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Mathematical knowledge for teaching: a focus on the mathematical work of a grade 8 teacher when introducing algebra

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DECLARATION

I declare that this research report is my own unaided work. It is being submitted for the degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

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(Signature of candidate)

5th day of June

ABSTRACT

This study investigates mathematical knowledge for teaching. One Grade 8 teacher was interviewed and observed for one week while teaching algebra to 8th graders. I focused on mathematical problems that confronted her as she went about her work, the mathematical work of teaching she grappled with, the knowledge resources she called on as well as the algebraic activities that she presented to her learners.

The analysis shows that she wrestled with explaining, representing, questioning, defining, working with learners' ideas while restructuring tasks was entirely absent in this practice. It was interesting to see that learners were exposed to representational, transformational and reading activities of algebra and not the generalizing and justifying activities of algebra. The analysis further shows that this teacher drew on mathematics, curriculum and her experience in order to sustain her practice. Based on the findings of this study I argue that mathematical knowledge for teaching in this practice is empirical, with a strong focus on conventions as well as terminology; and is characterized by representational, transformational and reading activities.

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KEYWORDS

Algebra

Mathematical knowledge for teaching

Pedagogical content knowledge

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Last but not least, I wish to express my sincerest thanks to my wife Puleng and our daughter Hlompho for the sacrifices they made, to my parents, brothers and sisters who supported and encouraged me throughout this academic journey. I am proud of you.

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DEDICATION

To my late mother, ‘Mafusi Alice Lerotholi-Talasi. Standing on your shoulders, I was able to see far.

To Puleng Mabejane-Talasi and Hlompho Talasi for their love, support and sacrifices.

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ABBREVIATIONS

ACE…….. Advanced Certificate of Education

B.Sc…….. Bachelor of Science

DoE…….. Department of Education

IRF…… Initiation Response Feedback

LOLT…… Language of Learning and Teaching

MfT…….. Mathematics for Teaching

MKfT…… Mathematical Knowledge for Teaching

PCK…….. Pedagogical Content Knowledge

PUMK…….. Pedagogically Useful Mathematical Knowledge

QUANTUM…….. Qualifications for teachers underqualified in mathematics

RNCS…… Revised National curriculum Statement

USA…….. United States of America

Wits…… University of the Witwatersrand

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MNEMONICS

BODMAS…….. Brackets, Of, Division, Multiplication, Addition, Subtraction

FOIL…….. First, Outside, Inside, Last

SOHCAHTOA…….. Sine = Opposite ÷ Hypotenuse;
Cosine = Adjacent ÷ Hypotenuse;
Tangent = Opposite ÷ Adjacent

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Chapter one: Introduction, Rationale and teaching algebra

1 Introduction

Recently, there have been curriculum shifts in mathematics in many countries that are driven by national policies, and influenced by research. There is advocacy for a transition from traditional ways of teaching that emphasize mastery of procedures and algorithms, to approaches which also foster mathematical reasoning in learners or learning mathematics as a set of practices e.g. justifying, representing, generalizing etc. It is believed that these new approaches to teaching mathematics would enable learners to re-invent mathematical ideas, procedures and algorithms when forgotten hence serve to aid retention of learnt facts. These curriculum innovations are also found in South Africa. Brodie & Pournara (2005) argue that these shifts are signaled by terms such as *educator*, *facilitator* and *mediator* (p.48) within the South African curriculum. According to them these terms indicate new roles for the teacher. The South African Department of Education (DoE, 2002), aligning itself with these curriculum shifts, advocates that learners should investigate patterns and relationships so as to "develop mathematical thinking skills such as generalizing, explaining, justifying, representing … predicting and describing" (p.63).

Curriculum change is not a smooth process. In support, Adler (2005) argues that more often there is a mismatch between the prior learning of in-service teachers and the intended curriculum. Hence the question: what mathematical and pedagogical knowledge do teachers need to know and know how to use in practice (Adler, 2005) in order to teach mathematics well? In addition, she argues that new topics that come with these curriculum shifts pose problems to in-service teachers because they have no working experience relating to what learners would find difficult, which activities work well and what misconceptions may arise in learners. I note that in this study, the topic (i.e. algebra) is not new but it is expected that it would be taught in the new ways as advocated by curriculum shifts. Adler (op cit) further points out that the

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mathematical knowledge that is needed to implement the curriculum is specific for teaching and so not the same as the kind of mathematics used in other fields such as Engineering, medicine, economics and the work place.

Informed by these changes in curriculum and the ongoing debate on the mathematical knowledge for teaching that teachers need to know and be able to use in practice, I set out to look at the mathematical problems that confront a Grade 8 teacher, the mathematical work[1] of teaching that she does as she goes about her work, and knowledge resources that are called on in this practice as well as the nature of the algebraic activities used. The assumption here is that if we understand what the teacher does, how and why, and consider how this relates to the goals of a new curriculum, we will be able to relate what she might need to know and use that is currently not in her repertoire.

1.2 Research problem and research questions

This study explores the mathematical work of teaching that a Grade 8 teacher does as she goes about her work, the mathematical problems that confront her and the knowledge resources that she calls on as she introduces algebra. This study is therefore located in the broad area of mathematical knowledge for teaching. In order to gain insight into the teacher's mathematical work I studied a teacher in practice. The following specific research questions guide this study:

1. What is the nature of the activities that the Grade 8 teacher uses to introduce algebra?
2. What mathematical problems[2] does the teacher confront as she introduces

[1]I use the term Mathematical work of teaching to describe what it takes mathematically to make mathematics accessible to learners (see also Ball & Bass, 2000, Adler & Pillay, In Press). Ball, Bass & Hill (2004) use the term mathematical problem solving instead.

[2] In the literature what I call mathematical problems is referred to as either learner's errors, misconceptions or learning difficulties. These compel teachers to grapple with mathematical work of teaching in order to move their learners on hence errors, misconceptions and learning difficulties

algebra in Grade 8?

3. What mathematical work does the teacher do as she goes about solving these mathematical problems in her teaching practice, specifically in order to introduce algebra to Grade 8 learners?
4. What knowledge resources does she call on for this work?

Together these questions enable reflection and consideration on mathematical knowledge for teaching particularly in relation to the new curriculum.

1.3 Rationale

Teaching encompasses both regularities and uncertainties. Teachers cannot always predict with certainty what learners know and how they know it; hence they always confront problems of practice. According to Ball, Bass & Hill (2004) mathematics teachers often analyze learners' responses for errors. At times, teachers find themselves appraising unexpected claims, solutions and methods, select and modify tasks (Ball, (unpublished work); Ball *et al.* (2004); Kazima & Adler 92006). Setting up activities/ tasks for learners and implementing them demand that the teacher is able to use both the language and concepts definitions that are appropriately pitched for the grade (Ball *et al*., 2004). Ball *et al*. (op cit) further argue that teaching mathematics demands that teachers give and evaluate explanations so as afford learners to gain access to the subject. Ball *et al*. (2004, p.59) refer to these activities and others which confront mathematics teachers as they go about their work as 'mathematical practices that involve mathematical problem solving' (see also Ball & Bass, 2000, Adler, 2005). Adler & Pillay (in press) have since described these activities of teachers as 'mathematical work' of teaching. In this study, I use 'mathematical work of teaching' instead of 'mathematical problem solving' so as to prevent confusion with the notion of problem solving commonly used in mathematics

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become problems of the practice that need to be solved. Researchers who have done some work in this area include MacGregor & Stacey, 1996a, 1997; Brodie, 2004; Wheeler,1996

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as well as the notion of mathematical problems that confront teachers which is a focus of this study.

Research that has been carried out in South Africa indicates that mathematics courses in teacher training programmes appear to be dominated by mathematical demands that emphasize procedural fluency and compressed mathematics (Adler, 2005; Adler & Davis, 2006). So where then do teachers learn to do the kind of mathematical work as described above? Furthermore, an emphasis on procedural fluency seems to be in contradiction with what is demanded by the Revised National Curriculum Statement (RNCS) for mathematics (DoE, 2002) which requires learners to engage with mathematical practices such as generalizing, explaining, justifying, representing, conjecturing, testing, analyzing, inferring, predicting, describing, and others. RNCS has redefined the mathematics content and how it should be taught and learned by including these practices which require teachers to work with unrefined forms of mathematics in order to make the subject accessible to learners. Consequently, teaching algebra has taken a new form which requires teachers to expose their learners to more open-end tasks so that learners can start to recognize the power of algebra over arithmetic with which they are conversant. They would realize that algebra is a tool for making generalizations and providing justifications i.e. it enables them to talk about calculations without actually performing any calculation.

The Revised National Curriculum Statement for mathematics (DoE, 2002) suggests introduction of algebra in the senior phase (Grade 7-9). The curriculum states that in Grade 7 learners should "describe, explain and justify observed relationships or rules verbally" p.74, while the 8^{th} graders are expected to either use "words or algebra" p.75. We see that the curriculum emphasizes the use of language and considers language to be an essential component of learning mathematics (algebra) as early as Grade 7. Just like other concepts, mathematical concepts are carried in a language; so are algebraic concepts. This raises some additional issues for the teacher. For instance, in mathematics some words (such as difference, product etc.) take on

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specialized meanings which may be a problem for some learners. This poses new problems of mathematics for teaching since if the teacher is to promote algebraic proficiency he or she has to listen interpretively (Davis, 1997) to learners' contributions so as to push their thinking forward.

According to the RAND study Panel (2002) algebra plays a gate-keeping role in school mathematics as it determines who will have access to higher education and other career opportunities. They argue that the reason for this is that algebra functions as a language system for ideas about quantity and space, and so serves as a foundation as well as prerequisite for all branches of mathematics. This could be the reason as to why the South African curriculum emphasizes the use of algebraic language as early as Grade 8. Proficiency in mathematics, of course, is not restricted to algebraic language only. Kilpatrick, Swafford & Findell (2001) argue that mathematics teaching and learning should address five interdependent strands/elements of mathematical proficiency (see section 1.3.2). According to Kilpatrick *et al.* (op cit) adaptive reasoning, one of the five strands, is a cognitive tool which enables one to *justify conclusions* made regarding the relationships among situations and concepts. They further argue that without adaptive reasoning one would have a problem in deciding about which procedures, concepts or solution methods are useful in a given situation.

School mathematics is divided into different branches such as geometry, arithmetic, statistics, algebra and others. RNCS has redefined these branches as "(i) Number, Operations and Relationships, (ii) patterns, Functions and Algebra, (iii) Space and Shape [Geometry], (iv) Measurement and (v) Data Handling (DoE, 2002). Algebra is one of the oldest branches of mathematics. Wheeler (1996) points out that full development of algebra took at least a millennium. He sees algebra as the completion of arithmetic as it answers some of the questions left out by arithmetic. Kilpatrick *et al.* (2001) argue that algebra builds on the proficiency that learners have been developing in arithmetic and develops it further. Arithmetic deals mainly with

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concrete numbers, empirical arguments and the actual execution of mathematical operations to obtain a result. Algebra on the other hand, is concerned mainly with making generalizations and justification of arguments. For example, in arithmetic one may show with examples that the sum of two consecutive numbers is odd but to show that this holds for all numbers becomes impossible. This problem is easily resolved using algebra by representing the problem as follows: $p+(p+1)$ where p is any whole number. This simplifies to $2p+1$which is the general form of an odd number. Wheeler (op cit) further points out that arithmetic could not be developed further without algebra due to its adherence to concrete numbers. In the primary school, learners mainly deal with arithmetic. On entering the secondary school learners have to make a significant shift as they move from the concrete in arithmetic to the more general in algebra which dominates the secondary school curriculum.

Bell (cited in Wheeler (1996)) contends that there is a multiplicity of algebra, not just one. These include abstract algebra, linear algebra, Boolean algebra and elementary (school) algebra. Wheeler (op cit) argues that elementary or school algebra is concerned with actions; "collecting like terms, factorizing, expanding, solving equations, simplifying and others" p. 319. Kilpatrick *et al.* (2001) refers to these actions as transformational or rule-based activities of algebra since their main focus is to transforming the 'form of an expression or equation into an equivalent form' through manipulation of the symbols and execution of a series of mathematical operations. For example, simplifying $3(x+1)+4(2x+3)$ requires one to replace $3(x+1)$and $4(2x+3)$ with $3x+3$and $8x+12$ respectively, and then grouping like terms to get $11x+15$ as an equivalent expression. While algebra builds on arithmetic, it differs from arithmetic in that algebra as a symbolic system does not depend on place-value which is central to arithmetic. In addition, when letters are used in arithmetic they represent unknowns. In algebra, however, they are variables – letters that can assume any value. Kilpatrick *et al*. (2001) point out that learning algebra is problematic to many learners although the "place-value numeration system used for

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arithmetic implicitly incorporates some of the basic concepts of algebra, and the algorithms of arithmetic rely heavily on the "laws of algebra"" p. 256. So then, what mathematical knowledge for teaching do teachers need to know and use in practice in order to manage the transition from arithmetic to algebra and consequently increase learners success in learning algebra?

1.3.1 Why is studying algebra important?

Generalizing and justification activities of algebra afford us the opportunity to express relationships between quantities. Algebra functions as a language and a tool for understanding other branches of mathematics (RAND study panel, 2002). Algebraic methods are used in other branches of mathematics such as geometry to solve geometric problems. Algebraic activities such as substitution play a vital role in problems involving calculus while factorization of algebraic expressions is useful in solving quadratic equations. Today's businesses rely heavily on the generalizing power of algebra as it enables them to produce mathematical models that can be used to predict future gains of the business. So knowledge and understanding of school algebra contributes to mathematical proficiency.

1.3.2 Attaining proficiency in mathematics, specifically in algebra

Teaching mathematics entails exposing learners to both the mathematical content and the language of mathematics. According to DoE (2002) content in mathematics consists of knowledge i.e concepts, operations and procedures, skills and values. In addition, DoE (2002, p. 4) points out that "mathematics uses its own specialized language that involves symbols and notation for describing numerical, geometrical and graphical relationships". Hence learners should be taught this language if they are to be proficient in mathematics. The debate on the relationship between language and mathematics dates back to Pimm (1981) who argues that the main role of a mathematics teacher is to assist learners in communicating their mathematical ideas. He further points out that, it is the teacher's responsibility to develop fluency in both spoken and written language of mathematics. It is clear that for Pimm (op cit) mathematics teaching is intended to develop communicative fluency in the language

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of mathematics. The question is; what mathematical knowledge does the teacher need to know and be able to use in practice in order to teach the language of mathematics?

Kilpatrick *et al.* (2001) coined the term mathematical proficiency to describe what mathematics teaching should develop in learners while in school. Mathematical proficiency as described by Kilpatrick *et al.* (op cit) consists of five interrelated elements or strands in their language viz:

- *Conceptual understanding* – comprehension of mathematical concepts, operations, and relations
- *Procedural fluency* – skills in carrying procedures flexibly, accurately, efficiently, and appropriately
- *Strategic competence* – ability to formulate, represent, and solve mathematical problems
- *Adaptive reasoning* – capacity for logical thought, reflection, explanation, and justification.
- *Productive disposition* – habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy [p. 116].

A closer look at these elements of mathematical proficiency shows that they incorporate the knowledge i.e. numbers, operations and relationships, skills such as representing, interpreting, analyzing, describing and others as well as values like love of mathematics and curiosity which are being advocated by DoE (2002, p.4). In addition, the notion of mathematical proficiency seems to bring together traditional and reform oriented approaches as it picks up the aspects that were ignored or deemphasized by each approach. For instance, traditional approaches emphasized mastery of procedures at the expense of skill such as representing, interpreting, analyzing, describing, problem posing etc. (DoE, 2002) while reform oriented approaches do the opposite. This notion gives me a more holistic picture of what the teacher may endeavor to develop as she introduces algebra as it includes both the

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practices and the procedures. However, this study employs these elements of mathematical proficiency with caution as they do not seem to completely help in describing this teacher's practice. For instance, they dot not say anything about the acquisition of mathematical language which seems to be central to this teacher's practice as the analysis will show. It should be noted that mathematical proficiency develops over time and that mastery of one strand has a direct influence on the other due to their interdependency.

Kilpatrick *et al.* (2001) and Pimm (1981) agree that fluency in mathematics is necessary although their perceptions of fluency in mathematics seem to differ. Kilpatrick *et al.* (op cit) advocates procedural fluency while Pimm (op cit) advocates communicative fluency. This study will show another form of fluency that is required in mathematics. I have termed this **conventional fluency**. In this form of fluency the teacher makes explicit meaning of mathematical notation (symbols and operations) and teaches accepted mathematical ways of writing using algebraic language and notation so as to make algebra accessible and sensible to her learners. The extract below sheds some light on this issue. The bold text shows how the teacher socialized learners into the mathematician's ways of writing using algebraic language and notation while the underlined text shows how the notation was made explicit in this practice.

Lineo[3]**:** Ok, **traditionally we always put up the numbers before the letter**. So $n2$ is correct but **we prefer to write the number before the letters**. Also we don't write the times between a number and the letter. When I write $2n$ what are we assuming?... 2 times n …
The same thing here with the x why did I write the curly x ?
Mpho: we do not want a straight x
Lineo: Don't call it out.
Thabo**:** inaudible
Lineo: Ok, as Thabo said, **if I were to write that as a straightforward x what does that look like? times times 2 or xx2 doesn't make sense. So guys I'm going to be very strict with you, when you write your x 's, it is the only letter that you have to change. I want you to write it as a curly x neatly**. You need to get into that habit. I will take off marks if you don't (Lineo, lesson 1, 07:32 – 09:00)

[3] Lineo is the name given to the teacher involved in this study. All the names that appear in this study are pseudonyms.

I began this study with the hope that the strands of mathematical proficiency would be adequate in describing this teacher's practice. However, on interacting with my data, I realized that the strands of mathematical proficiency were inadequate as they did not explicitly indicate how learners acquire fluency (conventional fluency) in handling mathematical symbols which is highly demanded in learning algebra as learners have to perform calculations quickly and effectively. The issue of how learners gain facility in handling mathematical symbols has also been left out by proponents of reform oriented teaching. Through my further reading on Kilpatrick *et al.*'s (2001) work, I found refuge in what they call activities of algebra. Kilpatrick *et al.* (op cit) point out that when working within algebra, the five elements of mathematical proficiency are subsumed in the following activities of algebra; "representational activities, transformational (rule-based) activities, and finally generalizing and justifying activities" p.256. It is these activities of algebra that I finally employ in studying this teacher's practice hence it is worth discussing each of them.

1.3.3 Activities of algebra

1.3.3.1 Representational activities

These activities require one to transform verbal information into symbolic expressions (Kilpatrick *et al.*, op cit). Since they demand one to engage in formulating and representing information with algebraic expressions or equations, representational activities facilitate development of 'strategic competence'. In order to formulate algebraic expressions, one needs to understand mathematical "concepts, operations and relations" (Kilpatrick *et al.*, op cit, p.116) as well as conventions and notation thereof. Consequently, representational activities also enhance the development of conceptual understanding.

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1.3.3.2 Transformational (rule-based) activities

Due to their rule-based nature these activities turn to assist in the development of 'procedural fluency'. Here one engages in transforming (changing) expressions and/ or equations into their equivalent forms by executing a series of operations on the original expression or equation. Symbol manipulation is central to transformational activities. This category comprises of what Wheeler (1996) terms actions of algebra.

1.3.3.3 Generalizing and justifying activities

These activities enable us to move beyond empirical arguments of arithmetic into making generalization and justifications. For instance, in arithmetic one can cite examples to show that the sum of two odd numbers is even but it is not easy to show that this holds for all numbers. Algebraically this could be generalized as $2p+1+2x+1=2p+2x+2$, where the key is recognition that x and p are whole numbers. The last expression could be transformed into an equivalent form $2(x+p+1)$ which is the general form of any even number. So this category of algebraic activities includes adaptive reasoning. Kilpatrick *et al*. (2001, p.258) argue that these activities

> can provide insight into, for example, the underlying mathematical structure of a situation… they encourage students to develop awareness of the role algebra can play in mathematical thinking. All strands of mathematical proficiency come together in these activities…

Hence meaning of algebraic symbols resides in generalizing and justifying activities as they afford learners the opportunity to create meaning for the rules or learn when to use them and why thus helping novices to "avoid forgetting, unsystematic errors, reliance on visual clues and poor strategic decisions" (Kilpatrick *et al*, op cit, p. 259).

The above-mentioned activities of algebra are important in this study in that they help me to realize that as I observe the teacher I may not directly see any of the elements of mathematical proficiency as stated earlier but may see these algebraic activities which encompass the strands of mathematical proficiency.

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Traditional teaching, also known as transmission teaching (Boaler, 1997), of algebra seems to have been over-emphasizing both representational and transformational activities of algebra at the expense of generalization and justification as it dealt mainly with symbol manipulation. Proponents of this teaching approach ignored the apparently most important feature of algebra – generalizing. On the other hand, proponents of reform oriented teaching pay more attention to generalization and justification activities of algebra. What I find interesting is that both approaches of teaching mathematics, algebra in particular, say nothing about acquisition of mathematical language which turns out to be critical to this study.

I find these activities of algebra to be critical to this study in that they do not only incorporate the strands of mathematical proficiency but they also include what Wheeler (1996) termed activities (actions) of school algebra such as collecting like terms, factorizing, expanding, substitution etc., algebraic notation and conventions thereof (Kilpatrick *et al*., 2001). For learners to be able to engage with these activities of algebra successfully they need to have a language so that they can communicate their thoughts in both written and spoken formats. Language facilitates learning in that it enables learners to share their thoughts and also functions as a tool for thinking. It is through the use of a language that teachers become aware of their learners mathematical (mis)understanding. The three activities of algebra indicate that learning and teaching mathematics is about manipulating symbols/numbers (found in representational and transformational activities) and also about using a (mathematical) language to represent ideas, to formulate and solve problems (found in generalization and justification activities). So how do teachers assist learners in acquiring the language of mathematics which seems to be critical to their engagement with generalization and justification activities of algebra?

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1.3.3.4 Reading/ Writing activities

In carrying out this study, I found that the algebraic activities discussed by Kilpatrick *et al.* (2001) do not fully capture this teacher's practice as they do not deal with how mathematical language is taught. To bridge this gap, I found it necessary to add another category which I have named **reading/ writing activities**. I had previously called this category conventional fluency [see page 8]. This category consists of all activities (excluded in the above activities of algebra) that are intended to make the mathematics (algebraic) register explicit to learners. In this case the teacher may define mathematical terms, explain algebraic notation (symbols and operations) or endeavor to attach mathematical meaning to the symbols or operations used in algebra. For example

> Lineo: Looking at number 2, n^2, is it the same as $n \times n$? Is that correct?
> What does n^2 mean?
> Learner: $n \times n$
>
> Lineo: $n \times n$ that would be perfect. (Lineo, lesson 1, 07:21-07:31)
>
> **What does that dot mean**? (Points at the dot between 2 and 4)
> Learners: times
> Lineo: times, you have seen that before?
> Learners: some say yes, others say no
> Lineo: yes, no. those people who possibly have not seen it before, again to **prevent confusion instead of writing the times because that might be confused with the x we just write a dot (writes $2 \bullet 4$) as soon as you see a dot, it will mean a times. So 2 times 4, 4 times** 2, the same difference. (Lineo, lesson 1, 23:20-23:49).

The underlined text indicates that the teacher is exposing learners to how to read algebraic notation and write algebraically by involving more than one representation of a square number. The bold text shows that the teacher attaches an algebraic meaning to the dot so that learners could read it as multiplication rather than a dot, a full stop or a decimal point. The **reading/writing activities** in algebra incorporates all activities regarding writing, reading, identifying, interpreting notation hence it is about being able to read and write.

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1.3.4 Algebra within the South African school curriculum

In South Africa, algebra is theoretically introduced to learners at Grade 8. Teachers could introduce algebra to their learners through generalizing number patterns, functions, problem solving and modeling (Usiskin, 1998; wheeler, 1996). The South African Department of Education (2002) encourages teachers to introduce algebra through number patterns. Osei (2005, p.5) contends that within the South African school curriculum algebra is made up of "simplifying expressions, factorizing, solving equations, functions and graphs, variables, word problems and others". At Grade 8, school algebra consists of "number patterns, solving linear equations, graphs, algebraic expression and algebraic vocabulary" (DoE, 2002, pp. 75-79). One can see that school algebra within the South African curriculum consists of representational, transformational, reading/writing as well as generalization and justification activities of algebra discussed in the previous sections as early as Grade 8. It is therefore important to find out what knowledge resources are put into use and the work of teaching that the teacher wrestles with when introducing algebra through a set of activities.

Curriculum shifts in South Africa require teachers to change their conception of algebra and their teaching strategies. Traditionally, learners were made to be passive recipients of knowledge in the form of rules and tricks of algebra aimed at getting the correct answer. While this developed procedural fluency in learners as advocated by Kilpatrick *et al*. (2001), it de-emphasized the other strands of mathematical proficiency hence products of traditional teaching were lacking in some aspects of mathematical proficiency as described earlier. Presently learners are expected to take a more active role in their learning by investigating patterns, making conjectures and justifying them (i.e. learners are to work with generalizing activities) under the guidance of the teacher. Teachers are expected to be more innovative and creative in designing learning programmes that will help learners to attain the expected outcomes (DoE, 1997).

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1.3.5 How is the notion of algebra developed in schools?

Arithmetic symbols and words are not as manipulatable as algebraic variables. The flexible use of variables makes algebra to be very important to humans in different spheres of life. Algebra is a powerful tool which can be used to express relationships accurately, clearly and concisely (Arcavi, 1995). The power and conciseness of algebra can be seen in cases where one needs to generalize patterns to get a formula that can be easily applied such as in finding the total number of league games a soccer team would play in a league with n teams if each team has to play with each other, home and away and hence to find the maximum number of points needed to win the league if a team gets three points for a win, one point for a draw and zero points for a loss.

According to Wheeler (1996) school algebra starts when an attempt is made to find an unknown number on which a given operation is preferred and a given result is obtained by making use of the symbols or letters (see also Stacey, Chick & Kendal, 2004 - fruit and salad algebra). Most mathematics textbooks use this approach to introduce learners to equations, in particular linear equations. MacGregor & Stacey (1997) point out that in Australia, learners first encounter algebra in Grade 7 or Grade 8. They argue that at this level learners are "taught to use letters to stand for unknown or generalized numbers, frequently in the context of writing formulas for number patterns. They/ *Learners* [italics added] are given the opportunity to learn how to write simple expressions and equations containing letters, numerals, operation signs and brackets" P.2. However, to most learners studying algebra, symbols appear to have little or no meaning. According to Wheeler (op cit) algebraic symbols are very important in mathematics, not only as means of expressing generalizations and as tools for manipulating problems that might be difficult to solve but also as a means of thinking and drawing conclusions. It is therefore important that learners are assisted in developing meaning for symbols employed in algebra and seeing their significance in communicating and expressing mathematical ideas. As learners investigate patterns and justify their conjectures i.e. work with generalizing activities, they construct

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meaning for the symbols. Hence meaning is linked with generalizing and justifying activities of algebra. The question that remains for this study is; what meaning of algebra are the learners in this study making?

1.3.6 Mathematical problems in algebra

Research shows that learners experience difficulties in learning algebra. Wu (2001) attributes these learning difficulties to poor learning of arithmetic in general and in particular to low levels of understanding of fractions. Wheeler (1996) argues that these difficulties arise from **language**. He contends that algebra employs words and symbols which learners are familiar with from arithmetic. He further states that this encourages the assumption that these variables have exactly the same meaning as in arithmetic. In support, Schoenfeld & Arcavi (1988, p. 420) argue that the concept of variable is central to teaching and learning elementary [school] mathematics and most importantly "understanding the concept provides the basis for the transition from arithmetic to algebra…"

MacGregor & Stacey (1997) point out that novices in algebra **conjoin** terms which can be linked with addition in arithmetic where symbols are superficially combined and replaced by another e.g. $3+5=8$ which does not hold in algebra. Learners difficulties in using the algebraic notation could also be attributed to analogies used in other school subjects, everyday life and other parts of mathematics (MacGregor & Stacey, 1996b, 1997). For instance, in Chemistry $C+O_2=CO_2$ but in algebra $ab \neq a+b$. Ferreni-Mundy *et al.* (unpublished work, 2006) point out that **conventions** that come with algebra often pose difficulties to learners until they learn how the convention is read. Consider the notation $g(x+2)$ for the function g. Novices often read this as g times $x+2$. So as learners move from arithmetic to algebra they have to cope with notational changes.

Pimm (1982) argues that learners' difficulties in learning algebra originate from **juxtaposition of symbols**. In arithmetic juxtaposition indicates place value whereas it

means multiplication in algebra. For instance if $p = 2,\ r = 5,\ q = 9$ then $7pqr$ means 7295 in arithmetic and $7 \times 2 \times 9 \times 5$ in algebra. Pimm (1982) further points out that within algebra itself **symbol interpretation** is context dependent since the symbol 2 is interpreted differently in $2x$ and x^2. Taking this point further MacGregor and Stacey (1996a) argue that novices often interpret x^2 as *twice* x. They named this kind of error **exponential notation for a product**. MacGregor & Stacey (1996a) point out that some learners in their study used **abbreviated words** in formulating equations/ *algebraic expression* [italics added]. They argue that this error indicates that learners have not grasped the difference between letters as abbreviated words and letters as representing quantities or variables. In support, Stacey, Chick & Kendal (2004) point out that in such case "the teacher/ *learner* [italics added] is viewing letters as objects rather than variable" p. 82. They point out that in a problem consisting of T-shirts and Sodas, T and S were used as abbreviations in formulating simultaneous equations.

The studies mentioned above (Wheeler, 1996; Pimm,1982; MacGregor & Stacey, 1996a; 1997) suggest that the road to learning and teaching algebra is pitted with diverse mathematical problems hence it is important for me to find out whether this teacher confronts these mathematical problems and/ or new ones, and how she works to solve them.

Both the curriculum and research seem to agree that algebra is an essential part of mathematics. However, one can see that attaining algebraic proficiency is not a straightforward process. It is filled with errors and misconceptions that demands engagement with the mathematical work of teaching from teachers. It is therefore important for the mathematics community to understand the mathematical knowledge which teachers need to know and know how to use in order to introduce algebra in a meaningful way to learners. The studies mentioned above help me to be aware of the

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kinds of mathematical problems that may be confronted by the teacher as she implements activities so as to introduce algebra to Grade 8 learners.

1.3.7 Categorizing mathematical problems that confront teachers of algebra

In section 1.3.6, I indicated that among the mathematical problems that confront teachers as they endeavour to teach algebra are **conjoining** and **juxtaposition of symbols**. Such mathematical problems crop up as a result of learners engaging with algebraic activities (section 1.3.3).

On the other hand, Wheeler (1996) argues that mathematical problems that confront teachers originate from the language used in teaching algebra [section 1.3.6]. MacGregor & Stacey (1996a; 1997) argue that these problems originate from lack of understanding of a variable, algebraic notation or from analogies used in other symbol systems, for example the Roman numeration system in which ix means 1 less than 10. So we see that in this case mathematical problems that confront teachers emanate from learners' inability to communicate their algebraic thoughts clearly. In this study, I have categorized such mathematical problems as **algebraic language** problems. This category includes writing algebraically, defining, meaning of algebraic symbols, use of letters (variables), interpreting/reading algebraic notation and identification of terms, constants, coefficient etc. These mathematical problems could also crop up as a result of learners attempting to attach meaning to the new algebraic notation (symbols and operations). The last category is **generalizing.** These arise as a result of learners engaging in more open-ended activities such as showing that the sum of two consecutive numbers is always odd.

Before I conclude this chapter, I would like to point out that a lot of research has been done on this topic [i.e. algebra]. For instance, Herscovics and Kieran (1980) carried out a study on teaching algebra and focused on how learners 'construct meaning for the concept of equation', Schoen (1998) looked at how word problems could be used in 'teaching elementary algebra', Pillay (2006) investigated MfT in a Grade 10 class

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learning functions, Blanton and Kaput (2005) studied a Grade 4 teacher in order to find out extent to which this teacher was able to incorporate algebra reasoning into her teaching. While my study has elements of these studies, it also focuses on the notion of nature of algebraic activities and mathematical problems.

1.4 Summary

In this chapter, I stated the research problem and the focus questions of this study. I highlighted current debates that influence mathematics teaching and research in mathematics education. I pointed out that there are two views on how mathematics should be taught. One view emphasizes teaching of rules and procedures – a 'traditional' approach; while the other focuses on teaching mathematics as a set of practices – a 'reform' oriented approach. I indicated that DoE (2002) seems to be advocating reform oriented teaching of mathematics. I further argued that Kilpatrick *et al.'s* (2001) notion of mathematical proficiency brings the two approaches together.

I argued, however, that I had to shift away from strands of mathematical proficiency in studying this teacher's practice, as they could not explicitly capture the teaching of mathematical conventions, what I termed conventional fluency as well as the language of mathematics which are critical to this practice. While the notion of algebraic activities covered conventions, it left out teaching of mathematical language. As a result, I invented another category – **reading/writing activities**. I concluded this chapter by categorizing mathematical problems.

In chapter 2, I discuss the literature pertinent to this study; in particular I focus on discussing studies on mathematical knowledge for teaching (MKfT) and knowledge resources that teachers draw on as they go about their work. I also discuss the theories that inform this study. I then describe the theoretical framework of this study, drawing from the theoretical field elaborated in Chapters 1 and 2.

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In Chapter 3, I discuss the methodological orientation of this study. I specifically focus on the selected case for this study, the instruments that were used in gathering data and their limitations, ethical issues, rigour in research and the data collection process.

Chapter 4 shows how I chunked the data. I also describe my recognition rules while at the same time providing the data analysis. I then engage in identification of the nature of activities used, mathematical work of teaching, mathematical problems and knowledge resources. I conclude this chapter by discussing how Lineo sees her teaching.

In Chapter 5 I conclude this study by pointing out the major findings of this study in relation to research questions posed in Chapter 1. I then discuss what MfT is in this practice. I relate what Lineo does to the goals of RNCS and reform oriented teaching. I also reflect on the study.

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Chapter two: Mathematics for teaching and theoretical framework

2.1 Mathematical knowledge for teaching (MKfT)

The idea that teaching mathematics requires a specialized knowledge dates back to at least Shulman (1986/1987) and his concept of Pedagogical Content Knowledge (PCK). Shulman pointed out that teaching requires knowledge that links content and pedagogy. PCK is distinguished from other forms of knowledge in that it brings together content and aspects of teaching and learning. Pedagogical content knowledge demands that teachers know the topics which learners find difficult or uninteresting and what *representations* are most helpful for teaching such problematic areas. Pedagogical content knowledge is general to teaching hence it may not address the specific problems that arise in particular subjects. For instance in mathematics the work of teaching that teachers often encounter is that of *choosing and using representations* that are appropriate to learners in a particular grade level. This may not be what a history or English teacher does in his/her practice. Taking the idea of PCK further Ball & Bass (2000) argue that mathematics teaching requires a specialized knowledge which they named Pedagogically Useful Mathematical Knowledge (PUMK). Unlike pedagogic content knowledge which requires teachers to anticipate what learners might not understand and have pre-formulated explanations and alternatives, PUMK affords teachers an opportunity to do on-the-spot *analysis of errors* in learners' mathematical ideas i.e. PUMK deals with uncertainties in practice and so cannot be easily planned for.

Polanyi (1958) cited in Ball, Lubienski & Mewborn (2001, p. 448) argues that "knowing mathematics in and for teaching requires one to transcend the tacit understanding that characterizes much personal knowledge". One has to know why a mathematical idea or algorithm is important and the kinds of situations in which the idea is applicable. Ball, Lubienski & Mewborn (2001) argue that research has previously focused on teachers' knowledge of mathematics to the extent of analyzing

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the courses taken by teachers. This was the result of the assumption that if teachers have more mathematical content their learners would gain more mathematical knowledge which would raise their achievement in high stake examinations. They further point out that a lot of research has been done on teachers' knowledge which seems to bear very little impact on learners' achievement hence there is a need to look more closely at teaching. They argue that it is not sufficient to look only at what teachers know but also to look at how they know it and what they are able to do in the course of teaching that would enable them to teach mathematics well. Informed by Ball *et al.'s* (2001) argument on studying teaching, this study endeavors to capture one teacher's everyday practice so as to describe the mathematical work of teaching that the teacher wrestles with, the mathematical problems confronted, the knowledge resources that the teacher calls on as she goes about her work and the nature of the activities that she uses in introducing algebra to her learners. So my study hopes to add a practice based notion of mathematical work of teaching by studying a Grade 8 teacher introducing algebra to her learners.

Ball *et al.* (2001) termed this knowledge which is required to teach mathematics well Mathematical Knowledge for Teaching (MKfT). MKfT is broader than PCK and PUMK in that it requires teachers to work with their content knowledge in its unprocessed form or unfinished state i.e. teachers should *unpack or decompress*[4] (Ball, *et al*., 2001, Adler, 2005) their knowledge. The proponents of MKfT argue that it is not only about how to do mathematics but also about knowing how to use it in the practice of teaching so that mathematics is accessible to all learners. Ball *et al*. (op cit) describe the teaching practice in general and provide examples of the problems that confront teachers as they go about their work. Adler (2005) develops Ball *et al.*'s ideas further by looking at in-service teacher upgrading programmes in South Africa and in the following section I focus on her study.

[4] Means transforming ones knowledge into simpler forms that could be easily understood by learners. Adler (2005) uses unpacking as synonymous to decompressing. It also refers to knowing why a mathematical idea is important and the situations in which it is valid.

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Adler (op cit) conducted a study in South Africa in order to synthesize a broad and robust pool of information on how and what mathematics was being privileged in teacher training programmes. The study was conducted in higher institutions that offered teacher training programmes in five provinces. The focus was specifically on mathematics for in-service qualifications and Advanced Certificate of Education (ACE). The survey incorporated the analysis of assessment tasks given to in-service teachers. The assessment tasks were intended to provide insight into which mathematics knowledge was being privileged and the type of mathematical and pedagogical competences which teachers were expected to display (Adler, 2005). She analyzed the courses and assessment tasks with which in-service teachers engaged to see whether *unpacking* was being advocated. She used the term *unpacking* mathematics to address skills such as explaining, conjecturing, justifying, and others which should be displayed by in-service teachers. The study revealed that *compressed*[5] (pure) mathematics (Adler op cit) dominated assessment tasks in mathematics teacher education programmes. Unlike the above mentioned study my study focuses on one Grade 8 teacher in practice in order to understand the mathematical work of teaching that the teacher does as she goes about her work.

The above mentioned study suggests that teachers need to *decompress [unpack]* mathematics while teaching. Decompressing is typically not taught to teachers (Adler & Davis, 2006; Adler, 2005). So it is important and interesting for me to explore how a Grade 8 teacher *decompresses* mathematics as she endeavors to introduce algebra. This would also afford me an opportunity to find out whether this teacher confronts any mathematical problems similar to those mentioned earlier or others and how she deals with them as she attempts to make mathematics accessible to her learners. In the section that follows I discuss the mathematical work of teaching which this teacher is likely to wrestle with as she introduces algebra.

[5] This is the mathematics which emphases knowledge of procedures and concepts, it is highly algorithmic. One does not necessarily need to know why the mathematical idea is important but must master the procedure.

2.2 Mathematical work of teaching

In the preceding sections I have argued that teachers confront a number of mathematical problems that require them to engage with the mathematical work of teaching (see chapter 1). Ball, Bass & Hill (2004) developed eight categories to describe the mathematical work of teaching that teachers wrestle with on daily basis and stated them as follows:

- *Design mathematically accurate explanations that are comprehensible and useful for learners*
- *Use mathematically appropriate and comprehensible definitions*
- *Represent ideas carefully, mapping between physical or graphical model, the symbolic notation and the operation or process*
- *Interpret and make mathematical and pedagogical judgements about students' questions, solutions, problems, and insights (both predictable and unusual)*
- *Be able to respond productively to students' mathematical questions and curiosities*
- *Make judgements about of instructional material and modify as necessary*
- *Be able to pose good question and problems that are productive for students' learning*
- *Assess students' mathematical learning and take the next step.*

(Ball, Bass & Hill, 2004, p.59)

This study draws on this work to analyze the mathematical work of teaching which a Grade 8 teacher did when introducing algebra. Kazima & Adler (2006) used a similar framework in studying the MKfT that is needed to teach probability in a South African context. In order to carry out their study, Kazima & Adler (op cit) condensed Ball *et al.*'s (op cit) eight kinds/elements of mathematical work of teaching into six. They argue that some of the elements in Ball *et al.* (2004) were overlapping and that

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in their study some elements occurred together. As a result they ended up with **explaining**, **questioning**, **representing**, **defining**, **restructuring tasks** and **working with learners' ideas** as elements of mathematical work of teaching that teachers grapple with. It is these six elements of mathematical work of teaching that I employ in studying the practice of a Grade 8 teacher within a South African context. In Kazima & Adler's (op cit) study the category **working with learners' ideas** entailed

- *Interpret and make mathematical and pedagogical judgements about students' questions, solutions, problems, and insights (both predictable and unusual)*
- *Be able to respond productively to students' mathematical questions and curiosities*
- *Assess students' mathematical learning and take the next step* (Ball *et al.*, 2004, p.59).

