production, finite-element analysis computer model. Predictions of failure loads and bracing behaviour are reasonably good with both procedures, in spite of the fundamental differences in their approach to the problem, some of which are listed below.

- The PANEL model is a two-dimensional frame, where the conditions at the connections are simulated by in- and out-of-plane moments, which are equivalent to the actions of the eccentric forces. By contrast, the ABAQUS model is a three-dimensional frame, where the connections between main legs and bracing are given by beam-elements, which have lengths equivalent to the actual eccentricities.

- The PANEL model is supported at the four nodes connecting the main legs and the diagonals, while the ABAQUS model is supported at the bottom of the main legs, as in the test cases.

- PANEL assumes that the connections sustain elastic deformations about the strut's x-axis, which are represented by a spring coefficient S_{XS}. By contrast, ABAQUS assumes that the bolt elements have infinite flexural stiffness, thus simulating a fixed condition about the strut's x-axis.

- 1-bolt connections are given in both models by a pin-ended condition. For 2-bolt connections, PANEL assumes full restraint about the strut's y-axis, while the ABAQUS model simulates this condition through the torsional stiffness of the bolt elements.

Notwithstanding the above, a comparison of both models could be considered as a validation of PANEL by ABAQUS. More importantly, examination of predicted results from both models under similar

Table 6.03: ABAQUS and PANEL - Comparison of results

Test Designation	Comparison of results								Test results
	PANEL				ABAQUS				
	S_{xs}	$\frac{f_{ult}}{f_y}$	S_{xs}	$\frac{f_{ult}}{f_y}$	I_{zB}	$\frac{f_{ult}}{f_y}$	I_{zB}	$\frac{f_{ult}}{f_y}$	$\frac{f_{ult}}{f_y}$
1	[illegible]	3	4	5	6	7	8	9	10
102	12.061	0.422	F	0.527	1.0E 3	0.517	F	0.577	0.491
602	12.061	0.394	F	0.500	1.0E 3	0.446	F	0.458	0.431
801	3.618	0.280	F	0.367	1.0E-2	0.274	F	0.274	0.288
802	12.061	0.337	F	0.436	1.0E 3	0.348	F	0.431	0.336
804	22.558	0.348	F	0.430	1.0E 3	0.343	F	0.431	0.364

conditions may improve confidence on the use of either PANEL or ABAQUS as an alternative theoretical design procedure.

Results of such a comparison are given in Table 6.03 for a few cases representing various test alternatives, where conditions in the two models are equivalent after the following changes:

a) For 1- and 2-bolt connections, the spring coefficient S_{XS} in PANEL was given a large value, thus simulating full restraint about the strut's x-axis, which is the default condition of the ABAQUS model. Failure loads and the spring coefficients for the initial and modified conditions are given in in Columns 2-5.

b) For 2-bolt connections, the bolt-elements in the ABAQUS model were given a large torsional stiffness, thus simulating full restraint about the strut's y-axis, which is the default condition of the PANEL model. Failure loads and the torsional constants for the initial and modified conditions are given in in Columns 6-9.

The rest of the assumptions being similar, the following conclusions can be drawn from Table 6.03, Columns 3, 5, 7, 9:

Except for Test 801 with ABAQUS, all calculated failure loads are higher in both models for the new equivalent conditions. The rate of increase of failure load is similar for both models at higher levels of slenderness (Tests 802, 804), while it becomes more uncertain at lower levels of slenderness (Tests 102, 602).

- The mechanism of failure (i.e. yielding of the extreme fibre at the heel of the angle in the longest subspan of the strut) is unchanged for Tests 801-804 with ABAQUS, and for Tests 802-804 with PANEL. For Test 801 (1-bolt connection) with PANEL, first yield occurs at the longest subspan in the strut, but at the horizontal toe of the angle instead.

- The mechanism of failure is also different for the case of lower slenderness, but with further implications: for Test 102 with L/r=100, both models predict yielding of the extreme fibre at the shortest subspan of the strut. According to the ABAQUS model, this is followed by yielding of the tie at the cross-over joint, and finally yielding of the strut at the longest subspan.

  This behaviour is not necessarily incorrect, in view of the increased stiffness in the bracing imposed by the new conditions. At low levels of slenderness and with increased rotational restrictions, the axial force and the torsional effect in the diagonals become dominant over the bending effect, which may well induce failure of the struts at the shortest subspan by a more complex mode of buckling.

- In all cases, the above failure characteristics are accompanied by a general reduction of in- and out-of-plane deflections, and also of nodal rotations.

It can be concluded, therefore, that the flexibility model PANEL can be used, within the range of parameters defined in the present investigation, for design of cross-bracing with 2-bolt connections. The model for 1-bolt connections requires further investigation.

6.5 - Use of ABAQUS for alternative analyses

It remains to perform analyses on a typical frame with cross-bracing in order to establish the behaviour and maximum resistance of the diagonals under several conditions. Many of these conditions were not included in the set of tests and in the computer analyses of this investigation, but are commonly found in most frames of actual steel transmission towers.

ABAQUS was selected to model these alternatives, for the following reasons:

- It is easier to introduce changes to the ABAQUS models. For example, the positions of the bracings can be changed simply by modifying the direction cosines of the section's minor axis, or by inverting the end eccentricity. PANEL, on the other hand, was programmed only for the position of the main legs and bracings shown in Figure 5.01 of Chapter 5. Any important changes to the model will require reprogramming of at least some of its modules and/or subroutines.

- It is of interest to examine the response of the models for various alternatives, and evaluate the applicability of ABAQUS as a tool for future design of steel transmission towers.

Several cases are examined in the following Sections, therefore, making use of ABAQUS finite-element models.

6.5.1 - Cases A and B: Conditions at the ends of the main legs for 1- and 2-bolt connections

As seen in the previous Chapters, the main leg supports of the frames in the present investigation were simulated, by definition, as pin ended in both the experimental and computer models. This, of course, is not the case on actual tower frames, where these members are connected either to foundations or to other parts of the structure. Tests by Behncke [11] and CIGRE [63] have simulated this, by connecting the main legs to transverse beams, or to rigid plates, see Figures 1.08 and 2.01.

These conditions have been simulated with ABAQUS, and the results are included in Table 6.04, where Case A1 in Column 2 indicates the base case (Test 801). The yield stress and maximum calculated torsional rotation of the main leg are given in Columns 3 and 4 respectively. The calculated failure loads are given in Column 5.

Case A2 indicates a frame with transverse beams at the top and bottom sections of the structure, as in Figure 1.08. The beams are rigidly connected to the main legs, and also prevent out-of-plane displacements of the outside cross-over joints. The main legs are not, however, constrained against torsional rotations by the supports. It is seen that the failure load increases by 12% with respect to the base case.

Table 6.04: ABAQUS - Case A: End condition, 1-bolt

Test Designation	ABAQUS				Test results
	Case	f_y (MPa)	θ_{zL} (°)	$\frac{f_{ult}}{f_y}$	$\frac{f_{ult}}{f_y}$
1	2	3	4	5	6
801	A1	333	2.420	0.274	0.288
	A2	333	1.034	0.307	
	A3	333	2.262	0.281	

Table 6.05: ABAQUS - Case B: End condition, 2-bolt

Test Designation	ABAQUS				Test results
	Case	f_y (MPa)	θ_{zL} (°)	$\frac{f_{ult}}{f_y}$	$\frac{f_{ult}}{f_y}$
1	2	3	4	5	6
802	B1	321	2.951	0.348	0.336
	B2	321	1.284	0.406	
	B3	321	2.826	0.354	

Case A3 corresponds to a frame without transverse beams, but the main legs are restrained against torsional rotations at the supports. The outside cross-over joints are free and can thus displace out of the plane, at the rate established in Chapter 3 for these tests. Column 5 of Table 6.04 shows an increment of failure load of less than 3% for Case A3.

For 2-bolt connections, the results of these analyses are given in Table 6.05, where Case B1 is the base case (Test 802), Case B2 indicates a frame with transverse beams, and Case B3 indicates main legs with full rotational restraint. It is seen that the failure load increases by 17% for Case B2, and by a lower 2% for Case B3.

Observing the main leg rotations in Column 4 of Tables 6.04 and 6.05, it can be seen that the transverse beams (and thus fixed cross-over joints in the outside panels of bracing in Cases A2 and B2) impose considerable torsional restraint on the main legs, resulting in an increase of failure load.

Cases A3 and B3 for 1- and 2-bolts, on the other hand, in which the main legs are fixed at the supports (but the outside bracings are able to move out of the plane), result in almost no increase of failure load.

The above deductions highlight the importance of the out-of-plane deflections of the cross-over joints in the outside panels of bracing. Since the transverse beams cannot impose a torsional restraint to the main legs higher than the fixed-support condition, it is concluded that the reduced rotations at the main leg-diagonal connection (and thus the improved strength of the bracing) are induced by the effect of out-of-plane deflections of the outside cross-over joints. This effect should be evaluated in future research.

6.5.2 - Case C: Inverted diagonals

Test 807, with inverted diagonals in the outside panels of bracing, is simulated with ABAQUS and compared with Test 801 in Table 6.06, where z_b in Column 4 indicates 1-bolt connection. These frames are shown in Figures 3.06-a and 3.06-b of Chapter 3.

The calculated failure loads in Column . show that the model can

Table 6.06: ABAQUS - Case C: Inverted diagonals

Test Designation	ABAQUS				Test results
	Case	f_y (MPa)	z_b	$\frac{f_{ult}}{f_y}$	$\frac{f_{ult}}{f_y}$
1	2	3	4	5	6
801	C1	333	1	0.274	0.288
807	C2	329	1	0.301	0.324

predict, with a good approximation, the increase of bracing strength produced by the change of position of the outside diagonals. An increase of 10% of failure load is anticipated, against an increase of 13% recorded in the test.

