

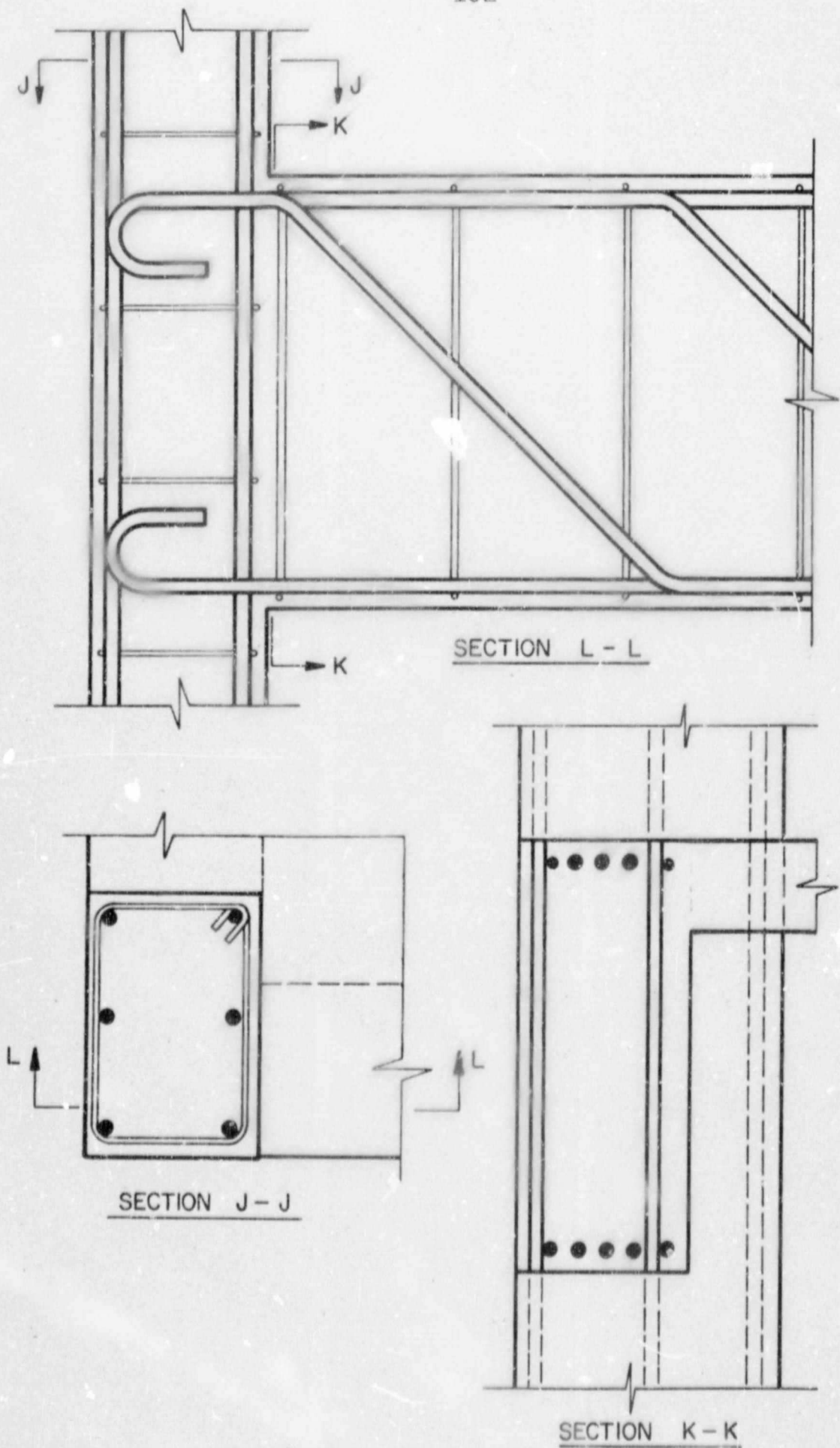


FIGURE 54

Presumed details of reinforcing steel at junction of beam BC and column B

the junction of a column and beam is the method of finite elements. If a problem can be simplified by reducing it to an analysis of stresses acting in a plane, a few computer programs are available which can determine the principal stresses in a relatively short computer time. Although, in the case of the cracks in the Cape Town building, the corner columns are subjected to two plane stress systems at right angles to each other, it is considered that the visible cracks on each of the outside faces of the corner column were the result of the principal stresses in the plane of that surface and internal planes parallel to it. The junction of column B and beam BC has been analysed as a plane stress problem. The probable shape of internal cracks was discussed in section 6.1.

6.3.1 Computer program

At the University of California, Edward L Wilson led a team who developed a general structural analysis program, SAP, based on finite elements. The program is a most powerful and efficient program for the linear elastic analysis of complex structural systems including the effects of gravity and expansions. An instruction manual²³ for users was written by Pells, Kovac and Traxler and the program for two-dimensional finite element analysis (matrix solution) was named WAXI.

Stresses obtained when using the WAXI program were compared with the stresses obtained by the iterative method of Richardson in the case of a simply supported deep beam loaded along its upper edge. Both the magnitudes and directions of the principal stresses were very similar.

The WAXI program is coded in FORTRAN IV and is practically machine independent. This was shown when converting the SAP program for use on the CDC computer. All the punched cards of the computer program used on the IBM system, 360 model 65 (CSIR) were used for the CDC with only the JCL (Job Control Language) cards and a couple of statements to do with

information /

information storage for the plot of the principal stresses having to be changed. This program can analyse systems composed of large numbers of elements. The capacity of the program is controlled by the number of nodal points defining the finite elements into which the plane structure is divided. Each joint in the system can have from 0 to 6 degrees of freedom as required. The number of joints that can be dealt with in the present program is 1 200 with a bandwidth of 39. Up to 12 materials of differing properties can be present in a system and these materials can undergo 8 different expansions. Cracking of a structure cannot be dealt with in this program.

The plane stress element included in the program was a general quadrilateral with a one-point integration of strain energy. The geometry of the quadrilateral element is defined by four points. The element must lie on a plane, therefore only three joints are used to define the plane. The fourth joint is assumed to be on the plane. This quadrilateral element does not have good bending characteristics; however bending stresses are not important within the material of the junction between a beam and column and this program gives quite acceptable results.

Triangular elements may also be used by making two joints of a quadrilateral element coincide at a nodal point, but my experience has been that triangular elements do not give principal stress directions very accurately. Changing the two nodal points that were considered as coinciding gave the same calculated principal stresses and directions.

The poor directions of principal stress with triangular elements has also been noticed in published work²⁴ done by others using finite element methods to solve plane stress problems, but using other computer programs.

For static analysis, the program is divided into four phases and a common storage area is used for each phase. These four phases are executed in the following sequence:

- (a) Data input - Joint co-ordinates and loads are read or generated. As element properties are read or generated, the element stiffness matrices are formed and placed on tape.
- (b) Formation of total stiffness is accomplished by reading the element stiffness tape and forming the joint equilibrium equations in blocks.
- (c) Equilibrium equations are solved for joint displacements; all load conditions are treated at the same time.
- (d) From the joint displacement, element stresses are calculated for all load conditions. Stresses are printed.

