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MODELING AND FORECASTING STOCK RETURN VOLATILITY IN THE JSE SECURITIES EXCHANGE

MASTER OF MANAGEMENT IN FINANCE AND INVESTMENT

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MODELING AND FORECASTING STOCK RETURN VOLATILITY IN THE JSE SECURITIES EXCHANGE

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A research report submitted to the Faculty of Commerce, Law and Management, University of the Witwatersrand, in partial fulfillment of the requirements for the degree of Master of Management in Finance and Investment.

Declaration of Authorship

I***, Zamani Calvin Masinga***, declare that this research thesis and the work presented herein are my own. This thesis or any part thereof has not been submitted to any other university for a degree or any other qualification. The submission of this document is in partial fulfillment of the requirements for Master of Management in Finance and Investment at the University of the Witwatersrand. The consulted work of other authors and resources is clearly acknowledged where appropriate.

_____ 09 September 2015

Z.C Masinga Date

Abstract

Modeling and forecasting volatility is one of the crucial functions in various fields of financial engineering, especially in the quantitative risk management departments of banks and insurance companies. Forecasting volatility is a task of any analyst in the space of portfolio management, risk management and option pricing. In this study we examined different GARCH models in Johannesburg Stock Exchange (JSE) using univariate GARCH models (GARCH (1, 1), EGARCH (1, 1), GARCH-M (1, 1) GJR-GARCH (1, 1) and PGARCH (1, 1)).

Daily log-returns were used on JSE ALSH, Resource 20, Industrial 25 and Top 40 indices over a period of 12 years. Both symmetric and asymmetric models were examined. The results showed that GARCH (1, 1) model dominate other models both in-sample and out-of-sample in modeling the volatility clustering and leptokurtosis in financial data of JSE sectoral indices.

The results showed that the JSE All Share Index and all other indices studied here can be best modeled by GARCH (1, 1) and out-of-sample for JSE All Share index proved to be best for GARCH (1, 1). In forecasting out-of-sample EGARCH (1, 1) proved to outperformed other forecasting models based on different procedures for JSE All Share index and Top 40 but for Resource 20 RJR-GARCH (1, 1) is the best model and Industrial 25 data suggest PGARCH (1, 1)

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Contents

Declaration of Authorship i

Abstract ii

Acknowledgments iii

List of Acronyms iv

List of Appendices v

List of Figures vi

List of Tables vii

CHAPTER 1: Introduction 1

1.1 Introduction 1

1.2 Problem Statement. 2

1.3 Objective of the study and Research Questions 3

1.4 Significance of the study 4

1.5 Limitations of the study 4

1.6 Outline of the study 4

Chapter 2: Literature Review 6

2.2 Empirical Literature 6

2.3 Theoretical framework 17

2.3.1 Autoregressive Conditional Heteroscedasticity (ARCH) Model 17

2.3.2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model 19

2.3.3 GARCH-in-mean (GARCH-M) 21

2.3.4 Exponential GARCH (EGARCH) 23

2.3.5 Glosten, Jagannathan and Runkle- GARCH (GJR-GARCH) or Threshold GARCH (TGARCH): 23

2.3.6 PGARCH 25

2.4 Summary and Conclusion 25

Chapter 3: Data and Econometric Methods 27

3.1 Introduction 27

3.2 Data 27

3.3 Volatility Models 29

Model diagnostic techniques 31

3.3.1.1 Investigating Stationary 31

3.3.2 ARMA Models 32

3.3.2.1 Statistical Tests 33

3.3.2.2 ARCH Test 34

3.3.2.3 Normality Test 35

Chapter 4: Methodology 37

4.1 In-sample estimation 37

4.2 Forecasting 38

4.3 Forecast evaluation statistics 38

Chapter 5: Empirical Results 41

5.1 Introduction 41

5.2 Error distribution and leverage effects 41

5.2.1 GARCH parameter estimates and their economic meaning 45

5.3 Diagnostics 51

5.4 Forecast valuation: GARCH out-of-sample 52

Chapter 6: Summary and Conclusions 55

References 57

APPENDIX A: Akaike Information Criteria 63

Conditional Variance Graphs 66

List of Acronyms

ADF	Augmented Dickey Fuller
ARCH	Autoregressive Conditional Heteroscedasticity
EGARCH	Exponential Generalized Autoregressive Conditional Heteroscedasticity
GARCH	Generalized Autoregressive Conditional Heteroscedasticity
GJR-GARCH	Glosten, Jagannathan and Runkle - Generalized Autoregressive Conditional Heteroscedasticity
GARCH	Generalized Autoregressive Conditional Heteroscedasticity in Mean
PGARCH	Power Generalized Autoregressive Conditional Heteroscedasticity
JB	Jarque-Bera
JSE	Johannesburg Stock Exchange
MAE	Mean Absolute Error
MAPE	Mean Absolute Percentage Error
RMSE	Root Mean Square Error
UK	United Kingdom
RC	Reality Check
SPA	Superior Predictive Ability

List of Appendices

- Appendix A: Akaike Information Criteria

List of Figures

Figure 1: Close prices of JSE Indices……30

Figure 2: Log returns of JSE Indices……..31

Figure 3: Conditional Variance of GARCH model ALSH……66

Figure 4: Conditional Variance of GARCH model Resource 20 Index……66

Figure 5: Conditional Variance of GARCH model Industrial 25……..67

Figure 6: Conditional Variance model of Top 40 Index……67

List of Tables

Table 3.1: Descriptive statistics for JSE Indices (log returns)……29

Table 3.2: ARMA Specifications for each JSE Index……..33

Table 3.3: Heteroscedasticity test ARCH……..35

Table 3.4: Breush – Godfrey serial correlation LM test……35

Table 3.5: ARCH test results……36

Table 5.1: Parameters estimates for JSE All Share Index……42

Table 5.2: Parameters estimates for Resource 20 Index……43

Table 5.3: Parameters estimates for Industrial 25 Index……43

Table 5.4: Parameters estimates for Top 40 Index……44

Table 5.5: Parameters estimates for PGARCH (1, 1)……45

Table 5.6: The model selection for the estimated models assuming t-student distribution……..50

Table 5.7: Box-Ljung Q-statistic test for squared standardized residuals, Engle's ARCH test and Jarque-Bera test for normality……51

Table 5.8: Error statistics forecasting daily volatility……...53

CHAPTER 1: Introduction

1.1 Introduction

The main characteristic of any financial asset is its returns, returns are typically considered to be a random variable. The spread outcomes of this variable known as assets volatility plays important role in numerous financial applications, economics, hedging, and calculating measures of risk. Volatility is defined as a measure of dispersion of returns for a given security or market index (Tsay, 2010). In simple terms, volatility can be defined as a relative rate at which the price of a market oscillates around its expected value. Volatility is one of the most important concepts in finance. The primary usage is the estimation of the value of market risk. Volatility is the key parameter for pricing financial derivatives. All modern option-pricing techniques rely on a volatility parameter for price evaluation, which first appeared in Black-Scholes model for option pricing (Black, 1976). Volatility is also used for risk management applications and in general portfolio management. It is crucial for financial institutions not only to know the current values of the volatility of the managed assets, but also to be able to estimate their future values.

Volatility makes investors more averse to holding stocks due to uncertainty; investors in turn demand a higher risk premium to insure against the increased uncertainty. A greater risk premium results in a higher cost of capital, which subsequently leads to less private investment (Emenike, 2010). Therefore, modeling volatility improves the usefulness of measuring the intrinsic value of securities and in the process it becomes easy for a firm to raise funds in the market. Additionally, the detection of volatility provides an insight for a better way to design an appropriate investment strategy. Traders (equity or financial derivatives known as options) and

investors need to know how the market behaves and volatility is the tool or the indicator that helps investors.

The theoretical framework for modeling volatility was traced back to the original ARCH model developed by Engle (1991), which captures the variability of time of the variance of returns by imposing an autoregressive structure on the conditional second moment of returns. In order to address the statistical requirement of a high-order autoregressive structure, a problem that is inherent in the formulation of ARCH, Bollerslev (1986) introduced the generalized ARCH (GARCH) model. The GARCH model extends Engel's model by including lagged conditional variance terms as extra regressors. Subsequently, many other ARCH-type processes have been developed to capture various dynamics which are the topics of this research.

While the imperative of understanding the risk profiles of emerging capital markets is well populated in the literature (Siourounis, 2002), but very limited work in this subject is reported for emerging markets, as acknowledged by Kasch-Haroutounian & Price (2001). The present work is motivated by the noticeable absence of work on African stock markets, of which the JSE is the most well-organized and active off them all. This study, therefore, contributes to the literature by providing evidence based of JSE data and specific sectors in the market.

1.2 Problem Statement.

With the increasing sophistication of emerging financial markets and complexities of the derivative instruments, the need for accurate volatility forecasting and estimation are becoming increasingly more important. This was reflected by the numerous studies, articles, books and papers written on the subject (Poon, S and Granger, C. 2003; Knight John and Stephen Satchell. 1998). Volatility impacts investment decisions, security valuation, risk management and even

monetary policy decisions. The deep understanding of the results produced by simple historical models and different types of GARCH models are needed for in-sample forecasting and out-of-sample forecasting in the South African market. In this study we examine and compare the forecasting accuracy based on the results produced by these models in different time horizon. Analysis of the output in these models is examined, studied and interpreted in different time horizon for different models with the aim of finding which model has a much predictive power than the other.

1.3 Objective of the study and Research Questions

The objective of this study is to forecast and compare return volatility using both in-sample and out-of-sample tests applied to daily returns of the Johannesburg Stock Exchange All Share index and analyzing the forecast performance of different volatility models. In addressing this issue, the study was focused on analyzing the three of the most popular used models proposed in the finance and economic literature, the historical volatility, different GARCH and implied volatility models. The objective is to determine which model best forecast and model JSE volatility returns on out-of-sample in short and long term horizon one day, one week and one month ahead forecast and the analysis of the models result. Specifically this study is guided by these research questions:

- Do the out-of-sample forecasts produce best accurately volatility forecasts?
- Which model best forecast and predict JSE volatility in and out-of-sample?
- How do GARCH (p, q) model and historical models perform in modeling and forecasting volatility?
- Which out-of-sample time horizon produces better estimates?

1.4 Significance of the study

This research contributes to the knowledge of forecasting and modeling volatility in JSE a lot of work have been done in the developed markets (Poon, 2005) but little in developing emerging markets. It informs all financial market participants on the JSE in South Africa, policy makers, portfolio managers, risk management, options pricing specialist and macroeconomics forecasters. The study assists the options contracts since volatility is the input when calculating price of options. Since most research into volatility forecasting has been done in the developed markets but less in emerging market this might help foreign investors who might like to invest in JSE market. It gives understanding in the broad understanding of volatility modeling and forecasting in emerging market.

1.5 Limitations of the study

This study is focused on performance of forecast volatility in the Johannesburg Securities Exchange (JSE) using All Share Index returns in the South African domestic market.

The study is conducted based on the univariate GARCH and variations of these model symmetric and asymmetric models.

1.6 Outline of the study

This study is divided into six chapters. Firstly, the paper provided an introduction background. Secondly, literature review will be conducted and summarize the findings. This entails the review of work that has been done before and its will help to align our study in the right direction. Thirdly, the paper captured the research methodology utilized in the analysis of the data and information of the study. Fourthly, the presentation of the empirical results and findings

provided. Lastly, inferences was drawn from the empirical results obtain with respect to the initial objectives of the study.

Chapter 2: Literature Review

2.1 Introduction

The good way of modeling the stock market volatility is imperative for various purposes. Portfolio managers, option traders, risk management and financial policy makers often require an adequate statistical characterization of volatility so that they can perform their duties well. Most of literature in modeling and forecasting financial volatility makes use of Bollerslev's (1986) GARCH model which became popular after Engle's (1982) ARCH model which is often found in the literature to be sufficient to model volatility(Brooks, 2008). Engle came up with this model when he was studying the variance of UK inflation in 1982. GARCH modeling alone with normal distribution of error term aren't found accurately compared to other models that account for asymmetries of the data in the conditional variance process. GARCH-GJR and EGARCH have been included to account for these asymmetries and other characterization of volatility stylized facts.

This section contains an in-depth review of the both theoretical and empirical literature review on volatility modeling and forecasting globally and domestically, relating to stock market volatility and selected models of conditional variance, with specific emphasis on GARCH models. To gain a comprehensively full understanding of the nature of stock return volatility, it is necessary to review various theoretical developments in this field.

2.2 Empirical Literature

After the recently national financial crisis (Poon and Granger, 2003) the academics and practitioners became more interested in the analysis of financial data especial the uncertainty of

the stock market. Therefore, a lot focus in research has been on forecasting and modeling stock volatility especial in the developed countries.

