

TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS

MASONDO, Thandolwakhe

Student Number: 2400193

Protocol number: 2021ECE083M



SUPERVISOR: PROF. JUDAH MAKONYE

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15 March 2022



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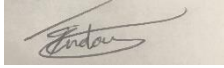
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Thandolwakhe Masondo

15/03/2022

ABSTRACT

This research report presents a qualitative study that was conducted at a primary school in Johannesburg, South Africa as part of an error analysis action research. The focus of the study is on the teaching of fractions, directed at teachers' resourcing of learners' mathematical thinking. Emphasis is put on the teachers' ability to professionally notice and intervene based on what they have observed. The study aims to investigate the extent to which the teachers' professional noticing was successful in improving the learners' performance on fractions. A group of 60 grade 7 learners were sampled to draw data for this study.

This study was grounded on the constructivist theory of learning and teaching and was supported by literature on professional noticing. The study's data is constituted from analyses of different methods, beginning with a pre-test which was used to identify the learners' errors, then interviews which were utilised to interpret learners' responses to the pre-test, thereafter an intervention lesson was used to address the errors observed and lastly a post-test to compare the performance results of learners in the pre-test with the post-test.

The learners' significant improvement on the post-test performance suggests that their cognitive level on fractions concepts had also enhanced. Thus suggesting that resourcing of learners' thinking when teaching fractions had a positive impact on their understanding. Moreover, the study confirms that teachers' ability to professionally notice creates a favourable environment in enhancing learners' conceptual knowledge.

KEY WORDS

- Noticing
- Fractions
- Errors
- Misconceptions
- Remedial
- Intervention

DEDICATIONS

This research report is dedicated to my son Nkanyezi Zanolukhile Khumalo. I know much of our time has been divided in fulfilment of this degree, but I promise that it is all going to be worth it someday son. I love you so much and I hope you take inspiration from this. I would also like to thank my entire family for the love and support you have given me during this time, without you, this would not be possible. Lastly, mom, I hope your daughter has made you proud.

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CHAPTER 1

1. STUDY BACKGROUND

1.1 INTRODUCTION

“The range of what we think and do is limited by what we fail to notice and because we fail to notice, there is little we can do to change until we notice how failing to notice shapes our thoughts and deeds” (Goleman, 1985, p. 24).

According to Goleman in the above citation, when we fail to notice we fail to improve. Therefore if we do not notice anything wrong with what we are doing, then we do not take time to correct it. This looks into the inner conscious of trying to help oneself from themselves. In other words, what you fail to notice influences the way you act. If you do not know that something is wrong with your doing, then you will not be able to act on it. This implies that you would not even know that you do not know. Thus there will not be any progress in what you do. This means that the range of what we do is affected by what we notice. The things you fail to notice cannot be acted upon, and because we fail to notice there is little we can do to change. Thus if you do not notice errors in your thinking then you will not be able to advance it. This considers asking a learner in the class to clarify what they mean when they give an incorrect answer, in order to get the greatest out of that learner’s thinking.

Too often unexpected or incorrect answers are pushed aside and ignored as an opportunity to learn. Instead, the focus is put on remediation and prioritizing the correct answer. This misses a rich opportunity to uncover the information that can guide a teacher to help a student who has made a mistake and use it as a resource for teaching. Furthermore, Nesher (1987) argues that learners’ incorrect answers should be used or extended as an opportunity to learn. This is because failing to notice results in poor quality of performance in mathematics as students are not encouraged to take advantage of errors as part of learning. “Most current views by teachers and learners is that making errors is perceived as negative” (Borasi, 1994, p.168). This comes from a behaviourist perspective where correct answers are rewarded in order to obtain higher grades by learners and incorrect answers lead to punishment. However, this study argues that errors should not be seen as obstacles but rather as a way where learners have the opportunity to reflect on their mistakes. This can be achieved through sound development of professional noticing skills.

Jacobs, Lamb, and Philipp (2011) define professional noticing as the, “teachers capacity to a) *attend* to students’ mathematical conceptions and practices as they occur b) *interpret* these conceptions and practices and c) *decide* upon a productive instructional course of action based on this interpretation” Jacobs, Lamb, and Philipp (2011, p.37). Over the years researchers such as Thomas, Eisenhardt, Fisher, Schack, Tassel, and Yoder (2014), Jacobs, Lamb and Philipp, 2010; Philipp (2014), Schack, Fisher, Thomas, Eisenhardt, Tassell and Yonder (2013), Sherin, Jacobs & Philipp (2011) have placed increasing emphasis on professional noticing of children’s mathematical thinking. Voicing that it must be every teacher’s competency in order to do their work properly. It is otherwise called “adaptive teaching” Jacobs et. al. (2010, p.173), which means responding to learners’ thinking on what you are teaching. “Professional noticing ability, then, is central to, and integrated within, teachers’ ability to provide instruction appropriately adapted to students’ immediate needs” (Gibson & Ross 2016, p.180). In turn, moving away from teaching as telling but rather teaching for conceptual understanding. As such this research study seeks to explore the capacity of teachers to elicit the thinking of students who have made mistakes and use it as a resource to teach fractions to grade 7 students. Although learners use the concept of sharing in most of their daily lives, fractions are particularly difficult for them to understand. One of the reasons for this is that learners seem to memorise procedural rules and techniques instead of developing a conceptual understanding of fractions (Geller, Son & Stinger, 2017), hence making most operational rules meaningless. This is why fractions are the most basic topic in the elementary school curriculum but also one of the most poorly understood concepts. This results from the constant rote memorisation of rules, formulas and procedures that learners do.

1.2 BACKGROUND

The South African curriculum encourages social constructivism as a theory of learning, wherein learners are expected to construct their own knowledge and strategies to solve a problem. Stears (2009, p.397) argue that “this curriculum not only advocated the development of knowledge and skills, but also emphasised education for democracy and citizenship”. He adds that the “curriculum is informed by principles of social, as well as critical constructivism” Stears (2009, p.397). This requires a teacher to be able to fully allow interactions and use the students’ errors to formulate resources to teach these learners. Similarly, a constructivist perspective views students’ misconceptions as an important part of learning. Moreover, Smith

et al. (1993, p.120) maintain that “misconceptions are normal part of learning, so educators must expect them as the difficulties of understanding mathematics”.

It is alluded that these misconceptions are a result of prior developed teachings, with Smith et al. (1993, p.120) claiming “that misconceptions originate in prior instruction as students incorrectly generalise prior knowledge to grapple with a new task”. As such, Olivier (1989) state that learners’ mistakes are not as a result of being stupid, they are rational and meaningful. In addition, Skemp (1970) constructivism perception of learning assumes that concepts are not taken directly from experience, but that a person’s ability to learn from and what he learns from experience depends on the quality of the ideas that he is able to bring to that experience.

As such teachers need to be able to dig deep into the root of the misconception by eliciting students’ thinking and use it as a resource to teach for understanding. This involves the teachers’ ability to professionally notice. Professional noticing as previously mentioned above is defined by Jacobs, Lamb, & Philipp (2010, p.169) as “teachers’ capacity to attend to student’s mathematical conceptions and practices as they occur, interpret these conceptions and practices, and decide upon a productive instrumental course of action based on this interpretation”. Moreover, Jacobs et al. (2010, p.169) argue that “we are interested in the extent to which teachers use what they have learned about their children’s understandings from the specific situation and whether their reasoning is consistent with the research on children’s mathematical development”.

Eliciting skill is important in teaching mathematics such that, when teachers evoke around errors and incorrect answer it gives them an overview of the students’ core thinking .it also has a significant role in their learning. Hence Battey and Franke (2015) argue that when learners’ thinking is limitedly elicited, it are not have a limited it can lead teachers to pursue less ambitious learning goals. Moreover, Teaching Works (2019, p.2) describes eliciting students practice as:

“Teacher pose questions that create space for students to share their thinking about specific academic content. They seek to understand student thinking, including novel points of view, new ideas, ways of thinking, or alternative conceptions. Teachers draw out student thinking through carefully chosen questions and tasks and attend closely to what students do and say”.

1.3 RESEARCH PROBLEM

South Africa as a whole is in a situation whereby learner performance in mathematics has raised a concern amongst educationists and the entire academia (Howie, 2001; Howie, 2003; Mji & Makgato, 2006). Thus the driving issue for this study is that teachers' instructional practice is generally weak "when not paired with detailed and comprehensive observation and understanding of students' immediate responses during instructions" (Gibson & Ross 2016 p. 180). This leads to learners' poor performance in mathematics and adds to their inability to cope with the subject in general. In my entire career as a mathematics teacher at senior phase, I also do not have a good strategy of dealing with learners' errors and misconceptions. Although it is possible for me to determine that the learner experienced a challenge working on the solution, coming up with the deduction of their thinking during that process becomes a challenge. This has impacted the mathematics results negatively as poor conceptual understanding resulted in poor performance in assessment tasks. This is what motivated me to conduct this study, in order to figure out what impact can the use of learners thinking have in the teaching of fractions. Instead of attempting to eradicate misconceptions, a teacher needs to incorporate them in their teaching strategies, which will enable them to become aware of their learners' errors and misconceptions (Olivier, 1989; Borasi, 1994; & Brodie, 2007). This highlights the importance of being aware of students' errors in order to enrich learning.

Like many other topics fractions are part of the topics that learners in senior phase struggle with. This is because teachers generally struggle to use the reasoning of the students to identify the learners' relational understanding of fractions. Studies by Jacob et al. (2010) found that "experienced teachers with little focused professional development demonstrated professional skills closer to those of pre-service teachers than their counterparts who participated in focused professional development" (Jacob et al. (2010, p.193). Therefore while some teachers have the competency to teach adaptively, they however have it at a lower level. Though teachers might be responding to learner's mathematical thinking, it could be happening at a limited responsiveness. Where else a teacher that can professionally notice has highly responsiveness when it comes to mathematical thinking of students. This is particularly evident in the topic of fractions, where different representations could be used to represent the same thing. Thus making it difficult for teachers to figure out where a learner made an error and how they came about it. This becomes evident when learners get probed to give detailed descriptions of what they have had done to get to their answer.

The notion of fractions appears to be one of the core mathematical aspects in South African curriculum, as it is one of the concepts that learners must grasp in order to move to the next level of mathematics learning at senior phase. According to Curriculum Assessment Policy Statement (CAPS) document Grade 7 learners are expected to

“convert mixed numbers to common fractions in order to perform calculations with them; use knowledge of multiples and factors to write fractions in the simplest form before or after calculations; use knowledge of equivalent fractions to add and subtract common fractions, and solve problems in contexts involving common fractions and mixed numbers, including grouping, sharing”. (CAPS, 2011, p.122).

This should be done in the allocated time of 18 hours (nine hours of common fractions and nine hours of decimal fractions). However the researcher has noticed that learners struggle to meet the above expectations. Therefore, this study seeks to investigate how well teachers can attend to students’ mistakes in order to understand the reason behind the errors they make. And also to get an understanding of their decision upon a productive instrumental course of action based on this interpretation to better their teaching.

Moreover, working with learners’ meaning is a complex task, this is because every teacher has their own cultural background and beliefs that act as resources they use when noticing. Thus Ernest (1997) argues that what students learn about mathematics depends almost entirely on the experiences that teachers provide every day in the classroom. This means that a teacher’s philosophy of mathematics has a bearing on how they teach mathematics. However, there is no agreement regarding what it means to know and understand mathematics in mathematical education. Theories such as behaviourism, constructivism, situated learning and sociocultural theory have influenced ways in which mathematics is taught. This is why we might be presented with the same phenomenon but notice it differently from our colleagues. Hence presenting a number of challenges in the way we teach and professionally notice.

Although pre-service teachers have pedagogical mathematics knowledge (PCK), they lack the mathematics knowledge for teaching (MKT). This is the integrated knowledge required to effectively teach mathematics. Thomas et al. (2017) state that human conceptions of pedagogical content knowledge provide the context for construction of this specialised framework, (MKT). This framework thus categorises “essential knowledge involved in the teaching and learning of mathematics” (Thomas et al. 2017, p.4). Ball et al. (2008) also

indicated the importance of this framework, stating that it is for categorizing and describing the different knowledge domains in mathematics, which consists of subject matter knowledge and pedagogical content knowledge. According to Ball et al. (2008, p.17) “subject matter knowledge consists of common content knowledge (CCK) (which talks to knowledge of the curriculum) and specialised content knowledge (SCK) (Skills and knowledge required for teaching purposes). On the other hand pedagogical content knowledge consists of knowledge of content and Students (KCS), knowledge of content and teaching (KTS) and knowledge of content and curriculum”. Thomas et al. (2017) hypothesized that “effective professional noticing occurs at the intersection of developed mathematical knowledge for teaching and a high level of responsiveness with respect to the mathematical activities of learners” Thomas et al. (2017, p.2). This draws a possible relationship between MKT and professional noticing. To professionally notice, teachers must use a set of integrated skills in order to respond to what learners are saying, what they are doing, their actions, understanding, and the way they make meaning in the classroom. However to notice and act to learners’ thinking is both complex and challenging, this is because as an individual we do not perceive the world objectively, but we construct what we see. Because we have different ways of interpreting things and making meanings depending on how we view the world we are exposed to, it makes it difficult to come to one particular way to professionally notice. Therefore, alluding to the point that there’s a relationship between MKT and professional noticing. Specialized mathematical knowledge is important but it is not enough for professional noting. Which is what pre-service teachers have in most times. Teachers do not only need to be taught content but they need to be taught how to make inference to what learners are thinking. It is however not always the case since development workshops emphasise the content more than the way to deliver the content. It is because of this that I see the need to investigate the importance of professional noticing in mathematics teaching. With the anticipation that the findings will influence emphasis of professional noticing in development workshops.

When teachers are not well equipped with noticing skills, or they are at a basic level of noticing, they miss a lot of opportunities to use learners’ misconceptions to enforce understanding. Hence when a teacher is very general in what they notice in learner’s responses, it makes it difficult for them to come up with a productive decision on how to help the learner. This is because they are unable to understand where the learner is coming from. As such, the learner loses interest in class and feels left out in the learning process. The aim of teaching mathematics

is not to direct learners to answers, but give them opportunity to explore their mistakes. Therefore, understanding learner's misconceptions is important. This is why teacher needs to be developed to professionally notice, when a teacher is able to notice, they are enabled to treat errors as opportunities for learning.

1.4 PROBLEM STATEMENT

Noticing the errors made while working on fractions

A lot of learners struggle to work on fractions in the senior phase. Even though fractions form part of the routine and procedures in CAPS, which contributes to an overall 45% of the cognitive levels. It requires the learner to have skills that enables them to “perform well-known procedures, simple applications and calculations which might involve many steps as well as derivation from given information” CAPS (2011, p.21). Fractions constitutes to over 15% of the assessment in grade 7 and is generally build up from grade four. However in my experience of teaching mathematics for five years, I have noticed that most learners find it particularly difficult to deal with procedures of multiplication of fractions as well as working with mixed fractions among other topics. This is the case even at grade 7 level, even though the learners are expected to have at least mastered these skills from as early as grade four. Teachers also find it particularly difficult to address the errors made by learners working with numbers because they analyse classroom interactions through different lenses which are usually informed by their own experiences, educational philosophies, cultural backgrounds, and their work experience contexts (Miller & Zhou, 2007). Some were taught fractions by following rules with lots of drill and repetition and less understanding of the rules and procedures, this implied that the rules and getting to the correct answer was more important than working at problems in an intelligent, reflective way with the express purpose of coming to a deeper understanding of the mathematics involved.

Therefore such teachers get influenced by those experiences and make procedural fluency more important than conceptual understanding when they teach. Thus making it difficult for them to notice learners' thinking when working with fractions. This is why some teachers have the competency to teach adaptively, and others have it at a lower level. Therefore, due to observed struggle faced by learners in working with fractions, the researcher has focused the investigation on how well teachers are able to attend to students mistakes in order to understand

the reason behind the errors made and the process they undertake to interpret their analysis to understand underlying issues in order to decide upon a productive instrumental course of action based on this interpretation to better their teaching.

1.5 PURPOSE OF THE RESEARCH

Firstly, the main aim of the study is to establish type of errors made on the topic of fractions at grade 7 level. From the identified errors, the study seeks to investigate the teachers' capacity to attend to student's mathematical conceptions and practices as they occur. Thereafter this study seeks to explore into the ability of teachers to notice students' understanding of relevant concepts to measure if the students' mistake is driven by minor mistake or reflective of a deeper conceptual understanding. This will give a teacher minimal information on how to base decision for instruction. On the other hand, teachers might be more familiar with students' patterns in mathematical thinking and might engage questioning to elicit more information about students' thinking before drawing conclusions and making instructional decisions. The focus will be on different approaches teachers use to interpret students' misconceptions based on reasoning employed by students.

In other words the purpose of this study is to explore and identify key issues relating to the ability of teachers to professionally notice and elicit thinking of students in order to use that as a resource to better their teaching and instructional procedures. Lastly the study also aims to determine the extent to which teaching interventions can uplift the achievement of mathematical topics such as fractions. The results of the study will be used to formulate conclusion on what still needs to be done in order to develop teachers to work with students' errors and misconceptions in South African mathematics classroom settings.

1.6 RESEARCH QUESTIONS

The main research questions of this study are:

- What errors and misconceptions do grade 7 learners make when working with fractions?
- How well do teachers notice these errors or misconceptions?
- To what extent does the use of errors and misconceptions noticed improve the learning of fractions?

The objective of the study is to figure out:

- How do teachers interpret the learners' misconceptions as they work on fractions?
- How do teachers respond to learner that has made a mistake while working on fractions?
- If there is any supporting evidence, from students' performance on pre and post-tests, that professional noticing had an impact on students' understanding of fractions?

It is hoped that the end results of the study will give a clear identification of the relationship between professional noticing and the understanding of fractions.

1.7 SIGNIFICANCE OF THE STUDY

Within the field of education, development of professional noticing skill is missing. This results in missed opportunities to develop understanding of concepts through the use of probing students. Novice teachers are sent out to the field without any guidance of how to notice. According to Ross, Vescio, Tricarico and Short (2011) although teachers are given many resources to guide them in their teaching, they however need to support to professionally judge and decide how to use them. "Expert teachers stressed that to become highly effective teachers, novices need to quickly learn to trust the professional decisions they make related to tailoring instruction to students' needs" Ross, Vescio, Tricarico and Short (2011, p. 69). In most cases expert teachers are also not developed to professionally notice themselves. Even if they are equipped with noticing skill they often are not willing to share that with the novice teachers. At times the novice teachers themselves are also not willing to learn from expert teachers. This is because each teacher has a learning theory that they ascribe to, and some are not willing to improve their practice. This highlights the importance of professional development in noticing. Even in content workshop development, there is a huge emphasis on development on MKT but no support is given to teachers on how to professionally notice. Self-reflection is very important so much that when teachers share with their colleagues their class experiences it enables them to improve the way they teach. Therefore content gained from the development of this study will be used to improve the practices of teachers in the way they professionally notice when teaching mathematics. It will also evaluate the academic performances of learners and their attitude towards mistakes and errors as a learning tool. Data gathered will also help teachers improve both academic and effective performance of learners through professional noticing. This study will foster new ways of enhancing knowledge, skills and attitude towards errors and mistakes in fractions. The documented outcomes of the study will also help improve the quality

of mathematics education and the way it is delivered. By focusing on the lack of attention paid to development of teachers' noticing skill, the study will highlight the need for support of teachers in professional noticing skills.

1.8 CONCLUSION AND STRUCTURE OF THE RESEARCH

This report consists of five chapters which are outlined below:

1.8.1. Chapter One: Introduction

This chapter discusses in detail the background of the study. This is done by breaking it down into a variety of subsections. The empirical settings, context, the research problem (whereby I introduce the research questions), the objectives and the methodology used. This gives an outline of the South African context.

1.8.2. Chapter Two: Literature Review

The literature review chapter gives an outline of the variety of literatures that support the study. Starting off with an introduction and further establishes the relationship between the objective of the study and its theoretical basis.

1.8.3. Chapter Three: Methodology

This chapter outlines the research method that has been used to collect data for this study. This chapter included research design method, the data collection method followed, sampling method, data analysis, reliability and validity, the ethical considerations and the research limitation of the study.

1.8.4. Chapter Four: Analysis of results and discussions

This chapter analyses the results of the qualitative data collected from the pre-tests, structured interviews and post-tests. It includes sub-chapters in accordance with the research question. This chapter also links the results obtained in chapter 4 to the literature review and interpretations were introduced in this chapter.

1.8.5. Chapter five: findings, Conclusions, limitations and Recommendations

The final chapter draw conclusions gained from the results, and offer recommendations for the future studies. It also highlighted the limitations and delimitations of the study.

CHAPTER 2

2. THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.1 THEORETICAL FRAMEWORK

Wallach & Even (2005), state that teachers who are well developed are most likely to notice learners' actions and act upon those actions. This means that development plays a major role in the teachers' practices and their effectiveness. This study's drive is to discuss professional noticing of learners' mathematical thinking and engagement of learners' errors and misconceptions in the topic of fraction at grade 7 level. It will further discuss the importance of professional development of mathematics teachers on professional noticing of learners' mathematical thinking as well as engaging learners' mathematics errors and misconceptions in their teaching.

