How to recognize 'hidden' mathematical thinking in the course of playing a game like *Tchadji*? There is a need to reconstruct these mathematical traditions. What methodology can be employed in researching such questions?

Gerdes (1988) and his research team have developed a complementary methodology that enables one to uncover in traditional material culture some hidden moments of, for example, geometrical thinking. When considering geometrical forms and patterns of traditional objects such as baskets, mats, pots, houses, and fish traps, one poses the question: *Why do these material products possess the form they have?* To answer this question, it proved to be necessary to learn the usual production techniques and try to vary the forms. This learning process occurs many times, in the initial stage, and during fieldwork among the people who produce the materials. Research shows that the form of these objects is almost never arbitrary, but generally represents many practical advantages and is the only possible or optimal solution of a production problem. Applying this method, Gerdes and his team have discovered quite a lot of 'hidden' or 'frozen' mathematics (See Gerdes: 1985a, 1985b, 1989, 1994a, 1994b, 1994c).

In fact, in order to be able to incorporate popular (mathematical) practices into the curriculum, it is first of all necessary to "*recognize their mathematical character*" (Gerdes. 1995:22). In this respect, there has been increasing talk, particularly with respect to developing countries, of mathematical activities identified within the everyday life of societies. The present phase of the study is aimed to study more deeply the mathematical elements embedded in the *Tchadji* - game, thus it is absolutely necessary to conduct field research.

For this phase of the study, the 'unfreezing' of 'frozen' mathematics, an ethnographic research was conducted by means of fieldwork among players of the *Tchadji*-game in Ilha

de Moçambique', Nampula Province. As Rubin & Babbie (1992) observes, "*field research, i.e., the direct observation of events in progress - is frequently used to develop theories through observation. A long and rich anthropological tradition has seen this method used to good advantage*"(p.358).

In fact, for this kind of research, it has to be a direct observation of everyday life as they naturally unfold in the real world. For this reason, the ethnographic research is sometimes also called *naturalistic research*. Sometime is also *ethnography*, which literally means 'writing about people', to emphasize the study of a culture from the point of view of the people who inhabit that culture.

In this study, as emphasized, it was also important to understand more about the *Tchadji*-players, as a group of people, for the purpose of describing their behaviours and socio-cultural activities, specially, the playing of *Tchadji* (See more in sections 1.3 & 1.4), and there ethnographic research showed to be the most appropriate (Cf. Burns, 1995; Rubin & Babbie, 1992).

4.3.1 Data Collection Techniques

Ethnographic 'fieldwork' is not a homogeneous method, but involves a variety of techniques of data collection. The most commonly employed approach is that of *participant observation*, whereby the field worker directly observes, and to some extent takes part in, everyday life in chosen setting. Rubin & Babbie (1992) observes that ethnographic research probably seems to be "*the most obvious way to learn about something: Just go and watch it happen, or experience it*" (p. 358).

4.3.1.1 The Researcher's role

What else could the researcher do in the field in order to collect relevant data? What really was his role in the field? Since participant observation is a broader concept with respect to the type of activities and role that the researcher can play in the course of the field research (Cf. Burns, 1995; Rubin & Babbie, 1992), it was important to exactly specify the role of the research in this phase of the study.

As Burns (1995) observed, "*whatever the purpose of study, four questions confront the observer: (1) What should be observed?, (2) How should observation be recorded?, (3) What procedures should be used to try to assure accuracy of observations? and (4) What relationship should exist between the observer and the observed, and how can such a relationship be established?*" (p.264).

In fact, depending on the kind of study conducted, a researcher may pretend to be a genuine participant, i.e., a **complete participant**, seen as a full participant, or, at the other extreme, may pretend to be only a genuine observer, i.e., a **complete observer**. Neither one extreme nor the other was appropriate for my research. Being relatively a stranger in the playing of *Tchadji* in this group of players, how would I as researcher be regarded as a genuine participant? On the other hand, how would it be possible to learn the complex moves and strategies of the game from the players without interacting with them?

Rubin & Babbie (1992) refer to the ideas of Raymond Gold (1969), who, besides the extreme cases, has discussed other two different positions on a continuum of roles that field researchers may play in this regard, a **participant-as-observer** role, where the researcher participates fully with the group under study, but makes it clear that she/he is also undertaking research; or the **observer-as-participant** role, where the researcher identifies himself or herself as a researcher and interacts with the but makes no pretence of actually being a participant. The later description is the most appropriate for this study for the reasons I mentioned above.

In fact, as a researcher during the fieldwork, it was clear that I was interested in learning more about the *Tchadji*-game. Most of the players knew me as a 'son of the land', since I was born on the Island and often used to go there to visit my parents. I did not participate fully in the activities of playing, but certainly did interact often with the players. I observed and recorded critical moments. So, my role, in the Gold's terms (1969, as in Rubbin & Babbie, 1992), was that of **observer-as-participant**. To support the process of data collection in this role, the observations were videotaped. In addition, fieldnotes were taken by myself to complement the analysis of data.

4.3.1.2 The *Tchadji* tournament

During the fieldwork it was necessary to record as many *Tchadji*-matches as possible. On the other hand, it was necessary to create a spirit of co-operation between myself and the players. Their thrust in me with regard to what I wanted to achieve, would certainly be important for data collection. Thus, in the first period of fieldwork, in February 1999, I organized a small tournament, where the matches were recorded for posterior analysis and some monetary incentives for the winners were provided.

4.3.1.3 Group Interview

As Burns (1995) stated, "*many fieldworks complement data from participant observation with information taken from interviews. In the course of an interview, the researcher can, among other things, investigate in more detail information's typification of persons and events. Informants may be asked to reflect and comment on events that have already been observed directly by the ethnographer. In addition though, they may be used to gain information about events which occurred in this setting before the ethnographer arrived and events within the setting to which he does not have access. 'Informal' interviewing' is often part of the ethnographic fieldwork. Such interviews, which are conversational in style rather than based on a fixed schedule of questions, are natural extensions of the social relationships established in the course of participant observation*" (p.265)

Thus, in the second period of fieldwork, in July 1999, not only *Tchadji*-matches, as many as possible, as in their natural setting were video-recorded, but five potential *Tchadji*-game players as suitable subjects for complementary interview research were selected. The interview took place on the last day of the fieldwork as open-ended interviewing (or in-depth interviewing), which is a type of interviewing which takes the form of a conversation between informant and researcher.

In fact, as stated in the aims of this research (See section 1.3), it was important to learn a little bit more from the players. So, I asked the players to respond on issues related to their background, and their contact with the game. According to Burns (1995), "*open-ended interview methods should be employed when it is (a) to obtain an individual's subjective*

experiences when life or oral history is being elicited, (b) to have access to events and activities that can not be directly observed by the researcher because perhaps they occurred in the past, and (c) it can also be used in a group interview context where the form would be an open-ended group discussion"(p.279).

Also, questions arising out of the analysis of the matches of initial period of fieldwork, were posed. The question were more of the following nature: *Why do the players behave in the way they do?, Why do they start the match in the way they start?, How do they know previously, if they know, where to capture seeds, where will end a move?, If they really count by moving in the board, how do they count?, Is a draw possible in Tchadji?, How do they judge the chances of winning against a certain player?, When do they say that a player wins a match?*

In addition to this, during the interview some variations of the game have been studied. For example, the players were asked to play a version of 4 rows of 7 holes of the game instead of the usual 4x8 version. Or, they were asked to play other version like the 4x9 holes, 4x11 holes, and so on. All these versions were video recorded as well.

It was also necessary to have a sense of the players thinking or the decision they made in playing in the way they played or capturing in the way they did it. Also, in order to make sense of a player's thinking processes, they were asked to play a *Tchadji*-match orally communicating their decisions and thoughts. Three of these matches were recorded for further analysis.

4.3.2 Factors undermining reliability and validity in the study

Reliability is based on two assumptions. The first is that the study can be repeated: the replicability (See also section 4.2.3.2). Because ethnographic research occurs in natural settings and is often undertaken to record processes of change, it is especially vulnerable to replication difficulties. The second assumption is that two or more people can have similar interpretations by using the same categories and procedures of the study. However, in ethnographic research it is difficult for an ethnographer to replicate the finding of another because of the flow of information is dependent on the social role assumed within the

group studied and the knowledge deemed appropriate for incumbents of that role to possess. Thus conclusions reached by an ethnographer are qualified by the social roles investigators hold within the research milieu (Burns, 1995).

Burns (1995) suggested that, "*reliability can be enhanced in these ways: (a) Investigators outline the reason for the research and the major questions they want to address; (b) They explicate their perspectives on the questions, stating their assumptions and biases; and (c) They explain their data-gathering procedures, including timing and time lines of observations, spatial arrangements of interviews, relationship with subject and categories developed for analysis*" (p.271).

In this study, attempts were made to maintain an acceptable level of reliability. For example, as a researcher, I was known to the players. The interaction between us was friendly and open and it took place in *Emakhuwa*, the language of Bantu origin spoken in this part of Mozambique (See figure 1.1 in Chapter 1). I might be seen as researcher but certainly not as stranger to them. The problem of observer-observed interaction, i.e., the presence of a researcher, which may change the behaviour of the people under study, can decrease reliability. As is suggested by Burns (1995), to overcome this problem, qualitative researchers try to interact with their subjects in a natural, unobtrusive and non-threatening manner. From the first period of fieldwork it became clear to the players what my intention in being with them was. This has most likely enhanced the reliability.

Do scientific researchers actually observe or measure what they think they are observing or measuring? This is the problem of internal validity. Solving it credibly is considered to be a fundamental requirement for any research design. Also, here attempts were made, where it was possible, to maintain an acceptable validity. For example, since the scenes were video-recorded for analysis purposes, any result can be further re-analysed for validity.

In field research, informants may dissemble, present an ideal self or tell the researcher what they think the researcher should, or wants to, hear. Parallel to this problem in observation is the credibility of informant's reports in interviewing. Informants may lie, omit relevant data or misrepresent their claims. Independent corroboration from multiple

informants or other fortuitous observers of the social scene, and residence in the field for a period long enough to reduce artificial responses from informants, improve validity. So, in this study, many matches were analysed to compare the consistency of any conclusion. The interview was a group interview with 5 players. This has certainly decreased the possibility of lying or of omitting information.

A second question is, to what extent are the abstract constructs and postulates generated, refined or tested by scientific research applicable across groups? This addresses the issue of external validity. External validity depends on the identification and description of those salient characteristics of phenomena for comparison with other similar types. Ethnographic studies are generally case studies from a single setting and results can be difficult in being translated to other similar settings (Burns, 1995).

4.3.3 Analysis of Data

In the course of the field research, data was collected through the use of a video-recorder. As many *Tchadji*-matches as possible were recorded for further analysis. There, I distinguished four types of matches, the tournament-matches, the regular-matches, the variation-matches and the interview-matches (See section 4.3.3.1). Also, during the fieldwork group interviews with some identified master players were conducted and these were video-recorded for description and analysis.

4.3.3.1 Analysis of *Tchadji*-matches

The *Tchadji*-matches were analysed by me by looking for any mathematical ideas involved in them as described in the ethnomathematical research methodology (See sections 3.4).

During the fieldwork a total of 57 *Tchadji*-matches were recorded. Of the total, 25 were regular-matches, i.e., matches that were recorded during the natural setting of the playing activity of the players as they usually do in their daily life. Eight matches were recorded in the first tournament organised by myself and 9 other matches were recorded in the second tournament (tournament-matches). 3 were interview-matches, where the players have been asked to comment on their moves and decision they made to play the way they played. 8

other matches were called extra-matches, they were recorded after the group interview, the players wanted to continue playing and so the matches they played were recorded. The last 4 matches have been of variations (variation-matches), i.e., the player were asked to play a different board version using *Tchadji*-rules, for example, a 7x4 board, a 9x4 board or a 11x4 board.

The analysis of the matches consisted of the following tasks: First, all matches were transcribed in two different ways, the moves with the number of seeds captured in each move and from which holes, and then, the critical moments of the matches were considered on worksheets representing the *Tchadji*-board (See Appendices B01-34 and C). For this process a wooden board (See figure 5.4a in Chapter 5) was used to reconstruct the moments of the match. So, while the matches were seen on the television screen, they were reconstructed and replayed, and so the moves were looked at more carefully and their principal objective and related possible player's thoughts were tentatively reconstructed.

Sometimes, there were comments on the match being played, made either by the player engaged in the match or by the people watching. These comments were also analysed to complement the description of the match.

4.3.3.2 Analysis of the Interviews

The data from the interviews with the players, which were transcribed and translated into English, also complemented the ideas raised from the analysis of the matches. Here, I focused on questioning the strategies of the master players and on the mathematics involved, which is related to the analysis of the game itself.

4.4 The Workshop (phase 3)

According to Gerdes (1998) *"It is particularly important for teachers to develop an awareness of mathematics as a universal activity in order to never underestimate the capacities, know-how, and wisdom of their students and the students' communities. It is important for educators to help pre-service and in-service teachers to see how they can give each other examples of mathematics in their life and history, and it is important for*

(teacher) students to see people like themselves doing mathematics and to reflect on their future in relation to mathematics " (p.47).

Within the concerns of the MERP, i.e., the incorporation of cultural traditions in the curriculum, the role of teacher is crucial. Gay & Cole, (1967) expressed this as follows: "*(...) the teacher should begin with materials of the indigenous culture, leading the child to use them in a creative way*" (as in Gerdes, 1988:138). Thus, this study also aimed to investigate in this third phase, by means of a pre-experimental study, a workshop with student teachers, whether or not with the *Tchadji*-game, at least, an awareness of one's own culture can be developed among them.

The intention was to investigate the acceptance and abilities of the use of the potentialities of cultural practices, e.g., cultural games, in the teaching of mathematics. So, a guided workshop with student teachers of the 2nd year 'licenciatura' degree in teaching mathematics and physics at 'Universidade Pedagógica' in Maputo took place, during 5 weeks, in September-October 1999.

4.4.1 The Structure of the Workshop

The establishment of causality is an important aspect in experimental research. In pre-experimental research "*there is a total absence of a control group, which makes them of minimal value in establishing causality.*" Here, "*no allowance is made for a comparison with the reactions of other students who were not exposed to that treatment.*" In fact, this kind of design is "*clearly deficient in terms of any reasonable controls of internal validity*" (Rosenthal & Rosnow, 1991:83).

The workshop study was essentially designed not to find out whether there was any effect of the exposure to the game. This study was designed to find out whether the student teachers, in this situation, are able to cope with the *Tchadji*-game, whether they were able to recognize the possible mathematical-educational potentialities of the game, and whether they were able to describe those recognized potentialities. In this case, the pre-experimental design was considered to be appropriate.

'Experimental' O1-----X----- O2----- O3

Figure 4.1 Structure of the intervention in the workshop

There was no randomised assignment. So, it was a pre-experimental design: the one group pre-test and post-test, which can be represented as in the figure 4.1 (Cohen & Manion, 1994).

O1, O2 and O3 (See figure 4.1) indicates the process of observation or measurement. In this case, O1 and O3 refer to the first and second administration of the questionnaire on attitudes towards mathematics. This was done at the beginning of the workshop and at the end. The students in the course of the workshop were asked to prepare group assignments on the main results of their study (O2). The X indicates the exposure of the group to an event or variable: in this case the group was to explore the mathematical-educational potential of the *Tchadji*-game.

4.4.2 Participants

An intact group of 24 (pre and in-service) student teachers participated in the study (See table 4.4). Their background, age, years of experience were recorded in the questionnaire (See tables 4.4 & 4.5).

	Frequency	Percent	Valid Percent	Cumulative Percent
High School	11	45.8	45.8	45.8
Teacher course	12	50.0	50.0	95.8
Other	1	4.2	4.2	100.0
Total	24	100.0	100.0	

Table 4.4: Participants' background

	Frequency	Percent	Valid Percent	Cumulative Percent
no experience	9	37.5	37.5	37.5
0 - 5	4	16.7	16.7	54.2
5 - 10	3	12.5	12.5	66.7
10 - 15	6	25.0	25.0	91.7
more than 15	2	8.3	8.3	100.0
Total	24	100.0	100.0	

Table 4.5: Participants' Teaching experience

The data in Tables 4.4 and 4.5 show that 50% of the participants had done formal training before (See section 1.1.4). 11 student teachers came from high school. Regarding their experience, the data indicate that 63.5% of the students, i.e., 15 students, had some experience as a teacher with 33.3%, i.e., 8 students, having more than 10 years experience. The group can be considered a mixed one with regard to these two aspects.

4.4.3 Instruments used

The main instrument used for data collection in the workshop with student teachers was an attitude questionnaire (See section 4.4.3.1). In addition, guidelines for preparation group assignments were given to the student teachers (See section 4.4.3.3). Also, some support materials were provided to the students for their work with the *Tchadji*-game (e.g., the worksheets, see section 4.4.3.2).

4.4.3.1 Attitude Questionnaire

According to Oppenheim (1992), an attitude statement is a single sentence that express a point of view, a belief, a preference, a judgment, an emotional feeling, a position for or against something. So, the items of the questionnaire are single sentences or statements

about mathematics in cultural daily life activities. The questionnaire is composed of 24 multiple choice item of Likert scale (See section 4.2.3.4 & appendices A31 & A32).

Some of the statements have been used in the Ethnomathematics project at RADMASTE Centre of the University of the Witwatersrand, Johannesburg (Laridon, 1998). Other were taken and readapted from the questionnaires used in the first phase of the study (See section 4.2.3).

To enhance consistency in the responses, some statements were modified. For instance, the statements 2, 7, 8 and 24 were modified in the 2nd administration (O3 in figure 4.2) of the questionnaire from a positive to a negative sense or visa-versa. So, for example, the statement 24 was changed as follows: *Maths is not about applications in the real life*.

The questionnaire has multi-dimensional aspects. It deals directly with attitudes at times but it also has statements related to practice and views (#5, 6, 12, 14, 15 and 21); of teaching and learning mathematics (#9, 10, 13 and 17); the relationship between mathematics and culture (#7, 8, 16 and 18); usefulness of mathematics (#2, 3, 4, 19, 20, 23 and 24) and confidence in learning mathematics (#1, 11 and 22). The statements 13, 15, 16, 17, 19, 20, 23, and 24 can also measure beliefs about the relationship between culture and mathematics.

Also here, attempts have been made to maintain an acceptable reliability. So, for the pre-and post-attitude questionnaire a Crombach's alpha reliability coefficient was computed to test their internal consistency (Cf. Cohen & Manion, 1994, Rosenthal & Rosnow, 1991; Wiersma, 1995). The first administration of the questionnaire indicated a reliability coefficient of 0.8431 (See Appendix F3a) and in the second administration it indicated a coefficient of 0.5095 (See Appendix F3b).

In addition to these items, in first administration of the questionnaire (O1 in figure 4.2), the students were asked to respond or comment on two aspects in an open-ended way. First, they were asked to comment on their answer to the item 16 (*There is no maths in board games practiced in daily life*), which allowed for a broader and deeper sense of what they meant with their response. Also, they were asked to make any comments about

the questionnaire itself. In the second administration of the questionnaire (O3 in figure 4.2), they were asked to comment on the relationship between mathematics and cultural elements.

4.4.3.2 Worksheets and other support material

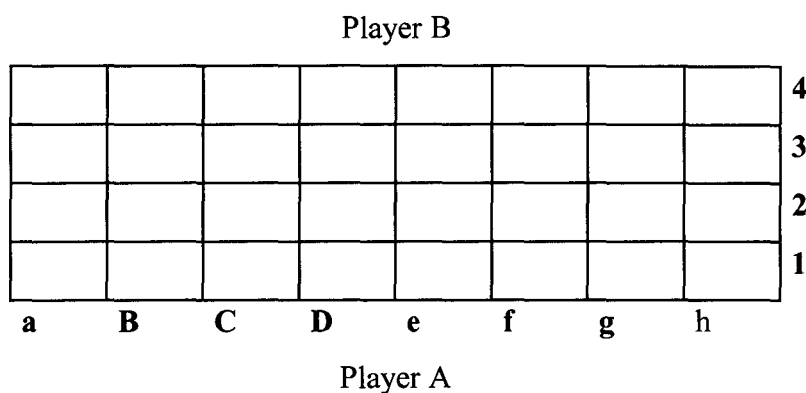


Figure 4.3: A representation of the 8x4 *Tchadji*-board

A wooden *Tchadji*-board was provided with original seeds to allow the students to practice the game. In addition some worksheets with the design of the game and beans for practicing it in groups were also provided. Many copies of the design of the game on A4 paper (See figure 4.3) were also provided to allow the students to describe their reflections on and analysis of the game.

4.4.3.3 The students' Group Assignments

To record the students' reflection on the game, they were asked in the course of the sessions to review their thoughts and results in the group' discussions according to the guideline questions (See section 4.4.4) in order to present them, some days after the workshop, in the form of a group assignment and to hand them to me.

4.4.4 Procedures

In terms of procedures, the workshop with student teachers was conducted as follows: the first session was devoted to familiarise with the game and its rules. The rules of the game were explained to them and a video made during fieldwork sessions was shown to the students. Students were asked to work and to practice the game in groups.

From the second session, the students were systematically asked to reflect on and analyse any mathematical ideas related to the *Tchadji*-game. To facilitate this process the following guideline questions were provided for reflection:

1. In all first moves, the objective of the player starting was, in most cases, that of capturing opponent's seeds. In which holes should the player start to be able to capture his opponents seeds and why? Which of these positions do you consider to be the best and why?
2. Is there any counting involved in playing of *Tchadji*-game? If yes, can you describe how it happens.
3. If counting is really involved in the playing of *Tchadji*, how do you explain that the moves are done very quickly by the players? Can they count so quickly? Do they count before, during or after the moves? How do they count?
4. Is a draw in the *Tchadji*-game possible? Which situations would be considered a draw in a match? How often do they occur? How can you describe them?

5. What is the minimum and the maximum number of moves in a Tchadji-match?

6. What is the minimum and maximum number of captured seeds in each move?

In the course of the sessions the students working in 6 small groups of 4 to 6 people, were asked to review their responses and positions and after the end of the workshop, they were given a week to complete their group assignments (See section 4.4.3.3) at home.

4.4.5 Analysis of Data

Data from the workshop consisted of the group assignments, where the students presented their thoughts on the exploration of the game, and responses to the pre- and post attitudinal questionnaire.

4.4.5.2 Analysis of group assignments

The analysis of the students' group assignments (O2 in figure 4.2) was an indication of the impact of the sessions. The mathematical ideas embedded in the *Tchadji*-game as indicated in students' group assignments were compared with those developed from the researcher and players' perspectives.

4.4.5.3 The questionnaire analysis

Data from the questionnaires were analysed using a statistical computer package for social sciences, the SPSS 10.0 for windows. Due to the type of data gathered by the questionnaires and the number of subjects involved in the analysis, descriptive statistics (mean, standard deviation, frequencies, percentages and charts) were used. There, the overall degree of agreement between the pre-questionnaire and the post-questionnaire, between findings in the overall average, on culture and mathematics statements and on the other beliefs statements are presented. Some selected items of the questionnaire were presented for comparison between the pre- and post questionnaire results.

The features that generally defined this analysis are the following:

1. The averages of both the pre- and post-questionnaires were statistically compared;
2. The averages for culture and mathematics statements on the pre- and post-questionnaires were statistically compared;
3. The averages of responses to the statements in all other dimensions, such as, the practice and view of mathematics; the methods of teaching and learning mathematics; the usefulness of mathematics and confidence in learning mathematics were statistically compared;
4. The averages for the statement 16 (board games and mathematics) on the pre- and post-questionnaires were statistically compared. Statements 10 and 13 (home experience and mathematics learning) were similarly treated.

Some features of the SPSS package were taken into consideration in the analysis. For example, some of the statements in the questionnaire were negative and others were positive. The responses were rated from 1 to 5 as from strongly disagree to strongly agree. For the positive questions, the higher the score the more positive is the attitude measured in the statements or group of statements. For the negative statements, the responses in the data were modified, i.e., the value 1 was transformed to 5, 2 to 4 and 3 was maintained.

4.6. Ethical considerations

Ethical issues have been increasingly used in medical research. Recently some ethical concerns have been raised in educational (Cf. Adler & Lerman, 2001) and in ethnographic research.

As Rosenthal & Rosnow (1991) observed, *"In studying human behaviour we are constantly in jeopardy of violating someone's basic right and it is prudent that we study and discuss the moral dimensions of our research. In the end, the behavioural scientist's ethical responsibilities are twofold. On the one hand, the researcher must protect the*

integrity of his or her work, in order to ensure that the work measures up to the standards of good scientific practice. On the other hand, the researcher must also respect the dignity of those he or she studies and the values that allow the pursuit of scientific knowledge in a free society" (p. 250).

Since this study involved human participants (The *Tchadji*-master players in 'Ilha de Moçambique', the 24 student's teachers at UP and the 162 students and 4 teachers at the schools in Nampula and Lichinga), it was important, as it is recommended in research, to take some measures in order to acceptably address ethical and moral aspects (see Cohen & Manion, 1994, Rosenthal & Rosnow, 1991; Wiersma, 1995). For this reason the measures listed below were taken.

1. The community of *Tchadji* - players at 'Ilha de Moçambique' township were all asked permission to undertake the study of their activity of playing. To avoid possible attempts to give false information on the main issue of the study the purpose of the study was made clear to the players.
2. The reason for undertaking the research was carefully explained to the group of student teachers before the workshop started. They were asked to participate actively in the workshops, but it was not compulsory to do so.
3. In some research studies it is extremely important to ensure the confidentiality and anonymity of participants, which I believe it was not the case in this study. Even so, names of *Tchadji*-players and their photographs taken in the course of the fieldwork, as important figures to illustrate the conditions and situations of the research in the field, were only included in the report after permission for this was given from the players. Names of participating student teachers were omitted in the report, but some photographs taken during the workshop session are presented in the appendices with the permission of the students.
4. The school students and the student teachers were assured that the questionnaires administered would be used exclusively for the research and not for other purposes. This aspect was explicitly written at the heading of the questionnaires. For example, it was

clarified in the heading of the questionnaires that they would not be used for evaluation purposes.

5. For debriefing the participants, both in the fieldwork and in the workshop, they were informed that the study is for the purpose of an academic study. In particular, the *Tchadji* - players were informed that the results of this study, if acceptable, would be published in the form of a book including some of the photographs taken and that a copy of the book would be sent to the group.

6. *"For reasons known only to themselves, some subjects may give false assurances. The researcher has a responsibility to alleviate the subjects' discomfort and restore a sense of well-being as much as possible"* (Rosenthal & Rosnow, 1991: 249). For this reason, during the fieldwork, the participants were treated as respectable persons in their activity as players of *Tchadji*. It was important for me, as researcher, to learn different aspects of the game. The fact that I was born in that town and spent my childhood there and that most of the player knew me helped very much in collecting data for research.

4.7 Limitations of the study

This study was carried out in Mozambique, a developing country. It required the interaction with students in their socio-cultural context in schools, with student teachers in their environment at University and with a particular socio-cultural group in Mozambican society, the *Tchadji*-players in their lived environment. The research methods employed in this study require, as do all research methods, a kind of stability and normality of the setting in which the research occurs. Developing countries are characterised by instability, due to the constant and abrupt reorganisation of political, social and economic forces.

It is therefore important to establish the limitations of the research for a better understanding and interpretation of the results. As Valero & Vithal (1998) stated, *"when the research process is obstructed by uncontrollable disruptions emerging from the very same unstable nature of the social context and of the research objects that are considered, then the whole process of research has to be reconceived to allow the disruptions*

themselves to reveal key problems that should be addressed in order to understand, interpret or transform the real issues of the teaching and learning of mathematics in developing societies” (p.157).

For the experimental study, which constitutes the first part of this research, no random selection was carried out for the reasons indicated earlier (See section 4.2). The lack of random selection can affect the validity of the experiment. As suggested by Wiersma, (1995), *"when considering problems of validity of quasi-experimental research, limitations should be clearly identified, the equivalence of the groups should be discussed, and possible representativeness and generasability should be argued on a logical basis"* (p.140).

Attitude questionnaires were used in the first and third interventions of this study. The measurement of an individual's attitude is unlikely to reveal her/his attitude perfectly, because, for example, people may respond to an attitude test in a way that makes them appear more favourable, more good than is true. According to Galfo (1975), subjects who are not eager to reveal her/his true feelings, the information collected can be wrongly interpreted, the statements may fail to measure the attitudes it seeks to measure, the instruments may not be appropriate for the intended group. This can constitute a limitation of the study.

The experimental study of the first intervention took place in six classes of two schools in two different towns in Mozambique. Apparently the two towns appear to be more or less comparable with regard to economic growth and cultural values. Other factors may exist, that are different in the towns. For example, the classes were taught by different teachers and the teaching style and other personality characteristics of teachers can act as confounding variables. These aspects could have introduced bias in the study.

According to Lunn (1970), many research findings have suggested that the type of teacher affects the pupil's behaviour and attitudes, particularly those concerning the pupil's relationship with the teacher, the pupil's motivation and desire to learn, the pupil's degree of anxiety in the classroom and self image.

Also, Aiken (1976) concluded, from an overview of many studies, that attitudes towards mathematics are fairly positive until the junior-high or middle school years, at which point they usually become less favourable. He also asserts that attitudes towards any subject can be affected by host of factors such as ability, developmental crisis, textbooks, teachers, school environment, etc

The fact that the experimental groups consisted of 3 classes from a particular school in Lichinga, the capital city of Niassa Province, considerably limits the generalisability of the conclusions to other classes and schools in Mozambique.

Since the Portuguese language is not the mother tongue of the majority of Mozambicans, (only 6,5% of the population have Portuguese as mother tongue and only 9,0% use it frequently at home¹), many students in both experimental and control conditions could have been weak in Portuguese. This could have interfered with the understanding of some words or phrases, and consequently have hindered comprehension of the questionnaire and tests. The topic of probability is related to a large variety of misconceptions, biases and emotions that may be caused, for example, by linguistic difficulties.

Field research, the methodology employed in the second part of the study among *Tchadji*-players, has a number of strengths. The chief strength of ethnographic research lies in the depth of understanding that result. Flexibility is another advantage of field research. In this paradigm, it is possible to modify details of methodology at any time. Moreover, you are always ready to engage such research, whenever the occasion should arise, whereas you could not as easily initiate a survey or an experiment.

Field research has a number of weaknesses as well. First, being qualitative rather than quantitative, it seldom yields precise descriptive statements about a large population. More generally, the conclusions drawn from qualitative field research are often regarded as suggestive rather than definitive. Data collected through field research, although in – depth, are also often very personal. Thus, the results obtained in this research cannot be generalised.

As Rubin & Babbie (1992) suggested, *"Generazability is a problem for field research. First, as we have already suggested, the personal nature of the observations and measurements made by the researcher can produce results that would not necessarily be replicated by another, independent researcher. Second, because field researchers get a full and in- dept view of their subject matter, they can reach an unusually comprehensive understanding. By its very comprehensiveness, however, this understanding is less generazable than results based on rigorous sampling and standardized measurements. Finally, there is often a problem of Generazability even within the specific subject matter being observed"*(p.365).

In the workshop with student teachers, which constitutes the third part, some aspects were observed which could constitute limitations for the research. For example, in order to get feedback on the content of the questionnaire from the students, they were asked to comment on it. Most comments were very encouraging (see Section 4.4.3.1).

However, two students commented on the way the questionnaire was administered. One wrote as follows: *"The content of the questionnaire is all good (...). We think that the anonymity should have been respected"* (My translation). The second wrote the following: *"Why did you ask for the name of the student? In a mathematics course like this, no one would disagree with the item 'I am sure I can learn mathematics'"* (My translation)

¹ 1997 Census (<http://www.inc.gov.mz/>)

One student queried whether the items were well formulated or not. He said that “*Some items were too implicit and they could have introduced a bias in the answer because I could not understand*” (My translation). However, he/she did not mention these items. On the other hand two other students contradicted this observation by writing that “*The questionnaire is very clear and objective (...)*” and that “*The questionnaire was very well formulated and contains important items (...)*” (My translation). These two aspects could have introduced a bias in the responses of the students.

CHAPTER FIVE: THE *TCHADJI*-GAME AND ITS PLAYERS

5.1 Introduction

The present chapter gives some historical background knowledge related to Mancala type games, in particular, the *Tchadji*-game. It presents the classification of these games according to Bell (1960), Junod (1962), Russ (1984) and more recently Silva (1995) and De Voogt (1995 & 1997). Additionally, it presents the background of *Tchadji*-players and some results regarding the tradition of playing *Tchadji* at Ilha de Moçambique, where the second phase of this study was conducted.

5.2 Games of the Mancala type

Mancala is the generic name, given by anthropologists to a class of board games. The word Mancala comes from the Arabic word *manqala*, or *ninqala*, from the verb *naqala*, which means ‘transfer’ or ‘move’ things about (Murray, 1952 & Russ, 1984).

These board games are played throughout the African continent as well as in many other parts of the world. For example, as a result of the slave trade to the “New World,” Mancala is also found in the Caribbean and on the eastern coast of South America (Cf Herskovits, 1930; Russ, 1984).

Murray’s book on boardgames (1952) define these games (or the German *Brettspiele*) as “Games, which resemble Chess, draughts, and backgammon in being played on a specifically arranged surface with pieces or ‘men’, whose powers of move and capture are defined by the rules of each game” (p.1). The board and the powers of move and capture of the pieces in the game therefore play a crucial role and are the aspects that differentiate them from other types of games. For example, dice, cards and dominoes are not considered board games. Roulette does have a board, but the pieces do not have powers of move and/or capture. Mancala games can be classified as board games (See Figure 5.1).



Figure 5.1: An example of a Mancala type game called Wari ¹

5.2.1 Brief History and Background

The history of the origin and subsequent development of Mancala is not known. It needs further research (Cf. De Voogt, 1997; Lumpkin & Strong, 1995). It is believed that these games are thousands of years old, and their origins are lost in antiquity and that they have originated either in Asia or Africa (De Voogt, 1997; Russ, 1984; Silva: 1994 & 1995). For example, “*Mankala boards have been found carved into stone in the pyramid of Cheops and the temples of Luxor and Karnak in Egypt*” (Hopson et al, 1996: 90).

There is some controversy surrounding the origin of the Mancala type games. Hopson et al (1996), for example, observed that, “*One source contends that the most popular version of Mankala was developed in the court of Shamba of the Bonnet, who was ruler of the Bushongo tribe in the Congo. He was well known for his peacekeeping efforts, for being an inventor, and for extensive travelling. Supposedly, he returned from a trip to Egypt*

¹ This game was exhibited at the Pedagogical, Interactive and Intercultural Exhibition for ‘Jeux, Mathématiques et Sociétés’, which took place at the French Cultural Center in Abidjan (Ivory Coast), in November 1994. At a seminar held on this occasion I gave a talk on research perspectives on ethnomathematics with special reference to traditional games

with a board game called Mankala, and he tinkered with it before introducing it to his people. Another source says that the game did not originate in any part of Africa but was brought to the continent from the Middle East” (p. 90).

Townshend (1986) argued that some kind of Mancala games, the four-row Mancala as the case of *Tchadji* (See section 5.2.4.3), had to be of African origin since it did not even occur in Asia, but subsequent research has shown that it also occurs in Sri Lanka and China (See, e.g., Eagle, 1995 & 1998). More evidence remains to be gathered on the games played in the Middle East and on the Indian subcontinent.

Modern anthropological evidence reveals that Asian Mancala is played mostly by women and children, while in Africa it is predominantly by men and that the rules in Asia have, in general, been less complicated and diverse than those in Africa. Since this kind of activity is predominately a men activity, this suggests that the Mancala games are of African origin (Townshend, 1986). Hopson et al (1996), for example, recognises that “*It is difficult to report with certainty when and where this game was created because it was played just about everywhere in Africa*” (p.90). In fact, “*they have never taken root in Europe*” (Bell & Cornelius, 1988: 22).

According to Walker (1990), there is also a physical evidence of two-row Mancala games (See section 5.2.4.1), which dates from the third century in Sri Lanka, the fourth century in Cyprus, and the sixth century in Ethiopia. Evidence of the three-row games is difficult to assess, as they are easily confused with the ancient Egyptian game of *senet*. However, a three- row board does appear on an eighteenth-century statue from Congo/Zaire (De Voogt, 1997). Also, there are several varieties played in Ethiopia (Cf. Bell & Cornelius, 1988).

5.2.2 Studies of Mancala type games

It is only in the eighties that Mancala games received due attention as a research topic and have been studied from many different perspectives. There have been within the study of

Mancala itself earlier general studies (e.g. Murray, 1952; Béart, 1955; Bell, 1960), which have been used as introductions to the more specialised recent studies written from art-historical, psychological, pedagogical and anthropological perspectives and on the use of Artificial Intelligence (See Reyssset, 1995).

In the last twenty years Mancala games have been the subject of three doctoral theses (Townshend 1986, Walker 1990, De Voogt, 1995) and other extensive studies (Retschitzki, 1990; Silva, 1994 & 1995 and Russ, 1984). Townshend's (1986) anthropological research, studies the roles that games play in various cultures taking the example of *Bao*, a game of Mancala type played in East Africa. Retschitzki (1990) studied African children's development as measured by their skill in *Awélé*, a West-African game classified as being of Mancala type and also known as *Wari* (See section 5.4).

More recently, De Voogt (1995) studied the limitations of human memory skill in the game of *Bao*, as played in Zanzibar Island. *Bao* itself has provided other earlier anthropological studies of symbolism in East-African Culture (See, e.g., Townshend, 1977, 1979 & 1986). As De Voogt (1995) stated in his assumptions, "*All competitive games, especially board games, presuppose human weakness and seek to exploit it. If there were no limits to the human mind, i.e., to human memory and problem-solving skills, there would be no championship board games nor masters. Therefore, this study characterises various weaknesses, in terms of limitations, in master skills in the game of Bao*"(p.2). De Voogt (1995) description of the limits of the mind by mastership in *Bao* also made a contribution to psychological studies in problem solving and memory.

5.2.3 The place of Mancala type games within boardgames

Board games can be classified in different ways and according to a variety of principles (Murray, 1952; Bell, 1960). But there is still a controversy, which leads one to conclude that there seems to be no theoretical bases for the classification of board games. For example, games that Murray (1952) considers hunt games, i.e., games with the objective of that the player with most pieces tries to blockade and annihilate his opponent, are

considered by Bell (1960) as war games, i.e., games in which the objective is to capture or immobilise the opponent's pieces.

Murray (1952) and Bell (1960) have a different class for Mancala games. As some board games depend on chance and some involve pure skill and strategy, most literature classifies board games as games of chance or games of skill of pure reflection. Murray (1952) considers this classification insufficient and prefers to adopt another based on the relation between the games and early activities and occupations of man.

The reasons for distinguishing games of skill or for pure reflection (Reysset & Pingaud, 1995) from games of chance, or addressing Mancala type games as games of strategy, seem to continue to be arbitrary. De Voogt (1997) made a reflection on the principles for classification of board games. Summarising, he mentioned that "*The purpose of the game, together with its main characteristics, will serve as an adequate indication to classify games*" (p.25).

Mancala games have long been considered a separate class within the group of board games, because they have a distinctive look and there are many kinds of them (Bell, 1960; Murray, 1952), but these two aspects are not the most relevant for classifying them. The objective of the game, the type of moves and the way of capturing, if there is any, should be of primary interest in classifying games. From this point of view it is not necessary to have a unique classification. Also, any classification has a purpose and perspective.

Other scholars, for example, classified Mancala games as strategy board game or "*games of calculations*" (Eagle, 1998). For example, "*Masters of an East African board game called Bao appear to be able to calculate extraordinary complex moves, easy for a computer to emulate but very difficult for humans*" (De Voogt, 1995: 1; see more in section 5.4).

Because the primary purpose of Mancala games is capturing seeds which leads to destruction (but not necessarily to winning a game), they appear to be war games (Cf. De

Voogt, 1995). In fact, the way the seeds are moved in Mancala games appear to be unique to them. This aspect could be considered the main difference between Mancala games and other board games. The way of capturing seeds in Mancala games can be found in other race or even war games (See Bell & Cornelius, 1988).

According to De Voogt (1995), Mancala games should be defined as board games with a particular move, that of spreading pieces, but they should not be classified as a separate class of board games. De Voogt (1995) gives a comprehensive classification of board games. He then classified Mancala games, as “*a class of war games of implied or pure destruction, where capturing in itself will lead to victory, with a particular move of spreading pieces*” (p.29). The term ‘war’ here appears inadequate with regard to what really happen on the board. Capture is taking place. I would rather call it a capture game.

5.2.4 The classification of Mancala type games

Since the way of moving seeds in all Mancala type games is almost the same, they are often classified according to the number of their rows. Thus, there are three different variants of Mancala type games: the two-row variant (Cf. Odeleye, 1977; Russ, 1984, Silva, 1994, 1995; Mve-Ondo, 1990; Reysset & Pingaud, 1992; Retschitzki, 1990; Provenzo & Provenzo, Jr. 1981), the three-row variant (Silva, 1995; Murray, 1952; Bell, 1960) and the four-row variant (Cf. Silva, 1995; De Voogt, 1997; Junod, 1962; Russ, 1984). All these variants are found in the African continent (See figure 5.2).

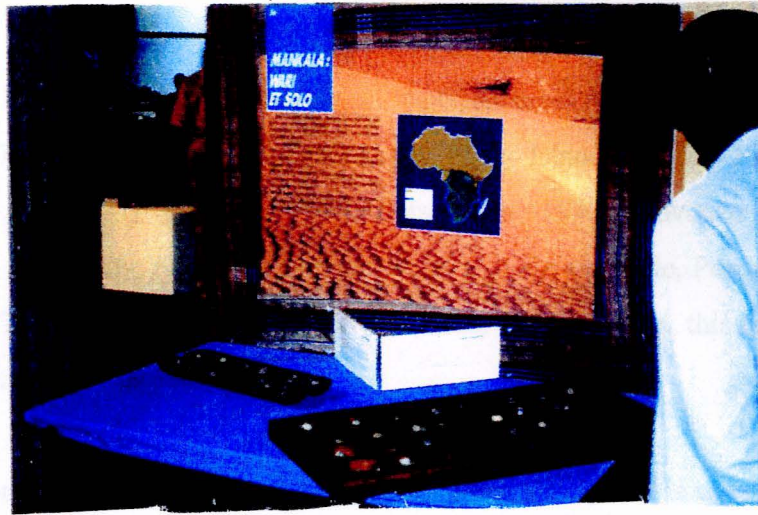


Figure 5.2: A sample of Mancala type games: Wari and Solo ²

On the other hand, considering the number of rows, it is usual to distinguish two families of Mancala type games, those in which the players engaged in the game have their own rows into which they spread their seeds (the four-row Mancala games) and those in which they share rows (the two-row and three-row Mancala games) (Cf. Eagle, 1995). This aspect appears not to be relevant to suggest another type of classification since it does not interfere in the initial objective of the game.

Also, in some cases, a classification of a game can consider the limiting of the game to a certain social setting or even geographical area. Here, we would consider, for example, African Mancala, but in Africa, as stressed earlier, there are many variations of Mancala type games. Therefore, this differentiation is not so relevant for the case of Mancala games.

² These boards were exhibited at the Pedagogical, Interactive and Intercultural exhibition for 'Jeux, Mathématiques et Sociétés', which took place at the French Cultural Center in Abidjan (Ivory Coast), in November 1994

5.2.4.1 The two-rows variant of Mancala

The two-row Mancala games are widely practiced in West African countries, like Nigeria (Odeleye, 1977), Ivory Coast, Senegal, Cape Verde Islands (Silva, 1994, 1995; Mve-Ondo, 1990; Reysset & Pingaud, 1992; Retschitzki, 1990; Provenzo & Provenzo, Jr. 1981) as well as in some other parts of the world like, for example, Portugal (Silva, 1994, 1995), India, The Philippines, The Caribbean (Russ, 1984). In fact, this is the variant most frequently found in other continents.

In its basic form a board of two-row Mancala or Mancala II, as it is sometimes called, is a board made of wood with two rows of either six holes, as there is in Ancient Egypt, seven in Syria, and some times 3 or even 5 holes cut into one side. For example, the *Wari* (See figure 5.2), has two rows of six holes each and uses 48 seeds (4 seeds in each hole). The direction of the moves is anticlockwise and the player with most seeds at end is the winner (Bell & Cornelius, 1988).

5.2.4.2 The three-rows variant of Mancala

As stressed above, there is little known about the three-row Mancala type games. In fact, Mancala games of this type are found only in restricted areas, North East Africa and Arabia. There are several varieties played in Ethiopia. *Gabata*, which contains 3 rows of six holes each, is one such example (Bell & Cornelius, 1988). *Gabata* is played with 54 pieces, 3 in each hole. The player with most pieces at the end of the match wins it (Bell & Cornelius, 1988: p.30)

5.2.4.3 The four-rows variant of Mancala

The four-row Mancala or Mancala IV (See figure 5.2) is very popular in Eastern and Southern Africa. A recent study revealed that it is also played in China (See Eagle, 1995). It appears under various names with the meaning varying according to name of the board

or the pieces used for playing it (See more in section 5.4). For example, *Bao* is the Swahili name for a variant of the four-row Mancala played in Zanzibar. Variants of four-row Mancala type games have also other names (See section 5.3.2).

The playing board is often made of wood. When wooden boards are not readily available the rows of holes are often dug out of the earth or sand (Russ, 1984). The four-row board contains either four, eight, ten, sixteen, twenty two or even thirty two holes in each of the four rows. The boards are often referred as to 4x4 board, 4x8 board, 4x10 board, 4x16 board or 4x22 board variants (Silva, 1995).

5.3 Other names of Mancala games

Mancala games are known by a great variety of names, which may refer to the material of the board used to play on, the playing pieces, the moves, the context of the game or even the sound of the play (De Voogt, 1997).

According to Deledicq & Popova (1977) *Wari* and *Solo* are the most common names for two-row Mancala and four-row Mancala on the African Continent. The word *Wari*, which is a common version of the two-row Mancala games, has many variants. In Cape Verde Island it is known as *Ouri*; In Senegal as *Ouré* or *Wari*; *Wouri* in Mali; *Ware* in Sudan, Togo and Gambia; *Woro* in Gambia; *Warri* in Sierra Leone, Togo and Barbados; *Wawi* in Antilles; *Awari* in Surinam; *Awèlè* in Ivory Coast and *Owela* in Angola (See Figure 5.2, See also Silva, 1995 & Murray, 1952).

Also for the four-row Mancala several names appear: *Solo* or *Lisolo* in Congo and Zimbabwe, *Kisolo* in Congo, *Adji* in Antilles, *Magi* or *Madji* in Nigeria, *Bao* in Zanzibar (Tanzania), *Libao* in Mozambique, also *N'tchuva* by the *Thongas* in the Southern Mozambican Provinces of Maputo and Gaza or the *M'palé* or *Tchadji* by *Emakuwas* in the Northern Province of Nampula (Junod, 1962).

