Techno-economic optimization of level and raise spacing in Bushveld Complex platinum reef conventional breast mining

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Transaction Paper

Synopsis

The Bushveld Complex is economically significant and strategically important to South Africa, thus it is imperative that optimal extraction of platinum group metal (PGM) resources on the Bushveld Complex is achieved. Optimal extraction broadly requires that the maximum amount of ore is extracted by excavating and hauling the minimum amount of waste in the shortest possible time, at the least cost, and in the safest and most environmentally acceptable manner. In open-pit mine planning this entails among other things, minimizing the waste stripping ratio, whereas in underground mine planning it includes minimizing the metres of waste development. In conventional mining, the main development that is in waste or partly in waste and defines the mining grid pattern, includes levels and raises. It was prudent to consider optimizing level and raise spacing in conventional mining because the method is a prevalent mining method on the Bushveld Complex accounting for nearly 70% of platinum production.

The techno-economic optimization of level and raise spacing is characteristically a multi-criteria decision analysis (MCDA) optimization process and should therefore be analysed using MCDA techniques. The analytic hierarchy process (AHP) was the most appropriate MCDA technique for this research study. By using an orebody code named OB1 based on real geological data that was typical of Bushveld Complex platinum reef deposits, the derived optimal range of vertical level spacing was 30 m–50 m, and the optimal range of raise spacing was 180 m–220 m. The research methodology used in this study and the results obtained were received positively by the South African platinum mining industry because for the first time in several decades, a holistic methodology and practically acceptable solution had been developed for the controversial debate of optimizing level and raise spacing for conventional mining.

Keywords
Techno-economic optimization, concentrated mining, backlength, multi-criteria decision analysis (MCDA), analytic hierarchy process (AHP), lagrange multiplier, valuation curve.

Introduction

The Bushveld Complex, previously known as the Bushveld Igneous Complex, is currently the only known source of economically mineable platinum group metal (PGM) or platinum group element (PGE) resources in South Africa. The Bushveld Complex is economically significant and strategically important to South Africa for a few reasons. Firstly, when platinum is considered alone, the Bushveld Complex hosts approximately 63% of all the known world platinum resources and reserves (Figure 1). However, when platinum is considered together with other PGMs, the Bushveld Complex is host to an estimated 87% of global PGM resources and reserves (Chamber of Mines, 2005; Chamber of Mines, 2006). In terms of production, South Africa produced about 77% of annual global platinum production in 2005 (Figure 2), a production figure that is in the same range as a most recent estimate of just above 75% of global output for 2008 (Research Channel Africa, 2009).

The high PGM production capacity and rich PGM mineral endowment attributable to South Africa give the country the enviable status of leading producer and resource base in the international platinum mining industry. However, in line with sustainable development principles, the sheer size of the resources and reserves obscures the fact that the PGM mineral resources are a wasting asset, and should therefore be extracted optimally in order to ensure sustainable production (Stilwell and Minnitt, 2006). In addition, a 2006 survey of research and development (R&D) needs of the South African platinum mining companies by the CSIR-Miningtek, identified that out of 19 possible R&D areas, layout optimization is one of the top four priority R&D focus areas (Singh and Vogt, 2006). Lastly, Section 51 of the Mineral and Petroleum Resources Development Act (MPRDA) of 2002, of South Africa, empowers the State with the discretion to force the holder

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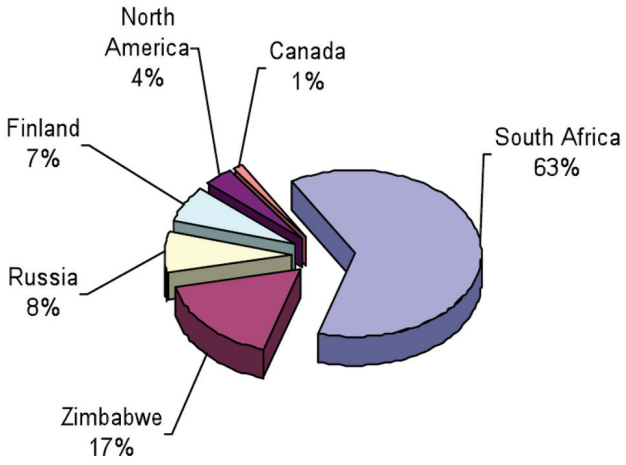

Figure 1—Relative proportion of global platinum resources and reserves by country in 2005 (adapted from Chunnett, 2006)

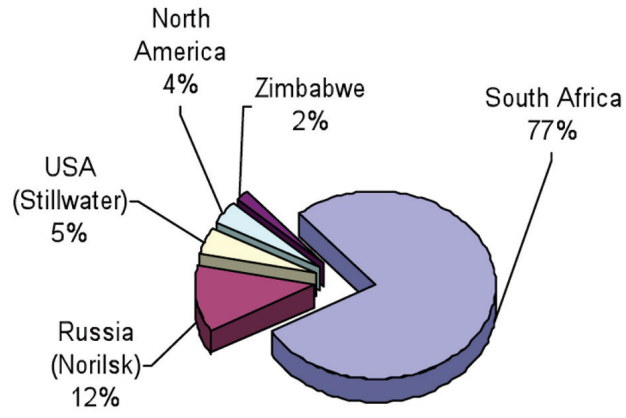

Figure 2—Relative proportions of global primary platinum production by country in 2005 (adapted from Chunnett, 2006)

of mineral rights to a development project to suspend operations if the State is of the opinion that the holder is not mining the mineral resources optimally.

The foregoing are imperatives for optimizing PGM mineral extraction on the Bushveld Complex. Optimal extraction or mining optimization broadly requires that the maximum amount of ore is extracted by excavating and hauling the minimum amount of waste in the shortest possible time, at the least cost, and in the safest and most environmentally acceptable manner. In open-pit mine planning this entails, among other things, minimizing the waste stripping ratio, and in underground mine planning it includes minimizing the metres of waste development. In conventional mining, the main development that is in waste or partly in waste and defines the mining grid pattern, includes levels and raises. In this study it was prudent to consider ways of optimizing level and raise spacing in conventional mining because the method is a prevalent mining method on the Bushveld Complex, accounting for nearly 70% of platinum production, whereas the remainder comes from open-pit, hybrid (mechanized access with conventional stoping), and mechanized mining methods (Figures 3 and 4).

Conventional mining is, and has been, used widely in the extraction of shallow-dipping, narrow tabular reefs in the gold, platinum, and chromitite sectors in the South African mining industry (York, 1999; Ryder and Jager, 2002; Egerton, 2004). In the platinum mining sector, this fact is supported by Figures 3 and 4, which indicate that conventional mining is likely to remain the principal platinum mining method in the medium-term, a paradigm that is further supported by the following cues:

- Moxham (2004) and Pickering, Smit and Moxham (2006) noted that despite efforts to mechanize the South African hard rock narrow reef mining industry in the last 40 years, almost all mechanized mines have converted to conventional mining.
- Lonmin announced its intentions about 10 years ago that by 2010 at least 50% of their PGM production would be coming from mechanized mining, thus contributing to the drop from 70% to 56% of platinum production from conventional mining (Figures 3 and 4). However, it is no longer certain if Lonmin is still on course since mechanization projects at their Saffy and Hossy shafts are now being converted to conventional mining.
- Northam, the deepest platinum mine with operations at between 1 300 m–2 300 m below surface, is still using conventional breast mining but has adapted the method