The SAP program is built around a large capacity linear equation solver, USOL, in which the procedure used to solve the equations is not significantly different from the method developed by Gauss. Immediately after the nodal joint data is supplied to the program, a relationship between each joint degree of freedom and the corresponding equation number is established.

It is appreciated that the finite element method of analysis is a tool and the large volume of seemingly accurate data should not delude one into believing that the answer is any more accurate than the quality of the input data permits. Nevertheless the method represents a very powerful analytical aid in solving plane stress and other problems.

6.3.2 Data input

A right-handed orthogonal co-ordinate system is used to

describe /

describe the geometry of the structure. All joint loads are defined with reference to this system.

Loads may be applied by means of both point loads acting at the joints and by element loading. By element loading is meant body forces due to the density of the material under the force of gravity, or body forces due to expansion of the material with a rise in temperature. By giving the brickwork a suitable thermal coefficient and rise in temperature, the moisture expansion of the brickwork can be brought into the calculations. Boundary compressive forces are put in as negative pressures.

In the program the number of digits that can be included after the decimal point is limited, e.g. the density of a material, the co-ordinates of nodal points and forces on nodal points can only have three such digits. For this reason units have to be selected so that the number of digits after the decimal point is not exceeded.

Boundary conditions at a nodal point are given in the program by a code number, i.e.

1 for nodal points that are free to move in any direction which can be loaded in directions parallel to the co-ordinate axes,

- 1 for nodal points that can move vertically and are fixed so as not to move horizontally; they can be loaded in the vertical direction,
- 2 for nodal points that can move horizontally and are fixed so as not to move vertically; they can be loaded in the horizontal direction,
- 3 for nodal points that are fixed and therefore do not have loads applied.

The internal nodes in the finite element models used in this thesis were free to move and the code number was 0.

When entering the numbers of the nodal points defining an element in the data, the numbers are listed counter-clockwise and the maximum difference between nodal point numbers of any one element must be less than the bandwidth of 39.

Element cards must be entered in element number sequence. The total number of element cards to be punched can be reduced by numbering the nodal points of identical elements in a sequence so that the numbers of all the nodal points of each succeeding element are one greater than the corresponding nodes of the preceding element. The first and last element card of such a sequence must always be supplied, but the rest can be omitted and the program will automatically generate the omitted cards. The material identification code for the generated cards will be the same as the first and last cards.

The execution time of the calculation of the six (see 6.4) theoretically loaded joints was an average of 5,6 minutes except for one which had to queue up at one stage of the computation and its execution time was 14 minutes. Only an average of 1 minute 7,7 seconds was actual computer time. In the case of the portion of the Cape Town building, the execution time was an average of 13,0 minutes of which only an average of 3 minutes 24 seconds was computer time.

The computer printed element stresses give tension as positive and the direction of principal stress is positive when measured anti-clockwise from the X-axis. The stresses parallel to the two axes and the associated shear stress at each centroid is also printed by the computer. The coding of the data cards is given in Table 10.

TABLE 10 CODING DATA CARDS (WAXI)

CARD GROUP	NUMBER OF CARDS	SYMBOLS	DESCRIPTION	FORMAT
1 CONTROL	1	NUMNP NUMEL NUMMAT ACELZ Q NP NPP	Number of nodal points Number of elements Number of different materials Acceleration in the X direction = 1 Reference temperature 1 for linear analysis ≠ 0 i.e. 1 for plane stress structure	3I5, F15.2, F10.2, 2I5
2 MATERIAL PROPERTIES	1 card for each material type plus 1 card for each temperature for which material constants are given	MTYPE NUMTC RO XXNO E(I, 1, MTYPE) E(I, 2, MTYPE) E(I, 3, MTYPE) E(I, 6, MTYPE)	Material number Number of different temperatures for which properties are given Weight density of material Modulus ratio. For linear analysis use 1 Temperature Young's modulus in plane XY Poisson's ratio in plane XY Coefficient of thermal expansion in plane XY	2I5 2F10.3 3F10.0 F30.0

TABLE 10 (CONT.) CODING DATA CARDS (WAXI)

CARD GROUP	NUMBER OF CARDS	SYMBOLS	DESCRIPTION	FORMAT
3	1 card for each nodal point	N CODE X Y UX UY T	Nodal point number Boundary condition X co-ordinate in plane problems Y co-ordinate (vertical) Specified load in the X direction Specified load in the Y direction Temperature developed	I5, F5.0, 5F10.3
4	1 card for each element	M IX(M, 1) IX(M, 2) IX(M, 3) IX(M, 4) IX(M, 5)	Element number Nodal point I Nodal point J Nodal point K Nodal point L Material identification number	

6.3.3 Plot of continuum and principal stresses

The information from the WAXI program, regarding the element numbers, co-ordinates of the centroids of the elements and the magnitudes and directions of principal stresses acting at these centroids, besides being printed by the computer, was also stored on tape so that it would be available for use with a program to plot this information. This program was named VECTOR and is included in the instruction manual²³ previously mentioned. The relative dimension statement in the program was increased, so that the program could deal with 2 200 elements instead of the 600 originally written into the program

A geometric scale factor, SCA, has to be calculated for each plot. The width of the paper in the plotter being used was 280 mm. The continuum was usually plotted not more than 200 mm high to allow for stress vectors being drawn past the boundaries of the continuum. This was done by making

$$SCA = \frac{\text{Difference in vertical co-ordinates of boundaries}}{8}$$

The scale factor for plotting the vectors of the principal stresses can be calculated automatically by the program and this has been done. If a principal stress is shown as positive by the computer, it is a tensile stress and the plotted vector is provided with an asterisk. The coding of the data cards is given in Table 11.

Where only a plot of the principal stresses over a limited area of the structure is required, the information can be punched onto cards and used instead of the information stored on the tape.

The execution time of the plot of the six theoretically loaded joints was about 3,1 minutes of which only 7,6 seconds was actual computer time. In the case of the

portion /

TABLE 11 CODING OF CARDS (VECTOR)

CARD GROUP	NUMBER OF CARDS	SYMBOL	DESCRIPTION	FORMAT
1	1	NUMEL NHOLE SCA SCB NO(1)	Number of elements Number of holes Geometric scale factor Scale factor for principal stress vectors Number of pairs of co-ordinates to describe structure	2I5, 2F10.2 I5
2	NO(1) ÷ 4	X(1,I), Y(1,I)	Co-ordinates defining outline of the structure	8F10.3, 8F10.3, etc.
3	1 for each 8 holes	NO(I)	Number of co-ordinate pairs to describe each opening within the structure	8I5
4	NHOLE set: of NO(I) ÷ 4	X(N,I), Y(N,I)	Co-ordinates describing the boundary of each opening	8F10.3
5	Tape or NUMEL cards	N PA(N) X(N) Y(N) XMAX(N) YMAX(N)	Element number Direction of major principal stress Co-ordinates of centroid of element Calculated value of major and minor principal stresses	I5, 5F10.2

portion of the Cape Town building, the execution time was an average of 2,2 minutes of which 9,9 seconds was computer time.