It is difficult to establish the true origin of any name, since each group of players is likely to have its own theories for the origin of the adopted version. Walker (1990) states that *Warri* was named after a town in Nigeria or taken from the Ghanaian word for 'far', a reference to the endless moves (See section 7.2.5.1 for the case of *Tchadji*) in the game. *Bao*, the name used for Mancala games in Zanzibar, refers to a wooden board, but *Solo* is both a Swahili word for the playing seeds and general word for the game in some African languages. It is believed that most of these names are from the same vocabulary, and that they vary according to the scholar, who described and studied the game (Silva, 1995).

5.4 Mancala type games played in Mozambique

5.4.1 General issues

In Mozambique only the four-row Mancala are found. The main boards found are the 4x4 board, the 4x8 board, the 4x10 board, the 4x12 board, the 4x16 board, the 4x22 board and the 4x32 board with two seeds in each hole at start of play (Wagner, 1927; Junod, 1962).

It seems that in Mozambique games of the Mancala type are only played by men (Cf. Junod, 1962, De Voogt, 1997), as it is generally the case in the African continent. However, in the case of *Wari*, Bell (1979:13) noted that "*Traditionally Wari is played by men rather than women. However, some women do play it but if a woman becomes too expert at the game, most men will refuse to play her, since it is considered humiliating to lose to a woman*" (Cf. Herskovits, 1930).

N'tchuva is the name of a four-row variant among the *Thongas* in the Southern Mozambican Provinces and *Tchadji* or *M'palé* (See figure 5.3) among the *Makhuwas* in the Northern Provinces of Nampula and Niassa. In Cabo Delgado, a province on the border with Tanzania, it is called *Libao*.

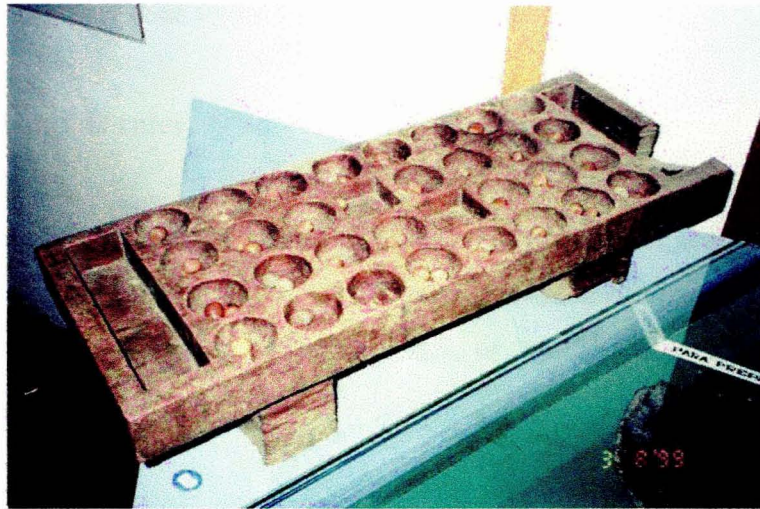


Figure 5.3: A *Bao*-board

Recognisable by the two squared holes and used to play *Tchadji*. It is exhibited at National Ethnographic Museum in Nampula (city)

5.4.2 The *Tchadji* - game at *Ilha de Moçambique*

Ilha de Moçambique is a small island situated on the northeast coast of Mozambique. The island has no industry and, actually, has about 12 thousand inhabitants mostly Moslem. The majority of adult males inhabitants of the island are mostly fishermen or do small business for living.

The playing of *Tchadji* at the Island has a long tradition. Presently, the Island is subdivided into small neighbourhoods. Actually the playing of *Tchadji* is organized in circles of players and many neighbourhoods have their own group of players.

Tchadji is usually played on wooden boards, which have four rows of eight holes each (4x8 board), carved into them. The pieces are fruit stones from *Caesalpinia Bonduc* bush: *Tchadji*, or seeds being used for counters. Very rarely wooden boards are not readily available. When this happens then the rows of holes are often dug out of the earth or sand

and for counters, cowry shells or even real stones are used. *Tchadji* is played by two people facing each other (See figure 5.4).

5.4.2.1 *Tchadji* and its rules

The playing rules for Mancala games in general, though simple, are rather difficult to explain. Essentially, the playing of Mancala games consists of one basic movement, which involves ‘sowing’ and ‘capturing’ playing pieces, or ‘seeds’, using a board comprising rows of holes.

Therefore, Mancala games are known as a ‘count and capture’ boardgames. They are generally played by two people or, on rare occasions, by two teams on a board containing two, three, or four parallel rows of cuplike depressions or holes. It is played with a predetermined number of identical counters or pieces. The goal is to immobilize or annihilate the other player(s) by the capture of the majority of seeds (Cf. Walker, 1990).

Each move in a Mancala game causes multiple changes in the position of the pieces on the board. If multiple sowings and captures are allowed, these changes become even more difficult to calculate. In fact, the art of these games is the calculation of multiple changes, a prerequisite for all good players since strategies can only be executed once this skill has been acquired.

5.4.2.1.1 General *Tchadji* rules

Tchadji is a game of movements some of which are complex. Perhaps a good way of introducing the rules is through the following illustration in which numbers indicate the number of seeds in a hole (See figure 5.4).

Tchadji is played by two people. The players face each other and each plays in the two rows of holes closest to him, the inner and the outer rows (See figure 5.4). There are 32

holes in a *Tchadji*-board, sixteen belonging to one player and sixteen to a second player (rule 1).

Player B								
2	2	2	2	2	2	2	2	4 (A outer row a4-h4)
2	2	2	2	2	2	2	2	3 (A inner row a3-h3)

2	2	2	2	2	2	2	2	2 (B inner row a2-h2)
2	2	2	2	2	2	2	2	1 (B outer row a1-h1)
Player A								
a	b	c	d	e	f	g	h	

Figure 5.4: A representation of *Tchadji*-board with two seeds in each hole

Generally, the players play alternately and, on his turn, a player begins a move by picking up the contents of any hole in his two rows which contains two or more seeds and moving in laps, **spreading** or redistributing them, placing one seed into each hole in a counter clockwise direction from the chosen hole, around his two rows until the last seed of a lap falls into an empty hole. The move is over, if the last seed of a lap falls into an empty hole in the player's outer rows (rule 2).

Single seeds can only be used to begin a move when no other hole on a player's side contains more than one seed. This situation is called *Ompwechá*, which literally means ‘divide into pieces’. In this case, only singles that move into adjacent empty holes can be chosen (rule 3).

Capture of seeds, *Odjá* in the terminology used for the game, which means ‘to eat’, is made only if the last seed of a lap falls into an empty hole in the player's inner rows and the opponent's inner row hole, directly opposite is occupied, i.e., it contains at least one seed. In this case, the player captures the contents of both his opponent's opposite holes (from the inner and outer rows) and the contents of one other of his opponent's holes: *Ossuwaree* in the terminology of the game, and removes them from the board (rule 2b). The winner is the player who first captures all of his opponent's seeds (rule 4).

If the move ends in an inner hole and the opponent's directly opposite inner row hole is empty, no captures are made (rule 2c). Or, if the last seed of a lap falls into an empty hole in the player's outer row, no captures are made (rule 2a) (See Appendix B for illustration).

5.4.2.1.2 *Tchadji* from the players' perspective

During the fieldwork on the *Tchadji*-game at 'Ilha de Moçambique', the tournament organised on February 6, 1999 and the regular matches recorded on August 1 and 4, 1999, many people, both spectators and those interested, in particular players, were present. Many players participated in the matches. However, when asked to provide more detailed information concerning the game, only 5 persons offered to be interviewed.

With the aim to learn more on specific aspects of the game, like the various names of the game, variants and history of the game, a group interview in form of a group discussion (Burns, 1995) took place in the afternoon of August 1, 1999 with 5 players. Next, I present some remarks based on some extracts of this interview. Since it was a group, I did not record specifically who was answering or commenting on any of the issues. The interview was conducted in the *Emakhuwa* language (See figure 1.1).

The interview started with the following comment from myself: "*Now, what is this game called? I have heard about M'palé or Tchadji, what is the real name for this game?*"

The players commented that, in principle, there is a question of preference regarding the name. They said that: "*There exist M'palé and Tchadji. M'palé is the one we play and Tchadji is the one played in the Province of 'Cabo Delgado', but we also used to call it here Tchadji.*" In fact, I have heard both names.

Since I was interested in knowing from their perspectives issues related to playing *Tchadji*, I suggested that we should discuss the game while two people are playing: "*Let us*

discuss the issues of Tchadji while playing it. So, two players, Momade and Anza, started playing. I could then interrupt them at any instant and ask questions concerning the match.

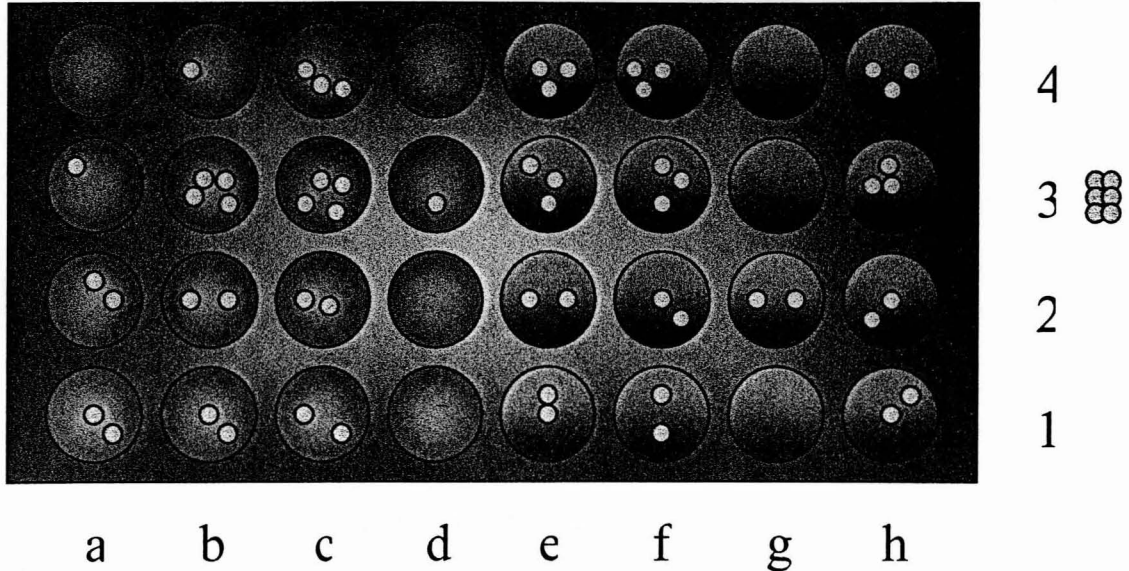


Figure 5.5a: The situation after the first move by Momade

Momade started playing in b4 and ended in d3 capturing the seeds in d2, d1 and g1 (See figure 5.5a). So, I asked him: *“Are there any specific holes where to start a match?”* He gave the following answer: *“Yes, some people created certain habits with respect of the way to start a match. I used to start here [pointing to b4; g1 is equivalent to b4 on the side of player A]. It does also depend on my opponent at this moment”* (See Appendix B26).

Then I asked, *“And the Ossuwaree [taking of the seed(s) of any other hole], what does it depend on?”* He said that *“It depends also on my opponent, his strategy and tactic make me do so.”* Then another player, Ali, explained: *“If he had not taken g1, his opponent would then play in e1 connect it to g1 and land in d2 (See figure 5.5a), where he could capture seeds.”* This explanation was not convincing at all, because if Anza, the opponent, had played even in h1 or f2 he would be able to capture seeds by landing in d2. There are also other options. For example, playing in e2 landing in b2 or, maybe the best option, playing in c1 and landing in c2.

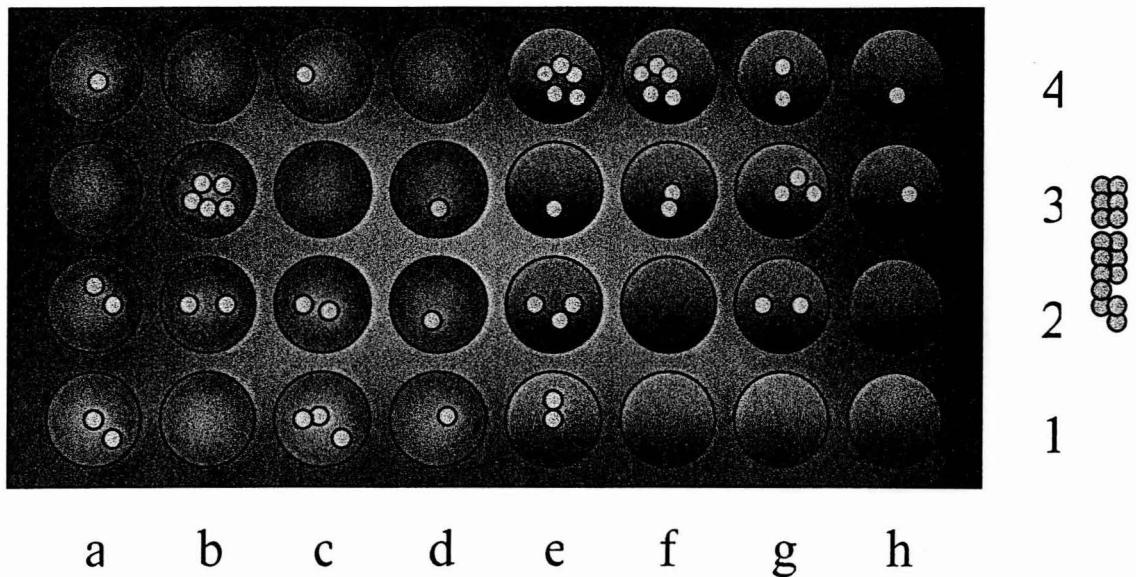


Figure 5.5b: The situation after Momade has played in e3 and landed in h3, capturing the seeds in h2, h1 and f1

Anza, on his turn, played in f2 and landed in d2, capturing the seeds in d3 and c4. Momade played in f3 to d4. Anza played from b2 and landed in d1. Momade, at his turn, played in e3 and landed in h3, capturing the seeds in h2, h1 and f1 (See figure 5.5b).

At this point I interrupted the match and asked the following: “*Why have you left these 4 consecutive holes empty?*” Momade answered as follow: “*So that he will not be able to get to his inner row.*” This means that Anza will not easily be able to capture Momade’s seeds at least for the following move (See more in section 7.2.7.2 & 7.2.7.3).

When the match was in the situation in figure.5.5c, Anza played in b2 and landed in c1, so I asked the following question: “*Did you know before where you would stop?*” Anza answered: “*Yes, I did. I counted so, if I take the seeds here [pointing to b2], I will connect here [pointing to b1] and end here [pointing to f1].*”

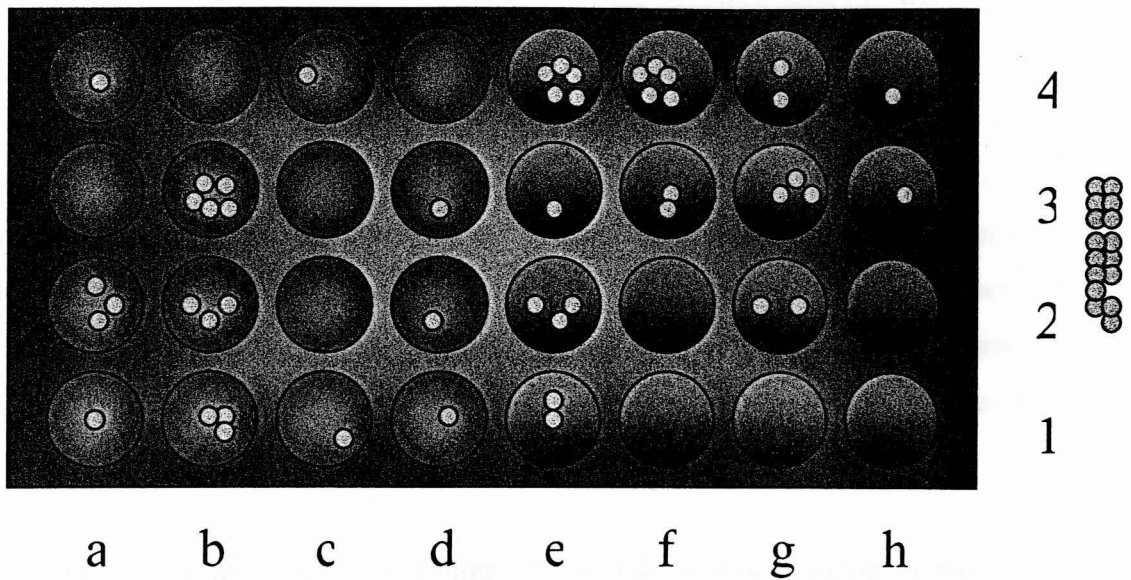


Figure 5.5c: The situation after some moves in the match Momade against Anza

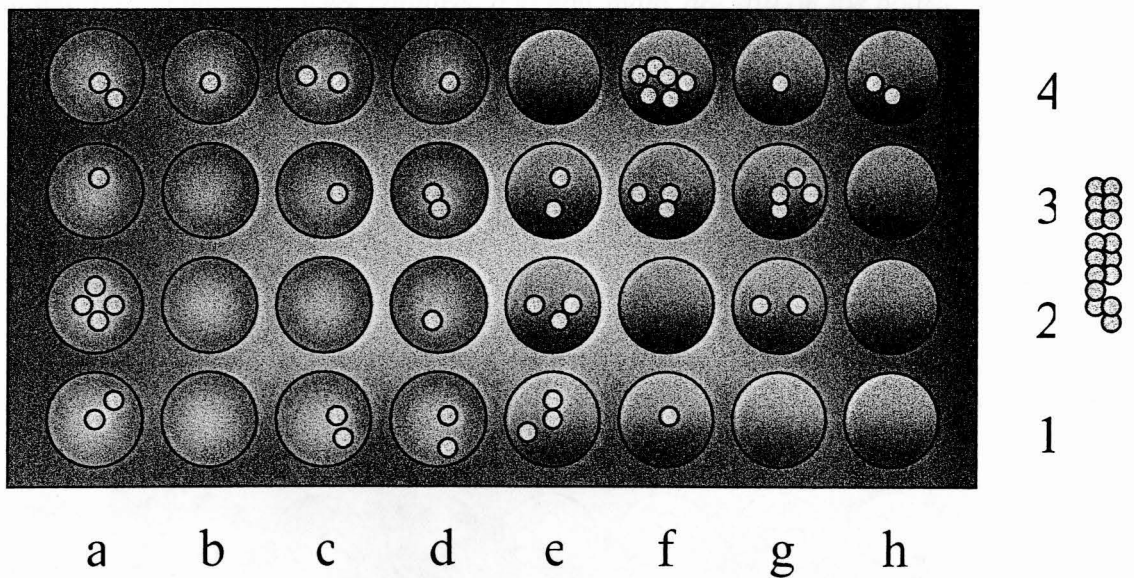


Figure 5.5d: 'Running away'

Momade played in g4 and ended in g4 after exactly one round. So, I asked him: *"Did you know that?"* He said, *"Yes, I did. It was also to run away from the corners"* (See figure 5.5d). So, I asked, *"How did you know?"* He said the following: *"I do counting here on my side and the side of my opponent"* (See more details in section 7.2.1).

I insisted on another question to make myself sure of his counting methods: “*Now tell me quickly, if you had played here [pointing to d3], where would you stop?*” He said, “*If I play here [pointing to d3], I will stop here [e4].*” I insisted again by asking, “*How do you know it?*” So he said, “*I know because I created a habit, I played many times.*” This is an indication that the players do not always count by moving the seeds, they might be in some situations in which they have memorised and thus play quickly knowing exactly where the move will stop. In other words, there might be some situations, which the players can simply memorise due to having been in that situation many times before (See more in section 7.2.2).

Momade himself explained further as follow: “*Look, I know how to count as part of my strategy. During the match, I memorise the number of holes, how the seeds are distributed in the holes, how many seeds were captured and how many are still on the board.*”

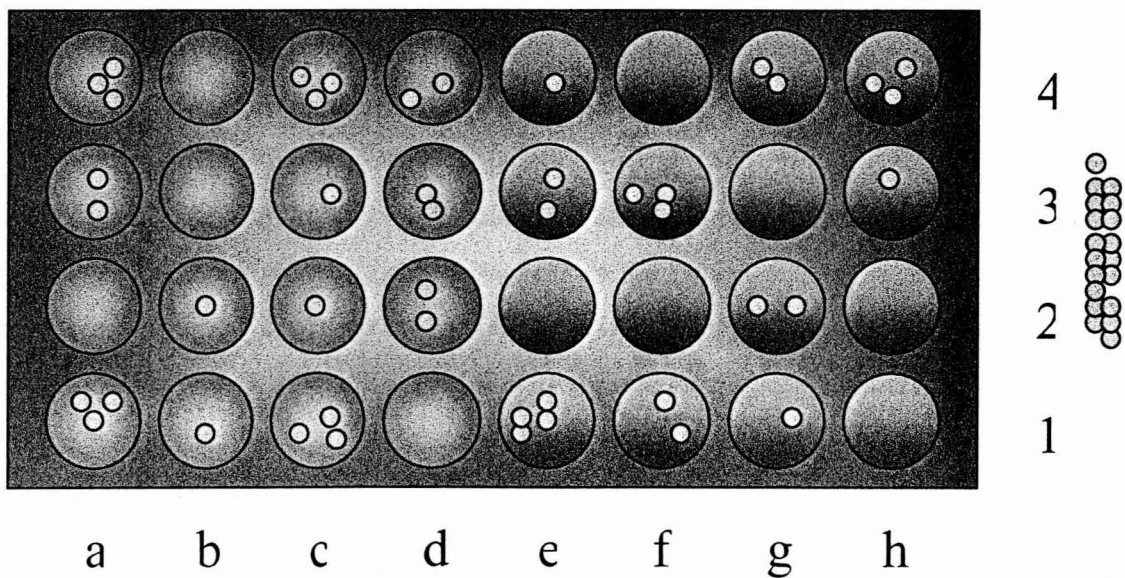


Figure 5.5e: ‘Running away’

Anza played in a2 and ended in g1. Momade, at his turn played from g3 to b3. Anza again played from e2, landed in b2, capturing the seeds in b3, b4 and when it came to capture the seeds of a third hole he commented (See figure.5.5e): “*I have to avoid being captured here [pointing to the column b], where I captured, or here [pointing to g-column].*” He

took then the seeds in c4 commenting: *“He [referring to his opponent] can capture me here [pointing to g3] by playing in here [pointing to e3] but then I will capture him any way.”*

Analysing this situation (See figure.5.5e), we see that from the perspective of Momade, c4 connected from the start in g4 or h4 would end in b3. This is exactly what Anza wanted to avoid. On the other hand, starting in c4, Momade would stop in g3, what Anza also wanted to avoid. So, in my opinion, c4 remains the best option to take.

At another moment during the group discussion, I asked the following questions concerning their knowledge of the origin of the game: *“Now, in your opinion, what is the origin of this game? Where does it come from? Look, we all were born here in the Island and have found it here and have seen people playing it. Where does it really come from?”*

One player answered: *“In my opinion, maybe the Arabs brought this game to our Island. This with 8, I only saw this here. The one with 10 [meaning 10x4 version] and the one with 12 [12x4 version], I have never seen it here before.”*

Another player supporting his colleague stated: *“Yes, here we had only the one with 8, others like the ‘12’ and the ‘15’, exist now here in the Island. Who brought these were other people who lived in Nampula city or Nacala and moved then to the Island and brought the other games with them.”* In fact, as a result of migration caused by the civil war during the period of 1980-1992, the population of the Island has increased from about 6 thousand to 12 thousand.

At another moment of the interview I concentrated on exploring strategies and tactics from the players’ perspective. So, I asked them: *“Now, look each player, as I realised, has his own strategy for playing. What is for example your strategy? What plan do you follow when you play Tchadji?”* (See more in section 7.2.7.2 & 7.2.7.3).

One player said, *“When I play Tchadji, my strategy is to win. I follow my strategy and I see how it will develop while playing on the board. It depends on the information I have about my opponent. If my opponent is a good player, I should play defensively to avoid losing.”* Another player said, *“Tchadji-strategy is as follows: when you start playing, you start thinking that if I play in this manner and capture there and here, my opponent will not have the opportunity to capture me in a certain hole. I will leave him to capture me in another place and then I will take the opportunity in capturing him also. So is the Tchadji.”*

At another time I asked, *“Now, tell me, do you usually train to play Tchadji? Do you usually in your homes reflect on how you will play in the future?”*

One player said, *“Here, as we are, we are all good players. We don’t need to do training, if we play the ‘8’ [meaning the 8x4 version]. We did our training when we were learning it.”* A second player added the following comment: *“We have memorised all of this game. Concerning our opponent, depending on his way of playing, we know that he will beat us or not. It is during the match itself that we change our way of playing, trying to avoid to be beaten at certain points. It is at that moment when the player plays or starts a move that we know that he will beat us at a certain place and so, we think to take another move to avoid it.”* A third player also indicated that each strategy depends on how the match is started. This again gave me an indication that they always play in more or less the same way. They may occasionally take risks in playing differently to what they usually play, but this does not happen often.

5.4.2.2 The Tchadji players' background

During the interview session with the 5 players, I asked them in addition to the interview on specific aspects of playing of the game, to give me information concerning their life and, in particular, how they came to learn to play Tchadji. In this section I present this information.

Ussene, the first player to be interviewed, was born at the Island and was at this time 49 years old. He has 8 children and went to school just up to grade 3. For a living he used to repair bicycles. When asked how he learned to play *Tchadji* he answered: *I learned Tchadji a long time ago by watching older people playing it. These people played it for long time. I learned when watching other people (who passed away) practising it.* He further said that he was 16 or 17 years old when he started to learn the game. He then added that, *“You can not learn how to play the game only by watching it. I started playing it against older people and losing many times but practising it, until I started winning against people with whom I always lost. No one taught me how to play Tchadji. Learning this game is only by practising, by losing and winning. It is by self learning.”*

Chomar, the second player to be interviewed was born in 1948. As profession he worked as tailor. He only completed the grade 2 in school. He said that he learned to play *Tchadji* by watching and practising it. *“A long time ago, when I was a kid, Tchadji was practised in the south part of the Island in the township of ‘Santo António’. There lived an old man called N’dhuma (who passed away). It was in his place that the people used to play Tchadji. There I watched the people playing until I started playing, losing and also winning until I learned. And it is so, when you start learning a game like this you first lose until you learn.”*

Momade was born in 1952, has 10 children, went to school up to grade 7 and is auto-electrician. When asked how he came across *Tchadji*, he explained as follow: *“a long time ago lived an old man called Naguibo. One day I had asked him to teach me Tchadji. He started teaching me this game. However, before I learned much how to play it, he died. So, I started looking for a place in which I could practise it. So, I found then the place N’dhuma. It was at this place that I watched people playing it and kept practising it until I got experience in the game. This old man then passed away and when this happened I took the board and transferred it to my place. It is there where most people come to play.”*

Anza, the fourth player to be interviewed, was born in 1960. He has 7 children and for a living he used to buy and sell fresh fish. He completed grade 3 in school. *“I learned to play this game, when I was in Nampula [the capital city of the province of Nampula*

which is 187 km away from Island]. However, there I first learned the 15x4 version. There I watched people playing until I learned it well. Years later I went to the army. When I came back from the army I returned back to my town [the Island]. Here, I used to see people playing this 8x4 version and I didn't know it. So, I started again watching and playing until I learned it."

The fifth player gave the following brief statement: *"My name is Ali. I was born in 1947, so I am 52 years old. I am a carpenter and went to school up to the grade 3. I learned to play Tchadji with my master, who also taught me carpentry."*

In general, I may say that *Tchadji*-player at Ilha de Moçambique did not attend the school for many years. They may have learned to count and do simple calculation partly in the few years they went to school but mainly from their own dealing with their environment and from their work for living. The *Tchadji*-game seems to be learned by practicing. There is no school for *Tchadji*. Becoming a *Tchadji*-master or a good player is a matter of having attained this status by playing well over time and being recognised as such.

CHAPTER SIX: PROBABILISTIC IDEAS IN *TCHADJI* AND OTHER CULTURAL GAMES AND THEIR USE IN THE CLASSROOM (PHASE 1)

6.1 Introduction

According to Garfield & Ahlgren (1988) there are at least three reasons for the difficulties that students appear to have in developing correct intuitions about fundamental ideas of probability at any level. *"First, many students have an underlying difficulty with rational number concept and proportional reasoning, which are used in calculating, reporting and interpreting probabilities; second, probability ideas often appear to conflict with students' experiences third, many students have already developed a distaste for probability through having been exposed to its study in highly abstract and formal ways"* (p.47).

Borovenik & Peard (1996) recommended teaching methods that take into account the social (and cultural) background of the students and the effect of this on the development of intuitions in probability. Also, Garfield & Ahlgren (1988) present recommendations to overcome difficulties in learning stochastics. Among these I emphasize the following:

"Introduce topics through activities and simulations, not abstractions; try to arouse in students the feeling that mathematics relates usefully to reality and it is not just symbols, rules, and conventions; use visual illustrations and emphasize exploratory data methods; create situations requiring probabilistic reasoning that correspond to the students' views of the world" (p. 48)

The variant of three-in-a-row game and the Mancala type game, both used in this study, can be considered as part of the students most daily experiences in Mozambique. Most of the students may have practiced these games or, at least, seen them before. Considering the issues raised by Garfield & Ahlgren (1988) above, this study is an attempt to avoid such a conflict between probability ideas to be taught and the students' experiences since the games used in the research are somehow part of the students' cultural environment.

The question of being familiar or not with the cultural artefacts to be used in the classroom seems to be crucial. It played an important role in the decision I made concerning which game to use in this research. For example, Peard (1995) reported different intuitions among secondary school students whose social background included extensive familiarity with track gambling. He reported that there were noticeable differences among these students in their understanding of the concept of mathematical expectation and its relationship with the concept of mathematical fairness.

On the other hand, this phase of the study was planned to include, as is was stated in aim 1.1 (see section 1.3), a mathematical investigation of the four types of games. Three of the games used in this study: a card game, cowry shell or coin tossing and the three-in-a-row game have been described by other researchers and they have showed their links with probability aspects (e.g., Zaslavsky, 1973a, 1993; Doumbia & Pil, 1992; Guik, 1989). For this study I took into account the results and suggestions of these authors' studies and I did further studies of links of the games with probability for the development of the teaching intervention.

For the *Tchadji*, several authors have implicitly referred to its mathematical and educational potential, but they have not described or analysed it (Cf. Zaslavsky, 1973a, 1989a, 1994, 1996; Cheska, 1987; Odeleye, 1977; Bell & Cornelius, 1988; Silva, 1994, 1995; Lumpkin & Strong, 1995). This work consisted of a mathematical analysis of aspects related to probability in the playing of the mentioned game. The investigation has been continued and concrete links with the different conceptual aspects of probability have been established.

From this previous research, I have developed a general view of how possible it would be to use games for teaching basic probability concepts. The aim 1.2 of this part of research depends on the results of the first aim. Without any preliminary ideas of how the four types of games could be used in the teaching of basic probability concepts the research would not be possible. The existing idea has been enriched with the further results of this investigation specially concerning as to when and how to use a certain game. So, in this

chapter, the results of the reflections and investigations for the construction of the probability teaching intervention of using the four types of games are described and the impact in terms of results of the implementation of this approach in secondary schools in Mozambique are presented and discussed.

6.2 Developing a teaching approach to probability based on games

6.2.1 Ideas on *Tchadji*

Tchadji is a four-row Mancala game played at Mozambique Island (See Chapter 5). Since the concept of probability implies, at least implicitly, the existence of a chance aspect, the question posed with regard to the *Tchadji*, has been whether or not there is a chance aspect in it. De Voogt (1995) referred to Mancala games as war games with complex moves (Cf. section 5.2.3). Herskovits (1930) classified Mancala type games in general as not being games of chance. Also, Cheska (1987) referred to Mancala as a game of strategy, "*where the attribute of chance is absent*" (p.15).

Nevertheless, when playing Mancala there is an amount of uncertainty involved. Also, Eigen & Winkler (1975:38) referring to deterministic games, such as 'damas' and chess, argued that these games, because of their complexity, are uncertain with regard to the final result. For example, one would guess that a certain player X should win. In fact, it can happen that X loses. A question that could be asked here is the following: 'What is for that person the basis for guessing that X wins?' or 'Why he thinks that X will win?' This person must be in possession of information that could make him decide to guess as he did (Cf. Ismael, 1996). The person guessing would probably be thinking that player X is more likely to win and not that player X certainly wins. Thus, it is clear that in a Mancala type game there is a **degree of uncertainty**, which cannot be quantified in the classical way, but it can foster and develop certain aspects of probabilistic thinking.

There were other questions posed that contributed to an important reflection on a classroom setting while playing these games. For example, considering the situations of a)

two people classified as "good" players, b) one good player and a medium player or c) two medium players, the following issues were raised in the group discussions: **'who wins the game?'**, **'who has a good chance of winning the game?'** or **'Is it possible to claim that a player will almost certainly win the game?'**.

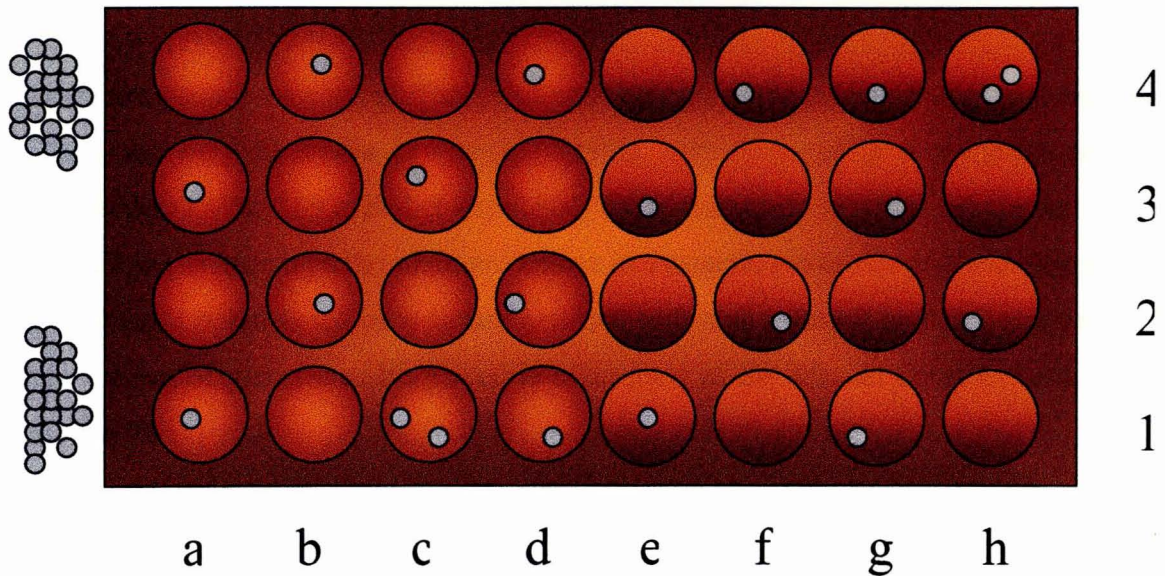


Figure 6.1: A *Tchadji* situation after a number of moves.

Some further questions arose from the use of this particular version of the Mancala game, **'What are the possible outcomes of a match?'**, **'Are they equally likely?'** For analysis of these issues an example from a previous study (Ismael, 1996) will be presented (figure 6.1).

The situation in figure 6.1 repeats again after $2n$ (n natural number) moves for each player. For example, it would appear again after B has played in b1 and ends in c1, A has played in h4 and ends in h3, B has again played in d1 and ends in d1 and A has played again in g4 and ends in g4. In all these moves no captures are made. It means that the match would never come to an end. For the students it became clear, after a little discussion, that the situation is a draw (See more in sections 7.2.5 and 8.2.2).

A concluding remark is that for the *Tchadji*-game it was possible to discuss issues related to **possible versus impossible outcomes, equally versus non-equally likely events** and also **random versus non-random phenomena**.

6.2.2 Ideas on Three stones - a Three-in-a-Row game

There are several versions of Three-in-a-row type games in Africa. There exist versions, which use even 12 stones in enlarged boards, and others using a triangular network (Zaslavsky, 1973a, 1993; Bell & Cornelius, 1988; Hopson et al, 1996). A simple version called three stones played by children, known in Mozambique as *Muravarava* (See section 9.4.3), has a board based on a square network with nine intersecting points (Figure 6.2). These types of games are also played in other parts of the African Continent and around the World.

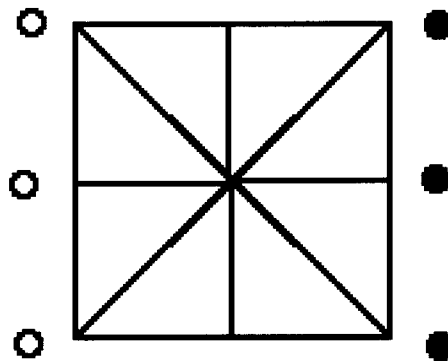


Figure 6.2: The Three stones game board

The three stones game is similar to the Three-Men's Morris game in the rules which apply (Cf. Bell & Cornelius, 1988). The board is similar to the one used for the *Achi*-game, which is played by schoolchildren in Ghana (See Bell & Cornelius, 1988; Provenzo & Provenzo Jr, 1981).

It is also known as Tic-Tac-Toe (Bell, 1978), as Tapatan from the Philippines (Zaslavsky, 1994) and Kwasida Frankaa in Ghana (Cheska, 1987). Generally, *Muravarava* belongs to

a group of games known as Three-in-a-row games or the Men's Morris games. A Somali version of the *Muravarava* is called *Shax* and in Lesotho and South Africa is known as *Morabaraba* (Jama, 1997 & 2000; Davies, 1988; Crane, 1982).

For the Three-in-a-row type games there is essentially one principle, which is that of making a row of three counters. The player who first makes a row, wins the match. The rules of the game can be described as follows:

1. Three stones is played by two people. Each player has three coloured stones.
2. The two players play in turns. One player places a counter of his colour on any point. Then six other player places a counter of another colour on any empty point. They take turns until all the counters have been placed on the game board;
3. One Player moves any of his counters along a line to the next empty point. Jumping over a counter is for a player not allowed. Then the other player does the same with any of his/her counters;
4. They continue to take turns moving their counters. The object of the game is to make a row of three counters of his colour, and to block the other player from making a row as in figure 6.3. A row can be made horizontally, vertically or along a diagonal (Cf. Zaslavsky, 1993).

For illustration of the rules I present in appendix I1 an example of playing Three stones.

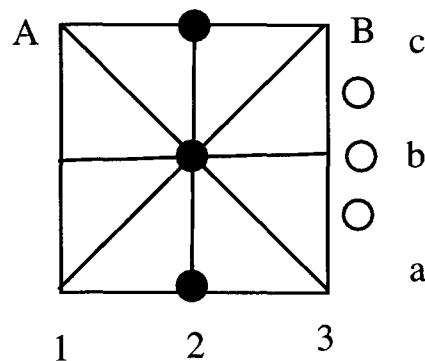


Figure 6.3: Example of a complete row (winning situation)

A preliminary study of the game shows that the player who starts has a strategy for victory when he/she places, in his first move, a counter in the centre spot. About this, Guik (1989: 249) mentioned that the player who start, has a strategy of victory, he must put his counter in the centre of the board. After that it is all very easy. In fact, with first counter in the centre a player has 4 possibilities of making a row. Further explorations of the game showed that there are two strategies, which will result in the player starting at the centre of the board winning (Cf. Bell & Cornelius, 1988). These strategies are explained in the appendix I2).

order	A (plays black)	B (plays white)	Situation in the match
1st	places in b2	Places in b3	A plays first in the centre
2nd	places in a1	Places in c3	B blocks A for winning
3rd	places in a3	Places in a2	A & B forced to play so
4th	moves a1 to b1	Moves c3 to c2	B try to block A
5th	moves b1 to c1		A makes a row in c1-c2-c3

Table 6.1: Description of a victory strategy I in three stones game

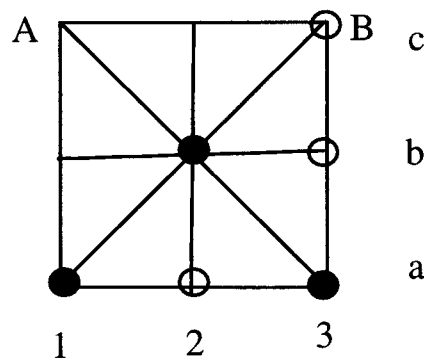


Figure 6.4: The situation after the counters have been placed on the board

order	A (plays black)	B (plays white)	Situation in the match
-------	-----------------	-----------------	------------------------

1 st	places in b2	places in c3	A plays first in the centre
2 nd	places in a2	places in c2	B blocks A for winning
3 rd	places in c1	places in a3	A & B forced to play so
4 th	moves a2 to a1	moves c3 to b3	B try to block A
5 th	moves b2 to b1		A makes a row in a1-b1-c1

Table 6.2: Description of a victory strategy II in three stones game

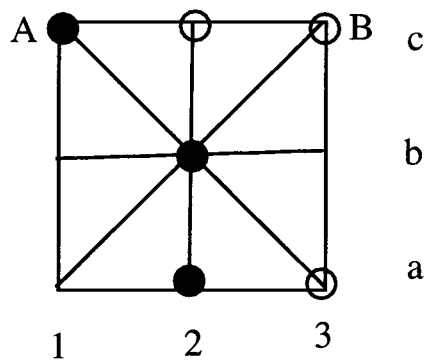


Figure 6.5: The situation after the counters have been placed on the board

The issue of strategy for a victory raised important discussions in the groups using games. There was many thoughts and activities in a classroom setting for analysing questions like **'Where should the first player place the first counter?'** and **'Is the person who goes first more likely to win than the second player?'**

The question **'What seems to be true is that the player starting in the centre has more chance to win and why is that?'** was analysed and important reflections were presented. One of the conclusions were that starting at the centre offers four different winning possibilities whereas playing at a point other than the centre a player has either only two or three different possibilities of completing a line. This seems to make sense and contributes to the discussion about who is likely to win which are aspects of probabilistic thinking.

Guik (1989) related the role of the central position of the Three stones' board to a draw. According to him, if the first player is not allowed to play the centre of the board, the match should end in a draw. Suppose now that, with this new condition, the second player plays exactly in the centre. How would the game end? The role of the centre is so important that the first player will only force a draw if s/he, in her/his two last moves, is able to occupy two neighbouring positions.

Also, the questions '**What are the possible outcomes of a match?**' and '**How likely are they?**' stimulated thoughts and activities in the classrooms. For the Three stones game only two possible outcomes were considered. A draw as in the *Tchadji* - game is not possible. Guik's position of a draw situation in the tree stones game is consistent. It is not an objectively determined situation. It depends more on persistence of the players. In every match, unless both players give up, it always happen that a certain player can win.

Bell & Cornelius (1988:19) suggest additional activities for the classroom, for example, to investigate the following questions by practicing the game "*Who should win the game? Where should the first player play? Should the first player always win? What positions are possible after 1, 2 and 3 moves? How many different positions are possible after all six pieces have been placed on the board?*" (See also Zaslavsky, 1993:118).

6.2.3. Ideas on the variant of the “donkey” card game

Card games are based on a pack of 52 cards (Figure 6.6). There exist various types of games that use all or some of the 52 cards. Some use all the 52 cards others use only of some of them. In Northern Mozambique, these games are usually played by adults as well as by children.



Figure 6.6: A sample of playing cards

According to Oldfield (1991d), card games can be adapted for the purpose of emphasizing the relevance of probability to occurrences in everyday life and simple games of chance. In fact, card games are classified as being games of chance.

It has been emphasized that there are many forms of manipulating cards, i.e., many types of card games, that relate to probability concepts, for example, the Bridge card game (See Beurtey, 1994). There are many possibilities to perform experiments with cards and so learn about different aspects of chance. Because of this chance characteristic, I chose to use these games as complementary activities.

Two variants of card games, played mostly by children in Mozambique, were used in the classroom as described below:

The first game used is called *burro*, which means donkey. This game is based on 52 cards. It is played by at least two people. The cards are placed face down in the centre. The game starts with a player taking the card on the top and turning it over. The second player does the same and then the third, and so on. The player who takes a card with the same figure

as the previously turned card, must take all the turned cards. The game continues until all 52 cards are shown. The game continues with the cards in the hands of the players with the same objective of not to match the figure of the previously exposed card. The player who at the end remains with cards loses and is considered a donkey.

By playing this game the students get a sense of the **classical definition of probability**. All 52 cards have an equal probability to be drawn. Questions like the following: '**Suppose that two tens and two kings have been drawn, what is the probability of getting a king in the following turn?**' or '**what is the probability of getting two tens in the following turns?**' have been discussed. The students in the classroom confronted with these situations, and considering the **equal likelihood**, calculated the probabilities.

The second game is called 'three cards' and is played by two people. A player takes three cards, two with the same colour, e.g., two red and one black, and moves them so as to hide the black card. After that the second player is asked to choose one of the three cards. He wins if he chooses the black card and loses by choosing a red card.

The cards were provided for use in groups. All exercises were solved through experimenting and counting the cards, i.e., in a practical way. The students were able to reflect on the likelihood of winning or losing and calculated the required probability. They also made important conclusions even when winning in situations in which they were more likely to lose. This aspect is of great importance for the development of the **concepts of chance and randomness**.

Another activity performed experimentally in the classroom with 40 cards involved making 20, 50 and 100 draws from the 40 cards with replacement and recording how often a 'red card' or 'black card' was drawn and then calculating the **relative frequency**. Here the students had the opportunity to make sense of the **objective data** for estimating **probability**, i.e., the **empirical approach to probability**. As observed by Runyon &

Haber (1991), the fundamental feature of empirical probabilities is that they are based on actual measurements rather than on theoretical proportions.

6.2.4 Ideas on coin and cowry shell games

People around the world often make a decision by tossing a coin. A fair coin has symmetry. It is just as likely to show a head as to show a tail (Eigen & Winkler, 1975:61). In the past, and even in recent times, some societies in Africa and Asia used cowry shells as money. Even considering that the cowry shell does not have symmetry, they were also used in fortune-telling and gambling games (See Figure 6.7).



Figure 6.7: Cowry shells and coins

For example, "*Igba-Ita, which translates to "pitch and toss", is one name for a cowry shell gambling game that is popular in many parts of Africa. The Mende people of Sierra Leone call the game Kpoyei. When the game is played with coins, the Igbo People of southeast Nigeria call it Igba-Ego (ego=money). The Gambling game was a favourite male recreation on market day*" (Zaslavsky, 1994: 116; see also Zaslavsky, 1993).

Coin tossing has often been used as an experiment to develop important concepts in probability such as the **concepts of chance, randomness, equally likelihood**. Cowry shell tossing, as suggested by Zaslavsky (1993) and Doumbia (1988 & 1992), has been

used alternatively for teaching aspects of probability in schools (Cf. Gerdes, 1996). For example, it has been used for teaching probability when the events are not equally likely, other than the **classical approach to probability**.

Coin and cowry shell tossing offers a variety of exercises and problems that can be performed experimentally in the mathematics classroom. Here, the same activity for both was performed experimentally in the three groups. The students were asked to toss a coin 20, 50 and 100 times and then a cowry shell (whole or not) and to record the number of the occurrences of *tail* and *head* with the coin (or *figura* and *emblema* as the Mozambican coins show) or 'up' and 'down' for the cowry shell. After that, they were asked to calculate the **relative frequency**.

