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An empirical evaluation of capital asset pricing models on the JSE

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AN EMPIRICAL EVALUATION OF CAPITAL ASSET PRICING MODELS ON THE JSE

A RESEARCH REPORT SUBMITTED BY

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ABSTRACT

The Capital Asset Pricing Model (CAPM), as introduced by Markowitz (1952), Sharpe (1964), Lintner (1965), Black (1972) and Mossin (1966), offers powerful and intuitively pleasing predictions about the risk and return relationship that is expected when investing in equities. Studies on the empirical strength of the CAPM such as Fama and French (1992), however, indicate that the model does not reflect the share return actually obtained on the equity market. Attempting to improve the model, Fama and French (1993) enhanced the original CAPM by incorporating other factors which may be relevant in predicting the return on share investments, specifically, the book – to – market ratio and the market capitalisation of the entity. Carhart (1997) further attempted to improve the CAPM by incorporating momentum analysis together with the 3 factors identified by Fama and French (1993). This research report empirically evaluates the accuracy of the above three models in calculating the cost of equity on the Johannesburg Stock Exchange over the period 2002 to 2012. Portfolios of shares were constructed based on the three models for the purposes of this evaluation.

The results indicate that the book-to-market ratio and market capitalisation are able to add some robustness to the CAPM, but that the results of formulating book – to – market and market capitalization portfolios is highly volatile and therefore may lead to inconsistent results going forward. By incorporating the short run momentum effect, the robustness of the CAPM is improved substantially, as the Carhart model comes closest to reflecting what, for the purposes of this study, represents the ideal performance of an effective asset pricing model. The Fama and French (1993) and Carhart (1997) models therefore present a step forward in formulating an asset pricing model that will hold up under empirical evaluation, where the expected cost of equity is representative of the total return that can be expected from investing in a portfolio of shares. It is however established that the additional factors indicated above are volatile, and this volatility may influence the results of a longer term study.

1 INTRODUCTION

1.1 PURPOSE OF THE STUDY

The purpose of this research report is to examine the extent to which the estimated expected return on an investment in shares on the Johannesburg Stock Exchange (JSE), determined using capital asset pricing models, accurately reflects the actual return that is earned by investing in such equities, for the period 2002 to 2012. The expected return on an investment is calculated by applying the following three asset pricing models: (1) the traditional Capital Asset Pricing Model (CAPM) as advocated by Sharpe (1964), Lintner (1965) and Black (1972); (2) the Fama and French (1993) three-factor model (F&F3); and finally (3) the Carhart (1997) four-factor model (C4).

This research report will take the following format; Chapter 1 will introduce the study and describe its context and significance, as well as setting out the research problems and definitions of important terms. Chapter 2 presents the background to the formation of the cost of equity and then explores the body of literature that exists on the subject of empirically testing the cost of equity internationally and in South Africa. Chapter 3 discusses the methodology of the study, describing in broad terms the approach that has been followed to empirically analyse the cost of equity. Chapter 4 presents the results and conclusions reached, while chapter 5 provides a summary of the results and concludes the study.

1.2 CONTEXT OF THE STUDY

Subrahmanyam (2008) describes investment ideology as being centered around the theories of (i) portfolio allocation based on risk and return and (ii) risk-based capital asset pricing models such as the traditional CAPM. A fundamentally important principle of finance is the tradeoff between risk and return. Logically, an investor is expected to demand a higher return for taking on excess risk. The traditional CAPM describes that an investor should obtain a higher return only where they take on higher systematic risk as reflected by the beta of the share (β). The F&F3 and C4 models also describe a positive relationship between risk and return, but contend that β is unable to accurately incorporate all of the information that the market adjusts for when analysing the actual return of a share on the stock exchange (Fama and French, 1993; Carhart, 1997). F&F3 and C4 contend that there are other factors that are important in describing the variability of share returns, which may or may not act as placeholders for other risks that are incorporated into share prices due to investor preferences (Fama and French, 1993; Carhart, 1997).

The CAPM remains the foundation of most finance courses and is therefore of great importance to the field of finance, while the F&F3 and C4 model are identified as the most recognized alternatives to the CAPM for the purpose of calculating expected cost of equity (Fama and French, 2004; Bello, 2008). It is therefore clear that when an investor makes a decision to invest according to traditional theories, emphasis is placed on the risk attributable to shares as reflected in each of the above equity pricing models, but may not be the only consideration that investors take into account (Womack and Zhang, 2003).

Fama and French (2004) contend that the empirical record of the CAPM is poor, indicating that the ability of CAPM to reflect actual returns on an investment in equity is questionable. Subrahmanyam (2008) asserts that the limitations of the CAPM cause investors to draw incorrect conclusions about which shares have value for investment purposes, which is supported by a variety of studies investigating the empirical robustness of the CAPM, both in the USA and South Africa (Fama and French, 1993; Van Rensburg and Robertson, 2003; Hoffman, 2012; Ward and Muller, 2012). Many of these studies describe factors that are better suited than β to explain the variability of share returns, including the book – to – market ratio (BTM) and the size of the company (Van Rensburg and Robertson, 2003; Hoffman, 2012). Although there have been many studies suggesting different factors that are best able to explain the returns on an investment in equity on the JSE, a test of the actual models that incorporate risk factors has not been extensively evidenced in a South African context.

1.3 PROBLEM STATEMENT

1.3.1 MAIN PROBLEM

The predictive accuracy of the CAPM, F&F3 and C4 models in predicting the cost of equity is as yet untested on the JSE.

1.3.2 SUB-PROBLEMS

The first sub-problem is to identify whether the Sharpe-Lintner-Black CAPM model is able to approximate the actual share return on the JSE.

The second sub-problem is to identify whether the addition of the book - to - market ratio and the size effect premium in the Fama and French (1993) 3 Factor model is able to approximate the actual share return on the JSE.

The third sub-problem is to identify whether the addition of momentum analysis in the Carhart (1997) 4 factor model is able to approximate the actual share return on the JSE.

1.4 SIGNIFICANCE OF THE STUDY

Although many studies have tested the explanatory ability of individual factors on share returns both internationally (Miller and Bromiley, 1990; Rouwenhorst, 1999) and in South Africa (Van Rensburg and Robertson, 2003; Auret and Sinclaire, 2006; Hoffman, 2012), there has been very little evidence in a South African context, of the ability of asset pricing models to accurately reflect the actual returns that are generated for ordinary shareholders. A reliable and accurate model that can predict the expected cost of equity is vital for applications in both theory and practice (Fama and French, 1993). This research report tests the practical applicability of three capital asset pricing models by measuring their accuracy in calculating the cost of equity in a South African context.

1.5 DELIMITATIONS OF THE STUDY

This research report tests the share price prediction value of the CAPM, F&F3 and C4 only. These models are identified in the literature as the most important alternatives for calculating cost of equity and are therefore of the most relevance to this study (Bello, 2008).

A significant discussion was noted in the capital asset literature regarding the additional factors that are incorporated into the F&F3. Fama and French (1993, 1996, 2004) argue that the additional F&F3 factors are indicators of systematic risk that are missed by the traditional CAPM β, meaning that the F&F3 model is still a strictly relevant traditional finance model, concerned with explaining the risk and return relationship. Lakonishok, Shleifer and Vishny (1994) however argue that these additional factors have been found to be significant in explaining share price movements due to irrational trading, and are not systematic risk indicators. This research report does not address this argument due to the methodology applied. This research report therefore only requires that the additional factors are good indicators of returns on shares, regardless of whether these factors represent un-diversifiable risk or not.

1.6 DEFINITION OF TERMS

Cost of equity: The cost of equity is defined as the return that an investor demands for taking on the risk that is inherent in a particular investment (Bartholdy and Peare, 2000). The term cost of equity will be used interchangeably with return on equity.

Book – To – Market ratio: The BTM ratio is calculated as the book value of ordinary equity, divided by the market value of the total ordinary share capital of the company (Fama and French, 1992). This ratio reflects a comparison of the value that investors place on the ordinary equity of a company, compared to the book value of equity reflected in the annual financial statements.

Short term price momentum: Short term momentum reflects the observed anomaly that where share prices have, over the previous 3 – 12 months, increased (decreased) in value, will continue to increase (decrease) in value over the next 3 – 12 months due to investor optimism (pessimism) (Jegadeesh and Titman, 2001).

Long term price momentum: Long term momentum reflects the observed anomaly where shares that have, over a period longer than 1 year, consistently increased in value, have increased past a reasonable share price and can be expected to decrease in value over the coming months due to the reversal of investor optimism which may have pushed a share price above what is a fair price (Baker and Stein, 2004).

State variables: A broad term for certain factors that influence how investors behave when they make investment decisions. This includes but is not limited to; available income from labour activities, existing expenditure required to maintain a lifestyle, the price of consumption goods and the investment opportunities that are available in the market.

Small-minus-big (SMB): SMB denotes a zero-sum investment portfolio of shares on the stock market that is chosen based on the size of a share. An investor short-sells a portfolio of large capitalisation shares, and uses the proceeds to purchase a portfolio of small shares on the stock market. SMB represents the return of such a portfolio, as the investor is expected to gain when small shares increase in value and loses when large shares increase in value.

High-minus-low (HML): HML denotes a zero-sum investment portfolio of shares on the stock market that is chosen based on the BTM ratio of a share. An investor short-sells a portfolio of low BTM shares, and uses the proceeds to purchase a portfolio of high BTM shares. HML represents the return of such a portfolio as an investor is expected to gain when investing in high BTM shares and loose when investing in low BTM shares.

Winner-minus-loser (WML): WML denotes a zero-sum investment portfolio of shares on the stock market that is chosen based on the short run momentum of a share. An investor short-sells a portfolio of loser shares and uses the proceeds to purchase a portfolio of winner shares. WML

represents the excess return of such a portfolio, as the investor gains when past winners continue to perform well, and past losers continue to perform badly.

Beta (β): β is a generic term that is used to describe the risk loading of an asset in relation to a particular factor that is being analysed. In this research report, references to β are to the traditional risk loading of a share in relation to the variability of the market portfolio of shares. The risk loadings relating to SMB, HML and WML are denoted as *SMBi, HMLi* and *WMLi* respectively, but in a strict sense these loadings also fall under the definition of β. A more in depth explanation of β is provided in the literature review and methodology sections.

1.7 ASSUMPTIONS

Investors are assumed to be rational utility maximisers, in line with the Markowitz (1952) concept of portfolio allocation, as an investor is interested in minimizing the risk for a particular return.

It is assumed that the 'best estimate' of the cost of equity is that which is best able to approximate the actual returns that an investor obtains by investing in a portfolio of shares (Miller and Bromiley, 1990).

A reasonable approximation of the market portfolio of Markowitz (1952) is assumed to be the top 160 shares of the JSE, adjusted for thin trading and beta estimation issues.

2 LITERATURE REVIEW

The literature review that follows is focused on providing a background to the study of capital asset pricing models and the results of their empirical evaluation, both internationally and in South Africa. This chapter will first provide an introduction to the concept of risk and return. Thereafter the development and testing of the CAPM is discussed, followed by the F&F3 model and the C4 model. Finally, areas for further research are described before moving on to the methodology that is to be applied in this study.

2.1 INTRODUCTION

Since the inception of the stock market**,** investors have attempted to develop theories to explain the historical returns on equities and to accurately predict the future expected returns from an investment in shares (Ward and Muller, 2012). Part of this endeavor has been the search for an equity pricing model that reflects the best estimate of the cost of equity.

Estimating the cost of equity has been an area for continued discussion between finance researchers and a variety of asset pricing models have been developed, including the CAPM, F&F3 and C4 models (Womack and Zhang, 2003). Traditionally, these models have their foundation in quantifying the risk that is inherent in an equity investment, as investors are expected to demand an increased return based on the riskiness of their investments (Sharpe, 1964; Lintner, 1965; Black, 1972).

2.2 RISK AND RETURN

If an investor is interested in obtaining a higher return, traditional finance explains that they will need to take on a higher level of risk (Sharpe, 1964). Intuitively, an investor would demand a higher *expected* return in exchange for taking on a higher level of risk. To a certain extent this risk-return relationship has been evidenced in practice, if one compares the historical long-run returns of equity, bonds and other securities (Womack and Zhang, 2003).

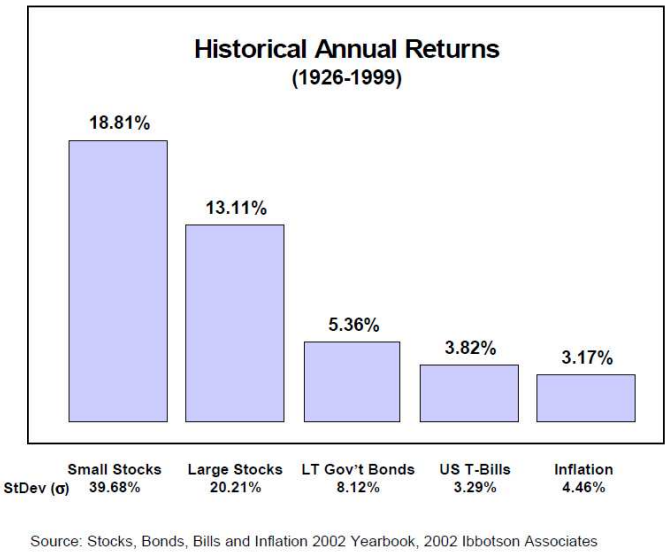

Figure 1

Historical Annual Returns on Various Asset Classes - (Womack and Zhang, 2003).

Figure 1 illustrates that equities have traditionally earned a higher return than government bonds. If one takes a look at the standard deviation of each portfolio which is detailed in Figure 1, the volatility of share returns can also be seen to be much higher than that of the other assets that are displayed. Therefore, it becomes evident that ordinary shares, as a riskier investment than government bonds, have historically provided a higher return to investors. It is interesting to note the evidence that small capitalisation shares have earned traditionally higher returns than large capitalisation shares. Based on the logic that is applied above, the higher return on small shares may provide some evidence that small capitalization shares are more risky than other assets. Whether it is enough to simply identify the volatility of assets as the reason for their return, or whether perhaps a factor such as the market capitalization of a share are also important to defining what return an asset will provide will form part of the discussion that follows when Fama and French introduce their capital asset pricing model.

2.2.1 QUANTIFYING RISKINESS

In order to understand the riskiness of an investment, it is important to assess the volatility of returns, as described by their standard deviation (Womack and Zhang, 2003). Consider an example where an investment is expected to generate R1 000 000 per annum into perpetuity, the price that an investor would be willing to pay for the investment is dependent on the uncertainty of the cash flows. Where the return of R1 000 000 is highly certain, the future cash flows would be discounted

at something close to the risk free rate. Where there is greater uncertainty regarding the timing and amount of future cash flows, the discount rate would be increased above the risk free rate as investors demand a higher return for taking on the increased volatility, and possibility of loss. Stated more simply, if the cash flows of an investment are more volatile, the investment is more risky and the cost of equity is increased as investors demand a higher return for taking on the excess risk (Womack and Zhang, 2003).

