

https://dspace.org/

Students' motivations and actions when they learn mathematics using CAS : a study using an activity theory approach.

Periasamy, Jeevasundarie

2012-02-29
http://hdl.handle.net/10539/11377

Downloaded from a DSpace repository. https://dspace.org/

Copyright Notice

The copyright of this thesis vests in the University of the Witwatersrand, Johannesburg, South Africa, in accordance with the University's Intellectual Property Policy.

No portion of the text may be reproduced, stored in a retrieval system, or transmitted in any form or by any means, including analogue and digital media, without prior written permission from the University. Extracts of or quotations from this thesis may, however, be made in terms of Sections 12 and 13 of the South African Copyright Act No. 98 of 1978 (as amended), for non-commercial or educational purposes. Full acknowledgement must be made to the author and the University.

An electronic version of this thesis is available on the Library webpage (www.wits.ac.za/library) under "Research Resources".

For permission requests, please contact the University Legal Office or the University Research Office (www.wits.ac.za).

STUDENTS' MOTIVATIONS AND ACTIONS WHEN THEY LEARN MATHEMATICS USING CAS: A STUDY USING AN ACTIVITY THEORY APPROACH

Jeevasundarie Periasamy

A thesis submitted to the Wits School of Education, Faculty of Humanities, University of the Witwatersrand in fulfilment of the requirements for the degree of Doctor of Philosophy

Johannesburg

2011

ABSTRACT

I explore students' motivations towards using Computer Algebra Systems (CAS) in learning mathematics, their associated actions and the relationship between their motivations and actions. Leontiev's (1978) philosophy of needs, motives and goals provides a powerful theory within which to understand students' motivations and actions in a learning context mediated by tools, thus the study is located within the framework of Leontiev's activity theory. It draws particularly on his notions of activity, actions and operations, as well as on Western motivational concepts of achievement goal theory and expectancy-value theory. Specifically, the notions of mastery approach goal-orientations, performance approach goal-orientations and social goal-orientations are examined, as are task value components (comprising 'importance' of task, intrinsic reasons and extrinsic utility value). I also discuss indexes of motivation, which comprise choice of tasks, effort and persistence. An important theoretical contribution of this thesis lies in the elaboration of the aforementioned Western motivational constructs to the activity theoretical construct of motives. The latter conception comprises three groupings: self-related, cognitive and social motives, as postulated by the activity theorist Lompscher (1999). Set within the qualitative research paradigm, this study utilises case study methodology. Methods of data collection include four interviews for each of the three participants, observations of the participants during two problem-solving sessions, and consequent computer screen analyses of these sessions. The mathematical problems involved the numerical solutions of differential equations using MATLAB. The participants were students studying towards vocational (diploma) qualifications in Mechanical and Electrical Engineering at a comprehensive university in South Africa. My research illustrates the relationship between the expressed needs, motives and goals of the individuals regarding their involvement in the activity of using MATLAB in mathematical learning within a third semester, Mathematics 3 service course.

DECLARATION

I declare that this thesis is my own unaided work. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

_____

Jeevasundarie Periasamy

_____ Day of _____ in the year _____

In loving memory of my father, Rungasamy Pathrachallam (Harry) Pillay, who taught me to value education, and that its true significance lies in disseminating knowledge.

ACKNOWLEDGEMENTS

My supervisor, Professor Margot Berger, gave me invaluable assistance right from the early stages of my conceptualising the study. Her support, guidance, patience and expertise are commendable. I appreciated the constant and speedy feedback, the many useful conversations we had, the telephonic appointments whilst I was abroad, during my sabbatical and vacations, and the encouragement that I had received, especially in the chosen path of qualitative research and using a theoretical framework.

I am grateful to the participants in this study who were generous with their time and effort.

This material is based upon work supported financially by the National Research Foundation. Any opinion, findings and conclusions or recommendations expressed in this material are those of the author and therefore the NRF does not accept any liability in regard thereto.

The University Research Council at the University of Johannesburg awarded me financial support, to which, I am most appreciative.

I am thankful to my sister, Nerinjeni Naidoo, who read parts of my findings chapter and the subsequent conversations helped stimulate my thinking.

My mum, Janakeeamma Pillay, for her patience and support.

My family in South Africa and Singapore, I am grateful for their continued support, encouragement and understanding. My father-in-law, Periasamy Poja Pillay, who in his eighties and an active scholar has been a great source of inspiration to me; I thank him for his support.

Last to be mentioned but first in my thoughts, my husband, Thirunavukkarasu s/o Periasamy, whose patience, emotional support, encouragement and faith in my abilities, has made a huge contribution to my student life. I am forever indebted to him, especially since this thesis would not have begun had it not been for his gentle persuasion.

LIST OF FIGURES

page

Figure 3.1: Vygotsky's mediation triangle 55

Figure 3.2: Engeström's activity system 56

Figure 3.3: Leontiev's activity levels 60

Figure 3.4: A model encompassing number of motives – one object – one activity 69

Figure 3.5: Link between motives (activity theory, achievement goal theory, and expectancy-value theory) 80

Figure 3.6: Model of the activity levels for learning mathematics using CAS 88

Figure 3.7: Outline of some constructs used in the study 89

Figure 5.1: Needs and motives from the analysis of Thembiso's interviews - reflections on the learning mathematics activity 173

Figure 5.2: Needs, motives and operations from the analysis of Thembiso's learning mathematics with CAS activity 174

Figure 5.3: Themes and actions from Thembiso's learning mathematics with CAS activity 175

Figure 5.4: Needs and motives from the analysis of Tumi's interviews – reflections on the learning mathematics activity 223

Figure 5.5: Needs, motives and operations from the analysis of Tumi's learning mathematics with CAS activity 224

Figure 5.6: Themes and actions from the analysis of Tumi's learning mathematics with CAS activity 225

Figure 5.7: Needs and motives from the analysis of Abraham's interviews – reflections on the learning mathematics activity 229

Figure 5.8: Needs, motives and operations from the analysis of Abraham's learning mathematics with CAS activity 230

Figure 5.9: Themes and actions from the analysis of Abraham's learning mathematics with CAS activity 231

LIST OF TABLES

page

7Table 2.1: Indexes of Motivation 37

Table 2.2: Summary: theoretical constructs of motivation 49

Table 3.1: The two perspectives of object of activity 66

Table 4.1: Summary of methods used to collect data 96

Table 4.2: Numbers of students doing Mathematics 3 surveyed for critical group by academic status, attitudes to computers, type of school and diploma registered 99

