GENERATION OF PROBABILISTIC STOPES USING MONTE-CARLO SIMULATION OF BLOCK ECONOMIC VALUES FOR USE IN MINE PLANNING UNDER UNCERTAINTY

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A thesis submitted to the Faculty of Engineering and the Built Environment, University of the Witwatersrand, Johannesburg, in fulfilment of the requirements for the degree of Doctor of Philosophy.

Johannesburg, 2021

DECLARATION

I declare that this thesis is my own, unaided work. Where use has been made of the work of others, it has been duly acknowledged. It is being submitted for the Degree of Doctor of Philosophy to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University. I have read the University Policy on Plagiarism and hereby confirm that no plagiarism exists in this thesis. I also confirm that there is no copying, nor is there any copyright infringement. I willingly submit to any investigation in this regard by the University of the Witwatersrand, and I undertake to abide by the decision of any such investigation.

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This **__20______** day of **___September_________ 2021**

“***The art of Managing is in foreseeing*.**”

French saying From Birge and Louveaux (1997)

ABSTRACT

Irrespective of the mining method, mining starts with prospecting and exploring for an orebody of economic interest from which geological data is gathered. Using the data, a three-dimensional (3D) model representing the orebody is created. The orebody model is then divided into regularly sized blocks to form a geological block model which is converted into an economic block model by calculating the net economic value of each block called the block economic value (BEV).

An economic block model is a key input for the mine design and planning process therefore, its realistic representation in 3D is important. However, its realistic representation is affected by uncertainty associated with input parameters used to calculate BEVs. It is known that uncertainty is inherent in mine design and planning and introduces risk into the process, something which mine planners must consider and plan for. Nonetheless, despite this known fact, most mine design and planning practices are deterministic including BEV calculation. Deterministic BEV calculation assumes that at the time of creating an economic block model, BEV input parameters are known with certainty, when they are not in reality. This invariably results in an incomplete representation of the orebody which subsequently leads to unrealistic mine designs and plans and subsequently to the destruction of shareholder value. Incomplete information due to uncertainty at the mine planning stage has been, and will continue to be, a challenge for the mine planning fraternity.

To address this challenge, this thesis developed a probabilistic BEV calculation approach that was applied on a synthetic geological block model for a gold deposit. The uncertainty associated with BEV input parameters (rock density, gold grade, gold price and processing recovery rate) was represented using different probability distributions. Density and recovery rate were assumed to follow the normal and uniform probability distributions, respectively. The Geometric Brownian Motion (GBM) was assumed for gold price. Uncertainty in gold grade was first analysed using conditional simulation and then the lognormal probability distribution was used to simulate gold grade. Average mining and processing costs were assumed in the BEV calculation. All these input parameters were then simulated simultaneously in Microsoft Excel using the Monte Carlo simulation method to determine the range of

all possible BEVs for each block under different future scenarios. Five thousand iterations were run and from the simulated BEVs, the probability of each block being positive, hence economic, was calculated.

One of the advantages of Monte Carlo simulation is that it can simultaneously consider the uncertainty of key input variables and generate a probability distribution of the output. Instead of a single deterministic economic orebody model and stope design, multiple possible economic orebody models which incorporated varying levels of risk were created. These models were then used to create multiple stopes and their associated conceptual development infrastructure designs at varying levels of risk. Finally, a simple Discounted Cashflow (DCF) analysis was done to calculate the Net Present Value (NPV) of a few selected designs. The aim of the thesis was to analyse the impact of uncertainty on the size of the deposit and subsequently on stope boundaries and NPV.

The thesis demonstrated that uncertainty in geological and economic factors significantly affect the economics of a mining project. Therefore, the integration of this uncertainty early in the mine design and planning process (that is, BEV calculation) is important to create optimal and robust underground mine designs and plans. The thesis found that as the confidence in BEVs increases, the size of the orebody and life of mine (LOM) decrease, while the average grade of the orebody increases. On the other hand, as probability increases the total economic value of the orebody increases to peak at approximately 50% probability and then starts to decrease. The same trend was observed with the total number of stopes, total stope tonnage, total stope economic value and ultimately NPV. The results also showed that the amount of primary development required remained the same regardless of the confidence associated with the orebody. However, the amount of secondary development required reduces as confidence in the orebody increases. The DCF analysis also confirmed that the design consisting of probability stopes with at least a 50% confidence level (that is, P50 stopes) in terms of economic value and chance of the stope's blocks being within the ultimate stope boundary (P50 design), provided the highest NPV of approximately US$29 m for the analysed designs compared with US$19 m for the deterministic design. Therefore, the results indicated that deterministic BEV calculation underestimated the true potential of the orebody by about 35%.

These results suggest that for any given orebody and after incorporating uncertainty, there is a level of confidence at which the total economic value and NPV of the orebody is maximum. Therefore, the orebody at this level should be used for planning purposes during underground mine design and planning. In this thesis, based on the case study orebody and assumptions made, P50 stopes should be considered for planning purposes.

However, it should be noted that the level of risk hence the optimal range of probability may vary from orebody to orebody depending on probability distributions underlying the input parameters used to calculate BEVs. Also, the ultimate decision on which confidence level to use, may vary from one company to another depending on the perception or tolerance of risk of the respective company. Therefore, it is recommended that actual specific project data be used so that the actual stochastic behaviour of the input parameters used in calculating BEVs is modelled to generate a realistic behaviour of BEVs specific to each orebody being evaluated.

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- My family for their moral encouragement and support.

May God bless you all!

DEDICATION

To my late father Joseph Mafuraha Tholanah whose presence would have been necessary to celebrate with me my successes.

May his soul continue to Rest in Peace.

LIST OF PUBLICATION EMANATING FROM THIS THESIS

The following publication has so far emanated from this thesis:

Tholana, T, Musingwini, C. and Ali, M. M. (2019), A stochastic block economic value model, *in Proceedings: Mine Planners Colloquium 2019: skills for the future – yours and your mine's*, 22-23 May 2019, Johannesburg, South Africa, The Southern African Institute of Mining and Metallurgy, pp. 35-50.

It is also important to note that the author contributed to the following journal paper, and some of the work in the journal was extracted from this thesis.

Nhleko, A., Tholana, T. and Neingo, P. (2018). A review of underground stope boundary optimization algorithms. *Resources Policy*, 56, 59–69.

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1 INTRODUCTION

1.1 Overview of Chapter 1

This chapter provides background information on mine design and planning. The key subprocesses of the design and planning process are briefly discussed in the chapter. These include geological and economic block modelling, mining method selection, production rate determination, cut-off grade determination and production scheduling. The chapter gives more detail on the geological block modelling process and the conversion of a geological block model into an economic block model through the calculation of block economic values (BEVs). This thesis focussed on economic block modelling because regardless of the mining method either surface or underground, an economic block model is the main input for the mine design and planning process (Nhleko *et al.*, 2018). It provides one of the first opportunities available to mine planners to optimise the long-term economic value of a mining project. An economic block model is a critical mine planning input that defines the quantity or size of the Mineral Resources that are economically available to mine and ultimately determines the NPV of a mining project. It also guides the optimum location of major underground development infrastructure such as shafts, declines, tunnels and raises, and subsequently the optimum production schedule. Gaps in the current deterministic BEV calculation and economic block modelling that result in static economic block models are outlined in this chapter.

To ensure a successful mine design and planning process, several aspects must be considered including optimisation. Therefore, the chapter also provides an overview of optimisation in general and stope boundary optimisation in particular, highlighting the lag in the development of algorithms for the optimisation of underground mines compared to algorithms developed for the optimisation of open-pit mines. An explanation on the lag in the development of underground optimisation techniques is also given in this chapter.

The aim or purpose of the research was to analyse the impact of uncertainty on stope designs and ultimately on NPV of a mining project. This was achieved by developing a probabilistic BEV calculation method for optimum underground stope design. The problem statement, research question and research objectives are also given in this chapter. The motivation for shifting from deterministic approaches to probabilistic approaches of calculating BEVs is also provided in the chapter. Finally, a summary of Chapter 1 is provided, and the structure of the thesis is outlined.

In general, the terms ‘stochastic’ and ‘probabilistic’ are synonyms both referring to the study of optimal decision making under uncertainty. However, the two have a subtle difference. Beichelt (2016 p. 230) mentioned that stochastic processes are classified based on “*their dependence on time, the statistical dependence of their developments over disjoint time intervals, and the influence of the history or the current state of a stochastically process on its future evolvement*”. In the entire thesis the terms ‘stochastic’ and ‘probabilistic’ are used interchangeably and the phrases ‘ultimate stope limits’, ‘stope layout’, ‘stope envelope’, and ‘stope boundary’ are also used interchangeably. There are many types of value, in this thesis value refers to economic value.

1.2 Mine design and planning

The South African Mineral Asset Valuation Committee (SAMVAL Code) (2016 p. 30) defined mine design as “*a framework of mining components and processes taking into account such aspects as mining methods used, access to the orebody, personnel and material handling, ventilation, water, power, and other technical requirements, such that mine planning can be undertaken*”. Mine planning is defined as “*production planning, scheduling and economic studies, within the mine design, can be undertaken, taking into account geological structures and mineralization, associated infrastructure and constraints, and other relevant aspects*” (SAMVAL Code, 2016 p. 30). Therefore, mine design sets the framework for mine planning which defines the long-term strategy to extract an orebody. Decisions that are made at the mine design stage affect the long-term economics of a mine. This makes mine design and planning to be important aspects that integrate various elements of the mine value chain. The mine design and planning process involves several components that are interrelated and interdependent. These components can be schematically summarised as shown in Figure 1.1.

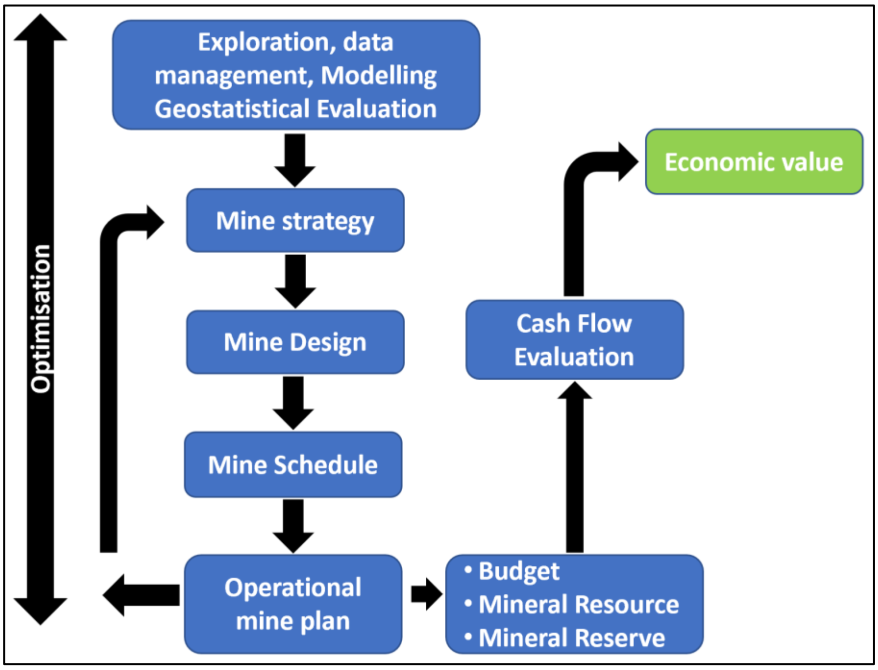

Figure 1.1: A simplified mine design and planning process

The process shown in Figure 1.1 starts with orebody evaluation, a process that characterises the orebody in terms of its size, spatial position, orientation, geochemical and other technical characteristics. This is the key input for the subsequent processes. The orebody evaluation process consists of two main processes; geological and economic orebody modelling. The aim of both processes is to define the characteristics of the orebody. After obtaining sufficient knowledge of the orebody, an optimum mining method is selected suited to the orebody characteristics. An optimum mining method is one that safely maximises economic value and recovery of the Mineral Resources. The mining method can be a surface mining method, underground mining method or a combination of both. The focus of this thesis was on underground mining. Once an optimum mining method is selected, the design of the layouts to access the orebody follows. This process depends on the characteristics of the selected mining method.

Mine design, stope design, layout definition and sequencing processes follow the mining method selection process. These processes are informed by the economic block model which defines the limit of an orebody that can be mined economically also called the optimum stope boundary. Once the stopes have been designed, production scheduling follows. The scheduling process defines the quantity, source, and timing of ore extraction. The main objective of production scheduling is to identify an extraction sequence that maximises NPV while satisfying constraints such as production rate, cut-off grade, stope geometry and materials handling capacities.

Cut-off grade determination, production rate determination, optimum equipment selection are the other key elements of the mine design and planning process. Asad (2002) defined a cut-off grade as the grade that distinguishes ore and waste. An optimum cut-off grade is influenced by parameters that include commodity price, operating cost and production rate.

The mine design and planning components mentioned above are interdependent. This interdependency increases the complexity of optimising the mine design and planning process. Musingwini (2016) described the circular logic of optimisation in underground mine design and planning illustrated in Figure 1.2.

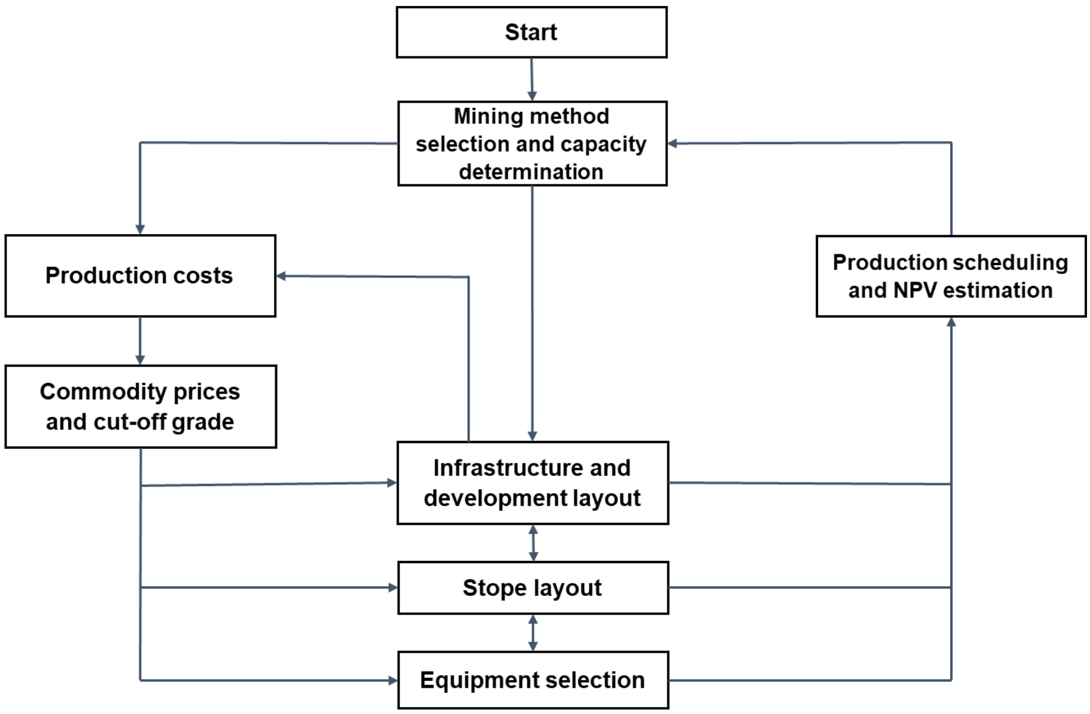

Figure 1.2: The circular logic in optimising underground mines

(Source: Musingwini, 2016)

Musingwini (2016) mentioned that the optimisation objective in underground mines is often to maximise the NPV of the mine design. However, the layout that maximises NPV for both development infrastructure and stopes can only be known after determining the operating costs and associated cut-off grade (Musingwini, 2016). However, determining the costs and cut-off grade requires first determining the mining layout and production schedule which in turn cannot be determined before the optimum mining method and production rate have been determined. In turn, to select the optimum mining method, production rate must be determined first (Musingwini, 2016). This circular logic makes defining an optimum mine design and plan a challenging exercise.

Whilst every component of the mine design and planning process described above is important, irrespective of mining method, the orebody is the key input for mine design and planning. Most value is created during the early stages of a mining project and can only be maintained or destroyed during the production phase (Garcia and Camus, 2011). This is why the focus of this thesis was on mine design starting at the orebody evaluation stage which is described in more detail in the following sub-sections.

1.2.1 Geological resource modelling

Nhleko *et al.* (2018 p. 2) mentioned that "*irrespective of the mining method, either underground or surface, mining starts with prospecting and exploring for mineral resources of economic interest*". From the collected geological data, a model representing the orebody in 3D is created and is called a geological block model. Wiederhold and The Burval Working Group (2009) defined a geological block model as a 3D representation of the distribution of minerals and rocks beneath the earth's surface. The model forms a context within which a mineralisation model of the orebody is built (Coombes, 2008). The orebody model created is then divided into equally sized blocks as illustrated in Figure 1.3 which shows an example of a geological block model.

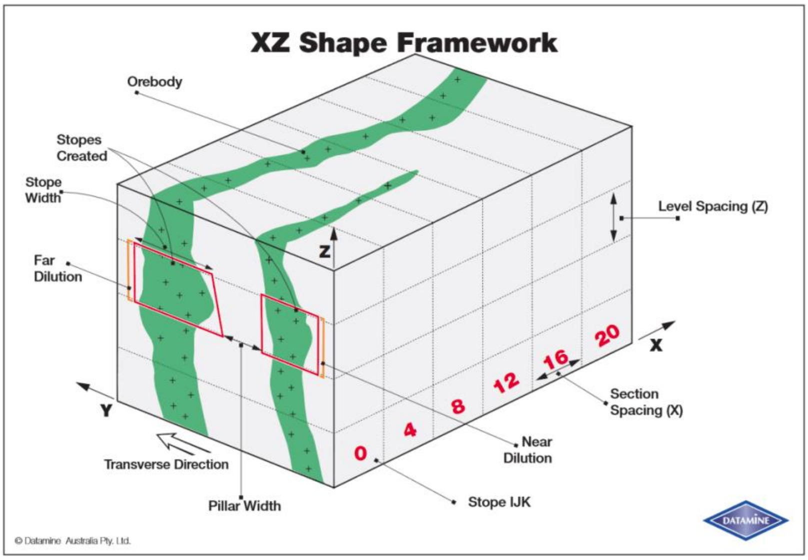

Figure 1.3: A 3D geological block model view

(Source: Datamine Corporate Limited, 2009)

The output of a geological block modelling process is a Mineral Resource. The SAMVAL Code (2016 p. 30) defined a Mineral Resource as "*A concentration or occurrence of material of economic interest in or on the Earth's crust in such form, quality, and quantity that there are reasonable and realistic prospects for eventual economic extraction. The location, quantity, grade, continuity, and other geological characteristics of a Mineral Resource are known, or estimated from specific geological evidence, sampling, and knowledge interpreted from an appropriately constrained and portrayed geological model. Mineral Resources are subdivided, and shall be so reported, in order of increasing confidence in respect of geoscientific evidence, into Inferred, Indicated, and Measured categories*." Turner and Gable (2007) mentioned that the geological orebody modelling process is technically challenging. This is because the earth mass is heterogeneous, and a geological block model is developed from isolated sample data which is most often insufficient to address all uncertainties.

The total quantity of blocks in a geological block model varies depending on the orebody size and distribution of grade within the orebody. Each block within the geological block model consists of its specific characteristic data, such as grade, volume, density and lithology (Ataee-pour, 2005a). These data are used to calculate the economic value of each block called BEV. Every block within the geological block model has a BEV associated with it. The process of calculating BEVs converts the geological block model into an economic block model which defines economically mineable blocks (Chung *et al.,* 2016). The conversion includes incorporating economic and technical factors such as commodity price, mining cost, processing cost and recovery rate (Chung *et al.,* 2016). Together with the economic evaluation, geological and geo-statistical techniques are used to estimate the quality and quantity of the mineral deposit, which are some of the inputs for estimating the economic value of the deposit (Bjørndal *et al.,* 2012).

Some authors including Goodfellow and Dimitrakopoulos (2016 p. 293) suggested that "*it is necessary to explore new models that shift the focus from the economic value of the blocks that are mined to the economic value of the products that are sold to customers*". Also, the concept of associating blocks to their corresponding BEVs is however mostly practised in the planning of open-pit mines and less in the planning of underground mines. It is worth noting that unlike in open-pit mine planning and optimisation, the standard in most underground mines is to identify blocks by their grade values instead of BEVs. Based on the cut-off grade it is then decided whether a block will be part of the stope envelope or not. A block is considered part of the stope envelope if its grade is equal to or above the cut-off grade, otherwise it is considered as waste and excluded from the stope envelope. However, there is now a gradual acceptance of using BEVs in underground mine stope optimisation as explored in literature in Chapter 3. This is because cut-off grade is calculated from economics of mining implying that each block must have an economic value.

This practice in some underground mines of identifying blocks by grade values instead of economic values has limitations. Ultimately, what determines whether a block is mined or not is not the grade only, but the interplay between economic factors that include mineral price, mining, and processing costs. Hence it is logical to use BEVs for planning purposes in underground mines. For example, it is impossible to decide whether a block should be mined if only the grade is known, but that decision can be made if the revenue attributable to that block and its associated costs are known.

In order to optimise stope boundaries blocks should be represented in economic terms because blocks may have the same grade value but different economic values due to several factors (Ataee-pour, 2005a). Ataee-pour (2005a) indicated that some of the factors that influence the BEVs include where the blocks are located, when they will be mined, and the mining method applied to mine the orebody. The location of a block in terms of depth determines whether that particular block will be economically mineable or not because the cost of mining increases with depth such that two blocks of the same size and with the same grade value will have different economic values. Also, different mining methods have different cost structures such that blocks may have the same grade value but different economic values depending on the mining method applied (Ataee-pour, 2005a).

Additionally, Whittle (2013) developed an enterprise optimisation approach to optimise the economic value of mining projects. The philosophy of the approach is to consider any mine as a ‘money mine’ and not a metal or coal mine with the aim of maximising economic value. Therefore, the traditional mind-set of focussing on grade block values to determine the mineability of a block distracts mine planners from this economic rationale. This emphasises the need to identify blocks in terms of their economic values (through a process of economic block modelling) instead of the traditional practice in some underground mines of identifying blocks by their respective grade values.

1.2.2 Economic block modelling

As mentioned in Section 1.2.1, the geological block model is converted into an economic orebody model through an economic block modelling process. The process results in an economic block model which is the key input to the mine design and mine planning processes. From the economic block model, ensuing mine design and planning processes are undertaken as described earlier in Section 1.2. These include, mining method selection, stope boundary definition, production rate determination, cut-off grade determination and layout definition. Ultimately, the output of the economic block modelling process is a Mineral Reserve. A Mineral Reserve is defined by The SAMVAL Code (2016 p. 30) as "*The economically mineable material derived from a Measured or Indicated Mineral Resource, or both. It includes diluting materials and allows for losses that are expected to occur when the material is mined. Appropriate assessments to a minimum of a Prefeasibility Study for a project, or a Life of Mine Plan for an operation, shall have been carried out, including consideration of, and modification by, realistically assumed mining, metallurgical, economic, marketing, legal environmental, social, and governmental factors*".

The economic block modelling process involves calculating the revenue obtained from each block and the cost of mining and processing each block, evaluating these two values on every block to get BEVs (Jamshidi and Osanloo, 2016). Ataee-pour (2005a) mentioned that various formulae have been developed to calculate BEVs, but the basic BEV calculation formula is as expressed by Equation 1.1 (Jamshidi and Osanloo, 2016):

$$\textit{Block economic value} = \textit{Block revenue} - \textit{cost} \qquad 1.1$$

The revenue of a block is a function of the metal content in the block and the unit metal price. In turn, the metal content is a product of block tonnage, block grade, and processing recovery rate (Ataee-pour, 2005a). Furthermore, cost include mining and processing costs. Therefore, Equation 1.1 can be re-written as shown by Equation 1.2 (Jamshidi and Osanloo, 2016):

$$\textit{Block economic value} = [\textit{Block tonnage x grade x recovery rate x price} - (\textit{Mining cost} + \textit{Processing cost})] \qquad 1.2$$

For each set of input parameters applied, the block economic values distinguish economic and uneconomic blocks (Nhleko *et al.,* 2018). A block will be economic to mine if its revenue is more than its mining and processing cost, that is, if the BEV is positive. Traditionally, the block revenue and cost parameters used to calculate BEVs are deterministic resulting with a deterministic economic block model, from which practical and optimum underground stope designs are then developed (Nhleko *et al.*, 2018). Figure 1.4 shows a simplified schematic illustration of a deterministic BEV calculation model.

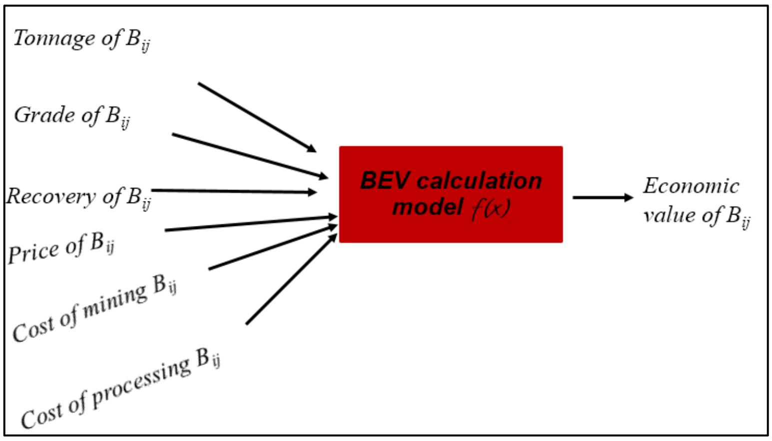

Figure 1.4: A simple deterministic BEV calculation model

(Source: Tholana *et al.*, 2019)

This deterministic approach assumes that economic, geological, technical input parameters, and the BEV results are known with certainty (Tholana *et al.*, 2019). However, this assumption ignores the reality that economic, geological, and technical parameters used to calculate BEVs carry considerable uncertainties (Tholana *et al.*, 2019). To address this uncertainty during mineral project valuation, a sensitivity analysis is usually done by varying economic and grade parameters without modelling the behaviour of these parameters. However, this does not correctly and completely account for geological and economic uncertainty. Therefore, uncertainty should be modelled using probabilistic methods. Due to the wasting nature of Mineral Resources and other constraints associated with the business of mining, optimum value must be created from the extraction of the resources. Therefore, optimisation is an essential component of the mine design and planning process.

1.2.3 Optimisation in mine design and planning

In general, optimisation involves either maximising or minimising an objective function against a set of constraints. Nhleko *et al.* (2018) mentioned that the objective can be minimising inputs into a process such as capital because they are scarce, minimising undesired outcomes such as dilution or maximising desired outputs such as economic value. The basis of optimisation is a mathematical model that represents the problem that is then solved using an algorithm (Nhleko *et al.*, 2018). According to Padmanabhan *et al.* (2012 p. 12), "*an* *optimisation algorithm is a procedure which is executed iteratively by comparing various solutions till an optimum or a satisfactory solution is found".* Little (2012) mentioned that, to date, optimisation algorithms have played a significant role in mine design and planning and decision making, although there is still a need for the development of more efficient and faster algorithms. The output of optimisation is an optimised mine plan which was defined by Musingwini (2016) as a plan that is robust with minimum deviations between actual outcomes and planned targets.

The development and application of optimisation techniques in the mining industry started in the 1960s with initial research development and application in surface mines (Topal and Sens, 2010). Little (2012) indicated that in open-pit mines, significant progress has been made in developing optimisation techniques for open pit optimisation using a range of Operations Research (OR) techniques. From the development of the commonly known Lerchs-Grossmann (L-G) algorithm in the 1960s by Helmut Lerchs and Ingo F. Grossmann, several other algorithms have been developed for the planning and optimisation of open-pit mines. Some of these algorithms have been embedded into software that is commercially available and being used for open pit optimisation.

On the contrary, Topal and Sens (2010) and Little (2012) highlighted that, fewer algorithms have been developed for optimising underground mining operations. This has resulted in most underground mines operating on mine plans that are sub-optimal particularly around optimisation of stope boundaries (Little, 2012). Examples of the few commercially available algorithms include the Floating Stope algorithm that was developed by Alford (1995) which is embedded in Datamine software, the Mineable Shape Optimizer (MSO) algorithm developed by Alford Mining Systems (2011) which is embedded in Datamine, Maptek and Deswik software packages, and the Stopesizer algorithm developed by Snowden Consultants for Snowden's in-house use (Nhleko *et al.*, 2018). Ataee-pour (2005b) mentioned that underground mining optimisation has only received attention in the last two decades prior to 2005, a fact also reiterated by Topal and Sens (2010). However, most of the available underground optimisation techniques do not guarantee optimum solutions (Nhleko *et al.*, 2018).

One of the challenges in developing a generic technique for the optimisation of underground mines, unlike in open-pit mines, is that there are several underground mining methods available, and their application varies from one mine to another, hence each deposit requires a specialised optimisation technique (Sandanayake, 2014; Alford *et al*. 2007). During underground mine optimisation, common constraints to consider include footwall and hangingwall angles, minimum and maximum stope dimensions all of which vary with the mining method (Bai *et al*., 2013). The other challenge is that underground mining involves other constraints that are not applicable to open pit mining (Nhleko *et al.,* 2018). These include ventilation and equipment size to fit into underground working areas. Additionally, in open-pit mines, for a given slope angle constraint there is one possible option to mine any given block (Sandanayake, 2014). Given these foregoing reasons, it is understandable that modelling of open pit mines can be generalised, but it is difficult to generically model underground mines since each mining method has its own unique properties, requirements, and limitations.

A combination of these challenges makes developing underground mine optimisation techniques more difficult, the main reason why there is a relatively small amount of research work currently available compared to open-pit mines (Musingwini, 2016; Alford *et al.* 2007). According to Alford *et al.* (2007 p. 574), "*the complexity of the underground mine design problem and the unique mine design solutions sought for each ore body suggest that there will never be an elegant solution method analogous to that which exists for open pit mining*".

1.3 Problem statement and research question

Bohr (n.d) said that "*Prediction is very difficult, especially if it's about the future*" and because mine planning is based on predicting the future, uncertainty is intrinsic to mine planning. All inputs for the mine planning process are estimated based on unknown future events and orebody characteristics that are known with a limited degree of certainty. Therefore, deterministic approaches to mine planning may lead to sub-optimal designs and plans, hence the need to incorporate uncertainty during mine design and planning. Ignoring uncertainty during mine design and planning can have consequences. According to Abdel Sabour *et al*. (2008, p. 54), "*In monetary terms, this may cause a loss of multimillion dollars due to either the foregone extra profits that could be gained during the periods of favourable market conditions or the net realised capital losses as a result of the unexpected weak market*". Therefore, the importance of incorporating uncertainty during economic block modelling and stope envelope optimisation cannot be overemphasised if robust mine designs and plans are to be developed and optimum value is to be created from the extraction of Mineral Resources (Nhleko *et al*., 2018).

A study by Vallee (2000) showed that geological information and Mineral Reserves estimation are some of the reasons that mining projects fail to meet their objectives. Of the projects analysed by Vallee (2000), 39% of these projects involved shortcomings in an aspect of mine design. This was most likely be due to incorrect definition of orebody boundaries among other factors. It can therefore be said that deterministic economic block modelling either over-estimates or under-estimates Mineral Reserves which affects subsequent processes in mine design and planning.

Deterministic approaches to stope boundary optimisation generally result in changes being made to mine designs later in the life of mine (LOM). Nhleko *et al.* (2018 p.10) mentioned that given the importance of accessing the orebody early for early cash flows and subsequently maximum NPV, together with "*the increasingly constrained mining business economic environment in terms of access to and availability of capital, economic value can be lost when designs are modified after the project has started. Some modifications to the mine design later in the life of mine (LOM) may require substantial capital which may not be available or may compromise the NPV of the project*". Musingwini (2016 p.816) stated that "*unlike in open pit mining, once an underground mine layout has been designed and development infrastructure put in place the mine will be 'locked-in' to that layout and any changes may be capital intensive*". Magagula *et al.* (2015) and Del Castillo and Dimitrakopoulos (2014) also highlighted the shortcomings of deterministic mine planning approaches mentioning that most mine design and planning approaches offer a static solution to a dynamic problem which causes a gap between planned outcomes and actual results.

To minimise all these limitations and ensure that uncertainty in economic, technical, and geological parameters is incorporated in mine plans, there is a need for a shift away from deterministic to probabilistic approaches in calculating BEVs. This ensures that, uncertainties associated with these parameters are modelled appropriately in the BEV calculation process. The application of probabilistic approaches in calculating BEVs and optimising stope boundaries will improve confidence levels in mine designs and plans. This therefore raises the research question:

> *"Can geological and economic uncertainty be simultaneously incorporated in the BEV calculation such that for any given stope dimension the probability that any particular block will be within the stope envelope can be established to produce probabilistic stope envelopes?"*

1.4 Research aim and objectives

It can be said from the previous sections that the current research on orebody economic evaluation is mostly deterministic, fragmented and limited mainly to optimising open-pit mine design. It was therefore the aim or purpose of this thesis to develop a probabilistic approach that integrates uncertainty in key input parameters used in calculating BEVs. The results are economic orebody models with associated probabilities that are then used to generate probability stopes to optimise the design and planning of underground mines. Probability stopes are stopes, each with an assigned confidence level, for example, a P90 stope would mean a stope with at least 90% confidence in terms of its economic value and in terms of the chance of its blocks being within the ultimate stope boundary. This is important during mine design to guide and improve confidence in the location of key underground development infrastructure and subsequently to create optimum production schedules which ultimately optimise economic value from the extraction of the finite Mineral Resources. The objectives of this thesis were therefore to:

- Review existing literature on probabilistic mine planning approaches.
- Review existing literature on the stochastic behaviour of economic and geological parameters used in BEV calculations.
- Review existing methodologies and literature on BEV calculations.
- Review existing literature on stope boundary optimisation algorithms.
- Develop a probabilistic BEV calculation approach.
- Use the probabilistic BEV calculation approach to create economic block models.
- Use the economic block models to generate probability stope boundaries.
- Analyse the impact of uncertainty on economic block models, stope designs, development infrastructure, and on NPV.

1.5 Research scope

Some of the data used in this thesis were obtained from an anonymous gold mine that uses the sub-level open stoping (SLOS) mining method and the Carbon-in-Pulp (CIP) processing method. The data include processing recovery rate, mine design criteria, development, mining, and processing costs. The data was slightly adjusted for this study. Due to confidentiality, the name of the mine could not be disclosed.

1.6 Research motivation

The global mining industry currently operates and is likely to continue operating in a challenging economic environment characterised by volatile commodity prices, subdued markets, and rising costs. Also, the industry is now more technically constrained; the high grade and shallow deposits have mostly been exhausted leaving only marginal and deep lying deposits available for extraction. These economic and technical factors together threaten the viability as well as the long-term value and competitiveness of the industry. Optimisation of mine plans is therefore not optional if mining companies are to sustain their operations and create long-term economic value from the available marginal and finite Mineral Resources.

As alluded to in Section 1.2.3, more effort is needed to develop underground mine optimisation techniques. This is because fewer greenfield open pit projects are being developed and in recent years a number of existing open-pit mines have been getting deeper necessitating transition from open pit to underground mining (Opoku and Musingwini, 2013; Chung *et al*., 2016). Examples include Venetia diamond mine in South Africa, Geita gold mine in Tanzania and Syama gold mine in Mali which are all planning and developing transition from open pit to underground mining. Chung *et al*. (2016) mentioned that worldwide, such mines with potential to transition underground or where transition has already been made also include Chuquicamata Mine in Chile, Grasberg Mine in Indonesia, Kanowna Bell Mine and Sunrise Dam Mine both in Western Australia. Moreover, existing underground mines are also getting deeper. On a comparable basis underground mines have a higher unit cost of production to open-pit mines and the cost increases significantly in ultra-deep mines which is a case for some current South African mines and other mines in future. Therefore, with fewer greenfield open-pit projects, as some existing open-pit mines reach transition to underground mining, as underground mines get deeper and as high-grade deposits get exhausted the future of mining is focussed on deeper underground mines. This means innovation is required to make marginal underground mines profitable. This implies that there is an increased need for underground optimisation techniques that guarantee optimum or satisfactory solutions (Little *et al*., 2013). The necessity of optimisation in mining in general coupled with the future concentration of mining underground motivated this research study.

Irrespective of the mining method, whether surface or underground, optimisation of the ultimate mine layout and production scheduling are the two main examples that are dependent on an economic orebody model (Ataee-pour, 2005a). Using the economic block model, succeeding processes are undertaken. These include stope boundary definition, mine design and production scheduling. Several optimisation techniques have been developed to date for optimisation of each of those areas in underground mines. According to Topal and Sens (2010), these techniques have been focussed on the optimisation of three broad areas namely:

1. Stope boundary and stope layout definition.
2. Placement of development and underground infrastructure.
3. Production scheduling.

This thesis focussed on stope boundary optimisation particularly economic block modelling. This is because economic block modelling provides one of the first opportunities available to mine planning practitioners to optimise the long-term value of a mining project (Nhleko *et al*., 2018). It is an important strategic mine design and planning area that determines the quantity or size of the Mineral Resources economically available to mine and ultimately the NPV of a mining project. Once the optimal stope boundaries are defined, major underground infrastructure such as shafts, haulage tunnels and orepasses can then be optimally located during mine design after which an optimum production schedule can be determined. Therefore, if stope boundaries are incorrect defined then underground development infrastructure may also be incorrectly designed and located. This may force mine designs to be altered later in the project implementation phase of a project with financial ramifications discussed in Section 1.3. In other words, optimum stope envelopes determine the efficacy of the mine design and subsequently, the long-term production plan, which then affects the cash flow profile and ultimately, the NPV of a mining project (Nhleko *et al*., 2018).

Since an economic orebody model is one of the key inputs for the mine design and planning process in general and stope optimisation process in particular, its real 3D representation (in terms of grade and size) should be as accurate as possible (Nhleko *et al*., 2018). This means that the calculation of BEVs is the first most opportunity available to optimise the mine planning process, justifying the need for an integrated probabilistic BEV calculation approach to ensure that uncertainty is addressed early in the mine design and planning process. Without a realistic economic block model, efforts to optimise later stages of the design and planning process may fail to maximise the economic potential of an orebody (Nhleko *et al*., 2018). Optimal and realistic representation of an economic orebody model and reliable definitions of stope boundaries can only be achieved when probabilistic BEV calculation approaches are used. Probabilistic approaches result in an economic block model that mirrors the real-life behaviour of the key input parameter used in the economic block modelling process resulting in a realistic representation of the orebody upon which reliable mine designs and plans can be done (Nhleko *et al*., 2018). This strongly motivated this thesis which supports the idea of shifting from deterministic modelling approaches with limitations discussed in Section 1.3 towards integrated probabilistic economic modelling approaches. Probabilistic modelling approaches enable the development of robust mine designs and plans that should reliably account for risks and uncertainties associated with an orebody throughout the LOM.

1.7 Significance of the research study

The key significance of this thesis was to develop a probabilistic model for calculating BEVs to address uncertainty intrinsic to mine planning at an early stage in the mine design and planning process. These values would then be used to create stope outlines with associated uncertainty incorporated. The importance of addressing uncertainty early in the mine design and planning process is to establish a solid foundation for creating robust mine designs and plans later in the design and planning process which is important for the economic success of a mining project. Probabilistic economic block models and stope designs will considerably improve confidence during mine design and increase reliability in the location and sizing of underground development infrastructure. As mentioned in Section 1.2.3 and discussed later in the literature review section of this thesis, very limited literature is available on integrated stochastic stope envelope definition, and this research contributes towards this important mine planning area.

1.8 Summary of Chapter 1 and structure of the thesis

The mining industry operates in a volatile economic, technically, and operationally challenging environment that often threatens its profitability and survival. The application of optimisation techniques is paramount if mining companies are to create optimal economic value from their finite and marginal Mineral Resources against a constrained technical and economic environment. Optimisation involves the best utilisation of scarce resources against a set of constraints. Even though optimisation techniques saw their application in the mining industry as early as the 1960s (Newman *et al.,* 2010), their application has been limited to open-pit mines such that some authors argue that the problem of open pit optimisation is saturated with optimisation techniques (Ataee-pour, 2000). In comparison, fewer techniques have been formulated to solve underground mine optimisation problems due to the technically, hence computationally complex nature of underground mines.

Stope boundary definition, placement of development infrastructure and production scheduling are the three main areas where underground mines optimisation has been focussed on, to date. This thesis focussed on stope boundary definition in general and economic block modelling in particular. This is because stope boundary optimisation is one of the first opportunities available to mining engineers to optimise value from the extraction of Mineral Resources. More so, an economic block model is one of the key inputs for optimising stope boundaries. Fundamental to creating an economic block model is the calculation of BEVs.

Traditional methods of calculating BEVs are deterministic in nature. They use single values of tonnage, grade, price, recovery rate and costs parameters in calculating BEVs. Significant uncertainties are associated with these input parameters but, these deterministic modelling methods do not incorporate those uncertainties. This has several consequences including overestimating or underestimating Mineral Resources and Mineral Reserves subsequently compromising the efficacy of optimisation efforts of subsequent mine design and planning processes. This also makes mines fail to create optimum value from extraction of their non-renewable Mineral Resources which may result in modification of mine designs later in the LOM, ultimately destroying economic value (Musingwini, 2016).

This thesis is structured in seven chapters. This chapter has provided a general overview and background of optimisation in mining, geological block modelling and economic block modelling. The problem statement, research aim and objectives, the research motivation are also discussed in this chapter. The remainder of the thesis is briefly elaborated in the next paragraphs and structured in Figure 1.5 which shows a schematic summary of the thesis.

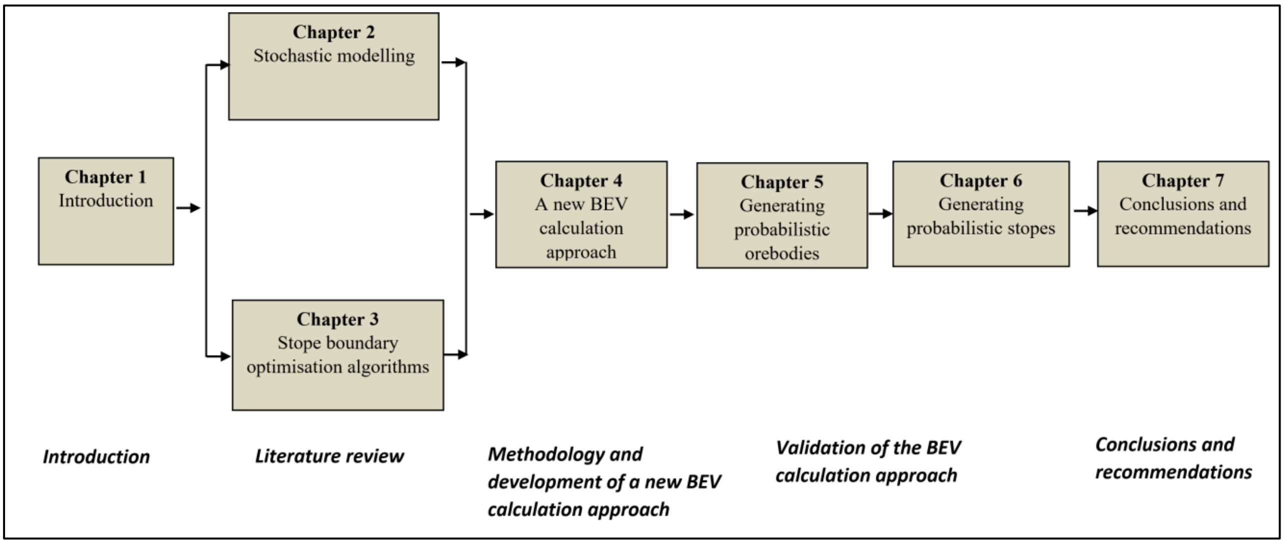

Figure 1.5: The structure of the thesis

Chapter 2 provides an overview of deterministic modelling, discusses risk and uncertainty in mining and reviews available literature on stochastic modelling in general then in particular to mine planning. The limitations of deterministic mine planning approaches compared to stochastic approaches are highlighted in this chapter. Further, the chapter reviews available literature on economic block modelling discussing research work that has been done to consider uncertainty in the economic orebody modelling process. The stochastic behaviour of commodity prices is also discussed in Chapter 2.