2.2.2 RISK, PORTFOLIO THEORY AND DIVERSIFICATION

What is described in the literature as "modern portfolio theory", but which dates as far back as the 1950's, takes the elements of risk and return and applies them to evaluate the return that an investor should demand where they hold a portfolio of shares.

Consider a situation where an investor can, without significant additional cost, invest in two shares with the same expected return, but with different volatility in returns (Markowitz, 1952). In such a scenario it becomes important to analyse whether the movements in share prices due to market events are usually in the same direction (when share price A increases, share price B also increases), or occur in different directions (when share price A increases, share price B decreases). This is termed the correlation of the shares, if they move in the same direction in response to an event, they are positively correlated, if they move in opposite directions they are negatively correlated. Two different share investments are not expected to have perfectly correlated returns and therefore the riskiness of the overall portfolio is reduced because the volatility of one share is offset to a certain extent by the volatility of the other (Markowitz, 1952). As long as assets are not perfectly positively correlated, an investor is able to reduce the portfolio risk without significant cost by reducing volatility, and therefore without affecting risk adjusted expected returns (Womack and Zhang, 2003). This concept is illustrated in Figure 2, where an investor places 50% of their funds in each of two shares with near perfect negative correlation, the volatility in returns of the portfolio are much less than that of the individual securities.

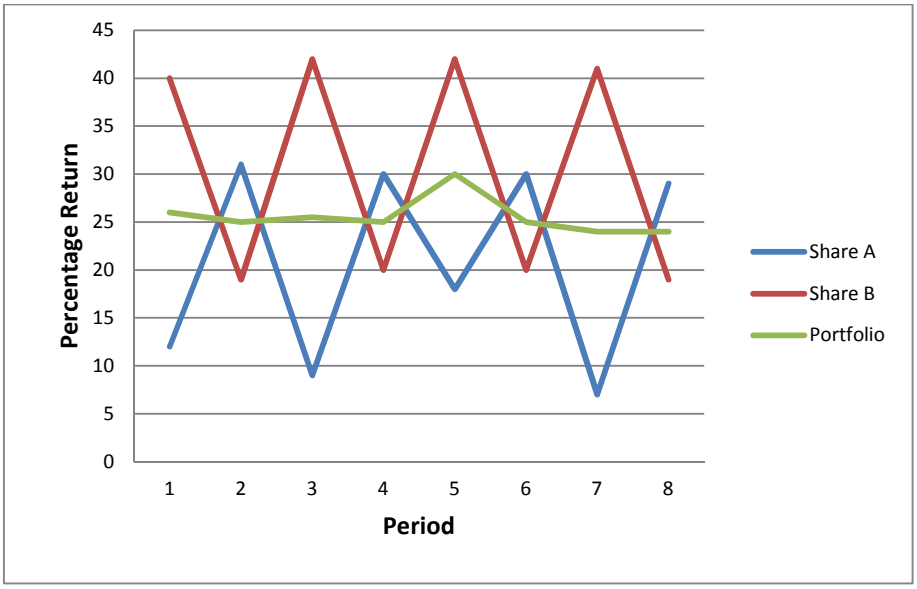

Figure 2

Return of a Portfolio of Shares with Near Perfect Negative Correlation

The above phenomenon, where volatility in portfolio returns is reduced by investing in more shares, is an example of the concept of diversification, and is one of the main tenets of modern portfolio theory (Fama and French, 1992). Intuitively, where an investor holds a portfolio of shares, risk is measured as the co-variance of all the assets that make up the portfolio, rather than the average of each assets individual variance. This is true because different assets will not normally be perfectly positively correlated, and therefore the variance of one asset may offset the variance of another asset, at least to some extent. Figure 3 shows that as more assets are added to a portfolio, the extent to which the riskiness of the portfolio can be reduced decreases. A common rule of thumb is that a portfolio containing 30 or more assets is well-diversified (Womack and Zhang, 2003).

If investors are able to diversify away part of the volatility of an individual share, then it makes sense that that investor should not be compensated for that part of share volatility that is asset specific and has no effect on the riskiness of a well diversified portfolio (Markowitz, 1952). Based on this logic, Markowitz (1952) describes that an investor can only expect to be compensated for that risk which cannot be diversified away, termed *systematic risk.*

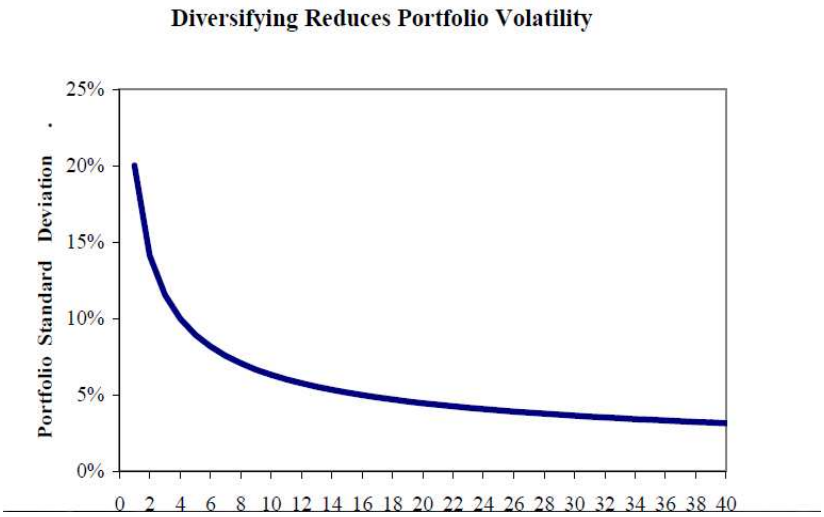

Figure 3

Illustration of the Effect of Diversifying - (Womack and Zhang, 2003).

2.2.3 FORMULATING THE MARKET PORTFOLIO

Markowitz (1952) states that investors will all hold a well-diversified portfolio of assets, which is called the market portfolio, because investors are rational and understand that they will only be compensated for taking on systematic risk. An interesting debate in the literature relates to the formulation of the market portfolio. In a strict sense, Markowitz (1952) intended that the market portfolio should include a market weighted inclusion of the entire universe of risky assets; including equities and bonds, but also including other assets; such as human capital, consumer durables and real estate. Therefore, it is extremely difficult to quantify the market portfolio, because items such as human capital will be very difficult to quantify. In line with this logic, all tests of the concept that investors should only be compensated for systematic risk have to a certain extent been invalidated by the inability to quantify the all encompassing market portfolio as a benchmark for testing what the market return should be (Roll, 1977; Fama and French, 2004). The response has been the use of a proxy for the market portfolio, which traditionally has been limited to the equity markets (Black, Jensen and Scholes, 1972; Fama and French, 2004).

2.2.4 QUANTIFYING SYSTEMATIC RISK

If it is assumed that investors are rational, and understand that they will only be compensated for taking on a higher level of systematic risk, the next important issue in traditional finance becomes how to compare the riskiness of an investment to the riskiness of the market, so that investors can make decisions regarding how much risk they wish to undertake (Markowitz, 1952). A widely

accepted measure of the systematic risk of a share is the market beta (β) of the share. β is defined in such a way that it reflects the variability of returns on a particular investment in relation to the return on a market weighted portfolio of assets that reflect the market portfolio (Fama and French, 1992).

2.2.5 THE SECURITIES MARKET LINE

The logic that underpins the risk and return relationship and the definition of systematic risk inevitably leads to researchers in finance becoming interested in quantifying the return that an investor could expect from investing in equities (Fama and French, 1992). The logical next step of the traditional finance literature was the development of a model for quantifying the risk-return trade off.

Figure 4 illustrates the logic behind the development of an asset pricing model (Fama and French, 2004). The x-axis measures the volatility of portfolio returns, while the y-axis measures the returns that are expected; therefore what is presented is a graphical analysis of the risk-return relationship. The curve *abc,* which is called the minimum variance frontier, shows the combination of risky assets that fall into portfolios that minimize risk for a given level of expected return (Fama and French, 2004). From Figure 4 it can be seen that if the investor wishes to obtain a high return, for example at point *a*, then he/she must accept higher risk. If an investor requires less risk, they can invest at point T and thereby accept a lower return. Therefore, an investor has the ability to choose a portfolio of shares that conforms to his/her risk appetite.

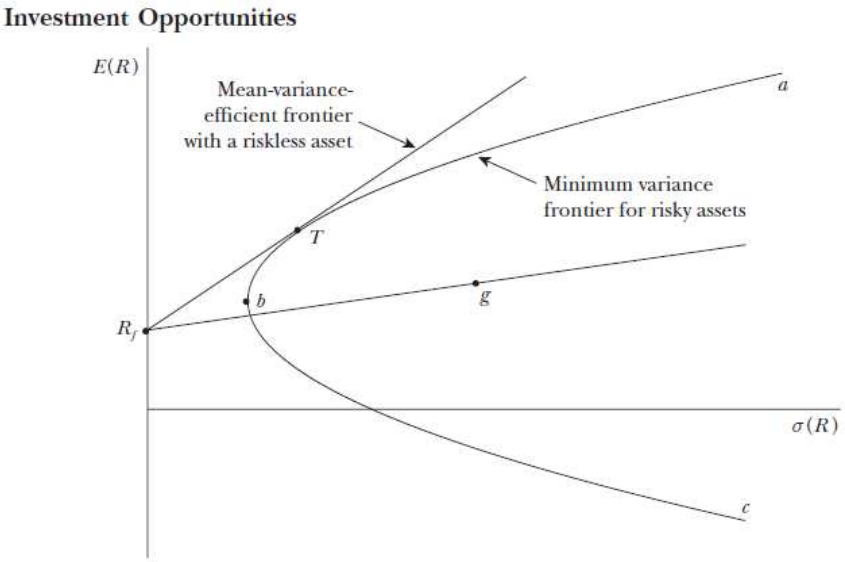

Figure 4 The Efficient Frontier - (Fama and French, 2004)

By adding the ability to borrow and lend at the risk free rate, the efficient set of portfolios turns into a straight line, as an investor is able to invest part of their funds in a risk free asset, at Rf, and the rest in a portfolio of risky assets that is as mean-variance efficient as possible (Sharpe, 1964). The most efficient portfolio of risky assets can be seen as point T, as this reflects the highest tangency point with Rf that falls along the efficient frontier, providing the highest return for the least possible risk.

Therefore, if it is assumed that all investors have homogenous beliefs about the returns on shares, they will all invest a portion of their funds in the risk free asset and the rest of their funds in portfolio T (Sharpe, 1964). All investors are interested in maximizing returns for the least possible risk, and portfolio T gives investors the best opportunity to do so. Since all investors now hold the same portfolio of risky assets, portfolio T must be the value-weighted market portfolio of all risky assets in the market (Sharpe, 1964).

The above theory leads to the formation of the Securities Market Line (SML) by Markowitz (1952), which describes that an asset is expected to earn the risk free rate, plus a premium for bearing the risk that is measured by the asset's β. The traditional CAPM leads on directly from the SML.



Figure 5

The Securities Market Line - Obtained from www.nobletrading.com

2.3 CAPM

Figure 5 is the graphical representation of the CAPM. The CAPM builds on the model of portfolio choice introduced by Markowitz (1952), and is jointly developed by Sharpe (1964), Lintner (1965), Black (1972) and Mossin (1966). The CAPM describes a simple linear model for estimating the expected return on an asset in terms of its systematic risk, termed β. Therefore the CAPM assumes that the return that should be expected from an investment is a linear function of the amount of risk

that is inherent in the investment, and any asset that can be purchased should plot on the SML line. Specifically, what is relevant is systematic risk in a well diversified portfolio of assets that makes up the market portfolio (Markowitz 1952).

2.3.1 CAPM ASSUMPTIONS

The CAPM incorporates a variety of restrictive assumptions that allow it to simplify the risk-return relationship to a linear equation that is reflected as the SML (Womack and Zhang, 2003). The most relevant of these assumptions relates to investor behavior and the presence of a single risk factor.

The first assumption is that when investors choose a portfolio of risky assets, they only care about expected return and the volatility of return (Markowitz, 1952). Therefore, as rational consumers, investors will always attempt to maximize expected return for a given level of expected volatility (Markowitz, 1952). This is the reason that the Markowitz (1952) approach is often termed the mean-variance model, as investors attempt to minimize variance while maximizing return. An extension of this assumption is that investors are only concerned with the wealth that their portfolio of shares will generate at the end of a single period, therefore investors are not concerned with how they can utilize the wealth that they have generated at the end of the period (Markowitz, 1952). This is a very unrealistic assumption that will be revisited later.

The next assumption is that only one risk factor is common to a well diversified market portfolio, *systematic* risk (Markowitz, 1952). Therefore investors will only be remunerated for taking on increased *systematic* risk and if a share's β is known, it is possible to calculate the expected return that the investor will demand (Womack and Zhang, 2003).

Sharpe (1964) and Lintner (1965) introduce the final two important assumptions. First is that all investors are expected to have homogenous beliefs about the risk/return tradeoff in the market, meaning that each investor has the same understanding of which shares are riskier than others, and that in order to increase return, an increased level of *systematic* risk needs to be undertaken (Sharpe, 1964). Second is that investors are able to borrow and lend unlimited amounts at the risk free rate (Lintner, 1965).

2.3.2 DISSECTING THE CAPM FORMULA

The systematic risk placeholder, β, is calculated as follows:

$$\beta_i = Cov(R_i, R_m) / Var(R_m)$$

This formula reflects the covariance of the returns of a particular share with the market portfolio, divided by the variance of the market portfolio, which measures the sensitivity of a share return to variations in the markets return (Fama and French, 2004). Another method for understanding the β is by more closely analyzing the variance of the market. The denominator in the β formula represents the variance of the market, or the riskiness of the market portfolio. This is calculated as the weighted average covariance of each asset that makes up the market portfolio. It has already been described that what is important in a diversified portfolio is the covariance of all asset in a portfolio, rather than the variance of each asset, because the riskiness of a portfolio is not equal to the average of the variances of each asset, but rather how those assets co-vary. Therefore, if the β of a share is equal to the covariance of the share with the market, relative to the variance of the market portfolio, then this reflects the proportional risk that each Rand invested in the particular asset contributes to the riskiness of the market portfolio (Fama and French, 2004).

The formula for the CAPM is as follows:

$$E(Ri) = Rf + \beta i(E(Rm) - Rf)$$

Where:

- E(Ri) is the expected return on the capital asset
- Rf is the risk-free rate of interest identified above as long term treasury bills
- Rm is the return on the market portfolio
- βi is the covariance of the share with the market, divided by the variance of the market

The CAPM reflects that the expected return on an asset is equal to the risk free rate, plus a premium for risk that is reflected by the asset's β multiplied by the excess return on the market over the risk free rate.

