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University of the Witwatersrand



Stochastic Volatility Models in Financial Econometrics: An Application to South Africa

Sambulo Malumisa

Thesis Submitted for the Degree of Doctor of Philosophy,

School of Economic and Business Sciences

Faculty of Commerce, Law and Management

University of the Witwatersrand

April 2015

DECLARATION

I declare that the thesis titled "**Stochastic volatility models in financial econometrics: an application to South Africa**" is my own work, that it has not been submitted for any degree or examination at any other university, and that all the sources I have used or quoted have been indicated and acknowledged by complete references.

Sambulo Malumisa April 2015

Signed:……

DEDICATION

This thesis is dedicated to my wife, Brenda; for your support during this trying phase. Your sacrifice humbled me throughout. To our toddler daughter Ophiweyinkosi Hannah, your bubbling smiles and inconsistent tantrums were the right dosage.

ACKNOWLEDGEMENTS

It is God who deserves all the credit for He daily sustains us. He is able to keep us from falling (Jude 24)

My gratitude also goes to my family at large for all the moral support and love. I am most grateful to my supervisor Professor Mthuli Ncube for the wise guidance during my studies. I am also indebted to Professor Busani Moyo and Dr Eliphas Ndou who also contributed immensely in my studies.

ABSTRACT

Stochastic volatility models in financial econometrics: an application to South Africa.

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PhD Thesis, Department of Economics, University of the Witwatersrand

The dissertation carries out a study to understand asset price behaviour in South Africa. This is investigated through the application of stochastic volatility models to trace the characteristics of high frequency financial data; daily temperature, exchange rates, interest rates, stock and house prices. Innovation in the derivatives market has seen the introduction of weather derivatives as a risk mitigation tool against adverse weather movements. Chapter Two applies three different time series models of temperature to estimate payoffs to determine which method offers the best hedging strategy in four South African cities. Results from the study suggest that the seasonality GARCH method of estimating payoffs for temperature based weather derivatives offers superior performance compared to the Cumulative Cooling Degree Days (CDD) and the historical method. This suggests that the seasonality GARCH method can be applied in these cities to hedge against adverse temperature movements. In Chapter three we consider the estimation methodology for jump diffusion models and GARCH models. Chapter four investigates volatility on exchange rate data. Use is made of the british pound/south african rand, euro/south african rand and u.s dollar/ south african rand exchange rates. The research introduces a jump diffusion model to trace the behaviour of exchange rate data. Estimation results are able to match the summary statistics in mean, variance, skewness and kurtosis. Results from the model can also explain the volatility smile for short and medium term maturities. A fat tailed GARCH model is introduced to capture the persistence in volatility on exchange rate data. Results from this chapter have an implication for pricing currency options to offer leverage to organisations affected by exchange rate risk. Chapter five extends the analysis to study the behaviour of short term interest rates, making use of the 90 Day Treasury bill (T-Bill) rate. The chapter considers a variant application of the Chan *et al*. (1992) model for short term interest rates wherein a jump diffusion model is introduced. The results match the summary statistics equivalent suggesting the capability of the model specification. Splitting the estimation period suggests that the jump size is highest post inflation target though with a smaller intensity. However, the 90 Day T-Bill shows higher volatility after inflation targeting though with a lesser intensity. These findings have a bearing on valuation of short term interest derivatives and also investigating multi factor models of interest rates. In chapter six four

sectors (banking, mining, media and leisure) are considered to explore movements in stock prices. A jump diffusion model is applied to get estimation results. The results confirm related studies that stock prices have incidents of volatility which can be captured by a jump diffusion model. The results also shed light on the importance of portfolio diversification considering the different results across the sectors investigated. The implication also lies in understanding market efficiency. Chapter seven applies the jump diffusion model on house prices to understand more on the drivers of volatility on house prices. The interesting results on this chapter can be summarised as follows; the four different house segments have almost similar jump sizes though the small house price segment has highest intensity. This can point to expectations and volatility from participants in this segment at a higher level than for other segments over different regimes over the study period. The estimated higher moments were not normalised as had happened for the three previous chapters after introducing the jump diffusion model. Results from this chapter have an application to valuing mortgage premium across different house price segments. It is recommended that rigorous research on asset prices using various approaches be considered as it goes a long way in informing policy makers and investors to mitigate risk in an environment of volatile asset prices. With the growing interest in weather derivatives world-wide, there is a need to educate farmers, government entities, potential counter-parties and other organisations affected by weather related risk on the importance of weather derivatives so that a foundation is laid for trading in this special type of insurance.

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CHAPTER ONE

INTRODUCTION

1.1 Background to the study.

Volatility and its measurement are important for the pricing of assets and risk management. For one, investors want to know the direction of the market as well as the velocity of such movements. This brings us to realise the importance of volatility in financial economics. Volatility even plays a big role in the valuation of derivative securities. If the stock has low volatility an option contract will have an equally low value. However, a high volatility in price movement means that the strike price will be exceeded, rendering the option expensive. Highlighting this importance of volatility, any successful method of forecasting will benefit all users affected.

This study explores the applicability of stochastic volatility models in South Africa to enhance understanding in volatile asset price movement and derive possible solutions for strategies amidst volatile financial data. One application of volatility modelling is in daily temperature data from which we can model weather derivatives. Weather derivatives are potential valuable tools for risk management against adverse weather conditions. They are designed as a "bet" on weather conditions with the only requirement being an observable objective variable agreed upon by both parties (Richards *et al.*, 2004). Their payoffs are contingent on weather indices based on climatic factors. They are useful for hedging risk in the agricultural, energy, entertainment, beverage, construction and even apparel industries. Instruments thereof include swaps, options and option collars with payoffs dependent on weather related variables like average temperature, heating and cooling degree days (HDD and CDD), humidity, maximum or minimum temperatures.

Another visible application of stochastic volatility models is in asset prices, perhaps popularised by the reverenced basic model for capturing the movements of asset prices is the Geometric Brownian Motion (GBM), which underpins the Black-Scholes (1973) option pricing model. The Black-Scholes model has been found to be insufficient in capturing typical characteristics of high frequency data sets; leptokurtic, volatility smile/skew. This motivated the development of models embedded with jumps as a better alternative to the purely diffusive processes. Some of the reasons supporting models with jumps include: (i)

Jump processes are able to model catastrophic events in the market; (ii) The high frequency data in finance indicates that the path of the asset price is not continuous in small time scale, (iii) Compared to purely diffusion models, jump diffusion models allow one to quantify and take into account the risk of strong stock price movements over short periods of time, thus filling the gap in the diffusion framework , and (iv) Jump diffusion models can better fit the existence of price jumps in the real world.

The study applies stochastic volatility models in five areas namely; (a) Estimating payoffs of temperature based weather derivatives (b) Model estimating the exchange rate returns, (c) Jump diffusion model for interest rates, (d) a Jump diffusion model fitting stock price data and (e) Volatility and jumps in house prices.

1.1.1 The generic model.

According to the Black-Scholes model, volatility is assumed to be constant. However, in practice volatility varies with time. If the volatility parameter in the GBM is a known function of time, the risk-neutral process followed by the asset price is represented thus

$$dS = \mu S dt + \sigma(t) S dz \tag{1.1}$$

Where; S is the price, μ the drift, and σ being the predictable instantaneous volatility and dz the Wiener process. In practice however, volatility varies stochastically. This has led to the development of many complex models with two stochastic variables: the stock price and its volatility, given by:

$$\begin{aligned} \frac{dS}{S} &= (r-q)dt + \sqrt{V}\, dz_s \\ dV &= a(V_L - V)dt + \xi V^{\alpha} dz_v \end{aligned} \tag{1.2}$$

a, V_L, ξ and α are constants, and dz_s and dz_v are Weiner processes. The variable V in the model is the asset's variance rate. The variance rate has a drift that pulls it back to a level V_L at rate *a* (Hull and White,1987). However, the above diffusion model does not capture all of the features of high frequency financial time series data. To better the model, we add jumps. In this study the researcher will not select the most appropriate jump diffusion specification and therefore selects one jump diffusion configuration to make all the forecasts. The model

draws from an approach close to the seminal work of Merton (1976)[1], specifying the risk neutral process followed by the asset price:

(1.3) $$\frac{dS}{S} = \mu dt + \sigma dz + kd\pi(\lambda)$$

where $S \sim N(\mu h, \sigma^2 h)$

μ is the drift

σ is the standard deviation of the diffusion

k is the jump size

π is a Poisson process with parameter λ

To compliment the jump diffusion specification there is also consideration of the GARCH (1,1)[2] representation to capture the persistence of volatility in asset price data. In this the GARCH (1,1) model is specified as follows

(1.4) $$Y_t = C + \varepsilon_t$$

(1.5) $$\sigma_t^2 = \kappa + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2$$

Equation (1.4) above represents the conditional mean model where the returns for the asset price, Y_t are explained by a constant and an uncorrelated, white noise disturbance. The conditional variance equation (1.5) consists of a constant plus a weighted average of last period's forecast, σ_{t-1}^2 , and the corresponding squared disturbance, ε_{t-1}^2 .

Equations (1.3) to (1.5) sum up the generic model in this study.

1.2 Research problem.

There is a growing research on understanding the behaviour of high frequency financial data more so in an era of financial crises witnessed over the past few decades. Studies carried out have shown the benefits of characterising asset price movements as precursor to understanding volatility and hence risk management. In this category, research studies by Clements *et al*, (2013; 2008), Cyr and Kusy (2007) Mykland (2007), Sorwar (2011), Yang *et al* (2009) and Mizrach (2008) form part of the literature having explored weather derivatives, exchange rate, interest rate stock and house prices respectively.

[1] see R.C Merton, " Option pricing when underlying stock returns are discontinuous", *Journal of Financial Economics*, 3 :125-144.

[2] GARCH models are used in forecasting and parameter estimation in stochastic volatility settings (Flemming, 2003)

However the picture changes when we look at the South African experience in documenting features of high frequency financial data as research studies have been limited for some asset prices and non-existent for others (see for example Scheepers, (2005), Geyser (2004), Taylor (2014), van Appel (2013), Duncan and Kabundi, (2013). This has not taken away insufficient understanding as some of these studies have involved complex asset prices (see for example Poklewski-Koziell (2012), Kalsheker (2009)) whilst there has not been investigation for some areas for example characterising house prices and temperature based weather derivatives.

This has left out an in-depth analysis on understanding what drives asset prices with the aim of informing academia, businesses, government and other stakeholders in making decisions amidst volatile financial data. This also comes at a time when there is a growing global move on applying the latest varying financial instruments in mitigating business risk. The thesis therefore seeks to fill this gap by carrying out a study on five different asset types so as to enhance decision making amidst volatile financial data through making use of an alternative characterisation of the features of asset prices. Use is made of stochastic volatility models mainly the jump diffusion model as an alternative process representing the behaviour of asset prices. This is done through investigating the unexplored areas in temperature based weather derivatives and house prices and also proposing alternative modelling of the other asset prices as specified below. This contributes to understanding drivers of volatility and hence serving as a precursor to decision making amidst high frequency asset prices in South Africa.

1.2.1 Estimating payoffs for temperature based weather derivatives.

The derivatives market has been innovative in developing products to meet the needs of market participants, even in weather risk. Weather derivatives are a growing derivatives sector the world over. They began to develop in 1997, as a result of the El Nino catastrophe. Companies with earnings tied to weather then, realised the importance of hedging.

This affirms that weather derivatives are valuable tools for risk management. Their payoffs are contingent on weather indices based on climatic factors. The objective of the study in this area is to enhance understanding of the benefits therein in applying weather derivatives in South Africa. This will be aided through an estimation of the payoffs of temperature based weather derivatives. Based on the availability of accurate data, the study focuses on four

South African cities, thus informing industry players affected about alternative strategies for safeguarding earnings against adverse temperature movements. This contribution fills the gap in the literature exploring the applicability of weather derivatives. Use is made of three different methods for estimating payoffs to determine the best method for use in making strategies. The objectives in this contribution include reviewing the benefits of a weather derivatives market with an application to South Africa. In this chapter we also consider some studies on weather derivatives in South Africa and determine as to which weather variables were employed. This will help us determine as to what extent the chapter will fill the existing gap. We also estimate payoffs for temperature based weather derivatives using three different methods. Based on availability of reliable weather data this will enable us to make recommendations as to the applicability of temperature based weather derivatives for the chosen cities.

1.2.2 Jump diffusion model for the exchange rate.

The success of financial economists over the past few decades is attributable to their ability to be able to predict with a certain level of accuracy the movement of asset prices. One approach to better understand asset price movement is the application of simple diffusion models which approximate the stochastic process for returns on financial assets. Though widely used, the "volatility smiles and smirks" derived using volatility in the Black-Scholes model reveal that a simple geometric Brownian motion process misses important features of the data. High frequency returns data display excess kurtosis (fat tailed distributions), skewness, and volatility clustering. To capture these features, there is a need to employ jump diffusion models. This will enable one to also compare the intensity and size of jumps in asset prices thereby offering leverage in making informed decisions to mitigate against financial losses.

Despite the benefits offered by understanding the asset price movements, there is a gap in South African literature in this regard detailing rigorous methods so that stakeholders affected by volatility in the exchange rate can make better informed decisions. It is in this context that this chapter applies stochastic volatility models for exchange rate data in South Africa. Studies on the volatile exchange rate in South Africa have come short of the robust models (for instance Mpofu, (2013), de Jager, (2012), Alpanda *et al.*, (2009)). Recent advances in computing and econometrics offer a better selection with studies carried out

having considered the applicability of jumps elsewhere in the world (Eraker *et al.*, (2003), Ramezani and Zeng (2002), Pan (2002) and Craine *at al.*, (2000)).

The chapter will test the applicability of going further beyond the Black-Scholes (1973) model and adding jumps in estimating the british pound/south african rand, US Dollar/south african rand and euro/south african rand exchange rates as a jump diffusion process using a simulation-based estimator. This fills the gap in the literature of modelling exchange rate movements in South Africa as it will consider the intensity and size of the jumps over the study period and will also expound on the noise behind different jumps sizes, intensity and the implications thereof. Furthermore this will contribute to improving forecasts and decision making on the impact of volatility on the exchange rate. The chapter objectives can be stated as to provide evidence of the relevance of jump diffusion models for exchange rate data in South Africa. This ensures that the chapter contribution identifies with literature and thereby adding validity of the research. Going further we estimate jump diffusion model and fat tailed GARCH model for exchange rate data. This enables us to compare with other findings in literature and deduce conclusions relevant to South Africa.

1.2.3 Jump diffusion model for interest rates.

Interest rates play important roles in financial markets; they can determine an organisation's profitability and they also affect the investment portfolios of organisations and individuals. As a result it is important to manage interest rate risk as the goals of investors and organisations alike hinge on interest rate movements. It is important therefore that a correct specification of interest rates is made in such a way that there is understanding of the implications of the risk after capturing the factors that drive interest rates. This has in fact led to a number of models being proposed to explain interest rate behaviour (Ahn and Gao, (1999), Andersen and Lund, (1997), Longstaff and Schwartz, (1992). However, the generic interest rate model as given in Chan *et al.*, (1992) does not capture sufficient characteristics, even after augmenting and adding other factors. A better alternative is to augment interest rate models with jumps (Sorwar, (2011), Attaoui and Six, (2008), Das (2002). Many studies on South African interest rates have not considered jump diffusion models (for example Kleynhans and Meintjes, (2013), Jordaan (2013), Maitland (2013), West (2008), Fernandes, (2005), implying weak understanding of the approaches for the management of interest rate risk. The chapter seeks to fill this gap by considering an advanced approach to understand interest rate movements, also paving the way for better risk management and forecasts on

interest rates. This will be explored through applying a variation of a linear factor model in the presence of jumps. The research goes further to test the applicability of the advanced approach model using as a proxy the 90 day treasury bill (T-Bill) rate for South Africa. The objectives in this chapter incorporate reviewing short rate models with implication for South Africa. This will enable us to identify the gap in the well-known models of short term interest rates and thereby justifying application of a jump diffusion model. Thereafter we proceed with estimating a jump diffusion model and fat tailed GARCH model for the 90Day T-bill. Here we will test the validity of results obtained and draw conclusions on result features in the interest rate specification. Furthermore we split the period to deduce parameter sizes pre and post inflation targeting and derive further analysis.

1.2.4 Jump diffusion model for stock prices.

Empirical studies have verified the existence of jumps in stock prices (Mykland, (2007), ELiu *et al.,* (2003), Eraker *et al.,* (2003), Andersen *et al.,* (2002). The jumps in stock prices, which are by nature large discontinuous price movements, are as a result of infrequent and large surprises to investors' information sets. For South Africa however, there has been minimal work focusing on jumps in individual stock prices (for example Kutu (2012) with the remaining visible studies not having considered jump diffusion models (Taylor, (2010), Moolman (2005). In this study the researcher is therefore carrying out an analysis documenting the benefits of considering jumps in selected Johannesburg Stock Exchange (JSE) sector stocks using Poisson distributed jumps. The analysis goes a long way in informing stakeholders like investors, government, of the behind the scenes reasons for fluctuating stock prices at one of the world's leading stock exchanges. This is also done through an analysis of the properties of individual stock price jumps and the jump intensity.

The main objectives in this chapter are to estimate jump diffusion models and fat tailed GARCH models for stock prices for selected sectors. This enables us to compare the findings with literature and deduce conclusions on the characterisation of stock prices across the different sectors.

1.2.5 Existence of jumps in house prices.

The housing market is an important source of investment and forms a large part of household wealth. The design of financial instruments for the housing market such as mortgages depends on the understanding of house price movements and dynamics. As such it is important to understand noise in house prices as captured by jumps in this form of household

wealth. Visible research on applying the jump diffusion model for house prices in South Africa to the best of this researcher's knowledge has not been explored (see for instance Clark and Daniel, (2006), Els and von Fintel, (2008)). This gives the researcher motivation to fill this gap in testing the existence of jumps in house prices and as well explain the impact of jumps on the house prices and how their distribution has been over the years. The main objective is to represent the house prices as a jump diffusion process and as well compliment with a fat tailed GARCH model. Since few studies have attempted the applicability of the jump diffusion model on house prices as a tool to understand volatility the chapter will fill this gap for South Africa, paving the way for rigorous analysis of house prices in South Africa.

1.3 Objectives of the study.

The following are the main objectives in the thesis:

Review the use and benefits of weather derivatives in South Africa. With the innovative derivatives market, new products are being introduced frequently as investors mitigate risk. Many global powerhouse countries have started trading in weather derivatives to mitigate weather related losses. This is also informed by empirical findings (for example Clements et al (2013, 2008), Cyr and Kusy (2007)). The aim will be to determine if South Africa can benefit from trading in weather derivatives. Having determined the usability of weather derivatives in South Africa the research will then model the process driving temperature and estimate the payoffs of temperature based weather derivatives in four South African cities. This will allow us to draw conclusions and policy recommendations on whether South Africa can pursue trade in weather derivatives as a risk management tool based on the chosen method.

The research will also provide evidence of the relevance of jump diffusion models for various asset classes. This contributes to understanding the features of asset prices in this application on exchange rate, interest rate, stock and house prices. Furthermore, the research estimates the Jump Diffusion Model for exchange rate, interest rate, stock and house prices. The parameter results for these different asset classes will enable us to draw conclusions, compare with related literature findings and provide policy recommendations on the applicability of rigorous methods in understanding volatility and hence risk mitigation.

Complimenting the jump diffusion model the research also estimates fat tailed GARCH models for exchange rate, interest rate, stock and house prices to test for persistent volatility on the high frequency data used. We also draw comparison with literature findings and conclusions and implications of the results in South Africa.

1.4 Results and benefits of the study.

The main findings of the study are as follows; Contribution to the understanding of the application of modelling to temperature based weather derivatives products in South Africa. This is motivated by the fact that weather derivatives are increasingly becoming valuable tools for risk management worldwide. Though the majority of weather derivative deals are carried out in the US, there is a growing market of participants and contract types all over the world. In this study the researcher carries out an estimation of the payoffs of temperature based weather derivatives in four south african cities. This enables us to inform industry players and academia alike about alternative strategies for safeguarding earnings against adverse temperature movements in the investigated regions.

In the analysis, the research presents three methods of estimating payoffs of temperature based european call options. For rigorous analysis, a data set comprising average daily temperature, spanning over fifty years for four South African cities is employed in carrying out the study. The results inform us which method performs best in estimating payoffs amidst volatile temperature data. The information is important for all organisations affected by adverse temperature movements as they will be in a position to hedge after determining the method applicable in their environment.

The “volatility smiles and smirks” derived using volatility in the Black-Scholes model reveal that a simple geometric Brownian motion process misses important features of the data. High frequency returns data display excess kurtosis, skewness, and volatility clustering. To capture these features, there is a need to employ a jump diffusion model. The jump diffusion models address the issue of “fat tails” and enable a better fit for “smiles”. The applicability of jump diffusion processes is seen in finance to capture discontinuous behaviour in asset pricing. The process is based on a Poisson distribution, which can also be applied for modelling systematic jumps caused by the surprise effect. This approach aids forecasts in these financial time series variables as results are more accurate when compared to purely diffusion models. Furthermore jump diffusion models play an important role in finance as they fit better the

jumps. All these factors enable us to arrive at conclusions crucial for policy makers and academia on the validity of jump diffusion models in South Africa.

The jump diffusion process is widely used to model financial time series to capture discontinuous behaviour in asset returns. However, a difficulty involved in the estimation of general jump-diffusion processes has prevented their implementation in empirical applications. This study estimates general parametric jump diffusion processes from discretely observed currency exchange rate data, with interesting results for analysis. The analysis is extended to interest rates, stock and house prices. The estimation results suggest that jumps are important components of the exchange, interest, stock markets and even for house price markets. As such the study contributes to understanding of asset price movements.

1.5 Limitations of the Study.

Chapter two considers four cities in South Africa due to the unavailability of daily data for longer horizon for other cities. Another limitation is the current lack of an organised market for trading in weather derivatives in South Africa.

The jump diffusion models and the stochastic volatility model can complement each other. As an example, for stochastic volatility models the kurtosis decreases as the sampling frequency increases. For jump diffusion models however, the instantaneous jumps are independent of the sampling frequency. This suggests that jump diffusion models may capture short-term behaviour better, while stochastic volatility models may capture long term behaviour better.

A common problem with jump diffusion models is that they do not capture the volatility clustering effects sufficiently, which can be captured by affine jump diffusion models, a combination of jump diffusion models with stochastic volatility models. Another consideration is that jump diffusion models are simpler than general affine jump diffusion models. Jump diffusion models have fewer parameters, making calibration easier. Therefore, jump diffusion models attempt to strike a balance between reality and tractability, especially for short maturity options and short term behaviour of asset pricing.

1.6 Organisation of the study.

The thesis has eight Chapters. Chapter one is the introduction to the study. Chapter two focuses on the dynamics of temperature based weather derivatives. Chapter three presents the methodology of jump diffusion models and GARCH models. Chapter four employs a jump diffusion model for estimating the returns of US$/ south african rand, euro/ south african rand and GBP/ south african rand exchange rates. Chapter five applies a jump diffusion model on interest rates and chapter six extends the model to stock prices. Chapter seven applies the jump diffusion model to house prices and chapter eight concludes the study.

CHAPTER TWO

TEMPERATURE RISK

2.1 Introduction.

The derivatives market has been known to take the lead in innovation through introduction of new products to mitigate risk. One such innovation has seen the emergence of weather derivatives to mitigate weather related risk. Weather derivatives are a growing derivatives sector the world over. They began to develop in 1997, as a result of El Nino, a prolonged warming of the ocean's surface temperatures. As a result of the El Nino calamity, companies that had earnings tied to weather realised the importance of hedging their seasonal weather. Weather derivatives are valuable tools for risk management. Their payoffs are contingent on weather indices based on climatic factors. They are useful for hedging risk in the agricultural, entertainment, beverage, construction and even apparel industries.

The majority of weather derivative deals are carried out in the US, but there is a growing market of participants and contract types all over the world. The growth within Europe is occurring mostly in France and the UK, with Scandinavia and Germany close behind. Most European deals continue to be over-the-counter (OTC)[3] rather than exchange traded contracts. Other parts of the world which have experienced growth include Asia, which has experienced rapid growth. The first deal on wind speed was carried out in Asia, for a wind power project. In addition, the Japanese have seen the majority of their deals coming from the non-energy sector, with banks acting as intermediaries between end users and weather risk management providers. In Australia, many deals have involved power retailers. The current size of the market is estimated at around US$20 billion (Broni-Mensah, 2012).

Though with wide spread evidence of the importance of weather derivatives in mitigating risk, their application to South Africa has not been well documented. Few studies have focused on weather derivatives (for example Scheepers, 2005, Geyser 2004) focusing on

[3] An over-the-counter product can be a forward or option contract providing a payoff dependent on the cumulative HDD or CDD during a month (i.e., the total of the HDDs or CDDs for every day in the month). .

rainfall as a weather variable. This has left out investigation on other important weather variables like temperature. This chapter therefore expounds on the benefits of applying temperature based weather derivatives as an instrument of mitigating risk in South Africa. In this application we make use of regions with reliable daily temperature data and therefore consideration is made of four South African cities.

The chapter applies three methods for estimating payoffs of temperature based weather derivatives; Historical method, Cumulative CDD and the Seasonality GARCH (SGARCH). The method which gives the best estimated payoffs is chosen for consideration in the cities as it outperforms the other methods in the optimal mean and standard deviation of payoffs. In this case the SGARCH method proves superior over the two remaining methods. The implication is that the SGARCH method provides better risk mitigation against adverse temperature movements in the four South African cities and as such should be considered in these cities.

2.1.1 Concepts and unique aspects of weather derivatives.

A measure of the volume of energy required for heating during the day is referred to as a Heating Degree Day (HDD) where as a day's Cooling Degree Day (CDD) is a measure of the volume of energy required for cooling during the day. The contracts are on the cumulative HDD and CDD for a month observed at a weather station. They are settled in cash just after the end of the month once the HDD and CDD are known. The buyer of the derivative is compensated by the under-writer for an amount that offsets the real business losses from adverse weather. To illustrate, an amusement park owner would buy a CDD put that pays out if there is a string of unusually cold days. The value accumulated with the long put position will help offset the lost revenue from customers who have stayed away during the cool weather period. If, on the other hand, the intervening period was unusually hot so that the CDD index rises well above the strike level, then the put expires worthless. The amusement park owner will have likely met desired risk management goals because increased business revenue compensates for the price of this "insurance policy" (Richards e*t al.*, 2004).

Instruments of weather derivatives include swaps, options, and option collars with payoffs dependent on weather related variables like average temperature, HDD and CDD, humidity, maximum or minimum temperatures. Temperature related contracts are more prevalent.

Weather futures and options offered on the Chicago Mercantile Exchange (CME) have similar characteristics to futures and options written on other types of assets. However, weather contracts have a number of characteristics that are unique. One unique aspect about the products is the ability of hedgers to construct seasonal weather contracts, as the amusement park owner example above illustrated. Since weather derivatives were initially launched, the CME has expanded their product offering, and now offers contracts on entire seasons (Jones, 2007). Another unique aspect of weather derivatives is the ability of a company to hedge volumetric risk. A third aspect of these contracts, that hedgers must be aware of, is basis risk. Basis risk is not unique to weather derivatives but since these contracts depend on the weather, understanding the nature of weather related basis risk is of particular importance.

2.1.2 Hedging volumetric risk.

A great advantage of weather derivatives is that they can be used to hedge an aspect of risk not often able to be hedged, volumetric risk. Many businesses are exposed to various types of risks, some of which are more difficult to hedge than others. Weather derivatives can be used to hedge volumetric risk as opposed to price risk. Volumetric risk is the risk that a company cannot generate enough volume to generate profit even though it is able to get the price that it may desire for its product (Pérez-González and Yun, (2010), Jones, 2007)). A farmer will not produce as much grain in times when the weather is damaging to his crops, thus he can use weather derivatives to hedge his yield risk, as opposed to his price risk. Agricultural companies have many approaches to hedge price risk, but these methods continue to leave the company exposed to low yield. Months of high HDD or CDD (extreme temperatures) can lower the yield of crops and squeeze a farmer's revenue even if he has hedged his price risk properly. For example, if a corn farmer is concerned with the potential of a large loss in revenue when the summer is hot, he could buy CDD futures or buy CDD calls. If the summer is hot and he produces a low corn yield, the revenue gained from the CDD weather derivatives will offset the loss from the lower yield.

2.1.3 Basis risk.

The OTC weather derivatives market includes most US cities because the derivatives are fitted to the needs of the individual buyers or sellers. However, the CME market only includes the cities listed on the exchange, which means that a company wishing to hedge weather for cities not listed by the CME faces basis risk, or risk that arises when the value of the asset used to hedge does not exactly equal the value of the asset being hedged (Considine, 2000). For weather derivatives, basis risk arises when the weather in one area does not exactly equal the weather used to value the futures or options contract. Agricultural risk managers must be aware of the basis risk inherent in the weather derivatives market as it can be substantial in this market. For South Africa, Geyser (2004) incorporated the basis risk for the rainfall related derivatives study.

The fact that the weather can be totally different in two relatively close locations, leads to a potentially high amount of basis risk. The information used to settle the weather contracts is gathered from equipment located at the end of airport runways in each respective city. Due to the geographic surroundings of a city, the weather of one area in a city for which a weather contract is written may differ from the "price" of the contract itself (Manfredo and Richards, (2005)). Thus, some basis risk may even exist in a hedge where a weather derivative is used to hedge weather in a different part of a city from where the temperature is recorded. Basis risk is largest in weather hedges where the hedger wishes to hedge the weather exposure in a city other than the city on which the weather contract is written. Basis can often be traded in the OTC market by writing a contract on the difference in the HDDs or CDDs between two cities. Numerous weather factors influence the correlation, which changes over time, between the HDDs and CDDs in two locations. Basis risk must be realised when a hedge is put on and monitored so that it does not get out of control. Even though the basis risk of a weather hedge may be large, the greater certainty of a firm's revenue is typically worth the basis risk taken, and the basis risk is usually never greater than the risk of an unhedged position where weather derivatives are not used.

2.1.4 Hedging with weather futures.

A company can use HDD and CDD futures contracts traded on the CME to hedge weather risk exposure. These trades like other contracts are "priced" based on degree day indexes. A company will buy the futures contract at a degree day index level traded in the market (Garman *et al.,* 2000). If the number of degree days in the month, turns out to be higher than the index level where the contract was bought, the buyer gains on the futures contract. In a related sense, the company can sell a HDD or CDD monthly weather futures contract. If the number of degree days in the month is lower than the index level where the contract is sold, then the seller gains on the futures contract.

Suppose a wheat farmer wishes to hedge against cold weather. He can buy an HDD contract, if the weather is very cold, he will receive the payment from the futures position because the HDD index went up. If however, the weather is only mildly cold, he will likely harvest a high yield crop and realise high revenues but lose money on the futures position, which is offset by revenues from the larger crop. In the same manner as HDD futures, CDD futures can be used to hedge the risk of decreased revenues when the weather is very hot.

2.1.5 Hedging with weather options.

The buyer of the weather option has unlimited profit potential on the upside, while only risking the premium paid for the option on the downside. Weather options derive their value based on the strike price, which is indexed as the number of degree days in a period. If a company wishes to hedge the risk in a period of extreme weather, they will buy a call option with a strike price equal to the number of degree days above which they want to realise a gain (Considine, 2000). The higher the strike number of degree days, the cheaper the option, but the more degree days in the period then they would have to accumulate for the company to realise a gain on the option position. This call option would increase value as the number of HDD or CDD in a period increases above the strike price. CME options on HDD and CDD futures are European style, which means that they cannot be exercised before their expiration date. The underlying instrument for each HDD or CDD option is one HDD or CDD futures contract.

2. 2 Literature Review

This section looks at existing literature work that has been conducted regarding weather derivatives. Such literature will help enrich our understanding of the weather derivatives as well as their application in South Africa. The deregulation of the energy industry in the US saw the first weather derivative security issue taking place in August 1996 between Enron and Florida Power and Light for a value of 40 billion US dollars (Geman & Leonardi, 2005). Growth in weather contracts has been largely influenced by the El Niño winter of 1997-98. During that time, warm weather conditions in the winter season resulted in significant earnings decline, thus compelling many energy companies to attempt to hedge their seasonal weather risk. The OTC market expanded rapidly driven largely by the energy sector and in September 1999 the CME started an electronic market on which standardized weather derivatives could be traded (Alaton *et al*, 2002).

It is interesting to note that $1/7^{th}$ of the industrialised economy is weather sensitive (Hanley, 1999) and that 30% of U.S GDP is exposed to some type and degree of weather risk (Finnegan, 2005). Temperature related contracts are more prevalent, accounting for at least 80% of transactions (Cyr & Kusy, 2007; Cao & Wei, 2004; Cao *et al.*, 2004; Garman *et al*, 2000), trading on the CME for major U.S cities.

There has also been a number of interesting applications developed for weather derivatives. One such application was when weather derivatives were used to hedge against low wine consumption in England. In May 2000, Corney & Barrow, a wine bar chain in London, England, entered into a temperature contract to hedge against low sales on cool summer days. In this application, they found that wine consumption declined when the temperature fell below 24°C. They purchased a derivative contract for the June-September season which entailed a payoff of £1000 x $(24°C - T_i)$[4] per day for the days when the average daily temperature was below 24°C (Cyr & Kusy, 2007; Wei, 2002).

