PRE-SERVICE TEACHER PROFESSIONAL LEARNING COMMUNITIES' INQUIRY INTO SOCIAL AND COGNITIVE DIMENSIONS OF LEARNERS' MATHEMATICAL ERRORS

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Declaration

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

Rolene Liebenberg, August 2019

Abstract

This study investigated pre-service teacher professional learning communities (PST-PLCs) as spaces to develop critical inquiry into practice. The purpose of pre-service teacher education programmes is to function as a support for pre-service teachers' professional learning. Professional learning communities are spaces in which new ways of thinking about practice can flourish and traditional practice be contested. Pre-service teachers in this study were encouraged to explore learners' mathematical errors from two entry points. The first entry point, defined as the social dimension, focused on inquiry into learners' mathematical learning experiences. The second entry point, defined as the cognitive dimension, prioritises learners' reasoning about their errors. The PST-PLCs were encouraged to inquire into these dimensions and the connections between these dimensions. This approach to inquiry into learners' mathematical errors takes on a non-deficit perspective of learners' mathematical errors and learners' mathematical learning experiences in class and beyond school.

The theoretical work of the thesis drew on the following: Cochran-Smith and Lytle's (1999) construct of 'inquiry as stance' which distinguishes between three different conceptions of teacher learning to locate sources of teacher knowledge; Zenios's (2011) construct of epistemic move in relation to deepening inquiry through knowledge and constructs within professional learning community theory and Wenger's (1998) concept of shared accountability in community of practice. A multiple-case study was conducted, consisting of five pre-service teacher professional learning communities. The inquiry within these groups was videotaped over the course of an academic year; this represented the main data source. Additional data included pre-service teachers' mathematical learning experiences as school learners, collected through individual interviews and group interviews at the end of the academic year, which focused on the pre-service teachers' reflections on their inquiry in the professional learning communities.

Findings from the research revealed that the stance towards the inquiry into a particular dimension of learning influenced the emergence of other dimensions and the potential for learning about the connections between the dimensions. Data analysis also highlighted how the stances differed in epistemic moves and their interactions that were generated as enablers of critical inquiry in relation to deepening the inquiry through knowledge. A significant contribution to knowledge that emerged from the findings is the importance of pre-service teachers' reflecting on their own learning experiences on a cognitive and social dimension of learning for making connections between these dimensions of learners' learning. The research makes a contribution to knowledge on collaborative inquiry practices and shared accountability towards these inquiry practices through introducing the concept of an inhibitor into the analysis of the PST-PLC inquiry.

The findings of the research provide insight into learning experiences for pre-service teachers in supporting them with developing collaborative inquiry practices for a critical stance on their teaching practice. The study suggests that PST-PLCs are a space for inquiry into new teaching practices and for developing a collaborative critical inquiry stance on practice.

Key words

Professional learning communities; pre-service teacher education; critical inquiry as stance; epistemic move; cognitive dimension of learning; social dimension of learning; learners' mathematical errors

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List of Abbreviations

B.Ed.	Bachelor of Education
CAPS	Curriculum and Assessment Policy Statement
CK	Content Knowledge
CPUT	Cape Peninsula University of Technology
DIPIP	Data-Informed Practice-Improvement Project
ISPFTED	Integrated Strategic Planning Framework for Teacher Education and Development
K-for-P	Knowledge-for-Practice
K-in-P	Knowledge-in-Practice
K-of-P	Knowledge-of-Practice
MSLC	Mathematics Service Learning Course
PCK	Pedagogical Content Knowledge
PLC	Professional Learning Community
PST	Pre-Service Teacher
PST-PLC	Pre-Service Teacher Professional Learning Community
SMK	Subject Matter Knowledge
WCD	Whole-Class Discussion

CHAPTER 1

Conceptualising the Study

1.1 Introduction and background to the study

I started my teaching career as a high school mathematics teacher at a school in a working-class suburb of Cape Town in 1984. In the second year of my teaching at this high school I recall a visit from a mathematics subject advisor. He was horrified that most of the learners who were in my Grade 11 mathematics class (Standard 9, as it was called then) had obtained a "G" symbol in mathematics (20-30%) in their June examination. He wanted to know what I was going to do about the situation. I responded that I was aware that the learners' prior knowledge partially explained their ongoing poor performance in mathematics which led to their disengagement from learning. I argued that the challenge was immense and that many of the learners disengaged for reasons other than struggling with mathematics. Willis's (1997) and Valero's (2002) research showed that low socio-economic status learners tend to disengage since they do not believe that investing in their learning would make much difference to their future lives.

The only solution I saw possible at the time was to support the Grade 11 learners who had performed so poorly in their June examination through providing additional after-school classes. My approach was to do an analysis of the learners' mathematical errors based on their written work and to determine where the errors were and to re-teach. I had not realised at the time that my approach was partially informed by a deficit perspective with regard to the learners' knowledge as I worked only from what they did not know and disregarded what they knew. I did not realise too that this approach of mine might have led to learners' silence and dependency on the teacher for the correct answers as their errors were always dealt with from a point of deficit. My interest in developing a deeper understanding of how to engage learners' mathematical errors from a non-deficit perspective was to determine whether learners would be encouraged to appreciate the challenges of mathematics when their mathematical errors were viewed as significant thinking on their part and as a valuing of their mathematical learning experiences. In the next section I review the literature on learners' mathematical errors that informed my practice as a teacher and teacher educator over the years and as I embarked on my study.

1.2 Review of research on learners' mathematical errors

Learners' mathematical errors are used extensively in mathematics education research to gauge the challenges that teachers and learners have with mathematics content (Radatz, 1979; Clements, 1980; Resnick, Nesher, Leonard, Magone, Omanson, & Peled, 1989; Smith, Di Sessa, & Roschelle, 1993; Tirosh, 2000; De Bock, Van Dooren, Janssens, & Verschaffel, 2002). Research studies on learners' errors that adopt a negative view consider learners' errors to be a result of the learners' confusion (Economou, 1995; Reason, 1977) and suggest teaching approaches such as avoiding dealing with the error and simply re-teaching. In earlier research studies where learners' mathematics errors are viewed positively (in the sense that they do not consider learners' errors as a result of ignorance), errors are related to the nature of the knowledge, independent of teaching approaches, or related to the teaching situation (Brousseau, 1984).

More recent evidence of common learner errors across many teaching situations, however, has led to the hypothesis that these errors are related to the nature of mathematical knowledge and how it is learnt, emphasising the epistemological and cognitive foundation of errors (Swan, 2000; Booth, Barbieri, Eyer, & Paré-Blagoev, 2014). The concept of a misconception originated within the epistemological frame of reasoning about learners' errors. Misconceptions are underlying conceptual frameworks that make sense to the learner and that produce errors (Nesher, 1987). From a constructivist learning theory perspective, misconceptions are produced when a learner is not able to transform his or her current knowledge to understand new knowledge (Nesher & Peled, 1986; Smith, Di Sessa, & Roschelle, 1993; Olivier, 1996). Resnick et al. (1989) argue that misconceptions are a normal part of learning mathematics and that learners' systematic errors should be used to diagnose learners' misconceptions. Both misconceptions and the errors they produce are often partially valid and thus can be built on (Brodie, 2013; 2014).

Smith et al. (1993) critique various approaches to engaging learners' mathematical errors which they argue are in conflict with constructivist theories that view errors as productive resources for learners' conceptual development. Many of these error engagement processes, they argue, work from a deficit perspective in which learners' errors are viewed as flawed knowledge that needs to be corrected. Smith et al. (1993) challenge the view that expert and novice conceptions can be treated as separable cognitive units which underpin the theory of 'removing' flawed conceptions from learners' cognitive structure. They draw on Clements's (1982) research and produce supporting evidence in their own research showing that learners can shift between correct and flawed strategies within the same problem-solving situation. Their research suggests that learners' cognitive structures can accommodate both expert conceptions and misconceptions.

Smith et al. (1993) argue that learners' evaluation of their conceptions is a powerful form of engagement with errors, providing learners the opportunity to reason about how their errors are related to limited constructions based on current knowledge. The shift from avoiding errors through correcting and re-teaching to instead placing an emphasis on engaging learners about the reasoning behind their errors is a move to support learners in developing a positive stance towards their errors; such an approach towards learners' mathematical errors has been referred to as an engaging approach (Brodie, 2013; 2014). The breaking of the learners' silence and of their dependency on teachers to correct their errors can result in the potentially enabling approach of engaging learners' reasoning about their errors.

An engaging approach to learners' errors may instil in learners the will to persevere when in difficulty, knowing that it is reasonable to make errors. Boaler's (2008) research provides evidence of how learners take up positions in which they learn to persevere through challenging mathematics content based on positive relations established through social interactions in the class. South African studies (Mhlolo & Schäfer, 2012; Khuzwayo & Bansilal, 2012), which focus specifically on the opportunities learners are afforded to give voice to their thinking within the teaching process in the classroom, provide evidence of the importance of social interactions in the classroom that facilitate learning. These studies show that learners' ways of thinking are often not explored or given sufficient recognition for the value learners' thinking adds to the learning environment. It can be inferred from these findings that the role of learners as co-producers of knowledge in the learning environment is obscured by a deficit approach. If learners' knowledge contribution to the learning is made visible, it may well minimise learner disengagement from learning.

Brodie's (2014) research in South Africa on engaging learners' mathematical errors provides evidence that to enhance a positive learner stance towards errors one can go beyond merely engaging learners' errors to embracing learners' errors. Brodie's (2014) concept of embracing learners' errors is developed from Borasi's (1987;1994) research on a form of engagement with learners' errors that has potential to generate inquiry into new mathematical directions and that may lead to new insights into mathematics.

The pedagogical approach that underpins an inquiry-focused view to learning involves a 'generative process of meaning-making that is personally constructed, informed by the context and the learning activity, and enhanced by social interactions' (Borasi, 1994, p. 167). Learning opportunities are generated through teaching which uses an inquiry focus on errors and demonstrates how errors can be a source of creativity and personal judgement. Brodie's (2014) research provides evidence that embracing learners' errors to explore mathematical ideas often involves lengthy conversations which are important for developing deep thinking.

Brodie's research (2014) suggests that the source of learners' mathematical thinking is not only the nature of mathematical content but that the classroom context also has an influence since social relations influence how learners' thinking is valued and how learners are recognised as part of the knowledge production. Her research also indicates that a focus on cognition and what happens in the minds of learners, in other words, understanding learners' misconceptions, while necessary is not sufficient for enhancing learners' mathematics learning. Having the opportunity to think through their learning experiences, having their reasoning made public in a safe and trusting learning environment and being recognised as a valued contributor within the learning are equally important for learners to learn (Brodie, 2014).

Learners' classroom learning experiences relates to their mathematical challenges. The important role that learners' classroom learning experiences play in how their mathematical errors come about suggested to me that a different starting point to engage learners about their mathematical errors is needed. A different entry point into engaging errors is having learners describe their mathematical learning experiences of the content in which their errors occur.

My sense of unease with restricting learners' mathematical errors to learners' classroom experiences was linked to my own experiences as a learner who often went beyond the classroom to seek support and engage with others about my mathematical challenges. In South Africa many learners from wealthier backgrounds who struggle with mathematics receive private tuition (Mogari, Coetzee & Maritz, 2009). This shows that learners do engage with others in their communities about their mathematical errors. Learners from South African poorer socio-economic backgrounds generally have fewer resources for private tuition but the assumption cannot be made that they do not seek support from family members or peers when they have challenges with their school mathematics (Mogari, Coetzee & Maritz, 2009).

I revisited the wealth of valuable research on engaging learners' mathematical errors in classrooms. I found that this research focused on classroom interactions in which learners' mathematical errors were engaged with as they unfolded in the teaching and learning process. Ingram, Pitt and Baldry's (2015) research provides evidence of the complex analytic and interpretive work required from teachers when engaging learners' mathematical errors. Ingram et al. (2015, p. 194) support Brodie's (2014) claim that teachers require support in recognising the reasoning behind learners' mathematical errors given the complex nature of the interpretive work: drawing on knowledge and expertise about learners' learning; sound understanding of the mathematics; the learning difficulties learners have with particular mathematics; the consequences of the interactions and the pedagogical goals involved.

A review of the literature revealed no research of teachers who, while engaging learners' mathematical errors, investigated, for example, how learners went about seeking support from others beyond the school when they made errors or when they were struggling with an aspect of mathematics. Likewise, there was no evidence of research on learners' mathematical errors where learners were asked to talk about how they experienced the learning of content in which their errors were located. In other words, the learners' reasoning behind their errors is the point of entry when engaging learners' mathematical errors, and not how learners experienced their learning of the mathematical content in which these errors occurred.

1.3 Motivation to explore learners' mathematical errors from a social dimension

In my study I was interested in understanding the engagement of learners' mathematical errors from both entry points. To distinguish between the two points of entry into learners' mathematical errors, I used the concepts of cognitive and social dimensions of learning. I considered a cognitive dimension of learning to be in the foreground when engaging learners' mathematical errors where the starting point of analysis is the understanding of the reasoning behind the error. I used the concept of a social dimension of learning to describe the starting point of analysis of learners' mathematical errors as eliciting learners' mathematical learning experiences in and beyond school related to the content in which their mathematical errors occurred.

The emerging body of literature in the South African context working with the concept of learners' 'funds of knowledge' as learners navigate between school and community contexts (Fataar & Du Plooy, 2012; Fataar, 2012) motivated me to engage with their mathematical errors from a social dimension of learning. Fataar's research (2012) provides evidence that we underestimate the potential of learners' learning experiences through their interaction with learning events in their everyday lives and hence do not give sufficient recognition to their learning experiences beyond the classroom.

The ‘funds of knowledge’ theory advocates the ethnographic study of learners’ households carried out by their teachers in order to analyse the accumulated knowledge developed within learner households. The identified ‘funds of knowledge’ are viewed as a means of connecting the curriculum with learners’ lives, affording learners more meaningful learning experiences and potential for enhancing learners’ learning in school (Delpit, 1988; Moll, Amanti, Neff, & Gonzales, 1992; González; Andrade, Civil, & Moll, 2001; Nasir & Hand, 2006; Nasir, Hand, & Taylor, 2008). This gives legitimacy to learners’ life experiences alongside their school experiences. Early ‘funds of knowledge’ research focusses primarily on the practices of adults and their social worlds but more recently this has shifted to a focus that includes children’s own social worlds as a source of their own ‘funds of knowledge’ (Moje, Chiechanowski, Kramer, Ellis, Carrillo, & Collazo, 2004; Subero, Vujansinović, & Esteban-Guitart, 2017). Moje et al. (2004) identified learners’ interest in television shows as an ‘everyday fund of knowledge’ which with ‘peer funds of knowledge’ works as potential sources for the teaching-learning process. The ways in which learners support each other in ‘doing’ school was viewed as one example of a ‘peer fund of knowledge’. The concept of learners’ ‘funds of identity’ (Esteban-Guitart & Moll, 2014) builds on the ‘funds of knowledge’ idea to include objects, activities and people who are part of the lived experiences of learners. Esteban-Guitart & Moll (2014) provide evidence of how identity artefacts connect school curriculum to learners’ prior experience, interests and knowledge. Esteban-Guitart & Moll (2014) argue that in a network, informational and digital age, a ‘knowledge-creation’ approach to learning as a ‘shared process of developing mediated artefacts such as knowledge practices or conceptual artefacts requires a deeper probing of learners’ ‘funds of identity’. The knowledge-creation metaphor of learning takes into consideration both social and epistemological perspectives in which the focus is not on the certainty of knowledge but rather how knowledge is used and developed (Paavola, Lipponen, & Hakkarainen, 2004). My hypothesis, which I locate within the knowledge-creation approach to learning, is, that learners’ mathematical learning experiences in class and beyond school can form part of the social dimension and thus provide a deeper understanding of knowledge about learners’ mathematical errors. This position, firstly, has the potential to extend learners’ mathematical knowledge and, secondly, can enhance teacher practice about engaging learners’ mathematical errors.

Unexplored in research that places value on learners' learning beyond school (Lange, 2009; Munns, Sawyer, & Cole, 2013; Fataar & Du Plooy, 2012) is the relationship between teachers' own learning experiences beyond school and their understanding of how these experiences connect with learning in school and how reflection on these experiences holds potential for deepening their practices in engaging learners' mathematical learning experiences. In my study which is situated in pre-service teacher education, the prior learning experiences of pre-service teachers in school and beyond school were explored to address this gap.

1.4 Situating the research in pre-service teacher education

I situated my research problem in the field of pre-service teacher education given the complex nature of the interpretive work needed to engage learners' mathematical errors and the ongoing support which teachers require to engage in this practice. The current four-year Bachelor of Education degree programme for pre-service teachers at South African universities includes a teacher practicum component in which pre-service teachers are placed in schools for up to seven weeks a year. There they engage in observation of teaching as well as in the teaching of lessons under the supervision of a mentor teacher.

The prescribed Curriculum and Assessment Policy Statement (CAPS) for South African government schools, which is the main pre-service teacher practicum context, demands strict adherence to pacing and content coverage. A teacher practicum within a CAPS context provides pre-service teachers with minimal space to engage with innovative classroom practices. These practices are perceived by some of the mentor teachers as potentially hampering the pace of content coverage. For example, engaging learners' mathematical errors through lengthy conversations may not be possible for pre-service teachers when doing their teacher practicum.

I chose to design a mathematics service learning course which allowed pre-service teachers to engage learners' mathematical errors from a social and cognitive dimension of learning in an after-school tutoring project. A detailed description of the mathematics service learning course is provided in Chapter 4. Pre-service teacher professional learning communities is one of the learning spaces in the mathematics service learning course provided to inquire into learners' mathematical errors with an explicit focus on drawing on learners' mathematical learning experiences in class and beyond school.

The Integrated Strategic Planning Framework for Teacher Education and Development of South Africa 2011-2025 (ISPFTED) recommends the participation of teachers in professional learning communities. This is based on strong theoretical arguments and empirical evidence that teaching practices and learner achievement can be improved when taking into consideration necessary contextual conditions (Louis & Marks, 1998; Grossman, Wineburg, & Woolworth, 2001; McLaughlin & Talbert, 2001; Vescio, Ross, & Adams, 2008; Earl & Temperley, 2009; Lomos, Hofman, & Bosker, 2011; Brodie, 2013). Literature on professional learning communities refers to the nature of engagement which aims for critical reflection and critique of current practice. Goodchild, Fugelstad and Jaworski (2013, p. 396) describe critical inquiry as 'disturbing practice on the inside, of challenging the status quo and of questioning accepted ways of being and doing'. Goodchild et al. (2013) argue that this form of inquiry starts as a mediating tool, for example, through the use of specific tasks to facilitate inquiry. Over time teachers in the community develop a habit of inquiry in their practice. Critical inquiry entails critical alignment with the practices of the community and is a necessary condition for growth of knowledge in the community (Goodchild et al., 2013).

Daniel, Auhl and Hastings (2013, p. 160) argue that through critical inquiry, a community develops a mutual understanding of practice and a shared conceptual language that enables the community to modify their practice and conceptual understanding. Nelson's (2009) research highlights teachers' difficulties in raising critical questions about their practice. In professional learning communities, where collaborative dialogue shifts from sharing practice to an inquiry stance, teachers raise critical questions that explore the relation between learners, learning and teaching. Nelson (2009) argues that teachers' learning is more clearly defined when there is an interplay among evidence of learners, learning and a change in teachers' understandings of content, pedagogy and assessment.

My reason for choosing to locate the research within pre-service teacher learning communities is to explore the potential of pre-service teacher learning communities to develop collaborative critical inquiry in preparation for participation in professional learning communities. In the next section I discuss the review of literature on pre-service teacher professional learning communities and identify a gap in the literature which my study addresses.

1.5 Review of research on pre-service teacher professional learning communities

Positioning pre-service teachers' professional experiences within professional learning communities promotes a shift from individualistic models of teacher practice to collaborative learning where pre-service teachers have opportunities to reflect, negotiate and share their developing knowledge of teaching practice (Manouchehri, 2002; Sim, 2006; Le Cornu & Ewing, 2008; Parks, 2008; Rigelman & Ruben, 2012; Santagata & Guarino, 2012; Youens, Smethem, & Sullivan, 2014; Hoaglund, Birkenfeld, & Box, 2014; Cavanagh & McMaster, 2015). In the research where pre-service teachers are interviewed about their learning experiences in professional learning communities, findings indicate that pre-service teachers develop greater confidence in sharing their experiences and that collaborative reflection supported their personal reflection (Sim, 2006; Hoaglund, Birkenfeld, & Box, 2014).

Manouchehri's (2002) research focusses on an analysis of pre-service teachers' collaborative reflection. Manouchehri (2002, p. 721) used a model that distinguishes between five levels of reflection to analyse the shifts in pre-service teachers' reflection: describing, explaining, theorising, confronting and restructuring. At the lowest level of reflection, descriptive information of an event is given. At the second level of reflection there is a move to explain connections between events. The third level of reflection, theorising, is when an explanation is supported by reasoning from data with references to research, previous experiences and coursework to justify an explanation. At the fourth level of reflection, 'confronting', there is a move to consider alternative ways of viewing an event. At the highest level of reflection, 'restructuring', the revisiting of an event is accompanied by a strategy to address a problem of practice. Shifts from the lowest to the highest levels of reflection were observed in Manouchehri's (2002) research.

In other research which focusses on collaborative pre-service teacher reflections (Santagata & Guarino, 2012; Youens, Smethem, & Sullivan, 2014; Cavanagh & McMaster, 2015) an improved ability to interpret and reflect on classroom practice was observed. In Santagata & Guarino's (2012) research pre-service teachers are given transcripts of an exchange between a teacher and learner. In these transcripts the solution of a mathematics problem is discussed and pre-service teachers are encouraged to focus on the teachers' questioning strategies. The transcript as artefact and the focus on the teachers' questions assisted the pre-service teachers to analyse the learners' thinking and to make inferences about learners' understanding. Santagata & Guarino (2012) found that few pre-service teachers were able to reason about teaching by considering the impact on learner learning.

Youens et al. (2014) encouraged pre-service teachers to review and reflect on video recordings of their classroom practice before meeting to discuss the video. Youens et al. (2014) found that the collaborative analysis of the video material assisted pre-service teachers to notice and deepen their awareness of the links between theory and practice.

Cavanagh & McMaster's (2015) research explored the breadth and depth of pre-service teachers' collaborative reflection on their observation of co-taught lessons. The breadth of reflection concerns the aspects of the lesson which are chosen as a focus and the depth of reflection refers to the complexity of the reflection. Cavanagh and McMaster (2015, p. 481) identified similar reflection levels to those of Manouchehri (2002) but restricted the levels to four:

1. Purely descriptive responses.
2. Descriptive responses containing an evaluative element.
3. Low-level reflective responses (makes a judgement and justifies it).
4. High-level reflective responses (explains possible causes and or imagines future responses).

Cavanagh & McMaster (2015) found that even though pre-service teachers attained high-level reflection they tended to focus on teacher's actions rather than learners' learning. Cavanagh & McMaster's (2015) research supports the findings of Santagata & Guarino's (2012) research that states that pre-service teachers need support when focusing on learners and learning. Both studies view pre-service teachers' collaborative reflection that involves explaining the reasons behind learners' thinking and making connections between teaching and learning as needing the most support.

The research reporting on pre-service teachers' collaborative learning is focused mainly on pre-service teachers' views of their learning through survey, interviews and written reflections (Sim, 2006; Rigelman & Ruben, 2012; Santagata & Guarino's, 2012; Hoaglund, Birkenfeld, & Box, 2014; Auhl & Daniels, 2014; Cavanagh & McMaster, 2015). Few research studies on pre-service teachers' collaborative learning in professional learning communities provide an analysis of pre-service teachers' talk and inquiry with a focus on what inhibits the inquiry and what has potential to extend the inquiry. Youens et al. (2014) research makes reference to how an analysis of the data from conversations between pre-service teachers, teacher mentor and university tutor generated different perspectives on a teaching situation with opportunities for pre-service teacher learning. Parks's (2008) analysis of pre-service teachers' conversations in the context of lesson study explored potential inhibitors of productive engagement such as pre-service teachers' prior learning experiences and assumptions which she argues are opportunities for challenge and learning.

My study thus addresses a gap in the research on pre-service teacher learning communities, namely the lack of focus on inquiry into learners' mathematical errors and learners' mathematical learning experiences with a view to exploring connections between these dimensions of learning. I build on the analysis of pre-service teachers' conversations, as within the Parks (2008) study, to explore the inhibitors and enablers of inquiry to gain deeper insight into how to support critical inquiry in pre-service teacher professional learning communities.

1.6 Defining the problem

Research on the thinking underlying learners' mathematical errors involves learners' written work, interviews with learners and observations/interviews on how teachers engage learners with their errors in classrooms or tutorials. Learners' written work has often generated insight into learners' errors that some researchers argue is not necessarily captured by the explanations which learners give for how their errors came about (MacGregor & Stacey, 1993).

Research which includes a focus on error-handling practices that teachers and learners enact in classrooms aims to gain greater insight into learners' errors as opportunities for learning. These studies address questions related to how teachers engage learners in reasoning about their errors, how learners use their errors as learning opportunities, and aspects of teachers' pedagogy that support learners' learning when dealing with errors (Brodie, 2005; Bray, 2011; Rach, Ufer, & Heinze, 2013; Gardee & Brodie, 2015).

There is little research that explores the role of social dimensions of learning which involve issues not limited to subject content in explaining learners' errors. There are, however, perspectives in mathematics education research that argue for understanding learners' participation in mathematics as a holistic, social and cultural experience, and not limited to cognitive matters related to content (Wyndham & Säljö, 1997; Valero, 2002; Nasir & Hand, 2006; Lange, 2009). These perspectives offer another way of interpreting learners' mathematical errors and situating them in the context of how learners experience mathematics learning in the classroom, including both teacher-learner relations as well as learners' mathematical learning experiences beyond the classroom.

The pre-service teacher professional learning communities in my study focus on both learners' written work and the reasons that learners give for their errors. Furthermore, in my study pre-service teacher professional learning communities inquire into learners' learning experiences of the mathematical ideas in which their errors are embedded, thus moving beyond classroom activities that address their errors. In addition, my study explores how pre-service teacher professional learning communities offer a space for critical inquiry into connections between learners' mathematical errors from both cognitive and social dimensions of learning.

1.7 Purpose of the study

The purpose of my study is to explore pre-service teacher professional learning communities' potential for critical inquiry into learners' mathematical learning experiences and learners' mathematical errors. The unit of analysis in my research is inquiry within pre-service teacher professional learning communities. In exploring the critical inquiry potential of the pre-service teacher professional learning communities, I analyse what enables and inhibits the inquiry into learners' mathematical learning experiences in classrooms and beyond school as well as learners' mathematical errors. The research question of my study is:

> What possibilities do pre-service teacher professional learning communities contribute to developing critical inquiry into the connections between learners' mathematical learning experiences and learners' mathematical errors?

This question was investigated through the following sub-questions:

1. How do pre-service teacher professional learning communities inquire into engaging learners' mathematical learning experiences?
2. How do pre-service teacher professional learning communities inquire into learners' mathematical errors?
3. What connections were made possible between learners' mathematical learning experiences and learners' mathematical errors through the pre-service teacher professional learning communities' inquiry?

1.8 Importance of the study

My research builds on knowledge about professional learning communities by focusing on pre-service teachers. Preparing pre-service teachers to engage in professional learning communities is important as professional learning communities are considered a space for ongoing learning about teaching practice. The research contributes to current work within the Data-Informed Practice-Improvement Project (DIPIP) on in-service teacher professional learning communities by adding a new site of practice, that of pre-service teacher education, and by adding a new focus, the inquiry into learners' mathematical learning experiences.

It may be considered a deficit perspective to work with learners' mathematical errors. My research eschews this deficit perspective and translates learners' mathematical errors into meaningful and generative experiences. My study encouraged pre-service teachers to engage mathematical errors from learners' perspectives through learners ascribing meaning to mathematical concepts and their mathematical learning experiences in classrooms and beyond school. This focus contributed to a deeper understanding of the connections between learners' mathematical errors and their mathematical learning experiences. The fine-grained analysis of the critical inquiry in pre-service teacher professional learning communities contributes to knowledge on how to support pre-service teachers' collaborative professional learning about engaging learners' mathematical errors from both cognitive and social dimensions of learning. My study explores the relationship between critical inquiry within pre-service teacher professional learning communities, practice and knowledge, and the shared accountability necessary to enhance this relationship.

My research therefore also contributes to knowledge about preparing mathematics pre-service teachers for engaging with the complexities of teaching and learning in challenging low socio-economic contexts through the focus on learners' mathematical learning experiences beyond the classroom.

1.9 Overview of the thesis

Chapter 1 introduces and explains the background and the focus of my study. I deal with the guiding questions, the purpose of the research and the situated nature of the research within a pre-service teacher education context.

Chapter 2 locates the study within situated learning. Wenger's (1998) social learning theory and more specifically his concept of community of practice provide a framework for aligning key elements of learning within professional learning communities. The overall conceptual framework developed for pre-service teacher professional learning communities includes Cochran-Smith and Lytle's (1999) concept of 'inquiry as stance' and Zenios's (2011) concept of epistemic moves as conceptual tools for analysing critical inquiry in professional learning communities.

Chapter 3 explores the literature concerning teachers' engagement with learners' mathematical errors and the analysis of learners' mathematical errors. I engage with literature that focuses on cognitive and social dimensions of learning in engaging learners' mathematical errors. I explore the literature on learners' mathematical learning experiences to extend the social dimension of learning in engaging learners' mathematical errors. I explore the literature on the practice of making connections between social and cognitive dimensions of learning to understand the challenges of making these connections.

Chapter 4 introduces the larger politico-historical institutional context in which my study has been conducted as a background to understanding the challenges of embracing new educational practices and of taking a critical perspective on knowledge. The mathematics service learning course in which the pre-service teacher professional learning communities of my study are situated is described to show how the pre-service teacher professional learning communities are connected to the other learning spaces of the mathematics service learning course.

Chapter 5 describes the research design and the methods of data collection. Strategies such as video recording of pre-service teacher professional learning communities; interviews; participant observations and reflection journals are used to gather data that are subjected to analysis and discussion in subsequent chapters. This chapter also discusses methodological issues that concern ethics and research rigour.

Chapter 6 presents the findings of the mathematical learning experiences of pre-service teachers as school learners within three contexts: in school classrooms; in their homes and in their broader communities. The analysis of the mathematical learning experiences of pre-service teachers as school learners provides insight into how these experiences influenced their inquiry into learners' mathematical learning experiences and learners' mathematical errors within the pre-service teacher professional learning communities.

Chapter 7 presents the findings, analysis and interpretation of the pre-service teacher professional learning communities' inquiry with a focus being on engaging learners' mathematical learning experiences. The identification and analysis of different stances towards the inquiry into learners' mathematical learning experiences demonstrate how the stances of inquiry influence practices of inquiry, teaching practices, knowledge to deepen inquiry and the learning opportunities provided by socio-cognitive connections.

Chapter 8 presents the findings, analysis and interpretation of the pre-service teacher professional learning communities' inquiry, and in this case the focus is on learners' mathematical errors. The identification and analysis of different stances towards the inquiry into learners' mathematical errors show how the inquiry stance influences practices of inquiry, teaching practices, knowledge to deepen inquiry and the social – cognitive connection learning opportunities.

Chapter 9 discusses the main findings, contributions and limitations of my study and suggest recommendations and avenues for further research.

CHAPTER 2

The Theoretical and Conceptual Perspectives

2.1 Introduction

In the previous chapter I presented the background to my study, its focus, the research questions and rationale. In this chapter I present the following key theoretical and conceptual perspectives which relate to the focus of the research: a situative perspective on learning (Wenger, 1998) is important in understanding the social aspects of inquiry and practice within the pre-service teacher professional learning communities; theory on learning within professional learning communities provides an understanding of how ‘representations of practice’ (Little, 2002) influence opportunities to learn; and Cochran-Smith and Lytle’s (1999) construct ‘inquiry as stance’ offers a means to understand the relationship between knowledge, practice and inquiry. Lastly, the concept of an epistemic move (Zenios, 2011) provides a means to explore the finer detail of the conversations that enable critical inquiry through knowledge in professional learning communities.

2.2 Teacher learning perspectives

Kelly (2006) distinguishes between two theoretical perspectives on teacher learning that are informed by different conceptions of, firstly, how learning takes place and, secondly, the nature of the relationship between learning and knowledge. The cognitive perspective on teacher learning places greater emphasis on how teachers’ knowledge is acquired through organising mental structures and advocates a view which separates the acquisition of knowledge, skills and understanding from teachers’ practice (Kelly, 2006, p. 506). Traditional views on cognitive perspectives on learning suggest that the mind is the primary site of human cognition and focus on the individual’s thinking (Gardner, 1985; Vera & Simon, 1993). Cognitive perspectives on learning view cognition as partly context-dependent and also partly context-independent (Anderson, Reder, & Simon, 1996). While the social as it relates to context is factored into cognitive perspective on learning, the focus is on how the social influences the thinking of the individual.

Within situative perspectives the locus of the learning process shifts from the mind of the individual learner to the participation framework in which learning takes place. From a situative perspective, knowing is viewed as deeply embedded in active participation within the social/material world (Lave, 1996; Barab, Cherkes-Julkowski, Swenson, Garret, & Shaw, 1999). Situative perspectives of learning view all learning as wholly tied to context and hence individual learning is social even when isolated from others (Greeno, 1997). Wenger's social learning theory (1998), which I discuss next, is a situative perspective which develops an understanding of learning as participation in the practices of social communities.

2.3 Wenger's social learning theory

Wenger's (1998) social theory of learning provides a framework for understanding the process of learning of participants that facilitates their becoming members of communities of practice. As with all theories of learning, assumptions about the nature of knowledge, knowing and knowers underpin what shapes and constitutes learning. Wenger's (1998, p. 4) theory is based on the following assumptions:

1. We are social beings and this is the central aspect in learning.
2. Knowledge is a matter of competence in valued enterprises.
3. Knowing is a matter of participating in the pursuit of such enterprises.
4. Meaning – our ability to experience the world and our engagement with it as meaningful – is ultimately what learning is to produce.

These assumptions indicate that the theory is based on the central premise that learning is social participation. Wenger (1998, p. 4) argues that 'participation shapes not only what we do, but also who we are and how we interpret what we do'. Social participation as a process of learning and knowing finds expression in four components in Wenger's (1998) social learning theory: meaning (learning as experience); practice (learning as doing); community (learning as belonging) and identity (learning as becoming). These components are interconnected and mutually defined. Meaning and practice are central to exploring the critical inquiry within the pre-service teacher professional learning communities in my study

Wenger (1998) introduced three dimensions of communities of practice to analyse how meaning is negotiated in practice: mutual engagement, joint enterprise and a shared repertoire. Mutual engagement involves the relations among people engaged in collective process of negotiating meaning about their practice. These relations involve mutual accountability to their practice which is a key element of the joint enterprise. Wenger (1998, p. 78) argues that the joint enterprise is 'not just a stated goal, but creates among participants, relations of mutual accountability that becomes an integral part of their practice.' Wenger (1998, p. 83) defines a shared repertoire as a resource for ongoing negotiation of meaning within a community of practice, such as: routines, words and ways of doing things, stories, gestures, genres, actions and concepts. Shared repertoires entail both the history of mutual engagement as well as being a resource to rethink practice.

Wenger's (1998, p. 5) concept of practice is defined as 'a way of talking about the shared historical and social resources, frameworks that can sustain mutual engagement in action'. The argument which he develops to connect meaning and practice is based on an exploration of the social production of meaning as an analytical construct to interpret practice. Wenger (1998, p. 5) defines meaning as 'a way of talking about our (changing) ability – individually and collectively – to experience our life and world as meaningful'. To develop the concept of meaning as an experience, three central arguments are developed (Wenger, 1998, p. 52):

1. Meaning as located in process called the 'negotiation of meaning'
2. The negotiation of meaning involves the interaction of two constituent processes: participation and reification
3. Participation and reification form a duality that is fundamental to human experience and thus to the nature of practice

The negotiation of meaning as process implies that there are multiple factors and perspectives involved that shape the process. Wenger's (1998) concept of participation is defined as a process of taking part both in the activities of a community and as well as in relationships with members of the community: participation thus involves both action and connection. Wenger (1998) identifies key characteristics of participation as constituent of negotiating meaning. Firstly, he argues that there is always the possibility of some form of mutual recognition as we engage. It is what we recognise that enables the mutual ability to negotiate meaning. In my study, pre-service teachers' past learning experiences become an important aspect of their mutual ability to negotiate meaning. For example, pre-service teachers' ability to recognise assumptions from past learning experiences and to challenge these assumptions contributes to how meaning is negotiated.

Secondly, Wenger argues that participation allows for all kinds of relations and that there is no need for agreement. The transformative potential, he further argues, is based on how participation in communities shapes individual experiences as well as those of the communities. Finally, Wenger (1998) argues that all participation is social, whether participants are interacting with each other or working individually. For example, in the process of writing-up their individual reflections on their professional learning experiences, pre-service teachers may not be interacting with their peers directly, however, the reflective writing process is also informed by their participation with peers and in communities and in learning. Wenger's (1998, p. 57) conceptualisation of participation was intended to capture the profound social character of our experiences of life, learning and meaning.

Wenger (1998, p. 58) uses the concept of reification to refer to 'the process of giving form to our experience by producing objects that congeal this experience into "thingness"'. Wenger (1998, p. 63) views participation and reification as complementary processes in that each has the capacity to address the limitations of the other. He explains that any community of practice will produce artefacts such as tools, procedures, stories and language, which reify some aspects of its practice. For Wenger (1998, p. 58), reification creates the points of focus around which negotiation of meaning is organised. Fernandez, Ritchie and Barker (2008, p. 197), for example, point out that a curriculum document as 'reified artefact of the partially shared meaning held by its writers and stakeholders' is a point of focus in a curriculum change process. Teachers will need to interpret the curriculum as a guide for practice. Different interpretations are likely to emerge based on teachers' personal experiences and perspectives on teaching and learning. Further participation in implementing the curriculum involves reification of the ideas underlying the curriculum, for example, lesson plans and assessment tasks. The incorporation of these reifications into actual practice in the classroom illustrates the potentially generative nature of reification, and its relationship with participation. -

Reifications are not always generative. Bourdieu's concept of habitus offers an explanation for the potential non-generative nature of reifications (Roberts, 2006). The concept of habitus is a way of thinking or disposition, which Bourdieu argued is unconsciously acquired and resistant to change. Roberts (2006, p. 629) argues that Bourdieu's concept of habitus suggests that communities of practice may be predisposed to the negotiation of particular types of meaning to the detriment or exclusion of others. When individuals become members of a community, their prior learning experiences and views of knowledge may influence their ability to take up new ways of thinking and doing. Mutch (2003, p. 27) argued that from an analytic perspective, what is important is the extent to which dispositions are challenged or altered by certain practices or to what extent dispositions remains resistant to change by influences in the community of practice. In my study it was important to understand the potential of professional learning communities to create opportunities to challenge assumptions on how to engage learners' mathematical errors to open up a new thinking about them.

2.4 Professional learning communities

In my study a professional learning community is viewed as a special kind (i.e. subset) of community of practice as it shares the key features of mutual engagement, joint enterprise and shared repertoires (Brodie & Borko, 2016). Professional learning communities have defining features that are different to other communities of practice. The first defining feature is the concept of a profession associated with key features as summarised by Brodie and Borko, (2016, p. 5):

- a knowledge base that guides professional decision-making and practices;
- methods for developing the knowledge base through research and practice;
- norms for acting in accordance with the knowledge base;
- a balance between autonomy and accountability for practitioners in relation to the knowledge base; and
- the expectation that actions can and will be justified in relation to the knowledge and norms of the profession.

Wenger's (1998) communities of practice is inclusive of many different facets of life. The learning across these communities of practice differs from each other. In professional learning communities, the second defining feature is the concept of professional learning which includes acquiring the competences and confidence to engage with the knowledge base of the profession (Brodie & Borko, 2016). The ability to use the knowledge base to enhance practices of the profession and thus to contribute to the knowledge base are fundamental to professional learning. Stoll and Louis (2007, p. 2-3) add that professional learning involves 'the sharing of practice and critically interrogating practice in an ongoing, reflective, collaborative, learning-orientated, growth-promoting way'.

Katz, Earl and Ben Jaafar, (2009) identify the importance of collaboration supported by productive relationships of trust for joint benefit as elements of professional learning communities. Katz et al. (2009) also found that successful professional learning communities must have a challenging focus, which is what the community collaborates to inquire into. The focus of inquiry in Brodie and Shalem (2011) and Brodie's (2014) research on professional learning communities is on learners' mathematical errors and provides evidence of how such a focus supports professional learning. The following are some of the professional learning opportunities afforded in Brodie's (2014, p. 236) professional learning communities:

> Sustained teacher enquiry into their assumptions about their own mathematical knowledge and pedagogical content knowledge.
>
> Enquiry into the valid reasoning behind learners' errors and how teachers began to understand the role of their own practices and knowledge in relation to learners' errors.
>
> Teachers collectively support each other to make sense of learners' errors, their current teaching practices and knowledge, and to think about possible new practices and new ways of thinking about mathematics.

Brodie and Shalem (2011, p. 422) argue that teacher learning in mathematics professional learning communities has to be seen as a practice that brings the practices of mathematics and mathematics teaching together. They emphasise two important criteria of practice: the criteria for what counts as appropriate acts within that practice and how the community that constitutes the practice defines what counts in the practice and holds people to account to the criteria of the practice (Brodie & Shalem, 2011, p. 421). They further argue that accountability to a practice therefore becomes a shared process in professional learning communities. Wenger (1998) frames mutual accountability in communities of practice as relations that arise through the negotiation of meaning around the joint enterprise (i.e. the focus of inquiry). Wenger (1998, p. 81) lists some of the following relations of accountability:

> … what matters and what does not, what is important and why it is important, what to do and not to do, what to pay attention to and what to ignore, what to talk about and what to be left unsaid, what to justify and what to take for granted, what to display and what to withhold, when actions and artefacts are good enough and when they need improvement or refinement.

The above list of relations of accountability suggests that the way in which a community develops mutual accountability will influence the commitment of individuals to their practice. Mutual accountability becomes a common practice which either produces relations that enhance the practice or simply monitor whether things are done in a particular way. A focus of my study is the ways in which mutual accountability develop in the pre-service teacher professional learning communities and influence critical inquiry into practice.

Little (2002) argues that if collaborative engagement in professional learning communities affords opportunities to learn in and from practice then it is important to identify the resources of teacher knowledge and classroom practice that are made visible and accessible through interactions. In her research, Little (2002, p. 931) identified certain analytic problems in locating teacher learning in emerging practices within community. The first problem relates to multiple interpretations that can be made about collaborative interaction which reflects the complexity of interpreting practice in community. The second problem is referred to as a 'boundaries of practice' problem and relates to what teachers select from their classroom practice and how teachers' classroom practice is represented. These two problems are interrelated.

Within professional learning communities, teacher learning will therefore be influenced by both the 'selective sampling' of teachers' classroom practice and the depictions of their classroom practice (Little, 2002). Little (2002, p. 934) argues that what is made visible from practice and the degree of 'completeness and specificity' with which the practice is depicted influence the opportunities for learning from practice.

The 'boundaries of practice' which Little (2002) raises has important implications for my study as the pre-service teachers made selections from their practice of engaging learners with their mathematical errors and mathematical learning experiences as well as the way in which their engagement with learners were represented in their professional learning communities. The flows of practice across 'activity settings' (Kazemi, 2008, p. 222) over time and social relations become a significant interrelated knowledge dynamic to consider. The ongoing selections from individual and collaborative practice influence both practice and learning. Practice within professional learning communities is influenced by what teachers bring to the community, the interactions within the community and the local and broader institutional context. Analysing collaborative teacher learning within professional learning communities through their interactions (i.e. the nature of the inquiry) therefore entails locating learning within emerging practices of inquiry in community.

Gau (2016) finds that the kinds of opportunities that support a culture of reflecting on established knowledge determine the ways in which emerging practices are shaped within a community of practice. Brown and Duguid's (1991) analysis of emergent innovative practice shows that cultures of innovation emerge through shared story-telling related to the identification of problems encountered in practice and where canonical accounts of work were hard to apply or to learn from. Brown and Duguid (1991) reference Orr (1990b, p. 178) who views story-telling as follows:

> The use of story-telling both to preserve knowledge and to consider it in subsequent diagnosis coincides with the narrative character of diagnosis.

This implies that through shared narratives and the negotiation of meaning and inquiry into practice, the potential for new knowledge creation is made possible. Brodie and Borko (2016, p. 6) view the broader community as a crucial reference point for teachers' professional learning in professional learning communities and argue that 'systematic reflection and inquiry into one's own practice is impossible without the reference to a broader base and to professional norms of judgement and accountability'. In my study, the pre-service teacher professional learning communities' recognition of when it is important for their learning to move beyond their local community becomes an aspect of developing a critical inquiry stance.

Horn and Kane (2015) argue that the complex asynchronous nature of teachers' practice accounts for the asynchronous nature of teacher conversations in professional learning communities. Unlike certain apprenticeship practices in Wenger's research, teachers' practice is predominantly individual, necessitating the reconstruction of practice when collaborating. Horn and Kane (2015) draw on Hutchins' (1995) construct of 'horizons of observation', which they describe as 'the aspects of practice that are available for individuals to see and experience in different contexts' (Horn & Kane, 2015, p. 377), to support their argument on the relationship between teachers' practice and teachers' conversations in community. They argue that the asynchronous nature of teachers' practice limits the 'horizon of observation' on one another's practice thereby raising the importance of attending to the conceptual resources constituted in teachers' talk that influence the opportunities to learn.

Horn and Kane (2015, p. 378) also make a distinction between representations and representational practices as further elaboration on the research of Little (2002) on the problems related to the depiction of practice:

> The opportunities to learn in any context rely in part on the representations to build collective understandings of practice. For instance, in research in the health sciences or physics, standardised representation such as statistical tables or time-graphs stabilise experimental work for collective examination, shaping a group's sense making. *Representational practices* refer to the ways in which people use these artefacts to interpret the world.

They argue that in teaching while there are commonplace representations (i.e. reifications and shared repertoires on reifications) such as textbooks and test results, there are not commonplace representational practices for using these (i.e. participation); the distinction here between representation and representational practice is located within teachers' individual practice. However, at the level of collective practice, the challenge around representation of practice is dependent on how teachers reconstruct their practice. Hence teacher representations of practice and the way they are engaged with in professional learning communities influence opportunities to learn.

This distinction between representations and representational practices is significant for my study in which representation of learners' mathematical work related to learners' errors and learners' mathematical experiences were a focus of the professional learning community. In the study, for example, both the actual written work of learners' mathematical errors (material artefact representations from teaching) and descriptions of learners' mathematical errors (talk-based representations from teaching) are representations of practice. How these representations are used to interpret and develop shared meaning about engaging learners' mathematical errors constitutes the representational practices. Representations and representational practices are conceptual resources for analysis of opportunities for learning within the pre-service professional learning communities. Wenger's (1998) conceptualisation of learning in practice as processes within each dimension of practice underpins the distinction made by Horn and Kane (2015). For example, in the shared repertoire dimension, Wenger (1998, p. 95) describe the processes as:

> *Developing their repertoires, styles and discourses:* renegotiating the meaning of various elements; producing or adopting tools, artefacts, representations; recording and recalling events; inventing new terms and redefining or abandoning old ones, telling and retelling stories; creating and breaking routines

Captured in this description is the ongoing engagement with repertoires of practice as learning: representations that when engaged with generate the representational practices. For Wenger (1998, p. 95-96) learning in practice is ongoing but he emphasises that this concept of learning does not mean that everything we do is learning:

> Significant learning affects these dimensions of practice … Such learning has to do with the development of our practice and our ability to negotiate meaning

Based on this I argue that sharing an experience from practice is necessary but not sufficient to be considered learning in practice. It is necessary for the community to adopt a critical inquiry stance of engaging with the shared experiences as in professional learning community research (Brodie, 2014) in which there was evidence of sustained inquiry into practice.

The way in which problems and activities are framed in professional learning communities is another conceptual resource in teachers' talk. Horn and Kane (2015) define framing as, 'how speakers define problems through activities and interaction'. Louie (2016) uses Greeno's (2009) two categories of framing to analyse opportunities for learning in professional learning communities: epistemological framing and positional framing. Greeno (2009, p. 271) defines epistemological framing as: 'the kinds of task that a participant or participants understand themselves to be engaged in, especially regarding the kind[s]) of knowledge that are relevant to and expected to be constructed in order to succeed in the task'. Positional framing is defined as the 'way in which an individual is entitled, expected or obligated to participate in the interaction in an activity system' (Greeno 2009, p. 272). These framings are adapted by Louie (2016) to analyse the teaching practice focus of the community and the collaborative interactions of community, as illustrated in Table 2.1:

Table 2.1: Framing in professional learning communities

Framing	Teaching practice focus	Teacher collaborative interactions
Epistemological (i.e. cognitive & social)	The nature of mathematical activity and the knowledge and skills that are relevant	The nature of collegial collaboration and the knowledge and skills that are relevant
Positional (i.e. social)	The ways in which learners are expected to participate in classrooms	The ways in which teachers are expected to participate in collegial conversations

The teachers in Louie's (2016) research framed professional knowledge as personal and framed collaboration as sharing which limited opportunities to learn: these teachers did not interrogate their practice or make connections between the key components of classroom mathematical learning. Louie (2016) argues that if collaboration framed as sharing limits opportunities to learn, then alternative framings of collaboration must be considered. I would argue that such alternative framings must be made explicit: collaboration, for example, framed as critical inquiry has potential to enhance opportunities to learn. My study explores the challenge of framing collaboration as critical inquiry within pre-service professional learning communities.

The next section explains how 'inquiry as stance' is conceptualised by Cochran-Smith and Lytle (1999) as a particular epistemological and positional framing of knowledge for teacher learning and its relation to practice within inquiry communities.

2.5 'Inquiry as stance' in professional learning communities

Cochran-Smith and Lytle (1999) draw a distinction among three different conceptions of teacher learning to locate the sources of teacher knowledge as knowledge-for-practice, knowledge-in-practice and knowledge-of-practice.

The first conception of teacher learning as knowledge-for-practice is based on the assumption that the knowledge teachers need to teach well is produced by university-based researchers and scholars in various disciplines. The importance of content knowledge of a discipline and the associated teaching strategies are considered important for becoming a highly skilled teacher. A knowledge-for-practice conception promotes the codification and dissemination of a distinct body of knowledge that provides teachers with a 'unique fund of knowledge' (Cochran-Smith and Lytle, 1999, p. 8) to apply in practice. Teachers are not the generators of knowledge but users of knowledge within this conception. However, Cochran-Smith and Lytle (1999) expand the knowledge-for-practice concept to include teachers' best practices as well as knowledge from research and theory.

The second concept of knowledge-in-practice challenges the idea that a formal body of knowledge is sufficient to address the problems of practice, and views knowledge as also produced in practice (i.e. referred to as local knowledge). Cochran-Smith and Lytle (1999, p. 262) explain the assumption on which the second conception is based as follows:

> ...teaching is, to a great extent, an uncertain and spontaneous craft situated and constructed in response to the particularities of everyday life in schools and classrooms. The knowledge teachers use to teach well under these conditions is manifested in their actions and in the decisions and judgements they make in an ongoing way. This knowledge is acquired through experience and through considered and deliberative reflection about or inquiry into practice.

In contrast to the first conception, teachers are not dependent on formal external theories to generate knowledge to enhance their practice. Within the knowledge-in-practice conception of teacher learning Cochran-Smith and Lytle (1999, p. 267) argue that teachers' learning is related to their understanding of their own actions which includes, for example, their own assumptions and their own reasoning and decisions. Fenstermacher and Richardson's (1993, p. 104) concept of 'practical arguments' as 'accounts of action that serve to explain or justify' what a teacher did in practice is important in supporting teachers to think more deeply about their actions. Practical arguments are conceptualised as two processes: elicitation, which involves the process of framing practical arguments through describing the reasons for actions, and reconstruction, which involves the process of assessing the elicited practical arguments (Fenstermacher & Richardson, 1993). The purpose of reconstruction in practical argumentation is to encourage teachers to rethink their practice and be open to changing it.

The third conception of teacher learning as knowledge-of-practice differs fundamentally from the first two conceptions in that it does not distinguish between formal knowledge and practical knowledge or create a hierarchical positioning of the expert-novice teacher relationship. Cochran-Smith and Lytle (1999) do not view their third conception of teacher learning as a synthesis of the first two conceptions because they reject the dualisms and make explicit a transformed and expanded view of practice. The third conception brings a critical perspective of practice into view as teachers' interpretative frameworks for practice foreground their practice in relation to the broader intellectual, social and cultural contexts of teaching (Cochran-Smith & Lytle, 1999, p. 273)

> The basis of this knowledge-practice conception is that teachers across the professional life span play a central and critical role in generating knowledge of practice by making their classrooms and schools sites of inquiry, connecting their work in schools to larger issues, and taking a critical perspective on the theory and research of others.

Teacher learning by constructing knowledge-of-practice involves systematic collection, analysis and interpretation of data sources within collaborative communities which draw on both local and global knowledge. Systematic inquiry, for example, entails the systematic observation and documentation of learners' work and sense-making. Collaborative inquiry within communities is the main context for teacher learning. The shared engagement necessitates taking into account multiple contexts and perspectives to generate new learning and to use this new learning to create change. The ways in which teachers connect their own experiences to broader contextual issues therefore become a central component of teacher learning within this third conception.

In an inquiry stance, teacher learning is associated more with uncertainty, posing of questions, identifying problems of practice and the recognition that inquiry both stems from and generates questions (Cochran-Smith & Lytle, 1999, p. 294). They highlight key aspects of the nature of discourse in an inquiry stance as calling into question assumptions about common practices and generating data that makes it possible to consider alternatives. Resources that are drawn on include 'reports and accounts of teacher researchers, action researchers and other practitioners, as well as from the extensive theoretical and research literatures related to teaching, learning and schooling' (Cochran-Smith & Lytle, 1999, p. 295). Teachers who take an inquiry stance within communities work to generate local knowledge, envision and theorize their practice, and interrogate the theory and research of others' (Cochran-Smith & Lytle, 1999, p. 289).

An inquiry stance to teacher learning has direct implication for my study in which pre-service teachers were supported and encouraged to inquire into engaging learners' mathematical errors through strategies that explored the connection between the local context (pre-service teachers' teaching context) and broader context of school and community (learners' experiences of learning in their classrooms and out-of-school learning experiences) as well as the pre-service teachers' own experiences of teaching and learning, and their knowledge of research and theory. Cochran-Smith and Lytle (1999, p. 292) argue that there are certain questions implicit in an 'inquiry as stance' framework that provide the lenses to interpret practice. For example, some of the kinds of questions that can be posed are: 'Who am I as teacher? What am I assuming about a learner? What sense are learners making of what happens in their classroom? Teachers' dialogue is therefore an important dimension in researching the development of teacher learning through an 'inquiry as stance' lens as teachers work towards epistemological framings of knowledge and engagement that move beyond the personal and local to exploring connections across knowledge fields and contexts.

2.6 Epistemic moves in inquiry

In this section I draw on research that focusses on a fine-grained analysis of the conversational patterns or moves that deepen inquiry through knowledge. Wenger's social learning theory provides an overarching lens for understanding learning as participation in social practice but does not provide the conceptual tools for the subtle ways in which knowledge is generated in professional learning communities.

I draw on Zenios and Holmes (2010) and Zenios's (2011) analytical approach for researching collaborative knowledge creation in online communities in which Ohlsson's (1995) concept of an epistemic activity is used. Zenios (2011, p. 266) defines epistemic activities as *epistemic moves* which 'help advance inquiry in reflective, indexical and contextual ways.'

Ohlsson (1995) provides a taxonomy of epistemic activities which guides the identification of these activities, i.e. describing; explaining; predicting; arguing; critiquing or evaluating; explicating and defining, in talk concerned with developing understanding. These epistemic activities were defined so as to show how they facilitate deepening of knowledge.

A complete list of Ohlsson's (1995) epistemic activities and their definitions is given in Table 2.2 and their potential relevance to my study is indicated.

Table 2.2: Ohlsson's epistemic activities (1995, p. 51)

Epistemic activities	Definitions as given by Ohlsson	Potential relevance to this study
Describing	Provides an accurate description of an object or event	In this study the object could be a mathematical error /challenge of the learner and the event is how the PST describes how the mathematical challenge or error was dealt with
Explaining	Able to help others understand why an event has occurred By drawing on inferences (Zenios)	In this study the PST could be explaining why the learner struggles – this could be related to the nature of the content or pedagogical approach
Predicting	To provide a reason for what may happen	In this study this could relate to the PST making predictions about learners' challenges based on their engagement with the learners
Arguing	To state reasons for or against a particular position	In this study the PST could, for example, argue for a particular teaching approach based on experiences from teaching practice or their own learning experiences
Critiquing/Evaluating	To point out the positive and negatives of an event or cultural product	In this study the PST could point out positive and negative approaches to teaching – often as specified by the curriculum
Explicating	To support the development of a clearer understanding of an event or idea/concept….through making connections with prior knowledge (Zenios)	In this study this could be related to a mathematical topic
Defining	To propose a usage for a term	In this study this could be related to a mathematical concept

Zenios (2011) argues that epistemic moves are not limited to the set identified by Ohlsson (1995). The following were some of the moves identified from her research: inferring; reasoning, comparing; negotiating and making connections to previously held knowledge. Some of her epistemic moves were in fact elaborations of Ohlsson's moves as illustrated in Table 2.2. Zenios (2011, p. 265) provides contextual examples of her moves and how they enable inquiry, for example:

> ‘Making inferences about each other’s written contributions enabled close examinations of concepts across settings and cultures as participants came from different disciplinary backgrounds and varied professional contexts and cultural contexts’

Working analytically with the concept of an epistemic move enables an understanding of which specific epistemic moves are linked to enhancing inquiry through knowledge-for-practice, knowledge-in-practice and knowledge-of-practice into the specific focus of learning communities. In my study, I employ a similar analytical approach to Zenios (2011), through identifying episodes of conversation that focus either on inquiry into learners’ mathematical learning experiences or inquiry into learners’ mathematical errors to identify the epistemic moves which enabled inquiry into teaching practices related to the inquiry, i.e. the social and cognitive dimensions of inquiry. It is important to understand reasons for why epistemic moves generated in conversation do not always lead to deepening inquiry. Hence, I identify critical moments in the PST-PLC conversations which inhibited the deepening of inquiry into learners’ mathematical learning experiences, learners’ mathematical errors and connections between learners’ mathematical learning experiences and learners’ mathematical errors.

2.7 Chapter summary

The central question of my study explores the potential of professional learning communities to support pre-service teachers’ critical inquiry into the connections between learners’ mathematical errors and learners’ experiences of learning mathematics, in the classroom and beyond the classroom. I have argued that Wenger’s social learning theory is important for my study as it provides a conceptual framework for understanding how meaning is negotiated in communities of practice. Wenger (1998) conceptualises the negotiation of meaning within communities of practice as the interaction between complementary processes, participation and reification. The relationship between participation and reification influences the nature of practice.

I have also shown how the three dimensions of practice, shared repertoires, mutual engagement and joint enterprise in Wenger’s (1998) community of practice align with the key features of professional learning communities, representations of practice, relations of mutual accountability and a focus of inquiry. I have argued that professional learning communities are a subset of communities of practice that have distinctive features, particularly the concept of professional learning. Within professional learning communities the systematic inquiry into practice is always with reference to a knowledge base and broader communities to enhance practice. Professional learning communities are conceptualised as spaces for developing a critical inquiry stance on practice and raise the importance of mutual or shared accountability to the development of a critical inquiry stance.

I have drawn on Cochran-Smith and Lytle's (1999) three different conceptualisations of teacher learning to explore the relationship between knowledge, practice and inquiry. Cochran-Smith and Lytle's (1999) three different conceptualisations of teacher learning provide a description of a critical inquiry stance on practice within professional learning communities as one in which theory *(knowledge-for-practice*) and practice (*knowledge-in-practice*) are mutually informing in the generation of new knowledge (*knowledge-of-practice*) through critical reflection. I have also argued that in order to explore the relationship between knowledge, practice and inquiry, a fine-grained analysis of epistemic moves (Zenios, 2011) within shared 'representations of practice' (Little, 2002; Horn & Kane, 2015) that can enable deeper inquiry through knowledge as well an analysis of inhibitors of inquiry become important in identifying practices of shared accountability for professional learning.

The next chapter explores the literature pertinent to this study. Included is an exploration of literature on critical inquiry into learners' mathematical errors; learners' mathematical learning experiences and the connections between these which is a focus of inquiry within the pre-service teacher professional learning communities.

CHAPTER 3

Literature Exploration: Errors and Experiences

3.1 Introduction

A focus of the pre-service teacher professional learning communities in my study is the inquiry into learners' mathematical errors. In this chapter I begin by exploring the literature on teachers' engagement with learners' mathematical errors. In Chapter One, I argued that the potential for exploring learners' mathematical errors from a social dimension of learning requires deeper insight into how to engage learners' mathematical learning experiences. In this chapter, I further explore the literature on learners' mathematical learning experiences to support my argument for a social dimension focus on learners' mathematical errors. My study aims to explore the social and cognitive dimensions of learning when pre-service teacher professional learning communities inquire into learners' mathematical errors and aims to understand the opportunities opened up through connecting these dimensions. I therefore review research on the knowledge and practices that support teachers to make connections between cognitive and social dimensions of learning.

3.2 Teachers engaging learners' mathematical errors

Teaching patterns have been found to reflect teachers' belief systems on the nature of subject knowledge, on the way learners learn and on teachers' engagement with learners in classrooms (Pajeras, 1992; Stigler & Hiebert, 1999). Teachers' engagement with learners' mathematical errors has been found to be influenced by a number of factors related to teachers' beliefs about teaching and learning, often based on their own learning experiences, as well as their subject knowledge (Gagatsis & Kyriakides, 2000; Bray, 2011). Gagatsis and Kyriakides's (2000) research found that teachers attributed learners' mathematical errors to the learners' attitudes towards their learning and their knowledge, and the nature of the subject knowledge, but rarely to their own pedagogical practices.

Bray's (2011) research suggests that teachers' beliefs and knowledge are related to three dimensions of teacher error-handling practices. The research findings indicate that teachers' beliefs are more strongly related to how they structure classroom discussion when errors arise whereas it is teachers' knowledge that has a greater influence on the quality of teachers' engagement with learners' errors. Bray's (2011) first dimension of error-handling practice concerns the extent to which there is an intentional focus on flawed solutions in whole-class discussion. An intentional focus on flawed solutions was viewed as an opportunity for learning. This view is supported by research in which activities are designed to give learners the opportunity to compare incorrect and correct solutions; guide learners to self-explain why a solution is incorrect and to explain how to solve a problem correctly (Isotani, Adams, Mayer, Durkin, Rittle-Johnson, & McLaren, 2011). The research of Isotani et al. (2011) in which learners engaged with errors in a computer-based environment found that not all learners benefited when encouraged to focus on flawed solutions. Isotani et al. (2011) argue that a potential reason is the nature of the activities which were designed by them. These activities were too wordy and learners were overwhelmed by the computer-based environment which does not allow for the same kind of interactive learning as in a classroom context. In Bray's research (2011) teachers who focused on learners' flawed solutions as part of whole-class discussion noticed that learners appeared to view errors as part of the learning process and were more confident in sharing their flawed solutions. In the first dimension Bray argued that teachers who engaged learners in the analysis of flawed solutions needed to draw on their knowledge about mathematical concepts and misconceptions.