Chapter 3 reviews available literature on underground optimisation in general and stope boundary optimisation in particular. Algorithms that have been developed for stope boundary optimisation are also reviewed specifically, the most common ones. Chapter 4 presents the proposed probabilistic BEV calculation approach and from algorithms reviewed in Chapter 3 one is selected to validate the proposed approach in Chapter 5 in which generated probabilistic economic block models are presented. Probabilistic stopes and their associated development infrastructure designs created using the generated economic block models are presented and analysed in Chapter 6. Conclusions and recommendations for future research are given in Chapter 7, as well as contribution to new knowledge arising from the research.

2 STOCHASTIC MODELLING

2.1 Overview of Chapter 2

Chapter 2 first provides an overview of deterministic modelling by highlighting the limitations of deterministic approaches that justify the need for stochastic approaches to mine planning. One of the unique characteristics of a mining business compared to other businesses is its high uncertainty and subsequent risk caused by the uncertainty. In general, there are many sources of geological and economic uncertainty in mining, particularly in mine planning. The main sources of uncertainty are discussed in this chapter, specifically focusing on uncertainty associated with grade and commodity price. The chapter also discusses the traditional techniques that are used to deal with these risks highlighting the limitations of these techniques that motivates the need to use stochastic approached to deal with risk and uncertainty.

To incorporate the uncertainty, stochastic approaches should be used during mine planning. Stochastic modelling is defined and discussed in the chapter. Monte Carlo simulation, which is one of the common techniques used to model uncertainty in mine planning, is also described in this chapter. Several research studies have been done to incorporate geological and economic uncertainty in the economic block modelling process. These advances in stochastic orebody evaluation are also reviewed in this chapter.

2.2 Overview of deterministic modelling and its application in mining

A deterministic problem also known as a static problem is a problem in which all input data is known or is assumed to be known with certainty. In other words, these problems do not consider any uncertainty associated with the input data. According to Birge and Louveaux (1997), static means that decisions are taken only once and deterministic means that the future is assumed to be completely and accurately known. Solving a deterministic problem is therefore referred to as deterministic modelling. A deterministic model is defined as a model that gives the same exact outcomes for a particular set of inputs, regardless of how many times the model re-calculates (Wittwer, 2004). Figure 2.1 shows a simplified deterministic model $f(x)$ with inputs x_1, $x_{2,}$ and x_3 and outputs y_1 and y_2.

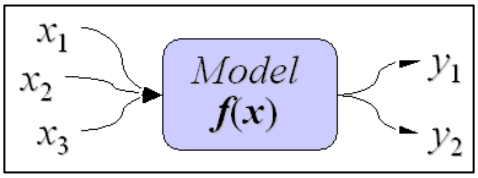

Figure 2.1: A simplified deterministic model

(Source: Wittwer, 2004)

In a deterministic model, single point fixed estimates (x_1, $x_{2,}\ldots x_n$) are combined to predict a single and static estimate of the output (y). Deterministic modelling is most suited to simple problems where input variables are not prone to variability and therefore are known with certainty. However, in reality input parameters to most problems cannot be known with certainty.

Mining in particular is a complex and dynamic process, so is mine planning; uncertainty and risk are inherent in mine planning processes. Despite the intrinsic uncertainty in mining, most current approaches to mine planning are deterministic, where fixed average values are used as input values in developing mine plans. This means that uncertainty is ignored in the mine planning process resulting in a static and deterministic mine plan which does not model the true and realistic behaviour of the mining process. For example, during the project phase of a mining project, estimates are made of project completion times and costs using the Critical Path Method (CPM). The assumption with the CPM is that activity durations and resources required can be estimated with certainty (Demeulemeester and Herroelen, 2006). Activity durations used to determine project durations are single time estimates obtained by calculating the mean or average and resources required are also fixed estimates.

This general practice during mine planning under uncertainty of replacing a range of possible outcomes with a single average or expected value is truly deterministic and has several implications to mine design and planning. Often times it leads to cost overruns and failure to complete projects in time because the initial estimates would have not incorporated uncertainty intrinsic to projects. It also results in below target, behind schedule and beyond budget outcomes. Savage and Markowitz (2009 p.11) mentioned that according to the concept of the 'flaw of averages', "*plans based on average assumptions are wrong on average*". Therefore, uncertainty of a random variable should not be represented by its average or expected value.

The main advantage of deterministic approaches is that they are simple to model and understand. However, Palisade (2016) argued that their disadvantage is that combined errors in each fixed estimate will result in significantly different real-life results. This was also reiterated by Magagula *et al*. (2015) who explained why there is always a gap between planned targets and actual production results in most mines. They alluded the difference to the fact that mine planning tends to be done deterministically while the mine planning process is actually stochastic in nature. To address this challenge, mine planning approaches should be stochastic. This idea of stochastic mine planning was also emphasised by Musingwini (2016) who mentioned the need to shift from deterministic to stochastic mine planning approaches, because of risk and uncertainty associated with mining.

2.3 Uncertainty and risk in mining

The term 'risk' has been defined differently in different contexts. The Corporate Finance Institute (2021) defined risk as the probability that actual results will differ from expected results. It is common to find the words risk and uncertainty being used synonymously but, they are different. Olivia (2011) outlined some of the differences between the two and stated that risk can be quantified and measured. On the contrary, there is no known way of measuring uncertainty. The other difference is that while risk can be minimised or eliminated uncertainty cannot. Nonetheless, both risk and uncertainty affect mine design and planning decisions.

All activities in life carry some risk, but some activities are inherently riskier than others (Olivia, 2011), for example mining. Mining is a risky business because it requires multiple technical, economic, financial, social, and environmental inputs and assumptions. In particular, mine planning relies on a significant amount of forecasting of these inputs and since planning is done for the future which is uncertain, uncertainty and risk are inherent in mine planning, something which mine planners must consider and plan for. Figure 2.2 shows the main sources of uncertainties in mining projects broadly classified as external (exogenous) and internal (endogenous) sources. Nonetheless, Kazakidis and Scoble (2003) mentioned that classification of uncertainties into these two groups depends on the type of analysis done and the characteristics of the mining project such that some uncertainties may be considered as external rather than internal, or vice versa.

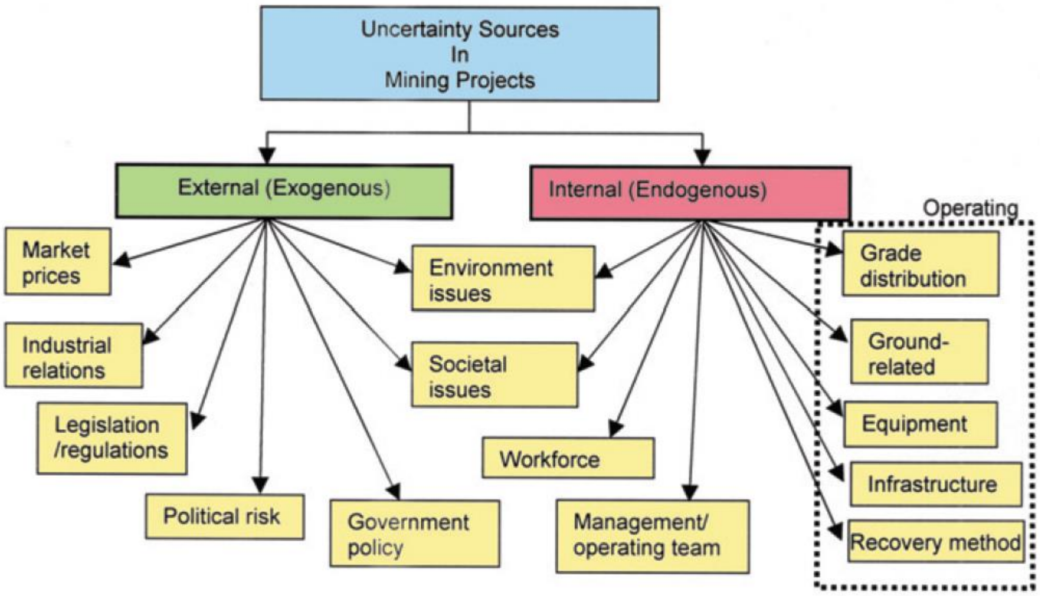

Figure 2.2: Sources of uncertainty in mining projects

(Source: Kazakidis and Scoble, 2003)

External sources of uncertainty are those that are driven by factors external to the company such as the business or market environment (Krantz and Scott, 1992). Risks from such uncertainties are best dealt with through strategic planning. Management of external risks involves strategies such as market engagement, diversification, business insurance, and implementation of inclusive solutions as well as the development of effective stakeholder engagement (Lane and Kamp, 2013).

On the other hand, internal sources of uncertainty are those that are related to the orebody (Krantz and Scott, 1992). Risks from such uncertainty are technical in nature and should be dealt with at their source. Dealing with risk at its source means that inputs to the mine plan must incorporate the variability of those input parameters. Accurate orebody evaluation is one of the first opportunities available to mine planners to deal with this risk since an orebody is one of the key inputs to the mine design and planning process. The efficacy of a mine plan will depend on the level of risks in a mine plan. Therefore, one of the objectives of mine planners should be to minimise both internal and external risks to a mining project.

Croll (1999) reviewed 11 mining projects and acquisitions spanning over a period of 30 years to determine how key geological and economic factors were estimated in the different feasibility studies. The study showed that grade estimation of Mineral Reserves and commodity price forecasts are the key factors that carried the greatest risk in mineral project evaluation.

Therefore, the sources of uncertainty and consequences for geological and economic factors are discussed in the next subsections.

2.3.1 Source of geological uncertainty

During orebody evaluation, the geology (grade) of a mineral deposit is estimated using geostatistical techniques by interpolating relatively limited data that is collected from exploration drilling results (Martinez, 2009). Since the data is not representative of the entire mineral deposit the geology of the deposit is a significant source of uncertainty in mining operations. However, interpolation based geostatistical techniques such as inverse distance and kriging do not consider the uncertainty associated with the grade estimates (Ersoy and Yünsel, 2006). Godoy and Dimitrakopoulos (2011) also mentioned that the main shortcomings with geostatistical estimation is that it fails to reproduce the *in-situ* spatial variability of geological parameters as inferred from the available sample data.

Geological uncertainty has severe consequences to the success of mining operations. According to Martinez (2009), the lack of information to represent the entire deposit may result in misclassification of some ore as waste and vice versa. Dimitrakopoulos *et al.* (2002) mentioned that geological uncertainty is the major contributor to projects failing to meet their targets. Vallee (2000) mentioned that a study conducted by the World Bank in the 1990s for failed projects in Canada and the USA indicated that a total economic loss of US$1106 million resulted from Mineral Reserve related problems. Subsequently, geological uncertainty leads to incorrect estimation of Mineral Resources and Mineral Reserves. Therefore, to minimise geological risk stochastic estimation techniques must be applied. These include conditional simulation.

2.3.2 Sources of economic uncertainty

In addition to grade uncertainty, the other major sources of risk in mining projects is the market behaviour of metal prices (Croll, 1999; Abdel Sabour and Dimitrakopoulos, 2009). For some mineral commodities mining companies enter into supply contracts where commodity price is fixed, and the future price is known with certainty. However, this is not the case with most commodities. Gentry (1988) mentioned that one unique characteristic of mining compared to other businesses is that most mineral commodities are fungible hence they are traded on international markets which means that the markets dictate the price. This makes mining companies to be ‘price takers’. Therefore, since commodity prices are dictated by global supply and demand fundamentals that are unpredictable, forecasting price accurately is impossible making it a significant source of risk in mineral project valuation.

Martinez (2009) found that usually commodity price is modelled as an average of the last three years particularly for precious and base metals. This can be misleading and may lead to overestimation or underestimation of the price subsequently leading to inaccurate valuation of mineral projects. According to Croll (1999), most feasibility studies assume that commodity price will remain constant in real monetary terms and escalated at prevailing inflation rates. This does not represent a realistic behaviour of these parameters.

Abdel Sabour and Dimitrakopoulos (2009) also found that in addition to commodity price uncertainties, the other major source of risk in mining projects is exchange rates, a fact also supported by Haque *et al*. (2014). Exchange rates are volatile making it speculative and erroneous to deterministically forecast them.

Cost is also another source of uncertainty in economic orebody evaluation. As mentioned earlier, mining and processing costs are some of the inputs in a BEV calculation. Like commodity price and exchange rate, the cost of most items required in mining projects fluctuates stochastically over time (Dixit and Pindyck, 1994). Therefore, costs are difficult to predict over the LOM because of the lack of engineering and economic information that is available at a planning stage. This makes mining and processing costs to carry a significant amount of uncertainty. In particular to orebody evaluation, the actual cost of mining and processing a certain ore block will only be known certainly after the block has been completely mined.

2.3.3 Traditional risk management

To model uncertainty during mine design and planning, traditional methods are used. One of the methods is sensitivity analysis. Sensitivity analysis involves varying one input parameter at a time to observe the impact to the outcome. In mineral project valuation, this is done by varying various economic, geological, and technical input parameters. However, this does not correctly and completely account for uncertainty. Higle and Wallace (2002) referred to sensitivity analysis as a mini-max technique that behaves in an extreme fashion. They argued that the approach has limitations in that it focusses on the rare events (best- and worst-case scenarios) at the expense of the remaining intermediate cases and that it ignores probabilities and treats all possible scenarios as equal. Palisade (2016) also stated the following disadvantages of risk analysis using sensitivity analysis mentioning that it:

- Can only vary one parameter at a time.
- Cannot capture the relationship between input variables.
- Cannot determine the probability between the best-case and worst-case scenarios.

The mining process is a complex system where inputs into the process are interrelated such that changing one input impacts other inputs in the entire system. For example, an increase in contaminants reduces recovery and increases processing cost. Therefore, varying one parameter at a time does not give a true behaviour of the system or outcome. Also, ignoring probabilities of scenarios happening fails to consider the fact that uncertainty is intrinsic to mining. The other weakness of sensitivity analysis as a risk assessment tool is that it mostly assumes linearity between input parameters and the output as shown in Figure 2.3 which shows a sensitivity analysis of NPV to revenue and costs.

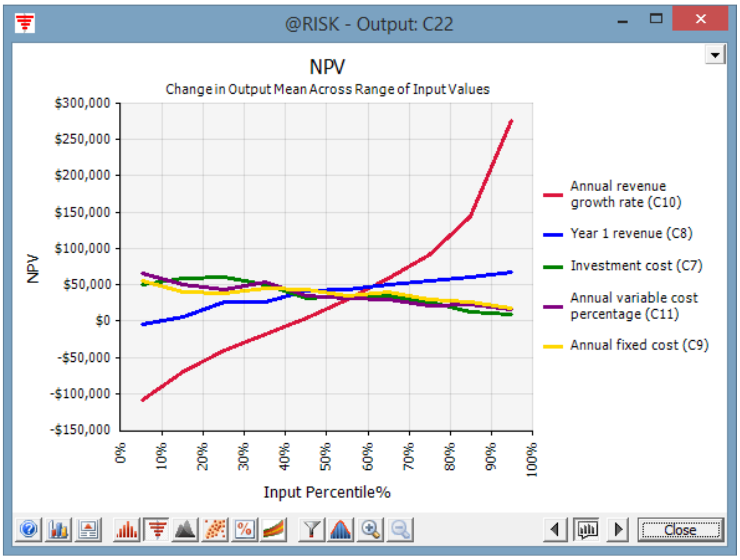

Figure 2.3: An example of sensitivity analysis results

(Source: Palisade, 2016)

Di Domenica *et al*. (2007) commented on the same concept by saying that sensitivity analysis does not represent the impacts of stochastic behaviour of the model's parameters. Therefore, it may give misleading conclusions in respect to the nature of the solutions.

The other and most common approach is the use of a risk adjusted discount factor as a default way of dealing with risk in mineral projects. All project risks are encompassed together into the discount rate to an extent that the rate can be too high to render a good project unviable. A discount rate does not deal with risk at source. Therefore, there is a need for more accurate stochastic orebody modelling methods that incorporate and deal with uncertainty at source (the orebody) and hence minimising the need for subjective judgements (Nhleko *et al.*, 2018).

These traditional risk management methods fail to correctly model uncertainty and risk. This may result in misleading estimates of the orebody in terms of its size and quality. To correctly account for risk and uncertainty in orebody evaluation, stochastic approaches must be used.

2.4 Stochastic optimisation

The word stochastic means that some or all data is uncertain which therefore means that a stochastic problem is a problem which involves uncertain input data. Solving a stochastic problem is synonymously referred to as stochastic programming, stochastic optimisation or optimisation under uncertainty. Stochastic programming was defined by Sima (2015) as programming in which some coefficients or parameter values incorporated into the objective function and constraints are usually uncertain. Corrigall (1998 p. 27) said that "*stochastic programming is the study of optimal decision-making for stochastic programs that deal with algorithmic optimisation procedures*". In stochastic optimisation, uncertainty is mostly represented by probability distributions or random numbers of the uncertain parameters (Corrigall, 1998). Dixit and Pindyck (1994) defined a stochastic process as a process that evolves over time in a way that is at least in part random. They mentioned a company's stock price as an example of a continuous time stochastic process. The price fluctuates randomly in the short term though in the long term it has a positive expected rate of growth that compensates investors for the risk in holding the stock (Dixit and Pindyck, 1994).

The aim of stochastic optimisation is to determine optimal decisions in problems that involve uncertain data. The underlying motivation for stochastic optimisation is that the future cannot be perfectly forecasted because of several reasons. Corrigall (1998) gave two reasons why the future cannot be forecasted. The first reason is the lack of reliable data or simple measurement error. The second and most important reason is that some data represent information about unobserved future events including product demand and price which cannot be known with certainty (Corrigall, 1998).

One of the advantages of stochastic approaches is that they enable decision makers to understand the effect of uncertainty on the output hence, enabling them to make informed decisions. However, Di Domenica *et al*. (2007) stated that a major challenge with any stochastic problem is the real representation of the underlying uncertain data. Otherwise, the famous statement ‘Garbage In-Garbage Out’ becomes true; the quality of output results is determined by the quality of the input parameters.

2.5 Advances of stochastic optimisation in mine planning

Di Domenica *et al*. (2007) mentioned that the future is naturally linked with uncertainty, hence the need for stochastic approaches to mine planning. This is because most mine planning problems have input parameters that are not known with certainty due to several reasons including the lack of reliable data and also because of data representing unknown or unobserved future events. At the time of planning, mine planners do not know with certainty the values of future commodity prices, the complete characteristics of the orebody and many other mine planning inputs. It is only as the orebody gets extracted that all its characteristics are known with improved confidence. In reality, the actual characteristics and the total quantity of Mineral Resources for any given orebody are known with better confidence at mine closure; before that stage only estimates are used in the mine planning process. This uncertainty associated with mine planning input parameters strongly necessitates the need for stochastic instead of deterministic modelling. Mining is a stochastic process, therefore it requires stochastic approaches to its problems. Del Castillo and Dimitrakopoulos (2014) mentioned that deterministic mine planning methods offer a static response to a dynamic reality. This was reiterated by Magagula *et al*. (2015) who mentioned that in mining, the dichotomy that planning is deterministic whilst the actual mining process is stochastic often results in mining operations failing to achieve their production targets. This, in some cases has led to litigation between the providers of capital and mining practitioners where capital providers felt that they were misled and misinformed into investing in projects that failed to achieve stated objectives (Magagula *et al*., 2015).

Finance, supply chain, transportation, environmental and telecommunications are some of the economic sectors where stochastic optimisation has been progressively applied (Di Domenica *et al*., 2007). In mining, the concept of stochastic mine planning is known but not well understood and practiced because stochastic mine planning problems are known to be computationally difficult to solve. This is evident from the intermittent research done to date on stochastic mine planning in general, lack of application of those few available stochastic techniques by industry and the absence of literature available in particular to stochastic BEV calculation. According to Birge and Louveaux (1997), to solve real-life problems many people naturally prefer solving the simpler deterministic problems by replacing all uncertain parameters with their expected values. Even though deterministic techniques are still mostly used in mining, considerable research has been done to develop stochastic techniques along the mining value chain. Nonetheless, the focus of this chapter is to review deterministic and stochastic techniques applied in orebody modelling only. Several techniques are used to model uncertainty in mine planning; the common ones only are reviewed in the next subsections.

2.5.1 Monte Carlo simulation

Monte Carlo simulation is one technique that is widely used in the mining industry to model uncertainty. According to Silverwood (2011), Monte Carlo simulation is a stochastic simulation technique that relies on repeated random sampling of data to approximate the stochastic behaviour of the actual system. The aim of the method is to determine how lack of knowledge, random variation, or error affects the sensitivity or performance of the system that is being modelled (Wittwer, 2004). Figure 2.4 shows a simplified Monte Carlo simulation model $f(x)$ with inputs x_1, x_2, and x_3 and outputs y_1 and y_2.

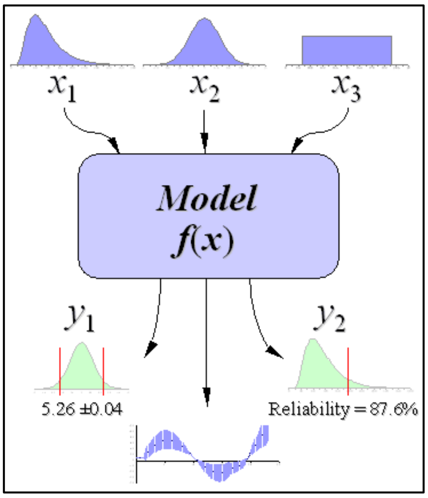

Figure 2.4: A simplified stochastic simulation model

(Source: Wittwer, 2004)

As shown in Figure 2.4, uncertainties associated with input variables (x_1, $x_{2,}\ldots x_n$) are represented by probability distributions of the uncertain variables as opposed to fixed single value estimates (Figure 2.1). Fitting probability distributions involve finding a mathematical function which best represents a statistical random variable (Ricci, 2005). Fitting a probability distribution to a set of data is a process of selecting a statistical distribution that best describes the randomness of the data. By considering probability distributions of input variables instead of single average values, Monte Carlo simulation helps to avoid the ‘flaw of averages’ problem (discussed in Section 2.2) in the interpretation of mine plans. By using random inputs, a deterministic model is essentially turned into a stochastic model (Wittwer, 2004). Multiple simulations are run to produce an output which is a probability distribution of the possible values of (y) which could occur. The more the simulation runs the more relibile the representation of the actual system becomes (Palisade, 2016).

According to Abdel Sabour and Wood (2009), one of the advantages of Monte Carlo simulation is that it allows the analysis of the effect of all key input variables simultaneously. However, Monte Carlo simulation does not give an optimum solution but, uses probability distributions to provide a reliable estimation of the actual behaviour of a real system. Results of a Monte Carlo simulation show a range of expected values of an output and the risk profile of each possible value of a problem at hand, thus, enabling informed decision making. For any given output, decision making is subjective depending on the risk appetite of the decision maker. An example of the results of a Monte Carlo simulation is shown in Figure 2.5 which shows the probability distribution of NPV from a discounted cash flow analysis.

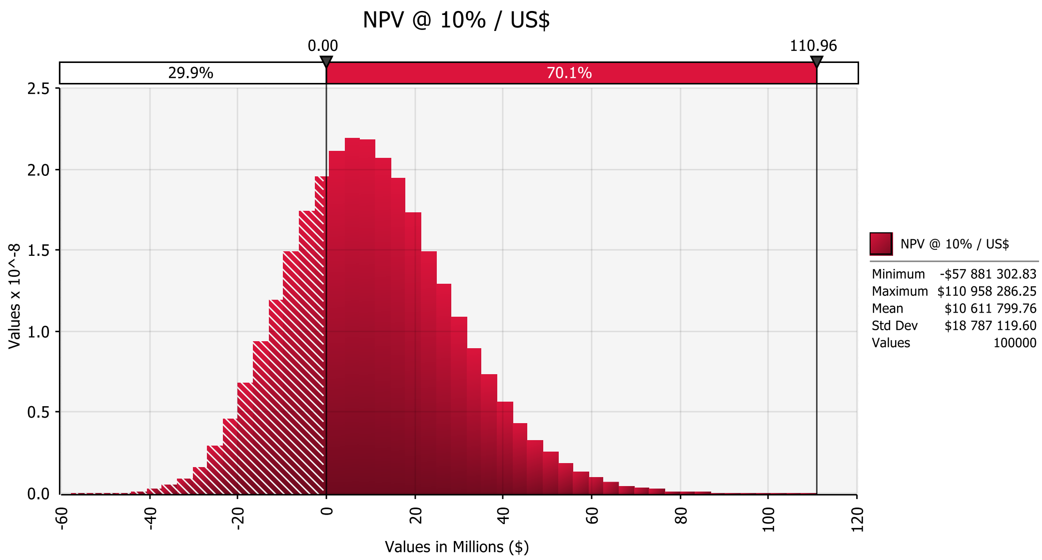

Figure 2.5: An example of a Monte Carlo simulation output

(Source: Example developed by author)

In the example in Figure 2.5, the project has an expected positive NPV of US$10.6m and deterministic approaches would automatically accept the project as feasible. However, a critical analysis of the results shows that the project has a 29.9% chance of a negative NPV which would significantly change the view of project owners because of the risk of failure. Therefore, the main advantage of Monte Carlo simulation is that one can make better informed decisions.

2.5.2 Stochastic orebody modelling using conditional simulation

To address the limitation of interpolation based geostatistical techniques mentioned in Section 2.3.1, conditional simulation algorithms were developed. Conditional simulation was defined by Dimitrakopoulos *et al*. (2002) as a Monte Carlo-type simulation technique for modelling uncertainty in spatially distributed attributes, such as for mineral deposits. Dimitrakopoulos (nd) mentioned that the application of conditional simulation in the mining industry started in the 1970s to simulate geological data for mineral deposits even though the concept has been developed and applied earlier in other industries. To date, several conditional simulation techniques have been developed to model geological uncertainty during orebody evaluation. These algorithms are categorised as Gaussian, non-Gaussian, non-stationary, non-parametric, multiple-point geostatistics, high-order spatial statistics and other categories (Dimitrakopoulos, nd). The basic idea with all the categories is to reproduce the actual *in-situ* variability and spatial continuity of geological attributes of interest (Ersoy and Yünsel, 2006). Irrespective of the algorithm used, the output of conditional simulation are multiple equally probable realisations of the orebody as opposed to a single orebody produced by traditional geostatistical estimation methods. The simulated orebodies are then used as inputs in subsequent processes such as mine design and scheduling.

Abdel Sabour *et al*. (2008) stated that Dimitrakopoulos *et al*. (2002) pioneered the integration of geological uncertainty in open-pit mine planning and optimisation using conditional simulation. Figure 2.6 shows a comparison of NPVs obtained from using one conventional orebody and 50 stochastic orebody realisations as inputs into pit optimisation. This was applied on a small gold mine by Dimitrakopoulos *et al.* (2002).

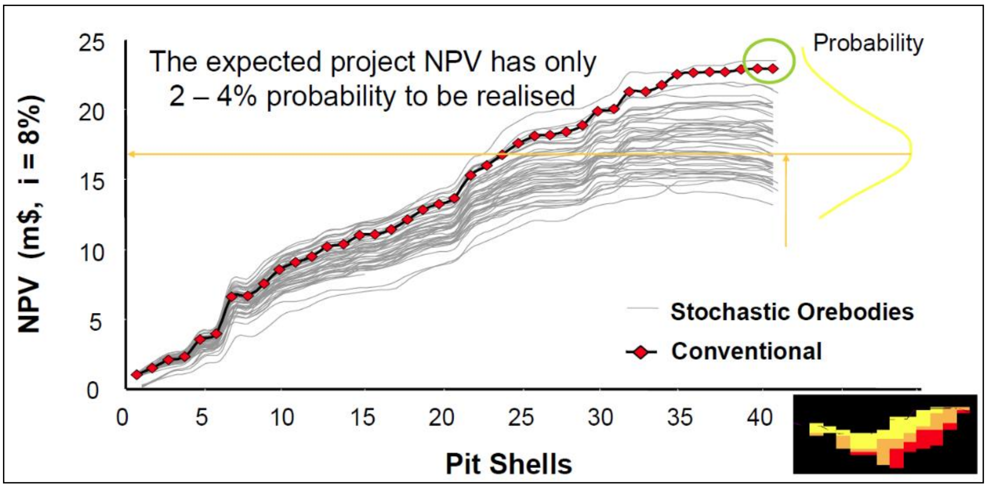

Figure 2.6: Deterministic versus stochastic pit shells

(Source: Dimitrakopoulos, 2010)

Figure 2.6 shows that the pit design from the conventional orebody was unlikely to meet NPV expectations compared to the pit designs from equally probable simulated orebodies. The conventional design had a 2–4% chance of achieving the expected NPV, which is about 25% less than the most likely NPV based on simulated orebodies (Dimitrakopoulos *et al.*, 2002). This shows that stochastic optimisation reduces the gap between planned targets and actual production. Figure 2.7 illustrates the outcomes from conventional Mineral Reserve estimation compared to conditional simulation approaches.

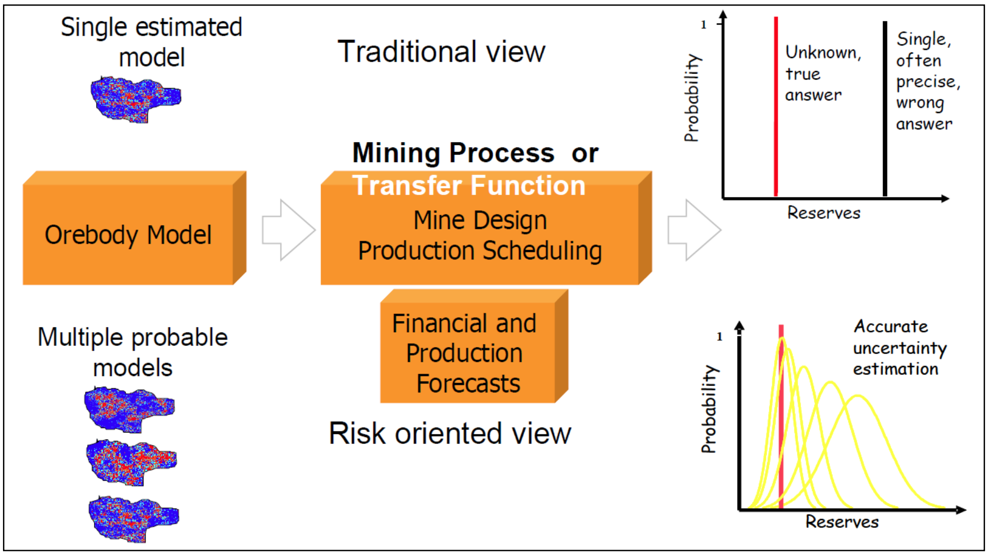

Figure 2.7: Deterministic versus stochastic orebody modelling

(Source: Dimitrakopoulos, 2010)

Figures 2.6 and 2.7 show that the conventional approach to the estimation of Mineral Reserves and project valuation results in a single unrealistic orebody model and NPV (with a lower chance of being achieved). In contrast, the stochastic method gives multiple equally probable simulated orebody realisations and a probability distribution of NPV. The use of multiple orebody models to represent uncertainty of the spatial distribution of grades improves mine design and planning. It assists mine planners to assess the sensitivity of mine designs and plans to grade uncertainty, and to produce mine designs and plans with higher NPVs through stochastic optimisation (Godoy and Dimitrakopoulos, 2011). Dimitrakopoulos (2010) mentioned that stochastic optimisation can distinguish between the maximum possible contained metal from the minimum possible metal content. In their study of stochastic optimisation on a particular gold mine, Godoy and Dimitrakopoulos (2011) found that stochastic optimisation led to a 28% improvement in NPV over the LOM, compared to conventional approaches that were applied at the mine.

The concept of conditional simulation in orebody evaluation includes the previous work of Grieco and Dimitrakopoulos (2007) and Dimitrakopoulos and Grieco (2009) among many others. The simulation method used in most of this work is based on the Sequential Gaussian Simulation (SGS) method. The SGS method (in Surpac software) was the method used in this thesis to model the uncertainty of gold grade. This is because SGS based methods are more recent, more established, and therefore extensively used in mining (Dimitrakopoulos, nd).

Nevertheless, the limitation of previous work on stochastic orebody modelling is that the work considers grade as the only uncertain parameter, ignoring the uncertainty of other parameters such as commodity price. Ignoring price uncertainty results in incorrect estimation of Mineral Reserves and LOM, subsequently leading to inaccurate plant capacity designs which ultimately destroys shareholder value (McCarthy and Monkhouse, 2002). In addition to grade uncertainty, commodity price uncertainty is an equally significant source of risk that must be considered during economic orebody modelling.

2.5.3 Stochastic modelling of commodity price

Several techniques have been developed to model uncertainty that is associated with commodity prices. Tapia Cortez *et al.* (2018) classified these techniques as econometric, time series, stochastic-gaussian and dynamic systems methods. Econometric methods include Vector Auto Regression and the Vector Error Correlation techniques (Tapia Cortez *et al.* 2018). Time series methods include the Autoregressive Conditional Heteroscedastic, Autoregressive Integrated Moving Average and Autoregressive Moving Average techniques (Tapia Cortez *et* *al.* 2018). The stochastic-gaussian techniques include the Stochastic Brownian Motion, Geometric Brownian Motion (GBM), and mean-reverting while the dynamic systems techniques include the Chaos theory and machine learning (Tapia Cortez *et al.* 2018). Nonetheless, research found that commodity price also behaves like stock price which tends to follow two main stochastic behaviours. The first is the Brownian motion also called a Wiener process (Dixit and Pindyck, 1994). Several authors have also supported this finding including Haque *et al.* (2014), Abdel Sabour and Poulin (2010), Lemelin *et al.* (2006) and Glasserman (2003). The second behaviour is the mean-reverting behaviour such as prices for base metals (Dixit and Pindyck, 1994; Schwartz, 1997; Lemelin *et al.*, 2006). The two main stochastic behaviours are discussed in the next subsections.

Stochastic modelling of commodity price using the Brownian motion process

The Brownian motion process was named after Robert Brown who observed that small particles from pollen grains immersed in a liquid move endlessly and randomly (Karlin and Taylor, 1975). Therefore, the Brownian motion process is a continuous-time continuous-state space process with three properties namely (Dixit and Pindyck, 1994 p.63):

1. "*It is a Markov process which means that the probability distribution for all future values of the process depends only on its current value, and is unaffected by past values of the process or by any other current information. As a result, the current value of the process is all that is needed to estimate its future value.*
2. *It has independent increments which means that the probability distribution for the change in the process over any time interval is independent of any other (non-overlapping) time interval.*
3. *Changes in the process over any finite time interval are normally distributed, with a variance that increases linearly with the time interval*".

These three properties of a Wiener process are the reasons why real-world problems that vary randomly through time including stock prices, and mineral commodity prices can be modelled using the Wiener process. Equation 2.1 shows a Brownian motion model. If $x\ (t)$ is a Wiener process, then any change in x, Δx, corresponding to a time interval Δt, satisfies the following conditions (Dixit and Pindyck, 1994):

$$\Delta x = \epsilon_t \sqrt{\Delta t} \qquad 2.1$$

where; ϵ_t is a normally distributed random variable with a mean of zero and standard deviation of 1. Assuming Δt to be infinitesimally small, Equation 2.1 can be represented as Equation 2.2 (Dixit and Pindyck, 1994):

$$dx = \epsilon_t \sqrt{dt} \qquad 2.2$$

The Brownian motion process model in Equation 2.2 can be modified to represent the actual movement of a stock or commodity price. Chance (1994) specified four properties of a stock price that necessitate the modification of Equation 2.2 and stated that:

1. A stock should have a positive expected return and in the long term the stock price should drift upward at a rate equal to its expected return.
2. A stock price should have a random, unpredictable component called the ‘noise’.
3. It should be difficult to forecast stock prices over the long term than it is in the shorter term, for example hours.
4. It should be impossible for the stock price to be negative.

Therefore, the future stock price is a combination of its gradual move at a rate equal to its expected return and its random noise (Chance, 1994). These properties are also true for mineral commodity prices and therefore the Brownian motion with drift can be used to model the stochastic behaviour of commodity price. Considering the above conditions, Equation 2.2 is modified to Equation 2.3 or 2.4 representing the Brownian motion with drift (Dixit and Pindyck, 1994):

$$\frac{dx}{x} = (r - \delta)dt + \sigma dz \qquad 2.3$$

Or

$$dx = \alpha dt + \sigma dz \qquad 2.4$$

where, x is the price, r is the risk-free rate of return, δ is the convenience yield, dt is an increment of time, σ is the standard deviation, $\propto$ is the drift parameter, and dz is the increment of the Wiener process. An example was done by Dixit and Pindyck (1994) using stock price data from 1950 to 1974 to forecast the price for 1975 to 2000 using a drift of 0.2 per year and standard deviation of 1 per year and the price paths are shown in Figure 2.8.

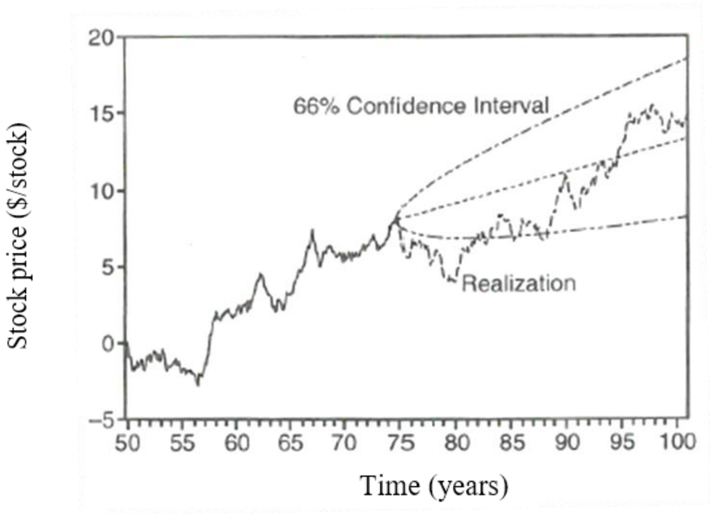

Figure 2.8: The Brownian motion with drift

(Source: Dixit and Pindyck, 1994)

The result in Figure 2.8 shows a 66 percent confidence interval in the forecasted price, that is, the forecasted trajectory for price, x, at time t, $x(t)$ ± one standard deviation. Figure 2.8 shows that the price is more volatile in the short term than in the long term.

Equation 2.3 representing the Brownian motion with drift assumes that the percentage changes in price, x, $\Delta x/x$ are normally distributed (Dixit and Pindyck, 1994). This means that the Brownian motion can take negative values such that it cannot be directly used to model price. Therefore, since these are changes in the natural logarithm of x, it can be assumed that the absolute changes in x, Δx are lognormally distributed such that $x(t)$ is given by Equation 2.5 (Dixit and Pindyck, 1994):

$$dx = \alpha x dt + \sigma x dz \qquad 2.5$$

Then $F(x) = log\, x$ is given by Equation 2.6 (Dixit and Pindyck, 1994):

$$dF = \left(\alpha - \frac{1}{2}\sigma^2\right) dt + \sigma dz \qquad 2.6$$

As $t \to \infty$, the change in the logarithm of price is normally distributed with mean $\left(\alpha - \frac{1}{2}\sigma^2\right) t$ and variance $\sigma^2 t$.

Therefore, given the current price, $x(0) = x_0$, the expected price at any time t, $x(t)$ is given by Equation 2.7 (Dixit and Pindyck, 1994):

$$\varepsilon[x(t)] = x_0 e^{\alpha t} \qquad 2.7$$

where, αt is the Brownian motion with drift. Equation 2.7 is the non-negative variation of the Brownian motion called the Geometric Brownian motion (GBM). Figure 2.9 shows the forecast of the GBM done by Dixit and Pindyck (1994) from 1975 to 2000 of the New York Stock Exchange (NYSE) Index expressed in real monetary terms.

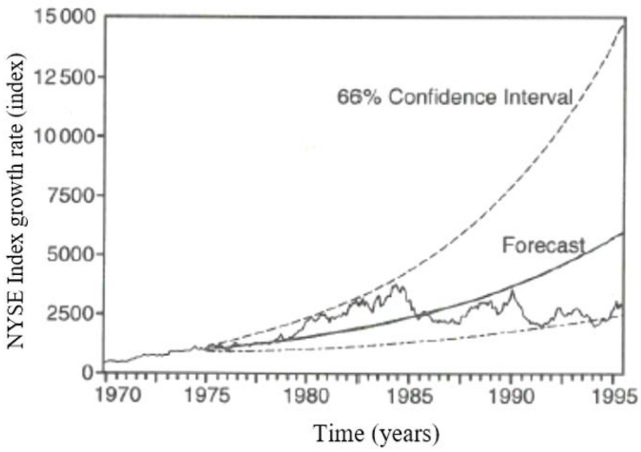

Figure 2.9: The Geometric Brownian motion

(Source: Adapted from Dixit and Pindyck, 1994)

The index value in 1974, that is, I (1974) was used to forecast index values from 1974 to 1995. In addition to stock price, Dixit and Pindyck (1994) mentioned that the GBM is also frequently used to model interest rates, wage rates and output prices. The GBM was used to model mineral commodity prices by Haque *et al.* (2014) and Lima and Suslick (2006) among many others. As mentioned at the start of Section 2.5.3, the second common stochastic behaviour of commodity price is the mean-reverting process, and this is discussed in the next subsection.

Stochastic modelling of commodity price using the mean-reverting process

Dixit and Pindyck (1994) mentioned that the Brownian motion tends to wander far from its starting point, and this was illustrated in Figures 2.8 and 2.9. This is it true for speculative asset prices but, unrealistic for some economic variables including commodity prices like copper or oil prices (Dixit and Pindyck, 1994). Such variables tend to follow the mean-reverting motion. This means that even though in the short term the variable may move randomly up and down in response to macro-economic factors, in the long term it tends to move back towards the historical mean or average. Therefore, unlike the Brownian motion, the mean-reverting motion process does not have independent increments (Dixit and Pindyck, 1994). Figure 2.10 and

Equation 2.8 show the mean-reverting behaviour for the price of an unspecified commodity. The mean reversion behaviour as described by Haque *et al.* (2014) is when the price of mineral commodities or stocks eventually move back towards its historical mean.