With the activities the students performed, they found experimentally that the probability of getting a *tail* is (more or less) equal to the probability of getting a *head*, i.e., these two events are equally likely. With the cowry shell the students have the opportunity to distinguish **equally likely events** from **non-equally likely events**. In these two experiments the students were dealing with the **empirical or frequentist approach to probability** (Runyon & Haber, 1991).

6.2.5 The Game Approach versus the Non-game Approach

The aim of this phase of study was to compare two groups of 3 classes each of grade 11 learners in two different schools in Mozambique. The subgroups in the school in the city of Nampula (a north-eastern province of Mozambique; see figure 6.8) were considered control groups and were essentially not using any kind of game, at least physically in class.

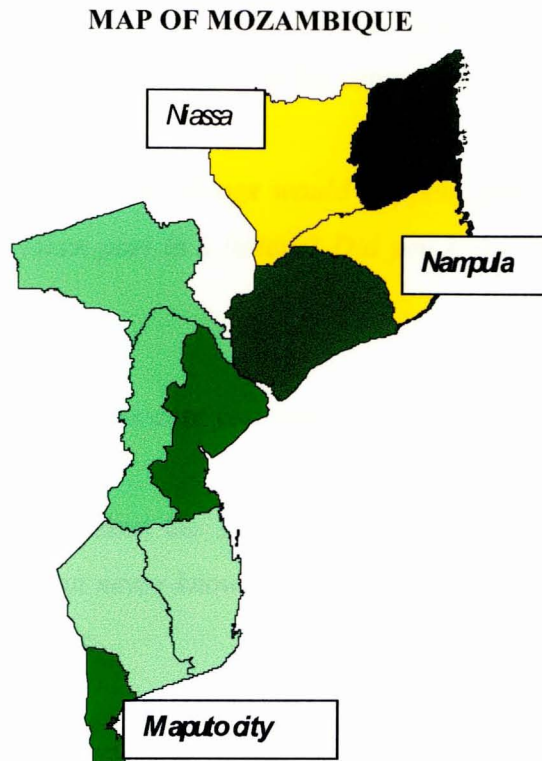


Figure 6.8: The two Northern provinces of Mozambique where the study took place

In the control groups, the probability unit, which is part of the curriculum, was taught as it has usually been done. The method was classified as being the *Non-game approach*. The groups in the school in Lichinga (a central northern province of Mozambique; see figure 6.8) were taught probability essentially using the four types of games mentioned above. These groups constitute the experimental part of the study. The way that the topic has been taught was classified as the *Games-approach*.

The main difference in the two approaches has been mainly that in the non-game the topics were exposed and learned theoretically whereas in the games-approach the topics were dealt with experimentally, with practical activities involving the games mentioned above.

In the control groups, for example, the teacher introduced the topics in a direct way. Some examples were used. Whether the students were familiar with the example or not it did not

matter. The teacher seemed to consider those examples as known by the students. Let us have a look at the following excerpt from one of the control group classes¹:

Teacher: *We know that in the world things would happen, whose outcome we cannot determine. Have you ever taken part in a lottery? Did you know whether you would win or not?*

Student(s): *Noooooo!*

Teacher: *Why didn't you know? Because you could not know the result (...).*

After this episode the teacher asked the students to write the following sentence: *Over many draws in a lottery we can never know in advance what the result will be.* And then the teacher concluded as follows:

Teacher: *Thus, there exist random or casual experiences. All those in which it is impossible to know beforehand what will be obtained.*

Even if the teacher recognizes that there is a need for the students to be familiar with the issue mentioned in the examples, he simple does nothing about it. The following passage illustrates this:

Teacher: *What is the probability of getting a king? Here it is necessary to know the cards, do you know them? (...) So, we must first know the game in order to solve this problem (...).*

This means that the teacher recognizes the need for the students to know a little bit more about the process of drawing a card out of the 52 cards if not to practice it.

¹ All the interactions took place in Portuguese language, which is the language of instruction in Mozambique. The following extract is of my translation.

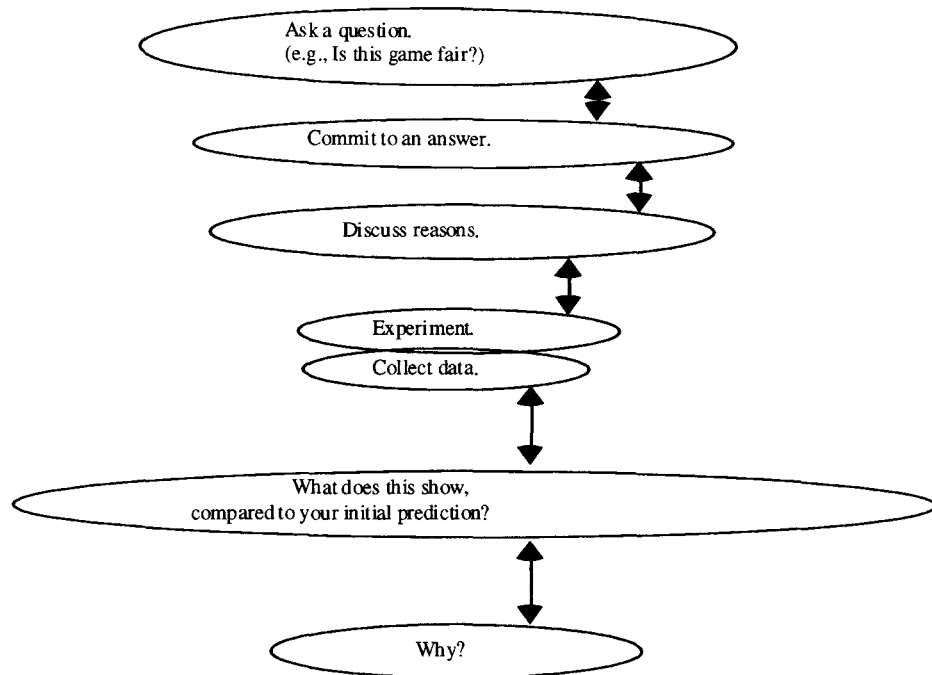


Diagram 6.1: Description of characteristics of the games-approach

The lessons of the game-approach were conducted in such a way to make as many activities as possible to explain the probabilistic ideas covered in the program (See section 4.2.1). For example, the concepts of randomness or chance were taught by permitting the students to play and analyse the *Tchadji*-game. Here, the students were able to recognise the degree of uncertainty latent in this game. And Although they could not relate it directly to the classical concept of probability, they were able to understand this important aspects of probabilistic thinking. The coin game, cowry shell game and the donkey game were also used in this stage to foster an understanding of chance aspects.

For the classical notion of probability, the students performed experimentally the coin and cowry shell games and the donkey game in order to gain a sense of the concept. In order to make sense of the concept of non-equally likely events (different probabilities), the game of cowry shell played an important role.

The lessons in the experimental classes can be described schematically according to diagram 6.1. For all tasks in the classroom the students had the opportunity of

experimenting, i.e., collecting necessary data by playing the game. Conclusions were reached on the basis of discussions in the classes and data collected.

6.3 The findings of the experimental study

The primary purpose of this study has been to investigate any underlying probability concepts in the playing of the four types of games. The first part of this chapter presented the results of these reflections and investigations for the construction of the probability teaching approach based on the four types of games as stated in aim 1.1 (see more in chapter 1). This part presents the results, both qualitative and quantitative, of the study on the use of this approach in secondary school mathematics classrooms for the probability syllabus, so as stated in aim 1.2, i.e., in order to determine its effect on

- (a) Attitudes towards the learning of mathematics,
- (b) Motivation for the study of probability,
- (c) Performance in a test on probability, and
- (d) The understanding of critical probability concepts (See more in Chapter 1).

Also, the results are discussed in relation to the stated research questions (See sections 1.3 and 1.4). Tables for Analysis of Covariance (ANCOVA), descriptive statistics, correlations and other statistics, which were obtained using SPSS 10.0 are presented

A total of 198 students and 4 mathematics teachers participated in the study, but for the purpose of analysis only 162 students have been considered. 75 students were from Niassa, the experimental treatment group, and 87 students from Nampula, the control treatment group. In the appendix E some photographs are presented taken during the lessons in Niassa (Lichinga Town).

6.3.1 Quantitative analysis

6.3.1.1 Motivation for learning mathematics and attitudes towards mathematics

Motivation for learning mathematics and attitude towards mathematics was measured through the pre- and post-questionnaire. The pre-questionnaire (see appendix A12) contained thirty Likert type items, single statements about mathematics chosen and readapted from Coulson (1992), Osei (1995) and Oskamp (1977), from 'Strongly Agree' to 'Strongly Disagree' (Burns, 1995). The questionnaire, with an internal consistency reliability measure of **0.7474** (See section 4.2.3.2), had also the aim to gather information on students' initial attitudes towards mathematics and their motivation for learning mathematics.

The post-questionnaire contained 32 items (see appendix A14). The items of this questionnaire were of the same nature as the pre-questionnaire with some exception. For example, two new items have been added and other items have been formulated more specifically for the probability topic. This questionnaire revealed an internal consistency reliability measure of **0.7458** (See section 4.2.3.2).

Both questionnaires contained negative and positive statements. For analysis purposes the answers of the negative statements has been reversed in order to create a general negative to positive analysis. This does not change their meaning at all, for example the statement #7 of the post-questionnaire (*I don't like absenting my self from a maths class*). Reversing it would mean *I like absenting my self from maths class*, and an agreement here would mean a disagreement in the initial statement, which does not change any thing.

6.3.1.1.1 Comparison of overall means

In general, I may say that there have been very small changes observed in motivation for learning and attitudes towards mathematics between pre and post-administration of the questionnaire in both experimental and control groups. For the control group, the overall mean in the pre-questionnaire was **2.94** (with a Standard deviation of 0.19), whereas in the post-questionnaire this group reached an overall mean of **3.16** (with a Standard deviation of 0.17). The experimental group had an overall mean of **3.02** (with a Standard deviation of 0.19) in the pre-questionnaire and **3.20** (with a Standard deviation of 0.17) in the post-questionnaire (See table 6.3).

A slight positive increase, about 0.2, was observed in the responses from both groups. However, in the pre- and post-questionnaire measures of both groups no statistically significant differences were observed between the groups (See Appendix F).

An interesting feature is the fact that the overall average on the post-questionnaire was greater than 3, i.e., above average. In the pre-questionnaire, the means were also around 3.

PROV		PREQUESTIONNAIRE	POSTQUESTIONNAIRE
Nampula	Mean	2.945977	3.162716
	N	87	87
	Std. Deviation	0.198199	0.175043
Niassa	Mean	3.024	3.200417
	N	75	75
	Std. Deviation	0.199666	0.164807

Table 6.3: Overall mean pre- and post- attitude questionnaire scores for control and experimental groups

Motivation was measured on 8 items included in the post-questionnaire (# 1, 2, 3, 4, 5, 6, 7, 8 in part iv). These items were designed to measure specifically issues related to motivation (See Appendix A14). The mean on motivation of the control group was **3.66**

with a standard deviation of **0.34**, whereas the mean of the experimental group was **3.69** with a standard deviation of **0.35**.

When the overall means for motivation of both treatment conditions are compared, no statistically significant differences were observed (See Appendix F). However, there has been a little different, about 0.03, between the two means more for the experimental treatment conditions (See table 6.4).

PROVINCE	Mean	N	Std. Deviation
Nampula	3.6695	87	.3405
Niassa	3.6950	75	.3543

Table 6.4: Overall average scores for items related to motivation under both treatment conditions

6.3.1.1.2 The Pearson correlations coefficients

How did the students respond to the questionnaires? The fact that there were no statistically significant differences observed in the overall means in the case of both questionnaires, does not mean that both groups responded to the questionnaires in the same way. It is of interest to find out how students in each of the two treatment conditions responded to the questionnaires. For this reason, Pearson correlation coefficients for the questionnaires in control and experimental groups were computed.

Tables 6.5a and 6.5b show the correlations between the pre- and the post-attitudinal motivational questionnaire. The overall mean correlation for the control group was **0.727**, which is a high positive correlation. This could mean that the students in the control treatment condition might have responded in more or less the same way to both questionnaires.

Class		N	Correlation	Sig.
1npl	Pre & Post-questionnaires	29	0.690	0.000
2npl	Pre & Post-questionnaires	18	0.739	0.000
3npl	Pre & Post-questionnaires	22	0.722	0.000
Overall		69	0.727	0.000

Table 6.5a: Paired Samples Correlations for control classes (Nampula)

The Overall Mean correlation for the experimental group was **0.414**. This could mean that, in contrast to the control treatment condition, the students of the experimental classes might have tended to respond differently to the questionnaires. They might have changed their opinion. This could be an indication that the students from the experimental group might have changed their attitudes, perhaps for the positive way, when confronted with the game-approach for learning probability.

Class		N	Correlation	Sig.
1nias	Pre & Post-questionnaires	22	0.454	0.034
2nias	Pre & Post-questionnaires	18	0.563	0.015
3nias	Pre & Post-questionnaires	20	0.317	0.173
Overall		60	0.414	0.001

Table 6.5b: Paired Samples Correlations for experimental classes (Niassa)

6.3.1.2 Performance on probability Test

A pre-test on previous mathematical knowledge useful for learning and understanding probability, consisting of twenty multiple-choice items was used (See appendix A11). The intention of using this test was to contribute, in the analysis of data, to minimisation of the initial differences of the intact groups since the research method chosen for this study is quasi-experimental. The test items have been mostly written by myself or adapted from

other assessment tests. The internal consistency reliability measure for this pre-test is **0.6166** and it was considered satisfactory (See section 4.2.3, Table 4.3).

At the end of the interventions in the two towns, the performance on probability was measured using a 12 multiple choice item construct response post-test (See Appendix A13). It includes aspects of understanding of probability concepts. This test has been entirely prepared by myself for this purpose. The internal consistency reliability measure of this post-test (**0.3079**) was weak (See section 4.2.3, Table 4.3).

For the analysis of data related to performance measure, ANCOVA was used to test for overall differences between the groups. The pre-test score was used as covariate. ANCOVA is, as it is sometimes described, a blend of Analysis of Variance (ANOVA) and regression. In terms of one dependent variable and one covariate, Analysis of covariance can be applied if the following three assumptions hold: (1). The dependent variable is linearly related to the covariate, (2). Treatments Groups have the same slop, and (3). The Covariate is unaffected by differences among the means of treatment groups (Rencher, 1998).

In the quasi-experimental design, the confidence that can be placed in the validity of results depends, in large part, on the case that can be made for the similarity of the groups (Wiersma, 1995). The groups used in this research were intact and are assumed, a priori, not to be equivalent.

In these cases, for the purpose of adjusting for initial differences between nonequivalent groups on the pre-test, the use of ANCOVA is recommended (Pedhazur & Schmelkin, 1991). As Pedhazur & Schmelkin (1991) stated, "*much of the controversy surrounding the validity of the comparisons among nonequivalent groups resolves around the validity of using ANCOVA for the purpose of adjusting for initial differences among the groups*" (p.574).

The mean performance score on probability for subjects taught through the game approach (experimental classes in Niassa school) was 11.55 (SD = 2.45), and the mean performance for the subjects of the non-game approach (control classes in Nampula school) was 9.21 (SD = 2.83) (See table 6.6a). When the treatment groups were compared, a statistically significant difference was found in their performance measures, $F(1, 159) = 23.850$, $p < 0.0005$ (See Tables 6.6b).

PROVINCE	PRE-TEST	POST-TEST
Nampula Mean	11.6322	9.2146
N	87	87
Standard Deviation	2.9378	2.8398
Niassa Mean	11.7333	11.5556
N	75	75
Standard Deviation	2.8868	2.4558

Table 6.6a: Summary of statistics per treatment
(Maximum score = 20.0)

The overall mean scores for two treatments in the pre-test on general mathematics turned out to be almost the same. However, the overall mean score of the control group on the probability test (post-test) revealed to be significantly less than the overall mean score of the experimental group. Thus, this difference, about 2.34, cannot be attributed to the initial group differences. The difference could be the result of using the game approach as a teaching strategy. However, there may be other acting variables which may explain the obtained differences.

The class 1npl in Nampula (control treatment) and the class 3nias in Niassa (experimental treatment) were taught by myself (See chapter 4). The differences in the means of the pre-test measure of these two classes (See table 6.7a and 6.7b) were found to be statistically significant ($t = 2.930$, $df = 52$ and $p < 0.005$).

Tests of Between-Subjects Effects

Dependent Variable: SOMAPOST

Source	Type III Sum of Squares	df	Mean Square	F	Sig.
Corrected Model	313.985 ^a	2	156.992	23.850	.000
Intercept	503.068	1	503.068	76.427	.000
SOMAPRET	93.252	1	93.252	14.167	.000
PROV	215.699	1	215.699	32.769	.000
Error	1046.595	159	6.582		
Total	18541.667	162			
Corrected Total	1360.580	161			

a. R Squared = .231 (Adjusted R Squared = .221)

Table 6.6b: ANCOVA results for post-test under both treatment conditions

When the two treatment groups were compared, a statistically significant difference was found in the performance measure, $F(1, 51) = 10.578$, $p < 0.0005$ (See tables F1c in appendix F).

The two classes (2npl & 3npl) in the control conditions and the two classes (1nias & 2nias) in the experimental conditions were taught by their own mathematics teachers. The differences in the means of the probability test (post-test) for these four classes, seen in treatment conditions, were also found to be statistically significant, $F(3, 103) = 15.707$, $p < 0.0005$ (See table F1b in appendix F).

CLASS		PRE-TEST	POST-TEST
1npl	Mean	13.1935	11.1290
	N	31	31
	Standard Deviation	3.2292	2.3722
2npl	Mean	9.9286	8.2143
	N	28	28
	Standard Deviation	2.0893	2.6030
3npl	Mean	11.6071	8.0952
	N	28	28
	Standard Deviation	2.3935	2.4727

Table 6.7a: Test statistics per class for the control group
(maximum score of 20.0)

CLASS		PRE-TEST	POST-TEST
1nias	Mean	13.0333	10.3889
	N	30	30
	Standard Deviation	2.6715	2.2609
2nias	Mean	10.9091	12.5758
	N	22	22
	Standard Deviation	3.0065	1.9739
3nias	Mean	10.8261	12.1014
	N	23	23
	Standard Deviation	2.4800	2.5730

Table 6.7b: Test statistics per class for the experimental group (maximum score of 20.0)

6.3.2 Qualitative analysis

From the nature of this research, it was also important to get an idea of the students thinking and understanding of some critical concepts related to probability. Since a multiple-choice test would not provide insight into this issue, I decided to carry out interviews.

Fifteen randomly selected students of the experimental groups (five from each class) were interviewed after the last session. The following three questions were asked in different ways: What is a random phenomenon?; What is probability? and Any comment about the previous sessions on probability unit'. In some cases the students were asked to justify their comments and to give examples clarifying their answers.

Unfortunately, only five randomly selected students from one of the control classes were interviewed. When it came to interview the students of the other two classes after the last session, they had left the school. For time reasons it was not possible to get them. Only two questions were asked to the 5 students interviewed and excerpts from the interviews are presented below.

6.3.2.1 Students' conceptions of randomness and probability

6.3.2.1.1 In control treatment condition

Asked to explain the concept of random phenomenon using their own words, the students of Nampula (control classes) gave the following comments:

Random phenomena are events... a set of events...it happens at random.

or

For example, the coin tossing is a random phenomenon, in card games there is a random phenomenon.

Here, the students were not able to explain properly the meaning of the concept. However, as it can be seen, they were able in some cases to give some examples, but not in a clear way. My understanding is that, because of the lessons having been of an expository nature, the students first listen and eventually write something. When asked to explain, they will remember some words and will answer in a disconnected way.

Concerning the question on the concept of probability, the students of the non-game approach gave the following answers:

We take the number of events p and divide by the number of events n .

or

Probability is the possibility for explaining random phenomena

A clear demonstration that the topics were mostly learned devoid of meaning or without reference to a specific context is provided by the following interview passage:

Interviewer: *What is the probability of getting a 3 when tossing a dice of 6 faces?*

Student: *A dice of 6 faces, to obtain how many?*

Interviewer: 3

Student: *The probability is equal to 2.*

Interviewer: *Is it 2? Think again.*

Student: *It is 18.*

6.3.2.1.2 In experimental treatment condition

The students of the experimental group were also asked to explain their understanding of random phenomena. By responding to the questions the students used examples experienced in the classes to explain their view. They also used examples of other games. The following are some of the comments of the students:

For example, in tossing a coin we cannot know if "figure" or "emblem" will appear. For this type of phenomena, at least for me, we call random phenomena..

If two master players are playing Tchadji, we can never know who is going to win. Here, it happens at random. Random phenomenon is uncertain...or is chance.

Random phenomena are those in which we can not predict the result...for example in tossing a coin we can not say that 'figure' or 'emblem' will appear

Are all phenomena that occur, random phenomena? For example, for a soccer match we can never know beforehand how many goals will be scored.

Student:....*in a soccer match we don't know which team wins*

Interviewer: *What if a team is much better than the other?*

Student: *We don't know how many goals will be scored*

Random phenomena are all these whose realization is uncertain (...) for example in a soccer match in not certain of always winning or loosing or a draw. A practical example is the Libao game (a name used for Tchadji-game)

Is an experience that we can make, but we cannot know its end.

We do not have absolute certainty of what is going to happen.

Concerning the probability concept some students from Niassa (Experimental groups) gave the following comments:

Interviewer: *What is the probability of drawing a red card from a pack of 40 cards?*

Student: *First, we know that a pack has 40 cards and there are 20 red cards, so the probability is ... 20/40, isn't it?(...)*

Interviewer: *What does the $\frac{1}{2}$ mean in this regard? Can you interpret it?*

Student: *It means half of the cards. For example, in a coin there are two faces. So, the probability of getting a figure or getting an emblem is 50%, which is $\frac{1}{2}$.*

Student: *Probability is the proportion of the favourable cases to possible cases (...)*

Interviewer: *Probability of $\frac{1}{2}$ of getting 'figure' by tossing a coin, what do you understand from it?*

Student: *I understand that there is an uncertainty, in this case it can happen or not.*

Interviewer: *If I tell you that the probability of a particular event is 0.5, what do you understand from it?*

Student: *I understand that it can happen or not*

Interviewer: *What is the chance of happening or not?*

Student: *The chance is 0.5 for happening. There are equal chances, 0.5 for one and 0.5 for the other.*

Interviewer: *What is the probability of getting a figure by tossing a coin?*

Student: *It is 1/2.*

Interviewer: *Why?*

Student: *Because the coin has two faces. So, it is nothing to prevent, it can happen emblem or figure, it is a random phenomenon.*

In these excerpts it is quite clear that some students explained the relevant concepts using the classical approach to probability. Other students, however, explained the concept using the frequentist way. The latter commented as follows:

Probability of an event is the number in which the relative frequency tends to stabilise
or

It is when the relative frequency of an event approaches a certain value

I was also interested to examine the students' ability to recognize probability concepts in their daily lives. The following interview passages of two different students illustrate this.

Interviewer: *If some one tells you that the probability of raining today is 0.9, would you take an umbrella when going out?*

Student: *Yes, I would*

Interviewer: *And if it is 0.01, would you take it?*

Student: *No*

Interviewer: *Why would you take an umbrella for the 0.9 and not take by the 0.01?*

Student: *Because it is more certain for 0.9 probability than for 0.01.*

Interviewer: *If the probability of raining is 0.999, will you take an umbrella when going out?*

Student: *No*

Interviewer: *Why not?*

Student: *But, look to these meteorologists. I cannot trust them.*

The answers of the students reflect two conceptions of probability: the classical and the frequentist approach. The examples given were more related to the coin tossing game, card games and in some cases the *Tchadji*-game. In general, it seems that the students from the experimental groups were more certain in their answers.

6.3.2.2 Students' comments on the use of games

For triangulation purposes I found it important to get oral feedback from the students of the games approach on their feelings when they were confronted with playing and analysing games in the classroom for the purpose of learning probability concepts. The following are their comments:

The last sessions were very nice. The game practice was very nice. We used to play this game at home without knowing what is essential in it.

For me, the lessons were very positive

I liked the lessons, they were very exciting because we were taught by doing...With this way of teaching you can learn really...other teachers should also teach us in this way if there is a possibility

With these lessons I opened my mind. I have now a base for investigating probability. I can now go to a library and read probability books.

I found the lessons very good and not complicated. I liked the lessons, because they were based on games, the games simplified the lessons.

I found the lessons very interesting. I learned a lot.

The lessons were excellent, I liked it (...) because I understood quickly the explanations from the teacher (...) it was a practical issue.

I liked much this topic, we learned probability well (...) for example in the practical games we had, the Libao, the probability of being a winner or having a draw is not certain, I did not know that the Libao game is a random phenomenon.

I liked it, I liked it very much (...) to know what is a random phenomenon and possible and impossible events, helps a lot.

First, the lessons I found very good. Second, I learned a lot (...) it was my first time to have lessons like that, I gained a lot of experience with the examples given here, they were practical lessons. Regarding other lessons I had, they were never given this way.

The lessons were good. I like the honesty as we treated the topic, without any pressure as in normal lessons, (...) not in a hurry (...) all problems were clarified. I gained experience ... because the lessons were practical, other lessons has been theoretical.

I did not imagine playing 'Libao' in the classroom. I knew the game itself is not strange for me. It was strange to have seen it in the classroom. This is an experience that I never had before.

I liked the lessons from the 'Libao' we had good explanations.

I liked more the random phenomena, (...) this with the game is a reality, also, in fact, to show how mathematics should be applied to reality.

It seems clear that the students enjoyed the lessons very much, had fun in playing the games and analysing the issues raised in the lessons.

6.4 General discussion of findings

One of the purposes of this study (See section 1.3) was to explore any embedded probability aspects, in the playing of the *Tchadji*-game, the three stones game, the *burro* (donkey) game, and coin and cowry shells games. The study showed that the concepts of randomness, chance, equally likely and non-equally likely events, and classical and frequentist notions of probability are involved (See section 6.2). The analysis of the *Tchadji*, previously unexplored in this regard, revealed that it is rich in activities that can foster understanding of probabilistic concepts (See section 6.2.1). A combination of the four types of games made it possible to develop game based activities for the learning of fundamental probability concepts in the classroom. This first phase of the study revealed that the probabilistic ideas related to these 4 types of games can successfully be introduced through suitable activities based on the games.

The analysis of the attitude questionnaire data indicated that there was not much difference in attitudes for both the control and experimental groups. However, the interview data showed a clear indication of enjoyment and satisfaction for students who were exposed to the games. Students commented that they had fun, had enjoyed the lessons, had become more aware mathematics in reality, had been surprised at seeing such games in the classroom for the first time and at having activity based lessons. These aspects reinforce the theoretical issue that games generate enthusiasm, excitement, and enjoyment by requiring the students to be actively involved in learning (Ernst, 1986; Wesson, Wilson, & Mandlebaum, 1988; as quoted in Klein & Freitag, 1991; See also section 2.3.2). This result is consistent with findings of other researchers who have reported that the use of games increases students' interest, satisfaction and continuing motivation (Sleet, 1985; Strauss, 1986; See also section 2.3.2).

Concerning performance in the post-test on probability the results suggest that the students in the experimental classes showed a better performance in the test as compared to the students exposed to the control treatment conditions. This result is consistent with the conjecture that games are effective for increasing student performance, because they make

practice more effective and students become active in the learning process (Ernst, 1986; Wesson at al., 1988). In fact, for these sessions the students were continuously active.

Nevertheless, there might still be a question of what it was about the intervention that led to the improvement. It is, in fact, characteristic of research that, when a new intervention is shown to be effective, many questions arise about what specific aspect of the intervention is producing the benefits. In the case of increased teaching, for example, we may wonder whether the improvement was due to the nature of the additional material; the increased attention from the teacher while presenting the additional materials; any accompanying increases in the eye contact, smiles, or warmth; or other possible correlates of teacher behaviour. There were indeed other factors that could not be controlled in this experiment (See section 4.7).

In fact, some of the possibilities have been reported on in the literature. The reported success of using additional concrete is sometimes dependent not so much on the pupils' learning ability as on the teachers' beliefs or expectations about their pupils' learning ability. The teachers' expectations about their pupils' performance may serve as a "self-fulfilling prophecy," in which the expectations become responsible for the outcome in behaviour (Rosenthal & Rosnow, 1991; See also Stacey & MacGregor, 1999; Pajares, 1992 and Thompson, 1992).

6.5 Summary

Generally, I might say that games like *Tchadji* and the three stones as a part of Mozambican culture are rich in providing opportunities for activities in the mathematics classroom. An ethnomathematical investigation of these games, which focused on the probability aspects and related concepts such as, for example, randomness, equally likely and non-equally likely events, or chance, together with the ideas from the study of the card game (donkey) and the coin and cowry shells game, were used to develop a teaching approach for the probability syllabus in the secondary school curriculum.

This approach was then used in a school in Lichinga (Northern Mozambique) to teach probability. The study in this school involved two mathematics teachers, who taught their own classes. A third class in this town was taught by myself. Three other classes in a school in Nampula (Northern Mozambique), two of them taught by their mathematics teachers and the third one by myself, were used as control groups. The study involved a total of 4 mathematics teachers and 162 students (75 from Lichinga and 87 from Nampula). The study was of quasi-experimental nature with the purpose of determining the effects of the game-approach on students' attitudes towards mathematics, students' motivation for learning probability and on performance in a test on probability.

Attitude towards mathematics and motivation for learning mathematics were measured using a 32-items questionnaire with a Likert scale. The quantitative analysis of the questionnaire data showed that there was no statistically significant differences in the overall mean scores of pre- and in the post-attitudinal motivational questionnaire and in for both the control and experimental groups. However, the qualitative data of the interviews revealed a clear indication of enjoyment and satisfaction for students who

were exposed to the games compared to the students of the control group (See section 6.3.1.1).

Performance on probability was measured using a 12 items multiple-choice test. The mean performance score of Niassa students was 11,55 (SD = 2,45) and the mean performance score of Nampula students was 9,21 (SD = 2,83). The overall mean score of the control group on the probability test was less than the overall mean score of the experimental group. When the treatment groups were compared, the difference was found to be statistically significant [$F(1, 159) = 23,850, p < 0,0005$]. The treatment could have been a major factor causing the differences. However, other factors may have also influenced the results.

CHAPTER SEVEN: *TCHADJI* AND MATHEMATICAL IDEAS FROM PLAYERS' AND RESEARCHER'S PERSPECTIVE: ANALYSIS AND DISCUSSION (PHASE 2)

7.1 Introduction

'Do the players really count while playing Tchadji?' This has been one of the questions that several anthropologists, mathematicians, educators amongst others have raised in dealing with games of the Mancala type for a variety of purposes. It seems that, generally, they have recognised that playing these games might involve some empirical 'mathematical' aspects, such as counting and logical thinking. In most cases, the scientists did not go so far as to analyse, describe and illustrate how this takes place (C.f. Zaslavsky, 1973a, 1989; Odeleye, 1977; Doumbia, 1989; Cheska, 1987; Doumbia & Pil, 1992; Silva, 1994, 1995; Lumpkin & Strong, 1995; Bell & Cornelius, 1988; Barta & Schaelling, 1998).

How would I come to know whether or not there are mathematical ideas in *Tchadji*-game? A need for ethnographic research emerged. Specifically, it was necessary to investigate mathematical ideas in the *Tchadji*-game among its players, i.e., the people who are in permanent physical contact with the game. These people are the best informants about issues related to *Tchadji*. Thus, in order to examine in more details the mathematical ideas embedded in the game, I had to do it among them.

For this investigation, I employed a particular different methodology inspired by ethnomathematical research. So, for example, the question *'If the players count when they play Tchadji, when do they count and how?'* (See more in section 7.2.1) seems to be still unanswered. It has been a purpose of this study to provide some evidence on this issue with regard to the *Tchadji*-game. In general, this phase of the study is more an exploratory research. I wanted to investigate the question of **'what mathematical ideas can be found in the *Tchadji*-game and in the procedures for playing it?'** In order to deal with this question it was necessary to explore more the environment of the *Tchadji*-

players. I further asked who are the *Tchadji*-players, how is *Tchadji* played and where (See section 1.4).

In conclusion, the challenge of the second phase of this study has been to investigate the mathematical ideas among the players of the *Tchadji*-game and to document them and explore their possible use in the mathematics classroom. The present chapter discusses the results of this investigation of the *Tchadji*-game. The mathematical ideas found in the game will be explicitly described.

7.2 Analysis of the *Tchadji*-matches

For this analysis, the crucial moments of each of the 57 matches were reconstructed with the help of a wooden board (See section 4.3.3.1). The moves have been replayed several times. They were studied more carefully and their principal objective and possible related player's thoughts were tentatively reconstructed. This section presents the results of this analysis in the form of ideas related to those crucial moments of *Tchadji*-matches.

7.2.1 Counting in *Tchadji*

Since the *Tchadji*-board consisted of 32 holes containing a total of 64 seeds at the start of a match and since the game involves spreading seeds around the row of holes in an anticlockwise direction (See section 5.3.2.1), it seems that **some kind of counting has to take place** when the seeds are distributed or even before. However, it has not been clear what is counted, when and how. The **holes may be counted and even the seeds to be distributed to the holes may be counted**.

The playing of *Tchadji* involves complex moves. Some moves may be classified as 'imaginable' and others 'unimaginable' with respect to their respective end results. For instance, if a move takes more than a complete round, i.e., if the process of spreading seeds covers more than 16 holes in a player's rows, then it might be difficult to predetermine its outcome.

On the other hand, a *Tchadji*-game, as played by the good players, is a very quick game. The players are able to perform more than 15 moves in about 90 seconds (See sections 7.2.6 and 7.2.7). Clearly, if mental counting takes place, then it must be performed very quickly. However, for an external observer or a learner, the structure and nature of the game necessarily imply counting. For this reason I shall analyse the counting issue in the *Tchadji*-game considering different phases of experience in playing *Tchadji*.

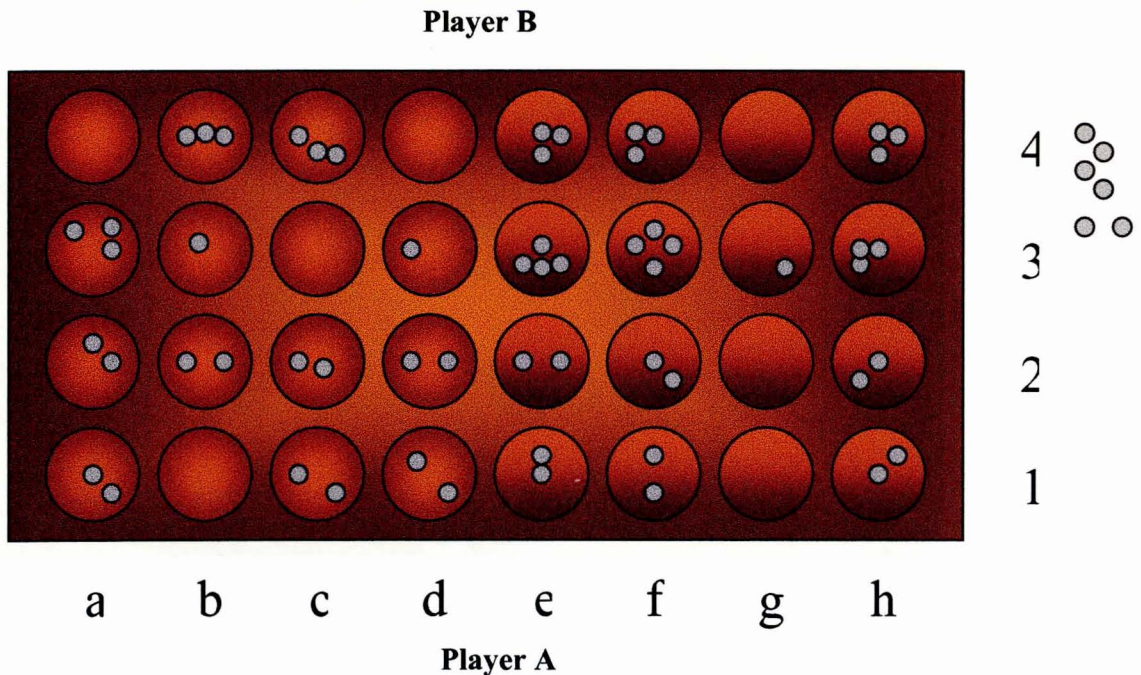


Figure 7.1: Simple counting in Regular-match 3

Simple counting takes place in several moments of the game. I will next present some moments when it occurs. In figure 7.1, a situation in the Regular-match 3 (See appendix B3) is presented after player B has played the first move. If Player A wanted, in his turn, to capture seeds, then he could simply play from h1 and end in g2. This in fact is the simplest move. With two seeds in a hole a player can advance two holes if it is not further connection with other holes ahead.

In case there are more than two seeds in a hole where a move is to be made, then several questions involving counting arise. How many seeds are in the hole? How many holes

need to be covered in order for a player to be able to capture seeds, safeguard his seeds from being captured in a next move, empty a hole, or connect with a non-empty hole in order to advance further? For example, in the following situation (See figure 7.2), player A, at his turn, can capture seeds in column h (h3 & h4), if he plays in a2. There he would be doing some counting. The 4 seeds in hole a2 only would not be enough to reach hole h2, but the 4 seeds in hole a2 with the seeds in the connecting holes d1 (1 seed) and f1 (2 seeds), i.e., with a total of 7 seeds in movement, leaving 2 empty holes, would cover the 9 holes from a2 to h2. Thus, **counting is taking place** here.

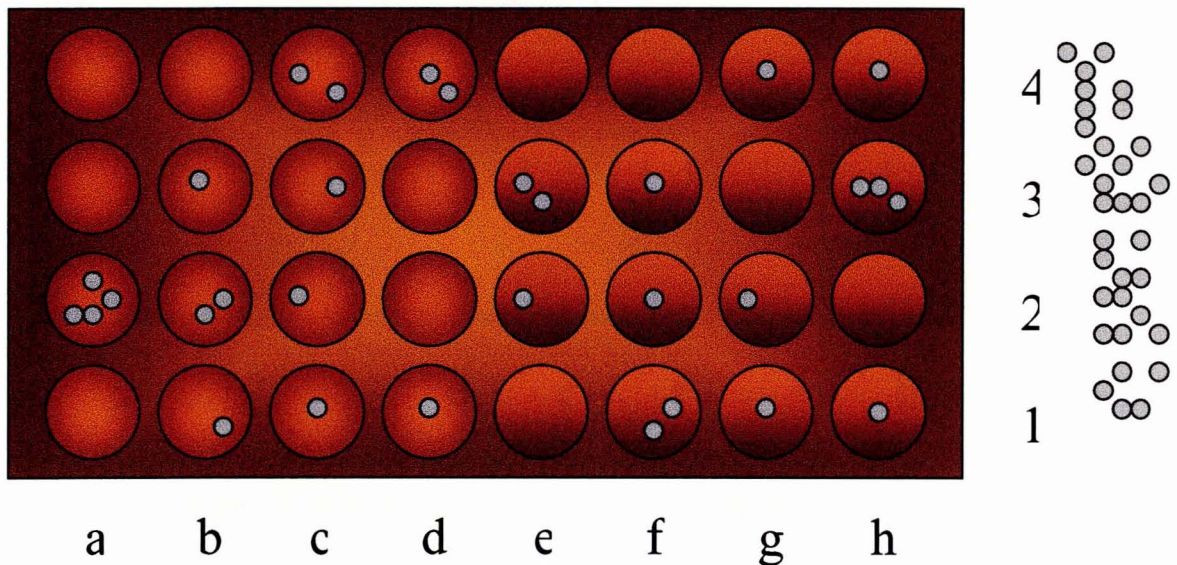


Figure 7.2: Counting in regular-match 2 (See appendix B2)

Also, a connecting operation is taking place here. The nature of moving seeds around the *Tchadji*-board, is such that this connecting operation cannot be classified solely as an addition of seeds. However, it is clear that the objective is to cover a certain number of holes in order to advance around the board. To accomplish this, certain number of seeds is needed. Thus, a kind of **addition** is taking place. If there were 9 seeds in hole a2 (See figure 7.2), it would be simple to advance to hole h2 (9 seeds cover 9 holes). However, it is rare to have this number of seeds in one hole.

For some one experienced in playing *Tchadji*, a master player for example, the operations are performed extremely quickly. The players must have memorised the game and its

structure, in such a way that facilitates the quick moving and thinking that precedes a move. In fact, *Tchadji*-board, in its structure, consists of four rows of eight holes each or, if seen differently, 8 columns of 4 holes each. The board has the form of a rectangle. Each player plays in the two rows closest to him. In this respect, each player has memorised that he has 16 holes arranged in $8 + 8$ positions.

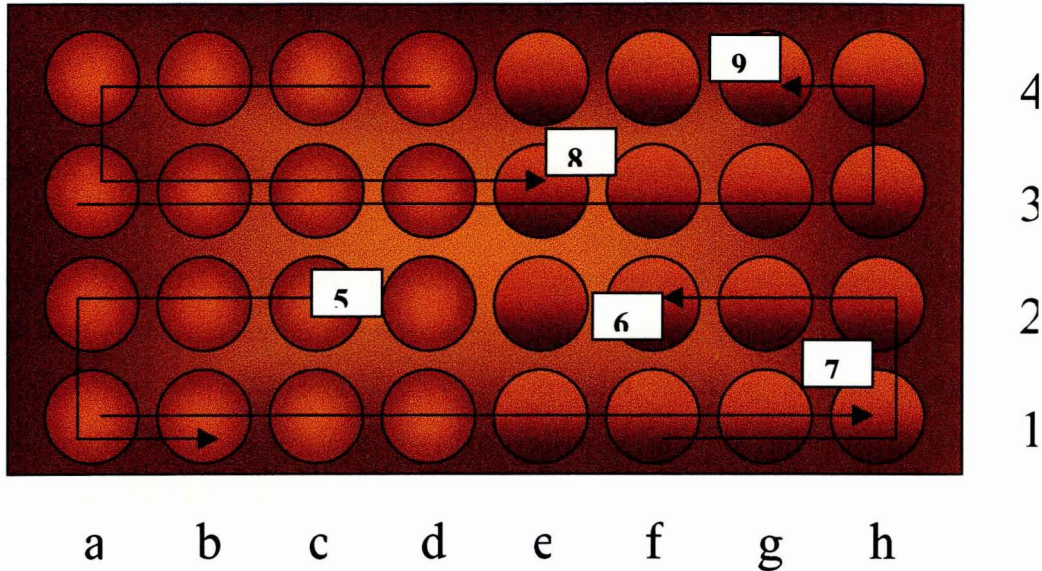


Figure 7.3: Memorised distances on the *Tchadji*-board

A player **memorises visually the positions of the *Tchadji*-board**. For example, the player no longer needs to count how many holes there are from a1 to h1 (See figure 7.3), or how many jumps, in terms of spreading seeds, he needs to make from a1 to h1. There are in fact 7 jumps. The player does not need to count these specifically. The number 7 might not even come into his mind. He might know that the effort you make to go from a1 to h1 is the same as going from b1 to h2, d2 to d1, e2 to c1 or b2 to f1, and so have other possible moves that cover certain holes (See figure 7.3).

Thus, by playing *Tchadji*, if there are enough seeds in the starting hole for the holes to be covered in moving or if the connecting holes contain sufficient seeds for the intention, the move is just performed. The counting is no longer taking place in a concrete way (See

also section 5.3.2.1.2). The fact that the *Tchadji*-board is in the form of a rectangle and not a circle may facilitate those visual memorisations.

7.2.2 The visualisation aspect

When played by master players, a match of *Tchadji* takes such a short time and the moves are performed quickly and accurately for their purpose, even the long moves (See section 7.2.4). In the process, the master players might be performing some kind of mental ‘calculations’ or, at least, as I believe, they must have developed some **visualisation abilities**, which permit them to play in this manner.

Visualisation is indeed an important issue in mathematics and in mathematics education. David Hilbert (1983, quoted in Zimmermann & Cunningham, 1991b) wrote, “...with the aid of visual imagination [*Anschauung*] we can illuminate the manifold facts to depict the geometric outline of the methods of investigation and proof ... In this manner, geometry being as many faceted as it is and being related to the most diverse branches of mathematics, we may even obtain a summarizing survey of mathematics as a whole, and a valid idea of the variety of its problems and the wealth of ideas it contains” (p.1).

In the section above, I showed that the players might have memorised the *Tchadji*-board in terms of the number of holes and the ‘distances’ between any two holes in the board, i.e., how many holes are there between any two holes (See figure 7.3), due to the structure and the form of the board, so that in certain situations direct counting no longer takes place.

Another situation that appears in *Tchadji* is related to the distribution of seeds in the holes. As I mentioned earlier, in order to advance from one position (hole) to another, the seeds must be disposed in such a way, that the connections for the holes ahead are guaranteed. Counting may certainly take place here, but the main issues on which the players concentrate, is the connection, i.e., the process that allows a move to continue.

Also, the objective behind any move in *Tchadji* is to surmise as quickly as possible where it ends and its possible outcome in terms of advancing in the board, capturing seeds or avoiding capture in any hole on the board. In this process, I believe that the players might have developed certain abilities to **recognise certain patterns** in the board, i.e., a certain distribution of seeds in the *Tchadji*-board, which allow the player to reach a certain objective. These patterns are **memorised to facilitate quick reflection**. To illustrate this, I now present some concrete examples.

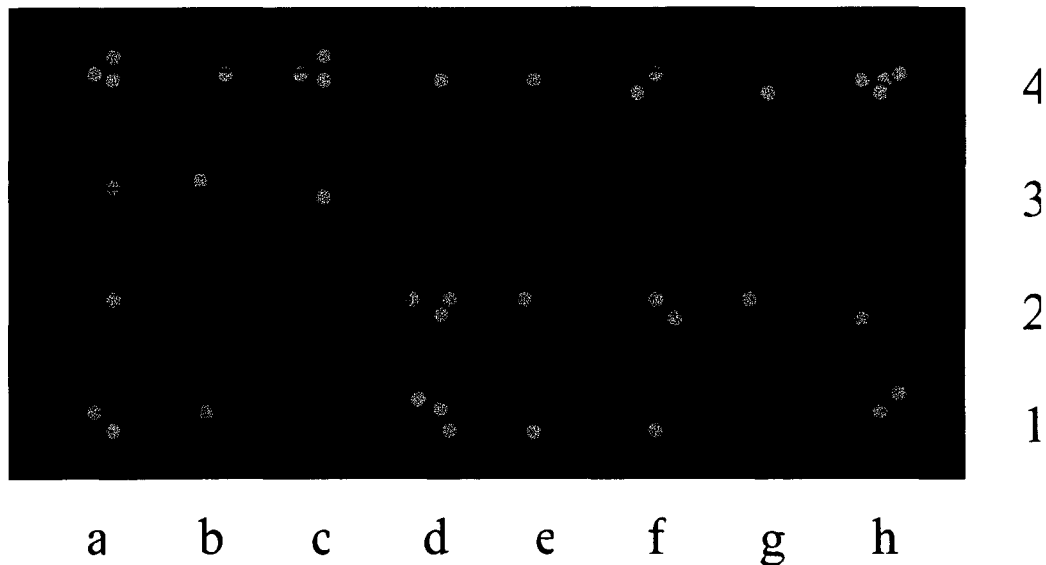


Figure 7.4: Visualisation patterns for *Tchadji*

Beginning with the most simple, like having two seeds and then one seed in the immediately following holes as in the hole a1 and b1 (See figure 7.4), the player would know that the move would advance only two holes ahead and that it does not matter how many seeds you have in the second hole (b1), the move ends in c1. The move in d1 (See figure 7.4), seen independently of other seeds in the board advances 3 holes and would end in g1. The move starting in h1 advances 4 holes ahead and would end in e2 or the move in f2 would advance 6 holes ahead, ending in a1. These patterns are memorised by the players.