In Bray's (2011) second dimension of teacher error-handling practice, teachers differed in the extent to which their response to learners' mathematical errors enhanced conceptual understanding. Bray (2011) found that teachers who believe that conceptual understanding of mathematical concepts was more generative than remembering procedures were more likely to focus on concepts associated with learner errors. Bray (2011) argued that teachers needed a strong knowledge of school mathematics to anticipate learner errors. For example, Bray (2011) provided evidence showing that when a teacher was not able to interpret a learners' strategy his/her response was that the learner guessed. The learner would be directed to a different strategy by the teacher rather than given an opportunity to justify his /her reasoning, build on an existing knowledge base and learn from a flawed solution.

In Bray's (2011) third dimension of teacher error-handling practice, teachers differed in the extent to which they responded by encouraging learners to collaborate. Teachers who believed less in learners' ability to support one another were less likely to encourage collaboration. Bray (2011) found that teachers who were less confident about learners' strategies were also less inclined to encourage discussion and encourage learners to address the errors as a community of learners. The teachers in Bray's (2011) research found it challenging to steer learners' discussions into productive discussion, which she argued was linked to teachers' interpretations of learner responses. Teachers' knowledge of subject content, misconceptions and analysis of learners' errors influenced the quality of learner discussion.

Rach, Ufer and Heinze (2013) suggest that teachers' avoidance of errors was related to their reluctance to interrupt their instruction process or to expose learners who made the errors. Keith and Frese's (2008) research indicates that dealing with learners' errors is more beneficial for learning than avoiding errors, and that the kind of feedback given and the generation of a classroom culture that promotes the value of engaging with errors for learning is crucial in this process. A classroom culture in which errors are overlooked or treated punitively has been found to be more likely make learners hide their errors instead of developing the confidence to talk about their errors (Rybowiak, Garst, Frese, & Batinic, 1999). Tulis (2013) and Heinze and Reiss (2007) have found that learners' perceptions of errors as learning opportunities is substantially influenced by their experiences of teachers' error-handling practices.

Santagata (2004) argues that the broader socio-cultural context in which learning takes place communicates a cultural 'logic of practice' (Bourdieu, 1990) that needs to be taken into account for teachers' practice of engaging learners' mathematical errors. The ways in which learners are socialised into a particular practice generates different meanings associated with the practice. For example, Santagata (2004) found that Italian learners viewed teachers' negative and harsh comments related to their errors as justified, necessary and showing care that they should be pushed to take responsibility for their learning, whereas the same approach in United States schools would be viewed as counterproductive to learning. These research findings indicate that socio-cultural norms established within classrooms and beyond the classroom present potential barriers or opportunities for optimising errors as resources for learning.

3.2.1 Learners' mathematical errors through cognitive and situative perspectives

Cognitive theories of learning acknowledge that social factors are important in learning but do not regard these factors as central to learning. Socio-cognitivist perspectives on learning view knowledge construction on two levels, firstly, knowledge construction that involves the use of prior knowledge and, secondly, knowledge construction through social interaction. It is argued that social interaction is necessary to generate cognitive conflict for an individual in the process of integrating prior knowledge and new ideas (Cobb & Bauersfeld, 1995). Even though interpersonal interaction is regarded as significant in generating 'socio-cognitive conflict' the focus remains on the nature of cognitive restructuring in conceptual development (Hatano, 1996).

Research with a cognitive focus on learners' errors provides analyses of learners' errors in terms of the mathematical content involved in the task in which the error is embedded as well as probing the difficulties that the learner has in accessing and applying the required mathematical knowledge for completing the task successfully. Eliciting learners' thinking in order to access the way in which their misconceptions arise is a priority in research with a cognitive focus.

Situative theorists argue that a focus on only the cognitive is not sufficient to account for learning, therefore they place a high premium on the participation of learners in the practices of a particular community as critical to an explanation of how learning takes place. Research on learners' errors with a situative focus needs to provide insight into the kinds of participation in mathematical and social practices that enable productive engagement with learners' errors to support their learning. Challenges that learners have in articulating their thinking and the ability of pre-service teachers to understand the reasons for these challenges from a social perspective is the additional focus of my study.

Brodie (2005) argues that both cognitive and situative perspectives are necessary to understand learners' errors and bases her argument on the premise that both perspectives have strengths and weakness that can be addressed through an integrated approach. Brodie (2005) provides evidence of learners' struggle to reason mathematically and their struggle to participate in mathematical practices intended to engage learners' errors. This suggests that a keen understanding of both learners' prior knowledge and participation in mathematical practices become necessary if one is to understand how best to support learners and build on their prior experiences. Santagata (2004) provided evidence of learners' socialisation into ways of being and the expectation they have of teachers in engaging their errors.

Cognitive research on learners' errors identifies patterns of thinking which account for learners' errors but merely using this knowledge to enhance learning has not always met with the desired success. Identifying patterns in learners' participation in mathematical and social practices and using this knowledge in conjunction with knowledge gained through cognitive perspectives holds potential for engaging learners' mathematical errors.

Thus far I have argued that teachers' beliefs, teachers' knowledge of mathematics, as well as teachers' knowledge of learners' engagement in the practices of mathematics are important for engaging learners' mathematical errors. Research on the specific nature of teachers' knowledge for engaging learners' mathematical errors is explored next, through research studies that focus on developing frameworks for analysing teachers' engagement with learners' mathematical errors.

3.3 Frameworks for analysing teachers' engagement with learners' mathematical errors

Thus far I have highlighted studies that theorise the nature of learners' mathematical errors with a focus on the origin of errors and studies that have explored various possibilities for engaging learners' mathematical errors. Few studies, however, have focused specifically on fine-grained analyses of teacher knowledge for understanding learners' mathematical errors. The frameworks developed by Peng and Luo (2009), Prediger (2010) and Shalem, Sapire and Sorto (2014) for analysing teachers' knowledge when engaging with learners' mathematical errors were informed by various conceptualisations of teacher knowledge (Shulman, 1986; Ball, Hill and Bass, 2005). Peng and Luo (2009) elaborated on Shulman's (1986) two teacher knowledge categories, subject matter knowledge (SMK) and pedagogical content knowledge (PCK) to analyse teachers' knowledge as used in error analysis. Shulman's SMK and PCK categories correspond to dimensions that Peng and Luo (2009) refer to as 'the nature of the mathematical error' and 'the type of error analysis'. In each dimension, analytical categories were defined to provide a fine-grained error analysis and Peng and Luo (2009) argued that their dimensions were related.[1] For example, they provide evidence of how teachers who were able to identify a learner's error but did not have sound mathematical knowledge of the nature of the error used incorrect mathematical knowledge to interpret the error. This led to an evaluation of the learner's performance and a teacher intervention strategy that was so ineffective as to be meaningless.

Prediger (2010) and Shalem et al. (2014) support Peng and Luo (2009) on the importance of teachers' SMK but provide different perspectives on SMK in their error-analysis process. Prediger (2010) provides a sequence of four steps in her analysis: (1) interest in student (learner) thinking; (2) interpretive attitude of understanding from an inner perspective; (3) general knowledge on learning processes, providing sensitising constructs for a deeper analysis; and

[1] Peng and Luo (2009) analytical categories for each dimension
The nature of the mathematical error dimension: mathematical; logical, strategic, psychological
The type of error-analysis dimension: identify, interpret, evaluate, remediate.

(4) domain-specific mathematical knowledge, with a focus on meanings. These four steps are considered to be constituents in developing 'diagnostic competence' which extends beyond a knowledge perspective to include an interest in how learners come to know. Prediger (2010) argued that if teachers' interest in learners' errors is based on studies of learners' errors and misconceptions from which a deficit position on learners' errors is developed, then the potential to explore learners' errors as resources for learning is at risk. Hence Prediger's (2010) next step focuses on practices which have the potential to challenge deficit perspectives on learners' errors. I view Prediger's (2010) focus on the development of systematic, joint reflection practice in which teachers learn to value what it means to do an in-depth analysis of learners' errors as a necessary constituent of developing a critical inquiry stance into learner' errors. Prediger (2010) also argues for the insertion of both domain-independent knowledge and domain-specific knowledge for understanding learning processes, and mathematical thinking and teacher strategies for supporting learners.

Shalem et al. (2014) have developed a framework for analysing teachers' explanation of learners' mathematical errors in standard assessments. They use the Ball, Thames and Phelps (2008) classification of teacher knowledge domains, arguing that recognition of learners' mathematical errors and the nature of learners' mathematical errors require different knowledges within the SMK domain. To recognise learners' errors, Shalem et al. (2014) consider an understanding of the correct solution as the first step in the error-analysis process. A procedural and conceptual understanding of a problem is required for error recognition. While procedural and conceptual understandings were identified as separate criteria within the common content knowledge domain, they acknowledge the importance of teachers making connections between these understandings when analysing mathematical tasks.

Shalem et al. (2014) further argue for knowledge on learners' reasoning about their errors which moves beyond simple identification of the error. To diagnose learners' reasoning, they identify knowledge about common errors in specific content supporting Prediger's (2010) argument for the importance of domain-specific knowledge related to learners' errors. The importance of being able to offer learners more than one explanation for an error is another criterion which Shalem et al. (2014) have identified which requires an in-depth understanding of domain-specific knowledge.

Shalem et al. (2014) also take into consideration the different social backgrounds of learners in relation to the teaching context and view teachers' knowledge of learners' everyday experiences as crucial in understanding learners' reasoning. How teachers use learners' everyday knowledge to gain insight into learners' mathematical errors is what defines this criterion of teacher knowledge for error analysis. A strength of the framework offered by Shalem et al. (2014) is that it has developed different levels across each of the categories when analysing teachers' explanations of learners' errors and provides insight into the extent of teachers' knowledge in specific error-analysis processes.

The analytical frameworks discussed thus far are based on teachers' engagement with learners' errors in written work and not on observed interactions with learners. Furthermore, the error-analysis process in these frameworks lead to remediation of learners' errors and does not account for teacher knowledge required to use learners' errors as a 'springboard for inquiry' (Borasi, 1994). Borasi (1994) argues for a form of engagement with learners' errors that generates inquiry in new mathematical directions that may lead to new insights into mathematics.

Brodie (2013) has developed a framework with four categories of teachers' engagement with learners' mathematical errors in classrooms: avoidance; correcting; probing and embracing, which includes Borasi's idea of working with learners' errors. The category, 'embracing learners' errors, is defined as, 'where teachers use learners' errors constructively to generate new knowledge for the learner' (Gardee & Brodie, 2015, p. 3), moving beyond mere remediation of errors. Gardee and Brodie (2015) argue that probing the learners' errors is an important part of error engagement in which learners have the opportunity to explain or justify their thinking. Teachers building on learners' justifications to provide access to new knowledge for the learner is what 'embracing an error' entails. What emerges from Gardee and Brodie's (2015) research is that embracing learners' mathematical errors depends on how teachers use domain-specific knowledge to negotiate meaning around important mathematical ideas. It is not merely about getting to the correct answer but to engage learners in mathematical practices where they value the process of questioning and the importance of being challenged mathematically.

The research of the DIPIP project extends the knowledge on engaging and analysing learners' mathematical errors through a focus on teachers' practice in classrooms as well as conversations in professional learning communities. The aim of the DIPIP project was to deepen teachers' knowledge and understanding of learners' mathematical errors and through learners' errors deepen their knowledge of teaching and learning mathematics (Brodie, 2014;

Molefe, 2016; Brodie, Marchant, Molefe & Chiamande, 2018). Brodie's (2014) analysis of professional learning community teacher conversations found that teachers made significant shifts in learning about learners' mathematical errors. Teachers shifted from identifying errors to interpreting errors to engaging errors which required them to question fundamental assumptions about how they work with learners' mathematical errors. Brodie (2014) found that through the collaborative inquiry into learners' errors teachers began to understand their own practice and knowledge in relation to learners' errors. The teachers in Brodie's (2014) research deepened their understanding of mathematics and the teaching of mathematics through their inquiry into learners' mathematical errors.

Molefe's (2016) research, situated within the DIPIP project, provides evidence of shifts in teachers' practice in engaging learners in classrooms and shifts in teacher conversations about learners' mathematical errors. Molefe (2016, p. 179) argues that the classroom and professional learning community as learning environments created 'pedagogical movement' which defined the teachers' learning in and between the two learning environments.

Brodie et al. (2018, p. 153) analyse the teacher knowledge focus in DIPIP professional learning communities and found that the type of PLC activity influences the knowledge focus. Content knowledge (CK) was the focus in lesson planning and error-analysis activities. PCK is the focus in lesson reflection activities. Since DIPIP design prioritises PCK, the amount of time spent on PCK was greater than CK. A deeper analysis of the PCK found that the PLCs spent more time on knowledge of instructional strategies than knowledge of learner understanding as conversations moved rapidly from identifying learners' errors and reasoning behind errors to teaching strategies to address learners' errors. Furthermore, DIPIP professional learning communities provide evidence that CK conversations can trigger PCK conversations and that PCK conversations can trigger CK conversations.

The importance of teachers' mathematical content knowledge and pedagogical knowledge for analysing learners' mathematical errors' is evident in the frameworks discussed. The frameworks differ in the extent to which there is a social dimension focus on knowledge about learners' participation in classrooms in relation to meaning-making and reasoning as part of PCK. Peng and Luo (2009) confine teachers' knowledge mainly to a cognitive dimension of learners' mathematical errors. Prediger (2010, 2017) argues for teachers' knowledge on what it means to hold a non-deficit view of learners' thinking through a focus not only on the cognitive dimension of learners' mathematical errors.

Shalem et al. (2014) include teachers' knowledge about how they use learners' everyday knowledge to support mathematical understanding by making the link between the everyday and the mathematical clear, and by starting to focus on learners' social learning experiences beyond the classroom. Gardee and Brodie (2015) and DIPIP research on engaging learners' errors, emphasise teachers' knowledge of learners' mathematical errors in relation to shifts in learner participation in the mathematics classroom which brings a stronger social dimension into focus.

Kramarski and Zolden's (2008) research on using learners' mathematical errors as springboards for analysing learners' mathematical reasoning identify three instructional approaches which support learners' engagement in metacognition. The first approach encourages learners to become involved in self-regulatory learning by asking questions to understand a problem; make connections between previous knowledge and new knowledge; think about appropriate strategies for solving a problem and reflecting on the solution; and trying to make sense of their thinking. The second approach which they refer to as the diagnostic errors approach, focusses directly on learners' analysis of their own errors. The third approach encompasses the first and second approaches which involve learners engaging with other learners, thus demonstrating a focus on encouraging learners' participation in mathematical practice both as individuals and collaboratively.

Kramarski and Zolden (2008) have found that learners' metacognitive knowledge to self-monitor errors is greater in the third approach which has a focus on self-regulatory learning and analysis of errors. Their research recommends developing teachers' knowledge on what they refer to as a metacognitive culture (Kramarski & Zolden, 2008). Such a culture would encourage learners to:

'(a) self-question and analyse errors

(b) make corrections

(c) formulate an action plan on how they have learned and understood the material and how they will remember the information' (Kramarski & Zolden, 2008, p. 148).

The focus on encouraging learners to think about their learning suggests a move beyond cognition which Watkins (2015, cited in Hargreaves, 2017, p. 62) refers to as meta-learning:

> it includes the learner's reflection and reaction in relation to cognition, affect, motivation and elements of the social and physical context. Metacognition could be seen as a subset of meta-learning.

A focus on learners' thinking about their mathematical learning experiences in relation to their mathematical errors is extending the focus on the social dimension which is not made explicit in the earlier studies (Prediger, 2010; Shalem et al., 2014; Brodie et al., 2018), and which pre-service teacher learning communities in my study were encouraged to make a focus of their inquiry.

3.4 Learners' mathematical learning experiences

The research explored thus far on engaging learners' mathematical errors from a cognitive perspective, shows that it is a necessary but not sufficient approach to provide learners full access to learn from their errors. To understand how learners reason, we need to get learners to explain their thinking and to feel valued as members of their classroom community. This brings a situative perspective to the fore, focusing on learners' participation in mathematical practices that explore social and cognitive dimensions of learning and on developing teachers' understanding of how these dimensions interact to support learning. In my study the social dimension of learning includes a focus on engaging learners' mathematical learning experiences in their classrooms as well as beyond school.

Hargreaves (2017) emphasises experience as the basis for learning to illustrate learning as taking into consideration a broad range of experiences of learners. The manner in which learners will respond to future experiences and relationships is informed by both past and present learning experiences and relationships and in particular by the knowledge constructed through these experiences (Hargreaves, 2017, p. 17). Hargreaves argues that when learners are encouraged to critique what they have learnt, reflect on what they have learnt and analyse what they have learnt about learning, they are given the opportunity to develop a sense of autonomy.

Lange's work (2009) explores learners' narratives of their mathematical learning experiences to gain insight into how their identities as mathematics learners connect with their agency in relation to learner difficulties in mathematics learning. Central to Lange's (2009, p. 30) argument for a learner narrative approach to understanding learner difficulties, is the shift from an 'essentialist or defect-oriented' account of learners' difficulties to an account that takes into consideration broader social structures which impact on teaching and learning, and learner participation in learning. Lange (2009, p. 50) developed the concept of learners' perspectives which he defines as 'meaning constructions: the meaning that children ascribe to their actions in the field of school mathematics learning'. Lange (2009) argues that learners' learning intentions and meaning constructions are connected to learners' lives, their background and foreground (Skovsmose & Valero, 2005) and not necessarily related to mathematics.

Learners' narratives in Lange's research highlighted that a great deal more than cognitive issues influences their learning of mathematics. Socio-political contexts of schooling and valorisation of mathematics emerged in learners' narratives of 'instrumental rationales' for learning mathematics (Lange, 2009, p. 77). Instrumental rationales included reference to the importance of mathematics in education and securing a job in later life. This research shows a notable lack of learner perspective of a social rationale for learning mathematics. Lange (2009, p. 77) describes a social rationale as evidence of how learners consider their life-worlds, and how active imaginations and thinking can contribute to learning mathematics. Lange's (2009) research demonstrates that the way in which learners experience their learning plays a significant role in their learning and that both cognitive and social dimensions of learning emerge in learners' narratives.

Munns (2007) and Munns, Sawyer and Cole (2013, p. 24) research provides evidence that learners' insight into their learning experiences reflect classroom cultures in which five interrelated 'discourses of power' and messages either enhance or inhibit learner participation. The 'discourses of power' are knowledge, ability, control, place and voice. Knowledge as a 'discourse of power' was reflected in learners' learning experiences in which they described their disengagement when they did not understand why they were learning certain content. Ability as a 'discourse of power' was reflected in learners' feeling of either being able or not to achieve which in turn influenced their expectations and aspirations.

Control and voice as 'discourses of power' both relate to learners' experiences of how teachers provide them with an opportunity to contribute to the learning process. Place as a 'discourse of power' concerns how learners' learning experiences reflect that they understand what it means to be recognised and valued as a learner from a particular social context. The latter discourse is of particular relevance for my study as in the case of Munns's research, the pre-service teachers in my study also engaged learners from poor working-class families. The pre-service teachers within my study were encouraged to be aware that the rural working-class learners whom they engaged may experience the same kind of feelings of fear, anxiety, inferiority and dependence in relation to schooling found in Reay's (2005) research on the embeddedness of social class in everyday interactions. In my study it is important for pre-service teachers to understand how these affective dimensions arise and not to view working-class learners from a deficit perspective in relation to learning.

Phelps, Nhung, Graham and Geeves's (2012) research in rural Vietnam primary schools explores learners' views on their learning experiences in and beyond school with the aim of gaining a deeper understanding of how to support learner participation in class. The learners in Phelps et al.'s (2012) research were asked about how they learn and also how their teachers teach mathematics. They found that these Vietnamese learners were not used to thinking or talking about learning, particularly learning beyond school, and also held narrow views on the meaning of learning. Learners were found to struggle to articulate their learning experiences of school and beyond, which Phelps et al. (2012) attributed to the lack of opportunities given to learners to reflect on their learning. Learners provided richer descriptions of their learning experiences beyond school and were more inclined to emphasise their independence in learning experiences beyond school (Phelps et al., 2012, p. 299). For example, they referred to the benefits of being taught 'step-by-step' and provided evidence of sophisticated help-seeking strategies when encountering a difficulty or gap in knowledge when describing their learning experiences beyond school. Learners' school learning experiences reflected passive and dependent forms of engagement. Phelps et al. (2012) argued that different kinds of learning experiences in school and beyond school could explain why the Vietnamese learners made no connections between learning in the two contexts. They further argue that greater recognition of learners' learning capabilities in contexts beyond school may contribute to developing a metacognitive learning culture in school.

An underlying assumption of research on the importance of eliciting learners' learning experiences is that learners can be encouraged to think about how they were taught; to critique what may have caused them to struggle; and through rethinking the teaching process may recognise gaps in their own knowledge. Reflected in this assumption is that teachers' PCK includes knowledge about learners' understanding of the content as well as learners' reflection on teachers' instructional strategies as well as learners' learning strategies.

Knowledge about learners' understanding of the content aligns with my concept of a cognitive dimension of learning as the focus is on how learners explain their reasoning or their thinking related to content. Knowledge about learners' reflection on how they were taught and learners' learning strategies align with my concept of a social dimension of learning as the focus is not directly on content but learners' experiences more broadly of their learning. The next section of the literature exploration focuses on teacher knowledge and practice for making connections between the social and cognitive dimension to support learners' learning.

3.5 Connecting cognitive and social dimensions of learning

In my study the cognitive dimension of learning focuses on learners' mathematical reasoning and the social dimension of learning focuses on learners' learning experiences in class and beyond school. Teachers' knowledge associated with my concept of a cognitive dimension of learning aligns with Aguirre, Turner, Bartell, Kalinec-Craig, Foote, McDuffie and Drake's (2012, p. 179) concept of understanding 'children's mathematical thinking' which include for example, being able to interpret learners' solution strategies, and understanding learners' misconceptions. Aguirre et al. (2012, p. 179) use the concept 'children's cultural funds of knowledge' to include, for example, the knowledge, skills and experiences from learners' homes and broader community. My concept of a social dimension of learning aligns with Aguirre et al.'s (2012) 'funds of knowledge' conception.

Aguirre et al.'s (2012) research focused on pre-service teacher learning about making connections to 'children's mathematical thinking' and 'cultural funds of knowledge', which were referred to as learners' 'multiple mathematical knowledge bases' by Aguirre et al. (2012). They found that there were three interrelated practices that formed the foundation to making connections between learners' 'multiple mathematical knowledge bases'. Drawing on the work of Mason (2002), the practices of attention, (what teachers notice), awareness, (how meaning is assigned to observations), and eliciting (questioning or other strategies to engage with learners and find out what multiple knowledges they have access to), are described as foundational to making connections between learners' multiple knowledges (Aguirre et al., 2012, p. 180).

Pre-service teachers' development in making connections between learners' 'multiple mathematical knowledge bases' were categorised into three stages involving different kinds of connections: emergent, transitional and meaningful (Aguirre et al., 2012). Emergent connections involve pre-service teachers designing lessons in which school mathematics tasks connected superficially to learners' community context with little attention to learners' funds of knowledge and or learners' prior knowledge, solution strategies or misconceptions. Transitional connections involve pre-service teachers paying attention to learners' solutions but finding a lack of specificity about the challenges learners experience. When making transitional connections, pre-service teachers did elicit ways in which mathematical practices are used in learners' communities but often missed opportunities to make authentic connections between mathematical practices used in community and in school. Meaningful connections involved anticipating strategies learners will use and noticing authentic ways in which learners and communities use mathematics beyond school. Meaningful connections were evident in lessons in which mathematical practices used in communities were used to design high cognitive demand problems in class (i.e. problems requiring complex thinking and reasoning involving conjecturing, justifying and interpreting).

The majority of the pre-service teachers in Aguirre et al.'s (2012) research made connections that were emergent and transitional. Aguirre et al. (2012) argue that to make meaningful connections pre-service teachers require in-depth understanding of learners' out-of-school experiences as well as learners' solution strategies and misconceptions. The importance of questioning to elicit learners' solution strategies and reasoning was emphasised as part of the cognitive dimension of learning.

The pre-service teachers in Aguirre et al.'s (2012) research were encouraged to enter the communities of learners to familiarise themselves with practices in communities in which mathematics is used. When pre-service teachers needed to rely on learners to describe their community practices in which mathematics is involved, the process of eliciting learners' learning experiences was much more difficult as learners were not always directly involved in many of the practices.

Other research which explores how pre-service teachers and in-service teachers make connection with learners' 'multiple mathematical knowledge bases' identify the challenges in making those connections (Parks, 2008; Garii & Rule, 2009; Bartell, 2013). For example, studies that focus on in-service teachers who integrated mathematics and social justice issues (Bartell, 2013) provide evidence of teachers' lessons that reflected a separation of mathematics and social justice components, indicating the difficulty to use the social justice components as a scaffold to develop mathematical ideas. Similar findings are reported in a study on primary school pre-service teachers who were encouraged to make connections between mathematics and social justice issues (Garii & Rule, 2009). In a pilot study which I conducted with primary school in-service teachers in school-based mathematics professional learning communities, similar challenges emerged in using children's learning experiences beyond the classroom to support mathematical conceptual development (Liebenberg, 2016). For example, teachers selected contexts from learners' everyday lived experiences to illustrate mathematical concepts but struggled to bring social justice issues to the fore or to recognise how some of the contexts may create conceptual challenges for the learners.

A study by Taylor (2012) found that teachers needed probes to think about the mathematics involved in learners' out-of-school learning contexts so as to design tasks that build onto the mathematics. Through a systematic analysis of teachers' talk in group discussion, Taylor (2012) identified four different approaches used by teachers for building onto mathematics in context. The approaches differed in their authenticity in terms of learners' actual participation in the activity in their out-of-school context and the degree to which the mathematics involved or built on was connected to the mathematics in the context.

Turner and Drake (2016) have reviewed the literature on theories of teacher learning related to the practice of connecting learners' mathematical reasoning and learners' funds of knowledge. A synthesis of the research reviewed led to the development of a framework to understand pre-service teacher learning about learners' mathematical reasoning and learners' funds of knowledge. The framework included pre-service teachers' identities, orientations towards learners, knowledge, beliefs and practice as interrelated elements that can shift through learning experiences that takes place across multiple spaces which include university course work, schools and classrooms and learners' homes and communities. Absent from Turner and Drake's (2016) framework are pre-service teacher professional learning communities which my study explores.

Several research studies in Turner and Drake's review used theories on teacher identities to explain how learning to connect learners' mathematical reasoning and learners' 'funds of knowledge' involves a shift in orientation towards learners, their families and communities. For example, Hand (2012) and Wager's (2014) research found teachers' prior lived experiences including their professional development, shaped their identities as teachers and thus impacted their disposition towards learners. Hand (2012) and Wager (2014) argued that dispositions influence what teachers notice in their classrooms and how they interpret what happens in their classroom, in turn influencing learners' opportunities for learning. Turner and Drake (2016) refer to the study of Andrews, Ching, Greenhough, Hughes and Winter (2005) that found that teachers' experiences with learners from diverse backgrounds can support dispositions that view diversity as resource to explore new ways of approaching mathematics. When pre-service teachers have limited experience of learners from diverse backgrounds, their belief and assumptions may undermine learners' learning experiences in out-of-school contexts.

The importance of scaffolds to support pre-service teachers learning about learners' mathematical reasoning and learners' 'funds of knowledge' has been highlighted in several studies. These scaffolds include opportunities for pre-service teachers to tutor or interview learners whose backgrounds were different from their own specifically to learn about learners' 'funds of knowledge'. Turner and Drake (2016) argued that while pre-service teachers' interaction with learners was important they needed support in how to elicit and interpret learners' thinking. McDuffie, Foote, Bolson, Turner, Aguirre, Bartell, Drake and Land (2014) used video to support pre-service teachers' noticing of learners' 'multiple mathematical knowledges'. The pre-service teachers in McDuffie et al.'s (2014) study were provided with prompts associated with four lenses to support their analysis of teachers' practice. A teaching lens focused on how a teacher elicits learners' thinking and responds to learners; a learning lens raised questions about specific mathematics understanding within learners' work, talk or behaviour as well as questions about the knowledge and resources that learners draw on to solve a problem; a task lens focused on questions about the knowledge or resource the task activates or connects to; and the power and participation lens focused on the classroom culture in terms of who participates and whether different approaches or perspectives are valued or not (McDuffie et al., 2014, p. 250). They found that with support in the form of prompts to enact noticing learners' 'multiple mathematical knowledges' and when pre-service teachers worked in small groups and engaged in whole-class discussion, they built on each other's ideas and higher levels of noticing were possible (i.e. the highest level was making connections between the lenses).

The research on how to support pre-service teachers in the practice of making connections between learners' mathematical reasoning and learners' learning experiences in and beyond school contexts provides evidence that pre-service teachers require opportunities to engage with learners from backgrounds different to their own. These opportunities provide a means for pre-service teachers to reflect critically on their assumptions about engaging learners' mathematical reasoning and learning contexts beyond school with potential to influence learners' learning. The practice of making connections between learners' reasoning about their mathematical errors and learners' learning experiences in and beyond the classroom therefore requires a conscious effort to explore learners' prior knowledge and experiences to scaffold the conceptual development of mathematics and participation in the practices of mathematics. In addition to pre-service teachers developing a critical stance on their own learning experiences the research suggests that such a critical stance requires a collaborative learning context (Civil & Planas, 2009).

3.6 Chapter summary

In this chapter, the literature reviewed on teachers' engagement with learners' mathematical errors shows that when teachers view learners' errors as learning opportunities for themselves and their learners, they probe learners' reasoning behind their errors by encouraging learners to justify their thinking. Probing the mathematical reasoning behind learners' errors is viewed as important in engaging learners in mathematical practices, providing a means to explore ways of enhancing learners' participation in mathematics, and involving teachers in creating opportunities in which to extend the mathematics learned through generating new insights into the mathematics.

The research reviewed on engaging learners' mathematical errors included a cognitive focus in which learners' reasoning behind learners' errors was key to diagnosing learners' misconceptions. Teachers' content knowledge and pedagogical content knowledge related to misconceptions and teaching strategies form an important part of a cognitive dimension of learning. The social dimension related to learners' errors was evidenced in research that prioritised understanding how learners' participation in mathematical practices shifted as a result of engaging their mathematical errors.

The research on connections between cognitive and social dimensions of learning when engaging learners' errors developed through a predominantly 'metacognitive culture' where teachers and learners are encouraged to think about their conceptual and procedural understanding of content. I have argued, based on the research reviewed on learners' mathematical learning experiences, that the social dimension of learning can be extended through a focus on 'meta-learning' (Hargreaves, 2017) in which teachers and learners are encouraged to think critically about how they have been taught.

The review of research on teachers' knowledge and practices for making connections between cognitive and social dimensions of learning provides evidence of the importance of teachers' knowledge of learners' learning contexts in and beyond school. Furthermore, the research reviewed argues that teachers need to engage in the practice of collaborative critical reflection of their own learning experiences to recognise assumptions within their own learning and assumptions about the learning contexts and learning experiences of diverse learners. In the next chapter, I elaborate on how the design of my mathematics service learning course included pre-service teachers' professional learning communities as one of multiple learning spaces to explore connections between social and cognitive dimensions in relation to learners' mathematical errors.

CHAPTER 4

Situating Pre-Service Teacher PLCs in Service Learning

4.1 Introduction

In this chapter I introduce the larger politico-historical institutional context in which my study was conducted as the background for understanding the challenges of embracing new educational practices and of taking a critical perspective on knowledge. The mathematics service learning course (MSLC) in which my study is located took up this challenge through affording pre-service teachers the opportunity to explore learners' mathematical learning experiences in their classrooms and beyond school and in so doing to develop an understanding of the potential of giving recognition to learners' learning experiences and knowledge gained in different learning spaces. The MSLC included PST-PLCs as a space for pre-service teachers to critically reflect on their prior learning experiences in order to identify their assumptions about learners, learning and knowledge. The pre-service teachers engaging learners about their mathematical learning experiences in class and beyond school and the critical inquiry in PST-PLCs open a space for new insights into the challenges of taking a critical perspective on knowledge and practice.

In the first part of the chapter, I introduce the development of service learning at South African universities as a response to the challenge of 'higher education for the public good' (Leibowitz, 2012). Secondly, I provide a brief overview of literature on critical pedagogy that informed the theoretical underpinnings of the conceptualisation the MSLC. Thirdly, an overview of the conceptualisation of the MSLC, including components of the first and second years of the B.Ed. mathematics curriculum course which served as preparation for the MSLC in the third year, is given. This is followed by a detailed description of the B.Ed. third-year MSLC for Intermediate-Senior Phase mathematics in which the pre-service teacher professional learning communities (PST-PLCs) in my study are located. A background of the school communities and the pre-service teachers who participated in the MSLC is presented to elaborate on the socio-political space of my study.

4.2 Service learning at South African universities

South Africa post-1994 has seen several policy changes in relation to teacher education. Waghid (2012, p. 101) argues that the 'most prominent conceptual and pragmatic change' in teacher education has been policy to address the development of teachers who enact their profession as democratic citizens. A central characteristic of the democratic teacher is understanding what it means to be a critical-reflective practitioner who strives for social justice through pedagogical action (Samuel, 2010, p. 5). It is against this socio-political background that university teacher education programmes were challenged to explore transformative pedagogies for social justice. The pre-service teacher professional learning communities in my study are a response to this challenge.

In 1997, Education White Paper 3 (South Africa, 1997), which is a government document for the transformation of higher education, spelled out the goals needed to promote and develop community engagement at tertiary level. This policy required universities to integrate the three core functions research, teaching and community engagement. Lazarus, Erasmus, Hendricks, Nduna and Slamat's (2008) analysis of a 1997-1998 survey of community engagement in higher education found that while community engagement was included in mission statements, it was not considered a core function of the university. Leibowitz (2012, p. xxii) argues that 'higher education for the public good' should be about preparing students to think and behave in ways which demonstrate ethical competencies, reflexivity, critical thinking, care, humanity and hope and are thus representative of 'pro-public-good thought and behaviour'. Student engagement with communities through service learning courses was recommended for curriculum within higher education in South Africa as a response to integrate the three core functions of the university.

To distinguish between service learning as a form of charity and service learning as a critical pedagogy, service learning literature emphasises the importance of power relations, critical citizenship and social justice (Butin, 2005; Mc Millan, 2013; Osman & Petersen, 2013; Bowen, 2014). McMillan (2013) argues that the issue of power relations between students and the community is one of the most important considerations in adopting a critical pedagogy approach to service learning. Students are encouraged to question assumptions and world views and how they impact on both learning and their service in a critical service learning approach. McMillan (2013) views students asking questions about the relationships being built through the service as part of the process of thinking about power relationships. In my study, the differences in the backgrounds of the pre-service teachers and the learners whom they tutored are a factor that could influence the power relations and learning.

Bowen (2014, p. 53) describes service learning as a critical pedagogy with a social justice orientation that allows students to explore the historical, sociological, cultural and political contexts of the social issues they address in communities. Bowen (2014, p. 53) argues that through this process, students develop civic knowledge and skills that prepare them for a role as agents of social change. In my view, a critical service learning pedagogy must allow both the university student and the members of the community to reflect on their experiences and to understand what it means to develop a critical stance in order for them to be 'agents of social change'. The design of my mathematics service learning course incorporates the development of a critical-reflective reciprocity in that pre-service teachers were to open up spaces for the learners to take a critical stance on their learning experiences, and pre-service teachers in their professional learning communities were supported to embrace a critical stance on their teaching practice.

4.3 Mathematics service learning course

The B.Ed. mathematics programme at the university in which my study is conducted comprises two mathematics courses in each year of the four-year programme. The one course focuses on mathematics content and the other on mathematics curriculum, i.e. theories of learning; teaching and assessment, with an emphasis on working towards an integrated approach of the foci of the two courses. The MSLC is the mathematics curriculum course of the B.Ed. mathematics third-year programme. This course ran from February to October of the academic year with a total of 25 weeks across the academic year. I will elaborate on how these 25 weeks were designed to include the pre-service teacher professional learning communities which are the focus of my research. Appendix A provides an outline of the 25-week programme of the MSLC.

4.3.1 Brief overview of the conceptualisation of the MSLC

The pre-service teachers are prepared for key components of the MSLC course in the B.Ed. first- and second-year mathematics curriculum courses. The B.Ed. mathematics programme has a teaching practice (the practicum) in each of the four years of the programme. The pre-service teachers did whole-class teaching for seven weeks split across two periods of the academic year during the practicum. In the first two years, as part of the mathematics curriculum courses, the pre-service teachers are required to do detailed descriptions of two learners' participation in the mathematics classes that they were assigned to on their practicum with the aim of exploring learners' participation in relation to the general culture of engagement and mathematical practices of the classroom. In the first two years the mathematics pre-service teachers do not have the opportunity to work with individual learners over an extended period to elicit their mathematical learning experiences and probe mathematical errors.

From the first year, the pre-service teachers were introduced to theory and research on mathematical errors and mathematical error analysis. During the second year, they were required to complete an analysis of a learner's mathematical errors in a test and to do an in-depth content analysis of the errors. The mathematical error analysis in the second year was done only on the basis of the learner's written work with no follow-up engagement with the learner.

So, on entering the third year, the pre-service mathematics teachers had experienced a theoretical introduction to learners' mathematical learning experiences in class and learners' mathematical errors, but no in-depth practical experiences of engaging learners' mathematical learning experiences and mathematical errors. The MSLC provided the pre-service teachers the opportunity to engage deeply with the practice of engaging learners' mathematical learning experiences and learners' mathematical errors through the tutoring of learners in after-school classes.

4.3.2 Conceptualisation of the B.Ed. third-year MSLC

The B.Ed. third-year MSLC was designed around three learning spaces: the lectures on the university campus; the after-school tutoring in schools and the pre-service teacher professional learning communities. These three learning spaces are interconnected and, as the lecturer, I was present in these spaces in various supporting roles. I will describe the course in relation to each of the learning spaces.

4.3.3 Lectures on the university campus

The aim of the lectures on the university campus was to have whole-class discussion to introduce the pedagogical theory of the MSLC and for feedback discussions on the tutoring and PST-PLC sessions. The pre-service teachers were introduced to the concept of service learning as a critical pedagogy. It was important for the pre-service teachers to understand some of the key assumptions underpinning a critical pedagogy. The school communities with whom the pre-service teachers were going to engage were working-class and farming communities, and existing deficit concepts about learners from these communities are often framed as ‘deficits in their culture’ or, as Lewis (1961) stated, ‘living in a culture of poverty’ (cited in Gorski, 2008). Gutiérrez, Morales and Martinez (2009, p. 218) argue that one must take into consideration that ‘in a stratified society, differences are never just differences; they are always understood, defined and ranked according to dominant cultural norms, values and practices’. Within a critical pedagogy the assumption is that the problem is not the learner but the learning experiences afforded to the learner. The pre-service teachers in my study were encouraged to think about assumptions which they have about learners and their learning contexts.

Breunig (2005) argues that based on school learning experiences, many learners come to accept that the meaning of a ‘good learner’ means to be one for whom the primary purpose is to learn the knowledge a teacher imparts in an unquestioning manner. Silence, passivity and acceptance can become the accepted norms of engagement through particular learning experiences. Pre-service teachers in my study were encouraged to rethink their own learning experiences at school and question what they might have come to accept as valued knowledge, norms of engagement and practices of mathematics and how this might influence the relations of power that would potentially be developed as they engaged the learners whom they would tutor.

Another key assumption that underpins critical pedagogy is that learners’ learning experiences in their homes and broader communities are important sources of knowledge for their success in formal schooling settings. Kincheloe (2008, p. 24) defines what he refers to as one of the basic concepts that constitute a critical pedagogy as follows, ‘critical pedagogy is enacted through the use of generative themes to read the word and the world and the process of problem-solving – generative themes involve the educational use of issues that are central to the lives of students as the grounding for a curriculum’. The knowledge embedded in learners’ lived experiences in their homes and communities has been referred to as their ‘funds of knowledge’ (Moll, Amanti, Neff, & Gonzales, 1992; Gale, Mills, & Cross, 2017). This knowledge, it is argued, can contribute value to their learning environment in school. This value is partly derived from enhancing access to specialised knowledge of disciplines taught in school. The ethical nature of the value is the importance of learners being recognised as having knowledge assets and learners knowing that they are able to contribute to knowledge based on their learning experiences beyond school. The fundamental argument is that to enhance learners’ learning requires a deep understanding of epistemological access to specialised knowledge of disciplines taught in school as well as recognition of learners’ knowledge assets. The challenge of the PST-PLCs in my study was to inquire into this connection. Pre-service teachers therefore engaged with the learners' mathematical errors to enhance their understanding of how to support learners' deep understanding of mathematical concepts and explored how this support can be enhanced through probing how learners' learning experiences influenced their learning.

The time-tabling of pre-service teachers' courses in their B.Ed. programme only made it possible to tutor learners after school. Pre-service teachers were not able to visit learners' communities nor their school classrooms. They had to engage the learners in conversation as part of the tutoring to access learners' lived experiences within school and beyond. I drew on the 'funds of knowledge' literature to include resources that learners draw on beyond school to support their learning. For example, I considered the practices involved in support with school work that learners receive from older siblings, parents or peers in their community as a 'fund of knowledge' as it connects with learners' learning in school (Moll et. al., 1992). How and what learners bring from their mathematics classrooms at school to the learning setting outside of school is a potential way of gaining insight into their learning inside their school classroom. Tapping into learning spaces beyond school opens up opportunities for accessing 'funds of knowledge' that are connected to what learners learn in school.

Within the whole-class discussion of the MSLC I shared with the pre-service teachers my observations from the tutoring, revisited key theoretical issues and provided pre-service teachers with reading materials related to challenges I had observed in the tutoring sessions as well as the PST-PLCs. In the lecture whole-class discussions, pre-service teachers raised issues or challenges that emerged from the tutoring sessions. For example, pre-service teachers raised concerns about learners' struggling with particular mathematical concepts, which learners were expected to have an understanding of, and which led to pre-service teachers sharing learners' errors with the concepts and their struggle to probe learners' reasoning.

4.3.4 After-school tutoring in school communities

The 10 one-hour after-school tutoring sessions constituted the community engagement component of the MSLC and took place throughout the academic year. The tutoring sessions took place in four schools within the surrounding community of the university. The pre-service teachers selected the school at which they wished to tutor learners and each school could accommodate five to seven pre-service teachers[2]. Pre-service teachers were allocated two

[2] In the next chapter, Table 5.1 PST-PLC Information, provides the number of pre-service teachers in each PST-PLC and the school at which they tutored.

learners selected by the teachers at the school. The tutoring took place in one of the school classrooms and every pre-service teacher was given a classroom in which only the two learners and the pre-service teacher were present. Before the start of the tutoring, the pre-service teachers met with the teachers of the learners to gain information about the learners' backgrounds which included their academic performance in mathematics. The teachers provided their year-plans which included assessments plans. The teachers were responsible for reminding learners to go to the tutor sessions and to take their note-books and class-tests with them.

The engagement with learners in the tutoring session was guided by the 'funds of knowledge' theory. Pre-service teachers were encouraged to tap into learners' social networks of support for learning in and beyond school as 'learning assets' (Zipin, 2009, p. 321) and to encourage learners to take a critical perspective on their learning experiences in class and to make their challenges known. The pre-service teachers were encouraged to demonstrate to learners a willingness to learn about them and from them. I argued in the previous section that I considered the practices involved in learners' learning from others in their community as a 'fund of knowledge' which a teacher can use to support learners with their learning at school.

I mentioned earlier that pre-service teachers in my study did not go into the homes or communities in which learners lived as the B.Ed. programme time-tabling did not allow for time for deeper engagement within learners' contexts beyond school. Hence, in the context of the MSLC, the pre-service teachers had to rely on eliciting narratives from learners about their learning experiences in their homes and broader communities. I considered the ways in which the learners 'moved' knowledge from school context to home or community context as a form of learner agency and encouraged the pre-service teachers not to assume that learners will simply ask those who support them to do the problem for them. It was important to elicit the nature of the interaction between the learner and persons from whom the learner received support. Pre-service teachers were also encouraged to ask learners about what new ways of thinking they gained from their support and whether they felt confident to share this in class. Rios-Aguilar, Kiyama, Gravitt and Moll (2011) argue for a better understanding of how 'funds of knowledge' are mobilised as a resource for learning in school. I encouraged the pre-service teachers to pay close attention to the ways in which learners navigated across school and home contexts with their knowledges and to view this as an opportunity to support learners to develop a systematic approach to their challenges, noting questions they would like to pose to their support network, and similarly when receiving support, to note ideas that could be taken back to the classroom.

I encouraged pre-service teachers to probe how learners' learning support systems become a 'fund of knowledge' to be harnessed to support learners' learning. The ways in which pre-service teachers engaged learners in eliciting their mathematical learning experiences was key to developing learner confidence in their ability to talk about their learning experiences. The assumption was that for many learners who were tutored, elaborating on their mathematical learning experiences may not have been a norm of classroom practice.

Pre-service teachers were encouraged to provide learners the opportunity to talk about their learning experiences of mathematics in their classes and beyond the classroom or school and to share their interests, joys, frustrations and challenges. The protocol in Table 4.1 was produced for this purpose.

Table 4.1: Recommended questions for eliciting learners' mathematical learning experiences

Tell me about how you participated in today's (or the past week's) mathematics lesson/s:

a) Did you ask questions in class? What were the questions? To whom did you ask the questions? Did you want to ask questions or comment but did not have an opportunity to do so?

b) Did your teacher ask you questions? What did she ask?

c) What did you enjoy most about the mathematics lessons?

d) What did you not enjoy about the mathematics lessons?

e) What mathematics did you cover? What were the main concepts dealt with?

f) Can you teach the concept the way in which the teacher taught it to you?

g) Did you share any of your mathematics learning experiences at home?

h) What or who in your home or community helps you with learning mathematics?

i) Did you find the support from your home or community helpful? Why or why not?

Learners often ask their friends, siblings or parents for support with mathematics. **Encourage the learners to explain how they asked for support and how a friend, sibling or parent supported them**. Encourage the learners to show you the learning materials that they received in class or from their learning contexts beyond school and ask them for their opinions about the learning material. Did they find it helpful? Why or why not?

Questions g) and h) were designed to access learners' 'funds of knowledge'. Question i) encourages learners to evaluate their learning experiences to enable pre-service teachers to identify the learning practices in learners' homes or communities that supports learners' learning.

The pre-service teachers were introduced to the concept of a social dimension of learning as involving the eliciting of learners' mathematical learning experiences that goes beyond a focus on learners' challenges or misconceptions in mathematics. As illustrated in the above protocol given to pre-service teachers, they were encouraged to elicit the ways in which learners experienced their learning to gain insight into the context in which learners' challenges arose.

The learners were encouraged to show the pre-service teachers their mathematical errors for the pre-service teachers to probe learners' reasoning in making these errors. The concept of a cognitive dimension of learning was introduced to the pre-service teacher as an explicit focus on understanding the nature of a learner error and the underlying reasoning involved in making the error. Pre-service teachers from the first year of the B.Ed. programme engaged in activities in which they analysed learners' errors based on written work. The focus was on the identification of the nature of the error, whether it was a 'careless' or random error or an error that was due to an underlying misconception. From first year, pre-service teachers were provided with research on misconceptions and the errors generated by these misconceptions. This research was used to develop an understanding of the possible reasons for the error. Pre-service teachers were required to do an in-depth analysis of the mathematics involved in the problem in which the error occurred. I drew on the protocols used in the DIPIP project (2011).

The challenge of probing learners' reasoning about their errors was modelled in class from first year. I often used pre-service teachers' errors in a test to illustrate how to do a mathematical error analysis and to probe pre-service teachers' reasoning behind their errors. Brodie (2014) argues for a systematic approach to engaging learners' reasoning that entails understanding what is valid and not valid in learners' thinking. I emphasised this systematic approach as key in avoiding a deficit stance on learners' thinking. It was also important to give learners an opportunity to see if they could identify their own errors and account for their errors. I discussed the importance of remediation of learners' errors but that re-teaching was not necessarily the best way as learners' mathematical errors often arise from prior learning (Brodie, 2014). It was for this reason that eliciting learners' previous mathematical learning experiences was an important aspect of the engagement with learners when engaging their mathematical errors.

Table 4.2 is a protocol given to the pre-service teachers to support them in the practice of probing learners' mathematical reasoning underlying their errors.

Table 4.2: Recommended questions for probing learners' mathematical error

a) Tell me about what you were thinking when you answered this question? b) Why did you do it in the way that you did it? c) Can you see where you possibly went wrong and why? d) Is there another way to do or approach the problem? Listen carefully to the learner explaining his/her thinking and look for opportunities to probe the learner's thinking. Try not to rush to a correct answer but spend time in trying to get to the **learners' underlying reasons for their errors**. Remember that you want **to build the confidence in these learners to have their voices come through** about what they have learnt and **to feel valued as learners who can think** and **can contribute to their learning** and **contribute to your learning.** Encourage the learners to ask questions about what they do not understand. Learners are inclined to say 'I do not know how to do this'. Encourage the learners to write what they know, and to not be afraid if errors emerge. When they have written something incorrectly, try not to use deficit language, for example, by saying 'this is not correct'. A non-deficit approach to engaging learners who made an error, for example, could be, 'This is an interesting start to this problem, tell me a bit more about why you started in this way'. You are encouraged to explore further problems on the learners' errors to extend the learners' thinking.

The first two questions were guided by research on the importance of using learners' errors to diagnose their misconceptions. Nesher (1987) argues that misconceptions are underlying frameworks that makes sense for learners. The first two questions probe learners' sense-making, providing pre-service teachers the opportunity to build on what may be partially valid in learners' thinking (Brodie, 2014). The third question gives learners the opportunity to evaluate their errors enabling pre-service teachers to experience whether learners are able to recognise that their reasoning was based on limited constructions of current knowledge (Smith et al., 1993). The fourth question extends learners' thinking and provides an opportunity for pre-service teachers to see how learners' errors are a means for deepening learners' mathematical knowledge (Borasi,1987; Brodie, 2014).

A key aim of the tutoring session was to afford the pre-service teachers the opportunity to engage deeply with learners' mathematical learning experiences and their mathematical errors or challenges in order to start thinking about how the social and cognitive dimensions of learning are connected. An exploration of these connections sought to gain an understanding of how a 'fund of knowledge' in relation to learners' mathematical challenges or errors can be used to support the development of mathematical concepts and challenges experienced in school.

I attended the after-school tutoring sessions, visiting each pre-service teacher's session for about twenty minutes so that I was able to visit at least three pre-service teachers in an after-school session per afternoon. In this way I was able to visit one tutor session of all the pre-service teachers and I visited two tutor sessions of some pre-service teachers. Over the tutor sessions across the academic year I observed one tutor session of 16 pre-service teachers and two tutor sessions of seven pre-service teachers, a total of 30 tutor sessions. The purpose of my visits to the tutor sessions was to observe the tutoring and to support the pre-service teachers with eliciting learners' mathematical learning experiences and probing learners' mathematical reasoning behind their errors. I would, for example, support the pre-service teachers by suggesting ways to get the learner to elaborate on their mathematical learning experiences. I supported pre-service teachers in probing learners' reasoning behind their errors by asking questions to the learners in the tutoring sessions that could potentially deepen understanding of learners' reasoning, thus modelling how to probe for errors. The engagement in the tutor sessions provided me with insight as to what pre-service teachers 'selectively sample' from their practice of engaging learners about their mathematical learning experiences and their mathematical errors to take into the PST-PLCs (Little, 2002).

Pre-service teachers were encouraged to keep a reflection journal on all the tutor sessions as part of the assessment for the course. The reflection templates in Appendices B1 and B2 were provided to support pre-service teachers with their reflections on these tutor sessions.

The pre-service teachers were encouraged to focus on one or two learner errors from each tutor session and do a detailed mathematical analysis of the errors. Table 4.3 was provided to guide the pre-service teachers with the mathematical analysis. The first step was to describe the mathematical content and processes involved in the problem and then to explore as many different ways of solving the problem as possible. The second step was to record the error and then to explore as many possible reasons for why the error was made. The last column of the table was to keep track of concepts with which learners developed misconceptions and to explore research on these misconceptions. The mathematical analysis of the errors formed an important part of pre-service teachers' preparation for inquiry into learners' mathematical errors in their PLCs.

Table 4.3: Guide to mathematical analysis of the mathematical error (DIPIP, 2011)

Problem	Description of mathematics content	Analysis of correct answers. Provide as many strategies as possible	Identify errors	Analysis of errors (reasons for the errors)	Key mathematics concepts

4.3.5 PST – Professional learning communities

The PST-PLCs were made up of four or five pre-service teachers who tutored learners from the same school. There were five PST-PLCs as 23 pre-service teachers enrolled for the MSLC. The PST-PLCs met the week after the tutor sessions and all the PST-PLCs met for eight PST-PLC sessions from March to October of the academic year in which the research was conducted. (See Appendix A for MSLC plan). Each PST-PLC met in their own room and I met twice with each PST-PLC to facilitate a one-hour session. As facilitator, I provided support with the inquiry into the practice of engaging learners' mathematical learning experiences and learners' mathematical errors. For example, one of the key elements of critical pedagogy is to recognise assumptions being made about a situation and to explore the implication of these assumptions. Pre-service teachers often made assumptions about learners' learning experiences which was evident in narrative accounts shared in the PST-PLCs. I was not always able to address all assumptions that arose within the PST-PLCs I facilitated. After a PST-PLC I too reflected on the inquiry that took place and would identify assumptions made by pre-service teachers and myself and share my reflections in a whole-class discussion. The importance of ongoing reflection on the PST-PLC's inquiry was emphasised.

Table 4.4 presents an excerpt is from the MSLC document given to the PSTs to follow in terms of mutual engagement and ways of moving the inquiry into practice beyond descriptions of practice in the PST-PLCs.

Table 4.4: Guidelines for engagement in PST-PLCs

The members of the PLC are encouraged to appoint a member who will facilitate the PLC. The facilitator is encouraged to have each member get a chance to share an aspect of their learners' mathematical learning experiences as well as the learners' mathematical errors. The members of the PLC are encouraged to bring samples of the learners' work to the PLC and share with the group how the errors were dealt with in the tutor session. It is the role of the facilitator to try and move the PLC beyond only describing experiences of engaging learners' mathematical learning experiences and mathematical errors. It is not only a show and tell and then move onto the next member to show and tell but rather a session in which members engage with one another's contributions. For example, once a member of the PLC has described the nature of a learner error and how the learner's reasoning was engaged with, it is recommended that the PLC inquire into the practice of engaging learners' mathematical errors by asking or elaborating, for example: a) Did you consider …….? b) When I worked with the learners I noticed that…. c) From my experience……… In this way your PLC is taking steps towards a critical inquiry stance on practice. You are encouraged to support each other to take the inquiry into the practice of engaging learners' mathematical learning experiences and learners' mathematical errors to a much higher level. For example, when engaging with how you probed the learners' reasoning behind their errors, try to explore how better questions may have been posed in engaging with learners' work and responses. You are encouraged to support each other in how learners' thinking could have been extended to explore deeper mathematics and new aspects of mathematics.

The PST-PLCs appointed a pre-service teacher to facilitate the PLCs in which I was not present. The guidance given to pre-service teachers to facilitate the PLC session was through the excerpt given in the MSLC document; my facilitation of PLC sessions; additional literature on facilitation of PLCs; and whole-class discussions of PLC sessions that had taken place.

The focus of the PST-PLC was inquiring into engaging learners about their mathematical learning experiences and mathematical errors and to explore the connections between learners' mathematical learning experiences and their mathematical errors. I encouraged the pre-service teachers to explore strategies to elicit learners' mathematical experiences. These strategies included asking learners to write about their learning experiences and to role-play teachers' teaching. These strategies provided the basis from which pre-service teachers were encouraged to initiate and support learners to reflect critically on their learning experiences. If, for example, learner narratives were shared in which learners said that they were bored with the mathematics they were being taught, the inquiry could include questions on whether pre-service teachers initiated a dialogue with learners about what kinds of problems they may have preferred. PST-PLCs were encouraged to share how they engaged learners about their mathematical reasoning and the challenges they encountered; to explore different ways in which the problem in which the errors occurred could be solved; and to identify as many possible reasons for why the error could be made, interpreting the learners' errors based on how the learners reasoned. Brodie (2014, p. 234) found that when teachers engaged in interpreting learners' errors, they were supported in rethinking their own knowledge. The critical inquiry into engaging learners' mathematical errors was a space for critical reflection on pre-service teachers' own knowledge and to identify opportunities when it was necessary to tap into others' knowledge, through for example, research to address the challenges they encountered.

4.4 Demographics: learners and pre-service teachers

In this section I firstly provide contextual background about the learners as it is important to understand that the learners who were tutored come from different socio-economic and cultural backgrounds from the pre-service teachers in the programme. The learners were mainly 'coloured'[3] and some were 'black'. During apartheid 'coloured' and 'black' learners attended separate schools but post-1994 there is some integration.

There were four schools involved in the MSLC, where the PSTs tutored, three in working-class neighbourhoods[4] and one was a farm school. One of the schools offered teaching in the medium of English and Afrikaans. Most of the 'black' learners whose home language is isiXhosa are in the English medium classes in this school. Most of the 'coloured' learners' have Afrikaans as a home language and some of these learners' parents have opted for them to attend the English classes. In the other two schools in which there were also 'coloured' and 'black' learners, only the medium of Afrikaans teaching was offered. The small farm school was attended by the children of farm workers who worked on farms nearby. These learners have Afrikaans as their home language which is also the language of learning and teaching at the school. The learners who attend the farm school live on the land owned by the farmers and their families do not have land of their own. Although my study does not go into the impact of how learners who are still living under a feudal system experience school, the pre-service teachers in this school were encouraged to engage with learners about their mathematical learning experiences in their context of living on farms.

[3] I use the inverted comma to indicate the apartheid classification and not my classification.
[4] These three schools were not farm schools.

Twenty-two of the twenty-three pre-service teachers in the MSLC are from families who are classified 'white' in South Africa and all of them come from Afrikaans-speaking homes. The 'white' pre-service teachers in the MSLC attended either private schools or schools now referred to as ex-Model C schools (i.e. school designated for 'white' learners during apartheid). One pre-service teacher is 'coloured' and is from an Afrikaans-speaking, middle-class home and attended schools previously designated for people classified as 'coloured' during apartheid.

In the B.Ed. programme, pre-service teachers had some degree of choice where they could do their teaching practicum and many of the pre-service teachers opted for schools located close to their homes. Many of the 'white' pre-service teachers indicated that the MSLC was a first-time experience for them to engage with farm schools and schools designated for learners classified as 'coloured' during apartheid. The mathematics service learning course and the explicit focus on engaging within a critical pedagogy paradigm was also a first-time experience for these pre-service teachers.

The pre-service teachers and most of the learners whom they tutored shared a common home language which is Afrikaans. All the pre-service teachers and the learners came from homes that practise Christianity. I mention this simply to provide some of the surface cultural commonalities[5] between the pre-service teachers and learners who participated in the course at the time of the study. The pre-service teachers acknowledged that they attended schools with far greater material resources and access to tutoring resources than the school communities in which they tutored. I elaborate in Chapter Six on the insight gained through individual interviews with all the pre-service teachers on practices in their homes with regard to engaging with mathematics.

What cannot be ignored is the impact of the historical socialisation of people and how this has shaped and continues to shape perceptions of superiority and inferiority. As part of a critical pedagogy, pre-service teachers were encouraged to think about the power relations between themselves and the learners and how these relations are impacted not only from a position of age or mathematical knowledge but also socio-historically. The importance of working towards

[5] I refer to 'surface' commonalities as I do not want to oversimplify the deeper power dynamics embedded in these cultures. For example, the particular expressions of Christianity that were practiced in certain groups under apartheid were exclusionary and discriminatory and formed the basis of the political ideology of apartheid. The point that I am making is that while on the surface there was a common religion and language as cultures, one cannot diminish the ways in which these cultures were used as forces of domination during apartheid and with the intention to develop beliefs of superiority and inferiority among people based on race classification.

relations of trust and respect was emphasised in the MSLC among pre-service teachers as well as between pre-service teachers and learners.

4.5 Chapter summary

In this chapter I have framed the research context within national policy, institutional history and at the micro-level of theoretical and conceptual course design. These different levels of the research context are significant as they provide insight into how what happens at the micro-level of practice is influenced by national and institutional dynamics. Furthermore, my research focuses on inquiry within the PST-PLCs which is one of three learning spaces in the B.Ed. third-year MSLC and detailed descriptions of all three learning spaces are necessary in order to understand the complex interconnectedness of these spaces.

I have explained the way in which I framed the concept of a ‘fund of knowledge’ of learners, which was central to my development of the concept of a social dimension of learning which is engaging learners’ mathematical experiences in classroom and beyond school. In Chapter Three the concept of a cognitive dimension of learning was defined as engaging learners’ mathematical errors in which understanding learners’ mathematical reasoning behind errors is the central focus. The elaboration of these ideas is central to understanding the focus of the PST-PLCs’ inquiry into engaging learners’ mathematical experiences and learners’ mathematical errors. I explained that the goal of the PST-PLCs was to gain insight into connections between the social and cognitive dimensions of learning through the inquiry into engaging learners’ mathematical experiences and learners’ mathematical errors.

In the following chapter, I provide the reasons for the choice of research design and methods to explore and analyse the potential of the PST-PLCs to critically inquire into engaging learners’ mathematical experiences and learners’ mathematical errors.

CHAPTER 5

Research Design and Methods

5.1 Introduction

My study focuses on gaining an in-depth understanding of the critical inquiry into learners' mathematical errors within PST-PLCs and hence a qualitative and interpretive research approach has been adopted. A multiple-case study design was chosen, given the five PST-PLCs in the mathematics service learning course, which enabled an in-depth analysis of the inquiry within each PST-PLC and a comparison across cases.

In this chapter I discuss the qualitative and interpretive research approach as it relates to my study. I describe the research design, research methods for data collection and analysis, and issues of research rigour. I raise ethical issues throughout the chapter and conclude with more general ethical issues related to the research.

5.2 Qualitative-interpretive research approach

Qualitative research is orientated towards understanding social phenomena in their temporal (or historical) and local particularity and starting from what people say and do in their local contexts (Flick, 2009). The aim of a qualitative research process is directed at providing an in-depth understanding of the 'social worlds' of research participants (Creswell, 1998). My research has a qualitative orientation in that it explores how PST-PLCs inquire into the connection between learners' mathematical learning experiences and learners' mathematical errors to gain an in-depth understanding of the potential of PST-PLCs to develop critical inquiry into the conditions for learning. Furthermore, Hatch (2002, p. 9) asserts that an underlying assumption of qualitative research is that the 'social settings are unique, complex and dynamic' and the research takes place in its natural setting. In my study the research can be considered to have taken place in its natural settings as the pre-service teachers' inquiry in the PST-PLCs involved their ongoing teaching practice through tutoring of learners at schools over an academic year.

Qualitative research includes various approaches based on ontological and epistemological assumptions. The ontological assumptions relate to the form and nature of reality (Guba & Lincoln, 1994, p. 108). I adopted an interpretive approach which assumes that reality is locally and specifically constructed by those involved in the research (Guba & Lincoln, 1994). The epistemological assumption consistent with this ontological view is that knowledge is produced by exploring and understanding the social world of the research participants, focusing on their meaning and interpretations. These assumptions are incorporated in Wenger's (1998) social learning theory which underpins the PST-PLC inquiry in my study.

5.3 Case study research design

A case study design is appropriate for my research since it seeks an in-depth understanding of the critical inquiry within PST-PLCs into an aspect of teaching practice, namely, the connection between learners' mathematical learning experiences and learners' mathematical errors. This purpose meets Yin's (2009, p. 18) definition of case study research as an 'empirical inquiry that investigates a contemporary phenomenon in depth and within its real-life context'. My study is by definition a descriptive case study as the intention is to describe and analyse the nature of the inquiry in PST-PLCs, making links with previous research in order to make a theoretical contribution. The intention of my study is not to test an existing theory as in the case of an explanatory case study (Yin, 2009).

