

**THE ANALYSIS OF TURBULENT
FLOWS USING A DIGITAL COMPUTER
WITH SPECIAL REFERENCE TO THE
PLANE MIXING LAYER**

N. L. BURNTON

THE ANALYSIS OF TURBULENT FLOWS
USING A DIGITAL COMPUTER
WITH SPECIAL REFERENCE TO THE PLANE MIXING LAYER

NEIL LORRAINE BURTON

A dissertation submitted to the Faculty of Engineering,
University of the Witwatersrand, Johannesburg, for the
Degree of Master of Science.

Johannesburg

1973

SUMMARY

Hot-wire anemometer measurements (single wire and X array) were made in the mixing region produced by two parallel streams of air moving at different velocities. Correlation function, power spectrum function and amplitude probability density function profiles were obtained across the mixing region using digital analysis.

The system developed for digital analysis of turbulence consisted of recording the analogue signals on magnetic tape, followed by digitising to produce a digital record on another magnetic tape for processing on an IBM 360/50 computer.

The computer programs can handle a single time series, two time series (to calculate cross correlations and cross spectra) and separate the two signals from an X array probe into the u and v velocity components. These time series were then analysed, using Fast Fourier Transform, to give correlation function, power spectrum function and Reynolds shear stress estimates.

Using a digital program developed by Dreyer (1973) an intermittency signal associated with a turbulence signal was obtained. The turbulence signal was separated (using a rough procedure) into signals for the turbulent and non-turbulent zones. Estimates of the correlation functions and power spectra in the two zones were then obtained which showed expected trends. This experiment further illustrates the versatility of the digital approach to turbulence analysis.

Finally a check on Phillips (1967) theory concerning the maintenance of Reynolds shear stresses was made. The value of 0.23 obtained for the constant A is in good agreement with that of 0.2 obtained by Wgnanski and Fiedler (1970).

ACKNOWLEDGEMENTS

My sincere thanks go to Dr. D.F. van der Merwe and Dr. S. Bloch for their help and guidance during the production of this dissertation.

Bursaries from the University of the Witwatersrand and the South African Council for Scientific and Industrial Research made the financing of the work possible.

The help from the University of the Witwatersrand Computer Centre and the Bernard Price Institute for Geophysical Research is gratefully acknowledged.

Lastly, thank you to Rosemary for the excellent typing and to Rosemarie for her help and moral support.

CONTENTS

	<u>Page</u>
1. INTRODUCTION	1
1.1 Arrangement of the Dissertation	1
1.2 The Aim and Scope of the Project	1
1.3 Previous Work Using the Digital Computer	5
1.4 Previous Work on the Two-Dimensional Mixing Region	7
2. INTRODUCTION TO DIGITAL FOURIER ANALYSIS	8
2.1 Introduction	8
2.2 Fourier Representation	8
2.3 Aliasing	10
2.4 Leakage	14
2.5 The Wiener-Khinchine Relationship	17
2.6 Method of Analysis	20
2.6.1 Single Series Analysis	20
2.6.2 Two Series Analysis	21
2.6.3 Zone Averages	22
2.7 Averaging Over Samples	23
3. STATISTICAL DESCRIPTION OF TURBULENT FLOW USING FOURIER ANALYSIS	26
3.1 Introduction	26
3.2 The Hot-Wire Anemometer	26
3.3 Correlation Analysis	30
3.3.1 Introduction	30
3.3.2 Longitudinal and Lateral Length Scales	31
3.3.3 The Autocorrelation	34
3.3.4 The Periodogram	36

Page

3.3.5	Moving Axis Autocorrelation:	39
3.3.6	The Cross Spectrum	47
3.3.7	The Cross Correlation	47
3.4	The Measurements Taken	43
3.4.1	Single Wire Measurements	43
3.4.2	Two Wire Measurements	43
3.4.3	Conditional Measurements	43
4.	THE COMPUTER PROGRAMS	44
4.1	General	44
4.2	Data	45
4.3	Results	45
4.4	The Programs	45
4.5	Method of Analysis	47
4.6	The General Program NLBONE	49
4.6.1	Quantities Calculated	49
4.6.2	Output of NLBONE	50
5.	EXPERIMENTAL APPARATUS AND PROCEDURE	52
5.1	The Turbulent Shear Layer	52
5.2	The Analogue Apparatus	54
5.3	The Experimental Procedure	59
6.	EXPERIMENTAL RESULTS	62
6.1	Calibration of the Hot-Wire Anemometer Probes	62
6.2	Directional Sensitivity of the Hot-Wire X-Array Probe	62
6.3	Measurements in the Free Shear Mixing Layer	65
6.3.1	General	65
6.3.2	Single Hot-Wire Traverse 30 cm Downstream of the Splitter Plate	66

	<u>Page</u>
6.3.3 Cross-Wire Traverse 30 cm Downstream of the Splitter Plate	80
6.3.4 Zone Properties of the Turbulent Shear Layer 30 cm Downstream of the Splitter Plate	96
6.4 Check of Phillips' Theory on the Maintenance of Reynolds Shear Stress	104
7. DISCUSSION ON THE ERRORS	111
7.1 The Analogue System	111
7.2 The Digitising Process	111
7.3 Sampling Errors	112
7.4 Analysing a Limited Time of Signal	113
7.5 Computational Errors	113
7.6 Characterising the Digital System	114
7.6.1 Resolution	114
7.6.2 Dynamic Range	115
7.6.3 The Minimum Detectable Signal (M.D.S.)	115
7.7 The Statistical Accuracy of the Results	116
8. CONCLUSIONS	121
8.1 The Analysis System	121
8.2 Turbulence Results	122
8.3 Suggestions for Further Work	124
9. NOMENCLATURE	125
10. REFERENCES	128
APPENDIX I : The Hot-Wire Response Equations	A.1
APPENDIX II : The Computer Programs	A.5
APPENDIX III : Analogue Circuit for u- and v- velocity fluctuations	A.63

LIST OF FIGURES

<u>Figure Number</u>		<u>Page</u>
2-1	Aliasing	11
2-2	Frequency Folding	13
2-3	Physically Realisable One-Sided Power-Spectral Density Function	19
2-4	Single Series Method of Analysis	21
2-5	Two Series Method of Analysis	22
3-1	Single Hot-Wire Sensor	25
3-2	Inclined Hot-Wire Sensor	27
3-3	X-Array Hot-Wire Probe	29
3-4	Longitudinal and Lateral Length Scales	32
3-5	Log-log Plot of Periodogram	37
3-6	Wire Configuration for Moving Axis Autocorrelation	39
3-7	Gross Correlation $R_{uu}(r, \tau)$	39
3-8	Moving Axis Autocorrelation	40
5-1	The Wind Tunnel	53
5-2	The Analogue Equipment	58
6-1	Calibration Curve for Cross-Wire Probe	63
6-2	Calibration Curve for Single-Wire Probes	63
6-3	Periodogram of the Analogue Sine Wave	67
6-4	Amplitude Probability Density Function of the Sine Wave	68
6-5	Autocorrelation of Analogue Sine Wave	68
6-6	Mean Velocity Profile and r.m.s. Values of the u Fluctuations (30 cm downstream)	70
6-7	Log-log Plot of Periodogram (Single Wire)	72

<u>Figure Number</u>		<u>Page</u>
6-8	Normalised Aut-correlation (Single Wire)	73
6-9	Taylor's Microscale Profile (30 cm downstream)	75
6-10	Skewness and Kurtosis Profiles (30 cm downstream)	75
6-11	Amplitude Probability Density Function (Single Wire)	78
6-12	R.M.S. Values of Signals from Two Wires of X-Array Probe	81
6-13	Calculated and Analogue Values of u - and v Velocity Fluctuations	86
6-14	Periodogram of the u - Velocity Fluctuations (Log-log)	87
6-15	Periodogram of the v - Velocity Fluctuations (Log-log)	87
6-16	Amplitude Probability Density Function of the u Fluctuations	89
6-17	Amplitude Probability Density Function of the v Fluctuations	90
6-18	Autocorrelations of the u - Velocity Fluctuations	91
6-19	Autocorrelations of the v - Velocity Fluctuations	91
6-20	Co-Spectrum, Quad-Spectrum and Cross Correlation (R_{uv}) at $y = -0.018$	92
6-21	Co-Spectrum, Quad-Spectrum and Cross Correlation (R_{uv}) at $y = 0.015$	93
6-22	Co-Spectrum, Quad-Spectrum and Cross Correlation (R_{uv}) at $y = 0.039$	94
6-23	Co-Spectrum, Quad-Spectrum and Cross Correlation (R_{uv}) at $y = 0.099$	95
6-24	Velocity Signal with Superimposed Intermittency Signal	98
6-25	Conventional Periodogram of u Fluctuations	99

<u>Figure Number</u>		<u>Page</u>
6-26	Turbulent Zone Periodogram of the u Fluctuations	99
6-27	Non-Turbulent Zone Periodogram of the u Fluctuations	100
6-28	Conventional Autocorrelation of the u Fluctuations	100
6-29	Turbulent Zone Autocorrelation of the u Fluctuations	101
6-30	Non-Turbulent Zone Autocorrelation of the u Fluctuations	101
6-31	Conventional Amplitude Probability Density Function	102
6-32	Turbulent Zone Amplitude Probability Density Function	102
6-33	Non-Turbulent Zone Amplitude Probability Density Function	103
6-34	Graph of Convected Integral Time Scale and Mean Velocity (30 cm downstream)	107
6-35	Cross Correlations ($R_{uu}(r, \tau)$) for Various Wire Separations at $y = 0.037$	108

LIST OF TABLES

<u>Table Numbers</u>		<u>Page</u>
6-1	Results of Directional Sensitivity Analysis	65
6-2	R.M.S. and Mean Velocity Profiles (30 cm downstream)	71
6-3	Table of Turbulence Measurements	76
6-4	Table of Skewness and Kurtosis Values	79
6-5	Analogue Measurements Using X-Array Probe	82
6-6	Analogue and Digital r.m.s. Measurements	83
6-7	Analogue Measurements of u- and v- Velocity Fluctuations	84
6-8	Comparison of Analogue and Digital Values of the u- and v- Velocity Fluctuations	85
6-9	Digital Values of the Reynolds Shear Stress	85
6-10	Calculations for the Convected Integral Time Scale	106
6-11	Least Squares Fit for the Mean Velocity Profile	106
6-12	Calculations for Check on Phillips' (1967) Theory	110
7-1	Results of Statistical Test on the Data from the Single- Wire Traverse	118
7-2	Results of the Statistical Test on the Cross-Wire Data	118
7-3	Comparison for u- Velocity Component	119
7-4	Comparison for v- Velocity Component	119
7-5	Comparison for the Reynolds Shear Stress	119

ANALYSIS OF TURBULENCE USING A DIGITAL COMPUTER

CHAPTER I

1. INTRODUCTION

1.1 Arrangement of the Dissertation

In this introductory chapter the aim and scope of the project are described with reference to certain investigations. The second chapter gives an introduction to digital Fourier analysis while chapter three discusses the parameters used to characterise turbulent flow. Chapter four describes the digital analysis system produced, while chapter five describes the experimental apparatus and procedure adopted. In chapter six the experimental results are presented, chapter seven indicates the various errors involved in the analysis and chapter eight presents the conclusions.

1.2 The Aim and Scope of the Project

Turbulence is a random process and its analysis is limited to a knowledge of relationships between statistical measures, such as averages of variables, products of variables, correlations and joint probabilities. The structure of turbulent flow is described by Mollo-Christensen (1971) in the following manner.

Turbulence is generated in intermittent bursts which are associated with distortion of the velocity profiles. The bursts contain velocity fluctuations of a range of scales and the intermittency (time fraction of no production) increases with Reynolds number.

It is therefore possible to have a situation where averages might hide rather than reveal information about the turbulence process. Intermittent events tend to be hidden in time averages unless properties occurring only in the intermittent event are measured.

Kibens (1968) discussed and showed the importance of separating an intermittent turbulent signal into two zones, turbulent and non-turbulent. The results for the two dimensional mixing region of Wagnanski and Fiedler (1970) show differences in the rms values of the velocity fluctuations and shear stress values in the turbulent bursts and the non-turbulent region between bursts. Mollo-Christensen (1971) showed that the production of turbulence is dominated by that generated in the turbulent bursts. Obviously it is essential to analyse the turbulent and non-turbulent regions separately when studying turbulence situations where the intermittency is significant. In digital analysis an intermittency signal usually consists of a series of ones or zeros, depending on whether the signal is turbulent or not at a particular point.

Intermittency signals and zone averages are generally obtained using a number of hot-wire probes and very sophisticated analogue circuits (Kibens (1968), Wagnanski and Fiedler (1969)). Measurements of zone correlation functions and zone power spectra are extremely difficult to obtain using analogue methods. Complex analogue systems such as those used by the authors referenced above are too expensive to construct or maintain in small University Departments.

Digital computer facilities are readily available and most students obtain training in computer programming. An investigation into the utilization of digital computers for the analysis of turbulence data is therefore of interest. This investigation is not into on-line digital analysis but describes the tape recording of relevant data, digitising it and then using digital methods of analysis.

There are of course various other considerations. In favour of analogue methods one can say that:

- i) Computer time may be expensive.
- ii) Large quantities of data are required to obtain statistically stable answers. This introduces problems of data handling as well as increasing the cost of analysis.
- iii) Analogue methods give the answer immediately.

However these advantages quickly disappear if a number of analogue instruments are used in series and the impedances are difficult to match. The maintenance of many different instruments soon requires the help of an electronics expert. In certain situations digital analysis can be an attractive proposition:

- i) With the introduction of the Fast Fourier transform algorithm by Cooley and Tukey (1965) accurate estimates of the periodogram and velocity correlations can be obtained in small computation times. The algorithm reduces the number of multiplications in a Fourier transformation from the conventional N^2 to $N \log_2 N$ where N is the size of the sample being transformed. Due to the decrease in multiplications there is a decrease in round-off error. Using the Fast Fourier transform it is quicker to calculate the autocorrelation going via the frequency domain than to convolve in the time domain. The frequency resolution possible using digital analysis is sometimes better than that using analogue filters.

- 4
- ii) When making comparative studies the same set of data can be used for each study. This generally makes interpreting the results much easier.
 - iii) When the analysis procedure is to be changed the associated program is updated. This eliminates modifying electronic equipment and dealing with the associated problems.
 - iv) Intermittency decisions are easier to make as the complete signal is available to be studied. Overall properties can be used to make the decisions. Once these have been made it is simple to separate the fluctuations into turbulent and non-turbulent regions. Conditional and point averages as described by Kibens (1968) can then be calculated.
 - v) When separating X-ray probe signals into the velocity fluctuations, differences in the directional sensitivity and calibration curves of the two sensors can be accounted for.

The scope of the project involved the following steps. Firstly, a system to tape record, digitise and store the resulting data in a form compatible to the IBM 360/50 computer at the University of the Witwatersrand was evolved. Secondly, a suite of programs to compute the various functions used in turbulence analysis was written. The programs are simple to use and can be combined to form any desired analysis path. A general analysis program utilising the above programs was also written. Thirdly, measurements were taken in the mixing region produced by two parallel streams of air moving at different velocities. The data was analysed using the prepared digital system and it compared favourably with results obtained by Wignanski and Fiedler (1970) in a similar flow situation. Using properties of the correlation functions it was possible to obtain an idea of the accuracy of the analysis and also the statistical stability of the averaged results.

Finally, using programs developed by Dreyer (1973) which determine the intermittency signal associated with a turbulent signal, the digital analysis approach was used to calculate the correlation functions and power spectra in the turbulent and non-turbulent zones. These particular quantities would be very difficult to obtain using analogue methods and serve to show the versatility of the digital approach to the analysis of turbulence data.

1.3 Previous Work Using the Digital Computer

Digital analysis procedures have been used to good advantage in a great variety of different situations.

The most comprehensive use of the digital computer for the analysis of hot wire anemometer signals is the system described by Kaplan and Laufer (1969). Other workers, Frenkiel and Klebanoff (1971) and Van Atta and Chen (1969) have used digital Fourier analysis for measuring correlation functions of turbulent quantities. Similar systems have also been used in the fields of acoustics and vibration analysis (Singleton and Poulter (1967) and Villasenor (1968)). A theoretical discussion on the application of the Fast Fourier transform is given by Cooley, Lewis and Welch (1970).

- 1) Kaplan and Laufer (1969) used a digital computer to analyse the intermittently turbulent region of the boundary layer. They used 10 hot-wire anemometers simultaneously and recorded the signals on an instrumentation tape recorder. An analogue to

digital converter connected directly to the digital computer made intermediate storage of the digitised data unnecessary. Programs were written to calculate the intermittency function for each hot-wire signal and then produce conditional and point averages. It was possible to perform calculations and digitise data simultaneously by using the computer's interrupt system. Care was taken not to let the calculations overtake the digitising and vice versa. In the above application the computer was used to perform relatively simple calculations on large quantities of data.

- ii) Van Atta and Chen (1968, 1969) used the digital computer to Fourier analyse turbulent velocity data and then calculate various time correlations of the longitudinal and lateral velocity fluctuations in grid generated turbulence. The Fast Fourier transform algorithm was used to obtain the Fourier series coefficients of the velocity or velocity squared etc. These were then combined and inverse Fourier transformed to give the correlation function. Obviously one particular series can be used for various different order correlations and time can be saved by careful designing of the analysis.
- iii) Frankel and Gebanoff (1971) used digital computing methods to study the statistical behaviour of turbulent velocity derivatives in a nearly isotropic turbulent field downstream of a grid. The first and second derivatives of the velocity fluctuations were obtained using analogue methods as numerical differentiation is not very accurate. The data was digitised at two different rates so that a reasonable range in time delays could be obtained.

- iv) A statistical reduction program, designed to reduce and analyse random data processes without regard to record length, is described by Krause et al (1970). Statistical certainty is constantly checked and updated until desired statistical accuracies are realised or the data source is exhausted. This discussion is mainly theoretical.
- v) Further applications of and discussion on Fast Fourier analysis can be found in the IEEE Transactions on Audio and Electro-acoustics (1967 and 1969).

1.4 Previous Work on the Two Dimensional Mixing Region

A well-known self-preserving shear flow was chosen to compare the results obtained. The two dimensional incompressible mixing region can be treated theoretically and has been studied by Liepman and Laufer (1947) and recently by Wignanski and Fiedler (1970). Townsend (1956) briefly discusses the above turbulent flow and compares it to other well-known flow situations.

Wignanski and Fiedler (1970), using some very elaborate analogue circuits performed a particularly comprehensive investigation. They took intermittency into account and measured conditional and point averages. Third and fourth order products of the velocity fluctuations, spatial derivatives of these fluctuations and space-time correlations were also made.

CHAPTER II

2. INTRODUCTION TO DIGITAL FOURIER ANALYSIS

2.1 Introduction

Chapters II and III try to clarify the problems and assumptions involved in the digital analysis of hot-wire anemometer signals. It was thought appropriate to discuss first the digital procedures and then move on to the more specific calculation of the various properties used to characterize turbulence flows.

A short theoretical discussion is presented to cover the proposed analysis. Various pitfalls encountered in digital Fourier analysis are discussed and the method of calculating correlations and spectra is described.

2.2 Fourier Representation

Any function $x(t)$, defined in the region $0 \rightarrow T$ can be represented by the Fourier series

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos \frac{2\pi n t}{T} + b_n \sin \frac{2\pi n t}{T})$$

where

$$a_n = \frac{2}{T} \int_0^T x(t) \cos \frac{2\pi n t}{T} dt \quad n = 0, 1, 2, \dots$$

$$b_n = \frac{2}{T} \int_0^T x(t) \sin \frac{2\pi n t}{T} dt$$

If $x(t)$ and $\frac{dx}{dt}$ are piecewise continuous and $x(t)$ is assumed periodic in t , with period T , then the series converges to $x(t)$ if t is a point of continuity and it converges to

$$\frac{1}{2}(f(t+0) + f(t-0))$$

if t is a point of discontinuity. A detailed analysis of the Fourier series can be found in Papoulis (1962).

Assuming that the function $x(t)$ is now sampled at times kh giving N discrete points.

Where

$$T = Nh$$

$$t = kh$$

and

$$k = 0, 1, 2, 3, \dots, N-1$$

the above expression becomes

$$x(kh) = \frac{a_0}{2} + \sum_{n=1}^{N/2} a_n \cos \frac{2\pi nk}{N} + \sum_{n=1}^{N/2-1} b_n \sin \frac{2\pi nk}{N} \quad (2-1)$$

For ease of manipulation the above can be represented as a complex function

$$X(kh) = \sum_{n=0}^{N-1} A_n e^{\frac{2\pi i nk}{N}}$$

where the time series must now be a complex series $X(kh)$. In practice only real series are obtained and hence the imaginary parts of $X(kh)$ will be zero. For a real time series only $N/2$ of the complex coefficients are unique, thus accounting for the different limits of n in the two summations.

It is useful to see the relationships between the coefficients in the two notations.

$$X(kh) = \sum_{n=0}^{N-1} A_n e^{\frac{2\pi i nk}{N}}$$

expanding the exponential:

$$X(kh) = \sum_{n=0}^{N-1} A_n \left(\cos \left(\frac{2\pi nk}{N} \right) + i \sin \left(\frac{2\pi nk}{N} \right) \right)$$

Now X and A are complex

$$\begin{aligned}\text{Therefore } X(kh) &= \sum_{n=0}^{N-1} (A_n^r + iA_n^i) \left(\cos\left(\frac{2\pi nk}{N}\right) + i\sin\left(\frac{2\pi nk}{N}\right) \right) \\ X(kh) &= \sum_{n=0}^{N-1} \left(A_n^r \cos\frac{2\pi nk}{N} - A_n^i \sin\frac{2\pi nk}{N} + i(\quad) \right)\end{aligned}$$

The sampled time series considered is real and hence the imaginary part of the right hand side is zero.

$$\text{Therefore } X^r(kh) = \sum_{n=0}^{N-1} \left(A_n^r \cos\frac{2\pi nk}{N} - A_n^i \sin\frac{2\pi nk}{N} \right) \quad (2-2)$$

Comparing equations (2-1) and (2-2) we have

$$\begin{aligned}a_n &= R(A_n) \\ b_n &= -I(A_n)\end{aligned}$$

NOTE: $A_0 = 2a_0$

Obviously it is much more compact to use the complex notation, and it will be used in this presentation.

We now move on to two of the problems that arise when dealing with digitised data.

2.3 Aliasing

The problem is most important if errors are to be avoided. It is best illustrated by the Sampling Theorem which will be explained below.

Obviously any frequency of a signal that has been digitised must have at least two points per cycle for that frequency to be observed. This is illustrated in the figure below.

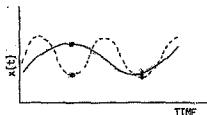


Fig. (2-1) Aliasing

The dotted signal cannot be identified at the sampling rate in the diagram. Unfortunately when one transforms into the frequency domain (Fourier transformation) these higher frequencies reappear as a frequency in the range being considered. Consider the continuous function $x(t)$ which can be Fourier transformed

$$x(t) = \int_{-\infty}^{\infty} A(f)e^{2\pi i f t} df$$

If this signal is now sampled giving N points, time h apart, the times of sampling are

$$kh \text{ where } k = 0, 1, 2, \dots, N.$$

Then

$$X(kh) = \int_{-\infty}^{\infty} A_n e^{\frac{2\pi i k n}{F}} dn \quad (2-3)$$

where

$$F = \frac{1}{h} = \text{sampling frequency}$$

F , the sampling frequency is twice the Nyquist frequency. This is the highest frequency that can be distinguished at the particular sampling frequency.

Equation (2-3) can be written as

$$X(kh) = \sum_{n=-\infty}^{\infty} \int_{-RF}^{(m+1)F} A_n e^{\frac{2\pi i k n}{F}} dn$$

Since n goes from $+\infty$ to $-\infty$ the integral (on summing) remains from $+\infty$ to $-\infty$.

The exponential $e^{\frac{2\pi i k n}{F}}$ is periodic with period F . Since each integration in the summation is over a period F one can write

$$\begin{aligned} X(kh) &= \sum_{n=-\infty}^{\infty} \int_0^F A(n + mF) e^{\frac{2\pi i k}{F} (n + mF)} dn \\ &= \sum_{n=-\infty}^{\infty} \int_0^F A(n + mF) e^{\frac{2\pi i k n}{F}} dn \end{aligned}$$

where $e^{\frac{2\pi i k n}{F}} = e^{\frac{2\pi i k}{F} (n + mF)}$

therefore

$$X(kh) = \int_0^F \left(\sum_{n=-\infty}^{\infty} A(n + mF) \right) e^{\frac{2\pi i k n}{F}} dn$$

Let

$$A_n^P = A_n + A_{n+F} + A_{n+2F} + \dots$$

Therefore

$$X(kh) = \int_0^F A_n^P e^{\frac{2\pi i k n}{F}} dn \quad (2-4)$$

Equation (2-4) is the version of (2-3) when one has a discrete time series and not a continuous series. It is important to notice that the integration is now over a fixed frequency range 0 to F whereas it was from $+\infty$ to $-\infty$. The Fourier coefficients in (2-4) are the *aliased Fourier coefficients* since they contain the contribution due to frequencies outside the range being considered.

To eliminate aliasing one must ensure that the signal is band limited so that the terms $A_{(n+F)}$, $A_{(n+2F)}$ are zero.

Assume that one has a signal containing frequencies of up to 520 HZ. The signal is sampled at a rate of 512 HZ. The Nyquist frequency at this sampling rate is 256 HZ. When one transforms into the frequency domain, where will the effect of the 520 HZ frequency be found?

$$\begin{aligned} A_n^p &= A_n + A_{n+F} + \dots \\ \text{therefore } A_B^p &= A_B + A_{B+512} + \dots \\ A_B^p &= A_B + A_{520} + \dots \end{aligned}$$

Therefore the 520 HZ frequencies will contribute to the 8 HZ frequency.

Aliasing is a result of not knowing the frequency content of the signal being analysed. To eliminate aliasing one should band-limit the signal by analogue filtering before digitising. Once the signal has been digitised the aliasing errors are present.

Graphically, aliasing can be illustrated by the following figure from United Geophysical Corporation (1965).

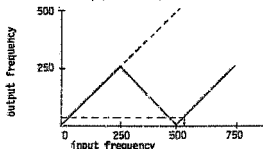


Fig. (2-2) Frequency Folding

The folding occurs at the Nyquist frequency 250 HZ. A frequency of 520 HZ (input) is equivalent to 8 HZ (output) because of the folding.

2.4 Leakage

This problem results from the fact that we are dealing with a discrete sample and not an infinite one. The infinite sample has been windowed by a window that is zero everywhere, except during the discrete sample where it is 1. Let the length of the sample be T .

If one Fourier analyses a time series of length T , Fourier coefficients at the discrete frequencies

$$\omega_n = \frac{2\pi n}{T} \quad n = 0, 1 \dots N/2$$

are obtained, where N is the number of points.

Consider the Fourier analysis of a single sine wave, the frequency of which is one of the calculated ω_n above. The only frequency coefficient obtained will be that for the frequency ω_n .

If on the other hand the frequency of the sine wave is not one of the ω_n , Fourier coefficients for all the ω_n are obtained. This effect is obtained when the sample length T is not a multiple of the period of the sine wave being analysed. The Fourier coefficients peak near (on the low frequency side) the frequency ω and then fall off as

$$\frac{1}{|W - \omega_n|}$$

When the signal contains many frequencies and the frequency resolution is not small enough, the accumulated leakage from each frequency can distort the power spectrum.