The experimental and calculated deflections at midspan and at the cross-over joint in the strut are shown in Figure 6.10-a. Note that the predicted in- and out-of-plane deflections are correct.

The strut-end rotations at node 16 in Figure 6.01-b are depicted in Figure 6.10-b, which shows that the calculated rotations are considerably smaller than the experimental records. Similarly, the main leg's torsional rotations at node 14, Figure 6.01-b, are smaller than in the tests, see Figure 6.10-c.

It is concluded that the ABAQUS model can be used to simulate this particular bracing arrangement, opening the way to further investigations on this interesting bracing arrangement.

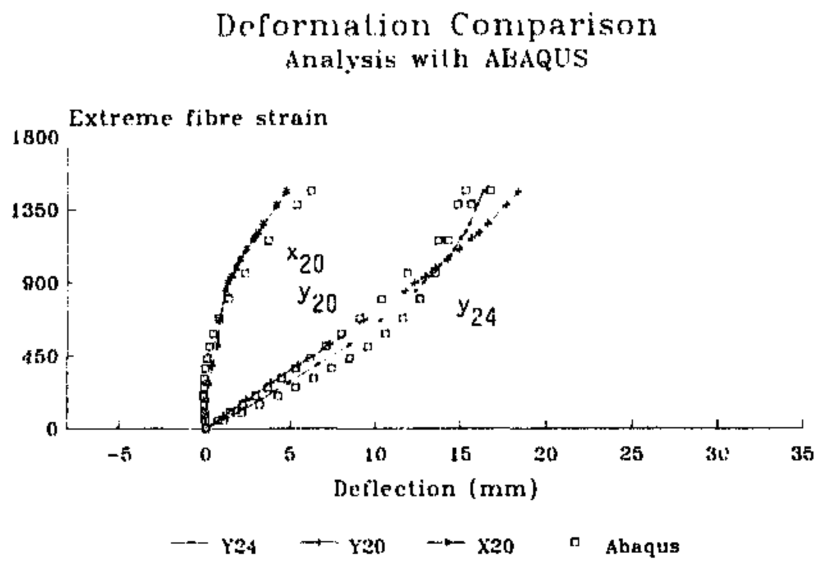


Figure 6.10-a: Vertical and horizontal strut deflections, see Figure 6.01
Test 807

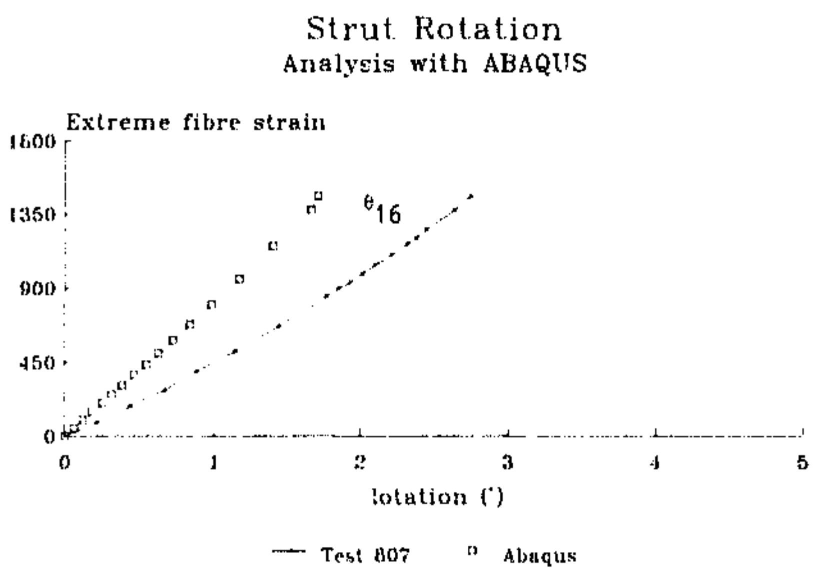


Figure 6.10-b: Strut's rotational response at node 16, see Figure 6.01
Test 807

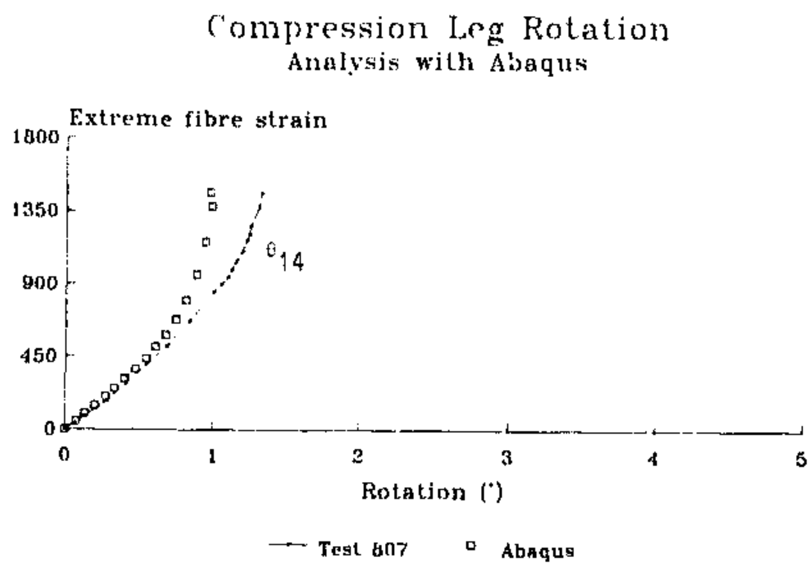


Figure 6.10-c: Main leg's rotational response at node 14, see Figure 6.01
Test 807

6.5.3 - Case D: Variable ratio of tension to compression forces in the diagonals

The experimental frames described in Chapter 2 were assembled with careful detail, in order to secure, at least in principle, equal and opposite forces in the diagonals. Figure 3.22 in Chapter 3 shows a typical distribution of forces in the tie and the strut for the present tests, where it can be seen that these forces were nearly equal (and of opposite sign) throughout the loading history, thus eliminating one unknown effect. Similarly, all the models, including PANEL and ABAQUS, were analyzed on the basis of a condition of equal-bracing-forces.

Evidence from tests of prototype towers exists [1], however, where the magnitude of diagonal forces was found to be non-equal and the bracing forces were of the same sign (i.e. both members in compression). This is a troubling situation, since the conditions of support to the strut at the cross-over joint, and therefore the stability of the bracing, depend on this ratio of tension to compression forces.

Consequently, an exercise was conducted on a typical frame modelled with ABAQUS, in which the ratio of tension to compression forces in the bracing was varied between 60% and 120% with respect to the base case, and the results are presented in Table 6.07 below.

Column 4 of Table 6.07 indicates 2-bolt connections in all cases, and the ratio of tie (P_T) to strut (P_C) forces is given in Column 5. Column 6 indicates the total horizontal force at the cross-over joint at the point of first yield in the strut for each case, and the failure loads are given in Column 7.

When the axial force in the tie is 20% higher than in the strut the strut's failure load decreases, as shown in Table 6.07, Column 7. However, the total force capacity of the bracing increases, as shown by the cross-over horizontal force in Column 6.

The out-of plane deflections at midspan and at the cross-over joint in the strut are shown in Figure 6.11-a. Note that both deflections are larger than in the base case, which explains the reduction of strength of the strut.

When the axial force in the tie is 15% less than the axial force in the strut, the strut's failure load decreases, and so does the total force capacity of the bracing. If the force ratio is

Table 6.07: ABAQUS - Case D: Variable ratio of tension to compression forces in the diagonals

Test Designation	ABAQUS						Test results
	Case	f_y (MPa)	z_b	$\frac{P_T}{P_C}$	ΣH (kN)	$\frac{f_{ult}}{f_y}$	$\frac{f_{ult}}{f_y}$
1	2	3	4	5	6	7	8
802	D1	321	2	-1.00	40.391	0.348	0.336
	D2	321	2	-1.20	43.836	0.346	
	D3	321	2	-0.85	37.135	0.345	
	D4	321	2	-0.70	35.130	0.347	
	D5	321	2	-0.60	32.073	0.357	

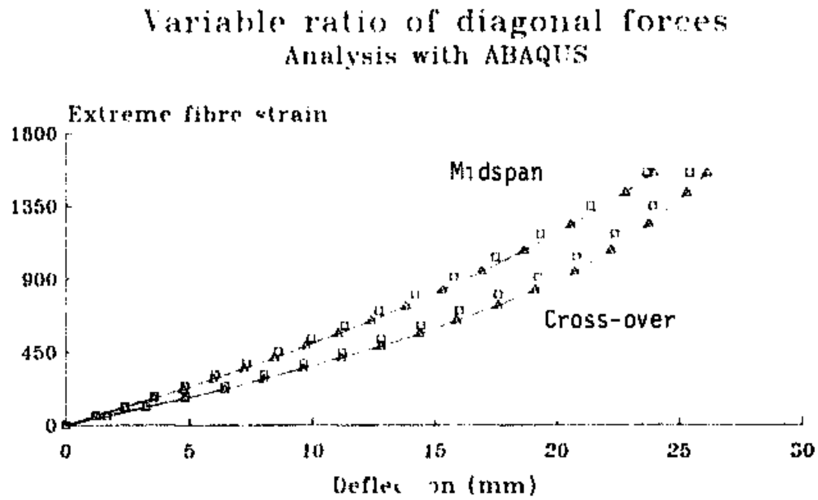
reduced again, say to 70 and 60% of the base case, the strength of the strut reduces to a minimum and then increases, while the total strength of the bracing continues to decrease.

The out-of plane deflections at midspan and at the cross-over joint in the strut are shown in Figure 6.11-b for the case of force ratio of P_T/P_C=-0.6. Note that both deflections are smaller than in the base case, which explains the increase of strength of the strut.

