

THE EVALUATION OF A LOW-ORDER PROPELLER PERFORMANCE AND WAKE
PREDICTION CAPABILITY FOR THE CALCULATION OF POWER EFFECTS

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CANDIDATE'S DECLARATION

I declare that this research report is my own, unaided work except where otherwise indicated. It is being submitted to the degree of Master of Science in Engineering to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other university.

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_____ day of _____ 2014

ABSTRACT

An analytical method was developed for the prediction of the performance of a propeller or wind turbine and the propagation of a vortical streamtube in the propeller's wake. The combined method is a product of compatibility between Blade Element Momentum Theory and Vortex Theory. The theoretical foundation of the combined method was discussed in sufficient detail to be translated into an algorithm for solving this type of problem. The algorithm was shown to be suitable for analysing the performance and trailing wake of an arbitrary propeller if the geometry of the blade is given. Published wind tunnel test data from various research institutions was used to compare the performance determined by the prediction model to the actual performance. A comparison with wind tunnel test data of an APC 8x8 Thin Electric propeller of simplistic geometry showed the propeller model to be adept at the prediction of thrust and adequate in predicting power. Further comparison with wind tunnel test results of a complex propeller developed at the CSIR indicated accuracy limitations in the propeller model primarily relating to insufficient consideration of 3-dimensional effects and a dependence on input data that accurately represents the physical conditions. The propeller model was extended to consider the performance of wind turbines and similarly provided useful predictions of the power extraction, particularly at higher advance ratios, in comparison with experimental results. An unsuccessful attempt was made to implement the propeller performance model results into a low-order panel method program. The intended combination was through the superposition of the propeller performance characteristics on the panel method inflow to enable the evaluation of propeller-airframe interactions.

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CONTENTS

CANDIDATE'S DECLARATION	i
ABSTRACT	ii
ACKNOWLEDGEMENTS	iii
CONTENTS.....	iv
LIST OF FIGURES.....	ix
LIST OF TABLES	xii
LIST OF SYMBOLS	xiii
1. INTRODUCTION.....	1
1.1. Background.....	1
1.2. Significance and Motivation	2
1.3. Objective	3
2. LITERATURE REVIEW.....	4
2.1. Introduction	4
2.2. The Panel Method.....	4
2.3. Propeller-Airframe Interaction	5
2.4. Historical Development of Propeller Theories	7
2.5. Concluding Remarks.....	14
3. RESEARCH METHOD.....	16
3.1. Introduction	16

3.2.	Analytical Procedure	16
3.3.	Panel Method Implementation Procedure	17
3.4.	Experimental Data	18
3.5.	Delineation and Limitations	19
3.6.	CMARC Integration of Propeller Performance and Wake Model Results...	20
3.6.1.	Introduction	20
3.6.2.	Wing-Propeller Wind Tunnel Tests	20
3.6.3.	Wind Tunnel Propeller Model	21
3.6.4.	Panel Method Wing Model	21
3.6.5.	CMARC Panel Method Wing-Propeller Combination	23
4.	ANALYTICAL MODELING	28
4.1.	Introduction	28
4.2.	Propeller Aerodynamics Analysis Method	29
4.2.1.	Introduction	29
4.2.2.	One Dimensional Momentum Theory	30
4.2.3.	Propeller Momentum Theory with Wake Rotation	33
4.2.4.	Blade Element Theory	36
4.2.5.	Blade Element Momentum Theory	38

4.2.6.	Propeller Vortex Theory	40
4.2.7.	Propeller Slipstream	43
5.	PROPELLER PERFORMANCE CALCULATION	49
5.1.	Introduction	49
5.2.	Required Inputs.....	49
5.3.	Aerofoil Characteristics	51
5.3.1.	XFOIL	51
5.4.	Calculation Process	53
6.	PROPELLER PERFORMANCE MODEL VALIDATION	57
6.1.	Introduction	57
6.2.	Wind Tunnel Tests	57
6.3.	Computational Propeller Model	58
6.4.	Results and Validation	59
6.4.1.	Thrust and Power Coefficient	59
6.4.2.	Blade Element Discretisation	62
7.	PROPELLER PERFORMANCE MODEL EVALUATION	64
7.1.	Introduction and Propeller Definition.....	64
7.2.	Propeller Performance Model	65
7.3.	Wind Tunnel Propeller Tests.....	65

7.4.	Results and Comparisons.....	66
7.4.1.	Initial Propeller Model Outputs.....	66
7.4.2.	Thrust and Torque Coefficient	68
7.4.3.	Power Coefficient.....	71
8.	PROPELLER SLIPSTREAM MODEL ANALYSIS	73
8.1.	Introduction and Delineation	73
8.2.	Results and Discussions.....	74
8.2.1.	Streamtube Contraction.....	74
8.2.2.	Velocity Adjustment	76
8.2.3.	Slipstream Deflection.....	77
9.	WIND TURBINE MODEL	80
9.1.	Introduction	80
9.2.	Betz Limit	81
9.3.	UAE Unsteady Aerodynamics Test	82
9.4.	Results and Comparisons.....	82
10.	CONCLUSIONS AND RECOMMENDATIONS	85
	REFERENCES.....	90
	APPENDIX A: MATLAB Simulation	94
	Propeller Analysis Setup Module.....	94

Blade Element Momentum Module	100
Slipstream Module	104
APPENDIX B: CMARC Input File.....	107
APPENDIX C: Propeller Geometry	128
APC 8x8 Thin Electric Propeller - Merrill (2011).....	128
CSIR Propeller - Monk (2010)	129

LIST OF FIGURES

Figure 1: APC Slow Flyer, Sport, and Thin Electric propellers. (Merrill 2011)	18
Figure 2: CMARC wing model.....	22
Figure 3: Comparison of predicted and experimental lift coefficient	23
Figure 4: Comparison of wing lift curve augmentation with propeller effects - "W" = Witkowski et al. (1989); "CM" = CMARC Model.....	25
Figure 5: CMARC Propeller model in the forms of an annular diaphragm.....	26
Figure 6: CMARC wing propeller combination	27
Figure 7: Propeller wake streamtube	31
Figure 8: Streamtube across a three-bladed propeller.....	34
Figure 9: Annular streamtube control volumes	34
Figure 10: Flow and force directions at a blade element	37
Figure 11: Propeller with trailing helicoidal vortex sheets	41
Figure 12: Vortex tube propagation with bound vorticity shed per unit time	44
Figure 13: Propeller at incidence	45
Figure 14: Slipstream geometry	47
Figure 15: Lift and drag coefficient comparison between wind tunnel (Abbot and von Doenhoff (1958)) and XFOIL (Benini (2004)) results - NACA 66-206.....	52

Figure 16: Lift and drag coefficient comparison between wind tunnel (Abbot and von Doenhoff (1958)) and XFOIL (Benini (2004)) results – NACA 66(3)-418.	52
Figure 17: Propeller performance prediction model algorithm	55
Figure 18: Propeller slipstream model algorithm.....	56
Figure 19: Thrust coefficient comparison between (Merrill 2011) wind tunnel results and propeller model predictions	60
Figure 20: Angle of attack across the Merrill (2011) blade radius at advance ratios $J = 0.25$ and $J = 0.58$	61
Figure 21: Power coefficient comparison between (Merrill 2011) wind tunnel results and propeller model predictions	62
Figure 22: Variation in performance prediction error with increasing number of blade elements.....	63
Figure 23: Monk (2010) Propeller model predictions without induction factor initialisation.....	67
Figure 24: Comparison between (Monk (2010)) wind tunnel and propeller model thrust coefficients (-8° to 0° pitch angles) – “M” = (Monk (2010)); “PM” = propeller performance model	70
Figure 25: Comparison between (Monk (2010)) wind tunnel and propeller model thrust coefficients (0° to 4° pitch angles) – “M” = (Monk (2010)); “PM” = propeller performance model	70
Figure 26: Comparison between (Monk (2010)) wind tunnel and propeller model power coefficients (-8° and -4°) – “M” = (Monk (2010)); “PM” = propeller performance model	72

Figure 27: Comparison between (Monk (2010)) wind tunnel and propeller model power coefficients (0° and 4°) – “M” = (Monk (2010)); “PM” = propeller performance model	72
Figure 28: Results of a numerical calculation of slipstream radius and velocity field in the plane of the propeller disc and far downstream (Kerwin (1994))	75
Figure 29: Slipstream contraction behind the Merrill (2011) propeller (propeller slipstream model).....	76
Figure 30: Axial velocity increase across the propeller	77
Figure 31: Vertical and lateral slipstream deflection in the intermediate wake downstream of the propeller (propeller slipstream model).....	78
Figure 32: Fully-developed slipstream with propeller tilt	79
Figure 33: Comparison between Lindenburg (2003) wind tunnel and wind turbine model power coefficients.....	84

LIST OF TABLES

Table 1: Propeller model inputs 51

LIST OF SYMBOLS

Symbol / Anagram	Description
a	Axial induction factor
$\acute{a}$	Tangential induction factor
B	Number of blades
C	Chord length [m]
C_L	Lift coefficient
C_D	Drag coefficient
C_T	Thrust coefficient
dr	Blade element and annulus width [m]
dQ	Torque of element or annulus [Nm]
dT	Thrust of element or annulus [N]
dP	Power of element or annulus [W]
D	Drag force [N]
F	Combined tip-loss and hub-loss coefficient
L	Lift force [N]
Q	Torque [Nm]
P	Pressure [P]
r	Local radius [m]
R_{hub}	Hub radius [m]
T	Thrust [N]
U	Resultant velocity at blade element [m/s]
α	Local angle of attack [rad]
β	Local element pitch angle [rad]
ψ	Yaw inflow angle [rad]
φ	Local inflow angle [rad]
λ_r	Local tip speed ratio
$\acute{o}$	Local solidity = $\frac{Bc}{s\pi r}$
ρ	Air density [kg/m ³]
J	Advance ratio

C_N	Vertical in-plane force coefficient
C_Y	Lateral in-plane force coefficient
D	Propeller disc diameter [m]
N	Propeller vertical in-plane force [N]
R	Propeller disc radius [m]
R_s	Radius of fully developed slipstream [m]
R_x	Radius of slipstream at station x [m]
r	Arbitrary radial position on propeller blade [m]
S	Propeller disc area [m ²]
S_s	Cross-sectional area of fully developed slipstream [m ²]
V	Axial velocity through propeller disc [m/s]
V_s	Velocity in fully-developed slipstream [m/s]
V_w	Rate of propagation of wake vortex tube [m/s]
V_x	Velocity in slipstream at station x [m/s]
V_{inf}	Freestream velocity [m/s]
x	Position downstream from propeller along thrust axis [m]
Y	Propeller lateral in-plane force [N]
y	Transverse displacement of wake laterally from propeller thrust axis [m]
z	Transverse displacement of wake vertically from propeller thrust axis [m]
α_p	Angle of attack of propeller thrust axis [rad]
α_s	Angle of fully-developed slipstream to propeller thrust axis [rad]
α_x	Angle of slipstream to propeller thrust axis at station x [rad]
β_s	Sideslip angle of fully-developed slipstream to propeller thrust axis [rad]
β_p	Sideslip angle of propeller to thrust axis [rad]
Γ	Strength of bound vorticity on propeller blade (m ² /s)
θ_p	Fully-developed slipstream incidence relative to freestream [rad]
θ_x	Slipstream incidence relative to freestream at station x [rad]
γ	Strength of distributed annular vorticity [m/s]
γ_s	Strength of distributed swirl vorticity [m/s]
σ_p	Lateral angle of fully-developed slipstream relative to freestream [rad]
Ω	Propeller angular velocity [rad/s]
n	Propeller rotational speed [rev/s]

ϕ	Angular coordinate around propeller disc [rad]
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1. INTRODUCTION

1.1. Background

The use of propellers as an aircraft propulsion system requires the consideration of aerodynamic interference due to the presence of a wing or fuselage. A propeller producing thrust by forcing air backwards produces a vortical wake or slipstream. This may be considered as a cylindrical tube of spiraling air propagating rearward over the fuselage and wings having both beneficial and detrimental effects. Flow within the slipstream is faster than the freestream flow resulting in increased drag over the parts exposed to its trajectory. The rotary motion of the slipstream also causes the air to strike the tailplane at indirect angles, having an effect on the stability and control of the aircraft. Kerwin (2001)

An understanding of the propeller slipstream is therefore important in the conceptual design phase of propeller-driven aircraft. For wing-mounted configurations, the integration aspect that will have the greatest impact on the overall aerodynamic performance of the aircraft is the interaction between the propeller and the wing. The installation design, during the conceptual and preliminary phases, thus requires adequate optimisation in integrating the propulsion system. Witkowski et al. (1989)

For the conceptual phase of aircraft design, low-order engineering codes are still the preferred method for designers. The iterative nature of aircraft design, particularly in the initial phases, requires the consideration of many versions of conceptual options. For each version the designer needs to obtain enough knowledge of the design's merits in order to evaluate it against a competitive option from which one is selected as superior according to the criteria under optimisation. Low-order codes are therefore powerful tools to be applied in the conceptual design stage. Their use in the early stages of design enables higher fidelity computational fluid dynamics (CFD) calculations or wind tunnel tests to be performed on more mature concepts and can trim many months off the design process.

The selection of analytical tools is made based on a number of factors, chief amongst these is the applicability of the method in consideration of its

simplifications. In the conceptual and preliminary design stages, simplistic modeling and low cost, rapid computation are strong requirements. In this regard a low-order propeller performance and wake prediction method is proposed as an addition to a surface panel method to enable the consideration of propeller power effects.

The aerodynamics of aircraft propellers are conceptually similar to those of horizontal-axis wind turbines. On this basis the theoretical model behind such a propeller performance and wake prediction tool may be useful in predicting the performance and wake characteristics of wind turbines.

1.2. Significance and Motivation

While jet technology is the more advanced aircraft propulsion technology of the day, propeller technology remains a widely used and popular solution for lower velocity applications. The benefits expected in terms of fuel consumption and the reduction of life-cycle costs have kept propellers as a viable propulsion alternative.

The comprehensive design process of an aircraft is typically comprised of three distinct phases: conceptual design, preliminary design, and detailed design. Each design phase is carried out in sequence and has different objectives and levels of complexity and thus uses different design and analytical methods. The fundamental objectives of the conceptual stage of aircraft design are the determination of the most effective shape, size, weight, and basic performance of the flight vehicle. In this pursuit the designer considers a multitude of configurations and specifications that are evaluated against a set of target parameters, for example: lowest weight, highest climb rate, or smallest takeoff distance. Merrill (2011)

A large variety of advanced propeller prediction methods have been developed since the invention of aerial vehicles. Generally, the methods can be categorised into inviscid and viscous models that use modern day CFD, and semi-empirical models based on published data.

In the early stages of aircraft design, designers typically opt for low-order analytical tools that provide moderately accurate results while having low computational times on standard desktop computers. These tools are mostly based on methods such as:

lifting line theory, the vortex lattice method, and the surface panel method. For the task of evaluating performance parameters against design targets and comparing the effects of specification variations, these low-order methods are generally adept at representing the real-world scenario. Hobbs (2008)

The afore-mentioned methods and tools typically do not provide for consideration of the aerodynamic effects of a powered propeller or the interaction between the propeller and other aerodynamic surfaces of the aircraft.

While the focus of this work is on propeller and aircraft applications, the methods developed in this regard are explored in the context of wind turbines. Whereas a propeller introduces energy into the flow, a wind turbine extracts energy. This is seen to be a useful extension and application of the predictive methods with the dramatic development of wind energy extraction technologies of the last few decades. The evaluation of the wind turbine capabilities will also form a prudent test of the theoretical model's limits.

The purpose of this research is the development of a low-order analytical method for the detailed analysis of the performance of advanced propellers and wind turbines and the resultant trailing wake or slipstream. The analytical model will be of a form that can be integrated into a unified analysis method applicable to propeller-powered aircraft where propeller power effects are included in the aerodynamic analysis of the aircraft. The investigation comprises a complementary computational and experimental comparison effort.

1.3. Objective

The primary objective of this work is the development of a low-order prediction method for analysing the performance of an arbitrary propeller or wind turbine and determining the nature of the fluid wake that emanates from behind the propeller/wind turbine. An important part of this development must involve the evaluation of the method's capabilities and limitations, and the validation of its accuracy through comparisons with published data.

2. LITERATURE REVIEW

2.1. Introduction

Through the past decades propeller theories have advanced considerably and several methods are available for propeller design and performance analysis based on different levels of sophistication. Of the understood methods, arguably the most complex are the 3-dimensional viscous flow models such as those of Michelsen (1994) and Lardeau and Leschziner (2005), where the 3-dimensional incompressible Reynolds-averaged Navier-Stokes (RANS) equations are implemented and solved iteratively. Advances in computer processing speed, memory capacity, and user interactivity have allowed numerical techniques in CFD to grow rapidly in the last decade. Unfortunately, even with the growing capabilities of CFD, solutions for rotating propellers are very difficult to achieve.

The problem faced when dealing with real flows is the fact that almost all flows are 3-dimensional. Without going into the details, it is sufficient to state that a 3-dimensional flow is considerably more computationally expensive to solve than the corresponding 2-dimensional flow. The tradeoff between ease of operation and predictive accuracy must satisfy the intended application in the conceptual design phase. In this regard, only propeller theories of an inviscid nature will be considered.

2.2. The Panel Method

For a panel method, the surface of the body over which the flow field is to be computed is represented by a set of panels. Singularities are distributed on the panel to perturb the flow field around the body surfaces. The assumed distributions - constant, linear, etc. - and panel geometry - quadrilateral, hyperboloid - depend on the method that is used. A commercially available example of a panel method code is CMARC, an inviscid fluid flow analysis package that uses a low-order panel method. Low-order, 3-dimensional panel methods, in which the calculated coefficient of pressure for each panel is applied uniformly over the entire panel, have been shown to provide effectively the same level of accuracy as higher-order methods in

which pressure gradients are calculated within panels. Panel method programs are able to model complex geometries and fluid flow conditions. Glatt et al. (1986)

CMARC uses constant strength source and doublet distributions over each panel, thus making it a low-order panel method. The boundary conditions (normal velocity component equal to zero) are satisfied at the control points with provision for relaxed flow conditions at the inlet/outlet panels resulting in a system of linear equations. Matrix methods are used to solve the system of equations for the unknown singularity strengths. Once the distribution of singularity strengths has been determined, the velocity components can be determined at any point within the flow field. AeroLogic (2005)

2.3. Propeller-Airframe Interaction

Since the 1930's, a large number of investigations have been performed on the effects of the propeller slipstream on a wing and/or other components of an aircraft considering both tractor and pusher configurations. Thompson (1951)

In almost all experimental studies [such as Aljabri and Hughes (1985) and Howard et al. (1985)] and numerical studies [such as Ting et al. (1972) and Rizk (1981)] devoted to investigating the propeller/wing/fuselage interaction problem, emphasis has generally been placed on the influence of the wake generated by the propeller either on the wing aerodynamic behaviour or on some part of the aircraft fuselage. Ting et al. (1972) used the method of asymptotic expansions to study the interference of a wing with multiple propellers by accounting for the images (and the mutually induced images) of a vortex distribution in the Trefftz plane due to the circular streamtube behind each propeller. The method was found to have high computational complexity. The method used by Rizk (1981) was developed to address the interactions between the slipstream and wing for transonic flight. The approach of Rizk (1981) was based on the panel method and was particularly adept at treating local patches of supersonic flow and shock waves, the method was found to be computationally expensive.

Computational codes that predict the optimum wing lift distribution for the tractor and pusher configurations have been developed by Kroo (1984) and Miranda and

Brennan (1986). Miranda and Brennan (1986) developed models that were particularly suited to study wingtip-mounted propellers or turbines using the PROPWING software program. Very little information was available on the computer program or its predictive accuracy. The approach of Kroo (1984) focused on the optimisation of net thrust for a propeller-wing combination with the wing designed to maximise this quantity. Both methods showed that significant wing drag reduction can be accomplished for an optimised tractor configuration, and significant propeller efficiency improvements can be obtained for the pusher configuration. The theoretical basis used in Kroo (1984) implied that the effect of the propeller slipstream on the wing-induced drag is identical to the effect of the wing wake on the thrust of a pusher propeller; the method was effective in addressing the interaction effects but lacked a base model for the calculation of the propeller forces.

The more modern analyses of Catalano and Maunsell (1997) and Catalano and Stollery (1994) demonstrated that laminar flow could be increased when pusher propellers are installed in convenient positions behind a wing, resulting in less friction drag. Catalano and Stollery (1994) focused on the investigation of a pusher propeller in high thrust conditions, such as during take-off and climb, while Catalano and Maunsell (1997) used an analysis based on the application of 'classic' propeller vortex theory to a 3-dimensional panel method program. Classic propeller vortex theory will henceforth be referred to as Goldstein's vortex theory and is discussed in more detailed in the following section.

The structural approach of Catalano and Maunsell (1997) succeeded in creating a functional propeller model that was successfully integrated into a more general fluid flow analysis program. The requirements of a suitable propeller prediction method include: low computational expense; accurate prediction of propeller characteristics for salient, steady-state flight conditions; and flexible interoperability with general aerodynamics analysis methods. The propeller vortex theory model used in their research requires fairly intensive computation in relation to other propeller theories (as is detailed in the following section), in this regard alternative solutions to this problem were explored.

2.4. Historical Development of Propeller Theories

The development of a rational theory of propeller behaviour begins with methods based on the actuator disc concept. The concept, which represents the propeller by equivalent forces distributed on a permeable disc of zero thickness in a flow domain, was introduced by Froude (1889) as a continuation of the work of Rankine (1865) on the momentum theory of propellers (Momentum Theory) in the 19th century. The work of Rankine and Froude developed the fundamental momentum relations governing a propulsive device in a fluid medium.

Momentum theory assumes that the flow being accelerated through the propeller disc is irrotational and incompressible. Momentum, being defined as the product of mass and velocity, makes the theory an energy method that therefore requires the determination of the momentum in the flow both ahead of and behind the propeller. Work energy is added to the flow by the rotating propeller as the streamtube passes through the propeller disc. Bernoulli's equation only applies to constant energy, incompressible flow which therefore requires the equation to be applied separately pre and post propeller. Induced velocities are calculated from the momentum gained in the flow in the axial and tangential directions. The change in flow properties across the propeller disc can be used to determine the thrust and efficiency of the propeller. Lan and Roskam (2003) states that the shortfall of Momentum Theory is its failure to account for energy losses due to slipstream rotation, blade profile drag, non-uniform flow, compressibility effects and propeller blockage due to a fuselage and/or nacelle. While Momentum Theory is arguably the most simplistic of the prediction methods, it's assumption of an infinite number of blades renders it unsuitable, on a stand-alone basis, for the evaluation of the optimum number of propeller blades. The assumption of a non-rotating wake disables the theory from usefully modeling the vortical streamtube in the propeller's wake.

Betz (1919) provided a description of a particular radial propeller blade loading that would minimise the energy loss in the wake. A follow up analysis by Lanchester (1915) and Betz (1920) theorised that the maximum extraction of energy possible by a turbine rotor is $16/27$ or 59.3% of the incoming kinetic energy.

Drzewiecki (1901) originated the blade element theory of screws (or screw propellers) considering the blades of the propeller as being composed of an infinite number of aerofoil sections. Blade Element Theory (BET), as described by Houghton and Carpenter (2003), involves analysing the local aerodynamic forces at incremental divisions along the propeller blade and for all azimuth of rotation. This theory determines the vector of the flow using a cross section of the propeller blade for each element - which operates aerodynamically as a 2-dimensional aerofoil whose aerodynamic forces may be calculated based on the local flow conditions. The speed of the flow (or alternatively speed of the aircraft as it moves through still air) varies according to the advance ratio and rotational speed of the propeller. This variation changes the local angle-of-attack, which directly affects the lift generated by the section. The local thrust and torque are determined using lift-curve data for the aerofoil shape. The integration of these elemental forces along the blade radius and over one propeller revolution yields a solution to the thrust and torque produced by the propeller.

Where BET becomes less accurate is in the determination of the elemental angle-of-attack due to its inability to predict the induced velocity. The rotational nature of the propeller produces axial and tangential velocity components. These two components of velocity combine to form the induced velocity. The induced velocity varies over the length of the blade due to the change in air velocity as a result of rotation. This induced velocity directly affects the elemental angle-of-attack which consequently affects the lift and torque. In isolation the BET is therefore not suitable. Its accuracy is furthermore dependent on the fidelity of the 2-dimensional aerofoil data used as input. While useful in providing simplicity to the theory, the assumption of the independence of blade elements provides limitations towards the consideration of cross-flow effects (Moriarty and Hansen (2005)). As is later discussed, the elemental nature of the BET creates a useful interface for the mapping of local blade conditions to annular streamwise conditions in the far wake of the propeller. An inclusion of induced velocity effects in the BET would make for a capable low-order predictive method.

The work of Wilbur and Orville Wright had no influence on the subsequent development of propeller theory, it is however remarkable that although their work was based on experimentation and not theory, they seem to have been the first to

combine blade element theory and momentum theory (Wald (2001)). They successfully used momentum theory to estimate the relative velocity and the angle of attack of blade elements to design efficient propellers. Glauert (1935) however, is recognised for the introduction of the Blade Element Momentum Theory (BEMT) which, along with the generalised momentum theory, initiated a major step forward in the modeling of flow through propellers and rotors. BEMT is the most common engineering model for the determination of aerodynamic loads and induced velocities on propellers. Borrowing from BET, the blades of a propeller are broken into elements along the radius. As these elements rotate in the propeller plane, they trace out annular regions across which the momentum balance takes place. These annular regions are also where the induced velocities from the wake change the local flow velocity at the propeller plane. BEMT can also be used to analyse streamtubes through the propeller disc, which can be smaller than the annular regions and provide more accurate results. Winarto (2004)

BEMT maintains the simplicity of its individual counter-parts but due to this simplicity, the theory does have its limitations. One primary assumption is that the calculations are steady and static; it is assumed that the flow field around the aerofoil is always in equilibrium and that the passing flow accelerates instantaneously to adjust to the changes in the vorticity in the wake. One other limitation is that BEM theory is based on the assumption that the forces acting on a blade element are essentially 2-dimensional, meaning that spanwise flow is neglected. This assumption also implies that there is very little spanwise pressure variation and the theory is therefore less accurate for heavily loaded rotors with large pressure gradients across the span (Moriarty and Hansen (2005)). BEMT effectively provides the induced velocity consideration commented above in support of the BET making it a useful theory for the estimation of propeller forces with very low computational requirements. The combination of the two methods, however, does not remove the irrotational flow assumption hindering the consideration of the rotational slipstream.

Propeller Vortex Theories

In propeller vortex theories the influence of the vorticity in the slipstream on the blade is included. This is achieved by prescribing the shape of the wake and calculating its induced velocity at the propeller disc. In the paragraphs below three methods are

described: the theory of Betz-Prandtl, the theory of Goldstein and the theory of Theodorsen. In all three methods the wake model that is used is based on a blade loading distribution for minimum induced loss. The theories differ in the detail in which the wake is described and in their complexity.

The propeller vortex theory of Betz and Prandtl (1927) determines the induced velocity field as a function of the vorticities of the blades and its wake. The propeller is assumed to have an infinitely large number of blades and the circulation along the bladespan is assumed to be constant. The wake of the propeller is assumed to consist of a system of rigid, cylindrical sheets. Vorticity leaving the blade trailing edge is therefore concentrated at the edge of the propeller disc and is transported downstream without contraction of the slipstream. The effect of this is that the theory is only valid in the case of lightly loaded propeller blades. Tip losses and the effect of multiple vortex sheets are ignored. Due to the assumption of an infinitely large number of blades there are no discrete vortex sheets that interact with the flow outside the streamtube. The radial velocity component will therefore not be present in the slipstream. The induced axial component of the velocity at the propeller disc is half the value of the axial velocity far downstream, as a result the induced velocities at the disc can be determined if the velocity distribution in the wake is known. Zondervan (1998)

The vortex theory of Goldstein (1929) takes account of the periodic nature of the slipstream. Each blade is modeled by a single bound vortex with a varying circulation distribution in the radial direction. The circulation is associated with the bound vorticity and a vortex sheet is continuously shed from the trailing edge. By considering a lightly-loaded propeller, the interference flow of the vortex system is small compared with the velocity of the blades. The vortex sheets take on a helical shape which follows the forward velocity and the rotational velocity of the propeller. No contraction of the streamtube or roll-up of the vortex sheets is considered. Zondervan (1998)

Goldstein (1929) succeeded in solving the problem of the ideal circulation distribution of single-rotating propellers with helicoidal vortices for small advance ratios. The circulation around an arbitrary blade section is assumed to be identical to the jump in the velocity potential at the corresponding point in the radial direction of the vortex

sheet. If the flow properties at a downstream station are known then the circulation distribution along the propeller blade can be calculated. The total propeller thrust, torque, and efficiency can be calculated by integration along the blade radius. With a good understanding, and accurate tabulation of the Goldstein circulation function, Goldstein's propeller vortex theory is capable of describing the behaviour of fluid through a propeller and in the wake in accurate detail. Wald (2006)

Theodorsen's theory is largely based on Goldstein's vortex theory. Theodorsen (1944) showed that the Betz condition for minimum energy loss may be applied to heavily loaded propellers. His method redirects focus to the ultimate wake far behind the propeller relating the conditions to those in the propeller plane through a mass coefficient. The radial distribution of the circulation in the far field of the wake behind a heavily loaded propeller is the same as that obtained by Goldstein's vortex theory for a propeller at light loading. This is the case if the results are compared for the same value of the helix angle. Like Goldstein (1929), Theodorsen (1944) defines a circulation function but instead of calculating the circulation and mass coefficient, Theodorsen (1944) solved them experimentally using an electrical analogy technique. Theodorsen (1944) proceeded to expand and refine the theory of propellers with ideal load distribution following this important realisation. The advancement was based on the analysis of helicoidal trailing sheets and the thrust and torque accompanying their form and displacement velocity. Cheung (1987)

The models of Betz-Prandtl (1927), Goldstein (1929) and Theodorsen (1944) are primarily used for the calculation of the conditions at the propeller plane. From far wake conditions the discontinuity in velocity potential is calculated representing the circulation around the corresponding blade section at the propeller plane and the velocities that are induced by the vortex system. While the propeller vortex theory of Goldstein (1929) is well known for its accuracy, the application of its complicated wake model, most notably the determination of the velocity induced by the vortex system, requires a considerable computing effort through the use of numerical methods (Aljabri (1978)). Theodorsen's theory computes in a similar fashion to Goldstein's vortex theory except that it does not compute azimuthal velocity variations nor does it represent the vortex wake by a finite number of helical vortex filaments (Crigler (1948)). In order to easily map the elemental blade forces to annuli

in the far wake slipstream, the wake would need to be represented in terms of a set of helical vortex filaments.