By multiplying the infinite series by a window that gradually changes from 0 to 1 instead of a step change the leakage can be reduced. Multiplying by a Hanning window reduces the effect to

$$\frac{1}{|N - n|^3} \quad (\text{Maling, Morrey and Lang (1967)})$$

When one uses a window to reduce leakage two other problems are introduced.

- i) the relative amplitudes of the frequency coefficients are altered
- ii) the statistical stability of the spectrum is reduced as the data has been altered.

The choice of data window used is a compromise between the leakage reduction and the resulting loss in statistical stability. Durrani and Nightingale (1972) give a discussion on the effects of data windows and also a comprehensive description of the effects of various different windows. Windows are discussed by Blackman and Tukey (1959).

In the frequency domain the Hanning window consists of multiplying the Fourier coefficients (sine and cosine terms) by a moving average with factors $1/4$, $1/2$ and $1/4$. For an analysis where the Fourier coefficients are centred about $N/2$,

$$\begin{aligned} a_1 &= \frac{1}{2} (a_1 + a_2) \\ a_n &= \frac{1}{4} a_{n-1} + \frac{1}{2} a_n + \frac{1}{4} a_{n+1} \\ a_N &= \frac{1}{2} a_{N-1} + \frac{1}{2} a_N \end{aligned}$$

Similarly for the sine coefficients

$$\begin{aligned} b_1 &= \frac{1}{2} (b_1 + b_2) \\ b_n &= \frac{1}{4} b_{n-1} + \frac{1}{2} b_n + \frac{1}{4} b_{n+1} \\ b_N &= \frac{1}{2} (b_{N-1} + b_N) \end{aligned}$$

It is important to note that the Fourier coefficients must be Hanned before they are combined to give the periodogram. Hanning the periodogram has no noticeable effect on reducing the leakage.

It can be shown (Villissener (1968)) that the time domain equivalent of the Hanning window is the function

$$x(kh) = \frac{1}{2} \left[1 - \cos \frac{2\pi kh}{H} \right] x(kh)$$

$$k = 0, 1, \dots, N-1$$

$$h = \text{time interval between samples}$$

It may save computer time to multiply in the time domain rather than convolve in the frequency domain.

Using a window changes the input series. According to Bingham, Godfrey et al (1967) a reasonable compromise (between changing the data and improving the resolution) for data stretching from $t = 0$ to $t = T$ would consist of two cosine bells, each extending for 1/10th of the signal. The centre 8/10th of the signal is left untouched. The equations representing the above in the time domain would be:

$$\begin{aligned} x(t) &= x(t) \times \frac{1}{2} \left(1 - \cos \frac{\pi t}{0.1T} \right) & 1 \leq t < 0.1T \\ x(t) &= x(t) & 0.1T \leq t < 0.9T \\ x(t) &= x(t) \times \left(1 - \cos \frac{\pi (T-t)}{0.1T} \right) & 0.9T \leq t < T \end{aligned}$$

When using the above windows it is important to remove any linear trends in the data. A varying mean does not allow a smooth transition at the ends of the data.

To reduce leakage effects as long a sample length as possible should be used (as many cycles of the lowest frequency as possible). Sample lengths large compared to the low frequencies in the signal are analysed in this investigation and so the leakage effects should be small. Although the option of using the Hanning window is available in the programs it was seldom used.

At this stage it is possible to represent the turbulent velocity signal by a discrete Fourier series. Some problems encountered in obtaining this representation have been discussed. The next problem is to calculate

- i) The periodogram
- ii) The autocorrelation of a single series, and
- i) The cross spectrum
- ii) The Cross correlation of two series.

2.5 The Wiener-Khinchine Relationship

The above indicates the relationship between the Fourier coefficients of two random stationary processes and their cross correlation. A brief explanation will be given here, while a more complete derivation can be found in Bendat and Piersol (1966).

Assume we have two continuous time series $x(t)$ and $y(t)$. Since the series are real we can represent them by their complex conjugates.

Using complex Fourier representation

$$Y(t) = \int_{-\infty}^{\infty} A(g)e^{12\pi g t} dg$$

$$X(t) = \int_{-\infty}^{\infty} B^*(f)e^{-12\pi f t} df$$

where the asterisk represents the complex conjugate.

The covariance is defined as

$$R_{xy}(\tau) = E[X(t)Y(t+\tau)]$$

Hence from definitions above:

$$R_{xy}(\tau) = \iint E[A(g)B^*(f)]e^{i2\pi(g-f)t}e^{i2\pi f\tau}dgdf$$

It can be shown, (Bendat and Piersol (1966)) that for a stationary random process the following relationship holds:

$$R_{xy}(\tau) = \iint S_{xy}(f)\delta(f-g)e^{i2\pi(g-f)t}e^{i2\pi f\tau}dgdf$$

where $S_{xy}(f)$ is the two-sided cross-spectral density function (two-sided since the limits are from $-\infty$ to $+\infty$). δ is Dirac delta function.

Integrating with respect to g , the integral is zero except when $g = f$. When $g = f$, $e^{i2\pi(g-f)t} = 1$.

Hence

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f)e^{i2\pi f\tau}df \quad (2-5)$$

where τ is the time delay.

From equation (2-5) we see that $R_{xy}(\tau)$ and $S_{xy}(f)$ are Fourier transform pairs. We therefore have the two relationships

$$X(t) = \int_{-\infty}^{\infty} A(f)e^{i2\pi ft}df$$

and

$$R_{xy}(\tau) = \int_{-\infty}^{\infty} S_{xy}(f)e^{i2\pi f\tau}df.$$

From the expression

$$E[A(g)B^*(f)] = S_{xy}(f)\delta(f-g)$$

when $f = g$

$$E[A(f)B^*(f)] = S_{XY}(f) \quad (2-6)$$

From equation (2-6) we see that by multiplying the Fourier series coefficients of a series by the complex conjugate of the Fourier coefficients of another series we get the cross-spectral density function of the two signals.

Using the cross-spectral density in equation (2-5) one is able to obtain the cross-correlation between the two signals. Obviously when $X = Y$ one obtains the periodogram and the autocorrelation.

When using analogue methods to measure the power spectral density one only measures positive frequencies. In that case the physically realisable one sided power-spectral density function $G_{XY}(f)$ is obtained where $G_{XY}(f) = 2S_{XY}(f)$

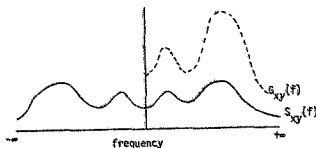


Fig. (2-3) Physically Realisable One-sided Power-spectral Density Function

The next section describes the method of analysis used for the analysis of a single sample.

2.6 Method of Analysis2.6.1 Single Series Analysis

Assuming that the signal is random and stationary, the following procedure is followed:

- i) The analogue signal is band limited so that aliasing errors are eliminated. This is done using analogue filters.
- ii) The signal is then digitised at a rate that will enable reliable information concerning the frequencies of interest to be obtained. *There is not much literature on the subject but it seems that sampling at least five times the highest frequencies of interest is adequate. Of course the sampling rate must also be high enough to prevent aliasing.*
- iii) The series is then Fast Fourier transformed using a digital computer to obtain the Fourier coefficients $A(f)$ $(a(f) + ib(f))$.
- iv) The Fourier coefficients $A(f)$ can be windowed to reduce leakage effects.
- v) An estimate of the periodogram $S_{xx}(f)$ is obtained as follows

$$\begin{aligned} S_{xx}(f) &= X(f)X^*(f) \\ S_{xx}(f) &= (a(f) + ib(f))(a(f) - ib(f)) \\ S_{xx}(f) &= a^2(f) + b^2(f) \end{aligned} \quad (2-7)$$

When the Fourier coefficients are Hanned before being combined to form the periodogram, the periodogram so formed is known as the *modified* periodogram. If the Fourier coefficients are not Hanned it is called a *raw* periodogram. From 2.6 we see that the periodogram is a real function. Obviously the autocorrelation will be an even function as from 2.5.1 it has only cosine terms in its Fourier series.

- vi) The autocorrelation is obtained by inverse Fourier transforming equation (2-7) according to equation (2-5)

$$R_{xx}(\tau) = \int_{-\infty}^{\infty} S_{xx}(f) e^{i2\pi f \tau} df$$

The figure below describes graphically the above procedure

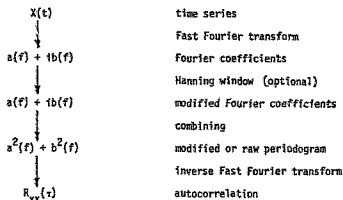


Fig. (2-4) Single Series Method of Analysis

2.6.2 Two Series Analysis

The cross-spectrum and cross correlation of two signals $X(t)$ and $Y(t)$ are obtained in exactly the same way as for the single series analysis except that the Fourier coefficients of the X and Y series must be combined. Assuming the two signals to be random and stationary the procedure is shown in the figure (2-5).

The cross-spectral density function is a complex function.

The real part $a^x a^y + b^x b^y$ is called the co-spectrum while the imaginary part $a^y b^x - a^x b^y$ is called the quadrature-spectrum.

Since the cross-spectral density function has both sine and cosine terms, the cross correlation is not necessarily a symmetrical function.

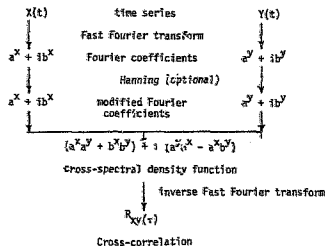


Fig. (2-5) Two Series Method of Analysis

2.6.3 Zone Averages

The conventional hot-wire anemometer signal is tape-recorded and digitised as described in 2.6.1. The signal is then processed to determine which points are in the turbulent zone and which points are in the non-turbulent zone. This was performed using digital programs developed by Dryer (1973). Briefly, a range of frequencies containing the turbulent fluctuations is chosen and a cut-off value for the amplitude then determined experimentally.

From the intermittency programs one obtains an intermittency signal consisting of a series of one's and zero's, depending on whether the associated point in the conventional signal is in a turbulent zone or non-turbulent zone. The next step is to form a signal containing velocity fluctuations only from the turbulent zone. In this analysis all the velocity values associated with the one's were added one after the other. The magnitude of the errors introduced will be dependent on the length of the individual turbulent bursts. This is discussed further

in Section 6.3.4.

Using the digital system available, the turbulent and non-turbulent zones were analysed according to 2.6.1. For comparison purposes, the analysis was also performed on the signal from which the turbulent and non-turbulent zone signals were obtained.

2.7 Averaging over Samples

The quantity of data obtained in the digitising process is far too large to analyse in one pass through the digital computer. Generally the data is divided into sections, the size being dependent on the computer available, and the results of the sections averaged to give a final answer.

In this analysis sections of 2048 data points were used and quantities such as the periodogram and autocorrelation calculated for each section. At the end of the analysis an average value is obtained.

The averaging of the periodogram in this manner is discussed by Welch (1967) who shows that the statistical accuracy of the results (for a stationary process) depends directly on the sample size and the number of samples analysed. An idea of the accuracy in this analysis was obtained by performing the following statistical test:

- i) For each sample analysed the standard deviation is calculated (X_i).
- ii) After each sample has been analysed the accumulated mean standard deviation is calculated

$$\bar{X} = \frac{1}{m} \sum_{i=1}^m (X_i)$$

where m = number of sections (2048 points).

- iii) Assuming the signal to be a random stationary series the quantity $\bar{X}$ should follow the student "t" distribution (a result of the central limit theorem).

- iv) The variance of the standard deviation is calculated

$$\text{var}(X) = \frac{1}{(m-1)} \sum_{i=1}^m (X_i - \bar{X})^2$$

- v) The standard deviation of the series of sample standard deviations is then calculated

$$SD = \sqrt{\text{Var}(X)}$$

- vi) The following confidence interval for the mean value of the sample standard deviation can be set up

$$\bar{X} - \frac{tSD}{\sqrt{m}} < \bar{X} < \bar{X} + \frac{tSD}{\sqrt{m}}$$

where

SD = standard deviation

t = students "t" coefficient

m = number of sections (2048 points)

If the confidence interval is narrow enough the calculation can be terminated.

The theory and possible errors involved in analysing a long section of data by dividing it into manageable sections is discussed in detail by Krause et al (1970). Two points should be considered.

An error (subdivision error) results when the signals being analysed have a mean value and the statistic being calculated cannot be obtained directly from the signals actually recorded. As an example consider the cross correlation of two signals $x(t)$ and $y(t)$ with mean values $\bar{x}$ and $\bar{y}$. The cross correlation $R_{xy}(\tau)$ is defined as

$$\begin{aligned}
 R_{xy}(\tau) &= \frac{1}{T} \int_0^T (x(t) - \bar{x})(y(t + \tau) - \bar{y}) dt \\
 &= \frac{1}{T} \int_0^T x(t)y(t + \tau) dt - \bar{x}\bar{y}
 \end{aligned} \tag{2-8}$$

If the total time T is divided into m sections of Δt and the cross correlation for each section calculated and then averaged.

$$\begin{aligned}
 R_{xy}(\tau) &= \frac{1}{m} \sum_{i=1}^m \left[\frac{1}{\Delta t} \int_{(i-1)\Delta t}^{i\Delta t} x(t)y(t + \tau) dt - \bar{x}_i \bar{y}_i \right] \\
 R_{xy}(\tau) &= \frac{1}{m\Delta t} \int_0^T x(t)y(t + \tau) dt - \frac{1}{m} \sum_{i=1}^m \bar{x}_i \bar{y}_i
 \end{aligned} \tag{2-9}$$

The subdivision error will then be (2-9) - (2-8)

$$= -\bar{x}\bar{y} + \frac{1}{m} \sum_{i=1}^m \bar{x}_i \bar{y}_i$$

For $m = 1$ the subdivision error is zero as expected.

Another error can result in calculating autocorrelations and cross correlations from segmented data. The statistical accuracy of coefficients with large time lags (relative to the segment length) will be less than those of smaller lags if the calculations move in only one direction.

To reduce the error, extra data points must be added to the segment when large time lags are being calculated. A second method is to continue the segment periodically. The Fast Fourier transform uses the latter method.

The applicability of Taylor's hypothesis is discussed by Lin (1953).
In this case

$$p_X(\tau) = f(\tau)$$

giving

$$\lambda_f = U_E$$

and

$$\Delta f = W_E$$

(3-10)

The importance of equation (3-10) is that U_E and W_E are measured using one hot-wire anemometer, while the measurement of λ_f and Δf requires two. Assuming Taylor's hypothesis λ_f and Δf can be obtained using a single hot-wire anemometer.

The experimental apparatus required for measuring the above time and length scales is described by Rasmussen (1966) and DISA (1968).

3.3.4 The Periodogram (Power spectrum)

If the discrete series of velocity fluctuations is represented as

$$\begin{aligned} X(kh) & \quad k = 1, 2, 3, \dots, N \\ h & = \frac{T}{N} \end{aligned}$$

where T = total time of the series. Then the complex Fourier coefficients can be obtained from

$$A_n = \frac{1}{N} \sum_{k=1}^N X(kh) e^{-2\pi i n k h}$$

Note: The series $X(kh)$ is written as a complex function although in reality the imaginary part is zero.

The periodogram is defined as

$$S(f) = A(f)A(f)^*$$

3. STATISTICAL DESCRIPTION OF TURBULENT FLOW USING FOURIER ANALYSIS

CHAPTER III

3.1 Introduction

This chapter will deal with the measurements used to characterise a turbulent field. Firstly the response equations of the hot-wire anemometer are discussed and then correlation analysis will be introduced. Lastly the various measurements made are mentioned.

3.2 The Hot-Wire Anemometer

The hot-wire anemometer is the most common instrument used in measuring turbulent velocity fluctuations. The structure and characteristics of the equipment used are described by Hinze (1959). In this section a description of what the hot-wire anemometer measures is given.



Fig. (3-1) Single Hot-wire Sensor

Figure (3-1) shows a hot wire in a turbulent flow, the wire being perpendicular to the mean velocity ($V = W = 0$). Velocity fluctuations u, v, w are superimposed on the stream. Effectively, the velocity measured by the hot wire is

$$U_{eff} = \sqrt{(U+u)^2 + v^2 + w^2}$$

for u, v, w small compared to U (i.e. less than 15%)

$$U_{eff} = \sqrt{(U+u)^2} = U + u$$

The wire essentially measures only the longitudinal fluctuations.

If the hot wire is placed at an angle to the mean velocity, then the effective velocity has components in both the u and v directions and hence the effective cooling velocity is a function of both the u and v velocity fluctuations (Fig. (3-2)).

Two hot-wire probes at a point in the stream, symmetrically displaced in an X array about the mean flow direction are affected equally by the u fluctuations, but oppositely by the v fluctuations. Using two wires as described above it is possible to separate the u and v fluctuating components (Fig. (3-3)).

Next one must obtain the relationship between the effective velocity and the u and v velocity components. Experiment has shown (Webster (1962), Champagne (1965)) that the tangential component of the velocity along the inclined wire does in fact effect the heat transfer from the hot wire. In general the cosine law is assumed which means that the amount of heat transfer depends only on the flow field through the normal component of the velocity across the hot wire.

$$\text{i.e. } U_{\text{eff}}(\theta) = U(0)\cos \theta$$



Fig. (3-2) Inclined Hot-wire Sensor

$$U(0) = \text{measured velocity at } \theta = 0.$$

A modified relationship suggested by Champagne (1965)

$$U_{eff}(\beta)^2 = U(0)^2(\cos^2\beta + k^2\sin^2\beta)$$

where

$U_{eff}(\beta)$ = velocity measured at $\beta = \beta$

$U(0)$ = velocity measured at $\beta = 0$

k = sensitivity coefficient

It has been found (Champagne (1965) and Jorgenson (1971)) that the value of k depends on:

- 1) the length to diameter ratio of the wire
- 11) the mean velocity

Generally the value of k varies between 0.1 and 0.3. The work by Jorgenson (1971) describes the errors and values of k involved when using the above relationship on probes manufactured by the DISA Company. It seems that for values of $\beta = 45^\circ$ it is advisable to use a sensitivity coefficient.

Having a well-substantiated relationship for the effective mean velocity, one requires a similar relationship for the fluctuating velocity components.

A complete derivation can be found in Appendix I. If second order velocity products (u^2 , v^2 , w^2 , uv) are neglected then the following relationship is obtained:

$$\frac{c_1}{U_{eff}} = \left[\frac{u + \frac{v \sin \alpha \cos \alpha (k^2 - 1)}{(\sin^2 \alpha + k^2 \cos^2 \alpha)}}{U} \right] \quad (3-1)$$

c_1 = instantaneous velocity fluctuation.

When using this relationship the value of $\bar{E}$ (average measured voltage) is underestimated by approximately

$$\frac{v^2 + w^2}{2U^2(\sin^2 \alpha + k^2 \cos^2 \alpha)} \bar{E}$$

note, $\bar{E} = KU_{\text{eff}}$

For this equation the effective cooling velocity and mean velocity are related by

$$U_{\text{eff}}^2 = U(0)^2(\sin^2 \alpha + k^2 \cos^2 \alpha) \quad (3-2)$$

Considering the X array in Fig. (3-3) one obtains an equation like (3-1) for each of the wires

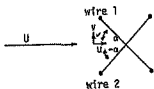


Fig. (3-3) X-Array Hot-wire Probe

$$\left. \begin{aligned} \frac{c_1}{U_{\text{eff}}} &= \frac{U}{U} + \frac{v \sin \alpha \cos \alpha (k_1^2 - 1)}{U(\sin^2 \alpha + k_1^2 \cos^2 \alpha)} \quad \text{for wire (1)} \\ \frac{c_2}{U_{\text{eff}}} &= \frac{U}{U} - \frac{v \sin \alpha \cos \alpha (k_2^2 - 1)}{U(\sin^2 \alpha + k_2^2 \cos^2 \alpha)} \quad \text{for wire (2)} \end{aligned} \right\} \quad (3-3)$$

Using equation (3-2) we have

$$\left. \begin{aligned} c_1 &= \sqrt{\sin^2 \alpha + k_1^2 \cos^2 \alpha} U + \frac{v \sin \alpha \cos \alpha (k_1^2 - 1)}{\sin^2 \alpha + k_1^2 \cos^2 \alpha} \\ c_2 &= \sqrt{\sin^2 \alpha + k_2^2 \cos^2 \alpha} U - \frac{v \sin \alpha \cos \alpha (k_2^2 - 1)}{\sin^2 \alpha + k_2^2 \cos^2 \alpha} \end{aligned} \right\} \quad (3-4)$$

The equations (3-4) are suitable for use in digital analysis. Records of c_1 and c_2 can be made at identical times on magnetic tape. These are then processed to give the u and v fluctuations. The sensitivities of the two wires are generally different and this fact can be incorporated into the solution of equation (3-4).

When using analogue methods for obtaining turbulent intensities and Reynolds shear stresses, equations (3-4) are not used. Champagne (1965) derived correction factors (a function of k and a) by which the quantities measured, assuming the cosine law, must be multiplied to take into account the directional sensitivity of the probe. It is also required that k_1 and k_2 be the same.

3.3 Correlation Analysis

3.3.1 Introduction

Usually the turbulent velocity is assumed to consist of a mean velocity and a superimposed fluctuating component whose average is zero. When this is used in the Navier-Stokes equation and the equation averaged, additional stresses, Reynolds stresses, are introduced, e.g. the xy -component of the Reynolds stress is defined by

$$\tau_{xy} = -\rho \overline{uv}.$$

These stresses represent the mean rate of transfer of momentum across a surface due to the velocity fluctuations.

To solve the resulting equations of motion a relationship between the Reynolds stress and the mean velocity is required. Another approach would be to obtain equations describing the behaviour of the Reynolds stresses. Any attempt to do this using the equations of motion results in the introduction of triple velocity correlations, and velocity pressure correlations. The latter can be eliminated but in so doing velocity correlations at different points are introduced.

To try and terminate the process of continually introducing higher order velocity correlations, workers such as Frenkiel and Klebanoff (1971) and van Atta and Chen (1969) have tried to find relationships relating, say, the two- and four-velocity correlations. The correlation functions between the derivatives of velocity fluctuations have been studied by Frenkiel and Klebanoff (1971).

The turbulent field can be characterised by length scales obtained from velocity correlations. They are also used to study the spatial configuration of the turbulent field. If one assumes homogeneity and isotropy of the turbulent field, theory can be applied to the form of the velocity correlations, and useful information concerning eddy sizes obtained.

Mathematically, correlation analysis is well covered in the literature - see Bendat and Piersol (1966), but only recently has it become possible to measure the correlation functions simply. A discussion of various methods available is given by Jenson (1970). Correlation analysis as applied to turbulence is discussed by Hinze (1959) and Townsend (1956).

3.3.2 Longitudinal and Lateral Length Scales

The scales are obtained from the longitudinal and lateral spatial correlations of the velocity at two different points. The normalised lateral spatial correlation is defined as

$$g(r) = \frac{u(x)u(x+r)}{\sqrt{u(x)^2 u(x+r)^2}}$$

and the normalised longitudinal correlation as

$$f(r) = \frac{u(x)u(x+r)}{\sqrt{u(x)^2 u(x+r)^2}}$$

where the velocities u and separation r are shown in figure (3-4).

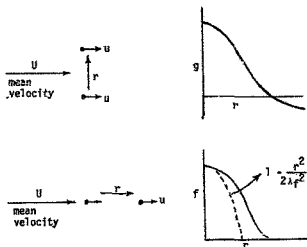


Figure (3-6) Longitudinal and Lateral Length Scales

The way in which the correlation behaves as $r \rightarrow 0$ depends on the turbulent field (periodicities, etc.). Assuming isotropy the shape of the correlation function in the region $r \approx 0$ can be described in terms of the velocity and its derivatives.

Considering the longitudinal spatial correlation we have (from a Taylor expansion)

$$f(r) = 1 - \frac{r^2}{2\lambda_f^2} \quad (3-5)$$

$$\text{where} \quad \frac{1}{\lambda_f^2} = \frac{1}{U^2} \left| \frac{\partial U}{\partial r} \right|_{r=0}^2 \quad (3-6)$$

Since this defines a parabola with its vertex at $r = 0$, $f(r) = 1$ where it oscillates the correlation curve. λ_f is found from the intersection of the parabola with the r axis. λ_f is known as the Taylor microscale.

The Taylor dissipative length scale λ_f is associated with the smaller eddies of the turbulence but is not the length scale associated with the small eddies responsible for viscous dissipation of energy. These eddies are associated with the Kolmogorov length scale which for isotropic turbulence is related to the Taylor length scale.

For homogeneous isotropic turbulence the rate of viscous dissipation is given by

$$\epsilon = 15\nu \left(\frac{\partial u}{\partial r} \right)^2 \quad (3-7)$$

ν = viscosity (kinematic)

The Kolmogorov length scale associated with the dissipating eddies is defined by

$$\eta = (\nu^3/\epsilon)^{1/4} \quad (3-8)$$

Using equations (3-6), (3-7) and (3-8) one obtains

$$\frac{\lambda_f^2}{\eta^4} = \frac{15U^2}{\nu^2} \quad (3-9)$$

λ_f is generally larger than η and at high Reynolds numbers it corresponds to an eddy size containing a negligible part of the total energy and is responsible for a negligible part of the total dissipation. It is a measure of the ratio of the total energy to the rate of energy loss and can therefore be regarded as a "reaction time" of the turbulence. A more complete discussion is given by Townsend (1956).

In general the lateral correlation will give a microscale λ_g where

$$\lambda_f \neq \lambda_g$$

For homogenous isotropic turbulence the relationship

$$\lambda_f = \sqrt{2} \lambda_g$$

exists.

Another length scale, giving the length of the largest correlation between two velocities can be measured. This length would be r^* where

$$f(r) = 0 \text{ for } r > r^*$$

In fact a slightly different length scale is defined

$$\Lambda_f = \int_0^\infty f(r) dr$$

where Λ_f = macro or integral scale of turbulence.

If $f(r)$ is a rectangular function then $\Lambda_f = r^*$. Periodicities or randomness change the shape of $f(r)$ and hence Λ_f can vary considerably from r^* .

3.3.3 Autocorrelation

The autocorrelation is defined as

$$R_X(\tau) = \frac{1}{T} \int_0^T u(t) u(t+\tau) dt$$

τ = time delay

T = total length of sample

and describes the general dependence of the values of the data at one time on the values at another time. As $T \rightarrow \infty$ the exact autocorrelation is approached. Useful properties of the autocorrelation are:

- i) $R_X(0) = \overline{u(t)^2} \geq |R_X(\tau)|$
- ii) $R_X(\tau) = R_X(-\tau)$

Usually one deals with the normalised autocorrelation

$$\rho_X(\tau) = \frac{R_X(\tau)}{R_X(0)}$$

The main application for autocorrelations (although it does not strictly apply to turbulence studies) is in finding deterministic data that has been disguised in a random background. Deterministic data (also periodic) have autocorrelations over all time displacements while random data has an autocorrelation that diminishes to zero for large displacements (Sengul and Piersol (1966)).

The normalised autocorrelation of the u fluctuations

$$\rho_X(\tau) = \frac{\overline{u(t)u(t+\tau)}}{\overline{u(t)^2}}$$

can, assuming homogeneity be represented by the parabola

$$\rho_X(\tau) = 1 - \frac{\tau^2}{2\tau_E^2}$$

near the origin $\tau = 0$.

$$\frac{1}{\tau_E^2} = \frac{1}{\overline{u^2}} \left(\frac{\partial \overline{u}}{\partial \tau} \right)^2_{\tau=0}$$

τ_E is a microscale, while a macroscale J_E , where

$$J_E = \int_0^\infty \rho_X(\tau) d\tau$$

can also be defined. The procedure just followed is identical to that for obtaining the Taylor microscale from the longitudinal spatial correlation.