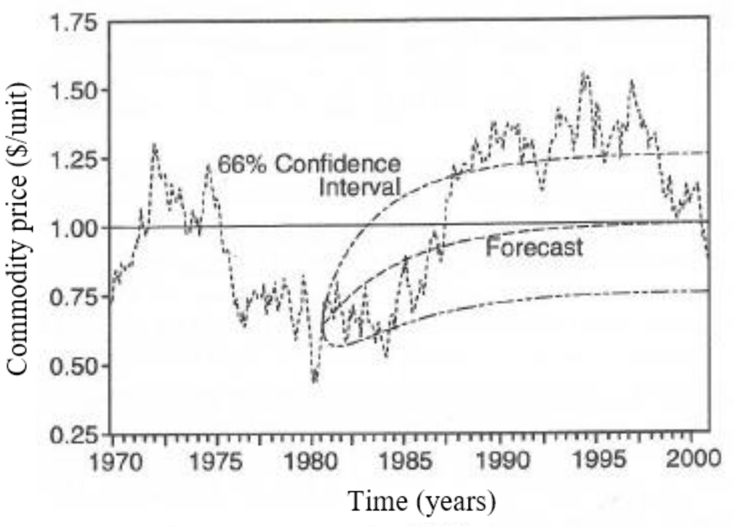

Figure 2.10: Mean-reverting motion

(Source: Dixit and Pindyck, 1994)

The mean-reverting motion model is represented by Equation 2.8 (Schwartz, 1997):

$$\frac{dx}{x} = K(\mu - \ln x)dt \ + \sigma dz \qquad 2.8$$

where x is the spot price, K is the reversion speed, μ is the logarithm of the long-term equilibrium level of metal price, σ is the standard deviation, dt is an increment of time and dz is an increment in a standard Weiner process.

It has also been found that the commodity price can follow a stochastic model that combines the two price behaviours as shown by Equation 2.9 (Haque *et al*., 2014; Cortazar and Schwartz, 1997; Cortazar and Casassus, 1998). Using this model, if the speed of reversion, μ, is zero then the price follows the GBM behaviour shown in Equation 2.3, otherwise the price follows a mean reversion behaviour (Haque *et al*., 2014).

$$\frac{dx}{x} = (r - \delta \ + \mu(x^{-} - x))dt \ + \sigma dz \qquad 2.9$$

where, x is the spot unit price of the commodity, x^{-} is the mean or average unit price of the commodity, r is the risk-free rate of interest, δ is the mean convenience yield on holding one unit of that commodity, σ is the volatility of returns of x, μ is the speed of the reversion, dt is an increment of time and dz is the Wiener increment of the mean reversion process (Haque *et al*., 2014).

However, most previous studies on orebody evaluation modelled commodity price in isolation and not in conjunction with grade or cost. Mine planning is an integrated process therefore, uncertainty in the process should be modelled in an integrated manner hence the use of Monte Carlo simulation in this study.

2.5.4 Integrated modelling of geological and economic uncertainty

Most of the work reviewed in the preceding sections model grade and commodity price separately. Research that has been done recently to integrate grade and price uncertainty during orebody evaluation includes the work of Dimitrakopoulos *et al*. (2007); Whittle *et al*. (2007); Abdel Sabour and Dimitrakopoulos (2011) and Del Castillo and Dimitrakopoulos (2014).

In the work of Dimitrakopoulos *et al*. (2007), stochastic simulation was only done to grade using the conditional simulation technique and, price was simulated in the later stages of the optimisation process through sensitivity analysis. The limitations of sensitivity analysis as a risk management tool were discussed in Section 2.3.3. In the case study, fixed economic (price and costs) and technical (recovery rate) parameters were used to generate 13 simulated orebodies and using the L-G algorithm together with the Milawa Scheduler 13 pit designs and push backs were generated. Only four pit designs that had less grade uncertainty measured by the quantity of ore in the pit were selected for further assessment. The upside potential and downward risk in terms of the discounted net value for each of the four designs were calculated and two designs that had high upside potential (discounted net values) were then selected and used to test the impact of price volatility (Dimitrakopoulos *et al*., 2007). This was done by running iterations, only changing the price of gold. However, this also only considers grade uncertainty and assumes certainty in other economic parameters and the authors indicated that the approach applies to open pit optimisation studies.

Whittle *et al*. (2007) also presented a robust mine planning approach that incorporated geological and economic uncertainty in ultimate pit design. This was done on a hypothetical case study. Probability distributions were defined for each input parameter as shown in Table 2.1 resulting in a probabilistic Mineral Resource model.

Table 2.1: Summary of input probability distributions and parameters

(Source: Whittle *et al*., 2007)

Parameter	Average	Standard deviation	Distribution	Comments
Cu grade factor	1.0	0.05	Normal	All Cu grades on the block model are multiplied by this factor to generate a distribution of Cu grades on each block.
Au grade factor	1.0	0.075	Log-Normal	As for above but applied to the gold grade.
Short-term (three-year) Cu price, $/t	$6195	$930	Log-Normal	The possible distribution of copper prices over the next three years.
Long-term Cu price, $/t	$4956	$1230	Log-Normal	As for the short-term Cu price but for longer-term periods (four years into future and onwards).
Short-term (three-year) Au price, $/oz	$664	$70	Log-Normal	As for copper short-term price but for gold.
Long-term Au price, $/oz	$730	$150	Log-Normal	As for gold long-term price but for gold.
Cu recovery factor	1.0	0.03	Normal	The metallurgical recovery estimate for Cu is multiplied by this factor to illustrate the impact of variability of copper recovery.
Au recovery factor	1.0	0.05	Normal	As for copper but applied to gold.
Selling cost, Cu, $/t	$2000	$250	Normal	Costs incurred in selling copper.
Selling cost, Au, $/oz	$8.20	$1.03	Normal	Costs incurred in selling gold.
Mining cost – $/t	$2.10	$0.32	Log-Normal	Mining operating costs.
Processing cost – $/t	$7.50	$0.94	Log-Normal	Processing operating costs.
Pit slope – east	45.0	2.0	Triangular	Distribution of pit slopes – eastern side of pit
Pit slope – west	40.0	2.0	Triangular	Distribution of pit slopes – western side of pit

The Lerchs-Grossmann algorithm was used to calculate blocks that made up the various phases of the optimal pit design. The result was a series of probability ultimate pit outlines as shown in Figure 2.11(a) and (b) which show the 3D and plan views of the final pit probability shapes, respectively.

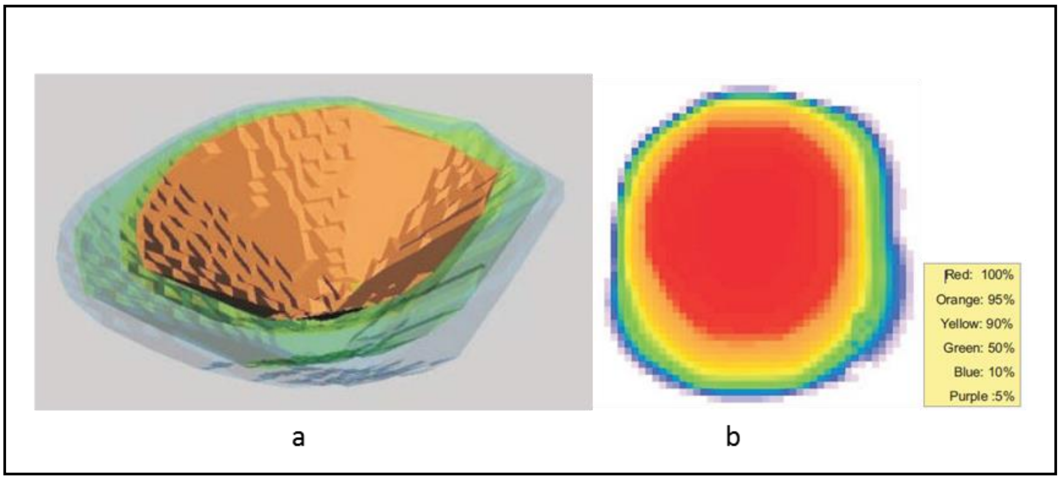

Figure 2.11: Probability pits

(Source: Whittle *et al.*, 2007)

The probability pit outlines guide the location of surface infrastructure. According to Whittle *et al*. (2007), for their case study the five percent outline in particular guided the location of infrastructure which ensured that future development options were not complicated by obstructing surface infrastructure. In comparison, deterministic open pit optimisation methods would generate a single pit with potential problems which may cause the operation to lose out on future opportunities when commodity prices increase as discussed earlier in Section 1.3. Such problems include placing permanent surface infrastructure in areas where the pit will possibly expand to when prices increase. Less equivalent work has been done in the design and optimisation of underground stope boundaries. In a review of underground mine optimisation techniques, Musingwini (2016) recommended the need to extend the work of Whittle *et al*. (2007) to underground stope design optimisation to improve confidence when locating and sizing underground development infrastructure. This is one of the reasons considered in motivating this research study.

To integrate geological and economic uncertainty, Abdel Sabour and Dimitrakopoulos (2011) extended the work of Dimitrakopoulos *et al*. (2007) to include price and exchange rate uncertainty in optimising open-pit mine designs. Conditional simulation was used to model geological uncertainty and the mean-reversion model was used to model metal price and foreign exchange rate uncertainty. Later, Del Castillo and Dimitrakopoulos (2014) developed a stochastic model that integrated geological and price uncertainty. The GBM together with the Poisson jump method were used to model price uncertainty and the direct block simulation (DBSim) method was used to model geological uncertainty (Del Castillo and Dimitrakopoulos, 2014). However, both cases were applied to open-pit mine design optimisation, hence the need to extend the concepts to underground mining optimisation in this thesis.

2.6 Summary of Chapter 2

Deterministic modelling is an approach that assumes that all input data into a model are known or assumed to be known with certainty. This modelling approach ignores uncertainty in the input variables. Therefore, it is suited to model simple problems in which all variables are known with certainty. In reality, geological, technical and economic input parameters encountered in most mine planning problems cannot be known with certainty because mining is a dynamic process. Most mine planning inputs are estimated based on unknown future events which makes uncertainty intrinsic to mine planning. Therefore, traditional deterministic mine planning approaches are inappropriate to solve mine planning problems because they offer a static solution to a dynamic or stochastic problem. The main advantage of deterministic approaches is their simplicity but, they result in outcomes that widely deviate from planned targets which can have significant consequences to optimum value creation from the extraction of Mineral Resources. Also, in a deterministic setup, decisions one would make on expected results may be different if one had a complete picture of all possible outcomes. Probabilistic approaches are therefore more appropriate than deterministic approaches to address stochastic mining processes.

Despite the known uncertainty intrinsic to mining, most approaches to mine planning are predominantly deterministic because stochastic models are complex to solve. Nevertheless, efforts have been made to develop stochastic models for orebody evaluation but, one their shortcoming is that most of the models only address uncertainty of grade and a few integrate grade uncertainty with economic uncertainty, *albeit* in a fragmented manner which therefore fail to provide true optimum solutions. Also, such efforts have been focussed on the optimisation of open-pit mine design and currently, no literature is available with reference to stochastic stope boundary definition.

It can therefore be concluded that the current research on economic orebody evaluation is mostly deterministic, fragmented and limited to optimising open-pit mine design. This, therefore, justifies the need to stochastically integrate geological and economic variables to quantify geological and economic risks more reliably in underground mine design and planning. This is achieved through probabilistic BEV calculation. Probabilistic BEV calculation and economic orebody modelling will improve decision making and achievement of results derived from the complex and dynamic mining planning process. It will result in robust mine plans and designs where actual outcomes are close or equal to planned targets. The next chapter reviews literature on algorithms that have been developed for stope boundary optimisation.

3 REVIEW OF STOPE BOUNDARY OPTIMISATION ALGORITHMS

3.1 Overview of Chapter 3

As explained earlier in Chapter 1 the aim of this thesis was to integrate geological and economic uncertainty in calculating BEVs to analyse the impact of these uncertainties on stope designs. The foundation for any optimisation process is Operations Research (OR), therefore an overview of OR is provided in this chapter. Fundamental to OR problem-solving techniques is the formulation of a model to represent the real-life problem and the development of an algorithm to solve the problem. Even though several algorithms have been developed for stope boundary optimisation, this chapter only reviews the common algorithms developed to date, some of which have been embedded in commercial software currently being used in the design and planning of underground mines. The application, strength and limitations of these algorithms are discussed in the chapter. Finally, from the algorithms that are reviewed, one is selected to be used in this thesis to generate probability stopes using economic block models created from the BEV calculation approach developed in this study.

3.2 Background on optimisation of underground mines

OR forms the foundation for optimisation in mine design and planning. Mishra and Sandilya (2011) defined OR as the application of mathematical models, statistics and algorithms to analyse and solve complex real-life problems with the goal of improving or optimising performance. Its fundamental objective is optimisation in the use of scarce resources, and it provides a rational basis for decision making in the absence of complete information; typical of mine design and planning problems. Mining involves the extraction of a finite Mineral Resource whose quantity decreases with each unit of extraction as it is an exhaustible natural resource. Therefore, there is only a single extraction strategy that optimises economic value from the extraction of this finite resource. Mine design and planning involves defining a single optimum strategy that considers scarcity of mineral and capital resources to optimise value from the extraction of Mineral Resources. Therefore, the application of OR techniques in mining is important to ensure the efficient use of these scarce resources.

OR is rooted in the United Kingdom pre-World War II in the British Royal Air Force and army where it was used to study the strategic and tactical problems involved in military operations and select the most effective strategies in the utilisation of limited ammunition resources (Mishra and Sandilya, 2011; Beasely, nd). After the end of the war in the 1940s, its application spread across different sectors of the economy associated with the utilisation of scarce resources. Even though it has been widely used in other sectors since the 1940s, the application of OR techniques in mining only started around the 1960s (Newman *et al.,* 2010) with initial application in open-pit mine optimisation. Optimisation in underground mines only started in the 1970s as an extension of open pit optimisation applications (Nhleko *et al*., 2018). Nonetheless, to date only a few optimisation techniques have been developed to solve underground optimisation problems compared to techniques for open-pit mining optimisation (Alford *et al*. 2007). This is because optimisation of underground mines is computationally more complex than open-pit mines as explained earlier in Section 1.2.3. Musingwini (2016) mentioned that because optimising underground mines is more computationally complex, most of the current work on in underground mine optimisation is mostly academic. Nonetheless, a number of optimisation opportunities are available with respect to underground mine design and planning. Alford (1995) identified the following as underground mine design problems amenable to optimisation:

- Location of primary development.
- Optimum mining method selection.
- Location and spacing of sub-levels.
- Definition of stope boundaries.
- Sequencing of stopes.
- Blending and transportation of ore.
- Scheduling of activities.
- Ventilation of workings.

However, according to Topal and Sens (2010), underground mine optimisation has been mainly focussed on the following three areas:

1. Optimisation of stope boundaries.
2. Optimisation of development and infrastructure placement.
3. Optimisation of production schedules.

These three are interrelated such that a change in stope boundaries will impact on the location of development infrastructure and on production scheduling. However, despite the interrelatedness of these three, no optimisation technique is available that integrates them. An initial attempt to integrate them was by Little (2012) who developed an algorithm that integrates stope boundaries and production scheduling but, development location was excluded. Except for the work by Little (2012) all available techniques for both open pit and underground have been focused on optimising these three areas separately. This is likely to result in incoherent and suboptimal mine designs and plans which do not consider the interaction and influence that mine design and planning elements have on each other.

The aim of development and infrastructure location optimisation is to minimise capital development and transportation costs over the LOM (Erdogan *et al*., 2016). Research done in this area includes the work of Brazil *et al*. (2002); Brazil *et al*. (2003); Brazil *et al*. (2005) and Brazil and Thomas (2007). On the other hand, optimisation of production schedules aims at sequencing stopes in a way that maximises NPV. Extensive research has been done to date in optimising production schedules including the work of Trout (1995); Topal (2003); McIssac (2005); Nehring (2011) and Little *et al*. (2013). However, as explained in Chapter 1 this thesis focussed on incorporating uncertainty on stope designs hence a detailed review of advances in optimisation of stope boundaries. Several stope boundary optimisation algorithms have been developed to date. These are discussed in the next section.

3.3 Algorithms in stope boundary optimisation

Nhleko *et al*. (2018 p. 2) defined a stope as "*a mining area in underground mining which consists of a number of mining blocks*". Figure 3.1 shows an example of a mining layout where underground stopes are labelled and indicated in green colour.

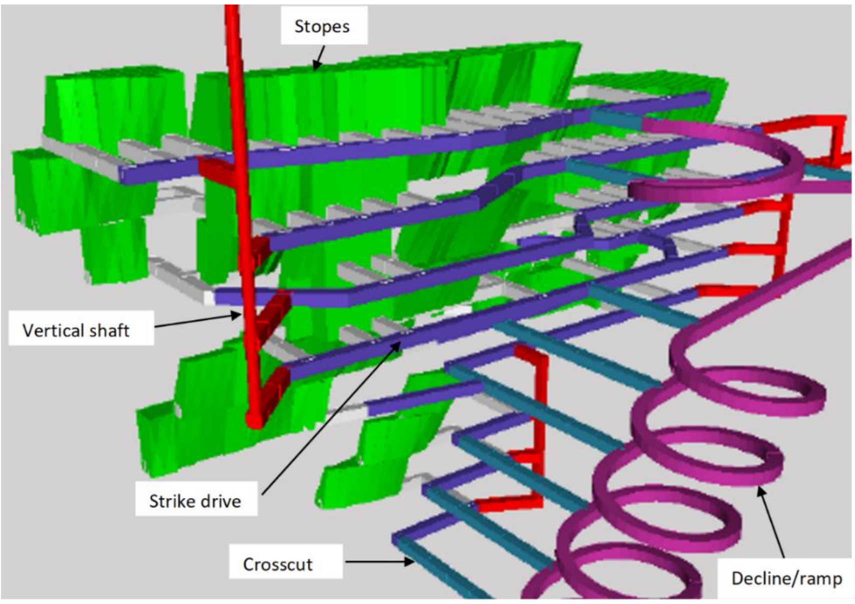

Figure 3.1: An example of a layout for an underground mine showing stopes

(Source: Adapted from Válio and Abilleira, 2017)

An optimum stope boundary is the limit for material in an orebody that can be mined economically using underground mining methods, equivalent to an ultimate pit limit in open pit mining (Nhleko *et al*., 2018). Defining an optimum stope boundary is important in optimising value from the extraction of Mineral Resources. It is an important component of strategic mine planning. According to Little (2012), the aim of optimising stope boundaries is to define the best combination of blocks to form a series of stopes based on value measures such as profit or grade while satisfying operating mining and geotechnical constraints. Satisfaction of physical constraints is to ensure practical and safe extraction of the orebody. Sandanayake *et al*. (2015) mentioned that the stope boundary optimisation process combines several blocks into a set of stopes, in a way that maximises economic value whilst satisfying any existing constraints.

Several algorithms have been developed to optimise stope boundaries. The algorithms are categorised as either exact/rigorous or heuristic (Ataee-pour, 2006). Exact algorithms are those algorithms that are based on a mathematical model hence, they guarantee an optimum solution. Examples of exact stope boundary optimisation algorithms include the Dynamic Programming (DP) algorithm, the Downstream Geostatistical approach, the Branch and Bound algorithm and Little (2012)'s algorithm (Nhleko *et al*., 2018). In contrast, heuristic algorithms are not based on any mathematical formulation, hence, they provide solutions which are not necessarily optimum but, solutions that are close to optimum solutions (Ataee-pour, 2006). A heuristic method is defined by Silver (2004 p. 936) as "*a method which, on the basis of experience or judgement, seems likely to yield a reasonable solution to a problem but which cannot be guaranteed to produce the mathematically optimal solution*". Examples of heuristic stope optimisation algorithms include the Floating Stope algorithm, the Octree Division algorithm, the Multiple Pass Floating Stope (MPFS) algorithm, Topal and Sens (2010)'s algorithm, the Maximum Value Neighbourhood (MVN) algorithm, the Network Flow algorithm and Sandanayake (2014)'s algorithm (Nhleko *et al*., 2018). These exact and heuristic stope boundary optimisation algorithms are reviewed in the next sub-sections in chronological order of development starting with the oldest.

3.3.1 Dynamic Programming algorithm

To define optimum stope boundaries for block caving mines, Riddle (1977) developed an algorithm based on the DP technique. Shahriar *et al*. (2007) mentioned that the algorithm is a modification of the DP algorithm for optimising open pit limits by Johnson and Sharp. There are two main differences between the optimisation process by the DP for underground mines and the one for open-pit mines as outlined by Riddle (1977). The first difference is that for open-pit mines, mining must be initiated from surface and the algorithm starts from block B_{11} in Figure 3.2 (a), ranging one block up or down, moving from column to column. On the contrary as pointed out by Riddle (1977), in a block cave mine a vertical boundary cut-off may be included such that mining may be initiated from any elevation within a fringe stope. Figure 3.2 (b) illustrates this.

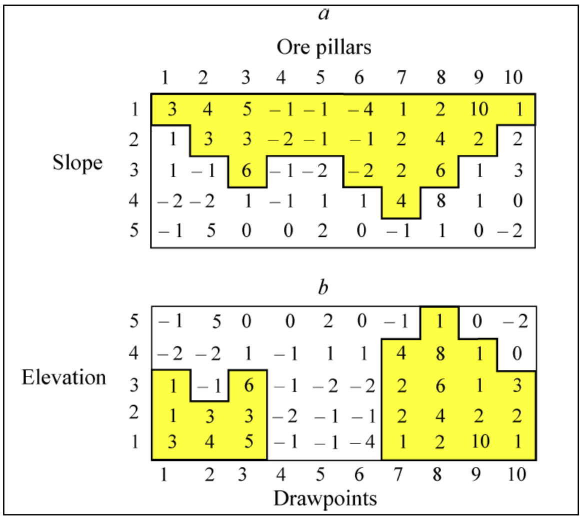

Figure 3.2: Optimum layouts for open pit and underground mines based on BEVs

(Source: Ataee-pour, 2005b)

The second difference is that in open-pit mines the relation of height mined is constrained by the pit slope angle (assumed to be 45 degrees) whilst it is constrained by draw control in the DP algorithm for underground mines. The other constraint considered by the algorithm is stope dimensions.

Riddle (1977) described the optimisation process by the algorithm. The first step of the algorithm is to assume that no footwall region exists in the mine layout. It then identifies a minimum number of draw-points that are adjacent to each other and should be mined in any discrete mining unit are then established (Riddle, 1977). The optimisation process continues by "*adding one footwall region within the section and then establishing a minimum number of adjacent draw-points that should be left in a discrete, non-mined footwall region. Each combination of mined and non-mined draw-points is investigated, and the profit of each combination is taken. If the profit obtained with a footwall case is greater or equal to the maximum profit obtained from the case without a footwall, then a more optimum layout exists* *with a footwall case*" (Nhleko *et al.*, 2018 p.2 citing Riddle, 1977). The process is then repeated to the two areas divided by a footwall (Riddle, 1977). For example, if the north block is more optimal with a footwall, the optimisation process is continued by adding the southern region to the previous optimal condition evaluating the north end for further footwall (Nhleko *et al.,* 2018 citing Riddle, 1977). The algorithm continues the process until no more optimum footwall cases can be found or a limit is reached such that it is no longer practical to add further footwalls (Riddle, 1977).

In a review of stope layout optimisation techniques Ataee-pour (2005b) highlighted the limitations of the algorithm. The first limitation is that the algorithm provides optimum stope layouts in two-dimensions (2D) but, fails to provide optimum stope layouts in 3D hence, it offers a 2D approximate solution that does not suit a 3D problem. Riddle (1977) thus, considered his algorithm to be a multi-section 2D approximation of a 3D problem. The other limitation is that the algorithm is only applicable to block caving mines and is not applicable to other mining methods. Additionally, this algorithm is applied on a fixed Mineral Reserve model and therefore, does not consider the uncertainty of geological and economic variables used in the Mineral Reserve calculation. All these limitations therefore mean that true optimality is not guaranteed by the algorithm.

3.3.2 Downstream Geostatistical approach

Deraisme *et al*. (1984) developed the Downstream Geostatistical approach which is based on downstream geo-statistics. Downstream geo-statistics was defined by Deraisme *et al*. (1984 p. 583) as a "*methodology for the study of the influence of mining constraints on mining recovery and ore quality*". The methodology is based on simulating a mining operation from *in-situ* ore to the plant, on an orebody model, varying economic and technical conditions to have several simulations on the orebody model. Therefore, this approach is somehow probabilistic since simulation is done to model grade variability which provides a numerical realisation of a random function, characterised by a variogram and histogram (Deraisme *et al*., 1984). However, since the approach only considers grade variability it can be argued that the algorithm is partially probabilistic because uncertainties of the other parameters are not modelled.

This algorithm was applied to 2D sectional numerical models at Ben Lomond Mine, an underground uranium mine in Australia to compare the feasibility of several mining methods according to their cost per tonne and selectivity (Deraisme *et al*., 1984). Geometrical constraints (stope dimension, hangingwall and footwall slope angle) are applied prior to economic optimisation. The algorithm has the following limitations noted by Ataee-pour (2005b). The first limitation is that it has been developed and implemented on cut-and-fill, and sub-level mining methods and the developers stated that further research was needed for its application to other mining methods. The other limitation is that the mine designer should manually complete the final design, which means the final design is subject to the designer's knowledge and skills. Ataee-pour (2005b) also argued that the algorithm is complex, has been implemented on a specific mine site and only produces optimum designs in 2D without guaranteeing optimal designs in 3D. Also, the algorithm does not model the uncertainty of all stochastic variables involved.

3.3.3 The Octree Division algorithm

Cheimanoff *et al*. (1989) developed the Octree Division algorithm, a rule-based algorithm that transforms Mineral Resources to Mineral Reserves while imposing underground mining constraints. Economic constraints and geometric constraints are the two main constraints applied by the algorithm (Cheimanoff *et al.*, 1989). Economic constraints applied are cut-off grade and mining cost while geometric constraints are those factors that define the dimensions of a practical working space based on the geotechnical characteristics of the mineral deposit. Ataee-pour (2005b) outlined the algorithm's three main steps below also schematically shown in Figure 3.3:

1. Gathering borehole data, analysis of geostatistical and geological data. This data is used to build a geometric model.
2. Translating Mineral Resources to Mineral Reserves.
3. Dividing the Mineable Reserves into sub-volumes for further economic evaluations.

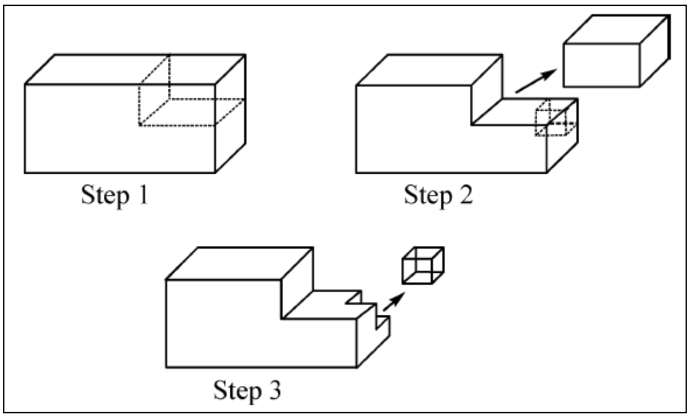

Figure 3.3: The sub-volume division process

(Source: Ataee-pour, 2005b)

Ataee-pour (2006) described the optimisation steps in detail indicating how the algorithm defines mineable blocks from the geometric model that is created in the steps above. The basic principle behind the process is to gather mineralised veins that are then used to form blocks and evaluate the economic values of each block. Ataee-pour (2006) mentioned that the process begins by identifying veins that are large enough to form mineable blocks on their own based on their grade and volume. Veins that are close to each other are combined into mineable blocks whilst veins too far apart are separated into different blocks. Small veins are evaluated individually to determine whether they have sufficient values and the evaluation continues until all veins are examined progressively surrounding the amalgamated vein with the initial large volume (Ataee-pour, 2006). The result of the vein amalgamation process is a mineable volume as shown in Figure 3.4.

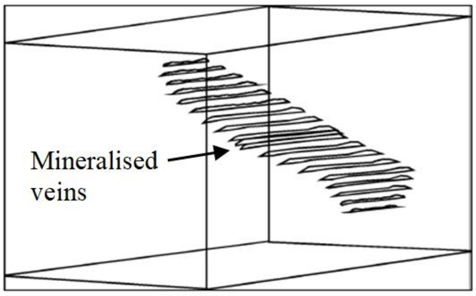

Figure 3.4: Vein amalgamation

(Source: Adapted from Ataee-pour, 2005b)

The Octree Division algorithm is based on the quad-tree division technique which creates a rectangular solid around an object, sub-dividing the solid into quarters, evaluating each quarter for further sub-division until further sub-division is no longer possible (Ataee-pour, 2005b). Applying this technique to a 3D mining problem, the volume shown in Figure 3.4 is divided into eight equal sub-volumes, then each of the eight sub-volumes is evaluated for mineralisation by sub-dividing it into eight other sub-volumes. The algorithm continues by limiting the evaluation into smaller and smaller sub-volumes and stops when there are no more sub-volumes to examine. This sub-division process is illustrated in Figure 3.5.

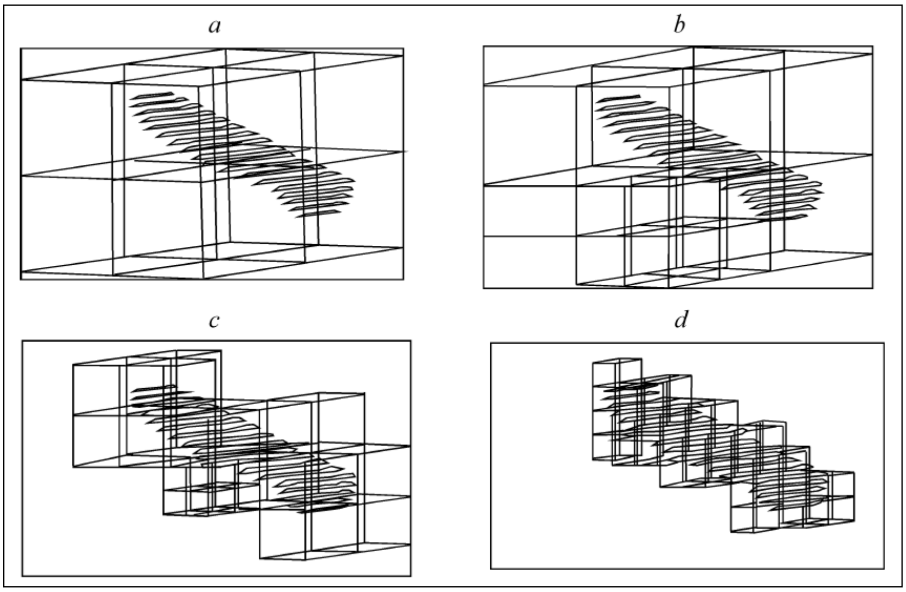

Figure 3.5: Sub-volume evaluation process

(Source: Ataee-pour, 2005b)

Each sub-volume is evaluated based on the following three rules (Ataee-pour, 2005b):

1. If a sub-volume contains no part of mineralised veins and satisfies the constraints, it is removed from the list of potential mineable blocks.
2. A sub-volume that lies completely within mineralised veins or meets the minimum allowable stope dimensions is stored in the list of future mineable blocks.
3. A sub-volume that is partially located inside and partially outside mineralised veins and does not satisfy the minimum allowable stope dimensions is further divided into new sub-sub-volumes.

The Octree Division algorithm is contained within the GEOCAD software package although Little (2012) mentioned that the package is yet to be commercialised. The advantage of the algorithm is that it provides a 3D solution to stope boundary problems but, has limitations. Ataee-pour (2006) mentioned that because the algorithm does not evaluate sub-volumes jointly more waste may be included into the final stope layout. Also, the geometric and economic constraints considered are deterministic, only constantly updated with time. Therefore, the algorithm does not guarantee optimal stope layouts.

3.3.4 The Floating Stope algorithm

The Floating Stope (FS) heuristic algorithm was developed by Alford (1995) to define optimal stope boundaries. The algorithm is analogous to the Moving Cone algorithm for defining optimal pit limits. The FS algorithm is applied on a fixed geological block model as well as an economic block model of the orebody using a calculated cut-off grade. The cut-off grade is applied to distinguish waste and ore blocks and the optimisation criteria includes maximisation of grade, contained metal or accumulated value or minimising waste. Alford (1995) mentioned that a stope shape of a minimum stope dimension is floated around the block model to locate the stope position of the highest grade, hence the name 'floating stope'. Two envelopes are defined during the floating process; the 'inner' and the 'outer' envelope as shown in Figure 3.6.

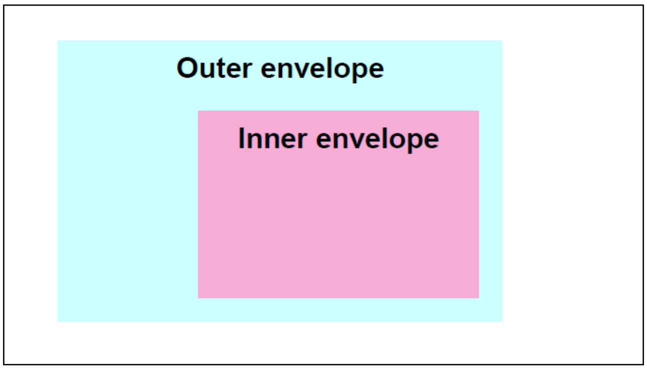

Figure 3.6: The inner and outer mining envelopes

(Source: Little, 2012)

Little (2012 p. 18) mentioned that "*the inner envelope contains all blocks above the cut-off that can be mined and is the union of all the best grade stope shapes. The outer envelope is the union of all possible stope positions for each block above the cut-off that can be mined*". The final stope design should then be within the 'outer' envelope and as close to the 'inner' envelope as practical as possible (Alford, 1995). The output of the algorithm is a block model that defines the volume of the envelope and the grade values of the best stope (Alford, 1995).

The algorithm has shortcomings. It generates overlapping stopes with shared blocks such that two overlapping stopes may individually be economic, but their union may be uneconomic if they share a high-grade block (Alford, 1995). Figure 3.7 shows a 2D illustration of this overlapping problem.

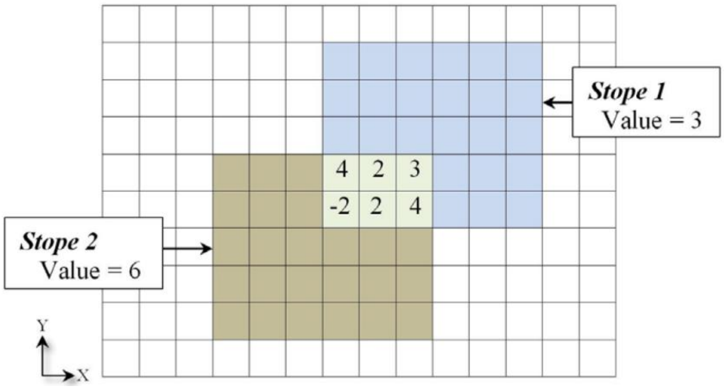

Figure 3.7: An illustration of overlapping stopes

(Source: Little, 2012)

In the figure, two stopes (Stope 1 and Stope 2) measuring 6 x 5 blocks, that is, six blocks along the x-axis and five blocks along the y-axis, overlap and share six mining blocks with BEVs shown on each overlapping block. Stopes 1 and 2 are assumed to have total economic values of 3 and 6, respectively. Therefore:

Total economic value of the two stopes calculated by the algorithm = 6 + 3 = 9

However, the actual total economic value of the two stopes = (6+3) – (4 + 2 + 3 – 2 + 2 + 4) = –4

The example above shows that the total economic value of the combined stopes is different from the actual economic values of Stopes 1 and 2. Therefore, the algorithm does not guarantee an optimal solution. Manual adjustment by the mine designer is required to produce practical stope designs. The use of a fixed geological or economic block model by the algorithm is another limitation which means that the resulting economic block model does not consider the uncertainty of economic and geological parameters that are used in the economic block modelling. The Floating Stope algorithm is commercially available in Datamine's Mineable Reserve Optimizer software package (Datamine Corporate Limited, 2017). Improvements to the algorithm were made resulting in the MPFS algorithm and the Vulcan stope optimizer that are embedded in Datamine and Maptek mine design and planning software packages, respectively (Musingwini, 2016).

3.3.5 Maximum Value Neighbourhood algorithm

The MVN algorithm was developed by Ataee-pour (2000). The algorithm is based on a heuristic approach and uses a regular 3D fixed economic block model of an orebody. The algorithm is based on the 'neighbourhood concept' which is based on searching for an optimum neighbourhood for any given single block against certain geo-technical and mining constraints. A neighbourhood is a sequence of blocks that should be mined to satisfy the minimum stope dimensions for any given block (Ataee-pour, 2000). An optimum neighbourhood, that is, the one that qualifies to be included in the final stope is one that has the maximum net economic value. The input data of the algorithm are the BEVs (Ataee-pour, 2000). Considering any given block B_{ij} with its BEV and for any minimum stope dimensions, possible neighbourhoods to block B_{ij} are determined to have a set of neighbourhoods. The economic value of each neighbourhood is calculated and from the possible sets of B_{ij} neighbourhoods, the algorithm locates the neighbourhood with the maximum economic value. The neighbourhood then forms a stope to be included in the final stope layout. Blocks in that stope are tagged, and the next block is selected for the same process. In this way the algorithm systematically moves throughout the block model identifying stopes to be included in the final stope layout. The process of selecting the neighbourhood with maximum value is demonstrated in Figure 3.8.

Given a minimum stope length of 10 m and a block length of 3 m, a neighbourhood has a minimum of four blocks as shown in Figure 3.8. Considering block B_j in the figure there are four possible neighbourhoods to block B_j, that is, NB_1, NB_2, NB_3 and NB_4 with different neighbourhood values of 6, 2, 4 and 7, respectively. NB_4 is the maximum value neighbourhood of B_j with an economic value of 7 which therefore means NB_4 is the optimum neighbourhood to block B_j. The next block, for example, block B_{j+1} is selected and the same process is repeated until all blocks in the block model are analysed.

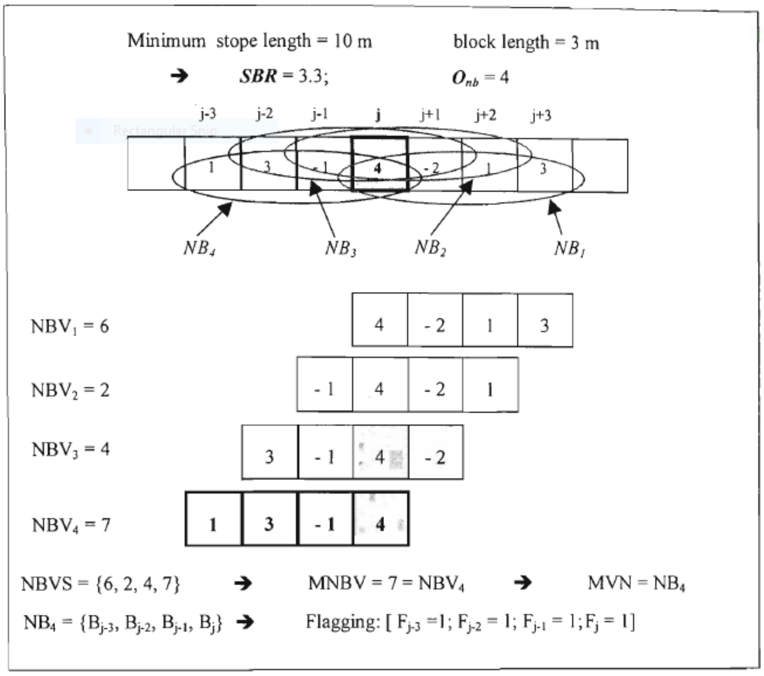

Figure 3.8: Locating the Maximum Value Neighbourhood based on BEVs

(Source: Ataee-pour, 2000)

The MVN algorithm has shortcomings. Erdogan *et al*. (2016) mentioned that if the starting location is changed the set of stope layouts generated from the same orebody also changes. Also, the algorithm only considers the costs of individual blocks instead of stope costing factors based on the size of the stopes (Erdogan *et al*., 2016). Additionally, like all algorithms reviewed the MVN algorithm is based on a deterministic economic block model. A deterministic economic block does not consider uncertainty of geological and economic data used in the BEV calculation. Therefore, it is likely that the algorithm does not always result in optimum stope boundaries.

3.3.6 Multiple Pass Floating Stope algorithm

The MPFS algorithm was developed by Cawrse in 2001 as an improvement of the Floating Stope algorithm (Sens, 2011). Like the Floating Stope algorithm, the optimisation process with the MPFS algorithm starts with the user defining parameters such as cut-off grade. Different sets of these parameters are used to create different economic stope envelopes, from which the algorithm identifies and selects all possible areas for stope designs. Sens (2011) mentioned that the MPFS algorithm is based on the same principles as the Floating Stope algorithm, the difference being that the MPFS algorithm delineates economic ore zones using multiple sets of input parameters such as head grade and cut-off grade. Iterations are run using different sets of input values such as head grade and cut-off grade to create stope envelopes (Sens, 2011).

Even though the MPFS algorithm is an improvement of the Floating Stope algorithm, Sandanayake (2014) mentioned that the MPFS algorithm does not eliminate all the shortcomings of the Floating Stope algorithm. Therefore, the algorithm does not guarantee optimum stope layouts. The other limitation is that it is applied on a deterministic block model of the orebody which results in the same consequences as the algorithms that were previously reviewed in terms of not incorporating uncertainty in the optimisation process. Even though the algorithm attempts to run a multi-optimisation process it still gives deterministic stope envelopes.

3.3.7 Topal and Sens' algorithm for stope boundary optimisation

Topal and Sens (2010) developed a heuristic-based algorithm for stope boundary optimisation. The algorithm starts by converting an irregular geological block model into a regular block geological model and recalculating block grade values. A range of all possible stope sizes, price per tonne, mining and processing costs per tonne are then applied on the new block model to determine the optimum stope boundary and layout for a given orebody (Topal and Sens, 2010). Starting with the smallest available stope size, the algorithm creates stopes and all stopes with positive profit values are noted in the stope list. This process is repeated for all the available stope sizes. A list of all possible stopes and their profit values is created (Topal and Sens, 2010). Stope envelopes are created around each stope and the profit and number of stopes in each envelope is calculated. The stope with the highest profit value is first selected, and its location, profit and size are noted in the final stope layout table and the algorithm then deletes from the stope table any overlapping blocks in different stopes (Topal and Sens, 2010). The next highest profit stope is selected for analysis and the process is repeated until all stopes are analysed, deleting from the table any overlapping stopes. The algorithm then defines the optimum stope boundary by selecting non-overlapping stopes based on their average envelope stope profit or on the highest stope profit per time unit (stope profit/total production time) (Topal and Sens, 2010).

The advantage of this algorithm is that it can select the best stope size from a given range of stope sizes to design a more profitable stope layout. Topal and Sens (2010) also mentioned that the major advantage of this algorithm is that it can find true optimum stope layouts in a 3D block model although it is not always able to find true optimal stope layouts. However, like all algorithms reviewed previously, this algorithm is also based on a deterministic block model.

3.3.8 Mineable Shape Optimizer

The Mineable Shape Optimizer (MSO) was developed by Alford Mining Systems (2011) as a tool to optimise stope designs. The algorithm defines the optimal size, shape and location of stopes for a range of stoping methods for underground mines. The MSO algorithm uses a block model with grade or BEVs as an input to the optimisation process (Datamine Corporate Limited, 2009). The optimisation process starts by creating strings on sections of the block model using user-defined geometrical parameters of the orebody such as dip and strike. The string slices are then linked to create wireframe 'seed' shapes that are evaluated against the block model. The final step involves annealing the seed shapes to create optimal stope shapes. The algorithm considers constraints such as minimum and maximum mining width, excepted mining wall dilution, minimum and maximum mining wall angles, separation distances between parallel stopes, and minimum and maximum stope dimensions (Datamine Corporate Limited, 2009).

The optimisation process generates three key outputs (Datamine Corporate Limited, 2009). The first output is the section strings used to create optimal stope shapes. The second output consists of wireframes for each optimised stope and the third output is a report with various fields for each stope that include volume, tonnage, optimisation field and other selected reporting fields. Figure 3.9 shows a combined snapshot of the three outputs.

