

Towards improving rural learners' mathematics achievement: The roles of social relationships and identities



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Signed

Kehinde Adedayo Adeniji

30 June, 2024

DEDICATION

To my late father, Oluwafemi S. Adeniji and late mother, Amope M. Adeniji

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ABSTRACT

This study initially starts as a research journey towards improving rural learners' mathematics achievement by interrogating the roles which teacher-learner social relationships and learners' mathematical identities could play. To make the journey productive, the study was framed with critical realist philosophy and social realist perspective because of their greater concerns for reality in general and social reality in particular, respectively. However, both realist assumptions were found not robust enough to fully explain the key findings that emerged from the evidential data gathered. The curiosity to fill this gap led to another research journey that re-examined the two realist positions. The new research journey eventually brought in new philosophical and sociological viewpoints maintained and by extension methodological approach employed in the study. That eventually led to the dominance of the second research focus and the suspension of three of four objectives of the initial research focus.

Employing mixed research designs (i.e., correlational and case study design) and mixed methods (i.e., questionnaires and interviews) inspired by the viewpoints and approach respectively, I was able to select four Nigerian rural schools and 237 learners for quantitative study, of which five of the learners participated in the qualitative study. All the data collected from the learners through the questionnaires and interviews were analyzed quantitatively and qualitatively respectively. The results/findings show that *generally* there is a strong relationships between teacher-learner social relationships and learners' mathematical identities for learners in rural schools in Nigeria. It also show that the nature of relationships can be *validly* considered as direct, indirect and neither direct nor indirect to provide deeper insights. At a much deeper level, it *practically* generated and explained why the relationships and the learners acted the way they did regarding mathematics.

These findings were used to justify the new philosophical, theoretical and methodological views maintained in the study. The fundamental view, being philosophical, maintains that reality is a whole which is analytical separable to entities that are necessarily dependable on one another. This novel and debatable position deviates from existing Bhaskar's CR philosophy and expected to shape any future study that has greater concerns for social reality such as mathematics learning.

Keywords: Critical realism, social realism, agency, mathematical identity and social relationships

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LIST OF ABBREVIATIONS

SREPs	Social Relationships Enhancement Practices
CR	Critical Realism (or Realist)
M/M	Morphogenetic/Morphostatic
OECs	Operational Emergent Cases
RECs	Relational Emergent Categories
PERs	Personal Emergent Reflexives
R-CR	Re-enforced Critical Realism
SRs	Social Relationships
MI s	Mathematical Identities
MA	Mathematics Achievement
EA	Emotional Attachment
SB	Social Belonging
SC	Socioemotional Connection
PS	Productivity Support
SA	Sociocognitive Ability
PD	Productive Disposition
SSSI	Senior Secondary School I
T-LSRs	Teacher-learner Social Relationships
LMI s	Learners' Mathematical Identities

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CHAPTER ONE

INTRODUCTION

1.1 Motivation for the study

The persistent low academic achievement of African rural learners in mathematics requires attention from stakeholders in education especially mathematics educators and researchers. The information available from Southern and Eastern African Consortium for the Monitoring of Education Quality (SACMEQ) and the Monitoring Learning Achievement (MLA) Project in Africa surveys since 1990s show the consistency in this low achievement when compared with their urban counterparts (Chinapah *et al.*, 2000; Chinapah, 2003; UNESCO-UNICEF, 2003; www.sacmeq.org). The analysis of eleven African countries' data that were available for 1999 MLA survey shows significant mean score differences – varying from 0.5 to 8.8 – between rural and urban learners, with the exception of nonsignificant result for Uganda (Chinapah *et al.*, 2000).

Also, the analyses of SACMEQ II, III and IV conducted between 2000 – 2015 indicate that for all countries surveyed – with the exception of Mauritius in SACMEQ IV – urban learners substantially and consistently outperformed their rural counterparts in mathematics. For example, SACMEQ III data show rural-urban mean score disparities that ranged from 9 – 76 (Department of Basic Education, 2017; Dwarkan, 2017; Chabaditsile, Galeboe & Nkwane, 2018; Shigwedha, Nakashole, Auala, Amakutuwa & Ailonga, 2017; Zhang, 2006; DEPRS, www.sacmeq.org). These data recorded in both MLA and SACMEQ surveys provide the basis to argue that in many mathematics assessments, urban learners have chances of doing substantially better than the rural learners in Africa. To explain the rural-urban achievement gap, UNESCO (2004) argues that income and education resources, which include teachers and teaching, play significant roles. In fact, UNESCO (2003) confirms that rural residence is an important factor for rural learners' underachievement, and the best explanation for educational inequality when compared with their counterparts in the non-rural areas lies in the type of instruction available to them.

Having recognized the disadvantaged effects of these achievement gaps on both human development and mathematics advancement, several mathematics education experts have called for studies which focus more on offering or proposing solutions rather than dwelling on reporting deficits of marginalized learners (Graven, 2013; Gutiérrez, 2002; 2008; Howley & Gunn, 2003;

Moletsane, 2012; Valero *et al.*, 2015). For example, Howley and Gunn (2003) suggest that researchers in the area of rural mathematics education need to direct and frame their studies towards silent questions “about structural features, equity and adequacy, and collective purpose” as concerns real “rural lifeways” instead of “measures of differences on conventional valued quantities” (p. 94). This suggestion is not limited to rural contexts, it is also relevant to all marginalized groups and underrepresented communities.

To better understand educational inequalities and offer appropriate strategies for solutions, researchers have outlined dimensions to consider for investigating inequality. Gutierrez (2007, 2008) emphasizes that any research that considers proposing solutions to close the gaps in mathematics education among marginalized groups needs to look at: access to and availability of resources to teach and learn mathematics; achievement in test scores and participation; identity in recognition of and maintaining learners’ experience, confidence, culture and language; and power to make changes in school and society. Gutierrez further categorizes these dimensions as either dominated or critical, where dominated dimensions are access and achievement, and critical dimensions are identity and power. The dominated categories are widely conceived of as means of measuring or assessing learners’ access to and attainment in mathematics; which is why they are more pronounced in the literature. The critical categories emphasize the learners’ self-realization and sense of belonging to analyze, critique and enhance the dominated categories and the society at large. Efforts directed at both categories may not only address the inequalities in every aspects but also improve mathematics attainment among marginalized communities. Therefore, addressing inequalities among marginalized learners requires equal attentions to both categories (Gutierrez, 2002; 2007).

Researchers have argued that very few empirical studies have focused on rural mathematics education especially as relevant to rural contexts (Howley, Howley & Huber, 2005; Mbhiza, 2021; Waters, Howley & Schultz, 2008). Using ERIC database search between 1985 – 2001, Waters *et al.* (2008) show that less than two dozen studies addressed issues concerning mathematics education in rural schools. Recently, a few more studies have been focusing on improved teaching strategies and pedagogical approaches to address the low mathematics attainments among under-served learners and disadvantaged communities by giving relevance to the critical dimensions. Some of these studies on under-served learners develop teaching strategies and initiatives drawing

from the strength of the learners' language, culture, experiences, ideas, and local context to close the attainment gaps (e.g., Brodie, 2006; Chiang, Clark & McConnell, 2017; Gutstein, 2003; Wendy, Lewis, & Heaton, 2013). But how the recommendations from some of these studies are implementable in the larger rural contexts in Africa and their adequacies for further empirical investigations still appear problematic. According to Howley *et al.*, (2005), very few strategy recommendations from the studies reviewed between 1993 – 2002 were adequate for further empirical rural mathematics education studies. Some of the recommendations provided by these studies – as observed by the teachers and researchers – come with one or more constraints. For example, some require serious curriculum shifts, more planning time and mathematical knowledge (e.g., Hill, Rowan, & Ball, 2005; Howley *et al.*, 2011; Marshall, Petrosino & Martin, 2010; Marshall, & Sorto, 2012), and various resources not often available in rural schools (e.g., Perrone *et al.*, 1999). So, many researchers have either expressed their concerns about how some successful teaching strategies are not finding their way into curriculum (e.g., Brodie, 2017) or observed the teachers' struggles to get new curriculum implemented in the larger classroom practices (e.g., Brodie, Jina & Modau, 2009). The time required to train teachers on the new teaching strategies and the financial implications are also possible impediments.

As a result, the focus of this study is to explore flexible social practices that emerge from teacher-learner social relationships in mathematics classroom that take the peculiarities of the rural contexts into considerations and at the same time to consider their implementation by the mathematics teachers in classroom settings. Flexible social practices include valuing and respecting learners' views and encouraging and motivating learners for success, which relate to Guitierrez's critical dimensions and are more implementable by the teachers of mathematics in respect of the available resources in a school and the teaching pedagogy employed by the teachers. Teacher-learner social relationships that afford rural learners these flexible social practices may support their positive identity development (Bibby, 2009; Darragh, 2013; Gardee, 2019a; Patrick, Ryan & Kaplan, 2007). My argument here does not consider these social practices as a stand-alone strategy for addressing low mathematics achievement among rural learners but rather emphasizes that their integration with traditional pedagogy may likely be a useful place to start thinking about closing the achievement gaps.

1.2 Statement of the Problem

Rural areas accommodate the majority population of Africans and rural areas all over the continent have the majority of their schools characterized with either shortages or below the expected standard in both human and physical resources (International Labour Office, 2016; UNESCO, 2004; UNESCO, 2016; United Nations, 2019). It is well documented that limited learning resources are common in several African rural schools (e.g., ILO, 2016; World Bank, 2000a). In Nigeria, rural schools attract less educational interventions from government (e.g., Anyaegbu, Christman & Jingpu, 2004; Odia & Omofomwan, 2007). In South Africa, similar rural schooling conditions exist (e.g., Graven, 2013; Spaul, 2019; Wolhuter, 2014). Letseka (2013) further affirms that many rural schools in South Africa are characterized with limited basic infrastructures for learning, inadequately qualified teachers and underperformance in mathematics. These conditions are almost the same in the eastern, northern and central African countries (UNESCO, 2016; United Nations, 2019; World Bank, 2000a). These contexts have significant influence on the type of teaching practices that are implementable – especially in the larger mathematics classroom settings.

Social relationships are teacher-learner interpersonal relationships (Rytivaara & Frelin, 2017) “which involve respecting and valuing learners” (Gardee, 2019a, p. 234). These relationships include positive social attention, verbal encouragement, and motivation for success and achievement offered by the teachers within and outside the classrooms. Felner, Aber, Primavera and Cauce (1985) argue that a supportive teacher-learner relationship has potential to build more academic efficacy among learners. An argument has also been made for teachers’ verbal encouragement in promoting students’ achievement, motivation and self-efficacy (Patrick, Ryan & Kaplan, 2007). In their longitudinal study, Fan, Lindt, Aroyyo-Giner and Wolters (2009, p. 37) confirmed that “when students feel liked and supported by teachers, instead of being ignored or made fun of, building confidence in mathematics” is likely to occur.

Despite these findings, little effort has been directed to explore what positive influence social relationships between teachers and learners could have in enhancing identity and achievement among rural learners, and the accompanying influence of enhanced mathematical identities and achievement on the social relationships between teachers and learners. Also, very little is known about the relationships between mathematics achievement and identities of rural school learners

in relation to teacher-learner social relationships in rural African schools. Identity is a growing research area in mathematics education, but still limited to exploring either learners' and teachers' identities in well-resourced and township schools (e.g., Brodie, 2018; Gardee, 2019a, 2019b; Gardee & Brodie, 2019; Gweshe & Brodie, 2019) or with less focus on inequality or disadvantaged situations of learning (e.g., Anderson, 2007). Therefore, there is a need to extend research in mathematics education to explore what influence social relationships between teachers and learners have on the mathematical identities constructed by and the achievement of the learners of rural schools. Concomitantly, how these identities and achievement influence the kinds of relationships between teachers and learners is also worth investigating.

At the heart of Gutierrez's dimensions is that conceptualizing and exploring identity as a socially constructed phenomenon may present different, flexible and powerful insights into enhancing marginalized learners' mathematics achievement. This enhanced achievement is expected to eventually present the learners the opportunities and powers to become major players and decision makers in the society. According to Walker (2012, p. 80), "experiences that promote mathematics socialization, identity development, and learning are critically important" for under-served learners. Similarly, Gweshe and Brodie (2019) suggest that understanding learners' identities and the development of strategies that transform these identities positively should be considered if underachievement in mathematics has to be addressed. In this current study, an intervention in form of social relationships enhancement practices (SREP) was implemented with selected mathematics teachers. The intervention is designed around possible teachers' social practices that enhance learners' identities (e.g., Bibby, 2009; Darragh, 2013; Fan *et al.*, 2009; Gardee, 2019a Patrick, Ryan & Kaplan, 2007), and by extension hypothesized to influence their achievement. Indicators of these social practices in relations to teacher-learner relationships are discussed in Chapter 4 and further exemplified in the methodology Chapter.

The purpose of this study is to explore the influence of teacher-learner social relationships on the mathematical identities and mathematics achievement of rural school learners, and how these identities and achievement influence the kinds of relationships offered by the teachers, with a view to suggesting teachers' social practices for addressing institutionalized rural learners' underachievement. By social practices, I refer to in and outside classroom practices that teachers enable for respecting learners' views, encouraging and motivating learners for success. The study

also aims to explore the influence of mathematics achievements and mathematical identities on each other, among the learners in rural schools. Overall, each of these influence explorations are going to be theoretically situated and comprehensively empirically investigated. The study is expected to be a source for formulating teachers' social practices that could be employed to disrupt gaps in learning outcomes of marginalized learners like those in rural schools. The study is also expected to contribute to the research base about teacher-learner social relationships and the mathematical identities of learners – especially as related to rural contexts.

1.3 Research Questions

There is a central research question and four research sub-questions for this study.

Central research question:

In rural schools, how do teacher-learner social relationships, learners' mathematical identities and achievement influence one another in the context of enhanced social practices?

Research sub-question 1:

What influence do social relationships between teacher and learners have on rural school learners' mathematical identities and vice versa?

Research sub-question 2:

What influence do rural school learners' mathematical identities have on their mathematics achievement and vice versa?

Research sub-question 3:

What influence do social relationships between teacher and learners have on rural school learners' mathematics achievement and vice versa?

Research sub-question 4:

How effective is social relationships enhancement practices (SREP) in improving the relationships between: teacher-learner relationships and learners' mathematical identities; learners'

mathematical identities and achievement; and teacher-learner relationships and learners' mathematics achievement in rural schools?

1.4 Justification of the research questions

Recent studies have shown how social relationships between teachers and learners influence mathematical identities constructed by the learners in their classroom community. When teachers present learning experiences which offer their learners opportunities to construct their identities in affiliation with or marginalization from classroom community, learners construct their mathematical identities in relation to what the teacher offers (Gardee, 2019b). Gardee finds that learners who are offered positive social relationships by their teachers tend to develop identities of affiliation with the mathematics classroom community while those who are offered negative social relationships by their teachers tend to develop identities of marginalization even if they experienced the same form of pedagogy (Gardee, 2019a). Positive or negative relationships between learners and teachers can influence the kinds of mathematical identities learners construct and concomitantly shape how they participate in the classroom community and learn mathematics. But how these relationships influence mathematical identity constructed by rural learners is yet to be explored. It also appears that no known intervention study focuses on social practices in that respect.

Research on how learners' mathematical identities influence mathematics achievement is still not clear. For example, the study of Gweshe and Brodie (2019) shows that relationships between learners' test scores and the kinds of identity constructed by the learners show inconsistent relationships between the two variables. The study could not relate learners' identities strongly with mathematics achievement; high, average and low achieving learners exhibited similar and overlapping identities. However, the positive relationships between the teacher-learner relationships and the identity constructed by the learners, and the influence of these relationships in sustaining learners' achievement (Gardee, 2019a) suggests the possibility for mathematical identity to influence mathematics achievement. There are arguments that improved mathematics knowledge stimulates positive identities, but another strong argument has claimed that enhanced identity realization also presents better opportunities for learners to learn mathematics by making sense of themselves and their learning (e.g., Varelas, Martin & Kane, 2012; Walker, 2012). This

argument emphasizes the learning experience that enables learners to translate their mathematical identity to achievement and empowerment. Efforts to find clarity in the relationships between mathematical identities and mathematics achievement, and to develop intervention that positively transforms them are important for rural learners.

As it is useful to explore the influence of teacher-learner social relationships on the learners' identities and achievement, so also is the influence of the achievement and identities on teacher-learner social relationships because the relationships may be bidirectional. Teachers are more likely to create good relationships with and provide more supports for learners they perceived as able to do mathematics and who they see as having good mathematical qualities (Gardee, 2019b; Walker, 2012). However, studies on how learners' actual mathematics achievement and teacher-learner social relationships influence each other are not only scarce, but the explanation for the mediating mechanism between the two variables is also not clear yet (Wang, Rubie-Davies & Meissel, 2018). There is also a concern that earlier studies in this area only give attention to individual mathematical qualities at expense of social aspects. Having a nuanced understanding of learners' individual and social mathematical qualities which influence social relationships offered by the teachers is a productive direction towards advancing mathematics education research (e.g., Rubie-Davies *et al.*, 2020), and to developing responsive intervention studies for marginalized learners.

1.5 Operationalizing identity

The above discussion shows that studies have related teacher-learner social relationships with learners' identity but how the latter relates with achievement still remains inconsistent (e.g., Gweshe & Brodie, 2019). It is suggested that to address learners' mathematics underachievement, nuanced conceptualization of mathematical identity is paramount (Gutierrez, 2002; Gardee & Brodie, 2019; Gweshe & Brodie, 2019).

Although a comprehensive definition of mathematical identity is still largely a challenge among researchers, there is a relative agreement on key aspects of mathematical identities of learners. These include persistence, sense of belonging, confidence or self-efficacy and beliefs (Bishop, 2012; Darragh, 2013; Gweshe & Brodie, 2019). So, one definition of mathematical identity,

according to Gardee (2019b, p. 3), is “constituted by relationships between personal identity, social identity and agency in relation to mathematics”. Personal identity relates to learners’ interests, emotions, aspirations and self-understandings regarding mathematics learning, and social identity relates to how learners position themselves and want to be positioned in the classroom community, while agency shapes the choices in how learners construct both personal and social identities (Gardee & Brodie, 2019; Gardee, 2019a, 2019b). This definition may offer nuanced understanding to navigate through and research learners’ mathematical identity.

Beyond the issue of definition, other challenges include nuanced operationalization of mathematical identity. This operationalization is key because how classroom practices and achievement are informed and improved by mathematical identity understanding cannot be built only on speculations, as it seems currently (e.g., Heyd-Metzuyanim, 2013). By extension, the different assertions about the ways in which institutionalized classroom practices and current conceptions of mathematics achievement distribute successes and reproduce or deny powers through mathematical identity lenses need to be well grounded theoretically and supported empirically. One of such assertions is the one from Darragh and Radovic (2018) which needs to be taken further. Darragh and Radovic (2018, p. 3) give insights about the potential of mathematical identity in the classroom community and beyond:

Identity has provided another way of looking at the practices within the mathematics classroom, beyond a focus on teaching and learning, to consider how the classroom culture may enable (or discourage) students from identifying with the subject of mathematics, how it distributes and reproduces power and may even propose an “alternative politics of possibility” (Chronaki, 2011, p. 210).

It then clear that despite its crucial position in understanding mathematics learning and achievement, mathematical identity is not only needed to be considered carefully but also situated in a well-grounded theorization and further operationalization.

1.6 Definition of terms

Besides the key terms that I used in this project, which must be defined during the course of the study, some other certain terms are also commonly used for the operation of the study. These terms

are not necessarily defined in the study but needed to because of the specific operational meaning they hold for the study. The following are the terms and their specific definitions for this study.

Reality or social reality: In general sense and the way it is used in this study specifically, this term means the true state of the (social) phenomenon of interest.

Elaboration: The term, in different places of arguments in the study, is used to show the need to further do something to bring better understanding to what it is being studied. In other words, it used to call for how true state (reality) of what is being investigated can be achieved.

Theorization: This relates to the generalized process of understanding the reality of interest. This is commonly used when I am trying to bring general understanding of a concept or different concepts to the fore.

Relationships: This term is commonly used to capture the influence of two concepts on each other.

Theoretical concepts or concepts: These terms are always used to represent three main focus of this study. These are mathematical identity, teacher-learner social relationships and mathematics achievement.

Mechanisms, dimensions or constituents: These terms are interchangeably used in the study to mean the compositions or elements of each of the three concepts of the study.

Individual: This is used with two different meanings in mind. In one hand it is used to capture individualist perspective rather than collectives, and in the other hand it is used to talk about persons as individual.

Emergent: It is always used to explain or describe the philosophical and by extension, theoretical idea of one thing coming out of another where the first and the second can exist together dependently.

Operationalization: This is an operational term strategically used to describe a concept or an idea from one philosophical or theoretical status to another (e.g., ontological to epistemological status or theoretical perspective to theoretical framework status).

1.7 Structure and overview of the thesis

This thesis ‘embraces’ emergentist (or realist) approach of conducting and structuring this study. The approach involves writing the thesis Chapters alongside with every aspects of the study conducted. This approach deviates from deterministic styles, which either allow completion of all the aspects of the study before writing the Chapters (pragmatist) or writing the Chapters before even conducting any part of the study (presentist). Few critical things emerged after the data collection and analysis which made what had already written, especially for the title, introduction and the literature review of the study appeared to be hanging somewhere; they are not. It is important to stress that emergence of this approach made the whole study more messy but very productive, and eventually insightful for me. To clear any possible mix-up for the readers, I quickly offer the following overview or summary of each Chapter of my study.

Chapter 2: Literature reviews. This Chapter solely reviewed the literature on the relationships between the key concepts of the study – social relationships, mathematical identity and mathematics achievement. The review specifically employed systematic review approach not only to establish the current positions on the relationship between the three concepts but also to establish the research perspectives and methodologies mostly embraced by the researchers.

Chapter 3: Philosophy and methodologies underpinning the study. This Chapter is the *heart* of this project. This is because it solely and critically interrogated the critical realist philosophy and realist social theory and by extension, morphogenetic/stasis (M/M) methodological approach chosen to inform the whole processes and stages of the study. Some crucial inconsistencies and non-realist methods/assumptions were respectively found in the critical realism (e.g., stratified ontology instead of emergent ontology and epistemology relativism instead of realism) and M/M approach (e.g., integrated instead of contingent social system and independent/flat instead of dependent/emergent structure-agency interaction) and the Chapter ‘fixed’ them by making new assumptions for the study. This emergent was not the initial plan. But after the data collection and analysis, some issues within the philosophy and methodology became clearer and new assumptions were made. Making such assumptions automatically resulted into the emergence of *two of one* study. That emergence *contingently* shaped my original and the newly emerged study.

Chapter 4: Study’s theoretical framework and methodologies. This Chapter is an offshoot of Chapter 3, it was initially combined with Chapter 3. Theoretical and methodological assumptions

that emerged in Chapter 3 were operationalized in this Chapter. Those assumptions are key to the main arguments in Chapter 3 because of their implications on the theorization of and realist methodology employed for the study's concepts, and by extension, the methodologies employed in Chapter 5 and the evidence presented in Chapter 6. In a nutshell, this Chapter is a *bridge* between what happened ontologically (or transcendently) and empirically (or evidentially), so it holds a refined epistemological position.

Chapter 5: Research design, methodologies and data. Research design and methodologies employed to access, collect, process and analyze the empirical (observed or measured) data about social relationships, mathematical identity and mathematics achievement were presented in this Chapter. The presentation involved multiple procedures and mixed methods employed at two different time points. Interestingly, all these procedures and methods were initially written for my original study based on my personal exposure to them, but later turned out to largely aligned with the philosophical and methodological arguments I made in Chapter 3. In other words, those procedures and methods were claimed by the critical and social realisms I argued for in Chapter 3. Even with the alignments, some adjustments were still made to the Chapter. The Chapter, as against the initial plans, presented how the research designs were reconsidered and the data were re-processed, re-analyzed and reported in line with the new development. Criteria for establishing explanatory powers of the evidence produced in Chapters 6 were also included.

Chapter 6: Findings, evidence and discussions. As expected, at least a Chapter must be dedicated to answer the questions raised from the beginning of and provide evidence for the philosophical, theoretical and methodological assumptions/claims made in this study. That is exactly what Chapter 6 did but with re-consideration. Only the results (quantitative) and findings (qualitative) regarding the relationships between mathematical identities and social relationships were considered and reported. The reasons for the re-consideration are three: philosophical, future studies and more importantly, time factors. Strong philosophical reasons were given in Chapter 3. Related to that is the opportunity for future studies or projects (e.g., post-doctoral studies), and lastly, the time has gone against me to complete this PhD studies. More closely, the substantial (i.e., philosophical) knowledge contributed by this thesis as it is, is far more than the partially suspended theoretical and practical/empirical knowledge that the rest of the reports will offer.

Lastly, the Chapter discussed the results/findings and evidence provided with the reviewed studies in Chapter 2 and other relevant ones.

Chapter 7: Contributions, limitations and future studies. The last Chapter, Chapter 7 discussed the contributions of the study and I systematically made the ‘academic or scientific sense’ out of all the complexities that accompany the study. Also presented in this Chapter are the limitations of the study and expectedly, recommendations for future studies.

CHAPTER TWO

LITERATURE REVIEW

2.1 Introduction

This study draws on three main bodies of literature: social relationships, mathematical identity, and mathematics achievement, in relation to rural mathematics teachers and learners. Generally, how the rural context is situated in the relationships among these bodies of knowledge still needs more attention. Thus, this section broadly reviews the existing studies on the relationships among social relationships, mathematical identity, and mathematics achievement, but with more attention given to those studies on underserved or marginalized learners, as well as focusing on inequality in teaching and learning conditions. There are studies in African rural context that consider some aspects of social relationships and mathematical identities which I also review in this section. Finally, I review the two considered influential works on social theories that are commonly used in mathematics education studies.

2.2 Social relationships and mathematical identities

There is almost no controversy among researchers regarding relationships between social relationships and mathematical identities: the accumulated research shows clearly that social relationships in and outside the classroom strongly influence the mathematical identity constructed. While some researchers considered the relationships in both ways in their studies (e.g., Gardee, 2019a, 2019b; Gardee & Brodie, 2019; Heyd-Metzuyanim, 2013), many focused only on the influence of social relationships on mathematical identities (e.g., Bibby, 2009; Darragh, 2013; Hodgen & Marks, 2009; Walker, 2012). It is noticeable that all these studies are predominantly qualitative studies that draw heavily from sociological perspectives. I start with the study done by Walker (2012).

Walker's (2012) study focused only on one-way relationship. Using website resources and a snowball sampling technique, she selected and interviewed 27 African American mathematicians, and analyzed four vignettes of six of them. The findings were categorized into three spatial experiences: out-of-school, within school, and in-between school. The findings are almost the

same in all the spaces, but findings reported for those within schools are pertinent for the current study. Three out of four vignettes captured the mathematicians' high school experiences that go "beyond the prescribed curriculum to engage students' interests and potential": extracurricular mathematics opportunities, supportive relationships that existed between the mathematicians and their teachers and peers, and their productive engagement with mathematics (p. 77). The message of high expectations, recognition and encouragement offered to the mathematicians when in high school substantially and positively impacted their mathematical identities and learning.

Based on the overall findings, Walker (2012) argued that "persons and relationships from multiple worlds formed academic communities (comprising family members, peers, teachers, and others) who had an important impact on the mathematicians' development" (p. 78). And he finally concluded that individuals' practices and actions in these relationships and the experiences they had were built on their expectations. So, if schools can create spaces where learners can engage with mathematics meaningfully and feel a sense of belonging as mathematics learners, positive mathematical identities may be developed. However, there is a need to explore the current experiences of younger marginalized students, to confirm whether this finding is similar in their case. The study is limited only to the one-way relationship and positive experiences of successful mathematicians. Therefore, there is also a need to extend the study to explore converse relationships and different relationship experiences for a better understanding of socialization in mathematical identity development.

Darragh (2013) explored the influence of social experience and the transition from primary to secondary school, on how learners constructed their mathematical identities and "new understanding of what it means to be a mathematics learner" among girls of minority ethnic groups in New Zealand (p. 216). Her discussion about how the 'exemplary practice' of a teacher greatly influenced the confidence and sense of belonging constructed by the girls in the study is particularly important for the influence of social relationships on mathematical identities (p. 226). She observed the lessons of a class, interviewed the teacher and six student volunteers. Darragh found from the interviews, that "the students demonstrated a huge appreciation of their teacher, and at times it seemed as though they were making a point of telling me how good she was at teaching them mathematics" (p. 226). She used this finding to emphasize the significance of social practices such as inclusiveness in developing students' sense of belonging and confidence within

the mathematics classroom. But how the observations conducted were used in the analysis were not reported which could have added more to our understandings of the girls' identities and the teacher's practices that accounted for their construction. And more importantly, how different relationship experiences or situations in different contexts – such as rural and co-educational schools – will play out are important areas of study.

Hodgen and Mark (2009) addressed one important limitation of Walker's (2012) and Darragh's (2013) studies: how does the relationship between different experiences influence mathematical identities? Two teachers were interviewed about the histories of their mathematical experiences. The finding showed that different relationship experiences with their teachers and with mathematics influenced the type of mathematical identity constructed when they were in school. One teacher's interviews indicated her transformation from someone "who could do mathematics" to "who couldn't get to grips with it at all". This is because she "didn't get on" with her teacher who used to make "some derogatory comment" about her (p. 38 & 39). The second teacher's interviews indicated the different relationships that existed between her and two mathematics teachers. She and her peers had good relationships with their first mathematics teacher because she made them develop interests in mathematics and made mathematics learning "a bit fun" which gave her more confidence. Some of the confidence disappeared and mathematics became "hard to understand", however, when a new teacher who had a "good degree" and "lots of mathematical knowledge" but lacked "good control" became their mathematics teacher (p. 39).

Hodgen and Marks (2009) emphasized the crucial roles of positioning, imagination and central role of agency in how teachers and learners act together to construct mathematical identities and a cycle of inequalities. They argued that creating relationships with learners without making these relationships mathematical is not sufficient enough to develop learners' mathematical identities. As it will be reviewed in the next section, Hodgen and Marks (2009) further argued that, like their pupils, teachers also constructed their mathematical identities based on 'ability' and assessment. And this has led to vicious cycle of "replicating and reproducing the inequitable mathematics that they themselves experienced" (p. 40). Building on these arguments, Hodgen and Marks (2009) asserted that the findings in the study suggested that reconstructing teacher-pupil relationships and relationship with mathematics "potentially offers a way of breaking out" of the institutionalized inequality cycles produced by our assessment approach .

Gardee is one of the few researchers who explored the relationships both ways in a study. She showed how learners' engagement with mathematics should not be limited to teachers' pedagogical practices but extended to their social relationships. Gardee's (2019a) directs our attention to the significant role of teacher-learner relationships and their implications on equity. She interviewed and observed four learners and their Grades 9 and 10 classroom teachers, in a well-resourced secondary school in South Africa. The two teachers were different in their social relationships with learners, but similar in their pedagogical practices. The analysis of the influence of social relationships on mathematical identities showed that all the learners constructed their identities in relation to the kind of identities offered by their teachers.

Gardee (2019a) argued that the Grade 9 teacher offered the learners social identities of affiliation, and the learners constructed their identities in affiliation with classroom participation. When their Grade 10 teacher offered them social identities of affiliation or marginalization, they constructed their identities in affiliation, compliance or marginalization. All the learners constructed their identities in affiliation when offered opportunities and treated equally but constructed their identities differently when treated inequitably. Gardee claims that learners' agency accounted for why they constructed identity similar to or differently from the social identities offered. The salience and novelty of this finding is that learners' identities are not reducible to the social identities or relationships offered by the teacher who treats them differently, unlike the teacher who treats them equitably. However, this finding does not provide a recognized/generalized rule to explicitly show or explain how the relationships take place, nor the agency objectivated to inform practice or intervention development. The finding has also not been tested yet with a quantitative study. Conducting such quantitative studies may be challenging because *how* agency can be operationalized (or objectivated) to play the central role in the construction process in different situation needs more elaboration than has currently been done.

Gardee's (2019a) analysis also showed that mathematical identity constructed by learners influenced the kind of social identities offered by the teachers, as mediated by their expectations. From the teachers' narratives, Gardee was able to show that the teacher who believed that all learners could do and participate in mathematics offered social identities of affiliation to every learner, while the teacher who believed otherwise, only supported the learners of high mathematical achievement and marginalized those with low mathematical achievement. This is

evident in her study when only one learner was offered opportunities to affiliate with the classroom in Grade 10, because he was “a smart guy” in mathematics. Gardee’s other work (Gardee & Brodie, 2019; Gardee, 2019b), from the same project, which combined the actual and narrative practices of teachers and which accommodated more teachers, also confirmed these findings. She concluded by emphasizing the implication of teachers’ social practices in maintaining equity among learners. The strength of these studies is that they addressed a substantial part of the weaknesses of the previous reviewed studies: more data collection methods were used; they explored the relationships in both ways and relationships between different experiences were investigated. However, her finding is silent on the actual influence of mathematical identity constructed on social identities offered. It only emphasizes how teachers mediate the influence of learners’ mathematical identities. Also, how all these findings could play out in rural school contexts are still to be investigated.

Tsanwani, Hardin, Engelbrecht, and Maree (2014) considered a rural school context in South Africa. The purpose of the study was to investigate the learners’, classrooms’ and schools’ contexts that facilitate mathematics achievement in rural communities. Learners and mathematics teachers of ten Limpopo rural schools participated in the study. The schools were categorized as low-performing school (LPS) and high-performing school (HPS), based on their three years successive performance. The findings were categorized into: teachers’ and learners’ expectations; work ethics and practices; and learning culture and motivation. In HPS, teachers’ and learners’ expectations were high, learners were more motivated by the teachers and the learners were more committed to learning mathematics, than in LPS. In HPS, teacher-learner relationships were positive, learners had a culture of working together, teachers offered more support to the learners, and all learners were treated equally, unlike in LPS. Most of the learners in HPS believed they could do mathematics and were self-motivated for success and higher education, while those in LPS saw mathematics as a difficult subject, they were less motivated and blamed the teachers for their failure.

Ndlovu, Pournara and Mwakapenda (2019) investigated the views of Grade 10-12 learners about the nature of mathematics, before and after participating in a mentorship programme. The mentorship programme was designed to reverse the trend of unsuccessful interventions that excluded rural learners, and that fostered the inequitable access to mathematics learning in South

Africa. Two rural schools in Eastern Cape and 1,200 learners participated in the study. Ndlovu *et al.* (2019) found an increase in the number of learners whose views about the nature of mathematics had shifted from who saw mathematics as a set of rules, procedures and formulae one needs to memorize to who saw “mathematics as relating to real life every day and also that mathematics is abstract in its very nature” (p. 119). The learners also articulated effective pedagogical practices as what are core and desirable for them to learn mathematics. Ndlovu and colleagues made an argument for classroom climate that fosters learners’ productive dispositions to mathematics by connecting teaching practices with “the practical issues of socialization” that learners “engaged in [during] mathematics tasks” (p. 120). Based on this argument, they asserted that how learners see and learn mathematics are directly linked. It is worth noting that Ndlovu *et al.*’s (2019) mentorship programme is mainly pedagogical in design. An effort to also explore the effectiveness of socially guided practices on mathematical identity development is important, especially for underserved learners.

The common finding in the literature reviewed above shows that relationships between learners and mathematical community, and learners’ engagement or relationship with mathematics influence how learners identify with mathematics. Although learners’ engagement with mathematics seemed to be achieved pedagogically, other evidence clearly shows that creating learners’ social relationship with mathematics is also important. It has also been shown that how learners identify with mathematics influences the teacher-learner relationships, as mediated by the teachers’ expectations. The role of agency in explaining these relationships is pertinent but needs more elaboration, both for theorization and intervention development. In addition to this elaboration, little is also known about the learners’ mathematical identities and social relationships in the context of rural schools in Africa in particular. And there is no known intervention study focusing on teachers’ social practices to improve learners’ identities, especially in Africa. Finally, studies employing both qualitative and quantitative research approach in this area of study are still scarce.

2.3 Mathematical identities and achievement

The findings from the relationships between mathematical identities and mathematics achievement are largely mixed and one direction of the relationships received more attention than other. Several

studies have investigated the influence of mathematics achievement on mathematical identities (e.g., Berry, 2008; Hodgen & Marks, 2009; Berry, Thunder & McClain, 2011; Gweshe & Brodie, 2019; Mbhiza, 2024), while only two studies were found to relate mathematical identities to mathematics achievement (Mbhiza, 2021; McGee, 2015). Other studies give some great insights into the relationships in both aspects (e.g., Gulemetova, Beesley, Fancsali, & Balakrishnan, 2022; Sfard & Prusak, 2005a; Varelas *et al.*, 2012). The majority of these studies are also predominantly qualitative studies and sociologically oriented.

Berry *et al.* (2011) examined the mathematical identities of mathematically successful Black boys in rural schools in the USA, with the aim of highlighting the factors or experiences that contributed to this identity development. Thirty-two Black boys from Grade 5-7 who were recognized as high achievers – scoring high in standardized tests, and who participated in a summer program, were the participants in the study. Their analysis showed three themes: construction of mathematics identities; construction of racial identities; and relationships between the two identities. Contributing factors for mathematical identities were computational fluency; engagement with the unique qualities of mathematics; extrinsic recognitions such as scoring high, getting good grades and tracking; and relationships with teachers, families, and out of school activities. What contributed to the boys' mathematical identities were the positive relationships with a mathematics community and with challenging mathematics content.

The boys' beliefs about unequal treatment from teachers based on gender, ability, and race accounted for their racial identities. Berry *et al.* (2011) reported that Black boys felt isolated and not comfortable because their teachers gave them less attention than other students. For the third theme, the boys “recognized important attributes in themselves that enabled them to be successful in mathematics” (p. 19) and because of that, redefined their racial identities of “lack of success in mathematics” by creating a sense of “otherness” which eventually led to being successful (p. 18). One critical concern about the finding of the first theme is that, can engaging with challenging mathematics content fundamentally account for how learners construct their identities or are there other basic personal and social underlying engagements that are responsible? As it can be argued that it is only when learners develop productive dispositions toward mathematics that they are likely to be identified with its challenging content.

Gweshe and Brodie (2019) explored what influences mathematical identity development among learners, to gain insights into how mathematics disengagement can be addressed. The findings showed that past and present experiences in mathematics classrooms, family members' and peers' influences clearly shaped how learners constructed their identities, but the evidence for achievement was mixed and complex. This is because high and low achievers overlapped in their mathematical identities. There was interconnectedness in the ways both low and high achievers demonstrated and reported their confidence, persistence and beliefs in mathematics. Gweshe and Brodie (2019) also found that pleasing others, gaining prestige, and accessing career opportunities in mathematics-related jobs in the future were some influences on the learners' identities. This finding provides valuable insights into the importance of meeting oneself and other peoples' expectations in learners' identity formation, which could be helpful in understanding the relationships between achievement and identity.

The study of Mbhiza (2024) which is situated within rurality context also confirms the findings of Gweshe and Brodie (2019). Mbhiza (2024) set out to "explore and understand the experiences and attitudes of Grade 10 mathematics learners within rural schools" (p. 2). Twelve learners from two different rural schools in South Africa were purposively selected and interviewed. The analysis of the interviews showed that one major theme – future prestige and two sub themes – gaining University admission and securing job employment are the rural learners' reasons for learning mathematics. Interestingly, a nuanced insight of what Gweshe and Brodie (2019) offered can be inferred from this findings. The study seems to recognize the aspect of 'meeting other peoples' expectations' as being accommodated in the aspect of 'meeting oneself expectation' as it recognizes the two sub-themes of its findings as "a dimension of introjected regulation" (p. 7). The study, Mbhiza concludes, "contributes to a deeper understanding of the underlying factors influencing rural learners' engagement with mathematics and the implications for educational policy and practice". (p.10).

Another finding in the literature, is the negative influence of mathematics achievement built on assessment and 'ability' on mathematical identity construction. Hodgen and Marks (2009) described the entrenched inequalities in our assessment approach. They observed mathematics lessons of three schools and interviewed 42 pupils in groups and individually. They found that instead of assessment serving the purpose of documenting attainment and informing learning, it

has formed the basis for which learners create for themselves groups of pupils with a fixed trajectory of mathematical identities and achievement. That is, despite the evidence of mathematical achievement and identity development as being continuously improving and declining, pupils easily recognized and labelled one another as a member of a fixed group of mathematical abilities which reinforce the effects of prolonged assessment. Hodgen and Marks (2009) further reported that it was surprising that most of the pupils saw themselves as “nonmathematical”, and that all the pupils appeared to have “limited engagement with the actual mathematics” (p. 37). Based on these findings, they suggested that assessment should be “manageable” and more significantly, “open[ed] up rather than close[d] down possibilities” (p. 40).

McGee (2015) examined what characterizes Black African Americans’ mathematical identities and their influence on how they achieved higher in mathematics despite being stereotyped and marginalized. The interviews of a Professor of mathematics and a PhD holder in engineering, that represented other 21 participants in the study were selected for analysis. McGee found that both participants developed coping strategies, motivation to succeed, and a disposition for success which defined the fragility and robustness of their identities at different periods. She reported further that, regardless of their mathematical identities, both were still scoring higher in their tests, though with different dispositions. Their disposition was to please others when operating from a fragile identity position, and for self-fulfillment when operating from a robust identity position. Though, the basis for it is not clear, this study is unique because it presents a different way of defining mathematical identity by considering the qualities of the experiences instead of just participants’ experiences, like other researchers always do.

Mbhiza (2021) extends the study of relationships between MIs and MA to another dimension that is mostly ignored by the other studies, rurality or local influences. The unique aim of the study was to explore the relationship between rural learners’ beliefs in witchcraft, a form of rural or local beliefs and their mathematics learning. The study from onset strongly maintains that such entrenched cultural beliefs is inextricably linked with the rural learners’ discourse of their academics, especially in mathematics. Mbhiza, like in the earlier reviewed study, selected and interviewed 12 learners of Grade 10 of two different rural schools in South Africa. The analysis of the interviews confirmed his position as all the learners held that: there are a lot of witches in their

schools; and with their witchcraft they can bewitch you for demonstrating that you know mathematics. Based on these findings, Mbhiza concluded that “learners have engendered undesirable attitudes towards learning mathematics, that they cannot exhibit particular levels of understanding in the subject because of fear of witchcraft”. As such, he suggested “that when we consider learners' experiences and attitudes towards learning mathematics in contexts such as Acornhoek [i.e., location of the study], it is essential to consider both the cultural context of the society as a whole and the subcultures of the families of the children”. (p. 34).

In their efforts to provide analytic tools for linking learning with identity, Sfard and Prusak (2005a) conducted interviews with students. Analyzing the interviews of two students that represented other students in the study, Sfard and Prusak (2005a) found that the students greatly differed in the way they learned; the first student’s learning was ritualized, and the second student’s learning was substantial. Ritualized learning is characterized by complying with the rules and pleasing others, while substantial learning is characterized by learning beyond classroom activities and for its own sake. Further analysis showed that difference in the way they learned did not differentiate the students’ mathematics achievement, but it substantially related to their identities. The first student negotiated largely within actual identities, the stories that indicated her current state of affairs with mathematics, while the second largely negotiated within designated identities, the stories that indicated her future state of affairs with mathematics. Sfard and Prusak (2005a) concluded that these findings provided clear evidence for linking learning with identity and filling the gap between actual and designated identities.

But what comes as a concern about these findings is the “tight correspondence between the way the two students learned and the effectiveness of their learning” and mathematical identities they constructed (Sfard & Prusak, 2005a, p. 22). It would be expected that substantial differences in the ways the two students learned and constructed their identities should have resulted into some degree of difference in their achievement. It appears, directly or indirectly, that there is insufficient power for learning inclination of the students to differentiate their learning outcome (i.e., achievement) despite its substantial power to differentiate their identities, another product of learning. So, it is then either that mathematical identity needs more elaboration, or what is considered as actual mathematics achievement needs to be reconsidered or both combined.

More complication around the observations above is noted by Varelas *et al.*'s (2012) study that provided a theoretical framework that views learning as a process of identity construction and content learning. They concluded that "they [identities] are not only the products of learning [but] also influence opportunities to learn" (p. 16). With these conclusions, the question that comes to mind is, does learning determine who we are or is who we are a function of how we learn or are these both integrally interrelated? Efforts to find answers to this complex question by exploring the role of agency may provide clearer relationships between mathematical identities and achievement.

Opolot-Okurut's (2005) study focused on rural school context in Uganda, with the aim of investigating the learners' attitudes towards mathematics. He selected 254 grade 9 learners and nine schools of which seven of them were in rural areas in Uganda. All the schools in the study were categorized as either high performing school (HPS) or low performing school (LPS) based on their average marks in public examinations. An attitude towards mathematics inventory comprising three subscales – confidence in mathematics, motivation in mathematics, and mathematics anxiety scales were administered to the learners. The results showed significant differences in all the three attitudinal factors between HPS and LPS. Learners of HPS showed higher and better attitudes towards mathematics than their counterparts in LPS. Opolot-Okurut (2005) explained what accounted for the differences by arguing for certain teachers' pedagogies and students' prior mathematical competence in HPS.

A similar study was conducted by Butakor, Ampadu and Cole (2017) in Ghana, using quantitative data from the Trend in International Mathematics and Science Study (TIMSS) 2011. The study, in part, aimed to provide an understanding of factors that best explain learners' mathematics achievement. The analysis involved 15 independent learner variables and mathematics achievement as the only dependent variable. The results showed that nine variables significantly accounted for the learners' achievement. The variables were level of aspiration, gender, safety in school, perception of the school, frequency of homework, time spent on homework and attitudinal variables – self-confidence, perceived difficulty of mathematics and value of mathematics. Although the results were significant, Butakor *et al.* (2017) noted that these variables only accounted for 20% of the achievement recorded in the study. Therefore, more nuanced students'

factors should be explored for more comprehensive explanations. Studies that employ qualitative research were suggested by Butakor and colleagues.

As shown above, the relationship between mathematical identity and achievement is still largely unclear. The literature review shows that the influence of mathematics achievement on mathematical identity is inconclusive, and the influence of mathematical identity on mathematics achievement is under-investigated. Though there is evidence that some rural learners' personal factors or beliefs (e.g., self-confidence, motivation, perceived difficulty of mathematics etc.) and cultural beliefs (e.g., witchcraft) influence their mathematics achievement, they are not separately sufficient enough to provide full explanations for the influence. One of the ways of illuminating clearer relationships between mathematics achievement and mathematical identity may be to employ rigorous and nuanced understandings of cultural, social and personal factors or beliefs that contribute to mathematical achievement and identity formations and utilizing both the qualitative and quantitative research approach.

2.4 Mathematics achievement and social relationships

Studies have positively related teacher-learner relationships to mathematics achievement among different learners and in different contexts (e.g., Adegoke, 2011; Fan *et al.*, 2009; Gamlem, 2019; Rubie-Davies, 2015). Within this literature, several studies considered teachers' expectations as the major determinant of achievement, with learners' socio-psychological factors – e.g., self-expectation, self-concept, self-perception, self-efficacy and sense of futility mediating the influence (Agirdag, Van Avermaet, & Van Houtte, 2013; Benner, & Mistry, 2007; Flunger, Nagengast, Jonkmann, & Trautwein, 2015; Friedrich, Lazarides, & Watt, 2015; Gilbert, *et al.*, 2014; Haraoka, 1991). Some other studies focused on the influence of the teachers' expectations on these socio-psychological factors to provide explanations for the mediation mechanisms (Bohlmann, & Weinstein, 2013; Chen, 2006; Pesu, Viljaranta, & Aunola, 2016; Upadyaya, & Eccles, 2015). But studies for the converse explanations are still scarce or overlooked. It also appears, in sharp contrast from the bodies of literature previously reviewed, that a quantitative research tradition and socio-/psychological perspective have dominated this area of study. I review the studies of Gamlem (2019), Fan *et al* (2009) and Rubie-Davies (2015) and few others that capture the nuance of the situation I am describing.

Gamlem (2019) aimed to contribute to the understandings of teacher-pupil interactions in the classroom and their impacts on pupils' achievement scores. Observing 115 mathematics lessons of 9th grade pupils in three schools in Norway, their teachers were scored high, medium, or low in the quality of their interactions with the pupils. Pianta and colleagues' (2012) observation tool, CLASS-S¹ which has four domains and 12-dimension variables was used. A standardized National Curriculum Mathematics Test was used to measure the pupils' mathematics score before and after seven months of observations. Gamlem (2019) found that only "positive climate" and "students' engagement" enabled by the teachers contributed significantly to the pupils' achievement scores in mathematics. Positive climate was described as the teachers' practices that promote good relationships and emotional connections between pupils and teachers (Pianta *et al.*, 2012 cited in Gamlem, 2019). However, the importance of being mindful of learners' expectations and maintaining equal treatment among them in the classroom are not accounted for in the study, because these were not captured in the observation tool used.

Similar and more expanded study is the work of Fan *et al.* (2009). Fan *et al.*'s (2009) major focus was students' interpersonal relationships with their teachers, parents, and peers, and the influence of the relationships on the students' self-efficacy and achievement in mathematics and reading. The data for the study were drawn from their Educational Longitudinal Study (ELS, 2002) where 15,362 10th graders in 752 rural and urban, and public and private schools participated. The results showed that all the interpersonal relationships, with the highest effect in teacher-student relationships, significantly influenced the students' self-efficacy and achievement in both subject domains. Based on the analysis conducted, Fan and colleagues argued that the resulted influence was mediated by the students' self-efficacy – a belief that students have the capability and ability to learn and do mathematics (Bandura, 1977 cited in Fan *et al.*, 2009). They concluded that when students get along and feel encouraged in their personal relationships, especially with their teachers, their self-efficacy increases which eventually influences their academic achievement. As stated earlier, several other studies support these results with various socio-psychological factors (e.g., Benner & Mistry, 2007; Friedrich, *et al.*, 2015; Gilbert, *et al.*, 2014; Haraoka, 1991; Lazarides, & Watt, 2015).