Two important issues arise from developing the logic that underlies the CAPM. Firstly it can be seen that the assumptions of the CAPM are relatively restrictive, and thus it will be interesting to identify whether these assumptions are able to hold under empirical evaluation, i.e. that the CAPM is able to accurately reflect the return that is actually generated for ordinary shareholders.

Secondly, it may be that a true test of the CAPM will be impossible to conduct, because of the way in which the market portfolio is formulated (Roll, 1977). It has thus far been impossible to quantify a market portfolio that contains an exhaustive list of the risky assets that are available in a market (Roll, 1977). Therefore all studies of the CAPM assume a proxy for the market portfolio, normally consisting of the equities market. Including a proxy for the market portfolio means that the actual market return can never be quantified with total certainty and therefore that any test that requires

the market portfolio is sample specific (Roll, 1977). Through attempts to increase the list of assets that are included in the market portfolio, it has become apparent that expanding the market portfolio beyond the equities market has led to very little change in the market portfolios return or the formulation of β, most likely due to the volatility of the market portfolio being dominated by the volatility of equity securities (Fama and French, 2004)[1].

Tests of the CAPM are concerned with three implications of the relationship between expected return and β (Fama and French, 2004). The first test relates to identifying whether applying cross-sectional and time-series regressions on the CAPM are able to accurately reflect the risk free rate as the intercept term and the excess return of the market over the risk free rate as the slope. The second test relates to identifying whether the assumption that β is the only risk factor that can explain the actual return on an investment is true, meaning that the expected returns on all assets are fairly close to the actual return that is generated. The final test is whether the β premium is always positive, meaning that a risky asset should always have a return that is higher than the risk free rate. Empirical findings of each of the three tests will be considered in the next sections.

The following sections are broken down into (1) early empirical studies, where the focus on testing was whether the CAPM was able to accurately reflect share returns and whether the equities market is a good proxy for the all encompassing market portfolio and (2) where a shift in focus towards identifying whether a model that is better able to reflect share returns than the CAPM is quantifiable. The shift in focus appears to have occurred somewhere around the late 1970's (Fama and French, 2004).

2.3.3 EARLY EMPIRICAL STUDIES

Early studies of the CAPM are concerned with the models ability to predict the slope and intercept terms of the CAPM formula (Fama and French, 2004). The CAPM formula can be restated as such:

$$E(Ri) - Rf = \beta i(E(Rm) - Rf)$$

The approach followed by early research studies is to perform a regression of the actual returns of an asset against the calculated β. The application of cross-sectional regression to the CAPM formula allows one to calculate what the model predicts the risk free rate (intercept) and excess market return (slope) to be. These regression values are then compared to the actual risk free rate and market risk premium to evaluate whether the CAPM is an accurate model (Fama and French, 2004).

[1] Please refer to 3.9 Limitations

A problem with testing individual assets became apparent to early researchers. Estimations of β for an individual asset are imprecise because of the way in which the statistics are formulated (Black et al., 1972). This creates a problem when that imprecise β is used to assess whether the CAPM accurately reflects the risk free rate and the market risk premium, because the researcher is attempting to validate a model based on an observation of a single asset in isolation (Black et al., 1972). To improve on this issue, Black et al. (1972) and Blume and Friend (1973) introduced portfolios, rather than individual assets, because the calculation of β for a portfolio is much more precise than for an individual asset.

Black et al. (1972) noted that the CAPM intercept term was consistently higher than the risk free rate, and that the CAPM market risk premium is consistently less than the average excess market return that is found on the market. They use these findings to conclude that low β portfolios on the New York stock Exchange underestimated the actual return that was generated, while high β portfolios overestimated the actual return that was generated.

Adding to these concerns, Blume and Friend (1973) found that the assumptions of unlimited borrowing at the risk free rate is an unreasonable assumption, and that the CAPM is unable to explain the actual returns that are generated by investing in shares.

Fama and MacBeth (1973) found that the above studies assessed a period in isolation, through the estimation of a single cross-sectional regression of average monthly returns on β. Fama and MacBeth (1973) extended testing of the CAPM by attempting to perform sequential cross-section regressions each month to evaluate more than a single period. The intercept terms and slopes that were calculated for each month were then used to test whether the CAPM was able to reflect share returns over more than a single period, in line with previous studies by Black et al. (1972). Fama and MacBeth (1973) come to the same conclusion, that the CAPM is unable to accurately reflect the risk free rate and the market risk premium.

With a wealth of early evidence indicating that the CAPM was ineffective in calculating the cost of equity, Gibbons, Ross and Shanken (1989) evaluated whether the proxy for the market portfolio that was applied by such early studies was reasonable. They applied time-series regressions of share returns on β to test the relevance of the ordinary equities market as a proxy for the all encompasing risky portfolio of shares. In a time-series regression, the CAPM was tested thus:

$$E(Ri) - Rf = \alpha + \beta i(E(Rm) - Rf) + \varepsilon$$

In cross-sectional regression, the intercept term is quantified as the risk free rate, however in time-series regression, the α represents the intercept term that is left unquantified and is found by regressing the actual returns of shares against the expected returns under the CAPM. If the α can be assessed as being reliably close to zero, then the expected returns are statistically close to the actual returns, and the CAPM has "good fit". An assumption of the test performed by Gibbons et al. (1989) was that the CAPM is an effective model for the calculation of expected returns, and that if the CAPM is unable to explain actual returns over a period, this is as a result of problems with the sample for the market portfolio that has been chosen, and not the CAPM. What is literally tested in this case is whether the market portfolio proxy that is chosen is a good fit of the true market portfolio at point T in figure 4 above (Gibbons et al., 1989). The result of the study by Gibbons et al. (1989) is that there is evidence that adding multiple classes of assets to the market portfolio might reduce the α but that quantifying the reduction of α was not possible at that time.

Gibbons (1982) attempted to perform a test of whether the equities market, which is commonly used as a proxy for the market portfolio, is a good fit for the true market portfolio shown at point T in figure 4. Gibbons (1982) added other factors to the CAPM in order to identify whether additional factors were able to better reflect the risk free rate and market risk premium. Gibbons (1982) found that a portfolio of shares was a good proxy for the market portfolio, as adding other factors such as human capital added very little to the explanatory power of the model.

Therefore, early tests of the CAPM indicated that the CAPM appears to be unable to accurately predict actual returns (Blume and Friend, 1973; Fama and MacBeth, 1973) but that the market for ordinary shares is a good approximation of the market portfolio (Gibbons, 1982).

2.3.4 RECENT EMPIRICAL STUDIES

Starting in the late 1970's, evidence begins to appear that there are other factors that are much better predictors of expected return than β. The second type of test of the CAPM relates to identifying whether there are other factors that explain the actual returns on shares (Fama and French, 2004).

One of the first of these studies was Banz (1981) who found that the size of a share is an important indicator of the future return that an investor could expect, because small shares consistently generate higher returns than what is explained by β. Similarly, Fama and French (1992) concluded that the traditional β was a poor reflector of the returns that investors actually obtained. They found that instead of β, the market capitalisation and book-to-market ratio of a company were

accurate in predicting the return that could be expected from investing in a portfolio of shares. This phenomenon is backed up by the research of Rosenberg, Reid and Lanstein (1998) who also found that the BTM ratio of a company indicated future returns that are not captured by β. Fama and French (1992) found that High BTM shares consistently outperformed low BTM shares in a way that is not reflected in the β of a share.

The above contradictions of the CAPM, relating to the size and BTM of shares, can be connected. The β of a portfolio appears to miss some information that is incorporated into ratios that involve the share price of a company (Fama and French, 2004). The market β of a share incorporates information about the expected cash flows that will be received. If a return of R1 000 000 per annum into perpetuity has a standard deviation of 10%, then it is possible to say that there is an equal chance that a return of R900 000, R1 000 000 or R1 100 000 will be the actual return at the end of period 1. Therefore, β will explain how returns may vary into the future (Fama and French, 2004). However, the price of a share is a reflection of the present value of the future cash flows that an investor expects to receive by investing in the share. Therefore the price of a share does not only include the variability of future returns, but has additional information about the expected return, because it explains the rate at which the market expects to discount the future cash flows to the present. The future return of R900 000, R1 000 000 or R1 100 000 can now be present valued to the beginning of the current year at an implicit discount rate (Fama and French, 2004). Therefore, incorporating ratios that look at the price of a share adds information about the rate at which the market expects the return on an asset to be present valued.

Therefore, recent tests of the CAPM have identified that there may be other factors that are useful in predicting the discount rate which investors are willing to use to present value the expected future returns of a share, at least in an American context (Fama and French, 2004). It was therefore concluded that the restrictive assumptions of the CAPM invalidate most of its applications in practice (Fama and French, 1992).

It is identified in Miller and Bromiley (1990) that if the issues that have been identified with the empirical robustness of the CAPM are to be fatal to its application in practice, then it is important to assess whether the same issues arise outside of an American sample.

2.3.5 THE CAPM IN SOUTH AFRICA

Studies of developing markets such as Rouwenhorst (1999) conclude that there is no significant relationship between average return and the β of a company, but rather that shares have

momentum effects and change in value based on price-earnings ratios and investor sentiment. These studies therefore dispute the relevance of the CAPM model in such countries as Brazil, Greece, Nigeria, Malaysia and Mexico.

Many studies have been performed in South Africa attempting to identify the individual factors that are best at predicting the return that an investor will obtain by investing in shares (Van Rensburg and Robertson, 2003; Basiewicz and Auret, 2009; Hoffman, 2012). This body of research finds that, similar to the American situation, there are other factors that are able to predict the return on shares more effectively than the β (Hoffman, 2012).

Hoffman (2012) performed a study focusing on disproving the efficient market hypothesis by indicating that shares will consistently change in value completely independently of their β risk. The paper concludes that the returns of the smallest shares on the JSE exhibit the most consistent anomalous returns and that the BTM effect is a very important indicator of share returns that are not explained by β.

The above South African studies have all been concerned with identifying specific factors that are able to explain the returns of shares, later studies develop an argument regarding the actual asset pricing models that are the focus of this research report.

In a study by Ward and Muller (2012), the CAPM is tested by sorting shares into portfolios based on β and plotting the cumulative value of portfolios over a long study period, it was found that the β of an equally weighted portfolio of shares actually has an inverse relationship to actual return, meaning that low β companies outperform high β companies (Ward and Muller, 2012). This finding contends that the application of CAPM in South Africa is unreasonable, as β is unable to accurately predict the actual returns that investors obtain.

2.3.6 CONCLUSIONS ON CAPM

Among those who conclude that the empirical testing of the CAPM is fatal to its application in practice, two distinct conclusions appear to arise (Fama and French, 2004). On the one hand is the behaviourists view; which contends that the anomalies that are not explained by β are an indication that investors act in a much more irrational way than the CAPM assumes. Therefore, in an efficient market, the size and BTM effects should dissipate (De Bondt and Thaler, 1987; Lakonishok et al., 1994).

The second view is that these anomalies are a placeholder for systematic risk factors that are not reflected by β (Fama and French, 1993, 1996). It has already been identified that the assumptions of the CAPM are restrictive (Fama and French, 2004). If it can be assumed that the above anomalies will persist, even when an efficient market is evident, then the assumption that all investors are concerned about is the variability of their return in a single period may be unreasonable (Fama and French, 1993). It may be more reasonable to assume that an investor is also concerned with the amount of disposable income that is available for investment and about the future prospects of an investment beyond one period, and therefore that the β of a share is not a complete description of an assets risk (Fama and French, 1993). This points to the need for a more complicated asset pricing model (Fama and French, 2004).[2]

Merton (1973) introduces the intertemporal CAPM, or ICAPM, which is an extension of the CAPM that has a different perspective on investors objectives. In the ICAPM, an investor is not only concerned with the return on a portfolio at the end of a period, but also with the opportunities that will be available to consume or invest the return (Merton, 1973). Therefore, when an investor invests at the beginning of a period, the ICAPM assumes that the investor will be concerned with how their wealth at the end of the period will vary with certain *state variables,* including their earnings from labour activities, the price of consumption goods and investment opportunities (Merton, 1973). As a result, optimal investment portfolios are multifactor efficient, and additional variables, over and above the market β, are required to explain expected returns (Merton, 1973).

It appears that from the conclusions that have been reached in both early empirical studies and more recent studies of the CAPM, that this asset pricing model is perhaps flawed in its applicability in practice. If the CAPM is unable to provide a cost of equity that accurately reflects the return that can be expected from investing in an asset, then its applicability in practice is invalidated. This issue forms part of what will be tested in this research report, but as the empirical robustness of the CAPM has already been assessed in a South African context, will also form a basis for identifying whether the methodology applied in this report agrees with the methodology of leading South African research on the subject of asset pricing models such as that by Ward and Muller (2012).

2.4 FAMA AND FRENCH 3 FACTOR MODEL

Fama and French (1993) take an indirect approach towards developing an efficient ICAPM portfolio, perhaps more in line with the Ross (1973) arbitrage pricing theory. Fama and French (1993) develop an asset pricing model that includes three factors to explain average return; the BTM ratio, size of

[2] Please see 1.5 Delimitations of the study

the firm and an overall market value factor similar to β. They argue that although size and the BTM ratio are not in themselves state variables, the effects of these two factors act as proxies for unidentified state variables that produce undiversifiable risk that is not reflected in the traditional β (Fama and French, 1993, 1996).

2.4.1 SIZE FACTOR

Beginning with the study of Banz (1981) and continuing with the research of Fama and French (1992, 1993, 1996) it is found that small capitalisation shares are able to outperform large capitalisation shares, in a way that is unrelated to the β of the share. It has been widely accepted that the size of a company may be a placeholder for the riskiness of the share, as it may reflect certain risk variables relating to the solvency and liquidity of the underlying company (Womack and Zhang, 2003).

In a South African context, early studies have found that the size effect is minimal when evaluating shares on the JSE, and that the size effect is most pronounced in the smallest shares on the JSE (Robins, Sandler and Durand, 1999; Van Rensburg and Robertson, 2003). In later studies that utilize a more robust methodology, it was found that size is an important factor on the JSE, albeit that the factor is still most prevalent in very small shares (Auret and Sinclaire, 2006; Basiewicz and Auret, 2009).