Mandelbrot (1963) and Fama (1965) played a major role in detecting that the uncertainty of stock prices as measured by variances that vary with time. Fama (1965) further observed that clustering of volatility and leptokurtosis are commonly observable in the financial time series data. Furthermore, Black (1976) noted another interesting phenomenon that is also often observable in the return series that is called leverage effect, which occurs mostly when stock prices are negatively correlated with changes in volatility. Leverage effect is the tendency for volatility to rise more following a large price fall than following a price rise of the same magnitude a definition by Brooks (2008, 380).

In order to model these stylized facts and to accurately forecast volatility, the different models were estimated consisting of GARCH, EGARCH, GJR-GARCH, and PGARCH models to capture all the dynamics of the volatility in the JSE stock exchange, some studies has make use of this models but the focus has been on developed countries.

Ladokhin (2009) in his study selected several methods that are heavily used in practice and testing the accuracy of this models using real data (S&P 500 stock index) where each family of methods has its advantages and disadvantages, which are describe in details in this study. They found that some methods are simpler but yield poor results (e.g. historical average models, random walk model) and other methods provide improved results but difficult to implement (e.g. Implied Volatility method). Exponentially Weighted and Simple Moving Average are both efficient and easy to implement. These results are also consistent with other published in the literature (McNei et. al, 2005; Samouilham and Shannon, 2008). The result suggested that

Moving Average can be used for a quick approximation or reference of the volatility forecast but can't be relied upon because no empirical evidence supports that claim.

Black (1976) found that the theories that changes in stock return volatility are partly caused by the volatility spikes called 'leverage effect'. From Black (1976) theories, a declined in the market value of a firm's equity, holding other things constant, is through time increase the debt/equity ratio (leverage ratio) of the firm and hence increases its inherent riskiness. The robustness of the negative relationship between return innovations and future volatility has been proven, and has led to a number of statistical models that incorporate leverage effects, such as GJR-GARCH model of Glosten, Jagannathan and Runkle (1993).

The Autoregressive Conditional Heteroscedasticity (ARCH) model was introduced by Engle (1982). He defined the ARCH model as the conditional variance of the current period's error term, which was a linear function of the previous period's squared error terms. Firstly Engle studied the variance of UK inflation which revealed that this model was designed to deal with the assumption of non-stationarity found in realized financial data returns. These ARCH models treat heteroscedasticity in the data as a variance to be model not as homoscedasticity as the past models did. After the publication of this method ARCH become popular such that other researches became interested in this model and started to propose the extension of this model, the first and foremost being using regular is GARCH by Bollerslev (1986).

The Generalized Autoregressive Conditional Heteroscedasticity (GARCH) model was introduced by Bollerslev (1986) as stated earlier, who generalized Engle's ARCH model by adding lagged conditional variance. Thus volatility was as additive function of lagged error terms. The GARCH model had become a popular way of modeling volatility because of its parsimonious characteristic and no need of estimating a lot of coefficient in building this model.

Then the GARCH models were introduced in the literature ranges from the simple GARCH model to more complex GARCH-type models such as EGARCH, GJR-GARCH, PGARCH, GARCH-M, CGARCH, APGARCG, FIGARCH etc.

The GARCH modeling and forecasting studies have focused on identifying which GARCH specification best models and forecasts volatility, opposing the symmetric versus asymmetric GARCH models. The GARCH and GRACH-M models captures leptokurtosis and volatility clustering and EGARCH, GJR-GARCH, APGARCH, FIGARCH and CGARCH captures leptokurtosis, volatility clustering, leverage effects and volatility persistence.

The Exponential Generalized Autoregressive Conditional Heteroscedastic (EGARCH) model of Nelson (1992) was an example of a more complex GARCH improved model. The EGARCH model did not make the assumption that positive and negative news had a symmetric impact on volatility like classical GARCH did but instead assumes that it had an asymmetric impact by Black (1976). Thus, the volatility became a multiplicative function of lagged innovations which can react differently from positive or negative news. The EGARCH model also caters for the effect of time, where recent observations carried more weight than older observations.

The existing literature regarding to the study on GARCH type models can be categorized into two, and that they are the investigation on the basic symmetric GARCH models and the GARCH models with various volatility specifications.

The predictability of ARCH (q) model on volatility of equity returns has been studied extensively in the literature. Nonetheless, the empirical evidence indicating the good forecast performance of ARCH (q) model are irregular. Studies done by (Franses and Van Dijk, 1996; Brailsford and Faff, 1996; Figlewski, 1997) showed that the out-of-sample forecast performance

of ARCH (q) models and the results produces conflicting conclusion. But the common ground of these studies is that the regression of realized volatility produces a quite low statistics of R^2. Since the average R^2 is smaller than 0.1, it's suggest that ARCH (q) model has a weak predictive power on future volatility.

The forecasting performance of ARCH models has a variety of restrictions influencing them. The one restriction is frequency of data, and it is an issue widely discussed in preceding papers. Nelson (1992) examined ARCH models and documented that the ARCH model using high frequency data performs well for volatility forecasting, even when the model is severely misspecified. However, the out-of-sample forecasting ability of medium-and long-term volatility is poor.

Pagan and Schwert (1990) analyzed an alternative models for conditional stock volatility focusing on U.S data from 1834-1925 because they believed that post-1926 has been meticulously analyzed by others in the past. The aim of the study was to compare various measures of stock volatility and from the results it emerged that the nonparametric procedures tended to give a better explanation of the squared returns than any of the parametric model. Both Hamilton's and the GARCH models produced weak explanations of data. The result of the study implied that the standard parametric models are not sufficiently extensive on their own. However, improving GARCH and EGARCH models with terms suggested by non-parametric methods yields significant increase in explanatory power. This fact allowed Pagan and Schwert (1990) to merge the two models for richer set of specifications, which was emphasized by the results.

Wilhelmsson (2006) investigated the forecast performance of the basic GARCH (1,1) model by estimating S&P 500 index future returns with nine different error distributions, and found that

allowing for a leptokurtic error distribution leads to significant improvements in variance forecasts compared to using the normal distribution. Additionally, the study also found that allowing for skew-ness and time variation in the higher moments of the distribution does not further improve forecasts.

Makhwiting *et. al*. (2012) examined the forecasting performance of different symmetric and asymmetric GARCH model. They modeled volatility and the financial market risk for JSE returns. The study makes use of two steps modeling process, which is first they estimated the ARMA (0, 1) models for mean returns, secondly fitted various univariate GARCH models for conditional variance (GARCH (1, 1), GARCH-M (1, 1), TGARCH (1, 1) and EGARCH (1, 1) models). The empirical results indicated the existence of stylized effects of ARCH, GARCH and leverage effects in the JSE returns over a give sample. The forecast evaluation indicated the model performed best in predicting out-of-sample returns for a period of three months is ARMA (0, 1)-GARCH (1, 1).

Niyitegeka and Tewari (2013) investigated both symmetric and asymmetric GARCH models GARCH (1, 1), EGARCH (1, 1) and GJR-GARCH (1, 1) in the study of volatility of JSE returns. The study also found the presence of ARCH and GARCH effects in JSE financial returns. But contrary to Makhwiting *et al.* (2013) failed to identify any leverage effects in return behaviour.

Chuang, Lu and Lee (2007) studied the volatility forecasting performance of the standard GARCH models based on a group of distributional assumptions in the context of stock market indices and exchange rate returns. They found that the GARCH model combined with the logistic distribution, the scaled student's t distribution and the Risk metrics model are preferable both stock markets and foreign exchange markets. However, the complex distribution does not always outperform a simpler one.

Franses and Van Dijk (1996) examined the predictability of the standard symmetric GARCH model as well as the asymmetric Quadratic GARCH and GJR models on weekly stock market volatility forecasting, and the study results indicated that the QGARCH model has the best forecasting ability on stock returns within the sample period.

Chong, Ahmad and Abdullah (1999) compared the stationary GARCH, unconstrained GARCH, non-negative GARCH, GARCH-M, exponential GARCH and Integrated GARCH models, and the study found that EGARCH performs best in describing the often-observed skew-ness in stock market indices and out-of-sample one-step-ahead forecasting.

Evans and McMillan (2007) examined the forecasting performance of nine different competing models for daily volatility for stock market returns of 33 economies. The empirical result of this study shows that GARCH models allowing for asymmetries and long-memory dynamics provide the best forecast performance.

Liu, H. and Hung J. (2010) on their study they explores the important of distributional assumption and the asymmetric specification in improving volatility forecasting performance through the superior predictive ability (SPA). This study investigates one-step ahead forecasting performance of asymmetry-type and distribution-type GARCH methods for the S&P 100 stock index. The results showed that GJR-GARCH generate the most accurate volatility forecasts, followed closely by EGARCH when asymmetric specification are taken into account. Secondly the analysis result indicate that asymmetric component modeling is much more important than specifying the error distribution for improving volatility forecast of financial returns in the presence of fat-tails, leptokurtosis, skew-ness and leverage effect. If asymmetric properties are neglected the GARCH model with normal distribution is preferable to those models with more sophisticated error distribution.

Hansen and Lunde (2005) compared 330 ARCH type models in their ability to describe the conditional variance. The aim of the study was to find out that there are volatility models that beat GARCH (1, 1) model using superior predictive ability and reality check (RC) for data snooping. The empirical analysis illustrated the usefulness of SPA test that it is more powerful than RC. The core findings of the study is that there are no concrete evidence showing that GARCH (1, 1) model is outperformed by other models when the models are evaluated using the exchange rate.

Despite extensive work on volatility forecasting of asset returns, very few had been done specifically to South Africa in terms of forecasting the volatility of stock market returns. The study was conducted by Samouilhan and Shannon (2008), where they used a small data set of 682 observations (01/02/2004 - 28/09/2006) of daily data for the TOP40 index of the JSE. The authors investigated the comparative ability of three types of volatility forecasts namely different autoregressive conditional Heteroscedasticity (ARCH) by Engle (1982), and as generalized ARCH by Bollerslev (1986) on one hand, a Safex Interbank Volatility Index (SAVI) for the options market, and measures of volatility based purely on historical volatility using a random walk and 5-day moving average forecasts. They found that GARCH (2, 2) specification provided the best in-sample fit of all the symmetric GARCH models. For their out-of-sample results the GARCH (1, 1) specification provided the best forecast of all the symmetric models as compared to GARCH (1, 2), (2, 1) and (2, 2) models.

Emenike and Aleke (2012) examined the volatility of Nigerian Stock Exchange in return series for evidence of asymmetric effects by estimating GARCH (1, 1), EGARCH (1, 1) and GJR-GARCH (1, 1) models. The GARCH (1, 1) model shows the evidence of clustering of volatility and the persistence of volatility in Nigeria. The study shows the evidence of volatility

asymmetric effect from the estimates of the asymmetric models (EGARCH and GJR-GARCH). But contrary to the theoretical sign of leverage effect, the result of EGARCH model estimate is positive suggesting that positive news increase volatility more than negative news. Similarly, the estimated results from the GJR-GARCH model show the existence of a negative coefficient for the asymmetric volatility parameter thereby providing support to the EGARCH result of positive news producing higher volatility in immediate future than negative news of the same magnitude.

The overall results from this study provide strong evidence that positive shocks have higher effect on volatility than negative shocks of the same magnitude. It also shows volatility clustering and high volatility persistence.

According to Babikir et al. (2012) investigated the empirical relevance of structural breaks in forecasting stock return volatility using both in-sample and out-of-sample tests applied to daily returns of the JSE All share Index from 02/07/1995 to 25/08/2010. Where the evidence of structural breaks were found in the unconditional variance of the stock returns series over the period, with high levels of persistence and variability in the parameter estimates of the GARCH (1, 1) model across the sub-samples defined by the structural breaks. The results show the relevance of structural breaks in JSE, but there are no statistical gains from using competing models that explicitly accounts for structural breaks, relative to GARCH (1, 1) model with expanding window.

By using the concept of McLeod and Li (1983), Engle (1982), Brock et.al (1996), Tsay's (1986), Hinich and Patterson (1995) and Hinich (1996), Alagidede (2011) conducted a study on the behavior of returns in Africa's emerging equity markets. This research aimed to provide evidence on the predictability of returns in Africa's emerging markets based on the behavior on the first and second moments of return behavior, risk trade off and mean reversion. The study

reveals that empirical stylized facts known as volatility clustering, leptokurtosis and leverage effect are present in the Africa data. The study also reveals also that risk/reward trade off does not always follows a known standard finance postulate that says high risk produce higher returns but also higher loses that has been shown in the case for Kenya. The study also shows that as these emerging African markets are growing and has low correlation with developed markets it can be used as agents for global risk reduction and potential investment avenues for investors seeking to diversify their portfolios. But however, there is lacking evidence regarding the behavior of returns. However, the results contradict the findings by Appiah-Kusi and Menyah (2003) concluded that returns for Kenya, Egypt and Morocco are not predictable. However, the results are contrast to Magnusson and Wydick (2002), Appiah-Kusi and Menyah (2003) and Smith and Jefferis (2005), they found the evidence that is mixed for weak form efficiency for some of the markets. In other different markets, the initial evidence was largely consistent with the view that developed stock markets are efficient.