In Africa, Mraoua and Bari (2005) considered temperature stochastic modelling and weather derivatives pricing in Morocco admitting the lack of development in weather derivatives in

[4] T_i is average temperature on a given day.

Africa save for attempts in South Africa. Literature on the application of weather derivatives to South Africa is thin. The first weather derivatives market deal in South Africa was structured to provide ZZ2 Ceres, a large producer of fruit and vegetables, protection from frost (Douglas-Jones, 2002). This saw the company being paid for days when temperature was very low, below 0 degrees Celsius. Some of the visible studies include works by Geyser (2004) who considered rainfall options as a yield risk management tool. In the study use was made of rainfall derivatives as an aid to agricultural production. The study concluded that use of weather derivatives is relevant to South Africa as weather related variables are source of economic risk in agriculture.

Trading in weather derivatives can benefit many industries in the economy including farmers, construction and even entertainment. As an example a farmer can hedge against low yield and prices by combining a rainfall option with insurance contract and agriculture futures (Scheepers, 2005, Geyser, 2004). A study related to Geyser (2004) is by Scheepers (2005) wherein rainfall options were considered as a yield management risk tool. The study recommended the use of weather derivatives as applicable to South Africa.

Also supporting the use of weather derivatives, Ladbury (2000) identified Agriculture and Construction as some of the sectors that could benefit from weather derivatives. Researchers who have focused on South African weather derivatives concur that there is need for awareness campaigns in order to encourage participation so that affected stakeholders can hedge against weather related risk (Scheepers, 2005, Geyser, 2004).

2.2.1 The contracts.

In addition to contracts traded on organised markets, there are also contracts which are concluded OTC. One of the most used contracts is the option, which can be of two types; calls and puts. To illustrate, the buyer of a HDD call pays the seller a premium at the beginning of the contract. If the number of HDDs for the contract period is greater than the predetermined strike level the buyer will receive a payout. The payout is determined by the strike level and the tick amount (monetary value for each HDD exceeding the strike level of the option). The parameters of a typical weather option are:

- The contract type (call or put).

- The contract period (e.g. January 2013).
- The underlying index HDD or CDD.
- A meteorological station from which the temperature data are recorded.
- The strike level or value.
- The tick amount or specification (e.g. x amount per Degree Day).
- A maximum payout (if there is any) (Mraoua, 2005, Cao and Wei 2004).

To illustrate, the option will acquire value when the cumulative CDD index rises above an agreed strike level. At the expiry date, the holder of the option receives payment if the CDD index rises above the strike level. The amount of the payment is thus:
(CDD Index - Strike Level) * Notional Rand value/unit of index.

The HDDs and CDDs are commonly referenced weather indices. The utility industry in the U.S found that 65 degrees Fahrenheit is a benchmark temperature to differentiate between transactions in the heating and cooling seasons. The standard came about because in the past, people often turned on their furnaces when the temperature fell to 65 degrees. Thus consumers will burn more energy to heat their homes for each degree below 65 and will use more energy for their air conditioners for each degree above 65 (Jones, 2007). The metric can be easily converted into degrees Celsius.[5]

2.2.2 Weather derivatives vs insurance

Insurance products have been involved in weather related risk (Geyser, 2004). Of importance is to point out that weather derivatives differ substantially from insurance in that insurance contracts require the filing of a claim. Furthermore, insurance is also generally intended to cover damages as a result of infrequent high-loss events rather than limited loss and high probability events as is the case for weather derivatives (Richards *et al*, 2004, Alaton, 2002). However, weather derivatives are designed as a "bet" on weather conditions (Richards *et al*, 2004).

Insurance also covers once-off risks which may and at time may not be proportional to the risk incurred. Weather derivatives on the other hand can compensate proportionally when

[5] Converting Fahrenheit to degrees Celsius. $C = (F - 32) * (5/9)$

circumstances satisfy the parameters in the contract, thereby delivering better hedging than an insurance product (Geyser, 2004, Dischel and Barrieu, 2002).

Although the use of weather derivatives has been successful in certain sectors and countries, it has met with little success in other countries. In particular exposure in the power and energy markets are almost linear with temperature; power demand increases steadily with both high and low temperatures. Furthermore, few exposures in other sectors of the economy can experience such simple measurement. As a result the exchange traded instruments such as the degree-day futures and options trading on the CME for major US cities are of little use for many other sectors if not countries. The fact that weather is a local phenomenon and can differ dramatically within a small geographic area results in significant "basis" risk for those agricultural producers wishing to use weather derivatives to hedge (Cyr & Kusy, 2007).

2.2.3 Studies on valuation of weather derivatives.

The valuation of temperature derivatives has unique aspects. Weather is not a traded asset. As a result, the conventional risk-neutral, arbitrage valuation does not apply. Furthermore, temperature being a meteorological variable follows a predictable trend, especially over a longer horizon. The nature of the temperature variable brings about two important issues: accurate modelling of the underlying and the assessment of the market price of risk (Cao and Wei, 2004).

The valuation methods in use can be classified into three categories: 1) insurance or actuarial valuation, 2) historical burn analysis, and 3) valuation based on dynamic models. Insurance or Actuarial methods are historically based statistical analysis methods widely used by insurance companies. A probabilistic assessment is attached to the insured event and a fair premium is calculated. For weather derivatives, this method is less applicable for most contracts since the underlying variable temperatures tend to follow a recurrent, predictable pattern. However if the contract is written on rare weather events such as extreme heat or coldness, then the actuarial valuation method will be useful.

The Historical Burn Analysis is simple to implement though prone to large pricing errors. The method evaluates the contract against historical data and takes the average of realszed payoffs as the fair value estimate (Dischel, 1998). The model rests on the assumption that the

past always reflects the future. The commonly accepted sample length in the industry appears to be between 20 and 30 years.

Both the insurance or actuarial method and the historical burn analysis methods are incapable of accounting for the market price of risk associated with the temperature variable. These methods are only useful from the perspective of a single dealer. It remains that there is a need for a dynamic and forward looking model to establish a unique market price which incorporates a risk premium. In contrast to previous valuation models, dynamic valuation models directly simulate the future behaviour of temperature as a continuous or discrete stochastic process.

Dornier and Querel (2000) fitted an Ornstein–Uhlenbeck stochastic process with constant variance to temperature observations at Chicago, O'Hare, airport, with temperature derivatives pricing in mind. Later, Alaton *et al*., (2002) extended the Ornstein–Uhlenbeck model to allow for a monthly variation in the variance, and fitted their model to temperatures observed on the Bromma airport outside Stockholm. They applied their model to derive prices for different temperature derivatives. Campbell and Diebold (2004) observed a clear seasonality in the autocorrelation function for the squared residuals when modelling temperature in several US cities with a higher-order autoregressive model. They proposed to use a seasonal autoregressive conditional heteroskedastic process to model this. However, in their study the pricing of temperature derivatives was not considered.

Mraoua and Bari (2005) suggest a mean-reverting model with stochastic volatility for the temperature evolution in Casablanca, Morocco. A temperature swap is priced based on their model. A different modelling approach using fractional Brownian motion to drive the noise in an Ornstein–Uhlenbeck temperature model is proposed by Brody *et al* (2002). Prices for different temperature derivatives are calculated by solving certain partial differential equations. Benth (2003) calculates an arbitrage-free price dynamic for temperature derivatives based on the fractional Ornstein–Uhlenbeck model in Brody *et al* (2002). In the temperature derivatives market there is a question of choosing the 'right' price among a continuum of possible arbitrage-free prices.

In most of the aforementioned papers a new parameter called the market price of risk is introduced, which can be calibrated to data and thereby, using the market to pin down the

price. This is the approach discussed in Brigo and Mercurio (2001) for interest rate theory. Davis (2001) proposed to use a marginal utility technique to price temperature derivatives based on the HDD index. A benchmark approach using the world stock index as the *numeraire* to price temperature derivatives is suggested by Platen and West (2005).

Other studies have applied jump diffusion models for weather derivatives, drawing from the literature in asset pricing for pricing derivatives for stocks; (Merton 1976; Ball & Torous 1983, 1985; Jarrow and Rosenfeld 1984; Jorion 1988; Bates 1991, Asea and Ncube 1998), exchange rates (Bates 1996; Asea & Ncube 1998) and commodity prices (Hilliard and Reiss, 1999). This can also be applied for temperature as the killer frost in California in December of 1998 demonstrated (Hilliard and Reiss, 1999). The weather derivatives model can be made more robust by allowing mean and variance to be time varying and stochastic. Other extensions have seen the method for pricing derivative securities on weather involving the use of an equilibrium approach based on a Lucas general equilibrium framework (see Cao and Wei, 2004; Lucas, 1976).

There is also a large pool of methods for estimating expected payoffs for weather derivatives. Of these, the general one computes the expected value of payoffs from historical records (Zeng, 2000; Platen and West, 2003). A more general method is to fit a model to the time-series of average temperature so as to capture seasonal variations in both temperature and its volatility (Platen and West, 2003; Campbell and Diebold, 2004). Thereafter, the model is used to simulate temperature outcomes over the period of the contract in order to construct the distribution of the temperature-based index on which the derivative is written. According to Benth and Saltyte-Benth (2005), there exist closed form solutions for expected payoffs which are not only simple to use but are viable for isolated cases.

A point to note is that widely available meteorological forecasts are not suitable for the purpose since these forecasts are made over relatively short horizons, such as 7 days, whereas temperature derivatives are often traded well before contracts can generate any payoffs (Wilks, 1995; Jewson and Caballero, 2003; Campbell and Diebold, 2004).

2.3. Tick values of temperature options

Allowing T to denote the temperatures in degrees Celsius [6]on a particular day at a weather station, the daily HDD and CDD indices at that station on that day are defined by:

$$HDD = \max(0,18.3333 - T_i)$$

(2.1) $$CDD = \max(0, T_i - 18.3333)$$

The term T_i is the daily average temperature of the daily maximum (T^{max}) and minimum temperatures (T^{min}) at a specified weather station, from midnight-to-midnight, measured in degrees Celsius.

(2.2) $$T_i = \frac{T^{\max} + T^{\min}}{2}$$

The degree days used in weather derivatives are calculated as the difference in the daily average temperature from 18.3333 degree Celsius. A HDD is calculated by subtracting the daily average temperature from 18.3333 degrees Celsius. A CDD is calculated by subtracting 18.3333 degrees from the daily average temperature. There cannot be both HDD and CDD within a single day, given that the daily average temperature can only be either above or below 18.3333 degrees. If T_i is less than 18.3333 degrees, HDD will accumulate where as if T_i is greater than 18.3333, CDD will accumulate. HDD can be thought of in terms of needing to use the heater where as CDD can be thought of needing to use the air conditioner (Jones, 2007).

In the southern (northern) hemisphere the HDD (CDD) season would be from May to September, while the CDD (HDD) season would be from November to March. These are obtained by summing up daily HDD/CDD indices over a period of N days to get

$$H_N = \sum_{k=1}^{N} \max(0,18.3333 - T_K)$$

(2.3) $$C_N = \sum_{K=1}^{N} \max(0, T_K - 65)$$

The CMEs HDD and CDD futures and options contracts are based on indexes of HDD and CDD. These are an accumulation of daily HDDs and CDDs, over a calendar month or an

[6] Converting Celsius to Fahrenheit. F = (9/5* Degrees Celsius) + 32

entire season. We let D be the strike price of a temperature based option defined as a particular value of the relevant cumulative index. Therefore, the buyer of a vanilla European call option pays an up-front premium and receives a pay-out if the value of the relevant index exceeds the strike price, D, at the maturity of the option. The tick value of a CDD call option with strike price D and duration N days thus becomes:

$T_N \max (C_N\text{-}D,0)$

The monetary payoff from the contract is the product of the tick value and the tick size, defined as the cash value of a tick. Given the cumulative index *x*, the probability density function, $f_N(x)$ of the relevant cumulative index over the period of the contract, a call option for N days with strike price D, will have as an expected tick-value the representation;

$$(2.4) \qquad E[T_N] = \int_D^{\infty} (x - D) f_N(x) dx$$

According to the Black-Scholes (1973) valuation model of options, expected payoffs are discounted at riskless interest. Such a discount rate is based on a zero-arbitrage argument involving the formation of a portfolio consisting of a riskless combination of an option and the underlying asset. However, when it comes to temperature-based weather derivatives, the underlying indices are not tradable, and therefore these derivatives cannot be priced by means of a zero-arbitrage argument.

2.4 Data

The data set comprises daily maximum and minimum temperature records in degrees Fahrenheit. The analysis is conducted on the time series of average daily temperatures computed as the arithmetic mean of the daily maximum and minimum values. Johannesburg (Jhb), Cape Town (Cpt), Durban (Dbn), Bloemfontein (Blm) were chosen as they are four major cities in South Africa and also because accurate temperature records of 50 years are available for these cities at comparable weather stations[7]. The construction of the temperature

[7] All the data were supplied by the South African Weather Services. For all the data sets, missing values were treated by averaging adjacent records. Following Campbell and Diebold (2004), all occurrences of the 29 February were removed.

record for each city is now discussed in detail. All data for each city has 18341 observations over the period 1/1/1960 ending 31/3/2010. However data is sourced from a different number of weather stations; two weather stations for Bloemfontein and Cape Town; three weather stations for Johannesburg and one station for Durban.

Table 2.1: Summary Statistics Temperature Data

T_i	BLOEM	CPT	DURB	JHB
Mean	15.9569	16.4401	20.8973	16.3936
Median	16.7	16.75	20.95	16.75
Maximum	30.13	29.85	30.45	27.71
Minimum	-0.4	0	0	-0.3
Std. Dev.	6.0273	4.3645	3.4354	3.9732
Skewness	-0.2443	0.069774	-0.0589	-0.4475
Kurtosis	2.5135	2.0720	2.2612	2.3072
Observations	18341	18341	18341	18341

From the table above, Durban is the hottest city on average and also records the lowest variability in average daily temperature. Bloemfontein has a relatively high variability in average daily temperature, suggesting that it is the coldest during the sample period. There are differences in all the cities between the sample means of temperature for the individual sample as shown above. Literature abounds with various models for pricing weather derivatives. With these results on the data behaviour a time trend can be considered as an important component of a model of average daily temperatures for the four cities.

Skewness quantifies how symmetrical the distribution is. A distribution that is symmetrical has a skewness of 0. In the case of the results presented in the table above, the data suggests closeness to symmetry perhaps with the exception of Jhb. Kurtosis quantifies whether the shape of the data distribution matches the Gaussian distribution. A Gaussian distribution has a kurtosis of 3. A flatter distribution has a kurtosis less than 3, and a more peaked distribution has a kurtosis greater than 3. For the four cities, the data are platykurtic. However, returns[8]

[8] A return is calculated as $\ln(\frac{y_t}{y_{t-1}})$

data are leptokurtic. The diagrams below indicate the volatile nature of the data and the density diagrams further confirm that the data are not normal distributions.

Figure 2.1: Average Daily Temperature 1960-2010

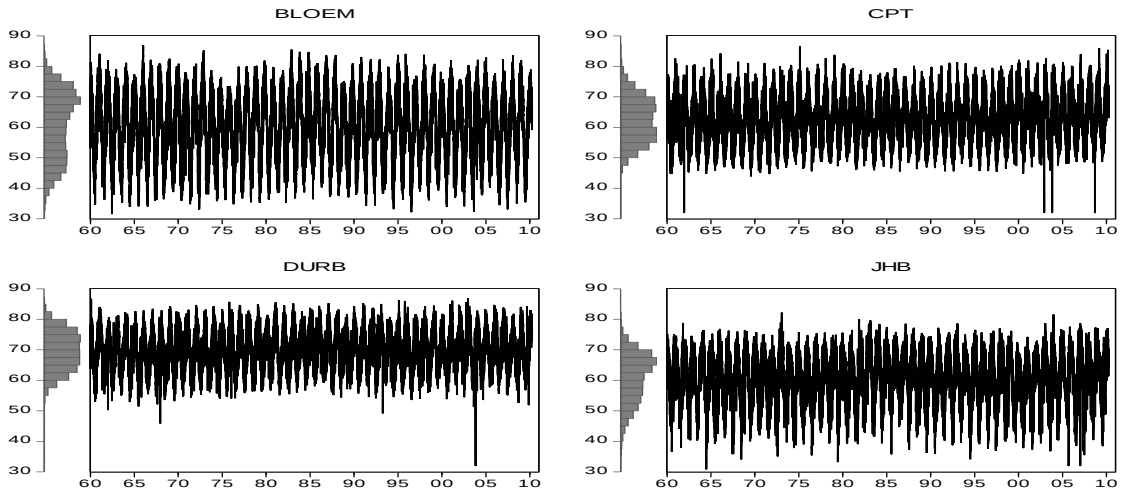
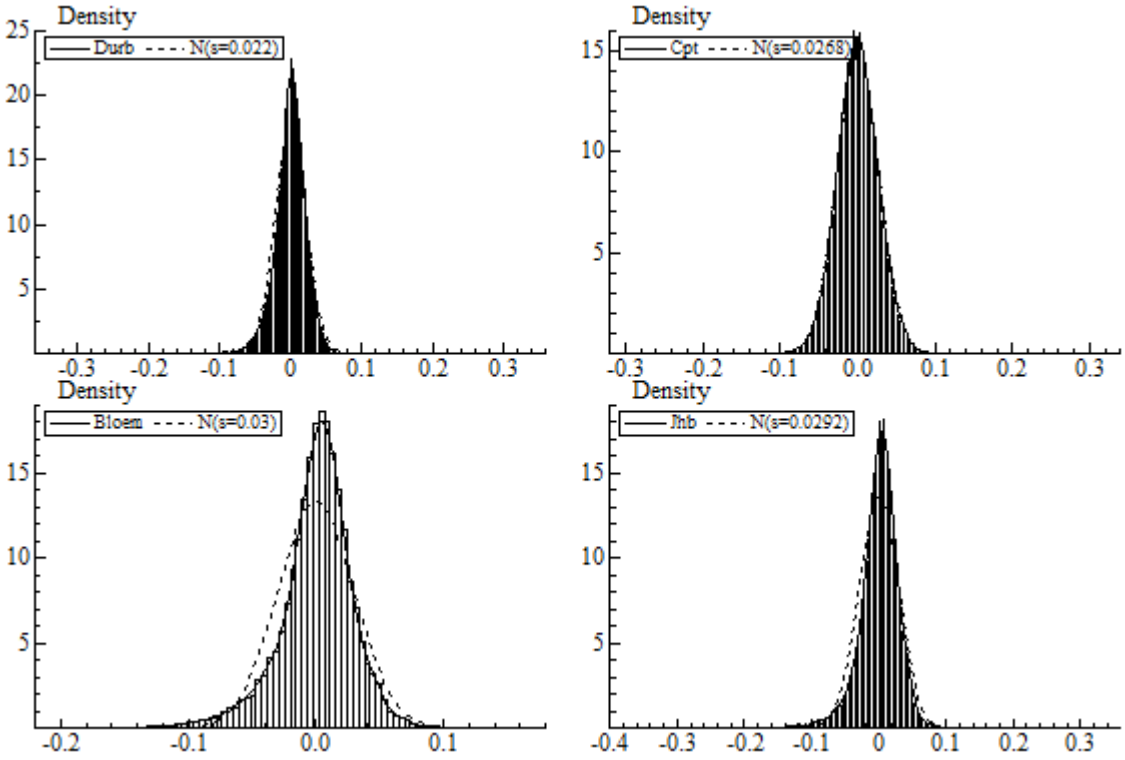


Figure 2.2 : Density Temperature



2.5 Statistical models for forecasting and pricing weather derivatives

The common practical approach employed in pricing temperature-based derivatives is termed the actuarial valuation method, discussed in Zeng (2000) and Platen and West (2003). This approach prices the derivative at the mean expected payout plus a premium for overhead expense. The simplest way of implementing this pricing scheme is to review historical records over the period of a contract in previous years and use these values to calculate the hypothetical pay-out of the contract had it been in place. The actuarially fair price for the derivative would then be the mean historical payoff. However the approach applies if the values are independent and identically distributed random variables.

2.5.1 Modelling Cumulative Degree Days

A straightforward approach to evaluating the expected tick value of a temperature derivative contract is to model cumulative CDDs directly, on the assumption that there exist historic temperature records for longer horizon (Clements *et al.*, 2008) as cumulative degree days exhibit behaviour closest to normality. Some studies have argued that the quadratic trend model can produce reliable index values (Van Keymeulen, (2013), Špička, (2011)). Should the simple quadratic model produce significant results then it can be used to obtain mean forecasts of CDD, thereby providing a basis in formulating a pricing model for the weather derivative. In this case the weather variable for the trend model is temperature.

The model is described as follows; We let C_1 , C_2, …….C_n be the time series of n historical observations of cumulative CDDs. Intuitively, in the absence of a trend in the temperature data from which the cumulative CDDs are derived, these observations will be independently and identically distributed realisations from the distribution of cumulative CDDs. A simple quadratic trend model is proposed for cumulative CDDs and is represented below;

(2.5) $C_t = \eta_0 + \eta_1 Trend_t + \eta_2 Trend^2{}_t + \in_t$

Where $\in_t$ represents the mutually independent error term; estimations for the parameters of this model yields:

$$E(Ct)_{\text{Cape Town}} = 385.241 \underset{(26.8873)}{-1.647\text{Trend}} + \underset{(2.4322)}{0.0992\text{Trend}^2} + \underset{(0.0462)}{0.3257e_t} \quad (0.1391)$$

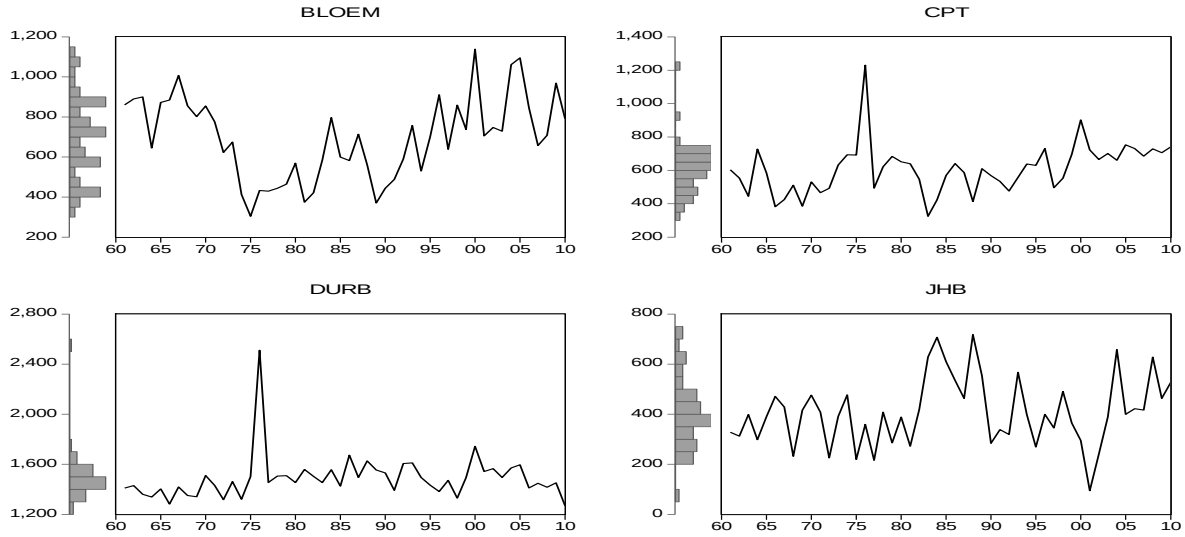
$$E(Ct)_{\text{Johannesburg}} = 213.9142 \underset{(48.3816)}{+2.5825\text{Trend}} - \underset{(4.3765)}{0.0322\text{Trend}^2} + \underset{(0.0832)}{0.2035e_t} \quad (0.1445)$$

$$E(Ct)_{\text{Durban}} = 908.602 \underset{(24.7283)}{+ 3.8248\text{Trend}} - \underset{(2.239)}{0.0438\text{Trend}^2} - \underset{(0.0426)}{0.1758e_t} \quad (0.1485)$$

$$E(Ct)_{\text{Bloemfontein}} = 550.4009 \underset{(32.6457)}{-32.744\text{Trend}} + \underset{(2.9557)}{0.5543\text{Trend}^2} + \underset{(0.0563)}{0.5904e_t} \quad (0.1231)$$

The figures in parenthesis are the standard errors. Results are not significant for all cities. Bloemfontein has the trend and quadratic trend terms significant. Cape Town has a quadratic term significant. Durban has trend significant. Johannesburg has a trace of a trend. The figure below presents the presents the cumulative CDD to complement the results above.

Figure 2.3: Cumulative CDD 1960-2010



2.5.2 Simple Option payoff pricing

Simple pricing models can also be constructed using a probability distribution fitted to a historical data set of monthly CDDs or HDDs. Thereafter, the next step will be to integrate the product of the probability distribution with the payoff of the option. The expected payoff of a CDD option, or its theoretical value, is simply determined by:

(2.6) $$M \int_{CDD=0}^{\infty} P(CDD)Q(CDD)dCDD$$

Where P(CDD) is the probability distribution of CDDs, Q(CDD) is the payoff of the option in units of CDDs, M is the rand amount specified in the contract per CDD, and d(CDD) is the differential. The expected value changes as a function of the strike, the probability distribution of CDDs, and the rand amount per CDD. A simple, and quite often sufficient, formula for pricing individual options can be derived for the case of a Gaussian distribution of CDDs or HDDs. Assuming that one knows the mean and standard deviation of CDDs or HDDs in a location, it is a simple exercise to approximate the price of an option. The algebraic expression relates the price of an option to three factors:

1. The standard deviation of the distribution;
2. The distance of the strike from the mean value;
3. The rand amount per degree day specified in the contract.

If we define a normalised strike in terms of the number of standard deviations of the strike away from the mean value, the cost of the option can be calculated from the relationship below

(2.7) $Y = -0.03X^3 + 0.22X^2 - 0.5X + 0.4$ (see Considine, (2000) for discussion on the algebraic expression)

Where Y is the expected value of an option and X is the standard deviation of the strike price from the mean.

Table 2.2: Estimate computation of Option Value using CDD

	Cpt	Jhb	Durb	Bloem
Mean	426.9	251.3	967	199.1
Standard Deviation	81.8	108.4	61.2	157.5
Strike Value	500	500	1000	500
X	0.89	2.29	0.54	1.91
Y	0.11	0.05	0.19	0.04
Specification	R 10 000/Degree [9]Day	R10 000	R10 000	R 10 000/Degree Day

[9] Assumption for demonstration purposes

Option Value: Cape Town = R10 000* 0.11*81.8 = R89 980

Johannesburg = R10 000 * 0.05* 108 = R54 200

Durban = R10 000* 0.19* 61.2 = R116 280

Bloemfontein = R10 000* 0.04*157.5 = R63 000

The expected value does not include the "risk premium" that the writer of the option charges for carrying the risk. Nevertheless, this simple formulation provides a baseline from which to price an option. The largest challenge facing the options market participant is determining the mean and standard deviation to use as the model inputs.

2.5.3 Modelling Temperature

Following the works of Clements *et al* (2008) and Campbell and Diebold (2004), this section seeks to determine whether the time series model for each city can provide an adequate model of temperature. If it does, repeated simulation of the model will allow accurate pricing of temperature based weather derivatives. For the cities in use, temperature, T_t is the average daily temperature defined in equation (2.2). The deviations of temperature from its long term average $\theta_t = T_t - \bar{T}_t$ are modeled as a low order autoregressive (AR) process

$$\theta_t = \bar{T}_t + \sum_{j=1}^{m} \alpha_j \theta_{t-j} + \sigma_t \varepsilon_t \tag{2.8}$$

Where m is the order of the AR process, ε_t is an iid(0,1) process and $\bar{T}_t$ is modelled as the sum of a trend and a periodic component by the expression

$$\bar{T} = \gamma_0 + \gamma_1 Trend_t + \sum_{j=1}^{k} \phi_j \cos\left(\frac{2\pi jt}{365}\right) + \sum_{j=1}^{k} \vartheta_j \sin\left(\frac{2\pi jt}{365}\right) \tag{2.9}$$

(see: Clements *et al.*, (2008); Caporin and Pres, (2008))

In order to capture both the observed seasonal pattern of the volatility of temperature and any persistence in volatility, a conventional GARCH(1,1) model (Clements *et al,* 2008, Bollerslev, 1986) is augmented by adding a constant seasonal pattern component as a forcing variable in the variance equation. As a result the SGARCH (1, 1) model for conditional variance thus becomes:

$$\sigma_t^2 = \beta_0 + \beta_1 \sigma_{t-1}^2 + \beta_2 \varepsilon_{t-1}^2 + \beta_3 S_t \tag{2.10}$$

Where;

$$(2.11) \quad S_t = \xi_0 + \sum_{j=1}^{m} \xi_j \cos\left(\frac{2\pi jt}{365}\right) + \sum_{j=1}^{m} \delta_j \sin\left(\frac{2\pi jt}{365}\right)$$

Table 2.3: below, reports estimation for the SGARCH (1, 1) model of temperature for the entire sample period. To reduce the dimension of the optimisation problem, the parameters of equations (2.9) and (2.11) are pre-computed by ordinary least squares using temperature and squared deviations of temperature from $\bar{T}_t$ as the other dependent variables respectively.

Table 2.3: Parameter Estimates for AR(7)[10]-SGARCH(1,1) models for average daily temperature data

	Cape Town	Johannesburg	Durban	Bloemfontein
α_0	0.0713	-0.05256	-0.0866	0.1102
	(0.0000)	(-0.0000)	(0.0000)	(0.0000)
α_1	1.3929	0.0240	8.6404	0.7496
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_2	2.7730	-3.7787	-4.8180	-0.3130
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_3	-6.0078	1.4801	1.2674	1.4315
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_4	2.7987	1.5391	-1.8871	4.6194
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_5	1.7720	-2.9692	0.5353	1.0756
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_6	-0.0123	-1.4355	0.2393	2.1747
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
α_7	0.0000	0.00298	0.0000	0.2425
	(0.0000)	(0.0000)	(0.0000)	(0.0000)
β_0	1.7843	-1.3228	0.6802	0.9652
	(0.6368)	(0.3500)	(0.1110)	(0.1863)
β_1	1E-7	0.0082	0.3034	0.0012
	(5.1855)	(0.0055)	(1.6383)	(0.0000)

[10] The optimal lag order was based on the Bayesian Information Criterion

β_2	0.0003	0.0003	0.0029	0.0035
	(0.0000)	(0.0000)	(0.3327)	(0.0000)
β_3	0.0084	0.0031	0.0049	0.0036
	(0.0000)	(0.0000)	(0.0014)	(0.0000)

* the figures in parentheses are the standard errors

The approach used a generic model taking similar structure across all four cities. The values in parenthesis denote the standard error. Results from the table show that most coefficients are statistically significant, hence generally meeting expectations. The βs are the coefficients of the variance equation. β_3 is the coefficient on the exogenous seasonal pattern in the conditional variance equation. Inclusion of this term can have a dampening effect on the persistence in volatility by comparison with the kinds of estimates usually obtained in GARCH models of financial asset returns, where the sum of β_1 and β_2 are close to 1. From the results in the table above, inclusion of the seasonality component has curtailed the persistence in volatility. β_3 seems to have had an enlarged dampening effect especially for Cape Town where β_1 is the smallest across cities. Cape Town and Durban specifically have weakened results for SGARCH.

When plots of the standardised residuals from the AR-SGARCH(1,1) model incline to standard normal, it may suggest that the model captures the characteristics of the dynamics of average temperature in these major cities (Clements *et al*. (2008). With parameter estimates of the AR(7)-SGARCH (1,1) model, equation (2.10) can be used to simulate realisations of average daily temperatures. In this way from a series of *k* simulations, realisations $\hat{C}_1, \hat{C}_2, \ldots \hat{C}_k$ of cumulative CDDs for the appropriate period can be obtained. These are regarded as *k* independent drawings from the distribution of cumulative CDDs for the period under consideration[11].

2.6 Computing Expected Payoffs

We first analyse the descriptive statistics of cumulative CDD before presenting the expected payoffs:

[11] See Clements *et al* (2008) and Caporin and Press (2008) for detailed discussion.

Table 2.4: Summary Statistics CDD 1960-2010

	Cape Town	Johannesburg	Durban	Bloemfontein
Mean	426.9	251.3	967	199.1
Median	423.5	236.9	978.6	188.5
Sdev	81.8	108.4	61.2	157.5
Max	571.5	526.4	1147.7	657.2
Min	254.3	53.6	829.7	4.47

The descriptive statistics for cumulative CDDs show that for Cape Town, Johannesburg and Bloemfontein, the distributions are slightly skewed to the right as evidenced by the mean which are greater than the median. For Durban, they are slightly skewed to the left. Durban records the least standard deviation, perhaps indicating the heat in the city whilst Bloemfontein has the largest standard deviation, pointing to the cold weather.

The next step is to compare the methods used and determine which one yields the best estimated payoffs. The methods to be compared are; the simple time series model of CDDs, the AR(7)-SGARCH model and the historical temperature records. The basis for comparison to be used here is the mean 'profit' of the call option contract over a period of years, following Clements *et al.* (2008). In computing the payoffs we specify a yardstick for two separate option contracts and incorporate results for the three methods written over the period 1 January to 31 March as reported in the tables below. Computation of the values involved pricing options for the year 1961 using data up to and including 1960. To illustrate, the actual payoff for 1960 is recorded, the profit or loss stored and the data set updated to include all the temperature records for the next year. These steps are repeated up to and including 2010, giving a total of 50 separate profits for each option. The call options used in the experiment have respective strike prices $D=\mu+0.5\sigma$ and $D=\mu+0.75\sigma$ where μ is the unconditional mean and σ is the unconditional standard deviation of CDDs over the period 1961-2010.

