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A statistical approach to some mine valuation and allied problems on the Witwatersrand

Krige, D G

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The line of regression of the true values on the indicated values is required. The average of all the indicated values ($\bar{y}$) is equal to the average of all the true values ($\bar{x}$) = 4.90 dwts/ton.

Parameter for distribution of x = a_x = 1.648

Parameter for distribution of the array of x's = a_{x_y} = 3.672

Parameter for distribution of y = a_y = 1.503*

$$\therefore \log x = \frac{a_y^2}{a_x^2}\log y + (\log \bar{x} + \frac{1}{4a_x^2})(1 - \frac{a_y^2}{a_x^2})$$

and $\therefore \log_e(\text{true value}) = .8323 \log_e(\text{indicated value}) + .28195$

and $\log_{10}(\text{true value}) = .8323 \log_{10}(\text{indicated value}) + .12245$

Solving this equation for the indicated values in row 1B of Table 17 yields the figures in row (ii) below.

TABLE 18

	Values in dwts/ton								
(i) Indicated value	2.05	2.76	3.45	4.43	5.42	6.41	7.41	8.41	11.20
(ii) True value (calculated)	2.41	3.09	3.72	4.58	5.41	6.22	7.02	7.80	9.90
(iii) True value from Table 17	2.42	3.19	3.70	4.57	5.40	6.17	6.97	7.89	9.73
(iv) True value from current stope sampling (Mine C)	-**	3.21	3.73	4.78	5.69	6.49	7.21	7.76	9.11

The slight differences between the figures in rows Nos. (ii) and (iii) are due to the fact that the figures in Table 17/...

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* From No. (40):-

$$\frac{1}{a_y^2} = \frac{1}{a_x^2} + \frac{1}{a_{xy}^2} = .3682 + .0742 = .4424$$

$\therefore a_y^2 = 2.2604$

$a_y = 1.503$

**Below pay limit.

Table 17 were practically entirely obtained graphically and cannot, therefore, be accepted as exact.

The figures in rows (i) and (ii) above are depicted on Diagram No. 5* in the form of the smooth curve ab (this curve plotting as a straight line on double logarithmic paper). The values in row (iv) are shown as small circles on this diagram and indicate that the practical results follow the same general trend (although with what appears to be a somewhat sharper curvature) than that represented by the curve ab. This diagram also clearly indicates the extent of under-valuation of low grade blocks and the over-valuation of high grade blocks.

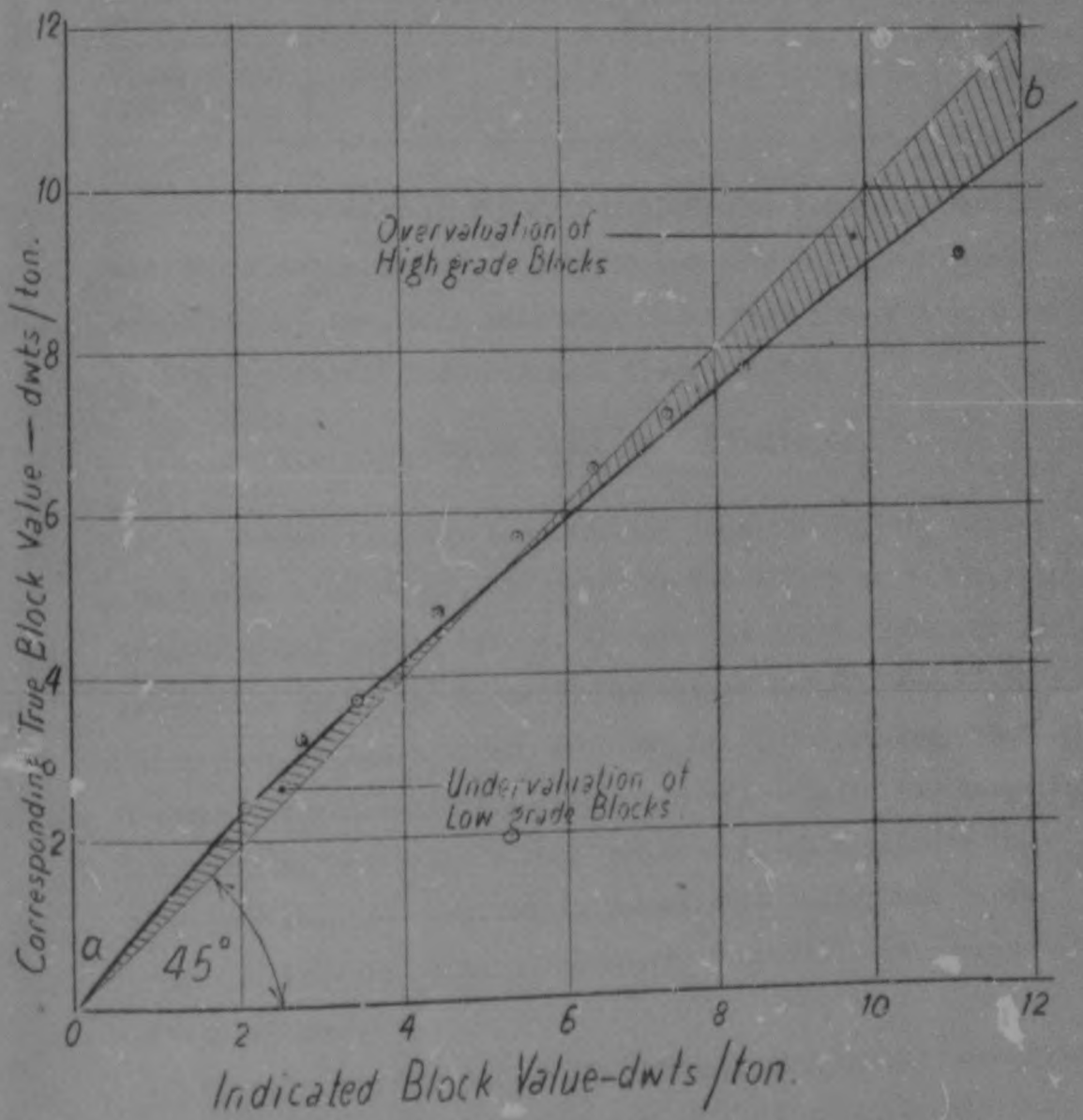
Irregular trend in Block Plan Factors: It is obvious from the foregoing, that where the number of samples per block is limited, and where no cutting of individual or block values is done, it is only natural and not due to wrong blocking policies, to expect Block Plan Factors exceeding 100% in the low grade ore categories and less than 100% in the high grade ore categories. The extent of the deviation either side will depend partly on the basic behaviour of the distributions of gold values in the mine and in individual blocks, but mainly on the average number of sample values available per block. The general trend of B.P.F.'s over the range of value categories can, however, be expected to be similar for all mines. This trend was, in fact, observed by Ross** in the case of the majority of the lease mines on the Witwatersrand. Where Block Plan Factors do not follow this general trend and are very irregular, the reason must be sought in the arbitrary cutting of individual and/or computed block values. On Mine B, e.g., cutting of block values is done, and the following average Block Plan Factors for this mine over a year's operation

clearly/...

*See over. **Ref. 2.

DIAGRAM No. 5

Showing the relation between Indicated Block Values, theoretically determined True Block values and Block values inferred from current stope sampling.



clearly indicate the irregular results obtained.

TABLE 19

Value category dwts	1.6 to 1.9	2.0 to 2.9	3.0 to 3.9	4.0 to 4.9	5.0 to 5.9
Block Plan Factor	122.5%	116.5%	111.3%	150.6%	125.7%
Value category dwts	6.0 to 6.9	7.0 to 7.9	8.0 to 8.9	9.0 to 9.9	10.0 & over
Block Plan Factor	108.5%	138.3%	94.2%	90.5%	126.1%

The average B.P.F. = 120.6%, and assuming that stope and block sampling are subject to the same degree of over-sampling, if any, this indicates an average cut of approximately 17% applied to the indicated block values,

$$\text{i.e., } \frac{\text{indicated block values}}{(100 - 17)\%} = 120.6\%$$

Had this cut been applied consistently throughout the whole value range, and ignoring the effect of the cutting of individual sample values, if any, the B.P.F.'s in the lower categories should have been in excess of 120.6%, and those in the higher categories less than 120.6%. The observed B.P.F.'s, however, do not exhibit such a trend and clearly indicate that the "cutting" of block values was done in an arbitrary and irregular fashion resulting in unreliable individual block estimates as well as an unsatisfactory overall ore reserve determination.

(c) Average Block Plan Factor above pay limit: Accepting the relative frequencies in row 1A of Table No. 17, and the indicated and true values in rows (i) and (ii) in Table No. 18 above, the average indicated and true values above any pay

limit/...

limit can be calculated, and is reflected in Table No. 20.