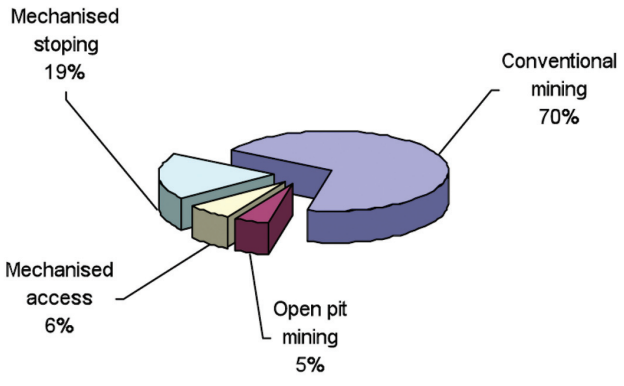

Figure 3—Distribution of PGM production output by mining method in 2005 (Pickering, 2007)

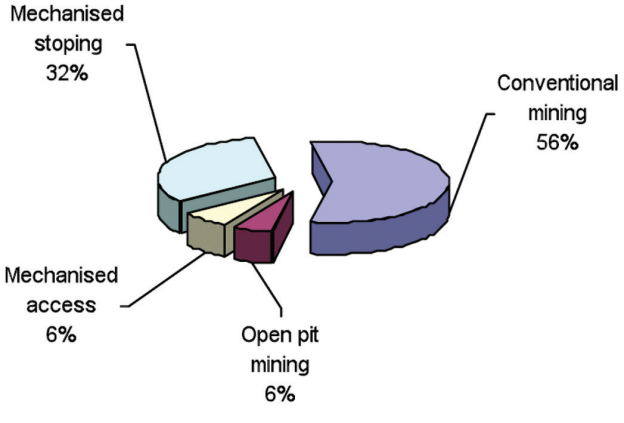

Figure 4—Forecast 2010 distribution of PGM production output by mining method (Pickering, 2007)

to use hydropowered equipment (HPE) instead of pneumatic equipment. This observation suggests that conventional mining can still be practised at deeper mining levels.

- The recent 'fourth generation' Impala Platinum 16# and 20# projects at deeper mining depths below surface were planned on conventional mining (Jagger, 2006; Zindi, 2008), suggesting that platinum mines might be expected to be practising some form of conventional breast mining at deeper mining levels.
- Egerton (2004) analysed eight different mining methods to mine the UG2 reef. Musingwini and Minnitt (2008) further analysed the results using the analytic hierarchy process (AHP) and noted that conventional mining ranked highest.
- The orebody will always dictate the mining method. The extreme hardness and high abrasivity of particularly the UG2 reef, due to the presence of chromite crystals making up the structure of the UG2, make it difficult to introduce rock cutting technology into the platinum mines (Moxham, 2004; Pickering, Smit and Moxham, 2006). The rolling reef nature of the UG2 and Merensky also makes it difficult to implement mechanized mining, thus favouring the continued use of conventional mining.
- The reality of mineral price cycles, intermittent strengthening of the ZAR/US$ exchange rate, and the associated cost-cutting measures have invariably resulted in occasional mothballing of mechanized mining projects because they are capital intensive and hence sensitive to such real changes which occur from time to time (Egerton, 2004). Again, this favours the continued use of conventional mining.

One way to optimize conventional mining is by minimizing waste development predominantly through increasing level and raise spacing. However, when level and raise spacing are increased, other factors such as productivity are negatively affected, thus requiring a delicate trade-off of contradicting optimization criteria. In order to investigate the behaviour of optimization criteria under variable level and raise spacing, a total of 15 conventional breast mining layouts were designed and scheduled on real geological data on the orebody code named OB1 that was typical of Bushveld Complex platinum reef deposits. The 15 layouts were selected such that there was a reasonably even distribution over the range of 180 m–400 m for raise spacing and 30 m–70 m for vertical level spacing (Musingwini, 2009). Ideally more layouts could have been designed and scheduled to improve the accuracy in estimating trends of optimization criteria with changing level and raise spacing, but the limit of 15 layouts was dictated mainly by time constraints because each layout took on average about 8 weeks to design, schedule, and construct its financial evaluation model.

MCDA nature of level and raise spacing optimization

Optimizing level and raise spacing in inclined narrow reef mining has been a subject of controversy for decades. This is noted in one of the feedback comments from industry on this research study in that, 'level spacing and raise line spacing has been a controversial topic in the mining industry for decades. No two mining engineers will agree on this issue as there has been no way to scientifically calculate the best option. The only way available to mining engineers previously has been to laboriously model these variables manually, with no conclusive decisions' (Impala Review Team, 2009). The underlying reason for the complexity of level and raise optimization in conventional breast mining is its characteristic nature of being a multi-criteria decision analysis (MCDA) optimization problem. Optimization problems of the MCDA type require decision-makers to select the best alternative or group of alternatives from a finite set of alternatives using two or more competing criteria based on objectives that are usually contradictory (Chen, 2006).

The basic structure of a generic MCDA problem (Table I) is premised on requiring a decision-maker to select an alternative, A_i, from a set of alternatives, $A = \{A_1, A_2, \ldots, A_m\}$, such that A_i gives the best or optimal trade-off among decision criteria defined by a set, $C = \{C_1, C_2, \ldots, C_n\}$. In total there are m alternatives and n criteria. The efficiency of alterative *1* against criterion *1* is expressed as the outcome O_{11}, that of alternative *i* against criterion *j*, as outcome O_{ij} and so on. The decision-maker is cognisant that each criterion has a greater or lesser degree of importance relative to other criteria in arriving at the the overall optimal decision. The relative importance or weight of C_i over C_j is denoted by w_{ij}.

The challenges faced in a MCDA optimization process include but are not limited to the following:

- The optimal decision must be one that carefully balances conflicting objectives (or criteria) by selecting the best trade-off among the competing objectives or criteria (Vieira, 2004; Ballington *et al.*, 2005; Chen, 2006). For example, by increasing raise spacing, the replacement factor (RF) increases and is a desirable outcome, while the productivity decreases and this is an undesirable outcome, thus resulting a conflict in objectives.
- The optimization criteria have different units of measure and the challenge is to integrate more than two different criteria that are measured in different units. For example, when raise spacing is increased, it is difficult to configure how to achieve an optimal trade-off between a decrease in productivity that is measured in centares/man/month with an increase in the RF that is measured in m²/m, unless the importance attached to either criterion, w_{ij}, is known.

Table I

The matrix structure of a generic MCDA problem

Alternatives	Criteria					
	C_1	C_2	...	C_j	...	C_n
A_1	O_{11}					
A_2		...				
...			...			
A_i				O_{ij}		
...					...	
A_m						O_{mn}

- The trade-offs are often too complex to configure if the criteria display a mixture of relationships that take other non linear forms. For example, it is difficult to configure a trade-off between two criteria if one is varying logarithmically while other one is varying quadratically with increasing level and raise spacing, as was the case with the optimization criteria in this study.
- The human brain can easily configure an optimal decision such as deriving maximum benefit or minimum loss when faced with a two-dimensional (2D) problem expressed as a quadratic function in an *xy* Cartesian plane, or when the decision problem is three-dimensional (3D) expressed as a surface in *xyz* space. When optimization decisions involve decision criteria that exceed 3D, humans have to rely on abstract thinking or attempt to simplify the problem back to 2D or 3D for easier configuration. However, as Saaty and Ozdemir (2003), Yavuz (2007), Yavuz and Pillay (2007a), Yavuz and Pillay (2007b) and Saaty (2008) noted, there are general limitations on human performance on abstract thinking.