The expressions $D=\mu+0.5\sigma$ and $D=\mu+0.75\sigma$ represent lower and higher strike price respectively. The goal is to determine how the different methods fare on these two strike prices. This we do by averaging the simulated payoffs over the period 1961-2010. The

method resulting with lower mean payoff and standard deviations suggests its usability for the region[12].

Table 2.5: Means and standard deviation of payoffs to temperature 90-day call option defined on CDDs with strike price D=μ +0.5σ, where μ and σ are the unconditional mean and standard deviation of available historical CDDs. The option is priced from 1961-2010

	Cape Town	Johannesburg	Durban	Bloemfontein
Historical				
Mean Payoff	74.47427	127.8784	128.9856	-356.206
SDev Payoff	835.252	889.4825	958.5658	867.79
CumCDD				
Mean Payoff	-28.362	11.20792	11.83155	-187.581
SDev Payoff	102.9843	91.74235	141.2208	126.2625
SGARCH(1,1)				
Mean Payoff	0.369418	3.412133	0.770571	1.059303
SDev Payoff	0.821563	4.891592	0.667595	1.483613

Table 2.6: Means and standard deviations of payoffs to temperature 90-day call option defined on CDDs with strike price D=μ +0.75σ , where μ and σ are the unconditional mean and standard deviation of available historical CDDs. The option is priced from 1961-2010

	Cape Town	Johannesburg	Durban	Bloemfontein
Historical				
Mean Payoff	43.807	106.1702	112.4158	-456.82
SDev Payoff	834.396	889.5702	959.0044	873.2694
CumCDD				
Mean Payoff	-51.9011	-6.63747	-4.38597	-230.593
SDev Payoff	102.9213	91.53025	141.809	131.7514
SGARCH(1,1)				

[12] See Clements et al., (2008), Garman *et al*, (2000) for more discussion

Mean Payoff	0.574809	4.635031	0.93747	1.430206
SDev Payoff	0.903719	5.380751	0.734355	1.631974

A number of conclusions can be derived from the two tables above. The ideal condition will be when profits are close to zero with lower standard deviation thus ensuring risk mitigation in the respective cities. Table 2.5 shows the expected payoffs when a lower strike price is used and table 2.6 uses a higher strike price. As per the findings of Clements *et al*., (2008), the historical pricing turned out to be the least robust of all the procedures for estimating payoffs, particularly for call options with higher strike prices. This is despite it being a common model (Platen and West, 2003). The implication is that it does not apply for the four South African cities over the study period. Results show that the model is over-priced for the other three cities and under-priced for Bloemfontein. This can be seen in the high mean payoffs in the historical method compared to Cumulative CDD and SGARCH in both tables above. Furthermore, the Historical method shows a negative payoff in Bloemfontein for both high and low strike prices.

The cumulative CDD model performs better when compared to the historical method as it has lower payoff and standard deviation values when compared to the Historical method. However, there appears to be a tendency to under-price the European call option particularly for Bloemfontein and Cape Town for both high and low strike prices. Interestingly the SGARCH(1,1) method of estimating payoffs offers superior performance relative to the other two methods as it has minimal payoffs and standard deviation for both high and low strike prices. It may imply that the mean of estimated payoffs from this approach is estimated relatively accurately, with the least standard deviation of the payoff across regions and therefore yielding reliable payoffs when compared to the Cumulative CDD and Historical methods. Results in this chapter confirm related works by Campbell and Diebold (2004) supporting the SGARCH method.

2.7 Conclusion

The contribution of the chapter was to consider the application of temperature based weather derivatives in four South African cities. With growing innovation in the derivatives market it

becomes necessary that businesses find ways of mitigating weather related risk and thus be cushioned against huge losses as a result of adverse weather. Different sectors of the economy experience weather related risk. The chapter applied estimation methods for temperature based weather derivatives.

Returns data on daily average temperatures revealed volatility clustering and the leptokurtic nature of the data. The simple trend model showed that Bloemfontein had trend and quadratic trend terms as significant whilst Cape Town had only quadratic significant and Durban trend significant. Johannesburg had a trace of trend. Another milestone was the demonstration of the computation of the option values using the CDD, adding more weight to the usability of weather derivative products in South Africa. The study reached its climax in comparing three different methods for estimating payoffs, the Historical method, Cumulative CDD and SGARCH.

The results may be summarised as follows; The SGARCH (1,1) model of estimating payoffs for temperature based weather derivatives offers superior performance compared to the more popular methods in literature, the Cumulative CDD and Historical methods. The SGARCH (1,1) turned out to have minimal payoffs and standard deviation across the four cities. This suggests that the SGARCH (1,1) method should be applied for the cities used in the study. It is important that organisations and authorities be educated about the benefits therein of weather derivatives as a risk management tool. As a general rule, data in use for estimation should be sizeable enough to capture the essential characteristics, thus not rendering poor resolution in the distribution of the relevant index. A successful model ought to be parsimonious enough in estimating payoffs as excessively complex models have poor predicting powers.

CHAPTER THREE

JUMP DIFFUSION AND GARCH MODELS METHODOLOGY

3.1 Introduction

The section presents methodology for the jump diffusion model and the fat tailed GARCH model which form basis for the results in chapters four to seven. The remainder of this chapter is organised as follows; section 3.1 highlights the jump diffusion specification. Section 3.2 incorporates the GARCH methodology and section 3.3 concludes.

3.2 GBM plus jump methodology

In this section the researcher presents the estimates from a popular specification in financial economics: GBM plus a jump process. Here the goal is to find sufficient estimates to the data with satisfying representation. Unlike the GBM which only matches the first two unconditional moments of mean and variance, the GBM plus jump can match higher moments of skewness and kurtosis.

Adding Poisson jumps to the GBM approximates the movement of stock prices subject to occasional discontinuous breaks. The returns process under the jump-diffussion model can thus be presented by the following differential equation

(3.1) $$\frac{dS}{S} = \mu dt + \sigma dW + k d\pi(\lambda)$$

where
- $S : N(\mu h, \sigma^2 h)$
- μ is the drift
- σ the standard deviation of the diffusion
- k is the jump size
- π is a Poisson process with parameter λ

Given any date $t \geq 0$ and period length $h > 0$, the returns $Z_t(h)$ over the period $[t, t+h]$ are given by the following

$$\ln S_{t+1} - \ln S \equiv Z_t(h) \equiv y_{t+1} = \begin{cases} x & \text{if } Q = 0 \\ x + k_1 + \ldots, + k_Q & \text{if } Q \geq 1 \end{cases}$$

where

- $k_1,\ldots,k_Q$ is an i.i.d. sequence with $k \sim N(\mu_k, \sigma_k^2)$
- Q is distributed Poisson with parameter λh

$\lambda > 0$ and for $q = 0,1,\ldots,Q$ we have $\text{Prob}(Q = q) = e^{-\lambda h}\dfrac{(\lambda h)^q}{q!}$

With the assumption that the first four moments of the jump size G exist and are finite. We denote the n th moment of $k : N(\mu_k, \sigma_k^2)$ by $\tau_n = E(y^n)$, where τ_2 is the variance.

We can write $Z(h) = Z_t(h)$ since $Z_t(h)$ does not depend on t. This means as well that conditional and unconditional returns coincide in a jump diffusion model (see Das and Sundaram, (1999) for more discussion on the model. The parameters of the jump diffusion process are estimated by maximum likelihood. The log-likelihood function is,

$$(3.2) \qquad l(\theta, y) = \sum_{t=0}^{T} \ln\left[\sum_{q=0}^{Q} \frac{e^{-\lambda}\lambda^q}{q!} \frac{1}{\sqrt{2\pi(\sigma_B^2 + q\bar{\sigma}^2)}} \exp\left(\frac{-(y_{t+1} - \mu_B - q\bar{\mu})^2}{2(\sigma_B^2 + q\bar{\sigma}^2)}\right)\right]$$

The sum of the logs of sums of exponentials weighted by the Poisson probabilities. The scores are messy nonlinear functions of the unknown parameters, θ[13]. The estimates are computed from the Optimization Toolbox in MATLAB. The likelihood function is not as well behaved as one would expect if there are occasional discontinuous jumps. The local maxima and the algorithm do not always converge. The parameters estimated from the log likelihood function in (3.2.4) are then fed into the equations below to obtain the jump diffusion moments for analysis.

the first four moments of $Z(h)$ are computed as follows:

$$(3.3) \qquad \text{Mean} = \mu_B + \lambda\mu_J$$

$$(3.4) \qquad \text{Variance} = \sigma_B^2 + \lambda(\mu_J^2 + \sigma_J^2)$$

$$(3.5) \qquad \text{Skewness} = \left[\frac{\lambda(\mu_J^3 + 3\mu_J\sigma_J^2)}{(\sigma_B^2 + \lambda\sigma_J^2 + \lambda\mu_J^2)^{3/2}}\right]$$

[13] A great number of jumps could occur during the day. The maximum number of jumps per day, Q, is set to ten, (Craine *et al*, 2000; Jorion 1988). The MATLAB code is included in the appendices.

(3.6) $$\text{Kurtosis} = 3 + \left[\frac{\lambda(\mu_J^4 + 6\mu_J^2\sigma_J^2 + 3\sigma_J^4)}{(\sigma_B^2 + \lambda\sigma_J^2 + \lambda\mu_J^2)^2} \right]$$

3.2 GARCH methodology

The GARCH (1,1) model which enables capturing of volatility clustering identified in high frequency data is specified as follows

(3.7) $$Y_t = C + \varepsilon_t$$

(3.8) $$\sigma_t^2 = \kappa + \beta\sigma_{t-1}^2 + \alpha\varepsilon_{t-1}^2$$

Equation (3.7) above represents the conditional mean model where the returns, Y_t , consist of a simple constant, plus an uncorrelated, white noise disturbance. The representation describes the conditional mean in financial data. The conditional variance equation (3.8) consists of a constant plus a weighted average of the last period's forecast, σ_{t-1}^2 , and the corresponding squared disturbance, ε_{t-1}^2 . The squared returns often indicate significant correlation and persistence, implying correlation in the variance process, therefore an indication that the data should be considered for GARCH modelling. In order to assume a distribution that reflects the features of the data better than the normal we include the Student t Distribution and the Generalized Error Distribution (GED). The likelihood functions for these distributional assumptions are:

The log-likelihood function for the GED;

(3.9) $$l_n = \sum_{t=1}^{n} \left\{ \left(\frac{v}{\lambda} \right) - \frac{1}{2} \left| \frac{X_t}{\sigma_t \lambda} \right|^v - \left(1 + v^{-1}\right)\log(2) - \log\left[\Gamma\left(\frac{1}{v} \right) \right] - \frac{1}{2}\log(\sigma_t^2) \right\}$$

$\Gamma(.)$ is the gamma function;

$$\lambda = \left[\frac{2^{-2/v}\Gamma(1/v)}{\Gamma(3/v)} \right]^{1/2}$$

For the Student-t distribution, the log-likelihood function becomes:

$$(3.10) \qquad l_n = \sum_{t=1}^{n} \left\{ \begin{array}{l} \log\Gamma\left(\frac{v+1}{2}\right) - \log\Gamma\left(\frac{v}{2}\right) - \frac{1}{2}\log\left(\pi(v-2)\right) - \frac{1}{2}\log\sigma_t^2 \\ -\left(\frac{v+1}{2}\right)\log\left(1+\frac{X_t^2}{\sigma_t^2(v-2)}\right) \end{array} \right\}$$

For the GED, the retail parameter v>0. If v=2, the GED becomes a normal distribution and is fat tailed when the degrees of freedom, v <2. For the Student-t, the degree of freedom v>2 controls the tail behavior. The distribution approaches normal as $v \to \infty$. The results are estimated using maximum likelihood.

3.4 Conclusion

The chapter highlighted the methodology employed to obtain results in chapters four to six. The jump diffusion model applied has constant jump intensity driven by a Poisson process. In generating the parameter results a program was run on MATLAB. For the GARCH model , two specifications of the fat-tailed GRACH were used, the GED and Student's t distributions.

CHAPTER FOUR

JUMP DIFFUSION MODEL FOR THE EXCHANGE RATE

4.1 Introduction

There is a growing need to better understand asset price movements with the objective of hedging against risk. For high frequency financial data the need to hedge is a necessity as failure can transmit to billions of rands in losses. This also comes at a time when there is much activity in the global economy and particularly for financial markets with companies big and small jostling for a better share of the market. Consequently there are exchange rate implications as both importing and exporting firms incur losses and profits during trade. Employment is also boosted through creating good forex hedging strategies, whilst some companies may have to downsize after failing to predict accurately exchange rate movements. At the same time, the domestic economy benefits most from exports via a positive current account. All these build the case for understanding what drives the exchange rate.

As such, there have been a number of studies carried out as researchers attempt to pin down the model offering the best strategy for businesses and investors. This has seen the elevation to some extent of financial economists as they build complex models making use of the latest developments in modelling high frequency financial data. Perhaps one such popular approach has been the application of simple diffusion models which approximate the stochastic process for returns on financial assets. However, the once reverenced method of the Black-Scholes (1973) model was ultimately shown to have shortcomings, as it failed to capture some important characteristics of high frequency financial data which were crucial for making accurate forecasts in the data. This has therefore motivated the need to employ more sophisticated models like the jump diffusion models as a better alternative with regards to understanding asset price movements.

Though there has been a wide application of jump diffusion models in many parts of the world for example (Figueiredo *et al.,* 2011; Yang *et al.,* 2009; Kou *et al.,* 2005; Cont & Tankov, 2004; Ramezani and Zeng, 2002), their application in South Africa has been reduced to only a few studies (van Appel, 2013; Poklewski-Koziell, 2012; Kalsheker, 2009). Moreover, these studies have looked at the pricing of options and thus have not given due

diligence to the importance of understanding jump diffusion models in exchange rates to complement decision making in financial econometrics. The remaining visible studies have not attempted to incorporate jump diffusion in exchange rate models (for instance de Jager, (2012), Alpanda *et al.*, (2009)). This chapter therefore bridges this gap as it provides and understanding of exchange rate characteristics. This is done by drawing from different studies the applicability of the jump diffusion model to exchange rate data. Furthermore, we estimate the parameters of a jump diffusion models for the three exchange rates; british pound/south african rand, US dollar/south african rand and euro/south african rand exchange rates to and draw conclusions thereof. This enhances knowledge for risk mitigation purposes through analysing the parameter values.

The results from the chapter confirm the presence of jumps and intensity across the three exchange rates. The higher moments of skewness and kurtosis were also obtained which turned out to be normalised after the incorporation of jumps. Another feature of the result was the dependence in jump size and intensity as confirmed in the literature. We deduce from the results the impact of the jump intensity and size for the exchange rate. This has a bearing for currency options and thus aids in improving forecasts, serving as important contributors to future exchange rate related decisions for companies and investors. This further establishes the contribution of jump diffusion models in financial economics. The outline for the rest of this chapter is as follows; Section 4.2 presents the literature on jump diffusion models, Section 4.3 the GARCH model, Section 4.4 Data and sample statistics. Section 4.5 Results from estimation and Section 4.6 concludes.

4.2 Jump diffusion models

The leptokurtic nature is observable in high frequency data. However, reverenced classical finance models like e Black-Scholes (1973) ignore this feature, positing the Brownian motion model as best fit, wherein the stock price is modelled as;

(4.1) $S(t) = S(0)e^{\mu t+\sigma W(t)}$,

Where the Brownian motion $W(t)$, has a normal distribution with mean 0 and variance *t*. In this setting, μ is the drift measuring the average return.

σ is the volatility which measures the standard deviation of the return distribution. In this model setting, the continuous compounded return, $\ln(S(t)/S(0))$, has a normal distribution,

which is not consistent with its leptokurtic nature. Many alternative models with jumps and stochastic volatility have been proposed as improvemensts to the Black-Scholes model.

This has seen several approaches proposed in modifying the Black-Scholes model to explain the three empirical stylised facts which are; the leptokurtic nature, volatility clustering effect and implied volatility smile. Some of the approaches include;

(a) Chaos theory and fractal Brownian motions: These models replace the Brownian motion by a fractal Brownian motion with dependent increments; instead of having independent increments (see Mandelbrot (1963)). Rogers (1997) however points out the arbitrage opportunities in these models.
(b) Generalised hyperbolic models; for example log t model and log hyperbolic model, and stable processes. Such models replace the normal distribution assumption with other distributions; such as Barndorff-Nielsen and Shephard (2001), Samorodnitsky and Taqqu (1994), Blattberg and Gonedes (1974).
(c) Lévy process-based models (as examples see Cont and Tankov (2004).
(d) Stochastic volatility and GARCH models (Fouque *et al.* (2000), Engle (1995), Heston (1993), Hull and White (1987). These capture the volatility clustering effect.
(e) Constant elasticity of variance (CEV) model (see Davydov and Linetsky (2001), Cox and Ross (1976).

(f) Jump-diffusion models first proposed by Merton (1976) and also seen in Kou (2002).

(4.2) $S(t) = S(0)e^{(\mu-\frac{1}{2}\sigma^2)t+\sigma W(t)}\prod_{i=1}^{N(t)} e^{Y_i}$

N(t) is a Poisson process. According to Mertons' (1976) model, Y has a normal distribution and Kou (2002) shows it as having a double exponential distribution. The double exponential distribution allows us to get analytical solutions for many path-dependent options, including barrier and look back options, as well as analytical approximations for American options. Asea and Ncube (1998) also expanded on the work of Merton (1976).
(g) Implied binomial trees; see, for example, Derman and Kani (1994) and Dupire (1994).

Further to the ones listed above, there are other types of models which combine many features, some of which are; stochastic volatility, jumps, and time changes. Two examples of these are;

(h) Time changed Brownian motions and time changed Lévy processes. In these models, the asset price S(t) is represented as

(4.3) $S(t) = G(M(t))$,

G here is either a geometric Brownian motion or a Lévy process, and *M(t)* is a non-decreasing stochastic process which models the stochastic activity time in the market. The activity process *M(t)* may link to trading volumes (Carr *et al.* (2003), Heyde (2000), Madan *et al.* (1998), Madan and Seneta (1990), Clark (1973).

(i) Affine stochastic-volatility and affine jump-diffusion models; Duffie *et al.* (2000), combines both stochastic volatilities and jump-diffusions.

In this discussion, one of the major motivations for jump diffusion models is that asset price returns reflect heavier tails than those from a normal distribution. Jumps in security prices are represented as rare events. Therefore, asset prices are modeled as Lévy processes having a nonzero Gaussian component with a jump component. Compared with stochastic-volatility models, jump-diffusion models are simpler yet can also capture the financial market phenomena, like the large fluctuations in asset prices. The representation is outlined below:

In a jump-diffusion model with a probability measure P the asset price, S(t), is modelled as

(4.4) $$\frac{dS(t)}{S(t)} = \mu dt + \sigma(t) dW(t) + d\left(\sum_{i=1}^{N(t)} (V_i - 1) \right),$$

Here *W(t)* is a standard Brownian motion, *N(t)* is a Poisson process with intensity λ, and {Vi} is a sequence of independent identically distributed (i.i.d.) non-negative random variables. In this setting, all sources of randomness, *N(t)*, *W(t)*, and Y's, are independent. However, essentially all models are not without shortcomings as they are approximations of reality. Here we briefly evaluate the jump-diffusion models by four criteria;

(1) self-consistent; In the finance context, a model has to be arbitrage-free and embedded in an equilibrium setting. Other alternative models may have arbitrage opportunities, thus are not self-consistent, as an example, the arbitrage opportunities for fractal Brownian motions as shown in Rogers (1997). In this way, Merton's jump-diffusion model and the double

exponential jump-diffusion model can be embedded in a rational expectations equilibrium setting.

(2) A model ought to capture important empirical phenomena. However, it should be pointed out that empirical tests are not the only benchmark to pass a model. By their nature, empirical tests favour models with more parameters. However, calibration is difficult for models with many parameters. This is one of the main arguments for practitioners' preference of less complex models. As a strong case for jump diffusion models, in their lesser complexity, are able to reproduce the leptokurtic feature of the return distribution, and the "volatility smile" observed in option prices (see Kou, 2002). Furthermore, the exponential jump diffusion model fit outperforms the normal jump diffusion model and both fit data better when compared to the reverenced Black-Scholes model (Ramezani and Zeng (2002).

(3) Simple enough to enable ease in computation. Similar to the Black-Scholes model, the double exponential jump-diffusion model not only yields closed-form solutions for standard call and put options, but also allows a variety of closed form solutions for path-dependent options, such as barrier options, look back options, perpetual American options (Kou *et al.*, 2005, Kou and Wang, 2004, 2003;).

(4) Relate with interpretation. As an example, motivation for the double exponential jump-diffusion model comes from behavioural finance. Many studies carried out deduce that markets tend to have both over-reaction and under-reaction to various good news or bad news (see, Barberis *et al*, 1998, Fama, 1965). The jump component of the model can be interpreted as the market response to outside news. In the absence of outside news the asset price follows a Geometric Brownian Motion (GBM). Good or bad news arrives according to a Poisson counter, and the asset price changes in response according to the jump size of the distribution. Since the double exponential distribution has both high peaks and heavy tails, it can be applied in modelling the over-reaction and under-reaction to outside news. Many models can satisfy at least some of the criterions listed above. The dominating attraction of the jump diffusion model is its simplicity and also the model attempts to improve the empirical implications of the Black-Scholes model whilst retaining its analytical tractability.

4.2.1 Shortcomings of jump-diffusion models

The main problem associated with jump diffusion models is that they cannot capture the volatility clustering effects sufficiently, which can be captured by other models with stochastic volatility, like the affine jump diffusion model. The jump diffusion model and the stochastic volatility model therefore complement each other. As an example, one phenomenon worth mentioning is that the daily return distribution tends to have more kurtosis than the distribution of monthly returns. Das and Foresi (1996) point out that this is consistent in models with jump diffusion models, but not so for stochastic volatility models. In stochastic volatility models the kurtosis decreases as the sampling frequency increases; while in jump diffusion models the instantaneous jumps are independent of the sampling frequency. This suggests that jump diffusion models may capture short-term behaviour better, while the stochastic volatility is aligned to long term behaviour.

Other general models combine jump diffusion with stochastic volatilities resulting in affine jump-diffusion models (Duffie *et al.* (2000)). Affine jump diffusion models incorporate jumps, stochastic volatility, and jumps in volatility. As special cases for these models we have the normal and double exponential jump diffusion models. However, as a result of the special features of the exponential distribution, the double exponential jump-diffusion model has difficult analytical solutions for path-dependent options. Overall, thejJump diffusion models are simpler than general affine jump diffusion models since jump diffusion models have fewer parameters, thereby making calibration easier. Thus, jump diffusion models attempt to strike a balance between reality and tractability, seen in short maturity options and short term behaviour of asset pricing.

To sum up, many alternative models may give some analytical formulae for standard European call and put options, however the solutions for interest rate derivatives and path-dependent options, like perpetual American options, barrier and look back options, are difficult. Therefore, jump diffusion models are more suitable for pricing short maturity options in which the impact of the volatility clustering effect is less pronounced. Furthermore, jump diffusion models can provide a useful benchmark for more complicated models.

4.2.2 Studies on application of jump diffusion models

The stochastic models of price behaviour used in the financial economics literature to value options are selected at least in part for their ability to yield either analytical solutions or efficient numerical procedures. The classical example is the lognormal diffusion process, from which follows the BS hedging argument and Cox & Ross' (1976) risk neutral valuations.

A number of researchers have established evidence further than the Black-Scholes application; volatility varies with time (Bollerslev *et al,* 1992). Other findings deduce that markets are characterised by the presence of jumps, especially with short frequencies data (Drost, Nijman, & Werker, 1995; Ball & Tourus, 1985; Jarrow & Rosenfeld, 1984). Literature has also documented that excess kurtosis in unconditional returns declines with time, (Kon, 1984). Even as far back, Fama (1965) deduced the presence of fat tails in stock returns in daily data. The work of Blattberg and Gonedes (1974) found that the normality assumption holds in stock returns with monthly data. (Jorion, 1988) also reports that excess kurtosis in the $/DM exchange rate and in the value-weighted CRSP index are, respectively, 3.29 and 2.92 under weekly data, but fall to 1.56 and 0.89 for monthly data.

From these findings we can assert that diffusion models are not robust enough to capture the appearance of jumps in underlying asset prices. As a result, jump diffusion processes have gained popularity for modelling in finance. On application, they can be calibrated to plain vanilla options and used to price and hedge exotic options. Other empirical works have deduced that a normal Poisson jump model provides a good statistical portrayal in exchange rate and stock returns data. A noteworthy application is by Bates (1991), who used Standard & Poor's 500 futures options and found systematic behaviour in expected jumps before the 1987 stock market crash. Most of the jumps in the exchange rate have been linked to the result of the actions of the central bank (Hull, 2006). Further than that jump diffusion models have been introduced in literature by many researchers who observed different paths for different assets and their prices.

The literature framework for jump diffusion models has shown to be consistent. We trace jump diffusion models from the seminal work of Merton (1976). Recent developments in mathematical finance have explored several approaches to model the underlying asset. This is

evidenced by the exposition carried out on the daily returns of a stock (Figueiredo *et al,* 2011; Yang *et al,* 2009; Craine *et al*, 2000; Eberlein & Keller, 1995; Eberlein & Jacod, 1997).

Related literature in Duffie *et al.* (2000) used a simulation-based estimator to estimate a complex model with conditional jump intensity and stochastic volatility. Bates (2000) estimated the stochastic volatility jump diffusion process for S&P futures prices implicit in futures option prices. Other applications supporting jump diffusion models have been proposed. A study by Heston (1993) employed the square root process for modelling the variance of the asset price, and showed how to apply the Fourier transform to solve the pricing problem for vanilla options. Taking this method further, Bates (1996) added to the Heston's stochastic volatility model log-normal price jumps governed by a Poisson process with a constant intensity rate. Fang (2000) extended the Bates' model by introducing a stochastic intensity rate. Duffie, Pan, & Singleton (2000) added to the Heston's model price and volatility jumps. All these studies support jump diffusion models.

It has been further shown that incorporating the jump component in both price and volatility in models is necessary. As examples, Bates (2000) and Pan (2002), considered models where prices follow a jump diffusion process. In their models, volatility was characterised by a correlated diffusive stochastic process. To sum it up, both authors show that these models are incapable of capturing empirical features of equity index returns or option prices, attributing this to the fact that volatility itself may contain jumps. Eraker *et al.* (2003) examined the jump in volatility models proposed by Duffie, Gray, & Hoang (1999) and provided a study that shows that the addition of jumps in volatility provide a significant improvement in explaining the returns data on the S&P 500 and NASDAQ 100 index, proving superior to a stochastic volatility model with just jumps in prices. Asea and Ncube (1998) used a doubly stochastic Poisson process to model the arrival of heterogenous information in a market. In their study they derived implications for the pricing of stock, index and foreign currency options.

Other related studies carried out have thus motivated continuous work on jump diffusion models. However some tests using option data disagree over the importance of jumps in prices: Bakshi *et al* (1997) found substantial benefits from including jumps in prices.

However, Bates (2000) found economically small benefits. Studies using the time series of returns unanimously support jumps in prices but disagree over jumps in volatility.

There are a few studies for South Africa which have incorporated jumps and stochastic volatility (for example Poklewski-Koziell, (2012), Kalsheker, (2009)). However, these studies have focused on calibration and option pricing without due consideration of expounding on benefits of jump diffusion models on asset prices like exchange rates. The remaining few studies on understanding the exchange rate have not considered at all the important application of jump diffusion models (for example de Jager, (2012), Gupta *et al.*, (2011),). The majority approach of these models has employed ordinary least squares and not maximum likelihood results (for example Gupta *et al.*, (2011); Dube, (2008); Ziramba, (2007)). The table below summarises some of the important applications of jump diffusion models. It can be deduced from the table below that well known applications of the jump diffusion model can be extended as well to various asset classes investigated in this thesis for example exchange rate, interest rate and stock prices.

Table 4.7: Other Jump Diffusion models in use

Jump Diffusion model	**Summary / Application**
Bernoulli	(Ball & Tourus, 1985) provided statistical evidence of log-normally distributed jumps for stocks
Gauss-Hermite	The process ensures efficiency properties for valuing compound options. Applied by (Omberg, 1988)
Jumps in Interest Rates	General equilibrium asset pricing model (Cox *et al*, 1985). Ahn & Thompson (1988) investigated the effect of the jump components of the underlying processes on the term structure of interest rates
Geometric	Bates (1996) showed that the exchange rate has geometric jump diffusion with conditional variance following a mean reverting square root process.
Jump Diffusion Models with Conditional Heteroscedasticity	Combine jumps with an ARCH/GARCH model in discrete time. Chan & Maheu (2002) developed a new conditional jump model to study jump dynamics in stock market returns. They present a discrete-time jump model with time varying conditional jump intensity and jump size

	distribution
ARCH/GARCH Jump Diffusion Model	Jorion (1988) considered a tractable specification combining both ARCH and jump processes for foreign exchange market
Doubly stochastic Poisson process (DSSP)	Applied DSPP on pricing for stock, index and foreign currency options (Asea and Ncube, 1998).

4.2.3 Estimation methods for jump-diffusion processes

Before considering estimation the coefficient functions of the jump diffusion process have to be well behaved for the Markov transition density functions to be well defined. Literature abounds with estimation methods for jump diffusion models. The arbitrarily chosen moments of jump diffusion models like the method of cumulants matching, such as the Generalised Method of Moments (GMM) all present problems stemming from distinguishing whether the movements in the underlying process belong to the continuous path or are part of the jump path dynamics.

Pure-jump processes have received much attention, (see Madan (2001); Carr *et al.* (2002). Literature has shown many different approaches to separate the variations in the state variable Xt attributable to the diffusive part from those due to jumps with a feasible econometric technique. For parametric models, the parameterised functional forms are imposed for the drift and diffusion functions and for the jump component. Related to this work Jiang and Oomen (2004) developed estimators based on a weighted sum of squared increments in an affine asset pricing model with the jump part having finite activity. In a related study, Aït-Sahalia (2004) reveals that it is possible to disentangle jumps from the continuous variations using a maximum likelihood approach. Furthermore, Aït-Sahalia and Jacod (2006) construct a threshold estimator of the volatility where the source of randomness is a stable Lévy process.

Barndorff-Nielsen, (2006) and Shephard, (2004), Woerner (2005), Mancini, (2004) and Jacod, (2006) estimate the integrated volatility using a nonparametric approach. Barndorff-Nielsen and Shephard, (2004) prove that when the volatility is independent of the brownian motion, the power variation of the state variable is a consistent estimator of the integral of the corresponding power of the volatility. This applies even in the presence of a finite activity jump process.

Woerner, (2005); Barndorff- Nielsen et al. (2006) and Jacod, (2006) allow an extension to the case of infinite activity jump component. Barndorff- Nielsen and Shephard (2006) develop the original theory of the bi-power variation, thus allowing the estimation of the integrated volatility in the presence of finite activity jumps. The bi-power variation theory has inspired new areas for research on financial data (see for example Andersen et al. (2006); Huang and Tauchen (2005); Tauchen and Zhou (2006)). Furthermore, it allows testing for the presence of jumps.

In summary the literature has shown that the jump diffusion model is a convenient form for pricing other derivatives, for example currency options ((Meyer & Yu, 2000); (Merton, 1976); (Yu *et al*, 2006), (Yang *et al*, 2009)). Also corroborating jump diffusion models, Kou (2002) discussed the option pricing, and Hu & Ye (2007) discussed the credit derivatives. At the same time, many empirical works (for example; (Payne, 2003); (Anderson et al, 1999); (Bates, 1996); (Vlaar, 1993); (Hsieh, 1988)) show that the financial time series have features of high peak and fat tail which results mainly from the occasional jumps. The literature considered above therefore supports that the exchange rate data frequently exhibits jumps.

4.3 GARCH Model

To complement the Jump Diffusion model, we introduce the GARCH specification to capture the persistence in volatility. The General Autoregressive Conditional Heteroscedasticity (GARCH) is credited to the work of Bollerslev (1986). The GARCH (1, 1) specification has been found to be sufficient in practice for modelling volatility in high frequency financial series data. GARCH modelling builds on advances in the understanding and modelling of volatility. It takes into account excess kurtosis and volatility clustering, the characteristics of financial time series. As such it can be applied to diverse fields like risk management, portfolio management and asset allocation, option pricing, foreign exchange, and the term structure of interest rates (Engle, 1982).

GARCH effects have a bearing in such areas as Value-at-Risk (VaR) and other risk management applications to do with the efficient allocation of capital. Furthermore, GARCH models are seen as a method for specifying and estimating a volatility filtration algorithm

(Flemming & Kirby, 2003, Nelson & Foster, 1994 and Nelson (1992)). Another motivation for GARCH models is that the family of GARCH models has proven successful in forecasting volatility. It should be stated that volatility clustering[14] can be captured in a GARCH representation. This is made possible by the distribution of the fat tailed GARCH process which can capture characteristics observed in high frequency asset prices.