The computer model shows, therefore, that reductions of the force ratio may increase the load capacity of the strut, through smaller out-of-plane deflections of the bracing. The opposite result is produced by increases of the force ratio. These results, however, need experimental confirmation.

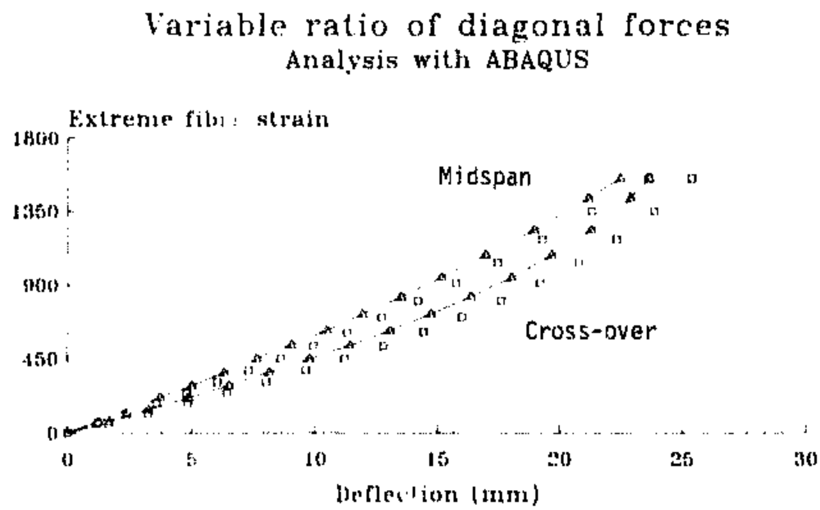
6.5.4 - Case E: Sensitivity analysis of conditions at the bolted connections

The important conditions at the bolted connections in frames with cross-bracing are simulated with ABAQUS through four



Pt/Pc=-1.0 Pt/Pc=-1.2

Figure 6.11-a: Out-of-plane strut deflections
Pt/Pc=-1.2



Pt/Pc=-1.0 Pt/Pc=-0.6

Figure 6.11-b: Out-of-plane strut deflections
Pt/Pc=-0.6

parameters, as indicated below:

- In-plane eccentricity e_{xB}, see Figure 6.03,
- Out-of-plane eccentricity e_{yB}, also shown in Figure 6.03,
- Flexural stiffness of the bolt elements, I_{xB}, and
- Torsional stiffness of the bolt elements, I_{zB}.

A sensitivity analysis was conducted on a typical ABAQUS cross-bracing case (Test 804), varying each of the above variables, one at a time, between 50 and 150% of the value calibrated for the analyses of tests in the previous Sections. The results can be summarized as follows:

- Failure loads are plotted in Figure 6.12-a against variations of the in-plane eccentricity e_{xB}, see also Figure 6.03. It can be seen that a reduction of eccentricity results in higher failure loads. The opposite occurs when the eccentricity e_{xB} is increased.

- Failure loads for variations of the out-of-plane eccentricity e_{yB}, see Figure 6.03, are given in Figure 6.12-b. As in the previous case, increments of e_{yB} result in lower failure loads, and higher failure loads result from reductions of eccentricity.

  These results are not surprising, since increments of end eccentricity induce larger in- and out-of-plane deflections in the bracings and larger nodal rotations, which in turn increase the bending effect, thus reducing the strength of the diagonals. Also, note that the eccentricities assumed for the analyses in the present investigation are the normal

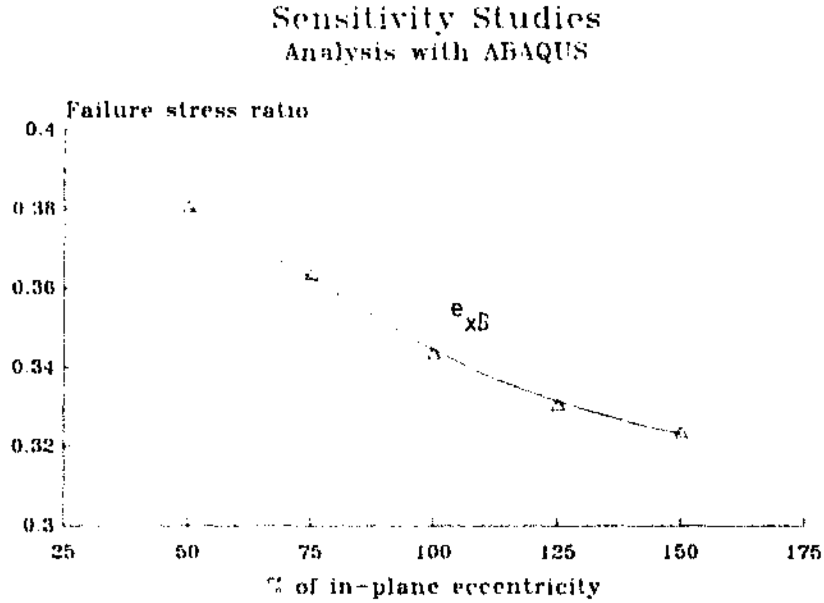


Figure 6.12-a: Effect of variable
in-plane eccentricity

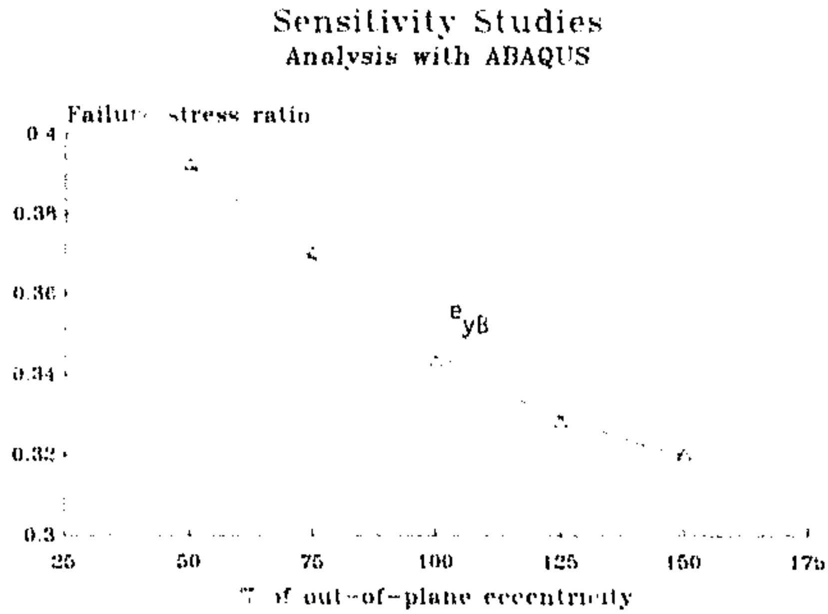


Figure 6.12-b. Effect of variable
out-of-plane eccentricity

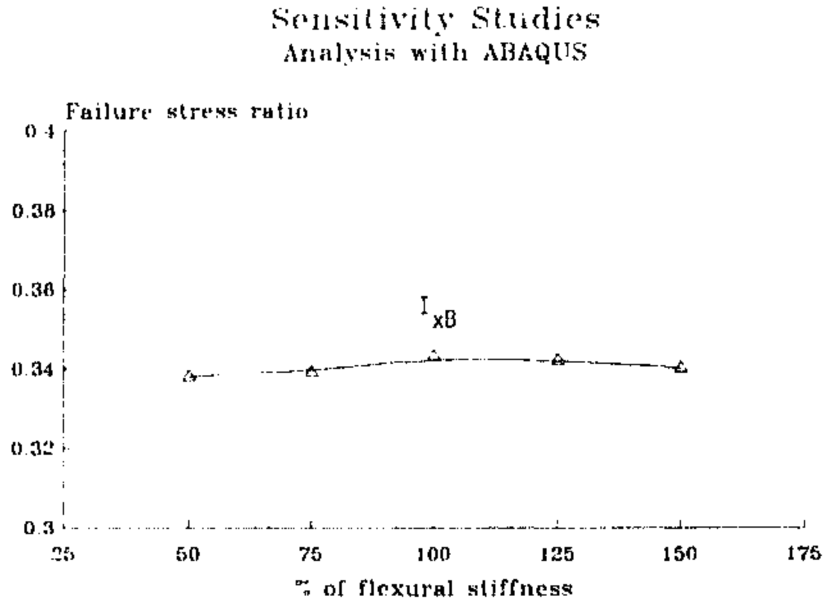


Figure 6.12-c: Effect of variable flexural stiffness (about x-axis)

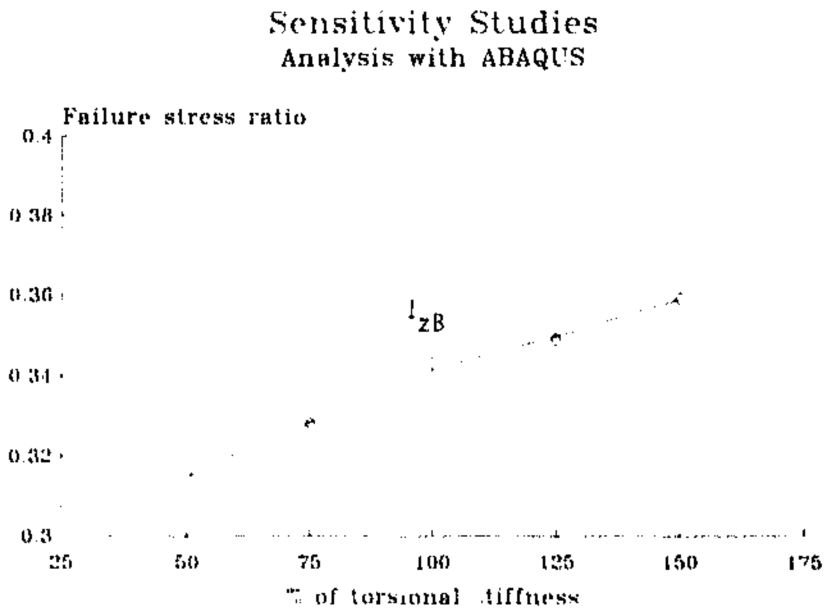


Figure 6.12-d: Effect of variable torsional stiffness (about the z-axis)

framing eccentricities, namely the distances between the section's centroidal axis and the design axis, or axis at which the bolts are located, see Figure 6.03.