Kilpatrick, Swafford and Findel (2001) state that, "In order for learners to be mathematically proficient, they need to be able to apply the five interwoven strands. These are conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition" Kilpatrick et al. (2001, p.5). As such, a learner that does not have conceptual understanding will lack procedural fluency (Kilpatrick et al. 2001). For the purpose of this study I will focus on conceptual understanding in saying that when learners are not fully furnished with conceptual understanding, they are more likely to lack in their procedural fluency as well as any other part of the five interwoven strands, which will develop errors and misconceptions in learning of mathematics. Therefore making it essential for a teacher to be able to notice and incorporate their learners' thinking in the teaching of mathematics. This means that a teacher needs to be able to attend to the needs of the learners, interpret their thinking and decide on appropriate manner to respond to them.

In general, schools have been known to assess mainly for grading purposes and teach learners to use correct procedures and rules, which in turn deprive such learners of the opportunity to develop deeper understanding of mathematics as they are taught to avoid making errors thus cannot learn from them. This however leads to more formation of learners' systematic errors and misconceptions as learners are not given the opportunity to get actively involved in the analysis of their errors and use that as enquiry level of understanding.

According to Olivier (1989) errors and misconceptions results from over generations of previous knowledge used where it does not apply. He adds that it happens as a result of learners trying to construct knowledge. This is further emphasised by Smith, diSessa and Roschelle (1993, p.120) who maintains that “misconceptions arise from prior learning, either in the classroom or from learners’ interactions with the social and physical world”. This view is also supported by Roselizawati et al. (2014) who adds that “misconceptions can be caused by failure to make connections with what students already know” Roselizawati et al. (2014, p3). Furthermore, some of the factors that add to errors made by learner include; lack of knowledge of the mathematical concepts or the students not understanding what is required of them in a mathematical task (Swan, 2001). Other factors can be pinpointed to learners’ “problem solving strategies, rules from previous learning experiences in the mathematics classroom or incompatibility with teachers’ instruction” (Roselizawati et al. 2014, p.4).

However, “errors and misconceptions can be used as catalysts for learning, whereby learners’ mathematical understanding is developed through identifying and addressing their errors” (Tan-Sisman & Aksu, 2016, p.6). It is thus suggested that “errors and misconceptions should be included in the mathematics curriculum to inform teachers beforehand about learning challenges that they can expect” (Ryan & Williams 2007, p.13). This is because capitalizing on errors and misconceptions will allow growth of the mathematical knowledge and it also provide a platform for learners to critically analyse and refine their errors (Borasi, 1994). Thus indicating that it is not teachers who can only benefit from errors, but learners can also benefit because they will engage in thought processes whereby, they start questioning their thinking and reflect on their understanding (Borasi, 1994). However, when looking at constructive epistemology under the springboard inquiry, Borasi (1994), argues that it has some radical implications, because the view that all knowledge is accepted as the truth at that point in time results in the conclusion that errors may not always be so clear cut. This means in a case where a mathematical response or contribution is considered as an error will be dependent on the context and the individual making the decision (Borasi, 1994).

This is emphasised in Ernest (1997) paper on the nature of mathematics and teaching claims that everyone has a philosophy which they change or modify. This influences how we think about mathematics and has consequences for how we teach it. He introduces two philosophies, the absolutistic view and fallibilist view. Absolutistic view perceives mathematics as having a life of its own, waiting to be discovered, while fallibilist perceives mathematics as something that was created by humans to solve their existential problems.

These philosophies have a bearing on how mathematics is taught. This is why teachers can be put in the same class to observe the same learners but manage to notice different meanings learners make. Professional noticing refers to the skills teachers use to identify and act upon important mathematical actions of learners (Thomas et al, 2017). The use of professional noticing skills effectively has positive impact on teacher's instructional practices, in other words, they engage in meaningful teaching practices (Thomas et al., 2017). Stated differently, professional noticing refers to the teacher's capacity to attend to learners' mathematical conception and "practices and decide upon a productive instructional course of action based on this interpretation" (Jacobs et al., 2010, p.38).

As such a teacher with an absolutistic value may not be able to professionally notice because they offer tasks "that involve the application of learnt procedures and stresses that every task has a unique, fixed and objectively right answer. This is coupled with disapproval and criticism of any failure to get answers correct" (Ernest, 1995, p.7). This type of teacher also sees mathematics as "separated" because it's pure, rigid and inhuman and exists on its own. Something that humans need to discover and be taught in way that there's no errors but only right answers. This makes it hard for this type of teacher to professionally notice because they cannot see why they need to attend to students' mathematical conceptions, and practices, interpret them and decide upon a productive instructional course of action from their interpretation. To them mathematics has unfeelingness, dispassionate reason and analysis Ernest (1995) this brings about anxiety and fear of mathematics to students as they fear being criticised when they make errors. . Making mathematics something you either know or do not know. Learners try to avoid errors because they see mathematics as a finished product ready to be served. This view is of a behaviourist theory perspective of learning, which is informed by "a traditional schooling and is not likely to invite students and teachers to see errors in a positive light. A behaviourist framework in fact assumes that learning is enhanced when correct responses are made (positive reinforcement) and incorrect ones are either punished or extinguished through lack of attention (withholding of positive reinforcement)" Ernest (1997, p.25).

On the other hand a teacher with a fallibilist view rejects the absolutistic image of mathematics and appreciates the errors made in the process of learning mathematics. This teacher views mathematics as something that is warm, human, creative and related to human situations (Ernest,1997). Hence allowing learners to fully engage and enjoy mathematics. This as well enables a teacher to be able to professionally notice, such that a teacher with a fallibilist view

is able to engage with learners' errors and misconceptions as they occur. Therefore is able to understand a student's approach to a problem and decide on an appropriate response to extend that student's thinking Ernest (1997). Such a teacher allows learners to generate discussions around their errors and develops their consistency of reasoning. Respecting students as mathematics students as well as creating and using community of practice in the mathematics classroom.

More importantly, the aim in teaching mathematics is not to direct learners to answers, but give them opportunity to explore their mistakes. As such, being able to attend and interpret learners' errors is equally important in deciding how to respond to them. This is why a teacher needs to be developed to professionally notice. When a teacher is able to notice, they are enabled to treat errors as opportunities to learn. Such that a teacher who professionally notices in class, has the ability to resource learners' thinking in order to remediate their misconceptions. Therefore a fallibilist views learners' errors as something that provide rich evidence of their thinking. This gives them a better understanding of what students know, thus allowing a teacher to plan more effectively and accommodate each students' learning needs. For this reason, teachers need to be developed so they can understand learners' thinking in a similar way.

Wallach and Even (2005), further assert that noticing what learners are doing and saying is a complex task, as such, this complexity also makes it difficult for teachers to understand learners' mathematical thinking and understanding. They further agree with Jacobs et al. (2010), that it becomes difficult for teachers to attend to the details of the learners' mathematical strategies (Wallach & Even, 2005). In other words, it tends to be a challenging task to describe strategies learners use in solving mathematical problems, because it requires teachers to have expertise skills of noticing learners' thinking. This includes but not limited to listening and hearing.

Listening and hearing as part of noticing

Studies reveal that these complexities that are associated with noticing how learners talk, show and act are referred to as to '*hear learners*' (Wallach & Even, 2005). The reason is that teachers find it difficult to attend to learners' strategies because their hearing of learners' thinking in mathematics is through their personal experience and social resources (Wallach & Even, 2005). As such, teachers hear learners using four interpretations; describing; this

describing what learners say and do, explaining; this explaining what learners say and do, assessing; this assessing what learners say and do and justifying; when teachers justify their decisions and choice of instruction (Wallach & Even, 2005). Through these interpretations, teachers will demonstrate how they hear learners during the lesson.

These interpretations are closely related to the three components of professional noticing as explained by Jacobs et al. (2010). The first component is attending to learners' strategies, which is concerned with the extent in which the teacher describes the strategies that are used by learners when they solve a problem. Wallach & Even (2005), assert that when teachers notice learners' strategies, they are describing the learners' thoughts and actions by quoting their written work or verbal responses. In other words, the teacher highlights and call outs the methods that are used by the learners in detail (Jacobs et al., 2010). Studies indicate that learners' individual strategies are complex and the teacher's ability to attend to these strategies is crucial because it provides a passage to learners' understanding (Jacobs et al., 2010). To give a detail description of learners' strategies it is challenging, hence, teachers need to have more expertise in learners' mathematical thinking to effectively attend to their strategies.

These components of professional noticing are interrelated in a way that they occur simultaneously, as such, teachers who can attend to learners' strategies are most likely to develop skills that will allow them to interpret learners' understanding through these strategies. This brings us to the second component; interpreting learners' mathematical understanding (Jacobs et al., 2010). This is about the explanation that the teacher gives with regards to what they have learned from learners' understanding by interpreting their strategies (Jacobs et al., 2010). In other words, teachers are explaining learners' strategies and the thinking that revolves around those strategies (Wallach & Even, 2005). The reasons provided by the teacher about learners' understanding needs to be evidenced based, however, this does not mean that teachers have to construct a complete picture of learners' understanding, the interest is on the extent to which teachers' reason about the learners' knowledge (Jacobs et al., 2010).

The last component is about deciding how to respond based on the learners' understanding (Jacobs et al., 2010). This component skill is interested on the reasons that teachers use when they decide how to respond to learners' strategies and understanding (Jacobs et al., 2010). In other words, it informs teachers' decisions on how to respond to learners' understanding, this

what other problems can the teacher pose to the learners to extend learners' understanding and effective use of their strategies (Jacobs et al., 2010). This component to be related to the interpretation of justifying, because when teachers decide on how to respond they are actually reflecting on their teaching (Wallach & Even, 2005), and try to come up with a way to further assess learners' thinking, in fact, this last component looks at both interpretations; assessing and justifying. This shows that professional noticing of learners' mathematical thinking is closely related to the theory of hearing.

Therefore when a teacher professionally notices, they need to be equipped with hearing and listening skills. Listening is as much as important as hearing in this perspective. Davis (1997, p.356) states that "attentiveness to how mathematics teachers listen may be a worthwhile route to pursue as we seek to understand and consequently help teachers better understand their practice". In other words each teaching episode is characterised by a particular manner of listening:

Evaluative listening

This type of listening according to Davis (1997) shows a behaviourism theory perspective, whereby a behaviourists listens to the answer and not the method that came about finding that answer. Evaluative hearing does not require understanding of the misconception instead it direct learners to a correct answer. Evaluative listening is also a type of listening that is practised by an absolutistic teacher. Thus this type of listening limits the learner's interaction class and places the teacher as the only source of knowledge. This is associated with behaviourist theory, which deals with memorization stating that learners can't explain why they are doing what they are doing but rather the associate or memorise the consequences or reward. This leads to learning at lower level. This can be seen represented in Blooms taxonomy lower level, remembering, where learners lack understanding and are simply equipped with skills such as listing, naming and matching.

Interpretive listening

In this type of listening there is interaction between the teacher and students as well as interaction between students, which according to Davis (1997, p.355) "demonstrates the importance of learners". Interpretive listening is linked to constructivism theory, whereby the listeners' duty is to repeat what the learner has said in a more acceptable way and maybe represent it differently. According to constructivism theory which is rooted in Jean Piaget's work, "students are not blank slates but rather creators and constructors of their own learning"

Smith (1994, p. 116). This theory have networks or cognitive schemas as tools with which additional new knowledge can be constructed. People build or give meaning to things they notice or think about. Whether listening inactively to a lesson or actively involved in creating findings, the brain is applying prior knowledge to existing schemas to make sense of new information.

Hermeneutic listening

This is the ideal type of listening in professionally noticing and working with learners' misconceptions. This seeks to have conversation that involves all learners and a teacher for justification. This form of listening is formed on a lesson according to interaction between everyone in the classroom (Davis, 1997). Hermeneutic listening plays a huge role in the professional noticing as a teacher that uses this type of listening is able to notice the learners' mathematical thinking in order to "attend, interpret and decide on how to respond" Jacob (2010, p. 37). They are also able to illuminate biasedness, overhearing and under hearing of learners, which are some of the complexities associated with professional noticing. Hermeneutic listening is associated with situated learning, whereby people learn content through activities and learning that is social, active authentic, thus allowing participants to all be part of the learning community. Learners come with results themselves and do not rely on the teacher as the source of knowledge.

Studies indicate that the link between theory and practice is an important aspect in education because it allows teachers to engage in the process of reflection (Wessels, 2018), and Wallach & Even (2005), also indicated that teachers who are able to reflect on instruction have better teaching qualities. Davis (1997), looks at the ability to make this link of theory and practice as teachers' way of teaching in relation to the insights of constructivism. Meaning that teachers go to class aware that learners bring their own knowledge in the classroom.

Furthermore, Davis (1997), discusses the types of listening that take place in the teaching process, and he argues that teachers need to use the correct form of listening to be able to reflect on their teaching. There are different situations that may occur in the classroom, the lesson can require immediate reshape of the lesson (Wessels, 2018) or can require the teacher to reflect after the lesson to improve instructional decisions (Jacobs et al. 2010). As a result, teachers need listening skills that allows them to hear these different views.

There are two notions of reflection that Wessels (2018), discusses in her paper, reflection-in-action and reflection-on-action. The first notion of reflection; reflection-in-action; comprises

of an in the moment situation, in other words, the teachers' ability to change what they are teaching during the lesson (Wessels, 2018). This implies that they engage in the process of thinking on their feet and use mathematical knowledge for teaching (MKT) as a reference to act upon a situation (Wessels, 2018). This type of reflection is more practical to teachers who prepare for the lesson and anticipate learners' response and errors since they are knowledgeable about their learners' thinking (Wessels, 2018).

However, this type of reflection also requires teachers who are able to engage learners and listen to their mathematical thinking. Davis (1997), refers to this type of listening as hermeneutic, because teachers will listen and explore learners' thinking capacity by allowing them to interact and construct multiple ways of presenting solutions. This implies that the classroom must be an environment that acknowledges that knowledge is subjective, because there are interactions that happen between teachers and learners, as a result, knowledge can change in those interactions (Davis, 1997)

In contrast, reflection-on-action refers to the teachers' ability to look back on the lesson and reflect on what happened and the reason why it happened (Wessels, 2018). This also goes back to Wallach & Even (2005), interpretation of justifying because this is where teachers need to reflect on their interpretation and their reflection-in-action to better understand their instructional methods so they can improve their practice. Wessels (2018), further states that noticing learners' mathematical thinking provide appropriate support for framing the development of productive reflection on learners mathematical understanding.

Wessels (2018), & Jacob et al. (2010), state that professional noticing progresses through three interrelated components. While, Wessels (2018), states that the three components also entail reflection-in-action during the lesson. For instance, for teachers to effectively reflect-in-action they have to be able to attend to learners' way of thinking and be able to interpret their thinking (Jacobs et al, 2010). In other words, the first two component of professional noticing are closely related to the notion of reflection-in-action, while on the other hand reflection-on-action goes hand in hand with the last component of deciding how to respond, because the teacher needs to reflect about what transpired in the lesson to make future references to their advantage so as to come with responses that will benefit and enhance learners' understanding of mathematics. Additionally, it is also clear that hermeneutic listening plays an important role in affording teachers the skills to notice efficiently.

Teachers who use hermeneutic listening are most likely to engage learners effectively and provide them with an environment that allows them to justify their thinking (Davis, 1997). On the contrary, it is most likely that during the lesson misconceptions will arise because they are part of a learning system (Nesher, 1987). Misconceptions have a negative connotation because they hinder learners from acquiring new knowledge. The reason for this is that misconceptions are persisting errors that show “a line of thinking that causes a series of errors all resulting from an incorrect underlying premise, rather than sporadic, unconnected and non-systematic errors” as cited in (Nesher, 1987, p.35). Which simply means they are errors that occur from learners’ faulty thinking or incorrect application of prior knowledge. Makonye & Khanyile (2015) also state that mathematical misconceptions and errors persist because conceptions are taught as if they are “isolated chunks of knowledge” Makonye & Khanyile (2015, p.55). In other words, teachers do not show learners the relationship that mathematical concepts have.

Furthermore, it is argued that when learners engage in rote learning it can result in memorizing the mathematical procedures and rules, because they will not be making mathematical connections of the concepts (Makonye & Khanyile, 2015). In other words, learners commit errors because they have incomplete knowledge that they acquired from the teachers’ previous instructions. These misconceptions are viewed as a process that is part of the learners’ learning process which can be used to the teachers’ advantage to understand how learners learn and correct their thinking (Makonye & Khanyile, 2015).

Davis (1997), would argue that teachers also need to move away from evaluative listening, i.e. listening for correctness, to more hermeneutic listening in order to engage misconceptions effectively. This is because the emphasis cannot be on teachers’ knowledge and examples only while there are certain beliefs that they perceive in their teaching. As a result, teachers’ beliefs and perception about teaching must also be altered for the betterment of the learners’ thinking processes (Wallach & Even, 20005).

Teachers who constantly make follow ups on learners’ errors and misconceptions can encourage learners to learn from these misconceptions and add on their knowledge. As mentioned before, the teacher will be probing learners’ thinking and this is an important aspect in their learning process. Makonye & Khanyile (2015) define probing as a teaching strategy that teachers’ use to follow up on learners’ responses by posing questions that will make learners think critically about their responses. They also propose probing as a

pedagogical strategy that will promote learners' mathematical understanding from errors (Makonye & Khanyile 2015).

Probing

While a teacher observes and listens to learners as they are work, they needs to probe. Probing according to Makonye and Khanyile (2015) is a teaching method that can be used to assist learners in understanding mathematical contents. After errors are captured, the teacher must then discuss with learners to understand their errors in depth, this allows the learners themselves to explain what they are thinking, instead of assuming or concluding on their prior thinking. During this discussions with learners, the tools of probing (funneling and re-voicing) are used to understand why errors were made. Funneling is a list of questions that directs the learners to preferred outcomes, these prompts can help a teacher notice what they know and are doing as part of funneling process:

- Tell me what you doing
- I see you have started to add these numbers can you tell me why?
- Can you tell me more about...
- Why did you..?
- How did you solve it?
- How does your picture connect to your equation?
- I'm not sure clear on what you have done here, will you explain it so I can understand?

Consequently, probing will involve interactions between the teacher and learners, in which learners will be required to reflect on their responses (Makonye & Khanyile, 2015). Teachers will be required to use hermeneutic listening in order to listen attentively to the knowledge learners construct on their own, because misconceptions originate from prior knowledge (Nesher, 1987). However, misconceptions can also be the result of teachers' generalizations, because teachers tend to make prior knowledge too generic (Hamza & Wickman, 2008). As a result, teachers' use of probing must take into consideration the variety of factors that resulted from learners' misconceptions. This will allow teachers to create a learning space where learners are constantly critiquing and clarifying their thinking, where learners' internal understanding of mathematical concepts is revealed (Makonye & Khanyile, 2015). This will

form as a basis for learners to be able to accommodate and assimilate new mathematical ideas in an interrelated way.

Subsequently, effective probing must consider the constructivist approach to learning, because it is important to understand that learners have internal mental networks which need to be guided by teachers and build on them to arrive at relational understanding of mathematical concepts (Makonye & Khanyile, 2015). Researchers assert that mathematical understanding can only be noticed through learners' relational thinking not procedural thinking (Makonye & Khanyile, 2015). The reason is that learners' procedural thinking is most likely to have errors and misconceptions because they do not have relational understanding. Hence, two forms of probing; funneling and re-voicing, can be used in combating learners' errors and misconceptions (Makonye & Khanyile, 2015).

Funneling as a form of probing occurs when teachers ask learners a series of question that will lead them to desired responses (Makonye & Khanyile, 2015). This form of probing can start with open questions to get learners to participate, but then move to restricted questions that will narrow the learners' thinking to the correct thinking (Makonye & Khanyile, 2015). Funneling is important because it allows learners to eventually realize where they went wrong, thus, helps them construct relational understanding (Makonye & Khanyile, 2015).

On the contrary, re-voicing refers to when teachers' re-utter learners' responses with the goal of getting them to pay attention to specific claims they made and get learners to elaborate and provide evidence for their claims (Makonye & Khanyile, 2015). In Makonye & Khanyile (2015), it is stated that learners get to refocus their claims when they hear what they said being repeated to them by the teacher. This is because when their responses are re-uttered to them, it provides learners the opportunity to critically think about their thinking and come up with better ideas to correct and enhance their thinking and mathematical understanding (Makonye & Khanyile, 2015).

These forms of probing can be used conjoined with Davis (1997) interpretive form of listening, because the teacher and learners will be engaging in interactions. Learners' knowledge contribution will be acknowledged since the teacher will repeat learners' contribution to get them to think critically towards an acceptable way of reasoning (Davis, 1997). Hence the teacher will play a role to ensure that learners' thinking is directed in the correct line of thinking, using re-voicing and funneling. In other words, they will capitalize on learner misconceptions for meaningful learning and teaching of mathematics.