In this study, it mainly consists of responding to learners questions and engaging with learners' responses. Ball *et al*. (2004) and Kazima & Adler's (2006) work played a pivotal role in this study by pointing out the kind of mathematical work of teaching that this teacher is likely to wrestle with. While these researchers acknowledge that teaching mathematics often confronts teachers with mathematical problems, for instance, Kazima & Adler (op cit) indicate that in their study involving probability the mathematical problem that confronted the teacher was 'working with unexpected answers' p. 52, they do not explicitly talk about what enables teachers to engage with mathematical work of teaching. Pillay (2006) and Adler & Pillay (in press) studied Nash's practice as he taught functions in a Grade 10 class. They develop Kazima & Adler's (op cit) work further by showing that Nash sustained his practice by appealing to a range of knowledge resources that they called **curriculum**, **mathematics** and **experience**. In their language Nash drew on these knowledge resources to sustain his practice. In the sections that follow, I discuss each of these knowledge resources that may avail themselves to the teacher in this study.

2.3 Knowledge resources

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2.3.1 Experience

As a knowledge resource it availed itself as everyday knowledge (here mathematics is integrated into the everyday life experience of learners and the teacher), professional knowledge and authority (Pillay, 2006). By professional knowledge, I refer to the knowledge of mathematics that is gained through learning and teaching mathematics. Everyday knowledge refers to that knowledge which is found in everyday life activities.

Unlike the other forms of experience, authority[6] is dictative in nature and gives the knowledgeable person (in this case a teacher) power over the less knowledgeable. S/ he is the sole custodian of the (mathematical) knowledge to be acquired. S/he dictates what is accepted in the community of practice. It is often indicated by the use of phrases like 'I want, I say, you should' etc. The question for this study is: does this teacher call on this experiential knowledge?

2.3.2 Mathematics

According to Pillay (2006), mathematical knowledge in Nash's practice availed itself in the form of rules and conventions, empirical arguments and definitions in mathematics. In mathematics, arguments are classified as empirical, theoretical and algebraic (Brodie, 2000). Empirical arguments use examples or instances of the concept under discussion to convince oneself or a sceptic. Theoretical arguments are usually based on a defining property or characteristic of the concept under discussion (Brodie, op cit), for example odd numbers end in 1, 3, 5, 7 or 9. Algebraic arguments employ variables, and refer to the general rather than the specific. The questions that remain for this study are (i) does this teacher draw on these mathematical resources or others? (ii) For what purpose does she call on these? and (ii) what categories does mathematics has in this practice?

[6] In this study authority has been placed under experience for convenience and ease of analysis

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2.3.3 Curriculum

Curriculum as a knowledge resource for this teacher availed itself in the form of assessment i.e. tests and exams (Adler & Pillay, in press) and teaching materials (textbooks and worksheets). When teachers draw on assessment to legitimate text for their learners they may talk about not awarding full marks or taking marks off as this is what is usually done in exams or tests. When referring to curriculum in the form of teaching materials teachers might refer learners to definitions or examples in the textbooks. In this case, the teacher fixes meaning by telling learners that this is what it is because it is required in the exam like that or the textbook says so. Teachers have considerable knowledge of what is in textbooks, syllabi, exams *etc*. which they can call on to fix meaning for their learners.

This study hopes to add to MKfT by applying the six-part framework in studying the work of teaching that a Grade 8 teacher does, the mathematical problems that confronts her and the knowledge resources that she calls on when introducing algebra in a South African context. The above mentioned knowledge resources inform me as to what is likely to come up in this study. I note however, that since I am dealing with a different teacher, in a different school and grade, the teacher might draw on other knowledge resources. In the section that follows I discuss the theoretical orientation that underpins this study.

2.4 Theoretical orientation

In this study I employed two broad educational theories namely situative learning theory and Bernstein's sociological theory of pedagogic discourse[7]. Situative theory allowed me to explain that I am looking at a specific practice. Bernstein's theory of pedagogic discourse provided me with the analytic tools to see this specific practice

[7] This study forms part of a large study on mathematics for teaching in teacher education and mathematics classrooms which falls under the QUANTUM project directed by Prof. Jill Adler at Wits University. Hence the analytic tools used in this study are borrowed from QUANTUM project. See Adler & Davis (2006)

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so that I could describe it. In particular, the notion of the evaluative rule was key to organizing and categorizing data.

The assumption underlying all studies discussed above is that mathematical knowledge for teaching is situated in teaching practice. Similarly, an assumption underlying my study is an orientation to knowledge as situated. This means that MKfT is specific and situated in teaching practice (Adler & Davis, 2006). It is shaped by the practice as the teacher interacts with both the mathematics she is attempting to teach and the learners as they engage with the mathematics and their learning of it. According to Lave and Wenger (1991) learning is increasing participation in a community of practice (see also Lave, 1993, Wenger, 1998). This entails having access to the resources offered by the community. In mathematics, learners need opportunities to engage with mathematical activities so that they can "talk within their mathematical practice" and "talk about their mathematics ideas" (Adler, 2001, p.96). Hence mathematics teaching involves mathematical work of teaching which is unique to teaching the subject. Situative theorists view mathematics as a set of practices which mathematicians and users of mathematics engage with. The RAND study panel (2002) points out that such practices include justifying, representing, generalizing, symbolizing, analysing, interpreting, describing and communicating ideas (see also Brodie, 2000, DoE, 2002). As learners engage in these practices they develop a deeper and more powerful ways of understanding and doing mathematics which could be used beyond the classroom (Boaler, 1997). So it is therefore clear that teaching mathematics is about affording learners opportunities to engage in these practices. Hence the teacher's teaching practice cannot be studied in a vacuum. It has to be studied within pedagogic practice.

As this study forms part of the larger QUANTUM project and is shaped by its methodology, I draw on the recontextualization of Bernstein's sociological theory of pedagogic discourse from the QUANTUM project in developing my analytic tools (Adler & Davis, 2006). Before I turn to Bernstein, I wish to discuss my research

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questions and some theoretical constructs that are used in this study. First, it is clear that I am trying to answer questions about the nature of activities. I have discussed this in section 1.3.3. They are representational, transformational, reading/writing and generalizing and justifying activities. These form part of my theoretical framework. Secondly, in sections 1.3.6 and 1.3.7, I discussed mathematical problems that confront teachers in algebra classrooms. I categorized these mathematical problems as either manipulating algebraic symbols problems, algebraic language problems or generalizing. Thirdly, I also need to talk about the mathematical work of teaching that this teacher grappled with as she introduced algebra. For this part of my framework, I drew on Ball *et al.* (2004) and Kazima & Adler (2006) [see section 2.2]. Lastly, one can see that as teachers endeavour to teach algebra, they need to convince learners as to why it is important that they learn what is being taught. In order to do these teachers appeal to some authority or what Adler and Pillay (in press) have since termed knowledge resources [section 2.3]. All these theoretical constructs form the lens that is used to look at this teacher's practice. In order to study teaching practice, one needs a theory of practice. As mentioned above, in this study such a theory is drawn from the QUANTUM project which recontextualized Basil Bernstein's sociological theory of pedagogic discourse.

According to Bernstein (1990, 1996) pedagogic discourse is composed of three interrelated rules viz. distributive rules, recontextualizing rules and evaluative rules. In this study, I find the notion of evaluative rules to be more illuminating and relevant in that in every instructional communication teachers find themselves evaluating learners' responses and contributions in order to make explicit the legitimate text to be acquired and to move them on. Singh (2002, p.573), in support argues that "*evaluative rules are concerned with recognizing what counts as valid acquisition of instructional (curriculum content) text"*. Pedagogic discourse is relayed through a 'specific code' that encompasses specialized contexts such as mathematics classrooms (Morais, 2002). In these classrooms teachers endeavour to equip their learners (acquirers) with recognition rules and realization rules (Bernstein, 1996).

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These rules enable the acquirer to produce the legitimate text. A text in Bernstein's (1996, p.32) language is "anything that attracts evaluation". Adler & Davis (2006) drawing on Bernstein (1996) points out that evaluation condenses meaning and transmits the criteria through which learners contributions and responses are judged in a pedagogic communication. In a way evaluation works to control both the transmission and acquisition of valid knowledge. Evaluation by the teacher alerts us to what is regarded as valid knowledge to be acquired. Hence in a pedagogic practice, teachers transmit criteria to their learners of what counts as valid text or knowledge. In order for teachers to do this, they make judgements from time to time. So according to Bernstein's theory of pedagogic discourse teachers convey to learners what is to be learnt, and the key into attaining this is evaluation. Evaluation helps teachers to ground or fix meaning (Adler & Davis, 2006) for the knowledge to be acquired. In this way teachers communicate (implicitly or explicitly) to learners as to why they are to learn what is being taught. My intention here is not to elaborate on Bernstein's theory of pedagogic discourse as this is done in the QUANTUM project but to point out that it forms the broad lens that is used in studying this practice and to show that I am not using the entire theory but move from the centrality of evaluation.

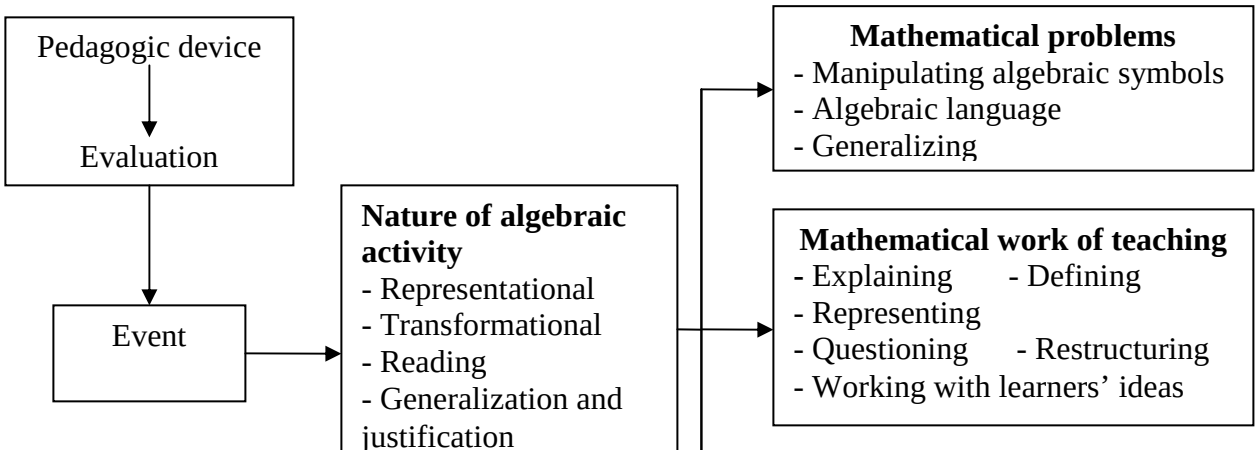
As pointed out earlier, my study demands that I identify the nature of algebraic activities, mathematical problems, work of teaching and knowledge resources. So what this demands of me is that I chunk the collected data into evaluative events (Adler & Davis, op cit). An event consists of a teacher attempting to transmit or teach some knowledge and grounding or legitimating that knowledge. In this study, what the teacher wanted learners to know was conveyed through algebraic activities. This implies therefore that in an evaluative event there is at least one algebraic activity involved in the transmission of knowledge. In order to manage these activities teachers engage with the mathematical work of teaching [section 2.2] and call on various knowledge resources [section 2.3]. So I identified instances where the teacher evaluated her learners. These marked the end of my evaluative event, i.e. my unit of analysis. In the section that follows, I present the theoretical framework for this study.

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2.5 Theoretical framework

In the previous sections, I engaged in discussion of the theoretical constructs that I use in this study. Figure 1, illustrates the theoretical framework that is used in this study. Central to this framework are mathematical problems that are confronted by the teacher as she went about her work, the mathematical work of teaching that she grappled with, the knowledge resources that she called on and the algebraic activities which learners were exposed to in relation to the research questions posed in section 1.2. I would like to acknowledge the partiality of this theoretical framework. While it enables me to study this teacher's practice with respect to the research question, it conceals the complexity of the teaching practice due to its linear nature. A closer look at this theoretical framework shows that for every algebraic activity, the teacher confronts mathematical problems, engages with the work of teaching and makes appeals. Practically, this is not the case since the teacher may choose to ignore an emerging problem in which case s/he may not wrestle with the work of teaching and does not call on any knowledge resources for that mathematical problem. Although the mathematical problems, work of teaching, knowledge resources and algebraic activities are distinct in this theoretical framework, they are contained in an event. The theoretical framework also does not to show that if the teacher grapples with the work of teaching s/he ends up drawing on the knowledge resources to fix meaning. In other words, the interdependency and connectedness of mathematical problems, work of teaching and knowledge resources and the cyclic nature of the practice are not explicitly shown in this tool. So this tool is a representation of what happens in practice and is used to examine particular questions hence it is partial.

Theoretical framework for this study



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Figure 1, Theoretical Framework

2.6 Summary

In this chapter, I discussed MKfT. I indicated that MKfT dates back to Shulman's (1986/7) notion of PCK. I also discussed one study on MKfT which I found pertinent to this study. I highlighted its importance and relevance to my study (see section 1.2). Studies in mathematics education have a theory that informs them, so too this study. This study assumes that mathematical knowledge for teaching is shaped by the interactions of learners and the teacher hence the theoretical lens used in this study is provided by Bernstein's sociological theory of pedagogic discourse. Lastly, I embarked on describing the theoretical constructs or recognition rules that I used in interrogating the data. In the following chapter, I discuss the methodological orientation of this study, the selected case, instruments and the data collection process.

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Chapter three: Methodology

In this chapter, I discuss the research methods and instruments that were used in gathering data for this study. I also deliberate on the sample that has been used and ethical issues. The limitations of both the research approach and instruments are also discussed.

3.1 Methodological orientation

Taking into consideration the research problem under investigation and the research questions, this study would best be classified under the *interpretive paradigm (Opie, 2000,* Cohen, Manion & Morrrison, 2000, Merriam, 1998*)*. According to Merriam (1998) researchers within this paradigm are interested in the experiences, interpretation or in discovering factors that distinguish people rather than testing hypotheses. Cohen *et al*. (2000) argue that

> the central endeavour in interpretive paradigm is to understand the subjective world of human experience, to retain the integrity of the phenomena being investigated, efforts are made to get inside the person and understand from within p. 22.

The purpose of this study was to gain insight into the Mathematical Knowledge for Teaching in use in introducing algebra to Grade 8 learners in a South African context. This was done by investigating the mathematical problems that confronted the teacher, the nature of algebraic activities used, the mathematical work of teaching that she grappled with as she went about her work as well as the knowledge resources called on by the teacher. A qualitative approach was found to be the most suitable approach as it assumes that meaning is entrenched in people's experiences (Merriam, op cit). In this study I looked at the actions (what was said and what was done) of one teacher as she introduced algebra to Grade 8 learners.

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The research method which seemed to best suite my approach was the case study. Merriam (op cit) argues that "a case study design is used to gain an in-depth understanding of situations and meaning of those involved" p. 19. She further defines a case study as "an intensive description and analysis of a single unit or a bounded system such as an individual, program…" p.19. Opie (2004) elaborates this point further by arguing that a case study can be viewed as an in-depth study of interactions of a single instance in an enclosed system. In this study, the case is the teacher being studied while the Grade 8 classroom and algebra form the boundary within which the teacher and the learners interact. In support, Leedy (1997) would argue that my study is intended to collect an extensive amount of verbal data from a small number (one Grade 8 teacher) of participants hence it qualifies to be a case study.

As mentioned earlier, this study is qualitative in nature. It focuses on one Grade 8 teacher in a particular school. Merriam (1998) would argue that this study seeks to provide a 'thick and rich description of the teachers' actions'. Merriam (op cit) further points out that the descriptive data collected would be "used to develop categories or to illustrate, support or challenge the theoretical assumptions held prior to data gathering" p.38. In this study data is used to illustrate the mathematical work of teaching that the teacher did as she introduced algebra.

Although this study is qualitative, I quantified some of the data in relation to the aspects of mathematical work of teaching described by Ball, Bass & Hill (2004), mathematical problems [see section 1.3.7], knowledge resources that the teacher draws on (Adler & Pillay, in press) and activities of algebra (Kilpatrick *et al.*, 2001) that were used to introduce algebraic concepts to Grade 8 learners. This quantification illuminates dominant trends in the mathematical work of teaching the teacher wrestled with, and those that were absent, the knowledge resources that were put to use and the activities of algebra that were likely to be enhanced.

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In this study, I looked at the actions of the teacher (what she did and said) when introducing algebra so as to assist Grade 8 learners in making a transition from arithmetic to algebra. This study was not meant to implement changes as a result of its findings but to help the researcher to understand, describe and explain the mathematical work of teaching that a Grade 8 teacher grappled with when introducing algebra. This study is therefore aimed at shedding light on the mathematical knowledge for teaching employed by this teacher when introducing algebra to Grade 8 learners.

3.2 Limitations of case study

Advantages of case studies are also their disadvantages. While they have depth, case studies lack breadth (Ary, Jacobs & Razaveih, 1990). Thus the dynamics of one individual [one teacher in one class in this case] may bear little, if any at all, relationship to dynamics of others in different contexts. This study could also be said to be limited in terms of sample size [involved only one teacher observed over a week] hence its findings cannot be generalized to other teacher or even to this teacher's overall practice as I only observed her for one week [four lessons in all]. However, Opie (2004) warns that a case study is a useful form of research particularly to teachers. Opie (op cit, p.5) substantiates this point by citing Bassey who argues that the value of any educational research is "the extent to which the details are sufficient and appropriate for a teacher working in a similar situation to relate his/her decision to that described". Merriam (1998) argues that although a case study is a rich, thick description and analysis of a phenomenon and might influence policy, it "may be too lengthy, too detailed or too involved for busy policy makers and educators to read and use" p. 42.

3.3 The selected case

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Another characteristic of a qualitative approach that was used in this study is purposeful selection of participants (Leedy, 1997). The sample (teacher) can be seen as opportunistic and *purposive sampling* (Cohen *et al.*, 2000, p.103). According to Cohen *et al*. (op cit) in purposive sampling, the researcher uses his/her judgement to handpick the sample that is typical to his or her interest. In support, Merriam (op cit) argues that this sampling strategy is used when the researcher wants to discover, understand, and gain insight into a phenomenon (e.g. the teacher's teaching practice). Patton (1990) cited by Merriam (op cit, p.61) argues that "the logic and power of purposeful sampling lies in selecting the information-rich cases for study in depth. Information-rich cases are those from which one can learn a great deal about the issues of central importance". The sample is opportunistic and purposive because I was linked to this teacher by a PhD student at University of the Witwatersrand (Wits) who happened to know my interest and that of the teacher. In addition, I wanted to 'gain insight' into the nature of algebraic activities that this Grade 8 teacher uses in introducing algebra, the mathematical work of teaching she grapples with, the knowledge resources she calls on as well as the mathematical problems that confronts her as she introduces algebra (without necessarily generalizing to other teachers). It is important to mention that this teacher is competent, highly qualified and works in a highly respected school. These characteristics of the teacher constrain generalization as teachers would have to be matched in almost all respects.

Moreover, the teacher was willing and interested in working with me in exploring her typical teaching of algebra to her Grade 8 learners. Furthermore, the school in which this teacher works was easily accessible to the researcher as it is closer to the researchers' residence and school. As it has been mentioned earlier, the main purpose of the study is not to evaluate her teaching but to learn from it by making analytic judgements of what mathematical work of teaching the teacher wrestled with when teaching algebra. I observed her for one week and conducted three interviews with the teacher.

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Merriam (1998) argues that in order to use purposive sampling one must determine the selection criteria. Cohen *et al.* (2000) expatiate this point further by arguing that the quality of any research writing is not only tainted by the exactness of methodology and instrumentation but also the precision of the sampling strategy employed. In the light of this and taking into account the purpose of this study and the motivation of the QUANTUM project, of which this study is part, I designed a selection criterion that was to assist me in hand picking the 'information rich case'. The criterion entailed; qualification [at least a degree with mathematics as a major], teaching experience of at least 3 years at this level, willingness to participate and the teacher was to be currently teaching Grade 8 mathematics. Grade 8 was included because that is where at least theoretically algebra begins (Department of Education, 2003).

Participants in this study came from a secondary school in Johannesburg. Data was collected from one teacher, in one class at a well functioning, reputable private secondary school. The selected case for this study consisted of a mathematically qualified teacher. Lineo[8] has a B.Sc. degree in Mathematics and Chemistry, B.Sc. (honours) in Chemistry, a teaching diploma, a Certificate in Adult Basic Education and she is currently furthering her studies in Bachelor of Commerce. Lineo's qualifications reveal that she is more qualified in Chemistry than in Mathematics though she has been teaching Mathematics for five years. In regard to this Lineo said:

> [TT = Interviewer]
>
> TT: Why are you not teaching Chemistry then?
> Lineo: It's funny; many people ask me that because I am more qualified in Chemistry than in Mathematics. Chemistry is a subject only in my head. I haven't figured it out. If somebody may come now with a question I have to think, but if somebody come with a Mathematics question I will be able to do it. (Lineo, post-teaching interview, L72 – L78[9]*)*

[8] Lineo is the pseudonym of the teacher involved in this study.
[9] The interview transcripts extracts that are used in this study are reference by name of interviewee, title of interview and lines, for instance, (Lineo, post-teaching interview, L111 – L119) means that the

The excerpt indicates that for one to teach a subject, qualifications alone are not enough. One should have understood the subject more than other users of the subject or in Lineo's description one must have 'figured out' the subject. Teaching as described by Adler (2005), Ball &Bass (2000) and Ball *et al*. (2004) demands that one unpacks the mathematical knowledge so that it becomes accessible to the novices.

Lineo is a White female teacher who speaks English as her first language. Lineo is an experienced teacher who believes that she uses reform oriented teaching. In her five years of teaching, Lineo has taught Mathematics from Grade 8 upwards and Additional Mathematics from Grade 10 to post Matric level. She teaches in a private boys' school. The language of instruction and communication in this school is English. The school can best be described as multicultural. There were 24 learners in Lineo's class.

Lineo's class is highly resourced. She has a computer, an overhead projector and a white board. Lineo also has access to curriculum documents, recent textbooks both local and international, for instance; she has geometry textbook and an algebra textbook from the United States of America (USA). In addition to these, Lineo seems to be a creative teacher. The walls in her classroom were decorated with posters for polygons, trigonometric identities, etc, what struck my attention most on the first day of data collection was the sequence: "Thursday, 4th May, the time and date will be 01: 02: 03: 04 / 05 / 06 this will never happen again" (Lineo, classroom poster). I asked Lineo what led her into thinking about this sequence. She explained that they were doing sequences and she thanked her boys for correcting her statement by indicating that this would happen in the next millennium.

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extract comes from the post-teaching interview conducted with Lineo and that the extract was taken from line 111 to line 119. in using these extracts, I emphasize certain points in Linoe's practice by underlining, italicizing, bolding or by using a combination of these to mark the illustrative text.

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The data collection process took place in the second term of the school calendar, from the 11th to the 14th July 2006. On the first day I interviewed Lineo twice, before observing her and after the lesson. During the post teaching interview she indicated that in the school each teacher was expected to be proactive and that they work independently most of the time (see section 4.1). Consequently, the findings of this study cannot be extrapolated beyond this case.

3.4 Instruments

Qualitative approaches usually use *triangulation* (Opie, 2004) i.e. use of more than one instrument in collecting data. This study was no exception as it employed interviews with the teacher, two of which were conducted on the first day of data collection while the third (follow up) interview was conducted when all the data had been collected. I also observed this teacher for a week. Interviews were audio-taped while all classroom observations were videoed.

3.4.1 Interviews

Opie (2004) argues that the kinds of interviews available to researchers lie in a continuum ranging from structured to unstructured interviews. Cohen *et al*. (2000) elaborate this point further by saying that interviews differ mainly in "degree of structure, which, itself reflects the purpose of the interview" p.270. In this study a semi-structured interview was employed due to its flexibility as it would enable the researcher to probe for more insight in an idea. Opie (op cit), in support, points out that a semi-structured interview allows for depth of feelings to be ascertained by providing opportunities to probe and expand the interviewee's response. He further points out that the main advantage of semi-structured interview is that it allows the researcher to clarify questions or change the wording, to deviate from the pre-arranged questions depending on the interviewee's response. Moreover, a semi-structured interview provides the overall shape of the interview and prevents aimless rambling (Opie, op cit).

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Cohen *et al*. (2000) argue that interviews are conducted for numerous purposes, such as "to evaluate or assess a person in some respect … to gather data as in surveys or experimental situations" p.268. In this study, the purpose of interview was multifaceted. First, interviews were conducted in order to probe the teacher on the mathematical problems that confront her, the mathematical work of teaching that the teacher wrestled with and the knowledge resources she called on when introducing algebra to Grade 8 learners. Secondly, Interviews were also aimed at finding the teacher's interpretations of the activities suitable for introducing algebra. Thirdly, interviews were used to collect biographical information of the teacher. Lastly, the information gathered through interviews was to corroborate my interpretation of what I observed.

The pre-teaching interview provided me with information about the activities that the teacher was going to use, in particular what she wanted to achieve by engaging her learners in these activities and how she was to use them. In addition, the pre-teaching interview was to probe the mathematical work of teaching which the teacher could have tackled and the knowledge resources that could have been called on by the teacher in anticipation of learners' errors. During the pre-teaching interview, I asked her, "what do you think will be interesting or challenging for the learners?" She said,

> … today is about differentiating between x and x^2. They get confused what the 2 resembles; x^2 *is* x *times* x. There are two lots of x and I will clarify today so that these boys do not struggle with these concepts. The x^2 *versus* $2x$. And the other goal is to try and make them understand that xy *will be the same as* yx (Lineo, pre-teaching interview, stanza 8).

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Lineo's response indicates that she anticipated that learners would have problems with what MacGregor & Stacey (1996) have termed exponential notation for a product and also the commutative property of multiplication. Hence she seems to have preformulated ways of explaining or clarifying as she puts it.

I also conducted a follow up interview (after all the data was collected) with Lineo. This was aimed at probing Lineo on the issues (e.g. use of definitions, misconception) that I found critical during observation and analysis of the collected data so that she could clarify thus refuting or confirming and interpretations of what I observed.

3.4.2 Observations

This study investigates MKfT with specific focus on the mathematical problems and mathematical work of teaching which Lineo grappled with as she went about her work. Since knowledge is situated in practice and is shaped by human interactions and context, I had to observe the teacher while teaching. I thought that this would afford me an opportunity to gain more insight into what mathematical knowledge came into play as the teacher interacted with the learners. In support, Patton cited in Cohen *et al*. (2000) argues that observation affords the researcher the "opportunity to look at what is taking place in situ rather than at second hand" p.305. Cohen *et al.* further point out that observation

> enables researchers to understand the content of the situation [own world]… to see things that might otherwise be unconsciously missed, to discover things that participants might not freely talk about in interviews… p.305.

Observation was corroborated with the pre-teaching interview (section 3.4.1) which might have been too artificial on its own since saying is one thing and doing is another. In actual fact, my experience as a teacher has taught me that what transpires in the classroom is a socially constructed reality, negotiated by the participants and

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the environment. Thus, in this study, observation was regarded as the main instrument as it was able to gather unrefined data.

Cohen *et al.* (op cit) indicate that the kinds of observations that can be carried out by a researcher lie in a continuum ranging from "unstructured observation to structured observation" p.305. They further point out that in a highly structured observation the researcher knows in advance what is to be observed and has pre-formulated categories. This study therefore employed structured observation in collecting data since the categories had been formulated in advance from the literature. Hence the observation schedule was developed in advance (See Appendix 1). However, I was aware that the categories formulated using the literature might not be exhaustive or could be too specific, so I was open to new ideas from my further reading of the literature or from the data itself. Also these new ideas could emerge from the piloting of instruments and from observing the teacher's practice. I constructed the observation schedule in such a way that I could tick the mathematical work of teaching which the teacher grapples with in each 10 minute interval. This gave me the opportunity to see which elements of the mathematical work of teaching were more dominant in each lesson. During the observation I made comments on the mathematical problems that confronted the teacher and the knowledge resources that she called on.

In order to carry out the observation successfully I had to assume a certain position during data collection. Cohen *et al.* (2000) suggest three roles which might be taken by the researcher. They argue that these roles lie in a continuum from "complete participant … to complete observer" p.305. Opie (2004) classified these roles into two main categories namely; participatory and non-participatory roles. Opie (op cit) further warns that a non-participatory role is difficult to maintain in a familiar environment. I found this to be the best role for me because firstly, I wanted to be as less intrusive as possible and secondly, the school environment was unfamiliar to me as I was there as a researcher only. So my role in the data gathering process could

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best be described as non-participatory-I had "no interaction with the participants during data collection" (Opie, 2004, p.128). This role gave me the opportunity to jot down incidences (mathematical problems, mathematical work of teaching and knowledge resources) as they occurred during the lesson. Assumption of this role seemed to have been facilitated by the fact that not all the learners were allowed to appear on the video which led the teacher into making a new sitting arrangement. In addition to this there were extra desks. This enabled me to sit at the back of the column with the fewest learners and apart from the learners. My sitting position at the back helped me to avoid being engaged in the class activities. Moreover, I did not show interest in what was going on in the class. Most importantly, I evaded eye contact since I believed that if I do not look around I might not be involved. When I was talked to I politely referred the learner to the teacher. Merriam (1998) argues that "knowing when and how to intervene is perhaps the most perplexing ethical dilemma facing qualitative investigators" p. 215.

Apart from using a structured observation schedule which was mainly used to help me focus attention during the lesson as I started quantifying (analyzing) the teacher's actions, I video recorded all the lessons (four lessons) observed. In support of video recording, Opie (op cit) argues that video recording offers the researcher the opportunity to make sense of the non-verbal behaviour. Cohen *et al.* (2000) point out that

> Comprehensive audio-visual *such as video-recording* [italics added] can overcome the partialness of the observer's view of single event and can overcome the tendency towards only recording the frequently occurring events. p.313.

Over-and-above, I felt that video recording would afford me the opportunity to see the events as they occurred long after leaving the site. Hence re-analysis of the lesson would be possible which I thought was essential for checking my interpretation,

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explanation and description of the MKfT needed in introducing algebra to Grade 8 learners. In this way, I endeavoured to ensure reliability and trustworthiness of the results. Also using a video helped me to capture most of the interactions and conversations that went on between the teacher and learner(s) which I might have missed during observation as I focused more on particular events, for example, the mathematical work of teaching as these were already well defined from the literature. Observation mainly sought to identify the mathematical work of teaching (refer to Appendix 1) wrestled with as the activities were being implemented. The observation was also focused on whether the teacher was able to implement the activities in a way that promoted the development of algebraic activities in learners, and if so how this was done and if not what mathematical (and other) problems constrained this.

3.4.3 Piloting instruments

Having developed the instruments, the next step was to check whether they would work as anticipated. According to Cohen *et al*. (2000) the purpose of piloting observation schedules is to ensure that "The categories for the observation are discrete, i.e. there is no overlap between them" p.306. Opie (2004) adds that one should carry out a pilot study to weed out any ambiguities in the questions and to check the length of the interview. Cohen *et al.* (op cit) takes this point further by arguing that as there would be many categories to be scanned very quickly during observation, "the researcher needs to practice completing the schedule until he or she becomes proficient and consistent in entering the data" p.306-07. I could not carry out a full study to pilot the instruments as this implied finding another school with similar conditions. However, I piloted the observation schedule by watching videoed lessons from past studies which were kept at the Marang Centre for Mathematics and Science Education. I observed two different lessons several times. One lesson was on Angles on a straight line while the other was on algebraic expressions.

The teacher in both lessons could be described as using reformed teaching because she encouraged learners to explain their methods, to give reasons for their responses.

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In fact these lessons were more of a discussion (learner-centred) rather than 'chalk and talk' (teacher-centred). It was during these observations that I realized that I had conflated ideas which should have been separated. For instance, I had categories such as 'interprets and responds to learners' ideas' and 'provides and evaluates explanations' which I broke into four simpler statements which could be easily observed. Interview schedules, on the other hand were piloted by interviewing one of my colleagues.

Piloting the observation schedule was helpful in that it indicated where I needed to make some modifications prior to collecting data. In addition, piloting enabled me to see that the categories in the observation schedule do not overlap. Further, the algebra lesson enabled me to think about the questions that I could include in the interview schedule.

As I was not able to operate the video recorder, my supervisor assisted me in finding a knowledgeable person to operate the video. He tested the video recorder with the sampled teacher a day before I came for data collection.

Piloting the instruments helped me in ensuring that all the equipment was functioning properly; in identify potential questions for the interview and to improve my interviewing skills. In addition, it helped me to see whether the interview questions were related to the specific research questions and the research problem.

3.4.4 Advantages and disadvantages of instruments

Since the study uses observations, the presence of the researcher might influence the results particularly if the researcher is new in the environment and stays for a short period as the subjects could have been on guard of what they say and do. In support to this, Opie (2004) argues that "people/***learners*** [bold added] consciously or unconsciously, may change the way they behave when being observed" p.122. According to Cohen and Manion (1980) the main advantage of observation is that the

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researcher “can discern ongoing behaviour as it occurs and is able to make appropriate notes on its salient points” p.103. On the other hand, videoing has technical problems of focusing and quality of sound. I indicated earlier that to minimize these videoing problems we asked a knowledgeable person to operate the video.

Interviews on the one hand, present challenges to the researcher in terms of interviewing and listening skills as potential information may be missed due to poor questions or listening skills (Opie, 2004). Moreover, interviews demand that the interviewer is able to maintain control of interview, to probe gently, be able to manage personal space and be non-judgmental (Opie, op cit). Interviews are time-consuming to transcribe due to too much data if recorded. The advantage of using interviews is that both verbal and non-verbal behaviour can be noted which cannot be achieved with a questionnaire. In addition the interviewer may rephrase questions or probe the interviewee to get a clear understanding which cannot happen when using a questionnaire.

3.5 Data collection process

As I mentioned earlier, three interviews were conducted. The pre-teaching interview with the teacher focused on how and why the activities were selected. After this the teacher was observed and videoed when implementing the activities. The purpose of the observation was to see what the teacher does and says in relation to the activities. Another interview was conducted with the teacher after the lesson. This interview was intended to provide insight into the mathematical problems that confronted the teacher during the lesson and the way forward. The last interview was conducted when all the data had been collected.

Classroom observations were conducted for one week (four lessons). These were enough for the teacher to cover at least one unit or subtopic. The assumption here is

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that the activities designed or selected by teacher reflect what she understood or considered to be important in learning algebra. As Cornbleth, (1990) has argued that what teachers do in classrooms communicates messages about their conception of curriculum and meaning of knowledge.

Prior to my data collection I checked all the consent forms to ensure that I was allowed to video record all the learners. I discovered that only one parent objected. I made Lineo aware of this. When I arrived at the school Lineo informed me that she had changed the seating arrangement so as to accommodate everybody. Each learner had his own desk. The desks were arranged so as to have six columns. The middle four columns were paired. The other two columns were against the opposite walls of the classroom. Two boys were seated right in front of the teacher's table. I realized that the learner who was not to be captured on video was seated in one of the columns alongside the wall. This seemed to be his new location since when he entered he went straight to this side. I sat at the desk behind this learner. This was a precautionary measure which indicated where the video was not to be directed.

3.6 Data analysis procedure

Data analysis is an on going cyclic process constituted in all phases of a qualitative research (Hatch, 2002). This process entails selecting, categorising, synthesizing and interpreting the data. All the data collected from videoing and audio-recording were transcribed by the researcher. Both inductive and typological analysis (Hatch, 2002) were used in this study. In order to do this I used the literature, for example the 8 kinds of mathematical work of teaching by Ball *et al.* (2004) and experience gained on continuous interaction with the data (Hitchcock & Hughes, 1995). I analyzed the lessons inductively by carefully reading and re-reading through the data in order to formulate themes or categories. Interview transcripts were analysed inductively. The emerging themes were then used to provide a description of the work of teaching that

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confronted a Grade 8 teacher when teaching algebra. Full details of how I chunked and analyzed the data are given in chapter 4.

3.7 Rigour in research

The issues of validity of the explanations made were ensured by either providing evidence from the transcripts [This is what Opie (2004) calls evidence based process] or by using the literature. In support, Opie argues that claims that are made should be backed-up by refereed source from journals and academic books. Another tool that I used is argument (Opie, 2004) based on either the data itself or statements from the literature. In using the literature one tried as much as possible to use data from the original sources i.e. avoiding citing sources cited by other researchers since different people interpret the same information differently.

Reliability refers to the extent to which an instrument gives consistent results under similar conditions. In contention, Opie (2004) views reliability as the property of the whole data gathering process. While reliability and validity are often used in quantitative research as measures of quality in a piece of research, LeCompete and Preissle (1993) in Cohen *et al.* (2000) argue that qualitative research appropriates selectively the tools of quantitative research. Consequently, issues of reliability, validity and generalizability turn to be unworkable in qualitative research. In qualitative research credibility is used instead of validity, confirmability as opposed to reliability and transferability instead of generalizability. As indicated earlier the study uses triangulation by procedures (Opie, 2004) as a means of ensuring credibility of the research.

3.7.1 Transferability

This study is qualitative in nature. It seeks to provide a thick description of a case. Therefore the issue of generalizing the findings is unworkable. Lincon and Guba

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(1985) cited by Cohen *et al.*(2000) point out that in qualitative research generalizability is interpreted to mean comparability and transferability of findings to other individuals working within similar conditions. Guba & Lincon (1983) argue that "some degree of transfer is possible if enough 'thick description' is available about both the sending and receiving contexts to make a reasoned judgement possible". It is therefore important that a researcher provides a clear and detailed description of the phenomenon so that others can decide the extent to which the findings of the research may apply to their situations. Opie (2004) draws on the work of Bassey to argue that the value of qualitative research lies in the adequacy, appropriateness and extent to which the description made would suite a teacher working in similar conditions to relate his or her decision making to those described. I am therefore aware that the outcomes of this research may not be generalized even to the overall practice of the case itself [the teacher] but I am optimistic that they could possibly highlight the mathematical problems that confront teachers as they endeavour to introduce algebra to Grade 8 learners. Consequently, it may open up new directions for curriculum developers, textbooks authors and the research community in mathematics education at large.

3.7.2 Confirmability

Cohen *et al.* (op cit) draws on the work of LeCompte and Preissle (1993) to argue that the 'canons of reliability in quantitative research are impractical in qualitative research'. Lincon and Guba (1985) cited by Cohen *et al.* (2000) take this point further by indicating that in qualitative research reliability is construed as dependability as it involves member checks or confirmability. Aligning myself with Lincon and Guba (op cit), I conducted follow up interviews with Lineo after all the data had been collected. This interview was to confirm or refute my assumptions and interpretation of Lineo's teaching practice. For instance, I noticed that Lineo's explanations emphasized mathematical conventions i.e. ways of writing using algebraic notation, reading this notation and she defined terms. These led me into describing her practice

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as teaching algebra as a symbolic language system. Lineo illuminated this point further during the follow up interview when she said (your attention is drawn to the bold text);

> … they need to know that umm **our terminology in mathematics is important**, umm when I am talking about umm a constant they need to know it is a number, it has got a role and they need to know umm that **mathematics language is important and that it will be to their own detriment in future if they do not learn to speak mathematics properly**. Umm **I will always emphasize the definitions and terminology and when they give me back an answer if they are not saying it correctly I will correct them**… (Lineo, follow up interview, L189 - l197)

In addition, the interview shed more light on what I had observed and amplified my observations as I was able to explicitly communicate my assumptions and interpretations with Lineo. I was aware that my interpretations of what I observed might change as I re-analyze the data or as I interviewed the teacher as she might agree or disagree with some of my assumptions.

