YIELDING PILLAR DESIGN IN SOUTH AFRICAN COLLIERIES

David Christopher Oldroyd

A project report submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering.

Johannesburg 1997

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ABSTRACT

Three cases of pillar failure on Southern African Collieries have been studied to analyse the behaviour of both the pillars and the overlying strata. Each of the cases shows a different type of pillar and strata behaviour during failure and thus provides an opportunity for back analysis.

In the first case, pillars failed in a controlled fashion while the overlying strata behaved in an elastic fashion. In the second pillars failed in a controlled fashion while the surrounding strata behaved inelastically. In the third failure was initially controlled but became uncontrolled.

Computer models have been run to determine the theoretical critical post peak pillar slopes and the results of these models have been compared with the actual pillar behaviour as derived from in-situ measurements during failure and that which might be predicted from the theory of controlled and uncontrolled pillar failure. Comparisons are also made with the expected behaviour implied from the results of in-situ strength tests carried out on small coal pillars to ascertain their load deformation characteristics. The results indicate that the behaviour of the pillars more closely resemble that predicted from in-situ tests carried out by Van Heerden[4]. The results also indicate inadequacies in using elastic methodologies to determine whether pillar failure will be controlled or uncontrolled.

DECLARATION

I declare that this project report is my own, unaided work. It is being submitted for the degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

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10th **day of** November **, 19**97 **.**

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ACKNOWLEDGEMENTS

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CHAPTER 1 INTRODUCTION

Interest in the topic of yielding pillars was created by the occurrence of pillar failure on 2 collieries, one in the Northern Natal and the other in Swaziland. Three different types of pillar failure occurred and all three have quite different characteristics but all have a yielding phase during failure. These failures, which occurred accidentally, give rise to the belief that yielding pillar design on collieries is possible.

Successful design of yielding pillar panels has eluded the coal mining industry over the last 25 years. To be fair, very few serious attempts at this type of mining have been tried since 1969 when Salamon[1] first studied the uses of yielding pillars and explored the requirements for stability in the post peak region of a pillar. In this project report, research has been conducted to evaluate yielding pillar designs in collieries and to assess the problem of pillar behaviour beyond the elastic limit of the overlying strata. This is a particular problem on collieries where the spans and depths are such that elastic convergence is very small and where, from the initiation of pillar failure to their eventual recompaction, strain values of up to 45% are required. The necessary convergence for final stability is then much greater than is usually elastically possible.

The need for yield pillar designs is apparent on some collieries where certain geological conditions prevail that may prevent successful stooping (the extraction of pillars on retreat). Some mines have high methane gas content in the thinner uneconomic seams that overlie the main mining horizon. If a method can be designed whereby some pillars can be extracted but allow those that remain behind to fail in a slow and controlled fashion significant ventilation benefits result. Further, because a full goaf does not develop improvements in productivity and a reduction in costs can result as breaker and fingerline support no longer need to be installed.

The current need for yielding pillar design is present on Ermelo mines, where high methane emissions and a number of explosions were experienced in the past. Other collieries which are deep and have relatively narrow seams have also been recently identified as being suitable for a yielding pillar method of pillar extraction.

The approach of this study has been to back analyse the three pillar failures which occurred at Emaswati and Zululand Anthracite Collieries (Z.A.C.)using the boundary element numerical model, BEPIL, developed by Ryder[2] and to compare the results obtained with in-situ results of pillar tests carried out by Wagner[3] and Van Heerden[4]. The BEPIL program has been found to be particularly suitable for the analysis of pillar stability in the post peak region and allows the determination of critical post peak pillar slopes for a specified geometry.

This project report contains a literature survey on several of the major developments that have led to the present day knowledge of yielding pillar design and the behaviour of pillars in the post peak region. Chapter 3 contains the descriptions of and observations made on pillar failures which have occurred at Emaswati and Z.A.C. collieries. In Chapter 4 the back analysis of these cases by BEPIL are discussed. Discussion and conclusions are presented in Chapters 5 and 6.

CHAPTER 2 LITERATURE SURVEY

2.1 INTRODUCTION

Many papers have been written and much research has been conducted on coal pillar stability and arguably it is one of the most greatly researched topics in coal mining rock mechanics. In bord and pillar workings pillars are ordinarily designed so that their strengths exceed the load of the overlying strata and most research has been conducted in the area of strength determination. Although pillar strengths are of prime importance when designing collieries, in yield pillar mining it is their behaviour during failure that is of greater significance and it is this particular aspect which is concentrated on here. Several researchers have contributed to yield pillar design, through both theoretical and experimental work. These include Salamon[1], Wagner[3], Van Heerden[4], Ryder and Ozbay[2]. Papers published by these authors and others are reviewed in the section that follows, extracts and analysis have been made where relevant to this study.

2.2 THE THEORY OF STABLE AND UNSTABLE FAILURE OF COAL PILLARS

In 1969, Salamon[1] identified and analysed mathematically two types of pillar failure, that which occurs gradually in a controlled manner and that which occurs very rapidly resulting in uncontrolled failure. Figure 2.1 shows what Salamon[1] describes as the complete stress strain curve of brittle rock and notes that it consists primarily of two regions OA and AB. In the portion OA increasing deformation of a pillar is accompanied by increasing load while in portion AB increasing deformation is accompanied by decreasing load. Salamon[1] goes on to equate pillars in a mine with a specimen in a testing apparatus having a stress strain curve similar to that described. The pillars being equivalent to the specimen while the overlying strata which loads the pillars being represented by a spring.

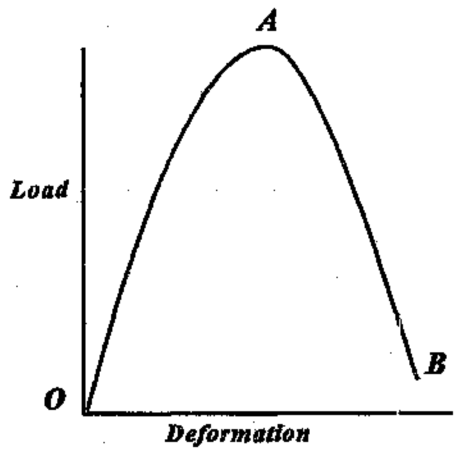

Figure 2.1 - Complete stress strain curve - after Salamon[1]

The slope of the graph after specimen failure has taken place, corresponding to portion AB in Figure 2.1, was called λ by Salamon[1] and is commonly expressed in the units of stiffness, MN/m. Salamon[1] showed that the mode of failure in a system of pillars, when the strata surrounding the pillars behaves in a linearly elastic fashion, is controlled by the two important factors, namely, the post peak stiffness of the pillar λ, and the stiffness of the loading system, k. Salamon[1] proves through two approaches that the mathematical requirement for stable failure is $k + \lambda > 0$. This is shown graphically in Figure 2.2 which in the first case shows stable pillar failure and in the second unstable pillar failure.

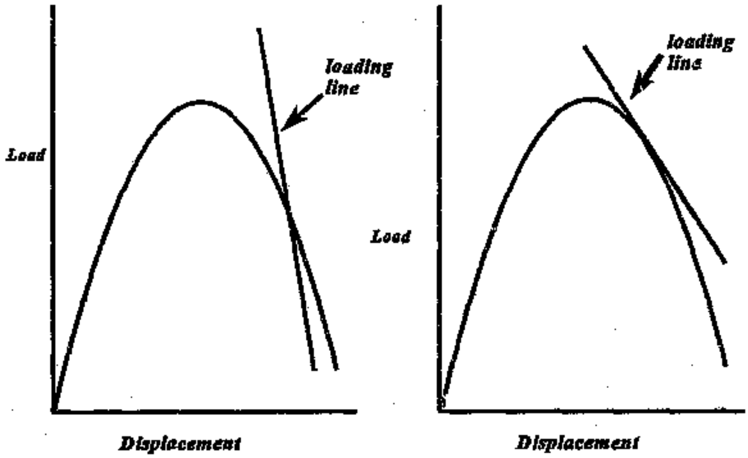

Figure 2.2. - Stable (a) and Unstable (b) pillar failure

He also showed that stiffness of the loading system k is a function of the pillar and panel geometry as well as the modulus of the overlying strata. Salamon[1] proposed the critical system stiffness λ_o for a panel of pillars. λ_o is the smallest given value of the stiffness matrix K of which the elements are stiffness coefficients. For perfect stability $\lambda_m > \lambda_o < 0$. Here λ_m is the minimum post failure slope of the pillars within the panels. Thus if pillars in the panels have post peak slopes greater than λ_o pillar failure will be in a stable fashion. Salamon[1] also calculates λ_o for relatively simple geometries by using local stiffness and matrix algebra. λ_o is graphed in Figure 2.3 for panels consisting of 1 to 5 pillars between two continuous and parallel barriers. This was derived from Salamon's work by dividing by the normalising factor where ν is poisons ratio and E is Youngs modulus. υ was assumed to be 0.2 and E 12 GPa,

$$\Omega = \frac{4\ (1 - \nu^2\ )}{E}$$

a typical value for coal measure strata. As can be seen there is rapid drop in the critical stiffness with increasing number of pillars in the panel, indicating that it will be more difficult to achieve yielding pillars in panels with a large number of pillars than in those with very few.

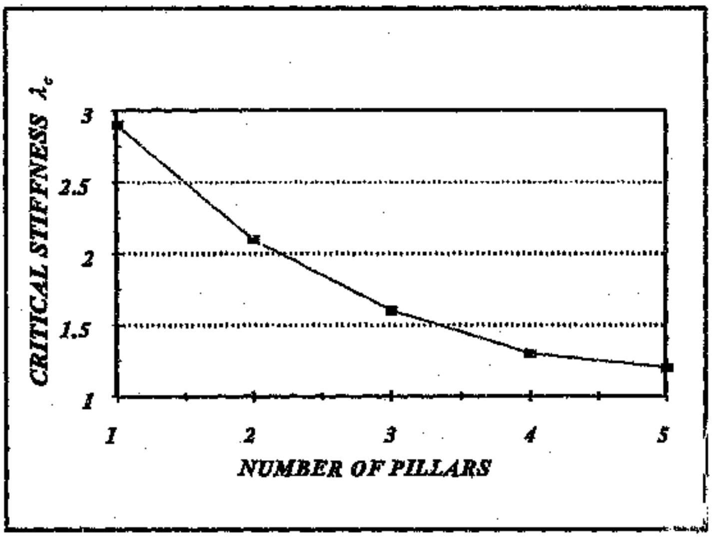

Figure 2.3 - Critical Slope λ_c for E = 12 GPa

As discussed λ, represents the slope of that portion of a load deformation graph after the peak strength has been reached. At the time that Salamon[1] had written his paper, very little work had been done on the post peak behaviour of pillars. Indeed, he suggested that research was necessary to determine the insitu complete load deformation characteristics of pillars. In this way values for λ could be experimentally determined.

Figure 2.2a and b show the "complete" load deformation curve and loading lines for two types of pillar failure one controlled and the other uncontrolled, respectively. In the first, the loading lines are steeper than the pillar curve. In the second the loading line is shallower than the pillar curve. These graphically illustrate the condition for stable and unstable failure, respectively $k + \lambda > 0$ and $k + \lambda < 0$. These two graphs or similar representations thereof continue to be used today when failure modes are discussed. Salamon[1] thus provided the framework for other researchers to design yielding panel pillars and in particular he suggested the experimental determination of the post peak characteristics of pillars.

2.3 INSITU COMPLETE LOAD DEFORMATION CHARACTERISTICS OF COAL PILLARS

Some 15 years after Salamon[1] first published w[illegible]able failure of pillars Wagner[3] and Van Heerden[4], in 1974 and 197[illegible] resp[illegible]lished papers on the insitu testing of coal pillars which was carried out to determine their "complete" load deformation characteristics. Each researcher used quite different methods. Wagner[3] choosing to load the pillar on the horizontal plane of symmetry, in effect creating two pillars while Van Heerden[4] chose the more conventional route of loading a small pillar from the top down. Both achieved a high degree of realism in their testing when compared to laboratory methods. Perhaps the most important of which, the end constraints at the top and bottom of the pillars were particularly well maintained. Wagner[3] determined post peak slopes (i.e. the value of λ) for pillar width to height ratios between 0.8 and 2.24 while Van Heerden[4] reported results for width to height ratios between 1.14 and 4.29. Their results are replotted in Figure 2.4.

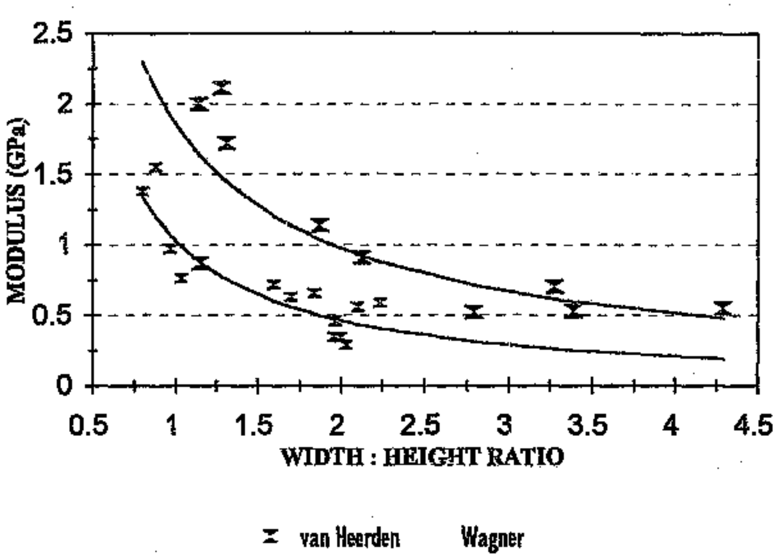

Figure 2.4 - Post Peak Moduli obtained by Wagner[3] and Van Heerden[4]

The trend is clear from these results, increasing width to height ratios result in shallower post peak slopes and correspondingly lower modulus. At pillar width to height ratios of around 2.5 to 4 the results are similar and tend towards a post peak modulus of approximately 0.5 MPa.

Graphs of the form

$$y = a\left(\frac{w}{h}\right)^{b}$$

were fitted to the data obtained by both authors. The following results were obtained:

Table 2.1 - Values of a and b determined from Wagner[3] and Van Heerden's[4] post peak slope data

	a	b	Correlation coefficient
Van Heerden[4]	1.861	-0.931	-0.848
Wagner[3]	1.033	-0.848	-0.852

The plots given in Figure 2.4 are described the equations :-

$$\lambda = 1.861\left(\frac{w}{h}\right)^{-0.931} \qquad -Van\ Heerden^{4}$$

$$\lambda = 1.033\left(\frac{w}{h}\right)^{-1.159} \qquad -Wagner^{3}$$

Salamon[1], Wagner[3] and Van Heerden[4] suggested the behaviour given in the graph in Figure 2.1 to be the complete stress strain characteristic of a slender pillar. Oldroyd and Buddery[5] in 1988 discussed pillar failure at Zululand Anthracite Colliery (Z.A.C.) and Emaswati and presented the stress strain characteristic shown in Figure 2.5. This shows the behaviour of a pillar up to the region where the vertical pillar compression is as much as 35% and re-compacts to once again carry the full overburden load. To enable the pillar to trace out this path the overlying strata failed and allowed inelastic convergence to occur. Salamon[1] performed analysis of unstable and stable failure of pillars on the assumption that the overlying and surrounding strata behave elastically. In this particular case the overlying strata deformed beyond the elastic limit.

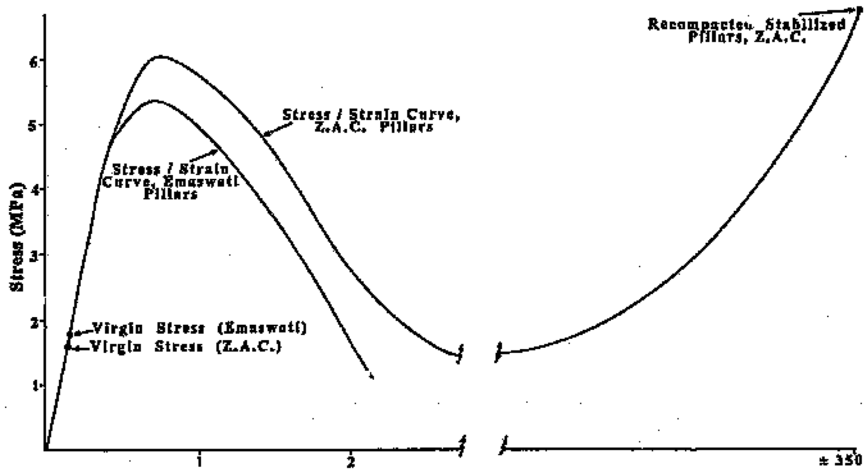

Figure 2.5 - Stress-strain curve after Oldroyd and Buddery[5]

In 1990, Ryder and Ozbay[2] also considered a more complete stress strain curve, reproduced in Figure 2.6. This has much similarity with that produced by Oldroyd and Buddery[5]. Both of these curves represent the complete stress strain curve of a pillar and both show an extended path where a large amount of strain occurs immediately following the post peak downward sloping portion. After this the pillar once again begins to carry more load as deformation increases. By following this upward trending portion the load on the pillars eventually matches the load of the overlying strata and ultimate stability is reached. Pillar failure at Z.A.C. suggests that it is not sufficient to simply examine the stability of pillars in a purely elastic fashion. What occurs should the overlying strata fail and behave inelastically must be taken into consideration.