Techniques for solving MCDA problems are structured to meet the above challenges. The next section briefly describes the four broad categories of MCDA techniques.

Overview of MCDA techniques and selection of AHP

There are four broad categories of MCDA methods. These are the French version *elimination et choix traduisant la réalité* (ELECTRE), which was translated into the English version elimination and choice translating the reality; preference ranking organization method for enrichment evaluation (PROMETHEE); multiple-attribute utility (MAUT); and analytic hierarchy process (AHP) and its subsequent generalization the analytic network process (ANP) (Almeida, Alencar and Miranda, 2005; Geldermann and Rentz, 2005; Saaty, 1980; Saaty, 2008). The methods are classified according to the type of information given by the decision-maker and the salient feature of the information depending on whether it is ordinal or cardinal scale information (Geldermann and Rentz, 2005). MAUT and AHP methods are most often applied when the available information is cardinal, whereas ELECTRE and PROMETHEE methods are applied to mostly ordinal scale information (Geldermann and Rentz, 2005). Data are ordinal when linguistic scales which are non-numerical scales, have to be assigned to it. An example of a linguistic scale is the rating from 'low' to 'medium' and 'high'. A linguistic scale can be assigned numerical values on a scale. For example, on a scale of 1–10, 'low' could take any values in the range 1–3, 'medium' 4–6, and 'high' 7–10. Data are cardinal if they are expressed as a real number.

The ELECTRE and PROMETHEE methods are founded on the outranking procedure. Outranking is done to account for the fact that preferences are not constant in time, are not ambiguous, and are not independent of the process of analysis (Geldermann and Rentz, 2005). Saaty (2008:7) concurred with the argument that human preferences are dynamic because, 'people, then, not only have different feelings about the same situation, but their feelings change or can be changed by discussion, new evidence, and interaction with other experienced people'. The outranking argument is that an alternative A_i outranks or is superior to alterative A_j if the decision-maker strongly perceives A_i to be at least as good as A_j. A comparison of two alternatives is called a pair-wise comparison. A comparison of the four categories of MCDA techniques is illustrated in Table II.

Choice of AHP methodology

The AHP was selected over the other MCDA methods for use in this research study for three main reasons. Firstly, the method has significant advantages which are:

- When compared with other MCDA techniques, the AHP can detect inconsistent judgements and provide an estimate of the degree of inconsistency in the judgements (Coyle, 2004; Saaty, 2008).
- The AHP is supported by an easy-to-use commercially available software package called Expert Choice® (Geldermann and Rentz, 2005) and more recently, the software DecisionLens® has become available for AHP problem analysis (Saaty, 2008). AHP can also be programmed easily in Microsoft Excel®.

Table II

Comparison of the four MCDA methodology categories (adapted from Geldermann and Rentz, 2005; Chen, 2006)

	MAUT	AHP	ELECTRE	PROMETHEE
Foundation	Classical MCDA approach	Hierarchical approach	Outranking procedure	Outranking procedure
Theoretical basis	Utility function additive model	Pairwise comparison (weighted eigenvector evaluation)	Pairwise comparison (concordance analysis)	Pairwise comparison (preference function)
Measurement of criteria	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to numerical scale)	Numerical (non-numerical data must be converted to numerical scale)
Determination of weights of criteria	Trade-off based weights (generate weights using swing, direct-ratio, or eigenvector methods)	Trade-off (generate weights using Saaty's eigenvector and geometric mean)	Non-trade-off (does not provide procedure to obtain weights)	Non-trade-off (does not provide procedure to obtain weights)
Result	Relative preference order	Relative preference order	A set of non-dominated alternatives	Partial and complete ranking order

- The AHP has the ability to rank alternatives in order of their effectiveness when conflicting objectives or criteria have to be satisfied (Coyle, 2004).

Secondly, the AHP has been used to solve successfully, a wide range of MCDA decision problems in the minerals industry and is gaining gradual recognition because most optimization and decision-making problems encountered in the minerals industry are of a multi-criteria nature (Musingwini, 2009). Lastly, the AHP was a preferred choice because the layout efficiency data in this research study were cardinal.

The AHP methodology

Saaty (1980) developed the AHP methodology. Matrix and vector algebra form the basis of the mathematical framework of the AHP methodology, thus AHP calculations can be easily performed in Microsoft Excel®. The books, *The Analytic Hierarchy Process,* by Saaty (1980) and *Decision Making for Leaders: The Analytic Hierarchy Process for Decisions in a Complex World*, by Saaty (2008) comprehensively explain the AHP mathematical theory. Musingwini and Minnitt (2008) and Musingwini (2009) also give a mathematical explanation of the AHP within the context of the minerals industry.

Criteria weights and consistency estimation

A survey was undertaken to establish the weighting attached to each of the criteria for selecting optimal level and raise spacing using a structured questionnaire to solicit expert opinion from mine planning and project planning practitioners in the local South African platinum industry. The questionnaires were completed by three independent divisions, namely Anglo Platinum Mine Technical Services, Anglo Platinum Strategic Long-Term Planning, and Impala Mining Projects. The responses from the survey were subsequently analysed in Microsoft Excel® to normalize them and estimate the degree of consistency of the decision-makers that were surveyed, as shown in Table III.

In Table III the symbol, λ, is the eigenvalue of the matrix of weights attached to the criteria in each of the three surveyed cases. If judgements are consistent in assigning the criteria weights, then $\lambda=n$ where n is the order of the matrix of weights. For inconsistent human judgements, $\lambda \geq n$ and the λ becomes λ_{max}. In the survey, 12 criteria were considered and therefore $\lambda_{max} \geq 12$ indicates a degree of inconsistency in judgement by the three divisions that were surveyed. The notation CI is for the consistency index, which is used to calculate the consistency ratio, CR. The CR values were then computed and all were less than or just equal to the allowable threshold limit of 0.1 (or 10%) that was derived by Saaty (1980). The respondents' judgements could therefore be considered reliable to proceed with the AHP since the all inconsistencies were below the 10% threshold. Table IV shows the aggregate weight of each criterion and NPV being ranked as the most important optimization criterion by the industry experts.

Orebody model and design process

In selecting a geological orebody model to work with, a cue was taken from Vieira, Diering, and Durrheim (2001), and Vieira (2003) who used the hypothetical Iponeleng orebody, based on data typical of the Witwatersrand ultra-deep level mining environment, to compare four different ultra-deep mining methods for the Deepmine project. However, in this paper real geological data based on the orebody code named OB1 for proprietary reasons (Figure 5), were used to capture the typical UG2 mining environment for conventional breast mining on the Bushveld Complex. The geological exploration work and geostatistical analysis carried out on OB1 qualified it into the measured resource category as defined by the SAMREC code.

The design process followed was the engineering circle or wheel of design developed by Stacey (2006) and Stacey, Terbrugge and Wesseloo (2007) but adapted to suit the requirements of this study, since this study had no implementation stage.

