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# Behaviour of single laced columns versus double laced columns

**Hasheela, Pamwenafye Fillemon.**

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# Behaviour of Single Laced Columns versus Double Laced Columns

Pamwenafye Fillemon Hasheela

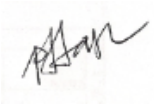
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A research report submitted to the Faculty of Engineering and Built Environment, University of the Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Science in Engineering

Johannesburg, 2013

# DECLARATION

I declare that this thesis is my own unaided work. It is being submitted to the Degree of Masters in Science to the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.



……

*(Signature of Candidate)*

11 day of July (year), 2013

## ABSTRACT

This study discusses the behaviour of built-up single columns versus double laced columns.

Finite Element Analyses was applied to evaluate buckling load, torsion resistance and modes of buckling. All simulations are performed using ABAQUS Version 6.8 (Dessault Systems, Inc.). An eight-node shell element was used for the nonlinear solution. To ensure the finite element solution was valid, a convergence study was concluded. The parametric study has considered different column widths, end supports and types of brace configuration. The behaviour has been analysed at varied load ratio. Two cases of different end supports have been investigated.

The results show less variability within different bracing configurations. The X-configuration showed best performance by 3-10% and 1-8% for single and double laced column respectively. A buckling load variation of 15%-25**%** and 1%-3% for single and double laced columns respectively was observed. A combination of maximum critical load and minimum degree of torsion is achieved at load ratio close to one.

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**Secondly**, I would like to extend my gratitude and appreciation to the School of Civil and Built Environment at University of the Witwatersrand for providing all necessary machinery and making this investigation possible.

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## LIST OF SYMBOLS

y: Deflection in y-direction

x: Varying distance along an x-axis

P: applied load

N: applied load

P/N: Load Ratio

L: Column Height

E: Elastic Young Modulus

I: Second moments of Area

G: Shear Modulus of elasticity

*n: shape factor*

δ: lateral displacements

θ: angle

a: is a panel height

h: Length diagonal member

d: Column Width

$A_{d:}$ Area of diagonal lacing members

$A_{b:}$ Area of battens

$P_{cr}$ : Critical Load

$P_e$: Euler Load

$P_d$: Shear Stiffness

ε: Strain

σ: Stress

$I_w$: Warping Constant

J: Torsion constant

$R_z$: Moment of inertia about local polar axis

$\Theta_x$ : Shear deformation

V: Shear Force

$\frac{dv}{dx}$ : Additional slope of deflection due to shear

$\delta_1$ and $\delta_2$ , are lateral displacements caused by batten and diagonal members

$P_d$: Shear Stiffness

$N$ Nodes in the element

$H$ Is the displacement interpolation matrix

$U^{el}$ is the displacement at any point in the element

$\alpha_i, \beta_i$ and $\gamma_i$ are referred to the generalized co-ordinates

F: Force

K: is the global structure stiffness matrix

U: displacement vector

*k*: effective length factor

λ: Slenderness ratio

η : Perry robertson factor

# CHAPTER 1

## 1.1 Introduction

The use of single laced columns has been observed in the construction industry, this has instigated this study to evaluate the buckling load, torsion and modes of buckling of single laced columns used in pratice, in comparison to traditional double laced columns. The objective of this research is to highlight the structural benefits of using the traditional double laced column incomparison to single laced column.

A built-up column is a frame which consists of two or more parallel main chords mostly hot rolled profiles that are connected at points along their length using transverse or diagonal connectors. The bracing of built up columns provides a system to resist shear and ensures that the column behaves as one integral unit capable of achieving maximum compressive capacity. The brace system helps to reduce the effective lengths of the main compressive chords, hence increasing the buckling capacity and torsion resistance of the column. Various types of connectors such as batten plates, lacing bars or perforated cover plates are used as bracing members of built-up columns. Built up columns can be made of sections of different sizes and shapes. Figure 1.1 shows various combinations of common build up columns.

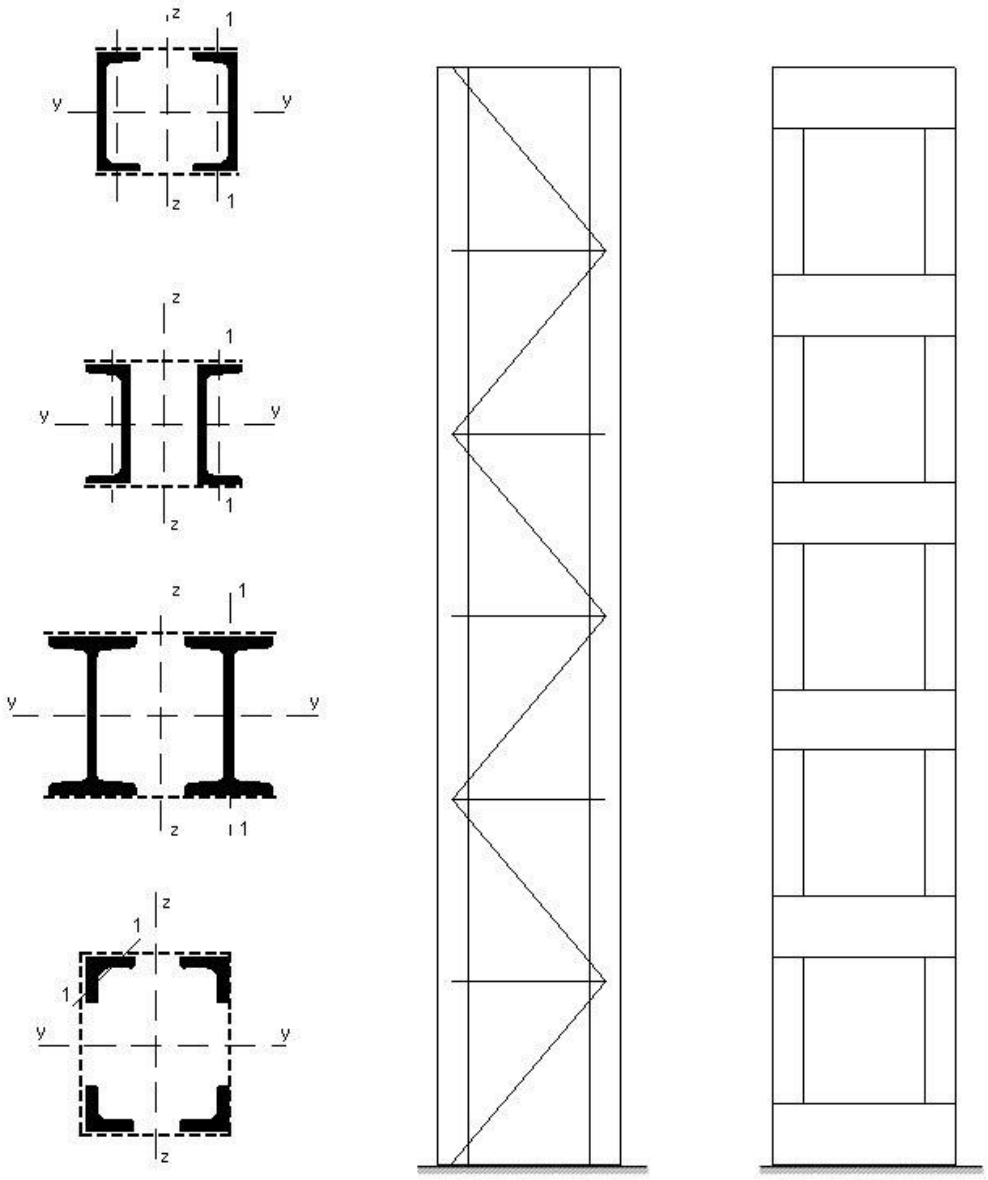



Figure 1:1 Types of built up columns (Ahmed Rahik, 2006)

Traditionally, the built-up column is doubly laced or battened with double layers, thus stabilizing the compression chords. Angle sections welded on both flanges are probably the common lacing type used. Figure 1.2 shows a typical double laced column.



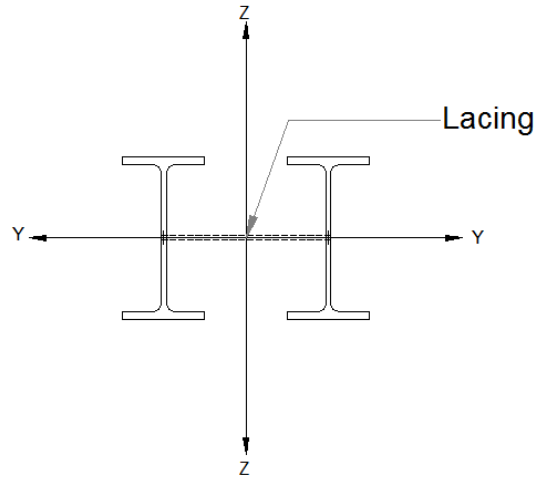
Figure 1:2 Traditional double laced built-up column

The singly laced column has started to be used in the construction industry. A typical example of single laced columns can be seen at O.R. Tambo International Airport, Johannesburg and in an industrial building with overhead crane girders used during Gautrain subway project in South Africa. Figure 1.3 shows a single laced column. Instead of bracing the column on both flanges, it is rather braced on web to web. For more photographs of built up columns please see section 8.3 under Appendix c.

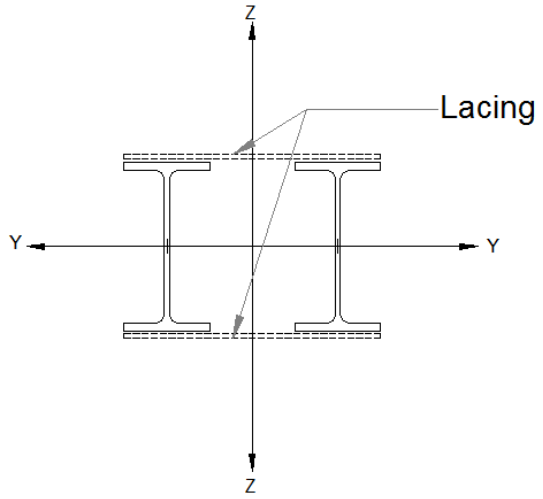


Figure 1:3 Single built-up column

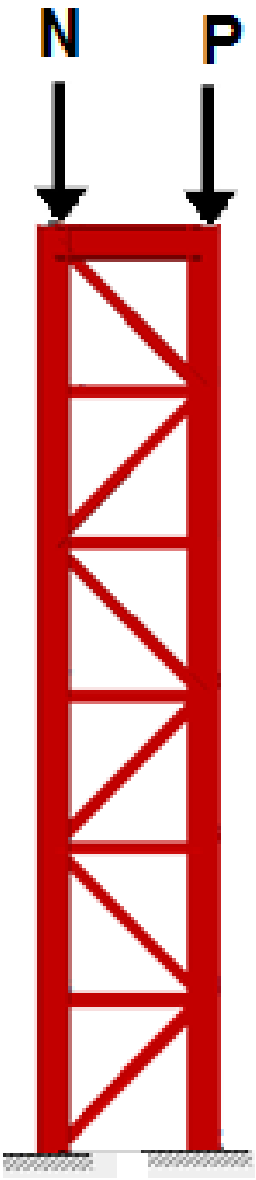
The behaviour of single laced columns are not well documented. In this study when a single layer of lacing or battens is used, it is termed a single laced column (SLC) as shown in Figure 1.4 (a). The traditional double laced column (DLC) refers to a built up column with two layers of lacing or battens on both flanges of the main compression chord Figure 1.4 (b).




a) Plan view of a built-up column singly laced (SLC): Web-to-Web




b) Plan view of a built-up column double laced (DLC) on both faces




c) Elevation of a built-up column

Figure 1:4 Built-up columns

Built up columns are commonly used in industrial buildings to support crane girders or both as crane columns and supporting roof structures. They are capable of attaining high compressive loads with minimum and effective use of materials. However, built-up columns are weak in shear as compared to solid columns (Timoshenko and Gere, 1961:135). Figure 1.5 below shows a practical setup of a

double laced column supporting a crane girder and roof structure for an industrial warehouse.




Figure 1:5 Built-up column with lacings in an industrial building

One of the factors which determine the strength of the column is the second moment of area. The second moment of area of the built-up column increases with the distance between the main compression chords (column width). Thus stiffness of the built up column increases with the column width. However the increase in stiffness is counterbalanced by the weight and cost increase of the connection between members (Ahmed, 2006).

Single laced columns require less material than double laced columns but require more attention to detailing. The overall benefits of these types of columns remains to be seen.

It is recommended by the building codes e.g. BS5950 and SANS10162-1, that the bracing system must comprise of an effective shear triangulated system on each face and the system should not have large varying length for force transfer. In addition, all lacing members should be inclined at an angle of between 45° and 70° to the longitudinal axis of the member. Lacing members may consist of bars, rods or sections. The crucial question that will be studied here is wether the bracing system provides adequate torsional resistance. It is an open question whether single laced columns provide the required torsional stiffness. It is important to note that building codes i.e. SANS 10162-1, BS5950 and EN1993 do not give a specific requirement for torsion stiffness.

It is important to note that by implication, single laced columns are not encouraged in the building codes as they do not provide adequate torsional restraint.

## 1.2 Literature Review

Although built up columns are well understood, there is a dearth of literature on the structural performance of single versus double laced columns. Literature to date has investigated the structural behaviour of double laced columns and derived formulae of estimating critical buckling loads and axial torsional buckling load.

An approach by F. Engesser (1891) has been used for design recommendations as a basis of methods for estimation of the elastic critical load in built-up columns. Engesser modelled a built-up column as an equivalent solid column. It has been shown that lateral displacement of the equivalent solid column is not only affected by the bending moment upon loading but also by the transverse shear forces.

Gere and Timoshenko (1961) discussed buckling of built up columns and derived equations of estimating carrying compressive capacity of a laced column. Their equations have taken into account additional deflection curvature due to shear forces. The approach is based on Engesser's approach. Timoshenko and Gere's work is an extension of Euler-Bernoulli theory. Their theory predicts the critical buckling load of built up column by incorporated effects of shear deformations in the members.

Hosseini Hashemi and M.A. Jafaria (2009) worked on the determining the elastic critical load of batten columns experimentally. They tested batten columns under axial compression and the elastic critical loads were determined using modified SouthWell theory (1932). Using the SouthWell plots, the elastic critical loads of the columns were calculated. They also considered other theoretical methods e.g. equivalent slenderness (with different equations for slenderness), Paul (1995) and structural stability research council (SSRC) methods.

Results show that theoretical methods are generally conservative in determining critical loads of columns and the equivalent slenderness method using the Engesser

equation is the most conservative. The critical loads predicted using the Paul method are closer to experimental critical loads (Paul, 1995).

Galambos (1998) had calculated the effects of shear on critical load for three basic boundary conditions. The shear flexibility of lacing or batten member is modified with a factor. The shear flexibility factor is read from the graph by using a value of column width to height ratio. The load ratio of column critical load to Euler buckling load ($P_{cr}$ /$P_e$) can then be read for the appropriate end conditions, hence the elastic critical load of the laced column.

Razdolsky (2008) solved a flexural buckling problem of a laced column as a statically indeterminate structure of a column with a crosswise lattice.

Razdolsky noted that, laced columns are highly redundant systems and the loss of column stability can occur by various buckling modes depending on a correlation between the chord rigidity and the lattice rigidity. Columns can lose stability in a manner that the joint cross-sections is not displaced.

Razdolsky concluded the critical force of the column is equal to the force which caused the buckling of the isolated chord panel. The critical force of columns with identical chords is a function of the number of panels and the lattice rigidity parameter of the column (Razdolsky,2008). Hence, the critical force for a column with any degree of static indeterminacy is determined as the smallest Eigen-value of fourth-order system of linear algebraic equations.

From deflection mode shapes, Razdolsky's work disproves the assumption of the sine-shaped deflection mode shape. Buckling mode shapes obtained for the column as a statically indeterminate structure take the form of the irregular curve consisting of several half-waves with un-equal amplitudes. The sine mode is the basis of Engesser assumption and in design manuals for steel-laced columns.

C.M.Wang and K.K. Ang (1988) determined the buckling loads of the column from minimizing the generalised Rayleigh quotient (derived from Timoshenko energy

approach) subject to some constraints on the deflection curve, with some lateral restraints. On the basis of convergence studies made on the energy solution, two terms of a trigonometric series are found to be sufficient for approximating the deflection curve of unrestrained columns. Consequently, simple approximate formulas can be obtained by solving the characteristic equation obtained from Rayleigh-Ritz energy approach. These formulae are more accurate and are simpler in forms than Kato's formulas (Kato,1971).

Bleich (1952) suggested a modified effective length formulae which is a conservative estimate of lacing at 60° to 45 °.

The literature to date deals with built up column with double lacing. To the author's knowledge, there is no literature that has compared the structural performance of single to double laced columns.

## 1.3 Objective of the study

1. To establish torsion and flexural buckling loads of built-up columns and compare failure modes at different load ratio.
2. To compare buckling loads of the single versus double built up columns of same design at different load ratios, over a range of column width.
3. To perform a parametric study of the variables which can affect the behaviour of built-up columns

## 1.4 Sequence of Project Completion

This study was approached in the following sequence:

1. A literature survey was conducted which broadly investigated the buckling load and torsion of double laced columns as well as various other parametric factors.

2. Review of recommendations on built up column from four different building codes i.e. EN1993:1-1, SANS10162-1, BS5950-1:, CAN/CSA (S6S-05). Furthermore, the theory which defines and determines the structural response of built up columns. The literature survey also gathered information on effects of load ratios on the buckling load.

3. Designing and running finite element analysis using the FEM software package ABAQUS, Version 6.8 (Dessault Systems, Inc.)

4. The comparison of theoretical results and FEM solution from ABAQUS.

5. Conclusion and recommendation on the behaviour of built up columns.

6. Highlighting subjects requiring further research.

## 1.5 Assumptions and Limitation to this study

- Attachment of the brace system onto the main compression members

It is assumed that all members are welded onto the flanges of the main compression chord.

- Angle to inclinations

All members are inclined at 45º to the main compression chord.

- Homogenous material

The material is assumed to be homogenous.

- All members have negligible imperfections

No imperfections were considered in these investigations.

- No temperature effects are present.

## 1.6 Organization of the report

This report is split into the following chapters:

- Chapter 2 provides theoretical background to buckling of built up columns in particular, the following is considered:
  - The effect of shear deformations on the elastic critical column load
  - Influence of brace arrangements on critical buckling load and torsion
  - Influence of end boundary conditions, columns width on critical load and torsion.
  - Variation of load ratio, critical load, and torsion.
  - Finite element method, Eigen-value problem, FEM modelling and Mesh density.

- Chapter 3 Recommendation and Specifications from building codes.
- Chapter 4 Behaviour of Single verses Double Laced Columns.
- Chapter 5 Comparison of  theory to FEM modelling.
- Chapter 6 Conclusions and Recomemdations.
- Chapter 7 Lists the reference.
- Chapter 8 Appendices.

# CHAPTER 2

## 2.1 Theoretical Background

A structure can be deemed unusable or can be considered to have failed under a number of conditions. A structure has failed when members or the entire structure have reached yield or ultimate strength, exceeding a specified maximum deflection and torsion or when fracture of members or collapse occurs. Buckling of members presents a stability issue that gives the limit of resistance of a member.

Buckling is a broad term that describes a wide range of mechanical behaviours. Generally it refers to an event whereby a compression member diverges from its linear elastic behaviour and large deformation accompanied by change of member shape due to a very small increase in loading.

For members with double symmetry, the load is equally likely to buckle in at least two directions of its symmetry. The load at which the column starts to deviate from the original geometry is called Critical Load ($P_{cr}$). Alternatively it can be defined as a compressive load causing the bowing of the column shape.

Considering the in-plane behaviour of a column and ignoring the possibility of local buckling, failure may occur in one of three basic ways:

1. Yielding of the cross-section: A compressive material failure, i.e., the material yeilds, cracks or crumbles. This type of column failure happens to columns that are short and non slender.

2. Elastic buckling: This type of column failure usually happens to columns that are long and slender, elastic buckling can also occur in plates and shells.

3. Another form of failure is a combination of both compressive and buckling failures. This type of column failure occurs when length and width of a column is in between a short and long column.

A column carries a hypothetical maximum axial load prior to failure in flexure. It is calculated assuming a linear elastic stress-strain relationship in the member. Critical load ($P_{cr}$) is independent of the magnitude of stress in the material and consequently is not affected by yield stress. Should the critical load be exceeded, deflection effects will cause instability of the entire compression member, as opposed to failures of specific zones that are subjected to high stresses.

Other forms of instability that are also commonly referred to as buckling are lateral-torsional buckling. Lateral-torsional buckling is a phenomenon common in slender laterally unsupported beams that can affect any laterally un-braced beam segment with a section height significantly greater than its width (Charles, 2003). When an un-braced beam is subjected to moments from vertical loads, the top portion of the beam reaches a critical load in compression and buckles laterally. This generally causes twisting of the beam section because the tension flange stays straight rather than moving laterally with the compression flange. Thus, both lateral and torsional stiffness must be considered when designing. This is also experienced in loaded un-braced columns but here it is called Axial-torsional buckling.

From literature, theoretical equations which estimate the column's critical load based on its end supports are well known and derived using the Euler-Bernoulli-beam assumption.

## 2.2 Buckling Load for a Primastic Pin-ended Column.

The Euler-Bernoulli's mathematical expression which estimates critical buckling load of an ideal column based on equilibrium, the mechanics of bending, geometry of the column, and material properties within the initial linear range is derived below. The

basic geometry and a free body diagram of a portion near one end of the column is shown in Figure 2.1.

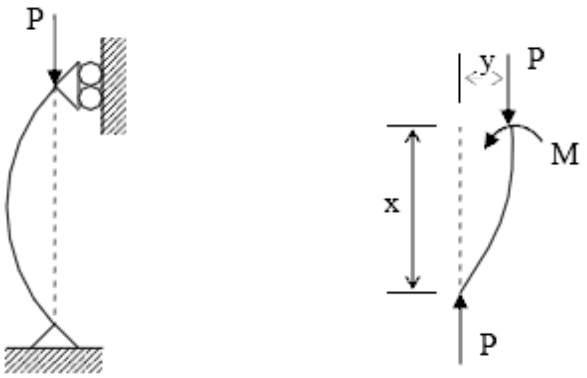



Figure 2:1 Pinned-pinned column (Charles,2003)

The moment at any section located at a certain distance “x” from the base of the column is given by:

$$M(x) = -EI\frac{d^2y}{dx^2} \tag{1}$$

Where E = Elastic Young Modulus

I = Second Moments of Area

y = Horizontal deflection

Equation (1) utilizes the approximate expression for curvature $\frac{d^2y}{dx^2}$ and not the exact expression

$$\frac{\frac{d^2y}{dx^2}}{\left[1+\left[\frac{dy}{dx}\right]^2\right]^{3/2}} \tag{2}$$

The bending moment at any cross section is given by:

$$M(x) = Py \tag{3}$$

Where P is the axial load

Combining equations (1) and (3) gives the governing differential equation for the pinned-pinned Euler column:

$$EI\frac{d^2y}{dx^2} + Py = 0 \tag{4}$$

By introducing the notation $w^2 = \frac{P}{EI}$, equation (4) may be written as:

$$\frac{d^2y}{dx^2} + w^2y = 0 \tag{5}$$

The general solution for this homogeneous linear differential equation is:

$$y = A\sin wx + B\cos wx \tag{6}$$

Constants A and B can be evaluated by considering the boundary conditions:

$y = 0$ at $x = 0$

$y = 0$ at $x = L$

Where L is the column height

The first condition requires that $B = 0$, leaving

$$y = A\sin wx \tag{7}$$

Evaluating Equation (7) under the second boundary condition yields

$$A\sin wL = 0 \tag{8}$$

In order to obtain a result that is not trivial, constant "A" must be non-zero. Thus

$$\sin wL = 0 \text{ Or } wL = n\pi \tag{9}$$

Where n = 1, 2, 3…

Substituting Equation (9) into Equation (3) yields the buckling load equation:

$$P_n = n^2\pi^2\frac{EI}{L^2} \tag{10}$$

The critical buckling load, or Euler load, can be evaluated with *n* =1 as this produces the smallest load for which instability of a column will occur:

$$P_{cr} = \pi^2\frac{EI}{L^2} \tag{11}$$

It should however be noted that higher modes of buckling can be evaluated from Equation (10). These higher modes of buckling are unlikely to happen unless forced to do so. The critical buckling load of a pinned-pinned column may be effectively quadrupled if deflection is held at zero at the centre of the column. This can be shown by examining the mode shape of the buckled column.

Substituting Equation (9) into Equation (7) yields the mode shape equation:

$$y = A\sin\frac{n\pi x}{L} \tag{12}$$

Equation (12) is used to evaluate any buckling mode and geometrically defines the bent shape of a column. The amplitude of the deflected shape is not defined due to the fact that buckling is an instability phenomenon.

As defined by Equation (12), the first three buckling modes of the pinned-pinned Euler column along with the buckling loads are shown in Table 2.1.

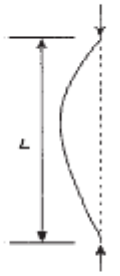
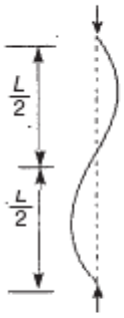
| Mode | 1 | 2 | 3 |
|---|---|---|---|
| Mode Shape |   |   |   |
| Critical buckling load | $\pi^2 \frac{EI}{L^2}$ | $4\pi^2 \frac{EI}{L^2}$ | $9\pi^2 \frac{EI}{L^2}$ |

Table 2:1 Modes of failure for pin-pin column (Charles,2003)

## 2.3 Buckling load for a prismatic fixed-free column

In the cantilever configuration the upper end of the column is free to move laterally and also to rotate. The expression for the buckling load of the ideal column is similar to that for the pinned-pinned case. The equations of equilibrium and boundary conditions are changed based on the different kinematic conditions as shown in Figure 2.2.

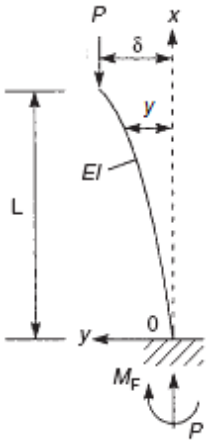



Figure 2:2 Fixed-free column support (Charles,2003)

Again, the relationship between bending moment and curvature, approximated by the second derivative, will be used:

$$M(x) = -EI\frac{d^2y}{dx^2} \tag{13}$$

The bending moment at any cross section at any section located at distance "x" from the base of the column is given by:

$$M(x) = P(\delta - y) \tag{14}$$

Equating Equations (13) and (14) gives the governing differential equation for the fixed-free Euler column:

$$EI\frac{d^2y}{dx^2} + P(\delta - y) = 0 \tag{15}$$

As before, by introducing the notation $w^2 = \frac{p}{EI}$, Equation (15) may be written as:

$$\frac{d^2y}{dx^2} + w^2y = w^2\delta \tag{16}$$

The general solution for this linear differential equation is

$$y = A\sin wx + B\cos wx + \delta \tag{17}$$

Constants A and B can be evaluated by introducing boundary conditions:

$$y = \frac{dy}{dx} = 0 \quad \text{at } x = 0$$

These conditions can be satisfied by:

$A = 0 \quad B = -\delta$

Thus

$$y = \delta(1 - \cos \omega x) \tag{18}$$

The condition at the free end of the column requires that

$y = \delta$ at $x = L$

This condition is satisfied if

$$\delta \cos \omega L = 0 \tag{19}$$

Equation (19) may be satisfied if either $\delta = 0$ or $\cos wL = 0$. If $\delta = 0$, then there is no deflection at the free end of the column and consequently no buckling. Thus, $\cos k\omega L$ must equal zero and the following relation must be true:

$$wL = (2n - 1)\frac{\pi}{2} \qquad \text{where n} = 1,2,3 \tag{20}$$

By re-introducing, $w^2 = \frac{P}{EI}$ this equation becomes

$$\mathrm{P} = (2\mathrm{n} - 1)^2 \frac{\pi^2 EI}{4L^2}$$

The critical buckling load will be obtained when n = 1:

$$\mathrm{P}_{\mathrm{cr}} = \frac{\pi^2 EI}{4L^2} \tag{21}$$

Substituting equation (20) into equation (18) yields the mode shape equation:

$$y = \delta\left[1 - \cos\left(\frac{(2n-1)\pi x}{2L}\right)\right] \tag{22}$$

Equation (22) can be evaluated for any buckling mode and defines the geometry of the buckled shape.

The First three buckling modes of a cantilever (fixed-free) prismatic column with the associate buckling loads are shown in Table 2.2. It should be noted that equivalent results may be obtained by analysing a guided-pinned column. The guided-pinned and fixed-free columns buckle in exactly the same shape and at the same load for a given mode.

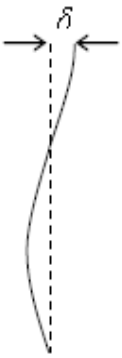
| Mode | 1 | 2 | 3 |
|---|---|---|---|
| Mode Shape |  |  |  |
| Critical buckling load | $\pi^2 \frac{EI}{4L^2} \approx 2.4674 \frac{EI}{L^2}$ | $9\pi^2 \frac{EI}{4L^2} \approx 22.2066 \frac{EI}{L^2}$ | $25\pi^2 \frac{EI}{4L^2} \approx 61.6850 \frac{EI}{L^2}$ |

Table 2:2 Failure modes of fixed-free (Charles,2003)

Derivation of critical buckling load with different combination of boundary condition is available in the literature (Megson,2005).

As mentioned in the preceding paragraphs, the Euler approach can only predict the critical load for slender/long columns. A lower limit to the slenderness for which the

Euler Equation is applicable can be found by substituting the stress at the yield limit ($\sigma_{e)}$ for $\sigma_{Euler}$ as shown.

The yield stress limits the Euler load. Slenderness ratio defined as length divided by radius of gyration (L/r) can be plotted as shown in Figure 2.3. The yield stress is included showing one limit. For a given the proportional limit stress, the Euler load can be limited to values of slenderness (λ) (In this case Slenderness ≥ 100) and this is represented on a stress/slenderness curve as shown in Figure 2.3.

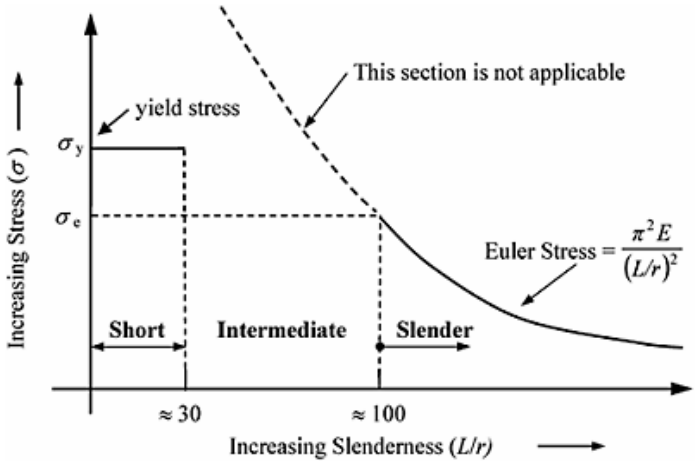



Figure 2:3 Stress vs slenderness for short and slender columns (McKenzie,2006)

The Euler Buckling Load has very limited direct application in terms of practical design because of the following assumptions and limiting conditions (McKenzie, 2005):

• The column is subjected to a perfectly concentric axial load,
• The column is pin-jointed at each end and restrained against lateral loading,
• The material is perfectly elastic,
• The maximum stress does not exceed the elastic limit of the material,
• There is no initial curvature,
•The column is of uniform cross-section along its length,
• Lateral deflections of the column are small when compared to the overall length,
• There are no residual stresses in the column,
• There is no strain hardening of the material,
• The material is assumed to be homogeneous.

In practice most columns are not short or long/slender but fall in the intermediate range .

An anlysis to overcome the limitations of the Euler curve (not applicable to stress greater than the elastic limit) is called the Tangent Moulus Theorem. This can be be applied for short and long columns. The stress/slenderness relationship is shown in Figure 2.4.

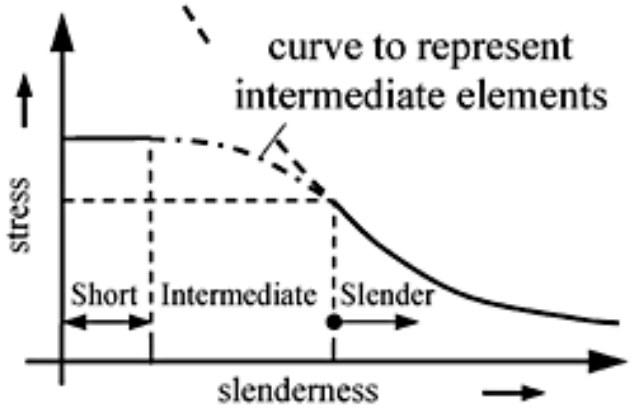



Figure 2:4 Stress/ slenderness for columns of all slenderness ratio (McKenzie,2006)

**Tangent Modulus Theorem**

The tangent modulus theorem is a modification of the Euler equation to establish the stress/slenderness relationship which allows the value of the modulus of elasticity to be determined at any given level of stress from the stress/strain curve for the material. The corresponding slenderness ratio can be then evaluated.

This can be shown by considering a column made of material which has a stress/strain curve as shown in Figure 2.5(a).

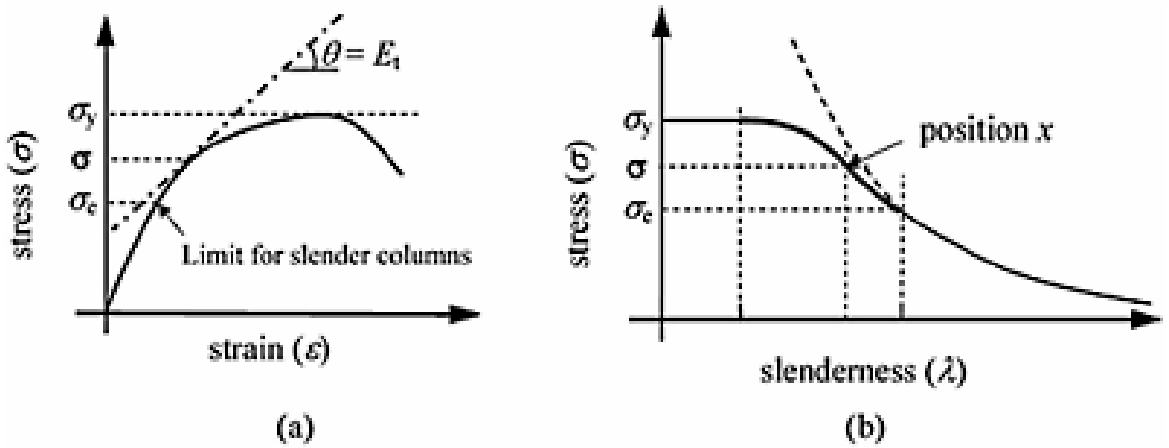



Figure 2:5 Stress / strain curve (McKenzie,2006)

The gradient of the tangent line of the stress/strain curve gives a tangent modulus of elasticity ($E_t = \sigma/\varepsilon$ ). It should be noted that this modulus is different from the elastic modulus. The value of $E_t$ can be used in the Euler Equation to obtain a modified slenderness corresponding to the value of stress σ as shown at position 'x' in Figure 2.5(b):

$$\sigma = \frac{\pi^2 E_t}{(l/r)^2} \qquad \therefore \textit{Slenderness Ratio at position } x = (l/r) = \sqrt{\frac{\pi^2 E_t}{(l/r)^2}} \qquad (23)$$

The curve representing the intermediate length columns can be developed by evaluating successively values of slenderness ratio (λ) when the stress lies between $\sigma_e$ and $\sigma_y$ plotted as shown in (Figure 2.5 (b)).

Perry and Robertson (1988) have established that this equation still has deficiencies similar to the original Euler equation. They developed an equation to account for such deficiencies. Perry-Robertson's formula evolved from the assumption that in practice there could be imperfections which could be represented by a hypothetical initial curvature of the column.

Perry and Robertson (1988) derived an equation of average value of stress in the cross-section which will induce the yield stress at mid-height of the column for any given value of a parameter η.

$$\sigma_{average} = \frac{\left[\sigma_y + 1(1+\eta)\sigma_e\right] - \sqrt{\left[\sigma_y + (1+\eta)\sigma_e\right]^2 - 4\sigma_y\sigma_e}}{2} \tag{24}$$

η is the Perry-Robertson factor and is dependent upon the assumed initial curvature

$$\eta = 0.3\left(L_{effective}/100r^2\right) \tag{25}$$

Where $L_{effective}$ is Effective buckling Length

This equation has been used for many years in design codes to determine the critical value of average compressive stress below which overall buckling would not occur. The curve of stress verse slenderness ratio for this method is compared to the Euler curve and the Tangent Modulus Solution in Figure 2.6.