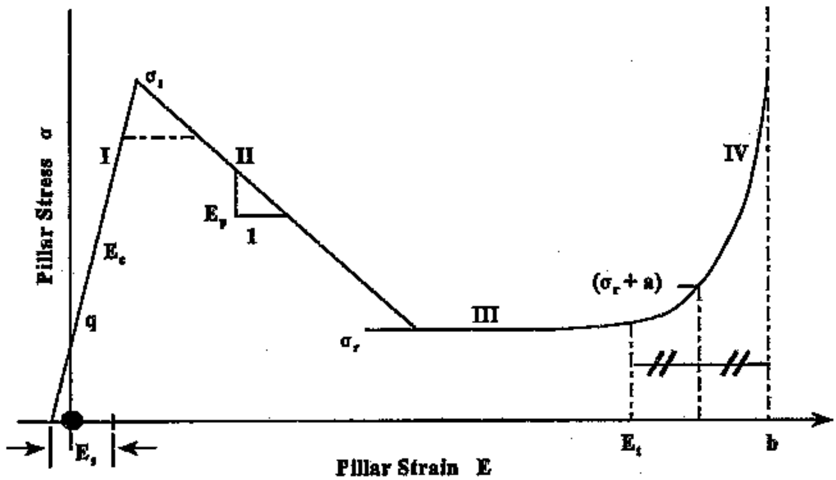

Figure 2.6 - Stress-strain curve after Ryder and Ozbay[2]

2.4 A METHODOLOGY FOR DESIGNING YIELD PILLAR LAYOUTS FOR SHALLOW MINING

In 1990 Ryder and Ozbay[2] published a paper on a methodology for designing yield pillar layouts for shallow mining and in particular introduced the program BEPIL which enables the 3 dimensional nature of pillars to be modelled from two dimensional input. The program is versatile in that several different types of pillar behaviour can be modelled. In particular pillars having only a post peak stiffness can be used enabling the mode of failure (controlled or uncontrolled) for a particular system of pillars and mining geometry to be determined. Furthermore the critical stiffness λ_c of the pillars corresponding to a certain mining geometry can be determined by varying the pillar stiffness in the post peak region. An example of this type of analysis was carried out by Buddery and Oldroyd[6] in 1994. Figure 2.7 shows the modulus of the overlying strata plotted against the number of iterations required to solve the model in BEPIL. In this example the post peak slope λ for individual pillars has been kept constant and the value of the elastic modulus of the surrounding strata reduced from 12 GPa until the number of iterations required to solve the model rapidly increase and becomes asymptotic at a Young's modulus of $\pm$ 1.5 and 2.1 GPa for post peak pillar moduli of -0.98 and -1.5 GPa respectively. The asymptote indicates the critical modulus, for convenience called E_c, at which the elastic modulus of the overlying strata causes unstable failure of the pillars. The post peak slopes in this example were obtained using equation developed by Ryder and Ozbay[2] discussed below.

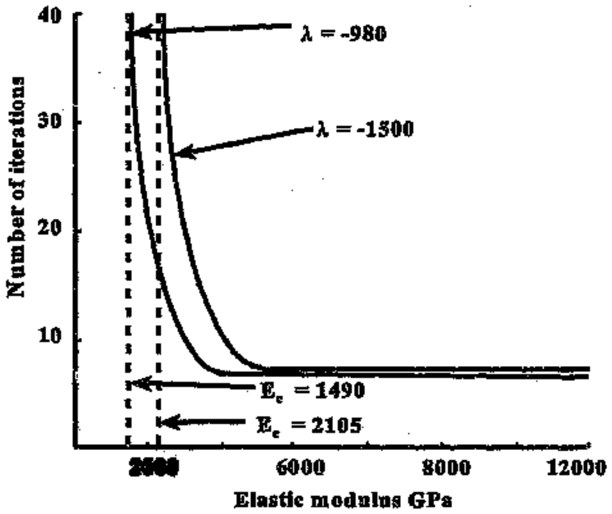

Figure 2.7 - Stiffness analysis using BEPIL output-after Buddery and Oldroyd[6]

In the same paper in which Ryder[2] introduced BEPIL, Ozbay[2] fitted an envelope to all the post peak moduli which had been determined from in-situ pillar testing. The assumption was made that the post peak modulus is related to the pre peak value. Norm[illegible] post peak moduli are plotted against width to height ratio in Figure 2.8.

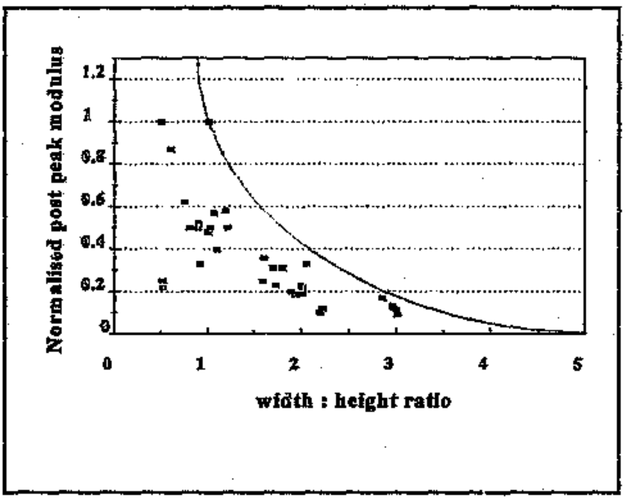

Figure 2.8 - Normalised post peak moduli and envelope after Ryder and Ozbay[2]

The equation of the envelope is given by

$$\frac{Ep}{E} = 0.23 \left[\frac{5}{\left(\frac{w}{h}\right)} - 1 \right]$$

Where Ep is the post peak modulus, E the modulus measured before failure, w is the pillar width and h the height. Interestingly, when the width to height ratio exceeds or equals 5 then the post peak slopes are found to be greater than or equal to zero. If this is true then pillars with width to height ratios exceeding 5 can only fail in a stable fashion as Salamon's[1] criterion $k+\lambda>0$ is always satisfied as k is always positive.

Wagner[3] and Van Heerden[4] while determining the complete load deformation curve for insitu pillars also found the modulus (E) of the pillars to be around 4 GPa. By using this value in the equation above, post peak moduli can be estimated.

The program BEPIL enables the determination of λ_{c}, the critical stiffness of a panel of pillars while the envelope of post peak moduli and equations fitted to Wagner[3] and Van Heerden's[4] data enable the calculation of the post peak pillar modulus and hence pillar stiffness in the failed region. Comparison of these values allows the determination pillar failure mode, either in a stable or unstable fashion. Various researchers have thus investigated the post peak moduli of pillars and system stiffness. A summary of results are presented in Figure 2.9. The envelope in Figure 7 is reproduced in Figure 2.9 for a Youngs modulus of 4.00 GPa, the average obtained from in situ tests by Wagner[3] and Van Heerden[4].

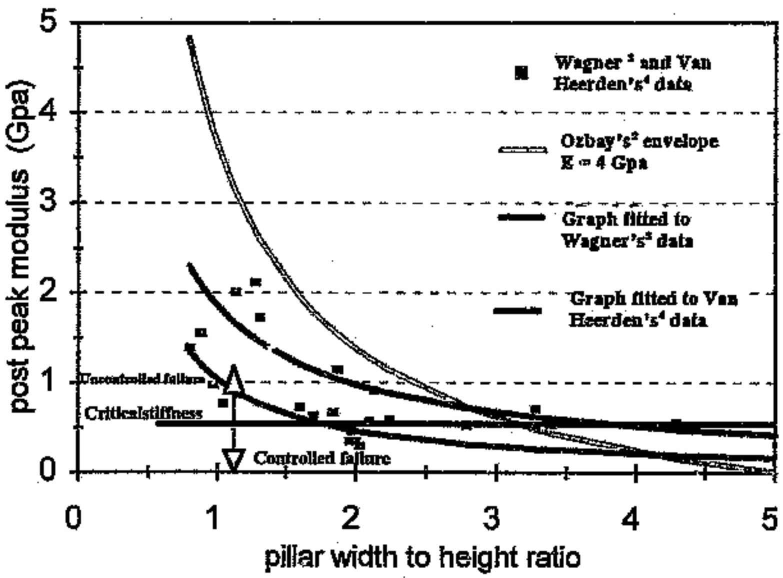

Figure 2.9 - Graph of post peak modulus vs pillar width to height ratio

A horizontal line representing the critical stiffness as a modulus of 0.6 GPa is shown. Stable failure is indicated when the post peak modulus lies below this horizontal line. Conversely uncontrolled failure maybe indicated by post peak moduli lying above the line.

2.5 PROGRESSIVE FAILURE OF COAL PILLARS

In 1988, Oldroyd and Buddery[5] published a paper on pillar failures at Z.A.C. in Northern Natal and at Emaswati in Swaziland. In each of these cases pillars had failed in a slow and controlled fashion. Those at Z.A.C. eventually tracing out the full load deformation graph for a pillar ultimately recompacting at strain values of the order of 41%. In the process the overlying strata cracked and converged inelastically along joints and faults. At Emaswati it was noted that pillars had failed but the strain values were low, being at less than 2%. In this case no fracturing of the overlying strata could be observed and the pillars remained stable at very low convergence values.

Amongst other conclusions, Oldroyd and Buddery[5] noted conditions that favoured the design of yielding pillars; stiff roof and structurally weakened pillars. They suggested that conditions at Z.A.C. were favourable to the eventual design of yielding pillars as the overlying strata consisted mostly of sandstone while the pillars are weakened (softened) by jointing. The average width to height ratios of the pillars in these two panels are 2.57 and 2.77, in both these cases there are six pillars in the panel.

In 1991, Buddery and Oldroyd[6] published a paper which included a number of items on cost savings in collieries. One of the items discussed the results from an analysis of pillar failures at Z.A.C. using the methodology proposed by Ryder and Ozbay[2] on determining critical stiffnesses were also presented. One of these suggested the design of a yielding pillar panel where pillars would be deliberately reduced in size on retreat or alternately every other pillar or row of pillars be extracted. The pillars left behind would be such that, should they fail, they would fail in a controlled fashion.

2.6 CONCLUSIONS

Salamon[1] in 1969 discussed two types of pillar failure, controlled and uncontrolled, and proceeded to demonstrate the requirements for each type Although at the time the critical stiffness for simple geometries could be determined very little information was available on the post peak stiffness of pillars.

In 1974 and 1975 Van Heerden[4] and Wagner[3] performed insitu tests on coal pillars during these determined both the elastic and post peak moduli for pillars of various width to height ratios. These results will be compared with the case histories and the computer analyses.

In 1990 Ryder and Ozbay[2] developed the program BEPIL and proposed a methodology for designing yield pillars. BEPIL features a facility for modelling square pillars of varying material behaviour. In particular post peak moduli can be modelled enabling the determination of the critical stiffness (λ_c) for a particular geometry. This feature was used extensively in the analyses of pillar failure mode.

In 1988 Oldroyd and Buddery[5] have documented pillar failures on two collieries which have occurred in a stable fashion and have postulated that it may be possible to design yielding pillars on collieries. During the failure at Z.A.C. strain of up to 41% was recorded and the overlying strata failed. This indicates that the stability of failing pillars beyond the elastic limit of the overlying strata may also have to be considered for their to be complete stability. This requires further investigation.

CHAPTER 3 OBSERVATION ON PILLAR BEHAVIOUR DURING FAILURE

3.1 INTRODUCTION

This study into yielding pillars on collieries was prompted by the occurrence of 3 areas of failed pillars on two mines. Two of these cases are published in a paper by Oldroyd and Buddery[5] while the 3rd case is unpublished but known intimately by the author. These three cases represent three different types of pillar behaviour during failure.

In the first case, pillars within a particular panel at Emaswati Colliery in Swaziland were observed to have failed under relatively low values of strain. Strain values of the order of 1 to 3% were estimated from broken mine poles. Convergence in the panel continued at ± 10mm per annum up until the last readings were taken in 1992.

At Zululand Anthracite Colliery (Z.A.C.) pillars were showing excessive fracturing in a small portion of a panel. Over a period of six months the area effected spread to the whole of the panel. Pillars continued to fracture and eventually failed leading to ultimate strain values of around 40% before convergence ceased.

In the third case pillars began fracturing along a main development at Emaswati Colliery in Swaziland and these were seen to be failing in a stable fashion. Some months after the onset of the pillar failure the rate of convergence in the panel increased rapidly and the pillars crushed in a sudden uncontrolled fashion which resulted in the trapping of a portion of the workforce.

3.2 EMASWATI CASE 1 - PILLAR FAILURE UNDER LOW STRAIN

In 1985 some of the pillars in a main access way at Emaswati Colliery were observed and recorded for the first time as being failed. Although some of the pillars had clearly failed the roof remained intact and the roadways open. The panel had been mined 7 years previously in 1978. Convergence metres were installed and these revealed that movement was occurring at a slow rate of 2.1mm per annum. The metres installed measured movement from the skin of the roof and the floor and thus may represent magnified values due to bed separation and not necessarily purely elastic movement.

The last time the panel was observed was in 1992, when it was found that the pillars were showing increased signs of fracturing and scaling. Although unmeasured, the strain values were assessed to be low as indicated by sticks which had not failed.

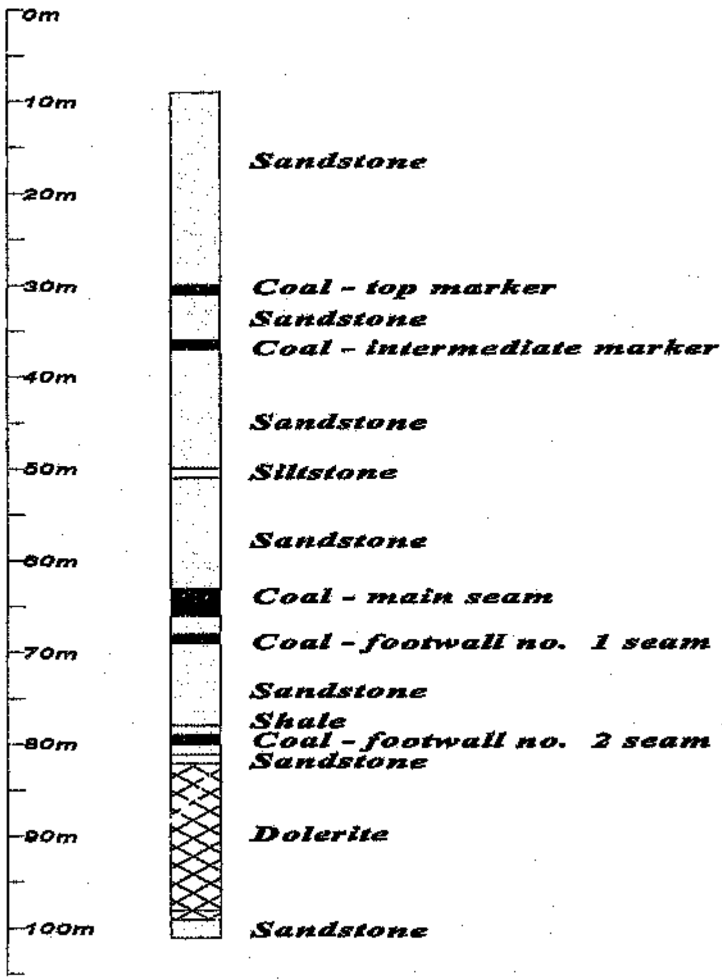

Figure 3.1 - Typical geological section at Emaswati Colliery

A typical borehole section of the area is shown in Figure 3.1 which clearly shows the massive and stroug sandstones which overly the main seam. The floor consists largely of sandstone but siltstone does occur in places and this has been seen, in this and other areas of the mine, to be prone to floor lift. The pillars consist almost entirely of coal except towards the top where a few shale layers are interbedded with the coal. Joints occur within the pillars and these are likely to have been induced by the two dolerite dykes which bound both the north and the south of the panel (see Figure 3.2). Also of significance is the slight dip at the southern end. The following table of parameters summarises the original mining dimensions and safety factors for this case and the two that follow.

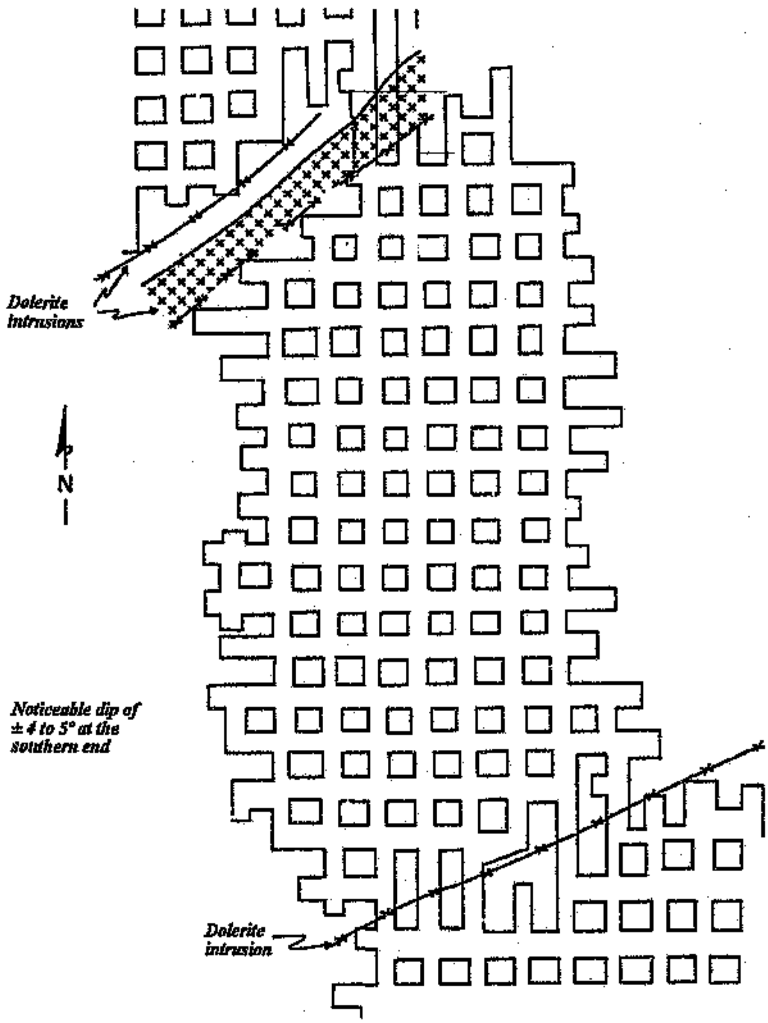

Figure 3.2 - Panel layout, Emaswati Case 1

Table 3.1 - Panel Geometries and parameters, Cases 1, 2 and 3

	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 3
Depth below surface	73m	50 to 66m	65m
Mining height	2.95 ± 0.3m	2.64m	1.9 to 3.1m
Pillar centres	13.5m	14m	16m
Original bord width	5.9m	6.7m	6.3m
Planned bord width	Unknown	6.5m	6.0m
Safety factor	1.56	1.56 to 2.06	2.19
Panel width	95m	91m	215m
Panel length	150m	273m	263m
Dip	4 to 5°	1°	1°
Width to height ratio	2.57	2.77	3.13 to 5.11
Number of Pillars across the panel	6	6	13

Based on the degree of fracturing in the sidewall a classification system was developed to estimate the state of pillars on a load-deformation curve. Five categories of pillar states were defined as shown in Table 3.2. Using this classification pillars were found to have failed in only a small area of the panel towards the southern end. The classification system and a plan of the classified pillars is shown in Figures 3.3 and 3.4.