2.4.2 BTM FACTOR

"The Book-To-Market ratio is a pandora's box, an enigma which may include indications of risk but may also be an indication of investor naivity in extrapolating returns too far into the future, and irrational market impulses being unraveled." (Lev and Sougiannis, 2003: p419)

Companies with a high BTM ratio have an accounting value of equity that is very similar to its market value and are described in the literature as value shares, while low BTM companies have share prices that are much larger than the accounting value of equity and are described as *growth,* or *glamour* shares (Fama and French, 1992). Previous studies have indicated that high BTM companies outperform low BTM companies in the market (Fama and French, 2008; Hoffman, 2012). One view posited for this effect is that (1) value shares are undervalued, as investors assume that those shares that have previously performed badly will continue to perform badly and (2) growth shares are overvalued, as their strong past performance causes investors to accept a price of the share that is above a fundamentally true price (Davis, Fama and French, 2000). When the overreaction is eventually corrected, the result is high returns for value shares, and low returns for growth shares (De Bondt and Thaler, 1987). The BTM effect has been found to be evident, both in developed

markets (America) and in developing markets (South Africa) (Fama and French, 1992, 1996; Basiewicz and Auret, 2009). However, there have been some arguments against the BTM effect as a factor in asset pricing over recent times.

The first argument for not including BTM in asset pricing models is that it is not an indicator of risk, but rather arises due to irrational trading (Lakonishok et al., 1994). It is argued that when a market operates efficiently, the BTM indicator in the F&F3 model will cease to have explanatory power of share prices (Lakonishok et al., 1994). This argument is invalidated for the purpose of this study as the BTM effect has been seen to be prevalent over a long period of time, and therefore has consistently proven itself to be an indicator of share returns (Fama and French, 1996).

Kothari, Shanken and Sloan (1995) argue that the BTM effect of Fama and French (1993) only exists because the statistical methods that are applied by Fama and French (1993) are more likely to include a large percentage of high BTM shares that manage to survive financial distress. Kothari et al. (1995) state that as these 'financially distressed' companies fall into liquidity and solvency problems, their share price drops as investors predict the failure of the company. When these companies manage to exit financial distress and continue to trade on the stock exchange, the returns on investing in these shares is high as investors change their perspective and again view the company as a going concern. Kothari et al. (1995) believe that this type of company is more likely to be included in the portfolios of shares that are formed by Fama and French (1993) than those companies who do fail. This argument is invalidated by Chan, Jegadeesh and Lakonishok (1995) who provide evidence that survivorship bias does not have a significant effect on the Fama and French (1993) findings.

Finally, MacKinlay (1995) proposes that anomalies that are not explained by the CAPM may simply be as a result of data snooping. They argue that the BTM effect may be as a result of researchers dredging through data in an attempt to find patterns that invalidate the CAPM. When these trends are applied to different samples, it may be found that the new factors that researchers find to explain share returns are no longer effective (MacKinlay, 1995). The research that has been conducted regarding the existence of the BTM effect internationally (Fama and French, 1998) and in South Africa (Hoffman, 2012) are evidence that the BTM effect is not sample specific, therefore MacKinlay (1995)'s argument appears to be invalidated.

Grundy and Martin (2001) propose that where an anomaly remains visible for a long period of time and across different samples, they become factors in financial literature. Therefore it appears that

due to the BTM effects continued existence in many markets across the globe, it may well become acceptable as a factor, rather than an anomaly (Fama and French, 1996).

In a South African context, Van Rensburg and Robertson (2003) found that the Price-Earnings (PE) ratio of companies is a good indicator of shares that fall under the category of *value* (low PE) and *growth* (high PE) and gives an indication of those shares that are undervalued (*value)* and those that are overvalued (*growth)* by the market. More recently, it is found that the BTM ratio is a better indicator of future share returns than the PE ratio (Basiewicz and Auret, 2009; Hoffman, 2012).

The literature above gives an indication that the BTM and PE ratios may provide some useful information regarding the return that should be expected from investing in equity. As a result of the recent conclusions by Hoffman (2012), this research report will test the applicability of the BTM ratio in a South African context for the purpose of evaluating the return on equity.

2.4.3 THE F&F3 FORMULA

Based on the above evidence, Fama and French (1993) propose a three-factor model for expected returns, which can be found under 3.5.2 F&F3 Cost of Equity and reflects the following:

$$E(Ri) - Rf = \beta i * (E(Rm) - Rf) + si * E(SMBt) + btmi * E(HMLt)$$

Where:

- E(Ri) is the expected return on the capital asset
- Rf is the risk-free rate of interest identified above as long term treasury bills
- Rm is the return on the market portfolio
- βi is the covariance of the share with the market, divided by the variance of the market
- *s*i is beta equivalent that relates to the covariance of the share with the SMB portfolio, divided by the variance of the SMB portfolio
- E(SMB) is the expected size premium at time t
- *btm*i is the beta equivalent that relates to the covariance of the share with the HML portfolio, divided by the variance of the HML portfolio
- E(HML) is the expected book-to-market premium at time t

The F&F3 is concerned with incorporating size and the BTM ratio into an explanation of the return that an investor should expect from investing in shares.

The calculations that accompany this model are described in section 3.6 Constructing Factors.

2.4.4 EMPIRICAL STUDIES OF F&F3

Tests of the ability of F&F3 to reflect actual share returns have been conducted in developed markets by Fama and French (1993, 1996), who consistently find that the F&F3 was able to explain the returns on shares better than the CAPM. However, the evidence does not all point to the F&F3 as the perfect model for the explanation of share returns (Fama and French, 1997).

Fama and French (1997) provide evidence that the F&F3 is inconsistent when comparing different industries; for some industries it has good explanatory power of share returns, while in others it is as ineffective as the CAPM. Fama and French (1997) propose that this inconsistency arises because (1) there is uncertainty regarding the true risk factors that size and BTM act as placeholders for and (2) the estimates of exactly how much of the total market risk comes from a specific industry is imprecise. Fama and French (1997) therefore conclude that both the CAPM and F&F3 will be imprecise when used to evaluate a specific industry in isolation, or for the purpose of discounting specific projects, because the β, size and BTM factors are more accurate for a large variety of shares together.

Fama and French (1998) performed a study of the F&F3 in Europe, Australia and the far east, and found that the F&F3 is more effective than the CAPM in explaining share returns in these developed areas, while L'Her, Masmoudi and Suret (2004) found that the F&F3 is a better indicator of share returns on the Canadian stock exchange.

Suh (2009) concludes that the F&F3 has explanatory power in highly volatile markets, but where market volatility is low, the CAPM is just as effective as the F&F3. This may be an indication that the F&F3 will be more effective in developing economies, where it is consistently found that markets are most volatile (Rouwenhorst, 1999).

South Africa may be classified into the grouping of developing economies (Bird and Vaillancourt, 2008). A study of the F&F3 on the JSE by Basiewicz and Auret (2010), finds that the F&F3 is a better predictor of actual share returns than the CAPM. It appears that the size and value premiums of the F&F3 should at least be long standing on the JSE, based on previous studies of what factors are able to explain the variability in share price return in South Africa (Van Rensburg and Robertson, 2003; Auret and Sinclaire, 2006; Basiewicz and Auret, 2009). The F&F3 has historically been tested by the application of cross sectional and time series regressions per Basiewicz and Auret (2010), it is however as yet untested in the South African market using the methods included in this research report.

2.5 CARHART 4 FACTOR MODEL

Fama and French (2004) contend that the size and BTM factors are indicators of systematic risk and will continue to provide a better explanation of share returns than the CAPM in an efficient market. They do concede, however, that one established factor that is missed by the 3 factor model is the short term momentum effect.

2.5.1 DEFINING MOMENTUM

Jegadeesh and Titman (1993) provide evidence that shares whose value has previously increased (decreased) can be predicted over a 3 to 12 month period to continue to increase (decrease). This is defined as the short term momentum effect of shares, and is described by Baker and Stein (2004) as an anomaly that arises due to investor optimism (pessimism). Baker and Stein (2004) also describe a situation where investor demand fuels share prices up past a point that can logically create value, leading to an inevitable correction where shares drop in value later. This leads to the anomaly where share value moves negatively due to high prior trading volume over the long term and is defined as long term momentum. Baker and Stein (2004) believe this to occur because investor optimism is reversing.

L'Her et al. (2004) identify that the F&F3 is able to capture most market anomalies, except for the short term momentum effect. Carhart (1997) made amendments to the F&F3 to incorporate the short run momentum phenomenon and thereby created the C4 model, which takes into account a market risk premium, size, the BTM effect and short run momentum.

2.5.2 THE C4 FORMULA

Carhart (1997) proposed a 4 factor model for the explanation of share returns, which can be found under 3.5.3 C4 Cost of Equity and reflects the following:

$$E(Ri) - Rf = \beta i * (E(Rm) - Rf) + si * E(SMBt) + btmi * E(HMLt) + wi * E(WMLt)$$

Where:

- E(Ri) is the expected return on the capital asset
- Rf is the risk-free rate of interest identified above as long term treasury bills
- Rm is the return on the market portfolio
- βi is the covariance of the share with the market, divided by the variance of the market
- *si* is beta equivalent that relates to the covariance of the share with the SMB portfolio, divided by the variance of the SMB portfolio
- E(SMB) is the expected size premium at time t

- *btm*i is the beta equivalent that relates to the covariance of the share with the HML portfolio, divided by the variance of the HML portfolio
- E(HML) is the expected book-to-market premium at time t
- *w*i is the beta equivalent that relates to the covariance of the share with the WML portfolio, divided by the variance of the WML portfolio
- E(WMLt) is the expected momentum premium at time t

This model is concerned with incorporating short run momentum into a capital asset pricing model, and the calculations that accompany this model are described in section 3.6 Constructing Factors.

2.5.3 EMPIRICAL STUDIES

The momentum effect has been found to be prevalent in the USA by Fama and French (1996, 2008) and Jegadeesh and Titman (2001), both in its short term and long term forms. Lee and Swaminathan (2000) and Jegadeesh and Titman (2001) conclude that the momentum effect may be found to be correlated to the size and BTM premiums of F&F3. Shares that previously performed well, termed winners, had a high past trading volume, increased in market capitalisation over a study period and therefore tended to be larger shares that had lower BTM ratios (Jegadeesh and Titman, 2001). Overall, Jegadeesh and Titman (2001) found that past losers were riskier than past winners because they were more sensitive to all three of the F&F3 factors, however, Jegadeesh and Titman (2001) did not go so far as to say that short run momentum would also act as a risk placeholder.

The source of momentum profits could arise from either (1) investors underreacting to positive trading results of a particular share, which means that the price of the share will take time to react to information and therefore arbitraguers have a chance to invest in the share as it increases in price, or (2) that past winners may in fact be riskier than past losers. A discussion of which viewpoint is most correct falls back to the Lakonishok et al. (1994) argument regarding whether the F&F3 factors are indicators of risk or irrational investor trading, and is not important for the topic at hand. On the whole, tests of developed markets make it clear that momentum has explanatory power in estimating the actual return that can be expected by investing in shares over both the short term and the long term (Jegadeesh and Titman, 2001; Zhang, 2006).

Tests of momentum have also been conducted in developing markets by Rouwenhorst (1999) who concluded that the momentum effect was pronounced in 20 developing markets including Argentina, Brazil and India, but not including South Africa. Tests of the momentum effect and of the C4 model have not been prevalent in a South African context, however, Fraser and Page (2000) and Hoffman (2012) did find that short run momentum was evident on the JSE and was an important indicator of future share returns. Therefore this research will be concerned with examining short run momentum, as it is incorporated in the C4 model.

2.6 CONCLUSION

Size, value versus growth, and momentum have consistently been found to have explanatory power with respect to share price movement, which has been consistent over at least the last 20 years, in the American equities markets (Fama and French, 1992). From a South African context, tests of the factors that explain how share prices change have been much more recent, but are consistent with the American conclusions, where size, the BTM ratio and momentum are indicators of future share price movements (Hoffman, 2012).

The share price of a company is a factor of the future cash flows that can be expected, as well as the discount rate that provides information regarding the value of those future cash flows in the present (Womack and Zhang, 2003). By bringing what has been identified as the most important factors that explain share price movements together, the CAPM, F&F3 and C4 have attempted to develop models that are able to explain to shareholders what return they can expect to receive by investing in ordinary equities, termed the cost of equity (Fama and French, 2004). Indeed, the cost of equity is important in many instances; to provide the company with information regarding the return that is required by shareholders, and as an input into the weighted average cost of capital for the purpose of evaluating the benefit of future projects that the company may enter into, or in determining optimal capital structure (Womack and Zhang, 2003).

Overall, as investors will be interested in identifying the return that they can expect to receive from investing in risky assets, it appears reasonable to be concerned with which model will most accurately calculate a cost of equity that reflects the actual return that investors obtain. The literature therefore indicates that the CAPM, F&F3 and C4 models are the most relevant capital pricing models, as investors demand a higher return for taking on a higher level of risk, shares do change in value based to some extent on their size and BTM ratios, and that momentum has an effect on share prices. These three models therefore form the base models which are tested in this research report.

2.7 AREAS FOR POSSIBLE FUTURE RESEARCH

Whether all of the above factors are indicators of *systematic* risk factors that cannot be diversified away, and what exact factors they act as placeholders for remains as an important question in finance, which is a possible area for future research.

3 METHODOLOGY

The next chapter of this report is focused on providing a description of the methodology that has been applied to empirically test the CAPM, F&F3 and C4 models. An introductory summary of the approach that has been followed is the starting point. Thereafter, a description of the research method, design and the data that has been utilized is provided. Once the data that is the population of this study is described in detail, it is possible to describe how each of the asset pricing models will be constructed, to provide an outline of the ultimate models that this research report has attempted to assess. Following the description of the 3 asset pricing models is more detail on each of the factors that make up each asset pricing model, along with how each of these factors has been calculated. Finally, the manner in which the 3 developed models have been tested is laid out by firstly, providing a description of the manner in which portfolios of shares have been chosen, secondly describing how the asset pricing models have been assessed once shares are classified into portfolios, and finally, laying out the identified limitations of the methodology that has been applied.

3.1 INTRODUCTION

The purpose of this research report is to assess whether the traditional CAPM, F&F3 or C4 models are accurate in calculating a cost of equity that reflects the actual return that is obtained by investing in equities, and to hence determine which model most accurately approximates actual returns.

The methodology first defines how the expected return on an investment in shares under each of the three equity pricing models identified above is calculated. Once the expected cost of equity has been defined and calculated, shares are classified into portfolios based on expected return for the purpose of comparison. Thereafter, the cumulative actual return on each portfolio is graphically represented, in line with Ward and Muller (2012).

The evaluation of each graph involves identifying whether each portfolio actually falls into a band of actual return that is predicted by the expected return, i.e. the highest expected return portfolio should outperform all others, the second highest expected return portfolio should outperform all other portfolios except for the highest expected return portfolio, and so on. This has been used as the test to evaluate whether each model is able to accurately reflect the actual return that is generated over the period under consideration and where mention is made of a portfolio that is accurate, the above definition of accuracy is intended.