Merriman defines case study research as an 'intensive, holistic description and analysis of a bounded phenomenon such as a programme, an institution, a process or a social unit' (Merriman, 1998, p. xiii). The 'social unit' or case in my study is the bounded set of PST-PLCs within a mathematics service learning course in which 23 pre-service teachers participated who were specialising in mathematics within the Intermediate/Senior Phase as part of their Bachelor of Education degree at a particular university at a specific time. The unit of analysis in my study is the inquiry within the PST-PLCs. The PST-PLCs was one of the three learning spaces of the mathematics service learning course and the inquiry in the PST-PLCs is connected to the other two learning spaces, as described in Chapter Four. In addition, the prior mathematical learning experiences of the pre-service teachers as school learners had potential to influence the PST-PLC inquiry, situating the unit of analysis in a complex research setting. Babbie and Mouton (2011, p. 282) argue that the unit of analysis in case study research is rarely unaffected by the context in which it is embedded which makes detailed descriptions of context important in understanding the phenomenon being researched. For this reason, a detailed description of the research context has been provided in Chapter Four. In this chapter I provide additional information about the research participants, the research setting, and further detail on the data collection and analysis in order to demonstrate the holistic nature of the contextual detail which is necessary to understand and interpret the phenomenon under investigation, namely the inquiry within PST-PLCs.

The selection of cases in case study research is determined on the basis of two issues, appropriateness and adequacy (Kuzel, 1999). Appropriateness relates to whether the case demonstrates a fit to the purpose of the research and the phenomenon of inquiry, while adequacy is concerned with the number of cases (Kuzel, 1999). Taking into consideration that all five PST-PLCs participated throughout the research, ensuring information-rich cases, all five were selected to gain an in-depth understanding of the challenges and potential within the PST-PLCs to develop critical inquiry into what was the focus of the PST-PLCs (Patton, 2002; Andrade, 2009).

5.3.1 Additional information on the research context

In Chapter Four I provided in-depth information on the socio-economic and cultural backgrounds of the pre-service teachers who participated in the research; the broader university context; and the design of the PST-PLCs within a mathematical service learning course. As stated, 23 pre-service teachers were enrolled in the MSLC offered in the third year of the B.Ed. mathematics programme. The 23 pre-service teachers agreed to participate in the research and, given the small number of pre-service teachers all were included in the research.

Table 5.1 below depicts the number of pre-service teachers in each PST-PLC, the gender of the pre-service teachers, the pre-service teacher facilitator and the schools at which they tutored learners. The pre-service teachers chose whom they wanted to work with in a professional learning community. The maximum number of pre-service teachers in a professional learning community was five to allow for greater participation of all pre-service teachers. There were only four schools in the MSLC and an agreement was reached with the schools that the 23 teachers would be divided between the four schools. School 1 had seven pre-service teachers, school 2 had five pre-service teachers, school 3 had six pre-service teachers and school 4 had five pre-service teachers. It was possible for four of the PST-PLCs to have all pre-service teachers assigned to the same school. The PST-PLCs chose the schools at which they wanted to tutor learners. It was only PST-PLC 3 where it was not possible to have all the pre-service teachers tutor learners at the same school because of the agreed upon allocation of pre-service teachers to the schools.

The learners whom the pre-service teachers tutored were in Grades 5, 6 and 7.

Table 5.1: PST-PLC information

PST-PLC # Number of PSTs	PST gender Facilitator of the PST-PLC	School at which learners in Grades 5, 6 and 7 were tutored
PST-PLC 1 (4 PSTs)	PST 1A – female (facilitator) PST 1B – female PST 1C – female PST 1D – male	School 1
PST-PLC 2 (5 PSTs)	PST 2A – female (facilitator) PST 2B – female PST 2C – female PST 2D – female PST 2E – female	School 2
PST-PLC 3 (4 PSTs)	PST 3A – female (facilitator) PST 3B – female PST 3C – female PST 3D – female	PST 3A – School 3 PST 3B – School 1 PST 3C – School 1 PST 3D – School 1
PST-PLC 4 (5 PSTs)	PST 4A – female (facilitator) PST 4B – female PST 4C – female PST 4D – female PST 4E – female	School 3
PST-PLC 5 (5 PSTs)	PST 5A – male (facilitator) PST 5B – male PST 5C – male PST 5D – female PST 5E – female	School 4 (Small Farm School)

5.3.2 Insider researcher status

In qualitative and interpretive case studies, the researcher is directly involved in the process of data collection and analysis which provides an opportunity to get deep insight into the social phenomenon from an insider perspective. In my study I was positioned as an 'insider-researcher' in the sense that I was connected to the research in multiple ways (Robson, 2002): as designer of the mathematics service learning course in which the research is located as well as the lecturer of the course. My research, however, was not on my own practice and hence my role as critical-reflective practitioner needed to be separated from my role as a critical-reflective researcher. In my role as observer of the PST-PLCs, I needed to take on a critical-reflective position on the research, for example, understanding how my position as lecturer potentially influenced the research. As a facilitator within the PST-PLCs my role as critical-reflective practitioner influenced how interactions unfolded in the communities. I noticed, for example, that when I facilitated a particular PST-PLC, a pre-service teacher who took on the role of facilitator in my absence engaged less spontaneously, and was more hesitant to take a critical position on an issue of practice. My presence could also have been viewed as evaluative. It is for this reason that in reporting on the research I focused mainly on PST-PLCs that were facilitated by pre-service teachers. I do, however, report on an episode which I facilitated in one PST-PLC to illustrate the support provided to deepen the inquiry.

Mercer (2007, p. 13), in her analysis of insider research within educational institutions, describes the research process as ‘wielding a double-edged sword’ in that perceived gains from insider research always need to contend with ‘delicate dilemmas’. My privileged access to the research site and the rapport I developed with research participants facilitated the logistics around setting up my research. The pre-service teachers were well informed about my research and were familiar with me as I had taught them the previous year. Mercer (2007) argues that an ethical dilemma within insider research linked to this level of familiarity is the research participants’ preconceptions about the researcher and that their participation could be shaped in a way that could generate bias. I interviewed pre-service teachers about their own mathematical learning experiences as school learners including their mathematical learning experiences within their homes and communities, and though I did not experience any overt reluctance from them to talk about their learning experiences, the possibility exists that the depth of information shared was influenced by my insider status as their lecturer for a period of three years of the four-year B.Ed. programme. As I stated earlier, the pre-service teachers’ interactions in the PST-PLCs were also influenced by my presence as a facilitator.

5.4 Data collection

The use of multiple methods in case study research intends to develop a deep understanding of the phenomenon in question and to converge lines of inquiry (Denzin & Lincoln, 2003; Yin, 2009). In a multiple case study, convergence is achieved by focusing on the phenomenon across cases (Babbie & Mouton, 2011). I motivate the use of video; semi-structured individual and group interviews; pre-service teacher reflection journals and observation as methods of data collection used in my study.

5.4.1 Video as data

Video data is the primary data for my research. Video as a tool in observational research is powerful in that it enhances the possibilities of capturing interactions as they unfold moment by moment. The subtle nuances in verbal and non-verbal interactions captured through video provide access to a greater depth of complexity of the interactions. Powell, Francisco and Maher (2003) refer to Grimshaw's (1982) and Bottorff's (1994) concepts of density and permanence to describe two potential advantages of video recordings over human observation note-taking in research. Powell et al. (2003) note that it is difficult when observing and taking notes to capture the different and simultaneous ongoing behaviours in interactions which the video camera has the capacity to do. The concept of density reflects the advantage of video data to capture much more than is possible through human observation. Rosenstein and Sheva (2002) argue that human observation and the use of video are both selective but the selectivity is greater in human observation. In the case of video, the selectivity is based on where the camera is placed. The possibility does exist that some information can be missed with video based on technical issues (Pirie, 1996). In my study, for example, I found that it was difficult to capture what pre-service teachers were saying when they all speak at the same time.

The concept of permanence reflects the capacity of video to enable the re-examination of what has been captured. New insights can emerge from reviewing the initial data and joint viewing can deepen understanding through reflective dialogue (Rosenstein et al., 2002, p. 13). In my study joint viewing of the data was not possible to check coding given resource constraints and this was one of the limitations of the study. I was able to use the video to check with pre-service teachers whether I heard correctly what they were saying and to request their journals for further information on mathematical problems shared in the PST-PLCs.

A number of methodological issues related to data collection though the use of video does however question the assumption of the completeness of video data. Powell et al. (2003) list Bottorff's (1994) three reasons why the incompleteness of video data has to be taken into consideration:

1. capability of selectivity because of mechanical limitation;
2. incapability of discerning the subjective content of behaviour being recorded; and
3. inability to convey the historical context of captured behaviour.

A video camera has mechanical limitations in terms of range and depth of what can be covered which limits the completeness of video data. In my study where the interactions of pre-service teachers as well as the artefacts which they brought to PLCs needed to be captured, a single video camera influenced the completeness of the data. While a single video camera was able to capture all the members of the PST-PLC, there were times when the entire PST-PLC was not in view. For example, when the pre-service teachers showed learners' work from their reflection journal or wrote in their books to do a problem, the camera would zoom in on one person. It was difficult at times to position the camera to zoom in on written work produced in the PST-PLCs. To address the mechanical limitations, I made notes in my reflection journal when viewing the videos and followed up with pre-service teachers on aspects of the conversation or written work that was not captured by the video camera.

The second reason for the incompleteness of video data is that the data cannot always explain why people behave in a certain way. For this reason, it is necessary to have follow-up conversations. I needed to address this form of incompleteness by asking pre-service teachers to elaborate on their experiences to gain a deeper understanding of their participation in the PST-PLCs. In a related issue Rosenstein et al. (2002) refer to as the 'issue of accurate representativeness' since the presence of a video camera in a learning setting can influence the authenticity of the responses of participants. Pre-service teachers and teachers need to develop a certain measure of trust before they acknowledge uncertainty or mistakes in professional learning communities. The fact that pre-service teachers were making their challenges with regard to issues of mathematics known suggests that they became familiar with the video camera. At times pre-service teachers were highly critical of the school contexts in which they worked which was a further indication of low impact of the video camera on the authenticity of pre-service teachers' responses in the PST-PLCs.

The third reason for video data incompleteness was the 'inability to convey the historical context of captured behaviour' as video captures specific moments in time. In my capacity as insider researcher I had knowledge about the context which was enhanced through the individual interviews with pre-service teachers on their mathematical learning experiences as school learners.

The five PST-PLCs in my research met for eight one-hour sessions which took place over the full academic year which produced 40 video recordings in total. I facilitated a total of fifteen of the PST-PLCs, meeting with each PST-PLC three times. The decision for the number of times that I facilitated the PST-PLCs was both logistical and ethical. Since the PST-PLCs met over two days for each session I was able to facilitate two PST-PLCs per session (one on each day). In my role as facilitator of the PST-PLCs, I was supporting pre-service teachers in how to critically inquire into their practice as teachers; and since the research was located in a course, I needed to provide the same amount of support across the five PST-PLCs.

The professional video team responsible for the video recording supplied me with the videos of each PST-PLC within a week of the recording. The videos were of a high quality which facilitated my watching the videos. I made notes of each video highlighting the issues that I wanted to follow up on in the PST-PLCs.

5.4.2 Semi-structured individual interviews

Kvale (2006, p. 481) states that the purpose of the qualitative interview is 'to investigate human experience ….an attempt to understand the world from the subjects' point of view and to unfold meaning of their lived world'. I argued in the previous section that the purpose of the individual interviews with pre-service teachers was to understand how their previous mathematical learning experiences as school learners influenced their inquiry into learners' mathematical learning experiences and mathematical errors. I chose a semi-structured individual interview format as I wanted to elicit the mathematical learning experiences of pre-service teachers as school learners in three contexts: school classrooms; homes and broader community. The semi-structured interview also allowed for the flexibility to follow new directions depending on pre-service teacher responses. The interview schedule in Appendix C illustrates the exploratory nature of the questions.

The individual interviews were conducted during the course of the academic year and each interview was scheduled for half an hour to one hour. The interviews with 22 out of the 23 pre-service teachers[6] took place in my office at the university and an audio-recorder was used for the entire interview. At the start of each interview I explained to the pre-service teacher the purpose of the interview, stated the three broad contexts and asked whether there were objections to answering questions on any one of the contexts. Pre-service teachers were reminded at the start of the interview that they could opt out of the interview at any time. All the interviews were transcribed for subsequent analysis.

Silverman (2013, p. 47) notes that an important methodological issue that needs to be justified and explained is whether 'interview responses can be treated as giving direct access to

[6] One of the pre-service teachers could not make the interview for personal reasons and due to time constraints, an alternative time could not be arranged for an interview.

“experience” or as actively constructed “narratives” involving activities which themselves need analysis’. In the interview process I needed to ask pre-service teachers to elaborate on their mathematical learning experiences. When a particular concept emerged in their narrative, for example, the concept of ‘teacher involvement’, I would provide time for reflection to explain the meaning of the concept. I also identified situations in the interview where it was appropriate to ask pre-service teachers about their emotions during particular learning experiences. In this way, I treated the interview as a process through which various narratives could be constructed about their mathematical learning experiences rather than considering the interview as giving direct access to their learning experiences.

5.4.3 Group interviews at the end of the academic year

At the end of the academic year on completion of the tutoring and PST-PLC sessions, I conducted two one-hour group interviews with the pre-service teachers. One group interview was made up of pre-service teachers from three PST-PLCs and the other group interview from pre-service teachers from the other two PST-PLCs. Both group interviews were video-recorded. The aim of the group interviews was to focus on pre-service teachers’ reflections on their learning experiences within their PST-PLCs. The group interview was guided by three questions (1) How did the PST-PLC inquiry support your learning about learners’ mathematical errors? (2) How did the PST-PLC inquiry support your learning about learners’ mathematical learning experiences? And (3) what did you learn from your PST-PLC inquiry about connecting the social dimension of learning, i.e. learners’ mathematical learning experiences, and cognitive dimension of learning, i.e. learners’ mathematical errors? I used the data from these interviews to support my analysis of the inquiry within the PST-PLCs and to understand challenges in making connections between the social and cognitive dimensions of learning.

5.4.4 Pre-service teacher reflection journals

As part of their coursework assessment, the pre-service teachers were asked to keep reflection journals in which they provided detailed descriptions of their tutoring sessions with the learners whom they tutored and the PST-PLC sessions. I provided them with reflection templates to support them with their reflections. (See Appendix B1 and B2.) All the journals were collected at the end of the academic year. The journals were used to identify details of learners’ work that was not clearly captured in the PST-PLC videos, and pre-service teachers’ reflections on aspects of tutoring learners. The journals were used to strengthen my interpretations of the inquiry process in the PST-PLCs.

5.4.5 Field notes from observations of tutoring sessions

I visited the tutor sessions of pre-service teachers over the academic year, visiting one school site on a particular tutoring day and spent approximately twenty minutes with each tutor/learner group. These observations provided insight into challenges pre-service teachers were experiencing in engaging learners' mathematical experiences and learners' mathematical errors. More importantly I wanted to observe what aspects of the tutor sessions pre-service teachers would bring to the PST-PLCs and how these aspects of tutoring practice were shared.

I was able to visit each pre-service teacher's tutoring session at least once during the research period, a total of 30 tutor sessions were observed. I was a participant-observer within the tutor sessions to provide support to the pre-service teachers on engaging learners' mathematical learning experiences and learners' mathematical errors learning. The support to the pre-service teachers was in the form of co-tutoring so as not to disrupt the tutor session. Immediately after the tutor sessions, I wrote field notes and reflections which I took along to PST-PLCs to support further inquiry. I used the field notes as supporting evidence to strengthen my interpretation and argument about aspects of the inquiry that unfolded in the PST-PLCs.

5.5 Data analysis process

In this section I provide a detailed description of the data analysis approach I adopted for the primary data of the study which is the video data of the 40 PST-PLC sessions (8 sessions for each of the five PST-PLCs) and the 22 semi-structured interviews. The data from pre-service reflection journals, field notes and group interviews were used as supporting evidence in the analysis of the PST-PLC inquiry.

5.5.1 Transcribing and translating the data

All the video and audio data were transcribed by professionals who were proficient in the language spoken, namely Afrikaans. Only sections of the data selected for reporting were translated into English. I did all the translations myself to avoid outsider misinterpretation of the data. My insider position allowed me to engage with the pre-service teachers throughout the research process and to clarify language issues. I checked all the transcriptions of the video and audio data through watching the videos again and listening to the audios, to identify areas that may not have been clear to the transcriber but which I could understand, especially where mathematical terms were used. In doing the analysis of the video data, I worked with the transcripts manually.

5.5.2 Analysis process for inquiry into PST-PLCs (video data)

I selected a qualitative content analysis approach to analyse the inquiry within the PST-PLCs as my interest was in the content of the conversations and less on the linguistic structures through which communication occurs. I followed a systematic process guided by the research questions to break down the data into smaller units. The levels of reduction in the data were informed by theory, i.e. a process of generating deductive categories, and through generating categories from the data, i.e. a process of generating inductive categories. The qualitative content analysis approach followed is referred to as 'type-building' by Mayring (2014) as it involves an in-depth analysis of types, i.e. PST-PLC stances towards inquiry, identified across the five PST-PLCs. I followed two analytic processes within this approach to explore the sub-questions linked to my main research question: *What possibilities do pre-service teacher professional learning communities contribute to developing critical inquiry into the connections between learners' mathematical learning experiences and learners' mathematical errors?*

The first analytical process, involving four stages, was used to explore the first two sub-research questions:

> How do pre-service teacher professional learning communities inquire into engaging learners' mathematical learning experiences?
>
> How do pre-service teacher professional learning communities inquire into learners' mathematical errors?

The second analytical process, involving two stages, was used to explore the third sub-research question:

> What connections were made possible between learners' mathematical learning experiences and learners' mathematical errors through the pre-service teacher professional learning communities' inquiry?

5.5.2.1 First analytical process

Stage 1: Identifying Episodes

The *first stage* involved reading all transcripts of each PST-PLC session and segmenting the transcript into episodes where the inquiry into learners' mathematical learning experiences was foregrounded, and episodes where the inquiry into learners' mathematical errors was foregrounded. The episodes of inquiry into learners' mathematical learning experiences were identified when the focus in a unit of PST to PST talk over several turns was on describing learners' narratives of the mathematical learning experiences and approaches to eliciting learners' mathematical learning experiences with limited focus on learners' errors. The episodes of inquiry into learners' mathematical errors were identified where the focus in a unit of PST to PST talk over several turns was on describing the mathematical problem, describing how learners' reasoning behind the error was probed, and describing how a learner reasoned.

Stage 2: Identifying Stances

In *stage two* I classified the episodes as stances towards inquiry into learners' mathematical learning experiences and stances towards inquiry into learners' mathematical errors based on key features that emerged within PST-PLCs across the sessions. In the inquiry into learners' mathematical learning experiences, I identified two stances based on the ways in which pre-service teachers described their engaging learners' mathematical experiences and whether the PST-PLCs continued with the inquiry over all the PST-PLC sessions. The two stances, embracing and compliance, were inductively generated.

The embracing stance was evident where there was ongoing inquiry into learners' mathematical learning experiences and where different strategies for eliciting learners' mathematical learning experiences were shared and experimented with.

In the compliance stance there was a willingness of all members of the PST-PLC to share their experiences of engaging learners' mathematical learning experiences, however the inquiry was not sustained. The inquiry in the compliance stance involved narratives of learners' mathematical learning experiences and pre-service teachers' struggles to elicit learners' mathematical learning experiences. The struggles to elicit learners' mathematical learning experiences in the tutoring sessions resulted in no further inquiry into learners' mathematical learning experiences beyond the first PST-PLC session in the compliance stance.

In the inquiry into learners' mathematical errors, I identified two stances derived from the literature on teacher error analysis (Brodie, 2013; 2014). The two stances: engaging and correcting were deductively generated. In the engaging stance the inquiry focused on probing learners' reasoning behind their errors, and in PST-PLCs where this stance was evident, detailed descriptions of engaging learners' mathematical errors were provided. In the correcting stance, the inquiry focused on correcting the error and where this stance was evident, the descriptions of engaging learners' mathematical errors was based mainly on written work of learners' errors.

Stage 3: Identifying Epistemic Moves and Inhibitors of Critical Inquiry in Stances

To gain an in-depth understanding of how the inquiry into learners' mathematical learning experiences and learners' mathematical errors unfolded in each of the stances, in the *third stage*, I identified epistemic moves as enablers of critical inquiry. I also identified inhibitors of critical inquiry in each episode. These epistemic moves and inhibitors of critical inquiry were dependent on the context of inquiry and could only be identified in relation to the context and stance towards the inquiry. I set up tables of the transcripts of episodes indicating the epistemic moves and inhibitors of critical inquiry into learners' mathematical experiences (see Chapter 7) and into learners' mathematical errors (see Chapter 8).

The epistemic moves of critical inquiry were generated deductively and inductively. The epistemic moves: describing, explaining, predicting, arguing, critiquing, inferring, and comparing, were epistemic moves identified in research (Ohlsson, 1995; Zenios, 2011). I adapted some of Zenios's (2011) epistemic moves. For example, critiquing is defined by Zenios (2011) as illustrating the strengths and limitations of a view or the flaw in an argument. In my study when a critique involved a limitation or negative aspect of an artefact or teaching approach, I considered it an epistemic move to probe for an alternative. I classified pressing as an epistemic move, as along similar lines as a push or challenge facilitation move in research on PLC inquiry (Brodie, 2016). The epistemic moves generated inductively in my research were raising uncertainty, making assumptions and providing further evidence.

Table 5.2 illustrates epistemic moves and an example of one subcategory of each move, identified to deepen critical inquiry into learners' mathematical learning experiences. With each epistemic move identified, I illustrate the potential of the move to deepen the inquiry. Nine categories of epistemic moves were identified in the inquiry into engaging learners' mathematical learning experiences in the PST-PLCs.

Table 5.2: Epistemic moves identified to deepen inquiry into engaging learners' mathematical learning experiences

Epistemic moves	**Description of move**	
EM 1: Describing	**To provide an accurate description of an object or event**	**Deepening the inquiry**
Examples	**Dialogue**	
Describing information about a learner's home context	And both of them said their mothers help them with mathematics in the afternoon	Into how learners receive support from parents
EM 2: Reflecting	**To question; notice; make suggestions based on what is being said or based on past experiences**	**Deepening the inquiry**
Example	**Dialogue**	
Reflecting on a PST's account of engaging a learner	So she is challenging you?	Into what the implications are for learning when learners take the initiative to challenge.
EM 3: Comparing	**To point out the similarities or differences in a situation**	**Deepening the inquiry**
Example	**Dialogue**	
Comparing of teaching approach when at school to the current teaching approaches and pointing out the difference	We learnt that step in Grade 4 and we used it until Grade 7, even now we are still using it. Whereas in Grade 4 they use a long method, next year a little bit shorter, then again shorter and in Grade 7 they do the short method.	Into the mathematical connections between the different approaches
EM 4: Critiquing	**To point out the positives or negatives of a situation**	**Deepening the inquiry**
Example	**Dialogue**	
Critiquing of curriculum	So they learn a lot of extra things, which is unnecessary in my opinion.	Into reasons for the different teaching approaches

EM 5: Inferring	To come to a conclusion based on evidence	Deepening the inquiry
Example	**Dialogue**	
Inferring about the learners' participation in class	I would like to add to this … it shows me they pay attention when the teacher explains something.	Into the implication of this inference about learners who are not confident to explain their learning experience
EM 6: Raising uncertainty	**To express the need for greater clarity on an approach or concept**	**Deepening the inquiry**
Example	**Dialogue**	
Raising uncertainty about the tutoring approach	My learners are also in Grade six like PST 4C's learners and so I asked them as they are also doing angles what is the definition of an acute angle and obtuse angle because that is where we begin right?	Into the teaching approaches for angles
EM 8: Making assumptions	**To draw conclusions without the necessary evidence**	**Deepening the inquiry**
Example	**Dialogue**	
Making an assumption about learners' parents	Not that they are necessarily good at it or in a business involved with it, but they at least get help.	Into how to use the support learners receive from parents to challenge assumptions of deficit
EM 9: Explaining	**To explain a reason for a situation**	**Deepening the inquiry**
Example	**Dialogue**	
Explaining a reason for learners' challenge	I still think the question was badly posed that is why	Into how questions could be phrased to support the learning.
EM 9: Pressing	**To press for specificity or to elaborate on a situation**	**Deepening the inquiry**
Examples	**Dialogue**	
Pressing for more information on how learner's mathematical learning experiences (MLExps) were elicited	Did you ask the learner why he explained it like that to you?	Through elaborating on how learners' MLExps were elicited

In Chapter 7, I elaborate on all the subcategories of these epistemic moves that emerged across the stances.

Table 5.3 illustrates epistemic moves and an example of one subcategory of each move, identified to deepen critical inquiry into learners' mathematical errors. Ten categories of epistemic moves were identified in the inquiry into learners' mathematical errors in the PST-PLCs.

Table 5.3: Epistemic moves identified to deepen critical inquiry into learners' mathematical errors

Epistemic moves	Description of move	
EM 1: Describing	**To provide an accurate description of an object or event**	**Deepening the inquiry**
Examples	**Dialogue**	
Describing how a learners' reasoning was probed	I asked him why he said seventy was a prime number	Into how learners' reason and reasons for learners' errors
EM 2: Pressing	**To press for specificity or to elaborate on a situation**	**Deepening the inquiry**
Example	**Dialogue**	
Pressing for reasons for learners' challenges with mathematics content	But why do you think she would understand it like that [pointing to flow diagrams] and not like that [pointing to the table]	Into reasons for learners' challenges
EM 3: Reflecting	**To question; notice; make suggestions based on what is being said or based on past experiences**	**Deepening the inquiry**
Example	**Dialogue**	
Reflecting on PSTs' challenges with mathematical content	But I will not be able sit and say take one to a hundred and work it out in my head…	Into reason for PSTs' challenges and the similar challenges of learners

EM 4: Explaining	To explain a reason for a situation	Deepening the inquiry
Example	**Dialogue**	
Explaining a reason for learners' errors	They are sometimes too hasty and then they cannot do it.	Through exploring other possible reasons for learners' errors
EM 5: Comparing	**To point out the similarities or differences in a situation**	**Deepening the inquiry**
Example	**Dialogue**	
Comparing of mathematical representations	Yes, because there [referring to table compared to flow diagram representation] they do not know what to multiply by.	Into learners' errors with mathematical representations
EM 6: Raising uncertainty	**To express the need for greater clarity on an approach or concept or situation**	**Deepening the inquiry**
Example	**Dialogue**	
Raising uncertainty about how a learner reasons	But is strange because he understands what it means	Into why learners do not always draw on their prior knowledge
EM 7: Making assumptions	**To draw conclusion without the necessary evidence**	**Deepening the inquiry**
Example	**Dialogue**	
Making an assumption about a teaching approach	And if they do it [referring to implementing the practising of multiplication tables] before class, then it will nicely get them thinking and to work nicely for the rest of the time	Into reasons for the benefits of the approach
EM 8: Arguing	**To state reasons for or against a particular position**	**Deepening the inquiry**
Example	**Dialogue**	
Arguing for a teaching approach	I still feel that multiplication tables must be practised, it must just – it must just come, And the thing is, it is not going to happen	Into the strengths and limitation of the teaching approach, in this case mental tests based on speed and memorisation

	within a period and that is why I say those speed tests work.	
EM 9: Predicting	**To raise concerns about a situation which leads to making a prediction about the situation**	**Deepening the inquiry**
Example	**Dialogue**	
Predicting learners' difficulties may be age related	They are only twelve years old and they only started with that.	Further reasons for learners' challenges
EM 10: Providing further evidence	**To provide further information on a situation that enables deeper understanding of a situation**	**Deepening the inquiry**
Example	**Dialogue**	
Providing further evidence of learners' work	And look they could do it there ([shows the PLC a flow diagram] like PST 4D	Further reasons for why learners find certain mathematical representations more challenging

In Chapter 8, I elaborate on all the subcategories of these epistemic moves that emerged across the stances.

Identifying the inhibitors of critical inquiry was important to explain why epistemic moves at times only generated potential for deepening inquiry, and did not actually deepen the inquiry. Table 5.4 provides a description of the inhibitor categories in the inquiry into learners' mathematical learning experiences, and Table 5.5 provides description of the inhibitor categories in the inquiry into learners' mathematical errors. An example of one subcategory of each inhibitor is given to illustrate how the inquiry is inhibited.

Table 5.4: Inhibitors of critical inquiry into learners' mathematical learning experiences

Inhibitor of critical inquiry	**Description of inhibitor**	
Limited description	**When limited or incomplete description of an event was given**	**Inhibiting the inquiry**
Example	**Dialogue**	
Describing a learner explaining how the learner's teacher taught	She explained and I understood what the teacher's approach is	Through limited detail of how the learners explained a teacher's approach
Non-reflecting on practice	**The absence of reflecting-on-practice and learning**	**Inhibiting the inquiry**
Example	**Dialogue**	
No reflecting on PSTs prior learning experiences	My learner he has a brother in high school and he helps him.	Through no reflection on own learning experiences of being supported in home in order to elicit specific aspects of learners' learning experiences in home
Non-recognition of epistemic moves	**Not recognising how epistemic moves deepen inquiry**	**Inhibiting the inquiry**
Example	**Dialogue**	
Non-recognition that the epistemic move referred to as making an assumption is an opportunity to challenge	Not that they [learners' parents] are necessarily good at it	Through no challenge on the assumption that learners' parents are not good at mathematics or with supporting their children

Table 5.5: Inhibitor of critical inquiry into learners' mathematical errors

Inhibitor of critical inquiry	**Description of inhibitor**	
Limited description	**When limited or incomplete description of an event is given**	**Inhibiting the inquiry**
Example	**Dialogue**	
PSTs describing how they were taught	I remember in our first year when we did prime numbers with lecturer King and I remember the	Through not elaborating in detail how they were taught

	table in the examination paper and I remembered how to do it Yes, the first hundred …Yes, like one yes, two no, and then you go on like that	
Limited reflecting-on-Practice	**When reflecting is limited to observation/noticing without elaboration**	**Inhibiting the inquiry**
Example	**Dialogue**	
Reflecting on mathematics concepts with which learners struggled	Gee, but one can see there is a three so it can divide into three Yes And there is a zero at the end So there are a few things…	Through making judgements about what learners ought to know rather than thinking more deeply about why they made the error
Non-recognition of epistemic moves	**Not recognising how epistemic moves deepen inquiry**	**Inhibiting the inquiry**
Example	**Dialogue**	
Non-recognition that the epistemic move referred to as raising uncertainty is an opportunity to further explore knowledge	I think because this [pointing to flow diagram] is more visual….I also do not know	Through not exploring further knowledge on learners' understanding of different mathematical representations

In Chapters 7 and 8 I elaborate on all the subcategories of inhibitors of critical inquiry into engaging learners' mathematical learning experiences and learners' mathematical errors respectively across the stances.

Stage 4: Analysis of deepening inquiry through knowledge

In *stage four* I analysed how the epistemic moves enabled the deepening of inquiry through knowledge. I used Cochran-Smith and Lytle's (1999) definition of knowledge-in-practice, knowledge-for-practice and knowledge-of-practice to analyse how epistemic moves deepen critical inquiry through this knowledge.

5.5.2.2 Second analytical process

The second analytical process involved a comparative analysis of stances towards inquiry into engaging learners' mathematical learning experiences (see Chapter 7) and towards inquiry into learners' mathematical errors (see Chapter 8).

Stage 5: Comparative Analysis: Inhibitors of Inquiry; Epistemic Moves and 'Representation of Practice' across Stances

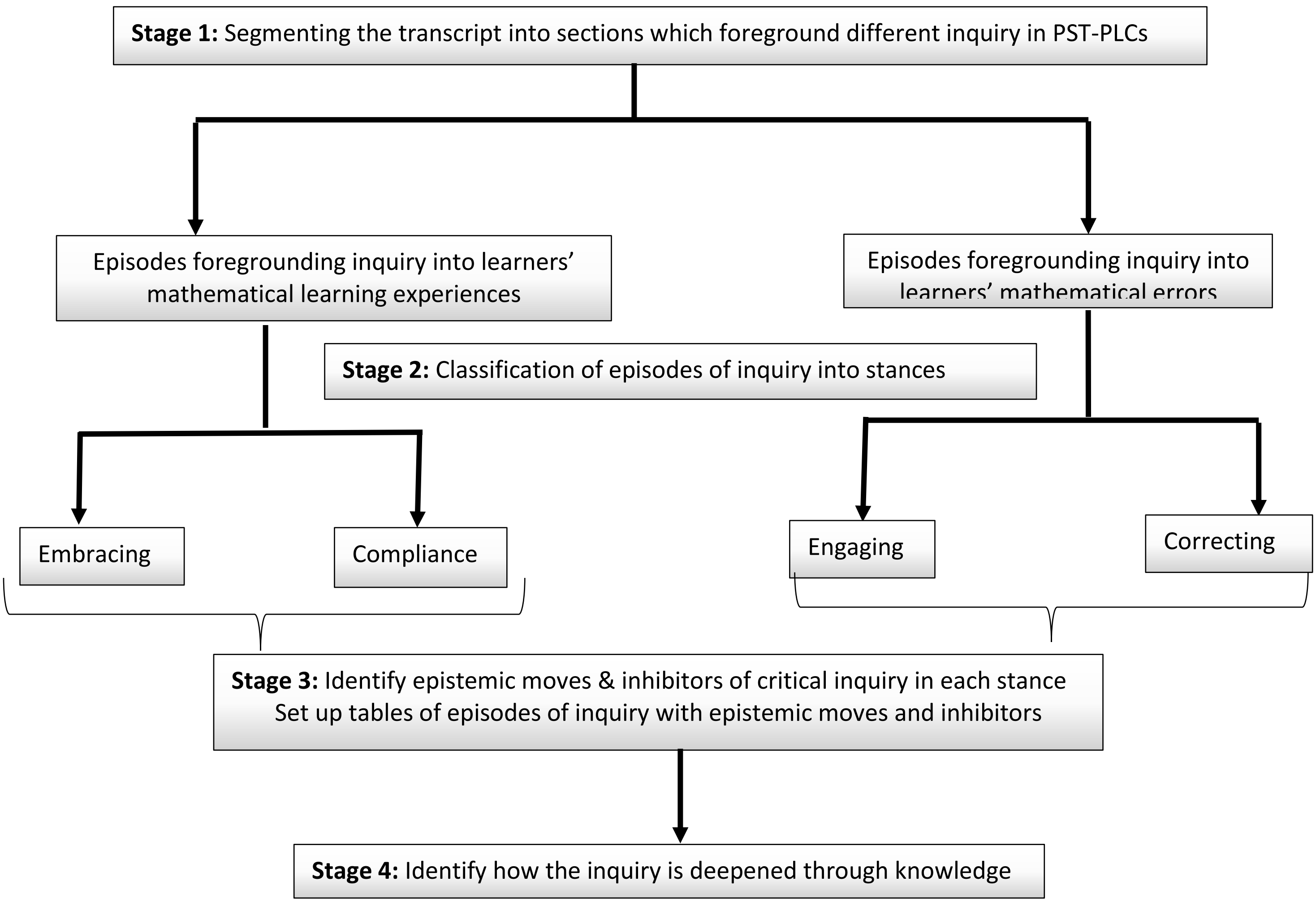
The different stances impacted on the social and cognitive dimensions that emerged from inquiry into learners' mathematical learning experiences and learners' mathematical errors. In order to gain a deeper understanding of this impact, *stage five* involved a comparison of the categories and subcategories of the inhibitors and epistemic moves which were representative of the stances towards inquiry. The inquiry was also influenced by what was selected from pre-service teachers practice of engaging learners' errors from both social and cognitive dimensions. Hence, I compared the descriptions of the tutoring practice which are representations of practice (Horn & Kane, 2015; Little, 2002) across the inquiry stances with the aim of understanding how the nature of the representations of practice influenced the flows of practice (Kazemi & Hubbard, 2008) across two learning spaces: the tutoring space and the PST-PLC space. This analysis provided deeper insight into how representations of practice, flows of practice and inquiry into engaging learners' mathematical learning experiences and learners' mathematical errors were related.

Stage 6: Comparative Analysis: Epistemic Moves in Social-Cognitive Connection Learning Opportunities Across Stances

In *stage six*, I analysed the epistemic moves across the stances to identify those with greatest potential for making connections between learners' mathematical learning experiences, i.e. the social dimension of learning, and learners' mathematical errors, i.e. the cognitive dimension of learning. This analysis made it possible to identify and explain differences in social-cognitive connection learning opportunities which emerged across the stances. In Chapter 7, I analyse the potential social-cognitive connection learning opportunities as a result of a cognitive dimension emerging when a social dimension was in the foreground. In Chapter Eight, I analyse the potential social-cognitive connection learning opportunities as a result of a social dimension emerging when a cognitive dimension was in the foreground.

Figure 5.1 provides an overview of the analytical approach followed.

First analytic process



Second analytic process

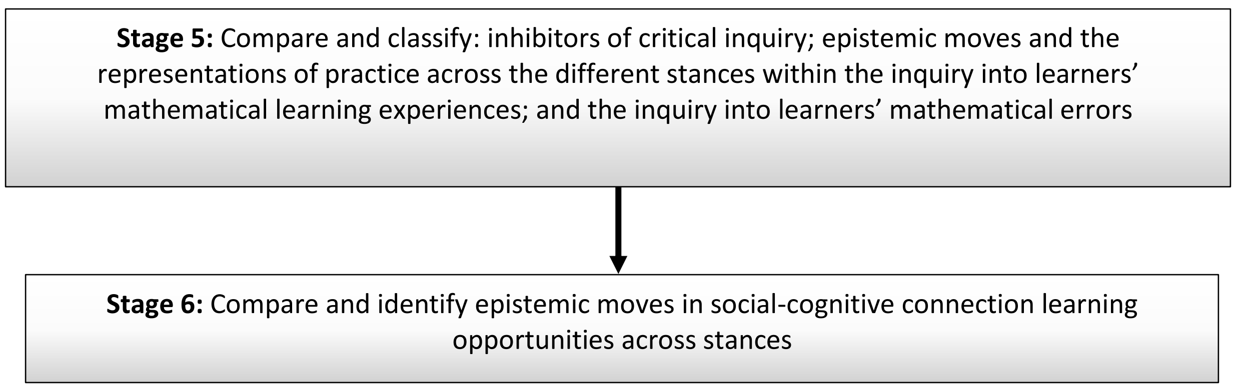


Figure 5.1: Overview of analytical process of video data

5.5.3 Analysis process for semi-structured interview data

The transcripts of the individual interviews of the pre-service teachers were imported into Atlas TI to facilitate the coding process. I applied an inductive approach to coding the interviews which generated the codes which I grouped according to themes related to each of the three contexts in which pre-service teacher mathematical learning experiences were explored: in mathematics classrooms at school, in their homes and their broader communities.

In the school context the teacher was central to how pre-service teachers described their mathematical learning experiences. I identified the first theme of 'teacher care' which emerged from codes related to teachers' teaching practice, namely, teachers' teaching approach and teachers' engagement with learners' difficulties. The second theme, 'mathematics content', emerged from codes related to pre-service teachers' understanding of mathematics content: motivated to learn and disengaging from learning. The third theme, 'emotions of pre-service teachers', emerged within codes related to teachers' teaching practices and pre-service teachers' understanding of mathematics content.

In the home context the parent was central in how pre-service teachers described their mathematical learning experiences. The only theme, 'kind of support', emerged from codes related to parents' interest in pre-service teachers' mathematics performance in school, namely, checking on homework and enquiring about areas of challenge.

In the context of the broader community the support with mathematics from others, mainly tutors, was central in pre-service teachers' descriptions of their mathematical learning experiences. The first theme, 'tutoring support', emerged from codes related to the tutoring context: individual or group tutoring and the tutors' approach. A second theme, 'mathematics in work context', emerged from codes related to pre-service teachers' understandings of mathematical learning experiences in the broader community, namely, application of mathematics in everyday situations, and reasons for not relating mathematics beyond school.

In Chapter Six, I provide extracts that are representative of the themes across the different contexts of pre-service teachers' mathematical learning experiences.

5.5.4 Analysis process for semi-structure group interview data

I analysed the transcripts of the two one-hour group interviews manually by setting up the transcripts in a table in which I coded the pre-service teachers' descriptions of their learning within their PST-PLCs.

The pre-service teachers' descriptions of their learning through the inquiry into engaging learners' mathematical learning experiences and the inquiry into learners' mathematical errors was coded either as 'content' or 'teaching practice'. The 'content' code included descriptions of mathematical content that was learnt through the PST-PLC inquiry. The 'teaching practice' code included descriptions of learning about how to teach mathematics content, how to probe learners' reasoning, how to elicit learners' learning experiences, learning about reasons for learners' errors and reasons for learners' mathematical learning experiences.

In the group interview, the pre-service teachers struggled to explain what they learnt about connections between the cognitive and social dimensions of learning in the PST-PLCs. I asked the pre-service teachers to reflect on how they understood the social dimension of learning first and then to think about connections with the cognitive dimension of learning. In response to this reflection the theme, 'social relations' emerged from codes related to pre-service teachers' understanding of teachers' teaching practice involving a social dimension: knowledge of learners and the role of the teacher in engaging learners. I drew on the analysis of the pre-service teachers' reflections on their learning in the PST-PLCs to interpret findings in the PST-PLC inquiry.

5.6 Rigour in the research

Mays and Pope (1995, p. 110) argue that to ensure rigour in all forms of research the basic strategy is a 'systematic and self-conscious research design, data collection, interpretation and communication'. A key challenge for qualitative researchers is to provide a convincing account given the vast amount of data usually in the form of text and the difficulty faced in summarising this data. In the next sections I discuss the concepts of validity and reliability that are central to ensuring research rigour and strategies that were used in my study.

5.6.1 Validity

The validity of research according to Silverman (2013) concerns the 'credibility of our interpretation of observations'. The definition of validity provided by Hammersley (1990, p. 57, cited in Silverman, 2013) refers to the 'extent to which an account represents the social phenomena it represents'. In my study, pre-service teachers' collaborative inquiry in PST-PLCs was captured through video and transcribed word for word which enhanced the accuracy of the data. The video data represented what was happening but not necessarily why, which is the reason for the additional data collection: pre-service teachers' individual and group interviews.

Seale (1999) references Cain and Finches (1981) argument that multiple data sources can help deepen understanding of different aspects of a phenomenon. In my study it was necessary to use data from different sources as critical inquiry into connections between learners' mathematical learning experiences and learners' mathematical errors is a complex phenomenon. The group interviews for example brought to the fore how pre-service teachers' understandings of the meaning of learners' mathematical learning experiences as a social dimension of learning influenced their inquiry. Through the use of multiple data sources, I was able to provide a more comprehensive account of the phenomenon of pre-service teacher critical inquiry within PST-PLCs which enhanced the validity of the data analysis.

Patton (2002) argues that credibility is related to the integrity of the researcher and calls for a critical-reflective stance (reflexivity) on the part of the researcher. Creswell and Miller (2000) view researcher reflexivity as making known the assumptions, beliefs and biases that may influence the research. I was aware that my position as an insider researcher may influence the validity of the data in the sense that pre-service teachers may have felt that they were being formally evaluated as the research was situated in their MSLC. I addressed this issue through ensuring the pre-service teachers that the research, which included their participation in the professional learning communities, the individual interviews and group interviews would not be a component of their assessment which would hopefully minimise their anxiety and contribute to the credibility of their responses.

A further issue of validity which I addressed was looking for disconfirming evidence when exploring emerging patterns in the data from which claims were made (Miles, Huberman, & Saldaῆa, 2014). Through paying attention to disconfirming evidence, only claims for which little or no contradictory evidence could be found within the data were presented as valid claims for my study.

To establish the validity of the epistemic moves, I used the list of epistemic moves from the literature as a guide (Ohlsson, 1995; Zenios, 2011). I found that not all of the epistemic moves from the literature were present in my study and that in my study there were epistemic moves not present in literature which supports Zenios's argument that the list of epistemic moves is context related.

5.6.2 Reliability

Hammersley (1992, p. 67, cited in Silverman, 2013) defined reliability as the degree of consistency with which instances are assigned to the same category by different observers or by the same observer on different occasions. In addressing the issue of consistency, I was firstly challenged by whether I provided completeness of all possible stances towards the inquiry into learners' mathematical learning experiences and learners' mathematical errors. I recognised that checking for completeness was a greater challenge in the case of the inquiry into learners' mathematical learning experiences as there was no prior research to reference and hence the stances were derived inductively. In the case of inquiry into learners' mathematical errors, the possible stances were based on research dealing with learners' mathematical errors. The next related challenge was with regard to representativeness of episodes of inquiry within a particular stance. The difficulty of finding a co-researcher to identify stances and epistemic moves as a means of cross-checking was related to funding. To address this challenge, I provided more than one instance within the embracing stance towards inquiry towards learners mathematical learning experiences and within the correcting stance towards learners' mathematical errors in which there were different nuances within the inquiry. I provide more than one episode in these stances as evidence for the reader to judge whether both instances fit the criteria for a particular stance.

5.7 Ethical clearance

This study has ethical clearance from the Wits University Ethics Committee, Protocol Number: 2014ECE025D. The third-year B.Ed. mathematics service learning course is registered as a community engagement project of the Cape Peninsula University of Technology (CPUT). Application for ethical clearance to work with learners in after-school tutoring sessions was granted from the following: the CPUT Community Engagement Committee and the Ethical Committee of the CPUT Faculty of Education, the CPUT Community Engagement Directorate; the Western Cape Education Department and the Department of Child Welfare. This was done because special permission is required to work with minors in schools or community centres.

5.8 Additional ethical considerations

I addressed ethical concerns in earlier sections of the chapter with regard to my dual role as lecturer of the MSLC in which the research is located and as researcher of the study. In this section, I focus on issues of informed consent and confidentiality and anonymity.

All 23 pre-service teachers of the B.Ed. third-year MSLC gave their informed consent to participate in all aspects of my research. The pre-service teachers were informed the year before that I was in the process of doing PhD research and that an aspect of the B.Ed. third-year mathematics service learning course would form part of my study. The pre-service teachers' participation in the research was explained to them in detail at the beginning of the academic year. I informed them from the beginning that they had the right of refusal and retained the right to withdraw from the research at any time with no impact in any way on their course participation. For example, I explained to them that should they choose to withdraw from the research, they would continue to be members of the PST-PLCs but that the community within which they were would no longer be video-recorded. None of the pre-service teachers withdrew from the research or the MSLC and the excellent course attendance minimised any negative impact on the research process.

The pre-service teachers granted their permission for the use of video to record their participation in the professional learning communities and group interviews. Permission was also granted for the use of an audio-recorder to record individual interviews and reflection journals of pre-service teachers.

I used numbers and letters for naming pre-service teachers and the PST-PLC groups to address the issue of anonymity and to minimise pre-service teachers' recognition of other PST-PLC members. I addressed issues of confidentiality with the persons who did the video recording and transcriptions of interview and video data. The video recordings were transcribed without the presence of other individuals and all videos were returned within a day on completion of the transcription.

The guarantee of complete confidentiality is difficult particularly when presenting findings to the research community as I acknowledge that the research is part of a course which I lecture. I avoided mentioning the year in which the research was conducted and blotted out the faces of research participants when a picture was shown of the PST-PLCs when presenting on my research.

5.9 Chapter summary

In this chapter, I have described how the methodological choices are related to the research aims. I have explained the choice of a multiple-case study design and provided the theoretical debates around the use of video recording which was the primary method for data collection in my study. The data analysis followed a qualitative 'type-building' content analysis approach

The next chapter presents the analysis of the individual interviews with pre-service teachers on their mathematical learning experiences as school learners.

CHAPTER 6

Pre-Service Teachers' Mathematical Learning Experiences

6.1 Introduction

The aim of this chapter is to present and analyse the findings of interviews with individual pre-service teachers about their mathematical learning experiences in three contexts: at school, in their homes and in their broader communities. I interviewed the pre-service teachers about their mathematical learning experiences in these three contexts as they themselves were being encouraged to engage learners about their mathematical learning experiences in these contexts. Research studies provide evidence of how pre-service teachers' prior teaching and learning experiences influence their beliefs about the nature of knowledge, teaching and learning (Lortie, 1975; Joram & Gabriele, 1998; Guillaume & Kirtman, 2010, Mansfield & Volet, 2010).

Conway and Jobson (2012) further argue that the experiences that people remember and narrate from their past have the potential to reveal specific events which influence their beliefs and actions. Pre-service teachers' current exposure to teaching and learning may also influence how they reflect on their past learning experiences. Whether pre-service teachers reflect on their own mathematical learning experiences when engaging learners on their mathematical learning experiences and how these reflections form part of the PST-PLCs' inquiry is explored in the next two chapters.

I identified themes within each of the three contexts in which pre-service teachers' mathematical learning experiences were explored. In the school context, teachers' teaching practice with a focus on care as a moral dimension was a central theme and the other two themes were the nature of the mathematics content and the emotions experienced in learning. In the home context, the kinds of support with mathematics was the only theme. In the context of broader community two themes, tutoring support with mathematics and mathematics in work contexts were identified. A comparative analysis of the pre-service teachers' mathematical learning experiences within the different contexts shows differences in social, cognitive and emotional dimensions of learning.

6.2 Mathematical learning experiences at school

The role of teachers in learners' participation in mathematics is the central theme in pre-service teachers' narratives on their mathematical learning experiences at school. The other themes were the influence of the mathematics content in learners' participation and learners' emotions linked with mathematics learning.

6.2.1 The role of teachers in creating a caring learning environment to enhance learner participation.

A central theme threading through the pre-service teachers' narratives about the influence of their teachers on their mathematical learning experiences is the concept of teacher care. Teachers' teaching approaches included narratives of teachers who made their lessons interesting. These teachers were described as being passionate about mathematics and there was a sense of care to make the lessons enjoyable for all learners. Pre-service teachers said that teachers who cared would try to generate a classroom culture which made learners feel that they were individuals with their own challenges.

Emerging from pre-service teachers' narratives is the view that teacher care involves ensuring that all learners are successful. Caring teachers are those who made great effort to diagnose what learners were struggling with and who were prepared to listen to learners. Pre-service teachers described their participation in their mathematics classrooms as being linked to their personality but the learning environment which the teacher created was by far the most important factor influencing their participation.

The extracts selected to represent this theme focus on pre-service teachers' narratives about the role their teachers played in relation to engaging them with their challenges or mathematical errors.

In extract 1, PST 1B's reason for enjoying mathematics at school is linked to a teacher who she described as being aware of what learners in class was struggling with:

Extract 1 from an interview with a PST 1B

R: Can you tell me when you enjoyed mathematics the most at school.

PST 1B: It was with my Grade 6 teacher. She made the mathematics interesting and it was the way in which she spoke that one knew that she understood what you were struggling with and this made you feel special. In her class she got to know you personally. I was also a friend of her son so perhaps on that level we got to know each other better. But it was the manner in which she, I do not know how to explain but in Grade 6 I enjoyed the mathematics.

PST 1B does not elaborate on a specific example of an aspect of the mathematics that she struggled with. She focusses on how the teacher spoke and highlights that the teacher got to know the learners personally. She highlights that being aware that the teacher was paying individual attention made her feel special which suggests that there is an emotional dimension to her learning experience.

In extract 2, PST 4D's narrative focuses on how her willingness to participate in class and ask questions when she did not understand was linked to the learning environment created by a teacher:

Extract 2 from an interview with a PST 4D

PST 4D: The teacher I had from Grade 10 to matric was very calm and she always made sure you understand okay. Do you understand… do you understand… and if you did not understand you felt comfortable to ask her even if she explained it sixty million times. One was never afraid to ask her because she was calm and she was very thorough with her work and I was never worried that I would not make it. But those learners who were worried could always go to her. And we had a learner, Chris[7] in our class who was very bright and she always asked him to support her when she struggled to get something across. She would say Chris you come and explain to her because maybe I would understand Chris's way.

PST 4D's exaggeration of the number of times the teacher would have explained and still address their questions illustrates her sense of the extent of the teacher's care to ensure that learners' understanding was prioritised. PST 4D's description of the teacher's practice of encouraging a learner to support other learners is evidence of a teacher practice in which the teacher does not view herself as the sole authority on supporting learners who struggle. PST 4D provided a reason given by the teacher for involving the learner Chris. Remembering the reason given by the teacher suggests that the teacher's practice and calmness made an impact on her mathematical learning experience.

PST 4D implied that she developed a sense of confidence based on the thorough approach of the teacher and knowing that the teacher would be available to support if she struggled. PST 4D's narrative of the teacher's practice evokes cognitive, social and emotional dimensions of her mathematical learning experiences. PST 4D's narrative also shows that these dimensions are interlinked in the teacher's practice. For example, the teacher's concern about the learners'

[7] This is a pseudonym

understanding and willingness to address their questions involved the encouragement of peer-learning support.

In extract 3, PST 2E's narrative describes a teacher's care as going beyond a focus on learners' performance in mathematics:

Extract 3 from an interview with a PST 2E

PST 2E: Because the teacher I had was really nice…wait that is not the way to describe it. She always went that extra mile.

R: Okay.

PST 2E: She knew each learner and she made an effort to speak to us. And not only on an academic level or focused on school. She tried to find out about your home circumstances. She took an interest in you as a person and not only in your school work. She did not always feel that to get high marks was the most important. It was more important for her that you understood and enjoyed it in her class. She even allowed you to eat but that you listen…

PST 2E noted that the teacher made an effort to speak to the learners about school and their lives beyond school. She viewed the effort the teacher made to speak to learners about their lives beyond the classroom as an interest in learners as holistic beings. Learners' lives beyond school as a 'fund of knowledge' for learning within school is not evident in PST 2E's narrative. PST 2E emphasised the teacher's view of the importance of ensuring that learners understood as well as creating a learning environment which learners enjoyed over that of attaining high marks. This emphasis is an indication of PST 2E's awareness of her teacher's values.

In extract 4, PST 3B's narrative of the teacher's engagement with mathematical errors portrays similarities to the pre-service teacher narratives in the previous extracts in that the teacher made an effort to support all learners who struggled. Her narrative shows how she develops an awareness of a diagnostic approach to learners' mathematical errors as one that involves different kinds of support to learners.

Extract 4 from an interview with a PST 3B

R: If you think back to when you made an error how did the Grade 12 teacher go about working with errors and how did you feel about it?

PST 3B: When we made simple errors like add or subtract she would simply help and make it right. But with more serious errors she would look whether most of the learners in the class struggled and she would then explain from the start, step-by-step. And she would tell the class if they still struggled then they are welcome to ask her after school where she provided one-on-one support.

R: And how did you feel about this way of engaging with errors compared to the way in which we are engaging with learners' errors?

PST 3B: I liked the way she handled it as it always felt to me that she was involved with what we are doing. And I felt I could always go to her if I struggled and she took careful notice of our marks. And she did take notice of errors that we made in a test and she would come and ask if we understand our error, which for me is very much the same to what we are doing now.

R: Could you elaborate a bit more on the idea of teacher involvement.

PST 3B: Okay, I feel, shoo…. that a teacher must at all times be involved with their learners because learners do not really have that self-discipline yet that we are expected to have in mathematics. Yes, learners must be able to some degree work on their own, but the teacher must at all times be aware of where learners are and their ability to do something.

PST 3B recognises the teacher's different approaches in dealing with different kinds of errors and how at times the teacher engaged with learners' errors with the whole class and at other times with individual learners. It is important to note that PST 3B mentions that the teacher *'took careful notice of their marks'* and recognised that when the teacher approached learners after a test on an individual basis the teacher focused on specific errors. In the case of whole-class error engagement, which was through a re-teaching approach, learners were invited to ask for individual support after school. PST 3B does not recognise that the approach to engaging learners' errors in the tutoring session or in a whole class was not based on re-teaching but the probing of learners' reasoning behind their errors.

PST 3B's response, when encouraged to elaborate on what it means for a teacher *'to be involved with what learners are doing'*, can be interpreted as a teacher's diagnosis of learners' difficulties as central to the concept of teacher involvement or care in learners' learning. It can also be inferred that PST 3B views the diagnosis of learners' challenges as important in supporting learners with understanding as well as developing a sense of self-discipline or taking ownership of their learning. PST 3B's reification of the concept of 'teacher involvement' has a predominantly cognitive dimension of learning in the teachers' recognition of the learners' difficulties followed by re-teaching. Yet PST 3B acknowledges that learners *need to take some responsibility for their learning* which indicated the opportunity to develop a social dimension of learning. PST 3B's reification of 'teacher involvement' therefore has elements of the cognitive and social dimensions of learning.

6.2.2 The role of mathematics content in learners' participation in mathematics

The mathematics content was either a motivator for enjoying the mathematics or for disengaging from learning mathematics. Many pre-service teachers indicated that they enjoyed mathematics at school when they understood the content. In some cases, the teachers were blamed for their lack of understanding and in other cases the teachers were not blamed and pre-service teachers acknowledged that they had difficulty in understanding the content.

There were pre-service teachers who indicated that they enjoyed mathematics when the content was challenging. Many pre-service teachers indicated that they enjoyed mathematics more at high school as the mathematics was more of a challenge in high school. I probed their interpretation of the concept of engaging in mathematically challenging problems. In extract 5 PST 1A presented a concept of engaging with challenging mathematical problems as an opportunity to persevere:

Extract 5 from interview with PST 1A

R: As a learner at school when did you enjoy mathematics the most?

PST 1A: I will say it was especially in Grade 11 and Grade 12 when the mathematics was difficult. Those problems were a page long. And you went home just before a test to study and you did those problems. You saw the answer but you just do not get it. You try three times, a fourth and fifth and that feeling that you can do the problem and when you then get it right

R: Wow and

PST 1A: That is the nicest

R: The challenge

PSTS 1A: Yes

R: The challenge

PST 1A: And it is nice if I see how many I actually can get right and for me to see what the right answer is. I hate it when I did a problem and I do not know whether my answer is correct or not.

A learning experience which involves a challenge and the opportunity to persevere and knowing *'how many I actually get right'* emerged as strong motivator to engage in learning mathematics for many pre-service teachers. Later in the interview PST 1A mentions that the mathematical challenge needed to be of such a nature where she could find problems of a similar type in which there is a step-by-step method of how to solve the problems.

Later in the interview PST 1A also mentions that she was struggling with the content in her current course lectures where she was expected to draw on her knowledge of circles to explore the ellipse. This is not the kind of challenge she appreciated and pointed out that first the work had to be explained and sample questions worked through step-by-step so that she could know what to do. This suggests that PST 1A's concept of challenging problems is limited to problems for which she knows a specific method or solution process.

In extract 6, PST 2A presents a different concept of 'a challenge' and what influenced her engagement with problems of a more challenging nature presented by the teacher.

Extract 6 from interview with PST 2A

R: Do you feel that you were challenged sufficiently at school?

PST 2A: Yes, because we were the academic class, the teacher always moved at a faster pace with us than the other classes.

R: Okay, for example

PST 2A: So with the trigonometry, that was when I started not enjoying the mathematics as it was already difficult for me what we normally needed to do.

R: Yes

PST 2A: And then the teacher gave us those A5 sheets with additional problems. I did not do it. At first I did a few and then I did it according to the way we were taught. If I got it incorrect I still tried but in the end I decided I will rather just focus on questions that we will get in the final examination.

PST 2A's concept of a challenge was related both to the pacing of the content as well as the nature of the mathematics problem. The disengagement from doing the more challenging problems was based on expectations of final examination questions and thus PST 2A's narrative provides evidence of how learner assessment influences a learner's learning disposition. In my study, pre-service teachers were encouraged to embrace learners' errors as 'springboards for inquiry' into mathematics which means extending learners' thinking into taking on new challenges in mathematics (Borasi, 1987). A learning disposition towards disengagement with challenging problems might influence the practice of embracing of learners' errors. The limitation placed on challenging problems by PST 1A might also influence her practice of embracing learners' errors.

At a point in the interview PST 2A mentions that she valued the importance of persevering when confronted with a challenge and not relying only on a textbook. Her concept of a challenge however also has boundaries. PST 2A and PST 1A and many pre-service teachers indicated that they appreciated a mathematical challenge but a challenge that aligns with what has been scripted in the artefacts of the curriculum: textbooks and formal/final examinations and clear final answers.

In extract 7, PST 4B's narrative about her worst mathematical learning experiences relates to content that was not given sufficient attention.

Extract 7 interview with PST 4B

R: What was one of the worst mathematical learning experiences for you at school?

PST 4B: Definitely word problems in mathematics. No attention was given to this. Truly miss from Grade 8 until matric they told us that if you do not understand word problems, it does not matter, it only counts five marks in all examinations anyway. So if you do not understand it, do not go and study it, as it only counts five marks.

PST 4B's narrative provides evidence that teachers reinforced a culture of disengaging with particular mathematical content based on value in terms of marks in a final assessment. Brown (2010) also provides evidence of pre-service teachers who describe their teachers' teaching to a test within high-stakes standards-based accountability reforms. It is significant that PST 4B did not critique the teaching approach of teachers when engaging with word problems as her re-collection of her learning experience is dominated by the lack of attention given to this aspect of mathematics.

Many pre-service teachers' narratives include experiences of reluctance to pose questions in class when they were faced with mathematical challenges. The reason for this form of disengagement was linked to the ways in which schools streamed[8] learners, making certain learners feel inadequate or feeling that teachers would view them as 'stupid'. This is illustrated in extract 8 below. The social structural aspects of schooling have been found to influence learners' learning experiences (Lange, 2009, p. 28). In some cases, teachers' explanations were considered incomplete by the pre-service teachers and prevented them from asking questions when they had challenges.

In extract 8, PST 2A who attended a high school where learners were streamed according to their performance in mathematics, describes her participation in mathematics within such a context:

[8] Schools that stream learners place learners in different classes based on their performance in mathematics.

Extract 8 from interview with PST 2A

PST 2A: Sorry, miss, I kind of just became more comfortable with the teacher and the more comfortable I became, the more questions I asked.

R: Okay

PST 2A: Then I was not scared, but at the beginning I was scared to ask questions as I thought the learners who were cleverer than I would think that I am asking a stupid question and wasting their time. But when they [*referring to the learners considered cleverer than her*] asked a question I would follow with an extra twenty questions.

R: Okay

PST 2A: I used the opportunity and as the year progressed, I used the opportunity more as I realised that there were times when the extra questions I asked, even the clever learners said but teacher we do not understand that either.

R: Yes

PST 2A: And then I realised that I do not have to be scared because sometimes they are just as scared when it is something that seems so simple and then…

PST 2A's narrative about her participation when she was confronted with mathematical challenges in her classroom reflects an important social dimension of learning: the influence of those with whom we learn. The classroom culture created by the teacher influenced her confidence to ask questions about the mathematics she did not understand. The broader school culture which promoted the streaming of learners according to performance impacted on the classroom culture. The impact is evident in the feeling of inferiority that PSTA 2A expressed, in the sense that she would only ask questions once the learners she categorised as 'cleverer' than her, asked questions. It was PST 2A's ability to recognise that the questions which she posed after the 'clever' learners, helped these learners to acknowledge their challenges. This was a critical incident within the social dynamics of the classroom which changed PST 2A's participation pattern in relation to her mathematical challenges. More importantly, the experience of PST 2A provides evidence of how learners' participation in mathematics is a struggle for understanding within the social configuration of learning environments that shape their learning dispositions. The learning disposition of learners in mathematics is a complex integration of the broader school culture, the micro social interactions of the classroom, as well as the interaction of learners with others beyond the classroom (Munns, 2007; Munns et al., 2013).