- Failure loads are given in Figure 6.12-c for the case of variations of the bolt elements' flexural stiffness. The curve in Figure 6.12-c shows that there is virtually no change in the strength of the bracings for these variations. Again, this is consistent, given the short length of these bolt members. The stiffness of 1.0E5 mm^4 given to these members clearly represents a fully-fixed condition about the x-axis, see Figure 6.03.

- Finally, failure loads are given in Figure 6.12-d for variations of the bolt members' torsional stiffness. The curve shows that failure loads increase for increments of the torsional stiffness of the bolts. This effect is consistent with the higher in-plane end restraint given by more rigid connections.

The problem of torsional stiffness of the bolt elements in the ABAQUS model, however, deserves further attention, as explained in the next Section.

6.5.5 - Further comments on bolted connections

Failure loads are indicated in Table 6.06, Column 5, for various in-plane end conditions simulated with ABAQUS. In the case of Test 801, with 1-bolt connections, the bolt elements were given a very small torsional stiffness, see Column 3, thus

Table 5.08: ABAQUS - Analysis of connection restraint

Test Designation	ABAQUS Analysis of connection restraint				Test results
	I_{xB} (mm^4)	I_{zB} (mm^4)	θ_{zB} (°)	$\frac{f_{ult}}{f_y}$	$\frac{f_{ult}}{f_y}$
1	2	3	4	5	6
801	1.0E 5	1.0E-2	9.7E-1	0.274	0.288
802	1.0E 5	1.0E 5	8.6E-3	0.431	0.336
802	1.0E 5	1.0E 3	6.6E-1	0.348	0.336

representing a pin-end condition about the y-axis, see Figure 6.03.

This might be a conservative assumption, since the actual 1-bolt connection may, depending on the torque given to the bolts, have a higher torsional restraint. But, from the design point of view, it is a safe condition, as shown by the calculated and experimental failure loads, Columns 5-6. Also, the angle rotated by the connection (between the strut and the main leg, nodes 14 and 16 in Figure 6.01-a) about the y-axis is given in Column 4 of Table 6.08.

The case of 2-bolt connections was originally modelled as a fixed-end condition, as in the second line of Table 6.08 for Test 802, by giving to the bolt elements a very large torsional stiffness, see Column 3. However, the calculated failure loads which resulted were considerably higher than the experimental results, see Columns 5 and 6. Also, note in Column 4 that the angle rotated by the connection is practically equal to zero.

As a consequence, the torsional stiffness of the bolt elements was calibrated using the recorded experimental failure loads and behaviour of the frames, obtaining a reduced torsional stiffness, see Column 3 for Test 802 in the third line of Table

6.08. Note that the connection also rotates about the y-axis, see Column 4.

The ABAQUS model indicates, through the above results, the influence of yet another variable of bolted connections: bolt slippage. Transmission tower assembly and erection commonly takes place in the field, with the help of sometimes untrained crews and rudimentary tools. Precise detailing and specification for bolt tensioning of the towers may render the field operations expensive or even impossible. The connections are therefore detailed with generous tolerances, in which the diameter of the holes is usually larger than the diameter of the fasteners, by 1.5 mm or more.

Also, the bolt-tightening operation is very difficult to control, and each bolt is likely to be given a different torque, even within the same connection. As a result of these particular characteristics of steel tower detailing and construction, the bolts and the connected members move at increasing loads, until they settle into a new position.

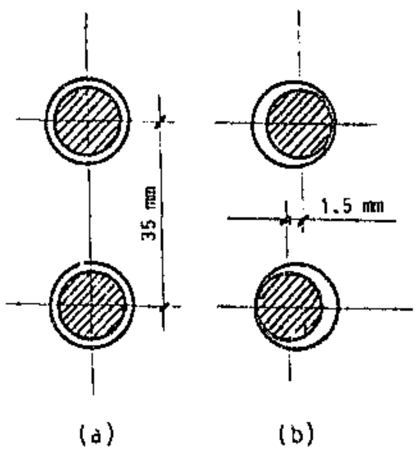


Figure 6.13: Bolt slippage: relative movements at the bolted connections, due to design tolerances. The holes are as a rule 1.5 to 2.0 mm bigger than the diameter of the bolts.

As an example, a connection is shown in Figure 6.13-a, where it is assumed that the bolts and the holes are initially concentric. The tolerance of the holes is 1.5 mm, therefore the total relative displacement of the connected members can be a minimum of 0.0 mm, and a maximum of 3.0 mm. Assuming an average relative displacement of 1.5 mm, as in Figure 6.13-b, the additional rotation of the member respect to the joint is about 2.5°.

It is apparent that these displacements of the connected members have a significant influence on the behaviour and resistance of the struts. It is generally accepted in industry that the resistance of the towers has, in the best of cases, a 10% dispersion due to design and construction tolerances. Bolt-slipping may explain, in no small amount, many of the differences observed in the experimental results.

6.6 - Summary

A non-linear model of cross-bracing, developed using the finite elements code ABAQUS, has been discussed in the previous Sections. Unlike most other standard packages for structural design, ABAQUS allows for the use of non-symmetric beam elements about non-orthogonal axes for non-linear frame analyses. This represents a significant development in structural design, and opens the way to the simulation of more complex bracing arrangements, with a minimum of simplifying assumptions.

Most of the design conditions of the test frames are included in the ABAQUS models. In particular, the bolted connections are represented by beam elements, with lengths defined by the normal framing eccentricities of the diagonals. The end conditions are introduced through the flexural and torsional stiffnesses of these elements.

Various alternative cases are studied, which show the behaviour of the frames under conditions not included in the experimental research. These additional studies show the importance of the out-of plane displacements of the cross-over joints of the adjacent panels of bracing. It is demonstrated that the strength of the bracing increases considerably when these nodes are fixed.

Similarly, variations of the ratio of forces in the diagonals influence the buckling capacity of the struts, while also affecting the total amount of force in the bracing, measured as the total horizontal load at the central cross-over joint. These results have been anticipated by Elmes [51].

Most importantly, the effect of bolt slippage due to typical steel tower bolt-hole clearances has been demonstrated by the ABAQUS models. It is apparent that in-plane rotations of the connections are partially due to this effect, which has an important influence on the behaviour of the struts. The formulation of these problems is relatively simple, although further investigation is required before being able to model these random variables for design purposes.

The predictions of failure load, and also deflections and nodal rotations, are fairly accurate, in view of the above uncertainties. As is the case of the flexibility model PANEL, however, there are problems associated with modelling the behaviour of the main legs. It is apparent that the simulation of the main leg requires further analysis. However, the conditions at the connections are already complex, refer to Figure 6.03, before introducing additional eccentricities about the main leg's principal axes.

The above analyses close the present investigation on cross-bracing in steel transmission towers. A final discussion and evaluation of results are included in the following Chapter.

CHAPTER 7

SUMMARY AND CONCLUSIONS

7.1 - Introduction

Experimental and theoretical investigations on cross-bracing typical of steel transmission towers have been presented. Transmission towers are amongst the simplest of steel structures. Their designs are simple because assembly and erection usually take place in remote areas. Also, members and connections are detailed so as to ease manufacturing, transport and delivery operations, thus reducing the costs.

However, many years of international experience on tower design and testing at various levels of voltage demonstrate that the behaviour of these structures is complex and difficult to predict with analytical models. The interactions between the tower members, usually steel angles connected eccentrically, induce non-linear effects at all levels of load. Further, as most of the members are critically loaded in compression, the secondary effects have considerable influence on the ultimate capacity of the bracings.

This complexity, as an analysis of existing research shows, means that adequate resolution of design problems for three-dimensional arrangements is unlikely to be achieved, especially if material, geometric and boundary non-linearities are considered together. Therefore, the major part of research in the past has concentrated on single angles under uniaxial or biaxial bending, with simulation of various boundary conditions.

Other investigations on simplified structures, as well as the accumulated experience of full-scale tower tests, have resulted in the development of design curves and design equations which are applied for different loading and end conditions of the tower members. These are, however, general solutions and for some conditions in the structures the models are unreliable. Studies reported by Kempner [72], for example, show differences between test results and predictions of buckling load from the ASCE [5] design curves for critical tower members. Similar differences have been observed by Behncke [11], and are also apparent from results of the present investigation, for the case of cross-bracing.

Accordingly, the objectives of the present investigation were to identify and quantify the effects of eccentricity of axial load and end restraint from connections on the behaviour of steel angle crossed-diagonals.

Of the possible bracing arrangements in wer panels, the cross-bracing system is the most interesting alternative from the research point of view, because of the interaction between the two diagonals at the cross-over joint, loaded with approximately equal and opposite forces. Thus in addition to the end-connection details the behaviour of the strut and, indeed, the stability of the system, depend on the distribution of forces in the racing.

As it has been observed in this and other investigations [11,20-22], steel angle members fail shortly after yielding of the extreme fibre. As a consequence, the bracings were analyzed within the elastic range, and it was assumed in all models that failure occurs at the axial load which causes the initiation of yielding in the extreme fibres.

No attempt was made to determine the influence of initial imperfections (e.g. residual stress, scatter of yield stress) on the mechanical properties of the material. Because of the predominance of end eccentricities in typical connections between steel angle members in transmission towers (reflected by the observed in- and off-plane deflection patterns of the bracing), the effects of these random factors were included in the models by means of empirical equivalent eccentricities, see [6,33,34].