In the case where the turbulent field has a constant mean velocity U in the x direction and the turbulence level

$$\frac{\sqrt{\overline{u^2}}}{U}$$

is low, Taylor's hypothesis may be assumed..

$$\text{i.e. } \frac{\partial}{\partial t} = -U \frac{\partial}{\partial x}$$

where * indicates the complex conjugate.

$$\text{i.e. } S(f) = (A^r(f))^2 + (A^i(f))^2$$

where

$A^r(f)$ = coefficient of the cosine term of the Fourier series
(real part of complex coefficient)

$A^i(f)$ = coefficient of the sine term of the Fourier series
(imaginary part of the complex coefficient)

When one calculates the Fourier coefficients using the Fast Fourier transform, the set of harmonics for which the periodogram is calculated are dependent on the total length of the sample analysed.

$$f = \frac{k}{T} \quad k = 1, 2, 3 \dots N/2$$

The Periodogram was first introduced by Schuster (1898).

Unfortunately, it could not be fully exploited because of the large quantity of calculation involved. Jones (1965) indicated that for moderate sample sizes (1 000 points) the modern computer made calculation of the periodogram feasible. In the same year the Fast Fourier transform appeared and made the periodogram a definite proposition.

Briefly, the periodogram describes the relative energy distribution amongst the frequencies of the turbulent fluctuations.

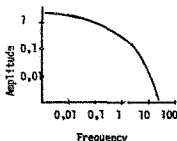


Fig. (3-5) Log-log Plot of Periodogram

Generally, the periodogram is plotted on a log-log scale and the form shown in figure (3-5) usually obtained.

In the case of a homogeneous, isotropic turbulent field the shape of the periodogram, also called the Taylor one dimensional energy spectrum, can be discussed theoretically (Lin and Reid (1963)). The name is such as only one component of the velocity is considered. Kolmogorov predicted that at sufficiently high Reynolds numbers of the turbulence (> 700) the energy spectrum function takes on the universal form

$$S(f) \propto f^{-5/3}$$

The Reynolds number used is defined in terms of the Taylor microscale and root mean square of the velocity fluctuations.

On the log-log plot one would therefore expect the slope of the periodogram to be approximately $-5/3$. This fact can be used to check the assumption of isotropy, even when the Reynolds number of the turbulence is not too high (Bradshaw (1967)). The Taylor micro and macro length scales can be obtained from the periodogram.

$$\lambda_T = \frac{U}{2\pi} \left(\int_0^\infty f^2 S(f) df \right)^{-1/2}$$

$$\lambda_f = \frac{U}{4} \lim_{f \rightarrow 0} f S(f)$$

This is possible as the autocorrelation and periodogram are Fourier transform pairs.

In general it is easier to calculate the Taylor length scales using the autocorrelation.

Besides describing the turbulent field the periodogram can sometimes be used to check if any vibrations due to external sources are present (fan speed, 60 cycle /min).

3.3.5 Moving Axis Autocorrelation

To apply Phillips (1957) theory to the Reynolds Shear stress profile one requires the convected integral time scale. In his theory Phillips indicates that the rate of change of Reynolds shear stress in a shear layer is proportional to the convected integral time scale.

For the hot-wire configuration in figure (3-6)

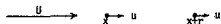


Fig. (3-6) Wire Configuration for Moving Axis Autocorrelation

the normalised cross correlation of the two u velocity fluctuations is defined as

$$\rho_{uu}(r, \tau) = \frac{1}{T} \frac{\int_0^T u(x, t) u(x+r, t+\tau) dt}{\sqrt{\int_0^T u(x, t)^2 dt \int_0^T u(x+r, t+\tau)^2 dt}}$$

For a turbulent field a curve similar to figure (3-7) is usually obtained

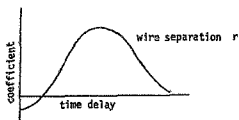


Fig. (3-7) Cross Correlation $\rho_{uu}(r, \tau)$

If the measurements are now made for various values of r the following series of curves is obtained

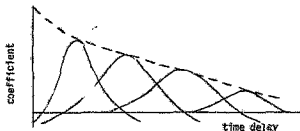


Fig. (3-8) Moving Axis Autocorrelation

Obviously the maximum values of each curve occurs at the time delay coinciding with the 'convection velocity' of the turbulence. The convection velocity should be the same for each wire separation. A discussion on the convection velocities is given by Wills (1964) and a possibly better convection velocity defined.

The dotted line, passing through the maximum of each curve, defines the autocorrelation of the turbulence in a frame of reference moving with the convection velocity of the turbulence.

According to Davies, Garratt et al (1963) the shape of the moving axis autocorrelation gives an indication of how fast the turbulence pattern is changing. If the turbulent field is regarded as frozen, then the maximum of each curve ($R_{uu}(r, \tau)$ vs τ) should be the same. The amount by which it decreases shows the amount by which the field has changed while being convected downstream.

The integral under the moving axis autocorrelation is defined as the convected integral time scale. It is an indication of the time it takes for the particular turbulence pattern to disappear.

3.3.6 The Cross Spectrum

The cross spectrum of two signals is a complex function. The real part is called the co-spectrum while the imaginary part is called the quadrature-spectrum.

Thus far the cross spectrum has not been used much in turbulence analysis, its physical interpretation being obscure, but is (in this analysis) calculated so that the cross correlation, which has a physical interpretation, can be obtained (Townsend (1956)).

Consider two random, stationary, complex series

$$X(kh) \text{ and } Y(kh)$$

where $k = 1, 2, \dots, N$.

One can calculate the Fourier series coefficients

$$A(f) \text{ and } B(f)$$

and obtain the cross spectrum

$$S_{XY}(f) = A(f)B^*(f)$$

Where * means complex conjugate.

Thus

$$\text{co-spectrum} = A^r B^r + A^i B^i$$

and

$$\text{quadrature-spectrum} = A^i B^r - A^r B^i$$

$$A_k^i = \text{imaginary part of the Fourier series coefficient of } X.$$

In this analysis the cross spectrum of the u and v velocity fluctuations is calculated.

3.3.7 The Cross Correlation

The cross correlation function indicates the dependence of one set of data on another set of data displaced by a time delay τ . For two continuous series $x(t)$ and $y(t)$, the cross correlation is defined as

$$R_{xy}(\tau) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_0^T x(t)y(t+\tau)dt$$

The cross correlation does not necessarily have a maximum at $\tau = 0$ and is also not an even function like the autocorrelation. Some useful properties of the cross correlation are listed below:

- i) $R_{xy}(\tau) = R_{yx}(-\tau)$
- ii) $|R_{xy}(\tau)|^2 \leq R_x(0)R_y(0)$
- iii) $|R_{xy}(\tau)| \leq \frac{1}{2}[R_x(0) + R_y(0)]$

A normalised cross correlation function may be defined as

$$\rho_{xy}(\tau) = \frac{R_{xy}(\tau)}{\sqrt{R_x(0)R_y(0)}}$$

The above only applies when the two series have zero mean values. When $R_{xy}(\tau) = 0$ for all time displacements then the two series are statistically independent. If $x(t)$ and $y(t)$ were statistically independent and had mean values of $\bar{x}$ and $\bar{y}$ then the cross correlation would be $\bar{x}\bar{y}$ for all time displacements.

In turbulence studies cross correlations of signals at different points in the flow are used to obtain information on eddy sizes and movement. They give a "picture" of the turbulent flow. Cross correlations of the various velocity components give an idea of the dependence of the fluctuations on one another, while the value at $\tau = 0$ gives values of the Reynolds shear stresses.

3.4 The Measurements Taken

3.4.1 Single Wire Measurements

Using a single hot wire the following quantities were measured in the turbulent shear layer described in Chapter V.

- i) Mean velocity
- ii) Turbulent intensity of the u velocity fluctuations

$$\text{Intensity} = \frac{\sqrt{u'^2}}{U}$$

where $\sqrt{u'^2}$ = root mean square of fluctuations

U = mean velocity

- iii) The periodogram of the u velocity fluctuations.
- iv) The autocorrelation of the u velocity fluctuations. From these the Taylor micro and macro scales were obtained and hence the Kolmogorov length scale.

3.4.2 Two Wire Measurements

Using an X-array probe the following quantities were measured:

- i) Mean velocity
- ii) Turbulent intensities of the u and v velocity fluctuations
- iii) The periodograms for the u and v velocity components
- iv) The autocorrelations for the u and v velocity components
- v) The Reynolds shear stress $\tau_{xy} = -\rho \overline{uv}$
- vi) The cross spectrum of the u and v components
- vii) The cross correlation of the u and v components

Using two hot-wire probes as described in 3.3.5 one obtained:

- viii) The moving axis autocorrelation
- ix) The convected time scale

3.4.3 Conditional Measurements

Using intermittency programs developed by Dreyer (1973), single-wire data was separated into turbulent and non-turbulent zones. The programs developed here were then used to calculate the quantities in 3.4.1 for each zone.

CHAPTER IV

4. THE COMPUTER PROGRAMS

4.1 General

A brief description of the available computer programs as well as the procedure for the analysis of data obtained from a single hot wire and data obtained from two hot wires is given. The general arrangement of the digital side of the analysis is described. More detailed descriptions are to be found in Appendix (II). The analogue side of the setup is described in Chapter V.

The system is not designed for on-line digital analysis. During the experiment, useful data is stored as an analogue signal on the instrumentation tape recorder. Having an analogue copy of the signal allows for versatility in the digitising process as the data can be redigitised at a different sampling rate if necessary. Recording a clock signal onto an extra channel of the tape recorder makes it possible to return to a particular section of data. This is necessary when two signals are to be cross correlated and they must therefore start at the same instant in time.

All the programs have been written in FORTRAN IV, to be processed by the I.B.M. 360/50 digital computer at the University of the Witwatersrand Computer Centre. Since the programs require disc space, magnetic tapes, private Libraries, and the Calcomp plotter, the Job Control Language can become fairly complex. Special 'In-line Procedures' have been written to overcome this as well as to make executing the same program a number of times in the same job, but with different data, a simple matter.

The majority of the programs and subroutines are stored on disc. In case of the Library being damaged they are all also available on punched cards.

4.2 Data

The main means of supplying data to the programs is from magnetic tape as the digitiser writes the turbulent velocity samples onto magnetic tape in Integer *2 form. Besides being used as an input device, magnetic tapes are also used to store the data.

Other data has to be supplied by means of punched cards, while the program NLBONE writes information onto temporary disc space. This can be used as data by subsequent programs.

4.3 Results

The results from the various programs are either printed on the line printer, plotted using the Calcomp plotter, written onto temporary disc space or else written onto magnetic tapes. A special program NLBSTORE has been written to transfer results calculated by NLBAUTO or NLBCROSS (which calculate autocorrelations and cross correlations respectively) onto magnetic tape. Temporary disc space is space available only for the duration of the Job.

4.4 The Programs

All the programs and subroutines mentioned in this section are stored in the Library ACM.NLB.LIB. Programs not used regularly are not on the Library, but it is no problem to put them there if necessary. The name by which each program or subroutine is referenced in the following sections is the same as the name of the module in ACM.NLB.LIB. that contains it.

STATSS

Subroutine STATSS calculates various statistics (mean, standard deviation, minimum, maximum, total) of an input series.

SCHECK

Subroutine SCHECK performs a Students "t" test on the mean of a set of values.

NLBONE

This is a large general purpose program, executed by NLBCROSS or NLBAUTO. It is discussed further later in this section.

PGRAM

Subroutine PGRAM calculates the periodogram of an input series.

STORE

Program NLBSTORE transfers data written onto disc by NLBONE, NLBAUTO or NLBCROSS onto magnetic tape.

NLBCONV

Program NLBCONV converts Integer *2 values supplied by the digitiser into Real *4 values.

CHECK

Program NLBCHK prints on the line printer the first block of any file on a tape containing Integer *2 values.

HAMMIN

Subroutine HAMMIN windows an input array using the Hamming coefficients.

HANNIN

Subroutine HANNIN windows an input array using the Hanning coefficients.

GRAPH

Subroutine GRAPH plots a single graph using the Calcomp plotter.

SMOOTH

Subroutine SMOOTH is used to smooth an input series using a triangular smoothing function.

REMOVE

Subroutine REMOVE calculates the mean and any linear trend in an input sample and then removes them if they are larger than a predetermined value.

AUTOC

Subroutine AUTOC calculates the autocorrelation of a time series, using the output from subroutine PGRAM as its input.

READS

Subroutine READS is used to read arrays larger than 1024 samples when the data being read is blocked in blocks of 1024.

RHARI

Subroutine RHARI performs a Fast Fourier transformation (or inverse transformation) on a real time series.

NBHIST

Subroutine NBHIST is used to plot a histogram using the Calcomp plotter.

MOMENT

Subroutine MOMENT calculates some higher order skewness factors for a time series.

4.5 Method of Analysis

Generally the following procedure is used to analyse data on an Integer *2 tape. See Appendix II for program descriptions.

- 1) The Integer *2 tape is checked by running NLBCHECK. One can see whether the values are reasonable. By digitising a sine wave one can check if the digitiser is functioning normally.

- ii) Using NLBCONV the files that are to be analysed are converted into Real *4 form and then stored on another magnetic tape.
- iii) The data on this tape can be examined by using either program NLBANY or NLBANYG.
- iv) NLBSMP is used on a few samples to check if the sampling rate is sufficient. If not, the analogue signal could be digitised again at a faster rate. See Appendix II.
- v) If the data consists of two simultaneous records, e.g. from a X wire, then it is possible to separate the two signals into the u and v velocity fluctuations. This is achieved using NLBSEP. The u and v signals are then stored on tape.
- vi) The signals are then in a form that can be analysed by one of the general programs NLBAUTO or NLBCROSS. NLBAUTO is used for analysis of single series, while NLBCROSS is used when cross spectra and cross correlations are required. Both programs in fact execute the program NLBONE. The difference is that NLBCROSS analyses two single series simultaneously, calculating cross correlations as well as the autocorrelations for each series.
- vii) NLNSTORE can then be used to transfer the results from the temporary disc space used by NLBAUTO or NLBCROSS onto magnetic tape. They can then be analysed further at a later stage.
- viii) The results from the magnetic tape can then be plotted on the Calcomp plotter. Graphs produced by these programs can be found in Chapter VI.
- ix) NLBZONE produces two signals, one in the turbulent zone and one in the non-turbulent zone, from the original signal and an intermittency signal.

4.6 The General Program NLBCROSS

The total signal (a complete file on the magnetic tape) is analysed using samples of 1024, 2048, 4096 or 8192 values. The sample size is specified by the user. If NLBCROSS is executed one of the signals is transferred to temporary disc space. This makes reading alternately from the two signals more efficient. The program then reads a sample from one of the signals and proceeds to analyse it.

4.6.1 Quantities Calculated

- i) Various statistics are obtained using subroutine STATSS. To avoid having to inspect the complete signal to find the minimum and maximum values, ± 4 times the standard deviation of the first sample is used as the range in amplitude of the signal.
- ii) The mean and any linear trend is calculated and removed if necessary.
- iii) The periodogram is calculated. It is Hanned if so required by the user.
- iv) The autocorrelation is calculated using the output from subroutine PROGRAM used above in ii).
- v) When more than one sample has been analysed, a Student's "t" test is performed on the standard deviations of each sample. If the confidence interval is narrow enough, calculations cease.
- vi) If NLBCROSS is being executed, the equivalent sample from the other signal is read and i) - v) performed.
- vii) The cross spectrum of the two samples is then calculated.
- viii) The cross correlation of the two samples is then calculated.
- ix) The various quantities calculated are added to those of the previous samples and then stored on a temporary disc data set.

- x) The program then reads the next sample from the file until the specified number of samples have been analysed.

4.6.2 Output of NLBONE

Output for each sample analysed is produced as well as overall and averaged properties of the complete signal.

For each sample the following is printed on the line printer:

- i) Total
- ii) Mean
- iii) Standard deviation
- iv) Minimum value
- v) Maximum value
- vi) Sample r.m.s. value
- vii) Sample slope
- viii) The number of samples outside the range considered for the amplitude probability density function.

The above quantities are useful for checking that the data is correct. It is easy to read 'garbage' from a magnetic tape or disc and occasionally the digitiser writes incorrect data. Improbable results can usually be picked out by following the minimum and maximum values of each sample.

Averaged values of the following quantities over the whole signal analysed are also printed out

- i) Overall total
- ii) Overall mean
- iii) Averaged standard deviation
- iv) Overall minimum

- v) Overall maximum
- vi) Overall amplitude probability density function
- vii) Averaged periodogram
- viii) Averaged autocorrelation
- ix) Overall r.m.s. value
- x) The confidence interval for 90% significance Student "t" test.

The quantities (vi), (vii) and (viii) are also written onto disc.

If NLBCROSS is executed, the above quantities are produced for each of the signals as well as the following:

- i) Averaged co-spectrum
- ii) Averaged Quadrature-Spectrum
- iii) Averaged cross correlation

These quantities are also written onto disc and can therefore be manipulated further by executing another program immediately after NLBAUTO or NLBCROSS.

CHAPTER V5. EXPERIMENTAL APPARATUS AND PROCEDURE

This chapter is in three sections, one dealing with the turbulent shear layer being studied, the second describes the measuring equipment while the third deals with the experimental procedure.

5.1 The Turbulent Shear Layer

A wind tunnel, discussed in detail by Dreyer (1973), was constructed so as to produce two streams of air with different but uniform velocities at the exit. These two streams then mixed in a perspex test section containing port holes to allow the hot-wire probes to be inserted. Fig. (5-1) shows the wind tunnel and indicates the physical dimensions of the shear layer produced.

Mean velocity and turbulence intensity profiles can be found in Chapter VI. The turbulence intensities indicate that it is possible to use the anemometers in the constant temperature mode.

More information concerning the shear layer is given below:

- i) At the entrance to the test section the fast side velocity is 17.5 m/sec and is flat to within 3%.
- ii) The slow side velocity is 7 m/sec and is flat to within 1.5%.
- iii) The layer was two dimensional to within 0.5% on the slow side, 0.6% on the fast side.
- iv) Down the length of the test section (1 metre) the mean velocity increased 8% in the slow region and 3% in the fast region. The increase is due to the expanding mixing region and the growth of boundary layers.

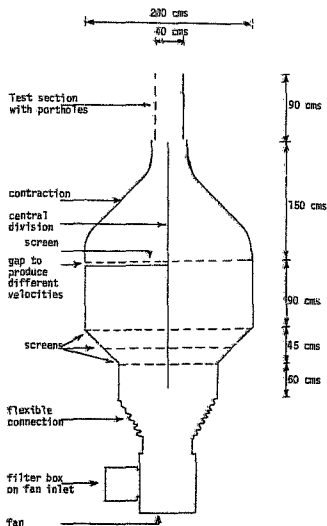


Fig. (5-1) The Wind Tunnel

- v) The layer obtains similarity less than 30 cms downstream of the splitter plate.
- vi) The approximate width of the shear layer is given in the table below:

Distance (cms)	16,9	31,9	45,9	59,9	73,9
Width (cms)	2	3	4	5	6

- vii) The traversing mechanism is accurate to within 0,5 mm.

5.2 The Analogue Apparatus

The analogue equipment used consisted mainly of DISA equipment. Because of its complexity, the setup is best described using figure (5-2). Basically the system consists of two hot-wire anemometers and their peripherals. An instrumentation tape recorder is used to record signal that is to be analysed. The convected integral time scale was measured using the analogue correlator as it was not intended to take intermittency into account.

The various units are discussed below, the reference numbers relating to figure (5-2).

1. All the hot-wire probes used were supplied by DISA. The models used in this work were:
 - i) 55F11 Gold plated probe with a straight sensor. The platinum plated tungsten sensor is approximately 3 mm. in length and 5 μ m. in diameter. The ends are gold plated leaving a sensing length of 1,25 mm. The widely spaced prongs ensure a low level of interference between the prongs and sensor ($\pm 2\%$).
 - ii) 55A38 X-array probes with the sensor plane parallel to the probe axis were used to measure u and v fluctuating components as well as the shear stress. The platinum plated tungsten probes

are 1,2 mm long and 5 μ m in diameter. The ϕ sensors are approximately 1 mm apart and the thermal wake interference is reduced to a negligible level (Jerome, Guitton and Patel (1971)).

2. The hot-wire anemometers used are DISA 55001, operated in the constant-temperature mode. The frequency response is dependent on the probe type and probe operating temperature. For probes similar to those used for this work the frequency response curve is flat up to 10 KHz.
3. Two DISA 55010 lineariser units are used to linearise the voltage versus velocity signal from the anemometers. The frequency response curves of the instruments are flat up to 10 KHz.
4. The digital voltmeter consisted of a Hewlett Packard 2212A voltage to frequency converter connected to a Hewlett Packard digital counter. Generally the counting time was 1 second although periods of up to 10 seconds are possible. The average value is displayed visually. The voltmeter could also be connected to a printer.
5. The root-mean-square voltmeter used was a DISA model 55035. Voltages proportional to the mean square, root mean square and squared value of the input signal are available at terminals. The input voltage range and integrator time constant can be varied.
6. DISA 55025 auxiliary units are used to band limit the signals. The high pass filter removed the dc component while the low pass filters were set at 10 KHz, the frequency limit of the tape recorder. The unit can be used to invert a signal or find the difference between two signals. The analogue filter roll-off is 6 dB/octave.

7. The instrumentation tape recorder used is a Philips Ana-log 7 with four channels plus a voice track. Signals can be recorded at one speed and played back at another without any distortion, the greatest speed being 76 cm/sec. Signals can be recorded either directly or on frequency modulation. The F.M. frequency response at the various speeds is

76 cm/sec	0 - 10 KHZ
38 cm/sec	0 - 5 KHZ
9,5 cm/sec	0 - 1,25KHZ
2,375 cm/sec	0 - 312 HZ

The input voltage range can be varied but the output range is constant at ± 1 volt.

8. A DISA 55075 time delay unit in conjunction with the analogue correlator are used to measure the cross correlation between the signals from the two anemometers. Its operation is based on the principle that correlation measurements do not require use of the whole analogue signal. It and the correlator are especially constructed to work together. Frequencies of up to 10 KHZ can be delayed for periods of up to 30 msec.
9. The DISA 55070 analogue correlator is used to measure cross correlation functions or normalised cross correlations of the hot-wire anemometer signals. For frequencies between 4 HZ and 10 KHZ the frequency response curve is flat. The input can be two signals (with or without a time delay between them) or else it can be connected to the time delay unit. The integration time for the time mean value amplifiers can be varied. Terminals on the correlator make it possible to obtain voltages proportional to the following functions:

- i) The correlation function
 - ii) Sum of amplified input signals
 - iii) Difference of amplified input signals
 - iv) The instantaneous squared value of the sum of the amplified input signals
 - v) Instantaneous squared values of the difference between the amplified input signals
 - vi) The average value of iv)
 - vii) The average value of v)
10. The digitiser used to convert the analogue input onto a digital tape that can be read by computer is situated at the Bernard Price Institute for Geophysical Research.

The input range has a maximum of ± 5 volts and the digitised values range between ± 2048 . The output is on magnetic tape in an I.B.M. compatible Integer *2 form. The data is recorded in blocks of 1024 values. Sampling rates of up to 10 KHZ can be obtained. Generally a slower playback speed on the analogue tape deck was used and the digitiser operated at about 4 KHZ.

By recording a clock signal on one of the channels on the analogue tape, it is possible to start the digitising process at a particular time. This is used when signals from two different channels are to be cross correlated.

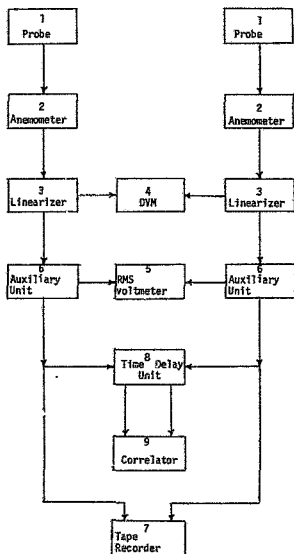


Fig. (5-2) The Analogue Equipment

5.3 The Experimental Procedure

Before doing an experimental run the analogue equipment must be allowed to warm up for a couple of hours. Generally the procedure then followed was;

- i) The probes to be used are calibrated using the calibration rig. The results are then fitted to King's law using a least squares technique and the necessary quantities for the lineariser calculated.
- ii) If an X-array probe is to be used it is calibrated in the position that it is to be operated in. This allows the true mean velocity to be obtained.
- iii) The linearisers are adjusted. If both anemometers are being utilised the voltage ranges for the two linearisers are made equal. This should give the two calibration curves similar slopes.
- iv) A series of voltages and velocities over the velocity range expected are read and the linearity of the calibration curves checked.
- v) The probes are then transferred to the traversing mechanism and lined up with the standard markings on the test section walls. A zero value, corresponding to the centre of the test section is read from the Vernier scale on the traversing mechanism.
- vi) The probes are connected to their corresponding anemometers, linearisers, auxiliary units, etc. One of the hot wires is monitored by the digital voltmeter and after the high pass filter, the root-mean-square voltmeter. The other channel is connected to the oscilloscope so that a visual check on the signal being measured can be made.

- vii) The probe is moved to the first position of the traverse and the instrument tape recorder started. The cassette contains 11 minutes of tape when recording at 76 cms/sec.
- viii) Using the voice track the start of the first reading is indicated. The digital voltmeter and root-mean-square voltmeter values are noted. On the voice track are noted the position in the shear layer, the tape recorder input level, digital voltmeter values and any other relevant information. This is invaluable later when checking to see if the correct reading is being digitised. After about 20 seconds the termination of the reading is indicated on the voice track.
- (ix) The traversing mechanism is immediately moved to the next position and the second reading taken, noting the details on the voice track. The tape recorder is not stopped during a traverse to avoid unnecessary stopping and starting which wastes tape space.
- x) It is important to note down which probe is connected to which anemometer and lineariser. The input level on the tape recorder should be noted as well as the channel being used. These are important when trying to obtain velocities from digitised values. When using a X-array probe one should further note the configuration of the wire with respect to the mean velocity and which channel is recording which probe signal. Without the above information the separation of the two X-array signals into velocity components is difficult.

- xi) When measuring the moving axis autocorrelation two anemometers connected to 55F11 probes are used. One probe is inserted into the traversing mechanism. The other is in a holder that can slide down the length of the test section. The position across the stream is determined by inserting perspex spacers between the side of the test section and the movable holder. The two probes are then lined up as close as possible (± 1 mm) and the position across stream read off the Vernier scale.
- xii) The downstream probe is then moved a known distance downstream and the cross correlation between the two signals measured for various time delays between them. The probes are then moved further apart and the procedure repeated.
- xiii) Another spacer is then inserted, the position across stream noted and then the readings in (xii) repeated.
- xiv) Once all the necessary readings have been taken, another set of velocity voltage readings are taken and the linearity of the calibration curves checked again.
- xv) The temperature is continually checked to see that it does not vary more than about 2°C .

CHAPTER VI

6. EXPERIMENTAL RESULTS

6.1 Calibration of the Hot-wire Anemometer Probes

The voltage vs. velocity relationship of the hot-wire probe generally follows King's law (Hinze (1959));

$$V^2 = A + BU^n$$

where

V = voltage

U = velocity

A, B, n are constants

Linearization of the hot-wire signals is performed using an analogue lineariser as applying King's law in the digital programs would be uneconomical. The linearizer used required a knowledge of the constants A, B and n and these were obtained by fitting the experimental velocity and voltage values to King's law using the least squares method.

Typical calibration curves for the single-wire and cross-wire probes used in this analysis are shown in figures (6-1) and (6-2). From the curves the following proportionality constants are obtained.