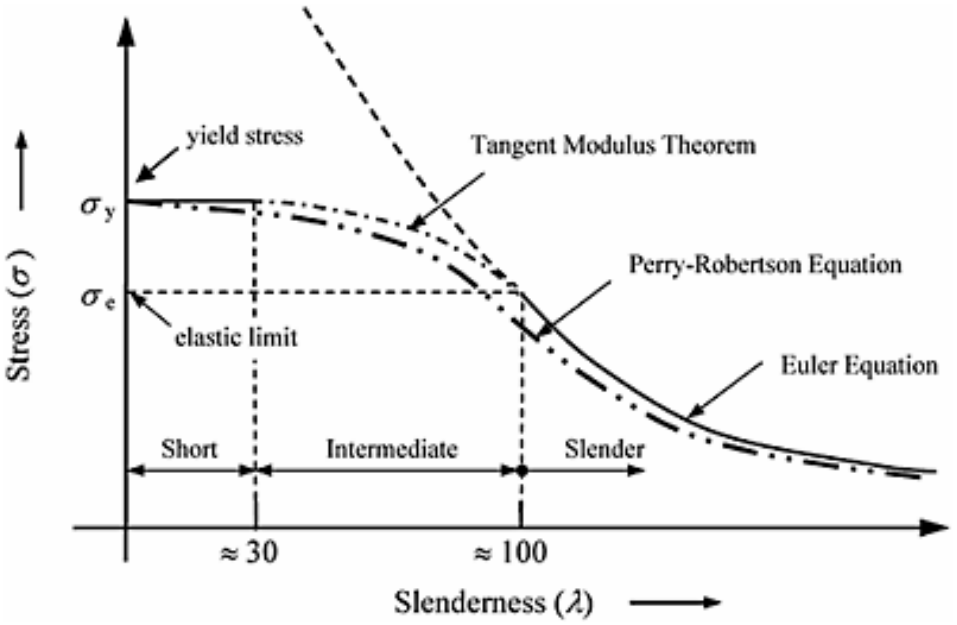



Figure 2:6 Typical stress/slenderness curve of columns (McKenzie,2006)

Although Perry-Robertson's formula does account for many of the deficiencies of the Euler and Tangent Modulus approaches, it is noted that it also does have shortcoming e.g. it does not consider all of the factors which influences column behaviour. Some of the shortcoming of Perry-Robertson's formula is that it does not taking into account

(a) Residual stresses effects induced during fabrication

(b) The type of section being considered (i.e. the cross-section shape)

(c) The material thickness

(d) The axis of buckling, the method of fabrication (i.e. rolled or welded)

This led to a refined formula of the critical load capacity of columns after extensive full-scale testing. This modification is referred to as Perry's strut formula in design codes and is given in the following form:

$$\left(P_e - P_c\right)\left(P_y - P_e\right) = \eta\ P_e\ P_c \tag{26}$$

From which the value of $P_c$ may be obtained using:

$$P_c = \frac{P_e P_y}{\phi + \left(\phi^2 - P_e P_y\right)^{0.5}} = \eta\ P_e\ P_c \quad where \quad \phi = \frac{P_y + (\eta + 1)P_e}{2} \quad and \quad P_e = \frac{\pi^2 E}{\lambda^2} \tag{27}$$

Where:

$P_y$ is the design strength

λ is the slenderness ratio

The Perry factor η for flexural buckling under axial force should be taken as:

$$\eta = a(\lambda - \lambda_0)/1000 \geq 0 \quad where \quad \lambda_0 = 0.2\left(\pi^2 \mathrm{E}/P_y\right)^{0.5} \tag{28}$$

where:

$\lambda_0$ is the limiting slenderness ratio below which it can be assumed that buckling will not occur.

The Robertson constant "a" in equation 28 should be taken as 2.0, 3.5, 5.5 or 8.0 as indicated in European design codes depending on the cross-section, thickness of material, axis of buckling and method of fabrication (See European design Code:EN1993:1-1)

## 2.4 Effective length factors

In Euler buckling theory, the fundamental buckling mode is dependent on the buckling length between points of contra-flexure. The actual column is replaced by an equivalent pin-ended column of the same strength that has an effective length ($L_e$):

$$\mathrm{P_{cr}} = \frac{\pi^2 EI}{(kL)^2}$$

$$\text{Effective Length } (L_{e)} = kL \tag{29}$$

Here, L is the actual length and *k* is the effective length ratio determined based on boundary end conditions. An alternative method to determine effective length is to determine the distance between points of contra-flexure of a deflected strut. The

effective length of a column is simply the length of a column, with given end conditions, required to fit into the Euler buckling load equation:

$$\mathrm{P}_{\mathrm{cr}} = \frac{\pi^2 EI}{(kL)^2} \tag{30}$$

The effective length factor takes on a more physical meaning when it is applied to the buckling mode shape. The effective length factor is the number of inverse half sine waves that occurs in the buckled shape.

Thus, for the pinned-pinned column, $k$ = 1, so there is a one half wavelength over the buckled column length. For the cantilevered column, $k$ = 2, so the buckled mode shape is one quarter wavelength. This can be seen graphically in Table 2.3. For design purposes, some of these effective length factors are reduced, due to practical difficulties in achieving fully fixed and other idealized boundary conditions.

Researchers have investigated the effective length factors that depend on whether the structures bracing. Yura (1971), Duan (1989) and Cheong (1997), proposed methods to determine factors of effective length.

Table 2.3 shows the critical loads and effective length factors for prismatic columns with various end conditions.

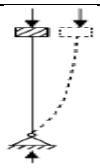
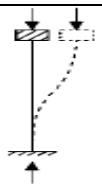
| **End Conditions** | **Critical Load** | ***k*** | **Deflected Shape** |
|---|---|---|---|
| Pinned-Pinned Support | $\frac{\pi^2 EI}{L^2}$ | 1 |  |
| Fixed-Free (Cantilever) Support | $\frac{\pi^2 EI}{4L^2}$ | 2 |  |
| Pinned - Guided Support | $\frac{\pi^2 EI}{4L^2}$ | 2 |  |
| Fixed-Pinned Support | $\frac{\pi^2 EI}{(0.6997L)^2}$ | 0.699 |  |
| Fixed-Guided Support | $\frac{\pi^2 EI}{L^2}$ | 1.0 |  |
| Fixed-Fixed Support | $\frac{4\pi^2 EI}{L^2}$ | 0.5 |  |

Table 2:2 Critical loads and effective length factors

## 2.5 Axial-Torsional Buckling

A column axially loaded may fail by twisting as it buckles. A schematic diagram below shows a typical column experiencing axial-torsional buckling.

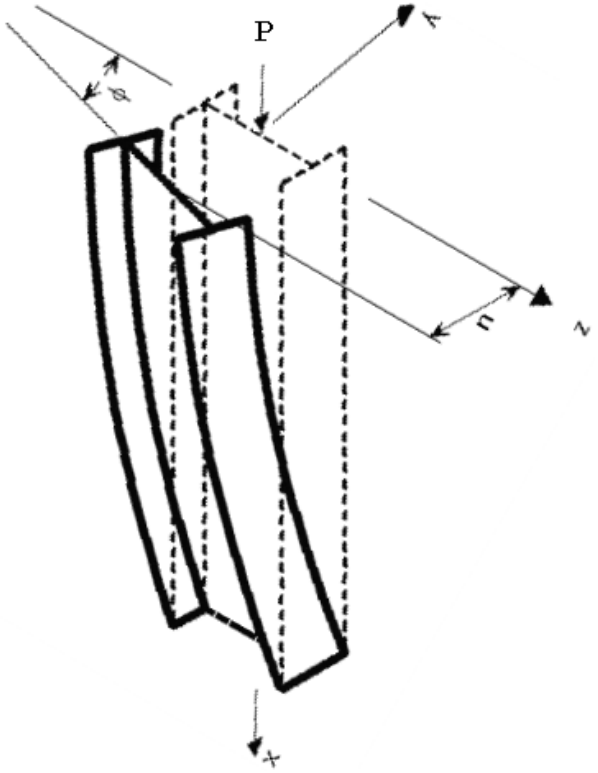



Figure 2:7 Column under axial torsional buckling

The axial torsional buckling load is found and given by Euler equation as follows (Timoshenko and Gere, 1961).

$$\mathrm{P}_{\mathrm{cr}} = \frac{1}{R^2}\left(GJ + \frac{\pi^2 EI_w}{(kL)^2}\right) \tag{31}$$

Where;

E = Young's modulus of elasticity

$I_w$ = The warping constant

G = Shear modulus of elasticity

J = Torsion constant

$R_z^{\,2} = I_y^{\,2} + I_z^{\,2}$ , Second Moment of area about local polar axis

From Euler's Equation (1) above, there exist critical loads $P_{cr\,x}$, and $P_{cr\,y}$ due to bending in the $xy$ and $xz$ planes respectively. The columns will buckle at the lowest critical load of $P_{cr\,x}$, $P_{cr\,y}$, $P_{cr\,\theta}$. These critical loads are influenced by shear deformation as discussed below.

## 2.6 The effect of shear deformations on the elastic critical column load

Euler-Bernoulli theory considers only deflection curvature due to flexural moments when deriving Equation (1). This is based on Kirchhoff assumptions that sections normal to the neutral axis remain normal before and after bending.

There is shear forces (V) acting on cross sections of the member (Figure 2.8) as buckling occurs, hence producing an additional curvature due to shearing force (Timoshenko and Gere , 1961).

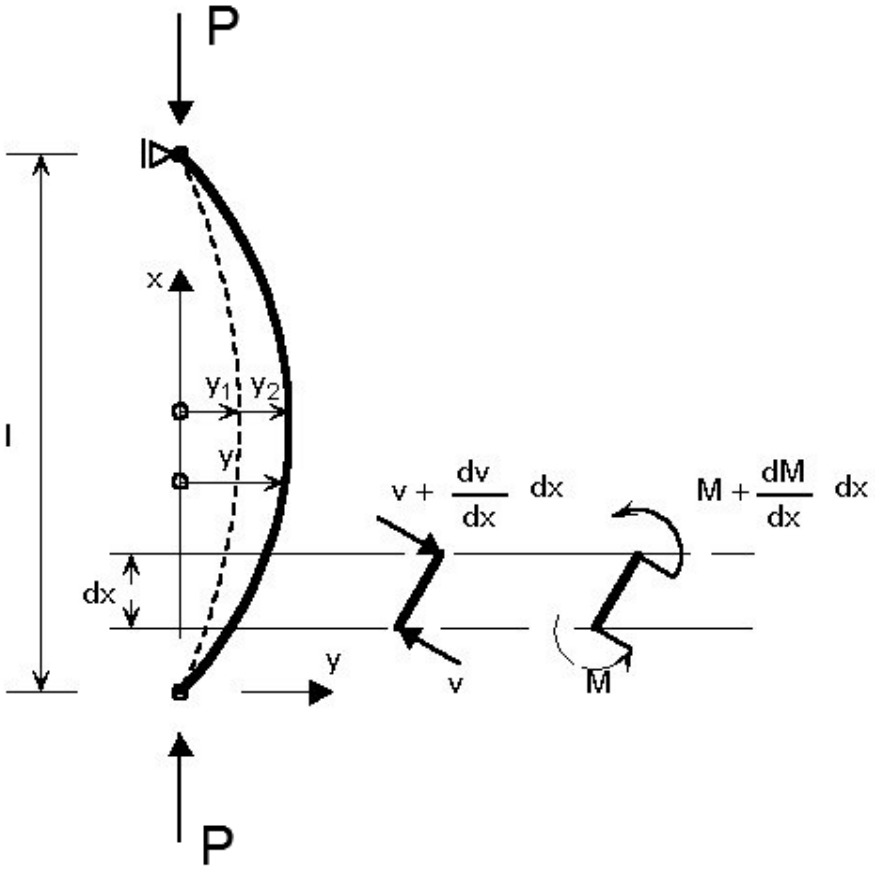



Figure 2:8 Effect of shear gorce (Ahmed,2006)

Timoshenko incorporated an additional slope due to shear forces acting on the cross sectional in the Euler's equation. Thus, Timoshenko's beam theory is an extension of the Euler-Bernoulli's beam theory which allows for the effect of transverse shear deformation. Timoshenko's beam theory loosens up the assumption of plane sections remaining plane and normal to neutral axis.

In the Timoshenko beam theory, plane sections still remain plane but they no longer have to remain normal to the longitudinal axis. The difference between the normal to the neutral axis and the plane section rotation is the shear deformation ($\Theta_x$). This relationship is shown in Figure 2.9.

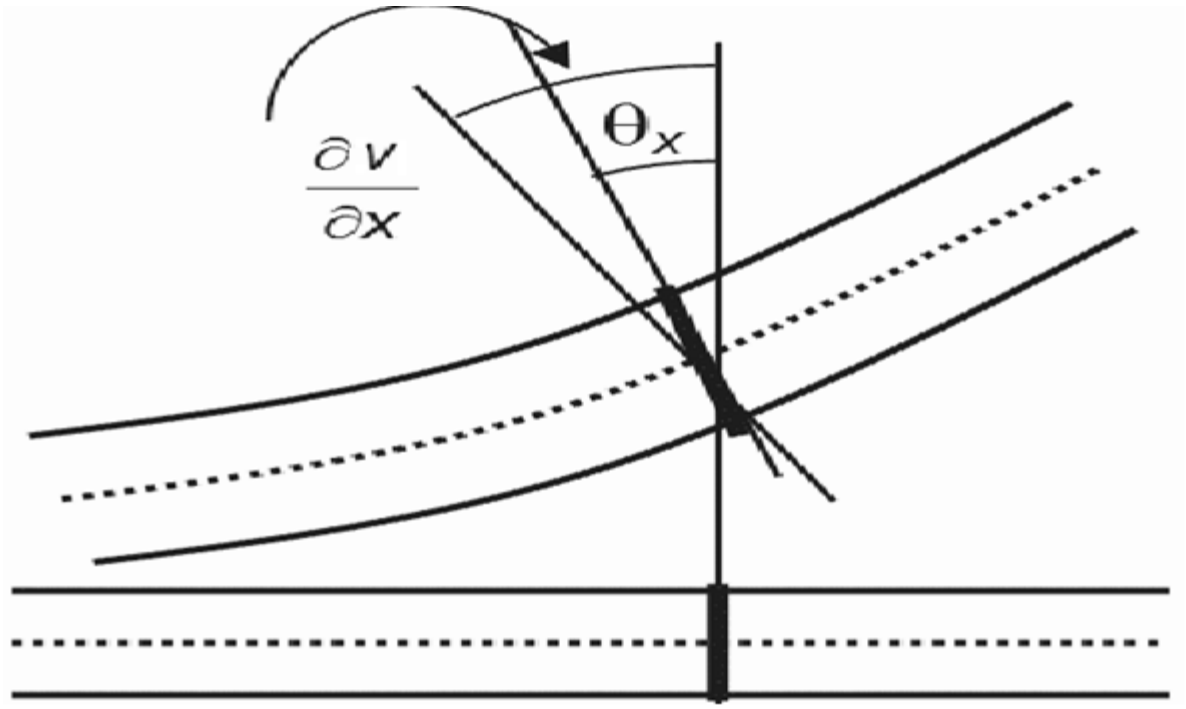



Figure 2:9 Effects of shear deformation

The total lateral deflection "*y"* of the centreline is the result of two components (Figure 2.8).

$$y = y_1 + y_2 \tag{33}$$

The bending moment M gives rise to the deflection $y_1$, and the shearing force V to the additional deflection $y_2$. According to elastic theory the curvature due to the bending moment M is as follows:

$$\frac{d^2 y_1}{dx^2} = -\frac{M}{EI} = -\frac{Py}{EI} \tag{34}$$

Where

E = The modulus of elasticity or Young's modulus.

I = The second moment area of the cross-section.

The magnitude of shearing force acting on the element $dx$, shown in Figure 2.8 above, is given by $V = \frac{dM}{dx} = \frac{Pdy}{dx}$, Where P is applied load. The slope due to the shearing force V is as follows:

$$\frac{dy_2}{dx} = n\frac{V}{GA} = n\frac{P}{GA}\frac{dy}{dx} \tag{35}$$

Where,

A = The total cross-sectional area of the column,

G = The shear modulus in shear of elasticity

$n$ = Shape factor of the column cross-section ($n$=1.11 for solid circular cross-sections; $n$ = 1.2 for rectangular cross-sections).

The rate of change in slope of the deflection curve produced by the shearing force represents the additional curvature due to shear and is given by:

$$\frac{dV}{dx} = n\frac{P}{GA}\frac{d^2 y}{dx_2} \tag{36}$$

By considering the total curvature produced by bending and shear force, the differential equation of the total curvature of the deflection curve is given by (Timoshenko and Gere, 1961):

$$\frac{d^2 y}{dx^2} = -\frac{Pdy}{EI} + \frac{n\,P}{AG}\frac{d^2 y}{dx^2} \tag{37 (a)}$$

Or

$$\frac{d^2 y}{dx^2} + \frac{P}{EI\left(1+\frac{nP}{AG}\right)}y = 0 \tag{37(b)}$$

Solving the differential equation, the critical load ( $P_{cr}$ ) for a solid column is found to be as given below. The formula of critical load of built up column's where shear effects is considered was derived by F. Engesser in 1891(Timoshenko and Gere, 1961).

$$P_{cr} = \frac{P_e}{\left(1+\frac{nP_e}{AG}\right)} \qquad 38(a)$$

Or

$$P_{cr} = \frac{P_e}{\left(1+\frac{P_e}{P_d}\right)} \qquad 38(b)$$

Where,

$P_e$ = Euler critical load.

$P_d$ = Shear stiffness.

Thus owing to the action of shearing forces, the critical load is diminished by the ratio of $\frac{1}{\left(1+\frac{nP_e}{AG}\right)}$. This ratio is very nearly equal to unity for solid columns such as a column of rectangular cross section. Hence in these cases the effect of shearing force can be neglected. The shear effect is of importance for built up columns consisting of struts connected by lacing bars or batten plates.

The critical load for laced column is always less than for solid columns having the same cross sectional area and the same slenderness ratio $(L/r)$, this decrease in critical load is due primarily to the effect of shear on the deflections. The influence of the shearing forces on the reduction of the critical load is very significant. The actual value of the critical load depends upon the arrangement and dimensions of the lacing bars.

These effects of shear on the critical load is represented by the additional slope of deflection due to shear as given by

$$\gamma = \frac{V}{P_d} = \frac{\delta_1 + \delta_2}{a} \qquad (39)$$

where:

*a*, is a panel height (Fig 2.10)

$\delta_1 and \delta_2$ are lateral displacements caused by batten and diagonal members. To determine the quantity ($1/P_d$) in any particular case, the elastic deformation produced by the shearing force, has to be investigated.

## 2.7 Evaluation of the Shear Stiffness ($P_d$) of Laced Columns

The shear stiffness ($P_d$) is derived from the elastic extension of the lacing members (diagonals and the horizontals); the elongation of the chords (the main components) is not taken into account, because they are already considered in the global flexural stiffness (EI) of the built-up column (Ahmed,2006).

Consider the brace arrangement of lacing, as shown in Figure 2.10. By considering the extension of one diagonal and of one horizontal, the shear stiffness can be derived as follows;

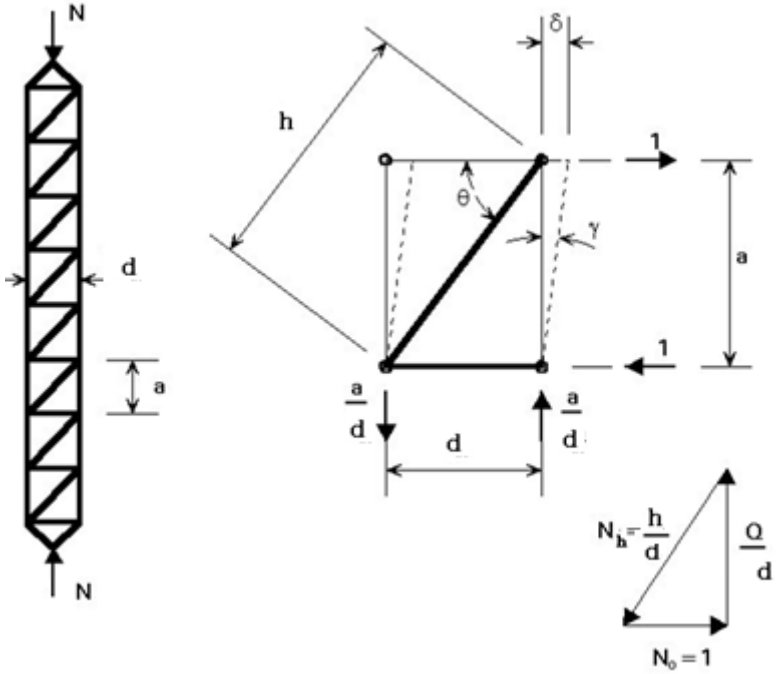



Figure 2:10 Effects of shear forces (Ahmed,2006)

$$\frac{1}{P_d} = \frac{\delta}{a} = \gamma \tag{40}$$

Where δ is the lateral displacement due to the unit shearing force.

The total displacement δ is the result of two components: $\delta_1$ is the contribution from the elongation of the diagonal; $\delta_2$ is the contribution from the shortening of the horizontal. From virtual work theory:

$$\delta = \delta_1 + \delta_2 = N_d \frac{h}{EA_h} + N_0 \frac{d}{EA_0} = \frac{h}{d}\frac{h}{EA_d}\frac{h}{d} + \frac{d}{EA_0} \tag{41}$$

Thus, for one plane of lacing:

$$\frac{1}{P_h} = \frac{1}{aE}\left[\frac{d}{A_0} + \frac{d^3}{h^2 A_d}\right] = \frac{1}{ah^2 E}\left[\frac{h^3}{A_0} + \frac{d^3}{A_d}\right] = \frac{d3}{ah^2 A_d E}\left[\frac{h^3 Ad}{d_3 A_0} + 1\right] \tag{42}$$

Shear stiffness of different brace arrangements are available in the literature (Ahmed,2006).

Table 2.3 shows some formulae of shear stiffness of different types of bracing.

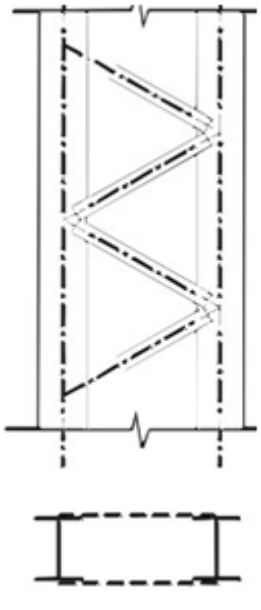
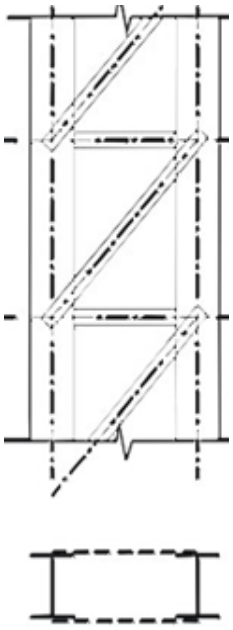
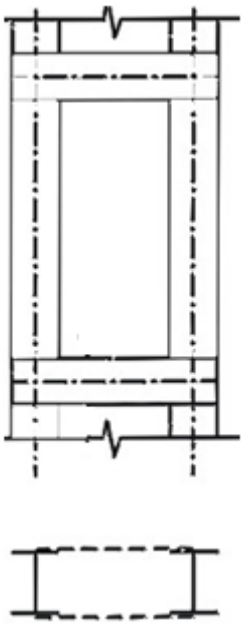
| Brace Type |  |  |  |
|---|---|---|---|
| Shear Stiffness | $P_d = \frac{a\ d\ E\ A_h}{h^3} 2$ | $P_d = \frac{a\ d^2\ E}{\frac{d^3}{A_0} + \frac{h^3}{Ah}} 2$ | $P_d = \frac{1}{\frac{a^2}{24\ E\ I_c} + \frac{ad}{12\ E\ I_b}}$ |

Table 2:3 Shear stiffness of various brace type

## 2.8 Finite element Analysis (FEA)

The finite element method was used to evaluate the behaviour of built-up columns. In principle, the finite element method is derived from computer methods/stiffness matrix method which is based on the principal of virtual work. The finite element method calculates the displacement field.

$$\int \sigma \, \delta \, \varepsilon \, dV = \int_s F \, \delta \, u \, dA$$

*Where*

$\sigma = stress\ tensor$

$\varepsilon = strain\ tensor$

$u = displacement\ \ field$

$F = Force\ \ field$

$\delta = \text{var}\, iation$

$V = Volume$

$A = Area$

The forces vector acting on the structure is given in the expression.

$$F = KU \tag{43}$$

K=Global stiffness matrix

U= Displacement vector

The finite element method uses displacement interpolation matrix to estimate displacements within the elements as $U^{el}$ given in the equation below

$$U^{el} = H^{el} \hat{U} \tag{44}$$

where

$\hat{U}$ = Vector displacement (global coordinates system) of the nodes of the element.

$$\hat{U} = (U_1 V_1 W_1 U_2 V_2 W_2 \ldots\ldots\ldots U_N V_N W_N) \quad (45)$$

$N$ = Nodes in the element

$H$ = Displacement interpolation matrix

$U^{el}$ = Displacement at any point in the element

The finite element method creates a stiffness matrix for individual elements in its local axes and assembles every stiffness matrix into a global stiffness matrix of the whole structure, boundary conditions are now applied. In non-linear analysis, as load changes, the structure stiffness matrix is updated to reflect the current structural geometry, stress conditions and possible changes at supports.

A finite element analyses uses a system of points called nodes, which forms a grid called a mesh. The deformation over each element is defined by a simple polynomial function through the nodes. The web of nodes which defines the element material helps to transfer the reaction from neighbouring nodes due to applied forces. The coefficients of these polynomial functions are known. As a result, the individual displacements of the entire structure may be calculated and consequently the behaviour of the structure may be fully described in terms of the displacements of the nodes(Charles,2003).

Nodes are assigned at a certain density throughout the structure depending on the level of accuracy required. Usually, regions where stress is varying rapidly receives a high density.

The dependent variables in the polynomial functions are the nodal displacements U and can be expressed as an Eigen-Value problem

$$\lambda^f [ K_{CG} ]\{U\} =[K_{CE} ]\{U\} \quad (46)$$

Where $\lambda^f$ is the load factor, $K_{CE}$ is the global elastic stiffness matrix corresponding to the nodes, $K_{CG}$ is the geometric stiffness matrix.

The first Eigen-Value, i.e. the smallest value of $\lambda^f$ at which the structure becomes unstable is termed the critical load factor $\lambda^f{}_{cr.}$

This classical Eigen-Value approach has been discussed by many authors among them Prezemieniecki (1968), Allen and Bulson (1980), Graves Smith (1983), Brebbia and Ferrante (1986), Coates and Kong, (1988), Galambos (1988) and Bathe (1996). Different techniques to obtain Eigen-Values and Eigen-Vectors are available; among them are Vector Iteration methods i.e. Inverse Iteration, Forward Iteration and Rayleigh quotient iteration, transformation methods such as Jacobi method and generalised Jacobi method and the subspace iteration method (Mahfouz, 1999).

Solving a problem using the FEM involves the user choosing a suitable type of element. Various element types exist for modelling i.e. solid, beams, plates, shells, truss, wires and so on. Elements have different number of degrees of freedom hence the choice depends on the type of problem.

Finite element analysis helps in producing stiffness and strength visualizations and in optimizing weight, materials, and costs. The behaviours of structure i.e. stress distribution and displacements can be visualized.

FEM increases accuracy, enhances designs and gives a better insight into critical design parameters, a faster and less expensive design cycle.

## 2.9 Finite element modelling

In this study, built up columns of different brace types, aspect ratio are simulated at defined load ratios. This simulation consists of an Eigen Value problem.

Eigen Value buckling analysis predicts the theoretical buckling elastic strength of an ideal structure given the loading and constraints. Eigen-Values are factors which if multiplied with the applied load will give a buckling failure load of the system. The corresponding Eigen-Vectors give modes of failure. However, structural imperfections and nonlinearities prevent most structures from reaching their Eigen-value predicted buckling strength (Lawrence ,2007).

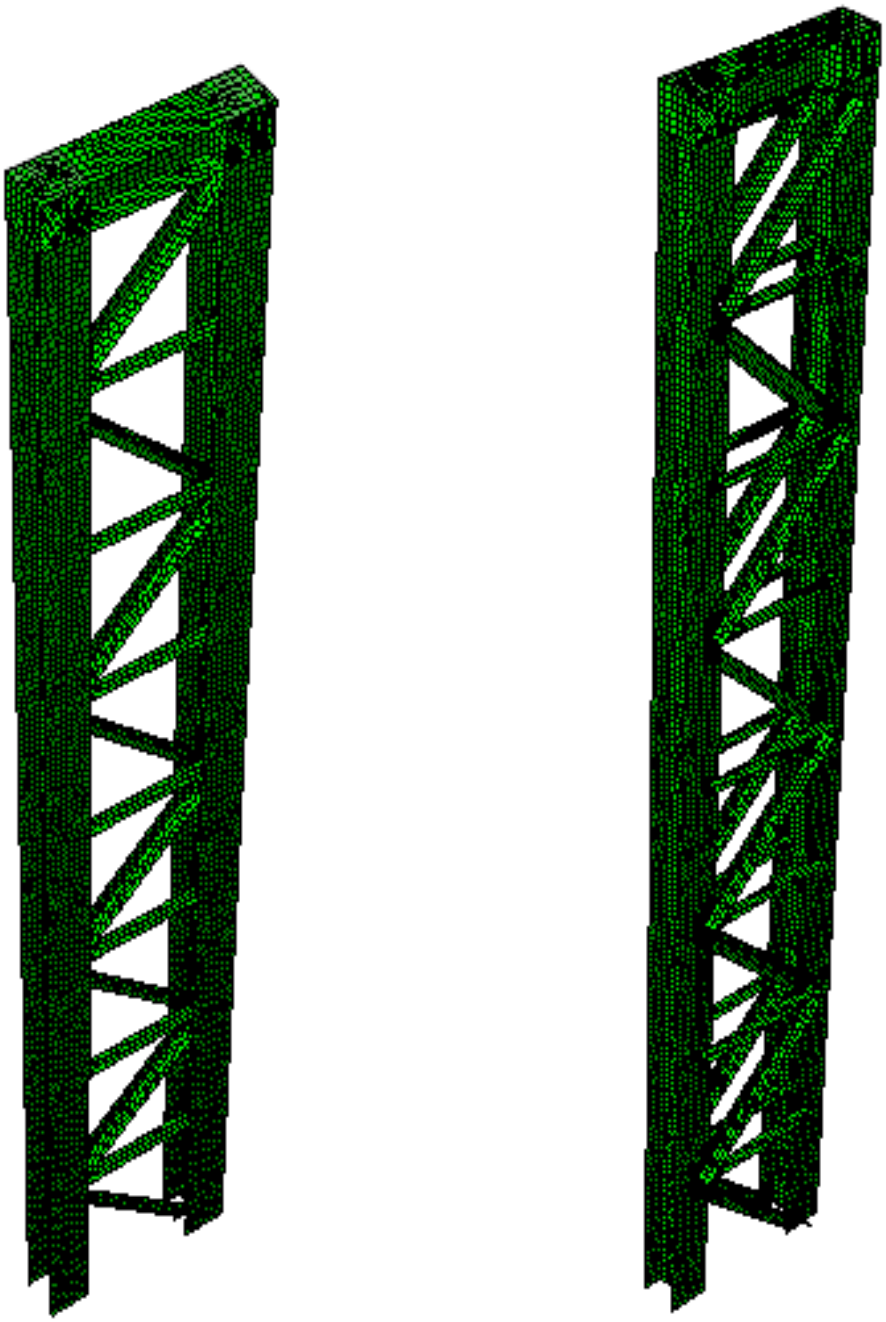
The type of analysis used in ABAQUS is linear perturbation. An analysis step during which the response is linear is called a linear perturbation step.

### 2.9.1 Critical Load of Columns Using the Finite Element Method

To compute the buckling, or critical load, of a structural system using the finite element method two approaches can be followed. A full geometric nonlinear analysis can be performed, or the effect of internal loads on the stiffness of members can be taken into account. In the latter approach, the Eigen-Value problem is solved to determine both the mode shape (Eigen-Vector) and the critical load (Eigen-Value). This latter approach is adopted in this study.

Here the built up columns are discretized using fully parabolic 8 node quadrilateral thick shell elements. Each node has six degrees of freedom. These elements not only capture bending but also shear deformation as discussed in lecture notes of (Amit and Whalem,2001) and (Alex Elvin, 2005). Capturing shear deformation is essential in non-solid built up columns. All simulations are performed using ABAQUS Version 6.9 (Dessault Systems, Inc.).

Typical finite element discretizations of single and double laced columns are shown in Figure 2.11 (a) and (b). The number of nodes is also given for each column type.



a) Single laced column; Elements: 14217 nodes: 45015 b) Double laced column; Elements: 17808; nodes: 56703

Figure 2:11 Finite element discretization of (a) single and (b) double laced columns

### 2.9.2 Type of elements

Thick shell elements (Figure 2.12) will be used in finite element simulation of built up columns. These elements accounts for the transverse shear flexibility within the shell during the analysis (Amit and Whalem, 2001). The elements are two dimensional shell quadratic elements comprising of 8 nodes with 6 degrees of freedom per node. The degrees of freedom are translation and rotational in the x,y,z directions. These elements are adequate and efficient in capturing shear stress in the member sections. Capturing shear stress is necessary to determine the critical load of a column correctly.

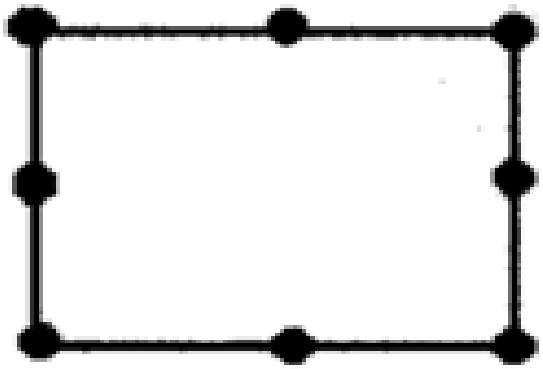



Figure 2:12 8 Node shell element

.

### 2.9.3 Element mesh size

A high mesh density will increase the accuracy of the results obtained at the expense of computation time, while low mesh density can lead to unacceptable errors. The mesh density is a trade-off between computational time and accuracy of results.

The acceptability of results was determined by performing a convergence study. Figure 2.13 shows a typical convergence study of the critical load with an increase of the number of degrees of freedom.

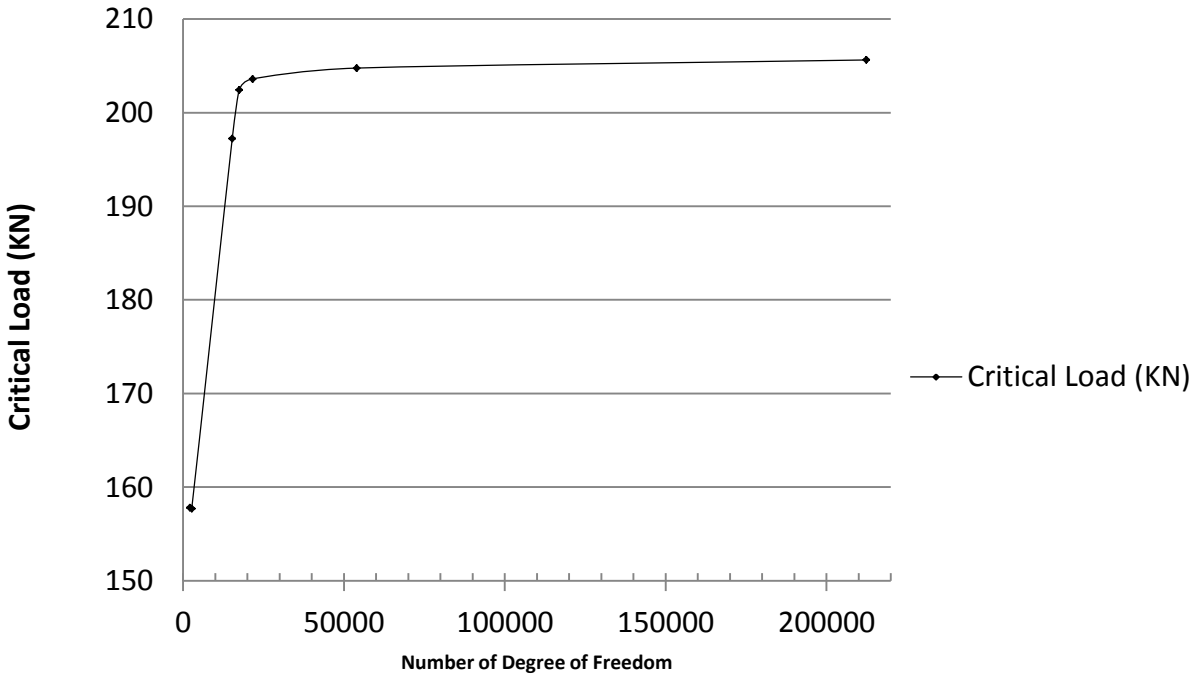



Figure 2:13 Convergence study

A typical finite element model of built up a column with a convergent mesh density is shown in Figure 2.14.