The Vortex Lattice Method (VLM) is a well-known way of predicting the steady aerodynamic performance of a finite wing and was first solved by Goldstein (1929). VLM, as applied to propellers, involves developing a shape function for the aerofoil while keeping the free-stream, rotational speed, and hub dimensions constant. In this method, the propeller blade and slipstream are represented by a system of segmented bound- and trailing-vortices, with the bound-vortex segments placed at the quarter-chord points of the blade. As in conventional lifting-line theories, the strengths of the bound- and wake-vortex filaments are determined using the Biot-Savart law and, additionally, the condition that the flow be tangent to the blade mean camber line along the three-quarter-chord line. Application of the Kutta-Joukowski law at any radial location then determines the elementary lift coefficient of the blade at that point. Blade drag is determined from empirical correlations based on camber, thickness, Mach number, and lift, including viscosity and compressibility effects. An implementation of the VLM by Chang (1980) and Sullivan (1977) showed the method to be effective and capable. The method represents the propeller wake as a finite number of vortex filaments and allows for variation of airfoil type along the blade radius. Glatt et al. (1986) listed shortcomings of the method to include: 1) simplified modeling of the wake geometry, 2) no provision for supersonic tip effects, and 3) no correction for finite thickness effects. The VLM is therefore seen to be a favourable option for consideration in the current study.

Larrabee (1979) presented a straightforward, practical procedure for optimum propeller design which has the virtue of convenience, but is limited by small-angle approximations and implicit light loading assumptions. Larrabee (1979) discovered compatibility between propeller design utilising lifting line theory with induced velocity distributions being induced by helical trailing edge vortex sheets and Glauert's radially graded momentum theory. The small angle approximation is seen to be a limiting factor to the theory and is therefore not favoured for inclusion in the current study.

Hess and Valarezo (1985) describe a surface panel method (SPM) analysis for propellers. In the method, the surface of the 3-dimensional propeller blade is

discretised into a large number of small, flat, quadrilateral panels. These panels lie on the surface of the blade, as opposed to an interior plane or camber surface, which is used by vortex-lattice or lifting-line methods. It is this feature of a SPM that enables it to calculate detailed surface pressure distributions. The method is able to model the rounded shape of a blade leading edge and becomes more accurate as the number of panels increases. Fluid dynamic singularities - source and dipole or vorticity - distributions are placed on the paneled surface and adjusted to satisfy the boundary conditions. The Kutta condition is applied to the trailing edge of the blade from which a trailing vortex wake emanates. In principle, a surface panel method can calculate flow about any body, including a propeller, and in this manner the problem has been solved for many years. As a practical matter, the application of the SPM – particularly in consideration of a rotational lifting surface - is computationally intensive and is therefore not suitable for use in the current study. Hess and Valarezo (1985)

The generalised actuator disc method can be seen as a straightforward inviscid extension of the BEM technique. The main difference is that the annular independence of the BEM model is replaced by the solution of a full set of Euler or Navier-Stokes equations. Axisymmetric versions of the generalised actuator disc method have been developed and solved either by analytical or semi-analytical methods such as Wu (1962) or Conway (1998), or by finite difference/finite volume methods such as Madsen and Rasmussen (1999). The actuator line concept introduced by Sorensen and Shen (2002) extends beyond the assumption of axial symmetry, where the loading is distributed along lines representing blade forces in a fully 3-dimensional flow domain. These approaches challenge the validity of the assumptions used in simpler methods and, in general, the 3-dimensional behaviour of the wake. These methods provide significant improvements in predictive accuracy but introduce exponential increases in computational complexity.

Phillips (2002) describes how other complimentary theories can be applied in conjunction with BET to give a more accurate prediction of induced velocity. Phillips (2002) recognised that Momentum Theory is one useful addition to BET, noting particularly that it's induced velocity calculation can be used in place of the more cumbersome process found in Goldstein's vortex theory. Momentum Theory assumes an irrotational flow when in reality this is not the case; the research conducted by Phillips (2002) used BEMT and introduced a further addition to the

theory that allows for rotation in the slipstream. The method removed the assumption of uniform pressure in order to apply an angular momentum term to the theory. This makes the theory more accurate and also more complicated.

The calculation of the velocity field induced by the helicoidal vortex sheet model of the slipstream is fairly complex. Gilbey and Green (2006) presents a simple approach towards modelling the overall induced effects representing the main features of a given propeller. Gilbey and Green (2006) develops a vorticity tube model for the cases of incidence and sideslip. The model relates the forces acting on a propeller to the characteristics of the far wake, in doing so it relies on compatibility between the theoretical vorticity tube and the propeller forces determined via the blade element and momentum theories. The helicoidal vortex sheets are replaced by two continuous distributions of vorticity: the axial vorticity and the tangential vorticity. The model assumes the propeller has a finite number of blades each carrying a constant bound vorticity that is shed at the tips. A further simplification is in ignoring the swirl component of rotational velocity in the wake on the basis of it being small compared to the propeller rotational velocity. The vorticity tube method of Gilbey and Green (2006) is shown to be suitable in producing the slipstream velocity components in a 3-dimensional domain while capturing the contraction and deflection of the slipstream for an inclined propeller. This method would therefore be suitable to the current study

2.5. Concluding Remarks

The panel method was described as a flexible and powerful, low-order aerodynamic modeling tool that is capable of providing accurate predictions of the performance of complete and complex aircraft geometries with low computational requirements.

The numerical evaluation of the propeller-airframe interaction problem is not new and has been studied extensively in various forms. Historical efforts to develop methods or form analyses capable of evaluating propeller-airframe interactions were discussed and found, in most cases, to require vast computational resources. A need was therefore identified for a different approach. Various propeller performance and wake prediction methods were discussed in terms of their underlying framework, capabilities, and computational complexity. Of the various methods that were

discussed, the main options including: Momentum Theory, Blade Element Theory, the Panel Method, the Vortex-Lattice Method, and several variants of Propeller Vortex Theory. Each of these theories was broadly discussed with emphasis given to their assumptions, accuracy, and computational intensity. Both the Vortex Lattice Method and the combined Blade Element and Momentum Theory appear to be suitable models for the rapid estimation of propeller performance. A preliminary selection of the Blade Element Momentum Theory was made on the basis of the wealth of literature that exists on the method, it's marginal simplicity benefit over the Vortex Lattice Method, and it's compatibility with other numerical methods.

The vortical wake method of Gilbey and Green (2006) provides a suitable method for the determination of the propeller slipstream taking into the consideration it's contraction and the resultant deflection in the case of an inclined propeller. The inherent compatibility between Blade Element Theory, Momentum Theory, and the vorticity tube model developed by Gilbey and Green (2006) reinforced the selection of Blade Element Momentum Theory as the propeller performance prediction method. This compatibility is made possible since the local lift at any radial station on a blade is associated with the circulation around the blade.

These combined theories, if correctly implemented, should be able to provide the necessary requirements towards a low-order propeller performance and slipstream model.

3. RESEARCH METHOD

3.1. Introduction

A propeller performance analytical method was developed into an algorithm and built into a working computer simulation capable of estimating the propeller performance parameters including: thrust, torque, power, efficiency, and slipstream characteristics including: velocity, radius, and deflection for a given geometry and operating conditions.

A basic validation of the algorithm was carried out by modeling a commercial propeller with a very basic geometry and comparing the simulated results with wind tunnel results over a range of advance ratios.

An evaluation was carried out of the limitations of the theoretical method and simulated algorithm by modeling a propeller of complex geometry across a wider range of operating conditions.

The algorithm was then adjusted to consider the extraction of energy from the incoming flow field. An evaluation of the method in predicting wind turbine performance characteristics is verified in a similar fashion to the propeller performance algorithm.

3.2. Analytical Procedure

The combined analytical method was based on compatibility between three stand-alone prediction methods including: blade element theory to predict the propeller forces, momentum theory to relate the flow momentum at the propeller to that of the far wake, and a vortical wake model to describe the slipstream characteristics referred to above.

The analytical method was developed into a computational algorithm and a computer simulation was built within the MATLAB environment. All simulations were performed on an Apple MacBook Pro laptop with a 2.53 GHz Intel Core 2 Duo processor and 4

GB of 1067 MHz DDR3 memory running Mac OS X 10.8.2. Simulation run times varied from 5 seconds per advance ratio setting for simple geometries to 15 seconds per setting for more complex geometries.

The calculation of aerofoil characteristics was carried out using XFOIL, a coupled panel/viscous code

(Drela (1989)). XFOIL is a collection of programs for aerofoil design and analysis for incompressible/compressible viscous flows over an arbitrary aerofoil.

3.3. Panel Method Implementation Procedure

The motivation for this work is the consideration of propeller power effects, these being the interaction between the propeller and the remainder of the aircraft. This analysis was thus applied to: the evaluation of the performance of a simple aircraft wing using a commercial low-order panel method package; the evaluation of the performance of a given propeller using the propeller performance and wake method referred to in Section 3.2; and the evaluation of the propeller induced effects using a method in which the velocities in the slipstream determined by the propeller performance and wake model are introduced together with the freestream as onset flow in the panel method. This study therefore focused on the separate contribution of the propeller and wing by uncoupling them during the numerical analyses.

Higher-order panel method codes such as PDAERO employ the propeller-slipstream envelope model wherein the wake is defined as a closed cylindrical shape enclosing a region with an energy level that differs from that of the undisturbed outer flow. This model makes a simplifying assumption in compacting all the vorticity of the slipstream in the outer edge of the streamtube by covering its surface with a continuous distribution of helical shaped vortex lines. This so-called “full-interaction” model is a sophisticated method that is generally reserved for higher-order panel codes. Droandi and Gibertini (2012)

A simpler method for the integration of a propeller wake is the “add on” model in which the velocity and total pressure within the streamtube is introduced to the onset flow. In lower-order panel codes this can be achieved through the definition of a

panel such as a diaphragm which models a cross-section of the streamtube at an intermediate section. The panel is allocated a normal velocity and pressure that represent the streamtube conditions at the station. Veldhuis (2005)

In keeping with the theme of simplicity and the use of low-order methods, the method of integration selected for this work was akin to the “add-on” requiring separate computation of the propeller wake, via a propeller performance prediction model, and the aerodynamic performance of the wing located in the wake of the propeller.

3.4. Experimental Data

Wind tunnel tests of propellers have been carried out extensively since their inception. Publicly available test data from reputable sources was used to validate the performance model algorithm confirming that it accurately represents the underlying mathematical model and it's solution.

Validation of the underlying propeller performance model was carried out using wind tunnel test results of an 8x8 APC Thin Electric propeller from Merrill (2011). This propeller has a very simple geometry and was modeled using a simplified lift curve slope based on an assumed and constant aerofoil profile rather than a more comprehensive lookup table.



Figure 1: APC Slow Flyer, Sport, and Thin Electric propellers. (Merrill 2011)

Very little experimental data was available in the literature pertaining to the measurement of phenomena within the extended slipstream of a powered propeller.

Quantitative data was therefore not made available for validation of the propeller slipstream prediction capabilities and the slipstream model results were discussed in relation to expected behaviour.

The propeller performance model was then validated by simulating a more complex propeller model over a wider range of test conditions and utilising 2-dimensional aerofoil data from Monk (2010) which was obtained using XFOIL.

Experimental data collected by the NREL in the NASA-Ames wind tunnel tests ((Lindenburg 2003)) was used to verify the propeller model's capabilities in predicting the performance and slipstream characteristics of a wind turbine.

Wind tunnel test data pertaining to the configurations of wing-propeller, and wing and propeller separately (Witkowski et al. (1989)) was considered for the validation of the panel method model to analyse the fidelity of the propeller power effect capability.

3.5. Delineation and Limitations

The model functions only for steady state conditions. The more general situation of the aircraft accelerating, climbing up, turning around or moving sideways as well as forward is not considered due to the complexity of the airflow that would exist in such situations. Even with these simplifying assumptions applied, the problem will still require significant computational resources.

Propeller performance model results are dependent on the accuracy of the input parameters obtained from external sources including: propeller geometry, propeller aerofoil characteristics, fluid properties, and flight settings.

The propeller slipstream model is limited to small angle considerations due to the simplifying assumptions of the vortex wake model that is used.

3.6. CMARC Integration of Propeller Performance and Wake Model Results

3.6.1. Introduction

The aerodynamic interaction effects of a wing operating in the slipstream of a propeller are simulated by super-imposing propeller wake velocities onto the uniform freestream flow ahead of a low-order 3-dimensional panel method wing model. The combined model predictions are evaluated in a comparison with wind tunnel tests of a model wing-propeller.

The two major assumptions of this model are that propeller performance is not altered by the presence of the wing and that propeller wake velocities are steady.

A combined experimental and computational investigation of the aerodynamic interaction between propellers and wings was conducted at the Purdue University Aerospace Laboratory. A wind tunnel test was performed using an untapered, semispan wing and a Purdue model propeller mounted in a tractor arrangement. Witkowski et al. (1989)

3.6.2. Wing-Propeller Wind Tunnel Tests

Witkowski et al. (1989) presents results of experimental testing performed at the Purdue University Aerospace Sciences Laboratory in the 3 x 4.5-ft Low-Speed Subsonic Wind Tunnel. The tests were conducted in the tunnel's closed test section where velocities of approximately 105 m/s can be attained. Test-section dynamic pressure was maintained at a preset value by an electronic feedback system. A pitot-static probe upstream of the test configuration measured the dynamic pressure. Total temperature was measured by a thermocouple in the tunnel stilling section.

All tests were conducted at low subsonic speeds, $M = 0.1$, and a Reynolds number of 470 000 based on a wing chord of 0.2m. All tests were conducted under approximately incompressible conditions at a dynamic pressure of 0.1 psi. This provided a freestream velocity of approximately 35 m/s. Propeller speeds were

varied from windmill (48 rps, $J = 2.4$) to 100rps at maximum power ($J = 1.1$). Wing angle of attack was varied from 0° to 13° . The tests covered thrust coefficients corresponding to free-stream velocities from zero forward speed to the normal range of cruising speeds.

3.6.3. Wind Tunnel Propeller Model

A two-blade propeller based on Purdue internal specifications was tested in the wind tunnel. The propeller had a constant chord of 0.051m, a diameter of 0.305m and an aspect ratio of 6. The propeller had a Goldstein's minimum induced-loss twist distribution. The propeller hub and nacelle diameters were of the same magnitude as the blade chord being 0.051m. The blade aerofoil section was a NACA 0010, and the pitch angle at the $\frac{3}{4}$ radius location was set at 45.4° . The propellers were driven by variable-frequency electric motors rated at 20 horsepower at 18 000 rpm.

3.6.4. Panel Method Wing Model

A CMARC model of the basic wing was created for a first-phase validation of the CMARC panel code prediction qualities against the wing-only wind tunnel test results of Witkowski et al. (1989). The input file is provided in Appendix B but may not intuitively be understood as it uses a Fortran NAMELIST format in which variable names are joined to values by equals signs.

An interesting aspect of the CMARC input is related to the definition of fluid (or alternatively aircraft) speed where both the fluid velocity and speed of sound must be defined. Values are either defined in real terms for both values or alternatively a value of 1.0 for the freestream velocity and the reciprocal of the design flight Mach number for the speed of sound entry. The Purdue report does not provide any information on the speed of sound at the test facility and at the time of the wind tunnel tests, a sea-level value of 340 m/s was therefore assumed.

The wing was modeled using 130 chordwise coordinates spread across the upper and lower wing surface. The $x = 0$ line of the CMARC coordinate reference runs through the quarter chord line of the wing. As the wing is straight and untapered, only

10 spanwise sections are defined albeit using a full cosine spacing to increase the section density at the spanwise extremities.

As the simulation involves the computation of lift, a wake is defined to prevent violation of the Kutta condition at the trailing edges of lifting surfaces. The wake is defined by identifying the wing trailing edge as the line of attachment of the wake and extending this attachment line some distance downstream. Intermediate sections are filled based on the specified number of panels.

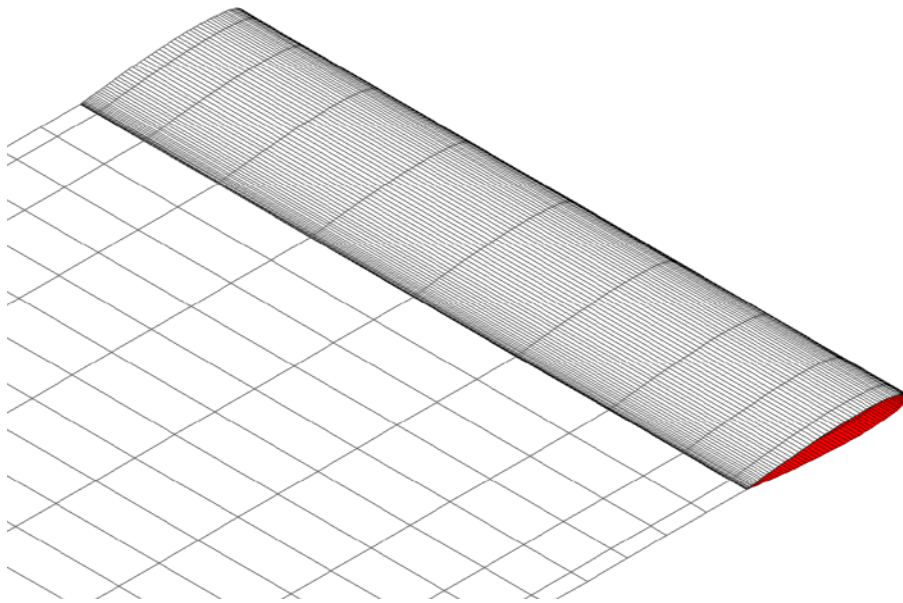


Figure 2: CMARC wing model

The equations used by CMARC to determine pressures and velocities over the model surface do not take viscosity into account. An important characteristic of an inviscid analysis is that in the absence of viscosity there is no drag. Consequently, no inferences about pressure drag can be made from a CMARC 3D analysis. This can be seen in Figure 3 below where the CMARC predictions fail to taper off towards the stall condition. Across the lower angle of attack range, the predicted values closely resemble those of the wind tunnel experiments.

The $J = 2.4$ condition at which the wind tunnel experiment was conducted refers to the windmill setting with the propeller unpowered but present within the incoming flow field and unlocked so as to allow it to spin due to the incoming air flow. Witkowski et

al. (1989) describes previous comparisons where the windmill data and the isolated wing data were virtually identical making the windmill data a useful comparison against the isolated wing predictions.

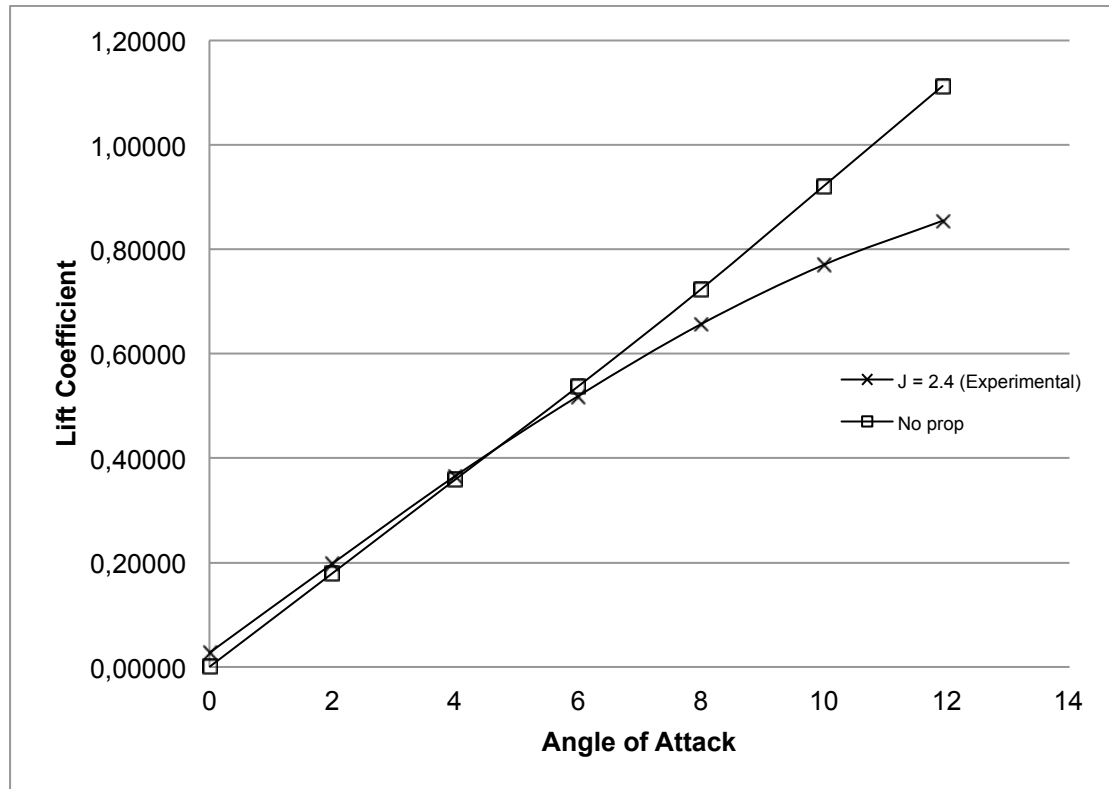


Figure 3: Comparison of predicted and experimental lift coefficient

3.6.5. CMARC Panel Method Wing-Propeller Combination

A suitable simulation was not achieved within CMARC using the addition of the slipstream velocities and pressures as a localized add on to the onset flowfield. A limitation of CMARC was discovered in its inability to model portions of the flowfield that have an elevated or different total pressure. The code was able to simulate a propeller as a disc with specified inflow and outflow velocities but in practice it was found to simply impede the onset flow rather than imitate an actuator disk resulting in a failed attempt to model the propeller-wing interaction in the late stages of this work.

The effect of interaction on wing performance is presented in Figure 4 below comparing the experimental results of Witkowski et al. (1989) to those of the CMARC

wing model with integrated propeller disk. Comparing the CMARC model results of Figure 3 with Figure 4 for the $J = 2.4$ windmill case it is immediately evident that the addition of the propeller disk does not suitably represent an unpowered propeller in the windmill setting. For this simulation the CMARC diaphragm representing the propeller disk was allocated a normal velocity equivalent to the freestream flow, the diaphragm resultantly impeded the onset flow with an average localized drag coefficient of $C_D = 53$. The 'propeller effect' resulting from the inclusion of this propeller disk diaphragm was a reduction in onset velocity and an increase in drag which effectively lowered the lift coefficient.

For the case of a powered propeller, Witkowski, et al. (1989) describes that the wing drag coefficient for this configuration can be reduced up to 155 counts at $C_L = 0.8$ and maximum propeller power ($J = 1.1$). Drag was reduced by a maximum percentage of 81% at $C_L = 0$ and $J = 1.1$. At the more typical operating lift coefficient of 0.4, drag was reduced up to 67%.

At an advance ratio of $J = 1.3$ the increase in lift can clearly be seen as compared to the windmill setting, this is to be expected. The CMARC model displays the opposite trend further reducing the lift coefficient and, in this case, producing an average drag coefficient around the propeller disk diaphragm of 55.

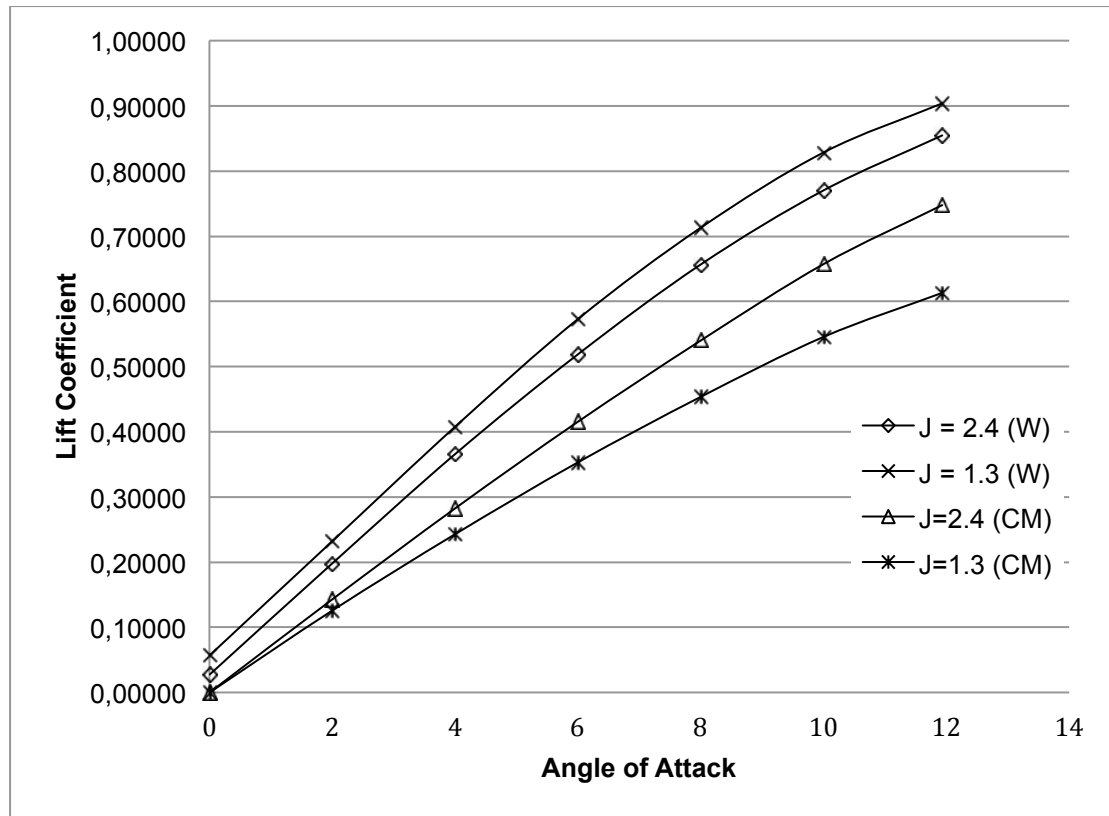


Figure 4: Comparison of wing lift curve augmentation with propeller effects - "W" = Witkowski et al. (1989); "CM" = CMARC Model

CMARC Propeller Disk Diaphragm

The CMARC literature describes a functional method as: define the propeller as a diaphragm that is able to simulate accelerated flow by applying normal velocities to the face. The diaphragm is modeled as two patches in the shape of a flattened toroid. They touch at their inner and outer edges and form a cross-section resembling a thin lens. Aerologic (1995)

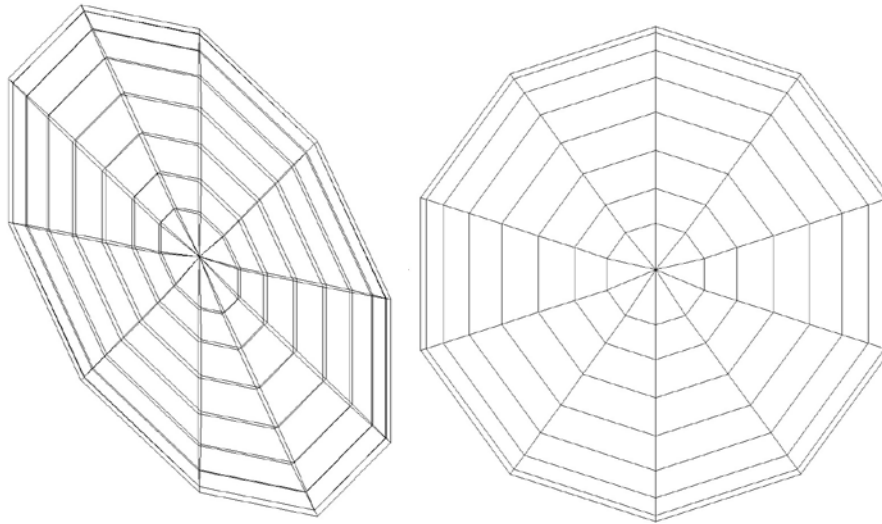


Figure 5: CMARC Propeller model in the forms of an annular diaphragm

As with the momentum theory actuator disc, the diaphragm must be modeled to have constant velocity across its inlet. To prevent the averaging of velocities between the diaphragm and neighbouring surfaces, the diaphragm is assigned to a separate assembly within the input file to ensure that the calculation is taking into account the specified inlet velocity.