6.2.3 Emotions linked with mathematical learning experiences

The narratives of pre-service teachers brought to the fore a range of emotions associated with their mathematical learning experiences at school. Pre-service teacher emotions linked to teachers' practice that emerged in the previous sections, for example, are feeling special, not being scared to ask questions, as well as being scared to ask questions. Pre-service teacher emotions are also linked to the mathematical content.

In extract 9, PST 1C's narrative provides evidence of deeply negative emotions linked to teachers' practice:

Extract 9 from interview with PST 1C

PST 1C: Yes she put one on the spot a lot by asking you to come and do a problem on the board and then I stood there like an idiot.

R: And you do not like the idea of doing work on the board?

PST 1C: No because one cannot go to her afterward to explain as she expected us to listen and will think but why did you not listen

PST 1C's description of the teacher's practice to call upon learners to do a problem on the board was viewed as being put on the spot rather than an invitation to show what attempt was made to solve a problem. While PST 1C's reluctance to go to the board may be related to her personality (Uusimaki & Nason, 2004), her finding the teacher inapproachable suggests that her negativity towards the teacher's practice is linked to how the teacher engages learners when learners need to ask further questions.

Many pre-service teachers' narratives revealed emotions of pride when they were able to do difficult mathematical problems as reflected in PST 1D's narrative in extract 10:

Extract 10 from interview with PST 1D

PST 1 D: We had a really difficult problem and I think my friend got it right but the top learner in the grade did not get it right. My friend asked me whether I got the problem right and I told him I got full marks for the problem. My friend was really impressed with me and he was the one who told me that the top learner could not do the problem. I am sorry that I cannot remember the detail about the problem now but it was one of those problems where one really had to think long. I must admit that my tutor prepared me well for problems like that.

R: How fantastic, I can hear the passion in your voice.

PST 1D: That was probably the only time that I really felt that I was able to handle a mathematical problem that was difficult for almost everyone but I was able to get it right.

PST 1D's narrative reflects the importance of being recognised and acknowledged for what is perceived to be a challenging mathematics problem. The depth of PST 1D's emotions reflects the impact that forms of recognition have on learners' learning. PST 1D's acknowledgement of his tutor for his achievement indicates that he reflected on the nature of the support beyond school.

6.2.4 Summary of mathematical learning experiences at school

Pre-service teachers' mathematical learning experiences at school reflect an awareness of influences on their participation in mathematics. Pre-service teachers' reflections on the influence of their teachers' teaching practice on their participation in mathematics relate to how teachers teach mathematics content; their relationships with learners; and teachers' values. The moral dimension relating to values concerning teacher care included the responsibility for understanding the challenges of all learners; the interests of learners; and recognition of learners' lives beyond school. The pre-service teachers' narratives reflect that teacher care as a moral dimension is integrated into cognitive, social, cultural and emotional dimensions of mathematical learning experiences at school. A similar finding on the moral disposition of pre-service teachers by Schussler, Stooksberry and Bercaw (2010, p. 359) indicates that 'responsibility and care' as a moral dimension cuts across cognitive, social, cultural and emotional dimensions of learning.

6.3 Mathematical learning experiences in the home

As within Guillaume & Kirtman's (2010) research, pre-service teachers' mathematical learning experiences in their homes involved school-related mathematics with parental support. In some cases, pre-service teachers' narrated experiences of supporting younger siblings with their mathematics.

6.3.1 Kinds of support in learning mathematics in the home

Many pre-service teachers indicated that their parents showed an interest in their schooling and regularly checked on whether they did their homework and how they performed on mathematics tests. Many parents of pre-service teachers sought support for them in the form of private tuition. Many pre-service teacher narratives about their parents highlighted a culture of discipline, hard-work and instilling a positive belief in their ability.

Many of the pre-service teachers mentioned that their parents did not, however, enquire deeply into how they experienced their learning at school or ask about how their teachers taught a specific concept or topic. In some of the pre-service teachers' homes there was a culture of deeper engagement with mathematics where parents wanted feedback, for example, of whether they approached their teachers for support. In extract 11, PST 5B described how his father always encouraged him to ask questions and would follow up on whether in fact he did ask his teacher:

Extract 11 from interview with PST 5B

PST 5B: Many times when I had to ask my teacher then that evening my father would ask me if I asked the teacher and if I did not ask I would have to explain why I did not ask.

R: Oh yes

PST 5B: My father always made sure. And he also learnt from the whole thing. So I had to ask and in the evening he would say, right, let me hear what you asked and what you understand now.

Emerging from the narrative of PST 5B is evidence that his father was not only interested in what he learnt and what he struggled with but also how he participated in class through asking questions. The cognitive and social dimensions of learning is evident in PST 5B's narrative of his mathematical learning experiences in his home. Furthermore, the insistence of PST 5B's father that PST 5B communicate his engagement with his teacher to him is a way of developing ownership of learning, another social dimension of learning.

Many pre-service teachers received support from a parent with mathematics while at primary school. The parents who supported pre-service teachers at primary school did not necessarily have secondary or tertiary mathematics education, but they still tried to help where they could as evident in extract 12:

Extract 12 from interview with PST 5A

PST 5A: And my mother, her mathematics was not necessarily that 'fresh' but she motivated me and she helped me to read time when I was young, and when we went shopping she would ask me by how much is the one product cheaper than the other or how much she would save if she chose the one and not the other. So my mother laid that solid foundation, not because she is a mathematician but just with the basic knowledge that she has.

R: So she was interested in your thinking?

PST 5A: Yes she created a nice learning environment in which I could ask questions…

The significance of PST 5A's narrative is the value he placed on his mother's knowledge and how she engaged him in mathematical learning experiences in various contexts. PST 5A's mother created a learning environment in which he could pose questions. PST 5A's reference to his learning experience with his mother as having '*laid that solid foundation*' suggests that while his mother's understanding of mathematics was not deep, her understanding of how to provide a context for her child's learning was. PST 5A's reflections on his early primary school mathematical learning experiences in his home illustrate how the social space for learning made both him and the mathematics important.

The parents of some pre-service teachers who had some form of tertiary mathematics education (for example, teaching or engineering) provided more specific support with mathematics. In extract 13, PST 1B narrates her experience of engaging with her father, an engineer, supporting her with mathematics problems. She indicated that she was extremely scared to ask questions in class as she was at a school where learners were streamed according to their mathematical performance. She said she was placed in a class with 'clever' learners but her own marks did not reflect that she should be in that class. It is significant to note that her mathematical learning experience involved her having to explain to her father the mathematics that they were currently doing and the mathematics done before.

Extract 13 from interview with PST 1B

R: Now can you explain to me how you asked your father for help with a mathematics problem?

PST 1B: When my father came home I told him that I could not do a mathematics problem. Then I had to start by explaining what the problem was about and what mathematics we covered before we started with the problem and many times as I explained this to him it actually 'clicked' what I needed to do … do you understand

R: Interesting

PST 1B: But most times when it did not click he showed me how to do it.

R: Okay, but you could not simply say father do this for me?

PST 1B: No. No. No.

R: And what did you learn from the approach followed by your father?

PST 1B: What I learnt. When he sat next to me he let me do all the work and if I struggled he would give me a clue. He would never give me the answer. For example, with factorisation he would say now go back and look and he would say, what do you see? He would provide leads but never give the answer.

PST1B's father's approach to engaging her with her mathematical challenges or errors was interpreted by her as leading her to identify an error by examining mathematical processes involved in solving the problem. The support her father provided for mathematical learning experiences did not include explaining the reasoning in her error. Through being given the responsibility to identify her own error, her father's approach to supporting her involved a social dimension of learning. PST 1B was encouraged by her father to revisit, for example, a factorisation process and to identify for herself how to proceed. This illustrates the importance placed on understanding mathematical processes, which is the cognitive dimension of learning.

In extract 14, PST 3A narrates a learning experience with her family and mentions the importance of communicating or explaining to another pre-service teacher.

Extract 14 from interview with PST 3A

R: Did you help your younger sister with mathematics?

PST 3A: I did help her. And I think through helping her it helped me a lot.

R: Interesting

PST 3A: Yes, and my boyfriend is doing his N2 and N3 that has mathematics which is basically matric mathematics. I explained to him as well which I found helped as one then has to sometimes see it in another way. I realised that when I have to explain to someone else then I begin to understand. That is why I and Samantha[9] in lectures communicate and she will sometimes explain it in a much simpler way than I would have explained

The learning experience of PST 3A illustrates the importance of engaging in the practice of explaining and supporting others with mathematics. PST 3A recognises that different ways of explaining to each other help to see different ways of thinking about a problem.

It is significant to note, however, that PST 3A did not mention that when she supported her sibling, she gave her the opportunity to explain how they were taught in class. There is evidence of a social and cognitive dimension of learning connect in PST 3A's narrative of her mathematical learning experiences. The social dimension of learning is through peers explaining to one another how they are thinking about a problem which supports the

[9] A pseudonym

understanding of mathematical ideas that had not been fully understood and became much clearer through the peer explanation.

The opportunity was missed in the interview to probe how PST 3A engaged with her sibling's reasoning or whether she simply re-taught the mathematics that her sibling was struggling with.

6.3.2 Summary of mathematical learning experiences in the home

The mathematical learning experiences of pre-service teachers in their homes focused on parents' interest in their participation at school and their understanding of mathematics as a source of motivation to learn. The narratives of pre-service teachers' mathematical learning experiences foregrounded social dimensions of learning such as the encouragement to pose questions and taking ownership of their learning. While the pre-service teachers' narratives did not include parents' probing their reasoning when they made errors, most pre-service teachers were encouraged to work through their mathematics and identify their errors. Many pre-service teachers valued explaining mathematics to others, as in the case of supporting siblings or when asked by a parent to explain what a mathematical problem was about. This was viewed as an opportunity to uncover aspects of the mathematics which they had not understood.

6.4 Mathematical learning experiences in broader communities

The mathematical learning experiences of pre-service teachers in their broader communities focused mainly on the forms of support for their mathematical challenges at school. The pre-service teachers found it difficult to respond to the question of identifying mathematical learning experiences other than tutoring support with mathematics in their broader communities. When the pre-service teachers were introduced to the 'funds of knowledge' literature, they acknowledged they had not thought about their mathematical learning experiences in contexts other than those related to school.

6.4.1 Tutoring support with mathematics

Many of the pre-service teachers received additional mathematics lessons either from a private tutor, which their parents arranged, or through the mathematics teacher at the school. This support translated into learning experiences which provided insight into their own understandings and views of the learning of mathematics. The support of peers beyond the classroom was also mentioned.

Emerging from the narratives of many of the pre-service teachers is that the people whom they approached for support first wanted to know what knowledge the pre-service teachers had that could help in solving a mathematics problem which they encountered.

In extract 15, PST 1C sought support from another teacher after school as well as friends.

<u>Extract 15 from interview with PST1C</u>

PST 1C: The teacher that I went to always first asked me what I knew even if it was just something small as he needed to know where to begin. And usually it was from the beginning and step-by-step. But he always first asked what do you know and did not just explain immediately.

R: And you had to say something?

PST 1C: Yes, yes, yes and if you did something, he will always say could you please do it again

R: Okay

PST1C: I like that approach

R: Okay

PST 1C: My friends simply explained straight away quickly or just gave the answer

R: Okay

PST 1C: So I have the answer but how do I get to the answer

PST 1C viewed the re-doing of a mathematical problem 'step-by-step' as a means to access her thinking. She indicated that her friends, whom she approached for support, went straight into solving the problem for her instead of understanding how she did the problem. When she remarks *'So I have the answer but how do I get the answer'*, it can be inferred that she is recognising the importance of being given an opportunity to explain how she reasoned to get her answer. It is important to note that no mention was made of being asked by her tutor to explain why she reasoned in a particular way especially when there was an error. The practice of re-doing a mathematical problem was evident in the way many pre-service teachers explained the nature of the tutoring support they received.

In extract 16, PST 3C provides further evidence of the practice of re-doing a mathematical problem in the tutoring.

Extract 16 from interview with PST 3C

PST 3C: So then I showed her how I did it but could not get to the answer and then she will explain or help me.

The practice of engaging learners with their mathematical errors through affording learners the opportunity to explain their reasoning was not evident in the pre-service teachers' mathematical learning experiences. Re-doing a mathematical problem provides partial access to the way a learner is reasoning about a problem.

In extract 17, PST 5D reflects on her learning experience of narrating to her tutor how her teacher taught the mathematics to her. PST 5D describes how the process of narrating how she was taught supported her learning.

Extract 17 from interview with PST 5D

R: When your tutor asked you to re-tell what happened in your mathematics classroom, did you find that the process of retelling helped you to understand something that you might not have?

PST 5D: Yes, for sure, for sure, as I needed to go through the entire lesson in my head and through repeating I believe certain things stuck.

R: Then you needed to think about how things happened?

PST5D: Yes, how the process went and through explaining I realised I might have missed something that was explained at a certain point by the teacher, and now I understand it.

R: So sometimes you identify your own gap?

PST 5D: Yes.

PST 5D mentions that she found the process of thinking through how a lesson unfolded helpful in identifying where a gap in her understanding may have originated. The approach which her tutor followed not only enabled access to the knowledge but also access to how she experienced her learning. In addition, the social dimension within this approach, which is the affordance given to narrate her mathematical learning experience, is a key factor in enhancing her learning of the mathematics. PST 5D was the only pre-service-teacher who experienced tutoring support through narrating her mathematical learning experience in class. However, PST 5D did not mention that her tutor encouraged her to explain her reasoning behind a mathematical error.

The tutoring experience of PST 4D in extract 18 was through group learning with learners from other schools. This was the only pre-service teacher who experienced mathematics support within a group context. PST 4D's reflection on her mathematical learning experiences indicated an emotional dimension of learning.

Extract 18 from interview with PST 4D

PST 4D: It was nice because I felt comfortable as you know you are here and that there are others who are also struggling. So you do not feel that you are stupid because you are sitting here. The other people are with you and it was nice as we met one hour per week and it was like we were a small family.

Embedded in PST 4D's narrative is how shared experiences of struggle are important in developing a sense of the problem not being with the individual learner but to recognise that aspects of mathematics present challenges for many learners. The developing of a community or 'family' as an important part of working through mathematical challenges is reflected in her narrative. Engaging with mathematical challenges as shared practice to build confidence and learning emerged within the mathematical learning experiences of PST 4D.

6.4.2 Mathematics in work contexts

In the interviews, pre-service teachers were prompted to reflect on mathematical learning experiences in other contexts. Pre-service teachers mention that mathematics is applied in many everyday contexts. They identify mathematics applications within contexts involving sports and shopping. They acknowledge that they did not think about their mathematical learning when engaged within these contexts.

For example, one of the pre-service teachers, PST 4C, recalls a conversation with her father who farms with fruit trees and tea trees in which they discussed how one would go about determining the price value of the farm. The different value of different trees and the area on which the trees were planted were some of the variables to be taken into consideration. PST 4C acknowledges that she had not thought about her conversation with her father as a 'mathematical learning experience' or 'fund of knowledge' to bring into the mathematics classroom at school.

In response to the question about mathematical learning experiences in the broader communities PST 2A, in extract 19, describes the teacher's response to her question about the relevance of some mathematics in school:

Extract 19 from interview with PST 2A

PST 2A: I asked my teacher why do we have to learn this? And she said you are never going to use it as it is just so to develop your thinking. And that is where I think we got the idea that mathematics is nowhere and one cannot see the relationships except for me as I am going to be a mathematics teacher and will need to use it again.

PST 2A's narrative and many other pre-service teacher narratives did not include their teachers asking them about their mathematical learning experiences beyond the classroom that could connect with their learning in school.

6.4.3 Summary of mathematical learning experiences in broader community

Pre-service teachers mathematical learning experiences of being tutored included different tutoring practices. The tutoring practice of learners involved learners providing information on the mathematics which they covered at school. This ranged from explaining what learners knew, to explaining how their teachers taught specific content. The latter tutoring practice was only evident in one pre-service teacher's tutoring experience. Pre-service teachers were mainly asked to redo a mathematics problem with which they struggled followed by a process of correction in which the tutor explained. There was an absence of probing reasoning behind mathematical errors in pre-service teachers' tutoring experiences. Pre-service teachers placed positive emotional learning experiences related to feelings of confidence, belonging and understanding of their mathematical challenges within their tutoring experiences.

While pre-service teachers were aware of the applicability of mathematics in everyday situations, they expressed uncertainty in what it means to talk about one's mathematical learning experiences in everyday contexts. Furthermore, there was an absence of school experiences in which learners were encouraged to think about their mathematical learning experiences beyond school. Phelps et al. (2012) argues that opportunities for learners to describe their learning beyond school is a move beyond mere recognition of their learning experiences but to develop 'metacognitive' insight into their learning experiences, in other words, how these experiences benefited their learning.

6.5 Comparative analysis of mathematical learning experiences in the three contexts

Pre-service teachers' mathematical learning experiences in the school context reflected a much stronger emotional dimension of learning linked to the teachers' practice or structural aspects of schooling than in the home and broader community context. The influences of teachers' practice or structural dimensions of school influenced pre-service teachers' participation within mathematics classes, a finding supported by research on learners' participation and learners' learning dispositions (Lange, 2009; Munns, 2007; Munns et al., 2013). The emotions expressed within mathematical learning experiences in school were positive and negative whereas in the context of the home and in the broader communities, positive emotional experiences were positive in terms of the support, interest, feelings of inclusion and confidence related to mathematical challenges.

In the school context a strong emphasis was placed on teachers as having a moral responsibility to care for all learners on a cognitive, social, cultural and emotional dimensions. In the homes and broader context, the parent or tutor was valued in their role as encouraging responsibility and discipline in learners as a form of motivation to learn but without placing a moral accountability on the parent or tutor. The more personal interaction that is possible in the home and broader community tutoring context, the more positive the accounts for the social and emotional experiences.

In the school context, pre-service teachers' learning experiences when dealing with their errors reflect a stronger emphasis on teachers re-teaching rather than probing the reasoning behind their errors. The re-teaching approach as a means of correcting errors prevents errors being used as learning opportunities (Smith et al., 1993). In the homes and broader community contexts where parents or tutors supported with mathematical challenges there was no evidence of probing reasons behind errors either. In the homes and broader community contexts there was, however, evidence of pre-service teachers being asked to explain what knowledge they do have that they may use to solve challenges or to identify their errors, reflecting a greater ownership encouraged in dealing with their mathematical challenges.

6.6 Chapter summary

In this chapter I have presented findings on the mathematical learning experiences of pre-service teachers as reflected in their narratives. I have provided evidence that pre-service teachers are able to reflect back on teaching practices and how this impacted on their participation in learning as school learners. Pre-service teacher narratives of their mathematical learning experiences provide evidence of emotional, social and cognitive dimensions of learning. Pre-service teachers' narratives did not include them drawing on their mathematical learning experiences beyond school as a 'fund of knowledge' for learning in school which may explain their difficulty in identifying mathematical learning experiences beyond school as a ‘fund of knowledge’.

Minimal evidence of practices that are the focus of my study, namely, probing learners’ reasoning behind their errors which is a cognitive dimension of learning, and engaging learners in detailed accounts of how their learning unfolded, which is a social dimension of learning emerged in pre-service teachers’ narratives.

The extent to which pre-service teachers draw on their mathematical learning experiences as school learners within the different learning contexts to reflect on their tutoring practice within PST-PLCs is explored in relation to the inquiry focus of the PST-PLCs in the next two chapters.

CHAPTER 7

Inquiry into Engaging Learners' Mathematical Learning Experiences

7.1 Introduction

This chapter focuses on how the PST-PLCs inquired into engaging learners' mathematical learning experiences. This foregrounds the social dimension of learning and addresses the first sub-question of the research:

> *How do PST-PLCs inquire into engaging in learners' mathematical learning experiences?*

The pre-service teachers in this study were encouraged, when tutoring learners in mathematics, to find out from learners how their teachers taught a lesson; whether learners posed questions when they had difficulty with the mathematics; and the nature of support learners received in classrooms and beyond the classroom. Engagement with learners' mathematical learning experiences in this way is an intentional focus to foreground the social dimension of learning since the focus is not directly on conceptual challenges learners have with the mathematics. My study explores the connection between the social and cognitive dimensions of learning and this chapter explores the connection when the social dimension is the entry point of analysis. While the focus is on the social dimension of learning I continue to explore how the cognitive dimension emerges and in so doing respond to the third sub-question of the research:

> *What connections were made possible between learners' mathematical learning experiences and learners' mathematical errors through the PST-PLCs' inquiry?*

In the next chapter I present the findings of the PST-PLCs' inquiry into engaging learners' reasoning about their mathematical errors thereby foregrounding the cognitive dimension of learning as the entry point of analysis.

I show that the PST-PLCs' critical inquiry into engaging learners' mathematical learning experiences is influenced by the stance adopted towards the inquiry. As a theoretical construct, stance refers to how we position ourselves in a certain context. In the context of the PST-PLCs, the stances towards inquiry into engaging learners' mathematical learning experiences reflected the pre-service teachers' struggles to elicit learners' mathematical learning experiences. I identified two stances towards inquiry into engaging learners' mathematical learning experiences in the PST-PLCs:

Table 7.1: Stance towards inquiry into engaging learners' MLExps[10]

Stance	Description of Stance
1. Embracing	Ongoing inquiry into engaging learners' mathematical learning experiences.
2. Compliance	Inquiry into engaging learners' mathematical learning experiences only in the first PLC session.

Table 7.2 below shows the stances towards the inquiry into engaging learners' mathematical learning experiences that I identified across the five PST-PLCs.

Table 7.2: Stance towards inquiry into engaging learners' MLExps across the five PST-PLCs

Stance	Pre-service teacher –PLCs				
Inquiry into engaging learners' MLExps in class	**PLC 1**	**PLC 2**	**PLC 3**	**PLC 4**	**PLC 5**
Embracing	✓	✓			
Compliance			✓	✓	✓
Inquiry into engaging learners' MLExps beyond school	**PLC 1**	**PLC 2**	**PLC 3**	**PLC 4**	**PLC 5**
Compliance	✓	✓	✓	✓	✓

An embracing stance towards inquiry into learners' mathematical learning experiences was only identified in PST-PLC 1 and PST-PLC 2 and was only evident in inquiry into learners' mathematical learning experiences in class. In PST-PLC 1, pre-service teachers elicited learners' mathematical learning experiences as a starting point but did not refer back to the learners' learning experiences when engaging their mathematical errors. In PST-PLC 2 learners' mathematical learning experiences were elicited as part of engaging learners' mathematical errors. In PST-PLC 3, 4 and 5 there was a compliance stance towards inquiry into learners' mathematical learning experiences in class, where learning experiences were engaged with only in the first sessions. Most of the inquiry in these PST-PLCs was around learners' mathematical errors and there was no ongoing inquiry into learners' mathematical learning experiences in these PST-PLCs.

[10] MLExps (mathematical learning experiences)

The non-inquiry into learners' mathematical learning experiences beyond school in the PST-PLCs is a reflection of the struggle of pre-service teachers to engage learners about their mathematical learning experiences beyond school in the tutoring session. Pre-service teachers' reasons for this struggle emerged within the group interviews.

In this chapter I select one episode from PST-PLC 1 to illustrate the embracing stance towards inquiry into engaging learners' mathematical learning experiences. I select one episode from PST-PLC 4 which I facilitated to illustrate how the embracing stance was supported but not taken up. I also select one episode from PST-PLC 1 to illustrate the compliance stance towards the inquiry into engaging learners' mathematical learning experiences beyond school which was typical of compliance stances in the other PST-PLCs.

In doing the fine-grained analysis of the inquiry in the different stances I identified epistemic moves as the enablers of critical inquiry (Zenios, 2011). I show how epistemic moves are connected to deepen inquiry and create potential for deepening inquiry in relation to knowledge. I also identify inhibitors of critical inquiry to understand why epistemic moves did not always lead to deepening inquiry. A deeper analysis of the inhibitors of critical inquiry across both stances provide insight into practices of inquiry within PST-PLCs that are necessary for developing a critical inquiry stance on teaching practice**.** These inquiry practices in PST-PLCs suggest that their development become a shared accountability of PLCs working towards a critical inquiry stance.

I use Cochran-Smith and Lytle's (1999) definition of knowledge-for-practice, knowledge-in-practice and knowledge-of-practice to analyse how epistemic moves deepen critical inquiry through this knowledge. Knowledge-for-practice includes knowledge of subject matter and theories of learning and pedagogy. Knowledge-in-practice is the 'knowledge embedded in practice and in teachers' reflection on practice' (Cochran-Smith & Lytle, 1999, p. 250). Knowledge-of-practice is constructed collaboratively and requires systematic inquiry into practice in which one draws on theory (knowledge-for-practice) where needed to deepen the inquiry into teaching practice. The PST-PLCs in this study focused on inquiry into engaging learners' mathematical learning experiences over a period of an academic year, which represents a systematic form of inquiry into a teaching practice. A challenge identified across all the PST-PLCs was eliciting learners' mathematical learning experiences, a key teaching practice in engaging learners' mathematical learning experiences.

A comparison of epistemic moves across the stances shows that representations of practice were used differently which resulted in different framings of collaboration and hence influenced the opportunities for learning (Little, 2002; Horn & Kane, 2015; Louie, 2016). I will show how inquiry in the embracing stance involved shared strategies of encouraging learners to talk about their mathematical learning experiences, a practice of the PST-PLC, which enabled deeper inquiry into engaging learners' mathematical learning experiences. The inquiry in the compliance stance did not involve these shared strategies, which inhibited further inquiry into learners' mathematical learning experiences.

I show how epistemic moves with potential for deepening inquiry into social-cognitive connection learning opportunities emerge across the stances. In the embracing stance, a cognitive dimension of learning emerged which involved mathematical methods in solving problems and different approaches to teaching. In the compliance stance there was only potential for a cognitive dimension of learning to emerge which involved learners' challenges within mathematics and learners' thinking. The opportunities for learning in the embracing stance point to knowledge of mathematics content, knowledge of learners and learners' learning and knowledge of how to teach. The opportunities for learning in the compliance stance point to knowledge of learners and knowledge of learners' learning.

7.2 Embracing stance

In all the PST-PLCs, the challenge in getting learners to elaborate on their mathematical learning experiences emerged. In the embracing stance within PST-PLC 1, shared strategies to encourage learners to talk about their mathematical learning experiences were part of the inquiry into engaging learners' mathematical learning experiences. The pre-service teachers in PST-PLC 1 shared their strategies of eliciting learners' mathematical learning experiences, experimented with each other's strategies and provided feedback on how these strategies worked. In the first session of PST-PLC 1, PST 1C shared her strategy of eliciting learners' mathematical learning experiences:

PST 1C: I made each of them a book. And before each tutor session they must write what they did in mathematics that week, what did they like, what did they not like (Turn 40).

Towards the end of the first session of PST-PLC 1, PST 1C continued to elaborate on her writing strategy. In the extract below, she does not only include questions which she asked learners but how she also encouraged learners to write down questions which they posed to her.

She encouraged learners to pose these questions to their teachers, write down their teachers' responses and then share their teachers' responses with her in the following tutor session:

PST 1C: I let my learners write in the back of the book which I gave them so this week if they ask me questions they write it because otherwise they forget and I told them to ask the teacher the questions too so that they can write what the teachers told them and come and tell me and then we can work from that. (Turn 135)

PST 1C: [*Shows the PLC the learner's book*] As they work they must make a margin line and as they think they must write so that I can also see how they are doing as one cannot read their thought. (Turn 137).

Learners were then also asked to show processes they applied in solving a mathematical problem. PST 1C recognised learners' unwillingness to speak about their mathematical learning experiences and, through the process of refining her writing strategy, attempted to develop learners' capability to describe their mathematical learning experiences.

In turns 34 to 38 from the third session of PST-PLC 1, PST 1A shared her experimentation with the strategy of encouraging learners to write about their mathematical learning experiences:

PST 1A**:** This time I'd let them write in their books for the first time. I wrote three questions for them on the board and told them to write down the question and the answer. I told them to write two, three sentences and when I got there it was like…. (Turn 34)

PST 1B**:** Nothing. (Turn 35)

PST 1A: Do you like it? Yes. I said don't just write yes or no, write something with it. I said to them, what work did you do? Here, I said what work did you do, what maths did you do this week? [*paging through journal brought to the PLC*]. They said, we are still busy with capacity and volume. Understand? (Turn 36).

PST 1C: They don't expand. (Turn 37)

PST 1A: They don't expand at all and I told them to, understand? So this writing thing still does not work for me. I do not get more out of them than I did when they talked. (Turn 38)

The inquiry into experimentation with the writing and talking strategy suggests that for some learners it might be more challenging to elaborate on their mathematical learning experiences through writing. An advantage of the talking strategy is pointed out by PST 1C in turn 41 from the third session of PST-PLC 1:

PST 1C**:** So when you talk to them you have to keep on asking follow-up questions? (Turn 41)

PST 1A**:** Yes, then they can [unclear 05:09]. So I think the talking thing is still working best. Not that it is working well but it is better than the other [*points to the books in which the learners wrote about their mathematical learning experiences and which were brought to the PLC]* (Turn 42)

PST 1B: They can't talk too much because if they talk too much, it takes too much effort, then we do not have enough information about what they are saying… then the time is gone and you can't work with them. (Turn 43)

Turn 43 illustrates how the inquiry into engaging learners' mathematical learning experiences raised concerns about spending too much time on getting learners to talk about their learning experiences. The concern reflected in turn 43 may be related to the pre-service teacher's own experiences of being tutored in which the norm is to move directly to a focus on the mathematical error.

In turn 75 from the third session of PST-PLC 1, the inquiry involved the sharing of another strategy, playing mathematical games, which according to PST 1C allowed the learners to become more comfortable with each other. PST 1C argued in turn 77 that learners needed to be comfortable with each other first before being comfortable with her and talking to her:

PST 1C: My two learners are very comfortable. They talk and they ask questions and when we had the four learners, PST 3B[11] and I decided they have to do their multiplication tables. Then we played a game, my learners against her learners, and then they climbed totally out of their boxes. (Turn 75)

PST 1A**:** Is it? (Turn 76).

PST 1C: So, because they are comfortable with each other and they are now comfortable with us, and when they played, I saw again that they cannot do multiplication tables. (Turn 77)

PST 1B**:** Yes. (Turn 78)

PST 1C**:** So, that was quite nice, but they are very comfortable. They talk and converse. (Turn 79)

In the embracing stance within PST-PLC 1, the shared strategies to encourage learners to talk about their mathematical learning experiences opened up not only new ideas for the community but also awareness (see turn 76) that it is possible for learners who at first may have found it

[11] PSTs had their own classroom in which to tutor learners but on this specific afternoon a room was not available for all PSTs. The situation of sharing classrooms did not happen again.

difficult to talk to gain confidence to do so. It also allows for a cognitive connection in noting learners' weakness in multiplying.

In the episode in Table 7.3, I show that the PST-PLCs inquiry into learners' mathematical learning experiences provides evidence of shifts in learners' confidence to talk about their mathematical learning experiences in class. I include inhibitors of critical inquiry and epistemic moves to show how both emerged through the inquiry. I placed the inhibitors first in the table as I first discuss the inhibitors of critical inquiry and thereafter the epistemic moves.

Table 7.3: Episode 1 from PST-PLC 1, session 3, facilitated by PST 1A

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
94	PST 1C	Mm, and they are [unclear 11:10]. Like last week the girl learner said to me, but miss, my teacher explained it like this, why don't you explain it like that?		Describing how a learner narrative of the learner's MLExps started
95	PST 1A	So she is challenging you?		Reflecting on a PST's account of engaging a learner
96	PST 1C	Yes		
97	PST 1A	That's nice.		
98	PST 1C	I then asked her to explain it to me like he explained it, as if I am the learner and she is the teacher. She explained and I understood what the teacher's approach is so I just adapted my explanation to his and they grasped it.	Describing that a learner explained how the learner's teacher taught Describing a response in supporting a learner	Describing approach to elicit learners' MLExps (role-play)
99	PST 1B	Similar to what you are saying about [unclear 11:41] the boy learner asked me, but Miss, we don't do the division like that, we do it like this. He began to explain to me, even without me asking him, he went to the board, took the chalk and started to explain. My girl learner came forward as well and she also helped him a little. I then also understood theirs is a longer method as opposed to when I was in primary school. We learnt that step in Grade 4 and we used it until Grade 7, even now we	Describing a shift in learners' confidence to talk	Describing how a learner narrative of the MLExps started. Reflecting on MLExps as school learner Comparing of teaching approach

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		are still using it. Whereas in Grade 4 they use a long method, next year a little bit shorter, then again shorter and in grade seven they do the short method.		when at school to the current one and pointing out the difference
100	PST 1A	So they could have learnt it in Grade 4 already.		Reflecting on another PST's reflection
101	PST 1B	You understand? So they learn a lot of extra things, which is unnecessary in my opinion, but I am very impressed that they came out of the blue and explained it to me, on their own initiative.		Critiquing of curriculum Reflecting on learners' shift in confidence to talk
102	PST 1C	I would like to add to this ….it shows me they pay attention when the teacher explains something because usually when you ask a learner how the teacher explained it …..		Inferring about the learners' participation in class
103	PST 1B	Then they don't know.		Describing the challenge of eliciting learners' learning experiences and reason for the challenge
104	PST 1C	…then they are unable to explain.		
105	PST 1A	My learners are like that. Why is it like this [12:58] why do you do it like this? Because that is what the teacher said. They are unable to say why. Or sometimes they can't even explain it to me.		

The inquiry in PST-PLC 1 entailed descriptions of how learners narrated their mathematical learning experiences as in turn 94 and 99. In turn 98, the inquiry entailed a description of an approach to encourage learners to talk about their mathematical learning experiences. The approach involved role-play in which the learner was encouraged to explain to the pre-service teacher and in so doing the learner takes on the role of teacher.

I considered describing an inhibitor when detail of an event was not provided as in turns 98 and 99. In turn 98, PST 1C provides no information on how the learner's teacher explained the mathematics to the learner which inhibited the community from gaining deeper insight into the teacher's approach as well as how the learner was able to explain. In turn 98, PST 1C's describing the nature of her support to the learner without information on how she supported the learner, inhibited the inquiry as the community could not evaluate her support strategy. In turn 99, PST 1B's describing learners' shift in confidence to talk and explain, provides no information on how the learners explained being taught to do the mathematics which as in turn 98 inhibits inquiry into how learners reconstruct their learning experiences.

In the next section I further analyse the inquiry that unfolded in PST-PLC 1 and provide reasons for why I considered the epistemic moves identified in Table 7.3 as enablers of critical inquiry.

7.2.1 Epistemic moves in embracing stance

The epistemic moves in this stance consist of five different kinds: describing, reflecting, comparing, critiquing and inferring. Describing is an epistemic move when account of an event is given which either deepens the inquiry directly or deepens the inquiry through generating other epistemic moves. For example, describing the strategies to elicit learners' learning experiences directly deepened inquiry into the challenges to elicit learners' learning experiences. In turns 94 and 99, describing as epistemic move deepened inquiry through generating further epistemic moves which held potential for deepening inquiry. I next elaborate on the interaction between the epistemic moves in this stance.

7.2.1.1 Epistemic moves - describing and reflecting

Describing as an epistemic move generated reflecting as epistemic move which in turn led to further describing. This interaction between describing and reflecting deepened inquiry and created potential for deepening inquiry further still. For example, the ongoing inquiry into strategies to elicit learners' mathematical learning experiences and the describing of challenges to elicit learners' mathematical learning experiences and reasons for the challenges (as in turns 103, 104 and 105) are epistemic moves for deepening inquiry into the reasons why learners struggle to explain or elaborate on their learning experiences.

In turn 94, PST 1C's describing a learner's questioning her about a particular way of explaining a problem that differed from the teacher's explanation generated PST 1A's reflection in turn 95 on whether the learner was challenging PST 1C. The reflecting in turn 95 as epistemic move held potential for deeper inquiry into shifts in learners' participation and into understanding how social relations in the learning situation entail power dimensions.

The reflecting in turn 95 deepens inquiry in that it leads to further describing in turn 98 where PST 1C shows how through role-play the learner is given an opportunity to develop confidence in explaining the way in which his teacher explained. PST 1C's describing role-play to elicit learners' mathematical learning experiences in turn 98 is an epistemic move which deepened the inquiry in that it led PST 1B in turn 99 to describe a learner's shift in confidence to explain how their teachers taught. PST 1B's reflecting on the learner's shift in confidence to explain to her how he was taught (turn 101) is an epistemic move to deepen inquiry into the underlying reason for this shift. The describing of different strategies of eliciting learners' mathematical learning experiences are epistemic moves which deepened inquiry into the challenges of eliciting learners' learning experiences.

7.2.1.2 Epistemic moves – Reflecting, comparing, critiquing and inferring

Reflecting as an epistemic move generated further epistemic moves such as comparing and inferring which deepened inquiry. The interaction between reflecting and comparing as epistemic moves created opportunities for deepening inquiry.

For example, PST 1B's reflecting on her mathematical learning experiences as a school learner led to her comparing how learners are currently being taught algorithms as opposed to when she was a school learner. Comparing, in this case, is an epistemic move for deeper inquiry into the different teaching approaches. PST 1B's comparing led to further reflecting by PST 1A in turn 100 on the possibility of introducing learners to the standard column algorithm earlier. The reflecting by PST 1A is an epistemic move to deepen inquiry into the strengths and limitations of early algorithm introduction. PST 1B's critiquing of the curriculum is an epistemic move generated through the epistemic move of comparing. Critiquing as an epistemic move is an opportunity to deepen inquiry through considering the positives and negatives which in this case would be related to the different teaching approaches to which PST 1B referred.

Reflecting on learners' shifts in confidence to talk in turn 101 by PST 1B led to PST 1C inferring about learners' participation in class in turn 102. I considered PST 1C's inferring about the learners' participation an epistemic move with potential for deepening inquiry into the reasons why learners struggle to explain or elaborate on their learning experiences.

Figure 7.1 provides a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry

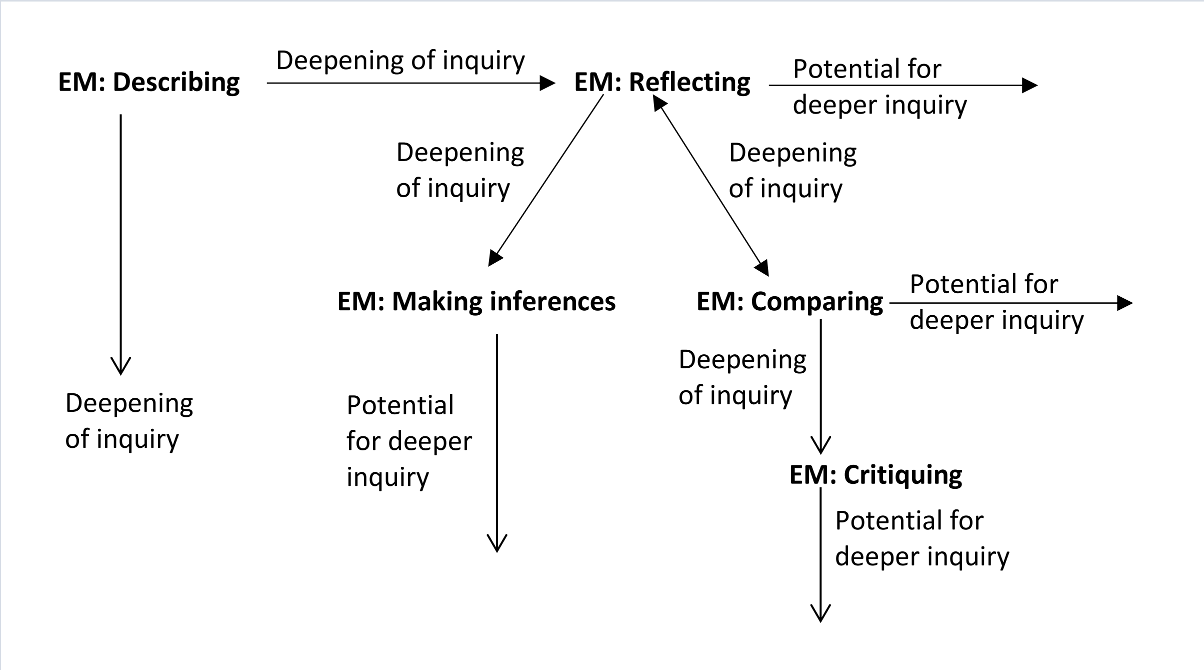


Figure 7.1: Interactions between epistemic moves influencing inquiry in embracing stance

Figure 7.1 shows that describing as an epistemic move deepens inquiry by itself and generates reflecting as an epistemic move. The interaction between describing and reflecting deepens inquiry and creates potential for deepening inquiry still further. Reflecting as an epistemic move generated further epistemic moves of which comparing led to a further epistemic move critiquing as well as further reflecting. The interaction between reflecting and comparing deepened inquiry and created potential for further deepening of inquiry. Inferring and critiquing did not lead to further epistemic moves but held potential for deepening inquiry.

In my comparative analysis of the stances I will discuss other inhibitors which influence deepening of inquiry of these epistemic moves. In the next section I elaborate on how describing and its interaction with reflecting as epistemic moves enable knowledge-in-practice and how the epistemic moves reflecting, making inferences, comparing and critiquing all lead to potential for deeper inquiry through knowledge.

7.2.2 Deepening inquiry through knowledge in embracing stance

In the embracing stance towards inquiry into engaging learners' mathematical learning experiences in class, describing key challenges in encouraging learners to talk about their mathematical learning experiences enabled knowledge-in-practice about these challenges. For example, in the third session PST 1A indicated that asking the learners whom she tutored to write down their mathematical learning experiences did not work. PST 1A indicated that she had to experiment with other strategies, for example, talking to the learners while engaging them in a mathematical problem. PST 1A's describing challenges to elicit learners' mathematical learning experiences affords deeper inquiry into the assumptions made about strategies for eliciting learners' learning experiences. The kind of support which learners require to describe their learning experiences in writing underlies PST 1A's observation of the learners' struggle to write about their learning experiences.

In the embracing stance, the describing of various strategies of eliciting learners' mathematical learning experiences, experimenting with each other's strategies, working with a range of different strategies and reflecting on implementing the strategies enables the building of knowledge-in-practice about eliciting and engaging learners' mathematical learning experiences. For example, in turn 98 PST 1C shared the success she experienced with the strategy of role-play through encouraging learners to talk about their mathematical learning experiences. It is significant to note that PST 1C shared with PST-PLC 1 that she indicated to the learner that he would take on the role as teacher. The learners were given the opportunity to view the tutoring context as a space in which both learners and teachers learn. Cole, Mooney & Nanlohy (2013) argue that teachers who encourage learners to ask questions about what they are learning, how they are learning and what will help them learn are building towards greater learner independence in learning. One can therefore, following Cole et al. (2013), conclude that having the learners explain how their teachers taught can be considered a way in which to develop confidence and taking ownership of their learning.

The describing of learners' shifts in confidence in talking about their mathematical learning experiences allowed PSTs to deepen reflection on how they engaged learners' mathematical learning experiences. PST 1C, for example, might not have considered the learners' questioning her method of doing a mathematics algorithm as her being challenged by the learner if PST 1A had not raised the question (in turn 95). The importance of raising the notion of 'being challenged' by the learner, is an epistemic move for deeper inquiry into the shifting power relation between learner and pre-service teacher. A systematic inquiry into eliciting learners' mathematical learning experiences has potential to deepen inquiry through knowledge-of-practice on these shifts. For example, a systematic inquiry into how learners' shifts in confidence to challenge or question is related to increased opportunity to elaborate on their learning experiences. And a systematic inquiry into how this relationship enhances learners' understanding of the mathematics generates the potential for knowledge-of-practice.

Describing learners' mathematical learning experiences generates pre-service teachers' reflections on their own learning experiences which deepen inquiry through comparing different ways of teaching. Comparing as an epistemic move holds potential for deeper inquiry through knowledge-for-practice into what is known about the strengths and limitations of various teaching approaches. Comparing led to critiquing of the current mathematics curriculum which created opportunity for revisiting the rationale for the teaching approach within the curriculum. The potential deepening of inquiry is through knowledge-for-practice about different teaching approaches.

In Zenios (2011), the drawing of inferences was an elaboration of explaining as an epistemic move to help others understand why an event had taken place. In my study making inferences about learners' participation was generated through reflecting on shifts in learners' confidence to talk about their mathematical learning experiences. These inferences create opportunities for systematic inquiry into how eliciting learners' mathematical learning experiences influence learner participation. Hence the potential to deepen inquiry is through knowledge-of-practice on connections between eliciting learners' MLExps and learner participation in class.

Figure 7.2 below builds on Figure 7.1 to provide a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in relation to knowledge in the embracing stance.

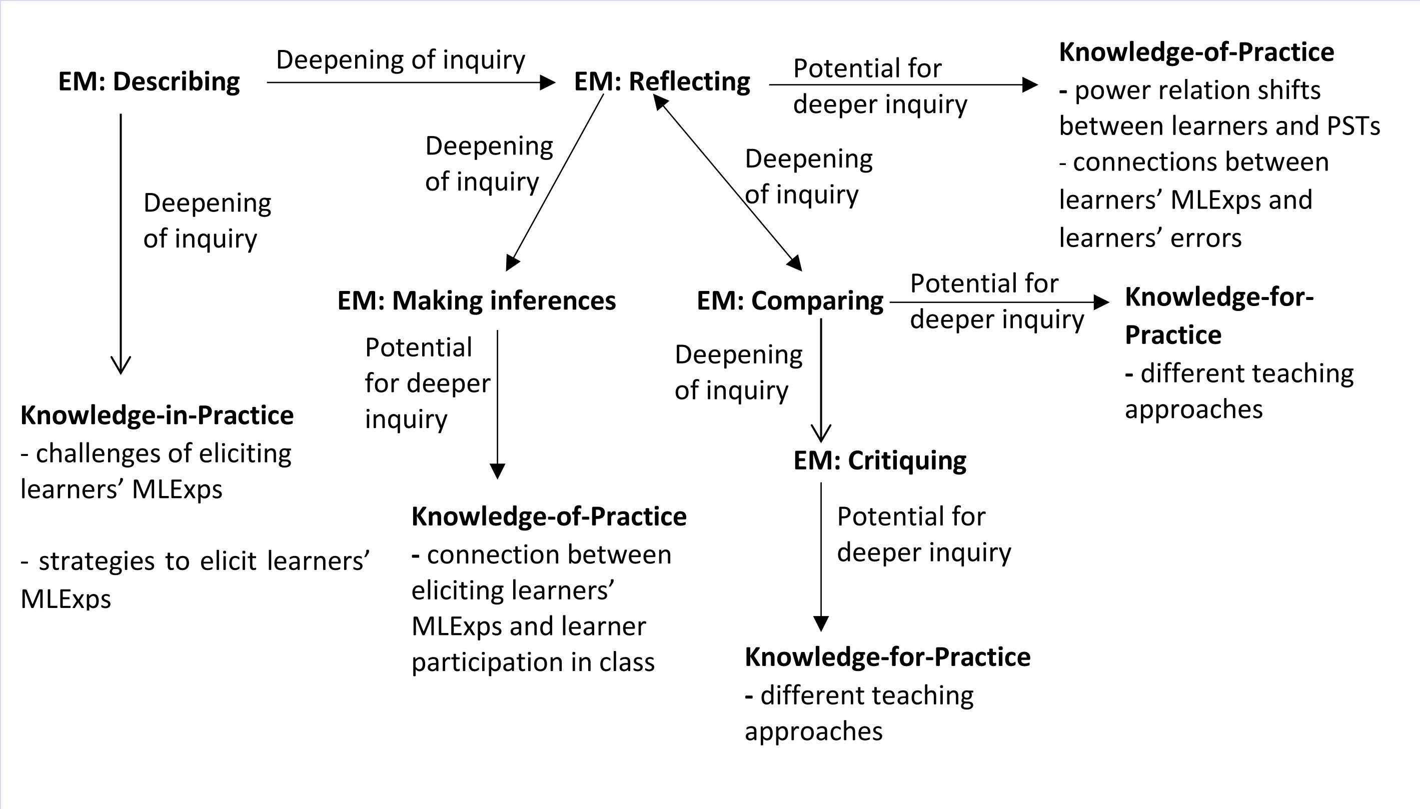


Figure 7.2: Interactions between epistemic moves influencing inquiry in relation to knowledge in embracing stance

Figure 7.2 shows that the interaction between the epistemic moves describing and reflecting enabled the epistemic move describing to deepen inquiry through knowledge-in-practice. Reflecting as epistemic move generated three pathways for the inquiry: one with potential to deepen inquiry through knowledge-of-practice and two pathways in which the inquiry was deepened through further epistemic moves, inferring and comparing. The epistemic move inferring created potential for deepening the inquiry through knowledge-of-practice. Comparing deepened the inquiry through generating a further epistemic move, critiquing, with potential for deepening inquiry through knowledge-for-practice.

7.2.3 Summary of embracing stance

Within the embracing stance there was ongoing inquiry into engaging learners' mathematical learning experiences in the PLC sessions. Furthermore, in the embracing stance inquiry enabled the developing of a professional learning culture of experimentation across the different 'activity settings', namely, the tutoring of learners and inquiry in the PLC. The descriptions of grappling to elicit learners' mathematical learning experiences generated a range of strategies that were experimented with and reflected upon. Describing and reflecting as epistemic moves enabled knowledge-in-practice on the challenges and practice of eliciting learners' mathematical learning experiences.

The reality that PSTs took the opportunity to view learners not as mere docile recipients of what teachers present to them but as capable of challenging and expressing what they know, emerged from a critical-reflective aspect of inquiry practised in the embracing stance. Epistemic moves which involved reflecting on shared experiences of engaging learners' mathematical learning experiences and PSTs' experiences as school learners were key in deepening inquiry through generating further epistemic moves. Comparing, an epistemic move generated through this reflecting, held potential for deepening inquiry through knowledge-for-practice on different teaching approaches.

Reflecting on a shared experience which involved a challenge by learners was instrumental in enabling elaboration on engaging learners' experiences. This reflecting held potential for deepening inquiry through knowledge-of-practice on connections between eliciting learners' mathematical learning experiences and shifts in power relations between learners and teachers. The potential for deeper inquiry through knowledge-of-practice signals the connections between the social and cognitive dimensions of learning which I elaborate on when doing my comparative analysis of the stances.

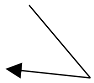
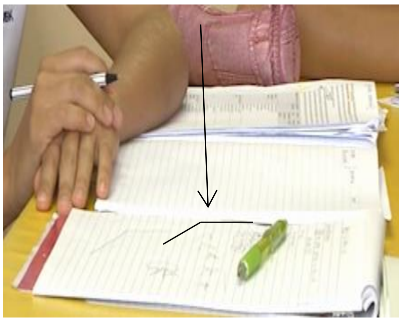
7.3 Supporting an embracing stance

In this section, I analyse an episode where I was the facilitator to show how pressing could be a significant epistemic move in encouraging PSTs to provide detailed descriptions of their engaging learners' mathematical learning experiences in an embracing stance. Through supporting an embracing stance, I was able to analyse the nature of critical moments in the inquiry when pressing as an epistemic move deepened inquiry. The supporting of an embracing stance did not lead to an embracing stance which signalled a deeper analysis of inhibitors other than describing. I elaborate on other inhibitors of critical inquiry when doing the comparative analysis of the stances.

Table 7.4 presents episode 2, facilitated by myself, depicting PST-PLC 4's potential to inquire into learners' mathematical learning experiences.

Table 7.4: Episode 2 from PST-PLC 4 session 1, facilitated by myself

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
1	R	PST 4C did you ask your learners what they are doing in class at the moment?		
2	PST 4C	Yes, the first week when I asked they said shapes and angles and then I asked if they struggled and they said they understood and the shapes they struggled to match the shapes with the names of the shapes. And then last week I gave them an activity and then they got the shapes right, but there was just one, they confused an acute and an obtuse angle	Describing learners' response to PSTs engaging learners' MLExps Describing how learners were engaged based on their MLExps Describing what learners were finding challenging based on observed errors	
3	R	Show us quickly.		Pressing for specificity on learners' challenges
4	PST 4C	[*Takes a pen and draws a sketch of the two angles.*] When we were in matric we were given a Cartesian Plane and quadrants and then I drew the angles in the plane and showed them the acute and obtuse angles in it.		Reflecting on MLExps as school learner to draw on a teaching approach to support learners
5	R	PST 4B if your learners struggled would you follow this approach?		Pressing for whether there is agreement on an approach to engaging learners' challenges
6	PST 4C	Can I say something; I took this problem out of a textbook and I think they should have asked it differently because you cannot expect a Grade 6 learner who is only beginning to recognise the difference. They were given right angle, obtuse, acute, reflex angles and they had to draw in the arrow to say in which direction the leg moved. [*PST 4C draws the angle and says]:* the arrow (angle leg) could have		Reflecting on tutoring practice Explaining a reason for learners' challenge Critiquing of the questions in textbook used in tutoring session

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		moved in either direction, so this was not a good question. 		
7	PST 4B	My learners are also in Grade 6 like PST 4C's learners and so I asked them as they are also doing angles what is the definition of an acute angle and obtuse angle because that is where we begin right? So the girl learner said to me, Teacher an acute angle is when two lines come together. Then I drew two lines and asked her whether that is an acute angle and she said no, it is an obtuse angle.  And I said to the learner but you said when two lines come together. Then the other learner came and said, no, no draw a scissors and he said as wide as a scissors can go that is an acute angle. And then I said but a scissors can open like this and that is not an acute angle:		Describing how learners' MLexps were elicited Raising uncertainty about the tutoring approach Describing how a learner explained Describing how learner MLExps were engaged Describing how a learner explained Describing how learner MLExps were engaged

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		 And then I also drew a plane like PST 4C, we were not supposed to give them the answer directly right?		Raising uncertainty about the tutoring approach
8	R	Did you ask the learner why he explained it like that to you?		Pressing for more information on how learner's MLExps were elicited
9	PST 4B	Yes, so I asked, he said firstly it was an acute angle where two lines meet and I said okay that is right but …and then he chipped in and said a scissors and I asked why do you say a scissors and he said because the teacher said the shape of a scissors (*Afrikaans:* a *skêr*) is an acute angle (*Afrikaans: skerp hoek*)…. *skêr* ….skerp hoek…….. *skêr* ….skerp hoek.		Describing how learner MLExps were elicited Describing learners' MLExps
10	PST 4C	To help them remember		Reflecting on the teacher's approach
11	PST 4B	And I asked why did you say it is where two lines meet and she said she can remember her teacher said when two lines cut each other and they make a sharp point then it is an acute angle and then I said, my dear, if you are going to do this *[PST 4B then redrew the angle and proceeded to share with the PLC her approach to developing a definition for an acute angle]*.		Describing how learner MLExps were elicited Describing learners' MLExps Making assumption about the learner's learning experience
12	PST 4B	I asked them, now what can you see on that ninety-degree angle about an acute and obtuse angle? And then the one learner said immediately because she is intelligent in mathematics and		Describing how learner MLExps were engaged Inferring about a learner being able to answer the question

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		she said, an acute angle is smaller than ninety degrees and an obtuse angle is greater than ninety degrees and then I said can you see you can come up with the answer… but then I said you must remember when it comes to a one hundred and eighty and they immediately said, teacher one hundred and eighty is a straight line and three hundred and sixty is a revolution, so they know this but they did just did not understand the definition of the concept of an acute and obtuse angle		Describing how a learner's MLExps was engaged Describing a finding about learners' understanding of a mathematical concept
13	PST 4C	Those things about the definitions, my learners knew their definitions, if you asked an acute angle and they knew it is between nought and ninety, but they could not identify the angles. But I still think the question was badly posed that is why.		Describing a finding about learners' understanding of a mathematical concept Explaining a reason for learners' challenge
14	PST 4B	My learners again, when they had to identify the shapes they could all say, obtuse, acute, reflex and even I forgot what reflex was and the learner could show me (*PST 4B moves her hands*). So they could identify but they did not have the definition of an acute and obtuse angle. And that is why I felt that I needed to go back in the way that I did.		Describing a different finding to PST4C about learners' understanding of mathematical concept Explaining a reason for approach to tutoring learners

I first inquired from PST-PLC 4 what the learners were currently doing in mathematics in class. In turn 2, PST 4C's describing indicates that she did engage learners about their learning experiences of angles, but the describing is an inhibitor of inquiry because PST 4C provides no specific information about the angles which learners found difficulty in identifying and naming which inhibits inquiry into the learners' errors. Also PST 4C provides no specific information on the activity which she designed to support the learners which inhibited inquiry into the support she provided. Hence, turn 3 illustrates a facilitation move to elaborate and for the pre-service teacher to provide a specific example of the confusion she noted that learners experienced between an acute and obtuse angle.

In turn 5, my facilitation move was intended to illustrate to the PST-PLC that when one teaching approach is presented, the community can be challenged whether another approach is possible (Brodie, 2016). Through encouraging more than one approach, the opportunity is generated for comparison of different approaches. In turn 8, my facilitation move is a press to elaborate with the intention of finding out how learners engaged their mathematical learning experiences and, in this case more specifically how learners' teachers taught the mathematical concepts that were discussed in the PST-PLC.

I coded facilitation moves as epistemic moves which enabled pre-service teachers to elaborate on their engaging learners which in turn generated epistemic moves for deeper inquiry into pre-service teachers engaging learners' mathematical learning experiences in class. I elaborate on these epistemic moves in the next section.

7.3.1 Epistemic moves in supporting an embracing stance

The epistemic moves generated in supporting an embracing stance consist of eight different kinds: pressing, reflecting, critiquing, describing, raising uncertainty, making inferences, explaining and making assumptions. I next elaborate on the interaction between the epistemic moves in supporting an embracing stance.

7.3.1.1 Epistemic moves – Pressing, Reflecting, Explaining and Critiquing

Pressing as an epistemic move deepens inquiry through generating further epistemic moves. The interaction between pressing and reflecting, an epistemic move generated by pressing, deepened inquiry through generating subsequent epistemic moves, namely explaining and critiquing. For example, my pressing in turn 3 for PST 4C to elaborate by giving a specific example of the learners' confusion led to her reflecting on her learning experiences of how the concept of angles was taught in Grade 12 and how she incorporated her Grade 12 learning of angles into supporting learners. I viewed PST 4C's reflecting on her Grade 12 learning experience of angles in turn 4 as an epistemic move for the PST-PLC to inquire into the approach for learners at Grade 6. Reflecting led to a further pressing in turn 5 for PST 4B to reflect on whether she would have followed the same approach. The pressing in turn 5 was intended to challenge the PST-PLC not merely to accept one teaching approach.

The pressing in turn 5 resulted in PST 4C reflecting on her tutoring practice, an epistemic move which deepened inquiry into the learners' challenges. For example, in turn 6 PST 4C critiqued the textbook she used in the tutoring and offered the PST-PLC a critique of the way in which questions on angles were structured. She believed this structure was responsible (see also turn 13) for the confusion the learners experience in identifying different angles. This critiquing of questions and explaining a reason for learners' challenges were epistemic moves for deeper inquiry into analysis of how questions around angles could be phrased to support the understanding of the concept of angles.

7.3.1.2 Epistemic moves – Pressing, describing, raising uncertainty, reflecting, making assumptions, making inferences and explaining

Pressing for further information was an epistemic move in that it generated describing where detailed information was provided of an event. Describing generated the epistemic moves raising uncertainty, reflecting, making assumptions, making inferences and explaining.

The describing which followed the pressing in turn 5 and turn 8 resulted in detail of how learners' mathematical learning experiences were elicited and engaged (turn 7) and learners' learning experiences (turn 9). I consider detailed describing of how learners were engaged on their mathematical learning experiences (turns 7 and 12) and raising uncertainty as epistemic moves for the PST-PLC to reflect on PST 1C's tutoring approach and whether they agreed that she was in fact adhering to the principles of the tutoring approach, i.e. to elicit the learners' mathematical learning experiences and to avoid simply re-teaching concepts with which learners had difficulty. Furthermore, describing learners' mathematical learning experiences and how learners explained mathematical concepts are epistemic moves for deepening inquiry into how learners' learning experiences relate to their challenges with mathematical concepts.

Describing learners' learning experiences led to another PST reflecting on a teacher's approach (turn 9). Reflecting was an epistemic move for deeper inquiry into the advantages and potential misconceptions which could arise from using practical tools in developing learners' mathematical concepts.

The detailed describing by PST 4B led to the making of assumptions about what the learner's teacher could have sketched when defining an acute angle to the class (turn 11). This assumption was a potential conceptual tool with which to have challenged the PST-PLC to consider other sketches the learner's teacher might have drawn in order to support the learner in refining her definition of an acute angle and rethinking how the teacher explained.

Making inferences about learners who are able to answer a question such as, *'the learner was intelligent in mathematics'* (turn 12) is an epistemic move with opportunity for deeper inquiry through reflection in the PST-PLC on what is implied about learners who found mathematics difficult.

Describing different challenges learners experienced with angles (turns 13 and 14) is an epistemic move with potential to generate deeper inquiry through reflection on how learners' knowledge of definitions does not necessarily mean they would be able to identify the angles correctly and vice versa. Describing different challenges led to PSTs explaining reasons for the learners' challenge and their tutoring approach. The explaining of reasons was an epistemic move for deeper inquiry into reasons for learners' challenges and approaches to engage learners with their challenges. In other words, the inquiry is deepened by being open to explore more than one reason for learners' challenges or approach in engaging learners.

Figure 7.3 below provides a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in supporting an embracing stance:

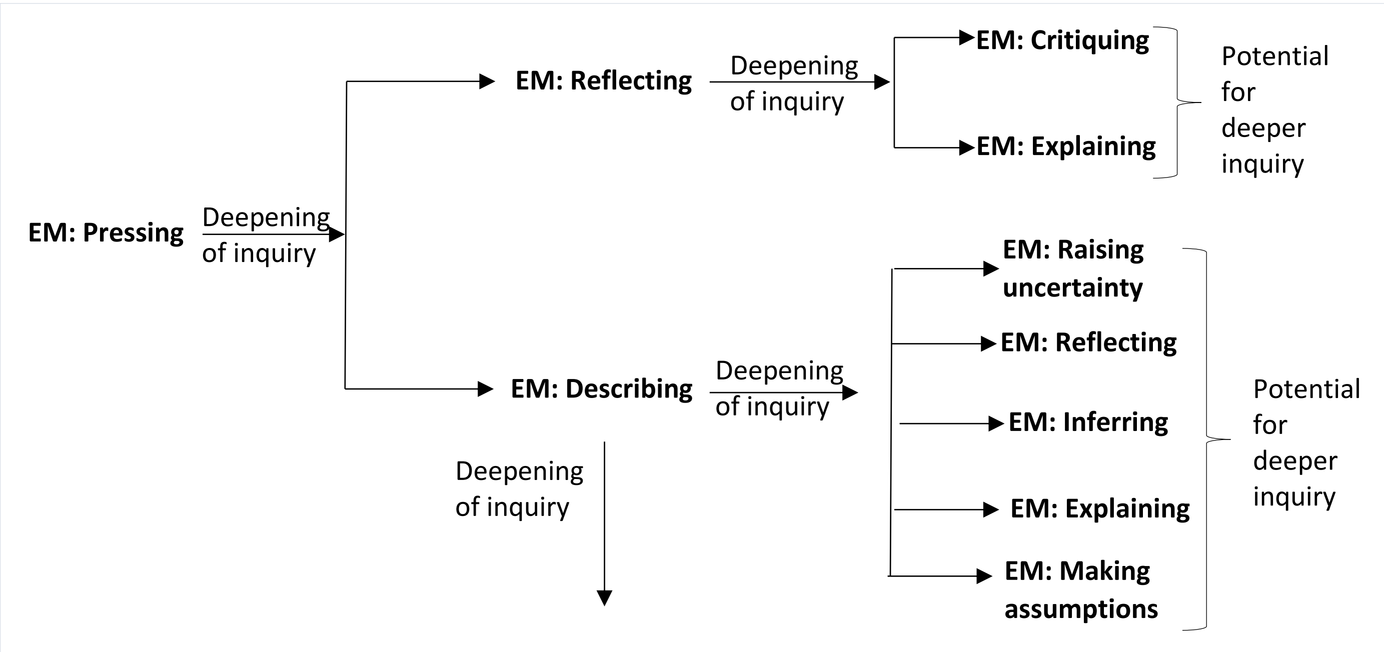


Figure 7.3: Interactions between epistemic moves influencing inquiry in supporting an embracing stance

Figure 7.3 shows that pressing is an epistemic move that was absent in the embracing stance facilitated by PSTs, is key to supporting an embracing stance in that it deepened inquiry through generating further epistemic moves describing and reflecting. Pressing as an epistemic move prevented limited descriptions which inhibited inquiry in the embracing stance. Describing as an epistemic move deepened inquiry by itself through the detail provided on learners' learning experiences and how learners were engaged. Furthermore, describing deepened inquiry through generating further epistemic moves: raising uncertainty, reflecting, making assumptions, making inferences and explaining with potential for deeper inquiry.