3.8 Ethical considerations

Cohen *et al.* (2000) argue that carrying out research demands that the researcher obtains consent and cooperation of the participants and the institutions that provide research facilities. In this study two institutions are involved; Wits University under whose name the study was conducted and a secondary school in Gauteng whose teacher and learners were actively involved in the study (see section 3.3, p. 36). Cohen *et al.* (op cit) further point out that the issue of access and acceptance is very crucial in pursuing a study. As the study was conducted under the name of Wits University, I requested permission from the Wits research ethics committee by submitting a proposal which was to provide the overall picture of the study. As the study was conducted in a private secondary school, I sought permission to conduct the study from the principal of the school concerned. I wrote and submitted a letter

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accompanied by a consent form to the principal of the selected school requesting his permission to involve his Grade 8 teacher and his learners in my study. Two more letters were written: one to the teacher while the other was for the parents of the learners (see Appendix 2). These letters were also accompanied by consent forms.

The letters indicated the research topic, purpose of the study and explained the need for videoing of lessons. Letters also addressed issues around confidentiality of results, anonymity of school and participants, privacy, freedom of participation, respect and fairness (Cohen *et al.*, 2000). In this regard, I decided to use pseudo names for the teacher and learners involved in this study. The name of the school is not mentioned in this study as I only refer to it as a private secondary school in Johannesburg. I also provided the principal with copies of letters and consent forms which were sent to the parents of the learners and the teacher regarding the same issues. The consent forms were to be completed by the principal, teacher and parents. The consent forms were to be returned before the study commenced.

This study used interviews and observations as data gathering instruments. Hence one needed to address ethical issues regarding each instrument. Ethical issues concerning observation were dealt with in the preceding paragraphs. It is important to note that this study focused only on the teacher therefore the teacher was the only person to be interviewed. Cohen *et al*. (2000) warns that interviews can be biased due to the characteristics of both the researcher and respondent. Kvale in Cohen *et al.* (op cit) identified informed consent, confidentiality and the consequences of interviews as problematic areas that need to be addressed in interviews. Oppehein cited in Opie (2004, 112) argues that bias in interviews can be minimized if "the interviewer… maintains control of the interview, to probe gently but incisively and present a measure of authority and assurance of confidentiality". Tuckman (1972) cited in Cohen *et al*. (2000) exemplifies the problems of interviews by saying that

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> … The interviewer should brief the respondent as to the nature or purpose of the interview (being as candid as possible without biasing responses) and attempt to make the respondent feel at ease. He should explain the manner in which he will be recording responses, and if he plans to record he should get the respondent's assent. At all times the interviewer must remember that he is a data collection instrument and try not to let his own biases, opinions, or curiosity affect his behaviour … p.279

I indicated to the teacher that the purpose of the interview was to confirm or refute my assumptions and interpretations of what I observed in the lessons and to assist me in understanding the mathematical problems that she encountered when selecting and implementing the activities as well as to inform me as to what mathematical work of teaching she wrestled with and which knowledge resources she called on to solve these problems.

I further asked her to allow me to tape record the interviews as this would enable me to re-listen to the interviews during data analysis. She had no objection to this and I tape recorded all interviews (Appendix 3). Tuckman cited in Cohen *et al.* (op cit) argues that interviews could be both threatening and stressful to respondents as they often do not know in advance the contents of the interview and the interview environment is usually unfamiliar. In response to this I conducted interviews in the teacher's classroom with the hope that she would feel more comfortable. I also began the interview with broad questions funnelling to the more specific questions. This afforded us time to settle. I tried as much as I could to be non-judgemental while being sensitive.

3.9 Summary

I began this chapter by indicating that the research questions and problem locates this study within an interpretive paradigm (Merriam, 1998). I also indicated that the study

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uses case study as its research methodology. I went further to show that interviews (3 in all) and classroom observations (over a week) were employed in collecting the data. I highlighted the limitations of both the methodology and instruments used in the data collection. Over-and-above, I indicated how I piloted my instruments and how piloting helped me in sharpening the instruments. Lastly, I indicated that the collected data was analyzed inductively as well as typologically. Although the collected data is qualitative, I quantified it in relation to the research questions.

With respect to issues concerning rigour in research, I pointed out that tools of quantitative research (reliability, validity and generalizability of findings) cannot be applied directly on qualitative research. To this end, I concerned myself with credibility instead of validity, transferability instead of generalizability and confirmability instead of reliability.

In the chapter that follows, I provide an overview of the lessons observed during data collection by indicating the main ideas of each lesson. Most important, I embark on the analysis of the data.

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Chapter four: Analysis and data interpretation

4.1 Introduction

In this chapter, I present the analysis of the data collected. As I indicated earlier, I used three interviews and four lesson observations as a window through which to look at Lineo's teaching practice so as to identify the mathematical problems that confronted Lineo, the mathematical work of teaching that Lineo wrestled with during the four lessons, the knowledge resources which she called on as she worked to legitimate meaning of algebra to the boys as well as the nature of algebraic activities that were presented to learners.

A crucial point in this study concerns a Grade 8 teacher's classroom activities and how she used such activities. Through a number of activities, representations and definitions, Lineo endeavoured to provide learners opportunities to develop algebraic concepts and to learn the language of algebra.

Conducting this study has required that I describe Lineo's practice. This inevitably involved making judgements about what Lineo did in the four lessons which I observed. These judgements are analytic, not evaluative. In effect they are descriptive statements about the mathematical work enacted in Lineo's teaching. Such judgements were not intended to describe Lineo's teaching practice as 'good', 'bad' or lacking in some ways. I have tried to leave this evaluative form of judgement aside so as to describe and analyse Lineo's practice in relation to my research questions (section 1.2) and theoretical framework (section 2.3).

In this chapter, I am going to discuss the following:

- Background to the lessons observed
- An overview of the observed lessons
- Chunking of data

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- ❖ Mathematical problems, mathematical work of teaching and knowledge resources used
- ❖ How Lineo describe her teaching

4.2 Background to the lessons observed

In what follows I present first the seating arrangement in Lineo's class and a brief overview of the lessons observed. I observed Lineo for one week. During this period she was teaching Grade 8 learners algebra, in particular algebraic expressions. The observation commenced on Tuesday, 11th July and ended on Friday, the 14th July, 2006. Each of the lessons I observed was 45 minutes long (Table 4.1). Altogether, they translate to three hours of teaching per week. All the lessons were single periods.

During the Post-teaching interview Lineo pointed out that there were six Grade 8 classes in the school which were taught by different teachers. She further told me that all the Grade 8 learners were taught mathematics at the same time. According to Lineo, teachers in this school work independently most of the time. She explained

> At this school we pretty much work by ourselves. I go by my own pace; I know that by the end of the term I need to have covered so much. I might be ahead or behind other teachers, umm, we don't share much of the resources… so we pretty much work independently. (Lineo, post-teaching interview)

However, she indicated that they do share some interesting or important materials. When asked how they work in relation to test and exams Lineo said: "we set our own tests… sometimes we have standardized tests where all the Grade 8 are writing the same test at the same time… other than that we set our own tests" (Lineo, post-teaching interview). Here Lineo emphasized that teachers in the school are expected to be independent. She indirectly acknowledges that there is collaboration amongst the Grade 8 teachers. In fact, she had indicated earlier that the worksheet which she

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was going to use had been made available to her by one of the teachers. The above quotes indicate that while Lineo finds herself in a very supportive environment she also does a lot of work individually. This also highlights the limitation of this study in that Lineo's practice cannot be extended even to teachers working within the same school.

Lineo had at her disposal a white board for written communication, graphing calculator, an overhead data projector, a computer and other supporting equipment. As I have mentioned earlier, Lineo was teaching algebra (e.g. collecting like terms, expressing as a powers, and expanding) during the observation week. In the next section, I provide an overview of what Lineo did in each of the four lessons.

4.3 Overview of lessons observed

The lessons observed focused on teaching of terminology (such as coefficient, term, constant, monomial etc.), conventions (e.g. writing numbers before letters) and concepts such as variable and like and unlike terms. This could be understood to be the result of introducing a new topic with its specific mathematical language, concepts and skills. The analysis will show that Lineo drew on various knowledge resources to ground what algebra is and to develop algebraic proficiency. She introduced algebra by recapping what they had done in Number patterns. She begun with 'guess my rule'. In this activity Lineo told the learners that she has a rule in her head and that they were to give her numbers on which to apply the rule. Learners randomly gave her numbers and Lineo gave back the corresponding answers. Learners wrote the numbers and their corresponding answers in their answer books. Lineo then asked the learners to write down the rules in each case. The three activities under 'guess my rule' are summarized as follows

4	1	5	3	2

5	7	3	1
25	49	9	1

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12	6	14	10	8

expression = $2n + 4$

1	2	6	3	0
-3	-5	-13	-7	-1

expression = n^2

expression = $-2x - 1$

Lineo used 'guess my rule' to lead learners into accepting and using letters other than n to represent numbers. After these activities Lineo asked the learners "what is algebra?" She employed various representations to help learners visualize or make some sense of algebra. These representations were also used to introduce learners to collecting like terms. Lastly, Lineo defined terms that are used in algebra.

Table 4.1 gives the overview of all the observed lessons.

Date	Duration of lesson	Topic/ subtopic	Main ideas (concepts/skills)	comments
Tuesday 11/07/06	45 minutes	-Number patterns -algebraic expressions	-Formulating expressions -collecting like terms -definitions	Lineo begins the lesson by recapping number patterns. She uses 'guess my rule'. She asks learners to give her numbers which she seems to be substituting into her rule to give back the answers. Learners are then asked to write the rule in each case. Lineo asks, 'what is algebra?' one learner responds, 'using letters to symbolize numbers' she uses this definition to enable learners to appreciate the use of letters in algebra. She then projects Italy's soccer jerseys, caps, socks, gloves, bags, flag and balls on the board. She asked learners, 'how many of the same things are there?' she then continued with other representations to introduce the language of algebra.
Wednesday 12/07/06	45 minutes	Algebraic expressions	-expression vs. equation -definitions -collecting like terms	Lineo starts by recapping the previous lesson. she clarifies the difference between an equation and expression. She writes $5x^3 + 3x^2 - 2x + 8$ on the board. she then engages learners in a discussion of the terms constant, coefficient etc. she then gives notes. The emphasis is on mathematics vocabulary. She defines monomial, binomial, trinomial, and polynomial. She asks learners to give examples. She asked learners to start working on the worksheet. She goes round and explains to individuals.
Thursday 13/07/06	45 minutes	Exponents: -expressing as powers -expanding	-simplifying expressions -expanding powers	Lineo talks about the structure of the exam. She then corrects the homework by reading out the answers while learners mark their own work. She then gives the following spot test. Simplify (1) $3x^2 - 5x + 2x^2 - x$ (2) $8x - 3y + xy$ (3) $(4x)(2x)(x)$ (4) $x.x^2.x$

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				(5) $(-2xy)(x^4y^3)$ Lineo explains the difference between expanding and expressing as a power $x^3 = x.x.x \rightarrow \text{exp}anding$ $x.x.x = x^3 \rightarrow$expressing as a power
Friday 14/07/07	45 minutes	Simplifying algebraic expressions	-Simplifying expressions -expanding -expressing as powers	Lineo starts by asking learners to read their homework answers aloud in turns. She confirms some and corrects others. She defines the term product. Lineo asks the learners to continue with the worksheet and she goes round explaining to individuals.

Table 4.1 Overview of observed lessons

4.4 Chunking data

In order to identify the mathematical problems that were confronted, the mathematical work of teaching which Lineo grappled with and the knowledge resources that were called on by Lineo, and working from an orientation of analyzing pedagogic discourse, I needed to chunk lessons into evaluative events – my unit of analysis. The end of an evaluative event/ unit of analysis was marked by the teacher establishing some ground for legitimating meaning for the ideas under discussion. For instance, in order to differentiate clearly between the multiplication sign and the letter x used in algebra Lineo said;

> … if I were to write that as a straightforward x, what does that look like? times times 2 or xx2, doesn't make sense. So I am going to be very strict with you when you write your x's, it is the only letter that you have to change. I want you to write it as a curly x neatly. You need to get into that habit. **I will take off marks if you don't** (Lineo, lesson 1, 08:40 – 09:03, emphasis in bold)

This extract marks the end of a discussion around 'what is algebra?' in which Lineo as the old-timer in the practice concluded the discussion by using her **authority** [see underlined text] and assessment (marks) i.e. **curriculum knowledge** [see bold text] in socializing the learners into the Mathematician's ways of writing. It is important to

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note at this juncture that Lineo used various knowledge resources (see section 4.8) to legitimate meaning of algebra to her learners.

Just as an event has an end marked by the teacher legitimating meaning so it has the beginning marked by the introduction of a new sub-topic, concept or activity. For example, in the extract below Lineo was explaining the use of x and y in mathematics and decided to move on to define a constant. I draw your attention to the bold text.

> I think, but I stand to be corrected, for me the *x* and the *y* are used more often because when it comes to plotting your points what do we use? The x-axis and the y-axis, so we will be developing … where we have everything in terms of the axes. So I think in the forefront of my brain I am always thinking x's and y's but there is no reason from stopping me from using the p's and the q's. If so, listen here if I say x is 3 then you have to stick with that 3, if I say to you hold on x is 3 you can't work with it as a variable and say I gonna make it 10. If I give it to you as a value you need to stick to that value…. **What do we call the numbers (writes 1, 2, 3, 4, 5 …) without any attachment to the x or the y, the p's and the q's? what do we call those? It start with a C. those guys who have done algebra last year** (Lineo, lesson 2, 32:47 - 33:20, emphasis in bold)

The extract marked the end of the discussion of a variable (underlined text) and also shows how Lineo initiated discussion of a constant. The sentences in bold indicate a new concept which is introduced through questioning by the teacher hence a new unit of analysis begins.

4.5 Using the analytic framework for data description and analysis

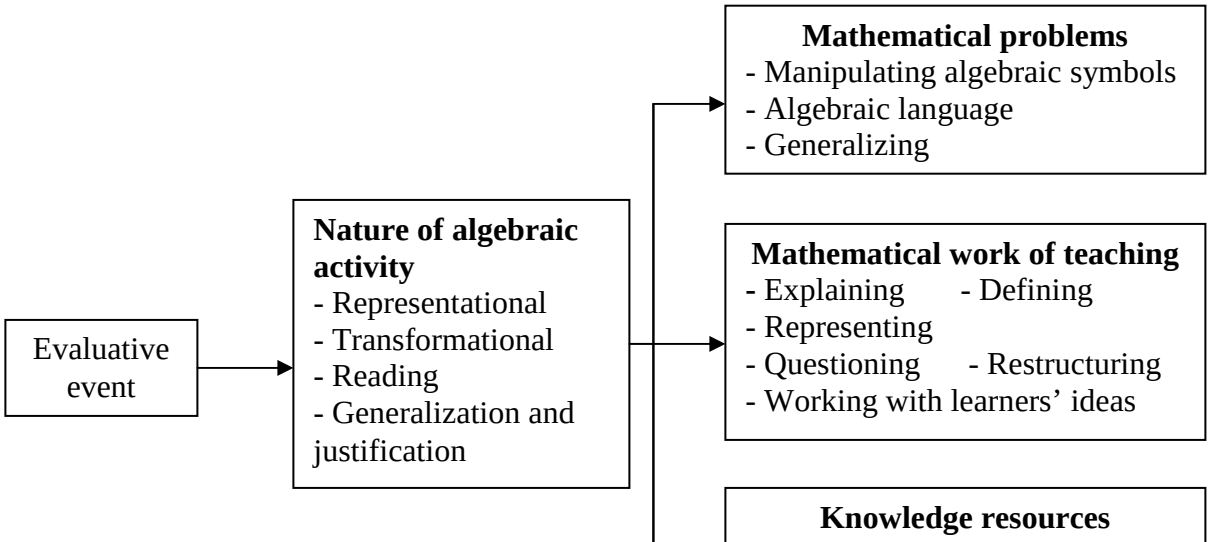
In this section I am going to present the overall quantitative and qualitative analysis of the lessons I observed. In order to do this, at the same time I show how I have done the analysis. I use extracts from the transcripts of the data I collected for this study to substantiate my identification of the mathematical problems that confronted Lineo, the mathematical work of teaching that she wrestled with and the knowledge

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resources that she called on as she introduced algebra to her boys. The extracts are further used to point out the nature of the algebraic activities that the 8th graders were exposed to in this practice. In order to do this, I first recap by presenting the analytic framework (see figure 2, p. 59) that summaries the lens which is being employed to look at the data.

I have indicated earlier in section 2.2 that my analysis of Lineo's practice is informed by Ball *et al.*'s (2004) 8 types of mathematical work of teaching that mathematics teachers wrestle with during their teaching. On interacting with my data I realized that Ball *et al.*'s categories are problematic in that they turn out to be too specific thus distinguishing what appears to be the same construct in practice. For instance the categories "interpret…students' solutions" and "be able to respond to students' …" (p.59) are practically the same since one does something as a result of the learners' contribution to the discussion. Other researchers, Kazima & Adler (2006), Pillay (2006) and Adler & Pillay (in press) have also found Ball *et al.*'s characterizations of teachers' mathematical work of teaching problematic thus leading them into condensing the 8 kinds of mathematical work of teaching into 6 types as follows: **defining**, **explaining**, **representing**, **working with learners' ideas**, **questioning** and **restructuring of tasks**. As I indicated earlier, it is these six categories of mathematical work of teaching that I finally employ in studying Lineo's practice. Bernstein's (1996) theory of pedagogic discourse, particularly the point that meaning is condensed in evaluation was critical to this study as it pointed out a way of chunking the data. The analytic framework for this study is shown below.

Analytic framework



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Figure 2: Analytic Framework

This analytic framework is used here to interrogate data so as to find answers to the research questions posed earlier in section 1.2. As I have indicated above, extracts from the transcripts have been chosen to show how the analytic framework was used in interrogating the data. The extracts also serve to illustrate that Lineo confronted diverse mathematical problems simultaneously, that she did not wrestle with the same mathematical work of teaching and that she drew on various knowledge resources to work with these problems. I also use this extract to demonstrate how I broke down the lessons. It is through this chunking that I started to see more clearly the mathematical problems and the mathematical work of teaching that Lineo wrestled with as she went about her work. Chunking the data in this way has also helped me to see the nature of algebraic activities which Lineo's presented to her learners.

Table 4.2 shows a chunked transcript extract from the first lesson which illustrates how I used the analytic framework. It is essential to note that time is given in minutes and seconds as 10:39 which means 10 minutes and 39 seconds elapsed i.e. into the lesson. It is important to note that my evaluative events or units of analysis are indicated by time intervals which show the beginning and end of the event, for example 06:04 – 06:47 shows that the event or unit of analysis began 6 minutes and 4

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four seconds ago and ended at 6 minutes and 47 seconds later. Table 4.3 illustrates how the transcript extract was quantified.

Table 4.2 (on p. 64) shows that concepts were introduced through different algebraic activities. In order to pick the category **representational,** Lineo would have to introduce the concept through a verbal statement, random distribution of terms [see page 17 and 95] or questions [such as simplify $2x+4y-3x+11y$] which learners have to transform into an equivalent algebraic expression. If she defined or explained the concept then I categorized that concept as being introduced through **reading/writing activities**. For concepts to be introduced via **generalization and justification,** the activity would have to allow learners to engage in either empirical, theoretical or algebraic arguments where learners engage in mathematical practices such as conjecturing, generalizing and justification. One should realize that this category includes the other activities of algebra and that in one event we may have more than one activity in use to drive the same concept home. in order to pick the category **transformational activities** Lineo would give her learners an activity that demands them to manipulate and change algebraic expressions into their simplest forms. The extract below illustrates how Lineo used transformational activities in teaching like and unlike terms and also to introduce expressing as power.

Lineo: ok question 1 (writes on the board)

Simplify

1) $3x^2-5x+2x^2-x$
2) $8x-3x+xy$

You don't need to write out the question, just go straight to the answer

3) $x \,.\, x^2 \,.\, x$
4) $(4x)(2x)(x)$
5) $(2xy)(x^2y^2)$

3, 4 and 5 I haven't taught but I want to see what you can make with them.

Learners work individually on the tasks

Lineo: I will give you one more minute (Lineo, lesson 3, 07:17 – 08:23, emphasis in bold).

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Here Lineo gave learners activities to solve individually. She explicitly points out that 3, 4 and 5 have not been taught (see bold text). This means that she was using them to introduce a sub-topic (expressing as powers) and to draw learners' ideas for discussion. We see that the activities are arranged in increasing order of difficulty and that the activities demand understanding of mathematical notation (e.g. when brackets are placed against each other mean × and that there is × between a coefficient and a variable) for successful completion.

The above activities demand knowledge of algebraic notation for successful completion. I categorized the knowledge resource called on in such activities as **mathematics,** specifically 'rules and conventions' (Pillay, 2006).

Tables 4.2 (p. 64) and 4.4 (p. 75) indicate that Lineo employed various knowledge resources to fix meaning for the algebraic concepts being taught. The excerpt that follows shows how Lineo fixed meaning for the mathematical concept - coefficient

> …what is the coefficient of x in the following… It is the stuff before the variable
> **In 3x is 3**
> **In – 15x is -15**
> **In 10ax is [10a]**
>
> (Lineo, lesson 1 34:06 – 37:03, emphasis in bold)

I categorized this appeal as **empirical mathematical knowledge** because examples of what a coefficient could be were used in the argument. The above example indicates that Lineo was not able to find a standard definition that could cater for all cases thus she opted for empirical cases/argument in order to convince her learners. The difficulty in finding a general definition of coefficient arises from the fact that coefficient is best defined in context as it could be a constant, a variable or both as the excerpt above illustrates (see underlined bold text above). Here, Lineo picks three different examples to illustrate what a coefficient is. It is important to note that at this point Lineo's teaching focused on reading/writing activities as it provided cues in a form of question for identifying or determining a coefficient. The extract also shows

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that although Table 4.4 indicates that Lineo used defining frequently, her definitions were not necessarily mathematically robust. In another instance, Lineo wanted learners to generalize the commutative property of multiplication which forms the base for collecting like terms. When she realized that some learners could not recognize xy as being the same as yx she asked them what is happening between the letters and then asked them to substitute numerical values (e.g. 3 for x and 16 for y were used) and to multiply out. This also is an empirical mathematical knowledge. The activities in this evaluative event are classified as **reading/writing activities** as learners are only required to identify the coefficient in each scenario.

For Lineo to be drawing on her **experience** which may either be professional, everyday or authority Lineo would have to use statements like (attention is drawn to the bold text)

> …So guys **I'm going to be very strict with you, when you write your *x* 's**, it is the only letter that you have to change. ***I want you to write it as a curly x neatly***… ***I will take off marks*** if you don't (Lineo, lesson, 1, 08:50 – 09:03)
> :
> :
> …that's why in South Africa we use a comma. ***If I want a decimal place I will put a comma*** (Lineo, lesson 1, 24:23 – 24:30)
> :
> :
> *Traditionally we always put up the numbers before the letter. So n2 is correct but we prefer to write the number before the letters* [emphasis in bold] (Lineo, lesson 1, 08:04 – 08:10)

The underlined and bold text in the above excerpt indicates that Lineo drew on her **authority** to instill a way of writing the x in mathematics and also for explaining the use of a comma in mathematics. In addition, Lineo appealed to the **curriculum** in a form of assessment, in particular, tests or exams (see bold text above). In this case Lineo talked about not awarding marks. The extract further shows that Lineo drew on **mathematics** in the form of **conventions** (see italicized and underlined text). She describes these conventions as the tradition of mathematics. Drawing on conventions

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afforded her the opportunity to legitimate a particular writing style (i.e. writing numbers before variables [letters]) to her learners. In the above extracts, Lineo explains the style of writing used in mathematics and the use of a comma in mathematics, in South Africa. I classified the work of teaching that Lineo wrestled with as **explaining** (08:04 – 08:10 and 08:50 – 09:03) **and working with learners' ideas** (see 24: 23 – 24:30).

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Event	Transcript	Knowledge resources in use	mathematical work of teaching	Mathematical Problem confronted	Nature of algebraic activities	What's the mathematics?
06:14 – 06:47	**Lesson 1** Lineo: ok now we are not doing number patterning with this new unit. We have done number patterning but this was a nice introduction to the algebra. What is algebra? Themba? Themba: using numbers to symbolize numbers Lineo: Thaabe, using letters to symbolize number. So … with question 1, with the issue that was raised. Lisebo said x everybody said nooo… it has to be n, we have always done finding the n-th term. Why is now that Lisebo comes up with his own letter, surely it must be wrong. Hello, you have put your hand up, Thabo? Thabo: it is because it does not matter what letter it is; it does not change its value	Mathematics (definition) Curriculum (books & worksheets)	Questioning Defining Working with learners' ideas	Language of algebra	Reading	variable
06:52 – 07:21	Lineo: ok, perfect remember with this letters, they can represent any number in the whole wide world and for the algebra we always choose little letters a, b, c, d …the whole alphabet to choose from. You have got 26 letters, I could have come up with $j \times 2 + 4$ because I think j is for Jacob and Jacob rules the world and that would be my choice of the letters. You could have had any letter.	Mathematics (empirical) Experience (everyday & professional) Curriculum (tests & exams, worksheets & books)	Explaining	Language of algebra	Reading	variable
07: 22 – 07:31	Looking at number 2, n^2 is it the same as $n \times n$, is that correct? What does n^2 mean? Learner: $n \times n$ Lineo: $n \times n$ that would be perfect.	Mathematical (rules & conventions) Experience (professional)	Working with learners' ideas	Language of algebra	Reading Representational	Mathematical convention
07:32 – 08:26	Back tracking, sorry (circles $n2$ in the expression) is that how we traditionally write out the notation? Thabo? Thabo: no Lineo: no, how do we traditionally write it. Thabo: we write n times 2 plus 4.	Mathematics (rules & conventions)	Questioning	Language of	Reading	Mathematical convention

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	Lineo: (writes $2\times n+4$) now which way is neater? That way (points at $n2+4$) or that way (points at $2\times n+4$) Learners: the first one Lineo: the first one, I really prefer that method (points at $n2+4$) of writing it but Learners: 2 times n plus 4 Lineo: (writes $2\times n+4$). Which way is neater? That way or this way? Learners: the first one Lineo: Ok, traditionally we always put up the numbers before the letter. So $n2$ is correct but we prefer to write the number before the letters. Also we don't write the times between a number and the letter. When I write $2n$ what are we assuming? 2 times n.		Working with learners' ideas Explaining	algebra	Reading	
08:27 – 09:03	The same thing here with the x why did I write the curly x? Mpho: we do not want a straight x Lineo: Don't call it out. Thabo: inaudible Lineo: Ok, as Thabo said, if I were to write that as a straightforward x what does that look like? times times 2 or xx2 doesn't make sense. So guys I'm going to be very strict with you, when you write your x's, it is the only letter that you have to change. I want you to write it as a curly x neatly. You neat to get into that habit. I will take off marks if you don't	Mathematics (rules & conventions) Experience (professional & authority) Curriculum (exams & tests, worksheets & books)	Questioning Explaining	Manipulating algebraic symbols	Reading	Mathematical convention
09:06 – 09:40	Lineo: next thing, question number 3 which one here is the problem one? (points at $n2+4$, $x\times 2+4$ and $n-2-1$) Out of these 3 choices, 1, 2 and 3 which one is the problem one? Thabo? Henry: n minus 2 minus 1 Lineo: (draws a rectangle around $n-2-1$) Ok he is the problem. What was the person trying to say? This is turning out to be n minus 3 (writes $n-3$) n minus 3 is different to saying n times minus 2 minus 1.	Mathematics (rules & conventions) Experience (professional)	Working with learners ideas	Language of algebra	Reading	Generalizing patterns
09:41 – 09:52	Now I like the way it's written (points at $-2x-1$) how do we write this one (points at $n\times -2-1$) in a neater way, John?	Mathematics (rules & conventions)	Questioning	Language of algebra	Reading	Generalizing patterns

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	John: minus 2 times n minus 1 Lineo: (writes $-2n-1$) hundred percent.		Working with learners' ideas			
09:53 – 14:10	Are you guys getting a feel of algebra? Ok you are familiar with this from your patterning. I want to explore further, keep your scrap papers out. I am going to switch on my data projector. Wily switch off the lights there. Lineo: projects the Italy soccer kit  What have we got there? Italyyy… I must admit that when I prepared this I had Italy and France in mind just in case but… France got defeated. Ok on your piece of paper, my question to you is and I want you to answer this, I want you to count how many of the same thing they are. How many of the same things are there? Not similar, same. Lineo: Have you added? Look at my question, how many of the same things are there? You count how many they are. Lineo: have you finished? Ok Paul, I haven't heard from you today. Give me the answer. What have you come up with? Paul: 4 hats Lineo: 4 hats, do you agree with that? (Learners agree) so hats are in one category. Then? Paul: 3 socks (interrupted) Lineo: correct, carry on	Experience (everyday & professional curriculum	Questioning Representing Questioning	Manipulating algebraic symbols	Representational Transformational	Collecting like terms

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	Paul: 5 shirts, 1 cap, 2 balls, 1 ball, 1 flag, 1 bag, 4 shorts. Lineo: 4 pairs of shorts, did everybody get the same idea? Did anybody do it differently? Khau? Khau: I also added the badges (unclear) the badges on the shorts, socks (others make some noise) Lineo: Ok, no no (unclear) thinking out of the box, I agree with you, that is another way of looking at it and then how many badges did you get? Lineo: you could nothing is stopping; I gave you an open ended question, a pretty arbitrary kind of pre-school exercise. What's the whole point of it? Yeah, all I am reminding you is that Italy won.					
14:11 – 15:10	Lineo: projects $a \quad x \quad x \quad x \quad y \quad y \quad x \quad y \quad x \quad y \quad y \quad a$ Do the same exercise as we did. Some are becoming faster than other. Mahomete, tell me what the answer is. Mahomete: $6y$ Lineo: yeah Mahomete: 5x plus $2a$ … Lineo: Everybody got the same answer.	Mathematics (rules and conventions)	Representing	Manipulating algebraic symbols	Representational Transformational	Collecting like terms
15:11 – 15:53	Ok, now my question is they are all letters, what made you distinguish the a 's apart from the x 's and apart the y 's, Hilda? Hilda: They are different letters Lineo: ok they are different letters. Mahomete, what would be a cleverer answer than that? Mahomete: they look different Lineo: they look different, Kele? Kele: different shapes Lineo: different shape of letters, now in terms of algebra, I ask you the question again, why did you classify them differently? Why did you put	Mathematics (rules and conventions) Experience (professional)	Questioning Questioning	Language of algebra	Reading	variable

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	the a 's in one group and the x 's and the y 's in another, Lethabo? Lethabo: they have different values					
15:54 – 16:20	Lineo: correct, the y 's and the a 's will represent different numbers. I am using letters two letters at the same time; I am assuming that those values will be different. a could be 10, x could be 5 and y could be – 100 , ok different values, any question so far.	Mathematics (rules & conventions)	Working with learners ideas Explanation	Language of algebra	Reading	Variable, collecting like terms
16: 22 – 16:56	Let's see the next thing, projects x^2 x x^2 x x^2 x^2 x x^2 Lineo: Ok, Rob? Rob: five x squared plus three x Lineo: so you have five x squared and three x (writes $5x^2 + 3x$) in other words what I have written there, 5 x squared, you are talking 5 lots of x squared, agreed? And continue Robert why did you classify the x squared and x differently? Robert: x^2 means a number times itself and x means a number	Mathematics (rules and conventions) Experience (professional) Curriculum	Representing Working with learners ideas Questioning	Manipulating algebraic symbols Language of algebra	Representational Transformational	Collecting like terms

Table 4.2 shows a chunked transcript extract from lesson 1

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Events (given in time)	Activities of algebra				Mathematical problems			Mathematical work teaching						Knowledge resources								
														Curriculum		Experience			Mathematics			
	Representational	Reading	Transformational	Generalizing & iustification	Manipulating algebraic symbols	Language of algebra	Generalizing	Working with learners' ideas	Restructuring tasks	Representing	Questioning	Defining	Explaining	Tests & exams	Textbooks & worksheets	Professional	Authority	Everyday	Empirical	Rules & conventions		
06:14 - 06:47	1	1	0	**0**	0	1	**0**	1	**0**	0	0	1	1	0	1	0	0	0	0	0		
06:52 - 07:21	0	1	0	**0**	0	1	**0**	0	**0**	1	0	0	1	0	1	1	0	1	0	1		
07:22 - 07:31	1	1	0	**0**	0	1	**0**	1	**0**	0	0	0	0	0	0	1	1	0	0	1		
07:32 - 08:26	1	0	0	**0**	0	1	**0**	0	**0**	1	0	0	0	0	0	1	0	0	0	1		
08:27 - 09:03	0	1	0	**0**	1	0	**0**	0	**0**	0	0	0	1	1	0	1	1	0	0	1		
09:06 - 09:40	1	0	0	**0**	0	1	**0**	0	**0**	1	0	0	0	0	0	1	0	0	0	1		
09:41- 09:52	1	1	0	**0**	0	1	**0**	0	**0**	1	0	0	0	0	0	0	0	0	0	1		
09:53- 14:10	1	0	1	**0**	1	0	**0**	0	**0**	1	1	0	0	0	1	1	0	1	0	0		
14:11- 15:53	1	1	1	**0**	0	1	**0**	0	**0**	0	0	0	1	0	0	1	0	0	0	1		
15:54 – 16:20	0	1	0	**0**	1	0	**0**	0	**0**	0	0	0	1	0	0	0	0	0	1	1		
16:22 – 16:56	1	1	1	**0**	0	1	**0**	1	**0**	1	1	0	1	0	0	1	0	0	0	1		

Table 4.3 shows how the above transcript extract was quantified

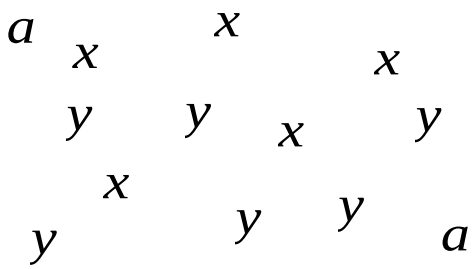
The transcript extracts (Table 4.2) show that amongst the concepts treated by Lineo in the first lesson were 'variable' and 'collecting like terms' or 'grouping terms' as described by Lineo. I have chosen this excerpt to illustrate the kind of mathematical work of teaching which Lineo tackled, the mathematical problems that confronted her, the knowledge resource that she called on as well as the nature of algebraic activities she engaged with as she endeavored to introduce algebra to Grade 8 learners. One would like to acknowledge that the selected extract is not complete in illustrating these constructs or even to fully show how I used the analytic framework. However it captures a larger continuum of concepts (For more details see Table 4.4 and Appendix 4).

In the above extract (Table 4.2) I identified 'algebra[10]' as the main concept with no sub-concepts. I begin by showing how this concept was introduced. During the period 06:16 to 07:28, we see Lineo posing the question "what is algebra?" this indicates that the concept 'algebra' is introduced through **reading/writing activities** as the question does not demand learners to formulate algebraic expressions or manipulate algebraic symbols. This underscores the fact that the mathematical work of teaching which Lineo wrestled with at this point in time is **questioning**. Lineo's question suggests that she wanted learners to define or explain what algebra is hence **defining** and ***explaining*** mathematical concepts are other elements of mathematical work of teaching that Lineo grappled with during her teaching of algebra in a Grade 8 class.

Algebra is also encountered through **representational and transformational activities** as Lineo projected a number of activities on the board for learners to work on. In this case the mathematical work of teaching that she grappled with was **representing.** The excerpt that follows sheds more light on these issues.

> Let's see the next thing.
> Lineo: projects

[10] Mathematically speaking algebra is a branch of mathematics just like geometry, statistics and others.



Do the same exercise as we did. Some are becoming faster than other. Mahomete, tell me what the answer is.
Mahomete: $6y$
Lineo: yeah
Mahomete: 5x plus $2a$ …
Lineo: Everybody got the same answer (Lineo, lesson 1, 16:18-17:29).

This extract, in particular the conversation between Lineo and Mahomete shows that learners were to collect like terms (**transformational activities**) and were also expected to write an algebraic expression (**representational activities**) from the given random symbolic distribution of terms.

Lineo drew on **everyday** knowledge and **experience** to fix the meaning of algebra to the boys. She argued that "…you have got 26 letters; I could have come up with $j \times 2 + 4$ because I think j is for Jacob and Jacob rules the world and that would be my choice of the letters. You could have had any letter" (Lineo, lesson 1, 06:52 – 07:21). Lineo's choice of letters (choice of j) is problematic in that while she endeavoured to make mathematics realistic, relevant and meaningful, her explanation obscures mathematics, as **letters are used as abbreviated words** (MacGregor & Stacey, 1996a) as opposed to variables. In support, Stacey *et al.* (2004) would argue that in this case Lineo views letters as objects which could lead to learning difficulties in future instead of understanding of the underlying mathematics. While Lineo's explanation about the choice of letters might be limited, it is synonymous with the use of acronyms such as SOHCAHTOA, FOIL, BODMAS etc. in mathematics which are often used to assist learners in recalling certain procedures, where each letter is an abbreviation and not a variable i.e. does not represent any number. In addition to this, Lineo being an English language first speaker restricts the choice of letters to the 26

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letter English alphabet. This could inhibit future learning since in mathematics letters from other alphabets e.g. θ and Φ from Greek are used.

It is essential to note at this juncture that Lineo seems to be teaching algebra as a symbolic language system (Wheeler, 1996) with its own grammar. In which case her main mathematical work was to ensure that learners are able to read and write using algebraic [mathematical] language. In this regard, Lineo engaged with two different elements of working with learners' ideas in her lesson. These are *interpreting learners' ideas* and *analyzing errors* as illustrated in "Looking at number 2, n^2 is it the same as $n \times n$, is that correct? What does n^2 mean?... Back tracking, sorry (circles $n2$ in the expression) is that how we traditionally write out the notation?" (Lineo, lesson 1, 07:22 – 07:31). This quote makes it clear that Lineo prefers a certain way of writing. In this case I have categorized the knowledge resource that Lineo employs to legitimate the style of writing as **mathematical conventions**. In support Lineo argues

> Traditionally we always put up the numbers before the letter. So $n2$ is correct but we prefer to write the number before the letters. Also we don't write the times between a number and the letter. When I write $2n$ what are we assuming? 2 times n. The same thing here with the x why did I write the curly x? (Lineo, lesson 1, 08:04 – 09:40)

Again we can see that Lineo embarks on **explaining.** She further engaged with another mathematical work of teaching, **working with learners' ideas** (appraising learners' solutions/ answers) as she acknowledges that $n2$ is correct. I have identified and classified the mathematical problems that confront Lineo in the above excerpt as she attempts to introduce algebra to her Grade 8 learners as **language of algebra** specifically *writing algebraically and reading algebraic notation.*

> When I write $2n$ what are we assuming? 2 times n. **The same thing here with the x why did I write the curly x? ... If I were to write that as a straightforward x what does that look like? times times 2 or xx2 doesn't make sense**. So guys *I'm going to be very strict with*

> *you*, when you write your x's, it is the only letter that you have to change. *I want you to write it as a curly x neatly*. You need to get into that habit. *I will take off marks if you don't* (Lineo, lesson 1, 08:40 – 09:03)

The above quote shows that Lineo confronted two mathematical problems namely language of algebra problems, in particular *meaning of algebraic notation* (see underlined text) and manipulating algebraic symbols, above all *symbol conflict* i.e. multiplication sign versus the letter x (see bold text) in algebra. Algebraic notation compels Lineo to decompress or interpret each algebraic (mathematical) statement. In order to address the manipulating algebraic symbols problem (symbol conflict) Lineo appealed to **authority** and the **curriculum in form of tests and exams** (see the italicized text). She told the boys that she wanted them to write their x's as curly x's and she would deduct marks if they do not do as she wanted. Lineo resolved language of algebra problems by unpacking algebraic notation. She pointed out that the absence of a mathematical operation be symbols would mean multiplication is implied [see underlined text in the above extract]. In this interval the algebraic activities are mainly reading/writing activities.