The diaphragm used in the attempted simulation is made up of 10 angular sections and 10 radial annuli with a full cosine spacing used in the radial direction. Figure 5 depicts the meshed geometric structure. The propeller appears to be modeled as a set of annular rings with a common centre-point. The centre-point is, in-fact, an overlay of an equal number of co-located nodes as there are angular sections.

As it has been assumed that the propeller performance is not affected by the presence of the wing, the combination of the propeller disc and wing models is in-effect the superposition of the accelerated velocity vectors from the propeller disc diaphragm outlet over the portion of the wing that sits directly behind the propeller.

While the propeller disc is assigned to a separate assembly than the wing, it maintains the same coordinate reference as the wing such that for finite angles of attack, the wing and propeller are pitched about the quarter chord line of the wing.

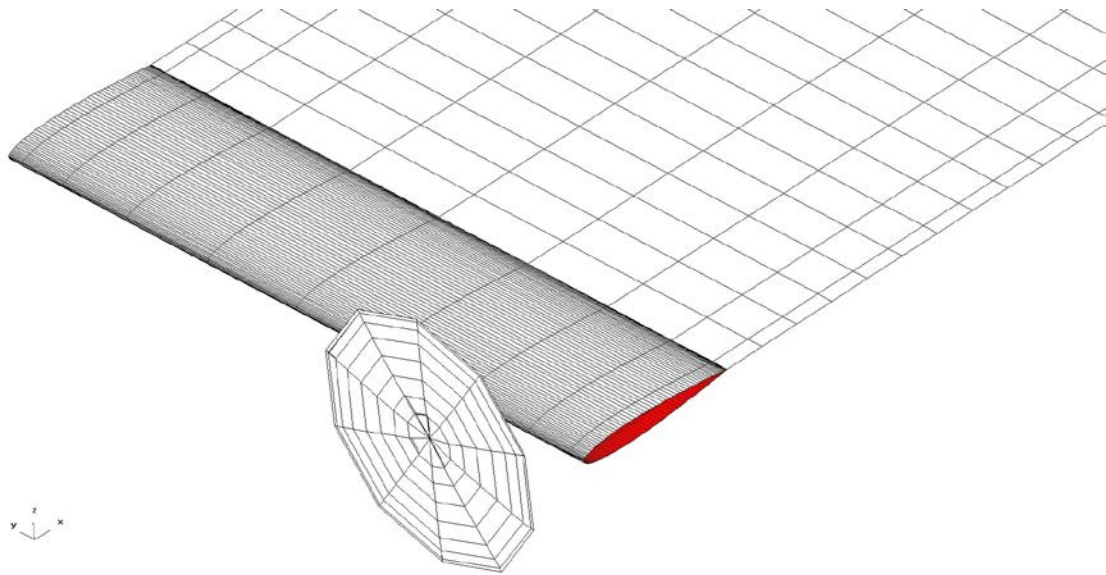


Figure 6: CMARC wing propeller combination

Due to the failure to successfully integrate the propeller performance and wake predictions into the intended panel method code, the scope of this work was adjusted to focus on the suitability of the underlying prediction method and its full set of capabilities as a stand-alone offering. Additional work was performed to evaluate whether the propeller model could be extended to consider the performance and wake of a wind turbine.

4. ANALYTICAL MODELING

4.1. Introduction

In modeling the performance of a propeller, the problem statement can be defined as follows. Given the geometry of the propeller blades, it should be possible to estimate, reasonably accurately, the performance of the propeller at any setting of the operating conditions. The propeller is assumed to be attached to an aircraft that is cruising at a certain altitude and a certain speed, or the aircraft may be stationary with the engine being at full power waiting to take-off.

The flow over the propeller is not interfered with in any way by it's proximity to other bodies and lifting surfaces even though the propeller is attached to an aircraft. The forward speed of the propeller as well as it's rotational speed must be given. The number of propeller blades is given and the blades are attached symmetrically around the hub. The radius of the propeller is given, as is the hub radius.

The attachment of the blade to the hub is such that the whole blade can be rotated about an axis perpendicular to the hub axis, which is known as the blade pitch axis. The geometry of the blade is given as the geometry of a number of blade elements that are obtained if the blade is cut up using planes perpendicular to the pitch axis.

The shape of an aerofoil is defined by the coordinates of a large but finite number of points on it's surface. The local blade twist angle is the angle subtended by the aerofoil's chord line and the plane of the propeller disc. The overall blade pitch angle is defined to be the local pitch angle of the aerofoil at a radial station of 70% propeller radius.

The performance of a propeller can be described in terms of the non-dimensional coefficients: thrust coefficient, power coefficient, and advance ratio. The efficiency of the propeller can be determined from these three parameters. The coefficients used in this analysis are:

Thrust coefficient $C_T = T/(\rho n^2 D^4)$

Torque coefficient	$C_Q = Q/(\rho n^2 D^5)$
Power coefficient	$C_P = P/(\rho n^3 D^5)$
Advance ratio	$J = V/nD$
Efficiency	$\eta = V/nd (C_T/C_P)$

In the following sections the development of the BEMT method with VT slipstream considerations is discussed in great detail along with the relevant assumptions. This is followed by the application of the theory in predicting the thrust, torque, and power of a propeller along with the behaviour of the wake propagating behind it. The discussion is sufficiently detailed to include the development of an algorithm for the simulation of the method and is then built into a working computer simulation to numerically compute those properties if the propeller's geometry and it's operating conditions are given as data.

4.2. Propeller Aerodynamics Analysis Method

4.2.1. Introduction

A propeller essentially consists of a series of identical blades attached axisymmetrically to a central and common hub. Each blade may be thought of as being similar to a high aspect ratio wing, thus the analysis may borrow from the aerodynamic analysis of an aircraft wing.

A propeller blade is often twisted geometrically such that the geometric angle of attack distribution of the airflow over the blade varies along the span. Furthermore, even if the blade is not twisted geometrically, the distribution of aerodynamic properties along the span may vary due to changes in aerofoil section. A propeller is therefore described by a number of parameters including: planform, aerofoil shape and distribution, rotational speed, and twist distribution as well as the angle of attack, Reynolds number, and Mach Number of the flow.

In order to represent a given flow-field, an airframe-fixed coordinate system is defined, with the x-axis coincident with the axis of rotation of the propeller. The origin

lies in the propeller plane that serves as the reference point for all axial dimensions of the propeller blade surfaces. The radial coordinate is denoted by r , and the angular coordinate by ϕ , which is measured in a clockwise (right-handed) sense when looking downstream with $\phi = 0$ being at 12 o'clock. Kerwin (2001)

4.2.2. One Dimensional Momentum Theory

The actuator disc or Momentum Theory description introduced in the Literature Review assumes a control volume in which the boundaries are the surface of a streamtube with entry and exit sections far upstream and downstream of the actuator disc. The only flow is across the ends of the streamtube. A simple schematic of this control volume is given in Figure 7.

The disc creates a uniform discontinuous change in total pressure to the fluid passing through the disc, which tends to accelerate the fluid in the positive axial direction and thus results in a force in the negative x direction. No tangential velocities, or swirl, are introduced by the disc and, thus by conservation of angular momentum, there is no torque supplied. Kerwin (2001)

The following assumptions are applied: Mikkelsen (2003)

- a) Flow is steady, homogeneous, and fixed in direction;
- b) Fluid is incompressible, inviscid, and irrotational;
- c) Both the flow and the thrust are uniform across the disc;
- d) Static pressure far upstream and downstream of the propeller is equal to ambient static pressure.

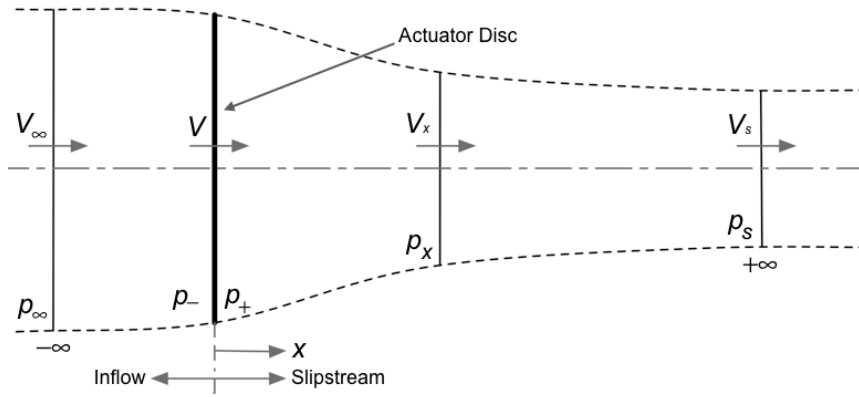


Figure 7: Propeller wake streamtube

The actuator disc is effectively the limit, in an ideal fluid, of a propeller with an infinite number of blades, zero chord length, and infinite rotational speed.

Upstream of the propeller disc the diameter of the streamtube is somewhat larger than the disc diameter, it then contracts to match the diameter of the disc as the flow passes through the disc, and continues to contract as it moves further downstream.

Since no fluid is created within the disc, axial and radial velocities are continuous. Between P_- and P_+ momentum is added to the flow and there is a change in pressure as a result. It is assumed that $P_\infty = P_s$ and that $V_- = V_+ = V$. The distributed forces on the propeller disc alter the local velocities through the disc and, in general, the entire flowfield around the disc. The balance between the applied forces and the changed flowfield is governed by the mass conservation law and the balance of momenta.

From the conservation of linear momentum, the thrust is equal and opposite to the rate of change of momentum of the flow-field. The thrust imparted to the air by the propeller is also equal to the mass flow rate within the streamtube multiplied by the difference between the air velocity leaving and that entering the tube. For steady flow, $(\rho AV)_\infty = (\rho AV)_s = \dot{m}$, (Kerwin (2001))

$$T = \dot{m}(V_s - V_\infty) \quad (4.1)$$

While the air velocity is regarded as changing gradually from V_∞ to V_s , the pressure of the flow immediately upstream of the disc is P_- and suddenly jumps to P_+

immediately downstream. The thrust is positive such that the velocity behind the propeller, V_s , is higher than the free-stream velocity, V_∞ . No work is done on either side of the propeller blade. By considering the thrust as the net sum of the forces on each side of the actuator disc and solving for the pressure on either side of the disc, the thrust can be expressed as, Winarto (2004)

$$T = \frac{1}{2} \rho A (V_s^2 - V_\infty^2) \quad (4.2)$$

By equating the thrust from Equations 4.1 and 4.2 and recognising that the mass flow rate $\dot{m} = \rho A V$, the flow velocity at the propeller plane, using this simple model, is the average of the upstream and downstream flow speeds. If one defines the axial induction factor, a , as the fractional increase in flow velocity between the freestream and the propeller plane, then the velocity can be described by (Mikkelsen (2003))

$$V = V_\infty (1 + a) \quad (4.3)$$

and,

$$V_s = V_\infty (1 + 2a) \quad (4.4)$$

It is evident that the definition of a is only valid if V is not zero. When V is not zero, the quantity $V_\infty a$ is the axial velocity induced by the propeller disc. The quantity, $V_\infty a$, is often referred to as the induced velocity at the propeller, in which case the velocity of the flow at the propeller is a combination of the free-stream velocity and the induced flow velocity. Winarto (2004)

The Power input, P , is considered to be the product of the thrust and the velocity at the disc. With the consideration of Equations 4.3 and 4.4,

$$P = \frac{1}{2} \rho A V_\infty^3 4a(1 + a)^2 \quad (4.5)$$

From Equations 4.2 and 4.4, the axial thrust on the disc is, (Manwell et al. (2009))

$$T = \frac{1}{2} \rho A V_{\infty}^2 4a(1+a) \quad (4.6)$$

4.2.3. Propeller Momentum Theory with Wake Rotation

In the one-dimensional momentum theory it was assumed that no rotation was imparted to the flow. Conserving angular momentum within the streamtube control volume necessitates rotation of the wake if the propeller is to impart useful torque. In the case of a rotating propeller, the flow behind the propeller rotates in the opposite direction to the propeller in reaction to the torque exerted by the flow on the propeller. An annular streamtube model of this flow, illustrating the rotation of the wake, is shown in Figure 8.

The Rankine-Froude actuator disc model is now extended to account for this wake rotation. To do this, assumptions (a) and (c) from the previous analysis are relaxed and three supplementary assumptions added: (Jonkman (2003))

- a) The control volume used in the previous one-dimensional model can be split into many non-interacting annular stream tube control volumes;
- b) The flow entering the control volumes(s) far upstream remains purely axial and uniform;
- c) The angular speed of the wake far downstream of the propeller is low so the static pressure far downstream can still be assumed to be that of the unobstructed ambient static pressure.

Assumption (a) disregards assumption (c) from the previous analysis and allows for the local pressures, axial velocities (and induction factors), wake rotation, angular velocities, thrust, and power to all be functions of the annular radius. Consistent with the uniform flow assumption of the previous analysis, the assumption of uniform axial velocity is continued but will not allow for a uniform angular velocity as well.

With the introduction of rotation in the slipstream, the Momentum Theory assumption of uniform pressure becomes invalid for all cases other than that of zero thrust.

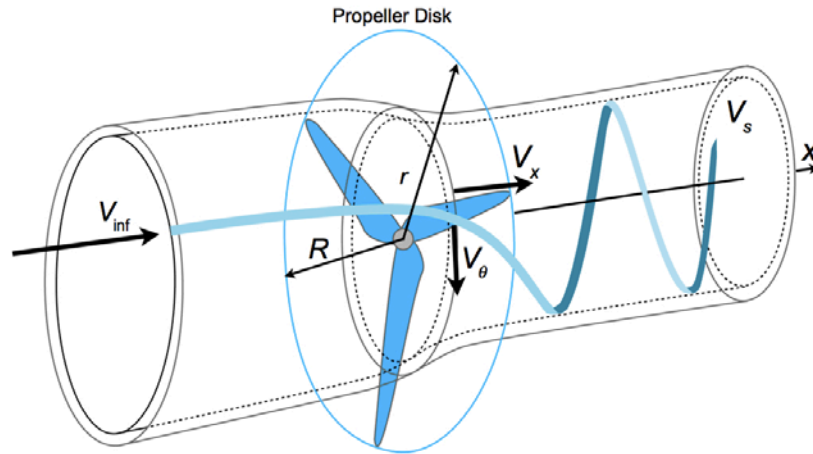


Figure 8: Streamtube across a three-bladed propeller

Although wake rotation is now included in the analysis, the assumption that the flow is irrotational has not been lifted. The definition of rotational flow is any flow with non-zero vorticity. A fluid particle that moves in a circular path but always retains its original orientation has zero vorticity and is considered to be irrotational.

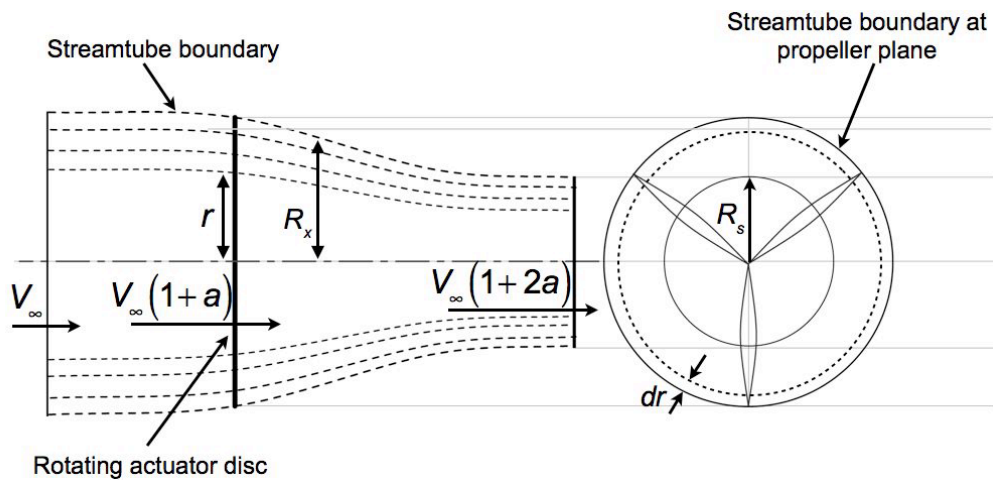


Figure 9: Annular streamtube control volumes

Figure 9 gives a schematic of the parameters involved in this analysis. The generation of rotational kinetic energy in the wake results in less propulsive efficiency than would be expected without wake rotation, Phillips (2002) estimates this to be in the order of 5%. If it is assumed that the angular velocity imparted to the flow-field is small compared to the angular velocity of the propeller blade, then it can also be

assumed that the pressure in the far wake is equal to the pressure in the freestream. Wilson et al. (1976)

The conservation of angular momentum about an axis consistent with the control volume's axis of symmetry is applied to obtain the differential torque at the propeller disc,

$$dQ = \rho V \omega_+ r^2 2\pi r dr \quad (4.7)$$

Since applying the conservation of linear momentum in the axial direction is unaffected by wake rotation, the differential thrust at the propeller disc, dT , can be found by differentiating Equation 4.6 with respect to r ,

$$dT = \frac{1}{2} \rho 2\pi r V_\infty^2 4a(1+a) dr \quad (4.8)$$

The pressure, wake rotation, and induction factors are now a function of the annular radius. This is necessary as the streamtube is now represented by a set of radial annuli.

To arrive at a description of the wake rotation, it is beneficial to find the differences in pressures P_+ and P_- not by using Bernoulli's equation, but by the energy equation. The approximate form of the energy equation is a replacement for Bernoulli's equation when viscous, heat transfer, and work insertion effects are more important.

Jonkman (2003) uses this form of the energy equation to provide an alternative relation for the differential propeller thrust can be described as,

$$dT = \rho \left(\Omega - \frac{1}{2} \omega_+ \right) \omega_+ r^2 2\pi r dr \quad (4.9)$$

An angular induction factor, $\acute{a}$, is now defined as,

$$a' = \frac{\omega}{\Omega} \quad (4.10)$$

where $\omega = \frac{\omega_+}{2}$ since the angular velocity of the flow stream at the plane of the propeller (at the jump discontinuity) is the average value of the angular velocity just upwind and downwind of the propeller. The differential propeller thrust, dT , and differential propeller torque, dQ , can now be written in terms of the two induction factors, a and a' , Jonkman (2003)

$$dT = \rho \Omega^2 r^2 4a'(1-a')\pi r dr \quad (4.11)$$

and,

$$dQ = \rho V_\infty \Omega r^2 4a'(1+a)\pi r dr \quad (4.12)$$

Equating the differential propeller thrust from Equation 4.11 with that of Equation 4.8, the two induction factors are related. With the necessary substitutions, the angular induction factor is written in terms of the axial induction factor and the local speed ratio,

$$a' = \frac{1}{2} \left[\sqrt{1 + \frac{4}{\lambda_r^2} a(1+a)} + 1 \right] \quad (4.13)$$

4.2.4. Blade Element Theory

The forces on the blades of a propeller can also be expressed as a function of lift and drag coefficients and the angle-of-attack. This state of flow is governed by the characteristics of the propeller blades, such as aerofoil shape and twist distribution. As the word element suggests, blade element theory again uses several annular streamtube control volumes.

At the propeller plane, the boundaries of these control volumes effectively split the blade into a number of distinct elements, each of length dr . The velocity at a blade

section cut by a cylindrical surface is illustrated in Figure 10. At each element, blade geometry and flow-field properties can be related to a differential propeller thrust, dT , and a differential propeller torque, dQ . If a number of assumptions are made:

- Just as the radial annuli used in the wake rotation analysis were assumed to be non-interacting, it is assumed that there is no interaction between the analyses of each blade element;
- The forces exerted on the blade elements by the flow-field are determined solely by the 2-dimensional lift and drag characteristics of the blade element aerofoil shape and orientation relative to the incoming flow.

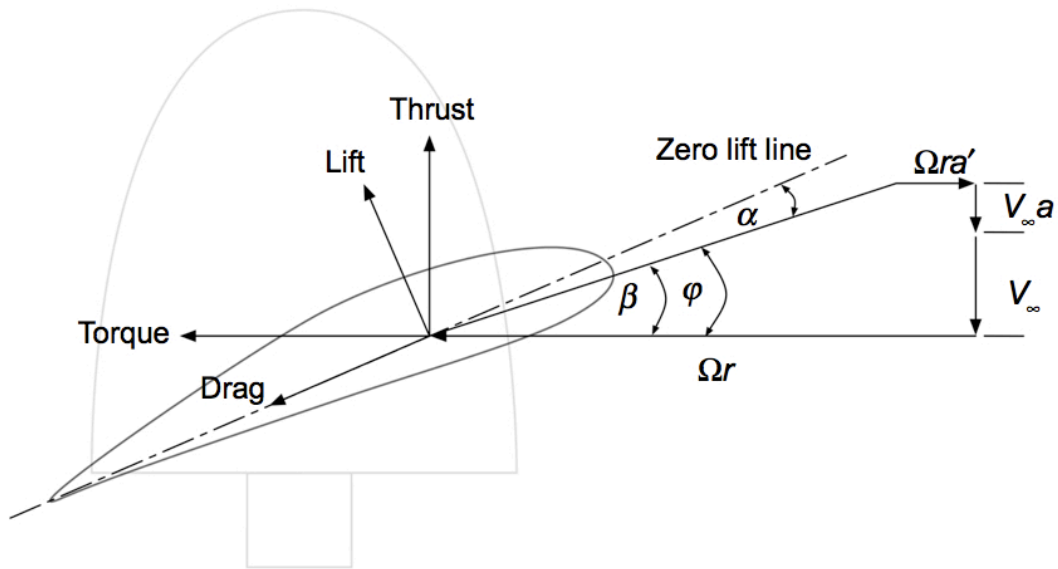


Figure 10: Flow and force directions at a blade element

The velocity of the incoming flow stream relative to the blade element, U , is the vector sum of the axial inflow velocity at the propeller plane, V , the inflow velocity caused by the rotation of the blade, Ωr , and the inflow velocity caused by wake rotation at the propeller plane, ωr , Wald (2006)

$$U = \sqrt{[V_{\infty}(1+a)]^2 + [\Omega r(1-a')]^2} \quad (4.14)$$

Additionally, using geometry, a relationship can be developed relating a , a' , and ϕ ,

$$\tan \varphi = \frac{V_{\infty}(1+a)}{\Omega r(1-a')} \quad (4.15)$$

The relationships between the resultant forces are, Kerwin (2001)

$$dT = dL \cos \varphi - dD \sin \varphi \quad (4.16)$$

$$dQ = dL \sin \varphi + dD \cos \varphi \quad (4.17)$$

The effective angle-of-attack is

$$\alpha = \beta - \varphi \quad (4.18)$$

To be precise, the lift, drag, normal, and tangential forces shown in Figure 10 and used in the equations represent the differential components of the force imparted by the flow on the blade element for which the cross-section is shown. In addition, all the differential forces shown in Figure 10 represent the differential forces acting on a single blade. If the propeller consists of B identical blades, Kerwin (2001)

$$dT = B \frac{1}{2} \rho U^2 (C_L \cos \varphi - C_D \sin \varphi) c dr \quad (4.19)$$

$$dQ = \frac{1}{2} B \rho U^2 (C_L \sin \varphi + C_D \cos \varphi) c r dr \quad (4.20)$$

dT and dQ now represent the total differential thrust and differential torque, respectively, acting on an annular ring of radius r and thickness dr .

4.2.5. Blade Element Momentum Theory

BEM theory is based on the fact that the differential propeller thrust, dT , and differential propeller torque, dQ , derived from momentum theory and blade element theory must be equivalent.

Before these equivalency relationships are applied, the momentum theory equations are customarily adjusted to account for the finite number of blades that are inherent in real aircraft propellers - the number of blades, B , cannot be infinite. The most common adjustment is the straight forward tip-loss - or correction - factor developed by Prandtl and Glauert (1935). The tip-loss factor is defined as,

$$F = \frac{2}{\pi} \cos^{-1} e^{-f} \quad (4.21)$$

where,

$$f_{tip} = \frac{B(R-r)}{2r \sin \varphi} \quad (4.22)$$

Prandtl's tip-loss factor is incorporated into the momentum theory equations such that,

$$dT = F \rho V_{\infty}^2 4a(1+a)\pi r dr \quad (4.22)$$

$$dQ = F \rho V_{\infty} \Omega r^2 4a'(1+a)\pi r dr \quad (4.23)$$

The tip-loss factor does not affect the blade element theory equations. By equating the differential propeller thrust from momentum theory, Equation 4.22, and blade element theory, Equation 4.19, and then incorporating the local solidity factor, σ , the axial induction factor can be estimated as follows,

$$a = \left[\frac{4F \sin^2 \varphi}{\sigma' (C_L \sin \varphi + C_D \cos \varphi)} + 1 \right]^{-1} \quad (4.24)$$

By equating the differential propeller torque from momentum theory, Equation 4.23, and blade element theory, Equation 4.28 and then including σ , the tangential induction factor may can be estimated as follows,

$$a' = \left[\frac{4F \sin \varphi \cos \varphi}{\sigma' (C_L \sin \varphi + C_D \cos \varphi)} + 1 \right]^{-1} \quad (4.25)$$

The BEMT developed above is known to perform well in representing the real world performance of a propeller while maintaining low computational complexity. This is largely due to the fact that flow over a 3-dimensional wing is considerably more difficult to solve numerically than that for the flow over a 2-dimensional aerofoil. As previously described, the BEMT assumes that the flow is locally 2-dimensional however, in reality the flow is strongly 3-dimensional with a cross-flow happening along the wingspan. To improve the accuracy of the BEMT as the model is extended into the far wake, this 3-dimensionality must be reintroduced into the flow-field using a suitable modeling technique from the literature. A method that has been found to be quite effective in modeling 3-dimensional flow is to assume that a vortex sheet is formed by the propeller blade and is shed at the trailing edge of the blade. Wald (2006)

4.2.6. Propeller Vortex Theory

The BEMT forces are uniquely related to the conditions some distance behind the propeller. This relationship is explored in the following sections to provide expressions for the behaviour of the wake behind a propeller. For this the vortical nature of the propeller wake is considered.

The flow field associated with each blade of a propeller rotating with its axis aligned to a uniform stream is similar to that of a wing. The local lift at any radial station on a blade is associated with the circulation around the blade. As with a wing, the circulation around the blade (the bound vorticity) varies along the blade from root to tip resulting in the shedding of a sheet of vortex filaments from the blade trailing edge (trailing vorticity), see Figure 11. Jones et al. (1985)

In modern propeller theory it is generally considered that, after an initial distortion, the vortex sheets shed from the trailing edges of the propeller blades form a set of interleaved helicoidal sheets which translate uniformly downstream parallel to the axis without further deformation as if they were rigid surfaces. In reality, these

helicoidal sheets will soon roll up into a set of helical vortex filaments and a central vortex filament of opposite sense along the axis of rotation. The helicoidal vortex sheets are floating freely in an irrotational field with equal velocity on either side of the sheet, hence equal pressure. The quasi-steady vortex sheets are uniquely related to the induced velocities and the distribution of bound vorticity on the propeller blades. Wald (2006)

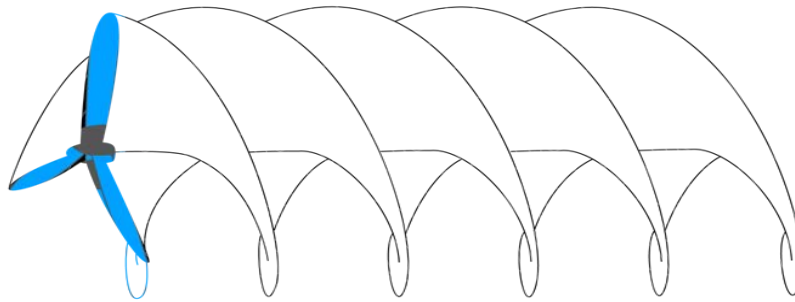


Figure 11: Propeller with trailing helicoidal vortex sheets

Blade Lift and Thrust

The influence of the wake on the flow in the propeller plane can be described as if it results from the vorticity in the wake. Although this approach is of practical use in the calculation model, a vortex can be used to describe the circulation in the flow. The mathematical relations for vorticity are based on the continuity relations and conservation of momentum for an incompressible fluid.