Some econometric tests for choosing between jump-diffusion and stochastic volatility models have been attempted ((Bates (1996); Jorion (1988)). Jorion (ibid) finds in his study that based on monthly data, there is minimal difference between the models he considers, but that this is not true under weekly data, supporting the Jump Diffusion model (Das & Sundaram, 1999).

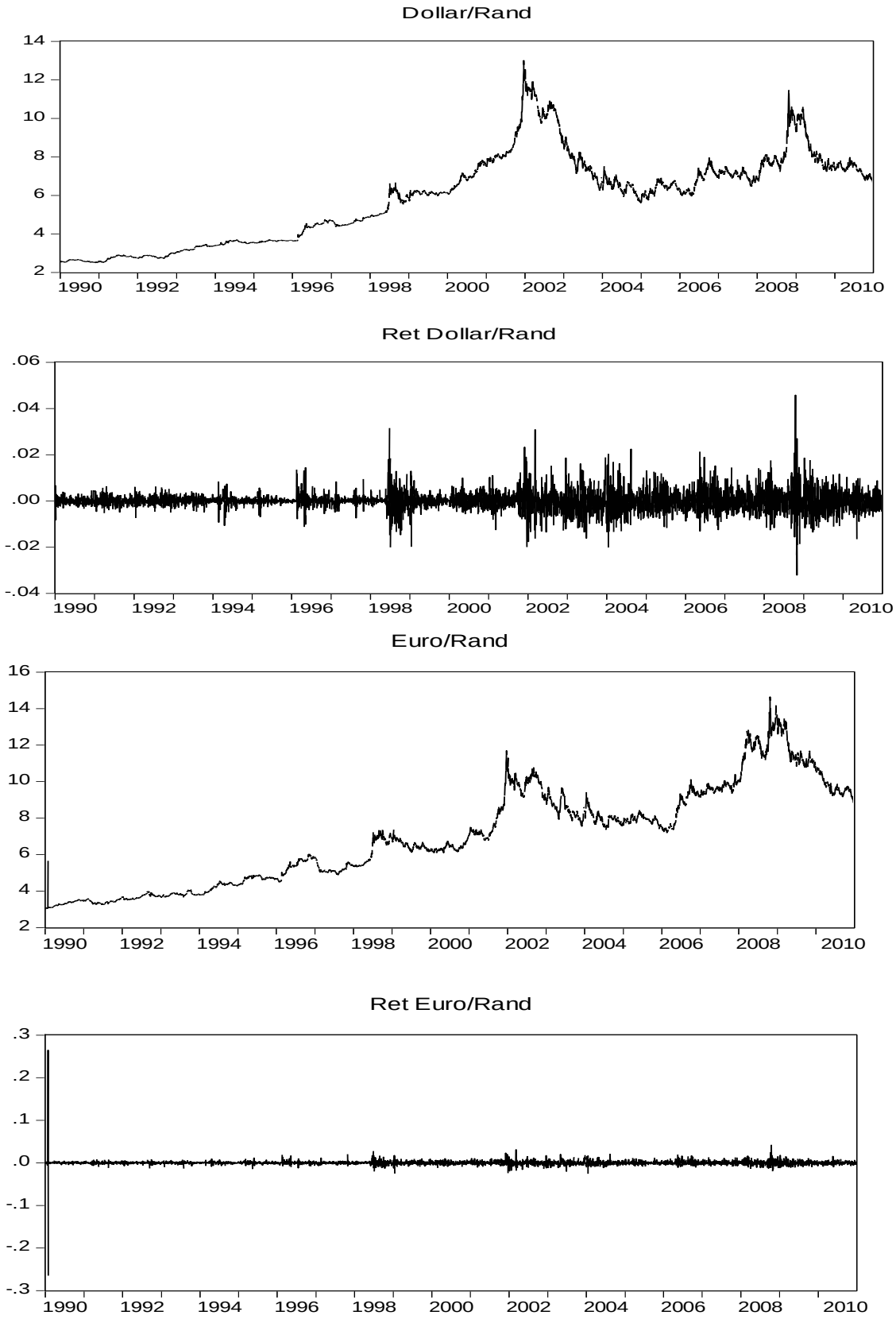
4.4 Data and sample statistics

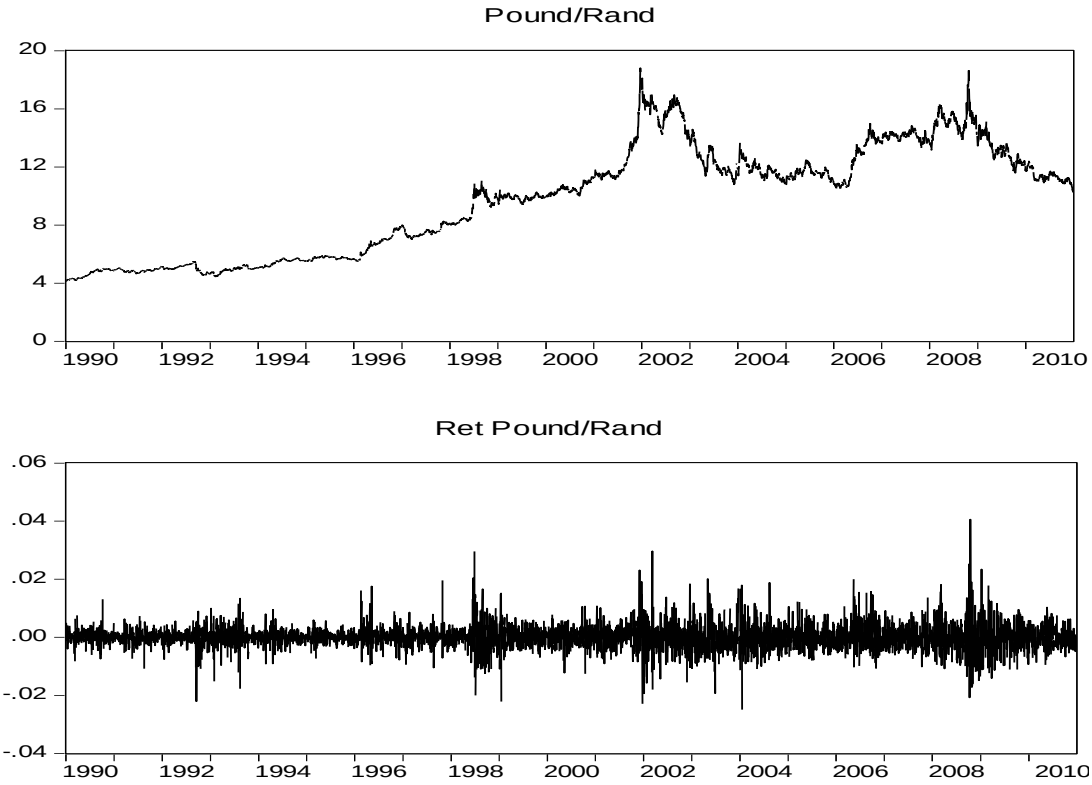
The data employed is daily exchange rate data across three currencies, the US dollar against the rand, the euro against the rand and the pound against the rand. The years of interest are 1990-2010, giving us 5478 observations for each exchange rate. We also divide the data into two periods 1990-2000 and 2001-2010 to determine the period with higher jumps and intensity. The diagrams below shows the daily exchange rate plots of South Africa's rand against major world currencies from January 1990 to December 2010[15]

[14] Observed when there are periods of high and low variance in the data and also where large changes are clustered around large changes and small changes clustered around small changes (Cont, 2007). In chapter three we specified a fat tailed GARCH capturing volatility clustering the results of which are presented in this chapter.

[15] The exchange rate is the price of a british pound in sa rand, united states dollar in rand and european euro in rand

Figure 4.4: Rand against major currencies

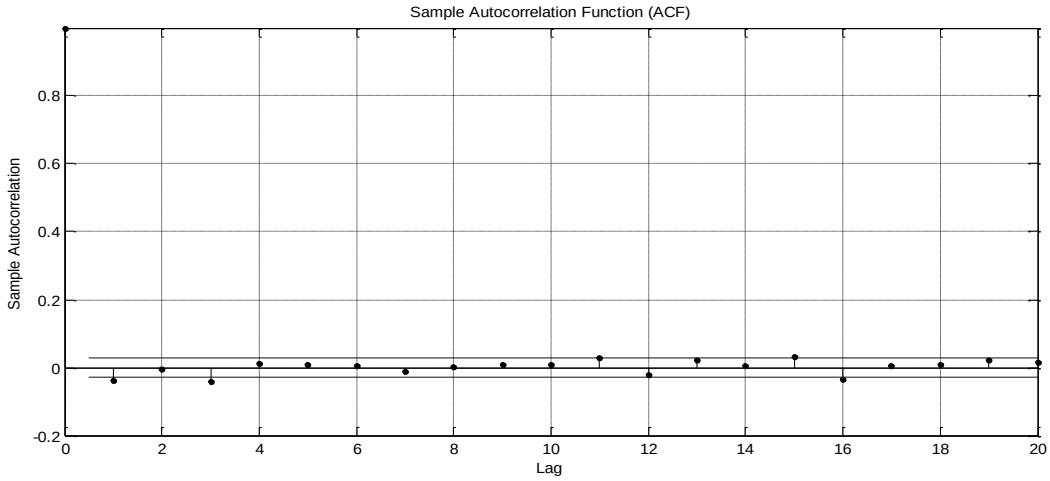




The figures above present exchange rate and returns data for the three major currencies. From the returns figures above, the exchange rate data is typical of characteristics seen in most foreign exchange rate data; skewness, kurtosis and potential jumps. Within the figures, the financial market crashes for 1997 and 2008 which exhibit huge jumps in the returns also feature. Furthermore, the returns diagrams display volatility clustering. Accordingly, Empirical Density Function diagrams for these currencies do not yield Normal Distribution returns[16]. Another graphical feature displaying the nature of the data is embedded in the autocorrelation of the returns in the diagram below.

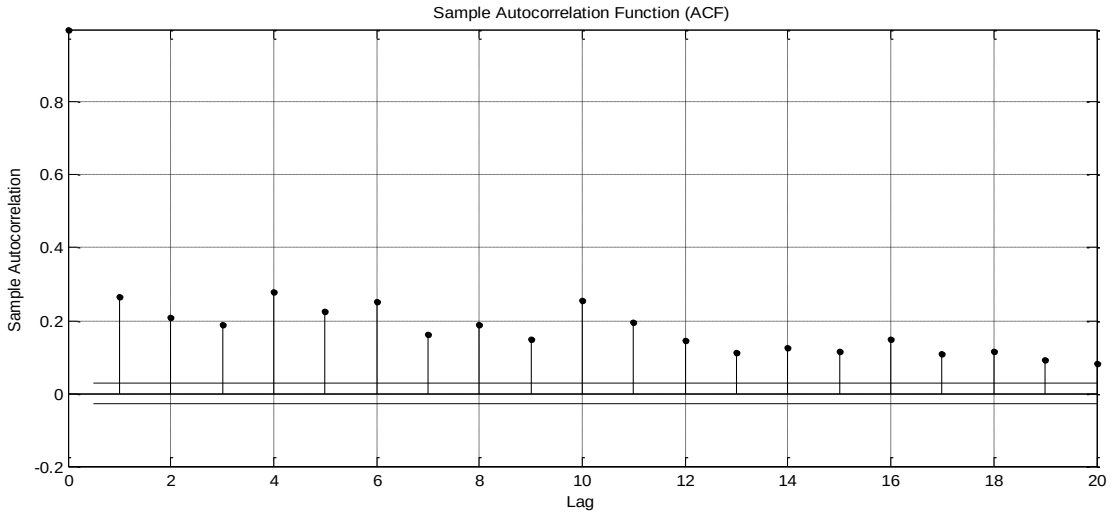
[16] The Normal fit diagrams are presented in the appendices.

Figure 4.5: Autocorrelation in returns[17] (US$/Rand exchange rate)



The autocorrelation function computes and displays the sample Auto Correlation Function (ACF) of the returns, along with the upper and lower standard deviation confidence bounds, based on the assumption that all autocorrelations are zero beyond lag zero. From figure 3.5 above there is no significant autocorrelation in returns. Although the ACF of returns shows little correlation, the ACF of the squared returns may indicate significant correlation and persistence in the second order moments. This can be checked by plotting the ACF of the squared returns as shown below.

Figure 4.6: Autocorrelation in squared returns (US$/Rand exchange rate)



[17] Returns are calculated as $\ln(\frac{y_t}{y_{t-1}})$

From the figure above, the ACF of the squared returns shows that, although the returns themselves are uncorrelated, the variance process shows some correlation. The ACF of the squared returns appears to die out slowly, indicating the possibility of a variance process close to being non-stationary. Therefore we assert that the data shows the generic characteristics of financial time series data-excess kurtosis, skewness, potential jumps and volatility clustering. There have been many studies documenting these features as mentioned in the literature review. Tables 4.8 and 4.9 below give estimates of the four unconditional sample moments

Table 4.8: Summary statistics exchange rate raw data

	1990-2010			**1990-2000**			**2001-2010**		
	Euro	Pound	Dollar	Euro	Pound	Dollar	Euro	Pound	Dollar
Mean	6.981	9.7373	5.8798	4.8074	6.7653	4.1944	9.373	13.0079	7.7345
Max	14.646	18.8125	13.0028	7.3683	11.3491	7.8453	14.646	18.8125	13.0028
Min	3.0351	4.094	2.5019	3.0351	4.094	2.5019	6.7704	10.2257	5.6148
Variance	7.1594	13.44	5.0965	1.43	4.4927	1.9948	2.5422	2.8753	1.9435
Skewness	0.339	-0.0031	0.258	0.3305	0.6825	0.7261	0.654	0.6041	1.1663
Kurtosis	2.1474	1.747	2.4049	1.7794	1.9403	2.3637	2.7903	2.3897	3.7997

Table 4.9: Summary statistics exchange rate returns data

	1990-2010			**1990-2000**			**2001-2010**		
	Euro	Pound	Dollar	Euro	Pound	Dollar	Euro	Pound	Dollar
Mean	8.56e-05	7.3e-05	7.6e-05	1.3e-04	1.5e-04	1.6e-04	3.8e-05	-1.6e-05	-2e-05
Variance	4.1e-05	1.6e-05	1.6e-05	5.8e-05	1e-05	6.5e-06	2.2e-05	2.3e-05	2.7e-05
Skewness	6.4e-02	0.522	0.724	-8.e-02	0.211	0.807	0.664	0.612	0.649
Kurtosis	1058	10.1	12.32	1007.9	12.358	21.05	8.38	7.99	8.097

For the raw data as presented in table 4.8, the mean for each currency is high for the period 2001-2010 compared to the other periods. The result is the same for the maximum and

minimum values, thus showing a downslide of the rand from 1990-2010. The general downward slide of the- rand could be attributed to a host of factors, some of which are; escalating global risk aversion as a result of investors' concerns over the spreading impact of the sub-prime crisis, and a general flight to "safe havens", away from the perceived risks of emerging markets and in recent years the unstable labour environment. Another factor to consider during the study period is the Eskom electricity crisis, which arose as the utility at times was unable to meet the country's rapidly growing energy demands. This had negative impacts on production and exports by south african companies and would worsen to a current account deficit. Negative sentiments towards emerging markets in late 2001 can also be cited as other pointers to the depreciation of the rand.

Kurtosis indicates the degree of "flatness" or "peakedness" in a distribution relative to the shape of normal distribution. By norm, a standard normal distribution has a kurtosis of 3, hence an excess kurtosis of 0. From table 3.8 above, for the period 1990-2010 the distributions for all the currencies are platykurtic, though with different levels with the dollar and euro closely trailing each other similar but both higher than the pound. When we split the period into 1990-2000 and 2000-2010, the latter period for the us$/r shows leptokurtic distribution with a positive skewness greater than 1. In this period, the skewness for the remaining currencies are relatively high and their kurtosis has risen as well, showing high volatility. In the period 1990-2000, all currencies are platykurtic, though with very high skews for the US$/R and the Pound/Rand. When we consider table 3.9 for the returns we can deduce the volatility as shown by the leptokurtic values ranging from 5 to 1057.

4.5 Estimation Results

This section presents results from estimation. We begin with stationarity results, followed by jump diffusion parameter estimates and finally GARCH (1,1) results.

Table 4.10: Stationarity results exchange rate

Variables	Dickey-Fuller		Augmented Dickey-Fuller		Stat. status
	Constant	Constant & trend	Constant	Constant & trend	
Results for unit roots tests on levels exchange rates					
LEuro	-0.0418	-1.9346	-1.6398	-2.0559	I (1)
LPound	0.2947	-0.8913	-1.8742	-0.898	I (1)
LDollar	0.3586	-1.021	-1.7504	-1.0818	I (1)
Results for unit roots tests on returns					
DLEuro	-2.2991**	-3.8872***	-27.182***	-27.179***	I (0)
DLPound	-1.7378*	-3.6820***	-31.338***	-31.335***	I (0)
DLDollar	-4.1371***	-8.0868***	-27.566***	-27.563***	I (0)

*** ** * represents stationary series at 1, 5, and 10% levels.

The table above presents stationarity results for the exchange rate. Results in levels for Dickey-Fuller (DF) and Augmented Dickey-Fuller (ADF) are not stationary whereas returns data are stationary, most at a 1 percent level of significance. Parameter estimates in the table below are generated from the returns data.

Table 4.11: Exchange rate parameter estimates for the jump-diffusion model

	1990-2010			**1990-2000**			**2001-2010**			
	Euro	Pound	Dollar	Euro	Pound	Dollar	Euro	Pound	Dollar	
μ_B	1.2e-04 (0.0000)	1e-04 (0.0000)	2e-04 (0.0000)	1e-04 (0.0000)	1e-04 (0.0000)	1.1e-04 (0.0000)	3.7e-04 (0.0522)	2e-04 (0.0000)	2e-04 (0.0000)	
σ_B	6.4e-03 (0.0000)	3e-04 (0.0005)	3.45e-03 (0.0001)	7e-03 (0.0000)	3.1e-03 (0.0000)	2.4e-03 (0.0005)	4.6e-03 (1.8e-15)	9.6e-03 (0.0000)	5.1e-03 (0.0000)	
μ_J	7.8e-04 (0.0000)	8e-03 (0.0000)	9.3e-03 (0.0000)	5e-04 (0.0000)	5.7e-03 (0.0000)	9.8e-.03 (0.0000)	7e-04 (0.0576)	4.5e-03 (0.0000)	7e-04 (0.0000)	
σ_J	2.4e-05 (0.0000)	3e-04 (0.0000)	3.5e-03 (0.0000)	2.5e-03 (0.0000)	2e-04 (0.0000)	1e-04 (0.0001)	1e-04 (0.0485)	2e-04 (0.0692)	4.3e-03 (0.0000)	
λ	2e-04 (0.0000)	1.6e-03 (0.0002)	1.64e-02 (0.0056)	1.9e-03 (0.0001)	1.5e-03 (0.0000)	3.2e-03 (0.0000)	2.7e-03 (0.0548)	1.6e-03 (0.0595)	2.3e-03 (0.0003)	
Mean	1.2e-04	1.13e-04	3.53e-04	1.1e-04	1.1e-04	1.38e-04	3.72e-04	2.15e-04	2.02e-04	
Variance	4.1e-05	9.1e-06	1.35e-05	4.91e-05	9.66e-06	6.01e-06	2.12e-05	2.04e-05	2.61e-05	
Skewness	3.6e-07	0.03	0.254	5.24e-04	9.29e-03	1.48e-01	1.01e-05	1.5e-02	6.8e-04	
Kurtosis	3	3.08	3.755	3	3.02	3.53	3	3.03	3.004	

The table above presents parameter estimates for the jump diffusion model. Using a likelihood ratio as a diagnostic, we reject the null hypothesis of no jumps at a 1% level. Jumps therefore help to explain some features in the exchange rate data.

Matching the moments in the jump diffusion model

The goal is to verify that the model can match the data characteristics. Table 4.11 above illustrates this by including the levels of skewness and kurtosis values for the jump diffusion model. The model was run for numerous iterations and deduced, overall, more positive jumps than negative ones. In the analysis we also allowed for higher jump size and variance to explore the effect on volatility. Overall, results confirmed related studies in that the jump diffusion model does trace levels of skewness and kurtosis for returns data with varying degrees of accuracy. On generating sample variance, the success rate ranged from 56% to around 96%. Thus the significant jump size and intensity results imply that the model can be used to trace the behaviour of exchange rate data, especially for short term maturities. The results can also serve in informing computation of currency options. Though having produced significant results, we have to state that the model does not satisfy all the character traits of returns data.

(1990-2010) Whole sample;. For the euro, the unconditional mean of the jump diffusion is 1.2e-04. The drift of the GBM portion matches the unconditional mean. The mean of the jump process is positive at 7.8e-04 and the estimate of the jump frequency is 2e-04 and significant. The unconditional variance of the jump diffusion matches the sample variance. An interesting find is that seemingly the inclusion of jumps normalises the data, as per findings of Sorwar (2011), thus bringing down the skewness to levels close to zero and the kurtosis close to 3.

For the pound, the GBM drift is 1e-04 and the unconditional mean of the jump diffusion is 1.13e-04. The drift of the GBM portion is less than the unconditional mean. The jump diffusion results produced a positive mean with a jump intensity of 1.6e-03. The Jump Diffusion model fails to generate a sufficient level of variance comparable with the sample moment at 57%. The generated level of volatility is lower than for the euro at 75%.

For the dollar, the unconditional mean of the jump diffusion is 2e-04. The drift of the GBM portion is less than the unconditional mean (2e-04<3.53e-04). The mean of the jump process is positive at 9.3e-03 and is highest across all currencies in the period. The estimate of the jump frequency is 1.64e-02 and significant. The unconditional variance of the jump diffusion generated 84% of the sample variance in Table 4.9. The effect of the jump diffusion process is seen in the normalised skewness and kurtosis values at 0.25 and 3.78.

1990-2000. **Split sample**; For the euro, the unconditional mean of the jump diffusion is 1.1e-04. The drift of the GBM portion is less than the unconditional mean (1e-04<1.1e-04). The mean of the jump process is at 5e-04, having slightly improved form the full sample equivalent of 1.2e-04; the estimate of the jump frequency is 1.9e-03, also higher than the full sample equivalent. The unconditional variance of the jump diffusion however produced 85% of the sample variance, with jump diffusion volatility coming at 92% of sample volatility

For the pound, the unconditional mean of the jump diffusion is 1.1e-04. The drift of the GBM portion is less than the unconditional mean (1e-04<1.1e-04). The mean of the jump process is at 5.7e-03 and the jump intensity each day is at 1.5e-03 and significant. The generated level of volatility is high at 98%.

For the Dollar, the unconditional mean of the jump diffusion is 1.38e-04. The drift of the GBM portion is less than the unconditional mean (1.1e-04<1.38e-04). The mean of the jump process is highest across all periods at 9.8e-03; the estimate of the jump frequency is also highest at 3.2e-03 and significant. The unconditional variance of the jump diffusion generates 92% of the sample variance. The levels of skewness and kurtosis have been normalised by the jump diffusion process to 1.48e-01 and 3.53.

2001-2010. **Split sample;** For the euro, the unconditional mean of the jump diffusion process is 3.72e-03 and is highest across all periods. The mean jump size is 2.7e-03 with generated level of skewness within normal distribution range as a result of the jump diffusion process. The jump diffusion variance generates close to 96% of the sample moment equivalent.

For the pound, the unconditional mean of the jump diffusion is 2.15e-04. The drift of the ump diffusion portion is less than the unconditional mean (2e-04<2.15e-04). The mean of the

jump process is positive with a jump intensity of 1.6e-03. The unconditional variance does generate 88% of the sample equivalent.

The unconditional mean for the dollar is 2e-04. The drift is less than the unconditional mean. The mean of the jump process is at 7e-04. The unconditional variance is equal to 97% of the sample moment.

The model was run again with assumed higher levels of the jump variance, and we deduced that excess kurtosis did increase. We deduced as well that the proposition for the moments given by the equations (3.3 to 3.5) imply that skewnesss and kurtosis of a jump diffusion model decrease as data becomes less frequent. Furthermore, as the variance of the jump component increases, the smile becomes deeper. However, the smile flattens out at a longer horizon unless the jump variance is high. From the simulations carried out we also noted that as the jump mean increases, excess kurtosis increases. The results have an implication on currency options and also on arbitrage opportunities across currencies. This also contributes to strategizing on currency risk management.

On the behaviour of implied volatility under jumps, there are variables which need adjustment for risk (Bates 1996). In the case of the jump diffusion model, the parameters to be adjusted for risk will be the jump intensity λ and the mean of the jump size. The variance of the jump size is not affected (Das and Sundaram, 1999). In a related application, after the appropriate variables have been adjusted for risk, options under a jump diffusion model can be priced using Merton's (1976) formula and implied volatility estimates backed out using the Black-Scholes model.

Also important to point out in this analysis is that for high parameter values, the difference in the implied volatility at one year and one month is low compared to the two to three percentage ponts in practice (see Campa & Chang, 1995), Derman & Kani, 1994).

Table 4.12: Exchange rate GARCH results

	GARCH Student's t			GED		
	Euro	Pound	Dollar	Euro	Pound	Dollar
C	6.07E-06	6E-05	3.47E-05	-1.52E-06	3.02E-06	1.37E-06
	(3.55E-05)	(3.49E-05)	(2.29E-05)	(3.21E-05)	(3.15E-05)	(2.24E-05)
K	5.181E-07	1.37E-07	1.86E-08	5.14E-07	1.53E-07	3.16E-08
	(7.03E-08)	(3.02E-08)	(6.47E-09)	(7.27E-08)	(3.34E-08)	(5.74E-09)
$GARCH(1)$	0.8126	0.8967	0.8635	0.8075	0.8915	0.8823
	(0.0106)	(0.008)	(0.0075)	(0.0115)	(0.009)	(0.1306)
$ARCH(1)$	0.1787	0.106	0.1266	0.1727	0.1068	0.13065
	(0.01478)	(0.0098)	(0.0075)	(0.0143)	(0.0098)	(0.0097)

The results above were generated from maximum likelihood with the number of iterations ranging from 18 to 20 for the student's t distribution and 21 to 330 for the GED distribution. From the results we have the coefficients of the variance equation; the intercept, ARCH(1), the lag of the squared return and GARCH(1), the lag of the conditional variance. The parameter α_1, which is represented by ARCH (1) also reflects the random deviations in the previous period whereas β_1, represented by GARCH (1) measure the part of variance from the previous period that is carried over into the current period. The parameter sizes determine the short run dynamics of the volatility in the exchange rate time series. Large values of α_1 imply that volatility reacts intensely to market movements whereas large values of β_1 imply that the shocks to conditional variance take a long time to die out, hence volatility is 'persistent'. From our results, β_1 is higher than α_1; for example for the euro/rand β_1 is 0.81 and α_1 is 0.18.

In this case this entails that shocks to the variance take time to die out, indicating volatility clustering. Furthermore, the sum capturing the persistence in volatility, is close to 1, indicating the presence of ARCH and GARCH effects in exchange rate data and also a required condition to have a mean reverting variance process. Related findings for South Africa are somewhat mixed. For example Farrel (2001) used a normal GARCH representation and deduced volatility persistent. However, Duncan and Liu (2009) deduced explosive GARCH results for a normal representation. They further went on to introduce a structural change GARCH model which produced non explosive results. In this chapter however we employed a fat tailed GARCH model which showed non explosive results and therefore becomes useful in explaining volatility and as well to compliment jump diffusion models.

4.6 Conclusion

The chapter illustrated modelling volatility in exchange rate data. Consideration was made of three different exchange rates. The goal was to understand the movement of asset prices through application of the jump diffusion model on exchange rate data. The graphical presentations of returns showed potential jumps and volatility clustering which were more pronounced during trying times in the financial market. Results from the jump diffusion model verified related studies in that the model incorporating jump diffusions is able to capture the high moments in skewness and kurtosis (Figueiredo *et al.*, (2011), Yang *et al.*, (2009), Kouet *et al.*, (2002) thus faring better than the GBM as it is able to trace jumps in the data. Results revealed significant levels of jump size and intensity across all three currencies. Furthermore, the volatility generated from the model in many cases matched the sample equivalent. A better estimation for volatility is crucial for hedging against adverse movement in the exchange rate, through currency options.

The implication for the results is that the jump diffusion model can explain the volatility smile for short and medium term maturities, but does not do the same for longer time horizons. As a result, the implied volatility smile in jump diffusions is seen for short maturities, dying out with time. This is important for pricing currency options as it offers leverage to businesses affected by currency risk as they can make informed hedging strategies by taking advantage of arbitrage opportunities that may lie therein. An interesting

result was the normalised moments in skewness and kurtosis after inclusion of the jump component.

However, the jump diffusion model does not satisfactorily capture all the characteristics of the data. The study also revealed dependence in jump intensity and jump size. To complement the jump diffusion results the GARCH models were applied to capture the persistence in volatility. GARCH(1,1) models take into account the autocorrelation in the volatility in returns and as such low order GARCH(p,q) models are preferred to high order ARCH(p) for reasons of parsimony and numerical stability in estimations. Using fat tailed GARCH distribution we captured presence of volatility clustering. As an improvement of the analysis in future we could consider the tracking the jumps sizes and intensity pre and post central bank announcements. Another future consideration is to merge jump diffusion models with stochastic volatility for South Africa and compare the generated parameters and deduce implications for currency options.

CHAPTER FIVE

JUMP DIFFUSION IN INTEREST RATES MODELS

5.1 Introduction

The understanding of the term structure of interest rates is important for practitioners in finance and policy makers alike. For one, the term structure can be used to determine the cost of capital and also to manage financial risk. Interest rates are also employed in a number of applications, for example; in pricing the risk of interest rate derivatives, and also importantly to bring about fiscal and monetary policies. It is therefore befitting that organisations manage financial risk brought about by volatility in the interest rate and consequently create value benefiting shareholders.

A correct specification of the short term interest rate model is one way of understanding the factors that drive interest rates. This has seen various models proposed to explain the term structure of interest rates. Some of the models explaining the significant role of interest rates include Ahn and Gao (1999), Chan *et al.* (1992), also known as CKLS after Chan, Karolyi, Longstaff and Sanders, Longstaff and Schwartz (1992). However the well-known models as specified in CKLS (1992) fail sufficiently capture the interest rate characteristics, even after augmenting and adding other factors (see Ahn *et al* (2002), Ball and Tourus (1985) and Andersen and Lund (1997)).

In this chapter we propose an alternative representation of the short term interest rate by augment interest with jumps as informed by recent literature (for example Sorwar, 2011, Xue *et al* (2011), Attaoui and Six (2008). This is the approach we introduce for South African data as there have been no studies incorporating jumps on short term interest rate models. This representation contributes to a better understanding the behaviour of interest rates and can therefore prove useful to policymakers and other stakeholders in understanding interest volatility in the South African context.

Results are significant for the jump diffusion specification for all the parameters and therefore expound on understanding the characteristics of short term interest rates and thus aiding forecasting of interest rates, hence risk mitigation. The remainder of the chapter is organised as follows; Section 5.2 incorporates jumps to the short term interest rate model examined in the chapter. Section 5.3 discusses the methodology. Section 5.4 looks at data characteristics. Section 5.5 presents estimation results and Section 5.6 concludes

5.2 Jump diffusion process for interest rate models

The generic term structure of single factor model takes the following stochastic process:

(5.1) $dr = \kappa(\theta - r)dt + \sigma r^{\gamma} dZ$

Which can also be presented as $dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ$, after allowing $-\kappa = \beta$ and $\kappa\theta = \alpha$ (Chan, Karolyi, Longstaff and Sanders, 1992)

The stochastic process above has drift and volatility. The volatility is heteroskedastic in that the variance is multiplied by the interest rate raised to a power thus the representation in (5.1) differs from equation (3.1) mainly as a result of the heteroscedastic volatility and exclusion of the Poisson distributed jump component. We can also assert that that the mean and variance of changes in the short term interest rate depend on r. The parameter θ in the model represents the threshold level of the interest rate r at which the drift is zero. Also to note is that the stochastic process in equation (5.1) shows a broad class of interest rate models. If we place restrictions on this equation, we obtain well known interest rate models as shown below with the corresponding parameter restrictions in the table.

(5.2)	Merton	(1973)	$dr = \alpha dt + \sigma dZ$
(5.3)	Vasicek	(1977)	$dr = (\alpha + \beta r)dt + \sigma dZ$
(5.4)	CIR	(1985)	$dr = (\alpha + \beta r)dt + \sigma r^{\frac{1}{2}} dZ$
(5.5)	Dothan	(1978)	$dr = \sigma r dZ$
(5.6)	GBM	(1973)	$dr = \beta r dt + \sigma r dZ$
(5.7)	Brennan-Schwartz	(1980)	$dr = (\alpha + \beta r)dt + \sigma r dZ$

(5.8) CIR VR (1980) $dr = \sigma r^{\frac{3}{2}} dZ$

(5.9) CEV (1976) $dr = \beta r dt + \sigma r^{\gamma} dZ$

Table 5.13: Parameter restrictions imposed on alternative short-term interest rates models

Model	α	β	σ^2	γ
Merton		0		0
Vasicek				0
CIR SR				½
Dothan	0	0		1
GBM	0			1
Brennan-Schwartz				1
CIR VR	0	0		3/2
CEV	0			

Chan, Karolyi, Longstaff and Sanders (CKLS) (1992) in their study compare the above eight models of short term interest rate with the aim of deducing as to which model best fits the short term riskless rate. The models are fitted within the same generic structure of equation (5.1) to allow for comparison among the models. Their findings indicate that models which best describe the dynamics of interest rates over time are those representing the conditional volatility of interest rate changes to be highly dependent on the level of the interest rates.