TABLE 20

Value limit = pay limit	Average indicated value above pay limit	Average true value above pay limit	Theoretical Block Plan Factor for all ore above pay limit
	Values in dwts/ton		
0	4.9	4.9	100%
2.55	5.33	5.28	99.1%
2.95	5.53	5.45	98.6%
3.95	6.32	6.11	96.7%
4.95	7.24	6.85	94.6%
5.95	8.18	7.59	92.8%
6.95	9.18	8.37	91.2%
7.95	10.22	9.17	89.7%
8.95	11.20	9.90	88.4%

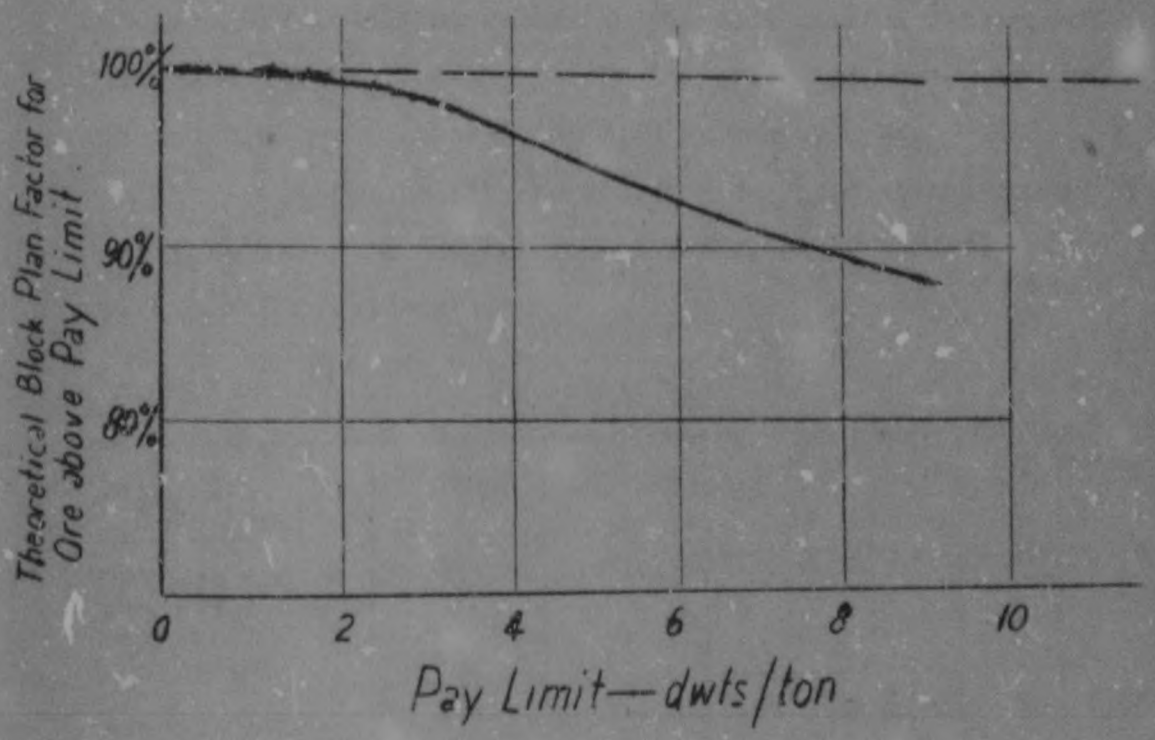
The overall theoretical Block Plan Factors for various pay limits are shown on Diagram No. 6,* and it is clear that as the pay limit is raised the Block Plan Factor will decrease.

(d). A partial explanation of the Mine Call Factor: The customary practice on the Rand in stoping to a pay limit is to base the policy of stopping and starting stope faces, i.e. of selecting the payable ore for stoping, largely on current stope sampling except, possibly, where an ore reserve block falls well within the payable category. In many cases portions of stope faces, which can from a mining point of view be stopped without upsetting the general layout, are stopped on the results of one or two, or at most, three unpay stope samplings, the total number of samples on which such a decision is based being as low/...

*See over.

DIAGRAM No 6

Showing Theoretical Block Plan Factors for Mine C for various Pay Limits.



low, possibly, as 10 or 15 (e.g., a 100' length of face sampled at 20' intervals). The average number of stope samples on which any such decision is based is probably larger but will in nearly all cases be considerably smaller than the average number of samples per ore reserve block. If, therefore, a table similar to Table No. 17 is prepared to allow for the selection of tonnages in value categories on, say, 30 sampling sections per working face, it is obvious that the variation of the average indicated values in the rows will be larger, and that the under-valuation in the low grade and over-valuation in the high grade categories will be accentuated.

On this basis, and adopting the theoretical procedure outlined above, a 2nd edition of Table No. 20 was prepared (Table No. 21 hereunder).

TABLE 21

Pay limit dwts/ton	0	2.55	2.95	3.95	4.95	5.95	6.95	7.95	8.95
Average indicated value above pay limit dwts/ton	4.9	5.42	5.68	6.49	7.41	8.39	9.41	10.45	11.49
Average true value above pay limit dwts/ton	4.9	5.31	5.51	6.08	6.71	7.35	8.00	8.63	9.27
Stope sampling factor	100%	98%	97%	94%	91%	88%	85%	83%	81%

From the above it is evident that in working to a pay limit, there will always be a consistent tendency for the ore selected for stoping to be over-valued and that in the case of a mine with a relatively low payability (i.e. a high pay limit in relation to the average value of all the ore in the mine) the percentage over-valuation in selecting tonnages of ore as payable on a limited number of sample values can be appreciable, and can thus account directly for a proportion of the difference

between/...

between the gold called for by sampling and that accounted for by the reduction works. The explanation of a Mine Call Factor of less than 100% can therefore to some extent be sought in the limited number of sample section values available per working face.

(e) The mine call factor and actual over-sampling: The question of over-sampling was ably dealt with in an experimental and theoretical manner by Sichel* who showed that for a narrow reef, over-sampling could amount to as much as 24%, due to the channelling out of too small a proportion of waste matter either side of the reef band. He concluded, however, that all sample values should therefore be adjusted by factors which vary from the one value category to the other. He based this conclusion on the assumption that the frequency curve for the mine, as a whole, could be adopted as a standard, and took no account of the fact that the sample values in every block of ore constitute a distinctive frequency distribution.

If, therefore, every sampling section is liable to yield any one of a range of errors as found experimentally by Sichel with an expected average error of +24%, then although the distribution of sample values in a block of ore resulting from such a range of errors will be different in relative shape than that of the actual values in the block (due to the introduction of an additional variance as well as an average bias error) the mean value of this observed distribution must still be equivalent to the actual mean value of the block, increased by the average error of 24%. No purpose is, therefore, served in adjusting individual sample values to varying extents if the resultant overall adjustment for the block as a whole (or stope face, or section of reef development) is a constant factor of 100/124. Sichel's so called "group

correction/...

*Ref. 3.

correction factors," therefore appear to serve no practical purpose, since the mine valuator is only interested in groups of sample values, which in every case have been drawn from a population covering a full range of values.

An average over-sampling of, say, 24% will however have a noticeable effect on the theoretical stope sampling factor discussed in the previous subparagraph (Table 21). Take, for example, the case illustrated in Table 21 with the introduction in addition of a 24% bias error as suggested. Adopting the procedure indicated in subparagraph (b) above, and based on formula No. (59a), where in this case

$$x = \frac{\bar{y}}{1.24}, \text{ i.e. } \bar{y} = 6.076, \quad a_x = 1.648, \text{ and } a_y = 1.395^*$$

$$\log_e(\text{actual value}) = .7165 \log_e(\text{indicated value}) + .32252$$

$$\text{i.e. } \log_{10}(\text{actual value}) = .7165 \log_{10}(\text{indicated value}) + .14007$$

As before, from the knowledge of the distributions of the actual and indicated block values, and from the above equation, the following relation between the average indicated value and the actual value above any pay limit was determined.

TABLE 22

Pay limit - dwts	0	3.16	3.66	4.90	6.14	7.38	8.62	9.86	11.10
Indicated value above P.L.-dwts	6.08	6.72	7.05	8.05	9.19	10.41	11.67	12.95	14.25
Actual value above P.L. - dwts	4.90	5.33	5.53	6.11	6.74	7.39	8.03	8.67	9.31
Theoretical stope sampling factor (above P.L.)	81%	79%	78%	76%	73%	71%	69%	67%	65%

It is, therefore, obvious that with an actual over-sampling averaging 24%, which would theoretically result in a

M.C.F./...

*a_{x_y} for 30 samples = 2.620 and a_y obtained by solving from No. (40).

M.C.F. of 81% where all the ore in the mine is stoped and no gold is lost in mining, the M.C.F. for Mine C can be expected to be lower than this figure due to the fact that the selection of pay and unpay stope faces is based on an average number of samples per face of only 30.

(f) The percentage of unpay ore included and of pay ore excluded in blocking out ore reserves: Taking again the case of Mine C and referring back to Table No. 17, it is evident that if the number of samples per block were to be unlimited, the mean values indicated for the individual blocks would be the true mean values for the blocks, and the distribution as shown in column 1A would be found in practice. In such an event the application of a pay limit of, say, 2.55 dwts/ton would result in the exclusion from the P.O.R. of the 9.55% of the total mine tonnage falling below this pay limit.