Table 4.3: Numbers of students selected from critical group as critical cases (for interviewing and observations) by academic status, type of school, attitudes to computers and diploma registered for 101

LIST OF ABBREVIATIONS AND ACRONYMS

CAS: Computer Algebra Systems

CAS (in maths): Computer Algebra Systems in mathematics

DEs: differential equations

CHAT: cultural-historical activity theory

Lab: laboratory

Labs: laboratories

B.Tech: Bachelor of Technology

M.Tech: Master of Technology

UJ: University of Johannesburg

T/S: township

RK II: Runge-Kutta II

RK IV Runge-Kutta IV

CONTENTS

Abstract *Page iii*
List of Figures *vii*
List of Tables *viii*
List of abbreviations and acronyms *ix*

CHAPTER 1: INTRODUCTION TO THE THESIS 1

1.1 Context of the study 2
1.2 Rationale for this study 3
1.3 Research questions 5
1.4 Outline of thesis chapters 7

CHAPTER 2: LITERATURE SURVEY OF CAS IN MATHEMATICAL LEARNING AND REVIEW OF MOTIVATIONAL THEORY

2.1 Introduction to the chapter 9

SECTION A: LITERATURE SURVEY OF CAS IN MATHEMATICAL LEARNING

2.2 Introduction to Section A 9
2.3 Instrumental Genesis 11
2.4 Calculus and computers 17
2.5 Affect in learning mathematics with CAS 21
2.6 My research and the current literature 25
2.7. Summary of Section A 32

SECTION B: REVIEW OF MOTIVATIONAL THEORY

2.8 Introduction to section B 34
2.9 The need to research motivation in learning contexts from a socio-cultural standpoint 34
2.10 The construct of motivation 36
2.11 Summary of section B 48

CHAPTER 3: THEORETICAL FRAMEWORK: AN EXPLICATION OFACTIVITY THEORY

3.1 Introduction to the chapter 51
3.2 Socio-cultural theory and Vygotsky's theory 52
3.3 The development of activity theory 55
3.4 A validation of Leontiev's version of activity theory 58
3.5 Constructs associated with Leontiev's activity theory 60
3.6 Establishing the link between motivations and motives 77
3.7 Learning at the levels of actions and of operations 81
3.8 The notions of mediation, consciousness and context 82
3.9 Development of model 86
3.10 Summary 87

CHAPTER 4: METHODOLOGY AND METHODS

4.1 Introduction 90
4.2 Methodology 91
4.3 Empirical domain 92
4.4 Methods 97
4.5 Research rigour 111
4.6 Data analysis procedures: open-, axial- and selective- codes 117
4.7 Summary 124

CHAPTER 5: A STRUCTURED ANALYSIS: EXPOSING STUDENTS' NEEDS, MOTIVES, OBJECT OF ACTIVITY, ACTIONS AND OPERATIONS

5.1 Introduction 125
5.2 Thembiso's interviews and problem-solving sessions 126
5.3 Analysis of Tumi's interviews and problem-solving sessions using the activity theory model 178
5.4 Analysis of Abraham's interviews and problem-solving sessions using the activity theory model 228
5.5 Summary 234

CHAPTER 6: DISCUSSION AND FINDINGS

6.1 Introduction 235
6.2 A comparison of student's object of activity and motives 236
6.3 A comparison of student's actions 247
6.4 A comparison of students' operations 272
6.5 Answering Research Question 3 274
6.6 Summary of Chapter 6 277

CHAPTER 7: CONCLUSION

7.1 Introduction 278
7.2 Contribution of thesis to new knowledge 278
7.3 Limitations of this study 281
7.4 Suggestions for future research 282
7.5 Recommendations for specific educational practices 283
7.6 Personal reflections 285

APPENDICES

A: Attitude to computers survey 286

B: Solutions to MATLAB tasks in Problem-solving Session 1 288

C: Solutions to MATLAB tasks in Problem-solving Session 2 290

REFERENCES 295

CHAPTER 1

INTRODUCTION TO THE THESIS

> "…The activity of every individual man depends on his place in society, on the conditions that are his lot, and on how this lot is worked out in unique, individual circumstances". Leontiev (1978, p.51)

The opening quotation from the pioneering Russian Marxist psychologist, Aleksei Leontiev, alludes to the individualistic nature of humankind, who are simultaneously positioned as active members of society and the social sphere. Each person's experience of the world is therefore unique, but all are shaped by the "conditions that are his [or her] lot" or the context in which he or she is operating. As Hollebrands, Laborde and Straber (2008, p.157) wrote: "We do not take the learner as an isolated individual facing the world, but take the learner as deeply embedded in his/her environment which is highly structured and defines the ways the individual is learning". From a socio-cultural perspective, the surrounding world of the individual exists as a result of historical-cultural development, and the existence of higher education institutions of learning is the result of these developments. Society deems it important that individuals seek education and in so doing become autonomous contributors to it.

The increased use of technology globally is having an influence on learning in general. Its entry into the mathematics classroom has stimulated a considerable amount of interest. Computer Algebra Systems (CAS), including software such as MATLAB, *Derive* and *Mathematica*, were initially developed for use by professionals, such as, mathematicians and engineers. In the last two decades there have been attempts to transform CAS into a pedagogical tool for use in the classroom, with importance placed on its investigative capacity in the learning of mathematics. CAS is believed to herald a new era of managing technical computations and lessening cumbersome mathematical manipulations. This should allow for emphasising and enhancing mathematical concepts.

Using CAS in mathematical learning is relatively new to South Africa, and in order to keep up with global trends in education it is necessary for research to be conducted here. With the introduction of MATLAB at the institution in which I lecture, I was attracted by the need to conduct a study relating to how students use this tool in the learning of mathematics and what informs the ways in which they use it. The skill of learning to use MATLAB is a valuable skill in its own right, but students' actions when using MATLAB in the mathematical learning process, and the reasons for studying it, are just as vital and should be illuminated.

This research aims to understand participants' learning of mathematics through their *motivations* and *actions* related to their "unique, individual circumstances" as well as "their lot" within the particular institutional context whilst using the MATLAB software. The issue of access to technology is bound to emerge in a study such as this due to the historical context of education in South Africa. Consequently my research can assist towards illuminating areas where the need for further investigation exists.