Single wires : 4.09 $\frac{m/s}{volt}$

Cross wire : 6.74 $\frac{m/s}{volt}$

The hot-wire anemometers were checked for linearity before and after each experimental run.

6.2 Directional Sensitivity of the Hot-wire X-Array Probe

The directional sensitivity of the sensors on the X array probe was measured at various mean-stream velocities for the angle to the mean stream at which the sensor was to be used. To obtain the angle between the sensors and the mean-stream velocity the following procedure was followed.

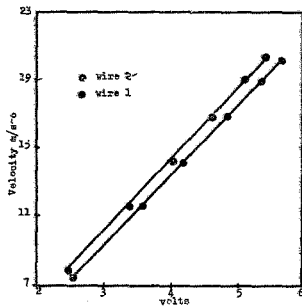


Fig. (C-1) Calibration Curve for Cross-wire Probe

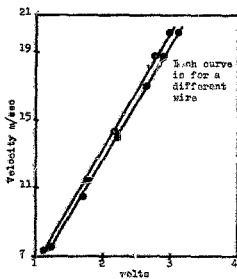


Fig. (C-2) Calibration Curve for Single Wire Probe

The sensor was rotated to obtain the maximum voltage on one of its wires and then rotated in the opposite direction to find the maximum voltage on the other wire. Assuming the wires to be symmetrical about the mean flow the required angle is half that between the two sensors. The directional sensitivity coefficient is then obtained from the equation:

$$U^2(\alpha) = U(0)^2(\sin^2 \alpha + k^2 \cos^2 \alpha)$$

where

$U(\alpha)$ = velocity with wire at angle α

$U(0)$ = velocity with wire perpendicular to the mean flow

k = directional sensitivity coefficient

α = angle between the sensor and the mean-velocity direction

The values of k obtained were unsatisfactory as the consistency was poor when one experiment was compared to the next. A short analysis was conducted to see the effect of various angles α and coefficients k on the root-mean-square values of the velocity fluctuations and also on the values of the Reynolds shear stress.

A particular section of tape recorded data was chosen and various values of k and α inserted into the above equation. Root-mean-square values of the resulting u and v velocity fluctuations as well as the Reynolds shear stress were calculated. The results are shown in Table (6-1).

The results show the following trends:

- i) varying only α does not introduce much error.
- ii) varying the values of k for the two sensors (i.e. different values for each sensor) introduces large variations in the values of the velocity fluctuations. The Reynolds shear stress is not affected to the same degree.

Because the values of k_1 and k_2 depend directly on the values of a_1 and a_2 , it is essential to measure a_1 and a_2 accurately. The experimental method used was not accurate enough and hence the cosine law was assumed when separating the u and v velocity fluctuations.

a_1^2	a_2^2	k_1	k_2	$\frac{\overline{u^2}}{\overline{v^2}}$ Arbitrary Units			$\overline{uv}$
47	47	0.00	0.00	36	25		-15
45	45	0.00	0.00	38	23		-15
55	39	0.00	0.00	40	21		-13
47	47	0.32	0.37	32	36		-16
45	45	0.32	0.37	34	34		-16

Table (6-1)
Results of Directional Sensitivity Analysis

6.3 Measurements in the Free Shear Mixing Layer

6.3.1 General

The results are presented in three sections:

- i) Results of a single-wire traverse 30 cm. downstream of the splitter plate.
- ii) Results of an X-ray traverse across the same section.
- iii) Zone results obtained by separating the conventional single-wire into turbulent and non-turbulent zones and then analysing each zone separately.

As an initial check, a sine wave was generated using a sine wave generator. It was tape recorded, digitised and processed using the digital system available. The frequency of the sine wave was not a harmonic of the total sample length.

Fig. (6-3) shows the periodogram which gives good resolution although smaller peaks are evident due to leakage. Fig. (6-4) shows the amplitude probability density function which is of the correct form. In Fig. (6-5) the first few cycles of the autocorrelation are plotted. The irregular peaks are a result of the frequency not being a harmonic of the total sample length, and that the Calcomp plotter joins the points on the graph by straight lines.

Only eight samples of 2048 points each were required to satisfy the program statistical check. At that stage the students "t" 90% confidence interval for the mean standard deviation was only 0.02% either side of the mean standard deviation. The signal to noise ratio obtained from the periodogram is approximately 60:1.

The results were written onto magnetic tape and later re-read and plotted using the Calcomp plotter. After smoothing, the periodograms and autocorrelations were decimated and 512 points plotted. This procedure gave adequate detail in the final results.

One concludes that the digital system gives good results on a single sine wave, and performs as it has been intended to work.

6.3.2 Single Hot-wire Traverse 30cms Downstream of the Splitter Plate

At each position marked in Fig. (6-6) a section of hot-wire anemometer signal was tape recorded and digitised at about 32 500 samples per second. Thirty sections each consisting of 2048 points were analysed for each position. The main analysis required 14 minutes of computer time. After averaging over the 30 samples, the Students "t" test on the standard deviation of the samples indicated that the 90% confidence interval was about 5% either side of the mean standard deviation.

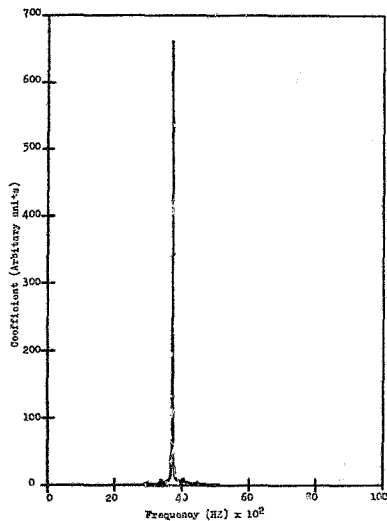


FIG. (6-3) Periodogram of the Analogue Sine Wave

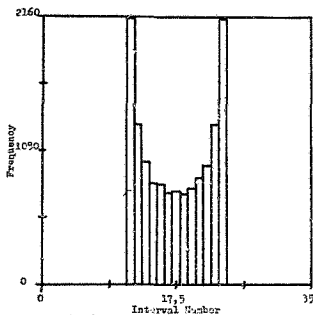


Fig. (6-4) Amplitude Probability Density Function of Sine Wave

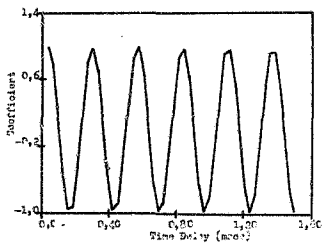


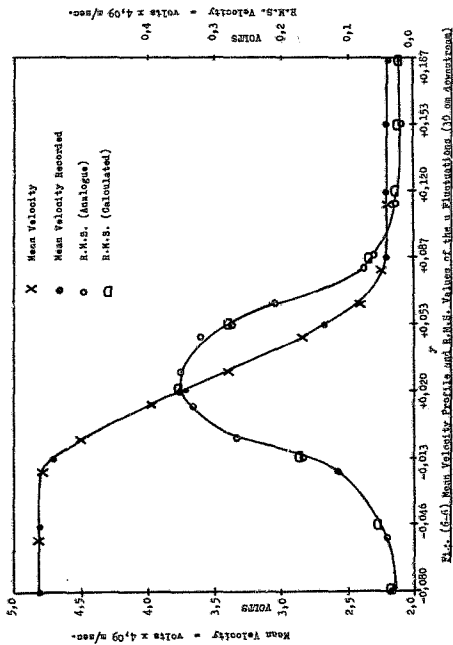
Fig. (6-5) Autocorrelation of Analog Sine Wave

The position across the shear layer was rendered dimensionless by using the variable y where $y = \frac{\Delta x}{30}$, $y = 0$ where the mean velocity is half the value of the two mean-stream velocities. (Δx in cms and 30 represents 30 cms downstream of the splitter plate).

Analogue measurements of the mean velocity and r.m.s. values of the longitudinal velocity fluctuations are shown in Table (6-2). Superimposed on the r.m.s. profile of Fig. (6-6) are scaled values of the r.m.s. values calculated from the digital records on magnetic tape. The two curves are close enough to show that there has been no change in the anemometer signal during the tape recording and digitising process.

From the recorded signals and using the digital methods previously described, the periodograms at various points across the shear layer were calculated. These are shown in Fig. (6-7). Interesting points to note are:

- i) On the edges of the mixing layer, particularly on the low velocity side, there are peaks in the periodograms occurring at frequencies of about 5.5 KHz, 6.7 KHz and 10 KHz. Peaks at these high frequencies are most unlikely in this turbulent flow. Using a wave analyser and a microphone to check for fan noise or vibration did not help in identifying the frequencies. The peaks were found to persist in the cross-wire results and hence are probably frequencies introduced by the digitiser. They are of a low intensity and only appear on the low intensity edges of the turbulent mixing layer. They could be removed by digital filtering techniques.
- ii) The periodograms calculated in the region where the mean velocity is flat but the r.m.s. values are still changing have a low



Position y	File No.	u^2 Arbitrary	$\sqrt{u^2}$ Units	$\sqrt{u^2}$ Volts (Scaled)	$\sqrt{u^2}$ Ana- logue Volts	Mean Volts
-0,080	18	$9,1 \times 10^2$	30,1	0,033	0,031	4,82
-0,046	19	$22,3 \times 10^2$	47,2	0,052	0,053	4,82
-0,013	20	$237,3 \times 10^2$	154,0	0,171	0,167	4,69
+0,020	21	$10,1 \times 10^4$	317,2	0,352	0,352	3,72
+0,053	22	$6,3 \times 10^4$	251,2	0,279	0,271	2,67
+0,087	23	$33,8 \times 10^2$	58,1	0,065	0,062	2,22
+0,120	24	$8,1 \times 10^2$	28,3	0,031	0,028	2,20
+0,153	25	$4,8 \times 10^2$	22,0	0,024	0,022	2,20

Table (6-2)
R.M.S. and Mean Velocity Profiles 30 cm Downstream

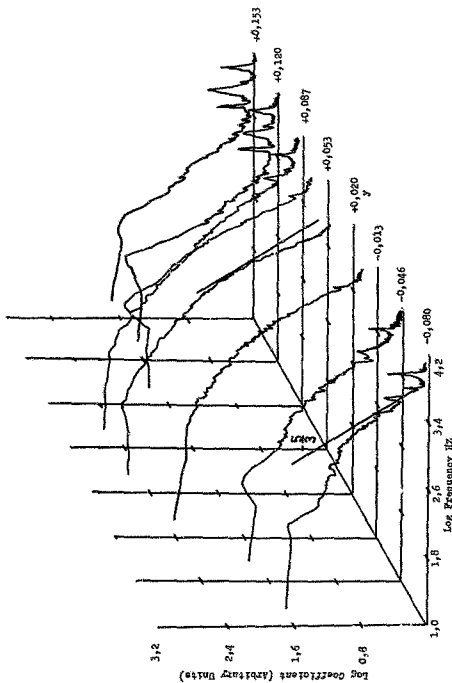


FIG. (6-7) Log-Log Plot of Periodogram (Single Wave)

frequency bulge. This is in the most intermittent region of the mixing layer and could give an indication of the frequency of the turbulent bursts passing the probe.

- 111) The periodograms all flatten out at high frequencies.

This is the noise level introduced by the digitisation process and the computational error in the computer programs. The signal was filtered at 10KHz using analogue methods, and playing the signal onto an oscilloscope showed very little frequency content above 4KHz.

The autocorrelations calculated as the inverse Fourier transform of the periodograms are shown in Fig. (6-8). A parabola of the form

$$y = 1 - \frac{x^2}{2\lambda^2}$$

where y = correlation coefficient

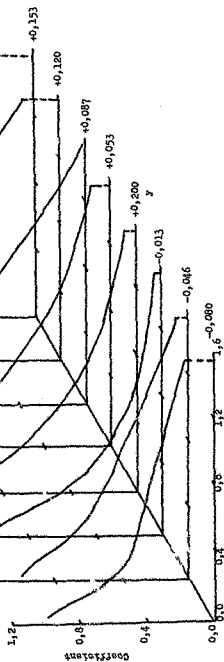
x = time delay

was fitted to the first few points and the Taylor's microscale at the various positions calculated (see Table (6-3) and Fig. (6-9)). The Taylor's microscale at the centre of the shear layer compares favourably with that obtained by Wignanski and Fiedler (1970).

Values for the Kolmogorov length scale were estimated using equation (3-9) and the values of the Taylor's microscale already calculated. These are shown in Table (6-3).

The Reynolds number based on the longitudinal microscale and the r.m.s. values of the u velocity fluctuations are shown in Table (6-3).

From the shapes of the autocorrelation curves the flow on the edges of the mixing region has a different structure to that in the mixing region. The values of $\lambda_f \times U$ in Table (6-3) show a reasonably



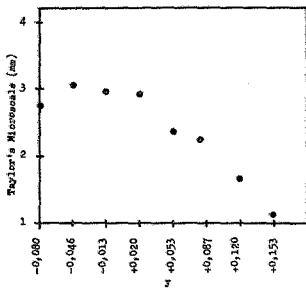


Fig. (5-9) Taylor's Microscale Profile (30 cm downstream)

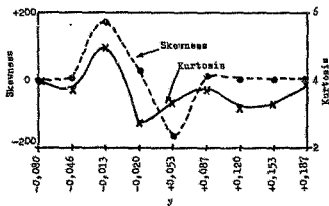


Fig. (5-10) Skewness and Kurtosis Profiles (30 cm downstream)

Position y	λ msec	Mean Velocity U m/sec	λu mm	A_T msec	η mm	Re_A	$A_T u$ mm
-0.060	0.140	19.7	2.76	0.699	0.210	44	13.8
-0.046	0.155	19.7	3.05	0.632	0.180	76	12.5
-0.013	0.155	19.2	2.98	0.495	0.087	285	8.7
+0.020	0.186	15.2	2.92	0.576	0.067	494	8.8
+0.053	0.217	10.9	2.36	0.717	0.068	317	7.8
+0.087	0.248	9.1	2.26	0.751	0.140	71	6.8
+0.120	0.123	9.0	1.67	1.033	0.170	25	9.3
+0.153	0.155	9.0	1.12	1.241	0.160	13	11.2

Table (E-3)

Table of the Following Turbulence Measurements

λ = Taylor's microscale

A_T = Taylor's macroscale

η = Kolmogorov length scale

constant value in the mixing region, with those on the edges increasing. A similar structural difference was observed by Wignanski and Fiedler (1970). A similar profile was observed in the cross-wire traverse (Fig. (6-18) and Fig. (6-19)).

From the recorded signals, amplitude probability density functions were obtained at the various points. The profile is shown in Fig. (6-11). Using this data the skewness and kurtosis profiles across the shear layer were calculated, where

$$\text{skewness} = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^3 f_i}{\left(\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 f_i \right)^{3/2}}$$

$$\text{kurtosis} = \frac{\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^4 f_i}{\left(\frac{1}{N} \sum_{i=1}^N (x_i - \bar{x})^2 f_i \right)^2}$$

and

N = number of intervals.

x_i = value of the midpoint of the interval.

$\bar{x}$ = mean value.

f_i = number of values in a particular range.

In this analysis $N = 35$. The various profiles are shown in Fig. (6-10). In the shear layer studied the kurtosis profile exhibited a similar shape to that obtained by Wignanski and Fiedler (1970). The different flow conditions at the edge of the shear layer probably account for the differences. Unusually large values of the skewness are obtained on each side of the shear layer where the mean-velocity profile flattens out. The profile is different to that of Wignanski and Fiedler (1970).

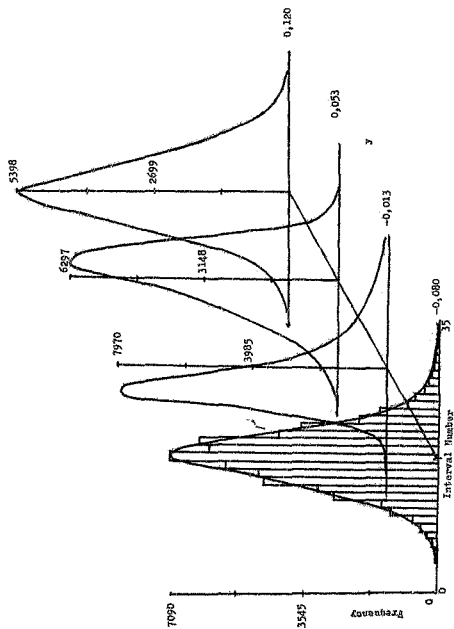


Fig. (c-11) Amplitude Probability Density Function (Single Wire)

Position y	Skewness	Kurtosis
-0,080	5,7	4,0
-0,046	16,1	3,7
-0,013	181,6	5,0
+0,020	34,2	2,8
+0,053	-174,4	3,4
+0,087	17,0	3,7
+0,120	2,5	3,2
+0,153	1,4	3,3
+0,187	5,0	3,9

Table (6-4)Table of Skewness and Kurtosis Values

6.3.3 Cross-wire Traverse 30 cms Downstream of the Splitter Plate

The signals from both wires were tape recorded and digitised at 32.5 KHz.

It was possible to obtain sections of both signals starting at the same time by means of tape recording a clock signal, that activated the digitiser, onto the third channel of the tape recorder.

It was found that the digitiser would sometimes write incorrectly onto the magnetic tape. Once the fault began it continued until the digitising stopped. By monitoring the r.m.s. values of the digitised samples it was possible to find 20 consecutive blocks of unaffected data at only four of the measured positions. With so few points available it was not possible to obtain a good profile across the shear layer.

Figure (5-12) compares the r.m.s. values of the two anemometer signals obtained by analogue measurement and those from the digital analysis. The scaling constants for the two signals differ by only 2%.

(Anemometer 1 = 1185, Anemometer 2 = 1214.) These results are also shown in tables (5-5) and (5-6). From these results it can be deduced that the signals considered were not part of a faulty digitising run.

Using a computer program, which separates the u- and v- velocity components from the cross-wire signals according to the equation (3-4), the signals were processed assuming the cosine law
i.e.

$$\begin{aligned} k_1 &= k_2 = D && \text{(directional sensitivity)} \\ \alpha_1 &= \alpha_2 = 45^\circ && \text{(angle the probe makes with the mean-stream velocity)} \end{aligned}$$

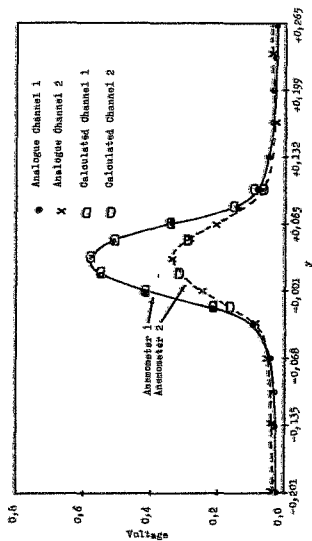


Fig. (6-12) R.H.S. Values, μ m, from Two Wires of X-ray Probe

Position y	r.m.s. Anem. 1 volts	r.m.s. Anem. 2 volts	Mean Vel. Volts
-0,201	0,026	0,034	2,617
-0,135	0,028	0,035	2,658
-0,101	0,033		2,655
-0,068	0,046	0,053	2,670
-0,035	0,092	0,090	2,695
-0,016	0,215	0,150	2,670
-0,001	0,420	0,250	2,510
+0,015	0,550	0,320	2,250
+0,031	0,580	0,340	1,950
+0,049	0,510	0,300	1,655
+0,065	0,340	0,205	1,420
+0,082	0,140		1,320
+0,099	0,080	0,071	1,310
+0,132	0,049		1,298
+0,165	0,039	0,032	1,295
+0,199	0,032		1,295
+0,232	0,025	0,027	1,297

Table (6-5)

Analogue Measurements Using X-Array Probe

Position y	r.m.s. Anem. 1 volts	Digital r.m.s. value volts	r.m.s. Anem. 2 volts	Digital r.m.s. value volts
-0,018	0,215	0,214	0,158	0,155
+0,015	0,550	0,551	0,318	0,319
+0,039	0,510	0,515	0,298	0,295
+0,099	0,060	0,080	0,071	0,071

Table (6-6)

Analogue and Digital r.m.s. Measurements

An attempt was made to measure the u- and v-velocity fluctuations using the simple analogue circuit described in Appendix III.

Table (6-7) gives the analogue measurements of the u- and v- velocity fluctuations while Table (6-8) shows the results obtained from the digital analysis. The very different scaling factors for the u and v fluctuations and the resulting bad fit of the v fluctuations would indicate a fault in the analogue measure of the v fluctuations. The results are plotted in Fig. (6-13).

Figs. (6-14) and (6-15) show the periodograms of the u- and v- velocity fluctuations at the four points considered. Peaks at frequencies similar to those found in the single-wire measurements are also noticeable here. The curves show a higher low frequency content for the u fluctuations.

The Reynolds shear stress was calculated from the digital records and the values obtained used to check Phillips'(1967) theory with satisfactory results. Shear stress values are given in Table (6-9).

Position y	Analogue $\sqrt{u^2}$ volts	Analogue $\sqrt{v^2}$ volts
-0,214	0,023	0,019
-0,147	0,025	0,021
-0,081	0,033	0,025
-0,047	0,047	0,037
-0,310	0,055	0,050
-0,140	0,150	0,082
+0,023	0,290	0,141
+0,016	0,360	0,220
+0,036	0,370	0,280
+0,053	0,320	0,260
+0,069	0,200	0,190
+0,086	0,080	0,104
+0,103	0,049	0,062
+0,135	0,029	0,036
+0,219	0,019	0,024
+0,286	0,017	0,022
+0,319	0,016	0,020

Tab. 2 (6-7)Analogue Measurements of u- and v-Velocity Fluctuations.

Position y	File No.	Digital $\sqrt{\frac{u^2}{T}}$ volts	Analogue $\sqrt{\frac{u^2}{T}}$ volts	Digital $\sqrt{\frac{v^2}{T}}$ volts	Analogue $\sqrt{\frac{v^2}{T}}$ volts
-0,018	5 & 6	0,130	0,131	0,151	0,075
+0,015	9 & 10	0,338	0,332	0,334	0,200
+0,039	13 & 14	0,332	0,332	0,299	0,090
+0,099	19 & 20	0,058	0,057	0,054	,079

Table (6-8)

Comparison of Analogue and Digital Values of the u- & v-Velocity Fluctuations

Position y	uv arbitrary units
-0,018	$-1,4 \times 10^4$
+0,015	$-1,4 \times 10^5$
+0,039	$-1,2 \times 10^5$
+0,099	$-6,0 \times 10^2$

Table (6-9)

Digital Values of the Reynolds Shear Stress

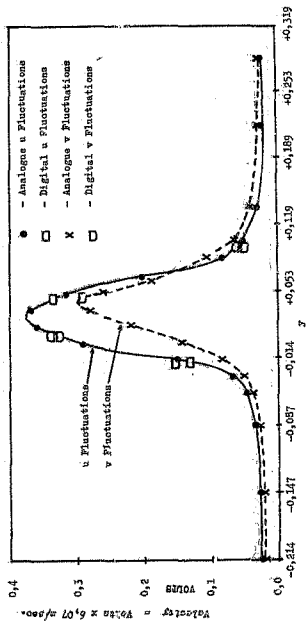


Fig. (2-13) Calculated and Analogous Values of u and v Velocity Fluctuations

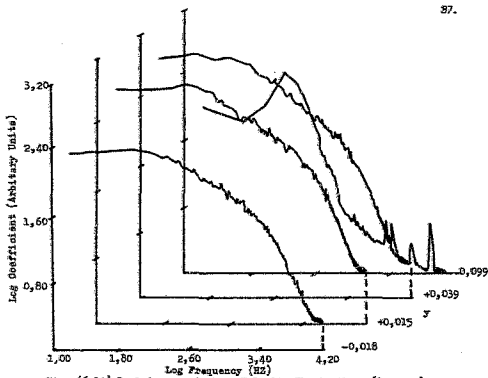


Fig. (6-14) Periodogram of the u Velocity Fluctuations (Log-Log)

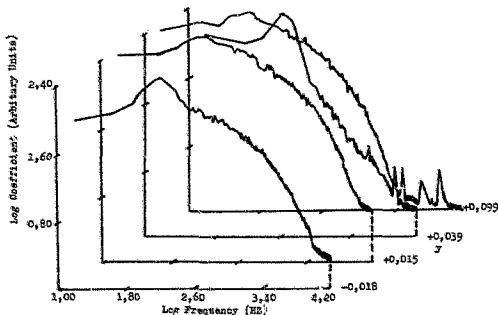


Fig. (6-15) Periodogram of the v Velocity Fluctuations (Log-Log)

The amplitude probability density functions are shown in Figs. (6-16) and (6-17). A similar profile was obtained for the u fluctuations using the single-wire probe.

In Figs. (6-18) and (6-19) the first section of the autocorrelation curves for the u - and v - velocity fluctuations are shown. The different shapes of the curves across the shear layer correspond to those obtained for the single-wire traverse (Fig. (6-8)). The v - velocity fluctuations reach zero correlation before the u - velocity fluctuations and also become negative. Both correlations have the form for turbulence consisting of two eddy ranges (Townsend (1956)). This form is also noticeable in the single-wire measurements.

The cross-correlation between the u - and v - velocity fluctuations is of interest as $R_{uv}(\tau = 0)$ is an estimate of the Reynolds shear stress. Comparing this value with that obtained by multiplying the fluctuations shows good agreement. See Table (7-5). The cross correlation, quadrature spectrum and co-spectrum at the four points considered are shown in Fig. (6-20), Fig. (6-21), Fig. (6-22) and Fig. (6-23).

The cross correlation is zero for time delays of 8msecs except for the high degree of periodicity at $y = +0.099$ on the low velocity side of the shear layer. The periodicity of approximately 180 Hz is of the same order as the low frequency bulges seen in all the calculated periodograms.

The co-spectrum and quadrature spectrum functions are not generally calculated in turbulence analysis and it is interesting to note the definite manner in which the correlation peaks at 180 Hz are shown.

In general the single-wire and X-ray probe results compare favourably, indicating a reliable analysis system.