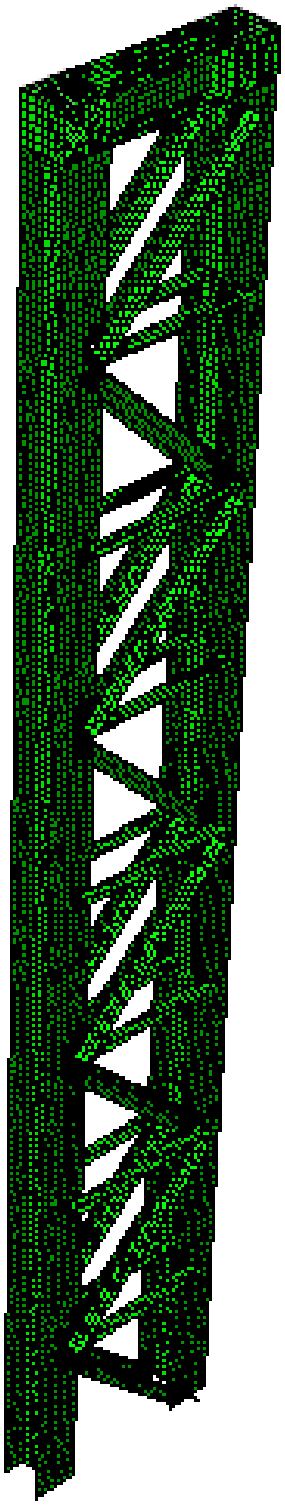


Figure 2:14 Typical finite element model

## 2.10 Parametric study

The objective of the parametric study was to investigate how the built up columns of different chord sizes and different geometries behave under varied load ratios. In addition.

To investigate the behaviour of built up columns, several variables were identified and varied. The variables that were considered are:

(a) Boundary conditions
(b) Lacing geometry
(c) Distance between the main chords of the column (column width)
(d) The ratio of the loads acting on the column's two main chords (Load ratio).
(e) Lacing Member sizes

Several parameters were kept constant throughout the simulations. The height of the column was set to six meters.

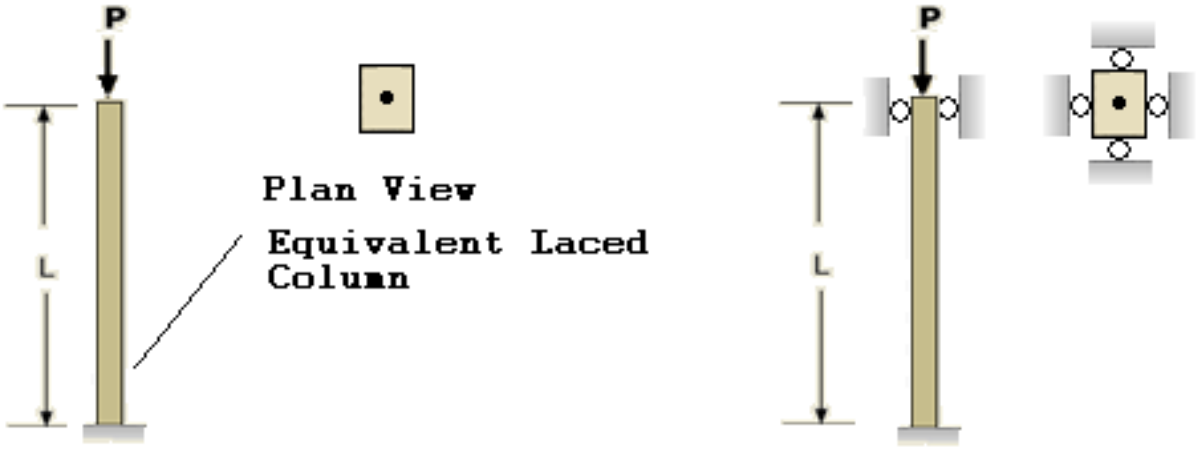
The main compression chords of the built up columns used in this investigation are made of 203x203x46 universal columns sections and the size of angles 70x70x6.

Based on literature and field observation, the main chord profile size used is considered realistic and practical for a built-up column supporting a crane.

In this study, the horizontal forces are not considered in the modelling of built up columns because horizontal forces in most cases are negligibly small as compared to the vertical forces applied. The built-up columns are assumed to be perfectly straight without geometry imperfections and materials are assumed to be homogeneous.

### 2.9.1. End supports

The end supports of all finite element models are chosen to simulate realistic end conditions. The two cases of end supports are investigated namely (a) Fixed-Free (Figure 2.15 (a)), (b) Fixed-Roller on top (Figure 2.15 (b)).




a) Fixed-Free (Cantilever) (b) Fixed-Roller

Figure 2:15 End supports

### 2.9.2. Column geometry

As part of the parametric study, the height of the all built up columns is kept the same at 6.0 m while the column width (D) (Fig 2.18) is varied as follows: 0.5m, 0.75m, 1.0m and 1.5m. The column dimensions are chosen based on field observation. Different brace types investigated in this study are shown in Figure 2.18. Recommendations from SANS10162 on the bracing system were followed.

In general, the lacing was kept as close to 45° to the vertical as possible. As the column width increases, the bracing panels decreases. Although not presented here

models of stiffened laced column are simulated and compared to un-stiffened column to make sure failure was not due to local buckling of the members.

The lacings members are welded onto the main compression chord flanges in the case of double laced column as in Fig 2.16.

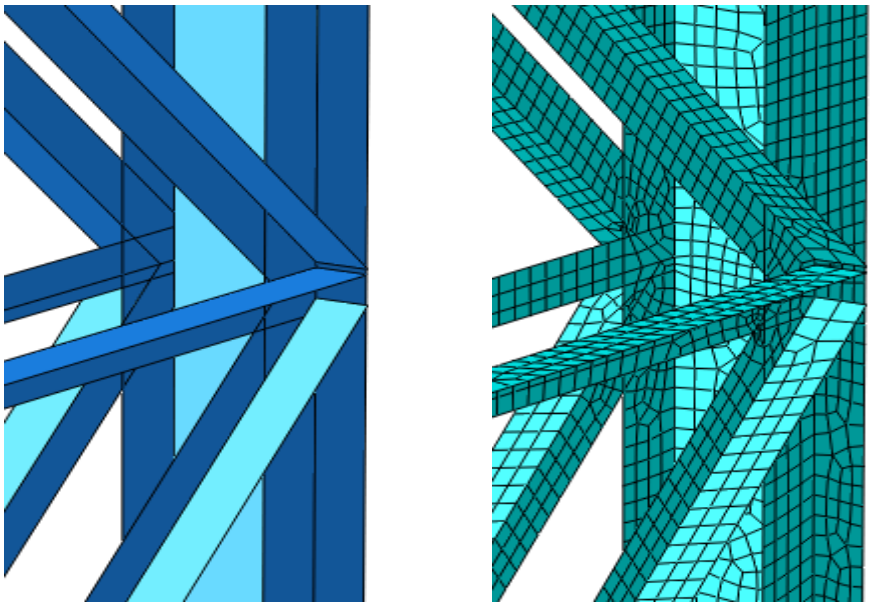


Figure 2:16 Connection detail of the lacing in the double laced columns

For the single lacing columns gusset plates were provided on the webs of the main compression chords in order to attach the bracing as shown in Figure 2.17.

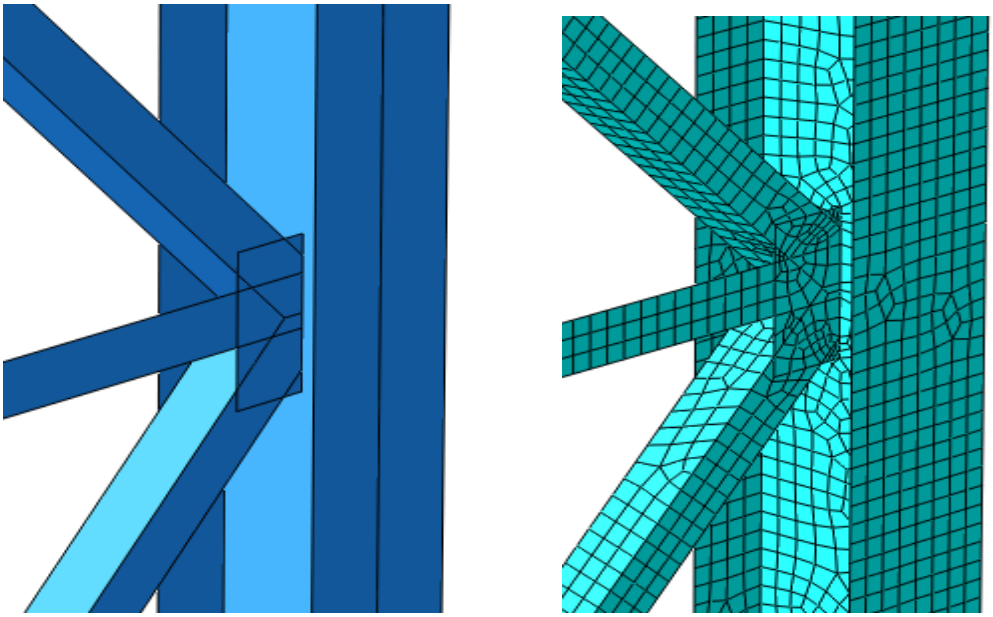
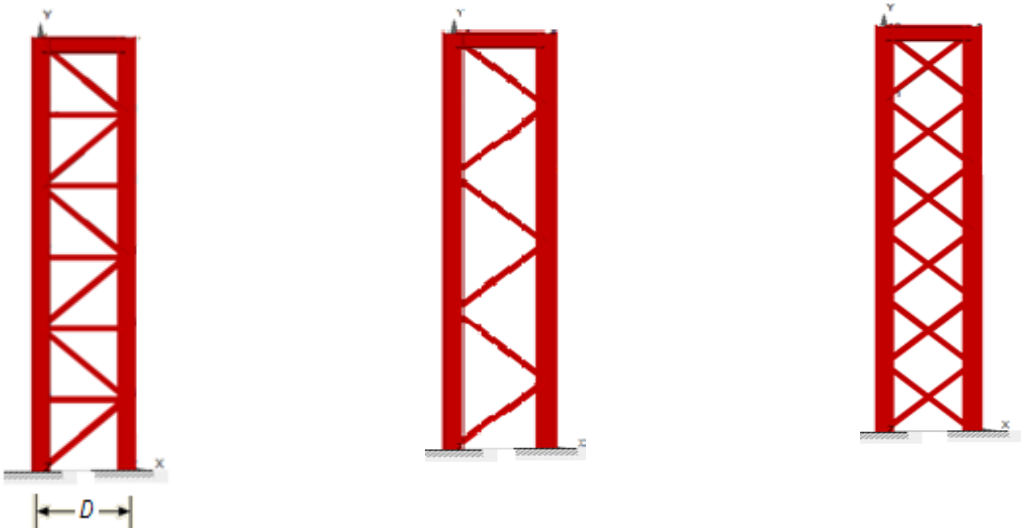


Figure 2:17 Connection detail of a gusset on single laced columns.

Although different types of bracing configuration are used in practice, this study has considered several common bracing types (Figure 2.18). In this study these lacing configurations are referred to as “W-brace” (Figure 2.18(a)), “V-brace” (Figure 2.18(b)), and “X-brace” (Figure 2.18(c)). These bracings types are simulated for both traditional double laced and single laced columns.




Type 1: W-Brace

Type 2: V-Brace

Type 3: X-Brace

Figure 2:18 Brace types

## 2.11 Influence of the end boundary condition on critical load and torsion

Boundary conditions have a considerable effect on the critical load of slender column. The end support conditions determine the effective length factor and mode of buckling of a column. The closer together the inflection points are, the higher the resulting buckling capacity of the column.

In Euler's expression of estimating critical load, it is known that the value of effective length factor *k* is entirely depending on the boundary conditions. Various effective length factors depending on different boundary conditions available and shown in Table 2.2.

## 2.12 Load ratio effects on critical load and torsion

The buckling and torsional performance of built up columns depends on the load ratio it is carrying in addition to brace type and end supports. In this study, the load ratio (P/N) is defined as the ratio of applied load on the main compression chords (Figure 2.19).

In principle, as the load ratio moves away from one, the loading system on the column becomes equivalent to a loading system with equal axial loads on both compression chords with one of heavily loaded chord carrying additional applied moment (Fig 2.19). The additional moment is equivalent to a difference of applied loads (P-N) multiplied with the lever arm (column width), (Fig 2.19).

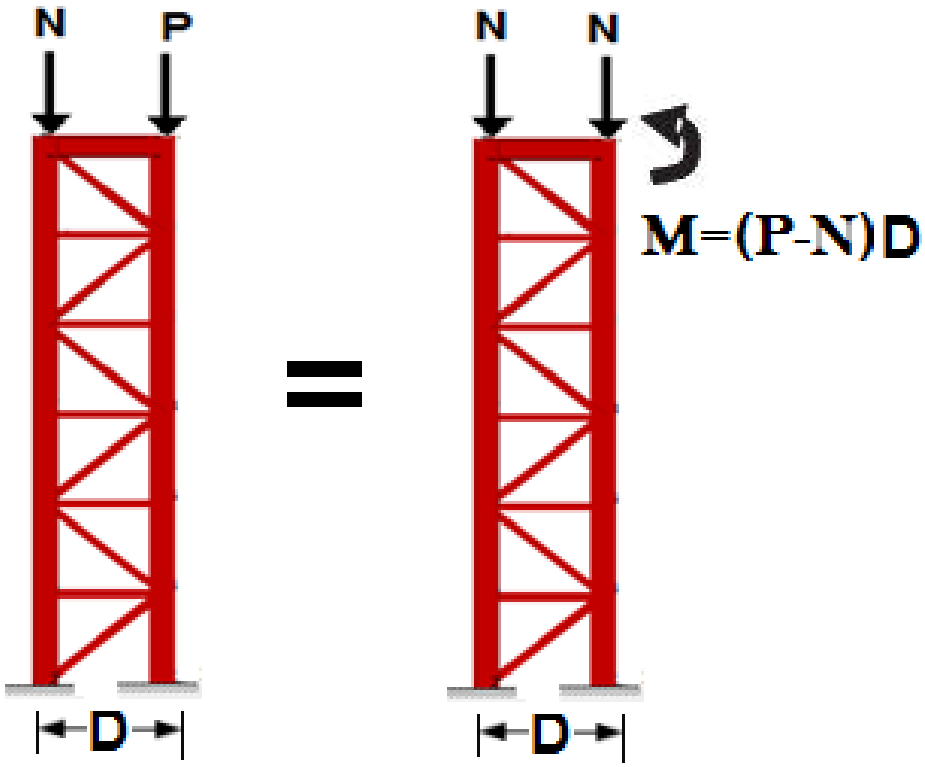



Figure 2:19 Load ratio on a built up column

To assess the efficiency of a column, a practical load ratio range is established from field observation. Efficiency is defined as the percentage of critical load at a certain load ratio over maximum critical load, achieved at load ratio one

In this investigation, the load ratio ranges from $\frac{1}{200}$ (lower bound) to 2/1 (high bound). Due to the anticipated symmetry of the column's performance curves below 1.0 and above 1.0, the buckling and torsional performance curves have only been plotted for one side (0.05 to 1)

# CHAPTER 3

This chapter highlights the recommendations from four steel design codes. The building codes considered are the South Africa Code, the European Code, the British Code and the Canadian Code.

## 3.1 Building Codes specifications on Built up Column

### 3.1.1 South Africa National Standards: SANS10162-1:

Lacing shall provide a complete triangulated shear system and may consist of bars, rods or sections.

SANS10162-1 (2005), lacing shall be proportioned to resist a shear normal to the longitudinal axis of the member of not less than 0.25 times the total axial load on the member plus shear from transverse loads in any member. Slenderness ratio of lacing shall not exceed 140. For single lacing, the effective length shall be the distance between the connections to the main components. For double lacing connected at the intersections, the effective length shall the 0.5times that distance. Lacing members inclined to longitudinal axis of the built-up member shall not be less than 45 degrees.

Lacing systems shall be in the plane of lacing and as near to the ends as practicable and at intermediate points where lacing has intersected. Such battens can be tie plates or sections. The thickness and length of a tie plate be should be at least 1/60 and less than the distance between lines of bolts or welds connecting them to the main components respectively.

Sections used as battens shall be proportioned to and connected to transmit from one main component to the other a longitudinal shear equal to 0.05times the axial compression in the member.

For sections that are symmetric relative to the plane of loading; Factored resistance of brace shall be at least 0.02 times of compressive force in the member subject to compression at the braced.

Braces connected effectively to restrain flanges and at interval not exceeding one-quarter of the span length and in a manner to prevent tipping at the ends and lateral deflection of either flange in either direction at intermediate braces.

This code gives the design guideline formulae to calculate lateral force on braces.

### 3.1.2 European Code: EN1993-1-1:2005

The code has given the design formulae of estimating a critical load of a built up column, laced on opposite faces of the main section components (traditional double laced column).

The code provides shear stiffness ($P_d$) formulae of lacing depending on the type of lacings system (Eurocode, EN 1993-1-1:2005: Table 6.9).

For structural analysis, the code gives an allowance of bow imperfections for column analysis, eccentricity $e_0 = H/500$

Double lacings systems in opposite faces of the built-up member with two parallel laced planes should be corresponding, as shown in Figure 3.1 below and arranged so that one is the shadow of the other.

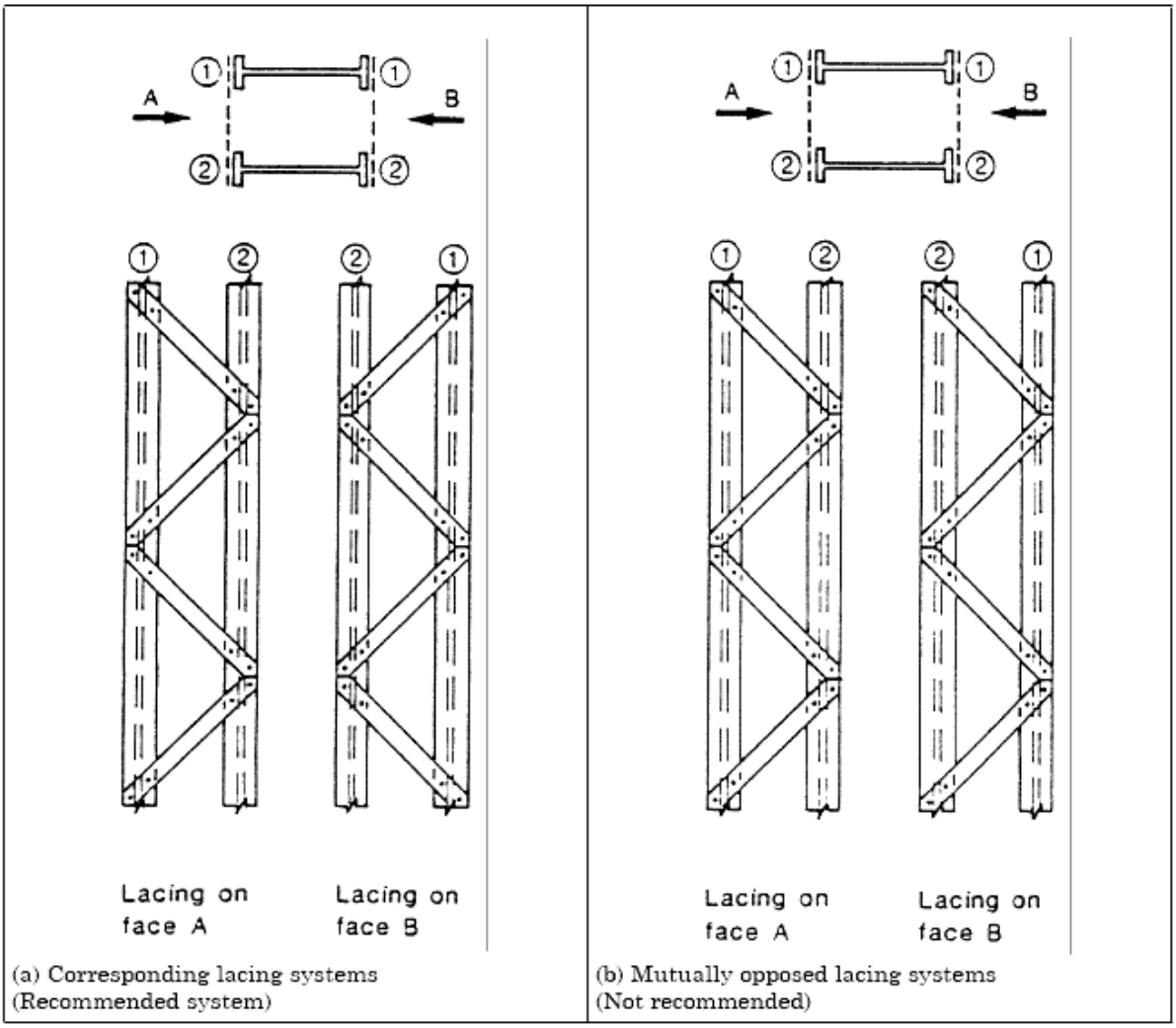



Figure 3:1 Lacing systems of on opposite sides of main components (EN 1993-1-1:2005:Table 6.9)

The checks for lacings of a built-up members or for the frame moments and shear forces of the battened panels or battened built-up members should be performed for the end panels taking account of the shear force in the built-up member.

When a double lacing system on opposite faces of a built-up-member with two parallel laced planes are mutually opposed in direction, the resulting torsional effects in the member should be taken into account.

Tie Panels should be provided at the ends of lacing systems, at points where lacing is interrupted and at the joints into account.

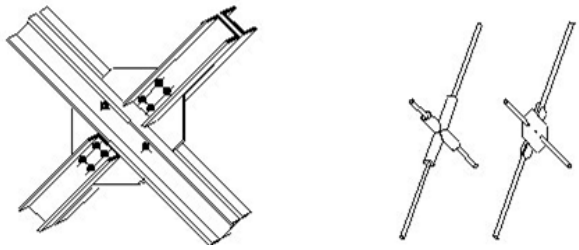


Figure 3:2 Recommend interruption detail of a Lacing systems(EN 1993-1-1:2005)

### 3.1.3 British Code: BS5950-1:200

The main components should be effectively restrained against buckling by a lacing system of flats or sections.

Lacing should comprise an effective triangulated system on each face as far as practicable; the lacing should not vary throughout the length of the member.

Doouble lacing systems mutually opposed in direction on opposite sides of two main components should not be used unless the resulting torsional effects are accounted for.

All lacing members should be inclined to the longitudinal axis of the built-up member at an angle between 45° and 70°.

Tie panels should be provided at the ends of the lacings systems, at point where the lacings is interrupted, and at connections with other members. Tie panels can take the form of battens. The tie panels should be designed to carry the loads for which the lacing system is designed.

The slenderness ratio of the main components between consecutive points where lacing is attached should not exceed 50. If the overall slenderness ratio of the member is less than 1.4 of the main components, then the design should be based on slenderness of 1.4 of the main components.

The lacing effective length should be taken as the distance between the inner end welds or bolts for single intersection lacing and 0.7 times this distance for double intersection lacing connected by welds at the intersection. The slenderness should not exceed 180.

The lacing and their connections should be designed to take 25% of axial force induced by transverse shear at any point in the length member, for a member carrying moments due to eccentricity loading. Lacing should be proportioned to resist the shear due to bending in addition to 2.5% of axial force.

### 3.1.4 Canadian Code: CAN/CSA-S16-01

The code defines a braced frame when it is five times stiffer than the frame without the bracing.

Allowable out-of-straightness ($\Delta_0$ ) should be less than 0.002times the distance between brace points. This shall be taken as the maximum tolerance.

Displacement ($\Delta_b$) of the member being braced at the brace point perpendicular to the member caused by the buckling load ($P_b$) and other external forces shall not exceed $\Delta_0$

The possibility of twisting of a member at brace points should be investigated and the bracing provided if necessary to prevent this. The top (tension) flange of a cantilever can deflect more laterally than the bottom, therefore bracing of the cantilever and tension flange should be considered.

Intermediate web stiffeners shall be required on only one side of the web for link beams less than 650mm in depth and both sides of the web for beams 650mm or greater in depth.

The stiffeners shall have the thickness of not less than web(w) or 10mm, whichever one is larger and the width shall be $0.5(b_f-2w)$, where bf is the thickness of the flange.

The column maximum slenderness ratio shall not exceed 60.

Formulae and Nomograph are provided for estimating column length (effective length).

The slender ratio (λ) of bracing members shall not exceed 200. Bracing members shall not exceed 40 m in height. Brace connections, eccentricities in connections to gusset plates or other supporting elements shall be minimized.

None of the above building codes has addressed the concept of single layer lacing of compressive chords, web-to-web.

# CHAPTER 4

This chapter presents the results of this study on the behaviour of single versus double laced columns.

## 4.1 Behaviour of Single versus Double laced column

The structural behaviour of both single and double laced columns for two case studies of end supports namely cantilever and pinned-roller case for each column type has been investigated. All columns have the same profile members. The result on structural behaviour is discussed below.

## a) Case 1-End Supports: Cantilever

Single and double laced columns are simulated for a cantilever case (fixed-free) at varied column widths (D) over a range of load ratios (P/N), as shown in Figure 4.1. The results of the variation of critical load, load ratio and column width is discussed. The data of these results are presented under Appendix A section 8.1.1. To show the trends, the results are plotted in this chapter.

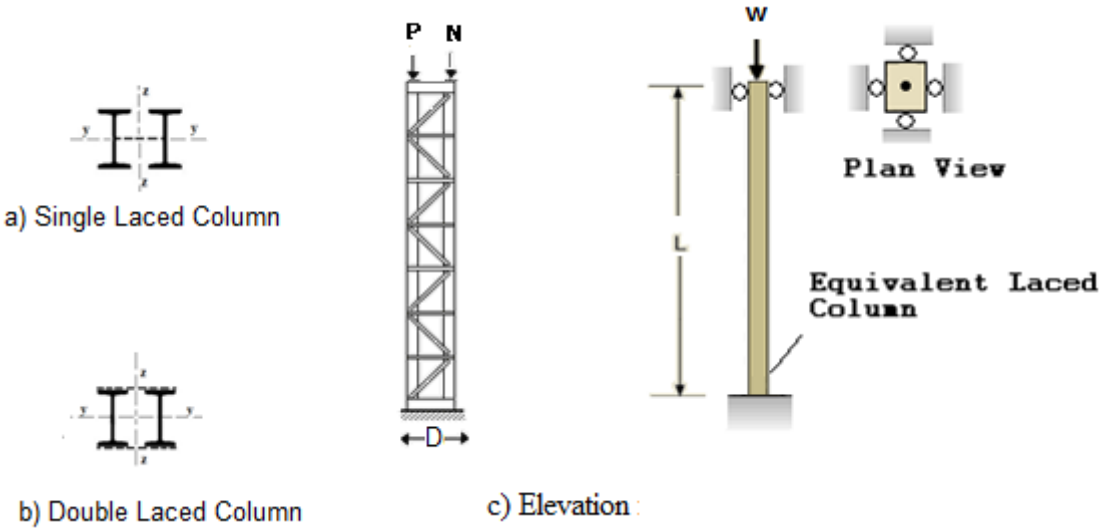



Figure 4:1 Bottom fixed and top free end support

To ensure that the behaviour does not include local buckling, built up column with stiffened compression chords and without stiffeners where simulated. The results both yielded almost equal critical loads with negligible differences assuring that the failure was not due to local buckling. To avoid plot congestion, only results of un-stiffened built up columns are plotted in these figures, Figure 4.2-4.5 show how critical load varies with load ratio for four column widths and various lacing configuration. Double laced columns (DLC) are compared to single laced columns (SLC).

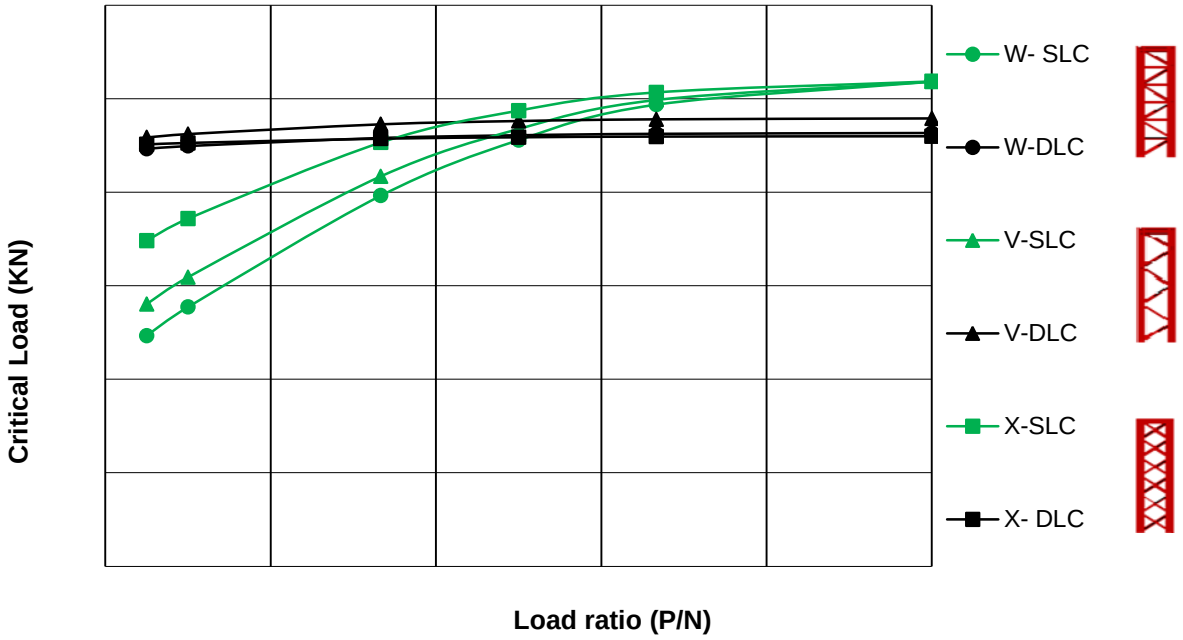



Figure 4:2 Critical load vs. load ratio of fixed-free columns of 0.5m width

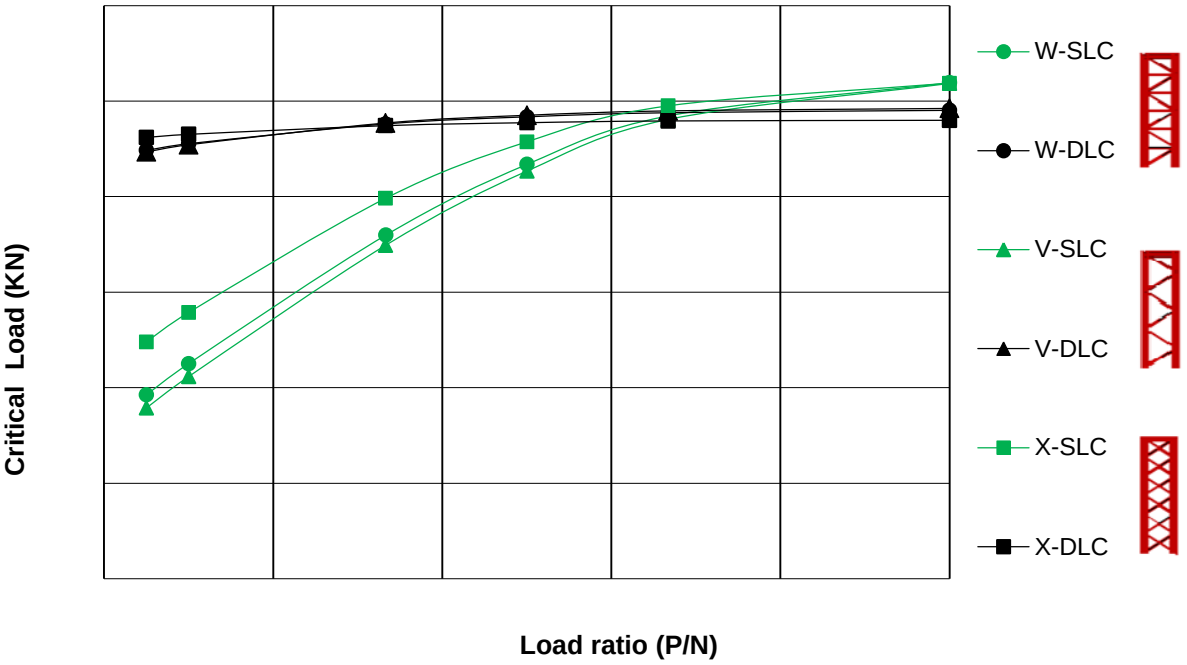



Figure 4:3 Critical load vs. load ratio of fixed-free columns of 0.75m width

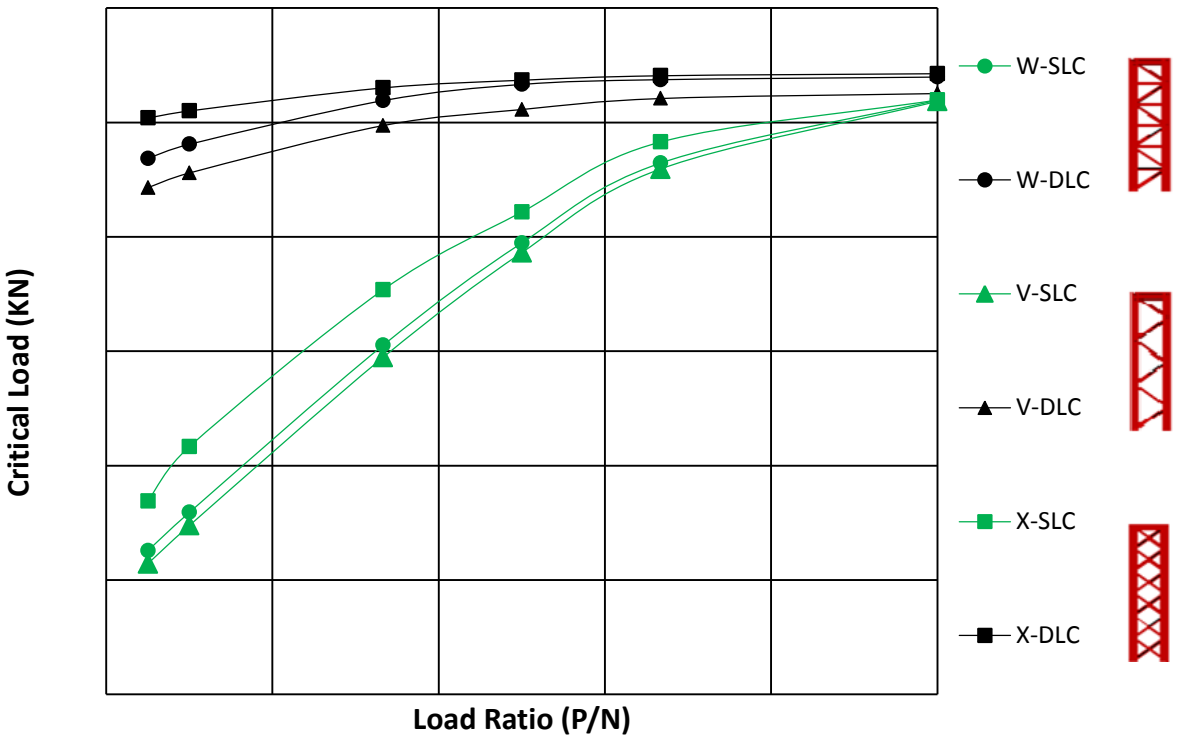



Figure 4:4 Critical load vs. load ratio of fixed-free columns of 1.0 m width

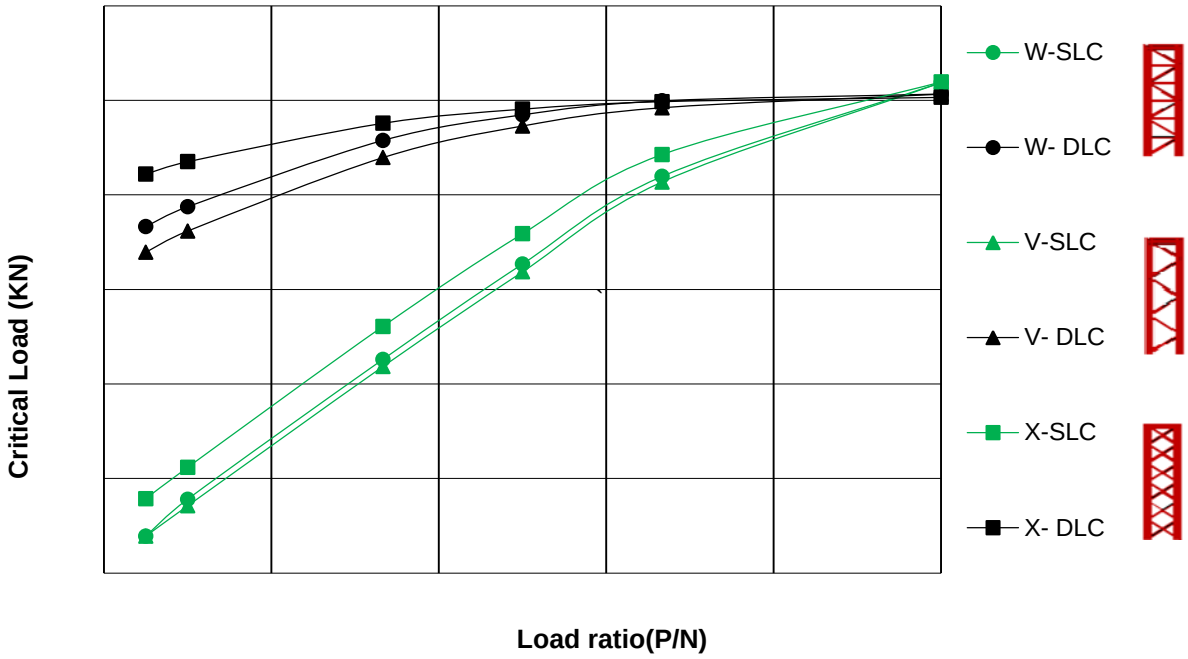



Figure 4:5 Critical load vs. load ratio of fixed-free columns of 1.5m width

From the results, double laced columns have shown less critical load variation over a range of load ratios than single laced columns. Double laced columns have yielded higher critical loads at low load ratio. Both built up columns have achieved maximum critical load at load ratio close to one respectively.