On the other hand when re-voicing, a learners' response is repeated by the teacher to emphasise certain responses. This will help the learner re-evaluate their answer. Probing allows learners to move from instrumental understanding to rational understanding, thus creating a deeper understanding of mathematics. It also helps a teacher to interpret where students are with their understanding of the concepts. However such a teacher must avoid statements such as "its easy" "let me help you" as these suggest to a child that they are not smart or they wouldn't be stuck or that you think the student cannot solve the problem without help. In contrast, correct funneling suggests that mathematics takes time and thinking while giving the sense of ease to a child. Before probing, a teacher however has to attend and interpret the learners' misconception. This is achieved by the teachers' ability to identify different errors posed by learners. Probing is a method that was used by Borasi, (1994) in a way to allow learners to build on their mathematics reasoning and their deeper mathematical knowledge.

So as learners negotiate their misconceptions, the teacher provides appropriate support. Such as changing the context to a more familiar one, or changing the mathematics to a similar problem with easier numbers. Probing, interviewing learners and showing interest in their work is some of the ways that makes learners view mathematics in a non-threatening subject.

Moreover, to understand mathematics, concepts need to be built upon from previous knowledge and applied to new information (Sarwadi & Shahrill, 2004). Thus a teacher who is well developed with professional noticing skills is able to interpret students' prior knowledge perfectly and use it to decide upon an appropriate way to respond to their misconceptions and errors. Thus the theory of constructivism comes into play in that knowledge is constructed from the past experiences into present structures, referred to by Piaget as assimilation and accommodation (Nesher, 1987). "If a learner is unable to assimilate or accommodate, it creates a learning gap, which can lead to errors and misconceptions" (Sarwardi & Shahrill, 2014, p.117).

As such a teacher who can professionally notice is able to capitalise on these learners' errors to ensure full understanding of the subject matter. Such teachers provide learners with opportunity to know how to use errors as means to learn mathematics "the idea of questioning your own work in order to understand mathematics deeper would be considered as inquiry" Borasi (1994). He further states that an inquiry approach allows the learner learns through questioning their own work in order for them to make links to their prior knowledge (Borasi,

1994, p.168). This views mathematics in a fallibilist approach as being humanistic, warm and full of joy. Because knowledge is seen as a “dynamic process of inquiry” (Borasi, 1994, p.167) and learning as something that is personally constructed by teachers that are supportive of learners’ explorations into understanding. It is encouraged that learners analyse their errors as they are able to do own introspection by self-questioning where they went wrong. Teachers then follow up on these errors and misconceptions and not be judgemental.

Hearing

Therefore, for a teacher to have better analysis of learner errors and better inform teaching and learning they need to professionally develop noticing skills in order to be able to take action to help learners based on what they notice. Without help, teachers will not be able to notice their own biases. From a constructivist view of learning, errors are seen as vital components of learning and as sources of information of how learners constructed knowledge. One form of noticing come through hearing students so to avoid overhearing students, under hearing them or have biased hearing.

What it means to hear students and interpret their talk and actions?

“Previously, student assessment were served mainly for evaluating students’ achievement at the end of the period of instruction. Today many leading mathematics educators call for student assessment that informs and guides teachers as they make instructional decisions” (NCTM, 2000, p. 22). Frequently, educators use student assessment to help design better learning tasks that will afford opportunity to hear students during contact time. However there is complexity associated with attempts to understand what learners are saying, feeling or doing. Hearing students would not allow the teacher to develop an understanding of the learner’s thinking and reasoning, (Ball, 1997). Studies by Even & Wallach (2005), Kazemi & Franke (2004) and Nicol (1999) show teachers’ lack of understanding of students’ thinking and reasoning in mathematics, and difficulties in attending to details of students’ strategies. Based on Wallach and Even (2005 p.394), Ruth’s presentation of the “shirt and numbers” problem. Analysis of Ruth’s hearing of her student indicated different characteristics of her hearing, which are reflected in the describing and explaining types of interpretation:

Over hearing: hearing things that were not said by the students

Under-hearing- Hearing where some parts are missing. Ruth “ignored” some of the things said or done by the students, such as the actual building of the two groups.

Non-hearing- Implies that the whole is missing.

Biased hearing- Volunteering answers without much thought.

This suggest a link between hearing, listening and noticing, such that when a teacher is able to listen (evaluative, interpretive, hermeneutic) Davis (1997), they are able to notice (attending, interpreting, deciding) Thomas et al. (2017) and hear “over hearing, compatible hearing, under hearing, non-hearing, biased hearing” Wallach and Even (2005, p.402). This allows them to explore deeper into mathematics (Borasi, 1994). Through enquiry learners have an opportunity to build on their mathematical reasoning and justification. This is done through further investigation of their mathematical errors.

Thus it is important for teachers to be fully developed with professional noticing skills to equip them with understanding of learners’ errors and misconceptions. Teaching is based on instructional understanding, Sarwadi and Shahrill (2014). This means that there is an emphasis on teaching of procedures and rules and not on building a deeper understanding. Therefore assumption on misconceptions cannot be made by just viewing patterns of errors but by interpreting learners’ misconceptions and how they are connecting them to new information. Thus teachers should not assume that learners have a fundamental understanding of concepts taught in previous grades. They should however critically examine the learners’ written work (Sawardi & Shahrill, 2014). For example teachers should provide assessment that have little variation to allow a good analysis to take place.

Furthermore Skemps’ theory (1976, p.18) suggests that “a concept is activated in the mind when examples of it is encouraged. He pointed out that in order to develop good concepts, good examples of the concepts are required”, conjuring up with Banduras’ social learning theory. Thus in a good mathematics learning is coupled with the teachers’ ability to use examples that are not confusing to learners and examples that are clear and precise. Ashlock (2002, p.4) further added that “misconceptions and erroneous procedures are results of overgeneralisation, and overgeneralisation of rules in an effort to make sense of new information”. Some students’ errors and misconceptions are resistant to change, this is because concept “and schemata are stable once they are formed and are held to be resistant to change. Thus good examples of concepts are required right at the beginning in order for good concepts to be established”

(Skemp, 1976, p.18). “By examining each of their written work diagnostically, teachers would be able to look for patterns and hence find possible causes for error and misconception”. Hence the need for professional noticing development. This development will enable a teacher not to see misconceptions and errors as “dead ends” rather regards them “as opportunity to reflect and learn” (Skemp, 1976, p.18).

2.2 Conceptual Framework on Teacher Noticing

It is very complex for a teacher to professionally notice and work with learners’ thinking. This is because not all learners’ responses lead to errors that can be analysed deeply. Therefore to understand learners thinking, it is important to understand the types of errors and misconceptions and how different authors define errors and misconceptions.

Based on Jacob et al. (2011) experiment with novice and expert teachers on professional noticing suggestions were “that teaching experience alone does not necessary result in the development of noticing skill” Jacob et al. (2011, p.193). Therefore alluding to a potential “connections between Professional noticing and Mathematical Knowledge for Teaching” (Thomas et al. (2017, p.8). Responsibility is with the educator to be involved in mathematical development and not only rely on their knowledge of mathematics in teaching learners the subject. Furthermore this will develop them with analysis on various levels to the learner, to promote reflection and discussion. Hence, the studies by Thomas et al. (2017), Davis, (1997) and Wallach & Even, (2005) indicate the importance of teacher development programs, stating that these programs will allow educators to acquire and develop noticing skills, listening skills and hearing skills respectively. Thus to fully work with learners’ errors, the following recommendations by Jacobs et al. (2010) are to be done simultaneously as one single move:

- *Attending to children’s strategies.*

This talks to identifying noteworthy aspects in a classroom situations and considering the extent which teachers attend to a particular aspect of instructional situation. It also entails describing in detail what you think each learner did in response to a given problem. Some teachers have evidence of this while others lack the evidence for doing this. Hence Jacobs et al (2010) concluded that experience alone is not enough to be competent in professionally noting.

- *Interpreting children's mathematical understanding.*

Which talks to using the knowledge about the context to reason about classroom interactions, considering the extent to which teacher's reasoning is consistent with both details of specific child's strategy. Moreover, explaining what you learned about these children's understanding through their strategies. Jacobs et al. (2010) came up with three codes to measure the evidence through which this was done: Robust, limited and lack of evidence

- *Responding based on the children's mathematical understanding.*

Making connections between the specific classroom events and broader principles of teaching and learning considering the test which teachers use what they have learned in a particular situation. This talks to what problem or problems might you pose next to extend the learner's understanding? This looks into the reasoning the teacher have in deciding on how to respond to learners' understanding and what problems they pose next in order to enhance learners understanding. This is measured again according to the three components by Jacobs et al. (2010). (Robust, limited and lack of evidence).

However Curriculum Assessment Policy Statements (CAPS) does not adequately empower teachers with the kind of errors and misconceptions they can expect when teaching fractions to their learners as well as how these errors and misconceptions arise. As such it is the objective of this study to explore how teachers go about professional nothing by eliciting thinking of a student who has made a mistake in mathematics in order to use it as a resource to teach fractions.

2.3 LITERATURE REVIEW

2.3.1 Background of literature

Fractions have traditionally been one of the most difficult mathematical topics to acquire for learners including learners with learning disabilities (Behr, Wachsmuth, Post & Lesh, 1984; Hiebert, 1985; McLeod & Armstrong, 1982; Ni, 2001). Dickson, Brown and Gibson (1984) argue that one of the difficulties of operating with fractions is that they have a multiplicity of

meanings. This means that fractions can be interpreted in many ways. Even though young children understand the concept of equal sharing, children only understand the part over whole approach to fractions, “Such that $\frac{3}{4}$ is not a number because a person cannot be given four parts of an object that is divided into three parts” (Fazio & Siegler, 2011, P.10). That however possess a great deal of a problem in South Africa as according to the National Mathematics Advisory Panel (NMAP) 2008, learners in grade 4 should be fluent in identifying and representing fractions. Then before they complete grade 5 they should be able to compare the sizes of fractions. Multiply and divide fractions in grade 6 and by end of seventh grade they should operate with negative and positive fractions. Thus highlighting a need for conceptual understanding of mathematical ideas, in order for learners to understand the concept of fractions. The problem that creates difficulties for learners in solving fraction problems is weak or non-existent of understanding of the fraction concept. Learners will be able to construct their own knowledge and strategies if they have conceptual understanding of fractions. They have to put their own thinking on solving the problem and not depend on procedures given or taught in class. Hence emphasising the need for a teacher to be equipped with professional noticing skills, such that they could be able to use the thinking of a learner who has made an error to direct their instructions and lessons for remediation of such errors in future. Hill, Rowan & Ball, 2005 suggests that, through professional development activities, teachers should learn about how students develop an understanding of fraction concepts and the difficulties that they face in proper understanding fractions. Other authors suggest that one way for teachers to gain a deeper understanding of how students learn is to represent fractions is to examine students’ written work and video tapes of them solving problems. Once they understand the problems then they can address the specific misunderstanding in the classroom.

However in most cases teachers teach fractions by following rules and procedures, with a lot of drill and repetition of the algorithms. Learners often trust answers obtained by applying faulty procedures drawn from traditional instruction without questioning whether the answers are suitable or not, because they not used to making sense of mathematics. This results in learners lacking conceptual understanding of the topic. Hallett et al. (2010) as cited in Bentley & Bosse, 2018 suggests that “When learning fractions, some students rely more on procedural understanding processes and algorithms to produce results” (Hallet, Nunes, & Bryant, 2010, p. 241). Moreover students who rely on conceptual knowledge, on the other hand, have more advantage over their colleagues who rely on procedural information. This is because they develop basic understanding of fractions if their conceptual knowledge of fractions is limited

in comparison to their procedural knowledge (Van Steenbrugge et al. 2014). This is more likely to result in errors and misconceptions in the long run.

2.3.2 Why students struggle with fractions?

Some of the most common misconceptions teachers spot in the classrooms are:

A whole number strategy: This is the most commonly noted error whereby learners employ the whole number strategies to solve fractions. Learners employ a number of ways to answer questions based on fractions, starting with simple to complex. However in certain cases, whole-number rules lead to the correct answer (for example, when comparing $\frac{2}{5}$ and $\frac{1}{3}$), and these fortuitous results support students' continued use of whole-number procedures. In young children, whole-number techniques are dominant and diminish in the face of education; however, in older children, whole-number strategies are dominating and reduce in the presence of instruction.

Treating fraction numerator and denominator as separate whole numbers: The common problem that is found in numerous research is that learners see fractions as different numbers “learners do not see the numerator and denominator as representing a single idea or object (the fractional part), but simply two whole numbers” (Lukhele et al, 1999, p.152). Fazio and Siegler, 2011 emphasises that many students view fractions as meaningless symbols or view the numerator and denominator as separate numbers rather than a unified whole. This lack of understanding makes learners use incorrect procedures to solve fraction problems. As a result, learners overgeneralize the properties of whole numbers in the topic of fractions. For example, when they multiply fractions, they expect the answer to become bigger, this is because of the whole number conception of multiplying. Another example can be noted when adding and subtracting fraction, learners subtract the denominators and then the numerators for example, $\frac{4}{8} - \frac{2}{4} = \frac{2}{4}$. Whereas, a learner shows that he sees the number as separate and not unified. This case can also be linked with the above misconception. The learner treats fractions as a whole number thus sees it necessary to apply whole number strategies in solving problems.

Leaving the denominator unchanged in fractions multiplication problems: Learners treat multiplication of fractions with the same mind set as addition of fractions, and forget that for example $\frac{3}{7} \times \frac{1}{7}$ is the same as $\frac{3}{7}$ of $\frac{1}{7}$

Misunderstanding mixed numbers: students ignore the fractions and concentrate only on the whole numbers for example, $6\frac{3}{9} - 3\frac{2}{9} = 6$

Connection between real world problems and fractions: students may give correct answers when they are presented with real life problems but may answer it incorrectly when you present it in a fraction context.

Since these misconceptions often go unresolved, students continue with them throughout the grades and consider them correct. This results in the continuous misunderstanding and struggle with fractions throughout the years. Thus this study is based on whether mathematical noticing skills of a teacher can help with the remedial teaching of some these misconceptions even at the seventh grade. This is often missing in research about fractions “often missing is the detailing precise nature of the mathematical errors, misconceptions and misunderstanding discoverable only through painstaking analysis of student work and communication regarding fractions and fraction operations” (Bentley & Bosse, 2018, p.241).

2.3.3 Effects of not understanding fractions.

Since fractions are continuously taught from grade 4 to secondary school leading to other topics in high school such as algebra and, trigonometry and calculus, the lack of understanding of fractions can lead to dislike and alternatively discontinuing of the subject which affects both mathematical development and career aspiration (Brantley & Bosse, 2018). Fractions also form part of the most assessed topics in senior phase, with about 15% of most question papers consisting of fraction, the misunderstanding of this topic can lead to low performance outcomes in students reports. Fractions are among one of the most important topics discussed in mathematics classroom and the understanding of fractions is critical to students’ success in more advanced mathematics courses (Booth & Newton, 2012; Braithwaite et al., 2017).

2.4 conclusion

This chapter begins by discussing existing literatures on teachers’ abilities to resource learners’ thinking in the teaching of mathematics. It also highlights the errors and misconceptions made by learners when working on fractions. The literature in this chapter shows the increasing need

to investigate whether resourcing the errors and misconceptions learners make in working in fractions can improve their understanding of the concepts taught. Thus the study seeks to establish if teaching focusing at the errors and misconceptions could be one of the ways of dealing with this predicament. Constructivism theory is used to discuss how learning takes place and how learners' misconceptions can be used to remediate understanding.

CHAPTER 3

3. METHODOLOGY AND RESEARCH DESIGN

3.1 INTRODUCTION

This chapter outlines the research method that was employed to collect data for this study. The purpose of this study is to first identify the errors and misconceptions which grade 7 learners display in working with fractions. Then establish the capacity of a teacher to professionally note and intervene based on errors displayed. This would yield possible learning opportunities to improve on the achievements of fractions as a topic in mathematics. This study will follow a qualitative research method to draw its results.

3.2 QUALITATIVE STUDY

For the purpose of this study I pursued a qualitative research methods. Neuman (2011, p. 424) states that “a qualitative researcher attempts to capture all the details of a social setting and the inner lives of the people in it”. This study seeks to discover patterns in answering questions such as “what”, “how, and “why”. The paradigm used in this study is constructivist in nature with an epistemological view that “knowledge is the product of a human mind, that understanding about knowledge and the world happens as a result from collaboration between the researcher and the participants” (Hatch, 2002).

3.3 RESEARCH DESIGN (Action Research)

According to Mason (2002) every act of teaching depends on noticing: noticing what students are doing, how they respond, evaluating what is being said or done against expectations and criteria, and considering what might be said or done next. Drawing on Mason’s philosophy this study involves an action research. This was a way to improve the quality of delivery in the way the investigator teach mathematics. Moreover it has also avoided breaking the rules and restrictions that are set by the presidency in line with the covid-19 regulations. The study uses a pre-test, remedial teaching and a post-test to draw its conclusion. Alternatively the researcher have used these to *attend* to general observations and also student mathematical thinking. From the pre-test, the investigator was be able to draw general observation of student mathematical

thinking, here was also able to tell generally where students have shown confusion on working with fractions. Thereafter, through the remedial teachings, the investigator was able to *attend* and *interpret* deeply the students' mathematical thinking. Hence the focus was on the level of interpretation observed in the investigators' teaching. As students are working on fraction, the investigator applied interpretation of whether they are generally confused or they are not understanding the problem or topic. Throughout the study the investigator attempted to identify possible reasons behind students' mathematical thinking, or what students might mean in relation to their understanding of mathematics. Lastly through remedial teaching and post-test the investigator was also able to *respond* to students' thinking, hence applying the *deciding* aspect of noticing. After the post-tests I interviewed the learner on their responsive nature. Thereafter the investigator discussed on the mathematics in order to address the students' confusion or misconception. This according to Wessels (2018) is a reflection in action stage. Therefore through remedial teaching, I was able to respond to things as they happen, which highlights the reflection on action.

The investigator initially started off by giving learners an hour pre-test on basic knowledge of fractions. Thereafter from the sample the investigator applied a non-probability sampling to select 6 learners' responses in a form of scripts. The investigator then analysed the 6 scripts to interpret common errors and misconceptions from the six learners. Then the investigator recorded the findings as well as reflected on errors the investigator have observed over the years. The investigator then used these findings in the remedial teaching, whereby the investigator was attending to them as well as interpreting new and arising issues through interviews. The interview has highlighted aspects such as listening skills, strategies of listening in teaching, the investigators' analytical and diagnostic approach while teaching, etc. Finally the investigator applied the reflection gained after the interview, as an intervention strategy to teach fractions to grade seven learners.

A post-test was then conducted on the same learners who were selected for the study. This was done to compare if there was any improvement in the learners' performance from the pre-test results.

3.4 SAMPLE

The study was conducted in South Africa, at a primary school in Johannesburg south district Gauteng province. It involved a sample of 60 learners enrolled in grade 7, ages 12-14. This

study was a purposive sampling, such that a mixture of learning abilities were presented so that the investigator can get more credible data. The sample group was requested to complete pre and post questionnaires. The questionnaires consisted of approximately 15 questions and took approximately 60 minutes to complete. The outline questionnaires includes sections addressing the students' errors and misconceptions.

3.5 RESEARCH INSTRUMENTS

Various research instruments are used for various purposes as mentioned above the researcher draw data for the study with the use of tests, observations schedules and interviews. Opie (2004) suggests that questionnaires are tools which can be used to answer the questions 'what', 'where', 'when', but not so easy to answer 'why'. Thus based on the fact that the study is not so much as concerned with providing the reasons as to 'why' learners have errors and misconceptions but 'how' they can be used to improve instruction. The limitations of questionnaires to provide reasoning as stated by Opie (2014) does not affect the study. As such the chosen instruments suit the study perfectly.

3.5.1 The written test (Pre-test and Post-test) included in Appendices

Tests, questionnaires and test results were used as part of the qualitative data collection tools. These allowed the research to be as objective as possible. Thus the type of design that was conducted by this study was current embedded design, wherein mixing occurs in research process either between phases or in discussion after analysis. In this design, one form of data collection is often embedded within another (larger) data collection procedure to analyse different questions or level of units Creswell et al. (2011).

Purpose of the tests

In order to get a full understanding of the learners' thinking I created a pre-test to identify the errors and misconceptions that learners display on fractions and to also determine what these errors might be attributed to. The identified errors and misconceptions in the pre-test were used to devise interventional teaching which was directed at the identified errors and misconceptions. After the teaching intervention I conducted a post-test which determined the nature of opportunities the interventional teaching had on the errors and misconceptions. This post-test was conducted with the same learners that took the pre-test and attended the

intervention process. This is important to note because this gives the researcher an opportunity to critically analyse the comparison of the learners' performance before and after applied interventions. This is to validate the second and third objectives of the study. The two tests were cognitive in nature, these tests measured what learners know, understand and can do on the topic of fractions. To create a standard of validity and reliability of the test the researcher has designed tests using past CAPS approved examination papers in the relevant grade. The pre and post-test is guided by the Annual Teaching Plan (ATP) from the Gauteng Department of Education (GDE). Tests were used as a primary tool to collect data for the study thus were designed in a good, reliable and valid manner such that a true reflection of the conceptual understanding will be provided by both tests.