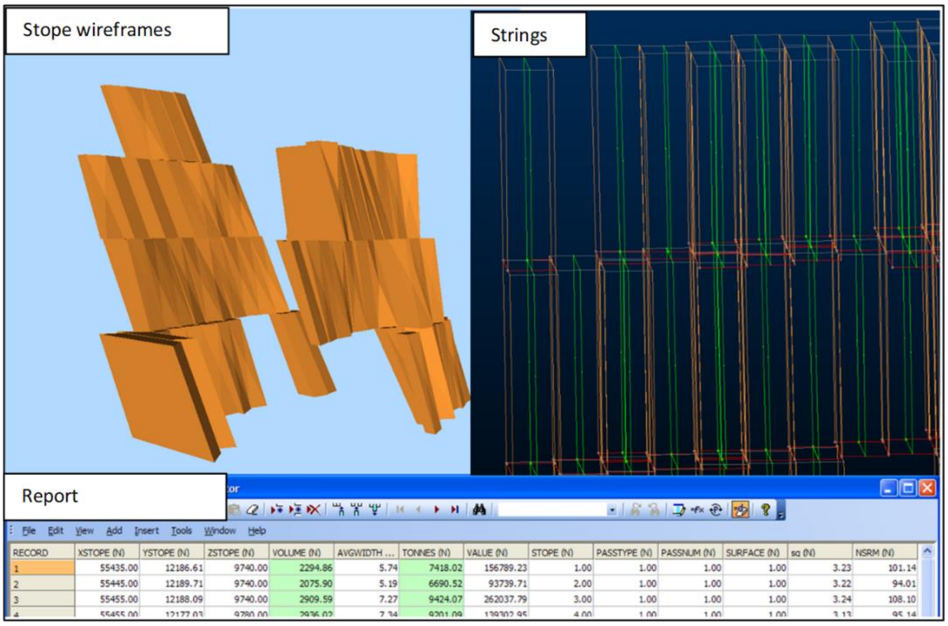

Figure 3.9: Typical output files for the Mineable Shape Optimizer

(Source: Adapted from Datamine Corporate Limited, 2009)

The MSO algorithm has several advantages including (Alford Mining Systems, 2011):

1. It can analyse different mining methods or stoping parameters quickly to enable development of a plan based on different mining scenarios.
2. It can specify irregular stope frameworks.
3. It can process thousands of iterations, considering applicable constraints to create the optimal stope shape.
4. In addition to its ability to identify the stope shapes to include in the stope boundary, the algorithm can also identify the stope shapes to exclude.

The MSO algorithm is commercially available in Datamine, Maptek and Deswik software packages. Since it is commercially available and has the advantages above, the MSO algorithm in Datamine software was selected and used in this thesis to create optimal stope outlines from the simulated economic block models.

3.3.9 A simultaneous stope layout and production scheduling optimisation algorithm

After recognising that most previous underground optimisation efforts were focussed on optimising stope boundaries and production schedules separately, Little (2012) developed a mathematical model that integrates the two processes for sub-level stoping mining operations. The algorithm is based on integer programming with the objective of maximising NPV. Little (2012) described the optimisation process of the algorithm as below.

The algorithm assigns a binary variable to each stope rather than to each block as suggested by other algorithms including the MVN algorithm. This helps to reduce the number of variables in the model thereby reducing solution time (Little, 2012). The algorithm starts by converting an irregular block model to a regular block model for easy of data processing. The Data Manager within the algorithm then creates stopes at every block increment in the x, y and z directions based on the size defined by the user. The stope creation process starts by identifying the first block location number in the block model. The stope’s start block is created after which the stope’s finish block is created. The process is repeated until the final block in the block model is reached (Little, 2012). After the stopes are created, each stope’s geological (weighted average grade, the extraction tonnage and the backfill volume), economic (cost and price) and technical (recovery factor) are applied to calculate the profitability of each stope. Only profitable stopes are included for consideration by the integer programming model. Information on constraints is then added to each profitable stope one at a time until all profitable stopes are finished. Scheduling parameters are also added during the optimisation process resulting in an integrated solution.

The algorithm's main advantage is the integration of stope layouts and production scheduling optimisation to create a robust optimisation model. Integrated mine planning is important because stope layouts and production scheduling are closely interrelated therefore, an integrated technique provides a more robust solution compared to fragmented techniques. However, Little (2012) stated that the model has only been applied to sub-level stoping mining operations which means it may not be applicable to other mining methods. Also, the algorithm is based on deterministically created economic block models. Therefore, true optimality is not guaranteed because BEVs are not deterministic but stochastic in reality.

3.3.10 Network Flow algorithm

The Network Flow algorithm was developed by Bai (2013) to optimise stope designs for the sub-level mining method. The algorithm is based on the network flow concept that has been applied in open pit optimisation. Its application in open pit optimisation is that mining is initiated from surface. In underground mining optimisation, mining is initiated from a vertical raise. The hangingwall slope angle, footwall slope angle and minimum stope width are the main constraints required for the optimisation process. The optimisation process consists of two main parts. The steps for the first part are described by Bai (2013) as:

1. The algorithm starts by defining a raise position from where mining is initiated.
2. Cylindrical coordinates (Figure 3.10 a) from the raise centerline are defined (expressed as r, θ, z) relative to the raise centerline where r is the radial distance of a block from the raise centerline, θ is the azimuth and z is the height of the block.
3. A graph based on the vertical arcs satisfying the constraints is constructed as shown in Figure 3.10 (b) with all the arcs receiving infinite capacities.
4. The maximum flow problem is solved and the sum of the residual capacities on the arcs defines the stope value which is regarded as conditionally optimal to the raise location and height.

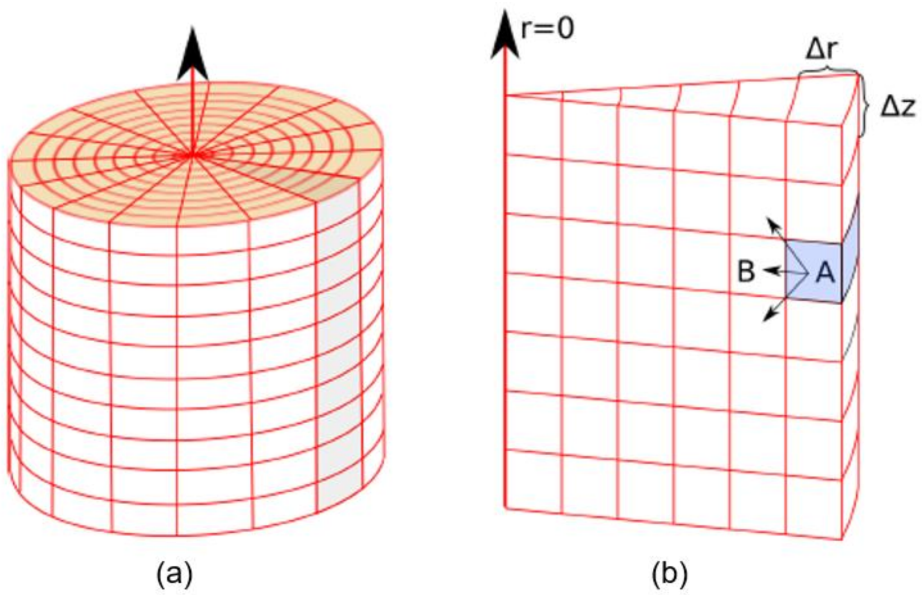

Figure 3.10: (a) Block model in cylindrical coordinate system. (b) Typical arcs based on constraints.

(Source: Bai, 2013)

The second part of the optimisation process searches for the best raise location and height through a global optimisation process of the stope profit which defines the overall optimal stope (Bai, 2013). One advantage of the algorithm is that it can result in optimal stope designs in 3D. However, as stated by Bai *et al*. (2013), its two main disadvantages are that it relies on the use of a vertical raise as a starting point which then limits its application to deposits mined by the sub-level mining method. They also mentioned that the algorithm is most suited to relatively small orebodies. A variation of the algorithm was developed by Bai *et al*. (2013) that considers multiple raises. The modified algorithm was found to result in better stope profits and less dilution as compared to the one for a single raise.

3.3.11 Sandanayake's heuristic algorithm

The algorithm was developed by Sandanayake (2014). The optimisation process starts by converting an irregular geological block model into a regularly sized geological block model. The geological block model is then converted into an economic block model (Sandanayake, 2014). The next step is to define the dimensions of the stopes in terms of number of blocks in the x, y and z directions. All possible stopes are then identified by floating the defined stope size in the economic block model along the x, y and z directions (Sandanayake, 2014). Figure 3.11 illustrates the floating process.

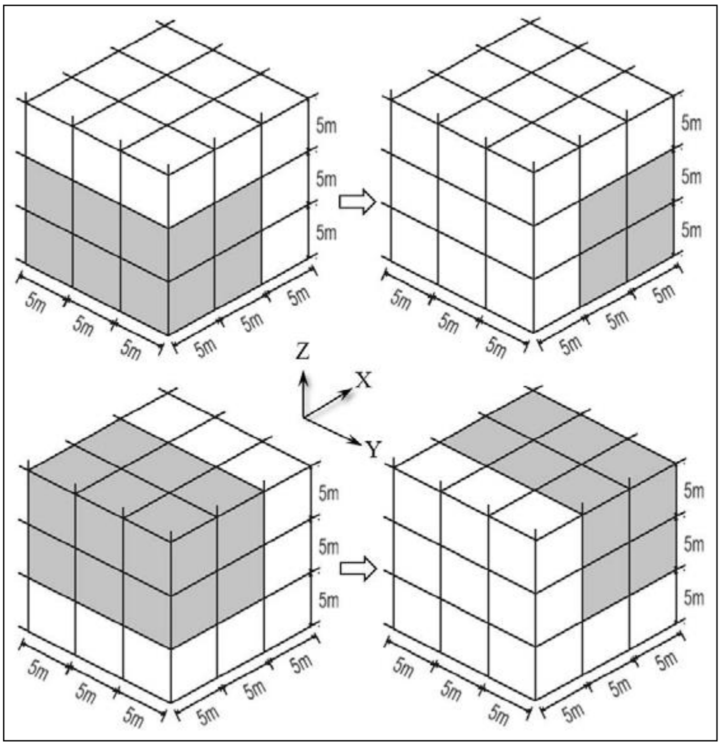

Figure 3.11: Stope generation process

(Source: Sandanayake, 2014)

Figure 3.11 shows that after floating the stope dimension for the given blocks, four stope position scenarios are possible. The stope economic values are then calculated and only stopes with positive stope values are selected (Sandanayake, 2014). Figure 3.12 provides a 2D illustration of the stope creation process. The BEVs for each block are written on each of the blocks.

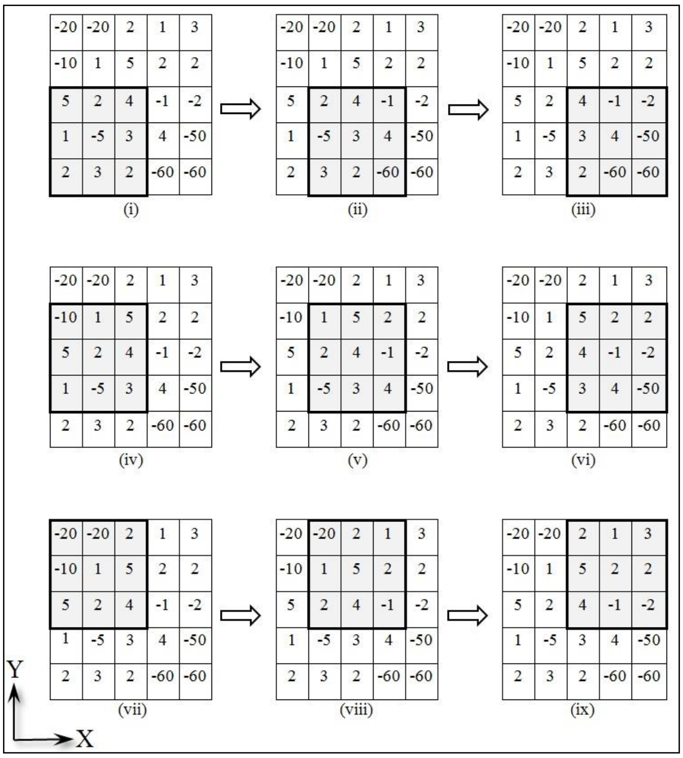

Figure 3.12: Generation of possible stopes in 2D based on BEVs

(Source: Sandanayake, 2014)

Figure 3.12 shows that after floating the stope dimensions of 3 x 3 blocks, nine stope scenarios are possible but, only stopes (i), (iv), (v) and (ix) have positive stope values of 17, 6, 15 and 16, respectively. From these scenarios, overlapping stopes are discarded resulting in a set of non-overlapping stope sets from which the set with the highest economic value is selected as the optimum solution (Sandanayake, 2014). One of the limitations of the algorithm is that it uses deterministic BEVs to create the economic block model (Nhleko *et al.*, 2018). This means that there is a possibility that some of the discarded stopes, that is, those with negative BEVs can in reality have positive BEVs if the uncertainty of the geological and economic parameters used in the BEV calculation is incorporated (Nhleko *et al.*, 2018). Therefore, the algorithm is also unlikely to generate reliable optimum stope boundaries.

3.3.12 Other stope boundary optimisation algorithms

The Stopesizor algorithm was developed by Snowden Consultants for their internal company use. The algorithm applies a range of cut-off grades to create a series of mining envelopes in 3D (Little, 2012). However, the limitation of the algorithm is that the final design is guided by the intervention of the design engineer which raises the question of true optimality of stope designs. Alford and Hall (2009) also developed an optimisation method that uses a range of cut-off grades to generate a series of nested stopes (equivalent to nested pits). However, both these algorithms are also based on deterministic orebody models which means that they are likely to give unreliable stope boundaries.

3.4 A summary comparison of the algorithms identified in the literature review

The previous sections show that the algorithms that have been developed to date vary in their application. The exact and heuristic algorithms are summarised in Table 3.1.

Table 3.1: A summary comparison of the stope optimisation algorithms

(Source: Adapted from Nhleko *et al*., 2018)

Algorithm	Classification	Mining method	Dimensional space	Optimisation criteria	Constraints
Dynamic Programming	Exact	Block caving	2D	Economic value	Stope dimension; draw control
Downstream Geostatistical	Exact	Cut-and-fill; sub-level stoping	2D	Geometry	Stope dimension; hangingwall and footwall slope angle
Branch and Bound	Exact	All	1D	Economic value	Stope dimension (stope length)
Little (2012)	Exact	Sub-level stoping	3D	Economic value	Metal content, production schedule related constraints
Octree Division	Heuristic	All	3D	Grade/vein	Stope dimension, cut-off grade and mining cost
Floating Stope	Heuristic	All	3D	Economic value	Stope dimension
Multiple Pass Floating stope	Heuristic	All	3D	Economic value	Stope dimension
Maximum Value Neighbourhood (MVN)	Heuristic	All	3D	Economic value	Stope dimension
Topal and Sens (2010)	Heuristic		3D	Economic value	Stope dimension
Network Flow	Heuristic	Sub-level stoping	3D	Grade/Economic value	Stope dimension, footwall angle and hanging wall angle
Mineable Shape Optimizer	Heuristic	All	3D	Grade/economic value	Mining width, dilution, hangingwall and footwall slope angle, stope dimensions
Sandanayake (2014)	Heuristic	Sub-level stoping	3D	Economic value	Stope dimension, pillar separation and level location

Table 3.1 shows that most of the underground stope boundary algorithms that have been developed to date are heuristic and unable to provide optimum stope boundaries in 3D. The table also shows that most of the algorithms' optimisation objective is to maximise economic value.

3.5 Summary of Chapter 3

OR formed the foundation of optimisation in mine design and planning. Given the finiteness of Mineral Resources and other operational and economic constraints affecting mining operations, optimisation is not optional if mining companies are to create optimum value from the extraction of Mineral Resources. Over the past three decades, optimisation of underground mines has been broadly focussed on three main areas. These are stope boundary definition, location of development and infrastructure and production scheduling; however, the focus of this thesis was on stope boundary definition.

At the core of OR is the formulation of a model which is solved by an algorithm. Several algorithms have been developed for the optimisation of stope boundaries which are categorised as rigorous and heuristic algorithms. Rigorous algorithms include the DP algorithm, the Downstream Geostatistical approach, the Branch and Bound algorithm and Little (2012)'s algorithm. On the other hand, heuristic algorithms include the Octree Division algorithm, the Floating Stope algorithm, the MVN algorithm, the MPFS algorithm, Topal and Sens (2010)'s algorithm, Network Flow algorithm and Sandanayake (2014)'s algorithm. However, these algorithms are unlikely to provide reliable and optimal stope boundaries. Their common limitation, except the Downstream Geostatistical approach which simulates grade variability is that they are based on deterministically created economic block models yet, input parameters used to create economic block model are uncertain. Algorithms that are based on deterministic economic block models are likely to result in sub-optimum stope boundaries because they do not consider uncertainty associated with quantifying orebodies (Nhleko *et el.*, 2018). This means that these algorithms provide a static solution to a dynamic problem. This has several limitations that were discussed in previous chapters. The MSO algorithm was the one selected for this thesis because of its commercial availability in Datamine software. The proposed probabilistic BEV model is presented in the next chapter.

4 A PROPOSED PROBABILISTIC BEV CALCULATION APPROACH

4.1 Overview of Chapter 4

This chapter presents a new probabilistic BEV calculation approach. The approach is summarised in Figure 4.1.

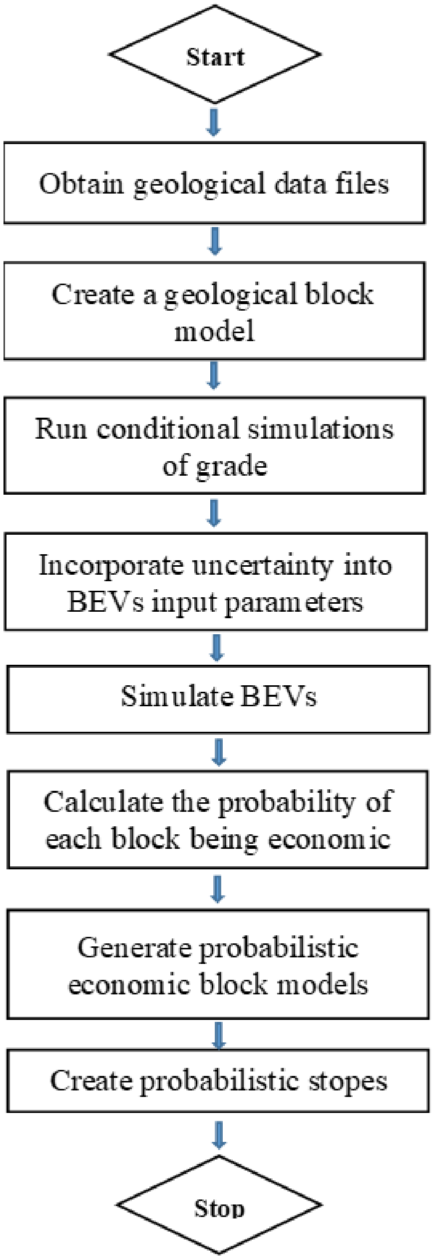

Figure 4.1: Process steps of the proposed BEV calculation method

The approach started by obtaining geological data files consisting of drillhole collar, lithology, assay, and survey data. A database was then created using the files from which a geological block model was created in Surpac software. To model uncertainty associated with gold grade, Sequential Gaussian Simulation (SGS) embedded in Surpac software was used to run conditional simulations. The grade values for the geological block model were then analysed in @Risk software to determine an appropriate probability distribution for gold grade. This was done to further simulate the gold grade (simultaneously with other BEV input parameters) using Monte Carlo simulation in Microsoft Excel. In addition to grade, the other input parameters used to calculate BEV are block dimensions, rock density, processing recovery rate, commodity price, mining cost and processing cost. Mining and processing cost were assumed to be constant, and uncertainty was modelled for grade, density, recovery rate and gold price. The chapter describes how uncertainty for the selected input parameters was modelled. Finally, the chapter presents the probabilistic BEV calculation model.

4.2 Block economic value formula

As shown previously in Equation 1.2, BEV is a function of block tonnage, grade, recovery rate, commodity price, mining cost and processing cost. Only density, gold grade, recovery rate and gold price were modelled for uncertainty. Mining cost and processing cost were assumed to be constant. A description of how uncertainty for each of the selected variables were modelled is provided in the next subsections.

4.2.1 Modelling uncertainty of block tonnage

The tonnage (T_{ij}) of a block (B_{ij}) is a product of the volume of the block and the density (D_{ij}) of the rock. The volume of a block is a product of the dimensions of the block. Therefore, T_{ij} is calculated using Equation 4.1:

$$T_{ij} = XINC_{ij} * YINC_{ij} * ZINC_{ij} * D_{ij} \qquad 4.1$$

where; XINC, YINC and ZINC are the dimensions of B_{ij} in the X, Y and Z directions, respectively. The density of a rock is a critical factor in estimating Mineral Resources and Mineral Reserves. It has a direct effect on the estimated quantities. These in-turn affect subsequent mine design and planning decisions and outcomes which include:

- Ore and waste tonnage estimates.
- Metal content estimates and consequently revenue estimates.
- Production schedules.
- Equipment selection.
- Plant design.

Whilst block dimensions XINC, YINC and ZINC can be estimated with certainty however, the density of rock varies. SRK Consulting (2015) mentioned that while great effort is put into accurate estimation of grade, the accurate estimation of density for Mineral Resource and Mineral Reserve estimation is often overlooked. This means that the uncertainty, hence the variability of density is often ignored during Mineral Resource and Mineral Reserve estimation despite its significant influence on the estimation. Often, an average density value is assumed for the whole block model (SRK Consulting, 2015; Abzalov, 2013; Reis *et al.*, 2021). This is because there is usually insufficient density data such that density values are assigned to blocks based on the position of blocks relative to geological contacts (for example weathering profiles) (Coombes, 2008).

An average estimate of density can be appropriate for deposits with low mineral content and simple mineralogy but can result in significant errors in tonnage and metal content estimates in deposits with more complex mineralogy (SRK Consulting, 2015). Realistically, density is a spatial variable whose uncertainty should be treated with equal importance as grade variability. Even though density is directly influenced by the lithology of each block it was not possible to get actual density data for the case study deposit since it is not usually included when analysing drillhole samples. Therefore, in this thesis density was arbitrarily assumed to follow a normal distribution with a probability density function given by Equation 4.2 (Beichelt, 2016):

$$f(D) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{D-\mu}{\sigma}\right)^2} \qquad 4.2$$

where D is density of rock, μ is the mean and σ is the standard deviation of the distribution. For this thesis μ and σ were assumed to be 3.0 t/m^3 and 0.05 t/m^3, respectively. Figure 4.2 shows the results for rock density simulations done in Microsoft Excel.

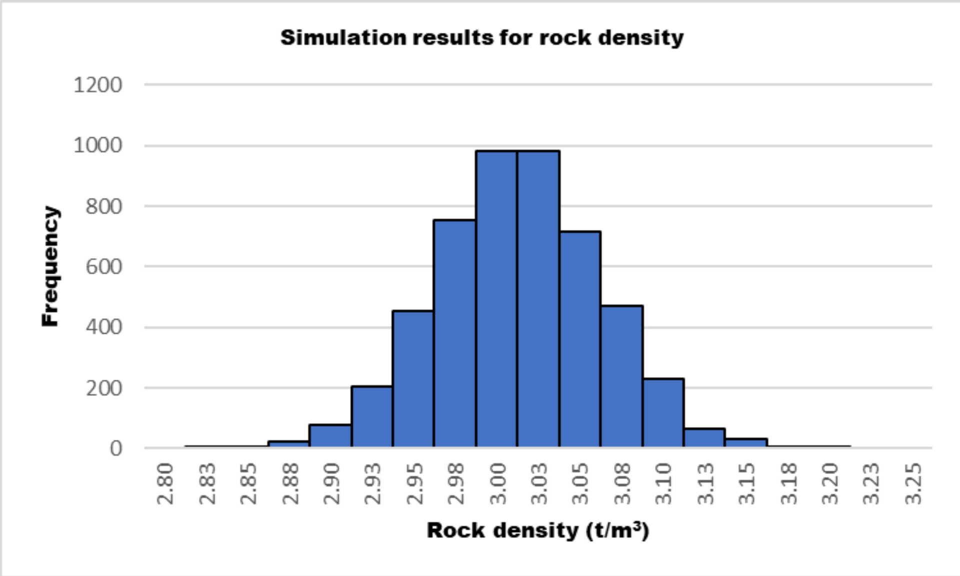

Figure 4.2: Simulation results for rock density

Since density was modelled as variable, the resulting tonnage estimates also varied reflecting the true behaviour of block tonnage. Figure 4.3 shows the resulting probability distribution for block tonnage.

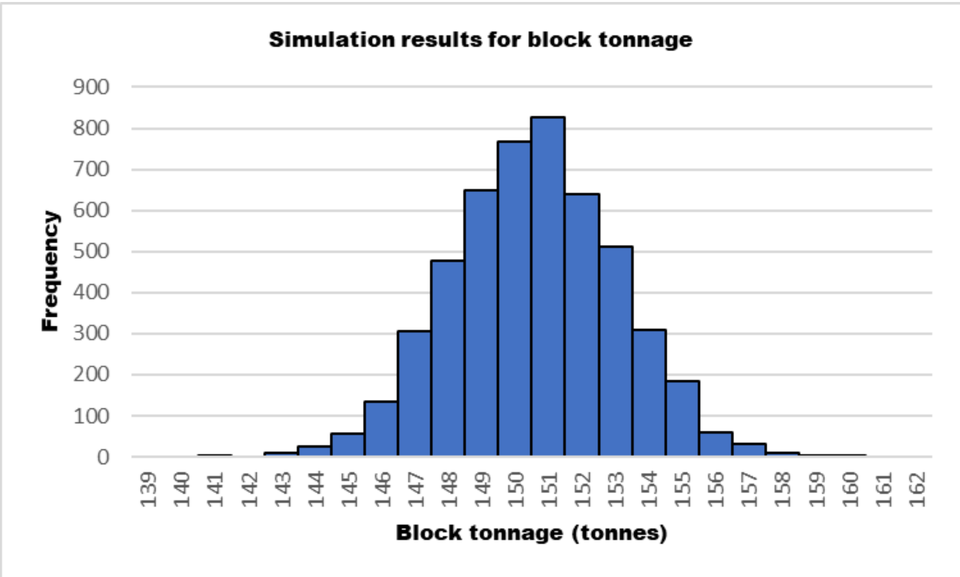

Figure 4.3: Simulation results for block tonnage

Figure 4.3 shows that like density, block tonnage also follows a normal probability distribution. The possible block tonnage ranges from 141 t to 160 t.

4.2.2 Modelling uncertainty of recovery rate

The recovery of gold during processing depends on the mineralogical and metallurgical characteristics of the gold ore being processed. According to Zhou *et al.* (2004), metallurgically, gold ores are classified into two main categories; free-milling also called non-refractory and refractory ores. Refractory literally means difficult to treat (Marsden and House, 2006). Unlike non-refractory ores, refractory ores have a low response to standard metallurgical processing methods therefore they require additional physical or chemical pre-treatment to maximise gold recovery (Aylmore and Jaffer, 2012). The recovery rate thereof is largely influenced by the mineralogical factors of the ore that include (Zhou *et al.*, 2004):

- The gold particles size.
- The association of the gold particles with other minerals such as silver which may hamper recovery of gold by flotation.
- The association of the gold particles with coatings and rimmings such as iron oxides or hydroxides formed by chemical reactions.
- The presence of cyanicides, oxygen consumers and preg-robbers, refractory gold minerals that negatively affect the recovery rate.
- Locking of sub-microscopic gold in sulphide mineral structure.

Marsden and House (2006) mentioned that all gold ores exhibit some refractory properties and that no treatment process will achieve 100% recovery. Since each ore has a unique mineralogical composition, an individual degree of refractoriness must be considered for optimum gold recovery. This uniqueness therefore means that the recovery rate cannot be estimated with certainty; it varies within the orebody depending on the mineralogical composition of each mining geo-zone. Despite its randomness, often in practice a single recovery rate is assumed for the whole orebody during mine design and planning processes.

To identify the probability distribution to use for processing recovery rate in this study, monthly historical recovery rate data for two years was collected for the anonymous mine mentioned earlier in Section 1.5. The data was analysed in @Risk software to identify an appropriate probability distribution of the recovery rate. Figure 4.4 shows the probability distribution results.

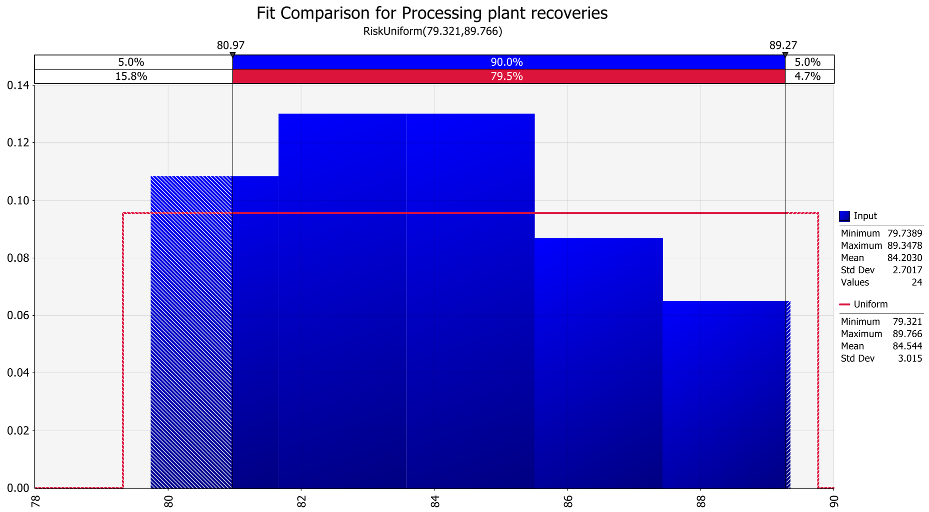

Figure 4.4: Probability distribution for processing recovery rate

Figure 4.4 shows that the processing recovery rate data analysed follows a uniform probability distribution. Therefore, a uniform distribution was used to model recovery rate in this thesis with a probability density function given by Equation 4.3 (Beichelt, 2016):

$$f(x) = \begin{cases} \frac{1}{\beta-\alpha}, & \alpha \leq x \leq \beta \\ 0, & otherwise \end{cases} \qquad 4.3$$

where *x* is recovery rate, α and β are the minimum and maximum possible recovery rates. Based on the data, α and β were found to be approximately 80% and 89%, respectively as shown on Figure 4.4. The assumption is that the recovery rate is randomly and evenly spread across the range interval (80%, 89%). Figure 4.5 shows the simulation of recovery rate results done in Microsoft Excel.

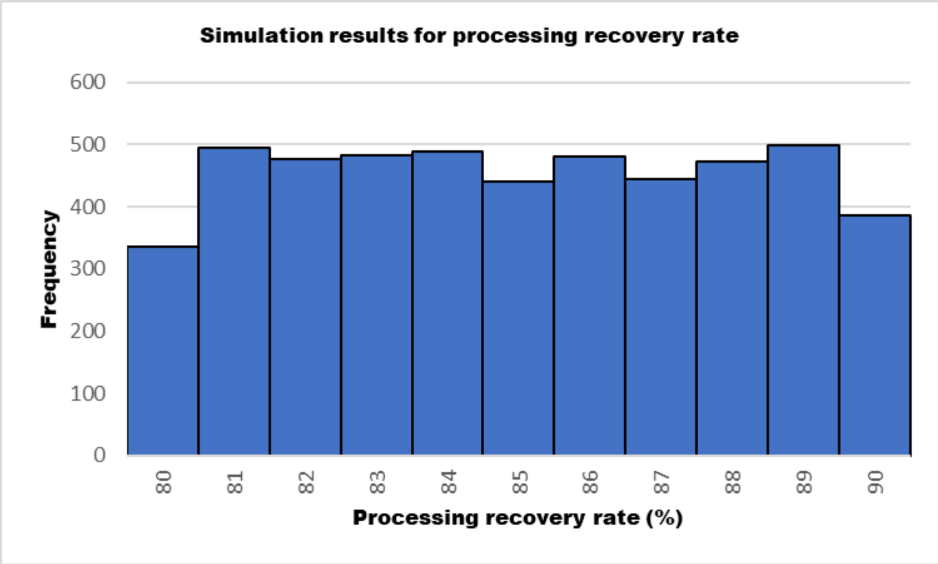

Figure 4.5: Simulation results for processing recovery rate

4.2.3 Modelling uncertainty of gold grade

The SGS technique embedded in Surpac software was used to run conditional simulations of gold grade as explained earlier in Sub-section 2.5.2. Twenty realisations were run resulting with twenty equally probable realisations of the orebody. Figure 4.6 shows five of the twenty equally probable realisations of the orebody. For illustration purpose only realisations 1, 5, 10, 15 and 20 are shown in Figure 4.6, which shows the different gold grade ranges.

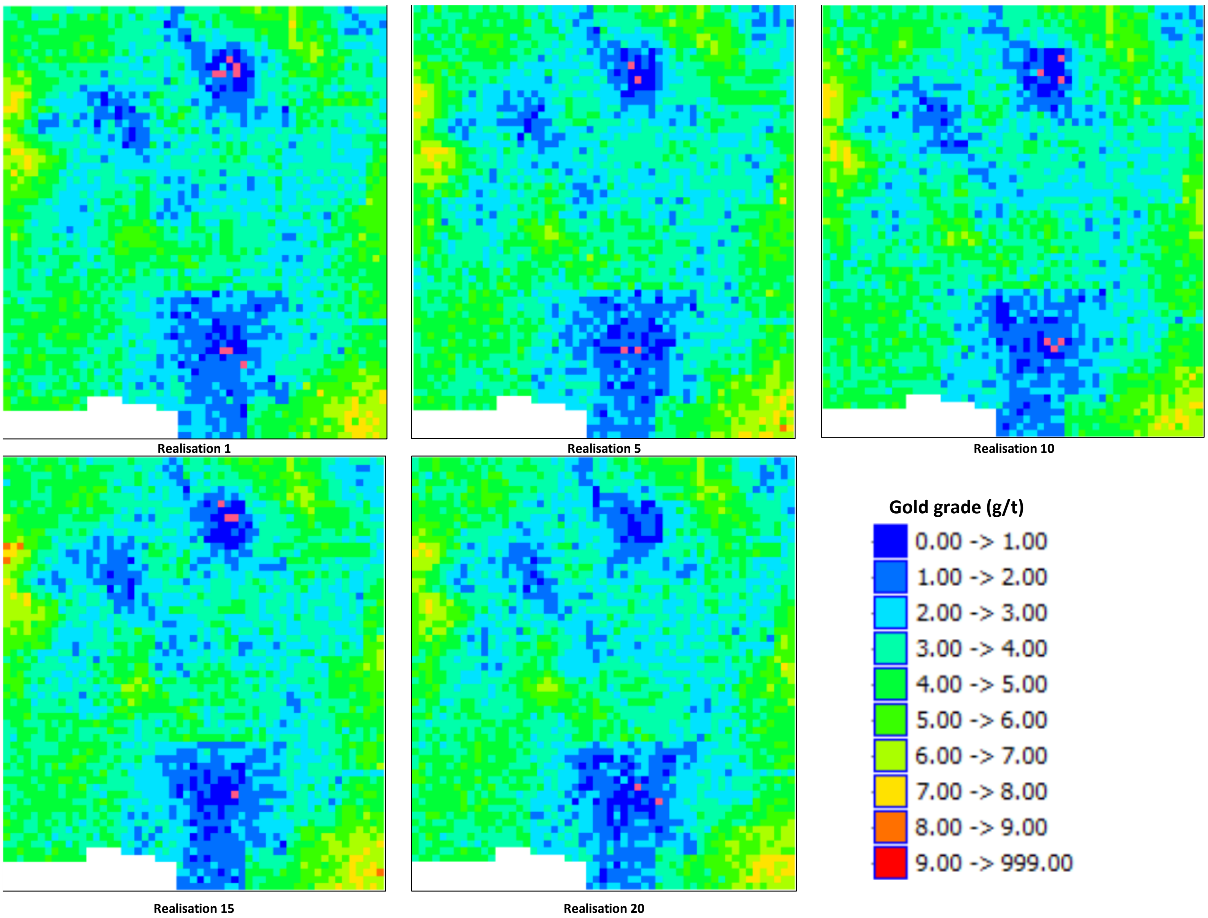

Figure 4.6: Equally probable realisations of the orebody showing different gold grade ranges

The mean and standard deviation of the gold grade for each block was calculated from the conditional simulation results of the geological block model. The statistical parameters were used to further run Monte Carlo simulations of gold grade in calculating BEVs. The use of conditional simulation in combination with other techniques is normal practice in evaluating orebodies. For example, Whittle *et al.* (2007) used conditional simulation in combination with Monte Carlo simulation, Rossi and Van Brunt (1997) used conditional simulation in combination with Whittle 4D. The gold grade data for the case study geological block model was analysed in @Risk software to determine an appropriate probability distribution for gold grade. The probability distribution results are shown in Figure 4.7.

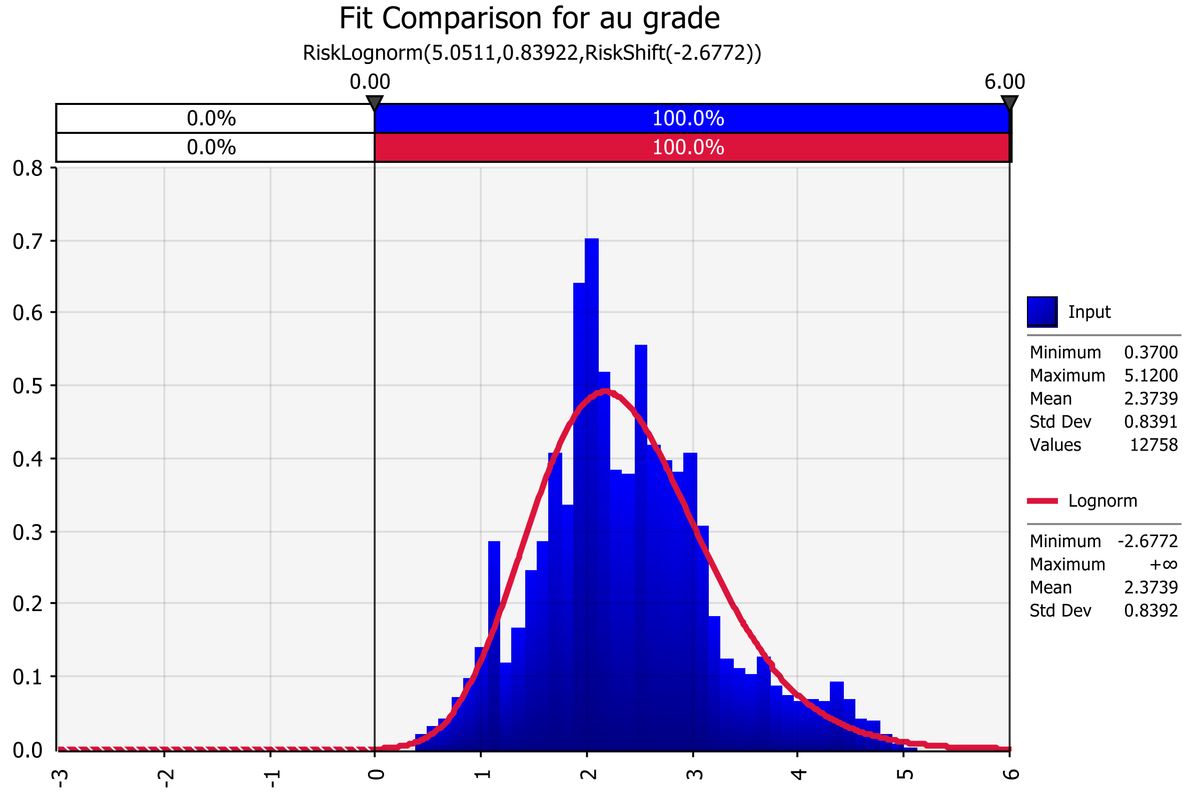

Figure 4.7: Lognormal distribution for the gold grade

Figure 4.7 shows that the gold grade for the deposit follows a slightly positively skewed lognormal probability distribution. These results confirm findings by Coombes (2008) that mineral commodities that constitute small proportions of the rock mass such as precious metals (that are measured in parts per million or grams per tonne) tend to be positively skewed. This means that most of the samples are low grades with a small percentage of extreme high grades. On the other hand, mineral commodities that constitute a large proportion of the rock mass like iron ore tend to show negative skewness (Coombes, 2008). Figure 4.8 shows the typical grade distributions for different mineral commodities.

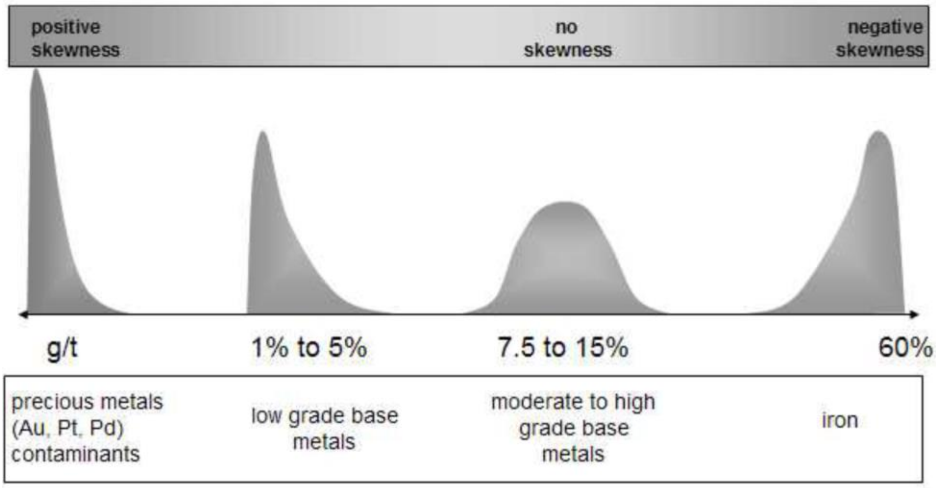

Figure 4.8: Typical distributions for grade for different commodity types

(Source: Coombes, 2008)

Therefore, the lognormal distribution was used to model gold grade with a probability density function given by Equation 4.4 (Beichelt, 2016):

$$f(g) = \frac{1}{g\sigma\sqrt{2\pi}} e^{-\frac{(\ln g - \mu)^2}{2\sigma^2}} \qquad g >= 0 \qquad 4.4$$

where g is gold grade, μ is the mean and σ is the standard deviation of the distribution. Figure 4.9 shows simulation results for gold grade for ten selected blocks in the geological block model. For illustration purpose only ten blocks are presented.

