

# **APPENDIX H CONSTANT VOLTAGE BEHIND TRANSIENT REACTANCE GENERATOR MODEL**

The improved two generator model uses the constant voltage behind transient reactance generator model. This model ignores magnetic saliency; assumes the operating reactance of the generator is the transient reactance,  $x'_i$ , and considers the voltage behind transient reactance to be a constant.

This appendix discusses the accuracy with which the constant voltage behind transient reactance generator model models the actual generator voltage and the actual generator reactance.

## **H.1 ACCURACY OF THE GENERATOR VOLTAGE**

In the Eskom network out-of-step tripping happens during the first slip cycle. Hence, to determine whether a constant voltage generator model could be used it should be shown that the voltage,  $\mathbf{E}' = E'_d + \mathbf{j} E'_q$ , behind transient reactance stays constant during the first slip cycle.

The voltage behind transient reactance decays according to [45, p114]:

$$\frac{dE'_d}{dt} = -\frac{E'_d}{T'_{qo}} \quad (\text{H.1-a})$$

$$\frac{dE'_q}{dt} = -\frac{E'_{ex} - E'_q}{T'_{do}} \quad (\text{H.1-b})$$

Typically  $T'_{do}$  is large and  $T'_{qo}$  is small. Hence,  $dE'_q/dt$  is small and  $dE'_d/dt$  is large (figure H.1).

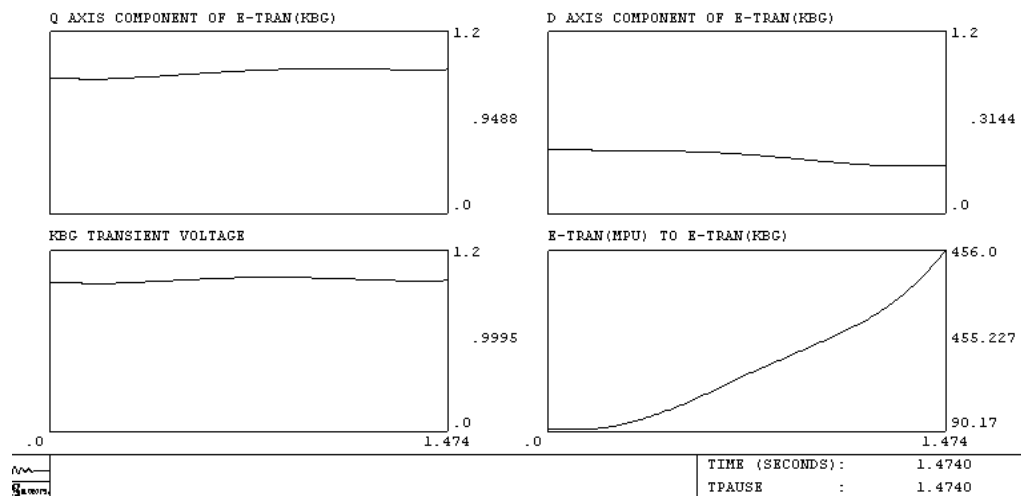
For normal steady state operation  $E'_q$  is larger than  $E'_d$  (Appendix F, figure F.1).

Hence, a rapid change in  $E'_d$  does not make  $|E'|$  change rapidly.

To show the decay in  $|E'|$  is small we trace the Koeberg generator voltage for the first slip cycle (figure H.1). The case considered is where Mpumalanga and the Western-Cape are slipping poles.

Table H.1 lists the values used for  $T'_{do}$  and  $T'_{qo}$ .

Figure H.1 shows it is reasonable to assume  $|E'|$  is constant for at least the first slip cycle. Therefore, when only the first slip cycle is considered it is acceptable to assume a constant generator voltage.



**FIGURE H.1** Decay in  $E'_d$  and  $E'_q$  measured at Koeberg

**Table H.1**  $T'_{do}$  and  $T'_{qo}$  of the Koeberg generator

TIME CONSTANT	
$T'_{do}$ seconds	$T'_{qo}$ seconds
10.9	1.6

## H.2 ACCURACY OF THE GENERATOR REACTANCE

The constant voltage behind transient reactance generator model computes armature reaction,  $x'_i$ , using (Appendix D, section D.7-a):

$$x'_i = \frac{x'_d + x'_q}{2} \quad (\text{H.2})$$

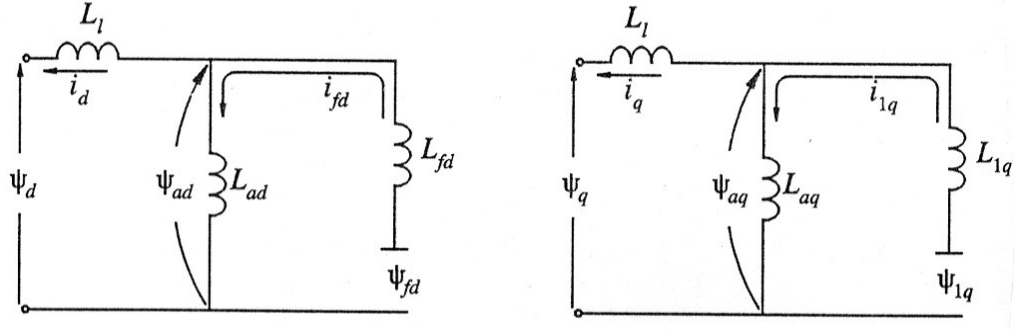
This section shows the accuracy with which  $x'_i$  represents armature reaction depends on the angle the generator current forms with the q-axis.