There are two ways to view knowledge: as created by authorities and isolated from students' personal feelings and lives; or socially negotiated and personally relevant (Johnston & Nicholls, 1995). The teaching of mathematics with CAS has been introduced by those in authority, but whether the individual student regards the knowledge as personally relevant may be revealed by their motives to learn mathematics (with CAS) and the goals that they formulate as the actions unfold in this activity. The importance of a learning task to the student provides the structure within which that student acts (Gordon, 1998).

The voices of students are not heard enough and this thesis provides a forum to listen to students enunciations. In this chapter I explain the context of the study, my rationale for researching this topic, and the research questions. In addition I provide an outline of the chapters in the thesis.

1.1 CONTEXT OF THE STUDY

The *empirical field* of this study is engineering studies in higher education. The empirical setting is a vocationally oriented Mathematics third semester (or second year) Engineering course at a South African university.

The institution is one of the two comprehensive universities in South Africa constituted as a result of mergers between universities and technikons[1] in 2005. Consequently, it offers a wide

[1] Technikons are in some parts of the world referred to as polytechnics.

range of qualifications, from diplomas to postgraduate degrees. In this country, the use of technology in learning tertiary mathematics is beginning to gain importance, but at the commencement of this research there was only one module in the university's department of mathematics that involved CAS in mathematical learning. It was therefore chosen for the study. At the time of completing the research there were a few modules consisting of CAS components in mathematical learning, albeit a slow increment over the previous few years.

Prior to the merger, the mathematics department in the ex-technikon, being part of the Faculty of Engineering, was mandated to introduce technology into the mathematics curriculum. MATLAB was selected as the software to learn mathematics because it was the belief amongst mathematics lecturers that MATLAB is used most commonly by engineers. They developed the MATLAB component and introduced MATLAB into the Mathematics 3 course of a few of the engineering diploma qualifications (Mathematics 3 being the name given to the third semester of Mathematics). In addition, the developers of the Mathematics 3 course made the decision to introduce MATLAB to only a few aspects of the syllabus. Consequently, MATLAB was used in just two sub-sections of the syllabus: to solve differential equations (DEs) numerically and to do basic linear algebra.

Students are taught how to programme MATLAB to find numerical solutions to DEs. It would have been interesting if CAS were used primarily as a tool to explore the learning of mathematical concepts, but this was secondary. Mindful of this, I set out to investigate how students approach and carry out the learning of mathematics using CAS and what motivations they have when they engage in this process.

1.2 RATIONALE FOR THIS STUDY

The scope of this study emerged from my experience as a past lecturer of the MATLAB (in maths) component, in which I noticed that students espoused various motives towards their study of MATLAB. For instance, a few believed that knowledge of the MATLAB tool would be useful in their future careers, therefore, might they be interested in learning to use a new tool as it adds value to their repertoire of skills? Others felt that finding solutions to DEs using MATLAB was just not related to their engineering course or to the wider world. Could it be that they studied this component because it was a requirement of the mathematics module? Various factors influence individuals to learn to use a new tool in mathematics, particularly when mathematics is a service subject to the Engineering diploma.

I observed that students solved DEs numerically by using a wide range of tools. For example, several would first begin writing out MATLAB code on paper before typing onto the MATLAB editor. Students appeared to struggle with the syntax of MATLAB statements and experienced difficulty interpreting error messages. Hence, I was curious to find out what actions they undertook to learn mathematics with MATLAB.

There are a range of experiences and actions regarding CAS, including its stimulation of the students' imagination and creativity, and helping them to view mathematics in ways that were not possible before. Also, students encounter technical obstacles with using CAS in learning mathematics, but there is a dearth of information on what motivates students to persevere with the use of CAS in learning mathematics. In particular, students who use CAS in mathematical learning within vocational[2] programmes at tertiary level have hardly been researched. This creates a gap in educational research at higher education level. Furthermore, in South Africa, much of the research around CAS in learning mathematics has been carried out amongst first-year university students studying towards degree programmes (see for example, Berger, 2006, 2010). There is a difference in the entry-level characteristics of vocational and degree students. For example, students entering vocational programmes typically have lower mathematics symbols than is the requirement for academic degrees in mathematics or engineering.

This thesis intends to fill the many gaps that exist in the field of the relationship between CAS and mathematical learning, particularly the following:

- there is a paucity of research on motivation and the learning of mathematics using CAS.
- the research takes place in a developing country as compared to the greater part of current research which is in developed countries
- this study draws on theories such as the use of socio-cultural frameworks in contrast to the usual exclusive focus on quantitative methods regarding motivation
- there is a lack of research into motivation and CAS in mathematical learning within the vocational stream as compared to academic degree programmes

[2] I remind the reader that the vocational programmes that I refer to are in South Africa called 'diplomas' and are career-focussed rather than academically focussed.

1.3 RESEARCH QUESTIONS

The aim of the research is to understand what happens when CAS is used by students as a tool for learning mathematics in this course. I am concerned with *how* students approach their learning of mathematics using this tool and how they go about actually solving DEs using MATLAB, in addition to *why* they engage in this activity. The wider context in which these students see themselves as CAS users may influence how they use CAS in mathematical learning. For instance, some students are motivated to explore mathematics using CAS, while others are troubled by the strict use of syntax and are put-off by its inflexibility. They then concentrate more on the CAS tool than the mathematics. How students use CAS in learning mathematics may be partly revealed by their engagement with the learning tasks.

In the component of solving DEs, students are taught to do basic programming to find the Euler, Runge-Kutta II and IV numerical solutions. My focus is on mathematical learning with CAS. I have collected data towards this focus from two major sources, namely, the problem-solving sessions with students solving DEs using MATLAB, and interviews in which students spoke more broadly about their actions and motives towards the use of CAS in mathematical learning. Accordingly, in this study I focus on the use of CAS in mathematical learning.

The research questions addressed by this study are:

1. What are the motives of students towards using CAS as a tool in mathematical learning?

2. How do students use CAS in mathematical learning (i.e. what are their actions and operations from an activity theory perspective?)