Where block valuations are, however, based on an average of 60 samples per block, the tonnage distribution indicated from such sampling results will be that shown in row 1A, which would indicate an apparent unpay tonnage of 12.57% consisting of 8.26% actually unpay and 4.31% of payable ore (see column 1). The Payable Ore Reserves as calculated on this basis will, therefore, contain an unpay tonnage equivalent to 1.29% of the total mine tonnage (or 1.29/87.43 = 1.5% of the P.O.R. tonnage) and will exclude 4.31% (of the total mine tonnage) actually payable.

The indicated average value of the P.O.R. as blocked will be 5.33 dwts/ton, a figure which will therefore be aimed at in stoping operations, whereas the true value will only be 5.28 dwts/ton (see Table No. 20). A more serious aspect, however, is that had the blocking been done on the basis of a sufficient number of samples per block so as to approximate the distribution in column 1A, Table No. 17, the true average value of the true P.O.R. would have been approximated, i.e.

5.19/...

5.19 dwts/ton (average value in column 1A of tonnages above 2.55 dwts/ton). In aiming at 5.33 dwts/ton the management would therefore in fact, but unknowingly, be practising over-mining.

It is probably due to the experience gained over many years that the average mine valuator on the Rand has sensed the somewhat obscure fact that a proportion of the payable tonnage is excluded from the P.O.R. tonnage in blocking out, and has consequently relied on subsequent stope samplings to guide him in finding such tonnages in blocks classed as N.I.R., and even blocked out and valued as unpayable. This approach probably accounts to a marked extent for the relatively large N.I.R. tonnages* stoped annually on most mines, and also for the attitude that the continued working of a stope face which yields a payable result on, say, only every 3rd or 4th consecutive sampling, is justified. The assumption in a case such as the latter, that the tonnage either side of the face position which yielded a payable result on a limited number of sample sections was payable, is unsound,**and results in relatively large unpay tonnages being stoped for the sake of an occasional small patch of ore which appears to be payable but may, in fact, in by far the majority of cases be unpayable.

The fact that a definite percentage of unpayable ore is actually included in the P.O.R. as blocked out is evident from the current stope sampling results in P.O.R. blocks, although such results inevitably largely exaggerate the actual percentage of unpayable ore in the P.O.R. In the case of Mine C, e[illegible]sampling of stope faces from the P.O.R. blocks as represented approximately in columns 2 to 9, Table

No. 17/...

*N.I.R. in this sense does not include ore which has been developed during the year and can be blocked out as P.O.R. before stoping commences.

**See paragraph 1 above.

No. 17, with an average of only 30 samples per face, will inevitably yield tonnage distributions in every row relatively even more spread out than those shown in Table 17, and therefore with a definite proportion falling in column 1, i.e. the apparently "unpay" category. Calculations as before indicate that of the P.O.R. tonnage in row No. 1 (i.e. 1.29%), 80%,, or the equivalent of 1.03% of the total mine tonnage will, on current sampling results, yield unpay values, i.e. values falling in column No. 1. Similarly, the P.O.R. tonnages in rows Nos. 2 to 5 will yield indicated unpay values (in column No. 1) amounting to the equivalent of 1.98%, 3.40%, 0.62% and 0.08% of the total mine tonnage respectively. The total percentage indicated as unpay on the current sampling results of P.O.R. blocks should, therefore, amount to 7.01% of the total mine tonnage, or 7.01/87.43 = 8% of the P.O.R. tonnage as blocked. This theoretical figure can be compared with the actual indicated 13% of unpay ore from P.O.R. blocks, based on the stope sampling results on Mine C for the year in respect of which the approximate P.O.R. tonnages in Table 17 were applicable. The difference of 5% between the theoretical and actual figures can be accounted for by the fact that the actual ore reserve distribution for Mine C is not exactly lognormal, but shows an abnormally high percentage of ore in the lowest payable categories.

4. Other Errors in Mine Valuation:

(a) Sampling errors: Errors introduced in the actual physical act of taking samples underground fall under the following heads:-

(i) Disproportionate channelling across the reef and waste bands.

(ii) Contamination of the sample material with gold particles/...

particles dislodged from the face while chiselling.

(iii) Incorrect measurements of the sampled width.

Sichel* investigated the errors under (i) in some detail on an experimental basis and found that the average bias introduced on account of such errors could under certain conditions result in over-valuation to the extent of as much as 24%.

Errors under (ii) above are probably not serious and can be avoided with a reasonable degree of care.

The incorrect measurement of sampled widths will always tend to result in over-valuation, since the correct width is always measured normal to the plane of the reef and any measurement not taken in this manner must, therefore, exceed the true width and result in too high an inch-dwt value. Where the face to be sampled is not normal to the plane of the reef, errors of this type should therefore be guarded against. It is as well to realise that an error of 10% in measuring a sampled width as, say, 6 inches, instead of 5.46 inches, will not be very apparent underground, but will if made consistently, result in a full 10% of over-valuation and a Mine Call Factor of 91%.

(b) Assaying errors: silver content of bullion: In assaying, any human errors in weighing off quantities as well as the final buttons, can on the whole be expected to be of an unbiassed nature.

Bias errors are, however, possible due to factors such as**

(i) the variable silver content of the ore, and the straight 10% deduction normally made for silver;

(ii)/...

*Ref. 3. **Refs. 21 to 23.

(ii) The silver content of the litharge used in assaying;

(iii) cupel absorption of gold and silver in the process of cupellation;

(iv) the adherence of cupel material to the bullion button when weighed.

Investigations carried out by Richards and Rubidge* at Rand Leases Gold Mining Company, Limited, indicated that the silver introduced in the litharge would be of the order of 0.08 dwts/ton, that cupel absorption ranges from 0.2% to 0.4%, and that contamination with cupel material could on average account for an over-statement of values to the extent of 1.8%. The fact that the percentage silver content of the buttons corresponding to reported gold values in the lower value categories is, in general, higher than the average silver content of all the buttons handled, with the reverse applicable in the higher value categories, was also confirmed.** Calculations based on the determinations of Richards and Rubidge indicate that after allowance is made for all the factors under (i) to (iv) above, the gold values as reported by the Assay Office (at Rand Leases) will, if anything, understate the actual values, the maximum extent thereof being about 3%.

It appears, therefore, that errors in assaying can on the whole be neglected for mine valuation purposes.

5. Estimating Face and Block Values more Efficiently from Available Sampling Results.

(a) Where the parameter "a" can be predicted with confi- beforehand (i.e. "a priori"): Where it is possible to determine with confidence, as suggested in Chapter V, paragraph 2, that the relative shapes of the distribution curves in respect

of/...

*Ref. 21. **Refs. 21 & 23.

of sample values within blocks of equal size are identical for a mine, or section of a mine, the straightforward method outlined in Chapter III, paragraph 10(b), will yield results far superior to those obtained from the customary arithmetic averages of sample values.

As previously stressed, the procedure is simple and will entail only the calculation of the geometric mean of all the available sample values for every block (i.e. the antilog of the mean of all the logs of the values) and the multiplication of this geometric mean by the factor

$$e^{\frac{1}{4a^2}(1 - \frac{1}{N})} \quad \text{..... See No. (34)}$$

$$= \text{antilog}_{10}(0.43429+5)(\frac{N-1}{4Na^2})$$

where "a" has a definite value determined for the particular size block concerned

and N = Number of samples involved.

The improvement in the results obtained in this fashion were listed in Table No. 2 and indicated that for equivalent result on this basis and on the orthodox arithmetic mean basis, the latter will, for the range of "a" values usually encountered, require from 1½ to 3 times as many sample values as the former. It is, therefore, evident that the time and effort required to be spent on research to establish the relevant values of "a," as well as on the calculation of the geometric means of all block values, will be more than compensated for by the improved results obtained, or alternatively, by the lesser number of samplers required to yield the equivalence of the results obtained under present methods.

Practical Illustration: The conclusions regarding improvements obtained by this method were tested in a practical way by taking the section of Mine B with 3,600 development sampling

values/...

values* as the parent population. Graphical determination of the "a" for this population gave a value of 0.651, and a mean value identical with the straight arithmetic mean of all the values, i.e. 477 inch-dwts.

Sixty sets of 10 equidistantly spaced samples were now selected from these 3,600 samples, and the mean and geometric mean of every set was calculated. The geometric mean of every set was then multiplied by the required factor

$$e^{\frac{1}{4a^2}(1 - \frac{1}{N})} = 1.7005$$

to yield the "improved" estimate "$\bar{m}_1$" of the true mean value (477 inch-dwts). Analysis of all the 60 sets of means and "improved" estimates of the true mean thus obtained indicated that the latter were distributed more closely round the 477 inch-dwt value than the former. This does not, however, mean that the "improved" estimate of the true mean value obtained from a set of 10 values was <u>in every case</u> closer to 477 inch-dwt than the ordinary mean value, the actual results indicating better results in only 36 out of the 60 cases. The average extent of the improvement in the 36 cases, however, amounted to 219 inch-dwts against an average worsening in the remaining 24 cases of only 33 inch-dwts, thus accounting for the overall advantage of this method.