Also, some long Patterns are often simplified in shorter pattern producing the same outcome. For example in figure 7.4, playing from c4 or from a4 the moves would end in e3, or playing from h4 or f4 would produce the same outcome with regard to the hole, where a player eventually wants to end.

We can imagine other patterns in the *Tchadji*-game. Having in 4 consecutive holes the following number of seeds,

3	2	0	1
---	---	---	---

or

3	2	1	1
---	---	---	---

whatever you play it produces the same outcome. Also, the issue of the distributions of seeds in producing the same outcome, is known by the players. The same would happen in the following pairs of Patterns:

4	1	2	1	1
---	---	---	---	---

or

4	0	2	0	1
---	---	---	---	---

and

3	1	3	1	1	1
---	---	---	---	---	---

or

3	0	3	1	0	1
---	---	---	---	---	---

There are other more complex situations involving more holes and implying even longer moves, like for example, the distribution¹

¹ Holes where a connection takes place are indicated in italics.

3	4	0	2	1	1	1	0	2	2	0	1	0	2	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

would advance 16 holes by starting in the first hole with 3 seeds. The number of seeds in the holes, where no connection takes place, does not matter and these are simply ignored. What is memorised is the pattern of the holes of connection.

The same would happen with the following distributions:

4	0	0	0	3	0	0	0	4	0	1	3	0	2	0	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

and

2	0	1	0	1	0	1	0	1	0	4	0	1	4	1	0
---	---	---	---	---	---	---	---	---	---	---	---	---	---	---	---

Sometimes we come across to moves that take more than a complete round (in the two rows of a player). Even so, it seems that the players are able to visualise, i.e., they know in advance where the move will end. These so called “long moves” are discussed in the next section.

7.2.3 The long move

The *Tchadji*-game involves complex moves. The moves are the principal characteristic of the game. A player might want to reach a certain position in the board, to capture seeds, to empty any endangered hole, to prepare space for a next move, and so on. All these objectives will depend on the situation encountered at the time the player wants to attain them. Thus, the skill of quickly knowing where a move ends is very important in the playing of this game.

Imagine that a player possesses 16 holes in a circle and he can play from whenever it is possible to play and stop whenever is possible. The majority of moves are short moves in terms of how far they can advance, i.e., each of these moves does not cover the whole 16 holes. Other moves are longer, i.e., such moves can cover more than 16 holes (See more in section 7.2.7.1).

A short move is much easier to predict, i.e., is easy to determine where it ends, since it only affects the existing number of seeds in the holes. In other words it touches only the seeds that the players can see in the board. For example in figure 7.5, if the player B would start play from h3 or g4, he would know immediately where these moves would stop. Short moves can also advance up to 16 holes. For example, if in the connection made you find only one seed in the 14th hole in the line, then it is clear that you will stop in the 16th hole, exactly where you started the move and it might be empty. For such moves, it is easy to determine where they stop.

For a long move it is quite 'difficult' to determine where it will stop. If a move goes beyond a complete round, i.e., it covers the whole 16 holes and goes further, it means that the connections to make after a round are not so easy to memorise since the initial positions on the board are affected and so changed. Some moves can advance more than two complete rounds, i.e., more than 32 holes ahead. This means that the initial distribution of the seeds in the holes is changed twice. So, if it is not easy to memorise the situation is the first change (in the 2nd round), it is much more 'difficult' to determine the situation after the second change (in the 3rd round).

Long moves do not happen very often and an analysis of some situations where they occurred showed that some players possess the **skills to determine exactly where a move stops even if it takes more than two rounds**. The following examples will illustrate this aspect.

In the extra-match 2 (See appendix B29), player A, in his 5th move (figures 7.5a & 7.5b), played in e2 and after more than 3 rounds landed in a2. He started in hole e2 with only two seeds and connected to the c2 with 5 seeds $(1+5)^2$, then to the hole d1 with one seed $(1+1)$ and so on. The last connection was made at the hole g2 with 5 seeds $(1+5)$ in the 3rd round and the move ended in the hole a2. In terms of calculation, this is what happened:

² Number 1 indicates the remaining seed in the hand. Number 5, in this case, indicates the seeds in the hole.

$$\begin{aligned}
 &2 + (1+5) + (1+1) + (1+1) + (1+2) + \\
 &(1+1) + (1+1) + (1+1) + (1+1) + (1+5) + \\
 &(1+2) + (1+1) + (1+1) + (1+7) + (1+1) + (1+5) = 52 \text{ holes} \\
 &52 \text{ holes} = 16 + 16 + 16 + 4 \\
 &3 \text{ rounds and 4 holes ahead}
 \end{aligned}$$

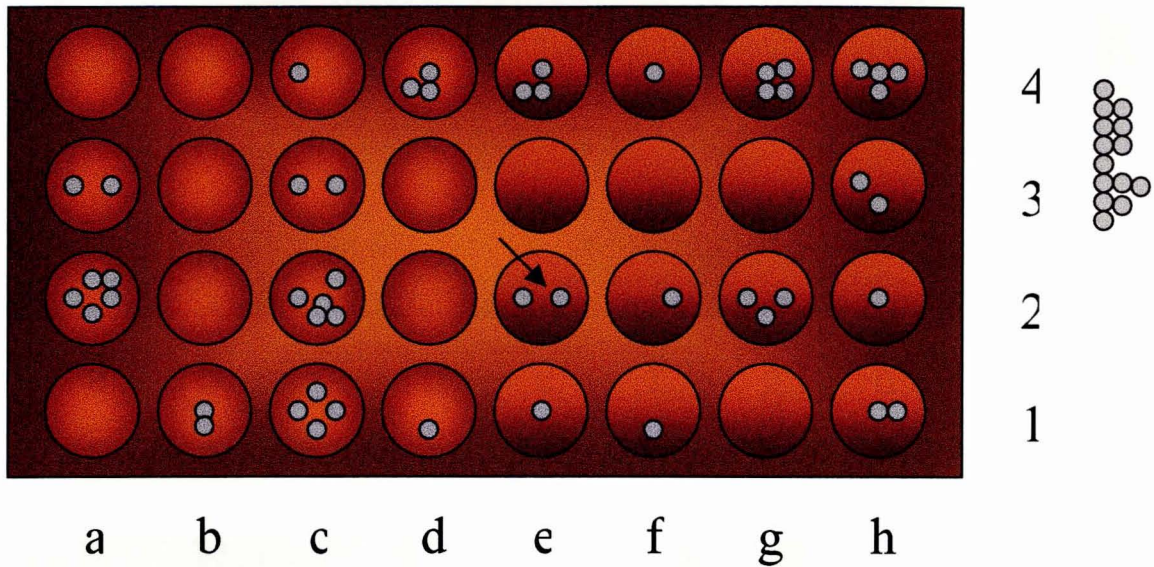


Figure 7.5a: Long move in the 5th move of player A in the extra-match 2

The question I pose here is the following: Could player A have predicted the outcome of this move? To answer this question, I tried to analyse the intention of the move and to see whether it was attained or not. In fact, there are players who demonstrated being able to do it. A question I did not answer was, ‘How do they do it?’ This needs further investigation. This investigation did not go so far to examine these aspects (See more in section 9.4.3).

For example, in his 11th move (See figure 7.6), player A might know that his opponent would land in g3 and that to avoid being beaten he has to maintain hole g2 empty. He has 3 possibilities for playing, c1, e1 and g1. He most probably will have quickly analysed all the three possibilities and then decided to take g1. In fact, c1 would land in c1 again and

would move a seed to g2 and e1 would land exactly in g2. The option g1 moved 26 holes ahead and in fact left g2 empty [$2 + (1+1) + (1+1) + (1+1) + (1+1) + (1+1) + (1+6) + (1+1) + (1+1) + (1+2) = 16 + 10$ holes ahead].

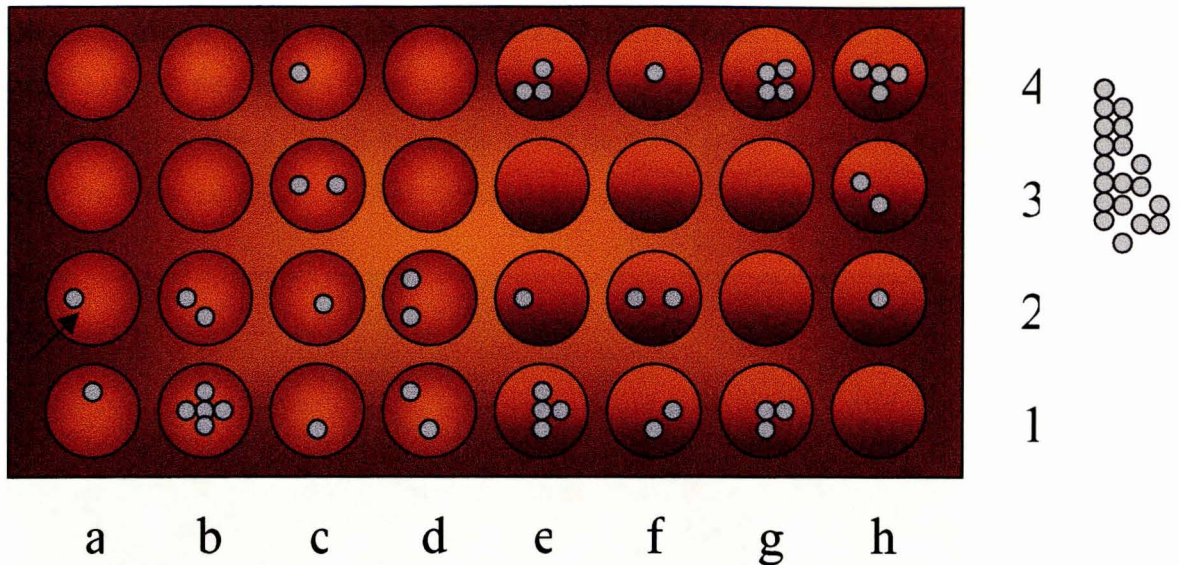


Figure 7.5b: A situation after the 5th move by player A in the extra-match 2

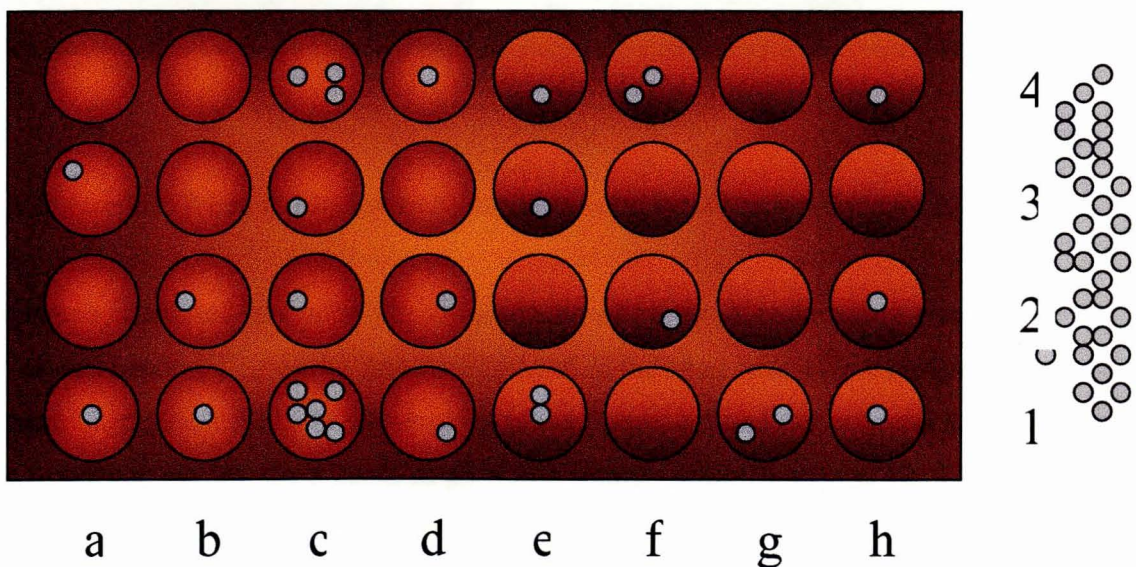


Figure 7.6: Taking best option to play in the regular-match 12

So, it was difficult to predict where it would stop since the move continued for more than one round. Did the player know beforehand of this outcome? It could be that since

the other two possibilities were not favourable to him he would have decided to take this, a risk in fact, or?

Sometimes things do not work as the intention. A situation (See figure 7.7) happened after player B played in g3 and after more than 2 rounds landed in f4. Here, player B apparently had not determined in which hole the move would stop, since his objective was not clear. If it was to avoid his seeds being captured it did not work, because the next move by A started in g2 to e2 and captured the seeds in e3, e4 and d4.

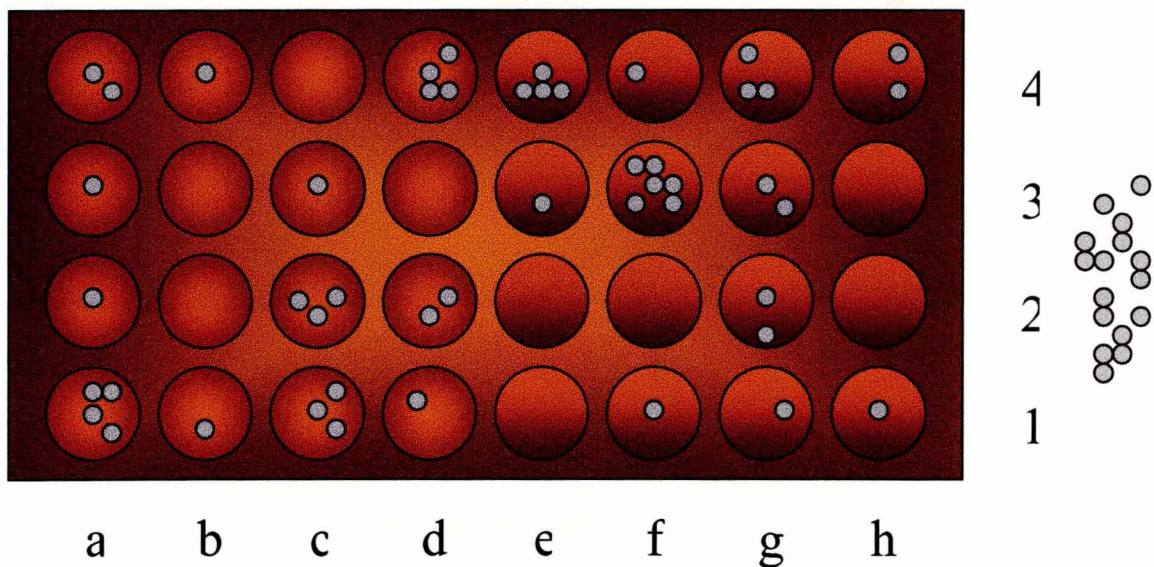


Figure 7.7: When it does not work (situation after the 4th move of player B)

In the match illustrated in figure 7.8, the positions of player A make many consecutive round moves happen, i.e., moves that start and end in the same hole after exactly a round (See more in appendix B30). In the situation in the figure, a round move would happen if player A plays in g2. So, the 5 seeds in g2 would connect with the seed in b2, 4 seeds in a1, 1 seed in f1 and finally 1 seed in h1. So, the move would advance exactly 16 holes. The same could happens by playing d1, i.e., $4 + 2 + 6 + 2 + 2 = 16$ holes.

In fact, in a round move we very rarely have 16 seeds involved. This would be a situation in which we would have 16 seeds in one hole. So, taking them in a move would advance

exactly 16 holes ahead and would stop exactly where it started. In most cases there are less than 16 seeds involved and the advance is, even so, 16 holes. What happens is that, some holes will remain empty. By taking seeds of a hole or connecting with seeds of another hole in the playing process, these holes remain empty. The **sum³ of the number of seeds** involved and the number of empty holes has to be 16.

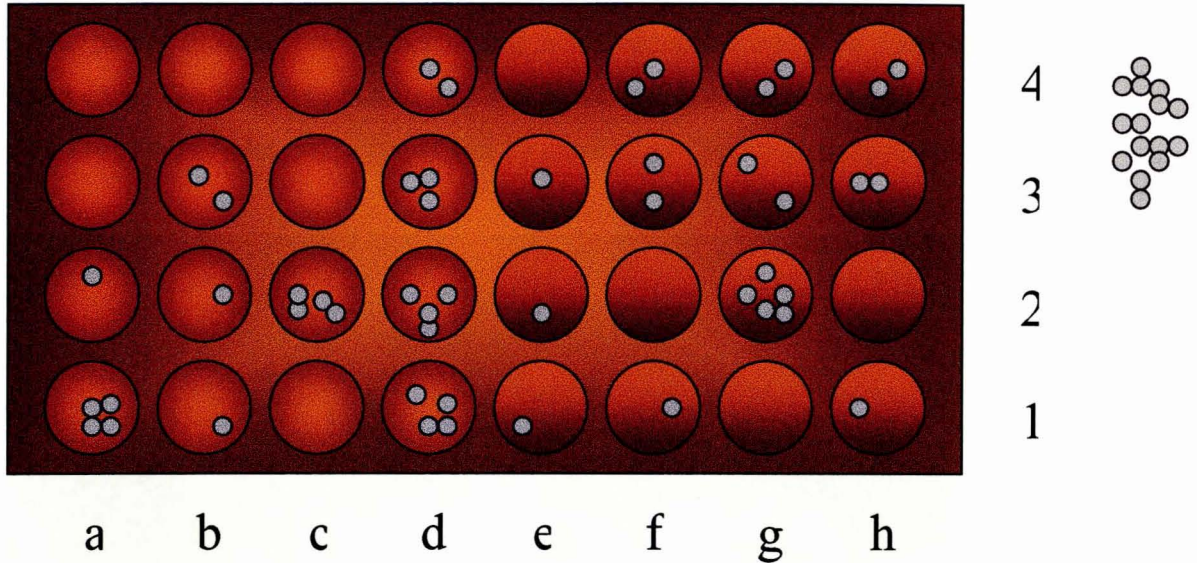


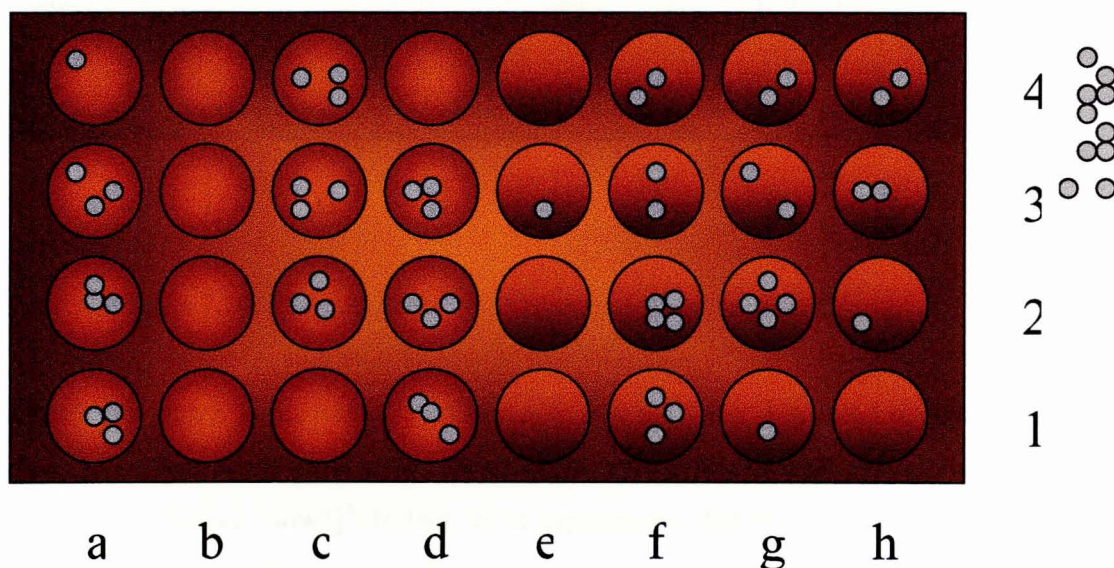
Figure 7.8: Consecutive round moves after the 2nd move of player A in the 1st match of the first tournament.

Now, I can ask, ‘**what is the maximum and minimum number of seeds involved?**’ and consequently ‘**what is the maximum and minimum number of empty holes that can be obtained?**’ As I referred earlier, we can have 16 seeds in one hole and playing there a move would advance 16 holes and would leave no empty holes. So, the maximum number of seeds involved for a round move is 16 and the minimum number of empty holes that can be obtained is zero. The more connections we have for a round move, the more empty holes are obtained. In fact, each connection corresponds to an empty hole. The less distance between two connections in the move we have, the less seeds involved

³ A further mathematical analysis of this situation, in terms of generalisation, revealed that, a move will advance z holes ahead, from which $z = x + y$ where x is the number of seeds involved and y is the number of empty holes.

would be. The minimum number of seeds to be directly involved is 9 and, in this case, 7 empty holes would be produced.

Another example happened in regular-match 13, where player A, who started the match, in his second move (See figure 7.9), played from f1 and landed in a2 exactly 26 holes ahead, capturing the seeds in a3, a4 and c4. This move happens very often in the first move. The player realises that the connecting holes are untouched after their opponent has played once. The following connection was made: $3 + (1+1) + (1+4) + (1+3) + (1+3) + (1+1) + (1+5) = 26$ holes, i.e., a round and 10 more holes (See more in appendix B13).



taking b4, player B could also land in c3, in an easier way, but he preferred the longest. Could he really **visualise** that he would stop there? I think so, since his intention was clear.

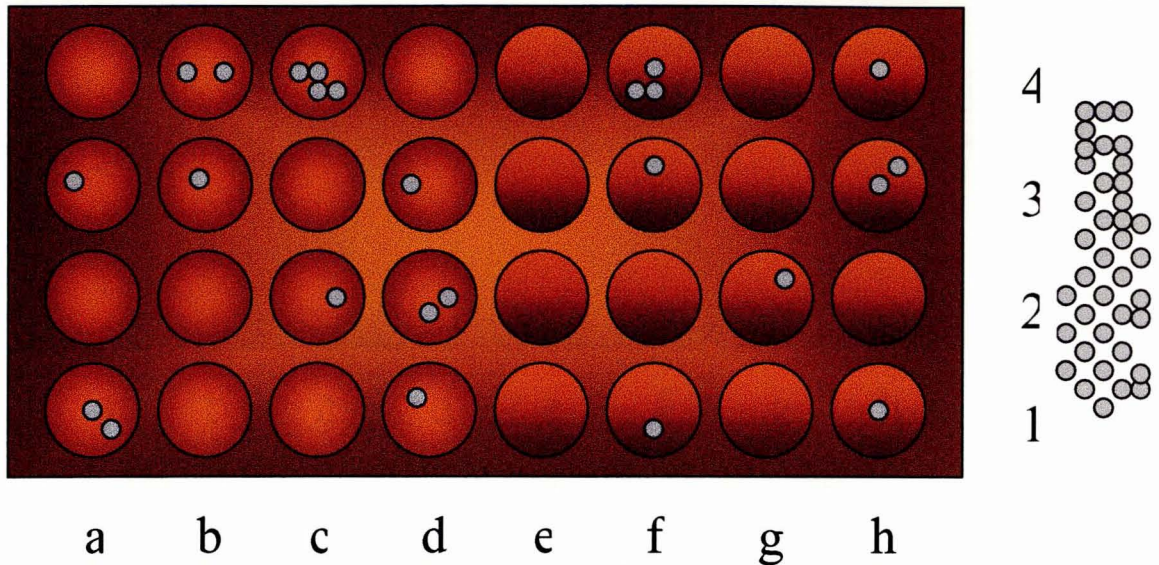


Figure 7.10: Long move in the 9th move of the regular-match 18

In the interview-match 2, the next move by player B at the point illustrated in figure 7.11, he played in e4 and while playing he commented that he would beat his opponent and that he knew that before [how?]⁴. In fact, what happened is that he was able to capture e2 and f1 by stooping in e3 after more than 2 rounds in his rows, exactly after 2 rounds and 9 holes. Certainly, the place where this move would land was not clear at the start since by making more than a round, the distribution of the seeds on the board changed. By 2 rounds it changed even more. Even so, he was able to **determine where he would land** (See more in appendix B27).

$$\begin{aligned}
 &4 + (1+1) + (1+1) + (1+4) + (1+5) + (1+1) + \\
 &(1+1) + (1+1) + (1+1) + (1+4) + (1+1) + (1+6) = 41 \text{ holes} \\
 &41 \text{ holes} = 16 + 16 + 9 \\
 &2 \text{ rounds and } 9 \text{ holes in a single move}
 \end{aligned}$$

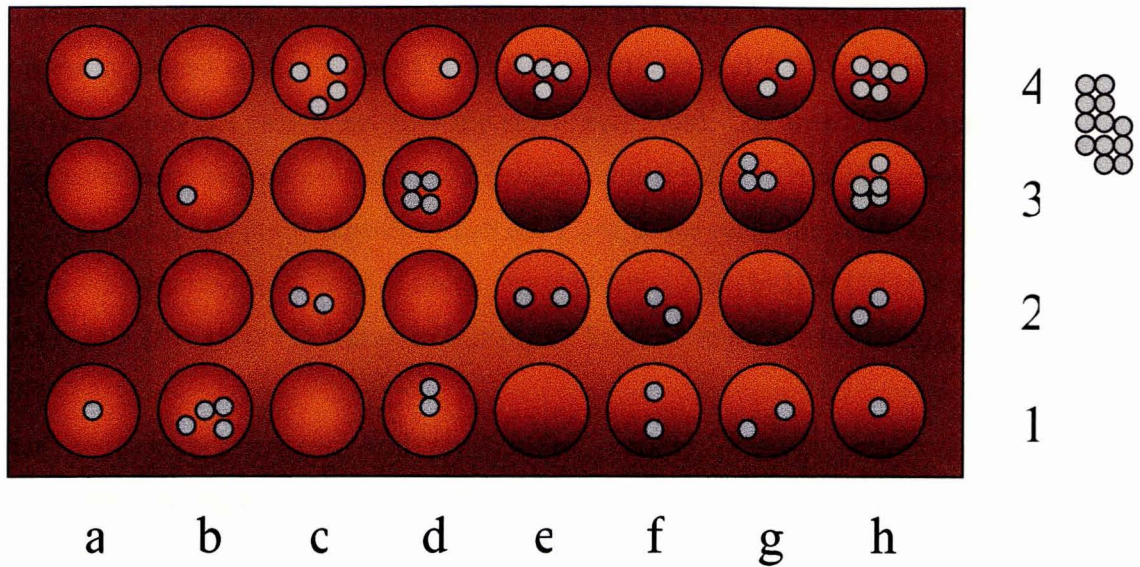


Figure 7.11: A long predicted move in the interview-match 2

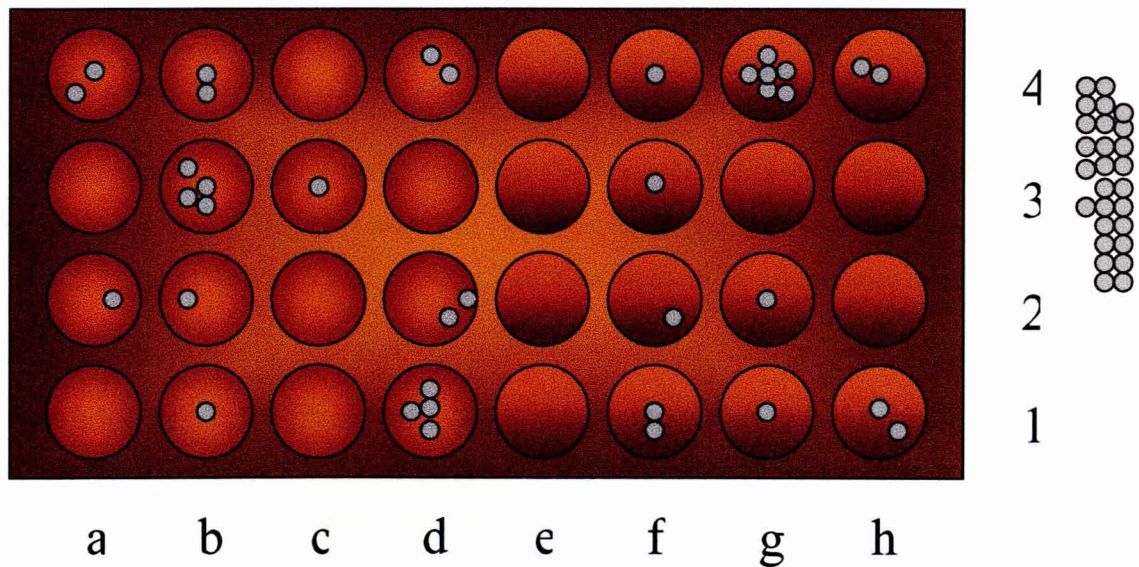


Figure 7.12: Another long move in interview-match 2

The last example of a long play happened again in interview-match 2. It was after player B has played in e4 and landed in c3 (See figure 7.12). After this he commented, *‘Now I will catch you in both sides.’* Player A responded, *‘You will beat me here (pointing to f3)’*

⁴ Here, there is a certain advantage for myself, when it comes to analyse the match. The players communicate

and I will eat you after that here (pointing again to f3)'. At this point I asked him, '*How do you know it?*' He said, '*I made calculations.*' In fact, what happened next, was that player A really played in f1 and ended in f2 after one round and 5 holes, eating in f3, f4 and b3. Here, it was not easy to see how the move would end. The hole f2 was not empty as can be seen here. The player must have recognised that the hole would become empty after a round, so that he could stop there and eat. Here, if we count the seeds involved we will see that by starting in f1 with 2 seeds, we will have $2 + (1+2) + (1+1) + (1+2) + (1+1) + (1+1) + (1+4) + (1+1)$ and so advancing **21 holes** from start to the landing.

The skills to **quickly recognise and predict** the outcome for long complex moves is related to **complex logical structured thinking** process, which the players of *Tchadji* experience while playing the game. The connections made from the players while performing a move imply necessarily **mental counting** and a **kind of addition**. These are mathematical aspects.

7.2.4 The opening play

In a *Tchadji*-match, the player who plays first (first move) will stop 5 holes ahead after having made a round in his two rows. So, for example, if player A (See sections 5.4.2.1, 6.2.1 & 8.2.1), starts in h2 he will stop in c2 or if he starts in g1 (frequently used) he will stop in e2.

There are holes in which a player starts and is able to capture seeds of his opponent (i.e., due to rule 2b, See section 5.4.2.1) if he stops in his inner row. There are other holes from which a player starts and makes no captures (i.e., if the player stops in the outer row according to rule 2a, See section 5.4.2.1). In fact, there are 8 starting positions (See figure 8.1) in which a player captures seeds and another 8 in which no captures are made. For the positions of player A, the 8 starting positions for capture are 5 in the outer row (d1, e1, f1, g1 and h1) and 3 in the inner row (f2, g2 and h2). For no captures there are 3 in the outer row (a1, b1 and c1) and 5 in the inner row (a2, b2, c2, d2 and e2). For no captures there are 3 in the outer row and 5 in the inner row (Cf. Section 8.2.1).

their intention and that it was a space for dialogue make it easier to understand.

In the group interview with *Tchadji*-players, I was interested to know about how they start the match, i.e., ways of playing by the player who plays first in a match. So, I asked the following question: “*Now, about the starting move? There are many ways of starting. Which of them do you classify as the best?*”

One player answered as follows: “*g1 [or b4] is certainly the best, because many people know and do it so.*” In fact, most matches recorded have started at those holes. I insisted on this issue by commenting as follow: “*I am asking you so, because I want to know it better. Look, imagine I am a player. If everybody knows my way of playing, I will never succeed in a match. The best thing to do is, if I am playing against Mr. Chomar, for example, I will have to do something that he doesn’t know, so that I can try to win, or?*”

One player answered as follow: “*Yes, if you start here [pointing to g1], you will think that you will beat Mr. Chomar, but he will make other tricks here with this start to change the situation.*” A second player said: “*Another way of starting is this [his start was in f2 and ending in a2]. Most people do not know this way of starting. We all here know it.*”

A third player concluded that, “*Every strategy of playing this game is determined by the way the match is started. Each playing strategy has its secret in the way of ‘ossuwaree’* (See section 5.4.2.1.1). *The way of ‘ossuwaree’ will determine my strategy.*” Indeed, the player starting has 14 possible holes for *Ossuary*, and so, according to the players, 14 different strategies.

For adding a comment another player said, “*The value of ‘ossuwaree’ goes down from left to the right. If you do ossuwaree’ in the inner row in your start you will lose the match in most cases.*” The final comment of the players was the following: “*Crucial is it that, by playing I know that my opponent will either beat me or run away [avoid to be beaten].*”

Following the first move by Player B in interview-match 2 (See figure 7.13, I asked the players, *'Here, you always know where to start in order to be able to capture seeds, isn't it?'* They answered, *'Yes, yes, yes!'* I further asked, *'Are there people who prefer to start by not capturing any seeds?'* The answer was, *'Yes, there are people who learned so not to capture seeds in the first move.'*

I insisted as follow: *'Do you always have to capture seeds when starting a match?'* they answered, *'No, no!'* I was interested to know more and so I insisted more as follow: *'Is it a good move to eat at the start of a match?'* The answer was, *'Yes, it is a good move, but there are people, depending on their habits, who start not by eating and even so they win.'* After this conversation, A seems to have been influenced and decided to play 3 times to avoid capturing seeds of his opponent.

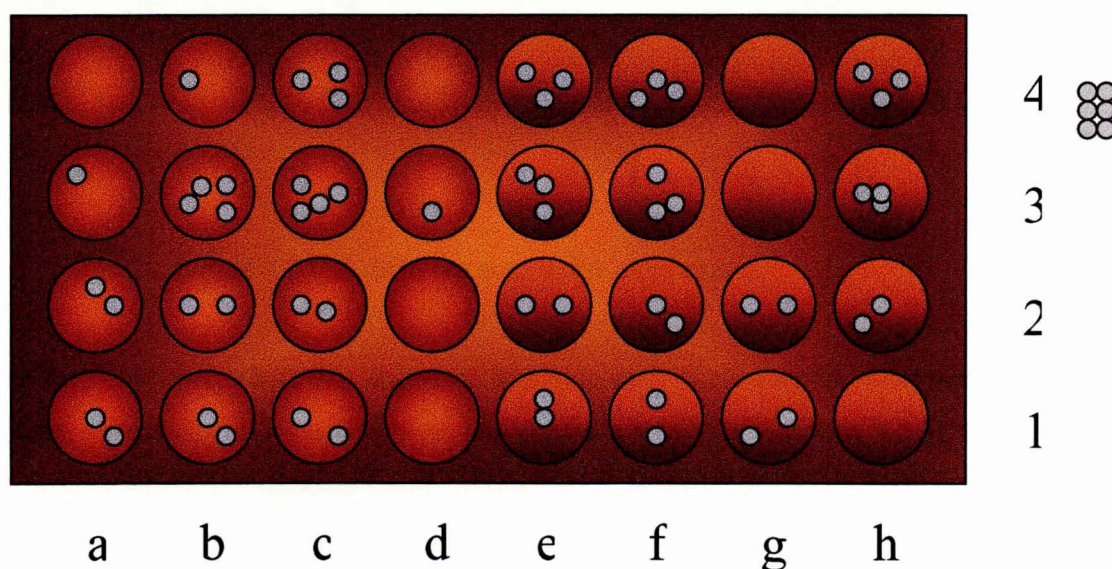


Figure 7.13: Captures at the starting move of interview-match 2

At the interview-match 1, player B started in b4 and landed in d3, capturing the seeds in d2, d1 and in h1. After this move I asked: *'It seems that this way of starting is quite popular here in this zone of Mutacani⁵?'* They answered, *'Yes!'* In fact, from the 49 matches recorded, 24 were started in g1 and 22 were started from the corresponding

position b4. Only three matches were started differently. These matches were the third match from the second tournament (a4), the third regular-match (b3) and the second extra-match (g2, see figure 7.14). Another aspect to consider here is for a player not to capture seeds at end of his first move. It is rare that in the first move a player does not capture any seed.

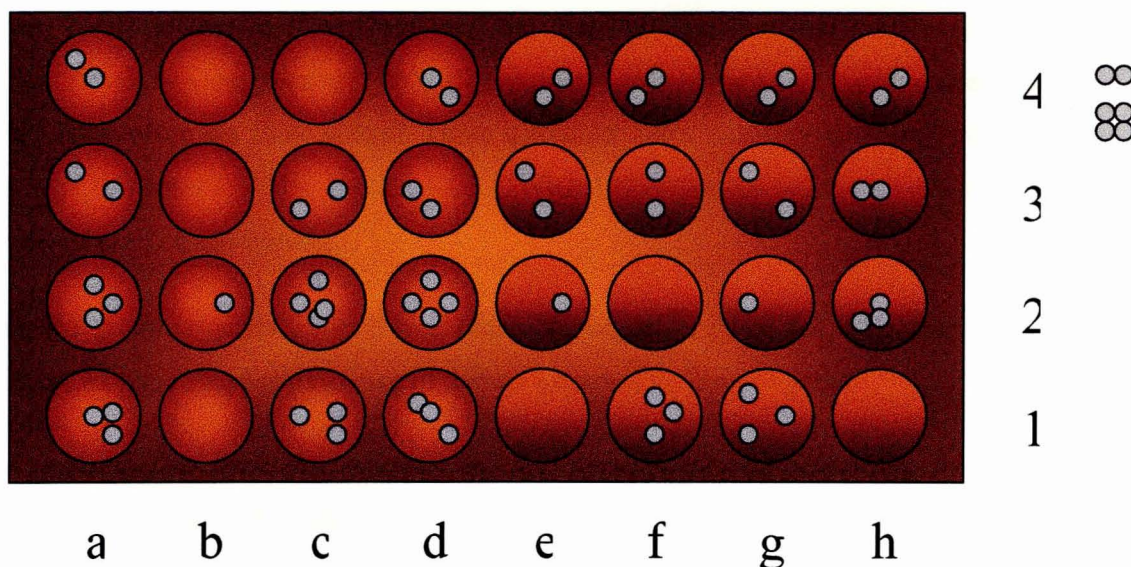


Figure 7.14: A different start in the extra-match 2

A player may calculate where his opponent will stop and, consequently, what seeds he will capture, sometimes after his opponent had started a move (See section 7.2.3), but he can not predict where his opponent will '*ossuwaree*.' In order for a player to plan his next move it is important to know about this. The place of '*ossuwaree*' is not determined by the move itself. It depends on the player and on his strategy or tactic. This makes the *Tchadji*-game more unpredictable and complex. The players are under constant pressure and are constantly forced to think in trying to logically predict the intention of their opponents.

⁵ '*Mutacani*' is the zone of the Island, where most of the players live and is near the place where the fieldwork took place.

7.2.5 The draw in the *Tchadji*-game

The *Tchadji*-game usually involves a considerable number of moves. The players generally concentrate on capturing their opponent's seeds and trying to annihilate their opponent. Thus, with this intention in mind it is difficult to think that it will not be reached by either of the players. If this happened, it would mean a draw in the game. The question I was interested to explore with the *Tchadji*-players was the following: **Is a draw in *Tchadji*-game possible? Which situations would be considered a draw in a match? How often do they occur?** These questions were explored by myself earlier in phase 1 of this research (See section 6.2.1). What I now wanted was to have a sense of the ideas of the *Tchadji*-players.

During the group interview, which took place on August 1, 1999, the following dialogue was recorded regarding the existence of a draw situation in *Tchadji*-game.

“Now, yesterday or even this morning we had commented that sometimes by playing Tchadji a draw occurs. What does a draw in Tchadji look like?”

A1: *“It is rare to happen. It happens very rarely. We play Tchadji for a long time, many days and many months, and then there comes a day where it happens.”*

A2: *“It is when no one wins or when the match does not end. It is not something we planned [Meaning: It happens by chance!].”*

A3: *“It is so, when you are trying to avoid to be beaten by your opponent, and so the game goes and goes until it never ends.”*

“Does it really happen?”

A4: *“Yes, it happens. If you have one by one⁶ by each player, it can really happen.”*

“No [I contested], if you have one by one by each player the moves will have to meet somewhere.”

⁶ Singled situation, i.e., there are only single seeds in the holes (This situation is called *Ompwechá*, which literally means ‘divide into pieces’; see section 5.4.2.1.1)

In fact, they have to meet. There are 8 holes in the inner row in which the seeds can meet. If the seeds are in two symmetrical positions, for example a1 and h4 or a2 and h3, they will meet after an exact number of moves.

Also, in a practical situation during a match recorded, the regular match 9 (See appendix B9), there was a comment by somebody in the audience that indicates that, the existence of this situation is acknowledged by the players.

After the 15th move by player B in regular match 9 and after player A has played in f2 ending in d2 in his 15th move, while player B was reflecting on how to perform his 16th move, there was a bit of concern by the audience questioning whether or not player B would be able by that time to move the seed in a3 (See figure 7.15), which was in danger of being taken, and to avoid it, since it became clear that his opponent would then in his next move necessarily end in a2.

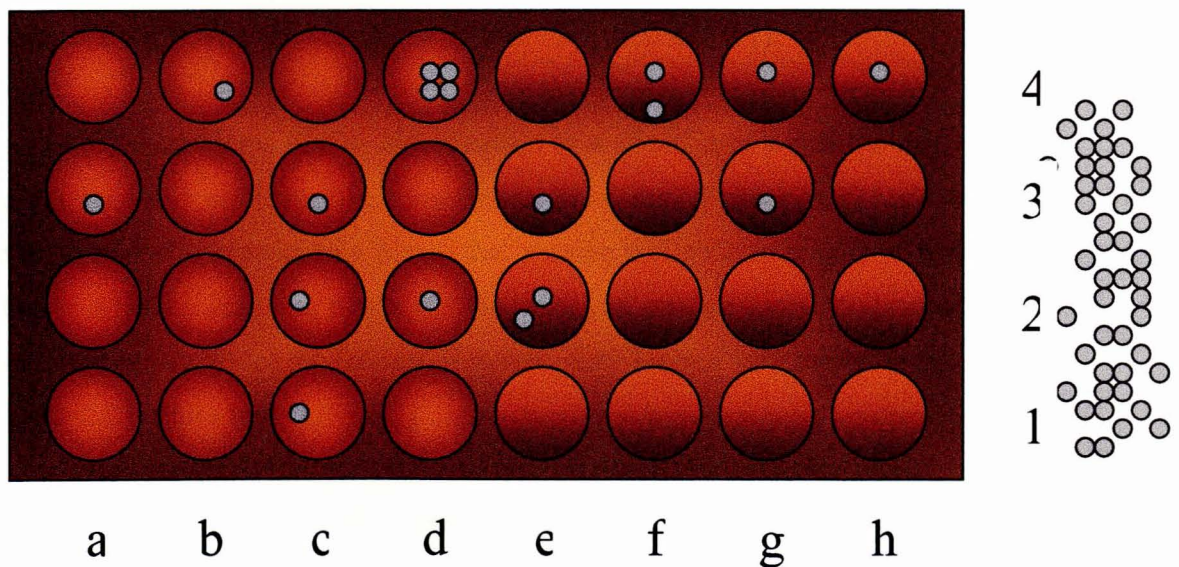


Figure 7.15: Suspecting a draw in the regular match 9 after the 15th move by player B

C1: *"Oh, so now it will not be possible to move it."*

C2: *"Unless he never stops."*

These comments revealed a bit of uncertainty about what really would happen, in particular, if the move goes beyond one complete round (see section 6.2.1) and, at the same time, it makes clear that the situation of a move that never ends really does happen. The move by player B ended again in d4 after exactly three rounds. It was really not possible to empty the hole a3.

Generally, the players recognised the existence of a draw in a match. They also recognised that a draw happens very rarely. In the players' comments, the **idea of randomness** can clearly be recognised (Cf. Section 8.2.2).

7.2.5.1 The infinite play as example of a draw

In the first phase of this study, when discussing the possible outcomes of the game, I had indicated that there are **three possible outcomes to a match**, which are **not equally likely** to happen. It can happen that player A wins the match or player B or it can happen that neither A nor B wins the match, i.e., a draw (See section 6.2.1).

A draw in a *Tchadji*-game can be characterised as a situation in which the match can no longer continue and, consequently, cannot come to an end. A match in *Tchadji* would not come to an end if it appears to a player that a move will go indefinitely, or if there is a situation in which neither one player nor the other can capture seeds. Both situations are likely to happen in a match of *Tchadji*. However, they happen very rarely. In this section, I deal with the **infinite move**.

My analysis of the regular-match 4 (see appendix B4) indicated that a situation could have occurred in which a move would never end. Since in *Tchadji*-game there are always various playing options for each player, this situation could really have happened. Player A, on his turn after certain moves, would have two possibilities for playing, from h2 or f2 (See figures 7.16a, 7.16b & 7.16c). By playing in f2 the move would never end, so it would be an infinite move.