3. What relationships are evident between the first two questions and how might these be explained?

I use the activity theory framework to gain an understanding of how students construct mathematical knowledge as they learn to use CAS (in maths). Leontiev's three-level scheme of an activity – the *activity* level, the *action* level and the *operation* level - provide different levels for investigation which I exploit in my analysis of activities. In this thesis, I explore a myriad of activity theoretical notions such as, but not limited to: learning context, conditions of tool-use, tools that mediate learning, students' needs, motives, goals, object of activity, and their consciousness and how these relate to the activity of using CAS to learn mathematics. In addition, an important part of the study is that I examine the Western motivational notions of

achievement goal theory and expectancy-value theory and link these to the activity theory construct of motives.

I argue that motivations of students are shaped by the social realm. For instance, the ways in which individuals set motives may well be informed by their experiences of the subject content, the tool (computer), the software (MATLAB), the lecturer, the tutor, the peer group, the classroom culture, the institution and society. I use the expression taken from Gordon (1998) in referring to individuals as 'beings' in context and within society, which is important as it sets the tone for the choice of my theoretical framework and analytical lens. Throughout my study I consider the activity of students learning mathematics using CAS as involving the whole person, inseparable from their actions, as in Gordon's (1998) considerations when she used activity theory to explore students' learning statistics.

The relationship between motivations and actions (or operations) is important because students' motives towards the use of CAS in learning mathematics may influence how they engage in using it. This will relate to the quality of their knowledge in mathematics and ultimately on how they exploit the use of CAS in the workplace, although a study of the last part will not be undertaken in this thesis. From an activity theory perspective, motives are intertwined with the context. An individual forms motives and goals as he or she engages with the subject matter and tools. This approach of examining students' thinking and learning suggests that the student discovers the power of the tool whilst being an active participant in the activity. It may also be that the individual formulates motives and goals based on experience with similar subjects. In an attempt to gain an understanding of motives and actions, students' prior experiences with mathematics were also investigated. In activity theory these are termed 'historical aspects'.

The formulation of the research questions and the discussion of the theoretical framework provide the parameters within which the study has been conducted.

1.4 OUTLINE OF THESIS CHAPTERS

In this chapter I have introduced the reader to the study and highlighted the purpose of this study. I have briefly described the context of this study and introduced the research questions. In addition, I have identified certain gaps which research is required to fill. In Chapter 2 I present and discuss the literature around the use of CAS in mathematical learning and the theory of motivation as portrayed by Western motivational theorists. This discussion leads to the portrayal of the gaps in the literature that were identified in Section 1.2.

In Chapter 3 the current debates around activity theory as proposed by Leontiev (1978, 1981) are developed; this is the theoretical framework underpinning the study. I discuss the suitability of Leontiev's framework as compared to Engeström's (1987) for this study. A link is made between Western motivation theory as explicated in Chapter 2 and the activity theory conception of motives and needs as postulated by Leontiev. I develop and elaborate the activity theory model which provides the principal analytical lens for data analysis in this study.

In Chapter 4 I describe and explain the methodology and research techniques utilised during the course of the research. This study is situated within the qualitative research paradigm and consists of multiple case studies, where each case is an individual. I explain the methods of data collection and the methods of analysis. The former include observations of participants whilst they solved MATLAB problems, screen recordings using *Bulent Screen Recorder* and interviews. The latter is based on producing themes by using the three levels of analysis as depicted by Neuman (2003), namely, open-, axial- and selective- coding. Important aspects pertaining to the credibility of this research, such as issues of validity, reliability and ethics, are considered.

In chapter 5 I present detailed analyses of two case study students by drawing upon a range of interview data as well as data pertaining to the screen recordings of MATLAB which was captured whilst participants solved DEs in two problem-solving sessions. I present insights into the third participant but not in as much detail, with only the analytic findings relevant to the data of this participant being portrayed. The major themes that emerge from the analyses include: actions relating to optimising the environment; actions relating to personal power; actions that could lead to learning MATLAB (in maths) operationally; actions relating to comprehension of mathematics (with CAS); actions related to general computer use; actions related to the use of commands in MATLAB; actions related to overcoming difficulties associated with mathematics whilst using CAS to solve DEs; and actions related to processes followed when solving a DE.

The motives are placed in three groupings: social (interrelations with others), self-related and cognitive motives, as postulated by the activity theorist Lompscher (1999).

In Chapter 6, detailed discussions of how the research questions are answered, as well as a comparison of findings related to the three participants' data, are presented. The chapter links the empirical data to the theoretical components of activity theory.

Chapter 7 reflects on the study as a whole. The contribution of the thesis to the field of education, the limitations of this study, and recommendations for further research and for specific educational practices are made. The chapter closes with personal reflections.

The necessity for the research study has been established in this chapter. The next chapter will review the literature relevant to the study.

CHAPTER 2

LITERATURE SURVEY OF CAS IN MATHEMATICAL LEARNING AND REVIEW OF MOTIVATIONAL THEORY

2.1 INTRODUCTION TO THE CHAPTER

I have divided this chapter into two sections, namely *Section A: Literature survey of CAS in mathematical learning,* and *Section B: Review of motivational theory*. Each section has its own introduction, sub-sections and conclusion. My theoretical framework, activity theory, is discussed in Chapter 3.

SECTION A:

LITERATURE SURVEY OF CAS IN MATHEMATICAL LEARNING

2.2 INTRODUCTION TO SECTION A

Currently, computer algebra systems, spreadsheets, graphing software, dynamic geometry systems and various new tools are available for use in the learning of mathematics. Recent developments include e-learning and online learning, wherein students can collaborate with peers and teachers in individualised digital learning environments. CAS was developed as a tool for use by professional mathematicians, scientists and engineers. Its use in the mathematics classroom, primarily as a tool for learning mathematics, has stimulated much discussion and remains the subject of ongoing debate and research. Arguably, the student may benefit from the numerous ways of experiencing mathematics, for example, by moving quite easily between the different representations offered by CAS. However, the body of literature around the complexities in its use as a learning tool in mathematics is still growing and cannot be ignored. In this section I pay attention to these aspects.

I argue that the chief power of CAS lies in its offering the user three instantaneous registers, namely symbolic, numeric and graphic. However, the availability of multiple registers does not guarantee that improved learning will occur. As Heid and Blume (2008b, p.73) have pointed out, "what matters is the action taken on those representations as well as the reflection on that action". Seen from an activity theory perspective, the actions that students perform, for example, with respect to the output from the different registers of CAS, are vital for internalisation of knowledge. Moreover, reflection plays an essential role in learning, as explained in the context of Leontiev's (1978) action level in Chapter 3.