The interval 10:46 to 15:55 indicates another episode taken from lesson 1. During this period I identified the main concept to be collecting like terms with sub-concepts like and unlike terms. Lineo projected Italy's soccer kit on the board and asked learners to count how many of the same things are there. Learners were given some time to work on this activity individually before it was discussed. This shows that Lineo wanted to work with learners' ideas. The activity itself shows that Lineo attempted to make mathematics relevant to learners by integrating mathematics and everyday in her teaching. In a follow up interview Lineo indicated that this activity helped some learners and not others. She further indicated that the activity helped to narrow the gap between arithmetic and algebra and to concretize algebra as it is too abstract for her learners. I have classified this activity as transformational [procedural] in nature because learners were to come up with a procedure or criteria which will be used in

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identifying the same things. Also, Lineo focused mainly on the outcome by asking questions such as "What have you come up with? ... and then how many badges did you get?" I identified the mathematical work of teaching that Lineo engaged with in this case as **representing** and **questioning.** The knowledge resource that Lineo uses to drive the concept of like and unlike terms is experience particularly **everyday knowledge** because her first activity (soccer kit) is closely related to everyday activities such as packaging as it involves putting the same things together and knowing how many of those things are in each pack. Once again we see that activities in this interval are representational and transformational in nature.

In the interval 15:55 to 17:41, Lineo drew on specific numbers to argue that two variables can only take on different values "When I am using letters, two letters at the same time; I am assuming that those values will be different a could be 10, x could be 5 and y could be – 100, ok different values, any question so far …" (Lineo, lesson 1, 15:54 - 16:20). This [i.e. use of specific numbers] enabled me to categorize the knowledge resource that Lineo drew on as **empirical mathematical knowledge**. While the quantitative analysis (Table 4, p.74) shows that Lineo grappled with explaining most frequently the above quotation suggests that such explanations were not always mathematically robust and could lead to difficulties in future learning or misconceptions. Again here we see that the mathematical work of teaching that Lineo grappled with is **explaining**. In this interval, I have identified and classified the mathematical problem that confronted Lineo as language of algebra [reading symbolic algebraic language and interpreting learners' ideas] as Lineo finds herself reading aloud algebraic notation. After writing an algebraic expression on the board Lineo said "(writes $5x^2 + 3x$) in other words, what I have written there, $5x$ squared, you are talking 5 lots of x squared, agreed?" (Lineo, lesson 1, 16:22 – 16:56). Here, Lineo teaches her learners how to read algebraic notation. Lineo also engaged in representing the learners' verbal responses algebraically. So in this interval Lineo attempts to develop conventional fluency by reading and interpreting the algebraic

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written language. I identified the activities in this interval as **representationa**l, **transformational** and **reading/writing** activities.

So far I have shown that Lineo drew on her authority as the teacher, curriculum and mathematics and that she used transformational, representational and reading/writing activities in introducing algebra to her boys. The problems that confronted Lineo were language of algebra and manipulating algebraic symbols.

4.6 Quantification of data

Section 4.5 provided an outline of how I used the analytic framework to look at Lineo's practice. In order to get a clearer picture of the mathematical work of teaching that Lineo wrestled with, the knowledge resources that she called on and the mathematical problems that confronted her during the four lessons that I observed. It was necessary to quantify all the lessons. Table 4.4 below shows the overall quantification of the lessons. Table 4.4 (page 78) shows that in Lineo's practice algebrais not introduced through generalizing and justifying activities and that restructuring of tasks is entirely absent. It further shows that Lineo's practice is dominated by mathematical demands that require her to engage with mostly with questioning, explaining and working with learners' ideas. the table reveals that Lineo introduced alegebra mainly through transformational and reading/writing activities and grounds meaning by appealing to mathematics, mostly in the form of rules and conventions.

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Construct(s)		# of occurrences
Nature of algebraic activities		
Representational		17
Transformational		48
Generalization & justification		0
Reading/ writing		45
Mathematical problems		
Manipulating algebraic symbols		20
Language of algebra		84
Generalizing		0
Mathematical work of teaching		
Explaining		44
Defining		13
Working with learner Reponses		38
Restructuring tasks		0
Representing		12
Questioning		87
Knowledge resources in use		
Mathematics	Rules & conventions	30
	Empirical	16
	Definition	21
Experience	Everyday	6
	Professional	9
	Authority	8
Curriculum	Tests & Exams	8
	Textbooks & worksheets	5

Table 4.4: Overall quantitative analysis of observed lessons.

In the preceding sections I described how I chunked the data and illustrated how the analytic tools were put into use. Chunking the data helped me to have smaller units which could be interrogated. Through interrogation of data I was able to identify and

quantify the activities of algebra that were likely to be developed in this practice, the work of teaching which Lineo engaged with and the knowledge resources which she employed. Table 4.4 above provides an overall quantitative analysis of the data. As I have indicated earlier, quantification of data helped me to see which elements of mathematical work of teaching were most prevalent in Lineo's practice as well as those that were absent. I noticed during the quantification of data that the elements of mathematical work of teaching that Lineo wrestled with occurred in varying amounts in this practice [see Table 4.4]. The reason for this could be that in the first lesson Lineo was introducing the concepts and the terminology which demanded more mathematical work of teaching than in the fourth lesson where she was consolidating concepts learnt in the previous lessons.

Table 4.4 shows that throughout the observation week Lineo afforded her learners an opportunity to gain access to the language of algebra and the techniques of algebra by engaging them in reading/writing and transformational activities of algebra respectively. However, these activities on their own may not enable learners to develop meaning of 'underlying structures' as argued by Kilpartrick *et al*. (2001) who associates meaning with generalizing and justifying activities which are absent in this practice [see Table 4.4]. The absence of generalizing and justifying activities denies Lineo the opportunity to confront generalizing problems which in turn does not allow her to wrestle with restructuring of tasks for a full week as learners are not exposed to algebra as generalizing (Usiskin, 1998). In order to manage these reading/writing and transformational activities which demand learners to know algebraic notation and conventions thereof for successful engagement, Lineo drew mainly on particular aspects mathematics to unpack algebraic notation and conventions hence the mathematical work of teaching that Lineo mostly grappled with is questioning, explaining and working with learners' ideas [see Table 4.4, p.74]. I note that what Lineo calls on at the moment seems to be adequate for her current practice. However, if she is to incorporate generalizing and justifying activities into her practice she might have to do more.

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It is essential to note that the six elements of mathematical work of teaching do not all occur in Lineo's practice. Table 4.4 [see p.74] shows that in Lineo's practice, restructuring of tasks is entirely absent. This is not surprising because Lineo used closed tasks focusing mainly on developing fluency in algebraic language. Her activities were highly structured and organized so as to lead learners into particular ways of writing and also to distinguish like and unlike terms. As discussed in section 2.2, in a similar study conducted by Adler & Pillay (in press) it was found that restructuring and working with learners ideas were totally absent. This led them to conclude that mathematical work of teaching is not only a function of a topic but also a function of the teacher's pedagogical approach. Kazima & Adler (2006) also working within Mathematics for Teaching (MfT) in a study of Probability reported that there was restructuring of tasks but other aspects or elements of mathematical work of teaching were not present. They point out that probability was new in the SA curriculum hence it posed problems for both the teacher and the learners hence restructuring was inevitable. They further argue that the teacher's approach was activity based thus affording learners the opportunity to wrestle with a variety of tasks.

It is worth mentioning at this point that restructuring tasks according to Kazima & Adler (op cit) entails "scaling down of the task if it is too difficult or scaling it up if is not challenging enough for the learners" p.51. In Ball *et al.'s* (2004, p.59) language, restructuring is about "making judgments about the mathematical quality of instructional materials and modify as necessary". These definitions of restructuring of tasks indicate that restructuring is dependent on the cognitive demands of the tasks/activities as set up by the teacher. It is therefore not surprising that it does not happen in Lineo's class as she seems to have set up activities with graded varying cognitive demands.

In the sections that follow, I set out to find answers to the following research questions which guide this study:

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Research questions

1. What is the nature of the activities that the Grade 8 teacher uses to introduce algebra?
2. What mathematical problems does the teacher confront as she introduces algebra in Grade 8?
3. What mathematical work of teaching does the teacher do as she goes about solving these mathematical problems in her teaching practice, specifically in order to introduce algebra to Grade 8 learners?
4. What knowledge resources does she call on for this work?

Together, these questions will tell us and reveal mathematics for teaching in Lineo's practice.

4.7 Identifying mathematical problems, the mathematical work of teaching and algebraic activities

Table 4.4 shows that in the four lessons observed Lineo grappled with **explaining**, **defining**, **questioning**, **representing** and **working with learners ideas**. It should be noted that these elements of mathematical work of teaching do not occur equally across the lessons. The most prevalent elements of mathematical work of teaching in Lineo's practice were **explaining**, **questioning** and **working with learners' ideas** (Table 4.4). While **defining** and **representing** are completely absent in lessons three and four (this cannot be seen in Table 4.4 as it give the overall quantification), I see them as critical in Lineo's practice hence they will be discussed. As I pointed out earlier **restructuring of tasks** (see Table 4.4) does not occur in Lineo's practice. The variation in the presence of these elements of mathematical work of teaching could be linked with the nature of activities presented to learners. The activities presented were mainly reading/writing and transformational activities and were closed. Successful

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completion of such activities demands learners to understand the algebraic notation (symbols and operations) and conventions thereof. This demanded that Lineo wrestled with questioning, working with learners' ideas and explaining most of the time so as to afford learners access to algebraic concepts, procedures and language by unpacking algebraic conventions.

I have identified extracts[11] from the four lessons to exemplify the work of teaching that Lineo wrestled with as she introduced algebra to Grade 8 learners. The excerpts also serve to illustrate the mathematical problems that confronted Lineo as she went about her work and the nature of algebraic activities that she used to introduce algebra to the boys. Over-and-above the quotations from different lessons demonstrate that Lineo does not confront the same mathematical problems and that she does not deal with the entire work of teaching in one episode or lesson. I have also noted that within a lesson there are instances where they occur in large numbers and also where they do not occur (see Appendix 4).

4.7.1 Explaining

In the late 1980's Shulman argued that teachers needed to know more of their subject since they were to transform such knowledge into forms that could be comprehended by learners. In line with this, Ball *et al.* (2004) argue that teaching entails providing explanations as to why the mathematical idea is important and valid. They point out that "knowing mathematics for teaching demands… explaining the basis for an algorithm and showing why it works" p.52. So explaining has mathematical entailments. Table 4.4 shows that in Lineo's practice explaining is the second frequently used with a count of 44 in four lessons. One may ask; what does explaining in Lineo's practice entail? I use the following extract to illuminate this:

[11] The extracts that are being used here are the same as those that I have used before or those that are used later. The reason for this is that some of the chosen extracts could shows more clearly more than one of the constructs that I am using to analyze this teacher's practice hence they are repeated but to illustrate a different construct or dimension of the study. Most importantly they are repeated for ease of reading as one could have referred to the page containing the extract.

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> Lineo: Ok, **traditionally we always put up the numbers before the letter**. So $n2$ is correct but we prefer to write the number before the letters. Also **we don't write the times between a number and the letter**. When I write $2n$ what are we assuming? 2 times n .The same thing here with the x why did I write the curly x ?
> Mpho: we do not want a straight x
> Lineo: Don't call it out.
> Thabo: inaudible
> Lineo: Ok, as Thabo said, if **I were to write that as a straightforward x what does that look like? times times 2 or xx2 doesn't make sense**. So guys I'm going to be very strict with you, when you write your x 's, it is the only letter that you have to change. I want you to write it as a curly x neatly. You neat to get into that habit. I will take off marks if you don't (Lesson 1, emphasis in bold, 08:04 – 09:00)

The excerpt above shows that in responding to learners' conventional way of writing Lineo grappled with **explaining.** Her explanation is based on stating the conventions. Lineo focused not only on the correctness of the response but also on how the answer is to be presented. So she is enculturating her learners into the mathematicians way of writing. It is essential to note that while Lineo's explanations focus on the conventions in mathematics, they are likely not to assist learners to realize that they [learners] are moving from the arithmetic representational system [which has place value] into an algebraic system [which does not have place value]. This is because she does not explicitly demarcate the two representational systems.

The extract further demonstrates that Lineo was faced with two mathematical problems viz. **language of algebra** (reading algebraic notation, writing algebraically) and **manipulating algebraic symbols** (symbol conflict). To resolve these mathematical problems Lineo engaged in explanations that focused more on **conventional fluency**. One notes that Lineo's explanations were focused on how to read and/ or write hence they were aimed at promoting **reading/writing activities of algebra**. The extract which follows demonstrates another dimension of Lineo's explanations.

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> Malawi: you always do the numbers then the variables
>
> Lineo: correct, the way I traditionally do the multiplying, Kori, Oscar you need to have a look at the board the process I usually do, **I first look at the signs, is my answer going to be negative or positive? Then I look at my numbers, and then I look at my variables (writes signs→numbers→variables**). **First thing, is my answer going to be positive or negative? Then I look at the big numbers and then I look at each of the letters separately**. (Lesson 2, emphasis in bold, 28:09 – 28: 54)

While Table 4.4 [p.74] shows that Lineo employed a lot of explanations during her teaching, the above excerpt sheds some light into the nature of these explanations. In responding to a procedural question from Malawi, Lineo explains a procedure that she normally follows when simplifying exponential terms. The use of arrows shows that Lineo wants her learners to execute the steps effectively and efficiently therefore this explanation is procedural in nature. From the excerpt above, it becomes apparent that Lineo explained in order to promote **transformational activities of algebra** (Kilpatrick *et al.*, 2001) [bold text].

Lineo also explained when learners seemed not to have grasped the concepts. In this case Lineo's explanation came as repetition which in my view does not mean that learners would understand the concept better. However, repetition afforded Lineo the opportunity to consolidate what was said, and the learners a second chance to hear and possibly reflect on the concept. Repetition by the teacher works more like chorusing which turns to favour instrumental understanding as opposed to relational understanding (Skemp, 1976). Instrumental understanding is only knowing how while relational understanding is about knowing how, when and why. Lineo's explanations are tied to her questions. I now look at questioning in Lineo's practice.

4.7.2 Questioning

Pillay (2006) points out that in his study of Nash's practice both questioning and defining were less dominant. This is interesting in that in Lineo's practice questioning

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was the most frequent element of mathematical work of teaching that was dealt with in the four lessons. Could this be the function of Lineo's pedagogical approach as Adler & Pillay (in press) argue? The data shows that Lineo employed questions when introducing a new concept, alerting learners of something as well as when probing their mathematical understanding. The extract that follows sheds more light on this:

> Ok, now my question is they are all letters, **what made you distinguish the *a*'s apart from the *x*'s and apart the *y*'s, Hil?**
> Hil: They are different letters
> Lineo: ok they are different letters. Mahomete, what would be a cleverer answer than that?
> Mahomete: they look different
> Lineo: they look different, Kele?
> Kele: different shapes
> Lineo: different shape of letters, **now in terms of algebra**, I ask you the question again, **why did you classify them differently? Why did you put the *a*'s in one group and the *x*'s and the *y*'s in another**, Lethabo?
> Lethabo: they have different values
>
> Lineo: correct, the *y*'s and the *a*'s will represent different numbers. When I am using letters, two letters at the same time; I am assuming that those values will be different. *a* could be 10, *x* could be 5 and *y* could be – 100, ok different values, any question so far. (lesson 1, emphasis in bold, 15:11 – 16:27)

The above extract indicates that Lineo wanted to see whether the learners understood the mathematical reason behind putting the a's, x's and the y's in three different groups. While learners were able to give correct answers, these in Lineo's view were void of mathematical understanding hence she did not take them as answers. This shows that Lineo foregrounded mathematical meanings/understanding rather than everyday knowledge in her teaching. We see that in this interval, Lineo attempts to attach a mathematical meaning to the symbols (letters) used in mathematics, thus the activities in this interval are **reading/writing activities**. The excerpt also shows that the mathematical problem confronted by Lineo is ***language of algebra*** particularly *variables as generalized numbers.* In order to combat this problem Lineo engaged in an explanation which drew on empirical mathematical knowledge to assist learners in

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conceptualizing letters as representing numbers. The extract below illustrates how Lineo used questioning to develop common understanding of mathematical symbols/ notation in her class.

> **What does that dot mean?** (points at the dot between 2 and 4)
> Learners: times
> Lineo: times, you have seen that before?
> Learners: some say yes, others say no
> Lineo: yes, no. those people who possibly have not seen it before, ***again to*** prevent confusion instead of writing the times because that might be confused with the x we just write a dot (writes $2 \bullet 4$) as soon as you see a dot, it will mean a times. So 2 times 4, 4 times 2, the same difference (Lineo, lesson 1, 23:20 – 23:49)

Here Lineo posed a question to draw learners' attention to the dot between 2 and 4 [see bold text above]. She immediately proceeds to explain that it means multiplication. In so doing, she embarked on developing a common understanding of the mathematical notation. She pointed out that the reason for using a dot is to prevent confusion of x and the multiplication sign [underlined text]. This turns to undermine the fact that the mathematical problem that Lineo confronts at this point is ***manipulating algebraic symbols*** in particular *symbol conflict*. Symbol conflict goes beyond mathematics since in everyday speech the same dot is interpreted as a full stop which marks the end of a statement. as a Grade 8 teacher, I have found the use of a dot in mathematics to be very problematic for novices as it could be interpreted in two different ways, as an operation (multiplication) in both algebra and arithmetic and also as an indicator of place value in arithmetic. What I can infer from Lineo's explanation is that learners bring different meanings of notations (symbols and signs) into mathematics which may differ from those found in mathematics hence the teacher has to address them.

These excerpts show that Lineo engaged in **reading/writing activities** in order to attach a mathematical meaning to a dot as used in mathematics and to the letters (variables) used in algebra. Again one can see that Lineo's questions are focused on

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transformational and reading activities because they provide cues as to what learners should do when they see dot [i.e. learners should be able to read it [dot] as a multiplication sign and be able to execute the procedure i.e. calculate the product]. Boaler (1997) refers to this cue-based teaching as 'pure' transmission. Through her comparative study at Amber Hill and Phoenix Park, Boaler (op cit) argues that 'pure' transmission method employed by teachers at Amber Hill resulted in *inert knowledge* [i.e. knowledge that did not transfer to contexts beyond where it was learned], in contrast, the problem solving approach used at Phoenix park resulted in *flexible knowledge* [i.e. transferable knowledge]. I do not intend to dichotomize Lineo's practice but to indicate that these dichotomies while they describe the end product of learning (knowledge gained) they do not tell us anything about the work of teaching that occurred in the two schools, the knowledge resources that the teachers drew on or the mathematical problems which the teachers confronted. In addition, the dichotomies do not say anything about developing fluency in mathematical language which seems to be critical in Lineo's teaching of algebra.

The previous extract further shows that Lineo posed questions in order to automate the execution of certain procedures in the presence of the dot. in doing this, she endeavoured to provide the learners with the recognition rules (Bernstein, 1996) so that they could produce the expected text in the presence of a dot. Consequently, Lineo assisted her learners in mastering both transformational and reading activities.

Lineo did not only use questioning for teaching mathematical notation. It was also used to teach properties of basic mathematical operations as illustrated in the following excerpt

> Lerato: $6xy$, (others laugh)
> Lineo: **You said there were six pairs of xy or yx. Let's back track, when I say** xy**, let's back track even further**, **when I say two** n **in guess my rule, what were you meaning, Henry?**
> Henry: 2 multiplied by n
> Lineo: 2 multiplied by n so when two things are next to each other with no sign, no plusing, minusing, multiplying, dividing, we are

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assuming timesing, 2 times n. **Now if I have** xy **what am I meaning?** x **times** y **,** yx **what am I meaning?**
Learners: y times x (chorusing)
Lineo: so now is xy and yx the same thing as separating the y's from the x's? … I separated them out. No because it's going to be the same answer. If I say 10 times 2 the answer is 20, it has got nothing to do with adding the 2 and the 10, I can' break them up. Ok? But now let's put in some numbers, umm Mthetwa give me a number.
Mthetwa: 16
Lineo: ok let's make your x 16. Mathe give a smaller number
Mathe: 3
Lineo: *let's make* y *3, so when I say* x *times* y *, what is the answer going to be, when* x *is 16 and y is* 3?
Learners: 48
Lineo: 48, so this is 48, so all the xy's are 48. *Now let's see if* y *is 3 because that's what we said y was and* x *is 16, what is 3 times 16?*
Learners 48
Lineo: *48, so the question is Wily, is* xy *different from* yx *or the same?*
Wily: the same

Lineo: the same, coming with the same answer, whether I say 2 times 3 or 3 times 2 it gives the same answer so algebraically that (ticks $6xy$) is the correct answer. (Line, Lesson 1, 19:05 – 23:10)

Here we see that Lineo employed questioning to check whether her learners could read the symbolic algebraic writing [bold text]. She then moved on to teach them that multiplication is commutative. This property is very important when simplifying expressions as it helps one to see things that appear to be different as being the same. The extract also shows that to drive the commutative property for multiplication home, Lineo relied on moving between arithmetic which learners are familiar with and algebra. While this could have been helpful to those learners who have come to terms with the use of variables it could have confused or created a barrier to others since $x \times y = xy$ but $2 \times 10 \neq 210$. However, this sheds light into the depth of Lineo's pedagogical content knowledge in particular her knowledge of her learners. It also suggests that Lineo sees algebra as generalized arithmetic. As I mentioned earlier Wheeler (1996) and Wu (2001) support this conception of algebra. The above quotation further shows that the mathematical problem that continued to dominate

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Lineo's practice was ***language of algebra*** (*reading algebraic notation and writing algebraically).* Questioning in Lineo's practice was also used in developing mathematics vocabulary and conceptual understanding as Lineo defined terms (product, constant *etc.*) that are often used in a specialized way in mathematics and mathematical concepts such as variable. One can see that questioning was used in order to engage learners in both **reading/wring activities** [see bold text] and **transformational activities** [italicized text]. In this practice questioning is tied to **defining,** I now focus my discussion on defining.

4.7.3 Defining

Skemp (1986) argues that definitions add precision to the boundaries of a concept once formed and state explicitly its relation to other concepts. Vinner & Dreyfus (1989) argue that most of these definitions are introduced to learners at high school or college though learners do not necessarily use them when responding to questions. Thompson (1994) defines concept definition as "a customary or conventional linguistic formulation that demarcates the boundary of a word's or phrase's application". Ball *et al.* (2004) argue that defining mathematical concepts is part and parcel of the teacher's daily work. They point out that teachers define concepts relative to the grade level of their learners. They stress that such definitions must be based on concepts that have been defined and already understood. In support, Skemp (op cit) would argue that "concepts of higher order than those which people already have cannot be communicated to them by definitions, but only by arranging for them to encounter a suitable collection of examples" and that "since in mathematics these examples are almost invariably other concepts, it must be ensured that these are already formed in the mind of the learner" p. 30. Skemp (1986) further argues that 'good' teachers intuitively back up definitions with suitable examples. Ball *et al.* (2004, p.58) argue that

> knowing mathematics definitions for teaching therefore requires more than learning mathematically acceptable definitions in course. Also needed is being able to understand and work with definitions in

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> classrooms, with pupils… a definition of a mathematical object is useless no matter how mathematically refined or elegant, if it includes terms that are beyond the prospective users' knowledge.

So there is consensus amongst the researchers in this field that definitions are critical to learning and teaching of mathematics. Researchers such as Ball *et al*. (2004) and Skemp, (1986) seem to concur that definitions should take into account what the learner already knows with the hope of developing such knowledge further. Consequently, defining has become a common practice in mathematics classrooms.

Table 4.3 shows that Lineo employed defining in her teaching. It is also noted (Appendix 4) that whenever Lineo defined she drew on **mathematical knowledge**. This exemplifies my earlier suggestion that Lineo seems to foreground mathematics in her teaching. The extract below explicates this point further:

> Lineo: Ok, first things first, let's make a note of what variables are (writes on the board variable). Ok, it's the first definition, what do I mean by a variable?
>
> Lineo: something that can change. *Tell me a bit more. Thabo, if you can … now that is English, it's a variable because there is a variation, not mathematically***,** I gonna still pick on *you Thabo mathematically it says you can … give me some more*
> Thabo: inaudible
> Lineo: Ok, Henry
> Henry: inaudible
>
> Lineo: Ok all of you are right, let's talk mathematically, **in algebra when I am talking about a variable I am talking about a letter of the alphabet that can assume any value (writes a, b, c, d, …, x, y, z) so any letter of the alphabet** and I make you write down your curly x (lesson 1, emphasis in bold, 30:50 – 31:26)

Here Lineo engaged in defining a variable [see the bold & underlined text]. She openly rejected learners' definitions on the grounds that they were not mathematical [see italicized text]. She pressed learners to respond mathematically but they failed, which eventually led Lineo to define a variable [see bold and underlined text]. Lineo's definition of a variable is accompanied by examples of the letters that could

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be variables. By stating these examples Lineo was assisting her learners to conceptualize a variable. The extract also demonstrates that Lineo listened evaluatively (Davis, 1997) to her learners' responses as she immediately dismissed most of them. The extract further shows that Lineo grappled with **questioning, defining** and **explaining.** The mathematical problem confronting her at this point is ***language of algebra*** (*defining).* Once again, we see Lineo drawing on empirical knowledge in order to fix meaning of variable. The excerpt also indicates that Lineo sees herself as the custodian of mathematics consequently she uses her authority to instil a particular way of writing x.

The extract further shows that Lineo regards the mastery of conventions and terminology in mathematics as critical as concepts are to learning mathematics. In a follow up interview I asked Lineo about the importance of defining terms and concepts. She said,

> … they need to know that umm **our terminology in mathematics is important**, umm when I am talking about umm a constant they need to know it is a number, it has got a role and they need to know umm that **mathematics language is important and that it will be to their own detriment in future if they do not learn to mathematics properly**. Umm I will always emphasize the definitions and terminology and when they give me back an answer if they are not saying it correctly I will correct them… (Lineo, follow up interview, L189 - L197)

Lineo stresses the importance of mathematical language (refer to bold lines). It is essential for future learning and communication in mathematics. Setati (2005) argues that "language is not only a tool for communication and thinking but also a political tool that can be used to gain access to educational and material resources"(p.464). It is therefore critical that learners are assisted in developing fluency in mathematical language so that they can have access to the educational and political opportunities in the global market as well as being contributing citizens. So Lineo uses **defining** to develop understanding of algebraic concepts and conventions and also for the development of mathematics register, in particular vocabulary. We see that Lineo

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considers communicative fluency to be a critical component of learning mathematics. This could be the reason why she engaged learners in reading/writing activities. The extracts above show that MfT in this case is characterized by terminology.

Lineo relied on examples when **defining** (Ball *et al*. 2004; Skemp, 1986). This could have helped learners to visualize and form mental image of the concepts thus developing a better understanding of the concepts than could be achieved with a definition alone. The use of these mathematical examples indicates that Lineo drew on empirical mathematical knowledge to legitimate meaning for the boys and that for Lineo meaning of algebraic notation [symbols and operations] lies in examples.

For Lineo mastering mathematical terminology is central to success in learning mathematics and this is what she emphasizes. The quote below sheds more light on this:

> Lineo: Ok term, what do I mean by a term? Anyone came across that word before?
> Lineo: Ok, let me give it to you before you come with all your English definitions. An example will be better; (writes on the board) they are separated by pluses and minuses (Lineo, lesson 1, 37:05 – 37:27)

Here, Lineo makes it explicit that learners' everyday language is of no use in mathematics (see underlined text). This is contradictory to well documented view that learners main language can be exploited in supporting learning. One can see that Lineo confronts a dilemma here. Allowing learners to participate offers her the opportunity to work with learners' ideas; however, it seems not to enhance acquisition of formal mathematical language or terminology which Lineo seems to be foregrounding as learners are not able to cross the boundary between everyday meaning of words and the mathematical meaning. Pimm (1991, p.21) alluded to this challenge by saying "one difficulty facing all *mathematics* teachers [italics added], however, is how to encourage movement in their learners from the predominantly informal spoken language with which they are pretty fluent, to the formal language

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that is frequently perceived to be the landmark of mathematical activity". In addition, Lineo seems to regard learners' Language of Learning and Teaching (LoLT) (Setati *et al*., 2002) as an obstacle rather than a resource that could be harnessed to promote learning. It is essential to note that mathematics does not have its own words or symbols but it relies on those found and used in natural languages. This is problematic in that it blurs the boundary between mathematical meaning and the everyday meaning of words, symbols and operations. In support, Wheeler (1996) argues that using the same words gives learners the impression that meaning has not changed. Hence teachers are to assist learners in distinguishing between everyday meanings of words and mathematical meanings otherwise some learners may not be able to recognize the speciality of the context in which the word is used, consequently may fail to produce the expected text (Bernstein, 1996). So through defining, Lineo endeavoured to provide her learners with both the recognition rules (Bernstein, 1996) so that they could spontaneously produce the legitimate text. In the process of **defining,** Lineo finds herself **working with learners' ideas** or blocking such ideas as the above excerpt reveals. The mathematical problem that confronted Lineo in this case is ***language of algebra*** *(e.g. defining).* The next section illustrates how Lineo worked with learners' ideas in her practice.

4.7.4 Working with learners ideas

A quick glance at Table 4.4 shows that working with learners' ideas was the third most common element of mathematical work of teaching which Lineo wrestled with. Working with learners' ideas in this study entailed: "analyzing errors and misconceptions, evaluating explanations, interpreting and responding to learners as well as appraising unexpected claims, solutions and methods" Ball (unpublished work). The excerpts that follow illustrate this element of mathematical work of teaching in Lineo's practice.

> Malawi: you always do the numbers then the variables
>
> Lineo: correct, the way I traditionally do the multiplying, Kori, Oscar you need to have a look at the board. The process I usually do, I **first**

> **look at the signs, is my answer going to be negative or positive? Then I look at my numbers, and then I look at my variables (writes signs→numbers→variables). First thing, is my answer going to be positive or negative? Then I look at the big numbers and then I look at each of the letters separately**. (Lineo, lesson 3, emphasis in bold, 30:37 – 30:50)

Malawi asked a procedural kind of question regarding the coefficients and variables when simplifying exponential terms. Lineo responds procedurally by saying that she usually deals with the signs followed by the coefficients and then the variables. She placed more emphasis on the procedure by indicating with arrows the sequence that has to be followed. The fact that Lineo talks about doing one thing at a time indicates that she attempted to develop strategic competence as she advocates that learners should have a plan whenever they tackle a mathematical task. The above extract also highlights teaching strategies that Lineo employed when introducing algebra to Grade 8 learners. She used repetition to foster understanding of the procedure (process) that she frequently uses when simplifying exponential terms. In a similar study of a Grade 10 teacher's practice conducted by Pillay (2006), it was found that Nash used recapping process through which he provided explanations to consolidate what was done and focus the learners' thinking. Working with learners' ideas afforded Lineo the opportunity to identify and tackle errors and misconceptions that learners brought into the lesson. The excerpt that follows exemplifies this point.

> Mahomete: mam, if it's like 3x plus 5x plus 2b would the answer be 10x (inaudible)
>
> Lineo: let's do that example, you said $3x + 5x + 2b =$, do that by yourselves, if you are not certain you can do that in pencil and come up with your answer. (Silence) August, what do you think your answer could be? It does not matter if it's wrong.
> August: $10xb$
> Lineo: **10xb tell me how you got the answer**.
> August: umm Mam, x and b can be the same value (interrupted).
> Lineo: can they?
> August: no
> Lineo: Ok, you are happy they are going to be different values. Let's have someone different, Mohapi?

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Mohapi: $8x + 2b$
Lineo: $8x + 2b$ (writes it on the board) **convince me you are right**.
Mohapi: we have 3x plus 5x, and 3x plus 5x is 8x them another term which is 2b

Lineo: Ok, Mohapi, this is right, the x on the board, remember when we had the x and the x^2 or the x and the y, the x you put in one group and the y in one group, the same thing here, the xs belong with each other and how many are they, there are only $8x$ and that b is a completely different story, he belongs by himself. $8x + 2b$, how many got that one correct? (Lineo, lesson 2, 13:38 – 15:27)

The above extract typifies the kind of mathematical problems that confronted Lineo, what I have described as ***manipulating algebraic symbols***. She confronted a well known misconception called *conjoining* (MacGregor & Stacey, 1997). I indicated in section 1.3.6, that this misconception could be the result of overgeneralization of arithmetic operations where novices turn to interpret addition to mean combine symbols and replace with another. I also indicated that it could be originating from balancing equations in Chemistry. In support, Nesher (1987) argues that misconception often derive from previous learning or experience which is overgeneralized. MacGregor & Stacey (1997) argue that some errors crop up as learners attempt to apply "algebraic manipulations to 'solve/*formulate* equations/ *algebraic expressions'* [italics added]. For cognitive theorists (Nesher, 1987; Hatano, 1996) misconceptions indicate that novices are constructing knowledge as these are not explicitly taught. On the other hand situative theorists see misconceptions as inappropriate or partial use of the tools in a practice. I note that the activity in this event is **transformational** in nature.

In the previous excerpt, Lineo seems not to agree with August's response but does not give the reason for her objection. Earlier on, Lineo had said "correct, the y's and the a's will represent different numbers. When I am using letters, two letters at the same time; I am assuming that those values will be different. a could be 10, x could

be 5 and *y* could be – 100, ok different values" (lesson 1, 15:11-16:16). this quote sheds some light into Lineo's objection. for her the use of different letters implies that the values of the variables are different hence August is incorrect. Lineo's explanation and objection to August's response is not surprising as Brodie (2005, p.35) points out that "many teachers, when they first introduce variables, talk about different letters representing different numbers and same letters representing the same number." Whether Lineo dealt with this misconception appropriately or not is beyond this study. My study is not aimed at making evaluative judgements but rather analytic judgements of Lineo's practice so that I could be able to describe the work of teaching which she embarks on as she goes about her work. However, the extract shows that Lineo's explanations were not always mathematically rigorous. The excerpt further shows that MfT is transformational in nature [see underlined text].

The above activity in the excerpt above [see page 89] although set up to engage learners in transformational activities, it was implemented in a way that enabled Lineo to assist her learners in developing reasoning skills as she asked the learners to convince her that they are right [see bold text]. Setting up activities to introduce algebra implies that one has to assist learners to develop mental images of what algebra is made up of. For Lineo this demanded engaging in various algebraic activities (Kilpatrick *et al.*, 2001). By embarking in these activities, Lineo endeavoured to assist her learner in developing both conceptual understanding, conventional and communicative fluency. In the section that follows, I discuss the use of representations in Lineo's practice.

4.7.5 Representing

A quick glance at Table 4.3 shows that representing, just like defining is least encountered by Lineo. Lineo mainly wrestled with representing only in lesson 1[see Appendix 4]. Given that lesson one was taken to be the first lesson in which algebra is formally introduced to the boys, one can understand that Lineo intended to help her learners to conceptualize algebra hence the use of different representations. In this

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lesson Lineo used pictorial, symbolic and algebraic representations. These representations enabled Lineo to gradually lead her learners from identifying 'same terms' [in soccer kit activity] into collecting like terms and writing down algebraic expressions [as in random distribution of terms, see page 93]. The South African Curriculum 2005 (C2005) encourages teachers to integrate learners' everyday into mathematics. Aligned with C2005, Lineo introduced algebra through the following activity



I want you to count how many of the same thing they are. How many of the same things are there? Not similar, same.
:
:
Khau: **I also added the badges (unclear) the badges on the shorts, socks (others make some noise)**
Lineo: Ok, no no (unclear) thinking out of the box, I agree with you that is another way of looking at it and then how many badges did you get?

Lineo: **you could nothing is stopping**; I gave you an open ended question, a pretty arbitrary kind of pre-school exercise. What's the whole point of it? Yeah, all I am reminding you is that Italy won. (Lineo, lesson 1, 09:52 – 10:05)

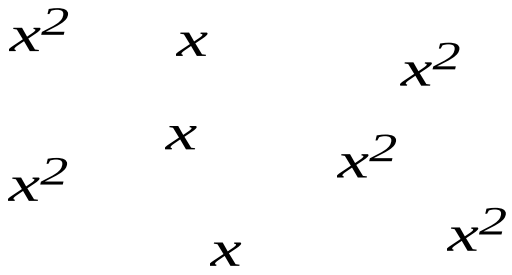
As the question indicates (see italicized text above) this activity was not challenging enough for the 8th graders because for most learners it could be regarded as packing clothes which has nothing to do with mathematics. Learners were expected to use

their everyday experiences in grouping same objects. However, this could have misled some learners as there is no specific property that could be used. For instance, Khau separated the badges from clothes. Algebraically, what Khau did is not allowed as it changes the form of the given term but the openness of the activity forced Lineo to ignore the mathematical sense of the response. Lineo's subordination of mathematics to the everyday is not unusual. Adler, Pournara & Graven (2000) [cited in Ensor and Galant (2005)] who studied Mrs. Shongwe's teaching practice argue that mathematics was backgrounded while the everyday took over. Sethole (2004) who studied two teachers in South Africa also made similar observations and argue that whenever mathematics is integrated in everyday contexts one discourse will dominate the other. The question is; what mathematical knowledge for teaching does Lineo need to know and know how to use in order to introduce algebra meaningfully while integrating with the everyday? Ensor & Galant (op cit, 291) point out that "only when learners have gained mastery over key aspects of mathematics can they reach beyond mathematics to mathematize other areas'. Although contexts such as everyday help to make mathematics less abstract, they may be very problematic for those learners who do not have the mathematical eyes as yet.

Lineo indicated during the follow up interview that the purpose of the soccer kit activity was to refocus the learners' thinking, to bridge the gap between arithmetic and algebra as well as to make algebra less abstract. It is interesting to realize that while the activity was aimed at helping learners to conceptualize algebra Lineo focused on the answer hence the activity ended up being procedural. I have mentioned earlier that the soccer kit activity brought up tension in Lineo's class in terms of how to identify like terms. The excerpt that follows illuminates this claim further:

Lineo: Let's see the next thing, projects

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Lineo: Ok, Rob?
Rob: five x squared plus three x
Lineo: so you have five x squared and three x (writes $5x^2 + 3x$) in other words what I have written there, $5x$ squared, you are talking 5 lots of x squared, agreed? And continue Robert why did you classify the x squared and x differently?
:
:
Tumi: (asks but inaudible)

Lineo: ok I do get what you are saying; now I want you to start thinking algebraically. If we gave this exercise to Grade 1's or Grade 2's they would do exactly, what you are saying (all laugh) I agree with you, sorry I didn't mean it. Tumi you understand what I am saying (others laugh) Ok. No, no, what I am saying is, is not to put you down Tumi, I promise. What I am saying is I want you to start thinking algebraically. He is 100% correct I would give him all the marks at this stage because all I have said to you is classify them and he is saying let's put those 2's in their own category. He is right. Why can't they be? But now we are starting to think algebraically, it's going to be an issue. (Lineo, lesson 1, 16:28 – 18:51)

Symbolic representations seem to have helped Lineo to automate symbol manipulation as learners were quickly verbalizing mathematical expressions. In addition, symbolic representations afforded Lineo the opportunity to transform verbal utterances into formal symbolic mathematical expressions which she wrote on the board. In this way she taught learners how to write using algebraic notation. The excerpt reveals that mathematical problem which confronted at this point Lineo was language of algebra, specifically interpreting learners' ideas. As Lineo confronted this mathematical problem she embarked on mathematical work of teaching. Engaging with the mathematical work of teaching in turn demanded that Lineo made appeals in order to fix meaning of algebra for the boys. In this extract Lineo drew on **empirical**

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mathematical knowledge to explain why x and x^2 would be placed in different groups. She pointed out that if x was 3 the value of x^2 would be 9 and that 9 and 3 would belong to two different groups. In addition, when responding to Tumi's question she explicitly indicated that she preferred algebraic thinking so she expected learners to draw on mathematics whenever they argued (see underlined text). Tumi just like Khau separated the exponents from their bases to form a group of twos alone. However, this time Lineo realizes the problem in Tumi's response and attempts to correct it by pressing Tumi to think algebraically. What I find interesting here is that Lineo did not ask Tumi what x^2 means as she did on previous occasions which I think could have helped Tumi to see the terms were unlike terms. The extract further shows that Lineo represented learners' verbal utterances algebraically. This seems to underscore the fact that the mathematical problem that confronted Lineo at this point was *representing.*

So far I have engaged in answering my first, second and third research questions. I have shown that Lineo employed representational, transformational and reading/writing activities when introducing algebra to Grade 8 learners. I identified manipulating algebraic symbols [*symbol conflict*, *conjoining*, *representing*] and algebraic language [*reading algebraic notation* (signs and symbols), *writing algebraically*, *defining*, and *interpreting learners' ideas*] as the mathematical problems that arise in Lineo's practice. These mathematical problems compelled Lineo to wrestle with **explaining**, **defining**, **representing**, **questioning** and to **work with learners ideas**. If these are the activities that Lineo uses, the mathematical problems that occur in Lineo's practice and the work of teaching that she grapples with, then what knowledge resources does she draw on when helping her learners to make sense of algebra? In the sections that follow, I discuss the knowledge resources that Lineo drew on as she legitimated meaning in her class with the hope that these would shed more light on Lineo's practice.