Therefore, the focus of this research report is not on absolute accuracy of each asset pricing model, but rather a reasonable approximation that the models are able to accurately show the outperformance of high cost of capital portfolios over low cost of capital portfolios.

3.2 RESEARCH METHOD

A purely quantitative method has been used in line with (Ward and Muller, 2012). This quantitative study will take the form of an empirical analysis of share returns on the JSE.

3.3 RESEARCH DESIGN

The research design follows the mathematical modeling of the CAPM, F&F3 and C4 models. The construction of the capital asset pricing models has been facilitated through microsoft excel, based on the methodology that will follow.

3.4 DATA

3.4.1 POPULATION AND SAMPLE:

Although there are typically 350 companies listed on the JSE over the time frame under evaluation, the All Share Index (ALSI) comprises only the largest 160 companies, representing approximately 99% of the total market capitalisation of the JSE main board. The population of this study is the total number of companies that are listed on the JSE (typically around 350), however, the majority of companies falling outside the ALSI are considered to be too small and too illiquid for the purposes of accurate evaluation, and are therefore excluded from the sample (Van Rensburg and Robertson, 2003; Ward and Muller, 2012). Therefore the sample for this research report includes, to begin with, only the top 160 companies ranked by market capitalisation. This sample is then adjusted to ensure that there are no variables missing in any of the sample companies, and to adjust for thin trading issues. Therefore the sample will always include 160 shares, but may exclude certain of the ALSI 160, and include a few shares from below the ALSI 160. The shares that fall into the ALSI 160 may vary over the period under study, therefore the shares included in the sample shall be re-evaluated at the beginning of each year to coincide with the rebalancing of portfolios that occurs at the end of December.

3.4.2 SOURCES OF DATA

12 years of JSE monthly share price data from July 2000 to June 2012, in combination with company financial statements reflect the data used in this study. Share price data is sourced from the JSE bulletin, while company financial data is sourced from McGregor BFA.

3.4.3 DELISTINGS AND NEW LISTINGS:

Where shares delist within a particular year, thereafter a zero return is used. These shares are excluded from the sample for the entire year in which delisting is achieved on the basis of the last price when the company ceased trading on the JSE in line with Ward and Muller (2012). Newly listed shares are included in the quarter after listing if this occurs during the quarterly period. This consideration is expected to adequately ensure that survivorship bias does not skew the results of the study (Ward and Muller 2012).

3.4.4 ACTUAL RETURNS:

Dividend receipts constitute a significant portion of the return an investor receives. Therefore dividends are included in share return by using the JSE bulletin historical time series of cash dividend pay-outs. Share buybacks are not adjusted for on the grounds that these are a form of capital reduction which only affects those shareholders that exit the company (Ward and Muller 2012). Share options issued to managers and directors are also ignored.

3.4.5 MARKET RETURN:

As this study is focused on the top 160 shares on the JSE, after adjusting for thin capitalisation and data collection issues, the market return for the purposes of calculating the expected return on equity is also based on the sample of companies that have been chosen each year. The use of a market return that is in line with the sample data is evident in both L'Her et al. (2004) and Ward and Muller (2012) and ensures that the sample data is consistently utilised in the study.

The chosen sample of shares is able to track the ALSI 160 with 98.34% correlation for the period 1 January 2003 to 30 June 2012. It is not possible to evaluate this relationship further as the J203T index was first developed in 2003. The ALSI 160 comprises 99% of the market capitalisation of the JSE, therefore the ALSI 160 is assumed to be an accurate approximation of the entire equities market return, as the return of any additional shares below the ALSI 160 adds very little to the weighted average market return (Ward and Muller, 2012).

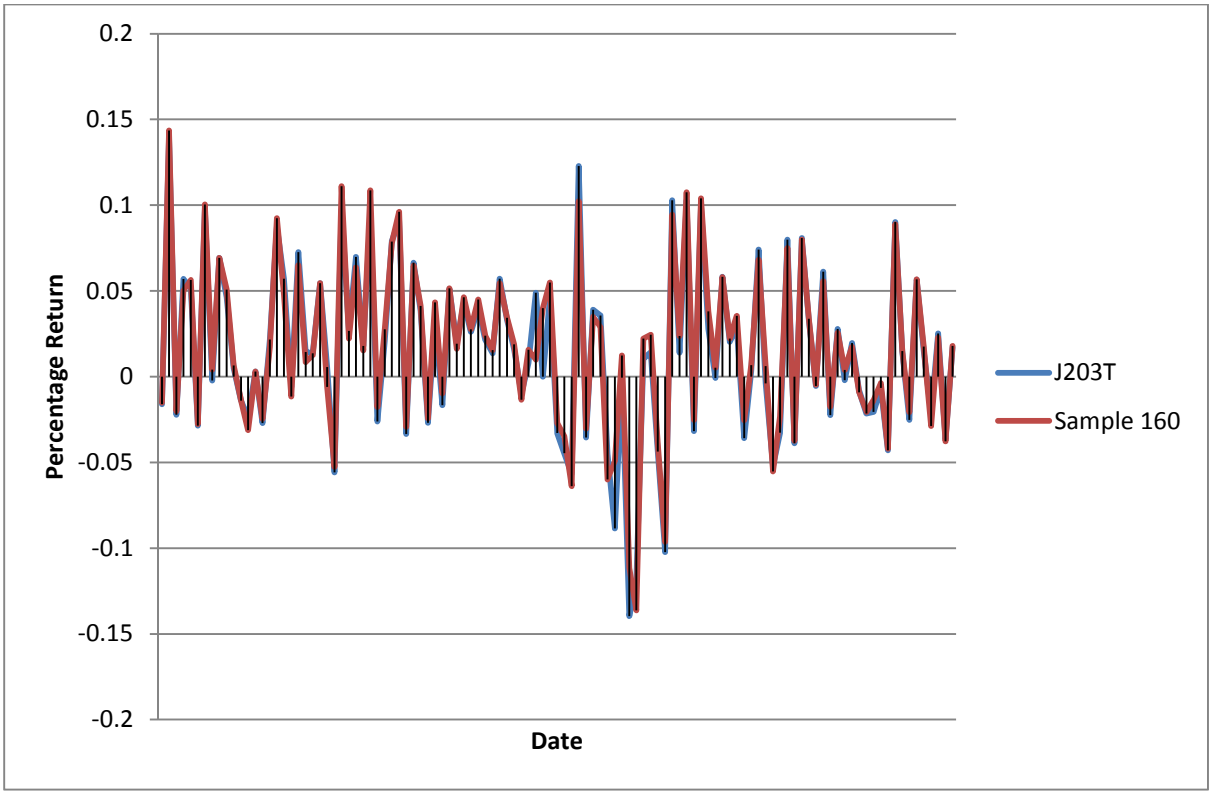

Figure 6

Correlation between Sample 160 and the ALSI 160 Total Return Index

3.4.6 RISK FREE RATE:

The risk free rate is taken as the South African 10 year domestic government bond rate, and is adjusted each year to ensure that the most up to date rate is applied.

3.4.7 BOOK-TO-MARKET RATIO:

The BTM ratio is calculated as the book value of ordinary equity divided by the market value of the total share capital of the company (Fama and French, 1992). The above values are lagged for 6 months to ensure that the information is readily available in the market, ensuring that the 'look-ahead' bias is avoided (Fama and French, 1992). Negative BTM companies are excluded from the sample.

3.4.8 THIN TRADING ISSUES

Basiewicz and Auret (2009) provide some indication that the JSE is an illiquid market, therefore thin trading is an issue that requires some attention in this research report.

Van Rensburg and Robertson (2003) attempted to adjust for illiquidity by only including shares whose turnover ratio is greater than 0.01%. They define the turnover ratio as the average number of shares traded daily for the month divided by the number of ordinary shares outstanding at the beginning of the month.

Although this measure takes into account the effect of infrequent trading, it does have one shortfall. Shares with a small market capitalization may be traded frequently but may only have a small value traded. Institutional and even some private investors may not be able to invest in these shares to a desired extent due to scale issues and it could therefore be argued that these shares should be classified as infrequently traded and excluded from the sample. The lack of a diverse set of investors trading such shares may mean that share prices will continue to be mispriced by the market for an unreasonable period of time, which may unnecessarily bias the results of this study due to thin trading.

The method applied in this research report attempts to take into account shares that are infrequently traded and those that may be frequently traded but are not large enough to warrant frequent trading by a diverse number of institutional and private investors. Any share with an average share price of less than R1 and an average market capitalisation of less than R100 million are excluded for the purposes of ensuring that illiquidity does not skew the results. Those companies that are excluded based on these criteria are replaced with those shares that fall outside the ALSI 160 but have the next largest market capitalisation, while meeting the above liquidity criteria.

3.5 MODEL CONSTRUCTION

The 3 asset pricing models that have been assessed are laid out in this section of the methodology. The traditional CAPM, the longstanding cornerstone of traditional finance literature, will first be introduced.

3.5.1 CAPM COST OF EQUITY:

The CAPM cost of equity, which has formed the basis for capital asset pricing theory is defined below:

$$E(Ri) = Rf + \beta i(E(Rm) - Rf)$$

Where:

- E(Ri) is the expected return on the capital asset
- Rf is the risk-free rate of interest identified above as long term treasury bills
- Rm is the return on the market portfolio
- βi is the covariance of the share with the market, divided by the variance of the market

3.5.2 F&F3 COST OF EQUITY:

In order to calculate the expected return on equity by applying the F&F3, it is necessary to construct each of 3 factors; the market risk premium (E(Rm) - Rf), the size premium (designated as Small minus Big or SMB) and the book-to-market premium (designated as High minus Low or HML).

The form of the equation to calculate the cost of equity under this model is:

$$E(Ri) - Rf = \beta i * (E(Rm) - Rf) + si * E(SMBt) + btmi * E(HMLt)$$

Where:

- E(Ri) is the expected return on the capital asset
- Rf is the risk-free rate of interest identified above as long term treasury bills
- Rm is the return on the market portfolio
- βi is the covariance of the share with the market, divided by the variance of the market
- *s*i is beta equivalent that relates to the covariance of the share with the SMB portfolio, divided by the variance of the SMB portfolio
- E(SMB) is the expected size premium at time t
- *btm*i is the beta equivalent that relates to the covariance of the share with the HML portfolio, divided by the variance of the HML portfolio
- E(HML) is the expected book-to-market premium at time t

3.5.3 C4 COST OF EQUITY:

The cost of equity according to the Carhart 4 factor model is calculated as follows:

$$E(Ri) - Rf = \beta i * (E(Rm) - Rf) + si * E(SMBt) + btmi * E(HMLt) + wi * E(WMLt)$$

Where:

- E(Ri) is the expected return on the capital asset
- Rf is the risk-free rate of interest identified above as long term treasury bills
- Rm is the return on the market portfolio
- βi is the covariance of the share with the market, divided by the variance of the market
- *s*i is beta equivalent that relates to the covariance of the share with the SMB portfolio, divided by the variance of the SMB portfolio
- E(SMB) is the expected size premium at time t
- *btm*i is the beta equivalent that relates to the covariance of the share with the HML portfolio, divided by the variance of the HML portfolio
- E(HML) is the expected book-to-market premium at time t

- *w*i is the beta equivalent that relates to the covariance of the share with the WML portfolio, divided by the variance of the WML portfolio
- E(WMLt) is the expected momentum premium at time t

3.6 CONSTRUCTING FACTORS

3.6.1 EXCESS MARKET RETURN

The excess return of the market is calculated as the total return of the sample that is utilized in each period, less the identified risk free rate in each period.

3.6.2 BETA:

Calculation of beta takes the form of the equation:

$\beta i = Cov(Ri,Rm)/Var(Rm)$

This reflects the covariance of the returns of a particular share with the market portfolio, divided by the variance of the market portfolio. Linear regression is applied to calculate the slope coefficient of the monthly growth in share value over the previous 60 months against the returns of the entire sample of companies for the quarterly period under evaluation. The use of 60 monthly periods is in line with most South African studies and is identified in Bartholdy and Peare (2005) as the best frequency of data to use for calculating a beta that reflects the actual return on shares. In those instances where less than 60 month return data is available, an estimate is made of β using the data points that are available. However, if less than 24 points are available the company is dropped from the sample (Ward and Muller, 2012).

3.6.3 SMB AND HML:

The factor SMB represents the excess return that is expected on a portfolio of only small capitalisation shares compared to the return on a portfolio of only large capitalization shares. Fama and French (1993) note that small capitalization shares consistently generate excess returns compared to large capitalization shares and that this is a reflection of small shares being more risky than large shares. SMB quantifies how much excess return should be provided when investing in small capitalization companies.

This factor is calculated by comparing the total returns of a portfolio of small shares, to the return on a portfolio of large shares. The amount invested in each of these two portfolios is equal, and thereafter the return of the *small* portfolio of shares is reduced by the return of the *large* portfolio

of shares. Therefore, the outperformance of the portfolio of *small* shares over the portfolio of *large* shares is shown.

Similarly, the factor HML represents the excess return that is expected on a portfolio of only high BTM shares compared to the return on a portfolio of only low BTM shares. Fama and French (1993) propose that BTM acts as a proxy for a number of unidentified risk factors that are not explained by β. High BTM shares are found to generate excess returns compared to low BTM shares, regardless of the β of the shares.

This factor is calculated by comparing the total returns of a portfolio of high BTM shares, to the return on a portfolio of low BTM shares. The amount invested in each of these two portfolios is equal, and thereafter the return of the *value* portfolio of shares is reduced by the return of the *growth* portfolio of shares, therefore showing the outperformance of *value* over *growth*.

The SMB and HML factors are constructed in line with Fama and French (1993). For each month *t* from July of year *y-1* to June of year *y,* the total share sample (ALSI 160) is separately ranked based on their market capitalisation and BTM ratio of June *y-1.* Rankings begin in the middle of the year as prior literature has found that the SMB and HML factors are most prevalent in January, which may skew the results of the study (L'Her et al., 2004). These rankings are then used to calculate a 50% breakpoint for size, a 30% and 70% breakpoint for BTM (Fama and French, 1993). Thereby, market capitalisation is split simply into small (80 shares) and big (80 shares), while the BTM classification is divided into low (48 shares), neutral (64 shares) and high (48 shares). The classification of some entities as having a neutral book-to-market ratio is in line with L'Her et al. (2004) and is a simplified version of the methodology applied by Fama and French (1993).

Six value-weighted portfolios, S/L, S/N, S/H, B/L, B/N and B/H are formed as the intersection of size and book-to-market groups.

SMB is the equally-weighted average of the returns on small capitalization share portfolios minus the returns on the large capitalization share portfolios:

$$SMB = \frac{[(S/L - (B/L)] + [(S/N) - (B/N)] + [(S/H) - (B/H)]}{3}$$

HML is the equally-weighted average of the returns on the high BTM share portfolios minus the returns on the low BTM share portfolios:

$$HML = \frac{[(S/H - (S/L)] + [(B/H) - (B/L)]}{2}$$

The SMB and HML factors are calculated independently in order to avoid scaling HML based on SMB and vice versa. It is evident in the literature that these two factors have some effect on each other and therefore need to be defined independently (L'Her et al., 2004; Auret and Sinclaire, 2006).