Table 3.2 - Classification of Pillars, Emaswati Case 1

S	Slight scaling, pillars generally undamaged
SC	Sidewalls slightly scaled, all pillar corners scaled and fractured
SP	Sidewalls and pillar corners severely scaled
P	Sidewalls and pillar corners severely scaled with open vertical fractures in the sidewall
PP	Sidewalls and pillar corners severely scaled. Pillars reduced in size by over 1.5m in the dip direction due to spalling

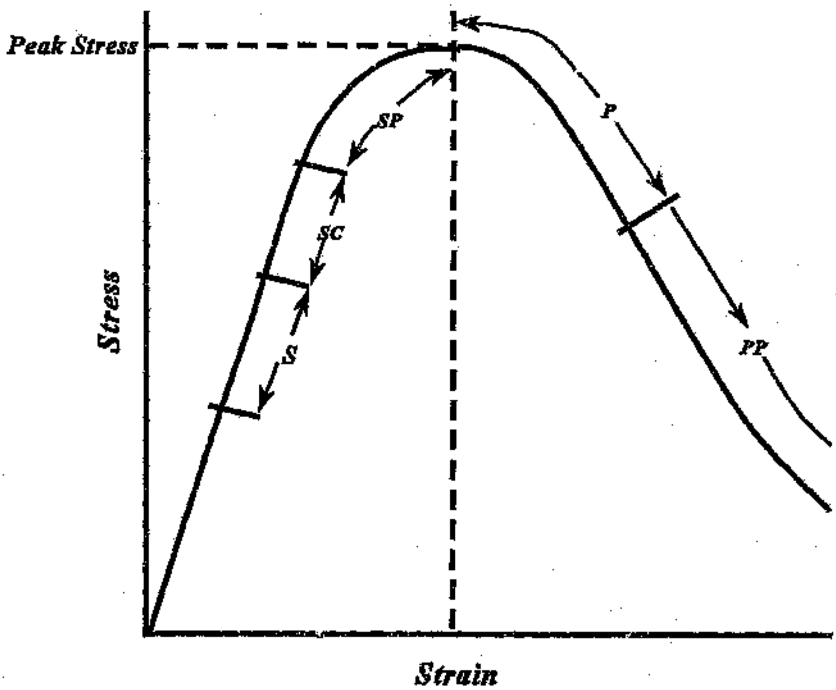

Figure 3.3 - Classification used by Oldroyd and Buddery[5] to describe pillars in Emaswati Case 1

It has been estimated from measurements taken from photographs of broken mine poles which had been installed in 1978 that strain of 3% had taken place since their installation. Table 3.3 below indicates the results from 3 poles.

Table 3.3 - Strain values of broken mine poles measured from photographs

POLE No.	AVERAGE STRAIN %
1	1.16
2	1.85
3	2.86

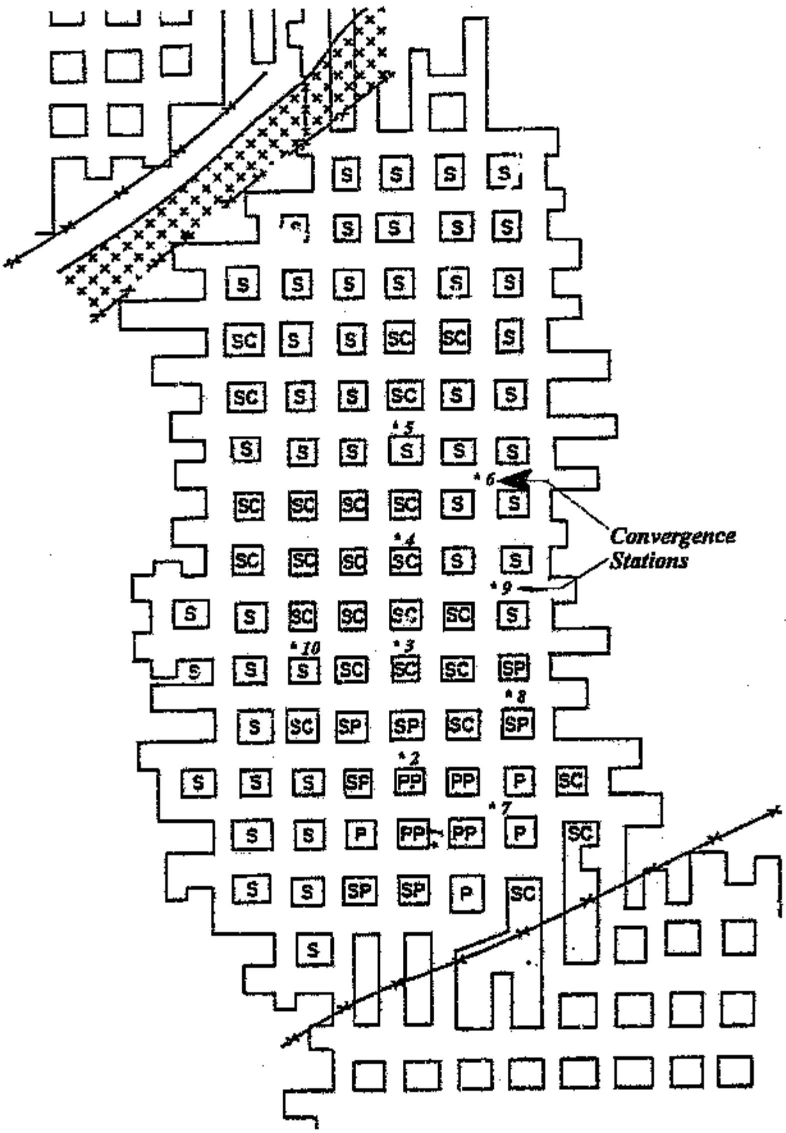

Figure 3.4 - Classified Pillars, Emaswati Case 1 also showing positions of Convergence Stations

Roberts and Reimann[7] showed that strain at failure for mine poles underground is typically of the order of 2 to 3%. Two of the values in Table 3.2 are close to or within this range while the other is significantly below this. If it is assumed that the poles were installed at the onset of pillar failure then the pillars are likely to have undergone no more than 4% strain from the time they were mined until recently. This is comprised of 0.1% elastic movement prior to attaining peak load and up to 4% post peak deformation. The elastic strain in this instance is approximated using the stress strain relationship

$$\epsilon = \frac{\sigma}{E}$$

assuming a coal pillar modulus of 4 GPa, a virgin stress σ_v of 1.825 MPa and pillar stress at failure, σ_p of 5.76 MPa.

Failure of the pillars appeared to be dip controlled as the dimensions of the pillars were dramatically reduced on dip from an initial 7.6m to as little as 2.0m in places while the strike dimension was lessened by only 0.5 to 1.0m (see Figure 3.5). While the pillars had indeed failed the solid sandstone roof remained intact and no falls were noted and no fracturing was observed. Floor heave of up to 0.05m was seen in places.

Figure 3.5 - Failed pillar corresponding o a PP classification, Emaswati Case 1

Convergence values as recorded by Oldroyd and Buddery[5] (1988) are reproduced in Figure 3.6, these indicate annual rates of 6.5mm at most. The closure was measured between the skin of the roof and floor and thus contains in-elastic deformation (e.g. bed separation) as well as elastic deformations. The true elastic convergence was probably not obtained. The pillar convergence would be less than the convergence measured in the bords as pillars would experience elastic convergence only. A maximum convergence of 33mm was recorded over the 5½ years that monitoring took place corresponding to a strain of 1.1%.

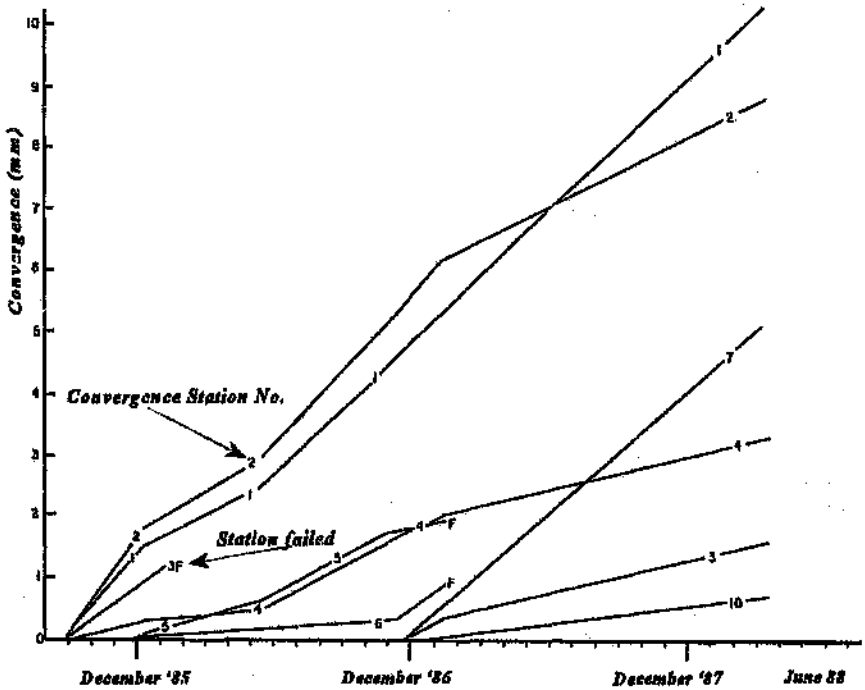

Figure 3.6 - Convergence graphs, Emaswati Case 1

In 1992, the panel was revisited and photographed. It was noted that spalling of pillars has continued and that pillar fractures had opened up further. Cluster stick packs, which were installed in 1988, had not failed indicating that very little convergence had taken place since that time. It is also significant that large roof falls had not occurred. Although small sandstone layers of about 0.1m wide had fallen, the non-existence of vertical or subvertical fracturing in the roof above these indicated that the beam forming the roof had not failed.

3.3 ZULULAND ANTHRACITE COLLIERY (Z.A.C.), CASE 2, PILLAR FAILURE UNDER HIGH STRAIN

A second case of pillar failure has also been extracted from the paper by Oldroyd and Buddery[5], (1988). This occurred at Z.A.C. during 1987 and continued until early in 1988. Z.A.C. is situated in Northern Natal approximately 200 kilometres south of Emaswati Colliery. Although the coalfields in which Z.A.C. and Emaswati are situated are not continuous, there are a number of geological similarities between the two coal mines.

Pillar failure at Z.A.C. occurred over a relatively short time period of eight months and stabilised with ultimate strain values in the vicinity of 40%. At Z.A.C., the roof of the main seam consists of massive sandstone extending 11m up into the roof before the weaker alternations zone is reached. The remainder of the overlying strata largely consists of sandstone, occasionally interbedded by coal and shale layers (see Figure 3.7). The floor consists of fine grained siltstones, micaceous sandstones and some thin coal layers.

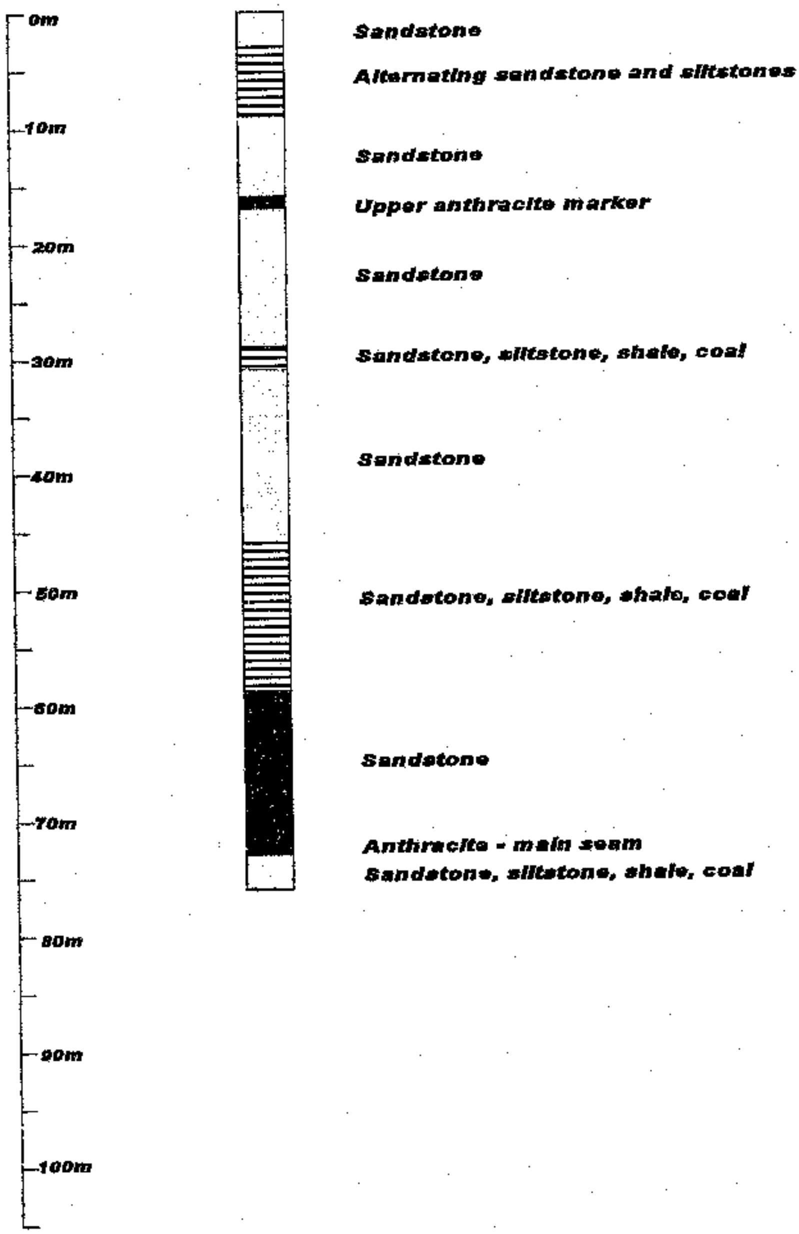

Figure 3.7 - Generalised geological section at Z.A.C.

Ir July 1987, production officials noted abnormal pillar spalling and joint dilation. This was confirmed during subsequent visits. Convergence metres were installed by both production and rock engineering personnel. Initially the failure was isolated beneath a ridge on surface, but this spread in the months that followed to cover the entire panel consisting of 82 pillars. Geometric parameters are given in Table 3.1.

Measured convergence in the panel was greater than it was in the Emaswati Case 1 and graphs of these are shown in Figures 3.8 and 3.9. During the early stages of failure the graphs indicate a linear rate of convergence of between 40 and 75mm after which the rate rapidly accelerates to a point where it became unsafe to take readings (±1000mm per annum). As the convergence increased the roof was found to break along joints which were previously cemented and closed (Figure 3.10). Accelerating convergence was accompanied by increased spalling of the pillars and some large diameter mine poles punched into the footwall. Although the roof cracked along joints and faults, roof falls did not occur with the exception of isolated areas located near the barriers.

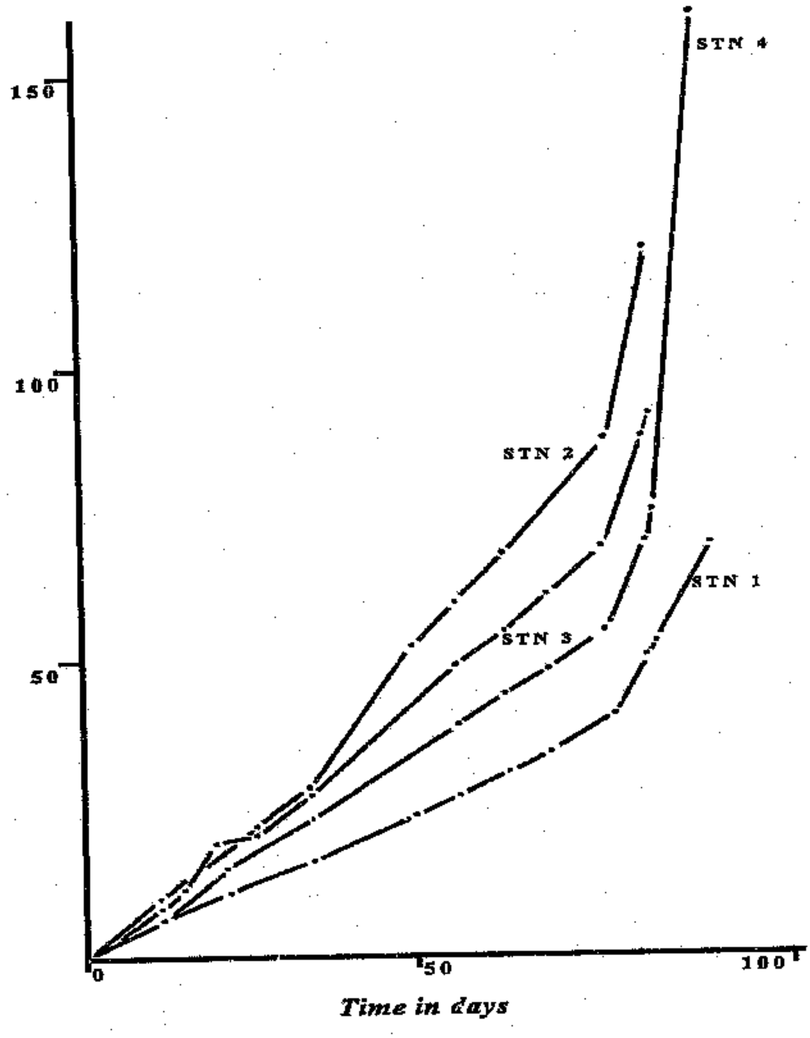

Figure 3.8 - Closure measurements at Z.A.C.