### H.2.1 Armature reaction

Armature reaction represents the voltdrop due to the armature, i.e. stator, current. The voltdrop is obtained by grouping all the stator current terms in the equation used to compute the terminal voltage. The case considered is where the generator is in the transient state.

The flux linking the d- and q-axis of the generator when in the transient state is shown in figure H.2.

It follows from figure H.2 that to obtain  $x'_i$  the equation used to compute the terminal voltage should be written in terms of the stator currents,  $i_d$  and  $i_q$ ; the q-axis flux linkage,  $\Psi_{lq}$ , and the field flux linkage,  $\Psi_{fd}$ .



**FIGURE H.2** The d- and q- axis equivalent circuits of a generator in the transient state [1, p184]. When in the transient state  $i_{1d} = i_{2q} = 0$

To obtain the equations used to compute the terminal voltage, we note the per unit stator voltage equations are [1, p86]:

$$e_d = p\Psi_d - \Psi_q w_r - R_a i_d \quad (\text{H.3-a})$$

$$e_q = p\Psi_q + \Psi_d w_r - R_a i_q \quad (\text{H.3-b})$$

$$e_0 = p\Psi_0 - R_a i_0 \quad (\text{H.3-c})$$

The transformer voltage terms,  $p\Psi_d$  and  $p\Psi_q$ , can be ignored [1, p170] and the per unit value of  $w_r$  can be set equal to 1 p.u. (hence  $w_r \Psi_q = \Psi_q$  and  $w_r \Psi_d = \Psi_d$ ) [1, p174]. Therefore, when  $i_0 = 0$  the stator terminal voltage can be computed using:

$$e_d = -\Psi_q - R_a i_d \quad (\text{H.4-a})$$

$$e_q = \Psi_d - R_a i_q \quad (\text{H.4-b})$$

To express  $e_d$  and  $e_q$  in terms of  $i_d$ ,  $i_q$ ,  $\Psi_{1q}$  and  $\Psi_{fd}$  we apply Kirchhoff's voltage law to the equivalent circuits shown in figure H.2. We obtain:

$$\Psi_{ad} = -L_{ad}i_d + L_{ad}i_{fd} \quad (\text{H.5-a})$$

$$\Psi_d = \Psi_{ad} - L_l i_d \quad (\text{H.5-b})$$

$$\Psi_{fd} = \Psi_{ad} + L_{fd}i_{fd} \quad (\text{H.5-c})$$

$$\Psi_{aq} = -L_{aq}i_q + L_{aq}i_{1q} \quad (\text{H.5-d})$$

$$\Psi_q = \Psi_{aq} - L_l i_q \quad (\text{H.5-e})$$

$$\Psi_{1q} = \Psi_{aq} + L_{1q}i_{1q} \quad (\text{H.5-f})$$

From equation H.5-f, we obtain:

$$i_{1q} = \frac{\Psi_{1q} - \Psi_{aq}}{L_{1q}} \quad (\text{H.6})$$

Substitution into equation H.5-d, gives:

$$\Psi_{aq} = -L_{aq}i_q + L_{aq}\left(\frac{\Psi_{1q} - \Psi_{aq}}{L_{1q}}\right)$$

$$\Psi_{aq}\left(1 + \frac{L_{aq}}{L_{1q}}\right) = -L_{aq}i_q + \frac{L_{aq}}{L_{1q}}\Psi_{1q}$$

$$\Psi_{aq}\left(\frac{L_{1q} + L_{aq}}{L_{1q}}\right) = -L_{aq}i_q + \frac{L_{aq}}{L_{1q}}\Psi_{1q}$$

$$\Psi_{aq} = \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}}(-i_q) + \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}}\left(\frac{\Psi_{1q}}{L_{1q}}\right)$$

$$\Psi_{aq} = L'_{aq}\left(-i_q + \frac{\Psi_{1q}}{L_{1q}}\right)$$

where:

$$L'_{aq} = \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}} \quad (\text{H.7})$$

The substitution of equation H.7 into equation H.5-e gives:

$$\Psi_q = L'_{aq} \left( -i_q + \frac{\Psi_{1q}}{L_{1q}} \right) - L_l i_q \quad (\text{H.8})$$