Merton's (1973) model for discount bond prices is represented as equation (5.2) and we can deduce the parameter restrictions from table 5.13 above for this model. Vasicek (1977) used the Ornstein-Uhlenbeck process to derive the equilibrium model of discount bond prices. This is represented in equation (5.3) and also shown by the parameter restrictions in the table above. Merton's model is nested with the Vasicek model by restricting the parameter $\beta = 0$. With this restriction, the models suggest a constant change in conditional volatility- of the

riskless rate. The square root process (SR) in the Cox, Ingersol and Ross (CIR) (1985) single factor general equilibrium term structure is represented as equation (5.4). The CIR model considers key factors in determining the term structure of interest rates such as; anticipation of future events, risk preferences, investment alternatives and even preferences about the timing of consumption.

Various valuation models for interest rate sensitive contingent claims have adopted this model; for example the yield valuation model by Longstaff (1990), swap pricing model by Sundaresan (1989) and for futures and futures option pricing models in Ramaswamy and Sundaresan (1986).

Application for Dothan's (1978) discount bond valuation model is seen in the study by Brennan and Schwartz (1977) for bond valuation. The model is shown above as equation (5.5). Equation (4.6) presents the geometric Brownian motion GBM from which follows the reverenced Black-Scholes (1973) model. On application, Marsh and Rosenfeld (1983) consider an interest rate model from the GBM. A model with fewer restrictions, equation (5.7) by Brennan-Schwartz (1980) is used for bond prices. Courtadon (1982) uses a similar approach for a model of discount bond option prices. If we impose $\alpha = 0$ on the Brennan-Schwartz model we get the GBM model.

The Cox, Ingersoll, Ross (1980) model is represented as equation (5.8). It was used in their study of the variable rate (VR) securities. Constantinides and Ingersoll (1984) use a similar model to CIR (1980) to value bonds with taxes. The last model, equation (5.9), gives us the constant elasticity of variance (CEV) process by Cox (1975) and Cox and Ross (1976). The model is also discussed by Marsh and Rosenfeld (1983). Imposing certain restrictions on the model we obtain the models of Dothan, Brennan-Schwartz and CIR VR.

Though possessing features of the interest rate models and ease of estimation, the stochastic process for these models as presented in equation (5.1), does not capture all the characteristics of the interest rate, as empirical studies have shown. In this regard, some of the factors considered for improving equation (5.1) include non- linear drift and a general form of the diffusion coefficient. Notably, Aït-Sahalia (1996) proposes the following model:

(5.10) $dr_t = [\kappa(\theta - r_t) + \alpha_0 r_t^2 + \frac{\alpha_1}{r_t}]dt + \sqrt{\beta_0 + \beta_1 r_t + \beta_2 r_t^{\beta_3}}\, dZ$ [18]

As an alternative, Conley *et. al* (1997) propose a model of the form;

(5.11) $dr_t = [\kappa(\theta - r_t) + \alpha_0 r_t^2 + \frac{\alpha_1}{r_t}]dt + \sigma r_t^{\gamma} dZ_t$,

with a constant elasticity of Variance (CEV) in CKLS but retaining the drift in the above Aït-Sahalia (1996) representation.

As a better alternative to Conley *et al.* (1997), Ahn and Gao (1999) proposed a parametric model from which we also deduce affine term structure models by placing restrictions on the parameters. The series of the interest rate they considered is represented by the stochastic differential equation. In their representation, the model features a nonlinear term structure generating dynamics of interest rate and market price of risk close to findings by CKLS (1992), Aït-Sahalia (1996), Conley *et al.* (1997), and Stanton (1997). The model used provided stationary interest rate process and a closed form expression for transition and marginal density of interest rate and bond prices. Their findings imply nonlinear relationship between interest rate and yields to maturity, thus outperforming the affine class models discussed above.

However, even with these modifications the models set above do not capture sufficiently all the characteristics of the interest rates. To better the model, we add a random jump component, giving us:

(5.12) $dr_t = [\kappa(\theta - r_t) + \alpha_0 r_t^2 + \frac{\alpha_1}{r_t}]dt + \sigma r_t^{\gamma} dZ_t + k dN_t$

The last term represents the jump component; this captures unanticipated market reaction, which can be as a result of macroeconomic news, endogenous and exogenous crashes (for example the Black Friday of 1987 and the September 11, 2001). N_t is a Poisson counter with constant intensity λ and k is the jump size which follows a normal distribution of the form $k \sim N(\mu_J, \sigma_J^2)$.

[18] β_3 is equivalent to γ in (4.1)

Therefore, adding jumps to the generic model in (5.1) improves its ability to capture movements in markets. Sorwar (2011) demonstrates that the behaviour of interest rates processes are better explained with the addition of jumps to the diffusion processes. He examined the performance of the linear and non- linear one factor CKLS model in the presence of jumps. The results support the presence of jumps as improving the linear CKLS model. Xue *et al.* (2011) apply the fractional jump-diffusion financial market model under stochastic interest rate. Giesecke and Smelov (2011) develop a method for the simulation of one-dimensional jump-diffusion with drift, volatility, jump intensity and jump size. Their analysis supports jump diffusion models on interest rates. Mancini and Reno (2009) deduce that adding jumps is important in interest rate models as evidenced by Monte Carlo simulations.

Jiang and Yan (2008) also studied a linear-quadratic term structure models with random jumps in the short rate process with a stochastic process jump arrival rate. They conclude that incorporating stochastic jump intensity improves model fit to interest rate and volatility term structure. Attaoui and Six (2008) extend the monetary equilibrium approach of Lioui and Poncet (2004) to a jump-diffusion setting to derive the jump-diffusion dynamics of a nominal short interest rate. Mora and Valdez (2007) examined jumps in a continuous-time short-term interest rate model for Mexico. They estimated jumps times and sizes in the time series and concluded that jumps in the diffusion model represents a better alternative than in a model without jumps.

Exposition carried out by Espinosa and Vives (2005) present a generalisation of the Levy–Merton jump-diffusion model, allowing discrete stochastic volatility, applying a quadratic variation to estimate jump sizes and jump times. They conclude that the model is more relevant for short term interest rate. Chiarella and Sklibosios (2005) considered term structure models with a parameterisation of the Shirakawa (1991) representation for jump-diffusion. In their study they considered the forward rate volatility structures that incorporate state dependent Wiener volatility functions and time dependent Poisson volatility functions. They obtained a class of jump-diffusion term structure models. Das (2002) adds jumps to the Vasicek (1977) model and finds evidence supporting jumps in the daily federal funds rate. Ahn and Thompson (1988) in their investigation of the term structure of interest rates when the underlying state variables and production technology follow jump diffusion processes, find evidence supporting jump-diffusion models.

With so many applications of Jump Diffusion models for interest rates, their application to South Africa has been limited. Maitland (2013) notes the insufficient studies in South Africa on stochastic models. A study by Nomoyi (2011) considers parameter estimation of a stochastic volatility model by Heston (1993) with implications for option pricing. The study estimates the Heston (1993) model through use of a Markov Chain Monte Carlo (MCMC) approach. In another related study on intrest rate volatility in South Africa, Fadiran and Ezeoha (2012) considered the error-correction model (ECM) together with the adjusted ECM-exponential generalised autoregressive conditional heteroscedasticity (ECM-EGARCH) (1,1)-M. They deduced that negative volatility impact and leverage effect are present in the symmetric deposit interest rate adjustment process.

Hassan and Aling (2012) used Gaussian estimation methods to obtain parameter estimates for non-linear representation for South Africa's short-term interest rates. The results suggested diffusions when interest rate volatility was moderately sensitive to the level of interest rate, particularly after the adoption of inflation targeting. The remaining visible studies on volatility of the interest rate do not cover much ground (see Fadiran and Ezeoha, (2012)), Kotze and Joseph, (2009)), Kahn and Farrel, (2002). In this chapter we seek to fill this gap by considering as a linear representation of the short-term interest rates and introduce a jump diffusion model as an alternative to understanding the short-term interest rate model. We complement this with a GARCH representation. This goes a long way in understanding mitigation of interest rate risk.

5.3 Methodology

In our specification however, we adopt a different approach from Sorwar (2011). We add jumps in a variation of the CKLS model with linear drift and assume jumps are normally distributed and that the average jump size is not zero. The data includes major periods in the South African economy, especially inflation targeting post year 2002. We estimate the unobserved jumps and jump sizes to determine the role of jumps in interest rate models. We estimate the parameters of the process from the log likelihood function. We analyse the ability of the model to capture jumps size and intensity and also draw lessons on the sufficiency of the model to trace higher moments from returns data.

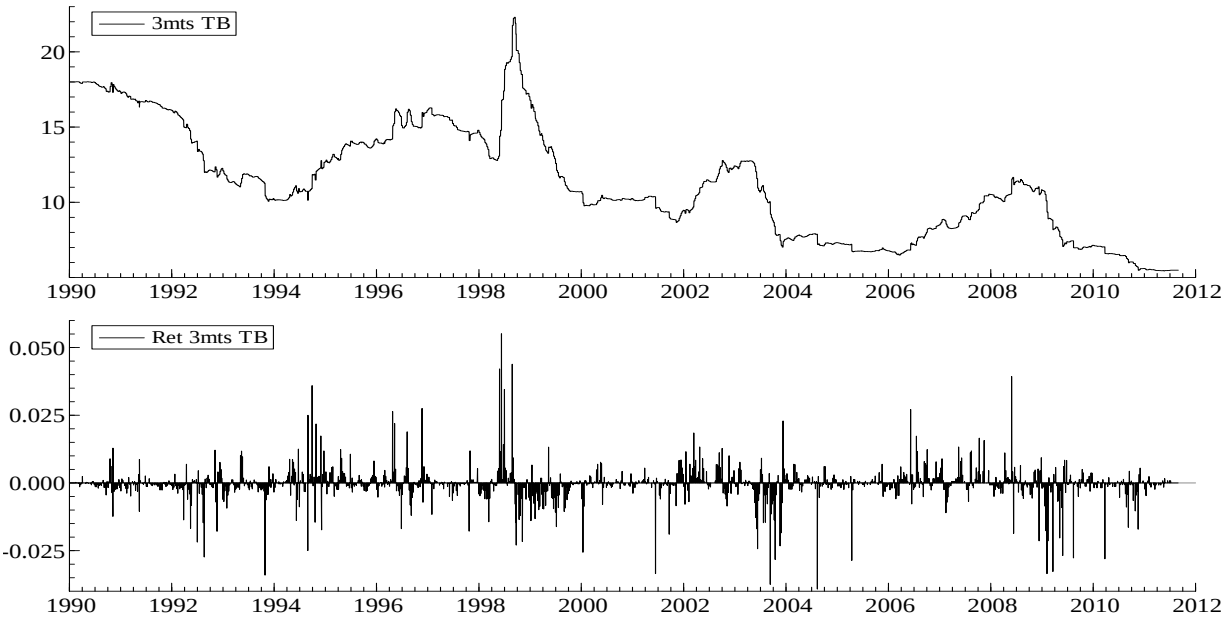
In our approach we add Poisson jumps to the stochastic process in equation (4.1) to obtain a process for short term interest rates. We specify the process as;

(4.14) $dr = (\alpha + \beta r)dt + \sigma r^{\gamma} dZ + kdq$, which is a variant of (4.13), allowing for a linear drift[19].

5.4 Data and sample statistics

The data we use is the 90 day T-Bill rate from January 1990 to August 2011, for South Africa. We also split the period into 1990-2002 and 20032011 to analyse the size and intensity of jumps before inflation targeting and post inflation targeting. A number of researchers support the use of the three month T-bill due to the advantages of liquidity, small bid ask spreads and also that it is free from peculiar effects which can lead to potential sources of non-normality (Sorwar, 2011). Even for South Africa, there is support for use of the three-month rate as a proxy for the short rate model (Jones, (2010)). The study period yields 6784 sample observations for the 3 months T-Bill rate. The figures below show the time series plots for the data. [20]

Figure 5.7: 3 months T-Bill January 1990-August 2011



[19] The methodology for the jump diffusion model and the GARCH model is detailed in chapter three
[20] The normal fit plots are shown in the appendix

From the figures above the 90 days T-Bill show traces of volatility clustering and potential jumps. This is more prominent for the returns graph for the periods 1998, 2004 and 2008/9. The accompanying normal fit plots for returns data are shown in appendix C portraying peak, skew and fat tail. The summary statistics in the table below reveal more on the volatile nature of the data in the graphs

Table 5.14: Interest rates summary statistics raw data

	1990-2011	**1990-2002**	**2003-2011**
	90DT-Bill	90DT-Bill	90DT-Bill
Mean	11.31	13.43	8.11
Max	22.3	22.3	12.76
Min	5.44	8.66	5.44
Variance	13.11	8.09	3.62
Skewness	0.35	0.35	78
Kurtosis	2.26	2.2	2.7

Table 5.15: Interest rates summary statistics returns data

	1990-2011	**1990-2002**	**2003-2011**
	90DT-Bill	90DT-Bill	90DT-Bill
Mean	-7.60E-05	-4.00E-05	-1.30E-04
Variance	8.20E-06	8.10E-06	8.40E-06
Skewness	0.202	3.2	-4.078
Kurtosis	94.6	104.4	80.5

The tables reveal interesting statistics for the data. For the raw data the mean are high for the period 1990-2002 compared to other periods. The analysis is the same for the maximum and minimum values, suggesting overall high interest rates before the advent of inflation targeting. At the same time however it should be noted that the period covers turbulences in the global economy like the Asian crisis and the global financial crisis of 2009. Other studies have verified instances of high volatility in the South African economy in periods of financial crises (see for example Duncan and Kabundi, (2013), Duncan and Liu (2009)) and also

considering that the maximum values are identified with a period with some questionable policies in the years 1996 and 1998 (Aron and Muellbauer, 2005).

The variance is highest for the full period and is least post inflation targeting. This can partly suggest that the phenomenal global financial market occurrences filter into interest rates space. The period of high volatility is also associated with the pre-1994 political instability. Perhaps the high volatility observed over the full period confirm some studies which deduced that South Africa is a small open economy prone to shocks (Steinbach *et al,* 2009). Volatility has been seen featuring prominently with emerging market economies (see for example Bae *et al*, (2003)

So research studies have identified gains attributed to inflationn targeting in 2000 (for example Aron and Muellbauer, (2009). Volatility for the raw data has decreased post inflation targeting. For the returns data in table 5.15 above, the distributions are leptokurtic for all the three different periods. The period 1990-2002 has a highest kurtosis of 104.4 suggesting more volatility clustering and potential jumps compared to other periods. However, both the full sample period and post inflation targeting also portray volatility clustering and potential jumps with kurtosis values of 94.6 and 80.5. The returns data statistics do confirm the non-normality observations in figure 5.7. The estimation results will compare the parameter sizes pre and post inflation targeting.

5.5 Estimation results

Below we present estimation results, starting with stationarity results, Jump diffusion parameter estimates and GARCH (1,1) parameter estimates

Table 5:16: Stationarity results interest rates

Variables	**Dickey-Fuller**		**Augmented Dickey-Fuller**		**Stat. status**
	Constant	**Constant & trend**	**Constant**	**Constant & trend**	
Results for unit root tests on levels interest rates					
L90DT-Bill	0.8076	-1.9169	-0.8490	-1.8855	I (1)
Results for unit roots tests on returns					
DL90T-Bill	-26.397***	-26.397***	-26.395***	-26.393***	I(0)

*** ** * represents stationary series at 1, 5 and 10% level of significance.

From the stationarity results above, data are stationary in returns at 1% level of significance.

Table 5.17 : Interest rates parameter estimates for the jump diffusion model

	1990-2011	**1990-2002**	**2003-2011**
	90DT-Bill	90DT-Bill	90DT-Bill
μ_B	7e-04 (0.0000)	4e-04 (0.0000)	2e-04 (0.0000)
σ_B	2.8e-03 (0.0000)	2.8e-03 (0.0000)	2.8e-03 (0.0005)
μ_J	6e-04 (0.0000)	7.e-04 (0.0000)	8.1e-03 (0.0000)
σ_J	1e-04 (0.0000)	4e-04 (0.0000)	2e-04 (0.0000)
λ	1.9e-03 (0.0007)	4.2e-03 (0.0000)	1.5e-03 (0.0005)
Skewness	2.03e-04	6.49e-06	3.57e-02
Kurtosis	3	3	3.103
Mean	7.11e-04	4e-04	2.12e-04
Variance	7.85e-06	7.84e-06	7.94e-06

The table presents results for the jump diffusion specification. The jump diffusion model is able to fit data better as it generates the higher moments[21]. Using the likelihood ratio test as a diagnostic, we reject the null hypothesis of no jumps at 1% level. Jumps therefore help explain some features in the data set. Not only do we draw jump sizes and intensity from the interest rate but we also compare from the two specifications as to which one is seemingly prone to jumps and volatility clustering.

[21] Unlike the Geometric Brownian motion, the jump diffusion model does not specify that returns are independent and identically distributed (see Ramezani and Zeng, (2007)) Richards *et al.*, (2002), Craine *et al.*, (2000) for more discusion)

1990-2011. Whole sample; the three month T-bill rate is used in this study. This allows us to generate jump diffusion model parameters and determine whether the model is able to trace the characteristics of interest rate returns data. Table 5.17 presents the parameter estimates for the period 1990-2011, 1990-2002 and 2003-2011, capturing phases in the monetary policy conduct of the central bank, with the prominent one being inflation targeting. The parameter estimates apply to daily changes in the interest rates. The drift is at 7e-04. The unconditional mean implied by the jump diffusion process lies at 7.11e-04 whereas the jump mean lies at 6e-04 and the jump intensity at 1.9e-03. All the parameters are significant.

The variance of the jump diffusion specification almost meets the sample moments for both rates at 96%. For skewness and kurtosis, the levels have been brought to normal by the inclusion of jumps, this we see in values for skewness at 2e-04 and 3 for kurtosis.

1990-2002. In this period the jump size has slightly increased from 6e-04 to 7e-04. Similarly, the intensity has also increased from 1.9e-03 to 4.2e-03. The jump diffusion specification generates 97% of the sample volatility. Compared to all periods, the unconditional mean of the jump diffusion is second highest at 4e-04. All parameter values are significant.

2003-2011. In this last period we see the highest jump size at 8.1e-03 perhaps indicating more jumps in the inflation targeting period, though with lower intensity. The jump probability has slightly declined to 1.5e-03. The model also generates 95% sample volatility and the higher moments have been normalised with the introduction of jumps

Table 5.18: Interest rates GARCH results[22]

	GARCH Student's t	GARCH GED
	90T-Bill	90T-Bill
C	9.89e-08 (1.52e-08)	1.3e-06 (4.88e-05)
K	8.72e-15 (6.52e-15)	1.6e-06 (1.48e-07)

[22] Student's t distribution d.o.f fixed at 10 and GED parameter fixed at 1.5.

$GARCH(1)$	0.9789 (5.86e-04)	0.674 (0.03)
$ARCH(1)$	2.55e-05 (1.69e-06)	0.0076 (0.0013)

Table 5.18 above presents the GARCH results for two scenarios; Students' t distribution and GED. The GARCH parameter sizes explain the short run dynamics of the data, in this case the volatility in the treasury-bill rate. In order to have a mean reverting variance process, the coefficients β_1 and α_1 should sum up to a number less than one. For GARCH Student's t, the result is very close to one, indicating a slow mean reversion process (Engle, 2001). Furthermore, the GARCH Student's t suggest that volatility shocks take a longer time to die out. Seemingly, ARCH effects are minimal considering the low coefficient values. The GARCH effect is less pronounced for GARCH GED wherein β_1 is 0.67 compared to 0.98 for GARCH Student's t although the persistence in volatility still holds. The results confirm a related study that T-Bill rate exhibits volatility persistence in South Arica (Strydom and Charteris, 2011)

5.6 Conclusions

Modelling the volatility of interest rates has gathered interest in academia and even from practitioners because of its importance in risk management, interest rate derivatives, option pricing. The chapter examined variation of the CKLS model augmented with jumps. Summary statistics for the returns data revealed the leptokurtic nature and skew associated with interest rates. This was also complimented by the returns graph which showed traces of jumps and volatility clustering. The period with more leptokurtic features seemingly is 1990-2002. Results incorporating the jump diffusion show the jump size having increased post inflation targeting at 8.1e-03. For the full sample period, the jump size is smallest at 6e-04. However, there seems to have been a lesser intensity post inflation targeting.

This interesting result perhaps points to information filtering after inflation targeting, a phenomenon which might have been less pronounced before. It is a safe assertion therefore that jumps help to explain important features of the interest rates as the generated results

fared well in producing sample equivalents, having generated sample volatility of 96%, 95% and 95% respectively for the three different periods analysed. Therefore results suggest that the model can capture unexpected movements in the market. Furthermore, this was proven by the generated levels of the higher moments in skewness and kurtosis. We can also conclude that generally the short term interest rate market suffers many more deviations than does the long term. Jumps can be traced more in the short term than in the long term. To capture the persistence of volatility we included scenarios for the fat GARCH model. Results from the parsimonious fat tailed GARCH(1,1) supported the specification in removing the ARCH effects, though indicating a slow mean reverting process. Findings in this chapter play a crucial role in interest rate forecasting, asset pricing, portfolio analysis and the valuation of short-term interest rate derivatives, especially where the one factor model has a yield curve determining variable. The application can also be extended to multi-factor models embedded with the stochastic behaviour on interest rates.

CHAPTER SIX

JUMP DIFFUSION IN STOCK PRICES

6.1 Introduction

A number of studies have confirmed the presence of jumps in asset prices, with implications for derivative pricing, risk management, and portfolio allocation (see for example, Aït-Sahalia *et al.* (2006), Eraker *et al.* (2003), Liu *et al.* (2003), Andersen *et al.* (2002), Duffie & Pan (2001), Bates (2000). This literature has enriched our understanding of return properties of various financial assets, such as exchange rates, interest rates, and stock markets.

Jumps in stock prices, which are by nature large discontinuous price movements, are as a result of infrequent and large surprises to investors' information set. Furthermore, there have been a number of tests developed to trace the existence of jumps in asset prices. In this category, Aït-Sahalia (2002), exploits the restrictions on the transition density of diffusion processes to assess the likelihood of jumps. A related study by Carr and Wu (2003) makes use of the decay of the time value of options with respect to maturity. Barndoff-Nielsen & Shephard, (2004, 2006b) propose a bi-power variation (BPV) measure to separate the jump variance and diffusive variance based on bi-power variation. Yet still, the volume of empirics for stock prices is not a lot when compared to other asset prices like the exchange rate and interest rate. This translates to insufficient tools for investors when considering acquiring investments across different sectors.

Moreover, studies incorporating South African stock price jumps are rare (for instance Kutu (2012), with the remaining studies on stock prices not considering jump diffusion models at all (for example Kulikova and Taylor, (2010), Bonga-Bonga and Makabule, (2010), Taylor, (2010), Moolman (2005). Evidently, there is a need to fill the gap of insufficient studies on understanding the stock price characteristics in South Africa. Furthermore, since South Africa's stock exchange features in the worlds' top 20 exchanges by market capitalisation (African Securities Exchanges Association, 2014); it compels the need to understanding stock

price properties. This will enhance portfolio diversification for investors and therefore encourage more participation in the stock exchange via economies of scale.

In this chapter we therefore carry out a study of price jumps for individual stocks, with an application to South Africa, making use of selected Johannesburg Stock Exchange (JSE) sector stocks. The results document properties of individual stock price jumps, thus bridging the gap in understanding the stochastic process for stock price movements in South Africa. The results also help us to compare with findings in literature. The significant results suggest that jumps are a feature for stock prices and vary across sectors. Furthermore, results from the study can be used to inform stakeholders of the importance of a diversified portfolio considering the different jump sizes and intensity across sectors. The remainder of the chapter is organised as follows. Section 6.2 presents the literature review. Section 6.3 presents the methodology. Section 6.4 describes data. Results are presented in section 6.5 and section 6.6 concludes.

6.2. Literature review

Volatility in stock markets can imply losses for some and profits for some. It is therefore important that there is a good understanding of stock price movement. This has seen continuous improvement in modelling asset prices. Finance literature has documented the leptokurtic nature of stock returns data with Merton (1976) laying foundations for the inclusion of jumps in asset prices. Since then, researchers have improved on the work of Merton (1976) in their quest to understand asset prices. As a result research studies by Jorion (1988) and Jarrow and Rosenfeld (1984) build the case for evidence of jumps in stock prices.

Another area considered under jump diffusion related with stock prices has been option pricing. In this work Pan (2002), Bates (200), Bakshi *et al.,* (1997) and Naik and Lee (1990) provide evidence that jumps are important for option pricing. Jumps and skewness have been known to be related; with positive jumps associated with positive skewness and negative jumps with negative skewness (Yan, (2008)). Some research findings have associated jumps with policy changes, market phenomena and social events (Lahaye *et al*., (2011), Lee and Na, (2005), Lee *et al.,* (2004)). Literature also documents the importance of detecting jumps in

prices noting consequences for financial risk management and pricing (Arshanapalli *et al.*,(2013), Carr and Wu (2010))

Aït-Sahalia and Jacod (2006) propose a family of statistical tests of jumps using power variations of returns. Works by Lee and Mykland (2008) exploit the properties of BPV and develop a rolling-based nonparametric test of jumps. Evans (2011), Barndoff-Nielsen and Shephard (2004, 2006a, 2006b apply non-parametric settings for detecting jumps. Lee and Mykland (2008) incorporated a Poisson-type jump test. Research findings by Figueiredo *et al.*, (2011), Sorwar (2011), Xue *et al* (2011), Yang *et al.* (2009), Das (2002), and Craine *et al.* (2000)) show the existence of jumps in stock prices. Another class of models has seen the emergence of the Generalized Method of Moments (GMM) carrying out known characteristic functions of affine models where a jump diffusion model is combined with a stochastic volatility model (Singleton (2001), Pan (2002), and Viceira & Chacko, (1999)). Related works include the method of simulated moments (Duffie & Singleton, (1993), indirect inference methods (Gourieroux *et al.* 1993) and the efficient method of moments (EMM), Gallant & Tauchen, (1996) amongst others.

The simulation-based methods offer several advantages. For instance, Andersen *et al.* (2002) use EMM to estimate jump diffusion models from equity returns. From the work for general state space models in Carlin *et al,*. (1992), Jacquier *et al,*. (1994) developed a method for estimating discrete time stochastic volatility models from returns data. From their study they deduced that the Markov Chain Monte Carlo (MCMC) is particularly well suited to deal with stochastic volatility models.

Further works into multivariate models can be found in Jacquier *et al*., (2004). Eraker (2001) proposes a general approach to estimating diffusion models with arbitrary drift and diffusion functions and possibly latent, unobserved state variables (for example stochastic volatility). Other studies on MCMC-based estimation of jump diffusion models on returns data include works by Eraker *et al,* (2003). Consideration has also been taken into account to determine the number of jumps and their sizes. In this work Andersen *et al* (2007) use an algorithm based on Barndorff-Nielsen and Shephard's (2004, 2006a, 2006b) work in extracting the jumps per day.

In a related application, Eraker, *et al.* (2003), Pan (2002) and Bates (2000) carried out studies deducing less frequent but large jumps in stock prices. Tauchen and Zhou (2006) in their

study applied daily measures of quadratic and bipolar variations in testing the number of jumps in a given day. The culmination of their study was in obtaining the jump intensity and size wherein they deduced at most one jump in a day with the price movement attributed to a jump. It is therefore beyond doubt that diffusion processes have been employed for stock market data[23]. This however contrasts the South African scenario (for example Kulikova and Taylor, (2010), Bonga-Bonga and Makabule, (2010). This chapter contributes to building empirical evidence of incorporating jump diffusion model for stock prices thereby offering a better method for understanding stock price movements in South Africa.

6.3 Methodology

Merton's (1976) work adds jumps to the GBM process in approximating the movement of stock prices. The continuous time model with instantaneous return thus becomes:

$$\frac{dS(t)}{S(t)} = \mu_B dt + \sigma_B dW(t) + k(t)dq(t) \qquad (t \geq 0) \tag{6.1}$$

μ_B is the drift and σ_B is the volatility. $W(t)$ is standard Brownian motion and $dq(t)$ is a Poisson counter with intensity λ [24].

6.4 Data

The data we used is the daily closing share price for indices over the period January 2008 to October 2011[25] on the JSE of South Africa. The selected sectors are banking, mining, media and travel and leisure. For each sector, a company is chosen for analysis. Based on data availability for the chosen companies, the sample size for media is 891 and 953 for each of the remaining three sectors. Omitting weekends and holidays has no impact on the data used (Jones, 2003). The data have typical features of high frequency financial time series data; skewness, kurtosis. The diagrams below show the characteristics of each sector.

[23] See Ait sahalia (2002) Duffie and Glynn (2004) ,Hansen & Scheinkman, (1995) and Conley *et al.* (1997) (moment based methods), and Ait-Sahalia (1996), Jiang and Knight (1997), Stanton (1997) and Bandi and Phillips (2002) (non- parametric methods)

[24] Methodology for jump diffusion model and GARCH model specified in chapter three

[25] Consistent daily data for the sectors gave us a sufficient sample to apply a jump diffusion process. Very long consistent daily data was not available at the time of estimation.

