UNIVERSITY OF THE WITWATERSRAND

DEPARTMENT OF SURVEYING

FACULTY OF ENGINEERING

THE DISTRIBUTION OF THEODOLITE OBSERVATIONS ASSOCIATED WITH OPEN PIT MONITORING SURVEY

VOLUME I

ALEXANDER ALBANIS

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THE DISTRIBUTIONS OF THEODOLITE OBSERVATIONS ASSOCIATED WITH OPEN PIT MONITORING SURVEYS

Alexander Altanis

A dissertation submitted to the Faculty of Engineering, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the Degree of Master of Science in Engineering.

Johannesburg, 1987

DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in Engineering in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

Alexander Altanis

Signature:

14th day of September 198[illegible]

ABSTRACT

For practical reasons, the dissertation has been prepared in two Volumes. Volume I includes the main body of the research, and Volume II the data and the computer programs.

The purpose of this dissertation is to investigate the distribution of the errors associated with theodolite observations in open pit monitoring surveys.

The research methods and procedure employed can be separated into two categories:

1. The reduction of the theodolite observations, in order to obtain the best estimates of the mean observations and consequently of their associated residuals, and

2. The Normal, Truncated Normal and Inverse Gaussian distributions are compared and tested (using the Chi-Square goodness-of-fit test) in respect of their appropriateness to describe the distribution of theodolite observation residuals.

The conclusions of this dissertation may be classified into two groups:

1. For *Control points*, the 99% Truncated Normal distribution should be used, and

2. For *Face points*, the Normal distribution approximates the distribution of the small sample residuals while the 99% Truncated Normal best describes the larger samples of residuals.

In general, a 99% Truncated Normal distribution should be used to describe the distribution of the residuals.

DEDICATION

To my father and mother

ACKNOWLEDGEMENTS

I would like to express my gratitude and appreciation to those people who have contributed to this dissertation in various ways.

I wish to thank Mr T. O. Crompton who supervised this dissertation. His knowledge, ideas and suggestions provided invaluable guidance. His assistance, criticism of and advice on the presentation of the subject matter led to the completeness of this dissertation.

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Also, I am indebted to the authors and publishers who are mentioned below for their contribution in the reproduction of some Figures and Appendix A Tables:

Figures

Fig. 1, 3 : Adapted from McCormac, J.C. (1983) *Surveying Fundamentals*, Prentice-Hall Inc., U.S.A.

Fig. 2 : Adapted from Richardus, P. (1966) *Project Surveying*,

North-Holland Publishing Company, Amsterdam.

Fig. 8	: Adapted from Wetherill, G.B. (1972) *Elementary Statistical Methods*, second edition, Chapman and Hall Ltd, London.
Fig. 11	: Adapted from Johnson, N.L. and Kotz, S. (1970) *Continuous univariate distributions*, Vol.I, J. Wiley & Sons Inc., New York.
Fig. 13, 14, 15	: Adapted from Mikhail, E.M. (1976) *Observations and Least Squares*, Harper and Row Publishers, New York.
Fig. 16, 17, 18, 19, 20	: Adapted from Watt, I.B (1980) "De Hoek Works", Piketburg Cape

Tables

Table A1	: Adapted from Crow, E.L., Davis, F.A. and Maxfield, M.W. (1960) *Statistics manual*, Dover publications Inc., New York.
Table A2	: Adapted from Mikhail, E.M. (1976) *Observations and Least Squares*, Harper and Row, New York.
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LIST OF SYMBOLS

σ^2 : population variance [= Var(x)]

s^2 : sample variance

σ : population standard deviation

s : sample standard deviation

λ : dispersion for Inverse Gaussian distribution

μ : population mean [= E(x)]

$\bar{x}$: sample mean

e : true error

U : residuals from theodolite observations

$\bar{U}$: residuals mean

p.e. : probable error

$f(x)$: probability density function (p.d.f.)

$H(x)$: cumulative distribution function (c.d.f.)

$f(z)$: p.d.f. of the standardised distribution [N(0,1)]

$H(z)$: c.d.f. of N(0,1)

μ'_r : rth moment about a constant or about the zero (origin)

μ_r : rth central moment

γ_1 : coefficient of skewness

γ_2 : coefficient of Kurtosis or excess

$M(t)$: Moment Generating Function

$n(t)$: Cumulant Generating Function

K_i : cumulants

c : confidence coefficient

λ : measure of dispersion for Inverse Gaussian Distribution

H_0 : null hypothesis

H_A : alternative hypothesis

α : level of significance

O_i : observed class frequency for Chi-Square goodness-of-fit test

E_i : expected class frequency for Chi-Square goodness-of-fit test

Q : statistic for the Chi-Square goodness-of-fit test

r : one parameter for a p.d.f.

$\hat{r}$: the maximum likelihood estimator of the parameter r

$\mathbf{r}$: vector of parameters for a p.d.f.

$\hat{\mathbf{r}}$: the maximum likelihood estimator of the parameter's vector $\mathbf{r}$

df : degrees of freedom

VOLUME I

CHAPTER 1

1.0 INTRODUCTION

1.1 GENERAL DISCUSSION

The Normal probability distribution can often be used to approximate the probability distributions of random variables. This follows from the Central Limit Theorem, which states that, under rather general conditions, sums and means of samples of random measurements drawn from a population tend to possess, approximately, a bell-shaped distribution in repeated sampling. The approximation will become more accurate as this repetition becomes large.

The random errors associated with theodolite observations have, traditionally, been regarded as having a standard normal distribution. Certain statistical tests used in the analysis and adjustment of survey data have been based upon this assumption.

There are some old studies which doubt the validity of the Normal distribution to be the best fitting curve of the observations. H.F. Rainsford (1968) says that

> *"H.R. Hulme states: 'We do find that series of observations which have been made under identical conditions, as far as the observer is aware, do not in general yield residuals following a Gaussian curve. Instead they give an excess of small and large errors with a deficiency of intermediate ones ....'."*

Also, initial studies in the Department of Surveying, University of the Witwatersrand, Johannesburg (personal communication, K.S. Milford) have indicated that observations may not follow a conventional Normal distribution, but may be non-Normal in nature.

The dissertation will, therefore, be directed at determining the distribution of the residuals associated with observed directions, and to determine distributions for general use. In addition to the Normal distribution, the Truncated normal and the Inverse Gaussian distributions will be tested for use as the stochastic model for the errors.

1.2 FURTHER DISCUSSION

The dissertation has been undertaken to test the validity of the following assumption made in adjustment theory.

In the adjustment of engineering surveys, the most common model used is the parametric adjustment. This modal has the form of

$$\mathbf{u} = \mathbf{A}\mathbf{x} + \mathbf{f} \qquad (1.1)$$

where:

- $\mathbf{x}$: vector of unknown parameters
- $\mathbf{A}$: matrix of coefficients or design matrix
- $\mathbf{f}$: vector of constant terms derived from the observations
- $\mathbf{u}$: vector of residuals

Many authors assume that the observations are normally distributed (Rainsford, 1968, pp. 5-7), (Pope, 1976, p. 9), (Mikhail, 1976, p.61) and (Cross, 1984, pp. 92-93). In general, the distribution of theodolite observations is accepted as Normal, denoted by $N(\mu,\sigma^2)$, where μ is the mean of observations and σ^2 the variance.

Following from this model, the distribution of the estimated parameters is then multivariate normal; $N(\mathbf{x},\mathbf{\Sigma})$. The estimate of $\mathbf{x}$ is given by $\hat{\mathbf{x}}=(\mathbf{A}'\mathbf{W}\mathbf{A})^{-1}\mathbf{A}'\mathbf{W}\mathbf{f}$ where $(\mathbf{A}'\mathbf{W}\mathbf{A})^{-1}$ is the dispersion matrix of the parameters. This dispersion matrix, the estimate of $\mathbf{\Sigma}$, will only be normal if the initial assumptions concerning the observations are valid. This follows directly, since

$$E[(\hat{\mathbf{x}}-\mathbf{x})(\hat{\mathbf{x}}-\mathbf{x})'] = (\mathbf{A}'\mathbf{W}\mathbf{A})^{-1}\ \mathbf{A}'\mathbf{W}\mathbf{W}^{-1}\ \mathbf{W}\mathbf{A}(\mathbf{A}'\mathbf{W}\mathbf{A})^{-1}$$

$$= (\mathbf{A}'\mathbf{W}\mathbf{A})^{-1}$$

where $\mathbf{W}^{-1}$ is the variance-covariance matrix of the observations, and hence $f \sim N(\bar{\mathbf{l}},\mathbf{W}^{-1})$.

Also, for the estimation of weights in the adjustment process, it is usually assumed that the observations are normally distributed, and hence $w_{ii}=1/\sigma_i^2$. If the observations are independent then $w_{ij}=0$. The methods developed by Kubik (1970), Schaffrin (1981) and Sjöberg (1981) employ maximum likelihood estimation for $\mathbf{W}$ with the assumption that the underlying distribution is Normal.

These methods would not be totally valid if the distribution is not Normal.

On the other hand, this dissertation will test selected distributions (which are referred into section 1.0) to determine which of them best fits the residuals and consequently to make conclusions and recommendations for the modelling of the distribution of the observations.

It is possible to show that, for an adjustment model in which particular errors are ascribed to the observations, the expected distribution of the residuals of adjustment is identical to the distribution of the errors of the observations.

The model for the parametric adjustment is a general linear model (Graybill, 1976):

$$\mathbf{l} = \mathbf{A}\mathbf{x} + \mathbf{e} \quad (1.2)$$

where:

- $\mathbf{l}$: observations
- $\mathbf{A}$: design matrix
- $\mathbf{x}$: vector of unknowns parameters
- $\mathbf{e}$: true errors

with

$$E(\mathbf{l}) = \mathbf{A}\mathbf{x} \quad (1.3)$$

Given the distribution of the observations as $\mathbf{l} \sim N(\mu, \mathbf{\Sigma})$, the distribution of the true errors is simply $e \sim N(0, \Sigma)$. This can be easily showed.

$$\begin{aligned} Var(\mathbf{l}) &= E[(\mathbf{l}-E[\mathbf{l}])'(\mathbf{l}-E[\mathbf{l}])] \\ &= E[(\mathbf{l}-\mathbf{A}\mathbf{x})'(\mathbf{l}-\mathbf{A}\mathbf{x})] \\ &= E[\mathbf{e}'\mathbf{e}] \\ &= Var(\mathbf{e}) \end{aligned} \quad (1.4)$$

1.3 ASSUMPTIONS

The following assumptions have been made and constraints applied in carrying out the research.

1. The data are derived from one type of survey project, namely "the monitoring of open pit mines" and have been acquired by one observer.

2. The conventionally used reduction procedure will be applied to the observations, in order to test the method currently used. The residuals from this reduction procedure will be analysed, in order to determine their distribution, and hence the validity of assuming that the errors of observations are "normal".

3. Systematic errors due to theodolite imperfections are eliminated or minimised by the observational procedure discussed in chapter 3, while the systematic errors due to atmospheric effects will be regarded as part of the modelling process during the subsequent adjustment of the mean observations. Thus, for the model $F(l)$, it will be assumed that the function $F(l)$ (which represents the mathematical model) will include those systematic errors remaining.

   This assumption conforms with A.J. Pope (1976) who states

   > ". . . *systematic effects are either corrected for, cancelled out by the design of the observational procedure, or modelled and estimated in the adjustment itself.*"

   This dissertation will only considers the "raw" (observed) data.

4. Since the main aim of this research is to obtain an estimate of the distribution of the residuals, all residuals have been included. An exception is made for gross errors (big mistakes or blunders) -such as an error of 10°- which have been excluded, if encountered.

   Outliers can be regarded as observations from an alternate distribution (Richardus, 1984, pp. 73-77) and have, therefore, not been evaluated or excluded. This enables the overall pattern of the distribution to be determined. It would be possible at a latter stage to search for outliers, but this would be dependent upon the distribution accepted as best describing the behaviour of the residuals.

In one sense, the problem is similar to the old chicken and egg story; which comes first? if any form of outlier detection is employed, the distribution based on the remaining residuals will be effected by the residuals rejected. Alternatively, if "outliers" are included, the distribution may also be biased.

It was, therefore, decided to include all the residuals (except blunders), so that the overall distribution could be determined.

1.4 PROJECT OUTLINE

The following main sub-divisions will be covered within the dissertation.

1. **Background and review:** This section provides a review of some of the definitions and concepts used in statistical distributions.

2. **Theodolite observations:** As these observations form the basis for the data used in the dissertation, theodolite measurements will be discussed in terms of errors and methods of reduction employed in order to obtain the best estimates of the mean observations and their associated residuals.

3. **Statistical distribution and the modelling of data:** Various statistical distributions, including Normal, Truncated Normal and Inverse Gaussian distributions will be considered. The Chi-Square

goodness-of-fit test will be employed for the fitting of the data (observational residuals) to these distributions.

4. Computer programs: For the purpose of the reduction of the theodolite observations and for the Chi-Square test, two computer programs have been written: the "THEO" and "HI DI TEST" programs, respectively.

5. Conclusions and recommendations: This section is based on the findings of the analysis carried out in the dissertation and identifies the distributions that best quantify the residuals of the observations.

For practical reasons, this dissertation has been prepared in two volumes (Volume I, Volume II)

- Volume I includes the theoretical background relating to this dissertation, statistical concepts (Chapter 1 to 5) and discussion and analysis of the results, conclusions and recommendations (Chapter 6 and 7). Tables e.g. for the theoretical values of the Chi-Square goodness-of-fit test are included in the Appendix A and, in addition, examples of the output of "THEO" and "HI_DI_TEST" computer programs are listed in Appendices B and C. The references and bibliography are cited in Appendix D.

- Volume II includes the data (Appendix E), flow charts and listing of the computer programs (in Appendix F for the "THEO" program and in Appendix G for the "HI DI TEST" program).

CHAPTER 2

2.0 GENERAL CONCEPTS AND REVIEW

2.1 GENERAL REVIEW OF SURVEY OBSERVATIONS

Measurement (British Standards Institute, 1980) is a set of experimental operations for the purpose of determining the value of a quantity.

Observation is an operation which consists in noting the indication of a measuring instrument.

However, it is common in both statistical and survey literature to use the term "observations" for measured quantities. This is particularly true of data relating to directions, where surveyors generally use observations as opposed to the term measurements, which is generally reserved for "distance" data. For the purposes of this dissertation both of these terms will be used interchangeably.

As far as surveying is concerned, measurements are carried out for a variety of purposes, such as the determination of the shape of the earth, to map at a certain scale part of the earth's surface, the setting out of construction works, and surveys for navigational and cadastral purposes.

The observations, being the recorded values resulting from the measurements, are generally not ready for use without first having been processed in some manner. For all surveys, the precision or repeatability of the observations should be known. This information is also generally required for the post-processing of the measured quantities.

However, it is well known that, in general, two or more measurements of the same quantity do not yield the same result but differ slightly. When a sufficiently large number of measurements is made under the same conditions, it appears that the observations follow a set pattern, which can be described by the following characteristics.

a. All observations are grouped about a central value.
b. Positive and negative deviations from this central value are almost equally frequent.
c. Small deviations are more frequent than large ones.

The residuals (being the differences between the observed and expected values) may be grouped according to their magnitude into classes and their distribution shown graphically in a **histogram.**

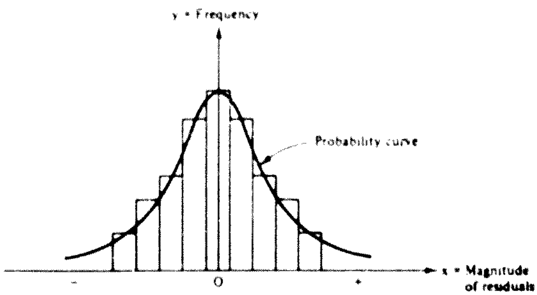

Figure 1. Histogram or frequency distribution diagram

Plotting the class boundaries at equal intervals along the abscissa, the frequency of the residuals in the interval is represented by the area of the rectangle with the base line equal to the class interval and the height equal to the number of observations (residuals) in the interval. An example of a histogram is given in Fig. 1.

The residuals associated with observations in surveying, when plotted in this manner, show very similar histograms: the class with the maximum number of residuals generally is situated near the centre of the figure. The lower frequencies are grouped approximately symmetrically about this centre.

2.2 THE NORMAL DISTRIBUTION IN THE FIELD OF SURVEYING

In survey it is generally accepted that the residuals associated with the measured quantities are distributed according to the **Normal (Gauss) distribution** (McCormac, 1983), (Richardus, 1984).

The general form of the probability density function (p.d.f.) is a bell-shaped curve which is symmetrical about the expected value (μ) as is shown in Fig. 1. Also, the curve is asymptotic to the abscissa at both right and left hand tails. This means that the theoretical range of the residuals is infinite. In practice, the tail regions, where $|(x-\mu)|>3\sigma$ (σ is defined below), appear to have little significance for the description of the distributions of the residuals. These regions usually, however, play an important part in outlier detection.

An "outlier" is an observation which does not appear to conform with the rest of the set. For a single univariate sample, an outlier is an extreme observation, rather larger or smaller than the rest of the sample. There may be several such measurements in a sample which do not appear to come from the same probability distribution as the rest of the observations. The actual problem arises in the decision as to whether or not some of the observations are outliers, and consequently, to decide if these observations could be taken into consideration for the probability distribution or whether they must be discarded. On the other hand, an obvious blunder has absolutely nothing to do with the determination of the distribution and it is discarded at once.

For further reference to the treatment of outliers see for example W. Baarda (1968), A.J. Pope (1976) and H. De Heus (1982).

2.3 VARIANCE AND STANDARD DEVIATION

The **variance of a population** (σ^2) of N measurements $x_1, x_2, \ldots, x_N$, is defined to be the average of the square of the deviations of the measurements about their mean (μ), and is given by the formula (Mendenhall, 1979):

$$\sigma^2 = \sum_{i=1}^{N} (x_i - \mu)^2 / N \tag{2.1}$$

The **variance of a sample** (s^2) of n measurements $x_1, x_2, \ldots, x_n$, is defined to be the sum of the squared deviations of the measurements about their mean x divided by (n-1). The sample variance is given by the formula (Mendenhall, 1979):

$$s^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 / (n-1) \tag{2.2}$$

where $\bar{x}$ is the sample mean.

The **standard deviation** of a set of n measurements $x_1, x_2, \ldots, x_n$, is equal to the positive square root of the variance (Mendenhall, 1979). The sample standard deviation is denoted by the symbol s, where

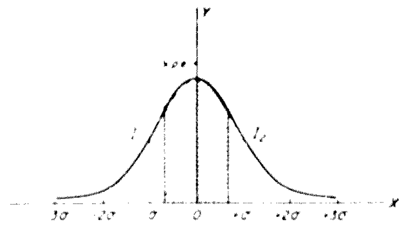

Figure 2. The p.d.f. of the Normal distribution

$$s = +\sqrt{s^2} \qquad (2.3)$$

and the population standard deviation by the symbol σ, where

$$\sigma = +\sqrt{\sigma^2} \qquad (2.4)$$

Consider the normal distribution curve as shown in Fig. 2.

The ordinate at the origin coincides with the axis of symmetry. The mathematics of statistics shows that the abscissa of the inflection points I_1 and I_2 are equal to the standard deviation (Richardus, 1984).

The nature of the relationship between the abscissa and the ordinate implies that the standard deviation, σ, is a measure of the scatter of the residuals. If the residuals cluster closely about the mean, the standard deviation will be small; conversely, if the scatter is large σ will be large.

Dispersion is also used as a measure of scatter. This term is defined as the phenomenon exhibited by a measuring instrument which gives different indications in a series of measurements of the same value of the quantity measured (British Standards Institute, 1980).

2.4 ACCURACY AND PRECISION

Accuracy may be defined as the extent to which an estimate approaches its true value. That is, it denotes how close a measurement is to the true value of the quantity (McCormac, 1983), (Mikhail, 1976).

Precision is the closeness of one measurement to another: it is the degree of conformity among a set of observations. An indication of precision is the spread of the probability distribution (McCormac, 1983), (Mikhail, 1976).

It does not necessarily follow that better precision means better accuracy.

Observations of a high precision belong to a distribution with a small standard deviation. Observations of a low precision belong to a normal distribution with a large standard deviation.

Historically, many authors in surveying indicated the precision of observations by the **probable error (p.e.)**. This gives the values $x_1=-p.e.$ and $x_2=+p.e.$ as limits between which an observation may fall with a probability of 50% (Fig. 3).

Statisticians have, for several reasons, abandoned the probable error in favour of the standard deviation, σ, as an indication of the precision of the observations. The most practical reason is that the standard deviation must be calculated in order to find the probable error. Therefore, σ is used for all computations concerning precision in this dissertation.

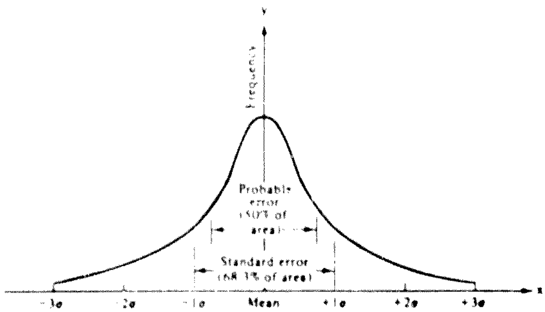

Figure 3. The probable and standard errors

This dissertation deals with the precision of the observations (which is directly related to the dispersion of a distribution and is represented by the value of the standard deviation σ), and not with accuracy (which reflects the closeness of a location statistic to the value of the parameter for which it is an estimate). Systematic errors directly associated with the accuracy of the observations and modelling errors will not be considered. These are generally non-normal in type.

2.5 ERRORS AND MISTAKES

The differences between measured quantities and the true values of those quantities are classified as either errors or mistakes (McCormac, 1983).

An **error** is a difference from a true value caused by the imperfection of a person's senses, by the imperfection of his equipment, or by environmental effects. Errors cannot be eliminated, but they can be minimised by careful work combined with the application of certain numerical reductions. Classically, errors are usually separated into two categories:

a. Systematic errors
b. Random errors

A **mistake** or **blunder** is a difference from a true value caused by the inattention of the surveyor. That is, mistakes are caused by the carelessness of the surveyor and this can be eliminated by careful checking

Mikhail (1976) says that

> *"From a statistical point of view, blunders are observations that cannot be considered as belonging to the same samples from the distribution in question. They should not therefore be used together with other observations."*

In other words, mistakes (blunders) should not be considered as a part of the observational sample, and for this reason the dissertation assumes that all mistakes (obvious blunders) have been eliminated from the initial observations used in the first computer program ("THEO") which deals with the reduction of observations.

2.5.1 SOURCES OF ERRORS

Errors in measurement are generally said to be personal, instrumental and natural. Some errors, however, do not clearly fit into just one of these categories (McCormac, 1983) and may be due to a combination of factors.

Personal errors occur because no surveyor has perfect senses of sight and touch. For instance, in estimating the fractional part of a scale, he cannot read it perfectly and the measurement will generally be either a little larger or smaller than the true value.

Instrumental errors occur because instruments cannot be manufactured perfectly and the different parts of the instruments cannot be adjusted exactly with respect to each other. Thus, as the observer cannot read the scale perfectly, similarly the manufacturer cannot make a perfect scale.

Natural errors are caused by temperature, wind, humidity, magnetic variations. As an example, a steel tape may increase in length because of the increasing of temperature (systematic errors). The surveyor cannot normally remove the cause of errors such as these, but he can minimise their effects by using good judgement and applying proper reductions.

2.5.2 SYSTEMATIC ERRORS

A **systematic** or **cumulative error** (McCormac 1983) generally, under constant conditions, remains the same as to sign and magnitude. For instance, if a steel tape is 50 mm too short, each time the tape is used the same error will occur. If the full tape length is used six times, the error accumulates and totals six times the error of one measurement.

The systematic influence may be reduced or eliminated by several methods. P. Richardus (1984) states:

"1. It may be reduced to a magnitude which is negligible by a better choice of the model or improvement of the instruments.

2. By a determination of the error independently of the measurements and the application of a corresponding correction to the observation.

3. The systematic influence may be derived by a special method from the processing of the data provided by the measurements.

4. By arrangement of the order of the measurements in a certain manner so that the systematic effect occurs an equal number of times with a positive and negative sign. The mean of these observations will then be free of this effect."

2.5.3 RANDOM ERRORS

A **random** or **observational error** is one whose magnitude and sign are attributed to factors beyond the control of surveyor, and can generally be attributed to personal or natural causes (see 2.4.1). Since these errors are just as likely to have one sign as the other, the mean of the measurements will approach the true value as the number of measurements increases. That is, for every measurement such as direction, elevation or distance, there is some uncertainty even if the measured values have been corrected for blunders and systematic errors. Random errors still remain which cannot be exactly determined or eliminated. Thus, the random errors may be described as the differences between the true values and the corrected measurements.

In statistics, the data (observations) are considered to be a sample selected from an infinitely large number of (possible) repetitions of a measurement for which the mean will be the true

"1. It may be reduced to a magnitude which is negligible by a better choice of the model or improvement of the instruments.

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In statistics, the data (observations) are considered to be a sample selected from an infinitely large number of (possible) repetitions of a measurement for which the mean will be the true value. The theory of probability can, therefore, be used to study the behaviour of random errors.

The model used to describe errors in surveying is generally restricted to the normal distribution, as the repeated observations usually display a "normal" frequency distribution. In principle, however, the term "random error" is not necessarily restricted to a normal distribution.

CHAPTER 3

3.0 SURVEY OBSERVATIONS

3.1 INTRODUCTION

In order to obtain the best estimates of the mean theodolite observations, it is necessary to first consider the theodolite errors associated with the measurements, and the methods of reducing the measurements in order to obtain both the mean and corresponding residuals.

3.2 THEODOLITE OBSERVATIONS

3.2.1 PURPOSE OF STATION ADJUSTMENT

The theodolite is used to take readings in order to determine directions and angles. For the reduction of these readings (observations) either the method of directions or that of angles is used. In South Africa, the commonly used method is that of directions. The adjustment of the observations is usually called a **station adjustment**.

In latter sections, two similar methods of reducing horizontal directions will be reviewed. A brief review of two methods of angles is given in sub-sections 3.4.3 and 3.4.4.

3.2.2 MEASUREMENTS OF HORIZONTAL DIRECTIONS

Consider the model in which the theodolite is set up at the station point K and the horizontal directions A,B,C,D are to be observed (Fig. 4).

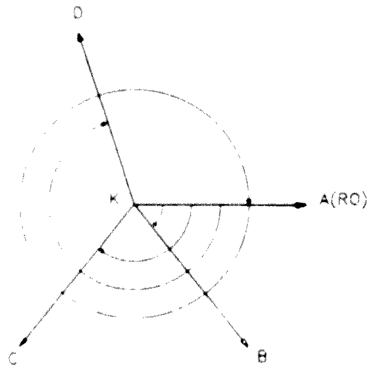

Figure 4. Observations from K to the points A,B,C,D

A signal or beacon is selected as the Reference Object (R.O.), and the theodolite horizontal circle readings to all other points observed are referred to this point. The R.O. may or may not be one of the stations A,B,C or D and should be selected so that, in general, accurate, repeated observations may be made to it. Assume A is selected. The theodolite is set up at K, centred, levelled and sighted on A, with the telescope on Circle Left (CL). The horizontal circle is set so that it is close to some predetermined value, e.g. 269° 39' 52.2". The remaining points are then observed in turn, in a clockwise sense, closing on the R.O. In general, this second R.O. reading will disagree with the initial reading by a small amount, due to observational and environmental errors. If the difference falls within the tolerance specified for the specific class of survey, the observations are accepted. If not, the error may be attributed to sub-standard observing or movement of the instrument, and the observations are possibly rejected. On completion of the circle left observations, the same procedure is repeated on the **C**ircle **R**ight (CR), but this time in an anti-clockwise manner. The difference in readings, between CL and CR, should be 180° (Olliver and Clendinning, 1978).

A set of observations on one face (CL or CR readings) to each of the pointings required constitutes a **round** (or tour). In Figure 4 this will give a single reading to each of A,B,C,D and A points (CL readings). It must be clarified that the point A is considered to be the R.O. in this example, and is the only point which is observed twice in the round. This will be re[illegible] by a reverse round on CR to A,D,C,B and A (the R.O. is again observed twice). The combination of these two rounds (CL and CR readings) constitutes an **arc**.

3.3 THEODOLITE ERRORS

Defects due to the mechanical and optical components of the theodolite affects measurements made with the instrument. These can be divided into two categories (Cooper, 1982):

- defects of adjustment (maladjustments)
- defects of construction (malconstructions)

The errors which result from these defects are usually referred as **theodolite errors**.

Generally speaking, the best values for the reduced observations can only be obtained by

1. eliminating blunders (mistakes) by comparing different readings of the same quantity.

2. applying corrections for all known sources of constant errors.

   Some authors do not separate the constant error from the systematic one but simply refer this as a sub-division of the systematic effects. Mikhail (1976) says that

   > *"Systematic effects take on different forms depending on the value and sign of each of the effects. If the value and sign remain the same all through the measuring process, we would have the so-called constant error."*

   as opposed to Rainsford (1968) who considers the constant errors in separate category from that of the systematic ones.

3. correcting for all sources of systematic errors, when this is possible.

A brief review of these defects for horizontal readings is outlined the next sub-sections.

3.3.1 MALADJUSTMENTS

It is quite impossible for a theodolite to be in perfect adjustment. Consequently, different types of maladjustments (Cooper, 1982, pp. 128-137) can be found since:

- the vertical (primary or rotation) axis is not really vertical.

- the line of collimation is not perpendicular to trunnion (secondary or horizontal) axis.
- the cross-hairs are inclined (orientation of the reticule).
- the existence of parallax
- the optical plummet is not in adjustment.

Good survey practice will eliminate most of the errors, the notable exception being non-verticality of the primary axis. However, even if the necessary adjustments have been carried out and the correct observing procedure followed, residual errors will be present. M.A.R. Cooper (1982, pp. 138-146) discusses these affects in detail.

It is assumed that the data used for analysis purposes have been observed and reduced correctly.

3.3.2 MALCONSTRUCTIONS

These defects contain all those errors which the user cannot normally eliminate by adjustments, but which can be reduced by suitable observing procedures or by computation of "correction" terms

Errors of this type can be caused by the following (Cooper, 1982):

- the trunnion axis is not perpendicular to vertical axis.
- the planes of the circles are not perpendicular to their respective axes (this error is, generally, negligible in practice).
- circle eccentricities.
- inclination of alidade axis in its sleeve.
- circle graduation errors.
- axis strain.

o errors in the optical micrometer.

See M.A.R. Cooper (1982, pp.147-169) for a detailed description of the effects, errors and the appropriate actions for this class of theodolite defects.

The surveyor is quite well aware of the existence of systematic effects and usually strives to minimise their presence through instrument calibration, observational procedures and modelling the atmospheric and other effects.

Some of the systematic errors due to instrumental factors can be eliminated by observing on CL and CR and using the mean value.

3.3.3 CONCLUSIONS CONCERNING THE THEODOLITE ERRORS

The data used in this dissertation will be assumed to be free from most of the errors discussed in the previous sub-sections. Professor I.B. Watt, who supplied the data, carried out the correct observational procedures in order to minimise the affect of instrumental errors. Systematic errors due to atmospheric conditions are considered to be an additional factor in the mathematical model, as outlined in the introductory chapter. The remaining errors will be considered as being significantly free from systematic effects.

3.4 REDUCTION OF THEODOLITE OBSERVATIONS

3.4.1 FIRST METHOD - DIRECTIONS (1)

This is the most commonly known method (Olliver and Clendinning, 1978).

1) Mean values: The object of changing face, direction of rotation and zero is to eliminate graduation errors of the circle and micrometer, and instrumental errors due to mechanical imperfections. Therefore, by taking the mean of the CL and CR readings the effects of the systematic errors are eliminated or minimised.

2) Orientation correction: When more than one arc has been taken at a station point, then the best possible value of the observed direction is the mean value obtained from the means of the individual arcs. As the means of the various arcs will relate to different parts of the horizontal circle, the means of the second and subsequent arcs are referred to the mean of the first arc.

Therefore, the orientation correction of the second, third or fourth arc will be found as the average of the differences of the mean values of the respective observation points between the first arc and second, or subsequent arcs.

3) Reduced mean value: Having found the mean of each arc and the orientation correction as outlined previously, the mean values of the second and the subsequent arcs are reduced to the first arc.

4) Final mean: The final mean value for each observed point, is obtained as the average of all the individual arc means.

The residuals have been abstracted at this stage (see sub-section 3.4.1.1). The following additional steps are often included for the reduction of theodolite observations.

5) **Horizon closure error**: In general, the final mean values relating to the R.O. point will not be in exact agreement. The difference is known as the horizon closure error.

The horizon closure error may be regarded as an observational check, or may be used to adjust the individual observed mean directions.

6) **Correction**: Since, theoretically, the final mean values to the R.O. should be identical, any correction is derived as the quotient of the of horizon closure erro. and the number of the different observation points.

7) **Reduced observations**: The horizon closure error is eliminated by applying to the final mean values a proportional correction.

It must be noticed that the reduced observations (step 7) are correlated since the horizon closure error (R.O. misclosure error) has been distributed according to steps 5, 6.

This method is applied to obtain the adjusted values for the means of the observations (directions).

3.4.1.1 Calculation of observational residuals

The residuals are calculated as the difference between the final mean values of the observations (step 4) and the reduced mean values (step 3); that is:

$$\text{Residuals} = \underset{(\text{ step } 4\ )}{\text{Final mean values}} - \underset{(\text{ step } 3\ )}{\text{Reduced mean values}} \tag{3.1}$$

At this point, it must be noticed that both step 3 and step 4, as discussed in the previous section, are independent of the influence of the R.O. misclosure error. However, the values of the observations calculated up to step 5 are slightly correlated within each arc for the small samples (<30 residuals) as opposed to large samples (>30 residuals) where the correlation tends to be zero, because of the orientation correction, and consequently, the residuals which are calculated in equation (3.1) are very slightly correlated (see example given in Appendix C).

The affect of correlation is examined in more detail in the subsection 3.4.2.1.

The total number of the observational residuals is:

$$\text{Tot} = m \times n \tag{3.2}$$

where:

m = number of arcs

n = number of observation points including R.O.

It is probably worthwhile to differentiate between the analysis of all residuals, and the statistics corresponding to an individual mean direction. Therefore:

1. The standard deviation of a direction as computed from the pooled sample of all directions is given by:

$$s = \pm \left[\frac{\sum_{j=1}^{m} \sum_{i=1}^{n} U^2(j,i)}{m \times n - (m+n-1)} \right]^{\frac{1}{2}} \tag{3.3}$$

2. The standard deviation of an individual direction computed from the sample pertaining to the specific direction is given by formula (3.4).

$$s = \pm\left[\frac{\sum_{j=1}^{m} (x_{j,i}-\bar{x}_i)^2}{m-1}\right]^{\frac{1}{2}} \qquad (3.4)$$

where:

$x_{j,i}$: the ith individual reduced mean direction of the arc j

$\bar{x}_i$: the ith final mean

3. In contrast to the statistics relating to an observation, it is also possible to obtain statistics associated with the residuals. The object of this dissertation is to analyse the distribution of the residuals (obtained as a difference between an individual observed direction and the mean direction for any specific line).

The residuals have, therefore, been combined into large samples and viewed as individual data points drawn from a common distribution of the residuals. From the manner in which the residuals have been calculated, the mean value of this distribution is known to be zero, and the standard deviation is unknown. The formula (3.5) has been used to estimate the standard deviation of the residuals (in the "HI_DI_TEST" program), and i given below:

$$s = \pm\left[\frac{\sum_{j=1}^{m} \sum_{i=i}^{n} [\, U(j,i)-\bar{U} \,]^2}{m \times n}\right]^{\frac{1}{2}} \qquad (3.5)$$

where:

$U(j,i)$: the ith residual for arc j

$\bar{U}$: the mean of the residuals ($\bar{U}$=0)

The "THEO" program (Reduction of Theodolite observations) has been written in accordance with the procedure described in sections 3.4.1 and 3.4.1.1 Some more specific information is given in the Volume II - Appendix F.

3.4.2 SECOND METHOD - DIRECTIONS (2)

This method is usually called: "*Adjustment of observations by the method of directions*" (Hazay, 1970).

Two sets of quantities are considered unknown: the directions and the orientation corrections.

The readings for the directions are different because of the errors of the measurements, and because each arc begins on a different position of zero. Therefore, these differences of "zeros", which are essentially the orientation corrections, must be considered as unknown values.

The observation equation for a direction i in an arc j is:

$$L_{ij} = D_i - z_j \qquad (3.6)$$

where:

L_{ij} : the reading of the ith direction for arc j

D_i : the adjusted value of the respective direction L_i

z_j : the adjusted value of the respective orientation correction for arc j

$i = 1,2,\ldots,n$ (n : number of directions)

$j = 2,3,\ldots,m$ (m : number of arcs)

Since the k ($n{*}m{=}k$, $n{+}m{-}1{=}p$) equations of (3.6) are inconsistent, and L_{ij} are the results of the measurements which have random errors, it means that the equations (3.6) may be written as follows:

$$D_i - z_j = L_{ij} + u_{ij} \tag{3.7}$$

Therefore, the equations (3.7) can be written using matrices,as follows:

$$\mathbf{A}_{p}^{k}\ \mathbf{x}_{1}^{p} = \mathbf{b}_{1}^{k} + \mathbf{u}_{1}^{k} \tag{3.8}$$

where:

$\mathbf{A}_{p}^{k}$: the matrix of the coefficients of the unknown quantities

$\mathbf{x}_{1}^{p}$: the vector of the estimates of unknown quantities $D_1,D_2,\ldots,D_n,z_2,\ldots,z_m$

$\mathbf{b}_{1}^{k}$: the vector of the observations $L_{11},L_{21},\ldots,L_{n1},\ldots,L_{1m},L_{2m},\ldots,L_{nm}$

$\mathbf{u}_{1}^{k}$: the vector of the residuals $u_{11},u_{21},\ldots,u_{n1},\ldots,u_{1m},u_{2m},\ldots,u_{nm}$

According to the theory of Least Squares the sum of the squares of the residuals must be minimised, that is:

$$[uu] = \mathbf{u}'\mathbf{u} = \text{minimum.} \tag{3.9}$$

This implies that:

$$\mathbf{A}'\mathbf{A}\hat{\mathbf{x}} = \mathbf{A}'\mathbf{b} \tag{3.10}$$

which are the normal equations. $\mathbf{A}'$ denotes the transpose of $\mathbf{A}$ and $\hat{\mathbf{x}}$ is the least squares estimate of $\mathbf{x}$.

The solution of (3.10) gives $D_1, D_2, \ldots, D_n, z_2, \ldots, z_m$.

The number of redundant measurements (degrees of freedom) is obtained as a difference between the total number of observations $(n*m)$ and number of unknowns $n+(m-1)$. Therefore, the standard deviation σ of all directions, individual directions, and all residuals are given by the formulae of (3.3), (3.4) and (3.5) respectively.

3.4.2.1 *Affect of the orientation correction*

Let 5 directions be observed on 2 arcs. This implies 6 unknowns (5 directions + orientation correction for the second arc), and according to observation equation (3.7) the matrix $\mathbf{A}$ and the vector $\mathbf{x}$ will be:

$$\mathop{\mathbf{A}}^{10}_{6} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ & & \vdots & & & \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 & -1 \\ 0 & 1 & 0 & 0 & 0 & -1 \\ & & \vdots & & & \\ 0 & 0 & 0 & 0 & 1 & -1 \end{bmatrix} , \qquad \mathop{\mathbf{x}}^{6}_{1} = \begin{bmatrix} D_1 \\ D_2 \\ D_3 \\ D_4 \\ D_5 \\ z_2 \end{bmatrix}$$

Therefore:

$$
(\mathbf{A}'\mathbf{A})_{6}^{6} = \begin{bmatrix} 2 & . & . & . & . & . \\ 0 & 2 & & . & . & . \\ 0 & 0 & 2 & . & . & . \\ 0 & 0 & 0 & 2 & . & . \\ 0 & 0 & 0 & 0 & 2 & . \\ -1 & -1 & -1 & -1 & -1 & 5 \end{bmatrix}
$$

and

$$
\left[(\mathbf{A}'\mathbf{A})^{-1} \right]_{6}^{6} = (1/10) \begin{bmatrix} 6 & . & . & . & . & . \\ 1 & 6 & . & . & . & . \\ 1 & 1 & 6 & . & . & . \\ 1 & 1 & 1 & 6 & . & . \\ 1 & 1 & 1 & 1 & 6 & . \\ 2 & 2 & 2 & 2 & 2 & 6 \end{bmatrix}
$$

where $(\mathbf{A}'\mathbf{A})^{-1}$ is the variance-covariance matrix, and therefore:

$$
\sigma_{D_1D_2} = \sigma_{D_1D_3} = \ldots = \sigma_{D_1D_5} = \ldots = \sigma_{D_4D_5} = 1/10
$$

$$
\sigma_{D_1}{}^2 = \sigma_{D_2}{}^2 = \ldots = \sigma_{D_5}{}^2 = 6/10
$$

Bearing in mind that the correlation coefficient is given by the formula

$$
\rho = \frac{\sigma_{D_i D_j}}{\sigma_{D_i}\ \sigma_{D_j}} \tag{3.11}
$$

where:

$i = 1,2,\ldots,n-1$
$j = 2,3,\ldots,n$

the correlation between the directions D_1, D_2, D_3, D_4, D_5 is:

$$
\rho = 1/4
$$

In general, if D_i $(i=1,2,\ldots,n)$ are the final reduced directions including R.O. and z_j $(j=2,3,\ldots,m)$ are the orientation corrections applied to the second and subsequent arcs, then **A** is a $(m \times n) \times (n+m-1)$ matrix, and it can be shown that the elements a_{ij} of the $(\mathbf{A}'\mathbf{A})$ matrix will be equal to:

$$
\begin{aligned}
a_{ij} = \; & m && \text{if} \quad i = j \quad ; \quad i,j \le n \\
& n && \text{if} \quad i = j \quad ; \quad i,j > n \\
& 0 && \text{if} \quad i \ne j \quad ; \quad i,j \le n \\
& -1 && \text{if} \quad i \ne j \quad ; \quad i \text{ or } j > n \\
& 0 && \text{if} \quad i \ne j \quad ; \quad i \text{ and } j > n
\end{aligned}
$$

The elements a_{ij}^{-1} of the inverse matrix $(\mathbf{A}'\mathbf{A})^{-1}$ will be given as:

$$
\begin{aligned}
a_{ij}^{-1} & = (n+m-1)/(n\,m) && \text{if} \quad i = j \quad ; \quad i,j \le n \\
& = 2/n && \text{if} \quad i = j \quad ; \quad i,j > n \\
& = (m-1)/(n\,m) && \text{if} \quad i \ne j \quad ; \quad i,j \le n \\
& = 1/n && \text{if} \quad i \ne j \quad ; \quad i,j > n \\
& = -1/n && \text{if} \quad i \ne j \quad ; \quad \text{and} \\
& && i > n\,,\; j \le n \quad \text{or} \quad i \le n\,,\; j > n
\end{aligned}
$$

Therefore, the correlation coefficient between $D_1,\ldots,D_n$ will be given by:

$$\rho = \frac{(m-1)/(n\,m)}{[(n+m-1)/(n\,m)]^{\frac{1}{2}}\,[(n+m-1)/(n\,m)]^{\frac{1}{2}}}$$

$$= \frac{m-1}{n+m-1} \qquad (3.12)$$

Figure 5 depicts the value of the correlation coefficient for both 2 and 4 arc samples. As can be seen from the graph, the correlation is small (generally less than 0.1 with more than 9 observed directions in the case of 2 arcs, and 27 sighted direction in the case of 4 arcs) and tends to be zero as the number of the observed directions, and therefore residuals, increases.

For the smallest sample used in the dissertation ($m=2$, $n=8$ - see section 6.2.1)the value of ρ is 0.11 (formula 3.12), and consequently, it can be considered that the correlation between the observations, and therefore, between the residuals, due to the orientation correction is generally small and for the large samples (>30 sighted directions) this approaches zero.

3.4.2.2 Comparison between first and second method

It is now possible to show that the first method is identical to the second one.

1. First Method: Suppose:

- n : directions be observed
- m : number of arcs
- L_i : mean values of the observation of the ith direction on the jth arc (sub-section 3.4.1 - step 1)
- z_j : orientation correction applied to the second and

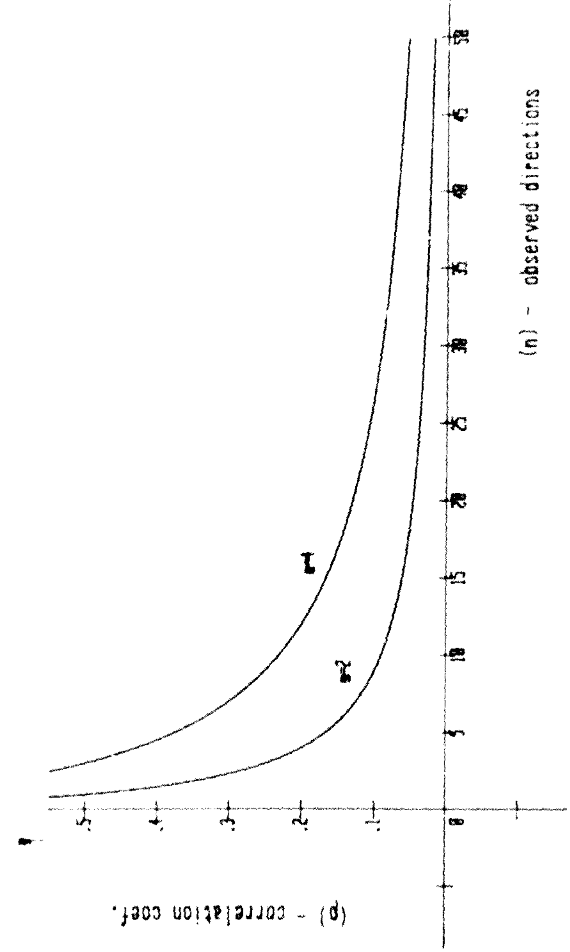

Figure 5. Correlation coefficient for m=2 and m=4 arcs.

subsequent arcs (sub-section 3.4.1 - step 2)

$\tilde{L}_{ij}$: reduced mean value of the observations of the second and subsequent arcs (sub-section 3.4.1- - step 3)

D_i : final mean values of the observations (sub-section 3.4.1 - step 4)

If Δ_{ij} is the difference of the mean values of the respective observation points between the first arc and second, or subsequent arcs, then:

$$D_{ij} = L_{ij} - L_{i1} \tag{3.13}$$

where:

$i=1,2,...,n$
$j=2,3,...,m$

Therefore:

$$z_j = \sum_{i=1}^{n} \frac{\Delta_{ij}}{n} \tag{3.14}$$

where:

$i=1,2,...,n$
$j=2,3,...,m$

Then the reduced mean values of the observations will be:

$$\tilde{L}_{ij} = L_{ij} + z_j \tag{3.15}$$

where:

$i=1,2,...,n$
$j=2,3,...,m$

and the final mean of them will be:

$$D_i = (1/m)\left[\sum_{j=2}^{m} \tilde{L}_{ij} + L_{i1}\right] \tag{3.16}$$

where:

$i=1,2,\ldots,n$

$j=2,3,\ldots,m$

Substituting (3.13), (3.14) and (3.15) into (3.16), D_i becomes:

$$D_i = (1/m)\sum_{j=1}^{m} L_{ij} + \{1/(m\,n)\}\sum_{j=2}^{m}\sum_{i=1}^{n}(L_{ij} - L_{i1}) \tag{3.17}$$

where:

$i=1,2,\ldots,n$

Second Method: The unknown quantities $D_1,D_2,\ldots,D_n,z_2,\ldots,z_m$, can be found according to sub-section 3.4.2 from the solution of (3.13) which is:

$$\hat{\mathbf{x}} = (\mathbf{A}'\mathbf{A})^{-1}\,\mathbf{A}'\mathbf{b} \tag{3.18}$$

where the quantities $\mathbf{A}$, $\hat{\mathbf{x}}$, $\mathbf{b}$ are explained into sub-section 3.4.2.

The elements a_{ij}^{-1} of the matrix $(\mathbf{A}'\mathbf{A})^{-1}$ are referred to in sub-section 3.4.2.1. On the other hand, the vector $\mathbf{A}'\mathbf{b}$ is as follows:

$$
(\mathbf{A}'\mathbf{b})^{p}_{1} = \begin{bmatrix} \sum_{j=1}^{m} L_{1j} \\ \cdot \\ \cdot \\ \cdot \\ \sum_{j=1}^{m} L_{nj} \\ \sum_{i=1}^{n} L_{i2} \\ \cdot \\ \cdot \\ \sum_{i=1}^{n} L_{im} \end{bmatrix} \tag{3.19}
$$

where p=n+m-1.

It can be shown -after somehow laborious work- that:

$$
z_j = (1/n) \left[\sum_{i=1}^{n} L_{ij} - \sum_{i=1}^{n} L_{i1} \right] \tag{3.20}
$$

where j=2,3,...,m

$$
D_i = (1/m) \left[\sum_{j=1}^{m} L_{ij} - \sum_{j=2}^{m} z_j \right] \tag{3.21}
$$

where i=1,2,...,n

Substituting (3.20) into (3.21), D_i becomes identical to that given by (3.17).

Therefore, it can be said that the first and second methods are equivalent and that the second method is the "mathematical expression" of the first method.

3.4.3 THIRD METHOD - ANGLES (1)

An alternative method to directions at a point is provided by horizontal angles.

The most commonly used method of angles (Balodimos, 1977) nowadays is the following one which is referred to the R.O. as zero:

Consider the angles (Fig. 4) at K between the points A (R.O.), B, C, D are to be measured. The theodolite is set up at K, and the horizontal circle reading is taken and recorded exactly as described in section 3.2.2.

The reduction of the observations is carried out by meaning the left and right face readings. If required, the mean discrepancy between the two R.O. readings can be adjusted proportionally to the number of angles, and finally the clockwise angle of each station can then be deduced.

The record of the observations at a point consists of the means of several such sets of angles, referred always to the same R.O., and this is the most economical way of presenting the results, since the value of any angle, or combination of angles, for example BKC or AKC, is given by a simple subtraction.

The method could be described as one of directions referred to the R.O. as zero direction.

3.4.4 FOURTH METHOD - ANGLES (2)

This method is usually called: "*Adjustment of observations by the method of angles in all combinations*", and is well known as "*Schreiber's method*" (Hazay, 1970).

This method is based on a completely different technique (the field procedure and observing techniques are different) compared with that of the previous sub-section, and consists in measuring all the angles in different combinations (Fig. 6). The number of the combinations and, therefore, of the angles to be measured is:

$$m = n(n-1)/2 \qquad (3.22)$$

where n is the number of individual points sighted.

Angles are generally measured in several sets the number of which is identical for each angle.

3.5 CONCLUSIONS

The first method (method of directions) has been applied in this dissertation because of the following reasons:

1. The first two methods for the reduction of theodolite observations (sub-sections 3.4.1 and 3.4.2) are identical. However, the first method is merely a convenient computational

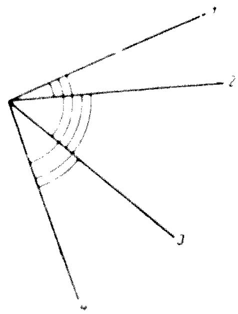

Figure 6. Angles in all combinations

model to obtain the Least Squares solution described in the second method.

2. The third method (method of angles) as has been described in sub-section 3.4.3 would have been applicable but not used, as the general practice in South Africa is to use the method of directions (first two methods).

3. Schreiber's method (second method of angles) is not applicable for dissertation purposes because of the field procedure adopted during the actual survey.

On the other hand, it is worth noting that the application of the orientation correction in the method of directions does not affect the calculation of the observational residuals since the correlation introduced by this is very small.

CHAPTER 4

4.0 DISTRIBUTIONS

4.1 MOMENTS

If some data are known to have been sampled randomly from a normal population, then the sample mean and variance do in fact summarise all the information contained in the data. More usually it may not be known that the population is normal, or perhaps it may be known to follow some other distribution. In such situations some measures may be needed to represent further shape characteristics of a distribution, such as skewness. That is additional moments can be determined from which characteristics of the distribution may be inferred. Population measures (although analogous definitions hold for sample data) will be considered below.

The **moments** and **cumulants** are a set of constants of a distribution which are useful for measuring its properties and, under certain circumstances, for specifying it.

The moment of order r ($r=1,2,3,\ldots$) is given by (Mikhail, 1976), (Patel, Kapadia and Owen, 1976),(Wetherill, 1972), (Bury, 1975) as:

$$\mu'_r = E[(x-b)^r] \tag{4.1}$$

and is called the **rth moment** of a variable x about a constant b. When $b = 0$, it is called the **rth moment about the origin** of the random variable x, and is denoted by μ'_r. When $b = E(x)$, it is called the **rth central moment** of the random variable x, and is denoted by μ_r.

For a continuous variable, the central moment is (Bury, 1975), (Mikhail, 1976):

$$\mu_r = \int_{-\infty}^{+\infty} (x-\mu)^r f(x)dx \qquad (4.2)$$

where f(x) is the p.d.f.