Table 3.1: Limitations and advantages of a written test:

Advantages of written tests	Limitations of written test
Tests are normally conducted in a controlled environment	Good tests are difficult to develop and consume time
Generally people are used to writing tests – it may not be a new activity for the respondents	Some candidates may leave some questions unanswered
Authenticity – the researcher would know exactly that the test was taken by the person it was meant for	May be subject to corruption through cheating
Can be scored in straight forward manner	Can cause anxiety and stress to those who are taking the test
Tests are generally accepted as a credible indicators for learning	Doesn't allow for candidates to express/ explain what they mean or how they came about the answer.
Provides an objective information about what the candidate knows and can do	May be time consuming.

Table 3.2: Analysis of the written test using Bloom's Taxonomy

Bloom's Taxonomy Category	Description of category	Pre-Test & Post-test Items/Questions	Marks
Knowledge	Exhibition of previously learned material recalling fundamental facts, terms, basic concepts, etc.	Question 1	12marks
Understanding	Ability to grasp meaning of learned material. Example, of translation of material from one form to another	Question 2 And question 3	18marks
Application	Ability to use learned material in new and concrete situations	Question 3	18 marks
Analysis	Identification of parts of material, analysis of their relationship and the organizational structure of the material	Question 3	12marks

3.5.2 Interviews (included in Appendices)

For the purpose of collecting data that was not possible collect with a test instrument, interviews were conducted with a sample that is purposive. According to Marshall, 1996, p.523 “many qualitative researchers use purposive sampling because they recognise that some participants are richer than others, and that they are likely to provide insight and understanding for the researcher”. Therefore as part of the qualitative tools for data collection this study has employed the use of interviews as the aid to interpret the learners' thinking.

In the interview section the investigator seeks to understand the level of success that the remedial lesson had on the learners. The interview had 6 sections, the first section the learner clarifies further the steps they made to come to their answers. This part of the interview will help me interpret the thinking that the learner used to get their answer. Questions such as, what did you do first? Why did you do this? Were used. In the second and third part of the interview the learners are required to rate their understanding of the topic before and after the intervention lesson. The fourth and fifth part entails learners discussing their favourite topic from both the pre and post-test. Lastly the interview investigates the level of lesson understanding from both the pre and post intervention lesson.

Opie (2014) argue that while questionnaires may not be good at answering the ‘why’ questions, an interview on the other hand can be structured to collect data which answers the ‘why’ questions. Therefore to ensure that the researcher got a full understanding of the learners’ thinking before deciding on the conclusion for the results, the researcher selected interviews as a research instrument. The teacher and the learners took part in each interview thus will reveal the reasons as to why certain decisions were made at each point of the lesson. I have selected to use one-on-one interviews that will have a mixture of open-ended and closed-ended questions. This will ensure that the interview is semi structured and follow up questions were used on an initial response.

3.6 LESSON INTERVENTION

The main task used in this study was the lesson intervention, two lessons took place a day after the pre-test was written, marked and analysed. This part presents the teacher’s ability to use the thinking of a learner who has made a mistake to remediate teaching for understanding. The objective of this lessons was to use the errors made by learners during the pre-test as a resource to reteach the topic of fractions. Using the errors, the investigator had an opportunity to address the thinking of the learners in order to help them understand what is required of them when they are dealing with the same topic. The other objective of this part is to draw links between learners’ thinking to understanding. The approaches that were utilized includes but not limited to: interactive, reflective and inquiry based learning. Learners became active participants and used models as well as other resources to construct meaning and understanding.

Lankshear and Knobel (2005, p. 266) claim that data means “bits and pieces of information found in the environment that are collected or generated in systematic ways”. However, this claim is underscored by David and Sutton (2004), who hold the view that “data should not be viewed as that which is out there to collect, but rather what the researcher ‘manufactures’ and records”. Thus for the purpose of this study data collection was guided by the selected research instruments above, as well as the conceptual framework, the investigator gathered data to answer the research questions in the following ways:

A pre-test was made for learners as a means of identifying their pre-existing errors and misconceptions. This was done qualitatively to analyse their understanding and the errors they

displayed. Thereafter, while grouping and clarifying the errors, an interview was conducted to clarify the learner's intentions in their responses. The focus was mainly on learners' errors and misconceptions as well as how the teacher uses the thinking of student who has made an error as a resource to teach fractions. The researcher then professionally attended to each of these learners' responses by giving comments separately next to each incorrect or partly correct answer as a way to identify the procedures followed by the learner to that conclusion. The findings were used to plan an intervention lesson that determined to address and remedy all these observed misconceptions. This used the interpreting and responding part of professional noticing. Thereafter students wrote a post-test, which determined the ability of the teacher to draw from, and use evidence of students' thinking in their teaching intervention.

Even in the context of Covid-19 regulation, the lesson were face-to-face, however all regulations including wearing of face masks, social distancing and frequent sanitizing were followed. The lesson unfolded in the school that the investigator is currently teaching at, this is to avoid going to an unfamiliar environment whereby hesitations of being allowed to come in contact with the staff or learners might be high.

3.7 REABILITY AND VALIDITY

This section discusses one of the most important considerations of the study. Addressing the reliability and validity of the data collection methods in order to ensure a reliable and valid degree of the study results.

Reliability as defined by Opie (2004) is the degree to which the instrument used in the collection of data can be trusted to yield consisted results if used in different research settings (with similar conditions), and following the same procedure and if used by different researchers. Validity as defined by Opie (2004) is the extent to which a research instrument measures what it is designed to measure. In other words are the findings and interpretations of results answering the research questions? Thus as mentioned above, to ensure maximum reliable and valid results in the study, the researcher has used multiple research instruments such that one supports and validates the other, the instruments are also reliable and valid in a way that they eliminate biasedness from the study.

Firstly a pre-test was conducted on the first day of data collection process. Pre-pairing to notice (Mason, 2012). The researcher then marked the pre-test and analyzed it for common errors and misconceptions. Thereafter an intervention was administered which addressed the identified

errors and misconceptions, this lasted for six hours which was spread out in three days with each day having a minimum of an hour period each. This enabled me to notice in the moment and pick up further misconceptions while trying to address the current ones that were being reflected on. Thereafter the researcher administered the post-test which lasted an hour and was done within a day shortly after the intervention lesson. This was done early so that the results would be reliable and credible.

Opie (2004) defines validity as the extent to which a research instrument measures what it is designed to measure. To prove validity of the study, the study adheres to the three research questions which are gathered by the pre-test, interviews and post-test. Questionnaires were designed according to CAPS document for grade alignment purpose.

3.8 ETHICAL CONSIDERATIONS

Ethical matters were eliminated by applying for ethical clearance from the Wits Ethics Committee. The Wits Ethics Committee granted permission to proceed with the study and the protocol number (2021ECE083M) was issued. Moreover the researcher has been granted permission by the Gauteng department of education as well as the principal of the school the research was conducted. The researcher has also gotten authorisation to observe the learners from the parents of the students taking part in the study that are below 18 years of age. The consent forms clearly stated that the participants will neither be advantaged nor disadvantaged by taking part in this study, and all partakers have been made aware that they can withdraw from participating in the study at any time. Pseudonyms were used throughout the study as a means of strict confidentiality. Data collected from the study is stored safely in protected area and confidentiality was maintained in any reporting of the study. The researcher has saved the data on a USB flash disk that has been encrypted with a secret password.

3.9 DATA ANALYSIS STRATEGY

Summarizing data techniques requires documentation utilized to gather data in the empirical set. “The documents includes written test responses, observation notes, interview notes, transcripts etc. As the follow up step from data condensation, data display is an organized assembly of information that allows conclusion drawing” (Miles & Huberman, 1994, p. 12). The study’s findings were derived from analyzing and comparing the pre and post-test responses as well as

the interviews. This is done to measure the extent through which noticing has improved learners' understanding of.

Table 3.3 below highlights the commonly found errors guided and informed the analysis of the study.

Table 3.3: Different types of errors

Code	Type of error	Description
C1	Ignorance of rules	Applying a rule where it does not apply (misapplication)
C2	Generalization over number	Disregarding the difference properties of different numbers
C3	Conceptual error	Lack of conceptual understanding (Existing schema in conflict with new schema)
C4	Generalization over operation	Disregarding the different properties of different operations
C5	Incomplete application of rule	Ability to apply a rule but failure to proceed
C6	Random	Non-systematic (unintended mistakes)

(Olivier, 1989; Luneta & Makonye, 2010)

These errors informed the study.

3.10 Conclusion

This qualitative study consisted of different phases through which data was collected using a wide variety of instruments. The investigator had to conduct a written pre-test to identify the errors and misconceptions learners have. Interviews were used to validate the learners' reasoning. Thereafter an intervention lesson was used remediate these errors and misconceptions. Lastly the learners were given the post-test to measure the success of the intervention lesson by the use of performances from both tests.

CHAPTER 4

Results analysis

4.1 INTRODUCTION

This chapter presents the analysis of data, whereby a thematic analysis was used. The process of analysing data involved taking raw data, examining, processing it and transforming it into meaningful information (Hatch, 2002). The study's data is established from the analyses of the data obtained from written tests and interviews with the participants as well as the general observations during the intervention process.

The chapter resumes with an analyses of the selected learners' responses to the pre-test questions. This provided the opportunity to identify and classify the learners' errors and misconceptions on fractions. From that the researcher carried an interview to further strengthen the understanding of the misconceptions, which enabled me to get a detailed analysis of whether what the researcher thought was noticing was what the learners were thinking. It also analyses the intervention as well as the learners' compared performances from the pre and post-test. This provided a difference between the two tests and the impact of the intervention after a lesson was directed on the noticing observed on learners' errors and misconceptions.

4.2 ANALYSIS OF LEARNER RESPONSES TO THE PRE-TEST

As mentioned above, the investigator has used various methods to draw data for this study.

This data was analysed in accordance to the following research objectives:

- to explore and identify key issues relating to the ability of teachers to professionally notice and elicit thinking of students in order to use that as a resource to better their teaching and instructional procedures.
- to determine the extent to which teaching interventions can uplift the performance in mathematical concepts such as fractions.

The objective was to check if there is any supporting evidence, from students' performance on pre and post-tests, that professional noticing had an impact on students' understanding of fractions.

A pre-test was used to identify the errors and misconception, thereafter interviews followed to get clarification on what the learners' responses. This provided a better analysis of the errors and misconceptions the learners had. Lastly an intervention lesson was conducted based on the findings and learners were given a post-test to complete.

4.2.1 Error identification, classification and analysis

From the pre-test these are the analysis of the learner's responses:

Table 4.1: Learners' correct and incorrect response analysis (n=60)

Question	Number of learners who gave correct responses	Number of learners who gave incorrect responses
Question 1	33	27
Question 2	24	36
Question 3	18	42

From this analysis it was clear that even though students seemed to have understood question 1 more than the rest of the questions, they still had a bit of a difficulty working on it. Question 2 and 3 however had more incorrect responses which highlighted that there were a lot misconceptions that students had pertaining the knowledge that was tested. These results were sufficient to start the noticing process. Hence I began by marking the students' scripts to identify the specific errors and misconceptions the students had. These are discussed below:

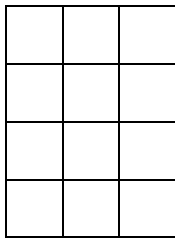
Task 1

This task was prepared to learners as follows:

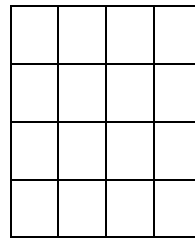
Consider the following figures:

1.1 In each figure shade the fraction that is indicated.

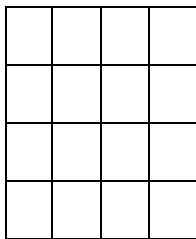
1.2 Express the given fraction in its simplest form.



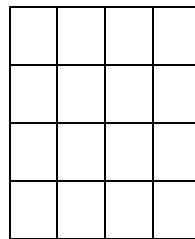
$$\frac{3}{12} = \frac{\quad}{\quad}$$



$$\frac{4}{16} = \frac{\quad}{\quad}$$



$$\frac{8}{16} = \frac{\quad}{\quad}$$



$$\frac{8}{16} = \frac{\quad}{\quad}$$

The activity in this task consists of low order questions, which meant that they were at an easy level. This is because students could easily use the illustrations to draw to their answers, thus most learners could solve this question.

Learners' responses to task 1

Most of the misconceptions in this question are associated with learner's lack of conceptual understanding of simplifying fractions. The misconceptions are as follows:

Type of error: Conceptual error

Misconception 1: Viewing the numerator and denominator as separate values:

(Case 1 of the first misconception)

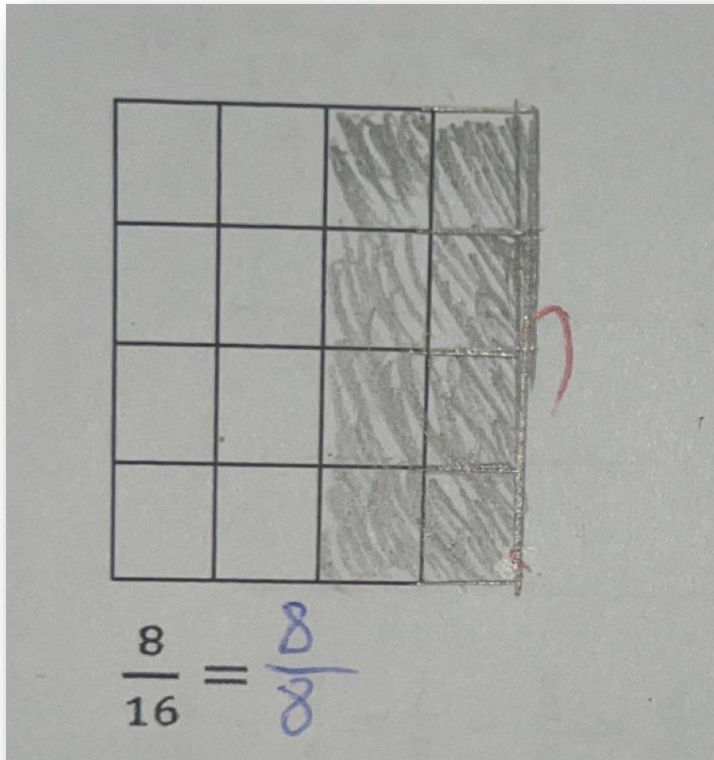


Figure 4.1: Learner response to pre-test question 1 (Case 1 of the first misconception)

In the example, Bheki has correctly represented $\frac{8}{16}$ by shading it in, however he struggled to simplify the fraction to its simplest form. It seems that he does not see connection between the numerator and the denominator. Which is why he has separated the fraction into two parts by counting every block of the shaded area and representing it as a numerator 8 and denominator 8. It is not clear to him that the 16 represented the total number of the blocks before 8 of them were shaded in. This shows that the learner sees these values as separate and does not see how they link. Furthermore this conceptual error has resulted in him not being able to identify that the fraction can be divided by 2 and that half of the total region has been shaded in. hence he could not identify that the fraction could be simplified as $\frac{1}{2}$.

(Case 2 of the first misconception)

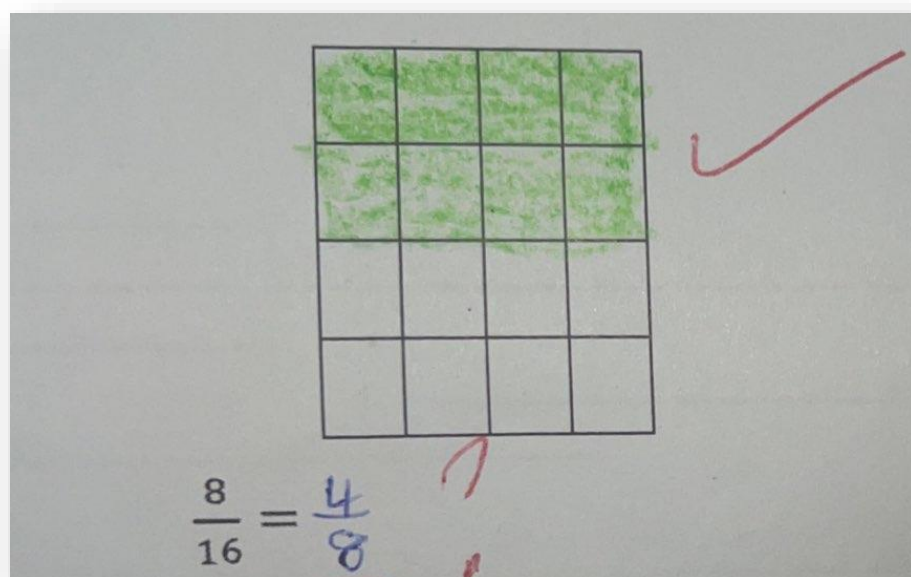


Figure 4.2: Learner response to pre-test question 1 (Case 2 of the first misconception)

Another student (Zanele) had the same perception as Bheki. In this example Zanele displays an understanding of modelling of fractions as she was able to also correctly represent it by shading it in. However she too has displayed a misconception in her method of simplifying the fraction. The fraction $\frac{8}{16}$ was simplified as $\frac{4}{8}$, it was not clear how she had come to this conclusion so the investigator had to interview and probe her to get a clear understanding of her thinking. Before this was done, the investigator tried to interpret her answer to get an idea on what to focus on when probing her. It seemed to that the four represents the number of shaded blocks in each row and the 8 represents the total number of the shaded blocks. Hence implying that she has completely forgotten to include the unshaded region in her answer. This could be that this learner does not know the meaning of the numerators and denominators hence they view them as separate values.

When the investigator interviewed the student to find out how she got to her answer these are some of the remarks she made:

Teacher: I'm not so clear on what you have done here, will you explain it so I can understand?

Zanele: The fraction says that I must shade in 8 squares of the 16 squares, right? (*Pause*)... so yah, I shaded in the 8 squares ma'am (*pause*). Then... uhm I looked for the LCF (lowest

common factor), by asking myself which number can divide into 8 and 16 without leaving a remainder? And I got 2

Teacher: Okay, tell me more

Zanele: I then divided 8 by 2 and got 4 (*long pause*)

Teacher: okay what did you do next?

Zanele: I then said $16 \div 2$ and the answer is 8 (*pause*) so that's how I simplified the fraction to $\frac{4}{8}$ ma'am.

From the interview, the learners' explanation was completely different from what the investigator has noticed. This proves that students' mathematical thinking is ambiguous and cannot be grouped and classified into one. The student in this case has used division to simplify the fraction, and did not use the illustrations as the investigator has anticipated it happening. This proves that the investigator was biased in the way she was hearing the students' response. This is because as teachers we prescribe methods to learners without even realizing it. The learners' answer is however still incorrect, but only this time it is because he has incorrectly used the LCF instead of the HCF (Highest Common Factor). To them, they simply needed to get the two numbers low, so they divided them with a LCF and they did not use the illustration to guide them as the investigator has anticipated it happening.

(Case 3 of the first misconception)

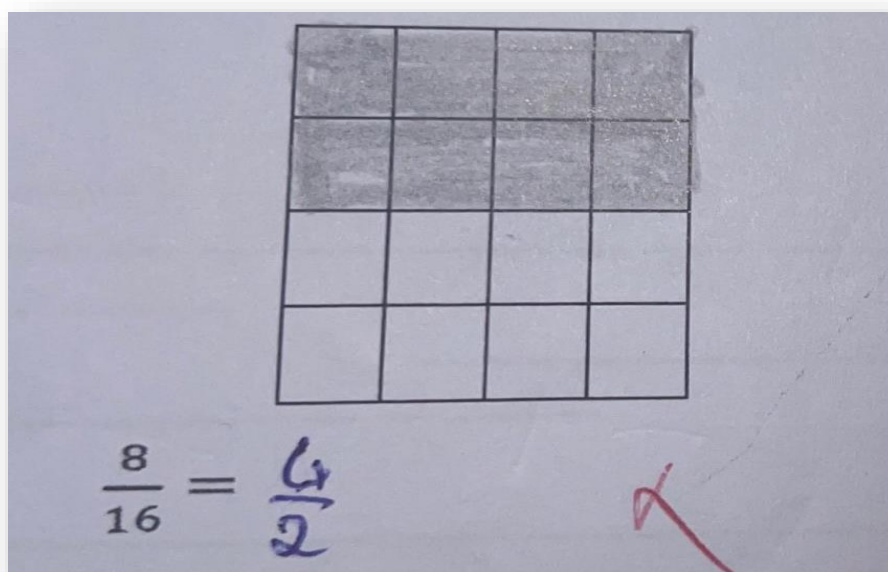


Figure 4.3: Learner response to pre-test question 1 (Case 3 of the first misconception)

In this case, Nosipho also correctly shaded in the fraction, however she simplified it incorrectly. This is due to the lack of conceptual understanding of denominators and numerators. Thus $\frac{8}{16}$ was simplified as $\frac{4}{2}$. It was not clear how she has done this so the investigator had an interview with her to probe her for clarification. The interview went as follow:

Teacher: Nosipho, you wrote $\frac{4}{2}$ as your answer, how did you get that?