The result show slightly less variability of critical load for columns of different bracing configurations. Double laced columns of X-configuration are the least affected by the load ratio. The X-configuration has shown the best performance by 3%-10% and 1%-8% for single and double laced column.

As load ratio moves towards one so as the buckling load increases with variations of between 15%-25**%** and 1%-3% for single and double laced column respectively. For a given column width and boundary conditions, load ratio has less effect on double laced columns than for single laced columns.

From the results, a relationship between critical load and column width was established. Figure 4:6-4.11 below shows how critical load varies with column width.

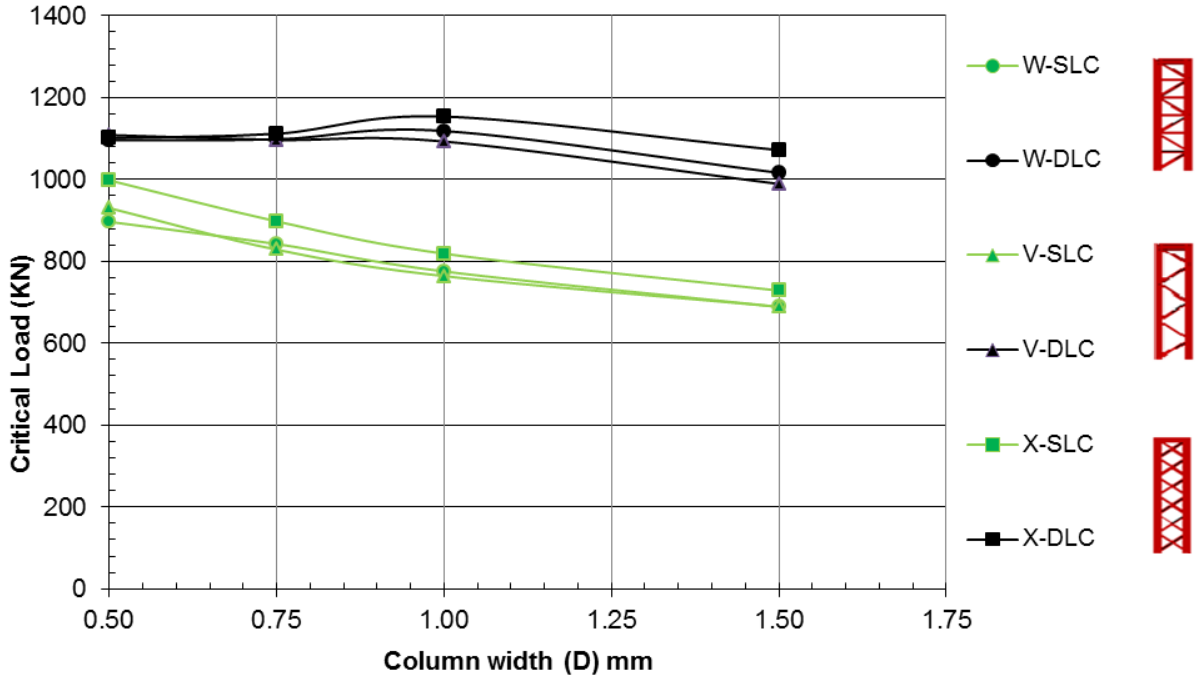



Figure 4:6 Critical load vs. column width for fixed-free columns at a load ratio of 0.05

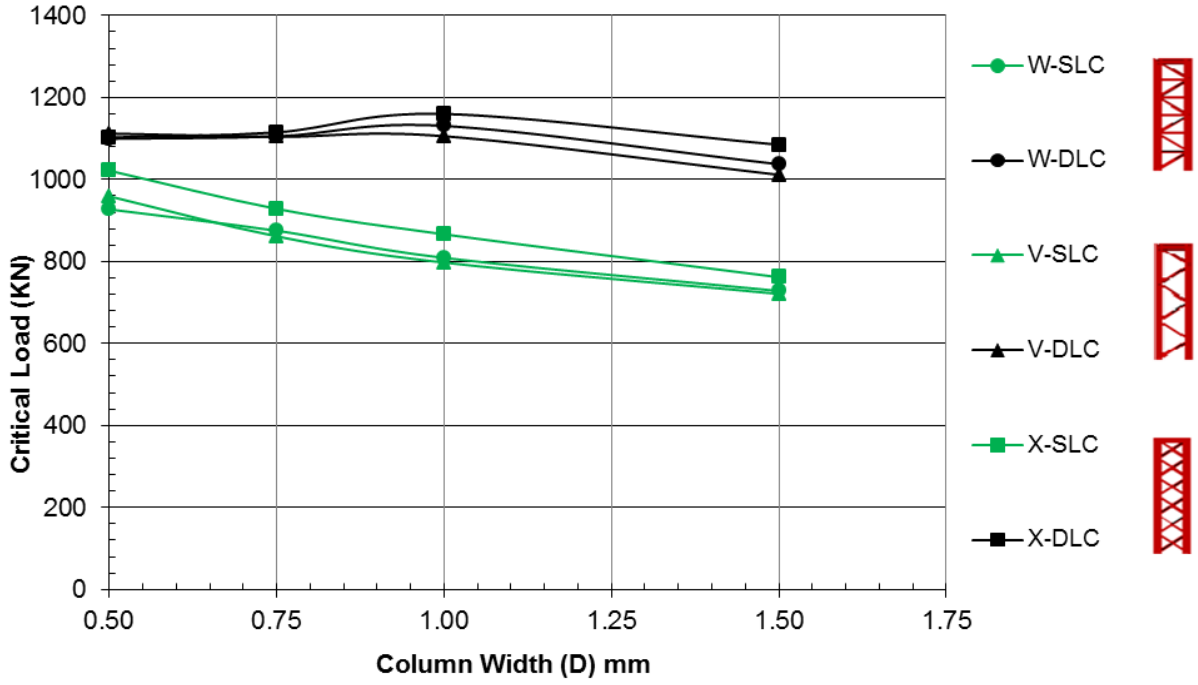



Figure 4:7 Critical load vs. column widthfor fixed-free columns at a load ratio of 0. 1

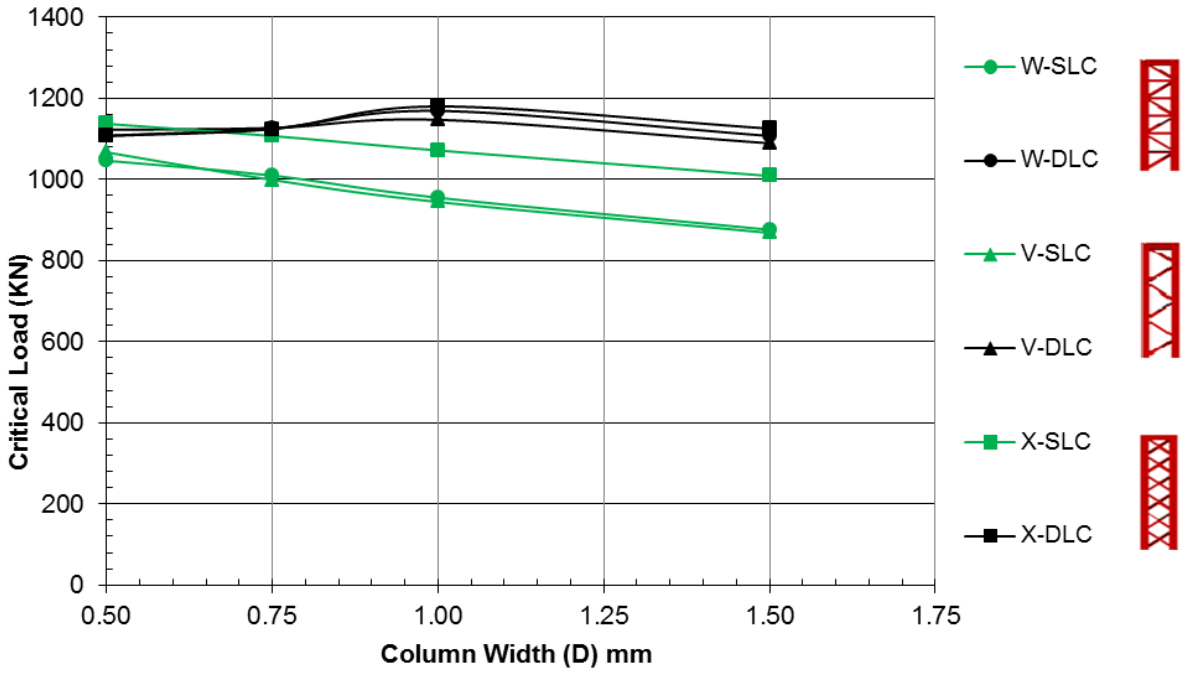



Figure 4:8 Critical load vs. column width for fixed-free columns at a load ratio of 0. 33

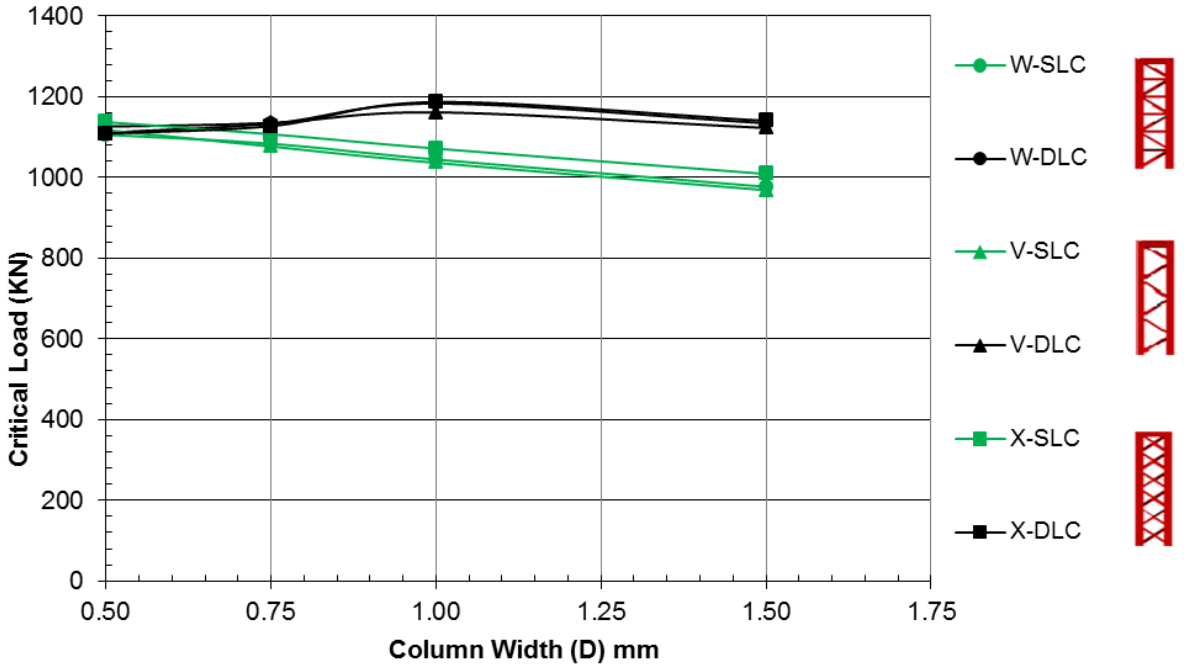



Figure 4:9 Critical load vs. column width for fixed-free columns at a  load ratio of 0. 5

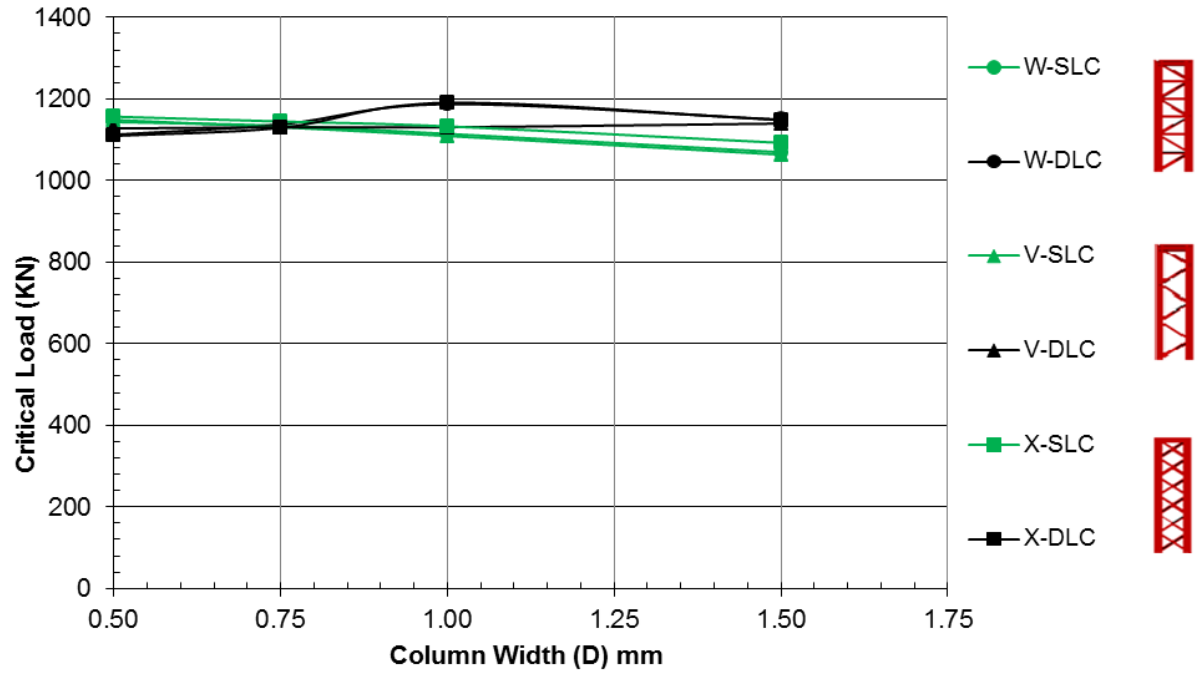



Figure 4:10 Critical load vs.column width for fixed-free columns at a load ratio of 0. 67

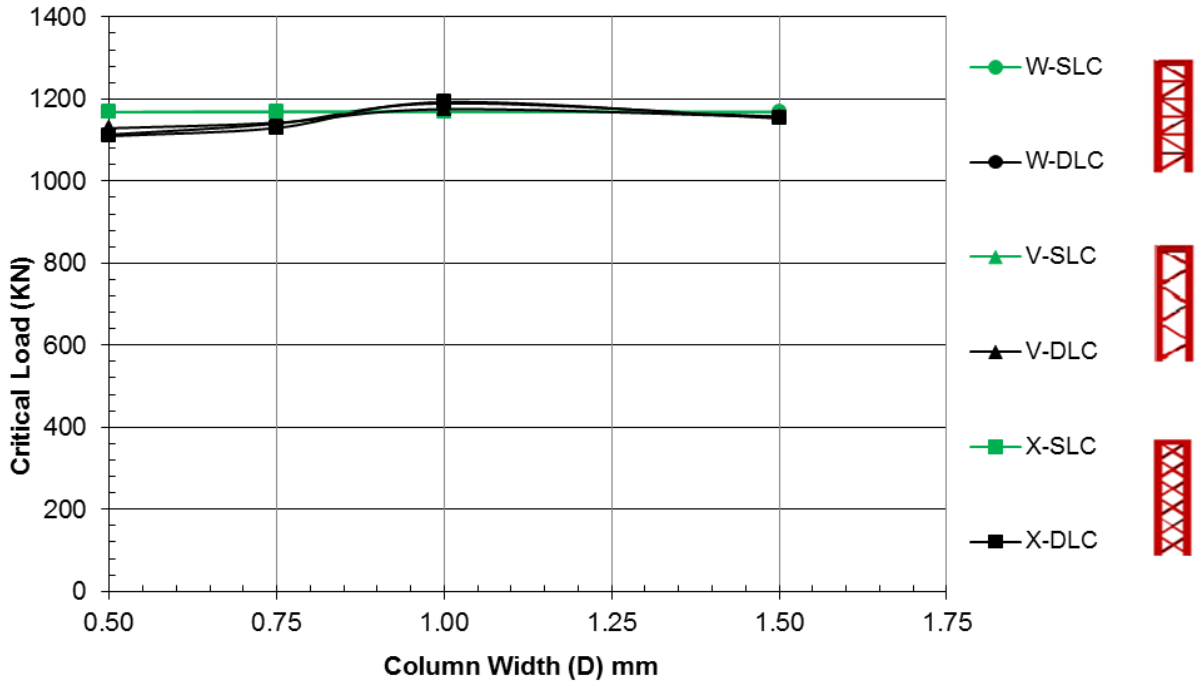



Figure 4:11 Critical load vs. column width for fixed-free column at load ratio of 1.0

The greater the column width, the lower the magnitude of critical load for a load ratio, this is due to reduced shear resistance of buckling of lacings caused by increased slenderness ratio of lacing members. This was also observed and explained by

Radzonsky (2008). Columns with X-brace configuration are superior for all considered column widths.

As the Load ratio move towards one, the gap of critical load between DLC and SLC narrows down which implies that SLC is adequate to carry almost the same load as DLC when the load ratio is one.

Generally, the critical load of a single laced column is ±1% - 26% less than a double laced column with the highest difference at the smallest load ratio (away from load ratio of one). However these difference decrease as column width increases. The double laced columns are more efficient because their performance curve have low gradient and variation and are thus more consistent over the varied range of load ratio.

## b) Case -2 End Supports: Pinned-Rollers

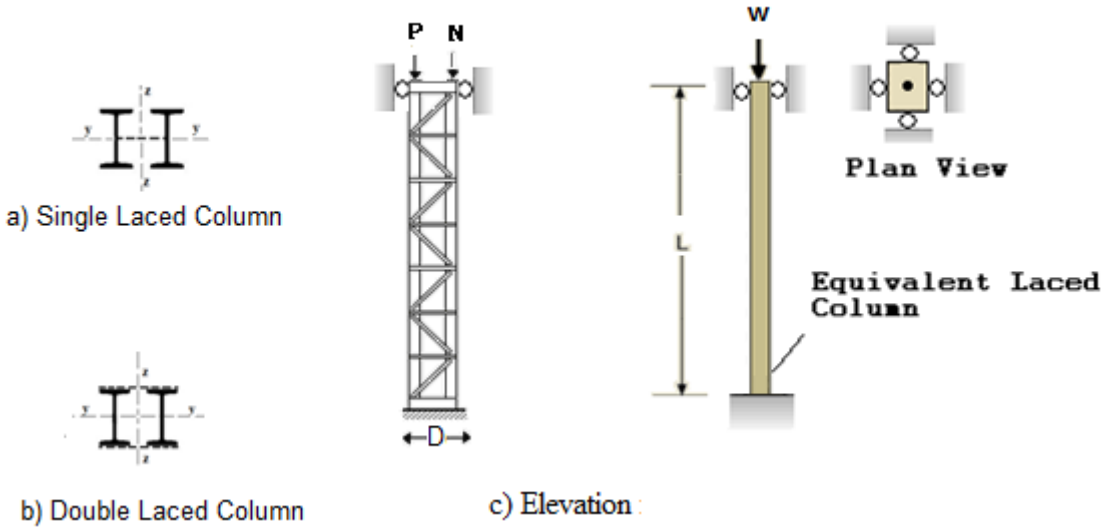



Figure 4:12 Pinned at bottom and roller support on top

Figure 4.13 to 4.16 plot critical load against load ratio when end supports of the columns is pinned-rollers. From the results, the magnitude of critical load has increased due to higher column stiffness due to fixed-roller end support. The variation of critical load for single and double laced column has a range of 9%-40% and 1%-22% respectively. The percentage of variation diminishes as the load ratio moves

closer to one. Double and single columns don't not yield the same critical load at any load ratio, unlike the cantilever case in the preceding set of results. However as before the results still have their maximum compression resistance at a load ratio of one. The columns with X-brace configuration are the least affected by the load ratio. This was also observed in the cantilever case.

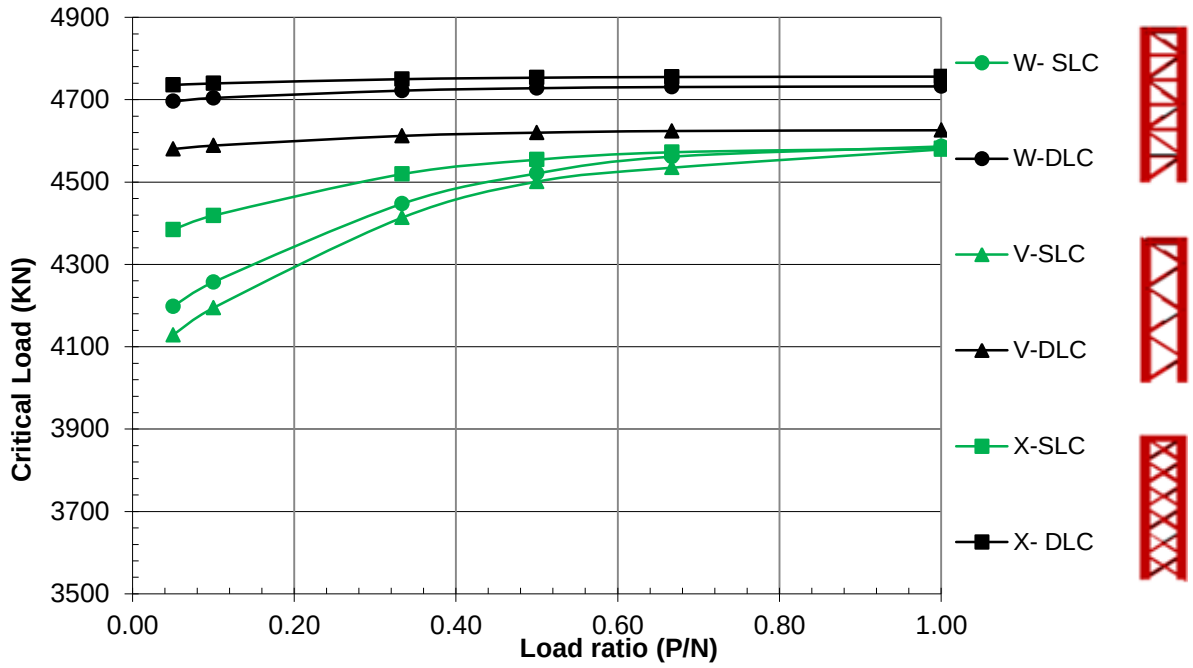



Figure 4:13 Critical load vs. load ratio for column widths of 0.5m

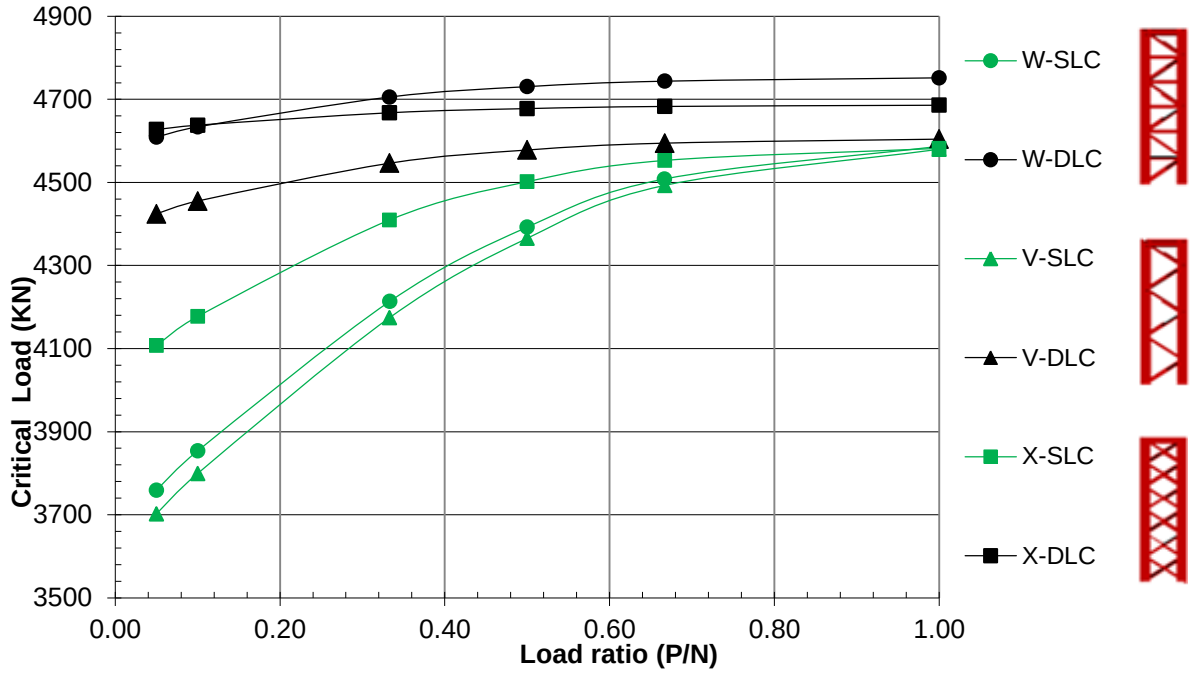



Figure 4:14 Critical load vs. load ratio for column widths of 0.75m

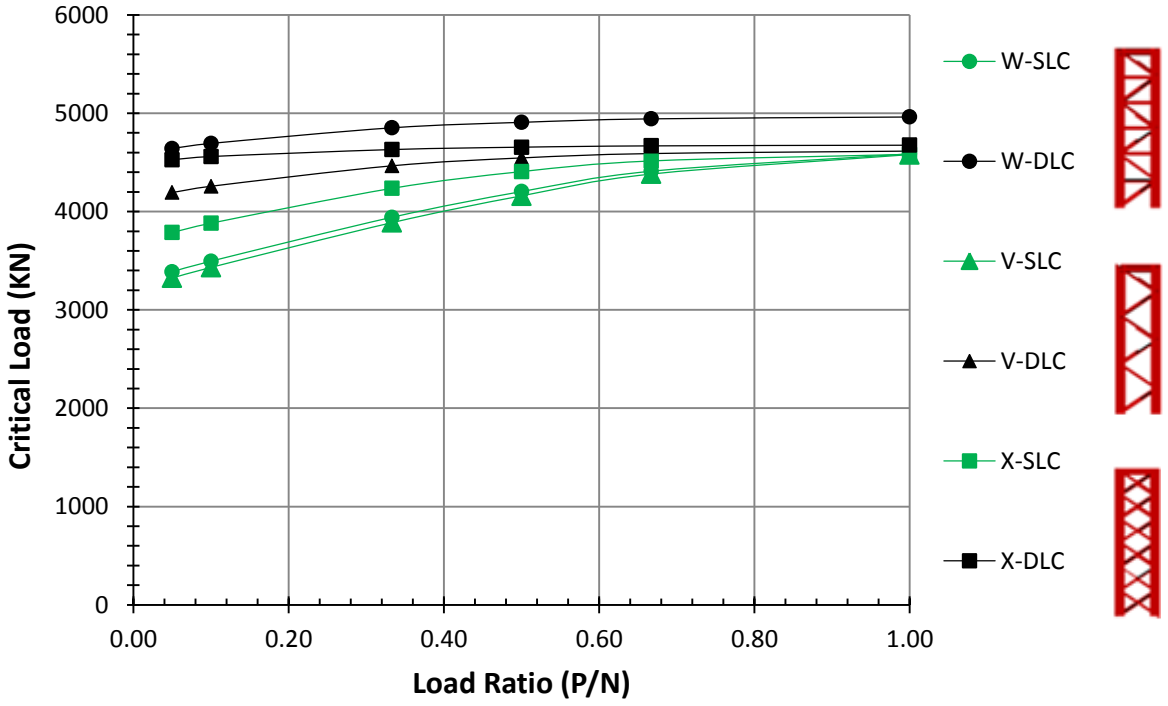



Figure 4:15 Critical load vs. load ratio for column widths of 1.0m

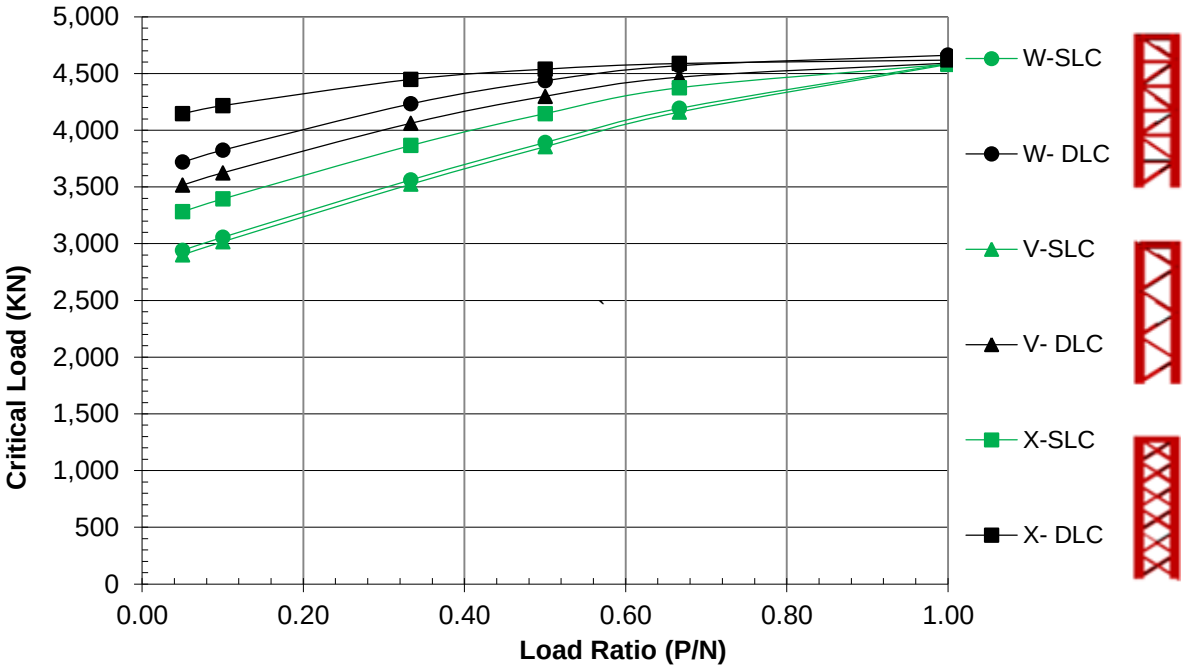



Figure 4:16 Critical load vs. load ratio for column widths of 1.5m

These results show that as column width increases, buckling load decreases. Figure 4.17-4.22 shows how critical load varies with column width.