The principal results of the experimental and analytical investigations in this thesis are summarized in the following Sections. The results are also applicable to other types of steel latticed-tower structures with angle members.

7.2 - Experimental research

A series of tests was carried out on various bracing and frame arrangements. The main variables were the slenderness ratio L/r (between 100 and 160), the length ratio L_s/L_g (between 0.7 and 1.0), and the end conditions of the diagonals (1- and 2-bolt connections, and two sizes of main legs).

The experimental results in this thesis yield the following observations:

- The strain-deflection characteristics of the bracings are typical of single-angle strut under biaxial loading, as follows. The deflection of the bracing occurs initially in the out-of-plane direction due to end eccentricity. Before failure, however, predominant deflections are about the minor axis of the section. The deflection patterns are further influenced by variations of the slenderness ratio L/r, the length coefficient L_s/L_g, and the end conditions.

- In all cases failure of the strut occurs at a load within 10% of the load causing yielding of the extreme fibre, thus indicating inelastic collapse. At this moment the strains in the tie are well below the yield limit for diagonals with slenderness of 130 and 160. At slenderness of 100, however, yielding in the tie at the cross-over joint occurs first, and the subsequent loss of strength in the bracing (and reduced out-of-plane support from the tie at the interconnecting joint) leads to premature failure of the strut.

- Variations of the bracing length ratio $r=L_s/L_g$ have influence on the behaviour of the diagonals. It is apparent that diagonals with low length ratio r are subject to a higher restraining action from the shortest subspan L_s, while diagonals with r=1 are subject to bifurcation between symmetrical and asymmetrical buckling loads.

- The resistance of the bracing increases when the end restraints are increased. Increasing the number of connecting bolts from one to two provides additional restraint about the v-axis. The same effect is observed about the y-axis for larger size of main leg relative to bracing (and therefore enhanced flexural stiffness) for 2-bolt connections. Consequently, the higher resistance of the struts seems to be related to lower bending effects about the minor axis at the critical section, through larger in-plane deflections.

- The rotation readings at the nodes indicate, for the conditions of the bracings in this investigation, that there is little or no torsional restraint from the main legs. More importantly, it is demonstrated that the actual nodal rotations are always smaller than the recorded strut-end rotations about the x-axis, thus indicating elastic deformation of the connections. These can be caused by local bending of the connected flange of the main legs. Bolt distortions and bolt slippage could also be significant factors.

- Addition of redundant members to the cross-bracing generally results in improvements of the failure loads. However, the deflection and rotation readings are uncertain, and are not suitable for use in development of theoretical or empirical models.

- It was not possible to perform complete tests on systems of locked-in diagonals with the existing test rig. Results from many trial tests are uncertain, and thus cannot be used to describe the main characteristics of the bracings.

These problems have been identified during previous research by Kemp [50] and Elmes [51], where it was demonstrated that some of the load carried by the legs is transferred to the diagonals in the locked-in cross-bracing, see Figure 3.07. As a result, the force in the compression diagonal is larger than in the tension bracing, and the node at the cross-over joint is enforced less effectively. Premature buckling of the strut may result from the additional axial force and the reduction of lateral support.

The uncertain distribution of force in the bracing and the reduced strength of the strut of locked-in systems have been highlighted in this investigation.

- The end eccentricity of the bolted connections relative to the centroid of the angles causes the cross-over joint of both diagonals to deflect in the same out-of-plane direction. While the cross-over joint in the central panel of bracing was always free to deflect, the deflections of the cross-over joints in the outside panels of bracing were simulated in the experimental investigation, see Figures 2.06-a and 2.06-c of Chapter

  Subsequent tests on frames with locked-in diagonals demonstrated that the assumed rate of out-of-plane deflections of the cross-over joints in the outside panels of bracing was incorrect. However, it was found that these deflections have an important effect on the behaviour of the main diagonals.

- The Southwell-plot procedure and the secant formula can be used to determine the end eccentricity of the struts, the elastic buckling load and an effective length coefficient. Since the deflection readings just prior to first yield are in some cases uncertain due to movements of the instruments or their supports, the Southwell-plot is modified, and is expressed as a function of axial forces and bending moments, which are obtained from strain readings.

7.3 - Design considerations

Based on results of the experimental investigation, a parametric study within the elastic range was conducted to determine effective length factors suitable for design of diagonal struts. Empirical equations are given for these factors, incorporating the important variables which influence cross-bracing behaviour, as follows:

- Relative sizes of diagonals and main legs,
- End conditions of the diagonals,
- Direction of the plane containing the main restraint,
- Length ratio Ls/Lg, and
- Slenderness ratio L/r.

The proposed solution is simple, and includes non-linear effects at increasing loads through the trigonometric factor in the secant formula.

Comparing test results and predicted results from the proposed model with usual design curves from the ASCE [5] and ECCS [7] design manuals, the following conclusion are drawn:

- The ASCE and ECCS design curves are increasingly conservative at higher slenderness ratios, while at lower slenderness ratios these curves give optimistic failure loads (as shown in Figure 4.04 of Chapter 4). These differences have created concern, and are being investigated, see Kempner [72].

- The ASCE and ECCS design curves do not include some important effects, such as relative sizes of connected members, and their inclinations.

The proposed solution gives acceptable predictions of failure loads for all the test alternatives. When the frame and bracing arrangements are too different from the models in this investigation, the errors in the calculated results are of the same order of those obtained with the ASCE and ECCS curves.

It is therefore concluded that the proposed equations can be used for design of steel angle cross bracing with slenderness ratios between 90 and 160, and for 1- and 2-bolt connections.

7.4 - Computer analyses

A computer model of planar frames with cross-bracing, PANEL, was developed based on flexibility equations which do not require an iterative analysis procedure. The non-linear effects are given through the inclusion of Berry stability functions. The models reproduce the conditions of the bracings in the experimental frames.

The following conclusions are derived from these flexibility analyses:

- The models give reasonable predictions of failure loads for all test alternatives.

- The computed in- and out of plane deflections are similar to the recorded deflections at both midspan and at the cross-over joints.

- Similarly, the calculated nodal rotations are very close to the recorded rotations, especially in frames with parallel legs.

- The predicted failure mode is in all cases correct for frames with inclined legs. For frames with parallel legs, however, failure of the struts occurs theoretically almost simultaneously at both sides of the cross-over joint, which is possibly correct for the conditions in the model, but is not in agreement with the experimental results.

Two problems are not completely resolved in this model:

- The predicted behaviour of bracings with 1-bolt connections is not accurate enough (although the 1-bolt test results are equally uncertain). Results are uncertain or incorrect for alternative frame configurations. It is apparent that the proposed modelling of 1-bolt connections between the main legs and the diagonals, in which it is assumed that there is no restraint from the main leg about the y-axis, is not the best approach and could be improved through further research.

- The computed torsional rotations of the main legs are considerably larger than the test records, particularly for the case of 1-bolt connections. These differences are attributed to the modelling of the main legs, in which no eccentricities were considered about any of the leg's principal or longitudinal axes. They may also be related to the problems mentioned above for 1-bolt connections.

In spite of the above, PANEL makes a valuable contribution to resolving most of the complex problems presented by cross-bracing. In particular, it has helped to explain how in-and out-of-plane conditions at the connections about orthogonal axes can be modelled in conjunction with conditions about the principal axes at midspan in the diagonals. The diagonals' interaction at the cross-over joints and evaluations of the cross-over forces in each of the panels are also successfully modelled.

An additional model was developed using the finite-element code ABAQUS. Like PANEL, this model incorporates most of the important characteristics of cross-bracing in transmission towers, allowing, perhaps for the first time for this type of general design code, for non-linear analysis of asymmetric members.

The predicted failure loads and bracing behaviour with ABAQUS are generally correct. Torsional rotations of the legs in frames with 1-bolt connections also show differences with respect to the experimental results. It is apparent that modelling of the main leg conditions at the connections deserves further attention

A comparison of both ABAQUS and PANEL cross-bracing models shows that they predict, under similar conditions, similar behaviour and failure loads of various bracing arrangements. These results thus validate the PANEL models for 2-bolt connections, and confirm the observed differences for the cases with 1-bolt connections.

Further analyses with ABAQUS show the importance of the following parameters:

- The out-of-plane deflections of cross-over joints in the adjacent panels of cross-bracing influence the torsional rotations of the main legs, and thus affect the resistance of the main diagonals.

- The ratio of tie-to-strut forces affects the buckling resistance of the struts, and also the total amount of force in the bracing, measured as the transverse in-plane force at the cross-over joint.

- Additional strength to the bracing is obtained when the diagonals in the outside panels of bracing are inverted with respect to their normal positions. It is apparent, by doing this, that moments at the ends of two connected diagonals have a balancing effect (see for example Figure 5.07-a in Chapter 5, where it is shown that the end moments M_{jxs} and M_{jxt} projected on the leg's longitudinal axis will have opposite directions if one of the diagonals is inverted). This effect induces lower levels of nodal rotations and midspan deflections in the bracings, which results in higher failure loads.

Most of the above alternatives, however, need further experimental confirmation.

Finally, the importance of bolt movements due to design tolerances and clearances are highlighted by the ABAQUS models. It emerges that bolt slippage has considerable influence on the in-plane end rotations of the diagonals, which in turn have an effect on the loading capacity of the bracings. Bolt slippage may well explain many of the differences observed in the experimental models, and which are so difficult to simulate by theoretical means.

In conclusion, these studies show that no matter which approach, theory or procedure is used, the analysis of cross-bracing diagonals under axial load and bending effects is an extremely complex issue. The proposed solutions have been formulated as functions of boundary conditions, size and strength of the members and loading. A more sophisticated approach would require au. ed methodology such as based on beam-column theory.