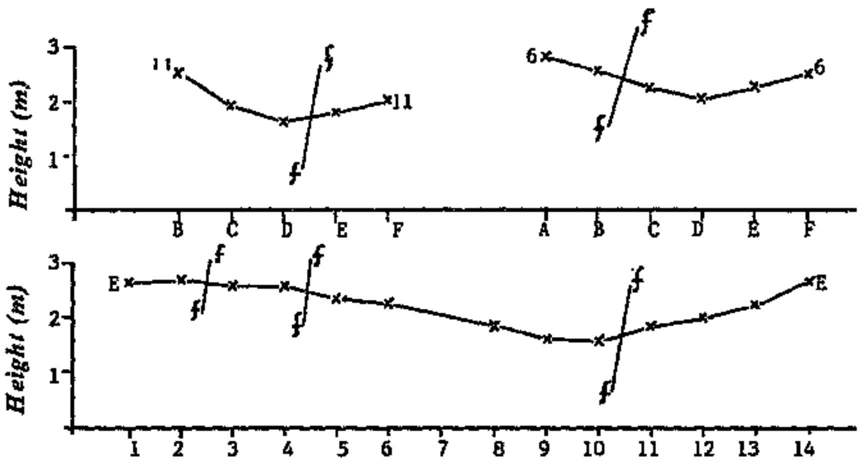

Figure 3.9 - Convergence across the panel after failure at Z.A.C.

Figure 3.10 - Roof joints opened during pillar failure at Z.A.C.

Figure 3.11 - Photograph of failed pillars at Z.A.C.

Initially pillar failure was characterised by movements along predominant joints which are a common pillar weakening feature at Z.A.C. As convergence progressed general spalling occurred. The slight dip of 1° did not appear to play a role in the failure of pillars. (In the Emaswati Case 1, there was a dip of 4 to 5° which appeared to control the mode of failure). This was probably because of the shallow dip and because spalled coal was not removed from around the pillars. Over a period of approximately 2 months rapid convergence continued in the panel resulting in strain values of up to 45%. As this figure was established from roof to floor measurements, actual strain values across pillars were likely to be somewhat less due to floor heave and bed separation.

Large strain was accompanied with severe pillar spalling to the extent that the toes of the inclined heaps of coal from the pillar sides met at the centre of the bord. Pillar edges could not be defined and the sidewalls consisted only of broken coal (see Figure 3.11). Floor heave of up to 0.7m was noted in places while roof falls were isolated. Fractures in the roof had dilated by up to 60mm.

In February 1988, convergence had ceased and the panel had reached stability allowing the conveyor belt structure to be removed. Oldroyd and Buddery[5] concluded that the pillars had recompacted and were carrying the full cover load (Figure 2.5).

3.4 EMASWATI CASE 3, PILLAR FAILURE UNDER INITIAL YIELD FOLLOWED BY SUDDEN COLLAPSE

A third case of pillar failure has been observed at Emaswati Colliery between 1990 and 1992 which was initially characterised by slow rates of convergence that were first recorded in January 1990. Geological conditions were similar to those in the first Case at Emaswati : The coal seam is overlain by a massive sandstone roof and dolerite dykes bound the north and south of the panel. The floor however, consisted of a weaker shaley sandstone. Furthermore the coal was weaker due to the slight metamorphic effects of dolerite intrusions. The panel width and length are substantially larger than either of the two previous cases.

Figure 3.12 shows the rate of convergence plotted against time from January 1990 when the first readings were taken. At first, convergence rates of up to 40mm per annum were measured which subsequently settled down in the 25 to 30mm per annum range. During this time pillar failure was indicated by slabbing and in places by blocks of coal sliding along joints. Footwall heave of about 0.1m was noticeable. The roof was stable and showed no signs of failure or opening up along joints. Early in 1991, additional convergence stations were established, and by March, high rates of movement were detected of up to 511mm per annum. Readings taken at more frequent intervals confirmed the trend. By May 1991, the rate of movement accelerated further and it was considered unsafe to take readings.

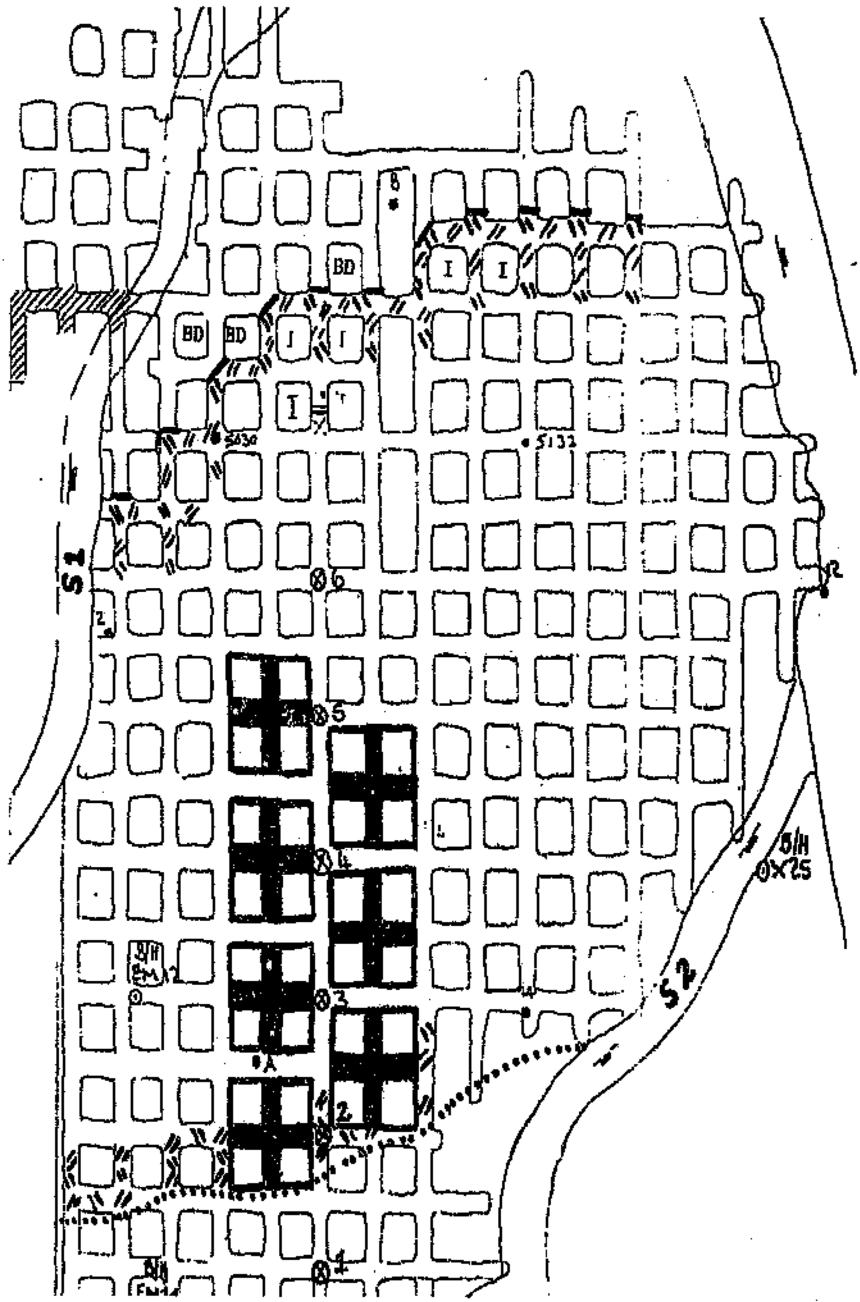

Figure 3.12 - Panel layout and Convergence stations, at Emaswati Case 3

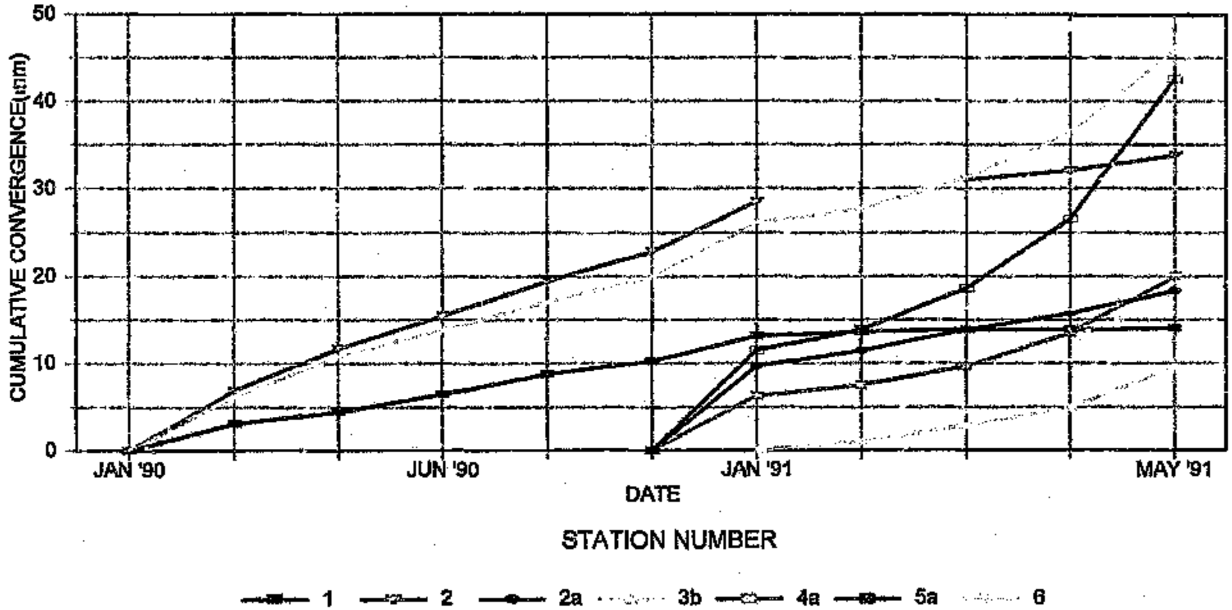

Figure 3.13 - Convergence measurements at Emaswati Case 3

In June 1991 the panel failed suddenly trapping 28 people underground who were fortunately later rescued unharmed. A 1.2m deep subsidence trough appeared on surface confirming that large movements had taken place underground. Underground, at the periphery of the failure high roof falls were evident and floor heave could be seen. Pillars along the edge did not appear to have failed. Deeper into the panel pillar observation was obstructed by roof falls. Before the sudden failure occurred floor heave was evident together with pillar spalling. It is likely that a combination of pillar failure and pillars punching into the floor occurred.

3.5 STRESS - STRAIN CURVES

Stress strain curves for the pillars in the three case histories were estimated by making some basic assumptions about their behaviour. The upward sloping portion prior to failure has been reproduced at an elastic modulus of 4 GPa, i.e. the value obtained by Van Heerden[4] from insitu testing. At failure the load on the pillars and hence their strength is assumed to be equal to that found by applying tributary load theory. Post-peak slopes have been determined by three different methods. Ryder and Ozbay's[2] envelope equation has been used as well as the equations fitted to Van Heerden's[4] and Wagner's[3] insitu test data. The results are tabulated below:

Table 3.4 Stress - Strain data for the case histories

	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 3 *
Pre failure slope (GPa)	4	4	4
Pillar strength (MPa)	5.76	6.07	4.42
Post peak slope after Ryder[2] (GPa)	-0.87	-0.74	-0.27
Post peak slope after Van Heerden[4] (GPa)	-0.77	-0.72	-0.53
Post peak slope after Wagner[3] (GPa)	-0.34	-0.32	-0.21

* For Emaswati Case 3 a width to height ratio of 3.88 has been used which is considered representative of the panel. The range in width to height ratios across the panel is significant.

These are also plotted in Figure 3.13 to 3.15. The graphs and the tables clearly show that using the "fit" to Wagner's[3] data consistently shows a lower post peak modulus than those values obtained by Using Ryder and Ozbay's[2] envelope or the fit to Van Heerden's[4] data.

Most stiffness analysis is done in units of MN/m, the defined units of stiffness. Table 3.5 below shows the data in Table 3 in units of MN/m. This can be obtained by re-arranging the stress strain relationship.

$$E = \frac{stress}{strain} \qquad E = \frac{\frac{F}{A}}{\frac{e}{h}} \qquad E = \frac{Fh}{eA}$$

$$\lambda = \frac{EA}{h}$$

where λ is stiffness, E is the post peak modulus, A is the area of the pillar and h the height of the pillar.

Table 3.5 - Post peak stiffnesses, λ

	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 3
Post peak stiffness after Ryder[2] (GN/m)	-16.9	-15.0	-10.0
Post peak stiffness after Van Heerden[4] (GN/m)	-15.1	-14.6	-19.8
Post peak stiffness after Wagner[3] (GN/m)	-6.8	-6.4	-8.1

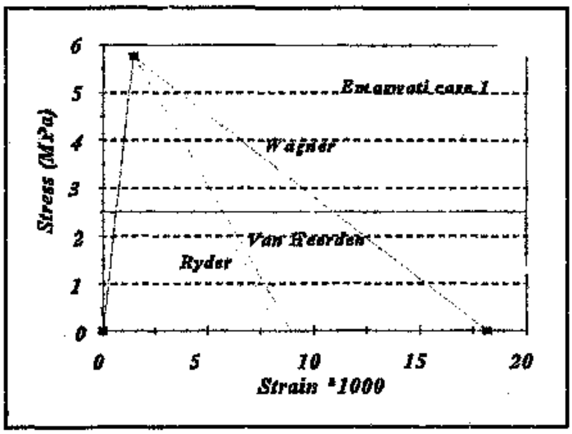

Figure 3.14 - Stress strain graphs Emaswati Case 1

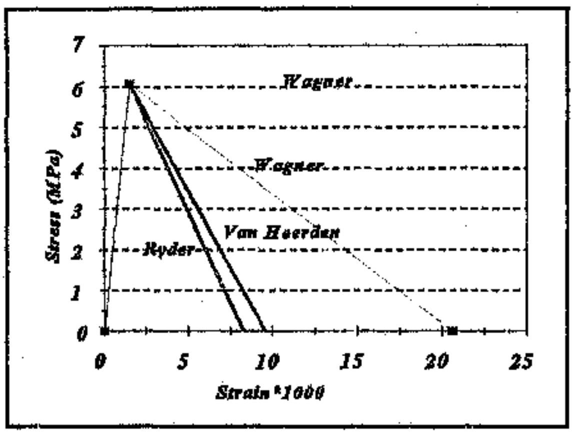

Figure 3.15 - Stress strain graph ZAC Case 2

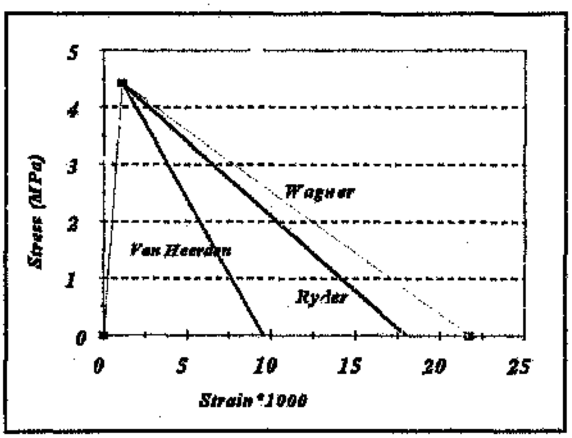

Figure 3.16 - Stress strain graphs Emaswati Case 3

3.6 ELASTIC CONVERGENCE

In all the models that have been discussed and used so far the assumption has been made that the surrounding strata is isotropic and behaves in a linearly elastic fashion. For the purposes of this analysis it is interesting to consider the allowable elastic convergence in open stopes having the same width as the panels in the three cases. That is as if there are no pillars in these panels.

Maximum elastic convergence at the centre of an open stope close to surface can be calculated using the following formula:

$$S = \frac{2\ (\ 1\ -\ \nu^2\ )\ \rho\ gHL\ f\,(\alpha)}{E}$$

where υ = poissons ratio

E = Young's modulus

ρ = density

H = depth below surface

L = panel span

$f(\alpha)$ = $1 + 0.41\alpha + 0.149\ \alpha^2 + 0.008\alpha^3$

α = Panel span/H

Table 3.6 shows the open stope elastic convergence calculated for the three cases. Table 3.7 shows these figures expressed as a percentage of mining height and this provides an indication of maximum elastic strain that maybe applied to the pillars while the surrounding strata behave elastically.

Table 3.6 - Span to depth ratio α and calculated elastic convergence

	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 3
α	1.46	1.52	3.31
Convergence for E = 12GPa	46.2mm	41.9mm	230.1mm
Convergence for E = 6 GPa	92.4mm	83.8mm	460.2mm

Table 3.7 - Elastic stope convergence expressed as a percentage of mining height

Host Rock Modulus	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 3
E = 12 GPa	1.6	1.6	7.7
E = 6 GPa	3.1	3.2	15.3

Calculated Elastic Strain at Emaswati Case 1 is consistent with the observed strain values on mine poles and the measured convergence. In the Z.A.C. Case 2 strain far exceeded that elastically possible as over 40% strain was estimated by Oldroyd and Buddery[5]. In Emaswati Case 3 the overlying strata broke up and failed and no final strain values were measured. It is obvious however that the elastically possible values were exceeded.