Typical cases of the improvements obtained for extremely low and high sets of values are the following (the true mean value being 477 inch-dwts).

<u>TABLE 23</u>

Sample set No.:	11	54	20	40	49
Mean value of set - inch-dwts	263.8	267.5	954.9	760.0	913.5
"Improved" estimate of true mean value - inch-dwts	346.9	394.0	754.5	581.4	572.9

This/...

*See paragraph 2(b) above for details.

This method also does away with the necessity of "cutting" any abnormally high value in a set of values, e.g.

Set No. 11:

Inch-dwt values: 41; 117; 132; 179; 367; 501; 675; 783; 979; 38,767.

Straight mean value: 4,254.1 inch-dwts.

"Improved" estimate of true mean value: 781.1 inch-dwts.

Set No. 53:

Inch-dwt values: 35; 74; 81; 112; 209; 266; 319; 528; 1,233; 18,928.

Straight mean value: 2,178.5 inch-dwts.

"Improved" estimate of true mean value: 503.1 inch-dwts.

Similarly, 30 sets of 20 samples each were taken and treated in the same manner, the factor $e^{\frac{1}{4a^2}(1 - \frac{1}{N})}$ in this case being equal to 1.7514. Here again, the "improved" values were on the whole clustered more closely around the true mean value of 477 inch-dwts and "improvements" were obtained in 21 out of the 30 cases,* the following being typical cases of how abnormally low and high mean values were "improved."

Set 24

Sample set	5	20	13	29
Mean value of set - inch-dwts	317.7	348.2	1,329.2	695.0
"Improved" estimate of true value - inch-dwts	340.7	380.6	466.7	472.5

Set ...

*Average extent of improvements in 21 cases = 189 inch-dwts.

Average extent of worsening in 9 cases = 34 inch-dwts.

Set No. 13:

Inch-dwt values: 35; 40; 66; 74, 81; 112; 146; 161; 202; 209; 244; 266; 290; 305; 319; 528; 1,004; 1,233; 2,341; 18,928.

Straight mean value: 1,329.2 inch-dwts.

"Improved" estimate of true mean value: 460.7 inch-dwts.

The following are the results of 6 sets of 100 samples each treated in the identical manner, the factor $\frac{1}{e^{\frac{1}{4}a^2}}(1 - \frac{1}{N})$ in this case being equal to 1.7947.

TABLE 25

Sample set No.:	1	2	3	4	5	6
Mean value of set - inch-dwts	403.3	454.2	445.9	832.6	517.2	682.4
"Improved" estimate of true mean value - inch-dwts	405.7	432.6	471.7	476.3	485.9	488.8

In the same way, one set of 600 samples gave a mean value of 555.9 inch-dwts. and an "improved" estimate of 461.0 inch-dwts, the former being subject to an error of +16.5% and the latter to an error of only -3.4%.

(b) Where the parameter "a" cannot be predicted accurately: In this case, as indicated in Chapter III, paragraph 13, the best mathematical estimate of the true mean of a population of lognormally distributed values from a limited number of observed values is provided by the Theory of Maximum Likelihood, but for practical purposes, the method outlined in paragraph 13(b), Chapter III, will yield results which are virtually as reliable.

An/...

As an illustration of the application of this method consider the following examples of sets of 10 samples each from the case dealt with under (a) above:-

Set No. 36:

1	2	3
Value - inch-dwts	Log_{10}value	Log_{10}value - mean of logs
75	1.87506	-0.60929
90	1.95424	-0.53011
99	1.99564	-0.48871
206	2.31387	-0.17048
330	2.51851	+0.03416
390	2.59106	0.10671
413	2.61595	0.13160
512	2.70927	0.22492
728	2.86213	0.37778
2,557	3.40773	0.92338
Mean = 540.0	Mean of logs = 2.48435	Mean of squares of 3 = 0.19960

From column 3:- Mean of square of deviations = 0.19960

$\therefore$ Mean of squares of deviations of logarithms to base e = 1.05826* $= \frac{1}{2a^2}$

$\therefore \frac{1}{4a^2} = .52913$

$e^{\frac{1}{4a^2}} = 1.6975$

Geometric mean = antilog_{10} 2.48435

= 305.3 inch-dwts

$\therefore$ Estimate of mean = 305.3 x 1.6975

= 517.8 inch-dwts

(Arithmetic mean = 540.0 inch-dwts)
(True mean value = 477 inch-dwts)

Set/...

*Multiplication factor = 5.3019 = $(\log_e 10)^2$.

Set No. 53:

1	2	3
Value - inch-dwts	Log_{10}value	Log_{10}value - mean of logs
35	1.54407	-0.92698
81	1.90849	-0.56256
74	1.86923	-0.60182
112	2.04922	-0.42183
209	2.32015	-0.15090
266	2.42488	-0.04617
319	2.50379	+0.03274
528	2.72263	0.25158
1,233	3.09096	0.61991
18,928	4.27712	1.80607
Mean = 2,178.5	Mean of logs = 2.47105	Mean of squares of 3 = .54513

As above:-

$$e^{\frac{1}{4a^2}} = 4.2423 \quad (a = 0.416)$$

Geometric mean = 295.84 inch-dwts

∴ Estimate of mean = 295.84 x 4.2423

= 1,255.0 inch-dwts

(Arithmetic mean = 2,178.5 inch-dwts)
(True mean value = 477.0 inch-dwts)

Although the basic assumption for this method is that the actual "a" is unknown, experience will in nearly all cases indicate the approximate limits of "a," say, from 0.5 to 0.8, and it will therefore in a case such as set No. 53 above, be possible to apply the lower limit of "a," i.e. 0.5, rather than that calculated from the sample values, i.e. 0.416, to arrive at a better estimate than the 1,255 inch-dwts above:-

$$e^{\frac{1}{4(.5)^2}} = 2.7183$$

and then geometric mean x 2.7183

= 804.2 inch-dwts.

Where/...

Where the approximate limits within which "a" must lie, are therefore known, it will be possible to improve in the case of extremely low or high calculated values for "a," the estimate of the mean value provided by the method considered above, this estimate in any case already being "better" than the straight arithmetic mean.

This method is naturally also very suitable for estimating the true mean value of a mine from a limited number of boreholes, since the "a" is generally not known beforehand within sufficiently close limits and therefore requires to be estimated from the observed borehole values.

As indicated previously, this method is virtually equivalent to the graphical method discussed in Chapter IV, but it does away with the human factor which must always be present in any graphical determination, particularly when the number of observed values is small.

CHAPTER VII/...

CHAPTER VII

SUGGESTED IMPROVED MINE VALUATION METHODS BASED MAINLY ON A STATISTICAL APPROACH

Before attempting to suggest any departures from existing methods of mine valuation, it is as well to recapitulate the main practical conclusions arrived at in previous chapters.

A sample section value cannot be regarded as having a so called "area" or "distance" of influence, except in so far as it is applicable to the cross-sectional area of the channel cut at the section.

The orthodox method of accepting the average of a number of available sample (or borehole) values as the true mean value of the relevant block or face (or mine) does not make use of the available data to the best advantage.

"Better" methods for estimating the true mean value based on the frequency distribution of gold values, require little additional effort, are relatively straight forward and render the "cutting" of exceptional values unnecessary.

The frequency weighting of values is unsound.

The practice of starting and stopping stope faces, or portions of faces, on the evidence of a small number of section values results inevitably in the stoping of a considerable tonnage of unpay ore and in the rejection of tonnages of payable ore. It also partially accounts for a Mine Call Factor of less than 100%.

Stope sampling could justifiably be discontinued, at any rate to a very large extent, with an advantageous transfer of effort to enable the number of sample sections per ore reserve block to be increased.

Bias errors are introduced unintentionally in blocking out ore reserves on the basis of a limited number of sample section values per block.

The practice of cutting individual or block values is unsound and should be discontinued.

Bias errors are introduced in underground sampling due to disproportionate channelling across the sample section and incorrect measurements of the sampled width.

Any assay errors can generally be neglected.

A/...

A complete understanding of the basic reasons on which the above main conclusions are based, is a prerequisite for any mine valuator who desires to improve the standard of valuation on a mine by the introduction of scientific methods based on statistics.

1. Practical Suggestions Concerning Sampling.

Over-sampling due to disproportionate channelling across the sampled width, and the incorrect measurement of this width, should be combated by proper instruction and regular inspection. In the case of the former malpractice, regular practical tests as initiated by Bichel* could serve as a general check on the reliability of individual samplers. The development and introduction of mechanical devices for taking samples not subject to such human errors should receive serious consideration.