The fundamental expressions for the forces developed by the propeller may be described most conveniently by applying the Kutta-Joukowski theorem, $dL = \rho U \times \Gamma dr$, where the cross-product relationship can be used to separate the two components of the resultant force, the thrust and the torque. Wald (2006)

$$dT = \rho \Gamma \Omega r (1 - a') dr \quad (4.26)$$

$$dQ = \rho \Gamma V_{\infty} (1+a) r dr \quad (4.27)$$

Equating Equations 4.26 or 4.27 with 4.22 or 4.23, respectively, the bound vorticity on a blade element can be related to the local angle of attack. Mikkelsen (2003)

Vorticity in the Far Wake

Since there is no pressure discontinuity across the vortex sheets, they may be seen to move axially backward without deformation. To approximate this it is assumed that the bound vorticity is transformed in the wake onto the surface of a vortex tube. There will be two components of this vorticity – a tube of elemental annular vortices that account for the thrust, and lines of elemental axial velocity along the tube that account for swirl. Gilbey and Green (2006)

At this stage only the annular vorticity is considered having strength, γ , and the following compatibility requirements:

- a) The total pressure within the wake is greater than that of the freestream by the propeller loading, $T/\pi R^2$. The inner and outer flows must, however, have the same static pressure,

$$\frac{1}{2} \rho V_s^2 = \frac{1}{2} \rho V_{\infty}^2 + \frac{T}{\pi R^2} \quad (4.28)$$

- b) Considering the conservation of mass, the radius of the fully-developed slipstream, R_s , is related to that of the propeller, R ,

$$VR^2 = V_s R_s^2 \quad (4.29)$$

- c) Considering vorticity, the velocity jump across the wake boundary,

$$\gamma = V_s - V_{\infty} \quad (4.30)$$

With these requirements we are able to develop a preliminary description of the propeller slipstream, this is discussed in the following section.

4.2.7. Propeller Slipstream

Equations are developed to describe the deformation and deflection of the propeller wake. The aerodynamic forces generated by a propeller under any deviation from uniform flight parallel to the thrust axis will lead to the deflection of the slipstream.

For simplicity the discrete element approach that was used in the previous sections is now abandoned. The case of the actuator disc is reconsidered but only now in conjunction with trailing wake vorticity. In this simplification each propeller blade is considered to have a constant bound vorticity of strength Γ shed at the tips. Wald (2006)

Vorticity Compatibility

In order to relate the bound vorticity to that distributed in the wake, it is necessary to consider the vorticity shed from the blade tips per unit time. Considering the swirl component of the wake in isolation represented by the vorticity, γ_s , the axial component of the bound vorticity shed from the tip of the propeller blades per unit time is related to the elemental axial vorticity lines along the tube by, Gilbey and Green (2006)

$$B\Gamma V\Delta t = \gamma_s 2\pi R_s V\Delta t \quad (4.31)$$

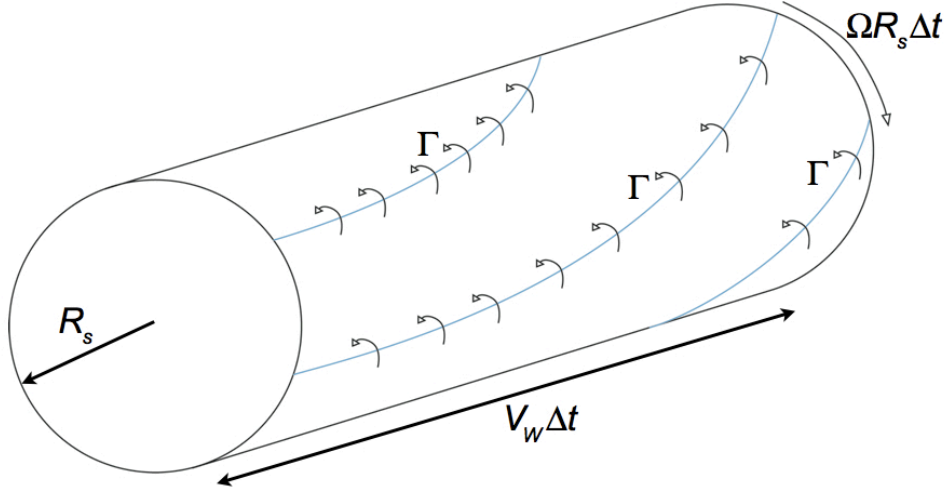


Figure 12: Vortex tube propagation with bound vorticity shed per unit time

This relation is depicted in Figure 12 and satisfies the condition that the total vorticity shed at the tips, $B\Gamma$, must also be shed in the opposite sense at the roots as a discrete vortex down the axis.

Behind the propeller the vortex tube propagates downstream as if it is pressed between the inner and outer flows with a velocity V_w .

$$V_w = \frac{V_s + V_\infty}{2} \quad (4.32)$$

Gilbey and Green (2006) states that the vortex tube is propagating at the mean rate of the inner and outer flows, which seems physically reasonable as it suggests it is acting as an intermediary between the two. This combined vortex model ensures that the swirl is contained within the streamtube and induces no flow outside the wake.

Propeller at Incidence

The case of a propeller at incidence to the axis of the wake is now considered. In addition to the generation of thrust, as an isolated blade of an inclined propeller rotates, the blade tip creates an in-plane force. Relative to the slipstream, the down-going blade will have a forward velocity component, whereas the up-going side will

be retreating. The forces developed by an inclined propeller are depicted in Figure 13.

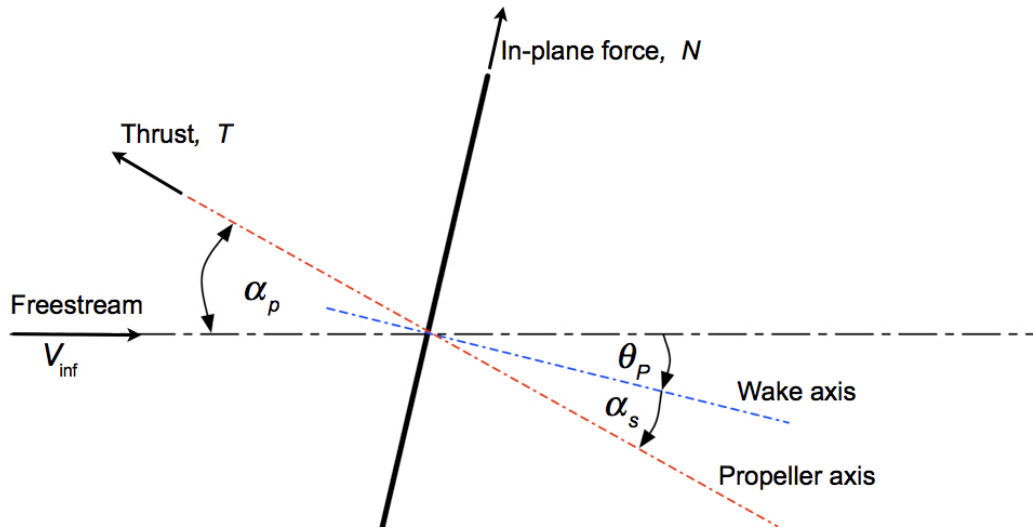


Figure 13: Propeller at incidence

Taking an angular position ϕ around the propeller disc, where $\phi = 0$ is at the top, then the vertical component of the tip velocity equals $\Omega R_s \sin \phi$, and the forward component of this is $\Omega R_s \alpha_s \sin \phi$ for small α_s . Equation 4.31 now becomes,

$$B\Gamma(V + \Omega R_s \alpha_s \sin \phi) \Delta t = \gamma_s 2\pi R_s V \Delta t \quad (4.33)$$

Considering the thrust contribution in isolation, the forward component of the tip velocity introduces an incremental sinusoidal variation of swirl velocity that will induce a downwash. This sinusoidal vorticity distribution will determine the deflection angle of the slipstream as it is counterbalanced by the upwash from the freestream. Gilbey and Green (2006)

For an idealised propeller blade with constant bound vorticity shed at the tip, the in-plane force acting normal to the blade will be $\rho \Gamma V R$ (for $\alpha_p = 0$). When $\alpha_p \neq 0$, to account for the net normal force, N , it is assumed that, in addition to any constant bound vorticity that may be present to generate thrust, there is a component of bound vorticity that varies sinusoidally around the propeller disc.

Integration around the propeller disc, allowing for the number of blades, B , gives an average of

$$N = \frac{1}{2} \rho V B R \Gamma \quad (4.34)$$

The annular component of the bound vorticity shed per unit time is related to the elemental annular vorticity smeared over the surface of the tube. This will add to the downwash in the wake, balanced by the upwash from the freestream, $V_\infty \theta_p$. The incremental effect is therefore, (Gilbey and Green (2006))

$$\Delta V_\infty \theta_p = \frac{0.5N}{V \pi \rho R R_s} \quad (4.35)$$

which can be compared with the thrust contribution. N is assumed to vary linearly with incidence. Expressed in terms of the propeller incidence, α_p ,

$$\frac{dC_N}{d\alpha_p} = \frac{\Delta V_\infty 4 R_s V}{\alpha_p R V_s^2} \quad (4.36)$$

The fully developed slipstream incidence is then described by,

$$\theta_p = \frac{\alpha_p (C_T + \frac{dC_N}{d\alpha_p})}{(1 + \sqrt{1 - C_T})^2} \quad (4.37)$$

This concept is mirrored perpendicularly for the sideslip case.

Slipstream Trajectory

The approach by Gilbey and Green (2006) to determine the trajectory of the wake, and hence it's displacement at any streamwise station, is to calculate it's angle to the freestream. The deflection of the streamtube from the freestream datum can then be obtained by integration. The longitudinal case is discussed as depicted in Figure 14.

Momentum is considered in a direction perpendicular to the resultant force F acting on the propeller at an angle of θ_F to the propeller axis. In this direction there is no force and the momentum must remain constant.

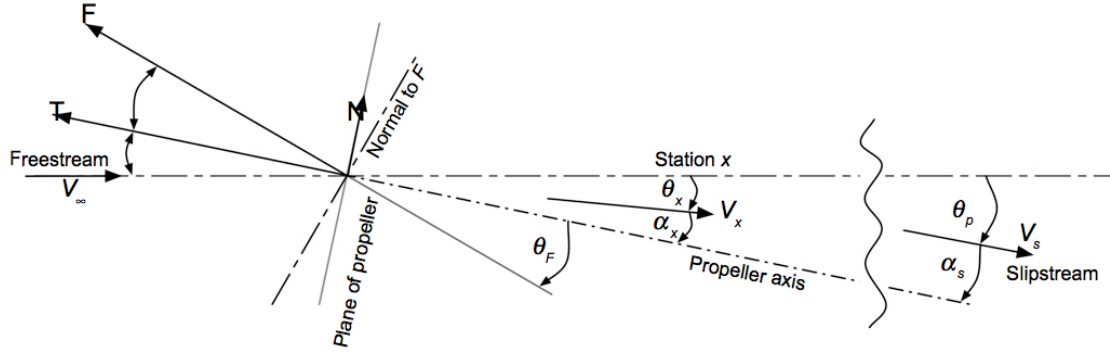


Figure 14: Slipstream geometry

A segment of wake is considered at station x downstream of the propeller, of elemental length dx , of cross sectional area, S_x , and with a slipstream velocity V_x . Within this segment the longitudinal momentum is $\rho S_x V_x dx$ and the transverse momentum due to the outer flow is $(\rho S_x V dx)\theta_x$ with all angles being assumed small.

Momentum is constant along the normal to F , from conservation of momentum the conditions at station x can be related to those at the propeller and in the fully developed slipstream through $S_x = \pi R_x^2$, $S_s = \pi R_s^2$ and $S = \pi R^2$, such that, (Winarto (2004))

$$S_x V_x = S_s V_s = SV \quad (4.38)$$

And

$$\rho S_p V dx \left[(\alpha_x + \theta_F) + \frac{V_\infty}{V_s} \theta_x \right] = \rho S_p V dx \left[(\alpha_s + \theta_F) + \frac{V_\infty}{V} \theta_p \right] \quad (4.39)$$

Now $\theta_x = \alpha_p - \alpha_x$ and $\theta_p = \alpha_p - \alpha_s$, such that, (Gilbey and Green (2006))

$$\alpha_x \left(1 - \frac{V_\infty}{V_x} \right) + \frac{V_\infty}{V_x} \alpha_p = \alpha_s \left(1 - \frac{V_\infty}{V_s} \right) + \frac{V_\infty}{V_s} \alpha_p \quad (4.40)$$

From the equations of motion the streamwise distribution of velocity can be derived as

$$\frac{V_s}{V_\infty} = 1 + s \quad (4.41)$$

Where s is the ratio of increase in axial velocity at a general point in the slipstream to freestream velocity, in which, (Jones et al. (1986))

$$s = a \left[1 + \frac{x}{\sqrt{(x^2 + d^2/4)}} \right] \quad (4.42)$$

which can be rearranged to give

$$V_x = V_s \left[\frac{V_\infty}{V_s} + \left(1 - \frac{V_\infty}{V_s} \right) f(x) \right] \quad (4.43)$$

Where $f(x) = \frac{1+\cos \xi}{2}$ and $\cos \xi = \frac{x/D}{\sqrt{(1/4 + (x/D)^2)}}$

Equation 4.40 can be re-written as, (Gilbey and Green (2006))

$$\alpha_x = \alpha_s \left[1 + \frac{V_\infty}{V_s} \left(\frac{1}{f(x)} - 1 \right) \right] - \alpha_p \frac{V_\infty}{V_s} \left(\frac{1}{f(x)} - 1 \right) \quad (4.44)$$

The diameter of the slipstream at any station x is given by

$$D_x = D \sqrt{\frac{1+a}{1+s}} \quad (4.45)$$

In order to calculate the displacement of the slipstream it is only necessary to integrate the angular deflection. For vertical displacement,

$$z = D \int_0^{x/D} \alpha_x d(x/D) \quad (4.46)$$

5. PROPELLER PERFORMANCE CALCULATION

5.1. Introduction

The propeller performance calculation is primarily based on the evaluation of the Blade Element Momentum Theory and Vortex Theory equations as they relate to the propeller performance and slipstream formation respectively. As previously mentioned, in this method the propeller is divided into a number of elementary strips which are mapped to an equal number of annular streamtubes along the radius, where a force balance is applied involving 2-dimensional profile lift and drag along with the thrust and torque produced within the element.

At the same time, a balance of axial and angular momentum is applied. This produces a set of second-order equations that can be solved iteratively for each blade element. The resulting values of elementary thrust and torque are finally integrated along the radius to predict the overall performance of the propeller.

The estimated performance parameters are then related to the flow behaviour in the far field wake to determine the nature of the fully developed slipstream. Additional force influences are then factored into the slipstream calculation in predicting the behaviour of the flow field immediately fore and aft of the propeller blade. The elemental streamtube deflection is then integrated across its length to determine the slipstream geometry as it propagates rearwards.

Once the equations for the proposed propeller performance method have been established, an iterative procedure is outlined to calculate the induced velocities, angles of attack, and thrust coefficients for each blade element along the span of a blade.

5.2. Required Inputs

The method aims to iterate for a and a' , thereby finding the forces acting on the propeller. A wealth of information is needed as it may be desirable to evaluate other conditions than the design point. A detailed description of chord length and twist

along the radius of the propeller is required, along with C_L and C_D values for a large range of angle of attack, where possible extending into the post stall region. At certain flight/test settings, portions of the blade may be stalled and can therefore result in the iterative procedure not producing a solution or having difficulty in converging rapidly. Where C_L and C_D values are not available from wind tunnel tests of aerofoil sections, the characteristics are obtained using XFOIL. In the process, several loss factors are included to give realistic results. Finally, the calculated values for the induction factors and the aerofoil coefficients are used to find the thrust and momentum force on the element. The elements are then added together to give the full estimate.

The hub radius in most cases occupies the bottom 10-20 percent of the blade radius. It is assumed that the attachment of the blade to the hub allows for the whole blade to be rotated in support of variable blade pitch. The geometry of the blade is typically provided in terms of elemental chord and twist angle across a range of radial locations.

The shape of an aerofoil is defined by a set of coordinate points along its surface. In evaluating aerofoil characteristics with XFOIL these coordinate points are referred to as nodes, the number of which affects the accuracy of the predictions and the ability of the solver to converge on a solution. This is discussed in more detail in the next section. The local aerofoil angle of attack, α , is the angle subtended by the aerofoil's chord line and the plane of the propeller disc.

The local angle of attack varies widely along the blade, due to the relatively severe geometric twist of the blade. It is possible that the pitch angle at the blade root may be close to 90° while the pitch angle at the propeller tip may be closer to 0° . The overall blade pitch angle is often defined to be the local pitch angle of the blade element at a radial station of 70 percent propeller radius. If the whole blade is then turned around through an angle of ζ about its pitch axis, then the geometry of all elemental aerofoils must be turned through the same angle about the pitch axis. Table 1 below lists the required inputs to the propeller performance model.

Table 1: Propeller model inputs

θ	Local pitch of blade
B	Number of blades
c	Chord length
r	Radius
δr	Length of element
ω	Rotational speed
V_{inf}	Wind speed
ρ	Wind density

5.3. Aerofoil Characteristics

5.3.1. XFOIL

Where necessary, the characteristics of aerofoils were predicted using XFOIL. The XFOIL analysis code consists of a second order panel method (linear vorticity distribution). In this code, a zonal approach is used to solve the viscous flow indirectly and an equivalent inviscid flow is postulated outside a displacement streamline that includes the viscous layer. Monk (2010) provides an evaluation of XFOIL against a competing aerofoil analysis code PROFIL citing inaccuracies in PROFIL's prediction if a laminar separation bubble or large separated flow region occurs. Monk (2010) concludes that while XFOIL's second order panel method is inherently not as accurate as PROFIL's third order panel method, it employs a more sophisticated boundary layer prediction method that takes the boundary layer into account while solving for the flow-field. Hepperle (2004) stated that his experiences with XFOIL "indicate that it tends to shift the polars to higher lift coefficient and that the simple panel methods in conjunction with the spline method) has some problem with leading edges, which often results in jaggy velocity distributions even for perfectly smooth airfoils". Figure 16 repeats the results of a comparison between measured and computed results regarding lift and drag coefficients of two modified NACA 66 profiles, i.e. 66(3)-418 and 66-206 taken from Abbott and von Doenhoff (1958), obtained using 120 panels for geometry discretisation of blade profile and a Reynolds number of 3×10^6 (Benini (2004)). The results confirm XFOIL's ability to provide accurate predictions on 2-dimensional flows around an aerofoil.

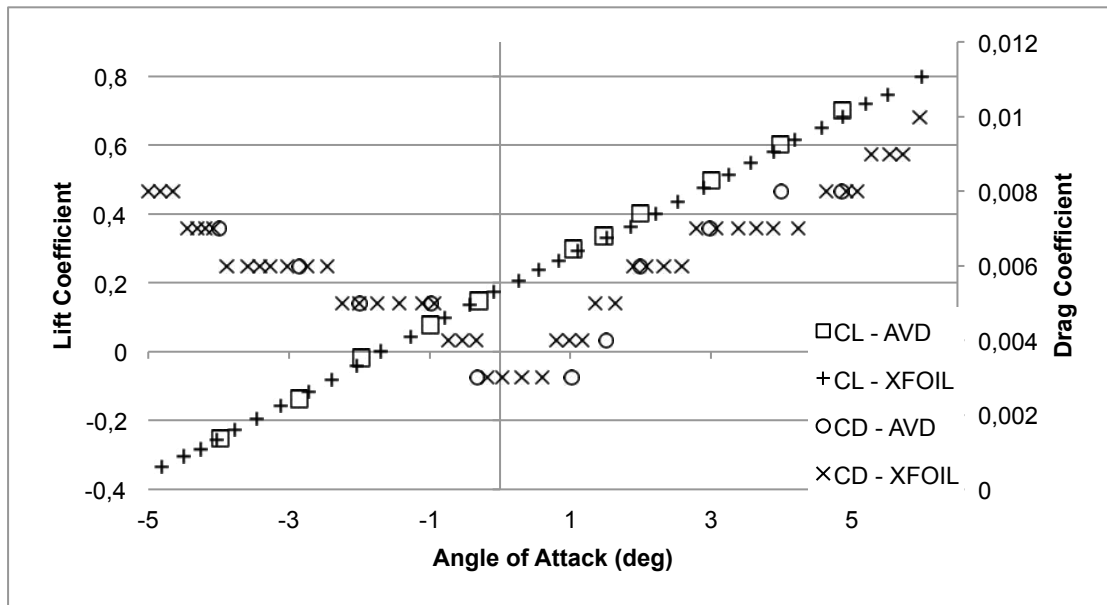


Figure 15: Lift and drag coefficient comparison between wind tunnel (Abbot and von Doenhoff (1958)) and XFOIL (Benini (2004)) results - NACA 66-206.

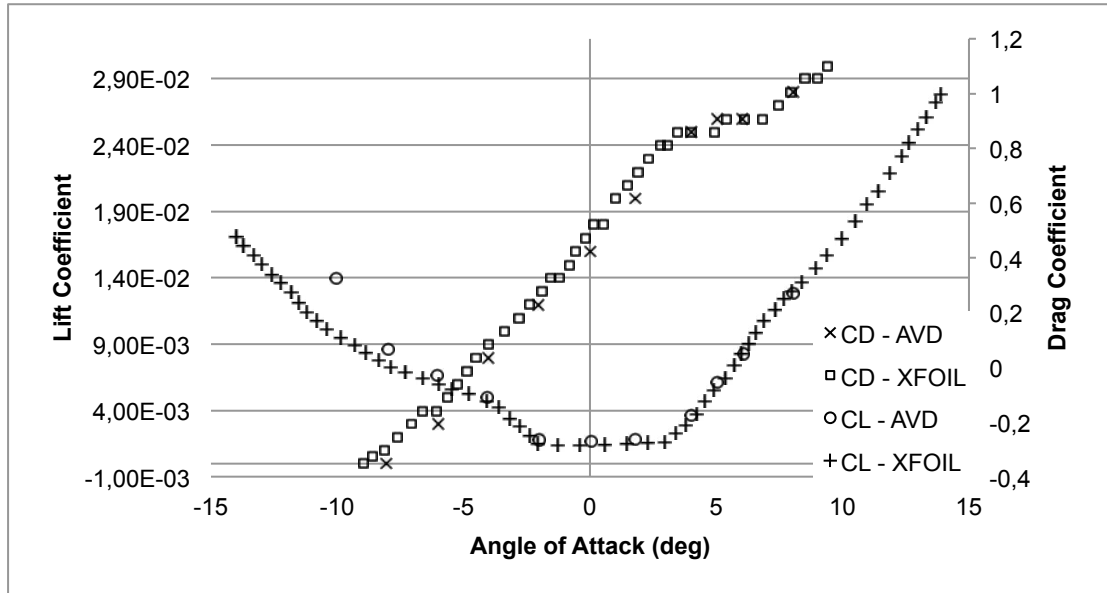


Figure 16: Lift and drag coefficient comparison between wind tunnel (Abbot and von Doenhoff (1958)) and XFOIL (Benini (2004)) results – NACA 66(3)-418.

In Figure 15 and Figure 16, “AVD” refers to the wind tunnel test results presented in Abbott and von Doenhoff (1958) and “XFOIL” refers to the XFOIL predictions obtained in Benini (2004). The number of nodes defining the aerofoil geometry used by the code is dependent on the number of coordinate points entered. XFOIL’s PPAR function enables the number of nodes provided to be increased within the programs limit of 240 points. It also allows for specialised spacing of the panels which, in some instances, needs to be close in order to capture the geometry of the boundary layer separation bubble on the upper surface.

In Benini (2004), XFOIL was able to capture the deviation of the lift coefficient from its relatively constant slope in the range from 7° to 8° of the incidence angle. The accuracy of XFOIL was confirmed in the analyses of the NACA 66-206 profile, where the discrepancies between experimental and calculated data never exceeded 5% in the angle of attack range between -4° and $+4^\circ$.

5.4. Calculation Process

A solution to the thrust and torque of a given propeller cannot be found directly from the BEMT equations as the final forms of the momentum balance equation still contain the variables for element thrust and torque. A second-order system of equations contains the four primary unknown variables $\delta T, \delta Q, a, \hat{a}$; an iterative solution is therefore required. The propeller performance model was simulated in the MATLAB environment on a personal computer, the solution scheme to obtain the propeller performance is indicated in Figure 17 and Figure 18

For each radial element, a loop is performed to obtain the values of the unknown variables of the system. The end parameters are obtained through radial integration of the elementary values. As the elements are assumed to be independent of one another, the calculation can be conducted outward from the hub or inwards from the blade tip. To begin the calculation, an initial value must be given to the inflow factors. A convenient form is to set the tangential induction factor $\hat{a}$ to 0 and to calculate the axial induction factor for an assigned initial angle of attack $\sim 2^\circ$, this form is described in more detail in Section 7.4.1.

The first step is then to estimate the inflow angle using Equation 4.15. Once this angle is known, the angle of attack is calculated through Equation 4.18. With the angle of attack known, the lift and drag coefficients can be obtained from the aerofoil characteristics to estimate the elemental thrust and torque, Equations 4.19 and 4.20. The solidity can then be calculated.

With these factors determined, Equations 4.24 and 4.25 are used to give improved estimates of the inflow factors a and $\hat{a}$. This process is repeated until the values for the inflow factors have converged within a specified tolerance. Once the induction factors have been determined for all elements along the blade radius, the thrust and torque can be calculated using Equations 4.26 and 4.27.

Once the thrust and torque of the propeller have been determined, they may be related to the fully-developed slipstream to determine its radius and internal velocity using Equations 4.28 and 4.29. For an inclined propeller, the in-plane force can then be taken into consideration with Equations 4.35 and 4.36. The angle of attack of the fully-developed slipstream may then be determined using Equation 4.37. This step can be mirrored horizontally for considerations of lateral propeller tilt orientations.

The velocity, orientation, and geometry of the wake at intermediate sections between the propeller and the far field are then determined using Equations 4.43 – 4.45. The displacement of the slipstream is integrated across its length with Equation 4.46.

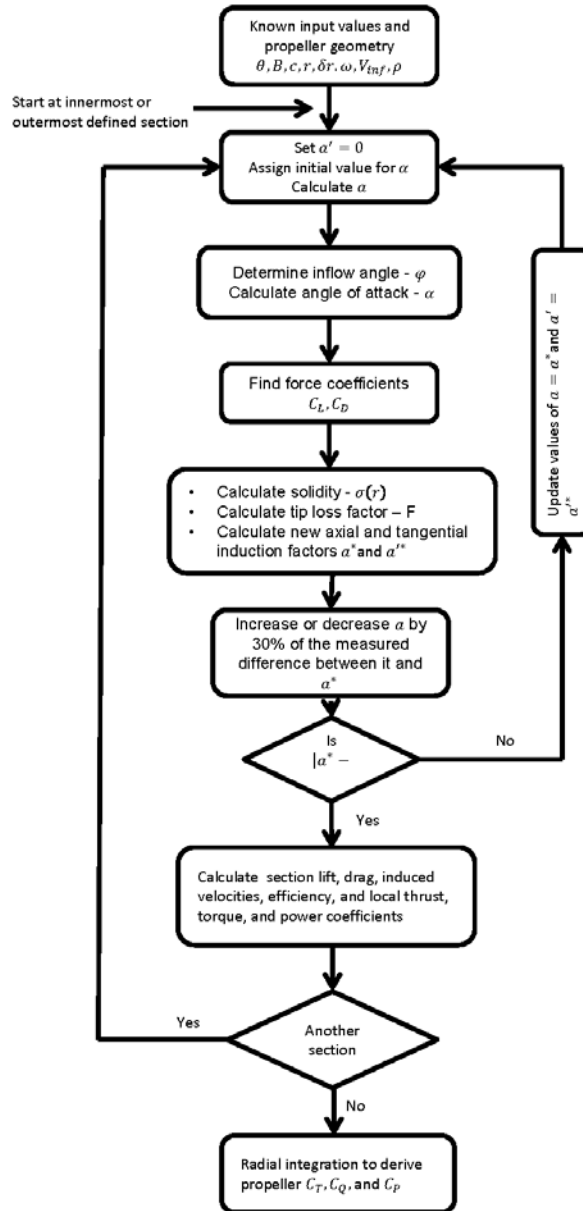


Figure 17: Propeller performance prediction model algorithm

The actual profile lift and drag coefficients are estimated as functions of the incidence angle and Reynolds number using XFOIL. A new value of α (named α^*) is determined and compared to the value assumed at the beginning of the iteration. If the difference falls within a predefined tolerance, the element forces as well as induction factors and local thrust and torque coefficients are calculated and stored, and another element is considered; if not, the value of α is adjusted by a fixed fraction of the difference and the process starts again until the convergence criterion is satisfied.