Nosipho: The question asked me to shade in $\frac{8}{16}$ ma'am, so after I shaded that in, then I had to simplify it.

Teacher: Okay, and how did you do that?

Nosipho: 2 rows of 4 blocks are shaded in ma'am, so that makes the answer $\frac{4}{2}$.

In this case the learner has completely ignored the unshaded part of the fraction and drawn to the answer by considering only the 8 shaded region. She has counted the 4 shaded blocks in each row as the numerator and the denominator was concluded by counting the total number of rows that are shaded in. It seems that the learner has no conceptual understanding of numerators and denominators. By also leaving out the second part of the fraction, Nosipho shows that she sees fractions as separated values such that, she can chose to work with one and forget the other.

(Case 4 of the first misconception)

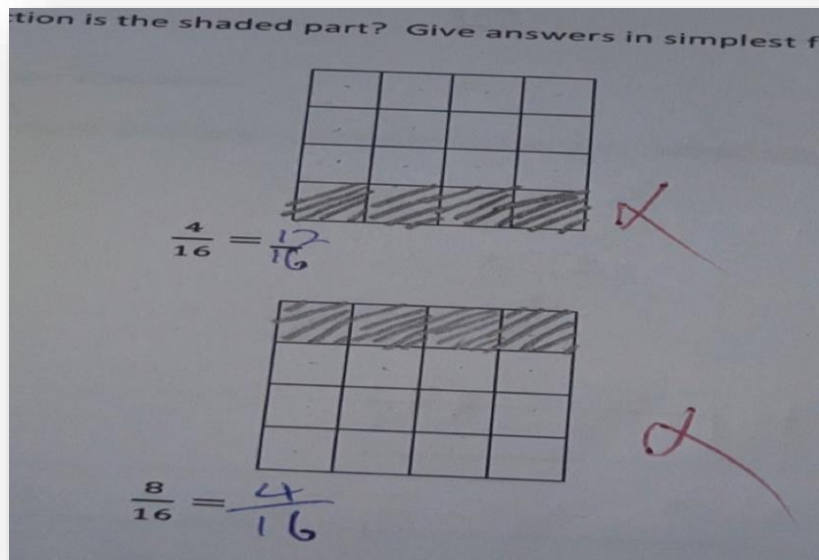


Figure 4.4: Learner response to pre-test question 1 (Case 4 of the first misconception)

Likewise, in this case Khosi also successfully shaded in the fraction however struggled to simplify it to its lowest form. In the example, Khosi simplified $\frac{4}{16}$ as $\frac{12}{16}$, it is clear that the learner has represented the numerator as the number of blocks that are not shaded and the denominator as the total number of blocks presented by the fraction. This shows that he a clue on what denominators are but lacks understanding of numerators. He also does not understand what it means to simply a fraction. The same can be observed in this his second answer where he simplifies $\frac{8}{16}$ as $\frac{4}{16}$, hence emphasising that this was not a random mistake but a conceptual error with denominators and numerators.

Task 2

Task 2 required the learners to:

Calculate:

a) $4\frac{1}{3} + \frac{5}{6}$

b) $7\frac{3}{4} - \frac{11}{12}$

c) $8\frac{3}{7} - 4\frac{5}{7}$

These questions consisted of the understanding, application and analysis Blooms taxonomy levels. They were intended to measure the learners' ability to grasp meaning of learned material, use learned material in new and concrete situations and Identification of parts of material, analysis of their relationship and the organizational structure of the material Anderson and Krathwohl (2001). Many learners got the questions incorrect, some of the errors associated with those incorrect answers are as follow:

Type of error: Random Errors

These errors are difficult to analyse because they are not linked to any previous knowledge, hence making them also difficulty to classify. Luneta and Makonye (2010) claim that some of them are what is referred to as unintended mistakes.

Handwritten student work for pre-test question 3c. The student is calculating $8\frac{3}{7} - 4\frac{5}{7}$. They find the LCM of 7 and 7 is 7. They convert $8\frac{3}{7}$ to $\frac{42}{7}$ and $4\frac{5}{7}$ to $\frac{28}{7}$. They subtract $\frac{28}{7}$ from $\frac{42}{7}$ to get $\frac{14}{7}$, which simplifies to 2. The final answer is 2.

Figure 4.5: Learner response to pre-test question 3c (Case 1)

In this example the student attempted to convert the mixed number $7\frac{3}{4}$ into an improper fraction by multiplying the whole number (7) and the numerator (3). However that was incorrect because they had to multiply 7 by 4 and then add 3. This mistake resulted in everything being incorrect, it is however evident that the learner has conceptual understanding of converting fractions, even though their procedural fluency was a bit distracted by this random mistake. Their second error was observed when they had to incooperate the LCM in order to make the denominators equal. Thus instead of multiplying by 3 to change the denominator 4 to 12, they multiplied by 2. It is thus not clear whether it was an honest mistake or the learner lacked procedural understanding of converting mixed numbers into improper fractions. Lastly the learner has chosen to completely ignore the fraction $\frac{11}{12}$ after looking for the LCM. This could be because they have an assumption that they do not have to use it in the calculation because it doesn't require converting, or that they have forgotten about it after working so long with the other fraction. The learner also did not convert the denominator to 12 in their answer even after looking for the LCM.

Interviewing the learner this is what the researcher discovered:

Teacher: Tell me what you did to get your answer?

Zanele: Ma'am, the denominators are not the same neh? So I had to make them equal. Manje (now) I said uhm (pause) ... I said 7×4 then I got 21. Then ...yoh I was supposed to add 3 ma'am, eish ma'am I forgot.

Teacher: Okay but what did you do after that?

Zanele: Uhm, eish, I uhm...I looked for the LCM then I got 12, then I had to change 4 to 12, so I multiplied by 2 (pause)

Teacher: Why 2?

Zanele: Because ma'am there are two numbers in the multiples of 4 before 12.

Teacher: Okay carry on.

Zanele: Then ma'am I got $\frac{42}{4}$

Teacher: Why 4?

Zanele: Because ma'am the fraction is over 4?

Teacher: But why did you have to look for the LCM?

Zanele: Because ma'am we have to look for the LCM if the fractions are not equal.

Teacher: Equal? How?

Zanele: Uhm ma'am you see the number below is 4 and 12, so we look for the LCM.

Teacher: Okay, but what happened to $\frac{11}{12}$?

Zanele: Eish ma'am I forgot it.

From interviewing the student the researcher could conclude that the learner has a clue on what needed to be done, however she rushed through the question and didn't pay attention to what she were doing, this lead to her making mistakes and alternately to the incorrect answer.

b) $7\frac{3}{4} - \frac{11}{12}$

$\frac{28}{4} - \frac{11}{12}$ LCM 4, 8, 12, 16, 20, 24, 36, 48 = 50/12 = 7 1/2

$\frac{28 \times 3}{4 \times 3} + \frac{11 \times 1}{12 \times 1}$

$\frac{39}{12} + \frac{11}{12}$

c) $8\frac{3}{7} - 4\frac{5}{7}$

$8 \times \frac{3}{7} - 4 \times \frac{5}{7}$

$\frac{56}{7} - \frac{5}{7}$ LCM 14, 21, 28

$\frac{56 \times 1}{7 \times 1} - \frac{5 \times 1}{7 \times 1}$

$\frac{61}{7} = 4 \frac{1}{6}$

Figure 4.6: Learner response to pre-test question 2 b and c (Case 2)

In question 2b of the example, the student forgot to add 3 when converting the mixed number $7\frac{3}{4}$ to an improper fraction, this resulted in every step being incorrect. However the solution is still correctly carried out with the wrong answer from the first step.

This is also observed in question 2c where the student successfully converts the first fraction $8\frac{3}{7}$ but forgets about the whole number 4 of the second fraction $4\frac{5}{7}$. He also looked for the LCM which was not relevant to this question as the denominators are the same. This confusion further moved him away from the main question they were solving, so he resorted

to just randomly writing anything as the answer as he was also confused by the mistake they previously made.

Handwritten student work for problem b) $7\frac{3}{4} - \frac{11}{12}$. The student correctly identifies the LCM as 12 and converts the fractions to $\frac{3}{4} = \frac{6}{12}$ and $\frac{11}{12}$. However, they incorrectly convert the whole number 7 to 3, leading to $3\frac{6}{12} - \frac{11}{12} = 3\frac{-5}{12}$, which is written as $3\frac{3}{12}$. A red 'msd' is written in the margin.

$$\begin{aligned} &\text{b) } 7\frac{3}{4} - \frac{11}{12} \\ &= \frac{3}{4} - \frac{11}{12} \quad \text{LCM} = 12 \\ &= \frac{3 \times 3}{4 \times 3} - \frac{11 \times 1}{12 \times 1} \\ &= \frac{6}{12} - \frac{11}{12} \\ &= \frac{5}{12} \end{aligned}$$

msd

$$\text{c) } 8\frac{3}{7} - 4\frac{5}{7}$$

Figure 4.7: Learner response to pre-test question 3c (Case 3)

In the example the question is supposed to be solved as follows: $7 \times 4 + 3 = 31$. However it seems like the student wrote the 3 and forgot the 1 of 31. This led to the mistake being carried out throughout their entire calculations. Even though the steps are still correctly followed with incorrect answer derived from the mistake, the learner still shows that he had both conceptual and procedural fluency of converting mixed number fractions.

Type of error: Ignorance of rule restriction

Examples of such errors is the inappropriate use of cancelling, learners like to cancel out numbers in fractions without the conceptual understanding of why that is done.

The image shows a student's handwritten work on lined paper. At the top, the problem $7\frac{3}{4} - \frac{11}{12}$ is written. Below it, the student has written $\frac{4}{3} - \frac{12}{11}$, which is an incorrect conversion of the mixed number. Further down, there is a calculation $3 \times \frac{2}{11} = \frac{6}{11}$. To the right, the student has written $= 8\frac{1}{2}$ with an arrow pointing to it. Below that, there is a crossed-out $8\frac{1}{2}$ and the word "next" written in red. At the bottom, the student has written $8\frac{3}{7} - 4\frac{5}{7}$ and then $= 12\frac{8}{7}$.

Figure 4.8: Learner response to pre-test question 3.2 (Case 1)

The question is $7\frac{3}{4} - \frac{11}{12}$, however the student inverted the numerator and denominator around and changed the fraction to $\frac{4}{3} - \frac{12}{11}$. He also dropped out the whole number (7) in the work out. It appears as if the learner applied the reciprocal relationship rule which is used when dividing fractions. This shows that the learner has no conceptual understanding of conversion of fractions thus lacking the procedural fluency as well.

Thus such examples were interpreted as ignorance of rule restriction. This learner doesn't pay attention at which rule is used when and where it has to be used. This highlights the lack of conceptual understanding the learner has. This lack of conceptual understanding has also made the learner lack procedural fluency. He ended up being confused and start applying every rule he remember in solving questions.

Type of rule: Generalization over operation

The image shows a student's handwritten work on lined paper. On the left, the problem is written as $c) 8\frac{3}{7} - 4\frac{5}{7}$. To the right of this, the number 12 is written. Further right, the same problem $8\frac{3}{7} - 4\frac{5}{7}$ is written inside a rectangular box. Below the boxed problem, the student has written the result $= 4\frac{2}{7}$.

Figure 4.9: Learner response to pre-test question 3 (Case 1)

The student in this example applied the correct operation and rule to get the answer of the whole numbers ($8-4=4$). However when he realised that the next step would them subtract a big number (which is 5) from a small number (which is 3), he flipped the numbers. So instead of saying 3-5 he said 5-3 to get that 2 as an answer. However this commutative rule only applies in addition and not in subtraction.

The image shows a student's handwritten work on lined paper. The problem $8\frac{3}{7} - 4\frac{5}{7}$ is written on the top line. On the line below, the student has written the result $= 12\frac{8}{7}$, indicating that addition was performed instead of subtraction.

Figure 4.10: Learner response to pre-test Q3 (Case 2)

In case 2 the learner added the fractions instead of subtracting. He completely ignored the subtraction sign and decided to add instead. It is not clear whether this is an honest mistake or the learner struggled to subtract and ended up resorting to doing addition.

Type of rule: Generalization over number

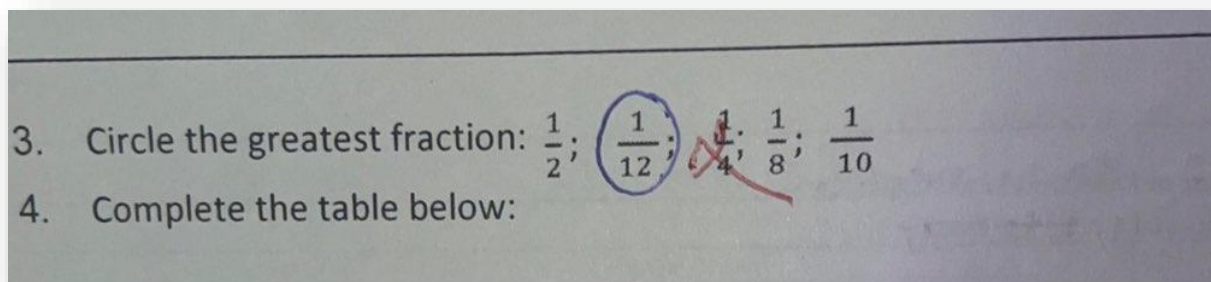


Figure 4.11: Learner response to pre-test Question 2 (Case 2)

In this example the learner lacks conceptual understanding of fraction concepts. The learner has an assumption that the bigger the denominator, the bigger fraction. This is however a whole number rule and does not apply in fractions. It can thus be concluded that the number was generalized.

Type of error: incomplete application of rule

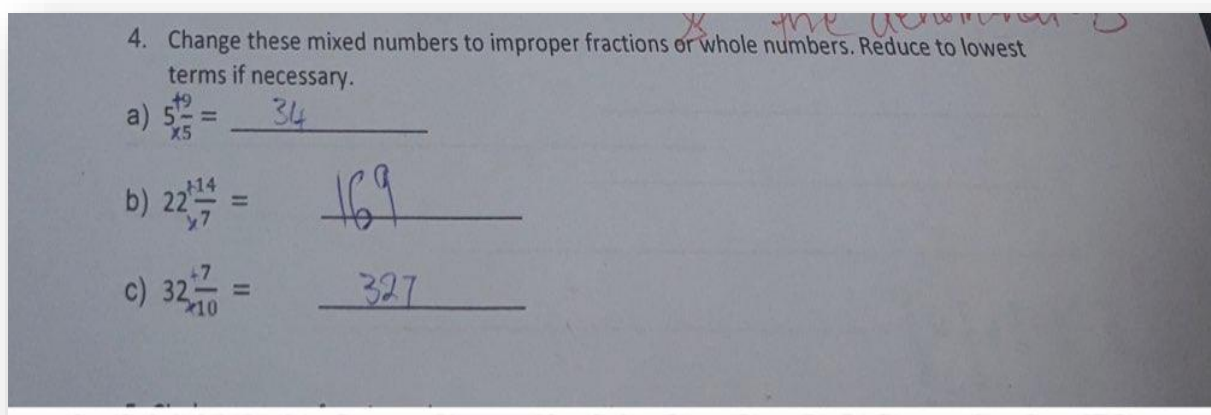


Figure 4.12: Learner response to pre-test question 3 (Case 1)

This example shows that the student has correctly converted the numerator but has completely ignored the denominator in their answering. It seems that her

misconception/assumption is that the denominator is no longer supposed to be included in the answer since it is not changed in the process of converting the fraction. The numerators are however all correctly calculated. This shows that the learner has conceptual understanding of the converting of fractions but it is not complete.

4.3 Analysis of lesson intervention

It is important to address misconceptions, because misconceptions are the root of errors. Therefore as mentioned earlier in the methodology, after identifying and analysing the learner's errors, the investigator arranged an intervention lesson to address the misconception that were identified while marking and interviewing some of the selected candidates. These selected scripts' errors were the ones that are shown, analysed, and discussed. An interview with a sample of the six students was also set up and conducted in order to gain a better understanding of their mathematical reasoning. Following the interview, notes were used to supplement the analysis of the inaccuracies and misconceptions discovered during the pre-test.

During the intervention the researcher noticed that some learners still display misconceptions and errors so I developed the following schematic approach to teach deal with them:

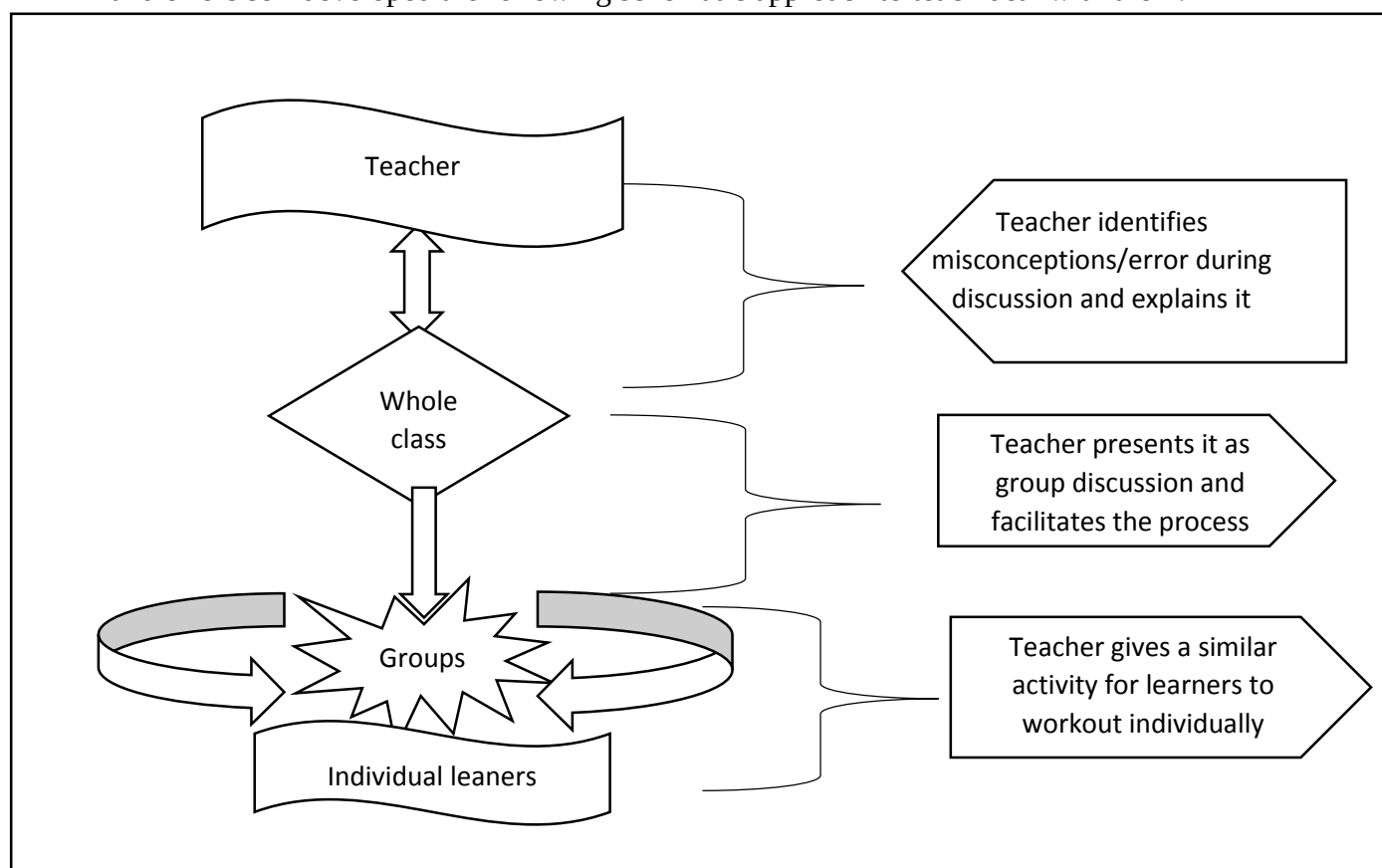


Figure 4.13: Schematic approach

Thus it was easy to work them out and solve them in a way that they would not build up to the next lessons to come.

Lesson 1: Describing and ordering fractions

The objective of this lesson was to teach learners how to compare and order common fractions in order for them to gain clarity that some fractions are equivalent and thus can become a simplified answer to the relevant fractions. This was part of the misconceptions most learners displayed in the pre-test task one. To begin with, the researcher had to teach the concepts of fractions again, this included explaining to students what a numerator and denominator is. The researcher had a few activities whereby learners were cutting whole objects into fractions and they were able to identify the numerator and denominator from that. The researcher proceeded the lesson by giving learners fraction strips and fraction walls and asked them to do a simple activity of arranging the fractions in ascending and descending order.