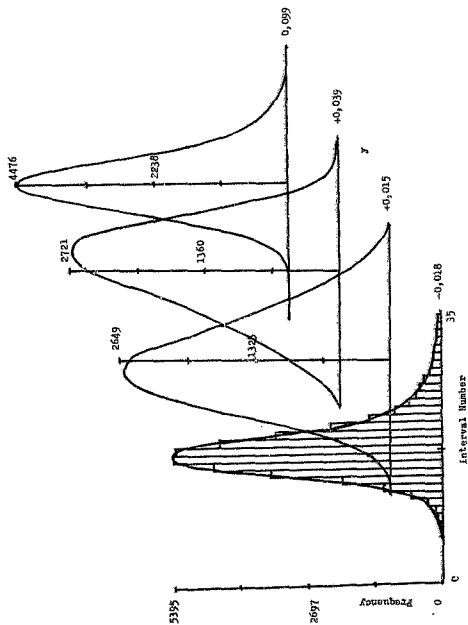


Fig. (6-16) Amplitude Probability Density Function of a Fluctuations

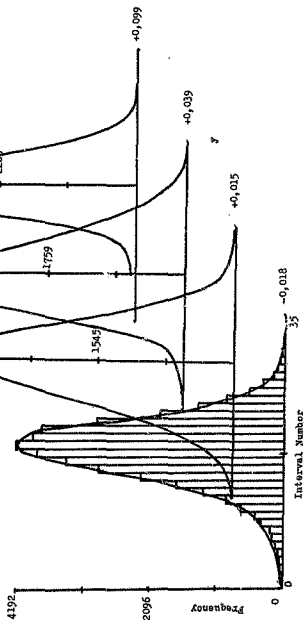


Fig. (6-17) Amplitude Probability Density Function of v Fluctuations

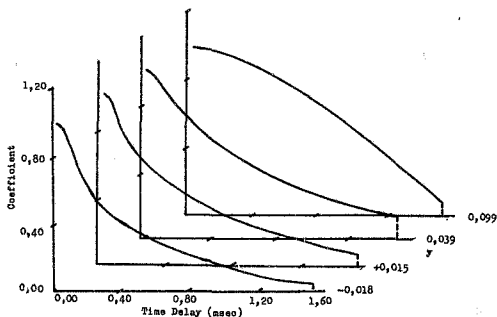


Fig. (6-18) Autocorrelations of the u-Velocity Fluctuations

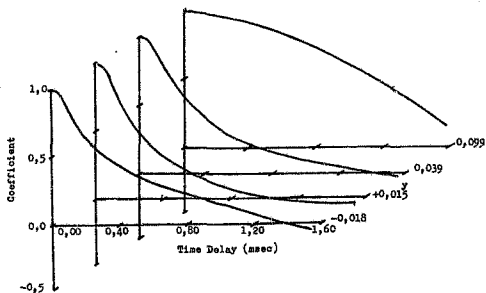


Fig. (6-19) Autocorrelations of the v-Velocity Fluctuations

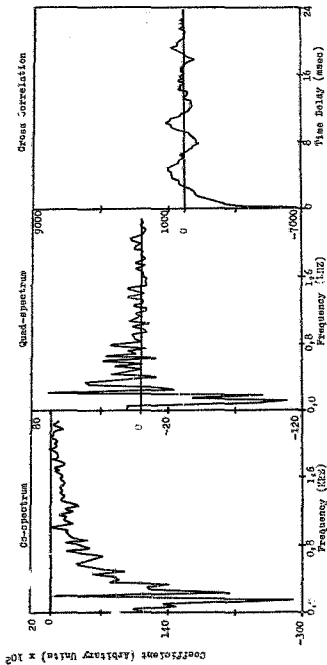


Fig. 1 (6-20). Os-spectrum, Quad-spectrum and Cross-correlation (S_{VV}) at $\gamma = -0.013$

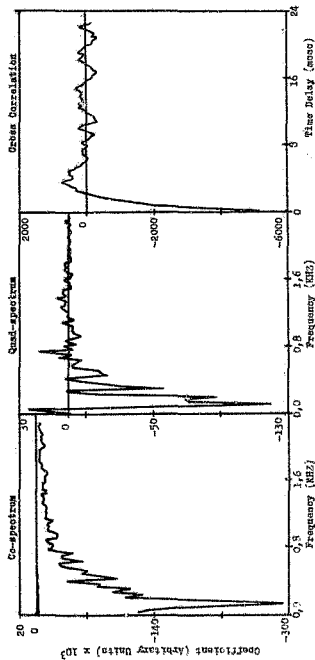


Fig. (5-21) Co-spectrum, Quad-spectrum and Cross Correlation (B_{xy}) at $\nu = 2.013$

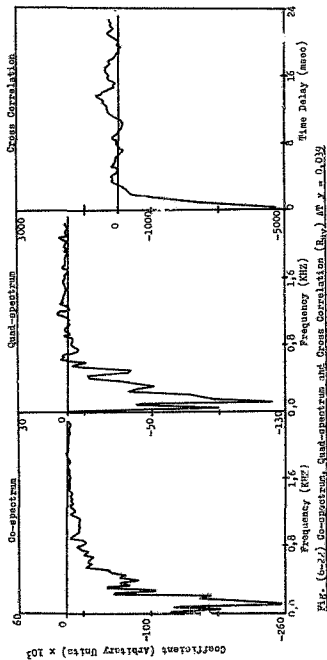


Fig. (6-27) Co-spectrum, Quad-spectrum and Cross Correlation (R_{xy}) AT $x = 0.039$

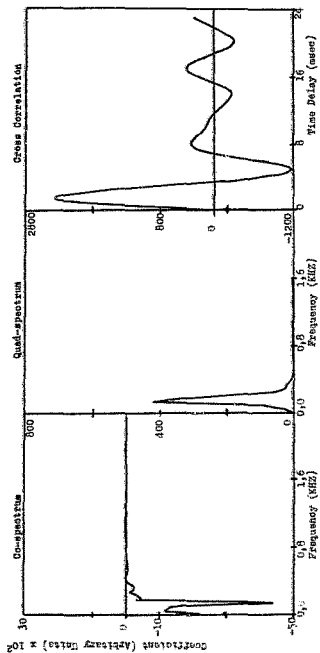


FIG. (5-33) Co-spectrum, Quad-spectrum and Cross Correlation ($\Delta t = 0.099$)

6.3.4 Zone Properties of the Turbulent Shear Layer 30cms Downstream of the Splitter Plate

A section of the single-wire probe signal measured in the intermittent region of the shear layer was tape-recorded and digitised at 20 KHz. The intermittency signal associated with the recorded velocity fluctuations was obtained using a slightly modified version of the intermittency program developed by Dreyer (1973). Basically the intermittency decision is made by considering both the frequency and amplitude of the velocity fluctuations. After deciding which points are turbulent, a data sequence is formed by adding the turbulent points one after the other.

The errors introduced will be a function of the number of points in a turbulent burst and the digitising rate used. In this analysis turbulent bursts ranged from two points to one hundred points. A reasonable average is twenty sample points. At the sampling rate of 20 KHz, this would give reasonable indications of the frequencies above 1 KHz, i.e. 3 on the log-log plot.

Comparing the periodograms to those from the single-wire traverse indicates that the general shape is probably correct. From Fig. (6-7) the slope of the periodogram does not increase above $\frac{5}{3}$ in the non-turbulent zone but does in the centre of the shear layer. The turbulent zone and non-turbulent zone results show the same trend. A better result could be obtained by selecting only long turbulent bursts or by smoothing the intermittency signal, i.e. when turbulent points are separated by only a few non-turbulent points, regard all the points as turbulent.

To obtain the 10 blocks of turbulent data it was necessary to process 44 blocks of the original signal. The program analyses 4 blocks of data at a time and gives the calculated intermittency factor for each set. The values (11 of them) varied between 0.17 and 0.28 with an average value of 0.23. Approximately 22 minutes of execution time on the computer were required to analyse the 44 blocks of data. This seems rather expensive as it is only 2.2 seconds of data real time.

The 10 blocks (making 5 samples of 2048 points) of turbulent data and the same quantity of non-turbulent data were analysed and compared with the results of the analysis of 10 sections (20 blocks) of the original signal.

A short section of the measured signal with the associated intermittency is shown in Fig. (6-24). It would seem that the intermittency signal leaves some turbulent sections in the non-turbulent zone.

In Fig. (6-25), Fig. (6-26) and Fig. (6-27) periodograms of the conventional signal, turbulent zone and non-turbulent zone are shown. The apparent large low frequency content in the non-turbulent zone and the almost isotropic form in the turbulent zone are the forms expected for the periodograms. All the periodograms pass through the Kolmogorov slope of $-\frac{5}{3}$ when plotted as a log-log plot. The high frequency content of the non-turbulent zone is partially due to the slightly incorrect intermittency determination.

The autocorrelation curves shown in Fig. (6-28), Fig. (6-29) and Fig. (6-30) also indicate some interesting results. That of the conventional signal has the classic form of two superimposed ranges of eddies. Separating the velocity fluctuations gives two curves showing only one range of eddies (Townsend (1956)). As expected, the large eddy scale is associated with the non-turbulent region while the

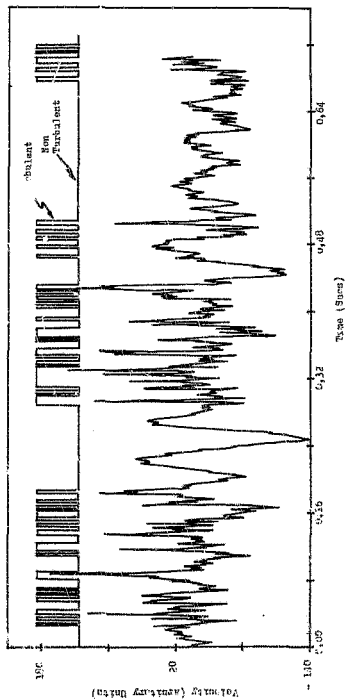


Fig. 13-21 Velocity Signal with Superimposed Intermittent Signal

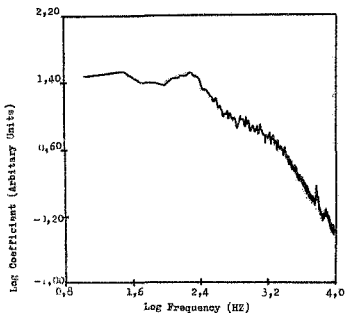


Fig. (6-25) Conventional Periodogram of a Fluctuation

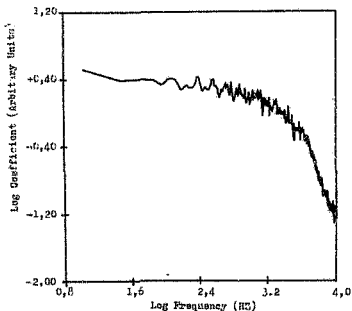


Fig. (6-26) Turbulent Zone Periodogram of the u Fluctuation

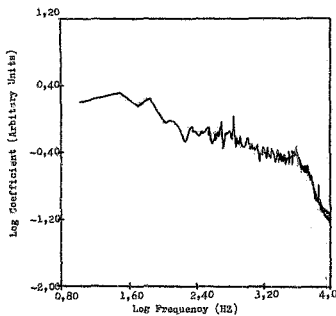


Fig. (6-27) Non-turbulent Zone Power Spectrogram of the u Fluctuations

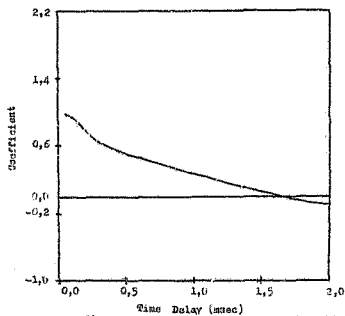


Fig. (6-28) Conventional Autocorrelation of the u Fluctuations

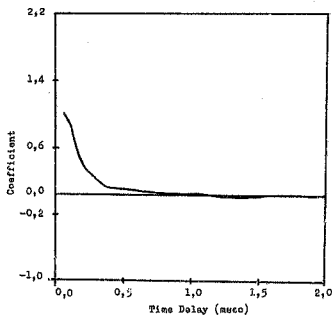


Fig. (6-29) Turbulent Zone Autocorrelation of the u fluctuations

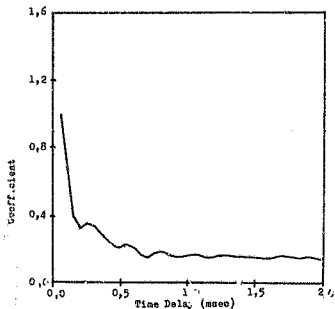


Fig. (6-30) Non-turbulent Zone Autocorrelation of the u fluctuations

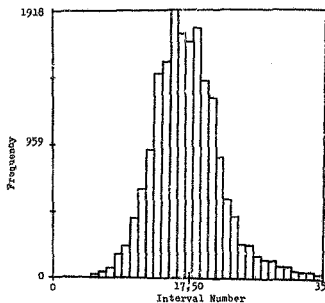


FIG. (6-11) Conventional Amplitude Probability Density Function

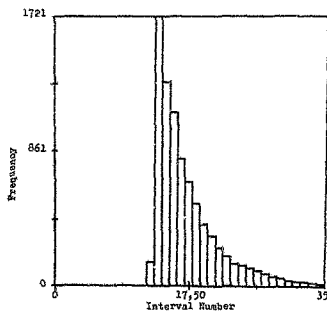


FIG. (6-12) Turbulent Zone Amplitude Probability Density Function

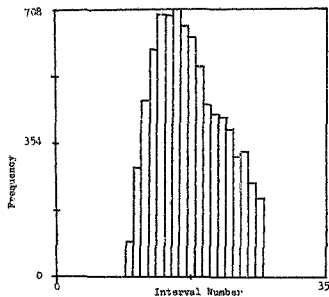


Fig. (6-13) Non-turbulent Zone Amplitude Probability Density Function

small eddy scale is associated with the turbulent region.

Studying the amplitude probability density functions in the various regions (Fig. (6-31), Fig. (6-32) and Fig. (6-33)) shows some interesting results. The fluctuations seem normally distributed in the conventional signal while on separating into zones, the turbulent zone velocity fluctuations are very skew with the large velocity fluctuations in the direction of mean flow. This agrees also with Fig. (6-24) which shows "turbulence" for the positive velocity fluctuations more than the negative fluctuations.

Although only a few samples were analysed, the results show the power of the analysis system and also verify conclusions drawn from the conventional measurements.

6.4 Check of Phillips Theory on the Maintenance of Reynolds Shear Stress

Phillips (1967) obtained the relationship

$$\frac{\partial \tau}{\partial x} = \nu_e \frac{\partial^2 U(x)}{\partial x^2}$$

where $\nu_e = A \rho u'^2 \theta = \text{"eddy" viscosity}$

$A = \text{constant}$

If the Reynolds stress vanishes at some point x_0 where the mean velocity gradient is zero, then the above expression can be integrated by parts.

$$\tau(x) = \nu_e(x) \frac{\partial U}{\partial x} - \int_{x_0}^x \frac{\partial \nu_e(x)}{\partial x} \frac{\partial U}{\partial x} dx$$

Assuming the second term to be small compared to the first

$$\tau(x) = v_E(x) \frac{\partial U}{\partial x}$$

$$\tau(x) = A_0 W^2 \frac{\partial U}{\partial x} \textcircled{B}$$

$$A_0 = \frac{\tau(x)}{W^2 \frac{\partial U}{\partial x}} \textcircled{B}$$

The convected integral time scale was obtained in the following manner. Using two hot-wire probes the cross correlation between the longitudinal fluctuations for various time delays and wire separations were measured. The maxima of the correlation curves at each point across the stream were used to fit an exponential of the form

$$y = \frac{e^{-ax} + e^{-bx}}{2}$$

where

y = correlation coefficient

x = time delay

Table (5-10) shows the following calculated quantities:

- i) Value of a in the exponential function.
- ii) Value of b in the exponential function.
- iii) Value of the time delay axis used as the limit in the integration of the fitted exponential function. (X)
- iv) The value of the correlation at the integration limit. (Y)
- v) The calculated convected integral time scale.

Fig. (5-34) shows a plot of the convected integral time scale and the mean velocity. Fig. (5-35) shows the shape of the cross correlations at one point in the shear layer. The slope of the velocity

Position y	a	b	X msec	Y	ϕ msec
0,071	0,09	0,41	58	0,0034	7,0
0,054	0,10	0,40	50	0,0034	6,2
0,039	0,04	0,60	125	0,0034	13,3
0,055	0,06	0,47	79	0,0035	9,7
0,005	0,07	0,41	73	0,0035	8,5
-0,012	0,08	0,59	67	0,0033	7,5

Table (6-10)

Calculations for the Connected Integral Time Scale

Position y	Actual Volts	Calculated Volts	Residual
-0,065	2,517	2,507	0,010
-0,048	2,519	2,547	-0,028
-0,032	2,516	2,520	-0,004
-0,015	2,474	2,441	0,033
+0,002	2,344	2,315	0,029
+0,018	2,160	2,181	-0,007
+0,035	2,025	2,040	-0,018
+0,052	1,885	1,908	-0,023
+0,068	1,783	1,798	-0,015
+0,085	1,753	1,727	0,026
+0,102	1,745	1,710	0,034
+0,118	1,738	1,765	-0,027

Table (6-11)

Least Squares Fit for the Mean Velocity Profile

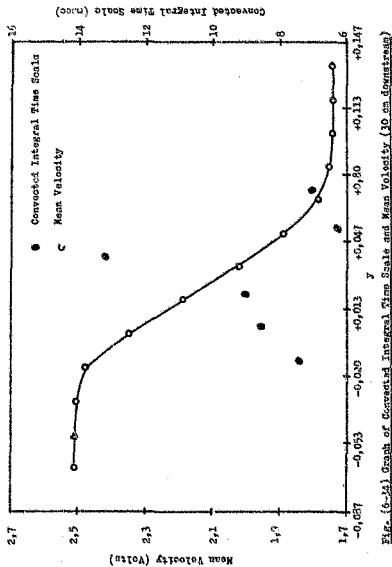


Fig. (a-14) Graph of Convected Integral Time Scale and Mean Velocity (30 cm downstream)

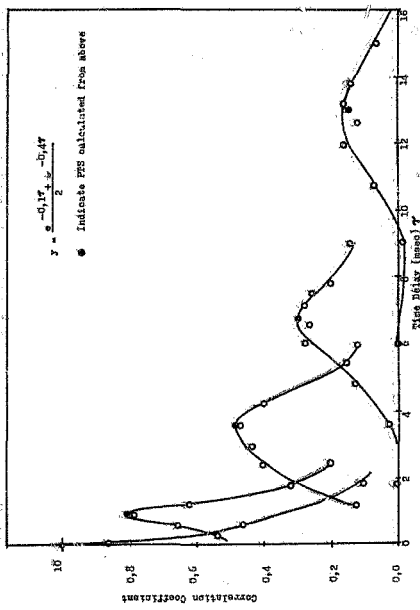


Fig. 1. Cross Correlations $R_{\text{int}}(\tau)$ for various wire derivations at $v = 0.031$.

profile was obtained by fitting an equation of the form

$$y = a + bx + cx^2 + dx^3$$

to the experimental mean velocity using the least squares method.

The following equations resulted

$$y = 2.23 - 0.28x - 0.021x^2 + 0.02x^3$$

$$\frac{dy}{dx} = -0.28 - 0.04x + 0.06x^2$$

$$\frac{d^2y}{dx^2} = -0.04 + 0.12x$$

The point of inflexion occurs at

$$x = 0.03 \text{ cms} \quad \text{i.e. } y = +0.015$$

The results are shown in Table (6-11). As the transverse-velocity fluctuations were not measured, the lateral fluctuations (y^2) were used. Table (6-12) shows the values used in the check. The value of the constant A obtained by Wagnanski and Fiedler (1970) was 0.2 which shows good agreement with the average value of 0.23 obtained in this analysis.

Previously Phillips (1969) applied the theory to the data of Davies et al (1963) for the mixing zone of a jet and obtained a value of $A = 0.17$. Balówin and Haberstrom (1966) calculated A for fully developed pipe flow of air at four different Reynolds numbers and obtained an average value of $A = 0.33$. Recently Atesman (1971) applied the theory to data from a nearly homogeneous shear flow and obtained $A = 0.55$.

Position y	T_{AV} Arbitrary Units	Slope of Mean Velocity	$\sqrt{v^2}$ Arbitrary Units	ϕ msec	A
-0,018	$-1,4 \times 10^4$	-0,19	$5,2 \times 10^4$	7,2	0,20
+0,015	$-1,4 \times 10^5$	-0,28	$2,6 \times 10^5$	9,0	0,21
+0,039	$-1,2 \times 10^5$	-0,26	$1,9 \times 10^5$	9,2	0,26

Table (6-12)

Calculations for Check on Phillips (1967) Theory

CHAPTER VII

7. DISCUSSION ON THE ERRORS

The errors involved result from the following steps in the analysis:

- i) The analogue instruments.
- ii) The digitising process.
- iii) Sampling of a continuous signal.
- iv) Analysing a limited time of signal.
- v) Computational errors.
- vi) Statistical errors.

An error in all the experimental measurements resulted from the fact that the mean-velocity direction was not perpendicular to the hot-wire probes. This was due to the expanding mixing region.

7.1 The Analogue System

All the analogue equipment is standard equipment and hence the limitations are specified. For all the units the frequency response curves are flat up to 10 KHz. The harmonic distortion of the analogue tape recorder used is less than 0.1%.

Some error will be introduced by the analogue filters in the auxiliary units. Both high pass and low pass filters have a roll-off of 6dB/octave. Some frequencies higher than 10 KHz will therefore be present.

7.2 The Digitising Process

The errors due to quantisation are discussed briefly by Bendat and Piersol (1966) and from a design point of view by Glisson, Black and Sage (1970).

Assuming the quantisation errors have a uniform probability density function over one scale unit, it can be shown that the errors will have a zero mean value and a standard deviation of approximately 0.29 units.

This can be regarded as the quantisation noise in the desired signal.

The digitiser used digitises an input voltage of ± 5 volts into ± 2048 units. Assuming only half the input voltage is utilized, the signal to noise ratio due to quantification will be

$$\begin{aligned} \text{SNR} &= \frac{2048}{0.29} = 7070 \\ &= 20 \log_{10} 7070 \text{ dB} \\ &= 18 \text{ dB} \end{aligned}$$

7.3 Sampling Errors

It can be shown (Glisson, Black and Sage (1970)) that if the signal has been sampled at twice the Nyquist Frequency and the signal is band limited, no errors are introduced. Aliasing errors, discussed in Section 2.3 result if these conditions are not met.

In this analysis the analogue filters were set at 70 KHz. Due to the digitising rate of about 32 KHz and the filter roll-off of 6 dB/octave, there will be small aliasing errors present. Filtering the signal and feeding it to an oscilloscope showed very little signal above 4 KHz. The aliasing errors should be minimal.

A sample of data was taken from the cross-wire analysis at $y = +0.039$ and processed by program NLBSMP (see Appendix II). The results

- (a) = 305
- (b) = 91
- (c) = 102
- (d) = 204

show that no aliasing should be present. Only $y = 0.039$ was chosen as it is in the centre of the shear layer and will contain the highest frequencies. All the signals were digitised at the same rate.

7.4 Analysing a Limited Time of Signal

As a result of truncating the signal length the resolution of the resulting spectrum is limited. If a total time of T seconds is analysed, the maximum resolution that can be obtained is $\frac{1}{T}$ Hz. The errors due to leakage (section 2.4) are proportional to the spectral resolution.

It can be shown (Glisson, Black and Sage (1970)) that when a sinusoid is not one of the frequencies calculated in the associated periodogram, the signal to noise ratio can drop by about 4 dB. This decrease must be taken into account when calculating the minimum input signal to noise ratio of the digitiser.

7.5 Computational Errors

The largest computational errors will be in the Fast Fourier transform routine and only this aspect will be considered. Various workers, Welch (1969), Glisson, Black and Sage (1970) and Graefe (1970) have discussed the computational errors obtained when using fixed point arithmetic in a digital computer. This aspect is important in the design of special purpose computers.

Using the lower error bound (i.e. no overflow occurs during the calculation) gives a result that increases at the rate observed for floating point calculations.

$$\frac{\text{r.m.s. (result)}}{\text{r.m.s. (error)}} = \frac{2^k}{0.3(M-2.5)1/2}$$

where

M = power of 2 i.e. $N = 2^M$

2 = number of bits used by the computer to represent the values i.e. 8 bits plus a sign.

The digitiser used for this analysis represents the data with 11 bits plus one for a sign. Assuming the above relationship, a pessimistic signal to noise ratio is obtained

$$\begin{aligned} \text{SNR} &= \frac{2048}{0,3 * \sqrt{8,5}} \\ &= 2345 \\ &= 20 \log_{10} 2345 \text{ dB} \\ &= 66 \text{ dB} \end{aligned}$$

It has been shown, Oppenheim and Weinstein (1969) that the errors in the Fourier transform and its inverse are not independent and hence a smaller ratio would be expected for the correlation functions. The computer uses floating point arithmetic with D = 15 and hence the errors should be considerably smaller than those calculated above.

7.6 Characterising the Digital System

Regarding the digitiser and digital analysis programs as a spectrum analyser one can characterise it by specifying the following quantities:

7.6.1 Resolution

The resolution is calculated as $\frac{1}{T}$ HZ where T is the sample length (in seconds) analysed. In all the analyses performed a sample size of 2048 points was used. For the single and cross-wire analyses the sampling rate was 32,5 KHz giving

$$\text{Resolution} = \frac{32500}{2048} = 15,85 \text{ HZ}$$

The signal used for zone averages was digitised at 20 KHz giving a resolution of 9,75 HZ.

7.6.2 Dynamic Range

The dynamic range is a rough measure of the ability of the digitiser to pass relative amplitude information contained in the signal. It can be related to the number of binary bits used by the digitiser

$$DR = 5x + 4 \text{ dB}$$

where

x = number of binary bits used by the digitiser to represent the data.

For this system

$$DR = 5 \times 11 + 4 = 70 \text{ dB}$$

7.6.3 The Minimum Detectable Signal (MDS)

The MDS is the smallest signal to noise ratio at the input that leads to a reliable detection at the analyser output. Generally one requires an output signal to noise ratio of about 10 dB (Glisson, Black and Sage (1970)).

Due to leakage, the input signal to noise ratio will drop by a small amount (≈ 4 dB) and this must be accounted for.

The processing gain (PG) of the analyser gives an indication of the signal enhancement obtained in the analyser

$$PG = 20 \log \left(\frac{SNR_{out}}{SNR_{in}} \right) \text{ dB}$$

and it can be related to the signal band width and length of the sample processed.

$$PG = 10 \log_{10} (Tb)$$

where T = total sample time

B = bandwidth of the signal

In this analysis

$$PG = 10 \log_{10} (10^4 \times \frac{1}{15.85})$$

$$PG = 28 \text{ dB}$$

Assuming the minimum output of 10 dB, the minimum possible signal to noise ratio at the input is

$$PG = 20 \log[\text{SNR out}] - 20 \log[\text{SNR in}]$$

$$PG = 10 + 4 - \text{MDS}$$

(10 = minimum output, 4 to compensate for leakage)

$$\therefore \text{MDS} = -14 \text{ dB}$$

A rough check is obtained by considering the sine wave analysis (Section 6.3.1). The signal to noise ratio there is about 60 i.e. 35 dB. The corresponding MDS would be 11 dB which is much larger than the minimum of -14 dB.

If the input noise is white noise, the dynamic range and processing gain should be about the same. If the noise is 1/f noise, then the dynamic range should be two or three times the processing gain. In this case

$$DR = 70 \text{ dB}$$

$$PG = 28 \text{ dB}$$

7.7 The Statistical Accuracy of the Results

The computational accuracy of the results has been dealt with in the previous section. An indication of the statistical accuracy i.e. the accuracy 4. to using a limited section of signal and then averaging a finite number of sections, should also be obtained.

As a result of the central limit theorem, the mean value of the standard deviation of each section (2048 points) analysed, should follow the Students "t" distribution and hence this was used to obtain a confidence interval for the mean value. A more detailed description of the test can be found in Section (2.7).

Table (7-1) shows the results of the test for the single-wire traverse. In this analysis the 90% confidence interval was used, so that Table (7-1) and Table (7-2) show the final value of the standard deviation, the percentage each side of the mean that the confidence interval extends and the value of the confidence interval. The 90% confidence interval is less than 5% of the mean value at the centre of the shear layer and increases to about 10% at the low intensity edges of the layer. For the present analysis this accuracy is acceptable.

Table (7-2) shows the results obtained for the X-array probe traverse across the shear layer. The percentages are slightly higher than those for the single-wire analysis but this is probably due to the fact that only 40 instead of 60 blocks of data were analysed. The digitiser was not operating properly when the records at $y = -0.001$, $y = +0.032$, $y = +0.065$ and $y = +0.082$ were digitised. This is shown up well by the far higher percentages i.e. 38% and 44% for the u fluctuations and 39.7% and 45.4% for the v -velocity fluctuations. The other percentages give an acceptable statistical accuracy.