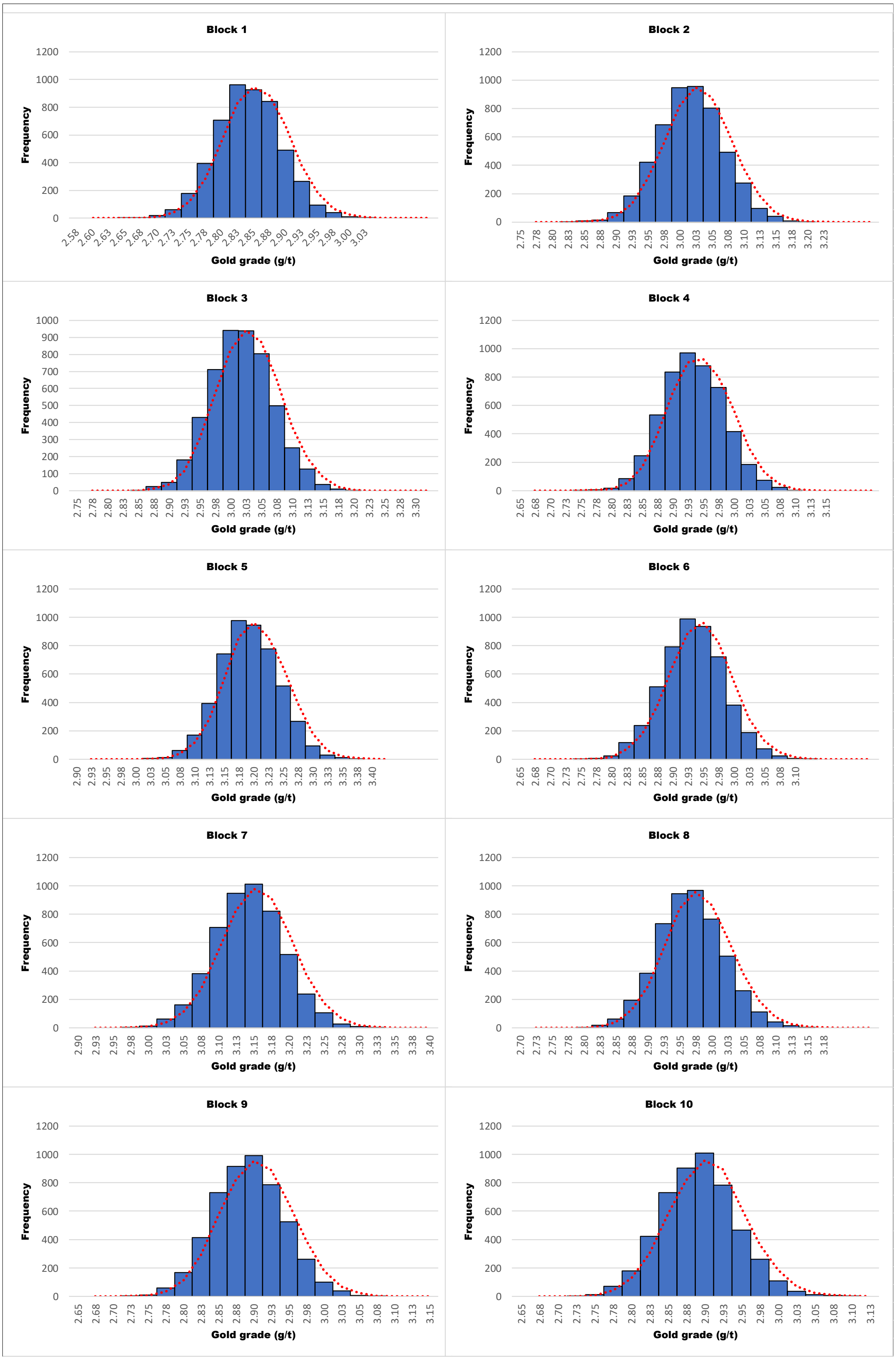

Figure 4.9: Simulation results for gold grade for ten selected blocks

4.2.4 Modelling uncertainty of gold price

Figure 4.10 shows historical gold prices from 2017 to 2021 to illustrate the stochastic behaviour of gold price.

Figure 4.10: Historical gold price in US$/Oz

(Source: GoldPrice, 2020)

Figure 4.10 shows that the gold prices, P, follow a non-stationary random process which means that the expected price grows without bound. The Brownian motion with drift behaviour explained in Chapter 2 was used to model the stochastic behaviour of gold price. This is because the behaviour of gold price shown in Figure 4.10 satisfies the properties of a Brownian motion discussed in Chapter 2 which are: that it is a Markov process and the price has a continuous sample path and the path is not differentiable at any point along the path. Lemelin *et al.* (2006) also supported that commodity prices such as gold price have a constant drift such that they can be modelled using the GBM. The assumption with the GBM with drift is that, it is expected that with time, t, commodity price will change by a factor, μ, in somehow an exponential manner starting from an initial price, P_0, such that, the price at any future time, P_t, can be modelled as shown in Equation 4.5:

$$P_t = P_0 e^{\mu t} \tag{4.5}$$

However, Figure 4.10 shows that realistically the gold price follows a stochastic motion as the price moves up and down with time. Therefore, the actual gold price motion follows an exponential Brownian motion shown by Equation 4.6:

$$P_t = P_0 e^{(\mu t + \sigma B(t))} \quad 4.6$$

where $\sigma B(t)$ is the noise of the gold price. Equation 4.6 can be re-written as Equation 4.7:

$$ln \left|\frac{P_t}{P_0}\right| = \mu t + \sigma B(t) \quad 4.7$$

Equation 4.7 can be re-written as a stochastic differential equation as shown by Equation 4.8:

$$dP = P\mu dt + \sigma P dB(t) \quad 4.8$$

where $B(t)$ is the standard Brownian motion, $\sigma B(t)$ is the noise with an expectation of zero. Equation 4.8 shows that gold price, P, follows an Ito process. In general, an Ito process states that if a variable x is assumed to follow an Ito process then (Dixit and Pindyck, 1994):

$$dx = a(x,t)dt + b(x,t)dz \quad 4.9$$

where dz is a Wiener or Geometric Brownian process, a and b are the drift and volatility, respectively which are both functions of x and t. The variable of x has a drift rate of a, and a variance of b^2. Therefore, it can be derived from Ito's Lemma that any function, G of x and t follows a process shown by Equation 4.10:

$$dG = \left(\frac{\partial G}{\partial x}a + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt + \frac{\partial G}{\partial x}bdz \quad 4.10$$

This means that Equation 4.8 can be modelled as an Ito's process. Applying the Ito's process, and assuming $z = \ln P_t$, and substituting a and b into Equation 4.10 it follows that:

$$dG = \left(\mu P\frac{\partial G}{\partial P} + \frac{\partial G}{\partial t} + \frac{1}{2}\frac{\partial^2 G}{\partial P^2}\sigma^2 P^2\right)dt + \sigma P\frac{\partial G}{\partial P}dB(t) \quad 4.11$$

$$dz = \left(\mu\frac{1}{\mathrm{P_t}}\mathrm{P_t} + 0 + \frac{1}{2}\frac{-1}{\mathrm{P_t}^2}\sigma^2\mathrm{P_t}^2\right)dt + \frac{1}{\mathrm{P_t}}\mathrm{P_t}\sigma dB(t) \quad 4.12$$

$$dz = \left(\mu - \frac{\sigma^2}{2}\right)dt + \sigma dB(t) \quad 4.13$$

Equation 4.8 leads to Equation 4.14:

$$\mathrm{P_t} = \mathrm{P_0} + \int_0^t P\mu\, dt + \int_0^t \sigma P dB(t) \quad 4.14$$

Since $z = \ln P_t$, it means the $P_t = e^z$

$$\mathrm{z} = \ln\left(\mathrm{P_0} + \int_0^t P\mu\, dt + \int_0^t \sigma P dB(t)\right) \quad 4.15$$

Therefore, the price at a point in time, t, is given by:

$$P_t = P_0 e^z \quad 4.16$$

$$P_t = P_0 e^{\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma dB(t)\right]} \quad 4.17$$

Equation 4.17 was used to simulate gold price paths used in the BEV calculation. Simulations were done in Microsoft Excel. As explained earlier in Chapter 2, the price is a Markov process such that only the current price is needed to forecast price. Therefore, the spot price as of 2 January 2020, P_0, of US$1520/oz was used to simulate gold prices (GoldPrice, 2020). Although the gold price has changed since then, updating the simulation model to consider the current gold price would have delayed competion of this thesis.

The daily logarithmic changes in the gold price over a three year historical period was calculated, and random numbers were generated to represent volatility. In reality, the blocks within the orebody will be mined at different time periods, t, but the periods are unknown because scheduling of the blocks was not part of this thesis' scope. The simulation gave different P_t representing the different prices at the different unknown times the blocks will be mined. Figure 4.11 shows gold price forecasts (50 simulations runs) for two years from 3 January 2020 using the GBM.

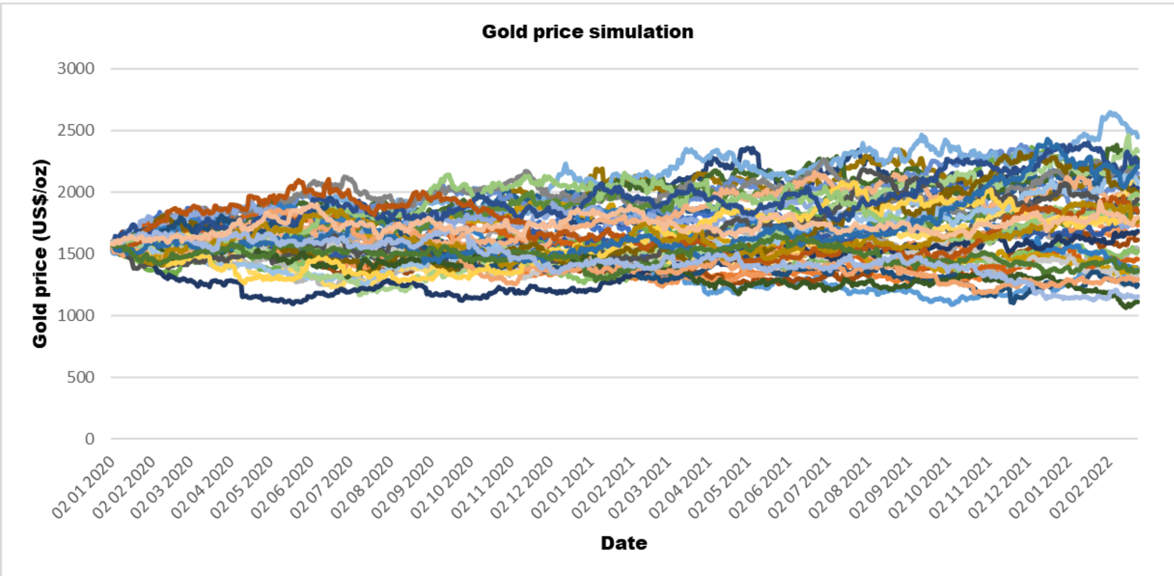

Figure 4.11: Sample gold price simulations

4.3 Probabilistic BEV calculation model

The different BEV input parameters were combined into a single model. The integrated BEV model is given by Equation 4.18 as follows:

$$BEV_{ij} = (G_{ij} * R_{ij} * P_t - C_{mij} - C_{pij}) * T_{ij} \qquad 4.18$$

where; BEV_{ij} is the economic value of block b_{ij} at time t,

$G_{ij} \sim LogN\ (\mu, \sigma^2)$ is the gold grade of block $b$ij,

$R_{ij} \sim U\ (\alpha, \beta)$ is the recovery rate of processing block b_{ij},

$P_t = p_0 e^{\left[\left(\mu - \frac{\sigma^2}{2}\right)t + \sigma dB(t)\right]}$ is the gold price at time, t,

C_{mij} = is the cost per tonne of mining block b_{ij},

C_{pij} = is the cost per tonne of processing block b_{ij},

$T_{ij} = XINC_{ij} * YINC_{ij} * ZINC_{ij} * D_{ij}$ is the tonnage of block b_{ij},

$XINC_{ij}$ is the dimension of block b_{ij} in the X direction,

$YINC_{ij}$ is the dimension of block b_{ij} in the Y direction,

$ZINC_{ij}$ is the dimension of block b_{ij} in the Z direction,

$D_{ij} \backsim N\ (\mu, \sigma^2)$ is the density of block $b$ij.

This model was then used in the simulation of BEVs in Microsoft Excel. After a series of trial and error from about 100 iterations initially, and considering the 366,792 blocks in the model, it ended up being practical to set up and run about 5000 arbitrary iterations over two weeks to generate reasonably stable BEVs distributions for each block. Figure 4.12 shows a schematic overview of the probabilistic model.

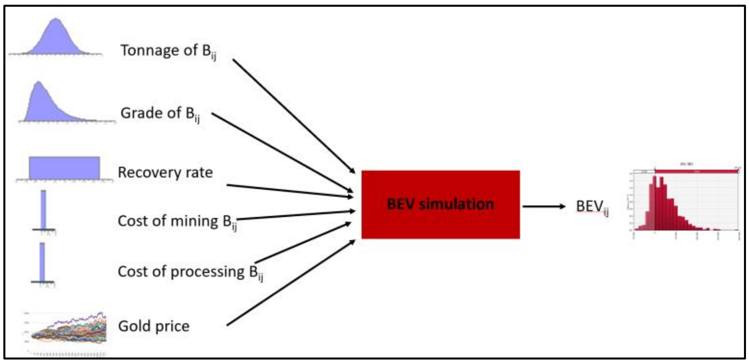

Figure 4.12: A probabilistic BEV calculation model

Figure 4.12 illustrates that instead of using fixed values for each input as in deterministic modelling (Figure 1.4), probability distributions were determined for each input parameter to model 5000 arbitrary possible BEVs. Using the simulated BEVs, the probability of each block being positive was calculated based on the number of times the BEV was positive over the 5000 iteration runs.

4.4 Summary of Chapter 4

The chapter described how input parameters used to calculate BEVs were modelled in this thesis. Density of rock and recovery rate were assumed to follow the normal and uniform probability distibution, respectively. Gold grade was assumed to follow the lognormal distribution while gold price was assumed to follow the Brownian motion with drift stochastic behaviour. These were combined into a single model that was then used to simulate BEVs in Microsft Excel. From the simulated BEVs, the probability of each block being positive was calculated from which different economic block models were generated. The generated probabilistic economic block models are presented in the next chapter.

5 PROBABILISTIC ECONOMIC BLOCK MODELS

5.1 Overview of Chapter 5

This chapter presents the application of the probabilistic method discussed in Chapter 4 to a geological block model for an underground gold deposit. The chapter starts by describing the block model and the characteristic of the orebody. From the geological block model, deterministic BEVs were calculated resulting in a deterministic economic block model. The deterministic economic block model is presented in this chapter. Uncertainty was then incorporated to the selected BEV input parameters as explained in Chapter 4. Using the assumed and identified probability distributions, BEVs for all blocks were simulated in Microsoft Excel using the Monte Carlo simulation method. Five thousand arbitrary iterations were run. From the simulated BEVs, the mean BEV and the probability of each block being positive was calculated. The probability was calculated from the number of times a block was positive in the 5000 iterations.

Using the probabilities, several economic block models with probabilities of BEV being positive were created at 5%, and 10% probability intervals from 10% to 100%. These economic block models are presented and analysed in this chapter. The 5% and 100% probability economic block models represent the models with blocks with low confidence and high confidence of being economic, respectively. The lowest confidence and highest confidence assume the worst-case values and best-case values of the range of input parameters sampled from the assumed underlying probability distributions of these input parameters, respectively. For example, a 100% probability economic block model does not mean that even when the gold price is zero, the blocks in the block model will be economic. It means that based on the range of inputs sampled from the various probability distributions, the blocks were found to be positive in all iterations. However, in reality 100% confidence cannot be achieved because there will always be some uncertainty involved as explained earlier in the 2nd paragraph of Section 2.4. In addition to simulations in Microsoft Excel, simulations were also done for a few selected blocks using @Risk software to visualise the probability distributions of BEVs. Five thousand arbitrary iterations were also run, and the results of the simulation are presented and analysed in this chapter for ten selected blocks only. All costs and gold price data in this chapter are in United States of American dollars (US$).

5.2 Geological and deterministic economic block models

The exploration data used for this thesis consisted of assay, survey, lithology, and collar files for 25 boreholes. The boreholes were then composited at 1.5 m intervals. Using the four files, a geological block model was created with 366,792 regular blocks of 5 m x 2 m x 5 m dimensions in the X, Y and Z directions, respectively. Since the original block model had block sizes of 5 m x 2 m x 5 m it was prudent to retain the same block sizes after converting the original block model to a synthetic block model. The following are the characteristics of the orebody:

- It is steeply dipping at a range of 60 to 80 degrees.
- It strikes for about 280 m.
- Gold grades are hosted in sulphides mainly quartz.
- The orebody is located at depths ranging from 300 m and 800 m below surface.
- The width of the orebody varies between 2 m to 15 m.
- The gold mineralization is mostly refractory gold with minor free native gold.

Twenty conditional simulations were then run as discussed in Section 4.2.3 and shown in Figure 4.6. The block model was then constrained by gold grade in Surpac software to display the mineralised blocks resulting in only 30 183 blocks. Figure 5.1 shows the X-Z view of the boreholes and geological block model coloured by gold grade attribute.

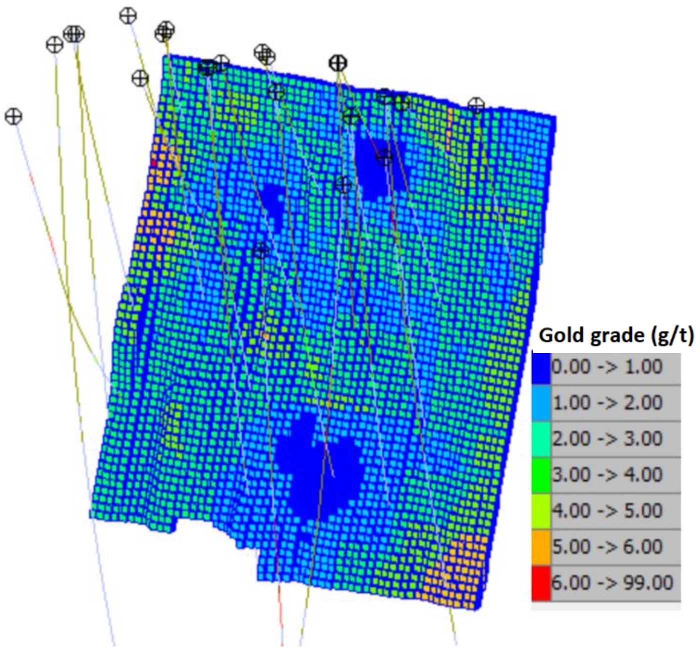

Figure 5.1: The geological block model for the case study deposit

The characteristics of the orebody and Figure 5.1 show that the orebody is a relatively small and low-grade deposit with gold grade varying from 0 g/t to about 6 g/t as also shown in Table 5.1 which shows the grade and tonnage report of the geological block model. The table is drawn from data shown in Appendix 9.1.

Table 5.1: Grade-tonnage distribution of the case study gold deposit

Grade range (g/t)	Amount of material (Tonnes)	Average grade (g/t)
0.01 -> 0.5	5 600	0.45
0.5 -> 1.0	70 880	0.82
1.0 -> 2.0	592 000	1.62
2.0 -> 3.0	916 960	2.45
3.0 -> 4.0	264 320	3.33
4.0 -> 5.0	65 280	4.40
5.0 -> 6.0	640	5.07
Grand Total	**1 915 680**	**2.32**

Based on the geometry and gold grade distribution of the orebody described above, it was assumed that the orebody is best mineable using the sub-level open stoping (SLOS) mining method. The parameters used to design the SLOS mine for this thesis are discussed in Chapter 6.

The geological block model was then converted into a deterministic economic block model by applying factors shown in Table 5.2. The input parameters are average values; the typical practise in deterministic BEV calculation. The economic block model coloured by BEV attribute is shown in Figure 5.2. Density of rock was based on the mean mentioned in Section 4.2.1, mining and processing costs were based on the anonymous gold mine mentioned earlier in Section 1.5, gold price was based on the spot price mentioned in Section 4.2.4 while recovery rate was based on the mean rate shown in Figure 4.4.

Table 5.2: Economic input parameters

Parameter	Value
Density of rock (t/m^3)	3
Mining cost ($/t)	50
Processing cost ($/t)	30
Gold price ($/oz)	1520
Recovery rate (%)	84

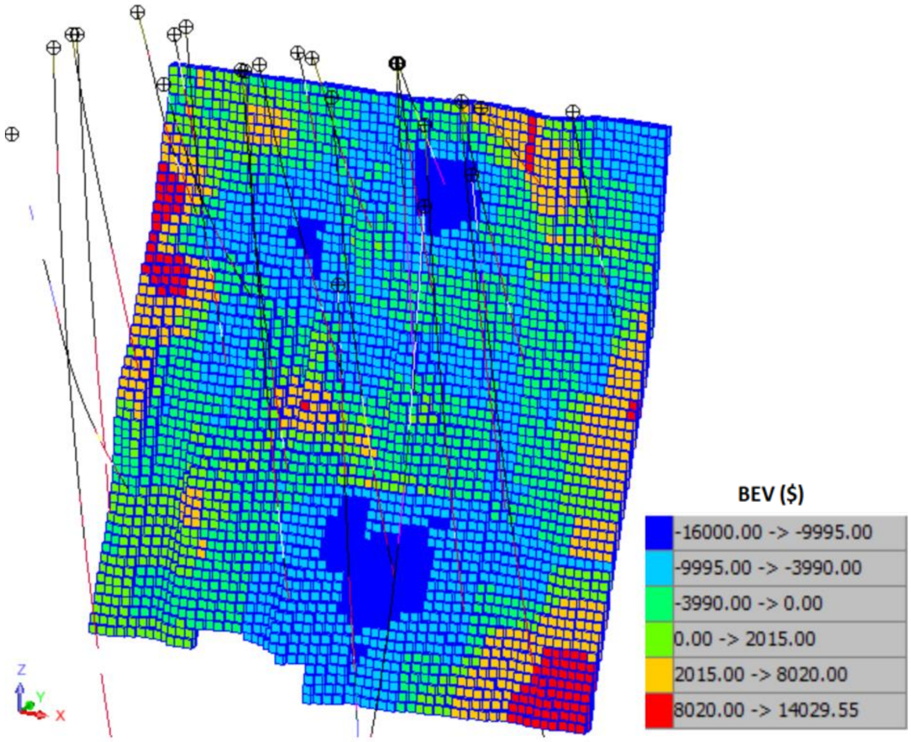

Figure 5.2: The deterministic economic block model for the case study deposit

Figures 5.1 and 5.2 display a similar distribution of gold grade and BEVs, respectively showing that grade has a significant influence on BEV. In deterministic economic block modelling, all blocks with negative BEVs are excluded from the final economic block model without evaluating the potential of the blocks to be economic under different future scenarios. The opposite is also true for blocks with positive BEVs; they are automatically classified as ore. Table 5.3 shows the total economic value, tonnage and grade of the final deterministic economic block model that excludes blocks with negative BEVs. Table 5.3 was drawn using data shown in Appendix 9.2.

Table 5.3: Total economic value, grade and tonnage of the final deterministic economic block model

Parameter	Value
Total economic value ($m)	17.71
Tonnage (mt)	0.71
Grade (g/t)	3.27

Table 5.4 and Figure 5.3 show the deterministic BEVs and their respective deterministic probability distributions, respectively for the ten selected blocks in the block model. Only 10 of the 30 183 blocks are presented and analysed for illustration purposes.

Table 5.4: Deterministic BEV results for ten selected blocks

Block No.	XINC (m)	YINC (m)	ZINC (m)	Density (t/m3)	Block tonnage (t)	Au_grade (g/t)	Recovery	Gold price ($/oz)	Min_cost ($/t)	Proc_cost ($/t)	Deterministic BEV ($)
1	5	2	5	3	150	2.19	0.84	1520	50	30	1478
2	5	2	5	3	150	2.19	0.84	1520	50	30	1478
3	5	2	5	3	150	2.19	0.84	1520	50	30	1500
4	5	2	5	3	150	1.70	0.84	1520	50	30	-1514
5	5	2	5	3	150	2.19	0.84	1520	50	30	1478
6	5	2	5	3	150	1.70	0.84	1520	50	30	-1514
7	5	2	5	3	150	2.19	0.84	1520	50	30	1478
8	5	2	5	3	150	2.21	0.84	1520	50	30	1598
9	5	2	5	3	150	1.71	0.84	1520	50	30	-1455
10	5	2	5	3	150	2.10	0.84	1520	50	30	902

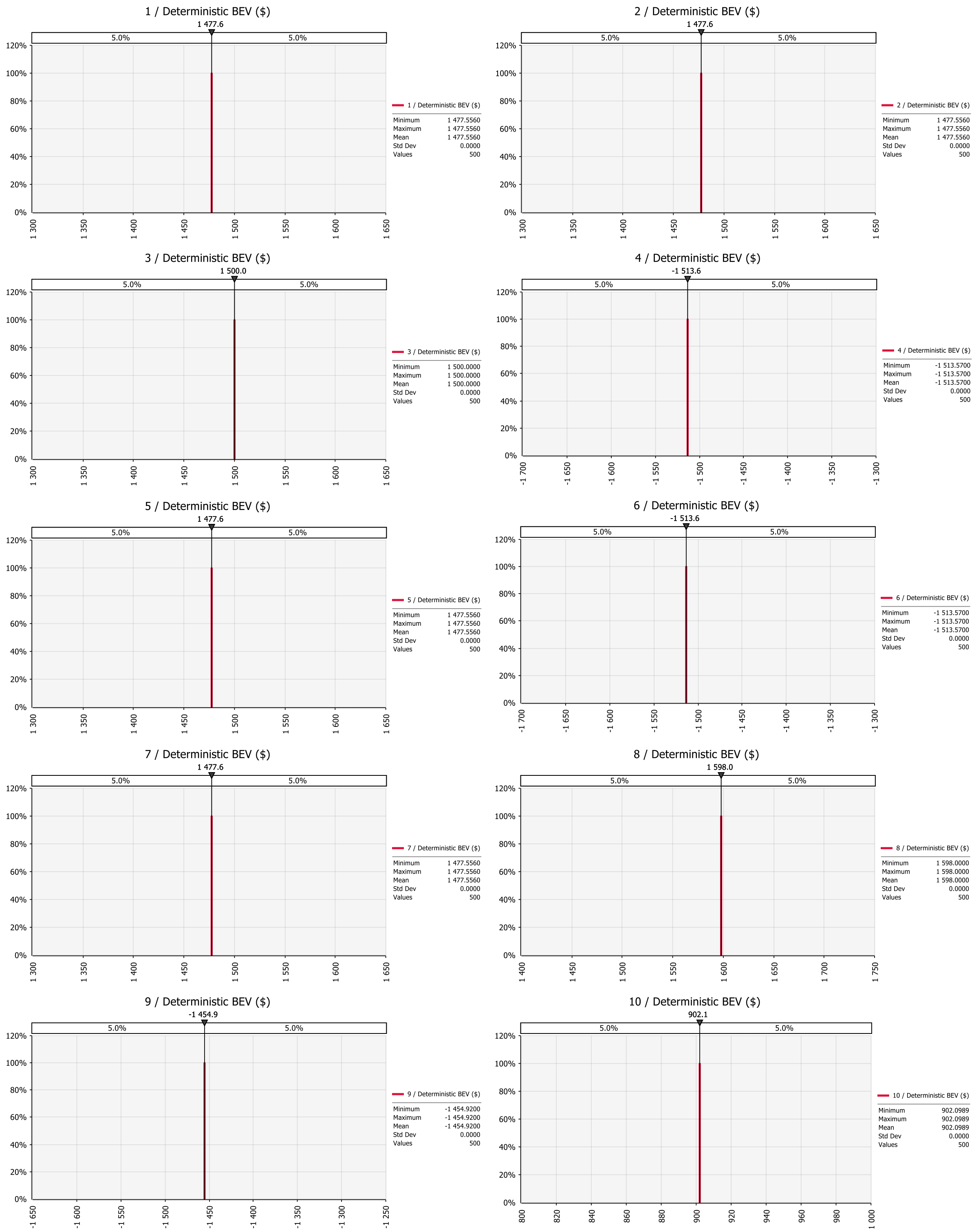

Figure 5.3: Deterministic BEVs' probability distributions for the ten selected blocks

As expected and mentioned earlier in the thesis, Figure 5.3 shows that deterministic BEV modelling results in single estimates of BEVs. Risk and uncertainty associated with the BEV for each block is not quantified. Analysis of risk is done in the next section.

5.3 Risk quantification results for the individual blocks

To quantify the risk associated with each block, uncertainty was incorporated to gold grade, processing recovery rate, gold price and rock density as explained in Chapter 4. Five thousand iterations were run for each block in Microsoft Excel. To visualise the impact of uncertainty on each of the ten selected blocks, BEVs were also simulated using @Risk software. Five thousand iterations were run for different combinations sampled from the probability distribution of each input parameter as described in Chapter 4. The simulation results and associated descriptive statistics for the ten blocks are shown in Figure 5.4.

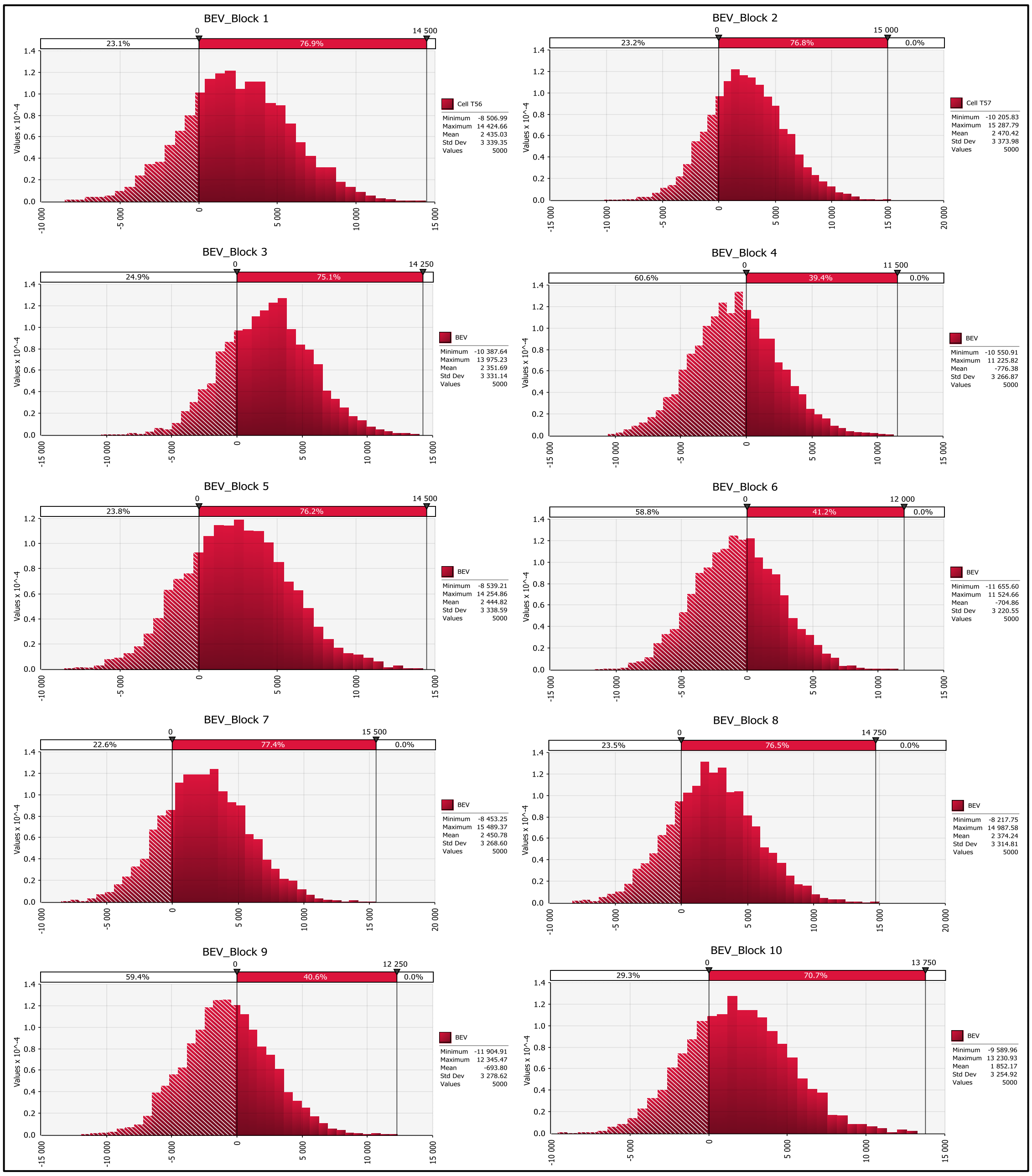

Figure 5.4: Probability distribution results for the ten selected blocks

Figure 5.4 shows that instead of a single deterministic economic value for each block (Figure 5.3), in reality each block has a range of possible economic values that are represented in the form of probability distributions. The probability distributions represent the levels of risk associated with each block. In this study, the level of risk is defined as the probability of BEVs being negative. This can then be used by mine planners to understand the impact of uncertainty on the orebody and subsequently guide mine designs and plans. The results show that all ten blocks have an associated level of risk which is a reality that is ignored by the static economic block modelling process. A comparison of deterministic BEV results for the ten blocks and the risk quantification after simulation of the same blocks highlight the importance of probabilistic BEV calculation. Table 5.5 shows the probabilities of the ten blocks of having a positive BEV compared with the deterministic BEVs. The deterministic BEV column is the last column in Table 5.4.

Table 5.5: Probability of ten selected blocks being positive

Block No.	Deterministic BEV ($)	Probability of being positive
1	1478	**0.78**
2	1478	**0.77**
3	1500	**0.75**
4	-1514	**0.39**
5	1478	**0.76**
6	-1514	**0.41**
7	1478	**0.77**
8	1598	**0.77**
9	-1455	**0.41**
10	902	**0.71**

As shown in Table 5.5, some of the blocks have positive BEVs from the deterministic calculation which would automatically qualify them as economic to mine. However, simulation results (Figure 5.4) show the true behaviour of the BEVs. Table 5.4 show that blocks with negative BEVs from deterministic calculation (Blocks 4, 6 and 9) also have the lowest probabilities of their BEVs being positive at 39%, 41% and 41%, respectively. The opposite is also true for blocks with positive BEVs. This shows that the BEV has a direct relationship with probability of BEV being positive as shown in Figure 5.5 which shows the relationship between BEV and probability of blocks being positive for the ten blocks.

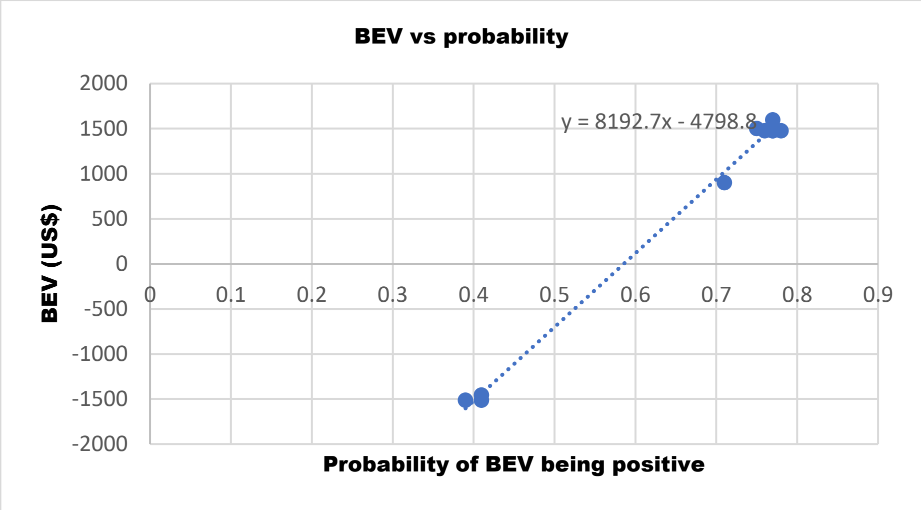

Figure 5.5: Relationship between BEV and probability of blocks being positive

Analysis of the simulation results also shows that even blocks with positive BEVs also have some varying level of risk of being negative, something that is ignored with deterministic BEV evaluation. This may result in loss of revenue due to misclassification of some ore and waste blocks. For example, Blocks 1, 2, 3, 5, 7, 8 and 10 (Table 5.5) with positive BEVs have different levels of risk that range from 23% to 29% as shown in Figure 5.4. This information would significantly change the decision to categorise these blocks as economically mineable blocks. Subsequently, this information will better inform the decision on which blocks have a higher probability of being included within a specific stope during stope boundary definition. Therefore, the main advantage of Monte Carlo simulation is that one can make better informed decisions.

5.4 Probabilistic economic block models

After the Monte Carlo simulation, each block in the economic block model was assigned its respective probability attribute. The economic block model (Figure 5.2) was then constrained by the probability attribute in Datamine software at 5%, and 10% probability intervals from 10% to 100% to analyse the impact of risk on the economic block model. The constrained economic block models are shown in Figures 5.6 to 5.16, which represent economic block models that are equally probable under different future scenarios. The economic block models are denoted as P5, P10 to P100 economic block model, respectivley in the thesis. It is important to note that the economic block models were constrained by a '*probability* $\geq x_i$' command expression (where x_i = 0.05, 0.1, 0.2…1) except for the 100% probability which was constrained by a '*probability = 1*' command expression. For example, a P5 economic block model was obtained after running a command expression '*probability* $\geq$ *0.05*'. This means that the P5 economic block model (Figure 5.6) consists of all blocks in the economic block model with a probability of being positive of at least 5% and, a P100 economic block model consists of blocks with a probability of being positive equal to 100% (Figure 5.16).