Reflecting as epistemic move deepened inquiry through generating further epistemic moves, critiquing and explaining with potential for deepening inquiry. In the next section I elaborate on how the epistemic moves deepened inquiry or held potential for deeper inquiry through knowledge.

7.3.2 Deepening inquiry through knowledge in supporting an embracing stance

Pressing for specificity on learners' challenges deepened inquiry through pre-service teachers' detailed describing of their tutoring approach to support learners, including how learners' mathematical learning experiences were elicited and engaged. Describing approaches to support learners often led to reflecting on resources that were used and the critiquing of these resources as underlying reasons for learners' challenges. For example, critiquing how questions were phrased in textbooks, created opportunity for systematic inquiry into the role of texts in influencing the challenges learners experience. Hence there is potential for deeper inquiry through knowledge-of-practice on the connection between teaching resources and learner challenges.

Describing learners' mathematical learning experiences and how learners explained mathematical concepts created opportunities for deeper inquiry into connections between learners' learning experiences and learner challenges with mathematical concepts. Furthermore, the making of assumptions about learners' learning experiences holds potential for deeper inquiry into a teacher's approach through affording the learner the opportunity to clarify the approach. For example, in turn 11, PST 4B indicated to the PST-PLC that the learner said that *'she can remember her teacher said when two lines cut each other and they make a sharp point then it is an acute angle'* which differed from the learner's first definition in that she, the learner, said *'an acute angle is when two lines come together'*. The opportunity for the PST-PLC to systematically inquire into how to support the learner's reflection on the learning experience is created; for example, the opportunity arose for the PST-PLC to suggest that the learner's reflection could be supported by asking the learner how the teacher drew the two lines which intersected. The reason that the inquiry did not happen is that while pre-service teachers' reflections on their own learning experiences at school (discussed in chapter 6) did involve trying to understand why they had challenges, their reflecting did not involve attention to the detail of how they were taught. Hence the potential for deeper inquiry through knowledge-of-practice into the connection between learner reflection on learning experiences and learner learning which may bring to the fore connections between social and cognitive dimensions of learning is a challenge.

The epistemic moves generated through describing created potential deepening of inquiry through knowledge-for-practice, in other words, the deepening of inquiry through further exploration of theory and not relying only on personal knowledge based on PSTs' experiences. Pre-service teachers did not recognise epistemic moves as moves for exploration of theory related to their practice. For example, emerging from the inquiry into learners' mathematical learning experiences are descriptions of teaching approaches to developing mathematical concepts: the concept of an angle, starting with concrete visual approaches (i.e. the use of scissor) and moving to more abstract representations. Describing generated reflecting on the learners' teacher's approach to developing the concept of an acute angle (see turn 10), and reflecting is an epistemic move for potential deeper inquiry into the teacher's approach. The potential for deeper inquiry is through knowledge-for-practice on the strengths and limitations of teaching approaches. Pre-service teachers did not recognise the epistemic move reflecting as involving the exploration of the strengths and limitations of teaching approaches.

Describing different learner difficulties with the same mathematical concepts created opportunities for the PST-PLC to recognise that the possibilities for learners' difficulties requires reflection on multiple factors influencing the teaching and learning context. Describing thus led to deepening inquiry through providing reasons for learners' difficulties which held potential for deeper inquiry through knowledge-for-practice on reasons for learners' difficulties, for example, drawing on theory about learners' challenges in understanding definitions.

In describing how learners' mathematical learning experiences were engaged, inferences were made about learners' abilities to understand mathematical ideas. The making of inferences did create potential for deeper inquiry into reasons for learners' difficulties in order to challenge the idea that learner 'ability' explains why some learners are able to produce correct answers and others not. The potential for deeper inquiry into how the development of mathematical ideas is inherently challenging is enabled through knowledge-for-practice into connections between learners' learning of concepts and learner challenges.

The detailed describing of learners' mathematical learning experiences and how learners were engaged created opportunities for the PST-PLC to see a different entry point to engaging learners who struggle with mathematics. The detailed describing enabled knowledge-in-practice on ways of supporting the learners' reflection on their mathematical learning experiences and in so doing identifying where they may have missed key information in the teaching and learning process. Hence I argue that there is more than one way of engaging learners' mathematical errors. Asking learners how they reasoned when they made an error is important but it is not the only point of entry to engaging learners. Starting with their learning experiences may enable PSTs to see how learners' errors arise within the context of their learning experiences.

Figure 7.4 below builds on figure 7.3 to provide a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in relation to knowledge in supporting an embracing stance.

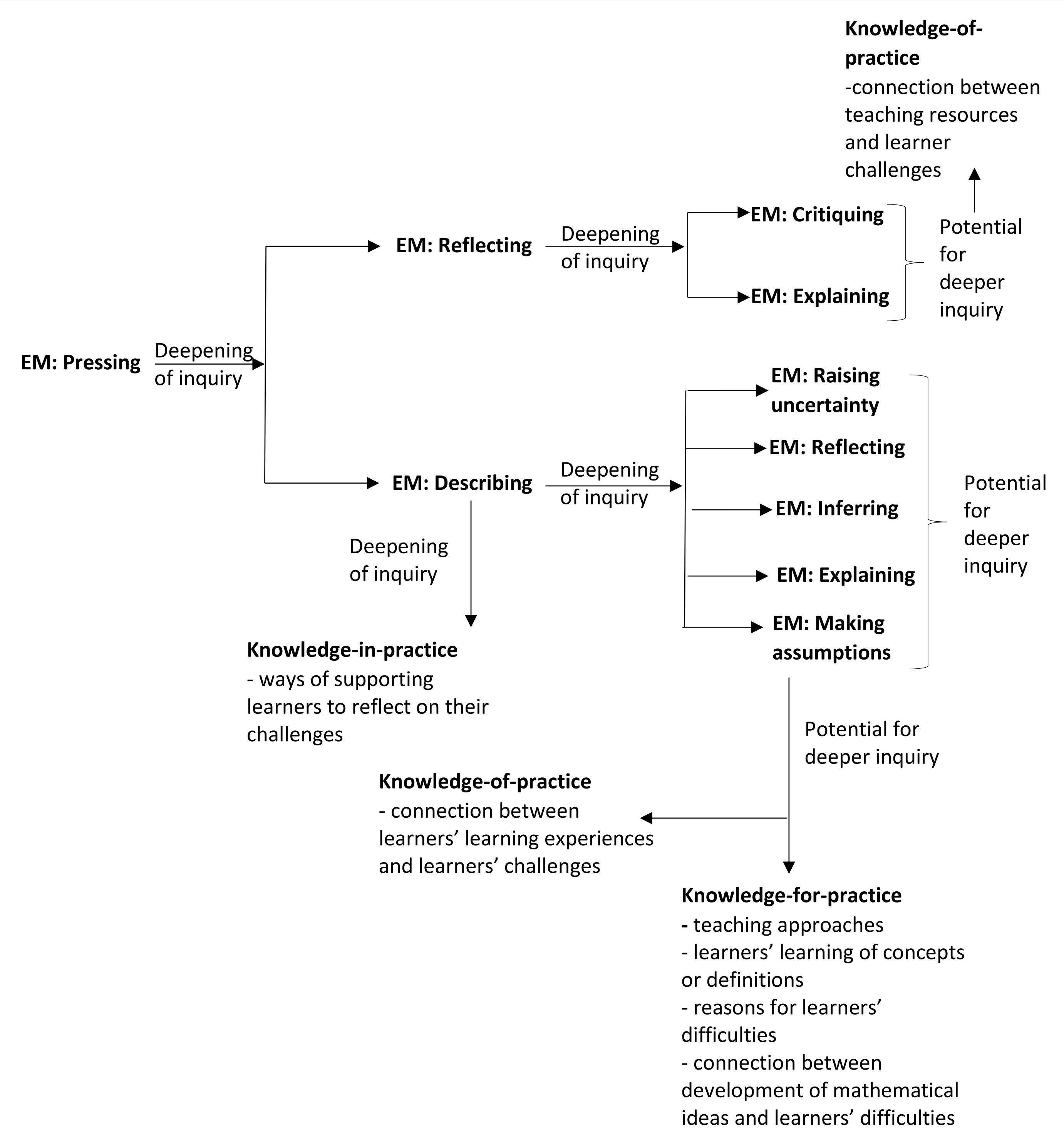


Figure 7.4: Interactions between epistemic moves influencing inquiry in relation to knowledge in supporting an embracing stance

Figure 7.4 shows how through supporting an embracing stance detailed describing generated through pressing deepened inquiry through enabling knowledge-in-practice and had potential to deepen inquiry through knowledge-for-practice and knowledge-of-practice. Where reflecting was generated through pressing, there was only potential for deeper inquiry through knowledge-of-practice.

7.3.3 Summary of supporting an embracing stance

In supporting an embracing stance, pressing generates detailed descriptions of how learners' mathematical learning experiences were elicited and engaged. The interaction between pressing and describing as epistemic moves enables knowledge-in-practice on how to elicit learners' learning experiences to enhance learners' reflection on their learning experiences.

Describing as an epistemic move deepened inquiry through generating other epistemic moves which held potential for deepening inquiry through knowledge-for-practice. The recognition of these other epistemic moves and how they deepen inquiry was key for exploring theory on teaching approaches; the learning of mathematical concepts; reasons for learners' difficulties and connections between the development of mathematical concepts and learners' difficulties. The non-recognition of these epistemic moves and how they deepen inquiry was the reason for the inquiry not being deepened through these epistemic moves.

Pressing for PSTs to elaborate on how they engaged learners' difficulties followed by PSTs reflecting and critiquing resources used holds potential for deepening inquiry through knowledge-of-practice on connections between teaching resources and learners' difficulties. Describing of learners' mathematical learning experiences and how learners explained mathematical concepts were key epistemic moves which held potential for deepening inquiry through knowledge-of-practice on connections between learner' learning experiences and learners' difficulties. The potential for deepening inquiry here signals the potential for connections between the social dimensions and cognitive dimensions of learning. In section 7.5.5 I will elaborate on the potential social –cognitive connection learning opportunities which emerged in this stance in my comparative analysis of the stances.

7.4 Compliance stance

In the compliance stance, PST-PLCs only inquired into engaging learners' mathematical learning experiences in the first PLC session. Three of the PST-PLCs reflected a compliance stance in relation to learners' mathematical learning experiences in class and all five PST-PLCs in relation to learners' mathematical learning experiences beyond school (see table 7.2). As discussed previously, one reason for the limited inquiry was the challenge in getting learners to elaborate on their mathematical learning experiences in class and beyond school in the tutoring sessions.

The inquiry into learners' mathematical learning experiences beyond school suggests that PSTs had struggled to understand the purpose of engaging learners' mathematical learning experiences beyond school. PSTs' limited understanding of the purpose of engaging learners' mathematical learning experiences beyond school may be another reason for not extending the inquiry after the first PLC session. The episode in Table 7.5 which I selected from the first session of PST-PLC 1 illustrates how PSTs shared experiences about engaging learners' mathematical learning experiences beyond school and that sharing experiences in relation to learners' mathematical learning experiences beyond school do hold potential for deeper inquiry into these experiences.

Table 7.5: Episode 3 from PST-PLC 1, session 1 facilitated by PST 1A

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
94	PST 1A	Is there something else you came across these learners? Oh yes, Miss asked us in our first session to find out how the learners, by their parents or who they live with, their family, brothers and sisters, how this influences that they are interested in mathematics or something, have you got something there?		
95	PST 1B	My boy learner he has a brother in high school and he helps him at home. And I can see if I ask him, oh this is nice where did you learn this and then he says by my brother. So I think his older brother plays an important role in his life especially with mathematics because I think that his brother enjoys mathematics and feels that he must also enjoy it, but I can see he really enjoys it. And the girl because she is so shy she does not ask around much….she figures out things for herself.	Describing information about the learners' learning experience Describing information about the learner	Describing information about the learners' home context. Inferring about a learner's attitude towards mathematics

Turn	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
96	PST 1A	Yes, I have two girls, but the one has I am not sure, a sister the other is probably a brother, also both in the high school, in Grade 12, and she shows her difficult maths to the little one this work and she is extremely interested in it. So I can see she is interested if she can go to a Grade 12 girl and say help me with this. And both of them said that their mothers help them with mathematics in the afternoon. Not that they are necessarily good at it or in a business involved with it, but they at least get help.	Describing information about the learners' home context.	Describing information about the learners' home context Inferring about a learner's attitude towards mathematics Making an assumption about learners' parents
97	PST 1C	My learners have younger brothers and sisters. But not that there is shortage of support in their homes.	Describing information about the learners' home context.	

In turn 94, PST 1A prompted the PLC to share experiences of their engagement in eliciting learners' mathematical learning experiences beyond the classroom, expressing uncertainty about the purpose of doing this. The primary reason for engaging learners about their mathematical learning experiences beyond the classroom or school according to her is to find out how these experiences influenced learners' interest in mathematics but she is not sure what else (in her words '*or something*'). PST 1A does not provide a reason for deeper insight into learners' learning to be gained from engaging learners' mathematical learning experiences beyond school.

The conversation which followed involved describing learners' home contexts with a focus on who the learners approached for support with their school mathematics. The limited describing in turns 95, 96 and 97 of learners' mathematical learning experiences in their homes were inhibitors of critical inquiry. For example, in turn 95 the description does not contain information on how the learner was helped by an older brother. In turn 95, PST 1B indicated that she found it interesting (in her words, *oh that is nice)*, which refers to the kind of mathematics the learner's brother was engaging with the learner. In turn 96, PST 1A provided information about one of the learners whose older sister in Grade 12 not only helps her with mathematics but also engages her with more difficult mathematics. The describing does not include information about what mathematics the learner asked for help with from her sister or what the mathematics was that the sister showed the learner. The limited describing inhibited PST-PLC 1 from gaining deeper insight into the learners' mathematical learning experiences in their homes.

As discussed in Chapter 6 many of the pre-service teachers' mathematical learning experiences in their homes involved parental support in which they were encouraged to explain the mathematical problems with which they struggled. Many of the pre-service teachers' learning experiences in their homes involved supporting younger siblings. While many pre-service teachers' narratives of these learning experiences indicate that these learning experiences supported their learning there was no indication in their narratives that they were asked to describe these learning experiences at school. While pre-service teachers were asked at home to explain mathematical challenges there was no indication in their narratives that they were asked to explain their mathematical learning experiences of these challenges in terms of how they were taught. Since pre-service teachers had no experience in describing their learning experiences and hence no support in doing so, explains their struggle to elicit learners' learning experiences in the tutoring. The struggle to elicit learners' learning experiences in the tutoring explains the limited describing of learners' learning experiences in their homes within PST-PLCs and inquiry into how learners' learning experiences in their homes could provide insight into learners' challenges and how these were overcome,

In the next section I discuss the epistemic moves generated in the episode in Table 7.5 with potential for deepening inquiry.

7.4.1 Epistemic moves as enablers of critical inquiry in compliance stance

The epistemic moves generated in a compliance stance consist of three different kinds: describing, making inferences and making assumptions. As within the embracing stance describing was both an inhibitor and epistemic move for deepening inquiry. I explained above that describing information of learners' home contexts was limited and hence inhibited deeper inquiry into learners' mathematical learning experiences in their homes. Describing as epistemic move generated further epistemic moves, namely making inferences and making assumptions.

7.4.1.1 Epistemic move – Describing and inferring

Making an inference was generated when describing information about learners' home context (turns 95 and 96). Making inferences illustrates that PSTs were trying to make connections between learners' attitudes to mathematics and their home mathematical learning experiences. For example, in turn 95, an older sibling's interest in mathematics is given as potential reason for the learner's motivation in trying to '*enjoy*' mathematics. The uncertainty of PST 1B in turn 95, when she mentioned that, *'she thinks the older brother plays a significant role*', is a significant dimension of uncertainty. This observation supports her contention that there is a connection between the learner's attitude towards mathematics and the role of the older sibling. And in turn 96, the learner's interest in mathematics is advanced as a potential reason for the learner seeking support from an older sibling. I consider the making of these inferences as epistemic moves for further inquiry into how learners' mathematical learning experiences in their homes influenced their learning of their mathematics.

7.4.1.2 Epistemic move – Describing and making assumptions

Making assumptions as an epistemic move is generated when describing learners' home contexts. Making assumptions was mainly about learners' parents' ability to support their children with mathematics. For example, in turn 96, PST 1A's assumption reflects her perception of one's ability in mathematics as being related to the kind of profession one is in. Making assumptions are epistemic moves as they have potential to ignite inquiry into why assumptions are made and can form the basis of the shift from reflection into critical reflection (Brookfield, 1995).

Figure 7.5 below provides a summary of the interaction between epistemic moves which led to the deepening of inquiry and potential deepening of inquiry in the compliance stance:

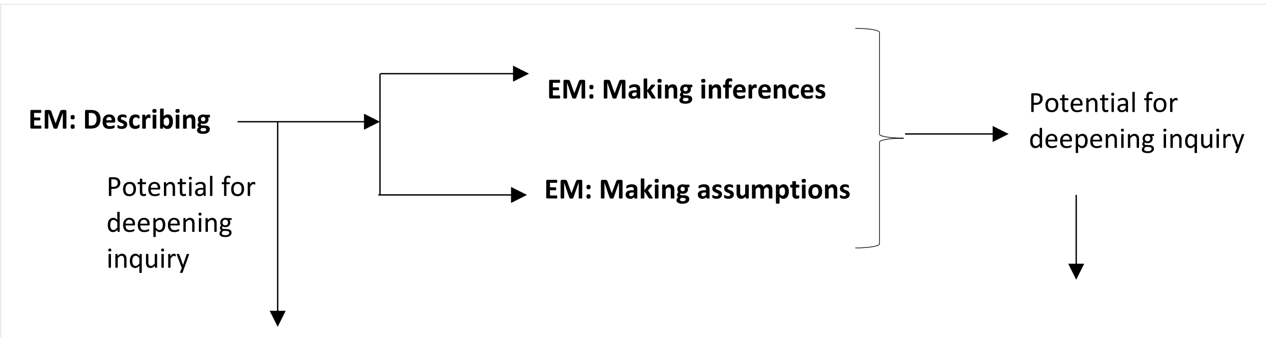


Figure 7.5: Interactions between epistemic moves influencing inquiry in compliance stance

Figure 7.5 shows that in the compliance stance, describing as an epistemic move deepened inquiry into learners' learning experiences through generating further epistemic moves, making inferences and making assumptions. Due to the limited describing of learners' learning experiences inquiry into these experiences were inhibited and hence describing by itself only held potential for deepening inquiry. The inferring about learners' learning experiences in their home is a move to make connections between learners' learning experiences and their attitude towards learning of mathematics which held potential for further inquiry into this connection. The making of assumptions about learners' learning experiences held potential for deeper inquiry into these assumptions and to challenge deficit perspectives of learners' learning experiences. In the next section I elaborate on how the epistemic moves created potential for deepening inquiry through knowledge.

7.4.2 Deepening inquiry through knowledge in compliance stance

In the compliance stance towards inquiry into engaging learners' mathematical learning experiences beyond school, describing learners' mathematical learning experiences in their homes reflected the diverse range of support which learners have within family structures. Potential for deeper inquiry into learners' mathematical learning experiences in their homes is through knowledge-of-practice, which involves the systematic inquiry into how learners convey their mathematical challenges to their siblings or parents and what and how learners learn through these learning experiences with parents or siblings.

The absence of support within a learner's broader community also provided evidence of how a learner who may feel alienated in class perseveres on her own. This was evident in the description of PST 1B in turn 57 from the first PLC session of PST-PLC 1 which shows her probing what the learner does when she struggles with mathematics:

PST 1B: When I ask the girl learner what work did you not understand and then she said she did not understand the right angles that well. They began with right angles. Then I asked her if she asked someone in the class. And then she said no she is too shy. So I said ok, so do you go to the teacher afterwards. And then she said no because she is too shy because she does not want that humiliation that she does not understand what she is doing. Do you understand? And then I asked her whether she feels more comfortable to ask someone in her home. Then she said the people at home cannot help her as they do not understand the work. Then I asked but is there at least someone that you can speak to about the work. And she said she finds it out for herself as the teacher moves along and as the work gets more difficult and then she gets it, what was mentioned earlier. (Turn 57)

This describing of the learner's perseverance when struggling with mathematics is an epistemic move as opportunity for the PST-PLC to deepen inquiry into how the learner learns. It can be inferred from PST 1B's describing of the learners' learning experience that the learner is able to make mathematical connections that enable her to understand mathematical ideas she struggled with previously. This describing provides the potential for deepening inquiry through knowledge-of-practice on how learners develop understanding of mathematical ideas as the work becomes progressively difficult.

Furthermore, describing learners' mathematical learning experiences in their homes provided information with potential to challenge deficit-based construction of learners in difficult circumstances on more than one level. Firstly, a common perception exists that learners from working-class contexts, as in the case with the learners who were tutored, have minimal resources; and secondly that these resources may not be of value to their learning in school (Swadener, 2010). The assumption made by PST 1A in turn 96 about the support the learner got from a parent/s is evidence of a perception of the learner's resource. One can conclude that PST 1A perceived the resource of little value. I argue that the assumption made by PST 1A is an epistemic move as it generates an opportunity for challenging a deficit-based construction of learners' support beyond school with potential for deepening inquiry through knowledge-of-practice about learners' resources for learning beyond school. Potential for deepening inquiry through knowledge-for-practice of learners' resources beyond school is necessary to support the challenge of deficit-based constructions of learners' resources beyond school. While PSTs were introduced to research and theory on 'funds of knowledge', the inquiry in the PST-PLCs did not draw on this knowledge. Theory on 'funds of knowledge' is knowledge-for-practice to deepen understanding of how to engage learners' learning resources beyond school. This knowledge-for-practice can support the deepening of inquiry into learners' learning resources through knowledge-of-practice: a systematic inquiry into how learners' learning resources support their learning.

Making inferences about learners' attitudes towards mathematics created potential for deepening inquiry through knowledge-of-practice on connections between learners' attitude to mathematics and their mathematical learning experiences in their homes.

Figure 7.6 below builds on figure 7.5 to provide a summary of the interaction between epistemic moves which led to the potential deepening of inquiry in relation to knowledge in the compliance stance:

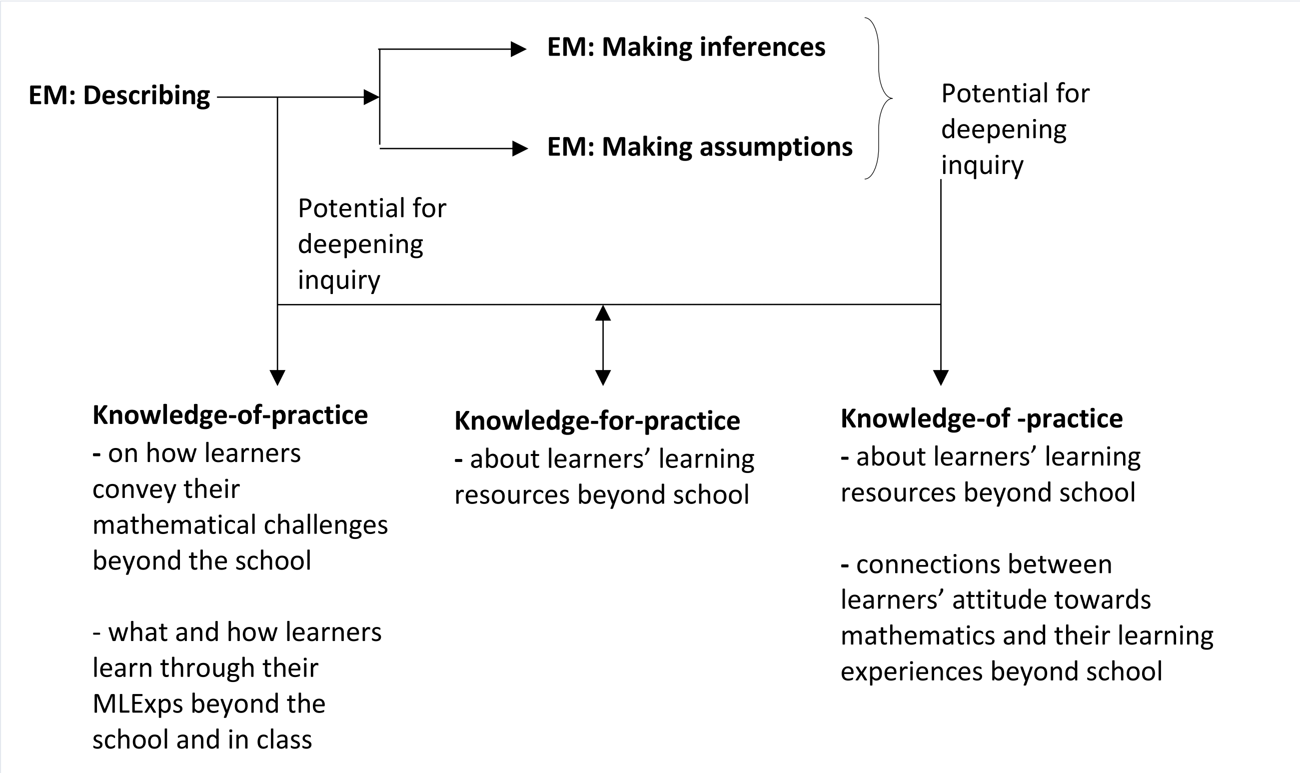


Figure 7.6: Interactions between epistemic moves influencing inquiry in relation to knowledge in the compliance stance

Figure 7.6 shows that in the compliance stance all epistemic moves created potential for deepening of inquiry through knowledge-of-practice. The inquiry into learners' learning experiences was confined to the first PLC session only and hence there was no deepening of inquiry through knowledge-in-practice. The assumptions made about learners' learning resources in their homes created the potential for deepening inquiry through knowledge-for-practice on learners' learning resources beyond school. The knowledge-for-practice interacts with knowledge-of-practice to deepen inquiry into learners' learning resources and experiences.

7.4.3 Summary of compliance stance

In the compliance stance towards inquiry into engaging learners' mathematical learning beyond the classroom, there was no evidence of experimenting with different strategies of eliciting learners' mathematical learning experiences as there was in the case of the embracing stance towards inquiry into engaging learners' mathematical experiences in class. Furthermore, the limited describing of learners' learning experiences and few epistemic moves explain why knowledge-in-practice was not enabled in this stance.

The PST-PLC's inquiry into learners' mathematical learning beyond the class did create epistemic moves with potential for disrupting deficit constructions of learners and communities. Pre-service teachers' descriptions of engaging learners' mathematical learning experiences in the compliance stance brought to the fore issues of motivation, challenge and perseverance as significant in understanding learners' learning experiences beyond school and in class. These descriptions of learners' mathematical learning experiences generated epistemic moves which held potential for deepening inquiry through knowledge-of-practice on learners' resources beyond school: firstly, systematic inquiry into how learners convey their mathematical challenges beyond school and what learners learn through their mathematical learning experiences beyond school and in class; and secondly, systematic inquiry into connections between learners' attitudes towards their mathematics and their mathematical learning experiences beyond school.

The potential for deepening inquiry through knowledge-of-practice signals the opportunity for further inquiry into connections between social and cognitive dimensions of learning as well as knowledge-for-practice required for knowledge-of-practice about these connections. In section 7.5.5 I will elaborate on the potential social-cognitive connection learning opportunities which emerged in this stance in my comparative analysis of the stances.

7.5 Comparative analysis of inquiry across stances

The different stances impacted on social and cognitive dimensions of learning that emerged from inquiry into learners' mathematical learning experiences. In order to gain a deeper understanding of this impact, I compared categories and subcategories of the inhibitors and epistemic moves that were generated in the episodes which are representative of the two stances towards inquiry.

Within both stances there was often a rapid shift from one shared narrative to another in the inquiry into engaging learners' mathematical learning experiences.

7.5.1 Comparing inhibitors of critical inquiry

To understand why epistemic moves do not always lead to deepening inquiry but only have potential to do so requires identifying the inhibitors of critical inquiry. I identified three broad categories of inhibitors.

The first category of inhibitor I defined as limited description when limited or incomplete description of an event was given. The five subcategories of this inhibitor all relate to the reconstruction of practice within the PLCs (Horn & Kane, 2015).

The second category of inhibitor is non-reflective practice. I defined this category as the absence of reflection on practice and learning. The three subcategories of this inhibitor relate to whether the reflection was on practice in teaching, reflection on the inquiry in the PLC or reflection on prior learning experiences, all of which relate to the reconstruction of practice.

The third category of inhibitor is not related to the reconstruction of practice. I defined this category as the non-recognition of epistemic moves as enablers for deepening inquiry. The four subcategories of this inhibitor were related to the following epistemic moves: raising uncertainty; critiquing and comparing; making assumptions or inferences; and explaining a reason. Table 7.6 shows the inhibitors of critical inquiry across the two stances:

Table 7.6: Inhibitors of critical inquiry across the stances

Inhibitor of critical inquiry category	Description of subcategories of inhibitors	Stance
Limited description When limited or incomplete description of an event was given .	Describing with limited detail of mathematical content in learners' narratives of their MLExps	Embracing and Compliance
	Describing with limited detail of how learner explained a teacher's approach	Embracing and Compliance
	Describing with limited detail of a learner's thinking	Embracing and Compliance
	Describing with limited detail of a learner's challenges	Embracing and Compliance
	Describing with limited detail of how PSTs engaged learners on MLExps	Embracing and Compliance
Non-reflecting on Practice The absence of reflection-on-practice and learning	No reflecting on teaching practice	Compliance
	No reflecting on inquiry in PLC	Compliance
	No reflecting on prior learning experiences	Compliance
Non-Recognition of EM Not recognising how EM deepen inquiry	Raising uncertainty to explore further knowledge	Embracing stance
	Comparing and critiquing to elaborate on strengths and limitations	Embracing stance
	Making an inference or assumption as opportunity to challenge	Embracing and Compliance
	Explaining a reason prompt to explore alternatives	Embracing stance

Table 7.6 shows that all the subcategories of describing and one category in non-recognition of epistemic moves as inhibitors were common across the two stances. Non-reflection-on-practice was an inhibitor only in the compliance stance.

The five subcategories of the inhibitor limited description point to the importance of noticing key elements within representations of practice, namely, the mathematical content, how learners explain, learners' thinking, learners' challenges and how PSTs engaged learners. In order to enhance the inquiry into engaging learners' mathematical learning experiences, an awareness of greater specificity in relation to these elements is required. The awareness of when greater specificity is required in the reconstruction of teaching practice is a first step in the direction to enhancing critical inquiry in PST-PLCs and relates to Wenger's (1998) concept of mutual accountability to the joint enterprise of a community of practice:

Wenger (1998, p. 78) made the following point about the joint enterprise that keeps a community of practice together:

> It is not just a stated goal, but creates amongst participants, relations of mutual accountability that becomes an integral part of practice.

In supporting the embracing stance, I showed how pressing for specificity was an epistemic move which enabled deeper inquiry into how learners' mathematical learning experiences were elicited and engaged through detailed describing. This press for specificity when describing one's teaching practice becomes a practice of inquiry of the PLC for which mutual accountability has to be taken in order to develop a critical inquiry stance.

The three subcategories of the inhibitor non-reflection-on-practice are only present in the compliance stance, suggesting that this inhibitor is related to the stance towards inquiry into engaging learners' mathematical learning experiences. A press for reflection on teaching practice and pre-service teachers' own learning experiences, as a well as a reflection on each other's practice, are practices of critical inquiry. These practices generate mutual accountability of a PLC developing a critical inquiry stance.

The four categories of the inhibitor non-recognition of epistemic moves point to pre-service teachers' understanding of how these epistemic moves deepen inquiry. I compared the four subcategories in terms of how the epistemic moves could deepen inquiry. For example, I argued that the recognition of assumptions and inferences create opportunities to challenge and question assumptions and inferences in order to disrupt deficit perspectives on learners' mathematical learning experiences. The recognition of when to explore further knowledge, to challenge assumptions and inferences, to elaborate on the strengths and limitations when comparing or critiquing and to consider as many possible reasons are practices of critical inquiry in PLCs. These practices become a mutual accountability of a PLC developing a critical inquiry stance. In section 7.5.5 I will elaborate on how these inhibitors influenced the deeper inquiry into potential connections between the social and cognitive dimensions of learning in the inquiry.

7.5.2 Comparing epistemic moves of critical inquiry

Comparing the categories and subcategories of epistemic moves across the stances explains the embracing stance's deeper inquiry into engaging learners' mathematical learning experiences and enabling of knowledge-in-practice. In Table 7.7 I compare the nine categories of epistemic moves, some of which were not present in Zenios's study (2011), namely, reflecting, raising uncertainty, making assumptions and pressing.

Table 7.7: Epistemic moves of critical inquiry across the stances

Epistemic move category	Descriptions of subcategories of epistemic moves	Stance
Reflecting	**1. Reflecting on inquiry in PLC**	
	Reflecting on PST's account of **engaging a learner**	Embracing
To question; notice; make suggestions based on what is being said or based on past experiences	Reflecting on **another PST's reflection on a teaching approach**	Embracing
	2. Reflecting on MLExps as a school learner	
	Reflecting on MLExps to draw on teaching approach to support learners	Embracing
	3. Reflecting on teaching practice	
	Reflecting on **learners' shift in confidence** to talk about their MLExps	Embracing
	Reflecting on learners' **teachers' teaching approach**	Embracing
Critiquing	**Critiquing of teaching resources**	
To point out the positives or negatives of a situation	Critiquing of curriculum	Embracing
	Critiquing of textbook questions	Embracing
Inferring	**Making inferences about learners**	

Epistemic move category	Descriptions of subcategories of epistemic moves	Stance
To come to a conclusion based on evidence	Making inferences **about learners** being able to answer questions	Embracing
	Making an inference **about learners'** participation in class	Compliance
	Making an inference **about learners'** attitudes towards mathematics	Compliance
Describing	**1. Describing aspects of teaching practice**	
	Describing the **challenge of eliciting learners'** learning experiences and probing their thinking	Embracing
	Describing how **learners' MLExps were elicited**	Embracing
To provide an accurate conception of an object or event	Describing how **learners' MLExps were engaged**	Embracing
	2. Describing aspects of learners	
	Describing how **learners explained mathematical concepts**	Embracing
	Describing a different finding to another PSTs about **learners' understanding of a mathematics concept**	Embracing
	Describing information about a **learner's home context**	Compliance
Raising uncertainty To express the need for greater clarity on an approach or concept	**Raising uncertainty about tutoring approach**	Embracing
Comparing To point out the similarities or differences in a situation	**Comparing teaching approaches when at school to current teaching approaches**	Embracing
Explaining To explain a reason for a situation	**Explaining a reason for learners' challenges**	Embracing
Making assumptions	**Making assumptions about learner**	
To draw conclusions without the necessary evidence	Making assumptions about **learners' learning experiences in class**	Embracing
	Making assumptions about **learners' parents**	Compliance
Pressing[12]	**1. Pressing for specificity on learners' challenges**	Embracing

[12] These epistemic moves were generated through my facilitation

Epistemic move category	Descriptions of subcategories of epistemic moves	Stance
To press for specificity or to elaborate on a situation	**2. Pressing for elaboration on teaching practice**	
	Pressing for whether there is agreement on an **approach to engaging learners**	Embracing
	Pressing for more information on **how learner's MLExps were elicited**	Embracing

Table 7.7 shows that in the embracing stance, the epistemic moves influenced inquiry into learners, pre-service teachers' learning, teaching practice (including teaching resources), and the practice of inquiry in the PST-PLC. In the compliance stance, the epistemic moves only influenced inquiry into learners with a focus on learners' participation, learners' attitudes towards mathematics and learners' parents.

The inquiry into learners' learning experiences in the compliance stance focused on the social dimensions of learning, whereas in the embracing stance the inquiry into learners' learning focused on cognitive and social dimensions of learning as well. The sub-category of the describing epistemic move, 'describing aspects of learners', shows the embracing stance applied to the cognitive dimension of learning of learners explaining mathematical concepts and their mathematical challenges. The sub-category of the reflecting epistemic move, 'reflecting on teaching practice', shows social dimensions of learning on learners' shifts in confidence to talk about their mathematical learning experiences. I will elaborate on how reflecting as an epistemic move was key in deepening inquiry into social and cognitive dimensions of learning in the embracing stance.

Table 7.8 provides a summary of the epistemic moves across the stances which deepened inquiry either directly through knowledge and/or through generating further epistemic moves and/or only with potential to deepen inquiry through knowledge.

Table 7.8: Epistemic moves that deepen inquiry and/or with potential to deepen inquiry

Stance	Epistemic Move (EM)	
Embracing	**Deepened inquiry (3)**	**Potential to deepen inquiry (6)**
	Describing – generating further EM – directly through K-in-P	Raising uncertainty
	Reflecting – generating further EM	Reflecting

	Pressing – generating further EM	Explaining
		Critiquing
		Making inferences
		Making assumptions
	Deepened inquiry (0)	**Potential to deepen inquiry (3)**
Compliance		Describing
		Making inferences
		Making assumptions

Table 7.8 shows that all three epistemic moves in the embracing stance deepened inquiry through generating further epistemic moves. However, in the embracing stance only describing deepened inquiry directly through knowledge-in-practice. I provided evidence and argued that in both stances the epistemic moves with potential to deepen inquiry was through knowledge-for-practice and knowledge-of-practice.

In the next section I draw on Horn and Kane's (2015) and Little's (2002) concept of representations of practice in PLCs. This is about how teaching practice is reconstructed and the way representations of practice are engaged to influence opportunities to learn. I show that the differences in the epistemic moves across the two stances is an indication of different framings of representations of practice (Horne & Kane, 2015; Louie, 2016). I draw on Greeno's (2009) concept of epistemological framing, namely, 'the kinds of task that a participant or participants understand themselves to be engaged in, especially regarding the kind(s) of knowledge that are relevant to and expected to be constructed in order to succeed in the task', i.e. to describe the different ways in which representations of practice were used in the two stances.

7.5.3 Representations of practice across the stances

A key element of PLCs is how participants draw on knowledge both from their classrooms as well and the professional learning community to mutually inform learning across both sites (Kazemi & Hubbard, 2008). Representations of practice are key in these transitions because they show what teachers select from their practice and how various aspects of practice are made visible in the move between classroom and professional learning community (Little, 2002, p.

934). The inquiry within PST-PLCs was influenced by the representations of practice that pre-service teachers selected to share with their community.

7.5.3.1 Representations of practice in embracing stance

In the embracing stance the representations of practice included descriptions of learners' narratives about their mathematical learning experiences as well as descriptions of strategies to elicit and engage learners' mathematical learning experiences in class. Emerging from the analysis of the inquiry within the PST-PLCs is that developing strategies to elicit learners' mathematical learning experiences is a key teaching practice when engaging learners' mathematical learning experiences.

The importance of developing the practice of eliciting learners' mathematical learning experiences became a key task of the inquiry within the embracing stance. This was shown in the ongoing experimentation with different strategies of eliciting learners' mathematical learning experiences. Kazemi and Hubbard's (2008) concept of learning that operates multi-directionally across contexts was a feature of the inquiry in the embracing stance. The ongoing feedback and reflection on the experimentation with different strategies of eliciting learners' mathematical learning experiences in the tutoring was a practice of the PST-PLC that generated knowledge about the teaching practice of eliciting learners' mathematical learning experiences. I describe the use of the representations of practice in the embracing stance as a *source of inquiry* in that knowledge development was enabled through epistemic moves. In the embracing stance, the representations of practice as sources of inquiry depicts a framing of collaboration as inquiry to enhance opportunities to learn (Louie, 2016).

Figure 7.7 shows that in the embracing stance representations of practice mainly on the challenge of eliciting learners' mathematical learning experiences, moved across the two learning spaces, i.e. tutoring class and PST-PLC, as a source of inquiry:

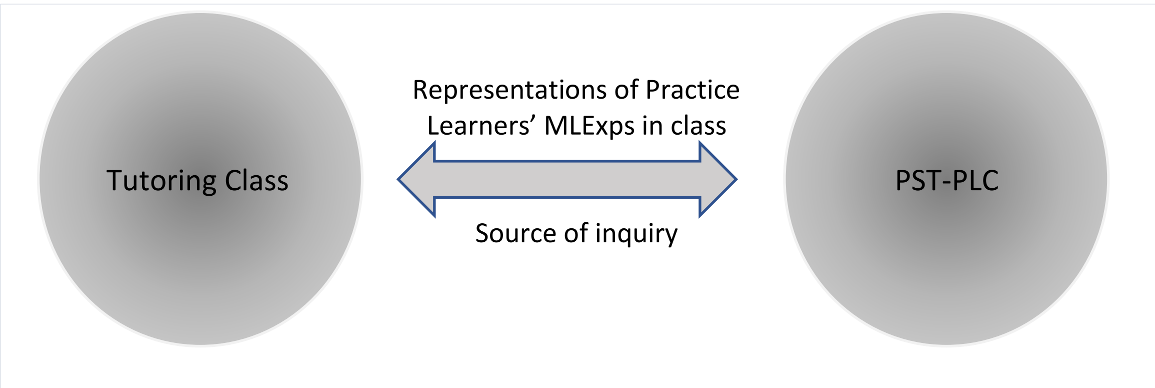


Figure 7.7: Bidirectional flow of practice in embracing stance

In the embracing stance, the ongoing enactment of strategies to engage learners' mathematical learning experiences in class generated new representation of practices in the PST-PLC. This involved shifts in learners' confidence in participation in the tutoring, which became sources of reflection within the PST-PLC and which created opportunities for generating new teaching practices, namely teaching practices of eliciting and engaging learners' mathematical learning experiences.

7.5.3.2 Representations of practice in compliance stance

In the compliance stance the representations of practice included descriptions of learners' narratives about their mathematical learning experiences but no descriptions of strategies to elicit learners' mathematical learning experiences in class or beyond school. The inquiry into engaging learners' mathematical learning experiences beyond school suggests challenges of eliciting learners' learning experiences but also raises the question whether pre-service teachers may have had a different understanding of the purpose of engaging learners' mathematical learning experiences.

The ways in which representations of practice were used in the compliance stance reflect limited inquiry into learners' mathematical learning experiences as is shown in only three epistemic moves generated with potential for deepening inquiry: describing, inferring and making assumptions. The representations of practice shared from pre-service teachers engaging learners in the tutoring session within the PST-PLC in the compliance stance did create opportunities for learning through the deepening of inquiry through knowledge-of-practice which requires a bidirectional flow of practice. In other words, systematic inquiry into connections between learners' mathematical learning experiences and how these experiences influence learners' learning is an opportunity to deepen inquiry through knowledge-of-practice. This systematic inquiry involves exploring in the tutoring session the connections between learners' mathematical learning experiences and how these experiences influences learners' learning and then sharing the learning from the tutoring sessions within PST-PLCs and to further refine the exploration within the tutoring session based on the inquiry with the PST-PLCs. This systematic inquiry enables the critical inquiry stance, namely, ongoing rethinking of both teaching practice and the practice of inquiry in the PST-PLCs.

I showed that no knowledge-in-practice was enabled in the compliance stance. I describe the use of representations of practice in this stance as a *source of information* which depicts a framing of the collaboration as sharing which limited the opportunities to learn (Louie, 2016). The use of representations of practice as a source of information explains the absence of multidirectional learning (Kazemi & Hubbard, 2008).

Figure 7.8 shows that in the compliance stance there is a unidirectional flow of a representation of practice as a source of information:

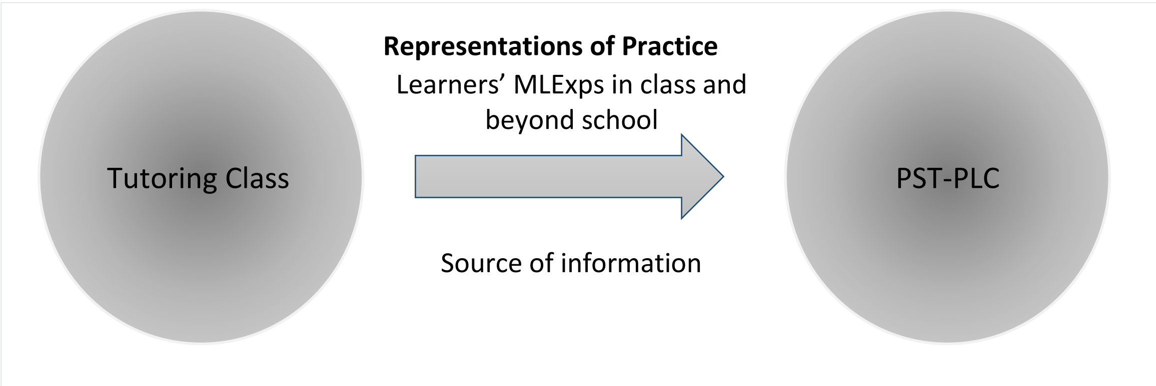


Figure 7.8: Unidirectional flow of practice in compliance stance

7.5.4 Other factors influencing the development and flows of practice

The reasons for the development and flow of practice across the inquiry stances has to take into consideration wider contextual issues. In chapter 6 I provided evidence of pre-service teachers' narratives of their mathematical learning experiences in class which shows that many considered their teachers to care about their learning if attention was paid to their mathematical challenges not only from a cognitive dimension. Pre-service teachers' narratives emphasised a social dimension of learning as care from teachers who took an interest in them holistically, in other words, learners needed to understand and enjoy the learning experience. The importance of creating a learning environment in which learners were not afraid to ask questions was raised in many pre-service teachers' narratives. The inquiry into learners' mathematical learning experiences in the PST-PLCs shows that pre-service teachers struggled to engage learners in this way.

As mentioned before, pre-service teachers within the inquiry stated that they struggled to elicit learners' mathematical learning experiences as learners struggled to talk about their learning experiences. It was only in the embracing stance in which the inquiry shows how PSTs persevered with different strategies to elicit learners' learning experiences in class. In both the embracing and compliance stances the inquiry into learners' mathematical learning experiences beyond school was limited to one PST-PLC session. In chapter 6 pre-service teachers' narratives of their learning experiences beyond school involved mainly support with mathematics in their homes or in after-school tutoring sessions which did not require of them to explain how they were taught. The main approach to tutoring that pre-service teachers experienced was one where their mathematical error was the starting point leading either to re-teaching as a means to correct the error or to asking them how they reasoned. Only one pre-service teacher's narrative included a starting point where she was asked about how she was taught the concept in class and to re-trace the teaching and learning process.

A meta-learning approach (Hargreaves, 2017) in which engaging learners' learning experiences are encouraged to help them to reflect critically on what and how they learn was not the norm for pre-service teachers in this study. The absence of a meta-learning approach in pre-service teachers' own learning experiences as school learners may explain the nature of the reflection which emerged through the inquiry into learners' mathematical learning experiences across all the stances.

In many pre-service teachers' narratives of their learning experiences beyond school in contexts other than after-school tutoring or in their homes they indicated that they did not think about mathematical learning when engaging in these contexts and many indicated that their teachers did not enquire about their learning experiences beyond school. This explains pre-service teachers' struggle to engage learners about their mathematical learning in contexts other than those directly related to their mathematical challenges in class. The group interviews with pre-service teachers at the end of their PST-PLC sessions in which their understanding of the social dimension of learning was probed shows a strong emphasis on the building of social relations as necessary for developing learners' confidence to communicate their mathematical learning experiences and the reasoning behind their errors. The respect for the language preference of learners, understanding learners' social circumstances, interests and aspirations were viewed as important in developing social relations but with little connection to learners' learning of mathematics and supporting learners' with their challenges in mathematics.

7.5.5 Social - Cognitive connections across the stances

A key task in this chapter is to show that inquiry into learners' mathematical learning experiences from a social dimension enables cognitive dimensions of learning to emerge. In the analysis of the stances, reflecting is a key epistemic move with the potential for deepening inquiry into connections between social and cognitive dimensions of learning. Reflecting is an epistemic move which only emerged in the embracing stance and explains the differences between the deepening of inquiry into these connections in the stances.

7.5.5.1 Potential social - cognitive connections in embracing stance

In the embracing stance the potential deeper inquiry into social-cognitive connections was greater because of the role of reflecting as epistemic move. I showed earlier how the interaction between describing and reflecting enabled knowledge-in-practice of the social dimensions of learning as well as deepening inquiry through further reflecting. For example, in PST-PLC 1 the inquiry brought to the fore a social dimension of learning in that it was noted learners developed the confidence to point out to pre-service teachers that their teacher's methods for solving a particular problem differed from those of the pre-service teachers. The reflection on this shift in learner's confidence generated further reflection on how pre-service teachers were taught as school learners. The problem that was discussed was different algorithms of multiplication. The learners were being taught the expanded method, using brackets and applying the distributive law of multiplication. The pre-service teachers were only taught the traditional column methods of multiplication as school learners. The opportunity was created for the PST-PLC 1 to inquire more deeply into the connections between their method of multiplication and those of the learners which is a cognitive dimension of learning. In other words, the inquiry afforded opportunities for the PST-PLC to focus on mathematics content in the inquiry through exploring connections within mathematics, namely, the connections between the different multiplication algorithms. The inquiry afforded opportunities for the PST-PLC to focus on how to teach through exploring within the inquiry how to encourage learners to compare different multiplication algorithms to support learners' learning of these algorithms. This shows how cognitive dimensions of learning that emerged from the social dimension within the inquiry generated learning opportunities for pre-service teachers about mathematics content and how to teach.

In PST-PLC 4, the inquiry brought to the fore a social dimension of learning in that a description was given of how a learner was encouraged to explain how their teacher taught the concept of an acute angle. The learner first provided a definition of an acute angle as '*how wide a scissors can go*'[13] and thereafter explained how the teacher taught the concept. It was clear that the learners' faulty definition of an acute angle was connected to the learners' experience of how the concept of an angle was taught. An opportunity was created for PST-PLC 4 to inquire into how the learner could be further encouraged to critique the teacher's approach and to rethink the definition provided. The move from the social to the cognitive involved the learner identifying his/her error or whether the teacher's way of explaining may have been misrepresented or incorrect.

Figure 7.9 below provides a summary of the interaction between key epistemic moves for deepening inquiry into social-cognitive connections in the embracing stance in relation to knowledge:

[13] Refer back to Table 7.4 in turn 7.

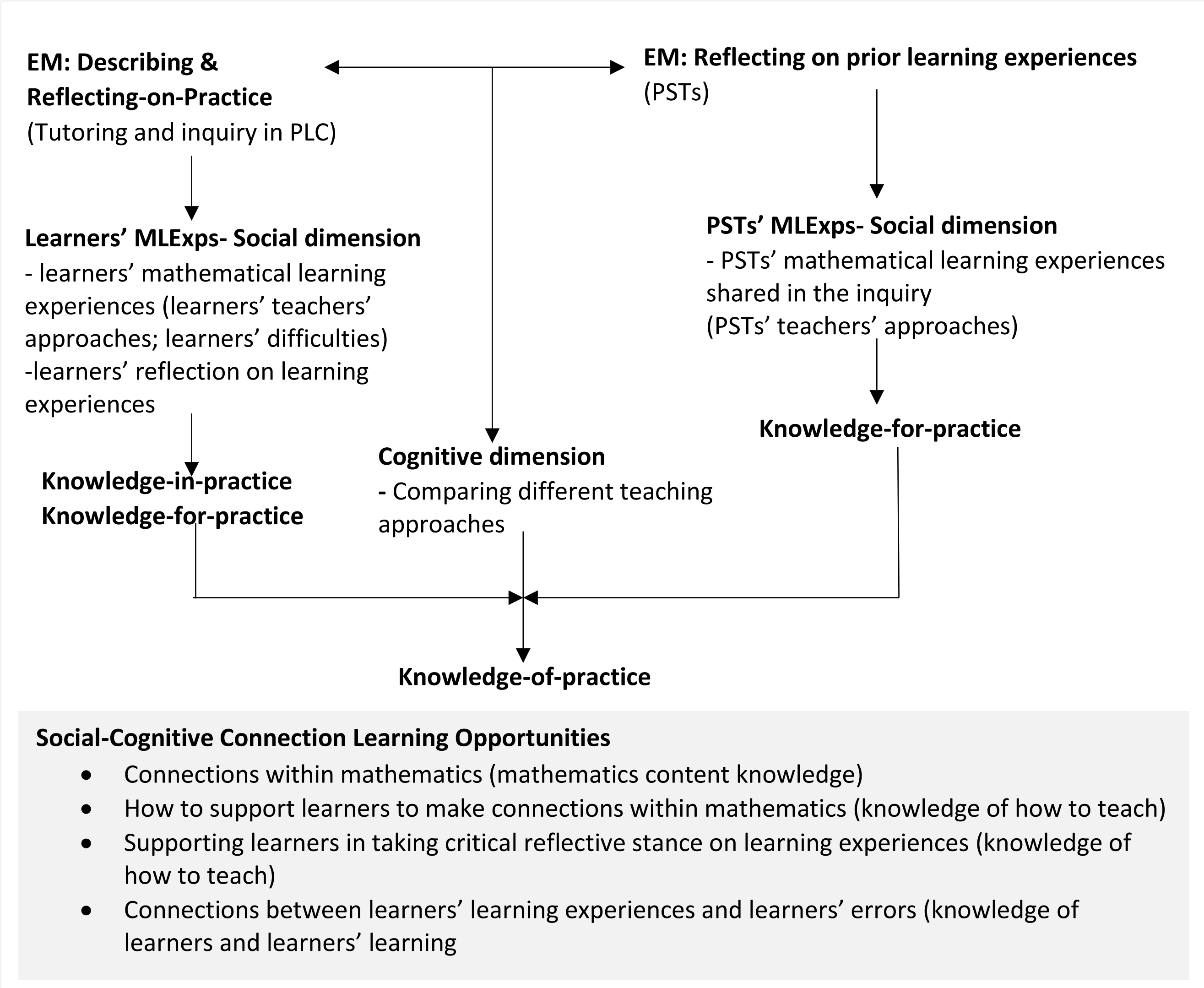


Figure 7.9: Interaction between epistemic moves for potential deepening inquiry into social-cognitive connections in relation to knowledge in embracing stance

Figure 7.9 shows that potential social-cognitive connection learning opportunities emerged through the inquiry which were related firstly to knowledge of mathematics content, secondly to knowledge of how to teach with a focus on the cognitive dimension, namely, how to support learners to make connections within mathematics; thirdly, knowledge of how to teach with a focus on the social dimension of learning, namely, supporting learners in taking a critical reflective stance on their learning experiences and fourthly, knowledge of learners and learners' learning with a focus on social and cognitive dimensions of learning, namely , connections between learners' learning experiences and learners' errors.

Hence potential knowledge-of-practice through the ongoing systematic inquiry into these socio-cognitive learning opportunities is created. The non-recognition of epistemic moves comparing and making assumptions were key inhibitors of deeper inquiry into the social and cognitive dimensions of learning. For example, in PST-PLC 1, the inquiry involved the comparison of the two different methods of solving problems without elaborating on the strength and limitations of the two methods.

In PST-PLC 4, the inquiry showed how in engaging learners' mathematical learning experiences opportunity was created for learners to rethink their learning experiences but opportunities for learners to clarify their thinking were often closed. The closing of opportunities was through responses to correct learners' explanations which led to assumptions about learners' learning experiences. These assumptions were not recognised and not challenged to deepen inquiry into connection between learners' learning experiences and their understanding of mathematical concepts.

The deepening of inquiry into social-cognitive connections is through knowledge-of-practice and knowledge-for-practice. Hence the non-recognition of the epistemic move, raising uncertainty, is an inhibitor to exploring knowledge for deepening the inquiry into the social-cognitive connections. Describing too is an inhibitor of deepening inquiry into the social-cognitive connections as descriptions often had limited detail on how learners explained a teacher's approach and the mathematical content in learner narratives.

7.5.5.2 Potential social-cognitive connections in compliance stance

In the compliance stance the inquiry brought to the fore the social dimension of learning in that it was noted that learners do engage with siblings and parents about their mathematical learning experiences in class. Deeper inquiry into the social dimension of learning held potential for the cognitive dimension of learning to emerge. For example, learners' narratives of their learning experiences included how they deal with their challenges in mathematics. Deeper insight into learners' challenges and learners' thinking is the potential cognitive dimension of learning. As within the embracing stance, limited describing was an inhibitor of deepening inquiry into social and cognitive dimensions of learning. Limited detail when describing learners' challenges and learners' thinking were inhibitors of a cognitive dimension of learning.

Figure 7.10 provides a summary of the potential of deepening inquiry into social-cognitive connections in the compliance stance in relation to knowledge:

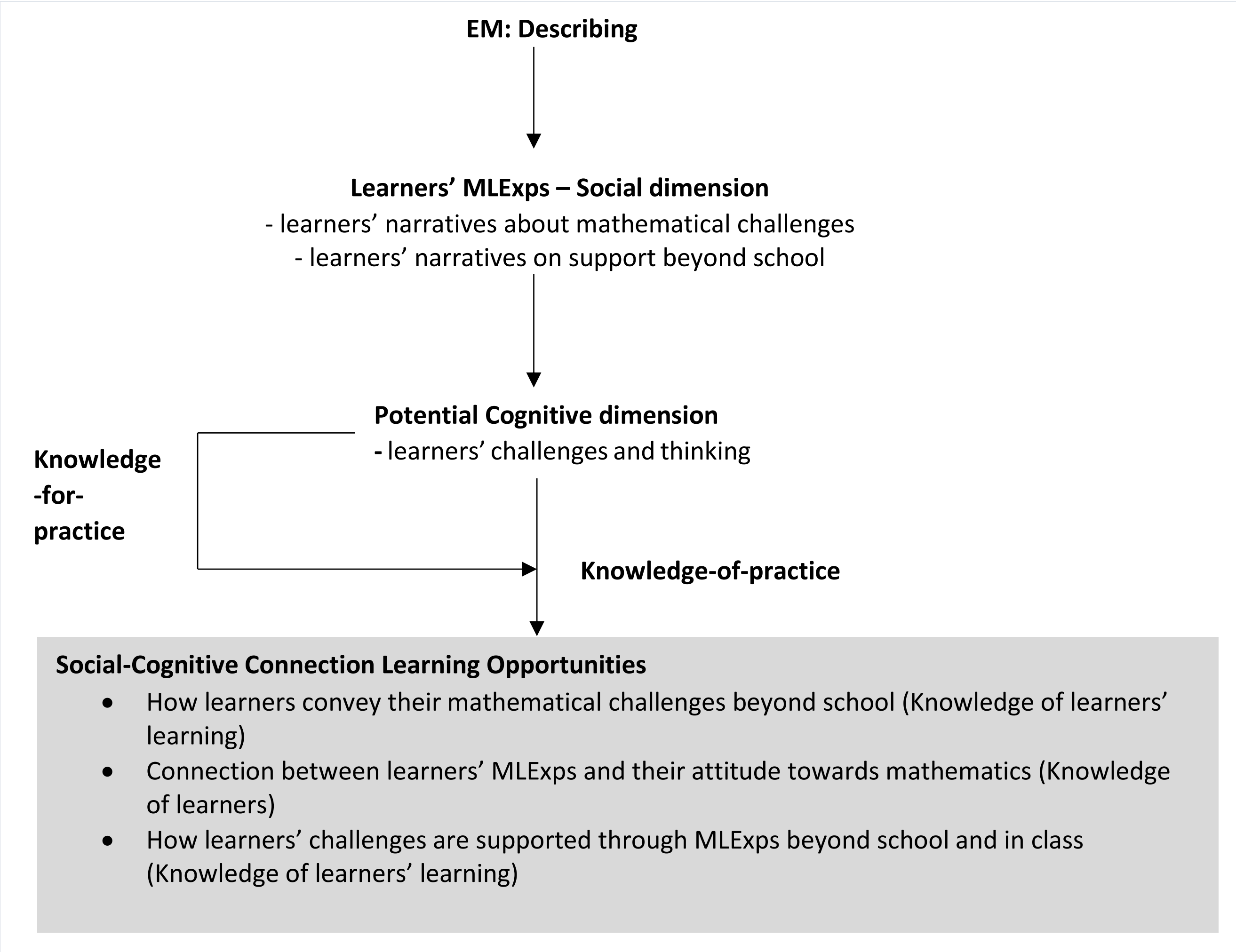


Figure 7.10: Potential deepening of inquiry into social-cognitive connections in the compliance stance in relation to knowledge

In Figure 7.10 I show that a difference between the embracing and compliance stance is that in the compliance stance, there was only potential for the cognitive dimension to emerge. In the embracing stance, reflecting enabled a cognitive dimension of learning to emerge. Non-reflecting-on-practice was a key inhibitor of deepening inquiry into the social and cognitive dimensions of learning and connection between these dimensions in the compliance stance. Non-reflecting-on-practice is an outcome of how representations of practice were used, namely as sources of information which limited the inquiry into sharing of experiences of engaging learners' mathematical learning experiences.

Social-cognitive connection learning opportunities did emerge in the compliance stance that were related to firstly, knowledge of learners' learning with a focus on social dimensions, namely, how learners convey their mathematical challenges beyond school; secondly, knowledge of learners with a focus on interaction between social dimensions, namely, connections between learners' mathematical learning experiences and their attitudes towards mathematics; and thirdly, knowledge of learners' learning with a focus on interaction between the social and cognitive dimensions of learning, namely, how learners' challenges are supported through mathematical learning experiences beyond school and in class. As within the embracing stance, the potential for deepening of inquiry into social-cognitive connection learning opportunities through knowledge-of-practice is created and requires knowledge-for-practice.

7.6 Chapter summary

In this chapter I have argued that the stance towards inquiry into engaging learners' mathematical learning experiences influenced the deepening of inquiry into learners' mathematical learning experiences. The analysis of the inquiry shows a relationship between the stance and whether the inquiry was on learners' mathematical learning experiences in class or beyond school. An embracing stance was only present when inquiring into learners' mathematical learning experiences in class. A compliance stance was present when inquiring into learners' mathematical learning experiences beyond school, and in class for three communities.

A fine-grained analysis of the inquiry involved identifying the epistemic moves as enablers of critical inquiry and the interaction of the epistemic moves in relation to knowledge. This analysis shows that knowledge-in-practice was only enabled through the inquiry in the embracing stance. The analysis shows that epistemic moves in both stances created potential for deepening inquiry through knowledge-of-practice and knowledge-for-practice.