Another check was made on the results by using the following relationships involving the autocorrelation and cross-correlation:

- i) $R_{uu}(\tau = 0) = \overline{u^2}$
- ii) $R_{uv}(\tau = 0) = \overline{uv}$

See Tables (7-3) to (7-5).

y	% of mean	Mean St. Dev.	90% Confidence Interval
-0,080	4,3	27,4	1,2
-0,047	3,3	45,6	1,5
-0,010	4,8	152,3	7,4
+0,020	2,4	315,6	7,4
+0,053	2,9	250,1	7,2
+0,087	4,6	57,0	2,6
+0,120	5,8	26,3	1,5
+0,153	8,8	19,1	1,7
+0,187	8,8	19,7	1,7

Table (7-1)

Results of Statistical Test on the Data from the Single-Wire Traverse

Number of blocks analysed = 60

y	u velocity fluctuations			v velocity fluctuations		
	%	Mean St. Deviation	90% Confidence Int.	%	Mean St. Deviation	90% Confidence Int.
-0,035	5,8	81,4	4,7	5,7	135,9	7,7
-0,018	8,1	216,9	17,5	3,6	224,7	8,2
-0,001	38,1	5323	2412	38,7	6281	2429
+0,015	3,6	570,4	20,7	2,67	491,4	13,2
+0,032	44,1	5283	2335	45,4	5194	2359
+0,049	3,5	551,9	19,3	2,3	434,6	9,9
+0,065	-	-	-	-	-	-
+0,082	-	-	-	-	-	-
+0,098	7,3	95,2	7,0	9,3	87,4	8,1

Table (7-2)

Results of the Statistical Test on the Cross-wire Data

Number of blocks analysed = 40

y	A	B	C	$\frac{ A-B }{B}$	$\frac{ B-C }{C}$	$\frac{ A-C }{C}$
	$R_u(\tau = 0)$	$\overline{u^2(40 \text{ Blocks})}$	$\overline{u^2(60 \text{ Blocks})}$	%	%	%
-0,018	$0,49 \times 10^5$	$0,51 \times 10^5$	$0,47 \times 10^5$	3,9	8,5	4,2
+0,015	$0,33 \times 10^5$	$0,33 \times 10^5$	$0,32 \times 10^5$	0,0	3,1	3,1
+0,049	$0,31 \times 10^5$	$0,31 \times 10^5$	$0,31 \times 10^5$	0,0	0,0	0,0
+0,099	$0,94 \times 10^4$	$0,94 \times 10^4$	$0,93 \times 10^4$	0,0	1,8	1,8

Table (7-3)

Comparison for u velocity component

y	A	B	C	$\frac{ A-B }{B}$	$\frac{ B-C }{C}$	$\frac{ A-C }{C}$
	$R_u(\tau = 0)$	$\overline{u^2(40 \text{ Blocks})}$	$\overline{u^2(60 \text{ Blocks})}$	%	%	%
-0,018	$0,51 \times 10^5$	$0,52 \times 10^5$	$0,52 \times 10^5$	1,9	0,0	1,9
+0,015	$0,24 \times 10^5$	$0,25 \times 10^5$	$0,26 \times 10^5$	4,0	3,9	7,7
+0,049	$0,19 \times 10^5$	$0,19 \times 10^5$	$0,19 \times 10^5$	0,0	0,0	0,0
+0,099	$0,80 \times 10^4$	$0,82 \times 10^4$	$0,68 \times 10^4$	2,4	20,0	18,0

Table (7-4)

Comparison for v velocity component

y	A	B	C	$\frac{ A-B }{B}$	$\frac{ B-C }{C}$	$\frac{ A-C }{C}$
	$R_{uv}(\tau = 0)$	$\overline{uv(40 \text{ Blocks})}$	$\overline{uv(60 \text{ Blocks})}$	%	%	%
-0,018	$-0,17 \times 10^5$	Not Calculated	-0,1	-	-	21,0
+0,015	$-0,14 \times 10^5$		$-0,14 \times 10^5$	-	-	0,0
+0,049	$-0,12 \times 10^5$		$-0,12 \times 10^5$	-	-	0,0
+0,099	$-0,65 \times 10^3$		$-0,60 \times 10^3$	-	-	8,3

Table (7-5)

Comparison for the Reynolds shear stress

The root-mean-square values of the signals are calculated before the data is modified, while the autocorrelation and cross correlation are the last quantities calculated. When calculating the auto- and cross-correlations, values for each section are averaged to give a final answer, while this is not the case when calculating the r.m.s. values. Comparing the two results will give an idea of the effects of averaging as well as the accuracy of the analysis procedure adopted.

It is interesting to note that the difference between the $R_u, R_v(r=0)$ and the $\bar{u}^2$ and $\bar{v}^2$ values does not exceed 5% (column $\frac{A-B}{B}$). The column $\frac{B-C}{C}$ shows that in some cases analysing 40 blocks of data is sufficient, whereas in others the r.m.s. value can change by 20%. At $y = +0.099$ one sees that analysing 40 blocks of data was sufficient for the u velocity fluctuations but not for the v fluctuations.

The results show that the statistical accuracy in the turbulent shear layer is good but decreases as one moves into the low intensity edges of the layer.

CHAPTER VIII

8. CONCLUSIONS

8.1 The Analysis System

i) A system for the digitising and digital analysis of analogue turbulent data has been completed. The digital programs are written in Fortran IV and are processed by the IBM 360/50 computer at the University of the Witwatersrand Computer Centre.

ii) A single time series can be analysed providing estimates of:

- a) The periodogram.
- b) The amplitude probability density function.
- c) The autocorrelation.

iii) Two signals can be analysed simultaneously to give the above mentioned properties for each time series as well as:

- a) The cross spectrum.
- b) The cross correlation.

Samples of either 1024, 2048, 4096 or 8192 values can be analysed and any number of samples averaged to provide the final answer.

iv) It is possible to separate the two signals from an X-array probe into the two velocity components. It is possible to produce turbulent and non-turbulent zone signals from the signal and its associated intermittency signals.

v) Comparing the root-mean-square values of the fluctuations and the value of the autocorrelation at $\tau = 0$ ($R_x(\tau = 0)$) gives values differing by 5%. The values of $R_x(\tau = 0)$ are obtained by averaging over 20 samples of 2048 points each.

- vi) Calculating the root mean square values of 30 samples of data and comparing it to $R_X(\tau = 0)$ calculated over 20 samples gives values differing by up to 20%. The Students "t" test on the mean value of the standard deviations of the samples showed a 90% confidence interval of at worst 9%, but generally 5% of the final mean value.
- vii) A short sensitivity analysis indicated the importance of correctly measuring the angles α_1 and α_2 between the hot-wire sensors and the mean-flow direction. Different values of the directional sensitivity coefficients k_1 and k_2 change the relative root-mean-square values of the u and v fluctuations considerably. The accuracy of the values of k_1 and k_2 depend primarily on the values of α_1 and α_2 .

8.2 Turbulence Results

- i) Using digital Fourier analysis it is possible to obtain turbulence quantities which are very difficult to measure using analogue methods. In this investigation estimates of the periodogram and autocorrelation of the turbulent and non-turbulent zones in the intermittent region of a free mixing layer were obtained.
- ii) The power spectra are of the expected form. The turbulent-zone periodogram is the classic turbulent-flow periodogram which passes through the Kolmogorov slope of $-\frac{5}{3}$. Indications are that the low frequency content is higher in the non-turbulent zone than in the turbulent zone.
- iii) The autocorrelation of the mixture (intermittent signal) indicates isotropic turbulence with two distinct eddy sizes (Townsend (1956)). From the zone calculations one sees that the large eddies are in the non-turbulent zone while the smaller eddy size is associated with the turbulent region. Both autocorrelations tail off as

expected for u velocity fluctuations.

- iv) The amplitude probability density function of the mixture shows the velocity fluctuations to be nearly normal. On separation there are two distinct distributions. Both are skew with their tails in the direction of mean flow.
- v) From the conventional measurements performed across the shear layer 30 cm downstream of the splitter plate, the following conclusions can be drawn:
 - a) Values of the Taylor microscale measured using the autocorrelation are similar to those obtained by Wignanski and Fiedler (1970). The average value across the shear layer is 1.4 times larger than that obtained by Leipman and Laufer (1947).
 - b) The single-wire traverse results show different flow structures as observed by Wignanski and Fiedler (1970). The different regions could also be deduced from the autocorrelation of the v -velocity fluctuations.
 - c) No linear relationship between the convected integral time scale and slope of the mean velocity profile was found. This supports the results of Wignanski and Fiedler (1970) but not those of Davies et al (1963).
 - d) Applying the available data to Phillips (1967) theory on the maintenance of Reynolds shear stresses gave an average value of the constant A to be 0.23. This compares favourably with the value of 0.20 obtained by Wignanski and Fiedler (1970).
 - e) The periodograms at all the points considered follow Kolmogorov's universal equilibrium theory over a short range of frequencies.

The Reynolds number based on the Taylor microscale and the root-mean-square value of the u -velocity fluctuations varies between 20 and 500. The results tend to support those of Bradshaw (1967).

8.3 Suggestions for Further Work

There are two important aspects of the work which should be investigated further.

- i) The accuracy of the result as a function of sample length analysed, and the number of samples averaged to give the final result.
- ii) The method of separating the intermittent signal into turbulent and non-turbulent zones must be improved to give better accuracy in the low frequency regions.

NOMENCLATURE

A	Constant in Phillips' theory and King's law.
A_n	Digital form of the complex Fourier coefficient.
A^i	Imaginary part of the Fourier coefficient.
A^r	Real part of the Fourier coefficient.
$A(f), B(f) \dots$	Continuous form of the complex Fourier coefficient.
$A^*(f), B^*(f)$	Complex conjugate of $A(f)$ and $B(f)$.
δ	Band width of the signal and King's law constant.
DR	Dynamic range.
F	Sampling frequency (twice the Nyquist frequency).
MDS	Minimum detectable signal.
N	Sample size.
PG	Processing gain.
$R_{xx}(\tau) = R_x(\tau)$	Autocorrelation
$R_{xy}(\tau)$	Cross-correlation function of the x and y series.
$S_{xx}(f) = S_x(f)$	Spectral density function.
$S_{xy}(f)$	Two-sided cross-spectral density function of the x and y series.
SNR	Signal to noise ratio.
T	Period of the Fourier series.
U, V, W	Mean velocity components.
U_{eff}	Effective mean velocity measured by the anemometer.
$X(z)$	Complex form of the continuous time series.
$X(kh)$	Complex form of the digitised time series.
a_n^x, b_n^y , etc.	Sine and Cosine terms of the n^{th} harmonic in Fourier series of x and y signals.

$a(f), b(f)$	Cosine and Sine terms in the Fourier series (continuous notation).
c, c_1, c_2	Instantaneous effective velocity fluctuations.
$f(r)$	Normalised longitudinal correlation.
$g(r)$	Normalised lateral correlation.
h	Time between samples.
i	$\sqrt{-1}$
k, k_1, k_2	Directional sensitivity coefficients of the hot-wire sensors.
k	Number of bits used by the digitiser.
n	Harmonics in the digital Fourier series $n = 0, 1, 2 \dots N$.
r	Separation between two sensors.
t	Student's "t" statistic.
u, v, w	Fluctuating velocity components.
$x(t)$	Continuous real time domain function.
$x(kh)$	Digitised real time domain function.
$y = \frac{\Delta x}{30}$	Dimensionless position across the shear layer $y = 0$ where $U = \frac{U_{fast} + U_{slow}}{2}$
$\alpha, \alpha_1, \alpha_2$	Angle between the hot-wire sensor and the mean stream velocity.
β	$90-\alpha$.
τ	Time delay
λ_f	Taylor dissipative length scale (microscale).
λ_g	Microscale obtained from the lateral correlation.
ν	Rate of viscous dissipation.
ν	Kinematic viscosity.
η	Kolmogorov length scale.
A, A_f, A_g	Macro or integral scale of turbulence.

$\rho_X(\tau)$

Normalised autocorrelation.

 τ_E

Microscale obtained from the autocorrelation.

 Δ_E

Macroscale obtained from the autocorrelation.

 $\rho_{XY}(\tau)$

Normalised cross correlation.

 (H)

Convected integral time scale.

dB

Decibels $20 \log_{10} \text{SNR}$ (SNR in volts) $(\quad)$

Average value of quantity in brackets.

REFERENCES

- ATESMAN, K.M., 'Phillips Hypothesis for Eddy Viscosity'. AIChE Journal, 17 no.5: 1256-1257, 1971.
- BALDWIN, L.V. and HABERSTRON, R.D., 'Test of Phillips Hypothesis for Eddy Viscosity in Pipe Flow'. AIChE Journal, 14 no.5: 825-26, 1968.
- BENDAT, J.S. and PIERSON, A.G., Measurement and Analysis of Random Data. Wiley & Sons : New York, 1966.
- BINGHAM, C., GODFREY, M.D and TUKEY, J.W., 'Modern Techniques of Power Spectrum Estimation'. IEEE Transactions on Audio and Electro-acoustics, AU-15 no. 2: 56-66, 1967.
- BLACKMAN, R.B. and TUKEY, J.W., The Measurement of Power Spectra. Dover Publications : New York, 1959.
- BRADSHAW, P., Conditions for the Existence of an Inertial Subrange in Turbulent Flow. National Physical Laboratory - NPL Aero Report 1220, 1967.
- CHAMPAGNE, F.H., 'Turbulence Measurements with Inclined Hot Wires'. Boeing Scientific Research Laboratories - Document D1-B2-0491, 1965.
- COOLEY, J.W., LEWIS, P.A. and WELCH, P.D., 'The Application of the Fast Fourier Transform Algorithm to the Estimation of Spectra and Cross Spectra'. Journal of Sound and Vibration, 12 no.3: 339 - 352, 1970.
- COOLEY, J.W. and TUKEY, J.W., 'An Algorithm for the Machine Calculation of Complex Fourier Series'. Math. Comput., 19: 297-301, 1965.
- DAVIES, P.O. and FISHER, M.J. and BARRATT, M., 'The characteristic of turbulence in the mixing region of a round jet'. Journal of Fluid Mechanics, 15 no.3: 327-367, 1963.

DISA. Instruction and Service Manual for Type 55070 Analog Correlator.

DISA Elektronik A/S, Herlev, Denmark, 1967.

DREYER, T.F., Turbulent Flow in the Mixing Region of Two Streams.

(M.Sc. Dissertation - not submitted yet).

DURRANI, T.S. and NIGHTINGALE, J.M., 'Data Windows for Digital Spectral Analysis'. Proceedings of the Institution of Electrical Engineers - Electronics and Power Control, 119 no.3: 343-352, 1972.

FRENKIEL, F.N. and KLEBANOFF, P.S. 'Higher Order Skewnesses in a Turbulent Flow', Department of the Navy - Translation 329, 486827, May 1966.

FRENKIEL, F.N. and KLEBANOFF, P.S., 'Statistical Properties of Velocity Derivatives in a Turbulent Field'. Journal of Fluid Mechanics, 48 no.1: 183-208, 1971.

GLISSON, T.H., BLACK, C.I. and SAGE, A.P., 'The Digital Computation of Discrete Spectra Using the Fast Fourier Transform'. IEEE Transactions on Audio and Electroacoustics, AU-18 no.3, 271-286, 1970.

GRAEFE, P.W.U., 'Fast Fourier Transform Programs for the EAI 640 Digital Computer'. National Research Council of Canada - NRC 11747, 1970.

HINZE, J.O., 'Turbulence: An Introduction to its Mechanism and Theory'. McGraw Hill Series in Mechanical Engineering, McGraw Hill: New York, 1959.

IEEE TRANSACTIONS ON AUDIO AND ELECTROACOUSTICS, Special Issue on the Fast Fourier transform. IEEE Transactions on Audio and Electroacoustics, AU-15, 1957.

IEEE TRANSACTIONS ON AUDIO AND ELECTROACOUSTICS, Special Issue on the Fast Fourier transform. IEEE Transactions on Audio and Electroacoustics, AU-17, 1969.

JENSON, J.O., 'A Survey of Methods for Measuring Correlation Functions'. DISA Information, 10: 10-14, 1970.

JEROME, F.E., GUITTON, D.E. and PATEL, R.P., 'Experimental Study of the Thermal Wake Interference between Closely Spaced Wires of an X-type Hot-wire Probe'. Aeronautical Quarterly, 12 no.2: 119-129, 1971.

JONES, R.H., 'A Re-appraisal of the Periodogram in Spectral Analysis'. Technometrics, 7 no.4: 531-542, 1965.

JORGENSEN, F.E., 'Directional Sensitivity of Wire and Fiber-Film Probes'. DISA Information, 11: 31-37, 1971.

KAPLAN, R.E. and LAUFER, J., 'The Intermittently Turbulent Region of the Boundary Layer'. Proceedings of the 12th International Congress of Applied Mechanics, ed. HETENYI, M. and VINCENTI, W.G., Springer-Verlag: Berlin: 236-245, 1969.

KIBENS, V., 'The Intermittent Region of a Turbulent Boundary Layer'. The John Hopkins University, Ph.D. Thesis, 1968.

KRAUSE, F.R., JONES, J.A., FISHER, M.J. and POOLEY, J.C., 'Digital Analysis of Random Data Records by Piecewise Accumulation of Time Averages'. NASA Technical Note. NASA TN D6073: 1970.

LIEPMAN, K.W. and LAUFER, J., 'Investigations of Free Turbulent Mixing'. National Advisory Committee for Aeronautics, Technical Note 1257, 1947.

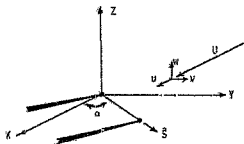
LIN, C.C., 'On Taylor's Hypothesis and the Acceleration Terms in the Navier-Stokes Equations'. Quarterly of Applied Mathematics, 10: 295-305, 1953.

- LIN, C.C. and REID, W.H., 'Turbulent Flow, Theoretical Aspects'.
Handbuch der Physik VIII/2, ed. FLUEGGE, S.,
Springer-Verlag: Berlin: 438-523, 1963.
- MALING, G.C., MORREY, W.T.(Jr), and LANG, M.W. 'Digital Determination
of Third-Octave and Full-Octave Spectra of Acoustical Noise'.
IEEE Transactions on Audio and Electroacoustics, AU-15 no.2:
99-104, 1967.
- MOLLO-CHRISTENSEN, E.L., 'Physics of Turbulent Flow'. AIAA Journal,
9 no. 7: 1217-1228, 1971.
- OPPENHEIM, A.V. and WEINSTEIN, C., 'A Bound on the Output of a Circular
Convolution with Application to Digital Filtering'. IEEE
Transactions on Audio and Electroacoustics, AU-17 no.2:
120-124, 1969.
- PAPOULIS, A., 'The Fourier Integral and its Applications'. Electronic
Science Series, McGraw Hill: New York, 1962.
- PHILLIPS, O.M., 'The Maintenance of Reynolds Stress in Turbulent
Shear Flow'. Journal of Fluid Mechanics, 27 no.1 : 131-144, 1967.
- PHILLIPS, O.M., 'Shear-Flow Turbulence'. Annual Review of Fluid
Mechanics, 1: 245-264, 1969.
- RASMUSSEN, C.G., 'The Measurement of Turbulence Characteristics'.
DISA Information, 3: 9-20, 1966.
- SCHUSTER, A., 'On the Investigation of Hidden Periodicities with
Application to a Supposed 26-day Period of Meteorological
Phenomena'. Terr. Magn., 3: 13-41, 1898.
- SINGLETON, R.C. and POULTER, T.C., 'Spectral Analysis of the Call of
the Male Killer Whale'. IEEE Transactions on Audio and Electro-
acoustics. AU-15 no.2: 104-113, 1967.

- TOWNSEND, A.A., 'The Structure of Turbulent Shear Flow'. Cambridge University Press: Cambridge, 1956.
- UNITED GEOPHYSICAL CORPORATION, 'A Pictorial Digital Atlas', Ed. PETERSON, R.A., 1965. Presented at the 35th annual meeting of the S.E.G., Dallas, Texas.
- VAN ATTA C.W. and CHEN, W.Y., 'Correlation Measurements in Grid Turbulence Using Digital Harmonic Analysis'. Journal of Fluid Mechanics, 34 no.3: 497-515, 1968.
- VAN ATTA, C.W. and CHEN, W.Y., 'Correlation Measurements in Turbulence Using Digital Fourier Analysis'. Proceedings of the International Symposium on High Speed Computing in Fluid Dynamics, The Physics of Fluids, Suppl. II, 12 no.12: II264-II269, December 1969.
- VILLASENOR, A.J., 'Digital Spectral Analysis'. NASA Technical note - NASA TN D4510: 1968.
- WEBSTER, C.A.G. 'A Note on the Sensitivity to yaw of a Hot-Wire Anemometer'. Journal of Fluid Mechanics, 13 no. 2: 307-312, 1962.
- MELCH, P.D., 'The Use of the Fast Fourier Transform for the Estimation of Power Spectra : A Method Based on Time Averaging over Short, Modified Periodograms'. IEEE Transactions on Audio and Electroacoustics, AU-15 no.2: 70-74, 1967.
- MELCH, P.D., 'A Fixed-Point Fast Fourier Transform Error Analysis'. IEEE Transactions on Audio and Electroacoustics, AU-17 no.2: 151-157, 1969.
- WILLS, J.A.B., 'On Convection Velocities in Turbulent Shear Flows'. Journal of Fluid Mechanics, 20: 417-432, 1964.
- WYGNAWSKI, I., and FIEDLER, H.E., 'Some Measurements in the Self-Preserving Jet'. Journal of Fluid Mechanics, 38 no.3: 577-612, 1969.
- WYGNAWSKI, I. & FIEDLER, H.E., 'The Two-Dimensional Mixing Region'. Journal of Fluid Mechanics, 41 no.2: 327-361, 1970.

APPENDIX IThe Hot-Wire Response Equations

A Hot-wire is assumed to be in a plane perpendicular to the Z direction. Its direction is given by the unit vector $\hat{s}$

Figure A1

The mean velocity is assumed to be in the xy plane, and without loss of generality can be assumed in the x -direction. The effective cooling velocity is assumed to be the velocity perpendicular to the wire plus a fraction k of the velocity parallel to the wire.

$$\underline{W}_e = \underline{W}_{\text{perp}} + k\underline{W}_{\text{parr}} \quad (I-1)$$

Now

$$\underline{W} = \begin{matrix} \text{instantaneous velocity vector} \\ \left(\begin{matrix} U + u \\ v \\ w \end{matrix} \right) \end{matrix}$$

also

$$\underline{W}_{\text{perp}} = \underline{W} \cdot (I - \hat{s}\hat{s})$$

$$\underline{W}_{\text{parr}} = (\underline{W} \cdot \hat{s})\hat{s}$$

where the direction vector $\hat{s}$ is written as

$$\begin{aligned}\hat{s} &= \hat{i} \cos \alpha + \hat{j} \sin \alpha \\ &= \hat{i}c + \hat{j}s\end{aligned}$$

Working in terms of components

$$(W_e)_i = N_{ij}W_j$$

where from (I-1)

$$N_{ij} = I_{ij} - s_i s_j + k s_i s_j \quad (I-2)$$

$$s_1 = c = \cos \alpha$$

$$s_2 = s = \sin \alpha$$

$$s_3 = 0$$

For constant temperature, linearised mode of operation, the hot-wire voltage E is proportional to the magnitude of the effective cooling velocity.

$$E = K|W_e| \quad (I-3)$$

$$|W_e| = \sqrt{W_e \cdot W_e}$$

From (I-1)

$$W_e = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} - \begin{pmatrix} c^2 & cs & 0 \\ cs & s^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} + k \begin{pmatrix} c^2 & cs & 0 \\ cs & s^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u + u \\ v \\ w \end{pmatrix}$$

Adding the first two matrices and using the relationship $s^2 + c^2 = 1$

$$W_e = \begin{pmatrix} s^2 & -cs & 0 \\ -cs & 1-s^2 & 0 \\ 0 & 0 & 1 \end{pmatrix} + k \begin{pmatrix} 1-s^2 & cs & 0 \\ cs & s^2 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} u + u \\ v \\ w \end{pmatrix}$$

Multiplying the two matrices and simplifying one obtains

$$W_e = \begin{pmatrix} (u + u)\{k + s^2(1-k)\} + csv(k-1) \\ (u + u)cs(k-1) + v(s^2(k-1) + 1) \\ w \end{pmatrix}$$

Now the magnitude of the effective velocity vector $\underline{W}_e$.

$$\begin{aligned} |\underline{W}_e|^2 &= \underline{W}_e \cdot \underline{W}_e = W_{e1}^2 + W_{e2}^2 + W_{e3}^2 \\ &= [(U+u)(k+s^2(1-k)) + cv(s^2(k-1))]^2 \\ &\quad + [(U+u)cs(k-1) + v(s^2(k-1)+1)]^2 + v^2 \end{aligned}$$

simplifying

$$\begin{aligned} |\underline{W}_e|^2 &= (U^2 + 2Uu + u^2)(s^2 + k^2c^2) + v^2(s^2k^2 + c^2) \\ &\quad + 2(U+u)vc(s^2k^2 - 1) + u^2 \\ |\underline{W}_e|^2 &= U^2(s^2 + k^2c^2) \left[1 + \frac{2u}{U} + \frac{2v}{U} \frac{(k^2 - 1)sc}{(s^2 + k^2c^2)} + \frac{u^2}{U^2} + \frac{v^2}{U^2} \frac{k^2s^2 + c^2}{(s^2 + k^2c^2)} \right. \\ &\quad \left. + \frac{u^2}{U^2(s^2 + k^2c^2)} + \frac{2uv(k^2 - 1)sc}{U^2(s^2 + k^2c^2)} \right] \end{aligned}$$

To obtain $|\underline{W}_e|$ use is made of the Binomial expansion

$$(1+a)^m = 1 + \frac{m}{1!}a + \frac{m(m-1)}{2!}a^2 + \dots$$

$$\begin{aligned} |\underline{W}_e| &= U \sqrt{s^2 + k^2c^2} \left[1 + \frac{u}{U} + \frac{v(k^2 - 1)sc}{U(s^2 + k^2c^2)} + \frac{u^2}{2U^2} + \frac{v^2(k^2s^2 + c^2)}{2U^2(s^2 + k^2c^2)} \right. \\ &\quad + \frac{uv(k^2 - 1)sc}{U^2(s^2 + k^2c^2)} - \frac{1}{8} \left(\frac{4u^2}{U^2} + \frac{4v^2(k^2 - 1)^2s^2c^2}{U^2(s^2 + k^2c^2)^2} \right. \\ &\quad \left. + \frac{8uv(k^2 - 1)sc}{U^2(s^2 + k^2c^2)} + \dots \text{higher order velocity terms} \right] \end{aligned}$$

$$\begin{aligned} |\underline{W}_e| &= U \sqrt{s^2 + k^2c^2} \left[1 + \frac{u}{U} + \frac{v(k^2 - 1)sc}{U(s^2 + k^2c^2)} + \frac{v^2}{2U^2(s^2 + k^2c^2)} \right. \\ &\quad \left. + \frac{u^2}{2U^2(s^2 + k^2c^2)} + \dots \right] \quad (1-4) \end{aligned}$$

Neglecting the contributions from second order terms in the velocities (u^2, v^2, uv) if $E = \bar{E} + \tilde{e}$ and using (I-3)

where E is the instantaneous measured voltage. On averaging (I-4)

$$\bar{E} = KU \sqrt{s^2 + k^2 c^2}$$

and

$$\tilde{e} = KU \sqrt{s^2 + k^2 c^2} \left[\frac{u}{U} + \frac{v(k^2 - 1)cs}{U \sqrt{s^2 + k^2 c^2}} \right]$$

The largest terms neglected in the mean velocity are

$$\frac{v^2 + w^2}{2U \sqrt{s^2 + k^2 c^2}}$$

If these terms are not $\ll 1$ then the above approximation is invalid.