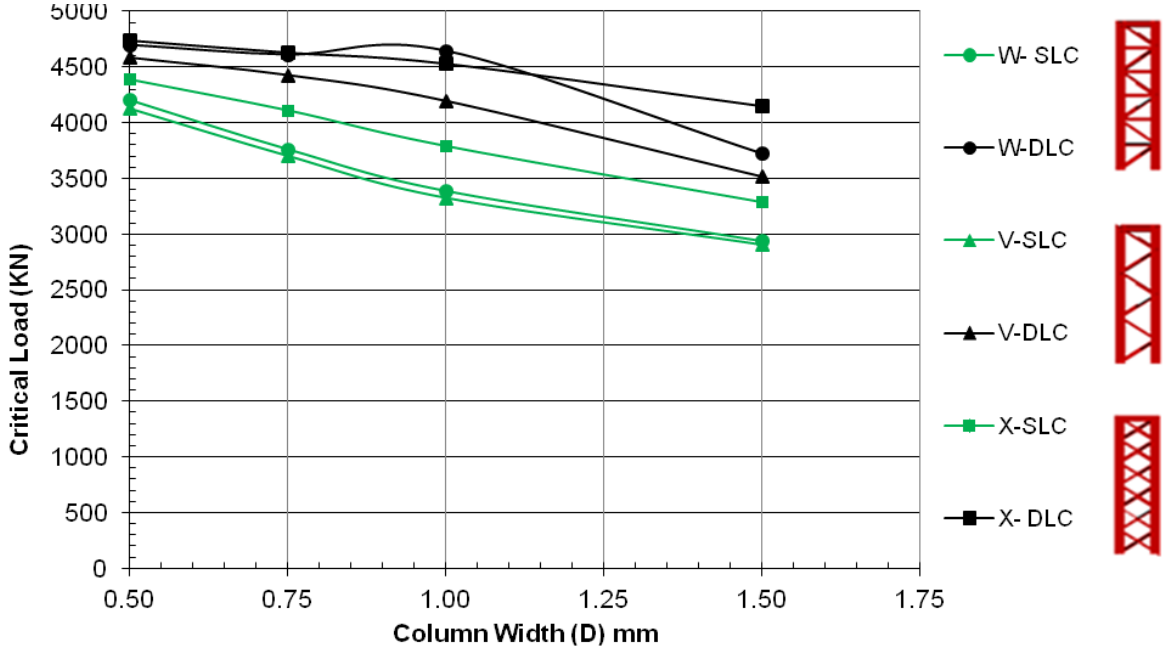



Figure 4:17 Critical load vs. column width for a fixed-free supports at a load ratio of 0.05

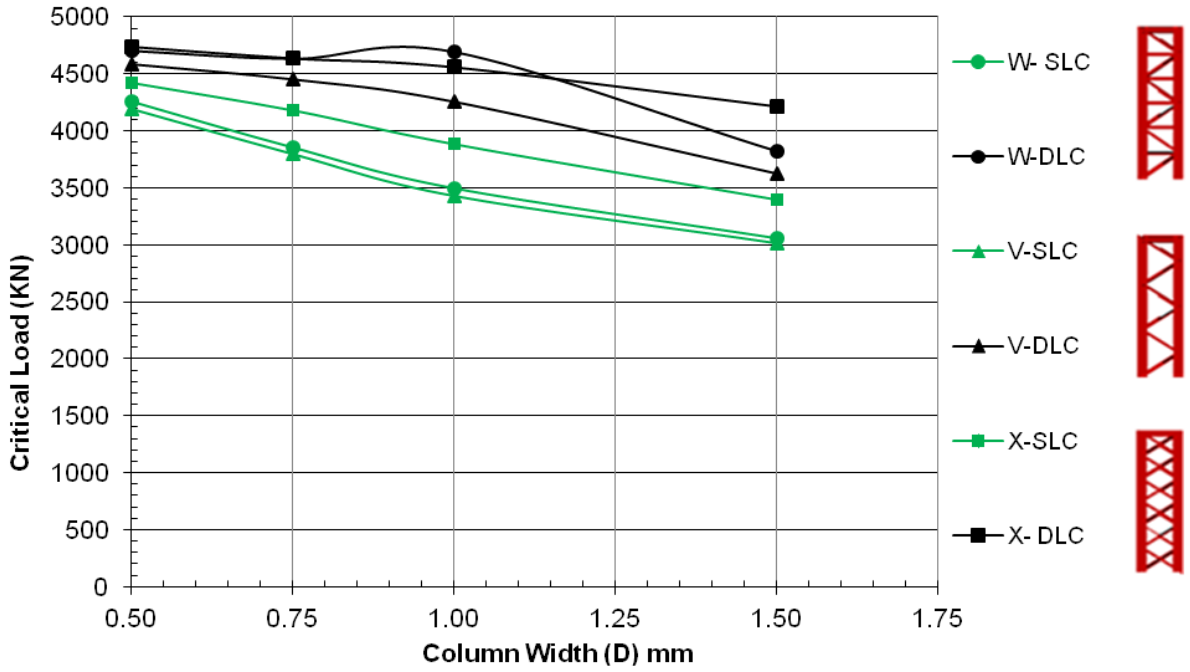



Figure 4:18 Critical load vs. column width for a fixed-free supports at a load ratio of 0.1

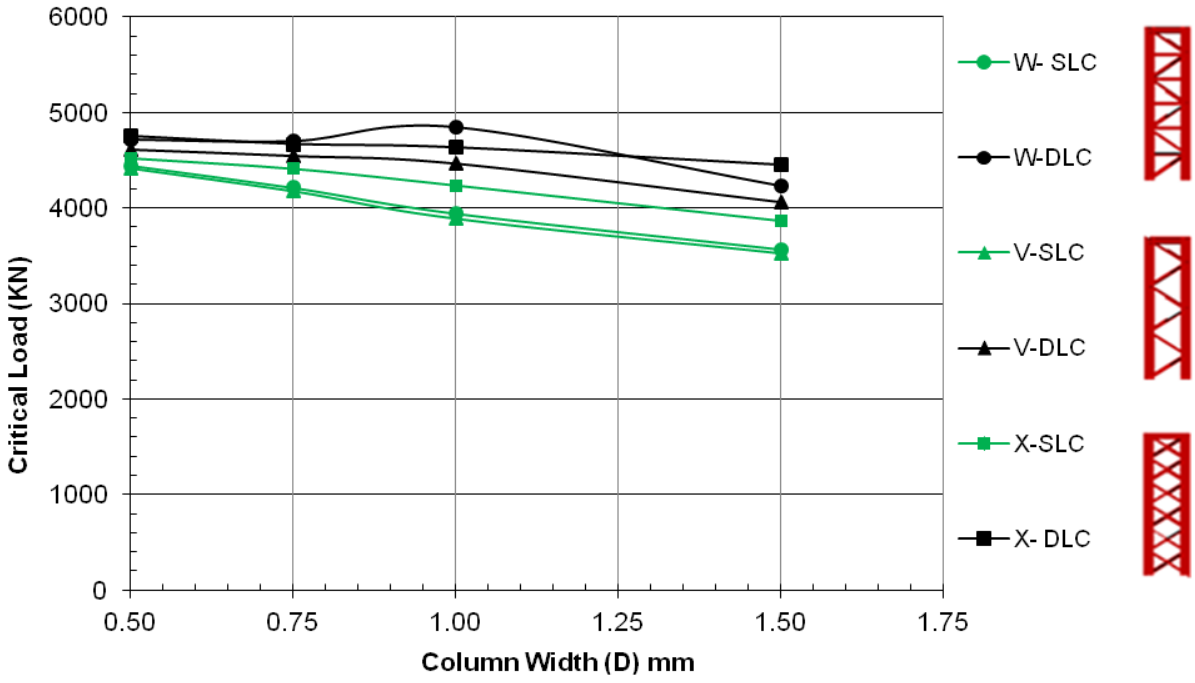



Figure 4:19 Critical load vs. column width for a fixed-free supports at a load ratio of 0.33

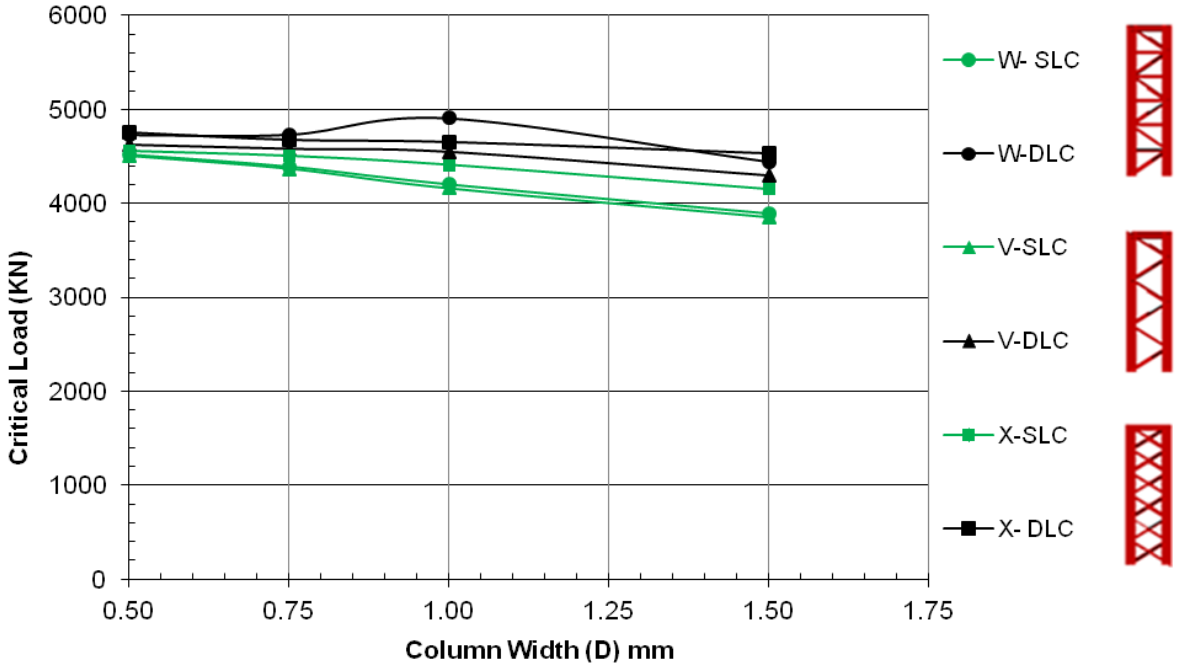



Figure 4:20 Critical load vs. column width for a fixed-free supports at a load ratio of 0. 5

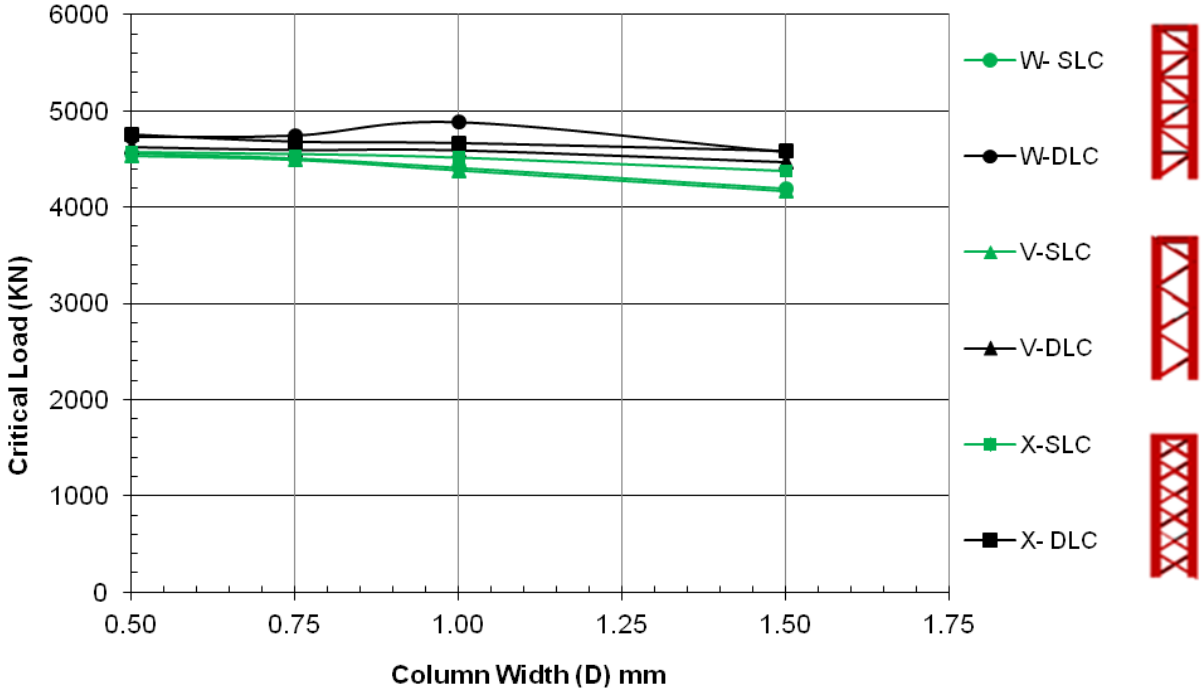



Figure 4:21 Critical load vs. column width for a fixed-free support at a load ratio of 0.67

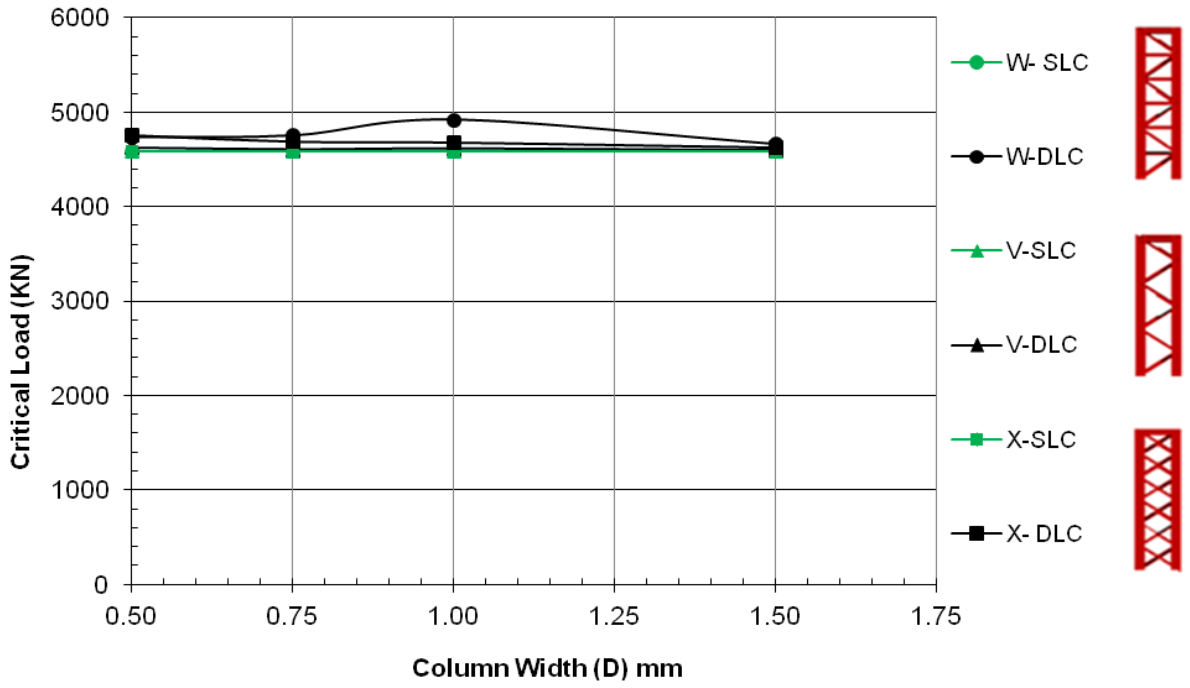



Figure 4:22 Critical load vs. column width for a fixed-free supports at a load ratio of 0.1

## 4.2 Torsion

### a) End Support: Fixed at bottom and free at top (Cantilever)

The torsion of built up columns in their buckled shape is measured by extracting the angle of rotation (T) of the member connecting the main compression chords. This is illustrated in Figure 4.23.

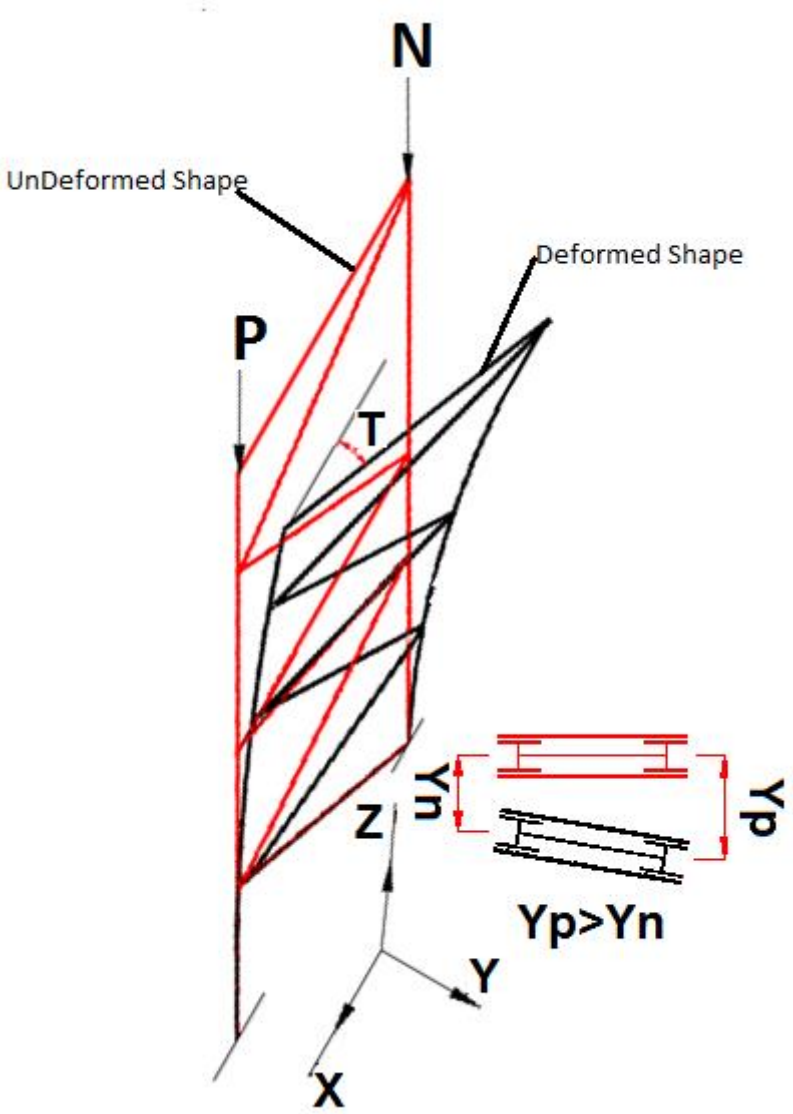



Figure 4:23 Measure of torsion

The results of torsion measured in radians at various column widths for the cantilever case are plotted below.

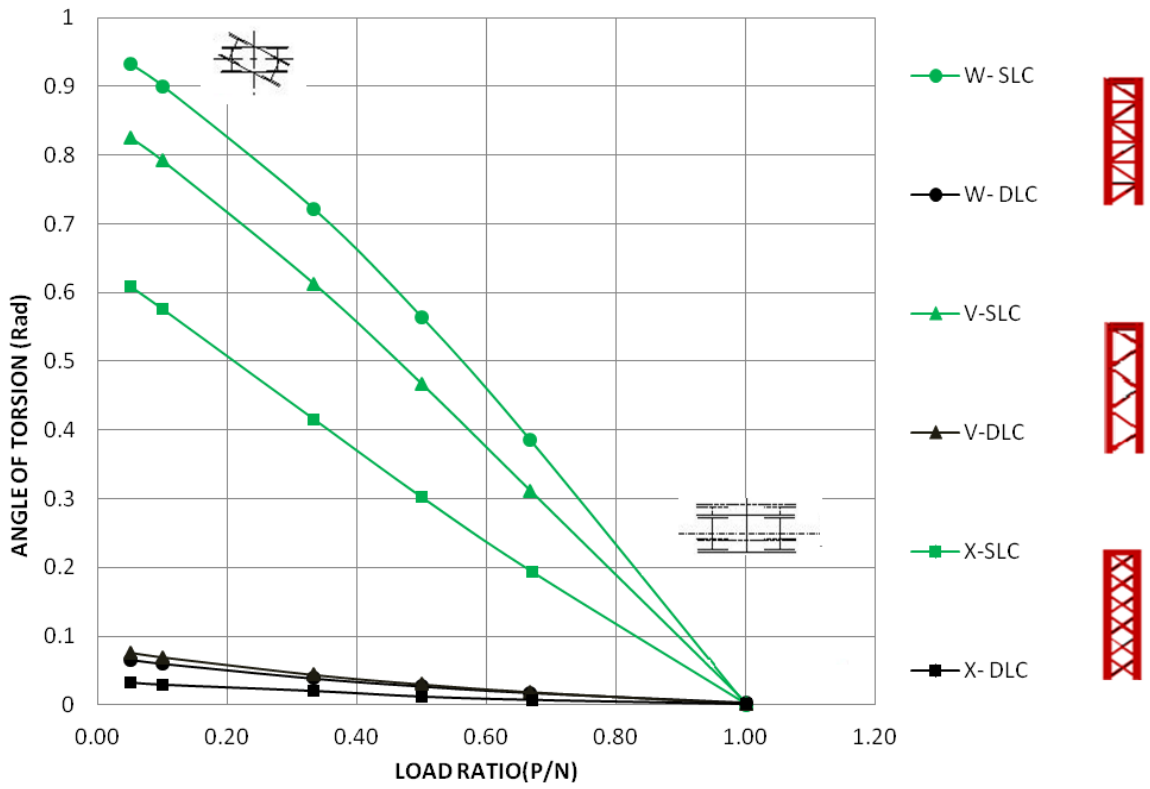



Figure 4:24 Torsion vs. load ratio for SLC and DLC at column widths of 0.5m

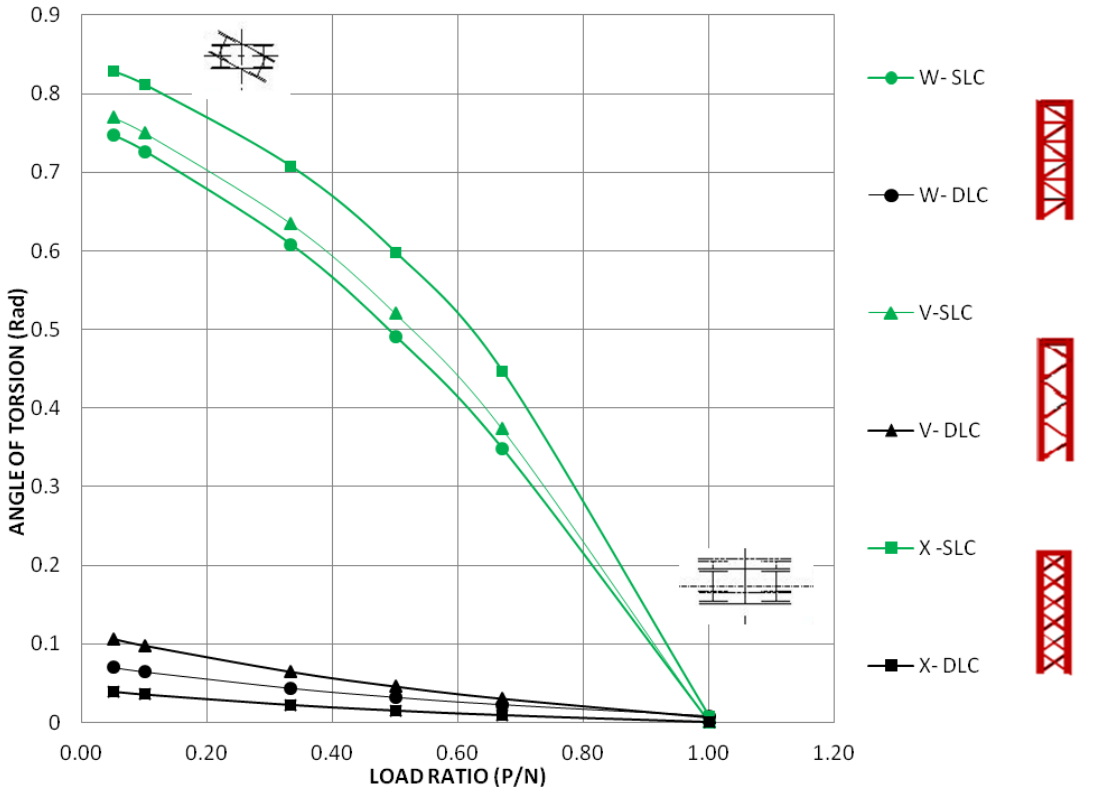



Figure 4:25 Torsion vs. load ratio for SLC and DLC at column widths of 0.75m

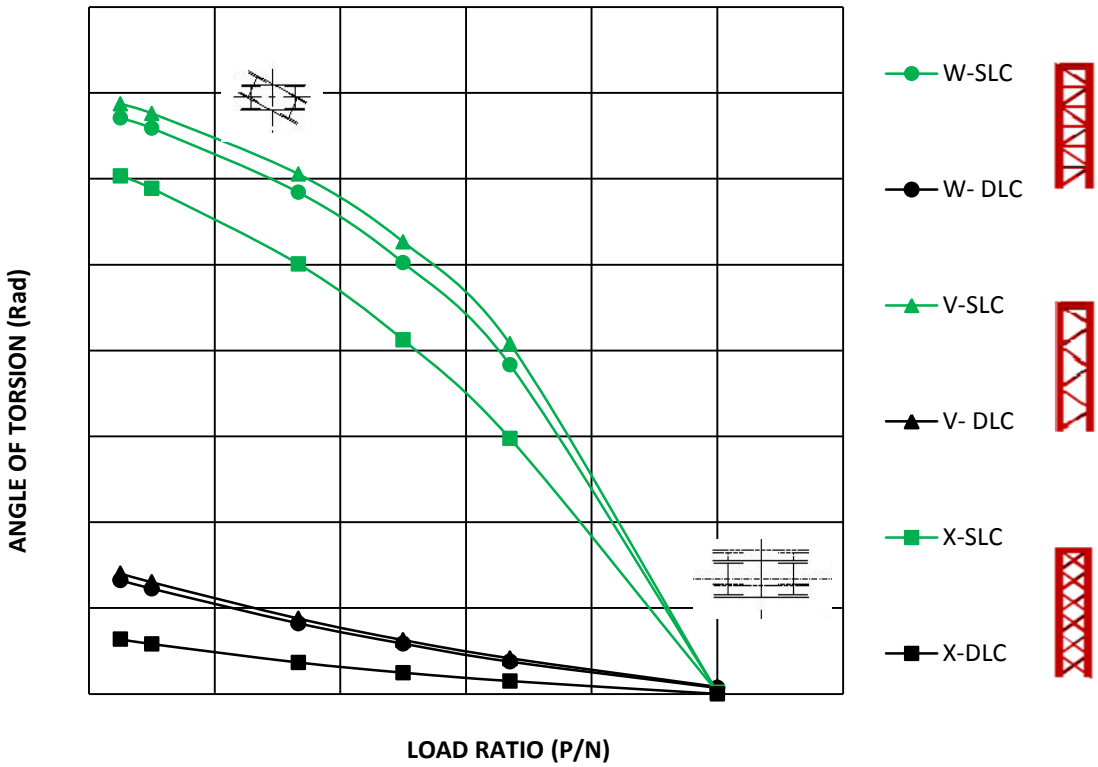



Figure 4:26 Torsion vs. load ratio for SLC and DLC at column widths of 1.0m.

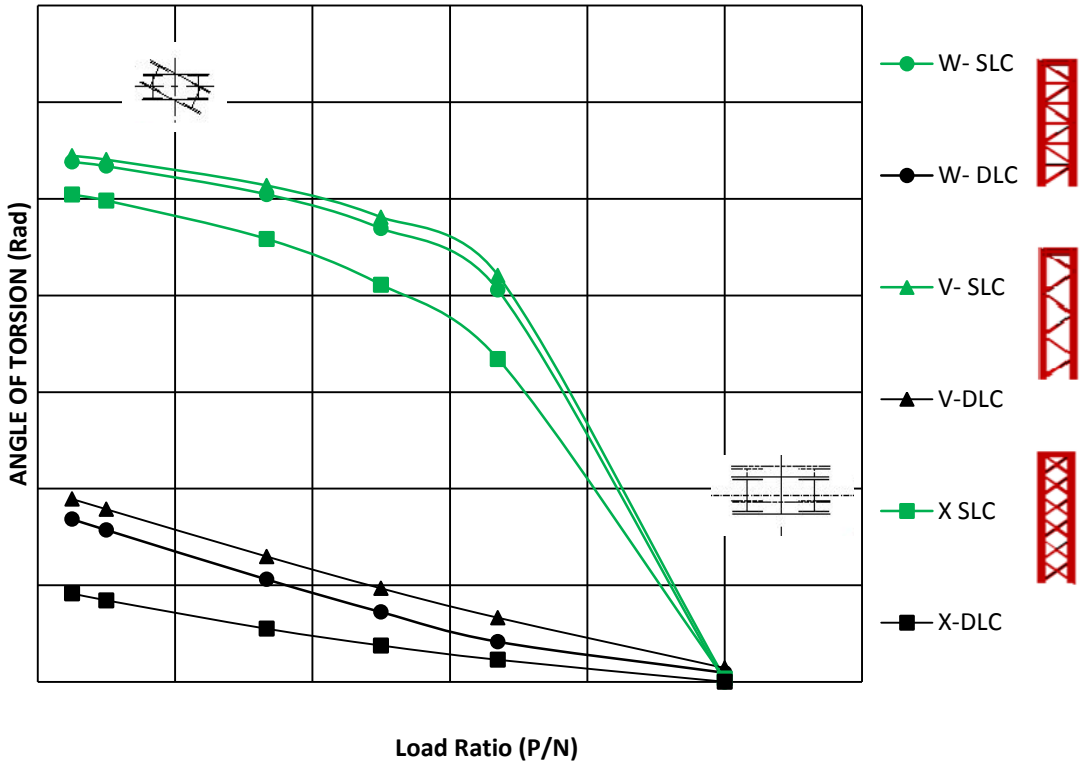



Figure 4:27 Torsion vs. load ratio for SLC and DLC at column widths of 1.5m

Away from load ratio of one, double laced columns have shown less torsional variation than single laced columns. As the load ratio moves towards one, the magnitude of torsion decreases to zero for both single and double laced columns.

The highest degree of torsion for each type of column is observed at the smallest load ratio point (away from one). This is due to the high axial load difference applied to the main compression chords. Columns with X-configuration lacing have produced the lowest torsional buckling. A combination of maximum critical load and minimum degree of torsion is achieved at load ratio close to one.

The torsional curves of single laced columns fall close to each other as column widths increase. Hence the bracing configuration has less effect on degree of torsion for wider column. The torsion of single laced columns decreases as the column width increases. In contrast, the degree of torsion for double laced columns increase with an increase in column width.

## b) End Supports: Pinned at bottom and free at top

The boundary condition; pinned at bottom and roller at top does not allow for global torsional buckling. Hence calculating the global degree of twist is not possible.

## 4.3 Mode of failures

## a) End support: Fixed-free

The modes of buckling in the case of fixed-free (Cantilever) end conditions (Figure 4.28) are discussed below.

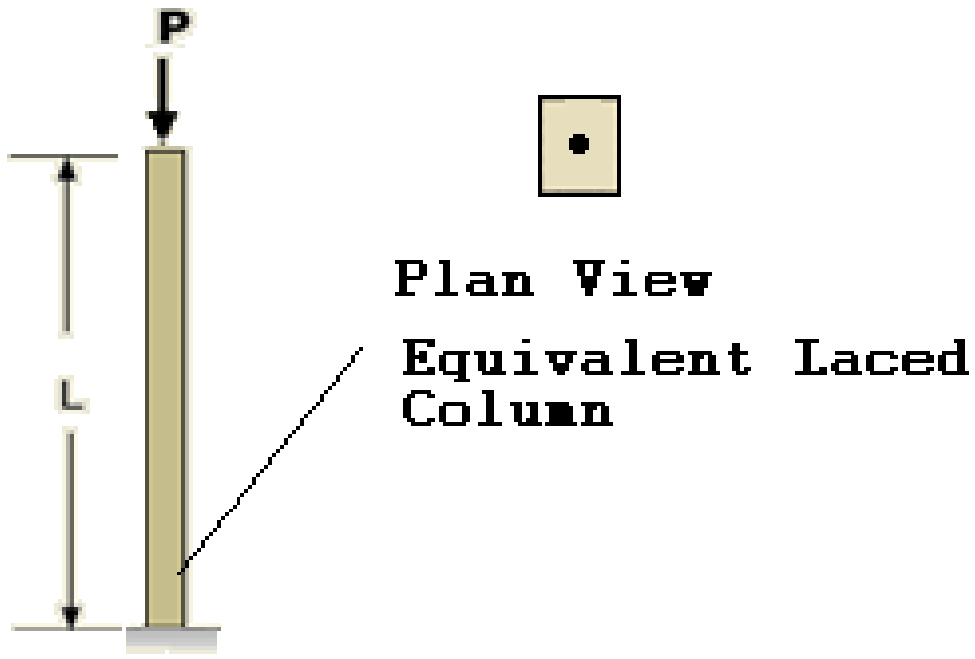



Figure 4:28 Fixed-free end supports

The modes of buckling observed with built up columns of different brace configuration under fixed-free end supports is typically a combination of torsion and bending. Away from load ratio one, the modes of buckling are dominated by bending and torsion. Figure 4.29-4.30 show the failure modes when the load ratio is less that one.

The highly loaded compression chord deforms and displaces differently (axially loaded differently); the whole column undergoes torsion.

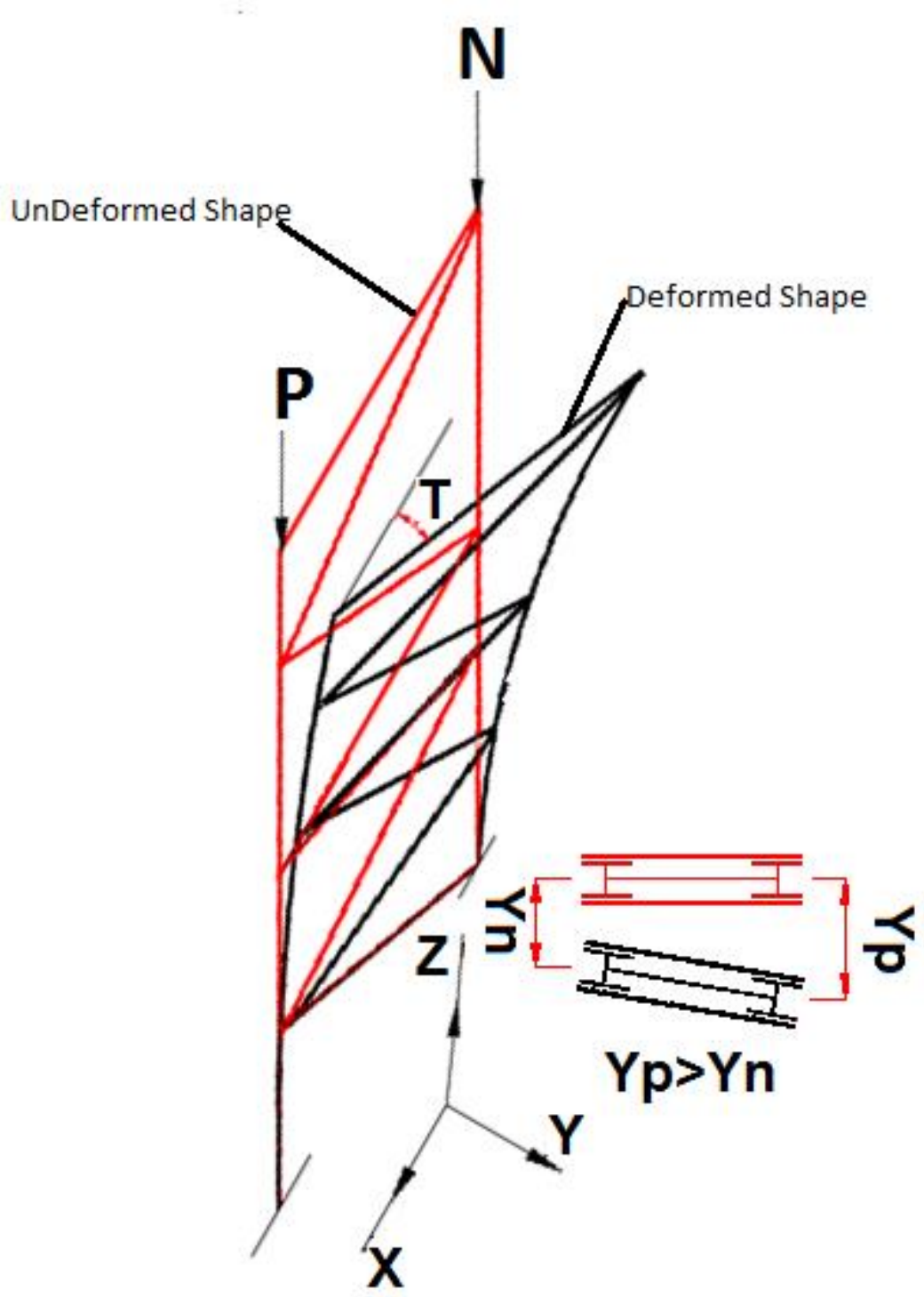



Figure 4:29 Sketch of mode of failure when load ratio (P/N) < 1

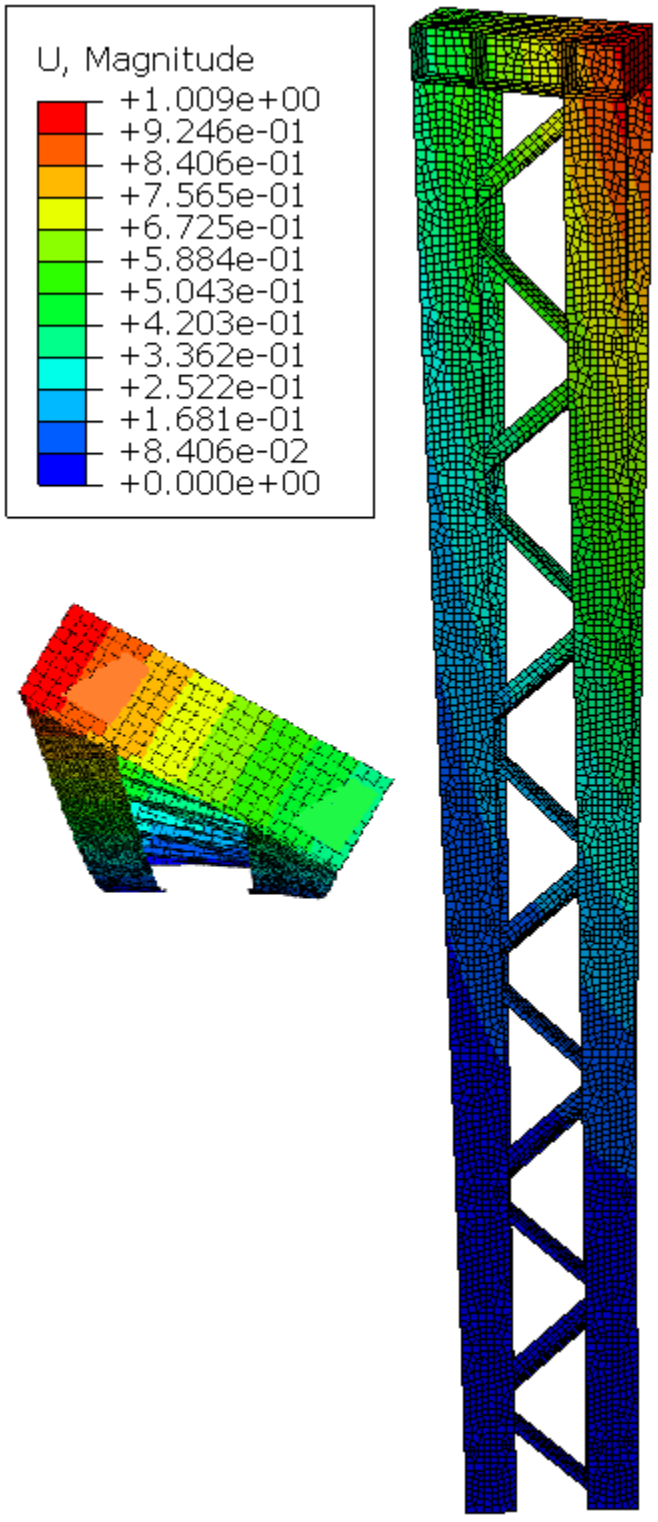



Figure 4:30 Finite element buckled mode shape when the load ratio (P/N) < 1

At a load ratio around one, the mode of buckling is mostly bending with negligible degree of torsion, Figure 4:31. The combination of maximum critical load with minimum torsion is achieved at this load ratio. The global mode shape is a portion of a sinusoid. Figures 4.32 shows the finite element buckled shape.