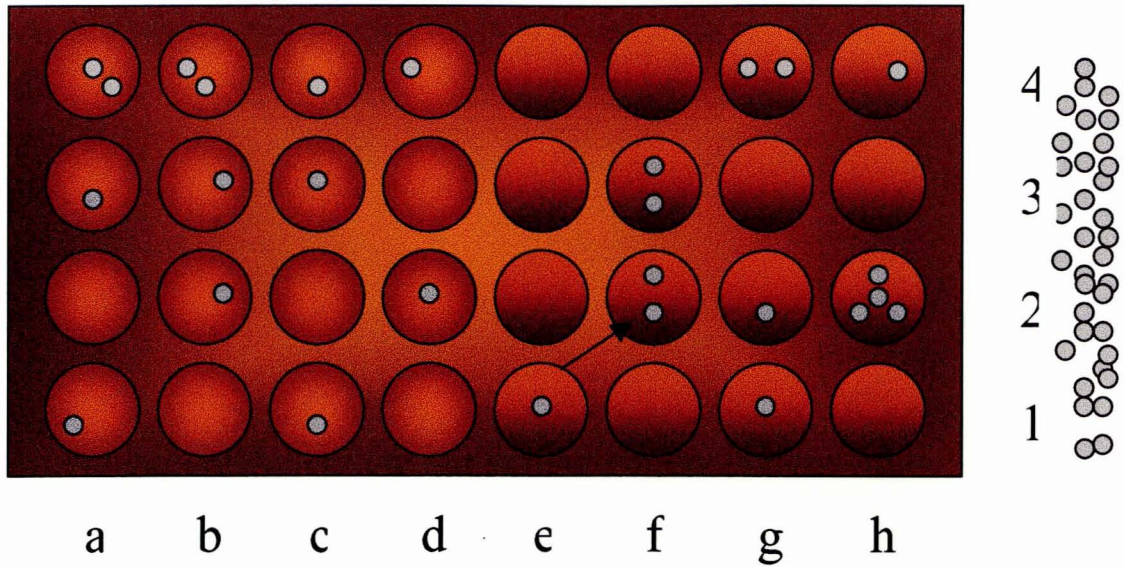


Figure 7.16a: If player A, at his turn, plays in f2, the move will never end

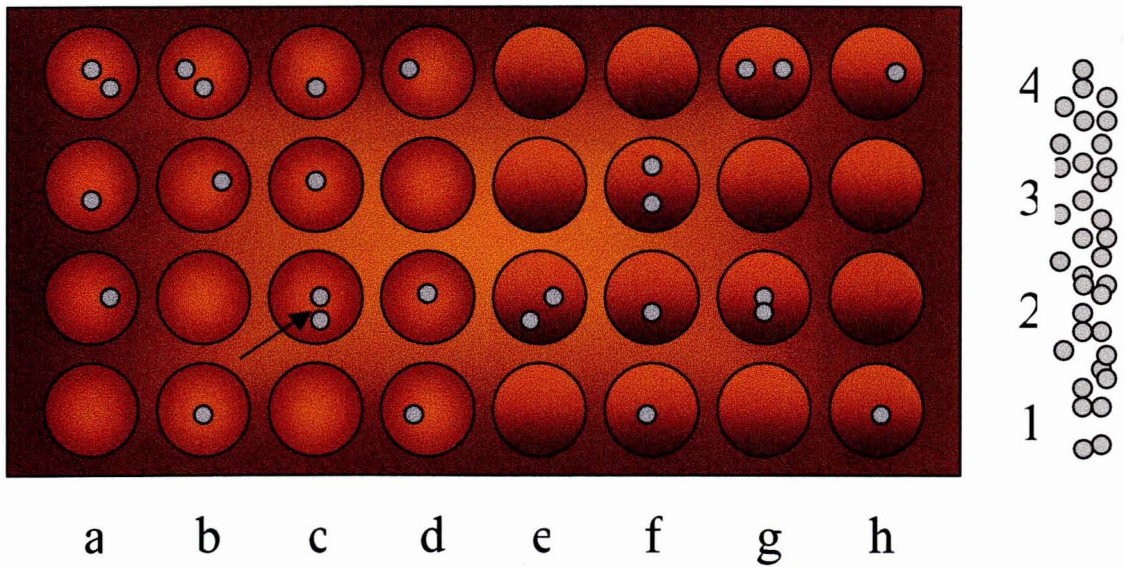


Figure 7.16b: The endless move in its second round with connection in c2

My further analysis (See figure 7.17) proved that it is really an infinite move. In fact, in the figure below, the 16 holes by player A are represented, in which the hole with 4 seeds is h2 and the one with 2 seeds is the f2. The movement would, as usual, be counter clockwise.

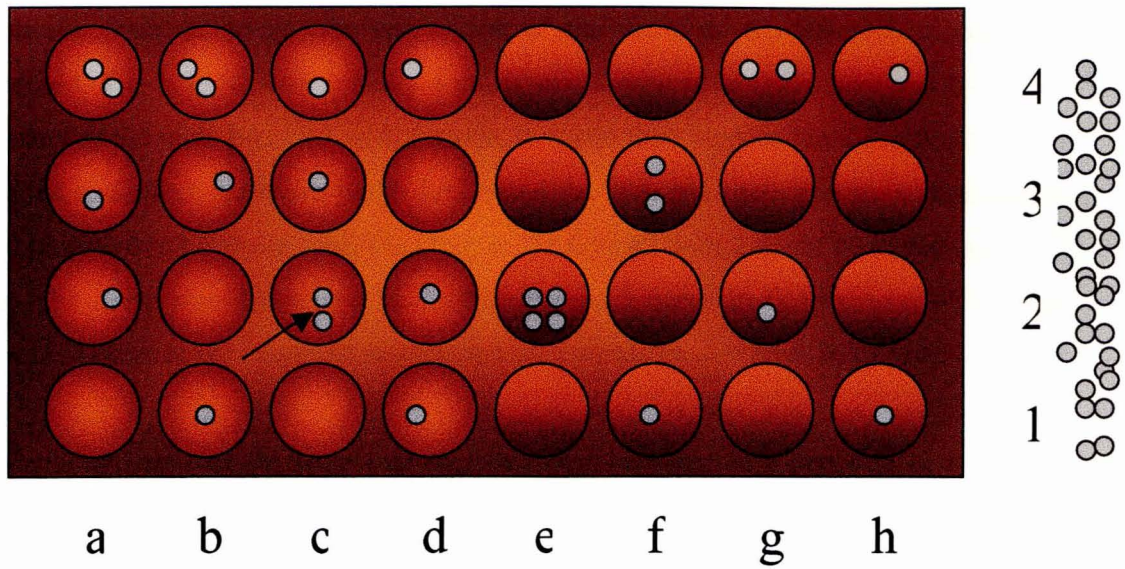


Figure 7.16c: The endless move in its 4th round with connection in c2 (Compare to 7.16a)

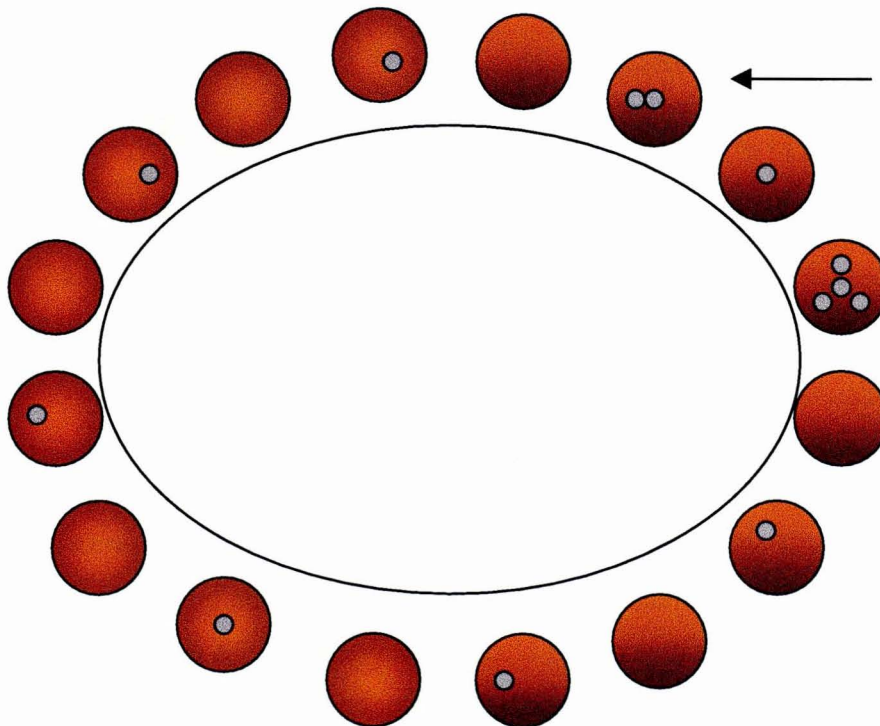


Figure 7.17: A representation of the 16 holes of the inner and outer row of player A of the regular match 4

By starting in f2 and because of the distribution of the seeds, 1 by 1, with an empty hole in between from d2 to g1, the connection of the move to the hole h2 (4 seeds) is guaranteed. The hole h2 with initially 4 seeds would then have one added and become 5 seeds during the play. From h2 with 5 seeds, it would then be possible to reach c2, which would, as a result of the move in the first round, be with one seed, and so the connection would be possible for the second round up to the hole g2 with 2 seeds be added one, so 3 seeds. The move would continue by distributing the 3 seeds in f2 (previous with 3 and after with 4) than in the empty e2 and the last seed would be deposited in d2 with 1 seed.

The initial situation would appear again, but this time with the 4 seeds in f2 and the 2 seeds in d2 (See figure 7.16c), so exactly 2 holes ahead. If then the move continues, after another three rounds the situation would appear again two holes ahead (See also table 7.1). This means that the move would never end. In other words, they would always be a connection or the last seed in the hand would never land in an empty hole.

H2	g2	f2	e2	d2	c2	b2	a2	a1	b1	c1	c1	e1	f1	g1	h1
4	1	2*		1		1		1		1		1		1	
	2	1	2	1	2*		1		1		1		1		1
1		2	3		1	2*		1		1		1		1	
	1		4	1	2*		1		1		1		1		1

Table 7.1: The distribution of seeds in the holes from h2. The (*) indicates the holes where the connection is made after 1st, 2nd, 3rd and so on. Here the situation in the 4th line is equivalent to that in the 1st.

The second example comes from the regular match 9 (see appendix B9). After some regularities were observed in preceding moves in terms of moves that take one round. From the 14th move by player B, I analysed what could have happened, if player B had played differently then he did. The match would have continued as follow (hypothetical moves). Playing in d4, instead of g4, player B would also make 3 rounds in his board and would land in d4 again. The situation in figure 7.18 would appear.

Assume player B, in his 15th hypothetical move, played in c4 to a4, and after that from a4 to a4 (a round move), in the 16th move. Then two different situations could emerge:

1. If then player B had played successively from b4 to d3 and from d4 to b3 respectively in his 17th and 18th moves, if untouched, playing then from a3 (19th move), the move would never end (See figure 7.18).

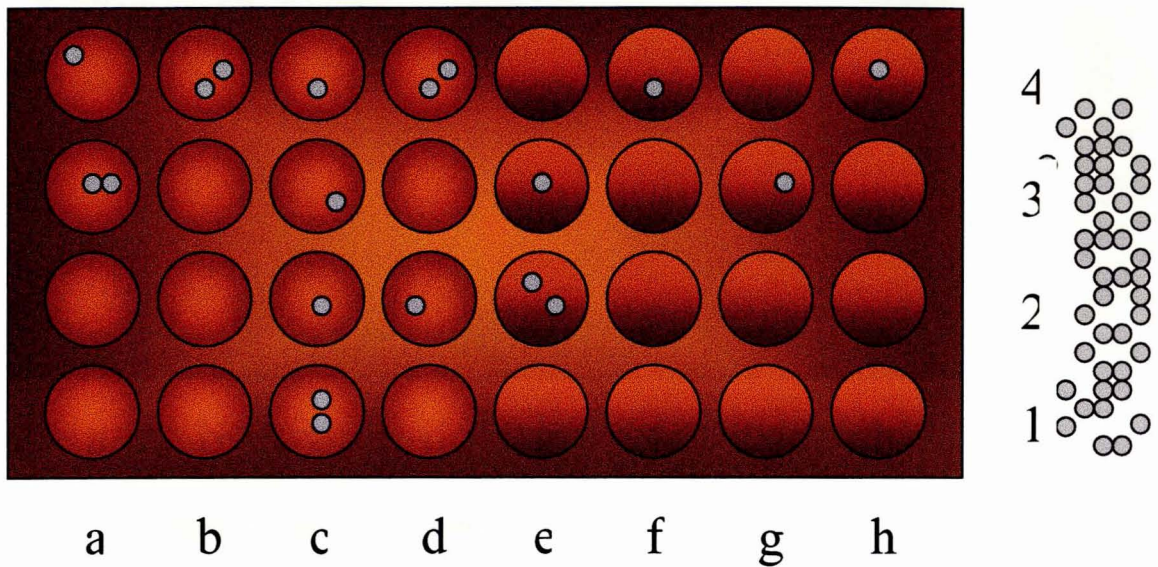


Figure 7.18: Situation after the 15th hypothetical move in the regular match 9 with a hypothetical situation of an infinite move by starting in a3.

2. A similar situation would have happened if player B had played successively in b4 to d3, c3 to c3, in his 17th and 18th moves. If he then had taken d3 the move would never end (See figure 7.19).

I did analyse the situation above once again, focusing on possible conditions that give rise to an infinite move. In the figure 7.18, by starting in a3, the same structure will repeat after exactly 3 rounds, but from hole d3, three holes ahead.

Again, the same structure is repeated in g3, 6 holes ahead and after 6 rounds, in g4 after 9 rounds in the board, then in d4 (after 12 rounds), in a4 (15 rounds and holes), in c3 (18

rounds and holes), in f3 (21 rounds and holes), in h4 (24 rounds and holes), e4 (27 rounds and holes), in b4 (after 30 rounds and holes), in b3 (33 rounds and holes), in e3 (36 rounds and holes), h3 (39 rounds and holes), in f4 (after 42 rounds and holes), in c4 (45 rounds and holes) and then again in a3 (after 48 rounds and holes).

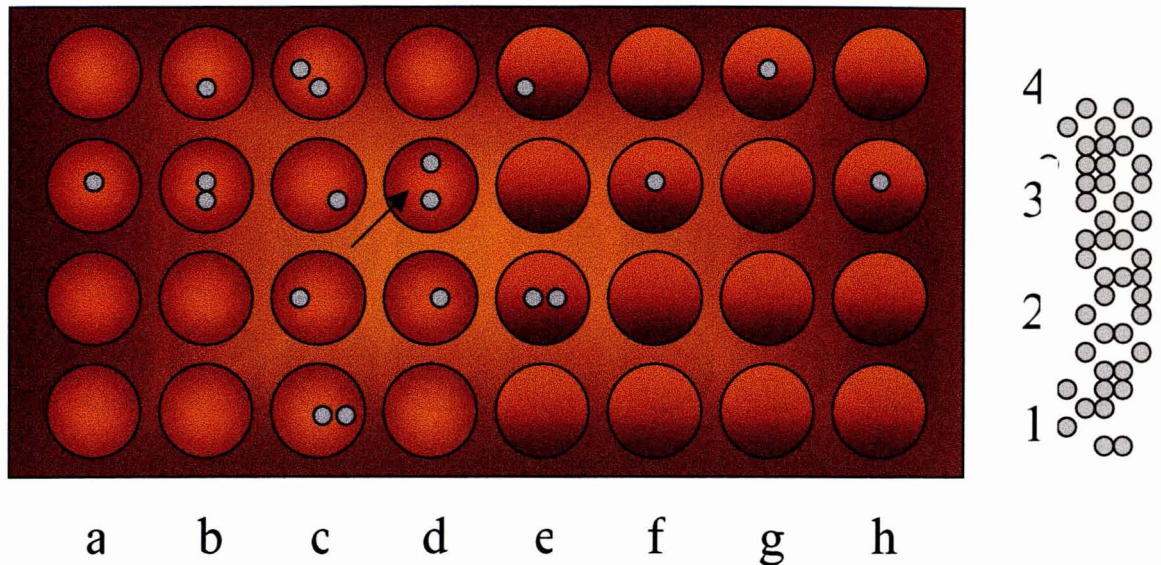


Figure 7.19: Situation after the 15th hypothetical move in the regular match 9 with a hypothetical situation of an infinite move arising by playing in d3

So, the situation would happen after 3 rounds and 3 holes ahead, this means that, we can only have the same situation again, in this case starting in a3, exactly after 48 rounds, because the advance is in 3 holes ahead and we have 16 holes. The position a3 will only be reached again after all holes have been covered once. So, in the first 16 rounds only 5 holes would be covered, in the 2nd 16 rounds another 5 and in the 3rd 16 rounds another 5, so after 48 rounds it could start again in a3.

In the figure 7.19, the second hypothetical situation of an infinite move is illustrated. By starting in d3, the same structure will repeat after exactly 4 rounds from h3, i.e., 4 holes ahead, then in e4 after 8 rounds and 8 holes ahead, in a4 after 12 rounds and 12 holes ahead and then as the initial move in d3 after 16 rounds and 16 holes ahead. So, after 16 rounds of the board the situation will repeat itself.

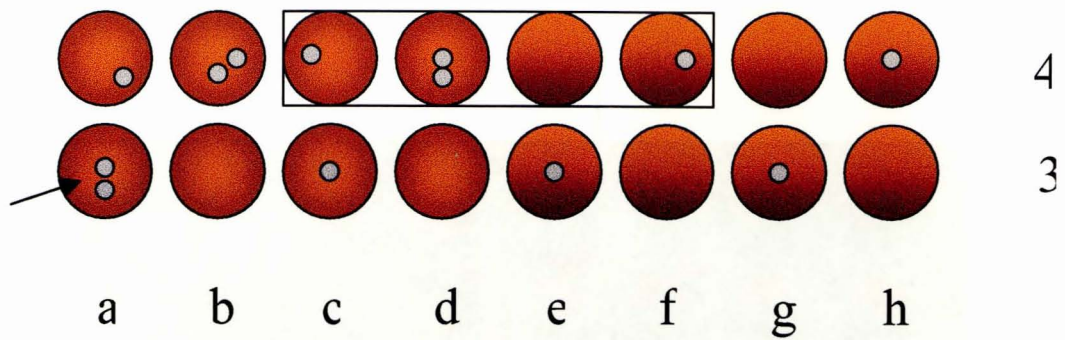


Figure 7.20a: First situations giving rise to an infinite move

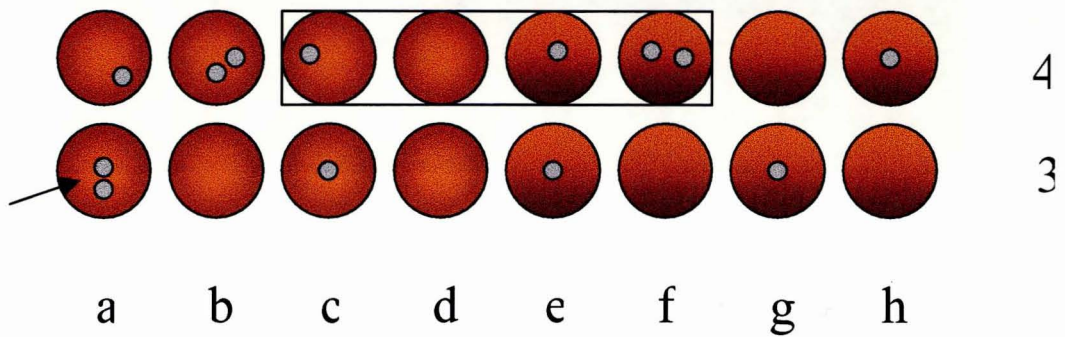


Figure 7.20b: Second situation giving rise to an infinite move

If we compare the two situations illustrated in Figures 7.20a and 7.20b, we see that the difference in the distribution of the seeds in the rows would be only in the 4 consecutive holes, i.e., c4, d4, e4 and f4. In both cases the endless connections are guaranteed. In the first situation, from f4 with one seed and one more seed coming from the move, the connection to d4 (two holes ahead) is guaranteed. In the second situation, from f4 with 2 seeds and one more coming from the move, the connection to c4 (three holes ahead) is guaranteed.

Mucavele (1998) had found a situation of a move that never ended, while analysing *N'tchuva*, a version of Mancala type games played in the Southern Mozambican Province of Gaza. In the subsequent analysis of a version of 4x5 type of Mancala game (See dividing line in the figure 7.21), if player B started in e4, then the move would go on indefinitely.

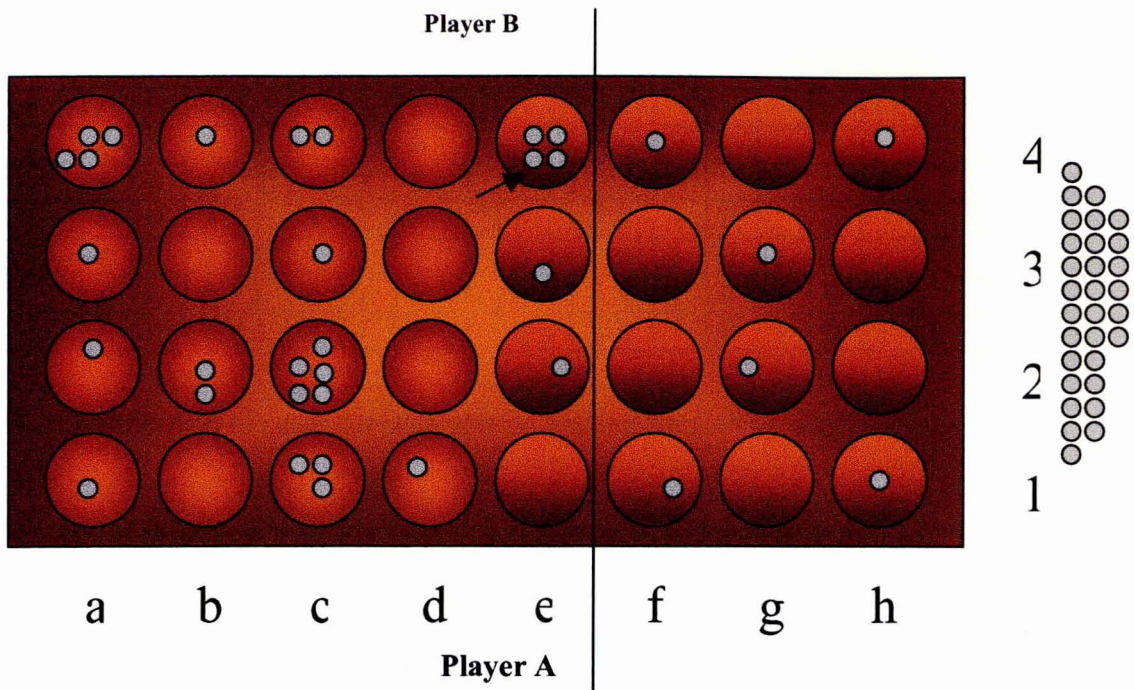


Figure 7.21: A similar situation generated by Mucavele (1998) in a 4x5 version of the *N'tchuva*-game

This situation can be generalised to other smaller or even larger version of the game. Since the connecting holes have to be an odd number it does not matter how many holes are in the rows. In each case, by adding respectively two holes with zero and one seed, or vice-versa, the connection is guaranteed. So, we can generalise it for shorter version, e.g., 4x4 version, or longer version, e.g., 4x6, 4x7, 4x8 (*Tchadji*), 4x9, 4x10, 4x11, 4x12, etc.

For a better illustration of the situation, I present the distribution of the seeds in the sequential holes, which would have the pattern in table 7.2. So, the move would be starting in the hole e4 of the player B. From one version of the board to any larger version, at least 2 new holes are added to the rows of each player (a column contains 4 holes). In order a move to continue two holes are needed with 0 and 1 seed respectively. So, the connection would be made with the 0 and 1 seed in consecutive supplementary

holes and this is valid for any other versions of the board with 4 rows. Once this is guaranteed, the move could continue indefinitely.

Version																	
4x4	4	0	2	1	4	1	0	1									
4x5	4	0	2	1	4	1	0	1	0	1							
4x6	4	0	2	1	4	1	0	1	0	1	0	1					
4x7	4	0	2	1	4	1	0	1	0	1	0	1	0	1			
4x8	4	0	2	1	4	1	0	1	0	1	0	1	0	1	0	1	
4x9	4	0	2	1	4	1	0	1	0	1	0	1	0	1	0	1	0

Table 7.2: Distribution of seeds for each version starting from e4 (or any).

The **concept of infinity** drawn from the *Tchadji*-game is, in practical sense, a bit different from the one drawn from the history of mathematics. However, in these examples, the infinity aspect can be clearly recognised and it constitutes an illustration to understand the idea behind the concept.

7.2.5.2 Other possible draw situations in *Tchadji*

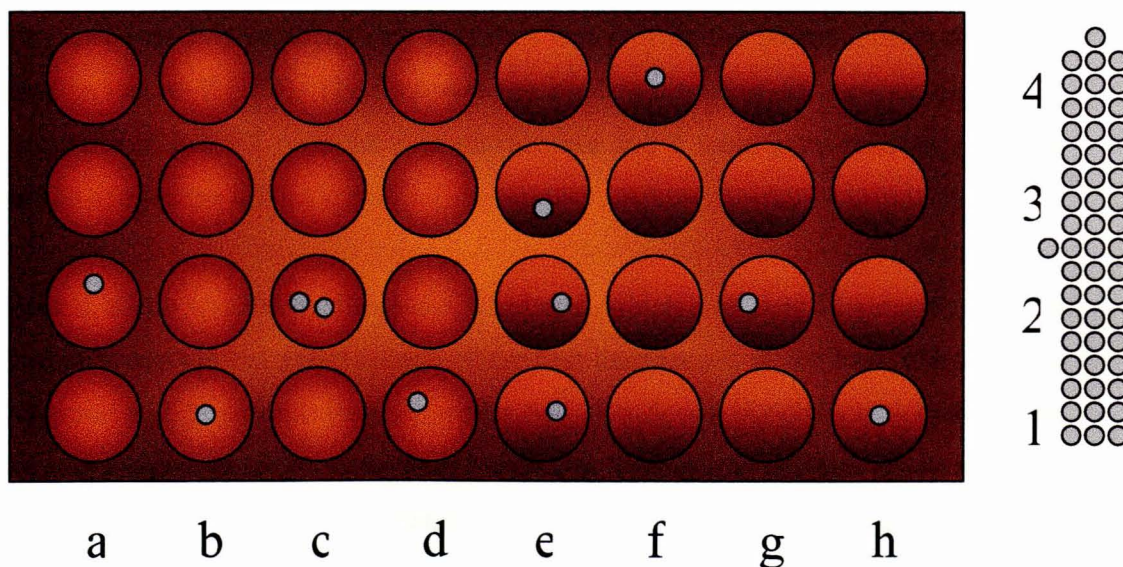


Figure 7.22: A possible match situation by player A. Playing in c2, returned to this situation after exactly 4 moves.

In the *Tchadji*-game there are some other situation, which indicate a draw in a match, not necessarily coming to an infinite move. These situation, as it is for the infinite move (See section 7.2.5.1), are very rare. They generally indicate a situation, in which neither one player nor the other is able to capture his opponent's seeds, which means that no one would be able to win a match (See figure 7.22).

The only possible move for Player A, from c2, would restore the initial distribution of seeds after exactly 4 moves. If the situation of player B is so arranged that he is also unable to make any captures, then a draw would happen.

7.2.6 Duration of play versus number of moves

How long does a *Tchadji*-match last? Taking into consideration the **mental operations** taken by the players it is important to know how much time they allow for thinking, planning moves and taking their decisions during a match. During the fieldwork I was concerned about this question and recorded the time that each of the matches took. I realised that the matches are played very quickly.

Also, the audience is always pushing for quick action from the players (about this issue see more in section 7.2.7.3). The time was recorded in a total of 48 matches played, 25 were regular-matches, 17 were tournament-matches and 6 were extra-matches (For this designation see section 4.3.3.1).

The shortest match was the regular-match number 5, which took only **62 seconds**, and the longest match was the last match (final) of the first tournament, which took **279 seconds**, i.e., 4 minutes and 39 seconds (See Appendix B35). There is a big difference between the shortest and longest match of more than 3 minutes. Taking into account all 48 matches, the **average duration** of a *Tchadji*-match came to **146 seconds** with a standard deviation, of 53 seconds. In this period the players are able to perform on **average a total of 38 moves**. In fact, the match with the most number of moves was the 3rd match of the second tournament with 73 moves performed in 161 seconds. In the 4th match of the first

tournament only 16 moves were performed and it took only 81 seconds. On average a move would take approximately 4 seconds to be performed, which is **very quick**.

7.2.7 The playing moves

Tchadji-game is a game of movement. The moves are the way of playing. The moves are related to several intentions of the players and it is to pursue their intention that the players decide to play in the way they play. Sometimes it is not even possible to pursue their objectives with the situation they encounter in the board. People move seeds with the intention of either capturing seeds or advancing in the board or even preparing another move, etc. It is important to know what kind of moves we can find in the *Tchadji*-game.

7.2.7.1 Type of moves in *Tchadji*

In the *Tchadji*-game we can distinguish various types of moves. I first consider two classifications, the first based on the outcomes of moves, and the second on how far on the board a move can go. Regarding the first classification, we find what I call **capture moves**, **tactic moves** or **strategy moves** (about these kind of moves see sections 7.2.7.2 & 7.2.7.3).

Regarding the second aspect, I shall distinguish **normal moves** and **single moves**. The normal moves would be ones, which advance at least 3 holes in the board. They can be **short moves** or **long moves**. Short moves are those, which advance a maximum of 16 holes, i.e., a complete round. Long moves advance more than 16 holes. Single moves are the ones moving only a single seed and advancing only a single hole, which only happens when the situation is of holes with single seeds only, as it is according to the rule 3 (See section 5.4.2.1.1).

Other classifications are also possible. Consider, for example, the two situations concerning the 13th and 15th moves of the regular match 9 (see appendix B9). From the

13th move originate the situation in figure 7.23a and from the 15th move originate the situation in figure 7.23b after he had played in f4 and in g4.

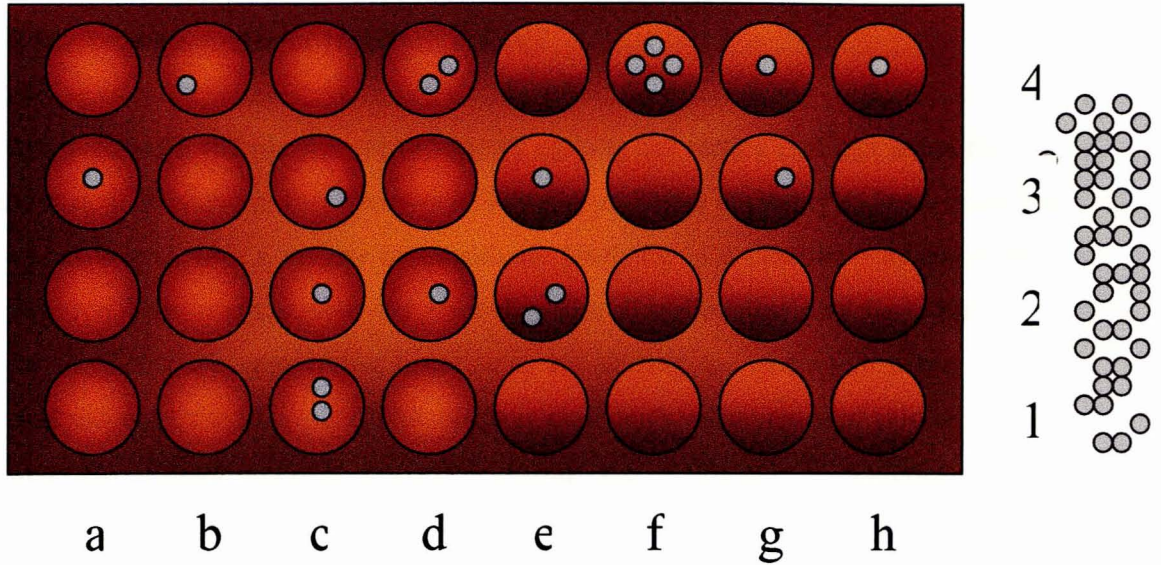


Figure 7.23a: A situation originated from the 13th move of player B.

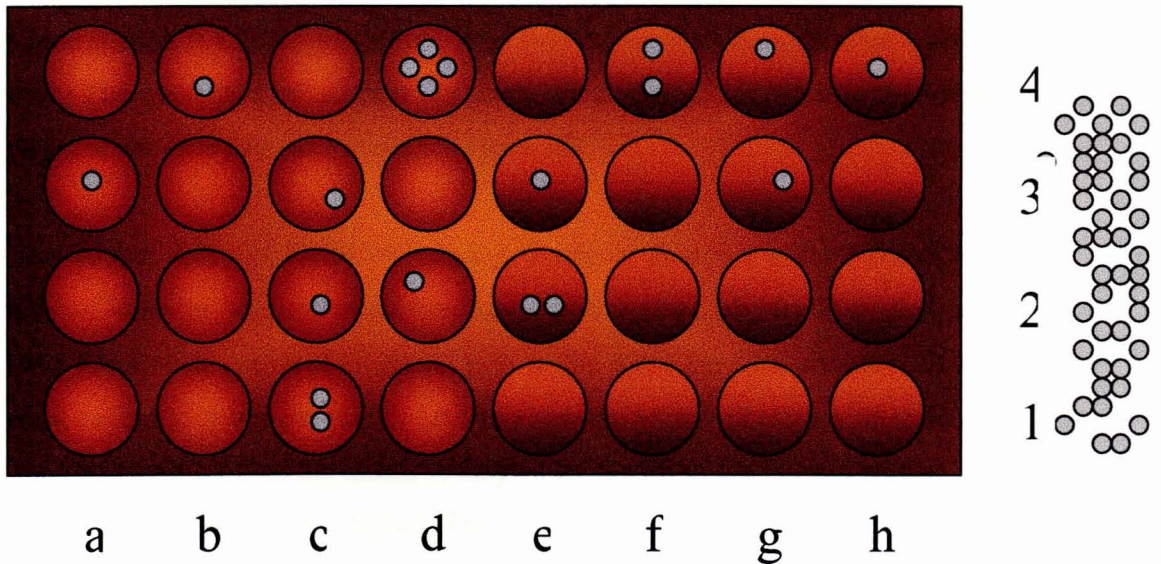


Figure 7.23b: A situation originated from the 15th move of player B.

If we confront both figures, we realise that in two moves the situation in (a) changed to the situation in (b). What happened is player B played his 13th move in g4 [= (a)], then in f4 and g4 again, which came to the situation in (b). Both situations starting in d4 would

also end in d4 after exactly 3 rounds. With exactly 22 connections made. These types of moves I may call ‘**equivalent moves.**’

7.2.7.2 Strategy measures or tactics in *Tchadji*

By playing *Tchadji*, the players are constantly looking for tactics that can assist them to win the match. Winning is the objective of any player. There are many of these measures that are taken and with a variety of intentions. Studying in a detailed way some these measures can help to understand better of the kind of **reflections and thinking** in the mind of the *Tchadji*-players. Also, some rules for playing the game will come out and other matters not easy to explain will become clear with examples. For this reason, by analysing the *Tchadji*-matches recorded during the fieldwork, I underlined some moments of the matches, which contain such matters, which I now will discuss.

The first tactic has to do with **winning and loosing**. Are there different forms of winning or loosing a match of *Tchadji*? If yes, what does count in these forms? According to the rule 4 (See section 5.3.2.1.1) the player, who first captures all his opponent’s seeds wins the match. The question here is, how the seeds are captured, i.e., what is the situation of each player at the time that the last seed is captured?

There are in fact different forms of winning. Some of them are quite difficult to reach. One form of winning happens when the match with the two players is singled, i.e., they only have holes with one seed in each. This is the most frequent situation. If a player wins so, it counts only one point. The second situation, which all players often look for, is when the player who looses the match is singled by the time that his last seed is captured and the player who wins is not singled. This is quite difficult to reach and for this reason its value in the punctuation is three⁷.

⁷ These situations have nothing to do with the physical playing of *Tchadji*. They are rather issues that the players have fixed to facilitate the change of person at the game. For example, two players will play as much as possible until one of them have reached 6 points.

The following examples illustrate the forms of winning. For example, in the Interview-match 3, in the last moves the situation in the figure 7.24 arose (See Appendix B28). Here player A, who won the match, could have directly played in g2 then landing in b2 and capturing the rest of his opponent's seeds. He preferred to take e2, because from there he just took the seeds of two from the three holes so immediately singling his opponent and making it easier to win the match for 3 points.

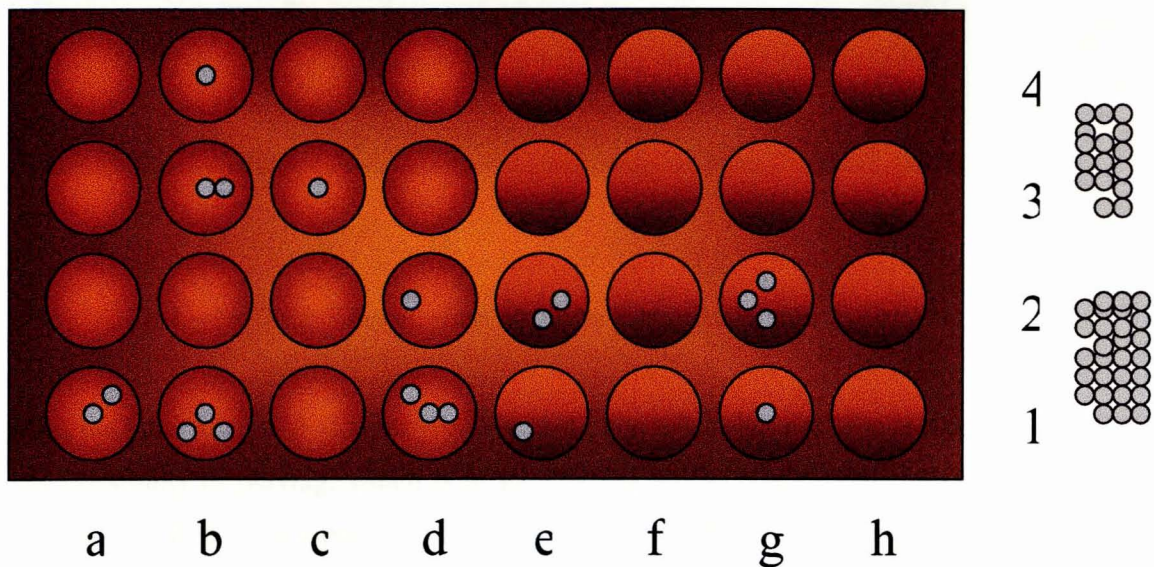


Figure 7.24: Winning for 3 points in Interview-match 3

Another situation happened in the regular-match 5. In the figure 7.25 the situation after the 9th move of player A is illustrated (See appendix B05). From there player B could certainly see that he would be able to capture seeds by landing in a3 from c4 and h4 or in d3 from e4. He preferred to play in c4 and land in a3, capturing seeds in a2, a1 and f2. Here, player A after two moves would be singled and apparently in a very good position to win. By taking this decision player B must have 'calculated' the possibilities very well. The intention is clear as to his chance in winning after his opponent has reached the single seeds situation.

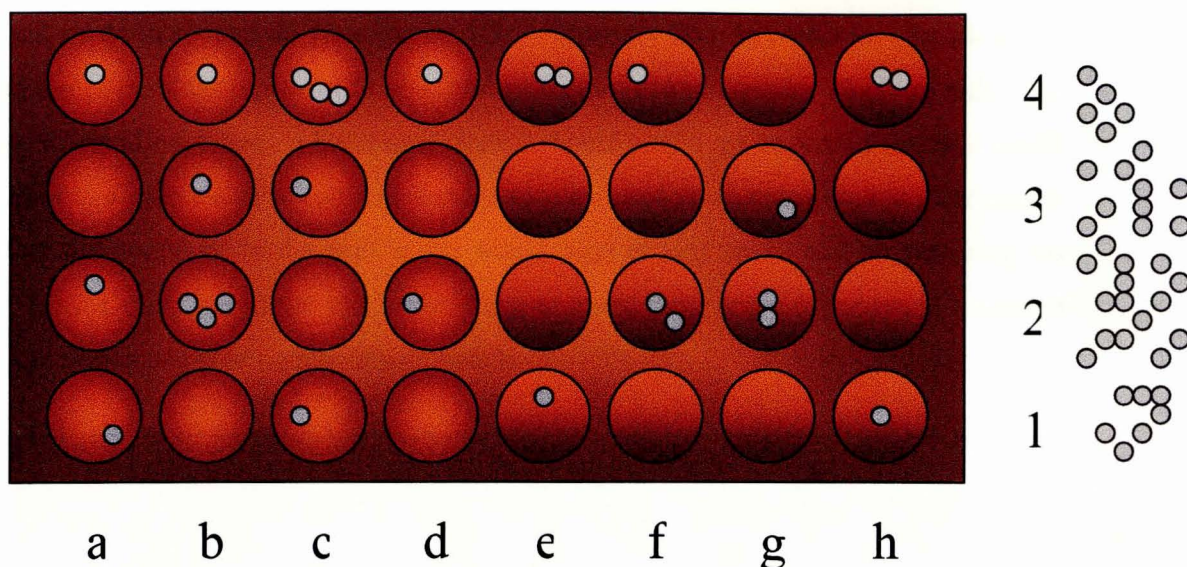


Figure 7.25: The question of points in regular-match 5

Analysing the situations after this move, I see that player A in his next move would have only one possibility of playing, which is in g2. Playing so, he would come to a situation of single seeds but in a very poor position for winning. In the move after this, player B would capture seeds in e3 (e2, e1 and perhaps f2) playing from c3. In the following two moves, player A would not be able to avoid being beaten in a3. So, B created a situation in which he knew exactly where his opponent would play next and so he could just control the match up to its end. This is what mostly happens. Players are constantly trying to single their opponents in such a way that they can win the match and get more points at the same time.

A second frequently used tactic, which is also related to the first, has to do with **avoiding to “single”** your opponent. In fact, once the player is singled, in most cases, more likely to win. On the other hand, the questions of winning for how many points is always in the mind of the players. They have to decide in most cases what is better. Singling your opponent while you still are not singled is also risk taking. Do the players take risks like these? The next example will illustrate such a situation.

In the regular-match 1, after player A has played in b1 ended in h2 (7th move) and captured seeds in h3 and d4 (See Figure 7.26, Appendix B1). In next move by player B, he had the possibility to play in f3 and land in h3 and capture seeds, which he didn't do. In fact, it would have been a bad move since it would single his opponent and this would put him into a position of losing. He decided to play in e4 to a3, even knowing (or maybe not) that his opponent in the next move would land exactly in a2 and so capturing his seeds. In this case, the player is clearly trying to avoid putting his opponent in a singled situation, which could bring him disadvantages.

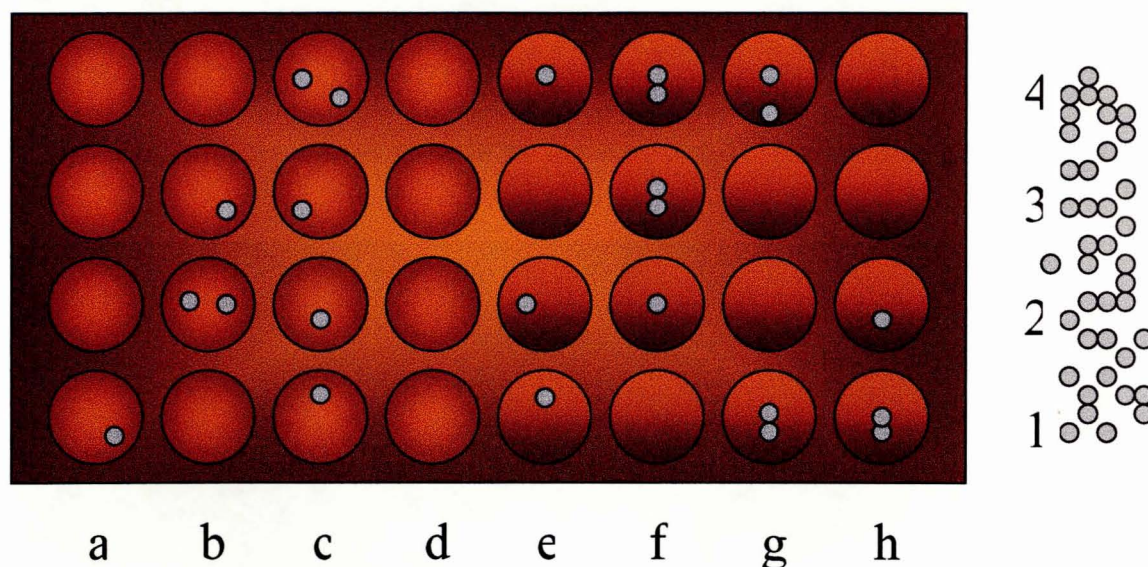


Figure 7.26: Avoiding to “single” the opponent in regular-match 1

On the contrary, players take risks. In the regular-match 16, for example, player A, in his 9th move, made a very courageous move, by playing in g1 to a2 and capturing a3, a4 and b4 and so singling his opponent. In fact, player B would be singled in at most three moves (See figures 7.27a & 7.27b). If we analyse more carefully the situation, we see that player A is in a very good position to win the match. His opponent would have as options play in e4, f3 and g4. His seeds in the holes b3 and c3 would be taken and so he would be annihilated. This is what happened (See appendix B16).

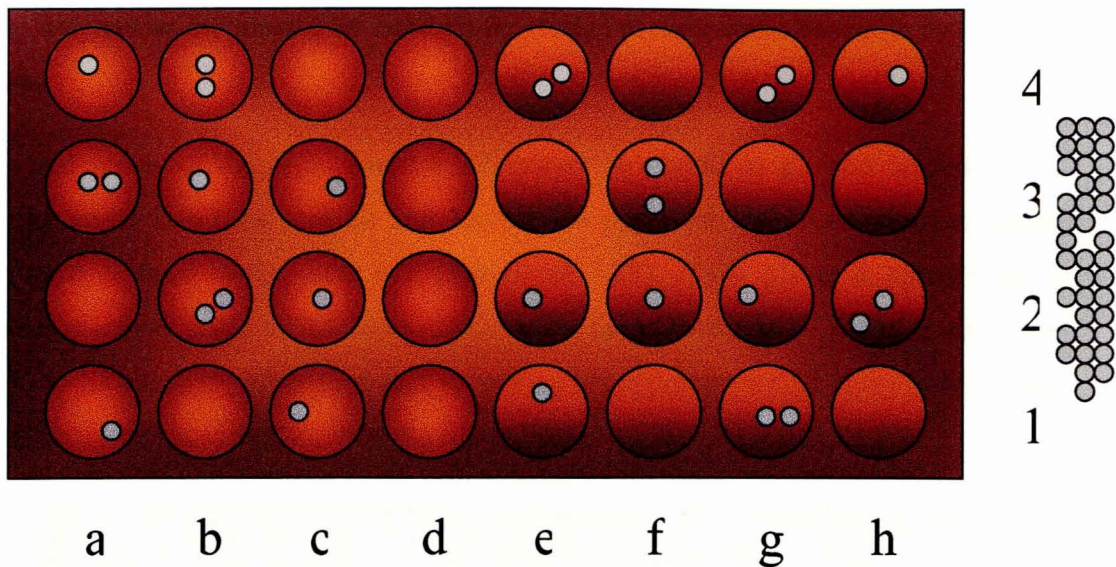


Figure 7.27a: A courageous move in the regular-match 16, playing from g1

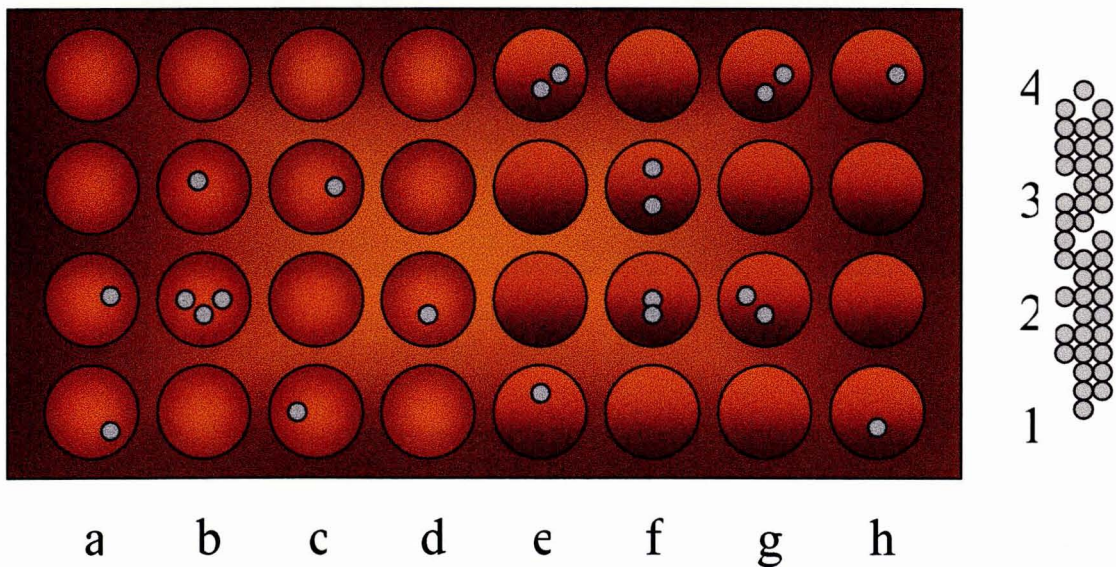


Figure 7.27b: A courageous move in the regular-match 16 with captures in a3, a4 and b4

A fourth tactic has to do with frustrating your opponent's efforts to capture seeds by **emptying consecutive holes of his outer row**, and thus giving oneself more space and time for acting. In fact, in a situation in which a player does not have holes with 3 or more seeds, three consecutive holes in the outer row will not lead to his defeat. This tactic

is very frequent in the playing of *Tchadji*. As next, I present some examples to illustrate the idea.