¹ Classroom Assessment Scoring System for Secondary Schools.

Rubie-Davies (2015) conducted an intervention study with mathematics and English teachers in New Zealand. The intervention was developed based on the evidence that teachers' expectations and psychosocial beliefs differentiate how they relate with and provide support for learners, which ultimately influence learners' self-concept and achievement. Generally, the intervention aimed to inculcate the practice of grouping, goal setting, and to modify the psychosocial supports and beliefs of the teachers, with the expectation of changing the students' achievement and beliefs positively. Twelve schools, 84 teachers and 2,500 learners of 4th through 8th grade participated in the three years' longitudinal study. The report of the first year of the intervention was categorized into four parts: teachers' expectation; students' achievements gain; students' perceptions of teachers' expectation; and teachers' and students' beliefs. All the results favored the intervention group – high expectation teachers and their students.

The findings of the teachers' expectations, students' achievement gains, students' perceptions of teachers' expectations and students' beliefs are important in this review. The teachers' expectations were stable among intervention teachers and declined drastically from their original levels among the control group – low expectation teachers. Rubie-Davies (2015) explained this finding with the argument that teachers' expectations are naturally and generally high at the beginning of a term, and that the intervention “had a buffering effect” for intervention teachers. The mathematics achievement gains for intervention students significantly outweighed their counterparts in the control group. On the students' perceptions of their teachers' expectations, intervention students were more aware of their teachers' expectations and the awareness increased across the year unlike those in the control group, where it remained the same. Finally, the students' self-concepts in and about the usefulness of mathematics increased among the intervention students compared to the control students.

Rubie-Davies's (2015) study and other studies have provided great insights for understanding the relationships between teacher-student relationships and mathematics achievement. How students' achievement influences teachers' relationships with learners as mediated by the teachers' and learners' expectations is no doubt complex, and it has not been empirically investigated (Wang *et al.*, 2018). To compound these complexities, Timmermans, De Boer and Van der Werf's (2016) study showed a non-significant association between teacher-student relationships and teachers' expectations when students' socio-psychological factors, socioeconomic status, achievements and

genders are considered. As evident in this study and several others, there are inconsistencies in the conceptualization and operationalization of the teachers' expectations in this field, as also observed by Wang *et al.* (2018). It appears that teachers' expectations are not considered by most researchers as both personal and social constructs that are central to an individual belief, but rather as a personal 'powerful' stand-alone construct.

Du Plessis's (2019) described the supportive school culture that positively impacted the poor mathematics achievement of some marginalized communities, compared to others in South Africa. Du Plessis (2019) recounted the practices employed by three high schools that served disadvantaged communities in Mpumalanga for closing this mathematics achievement gap. Du Plessis argued that these schools had successfully closed or narrowed down the achievement gap for four consecutive years. The findings reported showed that all the three schools developed unique supportive cultures that facilitated improvement in learners' mathematics achievement.

According to Du Plessis (2019), three practices were common among the schools. First, the schools had a culture of having high expectations for their learners. The teachers in the schools believed that their learners could achieve well, and they provided required supports for the learners to meet the expectation. Second, the schools shared common practices regarding curriculum and instruction. The teachers had a culture of adjusting the curriculum and their instruction to implement changes from the education department. Third, school leadership involved teachers and other administrative staff acknowledging that narrowing the achievement gap was not easy and quick, but they believed that it was achievable. One strong message taken from this study is that it takes some deliberate and strategic plans to positively influence learners' mathematics achievement.

In a similar study, Barasa (2020) investigated teacher quality, which included teachers' classroom practices that influenced learners' mathematics achievement in Kenya. Eight practices, six pedagogical and two social, were considered by 2012 Service Delivery Indicators (SDI) data used in the study. The study was conducted in 181 rural and 91 urban primary schools. For rural schools, the results showed positive effects for two pedagogical practices – using textbooks during instruction and asking application questions, and one social practice – individual attention to the learners. As expected, negative effect was reported for corporal punishment – the second social practice – for both rural and urban schools.

One surprising result in the study is that of positive association between individual attention and mathematics achievement that was only found for rural schools. Barasa explained this result by referring to individual attention as “a key strategy for instruction” that responds to learners’ different needs (p. 62). The question in mind is why the practice is not key for the learners in urban schools? The most likely answer to the question may be that learners in urban schools receive other forms of support that are not available to their counterparts in rural schools. In this study for example, assigning and revising homework, using local information, keeping achievement records all were positively associated with learners’ achievement in urban schools but not in rural schools. The weakness of the study, as stated by Barasa, are the unobserved teachers’ characteristics and by extension, teachers’ classroom practices in the SDI that may potentially have effects on learners’ achievement.

The literature in this section shows that teacher-learner relationships influence learners’ achievement. The reviews further focus on the teachers’ expectations and learners’ socio-psychological factors as mediating variables. But the influence of learners’ achievement on teacher-learner relationships receives little or no attention. Further, there is evidence of inconsistencies in how teachers’ expectations and learners’ socio-psychological factors were conceptualized. Therefore, the nuanced understandings of both constructs are required for better explanations of the mediating mechanisms involved. Other common findings in the literature are that various strategic teachers’ practices and school cultures accounted for learners’ achievement in rural schools, but no study considers the strategic roles of maintaining equity among the learners and being conscious of learners’ expectations. With all the indications, it appears that extending this area of the study to rigorous re-conception of mathematics achievement and the research approach that considers both quantitative and qualitative data is necessary.

2.5 Social influences on mathematical identity

Studies have established that identity is constructed in relation to social contexts. Two of them, Wenger (1998) and Sfard and Prusak (2005a), stand out as they influence other studies in that direction (Darragh, 2016). For Wenger (1998), identity is a negotiated experience as a member of a community of practice, formed through participation and reification. By participation, Wenger means both social and personal active “involvement in social enterprises” and “membership in

social communities” which serves as a “source of identity construction” (p. 55 & 56). By reification, he means ideas, concepts, knowledge that are usually written down so that we can continue to refer to them and they enrich our participation. Reification and participation are mutually constitutive – you cannot have one without the other. Wenger illustrates such ideas to include “a complex web of conventions, agreements, expectations, commitments, obligations” that one has to work with in a community of practice (p. 59). Wenger (1998) argues that the process of reification is central to the negotiation of meaning in any practice, because this can shape our experience from very less obvious ways to very concrete ways. Identity construction is a continuous interplay between participation and reification within a social context.

Wenger (1998) presents three modes of belonging to provide explanations for the processes of identity formation and learning; engagement, imagination, and alignment. These modes of belonging have received a lot of attention from many mathematical identity researchers. Engagement explains learners’ direct experience with mathematics learning and how this experience influences their identity construction as they relate with their classroom community. These relationships translate to ways a learner identifies her or himself, and is identified by the others, and whether s/he participates as a full member of a classroom community or not (Boaler & Greeno, 2000; Boaler, William & Zevenbergen, 2000; Anderson, 2007). Imagination explains learners envisaging themselves and how mathematics and their broader experience of life fit together (Wenger, 1998; Anderson, 2007). That is, through learners’ ideas about the relevance of mathematics in relation to their future dreams, careers and accomplishments, they positively or negatively construct their mathematical identities (Anderson, 2007). The learners’ decisions built around what is required of them by institutions and communities constitute their alignment. This means that learners construct positive identities by focusing their efforts to learn and participate in the mathematics classroom community if they believe they must meet certain requirements to achieve their future goals or certain benefits, and that they will construct negative identities if no interest is attached to the goals involved.

At the heart of Wenger’s position is the avoidance of distinctions between the social and the individual, and the emphasis on social influences on identity. The first position is evident in the assertion that “talking about identity in social terms is not denying individuality but viewing the very definition of individuality as something that is part of the practices of specific communities”

(p. 146). Therefore, “it is as misleading to view identities as abstractly collective as it is to view them as narrowly individual” (p. 146). Evidence for the second position is found in Wenger’s argument that “even our most private thoughts make use of concepts, images, and perspectives that we understand through our participation in social communities” (p. 146). In fact, based on this perspective, our internal conversations are exclusively social (Wenger, 1998). Thus, “identity is not equivalent to a self-image; it is not, in essence, discursive or reflective” (p.151).

Another influential study in mathematical identity which is based on social context, is the study by Sfard and Prusak (2005a, 2005b). They argue that learners’ identities directly represent the stories they tell about themselves and the stories told about them by others. Based on these arguments, they categorized a person’s identity into two: actual and designated identities. Actual identity is one’s actual narrative of who one is and the possible roles that could play in what one wants to be; and designated identity is what one wants to be, and how it is socially and collectively influenced by the narratives of other members of the community. Statements such as ‘I am good at mathematics’, ‘I love mathematics’ and narratives like ‘I want to become a mathematician’, ‘I have to study hard to be successful in mathematics’ are typical examples of actual and designated identities of a learner respectively.

The general position held by Sfard and Prusak (2005a) about identity is not different from the other proponents of exclusive sociological perspectives. That is, identity is entirely social and attempt to involve individual perspectives is not necessary. This is evident in their words that identity should be “thought of as man-made and as constantly created and re-created in interactions between people” (p. 15). They say:

As a preparation to the critical analysis of the current discourse on identity it may be useful to give thought to certain well-documented weaknesses of such widely-used motivational notions as *beliefs* or *attitudes* which, on the face of it, can compete with *identity* for the role of conceptual bridge between learning and its cultural setting (Sfard & Prusak, 2005a, p. 15, italics in original).

Sfard and Prusak (2005a), like Wenger (1998), also hold that identity is dynamic, fluid, and changed with social contexts. But, both Wenger (1998) and Sfard and Prusak (2005a) differ on the basis of participative and narrative understandings of identity.

As influential as the studies of Wenger, and Sfard and Prusak on identity are, they are not free of some criticisms based on their conceptualizations and implications for achievement and equity. Gardee (2019a, 2019b) points out the limitations of Wenger (1998) and Sfard and Prusak (2005a) in that they emphasized social influences, discounted self-understanding, and overlooked the crucial role of agency in identity construction. I return to this criticism later. Jorgensen (2014) also asserts that exclusive “social theory [or any mathematics education study that builds on it] has failed to make any substantive inroads into challenging the status quo in terms of equity, access and/or success. It is almost as if mathematics has become quite secondary to any analysis” (p. 314). These criticisms present challenges which researchers in the field still need to address, since most mathematical identity studies draw from sociological perspectives.

Another salient criticism not emphasized in the literature concerns the process of how both Wenger (1998) and Sfard and Prusak (2005a) link learning with identity. Both maintain that identity and learning are intimately related. But the notion of participation as a central idea of identity understanding appears problematic in mathematics learning. In mathematics, it is clear that identities could be constructed differently with learning and classroom participation (Varelas *et al.*, 2012; Tossavainen, Rensaa & Johanssona, 2019). In other words, the concepts of action-based or participative definitions and “objectification” of identity do not comprehensively and conceptually explain mathematics learning and identity (Sfard & Prusak, 2005a; Marks & O’Mahoney, 2014; Radovic, Black, Williams & Salas, 2018; Graven & Heyd-Metzuyanim, 2019). And in addition to my previous observation, identity as just an idea of collections of stories, on the other hand, limits the complexities involving the ‘reality’ of identity construction in social contexts. This is because who we are and what we are identified with is not limited only to our own stories nor others’ stories about us, but rather it is a starting point (e.g., Wenger, 1998; Archer, 2000, 2002a; Varelas *et al.*, 2012; Marks & O’Mahoney, 2014; Radovic *et al.*, 2018).

These critical observations might be responsible for the insufficient power of any framework or analytic tool that fully or partly builds on these central ideas to adequately define identity and elaborately explain the processes of identity formation (Graven & Heyd-Metzuyamin, 2019). These observations may also explain why the studies cannot clearly predict learners’ identities and achievements in different situations, or to ultimately address the issue of inequalities. So, my argument and as suggested by Gweshe and Brodie (2019), is that conception of mathematical

identity need to be revisited in more nuanced way for better understanding and clarity of its relationships with other components of the study.

2.6 Conclusion

I have reported what the existing studies currently hold for the relationships between learners' mathematical identities, teacher-learner social relationships and learners' mathematics achievement. There is a consensus that strong relationships exist between mathematical identities and teacher-learner social relationships among the studies reviewed. This conclusion cannot be established among the studies reviewed for the relationships between mathematics achievement and mathematical identities. This is because the influence of one on the other is either requires further studies or needs further re-conception. For relationships between mathematics achievement and teacher-learner social relationships, the influence of the latter on the former is well established but the converse needs more studies. In all these reports, what is so salient is that there is a need to consider the re-conceptions of these concepts and/or improve on the methods/methodologies being employed by researchers to better understand the way they relate with one another.

While the current conceptualizations or theorization of social relationship and mathematics achievement appear to be between sociological and psychological influences, several studies including those reviewed here, have established that mathematical identity is more in relation to social influences. However, the current perspectives on social theory being employed by researchers raises some concerns. This is because there are evidence in the literature that mathematical identity is variously conceived, defined and appeared not adequate or comprehensive enough (e.g., Sfard & Prusak, 2005a; McGee, 2015; Gardee, 2019). This concern can also be extended to the other two concepts. Then I suggest that there may be a need not just to make another 'social turn' but a strictly 'realist turn'. In the next chapter, I discuss the framework situated within the critical realists philosophy that my study builds on.

CHAPTER THREE

PHILOSOPHY AND METHODOLOGIES UNDERPINNING THE STUDY

3.1 Introduction

This chapter discusses critical realist philosophical and methodological frameworks which provides a lens to a nuance understanding of the three theoretical concepts of the study: mathematical identities, social relationships and mathematics achievement. Both frameworks primarily build on the contributions of Bhaskar (1978, 1998) to realist ontology within the critical realism philosophy, which is methodologically foregrounded by the works of Archer (1995, 2000, 2002) and further elaborated by the work of other realist theorists and methodologists. Other two principles of critical realism – epistemological relativism and judgmental rationality – which respectively foreground theorization and explanatory powers of the approach employed – are also discussed. All the principles discussion assumes the general emancipatory position of critical realism (CR), including how it informs intervention development in the study. I present morphogenesis/stasis approach of Archer (1995) to derive the ontological basis for analyzing and relating the three concepts to one another. Such basis involves presenting agency and structure and how they interact with social structures in ways that are relevant and strategic to my study.

For more clarification, CR informed framing employed in and for this study deviates from the dominant and commonly employed ones (i.e., positivist and interpretivist). Specifically, the Chapter first establishes the research framework for the study from the three principles of CR which are integrated with key assumptions of social realism. The framework is then used (later in Chapter 4) to inform the development of the three theoretical concepts of the study without engaging with data collection and analysis, contrary to the other common framings. This framing makes it inevitably necessary for CR to provide ontological methodologies for explaining the ‘abstract constituents’ or social objects (using structure and agency) of each study concepts and their *between* and *within* relationships of any social system, without working directly with specifics or empirical data. Applying these philosophy and methodologies contingently to achieve the ontological tasks stated make CR research and structuring different, messier but yet interestingly robust because of the greater concerns that CR has for reality than other worldviews.

3.2 Critical realism

There are several philosophical views about world reality. But three of them have shown to be more fundamental (Fryer, 2020). These are positivism/empiricism, interpretivism/constructivism² and critical realism. The first two are the dominant views and have influenced educational research, policy and practice, while the last view is relatively recent and strongly developing with greater potential for education and beyond (Tikly, 2015). Experts have judged critical realism as a “much more internally consistent and philosophically developed framework” than its alternatives (e.g., Gorki, 2013, p.659). Critical realism (CR), as a philosophy largely developed by Roy Bhaskar, basically stands on three principles: ontological realism, epistemological relativism and judgmental rationality (Bhaskar, 1978, 1998). Ontological realism provides a lens to transcendently explain the three concepts of my study; epistemological relativism provides the insights to theorize and explain/evaluate the concepts; and judgmental rationality offers the empirical basis to whether the transcendental and theorization explanations build logically on sound inferences and provides improved and/or greater explanations for the relationships between the three concepts. I discuss in detail each of these principles below.

3.2.1 Ontological realism

Assumptions for adopting or rejecting a philosophical view are mostly ontological and epistemological and these are also salient for this study. Positivists and interpretivists argue that what learners are identified with regarding mathematics is limited to observable and/or measurable actions or interactions. Specifically, positivists hold that reality (e.g., mathematical identity) is always observable, measurable, deterministic. Interpretivists hold that reality is ever a construct of peoples’ language, culture and practices – that is, reality is always objectified or considered as action-based. Critical realism (CR) maintains that observable reality is generated by unobservable interactions of mechanisms at the real domain of the reality. Specifically, CR makes a distinction between transitive dimension of reality – i.e., observable occurrences and intransitive dimension of reality – i.e., unobservable occurrences (Bhaskar, 1978). Building on this distinction, Bhaskar

² Many authors used interpretivism and constructivism interchangeably, but in this thesis the former will be used throughout.

further maintains that the other two views are reductionist, which always leads to epistemic fallacy. By reductionist, Bhaskar means a view that reduces reality to only the transitive dimension. Committing such reductionism is a fallacy which Bhaskar calls epistemic fallacy (Bhaskar, 1978).

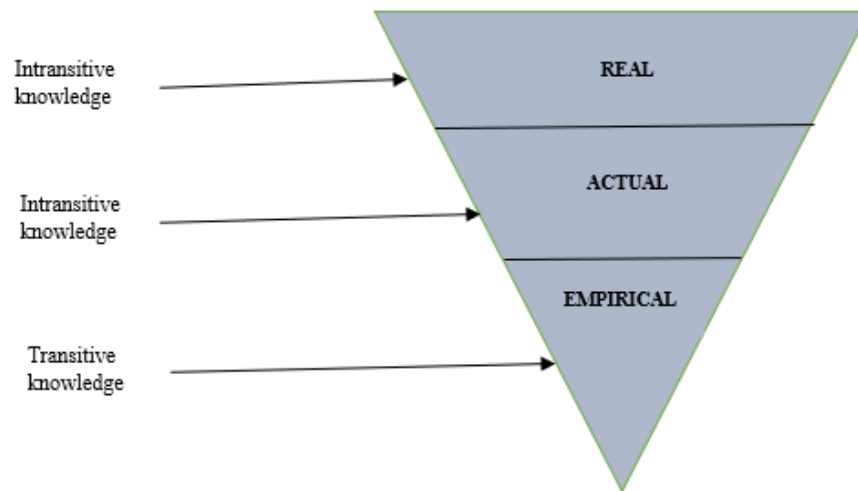


Figure 3.1: Bhaskar's stratified ontology

To illustrate this distinction between the two dimensions, Bhaskar stratifies reality into three domains: the real, the actual and the empirical (see Figure 3.1). According to Bhaskar (1978, 1979), the real domain captures the mechanisms – unobservable structures and agency (causal powers) – that explain what is experienced or not experienced at the other two domains. The main goal of CR is to explain observable social phenomena using inferred mechanisms that operate at the real domain (Fletcher, 2017; Tikly, 2015). The actual domain represents transient events that are experienced or not directly experienced, depending on our capability to capture or interpret their occurrences or existences (Danermark, Ekstrom, Jakobsen, & Karlsson, 2002). In other words, the actual domain captures the events that occur without human mediation (Fletcher, 2017). As a holder of ‘true’ events free of human mediations and interpretations, CRs build their theories from the actual domain (Leca & Naccache, 2006; Mukumbang, Kabongo & Eastwood, 2021). These two domains capture the intransitive dimension of reality. This ontological argument is crucial for the framing of my study. I pick it up later.

The empirical domain, which captures the transitive dimension, represents our direct experiences of the events, actions and conversation that are directly observable and/or measurable. So, this is

the dimension of reality which both positivism and interpretivism assume for reality. The main CR argument here is that the failure of both views to recognize the intransitive dimension does not reduce the reality to transitive dimension only. Because the higher-level mechanisms obtainable at empirical domain are always multiple and contingent and offer weak explanations for social phenomena, CRs always go for mechanisms at the real and the actual domains. This stratification³ is referred to as realist ontology, and the lack of such explanations in the earlier philosophies makes CR perhaps more robust and with less epistemic fallacy worldview. So, CR maintains *realist ontology*.

3.2.1.1 Realist ontology and research implications

This realist ontology has pertinent implications for social system framework and methods/methodologies in both intransitive and transitive realist inquiries. Social system framework, as a transcendental or abstract representation of social world, is a *two-of-one framework* (i.e., social and systemic frameworks), in which one *emerges* from another. By emergence, CR upholds that they are necessarily *dependent* (not independent) on each other but at the same time not conflationary. By dependence, they *contingently emerge*, implying that CR maintains that systemic framework emerges from the social framework. Social framework captures a transcendental, complex and contingent representation of the *explanatory* interactions between the social objects (i.e., structure and agency) of any social phenomenon and social structure (i.e., context) they exist in, causing their (i.e., social phenomenon and structure) emergence to new form. Systemic framework, which emerges from social framework, captures a transcendental but refined representation of the *explanatory* interactions between the social phenomenon and social structure they exist.

For clarity, social phenomena, like social structures, are social occurrences of our social world composed of structure and agency (more detail below). Beyond their compositions, social phenomenon and structure can take each other position depending on the situation of interest. When one takes the position of social structure, it is considered as a context or strategic context

³ This stratification implies separation into distinct layers and an action that Bhaskar did, not what necessarily happens. This is different from “emergence” which connotes “dependence” and non-human action.

with the former emerges from the latter. It becomes context or strategic context when indirectly (or unintentionally) and directly (or intentionally) employed by human to influence the social objects of the other that takes the position of social phenomenon respectively. Based on that clarification, both social and systemic interactions (i.e., two of one interactions) explained above also hold for strategic contexts. It is also important to state here clearly that the two interactions, say, *explanatory analytical approach*, occur contingently (i.e., dependently without conflation) at the actual and the real respectively. This explanatory analytical approach will be elaborated further later in section 3.3.

The above exposition suggests that emergence of social world (and possibly natural world) at the actual and the real within ontological realm comes from what structure and agency are, how they emerge from and interact with each other. Based on that suggestion, it means more elaboration of structure and agency is required. Structure and agency are transcendental representations of social objects of our social world. They are both recognized by CR as objects which take or attract different properties of social world to themselves. Therefore, they possess ontological and methodological properties or statuses which also emerge, giving them the powers to cause changes; after all, “causation is consistent with the emergence of newness” (Bunge, 1959 cited in Yucel, 2018, p. 415). Ontologically, structure and agency, as causative objects, can be conceptual, psychological or as stated before, social (Bunge, 2004; Mingers & Standing, 2017). According to CR, they both possess three methodological or analytical properties that are critical for social framework and by extension, systemic framework build up. First, agency emerges (not derived or stratified)⁴ from structure. Second, they depend (not independent) on each other but not conflationary. Third, they both necessarily emerge contingently (not constitutionally or dialectically)⁵.

So, when structure and agency interact they generate an *underlying explanatory mechanism* at the empirical which broadly captures what happens at the actual and the real domains respectively. By this interactions, more importantly, they further provide a *framework* which explains what happens at the actual domain of epistemological realm and offer transcendental evidence for what happens

⁴ Emergence implies non-human direct involvement while derivation and stratification imply human involvements.

⁵ This distinction is necessary; contingency implies dependency with explicit interaction, constitution implies integration with subsumed interaction and dialectic implies independence with no interaction. Bhaskar goes for the latter.

at the real of judgmental realm of our social world, leading to the production of *only* intransitive knowledge. These conclusions specifically suggest that interactions between structure and agency hold a causative property which generates underlying explanatory mechanism (UEM) for the happenings at the actual and the real domains of ontology and a framework, say, *explanatory framework* which extends through to the other two realms – epistemological and judgmental. More details on explanatory framework is provided under epistemology.

Realist ontology also implies that methods/methodologies for social system framework *have to emerge* (not to be stratified)⁶ and *realist* (not non-realist). Emergent because social system framework, as a *two-of-one framework* (i.e., social and system frameworks), necessarily contingently emerges. Realist because social system framework is contingently driven by both human and non-human activities (not one of them or one subsuming the other)⁷, which on their own also emerge contingently. Precisely, it means human activities emerge from non-human, both dependent on each other without conflation and emerge to another forms contingently. With all these properties, CR also maintains *realist methodologies*. This conclusion further suggests that realist ontology and realist methods/methodologies emerge contingently⁸. Transcendentalism, as a method for developing ontological methodologies for realist ontology, is crucial for CR. CR realizes that transcendental arguments may navigate between subjectivity and objectivity, since it involves human. To develop an ontological methodologies for analyzing the interactions between structure, agency and/or social structure, CR upholds transcendental procedures that balance between human subjectivity and objectivity.

This position implies that the methods do not only *re-describe* philosophers' or methodologists' specific experience and knowledge of our emerging social world but also *objectivates* and employs the experience and knowledge in a way that guides the emerging occurrences and happenings that *operationalizes* our social world that they want to understand. This is exactly the case with what I am doing here currently. My experience and knowledge of the social world (i.e., empirical data of my initial study and CR literature) is guiding every single argument as they emerge in this section

⁶ Meaning there is no ontological need to develop concepts to explain social world; it emerges.

⁷ Holding on to one implies non-realist position (e.g., interpretivist and positivist views) and subsuming them implies less CR position (e.g., 'conflationist' view). Conflationist position is a step away from non-realist but not realist yet.

⁸ Implication of this conclusion is that philosophical tenets (or being a philosopher) and appropriate methodologies (or being a methodologist) are dependent (resides in one mind), with the methodologies emerging from the tenets.

so as to have a better understanding of the topic at hand, CR. No concept or tool developed! For clarification, both operationalization and re-description have philosophers/methodologists indirect and direct-indirect involvement respectively while objectivation has their direct involvement, and they (the three methods) respectively emerge from one another. So, CR recognizes re-description method because of its nuances – it covers both operationalization and objectivation methods, without ignoring them.

This is the reality or realist methods/methodologies that cannot be reduced or overlooked by any realist philosopher and methodologist. This realist position will guide me throughout this work, and for more emphasis, the methodologies to be employed for the analytical purposes. But it seems Bhaskar chooses only extremely subjective transcendental arguments (Bhaskar, 1978, 1979). Such extreme position implies that except with the subjectively developed concepts, no interview, questionnaire and/or any other possible logical and tangible means or arguments can inspire access and analysis of the interactions between structure and agency (see Archer, 1995, 2000).

This extremely subjective transcendental position is the basis for ontological and methodological collaborations between Bhaskar's CR and Archer's methodological approach (or social realism) (Radulescu & Vessey, 2008). More striking implication of the collaboration is the suggestion of a separation of ontological realism of CR from its (realist) methodology, instead for both to emerge contingently, dependently without conflation and for the latter to emerge from the former all from Bhaskar's own arguments⁹, like I have done here. This observation is key for interrogating CR, morphogenetic approach and by extension realist social theory relationships, as that may explain why Bhaskar also assumes non-realist methodologies. To sum it up, Bhaskar's CR maintains realist ontology but non-realist methodologies, and by extension, the methodologies further foregrounded by Archer (further discussed in section 3.3). This conclusion shows internal inconsistency in Bhaskar's ontological realist assumptions for social world, and as it also suggests a philosophy that tends towards the creation of intransitive knowledge only.

⁹ I am aware that Bhaskar develops transformational model of social activity (TMSA) but it is not as popular as Archers' approach and he himself and his students even consented to the latter (Archer, 1995; Jessop, 2005).

3.2.2 Epistemological relativism¹⁰

Epistemology is the ways of knowing and accessing knowledge about reality. Beyond CR considering conflating epistemology with ontology as an epistemic fallacy, all the three philosophical views differ on epistemology. Positivists maintain that reality is only accessed by studying sequential and observable patterns and conducting some statistical analyses and procedures for generalization, with no researchers' interference (Mingers & Standing, 2017). Therefore, they do not only maintain a flat, absolute epistemology (a form of realism), embrace extremely objective methods for epistemology but also assumes flat causality. This position is rejected in some ways by interpretivists, who maintain a flat, relative epistemology, embrace extremely subjective methods and also flat causality, by emphasizing researchers' interpretations or meaning creation for or from observable events. This position is based on the argument that reality is mostly social, complex and context based. So, knowledge production should not be pre-determined procedures nor generalized without considering social means and contexts of accessing it.

For Bhaskar's CR, epistemology is assumed to be non-flat, relative and sustained by multiple extremely subjective methods therefore considers the means of accessing epistemological reality as always fallible. By fallibility, Bhaskar means that our knowledge or the methods of accessing reality is unlikely to be absolute or real (Bhaskar, 1978; 1979), making it more in agreement with interpretivism on epistemology. So, Bhaskar's CR maintains non-flat, relativist epistemology, non-realist methodologies/methods and by extension, flat causality. This position suggests an internal consistency (non-flat epistemology vs non-realist methodologies), external inconsistency (ontological realism vs epistemological relativism) and a problematic causality (flatness vs emergence). Combining these observations, one can raise a red flag on the epistemic credibility of Bhaskar's arguments for epistemological relativism.

For example, to free itself from epistemic fallacy it accuses others, Bhaskar's CR further argues that the transitive dimension of reality or knowledge, representing the empirical domain, covers epistemology (but not the whole of intransitive dimension), and that ontology accommodates epistemology (e.g., Isaksen, 2016). This argument, however, appears problematic as it holds

¹⁰ Some commentators have argued that fallibilism is more appropriate than relativism used by Bhaskar (see Yucel, 2018).

different critical implications for transitive and intransitive inquiries and at large, for CR arguments about stratified ontology. A quick critical observation one can make is the inconsistency about the stratification of ontology and not for epistemology, a means of accessing reality, or/and the dimension of knowledge itself. The key omission on the part of Bhaskar's arguments, I contend, is the failure to recognize that epistemology is also an emergent of ontology. In fact, the transitive knowledge dimension at epistemological realm, representing the empirical domain, also covers the real-empirical and the real domains, and by extension, ontology (diagrammatical representation is provided later). Also, intransitive dimension of knowledge, representing the real, *only* covers ontology. This sharp shift from Bhaskar's usage of stratification (implying staleness of causation) instead of emergence (implying newness of causation) and the broader understanding of the relationships between the knowledge dimensions, domains of reality and CR philosophical principles confirms the external inconsistency and flat causality in his arguments, as observed above.

3.2.2.1 Making relativist epistemology realist

Addressing such omission stated above will necessarily reveal another dimension of knowledge, transitive-intransitive (i.e., semi-intransitive and hyper-transitive)¹¹, which represents a new emergent domain of reality, real-empirical (i.e., semi-real and hyper-empirical) and still maintains an epistemology that holds CR principle (i.e., emergentist and realist) that were overlooked (or rather collapsed) by Bhaskar. This contention at a glance confirms the less robustness of epistemic credibility of Bhaskar's CR itself. So, the argument of epistemic fallacy should have been a matter of conflation between (or collapse of) the transitive and intransitive knowledge, overlooking the real-empirical domain of reality and/or the robust properties of epistemology, which led to his relativist and non-realist methodological positions. If this my transcendental argument holds as I proceed and further substantiated in Chapter 4, then epistemological relativism is better replaced with *epistemological realism*. Such replacement will adequately capture the external consistencies of the CR philosophy, at least within and between ontology and epistemology for now.

¹¹ Recall the "two of one" representation mentioned earlier. Such representation is unique with CR framing.

Moving on, the CR's positions as modified above (or as even initially conceived) means that CR generates its own epistemological methods/methodologies. However, the generation or claiming of these methodologies receives little attention from the CR loyalists (Bhaskar, 2007; Isaksen, 2016). For maintaining a realist epistemology, explanatory framework, social system framework and the methods for recognizing or accessing them at ontological stage should *necessarily contingently emerge* and remain *real* respectively. More specifically, the emergence of explanatory and social system frameworks are necessarily an inferential procedures while realist methodologies is through specification of the inferential procedures. By inferential procedure, it means *re-description* of the ontological status of explanatory framework and social system framework to epistemological status (i.e., from explanatory framework to realist descriptive mechanism, and from social system framework to theorized social phenomenon mechanism) and by extension, *objectivation* of the explanatory framework as explanatory-exemplified mechanism and *objectivation* of social system framework as explanatory/evaluative mechanism and eventual *operationalization* of the explanatory-exemplified mechanism and the explanatory/evaluative mechanism. By inference specification, it means choosing a nuance, logical and explicitly stated methodologies or procedures (i.e., deduction, induction and abduction) to identify, access and analyze the mechanisms of social phenomenon one is interested in. All these logical arguments are salient for showing the external consistency between the realist ontology and realist epistemology, and internal consistency within the realist epistemology and the realist methodologies that sustain the version of CR that I am explaining or exposing.

Drawing from the above logical arguments, three of two key points need further attention and/or elaboration. The first two have no direct human involvement (i.e., realist descriptive mechanism and explanatory/evaluative mechanisms), while the last has both human and non-human involvement (i.e., realist methods/methodologies stand). First, the emergence of *realist descriptive mechanism* comes with explanation-exemplification mechanism¹² and that needs more elaboration. The explanation-exemplification mechanism becomes point of attention because it represents the expected emergent theorization framework for any social phenomenon of the social world. By emergent theorization framework, it means a generalized descriptive mechanism for

¹² This mechanism draws from explanatory framework of ontological realm and exemplified framework of judgmental realm. Details about the former have been provided earlier while for the latter comes later.

explaining the transcendental nature of all the understandable¹³ social world at the empirical domain of ontological realm, leading to the production of intransitive knowledge, and at the same time for *exemplifying* all the observable or experienced social world at the real domain of judgmental realm for better decisions on practice, leading to the production of transitive knowledge.

That definition above confirms that emergent theorization framework, say, *realist social theory*, at the empirical-real domain of reality holds a dimension of knowledge which does not only retain the realist epistemological position of CR, its realist methods/methodologies standpoint (discussed under the third point) but also covers the dimensions of the transitive and intransitive knowledge. That dimension of knowledge I called *transitive-intransitive knowledge*. For more clarity, semi-intransitive and hyper-transitive knowledge production as a *two-of-one* representation at epistemology, covers the dimensions of both transitive and intransitive knowledge of social world (see Figure 3.2). So, overlooking the dimension or collapsing its two of one representation of knowledge production is simply an epistemic fallacy.

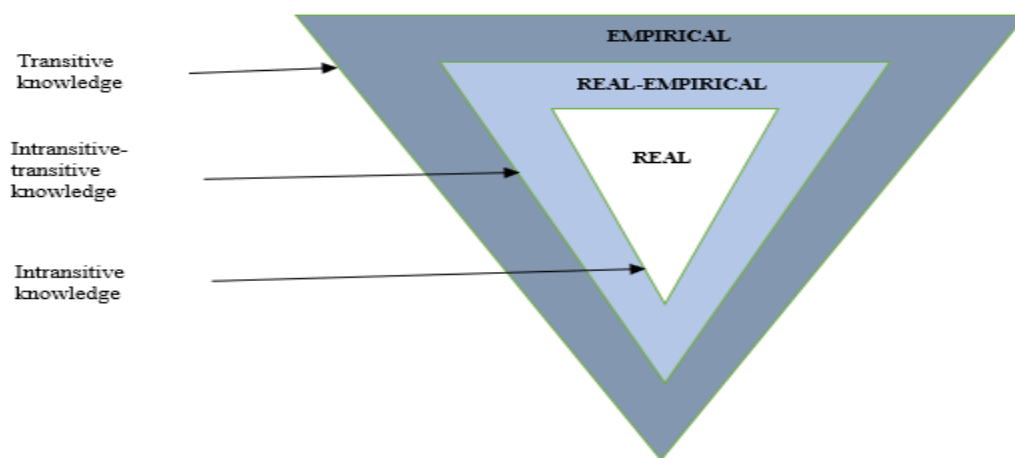


Figure 3.2: A critical realist epistemology

Second of the first point elaborates on the emergence of *social phenomenon mechanism* that comes with explanatory/evaluative mechanism¹⁴. Explanatory/evaluative mechanism receives attention

¹³ The use of “understand” here is deliberate as against “experienced” or “observed”. This is a great shift from how the empirical domain is understood in Bhaskar’s stratified ontology.

¹⁴ This mechanism draws direct from the social system framework of ontological realm only. The details have been provided earlier.

because it represents the expected emergent analytical approach of any theorized social phenomenon of our social world. By emergent analytical approach, it means an *explanatory/evaluative* contingent approach of *social interactions* between the constituents of a theorized social phenomenon one is trying to understand (e.g., constituents of theorized mathematical identity) and the constituents of a theorized social structure which the social phenomenon exists in (e.g., constituents of theorized teacher-learner social relationships/SREP programme) at the empirical domain of epistemology, leading to their possible emergence, and by extension, the approach of *systemic interactions* between the theorized social phenomenon (e.g., theorized mathematical identity) and the theorized social structure they exist in (e.g., theorized teacher-learner social relationships/SREP programme) at the real domain, leading to the better understanding of social phenomenon of interest (e.g., mathematics teaching and learning)

The above definition confirms that emergent analytical approach, say, *realist explanatory/evaluative analytical approach*, at the real-empirical domain of reality holds a mechanism which does not only retain the realist epistemological position of CR on domain of reality, its realist methods/methodologies standpoint but also covers the domains of the empirical and the real domains. That domain of reality I called *real-empirical domain*. For more clarity, semi-real and hyper empirical, as a *two-of-one* representation at epistemology, covers the domains of both the real and empirical of social world (see Figure 3.2). So, overlooking the domain or collapsing its two of one representation of realist way of understanding our social world is simply another form of epistemic fallacy.

Third point, the inferential methodologies or procedures – deduction, induction and abduction – to obtain the mechanisms or constituents of social phenomenon of our social world needs more elaboration because they involve both human and non-human engagement. The procedures become another point of attention because the realist option of the procedure are not commonly used by CRs and methodologists, despite its efficacy. That may be explained by inconsistency in Bhaskar’s ontological and/or epistemological standpoints and his ‘approval’ of non-realist method (i.e., only or multiple of subjective methods). Inferential procedure, as hinted earlier, not only emerges from each other but also that each varies in how they involve methodologists in their inferences: induction involves non-human engagement (i.e., engaging with existing explanatory frameworks only) and deduction involves both human and non-human (i.e., engaging with existing

explanatory frameworks and employing methodologists' thought experimentation/intervention) while the abduction involves human engagement (i.e., employing methodologists' thought experimentation/intervention only) with the mechanisms of social phenomenon under study. The development of these inferences have been associated with Peirce (1907) by several authors. Further details and operationalization of the inferential procedure, as a nuance or realist option, are presented in Chapter 4.

The three points above elaborate the inferential procedures earlier stated, with the exception of the operationalization. With those elaborations, they have also demonstrated that both realist social theory with explanation-exemplification mechanism, realist analytical approach with explanatory/evaluative mechanism and the realist methods/methodologies with both human and non-human direct engagement not only emerge contingently, but that realist social theory and realist analytical approach dependent on each other, without conflation and with the former emerging from the latter. The paragraphs further authenticate or validate the existence of another knowledge dimension, transitive-intransitive and the robust properties of epistemological realm, as contained in the elaborated domain of the empirical-real, which Bhaskar's CR seems to overlook. They also establish the validity of emergence of causality (rather than being flat) and its consistency with the new dimension of knowledge, domain of reality and understanding of epistemological properties. For the flow of my arguments, validity of causality is more elaborated under judgmental rationality. All these conclusions are critical for CR and that suggests a name, re-enforced CR, for the kind of CR I am explaining here. Both the realist social theory and realist analytical approach are operationalized in Chapter 4.

3.2.3 Judgmental rationality

The idea of adjudicating on (or making judgement about) competing theories or produced knowledge about reality is key to all philosophical views, as well as the studies that build on them. The ontological and epistemological grounds for establishing the soundness, comprehensiveness and more importantly the superiority of one theory or knowledge over another require different attention from different philosophers. While positivism and by extension interpretivism assume that there is no rational justification to consider one produced knowledge to be better than another, CR insists that examining the greater explanatory powers of such produced knowledge can be used

to rationally favored one over another (Bhaskar, 1998; Yucel, 2018). Therefore, both positivism and interpretivism maintain judgmental relativity but CR, in contrary, maintains judgmental rationality. CR acknowledges that judgmental rationality is a concept that argues for logical bases for which rival theories developed ontologically, knowledge produced epistemologically, and those explanations that connect the two are compared and/or chosen (Fryer, 2022; Isaksen, 2016; Yucel, 2018).

CR maintains rationalist judgmentality. This position unescapably implicates retroductive process or inference, since retroductive process is a bridge that links the intransitive and transitive insights (Yucel, 2018). According to Zachariadis, Scott and Barrett (2013, p. 858), retroductive reasoning “allows researchers to move between the knowledge of empirical phenomena as expressed through events to the creation of explanations (...) in ways that “hold ontological depth”. Unfortunately, this process is less developed and the judgmental rationality criteria itself is most times peripherally discussed or outrightly overlooked by CR literature (Bhaskar, 2007; Isaksen, 2016; Mukumbang, Kabongo & Eastwood, 2021). Nonetheless, there are still few methodology texts that have provided valuable but relative and divergent suggestions or approaches on how to go about the process (e.g., Danermark et al., 2002; Danermark, Ekstrom, & Karlsson, 2019; Jagosh, 2020; Mingers, 2000, 2005; Mukumbang, 2021; Mukumbang, et al., 2021; Zachariadis *et al.*, 2013). This observation collaborates Isaksen’s (2016, p. 248), argument that retroductive process is “local and transient rather than general and certain”. I beg to differ!

Retroductive process, according to Bhaskar (2009, p. 12), is a method or inference employed to explain or answer what the world must be like to act the way it does. This position has implications for knowledge production and of cause, mode of method employed for the explanation. While retroductive process is always consistent or implicated with the transitive and intransitive knowledge production by methodologists (e.g., Jessop, 2005; Yucel, 2018), Bhaskar’s question suggests production of only the intransitive dimension of knowledge (i.e., hyper-transitive). Also, while the question is free of direct human involvement, the process of retroduction, with the concept of immanent critique puts forward by Bhaskar, are purely extremely subjective arguments and both the world (or question) to be explained and the employed method are separated from each other (i.e., independent). So, the arguments for retroduction raises question on the epistemic credibility of CR; it needs to be looked into.

3.2.3.1 Retroductive process vs explanatory powers

To do this, Bhaskar's question must have the transitive dimension of knowledge production (i.e., semi-intransitive): 'what set of valid observations (or experiences) must look like to significantly represent what the world truly is' This move is necessary because the (social) world, methods/methodologies and causation emerge, involve both human and non-human and are contingent rather than flat, only human dependent and separated respectively. While both questions can only be answered epistemologically with *realist social theory* and *realist methodologies* (including realist analytical approach) which Bhaskar overlooks, his question can only be answered ontologically with the *underlying generative mechanism* that he sustains with non-realist methods/methodologies. Similarly, any worldview (including positivism and interpretivism) that holds on only to my emergent question can also only answer it judgmentally (evidentially) with the *exemplified framework* (that will be exposed later in this section) with non-realist methods/methodologies. So, there is no need to develop any concepts or tools to answer the two questions contingently, as they necessarily emerge in every social phenomenon.

For fairness, combination of interpretations and explanations of knowledge production, according to Jessop (2005), are strongly emphasized by CR. But how the combination of the two will work without recognizing the third dimension of knowledge, robust properties of epistemology and emergent properties of causation will still be a matter of further relative and divergent arguments. This is because there is an overlooked dimension of knowledge, unrecognized epistemological and nuanced causality properties right from the beginning of Bhaskar's CR assumptions. This observation is valid for the latter¹⁵, as the validity of first two have already been explained earlier. This is because Event causality obtained from the transitive insights, being vulnerable to mediating influences of human and contexts, "is never determinate [and 'never accurate'] but always multiple and contingent" (Mingers & Standing, 2017, p. 22), though still tends to offer weak and corrigible, practical or evidential interpretations of what happen in the social world. So also, the Generative causality obtained from the intransitive insights, being free from mediation by human and non-human influences, tends to always provide basic, robust and strong underlying explanations

¹⁵ Not only its validity but also its consistency with other two.

(Mingers & Standing, 2017), but fallibly determined and its practical importance always tends to be transient.

Bridging the gap between these two insights or knowledge calls for another nuance one that covers both event and generative causality (i.e., event-generative causality) and the dimension that has emerged earlier, *intransitive and transitive knowledge dimension*, and by extension *robust epistemological properties*. So, claiming or developing a valid and generally acceptable retroductive process will be an unending debate among or project for methodologists¹⁶. This is because, as argued earlier, there is a collapse of dimension of knowledge, an overlooked robust epistemological and nuanced causality properties right from the beginning. So, the process, rather the method or inference, is necessarily simply seemingly synergy explanations between realist ontology and judgmental realism (not rationality)¹⁷. This argument then suggests that the methods or inferences employed by theorists determine the explanatory power of the competing theories. For now, I am keeping the issue of unending debates aside for me to prepare the justification ground for the strong argument I just made and to also complete my own explanations to demonstrate the explanatory power of my CR.

3.2.3.2 Making judgment – rationalism vs realism

The far-reaching importance of my position is that re-enforced CR gives causation not only a strong ontological status, as described earlier, but also, like other two views, epistemological status (Mingers & Standing, 2017). But ontological status is transcendental, transient and not readily accessible for exemplification. And it has been argued that interpretivism and positivism hold weak views about causation compared to CR, but Bhaskar's CR has not convincingly proven it. Now CR can make this claim and convincingly prove it. Causality, in interpretivist view, is ontological status free and so it is ignored (Mingers & Standing, 2017), while causality is limited or reduced to the empirical domain of ontology in positivist view. For CR as I argued, causality does not only attain ontological and epistemological statuses combined, but it can as well be

¹⁶ I have also developed a retroductive process in the cause of this study because I was not 'pleased' with the existing ones.

¹⁷ While realism implies concerns for reality that involves both human and non-human involvement methods/methodologies, rationality only implies human involvement.

explained in the empirical domain of ontological realm (i.e., transcendentally) and *exemplified* in the domain of the real of judgmental reality realm (i.e., evidentially) of CR. For emphasis, and more importantly for this argument, it means that these explanations and exemplifications, through realist understanding of emergence, can be provided and justified empirically. What are the bases for these strong claims?

This is because judgmental realism, representing the transitive dimension also covers both epistemology and ontology (see Figure 3.3). So, CR can now claim the superiority over other views on causality as stated above. This superior stand of re-enforced CR implies that criteria for making judgment can be operationalized on three arguments or premises: rational, rational-irrational (realist) or irrational premises. But Bhaskar's understanding of judgmental rationality does not recognize it as a candidate of transitive knowledge, covering the epistemology and ontology with its own domain of reality – the empirical, the actual and the real. He then fails to conceive the criteria for making judgment on superior explanations and theories can also be made on the rational-irrational (realist) premise other than rational premise. The reason being that Bhaskar seems to consider judgmental rationality as an ontological matter that can only be addressed with rational arguments or argued on rational premise at the empirical domain.

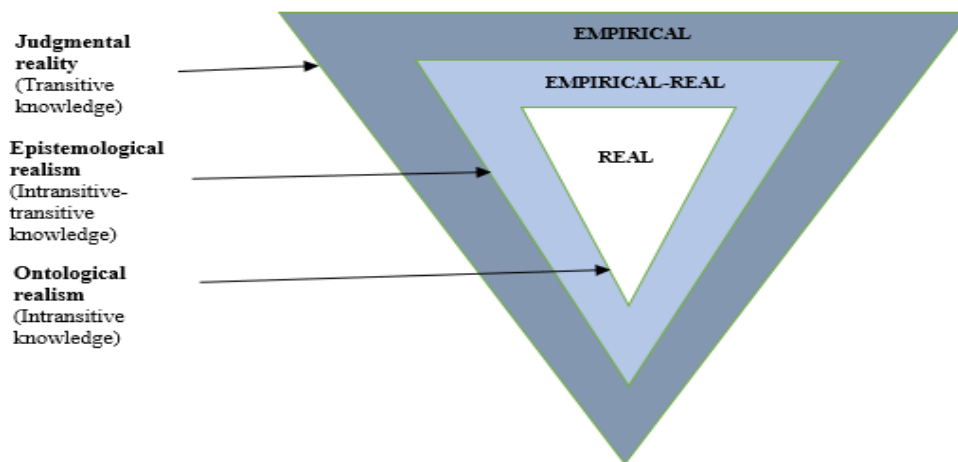


Figure 3.3: Re-enforced critical realist philosophy

It is clear to re-enforced CR that choosing one explanation or theory over another at judgmental realm cannot be convincingly argued on rational premise at the empirical domain because of their contingencies and weaknesses, but such choice can be based on the rational-irrational premise at

the actual domain, as against CR of Bhaskar, and more interestingly, on irrational premise at the real, as outrightly overlooked by him. That indicates a reductionist view, not even like a positivist or interpretivist¹⁸. So, retroductive process is simply a realist method or inference for explaining social world from the real domain of judgmental realm that forges through epistemological realm and seemingly synergizes with the empirical domain of ontological realm. This definition is different from Zachariadis, et al., (2013, p. 858) which sees the process as purely human activities, as against both human and non-human, and that conflates or ignores the key position of domains of reality within the process. It then confirms that retroductive inference or process is all about the *robustness* of explanatory power or more precisely, realist methods employed by competing theories to explain social phenomenon from judgmental realm through epistemological up to ontological realm, without ignoring the relevant domains of reality involved. This definition of robust explanatory powers also holds conversely. This robustness I have shown between realist epistemology and realist ontology. I am now picking up the explanations between judgmental realism and realist epistemology.