Design and scheduling results

Reasonableness checks were performed to check the validity of the results obtained from the design and scheduling process and so establish confidence to proceed to analyse the results. Three sets of checks were performed namely, centares discrepancy, tonnage discrepancy, and valuation checks. Since it was necessary to study the behaviour of each optimization criterion over the range of level and raise spacing limits noted earlier, yet only 15 data points were available for each criterion, it was necessary to perform interpolations using curve fitting techniques complemented by the expected behaviour of the criteria for level and raise spacing, in order to fill in gaps in the data between the set limits. The relationship that was considered most appropriate

Table III

Level of consistency in respondents

Consistency measure	AngloPlat SLTP	AngloPlat MTS	Impala Platinum	Industry aggregate
λ_{max}	12.28	12.59	13.57	12.19
CI	0.03	0.05	0.14	0.02
CR	0.02	0.04	0.10	0.01

Table IV

Optimization criteria weights

Criterion	Weight
NPV	0.129
PV_Dev_Cost	0.086
Payback period	0.079
RF	0.043
Shaft head grade	0.113
Overall dilution	0.099
Production rate	0.078
Productivity	0.076
Flexibility index	0.081
Life of raiseline or stope	0.050
LOM	0.068
Build-up period	0.098

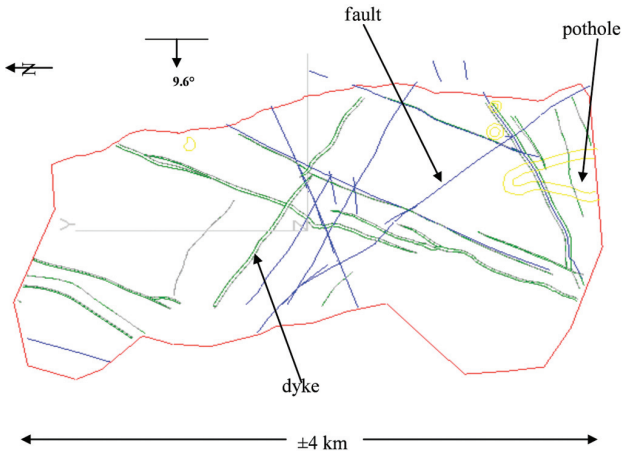

Figure 5—A Mine 2-4D® representation of OB1 structural geology

was the one where the set of curves obtained from curve fitting produced was the set having mostly highest values of the R^2 statistic. Results for only a few of the criteria are shown below for illustration of the kind of results obtained.

Reasonableness checks

Digitizing errors sometimes occur during the design process leading to overlap of boundaries of excavations or having boundaries that are common to two excavations but are separated by some gaps between them. This can lead to over-estimating or underestimating centares and tonnages. If the discrepancy in tonnages exceeds 10%, designs usually have to be rechecked for such errors. This is despite the fact that there is an in-built function in Mine2-4D® for checking overlaps and crossovers, but the software can miss small overlaps and minor crossovers, or adjust strings and points in a design to avoid data corruption arising from such errors. Therefore, it is not unusual for final designs still have discrepancies in centares and tonnages when compared with *in situ* estimates.

In order to check for centares discrepancy, the total *in situ* stope centares were added to the sum of areas left *in situ* as regional pillars and geological losses, and then compared to the original *in situ* centares for the OB1 wireframe. The summary is illustrated in Table V.

In order to check for tonnage discrepancy, the total *in situ* stope tonnes were added to the sum of tonnes left *in situ* as regional pillars and geological losses, and then compared to the original *in situ* tonnes for the OB1 wireframe. The summary is illustrated in Table VI.

Tables V and Vi show that the discrepancies obtained were within acceptable limits. A summary comparison of Tables V and VI, shows that on average the tonnage discrepancies are about 4 times the centares discrepancies, confirming the tonnage factor of 4t/m² for OB1.

The net present values (NPVs) from the discounted cash flow (DCF) valuation of each of the 15 layouts were normalized to US$/oz and plotted on the valuation curve as shown in Figure 6. The platinum valuation curve is an in-house valuation tool developed by Venmyn, a South African

Table V

Summary centares discrepancy for the 15 layouts

		Raise spacing (m)				
		180	200	280	360	400
Level spacing (m)	30	0.07%	0.10%	0.03%	0.33%	0.04%
Level spacing (m)	50	0.24%	-0.02%	0.07%	-0.99%	0.05%
Level spacing (m)	67	0.06%	0.01%	0.06%	0.12%	-0.23%

Table VI

Summary tonnage discrepancy for the 15 layouts

		Raise spacing (m)				
		180	200	280	360	400
Level spacing (m)	30	0.28%	0.30%	0.24%	0.23%	0.24%
Level spacing (m)	50	0.44%	0.19%	0.27%	-0.79%	0.26%
Level spacing (m)	67	0.27%	0.22%	0.26%	0.33%	-0.02%

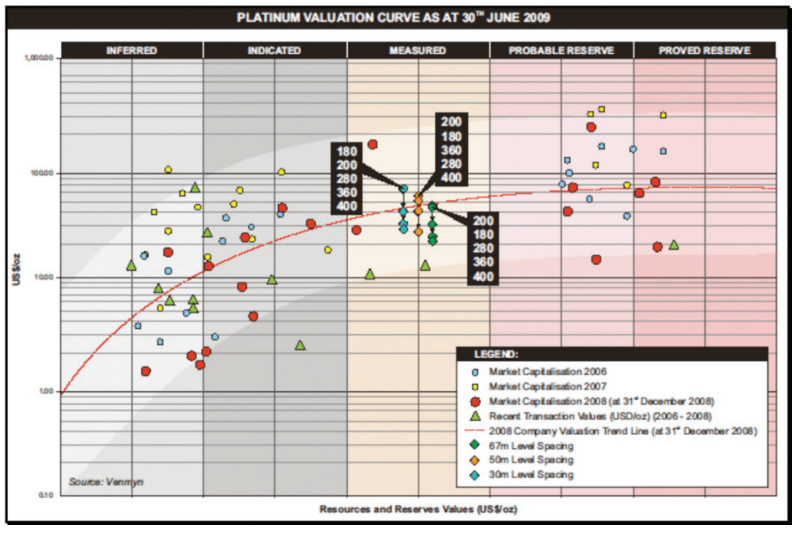

Figure 6—Relative positions of the 15 layouts on the platinum valuation curve (Courtesy of Venmyn)

advisory company specializing in mineral project valuation and statutory compliance. The layout values plotted on the lower band of the measured resource category, which is typical of UG2 properties on the Bushveld Complex. Figure 6 shows that the layouts at 180 m and 200 m raise spacing plot on nearly the same point, indicating that there is no significant change in resource value when raise spacing is altered by increments of up to 20 m. Therefore when considering raise spacing changes, it makes economic sense to change the spacing in increments of at least 20 m.

Trend in PV of development costs per centare

The PV of the development costs was reported in ZAR/m2. The variation of PV of development costs with level and raise spacing is illustrated by Figures 7 and 8. The nonlinear power relationship displayed in Figures 7 and 8 is expected based on the work by Lewis (1941), Brassell (1964), and Lawrence (1984).

Trend in project NPV

The best fit obtained for the relationship between project NPV and increasing level and raise spacing was quadratic as depicted in Figures 9 and 10. The quadratic trend can be explained using Eaton's (1934) argument that beyond a certain level spacing or raise spacing, the cost saving benefit associated with reducing the amount of development is more than offset by the cost of mining at longer distances.

If NPV were the sole optimization criterion, then Figure 9 shows that the optimal range of vertical level spacing for OB1 would be between 40 m and 50 m, whereas Figure 10 shows that the optimal range of raise spacing for OB1 would be between 200 m and 250 m, thus confirming Lawrence's (1984) findings on the economic optimal raise spacing for conventional breast mining.