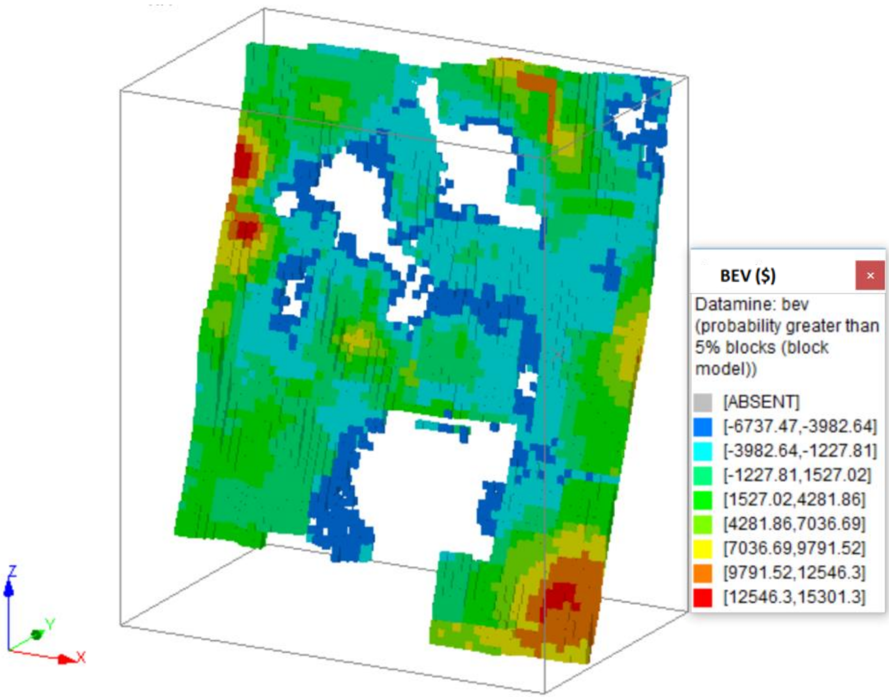

Figure 5.6: P5 Economic block model

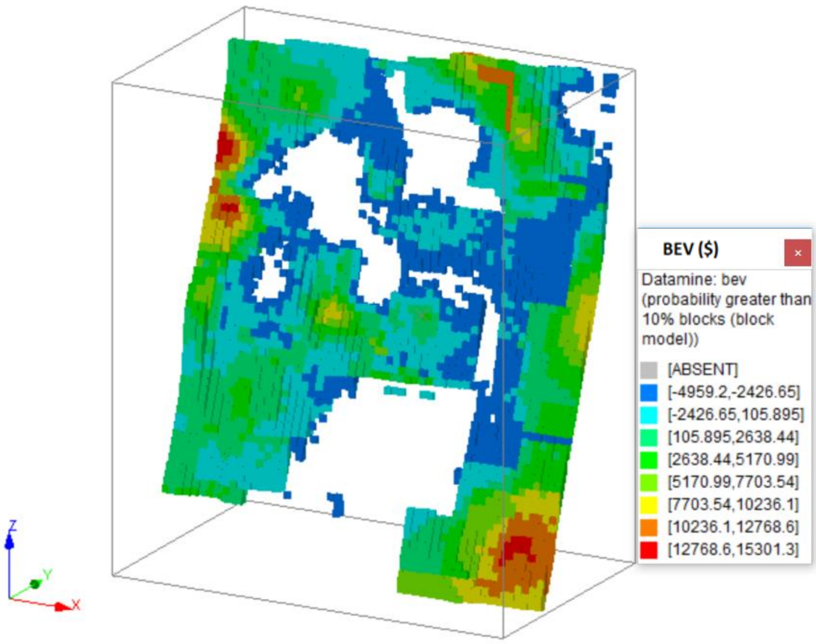

Figure 5.7: P10 Economic block model

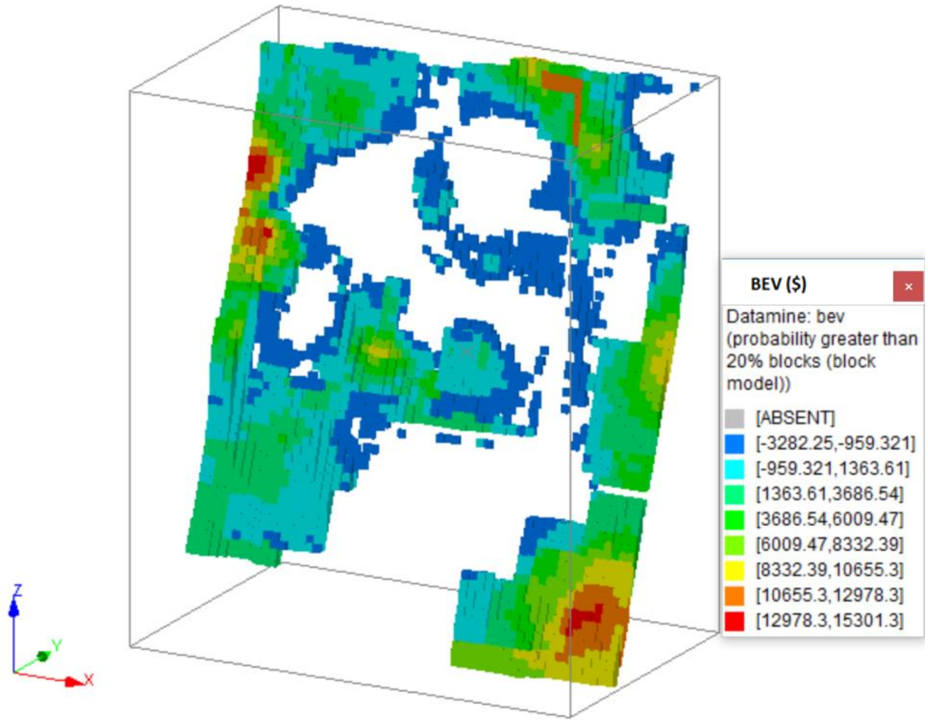

Figure 5.8: P20 Economic block model

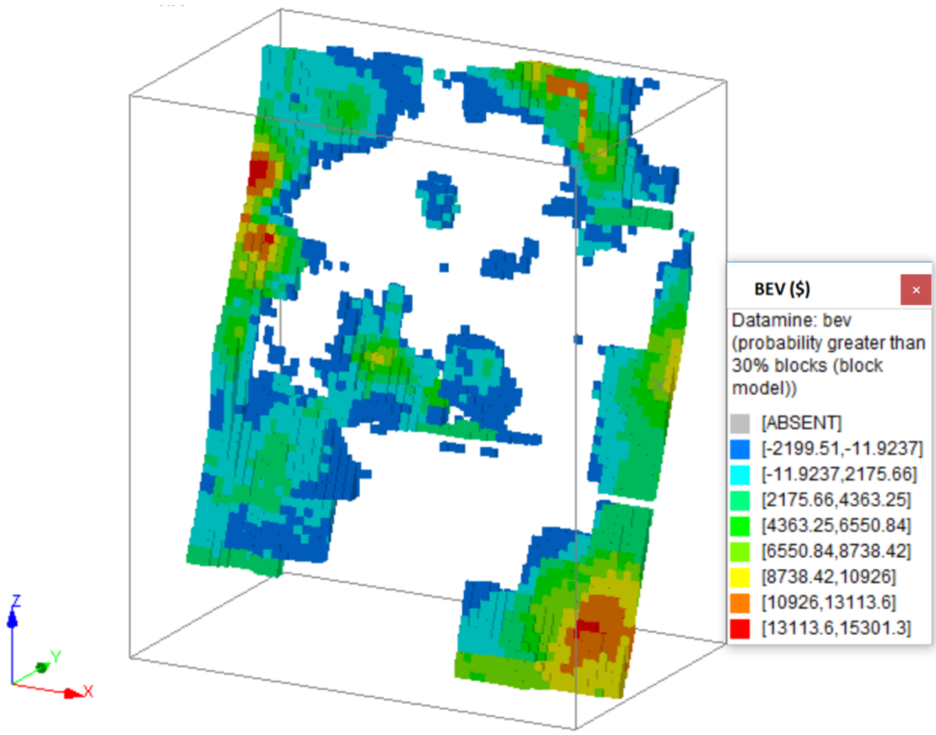

Figure 5.9: P30 Economic block model

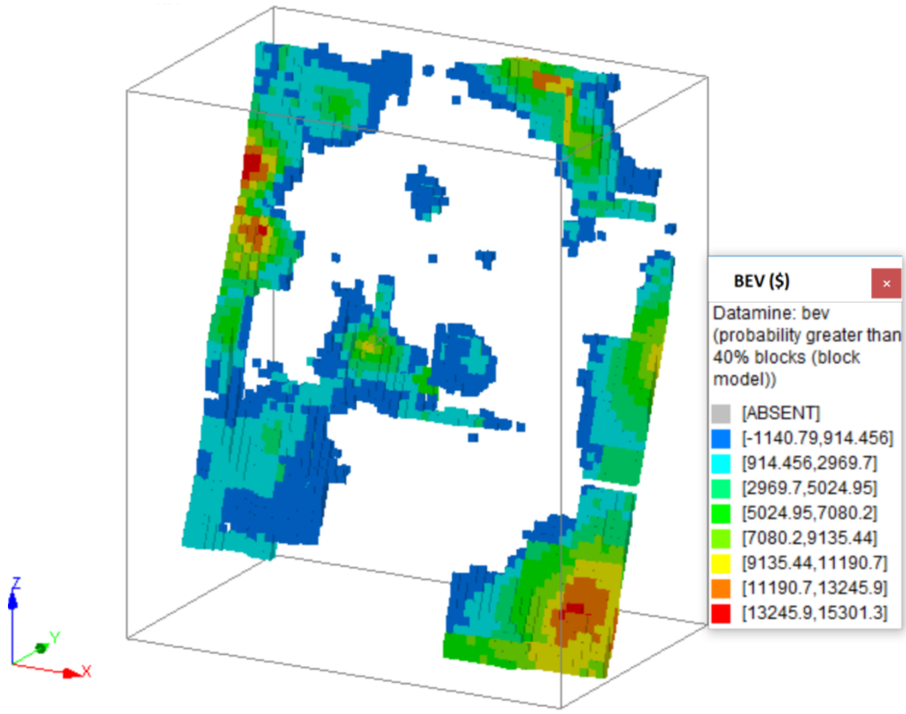

Figure 5.10: P40 Economic block model

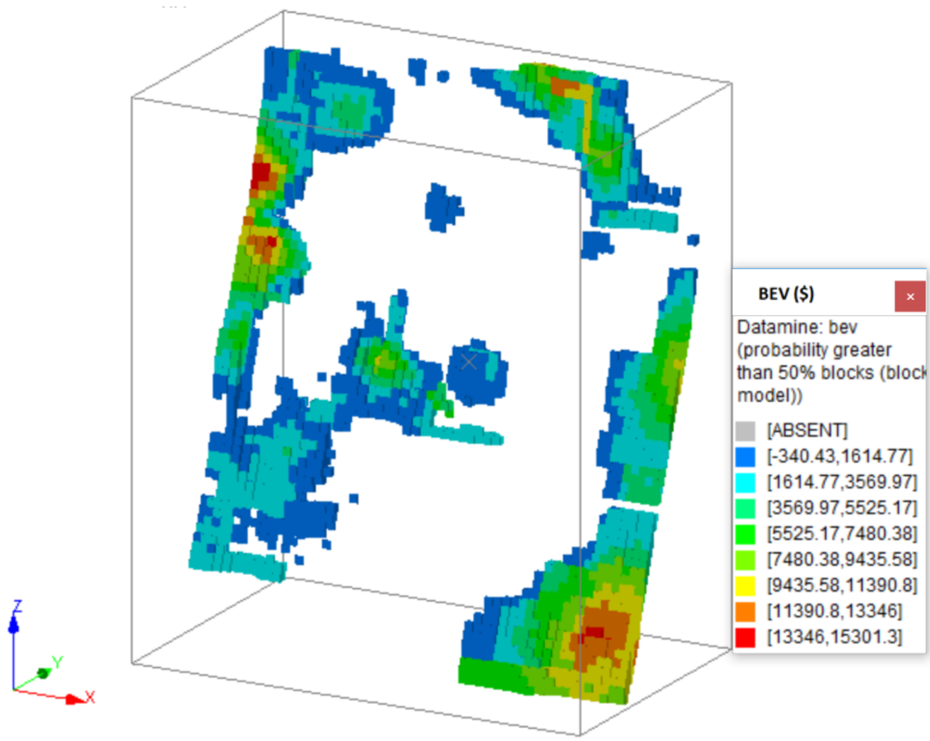

Figure 5.11: P50 Economic block model

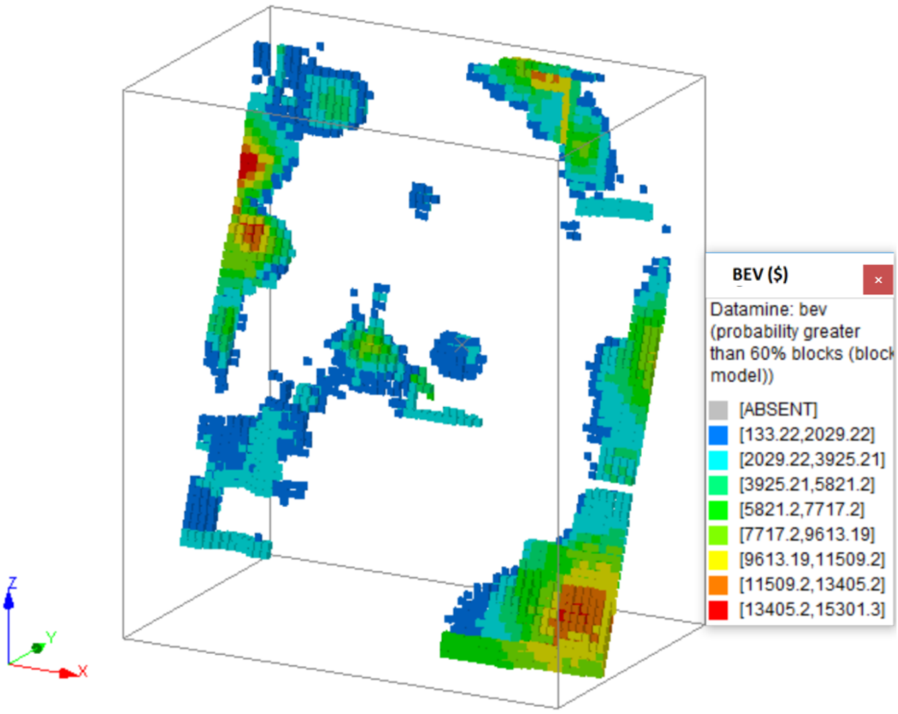

Figure 5.12: P60 Economic block model

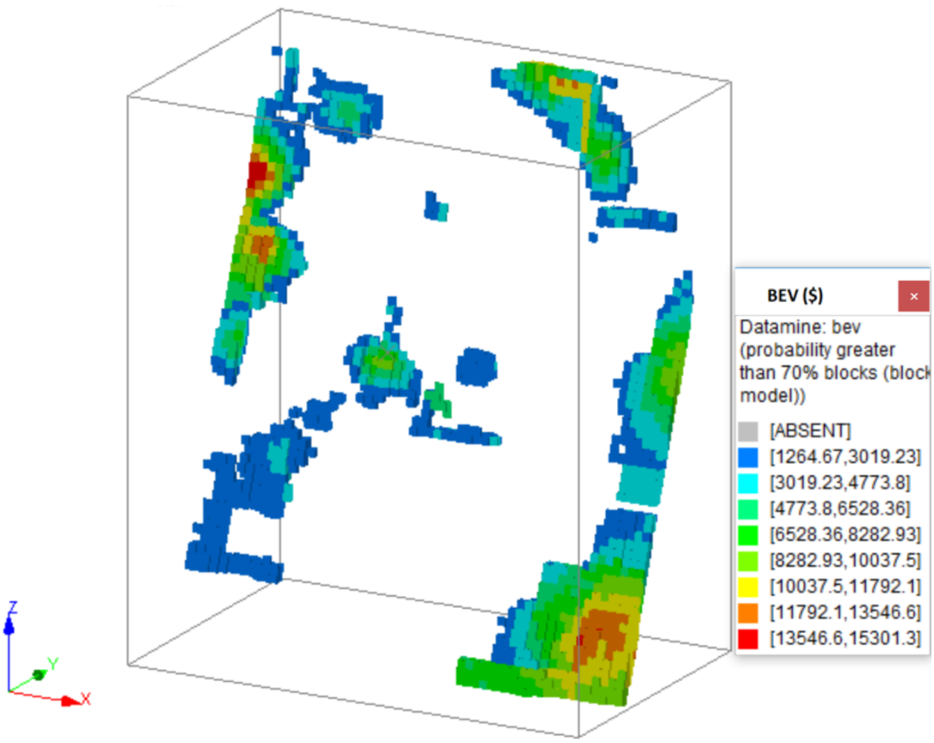

Figure 5.13: P70 Economic block model

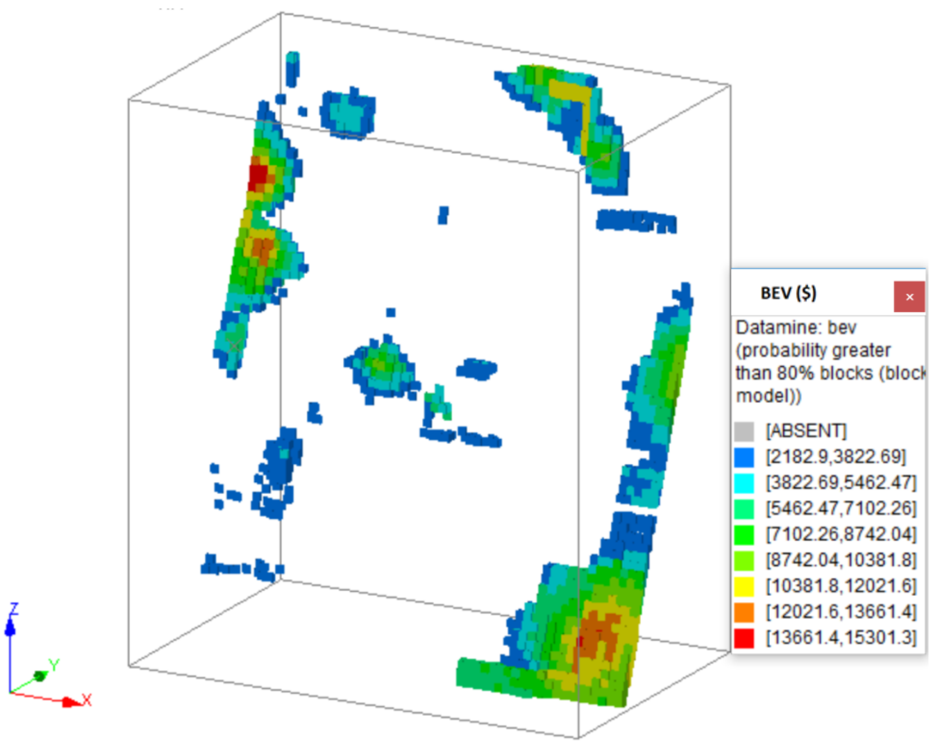

Figure 5.14: P80 Economic block model

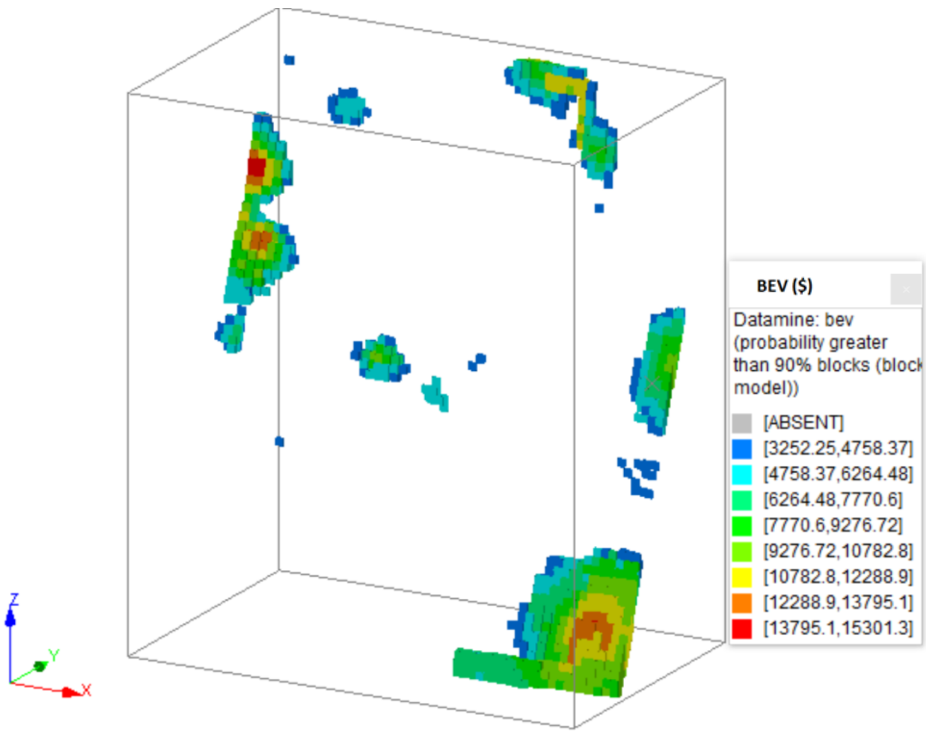

Figure 5.15: P90 Economic block model

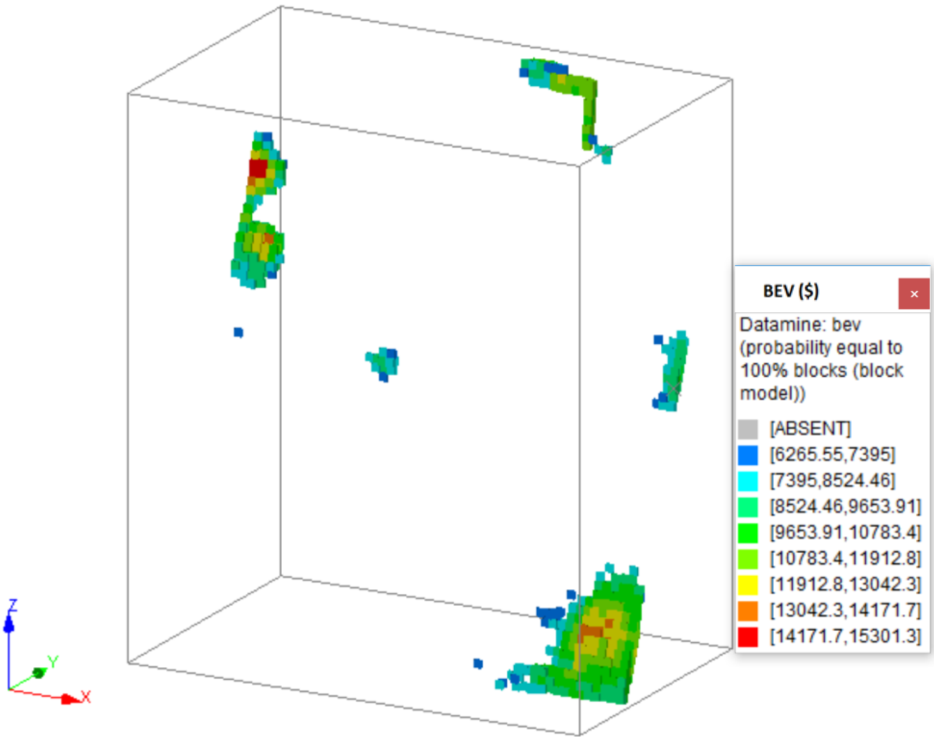

Figure 5.16: P100 Economic block model

5.5 Discussion of the simulation results

Figures 5.6 to 5.16 show that as the probability of BEV being positive increases the number of blocks, hence the size of the orebody decreases. This means that a significant amount of the orebody is sacrificed by increasing the confidence levels in BEVs. This information provide mines planners an opportunity to evaluate the likely ranges of total orebody tonnage, grade and contained metal, with respect to acceptable risk levels/tolerance that the mining company is prepared to take for mine planning. Uncertainty affect the overall economics of a mining project as discussed in the next subsections.

5.5.1 Impact of uncertainty on the total economic value of the orebody

The total economic value for each of the economic block models were analysed in relation to the respective probabilities. Figure 5.17 shows the relationship between probability of blocks having a positive BEV and the total economic value of each economic block model. The data used to plot Figure 5.17 is shown in Appendix 9.2 and Appendix 9.3 for the deterministic and probabilistic economic block models, respectively. The green line on Figure 5.17 represents the total economic value for the deterministic economic block model for comparison with economic values of probabilistic economic block models.

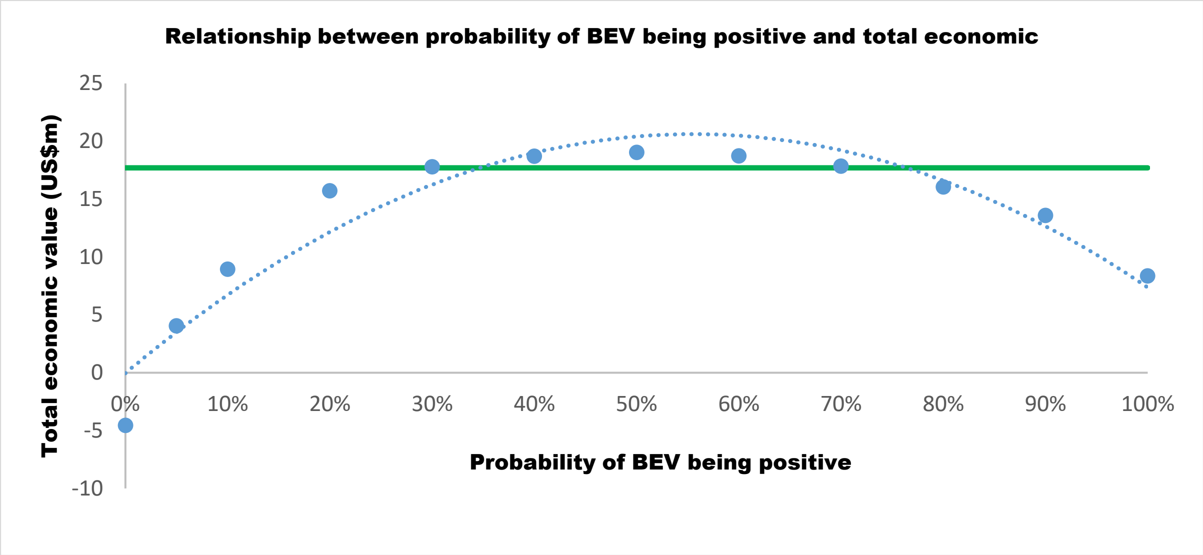

Figure 5.17 Relationship between probability of BEV being positive and total economic value

Figure 5.17 shows that as the probability of blocks having a positive BEV increases from 0% to 50%, the total economic value of the orebody increases to a peak point of about $19 million. This is because at the lowest confidence all blocks in the orebody are included including blocks with negative BEVs but, as the confidence increases the number of blocks with negative BEVs decreases, hence increasing total economic value. However, as the probability further increases from 50% to 100%, the total economic value of the orebody decreases until it reaches $8.3 million. This is because as the probability continues to increase all blocks with negative BEVs and low positive BEVs are excluded from the orebody as shown in Figure 5.18 which shows the ratio of negative and positive BEV blocks in the orebody. The data used to plot Figure 5.18 is also shown in Appendix 9.3.

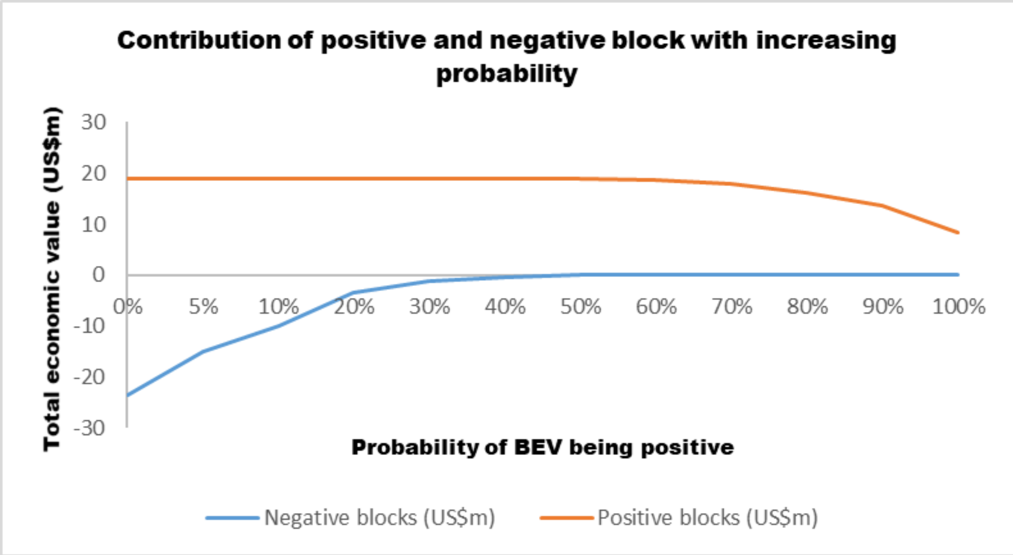

Figure 5.18: Ratio of negative BEV to positive BEV blocks

Figure 5.18 shows that the ratio of blocks with negative BEVs in the orebody decreases with increasing probability. From 40% probability and above all blocks with negative BEVs are excluded from the orebody. This is because BEV and the probability of BEV being positive are directly related as shown earlier in Figure 5.5. On the contrary, deterministic economic block model results in a single estimate of the total economic value of the orebody as shown by the straight line on Figure 5.17. This can result in underestimating or overestimating the total economic value of the orebody due to potential misclassification of some ore as waste and vice versa. Subsequently, this potentially results in loss of revenue.

5.5.2 Impact of uncertainty on the average grade and tonnage of the orebody

In addition to total economic value, uncertainty also affects the grade and size of the orebody. Figure 5.19 shows the impact of uncertainty on the grade and tonnage of the deposit.

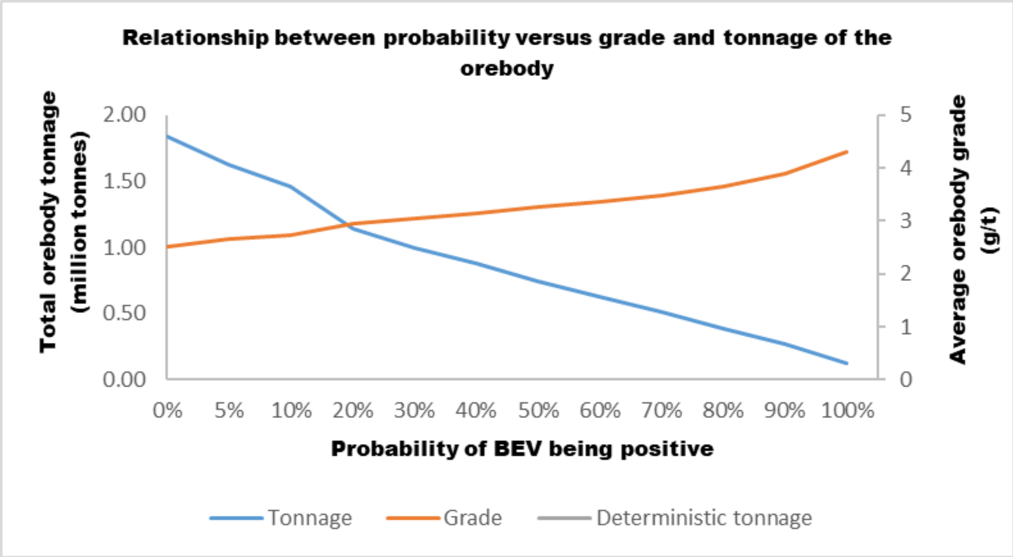

Figure 5.19: The relationship between the average grade and total tonnage of the orebody versus probability

Figure 5.19 shows that as probability increases, the average grade of the orebody increases, and the total orebody tonnage decreases. This is also seen in Figures 5.6 to 5.16 that as probability increases the size of the orebody gets smaller. The highest average grade is obtained with the P100 economic block model. This is the block model that would result in the most reliable stope design. However, extracting only the P100 orebody is what is considered to as 'high grading', a practise that can be unsustainable because it may result in sterilizing the orebody thereby destroying economic value. Also, the high average grade may not be able to offset the lower tonnage. This may also result in the mine failing to meet production targets, reduced LOM and subsequently destruction of value.

5.5.3 Impact of uncertainty on the LOM of a project

Even though total economic values for P10 and P100 economic block models (Figure 5.17) are approximately the same, the tonnages and subsequently LOMs of the block models are different. Assuming a run of mine production rate of 500 000 tonnes per annum, Figure 5.20 illustrates the relationship between LOM and probability.

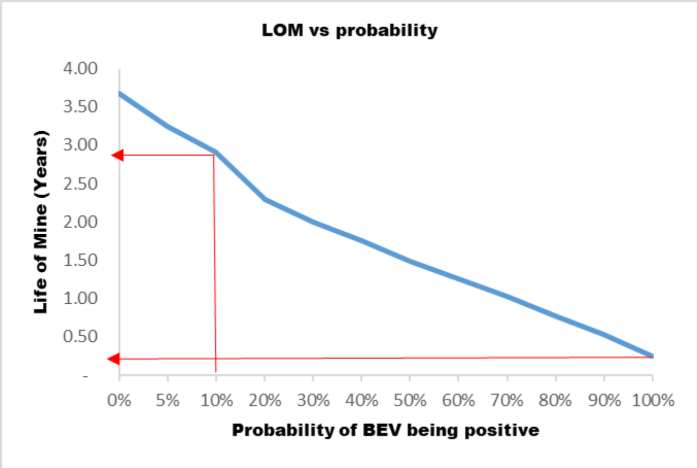

Figure 5.20: The relationship between LOM and probability

Figure 5.20 shows that the LOM for the P10 economic block model will be 2.92 years while the LOM for the P100 economic block model will be 0.25 years or 3 months. Therefore, to plan using an economic block model with the greatest confidence may be unsustainable and unrealistic. It results in sacrificing a significant portion of the orebody thereby reducing LOM which may render the time practically unrealistic and uneconomic. Moreover, as mentioned in the beginning of this chapter in reality, 100% confidence is unachievable. This is because all input parameters used in calculating BEVs can never be known with certainty. For example, a 100% confidence in the orebody geology requires sampling 100% of the orebody to have 100% confidence in the grade, density, lithology of the orebody, something which is not practically and economically possible. The confidence of 100% is only based on the ranges of uncertainties that were assumed for the input parameters as explained earlier in the thesis. Therefore, an orebody with an optimal confidence level must be targeted for, for mine planning purposes. Figure 5.17 shows that an orebody with a probability range of between 40% and 60% is the optimal one. However, this range may vary from orebody to orebody and the ultimate decision on which orebody to use for planning purposes may also vary depending on the risk tolerance of a company.

5.6 Summary of Chapter 5

This chapter has shown that the total economic value, total tonnage, and average grade of an orebody are highly uncertain. Therefore, ignoring such uncertainty can result in underestimation or overestimation of Mineral Reserves which subsequently results in significant loss of economic value. Quantifying the impact of uncertainty to grade, tonnage and economic value of an orebody allows mine planners to analyse the likely range of those key orebody parameters. One of the major impacts of uncertainty is on optimal stope boundaries. This is analysed in the next chapter.

6 PROBABILISTIC STOPE DESIGNS

6.1 Overview of Chapter 6

This chapter presents the approach taken in using the deterministic economic block model and the probabilistic economic block models presented in Chapter 5 to generate a deterministic, and probabilistic stopes, respectively. These stopes are presented and analysed in this chapter. These stope designs are based on varying levels of acceptable risk at 5%, and 10% probability intervals from 10% to 100% denoted as P5, P10 to P100 stopes, respectively. For example, a P5 stope refers to a stope design based on the P5 economic block model (Figure 5.6). Therefore, P5 and P100 stopes represent the lowest and highest-confidence stopes, respectively. All costs data in this chapter are in United States of American dollars (US$).

6.2 Mining method description

As mentioned earlier in Section 5.2, SLOS was assumed as the suitable mining method to extract the case study orebody. With the SLOS method, the orebody is divided into separate regular stopes with pillars left between stopes to support the roofwall and the hangingwall (Atlas Copco, 2008). Main levels are mined in waste and between them, sublevel drifts are mined within the orebody from where long-hole drilling of blast holes is done. In this thesis main levels were located at 60 m vertical intervals with 15 m sub-levels drives in-between. Figure 6.1 shows a snapshot of a SLOS mining method layout.

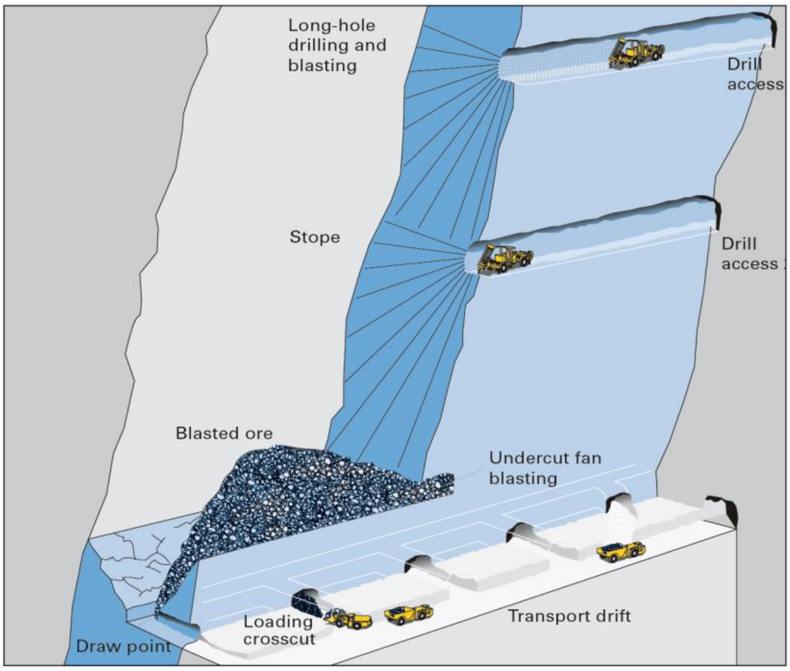

Figure 6.1: A snapshot of a sub-level open stoping layout

(Source: Atlas Copco, 2008)

6.3 Stope design parameters

As mentioned earlier in Section 3.3.8, Datamine software's MSO tool was used in this thesis to create the optimal stope boundaries from the deterministic block model and different simulated economic block models. To determine an optimal stope design in MSO, the optimisation objective can be either to maximise (Alford Mining Systems, 2011):

- Stope grade.
- Economic value.
- Total contained metal.

In this thesis the objective was to maximise total economic value (BEV). To achieve the optimisation objective, several stope layouts and mining constraints were considered. Table 6.1 shows a summary of the key input parameters (constraints) used in the stope optimisation process.

Table 6.1: MSO input parameters

Optimisation constraint	Value
Apparent dip (degrees)	80
Sub-level spacing (m)	15
Stope width (m)	5 - 10
Slicing interval (m)	0.5

6.4 Deterministic stopes

Figure 6.2 shows the deterministic stopes generated from the deterministic economic orebody model (Figure 5.2).

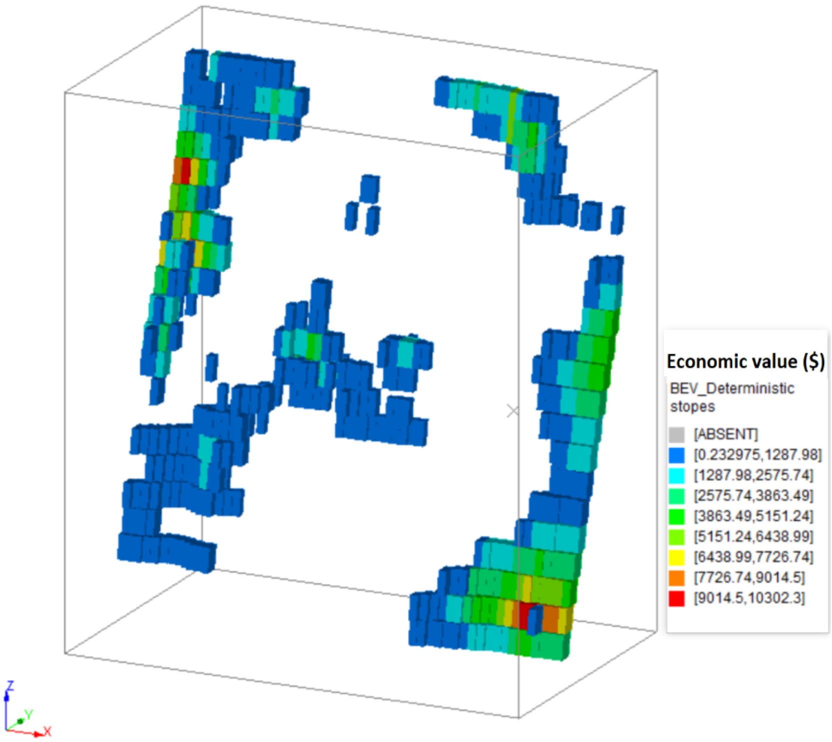

Figure 6.2: Deterministic stopes

The deterministic stopes in Figure 6.2 consist of a total of 463 stopes with a total amount of 937 465 tonnes of ore. Their total economic value is $714 614. However, like the deterministic economic block model results discussed in the previous chapter, these deterministic stope design results do not quantify the level of confidence or risk associated with the stopes, hence the need for probabilistic stope design analysis.

6.5 Probabilistic stopes

Figures 6.3 to 6.13 show the stopes that are equally probable under different future scenarios. The stopes denoted as P5, P10 to P100 stopes, respectively were generated using economic block models presented in Chapter 5. For example, P5 stopes were generated using the P5 economic block model (Figure 5.6).

