

Appendix D: Integrations of Velocity Distributions for Non-vegetated Zone Discharge

D.1: Method of Ikeda et al. (1991, 1994) and Tsujimoto and Kitamura (1994) for the Non-vegetated zone

The velocity for this method was given as:

$$V = V_{\inf} - \left(V_{\inf} - \sqrt{V_{\inf} \cdot V_{VegZone}} \right) e^{-Y/bm} \quad (28)$$

The non-vegetated zone discharge :

$$Q_c = \int_0^{bfo} V \cdot D \cdot dY = \int_0^{bfo} V_{\inf} \cdot D \cdot dY - \int_0^{bfo} \left(V_{\inf} - \sqrt{V_{\inf} \cdot V_{VegZone}} \right) e^{-Y/bm} \cdot D \cdot dY \quad (D1)$$

Noting that $\int e^{ax} dx = e^{ax}/a + c$

$$Q_c = \int_0^{bfo} V \cdot D \cdot [Y]_0^{bfo} - \left(V_{\inf} - \sqrt{V_{\inf} \cdot V_{VegZone}} \right) \cdot D \cdot \left[-Bm \cdot e^{-Y/Bm} \right]_0^{bfo} \quad (D2)$$

$$\therefore Qc = V_{\inf} \cdot D \cdot bfo + Bm \cdot D \cdot \left(V_{\inf} - \sqrt{V_{\inf} \cdot V_{VegZone}} \right) \left(e^{-bfo/Bm} - 1 \right) \quad (D3)$$

$$\therefore Qc = V_{\inf} \cdot D \cdot bfo - Bm \cdot D \cdot V_{\inf} \left(\sqrt{V_{VegZone}/V_{\inf}} - 1 \right) \left(e^{-bfo/Bm} - 1 \right) \quad (D4)$$

$$\therefore Qc = V_{\inf} \cdot D \cdot \left[bfo - Bm \cdot V_{\inf} \left(\sqrt{V_{VegZone}/V_{\inf}} - 1 \right) \left(e^{-bfo/Bm} - 1 \right) \right] \quad (36)$$

D.2: Method of Ikeda et al. (1991, 1994) and Tsujimoto and Kitamura (1994) for the Vegetation zone

$$V = V_{VegZone} + \left(\sqrt{V_{\inf} \cdot V_{VegZone}} - V_{VegZone} \right) e^{Y/Bs} \quad (28)$$

The Vegetation zone discharge:

$$Q_v = \int_{-Bv}^0 V \cdot D \cdot dY = \int_{-Bv}^0 V_{VegZone} \cdot D \cdot dY + \left(\sqrt{V_{\inf} \cdot V_{VegZone}} - V_{VegZone} \right) \cdot D \cdot \int_{-Bv}^0 e^{Y/Bs} dY \quad (D5)$$

$$\therefore Q_v = V_{VegZone} \cdot D \cdot [Y]_{-Bv}^0 + \left(\sqrt{V_{\inf} \cdot V_{VegZone}} - V_{VegZone} \right) \cdot D \cdot \left[Bs \cdot e^{Y/Bs} \right]_{-Bv}^0 \quad (D6)$$

$$\therefore Q_v = V_{VegZone} \cdot D \cdot Bv + Bs \cdot D \cdot \left(\sqrt{V_{\inf} \cdot V_{VegZone}} - V_{VegZone} \right) \cdot \left(1 - e^{-Bv/Bs} \right) \quad (D7)$$

$$\therefore Q_v = D \cdot Bv \left[V_{VegZone} + Bs/Bv \cdot \left(1 - e^{-Bv/Bs} \right) \cdot \left(\sqrt{V_{\inf} \cdot V_{VegZone}} - V_{VegZone} \right) \right] \quad (37)$$

D.3 Method of Nuding (1991, 1994) for Non-vegetated Zone limited to $V_{\max}$

$$\begin{aligned}
 Qc &= \int_{bm}^{bf} V \cdot D \, dy \\
 &= \int_{bm}^{TW} \frac{K \cdot u_v^*}{\log(V_{\max}/V_{\text{vegZone}})} \cdot \sqrt{\frac{h_T}{R_v \cdot bm}} \cdot D \cdot y^{1/2} \, dy + \int_{TW}^{bf} V_{\max} \cdot D \, dy \\
 &= \frac{K \cdot u_v^* \cdot D}{\log(V_{\max}/V_{\text{vegZone}})} \cdot \sqrt{\frac{h_T}{R_v \cdot bm}} \cdot \left[\frac{2}{3} y^{3/2} \right]_{bm}^{TW} + V_{\max} \cdot D \cdot [y]_{TW}^{bf}
 \end{aligned} \tag{D8}$$

$$Qc = V_{\max} \cdot D \cdot (bf - TW) + \frac{2 \cdot K \cdot u_v^* \cdot D \cdot [TW \cdot \sqrt{(TW/bm)} - bm]}{3 \cdot \log(V_{\max}/V_{\text{vegZone}})} \cdot \sqrt{\frac{h_T}{R_v}} \tag{20}$$