APPENDIX IITHE COMPUTER PROGRAMS

The various programs available for the analysis of digital turbulence data are described in this section. Rather than give the actual program a description is given (where necessary) of how the various quantities are calculated. A general description of the software is given as well as instructions on how to use each program or subroutine.

Primarily the object was not to write one enormous program calculating everything, but to write subroutines performing the various calculations which could later be combined to form more complex programs. This means that the user can write programs satisfying his own particular needs. There have also been a number of fairly general programs written.

The General Layout

The first time that computer programs are required is once the analogue signal has been digitised. It is informative to know what the digitised tape actually looks like.



The digitiser samples the analogue signal and then converts it into an Integer *2 number. The samples are then written in blocks of 1024 onto the magnetic tape. Integer *2 means that each value is represented as an integer using two BYTES. Each BYTE consists of 4 BITS. A bit is the basic on/off element used for binary representation. The magnetic tapes available can handle 1600 BPI (Bits per inch) although we operate at 800 BPI. When a particular section of data has been

digitised and written onto magnetic tapes, the FILE can be terminated by using a TAPE MARK. This is produced by the digitiser itself.

Summarising we have:

- i) FILES - The size of which we determine.
- ii) BLOCKS A group of 1024 values (2048 Bytes).

The magnitude of an Integer *2 variable is limited and so the data is converted to the Real *4 representation so that too larger values are not encountered in the calculations. After checking and then converting the data we can proceed to calculate turbulence characteristics.

Generally when debugging a program one only requires a small quantity of data. A catalogued data set on disc is available and programs to transfer about 4096 points have been written. Using this data has two advantages:

- i) Faster turnaround time of the programs
- ii) Less wear and tear on the magnetic tapes.

A Library has been formed on disc which contains all the subroutines and the programs that are executed by "In Line Procedures". This removes the need for large decks of cards and allows the same subroutines to be used in different programs at the same time. It is very simple to add more subroutines to the Library. All programs and subroutines are also on punched cards in case the Library is damaged.

The general programs are written in a manner that makes it possible to store the results on magnetic tape. This is most useful if one intends at a later stage to compare results or try different methods of plotting them.

Each of the programs and subroutines available will be discussed separately, and where necessary the theory discussed. Because of its importance the Fast Fourier Transformation will be discussed first.

SUBROUTINESSUBROUTINE RHAR1

This subroutine performs a Fast Fourier Transformation or Inverse Fourier Transformation on real data. The FFT algorithm (Cooley & Tukey 1965) is used so ensuring fast execution times compared to conventional methods e.g. Goertzel. SUBROUTINE RHAR1 converts the real data into a three dimensional complex array which is then processed by SUBROUTINE HARM. The output of HARM is then manipulated to give the Fourier Coefficients of the original time series. The subroutine can only handle samples that are a power of 2.

Name

SUBROUTINE RHAR1 (A, M, INV, S, IFERR, NN)

Purpose

Performs Fourier Transformations on real, one dimensional data.

Description of Parameters

A : As input contains the data to be transformed

1) Time Domain to Frequency Domain

The input consists $2^{**}(M+1)$ time samples. The output consists of $2^{**}(M+1) + 2$ values where:

A(1) = D.C. value (mean)

A(2) = 0.0

A(3) to A($2^{**}(M+1) + 2$) contain the cosine and sine terms

of the Fourier series

A(3) = Cosine term of 1st harmonic

A(4) = Sine term of 1st harmonic etc.

ii) Frequency Domain to Time Domain

The input A contains $A(2^{**}(M+1)+2)$ frequency domain values

$A(1) = \text{D.C.}$

$A(2) = 0.0$

$A(3) = \text{Cosine term}$

$A(4) = \text{Sine term, etc.}$

M: Integer constant which determines the size of A (in time domain
 $= 2^{**}(M+1)$)

INV: Work area, must be dimensioned to $1/\delta$ size of A

S: Work area, to be dimensioned as for INV

IFERR: Error indicator

i) If $NN = 0 + 1, -1$ then if IFERR = 1 the
 value of M is $20 < M < 3$.

ii) If $NN = +2, -2$ then IFERR = 1 means that the Sine and
 Inverse Tables are not large enough or have not been computed.

NN: This is an option parameter which may have the following values

i) 0 - Set up Sine and Inverse Tables only

ii) 1 - Set up the Sine and Inverse Tables and calculate the

Fourier Transform

$$X(j) = \sum_{k=0}^{N-1} A(k) e^{\frac{2\pi i}{N} j k}$$

iii) -1 - Set up the Sine and Inverse Tables and calculate the
 Inverse Fourier Transform

$$A(k) = \frac{1}{N} \sum_{j=0}^{N-1} X(j) e^{-\frac{2\pi i}{N} j k}$$

- (iv) 2 - Calculate the Fourier Transforms only (assume that the Sine and Inverse Tables exist).
- v) -2 - Calculate the Inverse Fourier Transform only. Assume Sine and Inverse Tables exist.

Remarks

N is limited to $3 < N < 20$

Subroutines and Functions RequiredHARM

Some timing information is given below:

<u>Sample Size</u>	<u>Approx. time (secs)</u>
1024	2.8
2048	6
4096	13.8

SUBROUTINE STATSS

This subroutine calculates various statistical properties of the sample. It is a simplified and modified S.S.P. routine. Calculation of the Amplitude Probability Density is optional.

If the input array is A, the following quantities are calculated:

- i) The maximum value of A.
- ii) The minimum value of A.
- iii) The total ΣA .
- iv) ΣA^2 .
- v) The mean $\frac{\Sigma A}{N}$.
- vi) The standard deviation $\sqrt{\frac{\Sigma(A - \text{mean})^2}{N - 1}}$.
- vii) The number of values outside range used for A.P.D.F.

Note:

$$\begin{aligned} \Sigma(x - \text{mean})^2 &= \Sigma(x^2 - 2x\text{mean} + \text{mean}^2) \\ &= \Sigma x^2 - 2x\text{mean} + \text{mean}^2 \\ \text{But mean} &= \frac{\Sigma x}{N} \\ &= \Sigma x^2 - \frac{2\Sigma x \Sigma x}{N} + \frac{N \Sigma x}{N} \frac{\Sigma x}{N} \\ &= \Sigma x^2 - \frac{\Sigma x \Sigma x}{N} \end{aligned}$$

Above is the quantity actually calculated

Name

SUBROUTINE STATSS (A, ND, UDC, FREQ, PCT, STATS, NYES)

Purpose

Calculates various statistics and the Amplitude Probability Density function of a sample.

DESCRIPTION OF PARAMETERS

A: The input array. It is not altered during execution of the subroutine.

NO: Number of points. The dimension of A.

UBO: An array of dimension 3

i) UBO(2) is the number of intervals required for the Amplitude Probability Density Function,

ii) If UBO(1) = UBO(3) then the subroutine calculates the minimum and maximum values of the array.

Otherwise:

UBO(1) = minimum value.

UBO(3) = maximum value.

FREQ: Output array containing the Amplitude Probability Density Function Dimension UBO(2).

PCT: FREQ/NO*100

STATS: Output array of dimension 7 containing the following:

STATS(1) = Total.

STATS(2) = Mean.

STATS(3) = Standard deviation.

STATS(4) = Minimum value.

STATS(5) = Maximum value.

STATS(6) = Sum of A*A.

STATS(7) = No of points outside range.

MYES: Option parameter:

i) MYES = 1 Causes Amplitude Probability Density Function to be calculated.

ii) MYES $\neq$ 1 Indicates that the A.P.D.F. is not to be calculated.

A.17

Remarks

None

Subroutines and Functions Used

None

SUBROUTINE REMOVE

This subroutine checks the sample to see if there is a mean or linear trend. Although the D.C. is filtered out of the signal, means and linear trends in each sample can still be found.

The method used is that described by Bendat & Piersol (1966). It only uses 2/3 of the data points and hence saves computation time.

Let the original signal be represented as

$$U(t) = \bar{U} + \bar{a}(t - \frac{T}{2}) + x(t) \quad 0 < t < T$$

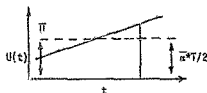
where

$\bar{U}$ = sample mean

$\bar{a}$ = average slope of $U(t)$

T = total time

Graphically



We need the quantities i) $\bar{U}$

ii) $\bar{a}$

Integrating between the limits 0 and $T/3$

$$\int_0^{T/3} U(t) dt = \int_0^{T/3} \bar{U} dt + \int_0^{T/3} \bar{a}(t - \frac{T}{2}) dt + \int_0^{T/3} x(t) dt \quad - I$$

Integrating between

$$\frac{2T}{3} \rightarrow T$$

$$\int_{2T/3}^T U(t) dt = \int_{2T/3}^T \bar{U} dt + \int_{2T/3}^T \bar{a}(t - \frac{T}{2}) dt + \int_{2T/3}^T x(t) dt \quad - II$$

Obviously

$$\int_0^{T/3} \ddot{u} dt = \int_{2T/3}^T \ddot{u} dt$$

Subtracting I from II

$$\int_{2T/3}^T u(t) dt - \int_0^{T/3} u(t) dt = \int_{2T/3}^T \left(\ddot{u}(t - \frac{T}{2}) \right) dt + \int_{2T/3}^T x(t) dt - \int_0^{T/3} \left(\ddot{u}(t - \frac{T}{2}) \right) dt - \int_0^{T/3} x(t) dt$$

The series $x(t)$ has no linear trend and hence

$$\int_0^{T/3} x(t) dt = \int_{2T/3}^T x(t) dt$$

Hence we have

$$\int_{2T/3}^T u(t) dt - \int_0^{T/3} u(t) dt = \int_{2T/3}^T \left(\ddot{u}(t - \frac{T}{2}) \right) dt - \int_0^{T/3} \left(\ddot{u}(t - \frac{T}{2}) \right) dt$$

Now

$$\int_{2T/3}^T \left(\ddot{u}(t - \frac{T}{2}) \right) dt = \left. \frac{\ddot{u}(t - \frac{T}{2})}{2} \right|_{2T/3}^T - \left. \frac{\ddot{u}(t - \frac{T}{2})}{2} \right|_{2T/3}^T = \frac{1}{9} \ddot{u}^2$$

$$\int_0^{T/3} \left(\ddot{u}(t - \frac{T}{2}) \right) dt = \left. \frac{\ddot{u}(t - \frac{T}{2})}{2} \right|_0^{T/3} - \left. \frac{\ddot{u}(t - \frac{T}{2})}{2} \right|_0^{T/3} = -\frac{1}{9} \ddot{u}^2$$

Therefore

$$\int_{2T/3}^T u(t) dt - \int_0^{T/3} u(t) dt = \frac{2}{9} \ddot{u}^2$$

Therefore

$$\ddot{u} = \frac{1}{2} \left(\int_{2T/3}^T u(t) dt - \int_0^{T/3} u(t) dt \right)$$

The corresponding version of III for use with discrete data would be:

$U(t)$ is represented as

$U(nh)$, $n = 1, 2 \dots N$

$t = nh$

$h =$ time interval between samples

$$\bar{u} = \frac{1}{N} \sum_{n=1}^N U(n) = \frac{h/3}{N} \sum_{n=1}^N U(n)$$

$$\text{Let } v = \frac{N}{3}$$

Therefore

$$\begin{aligned} N - v &= N - \frac{N}{3} = \frac{2N}{3} = 2v \\ &= \frac{1}{h} \sum_{n=1}^N U(n) - \frac{h/3}{N} \sum_{n=1}^N U(n) \\ &= \frac{1}{h} \sum_{n=1}^N U(n) - \frac{h/3}{N} \sum_{n=1}^N U(n) \end{aligned}$$

Finally,

$$\bar{u} = \frac{1}{N} \sum_{n=1}^N U(n) = \frac{v}{N-v} \sum_{n=1}^N U(n)$$

Since we use large samples it is generally sufficient to take v as the integer nearest $N/3$.

The series $x(t)$ can be calculated as follows:

$$x(nh) = U(nh) - \bar{u} = U(nh) - \bar{u}, n = 1, \dots, N$$

Name

SUBROUTINE REMOVE (X, H, N, RESULT)

Purpose

The subroutine calculates the mean and slope in a sample and then removes one, both or neither depending on the magnitude of them.

Description of Parameters

X: Input array of dimension N. As output it consists of the modified signal.

H: Time between samples of X

RESULT: Contains the mean and slope of the sample. Dimension is 2.

RESULT (1) = mean

RESULT (2) = slope

Remarks

By changing the values in the IF statements in the subroutine the critical values of the mean value and slope can be varied.

Functions and Subroutines Used

None.

For the size of sample used the routine gives reasonably accurate results as shown in the table below:

<u>Sample Size</u>	<u>Actual Slope</u>	<u>Calculated Slope</u>
100	0,2	0,22
200	0,2	0,21
500	0,2	0,20

SUBROUTINE HAMMIN

The subroutine HAMMS the input signal. In effect it is a moving average, the coefficients being 0.46; 1.08; 0.45.

Name

SUBROUTINE HAMMIN (A, N)

Purpose

The subroutine Hams the input array.

Description of Parameters

A: Input array. As output it is the Hanned array.

N: Number of points in the sample.

Remarks

None.

Subroutines and Functions Used

None.

A.18

SUBROUTINE HANNIN

The subroutine HANNIN the sample. In effect it is a moving average.
the coefficients being 0.25; 0.5; 0.25.

Name

SUBROUTINE HANNIN (A, N)

Purpose

The subroutine Hanns the input array A.

Description of Parameters

A: Input array. As output it is the Hanned array.

N: Number of points in the sample.

Remarks

None

SUBROUTINE PGRAM

The subroutine calculates:

- i) The Fourier series coefficients $a(f)$, $b(f)$.
- ii) The Periodogram coefficients $a(f)^2 + b(f)^2$.
- iii) The Phase angles $\text{Tan}(b(f)/a(f))$ of the input signal. The routine uses the Fast Fourier Transform Algorithm.

Name

SUBROUTINE PGRAM (DATA, N, AMP, D, NYES, INV, S)

Purpose

The subroutine calculates the Fourier coefficients of the input array using the Fast Fourier Transform. The coefficients can be Hanned if required. They are then combined to give the Periodogram (Power Spectrum). The Phase information is also calculated.

Description of Parameters

DATA: As input DATA is the Real array of N data points. Dimension

N + 4. The output array is of size N + 2 and consists of

DATA (1) = mean value

DATA (2) = 0.0

DATA (3) = 1st Cosine term

DATA (4) = 1st Sine term

. . . and etc.

AMP: Array of amplitude squared terms $\{a^2 + b^2\}$. Dimension N/2.

The D.C. term is not included.

N: Number of points in the input. It must be a power of 2.

D: Array of Phase information. Dimension N/2.

NYES: Is an option parameter. If NYES = 1 then the Fourier coefficients are Hanned. If NYES $\neq$ 1 then the coefficients are not Hanned.

INV: Work area required for subroutine RHAR1 . Dimension N/8.

S: Work area required for subroutine RHAR1 . Dimension N/8.

Remarks

The input array is destroyed on passing through the subroutine.

Functions and Subroutines Used

RHAR1.

SUBROUTINE AUTOC

The subroutine rearranges the output of subroutine PGRAM and then Inverse Fast Fourier Transforms it. It should be used in conjunction with PGRAM. The mean value of the array to be IFFT is made equal to zero. The Cosine terms are equal to the Periodogram coefficients while the Sine terms are all zero. The output consists of the autocorrelation coefficients with time lags being multiples of the time interval between samples. Because the autocorrelation function is an even function only half of the calculated coefficients are unique.

Name

SUBROUTINE AUTOC (DATA, AMP, N, INV, S)

Purpose

The subroutine rearranges the Periodogram coefficients and then IFFT them to give the autocorrelation function. In general the Periodogram coefficients are obtained using PGRAM.

Description of Parameters

DATA: Is the output array of autocorrelation coefficients, dimension

$N + 4$.

AMP: Input array of Periodogram coefficients. Dimension $N/2$.

N: Number of points in the autocorrelation (same as original time series).

INV: Work area for subroutine RHAR1, dimension $N/8$.

S: Work area for subroutine RHAR1, dimension $N/8$.

Remarks

Generally used in conjunction with subroutin PGRAM.

Functions and Subroutines Used

RHAR1.

SUBROUTINE READS

The data on the magnetic tapes is blocked in groups of 1024 samples. The maximum number of points any READ statement can read is one block i.e. 1024 values. If a sample of say 4096 points is to be read, each block that is read must be put into different areas of the sample array. If this is not done then the first 1024 points will be over-written by those from the next READ. The subroutine reads any sample size (a multiple of 1024) greater than 1024.

Name

SUBROUTINE READS (N, NA, DATA)

Purpose

To read an array ≥ 1024 points when the data is blocked in blocks of 1024.

Description of Parameters

N: Size of sample to be read.

NA: The data set reference number of the data to be used.

DATA: Output array of data, dimension N.

Remarks

Samples of less than 1024 points cannot be read.

Functions and Subroutines Used

None

SUBROUTINE SMOOTH

The subroutine takes the input array and smooths it using a moving average technique. The width of the triangular weighting function is variable. It is a useful method of smoothing the Periodogram (Singleton and Poulter 1967). An optional check is available to safeguard very sharp peaks which would otherwise be removed. It is a fairly time consuming process but useful for final "tailoring" of the results.

If the span of the smoothing function is S , then;

$$X(I) = \frac{1 \cdot X(I-2) + 2 \cdot X(I-1) + 3 \cdot X(I) + 2 \cdot X(I+1) + 1 \cdot X(I+2)}{M}$$

where

$$M = \frac{S+1}{2} = 3$$

when using a span of 3, the effect is identical to Hanning the sample.

Name

SUBROUTINE SMOOTH (X, IS, N, IT)

Purpose

The subroutine smooths an input array using a triangular smoothing function the span of which is variable.

Description of Parameters

- X: Input array of dimension N. As output it contains the smoothed data.
- N: Number of points.
- IS: Span of the smoothing function.
- IT: IT = 0, then sharp peaks are left.
- IT = 1, then sharp peaks are smoothed.

Remarks

The maximum span that can be used is 19.

Functions and Subroutines Used

None.

Some time information is shown in the table below:

<u>Sample Size</u>	<u>Span</u>	<u>Time (secs)</u>
1024	3	10
1024	7	19
1024	15	38

SUBROUTINE MOMENT

The subroutine calculates Higher Order Skewness factors of the input array. It is written so that if the sample is too large to be handled in one pass, smaller samples can be taken and the results averaged. The moments calculated are defined by,

$$S(NP) = \frac{[\overline{u(x, t+h) - u(x, t)}]^{NP}}{[\overline{u(x, t+h) - u(x, t)}]^{NP/2}}$$

where the overbar implies averaging and where $u(x, t)$, $u(x, t+h)$ are velocities at the same point in space, but different instants in time. The two quantities

$$[\overline{u(x, t+h) - u(x, t)}]^{NP}$$

and

$$[\overline{u(x, t+h) - u(x, t)}]^2$$

are the outputs since they can be averaged over successive samples.

Only one value of NP can be calculated at any time, but as many different values of the time delay as required can be calculated. The time delay must be multiples of the time interval between samples.

NOTE: this is not the optimum way to calculate the moments above.

The F.F.T. can be used as described by Van Atta and Chan (1969).

Name

SUBROUTINE MOMENT (X, N, NT, NI, NP, TEMP, BEMP)

Purpose

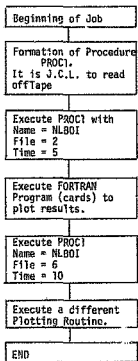
The subroutine calculates higher order skewness factors for a particular value of NP over as many time intervals as required.

PROGRAMS

The previous section described most of the subroutines available. They can be used by any program written by the programmer. This section describes some of the programs available to perform particular calculations. One general program that calculates cross correlations and autocorrelations has been written. Most programs are available in two forms, the difference being the form of the Job Control Language processing it.

The first form is the standard I.B.M. Job Control Language. In the second form In-line procedures have been utilised. The necessary Job Control Language for the program is formed into a standard procedure. By using substitution Job Control Language it is possible to change any particular parameter in the Job Control Language by over-riding it in either the PROC or EXEC statements.

To use In-line procedures the program must be compiled and Linkage Edited and stored in a Partitioned Data Set, whereas for standard Job Control Language the program can be in card form. The biggest advantage of the In-line procedure mode of execution is that the same program can be executed a number of times in the same JOB using the minimum of Job Control Language. Each time the routine is executed different Job Control Language parameters or data can be used. Graphically a JOB consisting of an In-line procedure is shown below:



The Parameters that can be Varied

- i) Name of tape (Name =),
- ii) File to be read (File =),
- iii) Execution time (Time =),

The actual program to be executed is stored on disc and is found by PROC1.

There is no reason why an in-line procedure cannot be executed alternately with various smaller Fortran programs which output results. If the same plotting program is used each time it would be advantageous to form another in-line procedure that executes the plotting routine.

Define File Statements have been used to transfer data from one step in the program to another. The disc data set formed by a Define File statement is only erased at the end of a JOB. Small programs doing specific calculations can therefore be executed after a large general program, the data being read off the data set that the general program wrote onto.

The first programs mentioned are for checking the data on the tapes and converting the data into a convenient form.

PROGRAM NLBCHECK

The program is used to check the data on the digitised tape containing Integer *2 data. It is possible to print the first block of any file on the tape to see if the values are those expected. The program reads right through the file and any error interrupting the reading routine will cause a message to be written. The program is available in two forms, one using standard Job Control Language; the other the In-line procedure NLBPROC1. The program is stored as the module CHEK on the Library ACM.NLI.LIB. In general it is easier to use the form using the In-line procedure.

The following deck of cards is required to execute the program:

//NLBCHECK JOB	
//NLBPROC1 PROC	formation of
↓	In-line
//NLBPROC1 PEND	Procedure


```
//ONE EXEC NLBPROC1, NAME = , TIME = etc.
      data cards for step ONE
//TWO EXEC NLBPROC1, NAME = , TIME = etc.
      data cards for step TWO
```

These are two parameters that can be changed:

- i) NAME = : name of tape to be read.
- ii) FROM = : file number to be read.

If all the steps use the same tape then the NAME = parameter can be specified on the PROC statement

```
//NLBPROC1 PROC, NAME = NLBGO
```

- iii) One data card giving the file number of the file to be read must be supplied for each EXEC statement

PROGRAM NLBCONV

The object is to convert the Integer *2 results on the digitiser tape into Real *4 results. The standard representation is Real *4 as the allowable magnitude of Real *4 is higher than that of Integer *2 variables.

Depending on the analysis to be performed it might be advantageous to have certain files of information next to one another. Files that were terminated because the digitiser had a Parity check, can be neglected completely. The program reads a particular file, converts it to Real *4, counts the number of blocks and then writes the complete file onto another magnetic tape.

The program is available with standard Job Control Language or using the In-line procedure NLBPROC.

The variable parameters are:

- i) TAPE = : Name of Integer *2 tape.
 - ii) ONTO = : Name of Real *4 tape.
 - iii) FROM = : Number of the file to be read.
 - iv) TO = : Number of the file to be written.
 - v) TT = : Execution time required.
- also
- vi) Integer *2 tape is NDLABEL.
 - vii) Real *4 tape is LABELLED.
 - viii) Each EXEC statement requires a data card. Any information can be written onto the first 20 characters. If no information is required place a blank card as the data card.
 - ix) Job Control Language messages are on SYSOUT = B while program message on SYSOUT = A.

Description of Parameters

X: Input array.

N: Number of points.

NT: An array, the members being the time delays to be calculated (multiples of the time interval between samples).

NI: Dimension of NT. It is the number of time intervals to be calculated.

NP: Power of the skewness factor.

TEMP: Array of dimension NI containing the values of

$$[u(x, t+h) - u(x, t)]^{NP}$$

for each time delay

BEMP: Array of dimension NI, containing the values of

$$[u(x, t+h) - u(x, t)]^2$$

for each time delay.

Remarks

A reference indicating the importance of the above type of skewness factors is: (Frenkiel and Klebanoff (1966)).

Functions and Subroutines Used

None.

SUBROUTINE NBHIST

The subroutine plots a Histogram on the Calcomp Plotter. The user must supply the frequencies to be plotted, the number of intervals as well as the minimum and maximum values of the array. The last two quantities are not essential as they are just used to calculate the ranges of each interval.

Name

SUBROUTINE NBHIST (XX, YY, YL, NUM, K)

Purpose

The subroutine plots a Histogram on the Calcomp Plotter.

Description of Parameters

XX: X co-ordinate of the left hand side of the Histogram (inches).

YY: Y co-ordinate of the left hand corner of the Histogram (inches).

XL: Width of the Histogram (inches).

YL: Maximum height of the Histogram (inches).

NUM: Input array of dimension K containing the frequencies to be plotted

NUM(K+1) = minimum value of array

NUM(K+2) = maximum value of array.

K: Number of frequencies to be plotted.

Remarks

At the end of the routine a new origin is defined, 2 inches past the end of the Histogram and at $y = 0.5$ inches.

Functions and Subroutines Used

The plotter subroutines are stored on the System Library SYS1.LOAD.

SUBROUTINE GRAPH

Given two arrays, this subroutine plots them on the Calcomp Plotter (rectangular co-ordinates). It is advisable to read the write up on the Calcomp Plotter supplied by the Computing Centre to be able to make full use of the routine.

Name

SUBROUTINE GRAPH (X, Y, XL, YL, NP, LT, INT, INC)

Purpose

The arrays X and Y are plotted on rectangular axes on the Calcomp Plotter.

Description of Parameters

- X: Input array (X axis). Dimension (NP + 2).
 Y: Input array (Y axis). Dimension (NP + 2).
 XL: Length of X axis (inches).
 YL: Length of the Y axis (inches).
 NP: Number of points in the X and Y arrays.
 LT: Equivalent to LINTYP in the Calcomp write-up. It describes the type of line to be drawn while the magnitude determines the frequency of the plotted points.
 INT: Equivalent to INTEQ in the Calcomp write-up. Its magnitude determines what type of symbol is to be plotted.
 INC: Equivalent to INC in the Calcomp write-up. It is used to select the data to be plotted. Sometimes only every n^{th} point is to be plotted.

Remarks

Provision has been made for headings along each of the axes and the writing of a title. They are supplied as data on punched cards.

The headings can be up to 20 characters in length. The data must be supplied as follows:

- i) X axis heading.
- ii) Y axis heading.
- iii) title.
- iv) 3F10.2 characters in the following order:
 - XC - X co-ordinate of the title.
 - YC - Y co-ordinate of the title.
 - SZ - height of the characters to be printed (in the title).

Functions and Subroutines Used

The plotter subroutines are stored on the System Library SYCL.LOAD.

COLUMNS ON DATA CARD

1	2	3	4	5	6	7	8	

File number (right justified integer)

iv) The tape being read must be a NOLABEL tape.

v) All Job Control Language messages are printed on SYSOUT = B
 while any messages written by the program itself are on SYSOUT = A.
 This keeps the two separate.

Ex/ Read the first block of files 6 + 10 on tape NLB00

//NLBCHECK JOB -

//NLBPROCI PROC NAME = NLB00

procedure

//NLBPROCI PEND

// EXEC NLBPROCI, FROM = 6

06001

// EXEC NLBPROCI, FROM = 7

07001

// EXEC NLBPROCI, FROM = 8

08001

// EXEC NLBPROCI, FROM = 9

09001

// EXEC NLBPROCI, FROM = 10

10001

//

Ex/ File number 14 on NLB0 is to be converted to Real *4 and written as File number 1 on NLB01. The estimated time required is 2 minutes, then File 23 is to be written as File number 2.