Shaft head grade

Shaft head grade is not affected by level spacing (Table VII) because all the material that is blasted from level development and off-reef development is trammed separately as waste. However, shaft head grade is affected by raise spacing because development ore from raises is scraped together with ore from production panels. The impact of

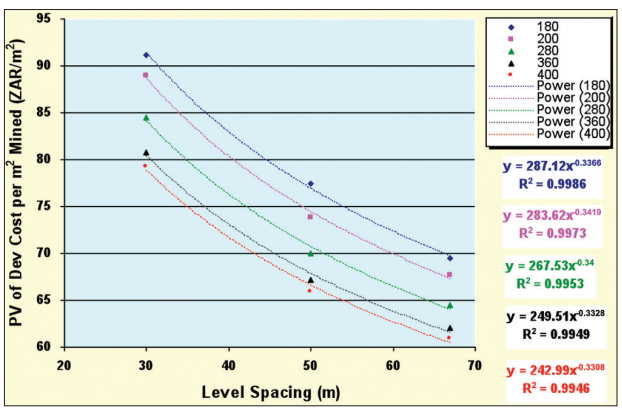

Figure 7—Variation of PV of development cost per centare mined in relation to level spacing

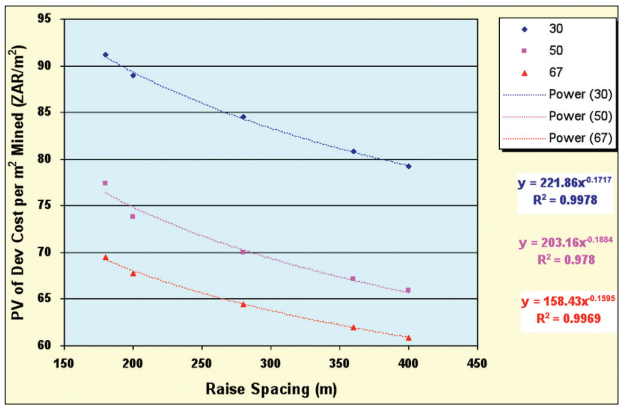

Figure 8—Variation of PV of development cost per centare mined in relation to raise spacing

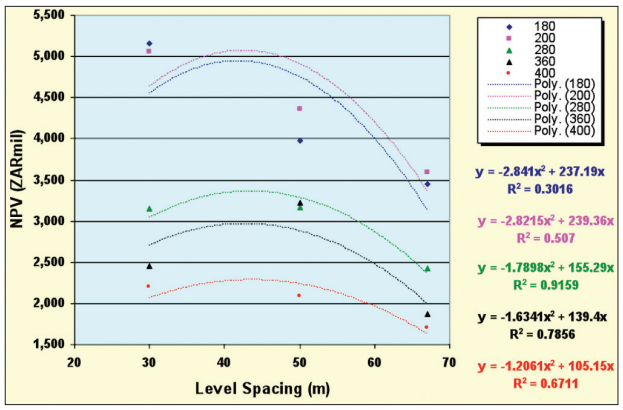

Figure 9—Variation of project NPV with increasing level spacing

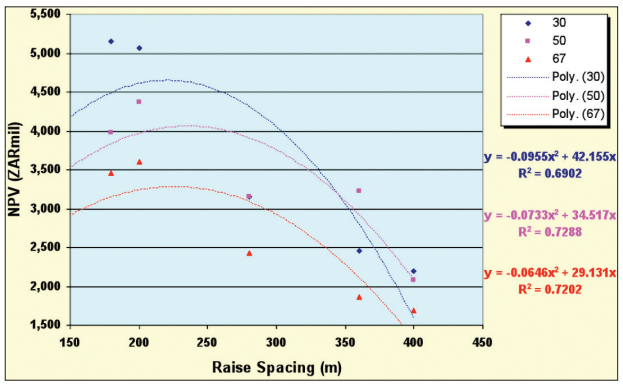

Figure 10—Variation of project NPV with increasing raise spacing

Table VII

Shaft head grade constant for increasing level spacing for all 15 layouts

Shaft head grade (g/t)		Raise spacing (m)				
		180	200	280	360	400
Level spacing (m)	30	5.50	5.50	5.25	5.53	5.53
Level spacing (m)	50	5.50	5.51	5.52	5.53	5.53
Level spacing (m)	67	5.50	5.51	5.52	5.53	5.53

reduced dilution from more spaced out raises is not very significant as Table VII shows that grade changes only from 5.50g/t to 5.53 g/t. Therefore, only the relationship between shaft head grade and raise spacing was analysed. The best fit for the relationship that was obtained was a logarithmic fit as indicated in Figure 11. It has not been possible at this stage to understand why the relationship should be logarithmic.

Techno-economic criteria summary

The summary techno-economic criteria values for the 15 layouts is indicated in Table VIII and all the results including the results for layouts for which criteria values were interpolated are shown in the Appendix. The criterion, RF, in Table VIII is the replacement factor or replacement ratio, which is a measure of the m^2 of stoping created by mining a metre of development.

Results and discussion

The weights (Table IV) were finally aggregated together with the layout efficiency scores (Appendix) against each criterion to get the overall AHP priority score. The efficiency scores were obtained from designs and schedules executed on OB1 using Mine 2-4D® and EPS® software suite. OB1 is an orebody based on real geological data that were typical of Bushveld Complex platinum reef deposits amenable to extraction by conventional mining methods. The only slight difference was that OB1's average dip is 9.6º, which, however, is close to the average regional dip of 10º for the Bushveld Complex. The AHP priority scores were plotted onto a 3D contour space as shown in Figure 12.

The highest AHP priority scores (in red) occur in the bottom left corner of Figure 12, indicating that for OB1, the optimal range of vertical level spacing is between 30 m and 50 m and that for raise spacing is between 180 m and 220 m. This suggests that optimization criteria such as productivity whose outcomes are desirable at shorter spacing far outweigh