3.7 DISCUSSION AND CONCLUSIONS

Three pillar failures have been described, these have a number of similarities, they are shallow and all at a similar depth varying between 50 to 73m. Mining heights were also similar being close to 2.8m, although in Emaswati Case 3 there was considerable variation in mining height across the panel. Pillar width to height ratios are also of the same order in case 1 and case 2, being 2.57 and 2.77 respectively, while in case 3 width to height ratios are higher varying between 3.13 and 5.11 due to the varying mining height.

In all three cases the seam dips slightly and the coal has been weakened, by joints in cases 1 and 2 and in case 3 by the metamorphic effect of a dolerite intrusion lying beneath the seam. Competent sandstone overlies the workings in all three areas. Significantly the panel width in cases 1 and 2 are similar, 91 and 95m respectively and they both have six pillars between major barriers, while in case 3 the panel width is substantially greater at 215m and the panel contains 12 pillars between major barriers.

Three failures have been documented which show 3 distinct outcomes. The Emaswati Case 1 is characterised by low pillar strain with slow deterioration of the pillars occurring over a 15 year period, this is currently a stable situation which may become unstable if the overlying strata fail. The eventual mode of failure is unknown.

In the second case pillar failure and strain occurred at a rapid rate and although at times it was felt unsafe to travel through the panel it always remained open. Roof falls were not a general feature. This case represents controlled pillar failure.

In the 3rd case pillars were failing slowly and the rate began to accelerate in a similar fashion to Z.A.C. However, this was not immediately accompanied by the opening up of joints in the roof. Twenty one months after the panel was mined the panel failed in an uncontrolled fashion resulting in roof break up which prevented the passage of people.

CHAPTER 4 ANALYSIS OF COAL PILLAR FAILURE MODES USING NUMERICAL MODELLING

The program BEPIL developed by Ryder[2] has been used to analyse various characteristics of the three case histories discussed in the previous section. BEPIL is a versatile two dimensional boundary element program and one which uses a "Z"correction introduced by Salamon[1] for determining of rectangular or square pillar stresses. It is particularly useful when examining the stability of pillars in the post peak region. It is in this fashion that the program has been used to analyse the sensitivity of the pillar failure mode to variability in the surrounding strata modulus and to the post peak modulus or stiffness of the pillar. BEPIL is written in Basic and requires a Quick Basic compiler to run. A full listing of the program is provided in Appendix 1.

4.1 INPUT REQUIREMENTS

Data for the model geometry is input within a small section of the listing. The input geometry and variables for the model of Case 1 at Emaswati Colliery are shown below. Briefly the input format is as follows :-

Line 1	Constants: TITLES = "Yielding Pillars - Single Panel"
Line 2	q = 1.86: g = 6.75: Sm = 3: E = 12000: Es = .333 * E: rep = 109.4
Line 3	Pattern: n = 17: FOR j = 1 TO n: READ Npat(j): NEXT j
Line 4	DATA 0,0,0,1,2,1,2,1,2,1,2,1,2,1,0,0,0
Line 5	Materials: m = 2
Line 6	M2: DATA K,7.6,7.6,13.5,-10 000

Line one contains t.. title of the particular run. The second line contains the parameter **q** - virgin stress (MPa), **g** is the grid size and is chosen in the models to be half the pillar centre distance (m). **Sm** is the mining height (m). **E** is the modulus of the surrounding strata (MPa), **Es** is modulus of the seam expressed as a multiple or fraction of **E** (MPa), **rep** is set either to the width of the model (number of elements × grid size) which then regards the model as isolated or if rep is set to a high number e.g. 10 000 then the panel geometry is repeated many times.

In the third line the variable n corresponds to the number of elements that the model consists of, in the example above n is 17. In the fourth line the panel layout is described, 0's denoting barrier pillars, 1's bords or mined areas and 2's are pillars which may have a user specified characteristic.

In the 5th, 6th and other lines of their kind that may follow the material types are specified. Material type data K was used extensively, this has the following input requirements:

M2 : DATA K, 7.6, 7.6, 13.5, - 10000

pillar dimensions on **dip** and **strike**, **pillar centres** on strike and **post peak stiffness** (λ) of the pillar. Examples of the input sheets for all three geometries modelled are shown in the Appendix 3.

4.2 MODELLED GEOMETRIES

Figure 4.1 is a section through the panel as modelled in BEPIL for Emaswati Case 1. Similarly Figures 4.2 and 4.3 represent sections through the Zululand Anthracite Colliery (Z.A.C.) Case 2 and Emaswati Case 3. It is important to note that the grid size in these examples has been set at half the pillar centre distance. According to Ryder and Ozbay[2] the model will give reliable results provided 0.5g < pw < 1.5g. Where pw is the pillar width specified in line 6.

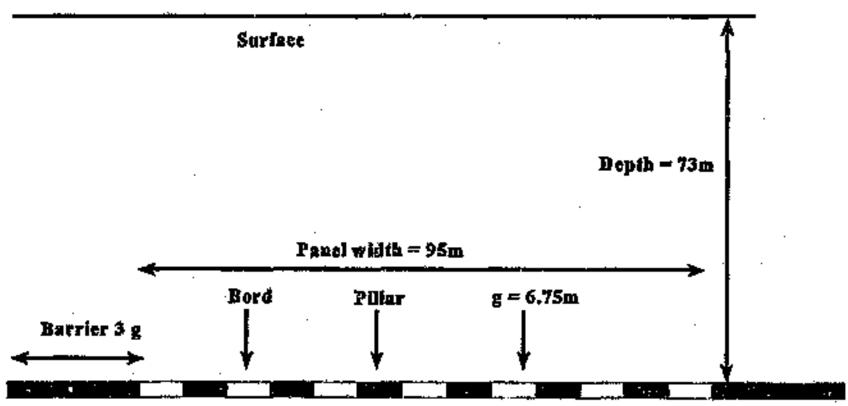

Figure 4.1 - Section through the BEPIL model of Emaswati Case 1 (Not to scale)

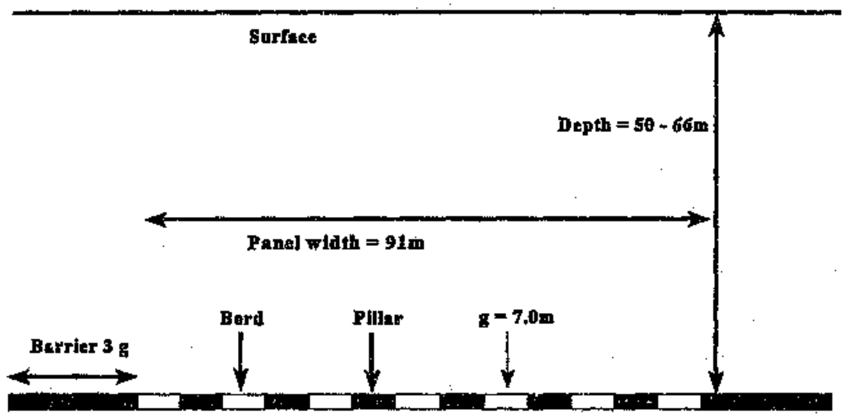

Figure 4.2 - Section through the BEPIL model of the Z.A.C. Case 2 (Not to scale)

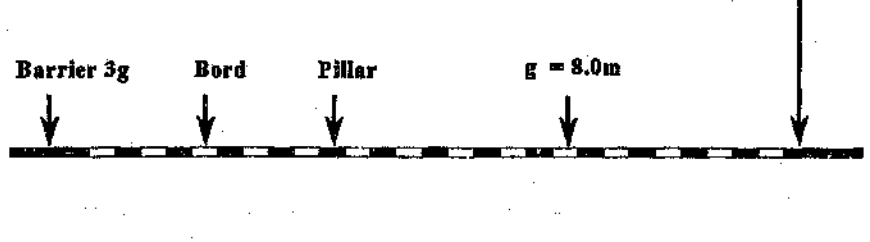

Figure 4.3 - Section through the BEPIL model used for Emaswati Case 3 (Not to scale)

4.3 DETERMINATION OF POST-PEAK STIFFNESS, λ_c

The critical value of post peak pillar stiffness for a mining geometry can be determined by varying λ for K type pillars in BEPIL and recording the number of iterations required to solve the model. When the post peak stiffness of the pillars approaches the critical value corresponding to $k + \lambda = 0$ (from Salamon[1]) the number of iterations, which are reported in the result list, rapidly increases. To find the critical value requires repeated runs with many different post peak stiffnesses. For each run the number of iterations required to solve the model is recorded. The number of iterations can be plotted against λ. The graph peaks at a point indicating the critical value of λ, λ_c. This methodology, although not taking a great deal of time for each run, is tedious and time consuming.

To automate the process a procedure was written in basic which runs BEPIL in preselected values of λ. For each run the procedure records the number of iterations required to run the model to a file.

This file can then be used to create graphs which indicate the critical values of λ. The basic listing required to perform BEPIL iteratively is listed in Appendix 2. As the program is no longer required to report pillar stresses, displacements and display graphs which it would ordinarily do. The portion of the program required to show these has been removed.

An example showing the determination of λ_c by this method is shown in Figure 4.4. The graph oscillates considerably as λ is varied between - 12000 and - 7000 MN/m. In this domain large changes in the number of iterations required to solve the problem are seen with small changes in λ. Nonetheless the number of iterations clearly peak at a little over - 3980 MN/m indicating the value of λ_c. It is interesting to note that the model "solves" when the post peak stiffness of the pillar is either above or below the critical value λ_c.

4.4 MODELLING RESULTS

4.4.1 λ_c vs Elastic Modulus

Figures 4.4 to 4.9 show λ vs number of iterations for all three case histories at moduli of 6, 9, 12 and 15 GPa. Table 4.1 summarises the variation of λ_c with the elastic modulus of the strata. These were determined by varying the value of post peak pillar stiffness using K type pillars in BEPIL as explained in the previous section.

TABLE 4.1 - Variation of λ_c with strata modulus

	λ_c (MN/m)		
Modulus (GPa)	**EMASWATI CASE 1**	**Z.A.C. CASE 2**	**EMASWATI CASE 3**
6	**-3.96**	**-4.06**	**-1.95**
9	**-5.79**	**-5.99**	**-2.93**
12	**-7.53**	**-7.85**	**-3.86**
15	**-9.22**	**-9.64**	**-4.78**

These are also plotted in Figures 4.16 to 18, which show that the results for each geometry plot on a straight line and confirm that if the geometry is fixed then λ_c is directly proportional to strata modulus. Furthermore extrapolating these results to a zero modulus indicates λ_c tends to 0. The following linear relationships between stiffness and modulus are obtained by fitting a straight line to the results.

$\lambda_c = -0.461 - 0.60E$ - Emaswati Case 1

$\lambda_c = -0.375 - 0.62E$ - Z.A.C. Case 2

$\lambda_c = -0.083 - 0.314E$ - Emaswati Case 3

The units of E are in MPa, and for Karoo sediments typically lie in the range 6000 to 15000 MPa, thus the value of the constant is very low in relation to the total value. The equations imply a nearly zero value for λ_c, for a zero value of modulus.

Expected results emerge from these graphs i.e. that λ_c is lower for the case 3 which has 12 pillars than the other two cases which both have 6 pillars. Similarly λ_c is almost identical for the first Emaswati Case and that at Z.A.C. Case 2 which is also to be expected as the geometries are similar.