It is possible, however, that a general, unique solution to these problems, even after considerable simplifications, should not be expected.

7.5 - Conclusion

Cross-bracing in panels of steel transmission towers have been examined in this thesis. The importance of the end eccentricities and restraints at the bolted connections have been demonstrated through an experimental investigation, a simplified design model and computer analyses. The pri. ipal contributions o this thesis are listed below:

- Very high quality data were obtained through an elaborate system of supports, assembly and adjustments of the test facilities, and data recording procedure. It was therefore possible to isolate the various parameters and their effects on the bracing under several conditions.

- End eccentricities and effective length factors were obtained from experimental results using the Southwell-plot and the secant formula.

- A simplified design model is introduced, which takes into account the parameters which influence cross-bracing behaviour. Many of these conditions are not considered in buckling curves given in design manuals currently in use. The proposed design model can be used to design steel angle cross-bracing of typical tower panels.

- A flexibility analysis demonstrates that it is possible to model non-linear cross-bracing systems inclusive of the interactions between the diagonals and the main legs, between the tie and strut diagonals, and the conditions of in- and out-of-plane restraint and biaxial loading at the bolted connections.

- An additional model is developed using a finite-element code, demonstrating that non-linear analyses can now be performed for more complex frames including asymmetric members.

Through this investigation, it has been noted that the following areas are opened for further research:

- Experimental data are required on the behaviour of the diagonals in the outside panels of bracing in terms of deflections and nodal rotations.

- It is recommended that the in-plane behaviour of the diagonals and the main legs be examined more carefully, including eccentricities in the main legs. This will possibly require that the test frames be assembled vertically instead of horizontally, due to the recording positions of the rotation transducers.

- The effects of bolt slippage should be examined. However, it is anticipated that modelling this variable accurately will be almost impossible, because of the random characteristics of these bolt movements.

- The effects of transverse beams between the main legs, and consequently the out-of-plane deflections of the outside bracings, should be examined. These conditions can be modelled with ABAQUS.

- Future investigations should be performed on full-scale frames. The effects of initial material imperfections in larger steel angle sections should be assessed.

- The effects of eccentricity of load and end restraint on the behaviour of two-dimensional frames made of steel angles should be examined separately. This will require simulation of main legs with reduced flexural and torsional stiffness.

It is suggested that further research be conducted on bracings with inverted diagonals. However, the design of adjacent lateral panels of the towers must be considered carefully.

APPENDIX A

Calculation of axial force and bending moment from strain readings

Strain gauges are located on the specimens in sets of three, as indicated in Figure A 01 below. Within the elastic range, stress at each gauge location can be calculated as follows:

$$\sigma_1 = Ek\epsilon_1 = P/A + M_u v_1/I_u - M_v u_1/I_v$$

$$\sigma_2 = Ek\epsilon_2 = P/A + M_u v_2/I_u + M_v u_2/I_v$$

$$\sigma_3 = Ek\epsilon_3 = P/A - M_u v_3/I_u - M_v u_3/I_v$$

where -E is the modulus of elasticity,
- -k is a strain gauge factor,
- -ϵ_i is the recorded strain at points i=1,2,3,
- -P is the axial force in the diagonal,
- -A is the cross sectional area,
- -M_u, M_v ar the moments about the principal u- and v-axis,
- -I_u, I_v are the moments of inertia about the u- and v-axis, and
- -u_i, v_i are the distances indicated in Figure A.01 for i=1,2,3.

The solution is given by solving the above system of simultaneous equations, where the unknowns are P, M_u and M_v. In matrix form, these equations are as indicated below:

$$Ek \begin{Bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_3 \end{Bmatrix} = \begin{bmatrix} 1/A & v_1/I_u & -u_1/I_v \\ 1/A & v_2/I_u & u_2/I_v \\ 1/A & -v_3/I_u & -u_3/I_v \end{bmatrix} \begin{Bmatrix} P \\ M_u \\ M_v \end{Bmatrix}$$

and finally:

$$Ek \{\epsilon_i\} = [M] \{P_i\} \qquad \ldots.(2.01)$$

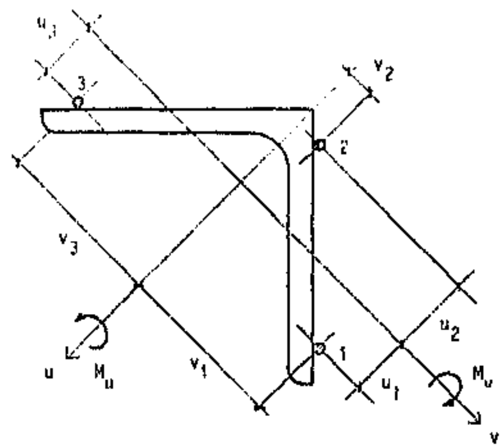


Figure A.01: Location of strain gauges in the angle section

APPENDIX B

Calculation of bracing deflections from test readings

Figure B.01-a shows the original (I) and deflected (II) positions of a section of an angle diagonal, where three instruments are used to measure the movements of the heel of the angle section: a) one rotation transducer (twisting Φ), b) one displacement transducer (horizontal displacement x_1-x_0), and c) one displacement transducer (vertical displacement y_1-y_0). A detail of the instruments' assembly is shown in Figure 2.13 in Chapter 2.

An auxiliary beam is used to support the displacement transducers. The beam is connected to the main legs at nodes a and d, see Figure 2.13 in Chapter 2, through a system of hinges and rollers, thus reducing interferences and distortions.

The displacement transducers are thus fixed, and their relative positions with respect to the diagonal cannot change during the test. Observing the detail in Figure B.01-b, the deflections of the heel of the angle can be expressed in terms of the measured twist and vertical and horizontal displacements as follows:

$$x_g = A \sin\Phi + B \cos\Phi$$
$$y_g = A \cos\Phi - B \sin\Phi$$

where

$$A = (y_1-y_0) \cos\Phi - b \sin\Phi$$
$$B = (x_1-x_0) \cos\Phi + b \sin\Phi$$

where b is the width of the angle section, which is consistent with the initial positions of the transducers displayed in Figure B.01-a, see also Figure 2.13 in Chapter 2.

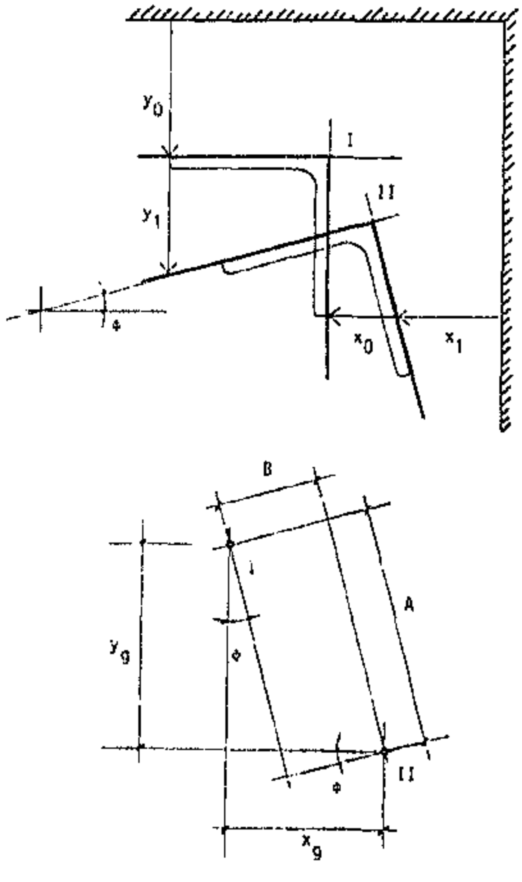


Figure B.01: a) Original (I) and deflected (II) positions of an angle diagonal. The vertical and horizontal transducers are placed perpendicular to the legs of the angle. b) Detail of the total movement of the heel of the angle.

Note that the heel of the angle is used to monitor the deflections, instead of the shear centre of the section. However, given the small size of the specimens, the error incurred is negligible. It is also assumed that the diagonals deflect at the cross-over joint only in the out-of-plane direction, and without twisting.

APPENDIX C

The Southwell-plot procedure

Actual structural members are not perfectly straight. They usually have initial deformations, or initial eccentricities, which increase the bending effect when axial loads are applied, thus increasing the stress at the extreme fibre.

Initial eccentricity and elastic buckling load of such members can be determined from deflection and strain readings obtained experimentally. The following load-deflection expression can be written for a member under compressive load:

$$\frac{u}{P} = \frac{a_0}{P_E} + \frac{u}{P_E} \qquad \text{....(C.01)}$$

Equation (C.01) is an expression of the form:

$$y = a + bx$$

thus representing a straight line which best fits the points determined from experimental readings of deflection and load, as in Figure C.01. The terms in Equation (C.01) are as follows:

- u is the recorded deflection,
- P is the recorded axial load,
- a_0 is the initial deformation, given by the intercept or initial ordinate in Figure C.01, and
- P_E is the Euler load, given by the reciprocal of the slope in Figure C.01.