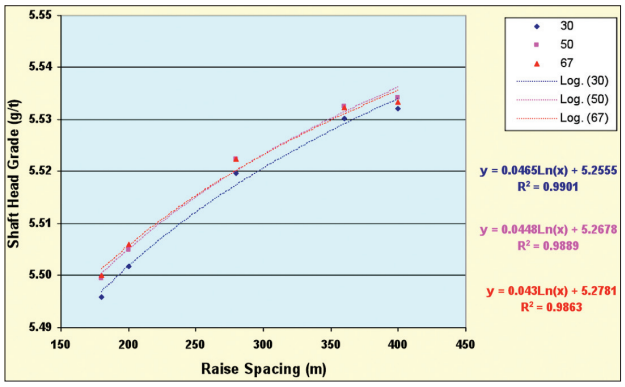

Figure 11—Variation of shaft head grade with increasing raise spacing

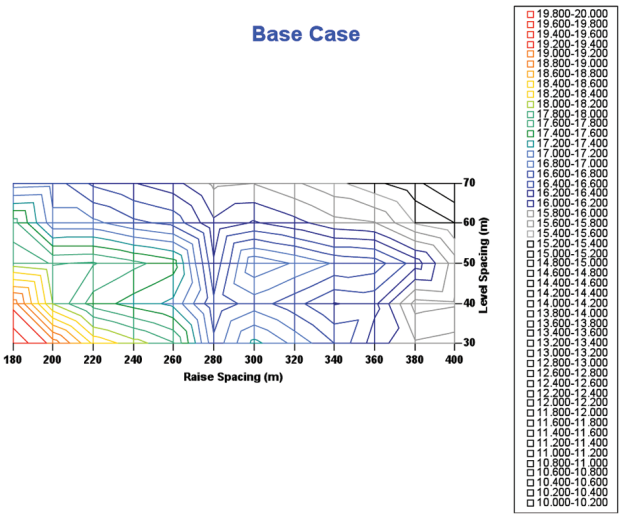

Figure 12—A 3D surface contour plot of the AHP priority score for the base case

Table VIII

Values for the 12 techno-economic criteria for each of the 15 layouts

	Criteria											
Layout	NPV (ZARmil)	PV_dev_cost (ZAR/m²)	Payback period (years)	RF (m²/m)	Shaft head grade (g/t)	Overall dilution (%)	Production rate (tpa)	Productivity (m²/stope employee)	Flexibiltiy index	Life of raiseline or stope (days)	LOM (years)	Build-up period (years)
Objective	max	min	max	min	max	min	max	min	max	min	max	min
30_180	5 155	91.19	5.3	21.64	5.50	9.90%	577 128	42	1.79	325	65	8
30_200	5 064	88.93	5.39	22.02	5.50	9.81%	545 859	39	1.77	369	69	9
30_280	3 148	84.5	6.72	22.75	5.52	9.51%	408 458	34	1.58	497	89	12
30_360	2 454	80.77	7.36	23.46	5.53	9.34%	335 472	35	1.27	570	109	11
30_400	2 191	79.21	7.21	23.78	5.53	9.31%	321 413	35	1.17	679	115	12
50_180	3 978	77.4	9.19	23.08	5.50	9.84%	518 945	37	1.88	444	71	9
50_200	4 362	73.85	7.16	23.67	5.51	9.75%	472 142	41	1.5	534	79	12
50_280	3 159	69.98	8.94	24.56	5.52	9.47%	408 482	34	1.14	750	89	13
50_360	3 220	67.12	6.49	25.32	5.53	9.30%	312 142	35	0.92	898	117	14
50_400	2 082	65.88	8.9	25.66	5.53	9.28%	313 047	30	0.92	1236	118	15
67_180	3 453	69.47	10.46	24	5.50	9.84%	447 999	34	1.29	587	83	10
67_200	3 594	67.72	9.45	24.45	5.51	9.74%	433 845	34	1.14	683	86	11
67_280	2 423	64.49	10.35	25.31	5.52	9.47%	358 307	29	0.93	880	101	11
67_360	1 869	62.01	10.46	26.06	5.53	9.31%	317 058	30	0.74	1152	114	14
67_400	1 695	60.91	10.83	26.4	5.53	9.29%	308 841	30	0.7	1223	119	20
Sum	47 849	1 103.44	124.21	362.14	82.77	143.16%	6 079 137.12	520	18.75	10 826	1 424	181

those criteria such as RF that are associated with desirable outcomes at wider spacing of levels and raises. This finding further suggests that contrary to the current practice of advocating wider spacing, conventional mining layouts should be planned at smaller spacing. Figure 12 also indicates that two layouts are associated with local maxima. These layouts are for 30 m vertical level spacing (≈180 m backlength at 9.6° dip) by 180 m raise spacing and 50 m vertical level spacing (≈300 m backlength at 9.6° dip) by 300 m raise spacing. This observation means that local optima are obtained for layouts in which the average backlength is nearly equal to the raise spacing, that is, the stope shape is almost a square shape. Current industry practice is that stope shapes are generally rectangular with the backlength usually exceeding the raise spacing.

As argued earlier in the paper, when optimization decisions involve decision criteria that exceed 3D, humans have to rely on abstract thinking for which they have great limitations. Saaty and Ozdemir (2003), Yavuz (2007), Yavuz and Pillay (2007a), and Yavuz and Pillay (2007b), therefore argued that the AHP produces reliable results when the number of criteria do not exceed 9 because of the general limitations on human performance in abstract thinking. Therefore, when faced with criteria that exceed 9, it is advisable to cluster criteria and perform the AHP analysis on clustered criteria. Where this is not possible, a sensitivity analysis must be done to check the stability of the solution obtained. In this survey a total of 12 criteria were identified and these exceed the recommended maximum number of 9. It was not possible to cluster the criteria, therefore sensitivity analyses were done to establish the stability of the optimal solution derived.

The sensitivity analyses are also necessary for two other reasons. Firstly, the process of assigning weights of importance to optimization or decision criteria is partly subjective depending on an individual's knowledge and experience or a company's policies and experiences. Secondly, as noted earlier, human judgements tend to have some degree of inconsistency, which the AHP methodology is able measure, as was done in this study, but cannot be eliminated. This implies that sensitivity analyses will aid in validating the solution against the slight inconsistencies inherent in the decision-making process.

Sensitivity analyses were done for several scenarios. The results of the sensitivity analyses confirmed the same trend towards smaller spacing and square stope configurations, although the range of spacing changes slightly in each case. The following cases are used to illustrate this observation:

- All criteria have equal weighting of 1, implying a case of indifference to the importance of each criterion (Figure 13).
- The importance of NPV should have been 10% more than what respondents thought it was (Figure 14).
- The importance of NPV should have been 10% less than what respondents thought it should have been (Figure 15).

The optima for square stope geometries could be explained as the result of the impact of the strong influence of the RF. A conventional breast mining stope is defined by the raiseline spacing and backlength. When the backlength is not equal to the raiseline spacing as currently practised on

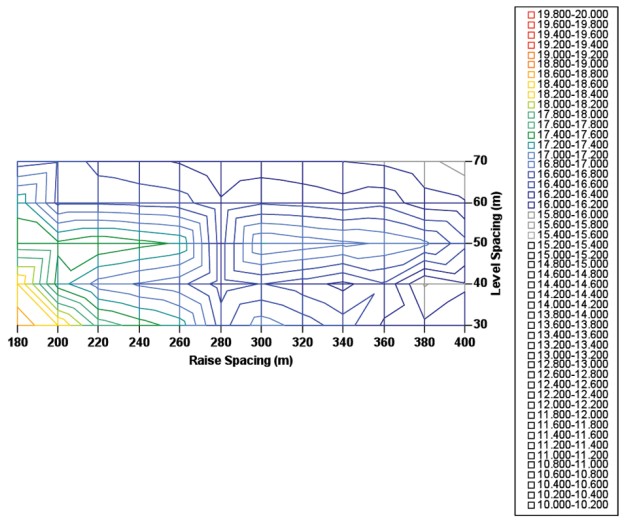

Figure 13—A 3D surface plot of the AHP priority score when criteria carry equal weighting

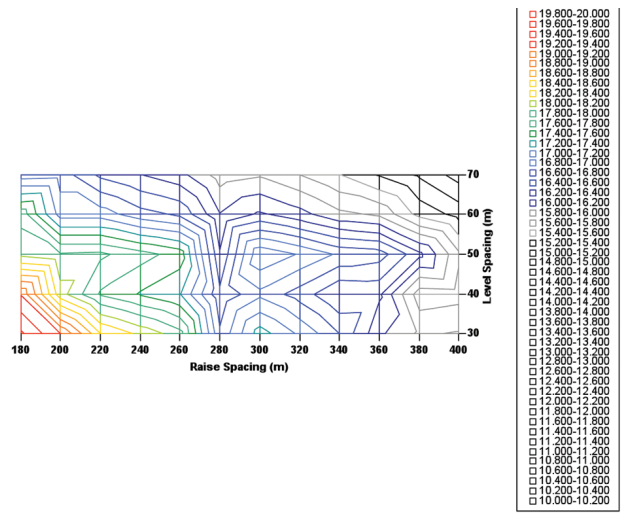

Figure 14—A 3D surface plot of the AHP priority score when NPV weight increases by 10%