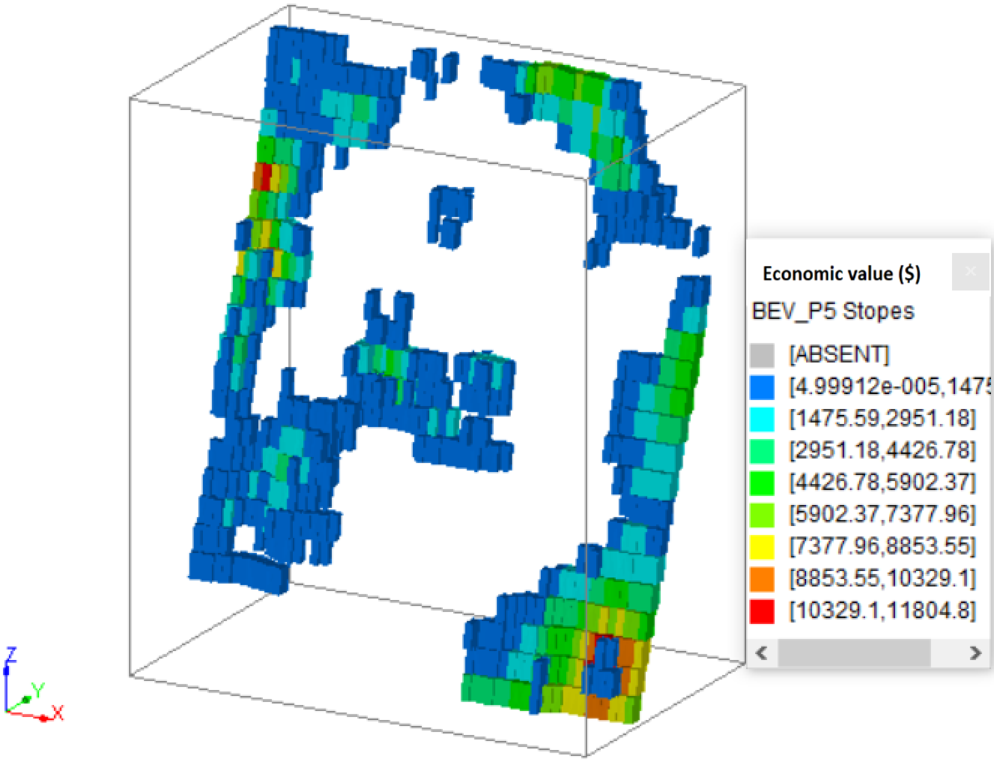

Figure 6.3: P5 stopes

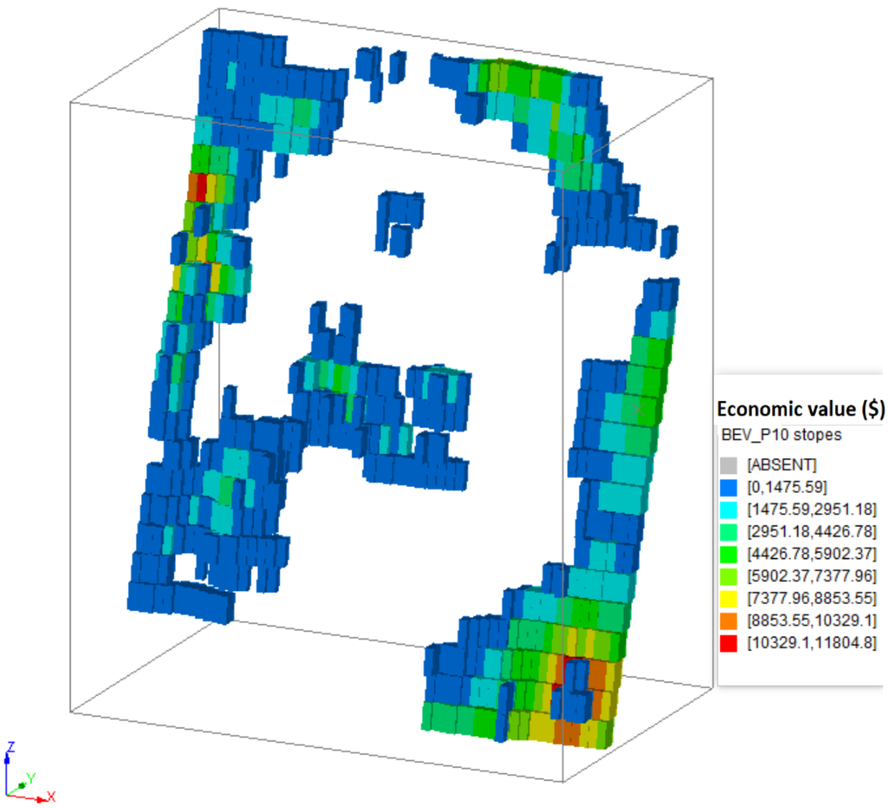

Figure 6.4: P10 stopes

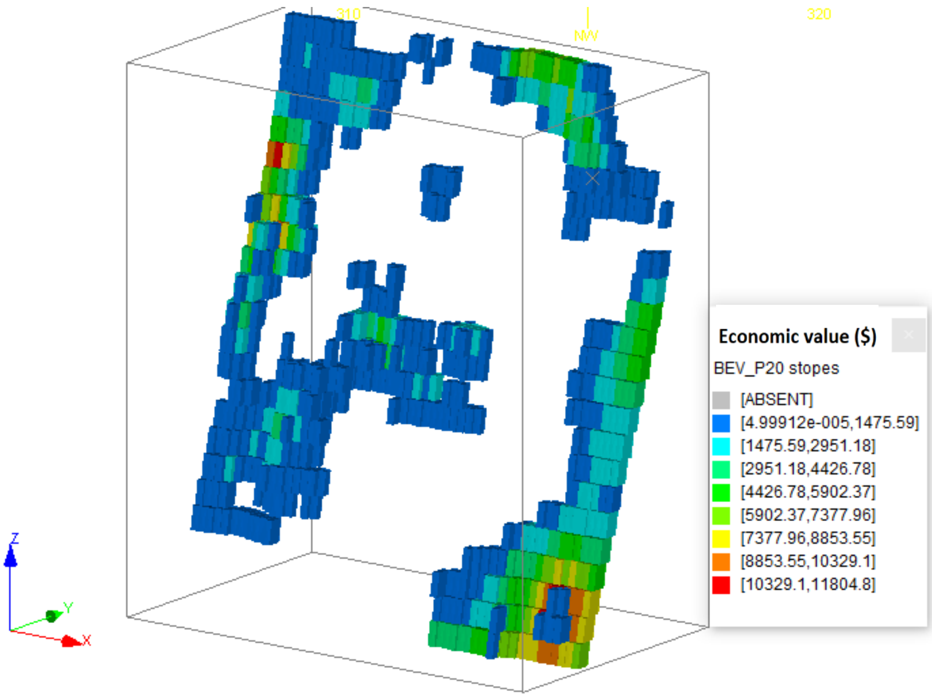

Figure 6.5: P20 stopes

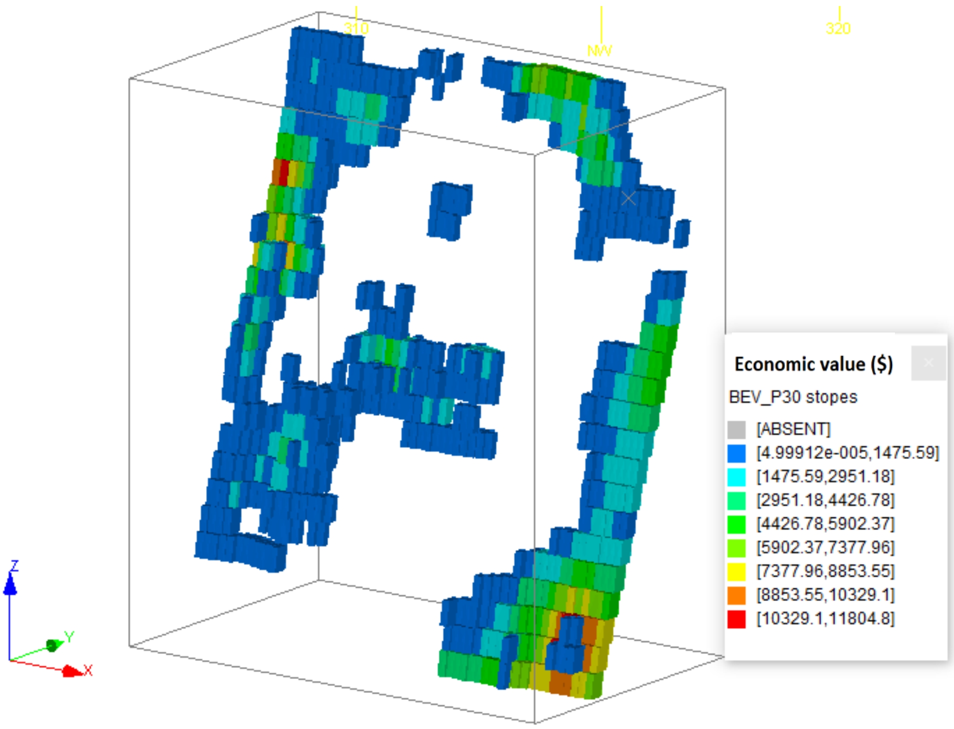

Figure 6.6: P30 stopes

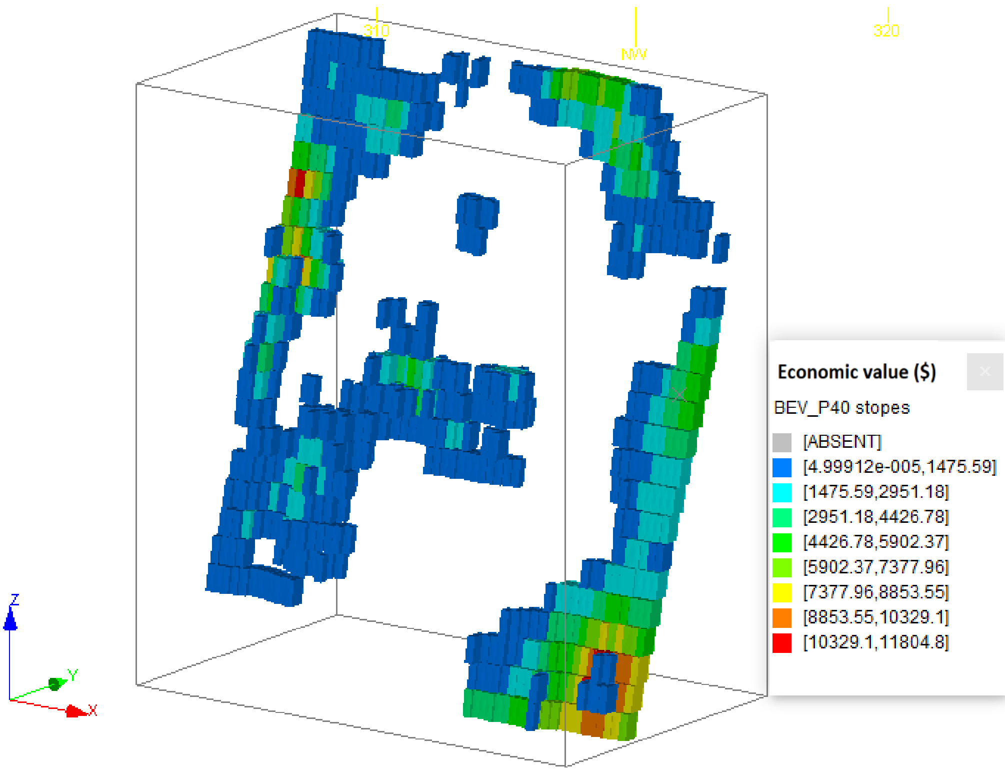

Figure 6.7: P40 stopes

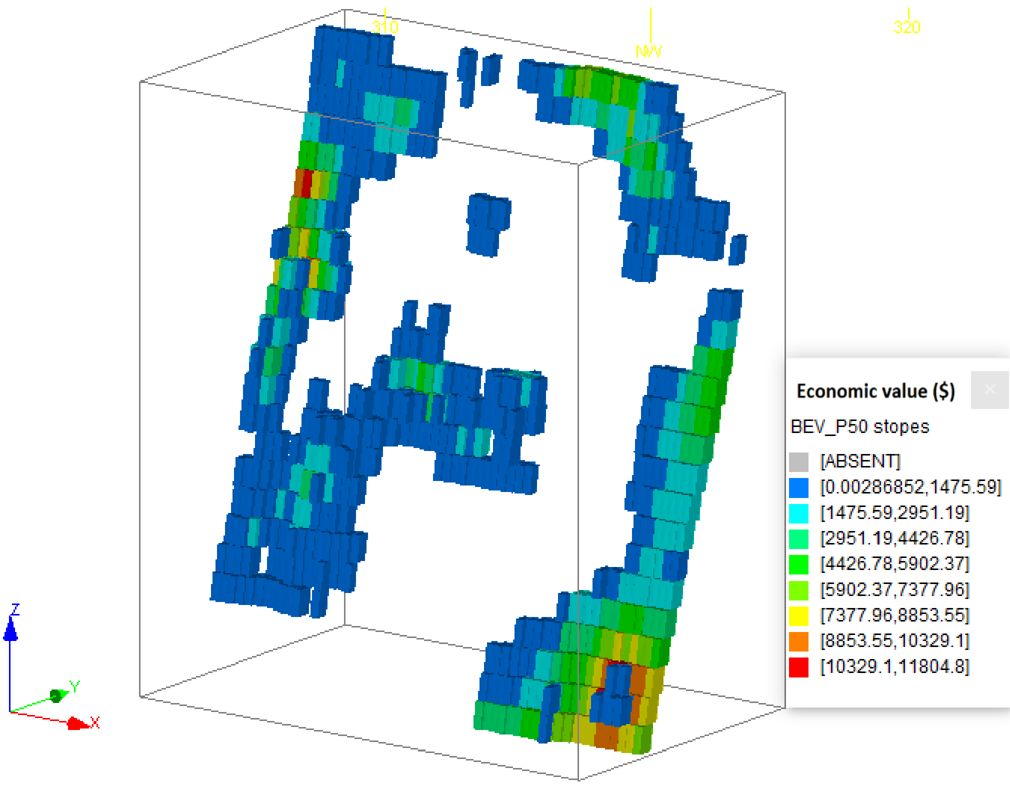

Figure 6.8: P50 stopes

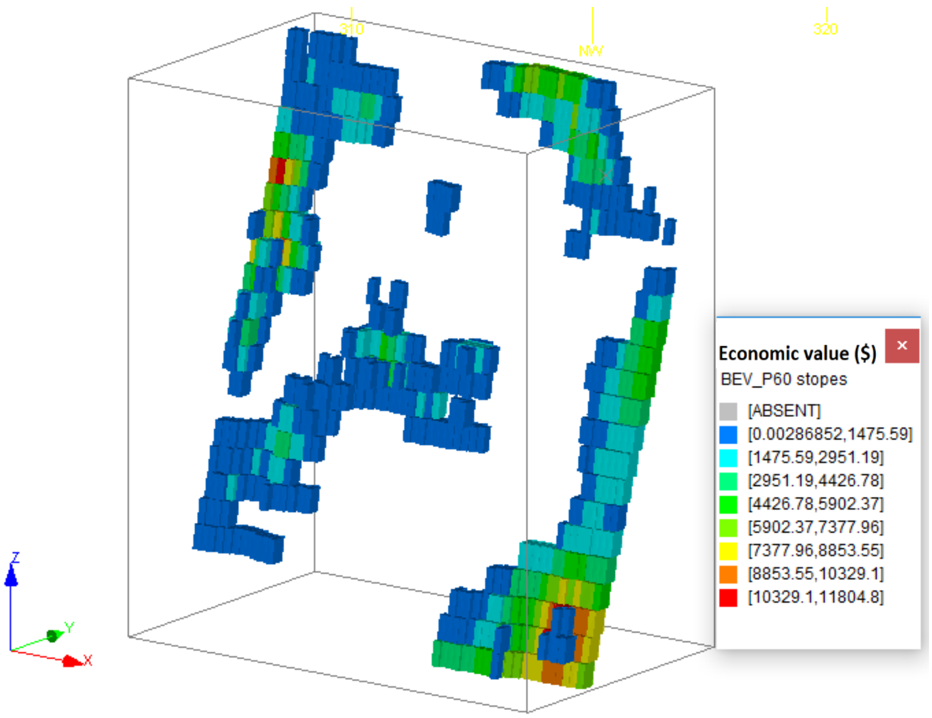

Figure 6.9: P60 stopes

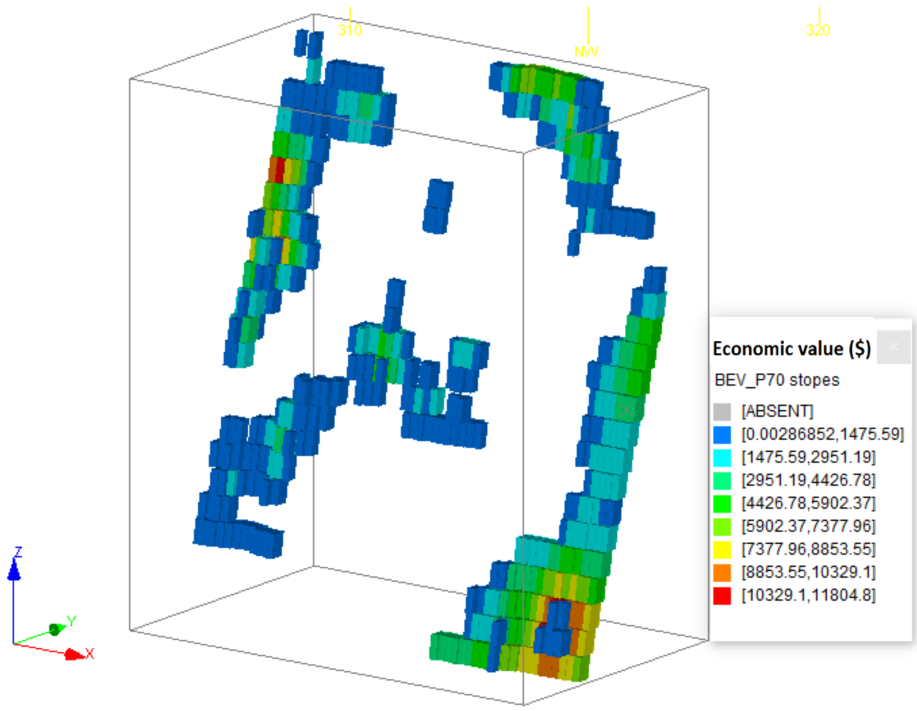

Figure 6.10: P70 stopes

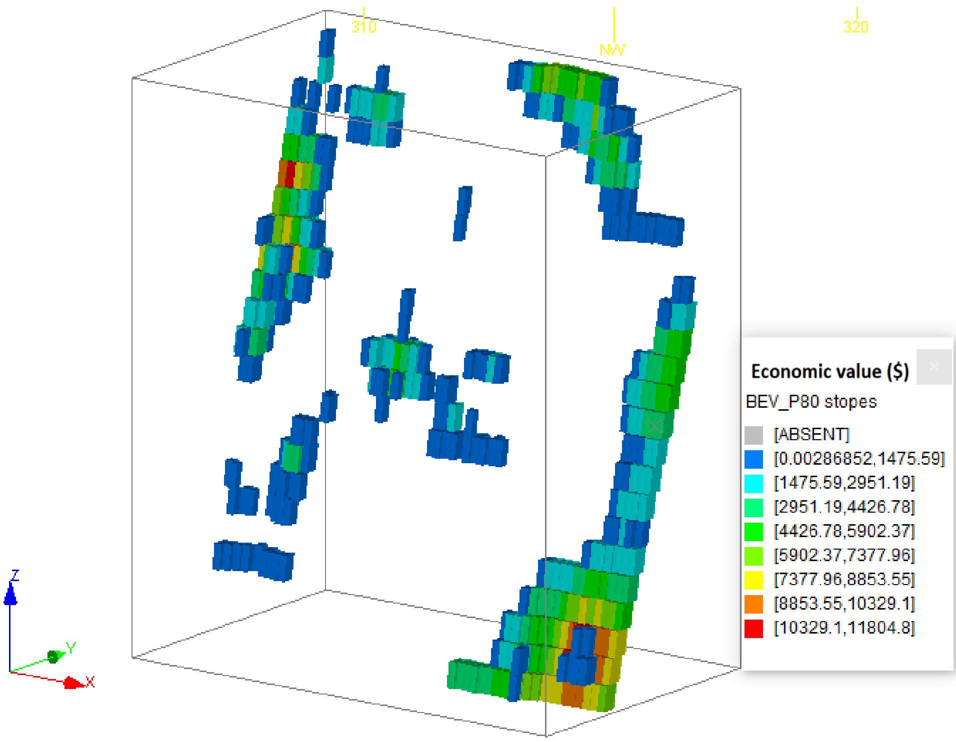

Figure 6.11: P80 stopes

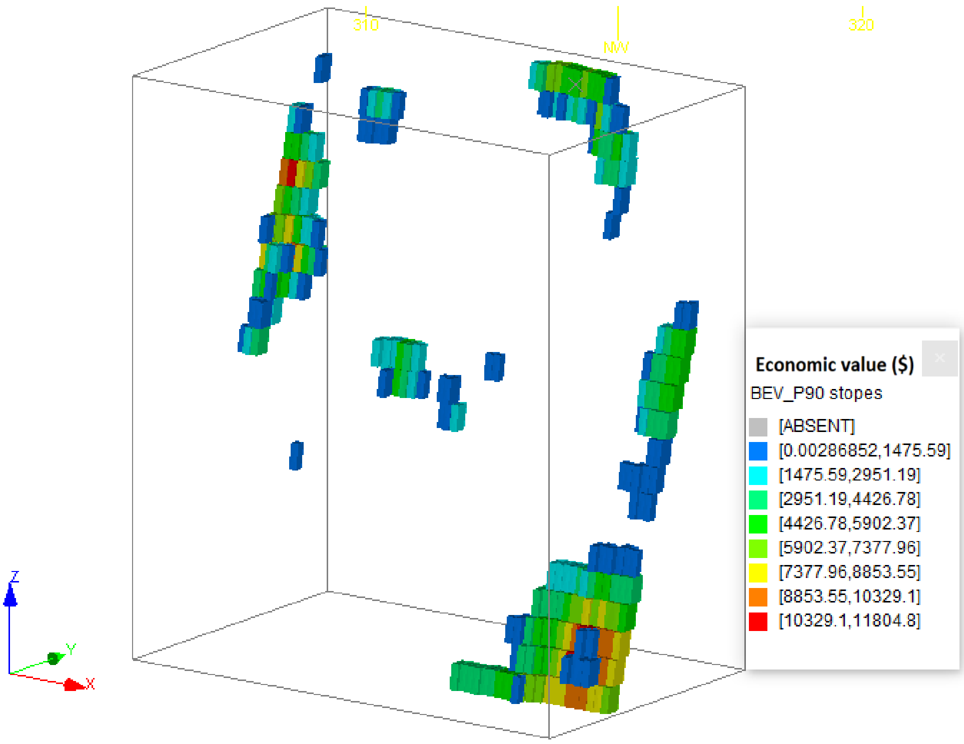

Figure 6.12: P90 stopes

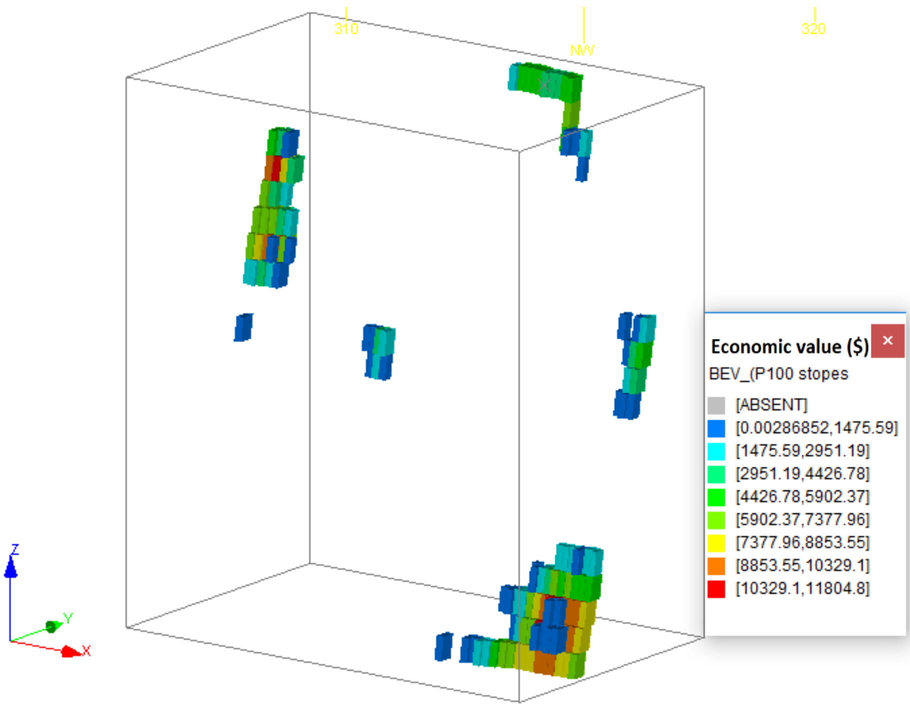

Figure 6.13: P100 stopes

6.6 Analysis of the stope design results

The stope design results are summarised in Figures 6.14 to 6.16. The figures were drawn using data in Appendix 9.4. Figure 6.14 shows the relationship between probability of BEV being positive and the total number of stopes. Figure 6.15 shows the relationship between probability of BEV being positive and the total stope tonnages.

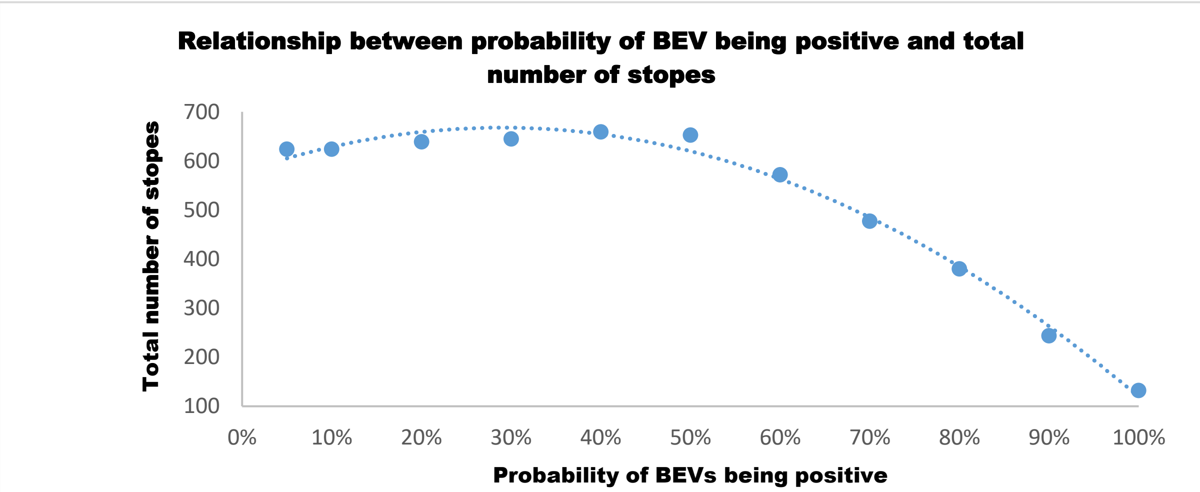

Figure 6.14: The relationship between probability of BEV being positive and the total number of stopes

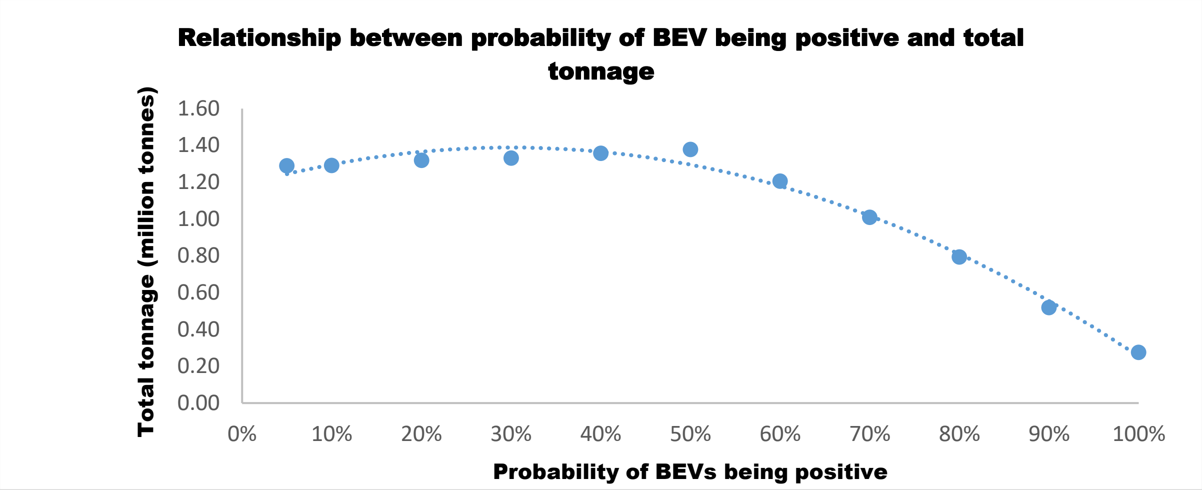

Figure 6.15: The relationship between probability of BEV being positive and the total stope tonnage

Figures 6.14 and 6.15 follow a similar trend displayed by the economic block models' results summarised in Figure 5.17. The figures show that as probability increases from 5% to approximately 50%, the number of stopes and the total tonnage increases *albeit* slightly. This is also evident on Figures 6.3 to 6.8 which show no significant visible difference in P5 to P50 stope designs. However, as probability increases from 50% to 100%, both the total number of stopes and total stope tonnage decrease gradually; a trend also visible on Figures 6.9 to 6.13 which shows a remarkable difference in P80 to P100 stope designs. Even though a stope design with 100% probability of its blocks being economic would offer the best confidence, it sacrifices a significant amount of ore tonnage, hence would not be practical and economic as explained in Chapter 5. It is only based on the ranges of input parameters sampled from the assumed probability distributions used to calculate BEVs. Figure 6.16 shows how the total economic value of stopes is affected by increasing confidence in the orebody.

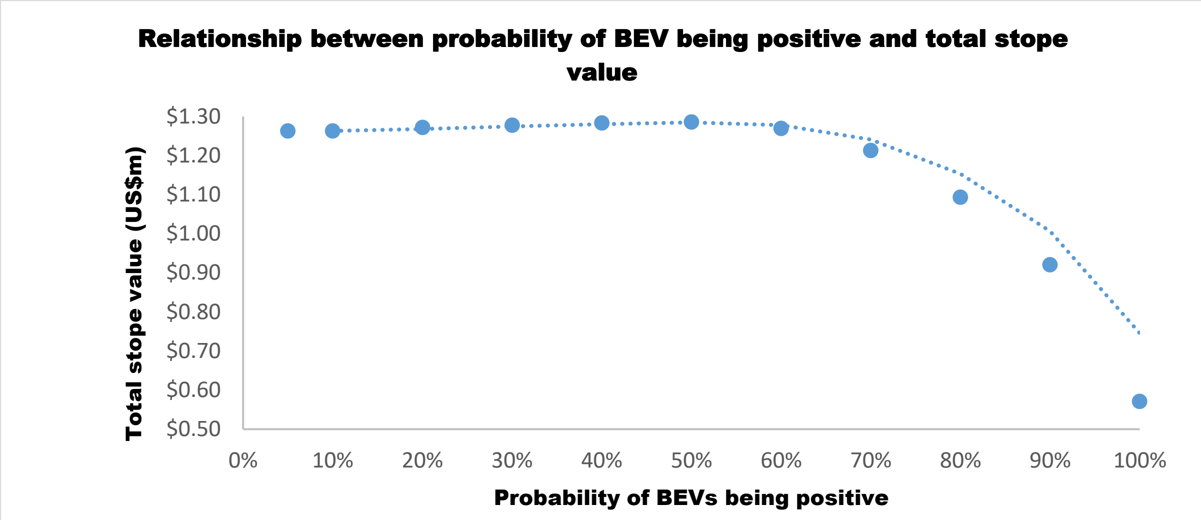

Figure 6.16: The relationship between probability of BEV being positive and total stope economic value

Figure 6.16 shows that a combination of reduced number of stopes (Figure 6.14) and reduced orebody tonnage (Figure 6.15) results in a significant reduction in the total economic value of the orebody. The figure shows similar trends (discussed in previous sections) of slightly increasing trends from 5% probability to 50% probability and a significant declining trend from 50% to 100% probability. In practise, it is expected that as confidence increases, the risk of the project failing to achieve its planning targets reduces. However, like economic block models explained earlier in Chapter 5, the higher confidence stopes may also be practically and economically unsustainable. The results in Figures 6.14 to 6.16 support the results in Figure 5.17 that the optimal stope design for planning purposes is obtained at a probability range of 40% to 60%.

Even though the time value of money was not considered in this thesis in calculating BEVs, the research finding can assist mine planners with deciding on which economic block model to use in the mine design and planning process. In addition to the impact of uncertainty on the total number of stopes, total stope tonnages and total economic value the different stope designs also show that uncertainty impacts the amount of development infrastructure required.

6.7 Impact of uncertainty on development infrastructure

Development in a typical SLOS mine is categorised into three main types which are primary, secondary and tertiary development. Primary development is the development done to access the underground workings from surface including shafts, ramps, and declines. According to O'Hara (1980), a production shaft should be located near the centre of gravity of the shape of the orebody (in plan view). Therefore, in this study, the shaft and ramp were located at the centre of the orebodies for all the evaluated stope designs. On the other hand, secondary development provides horizontal access to the orebody whilst tertiary development basically divides the orebody into mining areas. This thesis analysed primary and secondary development only. Basic conceptual SLOS designs were done for each stope design using Datamine software. Table 6.2 shows the input assumptions that were used in designing the primary and secondary development.

Table 6.2: Development design parameters

Excavation type	Value
Shaft dimensions (m)	3 x 2
Ramp gradient (degrees)	-10
Ramp dimensions (m)	5 x 3
Main level spacing (m)	60
Main level dimensions (m)	5 x 3
Sublevels dimension (m)	4 x 3
Sublevels spacing (m)	15
Strike drives dimensions (m)	4 x 3

Figures 6.17 to 6.23 show the side and front views of the development designs for the deterministic, P10, P30, P50, P70, P90 and P100 stopes. Each design consists of a ramp (blue), vertical shaft (red), main levels (green), sub-levels (turquoise), and strike drives (purple).

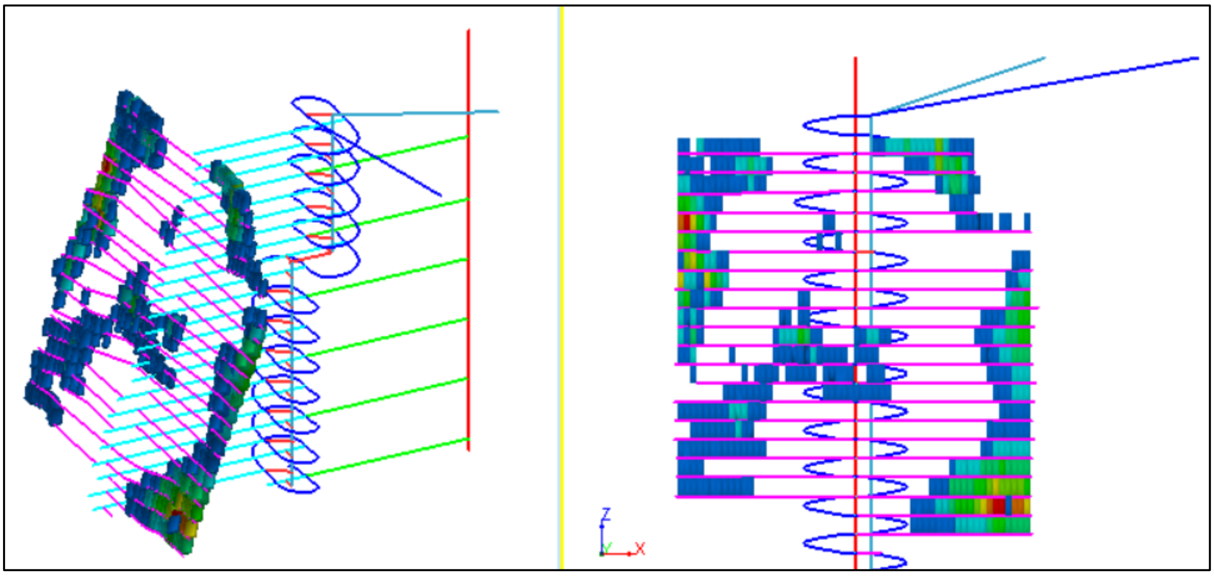

Figure 6.17: Deterministic development design

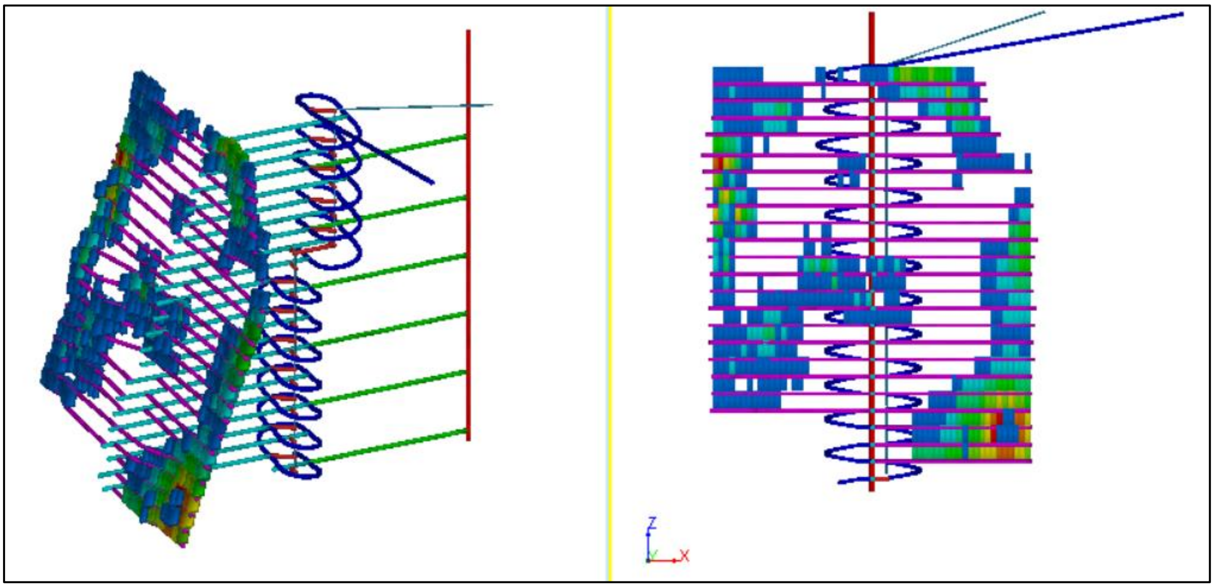

Figure 6.18: P10 development design

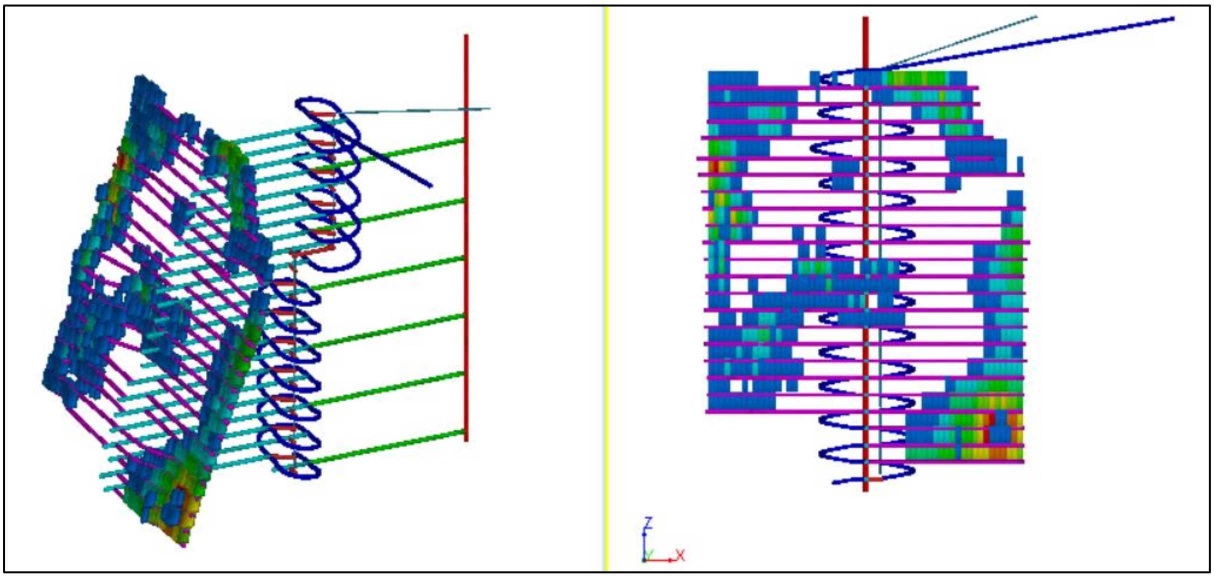

Figure 6.19: P30 development design

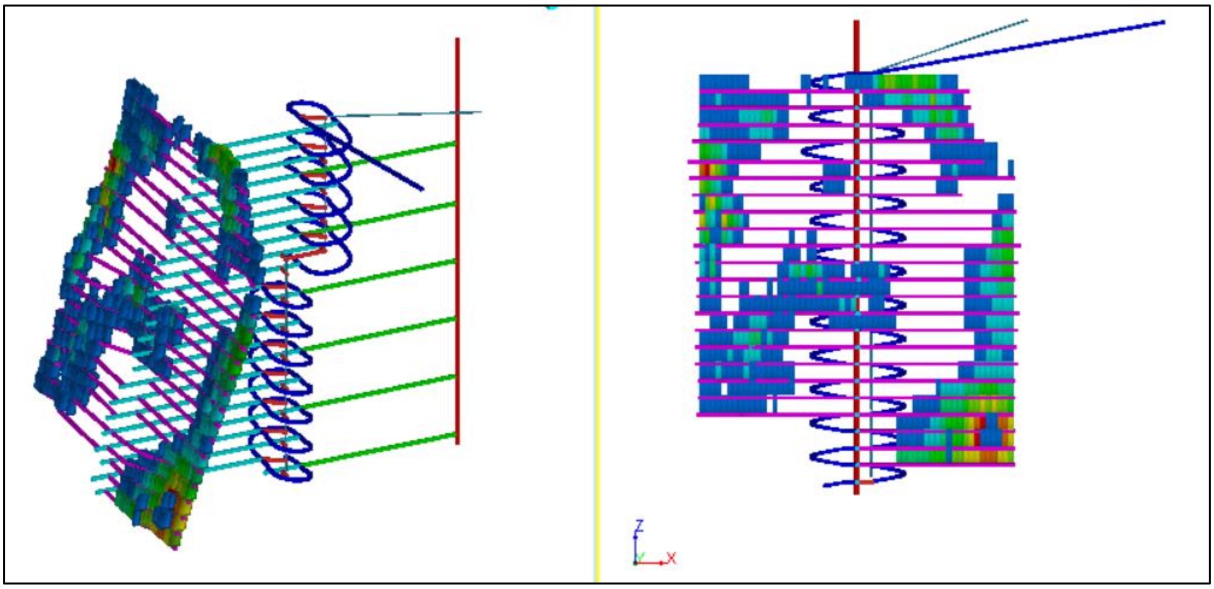

Figure 6.20: P50 development design

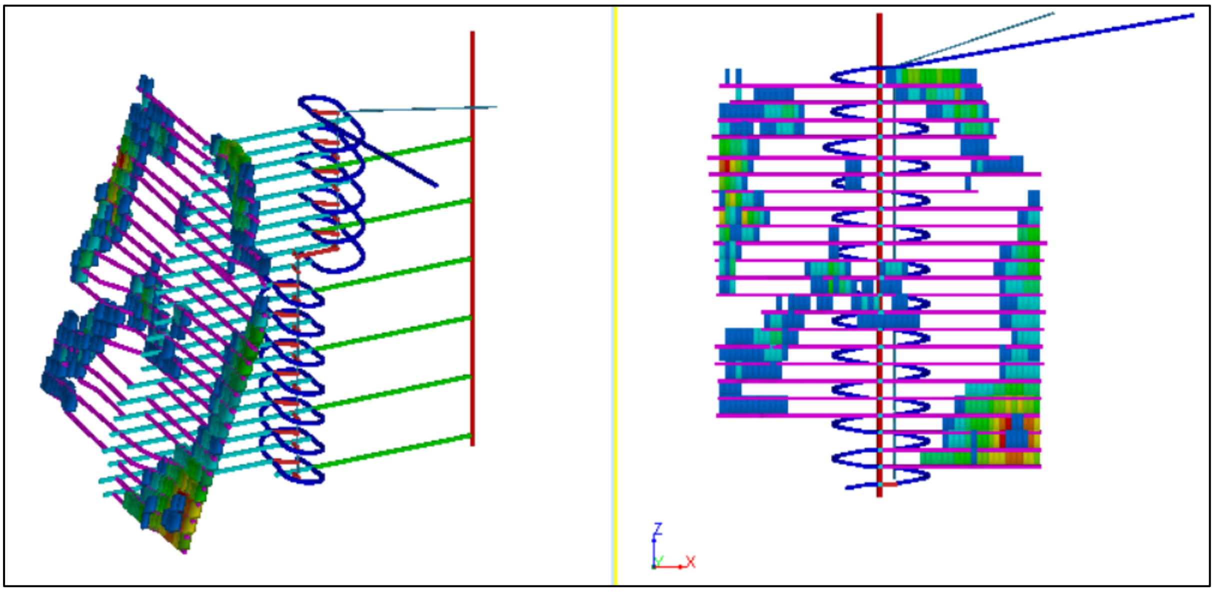

Figure 6.21: P70 development design

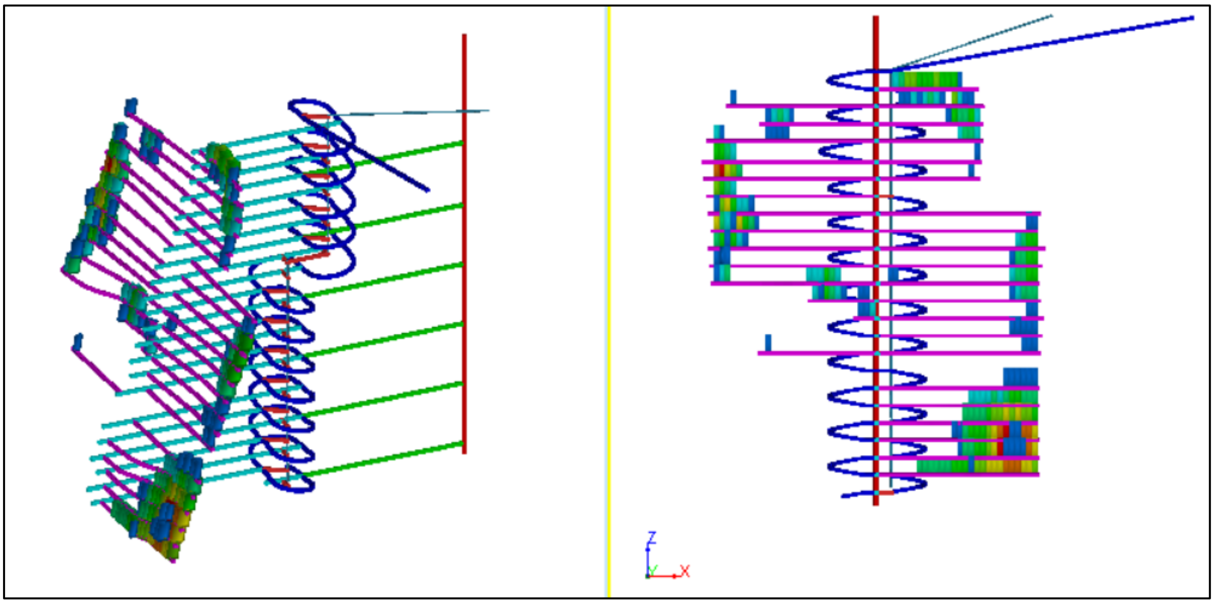

Figure 6.22: P90 development design

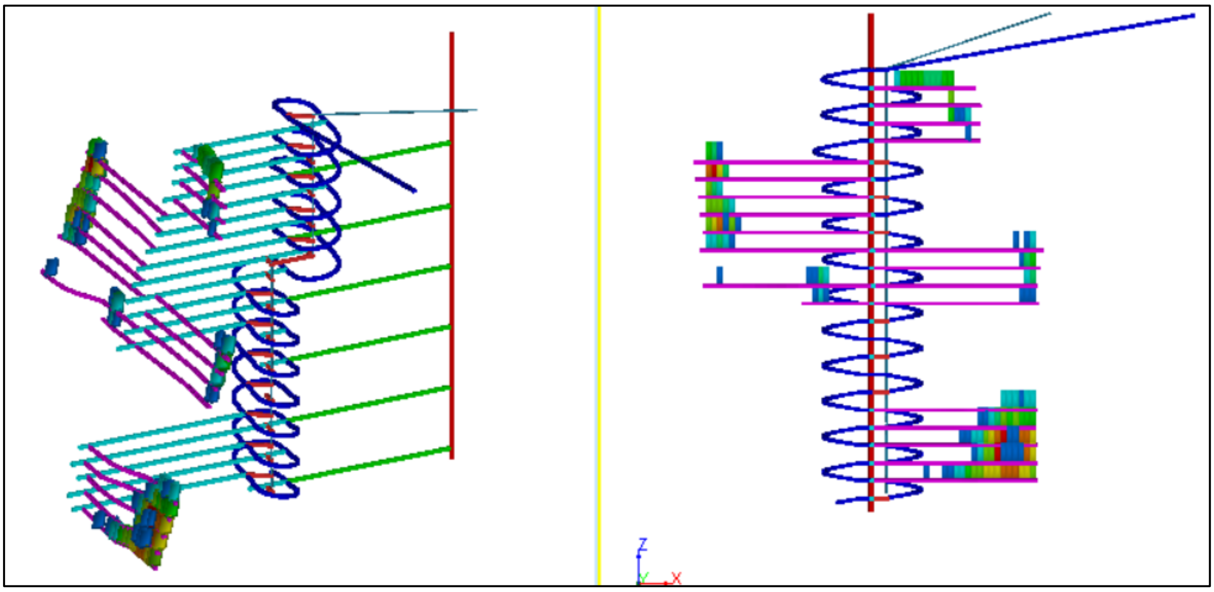

Figure 6.23: P100 development design

Figures 6.17 to 6.23 show that uncertainty has a significant effect on the amount of primary and secondary development required. As mentioned earlier, shafts and ramps were located at the centre of each orebody. This resulted in the position of the shaft and ramp being the same for all the designs because both the lateral and vertical extends of the orebodies were the same for the deterministic and all probabilistic economic block models. Therefore, these results strengthen the advantage of probabilistic orebody evaluation discussed in previous chapters that it provides confidence in the location of key underground development infrastructure. As shown in Figures 6.17 to 6.23 the mine design engineer would be confident that regardless of any future changes in economic and geological assumptions used during mine design, the location of primary development remains the same. From each design, the total development meters for all development excavations were calculated. Figure 6.24 shows the different types of development and their total lengths or depth (for the shaft) in relation to probability.