By affording students the simultaneous access to various representational registers, their preferences for one representation over another is revealed (Heid & Blume, 2008b), and could lead them to use multiple approaches to solve a problem. Yerushalmy (1991) contends that, based on her observations, there should be intentional teaching of ways to combine the visual and computational methods for both solving and crosschecking, so that students achieve the most from a multi-representational environment.

Previous arguments that technology would free students from procedures and cumbersome calculations, thereby enabling them to focus on developing conceptual understanding and concentrate on realistic applications, have become nuanced. The underlying implicit view was that technical skills and conceptual understanding could be separated in the learning (Drijvers & Trouche, 2008), but 'instrumental genesis' (see below, 2.3) has raised awareness of the relationship between the co-evolving technical and conceptual elements. Drijvers and Gravemeijer's (2005, p.164) assertion that "… this insight can be considered as the core of the instrumental approach to learning mathematics in a technological environment" is illustrated in studies by Kieran and her research group (Hitt & Kieran, 2009; Kieran et al., 2006; Kieran & Saldhana, 2008). Their research findings, amongst others, indicate the co-development of conceptual and technical knowledge in CAS activity, to be discussed below.

The research related to CAS in the learning of mathematics has been developing rapidly over the last decade, with the most recent developments involving descriptions of well thought-out tasks, for example, Berger (2010) and Hitt & Kieran (2009). It is envisaged that these may lead to more success stories in the use of CAS tools in mathematics classrooms.

In structuring Section A, I first describe the constructs related to instrumental genesis as postulated by French mathematics education researchers, and include a brief discussion on their earlier studies as well as more recent developments. I then focus on the research into calculus

and computers in developed countries and in a developing country like South Africa, particularly its effect on learning mathematics with CAS. The link between my study and the literature review completes this section of the survey.

2.3 INSTRUMENTAL GENESIS

The French school of research involving CAS in mathematical learning is primarily based on the instrumental approach, also known as 'cognitive ergonomics', influenced by the work of Verillon and Rabardel, and the anthropological approach influenced by Chevallard (Artigue, 2002). These approaches have been conjoined, even though the combination is viewed as problematic (Monaghan, 2005). Kieran et al. (2006) affirm that the instrumental approach to tool use include rudiments from both anthropological theory and cognitive ergonomics. The latter approach, as remodelled by Artigue (2002) and elaborated upon by Hitt and Kieran (2009), underscores three main imperatives from anthropological research:

task	a piece of work to be done
technique	a method for performing a task that may well include complex reasoning
theory	the discourse used to explain and justify techniques.

Within the instrumental approach the artefact is the material object used as a tool, for example, a calculator or computer. When there is a meaningful relationship between the artefact and the user for solving a certain type of task then the concept of an instrument is used (Drijvers & Gravemeijer, 2005). This process of the artefact becoming a functional mathematical instrument in the hands of the user is known as 'instrumental genesis' (Kieran et al., 2006), and involves the development of mental schemes, which in this context include the skills needed to use the artefact in a competent way, as well as the knowledge related to the circumstances under which it is useful: "In the case of a mathematical problem, a mental scheme involves the global solution strategy, the technical means that the artifact offers, and the mathematical concepts that underpin the strategy" (Drijvers & Trouche, 2008, p.369). Consequently, mental schemes play a vital role in organising the problem-solving process.

As defined by Artigue (2002), the term instrumental genesis means that the computer does not exist as a learning tool in itself but becomes an instrument through a process that involves the subject appropriating the computer or CAS for him/herself and integrating it with his/her mathematical activity. Instrumental genesis comprises two closely connected processes termed

'instrumentation' and 'instrumentalisation' (Artigue, 2002). The former is the process by which the computer or CAS shapes students' learning and developing conceptions, while the latter term refers to how they are able to exploit the potentialities of the computer so as to transform it for specific uses. In other words, students 'instrumentalise' a computer to become a mathematical tool and correspondingly 'instrument' mathematical actions through use of the tool (Ruthven, 2002). Artigue (2002) asserts that in order to comprehend instrumental genesis it is essential to study the constraints and enablement of the computer or CAS, while highlighting "the unexpected complexity of instrumental genesis, the mathematical needs of instrumented techniques and the problems arising from their connection with paper and pencil techniques" (Artigue, 2002, p.252). This will be addressed in more detail below.

The student develops mental schemes within a classroom community of learners and the influence of the teacher is seen as vital for the learning process involving CAS. Trouche (2004) introduced the term 'instrumental orchestration' to describe the actual organisation of the classroom environment by the teacher, and defined it as "the intentional and systematic organization of the various artifacts available in a computerized learning environment by the teacher for a given mathematical situation, in order to guide students' instrumental genesis" (Drijvers & Trouche, 2008, p.377). Such organisation includes projecting the screen of the teacher's graphical calculator or even a student's small calculator screen onto a larger screen. In this context, a 'sherpa-student' is identified as one who handles the overhead-projected calculator. Drijvers and Trouche (2008) claim that debates and discussions are encouraged when students' screens are displayed for all to view, an arrangement that can be usefully analysed as an activity system of Engeström (1987).

The instrumental approach has some components that originate from Vygotsky's ideas (Drijvers & Trouche, 2008), for instance, that mental processes and psychological aspects are important features in problem-solving tasks involving tool use. Indeed, from the exposition of instrumental genesis above, the notion of tool use is linked closely with the mental schemes that are formed as an individual engages in an activity involving artefacts and instruments. Instrumental genesis clearly distinguishes between an instrument and an artefact, the latter viewed by Drijvers and Trouche (2008, p.367) as "becoming part of a valuable and useful instrument that mediates the activity". Therefore, the artefact itself, even if it is a computer, is not a mediating tool, but rather the instrument takes on a mediating role, where it is described as comprising both the artefact and the associated mental schemes that the user develops for carrying out particular tasks (Drijvers & Trouche, 2008).