Activity 1

Use the fraction strip or wall to answer the following questions:

- a) Which is more $\frac{4}{5}$ or $\frac{6}{7}$?
- b) Which is more $\frac{72}{100}$ or $\frac{5}{10}$?
- c) Which is more $\frac{13}{14}$ or $\frac{1}{2}$?

Using the answers given, we then discussed numerators and demonstrated with students. This further emphasised to them that a fraction is one value and not separate values. Thus students could now easily identify that the numerator and denominator are a single value. This is because the fraction strips could easily be moved around so as to make comparing of the fractions more visible and easier. Thereafter the researcher gave them a few more activities to enhance their conceptual understanding. In this section the researcher used hermeneutic listening while probing the students for their understanding.

While they were still working with the fraction strips, the researcher asked learners to identify fractions that are equivalent. These were easy for them to identify as they were just using a ruler to measure the fractions' equivalence down the fraction strips or fraction wall.

In this activity the researcher included some of the questions that were in the pre-test task one. This helped learners to identify how those fractions could be simplified. They could also recognise that the modelling of the fraction did not matter, only the size of the fraction mattered.

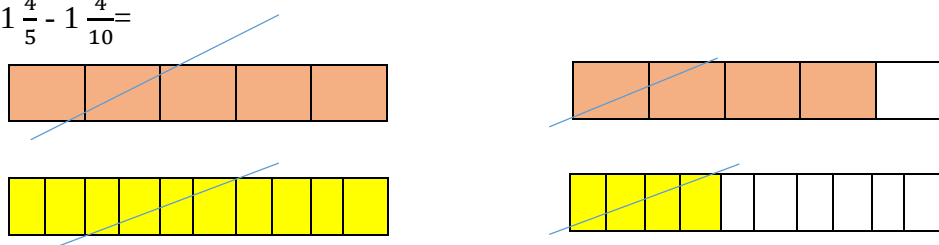
Students were reminded constantly that to make the fractions equivalent, the numerator and denominator must be multiplied by the LCM. This knowledge was important in order for them to understand that when they calculated mixed fractions the denominators need to be the same and how to do this. The first lesson ended and they had a class activity to complete.

Lesson 2: Calculating fractions

Now that they were equipped with skills to describe fractions and order them, the next lesson required a step further. Here the aim was to introduce addition and subtractions of mixed numbers. They had already mastered that adding and subtracting common fractions in which one denominator is a multiple of another in the previous lesson. The researcher then gave them a subtraction activity mixed numbers, however this time the researcher introduced it with fraction strips contained in activity 2).

Activity 2

$$1.1) 1\frac{4}{5} - 1\frac{4}{10} =$$



Instead of giving the steps for making the denominators the same, the above activity shows learners how to make them equal. By using visuals instead of just stating procedures the activity emphasised conceptual understanding of equivalent fractions. This allowed an opportunity to emphasis meaning so that when I introduce the numbers only instead of the strip they had an understanding of what was needed from them.

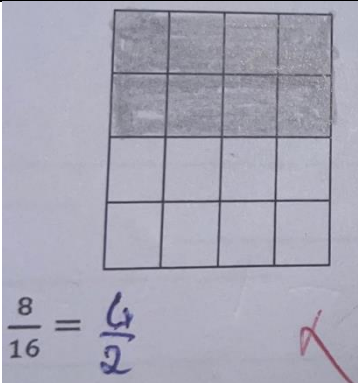
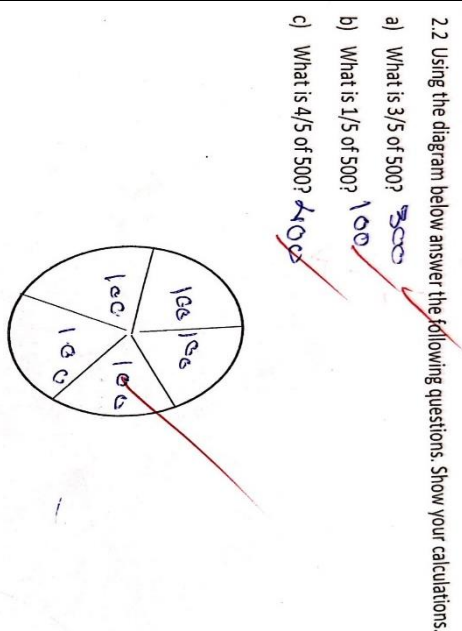
4.4 Analysis of the post-test in comparison with the pre-test learner responses

In this section the researcher compare the learner's performance in the post-test with the pre-test. This is to check if there are any improvement after the learners received an intervention lesson based on the errors they shown in the pre-test.

Task 1

Both the pre and post-test first tasks were measuring the learners' conceptual knowledge of fractions. In the pre-test learners proved to have a bit of this knowledge however with only 55% of them getting all the answers correct a detailed interpretation had to be done on where the misconceptions were arising from. With the identified errors, a section was included in the intervention lesson to address those errors. In the teaching of the concepts, the researcher had to start fractions from scratch with a bit of revision from grades 5 and 6. This was done so that they can have the basic level of answering the other questions thereof that required the conceptual understanding of fractions. The results of the post-test proved that the intervention was successful with 26% improvement from the pre-test. As mentioned earlier, the key to this study was to check if the researcher ability to notice the learner's mathematical thinking would help to improve the researchers' teaching of fractions and in turn improving learner performance.

Table 4.4: Learner' task 1 average performance

	Task 1 pre-test	Task 1 post-test
Percentage of correct responses	55%	81%
Sample -of work done by one individual	 Handwritten work for pre-test: A 4x4 grid with 8 squares shaded. Below the grid, the calculation $\frac{8}{16} = \frac{4}{2}$ is written. A red checkmark is next to it.	 Handwritten work for post-test: A circle divided into 5 equal sectors, each labeled $\frac{1}{5}$. To the right, three questions are listed with answers: a) What is $\frac{3}{5}$ of 500? 300; b) What is $\frac{1}{5}$ of 500? 100; c) What is $\frac{4}{5}$ of 500? 400. A red arrow points from the circle to the answers.

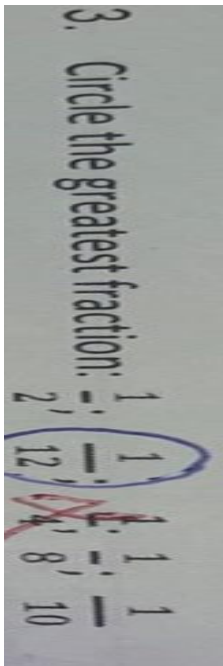
In the table 4.5 it is evident that the number of errors has substantially decreased from the pre-test to the post-test. In the example given, this particular learner has discovered that a fraction is a single value and not separate. He have also learnt the meaning of numerators and denominators, thus addressing the first conceptual errors that learners has in the pre-test task.

Task 2

This section was designed to measure the learner's application level of Blooms taxonomy. With the intervention lesson offered from the errors observed in the pre-test, it is evident that there was a cognitive shift in that leaners post-test responses proved a better understanding of the question, hence the correct solution with the correct representations of the fractions. Generalisation over numbers and ignorance of rule restrictions were errors noted and

discussed from the pre-test. However, in the post-test learners seemed to be vigilant of operations and wary of numbers. Which implies that learners got some form of classification of their errors they had. However it is important to note that the students' errors might be limited for only that period of time.

Table 4.5: Learner' task 2 average performance

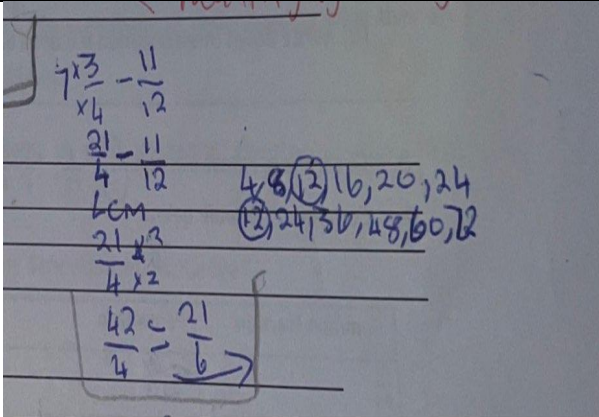
	Task 2 pre-test	Task 2 post-test
Percentage of correct responses	40%	61%
Sample of work done by one individual		1. Rewrite these fractions in an ascending order a) $13/14$; $1/12$; $24/28$; $4/7$ $\frac{1}{12}$ $\frac{4}{28}$ $\frac{13}{28}$ $\frac{14}{14}$ b) $26/28$; $14/28$; $25/28$; $16/28$ $\frac{26}{28}$ $\frac{14}{28}$ $\frac{25}{28}$ $\frac{16}{28}$ c) $1/2$; $4/7$; $26/28$; $13/14$ $\frac{1}{2}$ $\frac{4}{7}$ $\frac{26}{28}$ $\frac{13}{14}$

Task 3

As mentioned earlier, this task consisted of the understanding, application and analysis blooms taxonomy levels. They were intended to measure the learners' ability to grasp meaning of learned material, use learned material in new and concrete situations and Identification of parts of material, analysis of their relationship and the organizational structure of the material (Anderson & Krathwohl, 2001).

The learners did extremely well in the post-test questions that were similar to the pre-test. This proved that even though there are some errors that were different and sometimes difficult to trace back, it is still feasible to teach learners to understand. Remedial teaching addressed the errors and offered easier solutions and methods.

Table 4.6: Learner' task 3 average performance

	Task 3 pre-test	Task 3 post-test
Percentage of correct responses	30%	46,6 %
Sample of work done by one individual	 Handwritten pre-test work showing calculations for LCM of 4, 6, 8, 12 and a fraction subtraction problem.	c) $7\frac{3}{8} + 2\frac{3}{4} - 1\frac{1}{2}$ $7\frac{3}{8} + 2\frac{3}{4} - 1\frac{1}{2}$ ✓ $8 + 4 + \frac{11}{4} - 1\frac{1}{2}$ ✓ $\frac{59}{8} + \frac{11 \times 2}{4 \times 2} - \frac{2 \times 1}{2 \times 4}$ ✓ $\frac{59}{8} + \frac{22}{8} - \frac{5}{8}$ ✓ $\frac{76}{8} = 9\frac{1}{2}$ ✓

4.5 DISCUSSION

This section includes the discussion of results derived from this chapter aligning it to the objectives of the study. The results is then compared to the initial conceptual framework that was drawn in the literature.

The study begins by citing Goldman saying that if we fail to notice we limit our thoughts and deeds. As such, when we fail to notice we fail to improve. This notion is further supported by numerous other researchers such as Nesher (1987) who argue that learners' incorrect answers should be used or extended as opportunity to learn. Thomas et. al., (2017) also maintains that using noticing skills positively impacted teachers' instructional practices. Furthermore Jacobs et. al. (2010) concludes that professional noting should happen in three interlinked stages, attending, interpreting and deciding.

Development of professional noticing skills

Although this study supports this act, it also maintains that there still is a lot of work that needs to be done to develop teachers to notice. Wallach & Even (2005), state that teachers who are well developed are most likely to notice learners' actions and act upon those actions. This means that development plays a major role in the teachers' practices and their effectiveness. For this reason this study did not only focus on identifying the errors and misconceptions learners had, but also on the teachers' capability to resource those errors in teaching for understanding.

Moreover, the collected results by this study concurs with Jacobs et al., (2010) in saying that it is imperative to understand that learners have a unique way of constructing meaning. This was particularly evident in the teachers' interpretation of the learners' work in figure 2, where the learners' approach to solving the question was different from how the teacher had anticipated it being answered. Although the answer was still incorrect, however because the strategy used by the learner was unexpected by the teacher, it became hard for them to notice it. This suggests that while something as simple as noticing can be done swiftly, it can also take a lot of time and effort to do at a robust level. Therefore calling for a need of teacher's professional development of noticing skills, listening and hearing skills so they can work with learners' thinking in a meaningful way. This will further allow teachers to take advantage of

these anomalies and uncover what learners really know and how they have constructed such knowledge (Borasi, 1994). This supports Jacobs et. al. (2010) in their series of studies that found that teachers can improve their actions to students' conceptions and how they reason. Moving from mere reporting and generation to evaluating children's comments. Thus teachers need knowledge, experience and development. These three combined will lead to the effective use of component skills of professional noticing. Studies by (Thomas et al. 2017), (Davis, 1997) and (Wallach & Even, 2005) also indicate the importance of teacher development programs, these programs will allow teachers to acquire and develop noticing skills, listening skills and hearing skills respectively.

Constructivism, analysis of learners' errors.

Despite the observable improvement of learners' performances after the intervention lesson, it is worth noticing that not all learners' marks improved in the post test. Although many of them displayed a form of understanding, there were some cases where learners' misconceptions remained or developed into new errors. This proves that the idea of errors and misconceptions are closely related to the idea of constructivism (Nesher, 1987). This suggests that if a learner is unable to assimilate and/or accommodate new information, it creates a learning gap which leads to errors and misconceptions (Sawadi and Shahrill, 2014). Radatz cited in Sarwardi & Shahrill, (2014) states that systematic errors are errors that are usually a consequence of learners' misconceptions connecting new information with previously incorrectly understood knowledge. These errors are a result of teaching procedural fluency instead of conceptual knowledge. This was mostly noticed in this study to be the main root of errors that were observed in learners. As such it is the teachers' responsibility to mathematically analyse on various levels to improve reflecting and discussion among learners.

Borasi (1994) states that although teachers and researchers understand the value in analysing learners' errors, learners should be given an opportunity to know how to use errors as a means to learn mathematics. This suggests that it was not enough for the teacher to just notice the errors but they should be means of engaging learners in the discovery of their misconceptions. Although this might seem rather difficult, in the proceeding of this study a systematic approach was used to attempt doing this. This also proved successful even though a lot of intervention was needed by the teacher to address arising errors and misconceptions.

As stated in the methodology chapter, this study was divided into 3 phases.

Pre-paring to notice

Smith, diSessa and Roschelle (1993) argue that “misconceptions are normal part of learning, so educators must expect them as the difficulties of understanding mathematics”. Therefore, in the first phase of the study the investigator began by giving out a pre-test on fractions to learners. This was done so that the investigator could identify the existing errors and misconceptions that learners have on the topic. Thus enabling the investigator an opportunity to get something to attend to, and interpret so that the investigator could contrive strategies for lessons accordingly towards remediating the discoveries. This is the three stage process of professional noticing of learner mathematical thinking (Jacobs et al., 2010). Mason (2002,2011) referred to this practice as “a collection of techniques for (a) pre-paring to notice in the moment and (b) post-paring by reflecting on the recent past to select what you want to notice” (2011,p.37).

Errors and misconceptions made by grade 7 learners when working with fractions.

In response to the first objective of the study, which seeks to determine the misconceptions and errors that grade 7 learners have on the topic of fractions. A written test was used to collect this data and a follow up interview was conducted to get the exact interpretation of the misconceptions the learner has. The analysis above indicate that learners have a number of existing errors that have emerged from their misconceptions, these errors are common with the ones that were identified in table 3, chapter 3. They also resonate with the findings observed in section 2.2.3 about the reasons learners struggle with fractions in mathematics. Thus it is safe to say that the errors and misconceptions observed in this study are not only random but can be linked with previously learnt content and misinterpretations.

The following errors where observed:

Table 4.8: Type of errors and their description

Type of error	Description
Ignorance of rules	Applying a rule where it does not apply (misapplication)
Conceptual error	Lack of conceptual understanding (Existing schema in conflict with new schema)
Generalization over operation	Disregarding the different properties of different operations
Incomplete application of rule	Ability to apply a rule but failure to proceed
Random	Non-systematic (unintended mistakes)

(Olivier, 1989; Luneta & Makonye, 2010)

The discoveries of these errors was further emphasised by using interviews and probing techniques. In that stage hermeneutic listening was used in order to illuminate bias hearing and over hearing.

Interpreting and deciding

Jacobs et al. (2010) argues that the interest of professional noticing is in the extent to which teachers use what they have learned about their children's understandings from the specific situation and whether their reasoning is consistent with the research on children's mathematical development. This is also emphasised by (Borasi, 1994; Brodie, 2007 & Olivier, 1989) who sustain that instead of attempting to eradicate misconceptions, a teacher needs to incorporate in their teaching, strategies which enable them to become aware of learners' errors and misconceptions. Thus in the second phase of the study the investigator conducted an intervention lesson with the learners based on the misconceptions that were identified. This is the final stage of the components of professional noticing which entails deciding to a productive instrumental course of action based on the interpretation Jacob et al. (2010). This stage sought to address and remediate the identified errors and misconceptions through an intervention lesson.

The teachers' interpretation of the errors and misconceptions the learners had.

This question focused on determining the strategy the investigator had employed to notice the learners' errors and misconceptions. The first approach was using a pre-test to identify the learners' relational understanding of fractions. This was done to classify the errors and misconceptions the learners displayed, which according to Jacobs et al. (2010) is the first step of noticing. Despite the widespread of errors the investigator had identified, the interpretation process proved that sometimes what the investigator had thought the investigator noticed was totally different from what the student meant. This was why an interview was used in order to probe the students to get their true understanding of where their misconception lies. This formed part of the interpretation stage of noticing. The errors and misconceptions are those that the investigator have mentioned above. Thereafter those identified and interpreted misconceptions and errors were used to draw up a lesson to remediate in order to prevent them from possibly reoccurring in future. This was the final step to noticing which entailed deciding. It meant responding using a productive instrumental course of action based on the interpretation. Thus an intervention lesson was conducted. This lesson was addressing the errors and misconceptions the investigator had identified. During this stage the investigator had applied interpretative and hermeneutic listening skills to respond in action to the newly noticed and arising misconceptions the investigator applied a schematic approach (mentioned above) wherein the investigator was conversing with learners for justification of their thinking.

The process of noticing is very complex and cannot be classified and boxed. There are various dynamics that need to be considered, like the learners' uniqueness, diversity and past learning experiences. These contribute to how they participate in their learning space. For instance some learners' were willing to voice out their thoughts, while others were not as comfortable. While other learners were willing to freely participate in the discoveries of the reasons behind their misconceptions as a process of learning, others feared that their answers were incorrect and thus talking about them was considered as exposing them. Such learners might have had experiences of teachers that employed the absolutistic view. Those teachers view mathematics as having a life of its own, waiting to be discovered, as such they did not consider incorrect

answers as an opportunity to learn. Therefore trying to get these learners to reason out their thinking became difficult.

Post-paring

In the last and final part of the study learners wrote a post-test. This was done in order to determine the extent in which the intervention process had impact in improving learner's understanding of the topic from the post-test I was able to compare both performances of the pre and post-test. This gave me an overview of the extent in which noticing was able to assist in improving understanding of fractions in these grade 7 learners. This part informs the last objective and directs the findings of the study.

The extent in which the teachers' response to what they have noticed has improved learners' performance in fraction.

The focus of this objective was to determine if noticing had an impact in improving learners understanding and remediation of errors and misconceptions they had on fractions. Thus a post test was used after the intervention lesson was completed. This is because creating an enabling environment does not necessary guarantee that students will learn from it. Therefore conducting a post-test was necessary to draw comparison from pre-test and a post test in order to determine whether noticing was successful or not in improving the learners' misinterpretations in their understanding of a topic or mathematical method used. The following was observed:

Percentages of learners who responded correctly to test questions

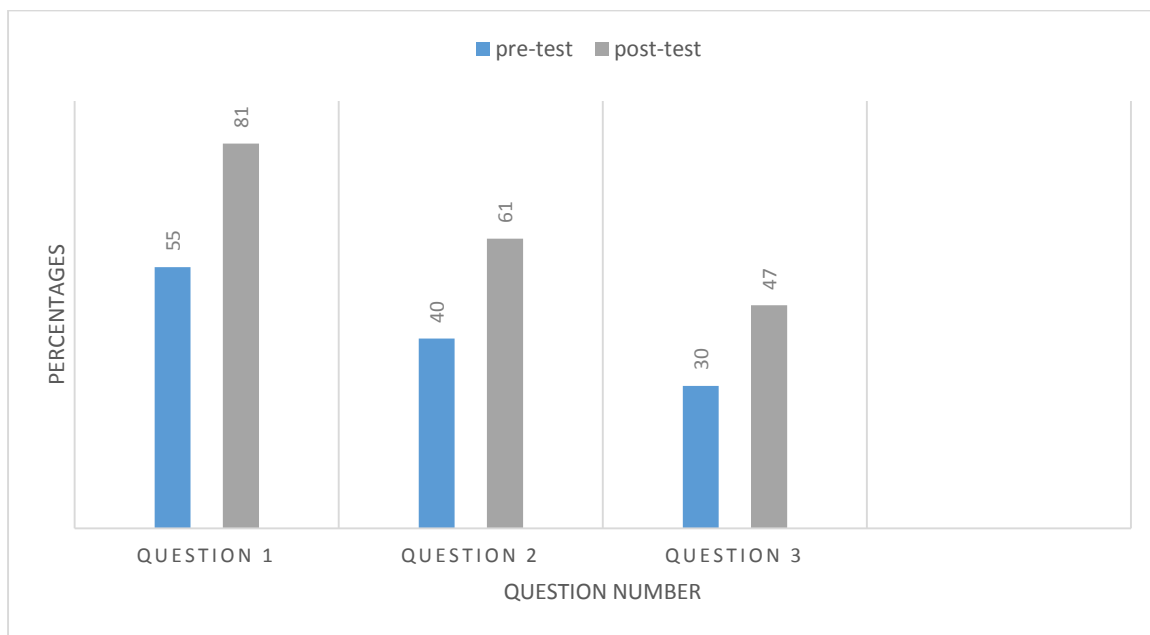


Figure 4.14: Comparison of learner's correct responses

In this case the use of intervention lesson based on the interpreted errors was successful, this is evident in figure 4.14 where learner's performance improved dramatically in the post-test when compared to the pre-test performances. More importantly was the percentage gains difference from question to question.