3.2.3.3 Judgmental realism

In line with my re-enforced CR arguments thus far, it is important to also stress that epistemological status of realist social theory and descriptive mechanism also undergo inferential procedure to emerge with a status of judgmental realism. With re-description, realist social theory and theorized social phenomenon at epistemological stage are re-described as exemplification mechanism and observed social phenomenon respectively, and by extension, objectivation of realist social theory as research design and objectivation of observed social phenomenon as explanatory/evaluative analytical model and eventual operationalization of both research design and explanatory/evaluative analytical model. Through specification of inference, the most nuance, logical and explicitly stated methods (i.e., quantitative, mixed and qualitative methods) and analytical approaches are selected (i.e., statistical, statistical-thematic and thematic approaches). As stressed severally, CR will always maintain a nuance method and analytical approach – mixed methods and approach without overlooking others. All these logical arguments are also key for

¹⁸ This assertion is made because both positivism and interpretivism have criteria, though flat and unrefined, for making judgement but Bhaskar's CR does not.

showing the external consistency between the realist epistemology and realist judgment, and internal consistency between emergent judgmental realm and the realist methodologies for re-enforced CR.

To elaborate the above three expositions, I reiterate that inferential procedures are non-human engagement for both exemplification mechanism and observed social phenomenon, while the inference specification involves methods/methodologies with both human and non-human engagement. First, emergence of exemplification mechanism that comes with research design criteria needs to be elaborated. Research design criteria receive attention because they represent the expected emergent research plan of a particular observed or experienced social phenomenon. By emergent research plan, it means an overall plans for how the observed or experienced social phenomenon of interest is accessed. Access to the observed social phenomenon include the type of methods/tools employed, methods of selection of locations that and participants who can provide information about the phenomenon and the time dynamic or dimension. For all these to be conducted in a realist way, researchers must hold on to the emancipatory (axiology) stand of CR. Emancipatory stands includes upholding and maintaining high standard of *research ethics and values* before, during and after accessing the observed phenomenon. Crewell and Creswell (2018) among others have done greatly in this aspect.

Second, elaboration of emergence of observed phenomenon that comes with explanatory/evaluative analytical model is also necessary. Explanatory/evaluative analytical model represents the expected contingent approach of social interactions between the constituents of an observed or measured social phenomenon under consideration (e.g., constituents of exemplified mathematical identity) and the constituents of an observed social structure which the observed phenomenon exists in (e.g., constituents of exemplified teacher-learner relationships/SREP) at the empirical domain of judgmental reality leading to their possible emergence and by extension, the interactions between the observed phenomenon (e.g., exemplified mathematical identity) and the observed social structure (e.g., exemplified teacher-learner relationships/SREP) at the actual domain, leading to the general understanding of a particular social world (e.g., mathematics teaching and learning in rural schools).

Emergence of analytical approach at the empirical and the actual further generates evidential framework at the real domain. By evidential framework, it means a generalized *evidential criteria*

for *exemplifying* the transcendental nature of all the understandable social world at the empirical domain of ontological realm through epistemology, leading to the production of *only* transitive knowledge. This exposition buttresses my earlier two arguments against Bhaskar's CR. The first argument regards Bhaskar's operational question for his philosophy and the need to have two dimensional questions to able to produce both intransitive and transitive knowledge in order to develop well-grounded theories and to eventually improve practices respectively. The second argument regards his position on only rational criteria for making judgement and the need to extend the criteria to rational-irrational (realist) criteria to be able to provide robust explanatory powers for the theories we develop and eventually, the intervention programme we implement. To provide such explanatory powers, I maintain that a realist inference that also maintains the realist methods/methodologies, which I have been employing in this philosophical exposition from onset, will be appropriate for the realist evidential criteria.

I propose *developmental* criteria and *practical* criteria as two different but still related realist evidential criteria at the judgmental realm. Specifically, *developmental* realist criteria capture the criteria for making judgement regarding studies that develop theories for its own sake or with the aim of improving practices (i.e., explanatory or descriptive studies), while *practical* realist criteria capture the criteria for making judgments regarding studies that directly apply theories or interventional programmes for practice improvement (i.e., experimental or evaluative studies). For *developmental* criteria, I propose *generality, validity and practicality*, while for *practical* criteria, I propose *inclusivity, universality and simplicity*. The main interest of developmental criteria is the practicability, validity and generality of the objectivation, re-description and operationalization procedures or inferences employed by the theorists or researchers respectively. For practical criteria, the interest is the simplicity, universality and inclusivity of the objectivation, re-description and operationalization procedures or inferences employed by the interventionists respectively. This is where the robustness of the explanatory powers of any theory or knowledge created lies not somewhere else¹⁹.

On their operationalization, *developmental* criteria and *practical* criteria have been nicely operationalized by different philosopher, methodologists and theorists. For example, Bhaskar

¹⁹ This conclusion answers Isaksen (2016) disturbing question on criteria for making (realist) judgement on two competing theories or explanations, without further creation of another concepts or tools.

(2009, p. 73 & 82) has operationalized *developmental* criteria nicely but needs to be refined or extended. The reasons for the need for the refinement and extension have been fully discussed in this section. Scott (2010), as highlighted in Tikly (2015, p. 247), has recognized these need and provided the refinement by extending the criteria to ‘practicality’ of the competing theories. As expected, however, little or no comprehensive work is known in relations to *practical* criteria. Available influential texts and studies, including Lincoln and Guba (1985), Merriam (1998, 2009), and Quraishi, Irfan, Purdie and Wilkinson (2022) only operationalize the criteria using research trustworthiness/ethical consideration on a rational premise. This is expected because most of the influential texts and studies on methods/methodologies at the moment are largely inspired by positivist or interpretivist views that have been around for so long. CR philosophy or more precisely, re-enforced CR is still ‘a baby yet to be delivered’.

Third, as for ontological and epistemological realism, realist methodologies and approaches are also sacrosanct for judgmental realism. This assertion implies that researchers must consider that re-enforced CR does not only support empirical data collection and analysis but also at different levels, with different multiple and mixed approaches. That is, data can be collected at different levels (i.e., individual, group and societal) and analyzed at different levels (e.g., analytic, systematic and generative) with multiple approaches or methods (i.e., qualitative, mixed and quantitative approaches). Some of these realist methods and approaches are well developed by methodologists, among which John Creswell is notable. Creswell, though without philosophical justifications, has greatly worked to develop these methods. Now, those work of his and other great methodologists, with possible realist improvement, can be claimed by re-enforced CR²⁰. However, texts on the analytical procedures that represent re-enforced CR position are still scarce. The reason stated above also apply here. Appropriate methods/methodologies and operationalization of emergent analytical model will be fully discussed in Chapter 5.

The key contention I make are broadly two. First, I maintained that CR principles emerge from one another. Second, I emphasized the strong connection between the principles of CR and the realist methodologies to maintain the internal and external consistencies of its assumptions. I maintained that to sustain the external consistency of CR philosophy, its principles necessarily need to be considered as emergent of one another, and appropriate realist methodologies should

²⁰ These methods will be difficult for Bhaskar’s CR to claim because of his non-realist methodological antecedent.

be employed. To maintain internal consistency of CR, realist analytical approach is emphasized. As further elaborated in the later sections, such moves are expected to re-enforce Bhaskar's CR and broaden the philosophy to offer a research framework that tends towards transitive and intransitive knowledge production.

So, understanding and applying CR in research requires a nuance consideration of all the key components of its philosophical assumptions and arguments. These are levels of reality – the real, the actual (or real-empirical) and the empirical, the dimensions of knowledge – intransitive, transitive-intransitive and transitive, and how the two components integrated with the general philosophical assumptions of CR – ontological realism, epistemological realism and judgmental realism. More importantly, such move has shown how realist social theory emerges with the realist analytical approach at the epistemological realm. These are key for social theory development and both will be operationalized in Chapter 4. Like realist explanatory analytical approach, which is taken up in the section below, emergent version of analytical approach at the judgmental realm also needs more elaboration. While realist explanatory/evaluative analytical framework is given full attention in Chapter 4, its operationalized version is attended to in Chapter 5.

3.3 Ontological methodologies – explanatory analytical approach

Methodologies of any ontological inquiry involve methods or inferences employed to *access* and *analyze* our social world – it has little or nothing to do with the identification of a specific social phenomenon. In the above section, I have extensively discussed (and practically utilized) the realist methods I employed for explaining the re-enforced CR, while the explanatory analytical approach that holds for both context and strategic context at ontological realm was only indicated. In this section, I elaborate on the analytical approach by also looking at the most popular approach among the existing ones. Among the great methodologists in the Bhaskar's ontological project, in relation to context situation, are Anthony Giddens, Bob Jessop and of course, Margaret Archer. Archer's contributions through her transcendental arguments for *analytical dualism* and *emergence* in particular and morphogenetic approach in general, both within social realism, is more prominent within the CR community (Knio, 2018, 2022; Radulescu & Vessey, 2008). In relation to strategic context, the great work of Pawson and Tilley (1997) is more popular (Radulescu & Vessey, 2008). They develop a context-mechanism-outcome (C-M-O) evaluative model to offer explanations of

social world phenomenon. Archer's morphogenetic approach, based on the earlier explanations that the latter situation emerges from the latter and because of its great background work that benefits my approach, is prioritized and considered.

3.3.1 Analytical approach – concepts vs explanations²¹

Drawing from the earlier explanations, it is clear that analytical dualism of Archer's morphogenetic/morphostatic (M/M) approach is sustained by abductive methods (i.e., subjective transcendental arguments), while explanatory analytical approach is maintained by inductive-abductive methods (i.e., both subjective and objective transcendental approaches). In what follows, I discuss the Archer's (M/M) approach which holds the concept of analytical dualism and the explanatory analytical approach that emerged earlier without any concept to sustain it. Because of its role in developing the M/M approach, social realism as inspired by Bhaskar's CR is also discussed.

3.3.1.1 Archer's morphogenetic/static approach

Archer (1995), drawing on Bhaskar's stratified ontology, develops morphogenetic cycle or approach. The approach, according to Archer (1995, p. 192), "attempts to bridge..." the fundamental methodological gap "between the explanatory power of practical social theory and the ontological strength of the realist philosophy". And to do that, the approach

makes no leap from the real to the actual, but rather dwells on the ground between them by analysing the generative mechanisms potentially emanating from structures (and cultures) as emergent properties and their reception by people, with their own emergent powers of self and social reflection. (p. 175).

The concepts of emergence and analytical dualism form the basis for analytical or more broadly, methodological and ontological collaboration between social realism and Bhaskar's CR. In other words, Archer's morphogenetic/static approach (M/M) offers an explanatory methodology or more precisely analytical model which, unlike the alternative ones, somewhat reveals the social

²¹ Analytical approach sustained with subjective concept as against the approach maintained by realist explanatory approach.

structure and agency-structure interplay and their changes or stability by drawing from Bhaskar's CR.

The key ontological claim of social realism lies in their arguments on emergence (Elder-Vass, 2007). Emergence is built on the argument that a whole can possess properties and powers which its parts do not. So, for an entity to be emergent, it has to have one or more emergent properties (Stephan, 1992 in Elder-Vass, 2007). Emergence, for social realists, is the possible outcome that may arise from the interactions between agency and structure, and the social structures in which they exist. The concept of emergence as it foregrounds the existing social realism appears less realist. This is because the conception of social realism itself is limited to ontological realm (which only focuses on intransitive knowledge or reality) and holds some ontological and analytical positions that are not realist enough (e.g., subjective explanatory arguments and separated/flat structure-agency interplay). This is against the realist social theory which emerged from more realist social theorization or perspective and covers both ontological realism (explanatory framework) and judgmental realism (exemplified framework) that I will be operationalizing in Chapter 4, the Chapter which situates the theoretical framework for the study. Based on these observations, some discussions regarding structure and agency will be presented later, because of their central roles in the existing perspective on social realism in general, and M/M approach and analytical dualism in particular.

Analytical dualism involves separation of agency (people) and structures/cultures (parts), and how they enable and constrain each other. Social realism maintains that structures and agency are irreducible or cannot be subsumed to one another because they are different ontologically (Archer, 2002; Danermark et al., 2002; Jessop, 2005). Since this position involves interaction or analysis of two components, different conflationary views are being held by different theorists (e.g., Adler, 1997; Gardee & Brodie, 2021; Gweshe & Brodie, 2023; Giddens, 1984; Wenger, 1998). Archer (2002) categorizes these conflationary views as downwards, upwards and central conflations. Downwards conflation is a determinists' view that agency is determined by the structure. Upwards conflation represents voluntarists' view that agency is unconstrained by the structures (Archer, 2002; Tikly, 2015). Central conflation, as elisionists' view, is whereby both structure and agency are merged, and their interactions ignored (Archer, 1995). These views fall into fallacy of conflation and/or offer simple approaches to a complex agency-structure interaction (Archer, 1995,

2002; Knio, 2022). For realists, such conflations do not only deny social theorists the robust knowledge of how agency and structure enable and constrain each other but also downplay the possible emergence that may arise from such interplay or interaction.

Another unique feature of M/M approach is the emphasis on time dimension of the agency-structures dynamics. According to Archer (1995, p. 89), “time is incorporated as sequential tracts and phases rather than simply as a medium through which events take place.” These sequential phases are represented in three temporal measure points between $T^1 - T^4$ (see Figure 3.4).

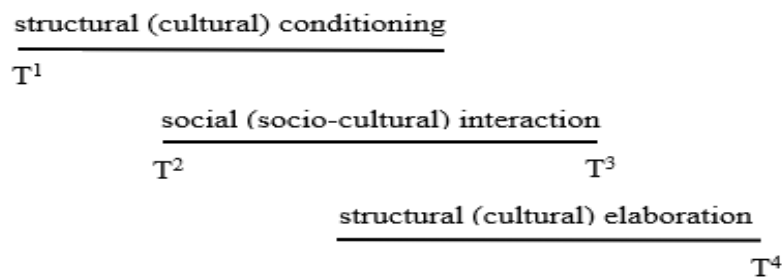


Figure 3.4: Morphogenesis/stasis of structure (culture)

Structural/cultural conditioning occurs at T^1 . This is a phase where social agents are constrained (or enabled) by the social structure which they find themselves or are born into. Social interaction occurs at $T^2 - T^3$. At this phase, agents respond to the conditions experienced and as a result mediate them but not without modifying themselves. The mediation process seems to encompass how agency, structure (or culture) interact and the emergence of new social agents with different structures and agency. The phase is centered on the interaction between agents and their transformation from certain type of agents (primary) to another agents (cooperate) to enable them to mediate the structural (or cultural) conditioning. At the last phase, T^4 , is when structural/cultural elaboration occurs. By elaboration, Archer means morphogenesis of the structure/culture – i.e., transformation/change of the constraining contexts or morphostasis of the structure/culture – i.e., reproduction/retaining the constraining contexts. The conditions available at T^4 , according to Archer (1995), becomes the starting context or condition at T^1 of the following morphogenetic cycle.

However, apart from the criticism that morphogenetic/static approach seems to overlook strategic contexts (Jessop, 2005), little or nothing is known about the contribution of structures to the

emergence through the approach (Vandenberghe, 2005) and it is not elaborated enough *how* the structure-agency interaction is analytically possible or accessible (Radulescu & Vessey, 2008; Turner, 1997). To buttress these criticisms, Archer's ideas of morphogenesis (transformation) and morphostasis (reproduction) suggest two different elaborations of structure/culture and two corresponding ways they are mediated, but where one of the elaborations shares the same *recognition rule* with another separate unrecognized elaboration which has its own mediation of a distinct ontological status. In other words, Archer suggests that agency mediates structure/culture rationally or irrationally (see Radulescu & Vessey, 2008) and correspondingly reproduces or transforms them (the contingent relation is separated from or external to the approach) but conflates the reproduction of structure/culture with another elaboration (i.e., 're-enforcement', unrecognized by Archer) and overlooks another ontological status of agency (i.e., 'eclectic', also unrecognized by her). So, the approach seems to be analytical but less categorical, because all the validly possible categories are not duly considered.

This Archer's position notwithstanding still makes her approach more elaborated (because it recognizes rationality and irrationality of agency) but at the same time, complicated (because it holds conflationary view, there is no explicit and emergent analytical explanations for agency-structure interactions and their emergences) compared to other existing approaches which assume that human agency can only be exercised rationally (e.g., Danermark *et al.*, 2002's and Pawson & Tilley, 1997's explanatory frameworks). And that eventually leads to a less robust and non-realist recognition rules for describing the elaborations of structure and modifications of agency, and to further assign them to how they are elaborated and modified respectively and *contingently*.

So, I contend that the mixture of elaboration and complication of her approach is due to her non-recognition of eclectic (rational-irrational) nature of agency, maintaining separated/flat analytical explanations for structure-agency interactions and what Knio (2022, p. 7) describes as "problematique of *objectivation*, or *the making of object itself*"²². Because of that, her position appears to emphasize *self* or *first-order* reflections (see Archer, 2002) and her structural/cultural conditioning appears to be more of *natural* and *practical orders* (see Archer, 2000), while *social* or *second-order* reflections and *social order* conditioning can also receive the same recognitions.

²² The problems of non-recognition of eclectic nature of agency and objectivation also applies to Jessop's (2005) strategic-relational approach.

For more details on the order of reality and Archer's concept of emergent properties because of their importance regarding the issues raised above, readers can consult these readings Archer (1995, 2000, 2002) and Knio (2022).

3.3.1.2 Re-examining social realism and M/M approach

The concepts of analytical dualism and emergence are key pillars that hold Archer's morphogenetic/morphostatic (M/M) approach, and both have great implications for or on structure, agency and structural conditioning and by extension, their elaborations. Structure and agency are two concepts that have taken different conceptions, diverse analytical stands with and various usages among social theorists and researchers. That is, different theorists and researchers have assigned different ontological (or theoretical) and analytical (i.e., methodological) statuses to agency and structure. This section briefly emphasizes on those two statuses because of their roles in the development of explanatory and analytical approaches for social theorization or perspectives (e.g., social realism). Building on these understandings, I then highlight the similarities and differences between my explanatory analytical approach and that of Archer's within social realist perspective.

Structure represents a set of internally related objects that bind people with social systems. Positions of social realists, especially Archer, have been discussed in different section before now. Nevertheless, some key ontological and analytical properties of structure are still emphasized here. It is crucial to emphasize that for Archer, structures are internal to agents while *necessarily* external to their minds (Archer, 1995; Knio, 2022). That further emphasizes that mechanisms of social phenomenon are derived from structures (Archer, 1995; Danermark et al., 2002). These positions are key to social realism and it is what binds different proponents of various perspectives of social realism together (see Knio, 2022). While this perspective foregrounds Bhaskar's CR arguments on transcendentalism and non-conflationary of the reality of social world than its alternatives, it seems to hold *structurationalist semiosis*, focusing more on structuration to make meaning of the social world (Knio, 2022, p. 9). This is a reductionist view which holds non-realist perspective on social theorization.

Agency, on the other hand, is a constituted and conceptual term to generate and provide explanations for social phenomenon manifestations (Archer, 2002a; Emirbayer & Mische, 1998; Littlejohn & Foss, 2009). But to provide these explanations “many theorists have failed to distinguish agency as an analytical category in its own right—with distinctive theoretical dimensions and temporally variable social manifestations” (Emirbayer & Mische, 1998, p. 963). Theorists who are social realist²³ do not only recognize the analytical but also its distinctive theoretical statuses of agency in their theorization of social world. These statuses were emphasized previously. For these theorists, agency is also reflexively generated, shaped by interrelated individual and social influences (Archer, 1995; Eteläpelto, Vähäsantanen, Hökkä, & Paloniemi, 2013; Manzo, 2012; Marks & O’Mahoney, 2014).

The above realist ontological and analytical positions hold three critical implications for perspective on social realism. First, that structure and agency are internal to agents and necessarily *contingently dependent* on each other. This means that structure and agency do not only share the same ontology but also analytically dependent. This position deviates from the earlier stated one which seems to consider only structure and independent ontology and/or interactions between structure and agency. The position is not only a new perspective on social realism but also shares the perspective between ontological and epistemological realms²⁴. Second, that agency cannot only be rational and irrational in its social manifestations (Archer, 1995; Manzo, 2012; Eteläpelto, et al., 2013) but also realistic or eclectic – a combination of both. For example, the possibilities of agents to consistently reflect or act (i.e., rationally or irrationally) in a certain way also suggests that they can as well be inconsistent in how they reflect and act (i.e., eclectically) in another situations. The third extends the second, in that agency, like structure, also emerges. So, agency is possessed, exercised and actualized and thus possess a distinct analytical status. That is, agency being necessarily possessed by human, always being the causes of their actions and mostly manifests as actions, holds a separate analytical status that is *necessarily have* to be operationalized because of its transient nature. These perspectives are operationalized in a non-conflationary way in Chapter 4.

²³ Theorists who are informed and inspired by re-enforced CR as explained previously.

²⁴ The key implication of this position is that we cannot only produce transcendental (or intransitive) knowledge but also evidential (or transitive) knowledge. This is a big shift for knowledge production within the CR philosophy and emerged realist social theory from it.

What implications do all these perspectives about social realism have for analytical approach? First, within the M/M approach, ‘agency emerges from structure and both constrain and enable each other’, suggesting no conflation between them. Second, Archer’s M/M approach maintains that ‘structural conditioning precedes social interaction’, which further implies that both structure and agency exist in different temporal dimension. Another salient analytical position is that structure has ‘emergent properties which itself has causal powers’. According to the M/M approach, interaction between emergent properties (SEPs, CEPs and PEPs) ‘makes no leap from the real to the actual, but rather dwells on the ground between them’, implying that both analytical dualism and emergence remain the same or flat within their temporal dimensions. To buttress and push this position further, Archer’s M/M approach argues to consider structure-agency interaction contingently and yet specifically singles out the elaboration of structure from the modification of agency (and vice versa), suggesting that analytical dualism and emergence are not contingent but independent.

While my explanatory analytical approach agrees with the non-conflationary interactions between agency and structure and the emergence of the former from the latter, it deviates from some other analytical assumptions of M/M approach. When M/M approach maintains “sequential tracts and phases” which suggests that structure and agency exist in different temporal dimension, I maintain that the sequential tracts and phases do not negate the fact that they, including structural conditioning, exist at the same temporal dimension. When M/M approach suggests a flat and independent ontology²⁵ for analytical dualism (interaction) and emergence (emergent properties), I contend that they are *necessarily* emerge and contingently dependent without conflation. And such emergence is maintained by three levels of interactions – social (at the empirical domain), systemic (at the actual domain) and reflexive (at the real domain) interactions which contingently produce another three emergent possibilities – operational emergent cases (OECs), relational emergent categories (RECs) and personal emergent reflexives (PERs) respectively. Full details come in the next section.

Such *realist* deviation are expected to necessarily extend or strengthen our: (i) ontological or theoretical understanding of agents exercising their agency (or reflecting) in different possible

²⁵ Jessop (2005)’s strategic-relational approach suggests stratified but independent ontology for agency and structure

options (i.e., rational, irrational and eclectic); (ii) analytical understanding of the contributions of both structure and agency in and the process of structure-agency interactions (i.e., non-conflationary, spatio-temporal, contingent and emergent); (iii) analytical understanding of how the structure-agency interactions result into their contingently and validly possible emergent (i.e., OECs, PERs and RECs); (iv) explanatory powers to explain how OECs, RECs and PERs respectively further emerge from three levels of interaction (i.e., social, systemic and reflexive) which correspond with the empirical, the actual and the real domains of reality contingently; (v) and finally foregrounding a simple underlying explanatory mechanism for all the emergences of structure and agency and *how* they emerge. This *explanatory analytical approach* (items i – v) is taken up in the section below, where they are fully considered.

3.3.2 Realist explanatory analytical approach

As highlighted earlier, there are few perceived issues with Archer's perspectives on social realism and M/M approach²⁶, in relation to re-enforced CR, but the underlying ones are the problem of objectivation²⁷ (Knio, 2022), non-recognition of eclectic nature of agency and the extreme subjectivity nature of analytical dualism and emergence. These issues are addressed by the explanatory analytical approach, by simply 'coordinating' and 'not partaking' in the *emergent interactions* and *emergent possibilities* as they emerge during structure-agency interaction and structural conditioning (i.e., context or strategic context). By coordinating, I only observed the possible logical interactions/conditioning and assigning or giving names to the occurrences. By not partaking, it means a total taking hands off the whole process of the interactions and emergences as they unfold. So, this section is dedicated to discussing in full details how these emergent possibilities and emergent relations are come by in a contingent and time-bound manner within the explanatory approach. The approach involves conditioning process that produces OECs RECs and PERs through explanatory social, systemic and reflexive interactions that happen at different analytical level.

²⁶ This perspective also include theorists and methodologists such Giddens (1984) and Jessop (2005).

²⁷ In social science, this word can be extended to the problem of *subjectivation*, the making of human itself.

During social interactions, OECs emerge when the social structure (i.e., context or strategic context) conditions the objects of social phenomenon and contingently, social phenomenon also conditions the objects of social structure in return. More specifically, in each situation the process of conditioning is the same but structure and agency (i.e., objects) of both social structure and social phenomenon are influenced differently, leading to the emergence of OECs of different characteristics. The conditioning has linear or stable influence on structure (i.e., if...then type) while the influence on agency results into further production of different possibilities (i.e., if...possibly type)²⁸. The analytical approach further holds that the possibilities of OECs in both situation depend on the levels of the conditioning. One level of conditioning (e.g., strong or positive) produces two possibilities of OECs so also two levels (e.g., strong and weak or positive and negative)²⁹ and three levels (e.g., strong, moderate and weak or positive, neutral and negative)³⁰ produce four and nine OECs respectively. So, all the emergent possibilities from the two contingent conditionings produce the OECs of social world we want to understand or study. However, between the conditioning process and the final emergence of OECs there are interactions that further foreground or explain why OECs emerge with different or diverse characteristics.

The main interactions happen when there are *full*, *partial* or *null alignment* between the conditioned structures and the conditioned agency. When there is a full, partial or null alignment, OECs respectively tend to possess or exhibit a stronger, moderate or weaker characteristics of the social structure and social phenomenon under study. It is important to emphasize that the contingent conditioning process, interactions between social phenomenon and (strategic) context and the final emergent of OECs of different characteristics all occur at the empirical domain and at a point in time. In short, there are three unique features of this approach during social interaction: extended conditioning (i.e., ‘if...then and if...possibly types’), level of conditioning and modes of alignment of or between structure and agency.

During systemic interactions which emerge from social interactions, RECs emerge when social structure and social phenomenon are related with each other. More specifically, all the diverse OECs are further assigned into a set of refined categories, RECs, by employing a set of well-defined rules. These rules are not readily or obviously available but rather emerge after a deliberate

²⁸ Archer (1995, p.192) also recognizes the two types but not specifically used as presented here.

²⁹ Such limited possibilities are maintained by positivism and interpretivism. This has great implication for axiology.

³⁰ Such comprehensive possibilities foregrounded by CR completes the limitations of other two views.

and careful observation by the researchers. The analytical approach also holds that the number of RECs available for grouping each pair of OECs depends on the comprehensiveness of the consideration/recognition given to the nature of human agency. When agency is only considered as rational, all OECs who meet ‘direct relationships’ rule become rationalist-strategist cases and are assigned into a REC³¹. When agency is only considered as irrational, all OECs who meet ‘indirect relationships’ rule become irrationalist-resistant cases and then assigned into another REC³². When agency is both considered as rational and irrational (i.e., eclectic), all OECs who meet neither ‘direct nor indirect relationships’ rule become realist-conformist cases and are as well assigned into another REC³³. The approach further maintains that the rationalist-strategist cases are always in the majority, relatively followed by the realist-conformist and the minority are always the irrationalist-resistant cases. So, for re-enforced CR, the validity of RECs depends on the robustness of recognition given to human agency and rules employed to allocate OECs of social world we want to understand or study. The main arguments of this interaction is that an approach cannot be considered analytical if it is not categorical, and being categorical requires that all the validly possible categories are considered.

The rationalist-strategist cases obey the ‘direct’ rule because they consider the influence of social structure reasonable enough to be exhibited as social actions and as well want the social structure to accordingly relate with such exhibited actions of theirs. So, social structure and social phenomenon are directly related for such group of cases. The irrationalist-resistant cases obey the ‘indirect’ rule because they consider the influence of social structure unreasonable to be exhibited as social actions and as well want the social structure to be changed in relation to their exhibited actions. So, social structure and social phenomenon are indirectly related for such group of cases. The realist-conformist cases obey the ‘neither direct nor indirect’ rule because they consider the influence of social structure both reasonable and unreasonable to be exhibited as social actions and as well want the social structure to nuancedly maintain such exhibited actions of theirs. So, social

³¹ This REC represents the positivist category. It holds reductionist views of human agency and emphasizes the influence of social structure (i.e., independent variable) on social phenomenon (dependent variable). The converse is mostly ignored and when it is considered, social objects (i.e., structure and agency) of both are always conflated.

³² This REC represents the interpretivist category. It also holds reductionist view of human agency and emphasizes the influence of social phenomenon on social structure (i.e., context). The converse is rarely considered and when it is considered, social objects of both are always conflated.

³³ This REC represents the CR category. It completes the deficiencies of other views on agency and considers the relationships both ways, without conflation of structure and agency.

structure and social phenomenon are neither directly nor indirectly related for such group of cases. Therefore, all the three groups of the cases represent the RECs that emerge at the actual.

At the real, the rationalist-strategist, irrationalist-resistant and realist-conformist cases are further considered and differentiated by the way of their reflexivity that governs or shapes their social actions at the empirical and the actual. Specifically, the mode of alignment that produces each case tends to generate different reflexivity about “what to do with the influence of social structure” (e.g., whether to accept, co-exist with or reject) and “what the influence of social structure should subsequently be” (e.g., whether to be re-enforced, reproduced or transformed) following the first reflection, which finally lead to the emergence of diverse OECs and recognizable rules observed at the empirical and the actual respectively. In accordance with the appropriate first- and second-order reflections, OECs that emerge from full, null or partial alignment respectively tend to belong to the three different RECs stated above respectively. The reflexive interaction specifically aims at generating explanations for why diverse cases observed at the empirical act the same way or otherwise towards social structure and at the same time engage in different projects that are personal or otherwise on it.

3.4 Conclusion

This Chapter initially adapts the Bhaskar’s CR framing to guide the original study, being adjudged as much more internally consistent philosophical and research framework. But getting to the stage of data analysis of the study, some emergences which were not well captured in the frame were observed. The key philosophical ones have to do with the methodologies employed by Bhaskar and the ways he uses emergence as a concept for his arguments. Non-realist methodologies and stratification which implies flat causation beyond ontology rather than emergence which suggests the opposite was employed. Such position makes Bhaskar’s CR not only to be externally inconsistent (though internally consistent as non-realist philosophy) but also as a philosophy that is grounded in a less realist perspective on social realism and on non-realist analytical approach of Archer’s M/M or analytical dualism.

The issues observed were addressed, at least to some reasonable extent, through the arguments that are necessarily realist explanations and/or methodologies rather than developing more subjective

concepts. The first key argument is that ontological, epistemological and judgmental realms also emerge from each other and that emergence involves causation of newness at subsequent realm they emerge. The second argument is that CR necessarily attracts realist methodologies, including analytical approach, not only at ontological realm but also at the epistemological and judgmental. The third argument connects with the second argument in that CR view on reality contingently emerge with the methods/methodologies (including analytical approach) employed. These key arguments were employed to re-examine Bhaskar's CR and Archer's perspectives on social realism and by extension M/M analytical approach. This re-examination finally 'give birth' to an alternative framing called *re-enforced CR* philosophy and *re-directed social realist* perspective and by extension *realist explanatory analytical approach*. The re-enforced CR does not only foreground the realist social theory for understanding social phenomena at ontological realm but as well will cover what goes on at the epistemological realm, which the next Chapter presents.

But before moving to the next Chapter, it is important to declare the implications of this new development on the rest of the study, especially as it concerns the study's research questions. In this Thesis, there are two forms of research questions: explanatory (or theoretical) and evaluative study questions. Explanatory study questions are captured in Sub-Research Questions 1 to 3, while evaluative study questions are captured in Sub-Research Questions 4. Based on my new position as presented above, the evaluative study (i.e., Sub-Research Question 4) and aspect of mathematics achievement of explanatory study (i.e., Sub-Research Question 2 and 3) are going to be suspended for later possible post PhD studies. So, aspects of social relationships and mathematical identities of explanatory study (i.e., Sub-Research Question 1) will be retained to allow me not only to interrogate my new philosophical and analytical positions but also to provide answers to the first key question of the study. These decisions still align with the broader aim or title of the study as the aspect of "the roles of social relationships and identities" needs to be firstly foregrounded to be able to move towards the next stage of "improving rural learners mathematics achievement" using my Thesis intervention program or suggestions. Having declared this implication, I now move to the next Chapter, the theoretical framework and methodologies Chapter.

CHAPTER FOUR

STUDY'S THEORETICAL FRAMEWORK AND METHODOLOGIES

4.1 Introduction

This Chapter operationalizes the realist social theory which emerged from the epistemological realm of re-enforced CR philosophy in Chapter 3. The theory necessarily provides the theoretical framework to theorize the three concepts of the study: mathematical identities, social relationships and mathematics achievement. Each concept in the study is firstly situated within the re-enforced CR understanding of social structure, structures and agency, and further theorizes the latter because of its unrecognized nature in the existing realist social theorization. As part of intransitive knowledge production argument in Chapter 3, this Chapter provides a realist theoretical framework for each concept for further pure theoretical studies. As also part of transitive knowledge production position maintained in Chapter 3, I as well operationalize the realist analytical approach for the ongoing explanatory and evaluative studies that elaborate the interactions between the three theoretical concepts to improve practice. As a strong argument of re-enforced CR, all these theorization have to be maintained with the realist methodologies, including realist analytical approach.

4.2 Epistemological methodologies

Generally, methodologies of any inquiry at epistemology realm involve methods, procedures or inferences employed to *identify (or develop)*, *access* and *analyze* social phenomenon of interest³⁴. While Bhaskar's CR seems to employ non-realist inference (subjective arguments) to identify the constituents of structure and agency³⁵, re-enforced CR employs realist procedures. Realist procedures involve engaging with existing explanatory or theoretical frameworks and employing researcher's thought experimentation/intervention. As it stands now, epistemological methodologies for CR are under-developed or receive less attention from the CR loyalists (Bhaskar, 2007; Isaksen, 2016). As explained earlier, this observation may be connected to the

³⁴ This is an indication that inquiry at epistemological realm is broad and tends towards production of both transitive and intransitive knowledge.

³⁵ Some Bhaskar's CR adherents have started acknowledging the reality by shifting from only abductive inference (subjective argument) to suggestion of addition of inductive inference from the existing framework (objective inference) (see Isaksen, 2016, p. 258).

inconsistency in Bhaskar's ontological and/or epistemological standpoints and his 'approval' of non-realist method (i.e., abduction only). For re-enforced CR, there is a logical and consistent basis to claim inferential procedures such as deduction, induction and abduction for its epistemological methodologies or more precisely for its analytical engagement with the available theoretical frameworks (see Fletcher, 2017; Mingers & Standing, 2017; Peirce, 1907). That is, re-enforced CR recognitions of the empirical, the empirical-real and the real domains, are sustained respectively by employing inductive, deductive (inductive-abductive) and abductive inferences.

Induction, through an explanatory inference at the empirical domain, only allows researchers to identify the objects of structure and agency from the existing relevant theoretical framework, without further (judgmental) exemplification from it. Deduction is a theory development process through what emerges from an inductive-abductive inference at the empirical-real domain. Inductive-abductive inference does not only allow researchers to ontologically explain the theoretical framework they engage with but also gives room for further (judgmental) exemplifications about it. Abduction, through an exemplification process at the real domain, only allows researchers to infer or generate a conclusion from the theoretical framework they engage with further (ontological) explanations about it.

4.3 Theoretical framework – Realist social theory

Realist social theory draws from the social realist perspective presented in Chapter 3, which itself inspired by the re-enforced CR philosophy. Social realism builds on three key premises: structure and agency are internal to agents and contingently dependent on each other, both structure and agency emerge contingently, with the latter emerging from the former, and agency can be rational, irrational and eclectic. These three key premises have crucial implications for theorizing the three concepts of the study. Specifically, these premises mean that the constituents of each of mathematical identities, social relationships (and mathematics achievement) are internally located for learners and teachers, dependent on each other without conflation. It further means that the constituents representing the agency of each concept (say, mathematical identities) emerge from the constituents of its structure. And further means that constituents representing agency of each concept (say, social relationships) should be considered as rational, irrational and eclectic.

However, while agency holds analytical and theoretical (or ontological) importance as discussed earlier, its practical manifestations are transient (Emirbayer & Mische, 1998). This ontological status makes agency random and probably too broad to be operationalized for strategic contexts or practices. And that further makes the theory of social realism at ontological realm less important for practice, which calls for practical realist social theory. This argument is further supported by Leca and Naccache (2006, p. 642-643) who observe that “although critical realist works consider that actors set structures in motion and develop their causal powers [i.e., agency], to the best of our knowledge they have never considered how actors can do so strategically”. Similar observation of randomness of “generalized taxonomies” and “facticity and fixity of structures [and agency]” in a strategic context is also emphasized by Jessop (2005. pp. 11 & 17). Therefore, to properly theorize the three concepts of this study, agency (or causal power) needs to be more realist grounded. To be more realist grounded, agency needs to be operationalized (or approximated) with a construct that holds both ontological (explanatory properties) and practical (exemplified properties) importance. That task is firstly taken up below, using the realist inferential procedures discussed above.

4.3.1 Agency and expectancy

The quest to properly conceptualize and operationalize agency, and eventually theorize the three concepts of the study, that addresses the complex relationships between the two, prompts the need for an understanding of expectations. Deductively, expectations are theoretical and analytical constituted constructs (Emirbayer & Mische, 1998; Webster & Whitmeyer, 1999) and they also hold practical importance for strategic practices (Emirbayer & Mische, 1998). Expectation in any strategic practice, including mathematics teaching and learning, is both personal and social constituted experience of what is likely to happen, and they are related to decisions on or of actions in such practices. In other words, the constituted experiences influence every teachers and learner to decide on what actions to take regarding mathematics, based on their expectations. Building on these assertions, I argue that teacher’s and learners’ expectations of teaching and learning mathematics operationalize their agency.

Inductively, Webster and Whitmeyer (1999) theorize how these experiences emerge from two interrelated forms of expectations: first-order expectation – a belief that one has about him/herself

in relation to others regarding a phenomenon/task, and second-order expectation – what one believes others think about him/her in relation to them, regarding a phenomenon/task. These reflectively generated expectations form the basis for causes of choices of actions and meaning making for learners and teachers. Analytically, if second-order expectation is consistent with the first-order expectation, only the latter shapes the next actions of the agents (e.g., learners). If the former conflicts with the latter, there are two possibilities: either both shape the subsequent actions or second-order expectation contribute more than first-order expectations (Troyer & Younts, 1997, cited in Webster & Whitmeyer, 1999).

By abduction, expectation states abstract agency. According to Webster and Whitmeyer (1999, p. 18), “[e]xpectation states are theoretical constructs, generating observable consequences [at the empirical domain] but not themselves directly observable [at the real domain]. They are created from either a unit of evaluation process (evaluation of performances) or a status generalization (inferential) process” (p. 18). Troyer and Younts (1997 cited in Webster & Whitmeyer, 1999) further provide some analytical insights into how these states are created. Expectation states become aggregated only by first-order expectations if both first-order expectations and second-order expectations are consistent. But both aggregate the expectation states if second-order expectations conflict with first-order expectations. So, learners’ and teachers’ evaluation/generalization of their expectation states generate how they exercise their first- and second-order expectations and eventually explain how they actualize or carry out different actions in and out of the classroom.

The analytical property stated above suggests that expectations are reflectively generated human experiences that are not only rational, irrational but also eclectic. Expectations become rational if first- and second-order expectations are both consistent positively and irrational if they are consistent negatively. But expectations become eclectic if first- and second-order expectations are inconsistent. The eclectic analytical property of expectation goes a step further than analytical property of agency has currently been recognized by many theorists. As discussed earlier, more social theorists believe that agency is only rational (e.g., Danermark *et al.*; Giddens) and few others further suggest it can also be irrational (e.g., Archer). But it seems the eclectic property of human agency is yet to gain recognition.

The difference between expectations and agency is that, while agency holds only analytical and theoretical importance, expectations hold analytical, theoretical and practical importance. Here, I provide the analytical and practical importance of how expectations strategically relate to experiences, while Webster and Whitmeyer (1999) provide the basic analytical and theoretical understanding of how the constituted experiences emerge from the expectations. I therefore argue that *first-order expectation* and *second-order expectation* provide the structurally oriented, strategically reflective and emergent calculation of learners' agency. This argument supports the realist position on human agency and its strategic, emergent, reflexive and relational properties to the structures (Archer, 1995; Jessop, 2005). Strategic calculation or reflection of these expectations may or may not be observed/actualized depending on the enabling or constraining structures.

The most salient argument here is the approximation of human agency with expectancy. By approximation, I mean an explanatory or theoretical language that deduce, induct and abduct agency without losing its ontological status. Also key to this argument is that expectations are not only rational and irrational but also eclectic. How these two expectations are *set* in general theorization of the three concepts of the study are discussed separately in the next three sections, using the same realist inferential procedures.

4.4 Theorizing the concepts of the study – an overall framework

This section explains how objects of structure and agency are obtained, with an emphasis on the theorizations from and operationalizations of the empirical-real domain mechanisms. In other words, I used this section to produce the intransitive knowledge (focusing on ontological explanation and theorization of the study concepts) and transitive knowledge (focusing on judgmental exemplification and practical utilization of the concepts). Working with the re-enforced CR ontology, the three concepts – mathematical identity, social relationships and mathematics achievement – are nuancedly theorized (see Figure 4.1). The theorizations draw on the existing studies that implicitly or explicitly provide theoretical understandings to build on them. Mathematical identity theorization draws on Marks and O'Mahoney (2014), Manzo (2012) and Archer (2002) to obtain emotional attachment, social belonging and expectation exertion as mechanisms that explain learners' identification with mathematics. Social relationship builds on Hagenauer and Volet (2014), Spilt, Koomen and Thijs (2011) and Rubie-Davies (2015) to obtain

socioemotional connection, productivity support and expectation activation as mechanisms that explain teacher-learner social relationships. Mathematics achievement draws mainly on Kilpatrick, Swafford and Findell (2001) and McGee (2015) to consider mathematics achievement as mechanisms explained by sociocognitive ability, productive disposition and expectation propensity.

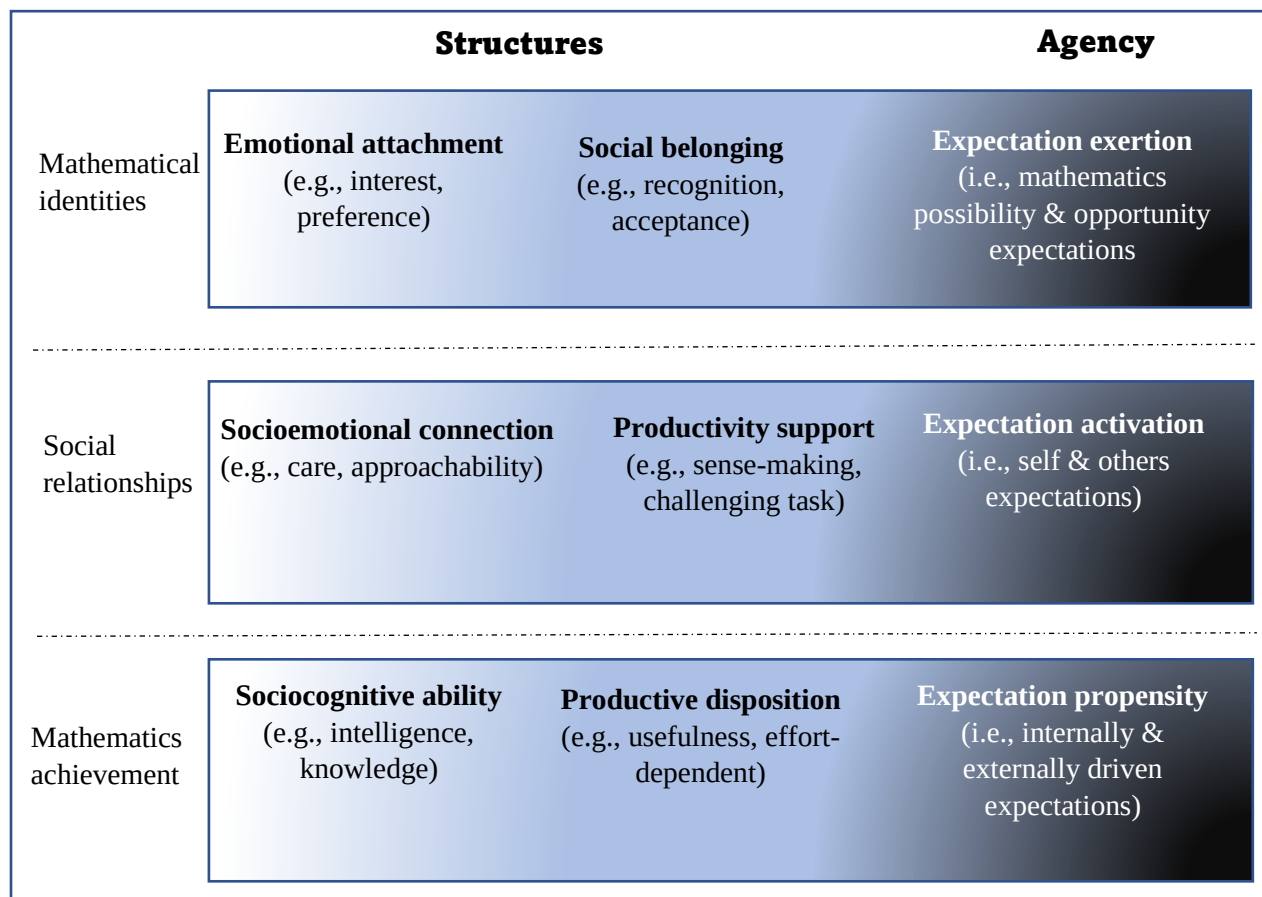


Figure 4.1: Overall theoretical framework for the study

4.4.1 Mathematical identities framework

The work of Marks and O'Mahoney (2014, p. 67) provides a 'broad sketch' or generalized taxonomies of realist identity-based explanation. They consider identity as relationships between personal identities, social identities and agency. To them, personal identity "emerges from embodied, reflexive self, in part forged through the interests and actions of that individual" (p. 71).

Social identity “is the navigated position between personal identities and the way which people believe they should be perceived in a social setting” (p. 72). Agency “is the point by which the person and social structure, and therefore, social identity and personal identity are reproduced and transformed” (p. 72). According to realists, personal and social identities as objects of structure and agency as an object of causal powers are only analytically distinct but remain interrelated. Working with this key argument, I narrow down the broad understanding of the three constructs to a learners’ mathematical identity conceptualization, as suggested by Marks and O’Mahoney (2014). They suggest that researchers will still need to come up with their own “domain-specific meso-level constructs or draw upon existing authors who have developed frameworks that are explicitly or implicitly consistent with a realist ontology”(p. 69). I work with the former because no known CR proponent³⁶ have strategically produced a framework to study learners’ mathematical identity.

Inductively, I theorize personal identity as emotional attachment (e.g., interest, perseverance, preferences), social identity as social belonging (e.g., recognition, acceptance, validation) and agency as expectation exertion (i.e., beliefs regarding mathematics possibility and opportunity expectations). Emotional attachment and social belonging involve learners’ reflective and recurring patterns of relationships with mathematics and in the mathematics community respectively, while expectation exertion involves learners’ reflections about their beliefs regarding mathematics possibility and opportunity expectations (see detail later). Deductively, I operationalize personal identity as attribute acquisition (e.g., creating extra time for mathematics, requesting for more mathematics tasks) that emerges from learners’ emotional attachment to mathematics. Social identity as community participation (e.g., asking and answering questions in the class) that emerges from learners’ social belonging in the mathematics community. And agency as expectation actualization (e.g., giving attention and having regular attendance in mathematics class) that emerges from learners’ expectation exertion for mathematics and mathematics community. Attribute acquisition and community participation involve learners’ direct experiences of self about mathematics, and of self and others in the mathematics community respectively, while expectation actualization involves various learners’ observable actions

³⁶ I am aware of Gardee and Brodie’s (2021) framework. It seems to reduce the mechanisms to the empirical domain, hold conflationary view, ontological status of agency appeared stripped off and the conception of their agency appeared too random to inform practice.

explained by their mathematics possibility and opportunity expectations. How does literature support these theorizations and operationalizations? Figure 4.2 represents the theoretical framework.

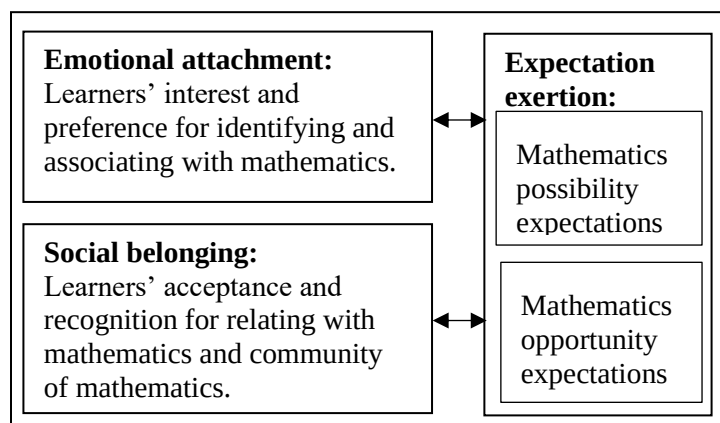


Figure 4.2: A learners' mathematical identity framework

Different literature reviews have largely shown that two major perspectives – *sociological* and *psychological* and observable mechanisms – *participation* and *acquisition* drive mathematical identity (e.g., Darragh, 2016; Radovic et al., 2018). The reviews provide the realist basis to begin to engage with theoretical perspectives commonly employed to study mathematical identity. For example, through induction, Darragh found that *sociological* – i.e., social influences and *psychological* – i.e., individual influences dominate the field. But being an area currently dominated by interpretivists, studies that engage with mathematical identity mechanisms at the actual domain are scarce (see Darragh, 2016; Radovic et al., 2018). Engaging inductively with Archer (2002) and Manzo (2012 reporting Pizzorno, 1986; Akerlof & Kranton, 2001), the objects of structure for learners' mathematics identity at the actual domain are obtained. According to Archer (2002), personal identity emerges from emotions which drive individuals' interests. And emotions are agents' (e.g., learners') concerns about social objects (e.g., mathematics) that are personally significant to them. Since people “work out their own *modus vivendi*” according to their concerns (Archer, 2002, p. 16), learners always, in different degree, reflexively develop their *emotional attachment* or relationship with mathematics.