In line with the affirmation by Drijvers (2000) and Lagrange (1999) that the introduction of technology as a tool for learning mathematics may be problematic, Trouche (2004) demonstrates the case of a student using a graphic calculator as the only tool, with no other resources (paper and pencil, calculator, multi-register work of algebraic and graphical representations). While the student does not gain any understanding of the concept of limit another uses all available artefacts and is able to do so. Students may face obstacles that have both a technical and mathematical character whilst working in a computer algebra environment and they may experience problems with the distinctive output of CAS (Drijvers, 2000). Other authors, not working within the instrumental genesis framework, such as Pierce and Stacey (2004) and Berger (2006), have noted the problems that students may face, for example, with the precise notation of CAS or its characteristic output, and these may impede the effective use of CAS: "Technical facility is essential for a student eventually to be able to focus on the mathematics rather than the machine" (Pierce & Stacey, 2004, p.66).

Having illustrated possible constraints of using a graphic calculator with respect to the graphing facility, Guin and Trouche (1999) also carried out an analysis of observations of students' behaviour using graphic and symbolic calculators, which they categorised into five profiles:

- *random work method*: characterised by students' difficulties in the graphic calculator setting similar to those in the traditional paper and pencil situation. The techniques used were mainly 'cut and paste' (from previously memorised solutions) as well as trial and error procedures.

- *mechanical work method*: characterised by information sources restricted to the graphic calculator investigations and straightforward manipulations. Students in this grouping based their reasoning on the accumulation of machine results and avoided any mathematical references.

- *resourceful work method*: characterised by the use of all available resources (graphic calculator, paper and pencil, and theoretical references). Students in this category made use of a variety of creative strategies in producing their solutions.

- *rational work method*: characterised by a decreased use of the graphic calculator and primarily working within the traditional paper and pencil setting. In this category, the emphasis was on the role played by inferences in reasoning.

- *theoretical work method*: characterised by drawing upon mathematical theory as a main resource. Students in this category made average use of verifying procedures of graphic calculator output.

The findings of Guin and Trouche (1999) reveal that transformations of the calculator into an efficient mathematical instrument vary from one student to another, and that they develop different relationships with their graphic calculators based on the behaviour profile mentioned above. The authors called for a full recognition of the constraints and potentialities of the artefact, as well as the various profiles of student behaviour when designing appropriate mathematical activities.

I have discussed the above research in detail because it is one of the early studies to incorporate personal characteristics of the user into the instrumental genesis framework. From an activity theory perspective it would be valuable to see how these characterisations of profiles could be linked with students' affective dimensions and in particular their motives and goals. I anticipate possible links between the behaviour profiles of students and their motivations towards the use of CAS calculators. Perhaps the students falling under the *resourceful work method* profile may espouse intrinsic motivation towards CAS use.

Having provided evidence of students learning mathematics in different and distinctive ways using technological means, the structure of mathematical learning tasks and the articulation of paper and pencil techniques with CAS techniques is the essence of Kieran et al.'s (2006) research. They demonstrated how techniques resulted in theoretical thinking and vice versa within the context of two carefully planned tasks created within the task-technique-theory triad of the instrumental approach. Two Grade 10 teaching experiments were described, one dealing with equivalence, equality and equations, the other involving generalising and proving within the area of factorising. Major findings indicate the importance of the co-emergence of technique and theory in both of these diverse sets of tasks.

Hitt and Kieran (2009) considered a peer interaction between a pair of Grade 10 students so as to gain a deeper insight into the methods of articulating paper and pencil techniques and CAS techniques in producing conceptual knowledge. From a theoretical perspective they examined the relationship between the task-technique-theory triad and how students constructed articulations between theories and techniques whilst engaging with the task. The tasks were formulated in such a way that students could first expand on their prior knowledge of factoring

and eventually construct more intricate mathematical knowledge with the support of CAS. The activity focussed on the factoring of the x^n *-1* family of polynomials (*n* being a positive integer).

In the *first sequence* of the task, students had to remember and recall the factorisations of various expressions involving differences between squares and cubes. This was written in a paper and pencil environment first and then verified using CAS. Some examples include the factorisation of a^2-b^2; a^3-b^3; x^2-1.

In the *second sequence* they were given various small tasks based on developing a theory around the telescoping expression $(x-1)(x^{n-1}+x^{n-2}+...+x+1)$ being equivalent to (x^n-1) for specific integral values of *n*. The first part of this sequence was to promote the development of technical knowledge in a paper and pencil environment by expanding $(x-1)(x+1)$ and $(x-1)(x^2+x+1)$, then predicting the product of $(x-1)(x^3+x^2+x+1)$, the latter being carried out without any algebraic manipulation. Students were then asked to verify this last product, first using paper and pencil and then CAS. They had to predict the factors for (x^5-1) and explain why $(x-1)(x^{15}+x^{14}+x^{13}+...+x^2+x+1)$ gives the result of $(x^{16}-1)$. Finally, they were asked if their explanations would also be valid for $(x-1)(x^{134}+...+x+1)=x^{135}-1$. In this way they were expected to "analyse relationships, notice structure, and generalise so as to predict the factorisation of expressions like $(x^{135}-1)$" (Kieran et al., 2009, p.132). A theory was formed around $(x-1)$ and $(x^{n-1}+x^{n-2}+...+x+1)$ being factors of x^n-1 for given integer values of *n*.

In the *third sequence* students had to confront the technique just learned, that is $(x-1)(x^{n-1}+x^{n-2}+...+x+1)=x^n-1$. Through a process of finding solutions to tasks they were introduced to the different representations obtained from using paper and pencil and CAS output. They were asked to factorise x^2-1; x^3-1; x^4-1; x^5-1; x^6-1 using paper and pencil as well as CAS, and to reconcile (where applicable) the two answers by performing calculations. The tasks instilled coordination between the techniques of CAS and paper and pencil.

The *fourth sequence* involved students conjecturing different values of *n,* the factors of (x^n-1) containing exactly two factors, more than two factors and $(x+1)$ as a factor. Here it was

expected that they would come up with a false conjecture, namely $(x^n - 1)$ contains exactly two factors $(x^{n-1} + x^{n-2} + ... + x + 1)$ and $(x - 1)$ for odd values of *n*. In anticipation of this, tasks were assigned to produce an immediate confrontation to this conjecture, for example, students were asked to factorise $(x^9 - 1)$ using paper and pencil techniques then CAS techniques, and finally to perform calculations that reconciled the two. This example containing more than two factors would provoke a rejection of their previous conjecture and invoke a state of cognitive conflict.