3.6.4 WML:

It is identified in Jegadeesh and Titman (2001) that analyzing short run momentum based on one year prior returns and a holding period of three months produces the most noticeable excess returns on a portfolio. As momentum will form one of the explanatory factors in the C4 model, it is considered most relevant to analyse momentum in line with a method that will yield consistently noticeable premiums for past winners versus past losers, therefore one year of prior share returns will be used to develop the momentum premium WML.

For each month t from July of year y - 1 (beginning in July 2001) to June of year y, the shares are ranked based on their size in July of year y - 1 and their performance between t − 12 and t – 2 and not t - 1 as the bid-ask bounce identified in Rouwenhorst (1999) can reduce the momentum effect that shares exhibit if the period used is close to a year or greater. The same break-point procedure is followed as for the BTM factor by dividing at 30% and 70%. Shares above the 70% prior performance breakpoint are designated W (for winner), the middle 40% are designated N (for neutral) and firms below the 30% prior performance breakpoint are designated L (for loser). Six value-weighted portfolios are then formed, S/L, S/N, S/W, B/L, B/N and B/W, at the intersection of size and prior performance groups.

WML is the equally-weighted average of the returns on the winner share portfolios minus the returns on the loser share portfolios:

$$WML = \frac{[(S/W - (S/L)] + [(B/W) - (B/L)]}{2}$$

3.6.5 INDIVIDUAL SHARE LOADINGS FOR SMB, HML AND WML

SMBi, HMLi and WMLi represent the covariance of the particular share with each of the portfolios that have been identified for SMB, HML and WML. These factors are calculated in the same way as

the traditional β, by performing a regression of the returns of a share with the returns of the above factors.

Multiple regression is applied to ensure that the loading of each factor incorporates the fact that a multivariate model is being utilized.

3.7 CONSTRUCTING PORTFOLIOS

3.7.1 PORTFOLIO FORMATION

Five equally weighted portfolios were constructed for the purpose of evaluating each of the models. A hypothetical R320 000 is invested in each portfolio on the 1st of July 2001. The choice of R320,000 per portfolio is a random figure that was chosen simply because this means that R10,000 is invested in each share at the start of the evaluation period, and therefore is easy to compare the growth of shares to. Shares are categorized into portfolios based on the expected return of the share. The use of equally weighted portfolios conforms to most international and South African studies of risk factors and is therefore applied in this study for the purpose of comparability (Suh, 2009; Basiewicz and Auret, 2010).

3.7.2 PORTFOLIO REBALANCING:

Portfolios were rebalanced every quarter. The choice of quarterly rebalancing is in line with Ward and Muller (2012)'s empirical testing of the CAPM, and is expected to ensure that the use of the most up to date risk factors in each asset pricing model is maintained. Quarterly rebalancing is also important to ensure that the momentum effect is incorporated in such a way that shares are held for a 3 month period, in line with the established pattern of short run momentum that is identified by Rouwenhorst (1999).

3.7.3 TRANSACTION COSTS:

Transaction costs have been ignored relating to the rebalancing of each portfolio on the grounds that these will be approximately the same between portfolios, and therefore immaterial to the methodology and results.

3.8 MODEL ANALYSIS

A separate analysis has been performed for the CAPM, F&F3 and C4 models. At the end of each quarter, the cumulative value of each portfolio was calculated from a base of R320 000 and graphically represented. Each of the 5 portfolios under each model were visually compared to

assess whether the portfolios that have the highest expected return actually generated the highest return over the 2002 to 2012 period. The use of cumulative returns and of visual comparison is in line with Ward and Muller (2012), who describe that this methodology is preferable to traditional statistical regression techniques.

It should be noted that it is not the purpose of this research report to specifically analyse whether the percentage returns that are actually generated by portfolios of shares are very close to the figures that have been generated as expected cost of equity by the asset pricing models. Therefore the concern of this research is whether the comparative riskiness of a high expected cost of equity portfolio over a low expected cost of equity portfolio is reflected in the actual returns of those portfolios over time.

3.9 LIMITATIONS

An identified limitation of this study is the use of a relatively short time horizon for the evaluation of the models. It is identified in Van Rensburg and Robertson (2003) that the period under evaluation should ideally be longer than 10 years, but for their particular study, data collection problems restricted the period that could be analysed. Van Rensburg and Robertson (2003) alert researchers who consider evaluating a longer period that they should be aware of the risk that survivorship bias may be incorporated into a more extensive study, therefore this study will not extend past a 10 year period, in order to attempt to ensure that survivorship bias does not taint the results to a significant extent.

An accepted limitation of this study is the broadness with which the various asset pricing models are evaluated. The purpose of this study is not to be entirely accurate regarding the ability of asset pricing models to reflect the actual returns on shares, but rather whether the model can be relied upon to ensure that shares that have a high expected return actually generate a higher return relative to shares that are expected to generate a low return.

There is some contention in the literature regarding the formation of the market portfolio, as the (Markowitz, 1952) concept of the market portfolio requires that the global portfolio of risky assets should be included. An accepted limitation of this study is the inability to incorporate all risky assets into the market portfolio.

4 RESULTS

The next chapter in this report lays out the results of the testing that was described in the methodology above. The results of this study are presented in two sections. First, the results of forming the SMB, HML and WML portfolios are presented. Second, the results of the share portfolios under each asset pricing model are presented.

4.1 FACTOR CONSTRUCTION

Annual returns on each of the portfolios: Market portfolio, SMB, HML and WML are presented in Table 1. In this research report, the growth of each portfolio was calculated for each month between July of 2000 and June of 2012, however it is superfluous to present each and every month's result for the abovementioned portfolios. Therefore, the growth of R1 invested in each portfolio for an entire year is presented, compounded monthly. It should be noted that by presenting compound annualized growth, the volatility of the data is enhanced dramatically, therefore the standard deviation and correlation of the annualized data is very different to the standard deviation and correlation of the monthly portfolio data. In the interest of completeness, the standard deviation of both monthly and annualized data has been presented below.

Cumulative Growth of R1 Invested per Annum				
	Market Portfolio	**SMB**	**HML**	**WML**
28 June 2012	6.543%	-10.110%	-14.116%	27.728%
28 June 2011	21.186%	-10.196%	-8.628%	4.296%
28 June 2010	25.551%	7.566%	7.577%	-5.360%
28 June 2009	-18.773%	19.812%	9.345%	-37.810%
28 June 2008	5.830%	-20.976%	-9.926%	29.492%
28 June 2007	35.489%	11.940%	-0.712%	5.404%
28 June 2006	56.419%	-18.030%	0.653%	10.117%
28 June 2005	41.157%	13.374%	26.287%	1.110%
28 June 2004	25.959%	13.459%	32.237%	4.427%
28 June 2003	-18.326%	27.113%	11.435%	0.393%
28 June 2002	38.753%	-5.126%	14.133%	10.791%
Correlation Matrix of Monthly Data				
Market Portfolio		-30.874%	-8.066%	-8.510%
SMB		100.000%	3.293%	-7.251%
HML		3.293%	100.000%	-6.563%
WML		-7.251%	-6.563%	100.000%

Table 1

Standard Deviation for monthly returns		
Market Portfolio:	5.132%	
SMB:	3.887%	
HML:	3.121%	
WML:	4.042%	
Standard deviation for annualised returns		
Market Portfolio:	22.637%	
SMB:	14.156%	
HML:	12.890%	
WML:	14.343%	

Table 2

4.1.1 FORM OF FACTOR ANALYSIS

The first section of the results is concerned with the performance of each zero sum portfolio SMB, HML and WML. The SMB portfolio is driven by the outperformance of small shares over large shares, the HML portfolio is concerned with the outperformance of value shares over growth shares and the WML portfolio is concerned with the outperformance of past winners over past losers.

The results are presented as follows:

(1) An analysis of the annualized return of each portfolio to provide information on the significance of portfolio outperformance.

(2) The standard deviation of monthly data over an 11 year period in order identify whether there is high volatility in the returns of each portfolio, as high volatility may indicate that the results are not consistent enough to be utilized over a long period.

(3) The correlation between the different portfolios. If two or more portfolios are highly positively correlated, then they incorporate the same information, and it is not necessary to include more than one of the factors in an asset pricing model, because by incorporating one of the factors, the others are already compensated for.

(4) The regularity with which each portfolio generates positive returns, both on a monthly basis and on a compounded annual basis. This provides information regarding whether the assertion that small outperforms big, value outperforms growth and past winners outperform past losers actually holds on the JSE.

4.1.2 SMB FACTOR

The SMB factor portfolio reflects annualized returns ranging from -21% in 2008 to 27% in 2003. The standard deviation of the SMB portfolio is relatively high, indicating high volatility in the monthly return data of this portfolio. The much higher volatility that is evident in the annualized data presented above is due to the compounding effect of condensing 12 months of monthly share growth data.

There is a medium negative correlation between the SMB portfolio and the market portfolio and there is very little correlation between the SMB factor and the HML and WML factors. The weak correlation between SMB and the other factors provides evidence that the inclusion of SMB as a factor in predicting share returns may lead to a different cost of equity compared to the traditional CAPM, as the returns of each portfolio are uncorrelated.

The weak correlation between SMB and other factors also provides an indication that the SMB factor incorporates information into share price prediction that is not provided by the traditional market portfolio, or the HML and WML portfolios. The negative correlation between SMB and the market portfolio is interesting, as it appears that portfolios of shares based on market capitalisation will perform in a dissimilar way to the market portfolio. As the SMB portfolio is driven by the performance of small shares, this provides evidence that the market portfolio return is dominated by the larger shares on the JSE, evidence that is backed up by the fact that the return of the JSE is dominated by the ALSI 160, and the return of the top 160 is dominated by the return of the top 40 shares on the JSE (Basiewicz and Auret, 2009).

The SMB portfolio generates positive growth, meaning that small shares have outperformed large shares in 63 out of 120 observed months, or 52.5% of the time. This gives an indication that the original Banz (1981) argument that small shares will consistently outperform large shares only holds around 50% of the time on the JSE. When considering the annualized returns of the SMB portfolio, there is still evidence that the SMB portfolio only performs positively around 50% of the time, in 5 out of 10 years. Based on this evidence it appears that the SMB factor is evident on the JSE stock exchange in a weak form, as the SMB portfolio does not consistently generate positive returns.

4.1.3 HML FACTOR

The HML factor portfolio reflects annualized returns that range between -14% in 2012 and 32% in 2004. The standard deviation of monthly returns of the HML portfolio is found to be relatively high, but slightly lower than the volatility of the SMB portfolio.

As mentioned above, there is very low correlation between the HML factor and the SMB factor of only approximately 3%. It can also be seen that the HML factor has weak correlation with all other factors as all correlations are below 10%, which provides some indication that the value effect is not already present in the market portfolio, nor the SMB and WML portfolios.

The HML portfolio generates positive growth in 60 of 120 observed months, or 50% of the time; however, if one analyses the annualized return of the HML portfolio, a different result emerges. The HML portfolio performs positively in 7 of 10 years, therefore indicating more consistent outperformance than the monthly data. This provides evidence that the positive growth that is observed in certain months is relatively larger than the negative growth that is evident in the remaining months.

Based on this evidence it appears that the HML factor is evident on the JSE in a moderately strong form, as the factor manages to generate positive returns on a relatively consistent basis if one assumes that 70% positive returns is an indication of consistency. A dampener on this conclusion is that the HML factor has a high standard deviation, meaning that the consistency with which the HML factor can be expected to generate positive returns cannot be forecast with a high level of certainty.

4.1.4 WML FACTOR

The WML factor portfolio is able to generate annualized returns that range between -38% in June of 2009 and 29% in June of 2008. The standard deviation of the WML monthly portfolio amounts to the highest of any factor that is being analysed in this report.

Past winners are found to outperform past losers in 72 of 120 observed months, a 60% success rate. When inspecting the annualized returns, again a different result emerges. In this case, the WML portfolio shows positive growth in 9 out of 10 years, indicating that the positive returns in 72 months is much greater than the decrease in value in the remaining 48 months. It is clear that the performance of the WML portfolio is much more positive than the other portfolios that form the basis for the C4 model. However it is also clear that there is a large amount of volatility in the data, which may reduce the reliability of the result. It may be that the large positive growth is a fortunate coincidence over the period under study, however, as the results span over a 10 year period, perhaps the positive result is sustainable.

4.1.5 OVERALL RESULT

Overall, the lack of correlation of the various factors provides some indication that the Fama and French (1992) principle that the size and value phenomenon are not incorporated in the original Markowitz (1952) risk and return model of the CAPM holds on the JSE. It can therefore be expected that adding these additional factors will create a more informative asset pricing model, which takes into account more than just the volatility of market returns.

There is, however, only a moderate amount of evidence in support of Fama and French (1992)'s belief that small shares will consistently outperform large shares, or that low BTM shares will consistently outperform high BTM shares.

There is strong evidence to support the Carhart (1997) principle that short run momentum is evident on the stock market, and that the Jegadeesh and Titman (2001) principle that portfolio rebalancing should occur on a quarterly basis is able to reflect consistent outperformance of the WML portfolio.

4.2 PORTFOLIO PERFORMANCE

The performance of each of the CAPM, F&F3 and C4 portfolios are separately assessed next. However before inspecting the results of these portfolios, an example of the ideal expected performance of an asset pricing model is presented.

4.2.1 IDEAL PERFORMANCE

For the purposes of this research report, a well formulated asset pricing model should be able to accurately predict the actual return that is obtained from investing in portfolios of shares (Fama and French, 2004). The result that could be expected for a well specified asset pricing model, is that the highest expected cost of equity portfolio generates the highest actual growth in value, and the lowest expected cost of equity portfolio reflects the lowest actual growth in value, at the end of a period. There should also be an equal spread of the 3 intermediate portfolios that shows a ranking that agrees with the expected cost of equity of each portfolio (Ward and Muller, 2012).