Conclusion

In summary it can be observed that resourcing of the learners' thinking was very effective in improving the learners' performance. As Smith et al. (1994) have pointed out that more needed to be done with errors and misconceptions identified to build on learners' conceptions. The research shows that teaching intervention focused on identifying errors not only improves learners' performance but also serves as a resource for improving performance of the subject. The main objective of this study was to check if identifying misconceptions or misinterpretations in a learner's understanding of a topic or mathematical method used can boost the teaching of the topic of fractions, and to what extent it can improve learning. The aim was to investigate if learners improve their understanding of fractions when a teacher attends to their misconceptions. The study was based on the idea that there would be arising errors from the pre-test responses which may be linked to misconceptions originating from previous knowledge. However the challenge was with interpreting the learners meaning, was that there

were instances where the responses were too ambiguous and sometimes not easy to classify. This challenge was addressed with the interview section as I was able to get clarity on how exactly the learner went about getting their answer.

The errors and misconceptions that were displayed by learners were almost similar, while others were totally different. So when planning for the intervention lesson I had to address all of the observed errors and when they would display other errors, a plan was put into place to address those. This included group discussion coupled with teacher intervention. The outcome of the post-test however did provide evidence of an improvement in learner's performance results. This supports that directing an intervention towards the identified errors has enabled learners to construct new knowledge. Hatano (1996) argue that "construction of knowledge is from individuals' experiences of solving problems from those they (individuals) create rather than those they are imposed with". As such students were able to create new experiences in the remedial class and could join the new knowledge to their previously developed schemas. Noteworthy that it is not always easy to correlate new ideas with existing schema due to the diversity of what has to be learned. Thus not every learner was able to improve.

Hence it was important for the investigator to probe every learner's response during the intervention process so that they get aware of what they were saying and why it is a misconception. The group discussions were also a strong strategy to remediate this because as learners were discussing, they got a chance to challenge each other with questions based on their responses. That could have easily drawn away from the main lesson, but the investigator was able to direct the probing to a point that the learners were able to interact and attach meaning to the concept.

Overall when noticing you get to know your learners' strengths and weaknesses. A lot of the time learners' move from one class to another and often the teacher faces the challenge of not knowing what knowledge and misconceptions they bring along with them. Even though they get to conduct a baseline assessment, these are not 100% effective as they are done once off and for only a limited time. In most cases they test for limited content. Thus a teacher only gets a glimpse of what already exists in the learner's schemas. The noticing activity can help not only with informing the current teacher but the information gained can be shared with the previous grade teacher to expand their attention on what misconceptions arise in their level.

Chapter 5

5. Findings, conclusion, recommendations, limitation and delimitations

5.1 Introduction

This chapter presents the discussion of findings derived in the previous chapter aligning it to the results which was researching the teachers' resourcing of learners' mathematical thinking in teaching grade 7 fractions. The findings were then compared to the initial conceptual framework that was drawn in the literature to formulate a conclusion. It also provides an overview of the answers for the research questions. Thereafter provides recommendations that can be pursued by mathematics teachers as well as policy developers of the subject.

As stated previously, the study was conducted at a primary school in Johannesburg central district as an action research to develop the researcher in the way mathematics is taught to grade seven learners. It employed qualitative research and utilised experimental method to draw data from a pre-test and a post-test used to compare the learners' understanding of fractions before and after noticing intervention lesson. Data was collected from 60 grade seven learners that were in the third term of their grade, the participants had previously been taught fractions before in the other terms and were currently doing other topics. The theoretical framework that informed the study was from Jacobs et al. (2010) conception that noticing can be classified into three components, attending, interpreting and deciding.

5.2. Findings

These findings are summarised in accordance with the research questions in chapter 1. The purpose of this study was to determine whether using professional noticing to identify learner's existing errors and misconceptions could improve their understanding of fractions. The objective was to explore the extent in which the teachers' response to what they have noticed has improved further learning of fractions in grade 7. The present study used data collected from learners' pre-test and post-test to compare the learner's performance before and after remedial teaching based on identified errors.

Three key findings emerged from this study based on the objections set prior. First being the errors and misconceptions that grade 7 learners make in fractions based on their lack of conceptual understanding. Secondly, what the teacher notices could be interpreted differently from what the learner was thinking, as a result of the learners' mathematical thinking being diverse. Such that different learners were observed to having different misconceptions from a similar question. Lastly the study has proven that professional noticing can improve the learners conceptual understanding of fractions, as their overall performance in the post-test has improved.

The study's objectives was to answer the following questions:

- **Which errors and misconceptions do grade 7 learners make when working with fractions?**

5.2.1 Errors and misconceptions identified

According to Jacobs et al. (2010, p.37) the first component of professional noticing refers to "the capacity of the teacher to attend to the students' mathematical conceptions and practices as they occur". This is why the study's first aim was to identify the errors and misconceptions that grade 7 learners have in fractions in order to resource those in the teaching of mathematics. This is because if teachers fail to address pupils' misconceptions in mathematics, they will only be accounting for failure to promote students' conceptual understanding. Nesher (1987) argued that learners' incorrect answers should be used or extended as opportunity to learn. Thus the learners' pre-test responses coupled with the interviews, were used in this study to best represent their thinking in order to identify their existing misconception for remedial purposes. The following 5 misconceptions and errors were thus identified from the pre-test and interviews:

➤ Conceptual error

According to Irham (2019) conceptual errors appear in the inability of students to describe the relationship between variables and cause errors in solving mathematical problems. This type of error was mostly observed in the study's first task, which required learners to have a conceptual understanding of fractions. This type of error as stated in the literature is caused by the lack of conceptual understanding of the work being assessed. This is caused by existing schemas being in conflict with the new schema. It is particularly evident in an event

where a student was taught the procedures but did not develop an understanding behind those procedures. Such a student will most of the time confuse the procedures and or make random errors. The students who had this type of error in study displayed a misconception of viewing a fraction as separate values instead of a single digit. This misconception lead to learners' incorrect simplification of fractions. Furthermore, the learners did not see any relationship between the numerator and denominator, thus they struggled with the second part of the question which required them to simplify the fraction to its lowest form. Therefore, when simplifying the fraction, they ended up just focusing on either the numerator or denominator and forgetting to implement the other in their simplification.

An example from the study is observed in figure 4 where a student simplified the fraction $\frac{8}{16}$ as $\frac{4}{12}$, the student thought that the numerator is 4 and the denominator is 12. Thus emphasising that he does not understand that one falls within the other such that $\frac{8}{16}$ means that 8 part of 16 equal parts is being considered.

This discovery by the study's data emphasises literature that state that students' conceptual errors are as a result of lack of conceptual understanding, thus influencing the procedural fluency of learners.

➤ Ignorance of rule restriction and Generalisation over operation

Olivier (1989) regards these as the deep level procedure from the two levels which guide cognitive functioning. These are type of errors that result from memorization of rules and procedures and not having a conceptual understanding of where and when they must be applied. As such the student doesn't form any schema around those rules taught, so they end up applying them where they are not applicable. This shows that the learner has previously focused on procedural knowledge and less on conceptual knowledge. Thus students fail to understand the bounds where a rule applies so they generalise it everywhere. Such errors were observed in this study when pupils were seen to be applying commutative property even in subtraction questions. They would swap around numbers if they could not subtract the bigger number from the smaller number.

An observed example in the study is in figure 4.10 and 4.11 when published the students used addition instead of subtraction, thus ignoring and generalizing the rules of addition. This was

mostly observed in cases where a learner had to convert a mixed number into an improper fraction so that they can be able to add them. However, because the students did not have the conceptual understanding of how to convert fractions to improper fractions, he would just resort to just adding whatever numbers they had and would in the process end up disregarding the properties of operations.

An example is when a student had to calculate $8\frac{3}{7} - 4\frac{5}{7}$. The student applied the correct operation and rule to get 4 ($8-4=4$) however when he realised that the next step they would have to subtract a big number from a small number, he flipped the numerators around. So instead of saying 3-5 he said 5-3 to get that 2 as an answer. This was incorrect to do in two ways, one that the rule of subtraction does not allow numbers to be flipped or swapped around and two, the student was supposed to have converted the mixed number to an improper fraction to avoid getting this confusion. so by ignoring that rule he later got stuck and chose to generalize the rule of addition in subtraction.

Applying a rule where it does not apply (misapplication). Examples of such errors is the inappropriate use of cancelling, learners like to cancel out numbers in fractions without the conceptual understanding of why that is done.

Another student was observed calculating $7\frac{3}{4} - \frac{11}{12}$, however the learner flipped the numerator and denominator around and changed the fraction to $\frac{4}{3} - \frac{12}{11}$. Then dropped out the whole number (7) in his working out of the answer. It appears as if the learner wanted to apply the reciprocal relationship rule, which is used when dividing fractions. This shows that the learner has no conceptual understanding of this procedural rule.

The learner still could not subtract 12 from 4, so he tried to look for the LCM which was also incorrectly calculated. The learner doesn't pay attention at which rule is used and where it has to be used. This highlights the lack of conceptual understanding the learner has. This lack of conceptual understanding has also made the learner lack procedural fluency. He seems confused and is applying each and every rule they remember in solving all questions.

➤ Incomplete application of rule

According to Olivier (1989) this is the ability to apply a rule but failure to proceed with it. An observed example in the study was when students were supposed to convert mixed numbers

into improper fractions. The student would correctly convert the numerator but completely ignore the denominator when converting mixed number into an improper fraction. The misconception here is that the student thinks that the denominator is no longer supposed to be considered since it was not changed in the process of converting the fraction. This type of error can be associated with random errors, whereby the learner has an understanding of what has to be done but would forget a step which messes up the answer. Or it could be as a result of not completely being equipped with conceptual knowledge, thus lacking the procedural understanding. Although there were only two observed cases throughout the study, they were informative in the study as they were addressed in the intervention process to avoid reoccurrences of them again.

➤ Random errors

According to Luneta & Makonye (2010) these are non-systematic or unintended mistakes. These errors could not be classified under any category. Even the learners themselves were not able to justify their actions when interviewed. This could be caused by lack of concentration to what is done or just general mistakes like forgetting to write a number in full and later working with the wrong number. This was particularly observed when a student forgot to write 31 and wrote 3 and ended up using 3 in the next step which lead to the correct procedure but the answer being incorrect

However these were also very informative in the study because they were highlighted in the intervention process in order for learners to be aware of them and also avoid doing them again. There were quite a lot of these errors observed in this study and this could be linked with the fact that they are spread across all the previously mentioned errors.

- **How did the teacher interpret the errors and misconceptions the learners had?**

5.2.2 Different interpretation of learners thinking

The next objective of the study pursued to identifying how a teacher was able to figure out the learners thinking. Equally important to attending to learners' errors is interpretation. "Professional noticing of children's mathematical thinking requires not only attending to children's strategies but also interpretation of the mathematical understanding reflected in those strategies" Jacobs et al. (2010, p.37). However this part presented a bit of a challenge. This is because despite the widespread of errors the investigator identified, the interview process sometimes proved that what the investigator thought and noticed was totally different from the students' strategies. Thus this process became a bit time consuming. Because there is no single way of doing mathematics, some errors became too ambiguous and could only be clarified by interviewing the student to get the exact mathematical thinking that was enforced in the strategy used.

This was not the only discovery the investigator made on this part of the component of the noticing. Another interesting revelation was that a number of learners would struggle with the same question but had different misconception and would make different errors from one another. This meant that the interpretation even difficult to do. This discovery can be associated with the learners' diverse learning styles and the fact that learners come to the learning situation with existing ideas about many phenomena. Thus their misconceptions are rooted differently according to their past learning experiences and which schemas they have developed.

- **To what extent did the teachers' response to what they have noticed improve the learners' performance in fractions?**

5.2.3 Summary of results

The focus of this study was to determine if noticing had an impact in improving learners understanding by remediation errors and misconceptions they had on fractions. This was determined by comparing the pre and post-test performance results. Thus a post test was used after an intervention lesson was completed based on errors and misconceptions identified from the pre-test.

It is evident from the graph on the previous chapter that professional noticing was successful in improving the learners' performance. This is observed in the performance improvement from one task to the next, with the first question having the most performance improvement of 26%. Task 2 having substantially gained 21% and task 3 improving by 16.6%. In light of these findings, it is safe to say that noticing the learner's errors and using it to reteach fractions improved the learners' understanding of them. This is mostly observable in cases that required conceptual understanding, nonetheless there was some noticeable improvement in procedural fluency of learners' as well. The investigator was however convinced that with more time and more practice activities for the learners, there could be even more improvement.

Hatano (1996) argue that construction of knowledge is from individuals' experiences of solving problems from those they create rather than those they are imposed with. Teaching fractions to the learners while highlighting their errors allowed them to reflect more on their thinking. This was one way in which the remedial teaching made the concept of learning concept available to learners (Watson, 2007). However it is also important to acknowledge that there were those areas where there was little or no improvement in the post-test results. These could be linked with the limited timeframe in which the study was concluded. The learners' experiences and existing schemas needed a little more intervention time to fit the new knowledge with the existing schemas.

5.3 Conclusion

The discussion made in this study contributes to the growing body of literature on professional noticing of students' mathematical knowledge. The study has followed the literature recommendations and theoretical frameworks in place to come to its' conclusion. It is noticeable from the findings of the study that the use of professional noticing was successful in improving the learners' performance on fractions. Although not all the errors and misconceptions noted in the pre-test were remediated, it is noteworthy that there was evidence of professional noticing being impactful in improving the performance in the learners' understanding of fractions in the post-test. The overall performance improvement from the pre-

test to the post test was 63.6%. This proves that the noticing of learners' mathematical thinking had a huge impact in assisting the teacher plan an intervention lesson and on addressing the misconceptions and errors learners make when working with fractions. Needless to say that attending to the learners' pre-tests has highlighted the errors and misconceptions that grade 7 learners have on fractions. These can develop further studies on how to avoid them with the coming grades.

5.4 Recommendations

The results shown that professional noticing was very effective in teaching mathematics, therefore in light of the study the following recommendations are hereby made:

- **Recommendation for the teachers:**

According to Olivier (1989) misconceptions are crucially important to learning and trenching because they form part of learners' conceptual structure that interacts with new concepts and influence learning. Thus being aware of the learners' misconceptions helps you to deal with them and redirect learners to correct procedures. Since the effectiveness of professional noticing has proven to be very successful in this class teachers should incorporate it into their classrooms to maintain quality education for the students and help build rational understanding of the mathematical thinking of their students. Professional noticing implementation is thus recommended to be embraced by all the educators in all the grades in the mathematics subject. Even though this method of teaching can be very time consuming but when implemented correctly it can provide the teacher with detailed thinking of learners. Such that they would know where to focus most and where learners are with their understanding of certain topics. This will help them design suitable tasks that will better learners' conceptual understanding of topics and provide learners with procedural fluency

- **Recommendations for policy makers:**

The study recommend that policy makers draw develop a separate slot for noticing in the curriculum planning. Such that, when designing annual teaching plans (ATP) they should allow teachers some time to conduct their own noticing right before an introduction of a new topic.

Also recommend that there be constant workshops on noticing skills. Teachers should not only be developed on the content they must teach, but also how to notice the learner's mathematical thinking and use it in their teaching of mathematics. . This is because not all teachers are equipped with such skills, despite the number of years the teacher has been teaching mathematics.

- **Recommendation for further research:**

As much as there was improvement on the learners' performance after noticing and intervention was applied, I however recommend that further research be conducted on a broader scale and on other topics and grades. This will provide a better overview of the learners thinking in mathematics.

Equally recommend that the same study be done on teachers that are teaching mathematics but have not measured in mathematics or are not qualified to teach mathematics. This is because a number of teachers are placed into the mathematics subject because of limited qualified mathematics teachers. The interest is whether they have the same ability to notice errors and misconceptions and relate them in their teaching the same way a qualified mathematic educator does. This is because studies by Jacob et al. (2010) suggests that novice teachers are as unskilled as their counter parties in their ability to notice. Thus suggestions for other researcher investigate if there is any difference in the way a mathematics qualified teacher notices as opposed to those that were placed in the subject because of limited numbers of mathematics teachers.

5.5 Limitations of the study

Although the study has proved the effectiveness of professional noticing in improving the learners' performances and mathematical understanding of fractions there were a number of limitations that hindered this study. These are as stated:

Covid-19 restrictions

The existing covid-19 restrictions which prevents visits in public sectors such as schools, this study could not be done on a broader scale. Therefore this study was only conducted in one

school and with only a small sample of 60 learners, thus the findings cannot be generalized as experienced by all grade 7 learners.

Limited timeframe

Another limitation was the time constrain, there was a limited time to conduct this study and it could not be pushed any further than the days the investigator used because it would have distracted the curriculum. This means that with enough time there could be an even detailed response and implementation of remedial teaching. Bywater, Chiu, Hong & Sankaranarayanan 2019. Maintain that effective noticing, interpreting and then responding to students' mathematical ideas can be quite challenging for teachers as they try to balance multiple, competing goals in an authentic classroom setting. This was particularly the case in this study as there wasn't enough time to deal with all the learners' responses. Although identifying misconceptions or misinterpretations in a learner's understanding of a topic can be done any time during a lesson or set of lessons, it was however very time consuming to follow. This is because of the ambiguous responses that needed to be followed up with interviews and probing to be clearly understood. Therefore with the demanding expectations of teachers in preparing for a meaningful lesson, this exercise proved to be putting more work strain on the teacher. Thus to implement it successfully to fit the specific learners being taught and not generalizing it on the common misconceptions experienced by other students, a teacher would have to do the exercise at least once every time before introducing a new topic. This would however present a challenge since the annual teaching plan does not include time for this practice to be done. Thus implementation of noticing can only be mostly effective in the current South African educational settings when done in the moment which requires a lot of training and development for teachers. This because studies by Jacobs et al. maintain that noticing is a skill that cannot be associated with the teachers' experience in the field, but rather acquires development and training of it.

Learners' uniqueness

What makes noticing to be more difficult is the learners' uniqueness. That is because every individual has their own thinking, and their own way of interpreting information. Therefore teachers cannot use every learner's thinking in a class more especially in a country like South

Africa where there are still an overlying issues of overcrowded classes and learner diversity informed by our history. Thus no single teaching pedagogy can be applied to fit all learners' needs. Learners also bring along different and misconceptions and errors with them. These are all difficult to address at once while trying to cover the prescribed syllabus curriculum. Therefore while addressing them individually might seem like a good idea, there wouldn't be enough time to do that. However addressing them in class also confuses some of the learners. For example one particular learner in the study had the procedural understanding of adding mixed numbers in the pre-test, but when the investigator tried to use the fraction strips to emphasise the conceptual understanding, the learner started getting confused in trying to add the new information to their existing schema. Fortunately this was identified early in the class activities and was quickly corrected. Nevertheless, researchers are convinced that there are moments that can be recognized and agreed upon by researchers, teachers, and teacher educators as being opportune for student mathematics learning Leatham, Peterson, Stockero, and Van Zoest (2015). However this limitation does not take away from the fact that noticing has helped most learners as their results after intervention was better than in the pre-test.

Not easy to interpret what the learner was intending.

One reason learning to use student mathematical thinking is so difficult seems to be the complexity of recognizing and interpreting such thinking (Leatham et al. 2015). Some instances required a follow up discussion in order to interpret the reasoning behind the learners' thinking. This is because other misconceptions are ambiguous such that you might interpret it differently from what the learner meant. Therefore students sometimes need to articulate their own mathematical thinking about a problem, which is often not easy in a realistic environment as they are sometimes not even aware that they have an existing misconception. In other cases the student coincidentally guess the answer correctly thus making it difficult to spot their existing errors.

5.6 Delimitations

In spite of the limitations that impacted this study, the results are still credible as they were designed to fit the desired outcomes. Although the study was done on a small group scale, it was however done successfully such that the results can be generalised as experienced by the

other group of learners in a larger scale. The limitation to time were on a realistic timeframe as aligned to the CAPS document, this means that no extra time was assigned to completing the tasks of this study, therefore proving that this can be fitted within the policy of mathematics teaching. The learners' uniqueness and diversity was what made the study to be more realistic in making the results credible. The use of interviews and probing also came in handy in getting the core meaning behind the learners' responses.