2. Practical Suggestions Concerning the Development and Stoping Policies.

(a) Prevention of bias in locating raises: On many mines, particularly low grade mines, it is the practice to select the location of a raise to coincide with the centre of the highest value stretches in the top and bottom drives, thus immediately introducing a bias which cannot be overcome by any subsequent methods of blocking or computation.

It should be the aim to locate raises irrespective of the values along the relevant drives, and wherever practicable, to develop a mine on a definite systematic grid pattern, so as to ensure that samples taken round the periphery of a block will approximate, as far as possible, "random" samples in a statistical sense.

(b) Regular size blocks: The above immediately leads on to the desirable policy of endeavouring to maintain ore reserve blocks/...

*Ref. 3.

blocks within as close a range of sizes as possible. This can be done, not only by developing the mine on a systematic grid pattern, but also by not being biassed in any way in deciding on the limit to which a block should be extended.

(c) Confinement of stoping operations to properly blocked out areas: The practice of nibbling at portions of stope faces where no proper block valuation is available, or where the block is shown as unpayable, but the stretch of face indicates payability on a small number of sampling sections only, should be entirely discouraged. The aim in the selection of tonnages for stoping should be to effect this on a block basis only, and once a block has been valued as pay or unpay on a sufficient number of samples* to retain the entire block tonnage in the relevant category, unless a larger number of samples available at a later date indicate a different value for the remaining tonnage.

3. The Discontinuation of all "Cutting" of Individual or Block Values.

"Cutting" in any manner whatsoever cannot be justified, and renders the scientific approach to the valuation of an ore reserve block, or of a mine as a whole, impossible. The extent to which arbitrary cutting of ore reserve blocks can yield misleading results was indicated in Chapter VI, paragraph 3(b).

4. The Introduction of Statistical Methods for Improving Block and Face Valuations.

The "improved" statistical methods for estimating the true block or face values from available data were fully discussed in Chapter VI, paragraph 5, and their application en masse on a mine should not present much practical difficulty. In this way, the overall standard of valuation on the mine will be improved considerably without enlarging the sampling or survey staffs.

5. Ore/...

*See Chap. VI, par. 1 above & par. 5 below.

5. Ore Reserve Computations Based on Defined Limits of Errors.

Once a regular size blocking has been adopted and the desired maximum limits of error decided upon, say, a 90% probability, or 9 in 10 chance, of not being in error by more than 10%, the number of sample sections per block to satisfy such limits of error can be determined,* and should be aimed at in every case. This can be done -

(a) theoretically by the employment of the "improved" methods referred to in paragraph 4 above;

(b) by reducing the sampling interval where necessary;

(c) by dispensing with current stope sampling as far as possible to enable all efforts to be concentrated on block sampling;

(d) by regarding a small remaining portion of a block previously valued on the required number of samples, as still being of the same mean value, i.e. the carrying forward where necessary of a block value until the block is entirely worked out;

(e) by planning main development and stope development in such a manner that as much as possible of the periphery of a block is exposed for sampling and valuation purposes, e.g., where a stope drive is in any case kept, say, 20 feet in advance of the stope face, it might to much better advantage from a valuation point of view be kept, say, 200 feet in advance. Where little or no driving on reef is normally carried out and blocking is done only off raises, the advantages to be gained by resorting to driving before any

blocking/...

*Chap. VI, par. 1 above.

blocking (and stoping) is done should be seriously considered; and

(f) by aiming at "random" samples for every block, i.e. more or less regularly spaced samples round the entire periphery of the block and by according equal weight to every sample. (Weighting based on the so called "distance or area of influence of a sample" cannot be justified scientifically and will not cancel any bias due to the disporportionate location of sample sections round the periphery.)

6. <u>The Introduction, where necessary, of Statistically Determined Block Value Correction Factors</u>.

Where a mine operates to a calculated pay limit, and the behaviour of the value distributions on the mine is known, the theoretical Block Plan Factors for the various value categories of ore can be calculated as indicated in Chapter VI, paragraph 3(b). These will all approach 100% as the average number of sample sections per ore reserve block is increased, and where this number is sufficiently large, the block values as calculated can be accepted. Where, however, it is impossible for practical reasons to obtain on average a sufficient number of sample sections per block, it will be advisable to calculate the theoretical B.P.F.'s as suggested and to apply these to the indicated block values in the respective value categories. This procedure of adjusting indicated block values by correction factors, has a scientific backing and where stope sampling is not discontinued, the actual B.P.F.'s in the relevant categories over a period of, say, 3 or 6 months, will serve as a practical check on the theoretical determinations.

7. <u>Conclusion</u>./...

7. Conclusion.

It is the author's hope that the Gold Mining Industry will put the above suggestions to the practical test, and in so doing provide further confirmation of the theories outlined in this thesis.

A short addendum dealing with some of the more specialised and entirely new problems of Uranium Ore Reserve valuation on the Witwatersrand and its Extensions is attached.

D. G. Krige

D. G. KRIGE.

JOHANNESBURG,

15th March, 1951.

ADDENDUM/...

ADDENDUM

THE FREQUENCY DISTRIBUTION OF URANIUM VALUES AND THE CORRELATION BETWEEN URANIUM AND GOLD VALUES

A public announcement made on 15th December, 1950, concerning an Agreement reached between the Union, American and British Authorities on the question of uranium production from certain South African gold mines, has confronted mine valuators on the Rand, and more particularly those connected with the mines already listed as potential producers, with entirely new and interesting mine valuation problems. Bearing in mind the security aspect and the specific provisions in the South African Atomic Energy Act regarding the maintenance of secrecy, a detailed and open discussion of the interesting results obtained by the author whilst engaged on research work on these problems in the Economic Branch of the Government Mining Engineer's Division, is impracticable. It is, however, possible to disclose the general nature of the conclusions reached in such a manner as to assist only those persons authorised to be in possession of the basic facts (e.g., uranium values in a mine, the cost of extraction, the price to be paid, etc.) and to be of purely academic interest to all others.

It is fortunate from a research point of view that a sample taken underground can be assayed for both gold and uranium (the uranium content being usually determined by Geiger counter on a small quantity of the crushed sample). Direct comparison between the gold and uranium contents of a sample is therefore possible.

Since/...

Since uranium values are measured in lbs/ton, the equivalent of the inch-dwt measure in the case of uranium is the inch-lb.

1. Distribution of Uranium Values.

The distribution of uranium inch-lb values was studied in some detail on Mine X, where the gold and corresponding uranium values of 1,073 sample sections, spaced more or less at random throughout the mining property, were available. The set of gold inch-dwt values was fitted on log probability paper with a lognormal curve having a parameter "a" = 0.667, and the uranium values with a lognormal curve with parameter "a" = 0.820.

The first interesting conclusion, therefore, is that uranium values are also distributed lognormally, and this was confirmed to some extent by the indicated lognormal distribution of uranium values on the Basal Reef in the Orange Free State from the limited number of available borehole assays. The general conclusions in this thesis regarding the valuation of ore to be mined for its gold content, therefore, also apply to uranium.

A second interesting conclusion is that the uranium values in comparison with gold values appear to be subject to less relative variation between values (since the "a" for uranium values is larger than that for gold/* i.e., they appear to be more consistent. This was confirmed at any rate for Mine X by dividing the developed portion of the property along a major dip fault into two distinct sections, and fitting the lognormal curves to the gold and uranium values in each section

separately/...

---

*See Formula No.(10(a)).

separately with the following results:-

Section	No. of Values Available	"a" for gold values	"a" for uranium values
East	510	.768	.856
West	563	.716	.820
Combined	1,073	.667	.820

In the case of some 89 pairs of gold and uranium borehole values on the Basal Reef in the Orange Free State, the graphical fitting of the lognormal curves suffers from the limited number of values available, but the indications are that here also the uranium values are more consistent than the gold values, the fitted gold and uranium curves having parameters "a" of 0.547 and 0.587, respectively.

2. Correlation between Gold and Uranium Values.

The principal problem as far as the mine valuator is concerned is to establish the relationship, if any, between corresponding uranium and gold values; i.e., whether uranium values in general increase with an increase in the gold values, and if so, whether the relationship is linear or curvilinear.

At the outset it can be stated that a cursory examination of some thousands of pairs of gold and uranium values on several properties indicated that, in general, the uranium values on a specific property and reef appeared to be higher where the corresponding gold values were high, and vice versa. It was, however, also immediately evident that a wide range of uranium values corresponded to the gold values in any particular value category, and therefore that any trend of relationship could only be established on averages.

From the knowledge that both gold and uranium values are distributed lognormally (i.e. normally, if the logs of the

values/...

values are plotted) it was inferred that the simplest relationship, if any, between these values would be more evident if the logarithms of the values were plotted jointly. The procedure adopted and type of results obtained will be evident in the following example.

Taking the 510 pairs of values available for the Eastern Section of Mine X above, the uranium values corresponding to gold values falling in particular value categories were noted and averaged to yield an average uranium value corresponding to the average for every gold value category. A graphical plot of the logs of these averages resulted in the 10 points shown with small circles on Diagram No. 7* (the uranium scale being omitted for security reasons). The straight line relationship between the logs of the relevant values is fairly evident from the diagram, and the same trend has been found in the case of several other properties and reefs including Basal Reef values in the Orange Free State field.