The moments μ_r can be expressed as functions of μ'_r. Noting that $\mu'_1=E(x)$ is a constant, the second moment about the mean can be written as:

$$\mu_2 = E(x-\mu)^2 = E(x^2-2\mu x+\mu^2) = E(x^2)-2\mu E(x)+E(\mu^2)$$

$$= E(x^2)-2E(x)E(x)+[E(x)]^2 = E(x^2)-[E(x)]^2$$

$$= E(x^2-0)-[E(x-0)]^2 = E(x-0)^2-[E(x-0)]^2$$

$$= \mu'_2 - (\mu'_1)^2 \qquad (4.3)$$

Similarly, it can be shown that:

$$\mu_3 = \mu'_3-3\mu'_1\mu'_2+2(\mu'_1)^3 \qquad (4.4)$$

$$\mu_4 = \mu'_4-4\mu'_1\mu'_3+6(\mu'_1)^2\mu'_2-3(\mu'_1)^4 \qquad (4.5)$$

Given that the expectation and variance of a random variable are E(x) and $E(x-\mu)^2$, then the third moment is zero in symmetrical distributions, and will be non-zero for skew distributions. The third moment will be positive for distributions as in Fig. 7(a) and negative for those represented by Fig. 7(b) respectively.

The third moment cannot be used in the form $E[(x-\mu)^3]$ as a measure of skewness, for it can be made arbitrarily larger or smaller by altering the scale.

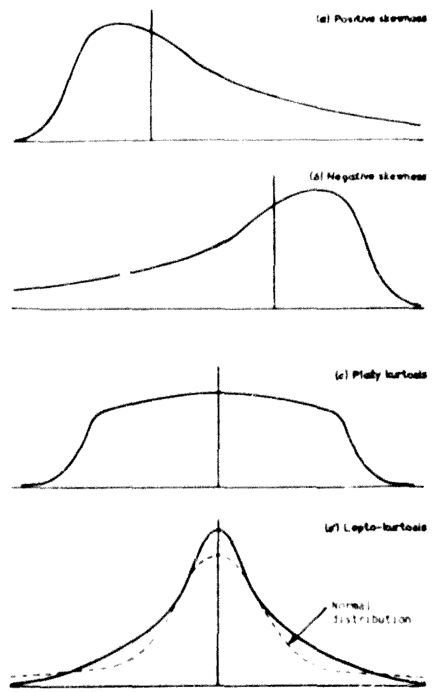

Figure 7. Some non-normal distributions

The following formula is invariant to scale and will be used as the **coefficient of Skewness** (Patel, Kapadia and Owen, 1976), (Wetherill, 1972), (Bury, 1975):

$$\gamma_1 = \mu_3/(\mu_2)^{3/2} \tag{4.5}$$

where:

$$\mu_2 = E(x-\mu)^2$$
$$\mu_3 = E(x-\mu)^3$$

Another coefficient which measures the shape of a distribution is the **coefficient of Kurtosis** or **excess**, which is defined (Patel, Kapadia and Owen, 1976), (Wetherill, 1972), (Bury, 1975) as:

$$\gamma_2 = \mu_4/(\mu_2)^2 \tag{4.7}$$

where:

$$\mu_2 = E(x-\mu)^2$$
$$\mu_4 = E(x-\mu)^4$$

The graph is more peaked (Leptokurtosis) for $\gamma_2>3$ (Fig. 7(d)), and becomes more flattened (Platykurtosis) for $\gamma_2<3$ (Fig. 7(c)) in the normal distribution.

The **Moment Generating Function** of a random variable x, denoted by M(t), is defined by (Patel, Kapadia and Owen, 1976), (Bury, 1975)

$$M(t) = E(e^{tx}) = \int_{-\infty}^{+\infty} e^{tx} f(x)dx \tag{4.8}$$

for all real values of t.

The derivatives of M(t) permit an alternative and simpler way to calculate the moments μ'_r. For example, the first derivative of M(t) is:

$$\frac{\partial M}{\partial t} = \int_{-\infty}^{+\infty} x e^{tx} f(x)dx \tag{4.9}$$

At t=0, this derivative is seen to be the expected value of x.

$$\left.\frac{\partial M}{\partial t}\right|_{t=0} = \int_{-\infty}^{+\infty} x f(x)dx = E(x) = \text{[illegible]} = \mu'_1 \tag{4.10}$$

Similarly, the second derivative is the μ'_2, and generally, the rth derivative is:

$$\left.\frac{\partial^r M}{\partial t^r}\right|_{t=0} = \int_{-\infty}^{+\infty} E(x^r) = E(x-0)^r = \mu'_r \qquad (4.11)$$

The **Cumulant Generating Function** of a random variable X, denoted by $\eta(t)$, is defined by (Patel, Kapadia and Owen, 1976), (Bury, 1975)

$$\eta(t) = \log M(t) \qquad (4.12)$$

for all real values of t.

4.2 NORMAL DISTRIBUTION

As indicated in section 2.2, the form of the probability density function is a bell-shaped curve (Fig. 1), and is given by the Gaussian probability distribution (Johnson and Kotz, 1970), (Wetherill, 1972), (Mikhail, 1976):

$$f(x) = L \cdot \exp[-1/2((x-\mu)/\sigma)^2] \qquad (4.13)$$

where:

$$L = 1/(\sigma\sqrt{2\pi}) \qquad (4.14)$$

It is obvious that the p.d.f. involves two parameters (denoted by the Greek letters μ and σ) which must satisfy

$$\sigma > 0 \quad \text{and} \quad -\infty < \mu < +\infty$$

for the case of a hypothetically infinite number of observations. A normal distribution is completely determined by the **population mean** (μ) or **expected value**, which is a "location" parameter, and **population standard deviation** (σ), which is a "dispersion" parameter. This distribution can be denoted by $N(\mu,\sigma^2)$. The symbol σ^2 is the **variance** of the population or the second moment about μ, the positive square root of which is defined as the σ. This is expressed in the same unit of measurement as the observed quantity. The deviations $x-\mu$ are usually called the **random** (accidental) or **observational errors**.

The limited number of actual observations (of independent measurements) will be considered as a **random sample** drawn from the infinite number of observations in the population.

The mean ($\bar{x}$) and the variance (s^2) of the sample are **estimates** of the population **mean** μ and the population variance σ^2 of this distribution. Different samples yield different estimates, which should not be confused with the theoretical values μ and σ^2.

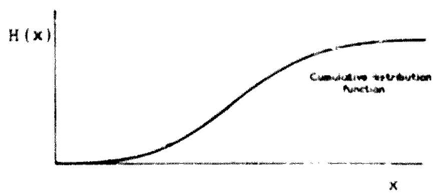

Figure 8. The c.d.f. of the Normal distribution

The **cumulative distribution function** (c.d.f.), which is sometimes called simply the *distribution function*, is a function (for continuous variables) which increases gradually from zero at the left of the range, to unity at the right of the range, as is shown in Fig. 8.

The c.d.f. of the normal distribution is given by the formula:

$$H(x) = \int_{-\infty}^{x} f(x)dx \tag{4.15}$$

where f(x) is given by (4.13).

4.2.1 THE STANDARD NORMAL DISTRIBUTION

If the parameters of the normal distribution take the values:

$$\begin{aligned} \mu &= 0 \\ \sigma^2 &= 1 \end{aligned} \tag{4.16}$$

then this is called the **normal standard distribution** or **standardised distribution** (Table A1 - Appendix A), and is denoted by N(0,1). The p.d.f. is symmetrical about the zero and is given by (Johnson and Kotz, 1970), (Wetherill, 1972), (Mikhail, 1976) as:

$$f(z) = (1/\sqrt{2\pi})\exp(-z^2/2) \tag{4.17}$$

The normal distribution $N(\mu,\sigma^2)$ may be transformed to a standardised normal distribution by the substitution:

$$z = (x-\mu)/\sigma \tag{4.18}$$

The c.d.f. of N(0,1) is written as follows:

$$H(z) = \int_{-\infty}^{z} f(z)dz \tag{4.19}$$

where f(z) is given by (4.17).

4.2.2 MOMENTS OF THE NORMAL DISTRIBUTION

Substituting the p.d.f of the Normal distribution (formula 4.13) into formula (4.8), the Moment Generating Function for the Normal random variable X becomes:

$$M(t) = \exp[t\mu + t^2\sigma^2/2] \tag{4.20}$$

[For a detailed description see Johnson and Kotz (1970)].

The first four derivatives of M(t) at t=0, in combination with the equations (4.11), (4.20) are given as:

$$\begin{aligned} \mu'_1 &= \mu \\ \mu'_2 &= \mu^2+\sigma^2 \\ \mu'_3 &= 3\mu\sigma^2+\mu^3 \\ \mu'_4 &= \mu^4+6\mu^2\sigma^2+3\sigma^4 \end{aligned} \tag{4.21}$$

Substituting the equations (4.21) into (4.3), (4.4) and (4.5), the moments about the mean become:

$$\begin{aligned} \mu_2 &= \sigma^2 \\ \mu_3 &= 0 \\ \mu_4 &= 3\sigma^4 \end{aligned} \tag{4.22}$$

Therefore, the coefficients of skewness and kurtosis for the case of the Normal distribution, according to equations (4.6), (4.7) and (4.22), become:

$$\gamma_1 = 0$$
$$\gamma_2 = 3 \qquad (4.23)$$

4.2.3 CONFIDENCE INTERVAL - CONFIDENCE LEVEL

The **confidence interval** is the region about a sample statistic containing the corresponding population statistic.

The probability that a confidence interval will enclose the estimated parameter is called the **confidence level** (Yamane, 1973).

The following confidence regions are commonly used with the normal distribution:

$$P[\ (\ \mu - c\sigma < x < \mu + c\sigma\ )\ |\ N(\mu, \sigma^2)\ ] \qquad (4.24)$$

c	Confidence interval	Confidence level
1.000	$P(\mu-\sigma<x<\mu+\sigma)$	0.683
1.960	$P(\mu-1.960\sigma<x<\mu+1.960\sigma)$	0.950
2.244	$P(\mu-2.244\sigma<x<\mu+2.244\sigma)$	0.975
2.576	$P(\mu-2.576\sigma<x<\mu+2.576\sigma)$	0.990

Table 1: Conf. interval - Conf. level of the $N(\mu,\sigma^2)$

For the standardised distribution, (4.24) becomes:

$$P\left[\frac{(\mu-c\sigma)-\mu}{\sigma} < z < \frac{(\mu+c\sigma)-\mu}{\sigma} \;\middle|\; N(0,1) \right]$$

$$= P[\ -c < z < +c \mid N(0,1)\ ]$$

$$= 2H(c)-1 \qquad (4.25)$$

Table 1 shows the different confidence levels which arise from different values of c.

4.3 TRUNCATED NORMAL DISTRIBUTION

The Truncated Normal distribution is defined in terms of a function of the p.d.f. of the Normal distribution, and the truncation points are based on the area enclosed within a Normal distribution.

If the distribution of a random variable x is doubly truncated at points A and B and is normally distributed within these limits (Fig. 9) (Johnson and Kotz, 1970), then the probability density function is:

$$f(x) = L\cdot\exp[-1/2\{(x-\mu)/\sigma\}^2]\left[\int_B^A L\ \exp[-1/2\{(t-\mu)/\sigma\}^2]dt\right]^{-1}$$

$$= L\cdot\exp[-1/2\{(x-\mu)/\sigma\}^2]\left[\int_{(A-\mu)/\sigma}^{(B-\mu)/\sigma} L\ \exp(-z^2/2)dz\right]^{-1}$$

This may be written as:

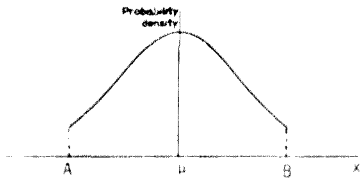

Figure 9. Doubly Truncated Normal distribution

$$f(x) = (1/\sigma\sqrt{2\pi})\exp[-1/2((x-\mu)/\sigma)^2]\left[H[(B-\mu)/\sigma]-H[(A-\mu)/\sigma]\right]^{-1}$$

$$= (1/W)(1/\sigma\sqrt{2\pi})\exp[-1/2((x-\mu)/\sigma)^2] \qquad (4.26)$$

where $A \leq x \leq B$ and

$$L = 1/(\sigma\sqrt{2\pi})$$
$$W = H[(B-\mu)/\sigma]-H[(A-\mu)/\sigma]$$

The $H[(B-\mu)/\sigma]$, $H[(A-\mu)/\sigma]$ denote the c.d.f. with $(B-\mu)/\sigma$ and $(A-\mu)/\sigma$ as argument.

The factor W is essentially a scale factor which depends on the truncation points. If $(B-\mu)/\sigma = (A-\mu)/\sigma = 2.5\%$, then W (which is calculated by the standardised statistic $(A-\mu)/\sigma$ with the contribution of the Table A1 given in Appendix A) is calculated as H(97.5) - H(2.5). Thus, W=0.95. It can be easily seen that the p.d.f. of the Truncated Normal distribution is similar to that of the Normal one, and it differs only by the factor (1/W). The shape of the Truncated Normal distribution will be symmetrical about the mean value ($\gamma_1=0$). However, since the factor $(1/W)>1$ and appears in the formula (4.17), it means that the p.d.f. will be more peaked [$\gamma_2>3$ - Figure 6(d)] than the shape of the Normal one.

If $A \to -\infty$, the p.d.f. is singly truncated from above (at point B) (Fig. 10a), and if $B \to +\infty$, the p.d.f. is singly truncated from below (at point A) (Fig. 10b).

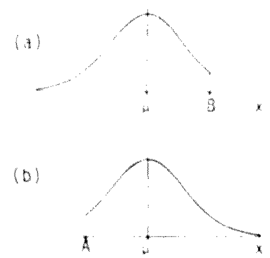

Figure 10. Singly truncated from: (a). above, (b). below

The expected value of x is given by the formula (Johnson and Kotz, 1970):

$$E(x) = \mu + \left[\frac{Z[(A-\mu)/\sigma] - Z[(B-\mu)/\sigma]}{H[(B-\mu)/\sigma] - H[(A-\mu)/\sigma]} \right] \sigma \qquad (4.27)$$

and the variance of x, by (Johnson and Kotz, 1970):

$$Var(x) = \left[1 + \frac{C \cdot Z(C) - D \cdot Z(D)}{H(D) - H(C)} - \left[\frac{Z(C) - Z(D)}{H(D) - H(C)} \right]^2 \right] \sigma^2 \qquad (4.28)$$

where:

$C = (A-\mu)/\sigma$ and $D = (B-\mu)/\sigma$

$Z(C)$, $Z(D)$ are the ordinates of the standardised Normal distribution.

In the particular case for

$$A-\mu = |-(B-\mu)| = \delta \qquad (4.29)$$

the equations (4.27),(4.28) become:

$$E(x) = \mu \qquad (4.30)$$

and

$$\mathrm{Var}(x) = \left[1 - \frac{2\delta Z(\delta)}{2H(\delta)-1} \right] \sigma^2 \qquad (4.31)$$

4.3.1 MOMENTS OF THE TRUNCATED NORMAL DISTRIBUTION

The Moment Generating Function is similar to that of the normal distribution.

The Truncated Normal distribution is symmetrical, and hence:

$$\gamma_1 = 0 \qquad (4.32)$$

The coefficient of Kurtosis is given by the formula (4.7), where $\mathrm{Var}(x)_T$ is the variation of the Truncated Normal distribution and, therefore, μ_2 in this case is:

$$\mu_2 = \mathrm{Var}(x)_T \qquad (4.33)$$

Considering that:

$$\mathrm{Var}(x)_T = k \cdot \mathrm{Var}(x) \tag{4.34}$$

and substituting (4.34) into (4.7)

$$\gamma_2 = \frac{\mu_4}{[\mathrm{Var}(x)_T]^2} = \frac{\mu_4}{[\mathrm{Var}(x)_N]^2}$$

$$= (1/k^2)\gamma_{2\,(\mathrm{Normal})} \tag{4.35}$$

Since γ_2 of the Normal distribution (see formula 4.23) is 3, then (4.35) becomes:

$$\gamma_2 = (1/k^2)\ 3$$

$$= [1/(\mathrm{Var}(x)_T)^2]\ 3 \tag{4.36}$$

because of (4.34) $[\mathrm{Var}(x)_N = 1]$.

Using the formula (4.31) the variation of the Truncated Normal distribution is calculated:

1. For 95% truncation points, where:

$\delta = 1.960$		(formula 4.29)
$Z(d) = Z(1.960)$	$= 0.0584$	(formula 4.17)
$H(d) = H(1.960)$	$= 0.9750$	(Table A1)

Therefore:

$$\mathrm{Var}(x)_T = 0.7590 \rightarrow$$

$$k^2 = [\mathrm{Var}(x)_T]^2 = 0.5761 \tag{4.37}$$

and using the formula (4.36)

$$\gamma_2 = 5.208 \qquad (4.38)$$

2. For **97.5%** truncation points, where:

$\delta = 2.244$ (formula 4.29)
$Z(d) = Z(1.960) = 0.0322$ (formula 4.17)
$H(d) = H(1.960) = 0.9876$ (Table A1)

Therefore:

$$Var(x)_T = 0.8518 \rightarrow$$

$$k^2 = [Var(x)_T]^2 = 0.7256 \qquad (4.39)$$

and using the formula (4.36)

$$\gamma_2 = 4.135 \qquad (4.40)$$

3. For **99%** truncation points, where:

$\delta = 2.576$ (formula 4.29)
$Z(d) = Z(1.960) = 0.0145$ (formula 4.17)
$H(d) = H(1.960) = 0.9950$ (Table A1)

Therefore:

$$Var(x)_T = 0.9245 \rightarrow$$

$$k^2 = [Var(x)_T]^2 = 0.8547 \qquad (4.41)$$

and using the formula (4.36)

$$\gamma_2 = 3.510 \qquad (4.42)$$

From the values γ_2 given in (4.38, 4.40 and 4.42), it is obvious that as the level of truncation points is decreased so the shape of the distribution becomes more peaked.

4.4 INVERSE GAUSSIAN (WALD) DISTRIBUTION

The following formula is a standard form of the p.d.f. of the Inverse Gaussian distribution (Johnson and Kotz, 1970):

$$f(x|\mu,\lambda) = [\lambda/(2\pi x^3)]^{\frac{1}{2}}\exp[-\lambda(x-\mu)^2/(2\mu^2x)] \qquad (4.43)$$

where x, λ, μ > 0.

This distribution involves two parameters: μ which is a measure of location and is in fact the population mean, and λ which is a measure of dispersion.

One of the alternative forms of (4.43) is:

$$f(x|\mu,\phi) = [\mu\phi/(2\pi x^3)]^{\frac{1}{2}}\exp(\phi)\exp[-1/2(\phi x/\mu+\phi\mu/x)] \qquad (4.44)$$

where $\phi=\lambda/\mu$.

4.4.1 ESTIMATION OF THE PARAMETERS

Considering n observations $x_1,x_2,\ldots,x_n$ which have an Inverse Gaussian distribution of the form (4.43), then (Johnson and Kotz, 1970), (Patel, Kapadia and Owen, 1976):

$$\bar{x} = (1/n)\sum_{i=1}^{n} x_i \qquad (4.45)$$

and

$$1/\lambda = (1/n) \sum_{i=1}^{n} (1/x_i - 1/\bar{x}) \qquad (4.46)$$

Johnson and Kotz (1970) gives an "*approximately unbiased*" estimator of $1/\lambda$ (similarly given in Patel, Kapadia and Owen, (1976))as:

$$1/\lambda = s^2/\bar{x}^3 \qquad (4.47)$$

where:

$$s^2 = 1/(n-1) \sum_{i=1}^{n} (x_i - \bar{x})^2 \qquad (4.48)$$

and $\bar{x}$ is given by (4.45).

4.4.2 MOMENTS OF THE INVERSE GAUSSIAN DISTRIBUTION

The Moment Generating Function is (Johnson and Kotz, 1970), (Patel, Kapadia and Owen, 1976):

$$M(t) = \exp\left[(\lambda/\mu) \; [\; 1-((1+\mu^2 t)/\lambda)^{\frac{1}{2}} \;] \right] \qquad (4.49)$$

and the Cumulant Generating Function is:

$$n(t) = (\lambda/\mu) \; [\; 1-(1+2\mu^2 t/\lambda)^{\frac{1}{2}} \;] \qquad (4.50)$$

Therefore, the first four cumulants according to (4.50) are:

$$K_1 = \mu$$

$$K_2 = \mu^3/\lambda$$
$$K_3 = 3\mu^5/\lambda^2$$
$$K_4 = 15\mu^7/\lambda^3 \quad (4.51)$$

However, the relation between the cumulants and central moments is:

$$K_2 = \mu_2$$
$$K_3 = \mu_3$$
$$K_4 = \mu_4 - 3\mu_2^2 \quad (4.52)$$

Using the equations (4.51),(4.52), the following central moments are:

$$\mu_2 = Var(x) = \mu^3/\lambda$$
$$\mu_3 = 3\mu^5/\lambda^2$$
$$\mu_4 = 15\mu^7/\lambda^3 + 3\mu^6/\lambda^2 \quad (4.53)$$

Therefore, the coefficient of skewness is:

$$\gamma_1 = \mu_3/\mu_2^{3/2}$$
$$= 3(\mu/\lambda)^{\frac{1}{2}} \quad (4.54)$$

and the coefficient of kurtosis (excess) is:

$$\gamma_2 = \mu_4/\mu_2^2$$
$$= 15\mu/\lambda + 3 \quad (4.55)$$

A special case of the Inverse Gaussian distribution with $\mu=1$, is the **Wald distribution** (Fig. 11) (Johnson and Kotz, 1970).

From the above Figure, it can be seen that as the ratio λ/μ (with μ fixed) is increased so the Wald distribution tends to a Normal distribution.

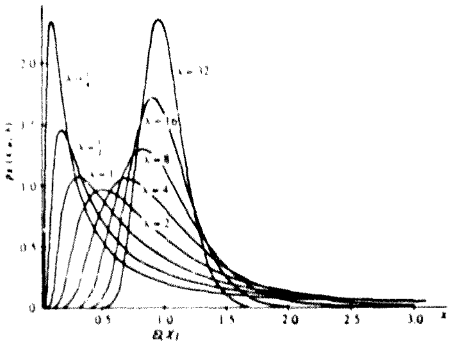

Figure 11. Wald Distribution

4.5 DISCUSSION ABOUT DISTRIBUTIONS

In this chapter, the mathematical forms of the following distributions have been given:

- Normal distribution
- Truncated Normal Distribution
- Inverse Gaussian (Wald) distribution

The reasons for the selection of these distributions are discussed below:

1. **Normal distribution**: The use of the *Normal* distribution (Figure 12) is very prominent in survey and is by far the single most important probability distribution in statistics for the following reasons:

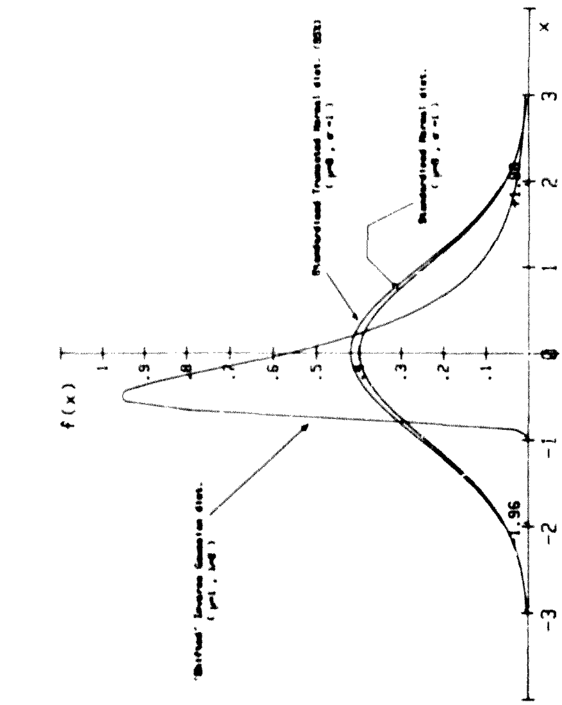

Figure 12. Normal - 95% Truncated Normal - Inverse Gaussian distribution

a This distribution has been thoroughly studied and tabulated so that it is very convenient, when data can reasonably be assumed to be normal, to use existing theory and tables. Therefore, the Normal distribution is, generally, the simplest to work with.

b. If a large random sample is taken from some distribution, then even though this distribution is not itself approximately normal, as a consequence of the Central Limit *Theorem, it can be noted that many important* functions of the observations in the sample will have distributions which are approximately normal. In particular, for a large random sample from any distribution that has a finite variance, the distribution of the sample mean will be approximately normal.

c. Many statistical techniques based upon the Normal distribution are available, and are applicable for data that are approximately normal. These include the X^2 test, the Student's test, the F test.

2 **Truncated Normal distribution**: The Normal distribution is generally used as a good approximation for the distribution of the residuals associated with survey measurements. This distribution displays both the central and symmetric properties exhibited by the residuals.

The Truncated Normal distribution is similar to the Normal distribution and permits the properties mentioned above. It has therefore been selected as a viable alternative to the full Normal distribution.

In other words the underlying distribution is at first "Normal", and secondly its tails are cut or truncated. This means that the Truncated distribution (Figure 12) carries more or less the same properties as that of the Normal, e.g. $\gamma_1=0$ for both of them, since the p.d.f of both them are approximately the same (see formulae 4.13 and 4.26). Consequently, some of the reasons which lead to test this particular distribution in this dissertation are similar to the reasons for the use of the Normal distribution.

The truncated distribution implies that data lying beyond the points of truncation are not accepted as coming from the population specifying the distribution. These data are, therefore, regarded as being inadmissible.

There is indeed precedence for this approach in surveying. For the measurement of directions with a single second theodolite a residual of one degree would be discarded as being a blunder. Similarly, a residual of thirty seconds may be treated in the same manner. These data points are excluded, as they are ascribed to a sample from data best described by an alternate distribution, or as blunders.

The selection of the points of truncation, or cut off points, is definitely a grey area in the determination of the truncated distribution. It will, therefore, be approached from the fit of the actual data to an hypothesised distribution in order to select the "best" distribution to describe the data.

3. **Inverse Gaussian distribution**: The shape of this distribution is showed in Fig. 12. It must be noticed again that, since the Inverse Gaussian is defined only for $x>0$, the data have been transformed by the addition of a constant, so that the zero represents the mean value of the this distribution.

This distribution seems to be quite useful because its parameters (μ, λ which are the measure of location and the measure of dispersion, respectively) are specified directly from the observations, and consequently the p.d.f. is determined thoroughly by them. This is similar to the case of both Normal and Truncated Normal distribution.

In addition to this, and as can be seen from the Figure 11, the Inverse Gaussian distribution looks like a French curve and is not, generally, symmetrical about the mean ($\gamma_1>0$ according to formula (4.54) since $\mu,\lambda>0$] as opposed to the other distributions.

At this point, it must be noted that in the analysis of data, it has been decided to study separately observations of 2 and 4 arcs. The residuals of 2 arcs, according to the described method of the reduction of theodolite observations (sections 3.4.1 and 3.4.1.1) are symmetrical about the mean, and hence the Normal or the Truncated Normal could be the best distribution/s. However, for the residuals derived from 4 arcs, although the mean of them is zero, symmetry is not implied. This means that there may be more residuals situated on the left or on the right side of the mean. In other words, since the coefficient of Skewness is not necessarily equal to zero, it was decided to test an alternate set of distributions that could exhibit asymmetrical properties. The distribution selected is the Inverse Gaussian (Wald) distribution, the cumulant generating function of which is found to be in inverse relationship with that of the Gaussian distribution (Johnson and Kotz, 1970, p.138).

After the above discussion, a question arises. Are these three distributions the only ones which could fit the residuals? The answer is negative because there are many other distributions which are considered to be a good approximation to the Normal distribution provided the number of the observations is large. For example, the X^2 distribution and the t-distribution may be examined as they are assymptotically normal. Apart of them, Johnson and Kotz (1970, pp. 53-59) suggest a series from distributions which may be good approximations to Normal distribution.

However, it was not possible to select a large number of distributions to be tested for the purposes of this dissertation. The selection of the the Normal, Truncated Normal and Inverse Gaussian distributions was decided upon in conjunction with K.S.

Milford of the Department of Surveying (University of the Witwatersrand).

CHAPTER 5

5.0 STATISTICAL INFERENCE

5.1 STATISTICAL TESTS

Statistical tests are increasingly applied in engineering and science.

The objective of a statistical test is to evaluate an hypothesis concerning the values of one or more population parameters. In testing, one seeks a judgment as to whether some estimator function is consistent with the assumption (hypothesis) that the sample was drawn from a population ..th specified parameter values, such as a normal distribution with a given standard deviation σ.

The **null hypothesis**, indicated symbolically as H_0 , describes the hypothesis to be tested. Thus, H_0 will specify hypothesised values for one or more population parameters.

To test whether the null hypothesis is consistent with certain data, some **alternative hypotheses H_A** are implied, although the knowledge of them is usually very vague.

The entire set of values that the test statistic may assume is divided into two sets or regions, one corresponding to the **rejection region** and the other to the **acceptance region**. If the statistic computed from a particular sample assumes a value in the rejection region the null hypothesis H_0 is rejected and the alternative hypothesis is accepted. If the statistic test falls in the acceptance region, the null hypothesis is accepted and the decision is in favour of the null hypothesis.

5.1.1 LEVEL OF SIGNIFICANCE AND TYPES OF ERRORS

The **level of significance** α is the probability (e.g. 5%) that the null hypothesis H_0 will be rejected even if it is correct.

The rejection of the null hypothesis when it is true (wrong rejection of a true hypothesis), is called a **Type I error** (Fig. 13). That is, the risk of Type I error is α (the level of significance). This level is arbitrary, but should be chosen when the experiment is designed.

When it is important to guard against Type I errors, a level of 1% or 0.1% is used.

The failure to reject a false hypothesis (wrong acceptance of a false hypothesis), is called **Type II error**, and is denoted by β (Fig. 13) (Crow, Davis and Maxfield, 1960), (Mikhail, 1976).

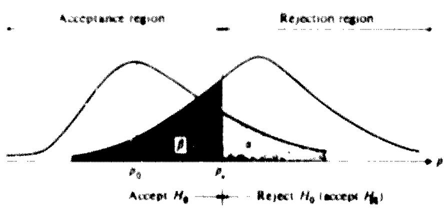

Figure 13. Type I error - Type II error - One-sided test

Figure 13 illustrates that for a certain alternative hypothesis H_A, it is not possible to make both α and β arbitrarily small. Increasing the probability for a Type I error decreases the probability for a Type II error and vice versa.

While the Type I error can be limited by properly choosing a level of significance, it is impossible in many practical cases to avoid risking Type II errors by simply not making them. In such cases there are graphs showing the probabilities of type II errors under various hypotheses which are called **operating characteristic curves** or **OC curves**. These indicate the **power of a test** (Spiegel, 1975), (Mikhail, 1976), which is designated by 1-ß and it shows the way to avoid making wrong decisions (to minimise Type II errors).

To test the null hypothesis H_0 against the alternative one H_A, two different tests are available:

1. The **one-tail** or **one-sided** test (Figure 13)
2. The **two-tail** or **two-sided** test (Figure 14)

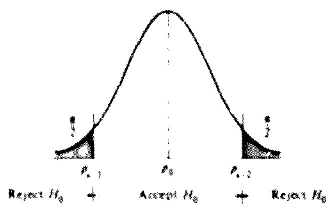

Figure 14. Two-sided test

5.2 DEFINITION OF A MAXIMUM LIKELIHOOD ESTIMATOR

The important concept of the maximum likelihood estimator will be briefly reviewed in this section, as it is required in the subsequent analysis.

Suppose that the random variables $X_1, X_2, \ldots, X_n$ form a random sample from a discrete or a continuous distribution for which the p.d.f. is $f(x|r)$, where the parameter r belongs to some parameter space Ω. For any observed vector $\mathbf{x}=(x_1, x_2, \ldots, x_n)$ in the sample, the p.d.f. will be denoted by $f_n(\mathbf{x}|r)$ and is called the **likelihood function**.

Furthermore, suppose that the probability $f_n(\mathbf{x}|r)$ of obtaining the actual observed vector $\mathbf{x}$ is very high when r has a particular value, say $r=r_0$, and is very small for every other value of $r \in \Omega$. In other words, for any given observed vector $\mathbf{x}$, a value of r is considered, for which the likelihood function $f_n(\mathbf{x}|r)$ is a maximum, and consequently this value (r) is used as an estimate of r. The estimator $\hat{r}$ is called the **M**aximum **L**ikelihood **E**stimator or **M.L.E.** of r (Bury, 1975), (Mendenhall, 1979).

The value of r which maximises the likelihood function $f_n(\mathbf{x}|r)$ is calculated by setting the first derivative of this function to zero. That is:

$$\frac{\partial[f_n(\mathbf{x}|r)]}{\partial r} = 0 \qquad (5.1)$$

provided

$$\frac{\partial^2[f_n(\mathbf{x}|r)]}{\partial r} < 0 \qquad (5.2)$$

It should be noted that in some problems, the maximum value of $f_n(\mathbf{x}|r)$ is obtained for some $r \notin \Omega$, and therefore, the $\hat{r}$ does not exist. In some cases, the maximum value may be obtained at more than one point in space Ω, and in such a case, the $\hat{r}$ is not uniquely defined. However, in most practical problems, the M.L.E. exists and is uniquely defined.

5.3 METHODS OF STATISTICAL INFERENCE

The techniques of statistical inference can usually be subdivided into two separate classes, namely parametric and non-parametric inference. These two approaches are described below.

If the observations which are available come from distributions for which the exact form is known, even though the specific values of some parameters are unknown, it is said that these observations come from a certain **parametric** family of distributions (Silvey, 1975) and a statistical inference must be made about the values of the parameters defining that family. In other words, the parametric problems, and consequently the parametric methods, are involved with situations where the appropriate probability distribution on the sample space of the mathematical model is known apart from the values of a finite number of unknown parameters.

On the other hand, problems in which the possible distributions of the observations are not restricted to a specific parametric family are called **non-parametric** problems, and the statistical methods that are applicable in such problems are called non-parametric methods (Silvey, 1975), (Lippman, 1971), (Blum and Rosenblatt, 1972), (DeGroot, 1975).

Non-parametric methods arise naturally when the statistical model is not well developed and it is not possible to select a specific parametric family. That is, the available observations do not come from a particular parametric family of distributions and, therefore, large families of possible distributions may be involved in the non-parametric theory.

The parametric method involves stronger assumptions about the family of possible distributions on the sample space than the non-parametric method. However, these assumptions may not be verifiable, and thus, the non-parametric approach is often more realistic. For example, it would be more realistic to assume simply that the observations form a random sample from a continuous distribution, without specifying the form of this distribution any further, and then to investigate the possibility that this distribution is, for example, a normal distribution.

Following from the previous discussion, the use of a non-parametric method for statistical inference in this dissertation is justified since the purpose of this research is to test not only the Normal distribution for suitability to describe the observational residuals, but to examine some other distributions (Truncated Normal and Inverse Gaussian) under the same assumptions.

5.4 TESTS FOR NON-PARAMETRIC PROBLEMS

Although many tests (Silvey, 1975), (DeGroot, 1975), (Blum and Rosenblatt, 1972) may be constructed for the testing of hypotheses, only the two most commonly used tests will be considered. These methods are:

1. The Chi-Square goodness-of-fit test
2. The Kolmogorov-Smirnov test

These tests may be applied to solve the following non-parametric problem which is essentially the purpose of this dissertation.

It is required to test the simple null hypothesis that the unknown distribution function $F(x)$ is a particular continuous distribution function $F'(x)$, against the general alternative that the actual distribution function is not $F'(x)$. That is:

$$H_0 : F(x) = F'(x) \qquad \text{for} \quad -\infty < x < +\infty$$

H_A : The hypothesis H0 is not true $\quad$ (5.3)

This problem is non-parametric since the unknown distribution f from which the random sample is taken might be any continuous distribution.

Tests which determine (Lippman, 1971)

"*whether or not the particular probabilistic mechanism ... under consideration is the appropriate mathematical model for a given random experiment*"

are called **goodness-of-fit tests**.

In the following sub-section the Kolmogorov-Smirnov test and the χ^2 goodness-of-fit test are briefly reviewed. The Chi-Square goodness-of-fit test has been applied ("HI_DI_TEST" computer program) and, consequently, will be considered in more detail.

5.4.1 THE KOLMOGOROV - SMIRNOV TEST

Suppose the random sample $x_1, x_2, \ldots, x_n$ is from a continuous distribution. Since the observations come from a continuous distribution, there is probability zero that any two of the theoretical values will be exactly equal. Order the sample to be $x_{(1)} < x_{(2)} < \ldots < x_{(n)}$, and let $F_n(x)$ be the sample (empirical) distribution function defined by [see for example (Silvey, 1975), (DeGroot, 1975), (Bury, 1975), (Blum and Rosenblatt, 1972)]:

$$F_n(x) = 0 \qquad \text{for } x < x_{(1)}$$

$$= k/n \qquad \text{for } x_{(k)} \leq x < x_{(k+1)} \quad , \quad (k=1,2,\ldots,n-1)$$

$$= 1 \qquad \text{for} \quad x_{(n)} \leq x \tag{5.4}$$

Then, this test uses the following statistic (Silvey 1975), (DeGroot, 1975):

$$D_n(x) = \sup_{-\infty<x<+\infty} \left| F_n(x) - F'(x) \right| \tag{5.5}$$

where:

the operator "sup" implies the largest values

$F_n(x)$: the sample distribution

$F'(x)$: the hypothesised distribution function

$D_n(x)$: the absolute maximum difference between $F_n(x)$ and $F'(x)$

From the above equation it follows that the value of $D_n(x)$ will tend to be small if the null hypothesis H_0 is true and will tend to be larger if the actual distribution function is different from $F'(x)$

M.H. DeGroot (1975) states that:

"It is convenient to express the test procedure in terms of $n^{\frac{1}{2}}D_n(x)$, rather than simply $D_n(x)$, because of the following result, which was established in the 1930's by A.N. Kolmogorov and N.V. Smirnov:

'It the null hypothesis H_0 is true, then for any given value $t<0$:

$$\lim \Pr\{n^{\frac{1}{2}}D_n(x) \leq t\} = 1-2\sum_{i=1}^{\infty}(-1)^{i-1}e^{-2i^2t^2} = H(t) \quad \text{'."} \tag{5.6}$$

Thus the proposed test procedure for the hypothesis (5.3) is to reject H_0, if (Degroot, 1975), (Bury, 1975)

$$n^{\frac{1}{2}} D_n(x) > c \qquad (5.7)$$

where c is an appropriate constant and can be chosen from the values of $H(t)$ (given in Table A3) to achieve, at least approximately any level of significance α.

From the above consideration and in combination with equation (5.4), it is evident that the Kolmogorov-Smirnov test is a one-sided test.

5.4.2 THE CHI-SQUARE GOODNESS-OF-FIT TEST

An alternative approach to the problem specified by the hypothesis given in (5.1) is to use X^2 goodness-of-fit test.

For better understanding of this test, it is necessary to review the X^2 distribution, related theorems, and K. Pearson's approximation which is used in the application.

5.4.2.1 Definition of the Chi-Square distribution

Given a sequence of normally distributed random variables $Y_1, Y_2, \ldots, Y_n$ with mean and variance:

$$E(Y) = \mu \quad , \quad Var(Y) = \sigma^2 \qquad (5.8)$$

The variables may be standardised using:

$$X_i = (Y_i - \mu)/\sigma \qquad (5.9)$$

where, X_i then will be normally distributed with mean and variance:

$$E(X) = 0 \quad , \quad Var(X) = 1 \tag{5.10}$$

The following statistic u

$$u = X_1^2 + X_2^2 + \ldots + X_n^2 \tag{5.11}$$

has a X^2 distribution, with the density function (Yamane, 1973) given by:

$$f(u) = [1/(n/2-1)!]2^{(-n)/2}u^{(n/2)-1}e^{-(1/2)u} \tag{5.12}$$

for $u \geq 0$, and $(n/2-1)! = \Gamma(n/2)$ (Mikhail, 1976); n is the number of independent random variables that are summed and the distribution has n degrees of freedom.

The mean and variance of a X^2 distribution of n independent random variables are given by (Mikhail, 1976), (Yamane, 1973):

$$E(X^2) = n \tag{5.13}$$

$$Var(X^2) = 2n \tag{5.14}$$

5.4.2.2 The Chi-Square table

The probability of the computed value lying within a given region can be obtained from the X^2 Tables (Table A2 - Appendix A) for a selected significance level.

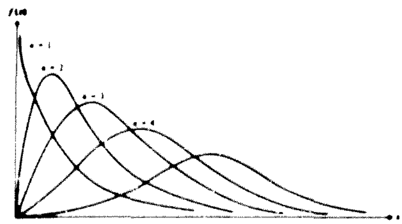

Figure 15. The Chi-Square distribution

5.4.2.3 Additive nature of Chi-Square distribution

The sum of K independent variables X_i, each being χ^2 distributed with n_i degrees of freedom. is also χ^2 distributed. Thus:

$$X_1^2+X_2^2+\ldots+X_K^2$$

will also be distributed as a χ^2 distribution with $n_1+n_2+\ldots+n_K$ degrees of freedom (Yamane, 1973).

5.4.2.4 K. Pearson's approximation

K. Pearson has shown that a discrete multinomial distribution may be transformed in order to approach a χ^2 distribution as $n \to \infty$.

Given an experiment with K events $E_1, E_2, \ldots, E_K$ and assume that the probability of an event E_i occurring is p_i. Let N_i be the number of occurrences of the event E_i. The joint distribution of $N_1, N_2, \ldots, N_K$ is called a *multinomial distribution* with density function (Yamane, 1973):

$$f(n_1,\ldots,n_k) = \frac{n!}{N_1!\ldots N_K!} p_1^{N_1}\ldots p_K^{N_K} \qquad (5.15)$$

K. Pearson has shown that if (Lindley, 1970),(Yamane, 1973):

$$X_i = \frac{N_i - np_i}{(np_i)^{\frac{1}{2}}} \qquad (5.16)$$

and

$$u = X_1^2 + X_2^2 + \ldots + X_n^2 \qquad (5.17)$$

then the distribution of u approaches a X^2 distribution with K-1 degrees of freedom as n becomes very large (Lindley, 1970), (Wolf, 1962), (Yamane, 1973), (Bury, 1975), (DeGroot, 1975).

How large must n be in order to assume that the u is approximately distributed as a X^2 distribution? Most authors (Crow, Davis and Maxfield, 1960), (Wolf, 1962), (Lindley, 1970), (Yamane, 1973), (DeGroot, 1975), (Bury, 1975) state that when $np_i \geq 5$, the X^2 distribution may be used as an approximation. When $np_i < 5$, it is necessary to combine the smaller groups into larger ones, so that the condition $np_i \geq 5$ is satisfied.

Pearson proposed the following statistic:

$$Q = \sum_{i=1}^{K} \frac{(n_i - np_i)^2}{np_i} \qquad (5.18)$$

which may be written as (Crow, Davis and Maxfield, 1960), (Lindley, 1970), (Yamane, 1973), (Bury, 1975), (DeGroot, 1975):

$$Q = \sum_{i=1}^{K} \frac{(O_i - e_i)^2}{e_i} \qquad (5.19)$$

where O_i is the observed and e_i is the expected class frequency.

Thus, the data are divided into class intervals. The number O_i of sample values falling in the ith class interval is recorded and the expected (theoretical) e_i value of the corresponding class interval is calculated using the p f of the hypothesised distribution. If $e_i \geq 5$, the quantity $(O_i - e_i)^2/e_i$ for the specific class interval is calculated. Alternatively, for $e_i < 5$, a "new" class interval (merging the smaller groups into larger ones) is created, so that $e_i \geq 5$. Thereafter the statistic Q is obtained using the formula (5.19). If $Q \leq d$ (where d is a constant taken from the Table A2 and is strongly associated with the selected level of significance), it implies that the null hypothesis is acceptable, otherwise the H_0 is not true.

As it can easily be seen, Q may be considered as a measure of discrepancy. Thus, if the sample distribution is identical to the hypothetical distribution), Q=0. As the discrepancy becomes larger, so the Q statistic increases. That is, the sample distribution is more consistent with the hypothetical distribution as the Q value calculated according to (5.19) becomes less.

The null hypothesis and the alternative hypothesis against which it is to be tested must be stated. As an example, it may be required to test that examination scores are normally distributed against the alternative that the distribution of examination scores is not a normal random variable, but some other given random variable.

It is, then, necessary to classify each possible outcome of the random experiment into one of K classes or cells: that is, the sample space of the random experiment is partitioned into K events. The partitioning of the outcomes may be dictated by the observations themselves. Alternatively, some criteria must be sought for the classification. For instance, in the case of the die, the sample space is partitioned into K=6 cells, since the number appearing on the die can relate directly to the cell.

The hypothesis can then be restated in terms of the partition. If the probability that the outcome of the random experiment is p_i in cell i, and p_i' is the probability when the null hypothesis is true, then (Lippman, 1971), (Yamane, 1973), (bury, 1975), (DeGroot, 1975):

$$H_0: \ p_i = p_i' \quad , \quad 1 \le i \le K \quad \text{and}$$

$$H_A: \ p_i \ne p_i' \quad \text{for some } i, \quad 1 \le i \le K \tag{5.20}$$

M.H. DeGroot states that

> *"This problem of testing hypotheses is not a nonparametric problem because the possible distributions of the observations are indexed by the parameter vector $(p_1, \ldots, p_K)$. Nevertheless, the procedure ... is often regarded as a non-parametric method because, as we shall see, many nonparametric problems of testing hypotheses can be reduced to problems in which hypotheses of the above form are to be tested."*

The acceptance or rejection of the null hypothesis is then based upon the significance level of the test. Due consideration is given to the probability of making a Type I or Type II error.

It is reasonable, therefore, to base a test of hypothesis (5.18) on values of the differences $(O_i - e_i)$ for i=1,2,...,K and to reject H_0 when the magnitudes of these differences are relatively large.

From the discussion which has already been presented, it implies that H_0 should be rejected, when $Q>d$. Thus, the X^2 test is carried out at a chosen level of significance α, d is the selected, so that $Pr(Q>d)=\alpha$, when Q has a X^2 distribution with K-1 degrees of freedom.

The test has been designed to assess the fit of the distribution and will not be concerned with the fit being "too good". As a result, only a one tailed-test will be used.

5.4.2.5 Degrees of freedom

In order to determine the degrees of freedom for the X^2 goodness-of-fit test, it is necessary to consider this concept in more detail.

Suppose that n independent observations $x_1, x_2, \ldots, x_n$ are to be taken and that the range of them is divided into K successive intervals with probability p_i that any particular observation will be of the sub-interval i $(i=1,2,\ldots,K)$. That is, each of these intervals includes N_i $(i=1,2,\ldots,K)$ observations.

Pearson showed that if the hypothesis H_0 of (5.20) is true, then the density function of Q converges to the density function of the X^2 distribution with K-1 degrees of freedom as the sample size $n \to \infty$ (Yamane, 1973), (DeGroot, 1975), (Bury, 1975).

Consider again the same problem, but instead of testing the simple null hypothesis that the parameters $p_1, p_2, \ldots, p_K$ have specific values, let H_0 specify that the parameters $p_1, \ldots, p_K$ can be represented as functions of a certain number of parameters. In other words, the probability p_i $(i=1,2,\ldots,K)$ is substituted by the function $p_i(\mathbf{r})$, where $\mathbf{r}$ is the vector of parameters

$(r_1, r_2, \ldots, r_S)$. Therefore, the null hypothesis (5.18) can be written in the following new form (Bury, 1975), (DeGroot, 1975):

$$H_0 : p_i = p_i(\mathbf{r}) \qquad \text{for} \quad i=1,2,\ldots,K$$

$$H_A : \text{The hypothesis } H_0 \text{ is not true} \tag{5.21}$$

In this case, the formula for statistic Q (5.18) is modified by replacement of the factor np_i by $np_i(\hat{\mathbf{r}})$, where $\hat{\mathbf{r}}$ denotes the M.L.E. (Maximum Likelihood Estimators) of the parameter vector $\mathbf{r}$ based on the observed numbers $N_1, N_2, \ldots, N_K$. That is (Bury, 1975), (DeGroot, 1975):

$$Q = \frac{[\,N_i - np_i(\hat{\mathbf{r}})\,]^2}{np_i(\hat{\mathbf{r}})} \tag{5.22}$$

R.A. Fisher [see for example, (Lindley, 1970), (Yamane, 1973), (DeGroot, 1975), (Bury, 1975)] showed that if the null hypothesis H_0 is true and certain conditions are satisfied, then the density function of Q converges to the density function of the X^2 distribution with (K-1)-S degrees of freedom as the sample size $n \to \infty$.

Therefore, the distribution of Q will be approximately a X^2 distribution when the sample size n is large and the H_0 is true. To determine the number of degrees of freedom for the hypothesis (5.21), the S must be subtracted from the number K-1 used for the hypothesis (5.20), because S parameters $(r_1, r_2, \ldots, r_S)$ are estimated when the observed number N_i compared with the expected number $np_j(\hat{\mathbf{r}})$. For example, in the case of Normal distribution, the parameter vector $\mathbf{r}$ is the two-dimensional vector $\mathbf{r}(\mu, \sigma^2)$ because the unknown parameters are the mean (μ) and the variance (σ^2).

It is important to note that in order to calculate the value of the statistic Q defined by (5.22) the M.L.E.'s μ and σ^2 must be found by using the numbers $N_1, N_2, \ldots, N_K$ of observations in the different sub-intervals, and should not be found by using the observed values of $x_1, x_2, \ldots, x_n$ themselves. However, this is not generally applied be[illegible] of its complexity. In addition, the M.L.E.'s of μ and σ^2 based on the n observed values $x_1, \ldots, x_n$ in the original sample are simply the sample mean $\bar{x}_n$ and the sample variance of the mean s_n^2/n.

If the null hypothesis H_0 is true and the vector **r** is estimated by maximising the likelihood function of **r**, then the statistic Q given by (5.22) will have approximately a X^2 distribution with

$$df = K-1-S \tag{5.23}$$

where:

K : the number of groups (cells)

S : the number of the unknown parameters (the number of restrictions on the theoretical distribution)

On the other hand, if H_0 is true and the M.L.E. of **r** is found from the n observed values $x_1, x_2, \ldots, x_n$ in the original sample, M.H. DeGroot (1975) states:

> "*In 1954, H. Chernoff and E.L. Lehmann established the following result. If the M.L.E.'s $\bar{x}_n$ and s_n^2/n are used to calculate the statistic Q, and if the null hypothesis H_0 is true, then as $n \to \infty$, the density function of Q converges to a d.f. which lies between the d.f. of the X^2 distribution with K-1-S degrees of freedom and the difference of the X^2 distribution with K-1 degrees of freedom*"

That is:

$$\text{If} : V_{(K-1-S)} \leq Q \leq V_{(K-1)} \rightarrow \text{distribution is consistent} \tag{5.24}$$

where:

$V_{(K-1-S)}$: the corresponding value for K-1-S degrees of freedom taken from Table A2 (Appendix A), for a given significance level α

$V_{(K-1)}$: the corresponding value for K-1 degrees of freedom taken from Table A2 (Appendix A), for a given significance level α

In other words, the appropriate tail area lies somewhere between the tail area found from the Table A2 of the X^2 distribution with K-1 degrees freedom, and the larger tail area found from the same Table of the X^2 distribution with K-1-S degrees of freedom.

The interpretation of (5.24) is that if the statistic Q is greater than the corresponding value of Table A2 with df=K-1-S, it does not mean that the postulate distribution is not consistent with the data. However, if Q is less than the corresponding value of df=K-1-S the hypothesised distribution is definitely consistent with the data.

It remains to determine S. As has been stated, S is the number of unknown parameters which defined the p.d.f of the distribution. If, before the experiment, the distribution function f(x) to be tested is completely specified, then S=0. Therefore, the degrees of freedom simply will be given by df=K-1. However, if the type or form of the distribution f(x) to be tested can be specified, but f(x) cannot be specified completely, then S stands for the number of quantities necessary to complete the specification.

In the case of this dissertation, where three different distributions are tested for their validity, the number of parameters for each of them is 2; the mean μ and the standard deviation σ (for Normal and Truncated Normal) or the dispersion λ (for Inverse Gaussian). Under this assumption, the value of S would be 2. However, since the mean ($\bar{U}$) of the residuals are known to be zero *a priori*, it implies that the number of the unknown parameters is reduced to 1 (S=1). In this research, the data has been analysed with:

$$S = 1 \quad , \quad df = K-2 \quad \text{and}$$

$$\text{If : } V_{(K-2)} \le Q \le V_{(K-1)} \rightarrow \text{distribution is consistent} \qquad (5.25)$$

At this point, it must be noticed that the Tables 4 and 5 compare only the statistic Q with the theoretical (expected) values which correspond to K-2 degrees of freedom, and not with those of K-1 degrees of freedom. In other words, the pessimistic value of df = K-2 is examined since in the most of the cases all of the distributions are consistent considering the worst case for the degrees of freedom. These Tables will be considered in more detail in Chapter 6.