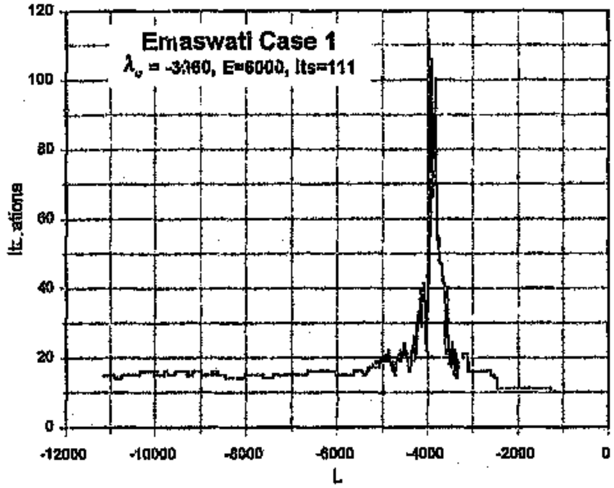

Figure 4.4 -Iterative solution of λ_c, Emaswati Case 1, E = 6000 MPa

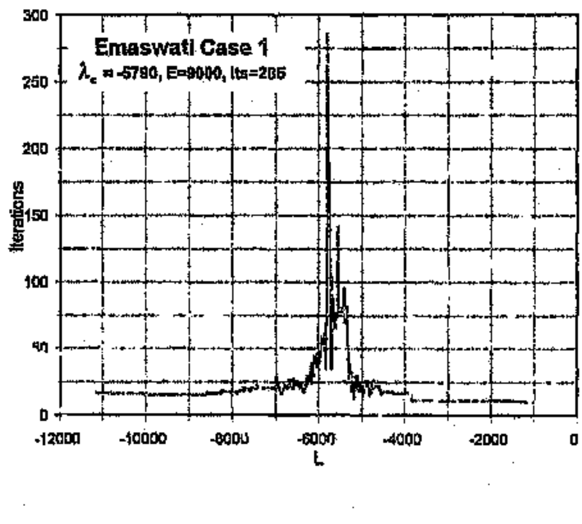

Figure 4.5 -Iterative solution of λ_c, Emaswati Case 1,E = 9000 MPa

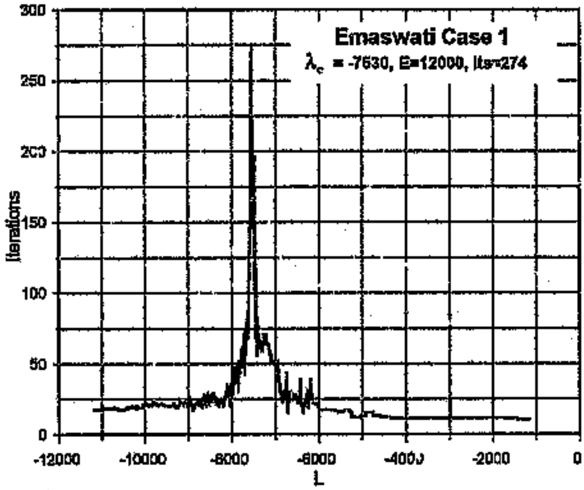

Figure 4.6 -Iterative solution of λ_c, Emaswati Case 1,E = 1200 MPa

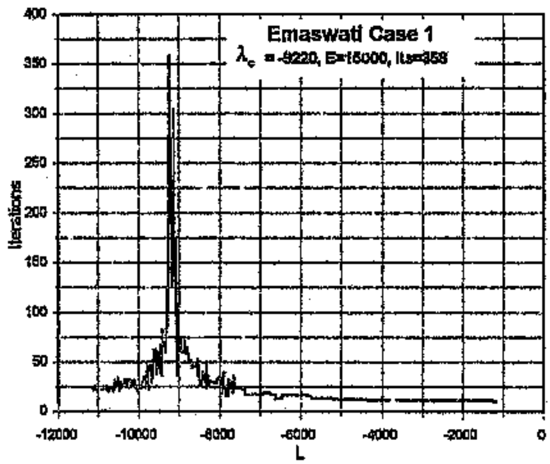

Figure 4.7 -Iterative solution of λ_c, Emaswati Case 1,E = 15000 MPa

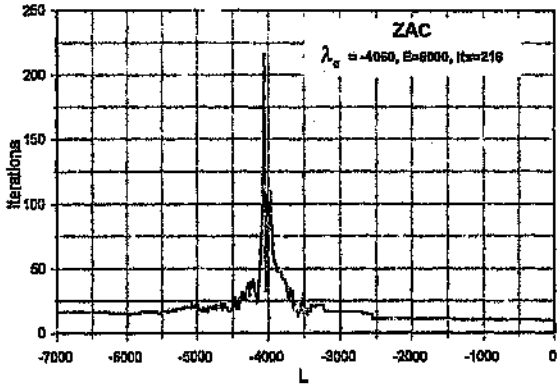

Figure 4.8 -Iterative solution of λc, Z.A.C. Case 2, E = 6000 MPa

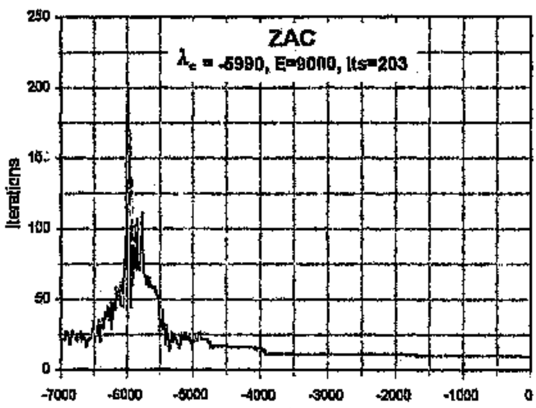

Figure 4.9 -Iterative solution of λ_c, Z.A.C. Case 2, E = 9000 MPa

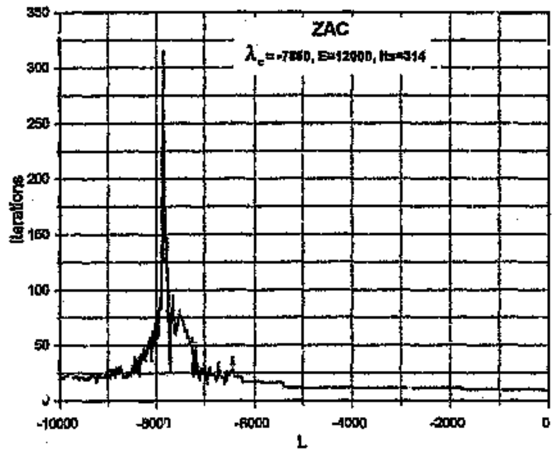

Figure 4.10 -Iterative solution of λ_c, Z.A.C. Case 2, E = 12000 MPa

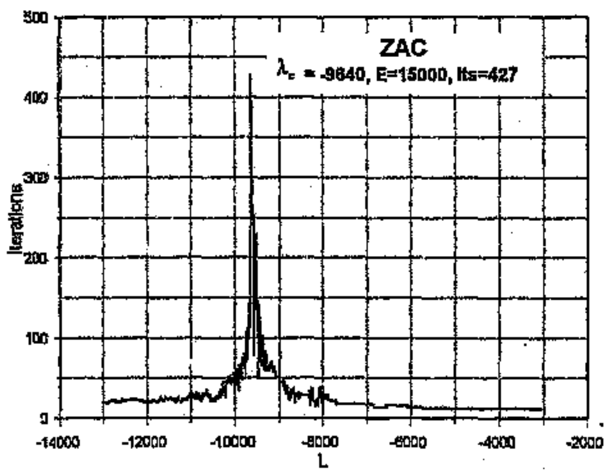

Figure 4.11 -Iterative solution of λ_c, Z.A.C. Case 2, E = 15000 MPa

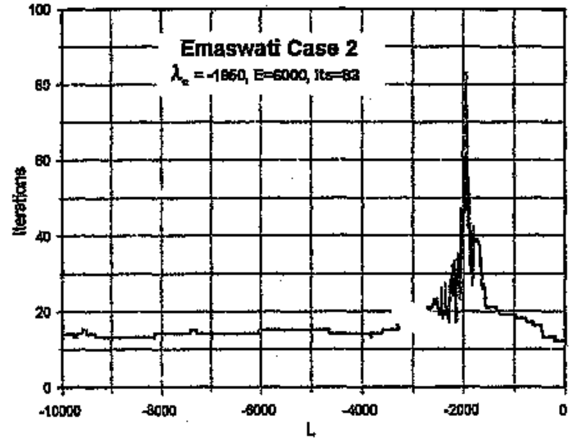

Figure 4.12 -Iterative solution of λ_{c}, Emaswati Case 3, E = 6000 MPa

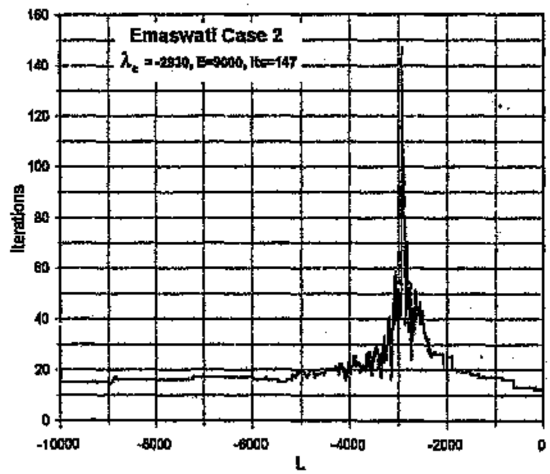

Figure 4.13 -Iterative solution of λ_{c}, Emaswati Case 3, E = 9000 MPa

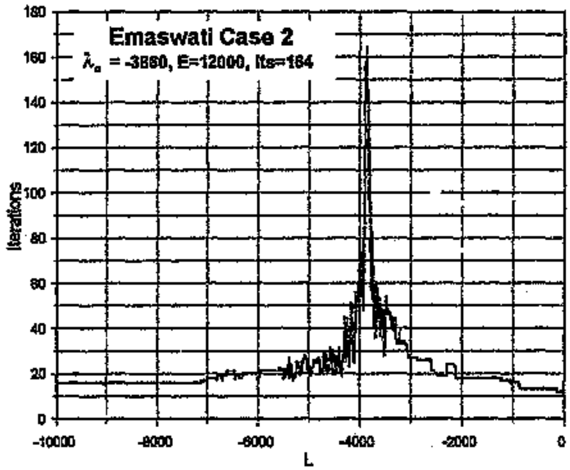

Figure 4.14 -Iterative solution of λ_c, Emaswati Case 3, E = 12000 MPa

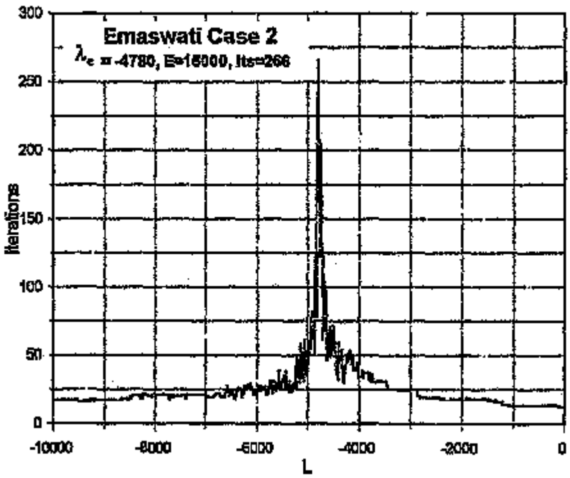

Figure 4.15 -Iterative solution of λ_c, Emaswati Case 3, E = 15000 MPa

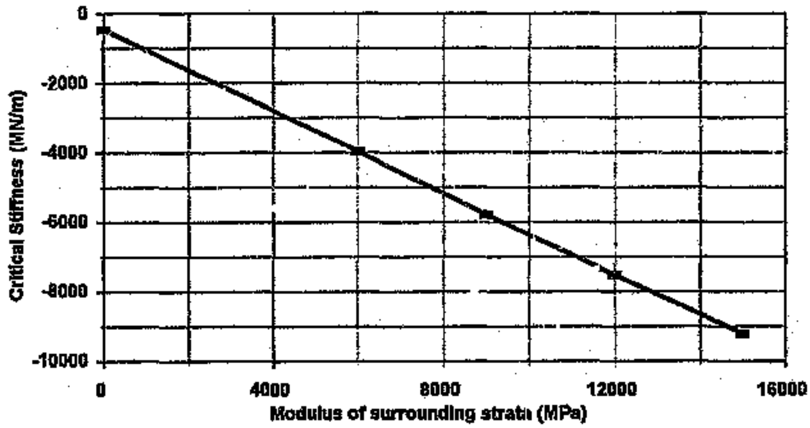

Figure 4.16 - Graph of modulus vs Critical stiffness, Emaswati Case 1

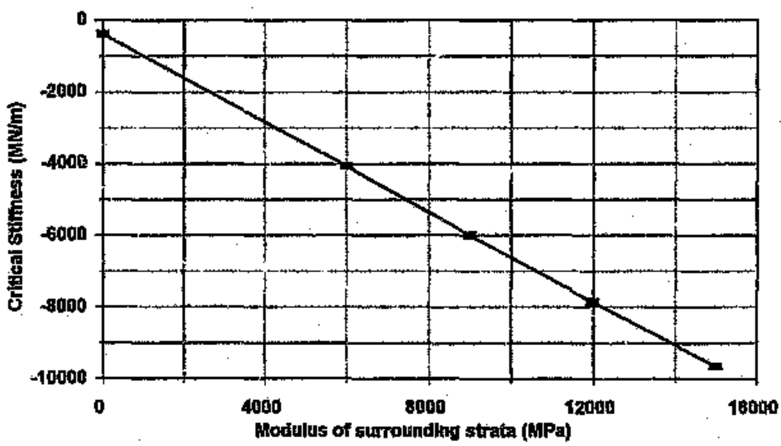

Figure 4.17 - Graph of modulus vs Critical stiffness, Z.A.C. Case 2

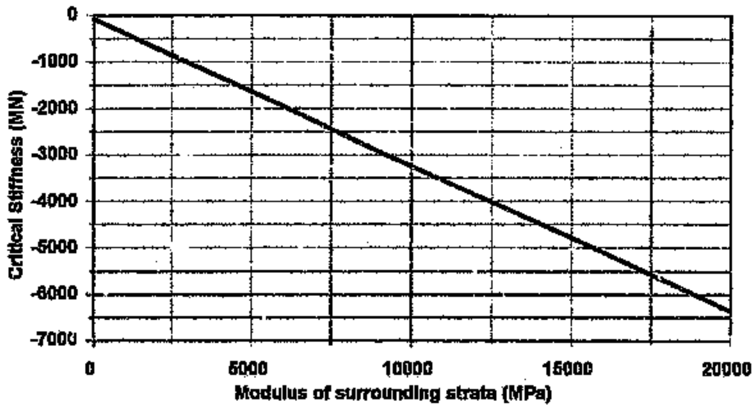

Figure 4.18 - Graph of modulus vs Critical stiffness Emaswati Case 3

CHAPTER 5 ANALYSIS OF RESULTS AND DISCUSSION

Three cases where pillars have failed have been studied by comparing their recorded behaviour during failure with that which may be predicted by elastic theory and empirical results from insitu pillar testing. The analysis of the results obtained is presented in this section.

5.1 Stability analysis

Table 3.4 (for convenience the table is reproduced below) shows three values of expected post peak slopes for the pillars in the three case histories. These were obtained using the formula describing Ryder and Ozbay's[2] envelope and from the equations fitted to Wagner[3] and Van Heerden's[4] insitu testing data obtained in Chapter 2. In Figure 5.1 the post peak slopes for the pillars in Emaswati Case 1, presented in table 3.4, are plotted as horizontal lines on a graph of surrounding strata modulus vs λ_o. The straight line of λ_o vs Elastic modulus for Emaswati Case 1, obtained from the BEPIL modelling in Chapter 4, is also shown.

" Table 3.4 - Post peak stiffnesses λ "

	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 2
Post peak stiffness after Ryder[2] (GN/m)	-16.9	-15.0	-10.0
Post peak stiffness after Van Heerden[4] (GN/m)	-15.1	-14.6	-19.8
Post peak stiffness after Wagner[3] (GN/m)	-6.8	-6.4	-8.1

The post peak moduli obtained by the three methods indicate values of local pillar stiffness λ. For stability $\lambda < \lambda_{u} < 0$, Salamon[1]. This is a conservative analysis as calculated values of λ_{u} are always less than the local values of stiffness of the pillars of which the panel is comprised.

Stable pillar failure on this graph would be represented by a point plotting above the sloping line representing the critical slope obtained from BEPIL. For example if the post peak slope of the pillars is estimated at -4 GN/m and the surrounding strata modulus is 12 GPa the point lies in the region of stable pillar failure for the particular geometry being modelled. However, if the post peak slope is - 8 GN/m and the modulus 10 GPa this would indicate an unstable geometry.

Stable pillar failure occurred in Emaswati Case 1 and as surrounding strata moduli of the order of 10 to 14 GPa are expected the actual post peak slope should lie in the region of -8.8 to -6.5 GN/m or more. These values can be read off the graph or determined from equations fitted to the data in 4.4.1. The post peak slope obtained from Wagner's insitu testing data fits this model and would indicate that stable pillar failure is possible.

Post peak slopes obtained from both the fit to Van Heerden's[4] data and the envelope of Ryder and Ozbay[2] indicate values of stiffness which are unlikely to result in a yielding pillar behaviour as these would require strata moduli in excess of 24.4 and 27.4 GPa respectively. These values of modulus are considered to be too high for the Karoo sediments found in the area.

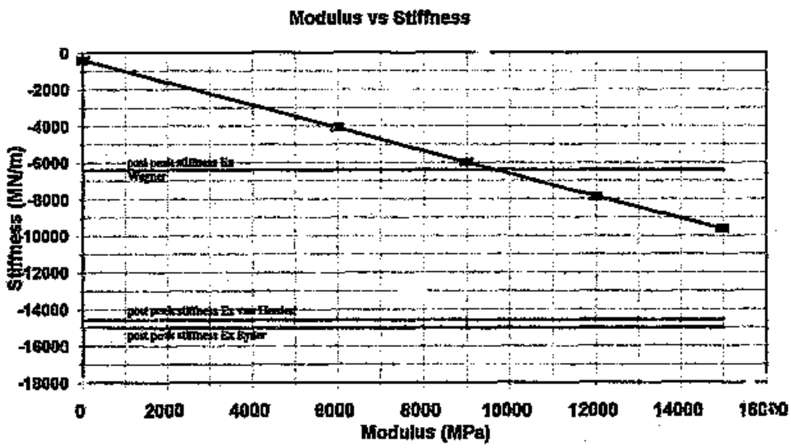

Figure 5.1 - Strata modulus vs Critical stiffness, Emaswati Case 1

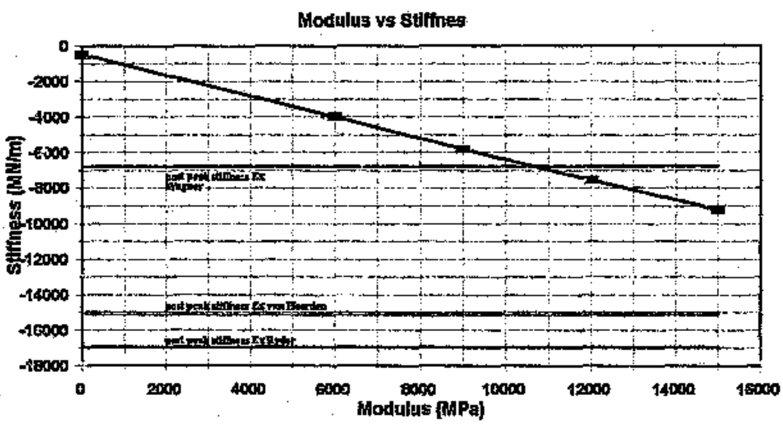

Figure 5.2 - Strata modulus vs Critical stiffne.s, Z.A.C. Case 2

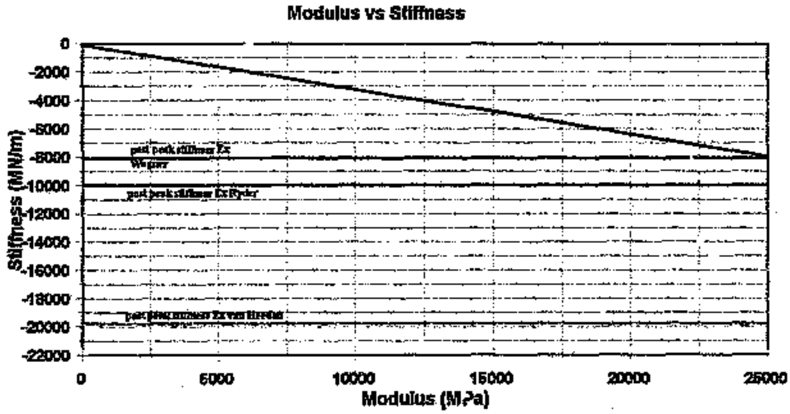

Figure 5.3 - Strata modulus vs Critical stiffness, Emaswati Case 3

Figures 5.2 and 5.3 show the Zululand Anthracite Colliery (Z.A.C.) Case 2 and Emaswati Case 3. Similar results are obtained for the Z.A.C. Case 2 as in Emaswati Case 1, i.e. the post peak slope obtained from Wagner's[3] data lies in a region where stable failure is possible and agrees with the observed pillar behaviour. Determinations of post peak slopes by Van Heerden's[4] data and the envelope of Ryder and Ozbay[2] require surrounding strata moduli greater than 22.9 and 26.7 GPa respectively for stable failure. Once again these values are far higher than those expected for the overlying strata.

In the Emaswati Case 3, shown in Figure 5.3, the situation is somewhat different in that the both post peak pillar slopes estimated by the envelope of Ryder and Ozbay[2] and the fit to Wagner's[3] data and that of Van Heerden[4] indicate unstable pillar failure. That by Wagner[3] indicating a modulus greater than 26.1 GPa for stable failure and that by Ryder and Ozbay[2] a modulus greater than 32.1 GPa. Van Heerden's[4] data indicating a very high GPa. The results are summarised below in Table 5.1.

Table 5.1 - Implied Strata Moduli for stable failure

	EMASWATI CASE 1	Z.A.C. CASE 2	EMASWATI CASE 3
Wagner[3] (GPa)	≥10.6	≥9.7	≥26.1
Van Heerden[4] (GPa)	≥24.4	≥22.9	≥63.3
Ryder and Ozbay[2] (GPa)	≥27.4	≥26.7	≥32.1

All the three cases had an initial phase of stable pillar failure and as overlying strata moduli are unlikely to greatly exceed 15 GPa it thus appears that the post peak slopes obtained by Wagner[3] more closely resemble the post peak pillar characteristics at Emaswati and Z.A.C. Collieries than the estimation of post peak slopes by Ryder and Ozbay[2] and by the data fitted to Van Heerden's[4] data. However, the Emaswati Case 3 results imply that stable pillar failure is unlikely to occur and indeed although there was an initial stage of yield and controlled failure, the collapse became uncontrolled.

The situation is further complicated by the large spread of mining heights in the panel and hence pillar width to height ratios. In the above analysis an average height has been assumed.

It is important to note that the above analysis is done on the assumption that the overlying strata behave in an elastic fashion. This was seen to be the case in Emaswati Case 1 but in the Z.A.C. Case 2 and in Emaswati Case 3 the overlying strata deformed inelastically.

5.2 Complete stress strain curves

Stress strain curves for the three case histories were produced in section 3.5, (Figures 3.13, 3.14, 3.15) based on assumptions for elastic modulus E = 4 GPa, pillar strength assumed to be equal to the tributary load at failure and post peak moduli using equations fitted to Wagner[3] and Van Heerden's[4] insitu data and from the envelope of all insitu tests determined by Ryder and Ozbay[2]. To these graphs have been added a shaded region indicating the possible range of elastic strain calculated with E varying from 6 to 12 GPa which was also determined in Chapter 3.