6.4 Analysis of Six Theoretical Conditions at a Junction

To get a picture of what to expect when a large horizontal force acts on a column immediately above where a horizontal beam joins it, six analyses were done on the representation of a junction between a column and beam with both the column and beam sectioned near the junction. Forces were applied to the cut ends to represent forces that might act across these sections in a structure. In most cases vertical loads were applied to the top of the beam and horizontal loads to the column. The computation of the forces acting on the nodal points for the different loading conditions are given in Appendix G. The finite element model had a uniform thickness and is shown on Figures 55 and 56.

The first three and the fifth analyses of the model were with the model under four different loading conditions and the fourth and sixth analyses had a crack simulated in the model.

CASE (a) : Only bending moments were applied to the model, both on the top of the column, 69 120 Nmm, and at the end of the horizontal member, 138 240 Nmm, while the nodal points on the bottom of the column had boundary conditions imposed on them to prevent the left-hand bottom corner from moving with respect to the axes and the other nodal points from moving vertically. The bending moment forces are shown lightly on Figure 57 with the resultant forces on the nodal points shown more heavily. The forces, on the nodal points, that are normal to AB and DE respectively, were calculated from the bending tensile and compressive stresses that made up the respective bending moments.

CASE (b) : Bending moments were applied as given in Appendix G

i.e. on the /

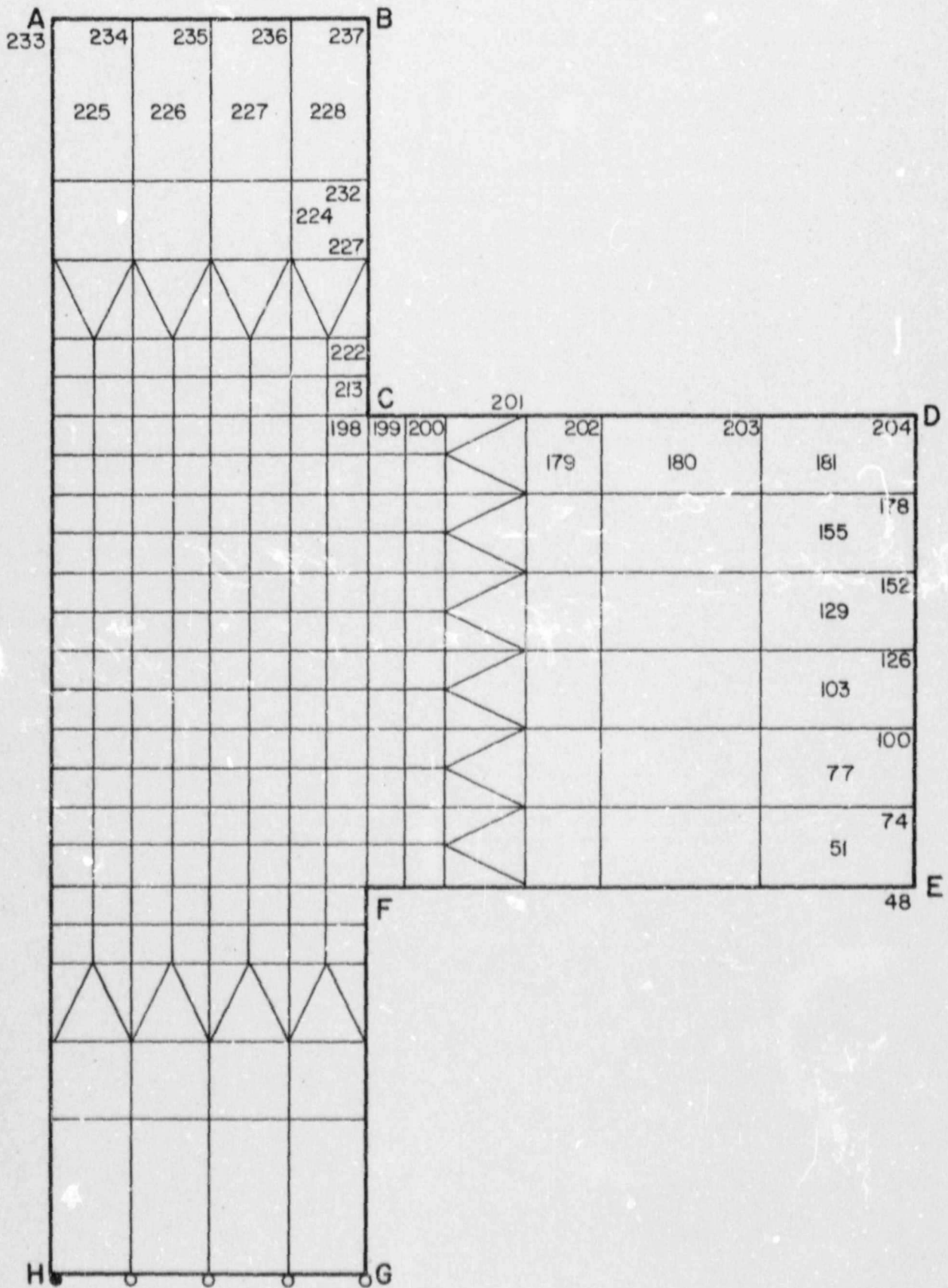


FIGURE 55

Finite element mesh for beam to column junctions

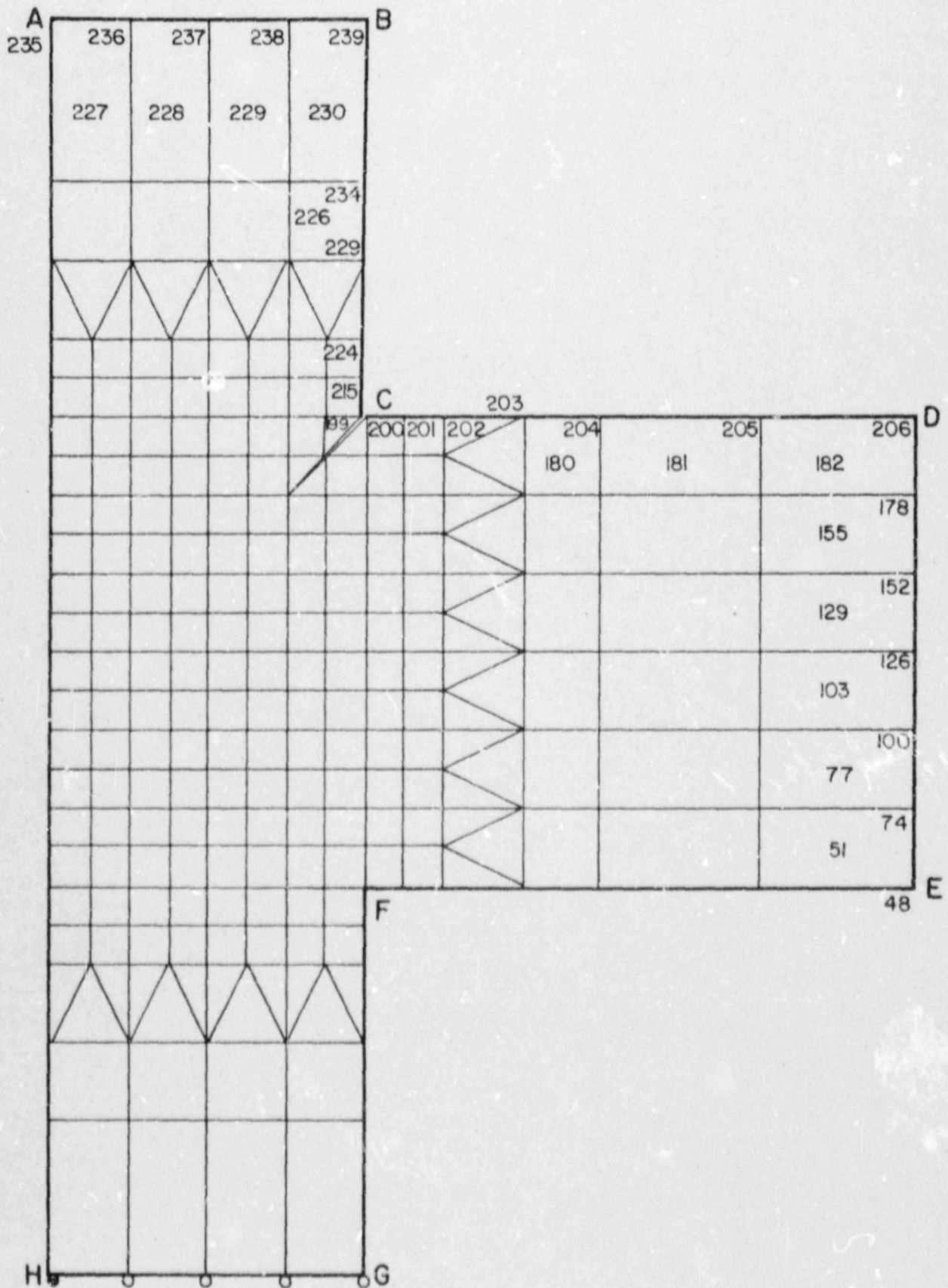


FIGURE 56

Finite element mesh for beam to column junctions with cracks.

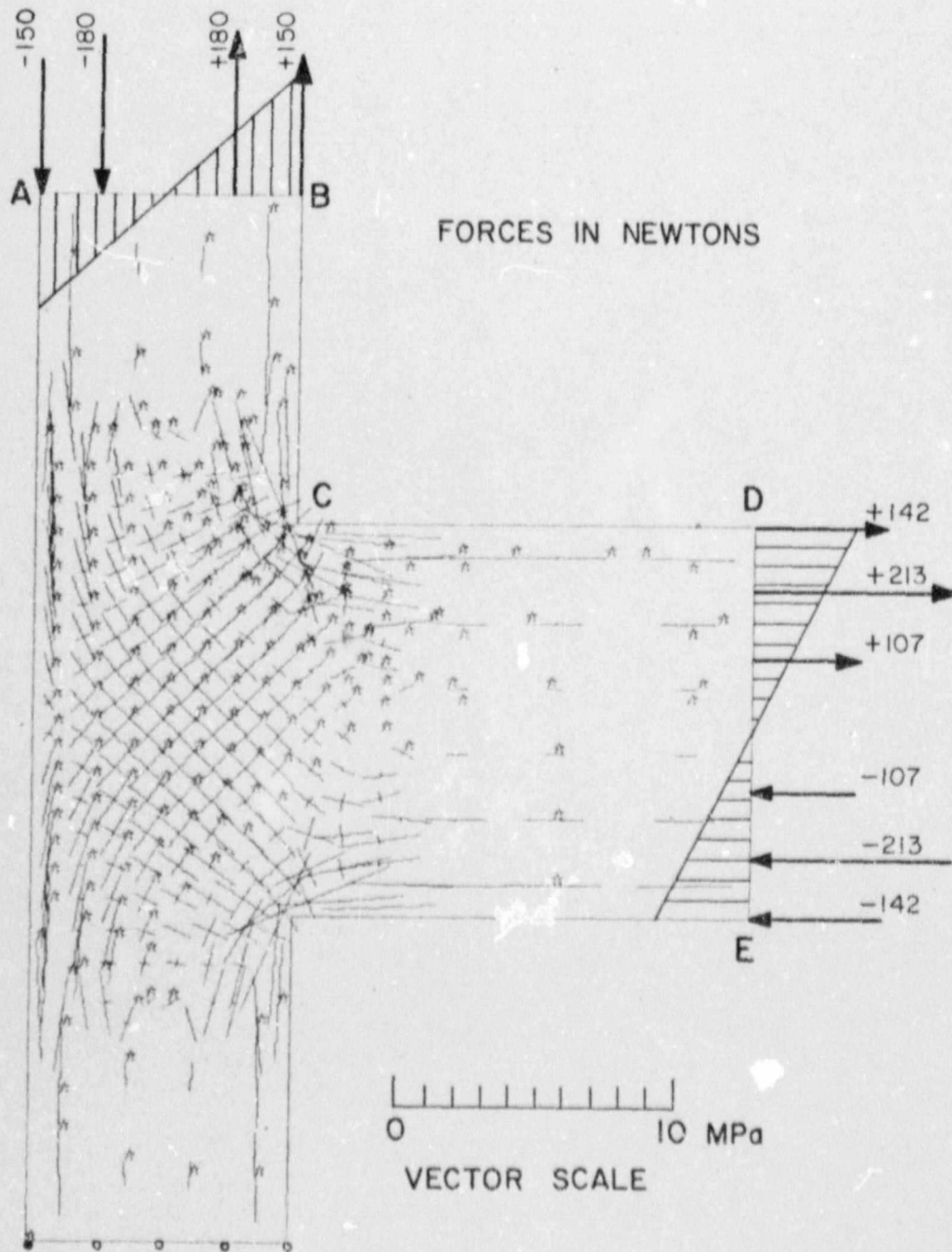


FIGURE 57

Principal stress vectors resulting from bending moments

i.e. on the top of the column 18 000 Nmm and on the beam 36 000 Nmm. In addition a vertical uniformly distributed load of 2,33 MPa on the top of the column was represented by forces on the nodal points along AB, a vertical uniform shear stress of 0,48 MPa was distributed between the nodal points along DE and vertical loads representing a uniformly distributed load of 0,023 N/mm were applied to the upper surface of the stub beam at the nodal points. The loads represented what could be expected in a small two-storey building and are shown on Figure 58. The bottom of the column had the same boundary conditions as in Case (a).