In his 3rd move of the regular-match 17, player A played in g2 and landed in c2, capturing seeds in c3, c4 and d4 and leaving his opponent with 6 consecutive empty holes, most of them in the outer row (See figure 7.28). Also, player A himself had 5 consecutive empty holes in his outer row. In his next move (the 4th), player played in from f3 to h3 capturing h2 and g1 and so leaving his opponent with 8 consecutive empty holes (See the captured seeds still in the figure 7.28).

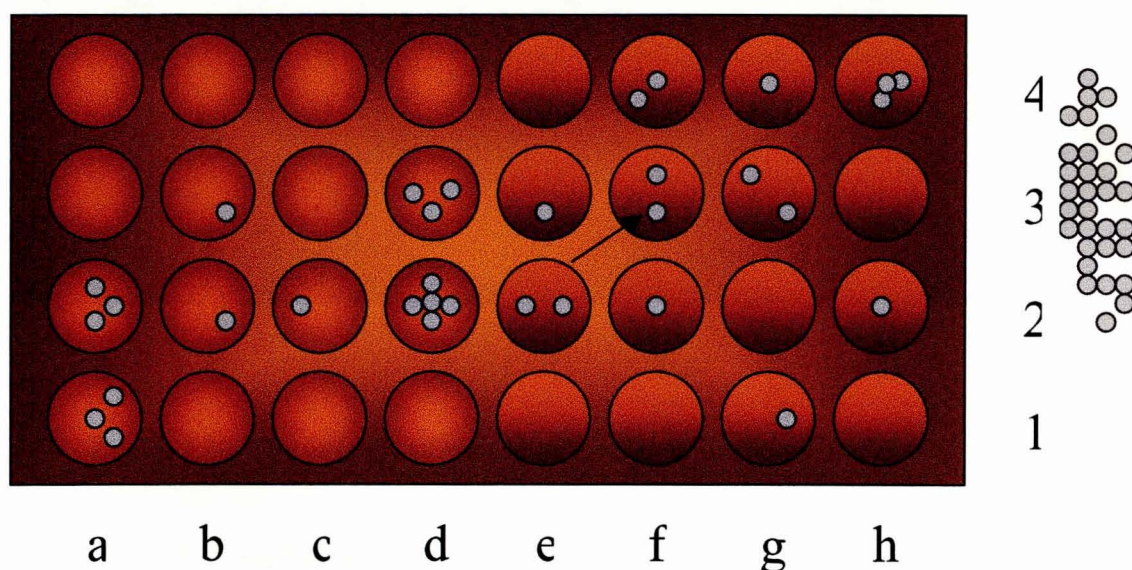


Figure 7.28: The strategy of consecutive empty holes in regular-match 17

In the interview-match 3, when player B had played in c3 ending in b4 as in the figure 7.29, he said, “*I am going to wait for him here (pointing to his right side, specifically the hole a3).*” After this move, player A, at his turn, played in e3 ending in h1 saying that the intention there was to put seeds in the empty holes g1 and h1. In fact, in his outer row, in the right side, two consecutive holes were completely empty. This would make it difficult to advance to the inner row in his next move.

Later, in the same match, player B by playing in f3 ending in h3 and capturing the seeds in h2 and g1, left 5 consecutive empty holes for A, 3 of them from the outer row. He said

then ‘*There (pointing to the right corner of player A) is no way.*’ In the following move, A was able to cover the empty holes by playing in b1 ending in g1. Even so he was still not able to capture any of his opponent’s seeds, only after another move could he then cover it all and reach his inner row. It took time for him to come again to the inner row in order to be able to capture seeds.

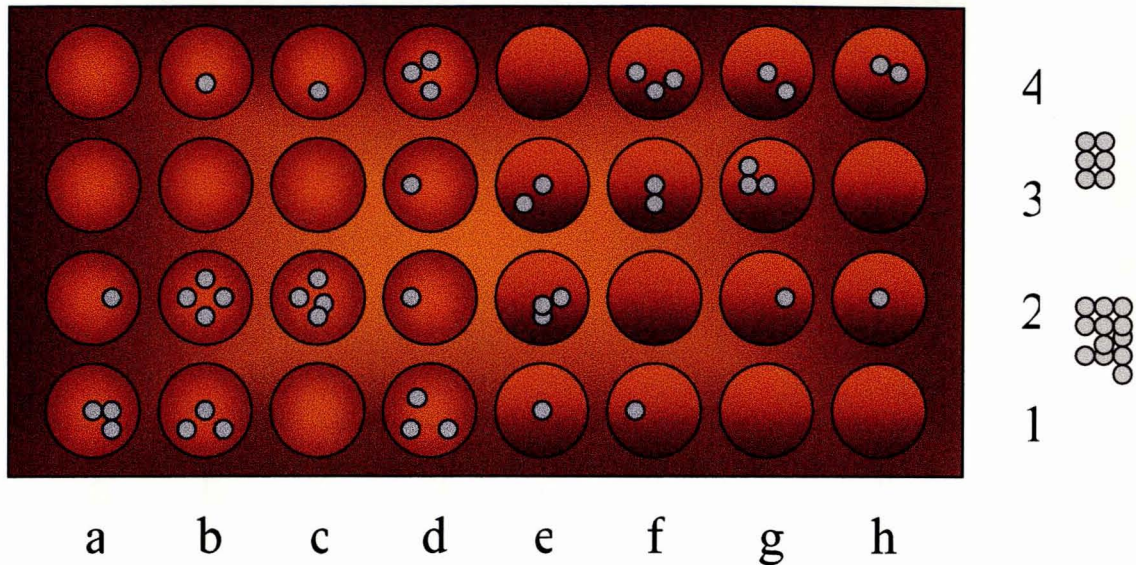


Figure 7.29: The strategy of consecutive holes in interview-match 3

In the second match of the first tournament, Player A having played in g2 and ended in c2 capturing the seeds in c3, c4 and d4. The strategy of creating consecutive empty holes in the outer row of the opponent was again recognisable (See more in appendix B28).

In the same match, in another situation, after player B has performed in f3 ending in h3, capturing the seeds in h2 and g1, I observed that he preferred to capture seeds in g1 in order to leave his opponent with 8 consecutive empty holes instead of capturing the most number of seeds, for example, by taking a1 or a2 instead of g1. Hole g1 with one seed would not represent any danger for him.

This situation shows that the tactic of leaving your opponent with many consecutive empty holes in the outer row is very powerful. Players will some time prefer it as to eat

the most number of seeds at once. In fact, if they leave their opponent in such a situation, they have space to move without worrying about losing their seeds and so they start capturing their opponents' seeds. Sometimes they are even able to capture more seeds than they would otherwise do in one move.

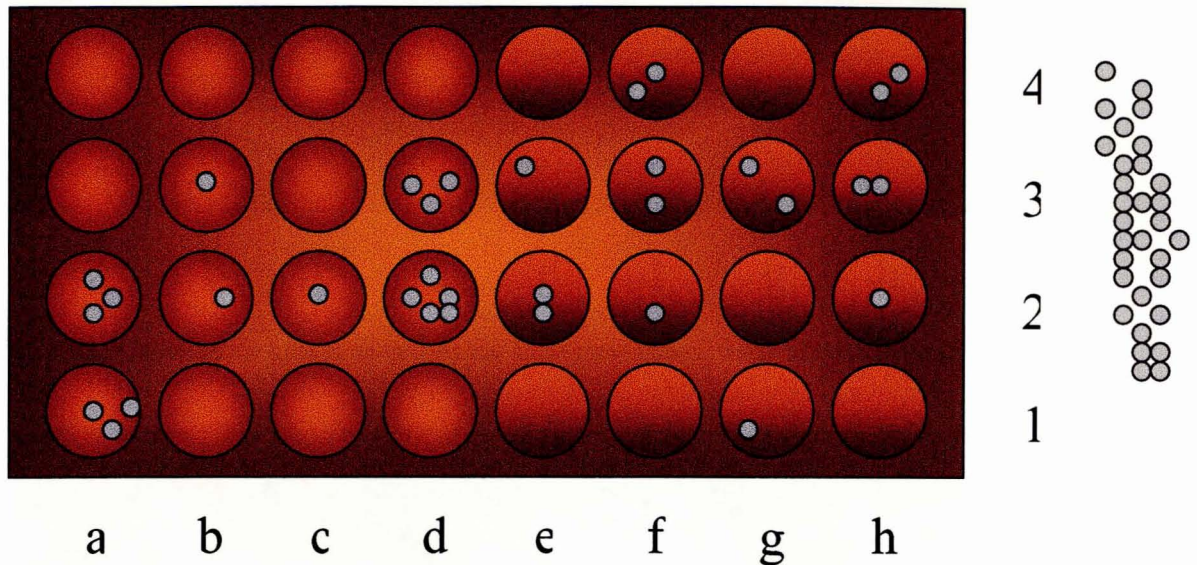


Figure 7.30: Consecutive empty holes in the 2nd match of the 1st tournament

A fifth tactic is related to **planning moves in advance**. For example, in some situations it may not be obvious that there is a move that results in capturing seeds. For this reason it is sometimes necessary to plan a sequence of moves that might lead to capture seeds.

For example, in the interview-match 1, Player B in the figures 7.31 and 7.32, at his turn, played exactly in c3 (4 seeds) and landed in g3. He captured the seeds in g2 (2 seeds), g1(2) and b1(2). So, I asked him: *'Why did you take b1?'* He gave the following answer: *'I am giving him the possibility to beat me here (pointing to f3), if he plays here (pointing to h2). If he does it, there will be ways to beat him from seeds coming from here (pointing to a3 and b3). Look, here is just a trick. I am just pushing him to make a mistake in beating me here (pointing to f2). If he does it, I will then be set up to properly defeat him.'* He was clearly planning a move.

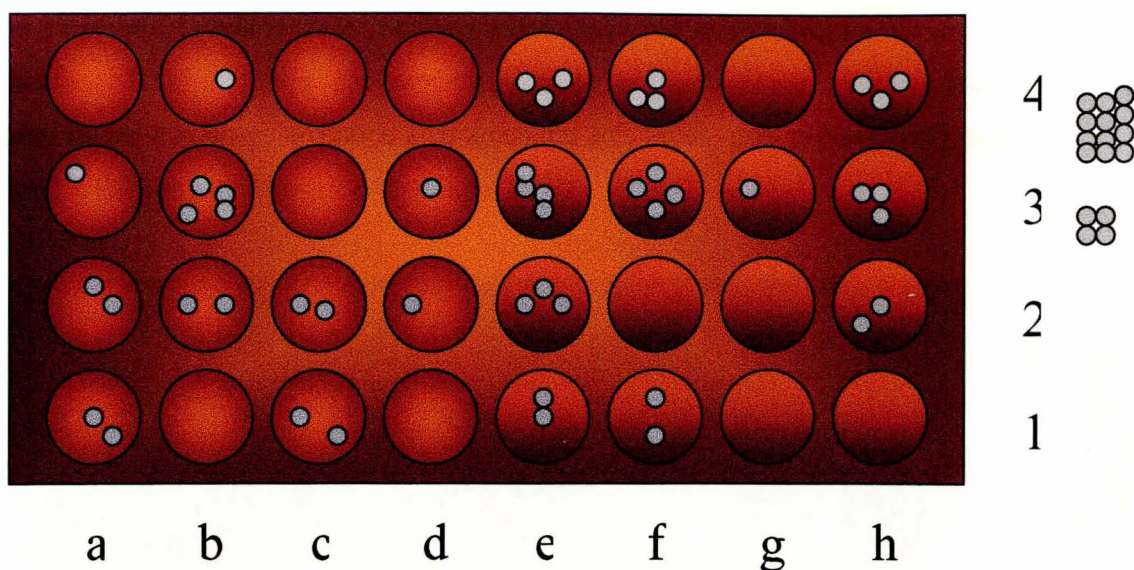


Figure 7.31: Captures in moves ahead in the Interview-match 1 (B played from c3 to g3)

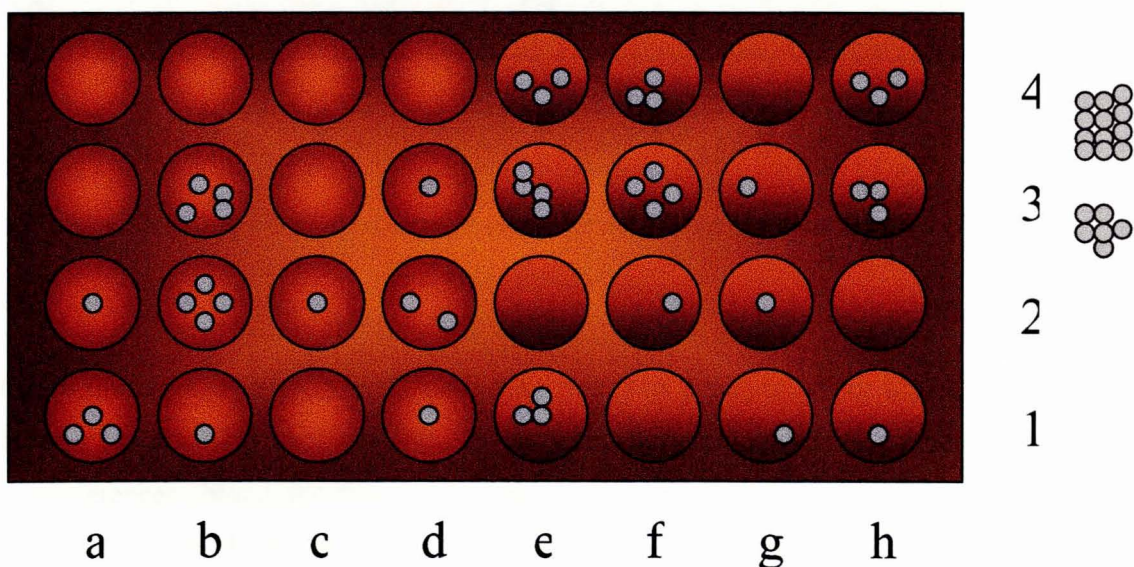


Figure 7.32: Planning moves ahead in interview-match 1

Another comment happened in the same match. A, after having played in c2 ending in a2 (See figure 7.32), explained, ‘*I play here to avoid being beaten (pointing to f2), where I could lose the match, and so I take here (a3 and b4 with one seed each). I opened here (pointing e3) and I want to beat him as next here (pointing to h3).*’

Similar situations, in which there is clearly a preoccupation in planning moves in advance by visualising future moves for captures, appear in many other situation (See, for example, figure 1 of Regular-match 1 in appendix B1, figure 1 in appendix B2, figure 1 in appendix B8 and also figure 2 of appendix B28).

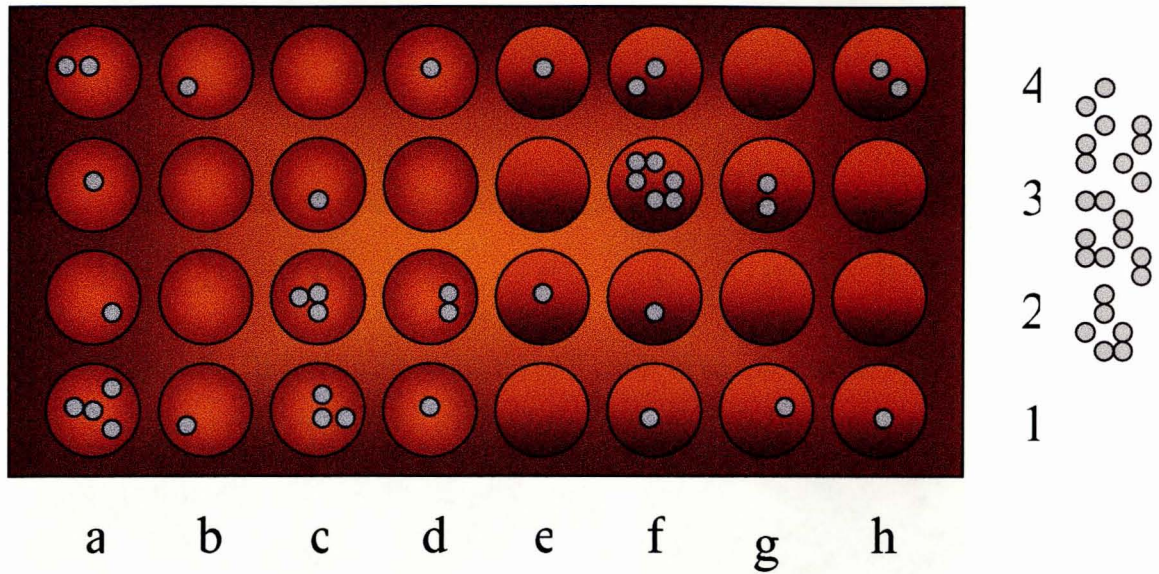


Figure 7.33a: Deciding whether to capture seeds by not taking the most number of seeds in regular-match 6

What is in fact more important in *Tchadji*, i.e., what sets a player up for a victory? For people who are learning, as I observed in the workshops organised with students and during conferences abroad, the first preoccupation is to capture the most number of seeds at once. It is indeed obvious, because of the objective of the game itself. **Captures** are necessary, of course, but what captures, always the most number of seeds at once. So, the sixth tactic showed by the player is that the number of seeds does not matter, players often look to holes, which endanger their game and not to holes with the **most number of seeds**.

In regular-match 6, for example, after player A has played in c1 to g2 (See figures 7.33a & 7.33b), he captured the seeds in g3 and in a4 and left intact f3 with 6 seeds.

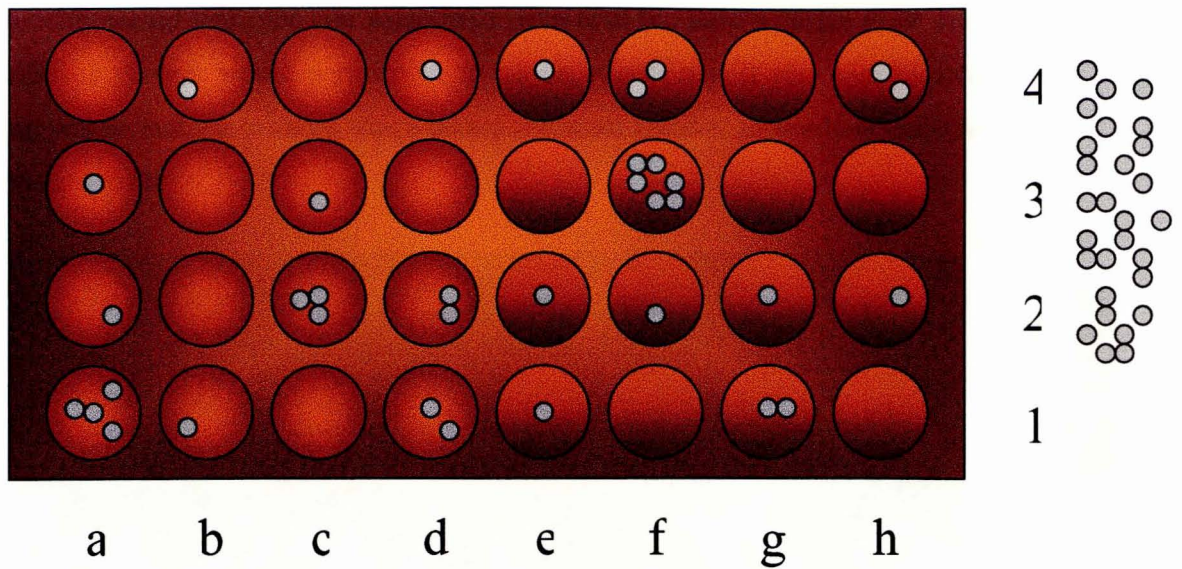


Figure 7.33b: Capturing 4 instead of 8 seeds in regular-match 6

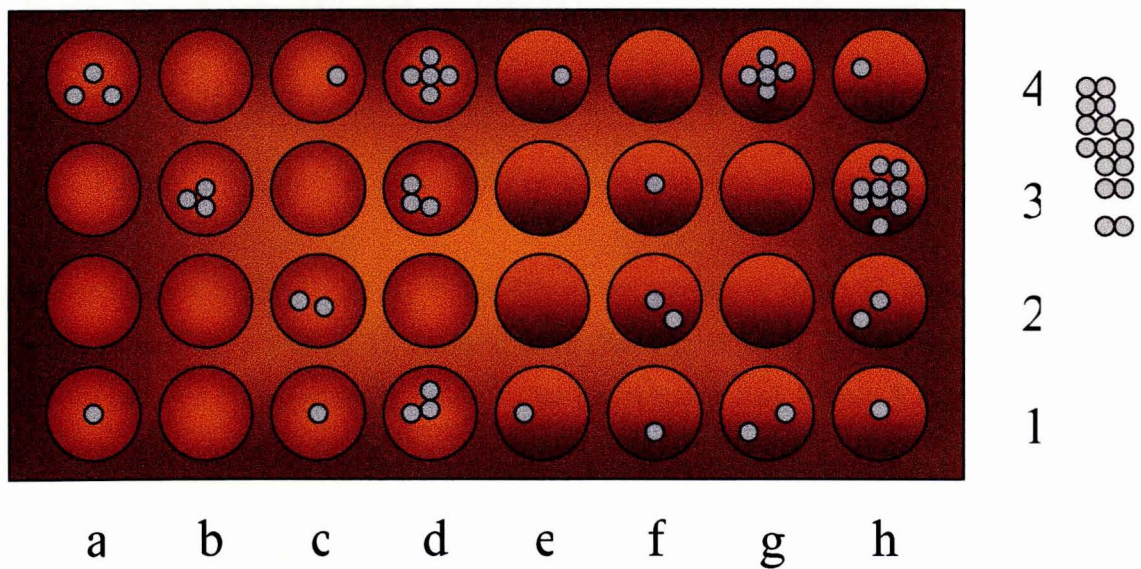


Figure 7.34: Captures versus number of seeds in interview-match 2

In the interview-match 2, where I had the possibility for asking questions, after player A had played in f2, ending in d2 and capturing the seeds in d3 (3 seeds), d4 (5) and a4 (3), and so leaving intact the seeds in h3 (9 seeds), I asked him, ‘*Why didn’t you take the big number of seeds in h3?*’ He said, ‘*No, this depends on your opponent.*’

He probably meant that, you can sometimes even take the hole with the most number of seeds. Some players would permit you to do so, others not. It also can be a trick for pushing you to make a mistake, since taking the most number of seeds at once seems to be attractive to all.

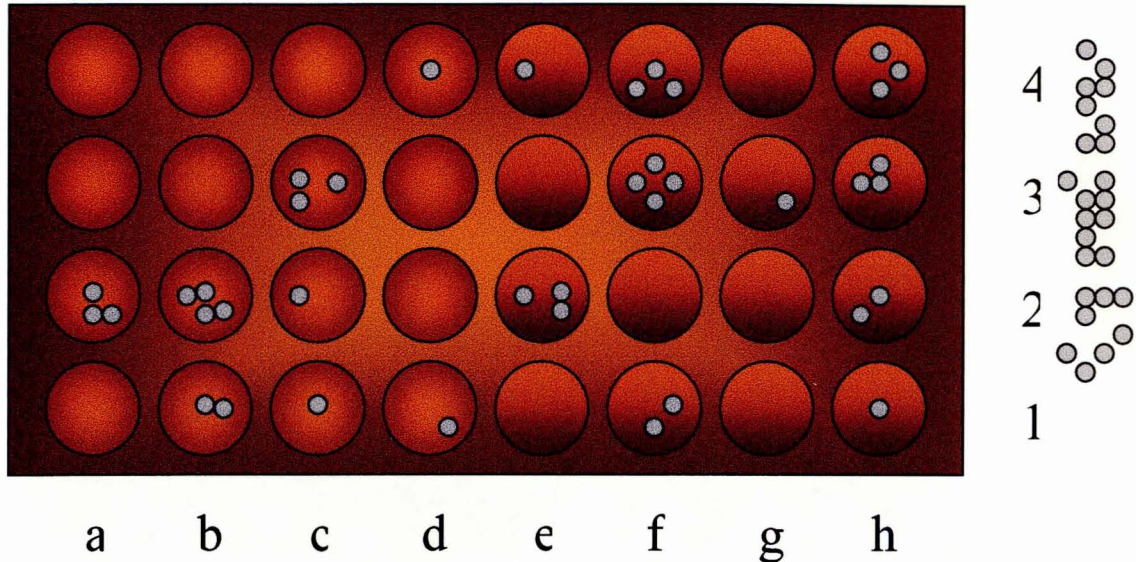


Figure 7.35: Captures versus number of seeds in regular-match 13

Also, in regular-match 13, Player A, in his 4th move, played in h2 to f2, captured the seeds in f3, f4 and c3, a total of 10 seeds at once (See figure 7.35). Here, player A had considerably reduced his opponent's seeds. What happened is that, player A surprisingly lost the match (See Appendix B13).

Having captured more seeds in a move does not necessarily mean that the player will win the match. It is a necessary condition but not a sufficient one. Situation similar to these happened many times in the matches recorded (See, for example, figure 4 of Regular-match 2 in appendix B2, figures 1 and 2 in appendix B4, figure 1 in appendix B11 and also figure 2 of appendix B19). So, when should the capturing of most number of seeds at once be a good strategy?

The seventh tactic has to do with what I call **‘invisible’ captures**. I mentioned earlier in the 5th tactic measure that it is sometimes necessary to plan a move for capture in a situation in which there are in fact no moves for making captures. Sometimes similar situations appear in which there are, in fact, moves for captures but they are ‘not visible.’ For good players, however, these captures are in most cases visible. Some examples are now presented to illustrate.

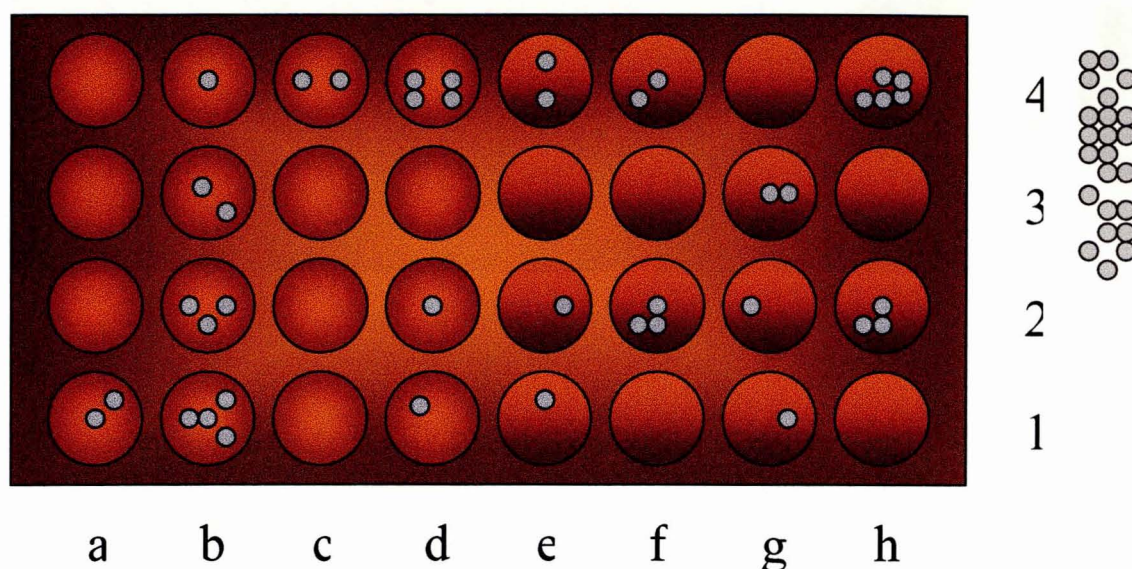


Figure 7.36a: ‘Invisible’ capture in regular-match 22

In regular-match 22, in the next move for player A (See figure 7.36a), it is not obvious, whether he would be able to capture seeds of his opponent or not, since the two possible holes in the opponent’s inner holes (b3 and g3) are apparently blocked, i.e., the direct opposite holes are not empty. So, a direct move to land in one of these two holes is not obvious, apparently so at least. Unless there exists a move, which goes one or more rounds. What happened is that, player A played then in b2 and landed in b2 again. So, he was able to capture seeds from his opponent in b3, b4 and d4.

In the regular-match R23, player B, in his 19th move, performed a very exciting move. The only place that he could capture seeds was f2 and this was blocked (See figure 7.36b). He then played in e4 and after more than one round (26 holes ahead) landed exactly there, capturing the seeds in f2 and h1. Similar situations happened also in

various other matches (See, for example, figure 1 in appendix B23 and figure 1 in appendix 24)

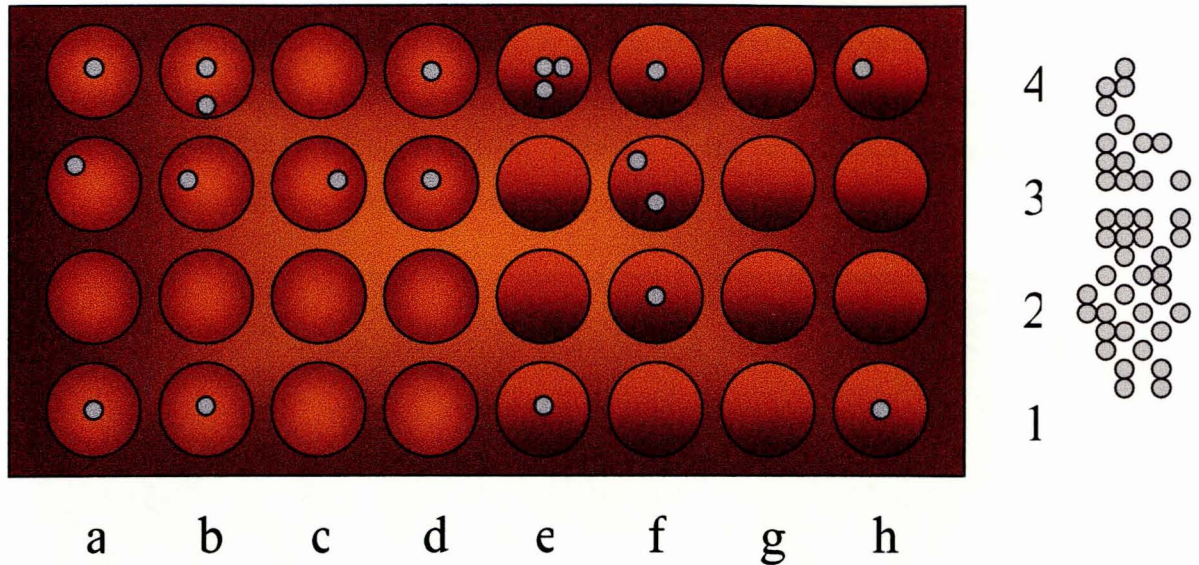


Figure 7.36b: 'Invisible' capture in regular-match 23

There are, of course, many other strategy measures or tactics in *Tchadji*. The player would follow them consciously or not. Many of them could have happened in the match I recorded. Also, such measures depend in most cases on the matches themselves and these are different from one another. I can imagine that analysis of more matches may reveal other tactics.

7.2.7.3 Other issues in *Tchadji*

Having interacted with the community of Tchadji-players at 'Ilha de Moçambique' during the period of the fieldwork, I was led to observe a variety of other issues related to the playing of *Tchadji*, which I would not classify as strategy or tactics but are extremely important to understand in order to appreciate more clearly how *Tchadji* is played. For example, it is difficult for a player to avoid making **mistakes**. What is a mistake? I do not mean that a player wins a match from the mistakes of the other player, a well used tactic might not have the intended effect, because the opponent played better to counter it. A

situation like this I would not classify as a mistake. Players will play irrespective of mistakes and even capture seeds wrongly.

For example, in the regular-match 7, player A in his 6th move, played in h1 to a2, capturing the seeds in a3, a4 and e4. Here, it seems to me that e4 with 2 seeds was taken by mistake (See figure 7.37). I think that, hole b3 would be the best to take, because it is the one, which permits his opponent to capture his seeds.

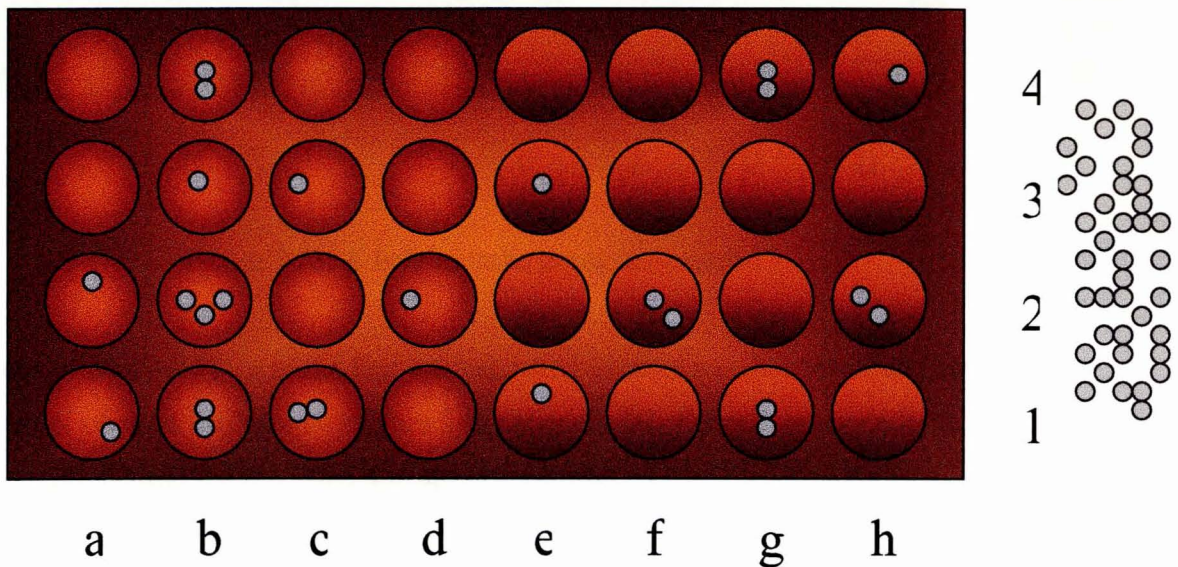


Figure 7.37: A mistake in regular-match 7

In his next move, the 9th in the regular-match 13, player B had two positions to capture seeds, playing in b3 or in g3 (See figure 7.38). He had to decide which was the best to take. He played in b3 to c3, capturing seeds in c2, c1 and b1, a total of 11 seeds. It is not usual that at this stage, in the 9th move, a capture of so many seeds happened.

Then, there was a comment at this stage: *'You took it for having many seeds?'* He answered: *'Yes, I am just waiting to beat him more.'* My analysis indicates that the best hole to capture would be the e2 instead of b1. The position e2 constitutes a danger for his match since it could capture his seeds next in column c (See more in appendix B13).

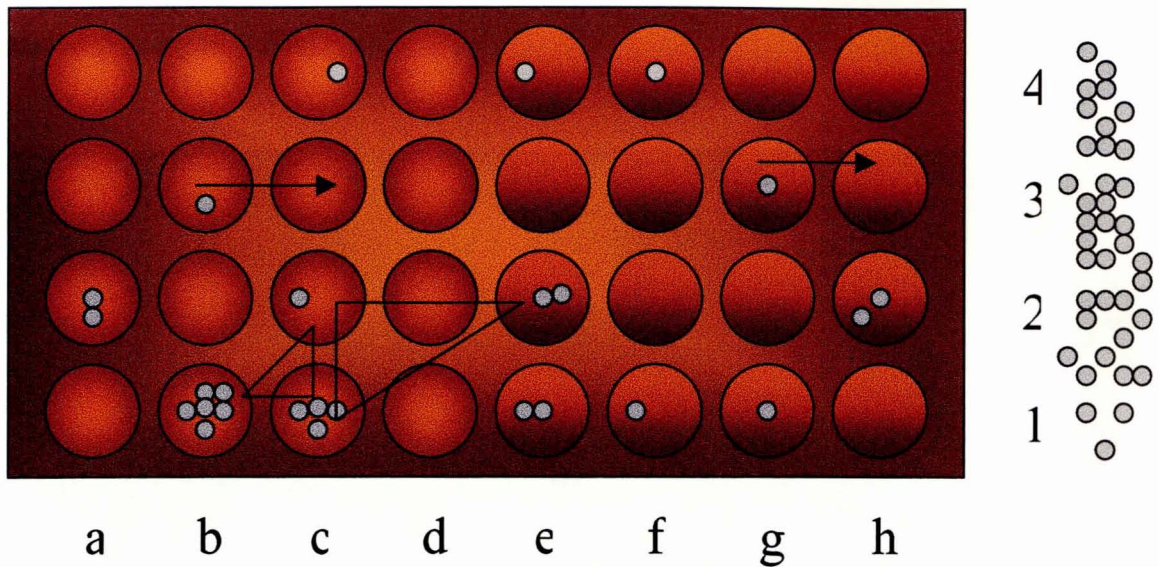


Figure 7.38: A little mistake in regular-match 13

Situations of clear mistakes in the first eye happened in other matches recorded (For more examples of obvious mistakes see figure 5 in appendix B1, figure 2 in appendix 2, figure 3 in appendix B3 and figures 3 and 4 in appendix B33).

Another kind of mistake is the **miscount**. As I referred earlier, *Tchadji*-players may count sometimes while playing the game. It is then obvious that they can miscount. And this also happens sometimes. For example, in the regular match 20, player B, in his 5th move, played in a4 and landed in b3 after 18 holes, capturing the seeds in b2, b1 and h1 (See figure 7.39a). While playing his opponent had said that he would end in a4, a miscounting perhaps. His opponent was convinced that he would stop at the hole where he started. This indeed happens often, the fact that the players, specially the less experienced, can count and follow with case and certainty only up to one round in the board makes it happen.

In regular-match 22, player B, in his 4th move, would be able to capture seeds in g3 by taking e3. Player A had apparently seen it and tried to escape. What happened is that he took exactly the seeds from g2 (hole in danger) and starts playing and landed again at the

same hole. So, in fact, he could not escape. If this was his intention, he probably miscounted (See figure 7.39b).

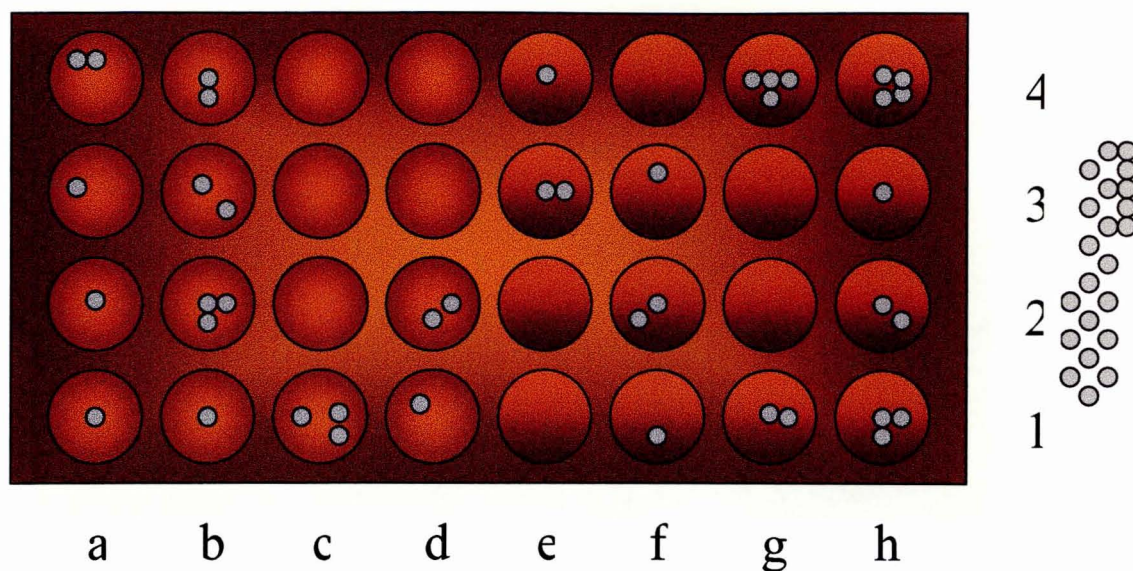


Figure 7.39a: A miscount by the opponent in regular-match 20

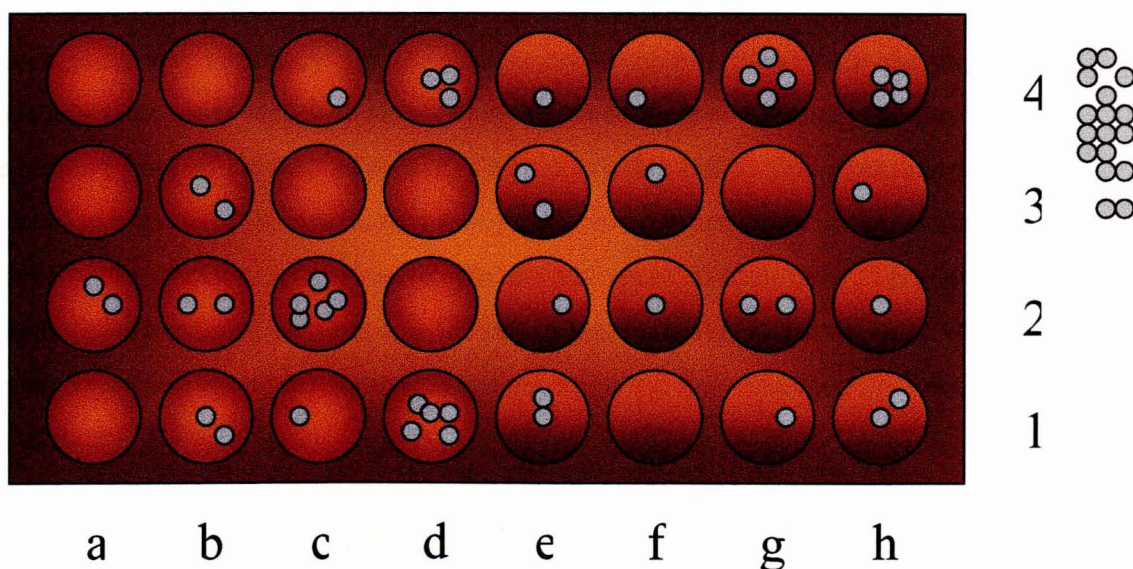


Figure 7.39b: A miscounting situation in regular-match 22

In regular-match 21, in his 14th move, player B must certainly have made a mistake, or must have miscounted, by playing in a4 to a3. He lost the match in the next move by A.

Would it be different if he had played in f4 instead of a4? I analysed this situation further and found that, if player B had played in f4 to e4, he would have lost the match anyway. Did he know this before?

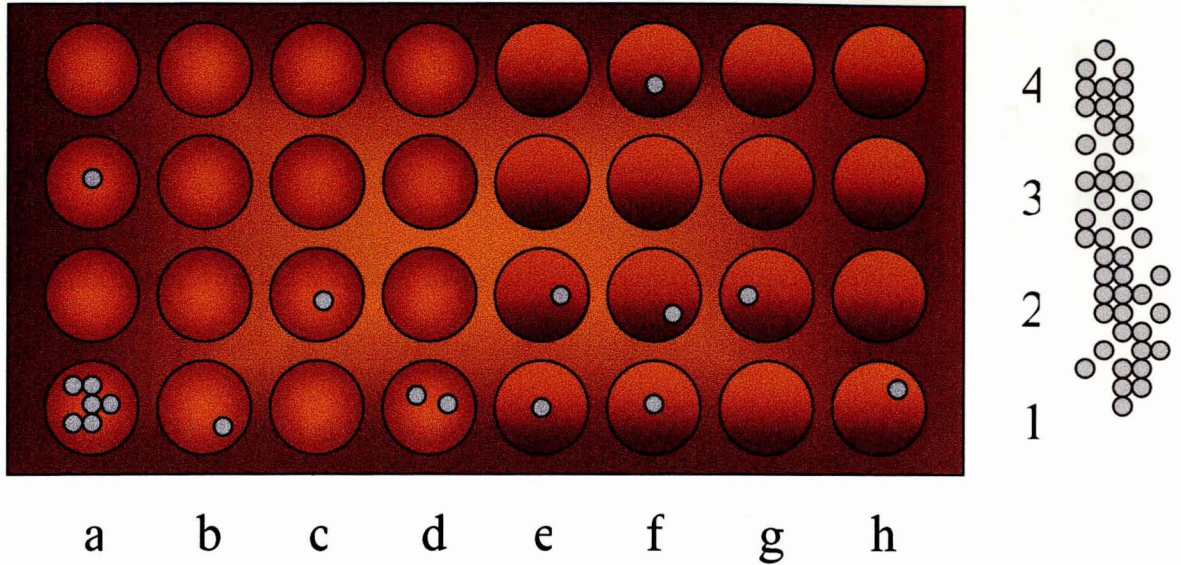


Figure 7.40: An apparent miscount in regular-match 21

For each game it is necessary that the players are in sintony. If only for the purpose of determining when each player plays a sintony between them is necessary. In the *Tchadji*-game another kind of sintony happens. I would say that the players even interact more than usual in a game. This **interaction among players** happens without interfering in the outcomes of the match or of any move. The players speak to each other. There is always a dialogue between them, whether it is for informing a possible intention, or to trick their opponent, or even a desperate expression of agony in trying to change a bad situation of either of them.

For example, in the regular-match 2, the intention of player A in the figure 7.41, of playing in f1 to end in e1, seemed to have been of emptying g2, which was in danger of being captured. Player A has pointed the hole c2 as his stop for the next move. In fact, by picking up to the hole g1 player A would then land in c2 and capture seeds and there is nothing that player B could have done to avoid it.