Figure 6.8: Daily closing prices and returns for banking sector index 2008-2011

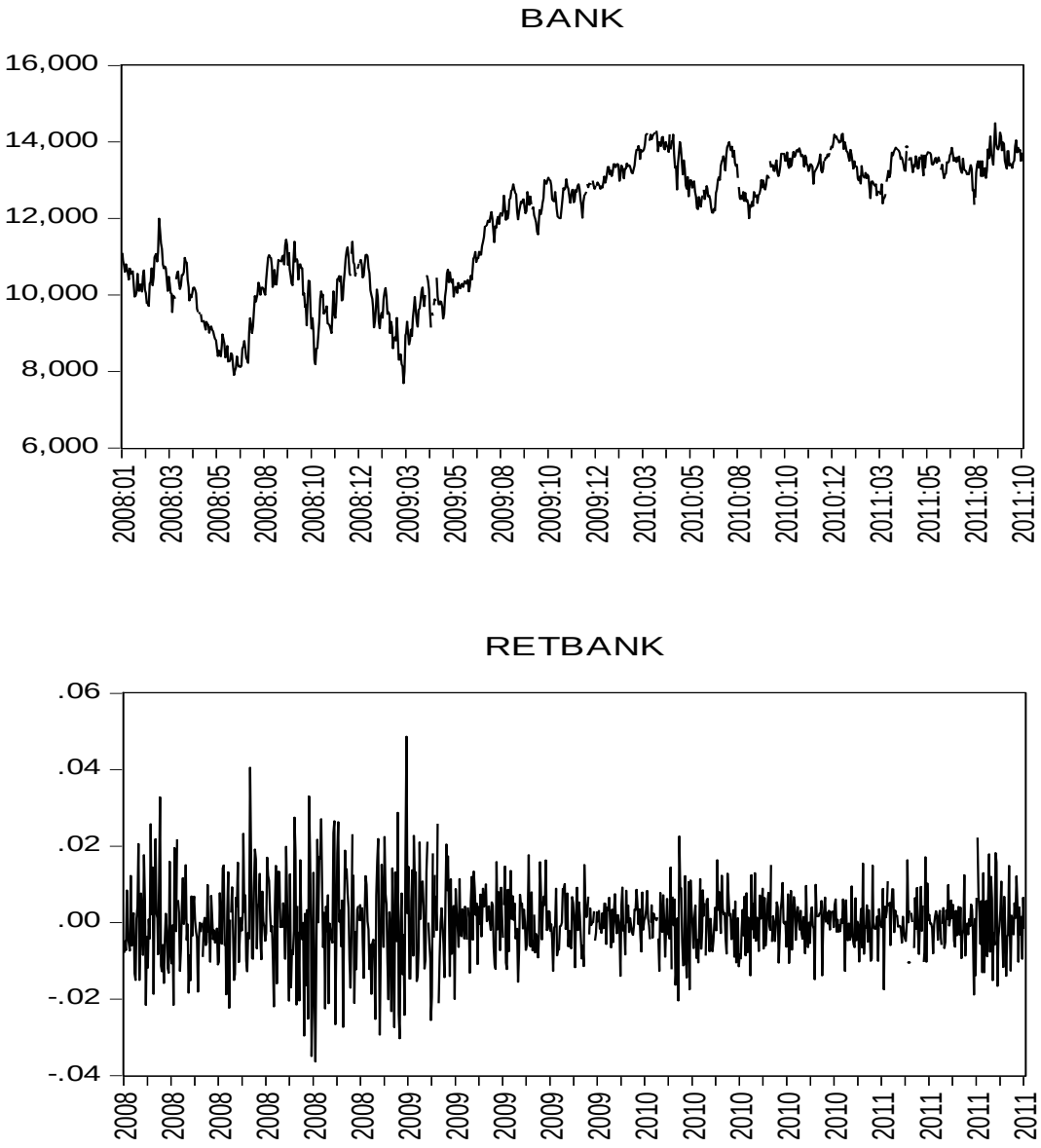


Figure 6. 9: Daily closing prices and returns for leisure sector index 2008-2011

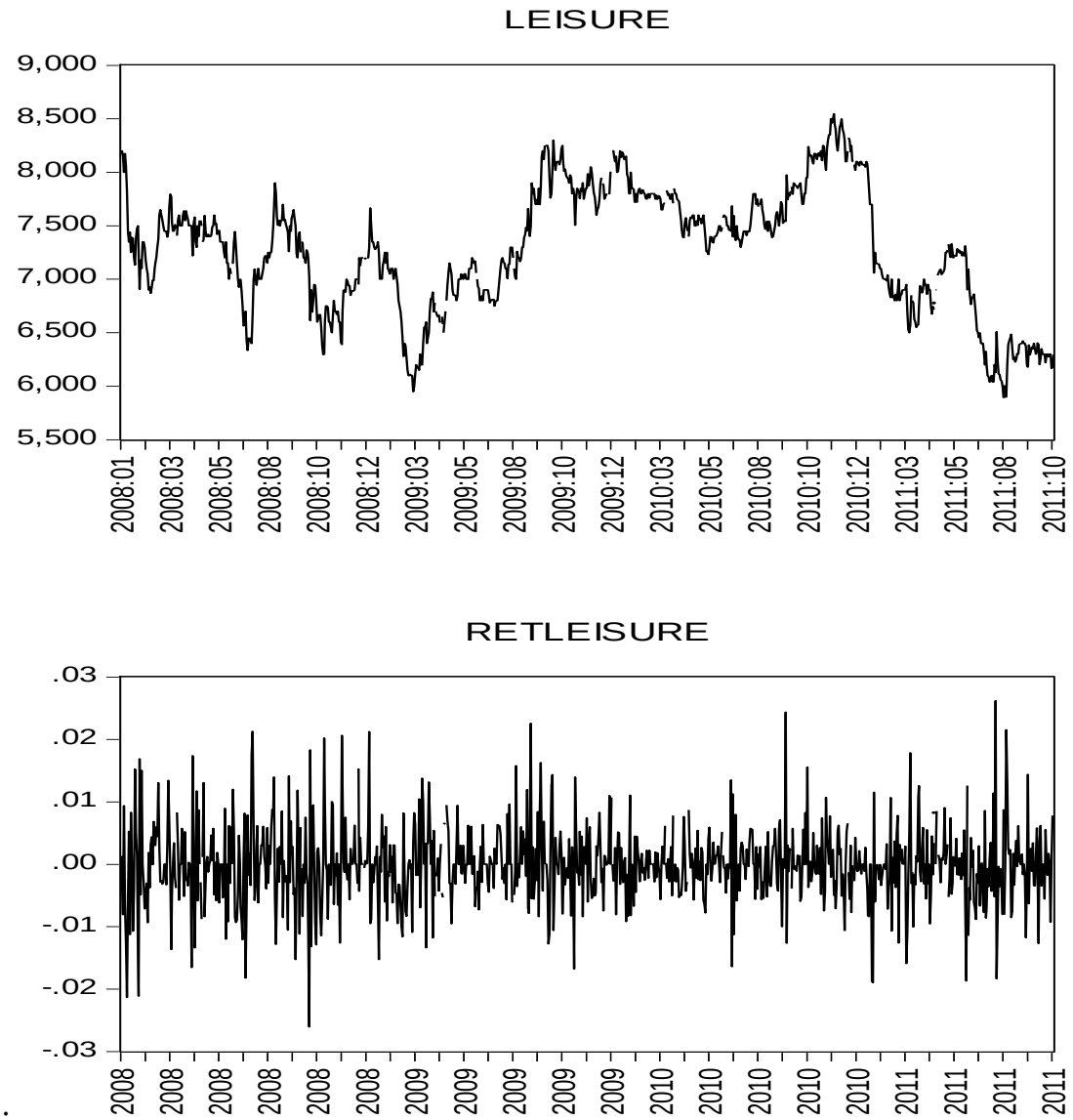


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Figure 6.10: Daily closing prices and returns for media sector index 2008-2011

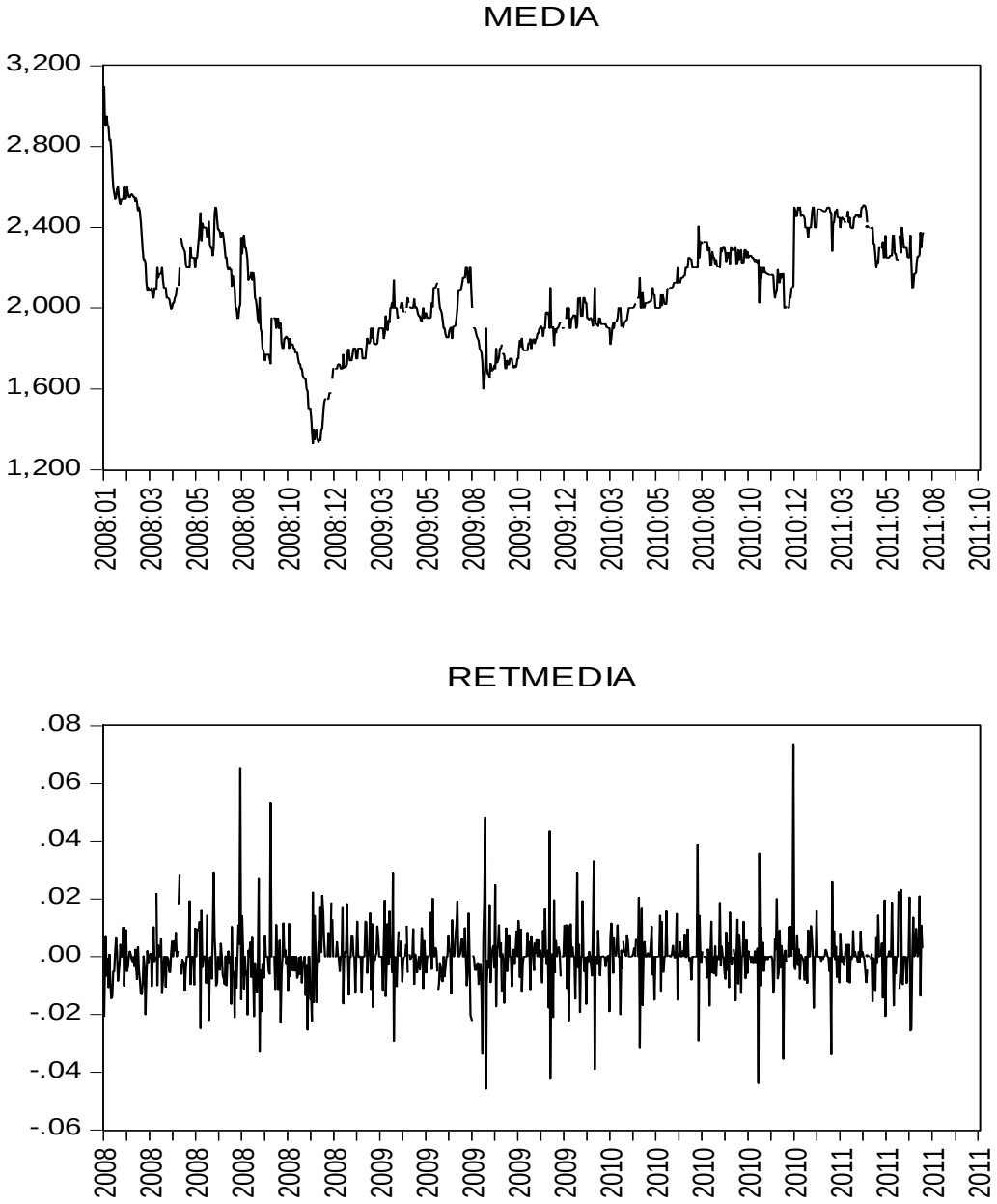
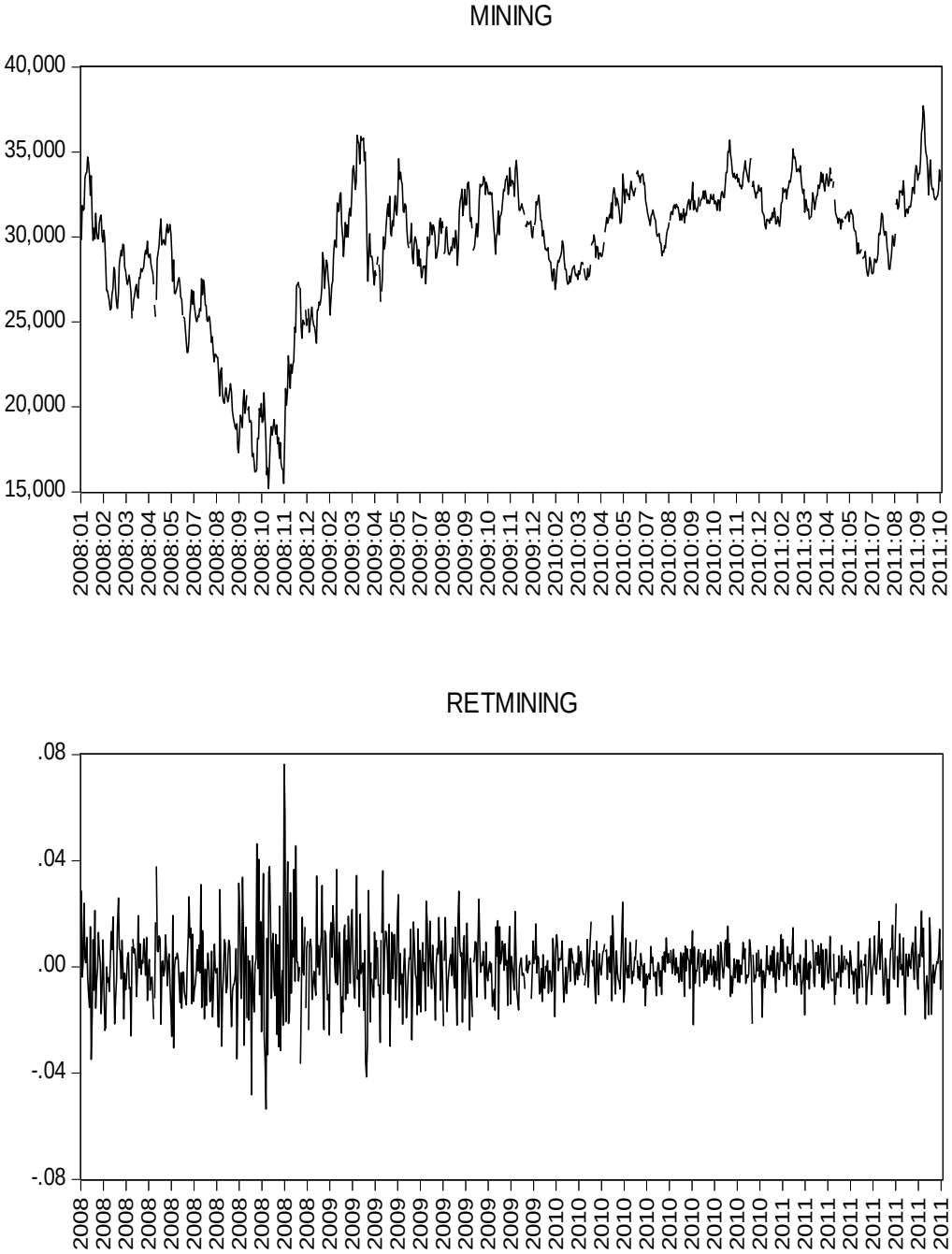


Figure 6.11: Daily closing prices and returns for mining sector index 2008-2011



From the figures above, the returns data for all sector stocks are typical of characteristics seen in most stock price data; skewness, kurtosis, potential jumps and volatility clustering. The accompanying logarithmic returns of stock price data reveal volatility clustering effects[26].

[26] Empirical density functions as shown in the appendices confirm that stock returns data do not follow a normal distribution

We further explore the data by looking at the summary statistics as shown in table 5.19 below.

Table 6.19: Summary statistics daily closing prices

	Mean	Max	Min	Variance	Std dev	Skewness	Kurtosis
Bank	11865.38	14494	7690	3013899.76	1736.0587	-0.4969	1.9049
Mining	29436.56	37714	15200	16706586	4087.369	-1.3264	4.5844
Media	2085.26	3100	1330	73511.89	271.1308	0.0896	3.0805
Leisure	7273.76	8547	5890	327360.5	572.1543	-0.2086	2.386

Table 6.20: Stock prices summary statistics returns

	Mean	Variance	Std dev	Skewness	Kurtosis
Bank	9.50E-05	9.30E-05	9.70E-03	0.144	4.79
Mining	5.30E-05	1.48E-04	1.20E-02	0.372	6.62
Media	-1.30E-04	1.05E-04	1.00E-02	0.759	11.13
Leisure	-1.20E-04	3.80E-05	6.20E-03	0.247	5.05

The maximum closing value for the mining sector index is at 37 714 with the minimum sector index at 3100 for media. In between we have banking and leisure at 14 494 and 8547 respectively. The mean values follow this order as well. It comes as no surprise that the highest variability is in mining where domestic labour developments in the sector continue to undermine productivity and hence profitability.

Mining is one of South Africa's most important sectors crucial for job creation and GDP growth. The companies in the sector are some of the most successful in the world. South Africa is the biggest producer of platinum in the world and one of the leading producers of gold, diamonds, base metals and coal. The major mineral categories in this sector are; precious metals and minerals, energy minerals, non-ferrous metals and minerals, ferrous

minerals, and industrial minerals. However, the sector has not been spared from volatility over the last few years, especially currency volatility.

Other factors which have worked against the sector include; falling gold prices, escalating input costs, strike action, taxation and royalty contributions. From the table we can deduce the leptokurtic nature of the mining sector with a kurtosis of 4.6 and skewness of -1.3. The returns data for mining show a higher kurtosis value of 6.62.

The banking system in South Africa compares well with many industrialised countries, this can be seen in the past decades' transformation through consolidation, technology and legislation. The sector volatility in the early 1990s paved the way for consolidation through the mergers of several banks. Despite volatility in the past, South Africa's banking sector remains solid and well regulated. Evidence of this is the number of foreign banks establishing branches or representative offices in the country and others acquiring stakes in major banks, for example the Industrial and Commercial Bank of China – Standard Bank and Barclays – ABSA deals. Furthermore, technology, products and the number of participants have changed the sector and injected high levels of competition. This we also see in smaller banks such as Capitec Bank and African Bank, which target the low-income and previously unbanked market.

The World Economic Forum Competitive Survey 2012/13 rated South African banks 2nd out of 144 countries for soundness, whilst occupying 3rd for financial sector development (Banking Association of South Africa, 2013). The outlook for South Africa's banking system is stable. Some of the pointers to this stability are attributed to low interest rates and sustainable inflation levels as a result of the monetary authorities' pursuit of inflation targeting. This in turn has led to a decline in borrowers' debt-servicing costs whilst consumer debt affordability has increased. These improved operating conditions augur well on the banking sector's performance. The current exchange control and restrictions provide some harbouring on financial market volatility. The banking sector data from the table is platykurtic with a significant negative skew. Returns data reveal volatility clustering leptokurtic distribution as shown in the graphs and accompanying statistics where kurtosis is at 4.8

Tourism is one of the fastest growing sectors in the South African economy. The sector contributes significantly to employment. The country has had continuing global exposure, with the FIFA World Cup in 2010 and also becoming a member of the five major emerging economies; Brazil, Russia, India, China, South Africa group (also known as BRICS). There have also been upgrades to transport and accommodation infrastructure. The mean closing price for Leisure is 7273 from the data used in the study period. The skewness is -0.2 and the raw data is platykurtic as presented by a kurtosis of 2.4. However, the returns data confirms the leptokurtic nature with a kurtosis of 5.

There is a vibrant media sector in South Africa. The sector is a central core in a network of industries and individuals in organisations involved in paper manufacturing, educational institutions, ink producers, authors, printers, designers, bookbinders, illustrators, booksellers, distributors and CD manufacturers. Undoubtedly, it contributes immensely to revenue and employment in the country. The mean closing price is 2085 with a skewness closest to zero when compared with other sectors. The kurtosis at 3.08 suggests the data is closest to normal distribution based on raw data when compared with other sectors in the table. The returns data for media however shows the highest kurtosis of 11, confirming the character trait of high frequency financial data.

6.5 Estimation results

Below we present the estimation results from the models. We begin with stationarity results, followed by jump diffusion parameter estimates and finally GARCH (1,1) results.

Table 6.21: Stationarity results stock prices

Variables	Dickey-Fuller		Augmented Dickey-Fuller		Stat. status
	Constant	Constant & trend	Constant	Constant & trend	
Results for unit roots tests on levels Interest Rates					
LBanking	-1.2266	-2.1179	-1.5043	-3.0711	I (1)
LMining	-1.8997*	-1.9914	-1.8703	-2.5498	I (1)
LMedia	-3.4115	-0.9578	-2.9898**	-2.9613	I(1)
LLeisure	-1.0601	-2.2567	-2.6772*	-2.6922	I(1)
Results for unit roots tests on returns					

DLBanking	-3.7644***	-6.3384***	-17.577***	-17.568***	I(0)
DLMining	-1.3755	-2.9769**	-21.75***	-21.741***	I(0)
DLMedia	-2.5081**	-5.0065***	-22.149***	-22.137***	I(0)
DLLeisure	-7.427***	-12.638***	-24.656***	-24.643***	I(0)

*** ** * represents stationary series at 1, 5 and 10% levels of significance.

Table 6.22: Stock prices jump diffusion parameter estimates

The parameter results below are generated from the return series

	Banking	Mining	Media	Leisure
μ_B	1e-04 (0.0000)	1e-04 (0.0000)	2e-04 (0.0000)	2e-04 (0.000)
σ_B	9.65e-03 (0.0000)	1.1e-02 (0.0000)	9.95e-03 (0.0000)	6.1e-03 (0.0000)
μ_J	1e-04 (0.0000)	6.66e-03 (0.0000)	8e-04 (0.0000)	8e-04 (0.0000)
σ_J	0.0025 (0.0000)	5e-04 (0.000)	2.1e-03 (0.0000)	0.5.8e-03 (0.000)
λ	1.91e-02 (0.0000)	9e-03 (0.0002)	3.7e-03 (0.0000)	1.8e-03 (0.0000)
Skewness	3.92E-05	1.97E-03	4.78e-05	6.12e-04
Kurtosis	3	3.001	3	3.004
Mean	1e-04	1e-04	2.03e-04	2.01e-04
Variance	9.32E-05	1.2e-04	9.03e-05	3.73e-05

Matching the moments in the jump diffusion model

Table 6.22 above presents the parameter estimates of a Jump Diffusion model. As a diagnostic, if we use a likelihood ratio test, it rejects the null hypothesis of no jumps at 1% level. Jumps therefore help explain some features in the data set.

For the Banking sector, the GBM drift is less than the unconditional mean of the jump diffusion specification. The jump size is least among the sectors at 1e-04 and significant. The jump intensity is highest at 1.91e-02. The volatility generated almost matches the sample. Introduction of jumps seems to have normalised the skewness and kurtosis.

For the Mining sector, the unconditional mean from the jump diffusion specification is 1e-04. The mean of the jump size is highest across sectors at 6.7e-03 and significant. The jump intensity is 9e-03 and also significant. Volatility for the jump diffusion generated 91% sample volatility.

The media sector's GBM drift is 2e-04 and almost matches the jump diffusion specification at 2.03e-04. The jump mean is second highest across sectors at 8e-04 with an intensity of 3.7e-03. The jump diffusion process generated 95% of sample volatility.

The model also performed fairly well in estimating parameters for the leisure sector. The generated unconditional mean is 2e-04 and is close to the sample moment. Also closely matching the sample equivalent is the generated volatility at 98%. The jump size is at 8e-04. The levels of skewness and kurtosis have been normalised by the inclusion of jumps.

In carrying out further simulations and allowing the jump size to increase, it was discovered that the kurtosis levels increased for all stocks. The jump intensity had the same effect as well on kurtosis. Related studies, for example, Andersen *et al.* (2002), and Chernov *et al.* (2003), find the mean jump size of the stock index to be negative whilst some found it positive (for example Campbell *et al,.* (1997)). Skewness can be a result of the presence of jumps, stochastic volatility and nonlinear drift. Findings by Andersen *et al,* (2002) suggest that jumps are important in explaining the negative skewness in market returns. From our findings, we can confirm the interdependence between kurtosis, skewness and jumps in stock prices.

Table 6.23: Stock prices GARCH results[27]

	GARCH Student's t				GARCH GED			
	Banking	Mining	Media	Leisure	Bank	Mining	Media	Leisure
C	0.00018	-1.61E-05	-0.00018	-0.00017	0.00015	-2.96E-05	-2.2E-05	-9.29E-07
	(0.00024)	(0.00029)	(0.00028)	(0.00016)	(0.00024)	(0.00028)	(0.0003)	(0.00015)
K	9.28E-07	4.84E-07	5.56E-05	1.44E-05	9.84E-07	5.1E-07	6.85E-05	1.29E-05
	(4.85E-07)	(3.19E-07)	(5.9E-06)	(3.99E-06)	(5.21E-07)	(3.36E-07)	(5.86E-06)	(3.49E-06)
$GARCH(1)$	0.9103	0.9532	0.0311	0.3829	0.9102	0.9511	-0.028	0.3842
	(0.0205)	(0.012)	(0.0737)	(0.1071)	(0.0220)	(0.0131)	(0.068)	(0.1187)
$ARCH(1)$	0.0805	0.042	0.2026	0.3449	0.0795	0.0437	0.1952	0.2904
	(0.0188)	(0.0117)	(0.0374)	(0.0978)	(0.0197)	(0.0127)	(0.0376)	(0.0796)

[27] Student's t distribution d.o.f fixed at 10 and GED parameter fixed at 1.5 for media

The parameter sizes from the table above determine the short run dynamics of the volatility for the selected JSE data. Results were produced from maximum likelihood with iterations ranging from between 11 and 19 for the fat tailed distributions. Large values of α_1 (represented by ARCH(1)) imply an intense volatility as a result of market movements whereas large values of β_1 (represented by GARCH(1)) imply that the shocks to conditional variance take a long time to die out.

Most of the values for β_1 and α_1 are non- negative and with their sum less than 1, indicating stationarity. From our results, for both Student's t and GED β_1 is high especially for banking and mining sector companies with the corresponding sum of $\beta_1 + \alpha_1$ capturing the persistence in volatility as it lies close to 1. For Banking, β_1 is 0.91 and α_1 is 0.08 for the Student's t distribution. For mining, β_1 is 0.95 and α_1 is 0.04.

It comes as no surprise that the persistence in volatility form mining is higher than for banking (0.995 > 0.990), considering movement of commodity prices and salient challenges in the mining sector in South Africa. For leisure, the persistence in volatility is considered sufficient at 0.73 (Depken II, (2001) though lower than for mining and banking. Seemingly there are no GARCH effects for media. However, the jump diffusion model for media produced significant results, complimenting the statistics of a high frequency data; leptokurtic and skewed.

6.6 Conclusions

There is strong evidence that stock prices contain jumps since returns data on stock prices series tend to show far too many outliers for a simple, constant-variance lognormal distribution to be valid. This was shown in the graphical presentation of returns data which portrayed potential jumps and also complimented by the statistics of returns data wherein for all sectors the data proved to be of a leptokurtic nature with skew. The high frequency data employed in many studies reveals these characteristics (see Figueiredo *et al.,* (2011), Evans (2011), Lee and Mykland (2008)). Simple graphs of price evolutions reveal the presence of changes that reveal traits different from purely diffusive processes. Over the past few

decades, the difficulty in the estimation of jump diffusion models has limited their use in empirical studies.

Using a database for the sectors in the JSE we reject the assumption of purely diffusive processes given the obtained average jump size and intensities. The jump size is highest for the mining sector, to some extent confirming the volatility in South Africa's mining sector, especially over the last few years. The major commodities hardest hit have been gold and platinum as a result of weakness in the eurozone and weak demand owing to the lacklustre growth in the global economy. Banking has the highest intensity, perhaps pointing to the sector's importance in the economy as it promotes economic growth via capital accumulation. It then follows that a fall in the banking sector can have catastrophic consequences as the recent global financial crisis demonstrated. Therefore identifying jump sizes and intensity is important as it can be used for understanding market efficiency and also allowing the financial regulators to consider optimal policies (Hanousek *et al.,* 2013, Tiniç, (1995)). Results from this chapter therefore contribute immensely in this regard for South Africa

Many researchers have noticed the heavy tails in stock prices as outlined in the literature. One major reason for the fat tails is the heteroskedasticity in the returns data. To capture these we employed GARCH models. Overall, GARCH(1,1) models take into account the autocorrelation in the volatility in returns and as such low order GARCH(p,q) models are preferred to high order ARCH(p) for reasons of parsimony and numerical stability in estimations. Results from the GARCH models confirmed the persistence in volatility for stock price returns data and the ARCH effects were absent for the Student's t and GED estimations for the banking and mining sectors. However, for the media and leisure sectors GARCH(1,1) results did not capture the persistence in volatility clustering. An important implication is that investors can reap optimal returns when they diversify their portfolios considering the different jumps and intensity across sectors. We also note that confirming findings in similar research work that stock prices are difficult to predict, more so in the short-run. A future study can look at the link between jumps and corporate disclosure and as well as the specific number of jump intensities across different frequencies.

CHAPTER SEVEN

VOLATILITY AND JUMPS IN HOUSE PRICES

7. Introduction

Investment in real estate is an important source of wealth for pension funds, investment banks and even for individuals. As such, understanding house price movements is crucial for the design of housing market portfolio. It is therefore important to understand the volatility of returns on house prices so that investors and individuals alike are enabled to make better decisions in managing their wealth. The house price volatility parameter is an important component for mortgage insurance therefore capturing jumps in the price change is crucial in the determination of mortgage insurance premiums.

Studies have been undertaken to understand house prices. LaCour-Little *et al.,* (2002), and Crawford and Rosenblatt, (1995) found that house prices affect mortgage default and prepayment. Other research work focused on the patterns of default and prepayment as explained by economic risk factors, like the interest rate and unemployment rate (Caselli *et al.* 2008, Lambrecht *et al.* 2003, Deng *et al.*, 2000 and Ncube and Satchell, 1992). Some studies have considered the methods which best represent house price characteristics. In this work Mizrach, (2008) and Chen *et al* (2010) deduced jumps in house prices.

For the South African context on studies relating to house price characteristics, to the best of this researcher's knowledge studies applying a jump diffusion model have not been explored (see for example Clark and Daniel, (2006), Els and von Fintel, (2008)). The implication thereof is that property analysts and investors alike have limited tools to make an in-depth assessment of the property sector. Even as Clark and Daniel (2006) admit, there is limitation of studies on the South African residential property market.

The contribution of this chapter is to enrich understanding of the property sector. This we do by first reviewing work done in characterising house prices. We also run a diffusion model to capture the behaviour of house prices in South Africa. The results show significant jump

sizes and intensity across different segments, with All price and Large price segments having slight higher jump sizes though the Small price segment has the highest jump intensity. The results have implications for insurance premiums determination. The rest of the chapter is organised as follows; Section 7.1 presents the housing market factors with a bearing for house prices. Section 7.2 gives an analysis of the literature. Section 7.3 details the methodology, Section 7.4 presents the data and the results from the estimates are analysed in section 7.5. Section 7.6 concludes.

7.1 Housing market factors

The section highlights some prominent factors relating to the house price movements, thereby contributing to volatility in the housing market. These factors incorporate affordability, building costs, interest rates, consumer confidence and even geographical location.

7.1.1 Affordability of housing

According to the ABSA Housing review (2012), housing affordability as indicated in the ratios of house prices and mortgage repayments to household disposable income, continued to improve up to the end of 2011. This was due to trends in house price and income growth in the previous quarter, while interest rates were still unchanged at year-end. However, many households' ability to take advantage of these affordability trends continued to be negatively affected by factors such as; an average debt-to-income ratio hovering above 70%, a significant percentage of credit-active consumers having impaired credit records; the impact of the National Credit Act (NCA); and banks' resultant lending criteria. As a result, a downward trend in the debt-to-income ratio implies that house prices and mortgage repayments are rising at a slower pace than household disposable income. This has impacted on the affordability of housing.

7.1.2 House price trends

The first quarter of 2012 saw a deflation trend in house prices, both at nominal and real terms in the middle segment of the market. In the affordable and luxury segments of the market year-on-year (y/y) price growth improved in the first quarter when compared with the

preceding quarter. The trends in house prices occurred against the background of macroeconomic developments and property market-related factors which impact household finances. These developments affect the demand for housing and may cause changes in property buying trends, which will impact market activity, transaction volumes and price trends in the various segments and regions of the market.

The average price of affordable houses[28] increased nominally by 2.8% y/y, from a previous increase of 10.2% in 2008. In essence, this was a decline of 4%. By the end of 2009, the average price of an affordable house increased by 2.9% y/y to reach R296 700.

Houses in the middle segment ($80m^2$-$400m^2$) were priced at R3.6m on average in the first quarter of 2012, up from R3.1m in the first quarter of 2010. This was a 1.4% decline y/y to around R1m, after an increase of 3.7% in the first quarter of 2011. There was a deflation of 7.1%, after a drop in real prices by 2.4% from the previous quarter. In summary the following price trends occurred in the middle-segment categories of the housing market in the first quarter of 2012:

- Small houses ($80m^2$-$140m^2$): -17.2% y/y (nominal) and -22% y/y (real)
- Medium Houses ($141m^2$-$220m^2$): 0.1% y/y (nominal) and -5.7% y/y (real).
- Large houses ($221m^2$-$400m^2$): 0.7% y/y (nominal) and -5.1% y/y (real)

For luxury houses, the average nominal price (houses valued at between R3,6 million and R13,4 million by the first quarter of 2012) increased by 3,3% y/y to R5m. In real terms, this was a 2.6% movement. This is a marked growth when compared to the first quarter of 2010 where the nominal price for this segment increased by 0.9% in 2009. After taking into consideration adjustments for inflation, prices went down 5.8% in 2009. By the fourth quarter of the same year, the nominal price declined 1.2% y/y

7.1.3 Regional house price movements

At regional level, prices varied on a nominal as well as on a real basis in the first quarter of 2012. Macroeconomic developments weighed on current and prospective homeowners, spilling over to the performance of the residential property market. Amongst the factors affecting the performance of the residential property market were; infrastructure-related

[28] Houses of $40m^2$-$79m^2$ and valued at R430 000 or less

aspects; the availability of serviced residential development land; regional economic performance; the extent of sectorial economic development; socio-economic conditions; the extent of property investment; location and the relative size and market activity per segment.

The general decline in real terms can be traced back to 2009 where economic factors had a huge impact on the decline. Area specific factors also contributed to this decline. The following table depicts the summary performance house prices at provincial level in 2009:

Table 7. 24: House price provincial performance in 2009

Province	Nominal Increase	Real Increase
Northern Cape	4.8%	-2.1%
Free State	2.6%	-4.2%
Gauteng	1.9%	-4.9%
Mpumalanga	1.4%	-5.3%
KwaZulu-Natal	0.6%	-6.1%
Limpopo	-0.1%	-6.7%
Western Cape	-1%	-7.6%
North West	-1.8%	-8.4%
Eastern Cape	-5.4%	-11.7%

Source: Absa housing review 2010

Further to the above table, the performance of the metropolitan areas was lacklustre as seen in house prices which weakened in nominal terms with the exception of Johannesburg which recorded an increase of almost 3%. As for the coastal regions, there was also a drop in real terms by 11.1%.

7.1.4 Building costs on new and existing house price trends

Building plans are one of the key indicators of house prices. The number of building plans approved is affected by current and future demand for property. When many plans are approved, it can serve as an indicator of a positive attitude towards residential property. Considering a brief history on building costs, the year 2009 saw a 6% increase in the cost of building a new house in the middle segment. When compared to the previous year, this was a marked decline as 2008 recorded an 8.3% increase from the previous year. This translates to

the average price of a new house costing R1.2m, a 5.5% increase from 2008. The average price for an existing house reached R0.96m in 2009, R0.26m cheaper than building a new one (ABSA, 2010).

By the end of 2009, the cost of a new house would increase by 5.5% in nominal terms, in line with consumer inflation which averaged 6% in the same period. By the first quarter of 2012, the cost of having a new house built increased by 4.2% y/y. As a result of rising building costs, the average nominal price of a new house increased by 7.2% y/y to R1.6m in the first quarter of the year. Adjusting for the effect of consumer inflation, the price of a new house increased by 1% y/y in the quarter. However, the average price of an existing house dropped by a nominal 2,3% y/y to about R996 600 in the first quarter, which comes to a real decline of 7,9% y/y, implying that it was R0.59m, or 37,2%, cheaper to buy an existing house than to have a new one built.

There are therefore a host of factors determining building costs some of which include; planning and professional services related to aspects such as rezoning where applicable, the drafting and approval of building plans. Other building related costs contributing to the house price include relevant professional and consulting fees, development finance, preparation of land for construction, providing road, electricity, water and sewage infrastructure, building materials, equipment, transport and labour.

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7.1.5 Mortgage finance

The value of outstanding household mortgage balances grew at less than 2% y/y in early 2012. The continued slow pace of growth in these mortgage balances is seen as a reflection of continued capital repayments on mortgage accounts and the state of household finances, affected by factors such as debt levels, impaired credit records and consumer price inflation, impacting real disposable income. Consumer confidence may have also played a role in the growth in credit extension, including mortgage finance.