CASE (c) : The bending moments and loads were as in Case (b), but in addition a horizontal load was applied to the upper stub column to represent horizontal forces that might be exerted by an expanding brick wall in the early stages of expansion. An equal and opposite horizontal load was applied along DE so that the stress was twice as high at D than at E. The resulting forces on the nodal points are given in Appendix G and are shown in Figure 59.

CASE (d) : The bending moments and forces on the nodal points were as in Case (c). A thin crack, 0,2 mm wide, was introduced into the model at C. This crack decreased in width as it proceeded downwards at about 45° into the junction for a short distance, as shown on Figure 60.

CASE (e) : The bending moments and loads were as in Case (b), but in addition a horizontal load, which was ten times higher than the horizontal load in Case (c) and is given in Appendix G was applied to the upper stub column. The resulting forces on the nodal points are shown in Appendix G and on Figure 61. These forces represent forces that may be caused by a brick wall that has expanded for quite a period. An equal and opposite horizontal load was applied along DE, so that the stress was twice as high at D as at E.

CASE (f) : The bending moments and forces on the nodal

points /

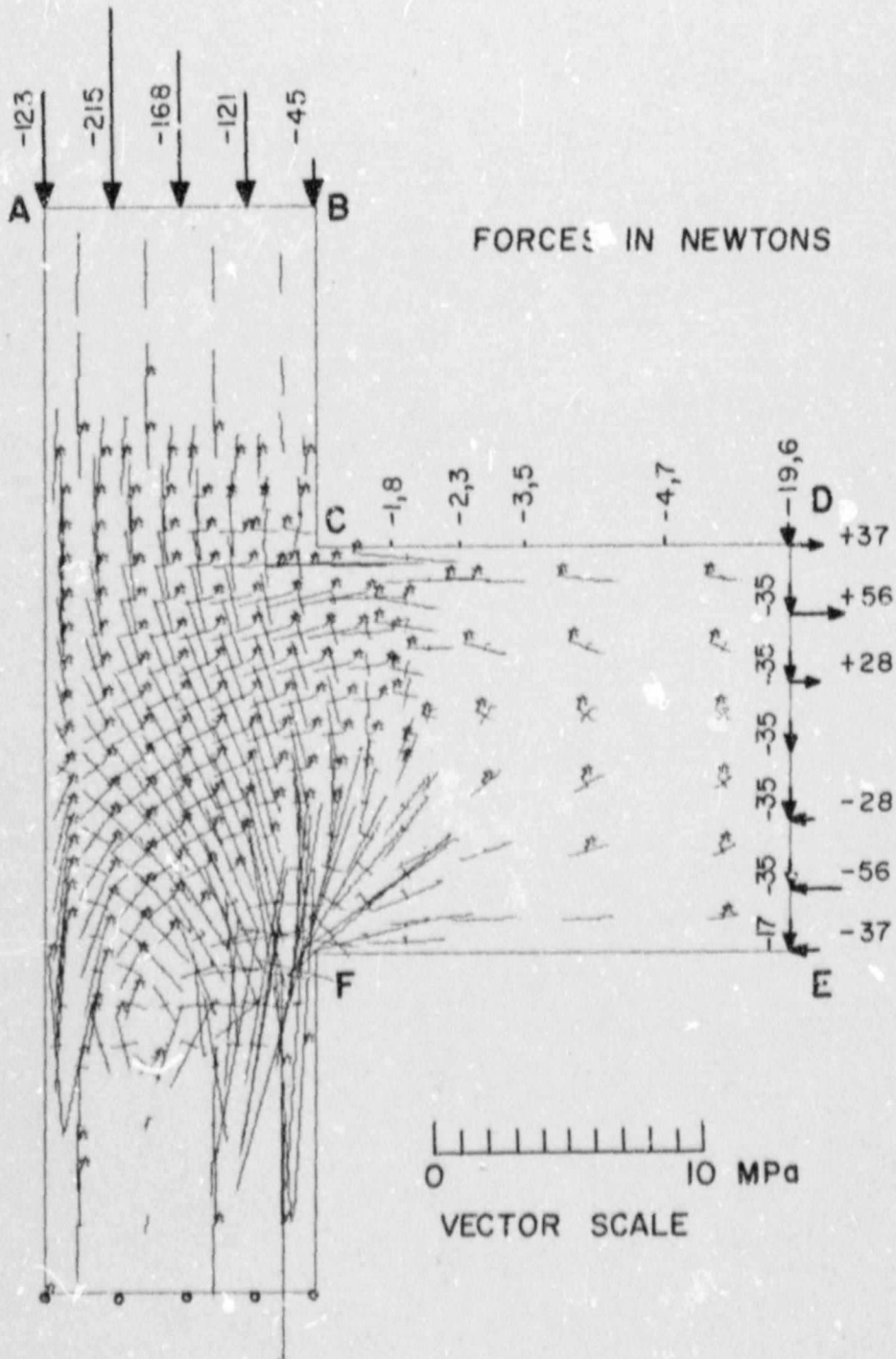


FIGURE 58

Principal stress vectors: case (b)