A feature that is common for GARCH-type models using daily financial data is that of a high level of persistence attributed to the shocks, so that the effect of a once off shock to volatility persists for many periods into the future. Many GARCH studies involving financial series have found that the estimated variance is generated by an approximate unit root process (Engle and Bollerslev, 1986; Susmel, 1999). Thus, this has led to the development of integrated GARCH (I-GARCH) model.

Alagidede, and Panagiotidis (2009) investigated the behavior of stock returns in Africa's largest markets. This has been done by employing the random walk and smooth transition models (STM) for the returns of each of the countries and tested for *i.i.d* through the uses of these following tests McLeod and Li (1983) and Engle (1982) test for (G) ARCH effects, BDS test for

randomness, bi-covariance test for third order non-linear dependence and the threshold effects in the data. The study reveals that the random walk hypothesis examined has been rejected by all battery of test that has been employed in the returns. Using smooth transition and conditional volatility models the empirical stylized facts of volatility clustering, leptokurtosis and leverage effects were found to be present in the African stock index returns.

Using the battery of ARCH-type models Mangani (2008) investigated the structure of volatility on the JSE. The first order GARCH (1, 1) formulation was found to be statistically preferred relative to higher order GARCH specifications for the forty-four securities studied, two of which were stock portfolios and the rest were individual stocks. The dummy GARCH specification was chosen before the exponential GARCH model to investigate the presence of asymmetric effects of shocks on volatility in the sampled series. To test whether volatility was priced on the market the GARCM-in-mean process was tested which yielded negative results, the findings suggested that volatility did not meet the criterion of a priced factor because only two individual stocks showed that volatility was positively priced which is insignificant. Therefore the study finds that there was no compelling evidence for the presence of leverage or even asymmetric effects of shocks on volatility. Secondly, there was no evidence that the volatility was priced on the market.

Floros (2008) conducted a study where he examined volatility in the Egyptian stock market using daily data for Egypt's CMA general index. Employing family of GARCH models, he found strong evidence of volatility clustering and noted the existence of leverage effect in the returns and that negative news increase volatility. A study by Samouilhan (2007) found that the evidence of large degree of persistence of volatility on equity returns on the JSE for the broad ALSI40 index and its various sub-sectors. Using a Component ARCH (CARCH) model, he found

significant evidence of volatility clustering over both the long and the short run for each series and for the broad index.

Olowo (2009) examined the volatility of Naira/Dollar exchange rate in Nigeria using GARCH (1, 1), GJR-GARCH (1, 1), EGARCH (1, 1), APARCH (1, 1), IGARCH (1, 1) and TS-GARCH (1, 1) models on a monthly data from January 1970 to December 2007. The study produce TS-GARCH and APARCH as the best fitting models.

The conclusion from this body of research is that modeling and forecasting volatility is a notoriously difficult task. Poon and Granger (2003) provided some useful insights into comparing different studies on this topic in their review about forecasting volatility in financial markets.

2.3 Theoretical framework

This section provides with some basic model description and a theoretical background on the financial econometrics models that have been proposed to model and forecast volatility especially those which was used in this study. Following the work of Samouilhan and Shannon (2008), Emenike and Aleke (2012), Magnus and Fosa (2006) this study focused on the following volatility models: (1) GARCH (1, 1); (2) EGARCH (1, 1); (3) GJR-GARCH (1, 1) and GARCH-M (1, 1). These models in literature are categorized as historical based volatility models. Below are the explanations of the theory behind these models as follows:-.

2.3.1 Autoregressive Conditional Heteroscedasticity (ARCH) Model

A time series and econometrics model relies on the premise of Ordinary Least Squares which assumes that the variance of the disturbance error term is constant (homoscedasticity). However, many economic and financial time series display period of unusually high volatility followed by

periods of relative quietness. In such scenarios the assumption of a constant variance is no longer appropriate.

The fundamental and very crucial model for financial time series with time varying volatility is the Autoregressive Conditional Heteroscedastic model of order one, ARCH(1). This model was developed by Robert Engle in 1982. It accommodates the dynamics of conditional Heteroscedasticity the assumption of varying variance. It has the advantage of simplicity in formation and easy estimation (Gourieroux and Jasiak, 2001).

An ARCH (1) conditional variance model is shown below

$$\sigma^2_t = \omega_0 + \alpha_1 \mu^2_{t-1}$$

A general ARCH model can be described as follows:

$$y_t = \mu_t + \varepsilon_t$$

$$\varepsilon_t = e_t \sigma_t$$

$$e_t \sim N(0,1)$$

This model consists of time varying dependent variable y_t which can be described by a conditional mean equation μ_t and residuals ε_t. The specification of the mean equation can take any form.

This model ARCH (q) specification seems to be comparable to the traditional moving average estimates of volatility. But Engle (1982) made the major advancement that the unconditional variance and weights attached to the innovations can be determined via maximum likelihood (ML) estimation, using information contained in the past data (Engle, 1982). Furthermore, lag lengths can be chosen using LRTs, residuals diagnostics, and relevant information criteria. Information criteria were the one method that was used to find lag length in this study.

The number one disadvantage of the ARCH model is that of the restriction of the model for the conditional variance to follow a pure AR (Autoregressive) process and henceforth it may require more adequately represent the conditional variance process in comparison with other more generalized models.

2.3.2 Generalized Autoregressive Conditional Heteroscedasticity (GARCH) Model

After the ARCH model became popular, Bollerslev (1986) introduced the generalization of an ARCH model called it GARCH, which generalizes the ARCH model to an autoregressive moving average model. The conditional variance of the GARCH model depends on the squared residuals and its past values. The generalization allows the model to avoid over fitting. It is the most used model today (Brooks, 2008). The GARCH model can be specified as follows:

Mean equation: $r_t = \mu + \varepsilon_t$ (2.3.2.1)

$$\sigma_t^2 = \omega + \sum_{i=1}^{q} \alpha_i \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_j \sigma_{t-j}^2 \quad (2.3.2.2)$$

Where the current conditional variance is parameterized to depend upon q lags of the squared error and p lags of conditional variance. The addition of the lagged conditional variance is important because the coefficient β_j allows for a smooth process, which evolves over a long time period. GARCH model also lets volatility depend on lagged conditional variances and squared errors that are farther in the past without the need for a large number of coefficients. By comparison, ARCH models, which include a limited number of lags in the conditional variance, are classified as more short memory models (Elyasiani and Mansur, 1998).

But in general a GARCH (1, 1) model was sufficient to capture the volatility clustering in the data, and any higher order model estimated in the academic finance literature is not even

entertained because there is a consensus that GARCH (1, 1) is sufficient to model all the clustering in volatility (Engle, 2004). Intuitively, the GARCH (1, 1) forecast of conditional variance at time t is a weighted average of three components; a constant term through which the unconditional variance is determined, the previous periods estimates of the conditional variance, and the new information obtained during the period t-1 (Engle, 2004:407).

ARCH effects can be supposed as the appearance of clustering in trading volume on the micro level. GARCH effects are due to volatility clustering according to Bollerslev et. al.(1992) which resulted from macro level variables such as dividend yield, margin requirement, money supply, business cycle and information patterns. Two reasonable explanation for volatility clustering as described by Engle *et al*. (1990): the arrival of news process and market dynamics in response to news.

The EGARCH, GJR-GARCH, PGARCH, GARCH-M models are compared in order to assess the impact of allowing for changes in differing types of volatility persistence. The evaluation of the performance of the models is based on likelihood ration tests (LRTs) and Ljung-Box Q-tests for autocorrelation in the squared standardized residuals for in-sample modeling. The forecast of the out-of-sample test performance of the models is measured through the use of the following loss functions measures, root mean squared forecast errors (RMSE), mean absolute forecast errors (MAE), mean absolute percentage error (MAPE) and Theil inequality coefficient (Theil's U).

The consequences of Heteroskedasticity are in general problematic, and as it is known that the consequences of Heteroskedasticity for OLS estimation are very serious. Even though the estimates remain unbiased but they are no longer efficient, thus they are no longer best linear unbiased estimators (BLUE) among the class of all linear unbiased estimators. For this reason

GARCH, EGARCH, GJK-GARCH models are being used in this study to account for autocorrelation, Heteroskedasticity, persistence and volatility clustering.

This study uses the GARCH framework for modeling and forecasting the stylized facts of volatility. The GARCH framework is an improved version of ARCH model that Engle (1982) developed, in which volatility is described through a specification for random behavior of returns where GARCH model include the lag variance of the previous estimate in the current variance. This process addresses the issue of Heteroscedasticity and volatility clustering frequently found in financial markets by specifying the conditional variance as a function of the past squared errors, allowing volatility to evolve over time.

Many econometric models operate under the constant error term variance assumption, it has been widely recognized that financial time series exhibit significant heteroscedasticity (Engle and Ng, 1993). Other market participants have dealt with this scenario through the use of simple moving average estimates of conditional variance (Engle and Ng, 1993).

2.3.3 GARCH-in-mean (GARCH-M)

The returns of a security may be influenced by its volatility. The symmetric GARCH-M, by Bollerslev *et. al*. (1988) is still a symmetric model of volatility, however the different from the classical GARCH model is that it's introduce the conditional term to the mean equation to account for the fact that the return of a stock security may depend on its volatility. The GARCH-M (1, 1) is written as:

Mean equation: $r_t = \mu + \lambda\sigma_t^2 + \varepsilon_t$ (2.3.3.1)

Variance equation: $\sigma_t^2 = \omega + \alpha_1\varepsilon_{t-1}^2 + \beta_1\sigma_{t-1}^2$ (2.3.3.2)

(r_t) represents the return of a security, and μ and λ (risk premium parameter) are constant parameters to be estimated. This model relaxes the assumption of constant average risk premium over the sample period. The GARCH-M specification relaxes this assumption by allowing volatility feedback effect to become operational (Brooks, 2002). λ, represent the conditional volatility term introduced in the GARCH mean equation to account for the fact that return for the security may sometimes depends on its volatility. If these parameter is positive and statistically significant, implies that the return is positively associated to its volatility and the increase in risk is given by an increase in conditional variance leads to an increase in mean return and vice versa. The implication of a statistically positive relationship would mean that an investor is compensated for assuming greater returns on the JSE equity market.

Since we are using high frequency data in this study, the models selected is restricted to incorporate only constant transition probabilities as opposed to time varying. This restriction is necessary for convergence of the maximum likelihood procedure.

ASYMMETRIC GARCH MODELS

The major disadvantage of the models explained earlier is the property of not being able to capture symmetries of the data. Like GARCH and GARCH-M can only capture two stylized fact of financial data Leptokurtosis (fat tails) and volatility clustering. The asymmetric models which are explained here captured the asymmetric of the data known as leverage effects. The leverage effects have been observed in the financial data in the previous studies (Black, 1976). These models include EGARCH, GJR-GARCH.

2.3.4 Exponential GARCH (EGARCH)

The EGARCH was proposed by Nelson (1991). The model has several advantages over the symmetric GARCH specification. Firstly no need to artificially impose non-negativity constraints. Therefore, conditional variance is always positive since it is expressed as a function of logarithm (Omwukwe *et al.* 2011). Secondly asymmetry is allowed in the formulation of EGARCH therefore negative and positive news are not treated as the same like the symmetric GARCH model assumes. The conditional variance as it was proposed by Nelson (1991) is specified as follows:

$$\ln(\sigma^2_t) = \omega + \beta\ln(\sigma^2_{t-1}) + \gamma \frac{u_{t-1}}{\sqrt{\sigma^2_{t-1}}} + \alpha \left[\frac{|u_{t-1}|}{\sqrt{\sigma^2_{t-1}}} - \sqrt{\frac{2}{\pi}} \right] \qquad (2.3.4.1)$$

The coefficient γ signifies the asymmetric effects of the shocks o volatility. These asymmetric effects can be tested by the hypothesis that γ=0. If the γ coefficient is zero, this would imply that positive and negative shocks of the same magnitude have the same effect on volatility of stock returns. If $\gamma \neq 0$ the effect is asymmetric. If the γ coefficient is positive, then positive shocks tend to produce higher volatility in the immediate future than negative shocks. The opposite would be true if γ were negative.