The South African reserve bank kept the monetary policy interest rate, the repo rate unchanged from August 2009, when the rate was cut by 50 basis points. The mortgage interest rate then became stable at a level of 10.5%. Monthly mortgage repayments were

26.3% lower in 2009 than their December 2008 equivalent, when the mortgage rate was 15.5%. In a global economy that was in dire need of recovery, the banking sector in 2009 relaxed selectively its mortgage lending criteria. This improved the affordability of housing and contributed to higher levels of market activity.

7.2 Literature review

Research on house price trends is not thoroughly documented when compared to other asset types like stock prices, exchange rates and interest rates. Studies on house prices have considered the implications of economic factors. An increase in interest rates results in increased costs of borrowing, implying high loan repayments on mortgage finance. To support this hypothesis researchers have found a strong negative correlation between real interest rates and housing prices (Kostas and Zhu (2004), Borio and Mcguire (2004), Sutton (2002) and Englund and Ioannides (1997)).

Related findings have confirmed the negative effect high interest rates have on the residential property market. Ling and Naranjo (1998); Chan *et al*,. (1990) deduced that the term structure of interest rates is an important consideration when modelling real estate returns. In a related study, Brooks and Tsolacos (1999) analysed the economic and financial impacts on the property sector in the UK. They concluded that the interest rate term spread is one of the significant drivers of the real estate series. Ncube and Satchell (1994) also deduced related findings. Sutton (2002) reported a significant effect of real interest rates, GNP and equity prices on house prices.

Attempts have also been made at deducing volatility on house prices. In their research work West and Worthington (2004) and Liow (2004) employed GARCH models taking account of conditional variance in returns and volatility clustering. Cont (2007) showed that housing market participants' inactivity can be linked to volatility clustering. As a result, GARCH effects could be traced in the housing market. In a related application, Gu (2002) confirms volatility in real estate indices.

Mizrach (2008) applied a jump diffusion model and found jumps in housing markets from the returns on the Chicago Mercantile Exchange (CME) futures. The results suggested that on

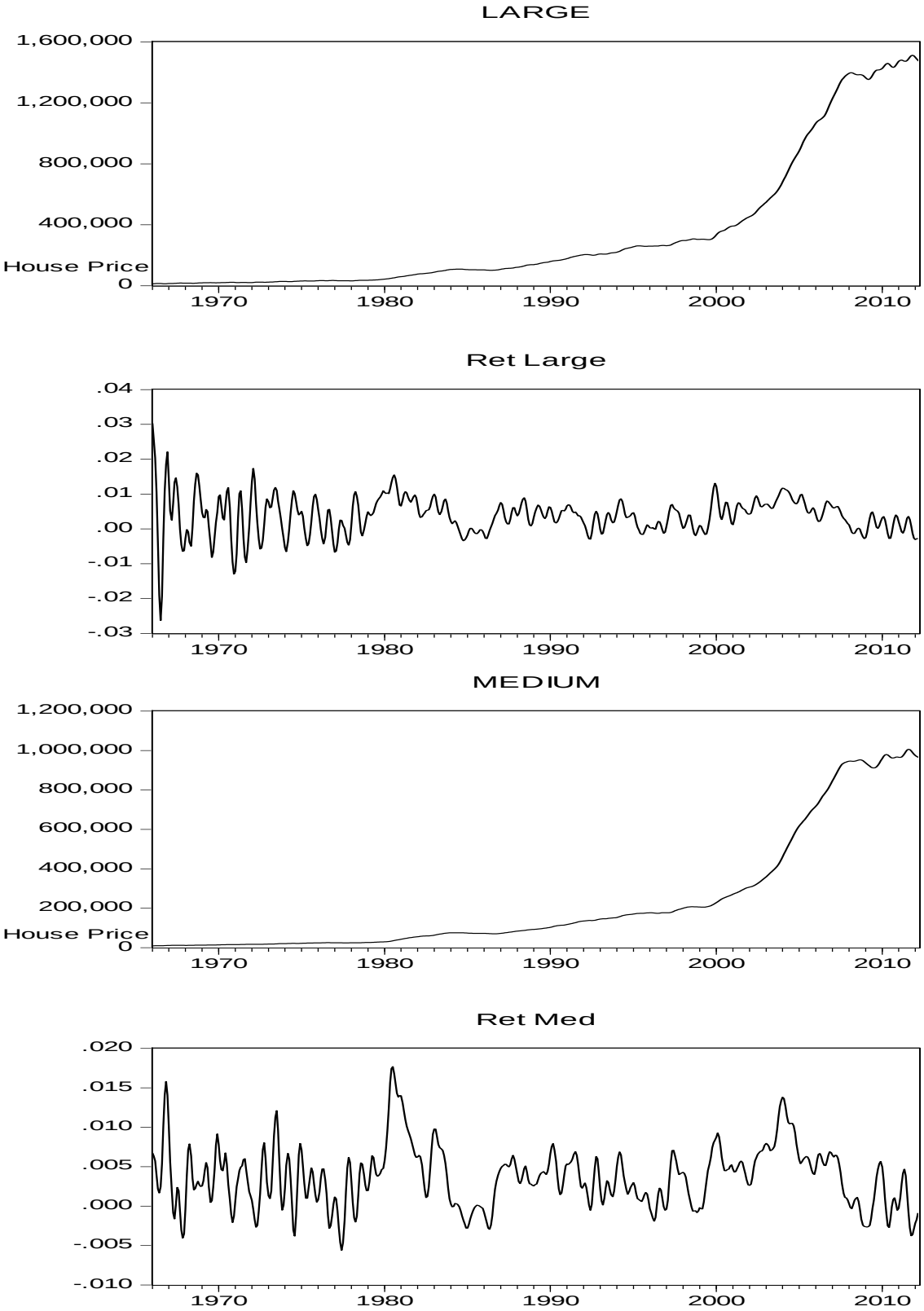
average, it required 69 jumps risks to be significant in the 315 day sample. Studies for South Africa have not accounted for volatility clustering on house prices (see Els and von Fintel, (2008), Clark and Daniel, (2006)). The contribution of this chapter argues for a better understanding of house prices in South Africa by applying a jump diffusion model.

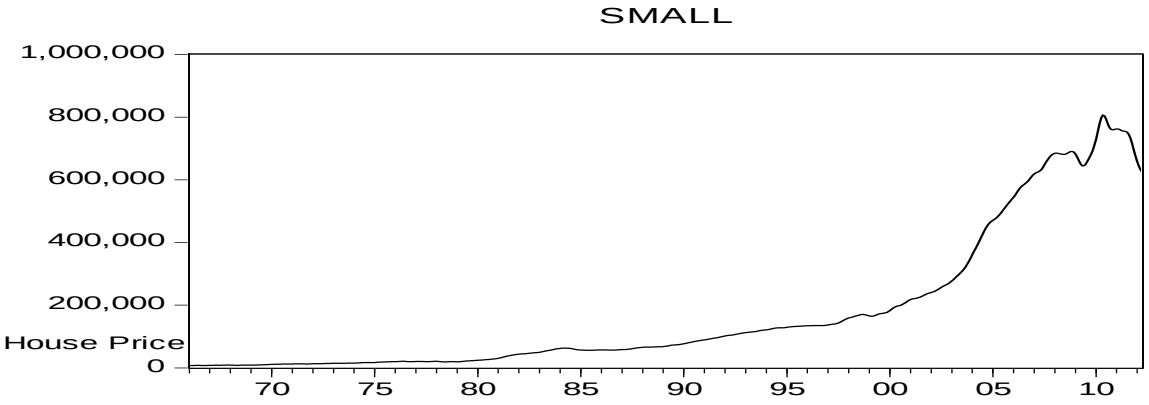
7.4.Data

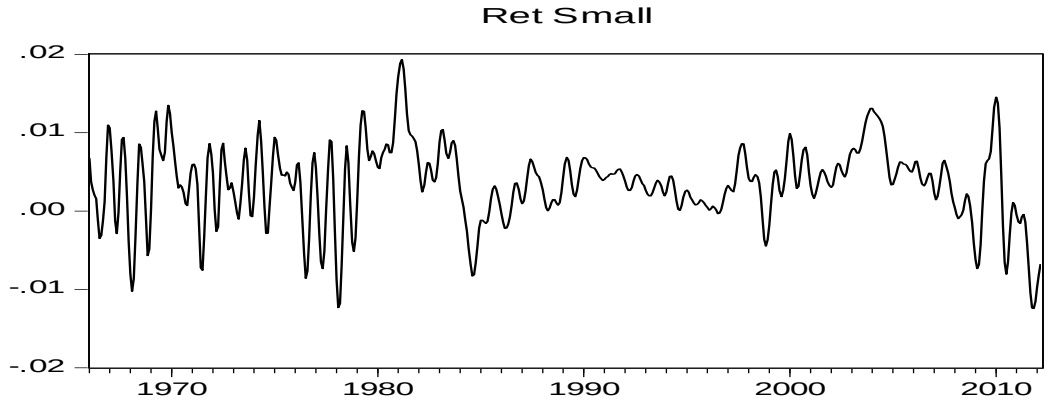
The data we used in the study is for monthly house prices for four different sub-group sizes; All, high, medium and low. The months of interest lie between, January 1966- April 2012, giving us 556 observations for each house size[29]. The figures below show the volatile nature of South Africa's house prices. The data reveals characteristics seen in most high frequency financial data; such as skewness, kurtosis, potential jumps and volatility clustering. Plotting the log returns data reveals volatility clustering as shown below.

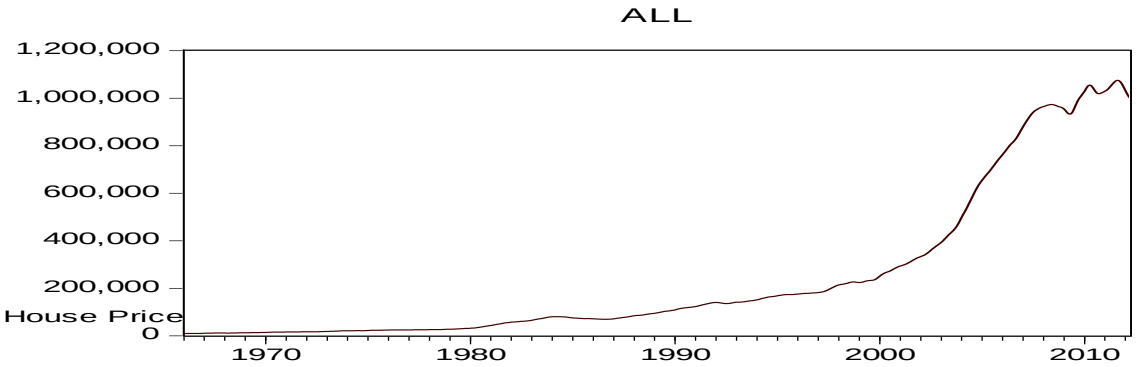
[29] The monthly SA house prices were supplied by ABSA. House prices are based on the total purchase price of all houses (including all improvements) in respect of which loan applications were approved by Absa Bank. Houses of which the prices exceed R3 600 000 in 2012 have been excluded from the calculations. Prices are smoothed for all houses between 80m² and 400m².

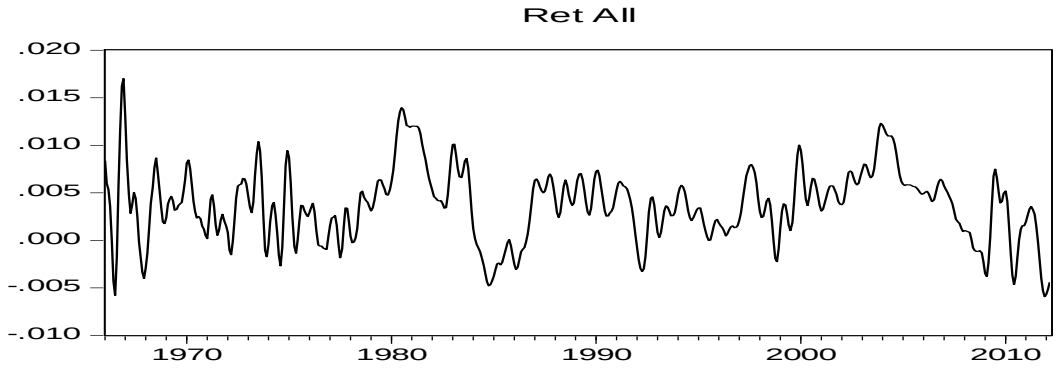
Figure 7.12: House price trend: 1966-2012











The graphs above reveal volatile house price data across all segments. Therein we can also deduce volatility clustering in the returns graphs. Data are also leptokurtic over the study period, indicating the impact of economic development on house prices. The current house price growth in the South Africa's residential property market is relatively low, affected by

economic developments impacting household finances, consumers' risk profile, levels of confidence and housing demand and supply. These factors are reflected in property buying patterns, the demand for and the affordability and accessibility of mortgage finance. Tables 7.25 and 7.26 below document the summary statistics for the data.

Table 7.25: House prices summary statistics

	1966-2012			
	All	Large	Medium	Small
Mean	246427	346370	234718	177751
Max	1074934	1512441	1006136	806685
Min	9114	12313	9045	7454
Variance	1.03E+11	2.06E+11	9.34E+10	5.14E+10
Skewness	1.525	1.55	1.54	1.51
Kurtosis	3.876	3.969	3.91	3.89

Table 7.26: House prices summary statistics returns

	1966-2012			
	All	Large	Medium	Small
Mean	3.7e-03	3.7e-03	3.7e-03	3.5e-03
Variance	1.4e-05	2.9e-05	1.5e-05	2.4e-05
Skewness	0.140	-0.20	0.57	-0.38
Kurtosis	3.34	6.82	3.74	4.04

The statistics from the tables are computed for the period January 1996-April 2012. The statistics confirm the volatility pattern observed in the figures above. From the table we can deduce the high skew for all the data sets, with the large house prices showing highest skew. The peakedness of large house price data is seen in the kurtosis value of 4 with the remaining house prices also showing varying degrees of kurtosis. Further than that, house price data show high levels of skewness with the large house price showing high skew at 1.6. For returns data, the statistics show the leptokurtic nature with varying levels of kurtosis from 3.7 to 6.8, again with large house price data having highest kurtosis at 6.8.

7.5 Estimation results[30]

Table 7. 27 Stationarity results house prices

Variables	Dickey-Fuller		Augmented Dickey-Fuller		Stat. status
	Constant	Constant & trend	Constant	Constant & trend	
Results for unit roots tests on levels House Prices					
LAll	1.229	-1.237	-0.634	-1.156	I (1)
LLarge	3.876	-2.351	0.178	-2.454	I (1)
LMed	1.705	-2.889*	-0.570	-2.890	I(1)
LSmall	0.491	-3.590*	-1.037	-3.591*	I(1)
Results for unit roots tests on first differences					
DLAll	-2.487**	-4.593***	-22.853**	-22.83***	I(0)
DLLarge	-2.139**	-4.077***	-17.581***	-17.554**	I(0)
DLMed	-3.317***	-4.559***	-6.584***	-6.592***	I(0)
DLSmall	-1.777*	-3.494***	-17.354***	-17.36***	I(0)

*** ** * represents stationary series at 1, 5 and 10% level of significance

The stationarity results show that data are not stationary in levels but stationary with first differencing. Below we present the summary results of a Jump Diffusion process using stationary data.

[30] The data suggests structural break in January 2004 for All price and Small price and March 2004 for Medium and Large price.

Table 7.28: House prices parameter estimates for the jump-diffusion model

	All	Large	Medium	Small
μ_B	0.0035 (1e-04)	0.004 (0.0000)	0.0034 (0.0000)	0.003 (1.93e-04)
σ_B	0.004 (1e-04)	0.004 (1e-04)	0.0035 (0.0000)	0.0035 (4.33e-03)
μ_J	0.009 (1e-04)	0.009 (0000)	0.008 (0.0000)	0.008 (0.0000)
σ_J	0.0005 (3.7e-07)	0.0045 (0.0000)	0.004 (0.0000)	0.0025 (1.33e-02)
λ	0.015 (2e-04)	0.0143 (0.0000)	0.0177 (6e-04)	0.0224 (3e-03)
Mean	3.64e-03	4.1e-03	3.54e-03	3.178e-03
Variance	1.73e-05	1.66E-05	1.33e-05	1.38e-05
Skewness	0.159	0.2634	0.3034	0.2868
Kurtosis	3.35	3.8929	4.01	3.7709

The table above presents parameter estimates for the jump diffusion model. Using a likelihood ratio as a diagnostic, we reject the null hypothesis of no jumps at 1% level. Jumps therefore help explain some features in the exchange rate data.

Matching the moments in the jump diffusion model

The aim is to verify that the model can match the data characteristics with observed levels of skewness and kurtosis at specified intervals. Table 6.28 illustrates this including the levels of skewness and kurtosis values for the jump diffusion model. We ran the model for numerous iterations and deduced overall more positive jumps than negative ones. We present results for positive jumps. We also allowed for higher jump size and variance to explore the effect on volatility.

All housing segment. The unconditional mean for the all housing price is 0.00364, this is quite close to the sample mean of 0.0037. The mean of the jump process is positive at 0.009 and the estimate of the jump frequency is 0.015 and is significant. The unconditional variance of the jump diffusion is also not too far from the sample variance for all house prices.

Large segment. For the large house prices, the unconditional mean of the jump diffusion is 4.1e-03, which is a close match to the sample mean. The mean of the jump process is positive with a jump frequency at 0.014, which is least across house prices. The volatility of the jump diffusion almost matches the sample at 76%. The levels of skewness and kurtosis have been normalised by the jump diffusion process.

Medium segment. The jump diffusion model generated the unconditional mean of 3.54e-03, which is not too far from the sample mean of 3.7e-03. The accompanying jump intensity is 0.018 and is significant. Volatility generated in the process almost matches the sample at 95%. The level of skewness has been normalised at close to 0.3.

Small segment. Finally, for the small housing price, the unconditional mean of 3.2e-03 is not too far from the sample mean of 3.5e-03, with highest intensity at 0.0224. The volatility of the process generates 99% of the sample equivalent and the result is significant.

From the results we deduce that house prices contain jumps and volatility clustering. Overall, the different house segments show similar jump sizes however the small house segment has highest intensity. The high intensity can serve to point to high volatility over the over certain regime periods. Furthermore it could imply that buyers of houses in this segment had high expectations and as the housing market recovers prices can rise at a higher percentage than for other segments. What is also striking with these results compared to other chapters in the study is that the higher moment of kurtosis closed matched the sample equivalents, complementing the significant jump diffusion parameters. This suggest that the model was able to trace house price characteristics

Table 7.29: House prices GARCH results

	GARCH Student's t				GARCH GED			
	All	Large	Medium	Small	All	Large	Medium	Small
C	0.0042 (8.4E-05)	0.0041 (1.16E-04)	0.0041 (1.04E-04)	0.0040 (9.45E-05)	0.0042 (9.47E-05)	0.0037 (1.24E-04)	0.0029 (1.02E-04)	0.0037 (1E-04)
K	1.01E-06 (1.64E-07)	1.867E-06 (3.60E-07)	1.46E-06 (3.41E-07)	8.31E-07 (2.29E-07)	1.22E-06 (1.2E-07)	1.67E-06 (2.63E-07)	4.75E-06 (2.61E-07	9.71E-07 (2.06E-07)
GARCH (1)	0.053 (0.031)	0.0444 (0.028)	0.0059 (0.0445)	0.196 (0.045)	0.1202 (0.021)	0.186 (0.027)	-0.634 (0.029)	0.232 (0.042)
$ARCH(1)$	0.926 (0.1739)	1.03 (0.152)	0.943 (0.156)	0.934 (0.143)	0.697 (0.0686)	0.774 (0.067)	0.742 (0.0486)	0.807 (0.0886)

The table above presents the GARCH results for South Africa's house prices. Most of the values for β_1 and α_1 are non-negative indicating stationarity. α_1, (represented by ARCH (1)) indicates the random deviations in the previous period whereas β_1, represented by GARCH (1) measure the part of variance from the previous period that is carried over into the current period. The results for house prices appear different from other asset prices investigated in the previous chapters. This we see in the values of α_1 which are higher than for β_1, indicating the intensity of volatility reacting to previous period, a feature more prominent in house prices. The sum of $\beta_1 + \alpha_1$, capturing the persistence in volatility, is close to 1, indicating the presence of ARCH and GARCH effects in house price returns. However for GARCH student's t results are explosive for both the large and small price segments. Explosive results can also be seen for GED distribution under the small segment. This can imply the effect of different regimes over the study period which to a greater extent affected the small price housing segment seeing as it is explosive for both specifications of the fat tailed GARCH model (see Medeiros and Veiga, (2009) for discussion).

7.6 Conclusion

The chapter presented a model characterising jumps in house prices in South Africa. An extension of the analysis introduced fat tailed GARCH scenarios; Student's t distribution and GED to capture the persistence in volatility. The overall contribution of the chapter was to trace the existence of Poisson distributed jumps and the existence of volatility. Our empirical results indicate that the volatility of house price data contains significant jumps, intensity and volatility clustering. Generally the different house price segments have more or less similar jump sizes. However, the small segment has highest jump intensity with the large segment having the lowest jump intensity. Perhaps the high intensity for the small segment points to high volatility over specific regime periods. The unconditional mean generated from the jump diffusion model for all, large, medium and small house price segments almost matched the sample equivalents. Furthermore, the volatiltity generated from the jump diffusion process almost matched the sample equivalents as well save for the large house prices which was a distant 76%. However, the model overestimated volatility for all house price segment.

Another interesting deduction from the results are the higher moments of skewness and kurtosis which turned out to be closer to sample equivalents, even after introducing jumps. This result is different from previous chapter results where the introduction of jumps normalised skewness and kurtosis. The different jump intensities in this chapter could be used as a precursor to understanding the mortgage premium for different housing segments. As an example, the intensity for the small price segment could suggest higher expectations from buyers during the study period which was not observed for other segments. Having this knowledge can be vital for determining the mortgage premium. Furthermore, another implication could be that when the housing market recovers, prices can rise higher in the small price segment compared to other segments.

For the GARCH findings, most parameter values confirm the persistence of volatility clustering. The small price house segment was explosive for both fat tailed GARCH specifications, implying the influence of different regimes over the study period.

CHAPTER EIGHT

CONCLUSION

8.1 Results and conclusion

The section summarises the findings in the thesis, highlighting all the five areas investigated to understand the asset price movement after incorporating volatility and jumps. Furthermore, the research also includes an implication of the results across different asset prices for South Africa.

8.1.1 Temperature risk

Continuous innovation in the derivatives market has seen the emergence of new products being introduced to mitigate against risk. One new interesting development has been in the area of weather derivatives which has grown as companies embrace the benefits of hedging against weather related factors. Thus weather derivatives are increasingly becoming valuable tools for risk management worldwide. Organisations always need to hedge risk to avoid the costs of failure and bankruptcy and therefore attain value for shareholders. Thus weather derivatives complement risk management tools like swaps, options and futures. Many companies for instance from agriculture, energy, construction and tourism all experience some form of weather related risk and therefore the importance of weather derivatives in South Africa can never be overemphasised.

Chapter two carried out an estimation of the payoffs of temperature based weather derivatives in four South African cities using three different methods; Autoregressive (AR) seasonality Generalised Autoregressive Conditional Heteroskedasticity (SGARCH), historical method and cumulative cooling degree day (CDD). This enables us to inform practitioners about alternative strategies for safeguarding earnings against adverse temperature movements.

Graphical presentation of the returns data complimented the summary statistics by portraying patterns in volatility clustering. Using a simple trend model, we found that Bloemfontein had trend and quadratic trend terms significant whilst Cape Town had only quadratic significant and Durban trend significant. Johannesburg had a trace of trend. Knowing the trend model contributes to developing pricing for temperature based weather derivatives of these cities. We also demonstrated the computation of option values to illustrate the monetary values of

the temperature products. If there was an existing weather derivatives market, the amounts involved could be enormous, as organisations benefit from investing in mitigating against weather related risks.

The chapter went further to estimate the AR (7) SGARCH (1,1) model which confirmed the dampening effect of the seasonality component as it reduced volatility persistence in the parameters beta and alpha as their sum became a distant from 1. Using three different methods in comparing estimated payoffs, the results revealed that the SGARCH (1,1) method of estimating payoffs performed best when compared to the other two methods. The mean of estimated payoffs from this approach was estimated relatively accurately, with values close to zero for the mean and lower standard deviation overall. This suggests that the SGARCH (1, 1) model can be applied to the regions: Cape Town, Johannesburg, Bloemfontein and Durban.

The historical method was rejected as it tended to be overpriced in three cities and under-priced in one city for both the high and low strike price. Though better than the historical price, the cumulative CDD also overpriced for the low strike price in two cities and under-priced for the remaining cities. Using a higher strike price the method under-priced for all four cities. The analysis in the chapter goes further as it can serve as a guide to enriching the understanding of pricing temperature related derivatives for other cities in the country.

8.1.2 Jump diffusion model for the exchange rate

Exporters, financial institutions, non-financial institutions, investors and government are all affected by exchange rate risk. As a result, an understanding of exchange rate movements will enable these players to make informed decisions when considering a hedge against adverse exchange rate movements. Chapter three modelled exchange rate movements in South Africa using a jump diffusion model as a contribution to understanding exchange rate fluctuations. The exchange rate data investigated were; british pound/ south african rand; euro/south african rand and U.S dollar/ south african rand.

Returns data showed high potential jumps in periods of financial market bubbles and crashes. The jump diffusion results were significant for the jumps and intensities. Inclusion of jumps was also shown to normalise the returns data, affecting skewness and kurtosis. The results

confirmed related studies in literature in the ability of the jump diffusion model to capture skewness and kurtosis whilst at the same time allowing the mean and volatility parameters match the sample equivalents. We can assert that the model explains the volatility smile for short and medium term maturities, but may not do the same for longer time horizons. The jump diffusion results also revealed dependence in jump intensity and jump size. The implication of the results is on currency options since a better estimate of exchange rate volatility is needed to enable hedging.

Complementing the jump diffusion model, we also applied the fat GARCH models to capture the persistence in volatility. Mostly, results for the GARCH specification showed that parameters were stationary and the beta parameter was higher than alpha with their sum less than 1 and confirming the persistence in volatility in exchange rate returns.

8.1.3 Jump diffusion in interest rate models

Various attempts have been made to understand interest rate behaviour considering its effect on investment portfolios of organisations and individuals alike. Even more, high interest bearing rates have dire consequences on company profitability via accessing loans for production and even investment purposes. Chapter four applied the Jump diffusion model in examining variation of the Chan *et al.*, (1992) model. We also considered a fat tailed GARCH specification. The study tested the model on the 90 day treasury bill (T-Bill) for South Africa. Our review for short rate models and results implied that a jump diffusion model can be considered to capture short term interest rate properties for South Africa.

Summary statistics portrayed the period 1990-2002 showing stronger leptokurtic features with a higher kurtosis than for other periods. Estimation results showed that the 90 day T-Bill rate had higher jump size for the post inflation targeting period though with a lower intensity. Seemingly, the jump intensity seems to have weakened after inflation targeting. We can infer form the results that jumps can be traced more in the short term than in the long term. Results from the chapter proved that the jump diffusion level can capture unanticipated market movements as seen in the generated levels of skewness and kurtosis which mirrored the sample equivalents.

Another contribution of the estimation was in sufficient faring in capturing volatility of the interest rates as estimation results matched sample moments satisfactorily. Generally the short term interest rate market exhibits more deviations than does the long term. The parsimonious fat tailed GARCH (1,1) supported the specification in removing the ARCH effects, indicating a slow mean reverting process. Findings in this chapter contribute to portfolio analysis, interest rate forecasting and also to the valuation of short term interest rate derivatives in single factor models. However, the analysis can be extended to multi factor models applying stochastic behaviour to interest rates.

8.1.4 Jump diffusion in stock prices

Surprises to investors' information sets have been attributed as causing unexpected movements in stock prices. Chapter five tested the applicability of this hypothesis to South Africa. Using a database of the sectors in the Johannesburg Stock Exchange we found that a pure diffusive process did not suffice. Parameter results captured significant jumps, intensity and stochastic volatility, verifying related studies on stock prices.

The jumps and intensities were significant in all four sector representatives; mining, banking, media and leisure. However, jump size was highest in the mining sector with intensity highest in the banking sector. The model generated levels of moments which matched the sample values and went further to generate skewness and kurtosis which were normalised as per related findings. This underscores the performance of the jump diffusion model in tracing returns data. Understanding jumps in stock prices sheds more light on market efficiency.

Nonetheless, jumps remain isolated events and days with more than one jump are few. The introduction of jumps normalises the higher moments. From a probabilistic viewpoint, the commonly assumed Poisson distribution seems to describe the rate of occurrence of discontinuities fairly. Fat tailed GARCH models captured volatility persistence for the banking and mining Sectors. For media and leisure volatility persistence was seemingly absent. The results underpin the importance of portfolio diversification when investing in the stock market.

8.1.5 Volatility and jumps in house prices

In Chapter Six, we extended the jump diffusion model to house prices to learn more about price movement in this important source of wealth. We considered four different house price segments; all, large, medium and small.

Empirical findings in the chapter indicate that that house price data contains significant jumps, intensity and volatility clustering. Jump sizes seem to be evenly distributed across all four segments. However, the small house price segment had a higher jump intensity whereas the large house price segment had the lowest intensity. The intensity for the small house price could have been influenced by different regimes over the study period at a higher level than for other segments. Higher expectations from participants in the small house price segment could be linked to the high intensity. The volatility generated was lowest for the large house price and yet matching the sample equivalent for the remaining segments.

More surprise came in the higher moments which were not normalised after introducing jumps. In the previous three chapters the moments skewness and kurtosis were normalised after adding jumps. On application, the mortgage premium calculation can draw from jump diffusion model results. The fat tailed GARCH generated persistence in volatility. For small house price however, the results were explosive for both specifications of the fat tailed GARCH.

8.2 Policy implications and areas for further research

This study provided insights into stochastic volatility and jump diffusion models. Furthermore, the study also contributed to the existing literature, more so in the South African context where there is a gap in understanding behaviour of asset prices. This we did through application of stochastic volatility models in South Africa. A number of implications can be derived from this study. There is a need to develop rigorous research in asset prices in the South African scenarios. Such studies will go a long way in informing policy makers and investors alike about the various strategies to consider in mitigating volatility in asset prices.

With the global interest in phenomenal weather, there are opportunities to consider in mitigating weather related risk, especially for businesses affected by weather components. Results from the study reinforce that trading in weather derivatives in South Africa is one possible risk management tool to be considered. South Africa has yet to have any organised trading in weather derivatives. The agriculture, tourism and energy sectors can benefit immensely from weather derivatives as they can combine for example rainfall or temperature options with their current insurance contracts. This will enable them to carry on business without fearing huge losses brought about weather catastrophes. There is therefore a need to educate farmers, government entities, potential counter-parties to weather related contracts and other organisations affected by weather related risk on the importance of weather derivatives so that foundation is laid for trading in this special type of insurance.

Another important consideration is availability and accessibility of lengthy historical records for all asset types. Measurability of variables for different assets enables various estimation models to be considered and allowing the most optimal method to be considered for each in hedging risk. As an example if the weather variable can be measured accurately for lengthy periods for most cities, it will allow the best method for mitigating weather related risk to be considered in each of those cities in South Africa.

8.3 Future research

1. Future research in stochastic volatility models in financial econometrics in the South African context can look at using time varying parameters and as well merging jump diffusion models with stochastic volatility.
2. Another interesting consideration is to look at the link between jumps and corporate disclosure as well as investigating jump intensities across different frequencies.
3. Tracking jump sizes pre and post central bank announcements can give more enlightenment to understanding asset price movements.