Social identity emerges from navigation between reflexive self and beliefs about one's social recognition or positioning. This navigation presents a difficult conceptualization. Nevertheless, the suggestion of theoretical idea of *social belonging* by Manzo (2012) attends to this difficulty. The

idea of social belonging is an integration of the concept of reason trans-subjectivity from reason-based explanation (see Boudon's "Analytical sociology"... cited in Manzo, 2012) and the ideas of recognition cycles and social conformism from identity-based explanation (see Akerlof & Kranton, 2001; Pizzorno, 1986, cited in Manzo, 2012). The former involves agents' strong beliefs about the reasons for their actions as being a shared reasons for other agents as well. The latter, for example, involves agents' need for reassurance of their identity by others with similar views. In both cases, according to Manzo (2012, p. 45), "actors need to search for potential sources of (more or less) local social consensus". And concludes that the identity-based explanations are able to "link social belonging, social identity and actors' beliefs" together. The theoretical idea of social belonging offers explanations for how learners always, in different degree, participate within and outside mathematics classroom. Therefore, both emotional attachment and social belonging become the two objects of structure at the actual domain.

Darragh (2016) also found, through deduction, that researchers commonly see mathematical identity as *participation/action* engage in and *acquisition* of some attributes by learners respectively. In line with CR, these reviews deductively offer the mechanisms for learners' mathematical identity at the empirical domain. So, I obtain the mechanisms (i.e., objects of structure) for learners' mathematical identity with little modifications as '*community participation*' and '*attribute acquisition*' respectively. In other words, as learners repeatedly reflect on their emotional attachment to mathematics as informed by their interest and preference, they translate that unobservable reflections to what can be observed by, for example, mostly creating more time for mathematics and requesting for more mathematical tasks or exercises. Such observable actions and many others are captured in the attribute acquisition. Similarly, as learners repeatedly reflect on their social belonging as informed by the recognition and acceptance they receive, they translate that unobservable reflections to what can be observed by, for example, frequently asking and answering questions in the classroom. Such observable actions and many others are captured in the community participation. Abductively, however, there is no evidence for the object of causal powers in the reviews, confirming that it is actually overlooked or conflated by the existing analyses or perspectives.

What I have done thus far is to obtain the empirical-real domain-based (or relational) and the empirical domain-based (or operational) mechanisms for learners' mathematical identity, but

without objects of agency or causal powers. Building on my earlier argument, I strategically set the first- and second-order expectations as *mathematics possibility* and *opportunity expectations* respectively. And I argue that *expectation exertion* is the object of causal powers for learners' mathematical identity at the empirical-real domain. So, learners' expectation exertion is captured by their personal and social mathematics past, current and future experiences. Learners do *exercise* their expectations, as informed by their possibility and opportunity expectations, and that allows them to strategically reflect on their initial possibilities about their emotional attachment to mathematics, and the current and future available or provided opportunities to socially belong to and for participating within a mathematics community respectively. If both mathematics possibility and opportunity expectations are consistent (rationally or irrationally), the exertion of such learners are solely shaped by their mathematics possibility expectations. But if they conflict or inconsistent (eclectically), the exertion of such learners are shaped by their mathematics opportunity expectations or both. These processes are unlikely to be observed or perceived. But, they have powers to cause or produce next observable actions learners *actualized* in and outside the mathematics classroom.

With *expectation actualization*, learners convey how they feel about their emotional attachment to mathematics and social belonging in mathematics community. The conveyance can involve myriad of reactions or responses that emerge from both mathematics opportunity and possibility expectations. Interestingly, these reactions or responses can be towards their teachers or peers within and outside the class. Studies have found that when learners believe that their potentials are considered minimal, that they are not getting enough learning opportunities or they are being marginalized by their teachers, they tend to react by deserting class and with contempt and volatile responses (e.g., Martin, 2000; McGee, 2015). These actions may be more visible among adult students, but they may also include being passive, making noise or being disruptive in the class among young learners. Similarly, when learners believe that there is no need for education or that mathematics offers no opportunities in real life, they are more likely to be harassing, teasing and disturbing their peers who are trying to concentrate in the class, and vice versa (e.g., Martin, 2000). Both Martin (2000) and McGee (2015) also found that when learners believe that they are capable and it is possible for them to do and learn mathematics, they become very attentive in the mathematics class, use their time and energy effectively and strive to be successful.

Overall, I argue that as mechanisms that operate at the empirical-real domain, ‘emotional attachment’, ‘social belonging’ and ‘expectation (of mathematics possibility and opportunity) exertion’, theoretically explain learners’ identification with mathematics. Also, I maintain that though they are analytically distinct, they still remain ontologically dependent and enable and constrain each other. So, as learners reflect on their mathematics possibility and opportunity expectations, it allows their emotional attachment and social belonging to be shaped respectively, leading to the development of observable robust, modest or fragile mathematical identities. Similarly, as learners evaluate their emotional attachment to mathematics and social belonging in the mathematics community, it also allows their mathematics possibility and opportunity expectations to be shaped respectively, leading to the development of observable robust, modest or fragile mathematical identities.

Learners of robust mathematical identities are likely to be more emotionally invested to learn mathematics, fully belong in mathematics classroom and they tend to hold positively consistent expectations about mathematics possibilities and opportunities. Learners of modest mathematical identities are likely to be moderately emotionally invested to learn mathematics, belong in mathematics classroom and they tend to hold inconsistent expectations about mathematics possibilities and opportunities. Learners with fragile mathematical identities are likely to be less emotionally invested to learn mathematics, belong in mathematics classroom, and they tend to hold negatively consistent expectations about mathematics possibilities and opportunities.

4.4.2 Social relationships framework

Social relationships are interpersonal relationships between two or more people, including teachers and their learners. The field of teacher-learner relationships is predominantly humanistic and constructivist driven (Cornelius-White, 2007). This is one concept that has witnessed different operationalizations from different researchers with almost similar results (e.g., Anderson & Cartafalsa, 2002; Bernstein-Yamashiro & Noam, 2013; Darragh, 2013; Gardee, 2019a; Spilt *et al.*, 2011; Yan, 2019). The lack of nuance operationalization has translated to a lot of overlaps, confluences or/and recursions in the attempts to obtain the mechanisms that empirically or theoretically explain the relationship between teachers and learners. However, three studies – Hagenauer and Volet (2014), Rubie-Davies (2015), and Spilt *et al.* (2011) – are instrumental in this current theorization

by providing initial foundations to build the realist theoretical framing on. While the first study directly aims to obtain the mechanisms or dimensions that explain higher education teachers-students relationships, the second study separately focuses on the implications of teachers' expectations on their practices, including how they establish relationship with learners. The last study explores the importance of teachers' wellbeing in their relationship with learners.

Drawing on their review, Hagenauer and Volet (2014) observed that teacher-learner relationships appear to be context dependent, multi-dimensional and they went further to provide two “tentative(..) super-ordinate dimensions” to be considered by researchers (p. 378). They argue that teacher-learner relationships are of two separate dimensions: affective and supportive dimensions. Affective dimension involves any personal or individual basis for building relationships between teacher and learners. Such relational basis includes respect, trust and care for learners. The concept of care, for example, has been emphasized for emotional connection with and a means of motivating learners (Komarraju, Musulkin & Bhattacharya, 2010; Marlowe, 2006). Supportive dimension involves certain kind of support or help provided by the teachers for their learners' success. According to the review, such supportive behaviour includes clear expectations, care, respect and approachability. But Hagenauer and Volet (2014) gives more emphasis to approachability.

As objects of structures, these two dimensions offer an initial opportunity to engage with the theorization of social relationships. However, being a tentative and agency or causal powers omitted theorization, I inductively make a refinement for more generalizable and nuanced mechanisms. First, I collapse the aspects of the two dimensions (not the dimensions themselves) to derive a broader dimension called *socioemotional connection* for the affective dimension. The reason being that there are overlaps in their aspects, as also reported and acknowledged by Hagenauer and Volet (2014). And more importantly, relationships are driven by emotional factors that are not limited to individual or personal influences but also social influences such as interaction and approachability (Hagenauer & Volet, 2014; Rubie-Davies, 2015). Social relationship is interpersonal; therefore, it also entails social aspects, as teaching and learning do (e.g., Essien, 2020). Second, because it has justification in the literature and support from my

experience³⁷, I retain the support dimension of social relationships, with modification as *productivity support*. It has been argued that establishing and sustaining relationship with learners is not enough if the relationship is not discipline-specific productive for the learners (e.g., Anderson & Carta-falsa, 2002; Hodgen & Marks, 2009). In other words, social relationship structures are not limited to persons-persons dimension but extend to persons-objects dimension as well. Figure 4.3 represents the theoretical framework for social relationships.

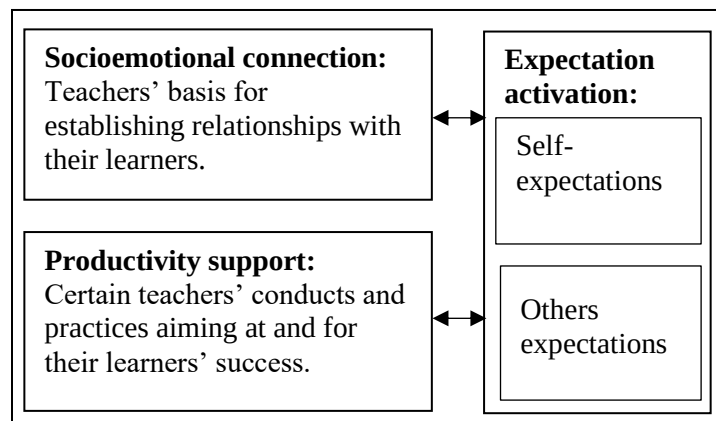


Figure 4.3: A teacher-learner social relationships framework

The above derivations are necessary to avoid overlap in and at the same time establish the relationship between the socioemotional connection and productivity support dimensions operating at the empirical-real domain. Socioemotional connection (e.g., care, respect, warmth, interaction, approachability) forms the foundation and commitment for establishing positive relationship climate between teachers and their learners (e.g., Anderson & Carta-falsa, 2002; Gallagher, Kainz, Vernon-Feagans & White, 2013; Redding & Walberg, 2012). In other words, if teachers are committed to develop respect for and build warm interactions with their learners, for example, they are likely to translate that unobservable commitment to what can be observed in their relationships. That is, teachers enact the commitment in form of *psychosocial support* they enabled for the learners in and outside the classroom (Kuklinski & Weinstein, 2001; Rubie-Davies, 2015). Such enactment includes giving attention to, valuing and getting familiar with their learners. Therefore, psychosocial support as an object of structure at the empirical domain operationalizes socioemotional connection of teachers with learners at the empirical-real domain.

³⁷ My experience as a mathematics tutor, teacher and teacher educator at all levels of education supports the dimension.

Productivity support (e.g., sense-making, challenging task, active engagement, cultural-based) describes any supportive practices (pedagogical or social) that are discipline-specific and deliberately employed by the teachers for learners' school success. Teachers dedication to support their learners for success is a strong factor for connecting the learners with the subject they teach, which in turn is likely to lead to positive teacher-learner relationships (Berry, Thunder & McClain, 2011; Redding & Walberg, 2012). Learners, for instance, are more likely to differentiate the relationships created for two closely related subjects and the teachers teaching them, (say physics and mathematics teachers), because of how the teachers differ in the ways they support and engage the learners. So, the dedication of teachers makes them to enact, in observable manners, their *professional practice or role* through various social (or pedagogical) practices that depict the relationship they provide for an individual learners in the classroom. Such enactment may include enabling learning opportunities, holding high expectations for and awarding/assigning rewards to their learners. Therefore, while productivity support operates at the empirical-real domain, the professional practice operationalizes it at the empirical domain.

As I pointed out earlier, Hagenauer and Volet's (2014) dimensions overlook the roles of causal powers in the social relationships theorization. To consider the causal powers, I draw on Rubie-Davies (2015) and Spilt *et al.* (2011) to suggest teachers' *expectation activation* as the third dimension at the empirical-real domain. Teachers' expectations are beliefs or implicit theories held by teachers which shape their current and future practices and relationships in a consistent and predictable way (Rubie-Davies, 2015; Spilt *et al.*, 2011). Spilt *et al.* (2011) assert that teachers hold "generalized expectations and beliefs about themselves as a teacher, about their various roles (...), self-efficacy beliefs, goals for interactions with students, and beliefs about how students *should relate* with teachers" to build relationships with learners (p. 463, italic added). Inductively, these implicit theories are two interrelated and they activate expectations of teachers: self-expectations and others expectations. Self-expectations describe teachers' beliefs about their own ability, knowledge and coping capacity to make impact in the learners' actions and achievement (Rubie-Davies, 2015; Spilt *et al.*, 2011). Others expectations describe teachers' beliefs about how their learners *should relate* with them, as teachers need to nurture their wellbeing in the relationship (Spilt *et al.*, 2011). However, this definition trivializes learners' agency, as they also need their wellbeing to be nurtured, being human as well. Therefore, I re-define others expectations as teachers' beliefs about what learners think about them, about mathematics and about the

relationship itself. This understanding is crucial and *must* be considered by any teaching approach that plans to put learners and their expectation (agency) at the center.

So, teachers' *expectation activation* captures both the personal and social experiences that theoretically explain how the quality of teacher-learner relationships will be. Building on this theoretical position, I strategically *set* the first- and second-order expectations as *self-expectations* and *others expectations* respectively. Deeper understanding of teachers' expectations and how they shape relationships with learners are better achieved through their beliefs than direct observation of their practices (Rubie-Davies, 2015). This conclusion aligns with CR ontological position. This is because when teachers *activate* their expectations, as informed by their self-expectations and others expectations, it enables them to choose the quality of the socioemotional connection they can establish with and productivity support they can offer to their learners. If both self and others expectations are consistent (rationally or irrationally), the expectation activation of such teachers are solely shaped by their self-expectations. But if there is conflict (eclectically), the expectation activation of such teachers are shaped by the others expectations or both. The interplay process presented here may not be directly observable but strongly manifests as what it is observed in and outside the classroom. The interplay becomes directly observable when teachers *enact* their expectations.

With *expectations enactment*, differences in how teachers provide psychosocial support and the quality of their professional practices towards individual learners are made directly observable and measurable. Deriving their powers from the self- and others expectations, such observable differences could be in many forms and of different quality. Several studies reported in Rubie-Davies (2015) found that when teacher believe that they have what it takes to positively impact their learners' behaviours and success, they tend to provide equal psychosocial support and learning opportunities for all the learners, contrary to those who believe otherwise and provide differentiated support (e.g., Ashton & Webb, 1986; Ross, 1998; Warren, 2002; Woolfolk Hoy, Hoy & Davis, 2009). Other studies have found that teachers who believe that intelligence or ability increases and decreases rather than fixed (e.g., Dweck, 2006; Jordan & Stanovich, 2001; Lumsden, 1998; Wilkinson & Townsend, 2000) and that every learner can learn (e.g., Gardee, 2019a, 2019b) are also likely to treat all learners equally, unlike those who believe otherwise. Also, the beliefs held by teachers about the malleability nature and overcoming difficulties in the teacher-learner

relationships has been argued as the basis for differentiating how they treat individual learners equally or otherwise (Chang & Davis, 2009 cited in Spilt *et al.*, 2011).

However, I contend that the teachers' differential treatment of learners is better ascribed to others expectations rather than self-expectations; though I still agree that the two are interrelated³⁸. It is more analytically and theoretically grounded to argue that when teachers believe that certain learners think that mathematics is too difficult, they cannot learn mathematics, their teachers are not good enough for them and other such thinking and they (the teachers) continue to work with those beliefs, they are more likely to enact differential treatment in how they attend to and support such learners, compared to their counterparts who they believe think contrary. For self-expectations, when teachers hold negative beliefs about their own ability and capability, about the general nature of ability/intelligence and about the malleability nature of relationships, they are more likely to be deserting or coming late to classes, avoiding and getting angry with questions asked by their learners and probably keeping distance from certain learners, and vice versa. By abduction, it seems these teachers' actions that derive their powers from self-expectations are overlooked by the existing analyses and perspectives, despite being salient in our mathematics classrooms. The omission is not a surprise as the existing analyses are mainly based on teachers' observations³⁹, rather than being based on theory engagement using realist inferential procedures.

Lastly, building on CR theorization processes presented thus far, I maintain that socioemotional connection, productivity support and expectation (of self and others) activation mechanisms theoretically explain the quality of social relationships between teachers and their learners. They are analytically separated but at same time ontologically related. So, as teachers reflect on their self-expectations and learners' relational expectations, it allows their socioemotional connection with and productivity support for individual learners to be shaped, leading to the development of observable robust, modest or fragile teacher-learner relationships. Similarly, as teachers evaluate their socioemotional connection and productivity support, it also allows their self-expectations and learners' relational expectations to be shaped, leading to the development of observable robust, modest or fragile teacher-learner relationships.

³⁸ What the studies could not capture in their analyses is that there was a confliction between self- and others expectations, and that the latter finally shaped the observed actions of the teachers.

³⁹ No teacher will enact any of these actions or practices while a researcher is planning or present to observe his/her classroom teaching.

In classrooms where teachers enable robust social relationships, learners perceive maximum socioemotional connection with and productivity support from their teachers and the teachers are more likely to hold positively consistent expectations about themselves and their learners. In classrooms where teachers enable modest social relationships, learners perceive moderate socioemotional connection with and productivity support from their teachers and the teachers tend to hold inconsistent expectations about themselves and their learners. But where teachers enable fragile social relationships, learners perceive minimum socioemotional connection with and productivity support from their teachers and the teachers tend to hold negatively consistent expectations about themselves and their learners.

4.4.3 Mathematics achievement framework

Mathematics achievement has different meanings for different researchers. Completing homework, studying mathematics related-courses, choosing mathematics based-careers, teachers' and peers' acceptance, mathematics excellence, participation rates and test scores have all been recognized as mathematics achievement (e.g., Gutierrez, 2002, 2007, 2008; Walker, 2007; Darragh, 2013; McGee, 2015). All these conceptions show the degree to which psychological and sociological perspectives, and by extension positivism and interpretivism have shaped researchers' understandings of mathematics achievement. Basically, learners' mathematics achievements are both socially and individually situated, and many studies have established this assertion from both teachers' and learners' perspectives (e.g., Adeniji & Abubakar, 2019; Gutierrez, 2008; Young, 1998). One view of mathematics achievement is the notion of mathematical proficiency of Kilpatrick, Swafford and Findell (2001), which this theorization builds on. The theorization also draws on other studies, McGee (2015) and Berry *et al.* (2011), that further offer other aspects of the CR framing.

Building on different sources – e.g., studies and theories in cognitive sciences, their experiences as learners and mathematics teachers – Kilpatrick and colleagues “provide a framework for discussing the knowledge, skills, abilities, and beliefs that constitute mathematical proficiency” (p. 116). The framework, according to Kilpatrick *et al.* (2001), has five interdependent and interwoven strands: conceptual understanding, procedural fluency, adaptive reasoning, strategic competence and productive disposition. Conceptual understanding is the comprehension of

mathematical concepts, operations and relations. Learners are more likely to recall, use and reconstruct methods they learn with understanding, and as a result “gain more confidence which then provides a base from which they can move to another level of understanding” (p. 119). Procedural fluency is the ability to skillfully carry out mathematical algorithms adequately and efficiently. The fluency in mathematical procedures or computations gives learners insights into how “mathematics is well structured (...), and how carefully developed procedures can be a powerful tool for completing routine tasks” (p. 121).

Strategic competence is the capacity to formulate, represent and solve mathematical problems. It involves the ability to detect relationships and formulate problem-solving techniques for nonroutine problems with flexibility. Adaptive reasoning is the ability to analyze, explain and justify mathematical arguments or problems. Adaptive reasoning holds all the other strands together because it involves “the capacity to think logically about the relationships among concepts and situations”, and the ability to make valid conclusions (p. 129). Productive disposition refers to the learners’ beliefs of their own efficacy and reflection on the significance of mathematics in their lives beyond classroom.

The strands of mathematical proficiency are instrumental to broaden and investigate the learners’ mathematical achievement because mathematical proficiency, according to Kilpatrick *et al.* (2001, p. 133),

goes beyond being able to understand, compute, solve, and reason. It includes a disposition toward mathematics that is personal. Mathematically proficient people believe that mathematics should make sense, that they can figure it out, that they can solve mathematical problems by working hard on them, and that becoming mathematically proficient is worth the effort.

However, what learners achieve mathematically depends not only on personal but also on social relationships in relation to mathematics. The ways learners are exposed to meaningful and common-sense mathematics, which relate them to their language, culture and social experiences, influence how they feel belonged, see importance and usefulness of mathematics beyond classrooms (e.g., Keitel & Kilpatrick, 2005; Martin, 2007) and concomitantly, influence their commitment to invest to learn and participate in mathematics activities (Berry *et al.*, 2011; Walker, 2012). How mathematical qualities and competence are also assessed, recognized or rewarded

have influence on learners' engagement with mathematics (Hodgen & Marks, 2009). It is then important to extend the mathematical proficiency strands to social influences.

Inductively, the first four strands are summed up into one dimension called *sociocognitive ability* and the last strand, *productive disposition*, with the extension made about it in mind, is retained as another dimension. So, sociocognitive ability (e.g., intelligence, knowledge, aptitude, skills) and productive disposition (e.g., usefulness, importance, making sense, exerting effort) are the interacting objects of structures at the actual domain. Berry *et al.* (2011), in a separate study of mathematically successful Black boys in rural schools, offer insights on how the two dimensions can be engaged deductively and operationalized. Berry and colleagues found that three broad factors are responsible for the learners mathematics success: engaging with unique/challenging mathematics and computational fluency, engagement with mathematics community and out of school activities, and extrinsic recognitions. While the first two factors are relevant for the operationalization here, the third factor is discussed later where it is more appropriate.

When learners can engage with unique or challenging mathematics, then they must have developed some sociocognitive abilities, leading to what some researchers (e.g., Graven, 2012) refer to as mathematical competence or what Gutierrez synthesizes as *mathematical excellence*. Mathematical excellence, according to Gutierrez (2008, p. 362), is “high performance on standardized tests and [demonstration of mastery of] broader notions of mathematical literacy”. When learners can engage with mathematics communities and out of school activities, they should have achieved productive disposition, leading to what I call *productive engagement* with mathematics. Productive engagement describes involvement in mathematical communities, whether by ‘benefiting’ from or ‘giving back’ to the communities. Learners benefit from the communities when they are engaged mathematically by the people around them and give back to the communities when they contribute to improve or change the mathematical situation in their communities (e.g., McGee, 2015). So, I operationalize both sociocognitive ability and productive disposition at the empirical-real domain with the mathematical excellence and productive engagement at the empirical domain.

Sociocognitive ability and productive disposition as interacting mechanisms at the empirical-real domain, according to CR, partially explain and theorize mathematics achievement. To include the dimension overlooked by the framework of Kilpatrick and colleagues, I inductively draw on other

studies to suggest *expectation propensity* as another dimension. Several studies have found that extrinsic recognitions or external expectations (e.g., high scores, proving oneself, pleasing others, career opportunities) are strong explanatory factors for mathematical success (Berry *et al.*, 2011; Gweshe & Brodie, 2019; McGee, 2015). McGee (2015) has also argued and subsequently found that mathematical success can as well be explained by learners' internally driven expectations (e.g., self-satisfaction, self-fulfillment, mathematics affinity, and pleasing oneself). Therefore, learners' *expectation propensity* depicts both human internal and external experiences that drive mathematics achievement. Figure 4.4 depicts the theoretical framework for mathematics achievement.

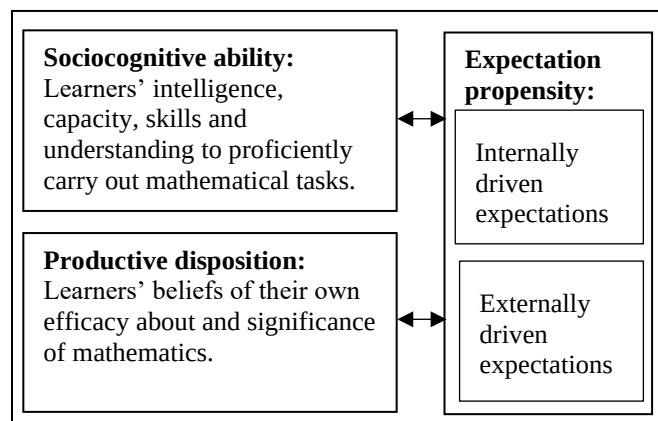


Figure 4.4: A mathematics achievement framework

Building on the above literature engagement, I strategically set the first- and second-order expectations as *internally driven expectations* and *externally driven expectations* respectively. Generally, learners' internal and external expectations drive their propensity for mathematics achievement. Specifically, when learners' propensity for achievement is driven by their internal and external expectations, it motivates them to invest more to build or develop their sociocognitive abilities for and further shape or enhance their productive dispositions towards mathematics. If both the internal and external expectations are consistent (rationally or irrationally), the expectation propensity of such learners are solely shaped by their internal expectations. But if there is a conflict or inconsistency (eclectically), the expectation propensity of such learners are shaped by the external expectations or both. All these processes are not directly observable, but they explain what the learners do or say in and outside the school. The processes become directly

observable when learners finally *realize* the expectations they have achieved or are likely to achieve from mathematics.

With *expectation realization*, learners show or demonstrate how they feel about their mathematical excellence and productive engagement statuses or successes. The demonstration manifests in different forms, including being happy or sad, confident or confused, enjoying or frustrated, feeling great or terrified/struggling. For example, it is a common report that students develop depression or even commit suicide because they have been scoring poorly in their school subject tests (including mathematics) or/and not meeting the academic expectations of their parents, guardians or teachers. More often, we do also hear cases of some students developing internal joy, becoming very happy, nicely relating with their teachers and friends or being associated with other positive behaviors because they have been able to achieve self-satisfaction or self-fulfillment just for learning mathematics. By abduction, but it seems researchers have not considered how achievement is strongly connected with such actions or behaviors in their analyses and perspectives. Thus, in this theorization, learners' expectation realization at the empirical domain operationalizes their expectation propensity at the empirical-real domain.

So, mathematics achievement of any learner lies in the continuous interplay among their mathematical excellence, productive dispositions – both personal and social and expectation propensity – both internally and externally driven for being successful in mathematics. For emphasis again, these mechanisms are only analytically separated but still ontologically related. As learners reflect on their internally and externally driven expectations, it enhances and develops their sociocognitive ability for and productive engagement with mathematics, leading to an observable of demonstration of high, average or low mathematics achievement. Similarly, as learners evaluate their sociocognitive ability for and productive engagement with mathematics, it also motivates and strengthens their internal and external expectations for achievement, leading to an observable of demonstration of high, average or low mathematics achievement.

High-achieving learners tend to be highly competent in, have strong productive dispositions towards and hold positively consistent internal and external expectations about mathematics. Average-achieving learners tend to be averagely competent in, have moderate productive dispositions towards and hold inconsistent internal and external expectations about mathematics.

Low-achieving learners tend to be less competent in, have weak productive dispositions towards and hold negatively consistent internal and external expectations about mathematics.

4.5 Theorizing the SREP Intervention

CR researchers, as social agents for change and transformation, always try to go beyond reporting research findings by suggesting well informed and grounded solutions to social issues. Social relationships enhancement practices (SREP) is an *emergent intervention* employed by the researcher to be able to make this social change. SREP is a set of *deliberate* social practices that emerge from engaging different theoretical frameworks which teachers can enable or demonstrate within and outside the classroom. The practices specifically emerge from learners' expectations (i.e., agency) and their personal and social experiences about mathematics and mathematics community, as theorized in sections 4.4.1 – 4.4.3 above. These social practices are four interrelated teachers' practices which I categorize as psychosocial support, agency sensitivity, productivity support and equity sensitivity. Psychosocial support describes the teachers' practices that are informed by learners' personal and social identities. These may take different forms which include valuing and respecting learners' views, verbal and nonverbal encouragement, recognizing and attending to learners' emotions and motivations for success. Agency sensitivity explains teachers' practices that are mindful of learners' agency; that is, learners' expectations of self and others in relations to mathematics are not ignored and they are influenced positively.

Teachers' practices that facilitate learners' social and personal productive engagement with mathematics are categorized as productivity support. Indicators of such productive supports include emphasizing the sense in, worth and usefulness of mathematics; awarding and recognizing learners' mathematical qualities and competence; and relating mathematics success with efforts and diligence. Equity sensitivity describes those teachers' practices that distribute learning opportunities equitably to learners in and outside the classroom. These may be demonstrated through being conscious of the type of social interactions teachers have with learners which can ignore, embarrass or prioritize, exalt some over others. So, SREP is an emergent flexible intervention employed to direct the teachers' awareness to the relevance of learners' identities in the learning processes and supportive responses to such learning processes, and by extension, to how learners' mathematical achievement could be broadened and improved.

4.6 Relating the theoretical concepts – explanatory analytical framework

Figure 4.5 captures an overall analytical framework for relationships between theorized social relationships (SR), mathematical identities (MI) and mathematics achievement (MA), as related to teachers and learners in mathematics classrooms⁴⁰. In the last section, I explained and exemplified each theoretical concept framework from a realist social perspective. In this section, I present their relationships by operationalizing the realist explanatory analytical approach earlier discussed in Chapter 3. With this approach, the relationships forges deep into the *why* and *how* of the interactions between the concepts. Analytically, acceptance, transformation or rejection of influence has been explained by individuals or agents exercising their agency or expectations (Archer, 1995; Fisek *et al.*, 1991; Gardee & Brodies, 2019, 2021; Marks & O'Mahoney, 2014). This non-reductionist approach is key to CR analyses and explanations of how social changes occur or do not occur. I am guided by this approach to provide the analytical framework for the relationships between theorized SR, MI and MA. For emphasis, only the relationships between SRs and MIs are re-worked based on the emergent re-enforced CR and realist social theory.

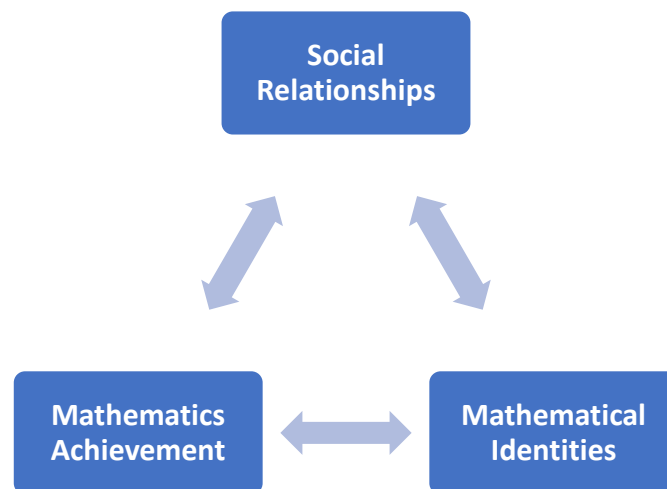


Figure 4.5: A framework for relationships between SRs, MIs and MA

⁴⁰ This is an initial overall framework for the relationships. The R-E/E analytical framework which later emerged from the R-CR was finally used for the relationships of the theorized concepts.

4.6.1 Relating social relationships and mathematical identities

Relationships between SRs and MIs can be comprehensively explained through learners' reflections. In classrooms where teachers repeatedly enable robust, modest or fragile social relationships, learners reflect whether to *accept, co-exist with or resist* the influence, which finally results in their emergence as either robust, modest or fragile MI learners. When a teacher enables robust SR, it strongly positively influences learners' emotional attachment (EA) and social belonging (SB), but both remain unenabled until the learners exercise their expectations – i.e., mathematics possibility and opportunity expectations. If both expectations aligned with EA and SB, they accept the influence, both EA and SB get fully enabled and then emerge as robust MI learners. If only one of the expectations aligned with EA or SB, they co-exist with the influence, one of EA and SB gets fully enabled and the other partially enabled and then emerge as modest MI learners. But if both expectations do not align with neither of EA and SB, they reject the influence, both EA and SB get partially enabled and then emerge as fragile MI learners.

When a teacher enables modest SR, it moderately positively influences learners' EA and SB, but both remain unenabled until the learners exercise their expectations. If both expectations aligned with EA and SB, they accept the influence, both EA and SB get fully enabled and then emerge as robust MI learners. If only one of the expectations aligned with EA or SB, they co-exist with the influence, one of EA and SB gets fully enabled and the other partially enabled and then emerge as modest MI learners as well. But if both expectations do not align with neither of EA and SB, they reject the influence, both EA and SB get partially enabled and then emerge as fragile MI learners. When a teacher enables fragile SR, it trivially positively influences learners' EA and SB, but both remain unenabled until the learners exercise their expectations. If both expectations aligned with EA and SB, they accept the influence, both EA and SB get partially enabled, and then emerge as fragile MI learners. If only one of the expectations aligned with EA or SB, they co-exist with the influence, one of EA and SB gets fully enabled and the other partially enabled and then emerge as modest MI learners. But if both expectations do not align with neither of EA and SB, they reject the influence, both EA and SB get fully enabled and then emerge as robust MI learners. These explanations do not only emphasize that learners' MIs, despite being influenced by social relationships, are not reducible to SRs enabled by teachers, but further show *how* learners' expectations (i.e., human agency) clearly and analytically contribute.

Contingently, learners' MIs also influence SRs enabled by the teachers. When learners repeatedly enact robust, modest or fragile MIs, they also reflect further to expect their teacher to *re-enforce, reproduce or transform* the SRs enabled for them, which finally inform their perceptions of either robust, modest or fragile SRs from their teacher. When learners enact robust MIs, it strongly positively influences how teacher enables socioemotional connection (SC) and productivity support (PS) for the learners, but both remain unenabled until the teacher activates his/her expectations – i.e., self and learners expectations. If both expectations aligned with SC and PS, the teacher re-enforces the influence, both SC and PS get fully enabled, and s/he enables and the learners perceive robust SRs. If only one of the expectations aligned with SC or PS, the teacher reproduces the influence, one of the SC and PS gets fully enabled and the other partially enabled, and s/he enables and the learners perceive modest SRs. But if both expectations do not align with neither of SC and PS, the teacher transforms the influence, both SC and PS get partially enabled and s/he enables and the learners perceive fragile SRs.

When learners enact modest MIs, it moderately positively influences how teacher enables SC and PS for the learners, but both remain unenabled until the teacher activates his/her expectations. If both expectations aligned with SC and PS, the teacher re-enforces the influence, both SC and PS get fully enabled, and s/he enables and the learners perceive robust SRs. If only one of the expectations aligned with SC or PS, the teacher reproduces the influence, one of the SC and PS gets fully enabled and the other partially enabled, and s/he enables and the learners perceive modest SRs. But if both expectations do not align with neither of SC and PS, the teacher transforms the influence, both SC and PS get partially enabled, and s/he enables and the learners perceive fragile SRs. When learners enact fragile MIs, it trivially positively influences how teacher enables SC and PS for the learners, but both remain unenabled until the teacher activates his/her expectations. If both expectations aligned with SC and PS, the teacher re-enforces the influence, both SC and PS get partially enabled, and s/he enables and the learners perceive fragile SRs. If only one of the expectations aligned with SC or PS, the teacher reproduces the influence, one of the SC and PS gets fully enabled and the other partially enabled, and s/he enables and the learners perceive modest SRs. But if both expectations do not align with neither of SC and PS, the teacher transforms the influence, both SC and PS get fully enabled and s/he enables and the learners perceive robust SRs. So, MIs of learners also conversely influence SRs enabled by teachers but cannot be reduced to the MIs.

Emerging relationships from the above interactions produce three set of RECs – *typical*, *interesting* and *crucial* and another three deeper set of PERs – *accept/re-enforce*, *co-exist/reproduce* and *reject/transform*. Typical category represents the set of emergent cases who have direct relationships between the SRs experienced and MIs enacted. Interesting category represents the set of emergent cases who have indirect relationships between the SRs experienced and MIs enacted. For crucial category, there is neither direct nor indirect relationships for the emergent cases. A further relationship between the categories holds that crucial category accommodates more diverse emergent cases followed by typical, with interesting category having the least. However, having more diverse emergent cases does not necessarily mean having the majority of the agent representatives under study, as typical category seems to relatively have representative than crucial and far more than interesting category. Table 4.1 depicts these emergent relationships.

SRs enabled	MIs emergent cases (as influenced by SRs)	MIs enacted	SRs emergent cases (as also influenced by MIs)	RECs	PERs
Robust	Robust	Robust	Robust	<i>Typical</i>	Accept and re-enforce
	Modest		Modest	<i>Crucial</i>	Co-exist and reproduce
	Fragile		Fragile	<i>Interesting</i>	Reject and transform
Modest	Robust	Modest	Robust	<i>Crucial</i>	Co-exist and reproduce
	Modest		Modest	<i>Typical</i>	Accept and re-enforce
	Fragile		Fragile	<i>Crucial</i>	Co-exist and reproduce
Fragile	Fragile	Fragile	Fragile	<i>Typical</i>	Accept and re-enforce
	Modest		Modest	<i>Crucial</i>	Co-exist and reproduce
	Robust		Robust	<i>Interesting</i>	Reject and transform

Table 4.1: SRs and MIs relationships and emergence of RECs and PERs

The accept/re-enforce reflexives represent the set of emergent cases (i.e., rationalist-strategist) who consider the SRs between them and their teacher acceptable and at the same time want such SRs to be further strengthened. Being diverse cases grouped together in a typical category, they tend to

consider and achieve the project of accepting and strengthening the SRs experienced from their teachers differently. The co-exist/reproduce reflexives are the set of emergent cases (i.e., realist-conformist) who has mixed feelings about the SRs between everybody learners (including themselves) and their teacher and despite that they still seem to prefer the SRs enabled for them reproduced. It appears that they only have mixed feelings with the SRs mainly because of their friends in the class and as such the SRs experienced don't necessarily influence how they enact their MIs. Though they belong to the same crucial category, their reflexivity works for them differently to maintain their co-existence/reproduction stand. The reject/transform reflexives are the set of cases (i.e., irrationalist-resistant) who have nothing against the SRs experienced from the teacher, but still reject and want the SRs transformed for them (including people like them). They seem to engage in a project that can change not only the SRs experienced but also some existing practices of the school and mathematics learning.

4.6.2 Relating mathematical identities and achievement

Contrary to what appears as just a speculation or unclear terrain in the literature, very strong relationships actually exist between MIs and mathematics achievement (MA). In support of this claim, a recent longitudinal study conducted by Gulemetova, Beesley, Fancsali, and Balakrishnan (2022) find positive and statistically significant relationships between learners' MIs and their MA⁴¹. Unlike the relationships between MIs and SRs which occur between teachers and learners, MIs and MA relationships are located within the individual learners and mathematics. No known study, including Gulemetova *et al.* (2022), has been able to provide an explanation on *how* these relationships occur. This unavailable explanations is understandable because studies that have established the relationships are still very few. In this section, I provide detailed explanations.

Generally, as learners recognize/are aware of their robust, modest or fragile MIs, it directly shapes how they emerge as high, average or low mathematics achievers respectively. In other words, the quality of learners' MIs *re-enforces* their level of MA. So, when learners recognize their robust MIs, it strongly improves their sociocognitive ability (SA) in and productive disposition (PD) for mathematics. Concomitantly, the improvement further strongly and positively influences the

⁴¹ Mathematics achievement obtained from the Departments of Education was only measured by test scores.

internal and external expectations they hold for mathematics success, leading to an emergence of mathematics high-achieving learners. When learners recognize their modest MIs, it moderately improves their SA in and PD for mathematics. Concomitantly, the improvement further moderately and positively influences the internal and external expectations they hold for mathematics success, leading to an emergence of mathematics average-achieving learners. When learners recognize their fragile MIs, it weakly improves their SA in and PD for mathematics. Concomitantly, the improvement further weakly and positively influences the internal and external expectations they hold for mathematics success, leading to an emergence of mathematics low-achieving learners.

Conversely, as learners are aware of their high, average or low achievement in mathematics, it directly shapes how they emerge as robust, modest or fragile MIs respectively. In other words, the level of learners' MA *re-enforces* the quality of their MIs. So, when learners are aware of their high MA, it strongly enhances their EA with mathematics and SB in mathematics classroom community. Concomitantly, the enhancement further strongly and positively influences the expectations they hold about mathematics possibilities and opportunities, leading to an emergence of a robust MI learners. When learners are aware of their average MA, it moderately enhances their EA with mathematics and SB in mathematics classroom community. Concomitantly, the enhancement further moderately and positively influences the expectations they hold about mathematics possibilities and opportunities, leading to an emergence of modest MI learners. When learners are aware of their low MA, it weakly enhances their EA with mathematics and SB in mathematics classroom community. Concomitantly, the enhancement further weakly and positively influences the expectations they hold about mathematics possibilities and opportunities, leading to an emergence of fragile MI learners.

4.6.3 Relating mathematical achievement and social relationships

As in the reports regarding relationships between SRs and MIs, several studies have also confirmed the influence of SRs on MA, but a few established the influence of MA on SRs. The latter happens, according to Hagenauer and Volet (2014), because many researchers do not consider teacher-learner SRs as a variable of interest or only consider it as an independent variable. Another important observation is that the majority of the researchers maintained a reductionist view on the

relationships between SRs and MA. That is, they maintain that SRs enabled or expectations held by the teachers always determine the learners' MA. The closest to the non-reductionist view is the analysis of Martin (2000) and McGee (2015) on the influence of teachers' practices (including social) on learners' achievement, but converse relationship not reported. I provide a comprehensive explanatory framework for these relationships in both directions.

In classrooms where teachers repeatedly enable robust, modest or fragile social relationships, learners reflect whether to *accept, co-exist with or resist* the influence, which finally results in the emergence of either high, average or low mathematics achiever learners. When a teacher enables robust SR, it strongly positively influences learners' SA and PD for mathematics, but both remain unenabled until the learners leaning toward their expectations – i.e., internally and externally driven expectations. If both expectations aligned with SA and PD, they accept the influence, both SA and PD get fully enabled and then emerge as high-achieving learners. If only one of the expectations aligned with SA or PD, they co-exist with the influence, one of SA and PD gets fully enabled and the other partially enabled and then emerge as average-achieving learners. But if both expectations do not align with neither of SA and PD, they resist the influence, both SA and PD get partially enabled and then emerge as low-achieving learners.

When a teacher enables modest SR, it moderately positively influences learners' SA and PD, but both remain unenabled until the learners leaning toward their expectations. If both expectations aligned with SA and PD, they accept or even transform the influence, both SA and PD get partially or fully enabled and then emerge as average- or high-achieving learners respectively. If only one of the expectations aligned with SA or PD, they co-exist with the influence, one of SA and PD gets fully enabled and the other partially enabled and then emerge as average-achieving learners as well. But if both expectations do not align with neither of SA and PD, they resist the influence, both SA and PD get partially enabled and then emerge as low-achieving learners. When a teacher enables fragile SR, it trivially positively influences learners' SA and PD, but both remain unenabled until the learners leaning toward their expectations. If both expectations aligned with SA and PD, they accept the influence, both SA and PD get partially enabled, and then emerge as low-achieving learners. If only one of the expectations aligned with SA or PD, they co-exist with the influence, one of SA and PD gets fully enabled and the other partially enabled and then emerge as average-achieving learners. But if both expectations do not align with neither of SA and PD,

they resist the influence, both SA and PD get fully enabled and then emerge as high-achieving learners.

Reciprocally, when learners repeatedly imply high, average or low MA, teachers reflect weather to *re-enforce, reproduce or transform* their SRs, which finally results in the enablement of either robust, modest or fragile SRs for the learners. When learners imply high MA, it strongly positively influences how teacher enables SC and PS for the learners, but both remain unenabled until the teacher activates his/her expectations – i.e., self and learners expectations. If both expectations aligned with SC and PS, the teacher re-enforces the influence, both SC and PS get fully enabled, and s/he enables robust SRs for the learners. If only one of the expectations aligned with SC or PS, the teacher reproduces the influence, one of the SC and PS gets fully enabled and the other partially enabled, and s/he enables modest SRs for the learners. But if both expectations do not align with neither of SC and PS, the teacher transforms the influence, both SC and PS get partially enabled and s/he enables fragile SRs for the learners.

When learners imply average MA, it moderately positively influences how teacher enables SC and PS for the learners, but both remain unenabled until the teacher activates his/her expectations. If both expectations aligned with SC and PS, the teacher reproduces or even re-enforces the influence, both SC and PS get partially enabled or fully enabled, and s/he enables modest or robust SRs for the learners respectively. If only one of the expectations aligned with SC or PS, the teacher reproduces the influence, one of the SC and PS gets fully enabled and the other partially enabled, and s/he enables modest SRs for the learners. But if both expectations do not align with neither of SC and PS, the teacher transforms the influence, both SC and PS get partially enabled, and s/he enables fragile SRs for the learners. When learners imply low MA, it trivially positively influences how teacher enables SC and PS for the learners, but both remain unenabled until the teacher activates his/her expectations. If both expectations aligned with SC and PS, the teacher re-enforces the influence, both SC and PS get partially enabled, and s/he enables fragile SRs for the learners. If only one of the expectations aligned with SC or PS, the teacher reproduces the influence, one of the SC and PS gets fully enabled and the other partially enabled, and s/he enables modest SRs for the learners. But if both expectations do not align with neither of SC and PS, the teacher transforms the influence, both SC and PS get fully enabled and s/he enables robust SRs for the learners.

4.7 Conclusion

This Chapter specifically discussed two kinds of frameworks – theoretical and analytical. The theoretical framework, which was informed by the social realist perspective and primarily by re-enforced CR, discussed the mechanisms that explain learners' MIs, teacher-learner SRs and learners' MA. In each case, the framework emphasizes the theorization at the actual domain of reality. The analytical framework, which re-direct morphogenetic approach of Archer, discussed why and how the process of structural elaborations go beyond transformation and reproduction to include reinforcement. The framework also argues that there are relationships between SRs, MIs and MA, with the teachers' and learners' expectations acting as mediators. By extension, it claims that learners' MIs and MA are influenced by the SRs enabled by their teachers, but such SRs cannot determine them. Conversely, SRs enabled by their teachers are influenced by the learners' MIs and MA, but such identities and achievement cannot also determine the SRs enabled by the teacher. However, while I still hold on to these claims, only the arguments regarding SRs and MIs are more elaborated and will be taken forward for further judgmental investigations.

Having understood the comprehensiveness and robustness of each theories and their relationships, I obtained an intervention program, SREP, to place myself, as CR researcher, in a position of a social agent for positive change. So, building on realist social theory in this chapter, I made strong claims regarding the three theoretical concepts of the study, and envisaged effectiveness of SREP on them. But as argued earlier, evaluating the SREP effectiveness will be suspended in this study for further study. Nevertheless, that suspension does not stop referring to the SREP where necessary for the flow of the study as initially written.

To empirically provide evidence (which resides within judgmental realist assumptions) for key philosophical, theoretical and analytical claims I made in this Chapter (or previously) and by extension to provide answer to Sub-Research Question One, both the theoretical and analytical frameworks which emerge from re-enforced CR will guide the research design, data collection methods and analyses approach I present in the next Chapter, Chapter five.

CHAPTER FIVE

RESEARCH DESIGN, METHODOLOGIES AND DATA

5.1 Introduction

Critical realists are inclined to multiple means of generating knowledge and to exemplify and/or apply such knowledge for social practice improvement (Marks & O'Mahoney, 2014; Tikly, 2015; Zachariadis *et al.*, 2013). In this Chapter, I exemplify three key emergent from Chapter 4. First, the theoretical concepts and SREP intervention of the study. Second, the mixed methods research design – embedded with descriptive and experimental studies. Third, the different methodologies that were employed for this study. The discussions on each research methodology include sample selection techniques, data collection tools/methods and data analysis techniques/procedures. The rest of the Chapter discuss the pilot study, trustworthiness strategies and ethical considerations as related to the study. It is also important to state that in certain sections of all these discussions, the CR emancipatory (axiology) position is emphasized.

The above introduction captures the initial research design and methodologies for my original study. While the introduction still captures the realist methods employed in the final study, I confirm that the research design and its time dimension slightly differ from the one that is finally reported. As hinted and philosophically justified in Chapter 3, this initial research design conflated experimental with descriptive research design instead to be contingent, therefore, the former study (or Sub-Research Questions 4) is stepped down in the final study. By time dimension, I mean a descriptive research design which assumes that social phenomenon is better understood 'at different point in time' (e.g., developmental design) rather than 'at a point in time' (e.g., causal-comparative and correlational design)⁴². These changes greatly re-shaped how the data already collected were finally analyzed and reported. However, the changes did not affect the original write-up of the Chapter, except data analysis section. This re-introduction (or re-description) is *necessary* for the emergent study and the reading of this Thesis moving forward.

⁴² This discussion arises as a result of the new social realist perspective on social theory discussed in Chapter 3 and then 4.