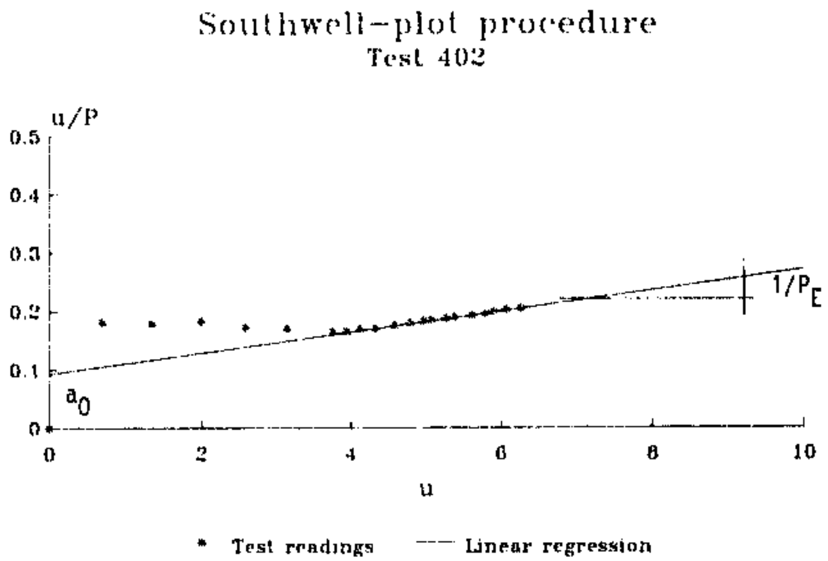


Figure C.01: Euler elastic load and end eccentricity calculated from Southwell-plot.

The above procedure is known as the Southwell-plot, and the background theory is well documented in Allen [70].

For the case of cross-bracing diagonals, deflection u in Equation (C.01) is the absolute deflection at midspan, including in-plane and out-of-plane deformations between the strut-leg connection and the cross-over joint, as indicated in Figure C.02-a. If it is assumed that the cross-over joint moves only in the out-of-plane direction, the deflection u about the minor axis in Equation (C.01) can be obtained from the following expression:

$$u = [(y_g - \tfrac{1}{2} y_c) + x_g]/\sqrt{2} \qquad \ldots (C.02)$$

in which y_g, x_g and y_c are calculated from test readings, as indicated in Appendix B and are shown in Figure C.02-b.

The Southwell-plot can also be expressed in a different format, as described next. If the strut has an initial deflection equal to a_0, the maximum deflection at midspan ($L_g/2$ in Figure C.02) can be calculated for any load P as follows:

$$u_{max} = \frac{a_0}{1 - \frac{P}{P_E}}$$

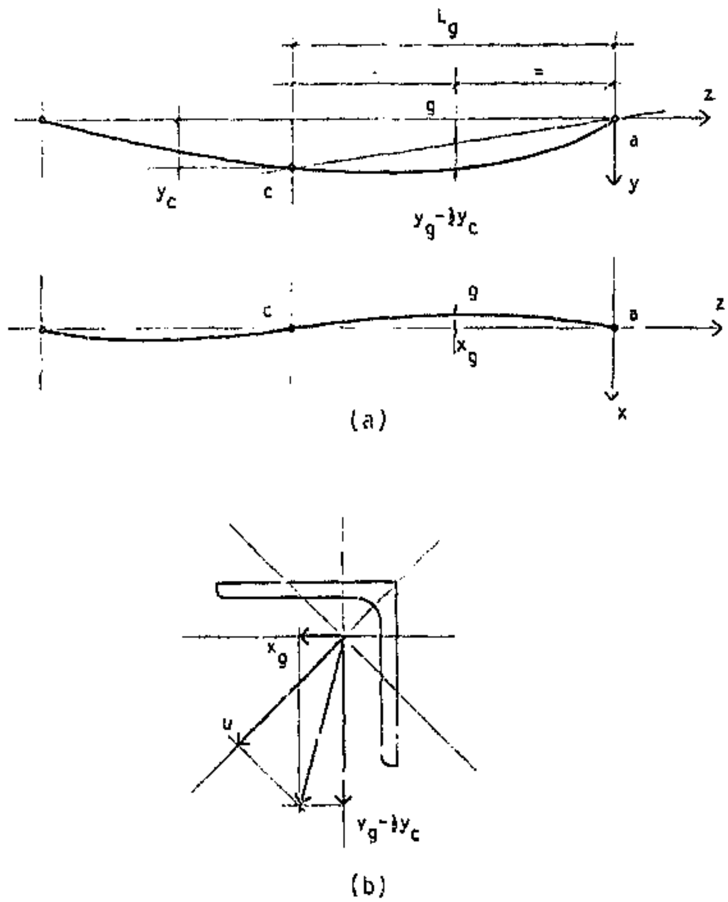


Figure C.02: a) In- and out-of-plane strut deflections, recorded during tests and used with the Southwell-plot procedure to determine experimental buckling loads. b) Detail of deflections at node g, see Appendix B.

and the measured displacement is $u = u_{max} - a_0$, from which:

$$u = \frac{a_0 P}{P_E - P}$$

The bending moment at midspan of the strut can be written as:

$$Mv_{max} = P\, u_{max} = \frac{Pa_0}{1 - \frac{P}{P_E}} = \frac{a_0 P P_E}{P_E - P} = u\, P_E$$

from which:

$$u = Mv_{max}/P_E$$

Replacing the new value of displacement u in Equation (C.01), the Southwell-plot equation becomes:

$$\frac{Mv_{max}}{P} = a_0 + \frac{Mv_{max}}{P_E} \qquad \text{.... (C.03)}$$

This is a modified expression of the Southwell-plot, in which:

- a_0 is the initial amplitude, or initial eccentricity, given by the initial ordinate in the plot.
- Mv_{max} is the bending moment about the minor axis of the section, and is obtained from strain readings.
- P is the recorded axial force, and is also obtained from strain readings.
- P_E is the Euler load, calculated as the inverse of the slope in the plot.

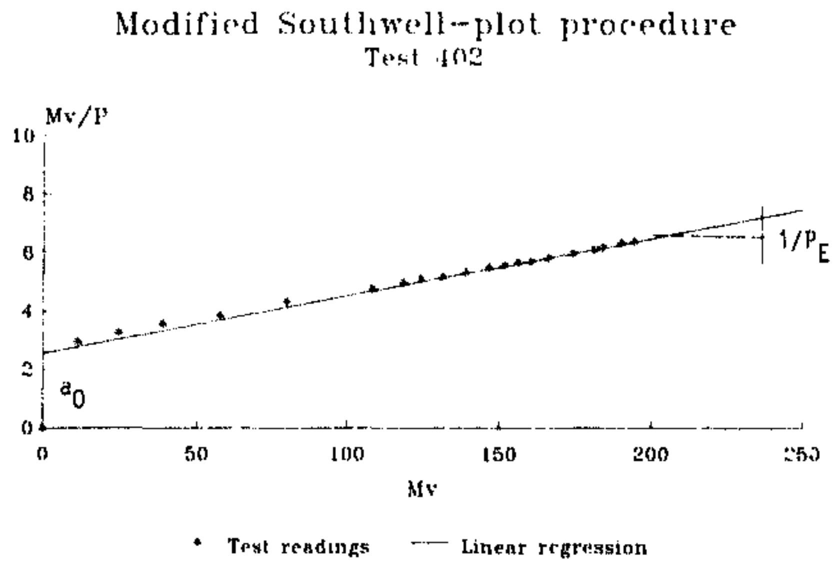


Figure C.03: Euler elastic load and
end eccentricity calculated from
modified Southwell-plot.

Equation (C.03) is solved for moment and axial force readings at and directly prior to first yield of each test, and the initial amplitudes and buckling elastic loads are obtained from the best-fit linear regression of these values, as indicated in Figure C.03.

APPENDIX D

Theory for frame analysis

The analytical approach used in this investigation (Chapters 4 and 5) for modelling cross-bracing systems, involves solution of equilibrium and compatibility equations. For example, to solve for n unknown moments at the ends of the n members meeting at joint j, see Figure D.01, the following conditions are required:

a) One equilibrium equation:

$$\sum_{i=1}^{n} M_{ji} = 0$$

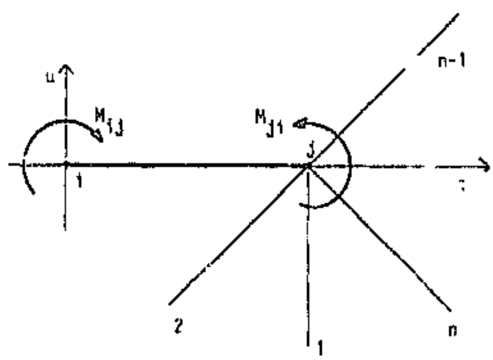


Figure D.01: Generic member ij in a structure. Rotations at nodes i and j can be expressed as functions of the end moments in the adjoining members.

b) (n-1) compatibility equations:

$$\theta_{j1} = \theta_{j2} = \dots = \theta_{ji} = \dots = \theta_{jn}$$

in which:

- M_{ji} are the ith member end moments at node j, and
- θ_{ji} are the rotations of the different members at joint j.

Nodal rotations θ_{ji} at the jth node of structural members, such as ji in Figure D.01, can expressed as a function of the end moments as follows:

$$\theta_{ji} = f(M_{ji}, M_{ij}) \qquad \dots\dots(D.01)$$

Definitions and basic theory for flexibility relationships such as (D.01) above can be found in Timoshenko [35], Pipard [62], or Chen [71].

Similar equations relating end moments and nodal rotations are written in this investigation for all nodes in the structure as a function of member geometry, applied loads and restraint conditions. These expressions are not dependent on the material properties.

To illustrate the rationale behind this approach, consider a generic element ij of length L, such as shown in Figure D.01 Following the notation and sign convention of Figure D.02, the rotations at nodes i and j can be expressed as indicated below:

$$\begin{Bmatrix} \theta_{ij} \\ \theta_{ji} \end{Bmatrix} = \frac{L}{EI_v} \begin{bmatrix} B_1/3 & -B_2/6 \\ -B_2/6 & B_1/3 \end{bmatrix} \begin{Bmatrix} Mv_{ij} \\ Mv_{ji} \end{Bmatrix} \qquad \dots\dots(D.02)$$

or $\{\theta\} = [F]\{M\}$

Equations (D.02) are fundamental in the proposed analysis. The matrices in Equations (D.02) relate the end rotations $\{\theta\}$ to the end moments $\{M\}$ of frame elements and are a generalized form of the flexibility equations. The flexibility matrix [F] of the member ij includes the functions B_1 and B_2, commonly known as Berry functions [35,62,66,71], which allow for the destabilizing and moment magnification effect of axial compressive forces acting on a beam-column through midspan deflections.