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4.8 Identifying knowledge resources in use in Lineo's practice

In chapter two, I discussed Lee Shulman's (1986/7) work and the term Pedagogical Content Knowledge (PCK) in order to describe the knowledge that teachers need to know and use. When teaching, mathematics teachers often rely on their mathematical knowledge in fixing meaning for their learners. Through their study of Nash's practice, Adler & Pillay (in press) found that mathematical knowledge had three sub-categories namely (i) rules and conventions, (ii) definitions, and (iii) empirical - *use of numbers and examples* [italics added]. They further indicate that teachers could also use curriculum knowledge in its various forms as well as their experience which could either be everyday knowledge, professional knowledge or authority in legitimating meaning. Now, the questions that I find interesting for this study are; does this teacher call on these knowledge resources or others as she goes about her work?; What does she use these knowledge resources for in her practice?

In the sections that follow, I discuss the knowledge resources which Lineo called on as she went about her work.

Table 4.4 shows that Lineo drew on **mathematical knowledge**, **curriculum knowledge** and **experience**. It is interesting to see that these knowledge resources were called on in varying amounts. Mathematical knowledge was the mostly used resource. Within mathematical knowledge, I noted that rules and conventions was mostly used followed by definitions. Empirical was the least called on. Curriculum knowledge was the least used resource. It came into play through assessment (test and exams) as well as in teaching materials (worksheets and textbooks). In Lineo's practice, experience consists of everyday knowledge, professional knowledge and authority. In the sections that follow, I discuss each of these knowledge resources found in Lineo's practice.

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4.8.1 Mathematics

I indicated earlier that whenever Lineo defined, she seemed to use definitions that are mathematical and she rejected responses that lacked mathematics. When dealing with the notion of a variable, Lineo pressed Learners to give a mathematical definition. To exemplify this point, I draw your attention to the underlined text in the extract below:

> Lineo: something that can change. Tell me a bit more. Thabo, if you can … now that is English, it's a variable because there is a variation, not mathematically, I gonna still pick on you Thabo mathematically it says you can … give me some more
> Thabo: inaudible
> Lineo: Ok, Henry
> Henry: inaudible
> Lineo: Ok all of you are right, let's talk mathematically, in algebra when I am talking about a variable I am talking about **a letter of the alphabet that can assume any value** (writes a, b, c, d, …, x, y, z) so any letter of the alphabet and I make you write down your curly x (lesson 1, emphasis in bold, 28:07 – 33:16)

Realising that learners are unable to provide a mathematical definition, Lineo drew on her **mathematical knowledge** (in particular **definitions**) to provide a mathematical definition of a variable (see bold text). In addition, Lineo wrote examples of the letters that could be used as variables. By stating the examples of a variable, I see Lineo as defining in ways that are appropriate to the learners (Grade 8) as argued by Ball *et al*. (2004). In this way she represented what a variable is or looks like. However, this was over-emphasized at the expense of what a variable stand for. Here we see that Lineo drew on **empirical mathematical knowledge** to legitimate what a variable is or looks like as opposed to what it stands for. In this way she was helping learners to conceptualize a variable hence defining coupled with examples was to enhance conceptual understanding. In support, Skemp (1986) argues that concept formation requires a number of experiences with a common property. He further points out that "naming can, also play a useful, sometimes essential, part in the formation of concepts" p. 23. So by writing letters of the alphabet on the board and naming them variables, Lineo afforded learners an opportunity to encounter an

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example of a variable. This shows that she endeavoured to develop conceptual understanding and also to automate the use of a variable (letters in particular) in mathematics by pointing out that a variable can take any value which implies that variables could be treated just like numbers.

Introduction of variables (letters) into mathematics confronts Lineo with manipulating algebraic symbols [*symbol conflict*] because there is a clash as to how one differentiates between the letter x and the multiplication sign. In order to overcome this problem, Lineo engaged in teaching that placed more emphasis on conventions that are used in mathematics. The extract shows that Lineo appealed to **mathematical conventions** to resolve symbol conflict. She encouraged her learners to use a curly x when writing algebraically. Lineo refers to these conventions as a tradition of mathematics.

> Lineo: Ok, **traditionally we always put up the numbers before the letter**. So $n2$ is correct but we prefer to write the number before the letters. Also **we don't write the times between a number and the letter**. When I write $2n$ what are we assuming? 2 times n .The same thing here with the x why did I write the curly x ? (Lineo, lesson 1, 08:04 – 08:30)

The excerpt indicates that she engaged in explaining conventions and mathematical notation (see bold text). What we see here is that Lineo drew on **mathematics,** in particular **mathematical conventions** to legitimate a particular writing style (where numbers come before letters and the multiplication sign is not written). It is interesting that the only reason given to learners is that it is a tradition. This contradicts the fact that most of her explanations and definitions were mathematical in nature. At times Lineo could not find precise words to demarcate the idea under discussion (I draw your attention to the bold text below) in which case she drew on **empirical examples in mathematics** to elaborate the idea and to fix meaning thus helping learners to understand as illustrated below

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> Lineo: coefficient, have you guys come up with coefficient before? (Writes, **it is the stuff before**) **bad English**. **The question is before what? Let's do an example and see whether it makes sense**. If I say to you what is the coefficient of x in the following. **It is the stuff before the variable**
> **In 3x is 3**
> **In – 15x is -15**
> **In 10ax is 10a** (Lineo, lesson 1, 34:06 – 37:07)

The above extract also demonstrates that while Lineo foregrounded mathematics she was not always mathematical as she used words like 'stuff'. This is not to imply that she lacked the correct mathematical language but that she could have been restricted by her learners' knowledge of the subject and its language. The extract further shows that defining in mathematics can be very problematic due to the hierarchical nature of mathematics. Some terms or concepts are best defined in relation to others or in context i.e. in relation to the question at hand. For instance, in the excerpts above coefficient was best defined in relation to the question. Defining a coefficient is complicated by the fact that one variable could be a coefficient of another variable. Ball *et al.* (2004) working within elementary mathematics argue that teachers should define concepts in relation to the grade level of the learner. It is assumed that such definitions would be robust enough to generalize as learners progress from one grade to the next. The extract above indicates that working within algebra presents more challenges to the teacher in finding definitions that are rigorous enough to generalize as some terms (e.g. coefficient) are best defined in context.

The work of teaching that she grappled with as she introduced algebra involved explaining these conventions i.e. how learners should write using mathematical notation. In this way learners were exposed to the grammar of algebraic language. In order to assist learners in making sense of this new language Lineo drew on various examples. For instance, to make sense of the commutative property of multiplication Lineo allowed learners to substitute numbers (in xy and yx) and compute the products. What the excerpts above suggest is that MfT in this practice is

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mathematical and composed of empirical knowledge, mathematical conventions and terminology.

4.8.2 Curriculum and experience

I indicated earlier that Lineo sees herself as the custodian of mathematics. She uses her professional knowledge to dictate which writing styles (numbers before letters) are acceptable in mathematics.

> Lineo: Ok, traditionally we always put up the numbers before the letter. So $n2$ is correct but we prefer to write the number before the letters. Also we don't write the times between a number and the letter. When I write $2n$ what are we assuming? 2 times n .The same thing here with the x why did I write the curly x ?
> Mpho: we do not want a straight x
> Lineo: Don't call it out.
> Thabo: inaudible
>
> Lineo: Ok, as Thabo said, if I were to write that as a straightforward x what does that look like? times times 2 or xx2 doesn't make sense. **So guys I'm going to be very strict with you, when you write your x 's, it is the only letter that you have to change**. **I want you to write it as a curly x neatly.** You neat to get into that habit. **I will take off marks if you don't** (Lineo, lesson 1, 08:04- 09:03)

The extract above indicates that the mathematical problems that confronted Lineo were language of algebra (reading algebraic notation, writing algebraically) and manipulating algebraic symbols (conflict). In order to address manipulating algebraic symbols, Lineo drew on her **professional knowledge** to unpack mathematical conventions for her learners (see the underlined text above). In unpacking these conventions Lineo drew on **mathematics**. The extract further shows that she used her **authority** as an old–timer in this community of practice and the **curriculum** in the form of assessment (test/ exam where learners are penalized for incorrect responses) to coerce learners into writing a curly x (I draw your attention to the bold text in the extract above). By drawing on this knowledge resources she also addressed what I termed language of algebra problems.

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Lineo's unpacking is not limited to conventional ways of writing in mathematics [e.g. x versus multiplication sign]. She also unpacked mathematical operations. In responding to Lefu's question about the use of a comma and the dot in mathematics, Lineo drew on her knowledge of the South African curriculum and said; "**Ok that's why in South Africa we use a comma**. If I want a decimal place I will put a comma. Ok, it's a good question that's why I wanted you to ask it (writes $2 \bullet 4$) that would mean times." (Lineo, lesson 1, 23:51 24:38). Here Lineo legitimates meaning for a mathematical operation by unpacking from the South African **curriculum** [see bold text] and **authority** [underlined text]. In contrast, Ball & Bass (2000), Ball *et al*. (2004) and Adler (2005) talk about unpacking from the mathematics.

As indicated earlier Lineo confronted a number of mathematical problems. To address these problems Lineo engaged in explicit teaching of mathematical conventions. Lineo employed her **professional experience** to teach mathematically accepted writing styles when using algebraic notation. For example, while writing $d5 + r3$ is mathematically correct it is not an approved way of writing in mathematics. In other words, Lineo taught the structure of algebraic sentences by pointing out that when writing algebraically numbers always precede variables.

Lineo's practice suggests that teaching mathematics goes beyond concepts and algorithms. There is notation, terminology as well as rules and conventions which should also be taught. These may demand that teachers draw on various knowledge resources in order to make the subject meaningful and comprehensible to their learners.

In section 4.7, I engaged with the identification and discussion of the mathematical problems that confronted Lineo, the nature of algebraic activities that she used when introducing algebra and the work of teaching which she wrestled with hence answering my first, second and third research questions. Section 4.8 focused on the knowledge resources which Lineo called on to legitimate meaning for the boys thus I endeavored to answer the fourth research question. In the section that follows, I

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discuss the interviews I conducted with Lineo regarding her teaching in relation to the research questions that guide this study.

4.9 How does Lineo describe her teaching?

Observing Lineo while she taught provided a window through which her actual work of teaching could be looked at, it does not shed light into how and why Lineo organized the activities as she did. In order to get insight into the work of teaching, knowledge resources called on and the mathematical problems that confronted Lineo at this stage I discuss the interviews that I conducted with her.

It is important to mention that I began this study with an assumption that algebra is introduced to learners when they get to Grade 8. Lineo indicated that this is theoretically true but in practice some schools introduce algebra to learners earlier (i.e. in Grade 7). Introduction of algebra into school mathematics is a hotly debated topic in mathematics education research. Lineo indicated that she was going to assume that all learners have not done algebra. When I asked her what she thought the boys would learn in the first lesson she said;

> Lineo: my goal is to make them familiar with using a variable, instead of a number; umm especially I wanted to do terminology, what is a term, coefficient and constant and then also umm **differentiating between like and unlike terms**. (Lineo, pre-teaching interview, L14 - L18)

The extract shows that Lineo had planned to teach mathematical concepts such as collecting like terms and the mathematical terminology, for example constant, coefficient, term etc. The above extract suggests that Lineo wanted to engage her learners in reading/writing activities of algebra [see underlined text], representational and transformational activities [bold and underlined text]. It is apparent that she wanted to automate the use of various symbols in mathematics as she talks about

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using a variable instead of a number. Kaput (1995) cited by Breitieg & Grevholm (2006) point out that using algebra, *in particular variable* [italics added] allows one to "talk about numbers and magnitudes without calculating anything" and also it enables "one to describe the calculations that one wants to do or get others do" p. 2-225. It (variable) enables one to generalize. It is therefore understandable why Lineo foregrounds familiarity with the use of a variable. Lineo explicitly pointed out that she would like her learners to master the mathematics register by singling out some technical terms that they were to learn. Pimm (1982) working from the stand point that mathematics is like a language, points out that mathematics does not have its own words but it uses the very same words that are found in natural languages but it attaches specialized meanings to each of them. As a result, if learners' attention is not drawn to these specialized meanings, the teacher may talk passed the learners as s/he may be using the technical meaning of the words while the learners use the everyday meaning (Moschkovich, 1996). Lineo's explicit teaching of technical terms indicates that she did not only aim at attaining mathematical competence but also mathematical communicative competence. Teaching technical terms may help learners in seeing the specialty of the context in which the terms are used, which in turn might help them in producing the legitimate text (Bernstein, 1996). During the third interview (follow up interview) Lineo reiterated the importance of knowing the language of mathematics. This is shown by the underlined text in the excerpt that follows:

> I think the boys, especially the boys and also the girls they need to know that umm our terminology in mathematics is important umm when I am talking about umm a constant they need to know it's a number, it has got a role and they need to know that umm mathematics language is important, umm and that **it will be to their detriment in future if they don't learn to speak mathematics properly.** Umm so I will always emphasize the definitions and the terminology and when they are giving me back an answer if they are not saying it correctly I will correct them. I think it's important that they don't malinger. (Lineo, follow up interview, L189 - L197)

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The extract shows that for Lineo learning algebra is about gaining access to the language of algebra. Lineo's emphasis on the language of mathematics is not unusual as algebraic ideas seem to connect to all areas of mathematics. Failure to master such a language implies that one has to take another educational direction. The bold text indicates that Lineo regards lack of precise mathematical language [terminology] as the main obstacle in the learning of mathematics and algebra, in particular. To her learning mathematics also involves speaking mathematics properly [see the bold and underlined text]. In order to address this problem she draws on **mathematics** in the form of **definitions**. The extract above also shows that Lineo operates from the stand point that algebra is a language hence it has a register that is to be made explicit to the newcomers. Her teaching of algebra as a language is interesting in that it does not focus on developing the language of learning and teaching (LoLT) but the language of mathematics. Lineo's explicit teaching of mathematical language is not unusual. Adler (2001) working in multilingual classrooms found that teachers working in suburban schools which had deracialized quickly used the same strategy and claimed that it benefited all learners. However, this strategy clashes with reformed teaching which demands that learners should be given opportunities to investigate and build meanings which teachers are to work with. Another study in a multilingual classroom conducted by Setati *et al*. (2002) indicates that teachers taught mathematics and LoLT. These studies, though they were conducted in multilingual classrooms assist me in situating Lineo's practice in a broad area of MKfT/ MfT. Hence Lineo's teaching of mathematics and the language of mathematics is a normal practice in mathematics classrooms. In addition, Lineo seemed to have organized the activities so as to address learners' inability to *read, write and interpret algebraic notation*. The extract below exemplifies this point.

> About, today is about differentiating between $2x$ and x^2, they get confused what the 2 resemble. x^2 *is* x *times* x, there are two lots of x and I will clarify today so that these boys do not struggle with these concepts. The x^2 *versus* $2x$ and the other goal is to **try and make them understand that** xy *will be the same as* yx. (Lineo, pre-teaching interview, L62 - L66)

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The quote above shows that Lineo was aware of the problematic areas in algebra for learners. Here Lineo confirms my earlier claim that she dealt with both anticipated and emerging learners' errors and misconception. As a mathematics teacher, I have observed that some learners read 5^2 as 2×5 instead of 5×5 which suggests that such learners have a problem in interpreting mathematical notation and/ or might have not understood the meaning of or the difference between coefficient and exponent. Over-and-above, the extract indicates that Lineo sees algebra as not only a language but a symbolic language system. This seems to compel her to ensure that her learners are able to interpret, read and write appropriately using the algebraic notation.

One can see that Lineo has set up her lessons, in particular lesson 1 so that she could address mathematical terminology, concepts and algebraic notation which could enhance learners' engagement with algebraic activities, in particular reading/ writing, representational and transformational activities thus affording them opportunities to develop a better understanding of algebraic concepts. Consequently, her teaching is centred on developing proficiency in algebra as a symbolic language system. In order to attain her goal Lineo sets up closed, short, highly structured and clearly sequenced activities for her learners. It seems that even at planning Lineo was wrestling with **defining** terms such as constant, term, coefficient etc. and concepts like variable. The extract also suggests that she grappled with **explaining** as she wanted to clarify the difference between x^2 *and* $2x$. This implies that Lineo was aware of the mathematical problem which MacGregor & Stacey (1996a) termed exponential form of product. The pre-teaching interview indicates that Lineo was going to draw on mathematics, most probably conventions and examples to make algebra accessible to the boys as she talked about drawing a distinction between a coefficient and an exponent [see extract on previous page]. In order to do this and expose learners to the symbolic language of algebra, Lineo employed initiation-response-feedback (IRF) approach in her teaching (Brodie, 2005). In this approach the teacher takes the leading role. S/he decides on which ideas are to be discussed. The conversation

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commences with the teacher posing a question or presenting an activity. Learners then, provide responses which are evaluated by the teacher. During this evaluation process the teacher draws on various resources to legitimate the intended meaning of the mathematical concept. I pointed out earlier that Lineo also used repetition in her teaching. Repetition is synonymous to chorusing (Adler *et al*. 1995) [cited in Brodie (2004)] in that it serves to mark what is important or to draw learners' attention to a particular idea. The above excerpt shows that in this practice mathematics is driven by conventions.

The interview suggests that Lineo sees herself as the custodian of mathematics and a mentor for the newcomers [see italicized text in the extract below]. Consequently, she sees her work as involving enculturation of learners into the mathematicians' ways of writing, speaking and doing mathematics. Lineo elaborated this further when she said [I draw your attention to the underlined text],

> Lineo: I think the boys, especially the boys and also the girls they need to know that umm… *our terminology in mathematics is important* umm… when I am talking about umm a constant, they need to know it's a number, it has got a role and *they need to know that umm mathematics language is important*, umm… and that it will be to their detriment in future if they don't learn to speak mathematics properly. Umm… so *I will always emphasize the definitions and the terminology* and *when they are giving me back an answer if they are not saying it correctly I will correct them. I think it's important that they don't malinger* [Lineo, follow up interview, L189 - L197, my emphasis underlined]

4.10 Summary

In this chapter, I provided a description of the lessons I observed. I also discussed how the data was chunked and illustrated how the analytic tool was used. I provided both the quantitative and qualitative analysis of Lineo's practice. The quantitative analysis indicated that restructuring of tasks was completely absent in Lineo's practice while **explaining, questioning, representing, defining** and **working with learners' ideas** were present. Through quantitative analysis, I showed that

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representational, **transformationa**l and **reading/writing** activities of algebra were prevalent in this practice while **generalizing and justification** activities were entirely absent although these were algebra lessons and this is a desired and advocated practice in reform oriented curriculum Like the RNCS. I also engaged with the identification of the mathematical problems that confronted Lineo and the knowledge resources that she called on as she went about her work. In this way, I endeavored to answer the research questions posed earlier in section 1.2. In the chapter that follows, I conclude this study by giving an overview of the study, its findings and my reflection on the study.

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Chapter 5: Findings, Conclusion and Reflections

5.1 Introduction

This study set out to investigate mathematical knowledge for teaching (MKfT). It focused mainly on the (i) mathematical problems that confronted a Grade 8 teacher as she introduced algebra, (ii) the mathematical work of teaching that this teacher wrestled with as went about her work and (iii) the knowledge resources that she drew on in order to legitimate meaning for her learners as well as (iv) the nature of the algebraic activities that were presented to learners.

Algebra was the topic under which the study was conducted. This topic was chosen for two reasons. First, I was interested in this topic because having taught 8th graders, I was aware that most learners find this area problematic. I thought by studying this area one would be able to identify the mathematical knowledge that a Grade 8 teacher needs to know and be able to use in practice in order to introduce algebra to learners so that they can master this branch of mathematics which also functions as a language for all other branches of mathematics. Secondly, algebra was the topic which the teacher was teaching at the time of data collection. This afforded me the opportunity to capture the actual practice of Lineo as she did not have to alter her schedule to suite me. This study was guided by the following questions:

1. What is the nature of the activities that the Grade 8 teacher uses to introduce algebra?
2. What mathematical problems does the teacher confront as she introduces algebra in Grade 8?
3. What mathematical work does the teacher do as she goes about solving these

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mathematical problems in her teaching practice, specifically in order to introduce algebra to Grade 8 learners?

4. What knowledge resources does she call on for this work?

In Chapter 1, I pointed out that algebra is an important component of school mathematics. It provides a language through which other branches of mathematics are communicated and learnt. RAND study panel (2001) argues that algebra plays a gate-keeping role for most learners, affording some access to mathematics based careers while keeping others out.

In Chapter 2, I indicated that in mathematics education, research debates are informed by theories of learning. This study was no exception. It used a situative theory and a sociological theory of pedagogy to guide it. Ball *et al.* (2004) 8 kinds of mathematical work of teaching formed the base for identifying the work of teaching in Lineo's practice. However, these were not used as they are (see section 2.2). Kazima & Adler (2006) condensed Ball *et al*. (op cit) categories into six kinds of mathematical work of teaching. It is these six kinds of mathematical work of teaching that I used to study Lineo's practice. My analysis of Lineo's practice was illuminated further by Adler & Pillay (in press) who through their study of Nash's practice argue that as teachers go about their work they often draw on knowledge resources to legitimate meaning for their learners. In the sections that follow I engage with the findings of this study by looking at each of the above mentioned research questions.

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5.2 Findings

5.2.1 What is the nature of the activities that the Grade 8 teacher uses to introduce algebra?

In section 4.7, I engaged with identification of the nature of algebraic activities that Lineo used as she introduced algebra to Grade 8 learners. Table 4.4 shows that these

were **representational**, **transformational** and **reading/writing activities**. Table 4.4 further indicates that Lineo mainly dealt with transformational and reading/writing activities. The table further shows that generalizing and justifying activities are completely absent in this practice. While this might be disturbing in view of the new curriculum as this is a prescribed and desired practice, it is one of the challenges facing teachers and the teaching of algebra in reform oriented curriculum such as RNCS. In chapter 4, I indicated that I only observed this teacher for one week hence this absence may not mean that she does not engage with generalizing and justifying activities in her overall practice. On the other hand, this study shows that this teacher does an excellent work on assisting learners to gain facility in mathematical symbols and language of mathematics, aspects of learning mathematics which have not been explicitly discussed by the proponents of reform oriented teaching. In fact, there are two issues which emerged from this study about this teacher's mathematical work of teaching. First, is working with mathematical conventions and reading/writing activities. This is what this teacher does, and so indicative of what she needs to know and use in practice. Secondly, we see that MKfT or MfT is about being able to read algebraic notation and write algebraically. In the literature, language issues are discussed quite generally and are not talked about through learning of mathematical conventions and reading/ writing activities which we learn from this teacher hence why defining is dealt with so briefly.

I would like to point out at this juncture that although I discussed reading/writing activities in section 1.3.3.4, they are actually the outcome of this study. As I struggled with the analysis, I realized that the algebraic activities suggested by Kilpatrick *et al.* (2001) did not indicate how learners gain facility in handling mathematical symbols (Pimm, 1992) and language of mathematics which were more prevalent in this study Hence they [Kilpatrick et al.'s (op cit) algebraic activities] could not completely capture this teacher's practice. This led me into inventing another category of algebraic activities which I termed **reading/writing activities**.

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Lineo's practice suggests that Kilpatrick *et al.* (2001) activities of algebra are limited in describing mathematical proficiency in algebra. It shows that in algebra learners are expected to understand not only the concepts, operations and relations (Kilpatrick *et al*., 2001) but also the conventions that are used. Lineo's practice suggests that meaning and understanding of what algebra is or consists of or comes from the use of different representations aimed at helping learners to form mental images of algebra, use of mathematical language and manipulation of symbols.

5.2.2 What mathematical problems does the teacher confront as she introduces algebra in Grade 8?

Researchers such as Ball *et al.* (2004) and Brodie (2004) in mathematics education argue that teaching presents teachers with a number of problems which they have to solve. Some of these problems can be predicted while others cannot be predicted. Some of these problems arise as learners attempt to incorporate new concepts into their existing knowledge structures. In the previous, section 4.7; I identified the mathematical problems that confronted Lineo as she introduced algebra to 8^{th} graders. These were manipulating algebraic symbols of which *conjoining* is well documented in mathematics education research (section 1.3.5) and language of algebra (*reading algebraic notation, writing algebraically, defining, etc.*). It is important to note that these mathematical problems confront the teacher as errors or misconceptions (Hatano, 1996) in the learner's response. By pointing out these mathematical problems, in particular *reading algebraic notation and writing algebraically* which I have categorized as language of algebra problems, this study contributes to the study of MfT as these are more likely to occur when introducing algebra, especially if algebra is taught as a symbolic language system as in Lineo's case. Table 4.4 shows that Lineo does not confront generalizing problems. As pointed out earlier, this is a direct consequence of the activities that were presented to learners. Lineo employed closed and highly structured activities when introducing algebra with more emphasis on reading/writing and transformational activities. This exemplifies my earlier claim that in this practice meaning of algebra or algebraic notation (symbols and

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operations) is linked to reading/writing activities and mathematical conventions. Activities related to mathematics as reasoning (Brodie, 2000) or algebra as generalizing and justifying (Kilpatrick *et al*. (2001) are missing in this practice.

In section 2.3, I pointed out that Ball *et al.* (2004) talk about the work of teaching (mathematical problem solving) that teachers do in practice. However, they do not elaborate as to why teachers engage in mathematical work of teaching. Lineo's practice sheds light into this as it shows that when Lineo confronted a mathematical problem she embarked on mathematical work of teaching. Section 2.3 below focuses on the mathematical work of teaching that Lineo wrestled with.

5.2.3 What mathematical work does the teacher do as she goes about solving these mathematical problems in her teaching practice, specifically in order to introduce algebra to Grade 8 learners?

In section 4.7, I discussed the work of teaching (**explaining, defining, questioning, representing** and **working with learners ideas**) that Lineo grappled with. These kinds of mathematical work of teaching occurred unevenly across the four lessons that I observed. **Representing** was mainly present in the first lesson and completely absent in the other three lessons. I also noted that not all the six kinds of mathematical work of teaching were present in Lineo's practice. **Restructuring of tasks** was completely absent (see section 4.6). It is worth mentioning here that Lineo used closed, textbook like activities. She mainly focused on how to read and write using algebraic notation. Mathematical conventions and structure of algebraic expressions (sentences) were over-emphasized. These suggest that Lineo sees algebra as a symbolic language system. While Ball *et al.* (2004) as well as Kazima & Adler (2006) point out that teaching entails engaging in mathematical work of teaching; they do not specify explicitly what enables teachers to do their work. By studying Nash's practice, Adler & Pillay (in press) argue that in the process of teaching mathematics, a teacher exercises some judgments in order to legitimate meaning of the concepts under discussion. Section 5.2.4 below looks at the knowledge resources that Lineo appealed to in her practice.

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5.2.4 What knowledge resources does she call on for this work?

Table 4.4 shows that Lineo drew on **mathematical knowledge, experience** and **curriculum knowledge**. These knowledge resources were called upon in varying amounts across the four lessons. In section 4.8, I provided a detailed description of how these resources were called on by Lineo as she went about her work. As I pointed out earlier, Lineo placed more emphases on writing and reading algebraic notation as well as acquisition of mathematical terminology during the observed lessons. Consequently, most of the work of teaching (explaining, questioning, defining, representing, and working with learners' ideas) which she did was centred on mathematical conventions, terminology and definitions. As a result, she mostly appealed to mathematics [see Table 4.4]. This seems to have been driven by her belief that algebra is a language which learners should master in order to be proficient in mathematics. Lineo explicated her belief during the follow up interview (draw your attention to the bold text) by indicating that

> I think the boys, especially the boys and also the girls **they need to know that umm our terminology in mathematics is important** umm when I am talking about umm a constant they need to know it's a number, it has got a role and **they need to know that umm mathematics language is important,** umm **and that it will be to their detriment in future if they don't learn to speak mathematics properly**. Umm **so I will always emphasize the definitions and the terminology and when they are giving me back an answer if they are not saying it correctly I will correct them**. I think it's important that they don't malinger. (Lineo, follow up interview, L189 - L197)

Lineo's practice cannot be classified as either traditional or reform oriented approach. Rather, it brings the two together by pointing out that both ignored acquisition of mathematical language which is an essential component of mathematical proficiency. Furthermore, Lineo's practice shows that gaining fluency in handling mathematical symbols (Pimm, 1992) is a necessary component of mathematical proficiency which has been neglected by reform oriented researchers.

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5.3 Conclusion

5.3.1 What constitutes mathematics for teaching in Lineo's practice?

The discussions in the previous sections indicate that MKfT or MfT in Lineo's practice is composed of mathematical conventions, terminology and what Adler & Pillar (in press) termed empirical/ *use of examples* [italics added] mathematical knowledge. As indicated earlier, MfT produced in this practice is much more driven by mathematics as conventions and reading writing activities than as reasoning (Brodie, 2000; Kilpatrick *et al*., 2001; Ball & Bass, 2003) or what Kilpatrick *et al.* (2001) have since described as generalizing and justifying activities. MfT in this practice seems to be a function of the topic – algebra which is highly symbolic and has a lot of conventions, and the teacher's belief (algebra is a symbolic language system with its own register which is to be made explicit to learners). Table 4.4 shows that in Lineo's practice transformational and reading/writing activities are more prevalent than others. This suggests that for Lineo 'meaning' and understanding of mathematical objects, in particular algebraic symbols resides in symbol manipulation [transformational activities] and from the use of language [reading/writing activities]. In contrast, Brodie (2000) and Kilpatrick *et al.* (2001) see meaning of algebra as residing in activities that afford learners the opportunity to engage in more open-ended activities which allow learners to conjecture and justify observed relationships.

Looking back at Kazima & Adler's (2006) 6 elements of mathematical work of teaching, we see that some of them viz. explaining, defining, questioning, representing and working with learners' ideas are present in Lineo's practice while restructuring is absent. I link this absence of restructuring to the nature of algebraic activities that were used in this practice. Restructuring just like generalizing and justification activities require the use of more open-ended tasks which were absent in this practice [see Table 4.4]. Lineo introduced algebra mainly through transformational and reading/writing activities. This required her to ascertain

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learners' (mis)understanding from time to time hence Lineo wrestled mostly with questioning, explaining and working with learners' ideas so to move them on and to make explicit the legitimate text or the valid knowledge to be acquired by the learners.

I also note that Kazima & Adler (op cit) do not talk about the knowledge resources that teachers call on as they go about their work. Table 4.4 shows that Lineo also appealed mathematics, experience and curriculum but with more emphasis on mathematics. This also could be linked to the nature of algebraic activities used by Lineo. As indicated earlier, Lineo's teaching emphasized transformational and reading/writing activities [see Table 4.4]. An emphasis on these algebraic activities implies that Lineo had to unpack algebra notation and conventions used in mathematics so as to afford learners an opportunity to make sense and understand this new notation and language of mathematics. Unpacking algebraic notation and conventions demanded that Lineo call on specific knowledge resource [in this case mathematics] to ground meaning of algebra and its notation. Hence the absence of both theoretical and algebraic [i.e. principled mathematics] based arguments in this practice. Engaging in such arguments require open-ended activities which were absent in this practice, as a result of which Lineo does not have to grapple with restructuring tasks.

Lineo engaged her learners with activities that demanded them [learners] to write algebraic expressions by either manipulating a given expression [such as $3x+2b-5x$] or a random distribution of terms [like

$$\begin{array}{cccc} & a & b^2 & \\ a^2 & & & a^2 \\ & b^2 & ab & \\ b & & & \\ ab & a^2 & ba & a \end{array}$$

(Lineo, lesson 1, 23:51 – 24:48)]

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this largely confronted her with language of algebra and manipulating algebraic symbols problems. Thus, what Lineo does [questioning, explaining and working with learners' ideas] seem to be adequate for her current practice. However, one can see that if she is to incorporate generalizing and justifying activities into her practice she might have to do more as this might bring new mathematical problems and she might have to engage with restructuring tasks which seems to be linked with these activities.

5.3.2 Reflections

When I began this study, I thought that the six kinds of mathematical work of teaching, the five strands of mathematical proficiency and knowledge resources (mathematics, experience and curriculum) would be adequate to describe Lineo's practice. On beginning my analysis I realized that the elements of mathematical proficiency were not as helpful as I thought (see section 1.3.2) as they do not explicitly capture the teaching of mathematical conventions and mathematical language. I then opted for what Kilpatrick *et al.* (2001) has since termed activities of algebra. I soon became aware that these activities were deficient in that they did not suggest how learners gain facility in the language of mathematics (see section 1.3.3.4). This prompted me to add what I termed reading/writing activities of algebra so as to incorporate activities that help learners in acquiring the language of mathematics.

I indicated that piloting of instruments for this study was done through observing videoed lessons from previous studies. This helped me in refining my instruments. Although interviews gathered data for this study, they became a minor part of this study but important as they indicate that there is consistency in what this teacher said and did in the lessons. Conducting interviews has been the most challenging part of this study in that in my culture looking someone in the eye is discourage as it is associated with disrespect while this is a desired and encouraged habit in conducting interviews. So in carrying out this study, I had to leave my old self and become someone who could look others in the eye. Eye conduct is believed to be a sign of

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interest in what is being said and indicates that the interviewer is paying attention. In addition, conducting interviews demanded that I to identify important points in what is being said that needed to be followed up and ask potential [on-the-spot] questions that can elicit more information. I found this difficult due to language issues such as accent and speed talking. While asking the interviewee to speak a bit slow helped me it seemed to frustrate her as she at time had to repeat sentences. Carrying out interviews demanded that one minimized the distance between the interviewer and interviewee. Again this clashed with my culture, particularly because I was interviewing a female. So I found myself being divided and uncomfortable. This is the part which I think I need to improve most.

In chapter three, I pointed out that this study involves one Grade 8 teacher and that only four lessons were observed. This implies that the findings of this study cannot be extended beyond this sample or to her overall practice as I only observed her for one week. In addition, this teacher does a lot of work individually, works in a very supportive environment and well established school with only 24 learners in the observed Grade 8 class which may not be the case in other schools.

As I come to the end of this study I keep on pondering whether I would have been able to carry out this study if Bernstein's sociological theory of pedagogy was not there. Bernstein's theory of pedagogic discourse assisted me in chunking the data into units of analysis by pointing out that in any pedagogic communication meaning is condensed in evaluation (Bernstein, 1996). I now realize that conducting research is not a straightforward undertaking. It is an interactive process which demands one to move back and forth between research questions, data analysis and the literature. Lineo's practice has been an eye opener to in terms of how to teach algebra. What I have learnt and found interesting in her practice was that in addition to teaching knowledge i.e. concepts, operations and procedures she taught the language of mathematics with particular reference to terminology, conventions and notation [symbols and operations].

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I have realized that curriculum implementation is open to different interpretations, dependent on teacher's beliefs and their preferences in terms of how the content is to be taught. RNCS and proponents of reform oriented-teaching advocate the use of generalizing activities while Lineo employs transformational and reading/writing activities. It seems that changing the curriculum document and availing it to teachers is likely not to be sufficient in transforming the teaching of mathematics in schools especially if the implementers are products of another curriculum. If the change is to be recognized by teachers it should also occur in assessment as this seems to influence what Lineo and possibly other teachers do. Also a mismatch between what practicing teachers know and what is advocated in the curriculum document may lead to partial implementation of the curriculum as we saw in Lineo's practice as teacher may turn to teacher what they know.

I indicated that Lineo is highly qualified and that she believes that she has 'figured out mathematics'. However, what this study reveals about Lineo's practice when introducing algebra is that the generalizing power of algebra which is being advocated by both the RNCS (DoE, 2002) and curriculum reforms is not apparent in her practice. In section 4.3, I showed that Lineo introduced algebra through number patterns which seems to have been aimed only at leading learners into accepting and using different letters to represent numbers rather than to lead them into recognizing the generalizing power of algebra. This study begs the question, if the new curriculum is to take off with all its force or if we want algebra to be invested with meaning for the learners, what are the challenges we face as teacher educators and policy makers to work with teachers like Lineo, who are successful but may be lacking in relation to the new curriculum? What form of intervention can we offer Lineo and other teachers so that they can realize this [generalizing] aspect of algebra and be able to incorporate it into her lessons?

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APPENDIX 1

1.1 Interview 1 schedule

Semi-structured interview conducted with a Grade 8 teacher in order to investigate the mathematical problem solving that she does when setting up and implementing tasks in order to promote development of algebraic reasoning.

1. Did you choose or design this task?

   _____

   _____

2. Why did you choose/ design this task?

   _____

   _____

3. Were there other tasks that that you designed or you could choose from?

   _____

   _____

4. What distinguishes this task from the other tasks that you could use?

   _____

   _____

5. What makes this task suitable for this lesson?

   _____

6. How did you plan to use this task with your learners? Why?

   _____

   _____

7. What do you think your learners will learn by engaging with this task?

   _____

   _____

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8. What problems do you think your learners will experience when engaging with this task?

_____

_____

9. How did you plan to deal with such problems?

_____

_____

10. What would you consider to be the features of a task suitable for teaching algebraic reasoning?

_____

_____

1.2 post-teaching interview schedule

1. Were you pleased with the way you implemented the task? Explain.

_____

_____

2. Did the plan work accordingly? Why?

_____

_____

3. What difficulties did the learners have in engaging with the task?

_____

_____

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4. What do you think is the source of learners' difficulties?

_____

_____

5. How do you plan to deal with these learners difficulties in future? Why?

_____

_____

6. What difficulties did you encounter when implementing this task?

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1.3 Observation schedule

Classroom observation conducted on a Grade 8 teacher in order to determine what problems confronts the teacher and how the teacher solves them

What is the teacher doing?	**Duration of lesson in minutes**				
	0-10	11-20	21-30	31-40	41-50
Provides explanations					
Evaluates learners' explanations					
Uses mathematical definitions					
Interprets learners' ideas					
Responds to learners questions					
Analyzes learners errors and misconceptions					
Modifies tasks					
Presses learners to justify					
Uses different representations					
Poses questions that challenge learners' thinking					

APPENDIX 2

LGD5 International House
Wits University
2050
17th May, 2006

The Principal
St John's College
Wits

Dear Sir/ Madam

Research title: Mathematical Knowledge for Teaching (MKfT): A focus on algebraic reasoning.

I, T. Talasi request your permission to involve your Grade 8 teacher and her learners in my study. I am enrolled for a Masters degree (Mathematics education) at the University of Witwatersrand under the supervision of Professor Adler.

The purpose of the study is to investigate the mathematical problem solving that a Grade 8 teacher does when teaching algebraic reasoning. I plan to work with this teacher and her learners who are willing to participate in the study. The study is important because it endeavors to understand the mathematical problem solving that confronts teachers as they attempt to teach algebraic reasoning. It will also shed some light into what types of task are preferred by teachers when teaching algebraic reasoning. This would be of great help to textbook authors and resource persons as they will know what would be of help to teacher.

During the data collection process I will interview the teacher and observe her while teaching. The classroom observations will be videoed. I hope to be within the school premises for a maximum of one week.

All the information collected in this study will be kept confidential and used only for the purpose of this study. No names or personal details of participants or the name of the school will be revealed in reporting the outcomes of the study. Participants are free to withdraw at any time during the study and where possible information collected on such participants will not be used in the study. If you require more information you can conduct me or my supervisor by email or phone.

Yours Faithfully
Talasi Tatolo
cell: 0725252851
Email: talasitatolo@yahoo.com

Supervisor

WISERD

Prof. Jillian Adler Telephone: (011) 717 3413
Email: jillian.adler@wits.ac.za

Consent form for the principal

Research title: Mathematical Knowledge for Teaching (MKfT): A focus on algebraic reasoning.

I (name of principal) _____of (name of school) _____, hereby allow (name of teacher and Grade of learners) _____to participate in the study conducted by Tatolo Talasi. I understand that the aim of the study is to investigate Mathematical problem solving that a Grade 8 teacher does when teaching algebraic reasoning.