Emaswati Case 1 represents a case of stable pillar failure with low strain. The pillars tracing out only a small portion of the possible strain range (Figure 5.4 shows the probable position of the pillars on the stress strain graph). Elastically, little or no further deformation can take place. To cause the pillars to trace out the complete path suggested by Oldroyd and Buddery[5] and Ryder, and Ozbay[2] would require that the overlying strata no longer behave in an inelastic fashion (and must fail).

Hence the use of BEPIL to determine the likely mode of failure would not be valid. If the overlying strata fail, the effective modulus of the overlying strata would drop and hence increase the likelihood of unstable pillar failure.

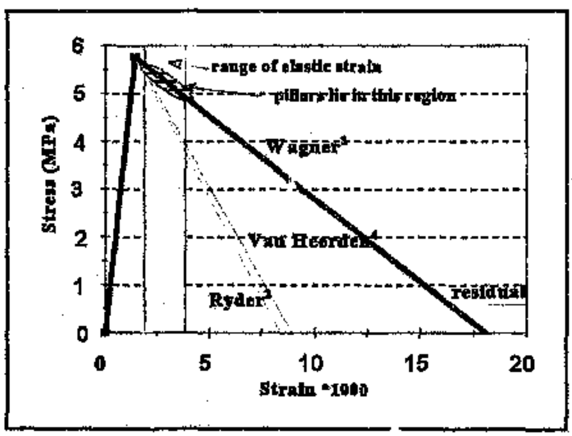

Figure 5.4 - Stress strain graph, Emaswati Case 1

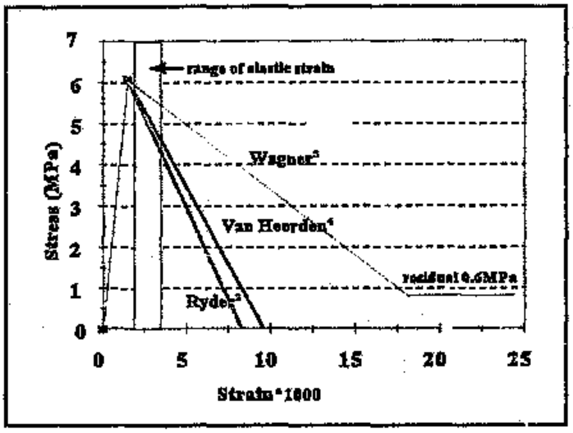

Figure 5.5 - Stress strain graph, Z.A.C. Case 2

In the Z.A.C. Case 2, shown in Figure 5.5, the overlying strata did in fact fail and passed beyond the elastically possible deformation. This allowed a far greater convergence and hence more strain on the pillars than is elastically possible. The failure remained stable throughout although the effective modulus of the overlying strata must have dropped considerably. If the residual strength of a failed pillars at Z.A.C. is 0.6 MPa (to be say 10% of the peak strength) then the stress strain curve may appear as it does in Figure 5.5. Open stope elastic convergence of between 41.9 and at the most 83.8mm corresponding to a strain of 1.6 to 3.2 x 10^{-3} is possible and this is also shown in Figure 5.5. It can be seen that the range of elastic strain is sufficient to cause pillar failure but not sufficient to allow the pillars to trace out the full load deformation graph and eventually re-compact and once again carry the full overburden load.

For the strata to converge by more than that calculated using the assumption that the panel is an open stope requires that the overlying strata fail, as it indeed did at Z.A.C. When this occurs the loading line must flatten as the effective strata modulus reduces and become shallower increasing the likelihood of unstable failures. At Z.A.C. the loading line flattened and the failure remained stable although at times convergent movement was relatively fast. The residual modulus of the broken strata was thus sufficiently high to allow this to occur. Assuming that the post peak pillar stiffness is currently determined by the fit to Wagner's[3] data then the modulus of the broken strata could be as low as 9.76 GPa.

In Emaswati Case 3 (Figure 5.6) the possible elastic strain is much higher than in the previous two cases due to the much wider panel width. In this case the pillars were seen to be failing in a stable fashion, which corresponds to the elastic region, but then they collapsed suddenly.

This indicates, and was observed, that the overlying strata failed. In this process the effective modulus was reduced to the extent that unstable failure resulted. In this last case the modulus of the overlying strata required for stable failure is much higher at 12.3 GPa than the 9.76 GPa than in the previous cases (Using the post peak stiffness from Wagner's[3] data).

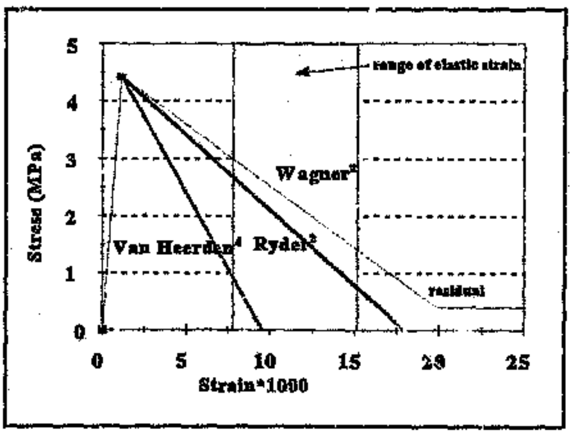

Figure 5.6 - Stress strain graph, Emaswati Case 3

5.3 Design of yielding Pillars

It would appear that yielding pillars can be designed on South African Collieries provided the overlying strata behave in an elastic fashion. The following methodology is proposed :-

1. Model the panel geometry or proposed geometry using BEPIL as modified in this thesis to determine the critical post peak stiffness of the pillars (λ_c).

2. Compare the value of the λ_c so obtained with the post peak pillar stiffness calculated from the fit to Wagner's insitu data λ. For stability $\lambda < \lambda_{cr}$

However, it has been observed in two of the case histo[illegible]rlying strata failed and behaved inelastically. In these cases the a[illegible]dered ineffective as BEPIL is an elastic model. In one of the cases unstab[illegible] pillars resulted while in the other failure was stable. It is thus possible even w[illegible]d overlying strata for pillars to fail in a stable fashion.

However, to be certain that unstable pillar failure does not occur would require two criteria to be satisfied :-

1. Stable failure of the pillars in the elastic region as determined by the methodology proposed above.

2. The possible elastic convergence obtained from open stope calculations exceeds that required for the pillar to trace out the entire downward trending portion of the stress strain graph. Thus should the overlying strata fail, they will do so when the pillar is tracing out the horizontal line representing the residual pillar strength and thus stable failure is assured.

CHAPTER 6 CONCLUSIONS

Three case histories of failed pillars have been documented and analysed. These have been modelled using BEPIL to determine the critical post peak slopes for the panel geometry. Results from this analysis have been compared with the actual pillar behaviour and insitu data on post peak pillar slopes obtained by Wagner[3] and Van Heerden[4]. They have also been compared with the envelope of post peak slopes proposed by Ryder and Ozbay[2]. These comparisons have enabled conclusions to be drawn about the suitability of insitu data to the application of yield pillar or design.

BEPIL proved to be a convenient programme for the analysis of pillar behaviour in the post peak region. It war successfully used to run sequentially to obtain the number of iterations required to solve the model .ence allowed the determination of the critical stiffness λ_c of the mining geometries. Comparison of these results with post peak stiffness calculated from three estimates of post peak slopes obtained from data fitted to insitu tests by Wagner[3], Van Heerden[4] and the envelope of all insitu pillar tests developed by Ryder and Ozbay[2]. These comparisons indicate that the post peak slopes given by the fit to Wagner's[3] data best fit the observed pillar failure mode. This result was obtained from back calculating to a strata modulus which would allow stable pillar failure. This implies that at Zululand Anthracite Colliery (Z.A.C.) and Emaswati Collieries the post peak moduli obtained from the fits Wagner's[3] data can be used to analyse pillar behaviour in the post peak region.

Within the range of width to height ratios considered the post peak slopes obtained from the fit to Van Heerden's[4] data do not match the observed pillar failure which predicts unstable pillar failure in the elastic region.

The envelope suggested by Ryder and Ozbay[2], which by the method of it's determination is a conservative approach indicates that unstable failure is indeed likely in the Emaswati Case 1 and the Z.A.C. Case 2. However, in Emaswati Case 3 where the pillar width to height ratio is greater than in the first two cases the value obtained is close to that which might be experienced.

The results obtained suggest that the design of yielding pillars is possible in the elastic region but as the allowable elastic deformation of the surrounding strata is small (in these examples) in relation to that required for the pillars to trace out the complete stress strain graph, design is indeed problematical in these cases. Design maybe possible where the possible elastic convergence is such that the whole post peak downward sloping portion of the pillar load deformation graph is contained within this region.

Further work is necessary on the post peak stiffness of coal pillars, particularly in the higher width to height range exceeding 3. In this respect the testing methodology employed bv Wagner[3] appears to offer results which more closely resemble actual behaviour than the method employed by Van Heerden[4].

Secondly there is a need to establish an acceptable methodology for determining the modulus of broken strata or to be able to model broken strata in a way which will allow stiffness analyses.

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APPENDIX 1 UNMODIFIED BEPIL PROGRAM LISTING FOR EMASWATI BASE CASE

Lines highlighted in yellow indicate those removed from the listing, shown in Appendix 1, to enable BEPIL to run sequentially.

```
'BEPIL - B.E.PILlar analysis - with Z-corrections & Replication.
SCREEN 9                                'EGA screen
DEFSNG A-H, O-Z: DEFINT I-N                      'FORTRAN convention
DIM AK(250), SS(240), Peo(240), EE(240), E0(240), Pa(240), NPat(240)
DIM Sp(9, 10), Pp(9, 10), Sn(9, 10), Pn(9, 10), Dn(9, 10), TYP$(9), ZZ(9), ZP(9)
 pi = 3.141593: PTYP$ = "HFKPX": PER$ = "00255075909599"
'-----------------------------------------------------------------
Constants: TITLE$ = "emaswati yield pillars - base case"
 q = 1.816: g = 6.75: Sm = 3: E = 7000: Es = .3571 * E: rep = 100000
Pattern: n = 17: FOR j = 1 TO n: READ NPat(j): NEXT j
 DATA 0,0,0,0,0,0,1,2,1,2,1,2,1,2,1,2,1
Materials: m = 2
M2: DATA P,7.6,7.6,13.5,2500,1000,-1000,1,.6,5,.8
Tolerance: tol = .001 * (q + 1): itmx = 30 * n: om = 1.4
'-----------------------------------------------------------------
T0: Sp(0, 0) = 3: Sp(0, 1) = 0: Pp(0, 1) = q: Sp(0, 2) = .2: Pp(0, 2) = .25 * Es + q
 Sp(0, 3) = 1: Pp(0, 3) = 1.5 * Es: TYP$(0) = "0": ZZ(0) = 0: ZP(0) = 1
T1: Sp(1, 0) = 3: Sp(1, 1) = 0: Pp(1, 1) = 0: Sp(1, 2) = 1: Pp(1, 2) = 0
 Sp(1, 3) = 1: Pp(1, 3) = 1000 * (q + 99): TYP$(1) = "1": ZZ(1) = 0: ZP(1) = 1
 FOR j = 2 TO m: READ TYP$(j): ityp = 8
 FOR i = 1 TO 8: IF TYP$(j) = MID$(PTYP$, i, [illegible]) THEN ityp = i
 NEXT i: READ pw, pb, py: r1 = pw / py: r2 = pb / py: r3 = pw / g
BEZCor: ' Generalized Salamon Z-correction via Boussinesq
 zed = 0: FOR k = -2 TO 2: FOR kw = 0 TO 1: QQ = 0
 ff = 1: fx = r1: fy = r2: IF kw = 1 THEN ff = -r2 * r3: fx = r1 / r3: fy = 1
 FOR y1 = -1 TO 1 STEP 2: FOR y2 = -1 TO 1 STEP 2: bet = r2 * y2 - (fy * y1 + 2 * k)
```

```
FOR x1 = -1 TO 1 STEP 2: FOR x2 = -1 TO 1 STEP 2
 alp = r1 * x2 - fx * x1: RR = alp * alp + bet * bet: R = SQR(RR) + .000001
 w = .5 * alp * bet * (bet * LOG® + alp) + alp * LOG® + bet)) - RR * R / 6
 QQ = QQ + y1 * y2 * x1 * x2 *
 NEXT x2, x1, y2, y1: zed = zed + ff * QQ: NEXT kw, k
 zed = zed / (8 * pi * r1 * r1 * r2 * r2): ZZ(j) = zed: ZP(j) = r2 * r3
   ON ityp GOTO TH, TF, TK, TP, TX
TH: READ Fw, a, B
 Sp(j, 0) = 9: al = Fw / Sm: ecr = 1 - al: bd = 1 - (1 - B) * al
 Sp(j, 1) = 0: Pp(j, 1) = 0: FOR k = 2 TO 8
 eps = ecr + .01 * (bd - ecr) * VAL(MID$(PER$, k + k - 3, 2))
 Sp(j, k) = eps: Pp(j, k) = a * (eps - ecr) / (bd - eps): NEXT k
 Sp(j, 9) = 1: Pp(j, 9) = 1000 * (Pp(j, 8) + q): GOTO Nxj
TF: READ Force: Sp(j, 0) = 2: Sp(j, 1) = 0: Pp(j, 1) = Force / (pw * pb)
 Sp(j, 2) = 10: Pp(j, 2) = Force / (pw * pb): GOTO Nxj
TK: READ Stiff: Sp(j, 0) = 2: Sp(j, 1) = 0: Pp(j, 1) = q
 Sp(j, 2) = 10: Pp(j, 2) = q + 10 * Sm * Stiff / (pw * pb): GOTO Nxj
TP: READ EE, qm, Ep, qr, et, a, B: Sp(j, 0) = 9
 Sp(j, 1) = q / EE: Pp(j, 1) = q: Sp(j, 2) = qm / EE: Pp(j, 2) = qm
 Sp(j, 3) = qm / EE + (qr - qm) / Ep: Pp(j, 3) = qr: FOR k = 4 TO 8: ew = .25 * (k - 4)
  Sp(j, k) = et + ew * (B - et): Pp(j, k) = qr + a * ew / (1.01 - ew): NEXT k
 Sp(j, 9) = 1: Pp(j, 9) = 1000 * q: ew = q / EE: FOR k = 1 TO 9
  Sp(j, k) = Sp(j, k) - ew: NEXT k: GOTO Nxj
TX: READ kmx: Sp(j, 0) = kmx: FOR k = 1 TO kmx: READ Sp(j, k), Pp(j, k): NEXT k
Nxj: NEXT j
Build: 'Builds replicd.kernels, & matl.rib closure/ext.stress tables
 pl = pi * g / rep: flam = E * SIN(pl) / (4 * .96 * rep): zfac = 2 * .96 * g / E
 FOR j = 0 TO n: AK(j) = flam / (SIN(pl * (j - .5)) * SIN(pl * (j + .5))): NEXT j
 ak0 = AK(0): ou1 = 1.5 * om * AK(1): SS(n + 1) = 0
 FOR j = 0 TO m: kmx = Sp(j, 0): Sn(j, 0) = kmx: FOR k = 1 TO kmx
  S = Sm * Sp(j, k) + zfac * ZP(j) * ZZ(j) * Pp(j, k): Pe = ZP(j) * Pp(j, k) - ak0 * S
  Sn(j, k) = S: Pn(j, k) = Pe: IF k > 1 THEN Dn(j, k) = (S - Sn(j, k - 1)) / (Pe - Pn(j, k - 1))
 NEXT k: S1 = Sn(j, 1): S2 = Sn(j, 2): Sn(j, 1) = 0
```

```
 Pn(j, 1) = (S2 * Pn(j, 1) - S1 * Pn(j, 2)) / (S2 - S1): NEXT j
 FOR j = 1 TO n: SS(j) = 0: Peo(j) = Pn(NPat(j), 1): NEXT j: its = 0
MLoop: 'Main itern.loop, including back-sub (double SOR) & Lanczos optimizn.
 its = its + 1: FOR j = 1 TO n: Pee = q: FOR i = 1 TO n
  IF j <> i THEN Pee = Pee + AK(ABS(j - i)) * SS(i)
 NEXT i: del = om * (Pee - Peo(j)): EE(j) = del: Pe = Peo(j) + del
 GOSUB GetS: SS(j) = S: NEXT j
BkSub: st = 0: su = 0: FOR j = n TO 1 STEP -1: del = EE(j) + ou1 * SS(j + 1)
 EE(j) = del: Pe = Peo(j): GOSUB GetS: S0 = S: Pe = Pe + del: GOSUB GetS: SS(j) = S - S0
 E0 = E0(j): w = E0 - del: st = st + w * E0: su = su + w * w: NEXT j: st = ABS(st)
L1D: RR = 0: IF its = 1 OR su = 0 THEN c = 1 ELSE c = c * st / su
 FOR j = 1 TO n: del = c * EE(j): RR = RR + del * del: E0(j) = EE(j): Pe = Peo(j) + del
 GOSUB GetS: SS(j) = S: Peo(j) = Pe: Pa(j) = Pe + ak0 * S: NEXT j
 RR = SQR(RR / n): PRINT its; : PRINT USING "######.###"; c; om; RR
  IF (RR > tol OR c < .1) AND its < itmx GOTO MLoop
 LOCATE 25, 60: INPUT "Hit ENTER", S$: IF S$ = "Q" OR S$ = "q" THEN END
ScFacts: 'Sets scaling factors for Closure/Stress plotting & printing
 zi = 2: nmx = 30: FOR i = 1 TO 3: IF n > nmx THEN zi = zi / 2
  nmx = nmx * 2: NEXT i
 lcmx = 10 - INT(-zi * n): smx = -999: pmx = -999
 FOR j = 1 TO n: ws = ABS(SS(j)): IF ws > smx THEN smx = ws
  wp = ABS(Pa(j)): IF wp > pmx THEN pmx = wp
 NEXT j: ws = .4344 * LOG(smx): ins = INT(ws): ws = 10 ^ (ws - ins): wins = 10 ^ (-ins)
  vjs = .2: fs = 70 * wins
  IF ws < 2 THEN vjs = .1: fs = 140 * wins
  IF ws > 4 THEN vjs = .5: fs = 28 * wins
 wp = .4344 * LOG(pmx): inpp = INT(wp): wp = 10 ^ (wp - inpp): winp = 10 ^ (-inpp)
  vjp = .2: fp = 70 * winp
  IF wp < 2 THEN vjp = .1: fp = 140 * winp
  IF wp > 4 THEN vjp = .5: fp = 28 * winp
```