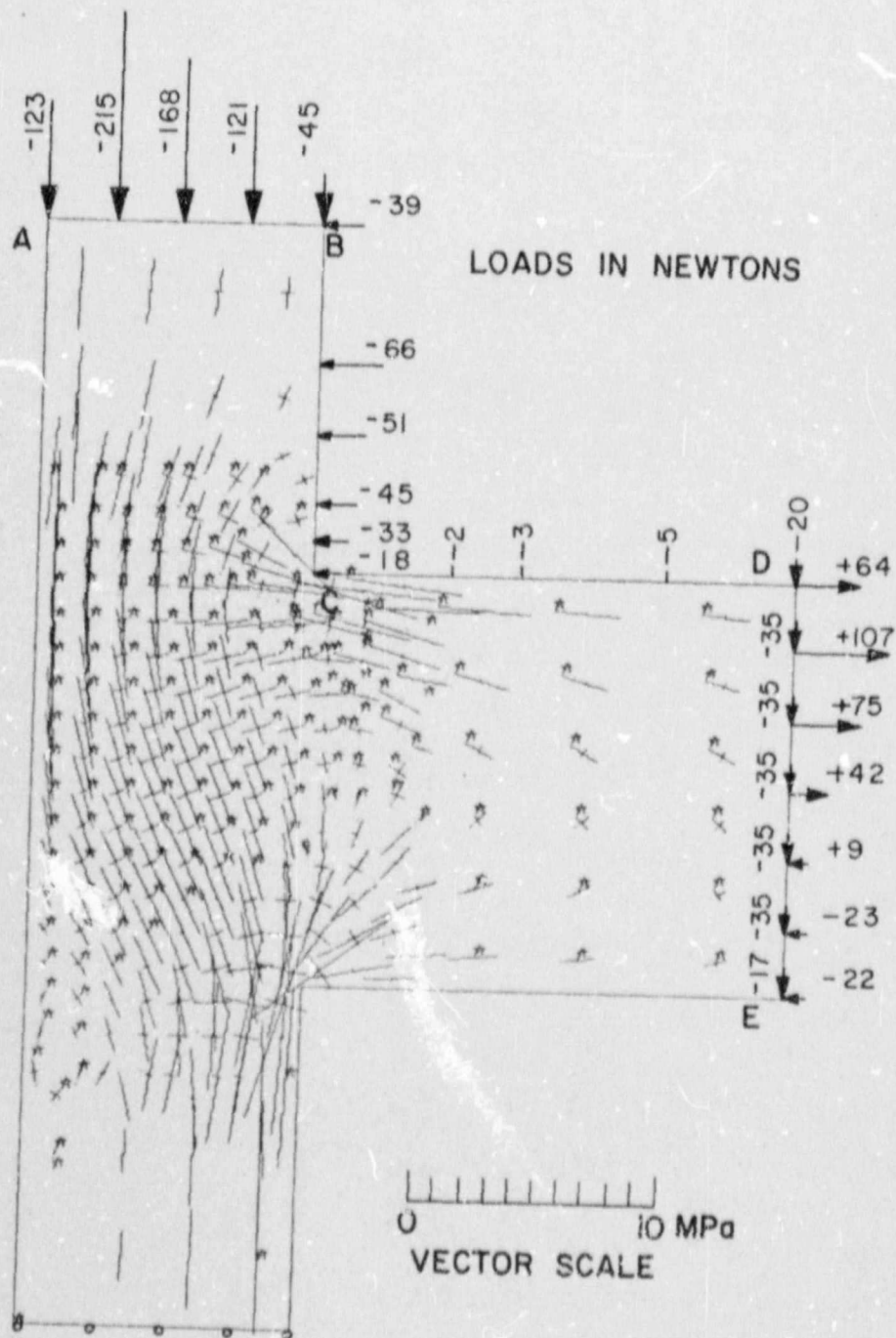


FIGURE 59

Principal stress vectors: case (c)

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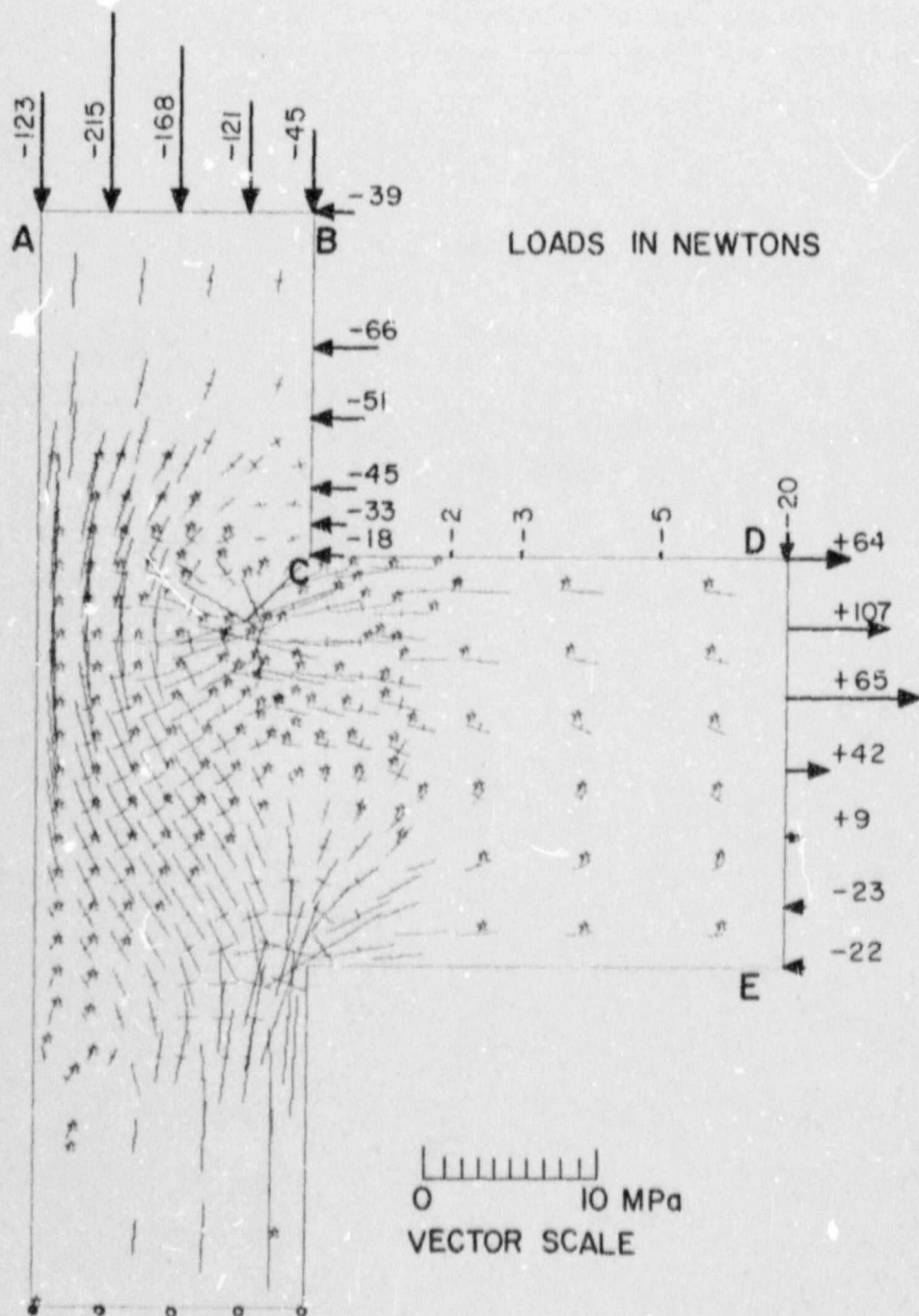