Please tick the most appropriate response

	Yes	**No**
1. Do you voluntarily allow your teacher to participate in the study?	[ ]	[ ]
2. Do you voluntarily allow your Grade 8 learners to participate in the study?	[ ]	[ ]
3. Do agree that your teacher could be interviewed and audio taped?	[ ]	[ ]
4. Do you agree that your teacher could be observed while teaching?	[ ]	[ ]
5. Do you agree that a video could be used during classroom observations?	[ ]	[ ]
6. Do you agree that the information collected from your teacher and learners be used for research purposes only?	[ ]	[ ]
7. Do you agree that you are free to withdraw either your teacher and/or your Grade 8 learners at anytime?	[ ]	[ ]

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8. Do you agree that information collected on the withdrawn participants will not form part of the study where possible? [ ] [ ]

9. Do agree that participation in the study should be voluntary and that no grades should be awarded? [ ] [ ]

10. Do you agree that copies of learners' work and the teacher's preparation may be taken [ ] [ ]

Signature of principal

_____

Date_____

LGD5 International House
Wits University
2050
Johannesburg

17th May, 2006

Dear parent

Research title: Mathematical Knowledge for Teaching (MKfT): A focus on algebraic reasoning.

I, T. Talasi request your permission to involve your child in my study. I am enrolled for a Masters degree (Mathematics education) at the University of Witwatersrand under the supervision of Professor Adler.

The purpose of the study is to investigate the mathematical problem solving that a Grade 8 teacher does when teaching algebraic reasoning. I plan to work with learners who are willing to participate in the study. The study is important because it endeavors to understand the mathematical problem solving that confronts teachers as they attempt to teach algebraic reasoning. It will also shed some light into what types of tasks are preferred by teachers when teaching algebraic reasoning. This would be of great help to textbook authors and resource persons as they will know what would be of help to teachers.

WitsETD

The study focuses on what the teacher says and does. I will observe her while teaching. During these observations a video will be used to capture what the teacher does and say consequently learners will also appear on video. However, learners' work and contributions will not form part of the study.

All the information collected in this study will be kept confidential and used only for the purpose of this study. No names or personal details of participants or the name of the school will be revealed in reporting the outcomes of the study. Participants are free to withdraw at any time during the study and where possible information collected on such participants will not be used in the study. If you require more information you can conduct me or my supervisor by email or phone.

Yours Faithfully

TALASI Tatolo

cell: 0725252851
Email: talasitatolo@yahoo.com

Supervisor
Prof. Jillian Adler

Telephone: (011) 717 3413
Email: jillian.adler@wits.ac.za

Consent form for parents

Research title: Mathematical Knowledge for Teaching (MKfT): A focus on algebraic reasoning.

I (name of parent) _____of (name of residence) _____, hereby allow (name of learner)_____ to participate in the study conducted by Tatolo Talasi. I understand that the aim of the study is to investigate Mathematical problem solving that a Grade 8 teacher does when teaching algebraic reasoning.

Please tick the most appropriate

	Yes	**No**
1. Do you voluntarily allow your child to participate?	[ ]	[ ]

WitsETD

2. Do you agree that the information collected should be used for research purposes only? [ ] [ ]

3. Would you want your child to appear on the video that is used in collecting the data? [ ] [ ]

4. Do you agree that you are free to withdraw your child at anytime and the information collected where possible should not form part of the study? [ ] [ ]

5. Do you agree that participation should be voluntary? [ ] [ ]

6. Do you agree that no grades should be awarded for participation? [ ] [ ]

7. Do you agree that a copy of the learners work could be taken? [ ] [ ]

Signature of parent
_____
Date_____

LGD5 International House
Wits University
2050
Johannesburg

17th May, 2006

Dear, teacher

WitsETD

Research title: Mathematical Knowledge for Teaching (MKfT): A focus on algebraic reasoning.

I, T. Talasi request you and your Grade 8 learners to participate in my study. I am enrolled for a Masters degree (Mathematics education) at the University of Witwatersrand under the supervision of Professor Adler.

The purpose of the study is to investigate the mathematical problem solving that a Grade 8 teacher does when teaching algebraic reasoning. The study is important because it endeavors to understand the mathematical problem solving that confronts teachers as they attempt to teach algebraic reasoning. It will also shed some light into what types of task are preferred by teachers when teaching algebraic reasoning. This would be of great help to textbook authors and resource persons as they will know what would be of help to teachers.

During the data collection I hope to interview you before and after the lesson. The interview will be related to the task(s) which you will be using. I will then observe you while teaching. The classroom observations will be videoed. I plan to work with you for one week.

All the information collected in this study will be kept confidential and used only for the purpose of this study. No names or personal details of participants or the name of the school will be revealed in reporting the outcomes of the study. If you require more information you can conduct me or my supervisor by email or phone.

Yours Faithfully
Talasi Tatolo

cell: 0725252851
Email: talasitatolo@yahoo.com

Supervisor
Prof. Jillian Adler

Telephone: (011) 717 3413
Email: jillian.adler@wits.ac.za

WitsETD

Consent form for the teacher

Research title: Mathematical Knowledge for Teaching (MKfT): A focus on algebraic reasoning.

I (name of teacher) _____of (name of school) _____, hereby agree to participate in the study conducted by Tatolo Talasi. I understand that the aim of the study is to investigate Mathematical problem solving that a Grade 8 teacher does when teaching algebraic reasoning.

Please tick the most appropriate response

	Yes	**No**
1. Do you agree to be interviewed and audio taped?	[ ]	[ ]
2. Do you agree to be observed while teaching and videoed?	[ ]	[ ]
3. Do you agree that the information collected should be used for research purposes only?	[ ]	[ ]
4. Do you agree that you are free to withdraw from the study at any time?	[ ]	[ ]

Signature of teacher

_____

Date_____

WitsETD

APPENDIX 3

Pre-teaching interview

L1 – L11

TT: can you describe what you are going to do today
Lineo: I am going to introduce algebra to the boys; they had seen uh some algebra and some umm number patterns. They are familiar with finding the n-th term. So they are comfortable with using n umm to generalize but they haven't been formally taught algebra, some of the boys might have seen it in the primary school because they are coming from different primary schools. I am going to start from the beginning and assume that everyone doesn't know.
TT: oohh they do algebra, part of algebra in primary schools
Lineo: some of the schools do
TT: They do, because I was thinking that it begins normally at Grade 8
Lineo: theoretically it does and I know that at St.Johns Prep. They do algebra

L11- L18

TT: so what do you anticipate…. Basically what will they learn?
Lineo: in todays lesson only?
TT: yes only today's lesson
Lineo: my goal is to make them familiar with using a variable, instead of a number, umm especially I waned to do terminology, what is the term, coefficient and constant and then also umm differentiating between like and unlike terms.

L19 – L29

TT: I think we should now look in more detail into today's lesson
Lineo: (takes one worksheet and gives me the other) today's lesson I am going to cover exercise 1, 2 and 3 is the more complicated exercise than 2. Whether they would start working on it at the end of today's lesson I would cover it
TT: tell me about these activities, did you develop them yourself?
Lineo: umm Stephen, he took photo copies of a section from Classroom Mathematics, because the new syllabus does not go into details with algebra and the textbooks we are using for the Removes do not cover algebra like this. This is from Classroom Mathematics, all that he chose is for all the Removes, and all the Removes are using the same worksheet. All of these are from Classroom Mathematics; I might develop my own worksheet to supplement depending on whether they need more practice.

L30 – L39

TT: do you normally develop them?
Lineo: it depends on the standard of the boys, the level at which they are , I find the lowest boys need more practice but the top boys do not need to practice but need more complicated ones to practice. sorry, Stephen only gives us worksheets like this only for some of the sections, we usually work from textbook and I believe that these textbooks need to be supplemented and so invariably I do supplement the text. Have you seen the textbook?
TT: no

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Lineo: I will show you

L40 – L45

TT: tell me what makes these ones particularly appropriate for today's lesson? These activities?

Lineo: umm I think what is nice about these activities is that they are starting off with all the definitions, what is a variable, coefficient, constant, grouping like terms, adding and subtracting and all these are not too advanced it will be manageable for the introduction

L46 – L49

TT: Have you ever used them before or this is the first time you use them?

Lineo: These worksheets?

TT: yeah

Lineo: I used this worksheet last year

L50 – L58

TT: what was good about it since you are using it for the second time?

Lineo: eeeh, last year I had the bottom six and now they are in Grade 9, all these exercises cover different types of questions, the simplifying, what is coefficient, writing as power so it covers all of the topic and these are enough for each of the question to cover all the work so it's a good foundation and I think most of the teachers, in my understanding will use nothing else, these will be enough.

TT: for the unit?

Lineo: yes

L59 – L66

TT: what do you think the learners will find interesting or challenging?

Lineo: about this?

TT: yeah

Lineo: … about today, today is about differentiating between x and x^2, they get confused what the 2 resemble x^2 *is* x *times* x, there are 2 lots of x and I will clarify today so that these boys do not struggle with these concepts. The x^2 *versus* $2x$ and the other goal is to try and make them understand that xy *will be the same as* yx.

L67 – L76

TT: what do you think will be the learners' problems as they engage with these activities?

Lineo I think the biggest problem would be being comfortable with working with letters as this will be the first time they do this exercise, that will be a big challenge for them, that's why I'm going to start slowly with what are like terms, what does it mean when I say $3x$?

TT: so you are saying that's how you will deal with the problem, you will start slowly

Lineo: I think another problem that will arise is with exercise 2(j) where you have $5a - 2a$ which term does the minus belong to?

L77 – L84

TT: in planning for today's lesson did you make any assumptions?

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Lineo: my assumption was that they are a bright group, I see myself spending more time walking around the class helping individuals rather than standing at the front most of the time.
TT: ok
Lineo: I foresee more of the lessons to be more on one on one because the boys are quite sharp, they grasp quite quickly. For your records the Grade 8, start from A_1, A_2, … A_6

L85 – L95

TT: you told me that Stephen gave you the worksheet and I think you made your own decision to include them so I want to know what informed your decision to include them in today's lesson?
Lineo: this worksheet?
TT: yeah
Lineo: Umm this is past experience of using this worksheet, I know it works and also I don't want to reinvent the wheel, this worksheet covers all the work umm… yeah and I will see from today's lesson whether I have to create something more and also is the textbook Classroom Mathematics that we have used for years and years… we know it work so it is reliable.

L96 – L106

TT: I just wanted to know from the curriculum, I tried to read it, it reads like learners have to learn to reason, I want to know what type of tasks will enable learners to reason?
Lineo: From these?
TT: yeah
Lineo: it's the introduction that I am going to put up, hopefully that will allow them to reason. You see that I usually cut out a whole lot of pictures which they put up into different categories for like terms and then they reason as why they put them into that category. I will be using the data projector to project them. I am going to project pictures on the board and then they can count….and also by the type of questions I ask will facilitate the reasoning.

Post-teaching interview

L1 – L20

TT: I think this one should be shorter, umm in the morning basically you highlighted some of the difficulties you anticipated that the learners would have, now that you have gone through the lesson ; what do you think have been their problems, you know in getting through?
Lineo: umm, I think one thing would be the incorrect order when they got that one with xy, yx and they did. I thought I had that sorted out until when one boy suddenly wanted to put everything together and take pieces and umm… I thought I had dealt with immediately in that slide but they are still having confused, there was still a problem to doing their own thing umm… what else?

WitsETD

TT: Umm it's Ok, it's fine, it's fine, umm… what could you attribute some of those problems to? You actually highlighted earlier that you tried to clarify some of them but you still [interrupted]
Lineo: you still have them doing their own thing, I think one boy answered correctly I think what I am going to do is [inaudible] and I know there might still be a couple, it's difficult when you are teaching and the majority of the class is understanding, you do not want to spend the whole 5 minutes dealing with the one issue like one boy trying to clarify why he got it wrong, I am going to keep my eye for … when I am doing
g the exercise when I can do one on one, because it's a difficult dilemma because if I pinpoint him I might omit others , as a weaker student in my class does like to be highlighted in the whole class so I think I clarify with him and then I will do a follow up.

L21 – L28

TT: suppose we come to you, you planned a lesson, you wanted to implement it did you find yourself encountering problems as you implemented the lesson?
Lineo: I did, some of the questions required, sorry [a learner enters and talks with Lineo]… it's a funny question like that … in terms of the conceptual umm, problem umm let me try to think, what could have been the problems that again that one boy who tried to combine it, if I had put in values for a and b I think that could have been another way of dealing with the problem.

L29 – L52

TT: this time I would like us to talk about the more personal information because I would definitely need it, like for how long have you been in this school?
Lineo: this one alone?
TT: yes
Lineo: 2 years
TT: which grades have you been teaching since you arrived?
Lineo: at St. Johns? Grade 8 to post matric.
TT: which subjects?
Lineo: mathematics and additional mathematics grade 10, 11, 12 and post matric
TT: how many Grade8 are in this school?
Lineo: they are 6
TT: and how many teachers?
Lineo: they will be 6 because all Grade 8 are taught mathematics at the same time
TT: so how do you work together because you have 6 classes and 6 teachers?
Lineo: at this school we pretty much work by ourselves, i go by my own pace, and I know that at the end of the term, I need to have covered these sections. I might be ahead or behind other teachers, we do not share much of the resources we pretty much independently.
TT: How do you work in terms of test?
Lineo: set our own tests
TT: individually?
Lineo: we do set standardized tests where all the Grade 8 write same test at the same time. Other than that we set our own.

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L53 – L61

TT: can you describe your role in the school, perhaps in the department or in sports?
Lineo: previously I was in a girls' school and I was involved in sports and cultural activities but in this school my role is a mathematics educator primarily
TT: how does the mathematics department assist you as Grade 8 teachers?
Lineo: in this department you need to be proactive, if you have a problem, you need to seek help from other teachers, other than that we assume everyone is working well and is fine. If you have something to share like a worksheet you can come forward and say can we share this worksheet.

L62 – L68

TT: you said you have taught in another school, for how long have been in that school?
Lineo: 3years
TT: and which subjects did you teach?
Lineo: maths and additional mathematics
TT: do you teach other subjects?
Lineo: no maths and additional mathematics

L69 – L78

TT: and tell me about the qualifications which you have?
Lineo: B.Sc in mathematics and chemistry, B.Sc (hons.) in Chemistry, teaching diploma, and one year certificate in Basic Adult Education and I am currently doing Become.
TT: why are you not teaching chemistry?
Lineo: it's funny people ask me that because I more qualified in Chemistry than in mathematics is the subject easier to explain but chemistry it's just a subject in my head. I have not figured it out. If somebody comes with a question in chemistry I will need to think about it but if somebody came with a mathematics question I would be able to explain

Follow up interview – interview 3

L1 – L16

TT: what do you think went well in the lessons you taught while I was here?
Lineo: yeah, umm in all the lessons?
TT: yeah, all of the lessons, what do you think went well?
Lineo: umm for me what went well is that I think I caught their attention with the soccer diagram at the beginning, umm I tried to make that not as umm that irrational or that conceptual, that move from arithmetic to algebra, I tried to link it whether I succeeded or not I am not sure. I am not convinced that is… all I think is that it helped some and not others. I think some were still not able to make the connection between what I was trying to convey and… so umm yeah I am undecided on that, umm [silence] I think my general style of teaching allowed them to ask (interrupted)
TT: questions
Lineo: questions hmmm… I am happy with their questioning though some boys were still not comfortable that they could ask questions, again may be I could address the

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problem from a different point of view and also the presentation… trying to do pictures, the numerical and also my algebra linking it all in, hopefully it helped some and also the algebra looking at …

L17 – L24

TT: Ok, is there anything that you think that this one did not go according to my plan and most probably why you think it didn't go well?

Lineo: there is one part that I realized afterwards that what I haven't taught correctly was umm the coefficient (interrupted)

TT: coefficient

Lineo: what does it mean to be a coefficient umm or what is a coefficient? Umm I realized then that there were… with the issue that got raised a couple of lessons down the line which I thought I had dealt with them at the beginning. That's the one thing.

L25 – L35

TT: that's the one thing; can you give me the examples of the issues that came up?

Lineo: when I say what is the coefficient of y in $10xy$, they were not certain whether they should give me $10xy$, $10x$ or 10 whether it was positive 10 or negative 10 or umm I don't know whether they had an idea of what a coefficient was or it's role umm I was talking about the stuff before the x, they also didn't know whether to include the entire expression if I have $5+10xy$ whether they need to say $5+10x$

TT: the 5 in front

Lineo: so they … (interrupted)

TT: did you rectify that later or what did you do?

Lineo: I think I did it later and the boys then became happier with the meaning being clarified

L36 – L42

TT: now the activities for that day, I remember you started with guess my rule, that's the first thing I remember we did. What I want to know is; why did you start with guess my rule??

Lineo: we did guess my rule first with number patterns, we used the letter n, finding the n-th term, so I was trying to link in, they have used n before and we have discussed what (unclear) of the n. I needed to put this in a context, they had seen before but now we are going to formalize it.

L43 – L53

TT: Ok Ok. So in that case what do you think is good about starting algebra in this way? Like starting with number patterning towards the simplifying, what is what, what do you think is good about this approach?

Lineo: about the order (pause) I think some of the boys had done number patterning in the primary school, a lot of them had seen it; they were familiar with generalizing the patterns, the n-th term. Whether or not they understood what it means I am not certain umm so I really needed to start with the section which they are exposed to and then (inaudible) its quite an isolated section with number patterning, finding the n-th term is not overwhelming like x's and the y's, they were happy with n-th term.

TT: with only one letter?

Lineo: yeah only one letter, so … (interrupted)

WitsETD

L54 – L60

TT: if you were to look at the activities of guess my rule, they were 3, what did you expect the boys to learn from these? Because they way you did it was give me the number and I will give back the answer, what will they have learnt after these activities?

Lineo: they wouldn't have learnt anything new because they have done this before, they know how to generalize in guess my rule, and this wasn't new. It served as a reminder

TT: so you were recapping

Lineo: there was nothing new at all

L61 – L67

TT: that is they way you introduced the algebra, are there other ways which you know that could be used to introduce algebra? [silence] you said you worked from number patterning, so what I want to find is; are there other ways that could be used in introducing algebra to the boys?

Lineo: mm, word problems could also be an introduction but umm that would be more for equations but could highlight the algebra Umm so word problems could be a good starting point and they don't rely on quick thinking.

L68 – L96

TT: Ok, let's look at the activities, the activities if I quoted them very well were 3, the first one resulted in $2n+4$, the other was n^2, and the last was $-2x-1$. So I want you to tell me about the sequencing, why did you sequence them like this?

Lineo: I can't remember (interrupted)

TT: what happened was that they were giving you numbers (interrupted)

Lineo: Oh for guess my rule

TT: the first expression was this one and then it was the square number and the last was this one of negative number

Lineo: Ok

TT: so I was interested in the sequence, did you plan it such that it goes this way?

Lineo: No, it was all spontaneous. I think I might have started with something a little bit easier like $n+5$. Umm probably because they are comfortable with number patterning so I started with this. Umm I wanted them to times and add, and then to jumble it a bit I suppose I thought, let's do n^2 and that is my logic for thinking and I was able to judge from their facial expressions, if they were not able to get that one right, I would have dumped it down a bit I also did not want them to have the mind set that they are all times and add.

TT: I am thinking about dumping it down, do you have the up as well? When do you dump it up?

Lineo: I don't know

TT: because what you are telling me is that if they had problems you would have simplified the expression, when do you decide that now I am moving it up?

Lineo: I think by looking at the general response, emotions and the physical expressions when they get it right or wrong. That will be my cue to say Ok we don't

WitsETD

want them to get bored, let's give them another challenge. Let's keep them on their toes.

TT: Ok, Ok, so rescaling up would mean putting more challenge to the boys

Lineo: you see this is not a new section. I don't want them to go away with an attitude that ooh number patterning is easy if they manage to get them all right. I need to make sure they are on their toes.

L97 – L106

TT: when you got to the last one you said "tricky one now" (interrupted)

Lineo: yeah

TT: tricky one now and I asked myself why you are saying this one is tricky as compared to the other ones. What do you think was tricky in this question?

Lineo: because now the numbers I was giving back were negative numbers and umm even if they are quite good… given a number I get a bigger number, with negative numbers they were getting exceptionally smaller numbers, so what am I doing? I get very small numbers so their thought process gets reversed. Umm (silence) they are so used to the n^2 and the $2n+4$ now suddenly when they give me a number, it suddenly becomes negative and very small, how do they explain what is happening?

L107 – L111

TT: Ok but then looking at the activity to me it had similar signs, two negatives; it looked like the first one. Do you think it was different from the first one ($2n+4$) if you look at it?

Lineo: umm just in terms of the positive coming out of the negative, I think it would not be different really. Umm…

L112 – L117

TT: what problems did you expect the boys would have in working with these activities?

Lineo: umm… I think the reason probably why I said trick one now is probably to make the boys feel at ease so that if they don't get it right it's Ok. It's a difficult one, let's give it a thought it's not going to come quickly and let's be patient and umm take time to think of it. Umm they … other boys would say Mam I can't think of it, umm I am stupid Mam, what I don't want them to do so

L118 – L129

TT: the next thing I would like us to talk about will involve what you see there. Tell me about the soccer kit activity, why did you put as the first activity?

Lineo: for visual, for them to umm to have a pictorial aspect in maths so that they can start to visualize, when I talk about x and x^2. It gives visual cues. Let's give them a visual cue so that they can think that it really makes sense that we can't group the shirts and shorts together in the same group, they are separate and the subject of the world cup was happening at the same time I think the world cup was on Saturday or Sunday and this was on Monday.

TT: so basically if you are to look at it are you saying [silence] what did you expect them to learn from this?

Lineo: mere grouping umm and identifying why are these different and they need to group them into different categories.

WitsETD

L130 – L138

TT: Ok, did you foresee any problems or any difficulties that the boys would have on working with this activity?

Lineo: soccer

TT: yeah

Lineo: I have done it before with pieces of paper but umm I found that kind of activity stupid. So I thought I can do a quick activity on the board so that they don't waste their time putting all the pieces together physically doing that. But I haven't used power point before.

TT: OK OK

L139 – L170

TT: If you look at these activities, these activities are quite interesting because I want us to look at the order of presentation. I tried to … I don't know whether you will see, they are going this way. They start there, come here and end there. Now why did you sequence them like this? It was first the soccer kit then the linear one, the quadratic, the numbers, the algebra, numbers and then the algebra. Why this particular sequencing?

Lineo: [laughs] it looks a bit odd but it is my thought process. Starting with the pictures into the umm the letters makes the link so the soccer kit will show the boys why we put the x's together and the y's together. so that was the reason for the simplification from the soccer kit into the variable, then the next one was merely to prompt the question what if we have the x^2 individual and the x's, are they the same things? So I didn't want to umm I don't want to give the answer because the 9 and 3 are obviously different. I wanted them to think about this before I make it obvious. So rather than saying x and x^2 are they the same or are they not the same? Umm what do we mean by x^2? May be we don't know the answer. obvious numbers of course, this one was I don't want them to be straightforward because here they would have said 3 and 9 of course are different and if I then put x and x^2 next , they would have said; oh no x and x^2 would be different but I needed them to think about what is the relationship with algebra? That's the reason (interrupted)

TT: why you sequenced them like that.

Lineo: like that and then again we have do the x and x^2, let's see what happens when we have xy and yx so that was the relationship between why that x^2 and x, and then the xy and then what do I mean by multiplication. Again pre-empting I don't want them to have the opposite. They need to think about the question. xy and yx, why is that and then numbers, 4 times 2 and 2 times 4 are they the same? I wanted them to start thinking algebraically before (interrupted)

TT: before they come to numbers

Lineo: had to start with (unclear) oh yeah if I had reversed it they would have said 4 times 2 and 2 times is the same and when I put up the next slide xy and yx, they would have said it is the same. That's what I was thinking (interrupted)

TT: Ok I see

WISER

Lineo: and again mixing them all up just to consolidate for the ideas that have come through, what do I mean when I have a^2, ab and ba?

L171 – L178

TT: now after this activity, what do you think the boys would have learnt?

Lineo: hopefully they would have learnt that algebra is not a bunch of letters where they can re-create their own rules, there is logic behind it and each letter represent a number, each letter represent a different number and why is xy and yx the same because of … just for them to conceptually understand that umm letters represent numbers, that umm what do I mean by like terms, to group and why x and x^2 would be different numbers. So I was trying to develop the relationship between the numbers behind the variables.

TT: Ok, behind,

L179 – L184

TT: do you think the boys experienced the any problems in going through these activities?

Lineo: I think they grasped very quickly, whether they remembered long enough for further exercises, it's another issue but I think up to this point I would say the boys got it, I think they knew, I think, it's my perception umm whether or not it had a deep enough impact I don't know and that's why I got some boys that are struggling

L185 – L197

TT: the other part which I would like us to talk about was where you defined term, you remember the variable the constant and the coefficient. Actually I tried to put the whole lesson in because it was interesting. So I wanted to find out from you why do you think these definitions are important in algebra or rather in introducing algebra to the boys? Why do you think definitions are very important?

Lineo: I think the boys, especially the boys and also the girls they need to know that umm our terminology in mathematics is important umm when I am talking about umm a constant they need to know it's a number, it has got a role and they need to know that umm mathematics language is important, umm and that it will be to their detriment in future if they don't learn to speak mathematics properly. Umm so I will always emphasize the definitions and the terminology and when they are giving me back an answer if they are not saying it correctly I will correct them. I think it's important that they don't malinger.

L198 – L207

TT: so in this case, do you think this approach of language, definitions helped the boys?

Lineo: I don't think it helped them, I think it's just that when I started my next lesson and I start talking about terms, constant and others like that at least they will know what I am talking about. I as a mathematics teacher I cannot umm for too many lessons talk about the stuff before the x or umm just a number with no x. They need to be familiar with it in my conversation with them that I am talking about the coefficient they need to know what I am talking about because also if I start talking about the stuff before for too long or the number then it creates a lazy attitude on their

WITSHED

part because that is the language they will adopt themselves, then they will start talking about the stuff before as opposed to coefficient.

L208 – L235

TT: OK, OK that was another… you know I like quoting you (interrupted)
Lineo: I don't like you quoting me (laughs)
TT: but basically, in fact to show you exactly what I am going to do so that you get a feel of (interrupted)
Lineo: how your project is
TT: yeah because I will be quoting you. At some stage you said "are you guys getting a feel of algebra" I don't know whether you can recall that (interrupted)
Lineo: laughs
TT: what did you mean by getting a feel of algebra?
Lineo: umm the boys didn't look at me oddly, they were comfortable with my question, so obviously I have used it in the past and they were familiar with what I am saying, umm for me I am always aware that algebra is so obscure for them, I mean all the way from the primary school up to now in Grade 8 they have been doing arithmetic really, problem solving whatever… The x's and the y's and umm I need to give them an understanding of, are you guys starting to comprehend the reason for algebra. What I mean by x's and y's. it's not just letters being uttered out of my mouth umm are you getting a sense of comfort rather than apprehension that this is completely new, are you starting to be comfortable… for me talking about ab, ba are you feeling Ok? (Interrupted)
TT: ooh, are you comfortable? (Interrupted)
Lineo: yeah
TT: because I don't think I got their answer after that, what did you expect them to say?
Lineo: there are a couple of boys that in the past would say, no I am not getting the feel, I am not, what are saying is I am not understanding umm and possibly I stand to be corrected also when I say, are you getting a feel for it? Is not saying are you understanding because if they say I am not understanding then they are saying I am not good at mathematics but if you are getting a feel, it's I could get it, I am in the right route, I am there, may be not I am not yet there but it's feeling Ok, instead of saying oh, no no, I am not good at mathematics, all others are good and they get it and I don't get it.

L236 – L262

TT: the other thing which I wanted us to talk about was the exam performance, how have the boys performed?
Lineo: terrible
TT: what was wrong? (Lineo laughs)
Lineo: that's my honest response. I will tell you the problems, the problem was this was the first exam they wrote and I never had time to test them on algebra formally before the exam, some boys are good at writing exams some are not. The send issue was the way we structured our exam papers, the first paper they wrote was problem solving paper and it was fairly straightforward for my boys because they are fairly bright, they did nicely in it. They were doing well for in mathematics, so when it

WitSED

came to writing the second paper they didn't do any work for it because they had this I am good at mathematics, I don't need to work with the mathematics.
TT: what was the second paper about?
Lineo: the second paper was about skills, the short questions, the predictable questions, they all got comfortable and felt good in the first paper, they did no work in the second paper umm that was the second issue, the third issue was that the skill paper was the very Last paper they wrote in their exam, umm so again there was no motivation to study, it was their last exam, they have really done well in the problem solving paper… And I spoke to the boys afterwards when they returned and they admitted, that they don't know how to study and they didn't have the motivation to study and unfortunately with the algebra being a new section for them, there are rules build into algebra, like terms. And they messed up
TT: with the rules?
Lineo: mm they did which was (unclear) I am generalizing but not all of them, they did fine but they could have done better, from where they were before the exam started they were getting it. They knew what algebra was about but 3 weeks or 2 weeks into the exam session they had lots of problems

L263 – L284

TT: I saw this kind of problem where they were combining everything, did this kind of a problem (2a + b =3ab) come up in the exam? The one I saw in the lesson was on 5x + 3x + 2b (interrupted)
Lineo: Ok and they made it 10xb, yeah
TT: Did it come up in the exam?
Lineo: yeah, question … (could not find the question) But I think if it was in the exam they might have mixed it up.
TT: I also wanted to find out what other approaches are you using to address that problem?
Lineo: when we got back in the third term, I discussed the exam and what went wrong and then I spend a week and half still revising algebra. I needed to make sure that the results which I thought were not great were not indicative of their understanding and this confusion did arise umm I just emphasized, I repeated a lot, is x and b the same? Yes/no I just added a lot more rule based. I emphasized the rules more for those who were not getting it if you are adding or subtracting ask yourself; are they the same? In case it does not come up what I did was a revision for a week and half and then I gave them the test, in this section (a) I extracted some questions from the exam. I needed to put more for my own sake to make sure that they knew what is happening in algebra. Umm because they are in term three, the middle of the road they need to carry along the stuff. I used the same questions and I here I added my own. They did well here.

L285 – L297

TT: it's like the condition; exam condition has something to do with the performance going down
Lineo: I think also if I had said boys come let's have revision they could have been perfect.
TT: this activity here, why did you think that this activity is tricky?

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Lineo: it's a lot difficult (unclear) first they needed to recognize that they need to add the x's and then the y's. Umm the second thing which was different for them is to umm, initially I think is umm the sign like in -3y, that negative belongs to the 3 they cannot forget about it. It is not positive 3y. They need to take into account the sign as well as the coefficient. So that was the big emphasis umm because they do turn to forget about the sign.
TT: so that is why it was said to be tricky.
Lineo: yes

L298 – L328

TT: the other activities, ooh they are there, that number 30. Why did you choose these activities in particular?
Lineo: these ones which were written on the board?
TT: yeah, these ones which were on the board. Why did you choose them? And the sequencing as well, they are quite interesting (Lineo laughs) you always use something, I don't know what that something is but there is something (interrupted)
Lineo: there is a reason in my head
TT: yeah, why, why this order?
Lineo: Ok, I said number 1 purely from adding and subtracting like terms and again emphasizing the negative to be included… Purely the x and x^2 we dealt with in previous like terms, they were familiar with it and the second one, umm the reason why I say it follows from number 1 is that they are fairly straightforward. Number 2 says hold on we don't always need to have a neat and simplified answer. It's going to remain the same and that's Ok, it was just to say think carefully, it's ok not to have a simplified answer, not always will the answer be simplified. umm number 3 Ok we have done the adding and subtracting, let's look at multiplication umm in number 1 we had x and x^2, adding them together, now we have x and x^2 in multiplication, what am I meaning by x^2 times x as opposed to x plus x^2. So this is the new rule, new scenario, what is happening with x^2 times x. Umm and then number 4 we say Ok we have done x^2 and x, now let's put in the numbers in the numbers with it, what happens now if I have got 4 and 2? How do they deal with each other and how do I simplify the coefficient? And then number 5 now dealing with the number and more than one letter.
TT: Ok, that's how (interrupted)
Lineo: that's my logic (laughs)

L329 – L338

TT: let's look at this, you know I got interested when I got here because to me it was like there was a change in the meaning of that word (simplify)
Lineo: mm
TT: so did you expect that the boys would see that change?
Lineo: what do you mean there was a change?
TT: like here it was collecting like terms but here it's like you expand and then re-group. That simplify is no longer collect like terms.

WITSRD

Lineo: I think they realized that simplify is make smaller so they know there is a process, like the first one we needed to gather like terms, it was a process in getting to the simplified answer and in number 2 we can't group and that's Ok.

L339 – L349

TT: so basically here, what was the expectation? What were the boys expected to learn?

Lineo: I think umm primarily umm to distinguish between am I adding or am I multiplying? If I am adding, I follow these rules or this thought process, if I am timesing I need to follow this thought process. I don't teach them the rules, I don't teach them umm x^a times x^b is x^{a+b} umm they need to think about it logically, think about the rules. I would rather… even now I still get them to work out, what is x times x^2, how many x's are being multiplied rather than just saying (inaudible) so the first thing was to distinguish between am I adding or multiplying, If I am adding what do I need to do and if I am multiplying what do I need to do? And the role of the numbers in each of those scenarios Like in adding I need to worry about the negative sign and when I am timesing what does… how do the numbers play their role?

L350 – L362

TT: what I realized when you were teaching which I think we agreed on earlier was that you used verification. Why did you use verification? Was there another way you could have approached that portion?

Lineo: the same (interrupted)

TT: like when you came to xy and yx you used 16 and 3 to check that xy and yx are the same. So that every time you give them xy + yx they say it's 2xy. To me its like you were moving between algebra and arithmetic. Why that?

Lineo: I think this is because this is their initial umm taste of algebra umm they need to know that again with the number behind and the letter that if they are stuck, they should put in numbers, see if the numbers work for you and is not only magic numbers that work, all the numbers work. So umm I needed them to make the link that this is not an obscure concept we need to go back to numbers and experiment to see if this makes sense.

L363 – L387

TT: suppose you were not using numbers; is there a way that you could go through this activity? Is it possible…? (Interrupted)

Lineo: to introduce algebra without number?

TT: numbers

Lineo: without the numbers?

TT: yeah

Silence

TT: is it possible? I was wondering because when I looked at the boys they seemed to enjoy the activity, it was easy, also I could see that that was the easiest way of doing it but I asked myself; is there another way of doing it?

Lineo: without numbers?

TT: yeah

WISERD

Lineo: possibly, I don't know, like pictorially, like… I don't know, this is catching me off guard (TT laughs)
TT: it's quite interesting because (interrupted)
Lineo: because you again with the soccer kit you should, I don't know. You know sometimes how I do it as well, I talk about aliens, I talk about x and x^2 being different aliens. x's belong to one planet and the x^2 have got growth out of their heads, this square so they have got this umm growth, now these aliens are different from the x aliens, do they belong to the same planet? No!! So the x's belong to one planet and the x^2 belong to another, that's why they are separate. Then the xy I have also done it in the past umm x is one part of the alien and the y is another part of the alien but they are interlinked, if you turn them around it is yx, are they of the same object? Are they not the same alien? Sometimes I refer to aliens and the square is the growth and the cube would be a different bigger growth.

L388 – L414

TT: I am coming to the last part, this is not related to the lesson I observed, I have to admit that that it is not from own interest. In fact I did not get it from a Grade 8 textbook. I got it from Grade 10 textbook, in one of the new books where they deal with number patterns. It may not be exactly like this in the textbook. So it is actually above Grade 8 (Lineo laughs) but it is believed that this kind of tasks promote algebraic reasoning or mathematical reasoning. So was my interest was to find out from you is whether you have ever come across this kind of task or discussed it with the other teacher?
Lineo: I have come across this before, well not this exact one but similar one, but umm honestly I would give it to my grade 9. It would be interesting discussion umm, a nice investigation actually umm (interrupted)
TT: so you think you can give it to the boys
Lineo: mm
TT: what, what do you think the boys would need to do in order to engage with this task?
Lineo: I think what they need is … for my boys would is… may be your top group I will leave it like that, for my boys a little bit of scaffolding umm part (a) would be when n is 1, is $n^2 + n$ be odd… when n = 2 if you start the process to start in the investigation that's all I would do. Just to start them off because they would say I don't know what's going on, how are we to know? So you train them which we do with our Removes (and name for Grade 8) with number patterning we always start with n = 1 and then build them up. Grade 9 and 10 we don't scaffold them. They must know to try (interrupted)
TT: their own way
Lineo: yeah
TT: with your top boys, you through them in the deep end (interrupted)
Lineo: although I don't know whether scaffolding would actually help, it might spoil your question. They can try any number they want

L415 – L422

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TT: if you were to give this task to the boys with or without scaffolding, what do you think they would learn by engaging with this task?
Lineo: I think they would have learnt that investigative techniques, sometimes your answers are not the same and so forth. Everyone would need to do some trial and error umm I think it would teach them some patience in doing trial and error umm so I just want them to explore that umm is the answer gonna be always even or odd, and if it is going to be even or odd why. So I think umm it requires a lot of natural thinking umm, yeah like this

L423 – L430

TT: what do you think you would need to do so that they could solve it completely?
Lineo: I would leave them to do the work and also I will need to do is that none may call the answer, that if they get the answer they could show me and I would say no or yes to see what their logic is. Because if they call their answers some might say aah man; they call out their answers, now I don't have to do it. So I think that will be the first thing, I don't need to show my intelligence to them. I think that's what teachers like to do, they interfere and show the boys that they clever too. So I will back off and then prevent them from calling out their answers. I think I would like to give them this in a test rather

L431 – L439

TT: but now if you put it in a test, it's like there has to be a total mark and then if there is a total mark now that portion… and I think that justification cannot be completed by all learners
Lineo: I think the marks would be allocated for the working and the thought process umm say out of 4 marks, 1 mark for your final answer and the other 3 marks for the working. Umm if they tried 1 example and came to a conclusion then they get 2 out of 4 if they have done a lot of examples and investigation then you can scaffold the marks. They also have a tendency… (Unclear) other people's knowledge in a group environment umm… I think it would be nice to see how each individual makes progress and I suggest you could do it in silence for 10 minutes or 15minutes. That might be another option.

L440 – L450

TT: I think I have taken too much time (interrupted)
Lineo: no not at all
TT: I have come to the end of what I wanted to ask unless there is something that you would like to talk about
Lineo: I talked to the other teachers about how their boys have performed in algebra in the exam and all complaint that the boys did terribly so I didn't take that personally as a reflection of my teaching, that's it.
TT: let me ask, were the problems the same?
Lineo: yeah
TT: I would like to thank you for your time and for allowing me to learn from your teaching. I hope you will pass my special thanks to the boys.