The algorithm to determine the behaviour of the slipstream is indicated in Figure 18. The slipstream algorithm is effectively a continuation of Figure 17 using the propeller performance parameters as inputs to the equation describing the conditions in the fully developed slipstream.

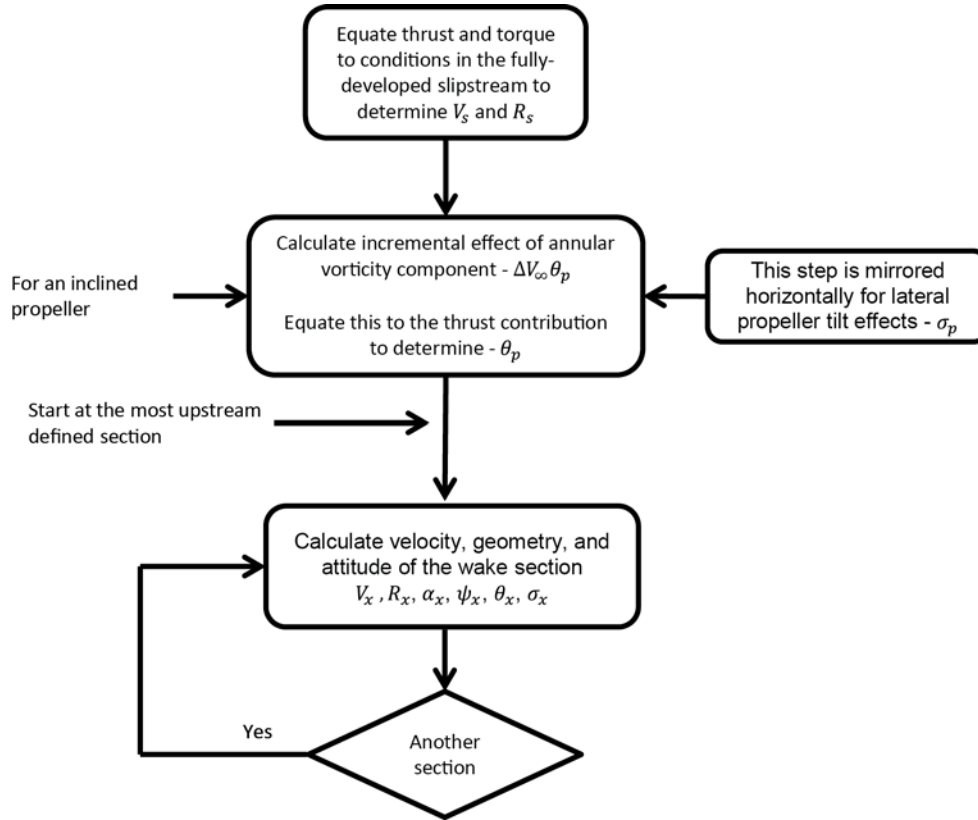


Figure 18: Propeller slipstream model algorithm

Elementary conditions at stations along the slipstream axis are determined in a loop for any number of axial stations that are defined. Parameters such as the slipstream deflection are integrated along the slipstream axis requiring the calculation loop to begin at or before the propeller plane and progress in the direction of wake propagation.

The MATLAB simulation of the model ran swiftly on the personal computer used in this work. Blade element simulations generally converged within 5 -10 iterations with a 40 element propeller model simulation taking approximately 10 – 20 seconds to complete.

6. PROPELLER PERFORMANCE MODEL VALIDATION

6.1. Introduction

Propeller performance at low Reynolds numbers has become increasingly important in the design and performance prediction of Unmanned Aerial Vehicles (UAVs). Many UAVs use propellers that must operate in the low Reynolds number range of 50 000 to 100 000, data on propellers such as these is fairly scarce. This chapter aims to verify the ability of the propeller performance model in estimating the performance of UAV applicable propellers and the resulting slipstream.

6.2. Wind Tunnel Tests

Experiments were conducted by the Mechanical and Aerospace Department of Utah State University, the results of which are presented in Merrill (2011). The propeller was an 8x8 APC Thin Electric propeller which was designed to be used solely with electric motors and has blade shapes that were optimised for efficiency and operational silence. The propeller had a diameter of 0.2 m and was tested in a low speed non-recirculating wind tunnel with test section dimensions of 0.406 x 0.406 x 1.219 m.

Merrill (2011) installed the 2-bladed propeller on a motor and gearbox assembly in the centre of the test section, mounted to one end of a pivot arm. The tests were conducted at a propeller rotational speed of 16 000 rpm with the wind tunnel speed being varied across a range of advance ratios.

An uncertainty analysis was performed on each of the measured values by Merrill, (2011). Two Omega PX143-01BD differential pressure transducers were used to measure the upstream and downstream velocity outside of the propeller wake. The bit resolution of these pressure sensors was found to be the primary cause of the measured uncertainty which was recording as, at least, an order of magnitude greater than: the random uncertainty, the instrument specifications, or the propagated uncertainties from atmospheric pressure or temperature.

Additional sources of experimental uncertainty were considered from: load cell calibration uncertainty that could not be removed through subsequent calibrations, power calculation uncertainties from the bit resolution of current measurement, ammeter instrument specifications, and uncertainty of the motor constants. The root mean square of all the considered parameters resulted in an overall 95% level of confidence in the measurements.

6.3. Computational Propeller Model

A range of data relating to the propeller was provided by the manufacturer and repeated by Merrill (2011) – see APPENDIX C. Propeller geometric parameters were provided including: chord, pitch, sweep, thickness, and twist however the aerofoil geometry was not provided making it impossible to determine the aerofoil characteristics using XFOIL or any other method. Based on the wind tunnel tests described above, Merrill (2011) was able to select a lift curve and a drag-to-lift curve, the inverse of the common aerofoil efficiency parameter L/D , such that they sufficiently matched the load cell measurements. The assumed characteristics of Merrill (2011) are used in this computational analysis.

The propeller was simulated using the performance algorithm described in Section 5 based on the settings above and the following additional input assumptions:

Hub radius: 0.24 m

Air density: 1.225 kg/m^3

Iteration error: 0.1%

Tip-loss correction: included

Results of a model validation analysis are presented below establishing that the performance algorithm is able to accurately predict the forces created by a powered propeller of simple geometry.

6.4. Results and Validation

6.4.1. Thrust and Power Coefficient

Figure 19 shows how propeller thrust coefficient varies with advance ratio as predicted by the propeller model and as measured in the wind tunnel tests of Merrill (2011). From the figure it can be seen that the agreement with the test data is good over a fairly wide range of advance ratio conditions. Based on the identified root mean square error, the Merrill (2011) wind tunnel data points are presented with 95% confidence interval error bars and a linear least squares fit trendline. The gradient of the propeller model thrust prediction curve is almost identical to that of Merrill (2011) and fits comfortably within the error interval.

Initial simulations showed difficulties in converging towards a solution. After an initial convergence the subsequent steps would tend to oscillate between two terminal values causing the simulation to reach its maximum iteration count. The algorithm was amended to include a fixed fraction adjustment based on the error between the current and previous axial induction factor estimates. This corrected the problem enabling the simulation to converge to within the minimum error in an average of 8 iterative steps. The algorithm provided in Figure 17 takes this into consideration in the innermost loop.

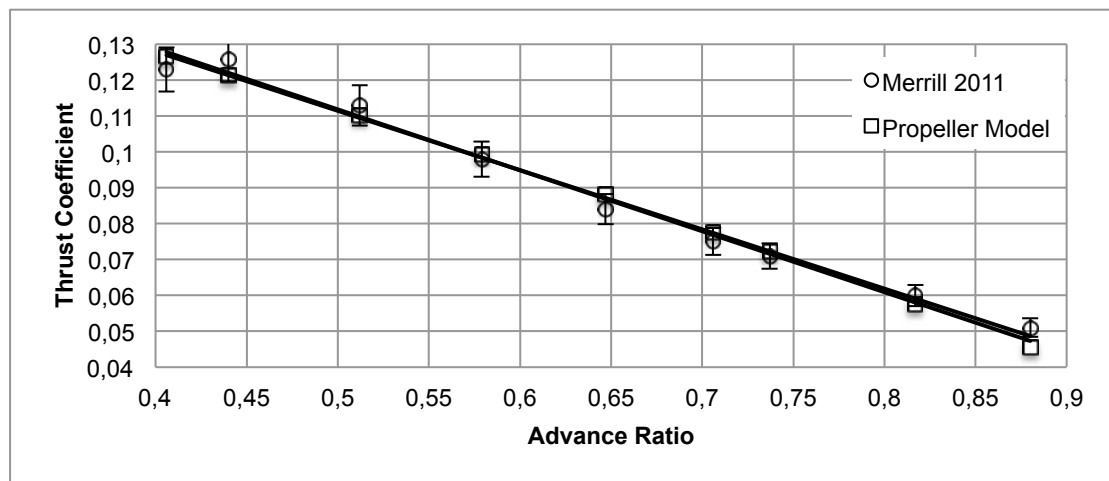


Figure 19: Thrust coefficient comparison between (Merrill 2011) wind tunnel results and propeller model predictions

Figure 20 shows the propeller model predicted angle of attack across the blade radius for a high and an intermediate advance ratio. For the lower advance ratio it is evident that approximately half of the propeller blade experiences an angle of attack that is likely to be out of its linear lift range. Without information on the aerofoil profile, it is difficult to determine exactly how wide the linear lift range is however, it is assumed that the blade is stalled above 15° .

At the higher advance ratio the angle of attack remains safely within the assumed linear lift range across the blade radius.

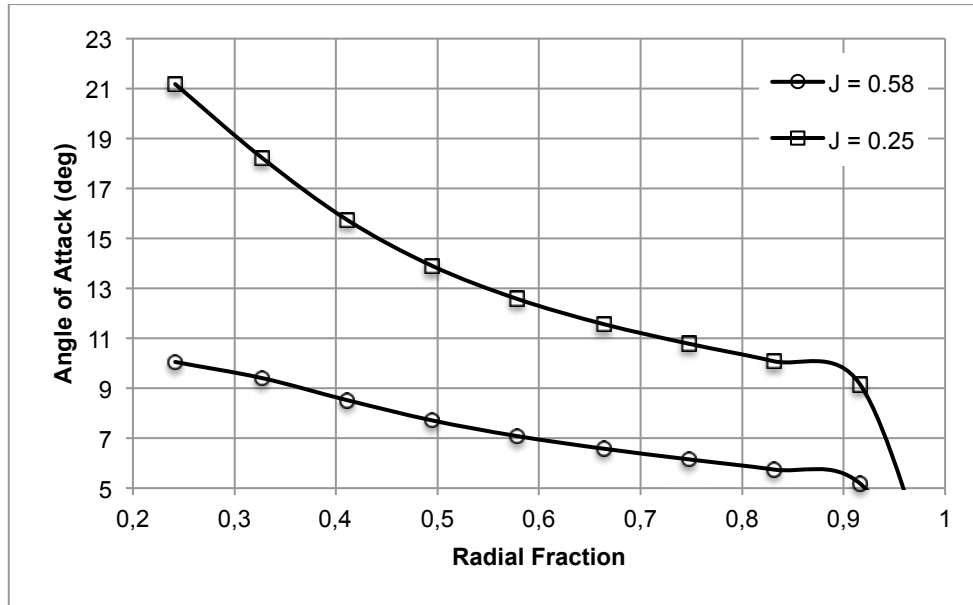


Figure 20: Angle of attack across the Merrill (2011) blade radius at advance ratios $J = 0.25$ and $J = 0.58$

The comparison of power coefficients between the propeller model predictions and wind tunnel tests of Merrill (2011) are presented in Figure 21. In this figure the comparative curves are represented by second order polynomial trendlines. The propeller model appears to do a good job of predicting the propeller power output. The two curves intersect to the effect that the propeller model under-predicts and over-predicts propeller power for lower and higher values of advance ratio respectively.

An average error of 6% across the advance ratio range was obtained between the two sets of results. In this instance the propeller model predictions fall outside of the error bounds for lower and higher and advance ratios. As the airspeed measurements exhibit a relatively high level of uncertainty, it is possible that the shortfall in power coefficient prediction is due to the additional power that would be required of the propeller to overcome the losses incurred by the higher velocity induced in the wind tunnel.

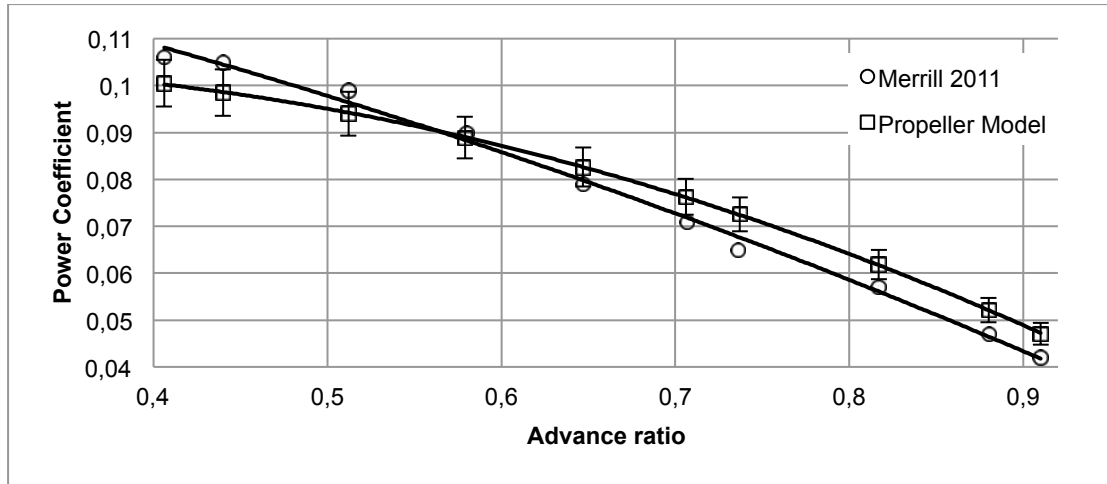


Figure 21: Power coefficient comparison between (Merrill 2011) wind tunnel results and propeller model predictions

6.4.2. Blade Element Discretisation

Figure 22 is provided to analyse the sensitivity of the propeller model to the number of blade elements that are modeled. A fundamental simplification of the blade element theory is the reduction of a 3-dimensional body into a series of 2-dimensional panels. It is therefore important to model a sufficient number of blade elements to effectively represent the 3-dimensional scenario.

The downside to modeling an increasing number of blade elements is that it is accompanied by increasing computational requirements. From Figure 22 it is clear that an optimum exists at 20 blade elements. Beyond 20 elements the gain in error reduction is minimal and does not warrant the additional computational expense.

An interesting observation is that the propeller model accuracy reduces for blade model resolutions above 10 and 20 elements based on thrust and power coefficient comparisons respectively. On this basis, propeller performance model simulations were carried out with 20 blade elements.

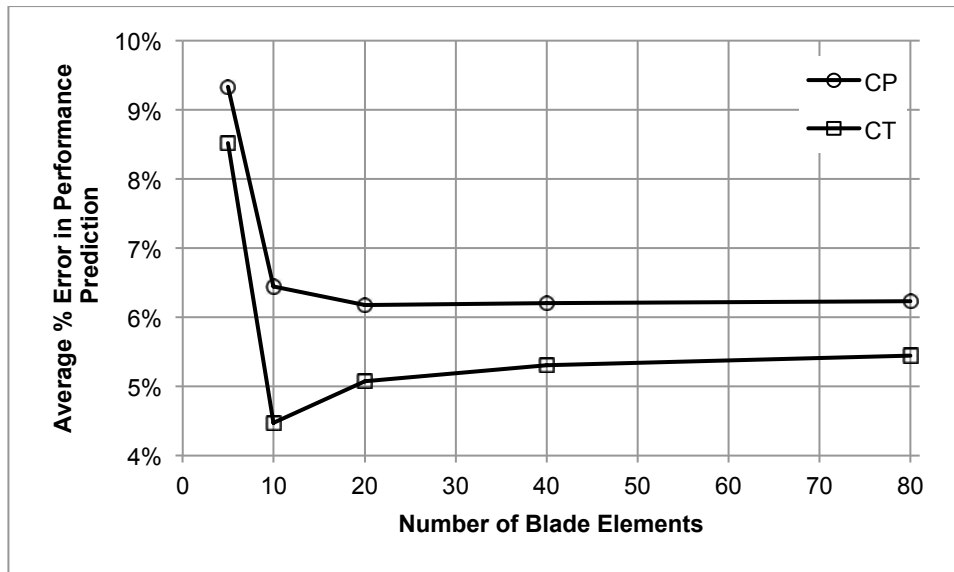


Figure 22: Variation in performance prediction error with increasing number of blade elements

7. PROPELLER PERFORMANCE MODEL EVALUATION

7.1. Introduction and Propeller Definition

The previous section validated the fundamental capabilities of the propeller performance model by simulating a basic propeller model across a limited set of advance ratios. The propeller model showed good agreement with wind tunnel correlations and was therefore shown, in its most basic form, to be a functional tool. The validation performed against the results of Merrill (2011) proves that the propeller model is capable across a very narrow band of operating conditions that a propeller can typically expect to encounter. In this regard it was necessary to perform further comparisons to evaluate the broader capabilities of the model and reveal any further limitations it may have.

The model is now extended to consider a propeller of more complex design along with variations to the propeller pitch angle. The propeller designed by Monk (2010) was optimised for operations ranging from take-off at sea level to efficient cruising at high altitudes. The propeller design point was an airspeed of 50m/s, 34kW input power and 15 000m altitude. Design for cruise flight at high altitudes results in a propeller of low Reynolds number and high Mach number.

Monk (2010) states that no single aerofoil section was found that satisfactorily met the operational requirements over the entire length of the propeller blade. As a result the propeller employs the Hepperle series of aerofoils (MH112, MH113, MH114, MH115, MH116, MH117, MH121) across its radius. An MH112 aerofoil section is used near the propeller hub ($r/R = 0.1$ to 0.3) with the intermediate sections being distributed along the blade length through to the MH121 tip section. The radial distribution of Hepperle aerofoil sections is tabled in APPENDIX C: Propeller Geometry.

7.2. Propeller Performance Model

The propeller performance model was adjusted to look-up the lift and drag characteristics of the relevant aerofoil for each radial location of the propeller blade. The model therefore accesses tables of C_L and C_D values as provided in Monk (2010).

Monk (2010) predicted the aerofoil characteristics using XFOIL at Reynolds numbers of 250 000, 500 000, and 1 million. The aerofoil characteristics were referenced from the aerofoil zero lift line, a sensitivity analysis revealed insignificant variations between the results at Reynolds numbers 500 000 and 1 million. An average of the two values was used in the propeller performance model.

The propeller performance model was not sophisticated enough to consider the blending of aerofoil sections towards the adjoining radial locations, the practical implementation is therefore an instantaneous switch from one dataset to another between two radial set-points.

As was done previously, the propeller was simulated based on the settings described above and using the following additional input assumptions.

Hub radius:	0.1 m
Air density:	0.984 – 1.02 kg/m ³
Iteration error:	0.1%
Tip-loss correction:	Included
Blades:	2
Diameter:	1.2 m

7.3. Wind Tunnel Propeller Tests

Wind tunnel tests were performed on the propeller described above by Monk (2010) over the range of conditions expected during the climb and cruise phase of a high-altitude UAV. The propeller was tested in the CSIR 2 metre open return wind tunnel.

Results of the Monk (2010) wind tunnel tests determining the effects of varying the blade pitch and propeller rotational speed are considered. The propeller was tested at blade pitch angles between -8° and 4° , propeller rotational speeds of 800, 1 000, and 1 200 rpm across a range of wind tunnel speeds from 4 m/s up to 30 m/s resulting in an advance ratio variation of between 0.068 and 0.40. This range is markedly different and complementary to the advance ratio range of 0.4 to 0.9 that was evaluated in the initial validation based on Merrill (2011). This study therefore extends the validation in an attempt to discover any limitation that may exist in the model for lower advance ratios.

Results are presented below towards an evaluation of the capabilities of the propeller performance algorithm and it's solution.

7.4. Results and Comparisons

7.4.1. Initial Propeller Model Outputs

An initial set of simulations produced performance predictions exhibiting unexpected behaviour at low advance ratios. An example of such results is presented in Figure 16 below. This behaviour was similarly evident in initial predictions of Torque and Power coefficient. A closer look at intermediate stages of the propeller model's iterations revealed extremely high inflow angles across the entire blade radius. The high inflow angles resulted in angle of attack values that fell outside of the provided range of airfoil data and were therefore treated as flat plate estimations. Tangler and Kocurek (2005)

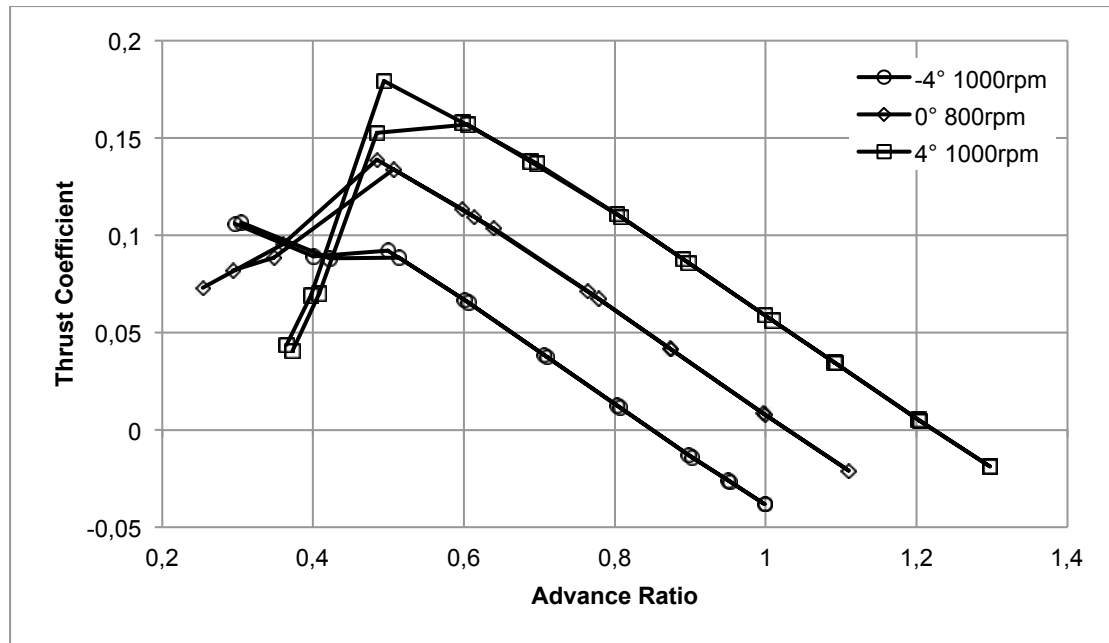


Figure 23: Monk (2010) Propeller model predictions without induction factor initialisation

The simulations were run through a range of advance ratios and then back across the range to the initial setting to evaluate repeatability. The results show good repeatability above advance ratios of approximately 0.5 but deviate significantly in the lower range where the prediction model seems to lose functionality. As the model is producing different results for two identical test settings, the likelihood is that the initial “guesses” for the axial and tangential induction factors have an impact on the convergent values.

A step was added to the performance model algorithm to assign the axial induction factor an initial value resulting in a finite angle of attack, e.g. 2° . This parameter initiation allowed the subsequent iteration to obtain a revised set of induction factor estimates based on an angle of attack within the linear lift range; this adjustment is included in Figure 17. More accurate estimations were provided within the low advance ratio range as a result of the initialisation step and are presented below.

7.4.2. Thrust and Torque Coefficient

Figure 24 and Figure 25 below provide a comparison between the results predicted by the propeller performance model and those obtained from the wind tunnel tests of Monk (2010).

From the figures it is evident that the propeller model reaches fair agreement with the wind tunnel results for lower advance ratios ($J < 0.65$). Across the blade pitch range the propeller model tends to over-predict the propeller thrust to larger degrees of error as the advance ratio increases. As a general indication, the accuracy of the thrust predictions depends on the value of the advance ratio.

For each blade pitch setting, the propeller model under-predicts the thrust at lower advance ratios. The thrust curve of the propeller model then intersects the Monk (2010) curve and begins to divergently over-predict the thrust for increasing advance ratio. Benini (2004) reports a similar comparative result between propeller wind tunnel measurements and BEMT predictions stating the further one moves away from the intersection point, the greater the error in the BEMT prediction. Benini (2004) attributes this discrepancy to 3-dimensional effects and justifies the assertion by adding a further comparative result based on a fully 3-dimensional Navier-Stokes calculation. The Navier-Stokes result achieves realistic results independent of advance ratio with the maximum error being in the order of 5% in the prediction of thrust and torque. The BEMT is therefore materially affected by the absence of secondary effects such as 3-dimensional flow velocities induced by angular acceleration due to the rotation of the propeller.

The data shows a mismatch between the experimental and computational thrust curve gradients with the predicted curves diverging from the wind tunnel results with increasing advance ratio. The results would therefore indicate that the thrust prediction accuracy increases as aircraft velocity is decreased. As this effect is consistent across the blade pitch range, it is likely attributable to the averaged dataset of aerofoil characteristics with the lower advance ratios resulting in localised Reynolds numbers more applicable to the set of lift and drag coefficients.

An additional source of error is possibly due to differences between the flow-field induced by the propeller in the wind tunnel test section and the unconstrained flow-field assumed in the propeller model. Monk (2010) suggests that the gradient of the velocities induced by the propeller in the wind tunnel could provide a different velocity reading from that through the propeller disc. As a result, Monk (2010) was unable to obtain experimental results at low advance ratios.

Monk (2010) does not present error bounds in the experimental results but describes certain isolated sources of uncertainty including:

- Tare drag measurements variations that remained within 5% of the mean for the test rig calibration; and
- Maximum measured errors of 1.6% and 1.8% above and below the mean respectively for the thrust load cell calibration.

On the assumption that the thrust measurement uncertainty is approximately 3.4%, the propeller model predictions remain an order of magnitude more inaccurate than the measurement uncertainty. Inaccuracies in the prediction of the propeller thrust will impact the prediction of propeller torque and, to a larger extent, power prediction as the calculation of these parameters is based on the same underlying elements of lift and drag coefficient, induced velocity, and inflow angles.

Small angle approximations built into the propeller performance model relate only to the slipstream deflection. Thrust coefficient prediction errors at higher blade pitch angles and angles of attack are therefore not affected by small angle approximations.