2.3.5 Glosten, Jagannathan and Runkle- GARCH (GJR-GARCH) or Threshold GARCH (TGARCH):

The GJR-GARCH this group of models is similar to the GARCH whereby the future variance depends on previous lagged variance values, but the different is that it includes a term that takes into account asymmetry. This model was named after these three scientists Glosten, Jagannathan and Runkle (1993). The model is the same as GARCH model with an additional term added to account for asymmetries. The conditional variance is given by:

$$\sigma^2_t = \alpha_{0+} \alpha_1 u^2_{t-1} + \beta\sigma^2_{t-1} + \gamma u^2_{t-1} I_{t-1} \quad (2.3.5.1)$$

Where $I_{t-1} = 1$ if $u_{t-1} < 0$

$= 0$ otherwise

This model can help us in detecting the leverage effect, if we see $\gamma > 0$ this tell us about the leverage effect in the series. There are two types of news: there is squared return and there is a variable that is the squared return when returns are negative and zero otherwise. The coefficients are now calculated in the long run average α_0, the previous forecast α_1, symmetric news β, and negative news γ. I is an indicator function. In this variance formulation, the positive and negative effects on the news on the conditional variance are completely different. The effect of the news is asymmetric if $\gamma \neq 0$. If the γ coefficient is positive, then negative shocks tend to produce higher volatility in the immediate future than positive shocks. The opposite might be true if γ were negative. β measures clustering in the conditional variance and $\alpha_1 + \beta + \gamma/2$ measures persistence of shocks on volatility. If the sum of this measure is less than one the shock is not expected to last longer but if it is close to one then the volatility can be predicted for some time. But if the sum of the coefficients is one then shock is going to affect volatility indefinitely.

The TGARCH model developed by Rabemananjara (1993) applied the same approach that used in GJR-GARCH model. The TGARCH model introduces a threshold effect in the form of a dummy variable into the volatility to account for leverage effects. TGARCH (1, 1):

$$\sigma_t^2 = \omega + \alpha_1 \varepsilon_{t-1}^2 + \gamma d_{t-1} \varepsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \quad (2.3.5.2)$$

Where dummy variable $d_{t-1} = \begin{cases} 1 \text{ if } \varepsilon_{t-1}^2 < 0, \text{bad news} \\ 0 \text{ if } \varepsilon_{t-1}^2 \geq 0, \text{good news} \end{cases}$

γ is the asymmetry component. If the leverage effect is present this means that the coefficient of asymmetry is positive and significant. The reasoning behind this is the same as that of EGARCH model, where negative news might have a greater impact on volatility than good news of the same magnitude.

2.3.6 PGARCH

The P-GARCH model was introduced by Bollerslev and Ghysels (1996) as a means of better characterizing periodic or season patterns in financial markets volatility. This model is similar to the GARCH model but now includes seasonally varying autoregressive coefficients.

The class of P-GARCH (p, q) processes can be defined as

$$\sigma_t^2 = \omega_{s(t)} + \sum_{i=1}^{q} \alpha_{is(t)} \varepsilon_{t-1}^2 + \sum_{j=1}^{p} \beta_{js(t)} \sigma_{t-j}^2 \qquad (2.3.6.1)$$

Where s(t) is the stage of the period cycle at time t. When estimating this model, the conditional variance, σ_t^2, must be positive in order for a plausible fit to be obtained. For a positive variance results the conditions may be needed in the following parameters $\omega_{s(t)}, \alpha_{is(t)}$, and $\beta_{js(t)}$ (Bollerslev and Ghysels, 1996). These conditions may be formulated on a case-by-case basis according to Nelson and Cao (1992) suggested that the condition of restricting the two seasonal coefficients to be non-negative, with seasonal intercept strictly positive.

2.4 Summary and Conclusion

This section of the chapter presented a review of various empirical and theoretical developments relating to forecasting and modeling volatility, with the emphasis on extended GARCH models. Since the influential studies of Engle (1982) and Bollerslev (1986), it is common to model the conditional variance of financial time series as following a single-regime GARCH process.

Throughout the literature reviewed, it is commonly found that GARCH (1, 1) model provide a better in-sample fit of data or more accurate forecast than any other models (Engle, 2004). But other models haven't been given much attention as these three popular models GARCH, EGARCH and GJR-GARCH are investigated in this study other models and its properties. The question that this study tries to answer is this. Is there any other models beside GARCH (1, 1), that can model and forecast volatility better, if there is how accurate is it?

In this regard, chapters 3, 4 and 5 are devoted to investigating whether a variety of GARCH models are better suited to modeling the African stock market data used. In chapter 3, the acquiring and explanation of data. relevant models are estimated, in sample goodness-of –fit test, and out-of-sample forecast are presented. Using likelihood test, liquidity ratio test (LRT) for testing diagnostics of the residuals. Chapter 4 presents the methodology used. Chapter 5 presents the empirical analysis results of the investigation. Lastly, conclusion in chapter 6 and future work.

Chapter 3: Data and Econometric Methods

3.1 Introduction

Before we proceed with the empirical analysis, it is crucial to present first and motivate the various data, models and econometric test used in this chapter. From the literature review in Chapter 2, this chapter aims to provide an understanding of the data and econometric methods used. As a groundwork analysis, sub-section 3.2 includes a descriptive summary of the data used in this study for the different sectorial indices returns, and take note of significant similarities and differences between models.

3.2 Data

To accurately compare the properties it is appropriate to ensure that the return series is compiled according to a standardized method. For this reason, the data used are daily returns on the JSE All Share Index (FTSE/JSE) which is about 90% of the market capitalization of the whole JSE, Industrial 25 Index, Resource 10 Index and Top 40 Index. All the data series are procured from JSE information data source Bloomberg data stream.

The Bloomberg data set comprises of daily readings for all FTSE/JSE indices. The sample period under investigation runs from 01/10/2002 to 30/12/2014 of daily close, which translates into 3067 actual trading days in total. The choice of daily returns is due to the finding that important information regarding volatility is lost at lower frequencies, especially during crisis periods (Edwards, 1998). Brooks (2002, p 389-390) stresses the point that for the models that is used in this study are more data intensive than simple regression, therefore they work better when the data are sampled daily rather than at lower frequency. Ensuring stationarity, the daily closing levels are transformed into daily continuous returns, defined in the standard way as the natural

logarithm of ratio of consecutive daily closing levels. The unit root test was used to test the null hypothesis of a root is stationary. The null hypothesis was rejected after transforming the daily prices to continuously returns.

Daily returns R_t are calculated as the first difference of the natural log of the index P_t, multiplied by 100, such that:

$$R_t = 100 * \ln(^{P_t}/_{P_{t-1}})$$

Where R_t denotes the continuously compounded percentage return at time *t*, P_t the price of an asset at time *t*.

The descriptive statistics for the log returns are shown in Table3.1. The mean returns across the board are close to zero. All the indices have slightly higher similar kurtosis value showing some fatness in the tail in the distribution (Gujarati and Porter, 2009). A normal distribution has a kurtosis of three. A kurtosis in all indices is greater than three which means that the indices results are peaked around their mean value (leptokurtosis). The standard deviation, which is the measure of the average volatility of the indices in 12 years 2 months, shows that the Resource Index has higher volatility of 1.8379 than other indices. Negative skew-ness is present in all series except from Resource 20 index. In addition based on Jarque-Bera statistic and the probability it is concluded that the normality assumption in the time series examined is rejected, supporting the non-normal distribution of the index returns that are being examined in this study. The large values for Jarque-Bera test statistic ranges from 1066.82 for Industrial 25 to 2341.644 for Resource 20 suggest the underlying non-normality in the return series.

The line graphs for the closing prices and log return data for each index is shown in the Figure 1. The closing prices of all indices show an overall exponential increase in the value of index between 2008 and 2014 while Resource 20 Index has been indecisive ever since financial global crisis. Since the event of the crisis indices Top 40, Industrial 25 and JSE All Share have seen a steady recovery.

The return graphs for all chosen indices appear to fluctuate around zero (Figure 2). Volatility pooling is evident in all the data with periods of large movements grouped together and small movements tending to also group together which is the same as volatility clustering.

Table 3.1: Descriptive Statistics for JSE Indices(Log returns)

	JSE All Share	Resource 20	Industrial 25	Top 40
Mean	0.0549	0.0246	0.0764	0.053
Median	0.0837	0.0189	0.1265	0.097
Maximum	6.8339	11.4998	7.1729	7.707
Minimum	-7.5807	-11.8154	-6.794	-7.959
Std. Dev.	1.2343	1.8379	1.158	1.352
Skewness	-0.1431	0.0335	-0.908	-0.081
kurtosis	6.6759	7.2801	5.884	6.529
JB	1737.202***	2341.644***	1066.82***	1594.692***
Observations	3067	3067	3067	3067

*** Denotes statistical significance at 1% level; JB is the Jarque-Bera test for normality;

3.3 Volatility Models

There are various different types of volatility models. However, only a limited number of those models are found to be useful for modeling financial data time series. The most popular used

ones are non-linear financial models known as ARCH or GARCH models and extension of these models, which are used to model and forecast volatility in this study. We are going to explain these models below.

The significant positions that volatility estimates and forecasts take in finance for example, in portfolio formation construction, risk management, and option pricing require the best available model of volatility to be used almost all the times.

Figure 1: Close prices of JSE Indices

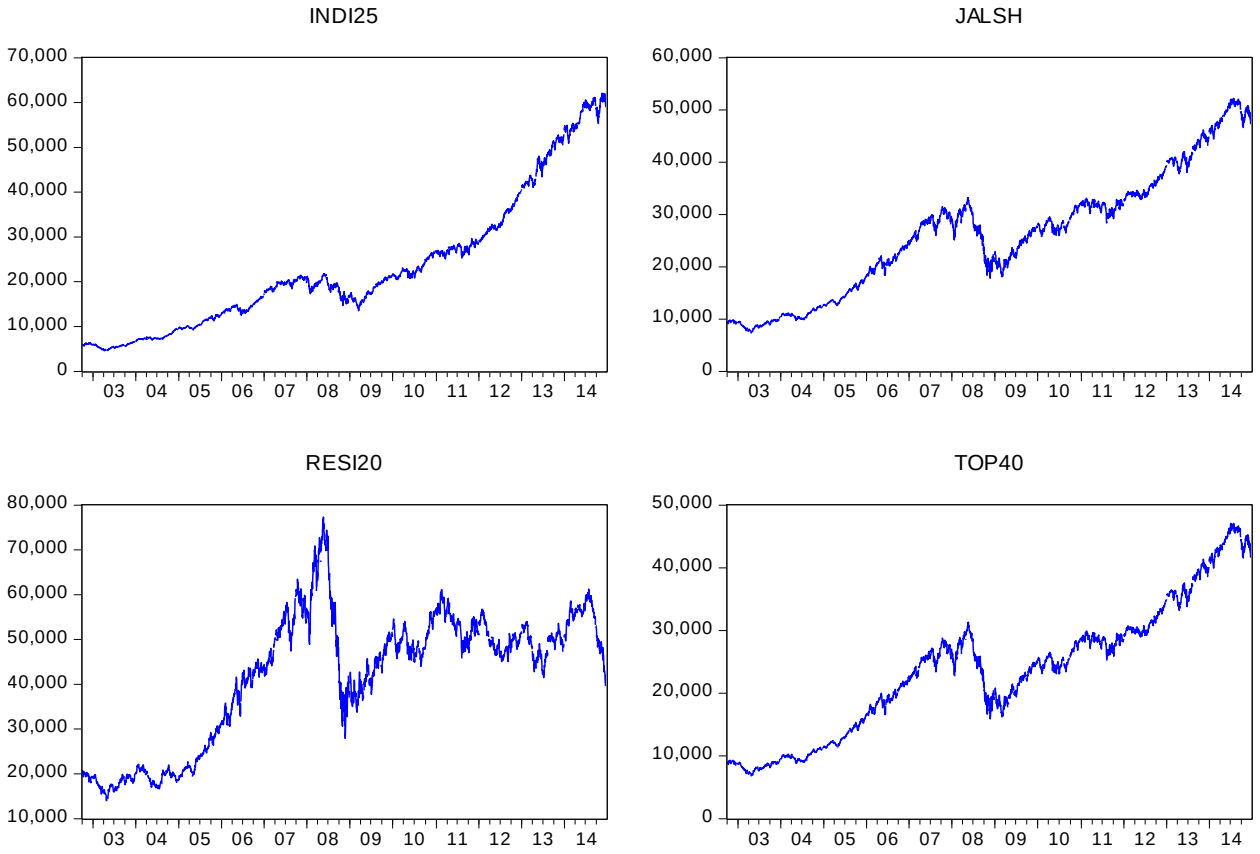
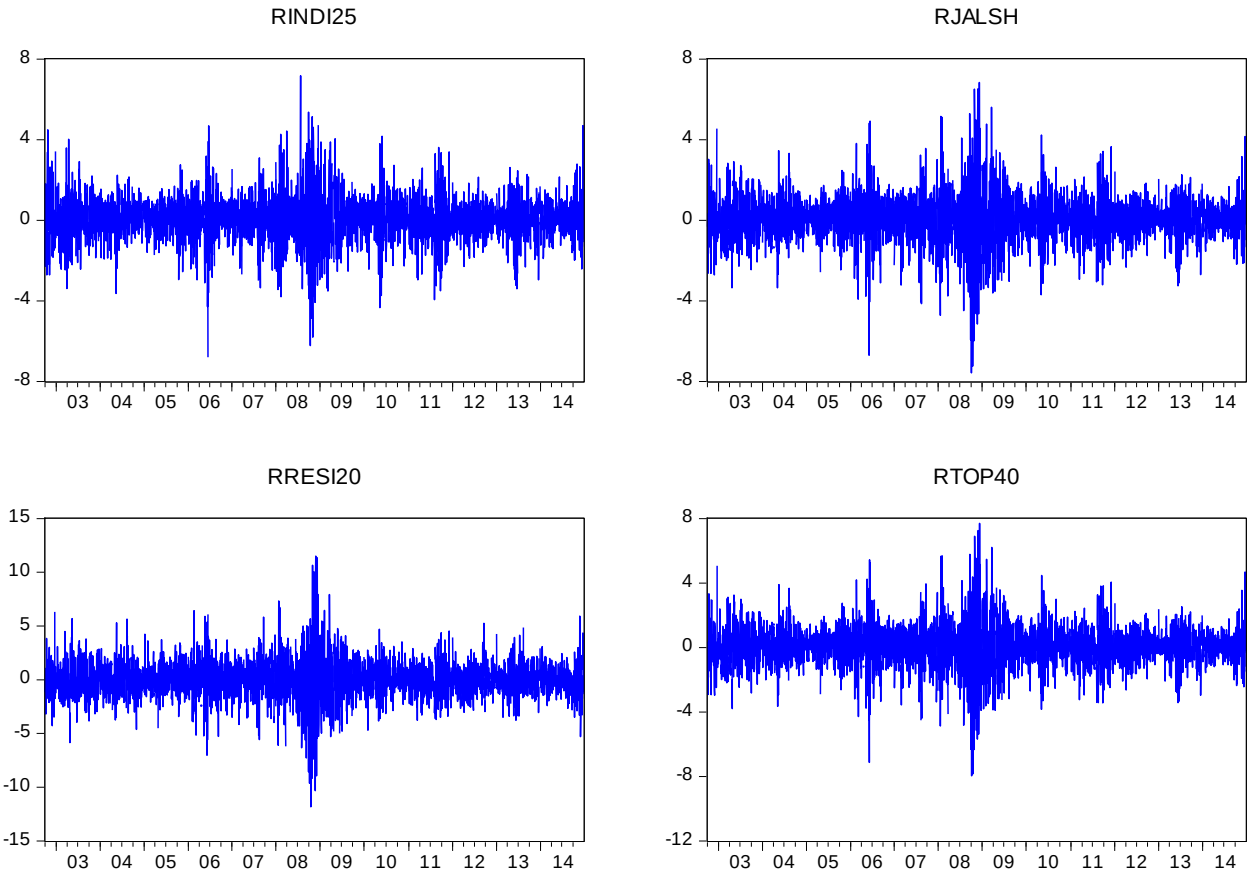