5.5 MORE DISCUSSION OF THE TEST PROCEDURE

There is often uncertainty as to whether or not a particular probability distribution is really appropriate. For these specific problems (non-parametric), a hypothesis as stated in (5.3) should be considered and a certain procedure test should be followed.

In this specific case, no alternative hypothesis is specified, and, as a result, it is the acceptance or rejection of this hypothesis that is under investigation. Simply, the test shows whether the selected distribution is consistent or not.

Two tests which relate directly to the solution of this problem have been referred to and discussed in the previous sections of this chapter. These are the "Chi-Square goodness-of-fit" test and the "Kolmogorov-Smirnov". Additional tests that could be used are the "Wilcoxon" or "Mann-Whitney" test (Silvey, 1975), (Blum and Rosenblatt, 1972), (Green and Margerison, 1978), the "Sign" test (Green and Margerison, 1978), (DeGroot, 1975), but these tests are not directly applicable for this dissertation.

The selection of the Chi-Square goodness-of-fit test (which requires that the observations to be grouped into a finite number of intervals) against that of the Kolmogorov-Smirnov test (which can be applied directly to any data with no need to discretise) has been based on the recommendations of several authors, namely E.L. Crow, F.A. Davis and M.W. Maxfield (1960) who refer to the "*supreme*" Chi-Square goodness-of-fit test, and P. Vaníček (1987) who states that

> "*The main objective of the χ^2 goodness-of-fit test is to test if the histogram is compatible with a postulated probability density function, usually the normal. This is a crucial test because all the other tests are based on the assumption of normality. But the test itself is equally valid for other probability density functions.*"

In addition, K.V. Bury (1975) states that:

> "*The two tests are comparable in terms of power, ...*"

The degrees of freedom are computed according to the formula (5.25) for S=1, since the unknown parameter of the distribution function f(x) is σ (Normal, Truncated Normal distribution) or λ (Inverse Gaussian distribution); the mean U=0. The σ or λ is obtained as an estimate from the experimental data.

The program for the Chi-Square test (HI_DI_TEST) is given in Appendix G (Volume II).

CHAPTER 6

6.0 RESULTS

6.1 DESCRIPTION OF SURVEY OBSERVATIONS

6.1.1 GENERAL INFORMATION

The data used in the computer analysis consisted of observations abstracted from the survey at the "de Hoek Works" (Piketburg, Cape). In total, eleven cycles of observations have been carried out at these works, but only the eighth cycle is used in this dissertation This cycle was observed in January 1980 by Prof. I.B. Watt.

The data are to be analysed separately in terms of:

1. Observations relating from and to the control network only
2. Observations from control beacon to face points
3. Size of samples

The discrimination between control and face points is necessary because of different properties in the targeting and centring relating to each of these aspects of the survey. In addition to this, both of these groups are sub-divided into observations of 2 and 4 arc samples.

The Figure 16 shows a part of the "de Hoek Works". It is important to note that the difference in height between the top, and the bottom of this pit is over 100m (Prof. I.B. Watt, personal communication). The atmospheric conditions in the lower part of the pit often differed from those across the top of the pit, and hence, it was decided to separate these two classes of observa-

Figure 16. A view of a part of "de Hoek Works"

tions. In addition, the targets used for the control points and face points were also different (see Figures 17, 18 and 19).

6.1.2 MORE SPECIFIC INFORMATION

1. **Instrumentation:** The equipment used to take the observations at the "de Hoek Works" was the following:

   - Directions (horizontal angles): KERN DKM3
   - Vertical angles: DKM2
   - Distances: Tellurometer MA100

2. **Targets:** Two different types of targets have been used for control points.

   - Zeiss target (Fig. 17)
   - Wild target (Fig. 18)

   These targets were placed on the concrete pillars as shown in figures 17, 18.

   For the face points a pattern (illustrated in Fig. 19) was used. For the pattern a hole is drilled into the rock and an iron bar was placed inside the hole. The iron bar is concreted into position and painted white, while the surrounding concrete is painted black. This allows for easy identification.

   The centring error for the theodolite is negligible for both the control and face point surveys as forced centring has been used.

Figure 1?. Zeiss target for control points

Figure 18. Wild target for control points

Figure 19. Target for face points

6.2 TECHNIQUES USED IN THE TREATMENT OF THE DATA

Two computer programs have been written for dissertation purposes. These are:

1. The "THEO" program for the reduction of theodolite observations and for the calculation of the observational residuals, and
2. The "HI_DI_TEST" which tests (using the Chi-Square goodness-of-fit test) some distributions for their validity for the residuals.

An analytical explanation of the treatment of the data used in the computer programs follows in the next sub-sections.

Figure 19. Target for face points

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An analytical explanation of the treatment of the data used in the computer programs follows in the next sub-sections.

6.2.1 DATA

The data used in the first computer program ("THEO") are part of the observations taken at the "de Hoek Works" by Prof. I.B Watt. These sets of observations are given in Appendix E - Volume II. Figure 20 is a diagram of the control points and shows the observed directions (a diagram with the topography of the area is not available). As has been referred to in sub-section 6.1.1, the observations have been divided into two categories (from and to control points, and from control points to face points).

It will be noticed that Figure 20 shows additional points, such as Y, W, V, which have not been included in the network as the data were not available. This dissertation deals with observations taken from control points:

A,B,C,D,E,F,G,N,O,P,Q

Many of the control points were occupied several times, and an additional number was given to these stations in order to distinguish between different samples of observations. Thus, for example, observations at A1, A2, ... denotes that measurements were made on different days or at different times from point A.

An additional point, POL, was used as an R.O. for the observations at many of the control points.

Briefly, the two categories are:

- From control points to control points:

  (A,B,C,D,E,F,G,N,O,P,Q) → (A,B,C,D,E,F,G,N,O,P,Q,POL)

- From control points to face points:

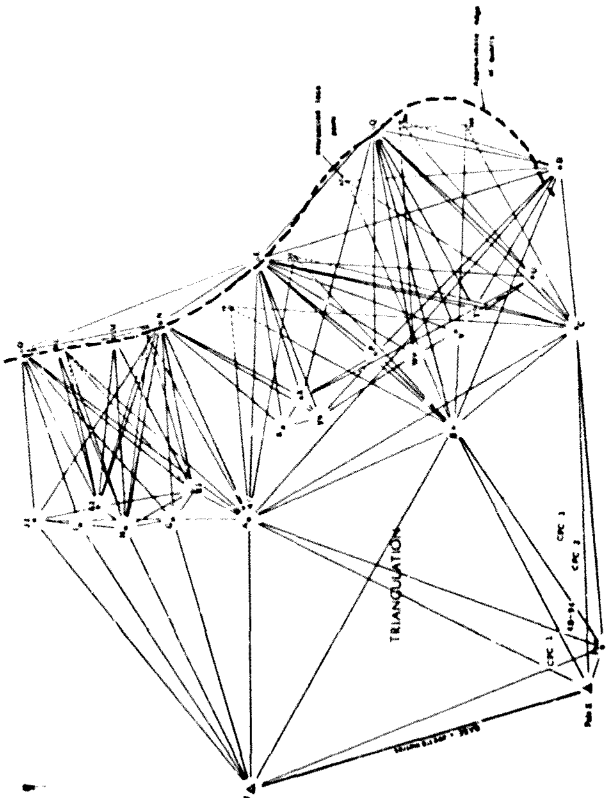


Figure 20. Diagram of the control points' position at "de Hoek Works"

(A,B,C,D,E,F,G,N,O,P,Q) *

(A,B,...,P,Q,Y,W,V,...,10,155,156,...)

Certain of the control points have been included in the subset of observations dealing with the face points. These control points were included in the arcs as points for orientation.

6.2.2 CALCULATION OF OBSERVATIONAL RESIDUALS

The theodolite observations are reduced according to the procedure described in section 3.4.1, and the residuals calculated using the formula (3.1). For this purpose the "THEO" computer program has been written (see example in Appendix B).

The observational residuals which have been calculated by "THEO" are the data for the "HI.DI TEST" computer program which is involved with the analysis of these residuals.

At this point, it must be emphasised again that this dissertation is involved with the analysis of the residuals of the theodolite observations because, having defined their distribution, the observations then follow the same distribution. This can be readily shown since a true error may be defined as:

$$e_{ij} = \mu_i - x_{ij} \tag{6.1}$$

where:

- e_{ij} : the ith true error for arc j
- x_{ij} : the ith observation for arc j

The result follows directly from equation (6.1), that is:

$$E(e_{ij}) = 0 \tag{6.2}$$

CONTROL POINTS		
Point	No of Residuals	$s_{\bar{U}}$
P	28	0.21
G	28	0.22
F	44	0.12
N	32	0.15
O	28	0.15
Q	36	0.13
E	44	0.11
A1	44	0.16
B2	28	0.17
C1	40	0.16
C3	36	0.16
D1	40	0.13
D2	32	0.12
2	94	0.06
4	460	0.04
2+4	554	0.04

Table 2 The standard deviation ($s_{\bar{U}}$) of the mean of the residuals for control points

F A C E P O I N T S		
Point	No. of Residuals	$s_{\bar{U}}$
P	52	0.14
G	60	0.14
F	76	0.09
N	60	0.15
O	56	0.14
Q1	34	0.12
Q2	80	0.11
E1	42	0.11
E2	84	0.08
A1	82	0.10
A2	66	0.09
A3	34	0.08
B2	60	0.10
B3	82	0.07
B4	82	0.07
B5	34	0.15
C1	28	0.16
C2	80	0.16
C3	64	0.13
D1	64	0.13
D2	84	0.08
D3	50	0.11
D4	84	0.07
2	394	0.02
4	368	0.05
2+4	1462	0.02

Table 3 : The standard deviation ($s_{\bar{U}}$) of the mean of the residuals for face points

since $E[x_{ij}] = \mu_i$.

Thus, the distribution of e_{ij} about zero is identical to the distribution of x_{ij} about its expectation μ_i.

Tables 2 and 3 show the quality of the data used (residuals) considering the standard deviation of the mean of "all" residuals for control and for face points, respectively. The calculation of the $s_{\bar{U}}$ is according to formula (3.5).

6.2.3 ESTIMATION OF DISTRIBUTION PARAMETERS

For the reasons discussed in section 4.5, three alternative distributions that have approximately similar properties are considered:

1. The ***Normal*** *(Gauss)* ***distribution*** which is the traditionally acceptable distribution (formula 4.13) for describing the statistical behaviour of the residuals. This distribution is symmetrical about its population mean. This mean is estimated from the sample under investigation as the arithmetic mean:

$$\bar{U} = (1/n) \sum_{i=1}^{n} U_i \qquad (6.3)$$

   However, strictly speaking for the case of the residuals, their mean is known *a priori* to be zero. The σ is approximated by the sample s according to formula (3.5).

2. The *Truncated Normal* distribution (formula 4.26) is also symmetrical, but the tail areas are truncated. This is examined for 5%, 2.5% and 1% levels of truncation. The values of

its parameters are exactly the same as those of the Normal distribution since the selected Truncated Normal is considered to be a distribution with statistics derived from the Normal distribution.

In particular, for the Truncated Normal distribution, the upper and lower truncated points are calculated according to the values given in Table 1.

3. The *Inverse Gaussian* distribution (formula 4.43) It should be noticed that as there are negative and zero residuals, the formula (4.43) cannot be applied directly (the quantity in the square root becomes negative). Therefore, an "equivalent" or "new" set of residuals are created according to the following transformation:

$$\text{"new" residuals} = K_x + \text{residuals} \qquad (6.4)$$

where K_x is an arbitrary positive quantity, so that the all residuals become positive. This means that the curve of the Inverse Gaussian distribution shifts, while the statistics remains exactly the same.

Concerning the estimation of its parameters, the population mean μ is found to be approximately the arithmetic mean of the *sample* values (formula 4.45 or 6.3). Obviously, since "new" positive residuals have been introduced, it implies that the mean is a positive number. On the other hand, the parameter of dispersion λ is given by the formula (4.47) and not by (4.46) since (4.47) is considered to be "more unbiased". At this point, it must be noticed that the calculation of the term s^2 in formula (4.47) is replaced by the equivalent formula in (4.48). However, in the case of the residuals, as the dissertation is attempting to identify their overall distribution, the denominator (n-1) is replaced by n.

The plotting of the Normal, Truncated Normal and Inverse Gaussian distributions is based on the p.d.f. given by the formulae (4.13), (4.26) and (4.43), respectively.

6.2.4 COEFFICIENTS OF SKEWNESS AND KURTOSIS

Each of the distributions under investigation has its own expected values for the coefficients of Skewness and Kurtosis. These values will be termed the "theoretical" values of γ_1 (Skewness) and γ_2 (Kurtosis).

The theoretical values for γ_1, γ_2 for the three distributions are tabulated in the Tables 12, 13 for the control and face points respectively, and have been calculated:

- For Normal distribution in sub-section 4.2.2, and are given by the formula (4.23).
- For Truncated Normal distribution (95%, 97.5% and 99%) in sub-section 4.3.1, and are given by the formulae (4.32), (4.38, 4.40, 4.42), and
- For Inverse Gaussian distribution in sub-section 4.4.2, and are given by the formulae (4.54), (4.55).

As can be easily seen for the first two distributions, the theoretical values of γ_1, γ_2 are **independent** of the sample, and in the third case (Inverse Gaussian distribution), the calculation of γ_1, γ_2 is based on the sample values, since γ_1 and γ_2 are **dependent** on μ, λ. This implies that the shape of the Inverse Gaussian distribution is strongly associated with the dispersion (considering the population mean μ to be fixed) of the sample values, as Figure 10 shows. Thus, the figures were separated into two Tables (12 and 13). The γ_1 and γ_2 for the Inverse Gaussian distribution have been calculated for each sample separately for both the control (Table 12) and face points (Table 13).

The "actual" values for γ_1, γ_2 for both the *Normal and Truncated* Normal distribution have been calculated from the samples values using the formulae (4.16) and (4.17), respectively (see Table 14).

6.2.5 CHI-SQUARE GOODNESS-OF-FIT TEST

For plotting the histogram and calculating the X^2 goodness-of-fit test, the residuals are divided in groups according to a selected class interval of 0".2.

This particular class interval is chosen in order to optimise the degrees of freedom and still satisfy the minimum number of data points per cell. The reason is that, as the class interval decreases, the degrees of freedom obviously increase, and consequently, the power of the test increases. It was, however, necessary to combine certain cells in order to obtain sufficient residuals per cell.

The application of the X^2 goodness-of-fit test is described below.

Since the residuals have already been grouped, there is a set of observed frequencies O_i (i=1,2,. .,K; where K is the number of the groups, classes or categories) which are recorded.

In the second stage of the test, the expected (theoretical) frequency E_i (i=1,2,...,K) for each category is calculated using the formula of the p.d.f. for the distribution under consideration.

Since the accepted convention regarding a safe minimum for any E_i is 5, categories in which $E_i<5$ are merged into larger groups, so that $E_i \geq 5$.

At this point it must be noted that, supposing K categories (classes) existed after the initial grouping, it is obvious that the observed frequency O_i in any class will be exactly the same for any of the distributions. However, on the other hand, the expected frequency E_i *is dependent on the* selected distribution since the value E_i is calculated each time from the formula of the p.d.f. which is under investigation, and hence differs from distribution to distribution. In other words, for a specific class (category) there are different values E_i for each of the distributions while the value O_i for this *specific* category remains the same. Therefore, the grouping of the categories (formed according to E_i) is not the same for all distributions.

Having grouped the residuals into classes, the statistic Q is calculated from the formula (5.19).

Table A2 provides values for the X^2 distribution at a significance level of 0.05.

Assuming that the theoretical value for the X^2 goodness-of-fit test is Th, then if:

$$Q \leq Th$$

the selected distribution fits the residuals. If not, it means that the hypothesised distribution is not consistent with the data (*sample values*). Obviously, if the sample is too small, resulting in the degrees of freedom becoming zero or negative (number of categories less than 3, as happens in the case of the control point for the B1 set with 20 residuals or for the D3 set with 18 residuals), the test cannot be applied. These specific sets of data are not tested, and consequently are not included in the relevant Tables.

The results of the X^2 test are listed separately in Tables 4 and 5 for the control and face points, respectively.

6.3 DISCUSSION OF THE RESULTS

Table 4 refers to the control points and Table 5 to the face points.

In these Tables, other sets of data could have been included, but, as they consisted of less than 25 residuals, the direct application of the Chi-Square test was not possible since the degrees of freedom would be zero or negative. However, these sets of data have been combined to form new sets of residuals. Therefore:

1. For the control points there are 13 sets of data (P, G, F, N, O, Q, E, A1, B2, C1, C3, D1, D2) observed on 4 arcs, and five more sets [A2(22), B1(20), B3(16), C2(18), D3(18)] comprise the 2 arc samples for which the X^2 test cannot be applied.

2. For the face points there are 6 sets of data (P, G, F, N, O, D1, D4) of 4 arcs, and there are 18 (Q1, Q2, E1, E2, A1, A2, A3, B2, B3, B4, B5, C1, C2, C3, D2, D3, D4) of 2 arcs. The B1 set includes 24 residuals and hence the X^2 test cannot be applied in this case.

The last three sets of data (2, 4, 2+4) in each Table represent the sets of the total residuals for two, four and two plus four arcs, respectively.

Table 4 : The results from the application of the Chi-Square test for the control points

CONTROL POINTS												
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Theor.	Actual	Theor.	Actual	Theor.	Actual	Theor.	Actual	Theor.	Actual
P(4)	28	[illegible]	[illegible].483	5.991	5.174	7.815	6.397	7.815	6.181	7.815	5.331
G(4)	28	[illegible]	[illegible].818	5.991	4.047	7.815	3.217	7.815	2.883	7.815	2.788
F(4)	44	[illegible]	4.102	11.070	10.461	7.815	0.160	7.815	0.178	7.815	0.160
N(4)	[illegible]	[illegible]	6.370	7.815	1.848	7.815	7.499	7.815	6.410	7.815	6.679
O(4)	[illegible]6	5.991	4.499	5.991	7.573	5.991	7.292	5.991	6.328	5.991	5.540
Q(4)	36	7.815	2.829	7.815	7.516	7.815	3.500	7.815	3.157	7.815	2.931
E(4)	44	7.815	4.229	11.070	5.[illegible]47	11.070	5.850	7.815	3.905	7.815	3.962
A1([illegible])	44	9.488	4.122	9.488	2.[illegible]18	9.488	4.051	9.488	3.873	9.468	3.846
A2([illegible])	22										
B1([illegible])	20										
R2([illegible])	28	5.991	0.285	7.815	5.779	5.991	0.355	5.991	0.540	5.991	0.334
B[illegible] 2)	16										
C[illegible]4)	40	9.488	6.712	9.488	12.390	9.488	7.083	9.488	6.825	9.488	6.261
C2(2)	18										
C3(4)	36	7.815	0.362	7.815	1.976	7.815	0.225	7.815	0.376	7.815	0.485
D1(4)	40	9.488	5.113	7.815	6.724	7.815	6.243	9.488	5.477	9.488	5.082
D2(4)	32	5.991	5.999	5.991	8.265	5.991	6.853	5.991	6.588	5.991	6.420
D3(2)	18										
2	94	14.070	3.942	14.070	10.529	14.070	6.265	14.070	4.738	14.070	3.599
4	460	30.140	18.698	28.870	81.122	25.000	14.442	27.590	15.392	27.590	14.122
2+4	554	30.140	25.265	27.590	67.586	25.000	18.488	25.000	18.389	27.590	18.174

Table 4 : The results from the application of the Chi-Square test for the control points

CONTROL POINTS											
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Theor.	Actual	Theor.	Actual	Theor.	Actual	Theor.	Actual	Theor.	Actual
P(4)	28	7.815	5.483	5.991	5.174	7.815	6.397	7.815	6.181	7.815	5.331
G(4)	28	7.815	2.818	5.991	4.047	7.815	3.217	7.815	2.083	7.815	2.788
F(4)	44	9.488	4.102	11.070	10.461	7.815	0.160	7.815	0.178	7.815	0.160
N(4)	32	7.815	6.370	7.815	1.848	7.815	7.499	7.815	6.410	7.815	6.679
O(4)	28	5.991	4.499	5.991	7.573	5.991	7.292	5.991	6.328	5.991	5.540
Q(4)	36	7.815	2.829	7.815	7.516	7.815	3.500	7.815	3.157	7.815	2.931
E(4)	44	7.815	4.229	11.070	5.347	11.070	5.850	7.815	3.905	7.815	3.962
A1(4)	44	9.468	4.122	9.488	2.718	9.488	4.051	9.438	3.873	9.488	3.846
A2(2)	22	—	—	—	—	—	—	—	—	—	—
B1(2)	20	—	—	—	—	—	—	—	—	—	—
B2(4)	28	5.991	0.285	7.815	5.779	5.991	0.355	5.991	0.540	5.991	0.334
B3(2)	16	—	—	—	—	—	—	—	—	—	—
C1(4)	40	9.488	6.712	9.488	12.390	9.488	7.083	9.488	6.825	9.488	6.261
C2(2)	18	—	—	—	—	—	—	—	—	—	—
C3(4)	36	7.815	0.362	7.815	1.976	7.815	0.225	7.815	0.376	7.815	0.485
D1(4)	40	9.488	5.113	7.815	6.724	7.815	6.245	9.488	5.477	9.488	5.082
D2(4)	32	5.991	5.999	5.991	8.265	5.991	6.853	5.991	6.588	5.991	6.420
D3(2)	18	—	—	—	—	—	—	—	—	—	—
2	94	14.070	3.942	14.070	10.529	14.070	6.265	14.070	4.738	14.070	3.599
4	460	30.140	18.698	28.870	81.122	25.000	14.442	27.590	15.392	27.590	14.122
2+4	554	30.140	25.265	27.590	67.586	25.000	18.408	25.000	18.389	27.590	18.174

Table 5 : The results from the application of the Chi-Square test for the face points

FACE POINTS											
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		[illegible].5% Truncated		99.0% Truncated	
		Theor.	Actual	Theor.	Actual	Theor.	Actual	Theor.	Actual	Theor.	Actual
P(4)	52	11.070	12.019	11.070	8.575	11.070	12.408	11.070	13.199	11.070	13.685
G(4)	60	12.590	4.046	12.590	16.403	12.590	5.425	12.590	6.368	12.590	5.298
F(4)	76	16.920	[illegible]	14.070	8.689	14.070	1.879	14.070	1.482	16.920	1.751
N(4)	60	12.590	9.530	12.590	9.694	12.590	9.290	12.590	9.479	12.590	9.817
O(4)	56	11.070	10.652	11.070	9.829	12.590	5.583	12.590	5.531	11.070	10.522
Q1(2)	34	7.815	0.518	5.991	1.531	5.991	0.882	7.815	1.018	7.815	0.436
Q2(2)	80	16.920	9.818	18.310	14.664	16.920	10.971	16.920	10.600	16.920	10.120
E1(2)	42	7.815	0.827	11.070	5.469	7.815	1.389	7.815	0.805	7.815	0.814
E2(2)	84	16.920	19.885	15.510	26.708	16.920	18.336	16.920	18.235	16.920	19.902
A1(2)	82	16.920	5.047	16.920	9.519	16.920	7.142	16.920	7.970	18.310	5.599
A2(2)	66	14.070	9.077	14.070	13.579	14.070	8.728	14.070	10.105	14.070	9.259
A3(2)	34	7.815	7.112	5.991	5.326	7.815	6.848	7.815	7.107	7.815	7.051
B1(2)	24	—	—	—	—	—	—	—	—	—	—
B2(2)	60	14.070	6.890	14.070	12.043	14.070	9.563	14.070	10.250	14.070	8.675
B3(2)	82	14.070	4.469	14.070	10.910	14.070	4.863	14.070	4.316	14.070	4.623
B4(2)	82	14.070	6.351	14.070	16.215	14.070	8.112	14.070	7.485	14.070	6.450
B5(2)	34	7.815	1.028	7.815	3.438	7.815	1.567	7.815	1.636	7.815	1.532
C1(2)	28	5.991	1.401	5.991	1.076	5.991	1.667	5.991	1.864	5.991	1.325
C2(2)	80	16.920	16.319	15.510	16.401	16.920	15 449	16.920	12.894	16.920	16.569
C3(2)	64	12.590	4.768	14.070	10.714	14.070	6.111	14.070	2.013	12.590	4.617
D1(4)	64	14.070	5.560	15.510	10.097	16.920	7.267	14.070	5.841	14.070	6.045
D2(2)	84	16.920	15.452	14.070	18.605	16.920	14.635	16.920	15.345	16.920	15.265
D3(2)	50	11.070	3.479	12.590	8.037	11.070	1.537	11.070	1.715	11.070	4.224
D4(2)	84	14.070	9.043	14.070	12.285	14.070	6.616	14.070	10.625	14.070	9.454
2	1094	30.140	23.033	28.870	107.803	22.360	14.222	25.000	14.205	27.590	18.018
4	368	30.140	14.991	28.870	39.128	27.590	16.128	30.140	16.191	30.140	16.030
2+4	1462	32.670	39.719	32.670	119.533	25.000	23.748	27.590	25.656	30.140	23 835

From both Tables 4 and 5 (which are the results of the "HI_DI_TEST" computer program), it can be seen that *in most of the cases*, all of the distributions are *consistent*, and hence these curves fit the residuals according to Chi-Square test, since:

$$V_{(K-2)} - Q_{\text{Observed frequency (Actual)}} \geq 0$$

where:

$V_{(K-2)}$: the corresponding value for K-2 degrees of freedom given by Table A2 (Appendix A), for $\alpha = 0.05\%$

Q : the (actual) value which comes from the application of the Chi-Square goodness-of-fit test

Following from section 5.4.2.5 the analysis of the test has been carried out with both K-2 and K-1 degrees of freedom. A distribution is only considered as inconsistent (that is, it does not satisfy the null hypotnesis) if Q is less than both $V_{(K-2)}$ and $V_{(K-1)}$. Only the values of $V_{(K-2)}$ have been listed in Tables 4 and 5, as these values are the pessimistic lower boundaries. For example, in Table 5, the point P has the following values for the Normal distribution:

$$V_{(K-2)} = 11.070$$

$$Q = 12.019$$

By considering these figures alone, the wrong conclusion will be drawn, because, in such a case

$$Q > V_{(K-2)}$$

However, considering the value of V for K-1 degrees of freedom as discussed in sub-section 5.4.2.5

$$V_{(K-1)} = 12.590$$

and hence:

$$V_{(K-2)} < Q < V_{(K-1)}$$

According to formula (5.25), this implies that the distribution is consistent.

On the other hand, if the last large sample (2+4) of the face points for the Normal distribution is considered, then:

$$V_{(K-2)} = 32.670 < Q = 39.719 \quad \text{and}$$

$$V_{(K-1)} = 33.920 < Q = 39.719$$

and hence, for this specific set of data, the selected (Normal) distribution does not fit the residuals.

The information can be summarised as follows:

1. In Table 4 (control points), the sets which are inconsistent with the distributions are:

    - Normal :
    - Inverse Gaussian : D2, 4, 2+4
    - Truncated Normal (99%) :
    - Truncated Normal (97.5%) :
    - Truncated Normal (99%) :

2. In Table 5 (face points), the sets which are inconsistent with the following distributions are:

    - Normal : E2, 2+4
    - Inverse Gaussian : G, E2, B4, D2, 2, 4, 2+4

- Truncated Normal (99%) : E2
- Truncated Normal (97.5%) : P
- Truncated Normal (99%) : P, E2

Therefore, it can be seen that the Normal and the three Truncated Normal distributions are, generally, consistent for both categories of the points (control or face points) and are also, generally, independent of the sample size (the only exception is the largest sample for the face points under the Normal distribution assumption).

On the other hand, it is quite obvious that the Inverse Gaussian distribution is by no means the best one for the large samples of either the control or face points. For the small samples, in the case of the control points, it is consistent with the data as opposed to small samples of the face points where the consistency is in doubt.

However, in order to determine the best fit, it is necessary to consider the ratio of the Actual (Q) to Theoretical $[V_{V(K-2)}]$ values (Tables 6 and 7). If this ratio is less than or equal to 1, it implies that the distribution is consistent. In addition to this, for a specific set of residuals, the smaller ratio implies that the specific distribution has a better fit than a distribution corresponding to a large ratio. For instance, the point D1 (Table 6), has the following ratios for each of the distributions:

a) Normal : 5.113 / 9.488 = 0.539
b) Inverse Gaussian : 6.724 / 7.815 = 0.860
c) Truncated (95%) : 6.243 / 7.815 = 0.799
d) Truncated (97.5%) : 5.477 / 9.488 = 0.577
e) Truncated (99%) : 5.082 / 9.488 = 0.536

This implies that all of the distributions are consistent as these ratios are ≤ 1. Also, it can be concluded that the Truncated 99%

is the *best* curve (distribution) for the residuals of this specific set of data (as the actual value O_i approaches the expected value E_i, this ratio becomes less, and hence, the selected distribution more closely models the distribution of the residuals).

The information contained in Tables 4 and 5 have been summarised in Tables 6 and 7, respectively.

In Tables 6 and 7, the column "Q/T" denotes the ratio of the Actual to the Theoretical values, and the column "Order" denotes the relative fit for each distribution with respect to the given sample. These Tables, therefore, rank the distributions for each set of data. For example, in Table 6 the

Normal	comes	first	4	times
Inverse Gaussian	"	"	3	"
Truncated (95%)	"	"	2	"
Truncated (97.5%)	"	"	0	"
Truncated (99%)	"	"	7	"

Based on the above, it can, generally, be seen that for the control points (Table 6), the Truncated 99% is the best distribution for most of the cases, as opposed to the face points (Table 7), where the conclusion is not very clear.

Although the ranking of the distribution provides an indicator of its goodness-of-fit, it is necessary to consider the selection from several points of view.

For this reason, four more Tables are produced. The Tables 8 and 9 refer to control points, and the Tables 10 and 11 to face points. Tables 8 and 10 refer to small samples, and Tables 9 and 11 to the pooled samples. The purpose of these Tables is to indicate the "weighted means" of the residuals, calculated as the sum of the products of the number of the residuals times the "Order" of the fit for a specific sample, divided by the sum of

Table 6 : Analysis of the Chi-Square test for control points

		C O N T R O L				P O I N T S					
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Q/T	Order	Q/T	Order	Q/T	Order	Q/T	Order	Q/T	Order
P(4)	28	0.702	2	0.864	5	0.819	4	0.791	3	0.682	1
G(4)	28	0.361	2	0.676	5	0.412	4	0.369	3	0.357	1
F(4)	44	0.432	4	0.945	5	0.020	1	0.023	3	0.020	1
N(4)	32	0.815	2	0.236	1	0.950	5	0.820	3	0.855	4
O(4)	28	0.751	1	1.264	5	1.217	4	1.056	3	0.925	2
Q(4)	36	0.362	1	0.962	5	0.448	4	0.404	3	0.375	2
E(4)	44	0.581	5	0.483	1	0.528	4	0.500	2	0.507	3
A1(4)	44	0.434	5	0.286	1	0.427	4	0.408	3	0.405	2
A2(2)	22	—	—	—	—	—	—	—	—	—	—
B1(2)	20	—	—	—	—	—	—	—	—	—	—
B2(4)	28	0.048	1	0.739	5	0.059	3	0.090	4	0.056	2
B3(2)	16	—	—	—	—	—	—	—	—	—	—
C1(4)	40	0.707	2	1.306	5	0.747	4	0.719	3	0.660	1
C2(2)	18	—	—	—	—	—	—	—	—	—	—
C3(4)	36	0.046	2	0.253	5	0.028	1	0.048	3	0.062	4
D1(4)	40	0.539	2	0.860	5	0.799	4	0.577	3	0.536	1
D2(4)	32	1.001	1	1.380	5	1.143	4	1.100	3	1.072	2
D3(2)	18	—	—	—	—	—	—	—	—	—	—
2	94	0.280	2	0.748	5	0.445	4	0.337	3	0.256	1
4	460	0.620	4	2.810	5	0.578	3	0.558	2	0.512	1
2+4	554	0.838	4	2.450	5	0.740	3	0.736	2	0.659	1

* * The Q/T denotes the ratio of the Actual to Theoretical values obtained from the Table 4.

Table 7 : Analysis of the Chi-Square test for face points

		F A C E P O I N T S									
From Point	No.of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Q/T	Order	Q/t	Order	Q/T	Order	Q/T	Order	Q/T	Order
P(4)	52	1.086	2	0.775	1	1.121	1	1.192	4	1.236	5
G(4)	60	0.321	1	1.103	5	0.431	3	0.506	4	0.421	2
F(4)	76	0.081	1	0.618	5	0.134	4	0.105	3	0.103	2
N(4)	60	0.757	3	0.770	4	0.738	1	0.753	2	0.780	5
O(4)	56	0.962	5	0.888	3	0.443	2	0.439	1	0.950	4
Q1(2)	34	0.056	2	0.256	5	0.147	4	0.130	3	0.056	1
Q2(2)	80	0.580	1	0.801	5	0.648	4	0.626	3	0.598	2
E1(2)	42	0.106	3	0.494	5	0.178	4	0.103	1	0.104	2
E2(2)	84	1.175	3	1.722	5	1.084	2	1.078	1	1.176	4
A1(2)	82	0.298	1	0.563	5	0.422	3	0.471	4	0.306	2
A2(2)	66	0.645	2	0.955	5	0.620	1	0.718	4	0.658	3
A3(2)	34	0.910	5	0.889	2	0.876	1	0.909	4	0.902	3
B1(2)	24	—	—	—	—	—	—	—	—	—	—
B2(2)	60	0.490	1	0.856	5	0.680	3	0.729	4	0.617	2
B3(2)	82	0.318	2	0.775	5	0.346	4	0.307	1	0.329	3
B4(2)	82	0.451	1	1.152	5	0.577	4	0.532	3	0.458	2
B5(2)	34	0.132	1	0.440	5	0.201	3	0.209	4	0.196	2
C1(2)	28	0.234	3	0.180	1	0.278	4	0.311	5	0.221	2
C2(2)	80	0.964	3	1.057	5	0.913	2	0.762	1	0.979	4
C3(2)	64	0.379	3	0.761	5	0.434	4	0.143	1	0.367	2
D1(4)	64	0.395	1	0.651	5	0.429	3	0.415	2	0.430	4
D2(2)	84	0.913	4	1.322	5	0.865	1	0.907	3	0.902	2
D3(2)	50	0.314	3	0.638	5	0.139	1	0.155	2	0.382	4
D4(2)	84	0.643	2	0.873	5	0.470	1	0.755	4	0.671	3
2	1094	0.764	4	3.734	5	0.636	2	0.568	1	0.653	3
4	368	0.497	1	1.355	5	0.585	4	0.537	3	0.532	2
2+4	1462	1.216	4	3.659	5	0.950	3	0.930	2	0.791	1

* * The Q/T denotes the ratio of the Actual to Theoretical values obtained from the Table 5.

the total residuals. For instance, in the Table 8 the weighted mean for the Normal distribution is:

$$(28\times2 + 28\times2 + 44\times4 + \ldots + 32\times1)/460 = 2.50$$

From these weighted means, the following conclusions can be drawn:

- **Control points**

  1. *Small samples* (Table 8): The Truncated 99% Normal distribution with a value of 2.00 for the weighted mean has the best fit, while the Normal distribution with 2.50 follows.
  2. *Large samples* (Table 9): The Truncated 99% with a weighted mean of 1.29 has the lowest value, followed by the Truncated 97.5% and 95% with means of 2.34 and 3.20, respectively.

- **Face points**

  1. *Small samples* (Table 10): The Normal and 95% Truncated Normal distributions with weighted means of 2.21 and 2.66, respectively, have the best fit. The Truncated 97.5% with a mean of 2.68 displays the third best fit.
  2. *Large samples* (Table 11): For the set of 368 residuals, the Normal distribution has the best fit, followed by the Truncated Normal 99%. For the set of 1094 residuals the Truncated 97.5% is the best distribution and the Truncated 95% follows. For the largest set (1462 residuals), the Truncated 99% comes first and the Truncated 97.5% second.

It could, therefore, be inferred that the results for the control points pertaining to the fit of the distributions are different to those of the face points. Summarising:

Table 8 : The weighted mean for each distribution for the control points

C O N T R O L P O I N T S											
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Order	P	Order	P	Order	P	Order	P	Order	P
P(4)	28	2	56	5	140	4	112	3	84	1	28
G(4)	28	2	56	5	140	4	112	3	84	1	28
F(4)	44	4	176	5	220	1	44	3	132	1	44
N(4)	32	2	64	1	32	5	160	3	96	4	128
O(4)	28	1	28	5	140	4	112	3	84	2	56
Q(4)	36	1	36	5	180	4	144	3	108	2	72
E(4)	44	5	220	1	44	4	176	2	88	3	132
A1(4)	44	5	220	1	44	4	176	3	132	2	88
B2(4)	28	1	28	5	140	3	84	4	112	2	56
C1(4)	40	2	80	5	200	4	160	3	120	1	40
C3(4)	36	2	72	5	180	1	36	3	108	4	144
D1(4)	40	2	80	5	200	4	160	3	120	1	40
D2(4)	32	1	32	5	160	4	128	3	96	2	64
SUM	460		1148		1820		1604		1364		920
Weighted mean		2.50		3.96		3.49		2.97		2.00	

* * P denotes the number of residuals of each set times the "Order" of the distribution.

Table 9 : The weighted mean for each distribution for the control points including the pooled samples

C O N T R O L P O I N T S											
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Order	P	Order	P	Order	P	Order	P	Order	P
P(4)	28	2	56	5	140	4	112	3	84	1	28
G(4)	28	2	56	5	140	4	112	3	84	1	28
F(4)	44	4	176	5	220	1	44	3	132	1	44
N(4)	32	2	64	1	32	5	160	3	96	4	128
O(4)	28	1	28	5	140	4	112	3	84	2	56
Q(4)	36	1	36	5	180	4	144	3	108	2	72
E(4)	44	5	220	1	44	4	176	2	88	3	132
A1(4)	44	5	220	1	44	4	176	3	132	2	88
B2(4)	28	1	28	5	140	[illegible]	84	4	112	2	56
C1(4)	40	2	80	5	200	4	160	3	120	1	40
C3(4)	36	2	72	5	180	1	36	3	108	4	144
D1(4)	40	2	80	5	200	4	160	3	120	1	40
D2(4)	32	1	32	5	160	4	128	3	96	2	64
2	94	2	188	5	470	4	376	3	282	1	94
4	460	4	1840	5	2300	3	1380	2	920	1	460
2+4	554	4	2216	5	2770	3	1662	2	1108	1	554
SUM	1568		5392		7360		5022		3674		2028
Weighted mean		3.44		4.69		3.20		2.34		1.29	

* * P denotes the number of residuals of each set times the "Order" of the distribution.

Table 10 : The weighted mean for each distribution for the face points

F A C E P O I N T S											
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Order	P	Order	P	Order	P	Order	P	Order	P
P(4)	52	2	104	1	52	3	156	4	208	5	260
G(4)	60	1	60	5	300	3	180	4	240	2	120
F(4)	76	1	76	5	380	4	304	3	228	2	152
N(4)	60	3	180	4	240	1	60	2	120	5	300
O(4)	56	5	280	3	168	2	112	1	56	4	224
Q1(2)	34	2	68	5	170	4	136	3	102	1	34
Q2(2)	80	1	80	5	400	4	320	3	240	2	160
E1(2)	42	3	126	5	210	4	168	1	42	2	84
E2(2)	84	3	252	5	420	2	168	1	84	4	336
A1(2)	82	1	82	5	410	3	246	4	328	2	164
A2(2)	66	2	122	5	330	1	66	4	264	3	198
A3(2)	34	5	170	2	68	1	34	4	136	3	102
B2(2)	60	1	60	5	300	3	180	4	240	2	120
B3(2)	82	2	164	5	410	4	328	1	82	3	246
B4(2)	82	1	82	5	410	4	328	3	246	2	164
B5(2)	34	1	34	5	170	3	102	4	136	2	68
C1(2)	28	3	84	1	28	4	112	5	140	2	56
C2(2)	80	3	240	5	400	2	160	1	80	4	320
C3(2)	64	3	192	5	320	4	256	1	64	2	128
D1(4)	64	1	64	5	320	3	192	2	128	4	256
D2(2)	84	4	336	5	420	1	84	3	252	2	168
D3(2)	50	3	150	5	250	1	50	2	100	4	200
D4(2)	84	2	168	5	420	1	84	4	336	3	252
SUM	1438		3174		6596		3826		3852		4112
Weighted mean		2.21		4.59		2.66		2.68		2.86	

** P denotes the number of residuals of each set times the "Order" of the distribution.

Table 11 : The weighted mean for each distribution for the face points including the pooled samples

		F A C E		P O I N T S							
From Point	No.of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Order	P	Order	P	Order	P	Order	P	Order	P
P(4)	52	2	104	1	52	3	156	4	208	5	260
G(4)	60	1	60	5	300	3	180	4	240	2	120
F(4)	76	1	76	5	380	4	304	3	228	2	152
N(4)	60	3	180	4	240	1	60	2	120	5	300
O(4)	56	5	280	3	168	2	112	1	56	4	224
Q1(2)	34	2	68	5	170	4	136	3	102	1	34
Q2(2)	80	1	80	5	400	4	320	3	240	2	160
F1(2)	42	3	126	5	210	4	168	1	42	2	84
E2(2)	84	3	252	5	420	2	168	1	84	4	336
A1(2)	82	1	82	5	410	3	246	4	328	2	164
A2(2)	66	2	122	5	330	1	66	4	264	3	198
A3(2)	34	5	170	2	68	1	34	4	136	3	102
B2(2)	60	1	60	5	300	3	180	4	240	2	120
B3(2)	82	2	164	5	410	4	328	1	82	3	246
B4(2)	82	1	82	5	410	4	328	3	246	2	164
B5(2)	34	1	34	5	170	3	102	4	136	2	68
C1(2)	28	3	84	1	28	4	112	5	140	2	56
C2(2)	80	3	240	5	400	2	160	1	80	4	320
C3(2)	64	3	192	5	320	4	256	1	64	2	128
D1(4)	64	1	64	5	320	3	192	2	128	4	256
D2(2)	84	4	336	5	420	1	84	3	252	2	168
D3(2)	50	3	150	5	250	1	50	2	100	4	200
D4(2)	84	2	168	5	420	1	84	4	336	3	252
2	1094	4	4376	5	5470	2	2188	1	1094	3	3282
4	368	1	368	5	1840	4	1472	3	1104	2	736
2+4	1462	4	5848	5	7310	3	4386	2	2924	1	1462
SUM	4362		13766		21216		11872		8974		9588
Weighted mean			3.16		4.86		2.72		2.06		2.20

* * P denotes the number of residuals of each set times the "Order" of the distribution.

1. In the case of the **control points**, for either *small* or *large samples* (and consequently for the *pooled samples*) the:

   - **99% Truncated Normal distribution best describes the stochastic properties of the residuals.**

2. In the case of the **face points** for the:

   - *small samples*: **the Normal distribution appears best.**
   - *large samples*: **the 97.5% and 99% Truncated Normal distributions appear to be best although, for the largest sample, the 99% Truncated Normal is the most appropriate.**
   - *pooled samples*: **the 97.5% Truncated Normal distribution best approximates the distribution of the residuals.**

3. It must also be noticed that for the case of the largest samples "2+4" for control (554 residuals) and face (1462) points, the Truncated Normal 99% is the best fitting curve. In addition to this, for the large samples ("2", "4", "2+4"), in the case of control points, the 99% Truncated Normal is the best. For the corresponding samples of the face points, the same distribution seems to be a good approximation in comparison to the other distributions (essentially, only the 97.5% Truncated Normal distribution could compete with the 99% truncation points). Therefore, in general, the 99% Truncated Normal distribution seems to fit better than the other distributions for the large samples (=>350 residuals) independently of the classification used (control or face points).

4. On the other hand, the **Inverse Gaussian** distribution seems to be the **worst** in comparison with the other distributions for all the cases (control or face points, small or large

samples). It must, also, be noticed that for the largest samples (n>350) it is completely inconsistent with the data.

The coefficients of Skewness and Kurtosis have been calculated in order to give an idea of the shape of the actual distribution of the data.

In the next pages, three more Tables (12, 13 and 14) are provided. Tables 12 and 13 refer to the theoretical values of the coefficients of Skewness (γ_1) and Kurtosis (γ_2), while Table 14 includes the actual γ_1 and γ_2 calculated from the residuals.

The actual values indicate the shape of the distribution of the residuals as opposed to theoretical values which show the shape of the selected distribution.

Essentially, Tables 12, 13 and 14 give three possibilities:

- The theoretical values of γ_1, γ_2 (Table 12 and 13) assist to form an idea about the shape of the hypothesised (Normal, Truncated Normal and Inverse Gaussian) distribution.
- Table 14 is able to show, using the actual values of γ_1, γ_2, the shape of the distribution which fits the residuals, and
- A combination of Tables 12 and 13 with Table 14 provides comparison between the actual and the hypothesised distribution, and therefore, assists in selecting the best fitting distribution. The best fitting curve (distribution) of the residuals should have coefficients of Skewness and Kurtosis that closely approximate actual values listed in Table 14.

Following from the previous paragraphs, the information can be summarised as follows:

1. Examining the results of Tables 12 and 13 concerning the theoretical values of γ_1, γ_2 it can be seen that:

- The Normal distribution is a symmetrical one ($\gamma_1=0$)
- The Truncated Normal distribution is symmetrical ($\gamma_1=0$). However, as the level of truncation changes from 99% to 95%, so the shape of the distribution becomes more peaked. For example, the 99% Truncated is less sharp than that of 97.5% and much less than the 95%. In all of these cases, $\gamma_2>3$ (see Figure 6(d)). On the other hand, the 95% Truncated is more eaked than the 99% because the latter distribution more closely approximates the Normal distribution.
- The Inverse Gaussian distribution has, for the case of both control and face points:

$$\gamma_1 >> 0 \quad \text{and} \quad \gamma_2 >>3$$

where the symbol ">>" indicates "much greater than". In this sense, ">>" will indicate that γ_1 is greater than 0.7 and γ_2 is greater than 3.8.

Thus, the Inverse Gaussian is not symmetrical ($\gamma_1\neq0$) and because the coefficient of Skewness is always a positive number, it implies that this distribution is positively skewed (see Figure 7(a) and the plot given in the example for the case of the Inverse Gaussian in Appendix C). Also, since $\gamma_2>3$ it means that the distribution is leptokurtotic in comparison with the Normal distribution (see Figure 7(d)).

Therefore, the Inverse Gaussian distribution is a peaked, non-symmetrical distribution with the main "body" of data lying on the left hand side of the population mean value.

2. Table 14 gives the actual values of γ_1 and γ_2 for the control and face points. The conclusions that can be drawn from this Table are:

Table 12 : Theoretical values for the coefficients of Skewness and Kurtosis for control points

CONTROL POINTS											
From Point	No. of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis
P(4)	28			1.299	5.815						
G(4)	28			1.176	5.305						
F(4)	44			1.189	5.356						
N(4)	32			1.342	6.005						
O(4)	28			1.082	4.954						
Q(4)	36			1.178	5 315						
E(4)	44			1.241	5.569						
A1(4)	44			1.253	5.620						
A2(2)	22			1.161	5.248						
B1(2)	20			1.332	5.959						
B2(4)	28	0	3	1.249	5.602	0	5.208	0	4.135	0	3.510
B3(2)	16			1.243	5.576						
C1(4)	40			0.943	4.484						
C2(2)	18			1.214	5.459						
C3(4)	36			1.462	6.564						
D1(4)	40			1.076	4.932						
D2(4)	32			0.868	4.255						
D3(2)	18			1.315	5.885						
2	94			0.961	4.539						
4	460			0.850	4.204						
2+4	554			0.804	4.078						

Table 13 : Theoretical values for the coefficient of Skewness and Kurtosis for face points

F A C E P O I N T S											
From Point	No.of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated	
		Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis
P(4)	52			0.973	4.580						
G(4)	60			0.915	4.396						
F(4)	76			1.349	6.033						
N(4)	60			1.012	4.707						
O(4)	56			1.023	4.745						
Q1(2)	34			1.092	4.990						
Q2(2)	80			0.973	4.578						
E1(2)	42			1.247	5.593						
E2(2)	84			1.153	5.216						
A1(2)	82			1.030	4.769						
A2(2)	66			1.219	5.477						
A3(2)	34			1.232	5.533						
B1(2)	24	0	3	1.301	5.821	0	5.208	0	4.135	0	3.510
B2(2)	60			1.131	5.135						
B3(2)	82			1.090	4.982						
B4(2)	82			1.079	4.943						
B5(2)	34			1.122	5.100						
C1(2)	28			1.09[illegible]	4.948						
C2(2)	80			1.050	4.838						
C3(2)	64			1.237	5.553						
D1(4)	64			1.069	4.906						
D2(2)	84			1.345	6.019						
D3(2)	50			0.939	4.471						
D4(2)	84			1.232	5.533						
2	1094			0.792	4.046						
4	368			0.871	4.265						
2+4	1462			0.717	3.857						

Table 13 : Theoretical values for the coefficient of Skewness and Kurtosis for face points

F A C E P O I N T S												
From Point	No.of Resid.	Normal Dist.		Inverse Dist.		95.0% Truncated		97.5% Truncated		99.0% Truncated		
		Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	Skewness	Kurtosis	
P(4)	52			0.973	4.580							
G(4)	60			0.915	4.396							
F(4)	76			1.349	6.033							
N(4)	60			1.012	4.707							
O(4)	56			1.023	4.745							
Q1(2)	34			1.092	4.990							
Q2(2)	80			0.973	4.578							
E1(2)	42			1.247	5.593							
E2(2)	84			1.153	5.216							
A1(2)	82			1.030	4.769							
A2(2)	66			1.219	5.477							
A3(2)	34			1.232	5.533							
B1(2)	24	0	3	1.301	5.821	0	5.208	0	4.135	0	3.510	
B2(2)	60			1.131	5.135							
B3(2)	82			1.090	4.982							
B4(2)	82			1.079	4.943							
B5(2)	34			1.122	5.100							
C1(2)	28			1.081	4.948							
C2(2)	80			1.050	4.838							
C3(2)	64			1.237	5.553							
D1(4)	64			1.069	4.906							
D2(2)	84			1.345	6.019							
D3(2)	50			0.939	4.471							
D4(2)	84			1.232	5.533							
2	1094			0.792	4.046							
4	368			0.871	4.265							
2+4	1462			0.717	3.857							

Table 14 : Actual values for the coefficients of Skewness and Kurtosis

CONTROL POINTS				FACE POINTS			
From Point	No. of Resid.	Skewness	Kurtosis	From Point	No. of Resid.	Skewness	Kurtosis
P(4)	28	0.610	3.749	P(4)	52	0.558	4.611
G(4)	28	-0.245	3.022	G(4)	60	-0.781	4.557
F(4)	44	0.246	3.146	F(4)	76	0.257	2.954
N(4)	32	0.997	3.676	N(4)	60	0.174	3.280
O(4)	28	-0.782	2.913	O(4)	56	0.191	4.477
Q(4)	36	-0.523	2.501	Q1(2)	34	0.000	3.004
E(4)	44	0.608	4.125	Q2(2)	80	0.000	3.995
A1(4)	44	0.581	4.149	E1(2)	42	0.000	2.750
A2(2)	22	0.000	2.085	E2(2)	84	0.000	2.560
B1(2)	20	0.000	1.469	A1(2)	82	0.000	2.993
B2(4)	28	0.249	2.889	A2(2)	66	0.000	2.273
B3(2)	16	0.000	2.945	A3(2)	34	0.000	2.045
C1(4)	40	-0.650	3.017	B1(2)	21	0.000	1.486
C2(2)	18	0.000	2.858	B2(2)	60	0.000	2.712
C3(4)	36	0.396	2.910	B3(2)	82	0.000	2.533
D1(4)	40	-0.309	2.939	B4(2)	82	0.000	2.741
D2(4)	32	-0.917	3.878	B5(2)	34	0.000	3.060
D3(2)	18	0.000	2.270	C1(2)	29	0.000	3.634
2	94	0.000	3.057	C2(2)	80	0.000	2.692
4	460	0.107	3.731	C3(2)	64	0.000	2.729
2+4	554	0.105	3.958	D1(4)	64	-0.178	2.950
				D2(2)	54	0.000	2.008
				D3(2)	50	0.000	4.106
				D4(2)	84	0.000	2.710
				2	1094	0.000	3.338
				4	368	-0.006	4.035
				2+4	1462	-0.003	3.981

- For both cases (control and face points), the coefficient of Skewness for two arc sets is equal to 0. This implies that the residuals of this type are situated symmetrically about the mean value. The explanation of this comes directly from the method of calculating the observational residuals as outlined in sub-section 3.4.1.1. Obviously, therefore, for this particular case (sets of two arcs) a symmetrical distribution such as Normal or Truncated Normal would be preferred.
- The coefficient of Skewness for the four arc sets of residuals is not exactly zero (it differs slightly from zero) which means that its distribution is a little bit negatively (see Figure 7(b)) or positively (see Figure 7(a)) skewed.