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Appendix A (Pre-Test)

Pre-Test
Mathematics grade 7
Fractions
2021

60 marks

1.5 hours

Name of learner: _____

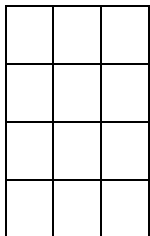
Gender : _____ Grade : _____ Age: _____

Name of subject teacher: _____

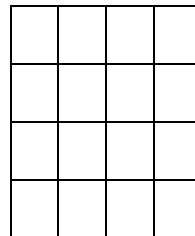
Name of school: _____

Question 1

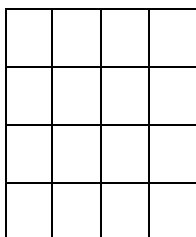
Consider the following figures. Shade in the fraction and give the answer in its simplest form.



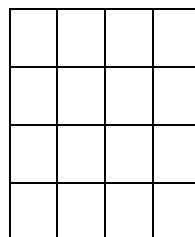
$$\frac{3}{12} = \frac{\quad}{\quad}$$



$$\frac{4}{16} = \frac{\quad}{\quad}$$



$$\frac{8}{16} = \frac{\quad}{\quad}$$



$$\frac{8}{16} = \frac{\quad}{\quad}$$

(12)

Question 2

1. Determine the lowest common multiple (LCM) of the denominators of $\frac{3}{4}$, $\frac{4}{5}$ and $\frac{5}{6}$.

2. Consider the fractions $\frac{5}{12}$ and $\frac{7}{18}$.

(a) Complete the following statement: $\frac{5}{12} = \frac{\quad}{24} = \frac{\quad}{36} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

(b) Complete the following statement: $\frac{7}{18} = \frac{\quad}{36} = \frac{\quad}{54} = \frac{\quad}{\quad} = \frac{\quad}{\quad}$

(c) Write down three common fractions bigger than $\frac{7}{18}$, and smaller than $\frac{5}{12}$.

3. Circle the greatest fraction: $\frac{1}{2}$; $\frac{1}{12}$; $\frac{1}{4}$; $\frac{1}{8}$; $\frac{1}{10}$

4. Complete the table below:

Common Fraction	Percentage	Decimal Fraction
$\frac{5}{10}$		
$\frac{2}{5}$		
$\frac{10}{50}$		

Addition and subtraction of common fractions:

Study the following calculations in each example and summarise your understanding below each example.

Addition:

Example 1: (same denomin

$$\text{(a) } \frac{12}{17} + \frac{5}{17} = \frac{17}{17}$$

$$= 1$$

$$\text{(b) } \frac{5}{11} - \frac{4}{11}$$
$$= \frac{1}{11}$$

$$\frac{7}{8} - \frac{3}{4} = \frac{7}{8} - \frac{3}{4} \times \frac{2}{2}$$
$$= \frac{7}{8} - \frac{6}{8}$$
$$= \frac{1}{8}$$
$$=$$
(18)

Question 3

6. Calculate:

a. $4\frac{1}{3} + \frac{5}{6}$

b. $7\frac{3}{4} - \frac{11}{12}$

c. $8\frac{3}{7} - 4\frac{5}{7}$

7. Which improper fraction is equal to $2\frac{3}{10}$?

$$\frac{33}{10}; \frac{16}{10}; \frac{21}{10}; \frac{23}{10}$$

8. Circle the mixed number that is equivalent to $1\frac{18}{12}$.

$$1\frac{2}{3}; 2\frac{1}{2}; 1\frac{1}{2}; 3\frac{1}{4}$$

9. Change these mixed numbers to improper fractions or whole numbers. Reduce to lowest terms if necessary.

a) $5\frac{9}{5} =$ _____

b) $22\frac{14}{7} =$ _____

c) $32\frac{7}{10} =$ _____

5. Circle a proper fraction whose numerator is a composite number.

$$3\frac{8}{6}; 4\frac{3}{5}; \frac{7}{13}; \frac{12}{13}$$

(30)

Appendix B (Post-Test)

Post-Test
Mathematics grade 7
Fractions
2021

30 marks

1 hours

Name of learner: _____

Gender : _____ Grade : _____ Age: _____

Name of subject teacher: _____

Name of school: _____

Question 1

1. Rewrite these fractions in an ascending order

a. $\frac{13}{14}, \frac{1}{12}, \frac{24}{28}, \frac{4}{7}$

b. $\frac{26}{28}, \frac{14}{28}, \frac{25}{28}, \frac{16}{28}$

c. $\frac{1}{2}, \frac{4}{7}, \frac{26}{28}, \frac{13}{14}$

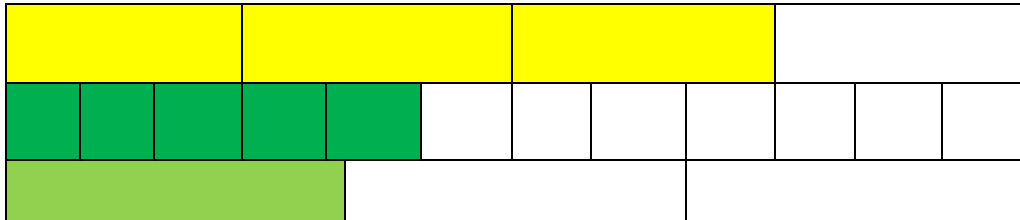
2. Rewrite the following fractions so that they have the same denominator and then rearrange the fractions in an ascending order

a) $\frac{4}{5}, \frac{1}{2}, \frac{3}{4}, \frac{7}{10}, \frac{19}{20}$

b) $\frac{3}{4}, \frac{11}{12}, \frac{5}{6}, \frac{2}{3}, \frac{2}{8}$

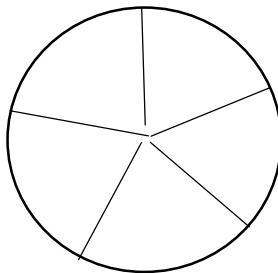
Question 2

2.1 is it true that $\frac{3}{4} = \frac{5}{12} + \frac{1}{3}$



2.2 Using the diagram below answer the following questions. Show your calculations.

- What is $\frac{3}{5}$ of 500?
- What is $\frac{1}{5}$ of 500?
- What is $\frac{4}{5}$ of 500?



2.3 Determine the LCM of the following fractions

- $\frac{2}{3}$, $\frac{1}{6}$ and $\frac{5}{12}$
- $\frac{4}{8}$ and $\frac{3}{6}$

Question 3

3.1 calculate the following. Show all your calculations.

a) $10\frac{1}{3} - 2\frac{5}{6}$

c) $7\frac{3}{8} + 2\frac{3}{4} - \frac{1}{2}$

B $3\frac{7}{12} + 4\frac{5}{6} - \frac{1}{6}$

d) $5\frac{1}{4} = 2\frac{1}{2} - \frac{7}{8}$

Good luck

Appendix C: interview schedule

Interview schedule

Name of interviewee: _____

Name of interviewer: MASONDO T

Date: _____

Duration: 1 HOUR

Tell me what you did here	
I see you have started to add these numbers can you tell me why?	
Can you tell me more about...	
Why did you..?	
How did you solve it?	
How does your picture connect to your equation?	
I'm not sure clear on what you have done here, will you explain it so I can understand?	

2. Rate your understanding of the topic before the intervention lesson?

Very poor (1)	Poor (2)	Moderate (3)	Good (4)	Very good (5)

3. Rate your understanding of the topic after the intervention lesson?

Very poor (1)	Poor (2)	Moderate (3)	Good (4)	Very good (5)

3 From the pre-test, what were your most favourite questions?

Why?

4 From the post-test, which were your most favourite questions?

Why?

5 Rate yourself on the scale of 1 to 5 on your understanding of fractions before the intervention lesson and after intervention. 1 being poor and 5 being very good.

Question	Before intervention	After intervention
Question 1		
Question 2		
Question 3		

ANNEXURE A: LEARNER ASSENT FORM



LEARNER ASSENT FORM

Please fill in the reply slip below if you agree to participate in my study called: **TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS.**

Researcher's name: THANDOLWAKHE MASONDO

I _____ the parent/ guardian of _____ in grade 7 give consent for my child to participate in this research project. This research has been explained to me and I understand what my child's participation will involve. I agree to the following:

(Please circle the relevant options below)

I agree that my child's participation will remain anonymous	YES	NO
I agree that the researcher may use anonymous quotes in her research report	YES	NO
I agree that the interview may be audio recorded	YES	NO
I agree that the information I provided by my child may be used in anonymised format after this project has ended for academic purposes by other researchers, subject to their own ethics clearance being obtained.	YES	NO

Informed consent

I understand that:

- My child's name and information will be kept confidential and safe and that my child's name and the name of the school will not be revealed.
- My child does not have to answer every questions and can withdraw from the study at any time.
- I can ask that my child is not to be audiotaped, photographed and/videotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of the project.

SIGNATURE : _____

NAME OF PARTICIPANT : _____

DATE : _____

ANNEXURE B: LEARNER PERMISSION LETTER



Learner permission letter

15 July 2021

Dear Learners.

My name is Ms.Thandolwakhe Masondo. I am a Master's in Education (Mathematics Education) student in the School of Education at the University of Witwatersrand, Johannesburg.

I am conducting a research project entitled: **TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS.**

I am interested in observing five introductory lessons focused on the teaching of fractions in two different grade 7 classes. Even in the context of Covid-19 regulation, the lesson will be face-to-face, however all regulations including wearing of masks, social distancing and frequent sanitizing will be followed. The lesson will partake in the school that I'm currently teaching at, this is to avoid going to an unfamiliar environment whereby hesitations of being allowed to come in contact with the staff or learners might be high. Having mentioned that this will be a self-study research whereby I will start off by giving learners an hour pre-test on basic knowledge of fractions. Thereafter analyse 6 scripts to interpret common errors and misconceptions from the 6 learners. Furthermore I will professionally attend to each of these learners' responses by giving comments separately next to each incorrect or partly correct answer as a way to identify the procedures followed by the learner to that conclusion. The findings will be used to plan an intervention lesson that seeks to address and remedy all these observed misconceptions.

I intend to videotape the lessons because my focus is on the notion of explanation and how teachers use the learners' thinking as a tool of teaching. The process of the research in the school should not take more than a week.

I am asking for permission to observe lessons instructed by your Mathematics teachers when they teach linear equations. I need you to assist me by attending classes where I will be observing and videotape the lessons.

Please understand that this is not a test or any task for your marks, it is voluntary, which means that you do not have to do it. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way.

I will not be using your own identity, but I will make one up so no one can identify you should your voice be captured in the videotape. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in the study.

I look forward to working with you!

Please feel free to contact me if you have any questions in relation to the research.

Thank you.

T Masondo

2400193@students.wits.ac.za

0833440179

ANNEXURE C: PARENT PERMISSION LETTER



Parent permission letter

15 July 2021

Dear parent/guardian

My name is Ms. Thandolwakhe Masondo. I am a Master's in Education (Mathematics Education) student in the School of Education at the University of Witwatersrand, Johannesburg.

I am conducting a research project entitled: **TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS.**

I am interested in observing one of the introductory lessons focused on the teacher's ability to use learners' thinking in the teaching of fractions in grade 7. Even in the context of Covid-19 regulation, the lesson will be face-to-face, however all regulations including wearing of masks, social distancing and frequent sanitizing will be followed. The lesson will partake in the school that I'm currently teaching at, this is to avoid going to an unfamiliar environment whereby hesitations of being allowed to come in contact with the staff or learners might be high. Having mentioned that this will be a self-study research whereby I will start off by giving learners an hour pre-test on basic knowledge of fractions. Thereafter analyse 6 scripts to interpret common errors and misconceptions from the 6 learners. Furthermore I will professionally attend to each of these learners' responses by giving comments separately next to each incorrect or partly correct answer as a way to identify the procedures followed by the learner to that conclusion. The findings will be used to plan an intervention lesson that seeks to address and remedy all these observed misconceptions.

I intend to videotape the lessons because my focus is on the notion of explaining how teachers use the learner's reasoning as a tool of teaching. The process of the research in the school should not take more than a week.

The reason I have chosen the school where your child's attends is because it is a school with multiple opportunities and good infrastructure. The culture that the school upholds is of interest to me. As such, I think this school is the ideal setting for my research project. These values will allow my research to be carried out in a conducive learning environment.

I am asking for permission to observe the teacher who will be teaching your child fractions.

Your child will not be advantaged or disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty.

There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will always be kept confidential and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

T Masondo

2400193@students.wits.ac.za

+27833440179

ANNEXURE D: PARTICIPANT TEACHER CONSENT FORM

UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG



PARTICIPANT TEACHER CONSENT FORM

Please fill in the reply slip below if you agree to participate in my study called: **TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS.**

Researcher's name: THANDOLWAKHE MASONDO

I _____ (name) give consent to participate in this research project. This research has been explained to me and I understand what my participation will involve. I agree to the following:

(Please circle the relevant options below)

I agree that my participation will remain anonymous	YES	NO
I agree that the researcher may use anonymous quotes in her research report	YES	NO
I agree that the interview may be audio recorded	YES	NO
I agree that the information I provide may be used in anonymised format after this project has ended for academic purposes by other researchers, subject to their own ethics clearance being obtained.	YES	NO

Informed consent

I understand that:

- My name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every questions and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/videotaped.
- All the data collected during this study will be destroyed within 3-5 years after completion of the project.

SIGNATURE : _____

NAME OF PARTICIPANT: _____

DATE : _____

ANNEXURE E: LETTER TO THE PRINCIPAL

UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG



University of the Witwatersrand,
BASANI PRIMARY SCHOOL. 0119805590

1781 MHINGA STREET, CHIAWELO EXT 2, 1818
MWALE BN
BASANI PRIMARY SCHOOL
1781 MHINGA STREET, CHIAWELO EXT 2, 1818

13 JULY 2021

Dear Madam,

Re: Permission to conduct research at BASANI PRIMARY SCHOOL.

My name is THANDOLWAKHE MASONDO.

I am studying for a MASTERS DEGREE (mathematics education) in the SCHOOL OF EDUCATION at the University of the Witwatersrand, Johannesburg. I am seeking permission to do research at BASANI PRIMARY SCHOOL.

I am conducting research on **TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS**. I am interested in observing five introductory lessons focused on the teaching of fractions in two different grade 7 classes. The research will entail collecting data from staff and grade 7 learners. I intend to videotape the lessons because my focus is on the notion of explanation and how teachers use the learners' thinking as a tool of teaching. The process of the research in the school should not take more than a week.

The reason in have chosen your school is because it is a school with multiple opportunities and good infrastructure. Moreover, I intend on being a participant of the study in order to limit the fear of spread of covid-19. This will thus be a self-study, as such, your school is the ideal setting for my research project since I work there. Even in the context of Covid-19 regulation, the lesson will be face-to-face, however all regulations including wearing of masks, social distancing and frequent sanitizing will be followed.

The quality of the research will also be enhanced since I have ample knowledge of the learner participants. Having that said, the cultural values of the school will allow my research to be carried out in a conducive learning environment.

I would like to invite your school to participate in this research because the results of the study will be beneficial to my career of teaching mathematics.

The research will not advantage nor disadvantage the participants in any way. They will be reassured that they can withdraw their permission at any time during the project without any penalty. There are no foreseeable risks in participating in this study and there is no remuneration that will be received.

Most importantly, the identity of the research participants and name of the school will always be kept confidential and in all academic writing about the study. Your individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

I therefore request permission in writing to conduct my research at your organization. The permission letter should be on your organization's headed paper, signed and dated, and specifically referring to myself by name and the title of my study.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,

THANDOLWAKHE MASONDO

0833440179

2400193@students.wits.ac.za

Supervisor:

Judah Makonye

0117173086

Judah.Makonye@wits.ac.za

ANNEXURE F: PARTICIPATION INFORMATION SHEET

UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG



PARTICIPATION INFORMATION SHEET

Dear Sir/ Madam

My name is Thandolwakhe Masondo, I am a Masters student in mathematics education at the University of the Witwatersrand, Johannesburg. As part of my studies, I have to undertake a research project, and I am investigating the **TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS** under the supervision of Prof. Judah Makonye. The aim of this research project is to discover how teachers engage with the learner's thinking in the teaching of mathematics. As part of this project, I would like to invite you to take part in an interview and answering of questionnaires, which will take an overall 30 minutes of your time. With your permission I would like to audio record the interview using a digital device. This recording will be stored in a password protected laptop and will be deleted after 3-5 years after the project.

There will be no personal costs to you if you participate in this project, you will not receive any direct benefits from participation but there are no disadvantages or penalties if you do not choose to participate or if you withdraw from the study. You may withdraw at any time or not answer any question if you do not want to. The interview will be completely confidential and anonymous as I will not be asking you for your name or any identifying information, and the information you give to me will not be shared with anyone else. I will be using pseudonyms (false names) to represent your participation in my final report. If you experience any distress or discomfort at any point in this process, we will stop the interview or resume another time. Even in the context of Covid-19 regulation, the lesson will be face-to-face, however all regulations including wearing of masks, social distancing and frequent sanitizing will be followed.

If you have any questions during or afterwards about this research, feel free to contact me on the details listed below. This study will be written up as a research report. If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za

Yours sincerely,

THANDOLWAKHE MASONDO

Researcher:

Thandolwakhe Masondo

2400193@students.wits.ac.za

+27833440179

Supervisor:

Prof. Judah Makonye

Judah.Makonye@wits.ac.za

0117173086

ANNEXURE G: ETHICAL CLEARANCE CERTIFICATE

SCHOOL OF EDUCATION ETHICS COMMITTEE

CONSTITUTED UNDER THE UNIVERSITY HUMAN RESEARCH ETHICS
COMMITTEE (NONMEDICAL)

CLEARANCE CERTIFICATE

PROTOCOL NUMBER: 2021ECE083M

PROJECT TITLE Teachers' resourcing of learners' mathematical thinking in teaching
grade 7 fractions

INVESTIGATOR

Thandolwakhe Masondo

SCHOOL/DEPARTMENT OF INVESTIGATOR

DATE CONSIDERED

18 October 2021

DECISION OF THE COMMITTEE

Approved unconditionally

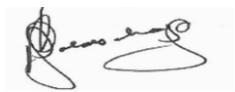
RISK LEVEL

EXPIRY DATE

Date of submission of the Research Report

ISSUE DATE OF CERTIFICATE

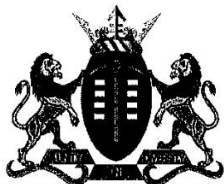
26 October 2021 CHAIRPERSON



Dr. Paul Goldschagg

cc: Prof Judah Makonye

ANNEXURE H: GDE APPROVAL LETTER



GAUTENG PROVINCE

Department: Education
REPUBLIC OF SOUTH AFRICA

8/4/4/1/2

GDE RESEARCH APPROVAL LETTER

Date:	14 September 2021
Validity of Research Approval:	08 February 2021– 30 September 2021 2021/285
Name of Researcher:	Masondo T
Address of Researcher:	4800 black and peacock spider street Leopards rest security estate Albertsdal
Telephone Number:	0833440179
Email address:	2400193@students.wits.ac.za
Research Topic:	TEACHERS' RESOURCING OF LEARNERS' MATHEMATICAL THINKING IN TEACHING GRADE 7 FRACTIONS.
Type of qualification	MASTERS IN EDUCATION
Number and type of schools:	1 Primary Schools
District/s/HO	Johannesburg Central

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

[Signature] 21/09/2021

The following conditions apply to GDE research. The researcher may proceed with the above study subject to the conditions listed below being met. Approval may be withdrawn should any of the conditions listed below be flouted:

1. Letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.

1

Making education a societal priority

Office of the Director: Education Research and Knowledge Management

7th Floor, 17 Simmonds Street, Johannesburg, 2001

Tel: (011) 355 0488

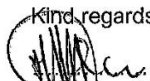
Email: Faith.Tshabalala@gauteng.gov.za

Website: www.education.gpg.gov.za

2. The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.
3. **Because of COVID 19 pandemic researchers can ONLY collect data online, telephonically or may make arrangements for Zoom with the school Principal. Requests for such arrangements should be submitted to the GDE Education Research and Knowledge Management directorate. The approval letter will then indicate the type of arrangements that have been made with the school.**
4. **The Researchers are advised to make arrangements with the schools via Fax, email or telephonically with the Principal.**
5. A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.
6. A letter / document that outline the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.
7. The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, and chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.
8. Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Director (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.
9. Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year. If incomplete, an amended Research Approval letter may be requested to conduct research in the following year.
10. Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.
11. It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.
12. The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.
13. The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.
14. On completion of the study the researcher/s must supply the Director: Knowledge Management & Research with one Hard Cover bound and an electronic copy of the research.
15. The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.
16. Should the researcher have been involved with research at a school and/or a district/head office level, the Director concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards



Mr. Gumani Mukatuni

Acting CES: Education Research and Knowledge Management

DATE: 21/09/2021

Office of the Director: Education Research and Knowledge Management

7th Floor, 17 Simmonds Street, Johannesburg, 2001

Tel: (011) 355 0488

Email: Faith.Tshabalala@gauteng.gov.za

Website: www.education.gpg.gov.za