Figure2: Log returns



3.3.1 GARCH Models

Model diagnostic techniques

3.3.1.1 Investigating Stationary

Time series is said to be strictly stationary if the joint distribution (r_{t1}, ….r_{tk}) is identical to that of ($r_{t1+\tau}$,…..,$r_{tk+\tau}$) for all τ, where *k* is an arbitrary positive integer and (t_1,…..,t_k) is a collection of *k* positive integers (Tsay, 2010).

A weaker stationarity is when the time series returns is when the mean of r_t and covariance between r_t and r_{t-1} are unchanging with time, where *l* is an arbitrary integer. Mathematically, a

series r_t is weakly stationary if the expected value of return μ (constant) and covariance is γ_l, which only depends on *l*.

3.3.1.2 Autocorrelation

To examine the linear dependence between present r_t and previous r_{t-1} returns we focus on correlation, if we have weakly stationary time series r_t the correlation coefficient between r_t and r_{t-1} is called the lagged-1 autocorrelation of r_t and is denoted by ρ

$$\rho = \frac{\mathrm{cov}(r_t, r_{t-1})}{\sqrt{\mathrm{Var}(r_t)\mathrm{Var}(r_{t-1})}}$$

Where, $cov(r_t, r_{t-1})$ is covariance and $var(r_t)$ is the variance. For a weakly stationary time series $var(r_t) = var(r_{t-1})$. Autocorrelation takes the values between (-1, 1). Thus $-1 \leq \rho \geq 1$.

3.3.2 ARMA Models

To determine the mean process for the returns, an iterative approach was used to minimize the Akaike Information Criteria (AIC). This is the model selection tool that has been proposed in the time series literature for selecting the order of the model among various possible choices for ARMA models. The ARMA specification for each of the returns is shown.

In the time series modeling it is imperative to identify the model that best fits the data from a set of models. There are other different kinds of Information Criteria that can be used which can produce a different specification for the mean process. The Schwarz Bayesian Information Criterion (SBIC) generally produces a specification that is small compared to Akaike as it impose a stiffer penalty terms. The residuals are extracted and used in a series of tests to better

understand the characteristics of the residuals and to help model volatility as accurately as possible. If the number of estimated parameter in the model is *k*, then the AIC is defined by

$$\mathrm{AIC} = \ln\left(\bar{\alpha}^2\right) + \frac{2\mathrm{k}}{\mathrm{T}}$$

Where $\bar{\alpha}^2$, the residual variance and T is the sample size.

But before we do that we used Augmented Dickey-Fuller (ADF) test and Phillips-Perron to test for stationarity of the time series if there is a unit root in the series. ADF test shows that all the indices close prices are not stationary because the null hypothesis of ADF test was rejected in favor of the alternative, but on continuously log returns the null hypothesis was rejected showing the stationarity of the time series data. And the Phillips-Perron tests produce the same inference about the stationarity of the series but highly significant than ADF test. The ARMA specification of each index is given in the table below.

Table 3.2: ARMA Specification for each JSE Index

Index	**ARMA (p, q)**
JSE All share	(8, 8)
JSE Top 40	(9, 6)
JSE Resource 20	(7, 4)
JSE Industrial 25	(9, 9)

3.3.2.1 Statistical Tests

From this test we determined volatility effects in a set of data that appeared in the residuals. In that way it is important to determine if there are indeed ARCH effects in the residuals. Once this

has been determined the distribution of the residuals need to be determined in order to ensure that the volatility models that are estimated are as accurate as possible. The tests that were performed on the residuals are as follows: 1 A Heteroscedasticity test (Engle's ARCH Test). 2 A normality test (Jarque-Bera Test).

3.3.2.2 ARCH Test

Engle's ARCH test involves a regression on the squared residuals and the null hypothesis that says all the coefficients are equal to zero.

The test that we implement is the usual Box-Ljung Q statistic test for auto-correlations in the series (McLeod and Li, 1983). The second test that can detect the ARCH effect in the time series returns is the Lagrange Multiplier (LM) test of Engle (1983). The Ljung-Box statistic, test for the null hypothesis that 'all *m* autocorrelation coefficients are zero' which occurs when ARCH effects are present. Under the Engle's LM test, the test is for autocorrelation in the squared residuals. The joint null hypothesis of LM test is that 'all *q* lags of the squared residuals have coefficient values that are not significant different from zero', which is the scenario when no ARCH effects present (Brooks, 2008).

The results of the ARCH tests are given in the Table 3. Its shows that residuals contain ARCH effects, therefore the null hypothesis in the ARCH tests in all indices can be confidently rejected. We rejected the null hypothesis of linearity at the 1% level. All the p-values are significantly zero indicating strong departures from the *i.i.d* condition. The result of these tests is below in this table. Therefore, this confirms that the series has ARCH effects. The test was conducted with 5 lags in the residuals.

Table 3.3: Heteroscedasticity Test ARCH

	ALSHI	RESORCE 20	INDUSTRIAL 25	TOP 40
F-statistic	151.7789(0.000)	149.7132(0.000)	102.0435(0.0000)	149.4170(0.0000)
LM	609.1217(0.000)	602.4627(0.000)	438.0792(0.0000)	601.5049(0.0000)

Table 3.4: Breusch-Godfrey Serial Correlation LM Test

	ALSH	RESOURCE 20	INDUSTRIAL 25	TOP 40
F-statistic	4.42812 (0.0005)	6.310757(0.0000)	3.178225(0.0073)	4.717191(0.0003)
LM	22.02470(0.0005)	31.29306(0.0000)	15.840004(0.0073)	23.45149(0.0003)

3.3.2.3 Normality Test

The Jarque-Bera test is used for testing normality in the time series data. This is the goodness-of-fit to test whether the residuals have the skew-ness and kurtosis that matches a normal distribution. From descriptive statistic in Table 1, all the indices have high Jarque-Bera (JB) values and low p-values. Hence, the null hypothesis that says the residuals are normally distributed is rejected to four significant values. Table 3 provides the result of the test for normality test in residuals. The Jarque-Bera statistics is calculated as

$$JB = \frac{T}{6}(S^2 + \frac{1}{4}(K-3)^2)$$

Where *S* is the skew-ness, *K* is the sample kurtosis and T is the sample size. Under the normality assumption *S* and *K*-3 are asymptotically distributed as normal with zero mean and variances $\frac{6}{T}$ and$\frac{24}{T}$, respectively. The decision rule is that, we reject the null hypothesis at α% significance

level if $JB > \chi^2_{1-\frac{\alpha}{2}}$, where $1-\frac{\alpha}{2}$ is the critical value of the chi-square distribution with 2 degree of freedom (Tsay, 2012).

Table 3.5: ARCH tests results

Index Name	F-statistic	TR^2
JSE All Share	90.295***	698.588***
JSE Industrial 25	56.798***	480.199***
JSE Resource 20	115.795***	841.505***
JSE Top 40	91.691***	706.832***

***, indicates significance at 1% level, TR^2 is a JB test statistics.

Chapter 4: Methodology

This section of the chapter outlines the methodology which was used to address the research questions that this study is based trying to answer. The first subsection explains the methods used for estimating the in-sample parameters of the various models using Eviews 7 software. The following subsection describes the forecasting approach that was followed in the study and the technique used to evaluate the GARCH model volatility forecasting.

4.1 In-sample estimation

4.1.1 Estimation of GARCH models

In the absence of homoscedasticity, Ordinary Least Squares (OLS) estimation is no longer appropriate to estimate the parameters of the model. The estimation will no longer be efficient. The estimate will violate the BLUE condition. So to estimate the GARCH model we use the Maximum Likelihood method for parameter estimation. This method simply chooses a set of estimated parameters that is most likely to generate the data observed, using an iterative computer algorithm (Marquardt algorithm), this is the built-in algorithm from the Eviews software. The conditional variance equations for the GARCH models was estimated and their mean equation have been specified in Chapter 2. The illustration would focus on GARCH (1, 1) model in outlining the Maximum Likelihood procedure, as the method is the same for all other various GARCH extensions models.

4.1.2 Diagnostic checks

After GARCH models have been estimated, we performed diagnostic tests on the residuals of each model estimated to find out if the model has been correctly specified. These diagnostic tests

check if there are any remaining non-linearity structures in the residuals. The number one commonly used principle behind these diagnostic tests is that once the model has been generated the non-linear structure has been removed from data and the remaining structure is from random noise of the unknown linear data generating process (Alagidede, 2011). In this study we examined the residual for non-linearity using two different tests; the Ljung-Box-Q and ARCH-LM test on residuals.

These tests have been already used in the study to check the presence of autocorrelation and the absence of homoscedasticity (heteroscedasticity) for the series data. We performed the same test on the standardized residuals and square residual of each GARCH model estimated (Magnus and Fosa, 2006).

4.2 Forecasting

We forecast 20 day a-head volatility. The 20 day a-head volatility was found by dynamic forecasts using Eviews. The in-sample data contains the first 3047 daily observations. The models are estimated over the in-sample data. The estimated parameters were used to forecast the 20 day a-head volatility forecasts of these companies.

4.3 Forecast evaluation statistics

The performance of these forecasting models are evaluated based on the statistical loss functions. The most commonly used loss functions in the literature which was used in this study are Root Mean Square Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Theil's U-statistic (Brailsford and Faff, 1996; Brooks, 2008).

$$RMSE = \sqrt{\frac{1}{T-(T_{1-}-1)}\sum_{i=T_1}^{T}(\sigma_i^2 - \tilde{\sigma}_i^2)^2}$$

$$MAE = \frac{1}{T-(T_{1-}-1)}\sum_{i=T_1}^{T}\left|\sigma_i^2 - \tilde{\sigma}_i^2\right|$$

$$MAPE = \frac{100}{T-(T_{1-}-1)}\sum_{i=T_1}^{T}\left|(\sigma_i^2 - \tilde{\sigma}_i^2)/\sigma_i^2\right|$$

$$Theil's\ U-statistic = \sum_{i=T_1}^{T}((\sigma_i^2 - \tilde{\sigma}_i^2)/\sigma_i^2) \Big/ ((\sigma_i^2 - \tilde{\sigma}_{bi}^2)/\sigma_i^2)$$

Where T represents the number of total observations both in-sample and out-of-sample and T_1 is the first out-of-sample forecast observation. The observation that the model is estimated for start from 1 to T_1-1 observations and T_1 to T are used for the out-of-sample forecasting. $\sigma_i^2$and $\tilde{\sigma}_i^2$denotes the actual and the estimated conditional variance at time *t*, respectively. $\tilde{\sigma}_{bi}^2$ is found from a benchmark model.