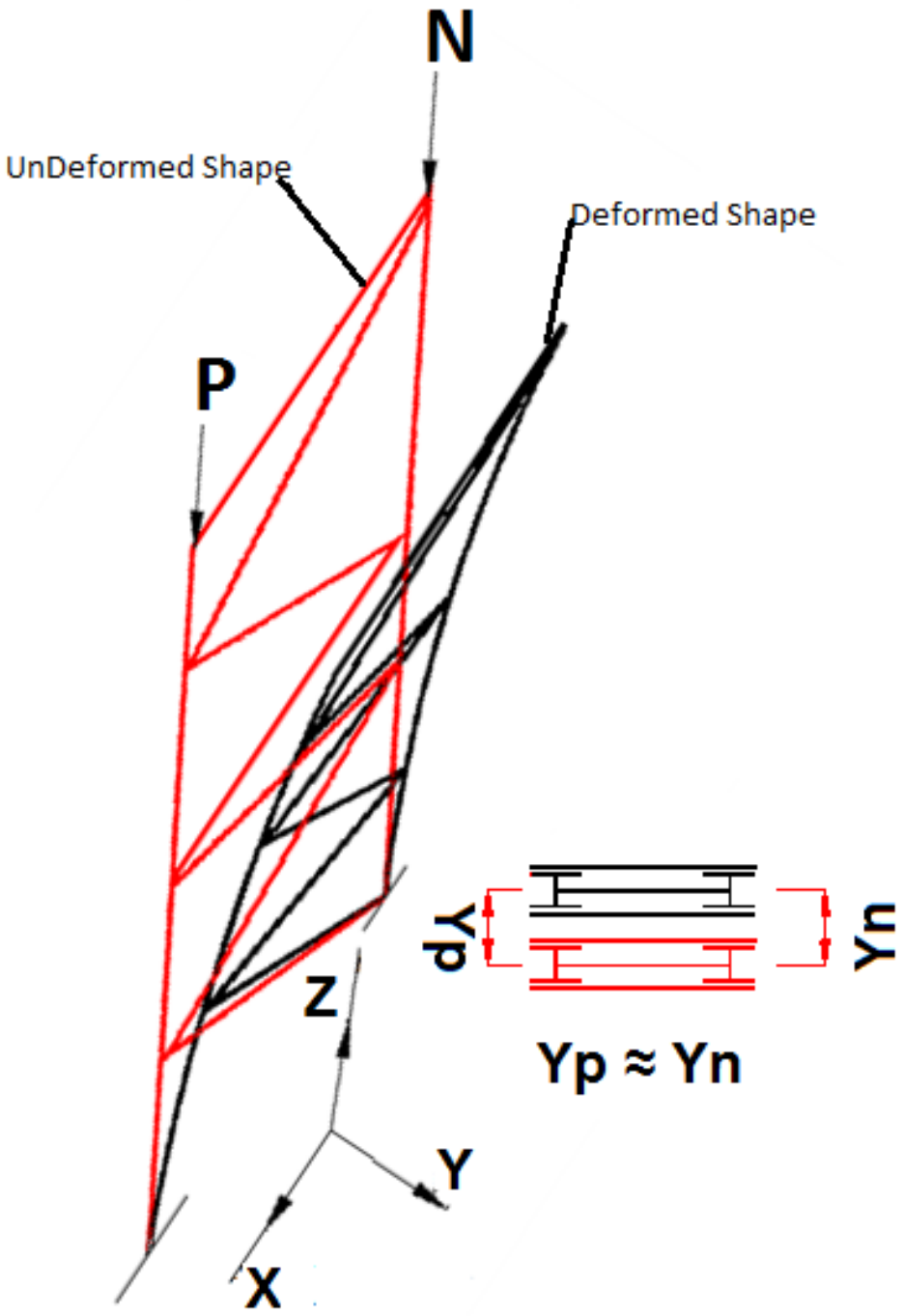



Figure 4:31 Sketch of mode shape when the load ratio (P/N) = 1.0

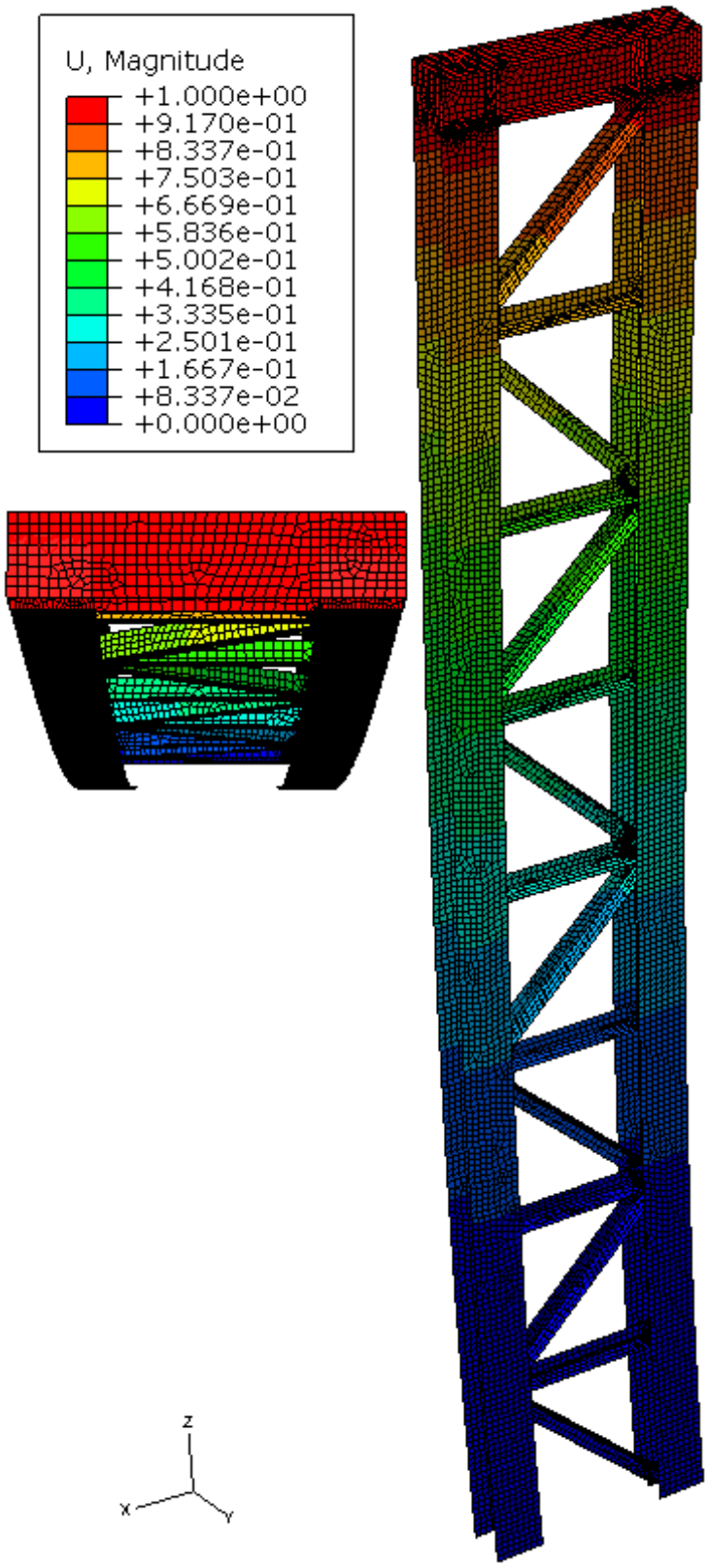



Figure 4:32 Finite element buckled mode shape of failure at load ratio (P\N) =1.0.

When the load ratio is greater than one, the inverse of the P/N<1.0 is experienced where the column bends and twists in the opposite direction. As before the heavily loaded compression chord deforms more resulting in a twist in the opposite direction. Figure 4.32 shows the finite element buckled shape.

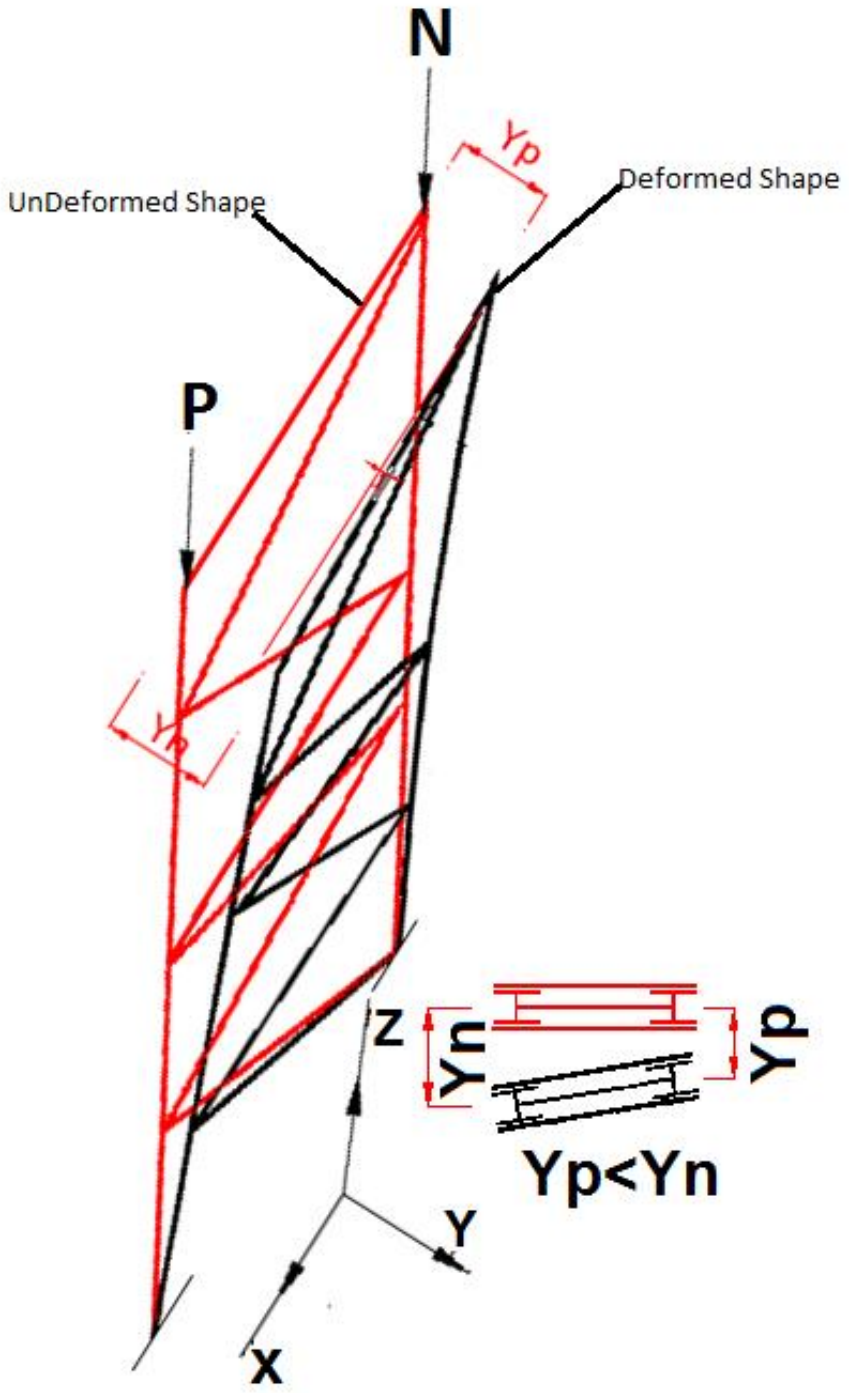



Figure 4:33 Sketch of mode buckled  when the load ratio (P\N) > 1

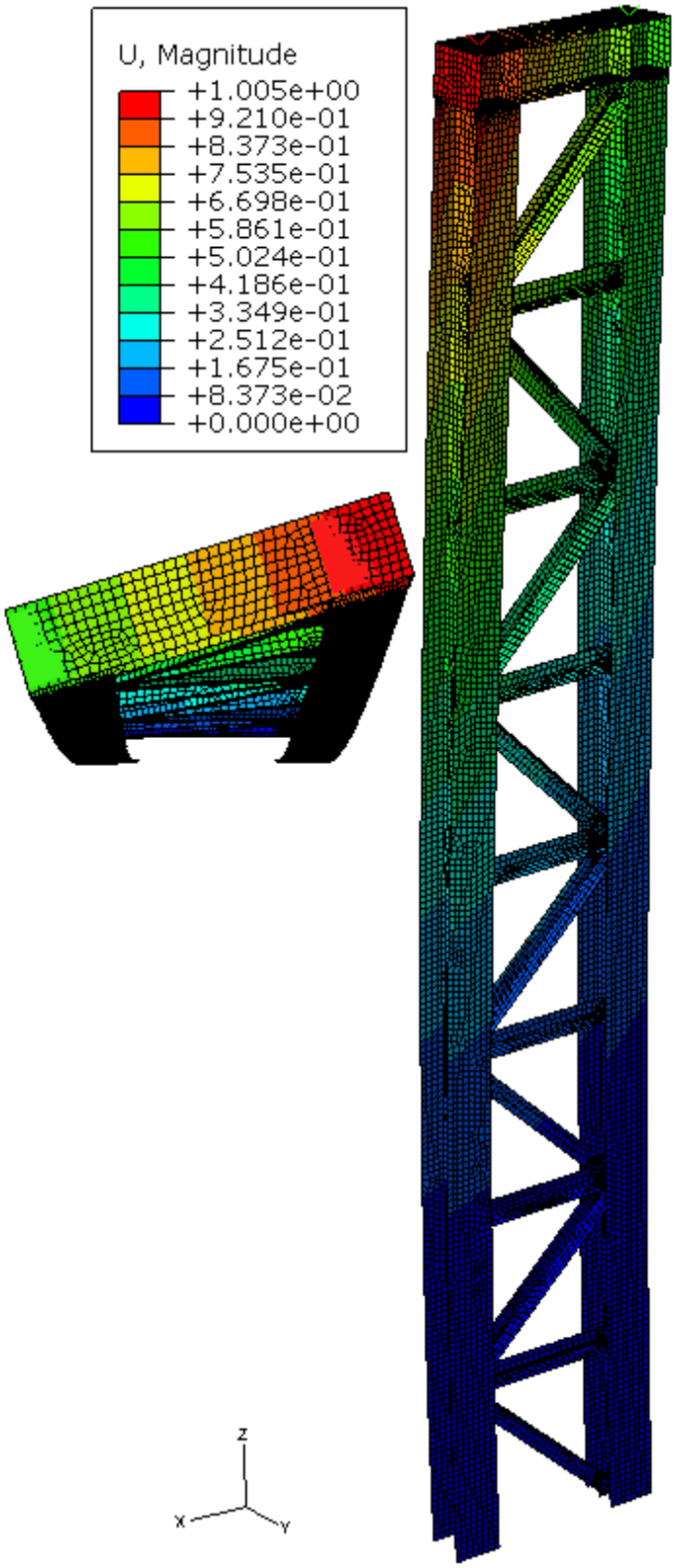



Figure 4:34 Finite element buckled mode shape when the load ratio (P\N) > 1

## b) End Support: Pinned at bottom and roller on top

The modes of buckling for the built up column pinned at the bottom and on rollers at the top (Figure 4.35) is discussed. The deformed finite element columns are shown in Figures 4.36 to 4.38.

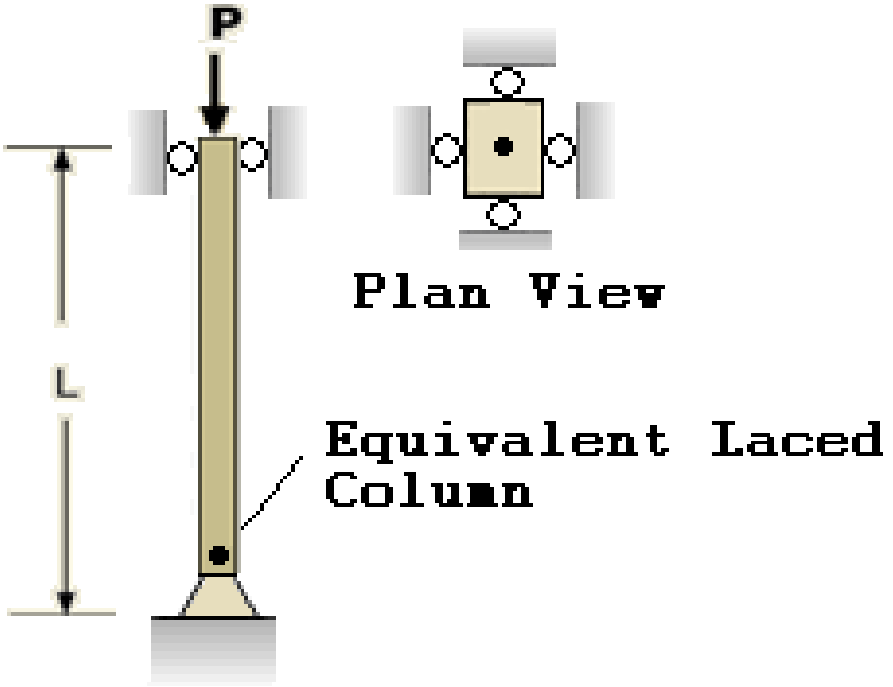



Figure 4:35 Pinned at bottom and roller on top

The overall built up column does not twist due to end supports restraining torsion. However, away from the load ratio of one, the heavily loaded chord displaces more with both chords following a sinusoidal deflected shape as in the preceding set of results. The sinusoidal takes a more pronounced shape when the load ratio is one and both chords carry equal amounts of load (Figure 4.37).

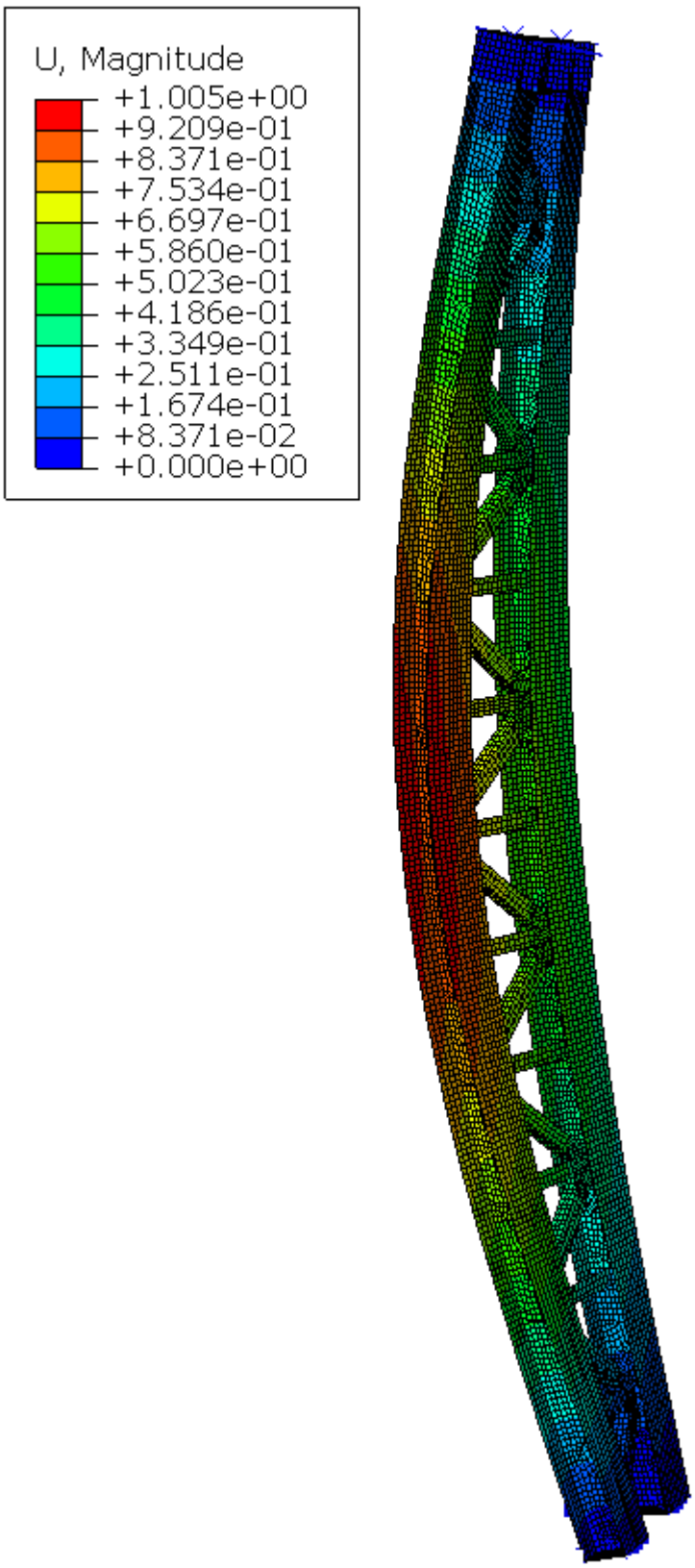



Figure 4:36 Buckled finite element model when the load ratio (P/N) <1




Figure 4:37 Buckled finite element model when the load ratio (P/N) is ±1




Figure 4:38 Buckled finite element model when the load ratio (P/N) >1

# 5. CHAPTER 5

## 5.1. Theory verse Finite Element Method solution (FEM)

From Chapter 2, theoretical derivation and expressions to determine critical load, axial-torsional buckling and expected mode of failures have been given.

It is has been established that the column will buckle at the lowest of buckling load in either direction (x or y).

The theoretical critical load of a column where shear effect is considered is given by the expression below:

Buckling load $P_{cr} = \dfrac{P_e}{\left(1 + \dfrac{P_e}{P_d}\right)}$

The finite element solution is compared to the theoretical calculations in an attempt to determine the accuracy of the finite element solution. The comparison of results is presented in Table 6.2 to 6.10.

The FEM and theoretical solutions are compared at a load ratio of 1.0 and over a range of column widths (i.e. 0.5m, .75m, 1.1m and 1.5m). This is due to the fact that the mode of failure at this load ratio buckling is primarily due to bending. The theoretical equation is derived based on flexural bending moments.

### 5.1.1. Results

### a) End Support: Cantilever

Figure 5.1 shows the geometric characteristics of the a built up column. Table 5.1 gives the geometric paramenters of the problem investigated.

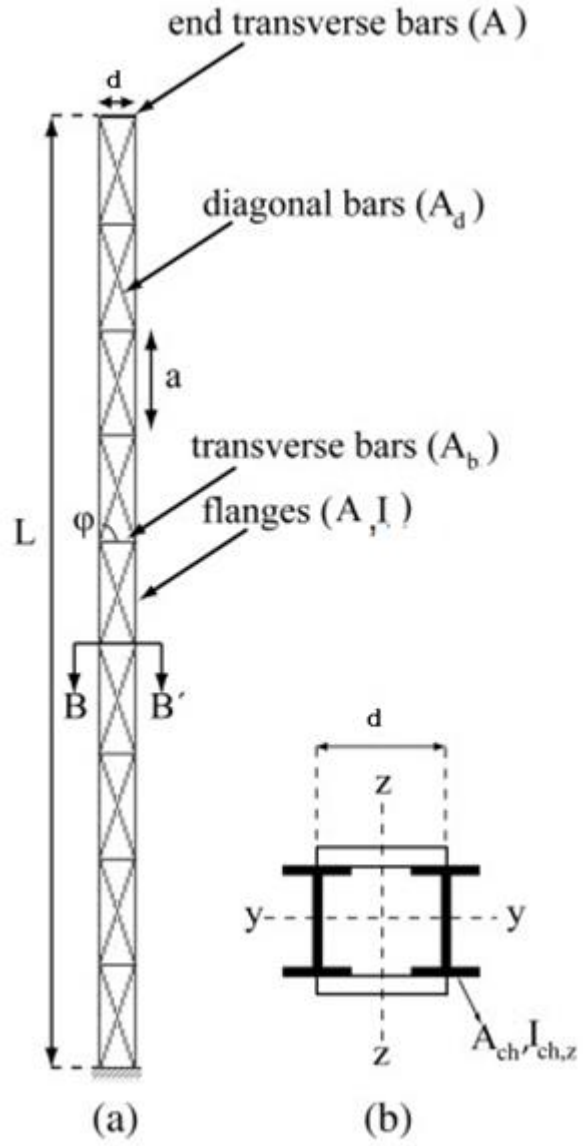



Figure 5:1 Geometric chraracteristics of a built up column

| Built up Properties | Symbol | |
|---|---|---|
| Profile | | H-203x203x46 |
| Area($mm^2$) | | 5880 |
| Lacing Angle of inclination (Rad) | $\phi$ | 0.79 |
| Length factor | K | 2 |
| Length (mm) | L | 6000 |
| 2nd Moment in X ($mm^4$) | $i_x$ | 4.56E+07 |
| 2nd Moment in Y ($mm^4$) | $i_y$ | 1.54E+07 |
| Area of Lacings ($mm^2$) | $A_d$ | 8.13E+02 |
| Area of Batten ($mm^2$) | $A_b$ | 8.13E+02 |
| Column Width(mm) | h | 5.00E+02 |
| Panel Height (mm) | a | 5.00E+02 |
| Lacing Length (mm) | d | 7.07E+02 |
| Young Modulus ($N/m^2$) | E | 2.00E+05 |

Table 5:1 Built up column prarameters

| Brace type | Column Width | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical L | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 12000 | 1250.15 | 58.412 | 1 | 1168.24 | 1.18E-05 | 1232.02 | -5.18 |
| W-Double | 500 | 12000 | 1250.15 | 55.669 | 1 | 1113.38 | 5.89E-06 | 1241.02 | -10.28 |
| V-SLC | 500 | 12000 | 1250.15 | 58.41 | 1 | 1168.2 | 8.70E-06 | 1236.70 | -5.54 |
| V-Double | 500 | 12000 | 1250.15 | 56.41 | 1 | 1128.2 | 4.35E-06 | 1243.39 | -9.26 |
| X-SLC | 500 | 12000 | 1250.15 | 58.425 | 1 | 1168.5 | 4.35E-06 | 1243.39 | -6.02 |
| X-Double | 500 | 12000 | 1250.15 | 55.494 | 1 | 1109.88 | 2.17E-06 | 1246.76 | -10.98 |

Table 5:2 FEM verses theoretical results at a column width of 0.5m

| Brace type | Column Width | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical L | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1250.15 | 58.621 | 1 | 1172.42 | 1.77E-05 | 1223.15 | -4.15 |
| W-Double | 750 | 12000 | 1250.15 | 57.016 | 1 | 1140.32 | 8.83E-06 | 1236.50 | -7.78 |
| V-SLC | 750 | 12000 | 1250.15 | 58.426 | 1 | 1168.52 | 1.30E-05 | 1230.09 | -5.01 |
| V-Double | 750 | 12000 | 1250.15 | 57.166 | 1 | 1143.32 | 6.52E-06 | 1240.04 | -7.80 |
| X-SLC | 750 | 12000 | 1250.15 | 58.438 | 1 | 1168.76 | 6.52E-06 | 1240.04 | -5.75 |
| X-Double | 750 | 12000 | 1250.15 | 56.494 | 1 | 1129.88 | 3.26E-06 | 1245.07 | -9.25 |

Table 5:3 FEM verses theoretical thoeritical results at a column width of 0.75m

| Brace type | Column Width | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical L | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1250.15 | 58.474 | 1 | 1169.48 | 2.35E-05 | 1214.40 | -3.70 |
| W-Double | 1000 | 12000 | 1250.15 | 59.484 | 1 | 1189.68 | 1.18E-05 | 1232.02 | -3.44 |
| V-SLC | 1000 | 12000 | 1250.15 | 58.424 | 1 | 1168.48 | 1.74E-05 | 1223.54 | -4.50 |
| V-Double | 1000 | 12000 | 1250.15 | 58.763 | 1 | 1175.26 | 8.70E-06 | 1236.70 | -4.97 |
| X-SLC | 1000 | 12000 | 1250.15 | 58.481 | 1 | 1169.62 | 8.70E-06 | 1236.70 | -5.42 |
| X-Double | 1000 | 12000 | 1250.15 | 59.631 | 1 | 1192.62 | 4.35E-06 | 1243.39 | -4.08 |

Table 5:4 FEM verses theoretical results at a column width of 1.0m

| Brace type | Column Width | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical L | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 1250.15 | 58.436 | 1 | 1168.72 | 3.53E-05 | 1197.29 | -2.39 |
| W-Double | 1500 | 12000 | 1250.15 | 57.833 | 1 | 1156.66 | 1.77E-05 | 1223.15 | -5.44 |
| V-SLC | 1500 | 12000 | 1250.15 | 58.836 | 1 | 1176.72 | 2.61E-05 | 1210.66 | -2.80 |
| V-Double | 1500 | 12000 | 1250.15 | 57.824 | 1 | 1156.48 | 1.30E-05 | 1230.09 | -5.98 |
| X-SLC | 1500 | 12000 | 1250.15 | 58.469 | 1 | 1169.38 | 1.30E-05 | 1230.09 | -4.94 |
| X-Double | 1500 | 12000 | 1250.15 | 57.655 | 1 | 1153.1 | 6.52E-06 | 1240.04 | -7.01 |

Table 5:5 FEM verses theoretical results at a column width 1.5m

Table 5:7 to 5:10 shows a comparison of FEM to theory. By comparison, shear effects reduces the column critical buckling load. The percentage error varies depending on the column width, brace configuration and column type. Double laced column has more percentage error than single laced columns. However the percentage error is less than 10%. The percentage error has decreases as column width increases.

For further comparison of finite element analysis verses theoretical solutions of built up columns the reader is reffered to Appendix B section 8.2.1.

### b) End Supports: Fixed-Roller

| Built up Properties | Profile | H-203x203x46 |
|---|---|---|
| | | |
| Lacing Angle of inclination | $\phi$ | 0.785398163 |
| $L_{ex}$ | $K_x$ | 0.85 |
| $L_{ey}$ | $k_y$ | 0.85 |
| Length (mm) | L | 6000 |
| 2nd Moment in X ($mm^4$) | ix | 4.56E+07 |
| 2nd Moment in Y ($mm^4$) | iy | 1.54E+07 |
| Area of Lacings ($mm^2$) | Ad | 8.13E+02 |
| Area of Batten ($mm^2$) | Ab | 8.13E+02 |
| Column Width(mm) | h | 5.00E+02 |
| Panel Height (mm) | a | 5.00E+02 |
| Lacing Length (mm) | d | 7.07E+02 |
| Young Modulus ($N/m^2$) | E | 2.00E+05 |

Table 5:6 Fixed-Pinned: Built up column properties

| Brace type | Column Width & height(mm) | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 5100 | 6921.25 | 229.32 | 1 | 4586.4 | 1.18E-05 | 6399.79 | -28.34 |
| W-Double | 500 | 5100 | 6921.25 | 236.61 | 1 | 4732.2 | 5.89E-06 | 6650.31 | -28.84 |
| V-SLC | 500 | 5100 | 6921.25 | 228.96 | 1 | 4579.2 | 1.74E-08 | 6920.41 | -33.83 |
| V-Double | 500 | 5100 | 6921.25 | 231.29 | 1 | 4625.8 | 4.35E-06 | 6719.01 | -31.15 |
| X-SLC | 500 | 5100 | 6921.25 | 229.91 | 1 | 4598.2 | 4.35E-06 | 6719.01 | -31.56 |
| X-Double | 500 | 5100 | 6921.25 | 237.8 | 1 | 4756 | 2.17E-06 | 6818.63 | -30.25 |

Table 5:7 FEM verses theoretical thoeritical results at load ratio is 1.0 at column width 0.5m

| Brace type | Column Width & height(mm) | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 5100 | 6921.25 | 229.35 | 1 | 4587 | 1.77E-05 | 6167.45 | -25.63 |
| W-Double | 750 | 5100 | 6921.25 | 237.39 | 1 | 4747.8 | 8.83E-06 | 6522.64 | -27.21 |
| V-SLC | 750 | 5100 | 6921.25 | 228.99 | 1 | 4579.8 | 1.30E-05 | 6348.04 | -27.85 |
| V-Double | 750 | 5100 | 6921.25 | 230.22 | 1 | 4604.4 | 6.52E-06 | 6622.26 | -30.47 |
| X-SLC | 750 | 5100 | 6921.25 | 229.1 | 1 | 4582 | 6.52E-06 | 6622.26 | -30.81 |
| X-Double | 750 | 5100 | 6921.25 | 234.29 | 1 | 4685.8 | 3.26E-06 | 6768.45 | -30.77 |

Table 5:8 FEM verses theoretical results at load ratio is 1.0 at column width 0.75m

| Brace type | Column Width & height(mm) | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 5100 | 6921.25 | 229.36 | 1 | 4587.2 | 2.35E-05 | 5951.40 | -22.92 |
| W-Double | 1000 | 5100 | 6921.25 | 245.58 | 1 | 4911.6 | 1.18E-05 | 6399.79 | -23.25 |
| V-SLC | 1000 | 5100 | 6921.25 | 228.98 | 1 | 4579.6 | 1.74E-05 | 6177.50 | -25.87 |
| V-Double | 1000 | 5100 | 6921.25 | 230.76 | 1 | 4615.2 | 8.70E-06 | 6528.26 | -29.30 |
| X-SLC | 1000 | 5100 | 6921.25 | 229.12 | 1 | 4582.4 | 8.70E-06 | 6528.26 | -29.81 |
| X-Double | 1000 | 5100 | 6921.25 | 233.75 | 1 | 4675 | 4.35E-06 | 6719.01 | -30.42 |

Table 5:9 FEM verses theoretical thoeritical results at load ratio is 1.0 at column width 1.0m

| Brace type | Column Width & height(mm) | Effective Length (KL) | Euler Load (Pe) | Abaqus | | | Shear stifness | Theoritical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | EigenValue | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 5100 | 6921.25 | 229.36 | 1 | 4587.2 | 3.53E-05 | 5561.73 | -17.52 |
| W-Double | 1500 | 5100 | 6921.25 | 233.04 | 1 | 4660.8 | 1.77E-05 | 6167.45 | -24.43 |
| V-SLC | 1500 | 5100 | 6921.25 | 229.01 | 1 | 4580.2 | 2.61E-05 | 5862.52 | -21.87 |
| V-Double | 1500 | 5100 | 6921.25 | 229.58 | 1 | 4591.6 | 1.30E-05 | 6348.04 | -27.67 |
| X-SLC | 1500 | 5100 | 6921.25 | 229.12 | 1 | 4582.4 | 1.30E-05 | 6348.04 | -27.81 |
| X-Double | 1500 | 5100 | 6921.25 | 230.91 | 1 | 4618.2 | 6.52E-06 | 6622.26 | -30.26 |

Table 5:10 FEM verses theoretical results at load ratio is 1.0 at column width 1.5m

The percentage error in the case of pinned-roller is more than 10%, which is considered high. This could be due to high shear effects induced by the end supports. The percentage error also decreases with increasing column width.