FIGURE 60

Principal stress vectors: case (d) after crack developed

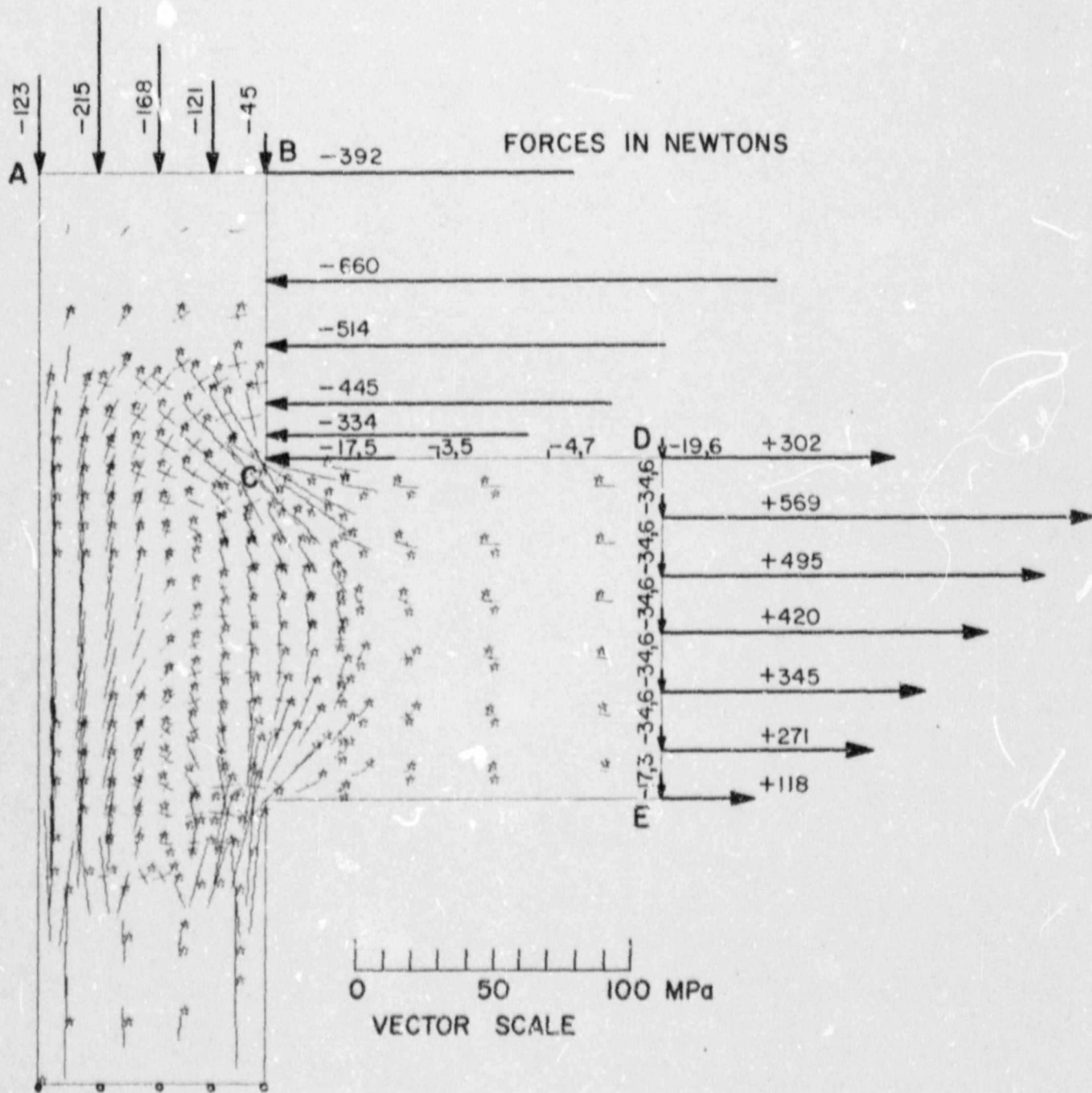


FIGURE 61

Principal stress vectors: case (e)

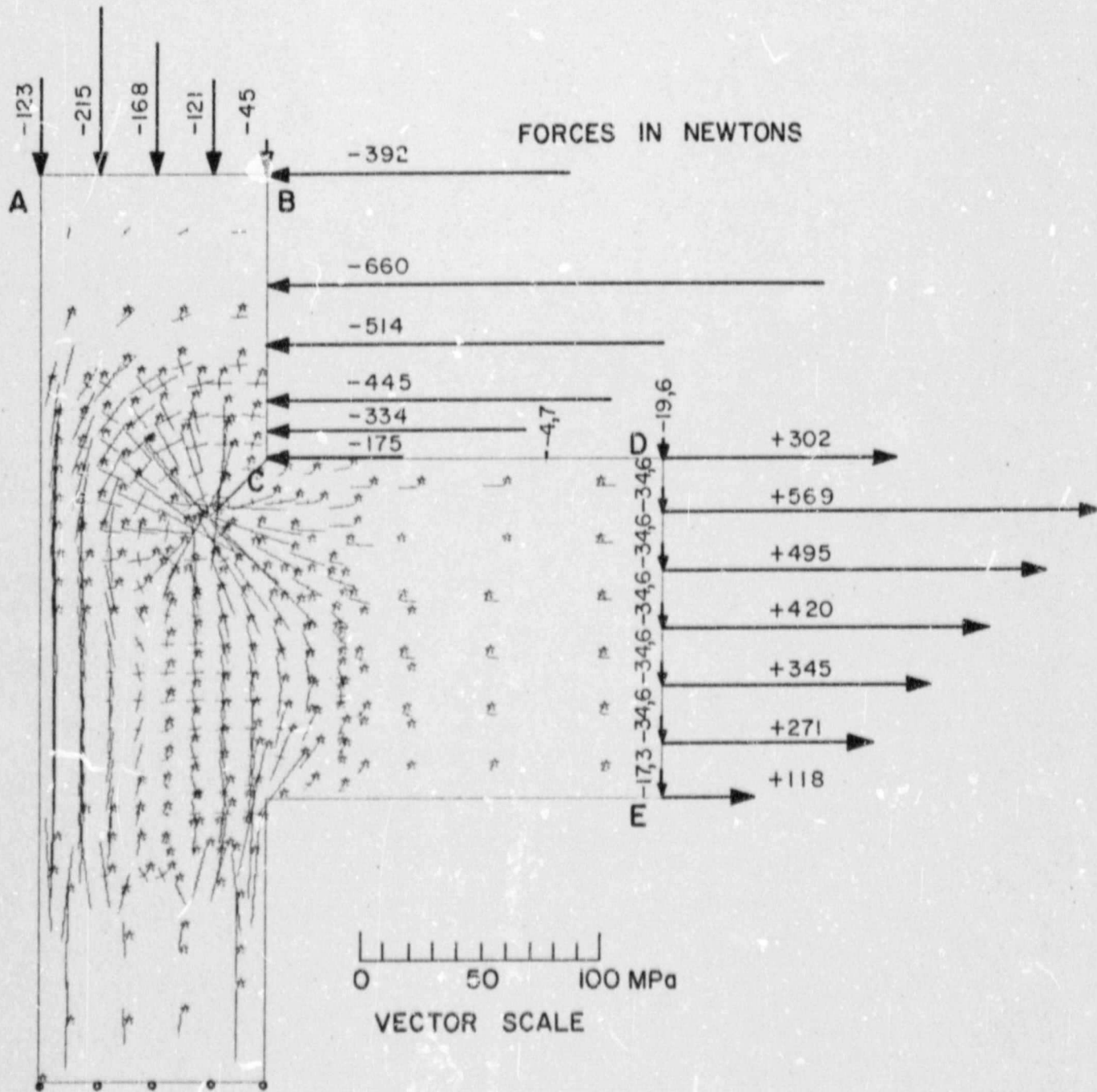


FIGURE 62

Principal stress vectors: case (f) after crack developed

points were as in Case (e). A thin crack, 0,2 mm wide was introduced into the model at C. This crack decreased in width as it proceeded downwards at about 45° into the junction as shown on Figure 62.