RMSE and MAE measures how close is the forecast from the actual data, thus the forecast which has the smallest RMSE and MAE is probably the most accurate forecast (Brooks, 2008).

RMSE is calculated based on the quadratic loss function (Brooks, 2008). It is advantageous in the sense that it is most sensitive to large errors of the four criterions because of the square of the errors. The most advantageous thing about it is that large estimate errors could lead to serious problems. But RMSE can lead to a tailback if the larger errors cannot lead to serious problems. MAE measures the average absolute forecast error (Brooks, 2008). The advantage of these loss functions is that it penalizes large errors less than the RMSE because of non-squaring errors.

RMSE and MAE are very simple loss function, and by that they are inconsistence to scalar transformation and symmetric which implies that they are not realistic in some cases (Brooks, 2008).

MAPE tells us about the ability to accounts for variability in the actual data from the forecasted output (Brooks, 2008). It measures the percentage error, ranges from [0, 100] percent. Therefore, the forecast with a MAPE closest to 100 is probably the most accurate. The advantage of MAPE is that it can be used to compare the performance of the estimate models and the random walk model.

The Theil's U-statistic compares the performance forecast of the model to the performance of a benchmark model. If the U-statistic = 1 this implies that the model under consideration has the same accuracy or inaccuracy as the benchmark model. The advantage of this error function is that comparing the Theil's U-statistic is constant to scalar transformation but symmetric.

Chapter 5: Empirical Results

5.1 Introduction

In this chapter, the volatility models such as GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1) and GARCH-M (1, 1) are compared, with the aim of identifying the most appropriate characterization of the conditional variance process for JSE African markets on the selected indices. These models are evaluated based on the economic implications of parameter estimates, *likelihood ratio* tests (LRTs), residual diagnostics, and forecast errors. As noted in the literature review GARCH (1, 1) model sufficiently models the volatility clustering in the financial data for stylized facts but cannot accommodate other facts like asymmetry which is being covered by other extension GARCH models explained before. The order of models is 1 for all models estimated. These calculated estimates are obtained by assuming a student *t*-distribution to account for fat tails.

5.2 Error distribution and leverage effects

All models are estimated using Student's *t* errors distribution to allow for fatter tails in the log returns shown from the Table 1. As illustrated in Table 5.1 to 5.5. The LRT are used to test the null of Student's *t* distributions in error against the alternative of normal error distribution in all indices. In each case we fail to reject null at 1% level of significant. Therefore, the study focuses on using the Student's *t* errors distribution.

Table 5.1: Parameter estimates for JSE All Share Index

	GARCH	EGARCH	GJR-GARCH	GARCH-M	PGARCH(1, 1)
μ	0.091***(5.372)	0.055***(3.321)		0.0268(0.481)	0.056***(3.29)
σ	0.017	0.017			0.017
δ				0.071 (1.213)	
γ		-0.095*** (-9.445)	0.131*** (8.645)		
ω	0.0145*** (3.243)	-0.091*** (-7.579)	0.018*** (4.96)	0.015*** (3.249)	0.015*** (4.5079)
α		0.116*** (7.684)			
α_1	0.0825*** (7.958)		7.78E-05 (0.009)	0.082*** (7.951)	0.05*** (2.923)
β	0.908*** (80.682)	0.986*** (332.234)	0.926*** (100.47)	0.908*** (80.504)	0.934*** (109.922)
Durbin Watson	1.946	1.948	1.948	1.942	1.948
log-likelihood	-4489.52	-4454.26	-4454.87	-4488.77	-4451.743
LBQ[12]	9.747[0.638]	10.512[0.571]	10[0.616]	9.35[0.672]	9.96[0.619]
LBQ^2[12]	12.305[0.422]	16.193[0.183]	16.684[0.16 2]	12.12[0.432]	16.82[0.157]
ARCH[12]	11.763[0.465]	15.633[0.207]	16.358[0.175]	11.63[0.476]	16.37[0.175]
T-dist	13.76 [4.317]	18.632[3.502]	17.622[3.659]	13.82[4.299]	18.25[3.588]
α + β	0.991	1.102	0.926	0.99	0.984

***, **,* indicates significance at 1%, 5% and 10% respectively. LBQ is the Ljung-Box statistic, Student-*t Test* statistic are reported in () while *p*-values in [].

Table 5.2 Parameter estimates for Resource 20 Index

	GARCH	EGARCH	GJR-GARCH	GARCH-M
μ	0.042 (1.587)	0.009(0.356)	0.0119(0.445)	-0.146(-1.3195)
σ	0.027	0.026	0.027	0.111
δ				0.129(1.76)
γ		-0.055***(-6.51)	0.0773***(6.509)	
ω	0.038*** (3.123)	-0.063***(-6.39)	0.032***(3.666)	0.037***(3.12)
α		0.092***(6.930)		
α_1	0.059***(6.667)		0.007 (0.9604)	0.059***(6.736)
β	0.929***(86.79)	0.991***(372.07)	0.944***(113.99)	0.929***(87.62)
Durbin Watson	1.905	1.9054	1.9055	1.902
log-likelihood	-5789.302	-5768.859	-5766.758	-5787.862
LBQ[12]	16.722[0.16]	15.826[0.199]	16.23[0.180]	16.51[0.169]
LBQ^2[12]	3.89[0.985]	6.799[0.871]	6.934[0.86]	3.756[0.987]
ARCH[12]	3.947[0.985]	6.879[0.87]	7.0999[0.851]	3.843[0.986]
T-distribution	12.218[4.563]	15.284[3.728]	14.826[3.93]	12.266[4.55]
α+β	0.988	1.083	0.951	0.988

***, **,* indicates significance at 1%, 5% and 10% respectively. LBQ is the Ljung-Box statistic, Student-*t Test* statistic are reported in () while *p*-values in [].

Table 5.3: Parameter estimates for Industrial 25

	GARCH	EGARCH	GJR-GARCH	GARCH-M
μ	0.1202***(7.297)	0.093***(5.665)	0.094***(5.669)	0.132(2.245)
σ	0.0165	0.016	0.017	0.059
δ				-0.014(-0.221)
γ		-0.091***(-7.46)	0.112***(6.234)	
ω	0.0234***(3.821)	-0.106***(-7.44)	0.024***(4.582)	0.0235***(3.818)

α		0.133***(7.34)		
α_1	0.091***(7.868)		0.016(1.391)	0.091***(7.866)
β	0.891***(64.42)	0.978***(220.8)	0.905***(72.12)	0.891***(64.38)
Durbin Watson	1.988	1.991	1.991	1.989
log-likelihood	-4381.793	-4363.712	-4360.692	-4381.771
LBQ[12]	15.465[0.217]	16.543[0.168]	15.813[0.2]	15.569[0.212]
LBQ^2[12]	8.895[0.712]	10.777[0.548]	9.029[0.7]	8.8439[0.716]
ARCH[12]	8.939[0.7081]	10.957[0.5326]	9.134[0.691]	8.879[0.7132]
T-distribution	11.339[5.443]	12.775[4.615]	12.773[4.768]	11.304[5.441]
α+β	0.982	1.111	0.921	0.982

***, **,* indicates significance at 1%, 5% and 10% respectively. LBQ is the Ljung-Box statistic, Student-*t Test* statistic are reported in () while *p*-values in [].

Table 5.4: Parameter estimates for Top 40

	GARCH	EGARCH	GJR-GARCH	GARCH-M
μ	0.089***(4.803)	0.049***(2.733)	0.052**(2.81)	0.017(0.277)
σ	0.019	0.018	0.017	0.062
δ				0.072(1.239)
γ		-0.099***(-9.69)	0.127***(8.447)	
ω	0.017***(3.207)	-0.085***(-7.21)	0.017***(4.261)	0.017***(3.219)
α		0.112***(7.47)		
α_1	0.082***(7.857)		-0.0029(-0.334)	0.081***(7.852)
β	0.9096***(80.59)	0.987***(339	0.928***(102.7)	0.909***(80.326)
Durbin Watson	1.973	1.974	1.974	1.969
log-likelihood	-4779.082	-4741.318	-4742.719	-4778.312
LBQ[12]	9.642[0.647]	11.209[0.511]	10.327[0.587]	9.121[0.693]
LBQ^2[12]	12.536[0.404]	16.932[0.152]	17.790[0.122]	12.426[0.412]

ARCH[12]	11.551[0.4824]	16.052[0.189]	17.412[0.1347]	11.426[0.4928]
T-distribution	13.662[4.294]	18.762[3.386]	17.49[3.604]	13.749[4.269]
α+β	0.9916	1.099		0.99

***, **,* indicates significance at 1%, 5% and 10% respectively. LBQ is the Ljung-Box statistic, Student-*t Test* statistic are reported in () while *p*-values in [].

Table 5.5: Parameter estimates for PGARCH (1, 1)

	JSE	RESOURCE 20	INDUSTRIAL 25	TOP 40
μ	0.056***(3.29)	0.011***(0.395)	0.092***(5.493)	0.049***(2.632)
σ	0.017	0.027	0.017	0.019
ω	0.015***(4.508)	0.025***(3.209)	0.025***(4.7128)	0.017***(4.503)
α_1	0.091***(2.923)	0.042***(3.99)	0.065***(4.626)	0.049***(0.759)
β	0.934***(109.92)	0.947***(116.31)	0.913***(78.9337)	0.936***(110.96)
Durbin Watson	1.948	1.905	1.991	1.974
log-likelihood	-4451.743	-5766.219	-4358.753	-4738.472
LBQ[12]	9.963[0.619]	15.953[0.193]	16.181[0.183]	10.528[0.57]
LBQ^2[12]	16.818[0.157]	6.745[0.874]	9.6024[0.651]	17.440[0.134]
ARCH[12]	16.374[0.175]	6.894[0.865]	9.78[0.635]	16.728[0.16]
T-distribution	18.254[3.588]	15.116[3.811]	13.009[4.557]	18.429[3.449]
α+β	1.025	0.989	0.978	0.985

***, **,* indicates significance at 1%, 5% and 10% respectively. LBQ is the Ljung-Box statistic, Student-*t Test* statistic are reported in () while *p*-values in [].

5.2.1 GARCH parameter estimates and their economic meaning

The estimates of GARCH, EGARCH, GJR-GARCH, GARCH-M and PGARCH models are presented in Table 5.1 to Table 5.5 for all indices and indices for sectors assuming student *t*-distribution. These coefficients estimates for JSE All Share, Resource 20, Industrial 25, and Top

40 indices are obtained using the maximum likelihood function over a sample period. The values in the parentheses are Student-*t* statistics. The numbers in square brackets are *p* values, log-likelihood represent the maximized log likelihood value. The values in LBQ [12], LBQ^2 [12] are Box-Ljung Q statistic calculated on standardized and squared standardized residuals of order 12 respectively. Durbin-Watson is the test statistic for autocorrelation.

These three constants of mean coefficient ω, ARCH term α_1 and GARCH term β for these models are highly statistical significant at 1% level. But the ARCH term for GJR-GARCH is insignificant from zero for JSE All Share index. This indicates that for these models, previous news about volatility has an explanatory power on current volatility because of the lagged conditional variance (GARCH term) and ARCH term have an impact on conditional variance. These results are consistence with the conclusion on the behavior of JSE returns by Makhwiting *et.al*, (2012) and Samouulhan & Shannon (2008).

The estimates of the coefficient of α_1 are statistically significant in GARCH (1, 1) at 1% level for all indices. They are all positive and statistically significance from zero for all models estimated except in GJR-GARCH (1, 1) they are all positive and insignificance except for Top 40 index which is negative. However, the superiority of GJR-GARCH (1, 1) model is that it allows asymmetric response to past positive or negative returns. The parameter estimate β measures the persistence of volatility shocks is positive and highly statistically significance. The asymmetric coefficient γ is negative and statistically significance at 1% level for EGARCH (1, 1) but negative for GJR-GARCH (1, 1) but significant. Therefore, the residual have asymmetric influence in the JSE indices, that the positive sign indicate that negative residuals increase volatility more than residual of the same magnitude.

The pattern of high persistence in the JSE data is observed. Refereeing to the coefficient estimates of GARCH (1, 1) (Table 5). The degree of persistence is measured by the sum of ARCH term (α_1) and GARCH term (β_1) for all model are close to unity except for EGARCH model, indicating that there is a high degree of persistence in the shocks to volatility and long memory in the conditional variance. This means that the shocks in the volatility - took time to decay. The EGARCH model has a sum more than unity which implies that there is a very long and explosive persistence in volatility suggesting an integrated (IGARCH) process in modeling volatility.

For all indices, the sum of these coefficients is relatively high, with the lowest of 0.982 for Industrial 25 index and the highest is 0.9916 for Top 40 which clearly shows the persistence of volatility. The entire coefficients are statistically significant for GARCH (1, 1) model at 1% since all the *p*-values are zero to 4 significant values for each GARCH estimate coefficients. This tells us that GARCH (1, 1) successfully remove heteroscedasticity in the data. And the corresponding standard deviation σ of the errors is small which indicates the good fit of a model.