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APPENDIX 4

Time	Transcript	Mathematical Problem solving	Mathematical Problem confronted	Nature of activity	Knowledge resource	Concept
00:00 – 02:06	**Lesson 1** Lineo: Put up your hand if you have done some algebra, put up your hand if you have done some …. For those guys who have done algebra some algebra, I am going to assume that the entire class has not. So just bear with me umm… hopefully I will be reminding you of some stuff. May be I will be teaching some of you some new stuff as well so bear with me for today's lesson especially. Lineo: now take out a piece of paper, scrap paper you are not going to hand it in, it's going to be rough, or your home work books somewhere for you put some answers. [silently moves towards the board turns and picks a paper on her desk, returns to her original position infront of the boys] Lineo: [standing infron of the middle columns] Ok, remember when we were doing number patterning, we were playing guess my rule. I am going to start today's lesson with guess my rule. It's just a recap, do you guys remember how guess my rule worked?... remember you gave me the number, [sits on her desk] and I have got the rule in my head and I give you back a number, remember, ok, let's see, we will just do 2 or 3 (points at a Lerato) number? Lerato: 4 Lineo: let me first come up with the rule (writes on a sheet of paper) Ok, your number? Lerato: 4 Lineo: ok 4 and I give you back 12. Number, Sithole? Sithole: 1 Lineo: 1 and I give you back 6. Thabo give me a number. Thabo: 5 Lineo: 5 and I give you back 14. Bheki, give me a number. Bheki: 3 Lineo: 3 and I give you back 10. Lethabo give me a number. Lethabo: 2	-Explanation Questioning Representing		Representational	Mathematics (rules & conventions)	Generalizing patterns

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	Lineo: 2 and I give you back 8. Ok think of the rule, if you have got it put it down					
02:17 - 03:38	Lineo: Ok next rule, I have got the rule in my head. Thato give me a number Lineo: 5 and I give you back 25. Peter? Lineo: 7 and I give you back 49. Khau? Lineo: 1 and I give you 1. Last one, Henry? Lineo: 3 and I give you back 9. [learners silently write the rule in their books] Lineo: ok tricky one now. Last rule. Mpho give me a number Mpho: 1 Lineo: 1 and I give you back minus 3 or negative 3. Ts'eli? Lineo: 2 and I give you back negative 5. Lisebo? Lisebo: 6 Lineo: 6 and I give you back negative 13. Benni? Lineo: 3 and I give you back negative 7. Lefty? Lefty: 0 Lineo: zero and I give back negative 1. Write down the rule. [learners write silently]	Questioning Questioning Representing	Representing Representing	Representational	Mathematics (rules & conventions) Mathematics (rules & conventions)	Generalizing patterns Generalizing patterns
03:39 – 04:46	Lineo: ok without shouting out, I will pick on people, the first rule. Who managed to work out the first rule? Themba? Themba responds and the teacher stands up and walks to the board. Lineo: what times 2? (Writes 2 on the board) tell me what you said before everyone told you what they thought you must say. What did you say? Ok you said x though, so you said … Themba: x times 2 plus 4 Lineo: (writes on the board) $x \times 2 + 4$. Is he right or wrong? Why? Lineo: you are saying he is right Lineo: Thembi, you are saying he is wrong. He is right. why? Why do you change your mind? Why do you think he might be right? Thembi: because x and n I don't think, they are (inaudible) Lineo: Ok let's have a look. Other people what did you say with the n ? One learner gives the answer in terms of n Lineo :(writes down the answer) $n2 + 4$ below $x \times 2 + 4$. Ok we shall come back to that	Working with learners' ideas Questioning	Use of other letters (than n)	Representational	Mathematics (rules & conventions)	Generalizing patterns
04:49 –	Lineo: question number 2, who managed to come up with that one? Themba?					Generalizing patterns

WISETD

06:16	Themba: n squared Lineo: [writes n^2 on the board] how many of you got number two correct? (Learners raise up hands) ok most of you. Well done (a learner suggests $n \times n$) Lineo: or $n \times n$, Thabo wrote $n \times n$. Eeeh number 3, Hilda? Hilda: n times negative 2 minus one. Lineo: (writes as the learner speaks) $n \times -2-1$. Like that? Ok correct, did anybody write that in a different way? Ok the minus one is correct. Khau how did you write you thing? Khau: n minus 2 minus 1 Lineo: $n-2-1$. Like that? Ok let's talk about that. (another learner suggests) Bongane: I wrote minus 2x minus one Lineo: you wrote (writes $-2x-1$) like that? Hape? Hape: I wrote n minus bracket n times 2 bracket minus 1 Lineo: (writes $n-(n \times 2)-1$) Lineo circles the n outside the bracket) I don't want to go into too much detail but if you treat it like one of my examples, this would be causing a grieve, that will spoil it. Other than that everything would be perfect. Lineo: ok now we are not doing number patterning with this new unit. We have done number patterning but this was a nice introduction to the algebra.	Questioning Working with learners ideas Questioning Working with learners' ideas	Writing algebraically	Representational Representational Reading/writing Transformational	Mathematics (rules & conventions)	
06:18 – 06:47	. What is algebra? Themba? Themba: using letters to symbolize numbers Lineo: Themba, using letters to symbolize number. So with Thaabe's question with 1, with the issue that was raised. as sooon as he said x everybody said nooo… it has to be n, we have always done finding the n-th term. Why is now that Thaabe comes up with his own letter, surely it must be wrong. Hello, you have put your hand up, Thabo? Thabo: it is because it does not matter what letter it is; it does not change its value	Questioning Defines Working with learners' ideas	Variable	Reading/writing Representational	Mathematics (definitions	Variable
06:52 – 07:20	Lineo: ok, perfect remember with this letters, they can represent any number in the whole wide world and for the algebra we always choose little letters a, b, c, d …the whole alphabet to choose from. You have got 26 letters, I could have come up with $j \times 2+4$ because I think j is for	Working with learners' ideas Explanation	Letters standing for numbers	Reading/writing	Mathematics (definitions) Experience	Variable

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	Jacob and Jacob is rules the world and that would be my choice of the letters. You could have had any letter.	Representing			(professional & everyday)	
07:21 – 07:31	. Looking at number 2, n^2 is it the same as $n \times n$, is that correct? What does n^2 mean? Naughtyboy: $n \times n$ Lineo: $n \times n$ that would be perfect.	Working with learners' ideas	Reading/ interpreting algebraic notation	Representational Reading	Mathematics (rules & conventions)	Generalizing patterns
	Back tracking, sorry (circles $n2$ in the expression) is that how we traditionally write out the notation? Thabo? Thabo: no Lineo: no, how do we traditionally write it. Thabo: we write n times 2 plus 4. Lineo: (writes $n \times 2 + 4$) now which way is neater? That way (points at $n2 + 4$) or that way (points at $n \times 2 + 4$) Learners: the first one Lineo: the first one, I really prefer that method (points at $n2 + 4$) of writing it but Disky: 2 times n plus 4 Lineo: (writes $2 \times n + 4$). Which way is neater? That way or this way? Learners: the first one Lineo: Ok, traditionally we always put up the numbers before the letter. So $n2$ is correct but we prefer to write the number before the letters. Also we don't write the times between a number and the letter. When I write $2n$ what are we assuming? 2 times n.	Questioning Working with learners' ideas. Explanation	Writing algebraically	Representational Reading/writing	Mathematics (rules & conventions) Experience (professional)	Mathematical convention
07:32 – 09:00	The same thing here with the x why did I write the curly x? Mpho: we do not want a straight x Lineo: Don't call it out. Thabo: inaudible Lineo: Ok, as Thabo said, if I were to write that curly x as a straightforward x what does that look like? times times 2 or xx2 doesn't make sense. So guys I'm going to be very strict with you, when you write your x's, it is the only letter that you have to change. I want you to write it as a curly x neatly. You neat to get into that habit. I will take off marks if you don't	Questioning Explaining	Symbol conflict	Representational	Mathematics (rules & conventions) Experience (professional & authority) Curriculum (assessment)	Mathematical convention
09:03 – 09:40	Lineo: next thing, question number 3 which one here is the problem one? (points at $n2 + 4$, $x \times 2 + 4$ and $n - 2 - 1$) Out of these 3 choices, 1, 2 and 3 which one is the problem one? Thabo?	Working with learners' ideas	Writing algebraically	Representational		Generalizing patterns

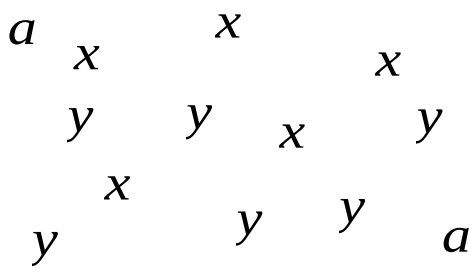
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	Henry: n minus 2 minus 1 Lineo: (draws a rectangle around $n-2-1$) Ok he is the problem. What was the person trying to say? This is turning out to be n minus 3 (writes $n-3$) n minus 3 is different to saying n times minus 2 minus 1.					
09:42 – 09:51	Now I like the way it's written (points at $-2x-1$) how do we write this one (points at $n\times-2-1$) in a neater way, John? John: minus 2 times n minus 1 Lineo: (writes $-2n-1$) hundred percent.	Questioning Working with learners' ideas	Writing algebraically	Representational Reading/writing	Mathematics (rules & conventions)	Mathematical conventions
09:52 – 14:10	Are you guys getting a feel of algebra? Ok you are familiar with this from your patterning. I want to explore further, keep your scrap papers out. I am going to switch on my data projector. Wily switch off the lights there. [pause] Ok usually I do this exercise with a whole lot of pieces of papers but because we have a nice data projector it will save on picking on pieces of paper off the floor. [short pause] it's going to get lighted don't worry. Lineo: projects the Italy soccer kit  Ok, What have we got there? [learners chorus Itally] Italyyy… I must admit that when I prepared this I had Italy and France in mind just in case but… France got defeated. So we stick with Italy. Ok on your piece of	Questioning Representing Questioning	Recognizing like terms Representing	Representational Transformational	Experience (everyday)	Collecting like terms

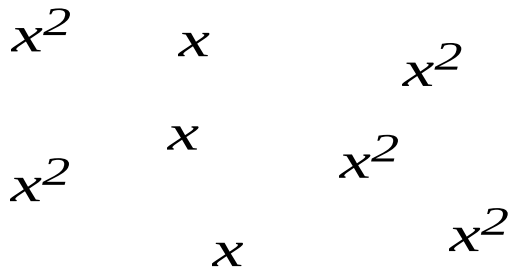
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	paper, my question to you is and you need you to answer this, I want you to count how many of the same thing they are . How many of the same things… how would you group the things [write it down] how many of the same things are there? Not similar, same. [learners write the answer silently] Lineo: Have you added? Listem to my question, Lineo: what was my question? Rethabile: how many [interrupted] Lineo: how many of the same items are there? You count how many they are. Lineo: have you finished? Ok Paul, I haven't heard from you today. Give me the answer. What have you come up with? Paul: 4 hats Lineo: 4 hats, do you agree with that? (Learners agree) so hats are in one category. Then? Paul: 3 socks (interrupted) Lineo: correct, carry on Paul: 5 shirts, 1 flag 2 balls1 cap, 2 balls, 1 bag, 4 shorts. Lineo: 4 pairs of shorts, did everybody get the same idea? Did anybody do it differently? Khau? Khau: I also added the badges (unclear) the badges on the shorts, socks (others make some noise) Lineo: Ok, no no (unclear) thinking out of the box, I agree with you that is one way of looking at it and then how many badges did you get? Lineo: you could nothing is stopping; I gave you an open ended question. Ok, a pretty, pretty arbitrary kind of pre-school exercise. What's the whole point of it? Yeah, all I am reminding you is that Italy won.					
14:11 – 15:10	Let's see the next thing. Lineo: projects	Representing	Recognizing like terms Representing	Representational	Mathematics (rules & conventions) Curriculum (teaching material)	Collecting like terms

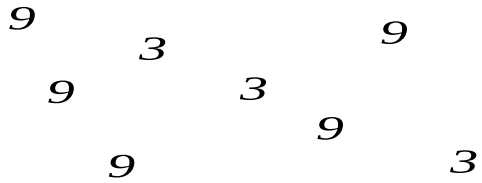
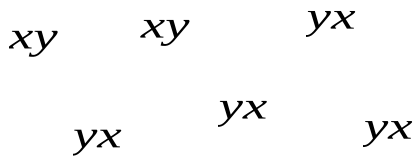
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	 Do the same exercise on that screen. [silence] Lineo: Some are becoming faster than other. Mahomete, tell me what the answer is. Mahomete: $6y$ Lineo: 6y's yeah Mahomete: 5x plus $2a$ … Lineo: Everybody got the same answer.	Questioning		Transformational		
15:11 – 16:16	. Ok, now my question is they are all letters, what made you distinguish the a 's apart from the x 's and apart the y 's, Hilda? Hilda: They are different letters Lineo: ok they are different letters. Mahomete, what would be a cleverer answer than that? Mahomete: they look different Lineo: they look different, Kele? Kele: different shapes Lineo: different shape of letters, now in terms of algebra, I ask you the question again but start thinking algebra, why did you classify them differently? Why were put the x 's in one group and the y 's in another and the a 's in a different, Lethabo? Lethabo: they have different values Lineo: correct, the y 's, x 's and the a 's will represent different numbers. I am using letters two letters at the same time; I am assuming that those values will be different. So a could be 10, x could be 5 and y could be –100 , ok different values, any question so far. [pause]	Questioning Working with learners' ideas Questioning Explanation	Visualizing letters as standing for numbers	Reading/writing	Mathematics (rules & conventions) Empirical	Variables
16:18 –	Let's see the next thing, projects					

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17:28	 [learners write the answer silently] Lineo: Ok, Rob? Rob: five x squared plus three x Lineo: so you have five x squared and three x (writes $5x^2+3x$) in other words what I have written there, 5 x squared, you are talking 5 lots of x squared, agreed? Now why did you Rob… continue Robert why did you classify the x squared and x differently? Robert: x^2 means a number times itself and x means a number. Lineo: ok when I am talking x^2 I am taking about a different value to the same old x.	Representing Working with learners' ideas Questioning Explains	Recognizing like terms Representing	Representational Transformational Reading/writing	Mathematics (rules & conventions)	Collecting like terms
17:29 – 18:51	Did everybody get the same answer or did anybody get something different? Tumi, you have a question. Tumi: (asks but inaudible) Lineo: ok I do get what you are saying; now I want you to start thinking algebraically. If we gave this exercise to Grade 1's or Grade 2's they would do exactly, what you are saying (all laugh) I agree with you, sorry I didn't mean it. Tumi you understand what I am saying (others laugh) Ok. No, no, what I am saying is, is not to put you down Tumi, I promise. What I am saying is I want you to start thinking algebraically. He is 100% correct I would give him all the marks at this stage because all I have said to you is classify them and he is saying let's put those 2's in their own category. He is right. Why can't they be? But now we are starting to think algebraically, it's going to be an issue. Just taking up on that (she projects)	Working with learners' ideas Representing	Recognizing like terms	Transformational Transformational	Mathematics (empirical)	Collecting like terms

UNISETD

	 That's the reason why you put x's and x^2 differently, if x was 3, what is the value of x^2 [pause] 9 and 3 would belong to different category to 9.	Explaining				
18:53 – 23:10	Ok, next exercise, the same question, think algebraically this time. I am forcing you to think in terms of algebra. Lineo Projects  Think algebraically this time I am forcing you to picture it in terms of algebra Lineo: Wily, you had an answer for me last time. Do you have an answer for me this time? Wily: $2xy + 4yx$ Lineo: writes Wily's answer on the board Lineo: How many got that one the same? Anybody came up with anything different? Mahomete? Mahomete: $6x + 6y$ (others: aah…aah) Lineo: Ok any other different answer? Lerato: $6xy$, (others laugh) Lineo: You said there were six pairs of xy or yx. Let's back track, when I say xy, let's back track even further, when I say two n in guess my rule, what were you meaning, Henry? Henry: 2 multiplied by n Lineo: 2 multiplied by n so when two things are next to each other with no sign, no plusing, minusing, multiplying, dividing, we are assuming timesing, 2 times n. Now if I have xy what am I meaning? x times y,	Representing Explanation	Recognizing like terms Splitting	Representational Transformational Reading/writing	Mathematics (rules & conventions)	Collecting like terms

WitsETD

	yx what am I meaning? Learners: y times x Lineo: so now is x times y and y times x the same thing as separating the y's from the x's? … I separated them out [pause]. No because it's going to be the same answer. If I say 10 times 2 the answer is 20, it has got nothing to do with adding the 2 and the 10, I can' break them up. Ok? But now let's put in some numbers, umm Mthetwa give me a number. Mthetwa: 16 Lineo: ok let's make your x 16. Mathe give a smaller number Mathe: 3 Lineo: let's make y 3, so when I say x times y, what is the answer going to be, when x is 16 and y is 3? Learners: 48 Lineo: 48, so this is 48, so all the xy's are 48. Now let's see if y is 3 because that's what we said y was and x is 16, what is 3 times 16? Learners 48 Lineo: 48, so the question is Wily, are xy different from yx or the same? Wily: the same Lineo: the same, coming with the same answer, whether I say 2 times 3 or 3 times 2 it gives the same answer so algebraically that (ticks $6xy$) is the correct answer.	Questioning Working with learners' ideas Explanation Questioning Questioning Explaining Questioning Working with learners' ideas	Reading algebraic symbols & notation	Transformational	Empirical Mathematics (rules & conventions)	Commutative property of multiplication
23:12 – 23:18	Ntando: asks a question but inaudible Lineo: ok I will speak more about that one. Ok … but hold on, who was I as the teacher to mess up your brains and start putting up yx because that is not the alphabetical order. The general rule is when you write your own answer down yes I it should be in alphabetical order. x should come before the y but sometimes the order can be mixed up and in the example that you are going to be doing the letters can be swopped around but when you write your answer yes I want you to write in alphabetical order. Ok, does that answer your question? Thabo, you had an issue. Thabo: no Lineo: ok, last one, projects	Explanation	Writing algebraically	Reading/writing Transformational	Mathematics (rules & conventions)	Commutative property of multiplication

WitsETD

	$2.4 \quad 4.2$ 4.2 $2.4 \quad 4.2$ 4.2 This one just explains, if I chose x to be 2 and y to be 4, 2 times 4 and 4 times 2 the same difference.	Representing Explanation			Empirical	
23:20 – 23:49	What does that dot mean? (points at the dot between 2 and 4) Learners: times Lineo: times, you have seen that before? Learners: some say yes, others say no Lineo: yes, no. those people who possibly have not seen it before, again to prevent confusion instead of writing the times because that might be confused with the x we just write a dot (writes $2 \bullet 4$) as soon as you see a dot, it will mean a times. So 2 times 4, 4 times 2, the same difference.	Questioning Explanation	Symbol conflict	Transformational Reading/writing	Mathematics (rules & conventions) Empirical	Mathematical notation/convention
23:51 – 24:38	Last one Lineo: projects $a^2 \quad a \quad b^2$ $a^2 \quad b^2 \quad ab$ b $ab \quad a^2 \quad ba \quad a$ Work out the answer to this. The last one and then we can do some notes on your theory books. Lineo: Lefu, it's a question … no no…. it's an important question. What is …. Lefu: now, because you say 2 point 4 (inaudible) Lineo: yeah with a dot Lefu: inaudible Lineo: with a comma? Lefu: yeah Lineo: Ok that's why in South Africa we use a comma. If I want a decimal	Representing Working with learners' ideas	Recognizing like terms	Representational Transformational	Mathematics (rules & conventions)	Collecting like terms

WitsETD

	place I will put a comma. Ok, it's a good question that's why I wanted you to ask it (writes $2 \bullet 4$) that would mean times.	Explanation	Meaning of symbols	Reading/writing	Experience (professional) Curriculum Empirical	
24:39 – 28:06	Ok, who can give me the answer there? Lineo: [silence] you are still busy. You are done. Disky, I haven't heard from you today. Disky: three ab plus three a squared plus two b squared plus one b plus two a. Lineo: (writes Teboho's response on the board) $3ab+3a^2+2b^2+1b+2a$. How many got that one write? The majority of you. One technical question before we do the notes. How could I improve that right thing? (Draws a line below the expression) one technical question, what could I do to improve that answer, Thato? Thato: put brackets Lineo: brackets, no they would make it more complex, I want to make it neater. Khau? Khau: Mam, you could have four ab plus 3 a squared plus 2 b squared plus a Lineo: wait… answer (writes $4ab+3a^2+2b^2+1a$) so you combined that b and a from there to get ab. Ok, right or wrong? All talk (some say correct, others say wrong) Lineo: Dave, you are quite convinced, share with me. Dave: you can't just add because this isn't the same thing. Three a squared and two b squared, you can't say is five a b squared. Lineo: Ok, let's pick up on what Dave is saying, Dave is saying why can't we combine those two 3 a squared and 2b squared (writes $5a^2$), what did you say $5ab^2$? Like that? Tell me more. So he is right? Who disagrees? Anyone to argue? Bob argue. Bob: inaudible Lineo: Ok from what does that mean? Ok (writes $5ab^2 \rightarrow 5*a*b$) now coming back to you Dave is that the same as that? Dave: no	Questioning Working with learners' ideas Working with learners' ideas	Conjoining Conjoining	Transformational Transformational	Mathematics (rules & conventions)	Collecting like terms

WitSETD

	Lineo: no can you see even visually it doesn't look similar, but I liked your approach … Rob? Rob: I can argue against that. Lineo: [unclear] (circles ab on 4ab) so I am not plusing the a and the b. perfect, Robert, good argument (crosses the incorrect expression) ok back to my question, you get full marks, I am not querying this answer, what is the technical thing that I am querying [interrupted]… the simplifying is correct, all I am saying is (removes 1 the coefficient of b). Ok, avoid saying one x, we always just write x because we assume it is just one x.	explanation			Mathematics (rules & conventions)	
28:07 – 33:16	sorry, take out your rules books, let's do some rules or your theory books. Wily switch on the lights. Lineo: Ok, brand new page, brand new heading (writes Algebra on the board) Lineo: Ok, algebra is the last section we are going to be dealing with until we write exams. It's the last section, we will be dealing with algebra for the rest of this week, next week we'll be doing revision for the exam. Dlamini: is it in the exam? Lineo: Ok, it is in the exam. Lineo: Ok, first things first, let's make a note of what variables are (writes on the board variable). Ok, it's the first definition, what do I mean by a variable? Thabo: something that can change Lineo: something that can change. Tell me a bit more. Thabo, if you can … now that is English, it's a variable because there is a variation, not mathematically, I gonna still pick on you Thabo mathematically it says you can … give me some more Thabo: inaudible Lineo: Ok, Henry Henry: inaudible Lineo: Ok all of you are right, let's talk mathematically, in algebra when I am talking about a variable I am talking about a letter of the alphabet that can assume any value (writes a, b, c, d, …, x, y, z) so any letter of the alphabet and I make you write down your curly x [interrupted] … all your letters should be with a cursive. Mokhele: why do they use…. Lineo: I think, but I stand to be corrected, for me the x and the y are used	Questioning Defining Representing Explanation	Defining	Reading /writing	Mathematics (definitions)	Variable

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	more often because when it comes to plotting your points what do we use? The x-axis and the y-axis so we will be developing … where we have everything in terms of the axes. So I think in the forefront of my brain I am always thinking x's and y's but there is no reason from stopping me from using the p's and the q's. If so, listen here if I say x is 3 then you have to stick with that 3, if say to you hold on x is 3 you can't work with it as a variable and say I gonna make it 10. If I give it to you as a value you need to stick to that value.	Explanation				
33;17 – 33:55	What do we call all the numbers then? (writes 1, 2, 3, 4, 5 …) without any attachment to the x or the y, the p's and the q's? What do we call those? It start with a C. those guys who have done algebra last year Lineo: constant, why? because its constant value, it does not change, it's not dependent on something changing. It'll always be 3	Questioning Representing Define	Defining	Reading/ writing	Mathematics (definitions) empirical	Constant
34:06 – 37:07	Lineo: coefficient, have you guys come up with coefficient before? [silence] (Writes the definition on the board) bad English. Your question is before what? Let's do an example and see whether it makes sense. If I say to you what is the coefficient of x in the following example. It is the stuff before the variable In 3x is 3 In – 15x is -15 In 10ax is 10a Lineo: if I gave you this question and I said what is the coefficient of 10ax, what is sorry x in 10ax? It would be 10a.	Questioning Define	Defining	Reading/writing	Mathematics (definitions) Empirical	Coefficient
37:05 – 38:26	Lineo: Ok term, what do I mean by a term? Anyone came across that word before? Lineo: Ok, let me give it to you before you come with all your English definitions (writes the definition) an example will be better; they are separated by pluses and minuses. Thabo: can I say (interrupted) Lineo: Ok give it to me Thabo: 10x plus 7 equal (interrupted) Lineo: (writes 10x + 7) I am going to stop you by the equal, equals will give me an equation, so I am not going to make it equal anything, I am just doing… what is the difference between…, I am picking upon you Duncan, he wanted to say equal and I stopped him, I said that's an equation so what am I dealing with then?	Questioning Defining Questioning	Defining	Reading/writing	Mathematics (definitions)	Term
38:28	It also starts with an E. Learners silent			Reading/writing	Mathematics (definitions)	Expression

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– 39:11	Lineo: I am dealing with an expression. An expression doesn't have your answer equal to anything. What we are dealing with are expressions or over here we can say algebraic expression next term we will deal with equations. How many terms are in (1)$10x+7$ Mpho: 2 terms	explanation Questioning	Defining			
39:13 – 39:49	Lineo: (writes (2)$16xy-3xy-5xy$). How many terms have I got there Mohapi: 3 Lineo; three even though I have got xy , xy , xy I still have 3 term.	Questioning	Defining	Reading/writing	Mathematics (definitions)	Counting terms
39:50 – 40:36	Lisebo I'm picking on you again, in question 1, what would be the coefficient of x in that expression? Lisebo: inaudible Lineo: question 1, let me read the question again Lisebo: 7 Lineo: why? Lisebo: inaudible Lineo: Ok, what is the meaning of coefficient? Lineo: aah aah, going back to the definition I said it can be a variable or a constant, tell me more Lisebo: a variable for x Lineo: it's a variable or constant before x, it's the stuff before x. Ok?	Questioning Defining Questioning Defining	Defining Identifying constants	Reading/writing	Mathematics (definitions	Coefficient
40:37 – 41:17	Lineo: in question 1, Wily what is my constant? Wily: silent Lineo: it's just the number by itself Thato: Mam why Lineo: because it's not dependent or attached to x, it is by itself	Questioning Defining Explanation	Defining Identifying constants	Reading writing	Mathematics (definition)	Constant
41:18 – 42:48	Lineo: Now, last one (writes $4(x+y)$) tell me about that one, how many terms are there? [silence] Ok, Mpho? Mpho: 2 Lineo: Mpho, 2, tell me, Mpho: x and y Lineo: x and y you got plus between them. I would turn to agree with you but (giggles) it is wrong. How many they are, Hlompho? Hlompho: one, ther is 4 times [inaudible] Lineo: ok, let's go through it again if I have number times something, that	Questioning	Defining Identifying terms	Reading/writing	mathematics (definitions)	Counting number of terms

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	something could have plusing and minusing inside it, its going to be 1 term. Lineo I will go through that again (puts brackets around $4(x+y)$) this is 1 term Wily: madam in 4x + 4y there will be 2 terms Lineo: In that expression you are 100% correct there will be 2 terms but in that given format if I say how many terms there will be 1 term. Henry and Oscar you are not concentrating. Ok, let's go through that again, if I have 4 times something, that something may have plus or minus inside it, it is going to be1 term. Lineo: ok, there is 4 times something; we don't care what is inside the bracket, 4 times something how many terms? 1 term	explanation				
42:49 – 43:46	Last example (writes $-3(x+y)-2(2x+y)$) take this down. Take it down and tell me how many terms are there. Don't shout it out. Close your eyes everyone, memorize that example and have your answer in your head, close your eyes, we need to go and put your hands in the air tell me with your fingers. How many terms? Put your hands in the air those people who said two may go those who said 3 we need to have a chat. I will deal with this tomorrow.	Questioning	Identifying terms	Reading/writing	Mathematics (definitions)	Counting number of terms
00:00 – 01:54	**Lesson 2** Lineo: ok, yesterday we looked at algebra; we started with looking at some terminology associated with algebra. We looked at how we group umm… the variables. In the meantime take out your theory books we are just going to do a couple more notes and for the remainder of the lesson I am going to you the worksheet to start working with some exercises. While you are taking out your theory books, let's just quickly recap a few things, Thabo (looks at the learner) talking. (writes $5x^3+3x^2-2x+8$ on the board silently) Ok, first things first, is that an expression or is that an equation? Hands up, Thato? Thato: an expression Lineo: an expression. What is the difference between an expression and an equation? Thato: an equation has an equal …(interrupted) Lineo: Ok Thato: an expression (inaudible) Lineo: Ok, an equation equals something; there will be a value, so that	Questioning Questioning Working with learners' ideas	Defining	Reading/writing	Mathematics (definitions) Empirical	Expressions & equations

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	expression will equal something and it turns into an equation. So that is an expression.	Explanation				
01:55 – 02:22	Lineo: second question, what is the value of the constant? What is the value of the constant, Mpho? Mpho: 8 Lineo: 8 remember your constant (circles +8 in the expression and writes constant) does not depend on the variable, there is no x attached to it or being multiplied by it.	Questioning Explanation	Defining Identifying constants	Reading/writing	Mathematics (definitions)	Constant
02:23 – 02:56	Lineo: next question, how many terms are in that expression? How many terms are in that expression, Lefu? Lefu: 4 Lineo: 4, give me the definition of how we count our terms. How did you come up with the value of 4? [silence] Lefu: I counted the terms separated by the pluses and the minuses Lineo: so the terms are separated by the pluses and minuses (underlines the terms) so there are 4 terms there.	Questioning Defining	Defining Identifying terms	Reading/writing	Mathematics (definitions)	Terms
02:57 – 03:54	Next question, what is the coefficient of x^2 . What is the coefficient of x^2 in that question, Neo? Neo: 5x squared and 3 (Lineo interrupts) Lineo: let's back track, what is the definition of a coefficient? Neo: before the number Lineo: before, Ok, let me clarify that because I was talking about the stuff before. When I am talking about the stuff before the variable or the constant is what is being multiplied by the x^2. what is the coefficient of x^2 in that question, eeer , Neo? Neo: [inaudible] … before Neo: Ok, let's back track, what is the definition of a coefficient? Neo: inaudible Lineo: Let me clarify it because I was talking about the stuff before, when I am talking about the stuff [unclear] the so variable or the constant is what is being multiplied by the variable. So when I say what is the coefficient of x^2 ? What will be the answer, Neo? Neo: 3 Lineo: 3 perfect, so 3 is my coefficient (points it with an arrow and writes	Questioning Explanation Working with learners' ideas	Defining Identifying coefficient	Reading/writing	Mathematics (definitions)	Coefficient

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	coefficient)					
04:00 – 04:44	What is the coefficient of x , Mathe? Mathe: 2 Lineo: have a look at the expression again, make sure you are 100% correct. What is the coefficient of x ? [Silence] Lineo: we want to give Mathe a little more cchance. It is negative 2, we are going to talk about now, that negative who does it belong to? You see that negative who does it belong to? It's not just sitting in the middle. It's attached to the 2 so that the coefficient of x there would be -2 (circles -2 and writes coefficient). Happy with that? So you include the sign as well.	Questioning Questioning explanation	Defining Identifying coefficient Reading algebraic notation	Reading/writing	Mathematics (definitions) Mathematics (rules & conventions)	Coefficient Mathematical notation
04:45– 04:58	Thato: do you include the plus then? Lineo: correct, plus 3 but when we say plus 3 what are we (interrupted) it's 3 but if it's negative 2 obviously we need to say negative 2.	Working with learners' ideas	Reading algebraic notation	Reading/writing	Mathematics (rules & conventions)	Mathematical notation
05:00 – 05:08	Lineo: last question what is the coefficient of x^3 , Dumi? Lineo: 5, ok. It's coefficient, constant variable and expression.	Questioning	Defining	Reading/writing	Mathematics (definitions)	coefficient
05:12 – 06:03	The thing that we ended with yesterday was $2(a+b)-6(a-b)$ I don't know what the question was but it was something like that and I said how many terms are in that expression and you got the answer correct and you were able to go home, if not you had to stay the rest of the afternoon. How many terms are in that expression, Vusi? Vusi: 2 Lineo: 2 and explain why for those people who are still not certain. Vusi: because what happens is you are saying 2 multiplied by everything in the bracket that's one term and then minus 6 multiplied by $a-6$ Lineo: (writes× between the 2 and the opening bracket and between 6 and the opening bracket) so there are 2 terms separated by a negative sign	Questioning Questioning Explanation	Identifying terms Reading algebraic notation	Reading/writing	Mathematics (rules & conventions)	Counting number of terms
06:11 – 06:25	Ok let us do some more notes in your theory books and then you can do some exercises. Mokhele: that will be negative 6 multiplied be $a-6$ Lineo: yes it is negative 6, but to count how many terms, we have got this individual pieces Mokhele: inaudible	Working with learners' ideas	Reading algebraic notation	Reading/writing	Mathematics (rules & conventions)	Number of terms

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	Lineo: correct, correct					
06:40 – 07:41	Lineo: under your terms we ended with counting terms, just some words you must be familiar with (writes on the board) if a expression contains one term we call that a monomial. (Interrupted) yeah, take it down in your theory books. Give me an expression with one term, Halala Halala: - 5 Lineo: (writes – 5 on the board) so – 5 we could have said – 5xyp but it would still be 1 term.	Defining	Defining	Reading/writing	Mathematics (definitions)	Monomial
07:43 – 08:44	Let's do two terms, who could guess what 2 terms would be? Learners: chorus binomial Lineo: binomial you know that as in bicycles- two wheels. Edward give me an example of an expression with 2 terms. Edward: 2 + 2 Lineo: $2 + 2$, umm, let's do a different example; let's introduce some algebra, some variables. Yeah $2ab$ that's one term, give me another term (writes $2ab - 6ab$) or we could say $2ab - 6pq$.	Questioning Defining	defining	Reading/writing	Mathematics (definitions)	Binomial
08:45 – 09:12	Lineo: Three terms (interrupted) Morena: Trinomial Lineo: trinomial, Lebo give me an example of 3 terms Lebo: Three bc minus one pq plus 2 Lineo: (writes $3bc - pq + 2$ on the board), hundred percent, instead of saying 1 I am just going to say pq,	Questioning Defining	defining	Reading/writing	Mathematics (definitions)	Trinomial
09:14 – 10:38	Now we are going to have a name for 4 terms. Now we are going to have a name for 4 terms, anything bigger than 4 terms, 4 terms, 5 terms, 6 terms … All the way to infinity. If you have got 100 terms or 4 terms, anybody knows what we would call that, Chipa? Chipa: polynomial Lineo: perfect, a polynomial, poly --------- many. John give me an example of a polynomial (writes $5cd + 8fg - 6ab + 3$ as John dictates it) we don't necessarily have to have ab, 2 letters next to each other, we could have $6a$ that also would have been fine but John wants $6ab$, that's your example.	Questioning Defining Working with learners' ideas	Defining	Reading/writing	Mathematics (definitions)	Polynomial
10:39 – 11:01	Chris? Chris: does $6ab$ mean 6 times a times b? Lineo: correct 6 times a times b. just to clarify Chris's question $6ab$ means	Working with learners' ideas	Reading algebraic notation	Reading/writing	Mathematics (rules & conventions)	Mathematical notation

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	6 times a times b, it does not mean 6 times a times 6 times b. ok its only the 6 times a times b.	Explanation				
11:02 – 13:24	Ok, last thing in your theory books for the moment. Heading (writes adding and subtracting) we saw this yesterday, so this should be fairly straightforward for you guys but let's see what happens. (Writes group *like terms*) ok whenever you are to add or subtract you have to group like terms. You remember yesterday on the Geometer's sketchpad you had x and x^2 and you put all the x together and the x^2 together. That was grouping your like terms. x Belong to one group and the x^2 in another. your instruction will always be simplify [pause]. (Writes simplify $3x+5x+2x$) Let's star with a straightforward one. Three x plus another five x another plus two x, Mpho, what do you think the answer could be? Mpho: $10x$ Lineo: explain to the class how you got that Mpho: umm I (inaudible) Lineo: $10x$, Ok, so all the x's belong with each other (circles 3x, 5x, and 2x). I am including the signs before the number, the sign always belongs with the number. 3 plus 5 plus 2 is ten x, Mpho?	Questioning Explanation	Interpreting algebraic language	Transformational Reading/writing	Mathematics (rules & conventions)	Collecting like terms
13:25 – 13:37	Mpho: Mam, if you have more than one x, 3 and 10 value Lineo; yes, if I said to you x is 2 then you would say 3 times 2, 5 times 2 and 2 times 2.	Working with learners' ideas Explanation	Substitution	Transformational	Empirical	Evaluating expressions
13:38 – 15:27	Mahomete: mam, if it's like 3x plus 5x plus 2b would the answer be 10x (inaudible) Lineo: let's do that example, you said [writes $3x+5x+2b=$ on the board], do that by yourselves, if you are not certain you can do that in pencil and come up with your answer. (silence) August, what do you think your answer could be? It does not matter if it's wrong. August: $10xb$ Lineo: 10xb tell me how you got the answer. August: umm Mam, x and b can be the same value (interrupted). Lineo: can they? August: no Lineo: Ok, you are happy they are going to be different values. Let's have someone different, Mohapi?	Question Questioning	Conjoining	Transformational Transformational	Mathematics (rules & conventions) Mathematics (rules &	Collecting like terms Collecting like terms

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	Mohapi: $8x + 2b$ Lineo: $8x + 2b$ (writes it on the board) convince me that you are right. Mohapi: we have 3x plus 5x, and 3x plus 5x is 8x them another term which is 2b Lineo: Ok, Mohapi, this is right, the x on the board, remember when we had the x and the x^2 or the x and the y, the x you put in one group and the y in one group, the same thing here, the xs belong with each other and how many are they, there are only $8x$ and that b is a completely different story, he belongs by himself. $8x + 2b$, how many got that one correct? (Learners raise up hands) Ok not too bad. Let's try another one umm (writes $-2x + 5x - 3y + 2x + 6y$) let's if you can come up	Working with learners' ideas Explanation			conventions) Mathematics (rules & conventions)	
15:29 – 17:31	Let's try another one umm (writes $-2x + 5x - 3y + 2x + 6y$) let's if you can come up with that answer. Tricky now because now I have some negatives in there. [Learners work silently] Lineo: Ok Dumi, you had your hand up first, tell me Dumi: $5x + 3y$ Lineo: $5x + 3y$, let's see if that makes sense, I am going to group all my x's and if you want, in your tests and in your exercises use colours, for the x I going to choose orange and when I group my x's I going to include the number and sign before it. Three is x (circles the x's in orange) another one and another. So minus 2 plus 5 is 3, 3 plus 2 is 5x. Let's do the y's minus 3y plus 6y obviously gives me positive 3y. How many have got that one correct? Ok we are making progress.	Questioning Explanation	Reading algebraic notation	Transformational	Mathematics (rules & conventions)	Collecting like terms
17:32 – 22:18	(Puts up $6x^2 - 8x - 3x^2 + -2x - x$) that's a long one. [Learners work on this problem individually] Lineo: how many of you are still busy with 4? Ok, Wily, you are closest to the board and do the groupings for me. Wily: number... Lineo: Ok, What are you going to group first? Ok go for it. Which one do you want to do first, the x squares or the x's? That minus who does it belong to (Wily says $8x$) Ok Wily: circle $6x^2$ and $3x^2$ leaving out the signs	Questioning	Reading algebraic	Transformational	Mathematics (rules &	Collecting like terms

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	Lineo: (makes some warning noise) that minus who does it belong to? Wily: includes the signs but seems to be puzzled. Lineo: $6x^2$ minus $3x^2$, what's the answer going to be? $6x^2$ minus $3x^2$, looking at the numbers (Learner silent) 6 minus 3? Wily: 3 Lineo: what's the answer going to be? What are we grouping? (Lineo writes $3x^2$) Wily: x Lineo: (x) square Lineo: next colour. Ok, guys just bear with Wily. Wily: circles -2x and –x Lineo: minus 8 minus 2 Wily: minus 10 Lineo: minus 11 what? Wily: x Lineo: then what? Wily: plus 4 Lineo: hundred percent. That 4 sitting by himself he had no one to pattern with so we must not forget him we write plus 4, any queries on that one? can you see how I strict I was with wily? Include the signs always before your number. Enoch? What did you come up with? Enoch: minus 5 Lineo: Ok, let's have a look. Did you group the x's like Wily did? Enoch: where is the minus 11 from? Lineo: We are looking at the signs as well minus 8 minus 2 is… Enoch: minus 10 Lineo: minus (points at the coefficient of x) what is the imaginary number there? 1. Minus 11. Teboho? Start again. Teboho: 2, it represents, it's like a letter… With the square	Working with learners' ideas Working with learners' ideas Explanation Questioning	notation		conventions)	
22:19 – 24:36	Lineo: Yes try the last one ($2pq - 3qp - p + 4p$), ok I will do this one. Still busy with question five. Ok, Mahomete is starting the question. $2pq$, I am going to do pq in orange (circles pq), are there any other pq's? Yes Kori? Kori: qp Lineo: yes remember yesterday xy and yx always came out to be the	Questioning Explanation	Reading	Transformational Reading/writing	Mathematics (rules & conventions)	Collecting like terms