  For the case of control points it would appear that γ_1 is predominantly positive, so that it could be said that the four arcs residuals of the control points are positively skewed as opposed to face points where no trend can be obtained.
- Another important point for the case of four arcs residuals for face points is that although the γ_1 of the small samples (P, G, F, N, O, D1) is more than $|0.174|$, the coefficient of Skewness tends to be zero for the case of the large sample "4" (γ_1=-0.006). The same applies to the case of the largest sample "2+4" for face points although this could be due to the influence of the residuals from two arcs (1094 in number) as opposed to those of four arcs (84). Therefore, it is quite valid, for the face points, to state that as the sample increases (≈>350), the actual distribution of the residuals becomes more symmetrically, and consequently the Normal or the Truncated Normal distribution would be more applicable.
- The values of the coefficient of Kurtosis varied about 3 for the small samples of both control and face points. As a result, no definite conclusions could be drawn ex-

cept to note that the general values were closer to the Normal family of distributions as opposed to the expected values for the Inverse Gaussian distribution.

- For the large samples ("2", "4", "2+4") for both cases (control and face points), $\gamma_2>3$ which implies that the actual distribution of the residuals is leptokurtotic. At this point, it must be noted that comparing the theoretical values of γ_2 of the 97.5% Truncated Normal distribution and of the Inverse Gausssian distribution given by Table 12 (control points) and Table 13 (face points) with the actual values given by Table 14 (control and face points), these distributions are better than the others since the actual γ_2 tends to 4.

3. Comparison of Tables 12, 13 and 14 cannot take place by examining the coefficients of Skewness and Kurtosis separately but rather by considering both γ_1 and γ_2 jointly. This is necessary because considering only γ_1 may result in a distribution which is different from that if γ_2 is considered. For example, in the case of the large sample "4" for face points (Table 14), the coefficient of Skewness indicates that the Normal or Truncated Normal distributions could be the best. However, by considering the coefficient of Kurtosis it could be concluded that the 97.5% Truncated Normal or the Inverse Gaussian distribution better approximate the distribution of the residuals. There is, obviously, a contradiction in the conclusions drawn by considering γ_1 and γ_2 separately. Thus, a combination of these coefficients must be used for analysis purposes.

   - **Small samples (control and face points)**: The actual coefficient of Skewness (γ_1) is zero (for the case of two arc residuals) or tends to be zero (for the case of four arc residuals). Also, the actual coefficient of Kurtosis (γ_2) is approximately 3. Therefore, the required distribution would have $\gamma_1 \approx 0$ and $\gamma_2 \approx 3$. These properties are exhibited by a distribution such as the the Normal and the Truncated Normal.

- **Large samples (control and face points)**: It is quite obvious that although the symmetry of the actual distribution is maintained ($\gamma_1 \approx 0$), the distribution becomes more peaked ($\gamma_2 > 3.3$) indicating that the distribution which combines these properties is of the Truncated Normal type.

6.4 RESULTS IN BRIEF

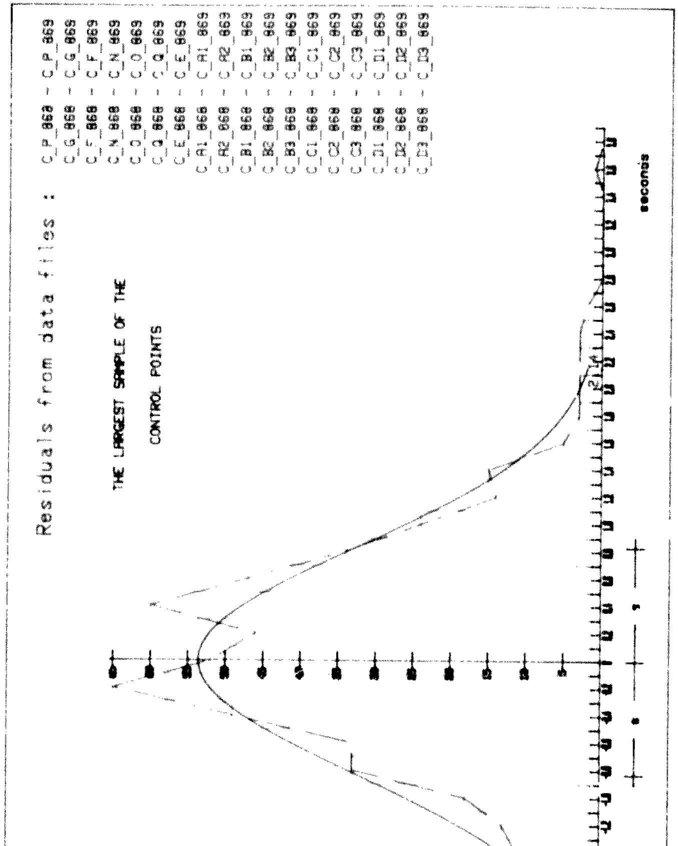
Combining the results from the Chi-Square goodness-of-fit test with those of the coefficients of Skewness and Kurtosis, similar conclusions can be drawn:

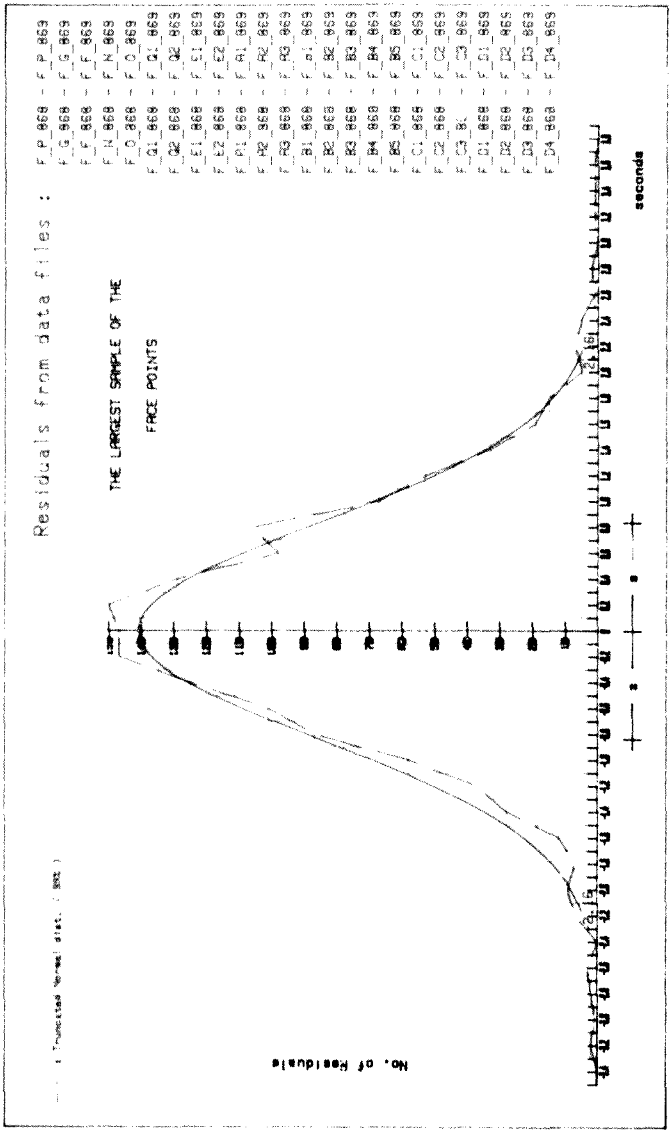
1. Both of them accept, in general, the predominance of symmetry as indicated by the Normal and Truncated Normal distributions as opposed to the Inverse Gaussian which is a non-symmetrical distribution. However, the Chi-Square goodness-of-fit test which is a better indicator than the coefficients of Skewness and Kurtosis, pin points the 99% Truncated Normal distribution as the best in the case of control points (for small and large samples) while for the face points a distinction must be made between small and large samples. The Normal distribution appears to be the best fitting curve for the small samples while the Truncated Normal 97.5% and 99% seem to be the most appropriate distributions for the large samples.

2. In particular, for the large samples, both of the analysis methods give priority to the Truncated Normal distributions which are more peaked than the Normal distribution. Specifically the 99% Truncated Normal seems to the best for the

control points, and the 97.5% or 99% one for the face points. On the other hand, the Inverse Gaussian distribution is totally rejected as not only does it fail to satisfy the symmetry of the residuals but, according to the Chi-Square goodness-of-fit test, is completely inconsistent.

The next two plots are referred to the largest samples for the case of the 99% Truncated Normal distribution for control and face points.


Residuals from data files :
C_P_868 - C_P_869
C_G_868 - C_G_869
C_F_868 - C_F_869
C_N_868 - C_N_869
C_O_868 - C_O_869
C_Q_868 - C_Q_869
C_E_868 - C_E_869
C_A1_868 - C_A1_869
C_A2_868 - C_A2_869
C_B1_868 - C_B1_869
C_B2_868 - C_B2_869
C_B3_868 - C_B3_869
C_C1_868 - C_C1_869
C_C2_868 - C_C2_869
C_C3_868 - C_C3_869
C_D1_868 - C_D1_869
C_D2_868 - C_D2_869
C_D3_868 - C_D3_869
THE LARGEST SAMPLE OF THE
CONTROL POINTS
No. of Residuals
seconds



Residuals from data files :
F_P_868 - F_P_869
F_G_868 - F_G_869
F_F_868 - F_F_869
F_N_868 - F_N_869
F_O_868 - F_O_869
F_Q1_868 - F_Q1_869
F_Q2_868 - F_Q2_869
F_E1_868 - F_E1_869
F_E2_868 - F_E2_869
F_A1_868 - F_A1_869
F_A2_868 - F_A2_869
F_A3_868 - F_A3_869
F_B1_868 - F_B1_869
F_B2_868 - F_B2_869
F_B3_868 - F_B3_869
F_B4_868 - F_B4_869
F_B5_868 - F_B5_869
F_C1_868 - F_C1_869
F_C2_868 - F_C2_869
F_C3_868 - F_C3_869
F_D1_868 - F_D1_869
F_D2_868 - F_D2_869
F_D3_868 - F_D3_869
F_D4_868 - F_D4_869
THE LARGEST SAMPLE OF THE
FACE POINTS
seconds
No. of Residuals
Truncated Normal dist.

CHAPTER 7

CHAPTER 7

7.0 CONCLUSIONS - RECOMMENDATIONS

7.1 CONCLUSIONS

While much research has been carried out in respect of the distribution of residuals, and the *a posteriori* analysis of survey networks, the relationship between the actual observations and the adjusted residuals of the observations prior to the network adjustment is often neglected. Pope (1976) has considered the statistics of residuals and has recommended the use of the tau distribution for the determination of outliers. Similarly, Baarda (1968) has proposed several methods for the testing of geodetic networks. Chrzanowski (1981), and Chrzanowski and Chen (1986) have described the standard procedures adopted for the analysis of deformation networks.

The common assumption underlying the methods quoted above is the acceptance that the residuals associated with the observations can be modelled via the normal distribution. All subsequent tests and inference are based on this premise. As shown by Milford (1986), the confidence regions pertaining to the parameters in an adjustment are directly related to the distribution assumed for the observational residuals and thus, for the observations themselves. This is briefly illustrated below.

The covariance matrix of the unknowns is directly dependent upon the weight matrix used in the adjustment. It is, generally, accepted that the inverse of the variance-covariance (dispersion) matrix of the observations is adopted as the weight matrix for the adjustment. For the parametric model, in which the observations are directly related to a function of the parameters (usually co-ordinates), the variance-covariance matrix of the parameters is given by $(\mathbf{A}'\mathbf{W}\mathbf{A})^{-1}$, (in which $\mathbf{A}$ is the design matrix, and W the weight matrix). This matrix will only be multivariate normal, if the matrix $\mathbf{W}$ is the corresponding covariance matrix for the observations.

However, it has been shown in this dissertation, that this assumption may not, generally, be true in open pit monitoring work.

The observations for the control framework have been shown to have a Truncated Normal distribution with the points of truncation at 99%. While it may be practical to use the complete Normal distribution for analysis purposes, the 99% limit has importance in the rejection of outliers in the observations. For large samples (and hence, large degrees of freedom) this would correspond to a figure of 2.58 times the standard deviation of the observations.

For the face points, it has been shown that the Normal distribution is the best approximation for the distribution of the residuals for the small samples, while for the larger samples the Truncated Normal distribution provides the best fit. It must be noticed, however, that as the size of the sample increases, so the Truncated 99% best describes the distribution of the residuals. Therefore, it can be said that for large samples, again, a figure of 2.58 σ can be used as the critical region, above which the residuals may be considered for rejection.

In general, the results of the dissertation concerning the Truncated Normal distribution agree with M.A. Zemel'man (1985) who states that:

> "*... practical probability densities for errors are truncated unimodal symmetrical ones.*"

On the other hand, the Inverse Gaussian distribution is much worse than the other distributions, and consequently, is not applicable.

Accepting that the distribution of the residuals may best be described by a Truncated Normal distribution, this will effect the calculation of confidence regions for parameters estimated in a subsequent least squares adjustment. It should, however, be noted that the estimate of the parameters will not change.

In order to examine the effect on the statistical inference, and to show that the estimates of the parameters will not change, it is convenient to express the variance from the Truncated Normal distribution as being directly proportional to the variance of the Normal distribution. Thus, formula (4.33) may be written as:

$$Var(x)_T = k \cdot Var(x)_N$$

where, $Var(x)_T$ is the variance for the Truncated Normal distribution and $Var(x)_N$ is the variance for the Normal distribution.

Similarly, the relationship between the variance-covariance matrices of the Truncated Normal and Normal distribution may be expressed as:

$$\Sigma_T = k \cdot \Sigma_N$$

for both Σ_T and Σ_N diagonal matrices.

Hence, for the estimate of parameters:

$$\hat{x} = (A'WA)^{-1} A'Wf$$

let the weight matrix be the inverse of the covariance matrix of the distribution of the observations. Then, for the Normal distribution:

$$\hat{x}_N = (A'\Sigma_N^{-1} A)^{-1} A'\Sigma_N^{-1}f$$

For the Truncated Normal distribution:

$$
\begin{aligned}
\hat{x}_T &= (A'\Sigma_T^{-1} A)^{-1} A'\Sigma_T^{-1} f \\
&= k\ (A'\Sigma_N^{-1} A)^{-1} A'(1/k)\Sigma_N^{-1} \\
&= (A'\Sigma_N^{-1} A)^{-1} A'\Sigma_N^{-1} f \\
&= \hat{x}_N
\end{aligned}
$$

The values of k can be obtained directly from (4.33), and are tabulated below:

- 95% Truncated Normal: $k = 0.759$
- 97.5% " " : $k = 0.852$
- 99% " " : $k = 0.925$

As can be seen from the figures, an eight percent reduction in the variance of an individual parameter is obtained by using the 99% Truncated Normal distribution statistics. Consequently, the scale of error ellipses calculated from the variance-covariance matrix $(A'\Sigma_T^{-1} A)^{-1}$ will be reduced by the square root of k.

An advantage of accepting the Truncated Normal distribution is that a simple natural boundary, given by the points of truncation, may be provided for the detection of outlying observations. Under the assumption that all residuals are drawn from the truncated population, values that fall outside the points of truncation may be identified readily and marked for further investigation.

However, the truncation points may not be well defined for small samples, as the degrees of freedom are low, and the confidence limits for the variance of the distribution are large. In this case, it is advisable to use the Student's "t" statistic to obtain the estimates for points of truncation. Although the Student t distribution has not been covered in this dissertation, the theory can be found in any elementary textbook on statistics [see for example, (Green and Margerison, 1978), (DeGroot, 1975)].

Thus, the value for the points of truncation for small samples would be:

$$\delta = t_{n-1,\alpha/2} \cdot s$$

where:

δ	: point of truncation as defined in section 3.4.1
s	: standard deviation of the sample mean
$t_{n-1,\alpha/2}$	: Student's statistic
α	: significance level
n	: number of residuals

The value of t may be obtained from statistical tables.

It is with relief that the Inverse Gaussian distribution was found to be inappropriate, as the statistics relating to this distribution are complicated, and, in general, non-parametric methods would have to be used for inference.

7.2 RECOMMENDATIONS

Based on the research work carried out in this dissertation, the following recommendations can be listed. These recommendations will, in the first instance, be valid for survey work carried out on projects similar to that analysed in this dissertation, and secondly, may be considered applicable to more general surveys.

1. Theodolite observations should be reduced using a method that will either eliminate or minimise the correlation between the final mean directions. In this context, the method outlined in section 3.4, in which the observations to the R.O. are regarded as checks, would be applicable. The practice of applying a horizon closure error would introduce correlation and should be discouraged.

2. The residuals obtained from the reduction of the observations should be tested against a Truncated Normal distribution. The Chi-Square goodness-of-fit test can easily be applied, and should become a routine part of the analysis of the measured quantities.

3. The calculation of both the coefficients of Skewness (γ_1) and Kurtosis (γ_2) will provide additional information concerning the shape of the distribution, and can be used as a guide for assessing the validity of distribution assumptions.

4. The nature of the project undertaken, and instrumentation used, may require that the observations are sub-divided into different classes for analysis. This is particularly important, if significant meteorological and topographical differences can be identified within one project.

5. The points of truncation from the distribution describing the residuals should be used to identify residuals that may be regarded as outliers. The observations pertaining to these residuals should then be considered in more detail.

6. Further research into the application of the Truncated distribution related to statistical inference for parameters estimated from a Least Squares adjustment is required. This topic is presently being investigated in the Department of Surveying at the University of the Witwatersrand, Johannesburg.

APPENDICES

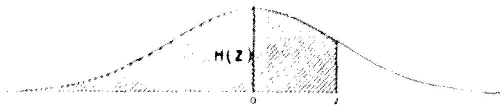
APPENDICES

APPENDIX A

APPENDIX A. TABLES

Three Tables are given in the following pages:

a. For the standardised Normal distribution, and
b. For the Chi-Square distribution
c. For the Kolmogorov-Smirnov test



$$H(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\, dz$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0	5000	5040	5080	5120	5160	5199	5239	5279	5319	5359
1	5398	5438	5478	5517	5557	5596	5636	5675	5714	5753
2	5793	5832	5871	5910	5948	5987	6026	6064	6103	6141
3	6179	6217	6255	6293	6331	6368	6406	6443	6480	6517
4	6554	6591	6628	6664	6700	6736	6772	6808	6844	6879
5	6915	6950	6985	7019	7054	7088	7123	7157	7190	7224
6	7257	7291	7324	7357	7389	7422	7454	7486	7517	7549
7	7580	7611	7642	7673	7704	7734	7764	7794	7823	7852
8	7881	7910	7939	7967	7995	8023	8051	8078	8106	8133
9	8159	8186	8212	8238	8264	8289	8315	8340	8365	8389
1 0	8413	8438	8461	8485	8508	8531	8554	8577	8599	8621
1 1	8643	8665	8686	8708	8729	8749	8770	8790	8810	8830
1 2	8849	8869	8888	8907	8925	8944	8962	8980	8997	9015
1 3	9032	9049	9066	9082	9099	9115	9131	9147	9162	9177
1 4	9192	9207	9222	9236	9251	9265	9279	9292	9306	9319
1 5	9332	9345	9357	9370	9382	9394	9406	9418	9429	9441
1 6	9452	9463	9474	9484	9495	9505	9515	9525	9535	9545
1 7	9554	9564	9573	9582	9591	9599	9608	9616	9625	9633
1 8	9641	9649	9656	9664	9671	9678	9686	9693	9699	9706
1 9	9713	9719	9726	9732	9738	9744	9750	9756	9761	9767
2 0	9772	9778	9783	9788	9793	9798	9803	9808	9812	9817
2 1	9821	9826	9830	9834	9838	9842	9846	9850	9854	9857
2 2	9861	9864	9868	9871	9875	9878	9881	9884	9887	9890
2 3	9893	9896	9898	9901	9904	9906	9909	9911	9913	9916
2 4	9918	9920	9922	9925	9927	9929	9931	9932	9934	9936
2 5	9938	9940	9941	9943	9945	9946	9948	9949	9951	9952
2 6	9953	9955	9956	9957	9959	9960	9961	9962	9963	9964
2 7	9965	9966	9967	9968	9969	9970	9971	9972	9973	9974
2 8	9974	9975	9976	9977	9977	9978	9979	9979	9980	9981
2 9	9981	9982	9982	9983	9984	9984	9985	9985	9986	9986
3 0	9987	9987	9987	9988	9988	9989	9989	9989	9990	9990
3 1	9990	9991	9991	9991	9992	9992	9992	9992	9993	9993
3.2	9993	9993	9994	9994	9994	9994	9994	9995	9995	9995
3 3	9995	9995	9995	9996	9996	9996	9996	9996	9996	9997
3.4	9997	9997	9997	9997	9997	9997	9997	9997	9997	9998

Table A1: Area under the Standard Normal density function

$$H(z) = \int_{-\infty}^{z} \frac{1}{\sqrt{2\pi}} e^{-z^2/2}\,dz$$

z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
.0	.5000	.5040	.5080	.5120	.5160	.5199	.5239	.5279	.5319	.5359
.1	.5398	.5438	.5478	.5517	.5557	.5596	.5636	.5675	.5714	.5753
.2	.5793	.5832	.5871	.5910	.5948	.5987	.6026	.6064	.6103	.6141
.3	.6179	.6217	.6255	.6293	.6331	.6368	.6406	.6443	.6480	.6517
.4	.6554	.6591	.6628	.6664	.6700	.6736	.6772	.6808	.6844	.6879
.5	.6915	.6950	.6985	.7019	.7054	.7088	.7123	.7157	.7190	.7224
.6	.7257	.7291	.7324	.7357	.7389	.7422	.7454	.7486	.7517	.7549
.7	.7580	.7611	.7642	.7673	.7704	.7734	.7764	.7794	.7823	.7852
.8	.7881	.7910	.7939	.7967	.7995	.8023	.8051	.8078	.8106	.8133
.9	.8159	.8186	.8212	.8238	.8264	.8289	.8315	.8340	.8365	.8389
1.0	.8413	.8438	.8461	.8485	.8508	.8531	.8554	.8577	.8599	.8621
1.1	.8643	.8665	.8686	.8708	.8729	.8749	.8770	.8790	.8810	.8830
1.2	.8849	.8869	.8888	.8907	.8925	.8944	.8962	.8980	.8997	.9015
1.3	.9032	.9049	.9066	.9082	.9099	.9115	.9131	.9147	.9162	.9177
1.4	.9192	.9207	.9222	.9236	.9251	.9265	.9279	.9292	.9306	.9319
1.5	.9332	.9345	.9357	.9370	.9382	.9394	.9406	.9418	.9429	.9441
1.6	.9452	.9463	.9474	.9484	.9495	.9505	.9515	.9525	.9535	.9545
1.7	.9554	.9564	.9573	.9582	.9591	.9599	.9608	.9616	.9625	.9633
1.8	.9641	.9649	.9656	.9664	.9671	.9678	.9686	.9693	.9699	.9706
1.9	.9713	.9719	.9726	.9732	.9738	.9744	.9750	.9756	.9761	.9767
2.0	.9772	.9778	.9783	.9788	.9793	.9798	.9803	.9808	.9812	.9817
2.1	.9821	.9826	.9830	.9834	.9838	.9842	.9846	.9850	.9854	.9857
2.2	.9861	.9864	.9868	.9871	.9875	.9878	.9881	.9884	.9887	.9890
2.3	.9893	.9896	.9898	.9901	.9904	.9906	.9909	.9911	.9913	.9916
2.4	.9918	.9920	.9922	.9925	.9927	.9929	.9931	.9932	.9934	.9936
2.5	.9938	.9940	.9941	.9943	.9945	.9946	.9948	.9949	.9951	.9952
2.6	.9953	.9955	.9956	.9957	.9959	.9960	.9961	.9962	.9963	.9964
2.7	.9965	.9966	.9967	.9968	.9969	.9970	.9971	.9972	.9973	.9974
2.8	.9974	.9975	.9976	.9977	.9977	.9978	.9979	.9979	.9980	.9981
2.9	.9981	.9982	.9982	.9983	.9984	.9984	.9985	.9985	.9986	.9986
3.0	.9987	.9987	.9987	.9988	.9988	.9989	.9989	.9989	.9990	.9990
3.1	.9990	.9991	.9991	.9991	.9992	.9992	.9992	.9992	.9993	.9993
3.2	.9993	.9993	.9994	.9994	.9994	.9994	.9994	.9995	.9995	.9995
3.3	.9995	.9995	.9995	.9996	.9996	.9996	.9996	.9996	.9996	.9997
3.4	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9997	.9998

Table A1: Area under the Standard Normal density function

df	α = 0.995	0.990	0.975	0.950	0.900	0.500	0.10	0.05	0.025	0.010	0.005
1	0.00	0.00	0.00	0.00	0.02	0.46	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	1.39	4.61	5.99	7.38	9.21	10.60
3	0.07	0.12	0.22	0.35	0.58	2.37	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	5.35	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	9.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	6.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	14.34	22.31	25.00	27.49	40.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.08	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	14.34	24.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.88	41.92	45.64	49.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
60	35.53	37.48	40.48	43.19	46.64	39.33	74.40	79.08	83.30	88.38	91.95
120	83.85	86.92	91.58	95.70	100.62	119.33	140.23	146.57	152.21	158.95	163.65

Table A2: Area under the Chi-Square density function

df \ α	0.995	0.990	0.975	0.950	0.900	0.500	0.10	0.05	0.025	0.010	0.005
1	0.00	0.00	0.00	0.00	0.02	0.46	2.71	3.84	5.02	6.63	7.88
2	0.01	0.02	0.05	0.10	0.21	1.39	4.61	5.99	7.38	9.21	10.60
3	0.07	0.12	0.22	0.35	0.58	2.37	6.25	7.81	9.35	11.34	12.84
4	0.21	0.30	0.48	0.71	1.06	3.36	7.78	9.49	11.14	13.28	14.86
5	0.41	0.55	0.83	1.15	1.61	4.35	9.24	11.07	12.83	15.09	16.75
6	0.68	0.87	1.24	1.64	2.20	5.35	10.64	12.59	14.45	16.81	18.55
7	0.99	1.24	1.69	2.17	2.83	6.35	12.02	14.07	16.01	18.48	20.28
8	1.34	1.65	2.18	2.73	3.49	7.34	13.36	15.51	17.53	20.09	21.96
9	1.73	2.09	2.70	3.33	4.17	9.34	14.68	16.92	19.02	21.67	23.59
10	2.16	2.56	3.25	3.94	4.87	9.34	15.99	18.31	20.48	23.21	25.19
11	2.60	3.05	3.82	4.57	5.58	10.34	17.28	19.68	21.92	24.73	26.76
12	3.07	3.57	4.40	5.23	6.30	11.34	18.55	21.03	23.34	26.22	28.30
13	3.57	4.11	5.01	5.89	7.04	12.34	19.81	22.36	24.74	27.69	29.82
14	4.07	4.66	5.63	6.57	7.79	13.34	21.06	23.68	26.12	29.14	31.32
15	4.60	5.23	6.26	7.26	8.55	14.34	22.31	25.00	27.49	40.58	32.80
16	5.14	5.81	6.91	7.96	9.31	15.34	23.54	26.30	28.85	32.00	34.27
17	5.70	6.41	7.56	8.67	10.08	16.34	24.77	27.59	30.19	33.41	35.72
18	6.26	7.01	8.23	9.39	10.86	17.34	25.99	28.87	31.53	34.81	37.16
19	6.84	7.63	8.91	10.12	11.65	18.34	27.20	30.14	32.85	36.19	38.58
20	7.43	8.26	9.59	10.85	12.44	19.34	28.41	31.41	34.17	37.57	40.00
21	8.03	8.90	10.28	11.59	13.24	20.34	29.62	32.67	35.48	38.93	41.40
22	8.64	9.54	10.98	12.34	14.04	21.34	30.81	33.92	36.78	40.29	42.80
23	9.26	10.20	11.69	13.09	14.85	22.34	32.01	35.17	38.08	41.64	44.18
24	9.89	10.86	12.40	13.85	15.66	23.34	33.20	36.42	39.36	42.98	45.56
25	10.52	11.52	13.12	14.61	16.47	14.34	24.38	37.65	40.65	44.31	46.93
26	11.16	12.20	13.84	15.38	17.29	25.34	35.56	38.88	41.92	45.64	48.29
27	11.81	12.88	14.57	16.15	18.11	26.34	36.74	40.11	43.19	46.96	49.64
28	12.46	13.56	15.31	16.93	18.94	27.34	37.92	41.34	44.46	48.28	50.99
29	13.12	14.26	16.05	17.71	19.77	28.34	39.09	42.56	45.72	49.59	52.34
30	13.79	14.95	16.79	18.49	20.60	29.34	40.26	43.77	46.98	50.89	53.67
40	20.71	22.16	24.43	26.51	29.05	39.34	51.81	55.76	59.34	63.69	66.77
60	35.53	37.48	40.48	43.19	46.34	59.33	74.40	79.08	83.30	88.38	91.95
120	83.85	86.92	91.58	95.70	100.62	119.33	140.23	146.57	152.21	158.95	163.64

Table A2: Area under the Chi-Square density function

t	$H(t)$	t	$H(t)$
0.30	0.0000	1.20	0.8878
0.35	0.0003	1.25	0.9121
0.40	0.0028	1.30	0.9319
0.45	0.0126	1.35	0.9478
0.50	0.0361	1.40	0.9603
0.55	0.0772	1.45	0.9702
0.60	0.1357	1.50	0.9778
0.65	0.2080	1.60	0.9880
0.70	0.2888	1.70	0.9938
0.75	0.3728	1.80	0.9969
0.80	0.4559	1.90	0.9985
0.85	0.5347	2.00	0.9993
0.90	0.6073	2.10	0.9997
0.95	0.6725	2.20	0.9999
1.00	0.7300	2.30	0.9999
1.05	0.7798	2.40	1.0000
1.10	0.8223	2.50	1.0000
1.15	0.8580		

Table A3 : Values of H(t) for Kolmogorov-Smirnov test

APPENDIX B

APPENDIX B. EXAMPLE OF THE "THEO" COMPUTER PROGRAM

****
REDUCTION OF THEODOLITE OBSERVATIONS
****

GENERAL INFORMATION

```
Date                 :  10 January 1988      Instrument :  KERN DKM3 #169608
Observer             :  I.B.W
Booker               :  I.B.W
Weather Conditions   :
```

* Code numbers to create/have access to these data: C, B2, B6
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR	MEAN
1	B2	F	126 24 56.2	306 25 13.0	126 25 04.6
2		N	205 39 40.9	25 40 01.4	205 39 51.2
3		E	226 29 51.7	46 30 07.1	226 29 59.4
4		Q	261 28 30.8	81 28 50.6	261 28 40.7
5		D	298 54 12.1	118 54 35.6	298 54 23.9
6		POL	60 52 40.6	240 53 00.7	60 52 50.7
7		F	126 24 55.6	306 25 13.0	126 25 04.7

ARC NO. 2

No	St point	Obs point	CL	CR	MEAN
1	B2	F	171 56 24.4	351 56 40.8	171 56 32.6
2		N	251 11 10.0	71 11 28.6	251 11 19.7
3		E	272 01 17.4	92 01 36.1	272 01 26.8
4		Q	307 00 00.7	127 00 15.1	307 00 07.9
5		D	344 25 41.5	164 26 01.9	344 25 51.7
6		POL	114 24 18.4	294 24 34.8	114 24 26.6
7		F	171 56 23.6	351 56 41.5	171 56 32.6

ARC NO. 3

No	St point	Obs point	CL	CR	MEAN
1	B2	F	216 57 51.9	36 58 09.3	216 58 00.6
2		N	296 12 38.8	116 12 56.1	296 12 47.5
3		E	317 02 54.0	137 03 04.4	317 02 59.2
4		Q	352 01 32.5	172 01 44.2	352 01 38.4
5		D	29 27 11.9	209 27 29.8	29 27 20.9
6		POL	159 25 50.3	339 26 06.4	159 25 58.4
7		F	216 57 52.4	36 58 10.6	216 58 01.5

ARC NO. 4

No	St point	Obs point	CL	CR	MEAN
1	B2	F	[illegible]	82 00 16.9	262 00 12.3
2		N	341 14 [illegible].1	161 15 04.9	341 14 59.0
3		E	2 05 [illegible].3	182 05 10.1	2 05 06.2
4		Q	37 03 39.7	217 03 49.5	37 03 44.6
5		D	74 29 24.0	254 29 36.5	74 29 30.7
6		POL	204 27 59.4	24 28 12.0	204 28 06.1
7		F	262 00 07.4	82 00 17.7	262 00 12.6

ORIENTATION CORRECTION OF ARC NO. 2 = 45 31 27.0

ORIENTATION CORRECTION OF ARC NO. 3 = 90 32 57.6

ORIENTATION CORRECTION OF ARC NO. 4 = 135 35 06.9

St point	Obs point	REDUCED MEAN VALUE FOR ARC NO. 2
B2	F	126 25 04.8
	N	205 39 51.9
	E	226 29 58.9
	Q	261 28 40.1
	D	298 54 23.9
	POL	68 52 58.8
	F	126 25 04.7

St point	Obs point	REDUCED MEAN VALUE FOR ARC NO. 3
B2	F	126 25 03.0
	N	205 39 49.8
	E	[illegible] 30 01.6
	O	[illegible] 28 40.7
	D	[illegible] 54 23.2
	POL	68 53 00.7
	F	126 25 03.9

St point	Obs point	REDUCED MEAN VALUE FOR ARC NO. 4
B2	F	126 25 05.4
	N	205 39 52.1
	E	226 29 59.7
	O	261 28 37.7
	D	298 54 23.7
	POL	68 52 59.2
	F	126 25 05.6

St point	Obs point	FINAL MEAN VALUE
B2	F	126 25 04.4
	N	205 39 51.2
	E	226 29 59.8
	O	261 28 39.8
	D	298 54 23.7
	POL	68 52 59.3
	F	126 25 04.7

HORIZON CLOSURE ERROR (sec) = 0.7000

CORRECTION (sec) = 0.0500

```
*********     **********     ********************
St point      Obs point      REDUCED OBSERVATIONS
*********     **********     ********************

   B2             F             126 25 04.4
                  N             205 39 51.2
                  E             226 29 59.7
                  U             261 28 39.7
                  D             298 54 23.5
                  F II           68 52 59.1
                  F             126 25 04.4
```

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............................................................................


              OBSERVATIONAL RESIDUALS (sec)
              *****************************

U(1,1) = 126 25 04.4   -   126 25 04.6   =   -0.2
U(1,2) = 205 39 51.2   -   205 39 51.2   =   +0.1
U(1,3) = 226 29 59.8   -   226 29 59.4   =   +0.4
U(1,4) = 261 28 39.8   -   261 28 40.7   =   -0.9
U(1,5) = 298 54 23.7   -   298 54 23.9   =   -0.2
U(1,6) =  68 52 59.3   -    68 52 58.7   =   +0.7
U(1,7) = 126 25 04.7   -   126 25 04.7   =    0.0
U(2,1) = 126 25 04.4   -   126 25 04.8   =   -0.3
U(2,2) = 205 39 51.2   -   205 39 51.9   =   -0.6
U(2,3) = 226 29 59.8   -   226 29 58.9   =   +0.9
U(2,4) = 261 28 39.8   -   261 26 40.1   =   -0.3
U(2,5) = 298 54 23.7   -   298 54 23.9   =   -0.2
U(2,6) =  68 52 59.3   -    68 52 58.8   =   +0.6
U(2,7) = 126 25 04.7   -   126 15 04.7   =    0.0
U(3,1) = 126 25 04.4   -   126 25 03.0   =   +1.5
U(3,2) = 205 39 51.2   -   205 39 49.8   =   +1.4
U(3,3) = 226 29 59.8   -   226 30 01.6   =   -1.8
U(3,4) = 261 28 39.8   -   261 28 40.7   =   -0.9
U(3,5) = 298 54 23.7   -   298 54 23.2   =   +0.4
U(3,6) =  68 52 59.3   -    68 53 00.7   =   -1.4
U(3,7) = 126 25 04.7   -   126 25 03.9   =   +0.9
U(4,1) = 126 25 04.4   -   126 25 05.4   =   -1.0
U(4,2) = 205 39 51.2   -   205 39 52.1   =   -0.9
U(4,3) = 226 29 59.8   -   226 29 59.3   =   +0.5
U(4,4) = 261 26 39.8   -   261 28 37.7   =   +2.1
U(4,5) = 298 54 23.7   -   298 54 23.7   =   -0.1
U(4,6) =  68 52 59.3   -    68 52 59.2   =   +0.2
U(4,7) = 126 25 04.7   -   126 25 05.6   =   -0.9


TOTAL NO. OF OBSERVATIONAL RESIDUALS =  28


STANDARD DEVIATION OF A DIRECTION - sigma(o) =  1.0918

STANDARD DEVIATION OF THE MEAN - sigma(x) =  0.2062


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APPENDIX C

APPENDIX C. EXAMPLE OF THE "HI_DI_TEST" COMPUTER PROGRAM

Note that, the data used are the results from the example given in the Appendix B.

The following example includes the Chi-Square test in the case of the Normal, Truncated Normal (95%) and Inverse Gaussian distributions.

After this, three plots of these distributions follow.

```
***************************************************************     **************

*  The CODF numbers for data files are :  C_B2_868  -  C_B2_869


                    OBSERVATIONAL RESIDUALS
                    -----------------------

                             -0.2
                             +0.1
                             +0.4
                             -0.9
                             -0.2
                             +0.7
                             +0.0
                             -0.3
                             -0.6
                             +0.9
                             -0.3
                             -0.2
                             +0.6
                             +0.0
                             +1.5
                             +1.4
                             -1.8
                             -0.9
                             +0.4
                             -1.4
                             +0.9
                             -1.0
                             -0.9
                             +0.5
                             +2.1
                             -0.1
                             +0.2
                             -0.9


TOTAL NO. OF OBS. RESIDUALS  =  28
---------------------------
```

%%

NORMAL DISTRIBUTION

THEORETICAL coefficient of SKEWNESS = 0

THEORETICAL coefficient of KURTOSIS = 3

ACTUAL coefficient of SKEWNESS = +0.249

ACTUAL coefficient of KURTOSIS = +2.889

CHI - SQUARE GOODNESS-OF-FIT TEST :

CLASS INTERVALS [sec]	CLASSIF. OF OBS. RESIDUALS	THEORETICAL FREQUENCY (e)	OBSERVED FREQUENCY (n)
-30 - -1.90(-)		.42	0
-1.90 - -1.70(-)	-1.8	.31	1
-1.70 - -1.50(-)		.49	0
[illegible]50 - -1.3[illegible]	-1.4	.71	1
-1.30 - -1.10(-)		1.00	0
-1.10 - -0.90(-)			

		-1.0	1.33	1
-0.90 -	-0.70(-)			
		0.9		
		-0.9		
		-0.9		
		-0.9	1.60	4
-0.70 -	-0.50(-)			
		-0.6	2.01	1
-0.50 -	-0.30(-)			
			2.30	0
-0.30 -	-0.10(-)			
		-0.3		
		-0.2		
		-0.3		
		-0.3		
		-0.2	2.40	5
-0.10 -	+0.10(-)			
		+0.0		
		+0.0		
		-0.1	2.54	3
+0.10 -	+0.30(-)			
		+0.1		
		+0.2	2.48	2
+0.30 -	+0.50(-)			
		+0.4		
		+0.4	2.30	2
+0.50 -	+0.70(-)			
		+0.6		
		+0.5	2.01	2
+0.70 -	+0.90(-)			
		+0.7	1.60	1
+0.90 -	+1.10(-)			
		+0.9		
		+0.9	1.33	2
+1.10 -	+1.30(-)			
			1.00	0
+1.30 -	+1.50(-)			
		+1.4	.71	1
+1.50 -	+1.70(-)			
		+1.5	.49	1
+1.70 -	+1.90(-)			
			.31	0
+1.90 -	+2.10(-)			
			.19	0

+2.10 - +2.30( )	+2.1	.11	1
+2.?0 - +?0		.12	0
		28.00	28

Since the Theoretical (Expected) frequency for each class interval should not be LESS than 5, we have the following 4 NEW categories:

CLASS INTERVALS [sec]	THEORETICAL FREQUENCY (e)	OBSERVED FREQUENCY (n)	(n-e)^2/e
-?0 - -0.70(-)	5.94	7	.188
-0.70 - -0.10(-)	6.79	6	.092
-0.10 - +0.30(-)	5.02	5	0.000
+0.?0 - +30	10.24	10	.006
	28.00	28	.285

....

The Table A2 (Appendix A) gives :

X^2(0.050, 2) = 5.991

Since :

X^2 <= X^2(0.050,2)

it means that at the 5% level of significance the sample distribution is:

C O N S I S T E N T

with the hypothesis that the parent distribution is NORMAL xxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxxx

****

TRUNCATED NORMAL DISTRIBUTION (95%)

****

THEORETICAL coefficient of SKEWNESS = 0

THEORETICAL coefficient of KURTOSIS = 5.200

ACTUAL coefficient of SKEWNESS = +0.249

ACTUAL coefficient of KURTOSIS = +2.889

****

+---+
! C H I - S Q U A R E GOODNESS-OF-FIT T E S T !
+---+

CLASS INTERVALS [sec]	CLASSIF. OF OBS. RESIDUALS	THEORETICAL FREQUENCY (e)	OBSERVED FREQUENCY (n)
-1.72 - -1.70(-)			
		.03	0
-1.70 - -1.50(-)			
		.51	0
-1.50 - -1.30(-)			
	-1.4	.75	1
-1.30 - -1.10(-)			
		1.05	0
-1.10 - -0.90(-)			
	-1.0	1.40	1
-0.90 - -0.70(-)			

			-0.9		
			-0.9		
			-0.9		
			-0.9	1.77	4
-0.70	-	-0.50(-)			
			0.6	2.12	1
-0.50	-	-0.30(-)			
				2.42	0
-0.30	-	-0.10(-)			
			-0.2		
			0.2		
			-0.3		
			-0.3		
			0.2	2.61	5
0.10	-	+0.10(-)			
			+0.0		
			+0.0		
			-0.1	2.68	3
+0.10	-	+0.30(-)			
			+0.1		
			+0.2	2.61	2
+0.30	-	+0.50(-)			
			+0.4		
			+0.4	2.42	2
+0.50	-	+0.70(-)			
			+0.6		
			+0.5	2.12	2
+0.70	-	+0.90(-)			
			+0.7	1.77	1
+0.90	-	+1.10(-)			
			+0.9		
			+0.9	1.40	2
+1.10	-	+1.30(-)			
				1.05	0
+1.30	-	+1.50(-)			
			+1.4	.75	1
+1.50	-	+1.70(-)			
			+1.5	.51	1
+1.70	-	+1.72			
				.03	0
				20.00	26

Since the Theoretical (Expected) frequency for each class interval should not be LESS than 5, we have the following 4 NEW categories:

CLASS INTERVALS (sec)		THEORETICAL FREQUENCY (e)	OBSERVED FREQUENCY (n)	(n-e)^2/e
-1.72	- -0.70(-)	5.51	6	.044
0.70	-0.10(-)	7.16	6	.187
-0.10	- +0.30(+)	5.30	5	.017
+0.70	- +1.72	10.04	9	.108
		28.00	26	.355

....

The Table A2 (Appendix A) gives :

X^2(0.050, 2) = 5.991

Since :

X^2 <= X^2(0.050,2)

it means that at the 5% level of significance the sample distribution is:

C O N S I S T E N T

with the hypothesis that the parent distribution is TRUNCATED NORMAL (95%)

%%

```
%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%%

                          N O T I C E
                          ***********

      Since the p.d.f of the Inverse Gaussian distribution

      is defined ONLY by POSITIVE numbers,

                     NEW  POSITIVE residuals

      are introduced.


                             N E W
                     OBSERVATIONAL RESIDUALS
                     -----------------------

                              +1.9
                              +2.2
                              +2.5
                              +1.2
                              +1.9
                              +2.8
                              +2.1
                              +1.8
                              +1.5
                              +3.0
                              +1.8
                              +1.9
                              +2.7
                              +2.1
                              +3.6
                              +3.5
                              +0.3
                              +1.2
                              +2.5
                              +0.7
                              +3.0
                              +1.1
                              +1.2
                              +2.6
                              +4.2
                              +2.0
                              +2.3
                              +1.2
```

TOTAL NO. OF OBS. RESIDUALS = 28

Mean value of NEW residuals = 2.1

Dispersion of NEW residuals = 12.1

****
INVERSE GAUSSIAN DISTRIBUTION
****

THEORETICAL coefficient of SKEWNESS = 1.249

THEORETICAL coefficient of KURTOSIS = 5.602

ACTUAL coefficient of SKEWNESS = +0.249

ACTUAL coefficient of KURTOSIS = +2.889

****

+---+
| C H I - S Q U A R E GOODNESS-OF-FIT T E S T |
+---+

CLASS INTERVALS [sec]	CLASSIF. OF OBS. RESIDUALS	THEORETICAL FREQUENCY (e)	OBSERVED FREQUENCY (n)
+0.05 - +0.20(-)		0.00	0
+0.20 - +0.40(-)	+0.3	0.00	1
+0.40 - +0.60(-)		.33	0
+0.60 - +0.80(-)	+0.7	.29	1

```
+0.00   -   +1.00(-)
                                1.00        0

+1.00       +1.20(-)
                     +1.1       1.90        1

+1.20   -   +1.40(-)
                     +1.2
                     +1.2
                     +1.2
                     +1.2       2.64        4

+1.40   -   +1.60(-)
                     +1.5       2.[illegible]        1

+1.60   -   +1.80(-)
                                3.07        0

+1.80   -   +2.00(-)
                     +1.9
                     +1.9
                     +1.8
                     +1.8
                     +1.9       2.88        5

+2.00   -   +2.20(-)
                     +2.[illegible]
                     +2.1
                     +2.0       2.60        3

+2.20   -   +2.40(-)
                     +2.2
                     +2.3       2.[illegible]7        2

+2.40   -   +2.60(-)
                     +2.5
                     +2.5       1.79        2

+2.60   -   +2.80(-)
                     +2.7
                     +2.6       1.45        2

+2.80   -   +3.00(-)
                     +2.8       1.16        1

+3.00   -   +3.20(-)
                     +3.0
                     +3.0        .92        2

+3.20   -   +3.40(-)
                                 .72        0

+3.40   -   +3.60(-)
                     +3.5        .55        1

+3.60   -   +3.80(-)
                     +3.6        .42        1

+3.80   -   +4.00(-)
```

				.32	0
+4.00	-	+4.20(-)			
				.24	0
+4.20	-	+4.40(-)			
			+4.3	.19	1
+4.40	-	+70			
				.56	0
				28.00	28

Since the Theoretical (Expected) frequency for each class interval should not be LESS than 5, we have the following 5 NEW categories:

CLASS INTERVALS [sec]			THEORETICAL FREQUENCY (e)	OBSERVED FREQUENCY (n)	(n-e)^2/e
+0.05	-	+1.40(-)	5.96	7	.220
+1.40	-	+1.80(-)	6.10	1	4.262
+1.80	-	+2.20(-)	5.56	8	1.066
+2.20	-	+2.80(-)	5.41	6	.064
+2.80	-	+70	5.08	6	.166
			28.00	28	5.779

....

The Table A2 (Appendix A) gives :

X^2(0.050, 3) = 7.815

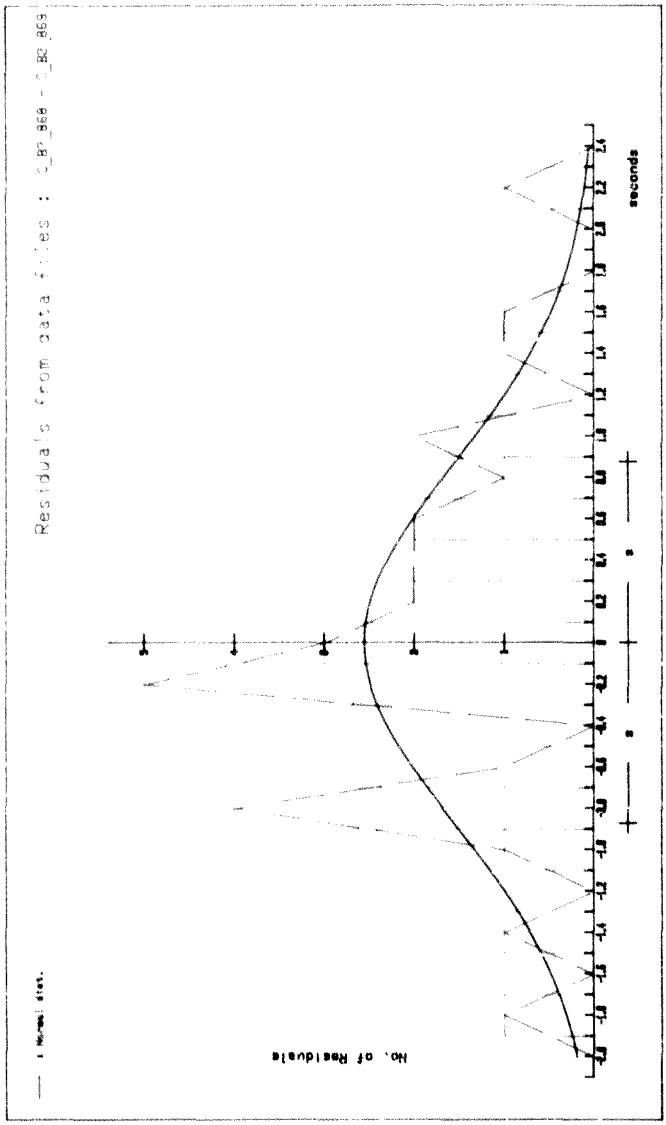
Since :

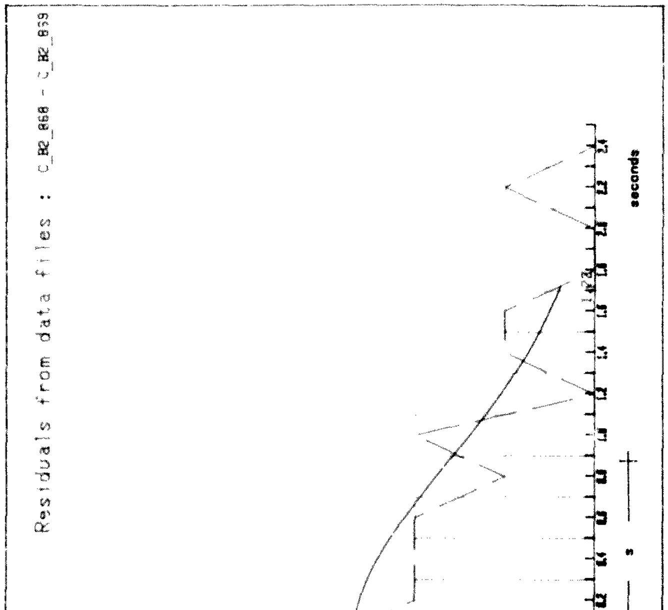
X^2 <= X^2(0.050,3)

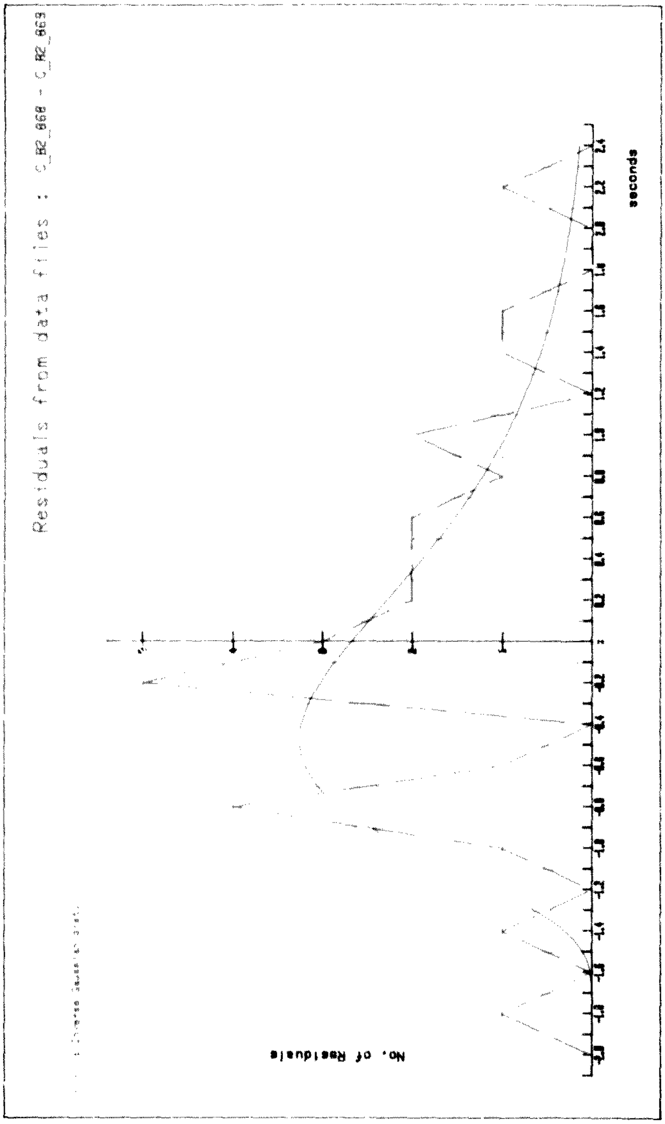
it means that at the 5% level of significance the sample distribution is:

C O N S I S T E N T

with the hypothesis that the parent distribution is INVERSE GAUSSIAN


Residuals from data files
No. of Residuals
seconds


Residuals from data files : C_B2_868 - C_B2_859
No. of Residuals
seconds


Residuals from data files : C_R2_068 - C_R2_069
seconds
No. of Residuals

APPENDIX D

APPENDIX D. REFERENCES - BIBLIOGRAPHY

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UNIVERSITY OF THE WITWATERSRAND

DEPARTMENT OF SURVEYING

FACULTY OF ENGINEERING

THE DISTRIBUTION OF THEODOLITE OBSERVATIONS ASSOCIATED WITH OPEN PIT MONITORING SURVEY

VOLUME II

ALEXANDER ALBANIS

Johannesburg, 1987

VOLUME II

APPENDIX E

APPENDIX E. DATA

This Appendix includes the data used for the first computer program ("THEO"). These data sets have been observed by Professor I.B. Watt.