The coefficients on both the lagged squared residual and lagged conditional variance terms in the conditional variance equation are highly statistically significant. Also, as is typical of GARCH model estimates for financial asset returns data, the sum of the coefficients on the lagged squared error and lagged conditional variance is very close to unity approximately 0.917309 for Resource 20 Index. This implies that shocks to the conditional variance were highly persistent. If the sum of these coefficients is large this implies that a large positive or a large negative return will lead future forecast of the variance to be high for an extended period. This is seen by considering the equations for forecasting future values of conditional variance in the next sub-section.

The most basic and popular model of conditional variance presented in this study is GARCH (1, 1) model for all indices chosen was estimated from the period of 01 October to 2002 to 10 December 2014 leaving 20 observations for out-of-sample forecast evaluation . This model yields the statistically significant on the lagged conditional variance at 1% level for all indices. The model characterizes the conditional variance as alternative between two constant, associated with lagged squared residual and lagged conditional variance.

The GJR-GARCH (1, 1) estimation for ALSH show that the series has leverage effect ($\gamma > 0$) 0.122649 which is highly significant signaling that positive news provide more positive returns then negative news of the same magnitude.

The GARCH-M models help us to understand the risk premium which is denoted by δ in the specification of conditional variance. The magnitude of this risk premium is important as the sign of it in the model. If the sign of δ is positive and statistical significant, this means that then the increased risk given by an increase in the conditional variance provides an increased in mean return. This tells investors that they are being compensated for a higher risk taken.

The GARCH-M interpretation for All Share the risk premium "δ" parameter is positive and insignificant showing no feedback from the conditional variance to the conditional mean. In All Share, Resource 20, Top 40 the risk premium parameter is positive and insignificant but Industrial 25 is negative but still insignificant theory tell us that there is no feedback from the conditional variance to the conditional mean. The insignificant of the risk premium suggest that investors in the JSE are not compensated for assuming greater risk.

The study applies two asymmetric models EGARCH (1, 1) and GJR-GARCH (1, 1) and the seasonal PGARCH (1, 1) model to investigate the existence of leverage effects in the returns and

seasonality in the JSE Stock market. The model estimated first is EGARCH (1, 1) which has the advantage of not imposing artificially negative restriction on its parameter while GJR-GARCH requires the satisfaction of positive condition (Nelson, 1991). All the estimates are highly statistical significant including the asymmetry coefficient (γ). The asymmetric coefficient is negative and statistical significant, this tells us that in the period studied the negative shocks or news on all indices estimated has a higher impact on volatility than the positive shocks with the same magnitude. Therefore, the EGARCH model indicates the presence of leverage effects in JSE markets.

Another alternative asymmetric model in the returns is the GJR-GARCH (1, 1). From Table 5.1-5.4, the estimated result of the model shows that the GARCH term, asymmetric term and constant term are statistically significant but ARCH term is insignificant which is in line with previous studies. These results show the presence of leverage effects on the data. The asymmetric term is positive implying that the negative shocks (bad news) have a larger effect on the conditional variance than positive shocks (good news) of the same magnitude.

ARCH test from residuals of the estimated model GARCH (1, 1) of JALSH Index is rejected meaning the GARCH model successful model all the specification of the series of volatility clustering in the data, F-statistic of 1.117566 with *p* value of 0.3487 which is highly insignificant. ARCH test for Resource 20 with F-statistic 1.005598(0.4127) was also rejected the null hypothesis that say series has ARCH effects on it; therefore the model was successful to model the volatility in the series. For Industrial 25 F-statistics is 0.871129 with *p* value of 0.4996 implying no ARCH effects in the residuals in the model estimated. For Top 40 index F-statistic is 1.418887 with *p* value of 0.2141 implying no effect remaining in the residuals. Therefore, after fitting the GARCH model the test for residuals and squared residuals indicated no ARCH effect

left in the series. The model was able to remove all Heteroscedasticity in the residuals and serial correlation.

Table 5.6: Model selection for the estimated models assuming t-student distribution

JSE ALSH	GARCH (1, 1)	EGARCH (1, 1)*	GJR-GARCH (1, 1)	GARCH-M (1,1)	PGARCH (1, 1)
AIC	2.9414	2.919	2.9194	2.9416	2.918
SBIC	2.9513	2.932	2.9312	2.9534	2.9318
HQIC	2.945	2.923	2.9237	2.9459	2.9230
Resource 20					
AIC	3.7921	3.7794	3.77798	3.7918	3.7783
SBIC	3.8019	3.7912	3.7898	3.8036	3.7921
HQIC	3.7956	3.7836	3.7822	3.7961	3.7832
Industrial 25					
AIC	2.8709	2.8598	2.8578	2.8716	2.8572
SBIC	2.8808	2.8716	2.8896	2.8834	2.871
HQIC	2.8745	2.8640	2.8620	2.8759	2.8621
Top 40					
AIC	3.1309	3.1069	3.1078	3.1311	3.10596
SBIC	3.1408	3.1187	3.1196	3.1429	3.1198
HQIC	3.1345	3.1111	3.1121	3.1354	3.1109

* chosen model

The log-likelihood values of the models suggest that the for JSE All Share index and all other indices GARCH (1, 1) model performs best in modeling volatility, while AIC,SBIC and HQIC suggest that EGARCH (1, 1) for ALSH and To40 indices. Resources 20 Index the results on data suggest that RJR-GARCH (1, 1) is the best model and so is Industrial 25 data suggested PGARCH (1, 1). This tells us that modeling volatility is not an easy task and that one model can

model JSE All share index does not necessarily mean that other indices in the JSE can be modeled with the same model this is in line with the studies of Poon and Granger (2003).

5.3 Diagnostics

After the specification of the GARCH model, it is important to investigate its adequacy. To examine the relationship between the residuals obtained from the fitted model, the conditional standard deviations and the observed returns are studied. Residual diagnostic check results for GARCH (1, 1) on JSE All share index are in the table 5.7.

Table 5.7: Box-Ljung Q-statistic test for squared standardized residuals, Engle's ARCH test and Jarque-Bera test for normality

Statistic	**value**	**p-value**
$LBQ^2[12]$	12.305	0.422
ARCH(12)	11.763	0.465
Jarque-Bera	74.037	0.0000

The standardized residuals and the standardized squared residuals of the GARCH, EGARCH, GJR-GARCH and GARCH model in Table 5.1-5.5. The large value of Ljung-Box statistics of 12 lags on each of the models finds that there is no evidence of serial correlations in the residuals. The ARCH LM test statistics reject the null hypothesis of ARCH effects in favor of no ARCH effects, therefore finds no evidence of conditional heteroscedasticity in the data. This tells us that the volatility models fitted better fits the data and successfully modeled any non-linear dependence in the series. For a properly specified model, the standardized residuals should form a sequence of independent and identical distributed (iid) random variables. If this model is correctly specified the residuals should have no serial correlation, conditional heteroscedasticity

or any type of non-linear dependence which is the result of this test tells us that the heteroscedasticity has been removed.

5.4 Forecast valuation: GARCH out-of-sample

Forecasting upcoming volatility is the most important aspect in the pricing of derivative securities. In the option pricing formula Black-Scholes volatility has to be estimated empirically and errors in calculating volatility may result in mispricing options. In order to test the forecasting performance of each model for a particular index, this study makes use of loss functions like MAE, MSE, MAPE and Theil inequality (Theil U) coefficient measures. Model with a smallest statistic is considered to be the best in forecasting these Indices. Tables 5.8-5.9 contain MAE, MSE, MAPE and U-statistic estimates for each index at one month (20 days) time horizon.

For all GARCH models the RMSE and MAE were low and MAPE for almost all indices is above 100. This shows that GARCH (1, 1) is still the best model generally to model stylized facts in the JSE stock market. Low RMSE and MAE statistics loss function tells us that forecast error were small in that model.

Industrial 25 index models have MAPE of less than 100 which suggest that EGARCH, GJR-GARCH and PGARCH models out-performed the benchmark. The Theil U-statistic was below 1 for all models, which means all models outperformed the benchmark model.

For JSE All share index. Using MAE measure, the GARCH (1, 1) provides the most accurate forecasts in 20 day horizon which is consistence with other findings in the literature. But MSE suggest that PGARCH (1, 1) is more accurate in 20 days' time horizon. Interesting thing to note here at a 1 month horizon all the models predict the same forecast.

MAPE and Theil-U statistics produce conflicting results in almost all models, however as we noted in the methodology chapter MAPE criterion cannot be relied upon if the series can take on absolute values less than one, which is always the case in forecasting volatility models.

For JSE Resource 20 index, EGARCH (1, 1) provides the most accurate forecast based on RMSE, MAE and Theil U-statistic. For JSE Industrial 25 GARCH (1, 1) forecast more accurately based on RMSE, MAPE and MAE but based on Theil U-statistic PGARCH forecast better.

***Table 5.8*:** Error statistics forecasting daily volatility

JSE All Share index	RMSE	MAPE	MAE	Theil's U
GARCH	1.2338	117.7703	0.9968	0.9374
EGARCH	1.4906	110.8013	0.9871	0.9607
GJR-GARCH	1.4903	115.5247	0.9881	0.9581
GARCH-M	1.4867	121.7089	1.0022	0.9249
PGARCH	1.4906	110.9166	0.9872	0.9603
JSE Resources 20	RMSE	MAPE	MAE	Theil's U
GARCH	2.0448	100.9436	1.6095	0.9788
EGARCH	2.046	100.2072	1.6065	0.9952
GJR-GARCH	2.04596	100.2644	1.6068	0.9939
GARCH-M	2.0442	102.6270	1.6166	0.9448
PGARCH	2.046	100.2337	1.6066	0.9946
JSE Industrial 25	RMSE	MAPE	MAE	Theil's U
GARCH	1.2828	127.6172	0.8820	0.9147
EGARCH	1.5950	94.3166	0.9797	0.9397
GJR-GARCH	1.5949	94.9147	0.9798	0.9386
GARCH-M	1.5933	104.9831	0.9846	0.9256

PGARCH	1.5950	94.104	0.9796	0.94
Top 40	RMSE	MAPE	MAE	Theil's U
GARCH	1.6717	113.0347	1.1084	0.9455
EGARCH	1.6744	107.2447	1.0976	0.9687
GJR-GARCH	1.6742	107.6654	1.0984	0.9669
GARCH-M	1.6704	117.9578	1.1164	0.93195
PGARCH	1.6745	107.1519	1.0974	0.9691

Chapter 6: Summary and Conclusions

In conclusion, the primary goal of this study was to examine and describe the stylized facts that appear more often in the financial data series (volatility clustering, leverage effects and leptokurtosis) and how to model these stylized facts on JSE stock market. And forecasting ability of GARCH model based against the extension of other GARCH models. The in-sample study period was from 01 October 2002 to 10 October 2014 and out-of-sample was done 20 days ahead. The purpose of the study was to determine which model best models the JSE ALSH volatility, which produce accuracy forecast as well and compare this finding with different GARCH models. The research also aimed to pin point one model that can be used in modeling and forecasting volatility on JSE stock exchange.

In the study a variety of univariate GARCH models were estimated, including GARCH (1, 1), EGARCH (1, 1), GJR-GARCH (1, 1), GARCH-M (1, 1) and PGARCH (1, 1) models. The estimated models reveal that the JSE stock market returns are characterized by volatility clustering, leptokurtosis and leverage effects. The estimated parameters also indicated that through EGARCH model the returns are highly persistent suggesting the integrated GARCH model. The results showed that the JSE All Share Index and all other indices studied here can be best modeled by GARCH (1, 1) based on log-likelihood function but EGARCH through AIC, SBIC and HQIC for JSE. But modeling out-of sample JSE All share Index data GARCH (1, 1) proved to be more accurately followed by EGARCH and GJR-GARCH. These findings of this study were generally in line with the conclusion made by other researchers about the characteristics of JSE returns (Makhwiting *et. al.,* 2012; Niyitegeka and Tewari, 2013; Samoulhan and Shannon, 2008).

Forecasting the performance of different GARCH model was evaluated. Forecasting on these indices provides different models with different forecasting evaluation procedures. Forecasting proved to be a difficult task to handle especial when you have 20 estimated models to forecast. For Industrial 25 the forecasting model that proved to be best from the rest was GARCH (1, 1) based on RMSE, MAPE and MAE but based on Theil U statistic suggest EGARCH (1, 1).

After modeling volatility on the JSE index, based on log-likelihood function GARCH (1, 1) shows superiority amongst other models for JSE All Share index. But based on AIC, SBIC and HQIC EGARCH (1, 1) proved to model better than GARCH, GJR-GARCH, etc. models for ALSH and Top 40 indices. While Resources 20 Index data suggest that RJR-GARCH (1, 1) is the best model and Industrial 25 data suggest PGARCH (1, 1).