5.2 Exemplifying the theoretical concepts – evidential data

My study being theory-led, necessarily needs to exemplify the three theoretical frameworks that emerged in Chapter 4 to be able to produce transitive knowledge. Earlier in that Chapter 4, I

Concepts	Mechanisms	Descriptors/indicators	Possible emergences
Mathematical identity	<i>Attribute acquisition</i>	e.g., creating extra time for mathematics, requesting for more mathematics tasks	Robust – demonstration (e.g., actions) or provision (e.g., narrations) of positive evidence for all the three mechanisms of mathematical identity; Modest – demonstration or provision of positive evidence for two mechanisms of mathematical identity; Fragile – demonstration or provision of evidence for only one mechanism of mathematical identity.
	<i>Community participation</i>	e.g., asking and answering questions in the class.	
	<i>Expectation actualization</i>	e.g., giving attention or concentration and having regular attendance in mathematics class; harassing, teasing or disturbing their attentive peers.	
Social relationships	<i>Psychosocial support</i>	e.g., teachers giving attention to, valuing, encouraging and getting familiar with their learners.	Robust – demonstration (e.g., actions) or provision (e.g., narrations) of positive evidence for all the three mechanisms of social relationships; Modest – demonstration or provision of positive evidence for two mechanisms of social relationships; Fragile – demonstration or provision of positive evidence for only one mechanism of social relationships.
	<i>Professional practice</i>	e.g., teachers enabling learning opportunities, holding high expectations for and awarding or assigning rewards to their learners.	
	<i>Expectation enactment</i>	e.g., teachers providing <i>equal/different</i> supports and opportunities for <i>all/certain</i> learners; deserting or coming late to classes, avoiding and getting angry with questions asked by their learners.	
Mathematics achievement	<i>Mathematical excellence</i>	Scores or performances in mathematics test or assessment, and demonstration or indication of mastery of mathematical literacy.	High – mathematically highly competent, strongly engage by/in communities and show positively consistent expectations; Average – mathematically averagely competent, moderately engage by/in communities and show inconsistent expectations; Low – mathematically less competent, weakly engage by/in communities and show negatively consistent expectations.
	<i>Productive engagement</i>	e.g., mathematically benefiting from, giving/aiming to give back to the communities and based careers.	
	<i>Expectation realization</i>	e.g., being happy or sad, confident or confused, enjoying or frustrated, feeling great or terrified/struggling, deliberate or random, proud or not proud about or with mathematics.	

Table 5.1: Mechanisms, descriptors and emergent outcomes for MI, SRs and MA

engaged with different existing theoretical frameworks to identify the constituents of the three emergent theoretical concepts of the study. These constituents have also been operationalized and their possible descriptors or example at the empirical level stated (see sections 4.4.1 – 4.4.3). I summarily recall these constituents and descriptors in Table 5.1.

5.3 Exemplifying the SREP intervention – activities and implementation

As I have discussed in section 4.5, the SREP intervention program is a set of emergent, deliberate, flexible and common practices that teachers can implement in their teaching practices. In this section, I discuss how the selected teachers were exposed to and implemented the activities of SREP. Using the SREP guide (see Table 5.2 below), four selected teachers for the treatment group (selection details, see section 5.5.2) were sensitized on how each practice could be effectively used in and outside the classroom. The sensitization ran for four weekends with four contact sessions. The first session was used to get to know and build trust with the teachers. The discussions about what the intervention is all about and the prevention of its leakage to non-participants were another aims of the session. The discussions emphasized the need to keep the information about the activities during the sensitization from other teachers.

Then, five case-study videoclips of about 15 minutes each developed from my pilot study and online classroom teaching were given to each teacher. The teachers were asked to document their thinking and reflections about the videoclips given, using some structured questions. The questions were used to prompt them to reflect on their learners' personal identities, social identities, expectations, disposition to mathematics and how to effectively improve these through classroom and after-school social practices. To properly guide the teachers on how to go about these tasks, I went through one of the case-study videos with them in this session and attended to their questions. This session lasted for about two hours. It is also important to state that, for the purpose of proper documentation and possible analysis of the activities that took place, all the sessions were video recorded. These recordings were carried out with the permission of the teachers.

In the second session, each teacher was asked to verbally present their initial documented thoughts about the remaining four videos. These presentations generated interactions between the teachers and me, and among the teachers themselves. Building on the interactions, I then gave a brief

presentation that drew from the teachers' presentations. The last part of this session was used to formally introduce the social practices, their dimensions and indicators to the teachers. This session lasted for about two hours. There is evidence that teachers tend to be excited about these activities and engage meaningfully (Rubie-Davies, 2015). This excitement was observed among the participants during the sensitization, except for one of the teachers who participated superficially. After the whole SREP activities, I decided to follow up with them within the period of SREP implementation to ascertain their commitment to the project. This engagement revealed a lot of valuable information. The information is documented in the next section.

Session	Activities
Session one (two hours)	(a) Formal familiarization, and presentation of the intervention as confidential and non-threatening activities.
	(b) Independent tasks: watching four case-study videoclips by using structured guiding questions; teachers' documentations of their reflections. One case-study example.
	(c) Answering possible teachers' questions regarding the tasks and study.
Session two (two hours)	(a) Teachers' presentations of their reflections on the case-study videos.
	(b) Interactions between the teachers and among the teachers and myself, and my brief presentation to summarize the ideas generated from the interactions.
	(c) My presentation: Introduction to each social practice, their dimensions and indicators.
Session three (three hours)	(a) Watching teachers' videoclips: Psychosocial support scenarios, teachers' ideas and suggestions on how it could be reinforced or transformed. <i>Interactions among the teachers and myself, questions and answers.</i>
	(b) Watching teachers' videoclips: Agency sensitivity scenarios, teachers' ideas and suggestions on how it could be reinforced or transformed. <i>Interactions among the teachers and myself, questions and answers.</i>
	(c) Watching teachers' videoclips: Productivity support scenarios, teachers' ideas and suggestions on how it could be reinforced or transformed. <i>Interactions among the teachers and myself, questions and answers.</i>
	(d) Watching teachers' video clips: Equity sensitivity scenarios, teachers' ideas and suggestions on how it could be reinforced or transformed. <i>Interactions among the teachers and myself, questions and answers.</i>
Session four (one hour)	(a) General discussion and summary about the study and intervention.
	(b) Attending to any possible question.
	(c) Formal closure of the intervention activities.

Table 5.2: SREP guide

The third session addressed the social practices drawing on different supporting studies (e.g., Hamre & Pianta, 2005, 2007; Babad, 2009; Rubie-Davies, 2015; Gardee, 2019a) and pre-recorded videos of the teachers during the first stage of the study. Activities in this session involved showing sections of the teachers' videos where they enabled robust/supportive social relationships, and other sections where they enabled fragile/limiting social relationships for participants'

interactions. The video selections and watching was carried out for all the four social practices, and it lasted for 5 minutes per video section for both robust/supportive and fragile/limiting social relationships for each teacher totaling two hours, 40 minutes. These activities led to exploring teachers' ideas on how their supportive and limiting social relationships could be reinforced and positively transformed respectively. The activities are also meant to initiate the ownership of the practices that teachers could employ in each situation. This session ended with my 20 minutes verbal presentation that built on the supporting studies, theoretical understandings of social relationships in this study, and the teachers' ideas. This session lasted for about three hours.

The last session was used to address the general discussions about the study and intervention. The teachers were informed that I may likely involve them when I want to sensitize the control group teachers on SREP activities (more detail under ethical consideration), and that the time would be communicated to them at an appropriate time. Other discussions included attending to any possible questions and the closure of the intervention sensitization. This session lasted for about one hour.

5.3.1 Implementation and follow-up meeting

The duration for SREP implementation by the teachers was six weeks. At the fourth week of SREP implementation, I followed up with the four teachers. The main aims of the follow-up meeting were to establish whether the SREPs were actually implemented, interrogate the initial successes recorded and more importantly to collectively find or suggest ways around any challenge they might be facing in the classroom implementation. All except the teacher mentioned earlier attended the meeting. The three teachers in attendance explicitly confirmed that they implemented the SREPs with some challenges. They also reported some initial successes both for themselves and for their learners. While the discussions on the successes recorded are not reported here for later presentation, the challenges they experienced are documented here.

One set of the challenges resonate around traditional, religious and administrative issues. These issues are at times interwoven or exclusive. For example, the teachers complained about restrictions of creating any kind of social relationships with learners, especially the female learners. In other words, teachers' interactions with their learners are limited to the classrooms. It is even an offense for teachers to invite their female learners to the staff rooms in some schools. I directly

had a share of this restriction as well. Only two of four schools that participated in the study freely allowed me to interview their female learners. The other two schools gave different conditions or restrictions such as learners must be interviewed in the classroom where other learners are present and/or one staff member must be present during the interviews. An exclusive case but purely administrative issue was reported by one teacher who specifically complained about a plan to change him to another class by his school administrators. This is because a new principal who was not aware of the ongoing research was posted to their school.

Other challenges mentioned by the teachers are overpopulated classrooms and lack of materials for utility engagement guide. Familiarizing with every learner and giving individual attention in a classroom of over 120 learners, according to the teachers, posed a big challenge to the implementation. The lack of materials which explicitly state and guide on implementing utility engagement dimension of productivity support practice also appeared to pose a challenge to the teachers. This dimension involves how teachers emphasis sense in, worth and usefulness of every mathematics topics they teach to learners. I suspected the teachers wanted special materials that would highlight this aspect of the practice in relation to different mathematics concepts and topics.

We all came to an agreement that there was little or nothing we could do to change these challenges, with the exception of the utility engagement materials/resources. As teachers and researcher, we cannot change or work against the traditions, religious beliefs of their community nor even the administrative rules of the schools. On the utility engagement resources, I suggested three ways of finding help: consulting with me anytime they require assistance, consulting mathematics teachers from other schools and surfing the internet. They all complained about the latter in that the network in the area was so bad. And throughout the implementation period none of them consulted with me on that challenge. I suspect the last option was employed; a teacher reported during the interview session that he always meets other teachers from other schools for assistance.

5.4 Research design

The study employed a transformative mixed methods design, a form of research design that holds the CR emancipatory (axiology) position dearly. The transformative framework is aimed at

changing social issues about marginalized and under-served groups through research (Creswell, 2012), and positions researchers as agents of social change (Mertens, 2012). In the heart of transformative mixed methods researchers is how the findings of their studies suggest solutions to social inadequacies and inequalities. Sweetman, Badiie and Creswell (2010) emphasize that:

this plan for change is critical, as it enables authors to be transformative, as opposed to reporting findings and moving on. This means that mixed methods transformative researchers should [and do] not stop at simply describing problems but suggest solutions for how these problems might be overcome. This section typically ends a transformative mixed methods study (p. 452).

Specifically, the study worked with the explanatory sequential design – built around a transformative framework – which is characterized by collections and analyses of quantitative data then followed by collections and analyses of qualitative data (Creswell & Plano Clark, 2011; Creswell, 2012). The design draws from complementary opportunities offered by both quantitative and qualitative data to present deeper understandings of learners’ mathematical identities and achievement, and their possible transformations as a result of teacher-learner social relationships. Mostly, explanatory sequential mixed design uses qualitative findings to explain and interpret the results of a quantitative study. However, sequential mixed design may use qualitative data beyond the explanatory to include exploratory purposes (Lodico, Spaulding & Voegtler, 2010). In this study, qualitative data were used for both explanatory and exploratory purposes.

The aim of the study is to investigate the interaction among social relationships enabled by the teachers, learners’ mathematical identities and achievement. To capture robust understandings of these relationships, using a mixed methods design is particularly necessary. Collecting and analyzing quantitative data allow for greater generalization and provide clearer results, including the degree of the relationships, between the teachers’ social practices, learners’ identities and achievement. These results may not be possible with qualitative data alone, because of the limited analytic potential of the data usually collected and less support for greater generalization of the findings they offer (Tikly, 2015; Lutovac & Kaasila, 2018).

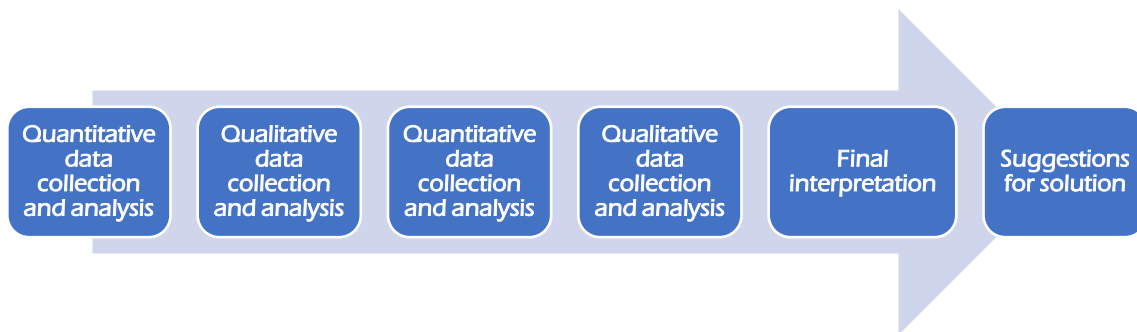


Figure 5.1: Schematic flow of each stage of the design

The need for robust explorations, thick descriptions and richer explanations for the findings in this study made collection of qualitative data also necessary. Collecting and analyzing qualitative data allow treating the sources of the data as *subjects* not as *objects*, as quantitative analysis does (Zachariadis *et al.*, 2013; Tikly, 2015). So, for researchers to present identity, for example, as “relevant for actual learning processes in the *mathematics* classroom, more particularly, in specific domains of mathematics” (Graven & Heyd-Metzuyamin, 2019, section 5), mixed methods designed studies could be more productive (Marks & O’Mahoney, 2014; Tikly, 2015; Lutovac & Kaasila, 2018). For the flow and connection of the two designs, the data and samples considered in the qualitative phase depend on the results of the quantitative phase (Gay, Mills & Airasian, 2006). Figure 5.1 represents the schematic flow of the stages of the design.

5.4.1 Quantitative design

In the quantitative phase, this study conducted descriptive and experimental studies. The descriptive study combined correlational and developmental designs. While correlational design investigates the direction and degree of the relationships between two or more variables at a point in time, developmental design accounts for the changes in the relationship over different periods of time (Cohen, Manion & Morrison, 2007; Olayiwola, 2007). Correlational design is particularly important for prediction between two or more variables, and developmental design helps to formulate or confirm theories by suggesting the processes that are going on over a period (Cohen *et al.*, 2007; Olayiwola, 2007). The combination of the two designs can be used as a confirmatory or refuting study in which a framework, model or hypothesis is tested (Cohen *et al.*, 2007). I planned to test the hypotheses and proposed frameworks in this study that: social relationships

enabled by the teachers influence rural learners' mathematical identities and achievement; learners' mathematical identities and achievement influence social relationships enabled by the teachers; and the learners' mathematical identities and mathematics achievement attained over time influence each other. In other words, the integrated design⁴³ is used to provide answers to Research Sub-Questions one – three. Both designs, however, only serve as an initial quantitative step to establish causal relationships between the concepts in the study. Therefore, it is important to introduce an appropriate intervention and further conduct an experimental study⁴⁴.

A comparative interrupted time series (CITS) design is particularly important for the experimental study⁴⁵. This is because it permits detection, projection, and comparison of the effects of an intervention, by using multiple pre-test and post-test scores (Creswell, 2012; Somers, Zhu, Jacob & Bloom, 2013). CITS has been an “especially rigorous” design (Somers, *et al.*, 2013, p. 31) and the interrupted time series is the “strongest quasi-experimental” design (Wagner, Soumerai, Zhang, & Ross-Degnan, 2002, p. 299). Generally, while quasi-experimental design is a nonrandomized experimental design that compares the treatment group with the control group to establish the influence of an intervention, an interrupted time series design investigates the trend and rate of change in a group, as a result of the intervention implemented (Handley, Lyles, McCulloch & Cattamanchi, 2018; Miller, Smith & Pugatch, 2020). In this experimental study, I planned to test the hypothesis that social relationships enhancement practices (SREP) improves how social relationships enabled by the teachers, mathematical identity developed and mathematics achievement attained by the rural learners over time influence each other. Therefore, the design provided the basis to answer Research Question Four.

Furthermore, issues associated with the internal validity of nonrandomized experimental designs, procedural threats related to instrumentation, especially the mathematical competency test in this study (Best & Kahn, 2012; Creswell, 2012; Somers *et al.*, 2013), and disruptions to the research site, are largely addressed with this design (Creswell, 2012). Validation studies at school level have established that nonrandomized studies are sound enough to produce true experimental results if a comparative interrupted time series (CITS) design is used, and pre-test scores for both treatment and control groups are obtainable or available (Somers *et al.*, 2013). This validation

⁴³ This integrated design now considered only correlational design to answer only the sub-question 1.

⁴⁴ This position clearly shows where conflation between initial experimental and descriptive designs occurred.

⁴⁵ Selection of this design makes study ill-designed (i.e., non-compliant with the social realist stance).

further confirms Fortson, Verbitsky-Savitz, Kopa and Gleason’s (2012) argument that nonrandomized experimental designs can only produce experimental results if rigorous designs and statistical analyses are used. The quasi-experimental design is often used by education researchers because it avoids major disruptions to the classroom settings (Cohen *et al.*, 2007; Best & Kahn, 2012; Creswell, 2012), and the involvement of the control group in the design has also curtailed the instrumentation threats (Best & Kahn, 2012; Somers *et al.*, 2013). Other strategies employed to address possible internal validity threats to the study beyond instrumentation are discussed under methodology.

In line with the aim of the experimental study, this quantitative phase incorporated SREP intervention to support teacher robust social relationships of learners – fully discussed in the next section, section 4.2. The design is schematically represented as:

CITS design:	O ₁ ,	X ₁	O ₂ ,	O ₃

	O ₁ ,		O ₂ ,	O ₃

Where:

X₁ = Intervention: social relationships enhancement practices (SREP) implemented for six weeks over two school terms;

O₁ = Pre-test scores measured within two weeks before the intervention for both treatment and control groups;

O₂, O₃ = Post-test scores measured at six and 12 weeks⁴⁶ respectively, after the intervention for both treatment and control groups.

5.4.2 Qualitative design

In the qualitative phase, a qualitative case study design was employed. The case study is appropriate for the study because it “is a way of organizing social data for the purpose of viewing social reality” (Best & Kahn, 2012, p. 259). A case study “penetrate[s] situations in ways that are not susceptible to numerical analysis” (Cohen *et al.*, 2007, p. 253). Rural schools and their contexts

⁴⁶ Time series design typically maintains equal spacing measurement between data points. And six weeks was chosen to avoid disruption to the schools activities (e.g., C.As and exams) that are highly likely after that week.

are unique and vary from one location to another, therefore, a design that has potential to “investigate and report complex dynamic and unfolding interactions of events, human relationships and other factors in a unique instance” such as a case study is required (Cohen, *et al.*, 2007, p. 253). All these features allow the case study researchers to “probe[.] deeply and analyze[.] interactions between the factors that explain present status or influence change and growth” (Best & Kahn, 2012, p. 259). The study, in this qualitative phase, aims to further explain, give a detailed account of, and explore the interactions among teacher-learner social relationships, learners’ mathematical identity development and mathematics achievement, that may not be possible with only quantitative data.

According to Yin (2009), research design on case studies has three major purposes:

- *descriptive*: to give a detailed account of the cases
- *explanatory*: to corroborate, falsify or generate theories
- *exploratory*: to generate hypotheses for future studies

Specifically, the quantitative findings of relationships between social relationships and mathematical identities (and achievement) were explored and further described and explained. The intention is to support the quantitative analysis with stronger descriptions and explanations offered by the case study, to support or falsify the frameworks proposed or hypotheses raised in this study, and to generate further hypotheses for future studies. Such further hypotheses, in this study for example, are to state and test the effectiveness of the SREP as stated earlier. The suitability of these purposes to the study makes the case study most appropriate.

For emphasis, the same research flow in the quantitative phase also applies in this qualitative phase. That is, using the case study design, both descriptive and experimental studies were also planned to be conducted with qualitative data collected at three different stages. And as for any other qualitative designs, case study also has some trustworthiness or validity issues that many methodologists have raised concerns about. Such issues, according to Best and Kahn (2012), include allowing researchers’ biases to set in, wrongly attributing effects to ‘mere factors’ and deceptively seeing case study as simple. Discussions on how these issues and other related ones were addressed in this study are presented under trustworthiness section, section 5.7

5.5 Making judgments – research methodologies

This study employed realist research methodologies at every stage of its inquiry⁴⁷, including this current one at the judgmental realm. It is important to stress that methodologies at judgmental realm involve methods, procedures or inferences employed to *identify (or apply)*, *access* and to *analyze* the specific social phenomenon of interest⁴⁸. In the previous sections of this Chapter, I have presented how the social phenomena in this study were identified and/or applied (through exemplifications of the study theoretical concepts and implementing SREP) and accessed (through emergent research design). But how the research design informed the accessibility has not been directly presented. These include selection of schools, participants, data collection tools and methods for and how the data collected were analyzed in the study. The following sections present how the data about the three concepts were empirically accessed and analyzed.

5.5.1 School and participant selection

I selected four schools, with two teachers and their learners from each school. The techniques that were employed to select the schools and the participants for the study, including the justifications for these techniques, are presented in this section. Also presented are the numbers and characteristics of the schools and participants.

5.5.1.1 Rural school selection

Being a study that focuses on rural schools, there is a need to clearly operationalize the definition of the rural area before the schools can be selected from such area. Chomitz, Buys and Thomas (2005) provide two convincing criteria to make this operationalization of rural areas: a location with low population density and remoteness to larger cities. The Nigerian statistical office defines low population density area as any location with less than 20,000 persons (cited in United Nations, 2019). Chomitz *et al.*, (2005) propose an operational definition of remoteness as an area with travel time of at least one hour to big cities of minimum of 100,000 persons. Using these definitions, I

⁴⁷ The previous Chapters 3 and 4 have conducted inquiries at the ontological and epistemological realms respectively.

⁴⁸ An indication that inquiry at the judgmental realm is complex and tends only to produce transitive knowledge only.

decided on the criteria I employed in locating rural areas for the study. But selection of the schools in these areas requires purposive criteria in this study.

Purposeful sampling allows sampling for specific research purposes, whereby researchers (subjectively) select samples that have characteristics of research interest based on some (objective) criteria or judgements (Cohen, *et al.*, 2007; Best & Kahn, 2012). In this study, four schools located in rural areas in north-western Nigeria based on the two criteria mentioned above, as well as meeting some further criteria, were purposively selected. These further criteria are that the schools should: be co-educational schools and run up to senior classes; be public or government owned schools; be accessible to the researcher; and allow their teachers and learners to participate and be videotaped in the study. The reasons for these criteria are to gain greater and robust insights into what are being studied, ensure compliance with the research procedures, facilitate permission to the schools and easy conduct of the study, and to address ethical issues.

The school and academic calendar arrangements in the north-western region of Nigeria, especially Katsina, favored this study. I also work in the north-western Nigeria which availed me better research opportunities than what any other region may offer. In addition to these arrangements and opportunities, Nigeria (and in particular Katsina state), has almost half of its population in rural areas (www.indexmundi.com/facts/indicators/sp.rur.totl.zs/map/africa) and little or no mathematical identity studies conducted to focus on the learners from such areas, making it a suitable country for the study. So, all the schools that participated in this study were selected from Katsina central zone of the state. As it will be elaborated in the next two section, the number of learners available in the four schools are sufficient for all the statistical assumptions related to the research designs and data analyses conducted for the quantitative study.

5.5.1.1.1 Research schools

Four schools, labelled school A, B, C and D participated throughout in the descriptive study. All the schools are located in Katsina central zone. While school A and B are located in an area of 0.709km² of about 59 minutes' drive from the state capital, with a population of 5,909 peoples in 2015 (6,658 peoples projected for 2020), school C and D are located in an area of 0.625 km² of about 1 hour, 16 minutes' drive from the state capital, with a population of 11,591 peoples in 2015

(13,060 peoples projected for 2020)⁴⁹. Like any other government schools in the state, all the schools except school C run on the free education policy of the state government. This means that learners and their parents pay little or nothing to attend these schools. In contrast, learners in school C pay token school fees and other administrative charges being a community established and funded public school. But all the schools are still under and regulated by the Katsina state ministry of education.

They all run co-educational and separated junior and secondary school system. By separated system, I mean the school management and teachers in junior sections are different from the ones in the senior sections, despite being in the same school compounds. The schools mainly serve the children of the indigenous and low socio-economic status parents – e.g., farmers, herders and other unskilled parents. Generally, the schools lack the basic necessary physical resources and conducive environment for effective teaching and learning. For example, learners always sit on their *Daduma*⁵⁰ they always bring to the school or on bare floor in their dilapidated classrooms.

For the experimental study, schools A and B were assigned to control group and schools C and D assigned to treatment group. The assignment was based on my initial knowledge about the schools and more importantly, to encourage easy participation in the SREP sensitization program. Control schools have two advantages over treatment schools: they are closer to the state capital and all their teachers except one have degree in mathematics/mathematics education unlike in the treatment schools. Being that schools C and D are disadvantaged with these factors and that they are located in the same local government area, they were assigned to the treatment group. As stated earlier, this decision eventually encouraged the attendance, familiarity and participation of the teachers during SREP sensitization. I envisaged that the teachers' attendance and familiarity might not have been possible if the teachers were drawn from different areas.

5.5.1.2 Teacher selection

The selection of the eight participating mathematics teachers, two teachers and their classrooms from each school, for the study was based on those who were teaching senior secondary school I

⁴⁹ Source: JRC (European Commission's Joint Research Centre).

⁵⁰ A mat-like material they always use for religious prayers.

classes (i.e., Grade 10). Senior secondary school I (SSS I) classes are more likely to be available for research than the more senior classes, as the latter are the revision, examination or exit class, which could limit their chances to participate in research over a long period. The following purposive criteria further guided the selection of the teachers: the teachers voluntarily accepted to participate in the study and were teaching in a full-time capacity in their schools. Most importantly, four of the teachers, two each from one school agreed to be guided by the SREP. The classes of such guided mathematics teachers participated in the study as the treatment group, and the other four classes as the control group. The purpose of these criteria in part was to address ethical issues, ensure compliance with the research procedures, and in part to strengthen the validity of the study.

5.5.1.2.1 Teacher participants

Eight male full-time teachers participated in the first stage of descriptive study. This number, as discussed above, comprises of two teachers each from school A, B, C and D (see Table 5.3). Mallam⁵¹ Mustapha, Mallam Ahmadu and Mallam Abdulhadi, Mallam Abati were selected from school A and school B respectively. Mal. Mukhtar, Mal. Sagir and Mal. Ali, Mal. Usman were selected from school C and school D respectively. Mal. Mustapha and Ahmadu are graduates of

Name ⁵²	School	Study stage	Qualification	Yr of TE	Age
Mal. Mustapha	A	1 st only (transferred)	BSc mathematics	7	30-35
Mal. Ahmadu	A	All	BSc mathematics	2	30-35
Mal. Kamal	A	2 nd only (joined)	BSc (Ed) mathematics	17	40-45
Mal. Abdulhadi	B	All	BSc (Ed) mathematics	3	30-35
Mal. Abati	B	All	BSc computer science	1	25-30
Mal. Mukhtar	C	All	NCE chemistry	10	30-35
Mal. Sagir	C	1 st only (withdrew)	BSc (Ed) mathematics	17	40-45
Mal. Ali	D	All	HND engineering	11	40-45
Mal. Usman	D	All	BSc (Ed) chemistry	3	40-45

Table 5.3: Characteristics of teachers for descriptive study

BSc mathematics with teaching experience of seven and two years respectively. Their ages range between 30-35. Mal. Abdulhadi and Mal. Abati have degrees in BSc (Ed) mathematics and BSc

⁵¹ It is a common title used for men (Mallama for women) as a sign of respect, and teachers are always given the title. It is usually abbreviated as "Mal."

⁵² All the names here and in the rest of the study are pseudonyms.

computer science respectively. Mal. Abdulhadi has three years of teaching experience while Mal. Abati has only taught for one year in secondary school. Their ages range between 27-33.

Mal. Mukhtar, as at the time of the study, has Nigeria certificate in education (NCE) with specialization in chemistry. He was also studying for his degree in BSc (Ed) chemistry during the same time. He was about 35 years and has taught for 10 years. Mal. Sagir has a degree in BSc (Ed) mathematics with 17 years teaching experience. He is over 42 years old. Mal. Ali is a holder of higher national diploma (HND) in engineering and Mal. Usman holds BSc (Ed) chemistry. A few weeks to the end of the of the second term, Mal. Usman replaced the initially selected teacher who was transferred to another school from the beginning of the term. Both Mal. Ali and Mal. Usman are both over 40 years. While Mal. Ali has 11 years of mathematics teaching experience, Mal. Usman has been teaching chemistry for five years in his previous schools but now mathematics in school D.

Unlike in the school selection where the same schools participated throughout in the study, there were cases of teacher transfer, swap and withdrawal during the study. Specifically, only seven of the eight teachers participated after the first stage of the study. At the beginning of the third term, Mal. Mustapha was transferred to another school and Mal. Ahmadu took over his class. Within a week into the term, another mathematics teacher, Mal. Kamal, was posted to the school to teach Mal. Ahmadu's previous class. Mal. Kamal is a graduate of BSc (Ed) mathematics with 17 years of teaching mathematics. He is over 42 years of age. For withdrawal case, as reported earlier, Mal. Sagir withdrew himself before the second stage of the study.

In line with any typical experimental study, the teachers were assigned to control or treatment groups. Being teachers selected from schools A and B, Mal. Mustapha/Kamal, Mal. Ahmadu, Mal. Abdulhadi and Mal. Abati were assigned to the control group. Mal. Mukhtar, Mal. Sagir, Mal. Ali and Mal. Usman were assigned to the treatment group, being teachers selected from schools C and D. As presented earlier, teachers in the treatment group also participated in the SREP intervention program for four weekends which those in the control group did not. It is important to emphasize that being assigned to a group did not permanently make the teachers to participate in that group. CR argues that a teacher, as a social agent, may be constrained, enabled or may even decide to implement or not to implement the SREP activities exposed to by the researcher (more detail later).

5.5.1.3 Learner selection

Learners of four SSS I intact classes taught by teachers in treatment group and another four taught by teachers in control group were selected to participate in the quantitative phase of the study. Specifically, 297 learners who comprise of 149 male and 148 female SSS I learners were selected to participate in the study. These learners included those who belong to the class of the selected teachers, agreed and whose parents/guardians allowed to partake in the study. There are reasons for these decisions. SSS I learners are expected to be articulate enough in describing their learning experiences compared with learners in the lower classes, and as I argued above, their classes are more likely to be available for research compared to more senior classes. An intact class allows learners to operate in their natural environment and prevents disruption to the classroom settings. The decision also prevents possible overlap in the overall analysis. Lastly, the sample of 297 learners is sufficient for the research designs and statistical analyses in the quantitative phase because they require “multiple measures on large samples” (e.g., Mitchell & James, 2001, p. 538).

At the qualitative phase, the learners were also selected using a purposive sampling technique. The preliminary analysis of the first phase of the quantitative data determined the 14 learners who were selected for the case studies. Qualitative case studies allow few typical or interesting cases for further in-depth studies of research interests⁵³ (Cohen *et al.*, 2007; Best & Kahn, 2012; Creswell, 2012). By typical, I mean cases that represent the majority in the study which could support generalization to other similar contexts (e.g., Best & Kahn, 2012), and by interesting, I mean cases that are infrequent or outlier from others (e.g., Cohen *et al.*, 2007). The trustworthiness of and ethical issues regarding the findings of a study are strengthened and addressed respectively, if interesting cases are analyzed and reported (Best & Kahn, 2012; Creswell & Creswell, 2018). Specifically, four learners each with consistent high, moderate and low scores in both mathematical identity and achievement instruments, and another one each with inconsistent scores in both mathematical identity and achievement instruments were selected to participate in this phase. For teachers to be part of this selection, I requested them to submit two names for each category mentioned above. The names submitted were finally selected for the study when their scores matched the criteria but dropped and substituted when they did not.

⁵³ Crucial cases have no clear definition or general means or rules of identification in the literature.

5.5.1.3.1 Learner participants – descriptive study

Out of the 297 learners selected to participate in the quantitative phase of descriptive study, only 237 and 234 learners made themselves available for the first and second stages of the study respectively. The third stage of the study could not hold due to the challenges of prevalent insecurity and completion of the session for the learners. I provide more discussions on these challenges later. The 237 learners are distributed across the four schools as follows: school A – 57, school B – 62, school C – 71, school D – 47. The 234 learners are distributed as follows: school A – 57, school B – 51, school C – 65, school D – 61. While a total of 127 male and 110 female learners participated at first stage, 114 male and 120 female learners participated in the second stage. During the study, their ages ranged from 14-17 years. Table 5.4 depicts the numbers of the learner participants in the quantitative phase of the descriptive study, according to their schools, gender and the stage of the study they participated in.

School	First stage		Total	Second stage		Total
	Male	Female		Male	Female	
A	32	25	57	24	33	57
B	35	27	62	31	20	51
C	32	39	71	24	41	65
D	28	19	47	35	26	61
Total	127	110	237	114	120	234

Table 5.4: Learner participants for quantitative-descriptive study

For the qualitative phase of the descriptive study, 14 and 11 learners participated in the first and

Criteria		First stage			Second stage		
		Name	School	Total	Name	School	Total
Consistent scores	High	Farida	A	2	Farida	A	2
		Abdullahi	B		Abdullahi	B	
		Ammar	C	2	Ammar	C	2
		Fiddausi	D		Fiddausi	D	
	Moderate	Zalifa	A	2	Zalifa	A	2
		Ibrahim	A		Ibrahim	A	
		Fauziyya	D	2	Fauziyya	D	2
		Maryam	D		Maryam	D	
	Low	Saifullahi	A	2	---	---	1
		Usaiba	B		Usaiba	B	
		Fa'iza	C	2	Fa'iza	C	1
		Samaila	D		---	---	
Inconsistent scores		Hadiza	B	1	Hadiza	B	1
		Aminu	C	1	---	---	---
Total				14			11

Table 5.5: Learner participants for qualitative-descriptive study

second stages of the study respectively (see Table 5.5). In other words, all the 14 learners who were selected participated in the phase of the study. But only 11 of the 14 learners participated at the second stage due to the absenteeism from schools and teacher withdrawal from the study. Saifullahi and Samaila represent the former case while Aminu represents the latter.

5.5.1.3.2 Learner participants – experimental study

The same 237 and 231 learners who participated in the quantitative phase of the descriptive study also participated in the quantitative phase of the experimental study. But their participation was based on the group they were assigned to. Based on the earlier stated criteria, learners who participated under the teachers from schools A and B were assigned to the control group while those who participated under the teachers from schools C and D were assigned to the treatment group. At the first stage, 119 learners (67 males and 52 females) and 118 learners (60 males and 58 females) participated in the control and treatment groups respectively. At the second stage, 108 learners (55 males and 53 females) and 126 learners (59 males and 67 females) participated in the control and treatment groups respectively. Table 5.6 summarizes this information.

Research group	School	First stage		Total	Second stage		Total
		Male	Female		Male	Female	
Control	A and B	67	52	119	55	53	108
Treatment	C and D	60	58	118	59	67	126
Total		127	110	237	114	120	234

Table 5.6: Learner participants for quantitative-experimental study

For the qualitative phase of the experimental study, the same 14 and 11 learners who participated in the qualitative phase of the descriptive study also participated. As discussed above, the learners were assigned to the research group based on the school they attended and the teacher who taught them. Working with CR argument as stated earlier, learners assignment at this stage, however, was not sacrosanct, as their teachers could still be swapped between the groups. So, at the first stage, seven learners each participated in the control and treatment groups. At the second stage, six and five learners participated in the control and treatment groups respectively. Table 5.7 presents the breakdown of the number of learner participants at each stage and in the both research groups of the study.

Criteria		First stage				Second stage			
		Name	School	Group	Total	Name	School	Group	Total
Consistent scores	High	Farida	A	Control	2	Farida	A	Control	2
		Abdullahi	B			Abdullahi	B		
		Ammar	C	Treatment	2	Ammar	C	Treatment	2
		Fiddausi	D			Fiddausi	D		
	Moderate	Zalifa	A	Control	2	Zalifa	A	Control	2
		Ibrahim	A			Ibrahim	A		
		Fauziyya	D	Treatment	2	Fauziyya	D	Treatment	2
		Maryam	D			Maryam	D		
	Low	Saifullahi	A	Control	2	---	---	Control	1
		Usaiba	B			Usaiba	B		
		Fa'iza	C	Treatment	2	Fa'iza	C	Treatment	1
		Samaila	D			---	---		
Inconsistent scores		Hadiza	B	Control	1	Hadiza	B	Control	1
		Aminu	C	Treatment	1	---	---	---	---
Total					14				11

Table 5.7: Learner participants for qualitative-experimental study

5.5.4 Data collection

As a mixed methods designed study, data collection methods in the quantitative phase are different from those in the qualitative phase. The discussions surrounding the data collection for each phase of the study, especially as relevant to the understandings of the relationships between social relationships enabled by the teachers and learners' mathematical identities and achievement, are presented. How the data collection instruments and methods were pre-administered/conducted is also discussed. As reported earlier, the intervention activities and follow-up meeting were conducted with four and three teachers respectively.

But in contrary to the earlier research design which aimed to collect the data at three different stages, the data were only collected at two different stages. The reasons, as stated earlier, are because of the frequent attacks and kidnapping cases in the participating school areas and the completion of the third term as at the time of the second stage of the study. The high possibility of attacks especially at the treatment schools, and the inevitable changing and promoting of the teachers and learner participants to another classes respectively made the third data collection difficult. Figure 5.2 presents when the intervention was conducted, the number of participants, and the sequential data collection methods for both quantitative and qualitative phases of both stages of the study.

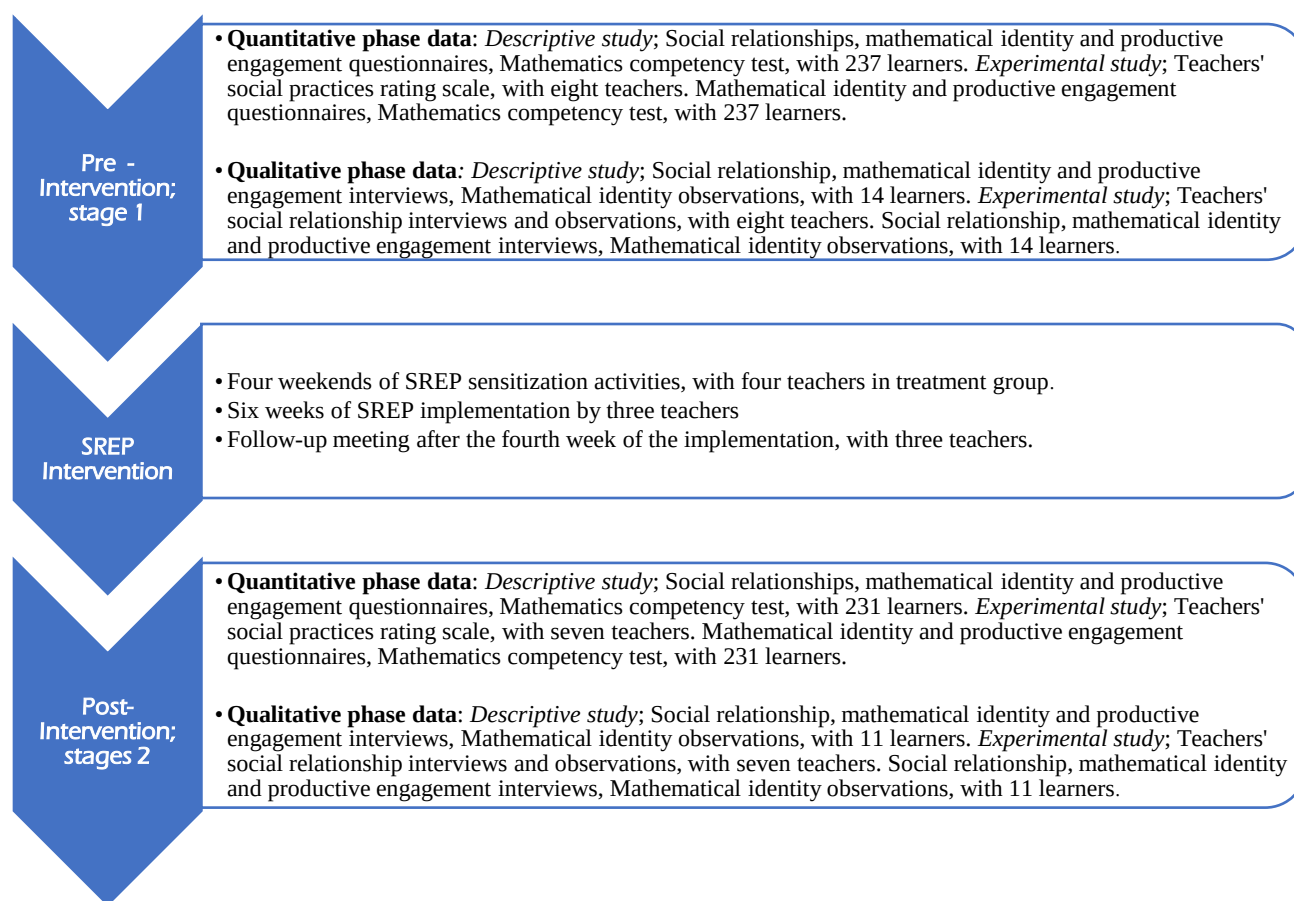


Figure 5.2: Data collection methods, participants, phases and stages of the study

5.5.4.1 Quantitative data collection

This section discusses the type of instruments, the sources of the data, the timing, the procedures, and the purpose of the data that were collected in the quantitative phase of the study for both pre- and post-intervention stages.

5.5.4.1.1 Teachers' rating scale

There could be teachers who participated in SREP activities and constrained/choose to or not to take up the practices in their teaching. Similarly, teachers who did not participate in the SREP activities may ordinarily choose/be influenced elsewhere to enact the SREP, being flexible and common social practices. So, since teachers' practices are likely to be a combination of both

limiting and supportive practices for both groups of the teachers, a social practice rating scale was used. Rating scales are particularly useful in planning interventions and monitoring their effectiveness (Hosp, Howell & Hosp, 2003; Campbell & Hammond, 2014). It is also useful for scoring present observable practices and “qualitative description of a limited number of aspects of a thing or traits of a person” (Best & Kahn, 2012, p. 310). However, according to Best and Kahn (2012), a rating scale is only considered an effective and valid observational tool if the traits to be rated are “carefully defined in observable (behavioral) terms” (p. 310). As presented earlier, the teachers’ social practices rating scale in this study contains carefully defined practices and clearly stated dimensions, and their indicators are more specific for easier rating (see Appendix A).

Based on the above CR argument, social practices in and outside the classroom of each teacher were rated and the teachers categorized as SREP or non-SREP teachers. Teachers whose social practices largely followed the SREPs were categorized as SREP teachers, while those whose practices are in a limited or contradictory form to the SREP were categorized as non-SREP teachers. As indicated in Figure 5.2, the classroom teaching of the eight teachers at the first (pre-intervention) stage and the seven teachers at the second (post-intervention) stage of the study were observed. On average, lesson of 40 minutes of each of the teachers were observed and rated. This gave 5 hours, 20 minutes observation at the first stage and 4 hours, 40 minutes at the second stage, leading to a total of 10 hours classroom teaching observations.

5.5.4.1.2 Learners’ questionnaires

There were three learners’ questionnaires: social relationships, mathematical identity and productive engagement questionnaires. The questionnaires follow a structured format, using a Likert scale. The social relationships questionnaire contains 21 items that were in part adapted from Fan *et al.* (2009) and Rubie-Davies (2015), and in part developed by the researcher. The questionnaire is structured to measure learners’ experiences of their teachers’ social relationships with them and how the relationships engaged them with mathematics. Learners’ experiences of their teachers’ psychosocial support, professional practices and expectation enactment were measured by seven items each respectively (see Appendix B). The identity questionnaire contains 30 items which were in part adapted from Gweshe and Brodie (2019) and in part, developed by the researcher. Items 1-10 aim at measuring learners’ acquisition attribute, items 11-20 target

community participation, and the remaining items focus on their expectation actualization (see Appendix C). The productive engagement questionnaire was developed to capture the exemplified and extended notion of productive disposition of Kilpatrick *et al* (2001). It was also used to capture my argument for expectation realization, as discussed earlier. So, the productive engagement questionnaire contains 30 items with the first 20 designed to measure learners' engagement with mathematics, and the last 10 items to measure their expectation realization (see Appendix D).

Administering the three questionnaires at two different stages in the study did not only provide the data to interrogate the theoretical claims of the relationships between teachers social relationships, learners' mathematical identity and productive engagement just at a shot but as far as twice. Also, it provided an opportunity to investigate the effectiveness of the intervention. That is, the administration of pre-intervention questionnaires provided opportunities to measure the baseline perceptions of social relationships, mathematical identities and productive engagement of the learners before the intervention, while the post-intervention questionnaires allowed for the gathering of data appropriate for analyzing the changes in the learners' perceptions of social relationships, mathematical identities and productive engagement after the intervention (e.g., Wagner *et al.*, 2002).

5.5.4.1.3 Learners' competency test

The mathematical competency test contains 40 multiple-choice items developed from the Nigerian mathematics curriculum for secondary school, as published by the Nigerian Educational Research and Development Council⁵⁴ (NERDC, 2007). Multiple-choice test allows wider coverage of subject contents, objective and easy scoring (Best & Kahn, 2012), and particularly provides room for measuring exemplified four strands of mathematical proficiency (earlier synthesized as mathematical excellence) of Kilpatrick *et al* (2001) (see Appendix F; table of specification). The test items, 10 each, were drawn from the number and numeration, algebra, geometry, and statistics curriculum of junior secondary school learners (see Appendix E). Being a study that focuses on SSS I learners, developing items which difficulty levels do not go beyond the curriculum of the

⁵⁴ The council is under the Ministry of Education and mandated to develop and review Nigerian curricula at all levels.

junior classes – especially junior secondary school III (i.e., Grade 9) – also strengthens the content validity of the test instrument.

Like in the questionnaires, the mathematical competency test was also administered twice of 12 weeks interval. With the combination of mathematical competency test and extended productive engagement scores to generate learners' mathematics achievement scores, the theoretical claims of this study were interrogated in a more comprehensive and elaborated manner. That is, mathematical achievement and mathematical identity scores were used to describe and explain how the kinds of social relationships enabled by the teachers influence learners' mathematical achievement and identities, and the converse influence of the learners' mathematical achievement and identities on social relationships. Using these scores again, how mathematical identities and achievement influence each other was also described and explained with two different data sets. Beyond the theories interrogations, effectiveness of the intervention administered to the teachers was also investigated through the learners. As stated above, the pre-intervention scores offered opportunities to analyze the baseline perceptions of social relationships, mathematical identities and the mathematics achievement of the learners before the intervention, while the post-intervention scores allowed for analyzing the changes in these learners' variables after the intervention.

I am aware that administering research instruments, especially mathematics test, more than once to the same participants poses internal validity threats to the study. But these threats have been adequately addressed in this study. Apart from the wide 12 weeks period (i.e., two weeks of SREP activities, four weeks of break and six weeks of SREP implementation) between the first and second administrations of the instruments, another measure I employed to address the threats was the usage of parallel questionnaire and test instruments, by reshuffling their items. These measures are in line with Creswell's (2012) suggestions that time series researchers should consider parallel items for their post-test and the distant usage of the instrument, but not by changing the instrument. Besides these measures mentioned, the introduction of a control group in the design also addressed the potential internal validity threats associated with the test instrument.

5.5.4.2 Qualitative data collection

This section discusses the methods of data collection, the source of the data, the timing, the procedures, and the purpose of the data that were collected in the qualitative phase of the study for both pre- and post-intervention stages.

5.5.4.2.1 Learners' and teachers' interviews

Interviews, especially open-ended, have unique advantages over other data gathering tools when the researchers' interest is to obtain information about the respondents' "actions, feelings and attitudes" (Best & Kahn, 2012, p. 337). The structure and procedures of interviews determine their validity (Best & Kahn, 2012). To maintain content validity, the interview questions targeted at the learners and teachers were carefully adapted from Gardee (2019a) or designed and structured to obtain enough and relevant information. Three face-to-face interviews were conducted with each learner and one with each teacher. Similar to the questionnaires, the three learners' interviews aim to capture social relationships enabled by their teachers, the narrative aspects of their mathematical identities and productive engagement (as part of achievement). Social relationship interviews probed the learners' stories about the kind of social relationships enable for them by their teachers and how the relationships engaged them with mathematics (see Appendix G). Mathematical identity interviews probed the learners' stories about themselves in relation to mathematics (see Appendix H). Productive engagement interviews probed the learners' expectation realization of, personal and social engagement with mathematics (see Appendix I).

The three learner interviews were conducted at the same time to elucidate the findings of the quantitative study and were only conducted after the completion of each phase of the quantitative study. More specifically, the interviews were conducted with each of the 14 learners at the first stage and the 11 learners at the second stage after the quantitative phase of the study. So, each learner responded to the three interviews twice, making 42 interviews at the first stage and 33 interviews at the second stage. Each of these interviews on average lasted for 15 minutes, making 10 hours, 30 minutes and 8 hours, 15 minutes of interviews at each stage respectively. This makes 18 hours, 45 minutes of interviews in total. Similar to the quantitative data collection instruments,

all the interviews remained the same throughout the study. However, in a few cases some additional questions were used to probe further what I noticed during observations.