The data have been separated into two categories:

1. From control points to control points, and

2. From control points to face points.

E.1 FROM CONTROL POINTS TO CONTROL POINTS

```
************************************************************************
                 REDUCTION OF THEODOLITE OBSERVATIONS
************************************************************************

                        GENERAL INFORMATION
                        ===================

Date                : 1 January 1980        Instrument :  KERN DKM3 #169608
Observer            : I.B.W
Booker              : I.B.W
Weather Conditions  :

*  Code numbers to create/have access to these data: C,_P ,_66
** The output of all the observations is in degrees, minutes, seconds


*****************************************************************************************

                                  ECHO
                                  ====

                             ARC NO.  1
                             ==========
```

No	St point	Obs point	CL	CR
1	P	N	187 32 19.5	7 32 30.3
2		E	217 52 26.4	37 52 35.6
3		Q	269 24 11.5	89 24 19.3
4		D	314 08 59.2	134 09 08.2
5		C	349 53 58.6	169 54 08.4
6		B	42 12 03.6	222 12 20.6
7		N	187 32 21.4	7 32 31.0

```
                             ARC NO.  2
                             ==========
```

No	St point	Obs point	CL	CR
1	P	N	232 33 50.5	52 34 00.0
2		E	262 53 55.0	82 54 06.4
3		Q	314 25 39.0	134 25 48.5
4		D	359 10 27.9	179 10 39.3
5		C	34 55 29.3	214 55 39.8
6		B	87 13 39.1	267 13 50.4
7		N	232 33 50.2	52 33 59.1

ARC NO. 3

No	St point	Obs point	CL	CR
1	P	N	277 35 18.5	97 35 26.9
2		E	307 55 22.7	127 55 33.2
3		Q	359 27 05.1	179 27 17.3
4		D	44 11 53.6	224 12 03.1
5		C	79 56 58.1	259 57 05.5
6		B	132 15 08.8	312 15 20.0
7		N	277 35 19.3	97 35 28.0

ARC NO. 4

No	St point	Obs point	CL	CR
1	P	N	322 36 53.0	142 37 03.1
2		E	352 56 57.3	172 57 07.5
3		Q	44 28 38.8	224 28 47.3
4		D	89 13 29.5	269 13 40.8
5		C	124 58 33.0	304 58 42.6
6		B	177 16 41.7	357 16 52.7
7		N	322 36 54.0	142 37 03.5

****

REDUCTION OF THEODOLITE OBSERVATIONS

****

GENERAL INFORMATION

Date : 1 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_8 ,_86
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	8	F	75 04 08.1	255 04 20.9
2		O	227 00 26.5	47 00 40.6
3		N	265 55 01.4	85 55 14.3
4		E	287 18 03.4	107 18 17.0
5		A	0 21 30.1	180 21 49.6
6		POL	21 17 57.7	201 18 17.1
7		F	75 04 08.4	255 04 20.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	8	F	120 05 40.1	300 05 51.9
2		O	272 01 58.5	92 02 10.4
3		N	310 56 31.9	130 56 43.7
4		E	332 19 34.6	152 19 45.2
5		A	45 22 57.7	225 23 10.9
6		POL	66 19 26.1	246 19 36.5
7		F	120 05 38.9	300 05 51.0

ARC NO. 3

No	St point	Obs point	CL	CR
1	G	F	164 42 48.2	344 43 00.7
2		O	316 39 08.3	136 39 18.0
3		N	355 33 40.5	175 33 50.0
4		E	16 56 45.4	196 56 54.8
5		A	90 00 11.9	270 00 18.5
6		POL	110 56 38.3	290 56 49.1
7		F	164 42 49.8	344 43 01.5

ARC NO. 4

No	St point	Obs point	CL	CR
1	G	F	209 44 21.6	29 44 31.4
2		O	1 40 41.5	181 40 49.5
3		N	40 35 15.0	220 35 24.5
4		E	61 58 16.5	241 58 27.4
5		A	135 01 42.6	315 01 53.1
6		POL	155 58 10.8	335 58 20.5
7		F	209 44 21.2	29 44 32.1

....

****

REDUCTION OF THEODOLITE OBSERVATIONS

****

GENERAL INFORMATION

Date : 1 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_F ,_86
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	F	POL	342 47 46.4	162 47 52.6
2		O	242 32 44.3	62 32 51.4
3		G	255 04 12.0	75 04 17.5
4		N	259 33 32.5	79 33 37.4
5		A	268 59 19.1	88 59 27.7
6		E	271 01 01.7	91 01 11.1
7		Q	280 47 26.3	100 47 34.1
8		D	296 33 35.6	116 33 42.2
9		B	299 35 05.4	119 35 18.3
10		C	304 59 59.0	125 00 03.9
11		POL	342 47 46.2	162 47 53.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	F	POL	27 49 40.9	207 49 47.2
2		O	287 34 38.2	107 34 45.3
3		G	300 06 06.9	120 06 11.4
4		N	304 35 28.7	124 35 33.7
5		A	314 01 15.6	134 01 22.1
6		E	316 02 57.3	136 03 05.5
7		Q	325 49 17.9	145 49 26.6
8		D	341 35 31.4	161 35 36.0
9		B	344 37 03.7	164 37 09.9
10		C	350 01 50.4	170 01 56.8
11		POL	27 49 41.1	207 49 47.0

ARC NO. 3

No	St point	Obs point	CL	CR
1	F	POL	72 50 56.5	252 51 00.0
2		O	332 35 50.6	152 36 00.1
3		G	345 07 20.1	165 07 24.4
4		N	349 36 39.5	169 36 45.4
5		A	359 02 26.9	179 02 33.8
6		E	1 04 10.2	181 04 16.4
7		Q	10 50 32.6	190 50 40.5
8		D	26 36 43.8	206 36 49.0
9		B	29 38 18.3	209 38 25.5
10		C	35 03 03.9	215 03 10.8
11		POL	72 50 56.0	252 [illegible]1 00.2

ARC NO. 4

No	St point	Obs point	CL	CR
1	F	POL	117 52 25.2	297 52 30.0
2		O	17 37 17.0	197 37 26.7
3		G	30 08 49.0	210 08 54.0
4		N	34 38 08.8	214 38 13.7
5		A	44 03 58.0	224 04 03.9
6		E	46 05 40.4	226 05 46.4
7		Q	55 52 05.3	235 52 08.5
8		D	71 38 14.9	251 38 18.6
9		B	74 39 47.1	254 39 54.1
10		C	80 04 36.7	260 04 37.7
11		POL	117 52 25.3	297 52 30.9

....

****
REDUCTION OF THEODOLITE OBSERVATIONS
****

GENERAL INFORMATION

Date : 2 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C, N , 86
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	N	POL	39 48 07.0	219 48 24.6
2		A	67 36 58.4	247 37 15.3
3		F	79 33 27.8	259 33 46.3
4		G	85 55 02.6	265 55 21.1
5		D	167 19 55.0	347 20 08.3
6		E	325 59 04.6	145 59 19.7
7		B	18 49 56.7	198 50 12.6
8		POL	39 48 08.0	219 48 25.2

ARC NO. 2

No	St point	Obs point	CL	CR
1	N	POL	84 59 43.8	264 59 58.6
2		A	112 48 34.5	292 48 49.0
3		F	124 45 08.7	304 45 23.8
4		G	131 06 38.8	311 06 53.7
5		D	212 31 27.1	32 31 39.2
6		E	11 10 42.3	191 10 53.4
7		B	64 01 32.8	244 01 46.4
8		POL	84 59 43.1	264 59 56.4

ARC NO. 3

No	St point	Obs point	CL	CR
1	N	POL	130 01 12.2	310 01 27.1
2		A	157 50 04.4	337 50 16.8
3		F	169 46 37.0	349 46 51.2
4		G	176 08 10.1	356 08 22.3
5		O	257 32 59.4	77 33 12.2
6		E	56 12 10.0	236 12 22.3
7		B	109 03 04.1	289 03 15.0
8		POL	130 01 14.0	310 01 27.8

ARC NO. 4

No	St point	Obs point	CL	CR
1	N	POL	175 02 46.7	355 03 00.0
2		A	202 51 37.8	22 51 48.8
3		F	214 48 09.6	34 48 25.8
4		G	221 09 41.4	41 09 31.5
5		O	302 34 30.3	122 34 46.3
6		E	101 13 43.1	281 13 58.0
7		B	154 04 34.7	334 04 49.5
8		POL	175 02 47.7	355 03 00.3

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 2 January 1980 Instrument : KERN DKM3 #167608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C, O , 86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	O	POL	29 51 15.4	209 51 31.8
2		A	36 49 55.3	216 50 07.1
3		B	47 00 31.8	227 00 46.7
4		F	62 32 46.4	242 33 00.3
5		E	337 50 44.5	157 50 57.6
6		N	347 19 53.1	167 20 07.1
7		POL	29 51 17.2	209 51 33.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	O	POL	74 52 51.0	254 53 05.3
2		A	81 51 32.6	261 51 38.1
3		B	92 02 05.7	272 02 14.7
4		F	107 34 22.5	287 34 29.0
5		E	22 52 21.5	202 52 27.4
6		N	32 21 28.5	212 21 35.7
7		POL	74 52 51.7	254 53 05.0

ARC NO. 3

No	St point	Obs point	CL	CR
1	O	POL	119 54 20.6	299 54 27.2
2		A	126 52 58.7	306 53 0[illegible].1
3		G	137 03 34.3	317 03 3[illegible]
4		F	152 35 52.3	332 35 56.7
5		E	67 53 50.0	247 53 52.1
6		N	77 22 56.5	257 23 03.3
7		POL	119 54 20.7	299 54 29.4

ARC NO. 4

No	St point	Obs point	CL	CR
1	O	POL	164 56 24.7	344 56 26.3
2		A	171 55 01.6	351 55 06.2
3		G	182 05 38.1	2 05 38.7
4		F	197 37 54.7	17 37 56.9
5		E	112 35 56.1	292 55 55.2
6		N	122 24 58.5	302 25 05.0
7		POL	164 56 24.0	344 56 28.3

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REDUCTION OF THEODOLITE OBSERVATIONS
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GENERAL INFORMATION
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Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C, Q , 86
** The output of all the observations is in degrees, minutes, seconds

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ECHO
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ARC NO. 1
==========

No	St point	Obs point	CL	CR
1	Q	POL	68 29 55.8	248 30 07.3
2		B	74 38 37.4	254 38 52.0
3		P	89 24 11.0	269 24 21.9
4		F	100 47 21.8	280 47 35.4
5		A	108 24 50.5	288 25 05.5
6		E	131 28 21.3	311 28 32.9
7		D	10 04 45.0	190 04 56.4
8		C	42 33 51.2	222 34 05.4
9		POL	68 29 55.6	248 30 07.4

ARC NO. 2
==========

No	St point	Obs point	CL	CR
1	Q	POL	113 31 29.8	293 31 40.3
2		B	119 40 11.6	299 40 22.9
3		P	134 25 38.6	314 25 53.0
4		F	145 48 52.8	325 49 06.4
5		A	153 26 23.8	333 26 35.1
6		E	176 29 50.8	356 30 05.7
7		D	55 06 16.6	235 06 28.4
8		C	87 35 26.7	267 35 40.0
9		POL	113 31 28.7	293 31 40.4

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C, _Q , _86
** The output of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	LR
1	Q	POL	68 29 55.8	248 30 07.3
2		B	74 38 37.4	254 38 52.0
3		P	89 24 11.0	269 24 21.9
4		F	100 47 21.8	280 47 35.4
5		A	108 24 50.5	288 25 05.5
6		E	131 28 21.3	311 28 32.9
7		D	10 04 45.0	190 04 56.4
8		C	42 33 51.2	222 34 05.4
9		POL	68 29 55.6	248 30 07.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	Q	POL	113 31 29.8	293 31 40.3
2		B	119 40 11.6	299 40 22.9
3		P	134 25 38.6	314 25 53.0
4		F	145 48 52.8	325 49 06.4
5		A	153 26 23.8	333 26 35.1
6		E	176 29 50.8	356 30 05.7
7		D	55 06 16.6	235 06 28.4
8		C	87 35 26.7	267 35 40.0
9		POL	113 31 28.7	293 31 40.4

ARC NO. 3

No	St point	Obs point	CL	CR
1	Q	POL	158 33 00.8	338 33 14.9
2		B	164 41 46.5	344 42 00.1
3		P	179 27 14.0	359 27 27.9
4		F	190 50 28.8	10 50 39.7
5		A	198 27 55.8	18 28 10.4
6		E	221 31 26.7	41 31 38.9
7		D	100 07 51.8	280 08 05.3
8		C	132 37 00.5	312 37 13.0
9		POL	158 33 01.9	338 33 14.4

ARC NO. 4

No	St point	Obs point	CL	CR
1	Q	POL	203 34 25.3	23 34 40.7
2		B	209 43 11.2	29 43 23.5
3		P	224 28 38.1	44 28 53.9
4		F	235 51 51.7	55 52 08.0
5		A	243 29 20.4	63 29 36.4
6		E	266 32 51.3	86 33 07.0
7		D	145 09 16.1	325 09 30.7
8		C	177 38 23.0	357 38 37.3
9		POL	203 34 26.5	23 34 40.6

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_E ,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	E	POL	51 45 24.8	231 45 31.7
2		F	91 11 01.6	271 11 04.1
3		A	93 16 09.7	273 16 13.5
4		G	107 28 01.1	287 28 06.8
5		N	146 08 58.7	326 09 03.2
6		O	158 00 38.1	338 00 44.0
7		Q	311 38 20.4	131 38 25.4
8		D	342 13 42.3	162 13 48.8
9		C	10 06 40.1	190 06 46.0
10		B	39 50 00.5	219 50 05.5
11		POL	51 45 25.4	231 45 32.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	E	POL	96 46 54.1	276 47 00.0
2		F	136 12 27.1	316 12 33.5
3		A	138 17 36.3	318 17 43.5
4		G	152 29 27.7	332 29 34.3
5		N	191 10 28.8	11 10 33.8
6		O	203 02 09.2	23 02 12.0
7		Q	356 39 45.6	176 39 53.5
8		D	27 15 10.6	207 15 18.1
9		C	55 08 09.1	235 08 14.8
10		B	84 51 28.7	264 51 32.4
11		POL	96 46 54.2	276 46 59.5

ARC NO. 3

No	St point	Obs point	CL	CR
1	E	POL	141 53 32.0	321 53 39.4
2		F	181 19 02.1	1 19 09.6
3		A	183 24 15.4	3 24 20.8
4		G	197 36 08.1	17 36 11.9
5		N	236 17 05.1	56 17 11.6
6		O	248 08 44.8	68 08 53.2
7		Q	41 46 24.3	221 46 33.4
8		D	72 21 48.1	252 21 58.1
9		C	100 14 47.7	280 14 54.5
10		B	129 58 08.3	309 58 13.8
11		POL	141 53 31.5	321 53 39.5

ARC NO. 4

No	St point	Obs point	CL	CR
1	E	POL	186 54 44.5	6 54 53.5
2		F	226 20 20.5	46 20 25.8
3		A	228 25 30.9	48 25 37.2
4		G	242 37 22.3	62 37 28.6
5		N	281 18 18.6	101 18 29.8
6		O	293 09 56.1	113 10 08.3
7		Q	86 47 37.7	266 47 48.0
8		D	117 23 01.9	297 23 12.2
9		C	145 16 01.8	325 16 09.0
10		B	174 59 20.3	354 59 28.3
11		POL	186 54 44.3	6 54 54.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_A1,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	A1	POL	24 51 46.6	204 52 08.0
2		F	88 59 13.3	268 59 34.1
3		G	180 21 19.3	0 21 44.5
4		O	216 49 46.4	36 50 07.4
5		N	247 36 55.4	67 37 13.7
6		E	273 06 06.9	93 06 29.8
7		Q	288 24 50.8	108 25 08.6
8		D	311 33 29.9	131 33 49.0
9		C	328 40 30.2	148 40 50.6
10		B	335 29 36.8	155 30 00.0
11		POL	24 51 46.4	204 52 07.3

ARC NO. 2

No	St point	Obs point	CL	CR
1	A1	POL	69 54 20.4	249 54 40.0
2		F	134 01 48.5	314 02 01.9
3		G	225 24 00.3	45 24 15.[illegible]
4		O	261 52 19.2	81 52 37.8
5		N	292 39 24.4	112 39 45.4
6		E	318 08 41.4	138 08 57.7
7		Q	333 27 24.4	153 27 40.4
8		D	356 36 00.8	176 36 21.4
9		C	13 43 00.3	193 43 21.3
10		B	20 32 11.5	200 32 25.7
11		POL	69 54 21.4	249 54 39.4

ARC NO. 3

No	St point	Obs point	CL	CR
1	A1	POL	114 56 54.1	294 57 07.1
2		F	179 04 14.9	359 04 30.2
3		G	270 26 29.1	90 26 44.7
4		O	306 54 49.7	126 55 07.6
5		N	337 41 54.7	157 42 11.6
6		E	3 11 09.5	183 11 26.7
7		Q	18 29 54.4	198 30 08.1
8		D	41 38 30.6	221 38 46.1
9		C	58 45 33.3	238 45 48.9
10		B	65 34 38.0	245 34 56.4
11		POL	114 56 52.4	294 57 08.5

ARC NO. 4

No	St point	Obs point	CL	CR
1	A1	POL	159 58 17.4	339 58 27.2
2		F	224 05 40.3	44 05 52.8
3		G	315 27 56.4	135 28 10.0
4		O	351 56 15.4	171 56 31.3
5		N	22 43 23.3	202 43 34.0
6		E	48 12 35.6	228 12 47.3
7		Q	63 31 18.4	243 31 32.9
8		D	86 39 59.5	266 40 11.2
9		C	103 46 59.3	283 47 13.7
10		B	110 36 05.4	290 36 20.6
11		POL	159 58 18.4	339 58 27.5

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REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_A2,_B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	A2	POL	24 51 47.8	204 51 56.6
2		F	88 59 12.5	268 59 21.0
3		G	180 21 28.5	0 21 36.6
4		O	216 49 44.9	36 49 57.7
5		N	247 36 54.1	67 37 03.4
6		E	273 06 06.1	93 06 18.9
7		Q	288 24 49.5	108 24 58.7
8		D	311 33 29.6	131 33 41.8
9		C	328 40 30.5	148 40 40.8
10		B	335 29 36.0	155 29 48.1
11		POL	24 51 [illegible].0	204 51 56.6

ARC NO. 2

No	St point	Obs point	CL	CR
1	A2	POL	114 54 34.0	294 54 39.4
2		F	179 01 55.9	359 02 02.5
3		G	270 24 10.8	90 24 19.6
4		O	306 52 28.8	126 52 41.5
5		N	337 39 37.4	157 39 45.5
6		E	3 08 50.2	183 09 00.4
7		Q	18 27 31.8	198 27 41.9
8		D	41 36 12.8	221 36 27.8
9		C	58 43 14.2	238 43 24.5
10		B	65 32 20.6	245 32 30.2
11		POL	114 54 33.0	294 54 39.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_B1,_B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	B1	F	119 35 17.8	299 35 22.0
2		A	155 29 44.1	335 29 51.7
3		N	198 49 57.0	18 50 06.9
4		E	219 40 05.8	39 40 13.1
5		P	222 12 25.6	42 12 31.9
6		Q	254 38 47.4	74 38 54.6
7		D	292 04 31.6	112 04 40.9
8		C	318 58 11.4	138 58 18.8
9		POL	62 03 11.7	242 03 19.7
10		F	119 35 16.6	299 35 21.8

ARC NO. 2

No	St point	Obs point	CL	CR
1	B1	F	164 36 51.7	344 36 57.8
2		A	200 31 21.5	20 31 28.9
3		N	243 51 35.7	63 51 43.4
4		E	264 41 44.1	84 41 50.1
5		P	267 14 00.8	87 14 08.4
6		Q	299 40 24.8	119 40 31.4
7		D	337 06 08.9	157 06 15.0
8		C	3 59 47.6	183 59 54.1
9		POL	107 04 50.4	287 04 55.1
10		F	164 36 50.8	344 36 58.2

CTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 10 January 1990   Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

Code numbers to create/have access to the.e data: C,_B2,_B6
The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	B2	F	126 24 56.2	306 25 13.0
2		N	205 39 40.9	25 40 01.4
3		E	226 29 51.7	46 30 07.1
4		Q	261 28 30.8	81 28 50.6
5		D	298 54 12.1	118 54 35.6
6		POL	68 52 48.6	248 53 08.7
7		F	126 24 55.6	306 25 13.8

ARC NO. 2

No	St point	Obs point	CL	CR
1	B2	F	171 56 24.4	351 56 40.8
2		N	251 11 10.8	71 11 28.6
3		E	272 01 17.4	92 01 36.1
4		Q	307 00 0' 7	127 00 15.1
5		D	344 25 4..5	164 26 01.9
6		POL	114 24 18.4	294 24 34.8
7		F	171 56 23.6	351 56 41.5

ARC NO. 3

No	St point	Obs point	CL	CR
1	B2	F	216 57 51.9	36 58 09.3
2		N	296 12 38.8	116 12 56.1
3		E	317 02 54.0	137 03 04.4
4		Q	352 01 32.5	172 01 44.2
5		D	29 27 11.9	209 27 29.8
6		POL	159 25 50.3	339 26 06.4
7		F	216 57 52.4	36 58 10.6

ARC NO. 4

No	St point	Obs point	CL	CR
1	B2	F	262 00 07.7	82 00 16.9
2		N	341 14 53.1	161 15 04.9
3		E	2 05 02.3	182 05 10.1
4		Q	37 03 39.7	217 03 49.5
5		D	74 29 24.8	254 29 36.5
6		POL	204 27 59.4	24 28 12.8
7		F	262 00 07.4	82 00 17.7

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

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Date                 :  13 January 1980      Instrument :  KERN DKM3 #169608
Observer             :  I.B.W
Booker               :  I.B.W
Weather Conditions   :
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* Code numbers to create/have access to these data: C,_B3,_B6

** The outout of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	B3	F	177 07 12.2	357 07 30.2
2		A	213 01 44.8	33 02 00.2
3		E	277 12 07.6	97 12 24.1
4		G	312 10 45.8	132 11 0[illegible].2
5		D	349 36 30.6	169 36 49.6
6		C	16 30 10.9	196 30 30.0
7		POL	119 35 07.0	299 35 23.1
8		F	177 07 13.4	357 07 30.3

ARC NO. 2

No	St point	Obs point	CL	CR
1	B3	F	266 59 55.9	87 00 12.1
2		A	302 54 29.5	122 54 44.8
3		E	7 04 47.4	187 05 03.5
4		G	42 03 28.3	222 03 45.8
5		D	79 29 13.7	259 29 30.7
6		C	106 22 52.3	286 23 12.4
7		POL	209 27 48.4	29 28 03.2
8		F	266 59 54.5	87 00 12.0

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REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 11 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_C1,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	C1	F	124 59 50.1	305 00 07.0
2		B	138 57 59.1	318 58 20.5
3		A	148 40 30.1	328 40 49.2
4		P	169 53 52.8	349 54 15.5
5		N	178 55 46.0	358 56 04.4
6		E	189 56 39.1	9 56 57.5
7		Q	222 33 50.7	42 34 08.3
8		D	263 38 47.5	83 39 03.4
9		POL	87 54 24.4	267 54 44.8
10		F	124 59 50.3	305 00 09.9

ARC NO. 2

No	St point	Obs point	CL	CR
1	C1	F	170 01 20.7	350 01 37.2
2		B	183 59 27.3	3 59 46.1
3		A	193 41 52.6	13 42 13.6
4		P	214 55 19.8	34 55 39.1
5		N	223 57 12.0	43 57 28.6
6		E	234 58 06.6	54 58 22.5
7		Q	267 35 19.2	87 35 35.8
8		D	308 40 12.3	128 40 31.7
9		POL	132 55 50.7	312 56 08.4
10		F	170 01 19.8	350 01 37.1

ARC NO. 3

No	St point	Obs point	CL	CR
1	C1	F	215 02 42.2	35 02 56.9
2		B	229 00 51.9	49 01 08.0
3		A	238 43 18.7	58 43 37.2
4		P	259 56 45.0	79 57 04.4
5		N	268 58 38.2	88 58 54.8
6		E	279 59 29.0	99 59 48.8
7		Q	312 36 43.3	132 37 00.7
8		D	353 41 35.6	173 41 54.7
9		POL	177 57 15.3	357.57 32.1
10		F	215 02 42.0	35 02 57.0

ARC NO. 4

No	St point	Obs point	CL	CR
1	C1	F	260 04 34.1	80 04 50.8
2		B	274 02 48.3	94 03 02.8
3		A	283 45 17.3	103 45 29.8
4		P	304 58 39.8	124 58 55.5
5		N	314 00 31.1	134 00 47.2
6		E	325 01 28.0	145 01 40.6
7		Q	357 38 36.7	177 39 50.0
8		D	38 43 33.9	218 43 46.1
9		POL	222 59 09.3	42 59 24.6
10		F	260 04 34.8	80 04 51.5

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

Date	: 12 January 1980	Instrument :	KERN DKM3 #169608
Observer	: I.B.W		
Bonker	: I.B.W		
Weather Conditions :			

* Code numbers to create/have access to these data: C,_C2,_B6
** The output of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	C 2	F	119 35 09.0	299 35 24.7
2		B	133 33 19.6	313 33 36.0
3		A	143 15 45.4	323 16 05.3
4		N	173 31 05.6	353 31 16.9
5		E	184 31 58.0	4 32 15.0
6		Q	217 09 08.3	37 09 26.3
7		D	25[illegible] 14 04.4	78 14 19.9
8		POL	82 29 42.4	262 29 57.3
9		F	119 35 09.7	299 35 23.8

ARC NO. 2

No	St point	Obs point	CL	CR
1	C 2	F	164 46 24.5	344 46 37.0
2		B	178 44 34.8	358 44 44.8
3		A	188 27 01.8	8 27 15.5
4		N	218 42 20.0	38 42 31.7
5		E	229 43 11.2	49 13 2[illegible].3
[illegible]		Q	262 20 24.0	82 20 37.1
[illegible]		D	303 25 18.1	123 25 32.5
8		POL	127 40 57.0	307 41 10.0
9		F	164 46 24.8	344 46 38.7

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

Date : 12 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_C3,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	C3	F	254 48 58.8	74 49 07.0
2		B	268 47 13.2	88 47 18.1
3		A	278 29 3[illegible]	98 29 49.5
4		N	308 44 51.[illegible]	128 45 03.5
5		E	319 43 [illegible]8.8	139 45 57.6
6		Q	357 22 57.7	172 23 10.0
7		D	33 27 53.6	213 28 02.1
8		POL	217 43 28.5	37 43 37.2
9		F	254 48 58.7	74 49 06.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	C3	F	299 50 20.3	119 50 24.5
2		B	313 48 27.5	133 48 35.1
3		A	323 30 58.4	143 31 05.1
4		N	353 46 15.0	173 46 20.6
5		E	4 47 08.6	184 47 13.7
6		Q	37 24 16.8	217 24 24.6
7		D	78 29 11.8	258 29 19.4
8		POL	262 44 50.5	82 45 00.4
9		F	299 50 19.2	119 50 24.1

ARC NO. 3

No	St point	Obs point	CL	CR
1	C3	F	344 53 01.0	164 53 04.8
2		B	358 51 12.9	178 51 17.7
3		A	8 33 40.9	188 33 45.7
4		N	38 48 57.9	218 49 01.9
5		E	49 49 51.3	229 49 53.9
6		Q	82 27 04.8	262 27 08.4
7		D	123 31 56.7	303 32 03.6
8		POL	307 47 35.1	127 47 38.5
9		F	344 53 01.0	164 53 04.5

ARC NO. 4

No	St point	Obs poin	CL	CR
1	C3	F	74 55 22.5	254 55 38.8
2		B	88 53 36.1	268 53 49.9
3		A	98 36 05.4	278 36 19.6
4		N	128 51 20.0	308 51 34.4
5		E	139 52 13.8	319 52 29.4
6		Q	172 29 26.5	352 29 40.0
7		D	213 34 19.6	33 34 35.5
8		POL	37 49 55.6	217 50 13.1
9		F	74 55 22.4	254 55 39.1

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

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Date                : 10 January 1980      Instrument :  KERN DKM3 #169608
Observer            : I.B.W
Booker              : I.B.W
Weather Conditions  :
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* Code numbers to create/have access to these data: C, _D1, _86
** The output of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D1	POL	86 38 10.1	266 38 20.6
2		B	112 04 26.5	292 04 33.3
3		F	116 33 34.5	296 33 42.3
4		A	131 33 33.9	311 33 39.7
5		P	134 08 57.3	314 09 07.7
6		N	157 50 19.9	337 50 27.0
7		E	162 03 43.6	342 03 52.8
8		Q	190 04 48.8	10 04 54.8
9		C	83 38 48.8	263 38 57.9
10		POL	86 38 11.7	266 38 21.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	D1	POL	131 40 07.5	311 40 14.1
2		B	157 06 20.3	337 06 28.6
3		F	161 35 28.5	341 35 34.6
4		A	176 35 29.1	356 35 34.2
5		P	179 10 53.9	359 10 58.8
6		N	202 52 11.6	22 52 20.0
7		E	207 05 39.5	27 05 44.9
8		Q	235 06 41 6	55 06 46.7
9		C	128 40 45.3	308 40 50.2
10		POL	131 40 03.0	311 40 14.1

ARC NO. 3

No	St point	Obs point	CL	CR
1	D1	POL	176 43 12.8	356 43 26.8
2		B	202 09 25.8	22 09 40.2
3		F	206 38 34.5	26 38 47.3
4		A	221 38 32.6	41 38 45.0
5		P	224 13 58.0	44 14 11.8
6		N	247 55 14.6	67 55 32.2
7		E	252 08 44.5	72 08 57.7
8		Q	280 09 46.7	100 10 00.6
9		C	173 43 53.5	353 44 04.6
10		POL	176 43 13.4	356 43 26.3

ARC NO. 4

No	St point	Obs point	CL	CR
1	D1	POL	131 41 41.5	311 41 53.7
2		B	157 07 51.4	337 08 04.2
3		F	161 37 00.8	341 37 13.6
4		A	176 36 59.1	356 37 12.1
5		P	179 12 22.8	359 12 37.7
6		N	202 53 41.0	22 53 56.8
7		E	207 07 11.6	27 07 23.1
8		Q	235 08 1..6	55 08 25.3
9		C	128 42 18.3	308 42 30.5
10		POL	131 41 41.8	311 41 53.7

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

Date : 11 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C:_D2,_86
** The output of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D2	POL	356 41 30.0	176 41 26.3
2		B	22 07 44.9	202 07 38.1
3		F	26 36 53.5	206 36 49.6
4		A	41 36 51.1	221 36 46.5
5		N	67 53 35.8	247 53 30.1
6		E	72 07 03.5	252 06 57.4
7		Q	100 08 08.4	280 07 59.8
8		POL	356 41 31.0	176 41 26.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	D2	POL	41 43 02.3	221 43 08.4
2		B	67 09 14.4	247 09 23.1
3		F	71 38 24.8	251 38 34.4
4		A	86 38 20.3	266 38 30.0
5		N	112 55 05.4	292 55 13.8
6		E	117 08 32.8	297 08 41.8
7		Q	145 09 34.5	325 09 41.3
8		POL	41 43 01.8	221 43 08.4

ARC NO. 3

No	St point	Obs point	CL	CR
1	D2	POL	86 38 28.8	266 38 41.1
2		B	112 04 40.3	292 04 55.5
3		F	116 33 51.6	296 34 03.5
4		A	131 33 49.5	311 33 59.8
5		N	157 50 32.5	337 50 43.0
6		E	162 04 00.4	342 04 10.8
7		Q	190 05 01.6	10 05 13.0
8		POL	86 38 29.0	266 38 41.8

ARC NO. 4

No	St point	Obs point	CL	CR
1	D2	POL	131 40 01.4	311 40 14.8
2		B	157 06 13.0	337 06 27.3
3		F	161 35 24.0	341 35 35.5
4		A	176 35 21.3	356 35 35.2
5		N	202 52 02.3	22 52 17.6
6		E	207 05 32.5	27 05 44.0
7		Q	235 06 32.5	55 06 46.0
8		POL	131 40 01.8	311 40 15.0

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

Date : 12 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: C,_D3,_86
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D3	POL	86 38 12.8	266 38 22.4
2		B	112 04 27.3	292 04 33.8
3		F	116 33 37.8	296 33 43.9
4		A	131 33 36.8	311 33 40.3
5		N	157 50 19.4	337 50 26.2
6		E	162 03 47.0	342 03 53.1
7		Q	190 04 50.4	10 04 55.2
8		C	83 38 54.5	263 39 00.3
9		POL	86 38 10.8	266 38 21.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	D3	POL	176 43 15.5	356 43 23.3
2		B	202 09 28.4	27 09 36.3
3		F	206 38 38.2	26 38 44.8
4		A	221 38 35.5	41 38 42.7
5		N	247 55 21.1	67 55 26.2
6		E	252 08 48.1	72 08 55.9
7		Q	280 09 49.8	100 09 59.6
8		C	173 43 52.6	353 43 59.4
9		POL	176 43 16.1	356 43 22.8

E.2 FROM CONTROL POINTS TO FACE POINTS

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                  REDUCTION OF THEODOLITE OBSERVATIONS
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                           GENERAL INFORMATION
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Date                 :  1 January 1980        Instrument :  KERN DKM3 #169608
Obser  er            :  I.B.W
Boc                  :  I.B.W
 eather Co ditions   :

*  Coda numbers to create/have access to these data: F,_P ,_86
** The output of all the observations is in degrees, minutes, seconds


******************************************************************************************


                                   ECHO
                                   ====


                                ARC NO.   1
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No	St point	Obs point	CL	CR
1	P	N	187 32 19.5	7 32 30.3
2		E	217 52 26.4	37 52 35.6
3		Q	269 24 11.5	89 24 19.3
4		D	314 08 59.2	134 09 08.2
5		U	333 07 17.6	153 07 29.0
6		T	334 31 20.7	154 51 29.7
7		C	349 53 58.6	169 54 08.4
8		W	0 30 07.9	180 30 20.0
9		B	42 12 03.6	222 12 20.6
10		Y	130 51 33.1	310 51 41.4
11		K	141 05 11.7	321 05 23.5
12		X	149 24 29.4	329 24 37.7
13		N	187 32 21.4	7 32 31.0

ARC NO. 2

No	St point	Obs point	CL	CR
1	P	N	232 33 50.5	52 34 00.0
2		E	262 53 55.0	82 54 06.4
3		Q	314 25 39.0	134 25 48.5
4		D	359 10 27.9	179 10 39.3
5		U	18 08 46.8	198 08 56.2
6		T	19 52 50.2	199 52 58.5
7		C	34 55 29.3	214 55 39.8
8		W	45 31 38.6	225 31 46.4
9		B	87 13 39.1	267 13 50.4
10		Y	175 53 01.9	355 53 11.6
11		K	186 06 43.6	6 06 53.6
12		X	194 25 59.5	14 26 08.3
13		N	232 33 50.2	52 33 [illegible]9.1

ARC NO. 3

No	St point	Obs point	CL	CR
1	P	N	277 35 18.5	97 35 26.9
2		E	307 55 22.7	127 55 33.2
3		Q	359 27 05.1	179 27 17.3
4		D	44 11 53.6	224 12 03.1
5		U	63 10 16.1	243 10 23.4
6		T	64 54 17.8	244 54 27.4
7		C	79 56 58.1	259 7 05.5
8		W	90 33 03.0	270 33 13.0
9		B	132 15 08.8	312 15 20.0
10		Y	220 54 28.7	40 54 35.6
11		K	231 08 10.1	51 08 18.4
12		X	239 27 26.1	59 27 32.9
13		N	277 35 19.3	97 35 28.0

ARC NO. 4

No	St point	Obs point	CL	CR
1	P	N	322 36 53.0	142 37 03.1
2		E	352 56 57.3	172 57 07.5
3		Q	44 28 38.8	224 28 47.3
4		D	89 13 29.5	269 13 40.8
5		L	108 11 52.4	288 12 00.5
6		T	109 55 55.6	89 56 00.7
7		C	124 58 33.0	304 58 47.6
8		W	135 34 39.8	315 34 51.0
9		B	177 16 41.7	357 16 52.7
10		Y	265 56 05.4	85 56 13.7
11		K	276 09 45.5	96 09 58.5
12		X	284 29 02.6	104 29 12.4
13		N	322 36 54.0	142 37 03.5

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date	: 1 January 1980	Instrument :	KERN DKM3 #169608
Observer	: I.B.W		
Booker	: I.B.W		
Weather Conditions :			

* Code numbers to create/have access to these data: F,_G ,_86

** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	G	F	75 04 08.1	255 04 20.9
2		H	168 [illegible] 40.5	348 14 51.8
3		I	175 13 25.4	355 13 36.0
4		J2	178 35 50.8	356 36 02.5
5		S2	188 32 44.5	8 32 56.6
6		O	227 00 26.5	47 00 40.6
7		L	235 11 16.8	55 11 30.1
8		M	251 23 45.8	71 24 00.9
9		N	265 55 01.4	85 55 14.3
10		E	287 18 03.4	107 18 17.0
11		R2	303 30 32.4	123 30 49.5
12		A	[illegible] 21 30.1	180 21 49.6
13		PO	15 45 34.5	195 45 48.3
14		POL	21 17 57.7	201 18 17.1
15		F	75 04 08.4	255 04 20.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	G	F	120 05 40.1	300 05 51.9
2		H	213 16 09.7	33 16 20.9
3		I	220 14 54.1	40 15 05.7
4		J2	223 37 19.3	43 37 36.9
5		S2	233 34 15.9	53 34 29.1
6		O	272 01 58.5	92 02 10.4
7		L	280 12 46.6	100 13 00.4
8		M	296 25 18.7	116 25 31.8
9		N	310 56 31.9	130 56 43.7
10		E	332 19 34.6	152 19 45.2
11		R2	348 32 04.3	168 32 16.5
12		A	45 22 57.7	225 23 10.9
13		PO	60 47 05.1	240 47 15.6
14		POL	66 19 26.1	246 19 36.5
15		F	120 05 38.9	300 05 51.0

ARC NO. 3

No	St point	Obs point	CL	CR
1	G	F	164 42 48.2	344 43 00.7
2		H	257 53 19.2	77 53 32.7
3		I	264 52 05.1	84 52 19.1
4		J2	268 14 31.1	88 14 40.2
5		S2	278 11 25.4	98 11 36.2
6		O	316 39 08.3	136 39 18.0
7		L	324 49 58.6	144 50 09.6
8		M	341 02 28.7	161 02 40.5
9		N	355 33 40.5	175 33 50.0
10		E	16 56 45.4	196 56 54.8
11		R2	33 09 16.6	213 09 26.7
12		A	90 00 11.9	270 00 18.5
13		PO	105 24 12.7	285 24 30.9
14		POL	110 56 38.3	290 56 49.1
15		F	164 42 49.8	344 43 01.5

ARC NO. 4

No	St point	Obs point	CL	CR
1	G	F	209 44 21.6	29 44 31.4
2		H	302 54 53.1	122 55 02.7
3		I	309 53 38.3	129 53 48.3
4		J2	313 16 03.1	133 16 12.7
5		S2	323 12 56.8	143 13 09.0
6		O	1 40 41.5	181 40 49.5
7		L	9 51 29.0	189 51 39.2
8		M	26 04 01.5	206 04 12.8
9		N	40 35 15.0	220 35 24.5
10		E	61 58 16.5	241 58 27.4
11		R2	78 10 52.3	258 11 00.9
12		A	135 01 42.6	315 01 53.1
13		PO	150 25 49.1	330 25 57.8
14		POL	155 58 10.8	335 58 20.5
15		F	209 44 21.2	29 44 32.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date	: 1 January 1980	Instrument :	KERN DKM3 #169608
Observer	: I.B.W		
Booker	: I.B.W		
Weather Conditions :			

* Code numbers to create/have access to these data: F, _F , _86

** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	F	POL	342 47 46.4	162 47 52.6
2		J2	231 43 23.6	51 43 31.3
3		I	237 56 17.1	57 56 22.4
4		S2	242 27 37.8	62 27 42.5
5		O	242 32 44.3	6[illegible] 32 51.4
6		H	245 16 34.1	65 16 40.4
7		L	246 35 39.3	66 35 44.8
8		M	253 34 38.6	73 34 45.5
9		G	255 04 12.0	75 04 17.5
10		N	259 33 32.5	79 33 37.4
11		R2	259 51 38.7	79 51 43.4
12		A	268 59 19.1	88 59 27.7
13		E	271 01 01.7	91 01 11.1
14		Q	280 47 26.3	100 47 34.1
15		D	296 33 35.6	116 33 42.2
16		B	299 35 05.4	119 35 18.3
17		C	304 59 59.0	125 00 03.9
18		PO	33[illegible] 37 55.9	157 38 02.0
19		POL	342 47 46.2	162 47 53.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	F	POL	27 49 40.9	207 49 47.2
2		J2	276 45 21.6	96 45 26.0
3		I	282 58 1[illegible].7	102 58 16.4
4		S2	287 29 32.1	107 29 38.8
5		O	287 34 38.2	107 34 45.3
6		H	290 18 30.6	110 18 35.4
7		L	291 37 33.5	111 37 3[illegible].9
8		M	290 36 36.6	118 [illegible] 41.3
9		G	300 06 06.9	120 [illegible]6 11.4
10		N	304 35 28.7	124 [illegible]5 33.7
11		R2	304 53 32.8	124 [illegible]3 38.1
12		A	314 01 15.6	134 [illegible]1 22.1
13		E	316 02 57.3	136 03 05.5
14		Q	325 49 17.9	145 49 26.6
15		D	341 35 31.4	161 35 36.0
16		B	344 37 03.7	164 37 09.9
17		C	350 01 50.4	170 01 5[illegible].8
18		PO	[illegible] 39 51.3	202 39 54.9
19		POL	27 49 [illegible]	207 49 47.0

ARC NO. 3

No	St point	Obs point	CL	CR
1	F	POL	72 50 56.5	252 51 00.0
2		J2	321 46 34.6	141 46 39.4
3		I	327 59 21.0	147 59 28.6
4		S2	332 30 44.4	152 30 50.8
5		O	332 35 50.6	152 36 00.1
6		H	335 19 43.5	155 19 47.2
7		L	336 38 45.7	156 38 50.3
8		M	343 37 48.2	163 37 51.8
9		G	345 07 20.1	165 07 24.4
10		N	349 36 39.5	169 36 45.4
11		R2	349 54 47.0	169 54 50.1
12		A	359 02 26.9	179 02 33.8
13		E	1 04 10.2	181 04 16.4
14		Q	10 50 32.6	190 50 40.5
15		D	26 36 43.8	206 36 49.0
16		B	29 38 18.3	209 38 25.5
17		C	35 03 03.9	215 03 10.8
18		PO	67 41 06.9	247 41 09.8
19		POL	72 50 56.0	252 51 00.2

ARC NO. 4

No	St point	Obs point	CL	CR
1	F	POL	117 52 25.2	297 52 30.0
2		J2	6 48 03.3	186 48 06.8
3		I	13 00 54.5	193 00 58.7
4		S2	17 32 13.3	197 32 18.4
5		O	17 37 17.0	197 37 26.7
6		H	20 21 14.5	200 21 15.0
7		L	21 40 15.7	201 40 19.5
8		M	28 39 16.2	208 39 23.0
9		G	30 08 49.0	210 08 54.0
10		N	34 38 08.8	214 38 13.7
11		R2	34 56 16.4	214 56 20.2
12		A	44 03 58.0	224 04 03.9
13		E	46 05 40.4	226 05 46.4
14		Q	55 52 05.3	235 52 08.5
15		D	71 38 14.9	251 38 18.6
16		B	74 39 47.1	254 39 54.1
17		C	80 04 36.7	260 04 37.7
18		PO	112 42 35.0	292 42 38.4
19		POL	117 52 25.3	297 52 30.9

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 2 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_N ,_B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	N	POL	39 48 07.0	219 48 24.6
2		A	67 36 58.4	247 37 15.3
3		R2	79 01 36.4	259 01 53.5
4		F	79 33 27.8	259 33 48.3
5		G	85 55 02.6	265 55 21.1
6		H	99 14 55.1	279 15 09.5
7		S2	108 03 50.7	288 04 05.1
8		I	110 55 07.9	290 55 25.6
9		M	161 26 24.6	341 26 45.6
10		L	166 05 30.5	346 05 48.4
11		O	167 19 55.0	347 20 08.3
12		E	325 59 04.6	145 59 19.7
13		B	18 49 56.7	198 50 12.6
14		PO	35 33 53.0	215 34 08.0
15		POL	39 48 08.0	219 48 25.2

ARC NO. 2

No	St point	Obs point	CL	CR
1	N	POL	84 59 43.8	264 59 58.6
2		A	112 48 34.5	292 48 49.0
3		R2	124 13 13.3	304 13 28.0
4		F	124 45 08.7	304 45 23.8
5		G	131 06 38.8	311 06 53.7
6		H	144 26 31.3	324 26 45.7
7		S2	153 15 25.8	333 15 42.2
8		I	156 06 41.8	336 06 56.0
9		M	206 37 57.8	26 38 18.6
10		L	211 17 00.6	31 17 21.4
11		O	212 31 27.1	32 31 39.2
12		E	11 10 42.3	191 10 53.4
13		B	64 01 32.8	244 01 46.4
14		PO	80 4' 29.6	260 45 42.4
15		POL	84 59 43.1	264 59 56.4

ARC NO. 3

No	St point	Obs point	CL	CR
1	N	POL	130 01 12.2	310 01 27.1
2		A	157 50 04.4	337 50 16.8
3		R2	169 14 44.7	349 14 56.7
4		F	169 46 37.0	349 46 51.2
5		G	176 08 10.1	356 08 22.3
6		H	189 28 02.7	9 28 18.2
7		S2	198 16 54.6	18 17 08.7
8		I	201 08 12.9	21 08 25.8
9		M	251 39 36.3	71 39 50.9
10		L	256 18 34.8	76 18 47.5
11		O	257 32 59.4	77 33 12.2
12		E	56 12 10.0	236 12 22.3
13		B	109 03 04.1	289 03 15.0
14		PO	125 47 00.6	305 47 11.9
15		POL	130 01 14.0	310 01 27.8

ARC NO. 4

No	St point	Obs point	CL	CR
1	N	POL	175 02 46.7	355 03 00.0
2		A	202 51 37.8	22 51 48.8
3		R2	214 16 15.5	34 16 27.7
4		F	214 48 09.6	34 48 25.[illegible]
5		G	221 09 41.4	41 09 51.5
6		H	234 29 35.1	54 29 49.5
7		S2	243 18 29.8	63 18 44.3
8		I	246 09 45.6	66 10 00.8
9		M	296 41 08.9	116 41 27.2
10		L	301 20 10.8	121 20 24.0
11		O	302 34 30.3	122 34 46.3
12		E	101 13 43.1	281 13 58.0
13		B	154 04 34.7	334 04 49.5
14		PO	170 48 29.2	350 48 45.8
15		POL	175 02 47.7	355 03 00.3

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ARC NO. 2

No	St point	Obs point	CL	CR
1	N	POL	84 59 43.8	264 59 58.6
2		A	112 48 34.5	292 48 49.0
3		R2	124 13 13.3	304 13 28.0
4		F	124 45 08.7	304 45 23.8
5		G	131 06 38.8	311 06 53.7
6		H	144 26 31.3	324 26 45.7
7		S2	153 15 25.8	333 15 42.2
8		I	156 06 41.8	336 06 56.0
9		M	206 37 57.8	26 38 18.6
10		L	211 17 00.6	31 17 21.4
11		O	212 31 27.1	32 31 39.2
12		E	11 10 42.3	191 10 53.4
13		B	64 01 32.8	244 01 46.4
14		PO	80 45 29.6	260 45 42.4
15		POL	84 59 43.1	264 59 56.4

ARC NO. 3

No	St point	Obs point	CL	CR
1	N	POL	130 01 12.2	310 01 27.1
2		A	157 50 04.4	337 50 16.8
3		R2	169 14 44.7	349 14 56.7
4		F	169 46 37.0	349 46 51.2
5		G	176 08 10.1	356 08 22.3
6		H	189 28 02.7	9 28 18.2
7		S2	198 16 54.6	18 17 08.7
8		I	201 08 12.9	21 08 25.8
9		M	251 39 36.3	71 39 50.9
10		L	256 18 34.8	76 18 47.5
11		O	257 32 59.4	77 33 12.2
12		E	56 12 10.0	236 12 22.3
13		B	109 03 04.1	289 03 13.0
14		PO	125 47 00.6	305 47 11.9
15		POL	130 01 14.0	310 01 27.8

ARC NO. 4

No	St point	Obs point	CL	CR
1	N	POL	175 02 46.7	355 03 00.0
2		A	202 51 37.8	22 51 48.8
3		R2	214 16 15.5	34 16 27.7
4		F	214 48 09.6	34 48 25.8
5		G	221 09 41.4	41 09 51.5
6		H	234 29 35.1	54 29 49.5
7		S2	243 18 29.8	63 18 44.3
8		I	246 09 45.6	66 10 00.8
9		M	296 41 08.9	116 41 27.2
10		L	301 20 10.8	121 20 24.0
11		O	302 34 30.3	122 34 46.3
12		E	101 13 43.1	281 13 58.0
13		B	154 04 34.7	334 04 49.5
14		PO	170 48 29.2	350 48 45.8
15		POL	175 02 47.7	355 03 00.3

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date	: 2 January 1980	Instrument :	KERN DKM3 #109608
Observer	: I.B.W		
Booker	: I.B.W		
Weather Conditions :			

* Code numbers to create/have access to these data: F,_O ,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	O	POL	29 51 15.4	209 51 31.8
2		A	36 49 55.3	216 50 07.1
3		R2	38 26 42.1	218 26 57.9
4		G	47 00 31.8	227 00 46.7
5		H	58 44 06.4	238 44 20.3
6		F	62 32 46.4	242 33 00.3
7		S2	62 42 10.8	242 42 21.4
8		I	70 19 00.5	250 19 11.4
9		E	337 50 44.5	157 50 57.6
10		N	347 19 53.1	167 20 07.1
11		M	350 24 35.[illegible]	170 24 47.6
12		L	351 04 05.5	171 04 27.3
13		PO	26 06 56.6	206 07 11.5
14		POL	29 51 17.2	209 51 33.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	O	POL	74 52 51.0	254 53 05.3
2		A	81 51 32.6	261 51 38.1
3		R2	83 28 17.9	263 28 27.9
4		G	92 02 05.7	272 02 14.7
5		H	103 45 42.1	283 45 50.7
6		F	107 34 22.5	287 34 29.0
7		S2	107 43 45.0	287 43 51.1
8		I	115 20 31.6	295 20 41.8
9		E	22 52 21.5	202 52 27.4
10		N	32 21 28.5	212 21 35.7
11		M	35 26 11.8	215 26 20.8
12		L	36 05 45.8	216 05 57.3
13		PO	71 08 31.0	251 08 42.8
14		POL	74 52 51.7	254 53 05.0

ARC NO. 3

No	St point	Obs point	CL	CR
1	O	POL	119 54 [illegible]0.6	299 54 27.2
2		A	126 52 58.7	306 53 02.1
3		R2	128 29 44.5	308 29 50.0
4		G	137 03 34.3	317 03 39.4
5		H	148 47 09.2	328 47 14.7
6		F	152 35 52.3	332 35 56.7
7		S2	152 45 15.5	332 45 16.0
8		I	160 21 59.4	340 22 06.0
9		E	67 53 50.0	247 53 52.1
10		N	77 22 56.5	257 23 03.3
11		M	80 27 40.8	260 27 45.8
12		L	81 07 05.6	261 07 16.8
13		PO	116 10 01.0	296 10 08.1
14		POL	119 54 20.7	299 54 29.4

ARC NO. 4

No	St point	Obs point	[illegible]	CR
1	O	POL	164 56 [illegible]4.7	344 56 26.3
2		A	171 5[illegible] [illegible].6	351 55 06.2
3		R2	173 31 [illegible]7	353 31 53.3
4		G	182 [illegible]5 [illegible]1	2 05 38.7
5		H	193 49 [illegible]	13 49 14.4
6		F	197 37 54.7	17 37 56.9
7		S2	197 47 17.2	17 47 18.7
8		I	205 24 05.2	[illegible] 24 05.3
9		E	112 55 56.1	[illegible] [illegible] 55.2
10		N	122 24 58.5	302 25 [illegible]3.0
11		M	125 29 41.5	305 29 44.0
12		L	126 09 16.1	306 09 17.8
13		PO	161 12 01.9	341 12 04.5
14		POL	164 56 24.0	344 56 28.3