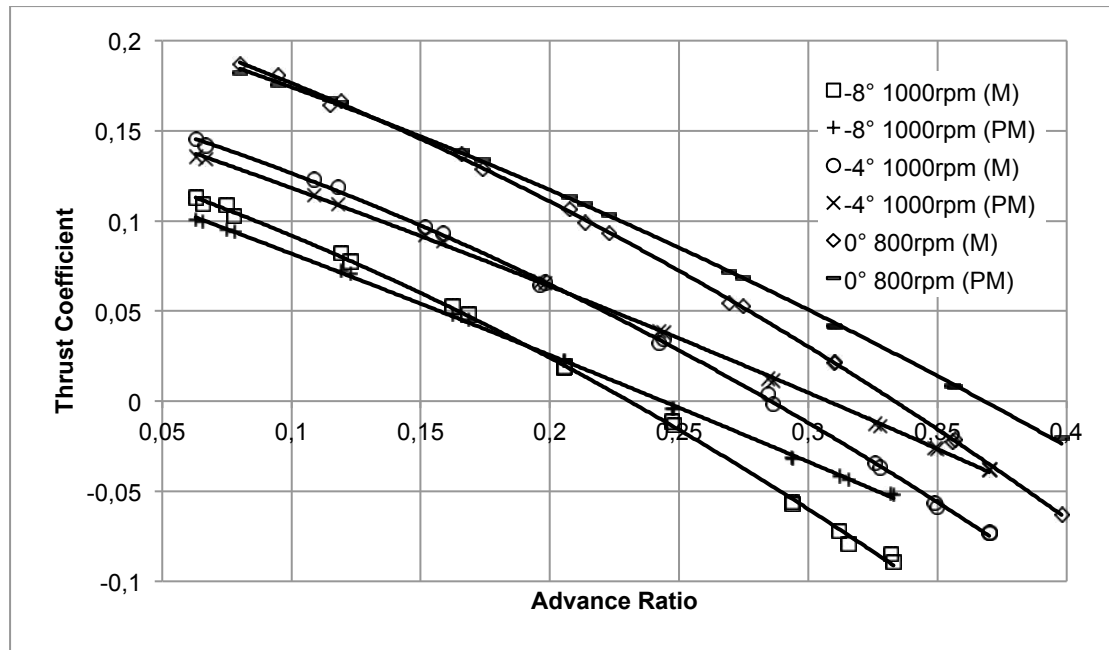


Figure 24: Comparison between (Monk (2010)) wind tunnel and propeller model thrust coefficients (-8° to 0° pitch angles) – “M” = (Monk (2010)); “PM” = propeller performance model

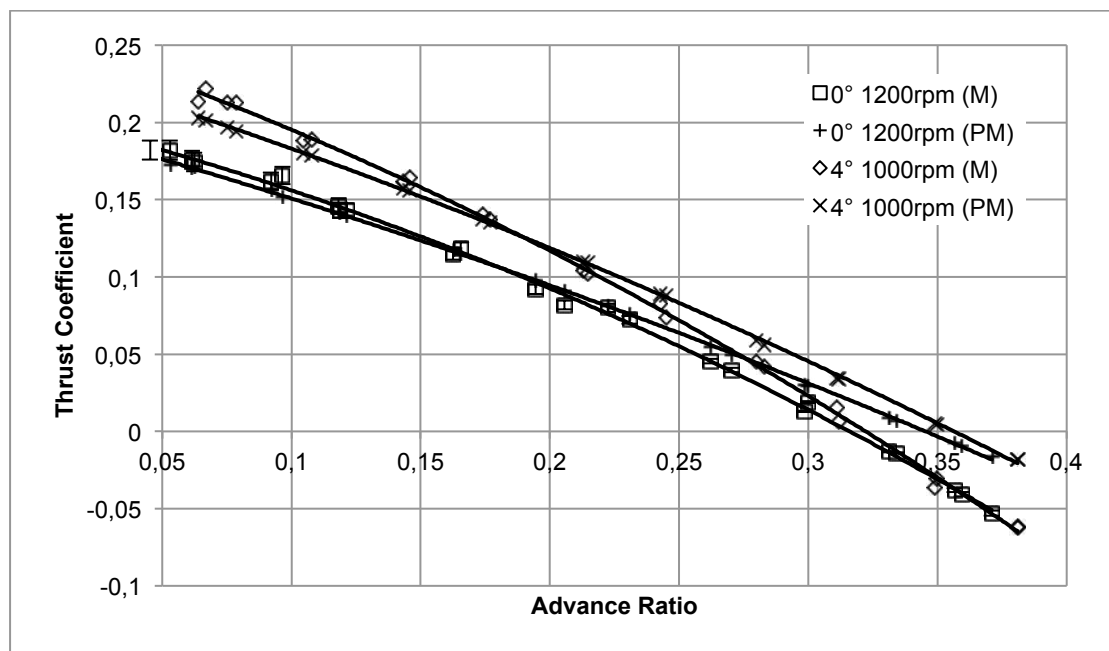


Figure 25: Comparison between (Monk (2010)) wind tunnel and propeller model thrust coefficients (0° to 4° pitch angles) – “M” = (Monk (2010)); “PM” = propeller performance model

Torque coefficient predictions were produced in the simulations of thrust and power coefficient. Due to failure of the torque load cell equipment, Monk (2010) does not provide any data that could be used for a meaningful discussion of the torque prediction accuracy. The torque coefficient predictions are therefore not presented here.

7.4.3. Power Coefficient

BEMT considers the propeller input power as the product of thrust and the freestream velocity. As with thrust and torque, the differential power input is calculated at each blade element and integrated across the tip speed ratio range. An unfortunate consequence is that the prediction errors made in determining thrust coefficient are magnified in the power predictions by the scale of the freestream velocity.

Comparative power coefficients are provided in Figure 26 and Figure 27 for the propeller model and wind tunnel tests of Monk (2010). A high degree of variation is shown between the propeller model and wind tunnel results. The magnification of thrust coefficient error is clear in that the areas of high and low error correspond between the thrust and power coefficient graphs. Accuracy of the propeller model's power prediction capability is therefore dependent on the factors affecting the accuracy of thrust coefficient prediction.

Following on from the description of a possible source of thrust coefficient error, the lower power results stemming from the wind tunnel tests of Monk (2010) may be ascribed to aerodynamic losses associated with the increase in air velocity induced by the propeller through the wind tunnel.

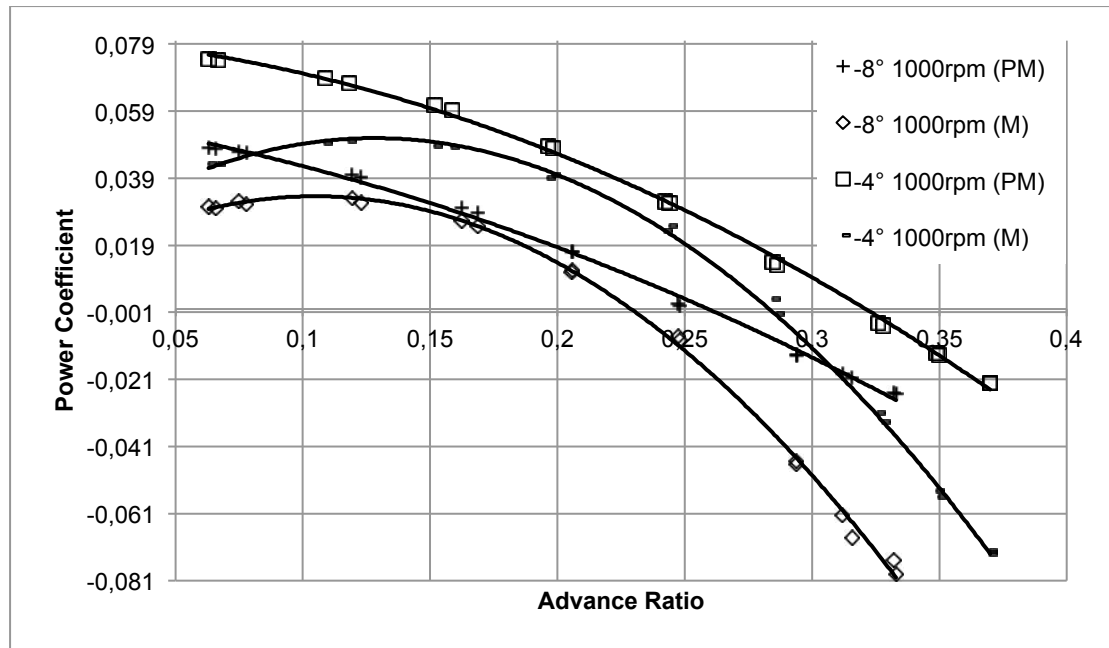


Figure 26: Comparison between (Monk (2010)) wind tunnel and propeller model power coefficients (-8° and -4°) – “M” = (Monk (2010)); “PM” = propeller performance model

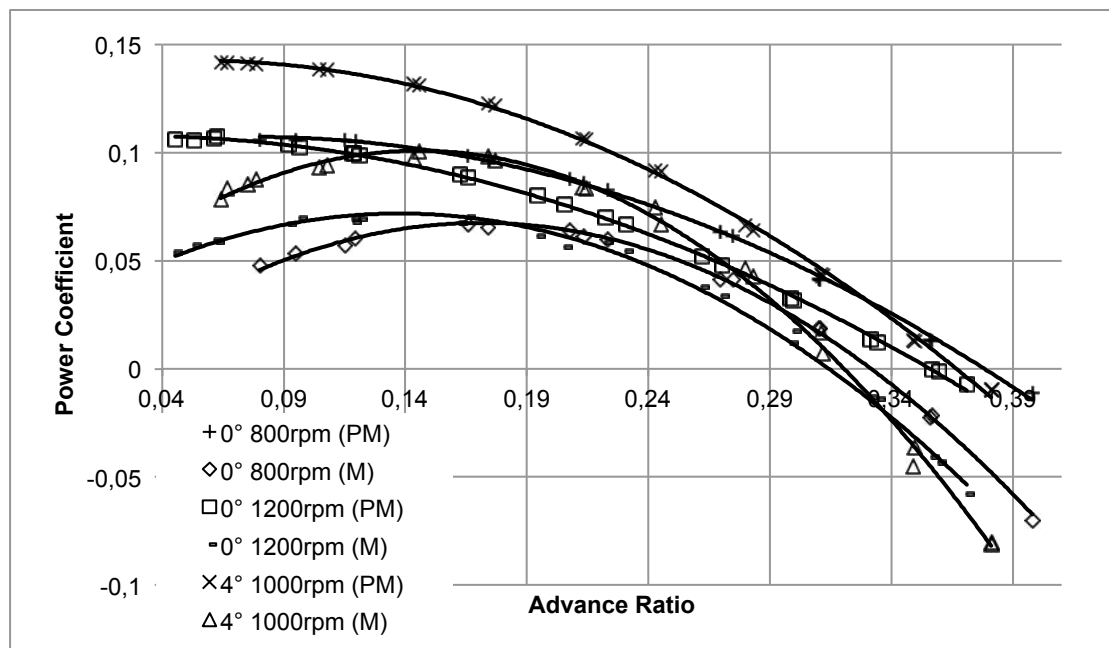


Figure 27: Comparison between (Monk (2010)) wind tunnel and propeller model power coefficients (0° and 4°) – “M” = (Monk (2010)); “PM” = propeller performance model

During an initial set of wind tunnel tests, Monk (2010) observed that the test rig drive belts were oscillating in the airflow which resulted in increasing drag with increasing

wind tunnel speed. The wind tunnel test results presented in Figure 26 and Figure 27 are of a revised set of tests where a larger vertical fairing was used to shield the airflow from the buffeting of the belt drives. This adjustment to the test rig reduced the magnitude of the tare drag corrections although a small increase in drag remained for increasing wind tunnel speed.

The result of this error is an under-prediction of blade section drag from XFOIL which has a large resultant effect on the power coefficient through the propeller torque. Monk (2010) further states that the predicted drag for the propeller aerofoils may be affected by the wind tunnel turbulence levels, blade skin smoothness, and aerofoil accuracy. These potential errors were not measured and are therefore more useful in describing possible sources of propeller model prediction error rather than quantitatively assessing the model's prediction error in relation to wind tunnel measurement error.

The results of the two validation analyses have not provided an obvious set of limitations. The comparisons with Monk (2010) suggest that the model performs well at low advance ratios, $J = 0.06 - J = 0.15$, and begins to experience difficulties in producing accurate estimations as the advance ratio increases above this range. The Monk (2010) comparison makes this suggestion within a limited advance ratio range of 0.068 – 0.4 which would go on to suggest that the model is inaccurate for advance ratios greater than 0.4. The comparison with Merrill (2011) conversely shows that the model is accurate across the advance ratio spanning 0.4 to 0.9 which would disagree with the results of the Monk (2010) comparison.

8. PROPELLER SLIPSTREAM MODEL ANALYSIS

8.1. Introduction and Delineation

Experimental testing does not form a part of the scope of this study. A method for the determination of the flow characteristics, geometry, and attitude of the slipstream trailing a powered propeller was detailed in Section 4.2.7. A comparative set of experimental outcomes in published literature that could be used in quantitative comparison could not be identified.

The wind tunnel experiments conducted as part of Merrill (2011) analysis included wake surveys to indirectly measure the thrust of the propeller with the wake profile being an indication of how much momentum was added to the flow from the propeller. The wake was measured using a sweeping pitot probe aft of the propeller. Total and static pressure were fed into a differential pressure transducer. The position of the probe was measured by a linear, pressure sensitive potentiometer. The combined measurement of the position and airspeed allowed the definition of a radial velocity profile aft of the propeller. In reality these tests produced unpredictable, noisy, and uncertain results in attempting to record the trailing velocity of the flow in the propeller's slipstream.

Simulation outcomes obtained with the propeller slipstream model are presented below based on the APC 8x8 Thin Electric propeller that was modeled in Section 6. The propeller model results are discussed based on physical expectations in an attempt to confirm that the model is broadly representative of reality. As the current work has adjusted it's focus away from the integration of such data into a panel code, the data produced by the propeller slipstream model can be made useful by outputting it to a simple format that is easily read as input to a suitable aerodynamic analysis package.

8.2. Results and Discussions

8.2.1. Streamtube Contraction

An assumption of momentum theory is that the propeller disc experiences uniform and averaged flow quantities and makes no distinction with the actual radial distribution therefore leaving the impression that the perturbation velocity at the disc is independent of radius. Detailed numerical computations in Kerwin (1994) show that the axial velocity is nearly constant over most of the disc but decreases significantly as the blade radius approaches it's tip value. This effect can be described as an effective inward pitch of the velocity vectors, and a resultant reduction in the streamtube radius, in the outer annuli of the streamtube immediately upstream of the propeller. Figure 28 depicts this phenomenon.

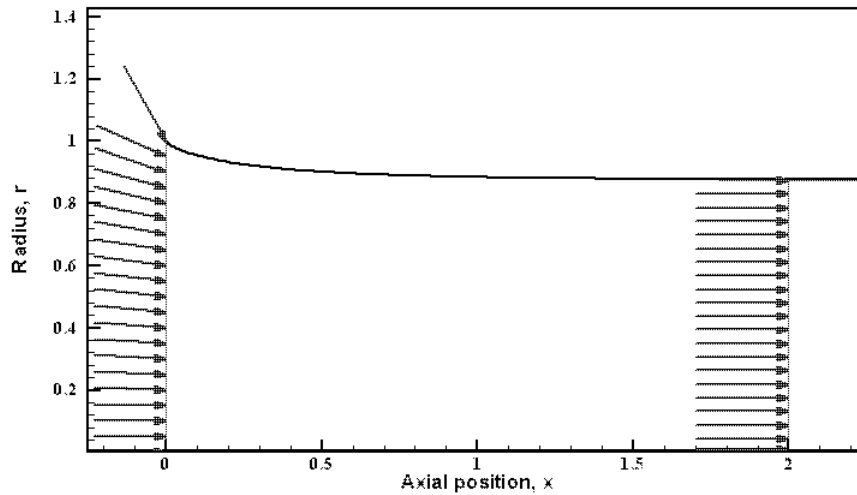


Figure 28: Results of a numerical calculation of slipstream radius and velocity field in the plane of the propeller disc and far downstream (Kerwin (1994))

The analytical model takes this effect into consideration from the equations of motion where the streamwise distribution of velocity in the slipstream is described by Equations 4.41 and 4.42 which are rearranged in to Equation 4.43, this effect is demonstrated in Figure 29. It is further evident from the figure that the radii of the streamtubes upstream and downstream of the propeller become larger and smaller respectively as the thrust coefficient increases. That is to say that, for a given thrust coefficient, the streamtube radius upstream of the propeller will be larger than that of a smaller thrust coefficient; and that the contraction of the streamtube across the propeller disc will be of a greater magnitude for the higher thrust coefficient. This is to be expected as the streamtube forms a closed control volume and propeller thrust is directly proportional to the mass flow rate.

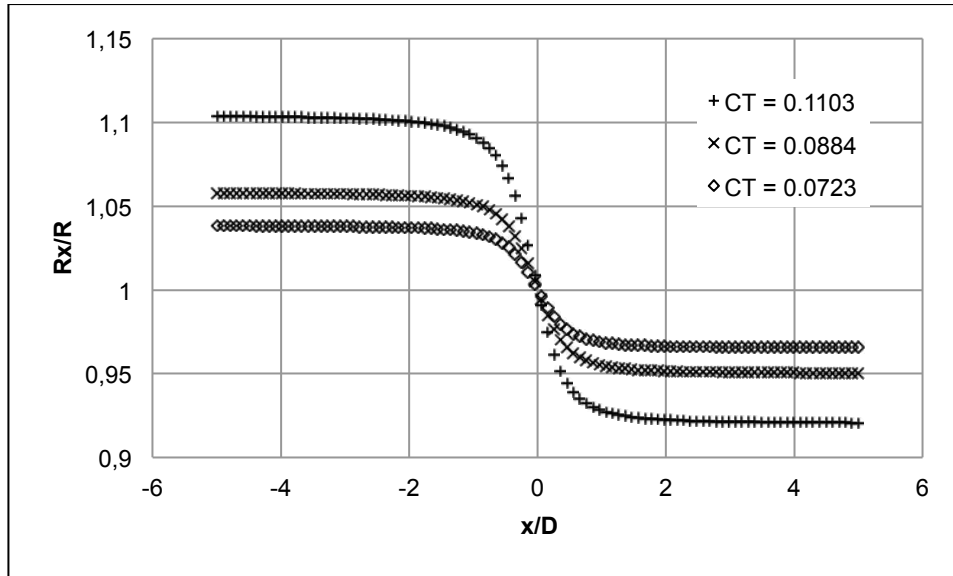


Figure 29: Slipstream contraction behind the Merrill (2011) propeller (propeller slipstream model)

8.2.2. Velocity Adjustment

The action of the propeller blades in producing a thrust is to increase the air pressure immediately downstream of the propeller and to decrease it immediately upstream. The axial velocity in the streamtube increases above the freestream value ahead of the propeller disc and continues to increase through, and downstream of it, attaining a maximum value some distance downstream. Kerwin (2001)

The increase in velocity is associated with the reduction in the streamtube diameter that was demonstrated in Figure 29, Figure 30 demonstrates this increasing velocity as described by Kerwin (2001). The results of the figure follow the logics that would consider a higher thrust coefficient to add a larger amount of energy to the flowfield which would result in a higher axial velocity.

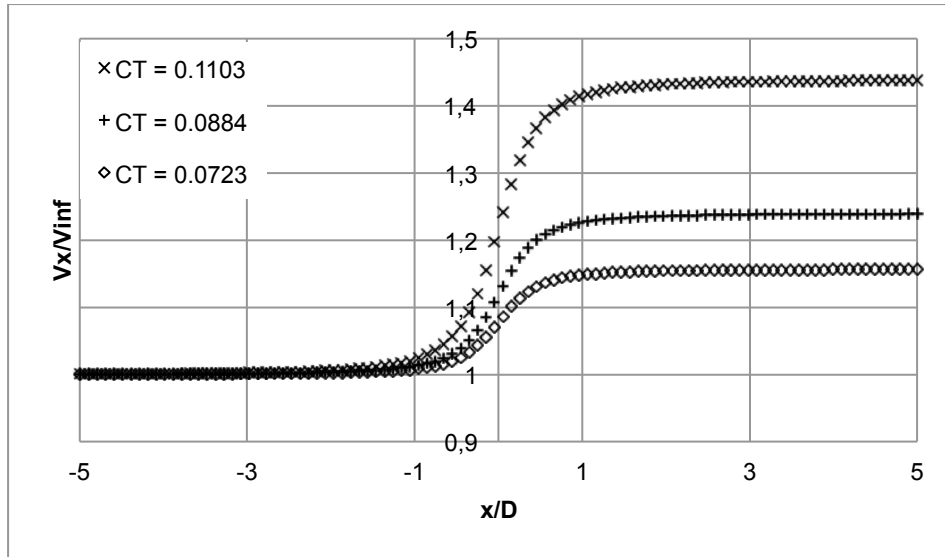


Figure 30: Axial velocity increase across the propeller

8.2.3. Slipstream Deflection

The following propeller model simulations were carried out at propeller pitch and sideslip angles of 4° each.

The thrust produced by a propeller at an incline to the onset flow is likely to induce a change in the direction of the flow - immediately upstream of the propeller as described in the previous section, and - downstream of the propeller. As described in the development of the slipstream model (Section 4.2.7) the slipstream will be affected by an inclined thrust and a transverse in-plane force. The logical argument is therefore that a positive tilt of the propeller (propeller inlet facing upwards) will result in a negative (downwards) deflection of the slipstream.

Figure 31 includes the results of a slipstream trajectory simulation behind the propeller for a 4° propeller pitch or sideslip angle over 10 diameter extensions of the slipstream behind the propeller. This figure clearly illustrates the anticipated trends of slipstream deflection. What is further evident is that the slipstream deflection reduces for increasing advance ratio as would be expected. For a given propeller speed, this is due to lower advance ratios having a larger thrust influence forcing the slipstream to align with the propeller thrust axis whereas a higher advance ratio will force the slipstream to align itself more with the freestream than the propeller thrust axis.

The concept of slipstream deflection for a propeller at an angle to the freestream applies for both vertical and lateral displacement. As the propeller sideslip angle and angle of attack are identical, the resultant planar deflection is also identical in magnitude and sense.

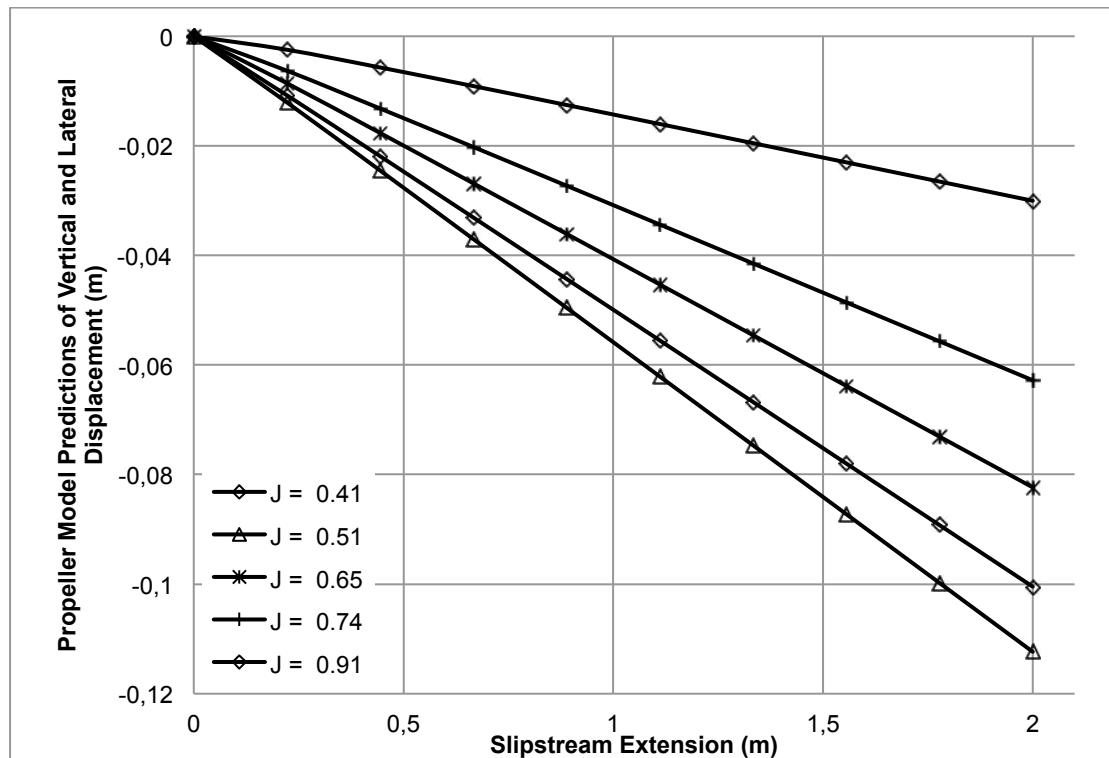


Figure 31: Vertical and lateral slipstream deflection in the intermediate wake downstream of the propeller (propeller slipstream model)

Aircraft that use thrust vectoring systems are always fitted with power plants of incredibly high capacity. The influence of a large mass of moving air over a body - or conversely a body moving through a large mass of air is significant and strongly resistive to change. It is therefore to be expected that, for an advance ratio of 0.5, a 4° propeller pitch will result in less than a 4° slipstream incidence angle relative to the freestream.

Figure 32 provides results of a propeller model simulation in support of the above argument. The figure describes the relationship between the propeller angle of attack and the incidence angle of the fully developed slipstream (relative to the freestream) for an advance ratio of 0.5. The relative flatness of the curve in relation to the

propeller angle of attack describes the tendency of the slipstream to align itself with the freestream flow θ_p as well as the fact that, for a propeller thrust axis in alignment with the freestream, no deflection or displacement of the slipstream will take place. The results describe a slipstream deflection of approximately 2% of the propeller pitch angle for the specific advance ratio setting.

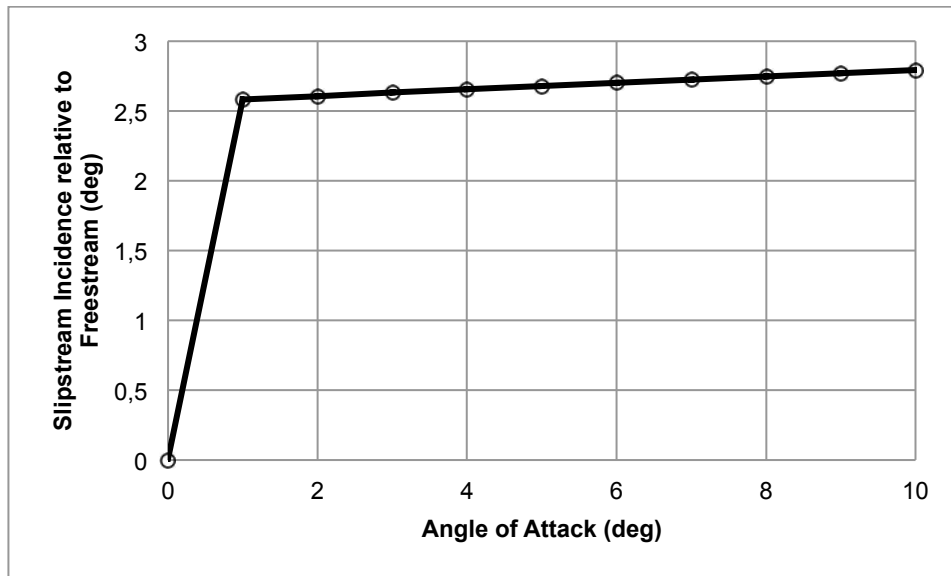


Figure 32: Fully-developed slipstream with propeller tilt

The propeller slipstream model has therefore shown indications of agreement with expected behaviour but would benefit greatly from quantitative comparisons with experimental results. The propeller slipstream model is able to predict a meaningful amount of detail pertaining to the wake behind the propeller for any number of axial stations, such data includes:

- Streamtube diameter
- Angle of the streamtube cross section to the freestream
- Local velocity

The model is readily able to output this data into a range of useful, transferable formats to enable it's implementation into existing or purpose-built aerodynamic analysis packages.

9. WIND TURBINE MODEL

9.1. Introduction

The BEMT propeller is now modified for application to wind turbines. This application is explored as a useful extension of the fundamental BEMT theory and its comprehensive ability to model the flow of fluids through an actuator disk. This analysis is provided as further validation of the underlying mathematical model considering the nuances of physical limitations to the amount of power that can be extracted from a passing fluid. Manwell et al. (2009) describes the fundamental differences between propellers and wind turbines as:

Propeller:

- Introduces energy to the air flowing through its blades
- Increases the speed of the air flowing through its blades
- Uses power to rotate its blades
- Generates thrust

Wind Turbine:

- Extracts energy from the air flowing through its blades
- Decreases the speed of the air flowing through its blades
- Generates power from the wind rotating its blades
- Creates drag

For most considerations, a wind turbine can be described by the blade element momentum theory in much the same way as has been detailed in this report for propellers. Wind turbines, however, have a theoretical maximum power coefficient for an idealized rotor accounting for wake rotation; this condition refers to the maximum amount of power that an ideal wind turbine is able to extract from the flow. The Betz limit is described in detail in the next section. Wald (2001)

9.2. Betz Limit

A version of Equation 4.5 can be written for wind turbines in coefficient form as $C_p = 4a(1 - a)^2$ describing the power coefficient as a function of the axial induction factor. The maximum value of this function can be determined from its first and second derivatives. This value is reached at $a = 1/3$ which leads to a maximum power coefficient of $C_{p,max} = \frac{16}{27} \approx 0.5926$.

From this it is understood that:

- A maximum of 59.26% of the available wind power can be converted to mechanical power at ideal conditions, regardless of the energy conversion device.
- The air velocity at the wind turbine is 2/3 of the freestream velocity, and the velocity in the fully developed slipstream is 1/3 of the freestream velocity when maximum power is extracted. Madsen and Rasmussen (1999)

The Betz limit introduces a limitation on the wind turbine equivalent of the axial induction factor equation utilized in the performance prediction model. Equation 4.24 is adjusted to represent the extraction of energy from the onset flow and, consequently, a reduction in flow velocity as follows: Manwell et al. (2009)

$$a = \left[1 + \frac{4F \sin^2 \phi}{\dot{\sigma}(C_L \cos \phi + C_D \sin \phi)} \right]^{-1} \quad (8.1)$$

Due to the Betz limit, Equation 8.1 only gives reliable results for axial induction factor values between 0 and 0.4. For axial induction factor values greater than 0.4, the rotor element is highly loaded and the modified Glauert (1935) correction is used instead:

$$a = \frac{18F - 20 - 3\sqrt{C_T(50 - 36F) + 12F(3F - 4)}}{36F - 50} \quad (8.2)$$

Outside of this change, the propeller performance model is adapted very slightly to consider energy extraction and the power coefficient becomes the dominant

parameter. This adjusted model is hereby referred to as the wind turbine performance model.