The *fifth sequence* involved a conceptual change and students would reject their prior conjectures and generate new ones. If they could postulate a new conjecture that $(x^n - 1)$ contains exactly two factors for *n* being prime numbers then this is considered as students constructing the related theory.

The *sixth* and *seventh sequences* were to distinguish among theories and justify their conjectures as well as use argumentation to deepen the theories produced.

Findings (based on the pair of students) indicate that the use of CAS enabled them to correct initial errors that they had made whilst working solely in a paper and pencil environment. They constructed a theory through the telescoping technique (see second sequence above) and reconciled equivalent expressions produced in a paper and pencil environment with CAS output. In this way they could articulate amongst representations and techniques and generate conjectures. Students faced a cognitive conflict in the fourth sequence which they tried to resolve. CAS allowed them to test extreme cases and eventually to theorise results: "In the construction of conjectures, and in their verification, the role of the calculator was crucial and permitted the students to construct an articulation among theories (rejecting old ones and generating new ones)" (Hitt & Kieran, 2009, p.149). They could construct theories that would not have been possible had they worked exclusively with paper and pencil. From an activity theory perspective, the use of technology shaped the activity through the immediacy of the feedback generated by the calculator.

The research by Hitt and Kieran (2009) draws attention to the importance of designing mathematical CAS-based tasks within the French task-technique-theory theoretical framework. In addition to the structure of the learning tasks, the setting in the social context of the classroom and working in pairs may have contributed to the students' successful creation of mathematical theory using CAS. From an activity theory perspective, knowledge is constructed

within a social and historical context and this construction incorporates multiple resources. Particularly, the research by Hitt and Kieran (2009) exploits peer interaction and discourse, integrating CAS and paper and pencil resources as well as drawing on students' historical or prior knowledge. (The relevance of instrumental genesis to my study is discussed in 2.6.2).

2.4 CALCULUS AND COMPUTERS

In the use of technology in tertiary mathematics, calculus has received considerable attention. Early studies, such as the experimental one carried out by Heid (1988), created optimism that CAS would be used in learning mathematics, her study having been on a teacher imposing a different practice on students. Heid (1988) was one of the first proponents of using CAS in mathematical learning, documenting its benefits as a tool for conceptual development of the derivative. The research was based on a comparative study between a traditionally taught calculus course and an experimental group. With the experimental group, Heid (1988) redesigned the course in such a way that the teaching of concepts preceded the teaching of skills in the sequence of the course as well as in priority. Working with first year university students studying calculus, Heid (1988) asked them to experiment with CAS to investigate the concept of the derivative, by means of using graphs. The routine algorithms for differentiation (skills) were taught only in the last three weeks of the course, after they had used software to handle routine calculations whilst focusing their attention and efforts on concepts. A major difference between the traditional and the experimental group of the calculus course was on knowledge of concepts, with results suggesting that students in the experimental group showed a better conceptual insight into the derivative and performed almost as well on a final exam of routine skills as a class of 100 who had practiced the skills for the entire 15 weeks. This is one of the ways that the computer can potentially be used at university level.

Tall, Smith and Piez (2008) have given in-depth summaries and syntheses of how learning calculus with technology has evolved, starting with the first microcomputer, with inbuilt *BASIC,* that allowed students to programme algorithms to find, for example, limits, rates of change and solutions of DEs. They report on studies involving some positive effects on conceptual understanding and problem-solving of this development, however, as more software environments in mathematics emerged, the use of programming specifically in the context of mathematics instruction appeared to decrease (Johnson, 2000). An exception was the work of Dubinsky (2010), who formulated a theory of cognitive development called Action-Process-Object-Schema (APOS), based on Piaget's theory of reflective abstraction and making use of some mathematical programming. The programming results in students generating examples

and constructing mathematical concepts, and the activities appear to facilitate reflection on those examples.

> We see the computer already proving a powerful tool in advanced mathematical thinking, both in mathematical research and in mathematics education at higher levels. The empirical evidence shows that it proves more successful in the educational process when it is used to enhance meaning, either through programming in a language embodying the mathematical processes or through the use of computer environments for exploration and construction of concepts. (Dubinsky & Tall, 1991, p.243)

These researchers were already arguing that the use of computers might enable or promote advanced mathematical thinking, while more recent research has indicated that low-ability students may benefit conceptually. For instance, Heid and Blume (2008b) describe how a well designed software environment can provide scaffolding which helps to structure mathematical problem-solving by freeing the student from some process tasks and simplifying the activity, resulting in conceptual understanding becoming easier for the low-ability students. Dunham and Hennessy (2008) point to a few studies within the last decade that have reported on how low-ability students make greater gains in mathematical achievement than middle- or high-achieving ones when taught and tested using graphing calculators.

The notion of conceptual versus procedural knowledge in calculus learning with technology is given prominence in the review by Tall et al. (2008), which is not surprising since most calculus courses are designed around developing algorithmic skills that involve finding derivatives, integrals and limits:

> The intent of changes in curriculum and inclusion of technology was to enable students to focus on development of conceptual knowledge through exploration of concepts via various representations and multiple examples, and by off-loading of procedural work. (Tall et al., 2008, p.230)

Some major conclusions of Tall et al.'s (2008, p.249) syntheses related to calculus and technology include:

- The development of the understanding of concepts in calculus can take place prior to the development of procedures (or skills) by using technology and an appropriate pedagogical approach.

- Results from quantitative comparison studies on questions examining *procedural* skills showed that, in general, students in the experimental studies (that is, with the use of technology) performed just as well as those in traditional settings. Note that these results are irrespective of the wide variety of approaches to teaching and learning using technology.

- Still within the quantitative comparison studies, results[3] that examined conceptual understandings were mixed. Students in the experimental sections performed either similarly to those in the control group (in four of the studies) or significantly better (in two of the studies). It is noteworthy that there was limited implementation of technology in those studies that reported no significant difference. However, in one of the studies that resulted in no significant difference between experimental and control group using test items, the interviews conducted by researchers revealed that students in the experimental sections showed better conceptual understanding in the sections of asymptotes, concavity, limits of functions, continuity and increasing/decreasing functions. These results suggest that using only test items to draw conclusions about students' conceptual understanding is problematic.

- Studies that used qualitative research methods (such as interviews) were more insightful with regards to students' understanding of mathematics in technology-enriched curricula. Results indicate that they developed better connections related to major concepts such as derivative and integral, were able to explain ideas using their own language and were in a position to solve problems drawing on their conceptual knowledge. They could also use procedural methods.