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APPENDICES
Appendix A: MATLAB Codes for Weather Derivatives

```
function main( );
global  newData1
clear all;
    clc;

    name = { 'Cape_Town'
             'johannesburg'
             'Durban'
             'Bloemfontein' };
   % Johannesburg
     % Looping over the cities

    for j = 1:4
       [Data]= importfile1( char( strcat(
'C:\Users\malumisa\Documents\MATLAB\',name(j),'.mat')));
  [Data];
   Ave = mean( Data,2 );



%strResponse = input(prompt, 's')
  %msgbox yyyy
        Nyears = (length(Ave)/365);

      %   Tmp1 = reshape(Ave(1:Nyears*365),365,Nyears);
       Tmp1 = reshape(Ave(1:Nyears*365),365,Nyears);
     % evalResponse = input(prompt)
        % input(prompt, 's')
      %    [ Sf,Sfv, ms ] = GetSeasFF( Tmp1 );
%%%[ ms ] = GetSeasFF( Tmp1 );



    % Mean
    tgrid = (1:1:length(Tmp1))';

    th    = (2*pi/365).*tgrid;
```

```
   % X     = [ ones(length(tgrid),1) tgrid cos(th)  sin(th) cos(2*th)
sin(2*th) cos(3*th) sin(3*th) ]
        X     = [ ones(length(tgrid),1) tgrid cos(th)  sin(th) cos(2*th)
sin(2*th) cos(3*th) sin(3*th) cos(4*th) sin(4*th) cos(5*th) sin(5*th) ];

    [x,se_x,mse]= lscov( Tmp1,X );
     Sm=mean( x )  %the ordinary least squares solution to the linear
system of equations A*x = b,
      Sv=mean(se_x) %the estimated standard errors of b
      mse  %   mse; %mean squared error.
      xx=[ Sm' Sv']
    %   input(prompt, 's')
      tg=size(tgrid);
    sX=size(X);
    Sn=size(Tmp1);
    m=size(Sm);
    v=size(Sv);

   Y  = xx(:,1) - xx(:,2);
        n = length( Y );
     %   strResponse = input(prompt, 's')
      %   n = size( Y )
       % Starting values
        load( 'SGARCHInitialVals.txt' );
        theta = SGARCHInitialVals(:,j);

         % Call optimizing routine
      %     Y = [Y mse'.^2]
        Y = [Y mse' ] ;
        %    strResponse = input(prompt, 's')
     %    options =
optimset('Display','iter','MaxIter',15000,'MaxFunEvals',10000,'TolFun',1e-
7,'TolX',1e-7);

        options =
optimset('Display','iter','MaxIter',15000,'MaxFunEvals',10000,'TolFun',1e-
7,'TolX',1e-7);
        [theta] = fminsearch( @glogl,theta,options,Y );

        % Saving parameters and standard errors
```

```
        theta(9) = abs( theta(9) );
        theta(10) = abs( theta(10) );
        theta(11) = abs( theta(11) );
        theta(12) = abs( theta(12) );



        % Compute standard errors
        g = numgrad(@logl,theta,Y)
        se  = sqrt(diag(inv(g'*g)))
        tstat = theta./se

        tmp1 = [ theta se tstat];
        SGARCHpms=tmp1
        % save( char( strcat(
'h:\Sambulo\CURRENT\WDeriv\Output\',name(j),'SGARCH.pms' ) ),'tmp1','-
ascii' );
       % save( char( strcat(
'C:\Users\Malumisa\Documents\MATLAB\',name(j),'SGARCH.pms' ) ),'tmp1','-
ascii' );
         save( char(
strcat('C:\Users\malumisa\Documents\MATLAB\',name(j),'SGARCH.pms') )
,'tmp1','-ascii' );
     % Residuals and conditional variance at optimal parameters
        [ Resid,CVar,Fval ] = dlogl(theta,Y);
        sr1 = Resid
     % Saving conditional variance and permanent component
        tmp2 = CVar
       % SGARCHvar=tmp2 ;
        %save( char( strcat(
'h:\Sambulo\CURRENT\WDeriv\Output\',name(j),'SGARCH.var' ) ),'tmp2','-
ascii' );
      %  save( char( strcat(  'C:\Users\malumisa\Documents\MATLAB\'
name(j),'SGARCH.var'  ) ),'tmp2','-ascii' );
        save(  char( strcat('C:\Users\malumisa\Documents\MATLAB\'
,name(j),'SGARCH.var') ) ,'tmp2','-ascii' )
      % Save standardised residuals
        sr1 = Resid./sqrt(CVar);

        SGARCHres=sr1
```

```
        %save( char( strcat(
'h:\Sambulo\CURRENT\WDeriv\Output\',name(j),'SGARCH.res' ) ),'sr1','-ascii'
);
     %  save( char( strcat(  'C:\Users\malumisa\Documents\MATLAB\'
name(j),'SGARCH.res' ) ),'sr1';'-ascii' );
         save(  char( strcat('C:\Users\malumisa\Documents\MATLAB\',
name(j),'SGARCH.res') ),'sr1','-ascii' );

 %input(prompt, 's')
%%%[ms]
          %  input(prompt, 's')
%%%         x1=mean(Tmp1);
        % X  = [ Tmp1' Sf Sfv];
%         X  = [ Sf' Sfv' ms'];
%%%          input(prompt, 's')
      %    X  = [ Sf Sfv];
        % Sf
        %Sfv



        % Computing ACF of standardised residuals

        % [ACF, Lags, Bound] = autocorr(sr1,730);
   %%%%
   %%[ACF, Lags] = corr(sr1',730)
 %%  input(prompt, 's')
      %  save( char( strcat( outfolder,name(j),'SGARCH.sr1' ) ),'ACF','-
ascii' );
    %  save( char( strcat(  'C:\Users\malumisa\Documents\MATLAB\'
name(j),'SGARCH.sr1' ) ),'ACF','-ascii' );
%%%%       save(  char( strcat('C:\Users\malumisa\Documents\MATLAB\',
name(j)) ),'SGARCH.sr1','ACF','-ascii' );

      % save( char( strcat( outfolder,name(j),'SGARCH.sr1' ) ),'ACF','-
ascii' );
     %%sr2 = sr1.^2;
      %[ACF, Lags, Bounds] = autocorr(sr2,730);
       %% [ACF, Lags] = corr(sr2',730);
       % save( char( strcat( outfolder,name(j),'SGARCH.sr2' ) ),'ACF','-
ascii' );
```

```
     %   save( char( strcat(  'C:\Users\Kenny\Documents\MATLAB\'
name(j),'SGARCH.sr2' ) ),'ACF','-ascii' );
%%%%%          save(  char( strcat(  'C:\Users\Kenny\Documents\MATLAB\',
name(j)) ),'SGARCH.sr2' ,'ACF','-ascii' );

        %================================================================
     %    (char( strcat(
'C:\Users\Malumisa\Documents\MATLAB\',name(j),'.mat')))
    %  char( strcat(name(j),'.mat'))

     %    ave=mean((char( strcat(  name(j)))),1)
   %     ave=mean((char( strcat(
'C:\Users\Malumisa\Documents\MATLAB\',name(j),'.mat'))),2)
    end
function [Ave]=importfile1(fileToRead1)
%IMPORTFILE1(FILETOREAD1)
%  Imports data from the specified file
%  FILETOREAD1:  file to read

%  Auto-generated by MATLAB on 17-Nov-2009 21:46:58

% Import the file
newData1 = load('-mat', fileToRead1);
%newData1
%are=mean(newData1)

% Create new variables in the base workspace from those fields.
vars = fieldnames(newData1);
for i = 1:length(vars)
    nn=newData1.(vars{i});
    assignin('base', vars{i}, newData1.(vars{i}));
end
Ave=nn;

%
% ================================================================

%function [ Sm,Sv, mse] = GetSeasFF( nn )
function [ xx ] = GetSeasFF( nn )
```

```
 %   results = ols(nn,X );
 %   Sm      = results.yhat;

   % Variance
 %   Svar = (se_x).^2
 %   X    = [ ones(length(tgrid),1) cos(th)  sin(th) cos(2*th) sin(2*th)
cos(3*th) sin(3*th) cos(4*th) sin(4*th) cos(5*th) sin(5*th) cos(6*th) ];
 %   X
 %   [x1, se_x1, mse1] = lscov( Svar,X' )
 %   Sv      = x1;
 %   input(prompt, 's')

%=========================================================================
%
%  Wrapper function to return log-likeliehood of AR(7)-GARCH(1,1) model
%
%=========================================================================
function lf = glogl( beta,data)

    lf = sum( logl(beta,data) );
%end

%=========================================================================
%
% Procedure to return log-likeliehood of AR(7)-GARCH(1,1) model
%
%=========================================================================
function lf = logl( beta,data )

    beta(9) = abs(beta(9));
    beta(10) = abs(beta(10));
    beta(11) = abs(beta(11));
    beta(12) = abs(beta(12));

    MaxLag = 7;
    % Set up data for Garch model
    y  = data(MaxLag+1:end,1);
    s  = data(MaxLag+1:end,2);
    n  = length(y);
```

```
    x  = ones(n,1);

    for i = 1:MaxLag;

            x = [x data(MaxLag+1-i:end-i,1)];
    end

    % Construct residuals
    u  = y - x*beta(1:8);
    u2 = u.^2;

    % Allocate memory
    lf = zeros(n,1);
    h  = zeros(n,1);

    % Starting value for volatility
    h(1) = data(MaxLag,2);

    for i = 2:n

        h(i) = beta(9) + beta(10)*u2(i-1) + beta(11)*h(i-1) +
beta(12)*s(i);
        lf(i)= (1/2)*log(2*pi)+0.5*log(h(i))+0.5*(u(i)^2/h(i));
    end

%end
%=========================================================================
%
% Procedure to return log-likeliehood of AR(7)-GARCH(1,1) model
%
%=========================================================================
function [u,h,logl] = dlogl( beta,data );

    MaxLag = 7;
    % Set up data for Garch model
    y  = data(MaxLag+1:end,1);
    s  = data(MaxLag+1:end,2);
    n  = length(y);
    x  = ones(n,1);
```

```
    for i = 1:MaxLag;

            x = [x data(MaxLag+1-i:end-i,1)];
    end

    % Construct residuals
    u  = y - x*beta(1:8);
    u2 = u.^2;

    % Allocate memory
    logl = zeros(n,1);
    h    = zeros(n,1);

    % Starting value for volatility
    h(1) = mean(u2);

    for i = 2:n

        h(i) = abs(beta(9)) + abs(beta(10))*u2(i-1) + abs(beta(11))*h(i-1)+
beta(12)*s(i);
    end

    % Log likelihood
    logl=-(1/2)*log(2*pi)-0.5*log(h)-0.5*(u2./h);
    logl = -sum(logl);
%end

% ========================================================================
% numerical derivative function
% calculates the numerical derivative of function f at the parameter value
% bd (uses central difference)
% inputs:   f, needs to be a function handle, ie if the function's name
%               is test you need to pass @test
%               f can be (columns) vector valued, ie its output
%               can be (q x 1)
%           bd, this is the parameter value at which to evaluate the
%             derivative  this vector has dim (k x 1)
%           y and x, further inputs into function f
```

```
%               eg function a = test(b,y,x)
% outputs:  der, a (q x k) vector of numerical derivatives
% ============================================================================
function der = numgrad(f,bd,varargin)

    k = size(bd,1);
    temp = repmat(feval(f,bd,varargin{:}),k,2);

    % compute stepsize (as in GAUSS' gradp)
    abd = abs(bd);
    if bd ~= 0
        dabd = bd./abd;
    else
        dabd = 1;
    end
    h1 = [abd (1e-2*ones(k,1))];
    h = 1e-8 * max(h1,[],2) .* dabd;
    temp = bd + h;
    h = temp - bd;

    for i = 1:k
        b_temp = repmat(bd,1,2);
        b_temp(i,1) = b_temp(i,1) + h(i,1);
        b_temp(i,2) = b_temp(i,2) - h(i,1);
        temp1(:,i) = feval(f,b_temp(:,1),varargin{:});
        temp2(:,i) = feval(f,b_temp(:,2),varargin{:});
    end
    q = size(temp1,1);
    der = (temp1 - temp2)./(2*repmat(h,1,q)');
%end




% ============================================================================
% PURPOSE: Computes finite difference Hessian
% ----------------------------------------------------
% Usage:  H = hessian(func,x,varargin)
% Where: func = function name, fval = func(x,varargin)
%           x = vector of parameters (n x 1)
%    varargin = optional arguments passed to the function
```

```
% ------------------------------------------------------
% RETURNS:
%           H = finite differnce hessian
% ------------------------------------------------------

% Code from:
% COMPECON toolbox [www4.ncsu.edu/~pfackler]
% documentation modified to fit the format of the Ecoometrics Toolbox
% by James P. LeSage, Dept of Economics
% University of Toledo
% 2801 W. Bancroft St,
% Toledo, OH 43606
% jlesage@spatial-econometrics.com
% ======================================================================

function H = hessian(f,x,varargin)

    eps = 1e-5;

    n = size(x,1);
    fx = feval(f,x,varargin{:});

    % Compute the stepsize (h)
    h = eps.^(1/3)*max(abs(x),1e-2);
    xh = x+h;
    h = xh-x;
    ee = sparse(1:n,1:n,h,n,n);

    % Compute forward step
    g = zeros(n,1);
    for i=1:n
        g(i) = feval(f,x+ee(:,i),varargin{:});
    end

    H=h*h';
    % Compute "double" forward step
    for i=1:n
        for j=i:n
            H(i,j) = (feval(f,x+ee(:,i)+ee(:,j),varargin{:})-g(i)-
g(j)+fx)/H(i,j);
```

```
            H(j,i) = H(i,j);
        end
    end
%end
```

Appendix B: Derivative Financial instruments traded on organised exchanges

By instrument and location

Notional principal in billions of US dollars

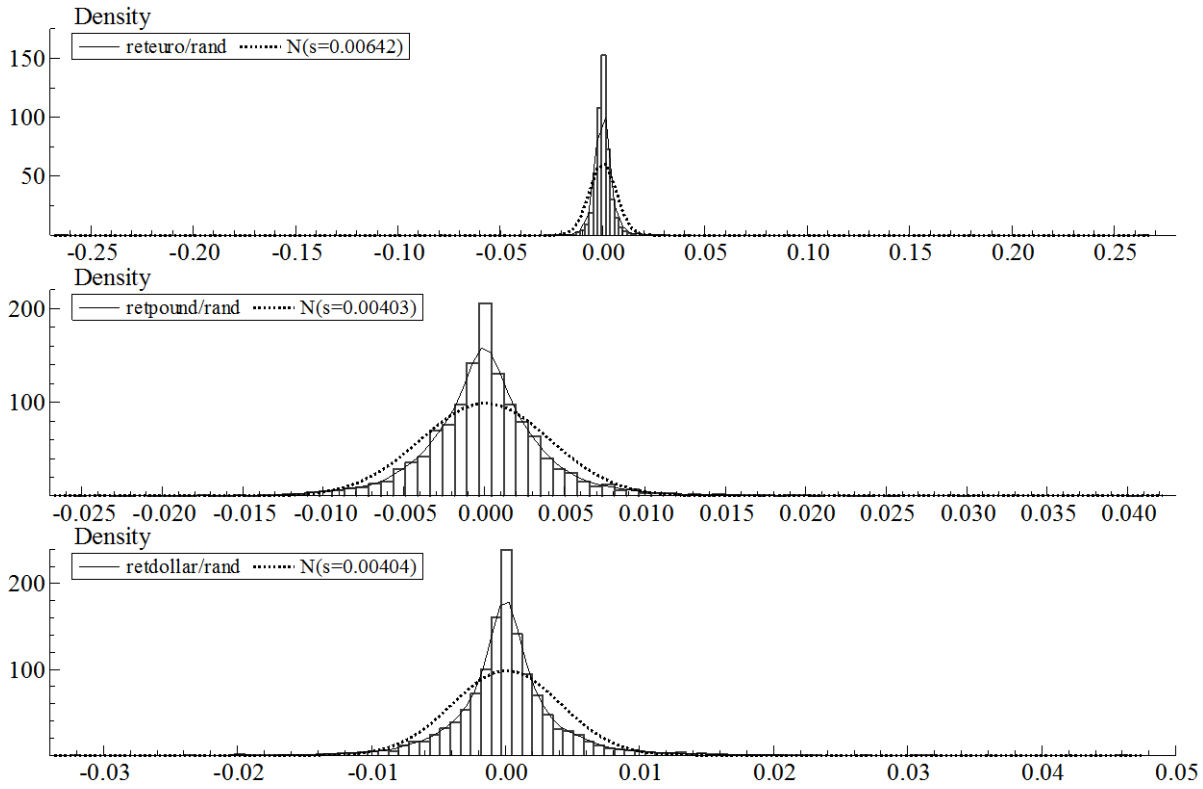
Instrument/location	**Turnover**			
	Q1 2010	Q2 2010	Q3 2010	Q4 2010
Futures				
All Markets	**346,816.4**	**384,716.5**	**305,361.8**	**343,493.1**
Interest rate	313,5656.2	344,455	270,635	307,181.1
Currency	8,457.7	9,825	8,306.8	9,120.2
Equity index	24,793.5	30,436.5	26,420.1	27,191.9
North America	**177,625.6**	**210,146.2**	**153,648.6**	**187,774.9**
Interest rate	161,033.8	189,903.7	136,806.6	170,450.3
Currency	6,794.1	7,824.1	6,736.	7,292.6
Equity index	9,797.7	12,418.3	10,106.1	10,031.9
Europe	**145,105.9**	**144,657.8**	**120,590.1**	**122,791.2**
Interest rate	136,405.8	135,108.9	112,784.2	114,463
Currency	19,1	36	56,9	83,7
Equity index	8,681	9,512.7	7,749	8,244.5
Asia and Pacific	**18,106.3**	**23,036.8**	**24,956.1**	**26,183.4**
Interest rate	12,316.5	15,057.7	16,352.3	17,181.7
Currency	335,6	420,6	379,9	458,2
Equity index	5,454.2	7,558.5	8,223.9	8,543.5

Other Markets	**5,978.6**	**6,875.9**	**6,167**	**6,6743.6**
Interest rate	3,809.2	4,384.7	4,691.9	5,086
Currency	1,308.8	1,544.2	1,134	1,285.6
Equity index	860,6	947	341	372
Options				
All Markets	**168,119.3**	**170,949.9**	**132,386.7**	**135,125.5**
Interest rate	136,820.5	135,190.9	100,874.2	95,957.6
Currency	790.3	837.3	725.5	696.2
Equity index	30,508.5	34,921.7	30,788	38,471.7
North America	**59,334.7**	**80,344**	**59,979.7**	**61,885.4**
Interest rate	50,290.	69,593.8	51,976.8	53,482.3
Currency	318.5	473.3	380.4	428.4
Equity index	8,726.2	10,276.8	7,622.5	7,974.7
Europe	**88,978.4**	**68,100.2**	**50,149.7**	**44,226.3**
Interest rate	84,172.3	63,463.5	46,382.6	39,884.9
Currency	1.4	1.5	1	1.2
Equity index	4,804.7	4,635.3	3,766.1	4,340.2
Asia and Pacific	**16,995.8**	**19,905.6**	**19,577.1**	**26,260.9**
Interest rate	558.8	568.2	718.2	760.5
Currency	-	-	0	6.2
Equity index	4,804.7	4,635.3	3,766.1	4,340.2
Other Markets	**2,810.4**	**2,600.1**	**2,680.2**	**2,752.8**

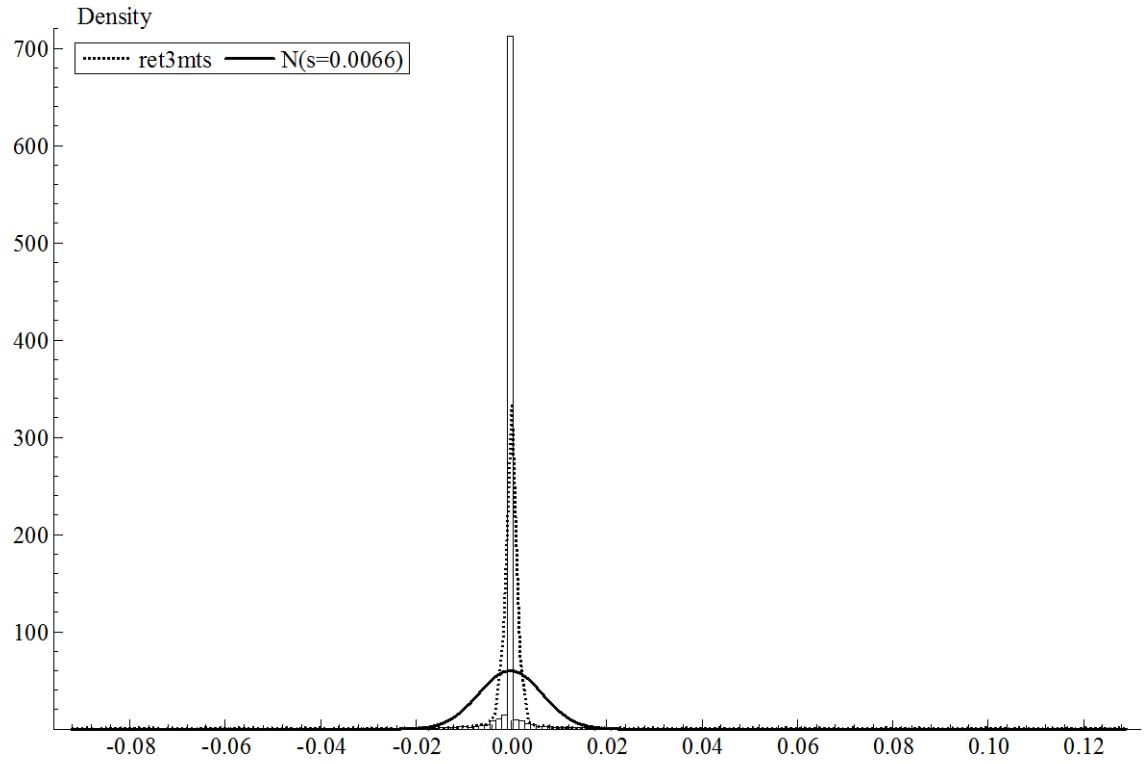
Interest rate	1,799.4	1,565.4	1,796.6	1,829.9
Currency	470.4	362.4	343.1	260.4
Equity index	540.7	672.2	540.5	662.5

Source: BIS www.bis.org/publ/cpss23a.pdf

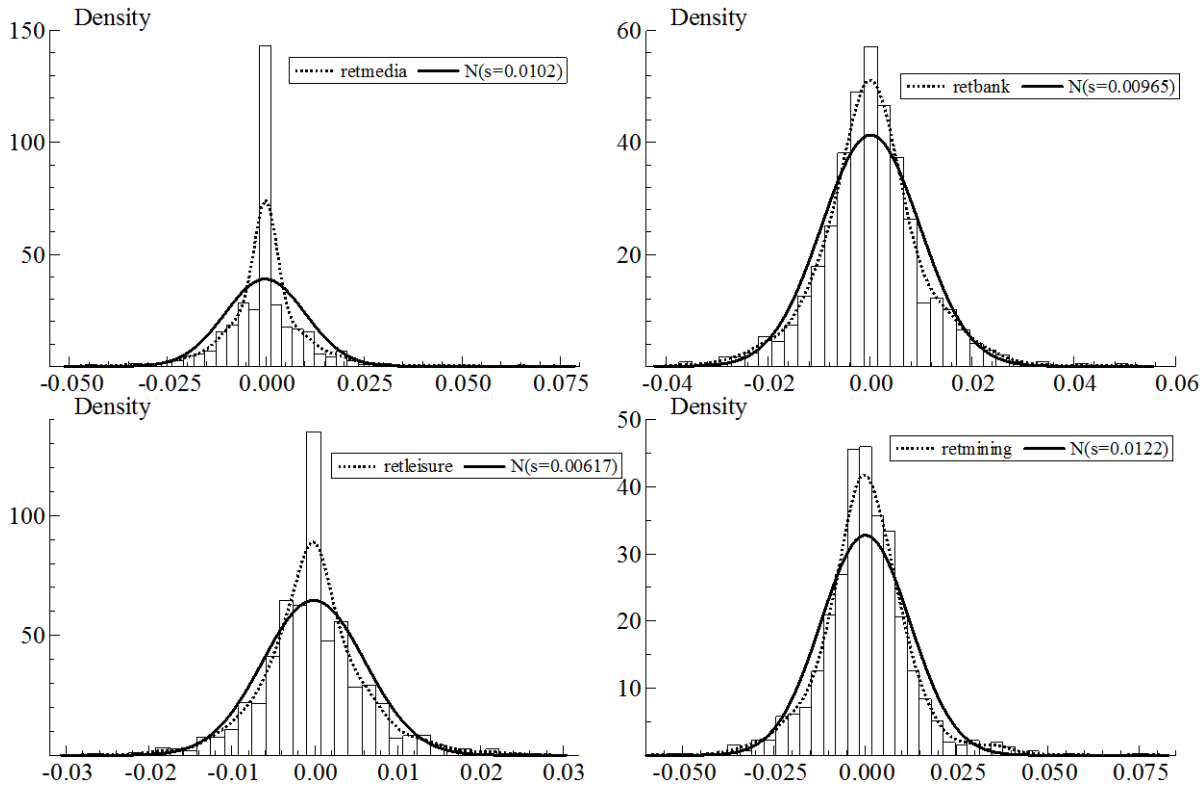
Appendix C: Empirical Density Functions for Exchange Rates



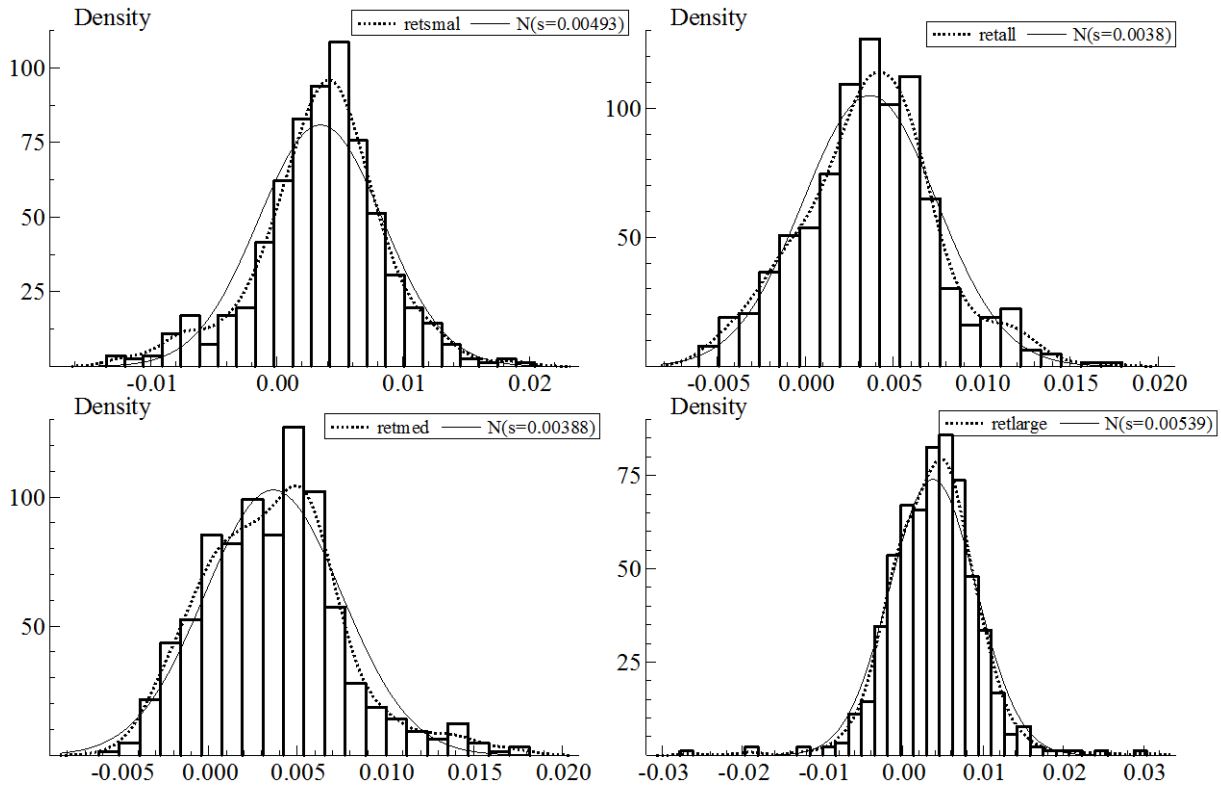
Appendix D: Empirical Density function for Interest Rate



Appendix E: Empirical Density Function for Stock prices



Appendix F: Empirical Density Function for House prices



Appendix G: Structural breaks test for House prices

All House price

Multiple breakpoint tests
Bai-Perron tests of L+1 vs. L sequentially determined breaks
Date: 12/09/14 Time: 13:59
Sample: 1966M01 2012M04
Included observations: 556
Breakpoint variables: C
Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05
Test statistics employ HAC covariances (Prewhitening with lags = 1, Quadratic-Spectral kernel, Andrews bandwidth) assuming common data distribution

Sequential F-statistic determined breaks:	1

Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	19.56104	19.56104	8.58
1 vs. 2	8.208377	8.208377	10.13

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Break dates:

	Sequential	Repartition
1	2004M01	2004M01

Large House price

Multiple breakpoint tests
Bai-Perron tests of L+1 vs. L sequentially determined breaks
Date: 12/09/14 Time: 13:58
Sample: 1966M01 2012M04
Included observations: 556
Breakpoint variables: C
Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05
Test statistics employ HAC covariances (Prewhitening with lags = 1, Quadratic-Spectral kernel, Andrews bandwidth) assuming common data distribution

Sequential F-statistic determined breaks:			1
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	17.02250	17.02250	8.58
1 vs. 2	8.611825	8.611825	10.13

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Break dates:

	Sequential	Repartition
1	2004M03	2004M03

Medium House price

Multiple breakpoint tests
Bai-Perron tests of L+1 vs. L sequentially determined breaks
Date: 12/09/14 Time: 13:54
Sample: 1966M01 2012M04
Included observations: 556
Breakpoint variables: C
Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05
Test statistics employ HAC covariances (Prewhitening with lags = 1, Quadratic-Spectral kernel, Andrews bandwidth) assuming common data distribution

Sequential F-statistic determined breaks:			1
Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	21.32367	21.32367	8.58
1 vs. 2	9.693994	9.693994	10.13

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Break dates:

	Sequential	Repartition
1	2004M03	2004M03

Small House price

Multiple breakpoint tests
Bai-Perron tests of L+1 vs. L sequentially determined breaks
Date: 12/09/14 Time: 13:46
Sample: 1966M01 2012M04
Included observations: 556
Breakpoint variables: C
Break test options: Trimming 0.15, Max. breaks 5, Sig. level 0.05
Test statistics employ HAC covariances (Prewhitening with lags = 1, Quadratic-Spectral kernel, Andrews bandwidth) assuming common data distribution

Sequential F-statistic determined breaks:	1

Break Test	F-statistic	Scaled F-statistic	Critical Value**
0 vs. 1 *	20.74897	20.74897	8.58
1 vs. 2	9.854830	9.854830	10.13

* Significant at the 0.05 level.
** Bai-Perron (Econometric Journal, 2003) critical values.

Break dates:

	Sequential	Repartition
1	2004M01	2004M01

Appendix H:Maximum Likelihood Estimation

We briefly characterise maximum likelihood estimation. If prices follow a diffusion process with constant drift parameter $E[\Delta P / P] = \alpha$ and constant variance $V[\Delta P / P] = \sigma^2$, the logarithm of price relatives $x_t \equiv \ln(P_t / P_{t-1})$ is normally distributed with mean $\mu \equiv \alpha - \frac{\sigma^2}{2}$ and variance σ^2. With T independent observations, the logarithm of the likelihood function $L(\theta, x)$, which is viewed as a function of the parameter vector $\theta = (\mu, \sigma^2)$ can be stated as

(A1) $$l_N = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\frac{1}{\sqrt{\sigma^2}} \exp\left(\frac{-(x_t - \mu)^2}{2\sigma^2}\right)\right]$$

If we consider a Process where λ is the mean number of jumps occurring per unit of time and the jump size Y having a distribution $\ln Y \sim N(\phi, \delta^2)$. The loglikelihood function for the mixed jump diffusion process is

(A2) $$l_J = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\sum_{j=0}^{\infty} \frac{\lambda^j}{j!} \frac{1}{\sqrt{\sigma^2 + j\delta^2}} \exp\left(\frac{-(x_t - \mu - j\phi)^2}{2(\sigma^2 + j\delta^2)}\right)\right]$$

To optimize the above log-likelihood function, the infinite sum is truncated after some value of N (Jorion, 1988). Ball and Torous (1985) derive a formula for the upper bound on the truncation error, it can be used to select desirable level of N. However, in practice, truncation at N=10 is considered satisfactory for all parameter values. The leptokurtic distributions can also arise as a result of time varying parameters instead of discontinuities. The ARCH process is one such approach (Engle, 1982). It is a tractable specification where the conditional variance h_t is modelled as non stochastic function of past squared deviations (Jorion, 1988). The simplest specification to choose from here is the first order ARCH model. The conditional volatility hereunder is written as:

(A3) $$h_t = \alpha_0 + \alpha_1 \in_t^2 = \alpha_0 + \alpha_1 (x_{t-1} - \mu)^2$$

Where $\in_t$, is defined as $x_t - \mu$. In the presence of heteroscedasticity, the parameter α tends to zero, in which case α_0 represents the variance of a stationary diffusion process. This can be explained to include a weighted average of a number of past observations, or in general predetermined information. For the ARCH model, the log-likelihood is

(A4) $$l_A = -\frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T} \ln\left[\frac{1}{\sqrt{h_t}} \exp\left(\frac{-(x_t - \mu)^2}{2h_t}\right)\right]$$

If the conditional distribution of $\in$ has a Poisson distribution, the log-likelihood function is written as

$$l_{AJ} = -T\lambda - \frac{T}{2}\ln(2\pi) + \sum_{t=1}^{T}\ln\left[\sum_{j=0}^{\infty}\frac{\lambda^j}{j!}\frac{1}{\sqrt{h_t + j\delta^2}}\exp\left(\frac{-(x_t - \mu - j\phi)^2}{2(h_t + j\delta^2)}\right)\right]$$

The likelihood function includes the jump process, the ARCH process, and the normal process as special cases. It can be used therefore to construct a generalised likelihood ratio $\Lambda = \sup_{\theta\in\Phi 0} L(\theta; x) / \sup_{\theta\in\Phi} L(\theta; x)$, where the likelihood functions have been maximised (1) over the parameter space $\theta \in \Phi_0$ under the null hypothesis and (2) over the parameter space $\theta \in \Phi = \Phi_0 \cup \Phi_1$, which includes the alternative Φ_1. Under the null hypothesis, the statistic -2ln Λ has a chi-square distribution with degrees of freedom equal to the number of parameters between the two models. This asymptotic result holds because the two hypotheses Φ_0 and Φ are nested in the parameter space (Jorion, 1988).