From Chapter 5 on forecasting evaluation the loss function measure were used. The data for JSE All share index suggested GARCH (1, 1) for forecasting volatility. For JSE Resource 20 index, EGARCH (1, 1) provides the most accurate forecast based on RMSE, MAE and Theil U-statistic. For JSE Industrial 25 GARCH (1, 1) forecast more accurately based on RMSE, MAPE and MAE but based on Theil U-statistic PGARCH forecast better.

For future research we can model volatility using multivariate GARCH models to assess the interrelation of volatility with other emerging stock markets like Nigeria, Botswana, Ghana, etc. Using multivariate models like Markov switching GARCH model, CCC-GARCH, BEKK-GARCH and GO-GARCH models with different distributions on the errors to study further the dynamics of the correlation between different financial markets.

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APPENDIX A: Akaike Information Criteria

						Rjalsh AIC						
	p/q	0	1	2	3	4	5	6	7	8	9	10
	0	3.2634	3.2591	3.2596	3.2561	3.2556	3.2551	3.2551	3.2555	3.2561	3.2567	3.2574
	1	3.2649	3.2600	3.256	3.2546	3.2553	3.2557	3.2558	3.2564	3.2569	3.2575	3.2582
	2	3.2602	3.256	3.2548	3.2553	3.2560	3.2489	3.2549	3.2548	3.2577	3.2583	3.2589
	3	3.2572	3.2547	3.2550	3.2558	3.2550	3.2489	3.2547	3.2553	3.2560	3.2555	3.2506
	4	3.2555	3.2550	3.2554	3.2547	3.2545	3.2551	3.2557	3.2463	3.2559	3.2484	3.2476
ARMA	5	3.2552	3.2554	3.2561	3.2524	3.2540	3.2546	3.2480	3.2477	3.2483	3.2489	3.2489
	6	3.2551	3.2558	3.2492	3.2491	3.2493	3.2484	3.2490	3.2496	3.2502	3.2460	3.2465
	7	3.2557	3.2545	3.2544	3.2550	3.2484	3.2492	3.2459	3.2476	3.2467	3.2485	3.2474
	8	3.2548	3.2552	3.2550	3.2555	3.2560	3.2486	3.2442	3.2451	3.2423*	3.2455	3.245
	9	3.2558	3.2559	3.2558	3.2485	3.2489	3.2489	3.2445	3.2472	3.2459	3.2463	3.244
	10	3.2551	3.2555	3.2562	3.2485	3.2461	3.2475	3.2451	3.2453	3.2456	3.2446	3.2426

						Rresi20 AIC						
	p/q	0	1	2	3	4	5	6	7	8	9	10
	0	4.059	4.0538	4.0544	4.0480	4.0486	4.0479	4.0472	4.0466	4.0463	4.0470	4.0476
	1	4.0588	4.0548	4.0509	4.0485	4.0484	4.0483	4.0464	4.0468	4.0472	4.0479	4.0485
	2	4.0548	4.0506	4.0501	4.0485	4.0489	4.0487	4.0464	4.0466	4.0473	4.0443	4.0458
	3	4.0494	4.0494	4.0491	4.0470	4.0457	4.0459	4.0466	4.0470	4.0476	4.0482	4.0454
	4	4.0502	4.0504	4.0497	4.0460	4.0457	4.0462	4.0472	4.0480	4.0481	4.0487	4.0430
ARMA	5	4.0487	4.0487	4.0494	4.0456	4.0471	4.0429	4.0426	4.0432	4.0439	4.0445	4.0451
	6	4.0485	4.0473	4.0465	4.0468	4.0474	4.0478	4.0433	4.0439	4.0442	4.0453	4.0413
	7	4.0485	4.0474	4.0465	4.0477	4.0476	4.0437	4.0443	4.0425	4.0438	4.041*	4.0448

	8	4.0471	4.0471	4.0473	4.0480	4.0486	4.0474	4.0446	4.0425	4.0398	4.0444	4.0407
	9	4.0466	4.0472	4.0477	4.0484	4.0432	4.0467	4.0473	4.0480	4.04059	4.0422	4.0411
	10	4.0476	4.0482	4.0487	4.0469	4.0467	4.0473	4.0458	4.0486	4.0423	4.0419	4.0429

						Rindi25 AIC						
	p/q	0	1	2	3	4	5	6	7	8	9	10
	0	3.1311	3.1317	3.1302	3.1305	3.1293	3.1292	3.1297	3.1302	3.130	3.1304	3.1309
	1	3.1307	3.1305	3.1298	3.1291	3.1287	3.1285	3.1290	3.1292	3.1295	3.1299	3.1290
	2	3.12899	3.1275	3.1280	3.1286	3.1287	3.1291	3.1239	3.1268	3.1273	3.1276	3.1267
	3	3.1297	3.1278	3.1258	3.1246	3.1259	3.1265	3.1201	3.1273	3.1275	3.1280	3.1270
	4	3.1277	3.1275	3.1193	3.1287	3.1196	3.1196	3.1209	3.1279	3.1284	3.1288	3.1262
ARMA	5	3.1278	3.1281	3.1201	3.1199	3.1205	3.1162	3.1161	3.1186	3.1192	3.1296	3.1264
	6	3.1284	3.12899	3.1206	3.1205	3.1167	3.1164	3.1166	3.1193	3.1162	3.1199	3.1174
	7	3.1291	3.12908	3.1273	3.1216	3.1288	3.1168	3.1179	3.1142	3.1145	3.1171	3.1176
	8	3.1293	3.1281	3.1291	3.1219	3.1168	3.1181	3.1200	3.1158	3.1162	3.1167	3.1189
	9	3.1282	3.1287	3.1293	3.1274	3.12097	3.1238	3.1212	3.1202	3.1203	3.1112*	3.1129
	10	3.12899	3.1283	3.1258	3.1264	3.1236	3.1177	3.1167	3.1219	3.1173	3.1130	3.1138

						Rtop40 AIC						
	p/q	0	1	2	3	4	5	6	7	8	9	10
	0	3.4421	3.4426	3.4429	3.4388	3.4382	3.4374	3.4374	3.4378	3.4384	3.4391	3.4397
	1	3.4429	3.4435	3.4384	3.437	3.4377	3.4380	3.4381	3.4389	3.4388	3.4401	3.4405
	2	3.4436	3.4384	3.4374	3.4379	3.4379	3.4317	3.4391	3.4396	3.4312	3.4377	3.4322
	3	3.4401	3.4374	3.4380	3.4383	3.4383	3.4363	3.4370	3.4374	3.4381	3.4382	3.4329

	4	3.4399	3.4380	3.4387	3.4372	3.4370	3.4372	3.4368	3.4308	3.436	3.439	3.4304
ARMA	5	3.4377	3.4379	3.4385	3.4375	3.4379	3.4294	3.4301	3.4299	3.4304	3.4305	3.4306
	6	3.4379	3.4383	3.4386	3.4380	3.4386	3.4304	3.4275	3.4316	3.4313	3.4283	3.4346
	7	3.4383	3.4389	3.4322	3.4374	3.4325	3.4315	3.4321	3.4309	3.4306	3.4286	3.4318
	8	3.4389	3.4378	3.4378	3.4384	3.4320	3.4322	3.4294	3.4285	3.4294	3.4299	3.4270
	9	3.4381	3.4384	3.4381	3.4311	3.4356	3.4310	3.4261*	3.4267	3.4274	3.4278	3.4275
	10	3.4388	3.4389	3.4390	3.4313	3.4297	3.4315	3.4269	3.4262	3.4273	3.4279	3.4288

Conditional Variance Graphs

Figures 1-4 presents the graphs of conditional variance for ALSH, Resource 20, Industrial 25, and Top 40 indices. The conditional heteroscedasticity is clearly visible from these graphs. We note the spike in the condition variance around late 2008 and beginning 2009 in all indices modeled. This spikes shows the period of much turbulence in the markets from the major financial crisis that took place on that period.

Figure 3: Conditional Variance of GARCH model ALSH

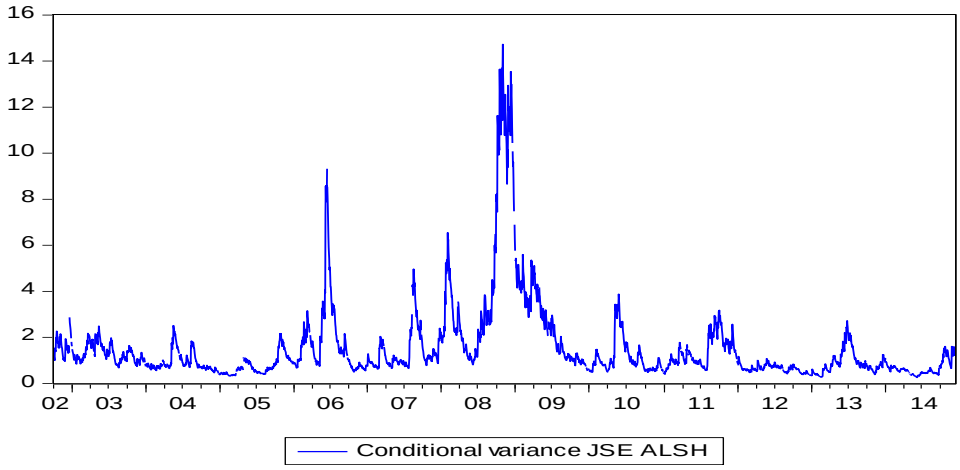


Figure 4: Conditional Variance of GARCH model Resource 20 Index

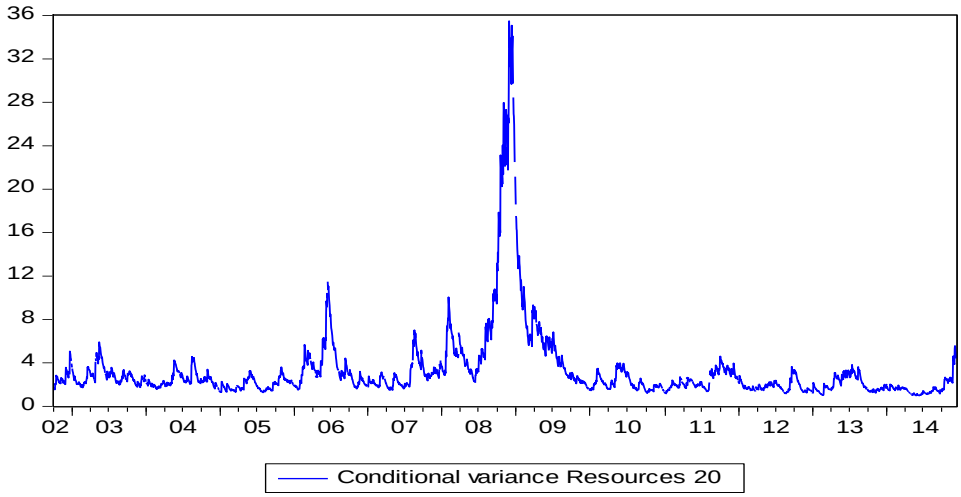


Figure 5: Conditional Variance of GARCH model Industrial 25

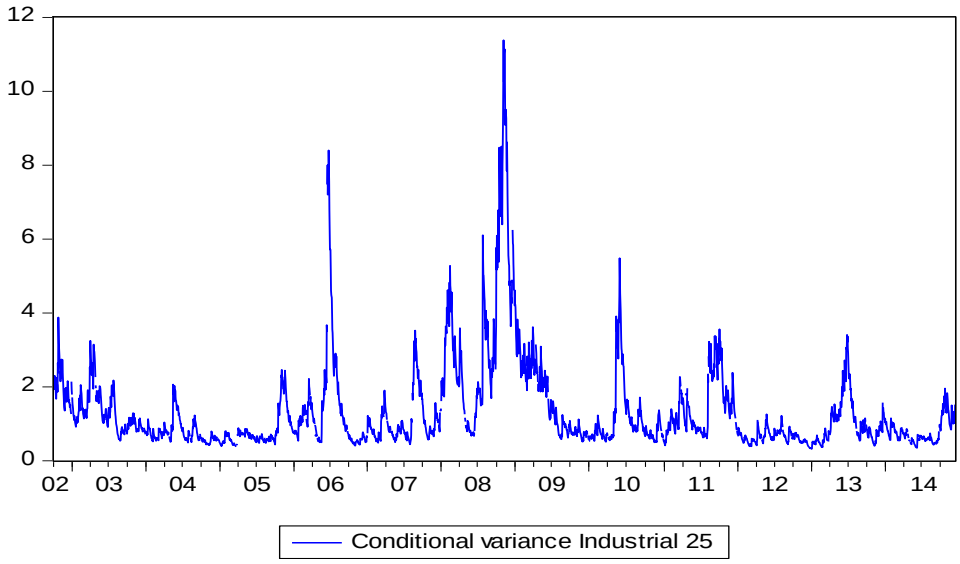


Figure 6: Conditional Variance model of Top 40 Index