Accepting line AB on Diagram 7 as a reasonable fit on the ten observed pairs of average values, the relationship between the logs of the uranium and gold values will be of the type

$$\log (\text{average uranium value}) = K \log (\text{gold value}) + C$$

$$\text{i.e. } \log (\bar{u}) = K \log(x) + (C)$$

where K and C are constants depending on the units of measurement and on the actual relationship between the gold and uranium values. and "$\bar{u}$" is the average uranium value corresponding to any gold value "x."

This formula implies that for a specific gold value, there will be a corresponding range of uranium values, but

that/...

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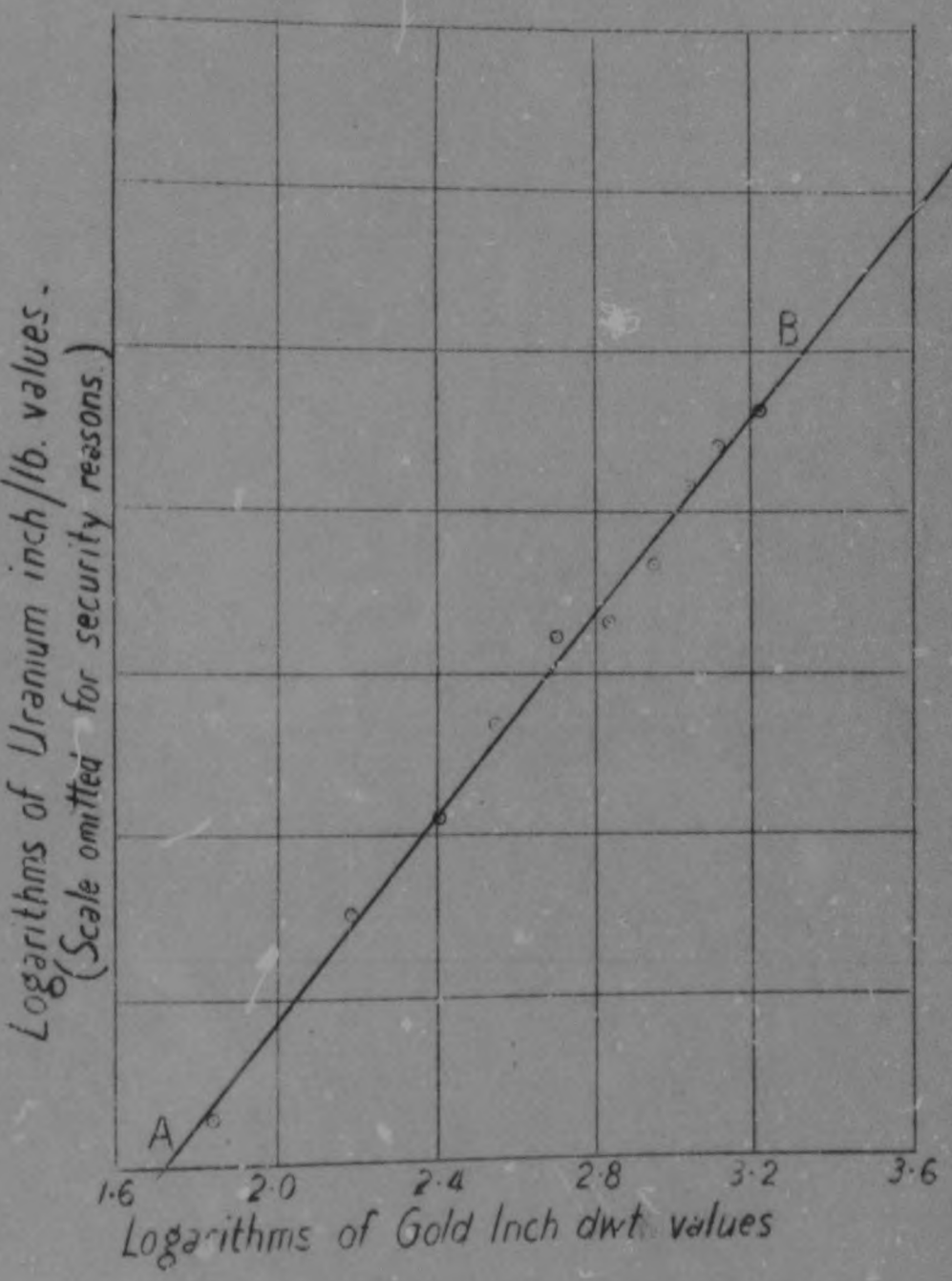
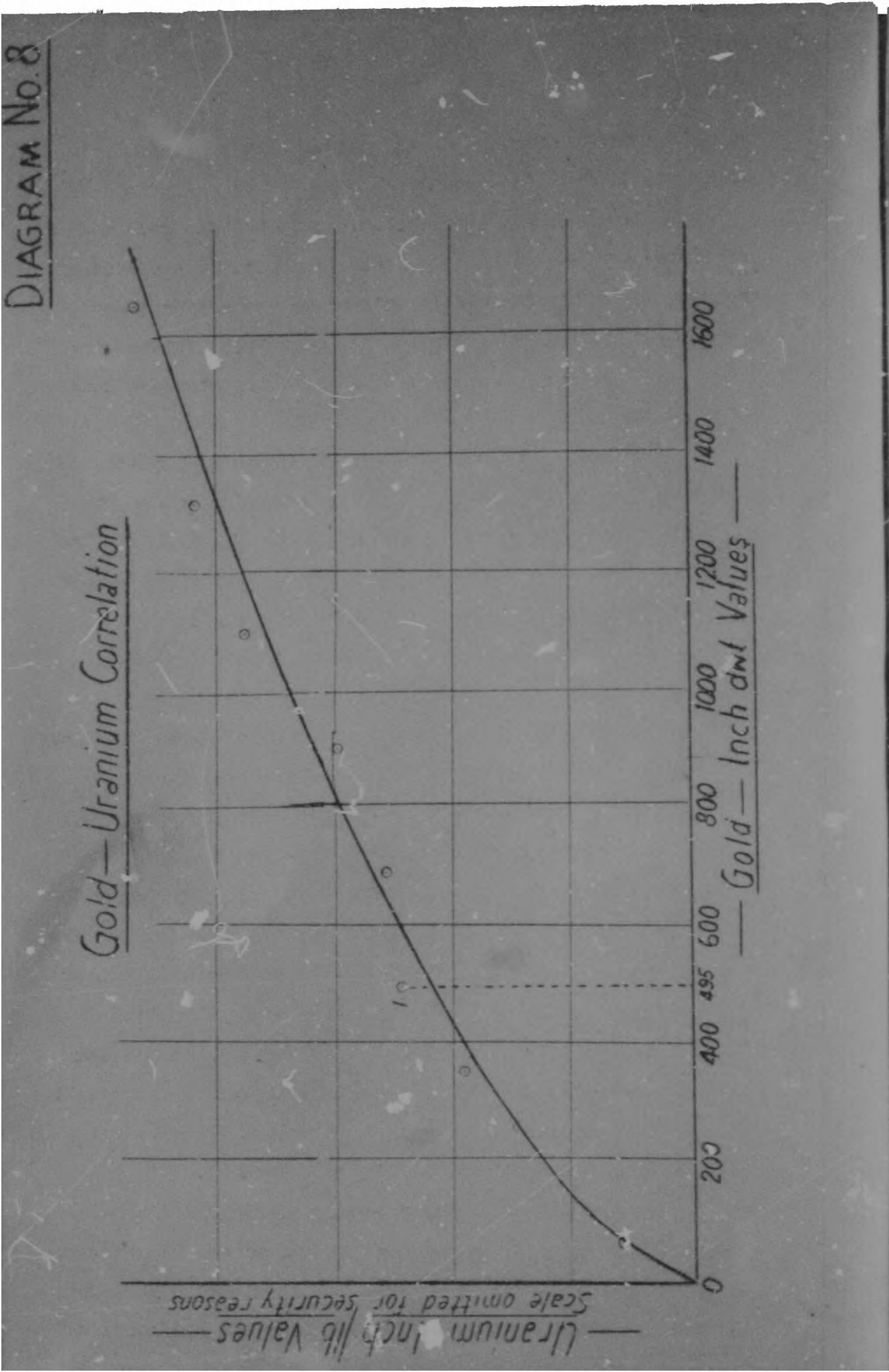
*See over.

DIAGRAM No. 7

Gold — Uranium Correlation on double logarithmic basis.





that the average of such values will be that
solving for "ū" in the formula.

The type of direct relationship be
uranium values expressed indirectly (in term
the values) by the formula is evident when t
Diagram No. 7 (corresponding to this formula
into the curve on Diagram No. 8,* by solving
for different values of x.