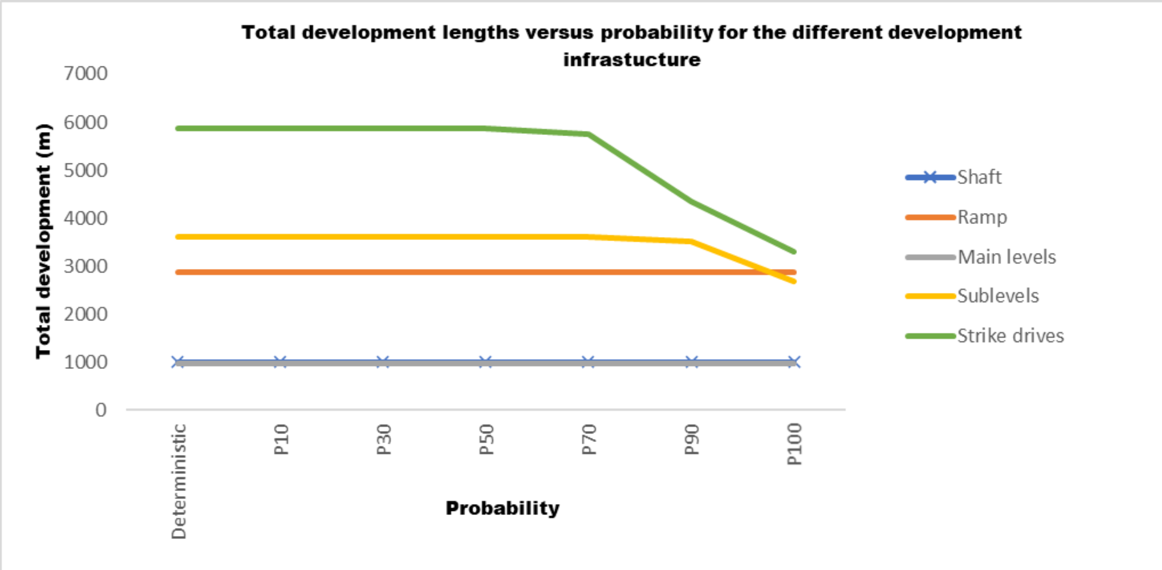

Figure 6.24: Type of development excavation versus probability

Figure 6.24 shows that the shaft, ramp and main levels have the same total depths or lengths for all the designs. Therefore, the same amount of development of these excavations is required regardless of the confidence in the economic value of stopes. This is because both the lateral and vertical extends of the orebody remained the same as probability increases. The same is true for strike drives and sublevels up to 50% and 70% probability, respectively. However, as the confidence increases from 50% for strike drives and 70% for sublevels, the total length of development reduces because the size of the orebody reduces significantly. As the size of the orebody reduces, some portions of the orebody become uneconomic such that no development is required to access those portions. These results are useful when estimating the range of capital required to develop a mine.

From the total development length or depth for each development excavation, the total development cost was calculated. Table 6.3 shows the cost assumptions used in calculating total development costs. The costs were based on the anonymous mine mentioned earlier in Section 1.5. The total cost of development for each design are shown in Appendix 9.5.

Table 6.3: Cost of development for each type of excavation

Item	Cost
Shaft development cost ($/m)	2 000
Ramp development cost ($/m)	1 800
Main level development cost ($/m)	730
Sublevels development cost ($/m)	500
Strike drives cost ($/m)	500

A conceptual DCF analysis was done for the respective designs to calculate the NPV for each design. This was done to analyse the combined effect on NPV of the observed results of decreasing tonnage, LOM and development cost, and increasing average orebody grade as confidence increase. The data used in the DCF analysis include total stope tonnage and average grade of the orebodies, total capital cost of development for the respective designs together with the average gold price, processing recovery rate, mining cost, and processing cost assumptions used earlier in the thesis. The royalty rate and tax rate formulae applicable to South African gold mines were used. Figure 6.25 shows the NPV results for the designs. The detailed discounted cashflows are shown in Appendix 9.6.

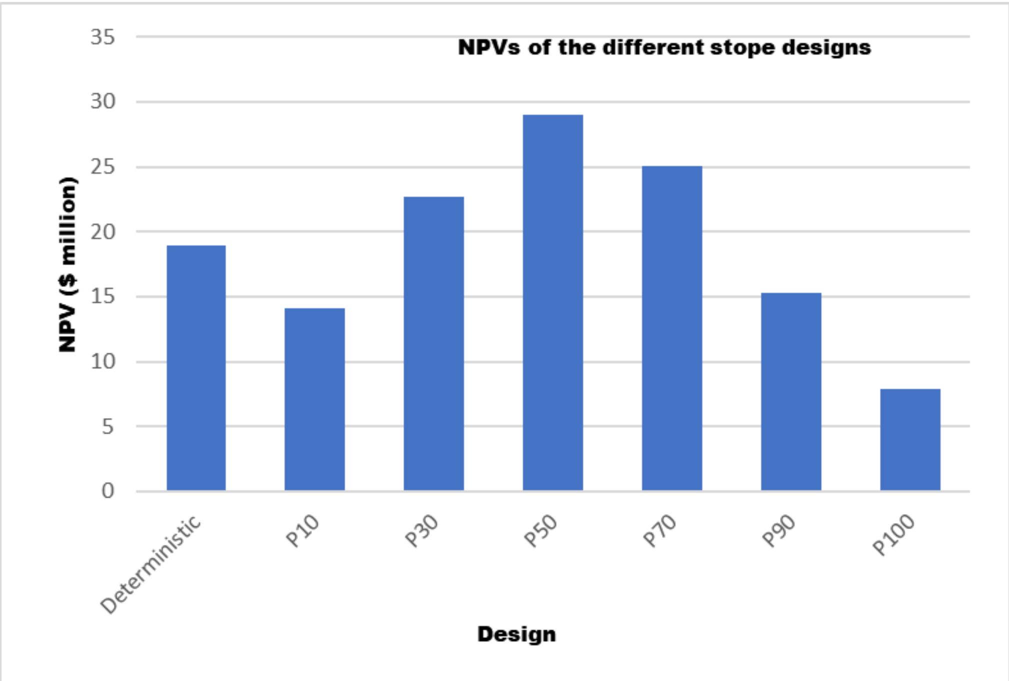

Figure 6.25: NPV results for the different conceptual stope designs

Figure 6.25 shows that as probability increases from 10% to 50%, NPV also increases to a peak of approximately $29 m at 50% probability then declines as probability further increases to 100% as explained in Chapter 5. These results confirm the results in Chapter 5 that the P50 design provides the highest NPV of approximately $29 m for the analysed designs compared with a NPV of $19 m for the deterministic design. Therefore, the results show that deterministic BEV calculation underestimated the true potential of the orebody because it does not quantify the upside or downside risk associated with BEVs. Based on these results for the particular orebody and assumptions made in this study, the orebody with approximately 50% of its blocks being positive should be used for planning purposes.

6.8 Summary of Chapter 6

The chapter has shown that the number of stopes and total tonnage of the stopes increase slightly as confidence in the orebody increases from 5% to approximately 50% but, decrease significantly as probability increases from 50% to 100%. The combined effect of the reduced number of stopes and tonnage beyond 50% confidence is the reduction in the total economic value of the orebody and NPV of the project. The chapter has also shown that regardless of the confidence in the orebody, the lateral and vertical extends of the orebody remains constant. This resulted in the same amount of primary development being required to access the different orebodies analysed. On the other hand, the amount of secondary development remains constant as probability increase from 5% to approximately 50%. However, as probability increases from 50% to 100%, the size of the orebody significantly reduces such that the amount of secondary development required also reduces. These results are important when estimating capital required during mine planning. They provide a range of capital that may be required to develop a mine under different future scenarios. The chapter has also shown that for planning purposes, the orebody with at least 50% probability of this blocks being economic should be used because it resulted in the highest NPV of $29 m compared with $19 m for the deterministic design. Therefore, the results show that deterministic BEV calculation underestimated the true potential of the orebody by 35%.

7 CONCLUSIONS AND RECOMMENDATIONS

7.1 Conclusions

Mine design and planning is a key process that defines an optimal strategy to exploit an orebody. The process starts with orebody evaluation, a process which defines the spatial representation of the orebody. The orebody is then divided into equally sized blocks called a geological block model. Each block within the geological block model contains its specific characteristic geological data; most importantly grade which is estimated using geostatistical methods. The geological model is then converted into an economic block model by calculating the net economic worth of each block called BEV. The economic block model is a key input for the mine design and planning process; subsequent processes that include stope design subordinate it. Therefore, its realistic representation in 3D in terms of its size and quality is important. However, traditional methods used to calculate BEVs are mostly deterministic. They assume certainty in BEV input parameters (such as grade, commodity price and costs). In reality, BEVs are a function of uncertain variables including grade, commodity price and costs. In practice during the mine planning stage, mine planners cannot know with certainty the material quantity and quality of an orebody as well as future commodity prices and exchange rates among other uncertain input parameters.

Literature has shown that considerable research has been done to incorporate geological and economic uncertainty in orebody modelling *albeit* in a fragmented way where either geological or economic uncertainty is considered in isolation. Also, the research has mostly been applied to open pit mine design optimisation. To address this, a probabilistic BEV calculation approach was developed in this thesis as summarised in Figure 4.1. The approach integrates uncertainty in some geological and economic factors used to calculate BEVs which are then used to create probabilistic orebody models and subsequently create probabilistic stope designs. The aim of the research was to demonstrate the impact of geological and economic uncertainty on the economic value of mineral deposits and ultimately on stope designs.

This research has found that geological and economic uncertainty significantly affect the size, average grade, and total economic value of an orebody and subsequently NPV of a project. It was found that as confidence in the BEV increases:

1. The size, hence, tonnage of the orebody, the number of stopes and LOM decreases.
2. The average grade of the orebody increases.

3. The total economic value of the orebody and NPV of the project increases to a peak point at approximately 50%. However, as the probability further increases to 100%, the total economic value of the orebody and NPV of the project decreases.
4. The amount of primary access development remains constant.
5. The amount of secondary and most likely tertiary development reduces since the orebody gets smaller.

7.2 Contributions to knowledge

The following are the thesis' contribution to knowledge:

1. The most important contribution is the development of a new probabilistic BEV calculation approach that simultaneously incorporate uncertainty associated with geological and economic parameters used in calculating BEVs. This ensures that uncertainty intrinsic to the orebody is addressed at an early stage in the mine design and planning process. The importance of addressing uncertainty early in the mine design and planning process is to establish a solid foundation for creating robust mine designs and plans later in the design and planning process which is important for the economic success of a mining project. As mentioned in Section 1.2.3 and discussed later in the literature review section of this thesis, very limited literature is available on integrated probabilistic stope envelope definition, and this research contributes towards this important mine planning area. It contributes towards the few available probabilistic underground mine design and planning optimisation approaches.
2. The other contribution is to show that planning using an orebody with the greatest confidence may not be sustainable and realistic. This is because a high confidence orebody sacrifices a significant portion of the orebody thereby reducing LOM and ultimately NPV.
3. The thesis showed that an orebody with a confidence level of at least approximately 50% probability is the optimal one for design and planning purposes. However, the level of risk hence the optimal range of probability may vary from orebody to orebody depending on probability distributions underlying the BEV input parameters. Also, the ultimate decision on which confidence level to use may vary from one company to another depending on the perception of risk of a company.

7.3 Research limitations

This thesis did not consider the time value of money in calculating BEVs. Ideally, BEVs depends on the time the blocks will be extracted. Also, the NPV calculated was not based on production schedule of stopes. Therefore, the BEVs and NPV results could be different if scheduling is integrated to the processing of calculating BEVs and stope design, respectively.

The other limitation is that some BEV input variables were simulated based on an assumed probability distribution for rock density because access to actual rock density data was not possible. Also, mining and processing costs were assumed to be deterministic yet they are actually not. Therefore, application of the BEV calculation approach requires defining actual probability distributions for each uncertain variable used to calculate BEVs. Nonetheless, the probabilistic approach developed in this thesis satisfied the research aim of highlighting the impact of integrating geological and economic uncertainty on economic block models, stope designs and subsequently NPV.

7.4 Recommendations and suggestions for future research work

Even though input parameters used in calculating BEVs can never be known with certainty, this certainty should be considered during economic block modelling. Moreover, the uncertainty should be incorporated simultaneously during mine planning. This will assist in providing a range of potential economically mineable portions of the orebody under different geological, economic and technical scenarios. This can then be considered in stope design resulting in robust design and subsequent production schedules.

Like every simulation, the quality of the results obtained in this thesis depends on the quality of the input data and probability distributions assumed. The following are recommendations to improve on the application of the BEV calculation model presented in this thesis:

1. Analysing actual data for input parameters used in calculating BEVs to model the actual behaviour of BEVs. For example, an accurate modelling of density requires a good understanding of how density varies with grade and geology of an orebody. This was not possible in this thesis therefore, future and practical application of this BEV calculation approach requires developing an actual density model of the orebody similar to the current practice in grade estimation.

2. Scheduling of blocks and stopes should be considered in the BEV and NPV calculations, respectively as part of future research work to have a more realistic impact of uncertainty on the economic value of orebodies and ultimately on NPV.
3. Sensitivity analysis was done on how variability in BEV input parameters individually impacted the total economic value of the orebody, and no noticeable pattern could be established. There is an opportunity for future research work to further investigate if any noticeable patterns can be identified and explained.

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9 APPENDICES

9.1 Grade and tonnage report of the geological block model

grade and tonnage report - Notepad

File Edit Format View Help

GEOVIA Apr 07, 2021

Block model report

Block Model: simulated block model.mdl

Constraints used

a. INSIDE CONSTRAINT ORE_CONSTRAINED_CLIPPED

Keep blocks partially in the constraint : False

Au	Volume	Tonnes	Au
0.01 -> 0.5	1750.00	5600.00	0.45
0.5 -> 1.0	22150.00	70880.00	0.82
1.0 -> 2.0	185000.00	592000.00	1.62
2.0 -> 3.0	286550.00	916960.00	2.45
3.0 -> 4.0	82600.00	264320.00	3.33
4.0 -> 5.0	20400.00	65280.00	4.40
5.0 -> 6.0	200.00	640.00	5.07
Grand Total	598650.00	1915680.00	2.32

1/1

9.2 Grade, tonnage and BEV report for the deterministic economic block model

deterministic economic block model grade, tonnage report - Notepad

File Edit Format View Help

```
GEOVIA                                                              Apr 27, 2021
                              Block model report
Block Model: deterministic.mdl

Constraints used
  a.  INSIDE CONSTRAINT DETERMINISTIC

  Keep blocks partially in the constraint : False




           Bev  Volume  Tonnes    Au           Bev
------------------------------------------------
0.0 -> 20000.0  221950  710240  3.27  17707730.40
------------------------------------------------
   Grand Total  221950  710240  3.27  17707730.40
------------------------------------------------
```

9.3 Grade, tonnage and BEV reports for probabilistic economic block models

P0 economic block model

probability greater than 0% report - Notepad

File Edit Format View Help

```
GEOVIA                                                              Apr 07, 2021
                                  Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 0% REPORT

  Keep blocks partially in the constraint : False



            Bev           Au  Volume   Tonnes    Au          Bev
-------------------------------------------------------------------
-20000.0 -> 0.0   0.0 -> 2.0  150200   480640  1.73  -15977603.54
                  2.0 -> 10.0 190850   610720  2.26   -7614188.29
-------------------------------------------------------------------
      Sub Total               341050  1091360  2.03  -23591791.83
-------------------------------------------------------------------
 0.0 -> 80000.0  2.0 -> 10.0  233000   745600  3.24   19054572.25
-------------------------------------------------------------------
      Sub Total               233000   745600  3.24   19054572.25
-------------------------------------------------------------------
    Grand Total               574050  1836960  2.52   -4537219.59
-------------------------------------------------------------------
```

P5 economic block model

probability greater than 5% report - Notepad

File Edit Format View Help

```
GEOVIA                                                              Apr 24, 2021
                                  Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 5% BLOCKS

  Keep blocks partially in the constraint : False



            Bev          Au      Volume      Tonnes    Au          Bev
-----------------------------------------------------------------------
-20000.0 -> 0.0  1.0 -> 2.0    83650.00   267680.00  1.87   -7376992.56
                 2.0 -> 3.0   190850.00   610720.00  2.26   -7614188.29
-----------------------------------------------------------------------
      Sub Total               274500.00   878400.00  2.14  -14991180.85
-----------------------------------------------------------------------
 0.0 -> 20000.0  2.0 -> 3.0   106850.00   341920.00  2.79    2720680.36
                 3.0 -> 4.0    92450.00   295840.00  3.36    8848710.10
                 4.0 -> 5.0    33500.00   107200.00  4.37    7425169.75
                 5.0 -> 6.0      200.00      640.00  5.07      60012.04
-----------------------------------------------------------------------
      Sub Total               233000.00   745600.00  3.24   19054572.25
-----------------------------------------------------------------------
    Grand Total               507500.00  1624000.00  2.65    4063391.39
-----------------------------------------------------------------------
```

P10 economic block model

```
probability greater than 10% report - Notepad
File  Edit  Format  View  Help
GEOVIA                                                                  Jan 28, 2021
                               Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 10% BLOCKS

  Keep blocks partially in the constraint : False



            Bev            Au  Volume   Tonnes    Au            Bev
---------------------------------------------------------------
-20000.0 -> 0.0  1.0 -> 2.0   33050   105760  1.95   -2565390.40
                 2.0 -> 3.0  189750   607200  2.27   -7528941.13
---------------------------------------------------------------
      Sub Total              222800   712960  2.22  -10094331.54
---------------------------------------------------------------
 0.0 -> 20000.0  2.0 -> 3.0  106850   341920  2.79    2720680.36
                 3.0 -> 4.0   92450   295840  3.36    8848710.10
                 4.0 -> 5.0   33500   107200  4.37    7425169.75
                 5.0 -> 6.0     200      640  5.07      60012.04
---------------------------------------------------------------
      Sub Total              233000   745600  3.24   19054572.25
---------------------------------------------------------------
    Grand Total              455800  1458560  2.74    8960240.71
---------------------------------------------------------------
```

P20 economic block model

```
probability greater than 20% report - Notepad
File  Edit  Format  View  Help
GEOVIA                                                                  Jan 28, 2021
                               Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 20% BLOCKS

  Keep blocks partially in the constraint : False



            Bev            Au  Volume   Tonnes    Au            Bev
---------------------------------------------------------------
-20000.0 -> 0.0  1.0 -> 2.0      50      160  1.99      -3159.28
                 2.0 -> 3.0  124350   397920  2.37   -3336406.66
---------------------------------------------------------------
      Sub Total              124400   398080  2.37   -3339565.94
---------------------------------------------------------------
 0.0 -> 20000.0  2.0 -> 3.0  106850   341920  2.79    2720680.36
                 3.0 -> 4.0   92450   295840  3.36    8848710.10
                 4.0 -> 5.0   33500   107200  4.37    7425169.75
                 5.0 -> 6.0     200      640  5.07      60012.04
---------------------------------------------------------------
      Sub Total              233000   745600  3.24   19054572.25
---------------------------------------------------------------
    Grand Total              357400  1143680  2.94   15715006.31
---------------------------------------------------------------
```

P30 economic block model

probability greater than 30% report - Notepad

File Edit Format View Help

```
GEOVIA                                                                   Jan 28, 2021
                                              Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 30% BLOCKS

  Keep blocks partially in the constraint : False



            Bev           Au  Volume  Tonnes    Au           Bev
--------------------------------------------------------------
-20000.0 -> 0.0  2.0 -> 3.0    78400  250880  2.46   -1255984.57
--------------------------------------------------------------
      Sub Total                78400  250880  2.46   -1255984.57
--------------------------------------------------------------
 0.0 -> 20000.0  2.0 -> 3.0   106850  341920  2.79    2720680.36
                 3.0 -> 4.0    92450  295840  3.36    8848710.10
                 4.0 -> 5.0    33500  107200  4.37    7425169.75
                 5.0 -> 6.0      200     640  5.07      60012.04
--------------------------------------------------------------
      Sub Total               233000  745600  3.24   19054572.25
--------------------------------------------------------------
    Grand Total               311400  996480  3.05   17798587.67
--------------------------------------------------------------
```

P40 economic block model

probability greater than 40% report - Notepad

File Edit Format View Help

```
GEOVIA                                                                   Jan 28, 2021
                                              Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 40% BLOCKS

  Keep blocks partially in the constraint : False



            Bev           Au  Volume  Tonnes    Au           Bev
--------------------------------------------------------------
-20000.0 -> 0.0  2.0 -> 3.0    41000  131200  2.52    -331359.47
--------------------------------------------------------------
      Sub Total                41000  131200  2.52    -331359.47
--------------------------------------------------------------
 0.0 -> 20000.0  2.0 -> 3.0   106850  341920  2.79    2720680.36
                 3.0 -> 4.0    92450  295840  3.36    8848710.10
                 4.0 -> 5.0    33500  107200  4.37    7425169.75
                 5.0 -> 6.0      200     640  5.07      60012.04
--------------------------------------------------------------
      Sub Total               233000  745600  3.24   19054572.25
--------------------------------------------------------------
    Grand Total               274000  876800  3.14   18723212.78
--------------------------------------------------------------
```

P50 economic block model

probability greater than 50% report - Notepad

File Edit Format View Help

```
GEOVIA                                                                       Jan 28, 2021
                              Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 50% BLOCKS

  Keep blocks partially in the constraint : False



           Bev           Au  Volume  Tonnes    Au          Bev
-------------------------------------------------------------
-20000.0 -> 0.0  2.0 -> 3.0    2700    8640  2.57     -6106.22
-------------------------------------------------------------
     Sub Total                 2700    8640  2.57     -6106.22
-------------------------------------------------------------
 0.0 -> 20000.0  2.0 -> 3.0  103700  331840  2.79   2714132.46
                 3.0 -> 4.0   92450  295840  3.36   8848710.10
                 4.0 -> 5.0   33500  107200  4.37   7425169.75
                 5.0 -> 6.0     200     640  5.07     60012.04
-------------------------------------------------------------
     Sub Total               229850  735520  3.25  19048024.36
-------------------------------------------------------------
   Grand Total               232550  744160  3.25  19041918.14
-------------------------------------------------------------
```

P60 economic block model

probability greater than 60% report - Notepad

File Edit Format View Help

```
GEOVIA                                                                       Jan 28, 2021
                              Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 60% BLOCKS

  Keep blocks partially in the constraint : False



           Bev           Au  Volume  Tonnes    Au          Bev
-------------------------------------------------------------
0.0 -> 20000.0  2.0 -> 3.0    69350  221920  2.86   2398924.92
                3.0 -> 4.0    92450  295840  3.36   8848710.10
                4.0 -> 5.0    33500  107200  4.37   7425169.75
                5.0 -> 6.0      200     640  5.07     60012.04
-------------------------------------------------------------
     Sub Total               195500  625600  3.36  18732816.81
-------------------------------------------------------------
   Grand Total               195500  625600  3.36  18732816.81
-------------------------------------------------------------
```

P70 economic block model

```
probability greater than 70% report - Notepad
File  Edit  Format  View  Help
GEOVIA                                                                  Jan 28, 2021
                              Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 70% BLOCKS

  Keep blocks partially in the constraint : False




           Bev           Au  Volume  Tonnes    Au          Bev
------------------------------------------------------------
0.0 -> 20000.0  2.0 -> 3.0    35150  112480  2.93   1521987.98
                3.0 -> 4.0    92450  295840  3.36   8848710.10
                4.0 -> 5.0    33500  107200  4.37   7425169.75
                5.0 -> 6.0      200     640  5.07     60012.04
------------------------------------------------------------
     Sub Total               161300  516160  3.48  17855879.87
------------------------------------------------------------
   Grand Total               161300  516160  3.48  17855879.87
------------------------------------------------------------
```

P80 economic block model

```
probability greater than 80% report - Notepad
File  Edit  Format  View  Help
GEOVIA                                                                  Jan 28, 2021
                              Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 80% BLOCKS

  Keep blocks partially in the constraint : False




           Bev           Au  Volume  Tonnes    Au          Bev
------------------------------------------------------------
0.0 -> 20000.0  2.0 -> 3.0     3300   10560  2.97    173847.93
                3.0 -> 4.0    83850  268320  3.39   8400950.02
                4.0 -> 5.0    33500  107200  4.37   7425169.75
                5.0 -> 6.0      200     640  5.07     60012.04
------------------------------------------------------------
     Sub Total               120850  386720  3.66  16059979.74
------------------------------------------------------------
   Grand Total               120850  386720  3.66  16059979.74
------------------------------------------------------------
```

P90 economic block model

```
probability greater than 90% report - Notepad
File  Edit  Format  View  Help
GEOVIA                                                                 Jan 28, 2021
                                         Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY GREATER THAN 90% BLOCKS

  Keep blocks partially in the constraint : False



           Bev            Au  Volume  Tonnes    Au           Bev
-----------------------------------------------------------------
0.0 -> 20000.0  3.0 -> 4.0   49700  159040  3.58    6105628.94
                4.0 -> 5.0   33500  107200  4.37    7425169.75
                5.0 -> 6.0     200     640  5.07      60012.04
-----------------------------------------------------------------
     Sub Total               83400  266880  3.90   13590810.73
-----------------------------------------------------------------
   Grand Total               83400  266880  3.90   13590810.73
-----------------------------------------------------------------
```

P100 economic block model

```
probability equal to 100% report - Notepad
File  Edit  Format  View  Help
GEOVIA                                                                 Jan 28, 2021
                                         Block model report
Block Model: simulated block model.mdl

Constraints used
  a.  INSIDE CONSTRAINT PROBABILITY EQUAL TO 100% BLOCKS

  Keep blocks partially in the constraint : False



           Bev            Au  Volume  Tonnes    Au          Bev
-------------------------------------------------------------
0.0 -> 20000.0  3.0 -> 4.0    6500   20800  3.86   1028988.53
                4.0 -> 5.0   32650  104480  4.38   7273233.59
                5.0 -> 6.0     200     640  5.07     60012.04
-------------------------------------------------------------
     Sub Total               39350  125920  4.30   8362234.16
-------------------------------------------------------------
   Grand Total               39350  125920  4.30   8362234.16
-------------------------------------------------------------
```

9.4 A summary of probability stope designs

Probability greater than (%)	Total stopes value (US$m)	Total tonnage (t)	Total tonnage (mt)	Total number of stopes
Deterministic	0.71	937 465	0.94	463
5	1.26	1 290 036	1.29	624
10	1.26	1 291 673	1.29	624
20	1.27	1 319 518	1.32	639
30	1.28	1 331 193	1.33	645
40	1.28	1 358 008	1.36	659
50	1.29	1 377 609	1.38	653
60	1.27	1 205 563	1.21	572
70	1.21	1 010 096	1.01	477
80	1.09	793 361	0.79	380
90	0.92	518 743	0.52	244
100	0.57	274 969	0.27	132

9.5 Development costs

	Deterministic	P10	P30	P50	P70	P90	P100
Shaft total depth (m)	1 000	1 000	1 000	1 000	1 000	1 000	1 000
Ramp total length (m)	2 877	2 877	2 877	2 877	2 877	2 877	2 877
Main levels total length (m)	973	973	973	973	973	973	973
Sublevels total length (m)	3 661	3 661	3 661	3 661	3 661	3 506	2 693
Strike drives total length (m)	5 875	5 875	5 875	5 875	5 757	4 351	3 302
Shaft development cost ($/m)	2 000	2 000	2 000	2 000	2 000	2 000	2 000
Ramp development cost ($/m)	1 800	1 800	1 800	1 800	1 800	1 800	1 800
Main level development cost ($/m)	730	730	730	730	730	730	730
Sublevels development cost ($/m)	500	500	500	500	500	500	500
Strike drives cost ($/m)	500	500	500	500	500	500	500
Shaft total development cost ($)	2 000 000	2 000 000	2 000 000	2 000 000	2 000 000	2 000 000	2 000 000
Ramp total development cost ($)	5 177 884	5 177 884	5 177 884	5 177 884	5 177 884	5 177 884	5 177 884
Main level total development cost ($)	710 310	710 310	710 310	710 310	710 310	710 310	710 310
Sublevels total development cost ($)	1 830 684	1 830 684	1 830 684	1 830 684	1 830 684	1 753 078	1 346 331
Strike drives cost ($)	2 937 641	2 937 641	2 937 641	2 937 641	2 878 494	2 175 408	1 651 073
Total development cost ($)	**12 656 518**	**12 656 518**	**12 656 518**	**12 656 518**	**12 597 372**	**11 816 680**	**10 885 598**

9.6 NPVs for the different stope designs

NPV for the deterministic design

Deterministic design Discounted cashflow					
	Calendar Year	2021	2022	2023	2024
	Project Year	0	1	2	3
	Units				
Production					
Available tonnes remaining in stopes		937 465	937 465	937 465	437 465
Tonnes Milled	Tonnes			500 000	437 465
Mill Head Grade	g/t			3.27	3.27
Milled Recovery	%			84%	84%
Gold Produced	g			1 373 400	1 201 629
Gold Produced	oz			44 161	38 638
Revenue					
US Gold Price	$/oz			1 520	1 520
Gross Sales Revenue	$ million			**67.12**	**58.73**
Costs					
Mining cost	$/t			50.00	50.00
Processing cost	$/t			30.00	30.00
Total unit cost	$/t			80.00	80.00
Total Costs	$ millions			**-40.00**	**-35.00**
Net Sales Revenue (operating profit)	$ millions			**27.12**	**23.73**
Tax Shield					
Capital Expenditure	$ millions		-12.66		
Gold Mine Capital Allowance (12%)	$ millions			-1.52	
Unredeamed Capex	$ millions		-12.66		
Net Sales Revenue after Capex	$ millions		**-12.66**	**27.12**	**23.73**
Royalties					
EBIT for Royalty Calculation	%		**0.00**	**12.95**	**23.73**
X=(EBIT Royalty /Gross Sales)			0.00	19.29	40.41
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	2.04%	3.73%
Royalty Paid	$ millions		**0.00**	**-1.37**	**-2.19**
Income Tax					
Taxable Income	$ millions		0.00	11.58	21.54
X=(Taxable Income/Gross Sales)	%		0.00	17.25	36.68
Gold Tax Rate (34-170/X)	%		0.00%	24.14%	29.36%
Income Tax Payable	$ millions		**0.00**	**-2.80**	**-6.33**
Cash Flow					
Actual Cashflow after tax	$ millions		**-12.66**	**22.96**	**15.21**
Discounted Cashflow	$ millions	**10%**	**-11.51**	**18.97**	**11.43**
NPV	$ millions	**18.90**			

NPV for the P10 design

P10 Discounted cashflow						
	Calendar Year	2021	2022	2023	2024	2025
	Project Year	0	1	2	3	4
	Units					
Production						
Available tonnes remaining in stopes		1 291 673	1 291 673	1 291 673	791 673	291 673
Tonnes Milled	Tonnes			500 000	500 000	291 673
Head Grade	g/t			2.74	2.74	2.74
Recovery	%			84%	84%	84%
Gold Produced	g			1 150 800	1 150 800	671 315
Gold Produced	oz			37 003	37 003	21 586
Revenue						
US Gold Price	$/oz			1 520	1 520	1 520
Gross Sales Revenue	$ million			**56.24**	**56.24**	**32.81**
Costs						
Mining Cost	$/t			50.00	50.00	50.00
Processing Cost	$/t			30.00	30.00	30.00
TotalUnit Cost	$/t			80.00	80.00	80.00
Total Costs	$ millions			**-40.00**	**-40.00**	**-23.33**
Net Sales Revenue (operating profit)	$ millions			**$ 16.24**	**$ 16.24**	**$ 9.48**
Tax Shield						
Capital Expenditure	$ millions		-12.66			
Gold Mine Capital Allowance (12%)	$ millions			-1.52		
Unredeamed Capex	$ millions		-12.66			
Net Sales Revenue after Capex	$ millions		**-12.66**	**16.24**	**16.24**	**9.48**
Royalties						
EBIT for Royalty Calculation	%		**0.00**	**2.07**	**16.24**	**9.48**
X=(EBIT Royalty /Gross Sales)			0.00	3.68	28.88	28.88
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	0.79%	2.81%	2.81%
Royalty Paid	$ millions		**0.00**	**-0.45**	**-1.58**	**-0.92**
Income Tax						
Taxable Income	$ millions		0.00	1.62	14.66	8.55
X=(Taxable Income/Gross Sales)	%		0.00	2.89	26.07	26.07
Gold Tax Rate (34-170/X)	%		0.00%	-24.92%	27.48%	27.48%
Income Tax Payable	$ millions		**0.00**	**0.40**	**-4.03**	**-2.35**
Cash Flow						
Actual Cashflow after tax	$ millions		**-12.66**	**16.20**	**10.63**	**6.20**
Discounted Cashflow	$ millions	**10%**	**-11.51**	**13.39**	**7.99**	**4.24**
NPV	$ millions	**14.11**				

NPV for the P30 design

P30 Discounted cashflow						
	Calendar Year	2021	2022	2023	2024	
	Project Year	0	1	2	3	4
	Units					
Production						
Available tonnes remaining in stopes		1 331 193	1 331 193	1 331 193	831 193	331 193
Tonnes Milled	Tonnes			500 000	500 000	331 193
Mill Head Grade	g/t			3.05	3.05	3.05
Milled Recovery	%			84%	84%	84%
Gold Produced	g			1 281 000	1 281 000	848 518
Gold Produced	oz			41 190	41 190	27 284
Revenue						
US Gold Price	$/oz			1 520	1 520	1 520
Gross Sales Revenue	$ million			**62.61**	**62.61**	**41.47**
Costs						
Mining cost	$/t			50.00	50.00	50.00
Processing cost	$/t			30.00	30.00	30.00
Total unit cost	$/t			80.00	80.00	80.00
Total Costs	$ millions			**-40.00**	**-40.00**	**-26.50**
Net Sales Revenue (operating profit)	$ millions			**22.61**	**22.61**	**14.98**
Tax Shield						
Capital Expenditure	$ millions		-12.66			
Gold Mine Capital Allowance (12%)	$ millions			-1.52		
Unredeamed Capex	$ millions		-12.66			
Net Sales Revenue after Capex	$ millions		**-12.66**	**22.61**	**22.61**	**14.98**
Royalties						
EBIT for Royalty Calculation	%		**0.00**	**8.43**	**22.61**	**14.98**
X=(EBIT Royalty /Gross Sales)			0.00	13.47	36.11	36.11
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	1.58%	3.39%	3.39%
Royalty Paid	$ millions		**0.00**	**-0.99**	**-2.12**	**-1.41**
Income Tax						
Taxable Income	$ millions		0.00	7.45	20.49	13.57
X=(Taxable Income/Gross Sales)	%		0.00	11.89	32.72	32.72
Gold Tax Rate (34-170/X)	%		0.00%	19.70%	28.80%	28.80%
Income Tax Payable	$ millions		**0.00**	**-1.47**	**-5.90**	**-3.91**
Cash Flow						
Actual Cashflow after tax	$ millions		**-12.66**	**20.15**	**14.59**	**9.66**
Discounted Cashflow	$ millions	**10%**	**-11.51**	**16.66**	**10.96**	**6.60**
NPV	$ millions	**22.71**				

NPV for the P50 design

P50 Discounted cashflow						
	Calendar Year	2021	2022	2023	2024	
	Project Year	0	1	2	3	4
	Units					
Production						
Available tonnes remaining in stopes		1 377 609	1 377 609	1 377 609	877 609	377 609
Tonnes Milled	Tonnes			500 000	500 000	377 609
Mill Head Grade	g/t			3.25	3.25	3.25
Milled Recovery	%			84%	84%	84%
Gold Produced	g			1 365 000	1 365 000	1 030 872
Gold Produced	oz			43 891	43 891	33 147
Revenue						
US Gold Price	$/oz			1 520	1 520	1 520
Gross Sales Revenue	$ million			**66.71**	**66.71**	**50.38**
Costs						
Mining cost	$/t			50.00	50.00	50.00
Processing cost	$/t			30.00	30.00	30.00
Total unit cost	$/t			80.00	80.00	80.00
Total Costs	$ millions			**-40.00**	**-40.00**	**-30.21**
Net Sales Revenue (operating profit)	$ Millions			**26.71**	**26.71**	**20.17**
Tax Shield						
Capital Expenditure	$ millions		-12.66			
Gold Mine Capital Allowance (12%)	$ millions			-1.52		
Unredeamed Capex	$ millions		-12.66			
Net Sales Revenue after Capex	$ millions		**-12.66**	**26.71**	**26.71**	**20.17**
Royalties						
EBIT for Royalty Calculation	%		**0.00**	**12.54**	**26.71**	**20.17**
X=(EBIT Royalty /Gross Sales)			0.00	18.79	40.04	40.04
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	2.00%	3.70%	3.70%
Royalty Paid	$ millions		**0.00**	**-1.34**	**-2.47**	**-1.87**
Income Tax						
Taxable Income	$ millions		0.00	11.20	24.24	18.31
X=(Taxable Income/Gross Sales)	%		0.00	16.79	36.34	36.34
Gold Tax Rate (34-170/X)	%		0.00%	23.88%	29.32%	29.32%
Income Tax Payable	$ millions		**0.00**	**-2.67**	**-7.11**	**-5.37**
Cash Flow						
Actual Cashflow after tax	$ millions		**-12.66**	**22.70**	**17.13**	**12.94**
Discounted Cashflow	$ millions	**10%**	**-11.51**	**18.76**	**12.87**	**8.84**
NPV	$ millions	**28.97**				

NPV for the P70 design

P70 Discounted cashflow						
	Calendar Year	2021	2022	2023	2024	
	Project Year	0	1	2	3	
	Units					
Production						
Available tonnes remaining in stopes		1 010 096	1 010 096	1 010 096	510 096	10 096
Tonnes Milled	Tonnes			500 000	500 000	10 096
Mill Head Grade	g/t			3.48	3.48	3.48
Milled Recovery	%			84%	84%	84%
Gold Produced	g			1 461 600	1 461 600	29 513
Gold Produced	oz			46 997	46 997	949
Revenue						
US Gold Price	$/oz			1 520	1 520	1 520
Gross Sales Revenue	$ million			**71.44**	**71.44**	**1.44**
Costs						
Mining cost	$/t			50.00	50.00	50.00
Processing cost	$/t			30.00	30.00	30.00
Total unit cost	$/t			80.00	80.00	80.00
Total Costs	$ millions			**-40.00**	**-40.00**	**-0.81**
Net Sales Revenue (operating profit)	$ millions			**31.44**	**31.44**	**0.63**
Tax Shield						
Capital Expenditure	$ millions		-12.60			
Gold Mine Capital Allowance (12%)	$ millions			-1.51		
Unredeamed Capex	$ millions		-12.60			
Net Sales Revenue after Capex	$ millions		**-12.60**	**31.44**	**31.44**	**0.63**
Royalties						
EBIT for Royalty Calculation	%		**0.00**	**17.33**	**31.44**	**0.63**
X=(EBIT Royalty /Gross Sales)			0.00	24.25	44.01	44.01
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	2.44%	4.02%	4.02%
Royalty Paid	$ millions		**0.00**	**-1.74**	**-2.87**	**-0.06**
Income Tax						
Taxable Income	$ millions		0.00	15.58	28.56	0.58
X=(Taxable Income/Gross Sales)	%		0.00	21.81	39.98	39.98
Gold Tax Rate (34-170/X)	%		0.00%	26.21%	29.75%	29.75%
Income Tax Payable	$ millions		**0.00**	**-4.08**	**-8.50**	**-0.17**
Cash Flow						
Actual Cashflow after tax	$ millions		**-12.60**	**25.61**	**20.07**	**0.41**
Discounted Cashflow	$ millions	**10%**	**-11.45**	**21.16**	**15.08**	**0.41**
NPV	$ millions	**25.06**				

NPV for the P90 design

P90 Discounted cashflow					
	Calendar Year	2021	2022	2023	
	Project Year	0	1	2	
	Units				
Production					
Available tonnes remaining in stopes		518 743	518 743	518 743	18 743
Tonnes Milled	Tonnes			500 000	18 743
Mill Head Grade	g/t			3.9	3.9
Milled Recovery	%			84%	84%
Gold Produced	g			1 638 000	61 401
Gold Produced	oz			52 669	1 974
Revenue					
US Gold Price	$/oz			1 520	1 520
Gross Sales Revenue	$ million			**80.06**	**3.00**
Costs					
Mining cost	$/t			50.00	50.00
Processing cost	$/t			30.00	30.00
Total unit cost	$/t			80.00	80.00
Total Costs	$ millions			**-40.00**	**-1.50**
Net Sales Revenue (operating profit)	$ millions			**40.06**	**1.50**
Tax Shield					
Capital Expenditure	$ millions		-11.82		
Gold Mine Capital Allowance (12%)	$ millions			-1.42	
Unredeamed Capex	$ millions		-11.82		
Net Sales Revenue after Capex	$ millions		**-11.82**	**40.06**	**1.50**
Royalties					
EBIT for Royalty Calculation	%		**0.00**	**26.82**	**1.50**
X=(EBIT Royalty /Gross Sales)			0.00	33.50	50.04
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	3.18%	4.50%
Royalty Paid	$ millions		**0.00**	**-2.55**	**-0.14**
Income Tax					
Taxable Income	$ millions		0.00	24.28	1.37
X=(Taxable Income/Gross Sales)	%		0.00	30.32	45.53
Gold Tax Rate (34-170/X)	%		0.00%	28.39%	30.27%
Income Tax Payable	$ millions		**0.00**	**-6.89**	**-0.41**
Cash Flow					
Actual Cashflow after tax	$ millions		**-11.82**	**30.62**	**0.95**
Discounted Cashflow	$ millions	**10%**	**-10.74**	**25.30**	**0.95**
NPV	$ millions	**15.28**			

NPV for the P100 design

P100 Discounted cashflow				
	Calendar Year	2021	2022	2023
	Project Year	0	1	2
	Units			
Production				
Available tonnes remaining in stopes		274 969	274 969	274 969
Tonnes Milled	Tonnes			274 969
Mill Head Grade	g/t			4.3
Milled Recovery	%			84%
Gold Produced	g			993 189
Gold Produced	oz			31 935
Revenue				
US Gold Price	$/oz			1 520
Gross Sales Revenue	$ million			**48.54**
Costs				
Mining cost	$/t			50.00
Processing cost	$/t			30.00
Total unit cost	$/t			80.00
Total Costs	$ millions			**-22.00**
Net Sales Revenue (operating profit)	$ millions			**26.54**
Tax Shield				
Capital Expenditure	$ millions		-10.89	
Gold Mine Capital Allowance (12%)	$ millions			-1.31
Unredeamed Capex	$ millions		-10.89	
Net Sales Revenue after Capex	$ millions		**-10.89**	**26.54**
Royalties				
EBIT for Royalty Calculation	%		**0.00**	**14.35**
X=(EBIT Royalty /Gross Sales)			0.00	29.57
Refined Royalty Rate (Y=0.5+(X/12.5)	%		0.50%	2.87%
Royalty Paid	$ millions		**0.00**	**-1.39**
Income Tax				
Taxable Income	$ millions		0.00	12.96
X=(Taxable Income/Gross Sales)	%		0.00	26.70
Gold Tax Rate (34-170/X)	%		0.00%	27.63%
Income Tax Payable	$ millions		**0.00**	**-3.58**
Cash Flow				
Actual Cashflow after tax	$ millions		**-10.89**	**21.57**
Discounted Cashflow	$ millions	**10%**	**-9.90**	**17.83**
NPV	$ millions	**7.93**		