APPENDIX 2 MODIFIED BEPIL PROGRAMME FOR ITERATIVE SOLUTION

The following highlighted code is omitted in the modified listing, as the plot routines are not required

```
Plot: CLS : PRINT TAB(5); "BEPil:  "; TITLE$: PRINT
 PRINT "    S"; TAB(lcmx); "     Q"
 PRINT "   (m)"; TAB(lcmx); "   (MPa)"
 PRINT "  *10^"; ins; TAB(lcmx); "  *10^"; inpp
 FOR l = 0 TO 20 STEP 5: LOCATE 22 - l, 6: PRINT USING "##.#"; vjs * l
  LOCATE 22 - l, lcmx + 1: PRINT USING "##.#"; vjp * l: NEXT l
 LOCATE 23, 6: PRINT "grid:": j0 = 10 / zi: FOR j = j0 TO n STEP j0
  LOCATE 23, 9 + zi * j: PRINT USING "###"; j: NEXT j
 LINE (80, 21)-(8 * lcmx, 301), , B: ip = 1: IF n > 60 THEN ip = 2
 FOR j = 21 TO 301 STEP 14: PSET (79, j): PSET (8 * lcmx + 1, j): NEXT j
 FOR i = 0 TO n STEP ip: PSET (80 + 8 * zi * i, 20): PSET (80 + 8 * zi * i, 302): NEXT i
 FOR i = 1 TO n: ws = fs * ABS(SS(i)): wp = fp * ABS(Pa(i)): nw = 80 + 4 * zi * (i + i - 1)
  LINE (nw - 4 * zi, 301 - ws)-(nw + 4 * zi, 301), , B: LINE (nw - 2, 301 - wp)-(nw + 2, 301 -
wp)
   IF Pa(i) > .01 THEN LINE (nw, 299 - wp)-(nw, 303 - wp)
 NEXT i: LOCATE 25, 60: INPUT "Hit ENTER", S$
Prt: CLS : PRINT "BEPil:  "; TITLE$: PRINT
 PRINT " q(MPa)="; q; " g(m)="; g; " Sm(m)="; Sm; " E(MPa)="; E
 PRINT "Es(MPa)="; Es; " rep(m)="; rep; " n="; n; " m="; m: PRINT
 FOR j = 1 TO n: PRINT USING "#"; NPat(j); : IF j MOD 10 = 0 THEN PRINT " ";
 NEXT j: PRINT : PRINT : PRINT " M T  zed [Pillar Strain(mm/m),Stress(MPa) pairs]"
  PRINT " ------ ------------------------------------------"
 FOR j = 0 TO m: kmx = Sp(j, 0): PRINT j; TYP$(j); : PRINT USING "##.##"; ZZ(j);
 FOR k = 1 TO kmx: PRINT " [" + STR$(FIX(1000 * Sp(j, k))) + STR$(FIX(Pp(j, k))) + "]";
  IF POS(0) > 65 THEN PRINT : PRINT "        ";
 NEXT k: PRINT : NEXT j: PRINT : PRINT "Its="; its; "  RMS=";
```

```
PRINT USING "##.###"; RR: INPUT "Hit ENTER", S$: CLS
PRINT "S(m) *10^"; ins: Vol = 0: FOR j = 1 TO n: S = SS(j)· Vol = Vol + S
PRINT USING "##.###"; wins * S; : IF j MOD 10 = 0 THEN PRINT
NEXT j: PRINT : PRINT "Vol(m^2) *10^"; ins; : PRINT USING "####.##"; wins * g * Vol

PRINT : IF n > 80 THEN INPUT "Hit ENTER", S$: CLS
PRINT "Q(MPa) *10^"; inpp: FOR j = 1 TO n
PRINT USING "##.###"; winp * Pa(j); : IF j MOD 10 = 0 THEN PRINT
NEXT j: PRINT
FOR j = 1 TO n: jj = NPat(j): zed = ZZ(jj): IF ABS(zed) < .0001 GOTO Nxxj
sigp = Pa(j) / ZP(jj): fact = zfac * ZP(jj) * zed
PRINT USING "###.###"; wins * (SS(j) - fact * sigp); winp * sigp; : PRINT "  ";
Nxxj: NEXT j: PRINT
INPUT "p=Plot, ENTER=Quit", S$: IF S$ = "p" OR S$ = "P" GOTO Plot
END
GetS: k = 0: jj = NPat(j)'Given ext.stress Pe, return cvg.S
GL: k = k + 1: IF Pe > Pn(jj, k) GOTO GL
S = Sn(jj, k) - Dn(jj, k) * (Pn(jj, k) - Pe): RETURN
```

Modified Bepil Program Listing for Emaswati Base Case

Lines highlighted in blue indicate those added to enable the program to solve sequentially and to write in file containing the number of interactions required to solve.

```
'BEPIL - B.E.PILlar analysis - with Z-corrections & Replication.
SCREEN 9                              'EGA screen
DEFSNG A-H, O-Z: DEFINT I-N                    'FORTRAN convention
DIM AK(250), SS(240), Peo(240), EE(240), E0(240), Pa(240), NPat(240)
DIM Sp(9, 10), Pp(9, 10), Sn(9, 10), Pn(9, 10), Dn(9, 10), typ$(9), ZZ(9), ZP(9), accum(5000,
2)
 pi = 3.141593: ptyp$ = "HFKPX": PER$ = "00255075909599"
'----------------------------------------------------------
Constants: TITLE$ = "emaswati yield pillars -base case modified"
 q = 1.825: g = 6.75: Sm = 2.95: rep = 94.5: eS = 4000: e = 6000
Pattern: n = 14: FOR j = 1 TO n: READ NPat(j): NEXT j
patd:  DATA 0,0,0,1,2,1,2,1,2,1,2,1,2,1
Materials: m = 2
m2: DATA K,7.6,7.6,13.5,
Tolerance: tol = .001 * (q + 1): itmx = 30 * n: om = 1.4
'----------------------------------------------------------
```

"L" is a variable used as a counter for construction of the matrix "accum" in which the results are stored. The matrix size is eventually 2 by "L"

```
L = 1
```

The variable "stiff" is the stifness of the loading system. The program is set to loop, incrementing "stiff" by ten each time until it is zero. Each time the number of iterations required for a solution is recorded and the maximum iterations required is recorded.

```
FOR stiff = -11150 TO 0 STEP 10
```

Each time the loop repeats, the data variable "patd" and "m2" are reset by the following two statements. The same input data is thus continuously used for varying values of stiffness.

```
RESTORE patd
RESTORE m2
```

```
T0: Sp(0, 0) = 3: Sp(0, 1) = 0: Pp(0, 1) = q: Sp(0, 2) = .2: Pp(0, 2) = .25 * eS + q
 Sp(0, 3) = 1: Pp(0, 3) = 1.5 * eS: typ$(0) = "0": ZZ(0) = 0: ZP(0) = 1
T1: Sp(1, 0) = 3: Sp(1, 1) = 0: Pp(1, 1) = 0: Sp(1, 2) = 1: Pp(1, 2) = 0
 Sp(1, 3) = 1: Pp(1, 3) = 1000 * (q + 99): typ$(1) = "1": ZZ(1) = 0: ZP(1) = 1
FOR j = 2 TO m: READ typ$(j): ityp = 8
 FOR i = 1 TO 8: IF typ$(j) = MID$(ptyp$, i, 1) THEN ityp = i
 NEXT i: READ pw, pb, py: r1 = pw / py: r2 = pb / py: r3 = pw / g
BEZCor: ' Generalized Salamon Z-correction via Boussinesq
 zed = 0: FOR k = -2 TO 2: FOR kw = 0 TO 1: QQ = 0
 ff = 1: fx = r1: fy = r2: IF kw = 1 THEN ff = -r2 * r3: fx = r1 / r3: fy = 1
 FOR y1 = -1 TO 1 STEP 2: FOR y2 = -1 TO 1 STEP 2: bet = r2 * y2 - (fy * y1 + 2 * k)
 FOR x1 = -1 TO 1 STEP 2: FOR x2 = -1 TO 1 STEP 2
  alp = r1 * x2 - fx * x1: rr = alp * alp + bet * bet: R = SQR(rr) + .000001
  w = .5 * alp * bet * (bet * LOG® + alp) + alp * LOG® + bet)) - rr * R / 6
  QQ = QQ + y1 * y2 * x1 * x2 * w
 NEXT x2, x1, y2, y1: zed = zed + ff * QQ: NEXT kw, k
 zed = zed / (8 * pi * r1 * r1 * r2 * r2): ZZ(j) = zed: ZP(j) = r2 * r3
   ON ityp GOTO TH, TF, TK, TP, TX
TH: READ Fw, a, b
 Sp(j, 0) = 9: al = Fw / Sm: ecr = 1 - al: bd = 1 - (1 - b) * al
 Sp(j, 1) = 0: Pp(j, 1) = 0: FOR k = 2 TO 8
 eps = ecr + .01 * (bd - ecr) * VAL(MID$(PER$, k + k - 3, 2))
 Sp(j, k) = eps: Pp(j, k) = a * (eps - ecr) / (bd - eps): NEXT k
 Sp(j, 9) = 1: Pp(j, 9) = 1000 * (Pp(j, 8) + q): GOTO Nxj
TF: READ Force: Sp(j, 0) = 2: Sp(j, 1) = 0: Pp(j, 1) = Force / (pw * pb)
 Sp(j, 2) = 10: Pp(j, 2) = Force / (pw * pb): GOTO Nxj
```

TK: A "READ" statement for "stiff" was omitted here, as the value of "stiff" is supplied by the loop. Sp(j, 0) = 2: Sp(j, 1) = 0: Pp(j, 1) = q

```
 Sp(j, 2) = 10: Pp(j, 2) = q + 10 * Sm * stiff / (pw * pb): GOTO Nxj
TP: READ EE, qm, Ep, qr, et, a, b: Sp(j, 0) = 9
 Sp(j, 1) = q / EE: Pp(j, 1) = q: Sp(j, 2) = qm / EE: Pp(j, 2) = qm
 Sp(j, 3) = qm / EE + (qr - qm) / Ep: Pp(j, 3) = qr: FOR k = 4 TO 8: ew = .25 * (k - 4)
  Sp(j, k) = et + ew * (b - et): Pp(j, k) = qr + a * ew / (1.01 - ew): NEXT k
 Sp(j, 9) = 1: Pp(j, 9) = 1000 * q: ew = q / EE: FOR k = 1 TO 9
  Sp(j, k) = Sp(j, k) - ew: NEXT k: GOTO Nxj
TX: READ kmx: Sp(j, 0) = kmx: FOR k = 1 TO kmx: READ Sp(j, k), Pp(j, k): NEXT k
Nxj: NEXT j
Build: 'Builds replicd.kernels, & matl.rib closure/ext.stress tables
 pl = pi * g / rep: flam = e * SIN(pl) / (4 * .96 * rep): zfac = 2 * .96 * g / e
 FOR j = 0 TO n: AK(j) = flam / (SIN(pl * (j - .5)) * SIN(pl * (j + .5))): NEXT j
 ak0 = AK(0): ou1 = 1.5 * om * AK(1): SS(n + 1) = 0
 FOR j = 0 TO m: kmx = Sp(j, 0): Sn(j, 0) = kmx: FOR k = 1 TO kmx
  s = Sm * Sp(j, k) + zfac * ZP(j) * ZZ(j) * Pp(j, k): Pe = ZP(j) * Pp(j, k) - ak0 * s
  Sn(j, k) = s: Pn(j, k) = Pe: IF k > 1 THEN Dr(j, k) = (s - Sn(j, k - 1)) / (Pe - Pn(j, k - 1))
 NEXT k: S1 = Sn(j, 1): S2 = Sn(j, 2): Sn(j, 1) = 0
 Pn(j, 1) = (S2 * Pn(j, 1) - S1 * Pn(j, 2)) / (S2 - S1): NEXT j
 FOR j = 1 TO n: SS(j) = 0: Peo(j) = Pn(NPat(j), 1): NEXT j: its = 0
MLoop: 'Main itern.loop, including back-sub (double SOR) & Lanczos optimizn.
 its = its + 1: FOR j = 1 TO n: Pee = q: FOR i = 1 TO n
  IF j <> i THEN Pee = Pee + AK(ABS(j - i)) * SS(i)
 NEXT i: del = om * (Pee - Peo(j)): EE(j) = del: Pe = Peo(j) + del
 GOSUB GetS: SS(j) = s: NEXT j
BkSub: st = 0: su = 0: FOR j = n TO 1 STEP -1: del = EE(j) + ou1 * SS(j + 1)
 EE(j) = del: Pe = Peo(j): GOSUB GetS: S0 = s: Pe = Pe + del: GOSUB GetS: SS(j) = s - S0
 E0 = E0(j): w = E0 - del: st = st + w * E0: su = su + w * w: NEXT j: st = ABS(st)
L1D: rr = 0: IF its = 1 OR su = 0 THEN c = 1 ELSE c = c * st / su
 FOR j = 1 TO n: del = c * EE(j): rr = rr + del * del: E0(j) = EE(j): Pe = Peo(j) + del
 GOSUB GetS: SS(j) = s: Peo(j) = Pe: Pa(j) = Pe + ak0 * s: NEXT j
 rr = SQR(rr / n): IF its > itmax THEN itmax = its: maxe = stiff
```

A print statement was omitted here - not required for functionality and halts the program unnecessarily. The "IF its >...." statements merely keeps track of the maximum number of iterations to date.

```
IF (rr > tol OR c < .1) AND its < itmx GOTO Mloop
```

A statement which halts the program here and awaits response from the user has been omitted here.

```
ScFacts:  'Sets scaling factors for Closure/Stress plotting & printing
 zi = 2: nmx = 30: FOR i = 1 TO 3: IF n > nmx THEN zi = zi / 2
  nmx = nmx * 2: NEXT i
 lcmx = 10 - INT(-zi * n): smx = -999: pmx = -999
 FOR j = 1 TO n: ws = ABS(SS(j)): IF ws > smx THEN smx = ws
  wp = ABS(Pa(j)): IF wp > pmx THEN pmx = wp
 NEXT j: ws = .4344 * LOG(smx): ins = INT(ws): ws = 10 ^ (ws - ins): wins = 10 ^ (-ins)
  vjs = .2: fs = 70 * wins
  IF ws < 2 THEN vjs = .1: fs = 140 * wins
  IF ws > 4 THEN vjs = .5: fs = 28 * wins
 wp = .4344 * LOG(pmx): inpp = INT(wp): wp = 10 ^ (wp - inpp): winp = 10 ^ (-inpp)
  vjp = .2: fp = 70 * winp
  IF wp < 2 THEN vjp = .1: fp = 140 * winp
  IF wp > 4 THEN vjp = .5: fp = 28 * winp
```

The following 6 lines are used to print the variable on the screen to keep track of what is happening through each loop.

```
LOCATE 10, 10: PRINT USING "#######.###"; its; c; om; rr; itmax; stiff
LOCATE 20, 10: PRINT USING "#######.###"; maxe
accum(l, 1) = its
accum(l, 2) = stiff
LOCATE 22, 10: PRINT accum(l, 1); accum(l, 2)
L = L + 1

NEXT stiff
```

End of the main loop.

A file called "accum.dat" is created on disk to store a text file containing the number of iterations required and the value for stifness of each loop. This text file can then be imported into another application to draw graphs or do other analysis on the data.

```
 OPEN "accum.dat" FOR OUTPUT AS #1
 FOR i = 1 TO 1000
   WRITE #1, accum(i, 1), accum(i, 2): NEXT i
   CLOSE #1
 END

GetS: k = 0: jj = NPat(j)'Given ext.stress Pe, return cvg.S
GL: k = k + 1: IF Pe > Pn(jj, k) GOTO GL
 s = Sn(jj, k) - Dn(jj, k) * (Pn(jj, k) - Pe): RETURN
```

APPENDIX 3 INPUT GEOMETRIES FOR BEPIL CASE ANALYSES

EMASWATI CASE 1

Constants: TITLE$ = "emaswati case 1"

q = 1.825: g = 6.75: Sm = 2.95: rep = 108: Es = 4000: E = 12000

Pattern: n = 16: FOR j = 1 TO n: READ NPat(j): NEXT j

patd: DATA 0,0,0,1,2,1,2,1,2,1,2,1,2,1,2,1

Materials: m = 2

M2: DATA K,7.6,7.6,13.5,

ZULULAND ANTHRACITE COLLIERY CASE 2

Constants: TITLE$ = "zac case history"

q = 1.45: g = 7.0: Sm = 2.64: Es = 4000: rep = 112: E=12000

Pattern: n = 16: FOR j = 1 TO n: READ NPat(j): NEXT j

DATA 0,0,0,1,2,1,2,1,2,1,2,1,2,1,2,1

Materials: m = 2

M2: DATA K,7.3,7.3,14,

EMASWATI CASE 3

Constants: TITLE$ = "emaswati case 3-uncontrolled"

q = 1.625: g = 8: Sm = 2.5: Es = 4000: rep = 240: E = 12000

pattern: n = 30: FOR j = 1 TO n: READ NPat(j): NEXT j

DATA 0,0,0,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1,2,1

Materials: m = 2

M2: DATA K,9.7,9.7,16,