Comparison of FEM and theoretical solutions at other considered load ratios are pressented in Appendix B 8.2.2 for pin-roller end supports.

# Chapter 6

## 6.1. CONCLUSIONS AND RECOMMENDATIONS

This chapter summarises the observations and findings of the research project. This dissertation has discussed the behaviour of single verse double laced columns. A summary of the objectives is repeated here for the reader's convenience:

1. To determine torsion and flexural buckling load of built-up columns and compare failure modes at different load ratios.

2. To compare structural integrity of the single versus double laced built up columns of the same design load at different load ratios.

### 6.1.1. Behaviours of built up columns

- Single built-up columns have poor structural performance e.g. biaxial strength and torsion resistance as compared to double laced columns.

- Single laced columns are more sensitive to load ratio than double laced columns.

- A combination of maximum critical load and minimum degree of torsion is achieved at load ratios close to one for both single and double laced built up columns.

- The X-braced columns have high buckling and torsion resistance for both single and double laced columns.

- An increase in column width does decrease the critical load of built up columns.

- For given end supports, both single and double laced columns made of the same member section yields similar critical load regardless of brace type at a load ratio of one (P/N=1).

- The buckling modes of some built up columns follow the sinusoidal shape.

- At load ratio away from one (P/N≠1), the mode of failure is a combination of bending and torsion.

- Higher critical loads are achieved when the column is exposed to low degree of torsion.

- Low (P/N<1.0) or high (P/N>1.0) load ratio produce high degree of torsion.

### 6.1.2. Recommendations and Future work

Future work should concentrate on the following:

- Evaluation of cost and effeciency of single verses double laced column

- Experimental verification

- Varying the support end conditions.

- Optimizing on the size of the steel sections in the chords and lacing.

- Considering other brace configuration.

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# APPENDICES

## APPENDIX A

# 8.1 Behavior of built up columns

## 8.1.1 Critical Loads and Torsion

### 8.1.1.1 Cantilever (Fixed at bottom and free at top)

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Fixed -free | 203x203x46 | 70x70x10 | Single | W | 0.03 | 500 | 85.397 | 84.289 | 78.476 | 73.717 | 68.619 | 58.412 | 45.751 | 36.865 |
| Critical Load (KN) | | | |  | | | 896.67 | 927.18 | 1043.73 | 1105.76 | 1143.88 | 1168.24 | 1143.78 | 1105.95 |
| Torsion (Rad) | | | | | | | 0.93 | 0.90 | 0.72 | 0.56 | 0.39 | 0.00 | 0.46 | 0.67 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 500 | 104.420 | 99.947 | 83.103 | 74.064 | 66.749 | 55.669 | 44.504 | 37.038 |
| Critical Load (KN) | | | | | | | 1096.41 | 1099.42 | 1105.27 | 1110.96 | 1112.71 | 1113.38 | 1112.60 | 1111.14 |
| Torsion (Rad) | | | | | | | 0.07 | 0.06 | 0.04 | 0.03 | 0.02 | 0.00 | 0.03 | 0.06 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | V | 0.03 | 500 | 88.608 | 87.192 | 80.027 | 74.511 | 68.918 | 58.410 | 45.966 | 37.282 |
| Critical Load (KN) | | | |  | | | 930.38 | 959.11 | 1064.36 | 1117.67 | 1148.86 | 1168.20 | 1149.15 | 1118.46 |
| Torsion (Rad) | | | | | | | 0.83 | 0.79 | 0.61 | 0.47 | 0.31 | 0.00 | 0.50 | 0.71 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 500 | 105.580 | 101.110 | 84.198 | 75.077 | 67.678 | 56.448 | 45.115 | 37.534 |
| Critical Load (KN) | | | | | | | 1108.59 | 1112.21 | 1119.83 | 1126.16 | 1128.19 | 1128.96 | 1127.88 | 1126.02 |
| Torsion (Rad) | | | | | | | 0.08 | 0.07 | 0.04 | 0.03 | 0.02 | 0.00 | 0.02 | 0.03 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | X | 0.03 | 500 | 95.064 | 92.881 | 82.738 | 75.820 | 69.406 | 58.425 | 46.282 | 37.925 |
| Critical Load (KN) | | | |  | | | 998.17 | 1021.69 | 1100.42 | 1137.30 | 1157.00 | 1168.50 | 1157.05 | 1137.75 |
| Torsion (Rad) | | | | | | | 0.61 | 0.58 | 0.42 | 0.30 | 0.19 | 0.00 | 0.45 | 0.64 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 500 | 104.870 | 100.250 | 83.045 | 73.918 | 66.571 | 55.494 | 44.373 | 36.949 |
| Critical Load (KN) | | | | | | | 1101.14 | 1102.75 | 1104.50 | 1108.77 | 1109.74 | 1109.88 | 1109.33 | 1108.47 |
| Torsion (Rad) | | | | | | | 0.03 | 0.03 | 0.02 | 0.01 | 0.01 | 0.00 | 0.01 | 0.02 |

Table 8:1 Critical load and torsion at column width of 0.5m

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Fixed -free | 203x203x46 | 70x70x10 | Single | W | 0.03 | 750 | 80.233 | 79.554 | 75.731 | 72.245 | 68.048 | 58.461 | 45.360 | 36.108 |
| Critical Load (KN) | | | |  | | | 842.45 | 875.09 | 1007.22 | 1083.68 | 1134.36 | 1169.22 | 1134.00 | 1083.24 |
| Torsion (Rad) | | | | | | | 0.75 | 0.73 | 0.61 | 0.49 | 0.35 | 0.00 | 0.48 | 0.63 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 750 | 104.610 | 100.490 | 84.458 | 75.569 | 68.255 | 57.016 | 45.559 | 37.861 |
| Critical Load (KN) | | | | | | | 1098.41 | 1105.39 | 1123.29 | 1133.54 | 1137.81 | 1140.32 | 1138.98 | 1135.83 |
| Torsion (Rad) | | | | | | | 0.07 | 0.06 | 0.04 | 0.03 | 0.02 | 0.01 | 0.03 | 0.06 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | V | 0.03 | 750 | 78.925 | 78.327 | 74.929 | 71.778 | 67.832 | 58.426 | 45.222 | 35.891 |
| Critical Load (KN) | | | |  | | | 828.71 | 861.60 | 996.56 | 1076.67 | 1130.76 | 1168.52 | 1130.55 | 1076.73 |
| Torsion (Rad) | | | | | | | 0.77 | 0.75 | 0.63 | 0.52 | 0.37 | 0.00 | 0.48 | 0.64 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 750 | 104.450 | 100.390 | 84.529 | 75.677 | 68.371 | 57.116 | 45.622 | 37.895 |
| Critical Load (KN) | | | | | | | 1096.73 | 1104.29 | 1124.24 | 1135.16 | 1139.74 | 1142.32 | 1140.55 | 1136.85 |
| Torsion (Rad) | | | | | | | 0.11 | 0.10 | 0.06 | 0.05 | 0.03 | 0.01 | 0.03 | 0.05 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | X | 0.03 | 750 | 85.517 | 84.423 | 78.617 | 73.824 | 68.687 | 58.438 | 45.776 | 36.893 |
| Critical Load (KN) | | | |  | | | 897.93 | 928.65 | 1045.61 | 1107.36 | 1145.01 | 1168.76 | 1144.40 | 1106.79 |
| Torsion (Rad) | | | | | | | 0.83 | 0.81 | 0.71 | 0.60 | 0.45 | 0.01 | 0.42 | 0.58 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 750 | 105.900 | 101.380 | 84.322 | 75.159 | 67.739 | 56.494 | 45.159 | 37.579 |
| Critical Load (KN) | | | | | | | 1111.95 | 1115.18 | 1121.48 | 1127.39 | 1129.21 | 1129.88 | 1128.98 | 1127.37 |
| Torsion (Rad) | | | | | | | 0.04 | 0.04 | 0.02 | 0.01 | 0.01 | 0.00 | 0.02 | 0.03 |

Table 8:2 Critical load and torsion at column width of 0.75m

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Fixed -free | 203x203x46 | 70x70x10 | Single | W | 0.03 | 1000 | 73.874 | 73.554 | 71.638 | 69.644 | 66.856 | 58.474 | 44.563 | 34.814 |
| Critical Load (KN) | | | |  | | | 775.68 | 809.09 | 952.79 | 1044.66 | 1114.49 | 1169.48 | 1114.08 | 1044.42 |
| Torsion (Rad) | | | | | | | 0.67 | 0.66 | 0.58 | 0.50 | 0.38 | 0.00 | 0.48 | 0.59 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 1000 | 106.530 | 102.810 | 87.682 | 78.883 | 71.229 | 59.484 | 47.416 | 39.223 |
| Critical Load (KN) | | | | | | | 1118.57 | 1130.91 | 1166.17 | 1183.25 | 1187.39 | 1189.68 | 1185.40 | 1176.69 |
| Torsion (Rad) | | | | | | | 0.13 | 0.12 | 0.08 | 0.06 | 0.04 | 0.01 | 0.04 | 0.06 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | K | 0.03 | 1000 | 72.789 | 72.512 | 70.844 | 69.081 | 66.547 | 58.424 | 44.244 | 34.386 |
| Critical Load (KN) | | | |  | | | 764.28 | 797.63 | 942.23 | 1036.22 | 1109.34 | 1168.48 | 1106.10 | 1031.58 |
| Torsion (Rad) | | | | | | | 0.69 | 0.68 | 0.61 | 0.53 | 0.41 | 0.00 | 0.52 | 0.62 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 1000 | 104.090 | 100.530 | 86.045 | 77.420 | 70.242 | 58.763 | 46.926 | 38.878 |
| Critical Load (KN) | | | | | | | 1092.95 | 1105.83 | 1144.40 | 1161.30 | 1170.93 | 1175.26 | 1173.15 | 1166.34 |
| Torsion (Rad) | | | | | | | 0.14 | 0.13 | 0.09 | 0.06 | 0.04 | 0.01 | 0.03 | 0.05 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | X | 0.03 | 1000 | 78.006 | 78.781 | 75.277 | 71.447 | 67.962 | 58.481 | 45.284 | 35.963 |
| Critical Load (KN) | | | |  | | | 819.06 | 866.59 | 1001.18 | 1071.71 | 1132.93 | 1169.62 | 1132.10 | 1078.89 |
| Torsion (Rad) | | | | | | | 0.60 | 0.59 | 0.50 | 0.41 | 0.30 | 0.00 | 0.45 | 0.56 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 1000 | 109.890 | 105.460 | 88.515 | 79.127 | 71.435 | 59.631 | 47.625 | 39.569 |
| Critical Load (KN) | | | | | | | 1153.85 | 1160.06 | 1177.25 | 1186.91 | 1190.82 | 1192.62 | 1190.63 | 1187.07 |
| Torsion (Rad) | | | | | | | 0.06 | 0.06 | 0.04 | 0.02 | 0.02 | 0.00 | 0.02 | 0.05 |

Table 8:3 Critical load and torsion at column width of 1.0 m

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Fixed -free | 203x203x46 | 70x70x10 | Single | W | 0.03 | 1500 | 65.617 | 66.187 | 65.701 | 65.124 | 64.168 | 58.436 | 42.778 | 32.567 |
| Critical Load (KN) | | | |  | | | 688.98 | 728.06 | 873.82 | 976.86 | 1069.68 | 1168.72 | 1069.45 | 977.01 |
| Torsion (Rad) | | | | | | | 0.54 | 0.53 | 0.50 | 0.47 | 0.41 | 0.00 | 0.45 | 0.50 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 1500 | 96.803 | 94.308 | 83.069 | 75.644 | 68.965 | 57.833 | 45.795 | 37.851 |
| Critical Load (KN) | | | | | | | 1016.43 | 1037.39 | 1104.82 | 1134.66 | 1149.65 | 1156.66 | 1144.88 | 1135.53 |
| Torsion (Rad) | | | | | | | 0.17 | 0.16 | 0.11 | 0.07 | 0.04 | 0.01 | 0.08 | 0.11 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | K | 0.03 | 1500 | 65.617 | 65.549 | 65.131 | 64.568 | 63.816 | 58.436 | 42.492 | 32.283 |
| Critical Load (KN) | | | |  | | | 688.98 | 721.04 | 866.24 | 968.52 | 1063.81 | 1168.72 | 1062.30 | 968.49 |
| Torsion (Rad) | | | | | | | 0.54 | 0.54 | 0.51 | 0.48 | 0.42 | 0.00 | 0.45 | 0.51 |
| Fixed -free | 203x203x46 | 70x70x6 | Double | | 0.03 | 1500 | 94.213 | 91.971 | 81.706 | 74.855 | 68.528 | 57.824 | 46.012 | 37.846 |
| Critical Load (KN) | | | | | | | 989.24 | 1011.68 | 1086.69 | 1122.83 | 1142.36 | 1156.48 | 1150.30 | 1135.38 |
| Torsion (Rad) | | | | | | | 0.19 | 0.18 | 0.13 | 0.10 | 0.07 | 0.01 | 0.05 | 0.09 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | X | 0.03 | 1500 | 69.399 | 69.250 | 68.318 | 67.255 | 65.555 | 58.469 | 43.637 | 33.568 |
| Critical Load (KN) | | | |  | | | 728.69 | 761.75 | 908.63 | 1008.83 | 1092.80 | 1169.38 | 1090.93 | 1007.04 |
| Torsion (Rad) | | | | | | | 0.50 | 0.50 | 0.46 | 0.41 | 0.33 | 0.00 | 0.43 | 0.49 |
| Fixed -free | 203x203x46 | 70x70x10 | Double | | 0.03 | 1500 | 102.090 | 98.638 | 84.427 | 76.041 | 68.914 | 57.655 | 45.937 | 38.013 |
| Critical Load (KN) | | | | | | | 1071.95 | 1085.02 | 1122.88 | 1140.62 | 1148.80 | 1153.10 | 1148.43 | 1140.39 |
| Torsion (Rad) | | | | | | | 0.09 | 0.08 | 0.06 | 0.04 | 0.02 | 0.00 | 0.04 | 0.06 |

Table 8:4 Critical load and torsion at column width of 1.5m

## 8.1.1.2 End supports: Pinned at bottom and Roller Support on top

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Single | W | 0.03 | 500 | 399.790 | 386.980 | 333.540 | 301.390 | 273.690 | 229.320 | 182.620 | 150.920 |
| Critical Load (KN) | | | |  | | | 4197.80 | 4256.78 | 4436.08 | 4008.49 | 3640.08 | 4586.40 | 4565.50 | 4527.60 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Double | | 0.03 | 500 | 447.210 | 427.650 | 354.150 | 315.190 | 283.840 | 236.610 | 189.200 | 157.550 |
| Critical Load (KN) | | | | | | | 4695.71 | 4704.15 | 4710.20 | 4192.03 | 3775.07 | 4732.20 | 4730.00 | 4726.50 |
| Fixed -free | 203x203x46 | 70x70x10 | Single | V | 0.03 | 500 | 393.170 | 381.300 | 331.010 | 300.060 | 272.080 | 228.960 | 182.090 | 150.180 |
| Critical Load (KN) | | | |  | | | 4128.29 | 4194.30 | 4402.43 | 3990.80 | 3618.66 | 4579.20 | 4552.25 | 4505.40 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Double | | 0.03 | 500 | 436.200 | 417.140 | 345.910 | 307.990 | 277.430 | 231.290 | 184.930 | 153.970 |
| Critical Load (KN) | | | | | | | 4580.10 | 4588.54 | 4600.60 | 4096.27 | 3689.82 | 4625.80 | 4623.25 | 4619.10 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Single | X | 0.03 | 500 | 417.560 | 401.690 | 338.940 | 303.610 | 274.330 | 229.100 | 182.830 | 151.730 |
| Critical Load (KN) | | | |  | | | 4384.38 | 4418.59 | 4507.90 | 4038.01 | 3648.59 | 4582.00 | 4570.75 | 4551.90 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Double | | 0.03 | 500 | 451.020 | 430.860 | 356.240 | 316.890 | 285.300 | 237.800 | 190.190 | 158.430 |
| Critical Load (KN) | | | | | | | 4735.71 | 4739.46 | 4737.99 | 4214.64 | 3794.49 | 4756.00 | 4754.75 | 4752.90 |
| | | | | | | | | | | | | | | |

Table 8:5 Critical load and torsion at column width of 0.5 m

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Single | W | 0.03 | 750 | 358.030 | 350.330 | 316.030 | 292.820 | 270.490 | 229.350 | 180.580 | 146.750 |
| Critical Load (KN) | | | |  | | | 3759.32 | 3853.63 | 4203.20 | 3894.51 | 3597.52 | 4587.00 | 4514.50 | 4402.50 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Double | | 0.03 | 750 | 438.960 | 421.250 | 352.920 | 315.370 | 284.620 | 237.590 | 189.830 | 157.800 |
| Critical Load (KN) | | | | | | | 4609.08 | 4633.75 | 4693.84 | 4194.42 | 3785.45 | 4751.80 | 4745.75 | 4734.00 |
| Fixed -free | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Single | K | 0.03 | 750 | 352.590 | 345.380 | 313.080 | 291.030 | 269.570 | 228.990 | 179.810 | 145.690 |
| | | | |  | | | 3702.20 | 3799.18 | 4163.96 | 3870.70 | 3585.28 | 4579.80 | 4495.25 | 4370.70 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Double | | 0.03 | 750 | 421.370 | 405.020 | 340.970 | 305.200 | 275.670 | 230.210 | 183.800 | 152.620 |
| | | | | | | | 4424.39 | 4455.22 | 4534.90 | 4059.16 | 3666.41 | 4604.20 | 4595.00 | 4578.60 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Single | X | 0.03 | 750 | 391.160 | 379.750 | 330.720 | 300.120 | 273.170 | 229.100 | 182.010 | 149.020 |
| | | | |  | | | 4107.18 | 4177.25 | 4398.58 | 3991.60 | 3633.16 | 4582.00 | 4550.25 | 4470.60 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Double | | 0.03 | 750 | 440.690 | 421.620 | 350.080 | 311.850 | 280.970 | 234.290 | 187.310 | 155.920 |
| Critical Load (KN) | | | | | | | 4627.25 | 4637.82 | 4656.06 | 4147.61 | 3736.90 | 4685.80 | 4682.75 | 4677.60 |
| | | | | | | | | | | | | | | |

Table 8:6 Critical load and torsion at column width of 0.75 m

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Single | W | 0.03 | 1000 | 322.690 | 317.780 | 295.630 | 280.220 | 264.560 | 229.360 | 176.830 | 140.700 |
| Critical Load (KN) | | | |  | | | 3388.25 | 3495.58 | 3931.88 | 3726.93 | 3518.65 | 4587.20 | 4420.75 | 4221.00 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Double | | 0.03 | 1000 | 442.090 | 426.670 | 363.910 | 327.170 | 292.980 | 245.840 | 197.870 | 163.930 |
| Critical Load (KN) | | | | | | | 4641.95 | 4693.37 | 4840.00 | 4351.36 | 3896.63 | 4916.80 | 4946.75 | 4917.90 |
| Fixed -free | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Single | K | 0.03 | 1000 | 316.560 | 312.020 | 291.530 | 277.310 | 262.840 | 228.980 | 175.320 | 138.940 |
| | | | |  | | | 3323.88 | 3432.22 | 3877.35 | 3688.22 | 3495.77 | 4579.60 | 4383.00 | 4168.20 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Double | | 0.03 | 1000 | 399.360 | 387.060 | 334.910 | 303.040 | 275.360 | 230.760 | 183.600 | 151.550 |
| | | | | | | | 4193.28 | 4257.66 | 4454.30 | 4030.43 | 3662.29 | 4615.20 | 4590.00 | 4546.50 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Single | X | 0.03 | 1000 | 360.680 | 352.880 | 317.710 | 293.770 | 270.840 | 229.120 | 180.410 | 146.720 |
| | | | |  | | | 3787.14 | 3881.68 | 4225.54 | 3907.14 | 3602.17 | 4582.40 | 4510.25 | 4401.60 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Double | | 0.03 | 1000 | 431.170 | 414.550 | 347.310 | 310.330 | 280.050 | 233.750 | 186.740 | 155.210 |
| Critical Load (KN) | | | | | | | 4527.29 | 4560.05 | 4619.22 | 4127.39 | 3724.67 | 4675.00 | 4668.50 | 4656.30 |
| | | | | | | | | | | | | | | |

Table 8:7 Critical load and torsion at column width of 1.0 m

| Load Ratio (P/N) | | | | | | | 0.05 | 0.10 | 0.33 | 0.50 | 0.67 | 1.00 | 1.50 | 2.00 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | 1/20 | 1/10 | 1/3 | 1/2 | 2/3 | 1 | 1.5/1 | 2/1 |
| Total Load Aplied (KN) | | | | | | | 10.50 | 11.00 | 13.30 | 15.00 | 16.67 | 20 | 25.00 | 30.00 |
| **End support** | **Profile** | **Brace** | **layer** | **Brace Type** | **Element Seed** | **Column Width** | **Eigen Value** | | | | | | | |
| | | | | | (m) | (mm) | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Single | W | 0.03 | 1500 | 280.090 | 277.880 | 267.160 | 259.410 | 251.550 | 229.360 | 168.210 | 130.410 |
| Critical Load (KN) | | | |  | | | 2940.95 | 3056.68 | 3553.23 | 3450.15 | 3345.62 | 4587.20 | 4205.25 | 3912.30 |
| | | | | | | | | | | | | | | |
| Pinned-Roller | 203x203x46 | 70x70x10 | Double | | 0.03 | 1500 | 354.140 | 347.690 | 317.430 | 295.740 | 274.160 | 233.040 | 183.190 | 148.380 |
| Critical Load (KN) | | | | | | | 3718.47 | 3824.59 | 4221.82 | 3933.34 | 3646.33 | 4660.80 | 4579.75 | 4451.40 |
| Fixed -free | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Single | K | 0.03 | 1500 | 276.310 | 274.310 | 264.290 | 256.980 | 249.620 | 229.010 | 166.750 | 129.060 |
| | | | |  | | | 2901.26 | 3017.41 | 3515.06 | 3417.83 | 3319.95 | 4580.20 | 4168.75 | 3871.80 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Double | | 0.03 | 1500 | 334.880 | 329.520 | 304.590 | 286.600 | 268.070 | 229.580 | 178.860 | 143.500 |
| | | | | | | | 3516.24 | 3624.72 | 4051.05 | 3811.78 | 3565.33 | 4591.60 | 4471.50 | 4305.00 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Single | X | 0.03 | 1500 | 312.590 | 308.580 | 289.870 | 276.420 | 262.480 | 229.120 | 174.820 | 138.080 |
| | | | |  | | | 3282.20 | 3394.38 | 3855.27 | 3676.39 | 3490.98 | 4582.40 | 4370.50 | 4142.40 |
| Pinned-Roller | | | | | | | | | | | | | | |
| Critical Load (KN) | 203x203x46 | 70x70x10 | Double | | 0.03 | 1500 | 394.880 | 383.360 | 333.610 | 302.550 | 275.290 | 230.910 | 183.630 | 151.420 |
| Critical Load (KN) | | | | | | | 4146.24 | 4216.96 | 4437.01 | 4023.92 | 3661.36 | 4618.20 | 4590.75 | 4542.60 |
| | | | | | | | | | | | | | | |

Table 8:8 Critical load and torsion at column width of 1.5m

**APPENDIX B: THEORY VERSES ABAQUS RESULTS**

# 8.2 Behavior of built up columns

## 8.2.1 End supports: Cantilever (Fixed at bottom and free at top)

## A) Load Ratio 0.05

| Brace type | Column Width | Effective Length (*KL*) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/Pd | KN | |
| W-SLC | 500 | 12000 | 1439.32 | 104.63 | 0.05 | 1098.615 | 1.18E-05 | 1.42E+03 | -22.38 |
| W-DLC | 500 | 12000 | 1250.15 | 104.42 | 0.05 | 1096.41 | 5.89E-06 | 1.24E+03 | -11.65 |
| V-SLC | 500 | 12000 | 1439.32 | 103.46 | 0.05 | 1086.33 | 8.70E-06 | 1.42E+03 | -23.58 |
| V-DLC | 500 | 12000 | 1250.15 | 105.58 | 0.05 | 1108.59 | 4.35E-06 | 1.24E+03 | -10.84 |
| X-SLC | 500 | 12000 | 1439.32 | 107.07 | 0.05 | 1124.235 | 4.35E-06 | 1.43E+03 | -21.40 |
| X-DLC | 500 | 12000 | 1250.15 | 104.87 | 0.05 | 1101.135 | 2.17E-06 | 1.25E+03 | -11.68 |

Table 8:1 Theoretical Critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1672.35 | 108.80 | 0.05 | 1142.4 | 1.77E-05 | 1.62E+03 | -29.67 |
| W-DLC | 750 | 12000 | 1250.15 | 104.61 | 0.05 | 1098.405 | 8.83E-06 | 1.24E+03 | -11.17 |
| V-SLC | 750 | 12000 | 1672.35 | 107.11 | 0.05 | 1124.655 | 1.30E-05 | 1.64E+03 | -31.28 |
| V-DLC | 750 | 12000 | 1250.15 | 104.45 | 0.05 | 1096.725 | 6.52E-06 | 1.24E+03 | -11.56 |
| X-SLC | 750 | 12000 | 1439.32 | 101.61 | 0.05 | 1066.905 | 6.52E-06 | 1.43E+03 | -25.18 |
| X- DLC | 750 | 12000 | 1250.15 | 105.90 | 0.05 | 1111.95 | 3.26E-06 | 1.25E+03 | -10.69 |

Table 8:2Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1439.32 | 99.28 | 0.05 | 1042.4085 | 2.35E-05 | 1.39E+03 | -25.12 |
| W- DLC | 1000 | 12000 | 1250.15 | 106.53 | 0.05 | 1118.565 | 1.18E-05 | 1.23E+03 | -9.21 |
| V-SLC | 1000 | 12000 | 1439.32 | 98.64 | 0.05 | 1035.6675 | 1.74E-05 | 1.40E+03 | -26.24 |
| V- DLC | 1000 | 12000 | 1250.15 | 104.09 | 0.05 | 1092.945 | 8.70E-06 | 1.24E+03 | -11.62 |
| X-SLC | 1000 | 12000 | 1439.32 | 91.50 | 0.05 | 960.7395 | 8.70E-06 | 1.42E+03 | -32.41 |
| X- DLC | 1000 | 12000 | 1250.15 | 109.89 | 0.05 | 1153.845 | 4.35E-06 | 1.24E+03 | -7.20 |

Table 8:3Theoretical critical load verses FEM (ABAQUS) at column width of 1.0 m

| Brace type | Column Width | Effective Length (*KL*) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 2091.81 | 106.46 | 0.05 | 1117.83 | 3.53E-05 | 1.95E+03 | -42.61 |
| W- DLC | 1500 | 12000 | 1250.15 | 96.803 | 0.05 | 1016.4315 | 1.77E-05 | 1.22E+03 | -16.90 |
| V-SLC | 1500 | 12000 | 2091.81 | 105.48 | 0.05 | 1107.54 | 2.61E-05 | 1.98E+03 | -44.16 |
| V- DLC | 1500 | 12000 | 1250.15 | 94.213 | 0.05 | 989.2365 | 1.30E-05 | 1.23E+03 | -19.58 |
| X-SLC | 1500 | 12000 | 1672.35 | 92.748 | 0.05 | 973.854 | 1.30E-05 | 1.64E+03 | -40.50 |
| X- DLC | 1500 | 12000 | 1250.15 | 102.09 | 0.05 | 1071.945 | 6.52E-06 | 1.24E+03 | -13.56 |

Table 8:4 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**a) Load Ratio 0.1**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 12000 | 1439.32 | 102.46 | 0.1 | 1127.06 | 1.18E-05 | 1.42E+03 | -20.37 |
| W- DLC | 500 | 12000 | 1250.15 | 99.947 | 0.1 | 1099.417 | 5.89E-06 | 1.24E+03 | -11.41 |
| V-SLC | 500 | 12000 | 1439.32 | 101.43 | 0.1 | 1115.73 | 8.70E-06 | 1.42E+03 | -21.51 |
| V- DLC | 500 | 12000 | 1250.15 | 101.11 | 0.1 | 1112.21 | 4.35E-06 | 1.24E+03 | -10.55 |
| X-SLC | 500 | 12000 | 1439.32 | 104.68 | 0.1 | 1151.48 | 4.35E-06 | 1.43E+03 | -19.50 |
| X- DLC | 500 | 12000 | 1250.15 | 100.25 | 0.1 | 1102.75 | 2.17E-06 | 1.25E+03 | -11.55 |

Table 8:5 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1672.35 | 107.53 | 0.1 | 1182.83 | 1.77E-05 | 1.62E+03 | -27.18 |
| W- DLC | 750 | 12000 | 1250.15 | 100.49 | 0.1 | 1105.39 | 8.83E-06 | 1.24E+03 | -10.60 |
| V-SLC | 750 | 12000 | 1672.35 | 105.97 | 0.1 | 1165.67 | 1.30E-05 | 1.64E+03 | -28.78 |
| V- DLC | 750 | 12000 | 1250.15 | 100.39 | 0.1 | 1104.29 | 6.52E-06 | 1.24E+03 | -10.95 |
| X-SLC | 750 | 12000 | 1439.32 | 99.866 | 0.1 | 1098.526 | 6.52E-06 | 1.43E+03 | -22.96 |
| X- DLC | 750 | 12000 | 1250.15 | 101.38 | 0.1 | 1115.18 | 3.26E-06 | 1.25E+03 | -10.43 |

Table 8:6 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width | Effective Length (*KL*) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1439.32 | 98.657 | 0.1 | 1085.227 | 2.35E-05 | 1.39E+03 | -22.05 |
| W- DLC | 1000 | 12000 | 1250.15 | 102.81 | 0.1 | 1130.91 | 1.18E-05 | 1.23E+03 | -8.21 |
| V-SLC | 1000 | 12000 | 1439.32 | 98.078 | 0.1 | 1078.858 | 1.74E-05 | 1.40E+03 | -23.17 |
| V- DLC | 1000 | 12000 | 1250.15 | 100.53 | 0.1 | 1105.83 | 8.70E-06 | 1.24E+03 | -10.58 |
| X-SLC | 1000 | 12000 | 1439.32 | 90.703 | 0.1 | 997.733 | 8.70E-06 | 1.42E+03 | -29.81 |
| X- DLC | 1000 | 12000 | 1250.15 | 105.46 | 0.1 | 1160.06 | 4.35E-06 | 1.24E+03 | -6.70 |

Table 8:7 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 2091.81 | 106.25 | 0.1 | 1168.75 | 3.53E-05 | 1.95E+03 | -40.00 |
| W- DLC | 1500 | 12000 | 1250.15 | 94.308 | 0.1 | 1037.388 | 1.77E-05 | 1.22E+03 | -15.19 |
| V-SLC | 1500 | 12000 | 2091.81 | 105.31 | 0.1 | 1158.41 | 2.61E-05 | 1.98E+03 | -41.60 |
| V- DLC | 1500 | 12000 | 1250.15 | 91.971 | 0.1 | 1011.681 | 1.30E-05 | 1.23E+03 | -17.76 |
| X-SLC | 1500 | 12000 | 1672.35 | 92.444 | 0.1 | 1016.884 | 1.30E-05 | 1.64E+03 | -37.87 |
| X- DLC | 1500 | 12000 | 1250.15 | 98.638 | 0.1 | 1085.018 | 6.52E-06 | 1.24E+03 | -12.50 |

Table 8:8Theoretical critical load verses FEM (ABAQUS) at column width of1.5m

**b) Load ratio 0.33**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 12000 | 1439.32 | 92.117 | 0.33 | 1225.156 | 1.18E-05 | 1.42E+03 | -13.44 |
| W- DLC | 500 | 12000 | 1250.15 | 83.103 | 0.33 | 1105.27 | 5.89E-06 | 1.24E+03 | -10.94 |
| V-SLC | 500 | 12000 | 1439.32 | 91.621 | 0.33 | 1218.559 | 8.70E-06 | 1.42E+03 | -14.28 |
| V- DLC | 500 | 12000 | 1250.15 | 84.198 | 0.33 | 1119.833 | 4.35E-06 | 1.24E+03 | -9.94 |
| X-SLC | 500 | 12000 | 1439.32 | 93.479 | 0.33 | 1243.271 | 4.35E-06 | 1.43E+03 | -13.08 |
| X- DLC | 500 | 12000 | 1250.15 | 83.045 | 0.33 | 1104.499 | 2.17E-06 | 1.25E+03 | -11.41 |

Table 8:9 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1672.35 | 100.68 | 0.33 | 1339.044 | 1.77E-05 | 1.62E+03 | -17.57 |
| W-DLC | 750 | 12000 | 1250.15 | 84.458 | 0.33 | 1123.291 | 8.83E-06 | 1.24E+03 | -9.16 |
| V-SLC | 750 | 12000 | 1672.35 | 99.743 | 0.33 | 1326.582 | 1.30E-05 | 1.64E+03 | -18.94 |
| V- DLC | 750 | 12000 | 1250.15 | 84.529 | 0.33 | 1124.236 | 6.52E-06 | 1.24E+03 | -9.34 |
| X-SLC | 750 | 12000 | 1439.32 | 91.149 | 0.33 | 1212.282 | 6.52E-06 | 1.43E+03 | -14.98 |
| X- DLC | 750 | 12000 | 1250.15 | 84.322 | 0.33 | 1121.483 | 3.26E-06 | 1.25E+03 | -9.93 |

Table 8:10 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | Abaqus | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1439.32 | 95.031 | 0.33 | 1263.912 | 2.35E-05 | 1.39E+03 | -9.21 |
| W- DLC | 1000 | 12000 | 1250.15 | 87.682 | 0.33 | 1166.171 | 1.18E-05 | 1.23E+03 | -5.34 |
| V-SLC | 1000 | 12000 | 1439.32 | 94.798 | 0.33 | 1260.813 | 1.74E-05 | 1.40E+03 | -10.21 |
| V- DLC | 1000 | 12000 | 1250.15 | 86.045 | 0.33 | 1144.399 | 8.70E-06 | 1.24E+03 | -7.46 |
| X-SLC | 1000 | 12000 | 1439.32 | 86.209 | 0.33 | 1146.58 | 8.70E-06 | 1.42E+03 | -19.34 |
| X- DLC | 1000 | 12000 | 1250.15 | 88.515 | 0.33 | 1177.25 | 4.35E-06 | 1.24E+03 | -5.32 |

Table 8:11 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 2091.81 | 105.02 | 0.33 | 1396.766 | 3.53E-05 | 1.95E+03 | -28.29 |
| W- DLC | 1500 | 12000 | 1250.15 | 83.069 | 0.33 | 1104.818 | 1.77E-05 | 1.22E+03 | -9.67 |
| V-SLC | 1500 | 12000 | 2091.81 | 104.21 | 0.33 | 1385.993 | 2.61E-05 | 1.98E+03 | -30.13 |
| V- DLC | 1500 | 12000 | 1250.15 | 81.706 | 0.33 | 1086.69 | 1.30E-05 | 1.23E+03 | -11.66 |
| X-SLC | 1500 | 12000 | 1672.35 | 90.561 | 0.33 | 1204.461 | 1.30E-05 | 1.64E+03 | -26.41 |
| X- DLC | 1500 | 12000 | 1250.15 | 84.427 | 0.33 | 1122.879 | 6.52E-06 | 1.24E+03 | -9.45 |

Table 8:12 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**c) Load ratio 0.5**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 12000 | 1439.32 | 84.804 | 0.5 | 1272.06 | 1.18E-05 | 1.42E+03 | -10.12 |
| W- DLC | 500 | 12000 | 1250.15 | 74.064 | 0.5 | 1110.96 | 5.89E-06 | 1.24E+03 | -10.48 |
| V-SLC | 500 | 12000 | 1439.32 | 84.554 | 0.5 | 1268.31 | 8.70E-06 | 1.42E+03 | -10.78 |
| V- DLC | 500 | 12000 | 1250.15 | 75.077 | 0.5 | 1126.155 | 4.35E-06 | 1.24E+03 | -9.43 |
| X-SLC | 500 | 12000 | 1439.32 | 85.767 | 0.5 | 1286.505 | 4.35E-06 | 1.43E+03 | -10.06 |
| X- DLC | 500 | 12000 | 1250.15 | 73.918 | 0.5 | 1108.77 | 2.17E-06 | 1.25E+03 | -11.07 |