The teachers' interview schedule was designed by the researcher – with a part being adapted from Gardee (2019b) – to explore the enabling of teachers' social relationships and any possible preference given to certain learners. Teachers' interviews probed the teachers' psychosocial support, professional practices and expectation enactment and views of their learners, as related to mathematics (see Appendix J). The teachers' interviews were also conducted after the completion of each phase of the quantitative study and the learners' interviews. Specifically, the interview was conducted with each of the eight teachers who participated in the first stage and the seven who participated in the second stage. So, each teacher was interviewed twice, making it eight and seven interviews at each stage respectively. Each of the interviews on average lasted for 40 minutes, making 5 hours, 20 minutes and 4 hours, 40 minutes of interviews at each stage respectively. This in total makes 10 hours of teachers' interviews. All other procedures that were observed for learners' interviews were also maintained for the teachers' interviews.

In addition to establishing good rapport with the respondents, which is crucial for effective interviewing (Cohen, *et al.*, 2007; Best & Kahn, 2012; Creswell, 2012), other recommended procedures for conducting effective interviews such as audiotape recording of the interviews, using probes during the interviews, and note-taking of non-verbal communications were also employed (e.g., Best & Kahn, 2012).

As stated before, the purpose of the data collected here is to elucidate the findings of the quantitative phase of the study. The data obtained from the learners' interviews offered an opportunity to further interrogate in an in-depth way the influence of social relationships, mathematical identities and productive engagement (i.e., mathematics achievement) on one another. The data gathered from the teachers' interviews were further used to help describe and explain the influence of learners' mathematical identities and achievement on social relationships enabled by the teachers. In line with the experimental study, the interviews conducted before the intervention allowed me to obtain data about the learners' entry experiences of their teachers' social practices, mathematical identities and productive engagement, while the interviews after the intervention allowed me to obtain data about any changes in these variables as a result of the intervention.

5.5.4.2.2 Observations of learners and teachers

The goal of any researcher using observations “is to fully understand the culture, setting, or social phenomenon” under study through the real players in such contexts (Hatch, 2002, p. 72). Best and Kahn (2012) explain further that observation is instrumental in collecting detailed information about “physical environment, social interactions, physical activities, nonverbal activities, planned and unplanned activities” (p. 265). Qualitative researchers choose to use observation because of its unique strength in capturing participants’ actions in the “act of doing” which may not be reported or revealed by the participants during interviews (Cohen *et al.*, 2007). Through direct observations and video recordings, the enacted aspect of learners’ identities (most times problematic) and the teachers’ social practices in the classroom were captured in this study.

To capture the enacted aspect of learners’ identities, their classroom activities were observed at two different time as well. There were two observation periods, and for each period, data collection and analysis for each quantitative phase have been conducted. The observations were conducted on one lesson of 40 minutes each, making five hours, 20 minutes of observations for eight classes in the first stage and four hours, 40 minutes of observations for seven classes in the second stage. This makes 10 hours of classroom observation in total. During the observations, the 14 or 11 learner participants in their respective classes for the qualitative study were the major focus. But in many occasions, focusing on them was not achieved (more detail below).

To capture the kind of social relationships enacted by the teachers, teachers’ social practices during their classroom teaching were observed. The teachers’ social practices watched out for are their psychosocial support, professional practices and expectation enactment towards their learners. In addition to earlier provided examples, these included but not limited to frequent calling of names, giving special attention to, praising and using certain learners as point of reference, rather than others in the class. The observations took place concurrently with the learners’ observations, after the teachers’ interviews at each stage had been conducted. The reason for this decision was to watch out for the match and mismatch between their actions and what they said during the interviews. But on a few occasions where teachers chose otherwise, the observations came before interviews.

Using video recorder and observation field note, I took accounts of the actions that demonstrate entry and changes in the learners’ identities, and the teachers’ preference or otherwise for certain

learners. Because of the nature of their classrooms, the video recorder in most of the time was placed at the back of the classroom to capture the class activities of the targeted learners and their teachers. But in many occasions, this was difficult to do; the video recorder only captured the teachers, as the learners always sit on the floor. This situation had no negative effect on the study as the realist methodologies employed emphasize mixed methods not multiple of a method. With my field notes, notwithstanding, I still captured actions that are mostly non-verbal communication and I fairly captured verbal communication/observable practices among the learners and between learners and their teachers.

The data obtained from the learners' observations, combined with the data from their identity interviews, were used to describe, explain, and explore the influence of social relationships enabled by the teachers on the learners' mathematical identities, and how mathematical identities and achievement influence each other. Also, the data obtained from the teachers' observations, combined with the data from their interviews, were used to describe, explain, and explore the influence of learners' mathematical identities and achievement on social relationships enabled by the teachers, and the converse influence of social relationships on learners' mathematical identities and achievement. For experimental study, the pre-intervention observations allowed me to observe the identities of the learners before the intervention, while the post-intervention observations allowed to observe any changes in their identities after the intervention implementation. These changes included but not limited to the ways learners participated, answered questions, interacted, and shared ideas with their counterparts and teachers.

5.5.4.2.3 Fieldwork Journals

According to Hatch (2002, p. 87), reflections assist researchers to make meaningful interpretations of data and also account for any personal biases and feelings. Using my own fieldwork journal, I documented my reflections about my experience, reaction to events, confusion, interpretations, mistakes, and feelings that arose during the study (Spradley cited in Hatch, 2002; Best & Kahn, 2012). Teacher participants were also requested to write their reflections in the form of ideas, feelings, experience, and interpretations about their learners, and their own experience about the study, every weekend from the beginning of the study. Unfortunately, none of the teacher participants complied with this request. So, no teacher's fieldwork journal was received.

So far in this Chapter, I have presented the exemplified version of the three theoretical concepts and the implementation of SREP intervention of the study. I have also discussed how mixed methods research design can be productive in obtaining elaborated findings in suggesting solutions to social issues in this study. The discussions cover how each phase of the quantitative study links up and connects with the qualitative study. This discussion has earlier been re-described to capture the new development in the study in terms of initial descriptive research design which now employed only correlational design. Furthermore, I presented and justified the realist sampling techniques as a useful technique for selecting the research sites and participants for the study. Section 5.5.4 discussed the structure of the data collection instruments and methods for both quantitative and qualitative studies.

I use Table 5.8 to provide the summary of the data collection methods according to the phases of the study and the research questions to be answered, which are recalled here:

Research sub-question 1:

What influence do social relationships between teacher and learners have on rural school learners' mathematical identities, and what influence do learners' identities have on the social relationships between teacher and learners?

Research sub-question 2:

What influence do rural school learners' mathematical identities have on their mathematics achievement, and what influence does learners' mathematics achievement have on their mathematical identities?

Research sub-question 3:

What influence do social relationships between teacher and learners have on rural school learners' mathematics achievement, and what influence does learners' achievement have on the social relationships between teacher and learners?

Research sub-question 4:

How effective is SREP in improving the relationships between: teacher-learner relationships and learners' mathematical identities; learners' mathematical identities and achievement; and teacher-learner relationships and learners' mathematics achievement in rural schools?

Research methods	Research question			
	Sub-question one	Sub-question two	Sub-question three	Sub-question four
Quantitative instruments	Questionnaires: - mathematical identity - social relationships afforded.	Questionnaires: - mathematical identity - productive disposition. Test: - test scores.	Questionnaires: - productive disposition - social relationships afforded. Test: - test scores.	Questionnaires: - mathematical identity - social relationships - productive disposition. Test: - test scores. Rating scale: - SREP.
Qualitative methods	Learner interviews: - mathematical identity - social relationships afforded. Learner observations: - mathematical identity. Fieldwork journals.	Learner interviews: - mathematical identity - productive disposition. Learner observations: - mathematical identity. Fieldwork journals.	Learner interviews: - productive disposition - social relationships afforded. Fieldwork journals.	Learner or teacher interviews: - mathematical identity - productive disposition. - social relationships enabled. Learner or teacher observations: - mathematical identity - social relationships enabled. Fieldwork journals.

Table 5.8: Data collection methods and research questions.

I also use Table 5.9 to summarize the volume of all the data collected in the study, according to the research phases, stages and participants.

Study Phase	Data collection Instruments or Methods	Data for Learners			Data for Teachers		
		First stage	Second stage	Overall	First stage	Second stage	Overall
Quantitative data	Rating scale	-----	-----	-----	5 hours, 20 minutes observations	4 hours, 40 minutes observations	10 hours classroom observations
	Questionnaires	711 scores	693 scores		-----	-----	-----
	Test	237 scores	231 scores		-----	-----	-----
Qualitative data	Interviews	10 hours, 30 minutes of audio records of 42 interviews	8 hours, 15 minutes of audio records of 33 interviews	18 hours, 45 minutes of audio records of 75 interviews	5 hours, 20 minutes of audio records of 8 interviews	4 hours, 40 minutes of audio records of 7 interviews	10 hours of audio records of 15 interviews
	Observations	5 hours, 20 minutes	4 hours, 40 minutes	10 hours videos of 15 lessons	5 hours, 20 minutes	4 hours, 40 minutes	10 hours videos of 15 lessons

		videos of 8 lessons	videos of 7 lessons		videos of 8 lessons	videos of 7 lessons	
	Fieldwork journals	-----	-----	----	Nil	Nil	Nil

Table 5.9: Summary of the data in the study

For emphasis, however, I have stated the reason why learners' interviews were the main source of qualitative data for any analysis regarding learners in the study. I have also stated and justified why only the analyses and findings of Research Question One would be conducted and reported. And by implication, I have ontologically indicated why the final analysis will be conducted at a single time rather than at (two) different time. These emphases are necessary before moving to the next section that presents how the data analysis was conducted.

5.5.5 Data analysis – a realist analytical procedures

Procedures for data analysis for both quantitative and qualitative data are discussed in this section. The procedures, as expected, followed a mixed analytical approach at three different levels of realist analysis. The mixed analytical approach involves employing statistical (quantitative) and thematic (qualitative) procedures complementarily to provide a set of evidence that corroborate or falsify earlier stated theoretical/analytical positions and at the same provide answers to research question raised. Employing statistical procedure is necessary because it involves a procedure with little or no researcher's involvement or inferences but with a comprehensive coverage of all the validly possible emergent cases (OECs) because of its great potential to generalize. Employing qualitative procedure, in the other hands, is also necessary because it involves a procedure that fully depends on researcher's involvement and inferences but likely to overlook some cases from OECs because it focuses mainly on the validation of earlier stated theoretical position(s). So, combination of both procedures is highly necessary for any realist inquiry to complement each other in providing convincing evidence and answers they are set out to provide.

As presented in Chapter 3, re-enforced CR recognizes the levels of analysis at the empirical, the actual and the real as analytic, systematic and generative respectively. Analytic analysis accounts for the complex relationships between two exemplified social phenomena which offer the direction, degree and testing for the generality of the such relationships. The analytical procedures

at the analytic level have little or no input or inference from the researchers. Systematic analysis involves inferential and categorical procedures that refine the outcomes of complex and complicated relationships that emerge from the analytic level. It specifically refines the complicated direction or nature of the relationships produced at the empirical level. The procedures are both researchers and non-researchers dependent. Dependent and independent because the rules for the refinement are necessarily embedded in the data but must be inferred/obtained and employed for the categorization by the researchers. Generative analysis involves procedures that provide evidence explaining why the relationships at the analytic and systematic levels are what they are. For generative analysis, the procedures are researchers' self-generated evidence through engagement with the data; so, they are mainly researchers dependent.

Researchers' ability to employ this analytical approach, and more especially systematic aspect, is a strong basis to finally answers the question 'so what' of any social inquiry. Providing answer to

SRs offered	MI's emergent cases (as influenced by SRs)	MI's enacted	SRs emergent cases (as also influenced by MI's)	RECs	PERs
Robust	Robust	Robust	Robust	<i>Typical</i>	Accept and re-enforce
	Modest		Modest	<i>Crucial</i>	Co-exist and reproduce
	Fragile		Fragile	<i>Interesting</i>	Reject and transform
Modest	Robust	Modest	Robust	<i>Crucial</i>	Co-exist and reproduce
	Modest		Modest	<i>Typical</i>	Accept and re-enforce
	Fragile		Fragile	<i>Crucial</i>	Co-exist and reproduce
Fragile	Fragile	Fragile	Fragile	<i>Typical</i>	Accept and re-enforce
	Modest		Modest	<i>Crucial</i>	Co-exist and reproduce
	Robust		Robust	<i>Interesting</i>	Reject and transform

Table 4.1: SRs and MI's relationships and emergence of RECs and PERs (Recalled)

'so what' question helps create a context for understanding phenomena in another broader and different situations. Research that answers the question has "the potential to make a difference, here and maybe elsewhere", because of the implications it has for both "policy makers and practitioners in education" (Morrison & Van der Werf, 2019; p. 333). To be able to make this

difference in the current study, the presented approach is fully informed by the analytical framework already foregrounded in Chapter 4. I recall the framework in Table 4.1.

In Chapter 4, the analytical approach was operationalized for the relationships between the three theorized concepts. Now as an analytical model at the judgmental realm, the influence of each theoretical concept as exemplified (recall section 5.2) and also captured by the research instruments/methods (recall section 5.5.4) earlier in this Chapter, can be analyzed. Employing the realist analytical model is not only necessary to answer the research question because it is grounded in re-enforced CR philosophy but also because it follows a different and more comprehensive framing. While other existing analytical models follow only the “*if ... then type*”, the model extend it with the “*if ... possibly type*” at the empirical domain. The model also does not only give primacy to reciprocal interplay between the two concepts of research interest, (in this case mathematical identities and social relationships at the empirical only) but further considers their interplay at the actual and much deeper at the real. Using the model also offers an avenue to test its efficacy, since theories are primarily tested and modified by being used in empirical studies, research practices and theory discussions (Danermark et al., 2019).

Recall that the realist explanatory analytical framework contains three key emergent – operational emergent cases (OECs), relational emergent categories (RECs) and personal emergent reflexives (PERs). These analytical emergent have been well discussed in Chapter 3, operationalized in Chapter 4 and will be exemplified here. OECs represents all the nine validly possible emergent cases from the conditioning and interactions between the constituents of learners’ mathematical identities and constituents of the teacher-learner social relationships at the empirical domain. RECs are the three emergent categories from the OECs as a result of the systemic interactions that exist between the learners’ mathematical identities and teacher-learner social relationships at the actual domain. OECs were further categorized as RECs – *typical*, *crucial* and *interesting*⁵⁵, because there are well-defined recognition rules between the two concepts under study for each of the categories. Typical category (of three diverse cases) represents the group of learners whose mathematical identities are enacted in *direct relation* to the kind of social relationships experienced from their

⁵⁵ Several methodologists always consider typical and interesting categories (see Best & Kahn, 2012; Cohen et al., 2007; Creswell, 2012; Creswell & Creswell, 2018), while few others who consider the third category (crucial) mainly consider it as deviant (see Gerring, 2009; Mukumbang, 2020). In both cases, they have no ontological/theoretical basis for them.

teachers and vice versa. Crucial category (of four diverse cases) represents the group of learners whose mathematical identities are enacted *neither direct nor indirect relations* to the kind of social relationships experienced and vice versa. For the interesting category (of two diverse cases), it represents the group of learners whose mathematical identities are enacted *indirect relation* to the kind of social relationships they experience and vice versa.

PERs are the learners' reflections that shape their actions about how they mediate the influence of teacher-learner SRs on them and how they also influence the SRs in return. Learners reflect whether to *accept*, *co-exist with* or *reject* the influence exerted by their teachers through their teacher-learner SRs and in the same vain, reflect on how their teacher should *re-enforce*, *reproduce* or *transform* the practices as a result of the MIs they enact in the mathematics classroom. How learners actualize their first- and second-order reflections determines how they are named as rationalist/strategist, realist/conformist or irrationalist/resistant. These PERs do not only provide evidence to validate the three rules of how the nine OECs of teacher-learner SRs and learner MIs interactions are assigned into the three RECs but further provide explanations for why they act the way do as measured/observed at the empirical. All these three forms of emergence are expected to be evidential during the quantitative and qualitative data analysis approach.

5.5.5.1 Quantitative data analysis

All the valid scores generated from questionnaires of 237 learners who participated in the first stage of the study were used to describe, analyze and exemplify the learners' experiences of their teachers' social relationships and developed mathematical identities quantitatively. The analyses were conducted by using contingency table within the statistical package for social science (SPSS) software. The choice of contingency table within the SPSS is informed by the analytical model and the efficacy of the chosen tool to accommodate the analytic procedure presented earlier. So, contingency table as a descriptive statistical tool was used to provide all the categorical possibilities of the relationships between the measured constituents of teacher-learner social relationships and learners' mathematical identities (i.e., OECs). To convert the measured constituents of each concepts to their categorical possibilities and assign them to the appropriate cell of the contingency table (i.e., diverse cases), some initial computations were carried out. Following the suggestion of Almeida and Freire (2003, cited in Ferrando et al. (2012)), I computed

for 25, 50 and 75 percentiles of all the scores obtained in the research instruments, which corresponded with the learners' actual scores that lied below 25, between 25 and 75, and above 75 percentiles. With the computations and as guided by Table 5.1 in section 5.2, learners' MIs were classified as robust, modest or fragile and teacher-learner SRs offered by the teachers were categorized as robust, modest or fragile (see Appendix K for full detail).

In addition to testing of the relationships between the constituents of two categorized concepts, Chi-square analysis which uses or builds on contingency table also allows researchers to interrogate trend analysis like theoretical frequencies and possibilities (Best & Kahn, 2012; Fraenkel & Wallen, 2006). On that basis, the Chi-square analysis allowed me to test the generality of the relationships between the constituents of the two categorized concepts under study and further offered me opportunity to test the validity of the theoretical possibilities of the OECs stated in Chapter 4⁵⁶. To quantitatively account for the generality of the relationships and the validity of the OECs, *p*-value at 0.005 and Chi-square statistic (i.e., χ^2 and contingency coefficient) were respectively reported. But because of the weaknesses of analysis at the empirical, the validity of the RECs (i.e., typical, interesting and crucial) at the actual were systematically inferred (i.e., employing the recognition rules stated earlier), cases assigned into the appropriate categories and reported by a *refined* Chi-square statistic (i.e., $R\chi^2$)⁵⁷.

To conduct the generative analysis aspect at the real, five learners who were selected for the qualitative analysis and their interviews were holistic considered. What this specifically means is that generative analysis slips into the qualitative analysis during quantitative data analysis. So, such quantitative analysis cannot be conducted only with statistical procedures (i.e., generality and validity) except in addition with the qualitative procedure (i.e., practicality) which only available to the qualitative data analysis. This is an indication of what makes the realist data analysis approach statistical-qualitative (i.e., mixed approach). And more importantly, it confirms that practicality as an evidential criterion only available to the qualitative data at the real domain. The full details of how the generative analysis aspect was conducted are presented under qualitative data analysis below.

⁵⁶ While regression analysis can also provide the relationships, testing the theoretical possibilities of OECs may be difficult because it only considers 'if... then type' model.

⁵⁷ To the best of my knowledge, no such descriptive statistic has been used or suggested before in any educational study or methodology texts respectively.

5.5.5.2 Qualitative data analysis

The data generated from the observations and most especially the interviews, were used to qualitatively describe, analyze and exemplify the learners' experience of their teachers' social relationships and their mathematical identities. Analyses of the transcribed interviews of five selected learners who participated in the first stage of the study were conducted qualitatively. The learners' selection was based on purposive selection that was guided by the nature of their mathematical identity scores and school representation criteria indicated in Table 5.5 earlier. Specifically, at least one learner who met consistent high, moderate and low and inconsistent scoring patterns⁵⁸ and simultaneously represented school A, B, C and D were purposively selected for the analysis. At the analytic level, the analysis was conducted with the help of the same Table 5.1 to establish the relationships between the constituents of teacher-learner social relationships (SRs) and learners' mathematical identities (MIs) and their OECs for each selected learner.

Specifically for MIs, I looked for the best and direct evidence for and coded them as attribute acquisition – e.g., how learners enact or report their dedication and commitment to mathematics; community participation – e.g., how learners enact or report their participation in mathematics classroom; expectation actualization – e.g., how learners enact or report their disruptive, volatile or attentive behaviors. Any selected learner who positively reported or demonstrated all the three, just two or only one of these MIs mechanisms was respectively labeled robust, modest or fragile MIs learner. For SRs, I looked for the best and direct evidence for and coded them as psychosocial support – e.g., how teachers give attention to, value or ignore their learners, or how the learners experience all these from their teachers; professional practice – e.g., how teachers offer learning opportunities, hold high expectations for and award rewards to their learners, or how the learners experience all these; expectation enactment – e.g., how teachers attend and treat all the learners equally, or how the learners experience all these. Also, any report or action from each selected learner which showed positive experiences of all the three, just two or only one of these SRs mechanisms was respectively labeled robust, modest or fragile teacher-learner SRs experience for that learner.

⁵⁸ For emphasis, these scoring patterns were sole informed by the theoretical arguments regarding RECs.

After then, I was able to analyze the qualitative data further by focusing on how the type of teacher-learner SRs offered by the teachers – labeled as robust, modest or fragile – influenced different enactment of MIs – labeled as robust, modest or fragile – among learners and vice versa. These analyses represent the analytic aspect to provide qualitative evidence which corroborate or falsify the pre-developed theories. The analysis was conducted to allow me establish the qualitative relationships between the constituents of teacher-learner SRs and learners' MIs and the theoretical possibilities of OECs. But unlike in the quantitative analysis, establishing the generality of the relationships and validity of the theoretical possibilities with the qualitative analysis was less comprehensive because it usually focuses only on a few OECs. So, to qualitatively account for the generality of the relationships and the validity of the OECs at the empirical, quantitative results were used to complement the qualitative findings (i.e., generality and validity). Also, because of the weaknesses of the analysis at the empirical, the validity of the RECs (i.e., typical, interesting and crucial) at the actual domain were systematically inferred (i.e., employing the recognition rules stated earlier), cases assigned into the appropriate categories and accounted for by *refining* the findings at the empirical.

To conduct the generative analysis, five learners representing three cases of typical, one case each of interesting and crucial categories who emerged from systematic analysis were further considered. The experiences of each of the learners as narrated through their interviews were used to *practically* generate the deeper explanations for the overall results/findings of the study. By overall results or findings, I mean the narration which holds the whole outcomes of the study at the empirical, actual and real levels.

5.6 Pilot study

The purpose of any pilot study is to establish the validity and reliability of the research instruments and methods before the main study is conducted (Yin, 2009; McMillan & Schumacher, 2010). Validity is the degree to which instruments measure and methods account for what they designed for (Best & Kahn, 2012; Creswell, 2012; Maxwell, 1992). I have discussed the procedures taken to maintain a high degree level of validity for the data collection methods in Section 5.5.4. Still, pilot testing the instruments and methods for improvement is important. Reliability is a measure of the degree of consistency of research instruments and procedures (Best & Kahn, 2012; Creswell,

2012). To establish the reliability and validity for all the instruments and methods in the study, one rural school and classroom that met all the criteria mentioned in Section 5.5.1.1 was selected for the pilot study. The study lasted for about four weeks: administration of the questionnaires and mathematical competence test twice with two weeks interval; interviews of three learners and their teacher within a week; and learners' and teacher's classroom observation within a week.

The data obtained from the four weeks' study were used to establish the reliability coefficients of the questionnaires and test. For the questionnaires, the reliability coefficient of internal consistency of each questionnaire in the study were determined. By calculating Cronbach's alpha (α) for the two sets of scores obtained from the both administrations through SPSS, substantial internal consistency of the items (i.e., $\alpha \geq 0.70$), which is considered very acceptable were obtained (see Table 5.10). Therefore, all the three questionnaires used in the study are not only conceptually valid but also substantially reliable.

Questionnaire	First Administration		Second Administration		Conclusion
	α	N	α	N	
SR	0.548	32	0.812	42	Acceptable⁵⁹
MI	0.746	31	0.667	30	Acceptable
PD	0.673	38	0.675	30	Acceptable

Table 5.10: Reliability coefficients of pilot-tested questionnaires

For the mathematical competence test, stability over time – test-retest and stability of items – internal consistency, are crucial (Best & Kahn, 2012). A test with good stability over time and items consistency stability is likely to have high test-retest and split halves reliability coefficients respectively (Best & Kahn, 2012). By high reliability coefficient, it means value of $r \geq 0.70$. Forty-six learners wrote the first round of the test, while only ten of the learners were allowed to write the second round of the test. The decision to randomly select only ten was made to control the learners copying from each other, as was observed in the first round. Being a test designed on both mathematical strands (e.g., conceptual understanding) and domains (e.g., algebra) and as expected, the coefficient of consistency was complex – seven items, items 2, 5, 10, 14, 19, 25 and 31, were removed from the analysis because they had zero variance. Zero variance here means that the learners had the same scores in the items. That is, the consistency is absolute; no need for SPSS to include them in the analysis for variance. So, the Cronbach's alpha (α) obtained for the

⁵⁹ The average coefficient, $\alpha = 0.700$, is considered very acceptable.

remaining 33 items in the first round is **0.674**, which is considered acceptable (Best & Kahn, 2012). Also, correlating the scores of the ten learners in the first and second test, a very high correlation coefficient of **0.906** was obtained. Therefore, the test items are substantially reliable and conceptually valid.

Study Phase	Data collection tools/methods	Procedures for	
		Validity	Reliability
Quantitative	Questionnaires	Development was based on the theoretical framework. Many of the items were adapted from other validated instruments. Pilot tested for further validation.	Pilot study; test-retest, internal consistency and item analysis.
	Test	Development was based on the theoretical framework. All the items were drawn from the NERDC curriculum. Pilot tested for further validation.	Pilot study; test-retest, parallel forms, and internal consistency.
	Rating scale	Development was based on the theoretical framework and SREP intervention. Pilot tested for further validation.	Pilot study; inter-rating.
Qualitative	Interviews	Development was based on the theoretical framework and SREP intervention. Protocols are largely adapted from Gardee's studies with modifications. Pilot tested for further validation.	Note taking, audio recordings and pilot study.
	Observations	The practices and their indicators were based on the theoretical framework and SREP intervention. Pilot tested for further validation.	Note taking, video recordings and pilot study.
	Fieldwork journals	-----	-----

Table 5.11: Summary of the validity and reliability procedures

For rating scale, I observed, video recorded and rated a classroom teaching of about 36 minutes of the selected mathematics teacher. I requested another mathematics teacher who is also a PhD student in one Nigerian University to be my second rater and he willingly accepted. I then compared my rating of the classroom practices with his rating to determine the level of our agreement. The inter-rater agreement of eighty-six percent, which is considered very reliable, was obtained. Therefore, the rating scale is also valid and reliable. Table 5.11 summarizes the procedures employed to maintain the validity and reliability of the data collection instruments and methods (as discussed below) in this study.

Available data from the four weeks' study were also used to determine the appropriateness of the interview and observation procedures. For the interviews, if the interview questions are slightly

reframed during the interview, the interviewees' responses will be evaluated for consistency (Best & Kahn, 2012). This technique was judiciously employed at different point of the interviews and consistencies were noticed in their responses. Effectiveness of other procedures such as time allowed for responses, questioning pace, and probe patterns were all evaluated. They are all largely found effective, especially for the teachers. However, I noticed that the learners' responses started getting low if the interviews took long, and as expected, they were free and more comfortable speaking their local language (i.e., Hausa).

For observations, the validity of the actions that respectively represented the teachers' and learners' enacted social practices and identities, as presented earlier, were watched out for. Effectiveness of the procedures such as positioning of the video recorders and recording of non-verbal communication also were evaluated. It was difficult for me to capture the intended learners. This is because I could only position my camera far away at the back of the classroom. Some of the learners were sitting on the few available chairs/tables and the rest sitting on the floor. I only focused on the teacher.

5.7 Trustworthiness

Trustworthiness involves different procedures employed by qualitative researchers to ensure the validity, reliability, and objectivity of their research findings. Terms commonly associated with trustworthiness in qualitative research include credibility, transferability, dependability and confirmability (Lincoln & Guba, 1985; Shenton, 2004; Loh, 2013). According to Shenton (2004, p. 64), credibility is the internal validity which researchers "seek to ensure that their study measures or tests what is actually intended". Ensuring credibility involves strategies undertaken by the researchers to maintain congruency between the participants' realities and the research findings in the study (Merriam, 1998, 2009). Transferability captures the external validity of research findings which ensures their application to similar situations (Lincoln & Guba, 1985; Merriam, 1998, 2009). Lodico *et al* (2010, p. 173) view transferability as the extent to which the "researcher has made it possible for readers to judge whether similar processes will be at work in their own or similar communities, by understanding in-depth how they occur at the research site".

Dependability is the degree of stability of the findings over time (Anney, 2014). Lincoln and Guba (1985) maintain that credibility and dependability are closely related; strategies that promote credibility directly enhance the dependability. So, the dependability can also be described as the extent to which “the findings of a study are consistent with the data collected” (Merriam, 2009, p. 221) and the procedures employed for data collection and analysis (Lodico *et al.*, 2010). Shenton (2004, p. 72) describes confirmability as “the qualitative investigator’s comparable concern to objectivity”. These concerns involve measures and strategies that prevent research findings from the researcher’s biases and preferences but rather demonstrate that the findings emerge from the participants’ ideas and experiences (Shenton, 2004; Anney, 2014).

Some recommended strategies to maintain the validity of the qualitative study will be used. Creswell and Creswell (2018) recommend eight strategies and emphasize their multiple use. These strategies are reporting disconfirming results; rich and thick descriptions; data source triangulation; prolonging the stay in the field; clarifying researcher’s bias; using external auditors; peer debriefing and member checking. So, the credibility, transferability, dependability, and confirmability of the qualitative study can also be improved when all or most of these strategies are used.

This study, by design, has incorporated all the strategies recommended, except the member checking. Both consistent and inconsistent cases were not only theorized but also that their evidence were provided in this study (recall the typical, interesting and crucial categories). Complementary opportunities offered by both quantitative and qualitative data collected and analyzed in this study confirm how thick and rich descriptions and robust explanations of the findings are achieved. Collecting the same data in both quantitative and qualitative forms using different data sources presents opportunities for triangulation in this study. Triangulation, when comprehensively employed, enhances the credibility and dependability of a study (Creswell & Creswell, 2018). The longer time the researchers spend with the participants, the more accurate the findings are likely to be (Creswell & Creswell, 2018). I spent over 32 weeks (January – September) with the participants; ensuring that the strategy for maintaining trustworthiness is exemplified in the study.

Using my fieldwork journal, as stated earlier, I documented my reflections, experiences, and biases as related to all the aspects of the study. Regarding my role in the study, I only played the roles of

a researcher who employed different data collection methods to obtain alternative accounts of a situation and utilized the quantitative methodology to limit my influence and bias on the outcomes of the study. External auditors who are unfamiliar with the researcher provide objective feedback about the whole procedures employed in the study (Creswell & Creswell, 2018). As part of the PhD studies, proposals and final reports are usually sent out for external examination, in addition to the internal critical reviewer's assessment. Throughout the study, feedback on the procedures and methods employed were sought from my supervisor, experts in the field and other people like mathematics teachers who checked my test instrument and helped with inter-rater reliability of my classroom observations. The peer debriefing strategy was also employed. This strategy involves one or two persons who are familiar with the research topic reviewing and asking questions about the study. I will request another PhD student working on mathematical identity and my supervisor to critically review my work and seek their feedback. Any suggestion that may arise from such reviews will be implemented, when and where necessary.

5.8 Ethical considerations

Ethical considerations involve sensitive issues researchers need to anticipate and find ways of addressing them before conducting any research. These issues may arise before, during and after the conduct of the study. Before starting this study, I submitted my proposal to the Wits University Ethics Committee (Human subjects) for ethical approval and it was approved (Protocol number, H21/07/01) (see Appendix L). I also sought for and got the permission to access the research sites and participants from the Ministry of Education and principals of the selected schools in Nigeria (see Appendix M). The principals of all the five schools also gave their approvals to conduct the study in their schools (approval letters are withheld for anonymity purpose). It is important to mention that all the letters sent to the Ministry of Education and the five principals clearly stated how the schools, learners and teachers would participate, how their identities would be anonymous and how the data would be collected in the study, according to the Wits University Ethics protocols.

When the study begins, it is important to disclose the purpose of the study and obtain the consent from the participants. Respecting the culture and norms of the participants at this stage of the study is also important (Best & Kahn, 2012; Creswell & Creswell, 2018). The selected SSS1 learners

were asked to voluntarily indicate their interest in the study by completing assent form after they have been informed among other information that they are free to withdraw from participating at any stage of the study (see Appendix N). Their parents or guardians were also informed about such information and were requested to complete consent form to allow their children to participate in the study (see Appendix O). For the teachers, the purpose of and how they would collaborate in the study were made clear to them through information sheets as well and their consents were also sought through consent forms (see Appendices P and Q). Being a study within rural environment, my familiarity with the cultural and religious differences among the participants in the study and consulting with the teachers in such environment prepared me to be mindful of and carefully navigate through any possible challenges these differences may bring.

Some ethical issues are directly related to data collection. Ethical considerations for data collection include causing minimal or no disruption to the research sites and introducing equal benefits to all the research sites and participants (Best & Kahn, 2012; Creswell & Creswell, 2018). As this study is designed, the final selected schools and their classrooms were free of any disruption; the study took place in the natural school and intact classroom settings. The control group and pilot schoolteachers will receive the intervention once the study is complete. Other ethical issues I avoided are collection of harmful information from, exploiting, or deceiving the participants (Best & Kahn, 2012; Creswell & Creswell, 2018), and as much as possible minimized the level of risk the participants may be exposed to. I made sure that I stuck to the interview protocols and appreciated the participants for the information given.

The ways and manners in which researchers analyze and use data also warrant ethical attention. Considerations were given to how to respect participants' privacy, avoid embracing participants' and researchers' views unethically, and avoid reporting only positive findings (Best & Kahn, 2012; Creswell & Creswell, 2018). All the teachers', learners' and schools' means of identification remain anonymous, and the information provided in the study was treated confidentially. Researchers act unethically when they only discuss favourable findings to their views and ignore the findings that contradict their expectations (Creswell & Creswell, 2018). Effort was made to avoid this issue by sharing the findings of this study with my supervisor, other PhD students and researchers working in the same area for their evaluation. The study is theoretically designed to

report what are considered positive and negative findings by selecting consistent and inconsistent cases for analysis.

5.9 Conclusion

I have provided a comprehensive summary of this Chapter before data analysis section. That summary specifically emphasized the earlier reasons why only Sub-Research Question One was analyzed and reported. That emphasis then allowed the study to directly moved into the realist analysis procedure employed. Section 5.6 was specifically used to give full details about the validity and reliability of the research instruments and methods used. Finally, the measures taken to maintain the trustworthiness of the study and the strategies employed to address ethical issues in the study were extensively discussed. The measures were not only for quantitative data collection instruments but also for the qualitative data collection methods. Likewise for the strategies employed for the ethical issues, the discussions captured before, during and after sensitive issues related to the data collection and how they were addressed in the study.

CHAPTER SIX

FINDINGS, EVIDENCE AND DISCUSSIONS

6.1 Introduction

This Chapter concurrently presents the results/findings on the influence of teacher-learner SRs experiences and enacted learners' MIs on each other (i.e., Research Question one) and provides evidence to corroborate or falsify the philosophical, theoretical and methodological claims made earlier. Based on that background and the discussions in Chapter 5, the data generated from the research instruments and methods were used for all the analyses conducted. Specifically, the questionnaires on the learners' experience of SRs enabled by their teachers and learners' MIs, and the interviews about the learners' experience of SRs enabled by their teachers and the learners' MIs provided the empirical data used in this Chapter. To reiterate, both the research instruments and methods were administered twice to the learners, at an interval of 12 weeks (Mid of March – Mid of June, 2021) but (based on my earlier justification in Chapter 5) only the data obtained at the first stage were analyzed and reported.

Drawing from the arguments made in Chapter 5, the data analyses in this Chapter were done by employing a realist analytical procedure. This procedure, in addition to establishing the direction, degree and their generality, further goes deep to account for the validity of the three emergent categories of teacher-learner SRs and learners' MIs (i.e., typical, interesting and crucial) when they interact at the actual. While contingency table within the SPSS software was used for quantitative data analysis, a qualitative procedure analysis was employed for qualitative data. The analytical procedure further extended to account for the interactions (i.e., relationships) at the real by practically exemplifying the learners' experiences of the social practices of their mathematics teachers and their own MIs to support or reject earlier claims. So, the combination of both quantitative and qualitative analyses within a realist analytical model allowed me to move beyond accounting for only empirical (surface) interactions between the two concepts under investigation to exemplifying *how* and *why* the interactions are theoretically (and analytically) corroborated or otherwise at the actual and the real levels.

6.2 Teacher-learner SRs and learners' MIs relationships

The analysis to provide answers to the question about the influence of teacher-learner SRs on learners' MIs and vice versa are fully presented here. The analysis builds on the earlier arguments that the question can be answered in three nuance ways and only through realist approach. Therefore, the analysis is based on three of four key theoretical claims inferred from Chapter 4 which retrospectively derive their inspirations from the philosophical assumptions made in Chapter 3. First, that *generally* the constituents/mechanisms of teacher-learner SRs experiences and enacted MIs of rural schools learners influence each other at the empirical domain, but neither of the two necessarily determines the other – that is, they determine and undetermined each other. Second, that the relationships that exist between teacher-learner SRs and MIs of rural school learners at the actual domain are *generally* direct (positive), indirect (negative) or neither (nor positive or negative). The third inference extends the first and second claims, that the relationships at the empirical is weak to make *valid* conclusion compared to the relationships at the actual. And fourth, that the relationships at both the empirical and the actual can be *practicalized* with the learners' reflections at the real. Having these claims in mind, the analyses to confirm or falsify the claims and answer the question are presented concurrently by directly working with the Research Question One:

What influence do social relationships between teacher and learners have on rural school learners' mathematical identities, and what influence do learners' mathematical identities have on the social relationships between teacher and learners?

6.2.1 Quantitative results

To empirically provide answers to the question stated above, a preliminary analysis was conducted. As stated in Chapter 5, classifications of all the emergent cases of both teacher-learner SRs experiences and learners' MIs were based on the total scores obtained by the learners in each of their constituents (see Appendix K for details) and their further categorization into the emergent categories were purely based on the emergent recognition rule which assigned each emergent case to either typical, crucial or interesting category. This further categorization is an argument which draws on the rational, eclectic and irrational nature of agency in Chapter 3. To do the analysis,

contingency table was used to classify all the possible emergent cases (OECs) obtained from the preliminary analysis stated above and that further produced the emergent categories (RECs) – typical, interesting and crucial⁶⁰.

The contingency table (see Table 6.1), through the analytical model of *if ... possibly type*, shows that the relationships between the teacher-learner SRs experiences and enacted learners' MIs are not only strong but also significant [$\chi^2(df= 4, N= 173, p = 0.000) = 34.402$; $CC = 41\%$]⁶¹. On the nature or direction of the relationships, the table further shows that while the relationship does determine what the emergent of learners' MIs and teacher-learner SRs experiences are for some learners, it does not for others. For example, while some learners experience robust teacher-learner SRs and enact robust MIs and vice versa, it is in contrary for others, also both ways. Also, there are learners who experience robust teacher-learner SRs from their teacher but enact modest MIs as opposed to enactment of robust or fragile MIs. Conversely, there are learners who enact robust MIs but experience modest teacher-learner SRs as opposed to experiencing robust or fragile teacher-learner SRs. Extending these examples to other cases in the contingency table should produce haphazard and contingent pattern of relationships as captured in the table. This is the result at the empirical.

Influence of T-LSRs on LMIs				Influence of LMIs on T-LSRs				Shared statistics		
	R	M	F		f	m	r	N	CC	Sig. (p-value)
f	2	20	21	R	2	21	21	173	0.407	0.000
m	21	46	18	M	20	46	22			
r	21	22	2	F	21	18	2			

Table 6.1: T-LSRs, LMIs relationship and emergent cases and categories – first stage. Note: f=fragile, m=modest, r=robust and R=Robust, M=Modest, F=Fragile are cases for T-LSRs and LMIs respectively. CC=Contingency coefficient.

With these observations, it is statistically concluded from the analysis that teacher-learner SRs experiences and enacted learners' MIs influence each other and that the influence is strong but the nature of the influence both ways is complicated to determine. According to the earlier argument,

⁶⁰ Emergent categories: Typical= Diagonal from right to left; Interesting= (f)(R) & (r)(F); and Crucial= others.

⁶¹ Relationships are strong if CC is above 35%, weak below 10% and moderate between the two (Hattie, 2009). Regression analysis also gives the results as positive, strong and significant relationships ($r = 0.502$, $R^2 = 0.252$, $N = 220$, $p = 0.000$). But the result holds determinist and reductionist views.

this result is weak and complicated as it does not provide generalized or recognizable rules to further refine the relationships between teacher-learner SRs and learners' MIs nor comprehensively considers the relationships. Therefore, the information provided may be less productive to further inform teaching practice. There is a need to go analytically deep into the level of the actual.

A deeper analysis of the contingency table at the actual level indicates that there are 21, 46 and 21 learners exemplifying the three cases of typical category; 2 and 2 learners exemplifying the two cases of interesting category; and the remaining 20, 21, 18 and 22 exemplifying the four cases of crucial category in this study [$R\chi^2(df=4, N=173, CC=41\%, p=0.000)=34.402$; $T(21, 46, 21)$, $I(2, 2)$, $C(20, 21, 18, 22)$]⁶². It then shows that: fragile, modest and robust teacher-learner SR experiences relate to fragile, modest and robust learners' MIs respectively for learners who exemplify the three kind of cases in the typical category; fragile and robust teacher-learner SR experiences relate to robust and fragile learners' MIs respectively for learners who exemplify the two kind of cases in the interesting category; and fragile, modest and robust teacher-learner SR experiences relate to learners' MIs in a way that is different from the other two stated relations for learners who exemplify the four kind of cases in the crucial category. These explanations simply represent positive, negative and neither positive nor negative relationships between SRs and MIs for typical, interesting and crucial categories respectively.

These analyses show that despite the strong influences of teacher-learner SR experiences and enacted learners' MIs on each other, the influences do not *always or necessarily* determine who their emergence is. While the teacher-learner SRs always determine the MIs of learners who exemplify typicality of relationships and vice versa, they are not for those who exemplify interesting and cruciality. This non-deterministic, non-reductionist and deeper analysis/result addresses the weakness of the existing analytical approach that only peripherally considers and reports results obtainable at the empirical level. Such weak approach has been argued earlier to also give less comprehensive recognition to the nature of human agency, leading to determinist and reductionist research findings. The realist approach employed for the current study does not

⁶² Where T, I and C represent typical, interesting and crucial categories respectively.

only consider analysis beyond the empirical to include actual level but as well extends the analytical model to include ‘if ...possibly type’.

However, the quantitative analysis still lacks the ability to cover and explain what goes on at the real level. Therefore, I mixed the analysis with the qualitative analysis as maintained by the realist approach. But moving to the qualitative analysis, I want to emphasis once again that quantitative result regarding teacher-learners SRs and learners’ MIs relationships is not only scarce in the existing literature but also rarely considered among teachers and learners of rural schools. No known study is available with which to compare the results of this study.

6.2.2 Qualitative findings

As a complement to the quantitative results presented above, the qualitative findings to answer Research Sub-Question One and to further interrogate the claims, as stated earlier, are also presented here. The findings are also obtained from the analyses conducted on the data collected at the first stage of the study. First, the analysis sought to present a much deeper understanding of the influence of teacher-learner SRs experiences on enacted learners’ MIs, and vice versa. To qualitatively do the analyses and provide rich findings, five learners and their interviews at the first stage of the study were purposively selected. The selection was justified by two reasons – to have at least one learner exemplifying a case of typical, interesting and crucial categories as operationalized in Chapter 4, and to represent each participating school as presented in Chapter 5.

6.2.2.1 Findings at the empirical level

The analysis of the learners’ interviews showed that teacher-learner SRs and learners’ MIs influence each other. The accounts of five learners, Zalifa, Hadiza, Fa’iza, Ammar and Fauziyya, who were purposively selected from schools A, B, C and D, which showed the influence of teacher-learner SRs experiences on their MIs and vice versa, are discussed. The learners variously accounted for the influence of psychosocial support, professional practices and expectation enactment enabled by their teachers on their MIs development, and the influence of the attribute

acquisition, community participation and expectation actualization they enacted on their teacher-learners SRs experiences.

At the first stage of the study, the learners' interviews showed that the ways their teachers encouraged, advised, played with and at times 'beat' or 'punish' them influenced their identification with mathematics. The following are few excerpts from their interviews showing their teachers' encouragement, support, attention and the way they relate in mathematics class influence their MIs.

He will encourage you with the words of advice, words of encouragement and sometimes *maa*⁶³ he can give you a gift for that [doing well in mathematics]. That always makes me feel happy [about mathematics] (Zalifa, School A).

Anytime he teaches us and I understand it very well, *gaskiya*⁶⁴, I used to feel like mathematics is simple to me (Hadiza, School B).

Gaskiya, I do enjoy the way he relates with me and everyone in the class and even outside the classroom (Ammar, School C).

I enjoy learning mathematics because the way he teaches us always impresses me (Fauziyya, School D).

Many also narrated the ways their teachers availed them opportunities, provided mathematical challenging environment and/or made them experienced difficult classroom practices to learn and identify with mathematics.

Anytime we are doing mathematics lesson, he always asks me questions and I will give him the answers (Zalifa, School A).

Yes mallam, I do feel happy if I am being taught mathematics but sometimes if I am tired, I used to feel like this mathematics is tiring (Hadiza, School B).

If I can see someone else who can teach me very well, I think that can make me have interest in mathematics (Fa'iza, School C).

⁶³ Hausa word to buttress or emphasis what they are saying. For example, "in fact".

⁶⁴ Hausa word meaning honestly or sincerely.

I do enjoy the way he poses challenging questions. Anytime he asks questions and you give him answers, he will now ask you why did you think that is the correct answer. Gaskiya, I do enjoy that (Ammar, School C).

Further, the learners reported that the way their teachers treated them ‘equally’ or differently, were serious with their teaching and rarely got angry when they asked them questions, also influenced their MIs.

Sometimes maa, he used to annoy me [when we have mathematics]. Because sometimes if he enters into the classroom, he will start calling someone’s name. If the particular person is not available, he will say that is why the class is boring; you who are all here are not sound, capable. That your “Jaruma” [i.e., overall best] doesn’t come that is why the class is somehow (Zalifa, School A).

... after the lesson..., I will stand up and ask him where I don’t understand and he will definitely explain to me (Fauziyya, School D).

I have never met him to assist me beyond classroom, because he said I am not the type to ask him questions and ... [audio not clear] (Fa’iza, School C).

Conversely, learners’ MIs also influenced their teacher-learner SRs experience. The interviews showed that enacted attribute acquisition, community participation and expectation actualization of the learners influenced the teacher-learner SRs they experienced. Though, there were weak and sketchy evidence (e.g., ‘He always asks me questions’, ‘His eyes are always on me’ etc.) pointing to how the attribute acquisition and expectation actualization influenced the teacher-learner SRs, several evidence were present pointing to the strong influence of community participation on teacher-learner SRs experience. Few excerpts from different learners’ interviews are presented below.

Also, if he gives us classwork or assignment and you didn’t do it, he will say come out and solve this question and you must. Because if you don’t do it, he will punish you (Zalifa).

He wrote a good question on the board and called me to come out to solve it [for not concentrating, as usual]. I solved it wrongly and he asked me to re-solve again and again

until I solved it correctly. He then told me that can you now see that you can do it when you are committed to it (Hadiza).

He also feels proud about me, because of the way I used to respond in the class (Ammar).

The learners' accounts showed that their teacher-learner SRs experience influenced their MIs development. In other words, each aspect or mechanism of the teacher-learner SRs contributed to the strong influence it has on the learners' MIs. In the other way round, the analysis showed that though learners' MIs influenced teacher-learner SRs experiences, not all the aspects of the former evidently contributed to the influence equally for the selected learners in the study. Therefore, teacher-learner SRs experiences and what learners enact as their MIs influence each other, but to different degrees. Overall, it is concluded that SRs enabled by the teachers and experienced by the learners strongly influenced the learners' identification with mathematics, and vice versa. This conclusion of strong influence further derived its justification from the quantitative result reported earlier (i.e., footnote regression analysis results⁶⁵), being an analysis that employed larger number of the learners and 'if ... then type'.

Nevertheless, this is still a weak conclusion because it is obtained from the analytical approach that uses only '*if ... then type*'. The conclusions corroborate the existing qualitative study findings which established that teacher-learner relationships strongly influence and/or influenced by learners' MIs (e.g., Darragh, 2013; Gardee, 2019a, 2019b; Gardee & Brodie, 2019; Hodgen & Marks, 2009; Walker, 2012). What my qualitative analyses are yet to provide, as a model that also uses *if ... possibly type* and considers only limited cases, is that the learners emerged with different characteristics. Also, some learners' MIs are irreducible to the kind of teacher-learner SRs experienced and vice versa, while for some learners it is reducible. Obviously, such refined categorization in terms of the relationships are not easily obtainable in the above analyses; one needs to go deep in the analysis. In the next section, I engage in such analysis to obtain the categorizations. With the exception of Gardee's studies (e.g., Gardee, 2019a, 2019b), all other existing studies overlooked this aspect of the realist analysis.

⁶⁵ 'If... then type' model (i.e., regression analysis) was not given full attention under the quantitative result reports because no known study on the topic used it. But it receives full attention here because several studies used its analytical procedures; so, there is a need to discuss the findings with them.