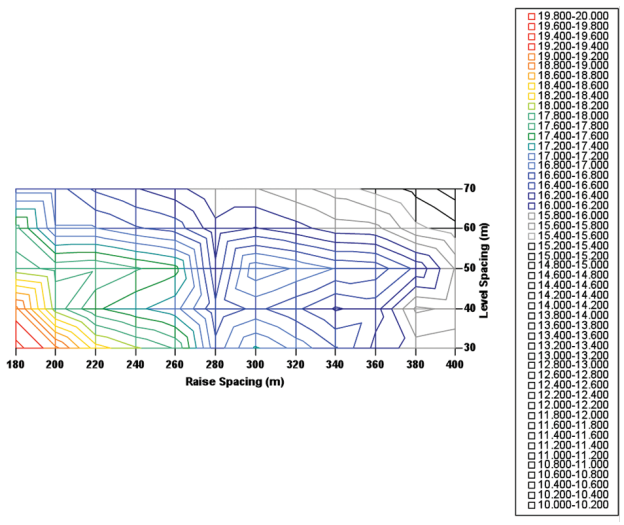

Figure 15—A 3D surface plot of the AHP priority score when NPV weight decreases by 10%

most platinum mines, the stope shape is rectangular. Considering the fact that the RF is an important factor in measuring the efficiency of conventional breast mining layouts, it is necessary to establish a stope shape that results in an optimal RF. An optimal RF is obtained when a fixed perimeter defined by a raiseline spacing and backlength produces a shape of maximum area, hence minimizing the amount of development required to expose and extract a m^2 of stoping. Such a problem is typically solved using the Lagrange multiplier method as explained below.

Consider a stope shape bounded by two consecutive raiselines each of length B and at a raiseline spacing of R as shown by the longitudinal section in Figure 16.

The problem can be summarized as a function of two variables R and B that requires maximizing the area of the stope, A, as given by Equation [1] and subject to the perimeter constraint given by Equation [2].

$$A = RB \quad [1]$$

$$P = 2R + 2B \quad [2]$$

In Equation [2], P is the fixed perimeter of the stope. The Lagrangian, L, for this system of equations is given by Equation [3].

$$L = RB + \lambda(P - 2R - 2B) \quad [3]$$

In Equation [3], λ is the Lagrange multiplier. By taking the partial derivatives for R, B, and λ and setting them to zero to determine the saddle points, the transform equations shown by Equations [4], [5], and [6] are obtained.

$$\frac{\partial L}{\partial R} = B - 2\lambda = 0 \quad [4]$$

$$\frac{\partial L}{\partial B} = R - 2\lambda = 0 \quad [5]$$

$$\frac{\partial L}{\partial \lambda} = P - 2R - 2B = 0 \quad [6]$$

The solution of the two transform Equations [4] and [5], is given by Equation [7].

$$\frac{R}{2} = \frac{B}{2} = \lambda \quad [7]$$

By back-substituting Equation [7] into Equation [6], Equations [8] and [9] are obtained:

$$R = B = \frac{P}{4} \quad [8]$$

$$\lambda = \frac{P}{8} \quad [9]$$

It can be concluded from Equations [7] and [8] that the maximum RF is obtained when the raiseline spacing is equal to the backlength, that is, the optimal stope shape is a square. Therefore maximum values of RF are obtained when raise spacing is almost equal to backlength, hence the local maxima observed in the 3D plots of AHP priority scores.

The research findings noted above, were initially presented to the two largest platinum mining companies in South Africa, Impala Platinum and Anglo Platinum, on 10 and 13 July 2009, respectively. Subsequently, I was invited to present the finding to the Association of Mine Managers of South Africa (AMMSA) and this was done on the 6th of August 2009.

Concluding remarks

This paper has demonstrated that, for decades, the narrow tabular reef mining industry in South Africa has been advocating longer backlengths (equivalent to wider level spacing), and wider raise spacing, instead of using smaller spacing that affords concentrated mining and higher productivities. This finding is in contrast to some traditionally held perceptions within the platinum mining industry that level and raise spacing optimization is achieved by increasing level and raise spacing. However, the finding was supported by one of the feedback comments from industry, which noted that, 'when conventional mining started in narrow reef mining, it started as 'concentrated breast mining' but as an industry we have over the years lost the plot by changing it to 'scattered' mining which does not afford us high productivities hence these findings make sense that we should be moving back towards smaller level and raise spacing' (Rogers, 2009). The drive for concentrated mining has previously been highlighted by Brassell (1964), Bullock (2001), and Vieira, Diering and Durrheim (2001). However, care must be taken that concentrated mining achieved through smaller level and raise spacing, is not a panacea for higher productivities because, as noted by Brassell (1964:461), the concentration of mining activities to improve productivity 'is no gimmick that can be introduced overnight with the introduction of new machines and techniques, but rather is the outcome of study, careful planning and the training of personnel, all of which takes much time and money to achieve.'

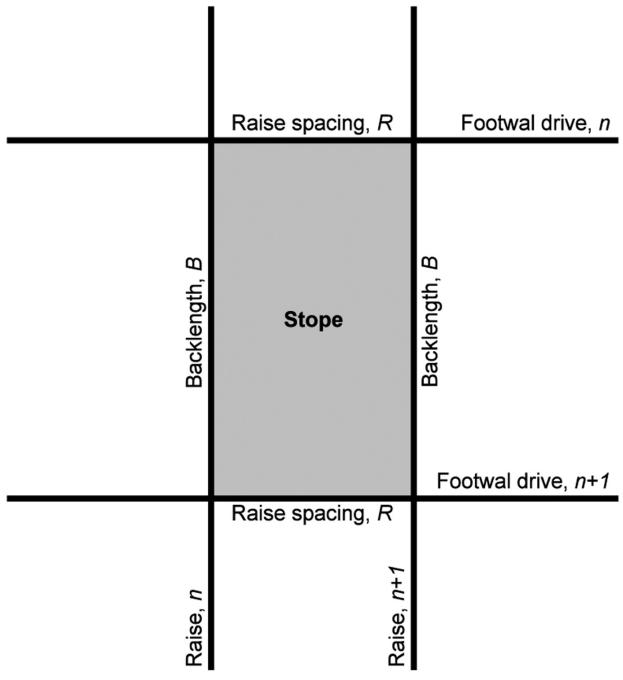

Figure 16—Stope blocked out by two consecutive raiselines and two consecutive levels

A second contribution coming from this research study is that mines should seriously consider using stope geometries that are approximately square because these are associated with overall local maxima as demonstrated by the 3D contour plots of the AHP priority scores and validated by the Lagrange multiplier method. A third contribution is that there is no economic merit in considering raise spacing increments of less than 20 m. Thirdly, the optimal vertical level spacing range of 30 m–50 m and optimal raise spacing range of 180 m–220 m derived OB1, which is a typical UG2 reef deposit, could be extended to apply to the rest of the Bushveld Complex since OB1 represents typical UG2 reef mining conditions. However, since each deposit has site-specific geological and geotechnical conditions, a more realistic range can be derived using the mine specific data with the methodology developed in this research study. This suggestion is reasonable because the methodology developed in this research study 'takes cognisance of the uniqueness of ore bodies by not providing a 'one size fits all' solution' (Impala Review Team, 2009). Lastly, the smaller spacing may also imply that there could be merit for greenfield projects to be designed at smaller spacing in the initial phases, to enable rapid build-up to full production and then space out the levels and raises a bit more in order to benefit from minimizing the development cost per centare mined.