The average uranium value which ca
correspond to any particular gold value can,
read off directly from the curve on Diagram
indicates clearly the nature of the fit of t
ten observed pairs of joint average values,
the gold values in the category 400 to
average 495 inch-dwts, and this value p
the average of the corresponding uraniu
in the point shown as No. 1. The obse
uranium value corresponding to this gol
therefore exceeds the theoretical value
the curve by approximately 10%. It is
percentage differences in the other va
all less, and that the observed values
the theoretical trend of the curve fair

The relationship between gold and
values can therefore, at this stage, be acce
line relationship when the logs of the value
i.e. when double logarithmic graph paper is

As a point of interest from a pure
angle, the 510 pairs of values were found, a
tion of the usual correlation table, to appr
lognormal correlation surface, and the logs

that the average of such values will be that obtained in solving for "ū" in the formula.

The type of direct relationship between gold and uranium values expressed indirectly (in terms of the logs of the values) by the formula is evident when the line AB on Diagram No. 7 (corresponding to this formula) is transformed into the curve on Diagram No. 8,* by solving the above equation for different values of x.

The average uranium value which can be expected to correspond to any particular gold value can, therefore, be read off directly from the curve on Diagram No. 8 which also indicates clearly the nature of the fit of this curve on the ten observed pairs of joint average values, e.g.,

> the gold values in the category 400 to 600 inch-dwts average 495 inch-dwts, and this value plotted against the average of the corresponding uranium values, resulted in the point shown as No. 1. The observed average uranium value corresponding to this gold value category therefore exceeds the theoretical value obtained from the curve by approximately 10%. It is evident that the percentage differences in the other value categories are all less, and that the observed values in general follow the theoretical trend of the curve fairly closely.

The relationship between gold and average uranium values can therefore, at this stage, be accepted as a straight line relationship when the logs of the values are plotted, i.e. when double logarithmic graph paper is used.

As a point of interest from a purely statistical angle, the 510 pairs of values were found, after the construction of the usual correlation table, to approximate the ideal lognormal correlation surface, and the logs of the individual values approximate the ideal case of the normal correlation surface with homoscedastic regression system, linear

regression/...

*See opposite.

regression,* and coefficient of correlation of approximately 0.7.

It may also be of interest to record that the correlation between the gold and silver values in the Witwatersrand reefs may prove to be of an identical nature to that between gold and uranium. This is suggested by the fact previously mentioned** that the percentage silver content of the samples corresponding to low gold values is above average, with the reverse applicable in the high gold value categories. This may result in a curve for average silver values corresponding to specific gold values of a type similar to that of the curve on Diagram No. 8.

3. Interpolation of Uranium Ore Reserves from Gold Ore Reserves.

From previous investigations conducted on Mine X, it was known that the distribution of individual gold inch-dwt values within the average ore reserve block, was lognormal with parameter "a" of approximately 0.85. Now, taking an ore reserve block with an average gold value of, say, 500 inch-dwts, the distribution of the individual sample inch-dwts represented by this block is known fully, since both the mean value and the parameter "a" is known, and can be calculated to be as follows:-

Value category inch-dwts	0 to 100	100 to 300	300 to 500	500 to 700	700 to 1,000	Over 1,000
% Frequency of values in value category	6%	35%	25%	13%	11%	10%

Now, the theoretical average uranium value corresponding to the average gold value in each of the above categories can be determined graphically for the Eastern Section of/...

*Ref. 4, p 209. **Chap. VI, par. 4(b).

of the mine from Diagram No. 8. It is, therefore, possible to substitute for the above gold values in a block, the corresponding average uranium values, and from the known percentage frequencies of values in the various categories to calculate the theoretical average uranium value for the block of ore with a known average gold value of 500 inch-dwts. Repeating such calculations for various average gold values of ore reserve blocks,a curve similar in trend to that on Diagram No. 8 was constructed indicating the average uranium value of the blocks of ore falling in any gold value category. Interpolation from this curve of relationship between gold and average uranium block values yielded the likely uranium grade for every ore reserve block in the Eastern Section of Mine X, and from these and the known tonnages of ore in the blocks, the average uranium ore reserve value for this section of the mine was estimated.

In a similar manner the average uranium value was estimated for the Western Section, and combined with that for the Eastern Section to yield the likely average uranium ore reserve grade for the mine. Subsequently, portions of most of the ore reserve blocks in the mine were resampled for the purpose of uranium assays, and the uranium ore reserves were computed in the orthodox manner. The average value thus obtained differed from that arrived at theoretically (by interpolation from gold values) to an extent less than 5%.

4. <u>The Operation of a Joint Pay Limit for Gold and Uranium</u>.

Since it is generally impossible to keep the ore obtained in mining from different blocks of ore separate in the process of transportation and eventual treatment in the reduction plant, it will not be possible to select specific tonnages of mined ore for uranium extraction purposes, i.e., if uranium extraction is undertaken in respect of the ore from

a/...

a certain mine, or section of a mine, all such ore will have to undergo both the gold and uranium extraction processes.

In considering the question of working to a pay limit, therefore, every ton of ore has to bear the full cost of mining, as well as of gold and uranium extraction, and the revenue from such a ton of ore will consist of the gold + uranium revenue as determined by the gold and uranium contents of this ton of ore. Where this joint revenue is equivalent to the total costs, the gold and uranium values of the relevant ton of ore will constitute the joint pay limit. It is also evident that various combinations of gold and uranium values can yield the same total revenue and, therefore, that the joint pay limit, when fully defined, will consist of a range of pairs of gold and uranium values. A block of ore will then be payable if its computed uranium value exceeds the theoretical uranium value corresponding to the computed gold value of the block in the joint pay limit range.

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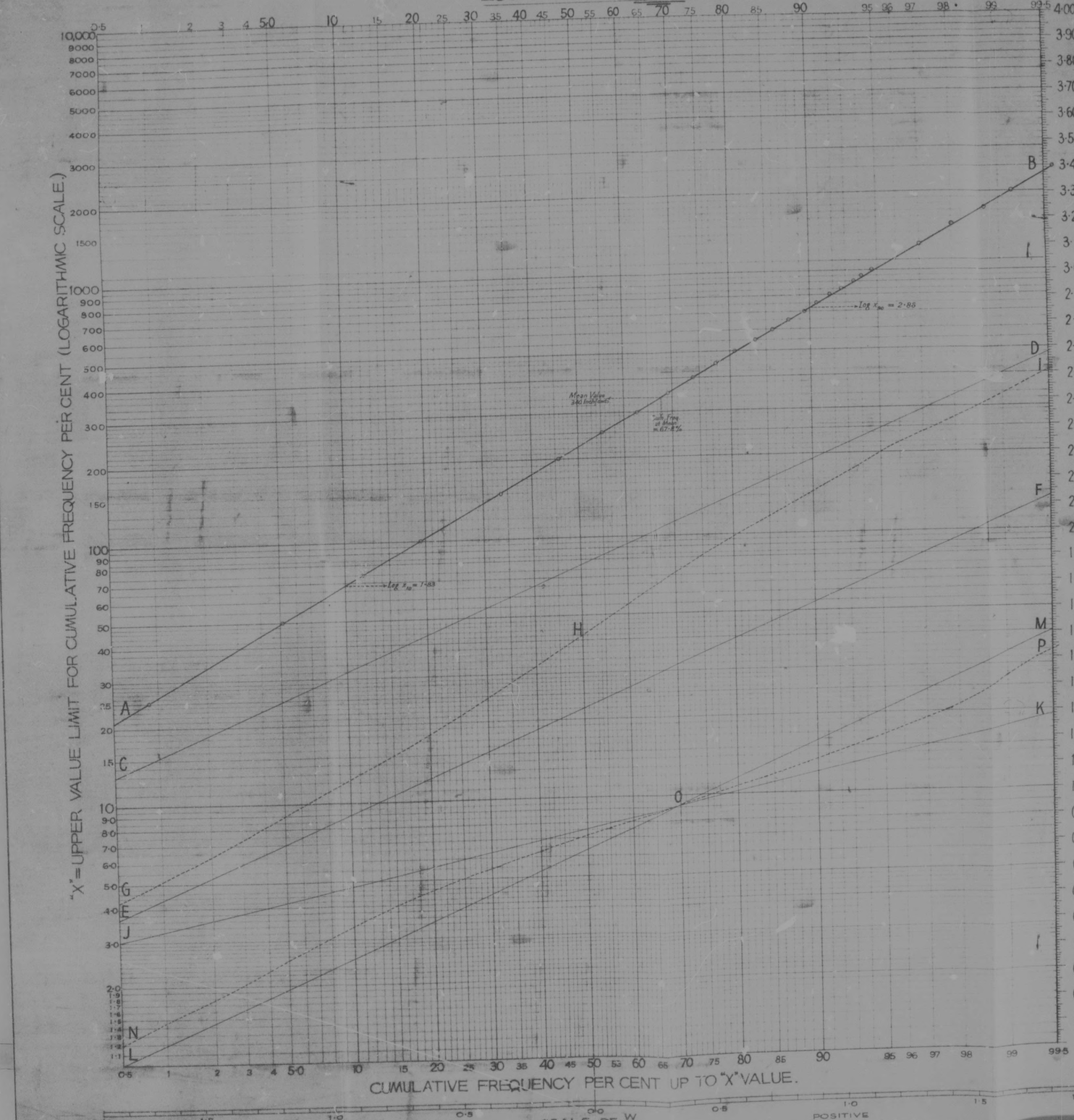
DIAGRAM N° 3