However, Gardee and her mentor's framework (i.e., Gardee & Brodie, 2019, 2021) that foregrounds her studies seems to have a few weaknesses. First, her non-realist accounts of how agency contributes to both learners' MIs development and the SRs enabled by teachers for their learners. This weakness may be responsible for her conclusion that learners' MIs are reducible to the social identities offered by the teachers who treat all learners equally, but irreducible to the social identities offered by the teachers who treat learners differently (see Gardee, 2019a). This conclusion contradicts the position of this study, as experiencing a robust SRs does not always reduce to an enactment of robust MIs. Second, despite arguing for learners constructing their MIs differently from the social identities offered by the teachers, no analytical (or theoretical) explanation for the learners' resistance, and more interestingly no such learner was found in all her studies (e.g., Gardee, 2019b). Third, her analysis maintains a flat and conflated analytical approach. In other words, her findings are always limited to the empirical level of analysis and social identities of affiliation and partial affiliation are conflated. This observation can be confirmed in her different studies where there are no recognized rules to explicitly distinguish each learners' MIs in relation to the kind of social identities offered to them by their teachers (e.g., Gardee & Brodie, 2019, 2021). To avoid such weakness, my study extends the analysis of the relationships between learners' MIs and teacher-learner SRs to the actual level.

6.2.2.2 Findings at the actual level

The findings at the empirical level are always overlap and difficult to relate further. To qualitatively engage these findings and provide refined findings, the accounts of the same five learners, Ammar, Zalifa, Fa'iza, Hadiza and Fauziyya were further interrogated. Ammar, Zalifa and Fa'iza were identified as different cases for typical category. Hadiza and Fauziyya were identified as a case for interesting and crucial categories respectively. Ammar, Zalifa and Fa'iza enacted their MIs as they experienced the teacher-learner SRs from their teacher, and vice versa. Specifically, Ammar, Zalifa and Fa'iza experienced robust, modest and fragile teacher-learner SRs and enacted robust, modest and fragile MIs respectively. Hadiza, in a sharp contrast, experienced robust teacher-learner SRs but enacted fragile MIs, and vice versa. In a different way from the two, Fauziyya experienced modest teacher-learner SRs and enacted robust MIs, and vice versa. In what follows, I provide series of evidence for this summary.

The typical category – cases of Ammar, Zalifa and Fa'iza

Ammar exemplifies a case of typical category because his experience of teacher-learner SRs from his teacher is directly related to the MIs enacted. His interview showed that is not only loved, cared for and appreciated, but also always given better opportunities to learn by his mathematics teacher. There were clear and strong indicators of good and direct or personal SRs between him and his teacher. I capture some of these assertions from his interviews.

Gaskiya, the relationship between me and my mathematics teacher is good.

If I don't do well in mathematics, he will call me and ask why you didn't perform well or what exactly the problem that made me perform low. He will also ask is it that you didn't understand the question *ne*, or you did not understand the topic when I taught you. If you don't understand them let me know, so I will explain it further for you.

He presents mathematics as the most simple and interesting subject among all subjects, which you can easily learn.

He also feels proud about me, because of the way I used to respond in the class.

In terms of his MIs, Ammar who, from his accounts, teacher's comments and the observation of other peoples around him, has demonstrated a great interest in and preference for mathematics by dedicating his time for practicing mathematics procedures and solving strategies. He participated very well with his teacher and peers, and very deliberate about mathematics. He has a clear goal in life – 'to study mathematics' so that he can 'teach others too'.

My favorite subject is mathematics.

I will sit down all alone because I don't want anybody to disturb me, revising, by working them out until I sort them out before I will sit up from where am sitting.

...I should say I am doing it already. ... we do have a group discussion after every breakfast. We discuss about mathematics, anything you don't understand in a particular topic.

I take mathematics as a simple subject in my life to the extent that anytime, any day and even if am wakening up from my sleep, I can do it.

The above excerpts from Ammar's interview showed that he experienced robust teacher-learner SRs and enacted robust MIs. The excerpts from the SRs interview showed that Ammar's mathematics teacher enabled positive psychosocial support, professional practices and expectation enactment for him. Ammar is loved, cared for and offers supportive learning opportunities as his teacher always monitors his progress. His teacher's expectation for him is high and he was proud of him. The excerpts from the MIs interview showed that attribute acquisition, community participation and expectation actualization were also all positively enacted by Ammar. He gives more attention to and concentrates more on mathematics tasks. He participates with his teacher and peers in and outside the classroom and he is very deliberate about mathematics. Having adequately accounted for his teacher-learner SRs experience and MIs enactment, Ammar's interview has provided the basis to maintain that he is a robust MI learner who as well experienced robust teacher-learner SRs, and vice versa. So, he is one of the three diverse cases of typical category.

Zalifa is another case representing typical category. Zalifa's interview showed her experiences of teacher-learner SRs from her teacher and the kind of MIs she enacted in mathematics classroom are also directly related. Her experiences of SRs represented a mixture type. In other words, she will at one time experience support, encouragement, learning opportunities and at another time differential treatment, emotional and corporal punishment. The following quotes present the evidence for the experience.

Gaskiya, what I don't like about him and even makes me not comfortable with his class is that excessive flogging. He always beats mercilessly to the extent that he will give you wounds [on your body].

Gaskiya, I like the way he teaches me and I will like him to continue teaching me. He teaches me well and I used to understand his lesson.

Anytime we are doing mathematics lesson, he always asks me questions and I will give him the answers.

Gaskiya, he is serious about his teaching, he doesn't take things for granted whenever he is teaching us. And he takes everybody as one and he loves everybody.

How Zalifa also demonstrated her MIs was obtained from her interview. The interview showed that she has passion and preference for mathematics as she sees it as her ‘favorite’ and an ‘important subject’, she participated well with her peers and teacher. But at sometimes she will refuse to do her classwork, come late to mathematics classroom [or school] and even prove stubborn to her mathematics teacher. I obtained the following quotes from Zalifa’s interview to buttress my points.

Gaskiya mallam, I take mathematics as an important subject. My favorite subject is mathematics because I do understand it very well.

Sometimes, if I see that I still don’t understand before he leaves the class, I will tell him I don’t understand this area, and he will put me through.

Yes mallam, you know sometimes if he enters into the classroom and maybe that time I am not feeling well, I will not even bother to concentrate and ready to listen to him... Gaskiya, there was a time he gave us a classwork and I didn’t do it. So, he eventually caught me and he beat me and he said I must do it. That I should come in front of the class... for everybody to see. No, I didn’t do it [she laughed].

Like now, if he finishes teaching us, after he leaves the class, I will go to one side of the class to revise my mathematics notes to practice it more and more for me to understand better.

The above quotes showed that Zalifa experienced modest teacher-learner SRs and enacted modest MIs. Excerpts from teacher-learner SRs interview showed that professional practices and expectation enactment were positively enabled while psychosocial support was negatively enabled for her by her mathematics teacher. She experienced positive learning opportunities, and equal treatment from her mathematics teacher. But she does not like having mathematics lesson because of the flogging and corporal punishment that she always experienced. Excerpts from MIs interview showed that she enacted positive attribute acquisition and community participation but negative expectation actualization. Zalifa’s favorite subject is mathematics, and she considers it very important subject to learn. She participates with her peers and mathematics teacher. However, her actions in the classroom sometimes indicate a negative expectation actualization, as she seems to enact deviant behaviours and prove stubborn to her teacher. Having experienced teacher-learner

SRs that are directly related to the enacted MIs, Zalifa's accounts have provided the basis to maintain that she experienced modest teacher-learner SRs which influence her to develop as a modest MIs learner, and vice versa. Therefore, she is also a case of the learners who belongs to the typical category.

Fa'iza also represents a case of typical category, drawing from her interview. Her teacher-learner SRs experience and enacted MIs are reasonably captured in her interview. She experienced low qualities of direct or personal relationship with and consistent conflicting SRs from her teacher. These experiences are evident in the following excerpts from the interview.

I have good interaction with my mathematics teacher. He always jokes with us and sometimes he does tell us stories.

I have never met him [mathematics teacher] to assist me beyond classroom, because he said I am not the type to ask him questions.

He presents mathematics as a simple subject; he used to teach us very well and he makes us to understand it.

The reason I don't understand is because most times he speaks Hausa language to teach us. Yes, I need somebody who can teach me better than my current teacher.

Though there were limited indicators in the interview, available evidence showed that her MIs was also enacted in fragile manner. She proudly accepted that she did not have interest in mathematics and convincingly considered Animal Husbandry as her favorite subject. She participated in mathematics, but only with her friends and mostly outside the class. There is no evidence of how she actualized her expectations for mathematics.

My favorite subject is Animal Husbandry, because I have interest in and understand it very well, more than any other subjects.

I just don't have interest [in mathematics].

Most times I go to her [a class member]. I even asked her questions two days ago.

I have never met him [mathematics teacher] to assist me beyond classroom...

The two sets of excerpts above showed that Fa'iza experienced fragile teacher-learner SRs and enacted fragile MIs as well. Teacher-learner SRs interview excerpts showed that there was no evidence where she had personal or direct positive psychosocial support, professional practices and expectation enactment with her mathematics teacher, but rather a general classroom relationship. Further evidence was that Fa'iza differentially experienced negative expectation enactment, as the teacher did not see her as who could get his assistance in mathematics. And this negative experience was further compounded by the use of local language, which Fa'iza does not understand very well, as mean of teaching by the teacher. MIs interview excerpts accounted for her negative attribute acquisition for mathematics, as she did not have interest in it and low-quality community participation, as she only participated with her friends outside the class. Her interview provided no evidence for expectation actualization. So, Fa'iza. makes a good example of the last case of learners in the typical category because she experienced fragile teacher-learner SRs and as well enacted her MIs as a fragile learner. The direct relationships between her SRs experience and MIs enactment indicate an influence of the former on the latter, and vice versa.

The findings thus far show that teachers-learner SRs and learners' MIs do not only influence but also determine one another. This assertion is based on the evidence that all the three learners presented enacted their identification with mathematics as they experienced the teacher-learner SRs enabled by their teachers, and vice versa. In other words, MIs of the three learners are reducible to the kinds of teacher-learner SRs experienced. For Gardee's studies (e.g., Gardee, 2019a, Gardee & Brodie, 2021, 2021b), they failed to recognize these cases as learners who belong to the typical category, even though it is easy to identify them from her framework with obvious conflation and non-realist analytical explanations. Specifically, there are evidence of learners who were offered social identities of affiliation or marginalization and constructed MIs of affiliation or marginalization respectively. But not only that social identities of partial affiliation was overlooked or conflated with social identities of affiliation and its corresponding constructed MIs, there was no explicit analytical approach that led to the direct relationships. However, making conclusion only on this key finding from my study is misleading as it reduces all the possible cases only to the typical category and eventually holds determinist and reductionist views. To CR, there are still other two analytically valid categories (interesting and crucial) that need to be considered to have comprehensive evidence of what is available in the empirical data.

The interesting category – a case of Hadiza

To further consider other possible cases in the data, a learner exemplifying interesting category was analyzed. Hadiza represents an example of such cases because her SRs experience is directly opposite of her MIs enactment. Though there were indications of moderate personal or direct SRs between her and her teacher, she said ‘there is nothing I don’t enjoy about him’. For Hadiza, her mathematics teacher is a person who ‘relates in good manners’ with her and ‘others’. She ‘enjoys’ the way he teaches them and how he ‘always points to her’ to attempt questions in the class, despite being a perpetual noise maker. The following quotes from her interviews buttress these assertions.

Like when he is teaching us and he notices that you [referring to herself] are making noise, he must point you out to solve questions on the board. If you cannot solve the question, he will start talking sense into you.

I do enjoy him anytime he is teaching and the way he always points at me to come out to solve questions, especially if he notices that I understand the lesson very well.

After the lesson if I understand the lesson fine and if I don’t, I will tell him and he will repeat the explanations.

When it is the time [for mathematics class] he will [always] come and teach us.

How Hadiza also enacted her MIs was obtained from her interview. She said she prefers Civic Education to mathematics and is more comfortable to ask her friends or seatmates questions than her teacher. Hadiza actualization of MIs was further established with her identity of always coming to the class ‘when mathematics teacher will be in the classroom already teaching’. Other related indicators of her MIs are presented.

Like, if he enters into the classroom and he has spent more time in teaching us, gaskiya I used to feel tired of sitting. You will be feeling hungry and nothing will enter into your brain anymore.

And sometimes I don’t want to ask him [the teacher] question so that the class won’t make fun of me...

No Mallam, I don’t go to his office for more explanations if I don’t understand something in mathematics. Just like that, gaskiya!

So, if I don't understand how to solve the questions [i.e., classwork] or some parts of the question, I used to ask my seatmate to explain for me.

Drawing from the excerpts, it is established that Hadiza experienced robust teacher-learner SRs and enacted fragile MIs. Excerpts of the SRs interview showed that all the three mechanisms of SRs were positively enabled for her by her mathematics teacher. Even though she was not always attentive in the class, the teacher would still mathematically engage and encourage her. Such a move by the teacher showed that he was indirectly encouraging and motivating her to be better in mathematics. When she did not understand something, he would still explain for her to understand. Excerpts of the MIs interview showed that Hadiza negatively enacted two of the three mechanisms of MIs. She is always tired and hungry during mathematics lessons and likes Civic Education most indicating her attribute acquisition for mathematics. She is also always coming late for mathematics class and playing along with teacher to get bonus marks demonstrating her negative expectation actualization. However, Hadiza enacted her community participation by asking her seatmates or sometimes her teacher questions. Building on these accounts, despite experiencing robust teacher-learner SRs from her teacher, Hadiza in sharp contrast still largely enacted fragile MIs. So, Hadiza is an example of one of the two kinds of cases who belong to the interesting category, as her enactment of MIs is indirectly related to her teacher-learner SRs experience, and vice versa.

This finding presents the first basis to empirically claim a theoretically and analytically grounded argument that the kind of SRs experienced by the learners does not determine the kind of MIs they enacted, and vice versa. For emphasis, this finding does not nullify the fact that the two concepts influence each other but further shows that the influence is not deterministic for either of the two. For Gardee and Brodie (2021, 2021b), such cases are considered as learners who marginalized themselves for resisting the supportive SRs. The second kind of the cases are the learners who resist marginalized SRs offered by the teacher. But why or what could make a learner marginalize him/herself was not analytically (or theoretically) accounted for, nor any evidence was available to support the second case, as I earlier observed. While the current study also argues for the same two kinds of cases in the interesting category, fundamental differences still exist between them. Though the cases selection criteria do not include any learner to provide evidence for the second

kind of the cases, the current study analytically and theoretically explains why and how the cases belong to the interesting category and there is no conflation in the SRs classifications.

However, providing evidential cases for interesting category has not satisfied the realist analytical (or theoretical) framing. So, I move to cases in crucial category which can also offer more basis to make such empirical argument from different recognition rule.

The crucial category – a case of Fauziyya

Fauziyya represents an example of a case of crucial category because her experience of teacher-learner SRs is neither directly nor indirectly related to the MIs she enacted. Her classroom experiences showed that her mathematics teacher ‘encourages’ her to do better, ‘laughs’ and ‘plays’ with the learners before he starts his lessons. Fauziyya always ‘enjoys’ and gets impressed by her teacher’s teaching approach. But the way the teacher enabled his expectation enactment to the learners was questionable, detrimental and differential.

Gaskiya mallam, no any bad interaction between us because ever since his coming to the class, you will never see him with a cane.

Gaskiya, my mathematics teacher doesn’t teach us in hard ways. He presents mathematics in simple ways to us and even during exams, he always sets simple questions for us, so that we can [all] answer it.

I like his teaching... Honestly, he teaches us well and we do understand his lesson.

Yes mallam. Like now, he knows you that you are trying your best in mathematics, you always copy notes anytime he writes note for us and you have a complete note and you always participate in the class anytime he is teaching mathematics. He could look at that to add some marks for you when he sees that you perform low.

Her accounts showed that she responded to the SRs experiences, as presented above, by enacting MIs that were different from the SRs experienced. Fauziyya, regardless of seeing her Hausa language as her favorite subject because she could ‘speak it very well’, she still took ‘mathematics as the most important subject to learn’, which she ‘always study’. She participated well with her

teacher and peers in and outside the class, and she always pays attention in the class and does her assignments on her own.

And sometimes *maa*, when he is teaching us, and I don't understand some parts of the workings, after the lesson... I will stand up and ask him where I don't understand and he will definitely explain to me.

Mallam, most time I do sit with my friends to solve so many questions and to practice together. Anything she [her friend] knows better than me, she will explain more for me and anything she does not understand too, I will explain to her too.

Mallam, like I told you earlier, I always sit and concentrate whenever he is teaching and I do group discussion with my friends.

And anytime I am back from school, I do practice it [mathematics exercises]. If we are given assignments I will just do my assignments by solving it; I don't do it at school.

Fauziyya's interview showed that she experienced modest teacher-learner SRs but enacted robust MIs. Excerpts from the SRs interview showed that Fauziyya experienced positive psychosocial support and professional practices but questionable or detrimental expectation enactment. As against the common practice of many teachers, her mathematics teacher does not beat and he always encourages her. The teacher also avails the learners learning opportunities by teaching them in simple ways. But he used to lower his expectation for her, as he used to compromise evaluative standard for her and others. Excerpts from the MIs interview showed that she enacted positive attribute acquisition, community participation and expectation actualization. Fauziyya always studies and practices mathematics because she sees it as an important subject and participates with her teacher and peers in and outside the class. She also always concentrates in mathematics class and does her assignment on her own at home, as against the practice of many learners who always copy the solution from their friends in the school. Having experienced modest teacher-learner SRs but enacted robust MIs, Fauziyya becomes a case of learners who belong to crucial category. Such neither direct nor indirect relationships between her teacher-learner SRs experience and MIs enactment is another basis for claiming that SRs experienced do not determine MIs enacted and vice versa.

Like in the other two categories, Gardee and Brodie's (2021) framework fails to recognize crucial category and the rule that governs it. Notwithstanding this failure, there were arguments for and evidence of learners who were offered social identities of affiliation or partial affiliation but constructed their MIs in compliance rather than in affiliation or marginalization (e.g., Gardee & Brodie, 2021b). Contrary to the position of my study which argues for four kinds or diverse cases for crucial category, it seems that Gardee and Brodie's framework only accounts for two. The weakness may result from the conflation of social identities of affiliation and partial affiliation and less robust analytical framing employed. Such failure is not peculiar to their framework as it appears that there is no known framework that has theoretically and analytically provided such robust or comprehensive explanations for all the three possible categories. An effort to provide similar explanations and selection techniques ended up creating more categories because it specifically trivializes the roles of elaborated theory and analytical approach in engaging such academic debate (see Seawright & Gerring, 2008).

6.2.2.3 Findings at the real level

The findings at this level also use the direct narrations of the five selected learners to capture what shaped their actions as observed and reported at the empirical and actual levels. Each case in the three categories reported above was reconsidered to generate what governs their observed actions. It is important to note here that analysis of the second stage interviews of the learners gave clearer pictures of their reflections compared to the first stage. So, the evidence used in that direction are largely from the data obtained at the second stage of the study⁶⁶. While there is sufficient evidence to generate the findings for the cases in the typical category, available evidence for the cases in the interesting and crucial categories may need more cases. This is largely because typical category has all the three cases to draw from while only one case each was available for the other two.

⁶⁶ It was at this point I truly understood what mixed methodologies meant in my study. It means only quantitative approach at the first stage then only qualitative at the second; not both at a time and/or in a repeated manner.

Rationalist-strategist learners

Ammar is a boy who appears so confident when it comes to mathematics. His peers and teacher, including other teachers in the school like him and confirm his ‘smartness’ in mathematics and other subjects. In his interview, he showed how the SRs between him and his mathematics teacher was a good experience. He further substantiated part of what his teacher was doing to make him have robust identities with mathematics. For example, he emphasized posing of ‘challenging questions’ and making him see mathematics as ‘a simple subject to learn, like the easiest subject ever and among all the subjects’. When Ammar considered his teacher’s robust practices reasonable to develop his robust MIs, he was also enacting some identities that were schemes to re-enforce the way the teacher was enabling SRs for him and more especially and strategically for other learners. For example, Ammar sometimes asked questions he actually understood very well just for his teacher to re-explain for his peers ‘who do not understand to also understand’. Apart from politely correcting his mathematics teacher anytime he makes mistakes on the board, Ammar said he also disliked the way ‘he always wants us to follow exactly the steps he gives us to solve questions instead to explore possible alternative methods’. He also frowns at the unprofessional manner his mathematics teacher always scores learners in assignment tasks, by always overlooking those who just copy solutions from their friends. Ammar was doing all these to strengthen the SRs enabled for himself and others, but at the same time seems that he was strategically foregrounding and institutionalizing his mathematical ‘superiority’ or ‘smartness’ over other learners.

Zalifa is a girl who seems to be moderately confident about mathematics and his mathematics teachers considered her as a good learner. Her interview showed that despite experiencing the mixture of positive leaning opportunities and physical or emotional punishment, she still ‘sincerely likes the way her mathematics teacher teaches her, and she likes him to continue teaching her’. This is because, according to her, ‘he teaches her well and she used to understand his lesson’. So, Zalifa sincerely accepted the influence of modest SRs enabled for her by the teacher to emerge as a modest MIs learner. But the acceptance also comes with her enactment of some identities to re-enforce the SRs as well for everyone, including herself. Because she believed that they (i.e., the learners) ‘always understand his lessons contrary to his belief’, which always makes him to ‘beat’ them ‘mercilessly’. To achieve the re-enforcement goal, she sometimes puts up some bad behaviors or actions such as ‘not doing assigned classwork or going out to solve problem on the

board' in order to disobey or prove her stubbornness to the teacher. She further gave an insight into why she might be acting in such way in the following excerpt:

I don't enjoy the way he always mentions those who are good and not [in mathematics], because I think those who are not good will be feeling bad or somehow. Gaskiya, I don't like it. There was a day he was in our class teaching us. There was a girl who is not good at mathematics. She raised her hand and asked a question and he answered and explained to her very well. But still, she didn't understand. With annoyance, he said to her "Janka" (Animal!). That statement made me felt somehow, gaskiya.

She seems to be strategic in enacting identities that can strengthen the SRs enabled by the teacher for everybody in the class, beyond herself alone.

For Fa'iza, she is also a girl who is much less confident about mathematics and her teacher barely knows or recognizes her. A further interrogation of her interview showed that she only received general learning opportunities from her teacher and rarely had direct relationship with him and where she did, it was a negative experience. Despite the fragile SRs, Fa'iza still accepted her teacher as 'a good man' and 'who used to teach them very well for them to understand' and developed fragile MIs. She even surprisingly likes being flogged because 'that will enable her do better another time'. In return, she enacted fragile MIs in order for the SRs to be re-enforced only for her. For example, she boldly enacted her non-interest identities in mathematics and passive participation by not asking her teacher questions or engaging with him in or outside the class. Fa'iza was so explicit about what she wanted – she needed her mathematics teacher to teach in English language rather than in Hausa. In short, Fa'iza wants the language of instruction in mathematics class to be in English. This is because that is language she understands better and probably because it is the official language of instruction in Nigeria, forgetting that Hausa is the major language of the environment where she is schooling.

Ammar, Zalifa and Fa'iza as diverse cases in the typical category are considered as rationalists-strategists because they *accept* the influence of teacher-learner SRs to shape their MIs and as well enact MIs to or to want to *re-enforce* the SRs enable by the teacher in return. Each of the learners appears to be reasonable and strategic. Ammar accepts robust SRs and enacts robust MIs to strategically strengthen the SRs for his superiority to be well pronounced, Zalifa accepts modest SRs and enacts modest MIs to strategically strengthen equal and more sensitive SRs for everybody

in the class and Fa'iza accepts fragile SRs and enacts fragile MIs to strategically strengthen the SRs for herself alone.

Irrationalist-resistant learner

Halima is a girl who appears nervous about mathematics but well recognized and always noticed by her teacher in mathematics class. Halima, the noise maker, as straightforwardly nicknamed by her teacher, narrated that she has absolutely 'nothing' against the SRs experienced from her teacher. Her teacher always encourages, engages and offers her opportunities to participate in and learn mathematics. Even though she always makes noise, the teacher still holds high expectations for her. But despite such robust SRs experience, she still enacts fragile MIs in contrary. Fragile MIs such as 'consistently making noise', showing that she is 'tired and hungry' and 'coming late when mathematics class is ongoing' seems to be a scheme to, in return, transform the SRs being enabled for her by the teacher. She gave few insights and what she really wanted:

... the problem is that if we have mathematics double period in the morning, after the lesson everybody will be saying I don't even understand the lesson and they don't enjoy it as well. But if the period is a single period, everybody will enjoy the class at least for 35 minutes. But for double period, everybody will be tired and they will be eager to leave the class whether they understand or not because of the long teaching. The moment the first period expires, all what he is saying will not enter anybody's brain because of the tiredness.

Halima appears to be enacting these fragile MIs to get some things changed, including duration of mathematics lesson. She actually prefers a single period of 35 minutes. She also seems to want to have the scoring standard compromised for her when she said that '...and during exam here, if I try to get some scores, the teacher can be impressed and add some marks to my scores'. And what she likes most is the way her teacher always overlooks her class disturbance and will still nicely engage with her. Halima as one of the two diverse cases in the interesting category is considered as irrationalist-resistant because she *reject* the influence of teacher-learner SRs to shape her MIs and as well enact MIs to or to want to *transform* the SRs enable by the teacher in return. Specifically, Halima has no reason to reject the robust SRs enabled for her to enact fragile MIs, but at the same time still enact the MIs to transform the SRs. Her project to transform appears not

to be limited to the SRs enabled for her alone but also to the established rules and arrangements of the school. She appears to engage in this project for herself and other small number of learners like her.

Realist-conformist learner

Fauziyya is also a girl who appears very composed and conscious and her mathematics teacher, who also knows her in her former school, describes her as a very good learner. Her interview showed that she has nothing against her mathematics teacher personally but rather how he relates with others. She said, ‘I don’t like the situation whereby learners haven’t committed any serious offense, and he has started beating them’. She also quickly provided an excuse for the teacher by emphasizing that ‘they [the learners; e.g., noise makers] know that after he [the teacher] warns them and they don’t listen, he will beat them’. However, such modest SRs experience do not make her enact any MIs other than a robust MI. More specifically, Fauziyya seems to be personally comfortable with the SRs experienced and at the same time shows concerns for others, but she does not allow both to totally shape her MIs. She seems to prefer the SRs reproduced since the experience does not perfectly directly or indirectly influence her MIs. So, Fauziyya as one of the four diverse cases in the crucial category is considered as realist-conformist because she *co-exists* with the influence of teacher-learner SRs to shape her MIs and as well enacts MIs to or to want to *reproduce* the SRs enable by the teacher in return.

These findings have not been presented by any known study on the topic.

6.3 Evidence for the key claims/assumptions

In addition to the main CR assumptions, the quantitative results and qualitative findings reported above have also complementarily confirmed all the key philosophical, theoretical and methodological arguments I made earlier. The first and key ontological assumption of CR is that reality exists in three stratified forms whether we are aware of it or not. This ontological position, with social realist arguments regarding structures and agency, guided my theorization of the relationships between SRs and MIs in Chapter 4. That theory has now been confirmed with the

empirical data in this Chapter. It is confirmed that teacher-learner SRs experiences strongly influence and influenced by the MIs enacted by the learners, but neither of the two necessarily determines the other. This is because the relationships between them can be direct, indirect or neither of the two for different learners. Any of these relationships becomes observable in mathematics classrooms because learners, through their reflections, accept, co-exist with or reject the influence of SRs and as well engage in a project to re-enforce, reproduce or transform the SRs. This presentation brilliantly captures the three levels of reality regarding the relationships between SRs and MIs. These results are not only *generalizable*, *valid* but as well *practicable* among rural learners.

However, this confirmation would not have been possible if the relativist position of Bhaskar's CR and Archer's M/M approach on epistemology and explanatory framework respectively were not re-considered as a realist position. That re-consideration maintains that methodologies are integral part of philosophical principles and theoretical perspective developments, which further led to the development of a realist explanatory analytical approach that differs from Archer's M/M approach. The realist explanatory analytical framework, like any other realist methods, mixes quantitative (objective) and qualitative (subjective) analytical approaches to establish a reality, unlike the relativists that use only or multiples of the latter. The former, only or multiples of the quantitative approach, is used by the absolutists. In this study, there is evidence that only or multiple of quantitative or qualitative data/analysis would not have confirmed the main assumptions of the CR. Only the mixture of the approaches can robustly do that, as provided in this study.

Further, the connection between the philosophical, theoretical and methodological (here specifically, explanatory analytical framework) in establishing this reality, also confirms my earlier argument about their dependency on each. So, the key assumptions of R-CR in this study are confirmed. While some previous studies that build on positivism (or psychological perspective) and interpretivism (or exclusive sociological perspective) were not bothered about the stratified and explanatory nature of the knowledge they create, others who bothered could not 'see' the robustness of the knowledge despite being present in their data because they build on CR.

6.4 Conclusion

The Chapter has provided the results/findings for one of the questions raised in the study and by extension, used the findings to provide series of evidence for the philosophical, theoretical and analytical claims made and finally discussed them. It has substantially shown the three levels at which reality can be studied and used the relationships between teacher-learner SRs and learners' MIs to demonstrate that. The Chapter further showed that *generally* there is a strong relationships between teacher-learner SRs and learners' MIs for learners in rural schools in Nigeria. It also showed that the nature of relationships can be *validly* considered as direct, indirect and neither direct nor indirect to provide deeper insights to the question raised. This conclusion is different from the common conclusions in literature in that relationships are mostly considered as only direct or indirect. At a much deeper level, it *practically* generated and explained why the relationships and the learners acted the way they did regarding mathematics. Finally, the Chapter discussed these claims and findings with the existing literature and found out that while the current study corroborated their claims and findings, it further went ahead to substantiate them in more comprehensive way.

CHAPTER SEVEN

CONTRIBUTIONS, LIMITATIONS AND FUTURE STUDIES

7.1 Introduction

The quest to interrogate and formulate set of flexible, adequate and easy-to-implement teaching social practices that can enhance mathematics achievement of rural learners was the main and initial focus of this study. However, this initial focus was finally extended to a more fundamental study which I will engage with later in this Chapter. The focus was borne out of a persistence and well documented disparity in mathematics achievement of rural learners and their counterparts in the urban areas. And despite this situation, mathematics education researchers still give rural mathematics education little attention. To have a strong basis before the formulation of the practices, I started with the two dimensions of Gutierrez (2007, 2008): *dominated* dimension – access and achievement and *critical* dimension – identity and power. The two dimensions gave the study a clear direction in that if identity can be nuancedly conceptualized as a socially constructed phenomenon, it is likely not only to provide a window to improve the learners' mathematics achievement but to also present them the opportunities to become major players in the decision making regarding their achievement, classroom environment and the society at large. Such earlier clear direction of my study translated to the generation of four interrelated research questions, which I recall below:

1. What influence do social relationships between teacher and learners have on rural school learners' mathematical identities, and what influence do learners' identities have on the social relationships between teacher and learners?
2. What influence do rural school learners' mathematical identities have on their mathematics achievement, and what influence does learners' mathematics achievement have on their mathematical identities?
3. What influence do social relationships between teacher and learners have on rural school learners' mathematics achievement, and what influence does learners' achievement have on the social relationships between teacher and learners?
4. How effective is social relationships enhancement practices (SREP) in improving the relationships between: teacher-learner relationships and learners' mathematical identities;

learners' mathematical identities and achievement; and teacher-learner relationships and learners' mathematics achievement in rural schools?

As a normal research practice, the research questions prompted the study to gauge the existing and current findings and positions in the literature. It was at this point of review of literature that I discovered that the key issues about the three concepts of the study, SRs, MIs and MA, were not their relationships but what they actually mean (definition or theorization), lenses being used (sociological perspectives), or how they were being studied (research methodologies). With the exception of the relationships between MIs and MA where inconsistent findings were reported, others reported consistent findings for the relationships. But for the definitions, perspectives and methodologies, different views and positions were noticed. Giving this gap an attention became necessary not only to have theoretical or conceptual coherence but also to address or improve on the observed weakness of the existing explanations for the relationships.

Critical realism (CR), a philosophy that is methodologically foregrounded by social realism caught my attention because of its greater concerns for reality. CR maintains that social reality exists in three stratified forms – the empirical, the actual and the real (fully explained in Chapter 3). Social realism seeks to identify and explain social phenomena using agency and structures that make them (fully explained as well in Chapter 3). I considered CR and morphogenetic/morphostatic (M/M) analytical approach from social realist perspective productive for the gaps noticed. That consideration was indeed productive as the three concepts of the study were defined or theorized in a more nuanced way. These definitions offered me viable grounds to investigate the relationships between the concepts by employing Archer's M/M analytical approach. However, I found among other things, as also observed by other researchers (e.g., Jessop, 2005; Radulescu & Vessey, 2008; Turner, 1997; Vandenberghe, 2005), that some aspects of the M/M approach were not clear enough for operationalization and has problem of objectivation because of its extremely subjective nature. With no better alternative, I proceeded with the study. Building on the theoretical explanations of the relationships as best available to me, I then developed an interventional program, SREP, with the intention to improve the teaching practices of mathematics teachers. All what I have introduced thus far happened in Chapter 1 – 4.

In Chapter 5, I directly engaged with the data collected from the learners. One interesting thing that happened in this Chapter was that, because of my great interest to access and analyze my data

in the best possible ways, I was employing comprehensive methodologies (finally called realist methodologies) which were not fully ontologically and epistemologically foregrounded by CR and social realism. By comprehensive methodologies, I mean a series of methodologies that are both objectively and subjectively executed – e.g., mixed methods for accessing, collecting and analyzing the data. There was no issue with that until I started analyzing my data, preparing for my Chapter 6. Emerging findings from both statistical (or quantitative) and thematical (or qualitative) analyses were not adequately captured or explained by the Archer's M/M approach. For example, I found that analysis at the actual level indicated *three rules* which could be used to further categorize all the emerging cases at the empirical. As key as this finding was, it was not captured nor explained by the M/M approach and that presented an unexpected experience.

This experience led to serious confusion for me and I decided to revisit and rework the chosen research frameworks – CR and social realism. The decision brought in another focus for my study and eventually situated the study between four big studies: pure/fundamental, theoretical, empirical and interventional studies. This development was overwhelming! The fundamental study eventually took the center stage being the 'heart' of others, the interventional study was suspended even though SREP was implemented and the data collected. The theoretical study only attended to and empirically answered Research Question One. The results of the Research Question were used to provide evidence for the new positions I maintained about the CR assumptions and Archer's M/M analytical approach and by extension, social realism. Bringing this study to a close, I used the rest of this Chapter to present the study's contributions, limitations and already identified future studies.

7.2 Contributions

As a fundamental study, I contend that this study has improved the philosophical understanding of CR principles and their methodologies. As a theoretical study, it has contributed to the robust perspective on social realism. Such perspective has led to nuanced theorizations of SRs, MIs and MA separately and the relationships between SRs and MIs in a more robust way. The latter contribution was not left at the theoretical level but as well supported with empirical findings. These empirical findings were further used to provide evidence for the philosophical positions I maintained in the fundamental study. As an interventional study, despite its suspension, it has also

contributed to the development of a well-informed intervention program, SREP. These contributions would not have been possible if the studies were not grounded in the principles of realist methodology that I argued for but overlooked by the existing arguments. I elaborate on each of these contributions below.

7.2.1 Philosophical contributions

The main contribution of Bhaskar's CR is in realist ontology – the most abstract level of theorization – which holds that reality is out there in respect of our knowledge of it (Bhaskar, 1975). The main contribution of social realism, especially through Archer's M/M approach, is methodological because it fills the gap between realist ontology and realist social theory (Archer, 1995). The key shared argument between the two is that the nature of the reality to be studied determines the kind of the theory and methodology to be employed (Craib, 1997). This shared argument implicates both Bhaskar and Archer as relativists and that further suggests two forms of inconsistencies within their realist philosophy: 1) Internal inconsistency – realist ontological positions of both Bhaskar and Archer are sustained *only* by extremely subjective arguments or methods, overlooking other valid possibilities; 2) External inconsistency – realist philosophical principles and/or the methodologies that sustain them are either inconsistent or one of the two is ignored. Relativism, like absolutism, suggests different subjective or objective methodologies and are inconsistent with the realist philosophical principles and/or their methodologies. Other thinkers have also shown concerns about the idea of relativism in Bhaskar's CR, suggesting fallibilism instead (see Yucel, 2018). However, the issue of inconsistency seems unlikely to be resolved with the mere suggestion of fallibilism because the initial argument regarding realist methodologies has not been addressed.

To address the inconsistent positions of the giants of CR and social realism, two key arguments must be made. This is where my philosophical contributions come in. I made two basic but general philosophical arguments: methodologies are integral part of (but not conflated with) philosophical principles and philosophical principles emerge from and depend on one another – i.e., ontology produces epistemology and that necessarily leads to or calls for judgmentality. These arguments have further specific implications for realist philosophy. Methodologies (plural is intentional) are considered as a *series of methods or inferences* realist researchers employed to *identify, access and*

analyze reality at different domains. First, “a series of methods or inferences” here does not imply multiple of subjective or objective methods/inferences only but also includes a mixture of both. This is because realist researchers are aware that reality may not avail itself to subjective or objective means alone but also to both (e.g., analytical procedures employed in the study). This is where the idea of relativism is misconceived, in my opinion. Second, while “to identify, access and analyze” here holds for the empirical, the actual and the real domains at the epistemological and judgmental realms, ontological realm problematizes the identification of reality at all the three domains. The problem of identification at the ontological realm makes our knowledge of reality fallible, as making of objects (i.e., objectivation) is not only highly transcendental but also on itself fallible. These are the arguments that underpinned the internal consistencies of the re-enforced CR I proposed and empirically justified in the study.

As part of my specific contributions to the internal consistency of realist ontology are the emphasis on the series of methodologies, called realist methodologies. While emphasis of realist methodologies was made for accessing and analyzing social reality, I went further to provide explanations for analytical approach that holds realist position. Providing such analytical approach was necessary because Archer’s M/M analytical approach within social realist perspective holds some critical weaknesses. These include flat/separated analytical explanations and non-recognition of eclectic nature of human agency (details in Chapter 3). These weaknesses were addressed without overlooking the basic principles of realist methodologies, suggesting another contributions of my study to social realist perspective (details later). So, re-enforced CR (R-CR) that came out from re-examining Bhaskar’s CR and Archer’s M/M approach maintains an internal consistence for ontological realism unlike theirs. The same basic principle of realist methodologies was applied for epistemological and judgmental realism.

For emergence of philosophical principles from one another, unlike Bhaskar and Archer who seems to separate them (and even themselves) from one another, the principles are considered as a whole that emerge from and depend on one another through a set of three inferences or processes – i.e., objectivation, re-description and operationalization. Objectivation, re-description and operationalization are respectively subjective, objective-subjective and objective inferences employed by realist researchers to transform reality of interest from one philosophical status to another (i.e., external transformation) and by extension from one domain to another (i.e., internal

transformation). External transformation, like internal transformation, causes a non-flat (i.e., non-conflated and non-isolated) but dependent form of transformation. Maintaining such form of external transformation across the realist philosophical principles, I was able to argue for the external consistence of the R-CR that emerged in the study. By external consistence, I mean maintaining ontological realism, epistemological realism and judgmental realism, unlike Bhaskar's CR. This realist position proves to be comprehensive as it does not only capture the processes of positivism and interpretivism but as well goes beyond them. While realist philosophy accommodates all the three processes, positivist philosophy only accommodates operationalization and re-description and for interpretivist, only objectivation and re-description are accommodated. This claim is evidently supported with the provisions of generality, validity and practicality of the results and findings presented in this study. This evidence is taken up further later.

Bhaskar clearly declared his relativist position for epistemology despite assuming realist ontology. Since epistemology concerns knowledge about reality, this external inconsistency was addressed by arguing for epistemological realism. The realist epistemology position was not only built on and sustained by the three processes mentioned above but also that realist epistemology accommodates both transitive and intransitive knowledge. As explained above, the three processes hold a non-flat but dependent transformation that foregrounded the basis to situate transitive, transitive-intransitive and intransitive knowledge within the realist epistemology. The emphasis on the transitive-intransitive dimension of knowledge at the actual domain led to the improved realist social theory that was employed in the study. By improved social realist theory, I mean the theory that was inspired by the realist epistemology and informed by the realist social perspective at ontological realm. Like in the realist social perspective, my study also contributed to the improvement of the existing realist social theory (details later).

At the judgmental realm, realist researchers provide evidence for and use well informed criteria to make judgement about the knowledge they have discovered or created at the epistemological and ontological realms. While the former weakly appears to be possible from Bhaskar's CR, the latter was not explicitly provided. These are two other key philosophical contributions of my study. First, by building on the research design and mixed methodologies that were informed by the realist social theory and realist methodologies respectively, I was able to provide robust evidence for the key philosophical assumptions/claims I made in the study (details in Chapter 6). Second, building

on the three processes that offered a greater explanatory power for my philosophical position, I was able to also provide the judgmental criteria for theoretical studies – i.e., practicality, validity and generality and for interventional studies – i.e., simplicity, universality and inclusivity. In other words, the practicality, validity and generality of the objectivation, re-description and operationalization processes respectively employed were proposed for making judgment on competing or available alternative theories. And the simplicity, universality and inclusivity of the objectivation, re-description and operationalization processes respectively employed were proposed for making judgment on competing or available alternative interventional programs or practices. I summarize the differences between Bhaskar’s CR and R-CR assumptions with Table 7.1

Philosophy	Ontology	Epistemology	Judgmentality
CR	<i>Realism</i> (conflated ⁶⁷ and isolated)	<i>Relativism</i> (methodology about reality emphasized)	<i>Rationality</i> (emphasizes superior methodology or inference – e.g., any retroductive process or inference)
R-CR (proposed)	<i>Realism</i> (non-conflated and non-isolated)	<i>Realism</i> (both knowledge about reality and methodologies emphasized)	<i>Realism</i> (emphasizes superior evidence and methodologies – e.g., explanatory power criteria such as generality, validity and practicality)

Table 7.1: Bhaskar’s CR and proposed R-CR assumptions

All these philosophical contributions had great implications on and guided every aspect of my study, including the theory and methodologies employed. As R-CR philosophy inspired, these contributions are expected to guide any future study that has concerns for social reality. This is because the contributions have in a nuanced way further foregrounded realist sociological perspective on reality and by extension realist social theory in CR philosophy. I elaborate a bit more on the contributions of my study to the realist sociological perspective generally, and realist social theory in particular.

7.2.2 Theoretical contributions

As it is possible for any theoretical perspective to emerge from R-CR so also an improved realist sociological perspective and by extension realist social theoretical framework can emerge from it. Social realism as a sociological (theoretical) perspective on social reality concerns the

⁶⁷ Conflation of both the nature of reality and methodology that sustains it.

identification and explanation of the unobservable mechanisms of agency and structures that generate observed social phenomena. Inspired by the Bhaskar's CR, two basic forms of social realism are noticeable: agency-structure mechanisms are separable and remain in-dependable and agency-structure mechanisms are separable and still subsumable. Among the prominent proponents of the former is Archer (1995) and while for the latter is Giddens (1984). The latter has been criticized for being a conflationist (see Archer, 1995), while the former has heavily been criticized for having problems of objectivation and (see Knio, 2022) and I add the problem of validity (of re-description), generality (of operationalization) and conflation to it. This begs the question: what position does my study maintain to provide a better alternative?

I maintained a perspective that agency-structure mechanisms are separable and yet dependable. As a key argument of my realist philosophy that methodologies are integral, I did not only maintain this perspective, but I also went further to provide an explanatory analytical approach to practically operationalize the ontological position, unlike the existing perspectives. The analytical approach⁶⁸, like other realist methodologies, is a robust, non-flat and time incorporated approach that accommodates both positivist and interpretivist analytical approaches and the approach that eludes them. For clarifications, the dependability or contingency of agency and structure on each other in the approach does not allow for conflation (like in Giddens') neither does the time incorporation exclusively separate their interactions (like in Archer's). The time is actually incorporated as non-flat phases through which social reality manifests itself differently. These clarifications buttress the point that the analytical approach deviates from Giddens' structuration theory and Archer's M/M approach. This is possible because of new ontological perspectives I brought into the realist understanding of human agency and structures. For example, both structure and agency were not considered only as a social but also as an individual dimension. Furthermore, the nature of human agency was not only considered as rational and irrational but also as eclectic (i.e., neither rational nor irrational). Also, structures were not only considered as reproduceable and transformable but can also be strengthened (details in Chapter 3).

The new realist sociological perspective at the ontological realm became the realist social theoretical framework at the epistemological realm. The theoretical framework, unlike the sociological (theoretical) perspective – the most abstract level of social theorization, presents an

⁶⁸ I may likely call it Algebraic approach in the future.

opportunity to engage directly with the social phenomena of interest – in this study’s case, teacher-learner SRs, learners’ MIs and MA and their relationships. Separation of sociological perspective from theoretical framework, though they are still dependent or contingent, has a salient implications for the nature or type of the knowledge one discovers or creates (i.e., transitive, intransitive or both) and by extension, the contributions of the current study. The separation is salient because both sociological perspective and theoretical framework seem conflated or complicated by the earlier theorists⁶⁹. For example, Giddens’ theory of structuration seems to conflate sociological perspective with theoretical framework because there is no analytical approach developed for the former for onward transformation into analytical framework. In other words, Giddens separates social reality or phenomenon from its methodologies. Archer’s M/M approach seems to complicate sociological perspective and theoretical framework because the analytical approach for the former presents a difficult transformation process for onward use as an analytical framework. The background to this observation is that realist researchers and enthusiasts find it difficult to clearly see the line that differentiates her analytical approach from theoretical perspective. Archer (2020) tries to explain this complication when she declares that M/M approach is not a theory but *preferred* it being considered as a methodology. Unfortunately, the analytical approach is not explicit enough for smooth transformation to analytical framework to be considered as one. In other words, she complicates social reality or phenomenon with its methodologies.

Opportunities provided by the realist social theoretical framework to engage directly with the social phenomena of interest offered me another ground to develop, in a more nuanced way, four different theories: teacher-learner SRs, learners’ MIs and MA theoretical frameworks and a framework for relationships between teacher-learner SRs and learners’ MIs (details in Chapter 4). One uniqueness of each of the framework is that all the three possible levels of realist social theorizations within realist philosophy were accommodated, instead of one level that is available in the other philosophies. Another is that the frameworks consider both social and individual aspects of social theorization, instead of the former alone. Overall, one salient implication of my position is that intransitive (i.e., sociological perspective) and intransitive-transitive (i.e., theoretical framework) knowledge dimensions are explicitly separated despite being dependent on

⁶⁹ The titles of Giddens and Archer’s work are clear evidence.

each other. While this realist position explains what existing perspectives also explain, it goes further to explain what they conflate/complicate or what eludes them. I summarize the differences between my social realist positions and the existing ones in Table 7.2.

Theorist	Intransitive knowledge		Intransitive-transitive knowledge		Nature
	Sociological (theoretical) perspective	Analytical approach⁷⁰	Theoretical framework	Analytical framework	
Giddens	Separable but subsumable	Not available	Readily available	Not readily available	Conflated
Archer	Separable and in-dependable	Available but not transformable	Readily available	Difficult to be available	Complicated
Adeniji (proposed)	Separable yet dependable	Available and transformable	Readily available	Readily available	Comprehensive

Table 7.2: Existing and proposed social realist positions

7.2.3 Empirical contributions

In the above sections, I have presented how my study philosophically and theoretically contributed to intransitive and intransitive-transitive knowledge. In this section, I will elaborate on the empirical contributions to transitive knowledge. Two forms of empirical contributions were made: provision of comprehensive answer to Research Question One and presentations of series of evidence for the key philosophical and theoretical assumptions made in the study.

The question ‘What influence do social relationships between teacher and learners have on rural school learners’ mathematical identities, and what influence do learners’ identities have on the social relationships between teacher and learners?’ was answered and answered comprehensively. The comprehensive answer did not only provide answer to the nature and the generality of the relationships between the teacher-learner SRs and learners’ MIs peripherally or superficially but also went deep to validly refine and categorize them and further explained what practically generated such relationships (details in Chapter 6). Such a comprehensive answer or finding has not been provided by any known single study.

⁷⁰ This extends to the other methodologies – identification and access.

On the provision of evidence, the current study, unlike the earlier philosophical and theoretical contributions, extends its contributions beyond intransitive and intransitive-transitive knowledge production. This feat may be difficult for the earlier contributions because, as indicated in the above section, they appear to be less robust about intransitive-transitive aspect of knowledge production or discovery. Such weakness is likely to make their produced intransitive knowledge much vulnerable to different operationalizations by researchers and by extension, making transitive knowledge production or discovery a bit more difficult and less comprehensive or coherent. One of the reasons why these weaknesses may be inevitable is the strong connection between intransitive-transitive knowledge and the research design that will guide the process of transitive knowledge production.

The feat was achieved in this study because I was able to work with the research design informed by the realist social theoretical framework (i.e., intransitive-transitive knowledge) to gather and analyze the empirical data regarding the Research Question and answer it accordingly. All the specific key theoretical arguments and those which have direct implications on the philosophical assumptions such as:

- i. agency can be rational, irrational and eclectic;
- ii. both agency and structures possess both social and individual aspects;
- iii. strong relationships exist between teacher-learner SRs and learner MIs; and
- iv. methodologies, especially analytical approach and social reality or phenomena are integral but not conflated;
- v. social phenomena are separable and yet dependable or emerge from one another on a timescale;

were empirically supported respectively (details in Chapter 6). Making all these claims with evidence to support them may likely present the R-CR and social realism I argued for in this study the most robust and consistent philosophy and sociological perspective at the moment.