9.3. UAE Unsteady Aerodynamics Test

Wind tunnel test data collected by the NREL in the NASA-Ames wind tunnel tests, as presented in Lindenburg (2003) is used to confirm the applicability of the wind turbine performance model in predicting wind turbine performance characteristics. Lindenburg (2003) reports on the results from the NOVEM project “*NASA-Ames Rotorblad[e] Aerodynamica[as]*”.

The wind tunnel test data provided in Lindenburg (2003) is based on the 2-bladed UAE phase-VI test turbine which has a 10m diameter and a twisted, variable chord rotor blade. The rotor blade uses the S809 aerofoil and was tested at various freestream speeds with a blade pitch angle of 3° and a constant rotational velocity of 72 rpm.

Lindenburg (2003) describes a collection of aerodynamic coefficient data provided by NREL from various wind tunnel aerofoil tests including:

- TU-Delft: measurements for a Reynolds number of 1×10^6
- Ohio State University: measurements for $Re = 0.75 \times 10^6$ and 1×10^6
- Colorado State University: measurements for $Re = 0.3 \times 10^6$, 0.5×10^6 , and 0.65×10^6

For a wind tunnel velocity of 15 m/s and a rotational speed of 72 rpm the Reynolds number is in the order of 0.9×10^6 to 1×10^6 , Lindenburg (2003) considers this result along with other literary references as suitable confirmation that non-rotating aerodynamic coefficients of the S809 aerofoil at a Reynolds number of 1×10^6 are adequate for this turbine blade

9.4. Results and Comparisons

Power coefficient variations with advance ratio are shown in Figure 33 as determined by NREL in Lindenburg (2003) and as predicted by the wind turbine performance

model. From the figure it can be seen that the wind turbine model is in general agreement with the wind tunnel results of Lindenburg (2003) although average prediction errors across the test range are in the region of 30%. If a focus is placed on the discrete data points as opposed to the exponential trendlines, it is evident that the two sets of results are very similar.

Above an advance ratio of 1.3, the wind turbine model is able to produce very accurate predictions. At the high end of the test range, above an advance ratio of 1.3, the wind turbine model is able to produce very accurate predictions of the power extraction. The gradient of the wind turbine model curve is greater than that of the Lindenburg (2003) wind tunnel tests in the lower half of the advance ratio range which may suggest that the wind turbine model consisted of a higher aspect ratio blade than that of the wind tunnel test and that the 3-dimensional effects on the blade geometry are sufficiently large to cause prediction errors.

The UAE turbine blade can have 2 different tips. The blade tip that was used in the NASA-Ames wind tunnel test (Lindenburg (2003)) included a rounded chord distribution at the leading edge. Lindenburg (2003) obtained measurements to the effect that the rounded platform gives a 0.0016m^2 reduction in blade area. This reduction in blade area was considered in the wind turbine model by cutting the chord linearly toward the blade tip while maintaining the overall blade radius. This results in a higher aspect ratio.

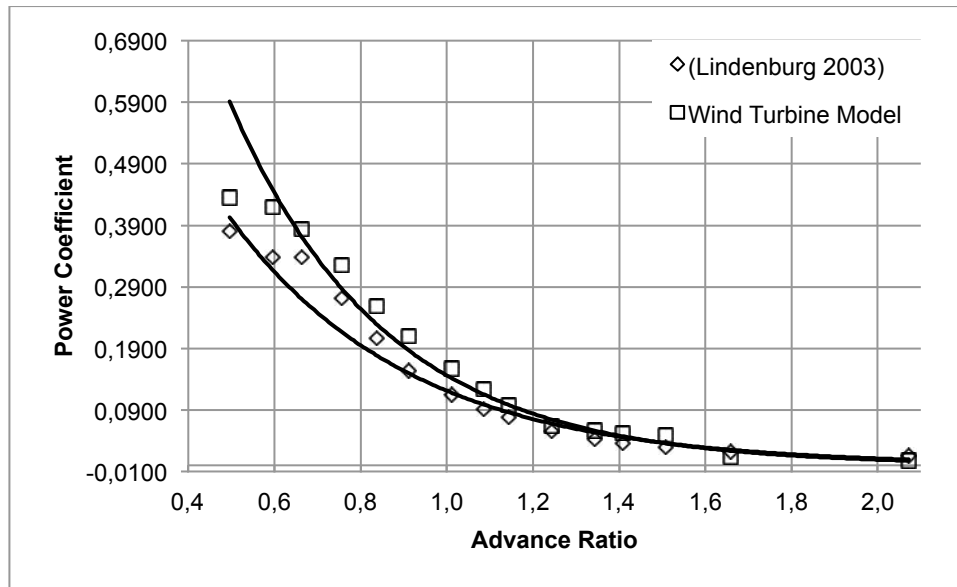


Figure 33: Comparison between Lindenburg (2003) wind tunnel and wind turbine model power coefficients

It is evident from the analysis above the method produced in support of the prediction of propeller power effects and wake analysis may be used, with minor adjustments, to analyse the performance of a wind turbine and the behavior of the flow in it's wake.

10. CONCLUSIONS AND RECOMMENDATIONS

This report details the development of a mathematical model and computational algorithm for the estimation of the performance characteristics of a given propeller and the propagation of a vortical streamtube in it's wake. In developing the model a set of suitable propeller theorems were selected based on their computational complexity and ability to provide suitably accurate predictions for the conceptual design phase of aircraft development.

The three methods that were selected include: blade element theory to predict the propeller forces, momentum theory to relate the flow momentum at the propeller to that of the far wake, and a vortical wake model to describe the slipstream deflection. The resultant model is a combination of the Blade Element Momentum Theory and Vortex Wake Theory with various simplifying assumptions being made for seamless compatibility.

The combined propeller model was developed into an algorithm and built into a working MATLAB simulation in order to evaluate the accuracy and capabilities of the model. The useful application of the model was then proven for a standard and commercial propeller design where comparison was made in Figure 19 and Figure 21 between performance predictions computed by the propeller model and those obtained from wind tunnel tests of an APC 8x8 Thin Electric propeller. Simulations were run at various advance ratios between 0.4 and 0.9; despite the propeller blade experiencing angles of attack outside of it's linear range for lower advance ratios, the thrust predictions were nearly identical to the wind tunnel correlations across the advance ratio range. Power coefficient predictions produced by the propeller model were shown to be suitably descriptive of the behaviour observed in the wind tunnel experiments with an average predictive discrepancy of 6% across the advance ratio range.

The blade element discretisation exercise presented in Figure 22 found the optimum number of blade elements to be 20 in order to ensure accurate modeling without superfluous analytical resolution. In addition to excess computational expense, increasing the number of blade elements beyond 20 lead to increases in prediction error.

The propeller performance model was then tested against a more complex propeller consisting of seven different aerofoil sections along its radius. Due to manufacturing complications, the propeller was of an unusual geometry and relied upon lookup tables of averaged aerofoil characteristic data for Reynolds numbers between 500 000 and 1 million based on the aerodynamic angles estimated in the model iterations. Propeller performance results were compared across a wider range of advance ratios (0.3 – 1.3) than the initial evaluation and also for various blade pitch angle settings ranging from -8° to 4° .

The thrust coefficient predictions provided in Figure 24 and Figure 25 proved useful for lower advance ratios but tended to over-predict increasingly as the advance ratio was raised, an effect likely due to insufficient consideration of 3-dimensional effects in the BEMT. The method used to determine the aerodynamic characteristics of the propeller aerofoils was not able to capture the influences of wind tunnel effects that the experimental propeller experienced.

The comparisons presented in Figure 26 and Figure 27 showed that the power coefficient was not accurately predicted with prediction errors being upward of 100% across more than half of the test range. The impact of the propeller model's inability to capture wind tunnel effects was more drastic in the prediction of propeller power. Inaccuracies in power prediction were exacerbated by the magnification of errors in predicting thrust with the BEMT power equation.

Across the two comparative tests, the propeller model yielded best results for advance ratios between 0.4 and 0.7 which corresponded with thrust coefficient values between 0 and 0.2 and a power coefficient values between 0 and 0.14.

This propeller geometry was then used to demonstrate the capabilities of the slipstream model in predicting the behaviour of the deflected and contracting streamtube in the wake of the propeller. Due to a lack of comparative data, the resultant streamtube was analysed to be in agreement with an understanding of the physical nature of a simplified propeller wake. In relating the forces determined at the propeller to those in the fully developed wake, the slipstream model was able to describe the enlarged streamtube ahead of the propeller as well as the streamtube contraction in the wake of the propeller (Figure 29). For the case of an inclined

propeller, Figure 29 and Figure 31 showed that the streamtube displacement increased in angle for increasing advance ratio. The slipstream was shown to deflect sharply immediately behind the propeller but quickly reached a far wake deflection angle of approximately 2% of the propeller pitch angle. For further study, the algorithm may easily be adjusted to output this data into a range of useful, transferable formats to enable its implementation into existing or purpose-built aerodynamic analysis packages.

The propeller model algorithm was then adjusted to describe wind turbine behaviour by considering the extraction of momentum from the flow. The adjusted model was evaluated by comparing simulated results of the UAE unsteady aerodynamics test with the NASA-Ames wind tunnel tests. The power coefficient comparisons given in Figure 33 indicate that the algorithm is able to suitably describe the power extraction behaviour of a wind turbine. The averaged prediction error across the blade radius was large ($\pm 30\%$) although the trend of the prediction curve was mostly in agreement with that of the wind tunnel results. The wind turbine over-predicted power extraction in the lower half of the advance ratio range with the predicted results being significantly more accurate at higher advance ratios; the literature was unclear on the exact geometry towards the tip of the blades, as a result the aspect ratio was uncertain. As an extension of the propeller model, the wind turbine model is similarly affected by the lack of a detailed consideration of 3-dimensional effects.

An unsuccessful attempt was made to combine the predicted propeller aerodynamic effects of a powered propeller with a steady state wing in a panel code. Fundamentally this was due to inherent limitations in the chosen panel method software package and while efforts were made to overcome these limitations, no workable solution was reached in this regard. In the practical implementation of the propeller disc in the tractor configuration to a wing, the CMARC model was not able to function in the desired manner. The objective of this analysis was to superimpose the propeller disc velocity vectors, normal to the propeller disc outlet, on the section of the wing located immediately behind the disc. In this implementation, a limitation of the CMARC panel code was met where the fluid flow from the rear face of the propeller disc, modeled with a normal velocity higher than the freestream, would not necessarily create a region of accelerated flow in its wake but rather disrupt the flow seen by the wing. The vastly increased drag trailing the propeller disk that was

produced by the CMARC propeller model resulted in a field of disturbed rather than accelerated flow over the wing portion in the propeller wake, this had the effect of reducing lift over the wing as opposed to the expected increase as was demonstrated in the comparative experimental data. It was therefore not possible to model the effects of propeller–wing interaction.

For the conceptual design of aircraft, low-order aerodynamic methods are recommended for designers. Without quantitative justification, the effects of a powered propeller upstream of a lifting body or control device can be material to the aerodynamic characteristics of the aircraft.

It is recommended that further research be carried out into the comparison of propeller performance predictions and wind tunnel experimentation including the evaluation of the propeller slipstream. This would require wind tunnel testing of propellers at various advance ratios and propeller rotational speeds, various blades pitch angles, and a range of incidence of yaw angles. A suitable and effective method for measurement of the propeller wake should be employed.

A useful extension to the propeller / turbine performance model would be for the consideration of multiple propellers or wind turbines located nearby to one another such that the interaction effects may be material.

Further work should be carried out on the implementation of the streamtube model into a low-order panel method such that the effects of the deflected and accelerated (and potentially rotational) flow are accounted for in the prediction of aerodynamic performance. This can be achieved through the superposition of experimentally correlated propeller wake velocities onto the uniform freestream flow ahead of a wing modeled in a similarly low-order aerodynamic method, e.g. the vortex lattice method.

A comprehensive propeller power effects analysis should be conducted on various propeller and aircraft configurations considering:

- Propeller
- Nacelle
- Wing
- Horizontal stabiliser

- Vertical stabiliser
- Aileron deflection
- Rudder deflection
- Elevator deflection

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APPENDIX A: MATLAB Simulation

Propeller Analysis Setup Module

```
%=====
%Propeller Analysis Setup Sheet (PAsSetup.m)
%
%Based on Merrill propeller and wind tunnel tests
%=====

clear all;
clc;
close all;
%% Initialisation

% Propeller Blade Geometry Import
fid = fopen( 'PropGeom.dat', 'r' );
propeller = textscan(fid, '%f %f %f %f', 40, 'headerlines', 2);
blade.radius = propeller{1,1};           % Radial position [m]
blade.chord = propeller{1,2};             % Local chord [m]
blade.sweep = propeller{1,3};             % Local blade sweep
[m]
blade.theta = propeller{1,4}*pi/180;      % Local blade twist
angle [rad]
blade.n_el = length(blade.radius);        % Number of blade
elements
blade.mac = mean(blade.chord);            % Blade MAC [m]

% Blade Element Coefficient Array Initialisation
blade.alpha = zeros(blade.n_el,1);       % Angle of attack
blade.ax = zeros(blade.n_el,1);          % Axial induction
factor
blade.at = zeros(blade.n_el,1);          % Tangential
induction factor
blade.U = zeros(blade.n_el,1);           % Mean velocity
blade.dCL = zeros(blade.n_el,1);         % Lift coefficient
```

```

blade.dCD = zeros(blade.n_el,1);           % Drag coefficient
blade.dCT = zeros(blade.n_el,1);           % Thrust coefficient
blade.dCN = zeros(blade.n_el,1);           % Normal force
coefficient
blade.dCQ = zeros(blade.n_el,1);           % Torque coefficient
blade.dCP = zeros(Blade.n_el,1);           % Power coefficient
blade.dT = zeros(blade.n_el,1);           % Thrust force
blade.dN = zeros(blade.n_el,1);           % Normal force
blade.dQ = zeros(blade.n_el,1);           % Torque

% Propeller Settings
ac.rpm = 16000;                             % Rotational speed
[rpm]
ac.n = ac.rpm/60;                           % Rotational speed
[rps]
ac.R = blade.radius(blade.n_el);            % Propeller radius
[m]
ac.D = 2*ac.R;                              % Propeller diameter
[m]
ac.alpha = 10;                             % Propeller angle of
attack [deg]
ac.alpha = ac.alpha*pi/180;                 % Thrust axis AoA
[rad]
ac.psi = 20;                               % Propeller sideslip
angle [deg]
ac.psi = ac.psi*pi/180;                    % Thrust axis
sideslip angle[rad]
ac.zeta = 0;                               % Blade pitch angle
[deg]
ac.B = 2;                                  % Number of blades
ac.R_hub = 0.24;                           % Hub radius [m]
ac.Omega = ac.n*2*pi;                      % Angular velocity
[rad/s]
ac.rho = 1.225;                            % Density [kg/m3]

% Wind Tunnel Data Import
fid = fopen( 'experimental.dat', 'r' );

```

```

wtunnel = textscan(fid, '%f %f %f', 40, 'headerlines', 1);
wtdata.J = wtunnel{1,1};
wtdata.CT = wtunnel{1,2};
wtdata.CP = wtunnel{1,3};

% Slipstream Setup
stream.ext = 1; % Streamtube rearward
extension[m]
stream.up = 0; % Upstream tube
extension [m]
stream.elements = 50; % Axial slipstream
stations
stream.radials = 20; % Radial slipstream
stations

% Error Resolution
res.err = 0.001; % Axial induction
factor error
res.min_it = 5; % Minimum iterations
res.max_it = 51; % Maximum iterations

% Aircraft Velocity Range Setup
experi.vel = wtdata.J*ac.n*ac.D; % Aircraft/wind-
tunnel velocities [m/s]
experi.CT = wtdata.CT; % Experimental Thrust
coefficient
experi.CP = wtdata.CP; % Experimental Power
coefficient

compu.vel = experi.vel; % Computational
velocity set [m/s]
compu.n_vel = length(compu.vel); % Velocity range
compu.CT = zeros(compu.n_vel,1); % Thrust coefficient
array
compu.CQ = zeros(compu.n_vel,1); % Torque coefficient
array

```

```

compu.CP = zeros(compu.n_vel,1);          % Power coefficient
array

% Test Case
opt.Ftip = 1;                             % 0 = excl tip loss.
1 = incl tip loss.
opt.Fhub = 0;                             % 0 = excl hub loss.
1 = incl hub loss.
opt.ren = 1;
%% Blade Element Momentum Analysis

% An iterative procedure is used to estimate the axial and
tangetial inflow
% factors. Aerofoil data (CL & CD vs. Alpha) is used to calculate
the propeller
% thrust and torque at each element along the blade radius.An
accepted error
% value is used to determine the accuracy of the predicted
induction.

fid = fopen('runs.txt', 'wt');
for k = 1:compu.n_vel
    ac.Vinf = compu.vel(k);
    T = 0;
    Q = 0;
    N = 0;
    P = 0;
    CT = 0;
    CN = 0;
    CQ = 0;
    CP = 0;
    A = 0;
    for i = 1:(blade.n_el-1)
        if i == 1,
            element.dr = blade.radius(2) - blade.radius(1);
        else
            element.dr = blade.radius(i) - blade.radius(i-1);

```

```

end;
element.radius = blade.radius(i);
element.chord = blade.chord(i);
element.beta = ac.zeta*pi/180 + blade.theta(i);
element.chord75 = interp1(blade.radius, blade.chord,
0.75*ac.R);

element = BEM(ac,element,res,opt,fid);

blade.alpha(i) = element.alpha;
blade.ax(i) = element.ax;
blade.at(i) = element.at;
blade.F(i) = element.F;
blade.U(i) = element.U;
blade.dCL(i) = element.dCL;
blade.dCD(i) = element.dCD;
blade.dCT(i) = element.dCT;
blade.dCN(i) = element.dCN;
blade.dCQ(i) = element.dCQ;
blade.dCP(i) = element.dCP;
blade.dT(i) = element.dT;
blade.dN(i) = element.dN;
blade.dQ(i) = element.dQ;
blade.dP(i) = element.dP;
blade.status(i) = element.status;

CT = CT + blade.dCT(i);
CN = CN + blade.dCN(i);
CQ = CQ + blade.dCQ(i);
CP = CP + blade.dCP(i);

T = T + blade.dT(i);
Q = Q + blade.dQ(i);
N = N + blade.dN(i);
P = P + blade.dP(i);

A = A + element.area;

```

```

        disp(sprintf('\n'))
    end;

    compu.ax(:,k) = blade.ax;
    compu.at(:,k) = blade.at;
    compu.status(:,k) = blade.status;

    compu.CT(k) = T/(ac.rho*ac.n^2*ac.D^4);
    compu.CN(k) = N/(ac.rho*ac.n^2*ac.D^4);
    compu.CQ(k) = CQ;
    compu.CP(k) = CP;

    compu.T(k) = T;
    compu.Q(k) = Q;
    compu.N(k) = N;
    compu.P(k) = P;
    compu.eff(k) = T*ac.Vinf/(ac.Omega*Q);
end;

%% Slipstream Determination
velset = 2; % Velocity setting
stream.T = compu.T(velset);
stream.N = compu.N(velset);
stream.Q = compu.Q(velset);
stream.CT = compu.CT(velset);
stream.Vinf = compu.vel(velset);

stream = slipstream(ac,stream);

%% Data Visualisation
bplot(blade,ac);
dataplot(experi,compu,ac);
sprof(stream);
stube(stream);

```

Blade Element Momentum Module

```
function [tmp] = BEM(ac,tmp,res,opt,fid);

%% Variable Initialisation
CL = 0; % Local Lift
coefficient
CD = 0; % Local Drag
coefficient
it = 1; % Iteration counter
ax = 0; % Axial induction
factor
at = 0; % Tangential
induction factor
cal = 1; % Iteration flag
Ftip = 1; % Tip-loss factor
Fhub = 1; % Hub-loss factor

%% BEMT Procedure Iteration
while cal ~= 0
    psi = atan((ac.Vinf*(1+ax))/(ac.Omega...
        *tmp.radius*(1-at))); % Local inflow angle
    [rad]
    alpha = tmp.beta - psi; % Local angle of
    attack [rad]
    % lookup lift and drag values
    CL = interp1(aero.foil(:,1),...
        aero.foil(:,2), alpha); % Lift coefficient
    lookup
    CD = interp1(aero.foil(:,1),...
        aero.foil(:,3), alpha); % Drag coefficient
    lookup

    sigma = (ac.B*tmp.chord)/...
        (2*pi*tmp.radius); % local solidity
```

```

if opt.Ftip == 1
    Ftip = 2/pi*acos(exp((-1)...
        *(ac.B/2*(ac.R-tmp.radius)...
        /(tmp.radius*sin(psi)))); % Tip-loss factor
end;
if opt.Fhub == 1
    Fhub = 2/pi*acos(exp((-1)...
        *(ac.B/2*(tmp.radius-...
        ac.R_hub)/(tmp.radius...
        *sin(psi)))); % Hub-loss factor
end;
F = Ftip*Fhub;
% find new induction factors
axn = real(1/((4*F*(sin(psi))^2)...
    /(sigma*(CL*cos(psi)-CD*sin(psi)...
    ))-1)); % Revised axial
induction factor
atn = real(1/((4*F*sin(psi)*cos(psi)...
    )/(sigma*(CL*sin(psi)+CD*...
    cos(psi))+1)); % Revised tangential
induction factor
% evaluate error
diff_ax = (axn-ax);
diff_at = (atn-at);
if abs(diff_ax) > res.err
    if diff_ax > 0,
        ax = ax + 0.3*diff_ax;
    else
        ax = ax + 0.5*diff_ax;
    end;
    if diff_at > 0,
        at = at + 0.3*diff_at;
    else
        at = at + 0.5*diff_at;
    end;
    it = it+1;
    if it == res.max_it

```

```

        ax = 0;
        at = 0;
        tmp.status = 0;
        cal = 0;
    elseif it < res.min_it
        cal = 1;
    end;
else
    tmp.status = (it - 1);
    cal = 0;
end;
end;

%% Function Passout Allocations

tmp.alpha = alpha;           % Blade element angle
of attack
tmp.ax = ax;                 % Blade element axial
induction factor
tmp.at = at;                 % Blade element
tangential induction factor
tmp.F = F;                   % Blade element loss
correction factor

tmp.U = sqrt((ac.Vinf*(1+ax))^2 + ...
    (ac.Omega*tmp.radius*(1-at))^2); % Mean velocity at
blade element

tmp.dCL = CL;                % Local Lift
coefficient
tmp.dCD = CD;                % Local Drag
coefficient
tmp.dCT = CL*cos(psi)-CD*sin(psi); % Local Thrust
coefficient
tmp.dCN = CL*sin(psi)+CD*cos(psi); % Local Normal force
coefficient
tmp.dCQ = (CL*sin(psi)+CD*cos(psi))*...

```

```

        tmp.radius;                                % Local Torque
coefficient
tmp.dT = 0.5*ac.B*ac.rho*tmp.U^2*...
        tmp.dCT*tmp.chord*tmp.dr;                % Local Thrust force
tmp.dN = 0.5*ac.B*ac.rho*tmp.U^2*...
        tmp.dCN*tmp.chord*tmp.dr;                % Local Normal force
tmp.dQ = 0.5*ac.B*ac.rho*tmp.U^2*...
        tmp.dCQ*tmp.chord*tmp.dr;                % Local Torque
tmp.dP = tmp.dQ*ac.Omega;                        % Local Power
tmp.dCP = tmp.dP/(ac.rho*ac.R^5*4*...
        ac.Omega^3/pi^3);                        % Local Power
coefficient

end

```

Slipstream Module

```

function [wake] = slipstream(ac,wake);

%% Far Wake Compatibility

wake.Vs = sqrt(wake.Vinf^2+(2*wake.T)...
    /(pi*ac.R^2*ac.rho)); % Velocity in fully-
developed slipstream [m/s]
wake.ax = 0.5*(wake.Vs/wake.Vinf-1); % Propeller disc
axial induction factor
wake.V = wake.Vinf*(1+wake.ax); % Axial velocity
through propeller disc [m/s]
ac.Gamma = 2*wake.T/(ac.B*ac.rho*...
    ac.Omega*ac.R^2); % Propeller bound
vorticity/circulation
wake.Rs = sqrt(wake.V/wake.Vs*ac.R^2); % Radius of fully-
developed slipstream [m]

%% Inclusion of In-Plane Force

dVinf_thetap = 0.5*wake.N/(wake.V*pi*...
    ac.rho*ac.R*wake.Rs); % In-plane force
incremental downwash effect
dCN_dalpha = dVinf_thetap*4*wake.Rs*...
    wake.V/(ac.alpha*ac.R*wake.Vs^2); % Normal force
coefficient / propeller AoA
dCY_dpsi = wake.N/(0.5*ac.rho*pi*...
    ac.R^2*wake.Vs^2)/ac.psi; % Side force
coefficient / propeller sideslip

theta_p = ac.alpha*(wake.CT+dCN_dalpha)...
    /(1+sqrt(1-wake.CT))^2; % Fully-developed
slipstream incidence relative
% to freestream [rad]

sigma_p = ac.psi*(wake.CT+dCY_dpsi)/...
```

```

        (1+sqrt(1-wake.CT))^2;                                % Fully-developed
slipstream sideslip relative                                  % to freestream [rad]

%% Slipstream Trajectory

% Far field aspects
alpha_s = (-1)*ac.alpha + theta_p;                          % Incidence angle of
fully-developed slipstream [rad]                             % to propeller thrust

axis [rad]
psi_s = (-1)*ac.psi+sigma_p;                                 % Sideslip angle of
fully-developed slipstream [rad]                             % to propeller thrust

axis [rad]
wake.length = wake.up + wake.ext;                            % Streamtube length
[m]
dx = wake.length/wake.elements;                             % Streamtube element
length [m]
wake.x = zeros(wake.elements,1);                             % Streamtube location
wake.x = [(-1)*wake.up : wake.length/...
          (wake.elements - 1) : wake.ext]';
% Streamwise aspects
for i = 1 : wake.elements

    f(i) = 0.5*(1+(wake.x(i)/ac.D)/...
              sqrt(0.25+(wake.x(i)/ac.D)^2));

    wake.Vx(i) = wake.Vs*(wake.Vinf/...
                          wake.Vs+(1-wake.Vinf/wake.Vs)*f(i)); % Slipstream velocity
at station x [m/s]
    wake.Rx(i) = sqrt(ac.R^2*wake.V/...
                      wake.Vx(i));                             % Streamtube radius
at station x [m]

    alpha_x(i) = alpha_s*(1+wake.Vinf/...
                        wake.Vs*(1/f(i)-1))-ac.alpha*...