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          REDUCTION OF THEODOLITE OBSERVATIONS
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                  GENERAL INFORMATION
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Date                 :  8 January 1989      Instrument :  KERN DKM3 #169608
Observer             :  I.B.W
Booker               :  I.B.W
Weather Conditions   :

*  Code numbers to create/have access to these data: F,_01,_86
** The output of all the observations is in degrees, minutes, seconds


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                             ECHO
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                          ARC NO.  1
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No	St point	Obs point	CL	CR
1	01	POL	68 29 55.8	248 30 07.3
2		B	74 38 37.4	254 38 52.0
3		W	79 26 45.4	259 26 57.0
4		P	89 24 11.0	269 24 21.9
5		Y	100 20 04.6	280 20 16.9
6		F	100 47 21.8	280 47 35.4
7		X	105 01 16.4	285 01 28.5
8		K	106 45 08.5	286 45 23.1
9		A	108 24 50.5	288 25 05.5
10		E	131 28 21.3	311 28 32.9
11		D	10 04 45.0	190 04 56.4
12		U	41 51 07.8	221 51 20.6
13		C	42 33 51.2	222 34 05.4
14		T	60 04 50.3	240 05 07.4
15		PO	65 39 11.0	245 39 21.1
16		V	67 18 52.0	247 19 05.8
17		POL	68 29 55.6	248 30 07.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	Q1	POL	113 31 29.8	293 31 40.3
2		B	119 40 11.6	299 40 22.9
3		W	124 28 16.8	304 28 28.8
4		P	134 25 38.6	314 25 53.0
5		Y	145 21 34.7	325 21 47.9
6		F	145 48 52.8	325 49 06.4
7		X	150 02 48.4	330 02 58.7
8		K	151 46 39.5	331 46 51.2
9		A	153 26 23.8	333 26 35.1
10		E	176 29 50.8	356 30 05.7
11		D	55 06 16.6	235 06 28.4
12		U	86 52 39.8	266 52 53.3
13		C	87 35 26.7	267 35 40.0
14		T	105 06 24.5	285 06 39.1
15		PO	110 40 42.8	290 40 55.0
16		V	112 20 23.3	292 20 38.2
17		POL	113 31 28.7	293 31 40.4

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

\# Code numbers to create/have access to these data: F, _Q2, _86
\## The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	Q2	POL	158 33 00.8	338 33 14.9
2		B	164 41 46.5	344 42 00.1
3		W	169 29 51.5	349 30 05.5
4		P	179 27 14.0	359 27 27.9
5		Y	190 23 09.3	10 23 24.7
6		F	190 50 28.8	10 50 39.7
7		X	195 04 20.3	15 04 33.7
8		K	196 48 13.3	16 48 27.8
9		A	198 27 55.8	18 28 10.4
10		E	221 31 26.7	41 31 38.9
11		95	69 39 20.1	249 39 36.0
12		94	71 59 55.0	252 00 09.4
13		95A	73 19 25.6	253 19 38.4
14		127	75 32 32.7	255 32 45.4
15		118	75 48 55.1	255 49 09.4
16		130	77 10 10.7	257 10 26.6
17		117	78 07 18.1	258 07 30.0
18		136	78 12 18.5	258 12 33.4
19		131	78 58 08.8	258 58 23.1
20		147	81 11 56.3	261 12 10.8
21		132	81 21 06.3	261 21 16.4
22		148	82 01 49.1	262 02 02.2
23		13A	82 13 13.5	262 13 26.7
24		149	83 57 51.4	263 58 04.8
25		133	85 12 40.1	265 12 58.7
26		168	85 20 30.0	265 20 43.8
27		151	89 32 22.1	269 32 36.6
28		169	91 12 17.5	271 12 29.8
29		152	91 30 02.3	271 30 17.5
30		114	94 57 43.5	274 57 55.7
31		175	95 55 01.3	275 55 16.4
32		T2	96 24 18.3	276 24 32.8
33		T1	98 39 11.9	278 39 25.7
34		D	100 07 51.8	280 08 05.3
35		U	131 54 16.6	311 54 28.4
36		C	132 37 00.5	312 37 13.0
37		T	150 07 58.4	330 08 12.8
38		PO	155 42 17.7	335 42 30.6
39		V	157 21 59.1	337 22 11.4
40		POL	158 33 01.9	338 33 14.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	Q2	POL	203 34 25.3	23 34 40.7
2		B	209 43 11.2	29 43 23.5
3		W	214 31 14.2	34 31 32.8
4		P	224 28 38.1	44 28 53.9
5		Y	235 24 32.4	55 24 50.0
6		F	235 51 51.7	55 52 08.0
7		X	240 05 47.7	60 06 06.2
8		K	241 49 37.5	61 49 52.5
9		A	243 29 20.4	63 29 36.4
10		E	266 32 51.3	86 33 07.0
11		95	114 40 46.5	294 41 01.1
12		94	117 01 18.0	297 01 30.8
13		95A	118 20 48.0	298 21 05.1
14		127	120 33 55.3	300 34 16.9
15		118	120 50 18.6	300 50 38.0
16		130	122 11 34.0	302 11 52.0
17		117	123 08 40.5	303 08 54.0
18		136	123 13 40.5	303 13 58.3
19		131	123 59 32.9	303 59 48.1
20		147	126 13 27.1	306 13 37.3
21		132	126 22 27.1	306 22 42.7
22		148	127 03 12.9	307 03 28.4
23		13A	127 14 38.1	307 14 50.6
24		1 9	128 59 16.4	308 59 30.0
25		133	130 14 06.4	310 14 20.7
26		168	130 21 58.5	310 22 11.8
27		151	134 33 40.6	314 33 58.3
28		1·9	136 13 38.6	316 13 51.3
29		152	136 31 30.1	316 31 43.5
30		114	139 59 10.8	319 59 16.3
31		1/5	140 56 23.8	320 56 41.0
32		T2	141 25 43.2	321 25 56.0
33		T1	143 40 32.3	323 40 49.4
34		D	145 09 16.1	325 09 30.7
35		U	176 55 38.4	356 55 52.4
36		C	177 38 23.0	357 38 37.3
37		T	195 09 22.7	15 09 33.3
38		PO	200 43 39.4	20 43 54.4
39		V	202 23 22.1	22 23 38.1
40		POL	203 34 26.5	23 34 40.6

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980   Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_E1,_86
** The output of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	E1	POL	51 45 24.8	231 45 31.7
2		Y	67 48 39.8	247 48 42.0
3		X	72 04 42.0	252 04 48.2
4		K	81 06 57.8	261 07 03.8
5		F	91 11 01.6	271 11 04.1
6		A	93 16 09.7	273 16 13.5
7		R2	105 13 39.5	285 13 46.0
8		G	107 28 01.1	287 28 06.8
9		N	146 08 58.7	326 09 03.2
10		O	158 00 39.1	338 00 44.0
11		Q	311 38 20.4	131 38 25.4
12		D	342 13 42.3	162 13 48.8
13		U	2 40 35.5	182 40 40.8
14		C	10 06 40.1	190 06 46.0
15		T	11 57 01.1	191 57 09.9
16		V	19 05 12.4	199 05 19.9
17		W	30 31 58.5	210 32 04.0
18		P	38 02 26.7	218 02 28.8
19		B	39 50 00.5	219 50 05.5
20		PO	47 43 38.5	227 43 43.9
21		POL	51 45 25.4	231 45 32.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	E1	POL	96 46 54.1	276 47 00.0
2		Y	112 50 06.7	292 50 15.0
3		X	117 06 08.6	297 06 13.6
4		K	126 08 27.4	306 08 32.7
5		F	136 12 27.1	316 12 33.5
6		A	138 17 36.3	319 17 43.5
7		R2	150 15 08.6	330 15 13.2
8		G	152 29 27.7	332 29 34.3
9		N	191 10 28.8	11 10 33.8
10		O	203 02 09.2	23 02 12.0
11		Q	356 39 45.6	176 39 53.5
12		D	27 15 10.6	207 15 18.1
13		U	47 42 01.8	227 42 10.0
14		C	55 08 09.1	235 08 14.8
15		T	56 58 30.4	236 58 38.0
16		V	64 06 45.3	244 06 50.5
17		W	75 33 27.5	255 33 32.8
18		P	83 03 53.3	263 04 01.8
19		B	84 51 28.7	264 51 32.4
20		PO	92 45 10.3	272 45 14.5
21		POL	96 46 54.2	276 46 59.5

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REDUCTION OF THEODOLITE OBSERVATIONS
****

GENERAL INFORMATION

Date : 9 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_E2,_86
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	E2	POL	141 53 32.0	321 53 39.4
2		F	181 19 02.1	1 19 09.6
3		A	183 24 15.4	3 24 20.8
4		R2	195 21 42.9	15 21 49.8
5		G	197 36 08.1	17 36 11.9
6		N	236 17 05.1	[illegible] 17 11.6
7		O	248 08 44.8	68 08 53.2
8		Q	41 46 24.3	221 46 33.4
9		91	46 22 04.8	226 22 14.5
10		94	46 42 50.4	226 42 57.6
11		95	48 20 12.2	228 20 19.9
12		89	49 37 45.7	229 37 53.6
13		115	49 39 17.9	229 39 26.6
14		94A	50 18 02.3	230 18 09.8
15		101	51 08 10.0	231 08 17.2
16		116	51 14 32.3	231 14 39.1
17		90	51 43 52.8	231 44 00.7
18		95A	52 10 22.2	232 10 29.7
19		117	53 03 34.9	233 03 44.5
20		137	53 52 57.9	233 53 06.0
21		126	53 57 05.5	233 57 12.0
22		118	54 15 24.4	234 15 29.3
23		127	54 20 05.8	234 20 12.7
24		136	54 39 31.4	234 39 40.5
25		130	54 56 58.4	234 57 04.3
26		147	55 00 11.0	235 00 20.5
27		11A	55 30 11.8	235 30 17.2
28		135	55 36 39.6	235 36 45.1
29		148	55 39 06.5	235 39 13.3
30		131	55 40 02.3	235 40 04.6
31		132	56 29 39.1	236 29 45.3
32		149	56 52 35.6	236 52 41.1
33		13A	57 07 23.7	237 07 30.8
34		119	57 19 22.3	237 19 27.3
35		133	58 14 05.8	238 14 12.6
36		151	59 12 44.7	239 12 50.5
37		152	60 06 53.1	240 07 03.3
38		D	72 21 48.1	252 21 58.1
39		C	100 14 47.7	280 14 54.5
40		B	129 58 08.3	309 58 13.8
41		PO	137 51 47.2	317 51 54.1
42		POL	141 53 31.5	321 53 39.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	F2	POL	186 54 44.5	6 54 53.5
2		F	226 20 20.5	46 20 25.8
3		A	228 25 30.9	48 25 37.2
4		R2	240 22 57.3	60 23 07.5
5		G	242 37 22.3	62 37 28.6
6		N	281 18 18.6	101 18 29.8
7		O	293 09 56.1	113 10 08.3
8		Q	86 47 37.7	266 47 48.0
9		91	91 23 21.7	271 23 29.1
10		94	91 44 01.4	271 44 13.5
11		95	93 21 28.0	273 21 36.4
12		99	94 39 01.7	274 39 09.8
13		115	94 40 33.9	274 40 42.0
14		94A	95 19 19.6	275 19 29.5
15		101	96 09 24.1	276 09 33.4
16		116	96 15 49.4	276 15 54.6
17		90	96 45 07.4	276 45 17.3
18		95A	97 11 35.0	277 11 43.9
19		117	98 04 51.7	278 04 58.7
20		137	98 54 15.3	278 54 22.3
21		126	98 58 19.8	278 58 29 8
22		118	99 16 38.3	279 16 46.7
23		127	99 21 19.4	279 21 29.1
24		136	99 40 46.9	279 40 56.6
25		130	99 58 14.5	279 58 22.2
26		147	100 01 25.7	280 01 35.6
27		11A	100 31 27.5	280 31 33.2
28		135	100 37 53.7	280 38 01.7
29		148	100 40 20.0	280 40 28.1
30		131	100 41 16.9	280 41 26.2
31		132	101 30 55.6	281 31 03.1
32		149	101 53 50.0	281 54 01.8
33		13A	102 08 38.2	282 08 47.6
34		119	102 20 37.8	282 20 47.4
35		133	103 15 21.5	283 15 32.4
36		151	104 13 57.2	284 14 09.0
37		152	105 08 08.5	293 08 18.6
38		D	117 23 01.9	297 23 12.2
39		C	145 16 01.8	325 16 09.0
40		B	174 59 20.3	354 59 28.3
41		PO	182 52 58.8	2 53 06.8
42		POL	186 54 44.3	6 54 54.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_A1,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	A1	POL	24 51 46.6	204 52 08.0
2		F	88 59 13.3	268 59 34.1
3		G	180 21 19.3	0 21 44.5
4		R2	210 29 55.9	30 30 15.0
5		O	216 49 46.4	36 50 07.4
6		97	217 26 17.7	37 27 07.4
7		98	220 40 19.6	40 40 37.5
8		103	221 25 09.9	41 25 28.6
9		L	222 38 33.1	42 38 55.4
10		108	223 21 56.3	43 22 15.0
11		110	225 43 14.8	45 43 34.1
12		101	227 51 32.9	47 51 54.2
13		106	228 21 59.4	48 22 19.8
14		111	229 46 22.9	49 46 45.6
15		102	230 37 38.6	50 37 59.4
16		107	232 17 05.8	52 17 26.4
17		1	233 07 32.0	53 07 52.1
18		112	233 40 51.8	53 41 14.1
19		M	234 46 38.7	54 46 59.7
20		10A	236 11 51.8	56 12 12.5
21		2	237 16 20.0	57 16 40.5
22		8	238 43 39.5	58 43 59.5
23		13	240 23 06.7	60 23 29.1
24		157	240 45 57.4	60 46 18.0
25		3	240 46 52.7	60 47 09.1
26		9	242 42 20.4	62 42 41.6
27		14	244 06 20.2	64 06 40.1
28		156	244 40 57.4	64 41 16.0
29		4	246 11 56.5	66 12 18.0
30		N	247 36 55.4	67 37 13.7
31		10	247 38 48.5	67 39 07.3
32		5	250 02 24.1	70 02 43.3
33		15	250 18 51.0	70 19 09.8
34		155	250 45 11.8	70 45 30.6
35		E	273 06 06.9	93 06 29.8
36		Q	288 24 50.[illegible]	108 25 08.6
37		D	311 33 29.9	131 33 49.0
38		C	328 40 30.2	148 40 50.6
39		B	335 29 36.8	155 30 00.0
40		PO	18 25 10.4	198 25 37.0
41		POL	24 51 46.4	204 52 07.3

ARC NO. 2

No	St point	Obs point	CL	CR
1	A1	POL	69 54 20.4	249 54 40.0
2		F	134 01 48.5	314 02 01.9
3		G	225 24 00.3	45 24 15.8
4		R2	255 32 30.4	75 32 47.3
5		O	261 52 19.2	81 52 37.8
6		97	262 29 21.5	82 29 40.0
7		98	265 42 49.7	85 43 07.1
8		103	266 27 42.9	86 28 01.7
9		L	267 41 06.3	87 41 30.2
10		108	268 24 28.7	88 24 43.3
11		110	270 45 48.6	90 46 06.0
12		101	272 54 07.0	92 54 24.5
13		106	273 24 34.3	93 24 52.8
14		111	274 48 59.8	94 49 17.3
15		102	275 40 15.6	95 40 31.6
16		107	277 19 38.4	97 19 58.0
17		1	278 10 08.8	98 10 23.5
18		112	278 43 24.8	98 43 42.1
19		M	279 49 12.5	99 49 31.4
20		10A	281 14 24.7	101 14 41.4
21		2	282 18 51.7	102 19 10.7
22		8	283 46 10.5	103 46 32.5
23		13	285 25 40.5	105 25 57.6
24		157	285 48 27.3	105 48 48.2
25		3	285 49 25.0	105 49 42.3
26		9	287 44 51.4	107 45 11.7
27		14	289 08 51 8	109 09 11.4
28		156	289 43 27.5	109 43 48.5
29		4	291 14 29.4	111 14 49.0
30		N	292 39 24.4	112 39 45.4
31		10	292 41 13.6	112 41 37.4
32		5	295 04 55.4	115 05 10.1
33		15	295 21 24.1	115 21 40.8
34		155	295 47 42.7	115 48 00.5
35		E	318 08 41.4	138 08 57.7
36		Q	333 27 24.4	153 27 40.4
37		D	356 36 00.8	176 36 21.4
38		C	13 43 00.3	193 43 21.3
39		B	20 32 11.5	200 32 25.7
40		PO	63 27 49.6	243 28 08.0
41		POL	69 54 21.4	249 54 39.4

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REDUCTION OF THEODOLITE OBSERVATIONS
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GENERAL INFORMATION

Date : 8 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_A2,_B6
** The output of all the observations is in degrees, minutes, seconds

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ECHO

Set No. 1

No	St point	Obs point	CL	CR
1	A2	POL	114 56 54.1	294 57 07.1
2		F	179 04 14.9	359 04 30.2
3		G	270 26 29.1	90 26 44.7
4		R2	300 35 00.0	120 35 16.9
5		O	306 54 49.7	126 55 07.6
6		L	312 43 37.3	132 43 55.2
7		M	324 51 41.9	144 52 00.0
8		N	337 41 54.7	157 42 [illegible]
9		19	356 44 29.3	176 44 47.4
10		21	357 48 10.7	177 48 26.8
11		22	358 02 36.7	178 02 51.2
12		23	0 [illegible] 44.5	180 37 59.2
13		29	1 50 08.3	181 50 20.4
14		E	3 11 09.5	183 11 26.7
15		25	3 16 24.4	183 16 40.8
16		26	4 27 28.6	184 27 42.7
17		27	6 37 11.2	186 37 25.4
18		28	8 20 24.8	188 20 40.8
19		29A	10 03 30.7	190 03 47.3
20		34	10 41 55.5	190 42 11.0
21		30	11 03 21.5	191 03 40.7
22		33	12 22 02.7	192 [illegible] 17.6
23		32	12 26 22.0	192 26 39.6
24		39	14 41 25.2	194 41 40.1
25		36	14 58 07.6	194 58 22.5
26		Q	18 29 54.4	198 30 08.1
27		D	41 38 30.6	221 38 46.1
28		T	47 21 30.5	227 21 31.4
29		U	49 38 05.8	229 38 20.4
30		C	58 45 33.3	238 45 48.9
31		B	65 34 38.0	245 34 56.4
32		PO	108 30 20.8	288 30 34.2
33		POL	114 56 52.4	294 57 08.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	A2	POL	159 58 17.4	339 58 27.2
2		F	224 05 40.3	44 05 52.6
3		G	315 27 56.4	135 28 ,0.0
4		R2	345 36 26.4	165 36 38.4
5		O	351 56 15.4	171 56 31.3
6		L	357 45 04.9	177 45 15.3
7		M	9 53 11.1	189 53 20.1
8		N	22 43 23.3	202 43 34.0
9		19	41 45 56.1	221 46 09.6
10		21	42 49 37.8	222 49 48.1
11		22	43 04 04.1	223 04 12.2
12		23	45 39 09.0	225 39 21.4
13		29	46 51 28.5	226 51 45.4
14		E	48 12 35.6	228 12 47.3
15		25	48 17 49.8	228 18 03.9
16		26	49 28 52.2	229 29 05.2
17		27	51 38 34.7	231 38 47.9
18		28	53 21 51.8	233 22 02.6
19		29A	55 04 58.9	235 05 11.4
20		34	55 43 21.9	235 43 35.6
21		30	56 04 48.4	236 05 02.8
22		31	57 23 28.8	237 23 38.6
23		32	57 27 46.0	237 28 00.7
24		39	59 42 52.6	239 43 04.3
25		36	59 59 34.4	239 59 50.0
26		Q	63 31 18.4	243 31 32.9
27		D	86 39 59.5	266 40 11.2
28		T	92 23 01.7	272 23 11.8
29		U	94 39 33.4	274 39 44.3
30		C	103 46 59.[illegible]	283 47 13.7
31		B	110 36 05.4	290 36 20.6
32		PO	153 31 43.8	333 31 55.4
33		POL	159 58 18.4	339 58 27.5

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

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Date                 :  8 January 1980      Instrument :  KERN DKM3 #169608
Observer             :  I.B.W
Booker                  I.B.W
Weather Conditions :
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* Code numbers to create/have access to these data: F,_A3,_B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	A3	POL	24 51 47.8	204 51 56.6
2		F	88 59 12.5	268 59 21.0
3		G	180 21 28.5	0 21 36.6
4		R2	210 29 53.6	30 30 04.9
5		O	216 49 44.9	36 49 57.7
6		L	222 38 34.4	42 38 43.3
7		M	234 46 40.8	54 46 49.0
8		N	247 36 54.1	67 37 03.4
9		E	273 06 06.1	93 06 18.9
10		Q	288 24 49.5	108 24 58.7
11		D	311 33 29.6	131 33 41.8
12		T	317 16 30.5	137 16 40.5
13		U	319 33 01.9	139 33 12.0
14		C	328 40 30.5	148 40 40.8
15		B	335 29 36.0	155 29 48.1
16		PO	18 25 15.5	198 25 24.0
17		POL	24 51 48.0	204 51 56.6

ARC NO. 2

No	St point	Obs point	CL	CR
1	A3	POL	114 54 34.0	294 54 39.4
2		F	179 01 55.9	359 02 02.5
3		G	270 24 10.8	90 24 19.6
4		R2	300 32 39.4	120 32 49.3
5		O	306 52 28.8	126 52 41.5
6		L	312 41 19.2	132 41 27.8
7		M	324 49 25.3	144 49 33.3
8		N	337 39 37.4	157 39 45.5
9		E	3 08 50.2	183 09 00.4
10		Q	18 27 31.8	198 27 41.9
11		D	41 36 12.8	221 36 22.8
12		T	47 19 13.7	227 19 23.3
13		U	49 35 43.2	229 35 54.6
14		C	58 43 14.2	238 43 24.5
15		B	65 32 20.6	245 32 30.2
16		PO	108 27 59.1	288 28 06.3
17		POL	114 54 33.0	294 54 39.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 8 January 1980   Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Wether Conditions :

* Code numbers to create/have access to these data: F, _B1, _B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
	B1	F	119 35 17.8	299 35 22.0
2		A	155 29 44.1	335 29 51.7
3		N	198 49 57.0	18 50 06.9
4		E	219 40 05.8	39 40 13.1
5		P	222 12 25.6	42 12 31.9
6		Q	254 38 47.4	74 38 54.6
7		T	280 00 16.6	100 00 22.7
8		D	292 04 31.6	112 04 40.9
9		C	318 58 11.4	138 58 18.8
10		PO	55 13 24.1	235 13 32.3
11		POL	62 03 11.7	242 03 19.7
12		F	119 35 16.6	299 35 21.8

ARC NO. 2

No	St point	Obs point	CL	CR
1	B1	F	164 36 51.7	344 36 57.8
2		A	200 31 21.5	20 31 28.9
3		N	243 51 35.7	63 51 43.4
4		E	264 41 44.1	84 41 50.1
5		P	267 14 00.8	87 14 08.4
6		Q	299 40 24.6	119 40 31.4
7		T	325 01 54.3	145 01 58.4
8		D	337 06 08.9	157 06 15.0
9		C	3 59 47.6	183 59 54.1
10		PO	100 14 59.5	280 15 07.4
11		POL	107 04 50.4	287 04 55.1
12		F	164 36 50.8	344 36 58.2

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 9 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_B2,_B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	B2	F	119 35 07.7	299 35 29.9
2		A	155 29 37.2	335 29 5[illegible].3
3		N	198 49 50.2	18 50 13.6
4		21	206 42 04.0	26 42 24.8
5		19	207 24 23.3	27 24 44.8
6		22	209 37 45.3	29 38 08.1
7		23	210 12 16.8	30 12 41.4
8		27	213 44 07.3	33 44 26.3
9		26	214 26 32.5	34 26 54.4
10		25	214 55 12.6	34 55 33.3
11		29	215 56 45.1	35 57 07.5
12		23A	216 59 24.8	36 59 47.1
13		29A	218 04 23.4	38 04 43.5
14		E	219 40 01.8	39 40 21.8
15		30	220 50 12.3	40 50 32.5
16		P	222 12 15.3	42 12 34.4
17		37	224 45 26.5	44 45 44.5
18		34	226 03 24.1	46 03 42.4
19		33	226 25 21.0	46 25 41.8
20		36	229 20 09.2	49 20 27.0
21		30A	231 33 09.1	51 33 33.4
22		40	232 57 11.4	52 57 32.1
23		39	233 20 08.0	53 20 26.8
24		Q	254 38 40.2	74 39 01.4
25		T	280 00 10.5	100 00 33.5
26		D	292 04 24.6	112 04 46.4
27		C	318 58 04.2	138 58 25.5
28		PO	55 13 15.9	235 13 37.6
29		POL	62 03 04.4	242 03 26.7
30		F	119 35 07.9	299 35 29.6

ARC NO. 2

No	St point	Obs point	CL	CR
1	B2	F	164 36 41.5	344 37 01.8
2		A	200 31 08.3	20 31 30.9
3		N	243 51 25.6	63 51 46.2
4		21	251 43 38.7	71 43 58.3
5		19	252 25 57.7	72 26 14.5
6		22	254 39 18.6	74 39 41.9
7		23	255 13 53.7	75 14 12.5
8		27	258 45 41.2	78 46 03.5
9		26	259 28 06.3	79 28 28.5
10		25	259 56 46.7	79 57 09.8
11		29	260 58 16.0	80 58 40.5
12		23A	262 01 02.0	82 01 21.4
13		29A	263 05 56.4	83 06 15.5
14		E	264 41 32.0	84 41 51.8
15		30	265 51 45.7	85 52 03.3
16		P	267 13 49.5	87 14 11.3
17		32	269 47 01.7	89 47 20.4
18		34	271 04 56.7	91 05 20.1
19		33	271 26 55.1	91 27 16.4
20		36	274 21 42.6	94 22 00.3
21		30A	276 34 47.3	96 35 09.9
22		40	277 58 47.5	97 59 05.7
23		39	278 21 41.9	98 22 02.2
24		Q	299 40 16.4	119 40 33.9
25		T	325 01 44.2	145 02 01.8
26		D	337 05 59.3	157 06 19.5
27		C	3 59 35.6	183 59 58.7
28		PO	100 14 50.2	280 15 11.4
29		POL	107 04 40.4	287 04 55.0
30		F	164 36 41.0	344 37 02.4

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REDUCTION OF THEODOLITE OBSERVATIONS

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GENERAL INFORMATION

Date : 9 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_B3,_B6
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR	MEAN
1	B3	F	209 59 24.0	29 59 40.5	209 59 32.3
2		A	245 53 52.7	65 54 11.7	245 54 02.2
3		N	289 14 09.4	109 14 26.4	289 14 17.9
4		E	310 04 17.9	130 04 33.8	310 04 25.9
5		P	312 36 32.8	132 36 55.7	312 36 44.3
6		41	323 27 46.1	143 28 01.8	323 27 54.0
7		42	324 32 35.1	144 32 54.4	324 32 44.8
8		44	324 40 44.6	144 40 59.8	324 40 52.2
9		45	327 05 35.4	147 05 54.8	327 05 45.1
10		51	327 13 37.6	147 13 54.6	327 13 46.1
11		30	327 45 13.3	147 45 31.4	327 45 22.4
12		47	328 13 51.8	148 14 05.7	328 13 58.8
13		58	329 05 44.0	149 06 01.0	329 05 52.5
14		59	329 36 16.9	149 36 31.4	329 36 24.2
15		54	330 13 14.8	150 13 31.4	330 13 23.1
16		53	330 18 39.8	150 18 58.7	330 18 49.3
17		52	330 58 25.2	150 58 42.4	330 58 33.8
18		55	331 40 17.6	151 40 33.1	331 40 25.4
19		56	332 05 05.0	152 05 21.5	332 05 13.3
20		62	332 46 52.5	152 47 10.1	332 47 01.3
21		63	333 26 48.1	153 27 05.5	333 26 56.8
22		64	335 46 53.8	155 47 10.3	335 47 02.1
23		67	336 14 24.5	156 14 42.3	336 14 33.4
24		69	336 52 36.2	156 52 54.1	336 52 45.2
25		63A	338 22 10.5	158 22 26.5	338 22 18.5
26		72	340 05 48.6	160 06 05.8	340 05 57.2
27		95	340 24 23.2	160 24 42.4	340 24 32.8
28		73	342 36 51.1	162 37 07.5	342 36 59.3
29		74	343 20 49.0	163 21 05.0	343 20 57.0
30		77	344 20 26.3	164 20 43.4	344 20 34.9
31		Q	345 02 58.6	165 03 12.8	345 03 05.7
32		T14	345 18 13.2	165 18 26.8	345 18 20.0
33		81	345 35 05.0	165 35 21.1	345 35 13.1
34		70	345 37 34.8	165 37 51.0	345 37 42.9
35		T13	346 30 29.8	166 30 45.9	346 30 37.9
36		T	10 24 22.8	190 24 43.5	10 24 33.2
37		D	22 28 42.2	202 28 57.5	22 28 49.9
38		C	49 22 20.6	229 22 37.3	49 22 29.0
39		PO	145 37 31.7	325 37 50.6	145 37 41.2
40		POL	152 27 20.2	332 27 38.3	152 27 29.3
41		F	209 59 24 5	29 59 41.5	209 59 33.0

ARC NO. 2

No	St point	Obs point	CL	CR
1	83	F	255 01 06.0	75 01 16.6
2		A	290 55 36.6	110 55 47.3
3		N	334 15 49.7	154 16 02.4
4		E	355 05 59.7	175 06 12.1
5		P	357 38 14.0	177 38 27.7
6		41	8 29 25.0	188 29 37.4
7		42	9 34 16.3	189 34 28.7
8		44	9 42 23.7	189 42 35.3
9		45	12 07 15.8	192 07 29.3
10		51	12 15 19.3	192 15 29.5
11		50	12 46 52.4	192 47 05.0
12		47	13 15 30.5	193 15 43.1
13		58	14 07 25.0	194 07 37.2
14		59	14 37 55.9	194 38 08.3
15		54	15 14 55.1	195 15 08.4
16		53	15 20 22.4	195 20 34.0
17		52	16 00 06.5	196 00 16.7
18		55	16 41 57.1	196 42 10.5
19		56	17 06 45.2	197 06 55.5
20		62	17 48 33.3	197 46 44.8
21		63	18 28 28.6	198 28 41.8
22		64	20 48 37.6	200 48 45.7
23		67	21 16 09.5	201 16 18.6
24		69	21 54 20.8	201 54 29.1
25		63A	23 23 53.8	203 24 04.0
26		72	25 07 31.8	205 07 40.7
27		95	25 26 08.3	205 26 17.5
28		73	27 38 33.3	207 38 43.9
29		74	28 22 29.7	208 22 40.5
30		77	29 22 06.5	209 22 21.6
31		Q	30 04 39.4	210 04 47.4
32		T14	30 19 56.4	210 20 05.9
33		81	30 36 46.2	210 36 57.1
34		70	30 39 15.4	210 39 25.5
35		T13	31 32 11.3	211 32 21.2
36		T	55 26 08.3	235 26 20.3
37		D	67 30 26.9	247 30 35.8
38		C	94 24 00.1	274 24 13.5
39		PO	190 39 14.7	10 39 26.0
40		POL	197 29 03.3	17 29 15.5
41		F	255 01 06.2	75 01 16.6

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REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 10 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F, B4, B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	B4	F	216 57 51.9	36 58 09.3
2		N	296 12 38.8	116 12 56.1
3		E	317 02 54.0	137 03 04.4
4		n	352 01 32.5	172 01 44.2
5		101	4 05 23.7	184 05 38.5
6		T5	4 16 42.4	184 16 53.9
7		143	24 19.1	184 24 33.5
8		159	4 25 56.8	184 26 11.3
9		94A	4 41 54.9	184 42 08.9
10		116	4 53 57.8	184 54 11.8
11		89	5 04 57.9	185 05 10.8
12		T4	5 12 03.4	185 12 18.8
13		144	5 47 52.8	185 48 06.1
14		166	6 56 45.0	186 56 57.4
15		158	7 02 28.6	187 02 41.9
16		136	7 02 51.2	187 03 03.2
17		145	7 27 22.3	187 27 35.4
18		95	7 45 30.6	187 45 43.7
19		T3	7 53 04.9	187 53 17.1
20		117	8 11 39.9	188 11 53.8
21		146	8 13 30.4	188 13 45.1
22		90	8 14 34.9	188 14 46.2
23		167	8 31 18.6	188 31 31.5
24		147	8 52 45.2	188 52 58.6
25		137	8 58 34.8	188 58 46.9
26		67	9 15 18.9	189 15 33.2
27		168	9 20 16.3	189 20 31.1
28		148	9 36 19.5	189 36 32.5
29		126	10 09 43.6	190 09 57.4
30		118	10 35 17.1	190 35 30.5
31		T2	10 37 58.0	190 38 10.4
32		149	10 44 50.6	190 45 05.8
33		127	10 56 24.5	190 56 38.0
34		135	10 57 49.0	190 58 05.9
35		130	11 09 14.7	191 09 30.2
36		174	11 14 07.1	191 14 25.0
37		132	11 35 05.9	191 35 20.8
38		175	11 35 10.6	191 35 21.6
39		D	29 27 11.9	209 27 29.8
40		POL	159 25 50.3	339 26 06.4
41		F	216 57 52.4	36 58 10.6

ARC NO. 2

No	St point	Obs point	CL	CR
1	B4	F	262 00 07.7	82 00 16.9
2		N	341 14 53.1	161 15 04.9
3		E	2 05 02.3	182 05 10.1
4		O	37 03 39.7	217 03 49.5
5		101	49 07 35.1	229 07 44.0
6		T5	49 18 49.3	229 19 05.7
7		143	49 26 30.5	229 26 43.4
8		159	49 28 08.5	229 28 17.1
9		94A	49 44 07.1	229 44 18.5
10		116	49 56 10.3	229 56 20.1
11		89	50 07 09.5	230 07 20.8
12		T4	50 14 15.8	230 14 28.0
13		144	50 50 03.2	230 50 14.9
14		166	[illegible] 58 54.2	231 59 06.2
15		158	[illegible] 04 39.7	232 04 49.1
16		136	52 05 00.3	232 05 12.0
17		145	52 29 32.4	232 29 43.5
18		95	52 47 40.2	232 47 50.0
19		T3	52 55 15.5	232 55 25.4
20		117	53 13 51.6	233 14 00.5
21		146	53 15 39.0	233 15 51.8
22		90	53 16 45.6	233 16 58.0
23		167	53 33 26.9	233 33 40.7
24		147	53 54 54.7	233 55 07.4
25		137	54 00 45.9	234 00 56.2
26		67	54 17 29.2	234 17 39.4
27		168	54 22 29.7	234 22 36.0
28		148	54 38 30.2	234 38 41.3
29		126	55 11 54.8	235 12 05.6
30		118	55 37 24.8	235 37 38.0
31		T2	55 40 07.6	235 40 17.7
32		149	55 47 02.8	235 47 12.7
33		127	55 58 34.8	235 58 44.5
34		135	56 00 00.0	236 00 10.6
35		130	56 11 26.6	236 11 37.9
36		174	56 16 21.5	236 16 27.3
37		132	56 37 18.8	236 37 27.0
38		175	56 37 22.9	236 37 27.8
39		D	74 29 24.8	254 29 36.5
40		POL	204 27 59.4	24 28 12.8
41		F	262 00 07.4	82 00 17.7

ARC NO. 2

No	St point	Obs point	CL	CR
1	B4	F	262 00 07.7	82 00 16.9
2		N	341 14 53.1	161 15 04.9
3		E	2 05 02.3	182 05 10.1
4		O	37 03 39.7	217 03 49.5
5		101	49 07 35.1	229 07 44.0
6		T5	49 18 49.3	229 19 05.7
7		143	49 26 30.5	229 26 43.4
8		159	49 28 08.5	229 28 17.1
9		94A	49 44 0[illegible].1	229 44 18.5
10		116	49 56 10.3	229 56 20.1
11		89	50 07 09.5	230 07 20.8
12		T4	50 14 15.8	230 14 28.0
13		144	50 50 03.2	230 50 14.9
14		166	51 58 54.2	231 59 06.7
15		158	52 04 39.7	232 04 49.1
16		136	52 05 00.3	232 05 12.0
17		145	52 29 32.4	232 29 43.5
18		95	52 47 40.2	232 47 50.0
19		T3	52 55 15.5	232 55 25.4
20		117	53 13 51.6	233 14 00.5
21		146	53 15 3[illegible].0	233 15 51.8
22		90	53 16 45.6	233 16 58.0
23		167	53 33 26.9	233 33 40.7
24		147	53 54 54.7	233 55 07.4
25		137	54 00 45.9	234 00 56.2
26		67	54 17 29.2	234 17 39.4
27		168	54 22 29.7	234 22 36.0
28		148	54 38 30.2	234 38 41.3
29		12[illegible]	55 11 54.8	235 12 05.6
30		118	55 37 24.8	235 37 38.0
31		T2	55 40 07.6	235 40 17.7
32		149	55 47 02.8	235 47 12.7
33		127	55 58 34.8	235 58 44.5
34		135	56 00 00.0	236 00 10.6
35		130	56 11 26.6	236 11 37.9
36		174	56 16 21.5	236 16 27.3
37		132	56 37 18.8	236 37 27.0
30		175	56 37 22.9	236 37 27.8
39		D	74 29 24.8	254 29 36.5
40		POL	204 27 59.4	24 28 12.8
41		F	262 00 07.4	82 00 17.7

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REDUCTION OF THEODOLITE OBSERVATIONS
****

GENERAL INFORMATION

Date : 13 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F, _B5, _B6
** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	B5	F	177 07 12.2	357 07 30.2
2		A	213 01 44.8	33 02 00.2
3		E	277 12 07.6	97 12 24.1
4		Q	312 10 45.8	132 11 06.2
5		131	331 36 37.7	151 36 54.0
6		T1	331 49 45.1	151 50 04.3
7		117	331 50 01.3	151 50 18.4
8		151	331 51 17.4	151 51 37.8
9		152	332 16 19.1	152 16 34.2
10		13A	332 25 12.1	152 25 29.6
11		133	332 39 36.5	152 39 55.6
12		119	333 08 12.7	153 08 28.0
13		D	349 36 30.6	169 36 49.6
14		C	16 30 10.9	196 30 30.0
15		PO	112 45 22.1	292 45 35.5
16		POL	119 35 07.0	299 35 23.1
17		F	177 07 13.4	357 07 30.3

ARC NO. 2

No	St point	Obs point	CL	CR
1	B3	F	266 59 55.9	87 00 12.1
2		A	302 54 29.5	122 54 44.0
3		E	7 04 47.4	187 05 03.5
4		Q	42 03 28.3	222 03 45.8
5		131	61 29 22.6	241 29 37.1
6		T1	61 42 32.4	241 42 41.8
7		117	61 42 43.6	241 42 59.0
8		151	61 44 05.3	241 44 22.8
9		152	62 09 01.6	242 09 17.0
10		13A	62 17 55.8	242 18 11.4
11		133	62 32 21.8	242 32 35.4
12		119	63 00 56.4	243 01 12.7
13		D	79 29 13.7	259 29 30.7
14		C	106 22 52.3	286 23 12.4
15		PO	202 38 00.6	22 38 16.1
16		POL	209 27 48.4	29 28 03.2
17		F	266 59 54.5	87 00 12.0

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 11 January 1980 Instrument : KERN DKM2 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F, C1, 86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	C1	F	124 59 50.1	305 00 07.0
2		B	138 57 59.1	318 58 20.5
3		A	148 40 30.1	328 40 49.2
4		K	159 26 31.9	339 26 50.6
5		X	163 52 45.8	343 53 03.3
6		P	169 53 52.8	349 54 15.5
7		V	174 11 34.1	354 11 54.4
8		N	178 55 46.0	358 56 04.4
9		E	189 56 39.1	9 56 57.5
10		Q	222 33 50.7	42 34 08.3
11		D	263 38 47.5	83 39 03.4
12		PO	85 11 22.7	265 11 43.4
13		POL	87 54 24.4	267 54 44.8
14		F	124 59 50.3	305 00 09.9

ARC NO. 2

No	St point	Obs point	CL	CR
1	C1	F	170 01 20.7	350 01 37.2
2		B	183 59 27.3	3 59 46.1
3		A	193 41 52.6	13 42 13.6
4		K	204 27 57.0	24 28 16.7
5		X	208 54 12.7	28 54 28.0
6		P	214 55 19.8	34 55 39.1
7		V	219 13 00.0	39 13 20.3
8		N	223 57 12.0	43 57 28.6
9		E	234 58 06.6	54 58 22.5
10		Q	267 35 19.2	87 35 35.8
11		D	308 40 12.5	128 40 31.7
12		PO	130 12 48.1	310 13 08.8
13		POL	132 55 50.7	312 56 08.4
14		F	170 01 19.8	350 01 37.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 12 January 1980 Instrument : KERN DKM2 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F, C2, 86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	C2	F	119 35 09.0	299 35 24.7
2		B	133 33 19.6	313 33 36.0
3		A	143 15 45.4	323 16 05.3
4		N	173 31 05.6	353 31 16.9
5		E	184 31 58.0	4 32 15.0
6		41	189 15 06.0	9 15 18.1
7		44	190 16 44.0	10 17 06.1
8		42	190 32 24.7	10 32 40.4
9		45	192 28 40.6	12 28 59.5
10		47	193 39 03.8	13 39 18.7
11		51	193 50 22.6	13 50 41.8
12		50	194 04 42.1	14 04 59.3
13		52	196 14 52.7	16 15 07.2
14		58	196 24 25.2	16 24 40.7
15		53	196 31 29.5	16 31 45.4
16		59	196 58 17.3	16 58 29.6
17		54	197 07 46.1	17 08 01.6
18		56	197 35 08.0	17 35 18.6
19		55	197 38 37.2	17 38 52.8
20		62	200 03 01.8	20 03 17.8
21		63	200 20 29.0	20 20 45.8
22		64	203 25 03.1	23 25 16.8
23		69	204 56 59.7	24 57 14.5
24		67	204 59 28.9	24 59 47.6
25		63A	205 29 09.5	25 29 24.8
26		95	207 49 10.3	27 49 24.8
27		72	208 45 00.8	28 45 18.1
28		73	211 27 48.9	31 28 08.3
29		T14	212 43 04.5	32 43 23.9
30		74	212 48 57.0	32 49 14.9
31		137	213 32 27.7	33 32 41.4
32		77	214 09 16.5	34 09 34.1
33		70	215 58 37.8	35 58 49.4
34		138	216 32 15.8	36 32 34.0
35		81	216 40 40.0	36 40 57.0
36		Q	217 09 08.3	37 09 26.3
37		D	258 14 04.4	78 14 19.9
38		PO	79 46 39.6	259 46 58.0
39		POL	82 29 42.4	262 29 57.3
40		F	119 35 09.7	299 35 23.8

ARC NO. 2

No	St point	Obs point	CL	CR
1	C2	F	164 46 24.5	344 46 37.0
2		B	178 44 34.8	358 44 44.9
3		A	188 27 01.8	8 27 15.5
4		N	218 42 20.0	38 42 31.7
5		E	229 43 11.2	49 43 23.3
6		41	234 26 18.1	54 26 32.0
7		44	235 28 04.6	55 28 18.9
8		42	235 43 41.3	55 43 50.8
9		45	237 39 58.4	57 40 14.0
10		47	238 50 19.5	58 50 32.7
11		51	239 01 42.2	59 01 54.5
12		50	239 16 01.5	59 16 12.6
13		52	241 26 09.6	61 26 19.7
14		58	241 35 39.1	61 35 52.6
15		53	241 42 48.2	61 42 59.8
16		59	242 09 32.4	62 09 45.0
17		54	242 19 03.1	62 19 14.6
18		56	242 46 18.5	62 46 32.1
19		55	242 49 52.3	62 50 09.4
20		62	245 14 20.4	65 14 32.0
21		63	245 31 42.1	65 31 55.1
22		64	248 36 15.8	68 36 28.4
23		69	250 08 15.3	70 08 29.7
24		67	250 10 47.6	70 11 00.8
25		63A	250 40 22.1	70 40 40.1
26		95	253 00 25.6	73 00 38.4
27		72	253 56 17.4	73 56 31.7
28		73	256 39 05.7	76 39 18.1
29		T14	257 54 22.1	77 54 36.4
30		74	258 00 11.2	78 00 24.6
31		137	258 43 42.4	78 43 56.2
32		77	259 20 31.2	79 20 44.3
33		70	261 09 47.4	81 10 03.2
34		138	261 43 41.0	81 43 46.5
35		81	261 51 54.7	81 52 08.5
36		D	262 20 24.0	82 20 37.1
37		D	303 25 18.1	123 25 32.5
38		[illegible]O	124 57 54.8	304 58 08.6
39		FOL	127 40 57.0	307 41 10.0
40		F	164 46 24.8	344 46 38.7

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REDUCTION OF THEODOLITE OBSERVATIONS
****

GENERAL INFORMATION

Date : 12 January 19C0 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_C3,_86
** The output of all the observations is in degrees, minutes, seconds

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ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	C3	F	254 48 58.8	74 49 07.0
2		B	268 47 13.2	88 47 18.1
3		A	278 29 39.2	98 29 49.5
4		N	308 44 51.7	128 45 03.5
5		E	319 45 48.8	139 45 57.6
6		Q	352 22 57.7	172 23 10.0
7		164	353 06 31.5	173 06 38.3
8		83	353 08 03.3	173 08 12.7
9		139	354 11 26.4	174 11 28.6
10		123	354 16 55.2	174 17 02.1
11		84	355 00 40.8	175 00 50.2
12		124	355 28 16.7	175 28 28.6
13		163	355 39 24.9	175 39 35.8
14		118	357 03 23.8	177 03 35.1
15		83A	357 11 46.6	177 11 55.1
16		86	357 26 45.1	177 26 56.7
17		162	357 56 55.7	177 57 00.8
18		119	359 26 18.4	179 26 23.0
19		140	359 46 38.4	179 46 45.1
20		161	359 54 45.1	179 54 54.2
21		94	0 16 44.7	180 16 54.4
22		114	0 22 50.6	180 22 57.7
23		91	1 02 19.6	181 02 29.8
24		141	1 04 41.2	181 04 50.6
25		160	1 18 12.8	181 18 24.1
26		142	2 36 36.8	182 36 45.8
27		130	2 40 05.6	182 40 11.6
28		95	3 16 06.8	183 16 16.2
29		D	33 27 53.6	213 28 02.1
30		PO	215 00 32.4	35 00 38.7
31		POL	217 43 28.5	37 43 37.2
32		F	254 48 58.7	74 49 06.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	C3	F	299 50 20.3	119 50 24.5
2		B	313 48 27.5	133 48 35.1
3		A	323 30 58.4	143 31 05.1
4		N	353 46 15.0	173 46 20.6
5		E	4 47 08.6	184 47 13.7
6		Q	37 24 16.8	217 24 24.6
7		164	38 07 49.9	218 07 54.1
8		83	38 09 22.2	218 09 29.2
9		139	39 12 41.7	219 12 45.0
10		123	39 18 10.6	219 18 20.1
11		84	40 01 57.2	220 02 07.1
12		124	40 29 35.7	220 29 44.9
13		163	40 40 45.7	220 40 53.4
14		118	42 04 46.1	222 04 54.5
15		83A	42 13 04.6	222 13 12.8
16		86	42 28 16.1	222 28 11.7
17		162	42 58 13.4	222 58 19.4
18		119	44 27 36.0	224 27 44.9
19		140	44 47 55.4	224 48 00.6
20		161	44 56 04.5	224 56 11.5
21		94	45 18 06.4	225 18 13.3
22		114	45 24 08.5	225 24 14.4
23		91	46 03 44.6	226 03 50.0
24		141	46 06 03.2	226 06 11.2
25		160	46 19 36.0	226 19 39.1
26		142	47 37 53.1	227 38 04.3
27		130	47 41 23.7	227 41 26.9
28		95	48 17 23.5	228 17 28.9
29		D	78 29 11.8	258 29 19.4
30		PO	260 01 47.6	80 01 58.5
31		POL	262 44 50.5	82 45 00.4
32		F	299 50 19.2	119 50 24.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 10 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create/have access to these data: F,_D1,_86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D1	POL	86 38 10.1	266 38 20.6
2		B	112 04 25.5	292 04 33.3
3		F	116 33 34.5	296 33 42.3
4		T	120 42 10.1	300 42 12.5
5		V	121 51 20.6	301 51 26.3
6		W	127 19 20.0	307 19 27.3
7		A	131 33 33.9	311 33 39.7
8		Y	133 21 43.0	313 21 48.8
9		P	134 08 57.3	314 09 07.7
10		K	136 17 16.5	316 17 22.1
11		X	137 52 12.5	317 52 20.5
12		N	157 50 19.9	337 50 27.0
13		E	162 03 43.6	342 03 52.8
14		Q	190 04 48.8	10 04 54.8
15		C	83 38 48.8	263 38 57.9
16		POL	84 41 34.6	264 41 40.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	D1	POL	86 38 11.7	266 38 21.5
2		B	131 40 07.5	311 40 14.1
3		F	157 06 20.3	337 06 28.6
4		T	161 35 28.5	341 35 34.6
5		V	165 43 59.1	345 44 08.3
6		W	166 53 14.0	346 53 19.6
7		A	172 21 14.5	352 21 18.1
8		Y	176 35 29.1	356 35 34.2
9		P	178 23 35.6	358 23 43.3
10		K	179 10 53.9	359 10 58.8
11		X	181 19 06.1	1 19 13.5
12		N	182 54 05.8	2 54 13.0
13		E	202 52 11.6	22 52 20.0
14		Q	207 05 39.5	27 05 44.9
15		C	235 06 41.6	55 06 46.7
16		POL	128 40 45.3	308 40 50.2

ARC NO. 3

No	St point	Obs point	CL	CR
1	D1	POL	129 43 26.9	309 43 32.7
2		B	131 40 08.0	311 40 14.1
3		F	176 43 12.8	356 43 26.8
4		T	202 09 25.8	22 09 40.2
5		V	206 38 34.5	26 38 47.3
6		W	210 47 00.8	30 47 16.4
7		A	211 56 18.4	31 56 31.5
8		Y	217 24 15.1	37 24 31.8
9		P	221 38 32.8	41 38 45.0
10		K	223 26 37.4	43 26 55.4
11		X	224 13 58.0	44 14 11.8
12		N	226 22 12.9	46 22 24.5
13		E	227 57 10.8	47 57 24.7
14		Q	247 55 14.8	67 55 32.2
15		C	252 08 44.5	72 08 57.7
16		POL	280 09 46.7	100 10 00.6

ARC NO. 4

No	St point	Obs point	CL	CR
1	D_1	POL	173 43 53.5	353 44 04.6
2		B	174 46 35.2	354 46 44.5
3		F	176 43 13.4	356 43 26.3
4		T	131 41 [illegible]1.5	311 41 53.7
5		V	157 07 [illegible]1.4	337 08 04.2
6		W	161 37 00.8	341 37 13.6
7		A	165 45 27.5	345 45 44.5
8		Y	166 54 43.8	346 54 57.3
9		P	172 22 44.9	352 22 55.8
10		K	176 36 59.1	356 37 12.1
11		X	178 25 10.0	358 25 21.8
12		N	179 12 22.8	359 12 37.7
13		E	181 20 38.3	1 20 51.0
14		Q	182 55 36.0	2 55 49.1
15		C	202 53 41.0	22 53 56.8
16		POL	207 07 11.6	27 07 23.1

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 11 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I B.W
Weather Conditions :

* Code numbers to create/have access to these data: F, _D2, _B6
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D2	POL	356 41 30.0	176 41 26.3
2		B	22 07 44.9	202 07 38.1
3		F	26 36 53.5	206 36 49.6
4		A	41 36 51.1	221 36 46.5
5		N	67 53 35.8	247 53 30.1
6		E	72 07 03.5	252 06 57.4
7		T11	93 39 04.4	273 38 54.0
8		T10	94 59 20.6	274 59 09.9
9		164	95 40 32.0	275 40 23.8
10		T9	97 27 54.8	277 27 47.5
11		139	97 56 41.6	277 56 33.3
12		B3	97 58 46.6	277 58 36.9
13		B1	98 04 53.6	278 04 45.5
14		163	98 48 20.2	278 48 10.9
15		B2	98 55 05.3	278 54 57.4
16		123	98 59 36.1	278 59 28.0
17		176	99 00 28.9	279 00 20.5
18		T8	99 49 52.4	279 49 45.1
19		D	100 08 08.4	280 07 59.8
20		124	100 28 43.3	280 28 34.7
21		B4	100 29 32.5	280 29 26.4
22		171	100 56 40.1	280 56 31.8
23		162	101 10 50.4	281 10 41.8
24		T7	101 46 48.1	281 46 40.6
25		118	102 41 20.4	282 41 09.6
26		172	103 13 16.9	283 13 07.8
27		T6	103 27 47.3	283 27 39.4
28		161	103 37 27.9	283 37 22.0
29		B3A	103 44 06.4	283 43 58.5
30		173	104 33 47.1	284 33 39.6
31		B6	104 38 29.3	284 38 19.6
32		140	104 39 05.7	284 38 56.9
33		119	105 11 23.8	285 11 15.9
34		160	105 19 46.4	285 19 32.6
35		141	106 30 42.8	286 30 35.4
36		114	106 47 23.8	286 47 15.3
37		94	107 47 48.8	287 47 35.7
38		142	107 50 03.6	287 50 00.0
39		139	103 52 42.6	288 52 37.3
40		91	109 10 44.4	289 10 36.7
41		95	111 04 43.8	291 04 36.6
42		POL	356 41 31.0	176 41 26.4