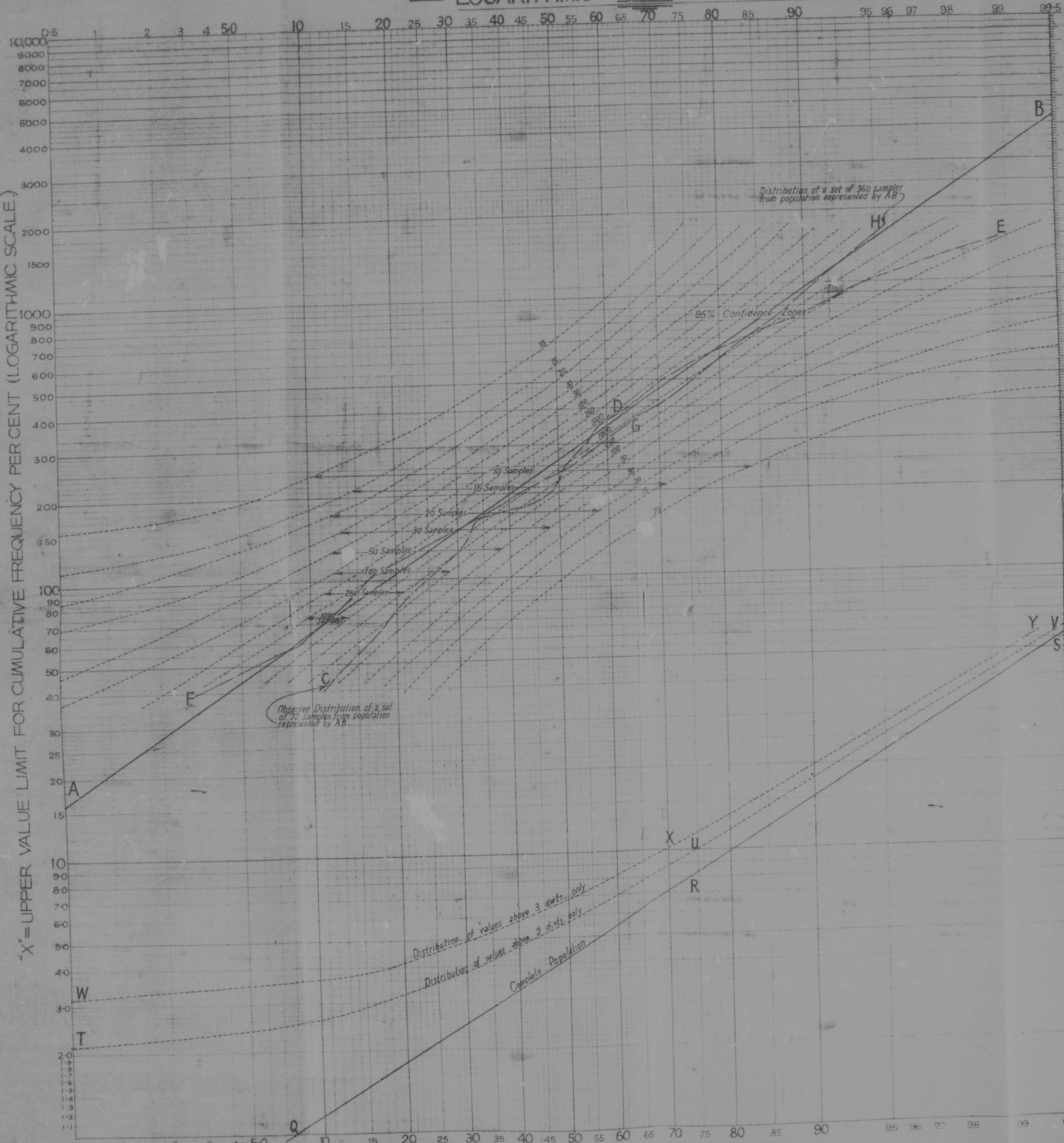
LOGARITHMIC—PROBABILITY PAPER.



Value of parameter 'a'	Difference in log. x between Cum. Freq of 10% & 90%	Cum. Freq per cent corresponding to Mean value
0·30	2·6237	88·07
0·35	2·2489	84·38
0·40	1·9678	81·16
0·45	1·7491	78·39
0·50	1·5742	76·02
0·51	1·5434	75·59
0·52	1·5137	75·17
0·53	1·4851	74·76
0·54	1·4576	74·37
0·55	1·4311	73·98
0·56	1·4056	73·61
0·57	1·3809	73·25
0·58	1·3571	72·89
0·59	1·3341	72·55
0·60	1·3119	72·22
0·61	1·2904	71·89
0·62	1·2695	71·57
0·63	1·2494	71·27
0·64	1·2299	70·97
0·65	1·2109	70·67
0·66	1·1926	70·39
0·67	1·1748	70·11
0·68	1·1575	69·84
0·69	1·1407	69·58
0·70	1·1244	69·32
0·71	1·1086	69·07
0·72	1·0932	68·83
0·73	1·0782	68·59
0·74	1·0637	68·36
0·75	1·0495	68·13
0·76	1·0357	67·91
0·77	1·0222	67·69
0·78	1·0091	67·48
0·79	0·9964	67·28
0·80	0·9839	67·07
0·81	0·9717	66·88
0·82	0·9599	66·68
0·83	0·9483	66·49
0·84	0·9370	66·31
0·85	0·9260	66·18
0·86	0·9152	65·95
0·87	0·9047	65·78
0·88	0·8944	65·61
0·89	0·8844	65·44
0·90	0·8746	65·28
0·95	0·8285	64·50
1·00	0·7871	63·87
1·10	0·7156	67·61
1·20	0·6559	61·58
1·30	0·6055	60·74
1·40	0·5622	59·97
1·50	0·5247	59·32
1·60	0·4919	58·75
1·70	0·4630	58·24
1·80	0·4373	57·79
1·90	0·4143	57·38
2·00	0·3936	57·02
2·50	0·3148	55·62
3·00	0·2624	54·69
3·50	0·2249	54·02
4·00	0·1968	53·52
5·00	0·1574	52·82
6·00	0·1312	52·34
7·00	0·1124	51·98
8·00	0·0984	51·76
9·00	0·0875	51·53
10·00	0·0787	51·39
15·00	0·0525	50·91
30·00	0·0262	50·43
50·00	0·0157	50·26

DIAGRAM Nº 4.

LOGARITHMIC—PROBABILITY PAPER.



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0·52	1·5137	75·17
0·53	1·4851	74·76
0·54	1·4576	74·37
0·55	1·4311	73·98
0·56	1·4056	73·61
0·57	1·3809	73·25
0·58	1·3571	72·89
0·59	1·3341	72·55
0·60	1·3119	72·22
0·61	1·2904	71·89
0·62	1·2695	71·57
0·63	1·2494	71·27
0·64	1·2299	70·97
0·65	1·2109	70·67
0·66	1·1926	70·39
0·67	1·1748	70·11
0·68	1·1575	69·84
0·69	1·1407	69·58
0·70	1·1244	69·32
0·71	1·1086	69·07
0·72	1·0932	68·83
0·73	1·0782	68·59
0·74	1·0637	68·36
0·75	1·0495	68·13
0·76	1·0357	67·91
0·77	1·0222	67·69
0·78	1·0091	67·48
0·79	0·9964	67·28
0·80	0·9839	67·07
0·81	0·9717	66·88
0·82	0·9599	66·68
0·83	0·9483	66·49
0·84	0·9370	66·31
0·85	0·9260	66·18
0·86	0·9152	65·95
0·87	0·9047	65·78
0·88	0·8944	65·61
0·89	0·8844	65·44
0·90	0·8746	65·28
0·95	0·8285	64·50
1·00	0·7871	63·82
1·10	0·7156	67·61
1·20	0·6559	61·58
1·30	0·6055	60·74
1·40	0·5622	59·97
1·50	0·5247	59·32
1·60	0·4919	58·75
1·70	0·4630	58·24
1·80	0·4373	57·79
1·90	0·4143	57·38
2·00	0·3936	57·02
2·50	0·3147	55·62
3·00	0·2624	54·69
3·50	0·2249	54·02
4·00	0·1968	53·52
5·00	0·1574	52·82
6·00	0·1312	52·34
7·00	0·1124	51·98
8·00	0·0984	51·76
9·00	0·0875	51·53
10·00	0·0787	51·39
15·00	0·0525	50·91
30·00	0·0262	50·43
50·00	0·0157	50·26

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