//NLBCHV JOB -

//NLBPROC PROC, TAPE = NLB0 , ONTO = K '91, TT = 2

↕
//NLBPROC PEND

// EXEC NLBPROC, FROM = 14, TO = 1

FILE 14 TO FILE 1

// EXEC NLBPVOC, FROM = 23, TO = 2

FILE 23 TO FILE 2 RUN 4

//

Obviously File number 2 of NLB01 cannot be written before File number 1 has been written. The Real *4 tape must therefore be written in sequence. The program is stored as NLBCON on the Library ACM.NLB.LIB.

PROGRAM NLBTEST

This program is used to check each digitised file for digitising errors and to calculate the mean value of the file. This is necessary when calculating the r.m.s. values of the file using program NLBSEP.

The in-line procedure NLBTEST executes the module NLOOK which is stored on the library ACH.NLB.LIB.

Parameters to be supplied

- i) Name of the magnetic tape to be used

TAPE =

- ii) Execution time required

T =

- iii) File number to be processed

FILE =

One data card is required to supply the file number. The format is IZ.

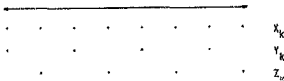
PROGRAM NLBSMP

This program reads a sample from the Real *4 tape and then checks if the sampling rate is at least four times the highest frequencies present. The method was suggested by Villaseñor (1968).

The user determines from where the sample is read. Sample size (a power of 2) is supplied as data. Briefly, the following procedure is followed:

- i) The sample X_k is read where $k = 1, 2 \dots N$.
- ii) The sample is divided into two series, one consisting of the odd values of X_k and the other consisting of the even values of X_k .

$T = \text{total time}$



- iii) The Fourier series coefficients

$X_k(f)$, $Y_k(f)$, $Z_k(f)$ are then calculated for the series X_k , Y_k , Z_k using the Fast Fourier Transform algorithm.

- iv) The following quantities are calculated and returned by the program:

$$(a) \text{ ODD-EVEN : } \sum_{k=1}^{N/4} (Y_k(f) - Z_k(f))^2$$

$$(b) \text{ ODD-BOTH : } \sum_{k=1}^{N/4} (Y_k(f) - X_k(f))^2$$

$$(c) \text{ EVEN-BOTH : } \sum_{k=1}^{N/4} (Z_k(f) - X_k(f))^2$$

$$(d) \text{ HARMONICS : } \sum_{k=1}^{N/4} (X_{N/2-k+1}(f))^2$$

The Y_k and Z_k series extend (less one time interval) for the same total time T as X_k so that the sampling rates for these series is $1/2$ that for X_k . If X_k has N points in the series then there are $N/2$ harmonics calculated. The Y_k and Z_k series have $N/2$ points and hence $N/4$ harmonics. Since all the samples extend over the same total time T , the first $N/4$ harmonics of each should be identical.

We now assume that there are no frequencies higher than the Nyquist frequencies of Y_k and Z_k (that is $1/2$ the Nyquist frequency of X_k).

(a) is the difference between the Fourier coefficients of similar series displaced by one time interval. The magnitude should give an estimate of allowable errors and can be used as a maximum possible value of the others.

Any frequency that is aliased will be added onto different frequencies in the X_k series compared to the Y_k and Z_k series. If this occurs, the sums (b) and (c) should be relatively large. A harmonic that will be aliased in the Y_k and Z_k series but not the X_k series will give values of (b) and (c) half as great as before, but its effect will be shown on (d) as well. In this case (b), (c) and (d) should all have approximately the same value.

The program is executed and the returned values of (a), (b), (c) and (d) examined to see if any of (b), (c) or (d) are larger than (a). If so, the sampling rate is not four times the highest frequencies present in the signal.

PROGRAMS NLBPL01 AND NLBPL02

These two programs plot the results of NLBAUTO and NLBCROSS respectively. The results must be in the form written by NLBSTORE.

No data cards, besides those required by subroutine GRAPH are required as all the necessary information is on the data sets produced by NLBSTORE. The job control statement

```
//GO.FT02F.. DD - ...
```

specifies from where the results are to be read.

The size of the axis, symbols printed, etc., are varied by changing the parameters in subroutine GRAPH.

(a) Information on each sample

The following is printed out for each sample analysed:

- i) Total.
- ii) Mean.
- iii) Standard deviation.
- iv) Minimum value.
- v) Maximum value.
- vi) Sample R.M.S.
- vii) Number of samples outside the range used for the Amplitude Probability Density function. As the overall range is not known before the analysis, the first sample is analysed and ± 4 *STANDARD DEVIATION * Mean is taken as the range for the A.P.D.F.. The A.P.D.F. for the first sample is not used. A check is made on the number of values outside this range,

viii) Sample size.

ix) Sample slope.

(b) Information on all samples

The following overall properties are printed:

- i) Overall average.
- ii) Overall minimum.
- iii) Overall maximum.
- iv) Overall R.M.S.
- v) Overall Standard deviation.
- vi) Minimum value used for A.P.D.F.
- vii) Maximum value used for A.P.D.F.

APPENDIX III

The analogue values of the u and v velocity components were measured using the circuit shown in Fig. (III-1).

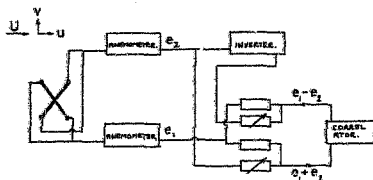


Fig. (III-1) Circuit for Obtaining the u and v Velocity Fluctuations and the Reynolds Shear Stress

A hot-wire transducer placed in a flow velocity field is primarily sensitive to the velocity component normal to the axis of the hot wire. It therefore only indicates the longitudinal component of flow velocity fluctuations when the wire axis is placed perpendicular to the mean flow direction. If the hot-wire axis is placed at an angle of 45° with respect to the mean flow velocity, then the transducer has equal sensitivity to the longitudinal component of flow velocity fluctuations u and the transverse component v . For measurement of the transverse component an X-probe is used. An X-probe consists of two mutually perpendicular wires placed at an angle of 45° with respect to the direction

- vii) Total number of points in the series that were outside the above range.
- ix) The Student "t" 90% confidence interval for the mean value of the Standard deviation.
- x) The percentage of the mean value that the above interval constitutes.
- xi) The overall Amplitude Probability Density function is printed as the number of the interval versus the frequency.
- xii) Either the Hanned or Unhanned Periodogram is printed. Values of 999 indicate that the particular value was not calculated.
- xiii) The normalised autocorrelation is printed. Once again any values of 999 indicate that the value was not calculated.

If the analysis was only a single series analysis above would constitute all the Output. When a two-series analysis is being performed each series is treated separately as above. After the output for each series is printed the combined output, consisting of the Cross-spectrum and Cross-correlation is printed.

- i) Co-spectrum is printed.
- ii) Quadrature-spectrum is printed.
- iii) Cross-correlation is printed.

(c) Information stored on temporary disc space

Two temporary disc data sets are defined using a DEFINE FILE statement, essentially one for each series to be analysed. Each data set consists of 7 records, each of which can be up to

The following cards are required when using the program:

```
//MLBSMP JOB -  
// EXEC FORTGCLG -  
// FORT.SYSIN DD *  
                program  
//LKED.SYSLIB DD  
                *
```

```
//GD.SYSIN DD *
```

Data card give sample size (Integer, Cols. 1 - 4)

```
//GD.FT03FD01 DD -
```

The DD card for FT03FD01 must direct the program to the sample to be read. As the program will not be used frequently it is only used with standard Job Control Language.

A sample of 4096 points takes approximately $1\frac{1}{2}$ minutes to process.

of mean flow velocity. The hot wires are connected to two constant-temperature anemometers as shown in Fig. (III-1).

The output signals from the anemometers are fed to the circuits which compute the sum and difference of input signals. The sum signal corresponds to twice the instantaneous value of the longitudinal component of the turbulence whilst the difference signals corresponds to twice the instantaneous value of the transverse component. This will be seen from the following:

Assuming the Cosine Law for the wires the output of hot wires 1 and 2

$$e_1 = (au + bv)$$

$$e_2 = (au - bv)$$

where a and b are sensitivity coefficients. The two anemometers should be adjusted to have equal sensitivities. The sum and difference signals are then:

$$e_s = e_1 + e_2 = 2au$$

$$e_D = e_1 - e_2 = 2av$$

The r.m.s. value of these signals is proportional to the longitudinal and transverse components of the turbulence.

4100 words long (1 word = 4 bytes = 1 Real *4 value). The average values of the correlations and Spectra are stored on these data sets.

When the analysis ends, the following quantities reside on each data set.

1) Single series analysis

The data set defined by DEFINE FILE 7(7,4100, U, IT) contains:

Record Number

1 : Contains the first N/2 values of the Fourier coefficients.

2 : Contains the last N/2 - 2 coefficients i.e. from N/2 + 1 to N + 2 where N is the sample size. The coefficients are divided into two so that sample sizes of 8192 can be analysed. The coefficients of each sample are written over those of the previous sample.

3 : Contains the N/2 Periodogram coefficients. The values for all the samples are added -

$$\text{value on disc} = \sum_{j=1}^{JJ} (a^2 + b^2)$$

 JJ = number of samples analysed.

4 : Contains the frequencies for the Amplitude Probability Density function as well as other information. If K1 is the number of frequencies in the A.P.D.F. the locations

1 + K1 contain the frequencies.

K1 + 1 contains 9999 (to be used as a marker).

PROGRAMS NLBANY AND NLBANYG

If the user is interested in any particular section of a file on the Real *4 tape these programs can be used to print it, transfer it to the disc data set ACM.NLB.HELLT or plot it.

NLBANY prints the required blocks and writes them onto an On-line disc data set. It is standard Fortran IV and requires one data card with two integer values

Columns 1 + 3 + 6 + 9
 M | N |

M = The number of the block before the first one to be read.

N = The number of blocks to be printed.

NLBANYG performs the same function except that the section read is plotted on the Calcomp Plotter. For space considerations only 8 blocks (maximum) of 1024 points can be processed. The number must also be an integer multiple of 1024. One data card with two integer values is required

Columns 1 + 3 + 6
 M | N |

M = The number of the block before the first one to be read.

N = The number of blocks to be read.

As written the program reads off NLB01 but this can be changed easily. The data set written onto is catalogued and called ACM.NLB.HELLT.

K1 + 2 contains N (sample size),
 K1 + 3 contains K1 (number of intervals),
 K1 + 4 contains DT (time interval between samples),
 K1 + 5 contains DVM (the mean value of signal fed in as data),

K1 + 6 contains JA(1) (File number of X series to be analysed),

K1 + 7 contains JA(2) (File number of Y series to be analysed),

K1 + 8 contains JA(3) (which indicates whether a single or two series analysis is performed).

5 : Contains the autocorrelation coefficients (totalled over each sample). There are $N/2 + 1$ coefficients.

6 & 7 : Are not used in the single series analysis.

ii) Two series analysis

There are two disc data sets defined by

DEFINE FILE 7 (7, 4100, U, IT)

and DEFINE FILE 8 (7, 4100, U, IT)

Records 1 to 5 are the same as for the single series analysis.

DEFINE FILE 7 containing the information on the X series and

DEFINE FILE 8 containing the information on the Y series. The

records 6 and 7 of each data set contain the following information.

DEFINE FILE 7

Record

6 : Contains the $N/2$ Co-spectrum coefficients.

PROGRAM NLBZONE

This program processes the output of NLBINT. It reads 4096 velocity values, the associated interwittency signal, and separates the velocity fluctuations into two signals, one containing the turbulent points and one containing the non-turbulent points. These two signals are written onto an output device. Execution is terminated when the end of the data file is reached. The output is written in blocks of 1024 values.

No data cards are required but the following Job Control Statements must be specified:

i) //GO.FT#3F091 DD ...

is the unit from which the data is read.

ii) //SYSUTZ DD in STEP 2 is the unit onto which the turbulent signal must be written.

iii) //SYSUTZ DD in STEP 3 is the unit onto which the non-turbulent points must be written.

The program writes the separated signals onto temporary disc space and the system program IEBGENER is then used to write from the disc onto the units specified by the user.

7 : Contains the $N/2$ Quadrature-spectrum coefficients.

DEFINE FILE b

Record

6 : Contains the first $N/2 + 1$ Cross-correlation coefficient.

7 : Contains the last $N/2 + 1$ to $N + 2$ coefficients.

iii) Data

Two data cards are required, providing the following information:

a) CARD 1

- i) Sample size N .
- ii) Number of samples (maximum) to be analysed, NS .
- iii) Number of intervals for the Amplitude Probability Density function, KI .
- iv) Indicator AS to whether Fourier coefficients must be Hanned or not, NH .
 $NH = 1$ Yes
 $NH \neq 1$ No
- v) The mean value of the signals (of use when analysing turbulence fluctuation, only), DVM .
- vi) The time interval (in sec) between values of the signal, DT .

b) CARD 2

- i) File number for X series $JA(1)$.
- ii) File number for Y series $JA(2)$.

PROGRAM INTBINT

Once the frequency range and amplitude cut-off have been decided upon, the program is used to obtain an intermittency signal. This is a modified version of a program written by Drayer (1973) and the particular write-up should be consulted.

Briefly, the program reads 4096 values of the recorded signal (//FT03F001) and then writes the section of signal, followed by its associated intermittency signal, onto an output device (//FT04F001).

The calculation is terminated in two ways:

- i) No more data is available to be read.
- ii) Enough turbulent and non-turbulent points have been calculated.

If one is only interested in obtaining, say 1024⁰ turbulent points and 10240 non-turbulent points then the IF statements in the main routine must be altered.

e.g. IF(NTURB.GT.10240) turbulent points

IF(NNTUB.GT.10240) non-turbulent points.

The .GT. is necessary as the program continues until the next multiple of 1024 points has been reached. The output is written in blocks of 1024 values.

The graph section has been by-passed in this version as so many points are analysed. No data cards are required for the program, but the following job control cards must be specified:

i) //G0.FT03F001 DD ...

is the unit from where the data is read

ii) //G0.FT04F001 DD ...

is the unit onto which the signal and its associated intermittency signal are written.

iii) Indicator showing whether a single or two series analysis is to be performed;

JA(3) = 0 Single series,

JA(3) = 1 Two series,

The figure below indicates how the data is to be arranged on the data cards.

Column	1	5	10	15	20	30	40
Card 1							
Quantity	N	NS	K1	NH	OVM	DT	
Format	I5	I5	I5	I5	F10,2	F10,2	
Column		2	4	6			
Card 2							
Quantity	JA(1)	JA(2)	JA(3)				
Format	I2	I2	I2				

Figure Giving Format of Data Cards

NOTE: Integer numbers must be right justified.

The input to NLBONE has been discussed as has its output and the path of analysis. It is now only necessary to describe the In-line procedures required to execute the program. The Program NLBONE contains all the Job Control Language to run the program in the conventional way.

PROGRAM NLSEEP

This program is used to separate the signals from the two wires of an X-array probe into the instantaneous values of the u and v velocity fluctuations. The user must supply the angles each wire makes with the mean-stream velocity as well as the directional sensitivity coefficient of each wire. Quantities calculated are:

- i) Number of blocks to be manipulated.
- ii) r.m.s. value u' the u component.
- iii) r.m.s. value v' the v component.
- iv) The Reynolds Shear Stress.
- v) r.m.s. value of the 1st signal before separation into components.
- vi) r.m.s. value of the 2nd signal before separation into components.

The instantaneous u and v fluctuation signals are written onto magnetic tape for later reference.

The program consists of 5 steps. To save time the program first transfers the blocks of data to be manipulated onto a temporary disc data set. Efficient system programs are used to perform this. The data are then read and separated into the u and v velocity components. These are written onto another temporary disc data set and then finally transferred to magnetic tape for storage purposes. The program assumes that the data read off magnetic tape is REAL*4 and blocked in blocks of 1024 values.

There is no In-line procedure for this routine and hence job control cards must be changed when analysing different sets of data.

Theory

The program reads 2 blocks of data, one from file 1 and the corresponding block from file 2. Equations (3-4) are then solved giving the instantaneous values u and v for the values of c_1 and c_2 used.

PROGRAM NLBAUTO

NLBAUTO is the name of the program as well as the In-line procedure used to execute NLBONE when a single series analysis is to be performed.

Parameters to be supplied:

- i) Name of Real *4 tape containing data.
TAPE =
- ii) Name of program to be executed.
PDS = NLBONE (in general)
- iii) File number to be used.
N =
- iv) Execution time required.
TT =
- v) Disposition of the temporary data sets (DEFINE FILE).
DF = NEW or OLD

NOTE:

- v) applies when the program is to be executed a number of times in the same JOB. The first time either NLBAUTO or NLBCROSS is executed DF = NEW. On all subsequent occasions DF = OLD.

The program uses 275 K of storage.

Ex/ Autocorrelations and Periodograms of the files 3 and 6 on NLB01 are to be calculated. Sample sizes are 2048, and the maximum number of samples to be analysed is 20. The execution time for each step is estimated at 15 minutes.

- i) 3 is to be Hanned. 6 must not.
- ii) Number of intervals is 30 in each case.

Values of k_1 , k_2 , α_1 and α_2 must be supplied by the user. The r.m.s. quantities are calculated from sample to sample but the taking of square roots is only done at the very end of the calculations. A parameter in the program is varied depending on the number of blocks to be analysed.

Job Control Statements

Steps ONE, TWO, FOUR and FIVE all execute the system program IEZGENER which transfers a file of data from one unit to another. The statement

```
//SYSUT1 DD ...
```

indicates the unit from which the data is to be read and

```
//SYSUT2 DD ...
```

indicates the unit onto which it must be written. In steps ONE and TWO the user must specify the correct parameters for //SYSUT1 and in steps FOUR and FIVE the parameters for //SYSUT2 DD ... must be specified.

The only parameter in step THREE that could need specification is the TIME.GO = parameter which depends on the number of blocks being analysed.

Data Cards

Only two data cards are required. The first contains 4 parameters in 4F10.2 format. They are:

- i) ALPHA1 = Angle between the hot-wire sensor and the mean-flow direction for channel 1 (in degrees).
- ii) ALPHA2 = same as above, but for channel 2.
- iii) SENS1 = Directional sensitivity coefficient for channel 1. It is defined in equation (3-2).
- iv) SENS2 = same as above, but for channel 2.

- iii) For 3 DVM = 1,7
DT = 0,002
- iv) For 6 DVM = 1,6
DT = 0,002

The JOB set-up would be as follows:

```
//NLBAUTO JOB -  
//NLBAUTO PROC QDS = NLBONE, TAPE = NLB01, TT = 15  
//NLBAUTO PENCJ  
//ONE EXEC NLBAUTO, N = 03, OF = NEW  
//SYSIN DD*  
2048 20 30 1 1,7 ,002  
030300  
//TWO EXEC NLBAUTO, N = 06, OF = OLD  
//SYSIN DD*  
2048 20 30 2 1,6 ,002  
060600
```

The second card contains 2 values in ZE12.5 format

AVE1 = Average value of channel 1.

AVE2 = Average value of channel 2.

PROGRAM NLBCROSS

NLBCROSS is the name of the program as well as the In-line procedure used to execute NLBONE when performing a two series analysis.

Parameters to be Supplied

- i) Name of Real *4 tape with both series.
DATA =
- ii) Name of program to be executed.
PDS =
- iii) Execution time required.
TT =
- iv) File number of X array.
X =
- v) File number of Y array.
Y =
- vi) Disposition of the temporary data sets.

DF = OLD or NEW (see NLBAUTO)

The data for NLBONE is as described in the section for NLBONE.

Ex/ Cross-correlations and Cross-spectra are required of Files 3 and 6 on NLB01. Both must be Hanned, otherwise the information is the same as the previous example. The JOB setup would be:

```
//NLBCROSS JOB -
```

```
//NLBCROSS PROC DATA = NLB01, PDS = NLBONE
```

```
//NLBCROSS PEND
```

```
// EXEC NLBCROSS, TT = 15, X = 03, Y = 06, DF = NEW
```

```
//SYSIN DD *
```

```
2048 20 30 1 1,7 ,002
```

```
030601
```

```
//
```

PROGRAM NLBOC:

This program is the only general routine written. It is fairly complex and will be discussed in some detail. The routine makes use of the subroutines already discussed and hence the theory behind them is not shown. The following functions are calculated:

- i) Various statistics of the samples.
- ii) The Periodogram.
- iii) The autocorrelation.
- iv) The Co-Spectra of two samples.
- v) The Quadrature-Spectrum of two samples.
- vi) The Cross-correlation.
- vii) The Amplitude Probability Density function.

The routine can be used to analyse a single time series in which case the quantities (iv), (v) and (vi) are not calculated. Briefly, the program reads a sample and calculates the quantities (i) to (vii) for the sample.

The results are then averaged with the results of previous samples and stored on a temporary disc data set. Depending on the number of samples already analysed or the result of the Student "t" test another sample is read or the program proceeds to print the results.

The results on disc can be read and stored on magnetic tape by NLBSTORE. NLBPLOT1 will plot single series analyses data (read off tape), while NLBPLOT2 will plot data from a two series calculation. The programs are discussed later.

The program is used in conjunction with two different Ir-line procedures, depending on whether one is doing a single series or two series manipulation. The single series procedure is called NLBAUTO

NOTE:

The program assumes that all the data applies to both signals
i.e. both have the same mean value.

The program uses 275K of storage.

Timing information:

<u>Sample Size</u>	<u>No. of Samples</u>	<u>Time</u>
2048	4	5 mins.

while that for two series calculations is called NLBCROSS. The program can also be executed using standard Job Control Language, a set of which is available.

Sample sizes are decided on by the user and can be either 1024, 2048, 4096 or 8192. The present form of the program cannot take samples larger than 8192 due to hardware limitations on the computer. Besides the final averaged results, some results for each sample are returned by the program. When calculating two series quantities, both signals must be on the same tape (but different files). Depending on the number of samples to be analysed, an appropriate quantity of the second file is transferred to disc to facilitate easy reading of the two files.

A simplified block diagram of the program is given below. All the necessary subroutines are stored in the Library ACM.NLB.LIB, as is the program NLBONE.

NOTE: Cross-correlations are given as $R_{XY}(\tau)$ where

X is the first series

Y is the second series

PROGRAM NLBSTORE

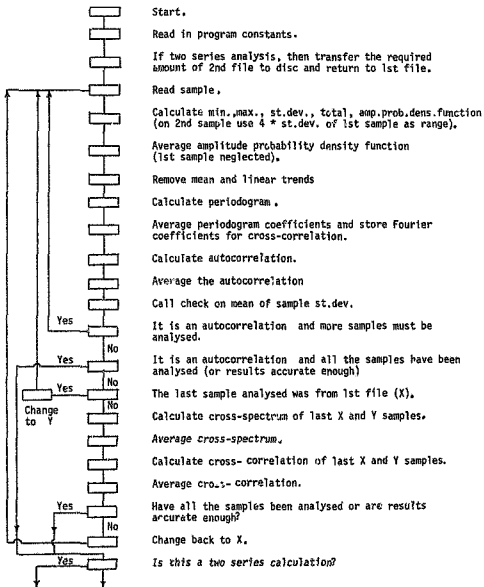
This program is designed to read the results on the temporary disc data sets and then write them onto magnetic tape. The results can then be manipulated or plotted at a later stage.

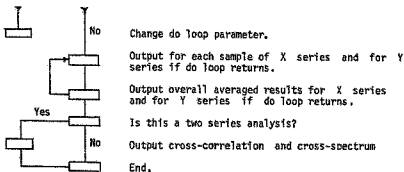
The in-line procedure NLBSTORE is used to execute the program. No data is necessary, as the program reads it off the disc. The Define File records of 4100 are read and depending on the sample size, divided into varying numbers of blocks of 1024. For convenience each record is divided into the same number of blocks, so that some of the blocks written contain nothing.

Ex/ N*BCROSS executed with $N = 4096$.

- i) $JA(3) = 1$ and hence two files to be read.
 - ii) 2048 Periodogram coefficients.
 - iii) Therefore 2 blocks of 1024 (sometimes 1 or 2 values in next block are neglected).
 - iv) 2 blocks from each record on DEFINE FILE 7 () are written onto magnetic tape, and then the same amount of DEFINE FILE 8 () written onto tape.
 - v) If $JA(3) = 0$ i.e. NLBAUTO then DEFINE FILE 8 () is neglected and records (6) and (7) on DEFINE FILE 7 () are not read since they would contain information from NLBCROSS.
- The parameters to be supplied in NLBSTORE are:
- i) Name of labelled tape.
 - TAPE =
 - ii) File onto which the results must be written.
 - FILE =
 - iii) Time of 2 minutes is specified.

The records containing the Fourier Coefficients are not transferred. As an example to illustrate the use of NLBAUTO, NLEBCROSS and NLBSTORE,

BLOCK DIAGRAM OF NLBONE

(1) Theory

The theory behind all the steps besides the decision making has been described elsewhere. To try and minimise the computer time used, a statistical check on the mean of the sample Standard Deviation is performed. For a more complete explanation of the theory and problems involved see F.R. Krause et al (1970). A maximum number of samples to be analysed is specified, but if the statistical test is satisfied before that stage then execution of the program is terminated.

In this program only the Standard Deviation of each sample is checked. The ideal situation would be where each of the quantities calculated has its own statistical test. As each test is satisfied so that quantity is not calculated any more. This would then give the lowest execution time.

(ii) Output

The program prints out information appertaining to each sample as well as overall properties. Various parameters as well as quantities such as the autocorrelation, Amplitude Probability Density function are stored on a temporary disc data set and can therefore be used by a program executed after MLBONE (as long as it is still in the same job). Firstly the information printed out will be described.

perform the example for NLBAUTO, writing answers of FILE 5 onto first file of RESULT. As the next step perform the example for NLBCROSS and write the results onto file 2 of RESULT.

The job setup would be:

```
//NLBALL JOB -
//NLBAUTO PROC PDS = NLBONE, TAPE = NLB01, TT = 15

//NLBAUTO PEND

//NLBCROSS PROC DATA = NLB01, PDS = NLBONE, TT = 15

//NLBCROSS PEND

//NLBSTORE PROC TAPE = RESULT

//NLBSTORE PEND

//ONE EXEC NLBAUTO, N = 03, DF = NEW
//SYSIN DD *
2048 20 30 1 1,7 ,002
030300

//TWO EXEC NLBAUTO, N = 06, DF = OLD
//SYSIN DD *
2048 20 30 2 1,7 ,002
060601

//FIV EXEC NLBSTORE, FILE = 01
//SIX EXEC NLBCROSS, X = 03, Y = 06, TT = 15, DF = NEW
//SYSIN DD *
2048 20 30 1 1,7 0,002
030601

//SEV EXEC NLBSTORE, FILE = 02
```

NOTE: NLBCROSS calculates the autocorrelation of 03 and 06 anyway.

Author Burnton Neil Lorraine

Name of thesis The analysis of turbulent flows using a digital computer, with special reference to the plane mixing layer.
1973

PUBLISHER:

University of the Witwatersrand, Johannesburg

©2013

LEGAL NOTICES:

Copyright Notice: All materials on the University of the Witwatersrand, Johannesburg Library website are protected by South African copyright law and may not be distributed, transmitted, displayed, or otherwise published in any format, without the prior written permission of the copyright owner.

Disclaimer and Terms of Use: Provided that you maintain all copyright and other notices contained therein, you may download material (one machine readable copy and one print copy per page) for your personal and/or educational non-commercial use only.

The University of the Witwatersrand, Johannesburg, is not responsible for any errors or omissions and excludes any and all liability for any errors in or omissions from the information on the Library website.