ARC NO. 2

No	St point	Obs point	CL	CR
1	D2	POL	41 43 02.3	221 43 08.4
2		B	67 09 14.4	247 09 23.1
3		F	71 38 24.8	251 38 34.4
4		A	86 38 20.3	266 38 30.0
5		N	112 55 05.4	292 55 13.8
6		E	117 08 [illegible].8	297 08 41.8
7		T11	138 40 [illegible]1.6	318 40 38.6
8		T10	140 00 44.5	320 00 53.1
9		164	140 41 56.6	320 42 06.0
10		T9	142 29 18.8	322 29 27.7
11		139	142 58 09.3	322 58 17.3
12		83	143 00 15.3	323 00 23.4
13		81	143 06 19.4	323 06 29.4
14		163	143 49 49.0	323 49 52.6
15		82	143 56 33.8	323 56 42.4
16		123	144 01 05.3	324 01 08.1
17		176	144 01 53.2	324 02 03.7
18		T8	144 51 19.6	324 51 26.6
19		D	145 09 34.5	325 09 41.3
20		174	145 30 12.7	325 30 18.5
21		84	145 31 00.5	325 31 07.2
22		171	145 59 06.9	325 58 12.8
23		162	146 12 15.0	326 12 24.1
24		T7	146 48 15.8	326 48 23.4
25		118	147 42 43.8	327 42 54.2
26		172	148 14 40.2	328 14 50.7
27		T6	148 29 14.6	328 29 21.8
28		161	148 38 58.1	328 39 04.2
29		83A	148 45 36.8	328 45 42.4
30		173	149 35 14.0	329 35 21.5
31		86	149 39 54.3	329 40 01.6
32		140	149 40 33.1	329 40 40.1
33		119	150 12 52.4	330 12 56.7
34		160	150 21 12.5	330 21 19.6
35		141	151 32 08.4	331 32 16.1
36		114	151 48 48.9	331 48 57.4
37		94	152 49 14.4	332 49 21.3
38		142	152 51 31.6	332 51 37.3
39		130	153 54 09.5	333 54 20.5
40		91	154 12 08.1	334 12 17.8
41		95	156 06 12.4	336 06 19.0
42		POL	41 43 01.8	221 43 08.4

****

REDUCTION OF THEODOLITE OBSERVATIONS

****

GENERAL INFORMATION

Date : 11 January 1980 Instrument : KERN DKM3 #169608

Observer : I.B.W

Booker : I.B.W

Weather Conditions :

* Code numbers to create/have access to these data: F,_D3,_86

** The output of all the observations is in degrees, minutes, seconds

****

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D3	POL	86 38 28.8	266 38 41.1
2		B	112 04 40.3	292 04 55.5
3		F	116 33 51.6	296 34 03.5
4		A	131 33 49.5	311 33 59.8
5		N	157 50 32.5	337 50 43.0
6		E	162 04 00.4	342 04 10.8
7		Q	190 05 01.6	10 05 13.0
8		T4	195 40 01.5	15 40 10.5
9		159	197 55 51.5	17 56 00.7
10		143	199 49 54.2	19 50 05.4
11		115	200 34 22.3	20 34 27.8
12		95	201 01 38.7	21 01 51.2
13		101	201 08 54.5	21 09 06.4
14		144	201 31 31.1	21 31 42.4
15		116	203 01 14.8	23 01 26.3
16		94A	204 23 24.3	24 23 39.2
17		136	205 44 10.6	25 44 20.5
18		89	206 48 44.1	26 48 51.8
19		117	208 28 56.8	28 29 08.4
20		137	208 55 18.5	28 55 28.4
21		95A	209 14 59.3	29 15 07.5
22		67	210 52 26.6	30 52 36.1
23		C	83 39 08.4	263 39 15.1
24		PO	84 41 48.7	264 41 59.0
25		POL	86 38 29.0	266 38 41.8

ARC NO. 2

No	St point	Obs point	CL	CR
1	D3	POL	131 40 01.4	311 40 14.8
2		B	157 06 13.0	337 06 27.3
3		F	161 35 24.0	341 35 35.5
4		A	176 35 21.3	356 35 35.2
5		N	202 52 02.3	22 52 17.6
6		E	207 05 32.5	27 05 44.0
7		Q	235 06 32.5	55 06 46.0
8		T4	240 41 32.7	60 41 46.4
9		159	242 57 21.8	62 57 32.5
10		143	244 51 28.1	64 51 40.6
11		115	245 35 50.8	65 36 03.1
12		95	246 03 11.6	66 03 24.7
13		101	246 10 28.9	66 10 41.1
14		144	246 33 00.0	66 33 14.4
15		116	248 02 47.3	68 03 01.1
16		94A	249 24 58.7	69 25 13.3
17		136	250 45 45.0	70 45 57.6
18		89	251 50 17.1	71 50 28.6
19		117	253 30 28.2	73 30 43.3
20		137	253 56 53.0	73 57 01.2
21		95A	254 16 29.1	74 16 41.8
22		67	255 53 58.1	75 54 13.1
23		C	128 40 36.3	308 40 49.7
24		PO	129 43 29.9	309 43 33.2
25		POL	131 40 01.8	311 40 15.0

REDUCTION OF THEODOLITE OBSERVATIONS

GENERAL INFORMATION

Date : 12 January 1980 Instrument : KERN DKM3 #169608
Observer : I.B.W
Booker : I.B.W
Weather Conditions :

* Code numbers to create have access to these data: F, _D4, _86
** The output of all the observations is in degrees, minutes, seconds

ECHO

ARC NO. 1

No	St point	Obs point	CL	CR
1	D4	POL	86 38 12.8	266 38 22.4
2		B	112 04 27.3	292 04 33.8
3		F	116 33 37.8	296 33 43.9
4		A	131 33 36.8	311 33 40.3
5		N	157 50 19.4	337 50 26.2
6		41	159 38 00.3	339 38 04.8
7		44	160 18 37.4	340 18 42.8
8		42	160 50 34.9	340 50 38.5
9		145	161 51 59.8	341 52 03.4
10		E	162 03 47.0	342 03 53.1
11		47	162 48 43.0	342 48 46.0
12		49	163 28 19.4	343 28 25.3
13		50	163 58 03.6	343 58 08.7
14		51	164 02 22.5	344 02 26.3
15		52	164 39 56.0	344 39 58.3
16		53	165 47 39.3	345 47 46.0
17		56	165 53 43.3	345 53 51.6
18		55	166 25 11.5	346 25 14.1
19		58A	166 46 17.2	346 46 23.7
20		54	166 57 17.3	346 57 28.9
21		59	167 16 55.7	347 17 00.7
22		58	167 26 41.5	347 26 49.6
23		63	169 27 49.5	349 27 52.2
24		62	169 38 35.4	349 38 40.6
25		64	172 41 20.6	352 41 25.3
26		63A	173 37 11.4	353 37 17.2
27		69	174 24 38.0	354 24 42.8
28		67	175 16 43.8	355 16 48.8
29		95	175 42 56.3	355 43 03.2
30		70A	176 05 29.8	356 05 35.4
31		72	178 06 26.6	358 06 31.1
32		T14	179 06 50.9	359 07 00.3
33		16	180 26 45.3	0 26 48.1
34		T13	180 51 35.8	0 51 40.8
35		74	182 24 20.7	2 24 24.6
36		77	183 58 00.3	3 58 05.8
37		70	186 14 25.8	6 14 30.8
38		81	188 01 37.7	8 01 42.5
39		O	190 04 50.4	10 04 55.2
40		C	83 38 54.5	263 39 00.3
41		PO	84 41 37.1	264 41 43.0
42		POL	86 38 13.8	266 38 21.5

ARC NO. 2

No	St point	Obs point	CL	CR
1	D4	FOL	176 43 15.5	356 43 23.3
2		B	202 09 28.4	22 09 36.3
3		F	206 38 38.2	26 38 44.8
4		A	221 38 35.5	41 38 42.7
5		N	247 55 21.1	67 55 26.2
6		41	249 42 58.7	69 43 07.6
7		44	250 23 40.0	70 23 45.5
8		42	250 55 33.1	70 55 41.3
9		145	251 56 57.8	71 57 08.8
10		E	252 08 49.1	72 08 55.9
11		47	252 53 41.3	72 53 47.3
12		49	253 33 21.5	73 33 30.1
13		50	254 03 03.5	74 03 12.8
14		51	254 07 22.0	74 07 30.2
15		52	254 44 52.8	74 45 01.4
16		53	255 52 40.6	75 52 46.4
17		56	255 58 45.4	75 58 52.4
18		55	256 30 07.7	76 30 16.5
19		58A	256 51 17.1	76 51 27.3
20		54	257 02 20.8	77 02 28.1
21		59	257 21 54.6	77 22 01.6
22		58	257 31 43.4	77 31 50.9
23		63	259 32 46.6	79 32 53.2
24		62	259 43 34.8	79 43 43.8
25		64	262 46 20.4	82 46 27.8
26		63A	263 42 08.7	83 42 21.0
27		69	264 29 34.8	84 29 45.1
28		67	265 21 42.4	85 21 53.4
29		95	265 47 56.7	85 48 05.2
30		70A	266 10 28.3	86 10 34.2
31		72	268 11 27.6	88 11 36.6
32		T14	269 11 53.1	89 11 58.6
33		16	270 31 44.4	90 31 51.9
34		T13	270 56 37.6	90 56 43.6
35		74	272 29 21.6	92 29 30.4
36		77	274 03 02.6	94 03 11.0
37		70	276 19 26.2	96 19 34.4
38		81	278 06 38.1	98 06 46.4
39		Q	280 09 49.8	100 09 59.6
40		C	173 43 52.6	353 43 59.4
41		PO	174 46 36.2	354 46 46.0
42		POL	176 43 16.1	356 43 22.8

APPENDIX F

APPENDIX F. "THEO" COMPUTER PROGRAM

F.1 REQUIRED BACKGROUND

The "THEO" program runs with BASIC 2.0 and BASIC 2.1 WITH EXTENSIONS (provided that there is enough memory) or only with BASIC 2.0. Also, this has been tested on a HP-9816 micro computer using a HP-9121 double disk drive.

The "THEO" program has been written to reduce theodolite observations according to the procedure of sub-sections 3.4.1. and 3.4.1.1.

This program can reduce observations of a maximum number of 4 arcs per set and 50 observation points per arc, and was designed specifically for the data relating to this dissertation. That is, 200 pointings can be reduced at the same time.

There are four checks in this program, in order to detect mistakes during the data entry from the data sheet. However, it is the user's responsibility to check that the point used as a reference object (R.O.) occurs at the beginning and at the end of each round. The checks are the following:

1) **Check 1**: The length of all entries must be according to the specified value (not more or less than 8 characters). That is, three digits for degrees, two digits for minutes, and two digits with one decimal for seconds.
2) **Check 2**: Every observation reading must be less than 360 degrees.
3) **Check 3**: The difference $|CL-CR|<180°\pm e$; e is a user speci -

fied maximum error in seconds (e.g. 10 sec.).

4) **Check 4:** A subsequent observation entry must always increase as theodolite turns clockwise.

The output of this computer program may be directed to either the screen or to both the printer and to the screen at the same time.

The print out includes the entries of the "General Information" (observer, booker, etc.), the entries of the data (station point, observation points, CL and CR readings), the mean value between CL and CR readings, the orientation correction and the reduced mean values of the second and subsequent arcs, the final mean of each observation, the horizon closure error, the correction and the reduced observations. Thereafter, the observational residuals and the standard deviation of both an individual observation and of the mean are displayed. The format of all observations is in degrees, minutes and seconds.

F.2 DATA FILES OF THE "THEO" PROGRAM

Nine random access data files are created automatically.

To create these data files a code number which is explained on the screen must be inserted just prior to the running of this program, to serve the purpose of automatic creation of data files or, if they have been already created, to enable recalling them. The individual data files are described below and the variable names are provided for reference:

File1: This contains all the CL readings of the complete data set. These entries are stored in the string variable Nari(200)[8].

File2: This contains all the CR readings of the complete data set. These entries are stored in the string variable Nde(200)[8].

File3: The numbers of the total arcs per set and of the different observation points are included in this file (Arcnum, Arcpoints).

File4: The set of "General information" (date, observer, booker, type of instrument, weather conditions) is included in this data file, the maximum length of each specific item of information being 20 characters. They are denoted by the string variables Da[25], Observe[25], Booke[25], Instrumen[25], Weathe[25].

File5: This contains the observation point(s) name(s) with maximum length of 3 characters (Point_nam(15)[10]).

File6: This contains the station point name having a maximum length of 3 characters (St_poin[10]).

File7: The maximum permissible error between CL and CR readings in seconds, which depends on the type of theodolite, is stored in this data file (N_n).

File8: The observational residuals are contained in this data file (U(200)).

File9: The total number of observational residuals is stored as the variable "Total" in this file.

All the above data files are associated by certain paths (as defined in Hewlett Packard Basic) for communication between the screen to diskette (OUTPUT) and vice-versa (ENTER). These paths are the following:

File1 → Emp
File2 → Emp_12
File3 → Path
File4 → Odos
File5 → Qaz
File6 → Plm
File7 → Akpibeia
File8 → Res

File'+ Top

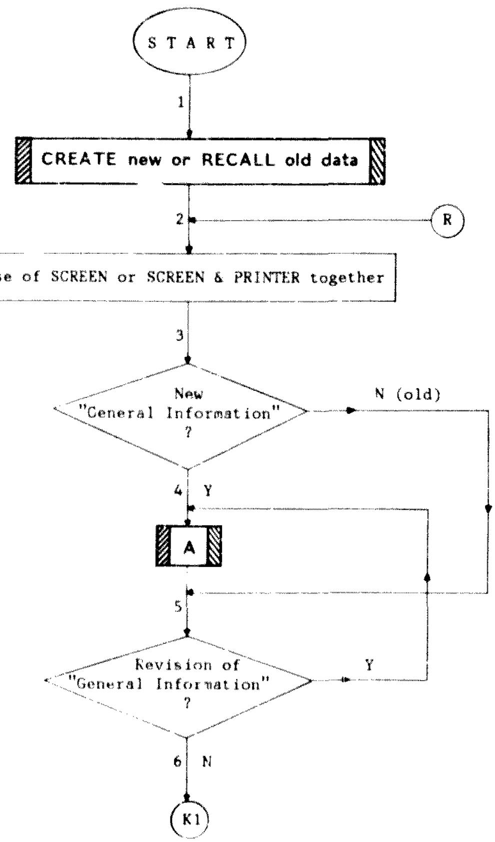
F.3 FURTHER INFORMATION

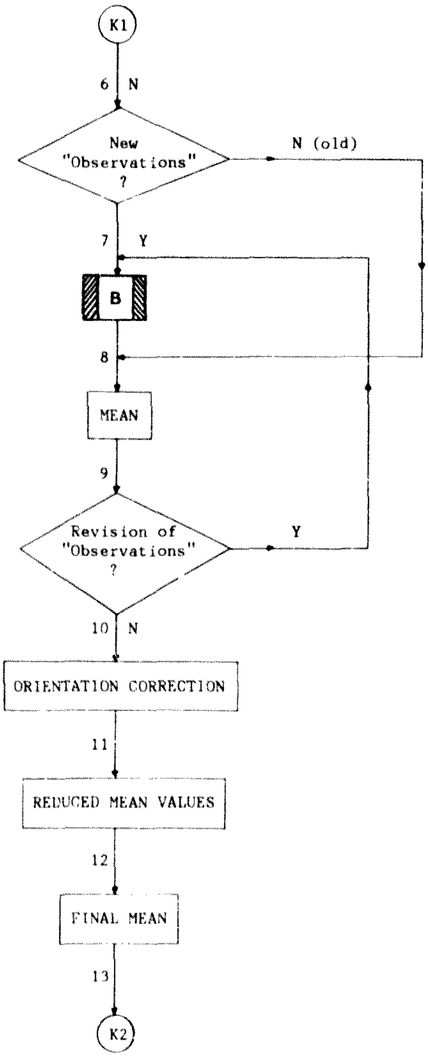
In addition to the variables described previously, the following variables are described below:

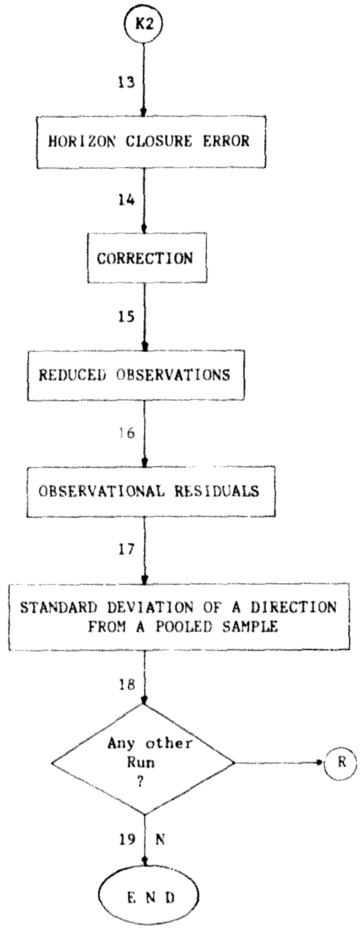
Mot(4,50)	:	It is a two-dimensional matrix for the mean value of CL and CR readings, in seconds.
A(4,50)	:	It stores results of the difference of the respective observation points between the values of the first and the subsequent arcs, in seconds.
Mesarc(4)	:	This is a one-dimensional matrix which stores the orientation correction of the second and the subsequent arcs, in seconds.
Redmo(4,50)	:	It stores the reduced mean value of the second and the subsequent arcs, in seconds.
Finmean(50)	:	It stores the final mean values for the observation points, in seconds.
Horiz_closure	:	This is required for the calculation of the horizon closure error, in seconds.
Correction	:	It stores the correction which is necessary to be applied proportionally to the final mean values, if and only if horizon closure errors (in seconds) exist.
Ro(50)	:	This is the matrix with the reduced values (in seconds), of the theodolite observations.
U(4,50)	:	This stores the observational residuals, in seconds.
Total	:	The total number of the observational residuals.
Typsfalma	:	The standard deviation of each observation.
Mes_o_sfalma	:	The standard deviation of the mean.

The flow chart of the "THEO" program and the associated program listing is given in the next sections.

F.4 FLOW CHART




K1
6
N
New "Observations" ?
N (old)
7
Y
B
8
MEAN
9
Revision of "Observations" ?
Y
10
N
ORIENTATION CORRECTION
11
REDUCED MEAN VALUES
12
FINAL MEAN
13
K2


K2
13
HORIZON CLOSURE ERROR
14
CORRECTION
15
REDUCED OBSERVATIONS
16
OBSERVATIONAL RESIDUALS
17
STANDARD DEVIATION OF A DIRECTION FROM A POOLED SAMPLE
18
Any other Run ?
R
19
N
E N D

CREATE new or RECALL old data

1

CREATE or RECALL 9 files

STORE in o RECALL from the 1^{st} file the CL readings

STORE in or RECALL from the 2^{nd} file the CR readings

STORE in or RECALL from the 3^{rd} file the number of the arcs and the number of the observed points excluding R.O.

STORE in or RECALL from the 4^{th} file the "General Information", e.g. date, observer, booker, etc.

STORE in or RECALL from the 5^{th} file the names of the observed points

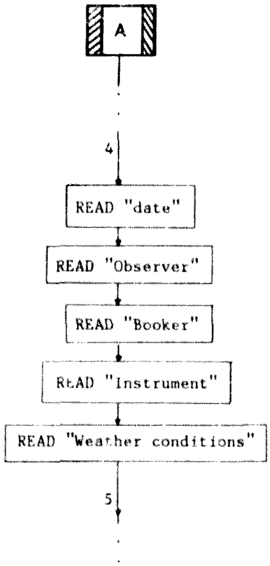
STORE in or RECALL from the 6^{th} file the name of the station point

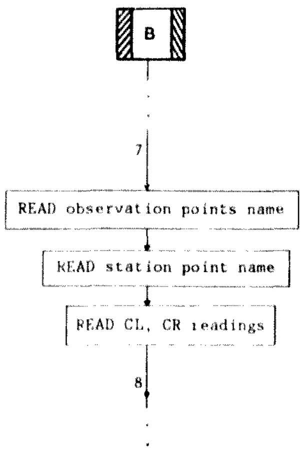
STORE in or RECALL from the th file the required accuracy (e) in seconds, so that $|CL-CR-180^\circ| \leq e$

STORE in or RECALL from the 8^{th} file the observational residuals

STORE in or RECALL from the 9^{th} file the number of the observational residuals

2


A
4
READ "date"
READ "Observer"
READ "Booker"
READ "Instrument"
READ "Weather conditions"
5