If it is to be concluded that an asset pricing model is well formulated for the purpose of this research, the following graph should be the result:

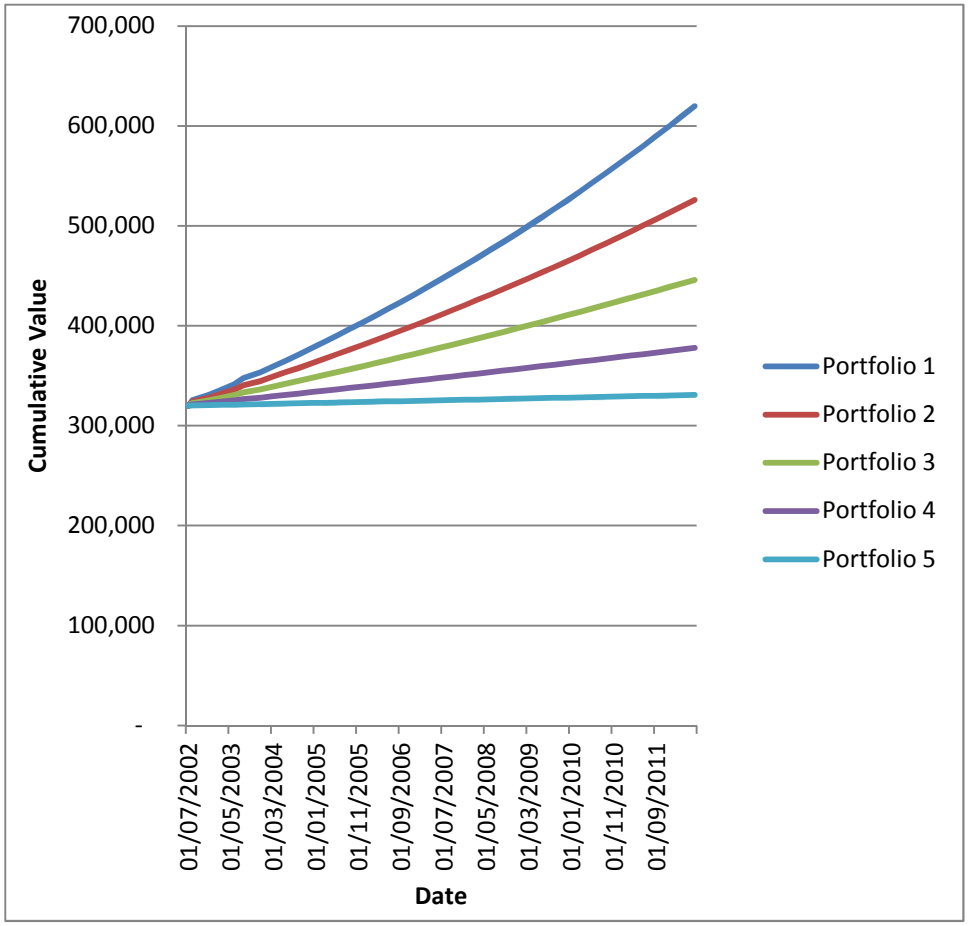

Figure 7

Ideal portfolio performance

In Figure 7 above, Portfolio 1 denotes the highest expected cost of equity portfolio whereas portfolio 5 denotes the lowest expected cost of equity portfolio. A highly successful asset pricing model would show a spread between the actual returns similar to that which is observed above, as the highest expected cost of equity portfolio has actually outperformed all other portfolios in the 10 year study period. It can also be seen above that the portfolios create a "fan type" graphical picture, where a portfolio that has a higher expected cost of equity than another portfolio does indeed outperform the latter portfolio; for example it is expected that portfolio 3 will outperform portfolio 4, and it does in fact do so (Ward and Muller, 2012).

4.2.2 CAPM PORTFOLIOS

Based on the literature review, evidence indicates that the CAPM should not be accurate in providing an expected return that is close to the actual return that is generated by investing in a portfolio of shares with very similar expected cost of equity (Ward and Muller, 2012).

Ward and Muller (2012) provided evidence that the CAPM in fact shows an inverse relationship between expected cost of equity and the actual return that is generated by share portfolios, meaning that those shares that have the highest risk (as measured by beta) and therefore are required by investors to have the highest expected return in the market in order for them to be attractive for investment, in fact generate the lowest of cumulative returns on the JSE. The methodology applied in this study has attempted to mimic the methodology applied by Ward and Muller (2012) in order to ensure that the results of this study will be comparable to their results. As such it is expected that if a similar result arises in this study, Ward and Muller (2012)'s methodology will have been consistently applied.

The cumulative growth for each of the 5 portfolios based on the CAPM cost of equity are presented in figure 8:

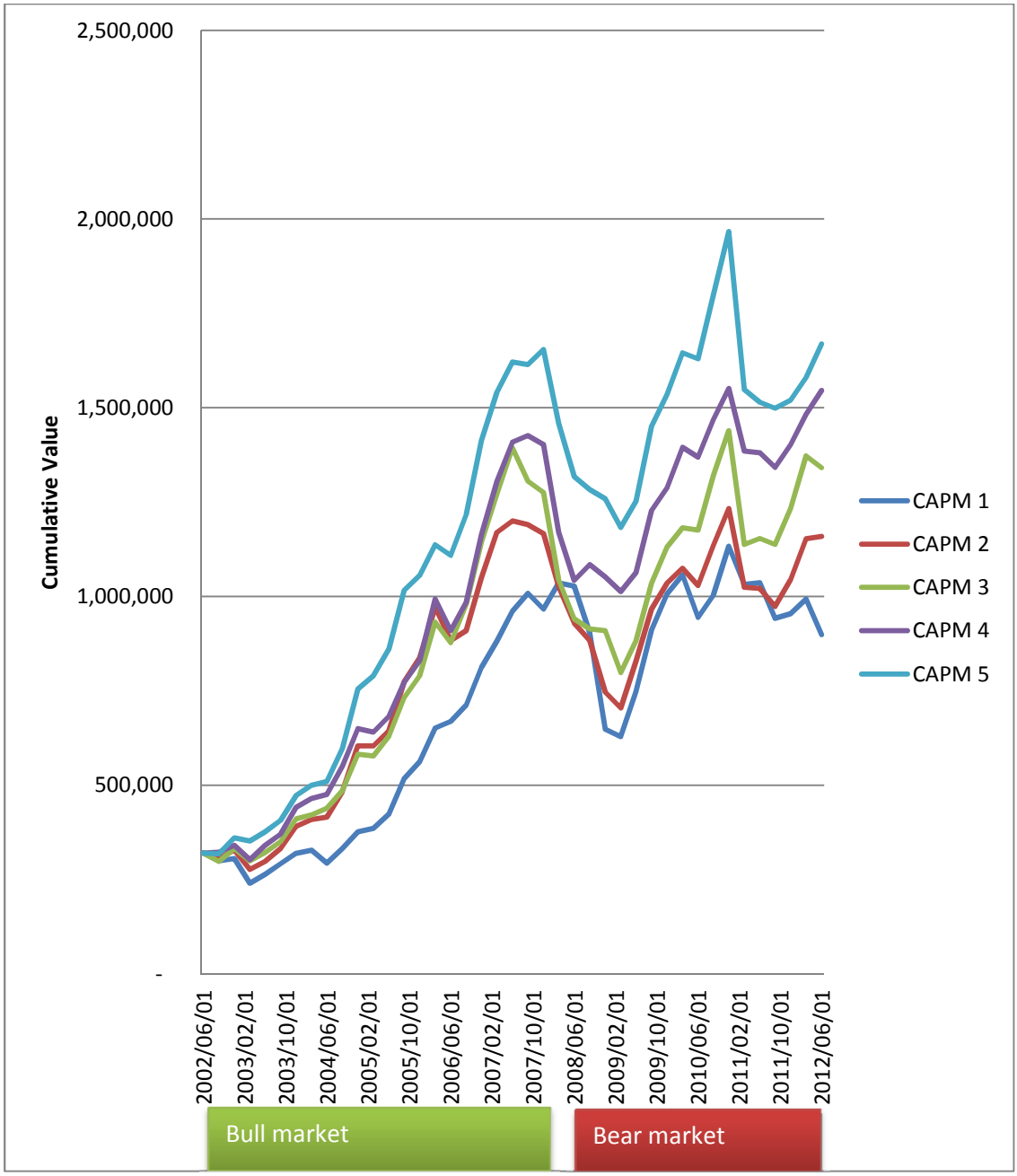

Figure 8

Cumulative Value of CAPM Portfolios

CAPM 1 denotes a share portfolio that is made up of the 32 shares with the highest expected cost of equity, CAPM 2 reflects the next 32 shares that have the highest expected cost of equity and so on until CAPM 5 which contains the lowest expected cost of equity shares.

As of 30 June 2012, it can be seen in figure 8 that the results of the portfolio performance is exactly the opposite of what is expected by Markowitz (1952)'s traditional portfolio theory. It is found that the lowest expected cost of equity portfolio, which investors should expect to generate the lowest

return, actually generates the highest return of any portfolio of shares. This indicates that those shares that could be seen to contain the least amount of risk, i.e. volatility in relation to the market, actually generate the highest return for investors. The cumulative value of each portfolio at the end of June each year is presented in Table 3 below.

	CAPM				
Date	Portfolio Balances				
	CAPM 1	CAPM 2	CAPM 3	CAPM 4	CAPM 5
Performance During Bull Market					
28 June 2002	320,000.00	320,000.00	320,000.00	320,000.00	320,000.00
28 June 2003	263,950.08	298,350.93	321,358.08	340,412.72	376,421.63
28 June 2004	293,242.02	415,438.27	438,765.31	475,395.00	509,356.40
28 June 2005	423,881.18	644,161.80	629,649.69	682,304.98	860,589.88
28 June 2006	669,094.64	883,618.58	876,939.04	909,266.90	1,108,409.21
28 June 2007	961,212.15	1,200,201.89	1,393,246.55	1,409,044.47	1,621,141.88
28 June 2008	1,027,404.22	929,245.25	940,842.88	1,042,650.70	1,317,299.89
Performance During Bear Market					
28 June 2009	747,396.21	828,534.99	882,899.39	1,063,194.31	1,252,081.42
28 June 2010	944,142.74	1,028,708.91	1,175,399.68	1,369,346.10	1,629,522.40
28 June 2011	1,036,248.89	1,020,948.30	1,153,558.82	1,380,798.84	1,514,658.21
28 June 2012	898,747.26	1,158,829.10	1,341,090.86	1,545,763.61	1,668,749.36

Table 3

CAPM portfolio values

These results are in line with Ward and Muller (2012), indicating that their methodology has been consistently applied in this study. More importantly, it is also established that the traditional finance principle that investors will demand a higher expected return for taking on excess risk does not actually pay off on the JSE, at least when one defines risk as the volatility of a shares price in relation to the market.

In September of 2008 the global financial crisis affected markets. It is interesting to consider whether the cumulative value of portfolios as at 30 June 2008 present a different result to the cumulative value at the end of the study period in June of 2012.

As of June 2008, the cumulative values of portfolios are not consistent with their expected cost of equity, there is in fact a random result as CAPM1, 4, 5 all have cumulative values that are relatively similar and have outperformed CAPM2, 3 to a larger extent.

The fact that the results of portfolio performance are different when one separates between Bull and Bear market periods gives some indication that the type of market an investors finds him/herself in may have an effect on the results of calculating cost of equity.

4.2.2 F&F3 PORTFOLIOS

Following the same methodology, figure 9 presents the cumulative values of the F&F 3 portfolios:

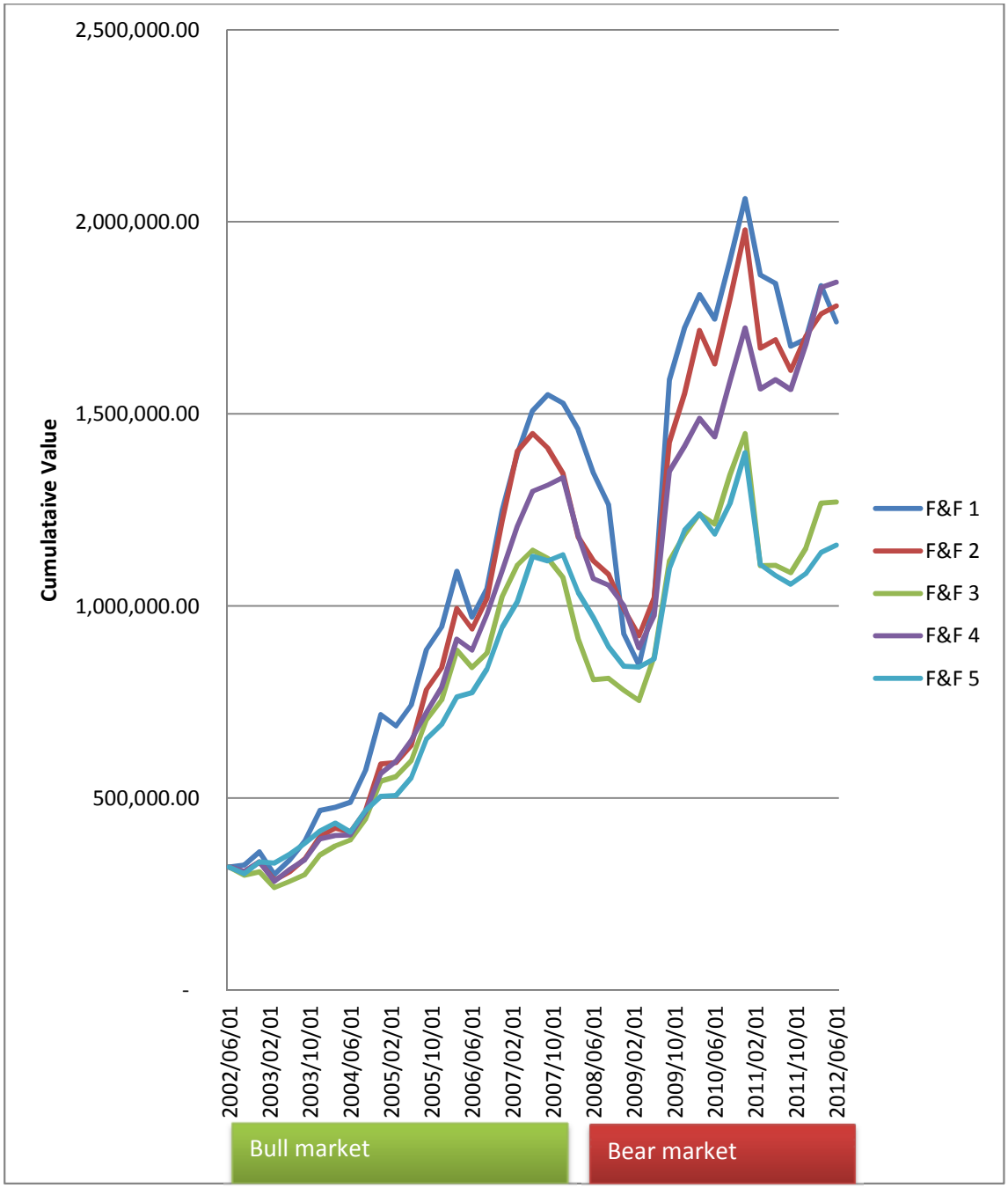

Figure 9: Cumulative Value of F&F3 Portfolios

In analyzing the graphical results of the F&F3 portfolios, it can be seen that there is very little difference in the cumulative values of F&F1, 2, 4 as at the end of the study period. There are periods where F&F1 and F&F2 outperform all other portfolios, which conforms to what was expected, however F&F4 consistently performs better than its expected return, which confounds the results. There is also very little difference in the values of F&F 3, and 5.