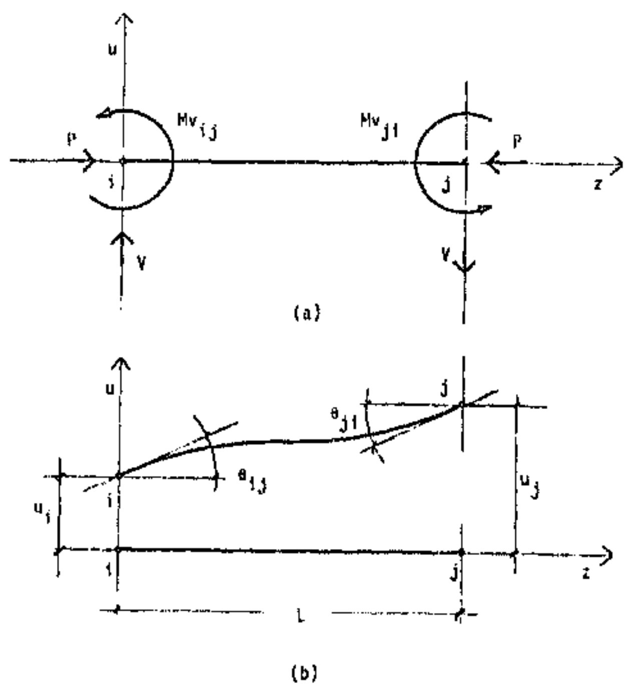


Figure D.02: Notation and sign convention adopted for the present investigation. a) Forces and moments. b) Deflections and nodal rotations.

Functions B_1 and B_2 are non-linear trigonometric factors, or stability functions, and are dependent on the ratio of forces P/P_E. They are, therefore, also related to the strut's slenderness ratio L/r. Figure D. 3 shows B_1 and B_2 plotted against the ratio P/P_E, from which we can derive the following conclusions:

- When the axial force P is very small the stability functions approach unity, and the relationship between the rotations θ and the end moments M become linear, since the flexibility coefficients in the matrix [F] are unaffected by P, depending only on the ratio L/EI_y. Equation (D.02) then takes the form of the linear flexibility equation.

- When the axial force approaches the elastic buckling load P_E, on the other hand, the stability functions tend to infinity. At this point, even the smallest end moment or lateral disturbance will induce considerable deflections and end rotations in the member ij.

It is clear, therefore, that the coefficients in the flexibility matrix [F] change with the axial load, resulting in a non-linear effect for increasing values of load. The relationship between rotations and moments, however, remains linear for a particular value of P and, as a result, the non-linear behaviour of the member ij due to this effect is calculated by a succession of linear analyses through increments of the axial force P.

In a rigorous second order analysis the axial forces are also unknowns, and have to be estimated for each iteration. However, for the bracing pair under consideration, it will be sufficient to calculate the member axial forces, at any load level, by considering equilibrium at the joints of the truss structure

Berry Stability Functions
Non-linear effect on beam-columns

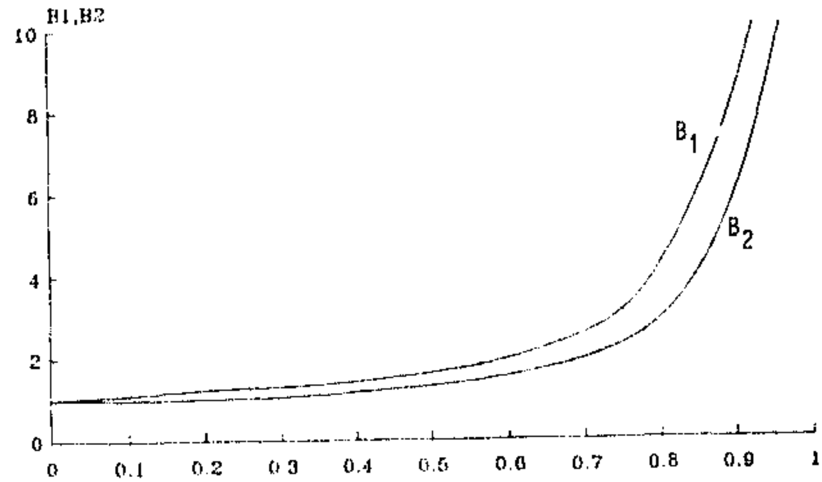


Figure D.03: Berry factors B1 and B2 as a function of the ratio P/Pe

without incurring significant error.

Extending this analysis to include a member ij having lateral joint deflection or sway, as shown in Figure D.02-b, Equations (D.02) can be extended simply to include the effect of these sways as follows:

$$\begin{Bmatrix}\theta_{ij}\\ \theta_{ji}\end{Bmatrix} = \frac{L}{EI_v}\begin{bmatrix}B_1/3 & -B_2/6\\ -B_2/6 & B_1/3\end{bmatrix}\begin{Bmatrix}Mv_{ij}\\ Mv_{ji}\end{Bmatrix} + \frac{1}{L}\begin{bmatrix}-1 & 1\\ 1 & -1\end{bmatrix}\begin{Bmatrix}u_i\\ u_j\end{Bmatrix}$$

or $\{\theta\} = [F]\{M\} + [f]\{d\}$ ....(D.03)

where [f] is a compatibility matrix relating end rotation to relative end deflection or sway (d), see Figure D.02-b. Similar analytical expressions can be developed for a member ij subjected to tensile forces.

A particular example of the above general case occurs when the beam-column ji of Figure D.01 is subjected to equal and opposite end moments, such as in the eventuality of equally eccentric loads. The maximum bending moment at half-span of element ji is given by the secant formula, see Chen [71], as indicated below:

$$Mv_{max} = P\ e_u\ \sec(w) \qquad \text{....(D.04)}$$

in which e_u is the eccentricity of load in the plane uz, see Figure D.01, P is the axial load, and the factor w is given as follows:

$$w = \frac{\pi}{2}\ \sqrt{(P/P_E)}$$

where P_E is the Euler critical load. The moment Mv_{max} is plotted against the ratio P/P_E in Figure D.04. Examination of Figure D.04 results in the following conclusions:

- When the axial force P is small, the factor sec(w) is equal to unity. In this case the bending moment is constant along the member ji, and equal to P e_u.

- When P approaches the elastic buckling load P_E, on the other hand, w becomes equal to $\pi/2$ and the bending moment at half-span of the member ij increases indefinitely. At this point, even the smallest end eccentricity e_u will induce instability and, consequently, large lateral deflections.

As in the case of the Berry functions B_1 and B_2, the trigonometric function sec(w) is related to the slenderness ratio L/r, and also acts as an amplification factor, thus allowing for non-linear effect at increasing loads.

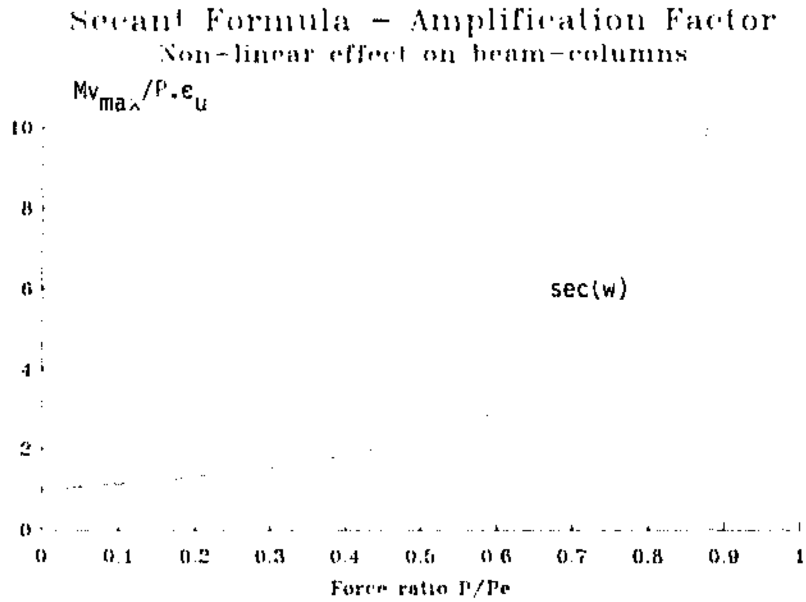


Figure D.04: Secant formula amplification factor sec(w)

One last important expression can be developed based on the equation of bending moment given by the secant formula. The stress at the extreme fibre in element ji of Figure D.01 can be calculated from axial force and uniaxial bending as follows:

$$f_{max} = \frac{P}{A} + \frac{M}{I_v} v$$

where:

- f_{max} is the stress in the extreme fibres of member ji.
- A is the cross-section area of member ji.
- I_v is the moment of inertia.
- h is the distance from neutral axis to extreme fibres.

Substituting the value of Mv_{max} given by Equation (D.04) into the expression of stress, we can now write:

$$f_{max} = \frac{P}{A} + \frac{h}{I_v} P \, e_u \sec(w)$$

Finally, considering that $I_v = A(r_v)^2$ and assuming that the maximum stress in the extreme fibres is equal to the yield stress f_y, the following expression results:

$$\left[\frac{f_y}{f_{ult}} - 1\right] \frac{r_v^2}{h} \cos(w) = e_u \qquad \text{....(D.05)}$$

where f_{ult} is the nominal axial stress at the moment of first yield. This equation gives an expression for the eccentricity based on the known value of yield stress and the measured value of P_E from the Southwell-plot, as explained in Appendix C.

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