The substitution of equation H.8 into equation H.4-a gives (  $w = 1$  p.u.)

$$e_d = \left[ (L_l + L'_{aq}) i_q - R_a i_d \right] - L'_{aq} \left( \frac{\Psi_{1q}}{L_{1q}} \right) \quad (\text{H.9})$$

The rotor base fluxes induce the speed voltage. Therefore, by grouping the rotor based fluxes of equation H.9 the equation for  $E'_d$  is obtained. The equation is:

$$E'_d = -L'_{aq} \left( \frac{\Psi_{1q}}{L_{1q}} \right) = -\frac{L_{aq} L_{1q}}{L_{aq} + L_{1q}} \left( \frac{\Psi_{1q}}{L_{1q}} \right) \quad (\text{H.10})$$

The similarity between the d- and q-axis in figure H.2 allows writing:

$$\Psi_{ad} = L'_{ad} \left( -i_d + \frac{\Psi_{fd}}{L_{fd}} \right) \quad (\text{H.11})$$

Where:

$$L'_{ad} = \frac{L_{ad} L_{fd}}{L_{ad} + L_{fd}}$$

By using mathematical manipulation similar to the manipulation used to obtain equation H.9, it can be shown (  $w = 1$  p.u.):

$$e_q = \left[ -(L_l + L'_{ad}) i_d - R_a i_q \right] + L'_{ad} \left( \frac{\Psi_{fd}}{L_{fd}} \right) \quad (\text{H.12})$$

Therefore, expressed in terms of the field and q-axis flux we have ( $w=1$  p.u.):

$$e_d = \left[ \left( L_l + \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}} \right) i_q - R_a i_d \right] - \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}} \left( \frac{\Psi_{1q}}{L_{1q}} \right) \quad (\text{H.13-a})$$

$$e_q = \left[ - \left( L_l + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}} \right) i_d - R_a i_q \right] + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}} \left( \frac{\Psi_{fd}}{L_{fd}} \right) \quad (\text{H.13-b})$$

Armature reaction represents the voltdrop due to armature current. Hence, by grouping the terms in equation H.13 containing armature current the d- and q-axis components of the armature reaction are obtained. Hence:

$$e_d^{armature} = \left( L_l + \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}} \right) i_q - R_a i_d \quad (\text{H.14-a})$$

$$e_q^{armature} = - \left( L_l + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}} \right) i_d - R_a i_q \quad (\text{H.14-b})$$

When resistance is neglected, we have [1, p146 and p185]:

$$\begin{aligned} e_d^{armature} &= \left( L_l + \frac{L_{aq}L_{1q}}{L_{aq} + L_{1q}} \right) i_q \\ &= x_q' i_q \end{aligned} \quad (\text{H.15-a})$$

$$\begin{aligned} e_q^{armature} &= - \left( L_l + \frac{L_{ad}L_{fd}}{L_{ad} + L_{fd}} \right) i_d \\ &= -x_d' i_d \end{aligned} \quad (\text{H.15-b})$$

When modelling the generator as a voltage,  $\mathbf{E}_i'$ , behind the reactance,  $x_i'$ , we have:

$$e_d^{armature} = \left( \frac{x'_d + x'_q}{2} \right) i_q \quad (\text{H.16-a})$$

$$e_q^{armature} = - \left( \frac{x'_d + x'_q}{2} \right) i_d \quad (\text{H.16-b})$$

Form equation H.15 and equation H.16 it follows for generators with  $x'_d = x'_q$  we have:

$$e_d^{armature} = \left( \frac{x'_d + x'_q}{2} \right) i_q = x'_q i_q \quad (\text{H.17-a})$$

$$e_q^{armature} = - \left( \frac{x'_d + x'_q}{2} \right) i_d = -x'_d i_d \quad (\text{H.17-b})$$

## H.2.2 Armature reaction and $x'_i$

Equation H.17 shows that equation H.2 describes armature reaction accurately when  $x'_d = x'_q$ .

When  $x'_d \neq x'_q$  the accuracy with which equation H.2 describes armature reaction depends on the angle that develops between the generator q-axis and the stator current. To illustrate, consider the case where  $\mathbf{I}$  is positioned with respect to the q-axis such that  $i_q = 0$  and  $i_d \neq 0$ . The armature reaction is then  $e_q^{armature} = -x'_d i_d$ . When  $\mathbf{I}$  is positioned with respect to the q-axis such that  $i_q \neq 0$  and  $i_d = 0$  the armature reaction is  $e_d^{armature} = x'_q i_q$ .

Which one of the two examples is best approximated by  $(x'_d + x'_q)/2$  depends on whether it is  $x'_d$  or  $x'_q$  that is numerically the closest to  $(x'_d + x'_q)/2$ .