The cumulative value of each portfolio is presented below.

	Fama and French 3 Factor				
Date	**Portfolio Values**				
	F&F 1	F&F 2	F&F 3	F&F 4	F&F 5
Performance During Bull Market					
28 June 2002	320,000.00	320,000.00	320,000.00	320,000.00	320,000.00
28 June 2003	338,638.30	307,392.86	282,951.37	313,489.35	352,999.43
28 June 2004	488,955.95	411,879.97	390,086.94	403,405.02	411,253.06
28 June 2005	741,585.29	636,559.70	596,791.73	649,846.65	552,565.52
28 June 2006	970,713.22	939,596.22	839,583.02	884,841.51	774,320.91
28 June 2007	1,508,122.27	1,448,454.72	1,145,115.15	1,298,361.03	1,129,094.92
28 June 2008	1,346,472.66	1,117,535.07	807,893.69	1,071,378.41	969,101.16
Performance During Bear Market					
28 June 2009	1,016,650.18	1,022,856.04	866,942.49	974,800.90	861,677.24
28 June 2010	1,746,444.19	1,629,809.70	1,212,845.08	1,440,013.40	1,186,748.68
28 June 2011	1,839,332.92	1,693,256.12	1,105,773.55	1,588,552.34	1,078,906.99
28 June 2012	1,738,818.48	1,780,915.20	1,270,259.84	1,842,900.77	1,158,099.34

Table 4

Cumulative value of F&F3 portfolios

Based on the results of the F&F3 portfolios, it can be seen that Fama and French (1993) make some progress in establishing an asset pricing model that allows investors to believe that if they take on the excess risk that is defined by the F&F 3 model, they will be rewarded for doing so. However it is still clear that there is not an even spread between the portfolios that could be expected to generate a higher return than less risky portfolios, as portfolio F&F4 appears to confound the results by generating the highest cumulative return of any portfolio over the study period.

4.2.3 C4 PORTFOLIOS

Following the same methodology, the results of the C4 portfolios are presented below in Figure 10:

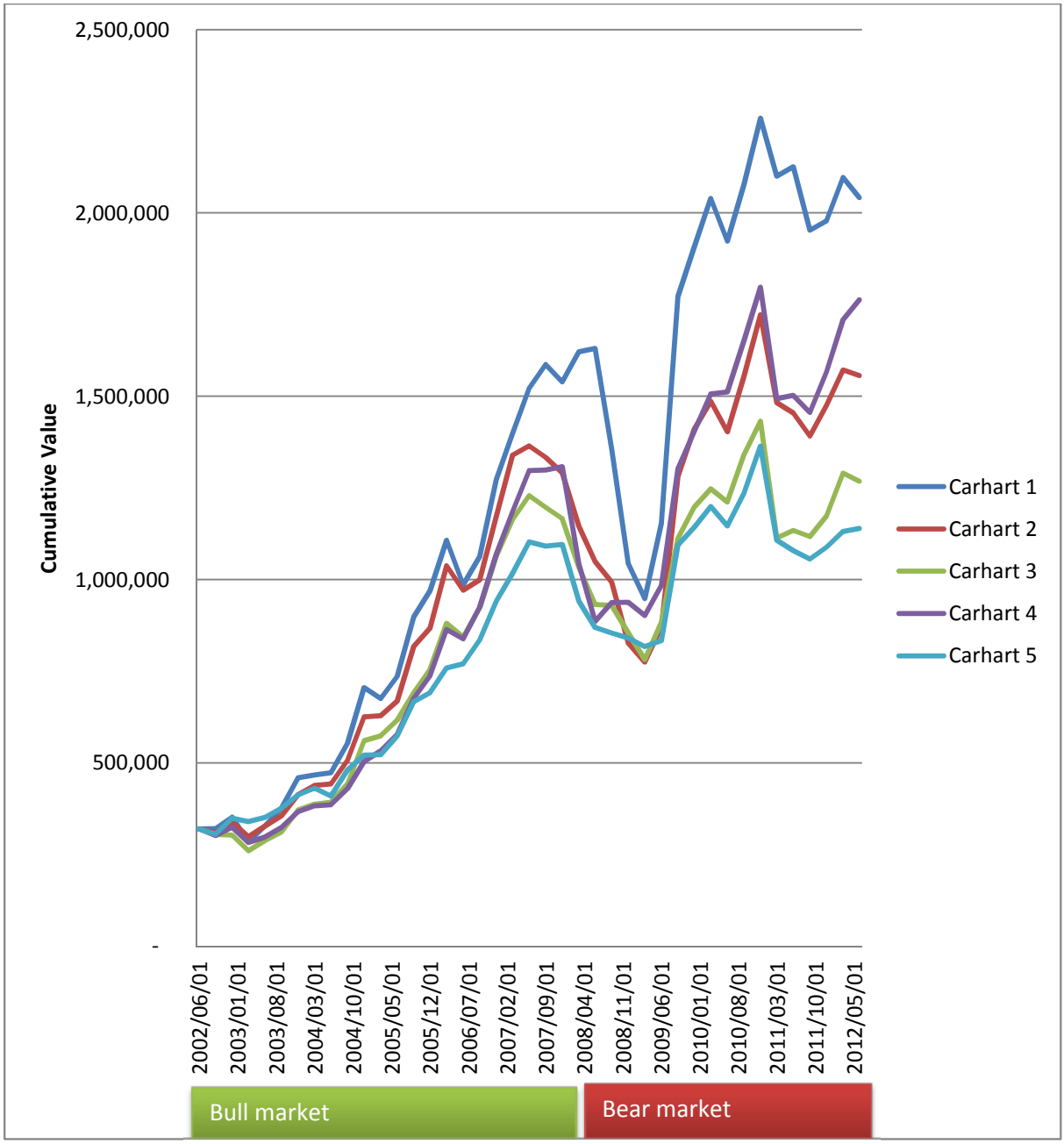

Figure 10

Cumulative Value of C4 Portfolios

It is interesting to note that Carhart 1 and Carhart 5 are performing just as is expected under the ideal performance graph described at the start of this section over the entire study period; the highest expected cost of equity portfolio (Carhart 1) outperforms all other portfolios significantly, while the lowest expected cost of equity portfolio (Carhart 5) performs the worst of any portfolio in total over the 10 year study period. There are however some inconsistent results in Carhart 2,3, and 4 which do create some doubt as to the validity of the model across the entire timeframe examined.

The results therefore establish the C4 model as being better equipped to reflect an `accurate cost of equity than either the CAPM or the F&F3 model when one considers the South African context.

In order to further analyse the results in terms of a bull or bear period, the C4 portfolio was broken down into two periods to assess the performance over a bull period only, in the time line of this study, this represents the results from the start up to 30 June 2008.

Figure 11 reflects the cumulative result of the C4 portfolios up until the 30 June 2008. As can be seen in the graph, the C4 portfolios behave in the way that is expected of a well formulated asset pricing model over the period 2002 to 2008. Any inconsistencies in portfolio performance therefore arise between 2008 and 2012, while the JSE is experiencing a bear market. Therefore it is possible that some of the reason for inconsistent cumulative performance of the C4 model at the end of the study period could relate to differences in investor behavior in Bull and Bear markets.

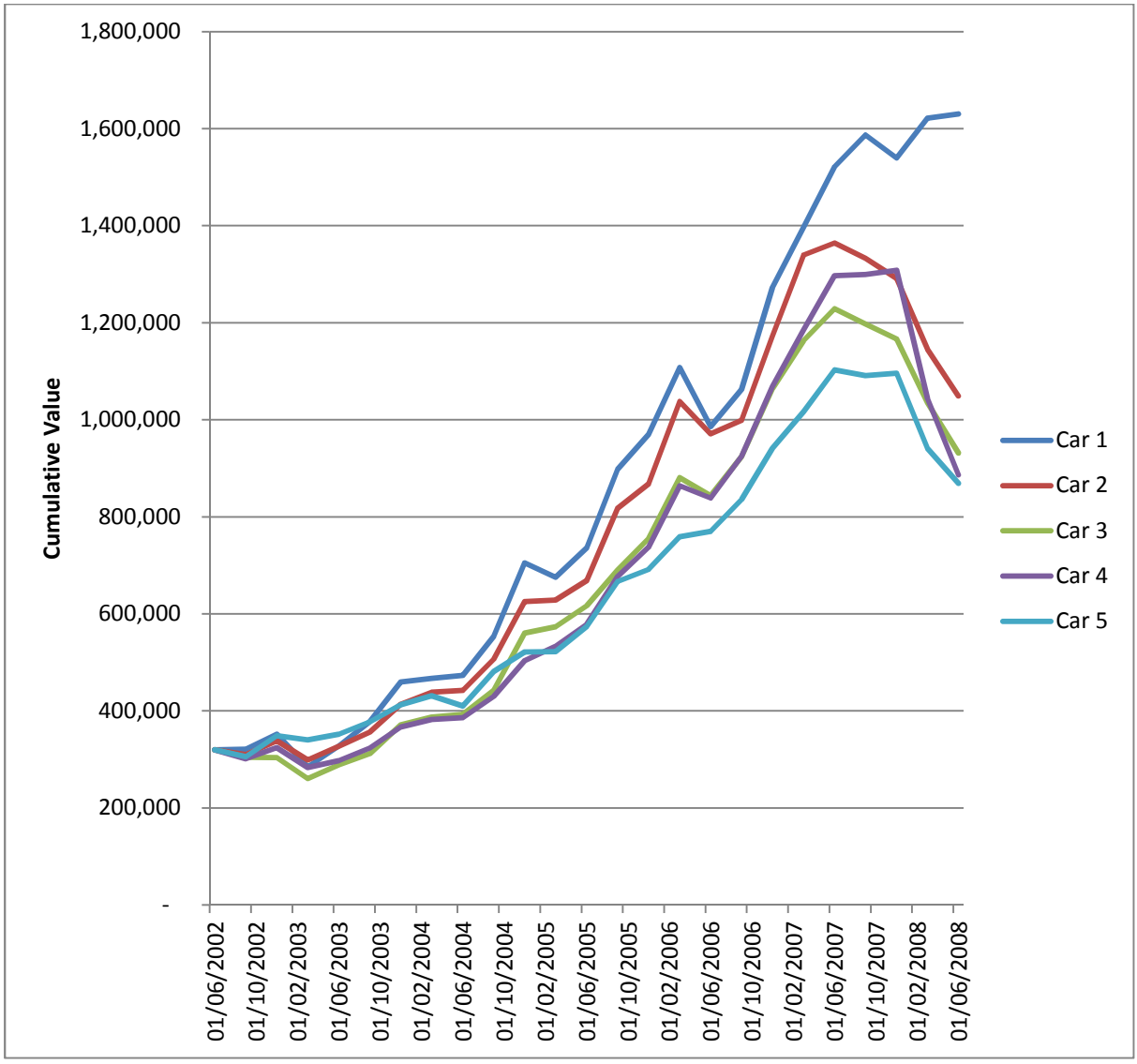

Figure 11

Cumulative value of C4 portfolios – Bull market

The cumulative values of C4 portfolios are presented in table 5 below:

	Carhart 4 Factor				
Date	**Portfolio Values**				
	Carhart 1	Carhart 2	Carhart 3	Carhart 4	Carhart 5
Performance During Bull Market					
28/06/2002	320,000.00	320,000.00	320,000.00	320,000.00	320,000.00
28/06/2003	327,773.99	327,386.22	288,821.47	297,650.22	351,766.40
28/06/2004	472,968.72	442,320.20	392,690.04	385,767.65	409,955.27
28/06/2005	736,097.28	668,355.46	616,472.62	578,248.25	573,856.17
28/06/2006	985,847.07	970,730.93	843,354.64	838,551.43	769,964.23
28/06/2007	1,521,672.63	1,364,554.75	1,228,703.57	1,296,924.78	1,102,444.64
28/06/2008	1,630,163.51	1,049,213.94	931,697.88	886,204.57	868,991.99
Performance During Bear Market					
28/06/2009	1,153,506.78	870,475.15	883,719.66	983,976.93	832,761.40
28/06/2010	1,923,220.34	1,402,819.73	1,211,146.67	1,511,351.51	1,146,535.85
28/06/2011	2,125,689.30	1,453,859.16	1,133,902.69	1,502,230.76	1,078,789.50
28/06/2012	2,041,692.51	1,555,759.38	1,268,439.32	1,762,726.91	1,139,197.85

Table 5

Cumulative value of C4 portfolios

From the results of the C4 portfolios, it can be seen that the C4 model has some inconsistencies due to portfolio C4 performing well above what is expected. However, it can be seen that the C4 model makes the most progress towards developing an asset pricing model that defines risk in such a way that investors are rewarded for taking on excess risk. It is also established that whether the market is experiencing a bull or bear phase has an influence on the ultimate results of the study.

5 CONCLUSION

International studies by the likes of Fama and French (1993) and Carhart (1997) indicated that the CAPM may not be reliable in providing a cost of equity that reflects the actual return that can be expected from an investment in shares. Local research by the likes of Van Rensburg and Robertson (2003) and Hoffman (2012) support the claims of the international body of research by identifying factors that are not incorporated into the CAPM, but which may explain how share prices move on the JSE.

This research report has set out to identify whether 3 well known asset pricing models can be relied upon to provide an indication of the actual return that should be expected from investing in a portfolio of shares. It was also the purpose of this research to identify whether the F&F3 or C4 models perform any better than the CAPM in indicating the return that should be expected from an investment in a portfolio of shares.

In order to address the objectives of this research, portfolios of shares were picked based on a ranking of the expected cost of equity. The method applied to develop cost of equity was informed by Ward and Muller (2012) in order to ensure that the results of this study are comparable to leading research in the field of finance in South Africa.

The results indicate that the CAPM is unable to reflect a cost of equity that is a good estimate of the actual return that can be expected from investing in shares. It is found that the F&F3 model and C4 model make great strides towards developing an asset pricing model that reflects a ranking where those portfolios that have the highest expected cost of equity will actually generate the highest return and those portfolios that have the lowest expected cost of equity will generate the lowest returns over the study period.

The results above provide support to the existing literature, indicating that the traditional CAPM is misapplied on the JSE, as it should not be relied upon to provide an indication of the return that should be expected from an investment in a portfolio of shares. The C4 model, by incorporating 3 additional factors to market risk, provides the most relevant model for application in assessing an expected return that conforms to reality. Finally, the results of this research reflect the concept that traditional finance principles may have assumptions that are not representative of the JSE.

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