```

```

        wake.Vinf/wake.Vs*(1/f(i)-1)           % Vertical angle of
streamtube to propeller

                                                    % thrust axis at
station x [rad]
        psi_x(i) = psi_s*(1+wake.Vinf/...
        wake.Vs*(1/f(i)-1))-ac.psi*...
        wake.Vinf/wake.Vs*(1/f(i)-1);         % Horizontal angle of
streamtube to propeller

                                                    % thrust axis at
station x [rad]

        wake.theta(i) = (-1)*ac.alpha -...
        alpha_x(i);                           % Streamtube
incidence relative to free-stream [rad]
        wake.sigma(i) = (-1)*ac.psi - psi_x(i); % Streamtube sideslip
relative to free-stream [rad]

        wake.z = zeros(wake.elements,1);      % Vertical streamtube
deflection
        wake.y = zeros(wake.elements,1);      % Horizontal
streamtube deflection
end;

for i = 2 : (wake.elements)
    wake.z(i) = wake.z(i-1) + wake.theta(i)*dx;
    wake.y(i) = wake.y(i-1) + wake.sigma(i)*dx;
end;

wake.centre = [wake.x wake.y wake.z]

```

APPENDIX B: CMARC Input File

```
WITKOWSKI LEE SULLIVAN WING
&BINP2 LSTINP=2 LSTOUT=0 LSTFRQ=0 LENRUN=0 LPLTYP=0 &END
&BINP3 LSTGEO=0 LSTNAB=0 LSTWAK=0 LSTCPV=0 &END
&BINP4 MAXIT=200 SOLRES=0.00005 &END
&BINP5 NTSTPS=3 DTSTEP=0.5 &END
&BINP6 RSYM=1.0 RGPR=0.0 RFF=5.0 RCORES=0.050 RCOREW=0.050 &END
&BINP7 VINP=35.052 VSOUND=340.0 &END
&BINP8 ALDEG=0.0 YAWDEG=0.0 PHIDOT=0.0 THEDOT=0.0 PSIDOT=0.0
&END
&BINP8A PHIMAX=0.0 THEMAX=0.0 PSIMAX=0.0 WRX=0.0 WRY=0.0 WRZ=0.0
&END
&BINP8B DXMAX=0.0 DYMAX=0.0 DZMAX=0.0 WTX=0.0 WTY=0.0 WTZ=0.000
&END
&BINP9 CBAR=0.2032 SREF=0.170322 SSPAN=0.4191 RMPX=0.00
RMPY=0.00 RMPZ=0.00 &END
&BINP10 NORSET=0 NBCHGE=0 NCZONE=0 NCZPCH=0 CZDUB=0.0 VREF=0.0
NNROT=0 &END
&BINP11 NORPCH=0 NORF=0 NORL=0 NOCF=0 NOCL=0 VNORM=0.0 &END
&BINP11A NROTPCH=0 NROTRF=0 NROTRL=0 NROTCF=0 NROTCL=0 ANGLE=0.0
VX=0 VY=0 VZ=0 &END
&BINP12 KPAN=0 KSIDE=0 NEWNAB=0 NEWSID=0 &END
&BINP13 NBLIT=1 &END

&ASEM1 ASEM1=0.0000 ASEM2=0.0000 ASEM3=0.0000
ASCAL=1.0000 ATHET=0.0 NODEA=5 &END
&COMP1 COMPM=0.0000 COMPY=0.0000 COMPZ=0.0000
CSCAL=1.0000 CTHET=0.0 NODEC=5 &END

&PATCH1 IREV=0, IDPAT=1, MAKE=0, KCOMP=1, KASS=1, IPATSYM=0,
IPATCOP=0, &END
WLS WING
&SECT1 STX=0, STY=0.0 STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4, TNODS=0, TNPS=0, TINTS=0, &END
0.121920 0.000000 -0.000256
```

0.121801	0.000000	-0.000273
0.121446	0.000000	-0.000322
0.120854	0.000000	-0.000405
0.120027	0.000000	-0.000520
0.118968	0.000000	-0.000667
0.117678	0.000000	-0.000844
0.116160	0.000000	-0.001050
0.114419	0.000000	-0.001285
0.112458	0.000000	-0.001547
0.110282	0.000000	-0.001833
0.107896	0.000000	-0.002143
0.105305	0.000000	-0.002475
0.102516	0.000000	-0.002827
0.099535	0.000000	-0.003197
0.096369	0.000000	-0.003583
0.093025	0.000000	-0.003984
0.089511	0.000000	-0.004396
0.085836	0.000000	-0.004819
0.082008	0.000000	-0.005249
0.078035	0.000000	-0.005686
0.073928	0.000000	-0.006127
0.069696	0.000000	-0.006569
0.065348	0.000000	-0.007012
0.060895	0.000000	-0.007451
0.056348	0.000000	-0.007886
0.051716	0.000000	-0.008314
0.047011	0.000000	-0.008733
0.042244	0.000000	-0.009140
0.037425	0.000000	-0.009533
0.032567	0.000000	-0.009908
0.027679	0.000000	-0.010265
0.022775	0.000000	-0.010599
0.017865	0.000000	-0.010909
0.012961	0.000000	-0.011192
0.008073	0.000000	-0.011444
0.003215	0.000000	-0.011664
-0.001604	0.000000	-0.011850

-0.006371	0.000000	-0.011997
-0.011076	0.000000	-0.012106
-0.015708	0.000000	-0.012172
-0.020255	0.000000	-0.012195
-0.024708	0.000000	-0.012174
-0.029056	0.000000	-0.012106
-0.033288	0.000000	-0.011990
-0.037395	0.000000	-0.011827
-0.041368	0.000000	-0.011616
-0.045196	0.000000	-0.011356
-0.048871	0.000000	-0.011048
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-0.058895	0.000000	-0.009846
-0.061876	0.000000	-0.009357
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-0.069642	0.000000	-0.007649
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-0.077038	0.000000	-0.004891
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-0.080806	0.000000	0.001713
-0.080214	0.000000	0.002540
-0.079387	0.000000	0.003347
-0.078328	0.000000	0.004131
-0.077038	0.000000	0.004891
-0.075520	0.000000	0.005625
-0.073779	0.000000	0.006331
-0.071818	0.000000	0.007006

-0.069642	0.000000	0.007649
-0.067256	0.000000	0.008256
-0.064665	0.000000	0.008826
-0.061876	0.000000	0.009357
-0.058895	0.000000	0.009846
-0.055729	0.000000	0.010292
-0.052385	0.000000	0.010693
-0.048871	0.000000	0.011048
-0.045196	0.000000	0.011356
-0.041368	0.000000	0.011616
-0.037395	0.000000	0.011827
-0.033288	0.000000	0.011990
-0.029056	0.000000	0.012106
-0.024708	0.000000	0.012174
-0.020255	0.000000	0.012195
-0.015708	0.000000	0.012172
-0.011076	0.000000	0.012106
-0.006371	0.000000	0.011997
-0.001604	0.000000	0.011850
0.003215	0.000000	0.011664
0.008073	0.000000	0.011444
0.012961	0.000000	0.011192
0.017865	0.000000	0.010909
0.022775	0.000000	0.010599
0.027679	0.000000	0.010265
0.032567	0.000000	0.009908
0.037425	0.000000	0.009533
0.042244	0.000000	0.009140
0.047011	0.000000	0.008733
0.051716	0.000000	0.008314
0.056348	0.000000	0.007886
0.060895	0.000000	0.007451
0.065348	0.000000	0.007012
0.069696	0.000000	0.006569
0.073928	0.000000	0.006127
0.078035	0.000000	0.005686
0.082008	0.000000	0.005249

0.085836	0.000000	0.004819
0.089511	0.000000	0.004396
0.093025	0.000000	0.003984
0.096369	0.000000	0.003583
0.099535	0.000000	0.003197
0.102516	0.000000	0.002827
0.105305	0.000000	0.002475
0.107896	0.000000	0.002143
0.110282	0.000000	0.001833
0.112458	0.000000	0.001547
0.114419	0.000000	0.001285
0.116160	0.000000	0.001050
0.117678	0.000000	0.000844
0.118968	0.000000	0.000667
0.120027	0.000000	0.000520
0.120854	0.000000	0.000405
0.121446	0.000000	0.000322
0.121801	0.000000	0.000273
0.121920	0.000000	0.000256
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END		
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0,		
THETA=0.0000, INMODE=4, TNODS=3, TNPS=10, TINTS=0, &END		
0.121920	0.838200	-0.000256
0.121801	0.838200	-0.000273
0.121446	0.838200	-0.000322
0.120854	0.838200	-0.000405
0.120027	0.838200	-0.000520
0.118968	0.838200	-0.000667
0.117678	0.838200	-0.000844
0.116160	0.838200	-0.001050
0.114419	0.838200	-0.001285
0.112458	0.838200	-0.001547
0.110282	0.838200	-0.001833
0.107896	0.838200	-0.002143
0.105305	0.838200	-0.002475
0.102516	0.838200	-0.002827
0.099535	0.838200	-0.003197

0.096369	0.838200	-0.003583
0.093025	0.838200	-0.003984
0.089511	0.838200	-0.004396
0.085836	0.838200	-0.004819
0.082008	0.838200	-0.005249
0.078035	0.838200	-0.005686
0.073928	0.838200	-0.006127
0.069696	0.838200	-0.006569
0.065348	0.838200	-0.007012
0.060895	0.838200	-0.007451
0.056348	0.838200	-0.007886
0.051716	0.838200	-0.008314
0.047011	0.838200	-0.008733
0.042244	0.838200	-0.009140
0.037425	0.838200	-0.009533
0.032567	0.838200	-0.009908
0.027679	0.838200	-0.010265
0.022775	0.838200	-0.010599
0.017865	0.838200	-0.010909
0.012961	0.838200	-0.011192
0.008073	0.838200	-0.011444
0.003215	0.838200	-0.011664
-0.001604	0.838200	-0.011850
-0.006371	0.838200	-0.011997
-0.011076	0.838200	-0.012106
-0.015708	0.838200	-0.012172
-0.020255	0.838200	-0.012195
-0.024708	0.838200	-0.012174
-0.029056	0.838200	-0.012106
-0.033288	0.838200	-0.011990
-0.037395	0.838200	-0.011827
-0.041368	0.838200	-0.011616
-0.045196	0.838200	-0.011356
-0.048871	0.838200	-0.011048
-0.052385	0.838200	-0.010693
-0.055729	0.838200	-0.010292
-0.058895	0.838200	-0.009846

-0.061876	0.838200	-0.009357
-0.064665	0.838200	-0.008826
-0.067256	0.838200	-0.008256
-0.069642	0.838200	-0.007649
-0.071818	0.838200	-0.007006
-0.073779	0.838200	-0.006331
-0.075520	0.838200	-0.005625
-0.077038	0.838200	-0.004891
-0.078328	0.838200	-0.004131
-0.079387	0.838200	-0.003347
-0.080214	0.838200	-0.002540
-0.080806	0.838200	-0.001713
-0.081161	0.838200	-0.000866
-0.081280	0.838200	0.000000
-0.081161	0.838200	0.000866
-0.080806	0.838200	0.001713
-0.080214	0.838200	0.002540
-0.079387	0.838200	0.003347
-0.078328	0.838200	0.004131
-0.077038	0.838200	0.004891
-0.075520	0.838200	0.005625
-0.073779	0.838200	0.006331
-0.071818	0.838200	0.007006
-0.069642	0.838200	0.007649
-0.067256	0.838200	0.008256
-0.064665	0.838200	0.008826
-0.061876	0.838200	0.009357
-0.058895	0.838200	0.009846
-0.055729	0.838200	0.010292
-0.052385	0.838200	0.010693
-0.048871	0.838200	0.011048
-0.045196	0.838200	0.011356
-0.041368	0.838200	0.011616
-0.037395	0.838200	0.011827
-0.033288	0.838200	0.011990
-0.029056	0.838200	0.012106
-0.024708	0.838200	0.012174

-0.020255	0.838200	0.012195
-0.015708	0.838200	0.012172
-0.011076	0.838200	0.012106
-0.006371	0.838200	0.011997
-0.001604	0.838200	0.011850
0.003215	0.838200	0.011664
0.008073	0.838200	0.011444
0.012961	0.838200	0.011192
0.017865	0.838200	0.010909
0.022775	0.838200	0.010599
0.027679	0.838200	0.010265
0.032567	0.838200	0.009908
0.037425	0.838200	0.009533
0.042244	0.838200	0.009140
0.047011	0.838200	0.008733
0.051716	0.838200	0.008314
0.056348	0.838200	0.007886
0.060895	0.838200	0.007451
0.065348	0.838200	0.007012
0.069696	0.838200	0.006569
0.073928	0.838200	0.006127
0.078035	0.838200	0.005686
0.082008	0.838200	0.005249
0.085836	0.838200	0.004819
0.089511	0.838200	0.004396
0.093025	0.838200	0.003984
0.096369	0.838200	0.003583
0.099535	0.838200	0.003197
0.102516	0.838200	0.002827
0.105305	0.838200	0.002475
0.107896	0.838200	0.002143
0.110282	0.838200	0.001833
0.112458	0.838200	0.001547
0.114419	0.838200	0.001285
0.116160	0.838200	0.001050
0.117678	0.838200	0.000844
0.118968	0.838200	0.000667

-0.1536	0.1834	-0.0453
-0.1537	0.1972	-0.0643
-0.1536	0.2110	-0.0834
-0.1533	0.2235	-0.1005
-0.1529	0.2334	-0.1142
-0.1525	0.2398	-0.1229
-0.1524	0.2420	-0.1260
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END		
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,		
INMODE=4,		
TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1533	0.1823	-0.0124
-0.1536	0.2025	-0.0189
-0.1537	0.2249	-0.0262
-0.1536	0.2473	-0.0335
-0.1533	0.2675	-0.0401
-0.1529	0.2835	-0.0453
-0.1525	0.2938	-0.0486
-0.1524	0.2973	-0.0498
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END		
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,		
INMODE=4,		
TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1533	0.1823	0.0070
-0.1536	0.2025	0.0136
-0.1537	0.2249	0.0209
-0.1536	0.2473	0.0282
-0.1533	0.2675	0.0347
-0.1529	0.2835	0.0399
-0.1525	0.2938	0.0433
-0.1524	0.2973	0.0444
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END		
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,		
INMODE=4,		
TNODS=0, TNPS=0, TINTS=0, &END		

-0.1524	0.1524	0.0000
-0.1533	0.1709	0.0227
-0.1536	0.1834	0.0399
-0.1537	0.1972	0.0590
-0.1536	0.2110	0.0780
-0.1533	0.2235	0.0952
-0.1529	0.2334	0.1089
-0.1525	0.2398	0.1176
-0.1524	0.2420	0.1206

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,

INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1533	0.1524	0.0287
-0.1536	0.1524	0.0500
-0.1537	0.1524	0.0735
-0.1536	0.1524	0.0971
-0.1533	0.1524	0.1183
-0.1529	0.1524	0.1352
-0.1525	0.1524	0.1460
-0.1524	0.1524	0.1497

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,

INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1533	0.1339	0.0227
-0.1536	0.1215	0.0399
-0.1537	0.1076	0.0590
-0.1536	0.0938	0.0780
-0.1533	0.0813	0.0952
-0.1529	0.0714	0.1089
-0.1525	0.0650	0.1176
-0.1524	0.0628	0.1206

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

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&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524    0.1524    0.0000
-0.1533    0.1225    0.0070
-0.1536    0.1023    0.0136
-0.1537    0.0799    0.0209
-0.1536    0.0575    0.0282
-0.1533    0.0373    0.0347
-0.1529    0.0213    0.0399
-0.1525    0.0110    0.0433
-0.1524    0.0075    0.0444
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524    0.1524    0.0000
-0.1533    0.1225   -0.0124
-0.1536    0.1023   -0.0189
-0.1537    0.0799   -0.0262
-0.1536    0.0575   -0.0335
-0.1533    0.0373   -0.0401
-0.1529    0.0213   -0.0453
-0.1525    0.0110   -0.0486
-0.1524    0.0075   -0.0498
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524    0.1524    0.0000
-0.1533    0.1339   -0.0281
-0.1536    0.1215   -0.0453
-0.1537    0.1076   -0.0643
-0.1536    0.0938   -0.0834
-0.1533    0.0813   -0.1005
-0.1529    0.0714   -0.1142
-0.1525    0.0650   -0.1229

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-0.1524      0.0628      -0.1260
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=3, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1533      0.1524      -0.0341
-0.1536      0.1524      -0.0553
-0.1537      0.1524      -0.0789
-0.1536      0.1524      -0.1024
-0.1533      0.1524      -0.1237
-0.1529      0.1524      -0.1405
-0.1525      0.1524      -0.1513
-0.1524      0.1524      -0.1551
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&PATCH1 IREV=-1, IDPAT=2, MAKE=0, KCOMP=1, KASS=1, IPATSYM=0,
IPATCOP=0, &END
PROP DISK AFT FACE %%2
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1524      -0.0341
-0.1512      0.1524      -0.0553
-0.1511      0.1524      -0.0789
-0.1512      0.1524      -0.1024
-0.1515      0.1524      -0.1237
-0.1519      0.1524      -0.1405
-0.1523      0.1524      -0.1513
-0.1524      0.1524      -0.1551
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1709      -0.0281

```

-0.1512	0.1834	-0.0453
-0.1511	0.1972	-0.0643
-0.1512	0.2110	-0.0834
-0.1515	0.2235	-0.1005
-0.1519	0.2334	-0.1142
-0.1523	0.2398	-0.1229
-0.1524	0.2420	-0.1260

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1515	0.1823	-0.0124
-0.1512	0.2025	-0.0189
-0.1511	0.2249	-0.0262
-0.1512	0.2473	-0.0335
-0.1515	0.2675	-0.0401
-0.1519	0.2835	-0.0453
-0.1523	0.2938	-0.0486
-0.1524	0.2973	-0.0498

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1515	0.1823	0.0070
-0.1512	0.2025	0.0136
-0.1511	0.2249	0.0209
-0.1512	0.2473	0.0282
-0.1515	0.2675	0.0347
-0.1519	0.2835	0.0399
-0.1523	0.2938	0.0433
-0.1524	0.2973	0.0444

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END

-0.1524	0.1524	0.0000
-0.1515	0.1709	0.0227
-0.1512	0.1834	0.0399
-0.1511	0.1972	0.0590
-0.1512	0.2110	0.0780
-0.1515	0.2235	0.0952
-0.1519	0.2334	0.1089
-0.1523	0.2398	0.1176
-0.1524	0.2420	0.1206

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,

INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1515	0.1524	0.0287
-0.1512	0.1524	0.0500
-0.1511	0.1524	0.0735
-0.1512	0.1524	0.0971
-0.1515	0.1524	0.1183
-0.1519	0.1524	0.1352
-0.1523	0.1524	0.1460
-0.1524	0.1524	0.1497

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,

INMODE=4,

TNODS=0, TNPS=0, TINTS=0, &END		
-0.1524	0.1524	0.0000
-0.1515	0.1339	0.0227
-0.1512	0.1215	0.0399
-0.1511	0.1076	0.0590
-0.1512	0.0938	0.0780
-0.1515	0.0813	0.0952
-0.1519	0.0714	0.1089
-0.1523	0.0650	0.1176
-0.1524	0.0628	0.1206

&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

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&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
  TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1225      0.0070
-0.1512      0.1023      0.0136
-0.1511      0.0799      0.0209
-0.1512      0.0575      0.0282
-0.1515      0.0373      0.0347
-0.1519      0.0213      0.0399
-0.1523      0.0110      0.0433
-0.1524      0.0075      0.0444
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
  TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1225     -0.0124
-0.1512      0.1023     -0.0189
-0.1511      0.0799     -0.0262
-0.1512      0.0575     -0.0335
-0.1515      0.0373     -0.0401
-0.1519      0.0213     -0.0453
-0.1523      0.0110     -0.0486
-0.1524      0.0075     -0.0498
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
  TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1339     -0.0281
-0.1512      0.1215     -0.0453
-0.1511      0.1076     -0.0643
-0.1512      0.0938     -0.0834
-0.1515      0.0813     -0.1005
-0.1519      0.0714     -0.1142
-0.1523      0.0650     -0.1229

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-0.1524      0.0628      -0.1260
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=5, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1524      -0.0341
-0.1512      0.1524      -0.0553
-0.1511      0.1524      -0.0789
-0.1512      0.1524      -0.1024
-0.1515      0.1524      -0.1237
-0.1519      0.1524      -0.1405
-0.1523      0.1524      -0.1513
-0.1524      0.1524      -0.1551
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

&WAKE1 IDWAK=1 IFLXW=0 ITRFTZ=8 INTRW=0 &END
    WING WAKE
&WAKE2 KWPACH=1 KWSIDE=2 KWLINE=0 KWpan1=0 KWpan2=0 NODEW=5
INITIAL=1 &END
&SECT1 STX=1.0 STY=0.0000 STZ=0.0 SCALE=1.0 ALF=0.0 THETA=0.0
INMODE=-1 TNODS=3 TNPS=20 TINTS=3 &END
    AFT FACE %%2
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1524      -0.0341
-0.1512      0.1524      -0.0553
-0.1511      0.1524      -0.0789
-0.1512      0.1524      -0.1024
-0.1515      0.1524      -0.1237
-0.1519      0.1524      -0.1405
-0.1523      0.1524      -0.1513
-0.1524      0.1524      -0.1551
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

```

```

&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
  TNODS=0, TNPS=0, TINTS=0, &END
-0.1524    0.1524    0.0000
-0.1515    0.1709   -0.0281
-0.1512    0.1834   -0.0453
-0.1511    0.1972   -0.0643
-0.1512    0.2110   -0.0834
-0.1515    0.2235   -0.1005
-0.1519    0.2334   -0.1142
-0.1523    0.2398   -0.1229
-0.1524    0.2420   -0.1260
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
  TNODS=0, TNPS=0, TINTS=0, &END
-0.1524    0.1524    0.0000
-0.1515    0.1823   -0.0124
-0.1512    0.2025   -0.0189
-0.1511    0.2249   -0.0262
-0.1512    0.2473   -0.0335
-0.1515    0.2675   -0.0401
-0.1519    0.2835   -0.0453
-0.1523    0.2938   -0.0486
-0.1524    0.2973   -0.0498
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
  TNODS=0, TNPS=0, TINTS=0, &END
-0.1524    0.1524    0.0000
-0.1515    0.1823    0.0070
-0.1512    0.2025    0.0136
-0.1511    0.2249    0.0209
-0.1512    0.2473    0.0282
-0.1515    0.2675    0.0347
-0.1519    0.2835    0.0399
-0.1523    0.2938    0.0433

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-0.1524      0.2973      0.0444
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1709      0.0227
-0.1512      0.1834      0.0399
-0.1511      0.1972      0.0590
-0.1512      0.2110      0.0780
-0.1515      0.2235      0.0952
-0.1519      0.2334      0.1089
-0.1523      0.2398      0.1176
-0.1524      0.2420      0.1206
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
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-0.1515      0.1524      0.0287
-0.1512      0.1524      0.0500
-0.1511      0.1524      0.0735
-0.1512      0.1524      0.0971
-0.1515      0.1524      0.1183
-0.1519      0.1524      0.1352
-0.1523      0.1524      0.1460
-0.1524      0.1524      0.1497
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1339      0.0227
-0.1512      0.1215      0.0399
-0.1511      0.1076      0.0590
-0.1512      0.0938      0.0780
-0.1515      0.0813      0.0952

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-0.1519      0.0714      0.1089
-0.1523      0.0650      0.1176
-0.1524      0.0628      0.1206
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
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-0.1515      0.1225      0.0070
-0.1512      0.1023      0.0136
-0.1511      0.0799      0.0209
-0.1512      0.0575      0.0282
-0.1515      0.0373      0.0347
-0.1519      0.0213      0.0399
-0.1523      0.0110      0.0433
-0.1524      0.0075      0.0444
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
-0.1524      0.1524      0.0000
-0.1515      0.1225      -0.0124
-0.1512      0.1023      -0.0189
-0.1511      0.0799      -0.0262
-0.1512      0.0575      -0.0335
-0.1515      0.0373      -0.0401
-0.1519      0.0213      -0.0453
-0.1523      0.0110      -0.0486
-0.1524      0.0075      -0.0498
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END
&SECT1 STX=0.0, STY=0.0, STZ=0.0, SCALE=1.0, ALF=0.0, THETA=0.0,
INMODE=4,
    TNODS=0, TNPS=0, TINTS=0, &END
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-0.1515      0.1339      -0.0281
-0.1512      0.1215      -0.0453
-0.1511      0.1076      -0.0643

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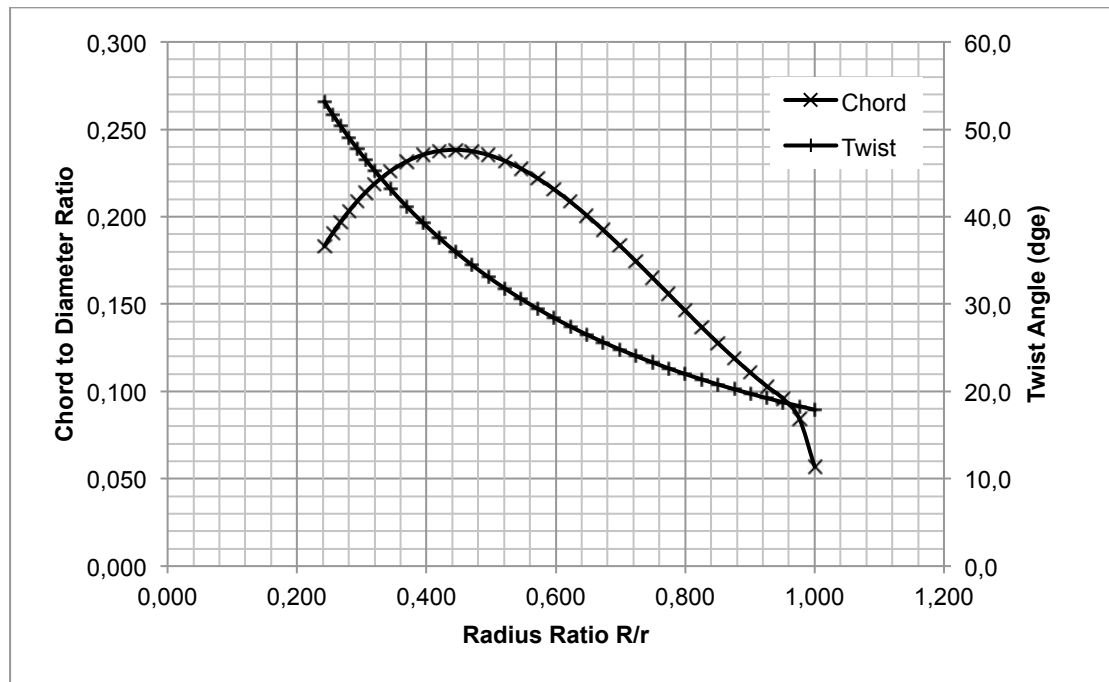
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-0.1512      0.1524      -0.0553
-0.1511      0.1524      -0.0789
-0.1512      0.1524      -0.1024
-0.1515      0.1524      -0.1237
-0.1519      0.1524      -0.1405
-0.1523      0.1524      -0.1513
-0.1524      0.1524      -0.1551
&BPNODE TNODE=3, TNPC=0, TINTC=0, &END

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    WING WAKE
&WAKE2 KWPACH=1 KWSIDE=2 KWLINE=0 KWPA1=0 KWPA2=0 NODEW=5
INITIAL=1 &END
&SECT1 STX=1.0 STY=0.0000 STZ=0.0 SCALE=1.0 ALF=0.0 THETA=0.0
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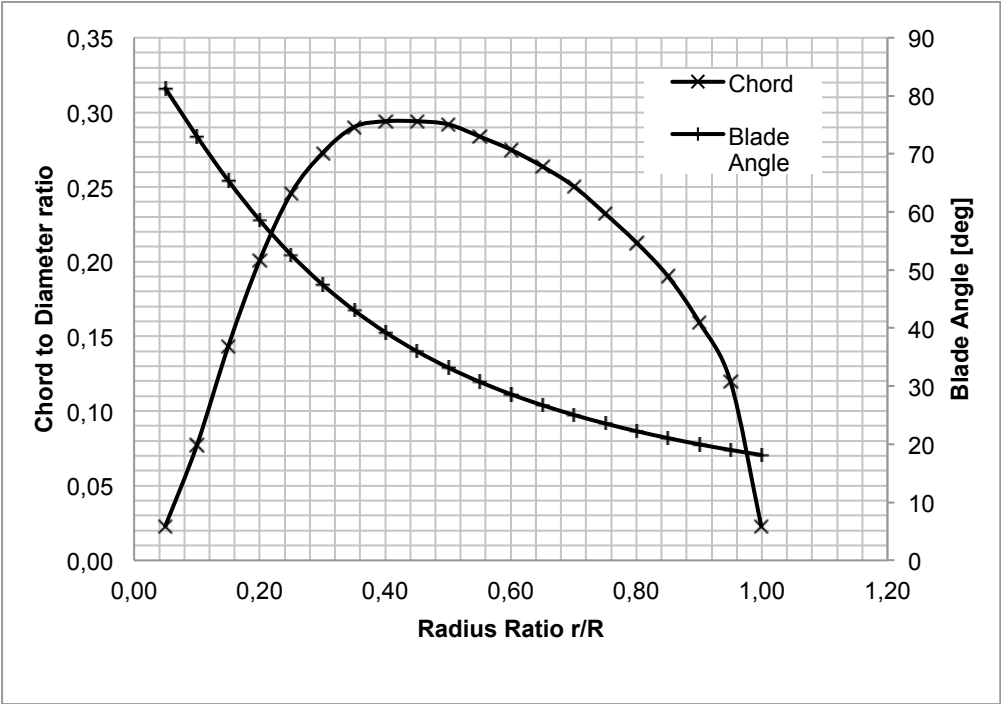
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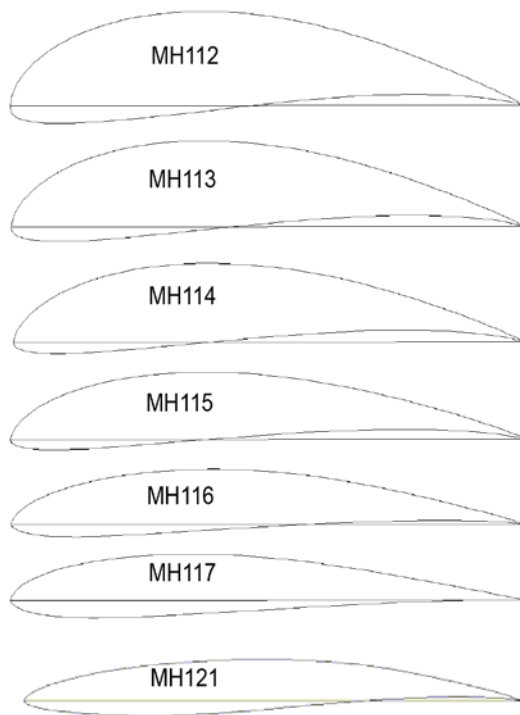
APPENDIX C: Propeller Geometry

APC 8x8 Thin Electric Propeller - Merrill (2011)



CSIR Propeller - Monk (2010)





Radial Position	Aerofoil
0.05	MH112
0.35	MH113
0.45	MH114
0.55	MH115
0.65	MH116
0.75	MH117
0.85	MH121
1.00	MH121