7.3 Limitations

The limitations of this study are not situated in the positions maintained, arguments made or findings presented to the best of my knowledge. Notwithstanding this ego, there are two types of

limitations that are broadly methodological – human and non-human situated. Non-humanly situated limitations are considered major while humanly situated limitations are considered minor limitations. This is because the former connects directly with the fundamental studies (i.e., philosophical and theoretical contributions) while the latter connects directly with the empirical study (i.e., evidential contribution). Non-humanly situated limitations were observed in both philosophical and theoretical studies or contributions. As a proponent of these philosophical and theoretical positions, I am yet to completely understand all the arguments I made in this study and their implications. Apart from the fact that I got to understand some assumptions and arguments when I was writing this Chapter, I still have one or two that are yet to be understood let alone their implications. For example, how “semi-intransitive” and “hyper-transitive” nicely fit in my study is still a big struggle for me. Also, at times I struggle to decide on which appropriate label or description to use for one or two arguments. For example, at one point I used “eclectic” to describe the nature of human agency that is not rational nor irrational, another time I used “realist”.

Similarly, I really wanted to make some of my arguments in this thesis to come out forcefully but I could not. This may be because of the template of the initial or original study which I could not completely rewrite to save my already lost time for the PhD. For example, I really wanted the process of internal and external transformations to come out forcefully than it was presented but I could not. It went beyond my capacity then. Also, I actually wanted my research design, as informed by my theoretical framework, to strongly come out but it was not. I considered these struggles as major limitations. Like for other ones, I only used this Chapter to make academic or scientific sense of it for readers to understand.

On the other hands, one minor limitation was recorded during the empirical study. I subjectively chose to work with both the quantitative and qualitative data collected at the first stage of the study only, while the quantitative data at the first and the qualitative data at the second stage of the study would have been better. The latter is in perfect compliance with the basic principle of realist methodologies I argued for in the study but that was not cleared to me then. It was when I was reporting for the last part of the qualitative findings that I discovered that more evidence was coming out more strongly in the data analysis of the second stage analysis than in the first, in conformity with the realist methodological position. Based on that clear understanding, I finally

reported the findings of that part from the qualitative data of the second stage. This limitation is considered minor because it did not contradict the theoretical findings earlier established.

7.4 Future studies

Two categories of future studies have already been identified: suspended theoretical studies captured in Research Questions 2 and 3, and interventional studies captured in Research Questions 4. Ordinarily the first two categories of the studies should have been completed in this study but suspended for the fundamental study that later emerged. Both categories were two studies of a broader project, in which the first category has two of its three Research Questions suspended and the whole Research Questions of the second category fully suspended. Research Question 2, ‘What influence do rural school learners’ mathematical identities have on their mathematics achievement, and what influence does learners’ mathematics achievement have on their mathematical identities?’ was raised to theoretically interrogate the relationships between learners’ MIs and MA in rural schools as a part of a larger study. Being a part of a larger study and broader project, it is important that I complete this study and the other part captured in Research Question 3.

Research Question 3, ‘What influence do social relationships between teacher and learners have on rural school learners’ mathematics achievement, and what influence does learners’ achievement have on the social relationships between teacher and learners?’, like the one stated above was raised to complete one larger study of the broader project. In other words, taking up the remaining Research Questions 2 and 3 in my future studies with already answered Research Questions 1 will allow me to wholistically understand the relationships between teacher-learner SRs, learners’ MIs and achievement. So, it already stated study that must be completed after my PhD studies.

The second category is an interventional study representing other parts of the broader project. Research Question 4 was raised to capture the interest of this part of the project. The study was set out to investigate ‘How effective is social relationships enhancement practices (SREP) in improving the relationships between: teacher-learner relationships and learners’ mathematical identities; learners’ mathematical identities and achievement; and teacher-learner relationships and learners’ mathematics achievement in rural schools?’. Like in the first category, this study was also suspended despite exposing the teachers to the SREP intervention, teachers implementing the

intervention and collecting appropriate data in the study. I would like to take this study up immediately after my PhD studies to evaluate the effectiveness of the developed intervention in improving learners' MIs and MA.

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Appendix A: Teachers' Social Practices Rating Scale

Basic Information:

Name: _____

Gender: _____

Lesson number: _____

Topic: _____

Start; _____; Stop; _____

Duration: _____

	Social Practices	Always	Rarely	Never
A.	PSYCHOSOCIAL SUPPORT:			
1.	Personal identity; <i>includes valuing, respecting, motivating, encouraging learners.</i>			
2.	Social identity; <i>enabling sense of belonging and full membership in the classroom.</i>			
B.	AGENCY SENSITIVITY:			
3.	Self-expectations; <i>recognizing and helping learners achieving their potentials.</i>			
4.	Others' expectations; <i>recognition and effective communication of teachers' and parents' high expectations.</i>			
C.	PRODUCTIVITY SUPPORT:			
5.	Personal engagement; <i>relating mathematics success and achievement with efforts and diligence.</i>			
6.	Social engagement; <i>awarding and recognizing learners' mathematical qualities and competence.</i>			
7.	Utility engagement; <i>emphasizing the sense in, worth, importance and usefulness of mathematics.</i>			
D.	EQUITY SENSITIVITY:			
8.	Equal opportunity; <i>distributing learning opportunities and classroom authorities equitably among learners.</i>			

Appendix B: Learners' Social Relationships Questionnaire

PART I

Instruction: Complete the following as applicable to you.

Gender: _____

Age: _____

Name: _____

PART II

This questionnaire is **voluntary AND confidential**.

You may skip any question that you do not wish to answer.

Carefully read each question and then **put a tick** in the appropriate box.

Please be as **TRUTHFUL** as you can.

Item	Statement	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree
1.	I get along well with my mathematics teacher.					
2.	My mathematics teacher is not interested in me.					
3.	I am motivated for success by my mathematics teacher					
4.	My mathematics teacher praises me for my hard work in schoolwork.					
5.	I often feel discouraged by my mathematics teacher					
6.	My mathematics teacher believes I can do mathematics					
7.	My mathematics teacher expects a lot from me in mathematics.					
8.	My mathematics teacher is helpful in shaping how I learn mathematics.					
9.	My mathematics teacher gives me less support in mathematics activities.					
10.	My mathematics teacher helps me feel confident in mathematics.					
11.	My mathematics teacher often says that only some people can do well in mathematics.					
12.	My mathematics teacher makes me believe all mathematical concepts are logical and sensible.					
13.	My mathematics teacher emphasizes effort for mathematics success.					
14.	My mathematics teacher discusses the usefulness of every mathematical topics.					
15.	My teacher rewards/commends me for demonstrating competence in mathematics.					
16.	My mathematics teacher treats all learners equally.					

17.	My mathematics teacher assigns more roles to certain learners in mathematics class .					
18.	My mathematics teacher gives me less attention than others in the class.					
19.	Only particular learners are called to answer questions in mathematics class.					
20.	My mathematics teacher favors some learners over others.					
21.	My mathematics teacher avoids questions from certain learners.					

Appendix C: Learners' Mathematical Identity Questionnaire

PART I

Instruction: Complete the following as applicable to you.

Gender: _____

Age: _____

Name: _____

PART II

This questionnaire is **voluntary AND confidential**.

You may skip any question that you do not wish to answer.

Carefully read each question and then **put a tick** in the appropriate box.

Please be as **TRUTHFUL** as you can.

Item	Statement	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree
1.	Learning mathematics means a lot to me					
2.	I see no need for learning mathematics					
3.	I feel bad if I do badly in mathematics tests					
4.	I do not give up on difficult mathematics problems					
5.	I am considering studying mathematics or related courses in future.					
6.	No situation should stop me from doing well in mathematics					
7.	I have lost hope in mathematics					
8.	I devote much time for mathematics					
9.	I do not want to do a job that needs mathematics					
10.	I will do better in mathematics this term					
11.	In mathematics, I feel important among my classmates					
12.	My contributions are valued in mathematics lessons.					
13.	I raise my hands to answer questions in mathematics lessons.					
14.	I am studying hard in mathematics to gain respect from my friends					
15.	I dislike being called to answer mathematics questions in the class					
16.	I am not afraid to ask my mathematics teacher questions.					
17.	I freely participate in mathematics classes					

18.	I often feel like I don't belong to my mathematics class.					
19.	I work with my classmates in mathematics exercises.					
20.	I request for help from my classmates if I encounter difficulties in mathematics.					
21.	I am as good in mathematics as my peers					
22.	Many of my classmates do better than me in mathematics					
23.	I grasp mathematical concepts easier than my peers					
24.	I find mathematics less interesting compared to my friends					
25.	I can outperform my peers in mathematics if I work hard.					
26.	My classmates know that I am good at mathematics					
27.	My parents have confidence in my mathematics performance					
28.	I have better future opportunities with mathematics					
29.	I am rarely called to answer questions during mathematics lessons					
30.	My mathematics teacher gives me little attention					

Appendix D: Learners' Productive Disposition Questionnaire

PART I

Instruction: Complete the following as applicable to you.

Gender: _____

Age: _____

Name: _____

PART II

*This questionnaire is **voluntary AND confidential**.*

You may skip any question that you do not wish to answer.

Carefully read each question and then **put a tick** in the appropriate box.

Please be as **TRUTHFUL** as you can.

Item	Statement	Strongly Agree	Agree	Not Sure	Disagree	Strongly Disagree
1.	I believe that mathematics is useful beyond the classroom.					
2.	Doing well in mathematics does not relate with future prospects.					
3.	Mathematics topics are connected and related to one another.					
4.	I am confident about my ability in mathematics.					
5.	Mathematics is only meant for learners with special talent.					
6.	Anyone could be successful in mathematics, if one studies hard.					
7.	Mathematics knowledge is less relevant after leaving school.					
8.	I learn mathematics for self-fulfillment.					
9.	I consider mathematics worthy of learning.					
10.	Putting more efforts leads to improved mathematics learning outcomes.					
11.	I participate in mathematics activities to succeed.					
12.	My mathematics teacher discourages my mathematics learning.					
13.	I feel that I belong in my mathematics classroom.					
14.	I am at ease to attempt mathematics questions in the class.					
15.	I enjoy working with my peers in mathematics.					
16.	My parents believe that I can do better in mathematics.					

17.	My mathematics teacher ignores me during class instruction.					
18.	I am not bothered if I miss mathematics class.					
19.	My classmates ask me mathematics questions they cannot solve.					
20.	I enjoy being praised when I do well at mathematics.					
21.	Scoring highly in mathematics gives me self-satisfaction.					
22.	Getting bad grades in mathematics makes me unhappy.					
23.	Solving mathematical problems is fun for me.					
24.	I often aim high in mathematics.					
25.	I rarely see the usefulness of mathematics.					
26.	I aim for good grades in mathematics to impress my teacher.					
27.	Doing well in mathematics shows my mental superiority.					
28.	I only try to pass mathematics exams because I must.					
29.	Passing mathematics exams is my main goal for learning mathematics.					
30.	My parent/guardian motivates me to study hard in mathematics.					

Appendix E: Learners' Mathematics Competence Test

PART I

Instruction: Complete the following as applicable to you.

Gender: _____ **Age:** _____ **Name:** _____

PART II

This is a non-evaluative assessment. Your performance in this test will not have any effect on your school grades in this subject.

Instructions:

1. Attempt all questions
2. Use the booklet provided to show all your workings/methods.
3. Circle ONLY ONE answer to each question.
4. Erase completely any answer you wish to change.
5. Time: 1hour 15Minutes

1. Find the Lowest Common Multiples of 24, 48, and 144.
(A) 6 (B) 12 (C) 20 (D) 24 (E) 30
2. Adamu plucked 24 oranges from three different trees. He can only keep six oranges in one Nylon, how many Nylon will he need?
(A) 24 (B) 18 (C) 12 (D) 8 (E) 4.
3. Simplify $7\frac{1}{3} - 2\frac{1}{2} - 1\frac{1}{4}$.
(A) $\frac{73}{12}$ (B) $\frac{53}{12}$ (C) $\frac{43}{12}$ (D) $\frac{33}{12}$ (E) $\frac{13}{12}$.
4. The place value of 4 in 637.41 is ____
(A) tenths (B) hundreds (C) units (D) hundredths (E) tens .
5. What is the sum of 111_2 and 110011_2 ?
(A) 111110_2 (B) 111010_2 (C) 111011_2 (D) 101101_2 (E) 111111_2 .
6. There are four Baskets labelled A, B, C, and D. If each Basket weighs $10\frac{1}{2}kg$, $10\frac{1}{4}kg$, $10\frac{1}{3}kg$, $10\frac{2}{3}kg$, respectively, which Basket is the heaviest and lightest?
(A) D and B (B) A and C (C) D and A (D) B and C (E) A and B.
7. If the cost of making 10mins call is #150.00, how much does it cost to make 3mins call? (A) #5.00 (B) #15.00 (C) #20.00 (D) #30.00 (E) #45.00

8. Bolu and Tunde attend the same school. If the distance between Bolu's and Tunde's houses and the school are 10km and 20km respectively, how fast will Bolu walk to reach her house the same time as Tunde?
- (A) As twice as Tunde (B) As thrice as Tunde (C) As half fast as Tunde
(D) As the same as Tunde (E) As double as Tunde
9. Find the sum of -23 and -9 .
- (A) -32 (B) -14 (C) 14 (D) 32 (E) 42 .
10. Arrange the following in descending magnitude order: $211_3, 110_4, 200_4$
- (A) $211_3, 200_4, 110_4$, (B) $200_4, 211_3, 110_4$, (C) $211_3, 110_4, 200_4$
(D) $200_4, 110_4, 211_3$ (E) $110_4, 200_4, 211_3$
11. What value of y satisfies this equation. $5y - 1 = 2y + 1$
- (A) $-1\frac{1}{3}$ (B) $-\frac{2}{3}$ (C) $\frac{2}{3}$ (D) 1 (E) $1\frac{1}{3}$.
12. Factorize $ab + 6a + 3b + 18$.
- (A) $(a - 3)(a + 6)$ (B) $(a + 3)(a - 6)$ (C) $(a + 3)(a + 6)$ (D) $(a - 3)(a - 6)$
(E) $(a + 6)(a - 6)$.
13. Given that $x + y = 3$ and $2x - y = 3$, find the values of x and y
- (A) $x = 2; y = 0$ (B) $x = 2; y = 2$ (C) $x = 0; y = 2$ (D) $x = -2; y = 1$
(E) $x = 2; y = 1$.
14. If 10 sweets cost 70 naira, how many sweets will 175 naira buy?
- (A) 25 (B) 27 (C) 30 (D) 35 (E) 46
15. The sum of ages of Abba and Bello is 30 years, and in five years Abba will be three times as old as Bello. What are their ages?
- (A) 10, 20 (B) 15, 15 (C) 7, 23 (D) 5, 25 (E) 4, 26
16. If the graph of $y = 7x - 2$ is drawn, which of the following is correct about the graph?
- (A) 7 is the gradient (B) -2 is the gradient (C) 7 is the intercept
(D) 2 is the intercept (E) -7 is the gradient.
17. Simplify $5x - 3y - 2x + 6y$
- (A) $2x + 3y$ (B) $3x - 2y$ (C) $3(x + y)$ (D) $3(x - y)$ (E) $3x + 2y$
18. If $a = 3$ and $b = -1$, evaluate $\frac{3a}{2} - 9b$.
- (A) $13\frac{1}{2}$ (B) $11\frac{1}{2}$ (C) $5\frac{1}{2}$ (D) $-5\frac{1}{2}$ (E) $-13\frac{1}{2}$

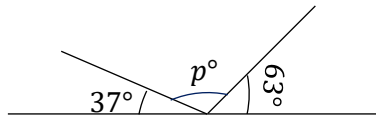
19. It takes a sportsman to get to the stadium in his town. He jogs 25km and travels another certain distance by train, and final travels as twice as far by bus. How far does he travel by train?

- (A) 10 (B) 15 (C) 20 (D) 25 (E) 30

20. A biro costs x naira and a cleaner costs y naira. How much in naira does it cost to buy three biros and two cleaners?

- (A) $3x + 2y$ (B) $2x + 3y$ (C) $6(x + y)$ (D) $3(x + y)$ (E) $5(x + y)$

21. What is the value of P in the figure below?



- (A) 80° (B) 90° (C) 60° (D) 50° (E) 40°

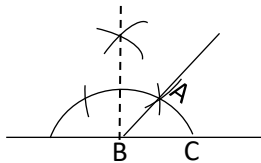
22. A 10m long ladder rests against a wall of 8m tall. What is the distance between the ladder and the wall on the ground?

- (A) 9m (B) 8m (C) 7m (D) 6m (E) 5m .

23. Given that the bearing of a point **B** from point **A** is 115° , what is the bearing of **A** from **B**.

- (A) 295° (B) 125° (C) 95° (D) 65° (E) 15°

24. Determine the value of angle $\hat{ABC}$ in the construction below.



- (A) 15° (B) 30° (C) 45° (D) 60° (E) 75° .

25. The diagram below represents the squared floor of a room. If the shaded portion is covered with carpet, what is the area of the carpet that needs be bought to cover the remaining portion of the floor?

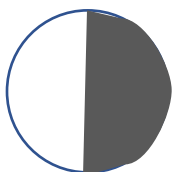


- (A) $36m^2$ (B) $24m^2$ (C) $20m^2$ (D) $12m^2$ (E) $10m^2$

26. If a Rod of 50m long is first used to construct a rectangular and later circular shaped objects, which of the following is **not** true about the two objects?

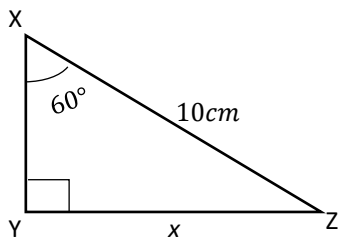
- (A) They have the same weight (B) They have the same perimeter (C) They have the same area (D) They are both plane objects (E) They have different areas

27. Calculate the shaded area in the figure below if $r = 7$ (take $\pi = \frac{22}{7}$).



- (A) $11cm^2$ (B) $22cm^2$ (C) $37cm^2$ (D) $77cm^2$ (E) $154cm^2$

28. Find the value of x to 1 d.p in the shape below .



- (A) $10.2cm$ (B) $9.3cm$ (C) $8.7cm$ (D) $9.8cm$ (E) $6.9cm$

29. In a competition of who has more amount of water in their Bowls. If each contestant has the same number of rounds but different shapes and sizes of containers to fetch water from the same Tap, who appears to win the competition?

- (A) One with the container of largest volume (B) One with the heaviest container
(C) One with the container of regular shape (D) One with the container of largest size (E) One with the container of largest total surface area

30. Assuming that the door of a classroom could be all wide opened and not a slide one, how wide can a student open the door to get angle 45° on the floor?

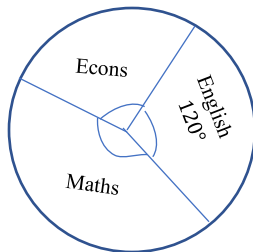
- (A) slightly (B) one-fourth (C) one-third (D) half (E) all wide

31. Use the frequency table below to find the mean score of the students in a Mathematics test.

Scores	7	6	5	4	3	2
No of students	2	4	5	3	2	2

- (A) 7 (B) 6 (C) 5 (D) 4 (E) 3.

32. The pie chart below represents the scores of Adamu in three subjects, if his total scores is 180. What did he score in English?



- (A) 40 (B) 47 (C) 60 (D) 73 (E) 82 .

33. The mode of a set of scores is the _____?

- (A) least score subtracted from the highest score (B) score with highest frequency
(C) score at the center when orderly arranged (D) sum of scores divided by number of scores
(E) difference between mean and median.

34. Statistical data are represented by all of the following except _____

- (A) Bar chart (B) Histogram (C) Pie chart (D) Pictogram
(E) Frequency.

35. The best place to collect data about prices of food product is _____

- (A) Schools (B) Farms (C) Markets (D) Ministries (E) Palaces.

36. Determine the median age of the following distribution: 10,20,11,12,18,16,17.

- (A) 14 (B) 16 (C) 17 (D) 18 (E) 20

37. If you are provided with different improved seeds of a grain, and you have a limited planting session for the year. What information will you request for you to make decision on which seed to plant?

- (A) methods of harvest (B) frequency of weeding (C) average germinating periods
(D) average cost of controlling pest (E) number of acres of land required

38. There are 20 fishes of different colors in a river. If five are red, two are black, two are pink, 10 are brown, and one is grey, which fish is most likely to be caught by fisherman?

- (A) red (B) black (C) pink (D) brown (E) grey

Use the information below to answer question 39 and 40.

Student age	12	13	14	15	16	17
No of Student	10	12	15	23	20	13

39. What is the modal age of the students from the above table?

- (A) 15 (B) 16 (C) 17 (D) 20 (E) 23

40. The range age is ____?

- (A) 3 (B) 4 (C) 5 (D) 6 (E) 7

Appendix F: Table of Test Items Specification

Mathematics Domain	Mathematical Proficiency strands*				Total
	Conceptual Understanding	Procedural Fluency	Strategic Competence	Adaptive reasoning	
Number and Numeration	1, 4 & 9	3 & 5	2 & 7	6, 8 & 10	10
Algebra	12 & 16	13, 17 & 18	15, 19 & 20	11 & 14	10
Geometry	23 & 24	21, 27 & 28	22 & 25	26, 29 & 30	10
Statistics	33, 34 & 35	36 & 40	31, 32 & 39	37 & 38	10
Total	10	10	10	10	40

*With the exception of productive disposition strand

Appendix G: Learners' Social Relationships Interview Schedule

Basic Information

Gender: _____ **Age:** _____ **Name:** _____

A. General questions to make learners feel comfortable talking to me:

- 1) What is your name?
- 2) How old are you?
- 3) What is your favorite subject in school?
- 4) What makes your favorite subject so special to you?

B. In and out classroom social relationships

What is your relationship with your mathematics teacher like?

Possible probing questions

- 1) How do you feel when you are sitting in your mathematics classroom? What makes you say that? Any examples?
- 2) How does your teacher make you feel about mathematics? What makes you say that? Any examples?
- 3) Share with me what you think about the way your mathematics teacher relates with you?

How do you know what your teacher expects from you in mathematics?

Possible probing questions

- 4) How does your teacher communicate what s/he wants from you in mathematics? Give examples please?
- 5) Can you please describe to me good/bad interactions that exist between you and your mathematics teacher? Any example to support your response?
- 6) Do you enjoy the way s/he interacts when teaching in the classroom? Give examples of a good/bad time.

What are your views about the way s/he relates with you in and outside the classroom?

Possible probing questions

- 7) What do you enjoy most about the way s/he relates with you during teaching? What about outside the classroom?
- 8) What do you enjoy most about how s/he relates with others during teaching? What about outside the classroom?
- 9) What do you enjoy least about the way s/he relates with you during teaching? What about outside the classroom?
- 10) What do you enjoy least about how s/he relates with others during teaching? What about outside the classroom?

C. Relationships with Mathematics

How does your mathematics teacher present mathematics to you?

Possible probing questions

- 1) How does your mathematics teacher make you like/dislike to learn mathematics? Any example(s) to support your responses?
- 2) How does s/he make you see mathematics? Why did you say that? Any example(s)?
- 3) How does s/he respond to your mathematical thinking and ideas? Why did you say that? Any example(s)?
- 4) How does your teacher make you feel when you don't do well in mathematics? What makes you say that? Any example(s)?
- 5) What does your teacher say about achieving highly in mathematics? Why do you think so? Any example(s) to support your point please?

Appendix H: Learners' Mathematical Identity Interview Schedule

Basic Information

Gender: _____ **Age:** _____ **Name:** _____

A. Identifying with mathematics

What is mathematics about for you?

Possible probing questions

- 1) What is learning mathematics like for you now? What makes you say that?
- 2) How do you see yourself as a mathematics learner? What makes you say that? Direct experience to support that?
- 3) What influences you to see yourself as that specific learner of mathematics? How does it influence you? Any example to support your point?
- 4) Do you enjoy learning mathematics? How do you know?

How do you learn mathematics?

Possible probing questions

- 5) What do you think is the best way to learn mathematics? How is it effective for you? Any example to support that?
- 6) How would a good mathematics lesson look like to you? What would make it good? Can you explain further please?
- 7) What do you think is expected of you in your mathematics classroom? How do you know? Any specific example(s) to support your point?

B. Agency

How capable are you to learn mathematics compared to other learners?

Possible probing questions

- 1) Who in your class is good at doing mathematics? How do you know? What makes them good?
- 2) Who in your class is not good at doing mathematics? How do you know? What makes them not good?
- 3) Are you good at doing mathematics? How do you know? What makes you good or not good? Any examples to support your response?

- 4) What do you expect of yourself regarding mathematics learning? Can you explain further please?

How do you think others see you as a mathematics learner?

Possible probing questions

- 1) How do your parents see you as a learner of mathematics? How do you know?
- 2) How does the way your parents see you as a learner of mathematics make you feel? Any further explanations on that?
- 3) How does your teacher see you as a learner of mathematics? How do you know? Any example to support that?
- 4) How does the way your teacher see you as a learner of mathematics make you feel? Can you explain further please?
- 5) How do your friends see you as a learner of mathematics? How do you know?
- 6) How does the way your friends see you as a mathematics learner make you feel? Can you explain further?
- 7) What do you think is expected of you for learning mathematics? Why? Can you explain further please?

Is mathematics necessary for your sense of being or future career?

Possible probing questions

- 1) How do think mathematics is important to your life? Why do think that way? Any example(s)?
- 2) What benefits are you currently enjoying as a mathematics learner? From who? How?
- 3) Would you like to be better at mathematics in the future? Why or why not? Explain further please.
- 4) Who would you like to see yourself becoming in the future (maybe in ten years' time)? What makes you say that?
- 5) How do you think mathematics can be helpful to you in the future? Why or why not? Can you give further explanations on that please?

Appendix I: Learners' Productive Disposition Interviews Schedule

Basic Information

Gender: _____ **Age:** _____ **Name:** _____

A. Disposition to mathematics (Personal)

Can you describe how you see mathematics and mathematics learning?

Possible probing questions

- 1) How do you see the usefulness of mathematics beyond mathematics classroom? Please, can you explain further?
- 2) How do you think efforts account for success in mathematics? Do you have any direct experience to support your response?
- 3) How do mathematical procedures and techniques make sense to you? Please, any example to support your response?
- 4) Do you believe you have the thinking or abilities it takes to learn and do mathematics? How do you know about that? Any example to support it?
- 5) Do you think mathematics is worthy of learning? Why do you think that way? Can you relate your response to any direct experience?

B. Disposition to mathematics (Social)

How can you describe how people make you see mathematics and mathematics learning?

Possible probing questions

- 1) How can you describe your mathematics teacher in the classroom? What makes you say that? Any scenario to support your response?
- 2) What do think your peers think about you in relation to mathematics achievement? Please, give more explanations on that. Can you give any example?
- 3) Do you think your parent believe in your mathematical ability? Why did you say that? Any further explanations and examples?
- 4) Do you think you have the same opportunity to learn mathematics like others? Any example to support your thinking?
- 5) Do you feel that you belong in the mathematics classroom? Can you explain further? Any direct experience to support that?

C. Mathematics performance

What does doing well mathematics mean to you?

Possible probing questions

- 1) How are you performing in mathematics tests? How does it make you feel? Any example(s)?
- 2) Do you think it is possible for you to perform better in mathematics? Why or why not? Are you doing anything about it? Can you tell me something more on this please?
- 3) What other means are you employing to help you do well in mathematics? Why those means? Are they effective? Why or why not?
- 4) What mathematical thinking do you believe is/are responsible for your current performance? Can you explain further please? Any direct examples to support your point?
- 5) How do you feel whenever you did or did not do well in mathematics? Why did you say that?
- 6) Can you describe how your teacher contributes to your current performance in mathematics? What makes you say that? Any examples to support your description?
- 7) Can you describe how your parents/guardians contribute to your current performance in mathematics? What makes you say that? Any examples to support your point?
- 8) Can you describe how your experience in the society influences your current performance in mathematics? What makes you say that? What are those experiences around you? Any examples to show the influence?

Appendix J: Teacher's Social Relationships Interviews Schedule

Basic Information

Gender: _____

Highest Degree: _____

Year of Teaching: _____

Date: _____

A. In and out classroom social practices

What characterizes your social relationships with learners in your class?

Possible probing questions

1. Can you please describe how you support your learners' social need in mathematics? Any example(s) to support your points?
2. Can you please describe how you support your learners' emotional needs in mathematics? Any example(s) to support your points?
3. How do you use your practices to support your learners' beliefs about themselves? Can you explain further? Any example(s) to support your response?
4. How do you communicate your expectations to your learners? Any direct example(s)?
5. Can you please describe how you mathematically engage your learners during teaching? Give me few example(s) please?
6. How is equity maintained among your learners in your classroom? Can you provide further explanations? Any example(s) to illustrate your submission please?

B. General views on learners' mathematical qualities and proficiency

What characterizes a successful mathematics learner in your class?

Possible probing questions

1. How do you think the way you relate with your learners promotes their mathematical qualities and proficiency? What makes you say that?
2. How do you recognize good mathematical qualities? Any example(s)?
3. How do you recognize bad mathematical qualities? Any example(s)?
4. How do you recognize mathematical proficiency? Any example(s)?

5. How do you recognize lack of mathematical proficiency? Any example(s)?

C. Views on specific learners' mathematical qualities and proficiency

How can you describe learner A (then B and C; one after the other) mathematical qualities?

Possible probing questions

1. Does s/he exhibit any mathematical qualities you usually notice? How do you know?
2. Does s/he feel composed and confident in your classroom? How do you know?
3. Does s/he see her/himself as capable as others in the class? How do you know?
4. Do you think s/he recognizes the opportunities that come with mathematics? How do you know?
5. Do you think her/his parent has influence on her/his mathematical qualities? How do you know?

How does Learner A (then B and C; one after the other) participate in the classroom with you, her/his peers and work?

Possible probing questions

1. Does s/he engage with you in the classroom? If so, how? If no, why? Any specific examples?
2. Does s/he ask or answer questions? Frequently or rarely?
3. Are his/her responses to questions in the classroom usually correct or incorrect?
4. Does s/he complete her/his classwork on time and correctly?
5. Does s/he submit her/his homework on time and usually correct?

Do you think that learner A (then B and C; one after the other) is good at doing mathematics? Why or why not.

Possible probing questions

1. What aspects or strands of mathematics is s/he good at? Why did you say so?
2. What aspects or strands of mathematics is s/he not good at? Why did you say so?
3. How do you know what s/he is good or not good at in mathematics? Any example?

4. How does s/he perform in mathematical assessments? Please, can you describe his/her usual feelings for such performances?
5. Do you think s/he recognizes that working hard improves performance in mathematics? Why do you think so? Any examples to support your points?

Do you think learner A (then B and C; one after the other) will pursue mathematics (or related courses) in his/her future?

Possible probing questions

1. Do you know what career choice s/he wishes to pursue? How did you know? When did you know about it?
2. Do you think s/he will pursue an academic degree? Why or why not? Any further explanations about these responses please?
3. Do you think that s/he could pursue an academic degree which is mathematically orientated? Why or why not?
4. Do you think s/he gets or does not get some kind of motivation from home to learn mathematics? Why or why not?
5. How can you describe his/her commitment to mathematical success? What examples can you give to support your response please?

Appendix K: Classification Criteria for Emergent Cases

Study stage	Concepts	Criteria	<i>P</i> ₂₅	<i>P</i> ₅₀	<i>P</i> ₇₅	Min	Max	N
FIRST	SRs	<i>Sores</i>	77	83	87	55	100	203
		<i>Range</i>	55-77	78-86	87-100			
		<i>Category</i>	fragile	modest	robust			
	MIs	<i>Sores</i>	116	125	133	87	147	190
		<i>Range</i>	87-116	117-132	133-147			
		<i>Category</i>	FRAGILE	MODEST	ROBUST			
	MA	<i>Sores</i>	128	135	144	112	165	192
		<i>Range</i>	112-128	129-143	144-165			
		<i>Category</i>	Low	Average	High			
SECOND (12 weeks after)	SRs	<i>Sores</i>	74	81	87	34	100	213
		<i>Range</i>	34-74	75-86	87-100			
		<i>Category</i>	fragile	modest	robust			
	MIs	<i>Sores</i>	116	124	131	84	150	202
		<i>Range</i>	84-116	117-130	131-150			
		<i>Category</i>	FRAGILE	MODEST	ROBUST			
	MA	<i>Sores</i>	130	136	143	99	166	199
		<i>Range</i>	99-130	131-142	143-166			
		<i>Category</i>	Low	Average	High			

Note: Small and capital letters are used to differentiate between SRs and MIs fragile and their corresponding modest and robust cases.

APPENDICES L AND M: APPROVAL LETTERS



Research Office

HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)
R14/49 Adeniji

CLEARANCE CERTIFICATE

PROTOCOL NUMBER: H21/07/01

PROJECT TITLE

Towards improving rural Learners' mathematics achievement:
The roles of social relationships and identity

INVESTIGATOR(S)

Mr K Adeniji

SCHOOL/DEPARTMENT

Wits School of Education/

DATE CONSIDERED

23 July 2021

DECISION OF THE COMMITTEE

Approved
Risk Level: Low

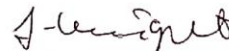
EXPIRY DATE

14 October 2024

DATE

15 October 2021

CHAIRPERSON


(Professor J Knight)

DECLARATION OF INVESTIGATOR(S)

To be completed in duplicate and **ONE COPY** returned to the Secretary at Room 10004, 10th Floor, Senate House, University. Unreported changes to the application may invalidate the clearance given by the HREC (Non-Medical)

I/We fully understand the conditions under which I am/we are authorized to carry out the abovementioned research and I/we guarantee to ensure compliance with these conditions. Should any departure to be contemplated from the research procedure as approved I/we undertake to submit an amendment of the protocol to the Committee. **I agree to completion of a regular progress report. For Minimal and Low studies, this is due annually on 31 December. For Medium and High Risk studies, this is due twice annually on 30 June and 31 December.**

Signature



Date

11, 11, 2021

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES



KATSINA STATE MINISTRY OF EDUCATION

STATE SECRETARIAT COMPLEX

No. 88 IBB Way, Kano Road, P.M.B. 2023, Katsina

Tel: 065-430581, 430725

E-mail: ministryofeducation@yahoo.com
katsinamoe@yahoo.com

KTS/MOE/DICTVE/257/VOL.I/57

21st October, 2021

Kehinde Adeniji

School of Education,
Mathematics Education,
University of the Witwatersrand,
Johannesburg,
South Africa.

RE: REQUEST TO CONDUCT RESEARCH IN FIVE RURAL SCHOOLS IN KATSINA

I am directed to inform you that your request to conduct research in five Rural Schools of Katsina State has been approved. Your study which is titled "*Towards improving rural learners' mathematics achievement: the role of social relationship and identity*" is timely and cannot be over emphasized, the research topic have potentials of suggesting alternative ways for effective teaching and learning of Mathematics and will contribute a lot in the Educational Sector.

2. Kindly accept the Honourable Commissioners best wishes on your studies.

Tanimu Nalado Kankia
DIRECTOR ICT & VOC. EDUC.
MINISTRY OF EDUC. KATSINA
DATE 22/10/21

Tanimu Nalado Kankia
Director ICT & Voc. Education
For Honourable Commissioner

"Home of Heritage and Hospitality"

Appendix N: Learners Information Sheet and Assent Form

Wits School of Education: Mathematics Education
27 Andrews Rd. Parktown. Private bag X3. Wits 2050.
South Africa. (011) 717-1000
October 2021



Dear Learner,

Re: Invitation to participate in a research

My name is Kehinde Adeniji. I am a lecturer in Federal University Dutsin-Ma in the Faculty of Education. I am studying for a degree of Doctor of Philosophy (PhD) in Education in the School of Education at the University of the Witwatersrand, Johannesburg. I am conducting research on how your teachers could be responsive to your mathematics learning and improve how you see, engage with and perform in mathematics. Overall goal of the study is for all learners to develop interest to choose mathematics related degrees and jobs in future. I titled the study as “*Towards improving rural learners’ mathematics achievement: the roles of social relationships and identity*”. The study focuses on senior secondary school one (SSS1) learners and their teachers.

I am inviting you to participate in this study, which is planned to start in October 2021 and ends in April 2022. Your participation in the study is voluntary. So, you may decide to participate and later withdraw at any time during the study, and there will be no penalty incurred nor will you be disadvantaged in any form. If you agree to be part of the study, you will be asked to complete three questionnaires and write a test at two or three different times within the period mentioned above. Each questionnaire will take about 20 minutes, and mathematics test will be 1 hour and 15 minutes. If you agree, I will also collect your SSS1 mathematics scores from the school and use it in the study. If you are selected for the second stage of the study, you will participate in three interviews on what you think about your teacher’s social interactions and how you see mathematics. The interviews will be audio-recorded, and each interview will last about 50 minutes. All these will take place in the school, and you can leave out any question you do not wish to answer. You will also be observed at two or three different occasions while learning mathematics in class, and these observations will be video-recorded. These recordings are important because they will allow me to revisit your responses and classroom activities in search for information that are crucial to the study. All the recordings and other information collected about and provided by you will be kept in password protected computers so that only my supervisor and I will have access to them, and they will all be destroyed five years after I complete my study.

As part of further re-assurance, all your information, activities and responses in the study will be treated confidentially, and your name and the name of your school will remain unknown to others outside your school. And you will also remain anonymous in all my research reports. I will share the findings of the study with you and your teachers, if requested. There are no expected risks associated with your participation in this study. And please note that no participant will be paid or given any gift for participating in the study.

If you want to participate in this research, please discuss with your parent or guardian first. After

that discussion, please complete the reply slip included in this letter and keep it for me for collection. Your reply will indicate to me whether you want to participate in this research or not. Your parent or guardian will be asked for their consent on behalf of you in a separate letter.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Please feel free to contact us if you require any further information by using the contact details listed below.

Your co-operation is greatly appreciated.

Yours sincerely,

Kehinde Adeniji, 1688574@students.wits.ac.za

Learners Assent Form

Title of the study: Towards improving rural learners' mathematics achievement: the roles of social relationships and identity

Researcher's name: Kehinde Adeniji

Dear Mr. Adeniji

I have read and understood your information sheet and invitation to participate in your research. After speaking to my parent/guardian, I have decided **to participate/ not to participate** in your research. My participation in the research activities is as follows:

To write a mathematics test at two or three different times

Choose: YES ☐ or NO ☐

To complete three questionnaires at two or three different times

Choose: YES ☐ or NO ☐

To allow my SSS1 mathematics marks to be used in the study.

Choose: YES ☐ or NO ☐

And if I am selected for the second stage of the study, I agree;

To be audiotaped during interviews at two or three different times

Choose: YES ☐ or NO ☐

To be videotaped during my classroom activities at two or three different times

Choose: YES ☐ or NO ☐

Informed Consent

I understand that:

- my name and information and the name of my school will be kept confidential and safe and will not be revealed outside my school and in any research report;
- I do not have to answer every question and can withdraw from the study at any time;
- all the data collected during this study will be kept in password protected computers and will be destroyed five years after the completion of the study.

Name of learner: _____

Signature of learner: _____

Date: _____

Appendix O: Parents/Guardians' Information Sheet and Consent Form

Wits School of Education: Mathematics Education
27 Andrews Rd. Parktown. Private bag X3. Wits 2050.
South Africa. (011) 717-1000
October 2021



Dear Sir/Madam,

RE: Request for your child to participate in a study

My name is Kehinde Adeniji. I am a lecturer in Federal University Dutsin-Ma in the Faculty of Education. I am studying for a degree of Doctor of Philosophy (PhD) in Education in the School of Education at the University of the Witwatersrand, Johannesburg. I am conducting research on the topic *"Towards improving rural learners' mathematics achievement: the roles of social relationships and identity"*. The purpose of this research, which focuses on senior secondary school one (SSS1) learners and teachers, is to improve learners' mathematics achievement and to change how they see and engage with mathematics.

I am requesting you to allow your child to take part in this study, which will take place between October 2021 and April 2022. If you allow your child to participate, s/he will complete three questionnaires and write a test at two or three different times within the period mentioned. Each questionnaire will take about 20 minutes, and mathematics test will be 1 hour and 15 minutes. I will also collect his/her SSS1 mathematics scores from school and use it in the study. If s/he is selected for the second stage of the study, and you agree, your child will participate in three interviews on what s/he thinks about her/his teacher's social interactions and mathematics. The interviews will be audio-recorded and each interview will last about 50 minutes. All these will take place in the school, and s/he can leave out any question s/he does not wish to answer. Your child will also be observed while learning mathematics in the class, and these observations will be video-recorded. These recordings are important because they will allow me to revisit their responses and classroom activities in search for information that are crucial to the study. All the recordings and other information collected about and provided by your child will be kept in password protected computers so that only my supervisor and I will have access to them, and they will all be destroyed five years after I complete my study.

The participation in this research is voluntary for all learners. So, your child, if allowed, may decide to withdraw at any time during the study, and there will be no penalty incurred nor will s/he be disadvantaged in any form. The study will have no effect on her/his school results. Your child's responses will be treated confidentially, and your child's name and the name of the school will be anonymous outside his/her school. And s/he will also remain anonymous in all my research reports. There are no foreseeable risks in participating in this study. And no participant will be paid or given any gift for participating in the study.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone

+27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Please, contact us if you require any further information by using the contact details listed below.

Please, complete and sign the consent slip attached and return it to me as soon as is convenient. Your co-operation is greatly appreciated.

Yours sincerely,

Kehinde Adeniji, 1688574@students.wits.ac.za

Parents'/Guardians' Consent Form

Title of the study: Towards improving rural learners' mathematics achievement: the roles of social relationships and identity

Researcher's name: Kehinde Adeniji

I, _____ the parent/guardian of _____, who is in SSS1, have understood the contents of the information sheet that was provided by Mr. Adeniji. I **give/ do not give** permission for my child to participate in the research. I also indicate my position for my child to participate in the following research activities:

To participate in writing one mathematics test at two or three different times

Choose: YES ☐ or NO ☐

To complete three questionnaires at two or three different times

Choose: YES ☐ or NO ☐

To collect and use his/her mathematics marks.

Choose: YES ☐ or NO ☐

And if s/he is selected for the second stage of the study;

To be audio-recorded during interviews at two or three different times.

Choose: YES ☐ or NO ☐

To be video-recorded during his/her classroom learning at two or three different times.

Choose: YES ☐ or NO ☐

Informed Consent

I understand that:

- my child's name and information and the name of his/her school will be kept confidential and safe and will not be revealed outside the school and in any research report;
- my child does not have to answer every question and can withdraw from the study at any time;
- all the data collected during this study will be kept in password protected computers and will be destroyed five years after the completion of the study.

Signature_____ Date_____

Appendix P: Information sheet and Consent Form – all other teachers

Wits School of Education: Mathematics Education
27 Andrews Rd. Parktown. Private bag X3. Wits 2050.
South Africa. (011) 717-1000
October 2021



Dear Sir/Madam,

Re: Invitation to participate in a research

My name is Kehinde Adeniji. I am a lecturer in Federal University Dutsin-Ma in the Faculty of Education. I am studying for a degree of Doctor of Philosophy (PhD) in Education in the School of Education at the University of the Witwatersrand, Johannesburg. I would like to invite you to take part in my research.

I am conducting research on the topic *“Towards improving rural learners’ mathematics achievement: the roles of social relationships and identity”*. The purpose of this research, which focuses on senior secondary school one (SSS1) learners and teachers, is to determine the influence of teacher-learner relationships on learners’ identification with mathematics and their mathematics achievement. The pilot study will start in October 2021 and the main study will take place between November 2021 and April 2022. If you agree to participate, you will volunteer for about 50 minutes interviews and to allow your classroom teaching to be observed. With your permission, you will be audiotaped and videotaped at two or three different times – two, 12 and 18 weeks into the study – when you volunteer to be interviewed about and observed during your teaching. These recordings are important because they will allow me to revisit your responses and classroom activities in search for information that are crucial to the study. Also, if you agree, you will be requested to document your ideas, feelings, experience and interpretations about your learners, and your own experience of the study in a fieldwork journal.

The participation in this research is voluntary. So, you may decide whether or not you would like to participate and if you decide to participate you may withdraw at any time during the study, and there will be no penalty incurred or have negative implication for you. Your responses and other activities during the study will be treated confidentially, and your name and the name of your school will be anonymous outside your school. And you will also remain anonymous in all my research reports. All research data will be kept in password protected computers so that only my supervisor and I will have access to them, and they will all be destroyed five years after I complete my study. There are no foreseeable risks in participating in the study. No participant will be paid for participating in the study. After the completion of the study, I will provide you, if you request it, a copy of the final research report.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Please, contact us if you require any further information by using the contact details listed below.

Please, complete and sign the consent slip attached and return it to me as soon as is convenient.

Your assistance is greatly appreciated.

Yours sincerely,

Kehinde Adeniji, 1688574@students.wits.ac.za

Teachers Consent Form

Title of the study: Towards improving rural learners' mathematics achievement: the roles of social relationships and identity

Researcher's name: Kehinde Adeniji

Dear Mr. Adeniji

I, _____, the mathematics teacher for SSS1 at _____, agree to have read and understood the contents of the information sheet that you provided. I have also read and understood the purpose of the research entitled: *Towards improving rural learners' mathematics achievement: the roles of social relationships and identity*. I therefore **agree/ do not agree** to participate in the research.

I also **give consent** to the following activities:

- To be audio-recorded during interviews at two or three different times.
Choose: YES ☐ or NO ☐
- To be video-recorded during my classroom teaching at two or three different times.
Choose: YES ☐ or NO ☐
- To document my reflections about the learners and the study in a fieldwork journal.
Choose: YES ☐ or NO ☐

Informed Consent

I understand that:

- my name and information and the name of my school will be kept confidential and safe and will not be revealed outside my school and in any research report;
- I can withdraw from the study at any time;
- I can ask not to be audiotaped and/or videotaped;
- all the data collected during this study will be kept in password protected computers and will be destroyed five years after the completion of the study.

Signature of teacher: _____

Date: _____

Appendix Q: Information Sheet and Consent Form – intervention teachers

Wits School of Education: Mathematics Education
27 Andrews Rd. Parktown. Private bag X3. Wits 2050.
South Africa. (011) 717-1000
October 2021



Dear Sir/Madam,

Re: Invitation to participate in a research

My name is Kehinde Adeniji. I am a lecturer in Federal University Dutsin-Ma in the Faculty of Education. I am studying for a degree of Doctor of Philosophy (PhD) in Education in the School of Education at the University of the Witwatersrand, Johannesburg. I would like to invite you to take part in my study.

I am conducting research on the topic “*Towards improving rural learners’ mathematics achievement: the roles of social relationships and identity*”. The purpose of this research, which focuses on senior secondary school one (SSS1) learners and teachers, is to determine the influence of teacher-learner relationships on the learners’ identification with mathematics and their mathematics achievement. I choose this topic because it has not been investigated in Nigeria. I plan to organize a four-week programme for you and some other teachers, to familiarize you with the learners’ mathematical beliefs about themselves and others and how they could be positively influenced individually and collectively. The programme will involve some videotaped activities such as discussions about and presentations on teaching and learning of mathematics and watching few selected videoclips of classroom teaching. These activities are planned to take place on weekends, for two hours per weekend, in the methodology laboratory of Federal University Dutsin-Ma.

The social relationships awareness activities will take place in November 2021 and the study will take place between November 2021 and April 2022. If you agree to participate, I will have with you audiotaped interviews of 50 minutes about your teaching and learners. I will also have videotaped observations of your classroom teaching. These interviews and observations will happen at three different times – two weeks into the study, and then six weeks and 12 weeks after the intervention programme – as suitable to you. These recordings are important because they will allow me to revisit your responses and classroom activities in search for information that are crucial to the study. Also, if you agree, you will be requested to document your ideas, feelings, experience and interpretations about your learners, and your own experience of the study in a fieldwork journal.

The participation in this research is voluntary. So, you may decide whether or not you would like to participate and if you decide to participate you may withdraw at any time during the study, and there will be no penalty incurred or have negative implication for you. Your contributions, presentations, responses, and other activities during the study will be treated confidentially, and

your name and the name of your school will be anonymous outside your school. And you will also remain anonymous in all my research reports.

All research data will be kept in password protected computers so that only my supervisor and I will have access to them, and they will all be destroyed five years after I complete my study. There are no foreseeable risks in participating in these activities or the study. No participant will be paid for participating in the study. After the completion of the study, I will provide you, if you request it, a copy of the final research report. And if the social practices indicate positive influence, I will be willing to scale up the programme for other mathematics teachers in your school, and you can be involved as a facilitator, should you wish to.

If you have any concerns or complaints regarding the ethical procedures of this study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za. Please let us know if you require any further information by using the contact details listed below.

Please, complete and sign the consent slip on the next page and return it to me as soon as is convenient.

Your assistance is greatly appreciated.

Yours sincerely,

Kehinde Adeniji, 1688574@students.wits.ac.za

Teachers Consent Form

Title of the study: Towards improving rural learners' mathematics achievement: the roles of social relationships and identity

Researcher's name: Kehinde Adeniji

Dear Mr. Adeniji

I, _____, the mathematics teacher for SSS1 at _____, agree to have read and understood the contents of the information sheet that you provided. I have also read and understood the purpose of the research entitled: *Towards improving rural learners' mathematics achievement: the roles of social relationships and identity*. I therefore **agree/ do not agree** to participate in the research.

I also **give consent** to the following activities:

- To be video-recorded during the social relationships awareness programme.
Choose: YES ☐ or NO ☐
- To be video recorded during my classroom teaching at three different times.
Choose: YES ☐ or NO ☐
- To be audio-recorded during interviews at three different times.
Choose: YES ☐ or NO ☐
- To document my reflections about the learners and the study in a fieldwork journal.
Choose: YES ☐ or NO ☐

Informed Consent

I understand that:

- my name and information and the name of my school will be kept confidential and safe and will not be revealed outside my school and in any research report;
- I can withdraw from the study at any time;
- I can ask not to be audiotaped and/or videotaped;
- all the data collected during this study will be kept in password protected computers and will be destroyed five years after the completion of the study.

Signature of teacher: _____

Date: _____