To gain a deeper understanding of why certain epistemic moves only held potential for deepening inquiry I analysed the inhibitors of critical inquiry across both stances. I identified three broad categories of inhibitors of critical inquiry which were related to epistemic moves, and two of the three inhibitor categories were related to how representations of practice were reconstructed. The analysis brought to the fore the inquiry practices that need to be developed to support PST-PLCs in developing critical inquiry: pressing for specificity when describing one's teaching practice; pressing for reflection on teaching practice and pre-service teachers' own learning experiences; as a well as reflecting on each other's practice and recognition of when to explore further knowledge, to challenge assumptions and inferences, to elaborate on the strengths and limitations when comparing or critiquing and to consider as many possible reasons for a situation. I argued with reference to Wenger's (1998) community of practice that developing these inquiry practices become a matter of mutual accountability of PLCs taking a critical inquiry stance

On further comparative analysis of the epistemic moves across the stances I drew on Horn and Kane's (2015) and Little's (2002) research on the reconstruction of teaching practice, Kazemi and Hubbard's (2008) concept of flows of practice, and Louie's (2016) concept of framing in collaborative inquiry to explain why the embracing stance enabled knowledge-in-practice and generated more epistemic moves than the compliance stance. I used Greeno's (2008) concept of epistemological framing to describe how representations of practice were used in the inquiry, either used as sources of inquiry in the embracing stance or as sources of information as in the compliance stance. Distinguishing between these different uses of representations of practice explains the bidirectional flows of practice within the embracing stance which enabled knowledge-in-practice on eliciting and engaging learners mathematical learning experiences in class. The different uses of representations of practice provide support to Louie's (2016) findings that the way in which collaboration is framed influences the nature of the inquiry. The framing of collaboration as mere sharing of experiences in the compliance stance limited the deepening of inquiry compared to the embracing stance.

Analysis of the epistemic moves across the stances show how the epistemic move reflecting enabled the emergence of a cognitive dimension of learning in the embracing stance when the social dimension of learning was used as an entry point into learners' difficulties. In the embracing stance the epistemic moves describing and reflecting on social dimensions generated the epistemic move comparing that focused on the comparing of different algorithms of multiplication which is the cognitive dimension of learning. In the compliance stance, there was only potential for a cognitive dimension of learning to emerge through describing learners' learning experiences with potential to inquire into how learners conveyed their challenges and thinking. The deeper analysis of learners' challenges and thinking is a cognitive dimension of learning. In the compliance stance this cognitive dimension of learning did not emerge which was due mainly to the limited describing of learners' learning experiences.

My analysis of the social-cognitive connection opportunities for learning as knowledge-of-practice, shows that the embracing stance includes knowledge on how to teach, knowledge about learners and learners' learning. The knowledge of how to teach includes teaching practices which support learners in taking a critical-reflective stance on their learning experiences. In the compliance stance, the social-cognitive learning opportunities include knowledge only on learners and learners' learning. The embracing stance therefore generated greater socio-cognitive learning opportunities to support learners' learning. In the next chapter I explore how PST-PLCs inquire into learners' mathematical errors as a cognitive dimension of learning and how different stances towards this inquiry influence the emergence of a social dimension of learning and socio-cognitive learning opportunities.

CHAPTER 8

Inquiry into Learners' Mathematical Errors

8.1 Introduction

This chapter focuses on how the PST-PLCs inquired into learners' mathematical errors thus foregrounding the cognitive dimension of learning in response to the second sub-question of the research:

How do PST- PLCs inquire into learners' mathematical errors?

The pre-service teachers in this study were encouraged to adopt an engaging approach to learners' mathematical errors when tutoring learners. The engaging approach involved probing learners' reasoning behind their mathematical errors in order to enable a direct focus on mathematical conceptual challenges. Since the mathematical conceptual challenges are brought to the fore explicitly, I considered the PST-PLC inquiry into engaging learners' mathematical errors as foregrounding the cognitive dimension of learning. In other words, the entry point into learners' difficulties with mathematics is the mathematical error and not the broader social context of learning captured through learners' narratives of their mathematical learning experiences. However, I do not view the cognitive and social dimensions of learning as discrete and see the integration of these dimensions as necessary to enhance learning. While the focus of this chapter is the PST-PLCs' inquiry into engaging learners' mathematical errors as the cognitive dimension of learning, I address the third sub-question again by exploring whether possibilities for the social dimension of learning emerged with potential to provide deeper insight into learners' mathematical errors. The third sub-research question is:

What connections were made possible between learners' mathematical learning experiences and learners' mathematical errors through the PST-PLCs' inquiry?

The five PST-PLCs in the study all inquired into learners' mathematical errors in all the PLC sessions. The inquiry into learners' mathematical errors was privileged over the inquiry into learners' mathematical learning experiences. As noted in Chapter 7, in the compliance stance the PST-PLCs only inquired into learners' mathematical learning experiences in the first PLC session.

I identified two stances towards inquiry into learners' mathematical errors in the PST-PLCs:

Table 8.1: Stances towards inquiry into engaging learners' mathematical errors.

Stance	Description of Stance
1. Engaging	Inquiry focused on how learners' reasoning behind their errors was probed.
2. Correcting	Inquiry focused on how to support learners to get to the correct answer and/or on how to teach the mathematics content in which the error occurred. Inquiry does not include how learners reasoned when making an error.

The wide range of errors which were brought to the PST-PLCs resulted in differences with regard to aspects of working with errors. In Table 8.2 below I provide an overview of aspects of working with learners' errors identified in the literature (Peng & Luo 2009; Prediger 2010; Brodie 2013, 2014; Shalem, Sapiro, & Sorto, 2014) in relation to the stances towards inquiry into learners' mathematical errors across PST-PLCs. I include an additional aspect, eliciting learners' mathematical learning experiences as part of dealing with learners' mathematical errors as I explore the connections between cognitive and social dimensions of learning when engaging learners' mathematical errors.

Table 8.2: Overview of aspects of working with learners' mathematical errors across the inquiry stances in the five PST-PLCs

Aspects of working with errors across the stances	Engaging stance	Correcting stance
	PLC 2	PLC 1,3,4,5
Identifying the error	✓	✓
Interpreting the mathematical problem		
No evidence of how problem was solved by PSTs		✓
PSTs solve problem correctly	✓	✓
PSTs solve problem but there is evidence of misconception		✓
PSTs explore more than one way to solve the problem	✓	✓
Interpreting the error		
One reason		✓
More than one reason	✓	✓
Dealing with the error		
Correcting		✓
Probing learners' reasoning behind the error	✓	
Eliciting learners' mathematical learning experiences	✓	

Table 8.2 shows that an engaging stance towards inquiry into learners' mathematical errors was only present in PST-PLC 2. Table 8.2 shows that in both stances the inquiry involved the identification of learners' errors. When interpreting the mathematical problem in which learners' errors occurred, the inquiry in the engaging stance involved PSTs solving the problem correctly and exploring more than one way to solve a problem. In the correcting stance, there were instances when the inquiry did not involve how a problem was solved and there was evidence of misconceptions when solving a problem. When interpreting learners' errors, the inquiry in the engaging stance involved exploring more than one reason for learners' errors whereas in the correcting stance there were instances when only one reason was given for learners' errors.

I will show that in PST-PLC 2, in which an engaging stance towards inquiry into learners' mathematical errors was present, greater opportunity was created for deepening knowledge about learners' mathematical errors through the inquiry into learners' mathematical learning experiences which emerged. Within the correcting stance in the other four PLCs, there was greater variation in how different aspects of working with errors emerged in the inquiry. I will show that in the PST-PLCs in which the correcting stance was evident, despite there not being inquiry into the learners' reasoning behind their errors, there was either deeper inquiry into aspects of the mathematics and/or how to teach learners who struggle with a mathematical concept.

The fine-grained analysis of the epistemic moves (Zenios, 2011) as enablers of critical inquiry facilitated a deeper analysis of the deepening of knowledge about engaging learners' mathematical errors. An analysis of the inhibitors of critical inquiry across both stances show similar categories of inhibitors as in the inquiry into learners' mathematical learning experiences. These inhibitors too point to practices of inquiry within PST-PLCs that are necessary for developing a critical inquiry stance on teaching practice.

As in Chapter 7, I used Cochran-Smith and Lytle's (1998) definition of knowledge-for-practice, knowledge-in-practice and knowledge-of-practice to analyse how epistemic moves deepen critical inquiry through this knowledge. The PST-PLCs in this study focused on inquiry into how to engage learners' mathematical errors over a period of an academic year, which represents a systematic form of inquiry into a teaching practice. A key challenge identified in the correcting stance was probing the reasoning behind learners' errors, a key teaching practice in engaging learners' errors. In the engaging stance the inquiry focused on how learners' errors were probed, leading to inquiry into learners' mathematical learning experiences.

A comparison of epistemic moves across the stances show that representations of practice (Little, 2002; Horn & Kane, 2015) influenced the teaching practices involved in the flows of practice (Kazemi & Hubbard, 2008). I will show how inquiry in both stances enabled teaching practices to support learners' learning but that in the correcting stance, some of the teaching practices were from pre-service teachers' own learning experiences and not through considering alternate ways of teaching which emerged in the PST-PLC inquiry. The teaching practices from pre-service teachers' own learning experiences emerged in the correcting stance where there was limited information on learners' written work and the mathematical problems in which learners' errors occurred. In the engaging stance, the teaching practices discussed were related directly to probing learners' reasoning and mathematical learning experiences.

I show the influence of epistemic moves in potentially deepening inquiry into social-cognitive connection learning opportunities across the stances. In the engaging stance, a social dimension of learning emerged which is the learners' mathematical learning experiences, whereas in the correcting stance, there was potential for this social dimension of learning to emerge. The opportunities for learning in both stances point to knowledge on how to teach with the engaging stance placing greater focus on developing learners' critical-reflective practice of their learning experiences. This results in the focus being not only on supporting learners' challenges with mathematical content as in the correcting stance.

8.2 Engaging stance

This stance was only present in PST-PLC 2. The inquiry into learners' mathematical errors included detail about the mathematical problems in which the learners' mathematical errors occurred; examples of learners' written work and descriptions of how learners' reasoning behind their errors were probed; and eliciting learners' mathematical learning experiences.

The mathematical error that is the focus of inquiry in Episode 1 in Table 8.3 is a learner's incorrect identification of prime numbers in the 1-100 table:

Table 8.3: Episode 1 from PST-PLC 2- session 3, facilitated by PST 2A

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
1	PST 2C	I gave them the prime number maze that they use to find the prime numbers. The Grade 6 boy he started – he used the pattern they were taught. I asked him why he said seventy was a prime number. Then he told that is the way he learnt it.		Describing the mathematical problem to be discussed Describing a learner's starting strategy to the mathematical problem Describing how a learner's reasoning was probed. Describing a learner's reasoning
2	PST 2B	Oh my world!		
3	PST 2C	Then I said – then I asked him, but can something ….can you see there seventy is circled [*shows to PLC]*. Then I asked him what can all divide into seventy. Then he started mentioning all that can go into seventy. Then I asked him, why do you say it is a prime number? Then he told me but in his table it says that it is. Then I came to realise that they do not know how to look for the prime number, or how to identify them, in the first row that just know there are so many and I……	Describing a learner's solution	Describing how a learner's understanding of mathematical content was probed. Describing how a learner's reasoning was probed. Describing a learner's reasoning Explaining the reason for the learner's error
4	PST 2B	But that is also how we learnt it. Can you –is it……		Reflecting on how prime numbers were taught when at school
5	PST 2E	I did not learn it that way… *[all start talking …not clear]*		
6	PST 2A	A prime number is number that can only be divided by one and the number itself…they tell you that….but they still gave them that (*referring to the table with prime numbers)*		
7	PST 2C	I remember in our first year when we did prime number with	Describing a way of being taught	

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		lecturer King[14] and I remember the table in the examination paper and I remembered how to do it.		
8	PST 2B	Yes, the first hundred… [*Everyone talked at same time*]		
9	PST 2A	Yes, like, one yes, two no, three no, four yes and then you go on like that.		
10	PST 2E	But I still will not be able to sit and say take one to a hundred and work it out in my head….		Reflecting on finding primes within the 1-100 table
11	PST 2A	I will not either		
12	PST 2C	I will not too.		
13	PST 2E	I will still take that number and if I am not sure then I will …what do you call that when you write it out next to (*indicated with hands doing prime-factor division)*		Describing how to check whether a number is prime
14	PST 2A	The prime factors		
15	PST 2E	Yes that.		
16	PST 2C	And the other thing the boy learner sat for half an hour with the one as he wanted to go through all of them. Then I asked him what a prime number is. Then he told me it is a number that can only divide by itself and one. And so I told him right now take all your numbers and divide them for me. Then I thought okay. Now he actually went and wrote each one's stuff in his book and then he got to thirty.	Describing a learner's solution	Describing how a learner's understanding of mathematical content was probed. Describing how a learner explained Describing how a learner was engaged
17	PST 2A	Gee, but one can see there is a three so it can divide into three	Reflecting on mathematical content learners ought to know	
18	PST 2E	Yes		
19	PST 2B	And there is a zero at the end		
20	PST 2A	And there, yes it can divide by ten too. So there are a few things…		

[14] Pseudonym for the lecturer who taught the pre-service teachers.

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
21	PST 2B	It can divide by five too		
22	PST 2E	This is what Mr King taught us	Describing a similar way of being taught	
23	PST 2D	I know that I was taught like this: one and itself, and then there is two, then there are two factors, okay, not factors, but okay and no even numbers are prime numbers because there is a two		Reflecting on how the process of finding primes in the one to one hundred table was taught
24	PST 2A	Yes, two		
25	PST 2D	Then the rest are the odd numbers and		
26	PST 2A	So you take all the odd numbers yes		
27	PST 2D	Then you rather continue with your odd numbers instead of a table.		
28	PST 2A	We were also taught like that.		
29	PST 2D	Yes. And then you look at those last numbers and so		
30	PST 2B	That – yes		
31	PST 2C	So I told him to draw his table in the way that their teacher taught them and then to check if it is the prime number on his list. Then he told me that he cannot do it out of his head (*meaning by remembering*). So I told him but you told me you circled the seventy because that is the way you were taught but you can explain to me how you were taught to draw the table (*meaning setting up the prime numbers in the table)*. Then he told me that he just has to look then he knows that is the number he was taught. Then I asked him now what is in the first number – numbers in your table. So he told me one to ten (*she means the learner could give the primes between one and ten)*. Then I asked him but how does seventy fall between one and ten		Describing how a learner's mathematical learning experiences in class were elicited Describing how a learner's mathematical learning experiences were engaged Describing how a learner explained Describing how a learner's reasoning was probed
32	PST 2D	Yes you just take…		

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
33	PST 2C	Then he told me that he just saw the seven and he knew the seven was in the first row and it did not matter to him if the seventy that has a zero as he knew seven is a prime number and that is why he circled the seventy.		Describing how a learner reasoned
34	PST 2B	But that is strange because he does understand what it means		Raising uncertainty about the learner's responses
35	PST 2C	Yes, he understands the concept of a prime number because later when I asked him to write everything down again so he got all the even numbers and stuff, but he could – he just could not understand		Raising uncertainty about the learner's responses

In turn 1, PST 2C starts the inquiry by describing the mathematical problem and a learner's starting strategy to solving the mathematical problem and the probing of the learner's reasoning behind the error. The learner's response to why he circled 70 as a prime number was that is the way he was taught and he did not elaborate, which may have been a way of avoiding to explain his thinking. This shows that learners do think back to how they were taught. The learner's response is an indication of how important it is to elicit learners' mathematical learning experiences to gain their understanding of how they were taught and whether through rethinking how they were taught learners can identify why they have challenges. As the inquiry unfolds, PST 2C shows PST-PLC 2 how, through a set of probing questions, she was able to gain a deeper understanding of the learner's error.

In Table 8.3 I indicate inhibitors of inquiry that emerged in this episode. In turn 3, PST 2C does not provide any detail on how the learner found the factors of 70 which inhibited further inquiry into whether the learner's difficulty in identifying primes was related to the process of finding the factors of a number. In turns 7 to 9, PSTs describing how they were taught show that they do not elaborate clearly on the teaching which inhibits the inquiry into how their teachers taught. In turns 17 PST 2A's response to a learner's understanding leads to a focus on the mathematical content the learner is expected to know. The reflecting on the mathematics that learners ought to know in turns 17 to 21, results in a judgement of learners' understanding, and inhibits inquiry into the underlying reason for the learner's challenge. The inquiry shifts to PST 2E in turn 22 describing how they were taught without any elaboration of the teaching approach, an inhibitor of inquiry too, as no detail is provided of PST 2E's understanding of the teaching approach.

The inquiry in episode 1 generated a number of epistemic moves through the participation by different members of PST-PLC 2. I next explain these epistemic moves and how they interacted to create opportunity for deepening critical inquiry into the learner's mathematical error.

8.2.1 Epistemic moves in engaging stance

The epistemic moves in this stance consist of four different kinds: describing, reflecting, explaining and raising uncertainty. Describing as an epistemic move deepened inquiry on its own and through the generation of further epistemic moves: explaining, reflecting and raising uncertainty.

8.2.1.1 Epistemic moves – Describing

Describing as an epistemic move deepened inquiry on its own, often through a sequence of describing moves in a single turn or consecutive turns. For example, in turn 31, PST 2C described to the PST-PLC how she elicited the learner's learning experiences in class by encouraging him to explain to her how the teacher taught him to use the 1-100 table to determine the prime numbers. The learner's response described in turn 31 indicated that he struggled to explain the way in which the teacher explained which could suggest that the learner did not follow the systematic approach of eliminating numbers that are not prime. In turn 33, the description of how the learner reasoned suggests that while he understood how to find the prime numbers less than ten, he does not apply his knowledge of the definition of a prime number when finding other prime numbers. This illustrates how the epistemic moves: describing how a learners' mathematical learning experiences were elicited; describing how a learner explained and describing the reasoning behind a learner's error, created opportunities to deepen inquiry into how the learner's mathematical error is potentially linked to his mathematical learning experiences in class.

8.2.1.2 Epistemic moves – Describing, explaining and reflecting

The epistemic moves describing how a learner's reasoning was probed and describing the probing of a learner's mathematical content knowledge, as in turns 1 and 3, deepened the inquiry through generating other epistemic moves: explaining the reason for a learner's error and enabling pre-service teachers reflecting on how they were taught (turn 4).

Explaining the reason for a learner’s error in turn 3 is an epistemic move to deepen enquiry into further reasons such as making connections with how learners were taught which could explain why the learner was not able to identify the prime numbers

Pre-service teachers reflecting on how they were taught deepened the inquiry through further reflecting on their own struggles with finding the primes in the 1-100 table. For example, in turn 9, PST 2A tried to remember finding the primes in the 1-100 table and in turns 10,11 and 12, PSTs 2E, 2A and 2C’s reflecting on their struggle in finding the primes in the 1-100 table, suggests they did not fully understand the systematic approach to eliminating numbers which are not prime in the 1-100 table. In turn 23, PST 2D reflected on how she was taught the systematic approach of finding primes in the 1-100 table and struggled to describe the systematic approach completely. PSTs 2A, 2C, 2D and 2E’s reflecting on their learning experiences were epistemic moves to inquire into why the learner struggled to find the primes in the 1-100 table. This shows how teachers reflecting on their prior learning experiences is an important aspect of teachers’ practice when engaging learners’ mathematical challenges.

The reflecting on how pre-service teachers were taught deepened the inquiry through describing mathematical process, such as checking whether a number is prime through prime factorisation. This shows how reflecting and describing interacted as epistemic moves to deepen inquiry into how important it is for learners to check their solutions as a way of addressing their mathematical errors on their own.

8.2.1.3 Epistemic moves – Describing and raising uncertainty

Describing how a learner reasoned generated the epistemic move raising uncertainty about a learner’s response. For example, in turn 34 and 35, PST 2E and PST 2C raised uncertainty in explaining how the learner was able to define a prime number correctly but was not able to identify a prime number in the 1-100 table. Raising this uncertainty was an epistemic move for deeper inquiry into why learners do not always draw on their prior knowledge.

Figure 8.1 below provides a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in the engaging stance:

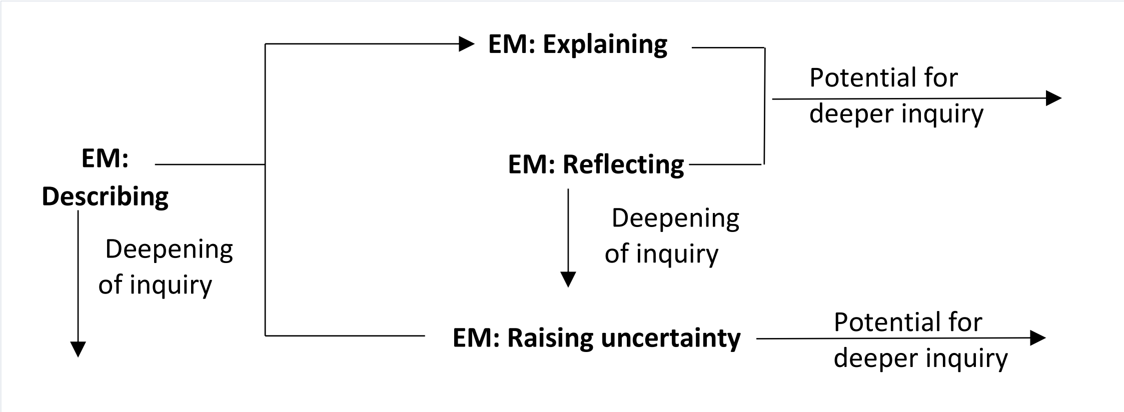


Figure 8.1: Interactions between epistemic moves influencing inquiry in engaging stance

Figure 8.1 shows how the epistemic move describing generated three categories of epistemic moves: explaining, reflecting and raising uncertainty. Describing enabled the deepening of inquiry on its own. The interaction between describing and reflecting enabled the deepening of inquiry. The potential for deepening inquiry was generated through the epistemic moves explaining, reflecting and raising uncertainty. In the next section I elaborate on how knowledge is enabled through these epistemic moves.

8.2.2 Deepening inquiry through knowledge in engaging stance

In the engaging stance towards inquiry into learners' mathematical errors, the detailed descriptions of how learners' reasoning behind their errors were probed and learners' responses to the probing enabled PST-PLC 2 to generate epistemic moves to further inquiry to enhance knowledge-in-practice about learners' errors.

The inquiry in PST-PLC 2 showed how PST 2C's sustained probing of the reasoning behind a learner's error included questions in which the learner was asked to explain how he reasoned (as in turns 1 and 3) but also questions on the learner's knowledge of mathematical content (as in turns 3 and 16). The response of the learner to why he said 70 was a prime number (in turn 1) suggests that the learner struggled to explain his reasoning from a mathematical perspective at first. The response of the learner to PST 2C's question in turn 1 and turn 3 provided no insight into his reasoning as he referred in a general way to how he was taught.

The questioning of the learner's mathematical content knowledge showed that the learner had knowledge about finding the factors of a number (turn 3) and that the learner could define a prime number (turn 16). The kinds of questioning that PST 2C described within PST-PLC 2 enabled knowledge-in-practice on how to probe learners' reasoning in situations where learners at first struggle to explain their reasoning from a mathematical perspective.

PST 2C engaged the learner's mathematical learning experiences of prime numbers through eliciting from him how he was taught (turn 31). Her detailed description of eliciting the learner's learning experiences provided PST-PLC 2 with evidence of the learner's struggle to explain how he was taught. Through PST 2C's ongoing questioning to deepen her understanding of the learner's learning experiences, insight was eventually obtained on how the learner reasoned to find the primes in the 1-100 table. The inquiry enabled knowledge-in-practice to interpret the learner's error, with a shift from a general reason for the learner's error, namely, that the learner did not know how to identify the primes (see turn 3), to a more specific reason for the learner's error related to how the learner identifies the primes (at turn 41, not in Table 8.3 – PST 2E: *Yes, they only look at one of the two numbers*).

The inquiry focused on the importance of affording the learner an opportunity to explain how he was taught to work with the 1-100 table which presented the opportunity to pose a further question on whether the learner understood the purpose of working with the 1-100 table in the context of identifying primes. The pre-service teachers remembered from their learning experiences that the purpose was to find a systematic way of eliminating the numbers that were not prime. The knowledge-in-practice enabled here is to engage learners around the purpose of a mathematical activity as part of the process of eliciting how they were taught. Understanding the purpose of the mathematical activity of identifying the primes in the 1-100 table is an aspect of interpreting the mathematical problem.

The epistemic moves in which the pre-service teachers reflected on how they were taught included ways in which the learner could have checked whether a number is prime. The process of factorisation until the unique set of prime factors for a number is found, for example, was suggested (see turn 13). Pre-service teachers' reflecting on their learning experiences as epistemic moves enabled knowledge-in-practice about the importance of encouraging learners at all times to use their definition of a prime number and/or to find the unique prime factors of a number to check whether a number is prime. The learner knew the definition of a prime but did not apply it when using the 1-100 table to identify the primes which suggests a culture of verification may not have been encouraged in the learners' learning experience.

The pre-service teachers found it challenging to explain the systematic approach of eliminating the primes in the 1-100 table, and the learner did not seem to understand the purpose of working with the 1-100 table. This presented an opportunity for knowledge-of-practice through the PST-PLC considering to further explore the underlying reasons why learners may struggle with identifying primes in the 1-100 table. The epistemic moves in turns 34 and 35 suggest the need for further exploration, namely, knowledge-for-practice, for the underlying reasons as to why learners may not apply their knowledge of the definition of a prime when working with the 1-100 table to identify primes.

Figure 8.2 below builds on figure 8.1 to provide a summary of the interactions between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in relation to knowledge in the engaging stance.

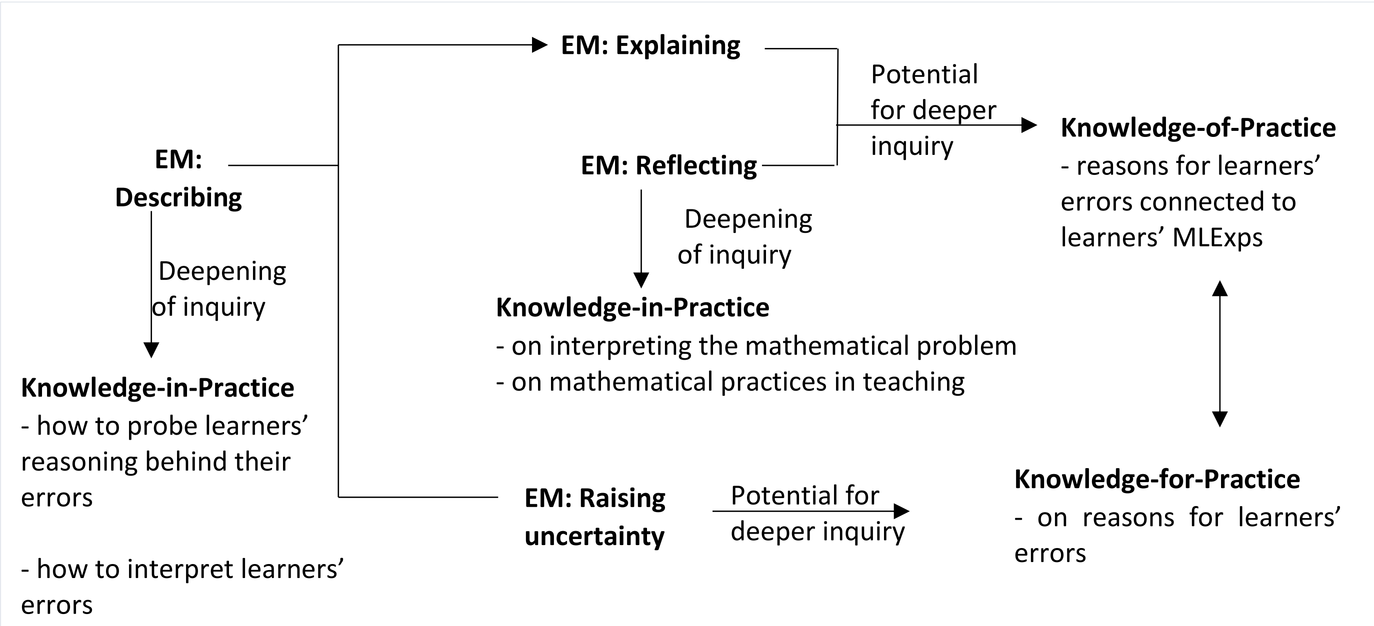


Figure 8.2: Interactions between epistemic moves influencing inquiry in relation to knowledge in engaging stance

Figure 8.2 shows describing as an epistemic move enabled knowledge-in-practice which was as a result of the detailed descriptions of how learners were engaged. Reflecting as an epistemic move enabled knowledge-in-practice and, in conjunction with explaining, held potential for deeper inquiry and knowledge-of-practice. The epistemic move raising uncertainty held potential for deeper inquiry and knowledge-for-practice which is key in enabling knowledge-of-practice.

8.2.3 Summary of engaging stance

In the engaging stance the focus was on the learner: the way in which the learner solved the problem; as well as probing the learner's reasoning in solving the problem and the reasoning behind the learner's error. When probing the learner's reasoning behind their errors was the focus, the urgency to correct errors or finding ways of re-teaching did not take priority.

Describing learners' responses to how they reasoned was a key epistemic move which deepened inquiry into how to probe learners' reasoning and interpret errors. The inquiry led to questions on how learners were taught which in turn created opportunity for reflection. This included reflection on how pre-service teachers were taught, the mathematical content in the problem as well as the teaching of the content. These reflections were key epistemic moves for deepening inquiry and enabling knowledge-in-practice about interpreting mathematical problems and mathematical practices to support learners' learning.

The raising of uncertainty regarding learners' responses was an epistemic move for further inquiry into learners' understanding of mathematical content and a signal for knowledge-for-practice into reasons for learners' challenges.

The describing of the eliciting of learners' learning experiences and the reasoning behind learners' errors created potential to deepen inquiry into the connection between learners' mathematical errors and their mathematical learning experiences. I showed how this connection is related to epistemic moves generated through the inquiry. In my comparative analysis of the engaging stance and the correcting stance, I will elaborate on how this connection is related to epistemic moves that were generated.

The inhibitors of inquiry point to practices of inquiry in the PST-PLCs that need to be developed to enhance the critical nature of the inquiry. I elaborate on these inhibitors in my comparative analysis of the stances.

8.3 Correcting Stance

A correcting stance towards the inquiry into learners' mathematical errors was evident in three out of the five PST-PLCs. Pre-service teachers in all the communities indicated that they had difficulty in getting learners to talk and explain their reasoning behind their error. In chapter 6 pre-service teachers' narratives about their experiences with mathematical errors indicate that they were not encouraged to explain the reasoning behind their errors. Where pre-service teachers received support in after-school tutoring sessions many were asked what knowledge they had about the mathematical content in which their errors occurred. The learning experiences of most pre-service teachers with mathematical errors in school and beyond school may explain their struggle in engaging learners' errors and the urgency to correct learners' errors.

I selected an episode each from PST-PLC 3 and PST-PLC 4 to show how variations in aspects of working with learners' mathematical errors as given in Table 8.2 influenced the inquiry into learners' errors. In the correcting stance, there was no evidence in the inquiry of questions that were asked to learners to explain the reasoning behind their errors as in the engaging stance. Whereas in the engaging stance the mathematical problem in which the learners' errors occurred was always shown to the PST-PLC, this was not always the case in the correcting stance, as in episode 2 taken from PST-PLC 3 (Table 8.4). Furthermore, in episode 2, no example of learners' work showing how they solved a word problem in which errors made was brought to PST-PLC 3. The absence of the mathematical problem in which the error occurred and examples of learners' work inhibited the inquiry as deeper insight into the learners' errors could not be gained from interpreting the mathematical problem and the learners' solutions and errors. I show how in episode 3 where mathematical problems in which learners' errors occurred and learners' solutions to the problem were shared, greater opportunity to move the inquiry towards an engaging stance was created but still not achieved.

Table 8.4: Episode 2 from PST-PLC 3, session 1, facilitated by PST 3A

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
144	PST 3D	Oh yes, and then word problems, the problem here it is the multiplication tables, because all their sums are basically multiplication tables. And this is why they cannot do it, because they cannot do multiplication tables that are mixed, as they are just used to multiply with the 'two-times' tables, but not mixed.	Describing the kind of mathematics problem in which the learners' errors occurred	Explaining a reason for why learners cannot do word problems
145	PST 3A	But how do you think multiplication tables…we can…		Pressing for different ways to teach multiplication tables
146	PST 3D	They must just practice it more…quick speed tests	Describing a teaching approach to support learners	

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
147	PST 3A	I know that we must not give them the answers, so I will say to him two multiply by three, I must not give him the answer, but if the learner does not know his multiplication tables, how do I probe, how do I ask him questions so that he gets to the answer? Understand? Because I now have to see three times or two three times and say to him, okay add the two together or….		Raising uncertainty about an engaging approach to errors Describing an understanding of an engaging approach to errors
148	PST 3D	The thing is, I do not know, I still feel that multiplication tables must be practised, it must just – it must just come. And the thing is, it is not going to happen within a period, and that is why I say those speed tests work. Set a time, give them five minutes for twenty sums, and there you go. Learners must just go and learn.		Raising uncertainty about other ways of dealing with learners who struggle with multiplication Arguing for a teaching approach Making assumption about learners' understanding
149	PST 3C	I think we must ask the lecturer whether we can do this with the learners. So, five or ten minutes before we begin to work with them, just quickly go through multiplication tables – then at least we know that once in two weeks they will get practice.	Reflecting on a recommended teaching approach	
150	PST 3D	I did…..no, you may speak.		
151	PST 3B	And if they do it before the class (*means tutor class)*, then it will nicely get them thinking and to work nicely for the rest of the time.		Making an assumption about the value of practising multiplication tables
152	PST 3D	Yes.		

The inquiry into learners' errors in episode 2 was brief with a focus on how to address learners' learning of multiplication tables, because learners' errors in the word problems were attributed to their not knowing their multiplication tables. The fact that PST-PLC 3 did not encourage PST 3D to present the word problem in which learners' errors occurred or to show the actual solution of the learners also inhibited the inquiry in interpreting the mathematical problem. A further inhibitor of inquiry was PST 3D's describing a teaching approach to support learners who struggled with their multiplication tables. The teacher approach described was from her own learning experiences as a school learner and influenced her lack of openness to exploring other ways of supporting learners with the learning of multiplication tables (turn 146). In turn 149, PST 3C's reflecting on the teaching approach of PST 3D and agreeing with the approach is an inhibitor as it influenced the rapid closure of inquiry into learners' challenges with word problems and the learning of multiplication tables.

I next present the correcting stance in episode 3 taken from PLC 4 (in Table 8.5) to show how the inquiry differed from that in episode 2. Episode 3 differed from episode 2 in that the mathematical problem in which a learner's error occurred and an example of the learner's solution to the problem were shown to PST-PLC 4. The mathematical problem shown to PST-PLC 4 was the completion of a function table:

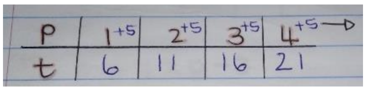
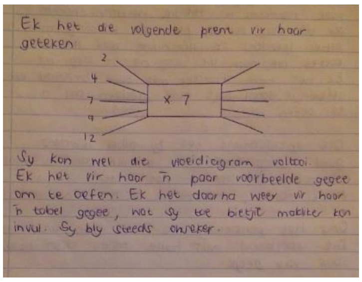
Given $t = p + 5$, Complete the following table:

p	1	2	3	4	5
t					

The showing of the mathematical problem in which the learner's error occurred and the learner's solution enabled the inquiry to focus on interpretation of the mathematical problem as well generating more than one reason for why learners may struggle with the function table.

Table 8.5: Episode 3 from PST-PLC 4, session 2, facilitated by PST 4A

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
1	PST 4C	They do not always know how to interpret the rule. Like the one learner she went and wrote *t* at the bottom and when she explained she did the first one correct but she did the second one incorrect.	Describing how a learner explained	Explaining a reason for a learner's error

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		 | p | 1 | 2 | 3 | 4 | | t | 6 | 11 | 16 | 21 | *[Taken from reflection journal of PST 4C]*		
2	PST 4B	And this is work they ought to be able to do as it is first term’s work	Describing the mathematics learners ought to understand	
3	PST 4C	Yes		
PST 4A shifts the conversation to another problem and in turn 15 the problem is revisited:				
15	PST 4D	The *p* and the *t*, I also did a table like that with my learners, with input and output values and the boy learner understood it completely. But the girl learner did not understand it and then I drew her a flow diagram and then she understood it. And then with the rest, she went to draw. But if she just sees that [*referring to the table]* then she does not know what is going on. And she knew how to work back [*pointing to the flow diagram]*.  *Taken from PST 4D’s reflection journal*. English translation of the writing below the flow diagram: *[I drew the following picture for her. She could only do the flow diagram. I gave her a few flow diagrams to practice. Thereafter I gave her a table again, which she could complete a little bit better. She remains uncertain.]*	Describing learners’ understanding.	Describing a teaching approach to support a learner
16	PST 4A	But why do you think she would understand it like that (*pointing to the flow diagrams)* and not like that *(pointing to the tables).*		Pressing for a reason why the learners understood flow diagram but not the table representation

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
17	PST 4D	I think because this *(pointing to the flow diagram)* is more visual….I do not also know.		Raising uncertainty about reason for why learner finds flow diagram easier to complete
18	PST 4C	I think what confused my one (*referring to the learner)*. Look I think it is the *p* and *t* that confuses them, it is not necessarily that they did not know what to do.		Explaining a reason for a learner's error
19	PST 4E	I think they do not always know where to substitute the values.		Explaining a reason for a learner's error
20	PST 4C	When I asked her. Where is it now? (*looks for a page*). She added the four, so they leave out steps.	Describing engagement with a learner	
21	PST 4B	They are sometimes too hasty and then they cannot do it		Explaining a reason for the learners' errors
22	PST 4C	And look they could do it there (*shows the PLC a flow diagram)*. Like PST 4D.		Providing further evidence of learners able to do flow diagrams
23	PST 4A	Because they can see it visually.		Explaining a reason for learners finding flow diagram easier
24	PST 4D	Yes because they can see what they must multiply by.		Explaining a reason for learners finding flow diagram easier
25	PST 4C	Yes because there (*referring to table*) they do not know what to multiply by.		Comparing the flow diagram with table representation of functions in analysing learners' errors
26	PST 4A	So how can we explain this differently to a learner		Pressing for new ways of handling the content
27	PST 4C	I think you need to begin with this (*points to flow diagram*) and then you take the *same example* and then you show them it is the same just in another representation		Describing a teaching idea
28	PST 4D	Yes, I agree.		
29	PST 4A	Yes the learners get into a comfort zone when they see one thing and when given another then it confuses them		Explaining a reason for learners' errors
Ongoing inquiry into why learners struggle with the function table				
40	PST 4E	Like that table method. I do not think there can be just one way of explaining, we must think of as many ways. Honestly, I do not know if I were in that grade and saw the table whether I would know that one must take the		Raising uncertainty about teaching table representation of functions Predicting learners' difficulties

Turn number	PST	Dialogue	Inhibitors of critical inquiry	Epistemic moves as enablers of critical inquiry
		bottom value and place it in there. And I am not sure whether they understand that. They are only twelve years old and they only started with that. I do not know.		
41	PST 4B	I think the easiest way for a teacher to explain to a learner of that age is to use pictures. Picture of ice-cream cones flip…just to get it to their level. So instead of the *p* and *t* used ice-cream cones. Okay in the examination there will not necessarily be an ice-cream cone. Or like the *p*, *p* can stand for a perd (*Afrikaans for horse)* and then you put a picture of a 'perd' there and the *t* stand for		Reflecting on own learning experiences at school about the role of letters.[15]
42	PST 4E	A 'tier' (*Afrikaans for tiger)*		
43	PST 4B	Whatever, 'tiere' and then you put a picture there to make it more visual for the learner. Then you say okay, *p* stands for 'perd' so if they make that connection then it will be so easy for the learners and they will know what to do		Arguing for a teaching approach
44	PST 4E	And *p* and *t* are actually place-holders, so there could have been an *x* and *y* also or *s* and *q*.		Describing a different role of letters

The inhibitors of inquiry in this episode were pre-service teachers' descriptions of their experiences of engaging learners with their errors, which often lacked specific information about how learners explained their thinking (as in turn 1), or, more specifically, did not specify what a learner did not understand (as in turn 15 and turn 20). These inhibitors accounted for the inquiry shifting to a focus away from the learner's error (as in turn 2), a move to a teaching approach to support the learner (as in turn 15), or providing a reason for learners' errors which implies that the errors were careless errors (as in turn 21). Describing the mathematics, the learners ought to know in turn 2 as a way of making a judgement about learners' understanding of content, inhibited the inquiry and explains the initial closure of the inquiry into the learner's error in turn 1.

The inquiry in episode 3 did generate epistemic moves to enable deeper inquiry into the mathematical problem in which the error occurred and teaching approaches which might

[15] In a follow-up PST-PLC which I facilitated, PST 4B confirmed that is how she remembers being taught about the role of letters in mathematics.

support learners' learning of the mathematics. The epistemic moves in episode 3 included facilitation moves which were epistemic moves to encourage PST-PLC 4 to think about reasons why learners find certain function representation easier than others, and for PST-PLC 4 to think about new ways of engaging the learners with the content. I next compare the epistemic moves generated in the two correcting stance episodes.

8.3.1 Epistemic moves in correcting stance

In this stance, fewer epistemic moves were generated when the mathematical problem in which the learner's error occurred and learner's work did not form part of the inquiry as in episode 2. There were six different kinds of epistemic moves in episode 2: explaining, pressing, raising uncertainty, describing, arguing and making assumptions. Where the mathematical problem in which the error occurred and learner's work were provided, as in episode 3, there were nine different kinds of epistemic moves: pressing, raising uncertainty, describing, comparing, explaining, providing further evidence, predicting, reflecting and arguing.

Reflecting, predicting, comparing and providing more evidence were epistemic moves which were generated only in episode 3. The epistemic move making assumptions was only generated in episode 2. I next compare the different interactions between epistemic moves in the different episodes.

8.3.1.1 Epistemic moves – Explaining, pressing, raising uncertainty, describing, predicting, arguing, comparing, providing further evidence and making assumptions

In episode 2 and 3, the epistemic move explaining deepened inquiry through the epistemic move pressing which in turn generated the epistemic move raising uncertainty. Raising uncertainty deepened the inquiry through generating the epistemic moves, describing and arguing (episode 2), and the epistemic moves explaining and predicting (episode 3). In episode 2, arguing deepened the inquiry through the epistemic move making assumptions. In episode 3 the epistemic move raising uncertainty deepened inquiry through generating the epistemic move explaining which further deepened the inquiry through generating the epistemic moves providing further evidence and comparing.

Explaining why learners made an error were epistemic moves in episode 2 and 3 to deepen the inquiry into exploring as many possible reasons for the error. In episode 2, there was limited inquiry into possible reasons for the learners' errors. For example, in turn 144, PST 3D's explaining a reason for why learners who could not do word problems because they struggled with their multiplication tables is an epistemic move for deeper inquiry into other reasons for learners' difficulties, although the inquiry was not deepened in this case. In episode 2, explaining the reason for a learner's errors created an opportunity to press for greater specificity about the mathematical problem in which the error occurred and examples of learners' solutions.

In episode 3, an initial explanation for a learner's error was an epistemic move that deepened inquiry into more than one reason for the error. One sees how PST 4C in turn 1 (in Table 8.4) first explains the learner's error very generally (*They do not always know how to interpret the rule)* and later, in turn 18, she explains that it is the letters p and t in the function rule that may confuse the learners. The latter explanation while still general moved the inquiry into another explanation for the learners' error in that the confusion may be related to not knowing what to substitute (PST 4E in turn 19). PST 4E's reason for the learners' errors is an epistemic move for deeper inquiry into learners' understanding of the variables. PST 4E's explanation for the error may be as a result of having had the opportunity to analyse the work of a learner (presented in turn 1) in which a learner only substituted $p = 1$ to find the value of t and thereafter added on the difference between the value of p and t. The importance of pressing for greater specificity in how the learner explained his thinking was critical at this moment to deepen the inquiry into the learner's error, as was the case in the engaging stance.

Explaining as epistemic move deepened the inquiry in both episodes through generating the epistemic move pressing. The nature of the pressing differed in the two episodes. In episode 2 the pressing was for new ways of teaching (turn 145) whereas in episode 3 the pressing was for the reason for learners' difficulty in understanding a certain mathematical representation (turn 16). This shows how the nature of the pressing in episode 3 created greater opportunity for the inquiry to get closer to the engaging stance but not achieve it as the inquiry did not include how learners' reasoning behind their errors were probed.

Pressing as epistemic move in both episodes deepened the inquiry through the raising of uncertainty. In episode 2 the raising of uncertainty was on what it means to engage learners' errors and alternative ways of teaching. In episode 2, I considered PST 3A's raising uncertainty about what it means to probe learners' errors, and her describing to the PST-PLC her understanding as not giving the learners the answer in turn 147, an epistemic move as it created opportunity for deeper inquiry into what it means to engage learners' errors.

Raising uncertainty about alternative ways of teaching generated the epistemic move arguing for a teaching approach. Arguing for a teaching approach is an epistemic move for deeper inquiry into assumptions on which the argument is based. In episode 2 in turn 151 an assumption about the teaching approach is made which is an epistemic move to deepen inquiry through challenging the assumption. Raising uncertainty about alternative ways of teaching mental calculation (turn 148) is an epistemic move for deeper inquiry into mental calculation teaching strategies. PST 3D arguing for a teaching approach based on giving learners opportunities in class to do speed tests to improve their memorisation is an epistemic move in that it created an opportunity for inquiry into strengths and limitations of the teaching approach. In episode 3, turn 148, PST 3D's assumption about learners who do not understand their multiplication tables as being related to their not practicing their tables is an epistemic move to deepen inquiry into this assumption. In episode 2, turn 151, PST 3B, made an assumption about the value of learners practising their multiplication tables, as a way to *"get them thinking"*. I considered the making of this assumption an epistemic move for deeper inquiry into firstly what PST 3B meant by *"getting them thinking"* and secondly learners' experiences of learning multiplication tables.

I will return to what potentially may have inhibited the deeper inquiry into understanding of what it means to probe learners' errors and assumptions about teaching approaches. PST 3A's suggestion in turn 147 to focus on the relationships between mathematical operations when supporting learners who struggle with their multiplication tables shows the potential to inquire into mental calculation strategies which do not rely solely on memorisation.

In episode 3, the raising of uncertainty was about why learners found difficulty in understanding a certain mathematical representation, deepening the inquiry through further explaining reasons for the learner's error in turn 1. For example, the explanation that PST 4E gave for learners' errors being related to understanding how to interpret the table (turn 40), coupled with her uncertainty as to why learners struggle with function tables, was an epistemic move for deeper inquiry as it led to her predicting that learners' ages and the stage at which they were introduced to tables may be a factor in why they struggle with the function table. PST 4E's predicting is an epistemic move for deeper inquiry into why learners struggle with tables and to draw on research which may have support for her prediction.

The ongoing inquiry through explaining reasons for learners' errors deepened the inquiry through the epistemic moves comparing and providing further evidence of learners' work. For example, PST 4A's pressing for a reason for why learners understood the flow diagrams but not the function table (turn 16) was an epistemic move to deepen inquiry into reasons for the learners' error. In turn 17 PST 4D's reason was that the flow diagram is a more visual representation. In turn 17, PST 4D's uncertainty about whether the visual nature of the flow diagrams could be a possible reason for learners finding the completion of flow diagrams easier than table representation of functions was an epistemic move that did generate deeper inquiry into what about the visual nature made the representation easier.

In turn 22, PST 4C's providing the PLC with further examples of learners' work that was correct when doing flow diagrams, was an epistemic move which deepened inquiry in that it led PST 4A in turn 23 to revisit the visual nature of flow diagrams as a possible reason for learners finding flow diagrams easier. PST 4A's reason in turn 23 was an epistemic move which led to further refinement of understanding why the visual nature of flow diagrams was easier for learners: in turn 24, PST 4D explained that in flow diagrams it is much easier to see what operation must be performed. PST 4D's explanation was an epistemic move which led to PST 4C comparing a flow diagram and table representation and pointing out the reason for a learner's error in a table representation. PST 4C's reason for learner's difficulties with table representations in turn 25 was an epistemic move for deeper inquiry into the learner's error in turn 1 which involved the transition from function rule to function table.

8.3.1.2 Epistemic moves – Explaining, pressing, describing, reflecting and arguing

In episode 3 the ongoing inquiry into possible reasons for the learners' errors led to pressing for new ways of teaching (turn 26). I considered this pressing an epistemic move as it did generate teaching ideas which in turn were epistemic moves for deepening inquiry into teaching approaches suggested. For example, the describing of a teaching approach to start with flow diagrams and to use the same example in a table representation (turn 27) is an epistemic move for deeper inquiry into how working with all three representations at the same time (equation, flow diagram, and table) can support the understanding of the various concepts involved in functions. Describing a teaching approach is also an epistemic move for deeper inquiry into how working with all three function representations at the same time supports learners in dealing with the kinds of errors made by the learner whose solution was shown in turn 1.

Describing as epistemic move deepened inquiry through further explaining of reasons for learners' errors (turn 29) which in turn deepened inquiry through PSTs reflecting on their learning experiences as school learners and arguing for a particular teaching approach. For example, the suggestion to support learners' understanding of letters in a function rule is based on a PSTs reflecting on own learning experiences in which letters were used to represent objects (turn 41). Arguing for a teaching approach based on prior learning experiences (turn 43) is an epistemic move to rethink the strengths and limitations of a teaching approach which in this case was based on flawed mathematical knowledge. A move to describing the role of letters in mathematics (turn 44) is an epistemic move for deeper inquiry into the mathematics and to challenge the teaching approach that was suggested. I return below to what may have inhibited deeper inquiry into the proposed teaching approach of supporting learners' understanding of the role of letters in function rules.

Figure 8.3 below provides a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in the correcting stance without mathematical problems and learners' work.

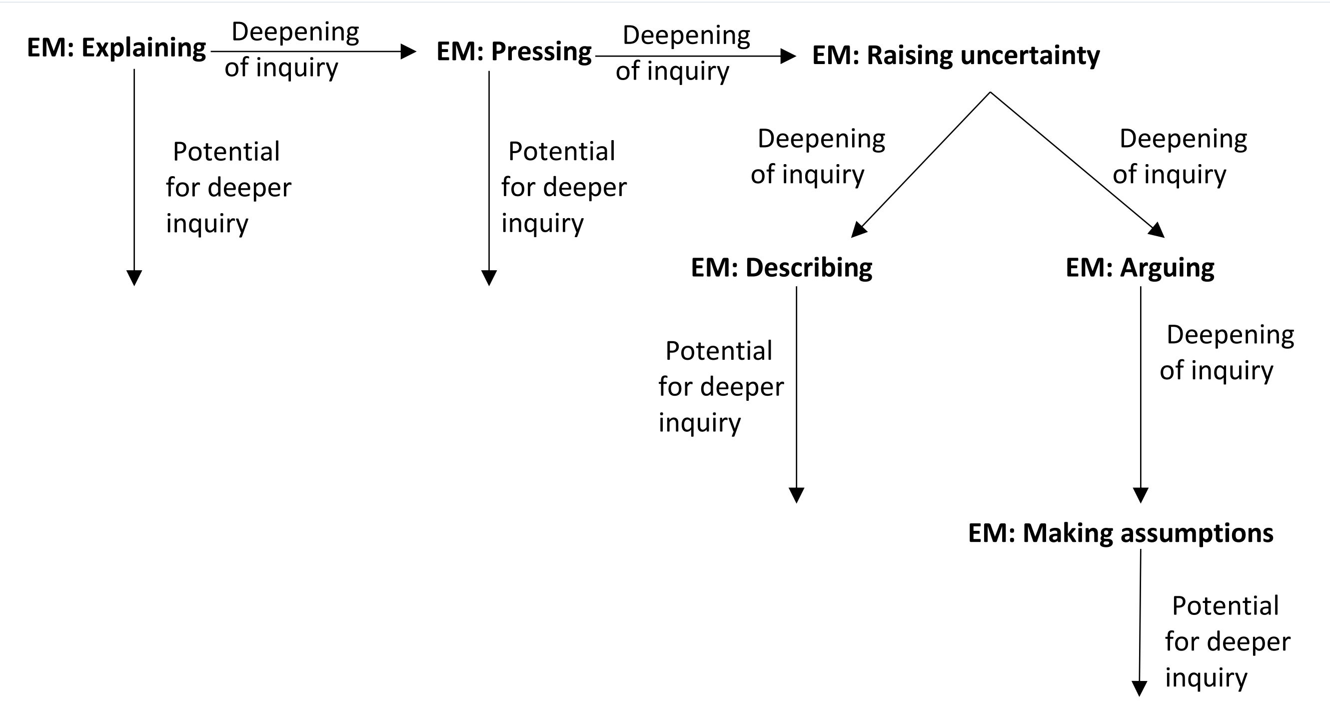


Figure 8.3: Interactions between epistemic moves influencing inquiry in correcting stance without mathematical problems or learners' work

Figure 8.3 shows how explaining reasons for learners' errors as an epistemic move deepened inquiry through the pressing for alternative ways of teaching. Pressing is an epistemic move which deepened the inquiry through raising uncertainty. Raising uncertainty in how to engage the learner's error deepened inquiry through further describing an understanding of the engaging approach to errors and arguing for a teaching approach. The arguing for a teaching approach as an epistemic move deepened inquiry through assumptions made about teaching approaches. Within the engaging stance the raising of uncertainty was about learners' responses when probing the learners' reasoning behind their errors.

Figure 8.4 below provides a summary of the interaction between epistemic moves to either deepen inquiry or lead to potential deepening of inquiry in the correcting stance with mathematical problems and learners' work.

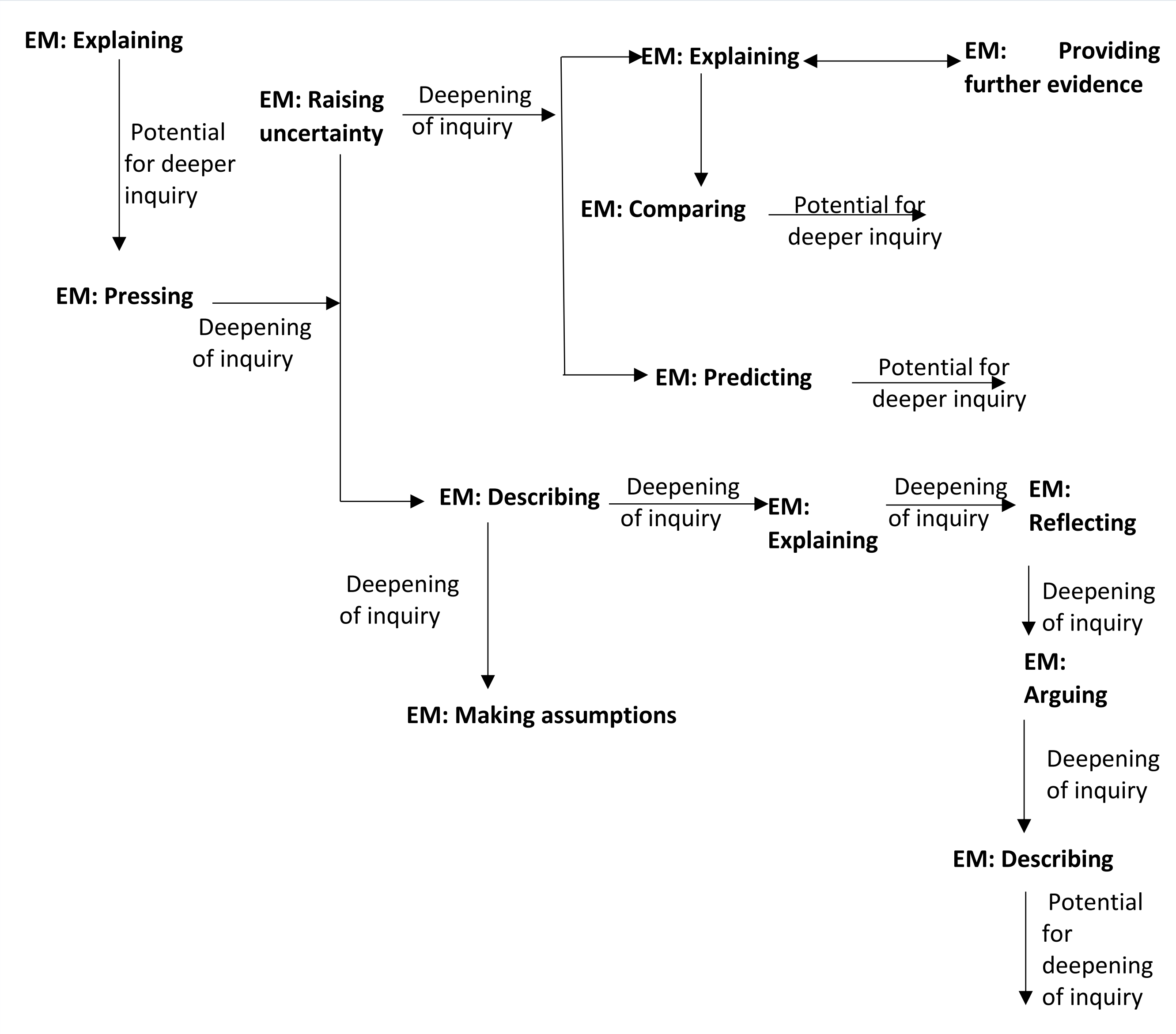


Figure 8.4: Interactions between epistemic moves influencing inquiry in correcting stance with mathematical problems and learners' work

Figure 8.4 shows that in the correcting stance with mathematical problems and learners' work, explaining as epistemic move deepened the inquiry through pressing for further reasons for learners' challenges as well as teaching approaches. The epistemic move raising uncertainty about why learners found certain aspects of mathematics more challenging deepened the inquiry through further explaining reasons for learners' challenges, comparing mathematical ideas, providing further evidence of learners' work and making predictions about learners' challenges. Describing teaching approaches deepened the inquiry into teaching approaches, further explaining reasons for learners' errors, reflecting on PSTs learning experiences as school learners and explaining the mathematical content.

In the next section I elaborate on how knowledge is enabled through these epistemic moves and the epistemic moves with potential for deepening inquiry.

8.3.2 Deepening inquiry through knowledge in correcting stance

In the correcting stance, the deepening of inquiry through knowledge was influenced by whether the mathematical problems in which learners' errors occurred and learners' work formed part of the inquiry.

Without mathematical problems and learners' work the epistemic moves with potential for deepening inquiry deepened inquiry through knowledge-for-practice only. Explaining reasons for learners' errors which was limited to a single reason held potential for deeper inquiry through knowledge-for-practice on more reasons for learners' errors. Explaining an understanding of an engaging approach towards learners' errors held potential for knowledge-for-practice on an engaging approach towards learners' errors. The epistemic move pressing for different ways to teach held potential for deeper inquiry through knowledge-for-practice on different teaching approaches. Making assumptions about teaching approaches is an epistemic move with potential for deepening inquiry through knowledge-for-practice about teaching approaches and how learners' understanding is enhanced.

Figure 8.5 below builds on figure 8.3 to provide a summary of the epistemic moves influencing inquiry in relation to knowledge in the correcting stance without mathematical problems or learners' work.

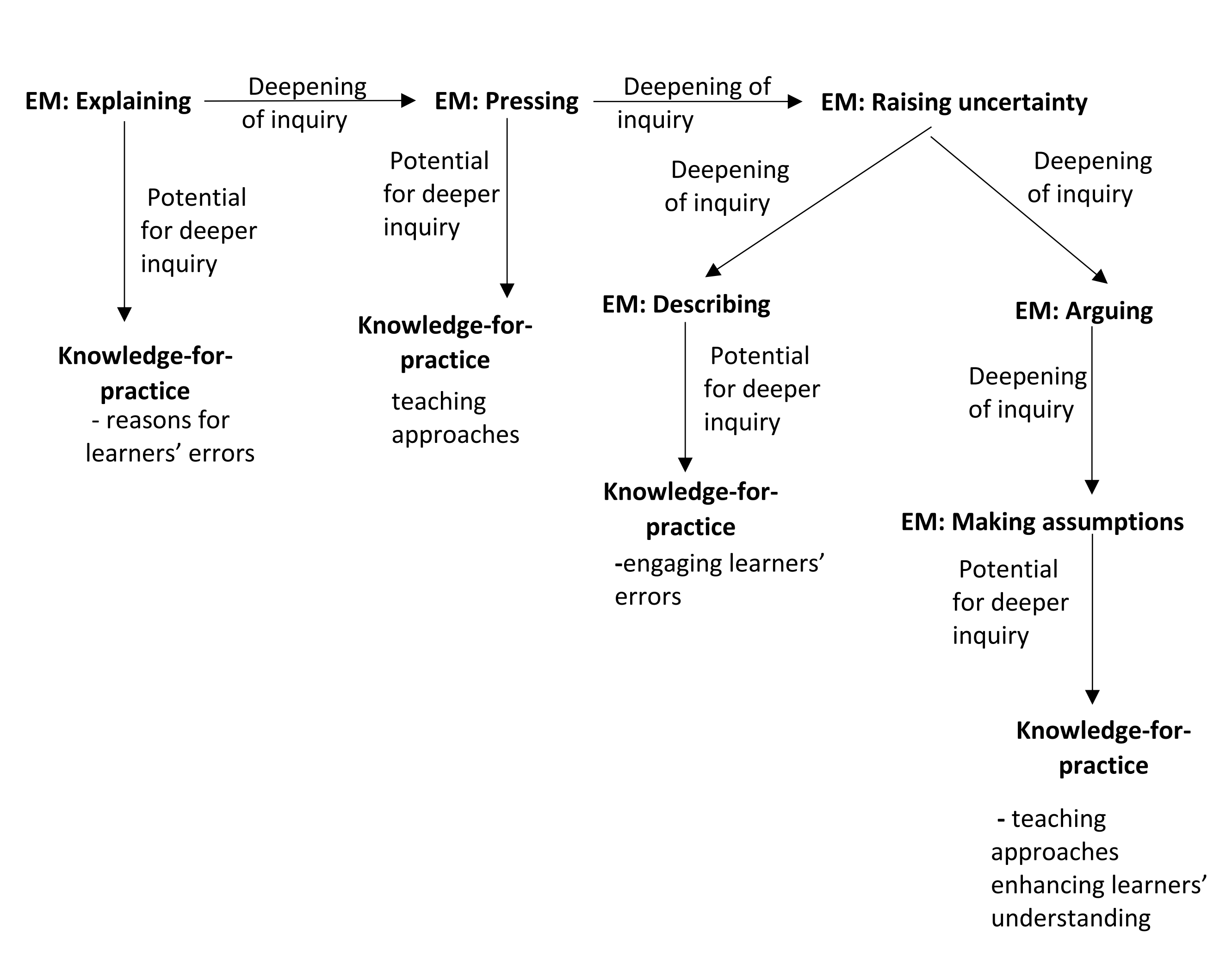


Figure 8.5: Interactions between epistemic moves influencing inquiry in relation to knowledge in correcting stance without mathematical problems and learners' work

Where mathematical problems and learners' work were included in the inquiry, the epistemic moves deepened the inquiry through knowledge-in-practice, knowledge-for-practice and knowledge-of-practice. Pressing deepened the inquiry through describing new ways of engaging learners with the mathematical content to address their challenges which in turn deepened the inquiry through knowledge-in-practice about teaching approaches. For example, the knowledge-in-practice that was enabled through the inquiry was the importance of recognising that the flow diagram is an easier representation of functions for learners to understand and that when shifting to other representations, it is best to start with the flow diagram so that connections can be made between the flow diagram and the function rule and then to move to a tabular representation so that connections can be made between all three forms of representation.