Table 8:13 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width | Effective Length (*KL*) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1672.35 | 94.891 | 0.5 | 1423.365 | 1.77E-05 | 1.62E+03 | -12.37 |
| W- DLC | 750 | 12000 | 1250.15 | 75.569 | 0.5 | 1133.535 | 8.83E-06 | 1.24E+03 | -8.33 |
| V-SLC | 750 | 12000 | 1672.35 | 94.342 | 0.5 | 1415.13 | 1.30E-05 | 1.64E+03 | -13.53 |
| V- DLC | 750 | 12000 | 1250.15 | 75.677 | 0.5 | 1135.155 | 6.52E-06 | 1.24E+03 | -8.46 |
| X-SLC | 750 | 12000 | 1439.32 | 84.575 | 0.5 | 1268.625 | 6.52E-06 | 1.43E+03 | -11.03 |
| X- DLC | 750 | 12000 | 1250.15 | 75.159 | 0.5 | 1127.385 | 3.26E-06 | 1.25E+03 | -9.45 |

Table 8:14Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1439.32 | 91.471 | 0.5 | 1372.065 | 2.35E-05 | 1.39E+03 | -1.44 |
| W- DLC | 1000 | 12000 | 1250.15 | 78.883 | 0.5 | 1183.245 | 1.18E-05 | 1.23E+03 | -3.96 |
| V-SLC | 1000 | 12000 | 1439.32 | 91.52 | 0.5 | 1372.8 | 1.74E-05 | 1.40E+03 | -2.23 |
| V- DLC | 1000 | 12000 | 1250.15 | 77.42 | 0.5 | 1161.3 | 8.70E-06 | 1.24E+03 | -6.10 |
| X-SLC | 1000 | 12000 | 1439.32 | 82.097 | 0.5 | 1231.455 | 8.70E-06 | 1.42E+03 | -13.37 |
| X- DLC | 1000 | 12000 | 1250.15 | 79.127 | 0.5 | 1186.905 | 4.35E-06 | 1.24E+03 | -4.54 |

Table 8:15 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 2091.81 | 103.59 | 0.5 | 1553.85 | 3.53E-05 | 1.95E+03 | -20.23 |
| W- DLC | 1500 | 12000 | 1250.15 | 75.644 | 0.5 | 1134.66 | 1.77E-05 | 1.22E+03 | -7.23 |
| V-SLC | 1500 | 12000 | 2091.81 | 102.94 | 0.5 | 1544.1 | 2.61E-05 | 1.98E+03 | -22.15 |
| V- DLC | 1500 | 12000 | 1250.15 | 74.855 | 0.5 | 1122.825 | 1.30E-05 | 1.23E+03 | -8.72 |
| X-SLC | 1500 | 12000 | 1672.35 | 88.504 | 0.5 | 1327.56 | 1.30E-05 | 1.64E+03 | -18.89 |
| X- DLC | 1500 | 12000 | 1250.15 | 76.041 | 0.5 | 1140.615 | 6.52E-06 | 1.24E+03 | -8.02 |

Table 8:16 Theoretical critical load verses FEM (ABAQUS) at column width of1.5m

**a) Load ratio 0.667**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 12000 | 1439.32 | 77.853 | 0.667 | 1297.81 | 1.18E-05 | 1.42E+03 | -8.30 |
| W- DLC | 500 | 12000 | 1250.15 | 66.749 | 0.667 | 1112.706 | 5.89E-06 | 1.24E+03 | -10.34 |
| V-SLC | 500 | 12000 | 1439.32 | 77.746 | 0.667 | 1296.026 | 8.70E-06 | 1.42E+03 | -8.83 |
| V- DLC | 500 | 12000 | 1250.15 | 67.678 | 0.667 | 1128.192 | 4.35E-06 | 1.24E+03 | -9.26 |
| X-SLC | 500 | 12000 | 1439.32 | 78.57 | 0.667 | 1309.762 | 4.35E-06 | 1.43E+03 | -8.43 |
| X- DLC | 500 | 12000 | 1250.15 | 66.571 | 0.667 | 1109.739 | 2.17E-06 | 1.25E+03 | -10.99 |

Table 8:17Theoretical critical load verses FEM (ABAQUS) at load ratio of 0.05

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1672.35 | 88.543 | 0.667 | 1476.012 | 1.77E-05 | 1.62E+03 | -9.13 |
| W- DLC | 750 | 12000 | 1250.15 | 68.255 | 0.667 | 1137.811 | 8.83E-06 | 1.24E+03 | -7.98 |
| V-SLC | 750 | 12000 | 1672.35 | 88.271 | 0.667 | 1471.478 | 1.30E-05 | 1.64E+03 | -10.09 |
| V- DLC | 750 | 12000 | 1250.15 | 68.371 | 0.667 | 1139.745 | 6.52E-06 | 1.24E+03 | -8.09 |
| X-SLC | 750 | 12000 | 1439.32 | 78.034 | 0.667 | 1300.827 | 6.52E-06 | 1.43E+03 | -8.77 |
| X- DLC | 750 | 12000 | 1250.15 | 67.739 | 0.667 | 1129.209 | 3.26E-06 | 1.25E+03 | -9.31 |

Table 8:18 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1439.32 | 86.898 | 0.667 | 1448.59 | 2.35E-05 | 1.39E+03 | 4.05 |
| W- DLC | 1000 | 12000 | 1250.15 | 71.229 | 0.667 | 1187.387 | 1.18E-05 | 1.23E+03 | -3.62 |
| V-SLC | 1000 | 12000 | 1439.32 | 87.205 | 0.667 | 1453.707 | 1.74E-05 | 1.40E+03 | 3.53 |
| V- DLC | 1000 | 12000 | 1250.15 | 70.242 | 0.667 | 1170.934 | 8.70E-06 | 1.24E+03 | -5.32 |
| X-SLC | 1000 | 12000 | 1439.32 | 77.236 | 0.667 | 1287.524 | 8.70E-06 | 1.42E+03 | -9.43 |
| X- DLC | 1000 | 12000 | 1250.15 | 71.435 | 0.667 | 1190.821 | 4.35E-06 | 1.24E+03 | -4.23 |

Table 8:19 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 2091.81 | 101.27 | 0.667 | 1688.171 | 3.53E-05 | 1.95E+03 | -13.33 |
| W- DLC | 1500 | 12000 | 1250.15 | 68.965 | 0.667 | 1149.647 | 1.77E-05 | 1.22E+03 | -6.01 |
| V-SLC | 1500 | 12000 | 2091.81 | 100.86 | 0.667 | 1681.336 | 2.61E-05 | 1.98E+03 | -15.24 |
| V- DLC | 1500 | 12000 | 1250.15 | 68.528 | 0.667 | 1142.362 | 1.30E-05 | 1.23E+03 | -7.13 |
| X-SLC | 1500 | 12000 | 1672.35 | 85.457 | 0.667 | 1424.568 | 1.30E-05 | 1.64E+03 | -12.96 |
| X- DLC | 1500 | 12000 | 1250.15 | 68.914 | 0.667 | 1148.796 | 6.52E-06 | 1.24E+03 | -7.36 |

Table 8:20 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**b) Load ratio 1.0**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 12000 | 1439.32 | 65.656 | 1 | 1313.12 | 1.18E-05 | 1.42E+03 | -7.22 |
| W- DLC | 500 | 12000 | 1250.15 | 55.669 | 1 | 1113.38 | 5.89E-06 | 1.24E+03 | -10.28 |
| V-SLC | 500 | 12000 | 1439.32 | 65.636 | 1 | 1312.72 | 8.70E-06 | 1.42E+03 | -7.65 |
| V- DLC | 500 | 12000 | 1250.15 | 56.448 | 1 | 1128.96 | 4.35E-06 | 1.24E+03 | -9.20 |
| X-SLC | 500 | 12000 | 1439.32 | 66.171 | 1 | 1323.42 | 4.35E-06 | 1.43E+03 | -7.48 |
| X- DLC | 500 | 12000 | 1250.15 | 55.494 | 1 | 1109.88 | 2.17E-06 | 1.25E+03 | -10.98 |

Table 8:21 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 12000 | 1672.35 | 75.484 | 1 | 1509.68 | 1.77E-05 | 1.62E+03 | -7.06 |
| W- DLC | 750 | 12000 | 1250.15 | 57.016 | 1 | 1140.32 | 8.83E-06 | 1.24E+03 | -7.78 |
| V-SLC | 750 | 12000 | 1672.35 | 75.424 | 1 | 1508.48 | 1.30E-05 | 1.64E+03 | -7.83 |
| V- DLC | 750 | 12000 | 1250.15 | 57.116 | 1 | 1142.32 | 6.52E-06 | 1.24E+03 | -7.88 |
| X-SLC | 750 | 12000 | 1439.32 | 66.017 | 1 | 1320.34 | 6.52E-06 | 1.43E+03 | -7.40 |
| X- DLC | 750 | 12000 | 1250.15 | 56.494 | 1 | 1129.88 | 3.26E-06 | 1.25E+03 | -9.25 |

Table 8:22 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 12000 | 1439.32 | 75.194 | 1 | 1503.88 | 2.35E-05 | 1.39E+03 | 8.03 |
| W- DLC | 1000 | 12000 | 1250.15 | 59.484 | 1 | 1189.68 | 1.18E-05 | 1.23E+03 | -3.44 |
| V-SLC | 1000 | 12000 | 1439.32 | 75.673 | 1 | 1513.46 | 1.74E-05 | 1.40E+03 | 7.78 |
| V- DLC | 1000 | 12000 | 1250.15 | 58.763 | 1 | 1175.26 | 8.70E-06 | 1.24E+03 | -4.97 |
| X-SLC | 1000 | 12000 | 1439.32 | 66.269 | 1 | 1325.38 | 8.70E-06 | 1.42E+03 | -6.76 |
| X- DLC | 1000 | 12000 | 1250.15 | 59.631 | 1 | 1192.62 | 4.35E-06 | 1.24E+03 | -4.08 |

Table 8:23 Theoretical critical load verses FEM (ABAQUS) at column width of1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 12000 | 2091.81 | 90.821 | 1 | 1816.42 | 3.53E-05 | 1.95E+03 | -6.75 |
| W- DLC | 1500 | 12000 | 1250.15 | 57.833 | 1 | 1156.66 | 1.77E-05 | 1.22E+03 | -5.44 |
| V-SLC | 1500 | 12000 | 2091.81 | 90.761 | 1 | 1815.22 | 2.61E-05 | 1.98E+03 | -8.49 |
| V- DLC | 1500 | 12000 | 1250.15 | 57.824 | 1 | 1156.48 | 1.30E-05 | 1.23E+03 | -5.98 |
| X-SLC | 1500 | 12000 | 1672.35 | 75.232 | 1 | 1504.64 | 1.30E-05 | 1.64E+03 | -8.07 |
| X- DLC | 1500 | 12000 | 1250.15 | 57.655 | 1 | 1153.1 | 6.52E-06 | 1.24E+03 | -7.01 |

Table 8:24 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

## 8.2.2 End supports: Fixed at bottom and Roller at top

### a) Load ratio 0.05

| Brace type | Column Width (mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 4200 | 6168.50 | 449.98 | 0.05 | 4724.79 | 1.12E-05 | 5769.48 | -18.11 |
| W- DLC | 500 | 4200 | 5357.79 | 447.21 | 0.05 | 4695.71 | 5.61E-06 | 5201.55 | -9.72 |
| V-SLC | 500 | 4200 | 6168.50 | 426.4 | 0.05 | 4477.20 | 8.28E-06 | 5868.64 | -23.71 |
| V- DLC | 500 | 4200 | 5357.79 | 436.2 | 0.05 | 4580.10 | 4.14E-06 | 5241.48 | -12.62 |
| X-SLC | 500 | 4200 | 6168.50 | 459.33 | 0.05 | 4822.97 | 4.14E-06 | 6014.84 | -19.82 |
| X- DLC | 500 | 4200 | 5357.79 | 451.02 | 0.05 | 4735.71 | 2.07E-06 | 5298.99 | -10.63 |

Table 8:25 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| **Brace type** | **Column Width** | **Effective Length (*K*L)** | **Euler Load (Pe)** | **FEM Solution (Abaqus)** | | | **Shear stiffness** | **Theoretical Load** | **Error %** |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | **Eigen-Value** | **Load ratio** | **FEM Load (KN)** | 1/pd | **KN** | |
| W-SLC | 750 | 4200 | 7167.21 | 452.12 | 0.05 | 4747.26 | 1.68E-05 | 6396.23 | -25.78 |
| W- DLC | 750 | 4200 | 5357.79 | 438.96 | 0.05 | 4609.08 | 8.41E-06 | 5126.81 | -10.10 |
| V-SLC | 750 | 4200 | 7167.21 | 444.69 | 0.05 | 4669.245 | 1.24E-05 | 6581.14 | -29.05 |
| V- DLC | 750 | 4200 | 5357.79 | 421.37 | 0.05 | 4424.385 | 6.21E-06 | 5185.19 | -14.67 |
| X-SLC | 750 | 4200 | 6168.50 | 438.23 | 0.05 | 4601.415 | 6.21E-06 | 5940.84 | -22.55 |
| X- DLC | 750 | 4200 | 5357.79 | 440.69 | 0.05 | 4627.245 | 3.11E-06 | 5270.08 | -12.20 |

Table 8:26 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 4200 | 7167.21 | 406.36 | 0.05 | 4266.78 | 2.24E-05 | 6174.82 | -30.90 |
| W- DLC | 1000 | 4200 | 5357.79 | 442.09 | 0.05 | 4641.945 | 1.12E-05 | 5054.18 | -8.16 |
| V-SLC | 1000 | 4200 | 7167.21 | 398.25 | 0.05 | 4181.625 | 1.66E-05 | 6406.52 | -34.73 |
| V- DLC | 1000 | 4200 | 5357.79 | 399.36 | 0.05 | 4193.28 | 8.28E-06 | 5130.11 | -18.26 |
| X-SLC | 1000 | 4200 | 6168.50 | 402.36 | 0.05 | 4224.78 | 8.28E-06 | 5868.64 | -28.01 |
| X- DLC | 1000 | 4200 | 5357.79 | 431.17 | 0.05 | 4527.285 | 4.14E-06 | 5241.48 | -13.63 |

Table 8:27 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 4200 | 5357.79 | 352.67 | 0.05 | 3703.035 | 3.36E-05 | 4539.67 | -18.43 |
| W- DLC | 1500 | 4200 | 5357.79 | 354.14 | 0.05 | 3718.47 | 1.68E-05 | 4914.92 | -24.34 |
| V-SLC | 1500 | 4200 | 5357.79 | 348.29 | 0.05 | 3657.045 | 2.49E-05 | 4728.26 | -22.66 |
| V- DLC | 1500 | 4200 | 5357.79 | 334.88 | 0.05 | 3516.24 | 1.24E-05 | 5023.38 | -30.00 |
| X-SLC | 1500 | 4200 | 6168.50 | 348.15 | 0.05 | 3655.575 | 1.24E-05 | 5729.38 | -36.20 |
| X- DLC | 1500 | 4200 | 5357.79 | 394.88 | 0.05 | 4146.24 | 6.21E-06 | 5185.19 | -20.04 |

Table 8:28 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**b) Load ratio 0.1**

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | Abaqus | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 4200 | 6168.50 | 435.86 | 0.1 | 4794.46 | 1.12E-05 | 5.77E+03 | -16.90 |
| W- DLC | 500 | 4200 | 5357.79 | 427.65 | 0.1 | 4704.15 | 5.61E-06 | 5.20E+03 | -9.56 |
| V-SLC | 500 | 4200 | 6168.50 | 414.93 | 0.1 | 4564.23 | 8.28E-06 | 5.87E+03 | -22.23 |
| V- DLC | 500 | 4200 | 5357.79 | 417.14 | 0.1 | 4588.54 | 4.14E-06 | 5.24E+03 | -12.46 |
| X-SLC | 500 | 4200 | 6168.50 | 443.75 | 0.1 | 4881.25 | 4.14E-06 | 6.01E+03 | -18.85 |
| X- DLC | 500 | 4200 | 5357.79 | 430.86 | 0.1 | 4739.46 | 2.07E-06 | 5.30E+03 | -10.56 |

Table 8:29 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| **Brace type** | **Column Width** | **Effective Length (*K*L)** | **Euler Load (Pe)** | **FEM Solution (Abaqus)** | | | **Shear stiffness** | **Theoretical Load** | **Error %** |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | **Eigen-Value** | **Load ratio** | **FEM Load (KN)** | 1/pd | **KN** | |
| W-SLC | 750 | 4200 | 7167.21 | 443.11 | 0.1 | 4874.21 | 1.68E-05 | 6.40E+03 | -23.80 |
| W- DLC | 750 | 4200 | 5357.79 | 421.25 | 0.1 | 4633.75 | 8.41E-06 | 5.13E+03 | -9.62 |
| V-SLC | 750 | 4200 | 7167.21 | 436.3 | 0.1 | 4799.3 | 1.24E-05 | 6.58E+03 | -27.07 |
| V- DLC | 750 | 4200 | 5357.79 | 405.02 | 0.1 | 4455.22 | 6.21E-06 | 5.19E+03 | -14.08 |
| X-SLC | 750 | 4200 | 6168.50 | 425.94 | 0.1 | 4685.34 | 6.21E-06 | 5.94E+03 | -21.13 |
| X- DLC | 750 | 4200 | 5357.79 | 421.62 | 0.1 | 4637.82 | 3.11E-06 | 5.27E+03 | -12.00 |

Table 8:30 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 4200 | 7167.21 | 400.77 | 0.1 | 4408.47 | 2.24E-05 | 6.17E+03 | -28.61 |
| W- DLC | 1000 | 4200 | 5357.79 | 426.67 | 0.1 | 4693.37 | 1.12E-05 | 5.05E+03 | -7.14 |
| V-SLC | 1000 | 4200 | 7167.21 | 393.13 | 0.1 | 4324.43 | 1.66E-05 | 6.41E+03 | -32.50 |
| V- DLC | 1000 | 4200 | 5357.79 | 387.06 | 0.1 | 4257.66 | 8.28E-06 | 5.13E+03 | -17.01 |
| X-SLC | 1000 | 4200 | 6168.50 | 394.08 | 0.1 | 4334.88 | 8.28E-06 | 5.87E+03 | -26.13 |
| X- DLC | 1000 | 4200 | 5357.79 | 414.55 | 0.1 | 4560.05 | 4.14E-06 | 5.24E+03 | -13.00 |

Table 8:31 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 4200 | 5357.79 | 350.4 | 0.1 | 3854.4 | 3.36E-05 | 4.54E+03 | -15.10 |
| W- DLC | 1500 | 4200 | 5357.79 | 347.69 | 0.1 | 3824.59 | 1.68E-05 | 4.91E+03 | -22.18 |
| V-SLC | 1500 | 4200 | 5357.79 | 346.22 | 0.1 | 3808.42 | 2.49E-05 | 4.73E+03 | -19.45 |
| V- DLC | 1500 | 4200 | 5357.79 | 329.52 | 0.1 | 3624.72 | 1.24E-05 | 5.02E+03 | -27.84 |
| X-SLC | 1500 | 4200 | 6168.50 | 343.96 | 0.1 | 3783.56 | 1.24E-05 | 5.73E+03 | -33.96 |
| X- DLC | 1500 | 4200 | 5357.79 | 383.36 | 0.1 | 4216.96 | 6.21E-06 | 5.19E+03 | -18.67 |

Table 8:32 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**c) Load Ratio 0.33**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 4200 | 6168.50 | 376.64 | 0.33 | 5009.312 | 1.12E-05 | 5769.48 | -13.18 |
| W- DLC | 500 | 4200 | 5357.79 | 354.15 | 0.33 | 4710.195 | 5.61E-06 | 5201.55 | -9.45 |
| V-SLC | 500 | 4200 | 6168.50 | 365.57 | 0.33 | 4862.081 | 8.28E-06 | 5868.64 | -17.15 |
| V- DLC | 500 | 4200 | 5357.79 | 345.91 | 0.33 | 4600.603 | 4.14E-06 | 5241.48 | -12.23 |
| X-SLC | 500 | 4200 | 6168.50 | 379.68 | 0.33 | 5049.744 | 4.14E-06 | 6014.84 | -16.05 |
| X- DLC | 500 | 4200 | 5357.79 | 356.24 | 0.33 | 4737.992 | 2.07E-06 | 5298.99 | -10.59 |

Table 8:33 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 4200 | 7167.21 | 402.63 | 0.33 | 5354.979 | 1.68E-05 | 6396.23 | -16.28 |
| W-DLC | 750 | 4200 | 5357.79 | 352.92 | 0.33 | 4693.836 | 8.41E-06 | 5126.81 | -8.45 |
| V-SLC | 750 | 4200 | 7167.21 | 398.49 | 0.33 | 5299.917 | 1.24E-05 | 6581.14 | -19.47 |
| V- DLC | 750 | 4200 | 5357.79 | 340.97 | 0.33 | 4534.901 | 6.21E-06 | 5185.19 | -12.54 |
| X-SLC | 750 | 4200 | 6168.50 | 372.65 | 0.33 | 4956.245 | 6.21E-06 | 5940.84 | -16.57 |
| X- DLC | 750 | 4200 | 5357.79 | 350.08 | 0.33 | 4656.064 | 3.11E-06 | 5270.08 | -11.65 |

Table 8:34 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| **Brace type** | **Column Width** | **Effective Length (*K*L)** | **Euler Load (Pe)** | **FEM Solution (Abaqus)** | | | **Shear stiffness** | **Theoretical Load** | **Error %** |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | **Eigen-Value** | **Load ratio** | **FEM Load (KN)** | 1/pd | **KN** | |
| W-SLC | 1000 | 4200 | 7167.21 | 375.53 | 0.33 | 4994.549 | 2.24E-05 | 6174.82 | -19.11 |
| W- DLC | 1000 | 4200 | 5357.79 | 363.91 | 0.33 | 4840.003 | 1.12E-05 | 5054.18 | -4.24 |
| V-SLC | 1000 | 4200 | 7167.21 | 369.82 | 0.33 | 4918.606 | 1.66E-05 | 6406.52 | -23.23 |
| V- SLC | 1000 | 4200 | 5357.79 | 334.91 | 0.33 | 4454.303 | 8.28E-06 | 5130.11 | -13.17 |
| X-SLC | 1000 | 4200 | 6168.50 | 356.64 | 0.33 | 4743.312 | 8.28E-06 | 5868.64 | -19.18 |
| X- DLC | 1000 | 4200 | 5357.79 | 347.31 | 0.33 | 4619.223 | 4.14E-06 | 5241.48 | -11.87 |

Table 8:35 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 4200 | 5357.79 | 338.94 | 0.33 | 4507.902 | 3.36E-05 | 4539.67 | -0.70 |
| W-DLC | 1500 | 4200 | 5357.79 | 317.43 | 0.33 | 4221.819 | 1.68E-05 | 4914.92 | -14.10 |
| V-SLC | 1500 | 4200 | 5357.79 | 335.44 | 0.33 | 4461.352 | 2.49E-05 | 4728.26 | -5.64 |
| V- DLC | 1500 | 4200 | 5357.79 | 304.59 | 0.33 | 4051.047 | 1.24E-05 | 5023.38 | -19.36 |
| X-SLC | 1500 | 4200 | 6168.50 | 324.4 | 0.33 | 4314.52 | 1.24E-05 | 5729.38 | -24.69 |
| X- DLC | 1500 | 4200 | 5357.79 | 333.61 | 0.33 | 4437.013 | 6.21E-06 | 5185.19 | -14.43 |

Table 8:36 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**d) Load Ratio 0.5**

| Brace type | Column Width | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 4200 | 6168.50 | 340.72 | 0.5 | 5110.8 | 1.12E-05 | 5769.48 | -11.42 |
| W- DLC | 500 | 4200 | 5357.79 | 315.19 | 0.5 | 4727.85 | 5.61E-06 | 5201.55 | -9.11 |
| V-SLC | 500 | 4200 | 6168.50 | 334.29 | 0.5 | 5014.35 | 8.28E-06 | 5868.64 | -14.56 |
| V- DLC | 500 | 4200 | 5357.79 | 307.99 | 0.5 | 4619.85 | 4.14E-06 | 5241.48 | -11.86 |
| X-SLC | 500 | 4200 | 6168.50 | 341.91 | 0.5 | 5128.65 | 4.14E-06 | 6014.84 | -14.73 |
| X- DLC | 500 | 4200 | 5357.79 | 316.89 | 0.5 | 4753.35 | 2.07E-06 | 5298.99 | -10.30 |

Table 8:37 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 4200 | 7167.21 | 374.82 | 0.5 | 5622.3 | 1.68E-05 | 6396.23 | -12.10 |
| W- DLC | 750 | 4200 | 5357.79 | 315.37 | 0.5 | 4730.55 | 8.41E-06 | 5126.81 | -7.73 |
| V-SLC | 750 | 4200 | 7167.21 | 372.31 | 0.5 | 5584.65 | 1.24E-05 | 6581.14 | -15.14 |
| V- DLC | 750 | 4200 | 5357.79 | 305.2 | 0.5 | 4578 | 6.21E-06 | 5185.19 | -11.71 |
| X-SLC | 750 | 4200 | 6168.50 | 338.93 | 0.5 | 5083.95 | 6.21E-06 | 5940.84 | -14.42 |
| X- DLC | 750 | 4200 | 5357.79 | 311.85 | 0.5 | 4677.75 | 3.11E-06 | 5270.08 | -11.24 |

Table 8:38 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 4200 | 7167.21 | 357.48 | 0.5 | 5362.2 | 2.24E-05 | 6174.82 | -13.16 |
| W-Double | 1000 | 4200 | 5357.79 | 327.17 | 0.5 | 4907.55 | 1.12E-05 | 5054.18 | -2.90 |
| V-SLC | 1000 | 4200 | 7167.21 | 353.49 | 0.5 | 5302.35 | 1.66E-05 | 6406.52 | -17.24 |
| V-Double | 1000 | 4200 | 5357.79 | 303.04 | 0.5 | 4545.6 | 8.28E-06 | 5130.11 | -11.39 |
| X-SLC | 1000 | 4200 | 6168.50 | 330.89 | 0.5 | 4963.35 | 8.28E-06 | 5868.64 | -15.43 |
| X-Double | 1000 | 4200 | 5357.79 | 310.33 | 0.5 | 4654.95 | 4.14E-06 | 5241.48 | -11.19 |

Table 8:39 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | Abaqus | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 4200 | 5357.79 | 330.42 | 0.5 | 4956.3 | 3.36E-05 | 4539.67 | 9.18 |
| W- DLC | 1500 | 4200 | 5357.79 | 295.74 | 0.5 | 4436.1 | 1.68E-05 | 4914.92 | -9.74 |
| V-SLC | 1500 | 4200 | 5357.79 | 327.39 | 0.5 | 4910.85 | 2.49E-05 | 4728.26 | 3.86 |
| V- DLC | 1500 | 4200 | 5357.79 | 286.6 | 0.5 | 4299 | 1.24E-05 | 5023.38 | -14.42 |
| X-SLC | 1500 | 4200 | 6168.50 | 310.29 | 0.5 | 4654.35 | 1.24E-05 | 5729.38 | -18.76 |
| X- DLC | 1500 | 4200 | 5357.79 | 302.55 | 0.5 | 4538.25 | 6.21E-06 | 5185.19 | -12.48 |

Table 8:40 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**e) Load Ratio 0.667**

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 4200 | 6168.50 | 309.61 | 0.667 | 5161.199 | 1.12E-05 | 5769.48 | -10.54 |
| W-Double | 500 | 4200 | 5357.79 | 283.84 | 0.667 | 4731.613 | 5.61E-06 | 5201.55 | -9.03 |
| V-SLC | 500 | 4200 | 6168.50 | 306.17 | 0.667 | 5103.854 | 8.28E-06 | 5868.64 | -13.03 |
| V-Double | 500 | 4200 | 5357.79 | 277.43 | 0.667 | 4624.758 | 4.14E-06 | 5241.48 | -11.77 |
| X-SLC | 500 | 4200 | 6168.50 | 309.84 | 0.667 | 5165.033 | 4.14E-06 | 6014.84 | -14.13 |
| X-Double | 500 | 4200 | 5357.79 | 285.3 | 0.667 | 4755.951 | 2.07E-06 | 5298.99 | -10.25 |

Table 8:41 Theoretical critical load verses FEM (ABAQUS) at column width of 0.5m

| **Brace type** | **Column Width & height(mm)** | **Effective Length (*K*L)** | **Euler Load (Pe)** | **FEM Solution (Abaqus)** | | | **Shear stiffness** | **Theoretical Load** | **Error %** |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | **Eigen-Value** | **Load ratio** | **FEM Load (KN)** | 1/pd | **KN** | |
| W-SLC | 750 | 4200 | 7167.21 | 347.58 | 0.667 | 5794.159 | 1.68E-05 | 6396.23 | -9.41 |
| W-Double | 750 | 4200 | 5357.79 | 284.62 | 0.667 | 4744.615 | 8.41E-06 | 5126.81 | -7.45 |
| V-SLC | 750 | 4200 | 7167.21 | 346.33 | 0.667 | 5773.321 | 1.24E-05 | 6581.14 | -12.27 |
| V-Double | 750 | 4200 | 5357.79 | 275.67 | 0.667 | 4595.419 | 6.21E-06 | 5185.19 | -11.37 |
| X-SLC | 750 | 4200 | 6168.50 | 308.92 | 0.667 | 5149.696 | 6.21E-06 | 5940.84 | -13.32 |
| X-Double | 750 | 4200 | 5357.79 | 280.97 | 0.667 | 4683.77 | 3.11E-06 | 5270.08 | -11.13 |

Table 8:42 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| **Brace type** | **Column Width & height(mm)** | **Effective Length (*K*L)** | **Euler Load (Pe)** | **FEM Solution (Abaqus)** | | | **Shear stiffness** | **Theoretical Load** | **Error %** |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | **Eigen-Value** | **Load ratio** | **FEM Load (KN)** | 1/pd | **KN** | |
| W-SLC | 1000 | 4200 | 7167.21 | 339.1 | 0.667 | 5652.797 | 2.24E-05 | 6174.82 | -8.45 |
| W-Double | 1000 | 4200 | 5357.79 | 296.53 | 0.667 | 4943.155 | 1.12E-05 | 5054.18 | -2.20 |
| V-SLC | 1000 | 4200 | 7167.21 | 336.71 | 0.667 | 5612.956 | 1.66E-05 | 6406.52 | -12.39 |
| V-Double | 1000 | 4200 | 5357.79 | 275.36 | 0.667 | 4590.251 | 8.28E-06 | 5130.11 | -10.52 |
| X-SLC | 1000 | 4200 | 6168.50 | 305.9 | 0.667 | 5099.353 | 8.28E-06 | 5868.64 | -13.11 |
| X-Double | 1000 | 4200 | 5357.79 | 280.05 | 0.667 | 4668.434 | 4.14E-06 | 5241.48 | -10.93 |

Table 8:43 Theoretical critical Load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 4200 | 5357.79 | 321.68 | 0.667 | 5362.406 | 3.36E-05 | 4539.67 | 18.12 |
| W-Double | 1500 | 4200 | 5357.79 | 274.16 | 0.667 | 4570.247 | 1.68E-05 | 4914.92 | -7.01 |
| V-SLC | 1500 | 4200 | 5357.79 | 319.22 | 0.667 | 5321.397 | 2.49E-05 | 4728.26 | 12.54 |
| V-Double | 1500 | 4200 | 5357.79 | 268.07 | 0.667 | 4468.727 | 1.24E-05 | 5023.38 | -11.04 |
| X-SLC | 1500 | 4200 | 6168.50 | 295.51 | 0.667 | 4926.152 | 1.24E-05 | 5729.38 | -14.02 |
| X-Double | 1500 | 4200 | 5357.79 | 275.29 | 0.667 | 4589.084 | 6.21E-06 | 5185.19 | -11.50 |

Table 8:44 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

**a) Load Ratio 0.667**

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 500 | 4200 | 6168.50 | 259.53 | 1 | 5190.6 | 1.12E-05 | 5769.48 | -10.03 |
| W-Double | 500 | 4200 | 5357.79 | 236.61 | 1 | 4732.2 | 5.61E-06 | 5201.55 | -9.02 |
| V-SLC | 500 | 4200 | 6168.50 | 258.81 | 1 | 5176.2 | 8.28E-06 | 5868.64 | -11.80 |
| V-Double | 500 | 4200 | 5357.79 | 231.29 | 1 | 4625.8 | 4.14E-06 | 5241.48 | -11.75 |
| X-SLC | 500 | 4200 | 6168.50 | 259.19 | 1 | 5183.8 | 4.14E-06 | 6014.84 | -13.82 |
| X-Double | 500 | 4200 | 5357.79 | 237.8 | 1 | 4756 | 2.07E-06 | 5298.99 | -10.25 |

Table 8:45 Critical load and torsion at column width of 0.5m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 750 | 4200 | 7167.21 | 295.62 | 1 | 5912.4 | 1.68E-05 | 6396.23 | -7.56 |
| W-Double | 750 | 4200 | 5357.79 | 237.59 | 1 | 4751.8 | 8.41E-06 | 5126.81 | -7.31 |
| V-SLC | 750 | 4200 | 7167.21 | 295.25 | 1 | 5905 | 1.24E-05 | 6581.14 | -10.27 |
| V-Double | 750 | 4200 | 5357.79 | 230.21 | 1 | 4604.2 | 6.21E-06 | 5185.19 | -11.20 |
| X-SLC | 750 | 4200 | 6168.50 | 259.31 | 1 | 5186.2 | 6.21E-06 | 5940.84 | -12.70 |
| X-Double | 750 | 4200 | 5357.79 | 234.29 | 1 | 4685.8 | 3.11E-06 | 5270.08 | -11.09 |

Table 8:46 Theoretical critical load verses FEM (ABAQUS) at column width of 0.75m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1000 | 4200 | 7167.21 | 295.69 | 1 | 5913.8 | 2.24E-05 | 6174.82 | -4.23 |
| W-Double | 1000 | 4200 | 5357.79 | 248.09 | 1 | 4961.8 | 1.12E-05 | 5054.18 | -1.83 |
| V-SLC | 1000 | 4200 | 7167.21 | 295.29 | 1 | 5905.8 | 1.66E-05 | 6406.52 | -7.82 |
| V-Double | 1000 | 4200 | 5357.79 | 230.76 | 1 | 4615.2 | 8.28E-06 | 5130.11 | -10.04 |
| X-SLC | 1000 | 4200 | 6168.50 | 259.33 | 1 | 5186.6 | 8.28E-06 | 5868.64 | -11.62 |
| X-Double | 1000 | 4200 | 5357.79 | 233.75 | 1 | 4675 | 4.14E-06 | 5241.48 | -10.81 |

Table 8:47 Theoretical critical load verses FEM (ABAQUS) at column width of 1.0m

| Brace type | Column Width & height(mm) | Effective Length (*K*L) | Euler Load (Pe) | FEM Solution (Abaqus) | | | Shear stiffness | Theoretical Load | Error % |
|---|---|---|---|---|---|---|---|---|---|
| | (mm) | (mm) | KN | Eigen-Value | Load ratio | FEM Load (KN) | 1/pd | KN | |
| W-SLC | 1500 | 4200 | 5357.79 | 295.62 | 1 | 5912.4 | 3.36E-05 | 4539.67 | 30.24 |
| W- DLC | 1500 | 4200 | 5357.79 | 233.04 | 1 | 4660.8 | 1.68E-05 | 4914.92 | -5.17 |
| V-SLC | 1500 | 4200 | 5357.79 | 295.27 | 1 | 5905.4 | 2.49E-05 | 4728.26 | 24.90 |
| V- DLC | 1500 | 4200 | 5357.79 | 229.58 | 1 | 4591.6 | 1.24E-05 | 5023.38 | -8.60 |
| X-SLC | 1500 | 4200 | 6168.50 | 259.33 | 1 | 5186.6 | 1.24E-05 | 5729.38 | -9.47 |
| X- DLC | 1500 | 4200 | 5357.79 | 230.91 | 1 | 4618.2 | 6.21E-06 | 5185.19 | -10.93 |

Table 8:48 Theoretical critical load verses FEM (ABAQUS) at column width of 1.5m

## APPENDIX C

# 8.3 Photographs of built-up columns



Figure 8:1 Built up column in an indurstrial ware house



Figure 8:2 Built up column in an indurstrial ware house supporting crane girder




Figure 8:3 Built up column in an indurstrial ware house supporting crane girder




Figure 8:4 Built up column in an indurstrial ware house supporting crane girder



Figure 8:5 A built up column of a crane girder used during Gautrain subway project




Figure 8:6 Single laced built up column of a crane girder used during Gautrain project