After studying the plots of the principal stresses that were computed for the different cases, the following conclusions were drawn:

(i) With the bending moments described in Case (a) acting on the structure, there are high tensile stresses at C. Such stresses will have been provided for in the design of such a concrete beam and column and any crack would not penetrate past the steel reinforcement. A little way into the column, the direction of the tensile principal stresses across the junction are shown, in Figure 57, as upwards at about 45° from left to right which means that cracks in the concrete of the junction would, if they occurred tend to be from the left at about 45° downwards towards the right.

(ii) The principal stresses, as a result of the bending moments and loads described in Case (b), are shown in Figure 58. A crack initiating near C would be stopped by the top horizontal steel in the beam that would run parallel to the top of the beam and thus the high tensile principal stresses. This reinforcement would have been designed to resist these tensile stresses in the top of the beam. If any cracks passed this steel, the cracks would tend to run downwards and slightly to the right, i.e. along the compressive stress trajectories in this zone and at right angles to the tensile stress trajectories. In this example, the concrete compressive stresses near F are higher than the permissible stresses in CP 114 but are not more than the usual crushing strength of concrete used in beams and columns.

(iii) The /

(iii) The principal stresses, that were caused by the bending moments and loads described in Case (c) acting on the continuum, are shown in Figure 59. The directions of these principal stresses can be expected to be similar to those occurring when an expanding brick wall exerts a light load on a column. The concrete would crack at C because the tensile stresses above and below C exceed the likely tensile strength of the concrete (2 MPa) but the crack or cracks would probably stop at the reinforcement, although it will have stresses in it higher than the normal service stresses.

(iv) Assuming that, because of some factor such as poor workmanship or accidental damage whilst the concrete was still green, the concrete was cracked from C downwards to the left at 45° before the service loads were on the structure, then the stresses shown in Figure 60 would develop when the frame became loaded, including a horizontal load from an expanding brick wall. The stresses in the vicinity of the toe of the crack are seen to be higher than those near C in Case (c), and the crack would propagate further.

(v) The principal stresses shown on Figure 61 are unlikely to occur as the concrete would crack seriously before these stresses could develop. As the horizontal loads increase from load case (c) to load case (e), the tensile stresses at C will increase until the column is probably sheared off above the beam steel, or the anchorages of the beam top horizontal bars, which end in the junction, fail. A crack initiating at C would therefore not be stopped by the steel reinforcement. Because the tensile stresses in this region reduce rapidly toward the centre of the junction, only the stress raising effect of the toe of the crack will cause the crack to propagate. It was decided therefore to introduce a crack starting at C in Case (f).

(vi) With /

(vi) With the bending moments and loads as described in Case (f) acting on the model, and after it had cracked at C, the computed principal stresses are given in Figure 62. This analysis clearly shows that if the displacement of the column accompanying the formation of a crack does not considerably reduce the forces causing the crack, the stress raising effect of the end of the crack will cause it to propagate very rapidly right through the continuum. In the case of an expanding material causing the high stresses that initiate a crack, as the crack opens it allows the member reacting with the expanding material to move in the direction of the expansion and the reactions are reduced sufficiently to lower the stresses at the end of the crack so much that the crack stops propagating.

These analyses demonstrate that expanding brickwork would cause a crack through a beam to column junction from the right downwards towards the left as observed on the actual building junction to be analysed, whereas excessive vertical loads or bending moments on the frame would tend to cause either vertical cracks or cracks tending to go from left to right downwards.

6.5 Analysis of the cracks where beam BC joins column B at the bottom of the second storey

The stresses causing the cracks seen from Durham Avenue at this joint will be analysed using finite elements and the computer program compiled by E.L. Wilson. The stresses, causing the cracks seen from the side street at the same height in this column, could be analysed using a similar method. The way these cracks have probably combined three-dimensionally within the joint was discussed in 6.1.

6.5.1 History of relative dimensional change of brick wall and reinforced concrete beam

(a) The concrete floor slab and beam would have been cast as one unit.

(b) No special curing would have been applied.

(c) Probably the brick walls were built about four weeks after casting the beam and slab. The concrete would have dried out considerably, therefore the residual shrinkage would not be too high.

(d) From the evidence, the bricks were of a type that would expand irreversibly whenever they became moist. After firing and cooling, the bricks would have started expanding slowly because of humidity in the air. The wetting they received when being laid would have caused expansion to accelerate immediately.

(e) The mortar of the brick wall would gain strength slowly. Initially the expansion of the bricks would to some extent be compensated for the drying shrinkage of the mortar, as well as the creep of the very weak mortar under even very low compressive stresses. After possibly twelve days, the continuing expansion of the bricks would start exerting a pressure on the column. That is to say, approximately 40 days after casting the concrete, the brick wall would start exerting forces on the concrete frame. The rate, at which the bricks expanded, would have depended on the expansive properties of the bricks, how moist the bricks remained because of the relative humidity and any further wetting from rain and other sources, and the restraint on expansion caused by the reactions of the beams and columns as they resisted deformation due to the expanding brickwork.

(f) As the force exerted by the brick wall on the columns

and /

and beams built up, the concrete would creep, slowing down the rate of increase of this force. The concrete would also continue shrinking which would increase this force.

(g) The concrete would continue to shrink steadily as the concrete beam dried out until the moisture in the beam became so low during a dry season that, during the next wet season, there would be an increase in moisture content and slight expansion of the concrete. From this stage on, there would be negligible further overall shrinkage, but there would be alternating dimensional changes in the length of the beam depending on the moisture changes because of weather conditions.

(h) The shrinkage of the concrete with loss of moisture would to some extent have been resisted by the reinforcement and, if other influences were not at work at the same time, the reinforcement would have become slightly compressed and the concrete would have gone into slight tension.

(i) The modular ratio in this beam for any transformed steel area = $\frac{E_s}{E_c} = \frac{20 \times 10^3}{2,6 \times 10^3} = 7,7$.

Considering the top portion of the beam above the neutral axis, the concrete cross-sectional area is 91 200 mm² and the transformed area of the two top steel bars over most of the length of the beam is 570 mm² x 7,7 = 4 390 mm² which is about a twentieth of the concrete area. The effect of the steel on the shrinkage would therefore have been small.

(j) The shrinkage of the concrete frame when not restricted by brick infilling walls was likely to have been of the order of 0,025 per cent during the first 40 days. After about ten years the moisture content of the concrete would probably be more dependent on weather conditions than on long-term drying-out, so that shrinkage would virtually cease to play an overall role in dimensional change after ten years except

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Author Boardman Vivian Reginald

Name of thesis The Effect Of Compressive Stress On The Expansion Of Brickwork And Its Implication In Buildings. 1977

PUBLISHER:

University of the Witwatersrand, Johannesburg

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