Pressing for reasons for why learners found certain mathematical representations challenging deepened inquiry through raising uncertainty about learners' challenges and led to making predictions about difficulties learners may encounter. Predicting was an epistemic move with potential to deepen inquiry through knowledge-for-practice on learners' challenges with mathematical representations. Reflecting on PSTs' own learning experiences at school, deepened the inquiry through arguing for a teaching approach from PSTs' learning experiences and revisiting the mathematical content. The potential for further deepening inquiry is through knowledge-for-practice on the teaching approach and mathematics.

When pressing for further reasons as to why learners found certain mathematical representations more challenging, the inquiry was deepened through the epistemic move comparing different forms of mathematical representation in relation to learners' errors. The potential for deepening inquiry through knowledge-of-practice is the ongoing systematic inquiry into comparing how learners' errors relate to mathematical representations.

Figure 8.6 below builds on figure 8.4 to provide a summary of the epistemic moves influencing inquiry in relation to knowledge in the correcting stance with mathematical problems and learners' work.

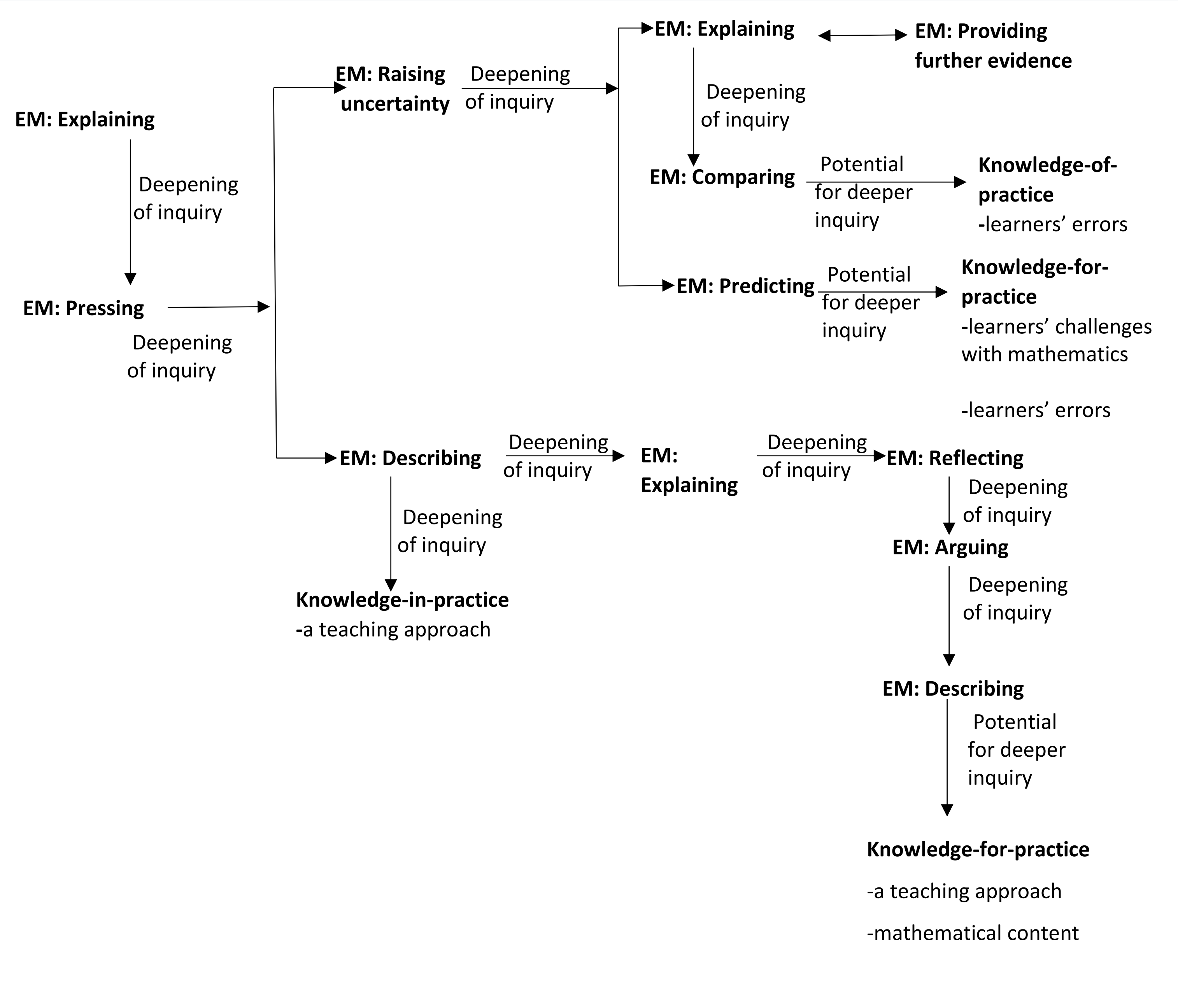


Figure 8.6: Interactions between epistemic moves influencing inquiry in relation to knowledge in correcting stance with mathematical problems and learners' work

8.3.3 Summary of correcting stance

The episodes in the correcting stance suggest that the absence of the PST-PLCs' inquiry into learners' reasons behind mathematical errors are mainly due to pre-service teachers' struggle to probe the reasoning behind learners' errors in the tutoring session. In the correcting stance, the inquiry moved from error identification, to providing a reason for the error, without inquiry into learners' reasoning, and then to how the error is either corrected or remediated (Brodie, 2014). In the correcting stance the interpretation of learners' errors was limited when learners' mathematical problems in which the error occurred and learners' solutions were not shown. In the correcting stance where the mathematical problems in which the error occurred and the learners' solutions were shown, deeper inquiry into learners' errors occurred but since learners' reasoning behind their errors were not part of the inquiry, the interpretation of learners' errors was limited when compared to that of the engaging stance. The knowledge-in-practice in the correcting stance was knowledge about ways to teach mathematics. In the engaging stance, the learners' reasoning behind their thinking required the posing of questions to get to the specifics of learners' reasoning, deepening the understanding of the learners' errors. This enhanced knowledge-in-practice about how to probe learners' reasoning behind their errors.

In the correcting stance while there was evidence of pre-service teachers drawing on their learning experiences, their own challenges with the mathematical content as learners did not come to the fore to deepen the inquiry into learners' errors. Since the learners' learning experiences were not elicited as part of the inquiry, opportunities to inquire into how misconceptions arise through teaching were not made possible. It can be inferred that if learners' learning experiences were elicited, deeper insight may have been gained into learners' classroom mathematical practices. In the engaging stance the eliciting of learners' learning experiences provided insight into learners' classroom mathematical practices.

In the case of the engaging stance, the focus of the inquiry into learners' mathematical content knowledge was part of the inquiry to understand the learners' error and not to reteach. This was not the case in the correcting stance where learners' understanding of mathematical content was assessed as a starting point to support them in making connections within mathematics as part of a teaching strategy.

In the correcting stance, as within the engaging stance, uncertainty arose with regard to how to probe learners' reasoning behind their errors and reasons for learners' errors. These were epistemic moves with potential to deepen inquiry into learners' errors through exploring knowledge-for-practice as part of enhancing knowledge-of-practice. In the correcting stance the move to teaching strategies from pre-service teachers' own learning experiences was an inhibitor to addressing uncertainty around ways of probing learners' reasoning behind errors.

8.4 Comparative analysis of inquiry across stances

The different stances impacted on social and cognitive dimensions of learning that emerged from inquiry into learners' mathematical errors. In order to gain a deeper understanding of this impact, I compared categories and subcategories of the inhibitors and epistemic moves that are representative of the two stances towards inquiry.

8.4.1 Comparing inhibitors of critical inquiry

I compared three broad categories of inhibitors of critical inquiry into learners' mathematical errors. The first category of inhibitor was limited description, which I defined in Chapter Seven. Five of the six subcategories of this inhibitor relate to the reconstruction of practice within the PLC and one category relates to how PSTs described their own learning experiences as school learners.

The second category of inhibitor that relates to the reconstruction of practice is limited reflection-on-practice. I defined this category as limited reflection because the reflection was at the level of observation without elaboration. Two subcategories of this inhibitor relate to whether the reflection was on practice in teaching or PSTs' reflection on prior learning experiences.

The third category of inhibitor was the non-recognition of epistemic moves as enablers of critical inquiry. The four subcategories of this inhibitor relate to the following epistemic moves: raising uncertainty, making assumptions, arguing and explaining a reason.

Table 8.6 shows the inhibitors of critical inquiry across the two stances.

Table 8.6: Inhibitors of critical inquiry across the stances

Inhibitor of critical inquiry category	Description of subcategories of inhibitors	Stance
Limited description	Describing with limited detail of mathematical content of the problem in which error occurred	Correcting
	Describing with limited detail of a learner's thinking	Correcting
	Describing with limited detail of a learner's challenges	Correcting
When limited or incomplete description of an event is given	Describing with limited detail of how PSTs engaged learners	Correcting
	Describing with limited detail of a learner's solution	Engaging and Correcting
	Describing with limited detail of how PSTs were taught as school learners	Engaging

Inhibitor of critical inquiry category	Description of subcategories of inhibitors	Stance
Limited reflecting-on-practice When reflecting is limited to observation/noticing without elaboration	Reflecting with limited elaboration on PST's prior learning experiences	Engaging
	Reflecting with limited elaboration on learners' understanding	Engaging and Correcting
Non-Recognition of EM Not recognising how EM deepen inquiry	Raising uncertainty to explore further knowledge	Engaging and Correcting
	Making an assumption as opportunity to challenge	Correcting
	Arguing to explore the positives and negatives	Correcting
	Explaining a reason as prompt to explore alternatives	Engaging and Correcting

Table 8.6 shows that only one sub-category of each of the inhibitors: limited description limited reflection-on-practice, and non-recognition of epistemic moves, were common across the two stances.

Five of the six subcategories of the inhibitor limited description relate to the importance of noticing key elements within representations of practice namely, the mathematical content of the problem in which an error occurred, learners' challenges, how PSTs engaged learners and learners' solutions to a mathematical problem. Limited description as an inhibitor was more prevalent in the correcting stance since learners' errors were not engaged through probing the reasoning behind them as within the engaging stance. Limited description as an inhibitor was only in the engaging stance in which PSTs described how they were taught as school learners. As pointed out earlier, PSTs struggled to provide a systematic account of the mathematics content they were taught in the engaging stance.

The first sub-category of the inhibitor limited reflecting-on-practice which relates to PSTs' limited reflection on their prior learning experiences as school learners, points to the challenge of PSTs reflecting on their own learning. For example, PSTs did not provide reasons why they struggled with the mathematical content the learners were struggling with. The second sub-category of the inhibitor limited reflection-on-practice, which relates to limited reflection on learners' understanding, mainly arose through judgements made about what learners ought to understand but without deeper reflection to enhance critical inquiry into why learners struggled with prior and current knowledge.

Three of the four subcategories of the third inhibitor, non-recognition of epistemic moves, were explained in Chapter Seven. Recognising that the raising of uncertainty is a prompt to explore further knowledge is an inhibitor of critical inquiry across both stances. Recognising that explaining a reason for learners' errors is a prompt to explore further reasons is an inhibitor of critical inquiry across both stances too. As within Chapter Seven, I put forward the idea that recognition of practices of inquiry in PLCs be developed as a form of mutual accountability of PLCs developing a critical inquiry stance.

In the correcting stance the rapid shift to focus on correcting learners' errors through re-teaching explains the epistemic moves, making assumptions, arguing and explaining reasons and non-recognition of these moves as inhibitors of critical inquiry. For example, PSTs argued for teaching approaches based on their prior learning experiences without taking a critical stance on the positive and negative aspects of the teaching approaches. Whether PSTs struggled with taking a critical stance as they were encouraged to do so and/or whether there was non-recognition of the epistemic move requires further research. I will elaborate on how these inhibitors influenced the deeper inquiry into potential connection between the social and cognitive dimension of learning in the inquiry.

8.4.2 Comparing epistemic moves of critical inquiry

Comparing the categories and subcategories of epistemic moves across the stances explains the deeper inquiry into learners' mathematical errors with a focus on how learners reason in the engaging stance compared to a deeper inquiry into teaching approaches in the correcting stance. Thus there was an enabling of knowledge-in-practice related to how to probe learners' reasoning about their errors in the engaging stance and an enabling of knowledge-in-practice on teaching approaches to support learners' challenges with mathematical content in the correcting stance.

In Table 8.7 ten categories of epistemic moves are compared, five of which were not present in the study of Zenios (2011), namely, reflecting, raising uncertainty, pressing, making assumptions and providing further evidence.

Table 8.7: Epistemic moves of critical inquiry across the stances

Epistemic move category	Description of subcategories of epistemic moves	Stance
Describing To provide an accurate conception of an object or event	**1. Describing aspects of teaching practice**	
	Describing how a learner's reasoning was probed	Engaging
	Describing how a learners' understanding of mathematical content knowledge was probed	Engaging
	Describing how a learner's MLExps in class was elicited	Engaging
	Describing a teaching idea with potential to connect mathematical representations	Correcting
	Describing how an engaging approach to learners' errors was understood	Correcting
	2. Describing aspects of learners	
	Describing how a learner reasoned	Engaging
	1. Describing mathematical content	
	Describing how a mathematical process works	Engaging
	Describing the role of letters in mathematics	Correcting
Explaining To explain a reason for a situation	**1. Explaining a reason for learners' challenges with mathematical content**	Correcting
	2. Explaining a reason for learners' errors	Engaging Correcting
Reflecting To question; notice; make suggestions on what is being said or based on past experiences	**Reflecting on MLExps as a school learner**	
	Reflecting on how mathematical content was taught	Engaging Correcting
	Reflecting on challenges with mathematical content	Engaging
Raising uncertainty To express the need for greater clarity on an approach, situation or concept	**1. Raising uncertainty about learners**	
	Raising uncertainty about how a learner reasons	Engaging
	Raising uncertainty why learners find certain mathematical content challenging	Correcting
	2. Raising uncertainty about teaching certain mathematical content	Correcting
Pressing To press for specificity or to	**1. Pressing for learners' understanding**	
	Pressing for reasons for learners finding certain mathematic content challenging	Correcting

Epistemic move category	Description of subcategories of epistemic moves	Stance
elaborate on a situation	**2. Pressing for teaching of mathematics**	
	Pressing for new ways of teaching content	Correcting
Arguing To state reasons for or against a particular position	**Arguing for a particular teaching approach**	Correcting
Making assumptions To draw conclusion without the necessary evidence	**Making assumption about teaching approach**	
	Making assumption about the value of teaching approach in enhancing learners' understanding of mathematical content	Correcting
Comparing To point out the similarities or differences in a situation	**Comparing mathematical representations**	
	Comparing mathematical representations to understand how differences influence challenges for learners and potential learner errors	Correcting
Predicting To raise concerns about a situation which leads to making a prediction about the situation	**Making predictions about learners' mathematical challenges**	
	Expressing concern about when certain mathematical representations are introduced and making a prediction about learner challenges related to age	Correcting
Providing further evidence To provide further information on a situation that enables deeper understanding of a situation	**Providing more examples of learners' written responses**	
	Providing more examples of correct and incorrect solutions to understand why learners find certain mathematical representations challenging	Correcting

Table 8.7 shows that four epistemic moves were generated in the engaging stance: describing explaining, reflecting and raising uncertainty. These four epistemic moves were also generated within the correcting stance. The subcategories of these four epistemic moves show differences in these moves across the stances.

For example, with regard to the first sub-category of the epistemic move describing, describing an aspect of teaching practice shows the deepening of inquiry into the practice of probing learners’ errors rather than a focus on teaching the mathematics content as in the correcting stance. The greater opportunity for deepening inquiry into learners’ thinking in the engaging stance is evident in the epistemic moves, explaining and raising uncertainty.

In the correcting stance where learners’ reasoning was not probed, the deepening of inquiry was on mathematical content challenges. The deepening of inquiry into mathematical content challenges was generated within the engaging stance but through the epistemic move reflecting. In both stances there was deepening of inquiry into teaching approaches through reflecting on PSTs’ own experiences as school learners but it was only in the engaging stance that there was reflection on PSTs’ challenges with mathematical content.

Pressing for learners’ understanding was an expected epistemic move in the correcting stance as there was no probing of learners’ reasoning and hence the reliance on learners’ written solutions to make predictions about learners’ challenges. The epistemic moves pressing, arguing, making assumptions and comparing in the correcting stance were all related to potentially deepened inquiry into teaching approaches. These epistemic moves held potential for deepening inquiry through knowledge-for-practice. Despite a lesser focus on teaching approaches within the engaging stance, the epistemic move reflecting in the engaging stance did deepen inquiry through knowledge-in-practice about mathematical practices that may enhance learners’ understanding.

Table 8.8 provides a summary of the epistemic moves across the stances which deepened inquiry either directly through knowledge and/or through generating further epistemic moves and/or only with potential to deepen inquiry through knowledge.

Table 8.8: Epistemic moves that deepen inquiry and/or with potential to deepen inquiry

Stance	**Epistemic Move (EM)**	
Engaging	**Deepened inquiry (2)**	**Potential to deepen inquiry (2)**
	Describing – generating further EM – directly through K-in-P	Raising uncertainty
	Reflecting – directly through K-in-P	Explaining
	Deepened inquiry (7)	**Potential to deepen inquiry (6)**
Correcting	Describing – generating further EM – directly through K-in-P	Describing
	Reflecting – generating further EM	Explaining
	Explaining– generating further EM	Pressing
	Pressing – generating further EM	Comparing
	Raising uncertainty– generating further EM	Predicting
	Arguing– generating further EM	Making assumptions
	Providing further evidence – generating further EM	

Table 8.8 shows that the correcting stance generated more epistemic moves than the engaging stance. However, six out of the seven epistemic moves in the correcting stance were moves that deepened inquiry through generating further epistemic moves and not directly through knowledge. I provided evidence and argued that the epistemic moves which held potential for deepening inquiry in the engaging stance was through knowledge-for-practice and knowledge-of-practice. In the correcting stance I argued that the epistemic moves which held potential for deepening inquiry was only through knowledge-for-practice.

In the next section, I draw on Horn and Kane's (2015) and Little's (2002) concept of 'representations of practice' in PLCs to analyse how reconstructions of practice were used across the stances and the influence on flows of practice and opportunities to learn.

8.4.3 Representations of practice across the stances

The inquiry into learners' mathematical errors in the PST-PLCs was influenced by what was selected from pre-service teachers' practice of engaging learners' mathematical errors in the tutoring. The mathematical problems in which learners' errors occurred, samples of learners' work or descriptions of how learners solved a mathematical problem and how learners' reasoning behind their errors were probed were key representations of practice that influenced the flows of practice or potential flows of practice (Kazemi & Hubbard, 2008).

8.4.3.1 Representations of practice in engaging stance

In Chapter Seven I described the use of representations of practice as a source of inquiry in that knowledge development was enabled through epistemic moves. In the engaging stance, the mathematical problems in which learners' errors occurred and learners' solutions to problems were sources of inquiry in that they were used to illustrate how learners' reasoning behind their errors was probed. The detailed descriptions of probing learners' reasoning behind their errors created opportunities to learn about the kinds of questioning that enables the deeper probing of learners' reasoning behind their errors. The ideas generated for probing learners' reasoning in the PST-PLC is a teaching practice to be taken back to the tutoring class and for further ongoing inquiry in the PLC as depicted in Figure 8.7 below.

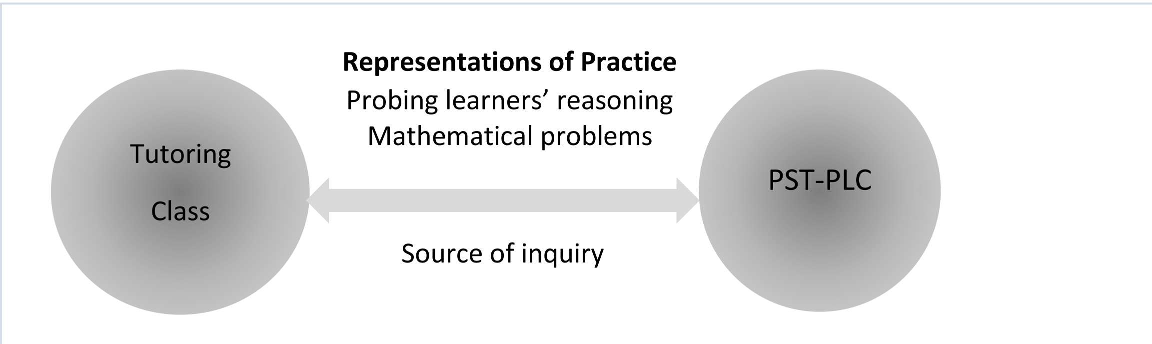


Figure 8.7: Bidirectional flows of practice in engaging stance

In the engaging stance mathematical problems were analysed in depth as part of the inquiry. For example, in episode 1, the inquiry focused on the meaning of a prime number and finding prime numbers in a systemic way. Focusing on both the conceptual and procedural aspects in an integrated way is an important part of working with mathematical ideas (Baroody, Feil, & Johnson, 2007; Long, 2005; Star, 2005). Through the analysis of the mathematics problem and learners' solutions the PST-PLC inquiry led to thinking about the importance of encouraging learners to revisit their understanding of mathematical concepts when solving problems. The teaching practice to be taken back to the tutoring class which emerged through the inquiry in this episode was the development of mathematical practices such as justification to support learners in making sense of their solutions to problems.

8.4.3.2 Representations of practice in correcting stance

In the correcting stance, representations of practice were used as sources of inquiry and sources of information. In Chapter 7, I described the use of representations of practice as a source of information which depicts a framing of collaboration as sharing with limited opportunities to learn (Louie, 2016). Mathematical problems in which learners' errors occurred were a source of information when the type of mathematical problem was mentioned, for example, a word problem but without showing the specific mathematical problem or when no in-depth analysis of the mathematical problem was done thus limiting opportunities to learn more about why learners struggled.

A conceptual understanding of the correct answer to the mathematical problem in which a learner's error occurred is considered an important factor in dealing with the errors. Prediger (2010, p. 77) argued that 'domain-specific mathematical knowledge, especially about meaning' was a key constituent in the diagnosis of learners' errors which has potential to shift from a deficit-orientated perspective in which learners are judged in terms of what they cannot do, to engaging learners' errors and building on from what they know. In the correcting stance, where mathematical problems were not analysed or used as a source of information, there was a focus on judging learners in terms of what they could not do and teaching practices were recommended based on PSTs' own learning experiences.

Where representations of practice, such as learners' solutions to the mathematical problem and the mathematical problems in which errors occurred, were used as sources of inquiry, a deeper analysis of the mathematics was involved and enabled the emergence of teaching practice which was not necessarily from PSTs' own learning experiences.

For example, the conceptual analysis of the mathematical problem,

Given $t = p + 5$, Complete the following table:

p	1	2	3	4	5
t					

generated inquiry that focused on learners' understanding of the table representation in relation to other function representations and the meaning of the letter as a potential reason for learners' errors. The inquiry into the teaching practice involved the exploration of how to support learners in developing a better understanding of different functions representations, namely, starting with flow diagrams and using the same example to move to tabular representations.

The inquiry into learners' understanding of the meaning of the letters revealed pre-service teachers' misconceptions of the role of the letter in algebra, when a widespread teaching practice using the letter to represent an object instead of the number of objects was proposed.[16] Peng and Luo's (2009) research provides evidence that teachers' knowledge of mathematics is important in error analysis and that incorrect mathematical knowledge can lead to meaningless evaluation of learners' errors. The teaching practice of the pre-service teachers that involved using the letter to represent an object is a teaching practice which is mathematically flawed. Pre-service teachers drew on their prior learning experiences as school learners in recommending this teaching practice. This could indicate that either pre-service teachers interpreted their teachers' explanations of using letters incorrectly or that their teachers taught incorrectly. Furthermore, the conceptual analysis of the mathematical problem did not focus on the learners' understanding of the meaning of an equation.

I have shown that in the correcting stance when representations of practice are used as a source of information or a source of inquiry there is a flow of practice as shown in Figure 8.8 below.

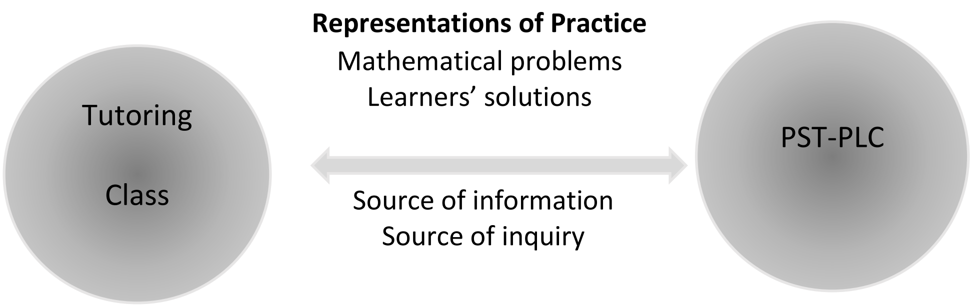


Figure 8.8: Bidirectional flows of practice in correcting stance

The teaching practice involved in the flow differ in that where the representations of practice was used a source of information, only teaching practices from PSTs' own learning experiences were involved. The deepening of inquiry through knowledge-in-practice only occurred when the representation of practice was used as source of inquiry in which teaching practices not from PSTs' prior learning experiences emerged.

[16] Refer to turns 41-43 in Table 8.5

8.4.4 Social-Cognitive connections across the stances

A key task in this chapter is to show that inquiry from a cognitive dimension into learners' mathematical learning errors enables social dimensions of learning to emerge. In the analysis of the stances, different epistemic moves were generated which influenced the potential connections between social and cognitive dimensions and generating knowledge about these connections.

8.4.4.1 Potential cognitive-social connections in engaging stance

In the engaging stance describing was a key epistemic move which influenced the potential social and cognitive connections. The cognitive dimension of learning was brought to the fore through detailed descriptions of how learners' reasoning behind their errors were probed and how learners' understandings of mathematical content were probed. To gain deeper insight into learners' errors, the social dimension of learning was brought to the fore by detailed descriptions of how learners' mathematical learning experiences were elicited. The eliciting of learners' mathematical learning experiences created the opportunity to make connections between the learners' errors and their learning experiences. For example, the analysis of episode 1 shows how the learner's error in identifying the primes in the 1-100 table was reflected in the learner's struggle to explain how his teacher taught the systematic approach to finding the primes in the 1-100 table.

The pre-service teachers experienced similar challenges in describing the systematic approach in their reflection on their own learning experiences. Based on the learner's learning experience it became evident that the learner relied on memorisation to find primes and in the process did not apply his knowledge of the definition of a prime. This example shows how engaging a learner's errors from a cognitive dimension through probing the learner's reasoning behind the error does enable engaging the learner's errors from a social dimension. This occurs through eliciting the learner's learning experiences which resulted in a deeper understanding of how the learner's errors are connected to the learner's learning experiences. Furthermore, deeper insight was also gained about a possible mathematical culture of teaching that the learner experienced as the learner did not indicate that his learning experiences included checking the primes he identified through the systematic method that was taught by drawing on his knowledge of the definition of a prime.

Figure 8.9 below provides a summary of the interaction between key epistemic moves for deepening inquiry into social-cognitive connections in the engaging stance in relation to knowledge.

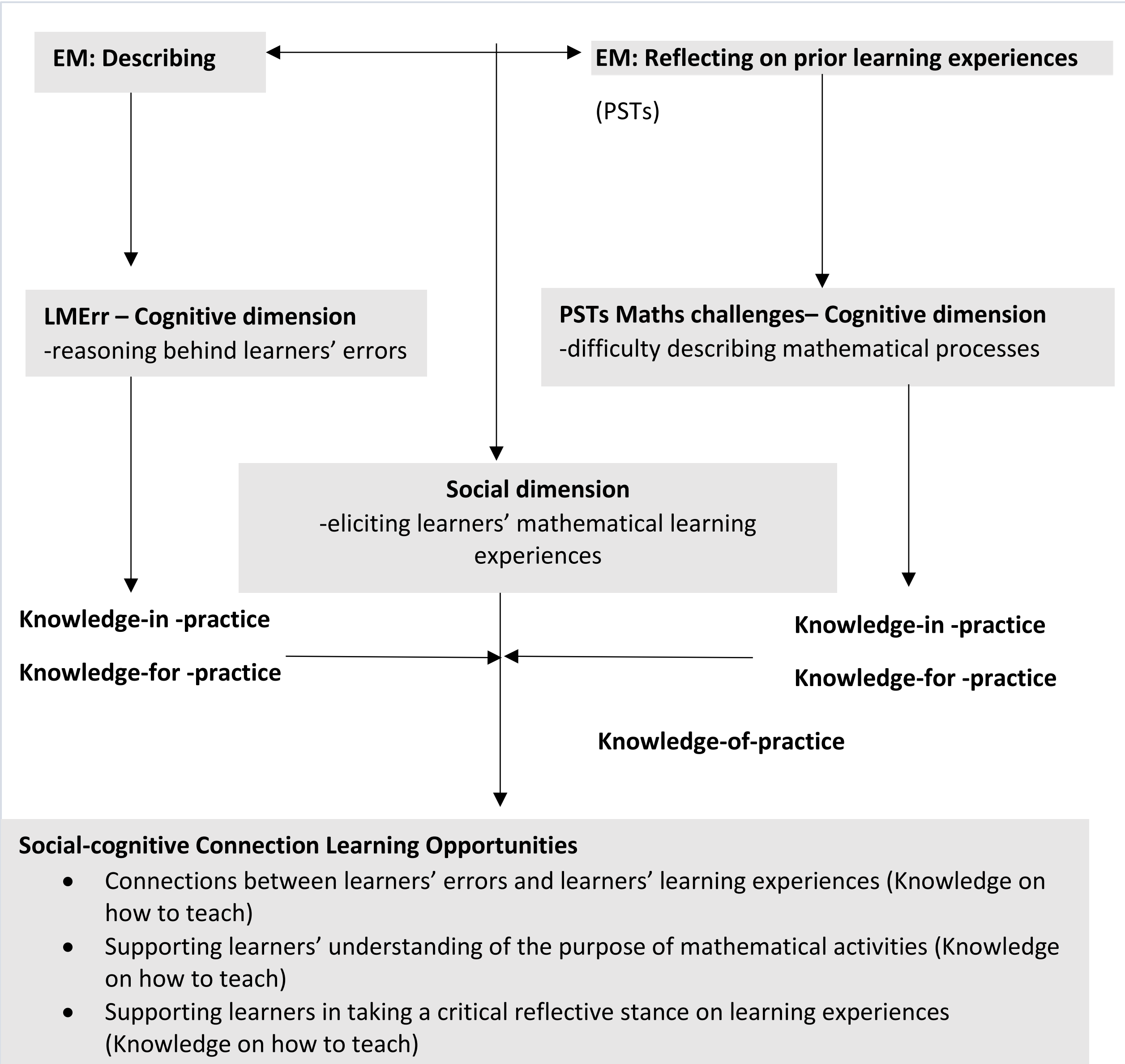


Figure 8.9: Interaction between epistemic moves for potential deepening inquiry into social-cognitive connections in relation to knowledge in engaging stance

Figure 8.9 shows that potential social-cognitive connection learning opportunities emerged through the inquiry which were related firstly to knowledge on how to teach with a focus on social and cognitive dimensions of learning, namely how to engage learners errors through eliciting their learning experiences; secondly, knowledge on how to teach with a cognitive dimension through supporting learners in understanding the purpose of mathematical activities and thirdly, knowledge on how to teach with a focus on a social dimension through supporting learners in how to take a critical reflective stance on their learning experiences. Hence potential knowledge-of-practice through the ongoing systematic inquiry into these socio-cognitive learning opportunities is created.

Inhibitors of critical inquiry explain why the systematic ongoing inquiry into social-cognitive connection learning opportunities in the engaging stance did not take place. In episode 1, I showed how the inquiry led to the raising of uncertainty about the learner's responses after the learners' explaining how they were taught which can be considered a first step in making connections between learners' errors and their learning experiences. The non-recognition of the epistemic move raising uncertainty is an inhibitor to exploring knowledge for deepening inquiry. Here there was opportunity to systematically inquire into why learners who understand mathematical concepts in one context do not apply their knowledge in another context and how learning experiences account for this disconnect.

The inquiry in episode 1 created opportunities for supporting learners in taking a critical-reflective stance on their learning experiences, for example, posing questions to the learner regarding why they knew the definition of a prime but did not apply their definition when identifying prime numbers in the 1-100 table. Limited reflection-on-practice was a key inhibitor of deeper inquiry into the social and cognitive dimensions of learning. For example, while PSTs reflected on their own challenges in understanding mathematical processes, they did not provide reasons for their challenges. There was often limited reflection on and elaboration of learners' understandings of mathematical concepts.

8.4.4.2 Potential cognitive-social connections in correcting stance

In the correcting stance, explaining and pressing were key epistemic moves which influenced the potential social and cognitive connections. An analysis of learners' errors based on their written work provided a cognitive dimension of the learners' errors through explaining potential mathematics-related reasons for the errors. Pressing for reasons as to why learners' errors were related to the nature of the mathematics led to the epistemic move comparing which provided a cognitive dimension of learning through comparing mathematical representations.

The potential social dimension emerged through the epistemic move pressing for new ways of teaching content. Through pressing for new ways to teach, opportunity is created for inquiry into how learners were taught and insight into learners' mathematical learning experiences. For example, the learner's error presented in turn 1 in Table 8.5 of episode 3 suggests that learners clearly did not make connections between function representations through checking whether the values of p and t in the learner's table make the function rule true. In turn 18 and 19 in Table 8.5 of episode 3 a reason for learners' incorrect completion of the table is given which points to learners' confusions as to where to substitute, suggesting the difficulty learners may encounter in making connections. This example shows how an analysis of learners' errors based on learners' written work provided a cognitive dimension of the learners' error through explaining potential reasons for the learners' error. This cognitive dimension created opportunities for exploring and making connections with learners' learning experiences as the social dimension of learners' errors. Pressing for new ways of teaching to make explicit connections between mathematical representations was an opportunity to pose question to learners about their learning experiences in making connections between mathematical representations.

Figure 8.10 below provides a summary of the interaction between key epistemic moves for deepening inquiry into social-cognitive connections in the correcting stance in relation to knowledge.

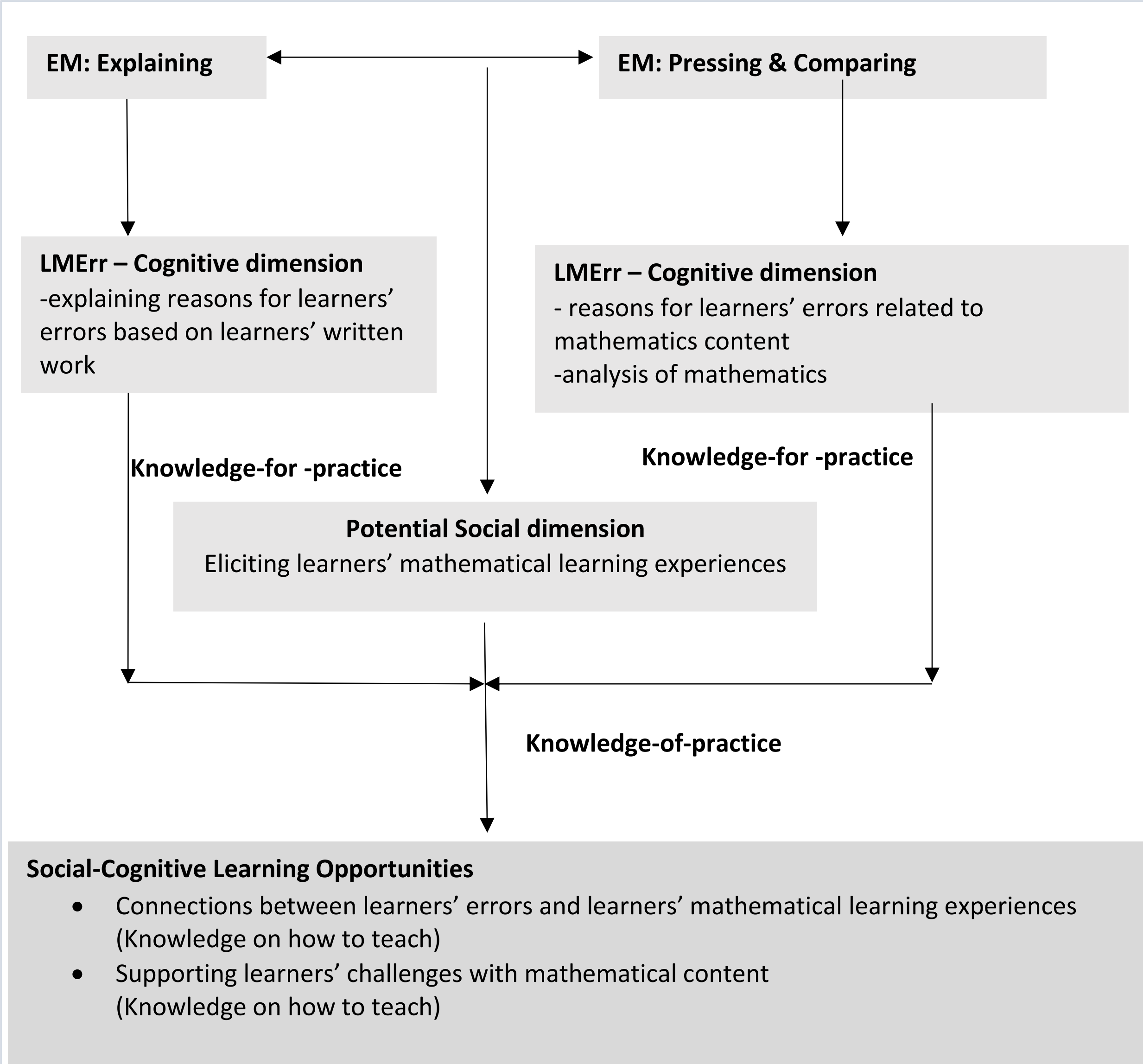


Figure 8.10: Interaction between epistemic moves for potential deepening inquiry into social-cognitive connections in relation to knowledge in correcting stance

Figure 8.10 shows that there was only potential for the social dimension of learning to emerge in the correcting stance as the inquiry did not involve describing how learners' reasoning behind their errors was probed or how learners' mathematical learning experiences were elicited.

The social-cognitive learning opportunities were all related to knowledge of how to teach with a focus on the cognitive dimension of learning and interaction between cognitive and social dimensions of learning. In the correcting stance, the analysis of learners' work led to an analysis of the mathematics with teaching ideas to support learners' challenges with content. The opportunity for knowledge-of-practice was created through the systematics inquiry into how new teaching and learning experiences impacted on learners' understanding. The inquiry brought to the fore epistemic moves of which raising uncertainty about learners' errors and learners' challenges with mathematical content provided knowledge-for-practice opportunities. The non-recognition of raising uncertainty as epistemic move was an inhibitor of the systematic inquiry into social-cognitive connection learning opportunities and deepening inquiry through knowledge-of-practice.

8.5 Chapter summary

In this chapter I have argued that the different stances towards inquiry into learners' mathematical errors influenced the deepening of inquiry into learners' mathematical errors. The engaging stance towards inquiry into learners' mathematical errors differed fundamentally from the correcting stance. In the engaging stance there was the ongoing probing of learners' reasoning behind their errors from which emerged the eliciting of learners' mathematical learning experiences in class.

The fine-grained analysis of the inquiry involved identifying the epistemic moves as enablers of critical inquiry and the interaction of the epistemic moves in relation to knowledge. This analysis shows that in both stances the epistemic move describing deepened inquiry through knowledge-in-practice but with a focus on different teaching practices. In both stances the epistemic move raising uncertainty created potential for deepening inquiry through knowledge-for-practice. In both stances the interaction of epistemic moves also created potential for deepening inquiry through knowledge-of-practice.

To gain a deeper understanding of why certain epistemic moves only held potential for deepening inquiry, I analysed the inhibitors of critical inquiry across both stances. I identified three broad categories of inhibitors. Limited description was an inhibitor more prevalent in the correcting stance related mainly to absence of probing learners' reasoning behind their errors. Limited reflection-on-practice and the non-recognition of epistemic moves were inhibitors of critical inquiry that emerged in both the engaging and correcting stances. The analysis brought to the fore the same inquiry practices as those in Chapter Seven that need to be developed to support PST-PLCs in developing a critical inquiry stance: pressing for specificity when describing one's teaching practice; pressing for reflection on teaching practice and pre-service teachers' own learning experiences as a well as reflection on one another's practice and recognition of when to explore further knowledge, to challenge assumptions and inferences, to elaborate on the strengths and limitations when comparing and to consider as many possible reasons for a situation.

The deeper analysis of the representations of practice within the inquiry across the stances shows that the engaging stance created a greater visibility of the practice of engaging learners' mathematical errors in that learners' mathematical problems, learners' written work, the probing of learners' reasoning behind their errors and eliciting learners' mathematical learning experiences formed an integral part of the inquiry. The analysis shows that representations of practice were used as sources of inquiry in the engaging stance and as sources of information in the correcting stances in which learners' mathematical problems and learners' written work were not incorporated as part of the inquiry. In the correcting stances, where the mathematical problems in which learners' errors occurred and learners' written work formed part of the inquiry, they were used as sources of inquiry to explain the reasons for learners' errors. The way in which representations of practice were used influenced the flows of practice across the tutoring and PST-PLC spaces.

In the engaging stance there was bidirectional flow of teaching practice designed to probe learners' reasoning behind their errors and further enable knowledge-in-practice. In the correcting stance when the 'representations of practice' were used as sources of information, there was bidirectional flow of teaching practice based on pre-service teachers' prior learning experiences. In the correcting stance when representations of practice were used as sources of inquiry there was bidirectional flow of teaching practice from pre-service teachers' prior learning experiences as well new teaching ideas generated through the inquiry which held potential for developing knowledge-of-practice.

The analysis of categories of epistemic moves across the stances show that while there were similar categories across the stances, the epistemic move within these categories differed in relation to what was prioritised in the inquiry. The interactions between different epistemic moves were involved in the social-cognitive connections across the stances. In the engaging stance, the interaction between the epistemic moves describing and reflecting on practice enabled the emergence of a social dimension of learning when the cognitive dimensions of learning, probing the reasoning behind learners' errors and pre-service teachers rethinking mathematical processes with which they struggled were entry points into learners' errors. In the correcting stance, the interaction between the epistemic moves explaining, pressing and comparing created potential for the emergence of a social dimension of learning when the cognitive dimension of learning focused on reasons for learners' errors based on learners' written work and related to the mathematics content as entry point into learners' errors.

My analysis of the social-cognitive connection opportunities for learning, shows that the engaging stance includes knowledge of how to teach. The knowledge of how to teach includes teaching practices to enhance the probing of learners' reasoning behind their errors, supporting learners in taking a critical-reflective stance on their learning experiences as well as supporting learners in understanding the purpose of mathematical activities in order for deeper connections with the mathematics to be made. In the correcting stance, the social-cognitive learning opportunities also include knowledge of how to teach but the focus is on teaching practices to enhance learners' understanding of the mathematics based solely on an analysis of the mathematical content. The engaging stance therefore generated opportunities for a deeper understanding of the learners' errors through teaching practices that focused on both the social and cognitive dimensions of learning.

In the final chapter, I provide an overview of the epistemic moves and inhibitors of critical inquiry across the stances towards inquiry into learners' mathematical learning experiences and the stances towards learners' errors to show how the teaching practice of making connections between the social and cognitive dimension of learning can be enhanced through practices of inquiry in PST-PLCs that enable a critical inquiry stance on teaching practice.

CHAPTER 9

Conclusions

9.1 Introduction

My final chapter draws together the research findings of the study, outlining the contribution it makes to knowledge and theory, and exploring its implications for teacher education and further research. As discussed in Chapter 4, which describes the research context, the study responds to ongoing challenges in education by exploring new teaching practices that contribute to a deeper understanding of the socio-political dimensions of these practices.

This chapter begins by discussing the empirical contribution of the study. It then moves to the theoretical work of the thesis, which drew on the following: Cochran-Smith and Lytle's (1999) construct of 'inquiry as stance' which distinguishes between three different conceptions of teacher learning to locate sources of teacher knowledge; Zenios's (2011) construct of epistemic move in relation to deepening inquiry through knowledge and constructs within professional learning community theory; and Wenger's (1998) concept of shared accountability in communities of practice. Given the significance of these theories for the methodological approach, reflections on methodological issues are included.

In the final section of this chapter, I discuss the implications of the thesis findings in terms of teacher education. The chapter concludes by acknowledging the limitations of the research and the implications for future research.

9.2 Empirical contribution

The study focuses on the inquiry practices within pre-service teacher professional learning communities, in which connections between learners' mathematical learning experiences and learners' mathematical errors were explored. The study makes a significant contribution to understanding pre-service teachers' inquiry practices when introduced to new teaching practices. For example, I argue that exploring learners' mathematical learning experiences provides a different entry point into engaging learners' mathematical errors, in which the learners' conceptual understanding of the mathematics is not in the foreground. This allows pre-service teachers to recognise learners' learning experiences in class and beyond school as a source of knowledge about learners' errors from a non-deficit perspective, and in so doing support learners' conceptual understanding of mathematics.

Most studies of pre-service teacher inquiry in professional learning communities have focused upon inquiry into other teachers' classroom practice, which include teachers' questioning strategies to provide an opportunity to analyse learner thinking and make inferences about learners' understanding (Santagata & Guarino, 2012; Youens et al., 2014; Cavanagh & McMaster, 2015). This study was based on pre-service teachers' inquiry into how they supported learners in a tutoring context in which they engaged only two learners, a context in which pre-service teachers were encouraged to focus on accessing learners' mathematical learning experiences as a social dimension of learning, to enhance their probing of reasoning behind errors as a cognitive dimension of learning, instead of merely re-teaching content when working with learners' errors.

The first sub-question pertained to the social dimension of learning: *How do pre-service teacher professional learning communities inquire into engaging learners' mathematical learning experiences?*

A contribution to knowledge that emerged from the empirical findings of this research was the importance of the role of stance, which refers to how we position ourselves within a certain context. The particular stance towards inquiry into a teaching practice provided insight into its influence on developing a critical inquiry stance on practice.

Two stances emerged in the inquiry into learners' mathematical learning experiences: an embracing and a compliance stance. These stances differed on the basis of whether the inquiry was ongoing and reflected the sharing of teaching practice to elicit learners' mathematical learning experiences, in the embracing stance; or whether the inquiry was abandoned after the first PST-PLC session, in the compliance stance, which reflected the PSTs' struggle to elicit learners' mathematical learning experiences.

The embracing stance was only present in the inquiry into learners' mathematical learning experiences in class and only in two communities. The non-inquiry into learners' mathematical learning experiences in class in three of the communities shows that these communities did not see value in ongoing inquiry into learners' mathematical learning experiences. I argued that the difficulty of eliciting learners' mathematical learning experiences could be related to learners' classroom culture, in which such a practice was not present. In chapter 6, I provided evidence that pre-service teachers' own classroom learning experiences did not involve them having to explain how they were taught, which might account for the lack of value placed on the practice of eliciting learners' mathematical learning experiences.

In the embracing stance, in which there was ongoing eliciting of learners' learning experiences, there was evidence that learners do develop the confidence to start talking about their learning experiences and challenge pre-service teachers' teaching approach, when it differed from that of their teachers. The ongoing eliciting of learners' mathematical learning experiences through different strategies, such as asking learners to role-play how their teachers taught, and learners writing about their learning experiences, generated conversations about the challenges of some of these strategies. Rios-Aguilar, Kiyama, Gravitt and Moll (2011) argue for a better understanding of how 'funds of knowledge' are mobilised as a resource for learning in school. Hence I argue that close attention should be paid to the ways in which learners navigate between school and home contexts with their knowledges. In the embracing stance, there was evidence of the pre-service teachers supporting learners to develop a systematic approach to their challenges, noting questions they would like to pose to their support network, and similarly, when receiving support, to note ideas that could be taken back to the classroom.

The non-inquiry into learners' mathematical learning experiences beyond school reflected the struggle of PSTs to elicit learner' learning experiences beyond school. A further explanation for the non-inquiry into learners' mathematical learning experiences beyond school is that pre-service teachers were not encouraged to think or talk about these learning experiences. Hence pre-service teachers could not draw on past experiences of eliciting mathematical learning experiences, or their own learning through narrating their mathematical learning experiences. Phelp's et al. (2018) argues that recognition of learners' learning experiences is not sufficient to support their learning unless there is support allowing learners to develop insight into how describing and reflecting on their learning experiences is of value to their learning.

In chapter 7, I noted that group interviews with PSTs revealed understandings of the social dimension of learning to include respect for the language preference of learners, and an understanding learners' social circumstances, interests and aspirations as important factors in developing social relations, but with little connection to learners' learning of mathematics and supporting learners' with their challenges. This may also explain their non-inquiry into learners' learning experiences.

This latter finding contributes to 'funds of knowledge' theory in relation to teacher education (Esteban-Guitart & Moll, 2014), as it signals the importance of supporting teachers in working through their own learning experiences within and beyond school in order to understand how their learning experiences beyond school may have contributed to their learning in school. Supporting pre-service teachers' learning in this way is a potential scaffold for accessing and working with learners' learning experiences.

My analysis of the inquiry across the stances revealed that the embracing stance generated greater potential for deepening inquiry into learners' mathematical learning experiences, through the ongoing sharing of teaching practice about challenges in eliciting learners' mathematical learning experiences in class. The ongoing sharing of teaching practice in turn generated the inquiry into pre-service teachers' own mathematical learning experiences in class, a focus on the social dimension of learning, and was instrumental in deepening inquiry into cognitive dimensions of learning.

In comparing the inquiry across the two stances, I found that *describing* as an epistemic move initiated the inquiry. In the embracing stance there was a strong focus on *describing* teaching practice and learners' thinking, compared to a focus on mainly *describing* the contextual aspects of learners' home contexts in the compliance stance. In the compliance stance, *describing* learners' home contexts provided information that learners were drawing on resources to support their learning from members of their family. In Chapter 6, I provided evidence that many of the pre-service teachers also had support from their own families to support their learning, but, as I have previously argued, reflection on the nature of this support in their learning was not evident.

Thus, the non-inquiry into learners' learning resources in the compliance stance led instead to the *making of inferences* about learners' participation in mathematics at school, and *assumptions* about the families who provided the learning support. Bowden (2014) argues that when one strives to develop a social justice orientation towards communities, there must be opportunities for students to explore the historical, sociological, cultural and political contexts of the social issues they wish to address in communities. The limited experience of many of the pre-service teachers in engaging with learners from working-class or farming communities, as well a limited exploration of the broader context of these communities, could account for the making of inferences and assumptions about the support learners received from their families. I identified how *making inferences* and *assumptions* as epistemic moves held the potential to deepen inquiry into learners' mathematical learning experiences, but because they were not followed through, inquiry was curtailed in these cases.

In the embracing stance, the inquiry generated *assumptions* of learners' learning experiences in class. The *making of assumptions* in this case, as an epistemic move, held potential for deepening inquiry into how providing learners with opportunities to elaborate on their learning experiences might enable learners to engage in critical reflection on their learning, and assist them in developing a sense of autonomy (Hargreaves, 2017).

A significant difference between the inquiry across the two stances is that while *describing* as an epistemic move deepened inquiry through the generation of further epistemic moves, it was only in the embracing stance that *describing* itself deepened inquiry through knowledge-in-practice, which included the challenges of eliciting learners' mathematical learning experiences, strategies to elicit learners' mathematical learning experiences and ways of supporting learners in reflecting on their challenges in learning.

The second sub-question pertained to the cognitive dimension of learning: *How do pre-service teacher professional learning communities inquire into learners' mathematical learning errors?*

Two stances towards the inquiry into learners' mathematical errors emerged, an engaging stance and correcting stance, that reflected the PSTs' error-handling practices in the tutoring sessions. These error-handling practices were found in previous research studies (Rybowiak, Garst, Frese, & Batinic, 1999; Bray, 2011; Brodie, 2013). In the engaging stance, the inquiry focused on how learners' reasoning behind their errors was probed; whereas in the correcting stance, the inquiry focused on how to teach the content in which the error was made and on assisting learners to arrive at the correct solution or answer.

A contribution to knowledge on inquiry into engaging learners' errors through probing the reasoning behind their errors, is the deepening of inquiry into learners' and pre-service teachers' learning experiences. The inquiry in the engaging stance provided evidence that learners often avoided explaining the reasoning behind their errors and responded that errors were a result of the way they had been taught. Rybowiak, Garst, Frese, & Batinic (1999) provides evidence of this avoidance in a classroom culture where errors are overlooked or treated punitively, which prevents learners from developing the confidence to talk about their errors. Brodie (2005) provides evidence of learners' struggle to reason mathematically, which also explains why it was difficult to probe learners' reasoning behind their errors. The non-inquiry into learners' errors across both stances was influenced by challenges in eliciting learners' prior knowledge and learning experiences.

In the engaging stance, the inquiry focused on *describing* questioning strategies to probe learners' reasoning, through focussing on learners' understanding of mathematical content. When the questioning strategies included eliciting learners' experiences of how they were taught, there was evidence that learners struggled to provide detailed systematic accounts. The challenge of eliciting learners' learning experiences on how they were taught was evident in the engaging stance and the inquiry did not focus explicitly on learners' challenges in providing detailed systematic accounts of their learning.

The non-inquiry into learners' challenges in providing detailed systematic accounts of their learning is linked to pre-service teachers' understanding of how their challenges are connected to their learning. In the engaging stance pre-service teachers struggled to provide detailed systematic accounts of how they were taught and they also did not take a critical-reflective stance on their learning experience. For example, pre-service teachers reflected on *what* they struggled with but not the reasons for *why* they struggled, and did not explore connections with their challenges and how they were taught.

A further implication of this finding is the importance of pre-service teachers' critical reflection on the nature of the mathematical content, as well as the teaching process involved in their own learning, which generates challenges in learning. Pre-service teachers were encouraged to reflect on their own learning experiences in school, but the findings suggest that they required much more time to develop critical reflection on their own classroom learning experiences, in order to engage learners critically on their learning experiences.

Further and unsurprisingly, learners did not intuitively take a critical-reflective stance on their learning experiences in school. The significance of this finding is that it shows that learners' errors are connected to how they were taught, and that errors are connected to learners' not always understanding the purpose of engaging in specific mathematical tasks. This demonstrates the importance of supporting learners in developing a critical-reflective stance on how they were taught, as part of learning to identify their challenges.

In comparing the inquiry across the stances, I found that in both the engaging stance and the correcting stance *describing* the mathematical problems and learners' solutions to the problems was a key epistemic move which deepened inquiry into learners' errors, when the inquiry was supported by artefacts of learners' solutions, mathematical problems or tasks given to the learners. Shalem et al. (2014) argue that engaging learners' errors require a procedural and conceptual understanding of the mathematical problem, as well as connections between these understandings. The non-inquiry into these understandings limited the inquiry into learners' errors in the engaging stance and shows the importance of domain-specific knowledge related to learners' errors (Prediger, 2010).

In the correcting stance in which learners' mathematical problems and solutions were not shared, the inquiry generated epistemic moves which only held potential for deepening inquiry into learners' errors. This finding supports the importance of artefacts such as learners' work in the inquiry into learners' errors. Bray (2011) argued that teachers needed a strong knowledge of school mathematics to anticipate learner errors. In the correcting stance, the absence of engaging with the mathematical problems in which errors occurred and learners' work resulted in non-inquiry into learners' existing knowledge base, misconceptions and teaching which the learners have received.

A significant finding of the inquiry is the conversations about teaching practice that were generated across the stances. In the engaging stance *describing* how learners reasoned as an epistemic move generated inquiry into the mathematics in which learners' errors occurred, reflecting on how the mathematics was taught and challenges with the mathematics. In the engaging stance pre-service teachers' *reflecting* on the mathematical content and *describing* how learners' learning experiences were elicited, generated teaching practice in which learners were encouraged to take ownership of their learning. *Reflecting* as an epistemic move did deepen inquiry into how learners can be supported to work with their existing mathematical knowledge in order to self-question and analyse their errors, as part of developing a metacognitive culture (Kramarski & Zolden, 2008).

The inquiry in the engaging stance focused on mathematical practices which generated conversations about teaching practice that was not necessarily part of pre-service teachers' own learning experience. In the correcting stance in which learners' mathematical problems and solutions formed part of the inquiry into the learners' potential mathematical challenges, conversations about teaching practices were generated that were not necessarily derived from pre-service teachers' own learning experiences.

In the correcting stance, where mathematical problems in which learners' errors occurred and learners' solutions did not form part of the inquiry, pre-service teachers' prior learning experiences informed teaching practices which focussed on correcting learners' errors.

Furthermore, there was no critical reflection on the strength or limitations of these teaching practices.

These findings show how generating conversations about teaching practice other than from pre-service teachers' own learning experiences requires inquiry into learners' errors focusing on the reasoning behind the errors, pre-service teachers' reflections on their own learning experiences, the mathematical content analysis of problems, and eliciting learners' mathematical learning experiences.

The third sub-question addressed the connection between the social and cognitive dimension of learning: *What connections were made possible between learners' mathematical learning experiences and learners' mathematical errors through the pre-service teacher professional learning communities' inquiry?*

The findings of this research confirm the difficulty which pre-service teachers and in-service teachers have in making connections between social and cognitive dimensions of learning (Parks, 2008; Garii & Rule, 2009; Bartell, 2013; Liebenberg, 2016). A significant contribution to knowledge on making connections between social and cognitive dimensions through inquiry is the finding that the stance towards the inquiry into a particular dimension influenced the emergence of the other dimension.

I found that it was only in the embracing and the engaging stances that the emergence of one dimension of learning from another was evident. In the deeper analysis into the epistemic moves, I found that in both the embracing and engaging stances *describing* and *reflecting* were key epistemic moves, and that their interaction influenced this emergence. These epistemic moves involved teaching practices which were identified as foundational to making connections between social and cognitive dimensions of learning (Aguirre et al., 2012). In the embracing stance, for example, the inquiry included the teaching practices of attention to learners' challenges in describing their learning experiences, which led to the teaching practice of eliciting learners' mathematical learning experiences and ongoing experimentation with this practice. As with McDuffie et al.'s (2014) research, pre-service teachers in my study were encouraged to focus on what McDuffie et al. (2014) referred to as a teaching lens, learning lens, task lens and power-participation lens to support pre-service teachers' noticing of their teaching practice. The teaching practice of eliciting learners' mathematical learning experiences, and ongoing inquiry into this practice, explains the noticing of connections between eliciting learners' mathematical learning experiences (i.e. a teaching lens), and shifts in learners' participation in which learners pose questions and challenge teachers when they are not in agreement (i.e. a power-participation lens) (McDuffie et al., 2014). In my study, the learning lens did not only include a focus on learners' learning but also the pre-service teachers' learning experiences, which explains what prompted the noticing of connections in their teaching practice.

In the engaging stance, the inquiry involved the teaching practice of attention to questioning strategies to probe learners' reasoning, which led to awareness of mathematical challenges for learners, supported by pre-service teachers' reflecting on their mathematical learning experiences and questions about knowledge that a task activated (i.e. a task lens). In the engaging stance there was therefore the connection between learning lens and task lens, and since questions about learners' mathematical learning experiences were elicited, there was a strong move towards connections between this lens as well.

A significant contribution to knowledge that emerged from the findings is the importance of pre-service teachers' reflecting on their own learning experiences on a cognitive and social dimension for making connections between these dimensions of learners' learning. Across all the stances, opportunities for social-cognitive connection learning opportunities emerged that hold potential for enhancing teachers' knowledge on teaching practices for connecting the social and cognitive dimensions of learning, and their knowledge of the connection between learners' errors and learners' mathematical learning experiences.

9.3 Theoretical contribution, and reflections on methodology

The study drew on Zenios's (2011) concept of an epistemic move to analyse the inquiry within PST-PLCs. From a methodological perspective, the concept of an epistemic move allowed me to do a fine-grained analysis of how critical inquiry can be enabled in the PST-PLCs. While Zenios's (2011) research worked with the epistemic move in categories, I found the process of identifying specific examples within each category of epistemic move, as well as identifying subcategories for epistemic move categories, useful to gain a deeper understanding of how the inquiry differed in the stances that emerged towards the inquiry. Another significant contribution to the study's analysis through working with the concept of an epistemic move was mapping the interaction between the epistemic moves, which showed how epistemic moves can deepen inquiry directly, or through generating other epistemic moves. This form of analysis is a shift away from a deficit perspective of PST-PLC inquiry, as epistemic moves were identified across all stances.

I used Cochran-Smith and Lytle's (1999) concepts of teacher learning as knowledge-for-practice, knowledge-in-practice and knowledge-of-practice to analyse how the epistemic moves deepened the inquiry through this knowledge. To gain a deeper understanding of how the epistemic move describing teaching practice deepened the inquiry through knowledge, I drew on Horn and Kane's (2015) and Little's (2002) idea of representation of practice by using Greeno's (2009) term 'epistemological framing' to describe the ways in which representations of practice were used. I identified and described two ways in which representations of practice was used, one as *sources of information* and the other as *sources of inquiry* which impacted on whether the 'flows of practice' (Kazemi & Hubbard, 2008) were bidirectional or not, and on the nature of the teaching practice conversations generated through inquiry in the different stances.

When representations of practice were used as sources of information, as in the compliance stance towards inquiry into learners' mathematical learning experiences, there was an absence of multidirectional learning (Kazemi & Hubbard, 2008). I found that even though representations of practice were used as sources of information in the compliance stance, epistemic moves were generated with potential for deepening of inquiry through knowledge-of-practice. A bidirectional flow of practice was needed for deepening the inquiry through knowledge-of-practice. In the embracing stance towards inquiry into learners' mathematical learning experience, and the engaging stance towards learners' mathematical errors, representations of practice were always used as *sources of inquiry,* which resulted in bidirectional flows of teaching practice, which then deepened inquiry through knowledge-in-practice. The inquiry in the embracing and engaging stance showed the importance of ongoing experimentation with new teaching practice into understanding learners' errors which generated knowledge-in-practice, as well raising uncertainty about knowledge integral to this teaching practice, and hence knowledge-for-practice, which in turn generated greater opportunities for systematic inquiry into this teaching practice across social and cognitive dimensions of learning as knowledge-of-practice.

Cochran-Smith and Lytle's (1999) knowledge-of-practice concept is based on the systematic inquiry into practice with a critical lens on practice through collaboration. Introducing the concept of an inhibitor into the analysis of the PST-PLC inquiry was important for the fine-grained analysis to gain a deeper understanding of the collaborative inquiry practices, which develop a critical lens on teaching practice.

From a methodological perspective, working with both the concept of an inhibitor of inquiry and epistemic move as an enabler of inquiry was valuable for the fine-grained analysis, as both inhibitors and epistemic moves often emerged in a single turn of talk. I was therefore able to identify which inhibitors were directly related to epistemic moves, and which were not. The three categories of inhibitor in the inquiry into learners' mathematical learning experiences and learners' mathematical errors were related to pre-service teachers' representations of their teaching practice; pre-service teachers' reflections on their teaching practice; and the epistemic moves generated through the inquiry. The analysis of the inhibitor categories provided insight into inquiry practices that enhanced the inquiry through knowledge, and into Wenger's (1998) concept of the shared accountability as developing 'specialised sensitivities': knowing what is important for the practice of a community.

The first inhibitor category, which I defined as *limited description,* involved describing different aspects of teaching practice and pre-service teachers' learning experiences, in which insufficient detail is provided for deepening the inquiry. An analysis of the sub-categories of this inhibitor within the inquiry into learners' mathematical learning experiences shows that all the sub-category inhibitors were present in the embracing and compliance stances. An analysis of the sub-categories of this inhibitor within the inquiry into learners' mathematical errors shows that within the correcting stance there was increased limited description in only one sub-category, namely limited description of a learner's solution common to both the engaging and correcting stance. Analysis of this inhibitor points to developing PSTs' inquiry practice in which greater attention is given to the specificity of teaching practice when reconstructing practice. The shared accountability to this inquiry practice requires recognition for opportunity where greater specificity can be provided to deepen the inquiry through knowledge.