WitsETD

	same x times y, y times x so grouping my pq's with the sign before. there is my pq [circles $2pq$] and he belongs with qp [circles $-3qp$], I group with the minus 3, no more pq 's, pq is one term [interrupted] Ntando: why can't you change the (unclear) can't you make p and 4q to be one? Lineo: how are you going to do that? Ntando: can't you make it 5qp? Lineo: What does pq mean? what's happening between q and p? (pause) Ntando: timesing Lineo: timesing, timesing is different from saying p plus q. Two minus three (referring to $2pq - 3qp$), Ntando: minus 1 Lineo: Minus one and instead of saying $-1pq$ I am going to just write $-pq$ in alphabetical order Khau: is $-1pq$ wrong? Would you get the question wrong if you write $-1pq$ Lineo: no, is fine for now. Yeah, just try to get into the habit of ignoring that 1. that minus p does he got anyone to belong to? We must not forget him and that plus $4q$ is by himself. Is that Ok, not so bad, you can put away your theory books, in the meantime take out your homework books,		algebraic notation Recognizing like terms Conjoining			
24:37 – 26:43	I have got a worksheet for you guys (distributes the work sheet, learners talk) firstly, Ok, let me tell you what I expect firstly I don't want to see any answers on the worksheet. You have to put your answers in your homework books. also I want you to write out the question equals then your answer. Ok, so write the question and the answer. It will not take you that much longer I promise. Exercise 1 umm Oscar, read out the question for me. Oscar: how many terms are there in each of the algebraic expressions? Lineo: Ok in number one all you need is just to count the terms. Number 2, umm Chris read out the question. Chris: write out the coefficient of x in the following algebraic expression Lineo: can you do that? Can you write down the coefficients? Exercise 2 Rod? Rob: simplify if possible					

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	Lineo: can you simplify the expressions? Yep, so start with exercise 1 and then move on to 2 and then we can see how far you can get. Chris?					
26:44 – 30:13	Chris: (unclear) Lineo: will walk around if there are queries. Retsy: Mam will this be 1 or none? Lineo: you tell me. What do you think? Retsy: I said is 1 but he says it's none Lineo: if you have abc to argue, can you hear? Retsy: a times b times c, and these are three terms, with abc together. It does not make a times b times c so there will be 3 terms Lineo: Ok, what's the definition of a term? Retsy: minus and plus Lineo: they are separated by pluses and minuses. So a times b times c, how many terms? One or two or none? Retsy: 1 Lineo: 1 term, now why did you say you needed a plus? Retsy: yes plus or minus Lineo: if you have abc plus, you have got plus something right? Retsy: yeah Lineo: plus two ?????? there will be two terms. Themba: if there is plus or minus separating them but is not a times b times c Lineo: it is Themba: so there is no term, ooh, it's plus abc. Lineo; it's a positive abc Themba: will that separate Lineo: Ok, let's do it this way $x + y$, how many terms? Themba: two terms Lineo: correct, if I take away that, how many terms are there? Themba: 1 term Lineo: 1 term Themba: so just letters can make terms Lineo: yeah Lineo: even, have a look over here umm that number (f), x sitting by himself its gonna be 1 term because also what the imaginary number before abc? Themba: 1 Lineo will that make you feel comfortable?	Working with learners' ideas Questioning Defining Questioning	Identifying terms	Reading/writing	Mathematics (rules & conventions, definitions)	Collecting like terms

WitsETD

30:14 – 31:10	Tobi: … with bracket how many terms is that? Lineo: you tell me Learner: I am not sure. Lineo: Ok that bracket basically we do not care what's going on inside that bracket because we are saying it's 3 times. There is no plusing or minusing. So it's just one term.	Explaining	Identifying terms	Reading/writing	Mathematics (rules & conventions)	Counting terms
31:12 – 32:14	Moshe: 3 times that number, there will be no terms. Lineo: I agree but umm Ok, let's go back to this x plus 2y, if I take away the 2y, how many terms have you got? Moshe: 1 Lineo: so why in abc did you say there were three terms? Moshe: I said there was times (interrupted) Lineo: but even x by himself is not timesing anyone. Moshe: ooh it's not next to another number Lineo: what is the number before that x? Moshe: I don't know, ooh 1 Lineo: 1 and again in the abc is one thing. So getting back to your question which one was that? Moshe: 3 times … it will be 1 Lineo: Have a delightful moment.	Questioning Questioning	Identifying terms and coefficients	Reading/writing	Mathematics (rules & conventions)	Number of terms
32:15 – 36:24	Lineo: there is on need but they have given you a bracket (interrupted). Ok it would be but why did they give you the bracket? What would be the number outside the bracket? The imaginary number outside. My question to you is what would be the imaginary number before the bracket? If there was to be a number, what would it be? Ok now I have got $2x, 3x, 4x$ and if I wrote just x what am I meaning? (learner quiet) $1x$ and that $1x$ means 1 times x so the same thing with that bracket 1 is the imaginary number outside the bracket so if you had written 1 immediately you would have said it's 1 term. It will help sometimes to clear your confusion but whether there is 1 or not there will still be one term. Kurt: … the coefficient, is it $5p+3$ or is it just (inaudible) Lineo: the coefficient is all the stuff is being multiplied before the x Kurt: Ok Mbuyi: when we were doing expression, is it one x with no coefficient? Lineo: what is a coefficient? Learner: shrugs his shoulders.	Questioning Explains Working with learners' ideas	Identifying terms, coefficients	Reading/writing	Mathematics (rules & conventions)	Coefficients, terms

WCSETD

	Lineo: you have just told me now. What is the number before the x ? Mbuyi: x Lineo: the coefficient is 1 Mbuyi: 1x Lebo: Mam, in x will the coefficient be 1? Lineo: what did timothy think it was? Lebo: zero Lineo: what would zero times anything be? Timothy: zero Mbuyi: in this expression $2x+1$ the coefficient will be plus 2 (inaudible) Lineo: it's just the 2. It's only when I was talking about multiplying the stuff, I think, I people misunderstood me, it's just the stuff being multiplied before that letter.	Questioning Explanation				
36:25 – 36:55	Semoli: (unclear) Lineo: if I said x , what is the imaginary number before the x ? Semoli: 1 Lineo: 1, if I say… Ok, let's do it this way, in question number (p), how many terms are there? Semoli: 2 Lineo: two, so in question number (m) Semoli: 1 Lineo: 1,	Questioning	Identifying terms	Reading/writing	Mathematics (rules & conventions)	Number of terms
36:56 – 38:10	Henry: (unclear) Lineo: how many terms did you say they are in number f? Henry: 1 Lineo: Ok, 1 umm (interrupted) Henry: is that the 1 term? Lineo: the whole thing is one term. Henry: why the whole? Ooh it's 3a you don't take away the brackets Lineo: but that bracket groups it we are not seeing what is inside that bracket, it's 3 times the bracket. So in number (n) we ha 3 on the outside, what would the imaginary number (interrupted) Henry: 1 Lineo: tell me why. Henry: it must be 1 term because it's the 1 times but now is it 1 times, not 1 plus or minus so you don't consider the times as a term	Questioning Questioning	Identifying terms, coefficients	Reading/writing	Mathematics (rules & conventions, definitions)	Number of terms

WitsETD

	Lineo: no Henry: it doesn't (interrupted0 Lineo: remember terms are separated by pluses or minuses. Henry: so it's only 1 term? Lineo: yeah					
38:12 – 39:24	Lineo: there was a hand down here. Thato; here I just wanted to know when we have $2x+5$, for the coefficient do we have to say plus 2? Lineo: no, because when you write 2 we assume its obviously positive 2 Thato: Ok, but if it's negative, then obviously you have to put it Lineo: yeah Lineo exercise 1 and 2 to be completed	Working with learners' ideas	Identifying coefficient	Reading/writing	Mathematics (rules & conventions)	Coefficients
00:00 – 03:30	**Lesson 3** Lineo: … two exams, both exams are examined on the same content. Ok, right from the beginning, data, geometry, the algebra, your number patterning. You have to study the same amount for both papers, one is a problem solving paper and the other one is just skills. I will talk [interrupted] Mpho: unclear Lineo: the problem solving one is your long one, I think is 1 hour and a half, and then your skills is just one hour. Ok, remember the only thing that is not going to be examined is your constructions. Ok, anyone now wants to admit that they have not done their homework exercise 1 and 2. Kele? No homework? Kele: no I did 1 Lineo: why didn't you go to 2 then? … anybody else? Ok let's mark the homework and then we can move on. Just from the time point of view I am going to read out some of the answers, I will then get you guys to read some of the other. Ok, are you ready? Ok, exercise 1, question number 1, how many terms? Number (a) there are 2, (b) 1, (c) 2, (d) 3, (e) 2, (f) 3, Am I going too quickly? (h) 1, (i) 4, (j) 1 (k) 1, (l) 4, (m) 1, (n) 2, (o) 1, (p0 2, (q) 2 and (r) is 3. Someone knocks			Reading/writing		Number of terms
03:31 – 06:00	Lineo: come in. where has you bag been? Why you couldn't you find the bag? Quickly, you have just made it under the five minutes deadline. Did you do your homework? Question number 2, the coefficient of the variable x, number (a) 3, (b) 2, (c) 3 … and (i) is 7					Collecting like terms

WCED

	Hlompho: Mam, (inaudible) why like c (not clear) Lineo; …. It's everything that is being multiplied by the x, I'll clarify that with you just now, Thabo. Lineo: exercise 2 let's go through all answers (calls on learners to read out their answers) number (a) Dlade?, number (b) Adam?.... (this continues until all questions are marked) Lineo: read out the question , the question was $ab \ \ plus \ \ ba$,first write out the question every time. Sam: mem, it's 2ab Lineo: 2ab Khau: mam, and if you say ab squared Lineo: that's different we 'll talk about that now. Ok 2ab Mpho: Mam, can you write like ab slash ba? Lineo: no, just 2ab, number (m), Wily? Wily: Mam, is $4x+8y+2m$ Setho: Mam can you say, like $4x+$ or you can say $4x8y$ Lineo: plus because if you put them together what does it look like? Setho: times Lineo: timesing, one thing. Number (n), Mpho? Mpho: ab + bc Lineo: number (o) Diski? Diski: 9mnp Lineo: 9mnp. Number (p) Ted? Ted: bac Lineo: the question was a + c + ac what do you think the answer would be? Have a look at it again. Is a the same as c and c the same as ac or are they all different? So what would the answer be? Ted: a + c+ ac Lineo: number (q) Kori? Kori: 4y Lineo: r, Oscar? Oscar: 8a Lineo: ok I will go through queries just now.	Working with learners' ideas Questioning	Reading algebraic notation Conjoining	Transformational Transformational	Mathematics (rules & conventions)	
06:01 – 12:00	While you have got your homework books out, I just want to do a quick spot on test, just 5 questions. You carry on where you last stopped your homework, 5 questions, sorry, Ok the test is not going to count for marks					

WitsETD

	but you need to do it by yourselves just put it at the back of your books. Sorry? Lineo: what's the date? Lineo: I don't know, 13 Lineo: ok question 1 (writes on the board) Simplify 1) $3x^2 - 5x + 2x^2 - x$ 2) $8x - 3x + xy$ you don't need to write out the question, just go straight to the answer 3) $x \,.\, x^2 \,.\, x$ 4) $(4x)(2x)(x)$ 5) $(2xy)(x^2y^2)$ 3, 4 and 5, I haven't taught but I want to see what you can make with them. [Learners work individually on the tasks] Lineo: I will give you one more minute. Lineo: you are still busy. [no response] Lineo: Ok can we mark? You are still busy. No one, Themba, you are done. Lineo: Ok you are going to mark your own work. It's going to be right or wrong, no half marks, no negotiations. Question number 1, where do we get up to Kori? What was your answer for number 1? Kori: minus 5x squared minus 6x. Lineo: let's check your x squares again [silence] Lineo: what did you say? Kori: 5x squared (interrupted) Lineo: Sorry, I thought I heard a minus (writes $3x^2 - 5x + 2x^2 - x = 5x^2 - 6x$) right or wrong. You could have put it in a different order if you wanted to, some of you might have written $-6x + 5x^2$ that would also be correct Learner: $5x^2 + -6x$ Lineo: plus minus $6x$, eeeh it's right but its not simplified I would…, give yourself a mark but just remember that plus minus becomes a minus.	Questioning Questioning Explanation	Writing algebraically	Transformational Reading/writing	Mathematics (rules & conventions) Mathematics (rules & conventions)	Collecting like terms Expressing as powers Collecting like terms
12:05	Lineo: question number 2, Themba?	Questioning		Transformational	Mathematics	Collecting like

WitSETD

– 12:16	Themba: stays the same Lineo: stays the same, nothing gets grouped, that's your answer.				(rules & conventions)	terms
12:17 - 12:51	Lineo: Ts'epo, question 3 remember 3, 4 and 5 I have not taught so if you make a mistake that's ok. Ts'epo: Mam, I got x cubed. Lineo: x cubed. Almost, let's see, what does that x^2 represent? Learners: chorus x times x Lineo: (writes below x^2, $x \ . \ x$) so how many x's are being multiplied by each other? Learners: 4 Lineo: so your answer will be Ts'epo: x to the 4 Lineo: x to the power of 4 (writes $x \, . \, x^2 \, . \, x \ = \ x^4$) right or wrong? That's it.	Questioning Questioning	Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Simplifying expressions
13:04 – 13:35	Lineo: mm…mm we gonna talk about that now. Question number 4, Tina? Lineo: ok let's have a look 4 times 2 is 8 (writes down 8) and then how many x's are being multiplied by each other? x times x times x, that will be x to the power of 3 (writes x^3 next to 8)	Questioning	Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Simplifying expressions
13:37 – 15:17	Ok question number 5, who wants to volunteer and answer? Am I getting weary, Hakan? Hakan: I said it's 6x Lineo: Ok, no, we will deal with this now, say it. Sam? Sam: $-2x^2y^4$ Lineo: almost (writes down – 2) just check your x's for me Sam: minus 2 x cubed [pauses] y to the power of 4 Lineo: (writes x^3 next to – 2) y to the power of 4 (writes y^4 next to $-2x^3$). Ok, I am going to do the close your eyes. The thing we have done in the past, except that I don't want your final mark out f 5, I am just going to say those who got question 1 right put up your hand if you didn't get it right keep your hand down. So you need to remember which ones you got right and which ones you got wrong. Ok, close your eyes. Who got number 1 right, number 2, number 3, number 4, and number 5? Ok not bad, thank you. Take out your theory books I want to look at this timesing and I want to deal with Themba's query as well and also looking at Ts'epo's	Questioning		Transformational	Mathematics (rules & conventions,)	Simplifying expressions

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	thing, which you picked up now.					
15:18 – 17:50	Lineo: heading in your theory books, we have looked at the definitions, what does it mean to be a coefficient, what is a variable, what is a constant? We then looked at adding and subtracting. Next thing, I want to look at is expressing as powers versus expanding (puts up the heading on the board). Ok, I think the best; I am not going to give notes I am just going to look at examples. If I wrote x to the power of 3 Oscar (writes x^3 on the board) and I wanted you to expand it in other-words make it bigger. What does x^3 actually mean? Oscar: x times x times x Lineo: writes it on the board (1) $x^3 = x.x.x$ So going in that direction (left to right) I have expanded it. If I gave you x times x times x and I ask you to express it as a power you would have said it's x^3. So expanding makes it bigger. What is the hidden meaning? Expressing as a power simplifies it, make it smaller.	Questioning	Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Expanding vs. expressing as powers
18:00 – 18:36	Lineo: (writes (2) x^4y^2 on the board) x to the 4 and y squared, expand that for me. Learners individually work on the task Lineo: Rob you had your hand up Rob: x times x times x times x times y times y Lineo: (writes $x^4y^2 = x.x.x.x.y.y$ on the board) ok no problem so far, I am assuming	Questioning	Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Expanding
18:44 – 19:27	Lineo: (writes (3) $2p^2$ on the board) 2 p squared, [short pause] Mathew?			Transformational	Mathematics (rules & conventions)	Expanding

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	Mathew: 2 times p times p Lineo: (writes (3) $2p^2 = 2.p.p$) correct, that square is only looking at p. If I wanted that square to look at the whole thing $2p$ then I would have to put in brackets.	Explanation	Interpreting algebraic notation	Reading/writing		
19:29 – 19:56	Have a look at this example (writes (4) $(3x)^2$) and I ask you to expand it. What would that answer be? Try, Mahomete? Mahomate: $3x$ times $3x$ Lineo: $3x$ times $3x$. So the square is saying I need you to repeat the whole bracket.	Explanation	Interpreting algebraic notation	Transformational	Mathematics (rules & conventions)	Expanding
20:00 – 22:10	Henry, coming back to your question when we were marking the homework I can't remember what it … there was something with an answer like $a + b$ and you said it was… can you remember? Henry: I said something about umm, but you said the answer is 2ab squared Lineo: ok let's have a look ab^2 Henry, expand that for me. What does that mean? Henry: a multiplied by b multiplied by b Lineo: (writes the answer as the Henry utters it) $ab^2 = a.b.b$. Multiplied by 2 Henry: multiplied by 2 Lineo: Ok have a look at the question, don't look at what you have written. ab square, what does that square mean? Lineo: Who is the square looking at? Henry: b Lineo: the b and is that a big 2 or a small 2 which means it's just the power saying I need to repeat b twice. (Writes $ab^2 = a.b.b$) is that going to be the same as $2ab$? No, can you see the difference? Kele?	Questioning Meaning of symbols	Transformational Reading/writing	Mathematics (rules & conventions)	Simplifying expressions

WitsETD

	Kele: Mam, if it was in bracket it would be ab by ab. Lineo: if it was (ab) squared, yes it would be ab times ab Lineo: this two over here it's a power (points at ab^2). Its only role, its function is to say I need to repeat b this twice. Learner: why only the b ? Lineo: why only the b ? Because if I wanted it to look at the whole thing then I would have to put up the whole thing into brackets. (Writes $(ab)^2$) can you see the difference? The same thing there (point at $(3x)^2$). So if there is no bracket you assume that the power is only looking at that letter.	Explanation Explaining				
22:12 – 24:31	Couple more (interrupted), Henry? Henry: umm, mam in that (unclear) Lineo: I will do it in this example (writes (6) $3x.2x^3.4x^2$) Ok, expand it. Learners work on the task individually [silence] Lineo: it's a bit of an odd question with those numbers lurking in between [silence], Kele Lineo: (writes as the learner responds $3x.2x^3.4x^2 = 3x.2x.x.x.4x.x$) Ok, the only thing I could do, can you expand that 4 a little bit more? Can you break it down? What can 4 are broken down into? Henry: 2 times 2 Lineo: 2 times 2. So we can break 4 into 2 times 2. Lefu? Lefu: can you break that 3x? Lineo: yeah you mean you just wanted a dot there (points between 3 and x). Yeah. Ok, are you all guys happy with expanding? Sephetho: can you put brackets on that? Lineo: this one here? (points at $(ab)^2$) Sephetho: no, no, number 6, in the answer, shouldn't you put brackets? Lineo: where do you want to put brackets? Sephetho: between 3 and) x Lineo: (writes $(3x)(2x.x.x)(2.2.x.x)$) like that? That is correct because you have expanded each portion but it's not necessary.	Questioning		Transformational	Mathematics (rules & conventions)	Expanding

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24:33 – 26:07	Ok, couple more going in the reverse direction. I am now going to give you the expanded form I want you to put it in terms of the power(writes (7) $x.x.x.y.y$ (8) $2.2.2.2$) I want you to work out question 8 but I want the answer, I don't want to know what is 2 times 2 times 2 times 2 is. I just want you to put it as a power. [silence] [Learners: silently work on the task individually] Lineo: Kori, question number 7? Kori: 2 to power of 4 Lineo: you are doing number 8 (writes $2.2.2.2 = 2^4$) that's fine, 2 to the power of 4. For number 8, 2 is being repeated 4 times and number 7, Limpho? Limpho: x cubed times y squared. Lineo: x cubed times y squared (writes x^3y^2), x cubed y squared. Any question before we go on with timesing a bit (silence)	Question	Interpreting algebraic notation	Transformational	Mathematics (rules & conventions)	Expressing as powers
26:12 – 28:54	next heading timesing you got a heading with adding and subtracting now let's look at timesing, Lineo: writes on the board Simplify (1) $4x^2.2x$ Lineo: Ok let's have a look now at question number 1, here we are timesing and I want you to simplify not to expand it, I want you to make the answer nice and neat. The simplest way we can (silence). Let me explain question number 1 and then you can try number 2 and the rest by yourselves. That 4 and the 2 they are acting, those 4's and 2's, the big 4 and big 2 (points at the coefficients) are they acting as powers or numbers as we traditionally know? Learners: numbers [chorus]	Questioning	Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Simplifying expressions

WitsETD

	Lineo: numbers, so looking at 4 and 2 what is happening between them I am asking you to multiply them. What is 4 times 2? Learners: 8 Lineo: 8. so dealing with the big numbers first, 4 and 2 act as proper numbers.	Explanation				
29:08 – 30:50	Then here I have got x^2 times x. How many x's are being multiplied by each other? Chomi:: 3 Lineo: three, because x^2 is the same thing as x times x. So I have got x times x and another x, final answer is $8x^3$ Lisema, you have a frown of a knod Lisema: I'll get… you know after you do more Lineo: you will get there, a couple more, Malawi? Malawi: you always do the numbers then the variables Lineo: correct, the way I traditionally do the multiplying. Kori, Oscar you need to have a look at the board. The process I usually do, I first look at the signs, is my answer going to be negative or positive? Then I look at my numbers, and then I look at my variables (writes signs→numbers→variables). First thing, is my answer going to be positive or negative? Them I look at the big numbers and then I look at each of the letters separately.	Questioning Explanation	Interpreting algebraic notation	Transformational Transformational	Mathematics (rules & conventions)	Simplifying expressions
30:53 – 33:00	Lineo: writes (2) $(-2x^4)(-3x^5)$ Minus 2 x to the four times, remember between the brackets is times minus 3 x to the five. Who can figure out what that answer could be?	Questioning	Interpreting algebraic	Transformational Reading/writing	Mathematics (rules &	Simplifying expressions

WitsETD

	Silence as they work on the task Lineo: Wily I am going to pick on you. Wily: $6x$ to the power of 9 Lineo: $6x$ to the power of 9. How many have got that one right? (Learners raise up hands) ok, first let's look at the signs, negative and negative, positive answer (writes (2) $(-2x^4)(-3x^5)$ = +) you obviously do not need to write that plus, the big numbers, 2 times 3, 6. x to the 4 times x to the 5, if you are not so sure on this one, let's see (writes $x \cdot x \cdot x \cdot x$) Lisema what does x to the 5 mean? Lisema: x times x times x times x times x Lineo: x times x times x times x times x. So how many x's are being multiplied by each other? Nine. Couple more questions and then you can practice Lineo writes (3) $(-4p^2q)(-5pq^2)(-2p)$ who can figure out that one out? Silently work on the task Lineo: John I'm going to pick on you John: minus 40 (inaudible) Lineo: minus 40, ok let's do the letters together, p and q are different letters so we need to look at them separately. Let's look at the first lot of p what does p^2 mean? John: p times (interrupted)	Questioning Questioning Question	notation Interpreting algebraic notation		conventions) Mathematics (rules & conventions)	

WitsETD

	Lineo: p times p them I have got p over there and another one over there all being multiplied by each other. How many p's? p to the [pauses] John: power of 4 Lineo; Perfect, let's see now if you can do the q's for me? John: q to the power of 4 Lineo: q the power of John:4 Lineo: have a look John: sorry, mam (inaudible) Lineo: q to the power of 3 are you all guys getting it?	Questioning Working with learners' ideas				
33:07 – 34:22	James: Mam is it minus 40 or 40 Lineo: negative and negative James: positive Lineo: and another negative James: negative Lineo: the last one what was that last question I gave you in the spot test. What did you say? Rob: I got negative 40p squared and then pq cubed I timesed pq Lineo: pq, yeah Rob: that pq is, I said times pq in the middle, so I got ppq cubed and then I have p squared. I said negative 40 p squared Lineo: hold, hold on. Minus 40 p squared (writes – $40p^2$) mm Rob: pq cubed Lineo: p q cubed (writes – $40p^2$ pq^3) like that? Rob: yes Lineo: (compares Rob's answer with the one given earlier; - $40p^4q^3$. she circles q^3 in both answers) that's right (then circles – 40) that's matching up nicely but your p square and…your p's look a bit funny. Let's have a	Working with learners' ideas Working with learners' ideas	Interpreting algebraic notation	Transformational	Mathematics (rules & conventions)	Simplifying expressions

WISETD

	look, you worked with that p squared (ticks p^2 in the original expression) and then you had this one (points at the last p in the expression) you said. Rob: yes Lineo: (points at the p in the $-5pq^2$) what happened to him? Rob: I said the q on the first place (interrupted) Lineo: yeah, Ok, wait Rob can you just wait, hold up for two seconds, just let put up the next question, get the class going on that. (Unclear) who got the last question of the spot on test that I gave them? Ted, what was it? Ted: bracket minus 2xy bracket, bracket x to the power of 2 and y to the power of 2 Lineo: y to the power of 3 Ted: 3					
34:52 – 35:47	Lineo: (writes (4) $(-2xy)(x^2y^3)$) guys do number 4 when I look at Rob's query. What did you say? Rob: you see that q next to the first p Lineo: yeah Rob: that made pq Lineo: correct Rob: That pq and the pq in the middle Lineo: yeah Rob: that gave pq^3 Lineo: but the first p there was p squared Rob: yes I get that like that Lineo: you have got p^2 and the last p Rob: and I timesed that by q Lineo: yeah Rob: it gives me pq Lineo: I am happy with that Rob: I added to that pq in the middle	Questioning Working with learners' ideas	Interpreting algebraic notation	Transformational	Mathematics (rules & conventions)	Simplifying expressions

WiSETD

	Lineo: you didn't, where is you other p ? Rob: which p ? Lineo: in the middle? Rob: that pq is one isn't?					
35:49 – 46:03	Lineo: ok Voyo, answer? Voyo: I wrote minus two x cubed and then y to the power 4 Lineo (writes it on the board $-2x^3y^4$) was that so bad? Learners: no Lineo: no. Ok, for the rest of the time, you have got about 12… 14 minutes I am going to put up homework for tomorrow, you gonna work on it now and the only thing, the only word of caution that I have for you guys, at the moment it's nice we are putting in boxes, we are adding, you know the rules, timesing know the rules, we gonna have to look at what happens when you mix it up. that's gonna be Ok Khiba: unclear Lineo; for exercise 3…I have a look now, [writes on the board]… Ok guys let me have you attention one last minute, for tomorrow exercise 3 and 4 number 1. if you are under time pressure and find ex. 3 too long to write out you can put up answers straight away. if though it will help to write it out and to group then write it out that will give you the choice. Ok exercise 3, exercise 4 number 1. I am Walking around dealing with queries, your coefficient queries. Lineo, Chris you had a question? Lineo: are you happy with that one you got wrong? when I asked you 8x plus 3y plus.. Chris: ooh, yeah this [interrupted] Lineo: the one you wrote when you were writing your homework Chris: but for this one (inaudible) Henry: mam 2n plus 3x plus 4, can we add? Lineo: but are they all the same? is ab the same as ac? Henry: no you cant add ab and ac Lineo: no			Transformational	Mathematics (rules & conventions)	Simplifying expressions

WCSETD

	Lineo: if you want me to check ex.3 ask me now before you get it wrong for homework. Lineo: Justice what you think the answer is gonna be? Justice: 4y plus y Lineo: what's the answer? Lineo: almost that 4y plus y, are they the same ? 4y and y Justice: no Lineo: why not? Justice: because 4y is 4 times y that is just y Lineo: Ok, but remember I am adding we just want to group the same things into the same categories, why did put the x's together into five x? Lineo: yeah [moves] Justice: because I got the same [unclear] Jimmy: … this in exercise 3 … with that could you add 3 and 2 and then x to the power 3? Lineo: Ok, if I said [interrupted]… what's the answer? Lineo: Kele? Kele: will that one be the same as five to the … yeah five to the power of two? [writes $5x^2$ on the homework book] Lineo: tell me why Kele: because that's x and we add on here… [cancel the x^2 in $5x^2$] that will be 4y two Lineo: but it's 4y plus y, what the imaginary number there? Kele: 1 Lineo: and why did you say y squared? Kele: I forgot there is one mam? Lineo: and the y squared where did he come from? Kele: I thought the and the y is y times y, then just square Lineo: but it's not timesing hee it's adding Lineo: just before the bell goes let's make sure that you have got at least the first 4 correct. 1) $5x+5y$ 2) $9a+5b$ 3) $5c+7d$ 4) $3x+8$ I do not want you to get everything wrong. guys for tomorrow minimum					

WitsETD

	homework ex.3 has got to be done if you have got time you can do ex. 4 number 1. Pheko where is you diary?...					
00:00 – 04:08	**Lesson 4** Lineo: except Sephetho and Mathe, anyone who has nnot completed exercise 3? Ching: mam I didn't do all. Lineo: why not? Ching I don't know Lineo: anyone besides MacNab? Lineo: Gary, I will hold on for detention, you are going for detention, first thing next term. Lineo: take out your homework, let's mark exercise 3. Minimum of your homework exercise 3, some of you might have done exercise 4 questions 1 as well. Ok I am ready to mark, not repeating. Question 1 $5x + 5y$ [interrupted] ... oh we already marked the first four, that's fine. question number 5, Edward, your answer for number 5 (learners reads out answer), number 6, who is there Poli? Poli: $9s + 8$ Lineo: if you put it in a different order its fine, if you say $8 + 9s$ that perfect. Number 7, Koari? Koari: three xy plus 4xx plus 3 xz Lineo: Ok, there was … no no no no, it was very clear with the printing printing so you read it as xx; some of you might have read it as xz, if that was the case you would have $7xz + 3xy$. Question number 8 Oscar? Oscar: $7pq + 12pr$ Lineo: 7pq + 12pr perfect, I will deal with all the queries at the end. Let's just get through the answers. eerr number 9 Timothy? (this continues until all questions have been marked)			Transformational	Mathematics (rules & conventions)	Simplifying expressions
04:09 – 05:08	Lineo: Ok, I will deal with queries individually, for the rest of the time you work on exercise 4, once you are done with 4 you go to 5, once you are done with 5 you go to 6, one you are done with 6 you go to 7. If you finish exercise 7 before the bell goes come and speak to me I will sort some revision for you, Ok. Lineo: you guys question number 4… exercise 4, question number 1, where it says write as the product of powers. Question (a). First of all, what does product mean?	Questioning	Defining	Transformational Reading/writing	Mathematics (rules & conventions,	Expressing as powers

UNISETD

	Learners: chorus timesing Lineo: timesing, what times what give us 4? If you are going to (interrupted) Lerato: 2 to power of 2 Lineo: 2 times 2. So question 1 (a) (writes on the board $4 = 2^2$) will be written as 2 to the power of 2.				definition)	
05:09 – 07:25	Lineo: let me speak with to Mahomete he had a query and then I will be with you, Sam. Mahomete: mam I had a query about the 4… [interrupted] Lineo: you are answered? Goes to another learner Lejone: mam [unclear] Lineo: where is yours? Lejone: I left it at home Lineo: you are borrowed and you return it with [unclear] conditions [moves to Sam] Sam: how do you … you, you write that three three x? Lineo: then when I am to read that what I am I going to read that as… Sam: because the x [interrupted] Lineo: no no if I'm going to read that how am I going to read it as? Lineo: Ok, no no what do I see when I see that? Learner: thirty-three Lineo: thirty-three, is this thirty-three? Sam: it's three, but you have got two threes Lineo: correct, but how do you write them as powers? [pause] Lineo: three to the power of two. Isn't three to the power of two three times three? Sam: ooh, three to the power of two, x to the power of three Lineo: correct [moves] Henry? Henry: I was just wondering that some of the answers [unclear] Lineo: you need to check with somebody else [moves]	Working with learners' ideas	Reading algebraic notation Writing algebraically	Transformational Reading/writing	Mathematics (rules & conventions)	Expressing as powers
07;28 – 12:30	Jimmy: would you put two to the power of two? Lineo: mm, do you see what I put on the board when you were talking to Chai? [moves] Lineo: Jimmy, why have you got ribbons on your hair? [learners laugh] MacNab: how long will the detention be, mam? Lineo: two hours, I [unclear] at my records, you just need to do your	Questioning	Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Expressing as powers

WitsETD

	homework for next week and you will be fine. but if you are like ching who does not do the homework [interrupted]… then you need to write your homework in your dairy. Lineo: Mahomete doesn't want to watch the video because he is being caght on camera Hlompho: mam when are you going to help with some hard stuff? Lineo: next week, it will be your time, I have lots of time next week. [moves] Khang: … to say four times four? Lineo: But you want to write it as powers, so how can… shhh, the four, how can you write 4 as a power? 2 times 2 …. So 8 is 2 times 2 times 2, so it will be 2 to the power of 3 Goes to another learner Lineo: do you want to catch some of the questions? [moves] Lineo: no noo, it's just that they are easy, do not worry about that its from a textbook, once you have done this exercise you need to catch up with your theory. borrow the rules books from someone [moves] Motong: unclear Lineo: it's fine, or you can even write ccccccc…. Jack: mam, with question five, simplify I was just wondering something like [interrupted] Lineo: now do the numbers, 3 times 2 is Jack: ooh 6 Lineo: so you write it as 6xy Tim: mam would this be $2x^2$ or …. Lineo: Let's back track and see if you are right, if I gave you $2x^2$ what am I meaning? If you are to expand it… with the two yeah Lineo: I am coming Edward Eli:… number b how do you simplify 3x times 2y?jone Lineo: these numbers the 3 and the 2 they are big numbers, they are like normal numbers, right? what would the answer there be? Henry: can I just put x times x times x times 6 times … Lineo: I think what they want you to do instead of doing that, though that is correct I agree with, is just to say is this times itself three times, so ti will be $3x^2y^2$ times $3x^2y^2$ times $3x^2y^2$ but I mean you are also right [interrupted] … that's real..ly expanded [moves]					

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	Lineo: Question number… Lejone: m Lineo: m, 2 times 2 times 2… write in the product of powers Lejone: it's 2 to the power of 4 Lineo: mm is the same instruction.					
12:31 - 12:58	Lineo: Mahomete, you are working, mm, question? Mahomete: no Lefu: mam how do you write [unclear] Lineo: you tell me what do you think about this? Lefu: two x squared equal … I do know, I do not know Lineo what is being repeated? Lefu: two x Lineo: correct, it's x times x that is being repeated twice Lefu: so there will be two x to the power two. Lineo: mm, if I say to you … if I gave you two x to the power of two, you would know is 2xx [moves] John: Ok mam, 20abxy Lineo: 20abxy [unclear] hundred percent. are you happy with all the others? John: number 4 if it's three [interrupted] … it will be $24x^2y^2$ Lineo: mm [moves] coming Henry: three xxyy three xxyy Lineo: mm [moves] Khau: number 4 mam, I don't understand this x times x times… (inaudible) Lineo: ok you were not here yesterday you missed out on the whole lesson, if I said to you umm x to the power of 3 what am I meaning? Khau: any number to the power of 3 Lineo: what does the power of 3 mean? Khau: times itself 3 times Lineo: ok, x to the power of 3 means x times x times x so this is going in the reverse direction, you need to catch up with the theory, heee, borrow Kele's book over the week-end. [moves] Lineo: xtimes y you want to make it into a neater way of writing it. just do number 2 for me, number b Jabu: it's three x twoy Lineo: Ok, the numbers can be multiplied by each other.	Questioning	Reading mathematical language Interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Expressing as powers

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	Jabu: so it's 6xy Lineo: mm [moves] Lefty: three to [unclear] times y to the power two and then brackets will that be like five x's and five y's Lineo: no how many… because what does this power of three mean? Lefty: it means whatever is inside the brackets [interrupted] Lineo: three times heee so I am gonna have $3x^2y^2$ $3x^2y^2$ $3x^2y^2$ Jabu: but then you won't have to… when you are writing, expand the notation like 3 times x times x times y times …. Lineo: have fun [moves] Sephetho: can we say it's 6xy Lineo: hundred percent [moves] Thando: unclear Lineo: like [interrupted] yeah, just do number b so that I can see you are on the right track. Thando: 3x 2y Lineo: it's correct but they want you to simplify it, is there anything that you can work out in that sum? [learner silent] what about the numbers? no no do not listen to him. what is happening between the three and the two [learner silent] timesing what is three times two? Thando: ohh 6 [lineo moves away] Lineo: Rob, are you Ok? Rob: yeah Lineo: what happened to the hair today? Rob it has always been like that, didn't you notice? Lineo: from the back it looks different. you didn't brush you hair. Chang: inaudible Lineo: with the power [moves] Lineo: question Ling: do we say a times b squared or [interrupted] Lineo: what does square mean? Ling: a times a times b times b [Lineo knods and moves away] Lineo: question? Hatla: two times a times c squared Lineo: so that becomes? [pause] no if you wanted to do it like that hee that means xy xy xy how many x's do I have? if you wanted to you could say xy to the power of 2 but I think what they want here is simplify it, you don't want any brackets. [moves]					

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	Tau: to simplify c would you say x cubed y cubed Lineo: no no y squared, x squared y squared [moves] Ching: when you do this x p to the power 4 is not two p Lineo: no no..no… [moves] Lineo that perfect excerpt i don't want the dot now because we are simplifying it so x times y is just xy Jack: is there anything we can do with this? Lineo: we can … is there anything you can do with the three and the two there? Jack: unclear Lineo: what's happening there? timesing what's three times two? … 6xy Shata: this will be two to the power 4 Lineo: mm Lineo can you ask from there? Thabo inaudible Lineo: what would 16 to the power of 50 mean? 16 times 16 times 16 times… is that the question? Thabo: no Lineo think abpout it or Rob will tell you [moves] Lineo: yeah					
13:02 – 13:23	Goes to another learner Lineo: x times y what is the neater way of writing it? Lefty: it's two x three y Lineo: The numbers can be multiplied by each other. Leaner: umm it's … $6xy$	Questioning Explaining	Writing algebraically	Transformational Reading/writing	Mathematics (rules & conventions)	Simplifying expressions
13:24 – 18:00	Goes to another learner Lineo: what does x to the power of 3 mean? Three times, hee, so I am going to have $3x^2y^2$ $3x^2y$ $3x^2y^2$ Lineo: do another number (b) so that I can see you are on the right track. It's correct but… Ntando: three x two y…. Lineo: it's correct but they want you to (interrupted)	Explaining	Interpreting	Transformational Reading/writing	Mathematics (rules & conventions)	Expanding

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	Lineo: I don't want the dot now because we are simplifying. Is there anything you can do with the 3 and the 2 there? What is happening there? Timesing, what is 3 times 2? Ntando: ooh 6 Lineo: 6xy	Questioning	algebraic notation			
18:04 – 19:36	Lineo: x y to the 4, like that, what does that mean? (Writes xy^4 on the learner's book) like that? If I ask you to expand that what would that mean? If I asked you to expand it. Learner: x dot y dot y dot y dot y Lineo: (puts brackets around xy so that it looks $(xy)^4$) if I do it like that, what would that mean? xy xy xy xy Let's see the tricky one (points at x^2y^2).	Questioning Explanation	interpreting algebraic notation	Transformational Reading/writing	Mathematics (rules & conventions)	Expanding

Table 5 transcript of the observed lessons

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Therefore, mathematics for teaching is composed of reading/writing activities i.e. it is about mastering conventions in mathematics.
Hence one can conclude that in this practice MfT is also transformational in nature. While Lineo still engages with reading/writing activities we see that MfT is now empirical.

MfT is now conventions.

In this case we see that MfT is empirical, transformational and conventions.

rather than in generalizing and justifying activities as argued by Kilpatrick *et al*. (2001). So far we see that for Lineo mathematics for teaching (MfT) is constituted by conventions in mathematics and empirical mathematical knowledge

as opposed to developing meaning for the underlying structures/*concepts, symbols* and *operations* [italics added] as argued by Kilpatrick *et al.* (2001) [see section 1.3.3.3].
rather than mathematics as reasoning (Brodie, 2000).
I have shown that in Lineo's practice MKfT or MfT is composed of empirical arguments, mathematical conventions and terminology.
Although, it is a desired practice in the new curriculum and reform oriented research.
For instance, in this study conjoining came up as learners engaged in transformational activities, where they were given $3x + 5x + 2b$ to work out. Throughout the observation week [see chapter 3], learners engaged with activities which demanded them to write algebraic expressions by manipulating either a given algebraic expressions or a random distribution of terms as in:

In this study, the mathematical problems that confronted the teacher occurred as a result of engaging learners in representational and transformational activities. Such mathematical problems were categorized as **manipulating algebraic symbols** problems. This category includes mathematical problems such as *conjoining, representing, symbol conflict, recognizing like term, splitting terms and substitution* [see Appendix 4].

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