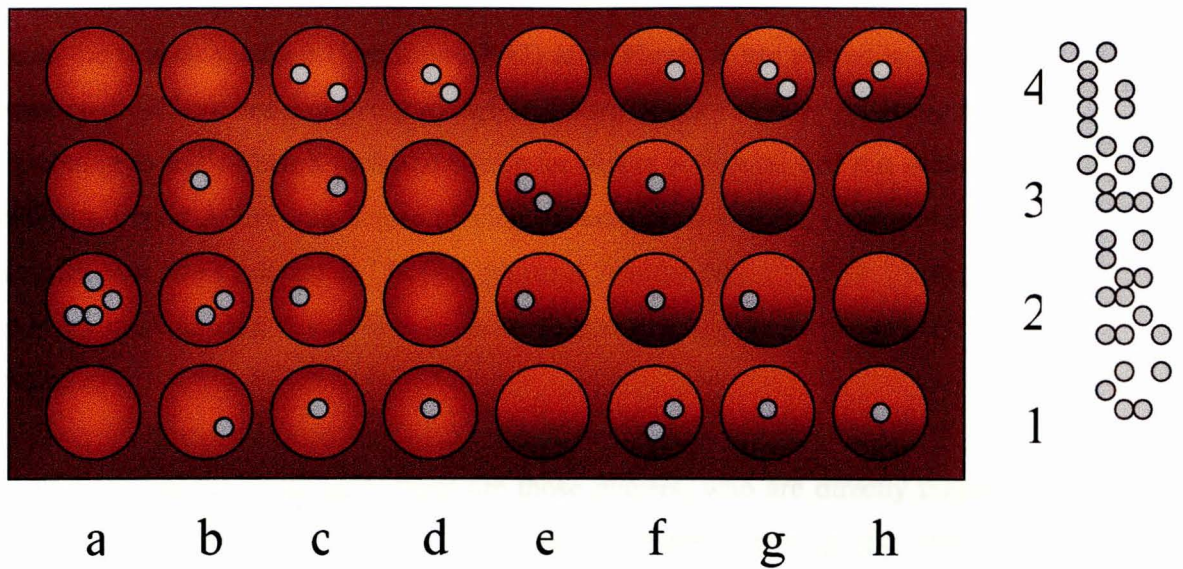


Figure 7.41: A telling of intention among opponents in regular-match 2

Was the move, the fact that there is nothing that B could have done to avoid being beaten, really **‘calculated’** before by player A? Certainly yes, as otherwise he would not predict it to his opponent. Here, it is clear that even knowing about it player B was not able to avoid it. So, player A feels in a position to inform his opponent without interfering with the match.

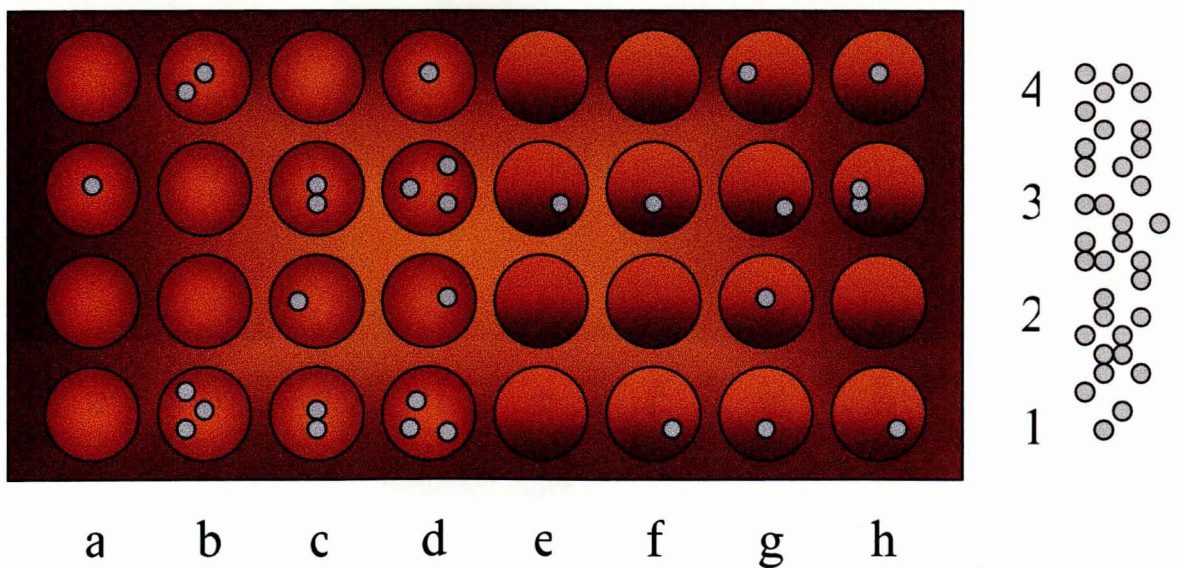


Figure 7.42: Interaction among players in regular-match 2

Again, in the regular-match 2 (See figure 7.42) after Player A has played in a1 to h1 his opponent said: *'Whom are you going to defeat, I want to eat you here (pointing to e3) or here (pointing to h3)'*. Then it took some time for player B to think and the audience was complaining. Player A has said before playing; *'I want to defeat you, I want to show you how to play, you don't respect us.'* In fact, player A is older than player B. Then, player B played in c3 to f4. In fact, A was able to show his younger opponent how to play. He won the match.

In the circle of players of *Tchadji*, there are those players, who are directly playing and there are many other players or even interested people watching the match (See photographs in appendix D). The environment in the place is of dialogue and constant comments among the players engaged in the game and among the audience. The audience do make its interventions and comments and most of them are related to the match being played, but these comments do not interfere in the course of the match itself. Most of the comments are related to past moves and not to forthcoming moves.

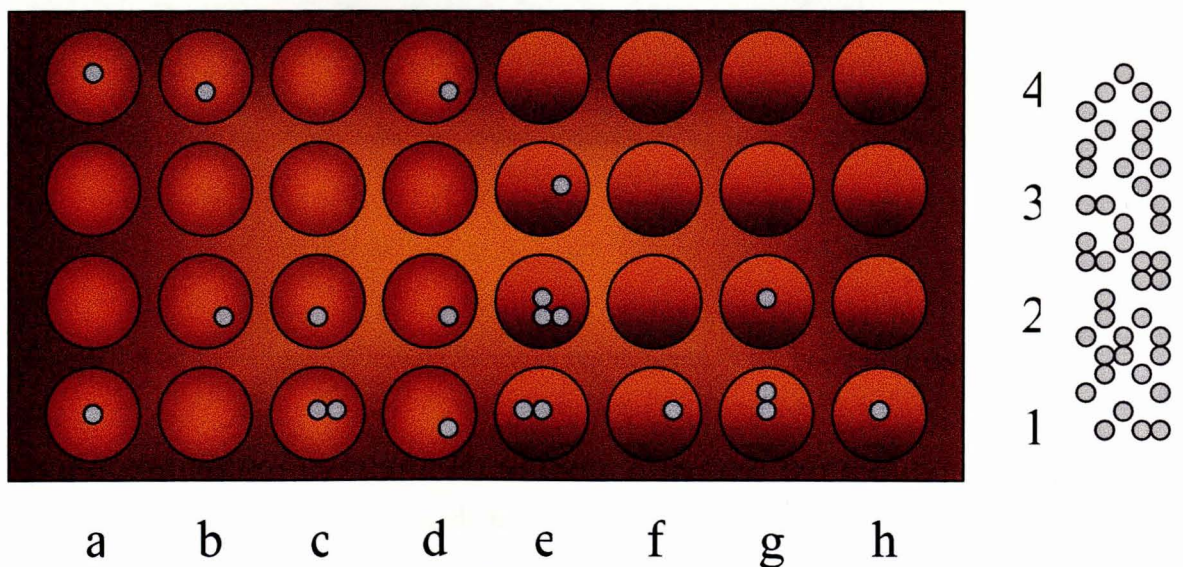


Figure 7.43: A contested move in regular-match 8

For example, the 16th move by player B, in the regular-match 8, was very contested. He played in e3 to f3 (See figure 7.43). I analysed why it was contested. I concluded that, playing so, player B most likely hoped to be able to capture seeds in g3, but several moves of player A could remove the content of the hole g2, e.g., from e1, c1 and e2. This is exactly what subsequently happened.

Another example comes from the regular-match 12. In his 12th move, player A played in d1 to e2 and removed his opponent's seeds from e3 (1), e2 (2) and a3 (1). Player B, in his turn (See figure 7.44a), had only c4 to play and so did and stopped in b3, taking the seeds in b2, b1 and g1 (See figure 7.44b).

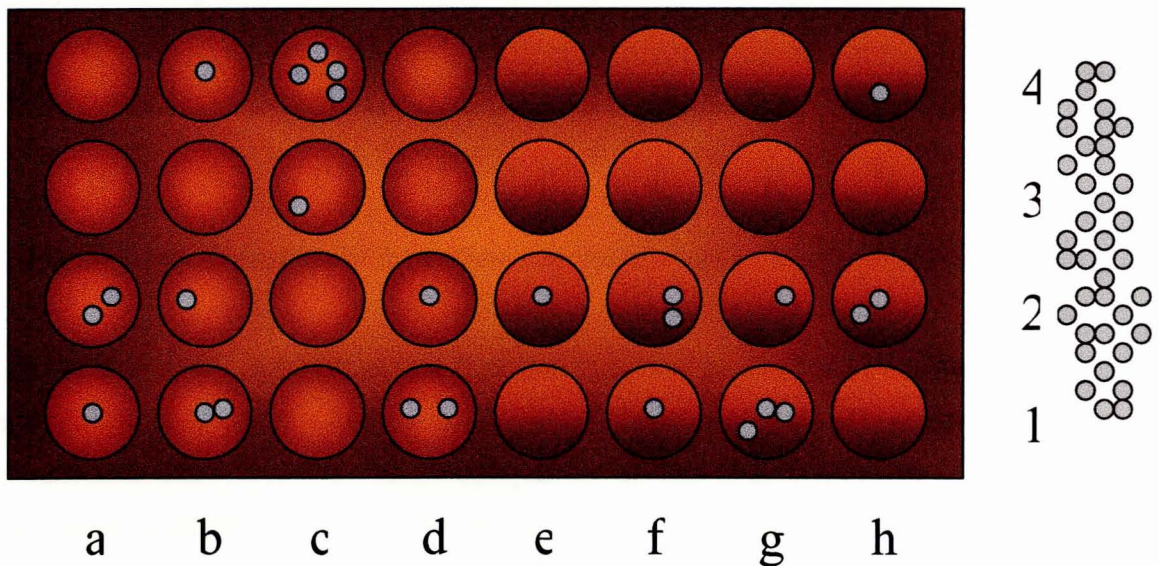


Figure 7.44a: Comments of the audience in regular-match 12, before the move

This decision was highly contested by the audience. They were indicating that the best thing to do would be to take the two seeds in a2 instead of three seeds in g1. My analysis indicates that the audience might have meant the following: 1st, that by taking g1 the player is not solving any problem, because even h2 would stop in c2, if his intention has been to avoid being beaten there. Second, it would be necessary to ‘destroy’ the situation of 2 and 1 seeds in the consecutive holes a2 and a1 respectively (See figure 3 in appendix

B12), because if it comes to wait for his opponent to advance to his inner row this position would make it to take longer and so play an important role. The player said: *'This is an offer.'*

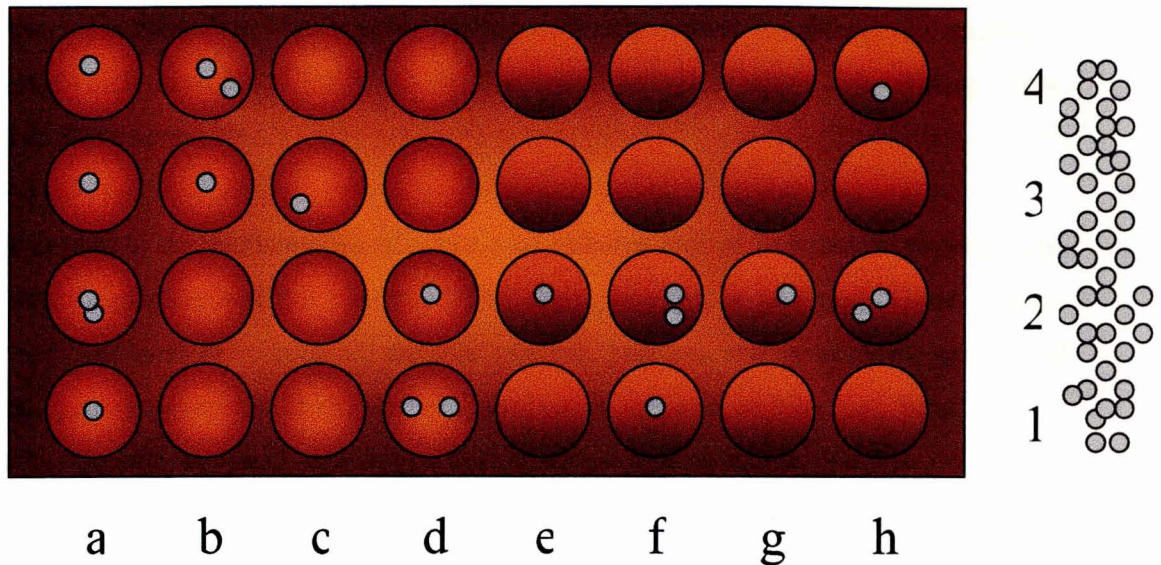


Figure 7.44b: Comments of the audience in regular-match 12, after the move

Also, in his next move, the 13th, player A played in h2 and landed in c2 (See figure 3 in appendix B12). He captured the seeds in c3 and h4. Once more a lot of contestant: *'Oh, you should have taken the seed here (pointing to a4) instead of here (pointing to h4).'*' In fact, taking a4 would limit the opponent on one possibility for playing during the rest of the match and so would be easy to beat him. There it was a possibility for the opponent for other possibilities. In fact, player A won the match in more 4 moves.

The comments and contests actions of the people watching the matches of *Tchadji* are very frequent and this makes the moment of playing *Tchadji* a moment of interaction. Certain values and ethics are also put in consideration. For example, when older players are playing against young players, the kind of comments made, are of more respect (For more comments see also figures 5 and 6 in appendix B2, figure 4 in appendix B7, figure 4 in appendix B11 and figures 1 and 4 in appendix B15).

7.3 Discussion of findings

The intention in this part of the study was to uncover mathematical ideas involved in the *Tchadji*-game and in the procedures for playing it. Since ethnomathematics has emerged as a research area, there have been studies on mathematical knowledge in traditional cultures, which have been informed by the *anthropological approach* (Bishop 1994; Vithal & Skovsmose, 1997).

In this research, the mathematics of traditional cultures and of indigenous peoples, who may have been colonized but continued with their practices, is analysed. These cultures have been explored in relation to: number systems, gestures and symbolism, games and puzzles, geometry, space, shape, pattern, symmetry, art and architecture, time, money, networks, graphs or sand drawings, kin relations and artefacts. Paulus Gerdes, in Mozambique, has been one of the major contributors of this research (see for example, Ascher, 1991; Zaslavsky, 1973a; Gerdes, 1985a, 1988, 1989, 1993, 1994a & b; Ismael & Soares, 1993; Ismael, 1994).

The results of this part of the study indicate that, in fact, there are mathematical ideas embedded in the playing of *Tchadji*. *Tchadji*-players do count, think and act logically, do calculations, predict moves, visualise situations, recognise different numerical patterns while playing *Tchadji*. It is like Bell & Cornelius (1988) argued, "*In trying to decide how best to play a particular game a player is forced into the need for logical thinking and thence, normally, to some mathematical thinking*" (p.2). This is also the case for the *Tchadji*-game.

These results are consistent with what Zaslavsky (1979) had argued that Mancala games, to which *Tchadji* belongs, require skills of counting and remembering numbers as well as higher-level strategic thinking and problem solving. Also, Sawyer (1949) referring to *Ouri*, another Mancala type game, commented as follows: "*It is, to the best of my knowledge, the most arithmetical game with a mass following anywhere in the world.*" (quoted in Mucavele, 1998: 63).

The game per se also involves mathematical aspects. For example, the existence of an ‘infinite’ move is directly related to the mathematical principle of infinity. In this respect, Zaslavsky (1996), referring to the Mancala type games in general, argued: "*It is based entirely on mathematical principles, and some experts rate it among the ten best games in the world*" (p. 189). In a similar view Odeleye (1977) describes *Ayo*, a Mancala type game, as "*A serious game involving a considerable skill and mental exertion. As an intellectual exercise it is on a level with chess. Experts can play it at great speed and it is not generally realized that it is a game requiring a considerable amount of skill at calculation*" (p.12).

With this investigation it was also possible to learn from the players other aspects of the *Tchadji*-game. For example, playing *Tchadji* has a long tradition and is a recreational activity. *Tchadji* is only played by men. Most players do not have much school education. They have learned a certain occupation, which they practise for a living. There is no school to learn *Tchadji*. People used to learn to play *Tchadji* by watching other people playing and by playing themselves. This result is consistent with Silva (1994), when he argued that Mancala type games are apparently very easy to play, but they require calculations, reflection and practice.

In playing *Tchadji*, different strategies and tactics are employed in trying to secure a victory. The knowledge about these aspects of playing *Tchadji* enriched this study. The idea of investigating the mathematical aspects of *Tchadji* through its players has proved to be fruitful. It would have been difficult to acquire this knowledge by other means.

7.4 Summary

This chapter presented and discussed the results of the ethnographic research done to explore the mathematical ideas embedded in the *Tchadji*-game and in the procedures for playing the game. The results included in this chapter reveal an empirical knowledge and

use of mathematics by the *Tchadji*-player, such as counting, logical thinking, calculation, visualisation, recognition of different numerical patterns, infinity, when playing the game.

These results are described as examples taken from different excerpts of the recorded *Tchadji* matches. This approach to the analysis of gives an insight into the complexity of the methodology used. It also uncovers different aspects of playing *Tchadji*, like rules, strategies and tactics. The value of continued investigations of this nature in order to uncover mathematical ideas in traditional culture is also illustrated by this research.

CHAPTER EIGHT: MATHEMATICAL IDEAS IN *TCHADJI* FROM THE PERSPECTIVE OF STUDENT TEACHERS: ANALYSIS AND DISCUSSION (PHASE 3)

8.1 Introduction

The idea of presenting a cultural instrument to student teachers has been reinforced by the arguments of scholars in the area of ethnomathematics, who stress that it is important to make teachers aware of the mathematical elements in their own culture(s).

Teachers are an important part of the whole process of mathematics learning and their role in the mathematics learning process is crucial. Thus, it is important to involve teachers for ultimate implementation of ethnomathematical ideas in the classroom. A good moment to do it is during teacher education. Gerdes (1998b) stressed the importance of this exercise as follows: "*The process of developing among future mathematics teachers an awareness of the social and cultural bases of mathematics, which contributes both to enhance their self-confidence and their capacity, and readiness and openness to work in a multicultural environment*" (p.33).

Acting in this way makes mathematics teacher education classrooms become a place of reflection, of experimenting with the raising of mutual cultural awareness, where the social-cultural tensions relate to the differences across rural versus city, manual work versus intellectual work, and models coming from abroad versus models with roots in the cultural environment (Cf. Gerdes, 1985a, 1986, 1998b). On the other hand, it is also important to ponder on the strategies that might stimulate and motivate mathematics teachers to multi-culturalise mathematics teaching (Cf. Gerdes, 1994a).

The developing an awareness, in teachers of mathematics in Mozambique, of the social and cultural bases of mathematics has to be an integral part of teacher education and training programs. The use of cultural instruments and activities such as cultural games is a continuation of this process and this is the intention of this part of our research. Teachers

need to be involved, since they are the implementers of ethnomathematical ideas in the classroom. Ethnomathematical research has to be seen not only as uncovering mathematical ideas from cultural traditions but also as looking for way to bring these ideas into the classroom in a way that it works. The role of teachers is crucial in the school implementation phase of ethnomathematics. Thus, the intention here was to have a sense of how it works with teachers in the phase when they are being trained.

This chapter discusses the results of example of continuation of the process of developing an awareness of the social and cultural bases of mathematics in teacher education. The example took the form of a workshop on mathematical and educational exploration of the *Tchadji*-game with student teachers.

In this chapter, the mathematical ideas and the possibilities of their use in the mathematics classroom resulting from the exploration of the *Tchadji*-game by the student teachers are described. A discussion of the attitudes and thoughts of student teachers when confronted with the game are also included in this chapter.

8.2 Mathematical ideas in *Tchadji* from the perspectives of student teachers

The idea of the workshop was to determine how the student teachers cope with the *Tchadji*-game as a cultural instrument from their environment, in terms of whether they are able to explore the game for mathematics teaching purposes. Also, from the affective perspective, it was intended to study the student teachers' attitudes towards mathematics in this process of working with the game, i.e., whether they enjoyed working with this game (See sections 8.3 & 8.4).

The student teachers were asked to explore specific questions such as, '*If the Tchadji-players count when they play the game, when do they count and how?*' (See more in section 7.2.1). The challenge presented to the student teachers was similar to the second phase of this study from the players' and researcher's perspective, i.e., to investigate the mathematical ideas of the *Tchadji*-game among the players and to document them with the

purpose in mind of their possible use in the mathematics classroom. This section describes the mathematical ideas in the game as revealed by the analysis of the student teachers.

8.2.1 The opening move

During the workshop, student teachers were provided guideline questions (See chapter 4) in order to prompt them for reflection on possible ‘mathematical’ ideas involved in the game. One of the questions was the following: In all first moves, the objective of the player starting has been, in most cases, that of capturing the opponent’s seeds. In which of the holes should the player start to be able to capture his opponents seeds and why? Which of these positions do you consider the best and why?

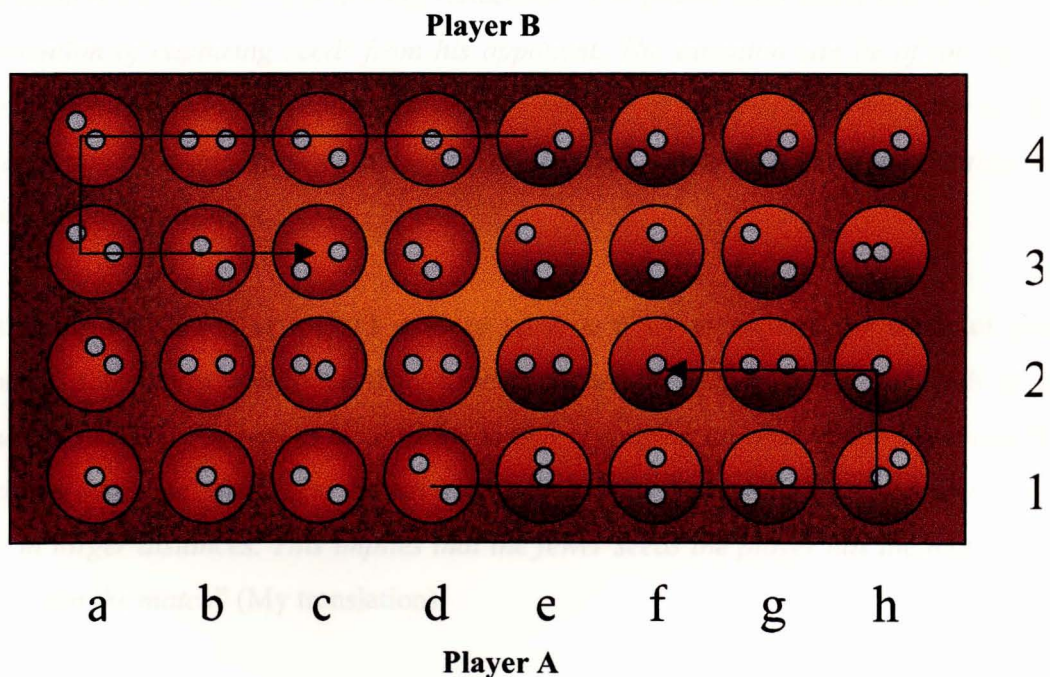


Figure 8.1: Starting positions for captures in the *Tchadji*-game

In fact, if we consider two players, A and B at the 8x4 board with 2 seeds in each hole at the start. The player who first plays (first move) will stop 5 holes ahead after having made a round in his two rows. So, for example, if player A (See figure 8.1) starts in h2 he will stop in c2 or if he starts in g1 (frequently used) he will stop in e2. Thus, there are holes in which a player starts and is able to capture seeds of his opponent (i.e., due to rule 2b, see

chapter 5) if he stops in his inner row. There are other holes from which a player starts and makes no captures (i.e., if the player stops in the outer row according to rule 2a, see chapter 5).

Here, the students, in general, were able to explore the board and find out that in the case of player A, there are 8 starting positions that lead to captures and another 8 that lead to no captures. The 8 starting positions for capture are 5 in the outer row (d1, e1, f1, g1 and h1) and 3 in the inner row (f2, g2 and h2). For no captures there are 3 in the outer row (a1, b1 and c1) and 5 in the inner row (a2, b2, c2, d2 and e2).

When asked about the intention of a player's first move, one group of students (group 1, see section 4.4.2) wrote the following reflection: *"The player who starts has not always the intention of capturing seeds from his opponent. The intention can be of forcing his opponent while playing to accumulate a large number of seeds in different holes so that he can capture a big number of seeds at once at some turn, thus debilitating his opponent"* (My translation).

Questioning whether it is a good strategy to always try to capture the seeds of your opponent, this group concluded that in some cases, *"A player allows his seeds to be captured, instead of avoiding it, until his match is singled, so that he can dominate the match. In this case, the player moves his seeds hole by hole while his opponent has to move in larger distances. This implies that the fewer seeds the player has the more likely he is to win the match"* (My translation).

Concerning these two issues, another group (group 2), stated: *"It is recommended that the player who first plays, starts in the 3 first holes of his inner row and in the last five holes of his outer row. So, this gives him the possibility of capturing seeds."* On another occasion, the same group concluded that, *"We should capture the seeds of our opponent as much as we can, because only in this way we can destroy the match of our opponent which is advantageous for us. However, in some positions we should even avoid to make captures. For example, we should not capture seeds leaving our opponent with only one*

seed in isolated holes (singled). This makes it difficult to control the moves of our opponent” (My translation).

Group 3 agreed with the position of the group 2, when they stated that, for general orientation, *“The player should capture the seeds of his opponent in order to be able to win the match.”* Elsewhere they gave the following explanation: *“The player who first plays has 16 distinct possibilities of initiating the match. For these, 8 give him the possibility of stopping in the inner row while another 8 possibilities allow him to stop in the outer row, i.e., in this case, without the possibility of capturing seeds. For the start of the first move the player seems to make the following calculations: $2 + (2+1) + (2+1) + (2+1) + (2+1) + (2+1) + (3+1) = 21$, i.e., he does a complete round and stops 5 holes ahead ($21 - 16 = 5$)” (My translation).*

The same group later made the following statement: *“From the 8 possibilities that the player has in his first move in order to capture seeds, there are some better than others, i.e., those from which the opponent, at his turn, will have more difficulties [less possibilities] of capturing seeds. They are the when player A starts from the holes d1 and f2, which end in the corners h2 and a2 respectively. In this case, capturing the contents of 3 consecutive holes, i.e., holes h3, h4 and eventually g4 by starting in d1 (a3, a4 and eventually b4 by starting in f2). The opponent will then have only one position in which he can capture seeds” (My translation).*

The students meant that, as in the figure 8.2a, B at his turn would have only one place to capture seeds by stopping in h3. To land in h3, player B can play either from f3, c3, a4 or d4. From d4, the longest move, and always 3n holes ahead ($n = 1, 2$ or 3), i.e., three holes ahead in a4, six holes ahead in c3 and nine holes ahead in f3. An equivalent situation for player A is presented in figure 8.2b.

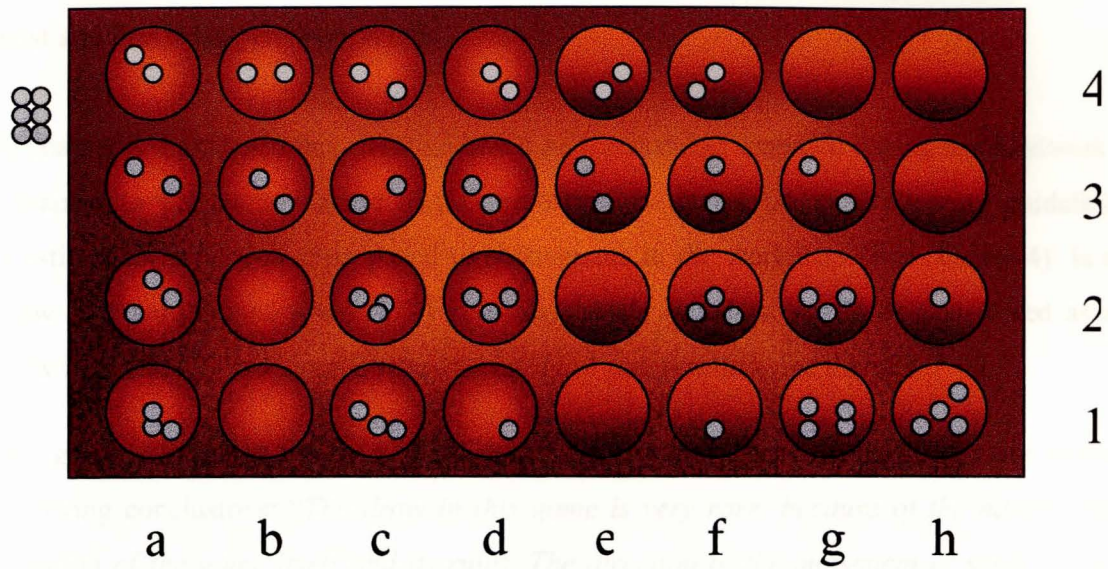


Figure 8.2a: Start in d1 by A

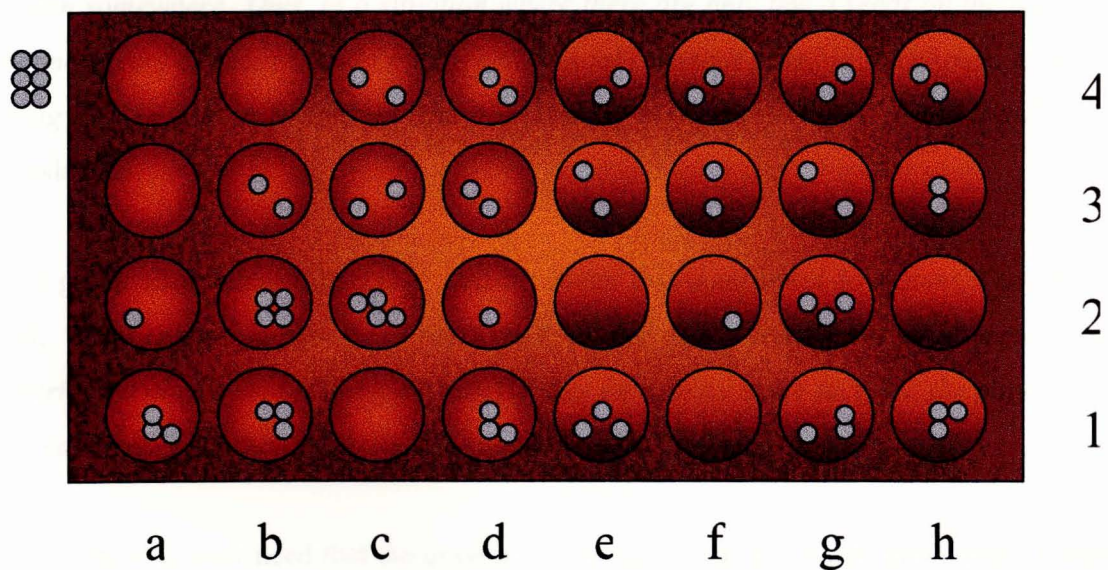


Figure 8.2b: Start in f2 by A

8.2.2 The draw in *Tchadji*

The explanations given by the *Tchadji*-players during the interview session at ‘Ilha de Moçambique’ (See section 7.2.5) was not clear with regard to the concrete situation in

which a draw happens in the *Tchadji*-game. However, the players recognised that it does exist and that it happens very rarely.

In chapter 6, I analysed this situation based on the matches recorded during the fieldwork. I wanted to get another sense of this aspect of the game. So, the following guideline questions for reflection were posed to participants in the workshop (See chapter 4): Is a draw in *Tchadji*-game possible? If possible, which situations would be considered as a draw in a match? How often do they occur? How can you describe them?

One group, discussing the issue of the reason for a draw in the *Tchadji*-game, came to the following conclusions: *“The draw in this game is very rare, because of the nature and situation of the game itself and its rules. The direction of the movement of seeds in the board cannot be inverted. This aspect imposes limitations of possibilities to avoid being beaten somewhere. Thus, in a situation where there are only few a seeds on the board, if the movement of seeds could be inverted, it would make it possible for players to avoid being beaten and consequently there would be more possibilities for a draw”* (My translation).

This group also mentioned another possible factor that contributed to a draw. They wrote that, *“The second factor that contributed to the quick conclusion of a match and contributed also to diminishing the possibilities for a draw is the fact that the players are forced to capture seeds according to the rules”* (My translation).

The group was convinced that the draw is a very rare event in *Tchadji*-game, since a draw seemed to be more of a theoretical construct. They wrote: *“There are situation of a draw which we think are more of a theoretical character. In a played game it is extremely rare to come to those situations”* (My translation). The student teachers in this group were able then to describe and define the draw depicted in figures 8.3a and 8.3b.

The same group of student teachers tried to determine and classify different draw situation in *Tchadji*. They explained that *“a conditional draw: a draw is only possible if the match*

initiative of one player is pre-determined. If, however, it is the second player to move, even in this situation, the draw does not happen. In the situation of figure 8.3a, the draw only happens if it is the player in position B to play. In figure 8.3b, in order for a draw to happen player B must play first. A non-conditional draw: here it does not matter which player plays first, a draw always happens (See figure 8.4)” (My translation).

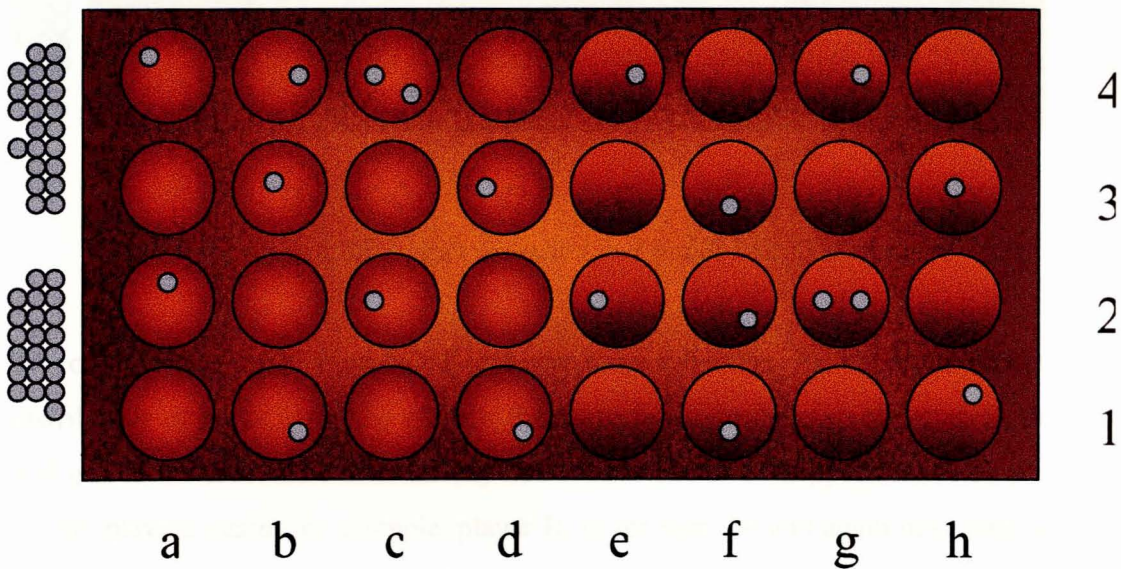
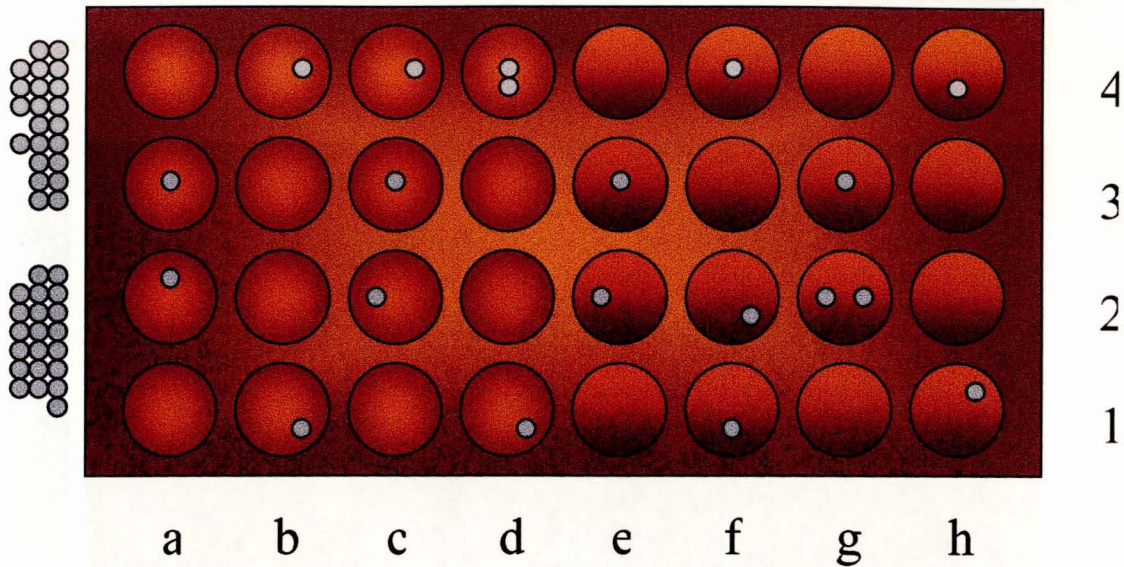


Figure 8.3a: A draw situation in *Tchadji* with A starting

The students in this group did not explain why they considered that these situations were draws. I did find the same situation in early 1990s when I first became interested in this matter by working with students (See Ismael, 1996). What happens there is that the distribution of seeds for both A and B are identical in all 3 figures (See figures 8.3 a, b and c). In figure 8.3b, for example, the match for B starts in c4, which due to the rule 3 (See Chapter 5) is the only possibility for playing, whereas the match for A starts in g2, but it could also be starting in f1, where it would be symmetrical to the match of player A.

Figure 8.3b: A draw situation in *Tchadji* with B starting

To complete this explanation I shall add here some reflection. An important aspect of this distribution of seeds is that by starting the move in d4, in the case of player B, the move will end in the same hole after exactly one round. The same happens for A by playing in g2. By playing again, for example, player B, in his turn, he will again have only one hole with 2 seeds in b4, playing there the move will again end in b4, another one round play, and the seeds will be distributed as in the figure 8.3a, i.e., after two moves by each player the initial situation arrived again. By thinking so, if the players continue playing, they will never come to the end of the match (an indefinite match), i.e., neither A nor B will be able to capture seeds, so it is a draw.

Only if the starting position is situated in the inner row of at least one player, as is the case for player A in Figures 8.3a and b, is there a possibility of this player capturing seeds of his opponent, and so destroying this composition of seeds which would then bring the match to an end. In the case of figure 8.3 a, if player A plays first, a draw happens anyway, because in his position, the move would begin and end in g2 after a round and because g3 is empty, no captures would be made. The next move by A would then be in f2 and stopping in d2, but B at his turn, before this move, would have removed the seed in f3, so that A would not make any capture, and so on. However, if B plays first, then A at his

turn would be able to capture seeds in g3, which would not be empty. A similar situation occurs (see Figure 8.3b), if player A plays first.

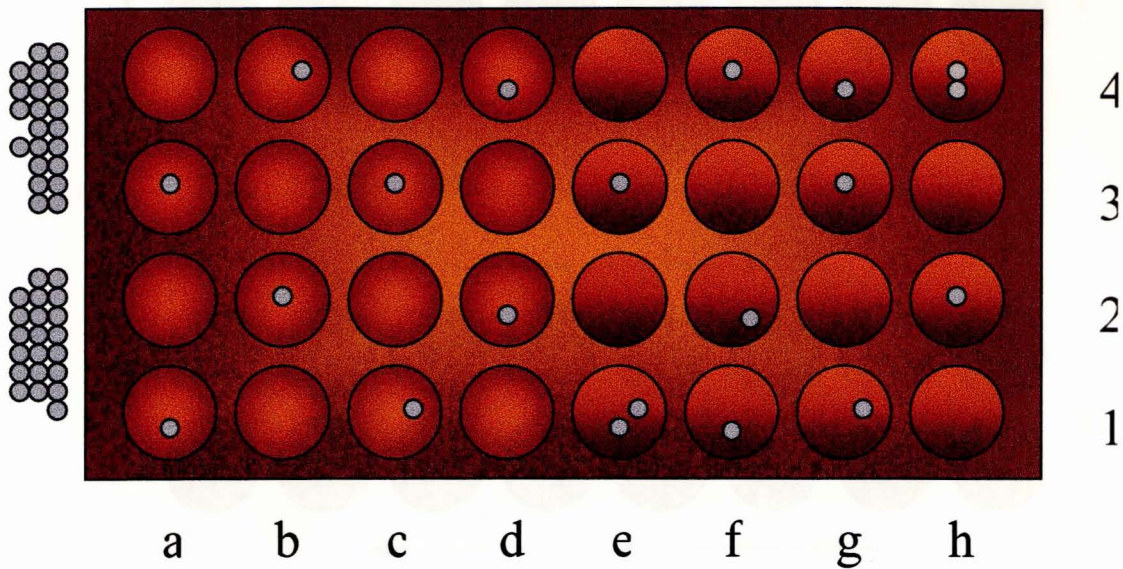


Figure 8.4 A non-conditional draw situation in *Tchadji*-game

In the figure 8.4, the non-conditional draw situation, it really does not matter who starts first, because the landing holes are always situated in the outer row. If we consider those situations always in symmetrical position, there are 8 symmetrical positions, in which a draw happens independently of who is playing first. Considering also the non-symmetrical position, there are $7 \times 7 = 49$ different positions.

If it is possible to consider both directions, clockwise and counter clockwise, in *Tchadji* it can be observed that after one move of the seeds on the board, the distribution of the seeds becomes inverted, i.e., after a round, if one looks in either direction, the distribution of the seeds are exactly in the same way. Playing again you come to the initial situation, in fact, a double inversion of a function is again the function. For example, in the figure 8.5a, the match in counter clockwise direction shows a start from the hole with 2 seeds and landing at the same hole, resulting the situation in figure 8.5b.

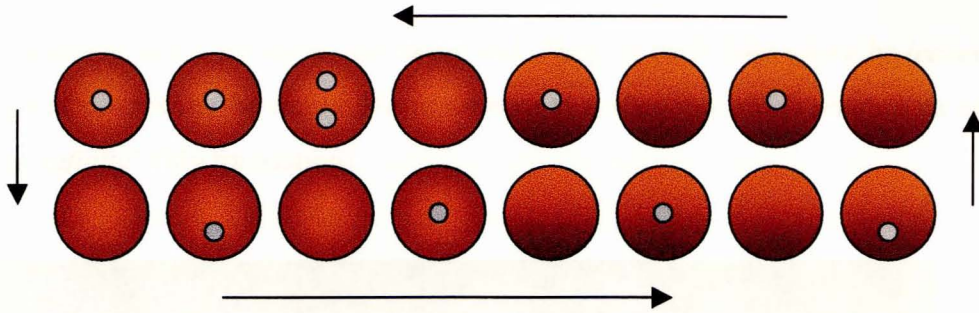


Figure 8.5a: The draw-move in a counter clockwise direction

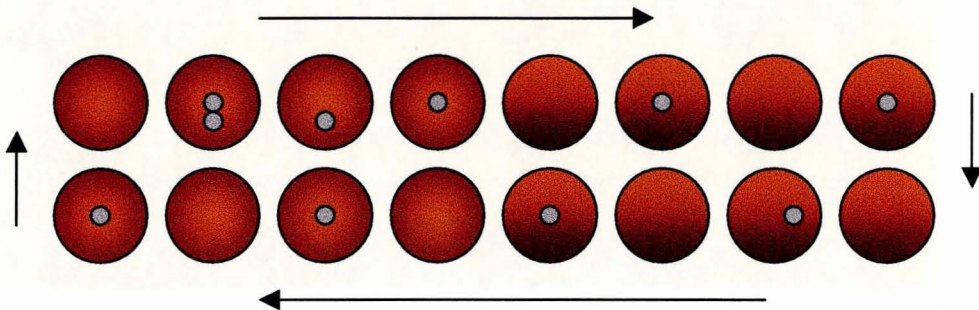


Figure 8.5b: The draw-move in a clockwise direction

Another group recognized a draw by referring to it in the following way: “*Fundamentally, there are two types of draw in Tchadji: first, when a move of one of the players never ends and second, when the existing seeds in the board do not allow for captures on both sides. The first situation happens with less regularity and when it happens, the players normally come to a consensus to jump a hole in order to be able to stop, so that the match can continue and come to an end. The second situation is more frequent*” (My translation). However, this group did not present illustration of the mentioned cases of a draw in the Tchadji-game.

A new aspect in the students’ perspective appears here. It is the case of a draw move, which never ends, i.e., an infinite move (See section 7.2.5.1). Another group presented the following example for this situation (See figure 8.6).

The students in this group wrote the following: “*A case, very rare, has been found with the seeds of the player in position B, for example, distributed as in the figure [figure 8.6].*”

So, for *B* starting in *b2*, the move will never end. Then, in each hole where he leaves the last seed will always not be empty. We come here to a situation which reminds us of the concept of infinity” (My translation).