B
7
READ observation points name
READ station point name
READ CL, CR readings
8

F.5 PRINT OUT OF THE "THEO" COMPUTER PROGRAM

```
10 ! RE-STORE "THEO"
20   REM
30   REM
40   REM**************************************************************
50   REM
60   REM        PROGRAM TO REDUCE THE THEODOLITE OBSERVATIONS
70   REM           THIS COMPUTER PROGRAM HAS BEEN COMPILED
80   REM                              BY
90   REM
100  REM                      ALEXANDER  ALBANIS
110  REM
120  REM
130  REM          MSc(Eng)    UNIVERSITY OF THE WITWATERSRAND
140  REM
150  REM**************************************************************
160  REM
170  REM
190  DIM Tell(4,50),Mesl(4,50),Arcl(4,50)
190  DIM Telr(4,50),Mesr(4,50),Arcr(4,50)
200  DIM Cl(4,50),Cr(4,50),Clt(4,50),Crt(4,50)
210  DIM Mo(4,50),Mot(4,50)
220  DIM Dat$[25],Observer$[25],Booker$[25],Instrument$[25],Weather$[25]
230  DIM St_point$[10],Nl$[16],Nr$[16]
240  DIM M(4,50),L(4,50),S(4,50),A(4,50),Mesarc(4),Redmo(4,50)
250  DIM Finmean(50),Ro(50),Q(50),R(50),E(50),U(4,50),Qfm(50),Rfm(50),Efm(50)
260  DIM Naris$(200)[8],Ndex$(200)[8],Point_name$(50)[10],V(4,50),V1(200)
270  DIM U1(200),R1(4,50),F1(4,50)
280  DIM Flp(50),Frp(50),Fle(50),Fre(50)
290  INTEGER J
300  PRINTER IS 1
310  Msus_0$=":HP8290X,700,0"
320  Msus_1$=":HP8290X,700,1"
330  Revin$[1,1]="N"
340  PRINT CHR$(128);
350  OUTPUT 2;"K";
360  PRINT USING 370
370  IMAGE 2/,8X,"        ******************************************"
380  PRINT TAB(17);CHR$(131);"PROGRAM TO REDUCE THE THEODOLITE OBSERVATIONS"
390  PRINT CHR$(128);
400  PRINT USING 410
410  IMAGE 8X,"       ******************************************"
420  PRINT USING 430
430  IMAGE /,18X,"THIS COMPUTER PROGRAM HAS BEEN COMPILED"
440  PRINT USING 450
450  IMAGE 6X,"                              BY",/
460  PRINT TAB(21);CHR$(130);"------>  ALEXANDER ALBANIS  <------"
470  PRINT CHR$(128);
480  PRINT USING 490
490  IMAGE /,16X,"MSc(Eng) - UNIVERSITY OF THE WITWATERSRAND"
500  PRINT USING 510
510  IMAGE /,4X,"            It has been tested on a HP-9816 mini computer",/,4
X,"                 using a HP-9121 double disk drive"
520  PRINT USING 530
530  IMAGE /,8X,"       ------------------------------------------"
540  DISP "Press <CONTINUE> to continue";
550  PAUSE
560  OUTPUT 2;"K";
570  PRINT USING 580
580  IMAGE 10/,4X,"       This computer program is running with BASIC 2.0",/,4X
,"       Provided that there is enough memory, it also"
```

```
590   PRINT USING 600
600   IMAGE 4X,"        runs with BASIC 2.0 & BASIC 2.1 with EXTENSIONS"
610   DISP "Press <CONTINUE> to continue";
620   PAUSE
630   OUTPUT 2;"K";
640   PRINT USING 650
650   IMAGE 9/,10X,"Insert the data disk in the right disk drive (700,1)"
660   PRINT USING 670
670   IMAGE /,5X,"Keep your disk with the program in the left disk drive (700,0)
",/,11X,"            Press <CONTINUE  to continue"
680   MASS STORAGE IS Msus $
690   PAUSE
700   OUTPUT 2;"K";
710   REM-----------------------------------------------------------------
720   REM
730   REM
740   PRINT USING 750
750   IMAGE 2/,6X,"***********************************************************"
760   PRINT USING 770
770   IMAGE 6X,"There are FOUR checks in this program, in order to eliminate",/,
              6X,"errors during data-reading from the data sheet.  However, it"
780   PRINT USING 790
790   IMAGE 6X,"is the user's responsibility to warranty, that the R.O point",/,
              6X,"  IS ALWAYS in the beginning and at the end of each round.",//
800   PRINT USING 810
810   IMAGE 6X,"CHECK 1: The length of all entries, must be according to the",/,
                   15X,"specified value."
820   PRINT USING 830
830   IMAGE 6X,"CHECK 2: Every observation reading must be less than 360 deg."
840   PRINT USING 850
850   IMAGE 6X,"CHECK 3: The difference (CL-CR) - 180 degrees + or - e; e is",/,
15X,"a specified error in seconds (e.g. 10 sec)."
860   PRINT USING 870
870   IMAGE 6X,"CHECK 4: A subsequant observation entry must always increase",/,
                    15X,"as theodolite turns clockwise."
880   PRINT USING 890
890   IMAGE 6X,"*********************************************************"
900   DISP "If you have read the notice, press <CONTINUE ";
910   PAUSE
920   OUTPUT 2;"K";
930   REM-----------------------------------------------------------------
940   REM
950   REM
960   REM*****************************************************************
970   REM     CREATE A NEW DATA FILE, IN CASE THERE IS NONE & ASSIGN PATH
980   REM ................................................................
990   !
1000  !
1010  OUTPUT 2;"K";
1020  PRINT USING 1030
1030  IMAGE 6X,"The: Day, Date, Time variables, must be inserted according to",/
,6X,"the following example listed below. Esp..lly, the Days must"
1040  PRINT USING 1050
1050  IMAGE 6X,"be printed using one letter exactly as it is shown below; the",/
,6X,"Date must occupy two digits and the time four digits (two for"
1060  PRINT USING 1070
1070  IMAGE 6X,"the hour and two for the minutes. NOTE '' You must write down",/
,6X,"these information on your sheet, and whenever you want to use"
1080  PRINT USING 1090
1090  IMAGE 6X,"these specific data you have to infore the computer by typing",/
```

```
,6X,"the particular Day, Date, Time written already on the sheet !",/
1100  PRINT USING 1110
1110  IMAGE 12X,"    DAY                DATE                TIME"
1120  PRINT USING 1130
1130  IMAGE 12X,"Monday    = M     27 = 27          13h 52m = 1352",/,12X,
                "Tuesday   = T      9 = 09           9h 41m = 0941"
1140  PRINT USING 1150
1150  IMAGE 12X,"Wednesday = W     31 = 31          20h  4m = 2004",/,12X,
                "Thursday  = H      7 = 07           2h  8m = 0208"
1160  PRINT USING 1170
1170  IMAGE 12X,"Friday    = F     15 = 15          23h 30m = 2330",/,12X,
                "Saturday  = S      3 = 03          17h 45m = 1745"
1180  PRINT USING 1190
1190  IMAGE 12X,"Sunday    = U      1 = 01          00  00m = 0000"
1200  DISP "Print Day,Date,Time (e.g. H,06,1215)";
1210  INPUT Day$,Hmer$,Ora$
1220  File$=Day$&Hmer$&Ora$
1230  File1$=File$&"1"
1240  File2$=File$&"2"
1250  File3$=File$&"3"
1260  File4$=File$&"4"
1270  File5$=File$&"5"
1280  File6$=File$&"6"
1290  File7$=File$&"7"
1300  File8$=File$&"8"
1310  File9$=File$&"9"
1320  ON ERROR GOTO 1340
1330  CREATE BDAT File1$,200,12      ! Naris$ - 200 records
1340  OFF ERROR
1350  ASSIGN @Emp TO File1$
1360  ON ERROR GOTO 1380
1370  CREATE BDAT File2$,200,12      ! Ndex$ - 200 records
1380  OFF ERROR
1390  ASSIGN @Emp_12 TO File2$
1400  ON ERROR GOTO 1420
1410  CREATE BDAT File3$,1,16        ! Arcnum, Arcpoints - 1 record
1420  OFF ERROR
1430  ASSIGN @Path TO File3$
1440  ON ERROR GOTO 1460
1450  CREATE BDAT File4$,8,30        ! General information - 8 records
1460  OFF ERROR
1470  ASSIGN @Odos TO File4$
1480  ON ERROR GOTO 1500
1490  CREATE BDAT File5$,50,16       ! Observation point names - 50 records
1500  OFF ERROR
1510  ASSIGN @Oax TO File5$
1520  ON ERROR GOTO 1550
1530  CREATE BDAT File6$,1,14        ! Station point name - 1 record
1540  OFF ERROR
1550  ASSIGN @Pla TO File6$
1560  ON ERROR GOTO 1580
1570  CREATE BDAT File7$,1,8         ! Required accuracy - 1 record
1580  OFF ERROR
1590  ASSIGN @Akpibeia TO File7$
1600  ON ERROR GOTO 1620
1610  CREATE BDAT File8$,200,8       ! Observational residuals - 200 records
1620  OFF ERROR
1630  ASSIGN @Res TO File8$
1640  ON ERROR GOTO 1660
1650  CREATE BDAT File9$,1,8          ! Number of obs. reiduals - 1 record
```

```
1660  OFF ERROR
1670  ASSIGN @Top TO File9$
1680  REM-----------------------------------------------------------------
1690  REM
1700  REM
1710  REM*****************************************************************
1720  REM          CLEANING  THE  SCREEN  OF  ALL  PREVIOUS  ENTRIES
1730  REM ...............................................................
1740  '
1750  '
1760  OUTPUT 2;"K";
1770  DISP "Will you use PRINTER & SCREEN (=PS) or only SCREEN (=S) ?";
1780  INPUT Question$
1790  OUTPUT 2;"K";
1800  IF Question$[1,2]="PS" THEN
1810    Pnr=1
1820  ELSE
1830    IF Question$[1,1]="S" THEN
1840      Pnr=0
1850      GOTO 1930
1860    ELSE
1870      GOTO 1770
1880    END IF
1890  END IF
1900  DISP "Print Printer s address [e.g. 701 etc.]";
1910  INPUT Pr
1920  ASSIGN @Prt TO Pr
1930  DISP "Do you want to insert new [N] or view the old general information [O
] ?";
1940  INPUT Decision$
1950  IF Decision$[1,1]="N" THEN
1960    GOTO 2040
1970  ELSE
1980    IF Decision$[1,1]="O" THEN
1990      GOTO 3400
2000    ELSE
2010      GOTO 1930
2020    END IF
2030  END IF
2040  DISP "How many Theodolite arcs have you measured ?";
2050  INPUT Arcnum
2060  DISP "How many different points per arc ?";
2070  INPUT Arcpoints
2080  OUTPUT @Path,1;Arcnum,Arcpoints
2090  DISP "Print the max sustained error between CL & CR readings in seconds";
2100  INPUT Nn
2110  OUTPUT @Akpibeia,1;Nn
2120  IF Revinf$[1,1]="Y" THEN
2130    DISP "Do you want to change the date (Y/N) ?";
2140    INPUT Chdate$
2150    IF Chdate$[1,1]="Y" THEN
2160      GOTO 2270
2170    ELSE
2180      IF Chdate$[1,1]="N" THEN
2190        GOTO 2300
2200      ELSE
2210        GOTO 2130
2220      END IF
2230    END IF
2240  ELSE
```

```
2250    GOTO 2270
2260  END IF
2270  DISP "What is the date ? [e.g. 21 May 1957]";
2280  INPUT Dat$
2290  X1=LEN(Dat$)
2300  IF X1>20 THEN
2310    BEEP
2320    DISP "The date name exceeds 20 chrs: Press <CONTINUE> & try again";
2330    PAUSE
2340    OUTPUT 2;"K";
2350    GOTO 2270
2360  END IF
2370  OUTPUT @Odos,1;Dat$
2380  IF Revinf$[1,1]="Y" THEN
2390    DISP "Do you want to change the observer's name (Y/N) ?";
2400    INPUT Chobse$
2410    IF Chobse$[1,1]="Y" THEN
2420      GOTO 2530
2430    ELSE
2440      IF Chobse$[1,1]="N" THEN
2450        GOTO 2640
2460      ELSE
2470        GOTO 2390
2480      END IF
2490    END IF
2500  ELSE
2510    GOTO 2530
2520  END IF
2530  DISP "Who is the observer ? [in total 20 characters]";
2540  INPUT Observer$
2550  X2=LEN(Observer$)
2560  IF X2>20 THEN
2570    BEEP
2580    DISP "The observer's name exceeds 20 chrs: Press <CONTINUE> & try again"
;
2590    PAUSE
2600    OUTPUT 2;"K";
2610    GOTO 2530
2620  END IF
2630  OUTPUT @Odos,2;Observer$
2640  IF Revinf$[1,1]="Y" THEN
2650    DISP "Do you want to change the booker's name (Y/N) ?";
2660    INPUT Chbook$
2670    IF Chbook$[1,1]="Y" THEN
2680      GOTO 2790
2690    ELSE
2700      IF Chbook$[1,1]="N" THEN
2710        GOTO 2900
2720      ELSE
2730        GOTO 2650
2740      END IF
2750    END IF
2760  ELSE
2770    GOTO 2790
2780  END IF
2790  DISP "Who is the booker ? [in total 20 characters]";
2800  INPUT Booker$
2810  X3=LEN(Booker$)
2820  IF X3>20 THEN
2830    BEEP
```

```
2840    DISP "The booker's name exceeds 20 chrs: Press <CONTINUE  & try again";
2850    PAUSE
2860    OUTPUT 2;"K";
2870    GOTO 2790
2880  END IF
2890  OUTPUT @Odos,3;Booker$
2900  IF Revinf$[1,1]="Y" THEN
2910    DISP "Do you want to change the type of theodolite (Y/N) ?";
2920    INPUT Chtheo$
2930    IF Chtheo$[1,1]="Y" THEN
2940      GOTO 3050
2950    ELSE
2960      IF Chtheo$[1,1]="N" THEN
2970        GOTO 3160
2980      ELSE
2990        GOTO 2910
3000      END IF
3010    END IF
3020  ELSE
3030    GOTO 3050
3040  END IF
3050  DISP "What is the type of the Theodolite [20 characters]";
3060  INPUT Instrument$
3070  X4=LEN(Instrument$)
3080  IF X4>20 THEN
3090    BEEP
3100    DISP "The instrument name exceeds 20 chrs: Press <CONTINUE> & trv again"
;
3110    PAUSE
3120    OUTPUT 2;"K";
3130    GOTO 3050
3140  END IF
3150  OUTPUT @Odos,4;Instrument$
3160  IF Revinf$[1,1]="Y" THEN
3170    DISP "Do you want to change the weather conditions (Y/N) ?";
3180    INPUT Chwhet$
3190    IF Chwhet$[1,1]="Y" THEN
3200      GOTO 3310
3210    ELSE
3220      IF Chwhet$[1,1]="N" THEN
3230        GOTO 3420
3240      ELSE
3250        GOTO 3170
3260      END IF
3270    END IF
3290  ELSE
3290    GOTO 3310
3300  END IF
3310  DISP "Description of weather conditions [20 char.]";
3320  INPUT Weatner$
3330  X5=LEN(Weather$)
3340 @IF X5>20 THEN
3350    BEEP
3360    DISP "The description of weather exceeds 20 chrs: Press <CONTINUE> & trv
again";
3370    PAUSE
3380    OUTPUT 2;"K",
3390    GOTO 3310
3400  END IF
3410  OUTPUT @Odos,5;Weather$
```

```
3420  OUTPUT @Odos,6;Day$
3430  OUTPUT @Odos,7;Hmer$
3440  OUTPUT @Odos,8;Ora$
3450  REM-----------------------------------------------------------------------
3460  REM
3470  REM
3480  REM*********************************************************************
3490  REM        ENTER OLD ENTRIES FOR THE GENERAL INFORMATION & PRINTOUT
3500  REM ....................................................................
3510  '
3520  '
3530  ENTER @Odos,1;Dat$
3540  ENTER @Odos,2;Observer$
3550  ENTER @Odos,3;Booker$
3560  ENTER @Odos,4;Instrument$
3570  ENTER @Odos,5;Weather$
3580  ENTER @Odos,6;Day$
3590  ENTER @Odos,7;Hmer$
3600  ENTER @Odos,8;Ora$
3610  PRINT USING 3620
3620  IMAGE "       ***********************************************************
***"
3630  IF Pnr=1 THEN OUTPUT @Prt USING 3620
3640  PRINT USING 3650
3650  IMAGE "                        REDUCTION OF THEODOLITE OBSERVATIONS"
3660  IF Pnr=1 THEN OUTPUT @Prt USING 3650
3670  PRINT USING 3680
3680  IMAGE "       ***********************************************************
***"
3690  IF Pnr=1 THEN OUTPUT @Prt USING 3680
3700  REM-----------------------------------------------------------------------
3710  REM
3720  REM
3730  REM*********************************************************************
3740  REM              PRINT, REVISE END REPRINT GENERAL INFORMATION
3750  REM ....................................................................
3760  '
3770  '
3780  PRINT USING 3790
3790  IMAGE ///,"                              GENERAL INFORMATION"
3800  IF Pnr=1 THEN OUTPUT @Prt USING 3790
3810  PRINT USING 3820
3820  IMAGE "                              *******************",/
3830  IF Pnr=1 THEN OUTPUT @Prt USING 3820
3840  PRINT USING 3850;Dat$,Instrument$
3850  IMAGE "Date                   :",2X,20A,1X,"Instrument :",2X,20A
3860  IF Pnr=1 THEN OUTPUT @Prt USING 3850;Dat$,Instrument$
3870  PRINT USING 3880;Observer$,Booker$,Weather$
3880  IMAGE "Observer              :",2X,20A,/,"Booker                :",2X,20A,/,"Wea
ther Conditions :",2X,20A
3890  IF Pnr=1 THEN OUTPUT @Prt USING 3880;Observer$,Booker$,Weather$
3900  PRINT USING 3910;Day$,Hmer$,Ora$
3910  IMAGE //,"*  Code numbers to create/have access to these data: ",A,",",3A,
",",4A,/,"** The output of all the observations is in degrees, minutes, seconds"
3920  IF Pnr=1 THEN OUTPUT @Prt USING 3910;Day$,Hmer$,Ora$
3930  DISP "Do you want to revise any entry (Y/N) ?";
3940  INPUT Revinf$
3950  IF Revinf$[1,1]="N" THEN
3960    GOTO 4040
3970  ELSE
```

```
3420  OUTPUT @Odos,6;Day$
3430  OUTPUT @Odos,7;Hmer$
3440  OUTPUT @Odos,8;Ora$
3450  REM----------------------------------------------------------------------
3460  REM
3470  REM
3480  REM======================================================================
3490  REM       ENTER OLD ENTRIES FOR THE GENERAL INFORMATION & PRINTOUT
3500  REM ......................................................................
3510  '
3520  '
3530  ENTER @Odos,1;Dat$
3540  ENTER @Odos,2;Observer$
3550  ENTER @Odos,3;Booker$
3560  ENTER @Odos,4;Instrument$
3570  ENTER @Odos,5;Weather$
3580  ENTER @Odos,6;Day$
3590  ENTER @Odos,7;Hmer$
3600  ENTER @Odos,8;Ora$
3610  PRINT USING 3620
3620  IMAGE "        ***********************************************************
***"
3630  IF Pnr=1 THEN OUTPUT @Prt USING 3620
3640  PRINT USING 3650
3650  IMAGE "                    REDUCTION OF THEODOLITE OBSERVATIONS"
3660  IF Pnr=1 THEN OUTPUT @Prt USING 3650
3670  PRI T USING 3680
3680  IMAGE "        ***********************************************************
***"
3690  IF Pnr=1 THEN OUTPUT @Prt USING 3680
3700  REM----------------------------------------------------------------------
3710  REM
3720  REM
3730  REM======================================================================
3740  REM            PRINT, REVISE END REPRINT GENERAL INFORMATION
3750  REM ......................................................................
3760  '
3770  '
3780  PRINT USING 3790
3790  IMAGE ///,"                          GENERAL INFORMATION"
3800  IF Pnr=1 THEN OUTPUT @Prt USING 3790
3810  PRINT USING 3820
3820  IMAGE "                          ===================",/
3830  IF Pnr=1 THEN OUTPUT @Prt USING 3820
3840  PRINT USING 3850;Dat$,Instrument$
3850  IMAGE "Date                  :",2X,20A,1X,"Instrument :",2X,20A
3860  IF Pnr=1 THEN OUTPUT @Prt USING 3850;Dat$,Instrument$
3870  PRINT USING 3880;Observer$,Booker$,Weather$
3880  IMAGE "Observer              :",2X,20A,/,"Booker               :",2X,20A,/,"Wea
ther Conditions :",2X,20A
3890  IF Pnr=1 THEN OUTPUT @Prt USING 3880;Observer$,Booker$,Weather$
3900  PRINT USING 3910;Day$,Hmer$,Ora$
3910  IMAGE //,"*  Code numbers to create/have access to these data: ",A,",",3A,
",",4A,/,"** The output of all the observations is in degrees, minutes, seconds"
3920  IF Pnr=1 THEN OUTPUT @Prt USING 3910;Day$,Hmer$,Ora$
3930  DISP "Do you want to revise any entry (Y/N) ?";
3940  INPUT Revinf$
3950  IF Revinf$[1,1]="N" THEN
3960    GOTO 4040
3970  ELSE
```

```
3980    IF Revinf$[1,1]="Y" THEN
3990      GOTO 2120
4000    ELSE
4010      GOTO 3930
4020    END IF
4030  END IF
4040  OUTPUT 2;"K";
4050  DISP "Do you want to insert new [N] or view the old data [O] ?";
4060  INPUT Decdata$
4070  IF Decdata$[1,1]="O" THEN
4080    GOTO 6310
4090  ELSE
4100    IF Decdata$[1,1]="N" THEN
4110      IF Decision$[1,1]="N" THEN
4120        OUTPUT 2;"K";
4130        GOTO 4330
4140      ELSE
4150        GOTO 4240
4160      END IF
4170    ELSE
4180      GOTO 4050
4190    END IF
4200  END IF
4210  REM-------------------------------------------------------------------
4220  !
4230  !
4240  OUTPUT 2;"K";
4250  DISP "How many Theodolite arcs have you measured ?";
4260  INPUT Arcnum
4270  DISP "How many different points per arc ?";
4280  INPUT Arcpoints
4290  OUTPUT @Path;Arcnum,Arcpoints
4300  DISP "Print the max sustained error between CL & CR readings in seconds";
4310  INPUT Nn
4320  OUTPUT @Akpibeia;Nn
4330  DISP "Print station point (up to 3 char eg. M24; the first letter alph.)";
4340  INPUT St_point$
4350  Y3=LEN(St_point$)
4360  IF Y3>3 THEN
4370    BEEP
4380    DISP "Your station name exceeds 3 chrs; Press <CONT> & try again";
4390    PAUSE
4400    OUTPUT 2;"K";
4410    GOTO 4330
4420  END IF
4430  OUTPUT @Plm;St_point$
4440  FOR I=1 TO Arcpoints
4450    DISP "Print the name of obs point no ",I,"(up to 3 characters, eg. MIL)"
;
4460    INPUT Point_name$(I)
4470    Y4=LEN(Point_name$(I))
4480    IF Y4>3 THEN
4490      BEEP
4500      DISP "Your obs point name exceeds 3 chrs; Press <CONT> & try again";
4510      PAUSE
4520      OUTPUT 2;"K";
4530      GOTO 4450
4540    END IF
4550    OUTPUT @Oaz,I;Point_name$(I)
4560  NEXT I
```

```
4570  CONTROL @Oaz,7;Arcpoints+1
4580  REM----------------------------------------------------------------------
4590  REM
4600  REM
4610  REM======================================================================
4620  REM              INSERT NEW DATA & STORE THEM IN THE DATA FILE
4630  REM ....................................................................
4640  !
4650  !
4660  FOR I=1 TO Arcnum
4670    Kp=1
4680    Flp(Kp)=0
4690    Frp(Kp)=0
4700    FOR I1=1 TO Arcpoints+1
4710      K=(I-1)*(Arcpoints+1)+I1
4720      !
4730      GOSUB Upodeigma
4740      !
4750      IF I1<Arcpoints+1 THEN
4760        DISP "Print circle LEFT reading for point no. ";I1;" of arc no. ";I;
4770      ELSE
4780        DISP "Print circle LEFT reading for RO (2nd time) of arc no. ";I;
4790      END IF
4800      INPUT Nl$
4810      Y1=LEN(Nl$)
4820      IF Y1<>8 THEN
4830        OUTPUT 2;"K";
4840        BEEP
4850        DISP "You have entered a no. with digits < 8 or > 8. Press CONT & tr
y again";
4860        PAUSE
4870        OUTPUT 2;"K";
4880        PRINT USING 5000
4890        !
4900        GOSUB Upodeigma
4910        !
4920        GOTO 4750
4930      END IF
4940      Narist$(K)=Nl$
4950      IF I1<Arcpoints+1 THEN
4960        DISP "Print circle RIGHT reading for point no. ";I1;" of arc no. ";I
;
4970      ELSE
4980        DISP "Print circle RIGHT reading for RO (2nd time) of arc no. ";I;
4990      END IF
5000      INPUT Nr$
5010      Y2=LEN(Nr$)
5020      IF Y2<>8 THEN
5030        OUTPUT 2;"K";
5040        BEEP
5050        DISP "You have entered a no. with digits < 8 or > 8. Press CONT & tr
y again";
5060        PAUSE
5070        OUTPUT 2;"K";
5080        !
5090        GOSUB Upodeigma
5100        !
5110        GOTO 4950
5120      END IF
5130      Ndex$(K)=Nr$
```

```
4570  CONTROL @Daz,7;Arcpoints+1
4580  REM-----------------------------------------------------------------------
4590  REM
4600  REM
4610  REM***********************************************************************
4620  REM              INSERT NEW DATA & STORE THEM IN THE DATA FILE
4630  REM .......................................................................
4640  '
4650  '
4660  FOR I=1 TO Arcnum
4670    Kp=1
4680    Flp(Kp)=0
4690    Frp(Kp)=0
4700    FOR I1=1 TO Arcpoints+1
4710      K=(I-1)*(Arcpoints+1)+I1
4720      '
4730      GOSUB Upodeigma
4740      '
4750      IF I1<Arcpoints+1 THEN
4760        DISP "Print circle LEFT reading for point no. ";I1;" of arc no. ";I;
4770      ELSE
4780        DISP "Print circle LEFT reading for RO (2nd time) of arc no. ";I;
4790      END IF
4800      INPUT Nl$
4810      Y1=LEN(Nl$)
4820      IF Y1<>8 THEN
4830        OUTPUT 2;"K";
4840        BEEP
4850        DISP "You have entered a no. with digits < 8 or > 8. Press CONT & tr
y again";
4860        PAUSE
4870        OUTPUT 2;"K";
4880        PRINT USING 5000
4890        '
4900        GOSUB Upodeigma
4910        '
4920        GOTO 4750
4930      END IF
4940      Naris$(K)=Nl$
4950      IF I1<Arcpoints+1 THEN
4960        DISP "Print circle RIGHT reading for point no. ";I1;" of arc no. ";I
;
4970      ELSE
4980        DISP "Print circle RIGHT reading for RO (2nd time) of arc no. ";I;
4990      END IF
5000      INPUT Nr$
5010      Y2=LEN(Nr$)
5020      IF Y2<>8 THEN
5030        OUTPUT 2;"K";
5040        BEEP
5050        DISP "You have entered a no. with digits < 8 or > 8. Press CONT & tr
y again";
5060        PAUSE
5070        OUTPUT 2;"K";
5080        '
5090        GOSUB Upodeigma
5100        '
5110        GOTO 4950
5120      END IF
5130      Ndex$(K)=Nr$
```

```
5140        Tell(I,Ii)=VAL(Nl$[6,8])
5150        Tell(I,Ii)=Tell(I,Ii)/10
5160        Mesl(I,Ii)=VAL(Nl$[4,5])
5170        Arxl(I,Ii)=VAL(Nl$[1,3])
5180        Cl(I,Ii)=Tell(I,Ii)+Mesl(I,Ii)*60+Arxl(I,Ii)*3600
5190        Telr(I,Ii)=VAL(Nr$[6,8])
5200        Telr(I,Ii)=Telr(I,Ii)/10
5210        Mesr(I,Ii)=VAL(Nr$[4,5])
5220        Arxr(I,Ii)=VAL(Nr$[1,3])
5230        Cr(I,Ii)=Telr(I,Ii)+Mesr(I,Ii)*60+Arxr(I,Ii)*3600
5240        G1=360
5250        G2=3600
5260        Gg=G1*G2
5270        REM
5280        REM
5290        REM--------------------------------------------------------------------
5300        !
5310        !
5320        IF Cl(I,Ii)>Gg OR Cr(I,Ii)>Gg THEN
5330          OUTPUT 2;"K";
5340          BEEP
5350          DISP "YOUR READING EXCEEDS 360 DEG ! Try again - Press <CONTINUE> to
 continue";
5360          PAUSE
5370          OUTPUT 2;"K";
5380          GOTO 4720
5390        END IF
5400        REM--------------------------------------------------------------------
5410        !
5420        !
5430        Diafora=ABS(Cl(I,Ii)-Cr(I,Ii))
5440        M1=180
5450        M2=3600
5460        M3=M1*M2
5470        IF ABS(Diafora-M3)>Nn THEN
5480          OUTPUT 2;"K";
5490          PRINT USING 5510;Ii,I
5500          BEEP
5510          IMAGE 6/,"   THE DIFFERENCE BETWEEN CL & CR READINGS IN POINT ",2D,"
 OF THE ARC NO. ",2D
5520          PRINT USING 5530;Nn
5530          IMAGE "                              IS GREATER THAN ",3D," SEC !!"
5540          PRINT USING 5550
5550          IMAGE //,"Do you want to continue the insertion of the data or to re
vise this entry ??"
5560          DISP "Print <C> to continue or <R> to revise.";
5570          INPUT Pao$
5580          IF Pao$[1,1]="R" THEN
5590            OUTPUT 2;"K";
5600            GOTO 4720
5610          ELSE
5620            IF Pao$[1,1]="C" THEN
5630              OUTPUT 2;"K";
5640              GOTO 5700
5650            ELSE
5660              GOTO 5480
5670            END IF
5680          END IF
5690        END IF
5700        REM--------------------------------------------------------------------
```

```
5710        '
5720        '
5730        Fle(Kp)=Cl(I,Ii)-Cl(I,1)
5740        Fre(Kp)=Cr(I,Ii)-Cr(I,1)
5750        IF Fle(Kp)<0 AND Ii<>1 THEN
5760          Fle(Kp)=Fle(Kp)+Gg
5770        END IF
5780        IF Fre(Kp)<0 AND Ii<>1 THEN
5790          Fre(Kp)=Fre(Kp)+Gg
5800        END IF
5810        Kritl=Fle(Kp)-Flp(Kp)
5820        Kritr=Fre(Kp)-Frp(Kp)
5830        IF Ii=1 THEN
5840          Kp=Kp+1
5850          Flp(Kp)=Fle(Kp-1)
5860          Frp(Kp)=Fre(Kp-1)
5870          GOTO 6260
5880        END IF
5890        IF Ii=Arcpoints+1 THEN
5900          GOTO 6260
5910        END IF
5920        IF Kritl>0 AND Kritr>0 THEN
5930          Kp=Kp+1
5940          Flp(Kp)=Fle(Kp-1)
5950          Frp(Kp)=Fre(Kp-1)
5960        ELSE
5970          OUTPUT 2;"K";
5980          BEEP
5990          PRINT USING 6000
6000          IMAGE 4/,11X,"WARNING ! ERROR HAS OCCURED AS YOUR ENTRY IS NOT IN OR
DER",/,11X,"A subsequent entry must always increase as the theodolite"
6010          PRINT USING 6020
6020          IMAGE 13X,"                rotates clockwise",//
6030          PRINT USING 6040;Ii-1,I
6040          IMAGE "TRY TO RE-INSERT THE CORRECT VALUES STARTING AGAIN FROM POINT
 ",DD,"  OF ARC NO. ",DD,/,29X," OR STOP THE RUN."
6050          DISP "Print <S> to stop the run or <RI> to re-insert the correct val
ue";
6060          INPUT Lathos_seira$
6070          IF Lathos_seira$[1,1]="S" THEN
6080            OUTPUT 2;"K";
6090            BEEP
6100            PRINT USING 6110
6110            IMAGE "THE RUN HAS BEEN DELIBERATELY STOPPED DUE TO WRONG ENTRIES.
 (A HORIZ. ANGLE WITH VALUE LESS THAN THAT OF THE PREVIOUS ENTRY HAS BEEN"
6120            PRINT USING 6130;Ii,I,Ii-1
6130            IMAGE "INSERTED IN POINT ",DD," OF ARC NO. ",DD," . CHECK ALSO POI
NT ",DD," OF THE SAME ARC."
6140            GOTO 12220
6150          ELSE
6160            IF Lathos_seira$[1,2]="RI" THEN
6170              OUTPUT 2;"K";
6180              Kp=Kp-1
6190              Ii=Ii-1
6200              GOTO 4710
6210            ELSE
6220              GOTO 5970
6230            END IF
6240          END IF
6250        END IF
```

```
5710      !
5720      !
5730      Fle(Kp)=Cl(I,Ii)-Cl(I,1)
5740      Fre(Kp)=Cr(I,Ii)-Cr(I,1)
5750      IF Fle(Kp)<0 AND Ii>1 THEN
5760        Fle(Kp)=Fle(Kp)+Gg
5770      END IF
5780      IF Fre(Kp)<0 AND Ii>1 THEN
5790        Fre(Kp)=Fre(Kp)+Gg
5800      END IF
5810      Kritl=Fle(Kp)-Flp(Kp)
5820      Kritr=Fre(Kp)-Frp(Kp)
5830      IF Ii=1 THEN
5840        Kp=Kp+1
5850        Flp(Kp)=Fle(Kp-1)
5860        Frp(Kp)=Fre(Kp-1)
5870        GOTO 6260
5880      END IF
5890      IF Ii=Arcpoints+1 THEN
5900        GOTO 6260
5910      END IF
5920      IF Kritl>0 AND Kritr>0 THEN
5930        Kp=Kp+1
5940        Flp(Kp)=Fle(Kp-1)
5950        Frp(Kp)=Fre(Kp-1)
5960      ELSE
5970        OUTPUT 2;"K";
5980        BEEP
5990        PRINT USING 6000
6000        IMAGE 4/,11X,"WARNING ! ERROR HAS OCCURED AS YOUR ENTRY IS NOT IN OR
DER",/,11X,"A subsequent entry must always increase as the theodolite"
6010        PRINT USING 6020
6020        IMAGE 13X,"                    rotates clockwise",//
6030        PRINT USING 6040;Ii-1,I
6040        IMAGE "TRY TO RE-INSERT THE CORRECT VALUES STARTING AGAIN FROM POINT
 ",DD,"  OF ARC NO. ",DD,/,29X," OR STOP THE RUN."
6050        DISP "Print <S> to stop the run or <RI> to re-insert the correct val
ue";
6060        INPUT Lathos_seira$
6070        IF Lathos_seira$[1,1]="S" THEN
6080          OUTPUT 2;"K";
6090          BEEP
6100          PRINT USING 6110
6110          IMAGE "THE RUN HAS BEEN DELIBERATELY STOPPED DUE TO WRONG ENTRIES.
 (A HORIZ. ANGLE WITH VALUE LESS THAN THAT OF THE PREVIOUS ENTRY HAS BEEN"
6120          PRINT USING 6130;Ii,I,Ii-1
6130          IMAGE "INSERTED IN POINT ",DD," OF ARC NO, ",DD," . CHECK ALSO POI
NT ",DD," OF THE SAME ARC."
6140          GOTO 12220
6150        ELSE
6160          IF Lathos_seira$[1,2]="RI" THEN
6170            OUTPUT 2;"K";
6180            Kp=Kp-1
6190            Ii=Ii-1
6200            GOTO 4710
6210          ELSE
6220            GOTO 5970
6230          END IF
6240        END IF
6250      END IF
```

```
6260        OUTPUT @Emp,K;Naris$(K)
6270        OUTPUT @Emp_12,K;Ndex$(K)
6280        OUTPUT 2;"K";
6290      NEXT I1
6300    NEXT I
6310    REM--------------------------------------------------------------------
6320    '
6330    '
6340    ENTER @Pith,1;Arcnum,Arcpoints
6350    ENTER @Akpibeia,1;Nn
6360    Syn=Arcnum*(Arcpoints+1)
6370    CONTROL @Emp,7;Syn+1
6380    CONTROL @Emp_12,7;Syn+1
6390    REM--------------------------------------------------------------------
6400    REM
6410    REM
6420    REM********************************************************************
6430    REM                    ENTER  OLD  DATA  FROM  DATA  FILES
6440    REM ...................................................................
6450    '
64(     '
64      ON END @Emp GOTO 6510
6480    FOR K=1 TO 500
6490      ENTER @Emp,K;Naris$(K)
6500    NEXT K
6510    Syn=K-1
6520    ON END @Emp_12 GOTO 6560
6530    FOR K=1 TO 500
6540      ENTER @Emp_12,K;Ndex$(K)
6550    NEXT K
6560    Syn=K-1
6570    ENTER @Plm,1;St_point$
6580    ON END @Oaz GOTO 6620
6590    FOR J=1 TO 500
6600      ENTER @Oaz,J;Point_name$(J)
6610    NEXT J
6620    Arcpoints=J-2
6630    REM
6640    REM--------------------------------------------------------------------
6650    REM
6660    REM
6670    REM********************************************************************
6680    REM                    PRINT  DATA  FROM  OLD  DATA  FILE
6690    REM ...................................................................
6700    '
6710    '
6720    PRINT USING 6730
6730    IMAGE 5/,"*****************************************************************
****************",3/
6740    IF Pnr=1 THEN OUTPUT @Prt USING 6730
6750    PRINT USING 6760
6760    IMAGE 35X,"ECHO",/,35X,"===="
6770    IF Pnr=1 THEN OUTPUT @Prt USING 6760
6780    FOR I=1 TO Arcnum
6790      PRINT USING 6800;I
6800      IMAGE ///,32X,"ARC NO. ",2D,/,32X,"===========",/
6810      IF Pnr=1 THEN OUTPUT @Prt USING 6800;I
6820      PRINT USING 6830
6830      IMAGE "No",5X,"St point",5X,"Obs point",12X,"CL",14X,"CR",12X,"MEAN"
6840      IF Pnr=1 THEN OUTPUT @Prt USING 6830
```

```
6850    PRINT USING 6860
6860    IMAGE "---",5X,"----------",5X,"----------",12X,"---",14X,"---",12X,"-----",/
6870    IF Pnr=1 THEN OUTPUT @Prt USING 6860
6880    FOR I1=1 TO Arcpoints+1
6890      Nl$=Naris$((I-1)*(Arcpoints+1)+I1)
6900      Nr$=Ndex$((I-1)*(Arcpoints+1)+I1)
6910      Tell(I,I1)=VAL(Nl$[6,8])
6920      Tell(I,I1)=Tell(I,I1)/10
6930      Mesl(I,I1)=VAL(Nl$[4,5])
6940      Arxl(I,I1)=VAL(Nl$[1,3])
6950      Cl(I,I1)=Tell(I,I1)+Mesl(I,I1)*60+Arxl(I,I1)*3600
6960      Telr(I,I1)=VAL(Nr$[6,8])
6970      Telr(I,I1)=Telr(I,I1)/10
6980      Mesr(I,I1)=VAL(Nr$[4,5])
6990      Arxr(I,I1)=VAL(Nr$[1,3])
7000      Cr(I,I1)=Telr(I,I1)+Mesr(I,I1)*60+Arxr(I,I1)*3600
7010      D=180
7020      Dd=3600
7030      Ddd=D*Dd
7040      IF Cr(I,I1)<Ddd THEN
7050        Mot(I,I1)=(Cl(I,I1)+(Cr(I,I1)+Ddd))/2
7060      ELSE
7070        Mot(I,I1)=(Cl(I,I1)+(Cr(I,I1)-Ddd))/2
7080      END IF
7090      Moiresl=Mot(I,I1)/3600
7100      Fractlepta=Moiresl-INT(Moiresl)
7110      M(I,I1)=Moiresl-Fractlepta
7120      Leptal=Fractlepta*60
7130      Fractdeft=Leptal-INT(Leptal)
7140      L(I,I1)=Leptal-Fractdeft
7150      S(I,I1)=Fractdeft*60
7160      IF (S(I,I1)*10-INT(S(I,I1)*10))>=.49 THEN
7170        S(I,I1)=(INT(S(I,I1)*10)+1)/10
7180      END IF
7190      W=I1
7200      IF W=1 THEN
7210        PRINT USING 7220;W,St_point$,Point_name$(W),Arxl(I,W),Mesl(I,W),Tell
(I,W),Arxr(I,W),Mesr(I,W),Telr(I,W),M(I,W),L(I,W),S(I,W)
7220        IMAGE 2D,8X,3A,10X,3A,10X,3D,X,2Z,X,2Z.D,5X,3D,X,2Z,X,2Z.D,2X,3X,3D,
X,2Z,X,2Z.D
7230        IF Pnr=1 THEN OUTPUT @Prt USING 7220;W,St_point$,Point_name$(W),Arxl
(I,W),Mesl(I,W),Tell(I,W),Arxr(I,W),Mesr(I,W),Telr(I,W),M(I,W),L(I,W),S(I,W)
7240      END IF
7250      IF W>1 AND W<Arcpoints+1 THEN
7260        PRINT USING 7270;W,Point_name$(W),Arxl(I,W),Mesl(I,W),Tell(I,W),Arxr
(I,W),Mesr(I,W),Telr(I,W),M(I,W),L(I,W),S(I,W)
7270        IMAGE 2D,9X,12X,3A,10X,3D,X,2Z,X,2Z.D,5X,3D,X,2Z,X,2Z.D,2X,3X,3D,X,2
Z,X,2Z.D
7280        IF Pnr=1 THEN OUTPUT @Prt USING 7270;W,Point_name$(W),Arxl(I,W),Mesl
(I,W),Tell(I,W),Arxr(I,W),Mesr(I,W),Telr(I,W),M(I,W),L(I,W),S(I,W)
7290      END IF
7300      IF W=Arcpoints+1 THEN
7310        PRINT USING 7320;W,Point_name$(1),Arxl(I,W),Mesl(I,W),Tell(I,W),Arxr
(I,W),Mesr(I,W),Telr(I,W),M(I,W),L(I,W),S(I,W)
7320        IMAGE 2D,12X,9X,3A,10X,3D,X,2Z,X,2Z.D,5X,3D,X,2Z,X,2Z.D,2X,3X,3D,X,2
Z,X,2Z.D
7330        IF Pnr=1 THEN OUTPUT @Prt USING 7320;W,Point_name$(1),Arxl(I,W),Mesl
(I,W),Tell(I,W),Arxr(I,W),Mesr(I,W),Telr(I,W),M(I,W),L(I,W),S(I,W)
7340      END IF
7350    NEXT I1
```

```
7360  NEXT I
7370  PRINT USING 7380
7380  IMAGE 3/,"...........................................................................
................"
7390  IF Pnr=1 THEN OUTPUT @Prt USING 7380
7400  REM-------------------------------------------------------------------------
7410  REM
7420  REM
7430  REM=========================================================================
7440  REM                             REVISE  DATA
7450  REM .........................................................................
7460  !
7470  !
7480  DISP "Will you revise the st point and/or the obs points and/or data (Y/N)
?";
7490  INPUT Revise$
7500  IF Revise$[1,1]="N" THEN
7510    GOTO 8950
7520  ELSE
7530    IF Revise$[1,1]="Y" THEN
7540      DISP "Do you want to revise the station point ? (Y/N)";
7550      INPUT R_st_p$
7560      IF R_st_p$[1,1]="N" THEN
7570        GOTO 7750
7580      ELSE
7590        IF R_st_p$[1,1]="Y" THEN
7600          DISP "Print the new name of the station point";
7610          INPUT St_point$
7620          Y5=LEN(St_point$)
7630          IF Y5>3 THEN
7640            BEEP
7650            DISP "The station name exceeds 3 chrs; press <CONT> and try agai
n";
7660            PAUSE
7670            GOTO 7600
7680          END IF
7690          OUTPUT @Pla,1;St_point$
7700          GOTO 7750
7710        ELSE
7720          GOTO 7540
7730        END IF
7740      END IF
7750      Op=Arcpoints
7760      DISP "Will you revise any names of obs points (up to point no.";Op;")
? (Y/N)";
7770      INPUT R_ob_p$
7780      IF R_ob_p$[1,1]="N" THEN
7790        GOTO 8000
7800      ELSE
7810        IF R_ob_p$[1,1]="Y" THEN
7820          DISP "Print the no. of entries for the obs points to be revised";
7830          INPUT No_r_ob_p
7840          FOR Tt=1 TO No_r_ob_p
7850            DISP "CHANGE NO ",Tt,"Print:(i) No point name, (ii)point name (u
p to 3 chrs)";
7860            INPUT Il,Point_name$(Il)
7870            Y6=LEN(Point_name$(Il))
7880            IF Y6>3 THEN
7890              BEEP
7900              DISP "The obs point name exceeds 3 chrs; press <CONT> and try
```

```
again";
7910             PAUSE
7920             GOTO 7850
7930           END IF
7940           OUTPUT @Oaz,I1;Point_name$(I1)
7950         NEXT I1
7960       ELSE
7970         GOTO 7760
7980       END IF
7990     END IF
8000     DISP "Do you want to revise any data entries ? (Y/N)";
8010     INPUT Revdata$
8020     IF Revdata$[1,1]="N" AND (R_st_p$[1,1]="Y" OR R_ob_p$[1,1]="Y") THEN
8030       GOTO 6710
8040     END IF
8050     IF Revdata$[1,1]="N" THEN
8060       GOTO 8950
8070     ELSE
8080       IF Revdata$[1,1]="Y" THEN
8090         GOTO 8140
8100       ELSE
8110         GOTO 8000
8120       END IF
8130     END IF
8140     DISP "How many entries do you want to change ?";
8150     INPUT Norevise
8160     FOR Mm=1 TO Norevise
8170       !
8180       GOSUB Upodeigma
8190       !
8200       DISP "CHANGE NO.",Mm,"Print: arc no, point no, left entry, right ent
ry";
8210       INPUT I,I1,Nl$,Nr$
8220       Y1=LEN(Nl$)
8230       Y2=LEN(Nr$)
8240       IF Y1<>8 OR Y2<>8 THEN
8250         OUTPUT 2;"K";
8260         BEEP
8270         DISP "You have entered a no. with digits < 8 or > 8. Press CONT &
try again";
8280         PAUSE
8290         OUTPUT 2;"K";
8300         GOTO 8490
8310       END IF
8320       K=(I-1)*(Arcpoints+1)+I1
8330       Naris$(K)=Nl$
8340       Ndex$(K)=Nr$
8350       Tl=VAL(Nl$[6,8])
8360       Tl=Tl/10
8370       Ml=VAL(Nl$[4,5])
8380       Al=VAL(Nl$[1,3])
8390       Cll=Tl+Ml*60+Al*3600
8400       Tr=VAL(Nr$[6,8])
8410       Tr=Tr/10
8420       Mr=VAL(Nr$[4,5])
8430       Ar=VAL(Nr$[1,3])
8440       Crr=Tr+Mr*60+Ar*3600
8450       G1=360
8460       G2=3600
8470       Gg=G1*G2
```

```
8480          IF Cl1>Gg OR Crr>Gg THEN
8490            OUTPUT 2;"K";
8500            BEEP
8510            DISP "YOUR READING EXCEEDS 360 DEG ! Try again - Press <CONTINUE>
to continue";
8520            PAUSE
8530            OUTPUT 2;"K";
8540            GOTO 8490
8550          END IF
8560          Diafora=ABS(Cl1-Crr)
8570          M1=180
8580          M2=3600
8590          M3=M1*M2
8600          IF ABS(Diafora-M3)>Nn THEN
8610            OUTPUT 2;"K";
8620            PRINT USING 8640;I1,I
8630            BEEP
8640            IMAGE 6/,"   THE DIFFERENCE BETWEEN CL & CR READINGS IN POINT ",2D
," OF THE ARC NO. ",2D
8650            PRINT USING 8660;Nn
8660            IMAGE      "                          IS GREATER THAN ",3D," SEC !!"
8670            PRINT USING 8680
8680            IMAGE //,"Do you want to continue the insertion of the data or to
revise this entry ??"
8690            DISP "Print <C> to continue or <R> to revise.";
8700            INPUT Pao$
8710            IF Pao$[1,1]="R" THEN
8720              OUTPUT 2;"K";
8730              GOTO 8490
8740            ELSE
8750              IF Pao$[1,1]="C" THEN
8760                OUTPUT 2;"K";
8770                GOTO 8830
8780              ELSE
8790                GOTO 8610
8800              END IF
8810            END IF
8820          END IF
8830          OUTPUT @Emp,K;Naris$(K)
8840          OUTPUT @Emp_12,K;Ndex$(K)
8850          OUTPUT 2;"K";
8860        NEXT Mm
8870        GOTO 6310
8880      ELSE
8890        GOTO 7480
8900      END IF
8910    END IF
8920    REM-----------------------------------------------------------------------
8930    !
8940    !
8950    Sm=0
8960    FOR I=2 TO Arcnum
8970      B=360
8980      Bb=3600
8990      Bbb=B*Bb
9000      FOR I1=1 TO Arcpoints+1
9010        A(I-1,I1)=Bbb-Mot(1,I1)+Mot(I,I1)
9020        IF A(I-1,I1)>Bbb THEN
9030          A(I-1,I1)=A(I-1,I1)-Bbb
9040          GOTO 9080
```

```
9050        ELSE
9060          GOTO 9090
9070        END IF
9080        Sm=Sm+A(I-1,I1)
9090      NEXT I1
9100      Mesarc(I-1)=Sm/(Arcpoints+1)
9110      Sm=0
9120      Moiresl=Mesarc(I-1)/3600
9130      Fractlepta=Moiresl-INT(Moiresl)
9140      Q(I-1)=Moiresl-Fractlepta
9150      Leptal=Fractlepta*60
9160      Fractdeft=Leptal-INT(Leptal)
9170      R(I-1)=Leptal-Fractdeft
9180      E(I-1)=Fractdeft*60
9190      IF (E(I-1)*10-INT(E(I-1)*10)) =.49 THEN
9200        E(I-1)=(INT(E(I-1)*10)+1)/10
9210      END IF
9220    NEXT I
9230    PRINT USING 9240
9240    IMAGE 3/
9250    IF Pnr=1 THEN OUTPUT @Prt USING 9240
9260    FOR I=2 TO Arcnum
9270      PRINT USING 9280;I,Q(I-1),R(I-1),E(I-1)
9280      IMAGE "ORIENTATION CORRECTION OF ARC NO. ",2D," = ",3D,X,2Z,X,2Z.D,/
9290      IF Pnr=1 THEN OUTPUT @Prt USING 9280;I,Q(I-1),R(I-1),E(I-1)
9300    NEXT I
9310    FOR I=1 TO Arcnum
9320      FOR I1=1 TO Arcpoints+1
9330        IF I=1 THEN
9340          Redmo(I,I1)=Mot(I,I1)
9350          GOTO 9670
9360        ELSE
9370          GOTO 9390
9380        END IF
9390        IF Mot(1,1) Mot(I,1) THEN
9400          Redmo(I,I1)=Bbb-Mesarc(I-1)+Mot(I,I1)
9410          IF Redmo(I,I1)>Bbb THEN
9420            Redmo(I,I1)=Redmo(I,I1)-Bbb
9430          END IF
9440        ELSE
9450          IF Mot(1,1)<Mot(I,1) THEN
9460            Redmo(I,I1)=Mot(I,I1)-Mesarc(I-1)
9470            IF Redmo(I,I1)<0 THEN
9480              Redmo(I,I1)=Bbb+Redmo(I,I1)
9490            END IF
9500          ELSE                  ! Mot(1,1)=Mot(I,1)
9510            PRINT USING 9520
9520            IMAGE "The execution of the programme has been stopped. CHECK YOUR
 DATA FILE. You should have started from another point on the horiz. circle !!!"
9530            IF Pnr=1 THEN OUTPUT @Prt USING 9520
9540            GOTO 12010
9550          END IF
9560        END IF
9570        Moiresl=Redmo(I,I1)/3600
9580        Fractlepta=Moiresl-INT(Moiresl)
9590        M(I,I1)=Moiresl-Fractlepta
9600        Leptal=Fractlepta*60
9610        Fractdeft=Leptal-INT(Leptal)
9620        L(I,I1)=Leptal-Fractdeft
9630        S(I,I1)=Fractdeft*60
```

```
9640       IF (S(I,I1)*10-INT(S(I,I1)*10))>=.49 THEN
9650         S(I,I1)=(INT(S(I,I1)*10)+1)/10
9660       END IF
9670     NEXT I1
9680   NEXT I
9690   FOR I=2 TO Arcnum
9700     PRINT USING 9710;I
9710     IMAGE 4/,"St point",4X,"Obs point",3X,"REDUCED MEAN VALUE FOR ARC NO. ",
2D,/,"---------",4X,"---------",3X,"--------------------------------------",/
9720     IF Pnr=1 THEN OUTPUT @Prt USING 9710;I
9730     FOR I1=1 TO Arcpoints+1
9740       IF I1=1 THEN
9750         PRINT USING 9760;St_point$,Point_name$(I1),M(I,I1),L(I,I1),S(I,I1)
9760         IMAGE 3X,3A,9X,3A,17X,3D,X,2Z,X,2Z.D
9770         IF Pnr=1 THEN OUTPUT @Prt USING 9760;St_point$,Point_name$(I1),M(I,I
1),L(I,I1),S(I,I1)
9780       END IF
9790       IF I1>1 AND I1<Arcpoints+1 THEN
9800         PRINT USING 9810;Point_name$(I1),M(I,I1),L(I,I1),S(I,I1)
9810         IMAGE 15X,3A,17X,3D,X,2Z,X,2Z.D
9820         IF Pnr=1 THEN OUTPUT @Prt USING 9810;Point_name$(I1),M(I,I1),L(I,I1)
,S(I,I1)
9830       END IF
9840       IF I1=Arcpoints+1 THEN
9850         PRINT USING 9860;Point_name$(1),M(I,I1),L(I,I1),S(I,I1)
9860         IMAGE 15X,3A,17X,3D,X,2Z,X,2Z.D
9870         IF Pnr=1 THEN OUTPUT @Prt USING 9860;Point_name$(1),M(I,I1),L(I,I1),
S(I,I1)
9880       END IF
9890     NEXT I1
9900   NEXT I
9910   PRINT USING 9920
9920   IMAGE 10/,"St point",4X,"Obs point",3X,"FINAL MEAN VALUE ",/,"---------",4X
,"---------",3X,"----------------",/
9930   IF Pnr=1 THEN OUTPUT @Prt USING 9920
9940   FOR I1=1 TO Arcpoints+1
9950     FOR I=1 TO Arcnum
9960       Smfin=Smfin+Redmo(I,I1)
9970     NEXT I
9980     Finmean(I1)=Smfin/Arcnum
9990     Smfin=0
10000    Moiresl=Finmean(I1)/3600
10010    Fractlepta=Moiresl-INT(Moiresl)
10020    Qfm(I1)=Moiresl-Fractlepta
10030    Leptal=Fractlepta*60
10040    Fractdeft=Leptal-INT(Leptal)
10050    Rfm(I1)=Leptal-Fractdeft
10060    Efm(I1)=Fractdeft*60
10070    IF (Efm(I1)*10-INT(Efm(I1)*10))>=.49 THEN
10080      Efm(I1)=(INT(Efm(I1)*10)+1)/10
10090    END IF
10100    IF I1=1 THEN
10110      PRINT USING 10120;St_point$,Point_name$(I1),Qfm(I1),Rfm(I1),Efm(I1)
10120      IMAGE 3X,3A,9X,3A,9X,3D,X,2Z,X,2Z.D
10130      IF Pnr=1 THEN OUTPUT @Prt USING 10120;St_point$,Point_name$(I1),Qfm(I1
),Rfm(I1),Efm(I1)
10140    END IF
10150    IF I1>1 AND I1<Arcpoints+1 THEN
10160      PRINT USING 10170;Point_name$(I1),Qfm(I1),Rfm(I1),Efm(I1)
10170      IMAGE 15X,3A,9X,3D,X,2Z,X,2Z.D
```

```
10180      IF Pnr=1 THEN OUTPUT @Prt USING 10170;Point_name$(I1),Ofm(I1),Rfm(I1),
Efm(I1)
10190    END IF
10200    IF I1=Arcpoints+1 THEN
10210      PRINT USING 10220;Point_name$(1),Ofm(I1),Rfm(I1),Efm(I1)
10220      IMAGE 15X,3A,9X,3D,X,2Z,X,2Z.D
10230      IF Pnr=1 THEN OUTPUT @Prt USING 10220;Point_name$(1),Ofm(I1),Rfm(I1),E
fm(I1)
10240    END IF
10250 NEXT I1
10260 WAIT 2
10270 IF Finmean(1)<Finmean(Arcpoints+1) THEN
10280    Horiz_closure=ABS(Finmean(Arcpoints+1)-Finmean(1))
10290    Correction=ABS(Finmean(Arcpoints+1)-Finmean(1))/Arcpoints
10300    IF Finmean(1)<Finmean(Arcpoints+1) THEN
10310      FOR I=1 TO Arcpoints+1
10320        IF I=1 THEN
10330          Ro(1)=Finmean(1)
10340          GOTO 10390
10350        ELSE
10360          GOTO 10380
10370        END IF
10380        Ro(I)=-((I-1)*Correction)+Finmean(I)
10390      NEXT I
10400    ELSE
10410      FOR I=1 TO Arcpoints+1
10420        IF I=1 THEN
10430          Ro(1)=Finmean(1)
10440          GOTO 10490
10450        ELSE
10460          GOTO 10480
10470        END IF
10480        Ro(I)=(I-1)*Correction+Finmean(I)
10490      NEXT I
10500    END IF
10510 ELSE
10520    FOR I=1 TO Arcpoints+1
10530      Ro(I)=Finmean(I)
10540    NEXT I
10550 END IF
10560 PRINT USING 10570;Horiz_closure
10570 IMAGE 5/,"HORIZON CLOSURE ERROR [sec] = ",2,4D,/,"------------------------------
-------------"
10580 IF Pnr=1 THEN OUTPUT @Prt USING 10570;Horiz_closure
10590 PRINT USING 10600;Correction
10600 IMAGE "CORRECTION [sec] = ",2,4D,/,"-----------------------------------",/
10610 IF Pnr=1 THEN OUTPUT @Prt USING 10600;Correction
10620 PRINT USING 10630
10630 IMAGE ,"========",4X,"=========",3X,"===================="
10640 IF Pnr=1 THEN OUTPUT @Prt USING 10630
10650 PRINT USING 10660
10660 IMAGE "St point",4X,"Obs point",3X,"REDUCED OBSERVATIONS ",/,"========",4X
,"=========",3X,"====================",/
10670 IF Pnr=1 THEN OUTPUT @Prt USING 10660
10680 FOR I=1 TO Arcpoints+1
10690    Moireel=Ro(I)/3600
10700    Fractlepta=Moireel-INT(Moireel)
10710    Q(I)=Moireel-Fractlepta
10720    Leptal=Fractlepta*60
10730    Fractdeft=Leptal-INT(Leptal)
```

```
10740    R(I)=Leptal-Fractdeft
10750    E(I)=Fractdeft*60
10760    IF (E(I)*10-INT(E(I)*10))>=.49 THEN
10770      E(I)=(INT(E(I)*10)+1)/10
10780    END IF
10790    IF I=1 THEN
10800      PRINT USING 10810;St_point$,Point_name$(I),O(I),R(I),E(I)
10810      IMAGE 3X,3A,9X,3A,10X,3D,X,2Z,X,2Z.D
10820      IF Pnr=1 THEN OUTPUT @Prt USING 10810;St_point$,Point_name$(I),O(I),R(
I),E(I)
10830    END IF
10840    IF I>1 AND I<Arcpoints+1 THEN
10850      PRINT USING 10860;Point_name$(I),O(I),R(I),E(I)
10860      IMAGE 15X,3A,10X,3D,X,2Z,X,2Z.D
10870      IF Pnr=1 THEN OUTPUT @Prt USING 10860;Point_name$(I),O(I),R(I),E(I)
10880    END IF
10890    IF I=Arcpoints+1 THEN
10900      PRINT USING 10910;Point_name$(I),O(I),R(I),E(I)
10910      IMAGE 15X,3A,10X,3D,X,2Z,X,2Z.D
10920      IF Pnr=1 THEN OUTPUT @Prt USING 10910;Point_name$(I),O(I),R(I),E(I)
10930    END IF
10940 NEXT I
10950 WAIT 2
10960 REM----------------------------------------------------------------------
10970 REM
10980 REM
10990 REM**********************************************************************
11000 REM                  CALCULATE   OBSERVATIONAL   RESIDUALS
11010 REM .....................................................................
11020 !
11030
11040 PRINT USING 11050
11050 IMAGE 5/,"...........................................................................
."
11060 IF Pnr=1 THEN OUTPUT @Prt USING 11050
11070 PRINT USING 11080
11080 IMAGE 3/,10X,"OBSERVATIONAL RESIDUALS [sec]",/,10X,"*********************
*******",/
11090 IF Pnr=1 THEN OUTPUT @Prt USING 11080
11100 Typefalma=0
11110 S1=0
11120 FOR I=1 TO Arcnum
11130   FOR I1=1 TO Arcpoints+1
11140     U(I,I1)=Finmean(I1)-Redmo(I,I1)
11150     IF U(I,I1)>=0 THEN
11160       IF (U(I,I1)*10-INT(U(I,I1)*10))>=.445 THEN
11170         V(I,I1)=(INT(U(I,I1)*10)+1)/10
11180       ELSE
11190         V(I,I1)=INT(U(I,I1)*10)/10
11200       END IF
11210     ELSE
11220       IF ABS((U(I,I1)*10)-(INT(U(I,I1)*10)+1))>=.445 THEN
11230         V(I,I1)=INT(U(I,I1)*10)/10
11240       ELSE
11250         V(I,I1)=(INT(U(I,I1)*10)+1)/10
11260       END IF
11270     END IF
11280     S1=S1+1
11290     U1(S1)=U(I,I1)
11300     OUTPUT @Res,S1;U1(S1)
```

```
11310        Typsfalma=Typsfalma+U(I,I1)^2
11320        IF Arcnum<=9 AND Arcpoints+1<=9 THEN
11330          IF V(I,I1)<>0 THEN
11340            PRINT USING 11350;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I1),S(I
,I1),V(I,I1)
11350            IMAGE 11X,"U(",D,",",D,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X,2Z,X,2Z
.D,"  =  ",SZ.D
11360            IF Pnr=1 THEN OUTPUT @Prt USING 11350;I,I1,Qfm(I1),Rfm(I1),Efm(I1)
,M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11370          ELSE
11380            PRINT USING 11390;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I1),S(I
,I1),V(I,I1)
11390            IMAGE 11X,"U(",D,",",D,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X,2Z,X,2Z
.D,"  =  ",X,Z.D
11400            IF Pnr=1 THEN OUTPUT @Prt USING 11390;I,I1,Qfm(I1),Rfm(I1),Efm(I1)
,M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11410          END IF
11420        ELSE
11430          IF Arcnum<=9 AND Arcpoints+1>9 THEN
11440            IF V(I,I1)<>0 THEN
11450              PRINT USING 11460;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I1),S
(I,I1),V(I,I1)
11460              IMAGE 11X,"U(",D,",",DD,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X,2Z,X
,2Z.D,"  =  ",SZ.D
11470              IF Pnr=1 THEN OUTPUT @Prt USING 11460;I,I1,Qfm(I1),Rfm(I1),Efm(I
1),M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11480            ELSE
11490              PRINT USING 11500;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I1),S
(I,I1),V(I,I1)
11500              IMAGE 11X,"U(",D,",",DD,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X,2Z,X
,2Z.D,"  =  ",X,Z.D
11510              IF Pnr=1 THEN OUTPUT @Prt USING 11500;I,I1,Qfm(I1),Rfm(I1),Efm(I
1),M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11520            END IF
11530          ELSE
11540            IF Arcnum>9 AND Arcpoints+1<=9 THEN
11550              IF V(I,I1)<>0 THEN
11560                PRINT USING 11570;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I1)
,S(I,I1),V(I,I1)
11570                IMAGE 11X,"U(",DD,",",D,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X,2Z
,X,2Z.D,"  =  ",SZ.D
11580                IF Pnr=1 THEN OUTPUT @Prt USING 11570;I,I1,Qfm(I1),Rfm(I1),Efm
(I1),M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11590              ELSE
11600                PRINT USING 11610;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I1)
,S(I,I1),V(I,I1)
11610                IMAGE 11X,"U(",DD,",",D,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X,2Z
,X,2Z.D,"  =  ",X,Z.D
11620                IF Pnr=1 THEN OUTPUT @Prt USING 11610;I,I1,Qfm(I1),Rfm(I1),Efm
(I1),M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11630              END IF
11640            ELSE
11650              IF Arcnum>9 AND Arcpoints+1>9 THEN
11660                IF V(I,I1)<>0 THEN
11670                  PRINT USING 11680;I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I
1),S(I,I1),V(I,I1)
11680                  IMAGE 11X,"U(",DD,",",DD,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X
,2Z,X,2Z.D,"  =  ",SZ.D
11690                  IF Pnr=1 THEN OUTPUT @Prt USING 11680;I,I1,Qfm(I1),Rfm(I1),E
fm(I1),M(I,I1),L(I,I1),S(I,I1),V(I,I1)
```

```
11700             ELSE
11710               PRINT USING 11720:I,I1,Qfm(I1),Rfm(I1),Efm(I1),M(I,I1),L(I,I
1),S(I,I1),V(I,I1)
11720               IMAGE 11X,"U(",DD,",",DD,") = ",3D,X,2Z,X,2Z.D,"  - ",X,3D,X
,2Z,X,2Z.D,"  =  ",X,Z.D
11730               IF Pnr=1 THEN OUTPUT @Prt USING 11720:I,I1,Qfm(I1),Rfm(I1),E
fm(I1),M(I,I1),L(I,I1),S(I,I1),V(I,I1)
11740             END IF
11750           END IF
11760         END IF
11770       END IF
11780     END IF
11790   NEXT I1
11800 NEXT I
11810 CONTROL @Res,7;S1+1
11820 Total=Arcnum*(Arcpoints+1)
11830 OUTPUT @Top,1;Total
11840 PRINT USING 11850;Total
11850 IMAGE 7/,11X,"TOTAL NO. OF OBSERVATIONAL RESIDUALS = ",3D
11860 IF Pnr=1 THEN OUTPUT @Prt USING 11850;Total
11870 Tvpsfalma=SQR(Tvpsfalma/(Total-(Arcnum-1+(Arcpoints+1))))
11880 Mes_or_sfalma=Tvpsfalma/SQR(Total)
11890 PRINT USING 11900;Tvpsfalma
11900 IMAGE 5/,11X,"STANDARD DEVIATION OF A DIRECTION - sigma(c) = ",X,Z.4D,/
11910 IF Pnr=1 THEN OUTPUT @Prt USING 11900;Tvpsfalma
11920 PRINT USING 11930;Mes_or_sfalma
11930 IMAGE 11X,"STANDARD DEVIATION OF THE MEAN - sigma(x) = ",X,Z.4D
11940 IF Pnr=1 THEN OUTPUT @Prt USING 11930;Mes_or_sfalma
11950 PRINT USING 11960
11960 IMAGE 4/,"************************************************************
***************",5/
11970 IF Pnr=1 THEN OUTPUT @Prt USING 11960
11980 DISP "Press <CONTINUE> to continue";
11990 PAUSE
12000 OUTPUT 2;"K";
12010 REM--------------------------------------------------------------------
12020 REM
12030 REM
12040 REM********************************************************************
12050 REM              ANOTHER  RUN  WITH  THE  SAME  DATA  FILE
12060 REM ...................................................................
12070 !
12080 !
12090 DISP "Do you want another run with the same data ? (Y/N)";
12100 INPUT Newrun$
12110 IF Newrun$[1,1]="N" THEN
12120   OUTPUT 2;"K";
12130   GOTO 12220
12140 ELSE
12150   IF Newrun$[1,1]="Y" THEN
12160     PRINTER IS 1
12170     GOTO 1770
12180   ELSE
12190     GOTO 12090        ! Mistaken entry - Try again
12200   END IF
12210 END IF
12220 PRINT USING 12230
12230 IMAGE 12/,"                IF YOU WANT ANOTHER RUN YOU MUST PRESS <RUN>",/,
                "                         Please Wait"
```

```
12240 WAIT 4
12250 OUTPUT 2;"F";
12260 REM ----------------------------------------------------------------
12270 REM
12280 REM
12290 REM
12300 REM ****************************************************************
12310 REM **************                                    **************
12320 REM                S  U  B  R  O  U  T  I  N  E  S
12330 REM **************                                    **************
12340 REM ****************************************************************
12350 '
12360 '
12370 REM*****************************************************************
12380 REM                  SUBROUTINE      Upodeigma
12390 REM .................................................................
12400 REM
12410 STOP
12420 Upodeigma:
12430 '
12440 PRINT USING 12450
12450 IMAGE 10X,"-------------------------------------------------------------"
12460 PRINT USING 12470
12470 IMAGE 10X,"PLEASE ENTER THE FOLLOWING DATA USING 3 DIGITS FOR DEGREES,"
12480 PRINT USING 12490
12490 IMAGE 10X,"2 DIGITS FOR MINUTES & 3 DIGITS FOR SECONDS. FOR EXAMPLE :"
12500 PRINT USING 12510
12510 IMAGE 10X,"The number 0020861 represents 2 degrees, 8 min & 6.1 secs."
12520 PRINT USING 12530
12530 IMAGE 10X,"That is, the seconds are always represented by one decimal,"
12540 PRINT USING 12550
12550 IMAGE 10X,"even if this is zero, e.g. 27.0 seconds."
12560 PRINT USING 12570
12570 IMAGE "-------------------------------------------------------------"
12580 RETURN
12590 '
12600 REM//  %%%%%%/////%%%%%%                   %%%%%%/////%%%%%%////
12610 '
12620 REM-----------------------------------------------------------------
12630 REM
12640 REM
12650 REM*****************************************************************
12660 REM                        CLOSE  PATHS
12670 REM .................................................................
12680 '
12690 ASSIGN @Prt TO *
12700 ASSIGN @Emp TO *
12710 ASSIGN @Emp_12 TO *
12720 ASSIGN @Path TO *
12730 ASSIGN @Odos TO *
12740 ASSIGN @Oaz TO *
12750 ASSIGN @Plm TO *
12760 ASSIGN @Akpibeia TO *
12770 ASSIGN @Res TO *
12780 ASSIGN @Top TO *
12790 MASS STORAGE IS Msus_0$
12800 REM-----------------------------------------------------------------
12810 '
12820 END
```

APPENDIX G

APPENDIX G. "HI_DI_TEST" PROGRAM

G.1 REQUIRED BACKGROUND

The "HI_DI_TEST" program runs with BASIC 2.0 and BASIC 2.1 WITH EXTENSIONS (provided, that there is memory of ~786 Kbytes, otherwise the user should decrease the size of the matrices which are specified at the beginning of the program), or only with BASIC 2.0. Also, this has been tested on a HP-9816 mini computer using a HP-9121 double disk drive. In addition, a HP-9872B plotter with four different colour pens has been used to plot the histograms and distributions.

This program can accept up to a maximum number of 2000 residuals (provided the memory is ~786 kbytes). The data files for this program are the eighth and ninth files of the "THEO" program, which are connected by the following paths (see Appendix F):

$$File^{8} \rightarrow Res$$
$$File^{9} \rightarrow Top$$

With "HI_DI_TEST" program, the user has the possibility to run either each set separately, or any combinations of the sets which have already been created by the "THEO" program. Also, there is flexibility to generate the histograms of the residuals in any desired class intervals. A menu is provided so that the user can select any of the following distributions:

i. Normal (Gauss)
ii. Truncated Normal (95%)
iii. Truncated Normal (97.5%)
iv. Truncated Normal (99%)
v. Inverse Gaussian

After plotting the histograms, the Chi-Square test with the predetermined class intervals (equal intervals) is carried out on the specific distribution. If one cell has a theoretical frequency less than 5, this is merged with the adjacent cell so that a theoretical frequency ≥5 is achieved. If this minimum theoretical frequency is not achieved the two previous cells are merged with the following successive cell, and so forth. At the end of the Chi-Square test, there is an automatic comparison between the $X^2_{\text{Observed frequency}}$ and the $X^2_{\text{Theoretical frequency}}$ (for 5% significance level according to the Table A2 in Appendix A). Based on the comparison, the program indicates the acceptance or rejection of the tested distribution.

Finally, a plot which shows the differences between the observed and the theoretical values of the selected distribution is available.

The results of this computer program may be obtained on the screen only, or directed to both the printer and the screen at the same time. Plots of the histograms, distributions and differences between actual and theoretical values may, therefore, be obtained on the screen, or on the plotter.

G.2 SPECIFIC INFORMATION

i) Histograms: The class interval is chosen by the user. The line (x axis) is divided into cells $x_1<x_2<\ldots<x_S$, so that S+1 intervals are created. That is:

- For *Normal* distribution:

$$\begin{aligned} I_0 &= (-30, x_1) \\ I_1 &= [x_1, x_2) \\ &\ \ \vdots \\ I_{S-1} &= [x_{S-1}, x_S) \\ I_S &= [x_S, +30) \end{aligned}$$

- For *Truncated Normal* distribution:

$$\begin{aligned} I_0 &= (-g, x_1) \\ I_1 &= [x_1, x_2) \\ &\ \ \vdots \\ I_{S-1} &= [x_{S-1}, x_S) \\ I_S &= [x_S, +g) \end{aligned}$$

where:

$g = \mu \pm c\ \sigma$

$c = \pm 1.960$ (95%), $c = \pm 2.244$ (97.5%) or

$c = \pm 2.576$ (99%)

- For *Inverse Gaussian* distribution:

$$\begin{aligned} I_0 &= (+0.05, x_1) \\ I_1 &= [x_1, x_2) \\ &\ \ \vdots \\ I_{S-1} &= [x_{S-1}, x_S) \\ I_S &= [x_S, +30) \end{aligned}$$

For the case of Normal and Inverse Gaussian distributions, the lower and upper limit should have been $(-\infty, +\infty)$ instead of $(-30, +30)$, and $(0, +\infty)$ instead of $(0, +30)$, respectively. However, this does not change the results since the limits -30, +30 are much bigger than the residuals (the largest of them does not exceed the value of 3".8). In the case of the Inverse Gaussian distribution, for practical reasons the integration starts from the value of +0.05, since this distribution [see formula (4.43)] is not defined for values less than or equal to zero.

ii) **Numerical integration**: Numerical integration has been used to calculate the area under the p.d.f. curve. Specifically, since certain limits have been given, an approximate integration over a finite interval has been developed.

Simpson's rule which is by far the most frequently used in obtaining approximate integrals has been considered. Some basic information for Simpson's rule is given below (Bakopoulos, 1979):

Suppose that the integration will take place for a certain probability density function f(x) between the lower a and upper b limits. Simpson's approximation is:

$$\int_a^b f(x)dx = \frac{b-a}{6}\left[f(a) + 4f((a+b)/2) + f(b) \right] \qquad (A.1)$$

However, Simpson's rule is most frequently applied in its *extended* or *compound* form. The interval [a,b] is divided into a number of equal sub-intervals or panels and Simpson's rule is applied to each. In order to describe the final result, it is convenient to employ a somewhat different notation than that of (A.1). Let:

$$a = x_0 < x_1 < \dots < x_{2n-1} < x_{2n} = b$$

be a sequence of equally spaced points in [a,b]:

$$x_{i+1} - x_i = h \quad , \quad i=0,\dots,2n-1 \qquad (A.2)$$

Set $f_i=f(x_i)$. Then, the compound Simpson's rule is:

$$\int_{x_0}^{x_{2n}} f(x)dx = (h/3)\Big[f_0 + 4(f_1+f_3+\ldots+f_{2n-1})$$

$$+ 2(f_2+f_4+\ldots+f_{2n-2})+f_{2n}\Big] + E_n \qquad (A.3)$$

The remainder E_n is given by:

$$E_n = -\frac{n\,h^5}{90} f^{(iv)}(\xi) \quad , \quad a < \xi < b \qquad (A.4)$$

If N designates the even number of sub-divisions (as it has been designed in this program) of [a,b], the N=2n and h=(b-a)/N, so that:

$$E_n = -\frac{(b-a)^5}{180\,N^4} f^{(iv)}(\xi) \quad , \quad a < \xi < b \qquad (A.5)$$

Chi-Square goodness-of-fit test: This test has been programmed according to the formula and methods outlined in chapters 5 and 6.

G.3 FURTHER INFORMATION

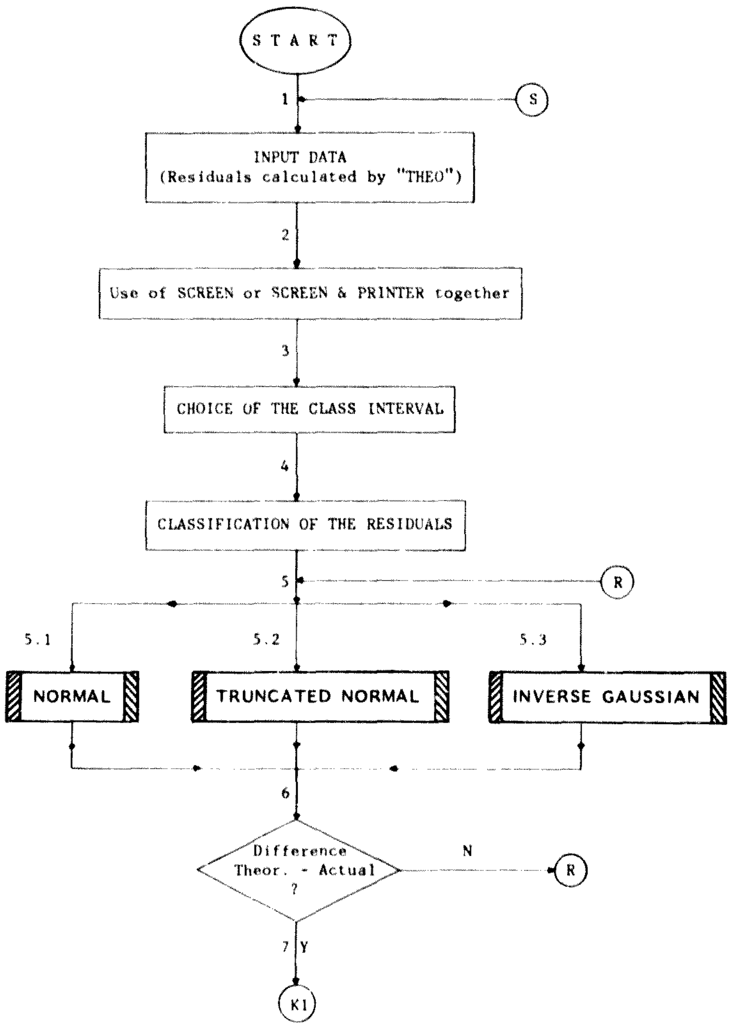
Some of the most important variables will be cited below:

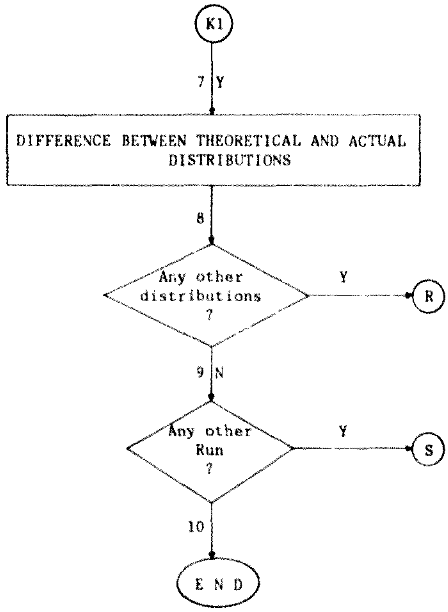
L(5000) : This matrix contains the residuals of a specific set which has already been calculated by the "THEO" program. These residuals are obtained by the File8of "THEO" program using the path "Res".

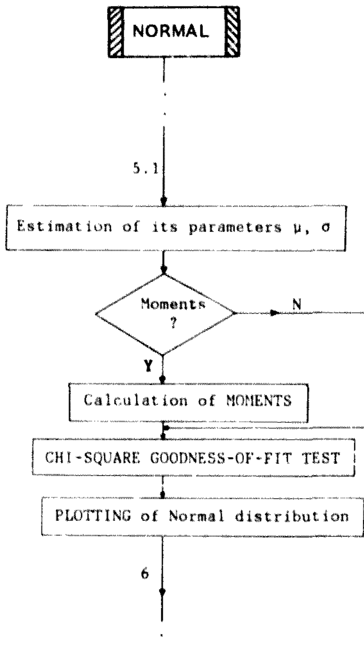
Res_1(5000)	: This matrix contains residuals of the com - bination of several sets, which have been created by the previous program.
Res(5000)	: This matrix includes the rounded off resi - duals of the combination of several arcs, which have been created by the previous program.
Th_fr_e(500)	: It contains the theoretical frequencies of the selected distribution.
Obs_fr_n(500)	: It contains the observed frequencies values of the selected distribution.
Th_fr_e_tel(500)	: It contains the summed values for the theo - retical frequencies, when there is one or more cells with theoretical frequencies less than 5.
Obs_fr_n_tel(500)	: It contains the summed values for the obser- ved frequencies, when there is one or more cells with theoretical frequencies less than 5.
Test_x_2_tel(100)	: This contains separately each term of the Chi-Square test.
A(30)	: It contains the values of the Table A2 of the Appendix A for the 5% significance level.

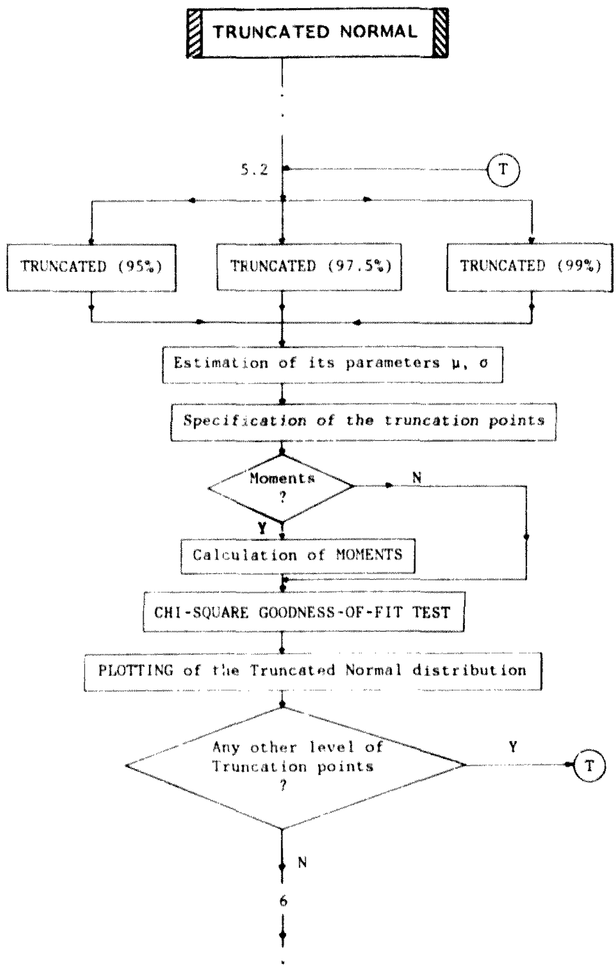
The flow chart of the "HI_DI_TEST" program is given below and the print out of this program is also provided.

G.4 FLOW CHART




K1
7
Y
DIFFERENCE BETWEEN THEORETICAL AND ACTUAL DISTRIBUTIONS
8
Any other distributions ?
Y
R
9
N
Any other Run ?
Y
S
10
E N D


NORMAL
5.1
Estimation of its parameters μ, σ
Moments ?
N
Y
Calculation of MOMENTS
CHI-SQUARE GOODNESS-OF-FIT TEST
PLOTTING of Normal distribution
6


TRUNCATED NORMAL
5.2
T
TRUNCATED (95%)
TRUNCATED (97.5%)
TRUNCATED (99%)
Estimation of its parameters μ, σ
Specification of the truncation points
Moments ?
N
Y
Calculation of MOMENTS
CHI-SQUARE GOODNESS-OF-FIT TEST
PLOTTING of the Truncated Normal distribution
Any other level of Truncation points ?
Y
T
N
6