In Chapter 2, I discussed the knowledge-in-practice conception of teacher learning which Cochran-Smith and Lytle (1999, p. 267) argue is related to teachers' understanding of their own actions. These include for example, their own assumptions, and their own reasoning and decisions. My study indicates that unless there is a challenge for greater specificity of representations of practice, the deepening of inquiry through challenging hidden assumptions and implications of learners' learning is inhibited. Describing was a key epistemic move in generating further epistemic moves to deepen inquiry, through knowledge-for-practice;

knowledge-in-practice and knowledge-of-practice. Hence limited describing is an inhibitor to generating further epistemic moves to deepen inquiry through this knowledge and developing a critical inquiry stance on practice.

The second inhibitor category, which I defined as *non* and/or *limited reflecting-on-practice,* involved either no reflecting on practice or pre-service teachers' learning experiences, or where the reflecting was limited to observations of practice, without elaboration for deepening the inquiry. An analysis of the sub-categories of this inhibitor within the inquiry into learners' mathematical learning experiences shows that in the compliance stance, there was no reflecting-on-practice, mainly due to representations of practice used as sources of information. An analysis of the sub-categories of this inhibitor within the inquiry into learners' mathematical errors shows that in the engaging and correcting stances, there was limited reflecting on learners' understanding, and in the engaging stance there was limited reflection on the pre-service teachers' learning experiences. Analysis of this inhibitor points to developing PSTs' inquiry practice, so that in reflecting on practice and their learning experiences, greater attention is given to posing questions which provide reasons or explanations for teaching practice, or observations within teaching practice, or recommending alternative teaching practice. Further, when describing learners' mathematical learning experiences and reflecting on how these experiences occurred, it was equally important to raise questions about the implications of learners' mathematical learning experiences, to deepen knowledge about these experiences. Limited reflecting on practice inhibited inquiry into connections between learners' learning experiences learners' and mathematical errors. To deepen inquiry through knowledge-of-practice into these connections required thinking about how to systematically investigate these connections.

The third category of inhibitor, which I defined as the *non-recognition of epistemic moves,* refers to epistemic moves that did not deepen inquiry, as pre-service teachers were either not aware of these moves when they emerged and/or how they deepen inquiry. An analysis of the sub-categories of this inhibitor within the inquiry into learners' mathematical learning experiences and learners' mathematical errors shows that all the epistemic moves that were generated, other than describing, reflecting and pressing, were not recognised for how they deepened inquiry. Analysis of this inhibitor points to developing PSTs' inquiry practice, in which there is awareness of epistemic moves and how they deepen inquiry through knowledge. In the embracing, engaging and correcting stances, non-recognition of the epistemic move raising uncertainty inhibited deepening of inquiry through knowledge-for-practice. The non-recognition of other epistemic moves inhibited deepening of inquiry through knowledge-for-practice and knowledge-of-practice.

The analysis of the inhibitors to critical inquiry provide evidence of inquiry practices that PLCs can develop, which are necessary for growth of knowledge in communities.

9.4 Implications of the research for teacher education

My research contributes to a growing body of literature which explores the potential of pre-service teachers' participation in professional learning communities as spaces to deepen their knowledge about teaching and learning. Previous research on in-service teacher education has suggested that professional learning communities have both the potential to support shifts in teachers' practice, and to enhance their content and pedagogical knowledge (Chauraya & Brodie, 2017; Molefe, 2016). Chauraya and Brodie's (2017) research shows a link between the kinds of teaching and learning activities which teachers focus on in the professional learning communities, and their shifts in practice. Teachers' shifts in practice, which take place through their ongoing inquiry within professional learning communities, have been found to require a deep reflexivity with how their past experiences influence their current thinking and practice (Feldman & Fataar, 2017).

In my study the epistemic move as a conceptual tool was critical in the analysis of the PST-PLCs' inquiry, as it enabled the identification of the potential of these communities to deepen inquiry through knowledge. My research indicated that the epistemic moves were not always activated to deepen inquiry, and were often linked to specific inhibitors of critical inquiry. For example, the lack of specificity and awareness of the implications of general descriptions of their tutoring practice in deepening inquiry suggests that pre-service teachers required further opportunities to reflect on their PST-PLC inquiry. For this reason, I included a component within the B.Ed. mathematics fourth-year course, in which pairs of pre-service teachers revisited their PST-PLC inquiry through watching videos of their inquiry. This practice helped them to identify critical incidents within the inquiry where greater specificity in their descriptions of their tutoring practice was important for deepening the inquiry. The value of video in collaborative professional learning has been found to be supportive of reflective practice (Borko, Jacobs, Eiteljorg, & Pitman, 2008; van Es & Sherin, 2010; van Es, Tunny, Goldsmith, & Seago, 2014; Youens et al., 2014). The practice of videotaping to support learning has focused mainly on video of classroom practice, whereas in the B.Ed. fourth-year mathematics course pre-service teachers used the video recordings of their PST-PLC inquiry to engage critically with their inquiry. I envisaged that the pre-service teacher pairs might need additional support in identifying critical incidents, even after whole-class discussions, and set up what I called 'critical conversations' with them on their inquiry, particularly within the PST-PLCs which were facilitated by the pre-service teachers.

My study confirms the importance of the PST-PLCs' inquiry into pre-service teachers' past learning experiences, and supports Feldman and Fataar's (2017) argument that pre-service teachers' learning requires a move beyond 'cognitive learning' to understand and challenge pre-service teachers' assumptions about learners, learning and teaching practice. The findings of my study suggest that the PST-PLCs' inquiry brought to the fore assumptions about learners' mathematical learning experiences and about teaching practice. Pre-service teachers' assumptions based on their own mathematical learning experiences, if recognised, had potential to generate the reflexive depth required to deepen inquiry through knowledge. However, when these assumptions were not recognised, it inhibited the inquiry. Hence, as part of the reflection on the PST-PLC inquiry in the B.Ed. fourth-year mathematics course, the pairs of pre-service teachers were encouraged to recognise incidents in the inquiry where assumptions were made about learners, learning and teaching practice, and identify how these assumptions could become epistemic moves to deepen inquiry.

The PST-PLC inquiry in the B.Ed. third-year mathematics service learning course generated a learning space for deepening knowledge about professional learning within professional learning communities in the fourth year. A key finding of both previous PST-PLC research and this study is the difficulty pre-service teachers have in integrating knowledge-for-practice (i.e. theory) into their inquiry of practice (Sim, 2006; Youens et al., 2014). The research of Youens et al. (2014) provides evidence that video material of classroom practice could be used to assist pre-service teachers to notice and deepen awareness of the links between theory and practice. Kwenda, Adendorff, & Mosito, 2017, p. 152) argue that when the pre-service teachers in their study spoke about theory becoming second nature to them, they were demonstrating how they developed pedagogical concepts and principles that enabled them to create personalised beliefs about teaching and learning, thus noticing and deepening awareness of the links between theory and practice. They further argue that pre-service teachers' personal beliefs are partially related to critical incidents in which they encounter challenges that require them to reflect in practice, using insights gained from what they have learnt informally and formally.

My study indicates that the PST-PLC inquiry generated many opportunities in which they raised uncertainty about aspects of the tutoring practice or understanding of mathematical concepts. I viewed the raising of uncertainty as an epistemic move to deepen inquiry through knowledge-for-practice (i.e. theory) exploration. I argued that the *non-recognition* of raising uncertainty as an epistemic move is an inhibitor of deepening the inquiry through knowledge-for-practice. In the B.Ed. fourth-year course, pre-service teachers were therefore encouraged to identify critical incidents where they raised uncertainty related to their teaching practice in PST-PLC inquiry, and to identify the knowledge-for-practice to be explored in order to enhance their practice. I argued that when there is the raising of uncertainty about an aspect of practice within a collaborative learning space, it should signal shared accountability to move beyond one's local and personal experiences so as to explore knowledge that may be of value to enhancing practice.

My study involved a new entry point into learners' mathematical errors as a focus of the tutoring practice and the PST-PLC inquiry, through exploring learners' mathematical learning experiences as a social dimension of learning. The PST-PLCs' challenge to inquire into learners' mathematical learning experiences was related to eliciting these mathematical learning experiences from the learners. The pre-service teachers' own learning experiences did not involve teachers eliciting their mathematical learning experiences, or reflecting on these experiences as part of their learning, which may account for their challenge. To be prepared to engage with learners' mathematical learning experience, pre-service teachers need to spend greater time engaging with learners on their learning experiences in different contexts. The B.Ed. mathematics second-year course is where pre-service teachers need to explore learners' mathematical learning experiences within different contexts, as well exploring ways of supporting learners to elicit their mathematical learning experiences. This kind of preparation holds potential for a deeper understanding of how learners' mathematical learning experiences are of value as a different entry point into learners' mathematical errors from a social dimension.

9.5 Limitations of the study and implications for future research

My study involved the introduction of PST-PLCs within a service learning mathematics course, in which pre-service teachers were afforded the opportunity to explore new ways of engaging with learners' mathematical errors during the course of the academic year. Traditional teacher practice sessions do not allow for this, given the pressure on in-service teachers to adhere to the CAPS pacing guidelines. The limitations of my study are linked to the PST-PLC's design within both the mathematics service learning course and the research. In this section I present these limitations followed by implications for future research.

The first limitation is related to the institutional teaching practice model, which does not always enable alignment with courses in the B.Ed. programme. Ideally the pre-service teachers in the B.Ed. mathematics service learning course needed to be assigned to the teaching practice schools at which they did their after-school tutoring. Pre-service teachers would have been afforded greater opportunity to gain insight into learners' mathematical learning experiences in their classrooms, and opportunity to explore learners' mathematical experiences beyond the classroom, through ongoing conversation with learners and input from learners' teachers. If the pre-service teachers were based at the same schools during the teaching practice session, it would have been possible for them to meet as a PST-PLC to develop inquiry into learners' mathematical errors based on insights gained in learners' classrooms. The initial design of my study included a component in which pre-service teachers visited the classrooms of learners. However, due to time-tabling constraints, this was not possible, resulting in a limitation beyond my control.

The second limitation is related to my responsibility to ensure that all the PST-PLCs received the same support from me, and that I observed all the pre-service teachers tutoring. As a result of limited time and the number of pre-service teachers, my support of the PST-PLCs was limited to two sessions with each PST-PLC, and one visit per pre-service teacher to observe a tutoring session. The support of a co-researcher with facilitation skills as part of the PST-PLCs' inquiry, who could have supported the same PST-PLCs throughout, would have provided an opportunity for comparison of PST-PLCs with ongoing support for those with limited facilitation support. Furthermore, ongoing observation of the same pre-service teachers' tutoring practice would have generated greater insight into the selections of their engagement with learners that were made from the tutoring practice for the PST-PLCs and to probe aspects not selected or elaborated on within the PST-PLC.

Further limitations of the research relate to the issue of generalisability. The multiple-case study was limited to PST-PLCs at one institution, within a specific programme, which may not make the findings generalisable to all PST-PLCs. Despite this limitation, my study does contribute to knowledge about a PST-PLC's inquiry into a specific focus, namely the engaging of learners' mathematical errors from social and cognitive dimensions. The capturing of PST-PLCs' inquiry over the academic year; the individual interviews with pre-service teacher about their mathematical learning experiences as school learners; the group interviews at the end of the academic year; and observation of all pre-service teachers tutoring allowed for an in-depth analysis of the PST-PLCs' inquiry into the specific focus. My study makes a contribution to knowledge about learning opportunities within PST-PLCs, as well as the challenges and implications for developing teacher education courses that focus on developing pre-service teacher participation in professional learning communities.

Another limitation of the research is related to data about how individual teachers experienced the PST-PLC inquiry. The group interviews at the end of the academic year allowed for individual perspectives, but the limited time of one hour for the group interviews did not allow for a deeper probing of individual perspectives. The written reflections of teachers were gathered at the end of the year, and these reflections included descriptions of what learning took place through the PST-PLC inquiry and held potential for follow-up probing to deepen insights into the PST-PLCs' inquiry in relation to the different inquiry stances towards learners' mathematical errors from social and cognitive dimensions of learning. The day after the last group interview was held, the student uprising around 'fees must fall' prevented any further contact with the pre-service teachers in order to organise individual follow-up interviews based on their experiences of the PST-PLCs' inquiry into the specific focus.

My study presents several opportunities for further research into PST-PLC inquiry. For instance, it would be a potentially rich area of research to develop a longitudinal case study, in which pre-service teachers who participated in PST-PLCs in the B.Ed. third year are followed into the B.Ed. fourth year, as they revisit their PST-PLC inquiry and identify opportunities for enhancing their inquiry. The implementation of professional learning communities as part of teacher professional development generates opportunities to do further longitudinal studies of pre-service-teachers who have been prepared for participation within pre-service teacher education, and to follow their participation within in-service teacher professional learning communities, in order to study shifts in their participation and their practice in their classrooms.

In order to provide more comprehensive data on the learning which takes place through PST-PLC inquiry, we need research which involves a more detailed tracking of pre-service teachers' tutoring practice in which they engage with learners' mathematical errors from a social and cognitive perspective, and how they select from this tutoring context into PST-PLC inquiry and back to the tutoring context. This research holds potential to deepen understanding on the challenges of working with learners' mathematical errors from a social and cognitive dimension, and their understanding of the connections between these dimensions.

9.6 Conclusion

Through the research presented, I have explored and analysed PST-PLCs' inquiry into teaching practices that are new to pre-service teachers' prior mathematical learning experiences. This research study is based on empirical data of PST-PLCs' inquiry into learners' mathematical learning experiences as a social dimension of learning and inquiry into learners' mathematical errors as a cognitive dimension of learning, and the potential connections to be made between these dimensions of learning. I endeavoured to make sense of this data by drawing upon Zenios's (2011) concept of an epistemic move as an enabler of inquiry; introducing the concept of an inhibitor of inquiry; Cochran-Smith and Lytle's (1999) three teacher learning concepts associated with developing a critical inquiry stance on practice; and various concepts associated with learning that is collaborative, particularly in PLCs and communities of practice (Horn & Kane, 2015; Little, 2002; Wenger, 1998). In so doing, this work has built upon the work of others in professional learning communities, working with learners' errors and the making of connections between social and cognitive dimensions of learning (Brodie, 2013; Brodie, 2014; Fataar, 2012; Aguirre, Turner, Bartell, Kalinec-Craig, Foote, McDuffie, & Drake, 2012). I have sought to build on insights provided through these educational researchers.

The purpose of this work has been to build knowledge of PST-PLCs' inquiry practices that enhance their critical stance on their practice. My hope is that this knowledge might be used to support pre-service teachers and in-service teachers in developing inquiry practices in professional learning communities, in which the generating of new teaching practice will be systematically explored, with the goal of addressing deficit perspectives of learners and often ourselves as teachers, who can be contributors to knowledge of our practice.

References

Andrade, A.D. (2009). Interpretive research aiming at theory building: adopting and adapting the case study design. *The Qualitative Report, 14*(1), 42-60.

Anderson, J. R., Reder, L. M., & Simon, H. A. (1996). Situated learning and education. *Educational Researcher, 25*(4), 5-11.

Aguirre, J.M., Turner, E.E., Bartell, T.G., Kalinec-Craig, C., Foote, M.Q., McDuffie, A.R., & Drake, C. (2012). Making connections in practice: how prospective elementary teachers connect to children’s mathematical thinking and community funds of knowledge in mathematics instruction. *Journal of Teacher Education, 64*(2), 178-192.

Andrews, J., Wan Ching, Y., Greenhough, P., Hughes, M., & Winter, J. (2005). Teachers’ funds of knowledge and the teaching and learning of mathematics in multi-ethnic primary schools: two teachers’ views of linking home and school. *ZDM. International Journal on Mathematics Education, 37*(2), 72-80.

Auhl, G. & Graham, R. D. (2014). Preparing pre-service teachers for the professions: creating spaces for transformative practice. *Journal of Education for Teaching, 40*(4), 377-390.

Babbie, E., & Mouton, J. (2011). *The practice of social research.* Southern Africa: Oxford.

Ball, D. L., Hill, H.C., & Bass, H. (2005). Knowing mathematics for teaching: who knows mathematics well enough to teach third grade and how can we decide? *American Educator,* 14-46.

Ball, D. L., Thames, M.H., & Phelps, G. (2008). Content knowledge for teaching. What makes it special? *Journal of Teacher Education, 9*(5), 389-407.

Barab, S.A., Cherkes-Julkowski, M., Swenson, R., Garrett, S., Shaw, R. E. & Young, M. (1999). Principles of Self-Organisation: Learning as participation in autocatakinetic systems. *The Journal of the Learning Sciences, 8*(3 & 4), 349-390.

Baroody, A.J., Feil, Y., & Johnson, A.R. (2007). An alternative reconceptualization of procedural and conceptual knowledge. *Journal for Research in Mathematics Education, 38*(2), 115-131.

Bartell, T.G. (2013). Learning to teach mathematics for social justice: negotiating social justice and mathematical goals. *Journal of Research in Mathematics Education, 44*(1), 129-163.

Boaler, J. (2008). Promoting ‘relational equity’ and high mathematics achievement through an innovative mixed-ability approach. *British Educational Research Journal, 34*(2), 167-194.

Booth, J.L., Barbieri, C., Eyer, F., & Paré-Blagoev, E.J. (2014). Persistent and pernicious errors in algebraic problem-solving. *Journal of Problem-Solving, 7*, 10-23.

Borko, H., Jacobs, J., Eiteljorg, E. & Pitman, M. (2008). Video as a tool for fostering productive discussion in mathematics professional development. *Teaching and Teacher Education, 24*, 417-436.

Borasi, R. (1987). Exploring mathematics through the analysis of errors. *For the Learning of Mathematics, 7*(3), 2-8.

Borasi, R. (1994). Capitalising on errors as "springboards for inquiry": a teaching experiment. *Journal for Research in Mathematics Education, 25*(2), 166-208.

Bottorff, J. L. (1994). Using videotaped recordings in qualitative research. In J. M Morse (Ed.), *Critical issues in qualitative research methods,* (pp. 244-261). Thousand Oaks, CA: Sage.

Bourdieu, P. (1990). *In other words: essays towards a reflexive sociology*. Stanford, CA: Stanford University Press.

Bowen, G.A. (2014). Promoting social change through service learning in the curriculum. *The Journal of Effective Teaching, 14*(1), 51-62.

Bray, W. S. (2011). A collective case study of the influence of teachers' beliefs and knowledge on error-handling practices during class discussion of mathematics. *Journal for Research in Mathematics Education, 42*(1), 2-38.

Breunig, M. (2005). Turning experiential education and critical pedagogy theory into praxis. *Journal of Experiential Education, 28*(2), 106-122.

Brodie, K. (2005). Using cognitive and situative perspectives to understand teacher interactions with learner errors. In H. L. Chick, & J. L.Vincent (Eds.), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, 2* (pp.177-184). Melbourne: PME.

Brodie, K., & Shalem, Y. (2011). Accountability conversations: mathematics teachers' learning through challenge and solidarity. *Journal for Teacher Mathematics Education, 14*(6), 419-439.

Brodie, K. (2013). The power of professional learning communities. *Education as Change, 17*(1), 5-18.

Brodie, K. (2014). Learning about learner errors in professional learning communities. *Educational Studies in Mathematics, 85*(2), 221-239.

Brodie, K., & Borko, H. (2016). *Professional learning communities in South African schools and teacher education programmes.* Human Sciences Research Council Press: Cape Town.

Brodie, K. (2016). Facilitating professional learning communities in mathematics. In K. Brodie & H. Borko (Eds.), *Professional Learning Communities in South African Schools and teacher education programmes*, (pp. 155-176). Human Sciences Research Council Press: Cape Town.

Brodie, K., Marchant, J., Molefe, N., & Chiamande, T. (2018). Developing diagnostic competence through professional learning communities. In T. Leuders., K. Phillip., & J. Leuders. (Eds.), *Diagnostic competence of mathematics teachers. Unpacking a complex construct in teacher education and teacher practice* (pp.151-172). Cham: Switzerland: Springer.

Brookfield, S.D. (1995). *Becoming a critically reflective teacher.* Sab Francisco: Jossey- Bass.

Brousseau, G. (1984). The crucial role of the didactical contract in the analysis and construction of situations in teaching and learning mathematics. In H. G. Steiner (Ed.), *Theory of mathematics education* (pp.110–119). Occasional paper 54. Bielefeld: IDM.

Brown, C.P. (2010). Children of reform: the impact of high-stakes education reform on preservice teachers. *Journal of Teacher Education, 61*(5), 477-491.

Brown, J.S., & Duguid, P. (1991). Organisational learning and communities-of-practice: toward a unified view of working, learning and innovation. *Organisation Science, 2*(1), 40-57.

Butin, D. (Ed.). (2005). *Service learning in higher education: critical issues and directions.* New York: Palgrave Macmillan.

Cain, M., & Finch, J. (1981). Towards a rehabilitation of data. In P. Abrams, R. Deem, J. Finch, & P. Rock (Eds.), *Practice and progress: British sociology 1950-1980* (pp.105-119). London: George Allen and Unwin.

Cavanagh, M. & McMaster, H. (2015). A professional experience learning community for secondary mathematics: developing pre-service teachers' reflective practice. *Mathematics Education Research Journal, 27*(4), 471-490.

Chauraya, M & Brodie, K. (2017). Learning in professional learning communities: shifts in mathematics teachers' practices. *African Journal of Research in Mathematics, Science and Technology Education, 21*(3), 223 -233.

Civil, N. & Planas, M. (2009). Working with mathematics teachers and immigrant students: an empowerment perspective. *Journal of Mathematics Teacher Education, 12*, 391-401.

Clements, M. A. (1980). Analysing children's errors on written mathematical tasks'. *Educational Studies in Mathematics*, *11*(1), 1-21.

Clements, M.A. (1982). Careless errors made by sixth-grade children on written mathematical tasks. *Journal for Research in Mathematics Education*, *13*(2), 136-144.

Cobb, P., & Bauersfeld, H. (1995). *The emergence of mathematical meaning: interaction in classroom cultures.* New York: Lawrence Erlbaum Associates.

Cochran-Smith, M., & Lytle, S. (1999). Relationships of knowledge and practice: teacher learning in communities. *Review of Research in Education*, *24*, 249-305.

Cole, B., Mooney, M., & Nanlohy, P. (2013). Student voice(s). In G. Munns, W. Sawyer & B. Cole (Eds.), *Exemplary teachers of students in poverty* (pp. 174-186). Routledge: London and New York

Conway, M. A & Jobson, L. (2012). On the nature of autobiographical memory. In D. Berntsen & D.C. Rubin (Eds.), *Understanding autobiographical memory: theories and approaches* (pp. 54-69). New York: Cambridge University Press.

Creswell, J.W. (1998). *Qualitative inquiry and research design: choosing among five traditions*. Thousand Oaks, CA: Sage.

Creswell, J.W., & Miller, D.L. (2000). Determining validity in qualitative inquiry. *Theory into Practice*, *39*(3), 124-130.

Daniel, G. R., Auhl, G., & Hastings, W. (2013). Collaborative feedback and reflection for professional growth: preparing first-year pre-service teachers for participation in the community of practice. *Asia-Pacific Journal of Teacher Education*, *41*(2),159-172.

De Bock, D., Van Dooren, W., Jannssens, D. & Verschaffel, L. (2002). Improper use of linear reasoning: an in-depth study of the nature and the irresistibility of secondary school student errors. *Educational Studies in Mathematics, 50*(3)**,** 311-334.

Delpit, L. (1988). The silences dialogue: power and pedagogy in educating other people's children. *Harvard Educational Review*, *58*, 208-298.

Denzin, N.K., & Lincoln, Y. S. (2003). *Collecting and interpreting qualitative data* (2nd ed.). Thousand Oaks, California: Sage.

Department of Education. (1997). Education white paper 3: a programme for the transformation of higher education. *Government Gazette,* 386(18207), 15 August. Pretoria: Government Printer.

Department of Higher Education and Training. (2011). Integrated strategic planning framework for teacher education and development in South Africa 2011-2025. Pretoria: Government Printer.

Earl, L., & Temperley, H. (2009). *Professional learning conversations: challenges in using evidence for improvement.* Dordrecht: Springer.

Economou, P. (1995). How teachers of mathematics confront students' errors. In G. Philippou, C. Christou, & A. Kakas (Eds.), *Proceedings of the Second Panhellenic Conference on Mathematics Education and the Informatics in Education* (pp.383-400). Nicosia: Sighroni Epoxi.

Esteban-Guitart, M., & Moll, L.C. (2014). Funds of identity: a new concept based on the funds of knowledge approach. *Culture & Psychology, 20*(1), 31-48.

Fataar, A. (2012). Pedagogical justice and student engagement in South African schooling: Working with the cultural capital of disadvantaged students. *Perspectives in Education: Rethinking citizenship and social justice in education: Special Issue, 30*(4), 52-75.

Fataar, A., & Du Plooy, L. (2012). Spatialised assemblages and suppressions: the learning 'positioning' of grade 6 students in a township school. *Journal of Education, 55,* 11-36.

Feldman, J., & Fataar, A. (2017). Embodying pedagogical habitus change: a narrative-based account of a teachers' pedagogical change within a professional learning community. *Journal of Education, 70,* 73-88.

Fenstermacher, G.D., & Richardson, V. (1993). The elicitation and reconstruction of practical arguments in teaching. *Journal of Curriculum Studies, 25*(2), 101-114.

Fernandez, T., Ritchie, G., & Barker, M. (2008). A socio-cultural analysis of mandated curriculum change: the implementation of a new senior phase physics curriculum in New Zealand schools. *Journal of Curriculum Studies, 40*(2), 187-213.

Flick, U. (2009). *An Introduction to qualitative research*. London: Sage.

Gagatsis, A., & Kyriakides, L. (2000). Teachers' attitudes towards their pupils' mathematical errors. *Educational Research and Evaluation, 6*(1), 24-58.

Gale, T., Mills, C., & Cross, R. (2017). Socially inclusive teaching: belief, design, action as pedagogic work. *Journal of Teacher Education, 68* (3), 345-356.

Gardee, A. & Brodie, K. (2015). A teacher's engagement with learner errors in her grade 9 mathematics classroom. *Pythagoras, 36*(2), 1-9.

Gardner, H. (1985). *The mind's new science: a history of the cognitive revolution*. New York, NY, US: Basic Books.

Garii, B., & Rule, A. (2009). Integrating social justice with mathematics and science: an analysis of student teacher lessons. *Teaching and Teacher Education, 25*(3), 490-499.

Gau, W. (2016). How to construct shared repertoires in older adults' communities of practice. *Journal of Adult Development, 23*, 129-139.

González, N., Andrade, R., Civil, M., & Moll, L. (2001). Bridging funds of distributed knowledge: creating zones of practices in mathematics. *Journal of Education for Students Placed at Risk, 6*(1& 2), 115-132.

Goodchild, S., Fugelstad, A., & Jaworski, B. (2013). Critical alignment in inquiry-based practice in developing mathematics teaching. *Educational Studies in Mathematics, 84*(3), 393-412.

Gorski, P. (2008). The myth of the 'culture of poverty". *Educational Leadership, 65*(7), 32-36.

Greeno, J.M. (1997). On claims that answer the wrong questions. *Educational Researcher, 26*(1), 5-17.

Greeno, J.M. (2009). A theory bite on contextualising, framing and positioning: a companion to Son and Goldstone. *Cognition and Instruction, 27*(3), 269-275.

Grimshaw, A.D. (1982). Sound-image data records for research on social interaction: some questions and answers. *Sociological Methods and Research, 11*, 121-144.

Grossman, P. L., Wineburg, S., & Woolworth, S. (2001). Towards a theory of teacher community. *Teachers College Record, 103*(6), 942-1012.

Guba, E.G., & Lincoln, Y.S. (1994). Competing paradigms in qualitative research. In N. K Denzin & Y. S Lincoln (Eds.), *Handbook of qualitative research* (pp.105-117). Thousand Oaks, CA, US: Sage.

Guillaume, A.M., & Kirtman, L. (2010). Mathematics stories: pre-service teachers' images and experiences as learners of mathematics. *Issues in Teacher Education, 19*(1), 121-143.

Gutiérrez, K.D., Morales, P.Z., & Martinez, D.C. (2009). Re-mediating literacy: culture, difference and learning from non-dominant communities. *Review of Research in Education, 33*, 212-245.

Hammersley, M. (1990). *Reading ethnographic research: a critical guide*. London: Longman.

Hammersley, M. (1992). *What's wrong with ethnography? Methodological explorations.* London Routledge.

Hand, V. (2012). Seeing culture and power in mathematical learning: toward a model of equitable instruction. *Educational Studies in Mathematics, 80*, 233-247.

Hargreaves, E. (2017). *Children's experiences of classrooms.* London: Sage.

Hatano, G. (1996). A conception of knowledge acquisition and its implications for mathematics education. In P. Steffe, P. Nesher, P. Cobb, G. Goldin, & B. Greer (Eds.), *Theories of mathematical learning* (pp. 197-217). Mahwah: NJ. Lawrence Erlbaum.

Hatch, J.A. (2002). Doing qualitative research in education settings. New York: SUNY.

Heinze, A., & Reiss, K. (2007). Mistake-handling activities in the mathematics classroom: effects of an in-service teacher training on students' performance in geometry. In Woo, J.H., Lew, H.C., Park, K.S., & Seo, D. Y. *Proceedings of the 31st Conference of the International Group for the Psychology of Mathematics Education, 3*, 9-16. Seoul: PME.

Hoaglund, A.E., Birkenfeld, K., & Box, J.A. (2014). Professional learning communities creating a foundation for collaboration skills in pre-service teachers. *Education, 134*(4), 521-528.

Horn, I.S., & Kane, B.D. (2015). Opportunities for professional learning in mathematics teacher workgroup conversations: relationships to instructional expertise. *Journal of the Learning Sciences, 24*(3), 373-418.

Hutchins, E. (1995). *Cognition in the wild.* Cambridge, MA: MIT Press.

Ingram. J., Pitt. A., & Baldrey. A. (2015). Handling errors as they appear in whole-class interactions. *Research in Mathematics Education, 17*(3), 183-197.

Isotani, S., Adams, D., Mayer, R.E., Durkin, K., Rittle-Johnson, B., & McLaren, B.M. (2011). Can erroneous examples help middle-school students learn decimals? Paper presented at *Sixth European Conference on Technology Enhances Learning: Towards Ubiquitous Learning.* Retrieved from https://pdfs.semanticscholar.org/b172/f15c553669c36cbecff2e8b7eb33a468bec1.pdf

Joram, E. & Gabriele, A.J. (1998). Pre-service teachers' prior beliefs: transforming obstacles into opportunities. *Teaching and Teacher Education, 14*(2), 175-191.

Katz, S., Earl, L., & Ben Jaafar, S. (2009). *Building and connecting learning communities: the power of networks for school improvement*. Thousand Oaks, CA: Corwin.

Kazemi, E., & Hubbard, A. (2008). New directions for the design and study of professional development. Attending to the coevolution of teachers' participation across contexts. *Journal of Teacher Education, 59*(5), 428-441.

Kazemi, E. (2008). School development as a means of improving mathematics teaching and learning: towards multidirectional analyses of learning across contexts. In K. Krainer & T. Wood (Eds.) *Participants in mathematics teacher education: individuals, teams, communities and networks* (pp.209-230). Sense Publishers. Rotterdam/Taipei.

Keith, N., & Frese, M. (2008). Effectiveness of error management training: a meta-analysis. *Journal of Applied Psychology*, *93*(1), 59-69.

Kelly, P. (2006). What is teacher learning? A socio-cultural perspective. *Oxford Review of Education*, *32*(4), 505-519.

Khuzwayo, H.B., & Bansilal, S. (2012). Granting learners an authentic voice in the mathematics classroom for the benefit of both teacher and the learner. *Pythagoras*, *33*(2), 1-7.

Kincheloe, J. L. (2008). *Knowledge and critical pedagogy. An introduction*. Springer

Kramarski, B., & Zolden, S. (2008). Using errors as springboards for enhancing mathematical reasoning with three metacognitive approaches. *The Journal of Educational Research*, *102*(2), 131-151.

Kuzel, A.J. (1999). Sampling in qualitative inquiry. In B.F. Crabtree & W.L. Miller. (Eds.), *Doing qualitative research* (pp. 33-45). Thousand Oaks, CA: Sage.

Kvale, S. (2006). Dominance through interviews and dialogues. *Qualitative Inquiry, 12*(3), 480-500.

Kwenda, C., Adendorff, S., & Mosito, C. (2017). Student teachers' understanding of the role of theory in their practice. *Journal of Education*, *69*, 139-159.

Lange, T. (2009). *Difficulties, meaning and marginalisation in mathematics learning as seen through children's eyes.* (Unpublished PhD thesis), Aalborg University, Denmark.

Lazarus, J., Erasmus, M., Hendricks, D., Nduna, J., & Slamat, J. (2008). Embedding community engagement in South African higher education. *Education*, *Citizenship and Social Justice Journal*, *3*(1), 59-85. Sage Publication.

Lave, J. (1996). Teaching, as learning, in practice. *Mind, Culture and Activity, 3*(3), 149-164.

Le Cornu., R. & Ewing, R. (2008). Reconceptualising professional experiences in pre-service teacher education….reconstructing the past to embrace the future. *Teaching and Teacher Education, 24,* 1799-1812.

Leibowitz, B. (2012). Reflections on higher education and the public good. In B. Leibowitz (Ed.), *Higher education for the public good: views from the south* (pp. xvii- xxvii). Sun Press and Trentham Books, Stellenbosch, Stoke on Trent, UK and Sterling, USA.

Lewis, O. (1961). *The children of Sanchez: autobiography of a Mexican family*. New York: Random House.

Liebenberg, R. (2016). Teachers' critical inquiry into cognitive and social dimensions of learners' mathematical errors in a professional learning community. In K. Brodie, & H. Borko (Eds.), *Professional learning communities in South African schools and teacher education programmes* (pp. 57-78). Cape Town: HSRC Press.

Little, J. W. (2002). Locating learning in teachers' communities of practice: opening up problems of analysis in records of everyday work. *Teaching and Teacher Education, 18,* 917-946.

Long, C. (2005). Maths concepts in teaching: procedural and conceptual knowledge. *Pythagoras, 62*, 59-65.

Lortie, D. (1975). *Schoolteacher: a sociological study*. London: University of Chicago Press.

Lomos, C., Hofman, R. H., & Bosker, R.J. (2011). The relationship between departments as professional learning communities and student achievement in secondary schools. *Teaching and Teacher Education, 27*, 722-731.

Louis, K.S., & Marks, M. (1998). Does professional community affect the classroom? Teachers work and student experiences in restructuring schools. *American Journal of Education, 106*(4), 532-575.

Louie, N.L. (2016). Tension in equity- and reform-orientated learning in teachers' collaborative conversations. *Teaching and Teacher Education, 53*, 10-19.

MacGregor, M., & Stacey, K. (1993). Cognitive models underlying students' formulation of simple linear equations. *Journal for Research in Mathematics Education, 24*(3), 217-232.

Manouchehri, A. (2002). Developing teacher knowledge through peer discourse. *Teaching and Teacher Education,* 18, 715-737.

Mansfield, C.F., & Volet, S.E. (2010). Developing beliefs about classroom motivation: journeys of pre-service teachers. *Teaching and Teacher Education, 26* (7), 1404-1415.

Mayring, P. (2014). *Qualitative content analysis: theoretical foundations, basic procedures and software solution.* Klagenfurt, Germany. URN: http://nbn-resolving.de/urn:nbn:de:0168-ssoar-395173.

Mason, J. (2002). *Researching your own practice: the discipline of noticing*. London: Routledge Falmer.

Mays, N., & Pope, C. (1995). Rigour and qualitative research. *BMJ, 311*, 109-112.

McDuffie, A.R., Foote, M.Q., Bolson, C., Turner, E.E., Aguirre, J.M., Bartell, T.G., Drake, C., & Land, T. (2014). Using video analysis to support prospective K-8 teachers' noticing of students' multiple mathematical knowledge bases. *Journal of Mathematics Teacher Education*, *17*, 245-270.

McLaughlin, M.W., & Talbert, J.E. (2001). *Professional communities and the work of high school teaching*. Chicago: University of Chicago Press.

McMillan, J. (2013). 'Service learning' or 'learning service'? In R. Osman, & N. Petersen (Eds.), *Service learning in South Africa* (pp. 33-58). Southern Africa: Oxford University Press.

Mercer, J. (2007). The challenge of insider research in educational institutions: wielding a double-edged sword and resolving delicate dilemmas. *Oxford Review of Education, 33*(1), 1-17.

Merriam, S. (1998). *Qualitative research and case study applications in education*. San Francisco (USA): Jossey-Bass.

Mhlolo, M.K., & Schäfer, M. (2012). 'Towards empowering learners in a democratic mathematics classroom: to what extent are teachers' listening orientations conducive to and respectful of learners' thinking?' *Pythagoras*, *33*(2), 1-9.

Miles, M.B., Huberman, A.M., & Saldaῆa, J. (2014). *Qualitative data analysis: a methods sourcebook* (3rd ed.). Thousand Oaks, California: Sage.

Moje, E. B., Chiechanowski, K. M., Kramer, K., Ellis, L., Carrillo, R., & Collazo, T. (2004). Working toward third space in content literacy: an examination of everyday funds of knowledge and discourse. *Reading Research Quarterly, 39*(1), 38-70.

Molefe, N. (2016). Shifts in practice of mathematics teachers participating in a professional learning community. In K. Brodie & H. Borko (Eds.), *Professional learning communities in South African schools and teacher education programmes* (pp.177-195). Cape Town: HSRC Press.

Moll, L.C., Amanti, C., Neff, D & Gonzales, N. (1992). Funds of knowledge for teaching: using a qualitative approach to connect homes and classrooms. *Theory into Practice*, *31*(2), 132 -141.

Mogari, D., Coetzee, H., & Martiz, R. (2009). Investigating the status of supplementary tuition in the teaching and learning of mathematics. *Pythagoras*, 69, 36-45.

Munns, G. (2007). A sense of wonder: pedagogies to engage students who live in poverty. *International Journal of Inclusive Education*, *11*(3), 301-15.

Munns, G., & Sawyer, W., & Cole, B. (2013). *Exemplary teachers of students in poverty*. Routledge: London and New York

Mutch, A. (2003). Communities of practice and habitus: a critique. *Organisational Studies, 24*(3), 383-401.

Nasir, N.S., & Hand, V.M. (2006). Exploring socio-cultural perspectives on race, culture and learning. *Review of Educational Research*, *76*(4), 449-475.

Nasir, N.S., Hand, V.M., & Taylor, E.V. (2008). Culture and mathematics in school: boundaries between "cultural" and "domain" in the mathematics classroom and beyond. *Review of Research in Education*, *32*, 187-240.

Nelson, T. H. (2009). Teachers' collaborative inquiry and professional growth: should we be optimistic? *Science Education*, *93*, 548- 580.

Nesher, P., & Peled, I. (1986). Shifts in reasoning. *Educational Studies in Mathematics*, *17*, 67-79.

Nesher, P. (1987). Towards an instructional theory: the role of student's misconceptions. *For the Learning of Mathematics*, *7*(3), 33-40.

Ohlsson, S. (1995). Learning to do and learning to understand: a lesson and a challenge for cognitive modelling. In P. Reimann & H. Spada, (Eds.), *Learning in humans and machines: towards an interdisciplinary learning science* (pp.37-62). Pergamon, London.

Olivier, A. (1996). 'Handling pupils' misconceptions'. *Pythagoras*, *21*, 10-19.

Orr, J.(1990b). Sharing knowledge. Celebrating Identity: war stories and community memory in a service culture. In D.S. Middleton & D. Edwards (Eds.), *Collective remembering: memory in society*, Beverley Hills, CA: Sage Publications.

Osman, R., & Petersen, N. (2013). *Service learning in South Africa.* Southern Africa: Oxford University Press.

Paavola, S., Lipponen, L., & Hakkarainen, K. (2004). Models of innovative knowledge communities and three metaphors of learning. *Review of Educational Research, 74*(4), 557-576.

Pajeras, M. F. (1992). Teachers' beliefs and educational research: cleaning up a messy construct. *Review of Educational Research, 62*(3), 307-332.

Parks, A.N. (2008). Messy learning: pre-service teachers' lesson study conversations about mathematics and students. *Teaching and Teacher Education, 24*, 1200 -1216.

Patton, M.Q. (2002). *Qualitative evaluation and research methods* (3rd ed.). Beverly Hills, CA: Sage.

Peng, A., & Luo, Z. (2009). A framework for examining mathematics teacher knowledge as used in error analysis. *For the Learning of Mathematics, 29*(3), 22-25.

Phelps, R., Nhung, H. T. T., Graham, A., & Geeves, R. (2012). But how do we learn? Talking to Vietnamese children about how they learn in and out of school. *International Journal of Educational Research, 53*, 289-302.

Pirie, S.E.B. (1996). Classroom video-recording: when, why and how does it offer a valuable source for qualitative research? *International Group for Psychology of Mathematics Education* (pp.2-17). Panama City, FL. Retrieved from https://files.eric.ed.gov/fulltext/ED401128.pdf

Powell, A.B., Franscisco, J.M., & Maher, C.A. (2003). An analytical model for studying the development of learners' mathematical ideas and reasoning using videotape data. *Journal of Mathematical Behavior, 22*, 405- 435.

Prediger, S. (2010). How to develop mathematics-for-teaching and understanding: the case of meanings of the equal sign. *Journal of Mathematics Teacher Education, 13*, 73-93.

Rach, S., Ufer, S., & Heinze, A. (2013). Learning from errors: effects of teachers' training on students' attitudes towards and their individual use of errors. *PNA, 8* (1), 21-30.

Radatz, H. (1979). 'Error analysis in mathematics education' In *Journal for Research in Mathematics Education, 10*(3), 163-172.

Reason, J. (1977). *Skill and error in everyday life, adult learning: psychological research and applications*. New York: Wiley.

Reay, D. (2005). Beyond consciousness? The psychic landscape of social class. *Sociology, 39* (5), 911-928.

Resnick, L. B., Nesher, P., Leonard, F.P., Magone, M., Omanson, S., & Peled, I. (1989). Conceptual bases of arithmetic errors: the case of decimal fractions. *Journal for Research in Mathematics Education, 20*(1), 8-27.

Rigelman, N.M., & Ruben, M. (2012). Creating foundations for collaboration in schools: utilising professional communities to support teacher candidate learning and visions of teaching, *Teacher and Teacher Education, 28*, 979-89.

Rios-Aguilar, C., Kiyama, J.M., Gravitt, M., & Moll, L.C. (2011). Funds of knowledge for the poor and forms of capital for the rich? A capital approach to examining funds of knowledge. *Theory and Research in Education, 9*(2), 163-184.

Roberts, J. (2006). Limits to community of practice. *Journal of Management Studies, 43*(3), 623-639.

Robson, C. (2002). *Real world research: A resource for social-scientists and practitioner-researchers*. Massachusetts, Blackwell Publishers.

Rosenstein, B., & Sheva, B. (2002). Video use in social science research and programme evaluation. *International Journal of Qualitative Methods, 1*, 22-43.

Rybowiak, V., Garst, H., Frese, M., & Batinic, B. (1999). Error orientation questionnaire (EOQ): reliability, validity and different language equivalence. *Journal of Organizational Behavior, 20*, 527-547.

Samuel, M. (2010). Searching for a 'pedagogy of hope': teacher education and the social sciences, *Perspectives in Education*, *28*(1), 5-15.

Santagata, R. (2004). Are you joking or are you sleeping? Cultural beliefs and practices in Italian and U.S teachers' mistake-handling strategies. *Linguistics and Education, 15*(1&2), 141-164.

Santagata, R., & Guarino, J. (2012). Preparing future teachers to collaborate. *Issues in Teacher Education, 21*(1), 59-69.

Schussler, D. L., Stooksberry, L. M., & Bercaw, L.A. (2010). Understanding teacher candidate dispositions: reflecting to build self-awareness. *Journal of Teacher Education, 61*(4), 350-363.

Seale, C. (1999). Quality in qualitative research. *Qualitative Inquiry, 5*(4), 465-478.

Shalem, Y., Sapire, I., Welch, T., Bialobrzeska, M., & Hellman, L. (2011). University of the Witwatersrand School of Education, 2011. *Professional learning communities for teacher* *development*: *the collaborative enquiry process in the data-informed practice improvement project*. Johannesburg: Saide.

Shalem, Y., Sapire, I., & Sorto, M.A. (2014). Teachers' explanations of learners' errors in standardised mathematics assessments. *Pythagoras, 35*(1), 46-55.

Shulman, L.S. (1986). Those who understand: knowledge growth in teaching. *Educational Researcher, 15*(2), 4-14.

Silverman, D. (2013). *Doing qualitative research* (4th ed.). Thousand Oaks, California: Sage.

Sim, C. (2006). 'Preparing for professional experiences—incorporating pre-service teachers as 'communities of practice'. *Teaching and Teacher Education, 22*, 77-83.

Skovsmose, O., & Valero, P. (2005). Mathematics education and social justice: facing the paradoxes of the informational society. *Utbildning & Demokrati, 14*(2), 57-71.

Smith, J. P., DiSessa, A. A., & Roschelle, J. (1993). 'Misconceptions reconceived: a constructivist analysis of knowledge in transition'. *The Journal of the Learning Sciences, 3*(2), 115-163.

Star, J.R. (2005). Reconceptualising procedural knowledge. *Journal for Research in Mathematics Education, 36*, 404-411.

Stigler, J.W., & Hiebert, J. (1999). *The teaching gap: Best ideas from the world's teachers for improving classroom practice*. New York: The Free Press.

Stoll, L., & Louis, K. S (2007). *Professional learning communities: elaborating new approaches.* Maidenhead: Open University Press.

Subero, D., Vujansinović, E., & Esteban-Guitart, M. (2017). Mobilising funds of identity in and out of school. *Cambridge Journal of Education, 47*(2), 247-263.

Swadener, B. B. (2010). "At risk" or "at promise"? From deficit constructions of the "other childhood" to possibilities for authentic alliances with children and families. *International Critical Childhood Policy Studies, 3*(1),7-29.

Swan, M. (2000). Making sense of algebra. *Mathematics Teaching, 171,* 16-19.

Taylor, E.V. (2012). Supporting children's mathematical understanding: professional development focussed on out-of-school practices. *Journal of Mathematics Teacher Education, 15*(4), 271-291.

Tirosh, D. (2000). Enhancing prospective teachers' knowledge of children's conceptions: the case of division of fractions. *Journal for Research in Mathematics Education, 31*(1), 5-25.

Tulis, M. (2013). Error management behaviour in classrooms: teachers' responses to student mistakes. *Teaching and Teacher Education, 33*, 56 -68.

Turner, E. E., & Drake, C. (2016). A review of research on children's mathematical thinking and cultural funds of knowledge. *Journal of Teacher Education, 67*(1), 32-46.

Uusimaki, L., & Nason, R. (2004). Causes underlying pre-service teachers' negative beliefs and anxieties about mathematics. *Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education PME 28*, 4 (pp.360-376). Bergen, Norway, University College.

Van Es, E. A, & Sherin, M.G. (2010). The influence of video clubs on teachers' thinking and practice. *Journal of Mathematics Teacher Education, 13*, 155-176.

Van Es, E.A., Tunney, J., Goldsmith, L.T., & Seago, N. (2014). A framework for the facilitation of teachers' analysis of video. *Journal of Teacher Education*, *65*(4), 340 -356.

Valero, P. (2002). The myth of the active learner: from cognitive to socio-political interpretations of students in mathematics classrooms. In P. Valero & O. Skovsmose (Eds.) *Proceedings of the 3rd International MES Conference* (pp 1-13). Copenhagen: Centre for Research in Learning Mathematics.

Vera, A.H. & Simon, H.A. (1993). Situated action: a symbolic interpretation. *Cognitive Science*, *17*, 7-48.

Vescio, V., Ross, D., & Adams, A. (2008). A review of research on the impact of professional learning communities on teaching practice and student learning. *Teaching and Teacher Education*, *24*, 80-91.

Wager, A. (2014). Noticing children's participation: insights into teacher positionality towards equitable mathematics pedagogy. *Journal for Research in Mathematics Education, 45*(3), 312 -350.

Watkins, C. (2015). Meta-learning in classrooms. In D. Scott & E. Hargreaves (Eds.), *The SAGE handbook of learning* (pp.321-330). London: Sage.

Wenger, E. (1998). *Communities of practice. Learning, meaning and identity*. Cambridge: Cambridge University Press.

Willis, P.E. (1997). *Learning to Labour. How working-class kids get working-class jobs.* Columbia: Columbia University Press.

Waghid, Y. (2012). Hopeful teacher education in South Africa, towards a politics of humanity. In B. Leibowitz (Ed.), *Higher education for the public good: views from the South* (pp.101-112). Sun Press and Trentham Books, Stellenbosch, Stoke on Trent, UK and Sterling, USA.

Wyndham, J., & Säljö, R. (1997). Word problems and mathematical reasoning – a study of children's mastery of reference and leaning in textual realities. *Learning and Instruction, 7*(4), 361-382.

Youens, B., Smethem, L., & Sullivan, S. (2014). Promoting collaborative practice and reciprocity in initial teacher education: realising a 'dialogic space' through video capture analysis. *Journal of Education for Teaching, 40*(2), 101-113.

Yin, R.K. (2009). *Case study research: design and methods* (4th ed.). Thousand Oaks, CA: Sage**.**

Zenios, M. (2011). Epistemic activities and collaborative learning: towards an analytical model for studying knowledge construction in networked learning communities. *Journal of Computer Assisted Learning, 27*, 259-268.

Zenios, M. & Holmes, B. (2010). Knowledge creation in networked learning: combined tools and affordances. In L. Dirckinck-Holmfeld, V. Hodgsen, C. Jones, M. de Laat, D. McConnell & T. Ryberg. (Eds.) *Proceedings of the 7th International Conference on Networked Learning* (pp.471-479). Aalborg, Denmark. Retrieved from https://www.lancaster.ac.uk/fss/organisations/netlc/past/nlc2010/abstracts/PDFs/Zenios.pdf

Zipin, L. (2009). Dark funds of knowledge, deep funds of pedagogy: exploring boundaries between life-worlds and schools. *Discourse: Studies in the Cultural Politics of Education, 30*(3), 317-331.

Appendices

Appendix A: Mathematics Service Learning Course Plan

Week	Date	Course Plan	Articles
1	2 - 6 Feb	**Whole-Class Discussion(WCD) 1: Introduction to service- learning and funds of knowledge**	Article 1: Moll, L. (1992). Funds of knowledge for Teaching: Using a Qualitative Approach to connect homes and classrooms. *Theory Into Practice*
2	9 – 13 Feb	**Community Orientation** Visit schools: Meet Teachers **WCD 2: Converting funds of knowledge into capital**	Article 2: Civil, M (1998). Bridging In-School Mathematics and Out-of-School Mathematics: A personal reflection. AERA. Article 3: Fielding, M & Ruddick, J (2002). The Transformative Potential of Student Voice: Confronting the Power Issues
3	16 – 20 Feb	**WCD 3: Noticing Learners' Participation**	Article 4: Jacobs, V.R (2004). Mathematical Thinking: Helping Prospective and Practising Teachers Focus. *Teaching Children Mathematics* Article 5: Wager, N. (2014). Noticing Children's Participation: Insights to teacher positionality toward equitable mathematics pedagogy. *Journal of Research in Mathematics Education*
4	23 - 27 Feb	**WCD: 4: Dialogue in Mathematics Classroom**	Article 6: Brodie, K (2007). Teaching with conversations: Beginnings and Endings. *For the Learning of Mathematics* Article 7: Brodie, K Dialogue in mathematics classrooms: beyond question and answer methods. Wits University
5	2 - 6 March	**Tutor 1**: PST first meeting with learners	
6	9 – 13 March	**WCD 5: Engaging Learners in Challenging Tasks**	Article 8: Mc Fadden & Munns (2002) Student Engagement and the Social Relations of Pedagogy. *British Journal of Sociology of Education*

		Feedback from first tutoring with learners	
7	16 – 20 Mar	**Tutor 2** Lecturer to visit tutor sessions	Article 9:Threlfall, J (2009) Strategies and flexibility in mental calculations
8	23 – 27 Mar	**PST-PLC 1** Lecturer facilitates 2 PST-PLCs	Article 10: da Ponte, J. P (2016) Teachers' professional practice conducting mathematical discussion. *Educational Studies in Mathematics*
9	30 Mar – 2 Apr	**WCD 6: Learning communities rooted in inquiry** Feedback from PLC and tutoring with learners	Article 11: O' Brien (2015) How enquiry enables teachers to facilitate growth mind-sets in mathematics classrooms. MERGA. Article12: Jaworski, B (2010) Teaching Better Mathematics: what, how and why?
		VACATION 3 April - 12 April	
	13 – 17 Apr	Teaching Practicum	
	20 – 24 Apr	Teaching Practicum	
	28 – 30 Apr	Teaching Practicum	
10		**Tutor 3** Lecturer to visit tutor sessions	Article 13: Roche, A (2013). Strategies for encouraging learners to persist on challenging tasks. Some insights from work in classrooms. *APMC*
11	3 – 6 May	**PST-PLC 2** Lecturer facilitates 2 PST-PLCs	
12	9 -13 May	**Tutor 4** Lecturer to visit tutor sessions	Article 14: Brodie, Shahan & Boaler. Teaching Mathematics and social justice: multidimensionality and responsibility. *Stanford University*
13	16 - 20 May	**PST-PLC 3** Lecturer facilitates 2 PST-PLCs	
14	23 – 27 May	**Tutor 5** Lecturer to visit tutor sessions	Article 15: Phillip, R. (2000) Unpacking a conceptual lesson: The case of dividing fractions.
15	30 May – 3 Jun	**PST-PLC 4**	Article 16: Caliandro, C. K (2000) Children's Inventions for Multi-

		Lecturer facilitates 2 PST-PLCs	digit multiplication and division. *Teaching Children Mathematics*
16	6 - 10 Jun	**Tutor 6** Lecturer to visit tutor sessions	Article 17:Wong, M. (2013) Locating fractions on a number line
17	13 - 17 Jun	**PST -PLC 5** Lecturer facilitates 2 PST-PLCs **WCD 7 : Feedback on tutor sessions and PLCs**	
		VACATION 27 June – 19 July	
	20 – 24 Jul	Teaching Practicum	
	27 – 31 Jul	Teaching Practicum	
	3 – 7 Aug	Teaching Practicum	
	10 – 14 Aug	Teaching Practicum	
18	17 – 21 Aug	**Tutor 7** Lecturer to visit tutor sessions	
19	22 - 26 Aug	**PST- PLC 6** Lecturer facilitates 2 PST-PLCs	Article 18: Day, L. (2015) An explanation for the use of arrays to promote the understanding of mental strategies for multiplication.
20	29 Aug - 2 Sep	**Tutor 8** Lecturer to visit tutor sessions **WCD 8: Feedback on tutor sessions and PLCs**	
		CPUT Vacation 5 – 13 September	
21	14 – 18 Sep	**Tutor 9** Lecturer to visit tutor sessions	Article19:Hunt, J (2015).How to better understand the diverse mathematical thinking of learners
22	21 – 25 Sep	**PST- PLC 7** Lecturer facilitates 2 PST-PLCs	
23	28 Sep – 2 Oct	**Tutor 10** Lecturer to visit tutor sessions	Article 20: Warren, E. (2002) Investigating the functional thinking in elementary classrooms; Foundations of early algebra reasoning. *Journal of mathematical Behavior*

24	5 – 9 Oct	**PST- PLC 8** Lecturer facilitates 2 PST-PLCs	
25	12 – 16 Oct	**WCD 9: Feedback on tutor sessions and PLCs**	

Appendix B1: Reflection Schedule for Engaging Learners Mathematical Learning Experiences

Reflection theme	Questions to think about	What you have learnt? Give examples from the tutor session (evidence)	How will you use what you have learnt in your teaching?
Learners' learning experiences and participation in class	What can you deduce about the learners' learning experiences and participation in class?		
	What do you think influences how the learners participate in class?		
	Do you think that the learners' participation influences what and how they learn? Why		
Learners' funds of knowledge (home and community learning experiences as a resource for learning in classrooms)	What kinds of home and community learning experiences did the learners talk about? Do you think that one could draw on learners' learning experiences beyond school when teaching mathematics? How?		
	Do your learners think that their experiences outside of school are valuable?		
Learners' Mathematical Identities	Do the learners have a positive or negative attitude towards mathematics? What influenced this? Why do you think so? What do you think these learners' aspirations are?		

Appendix B2: Reflection Schedule for Engaging Learners' Mathematical Errors

Reflection theme	Questions to think about	What you have learnt? Give examples from the tutor session (evidence)	How will you use what you have learnt in your teaching of the mathematics content
Learners' explanations for their errors	What did you notice about the way in which learners explain their thinking about their errors? Do they struggle? Why? What are learners' attitudes towards their errors? Do they depend on their teachers to give them answers?		
Probing learners' errors	What kinds of questions did you use to probe the learners' thinking?		
	Which questions were good questions that enabled you to gain greater insight into the learners' conceptual understanding		
	Were you able to extend your learners' thinking? How and why did this happen?		
Underlying reasons for the learners' errors?	Are the errors related to content?		
	Are the errors related to factors less related to content? What could these factors be? Why do you say so?		

Appendix C: Individual Interview Guide with Pre-Service Teachers

Interview Theme	Questions
1. PST mathematical learning experiences at school	1.1 When did you enjoy doing mathematics at school?
	1.2 Describe your worst and best experience in a mathematics classroom at school
	1.3 What did you do when you did not understand the mathematics?
2. PST mathematical learning experiences in home	2.1 What mathematics do you talk about in your home?
	2.2 What mathematics did you learn in your home?
	2.3 How did you experience learning mathematics in your home?
3. PST mathematical learning experiences in community	3.1 What mathematics did you learn through activities you were involved in your community?
	3.2 How did you experience the learning in your community?
	3.3 Did you draw on this learning or apply this learning when engaging with mathematics at school?

Appendix D: Research Participants' Consent Forms

Principal's Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

<u>Inducting pre-service teachers into professional learning communities that integrate the cognitive and social dimensions of mathematics learning</u>

My name is: ________________________

Permission to review/collect documents/artefacts	**Circle one**
I agree that my learners' mathematics tests and class workbooks can be used for this study only.	YES/NO

Permission to observe you in class	
I agree to my learners and teachers be observed in class.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw my school from the study at any time.
- all the data collected during this study will be destroyed within 3-5 years after completion of the project.

Sign_____________________________ Date___________________________

Learner's Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

Inducting pre-service teachers into professional learning communities that integrate the cognitive and social dimensions of mathematics learning

My name is: ________________________

Permission to review/collect documents/artefacts	**Circle one**
I agree that my mathematics tests and class workbooks can be used for this study only.	YES/NO
Permission to observe you in class	
I agree to be observed in class.	YES/NO
Permission to be audiotaped	
I agree to be audiotaped during the interview or observation lesson	YES/NO
I know that the audiotapes will be used for this project only	YES/NO
Permission to be interview	
I would like to be interviewed for this study.	YES/NO
I know that I can stop the interview at any time and don't have to answer all the questions asked.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.

- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of the project.

Sign______________________________ Date____________________________

Parent's Consent Form

Please fill in and return the reply slip below indicating your willingness to allow your child to participate in the research project called : Inducting pre-service teachers into professional learning communities that integrate the cognitive and social dimensions of mathematics learning

I, _________________________ the parent of _______________________

Permission to review/collect documents/artefacts	**Circle one**
I agree that my child's mathematics tests and workbooks can be used for this study only.	YES/NO
Permission to observe my child in class	
I agree that my child may be observed in class.	YES/NO
Permission to be audiotaped	
I agree that my child may be audiotaped during interview or observations.	YES/NO
I know that the audiotapes will be used for this project only	YES/NO
Permission to be interview	
I agree that my child may be interviewed for this study.	YES/NO
I know that he/she can stop the interview at any time and doesn't have to	

answer all the questions asked. YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- he/she does not have to answer every question and can withdraw from the study at any time.
- he/she can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of the project.

Sign______________________________ Date____________________________

Teacher's Consent Form

Please fill in and return the reply slip below indicating your willingness to be a participant in my voluntary research project called: Inducting pre-service teachers into professional learning communities that integrate the cognitive and social dimensions of mathematics learning

I, _________________________ give my consent for the following:

Permission to review/collect documents/artefacts	**Circle one**
I agree that my learners' mathematics tests and workbooks can be used for this study only. YES/NO	
Permission to observe you in class	
I agree to be observed in class.	YES/NO
Permission to be audiotaped	
I agree to be audiotaped during the observation lessons	YES/NO
I know that the audiotapes will be used for this project only	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of the project.

Sign______________________________ Date____________________________

BED Student's Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

Inducting pre-service teachers into professional learning communities that integrate the cognitive and social dimensions of mathematics learning

My name is: ________________________

Circle one

Permission to be audiotaped

I agree to be audiotaped during the interview or observation lesson	YES/NO
I know that the audiotapes will be used for this project only	YES/NO

Permission to be interview

I would like to be interviewed for this study.	YES/NO
I know that I can stop the interview at any time and don't have to answer all the questions asked.	YES/NO

Permission to be videotaped

I agree to be videotaped in class.	YES/NO
I know that the videotapes will be used for this project only.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my university will not be revealed.

- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of the project.

Sign_______________________________ Date_____________________________

Appendix E: Ethics Approval Letter

Wits School of Education

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

27 November 2014

Student Number: 905531

Protocol Number: 2014ECE025D

Dear Rolene Liebenberg

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

Pre-Service Teachers' Engagement with Learners' Mathematical Errors and Mathematical Learning Experiences in Professional Learning Communities

The committee recently met and I am pleased to inform you that clearance was granted.

Please use the above Protocol Number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

Wits School of Education

011 717-3416

cc Supervisor: Prof K Brodie

Appendix F: Western Cape Education Department Approval Letter

Western Cape Government
Education

Audrey.wyngaard@westerncape.gov.za

tel: +27 021 467 9272

Fax: 0865902282

Private Bag x9114, Cape Town, 8000

wced.wcape.gov.za

REFERENCE: 20140716-33173

ENQUIRIES: Dr A T Wyngaard

Ms Rolene Liebenberg
3 Caxton Close
Meadowridge
7806

Dear Ms Rolene Liebenberg

RESEARCH PROPOSAL: Pre-service Teachers' Engagement with Learners' Mathematical Errors and Mathematical Learning Experiences in Professional Learning Communities

Your application to conduct the above-mentioned research in schools in the Western Cape has been approved subject to the following conditions:

1. Principals, educators and learners are under no obligation to assist you in your investigation.
2. Principals, educators, learners and schools should not be identifiable in any way from the results of the investigation.
3. You make all the arrangements concerning your investigation.
4. Educators' programmes are not to be interrupted.
5. The Study is to be conducted from **08 April 2015 till 28 August 2015.**
6. No research can be conducted during the fourth term as schools are preparing and finalizing syllabi for examinations (October to December).
7. Should you wish to extend the period of your survey, please contact Dr A. T Wyngaard at the contact numbers above quoting the reference number?
8. A photocopy of this letter is submitted to the principal where the intended research is to be conducted.
9. Your research will be limited to the list of schools as forwarded to the Western Cape Education Department.
10. A brief summary of the content, findings and recommendations is provided to the Director: Research Services.
11. The Department receives a copy of the completed report/dissertation/thesis addressed to:

    The Director: Research Services
    Western Cape Education Department
    Private Bag X9114
    CAPE TOWN
    8000

We wish you success in your research.

Kind regards.
Signed: Dr Audrey T Wyngaard
Directorate: Research
DATE: 16 July 2014