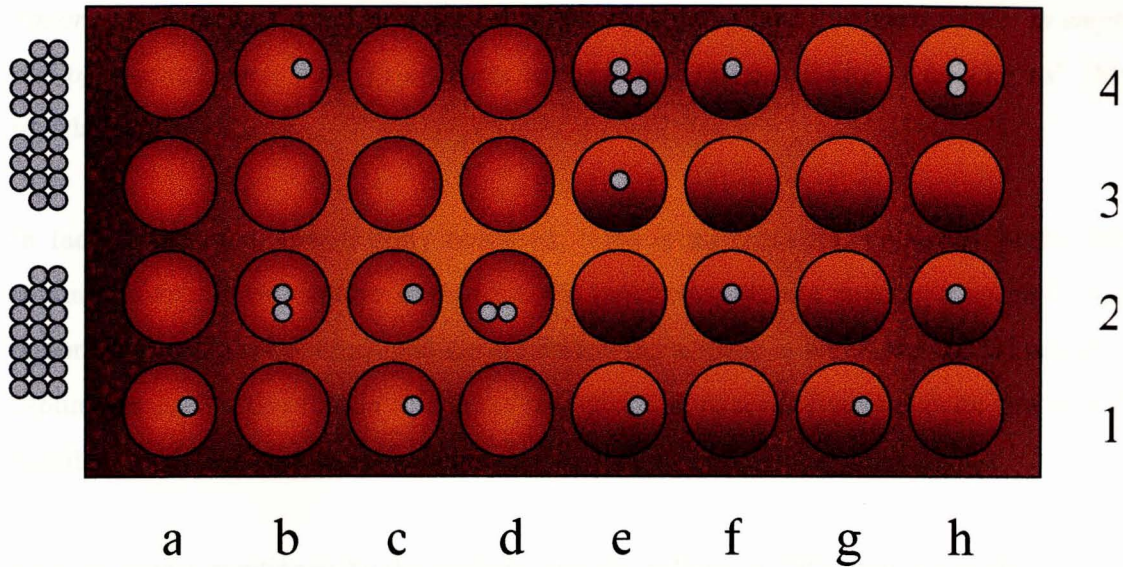


Figure 8.6: An infinite move with conditional start

8.2.3 Strategies for victory

Practising the game excited the students who participated in the workshop. They wanted always to win the matches against their colleagues. So, they become engaged in reflecting on factors that contribute to a victory in the *Tchadji*-game. Of course, either of the players naturally have victory as objective. For this reason it is important to analyse the strategies that one develops to come to a victory or, at least, to try to come to a victory. The students were interested to understand what the master players of *Tchadji* may do in order to secure a victory. The following are the results of the students' independent reflections without knowing what the *Tchadji*-players did.

Generally, the students seemed to have agreed that the game is complex and, for this reason, “*There are various factors that contributed to a player having a victory in a match of Tchadji.*” Some groups tried to refer to the strategies: “*Among these factors we*

shall distinguish the following: first, the player is preoccupied with capturing the most number of his opponents' seeds, taking mostly seeds from his opponent's outer row, trying to empty most holes of this row because from there he can counter the best moves of the opponent and secondly, the player has to avoid his seeds being captured by the opponent." A second group supported this idea by arguing that: *"we must try not to empty our opponents' inner row. From there we give him the possibility of beating us"* (My translation).

In fact, if the inner row is fully occupied, there is no room for any easily predictable capture moves. If we capture seeds from the inner row, we are opening a space for our opponent to capture our seeds and we are destroying a space in which we could eventually capture seeds. For this reason it is better to capture more seeds from the outer row and, if possible, seeds from consecutive holes.

Another aspect mentioned by the students was the following: *"We shall try to destroy any possible connections that our opponent can make in order to be able to beat us, by emptying the most number of consecutive holes in our opponent's outer row"* (My translation). In fact, a player with several empty holes in the outer row may require two or more consecutive moves before he can capture any seeds. Such a situation is to the advantage of his opponent (See also chapter 7).

The students also recognised that there was some thinking involved in the game as it is in every game. Concerning this issue one group wrote the following: *"There is also one aspect to take into account while playing Tchadji as strategy for a victory, which is to try to understand your opponent thinking with regard to the construction of his moves in the match."* They also were convinced that it is necessary not only to think before you play but also that it is important to think quickly. As it appeared in one group assignment, *"Another determining aspect for victory is quick thinking and quick acting of the players"* (My translation).

Insofar as the rule of capturing the seeds specifies only two (or some time only one) of the three holes from where captures are made (See section 5.4.2.1.1), the choice of the third hole is of particular importance. Some groups of students recognised this aspect when stating that, *“How and where a player captures seeds, in particular, the capture of the seeds of a third hole is a determinant of a victory”* (My translation).

In order to become a good player, in any game, requires practice. Recognising this, the students, with regard to *Tchadji*-game, noted that, *“The memorisation of most moves in the game is also a determining factor to get a victory. Thus, good players are those who regularly practise the game. It is in the regularity of practising the game that the master players develop certain abilities and some master players automate most moves. These players can easily discover the thinking of their opponents from their moves and so they can prevent the next moves”* (My translation).

8.3 Student teachers' attitudes towards mathematics

Practising the *Tchadji*-game has been an exciting activity for the students during the workshop. The game was familiar to the students. Most of the students, as they said at the first session of the workshop, have at least seen people playing one of the variants of the 4 rows type of Mancala games. However, the idea that *Tchadji*-game could be useful in teaching mathematics was totally new to most of them. Thus, the idea of analysing mathematical aspects of this kind of game as part of Mozambican culture was a surprise for most of the students.

Thus, in order to examine the level of the students' attitudes towards mathematics a questionnaire (See section 4.4.3.1 & Appendix A) has been administrated at the beginning of the workshop and at end. The items of the questionnaire were single statements about mathematics and the daily life activities including games. The questionnaire is composed of 24 Likert scale type items with a rating from 'Strongly Agree' to 'Strongly Disagree' scaled 5 to 1 (Burns, 1995).

The data obtained from the questionnaires were analysed by using SPSS 10.0 for windows. Due to the type of data gathered by and the number of subjects involved, the analysis was done by means of descriptive statistics. In the questionnaires, both the pre- and post-questionnaire, some open questions were also included for the students to comment on. In this section I present the analysis of data from the questionnaires.

The analysis of the questionnaire data has followed the features defined in the methodological chapter (See chapter 4). Thus, in order to find out what the attitudes towards mathematics are among the student teachers and what the overall changes in attitudes that take place during the workshop are, the overall average (mean) of both the pre- and post-questionnaires and their difference has been examined.

The overall mean of the pre-questionnaire (See appendix A31) items was of **4.13** with a standard deviation of **0.34**, whereas the overall mean of the post-questionnaire (See appendix A32) has been **4.01** with a standard deviation of **0.21** (See table 8.1).

		Pre-items	Post-item
N	Valid	24	22
	Missing	0	0
Mean		4.1267	4.0114
Std. Deviation		.3386	.2070

Table 8.1: Overall statistics for attitudes towards mathematics scores from the Pre- and Post-questionnaires

In general, I may say that very small changes were observed. The differences in the means were not statistically significant with $t = 1.826$ and $p > 0.05$ (See more in figure F9 in Appendix F). An interesting feature is the fact that the overall average on the pre- and post-questionnaire are greater than 4, i.e., above average. This fact may imply that the student teachers, as students of a university, where several activities related to the area of

ethnomathematics have taken place and through publications from the MERP, even though they were not directly involved, they may have become familiar with activities of the project, and might have agreed with the approaches even before the workshop.

8.3.1 Student teachers' attitudes towards the relationship between mathematics and culture

8.3.1.1 Quantitative data

The items of the questionnaire were classified as being related to many aspects of attitudes (See Chapter 5). The questionnaire deals directly with attitudes at times but it also has statements related to practice and views of mathematics (#5, 6, 12, 14, 15 and 21); methodology of teaching and learning mathematics (#9, 10, 13 and 17); relationship between mathematics and culture (#7, 8, 16 and 18); usefulness of mathematics (#2, 3, 4, 19, 20, 23 and 24) and confidence in learning mathematics (#1, 11 and 22). The statements 13, 15, 16, 17, 19, 20, 23, and 24 can also measure beliefs on the relationship between culture and mathematics, i.e., whether or not it is believed that mathematics is a culture-free subject.

Considering an alternative classification, we can say that there were 13 items directly related to the issue of the relationship between mathematics and culture. In these items, a little change has been observed but not a statistically significant one (See figure F10 in Appendix F), since the overall average of these items in the pre-questionnaire has been of **4.2** with a standard deviation of **0.37**, whereas the overall mean of the post-questionnaire has been **3.97** with a standard deviation of **0.20** (See table 8.2). The students seemed not to have changed their view on the relationship between mathematics and culture. They are strong convinced that there is a relationship between mathematics and culture. This can be observed in the average in these items.

		Maths & CULTURE Pre-items	Maths & CULTURE Post-items
N	Valid	24	22
	Missing	0	0
Mean		4.2051	3.9755
Deviation		.3704	.2020

Table 8.2: Statistics for “Mathematics & Culture” attitudes from the pre- and post-questionnaires

A further 11 items of the questionnaire were regarded as being related to other areas of general attitudes towards mathematics. There, the average has been greater than 3, i.e., above average, both in the pre- and post questionnaire (See table 8.3).

		OTHERS Pre-items	OTHERS Post-items
N	Valid	24	22
	Missing	0	0
Mean		4.0341	3.6405
Std. Deviation		.3270	.2798

Table 8.3: Statistics for general attitudes from the pre- and post-questionnaires

Also, in general attitudes, they were no significant differences observed (See figure F10 in Appendix F). It seems that the student teachers have, generally, positive attitudes towards mathematics and also towards use of other instruments for the purpose of learning mathematics. For mathematics student teachers, it is normal that they already have positive attitudes towards the subject they are studying.

8.3.1.2 Qualitative data

What was the students’ opinion or view about the relationship between mathematics and culture? What were their views after having experienced a work of exploration of a cultural element? It was important to have an idea about this. For this reason, in the post-

questionnaire, after the students have been involved in the workshop with the *Tchadji*-game during a period of two and half months, they were asked to write any comment concerning the relationship between mathematics and cultural elements.

The student teachers involved in this project, in general, recognised that in one or other way mathematics is related to culture. The relationship in many cases is, lets say, not visible, i.e., is implicit. There is always talk of application of mathematics in the culture, as a student wrote, “*Mathematics is strongly related to cultural aspects because mathematical knowledge is implicitly applied in various cultural activities*” or as another student wrote, “*The human being applies mathematics in his daily life activities. A peasant needs to know how much material is necessary for the construction of his house*” (My translation).

About this issue another student wrote that, “*There is a straight relationship between mathematics and cultural elements. Mathematics can be applied in board games without the person knowing that he is applying mathematics. Implicitly there is a use of mathematics there. For me the application of mathematics in cultural elements is evident*” (My translation). Another student attempts to explain this by writing that, “*Mathematics is related to culture because most practical activities require the application of mathematics. What happens is that, many people apply mathematical knowledge unconsciously. Generally, the people living in the countryside, according my experience, are able to control their products and animals.*” “*Mathematics is a science. Cultural elements are related to science, but not explicitly (...). When a cultural activity is idealised, there are important aspects related to the questions like ‘how to do?’ or ‘how many elements are necessary?’ and so mathematical science is used but implicitly*” (My translation).

Some students went much further when they argued that, “*Mathematics is a mother subject for solution of social, technical and cultural problems. Mathematics is sometimes applied in an indirect way, specially, in the countryside. A peasant controls his production in a satisfactory manner, but he never went to school to learn mathematics.*

Another example is that he uses mathematics in practising traditional games.” Another student wrote that, *“Starting from the point that mathematics is about applications in real life, it is evident that there is a relationship between mathematics and cultural elements”* (My translation). Here the role of mathematics in the real life activities of society in general is clearly recognised.

About the power of mathematics in real life, another student wrote that, *“Mathematics, as science, is in all aspects of life and its concepts and laws are as basic rules and guidelines for the real life. Culture being one constituent of our life, there mathematics is evident”* (My translation).

Other students remembered the role of mathematics within history, when they argued that, *“It is obvious that there is a relationship between mathematics and cultural elements. Look, if we go back in time, we will see that some of the biggest mathematical achievements in history, like the Egyptian Pyramids and the Pythagorean theorem, appear within cultural elements”* (My translation).

Concerning the *Tchadji*-game, motivated from the results they got (See section 8.2), the students were convinced of its relationship with mathematics, as is the case with several other scholars. They wrote that, *“In the Tchadji-game, for example, we find some mathematical ideas involved. When we count the number of holes for each move, the distribution of seeds in the board, the situation of a draw, localisation of holes to stop and principles for moving seeds and capturing seeds”* or that, *“In Africa, we practice board games, which show us issues of counting. In the construction of traditional houses people used to apply various geometrical conceptual rules which apply to circles, rectangle and triangle. For this reason I think that there is a relationship between mathematics and culture”* (My translation).

One student made reference to other type of games as having something to do with mathematics. He wrote as follow: *“There is a relationship between mathematics and culture, for example, in the playing of games such as Tchadji and Muravarava and in the*

production of cereals” (My translation). *Muravarava* is a three-in-a-row game played in Africa (See section 6.2.2), which seems to be also very rich in mathematical activities (See, e.g., Zaslavsky, 1989).

8.3.2 Student teachers’ attitudes towards cultural games and mathematics

8.3.2.1 Quantitative data

Because of the fact that, the student teachers were working with a particular game, it was important to know what their views were on the issue. Question 16 of the questionnaire (*There is no maths in board games practised daily*) was directly related to the relationship between cultural games and mathematics. Its mean in the pre-questionnaire was **4.33** with a standard deviation of **0.63**, whereas in the post-questionnaire the mean was of **4.72** with a standard deviation of **0.63**, so an increase 0.39 has been observed (See table 8.4 & figure 8.7).

Also, it can be observed here that, before the game was presented to the students to explore, 8.3% of them were not sure whether there was mathematics in board games or not, 45.8% agreed (A) and 41.7% strongly agreed (SA). In the post-questionnaire, however, 9.1% of the students were not sure, 9.1% responded that they agreed that there was mathematics in board games and the large majority of 81.8% strongly agreed about it (See also figure F11 in Appendix F).

		Pre	Post
N	Valid	24	22
	Missing	0	0
Mean		4.3333	4.7273
Std. Deviation		.6370	.6311

Table 8.4: Statistics for items 16 of the pre- and post-questionnaire

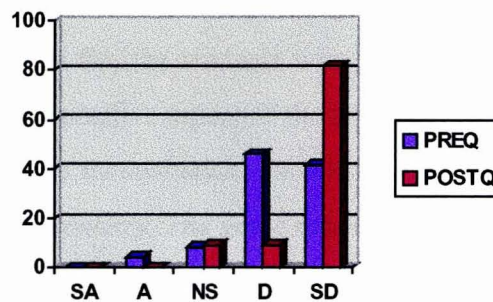


Figure 8.7: Bar chart of percentages of responses in all categories of item 16

The large scale difference on the average of this item in the pre-questionnaire is not surprising at all. It could mean that the students did believe that there was some mathematics in the games, but that they were not sure about this. It means that by the end of the workshop, after they had the opportunity to explore the *Tchadji*-game from a mathematical point of view, there were more students convinced about it than before.

Before the students knew that they would be working with a particular game, I asked them in the pre-questionnaire, to comment on the following issues: firstly, to give more comments on the response to item 16, i.e., the question which was directly related to mathematics and cultural games, and secondly, in order to get from the students some feedback on the content of the questionnaire, they were asked to comment on it. The results showed that 2 students, i.e., 8.3%, did not answer the question which could mean that they were probably not sure, 12 students, i.e., 50%, responded ‘disagreed’ (D), and 41.7%, i.e., 10 students ‘strongly disagreed’. Since the item was negative stated, this means that 91.7% of the student teachers were sure that there is mathematics in board

games. The students who were probably not sure about this statement did not give any comment.

8.3.2.2 Qualitative data

I was interested to know about their comments on the responses to the question 16 of the questionnaire (*There is no maths in board games practised daily*) and also in form of clarification of their positions. So, the students who responded 'strongly disagree' (SD), i.e., who fully agreed with the positive aspect of the statement, since the item was negative stated, seemed to recognise the mathematical knowledge in board games. As one argued, *"The board itself is already geometry. The game requires logical thinking. This is what to expect from a player. In order to win a game a player has to reflect on solutions for the various moves he encounters in the match."* Here, it is interesting to realise that, this student was thinking on a particular type of board games, whose board has a geometrical shape. It could even be the Mancala type games.

Generally, the students recognised that, board games involved a kind of mathematical thinking. One student argued that, *"This kind of games develop abilities of abstraction and quick and creative reflections"* or another student, who wrote that, *"This kind of game requires quick memorisation and mathematical calculation"* (My translation).

Also, there was a remark on logical thinking as part of general mathematical thinking. Some students argued that, *"The fact that you have to think logically is a mathematical exercise."* Another commented that, *"The rules of the game are, as I can say, mathematics norms"* (My translation).

The students who have chosen to respond 'disagree', i.e., who just agreed with the positive side of the statement, since the item was negative stated, gave similar explanations about their position. There were no differences in the responses. For example, a student wrote about a mathematical language, *"I think that in any type of game there is mathematics"*

involved. Mathematics itself is a language. When playing a game you are using a language, a mathematical language, in one or other form" (My translation).

Other students referred to the calculations involved, *"I think that it is necessary to apply some calculations related to geometry in board games,"* or as another student similarly wrote, *"Every move in a match is made after some calculations in order to make sure of its success"* (My translation).

The rules of the game imply mathematics, or as a student wrote, *"The board games have rules and all these rules are related to mathematics."* In fact, as another student argued, *"There is no game without logical thinking"* (My translation).

The fact that, as another student wrote, *"Generally, in the board games the player is forced to think in order to prevent disadvantages of any action in the game and this thinking is done within mental calculations and logical reflections"* (My translation).

Recognising the empirical mathematical skills of the players of these games some students argued that, *"The people who practice these games possess certain mental abilities in terms of calculations and this is mathematics."* They *"use mathematics without knowing that they are using it. They need to know, for example, how many holes they have to jump in order to be able to capture seeds."* Also, *"Sometimes there are situations of interconnection of seeds and of the distribution of seeds in the board which require quick mathematical thinking for the success of the move"* (My translation).

8.3.3 Student teachers' attitudes towards home experience and mathematics in school

Another important issue related to this work was to have the students' views about whatever the advantages are in taking into consideration the home experience in the

learning of mathematics in school. So, a question of using the experience that the students bring from home to help in learning mathematics was included in the questionnaires. The items 10 and 13, which were directly related to this issue were closely examined.

From item 10 (*Home experience can help to learn maths in school*), the mean in the pre-questionnaire was **4.45** with a standard deviation of **0.58**, whereas in the post-questionnaire the mean was of **4.27** with a standard deviation of **0.45** (See table 8.5 & figure 8.8). There were no statistically significant differences observed (See also figure F11 in Appendix F).

		Pre	Post
		<i>Home experience can help to learn maths in school</i>	<i>Home experience can help to learn maths in school</i>
N	Valid	24	22
	Missing	0	0
Mean		4.4583	4.2727
Std. Deviation		.5882	.4558

Table 8.5: Statistics for items 10 of the pre- and post-questionnaire

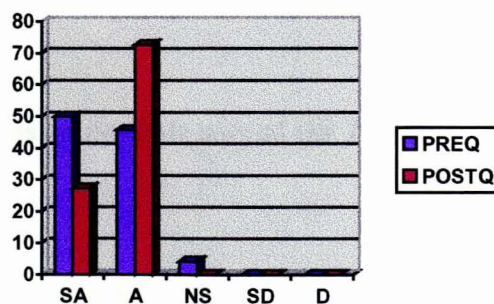


Figure 8.8: Bar chart of percentages of responses in all categories of item 10

For the statement 13 (*There is a relation between maths in school and at home*), the mean in the pre-questionnaire was **4.04** with a standard deviation of **0.85**, whereas in the post-

questionnaire the mean was of **3.95** with a standard deviation of **1.04** (See table 8.6 & figure 8.9). Also in this item, they were no statistically significant differences observed (See also figure F11 in Appendix F).

		Pre	Post
		<i>There is a relation between maths in school and at home</i>	<i>There is a relation between maths in school and at home</i>
N	Valid	24	22
	Missing	0	0
Mean		4.0417	3.9545
Std. Deviation		.8587	1.0455

Table 8.6: Statistics for items 13 of pre- and post-questionnaire

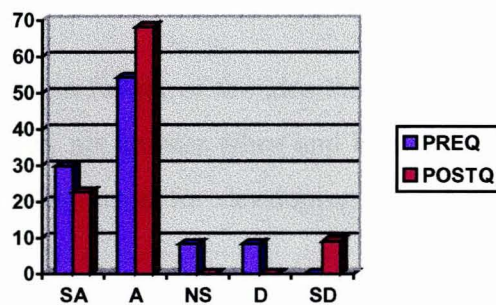


Figure 8.9: Bar chart of percentages of responses in all categories of item 13

8.4 Student teachers' ideas on possible uses of the *Tchadji* in the mathematics classroom

The workshop with students was set to find out how the students, in the situation where they are, are able to cope with the *Tchadji*-game, in terms of, whether they are able to recognise the possible mathematical-educational potentialities of these kind of games, and whether they are able to describe those recognised potentialities. Thus, apart from the exploration of mathematical ideas embedded in the *Tchadji*-game, it was recommended to

the students to reflect also on possible uses of ideas from the game in the mathematics classroom. In this section I present the ideas given by the students.

One group presented the following reflection: *“In the first years of the primary school, the children could learn more effectively by using a Tchadji-board.”* They further commented that, *“if the children want to reach a certain position in the board, they are forced to count and so they consolidate their counting skills.”* They also wrote that, *“Tchadji can develop the logical thinking in children”* (My translation).

Another group wrote the following, *“In our opinion, this game should be introduced in our schools for development of cognitive qualities of the pupils, development of a quick logical thinking and mental operations. It would also be a form for acquisition of moral values and social experience”* (My translation).

In general, the students were in favour of the idea that, Tchadji can be introduced in our schools. They were, however, not so sure when and how this should be done. They also recognise the potential of this game for developing logical thinking and other mental operations.

8.5 Discussion of findings

The purpose of the work with student teachers on the *Tchadji*-game was to find out how able they were in analysing the mathematical ideas embedded in the *Tchadji*-game. In other words, having been given a mathematically rich cultural resource, was there an indication or not that they were able to transform it into a resource for teaching mathematics.

Generally, I can say that the student teachers who participated in the workshop with the *Tchadji*-game, were quite excited to know and experience the richness of the game in terms of the mathematical ideas and mathematical activities, which could be used in the

mathematics classroom. Shirley (1995) recognised this, when he argued that *"Recreational mathematics is informal, often coming from games and puzzles, and it is pure, usually without a direct practical application beyond enjoyment and intellectual beauty, (but) it is rich in opportunities for classroom activities. One that is appropriate for all levels of school is the game generically called Mancala"* (p. 39).

On the other hand, the student teachers were able to analyse the game and results showed that they were able to identify embedded mathematical ideas in the game. For example, in the analysis of certain aspects of the game, like the opening play in *Tchadji*, the draw and other strategies in the game, they were able to identify some measures which related to logical thinking, counting and empirical and mental calculation.

In this regard, Gerdes (1996) argued that the *"Incorporation into the curriculum of elements belonging to the socio-cultural environment of the pupils and teachers, as starting point for mathematical activities in the classroom, is likely to increase the motivation of both pupils and teachers"* (p.928).

Also, Abreu (1993) stressed the importance of concrete action in order to bring changes in teachers and that research needs to be done in order to find out the best ways for achieving this. She argued that, *"teachers need to be able to cope with different mathematical tools and to take on an analytical role which contrasts with the traditional prescriptive role"*(p. 271).

Another intention of this part of the study was to research the potential impact, in terms of attitudes towards mathematics and the mathematical ideas developed, of interaction with the *Tchadji*-game on teacher education in Mozambique. Would the *Tchadji*-game be rejected by the student teachers or accepted as a useful resource for activities in the mathematics classroom, or, at least recognised as such. So, what would their views be on the use of the *Tchadji*-game as starting point for doing mathematics in schools?

In fact, it is important, first of all, to make teachers aware (Cf. Gerdes, 1998b) of cultural diversity and that this diversity can lead to different developments in mathematics. This can happen during a teacher education course. It is in this process that teachers can develop favourable views and attitudes towards mathematics. It is indeed important that the teachers themselves become interested and motivated because, in this way, they will be much better prepared for motivating pupils' interest in mathematics. As Sellinger (1994) stated, *“one of the most crucial aspects of the teacher's role is to motivate young people's interest in mathematics. One way to achieve this is by appealing to the pupils' own experience”* (p.113).

The results indicate that, in general, there were no statistically significant changes in affect observed over the duration of the intervention. The attitudes of the student teachers were quite positive with regard to different aspects included in the questionnaires. Their views, as they were observed in the written comments they made, were strongly positive. They all accepted the game and worked enthusiastically with it.

There were some factors that could have contributed to these results. In the first instance the second administration of the questionnaire was done after 2 months, during which time 10 sessions of the workshop with the game took place at irregular intervals. I conclude that, because of the short period of time, I could not expect big changes in attitudes.

A second aspect, which can be related to the student teachers' responses, is the possibility that they were concerned about their final results and so wanted to please their teacher. So, they could thus have responded to the questions so as to impress the researcher as opposed to being forthright. Also, the fact that I asked them to write their names on the questionnaires could have contributed negatively to the process.

8.6. Summary

The analysis of data carried out in this chapter indicates that the 24 student teachers, who participated in the research, showed enthusiasm, satisfaction and excitement in

experiencing the richness of the *Tchadji*-game in terms of mathematical ideas and of mathematical activities, which could be performed in the mathematics classroom. They were able to analyse the game independently and to identify some mathematical ideas embedded in the game.

CHAPTER NINE: CONCLUSIONS AND IMPLICATIONS

9.1 Introduction

This chapter presents a general review of the study. The chapter reflects on what can be concluded from the study with respect to its research questions and on possible implications and recommendations for activities in the professional development of teachers, for teaching mathematics in school and for further research in cultural issues related to mathematics education. The possible empirical, theoretical and methodological contributions of the study are also presented in this chapter.

In its theoretical, empirical and methodological contributions, its implication for mathematics education and suggestions for further research, this thesis offers generative possibilities in the field of ethnomathematics.

9.2 Conclusions

The study consisted of three interventions in the area of ethnomathematical research by means of a cultural instrument, the *Tchadji*-game. The first intervention was in the school mathematics classroom. This is also the most important one, because one of the goals of ethnomathematics research is to improve the quality of mathematics education (Ale, 1989; Doumbia, 1989; Eshiwani, 1979; Jacobsen, 1984; D'Ambrosio, 1985a; Bishop, 1988a; Zaslavsky, 1989a; Gerdes, 1985b, 1988, 1995, 1996).

This intervention had to be carefully prepared. A mathematical exploration of the cultural instrument had to be carried out to identify the ideas related to the topic under consideration (probability) before the intervention took place (See research questions 1.1, 1.2 and 1.3 in section 1.4.1). The general idea was to gain insight into how using cultural artefacts (in particular, games involving chance) in the mathematics classroom could enhance different aspects of learning.

Regarding the issue of *Tchadji* and other cultural games (the three stones game, coin or cowry shell game and the donkey (*burro*) card game) and probability related concepts, it can be concluded that, in fact, there are probabilistic ideas, such as certain, possible and impossible outcomes, random and non-random phenomena, equally versus non-equally likely events, concepts of chance and randomness, the relative frequency, empirical, classical and frequentist approaches to probability (Runyon & Haber, 1991) embedded in the playing of these games. These games can be used effectively in the classroom for the purpose of teaching elementary probability concepts. These ideas are not easily recognisable. There was a need for a mathematical exploration of them (See section 6.2), particularly, the *Tchadji*-game.

In terms of outcomes (See research question 1.4 in section 1.4.1) of this intervention, performance and understanding of critical probability concepts was measured by means of a post-test on probability. The results suggest that the students in the game-approach showed a better performance in the test as compared to the students of non-game approach (See sections 6.3.1.2 & 6.3.2.1). In general, I conclude that using such games in the mathematics classroom is suitable for improving students' performance in mathematics, because the students make practice more effective and become active in the learning process (Cf. Ochepa, 1997; Barta & Schaelling, 1998; Ernest, 1986; Wesson et al., 1988).

Concerning attitudes towards mathematics and motivation for learning mathematics, the quantitative analysis of questionnaires indicated no statistically significant difference in attitudes across the control and experimental groups (See section 6.3.1.1). However, from the qualitative analysis of the interviews, the students gave comments as having had fun, having enjoyed the lessons, having seen the mathematics embedded in the games, having been surprised at seeing such games in the classroom for the first time and having experienced lessons which were practical (See section 6.3.2.2). Enjoyment and fun are some indicators of attitudes. Enjoyment and fun play an important role in learning mathematics (Cf. Oldfield, 1991a; Ernest, 1994). Therefore, I conclude that the impact of this intervention on attitudes and on motivation was considerable.

From these results, it may be concluded that the use of games can increase students' enthusiasm, excitement, interest, satisfaction and continuing motivation by requiring the students to be actively involved in learning (Cf. Klein & Freitag, 1991; Doumbia, 1989; Ernest, 1986; Wesson, Wilson, & Mandlebaum, 1988; Sleet, 1985; Strauss, 1986). However, over a short period of time it is unlikely that there could be considerable or permanent change in the attitudes of the students (McLeod, 1992).

The second phase of this study was directed to uncover further mathematical ideas involved in the *Tchadji*-game and in the procedures for playing it by means of ethnographic research among the *Tchadji*-players (See research question 2.1 in section 1.4.2). This is one of the major research directions in ethnomathematics. There are numerous studies in the area of uncovering mathematical ideas from traditional cultural artefacts (See, for example, Ascher & Ascher, 1981; Zaslavsky, 1973a; Gerdes, 1985a, 1988, 1989, 1993, 1994a; Ismael & Soares, 1993; Ismael, 1994).

The results of this second phase indicated that the *Tchadji*-players, as other scholars suspected in the case of Mancala games in general (See, e.g., Zaslavsky, 1979; Bell & Cornelius, 1995, Silva, 1994, 1995), perform count, think and act logically, do calculations, predict moves, visualise situations, recognise different numerical patterns while playing *Tchadji* (See section 7.2). Therefore, I conclude that there is mathematical thinking involved in the playing of *Tchadji*. This mathematical thinking appears in the form of mathematical concepts, processes and principles and is not easily recognisable at all. The description of the processes involved in this mathematical thinking represents a contribution to the field of ethnomathematics.

For example, in the opening move the *Tchadji*-players never count, they have memorised from their experience that certain moves will land in certain holes. Counting does take place while playing the *Tchadji*-game. The seeds in the holes are counted. When it comes to move seeds, connections have to be made in order to advance round the board, especially for long moves. These connections necessarily imply some

counting and addition of seeds and recognition of positions on the board. For example, a player might know how many jumps or positions there are between position g1 and g2. A direct counting takes place for shorter moves (See section 7.2.1).

There are also certain positions, from which the players often start their move, most used position is the g1 for an A player, or b4 for a B player (See section 7.2.4). During the course of the match, there are also some moves, for which the players will know, from their experience, where they will land. Some of these moves are even long moves, taking a round or more (See section 7.2.3). Because of the short time that a move like this takes, I surmise that, in these moves, the players concentrate their attention on the connecting holes and no direct counting is taking place.

If one of the players at a certain moment makes an unusual move, which happens, the other player starts to think and possibly calculate. This can result in a break and many protests from the audience. However, there are also certain distributions of seeds on the board, from which the player will know where the move stops without, I surmise, any further mental calculation.

The fact that the wooden board of *Tchadji* has the form of a rectangle with four rows of eight holes each (and not a row of 32 holes or a circle of 32 holes), make it easier for the players to memorise the positions. Due to the structure and the form of the board in certain situations direct counting no longer takes place. The players have memorised the *Tchadji*-board in terms of the number of holes and the ‘distances’ between any two holes on the board, i.e., how many holes are there between any two holes (See section 7.2.2). This fact helps the players to visualise different moves in *Tchadji*. Visualisation, like geometry, is related to the most diverse branches of mathematics (See more in Zimmermann & Cunningham, 1991a)

The players have developed certain abilities that enable them to recognise certain patterns in the board, i.e., a certain distribution of seeds in the *Tchadji*-board, which allow the player to reach a certain objective. These patterns are memorised to facilitate quick

reflection. In most cases it makes it easier to even determine the actual position where a certain complex move will end, without, I surmise, further complex calculations (See section 7.2.2).

There are also some occasions where a certain move goes beyond the limits of a simple calculation. This aspect was easy to see in the video-tapes (See 7.2.3). The players become suddenly surprised about the outcome of the move, especially when the move lasted beyond one complete round of the board. However, some players know exactly where the move stops. There are some simple calculations in the last move of a match, especially, if both players are “singled” (See section 7.2.7.2), in terms of knowing how many holes one has to play. When “blocked”, winning or losing depends on how many holes you have ahead up to the blocking position. This aspect can result in the player with fewer seeds winning because of his positioning on the board.

From this study, it can also be concluded that (See research questions 2.1.1 to 2.1.7 in section 1.4.2), playing *Tchadji* is a recreational activity and has a long tradition. The players learn to play *Tchadji* by watching other people play and then by playing themselves. In playing *Tchadji*, different strategies and tactics are employed in trying to secure a victory. The knowledge about these aspects of the activity of *Tchadji*-playing enriched this study.

The third phase of this study was a case study based on an intervention which was designed to work with student teachers in order to find out to what extent they were able to analyse the mathematical ideas embedded in the *Tchadji*-game? In other words, having given them a mathematically rich cultural resource, was there an indication or not that they were able to transform it into a resource for teaching mathematics. Would it be worthwhile to incorporate the *Tchadji*-game as a cultural activity in mathematics teacher education courses in Mozambique (See research question 3.3 in 1.4.2)?

The results showed that the student teachers were quite excited to know and to experience the richness of the game in terms of mathematical ideas and of mathematical activities

(See section 8.2). The student teachers were able to independently analyse the game and results showed that they were able to identify some embedded mathematical ideas in the game. For example, in the analysis of certain aspects of the game, such as the opening move, the “draw” and other strategies in the game, the student teachers were able to identify some measures which related to logical thinking, counting and mental calculation.

The attitudes of the student teachers were quite positive with regard to different aspects included in the questionnaires (See research questions 3.1 and 3.2 in section 1.4.2). The student teachers showed positive views. They all accepted the game and worked enthusiastically with it (See section 8.3). From these results, I conclude that these student teachers were able to cope with *Tchadji* as a mathematical tool and they enjoyed being challenged to explore this tool.

From my experience with teachers in Mozambique, they will not analyse such potential mathematical tools on their own initiative. It is usually necessary to bring to their attention such possibilities and one place for doing this is during teacher education courses both pre-service and in-service. *Tchadji* or, at least some other Mancala type game, practiced in Mozambique, is familiar to most of the mathematics teachers in the country (See section 4.4.2). Because the probability topic is usually seen as difficult to teach and, in a country like Mozambique, there is a lack of teaching resources. Other mathematics teachers would thus most probably be interested in the activities developed from the games as were those in this case study.

9.3 Research contributions

As argued earlier, games in general, have been used for a variety of purposes in the mathematics classroom. An important finding of this study was that traditional (non-western) games, for example *Tchadji*, are eminently suitable for use in the classroom for the purpose of teaching mathematics. I have shown that *Tchadji* involves probabilistic concepts and thinking (See section 6.2.1) and that activities related to *Tchadji* can

effectively be used in the classroom for the purpose of teaching certain probability concepts. Mancala type games, to which *Tchadji* belongs, are played in most African countries. In most of these countries there is a lack of teaching resources. The use of traditional games in general can be seen as having potential for effective teaching.

Another interesting finding was that *Tchadji*, previously unexplored, is related to a variety of mathematical concepts, processes and principles (See section 7.2). From this research, it was possible to analyse these mathematical ideas and to some extent document them. There have been other studies in Mancala games (See, for example, Murray, 1952; Béart, 1955; Bell, 1960; Reysset, 1995; Russ 1984, Townshend 1986, Walker 1990, De Voogt, 1995; Retschitzki 1990; Silva, 1994 & 1995; see also section 5.2.2). However, this is the first time that the *Tchadji*-game has been investigated specifically for the purpose of uncovering its mathematical ideas.

Student teachers showed that they are capable of capitalising on the *Tchadji*-game. They worked with the game in an enthusiastic manner. Teachers, in themselves, would have little confidence in using such a game in teaching. Teachers usually refer to other textbooks for support or for ideas. No book that they could have consulted would support the use of *Tchadji*. This did not happen, the student teachers enjoyed working with the game and most importantly, they showed themselves to be able to analyse the game and suggest ways of incorporating it in their teaching. This is a contribution to ethnomathematics in that mathematics teachers generally should be in a position to use the game effectively in their teaching.

The role of teachers in implementing ethnomathematics in the school is seen to be crucial. Before, for implementation to take place, it is important that the teachers themselves accept it. This research showed that the student teachers accepted the use of *Tchadji* in their teaching and therefore it is highly likely that the teachers would do the same.

On the other hand, in order to carry out this study, different methodological approaches were employed. For example, for the study of the mathematical ideas in the *Tchadji*-game among its players, ethnographic research methodology was employed (See section 4.3). It would not be possible to analyse mathematical ideas embedded in the *Tchadji*-game without the presence of the researcher in the environment of the players. Thus, the researcher was in some way a participant (See section 4.3.1.1).

Also, a knowledge of the language of the players was shown to be beneficial, otherwise, how would I have communicated with them, understood and interpreted their assumptions. Thus, in the study of mathematics of traditional culture, it is crucial that the researcher interact directly with the culture.

I used video to records the events in the fieldwork (See section 4.3.1). However, my descriptions of the data and my analysis went beyond what I was watching in the videotapes only. It was necessary to analyse other issues in the environment of the players. For example, the video recorded mostly the matches, it did not record how the players were physically interacting. It did record comments of the players and of the audience, but it did not recorded which person of the audience made the comment. It did not record whether they were pointing to any person or any specific hole while making the comment. However, it was necessary to describe most of these accompanying aspects of *Tchadji*-playing (See, for example, sections 7.2.7.2 and 7.2.7.3).

My presence and my physical observations have been very useful. In this case, the subsequent analysis of the situation through the recorded videotapes and being able to replay the matches in their environment (See section 7.2) has played a crucial role in this research. So, the results are not directly taken from what is observed. There is a supplementary analysis necessary to come to results. In other words, for uncovering mathematical ideas in traditional cultures it is necessary to do field research in order to observe the culture in its natural setting. After the fieldwork, repeated analysis through the use of the video material adds considerably to verifying what observation and

interview can reveal. In the field of ethnomathematics, this is an important methodological contribution.

The nature of the research required interviews with the *Tchadji*-players. This is indeed recommended in the literature. However, in my particular case, in order to prepare for my subsequent analysis of the recorded *Tchadji*-matches, it was necessary to visually record the situation, thus interview, in which the players were playing a match (Interview-matches, see appendices B26-B28) and were commenting on their decisions to play in the way they played was also crucial. This has been an innovation in data collection and analysis in the ethnographic aspects of ethnomathematical research.

The mathematical ideas found in the analysis of the *Tchadji*-game have most to do with the players themselves, in spite of them not discussing the game in mathematical terminology. It is what the players were doing and saying, and also how they were acting that counts in this research. It was indeed necessary to further analyse all these situations, but it is clear that these ideas are of the players.

9.4 Implications for mathematics education

In this research, *Tchadji* showed a richness in manipulations with numbers. For example, as shown, counting takes place in playing *Tchadji* (See section 7.2.1 and 8.2). A richness in probabilistic ideas is also embedded (See section 6.2.1). Students working with *Tchadji* and other games in schools in my experimental study showed excitement and enjoyment. They were motivated. Also, their performance, as compared to the groups not using these games, was much better (See section 6.3).

From these results, I conclude that *Tchadji* could be effectively used in schools in Mozambique, in lower grades for counting and learning of numbers and for higher grades in learning elementary probability. Also, other African countries could benefit from the Mancala games in general, because of the common playing principles these games share in common with *Tchadji*. *Tchadji* is classified as being a ‘Non-Western’ game. Western

countries could also benefit from this game, once introduced there. About these issues, Sawyer (1949) has argued that " *African children must benefit from the familiarity with numbers that this game [Ouri] gives, and it may have classroom use in other countries*" (quoted in Mucavele, 1998: 63). No other study, to my knowledge, has shown this.

Powell (1993) argued, "*the role of the teacher involves structuring an environment, presenting situations, creating constraints, stimulating dialogue, and posing questions, in short, providing experiences, with the purpose of drawing students' attention to aspects of experiences that contain mathematical particularities that ought not to escape their attention, be ignored, or overlooked. A corresponding role of teachers is to provide further experiences for students to obtain facility*" (p.81). The research showed (See section 6.2.1) that *Tchadji* can generate exciting activities. The use of such activities presents a good opportunity for teachers to enhance their role.

It is also an opportunity for students to enhance their awareness of social and cultural bases of mathematics. Indeed, "*students must give of their time and use their will to focus their attention and enter into a dialogue with situations to educate their awareness*" (Powell, 1993:81). The use of direct cultural artefacts as in the case of *Tchadji*, gives the students an opportunity to understand different phenomena in their environment.

Also, the knowledge acquired, through the use of concrete everyday materials such as games, is more discursively elaborated knowledge than procedurally elaborated knowledge. Knowledge, which hides the articulations of its elements, is called procedurally elaborated knowledge. It largely presents mathematics as a collection of unconnected rote-learned, mysteriously determined facts and algorithms (Coombe & Davis, 1994).

The students working with *Tchadji* have shown considerable interest in the activities involved and were sometimes surprised to learn mathematics from cultural aspects of their environment (See section 6.3.2.2). The use of *Tchadji* in mathematics classroom most probably contributed to enhance such awareness in these students. For these reasons, the use of *Tchadji* should be encouraged.

9.5 Suggestions for further research

9.5.1 Variations of *Tchadji*

It was not the intention of my research to investigate types of Mancala games other than *Tchadji*. In particular the scope of the research did not encompass any possible variation of the game itself or even other forms of the game with regard to the board or rules. However, some ideas have emerged during the study that I would like to present here in the form of suggestions for further research and activities in this area.

9.5.1.1 Variations in use

For example, during the interview sessions (See section 7.2.7.2 and 7.2.7.3), different aspects of *Tchadji* were explored, but the data collected from the videotapes led me think that some other aspects can be explored with regard to the thinking process of the players. One aspect is related to the playing time. As I mentioned earlier, a game of *Tchadji* is played very quickly (See session 7.2.6). During these short periods of time it would be interesting to know, for example, what is the **thinking time** and what is the **playing time**. Another aspect is related to concrete moves. It would be important to know how expert players are able to determine the outcome of, for example, the **long moves** (See section 7.2.3). The *Tchadji*-players memorise certain moves. This could possibly be because they have memorised the outcomes of these moves. Is this case?

9.5.1.2 Variations of the form of the board

Another area for research has to do with the number of columns in the *Tchadji-game*. As remarked earlier, *Tchadji* is played on a wooden board consisting of four rows of eight holes each, which makes a 4x8 board, or if we see it from another perspective, 8 columns of four holes each, (See section 5.2.4.3). *Tchadji* is thus a Mancala games of type IV, i.e., with 4 rows of holes.

In some regions of Mozambique, e.g., in the south, different versions of type IV are often seen with regard to the number of columns. The numbers vary from 5 columns, the shortest ever seen (C.f. Junod, 1962; Mucavele, 1998), to 22 columns or more as I observed. It involves either 2 players or two teams of players. Each team is constituted of 2 or 3 players depending on the size (number of columns) of the board. The following research possibilities arise.

Concerning the first move (see section 6.2.7.2) in any match of *Tchadji*, the person who plays first in a match ends 5 holes ahead from his starting position after one round in his two rows. On a 4x7 board, the person starting ends exactly in the same hole after exactly a round. The situation with other versions can easily be found experimentally. Another situation would arise if we consider other than two seeds in each hole at the start of the match. These could be activities to be performed in the classroom. Once, the counting and calculation in *Tchadji-game* are established, what can be generalised to other version?

Tchadji involves only independent players, i.e., the players think and decide on the moves to make on their own. In the case of versions, larger versions, that involve teams of players, two or three people play together and take decisions concerning how to play next. My suggestion here is to investigate the thinking process within a team. How do the team communicate? How do they take decision? Is there a moment in the match where they share ideas? If yes, when and how do they do it? Are all the players thinking or is it only one player who takes the decisions? If yes, which player? What is the fun in it?

By playing in a team, it certainly may involve aspects of counting, memorisation and visualisation of moves, in a much more complex situation than in a *Tchadji*-board. It would be important to find out how these aspects of mathematical thinking take place in this process of team playing and how comparable these thinking processes are to what happens in *Tchadji*?¹

On the basis of the knowledge about the *Tchadji*-game and its rules, one could even think of inventing other versions of Mancala by changing the form of the board or some of the rules involved.

For example, by changing the general structure of the board, one could think of considering more than 2 people or teams playing at the same time, i.e., instead of 2 people, one against the other, what about 3 people or, instead of two teams, what about three teams. A version of Mancala with 3 positions instead of 2 could be considered. In this version, the general rules for Mancala type IV would apply with a few additional rules (See figure 1 in appendix K). For example, a player would be able to capture seeds from both his opponents in the same columns and positions.

Another rule to be adapted could relate to deciding on winner being the player who is able to eliminate not only the second player but also the 3rd player. Or, it could change in terms of scoring, i.e., a player who eliminates both his opponent could have more points than he one of his opponent eliminates a second player and he eliminates him. Alternatively a version of 4 positions could be considered, in which two teams of 2 people play (See figure 2, in appendix K).

9.5.1.3 Variations in the rules

One could think of changing a little the basic rules of Mancala type games. For example, by eliminating the rule (see rules in section 5.4.2.1) or by capturing seeds of the opponent

removing also the contents of a third hole. By imposing a rule of being able to capture seeds of the opponent, even if the hole direct opposite to holes where a player land is empty. Or by imposing a rule that a player can also capture seeds from his opponent even if he lands in his outer row (see rule in section 5.4.2.1).

One could even think to change basic features of Mancala type games. For example, by considering other than 2 seeds in each hole by the start of a match, e.g., 3 seeds or even 4, or by considering two or more of these little changes at the same time.

Different research questions can be generated from these changes. For example, what kind of mathematical activities could be generated as a result of these changes? How could these activities be incorporated in the teaching of mathematics in school? What would the students benefit from this incorporation? What are the mathematical concepts involved in these versions of the game?

It is not surprising, as stressed in chapter 5, that there are other variants of Mancala type games, which involve specific rules or features. Did the people who invented such variations reflect in the same direction? Was it a matter of changing the routine of always playing in the same way? What makes people invent other versions of Mancala type games?

Mancala type games in general may involve more mathematical ideas than those uncovered in this research. As Bell & Cornelius (1988) argued “*Mancala games lend themselves easily to interesting analysis, and endless possible investigations could be carried out at many different levels of mathematical sophistication*” (p.24). Therefore, a need for further research. This reinforces the issue that mathematical can no longer be seen as culture free.

¹ Currently I am advising a student at ‘Universidade Pedagogica’ investigating mathematical thinking

9.5.2 Other traditional games

In the first phase of this study a game called three stones was included to complement the study on probabilistic ideas in games (See section 6.2.2). This game belongs to class of games known in the literature as Three-in-a-row games or Three-Men's Morris.

In Southern Mozambique, a larger version for the three stones game is known as *Muravarava*. In the literature, this game is known as Twelve Men's Morris (Bell & Cornelius, 1988). This game is played mainly by adults. The game is very old. A board of this kind of games has been found in Egypt (c. 1400 BC). For playing, each player has 12 counters (See Figure 3 in Appendix K). The players place the counters alternately at the intersection points until all are on the board. Then, they take turns to move a piece along a straight line to neighbouring empty point. When a player succeeds in getting three pieces in a line, he can remove one piece of his opponent. If a player is reduced to two pieces or is blocked and unable to move, he has lost the game. In Mozambique, this game is also unexplored with regard to its mathematical and educational potential. A research on this game employing a methodology similar to the one employed in this study could produce exciting results.

involved in the playing of *N'tchuva*, a game of type Mancala played in Southern region of Mozambique.

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