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Appendix

Layout efficiency scores

						Criteria						
Layout	NPV (ZARmil)	PV_Dev_Cost (ZAR/m^2)	Payback Period (years)	RF (m^2/m)	Shaft Head Grade (g/t)	Overall Dilution (%)	Production Rate (tpa)	Productivity (m^2/stope employee)	Flexibility Index	Life of Raiseline or Stope (days)	LOM (years)	Build-up Period (years)
Objective	max	min	min	max	max	min	max	max	max	max	max	min
30_180	5,155	91.19	5.30	21.64	5.50	9.90%	577,128	42	1.79	325	65	8
30_200	5,064	88.93	5.39	22.02	5.50	9.81%	545,859	39	1.77	369	69	9
30_220	4,652	87.88	5.83	22.12	5.51	9.73%	477,357	38	1.67	374	74	9
30_240	4,616	86.58	6.07	22.31	5.51	9.67%	446,512	38	1.59	408	79	10
30_260	4,505	85.39	6.29	22.50	5.51	9.61%	419,900	37	1.53	442	84	10
30_280	3,148	84.50	6.72	22.75	5.52	9.51%	408,458	34	1.58	497	89	12
30_300	4,052	83.32	6.68	22.87	5.52	9.50%	376,214	36	1.41	510	93	11
30_320	3,710	82.40	6.86	23.06	5.52	9.45%	358,029	35	1.36	544	98	11
30_340	3,293	81.55	7.02	23.25	5.53	9.40%	341,747	35	1.32	578	103	11
30_360	2,454	80.77	7.36	23.46	5.53	9.34%	335,472	35	1.27	570	109	11
30_380	2,229	80.01	7.33	23.62	5.53	9.32%	313,777	34	1.24	646	112	12
30_400	2,191	79.21	7.21	23.78	5.53	9.31%	321,413	35	1.17	679	115	12
40_180	4,942	82.95	6.75	22.33	5.50	9.84%	653,560	39	1.72	361	69	9
40_200	5,060	80.35	6.23	22.78	5.51	9.67%	348,700	39	1.57	423	74	10
40_220	4,669	79.60	6.84	22.88	5.51	9.67%	340,411	37	1.50	447	78	10
40_240	4,655	78.40	6.99	23.09	5.51	9.67%	322,346	36	1.41	488	82	11
40_260	4,568	77.31	7.13	23.29	5.52	9.51%	306,575	36	1.34	528	87	11
40_280	3,348	76.33	7.67	23.58	5.52	9.51%	399,636	33	1.31	565	90	12
40_300	4,176	75.40	7.37	23.69	5.52	9.51%	280,277	35	1.21	609	96	12
40_320	3,870	74.56	7.48	23.89	5.53	9.34%	269,167	34	1.16	650	100	13
40_340	3,491	73.77	7.59	24.10	5.53	9.34%	259,133	34	1.11	691	105	13
40_360	2,961	73.10	7.30	24.31	5.53	9.34%	325,226	34	1.05	706	112	12
40_380	2,514	72.35	7.78	24.50	5.53	9.34%	241,685	33	1.03	772	114	14
40_400	2,276	71.72	8.03	24.63	5.53	9.34%	316,732	33	1.00	832	116	14
50_180	3,978	77.40	9.19	23.08	5.50	9.84%	518,945	37	1.88	444	71	9
50_200	4,362	73.85	7.16	23.67	5.51	9.75%	472,142	41	1.50	534	79	12
50_220	4,046	73.54	8.25	23.75	5.51	9.68%	589,510	37	1.46	607	81	11
50_240	4,062	72.34	8.20	23.98	5.51	9.62%	556,929	37	1.36	662	85	12
50_260	4,019	71.26	8.16	24.20	5.52	9.56%	528,550	36	1.26	717	89	12
50_280	3,159	69.98	8.94	24.56	5.52	9.47%	408,482	34	1.14	750	89	13
50_300	3,758	69.37	8.08	24.65	5.52	9.45%	481,370	34	1.12	827	98	13
50_320	3,540	68.53	8.04	24.87	5.53	9.41%	461,493	34	1.06	882	103	14
50_340	3,262	67.75	8.01	25.10	5.53	9.36%	443,569	33	1.00	937	107	14
50_360	3,220	67.12	6.49	25.32	5.53	9.30%	312,142	35	0.92	898	117	14
50_380	2,532	66.34	7.94	25.54	5.53	9.28%	412,477	32	0.91	1048	116	15
50_400	2,082	65.88	8.90	25.66	5.53	9.28%	313,047	30	0.92	1236	118	15
60_180	4,004	72.37	9.79	23.61	5.50	9.84%	577,956	36	1.49	542	78	10
60_200	4,204	69.95	8.43	24.11	5.51	9.67%	310,570	37	1.27	635	83	11
60_220	3,864	69.40	9.87	24.20	5.51	9.67%	421,219	35	1.22	670	86	11
60_240	3,858	68.38	9.90	24.41	5.51	9.67%	403,550	34	1.14	731	90	12
60_260	3,793	67.45	9.94	24.63	5.52	9.51%	387,951	33	1.07	792	94	13
60_280	2,874	66.50	9.69	24.98	5.52	9.51%	376,863	31	1.01	847	96	12
60_300	3,484	65.82	9.99	25.06	5.52	9.51%	361,551	32	0.95	914	101	14
60_320	3,240	65.10	10.02	25.27	5.53	9.34%	350,239	32	0.91	975	105	14
60_340	2,936	64.43	10.04	25.48	5.53	9.34%	339,937	31	0.86	1036	109	15
60_360	2,481	63.87	8.74	25.73	5.53	9.34%	315,075	32	0.80	1059	115	14
60_380	2,151	63.22	10.09	25.91	5.53	9.34%	321,816	30	0.79	1158	116	16
60_400	1,967	62.72	10.01	26.06	5.53	9.34%	310,388	30	0.78	1248	119	18
70_180	3,453	69.47	10.46	24.00	5.50	9.84%	447,999	34	1.29	587	83	10
70_200	3,594	67.72	9.45	24.45	5.51	9.74%	433,845	34	1.14	683	86	11
70_220	3,282	67.02	10.12	24.57	5.51	9.67%	425,328	33	1.09	696	90	11
70_240	3,270	66.10	10.20	24.78	5.51	9.61%	407,551	32	1.02	759	93	12
70_260	3,207	65.26	10.27	25.00	5.52	9.55%	391,854	32	0.96	822	97	13
70_280	2,423	64.49	10.35	25.31	5.52	9.47%	358,307	29	0.93	880	101	11
70_300	2,925	63.79	10.40	25.42	5.52	9.45%	365,282	31	0.87	949	103	14
70_320	2,707	63.13	10.46	25.63	5.53	9.41%	353,895	30	0.83	1012	107	15
70_340	2,437	62.53	10.52	25.84	5.53	9.36%	343,522	30	0.79	1075	110	15
70_360	1,869	62.01	10.46	26.06	5.53	9.31%	317,058	30	0.74	1152	114	14
70_380	1,742	61.43	10.62	26.27	5.53	9.29%	325,276	29	0.73	1202	117	16
70_400	1,695	60.91	10.83	26.40	5.53	9.29%	308,841	30	0.70	1223	119	20
sum	205,081.41	4,344.02	503.23	1,453.97	331.19	570.72%	23,439,841.47	2,048.49	71.05	44,200.17	5,777.73	744.77