In Tall et al.'s (2008) syntheses there were very few studies that reported specifically on solving differential equations. An exception is a study that made use of *Maple* (in a calculus context) in two classes that studied differential equations over a seven-week period (Aldis, Sidhu & Joiner, 1999 cited in Tall et al., 2008), where it was found that both classes experienced difficulty with *Maple* syntax in the time available. The authors propose that a longer exposure to *Maple* may reduce the problems regarding syntax.

[3] Note that the instruments used to gather data in all six studies consisted of tests with a subset of questions that assessed conceptual understanding.

Tall et al.'s (2008) comprehensive summary of over a decade's research into using technology in the learning of calculus has provided various valuable insights into the cultural contexts of calculus and its changing nature in a technological society, as well as into a wide variety of approaches (wherein technology is used to enact visualisation, conceptual understanding and the practical use of technology involving symbolic, numeric and graphic representations), the roles of the teacher and learner and analysing empirical research. However, absent from most of these syntheses is the vital aspects of affect, students' motives and the goals that they formulate as they go about their learning of mathematics with technology (see 2.5. below).

Cretchley, Harman, Ellerton and Fogarty (2000) found that early difficulties with syntax were easily overcome, with more evidence than counter-evidence of increased interest and enjoyment in students' experiences of CAS. Meanwhile, Berger (2006, 2010), in a well-known university in South Africa, studied the affordances regarding the technical, mathematical (in the sense of CAS mediating learning) and affective use of CAS in mathematical learning in a Mathematics 1 major course. The course largely consisted of calculus and some early linear algebra components, with students expected to learn and use *Mathematica* as a tool for supporting or enriching their knowledge of various concepts in mathematics. During fortnightly tutorial sessions they used worksheets comprising technology-oriented exercises related to sections covered in class, as well as anticipated topics that would have been covered in upcoming lectures.

In a study of two assignments on Maclaurin polynomials completed by 108 groups (comprising three or four students), Berger (2006) found that at least 90% of students were able to use the software as a tool to facilitate algorithmic tasks but only 56% were able to exploit it as a tool for conceptual insight. It is promising to note that students may well be in a position to use the tool effectively for computations or to carry out tasks of a repetitive nature. However, it is of concern that a relatively large percentage were "unable to exploit the cognitive reorganising affordances at all" (Berger, 2006, p.16). These studies bring to the fore much needed results around tertiary use of CAS (in maths) in South Africa and indicate that more research is needed around using varied CAS tasks in mediating mathematical learning. Using data gleaned from 108 questionnaires administered on the technical affordances of CAS, Berger (2006) concluded that many students experienced technical difficulties using CAS, including problems with syntax and interpreting output. The inclusion of CAS in mathematical learning is thus problematic and some of the difficulties reported here resonate with those experienced by students in the developed world, see Drijvers (2000).

Current research by Berger (2010) in South Africa around the use of CAS with first-year mathematics university (degree) students, involves qualitative analyses of solutions to mathematics tasks (on the Maclaurin polynomials) they attempted, with semiotics used as a theoretical framework. One finding was that 61% (i.e. 123 pairs) of the students were able to locate the points of intersection of two graphs but that 39% (80 pairs) took these points to be endpoints of the required interval. This shows the need for widespread research into the distinct challenges of interpreting CAS outputs with respect to the requirements of particular tasks. Another important finding related to non-computer literate (NCL) students being able to construct CAS-based signs in their solutions to the allocated task but struggling much more than computer-literate (CL) students with the interpretation of CAS output. In addition, Berger (2010) argues that these differences in performance between NCL and CL students could be attributed to the larger context of the inequities in the schooling system (a legacy of apartheid) rather than to reasons directed primarily at the lack of computer skills per se: "I have supported this argument with evidence that NCL students perform worse on paper-and-pencil tests (requiring conceptual/ interpretative and procedural thinking) as compared to CL students" (Berger, 2010, p.12).

2.5 AFFECT IN LEARNING MATHEMATICS WITH CAS

In this sub-section I review literature related to affect (and mathematics with computers) as it relates directly to my first research question. I also consider issues of equity and access, which although not the focal point of my research do overlap with the affective domain, particularly attitudes, emotions and confidence. Research in the category of affect when using a computer to learn mathematics, includes motivations, attitudes, confidence, interest and beliefs. Whilst Cretchley and Galbraith (2002), Galbraith and Haines (1998), Pierce and Stacey (2004) and Machin and Rivero (2002), amongst others, have researched students' attitudes towards mathematics and computers when using CAS in the learning of calculus, many of these investigations took place in the developed world, where computers are ubiquitous.

Locally, I draw on the research carried out by Berger and Cretchley (2005) into students' attitudes towards CAS in learning calculus amongst first-year university students. In a pilot study, 57 randomly selected students from the Mathematics 1 major course were exposed to the use of *Mathematica* during tutorial sessions, with the sample attending the same lectures and writing the same tests or exams as the others. Their findings reveal that the attitudes of the pilot group held similar (positive) attitudes to those found in an Australian large group study, measured using similar instruments. Those students who felt they were not computer literate at

the beginning of the course did not have significantly lower attitudes than those who felt they were. The majority of students who did not have prior exposure to computers on entering university or did not have access to them at home in South Africa did not feel threatened by the inclusion of CAS in learning mathematics: "Indeed, it seems that these less-resourced students are more appreciative of the opportunities afforded by the technology than other students" (Berger & Cretchley, 2005, p.104), which is reassuring for users of CAS in developing countries.

Berger (2006), in a follow-up study with a larger cohort of Mathematics 1 major students (108), found their attitudes were similar to those of the pilot project group mentioned above and to those found in the Australian large group study, that is, generally positive. The implications are that South African mathematics students studying at tertiary level may be able to take full advantage of technological resources in spite of not having been exposed to such resources at school level (Berger & Cretchley, 2005). These findings are encouraging and I propose that a useful follow-up study to this survey approach would be to undertake in-depth qualitative research into attitudes and computer use in mathematical learning in South Africa.

When one introduces a new tool into the mathematical learning process, apart from investigating how students feel towards using it, other relationships of affect, for example, attitudes to mathematics and to computers, may be regarded as an important part of the investigation. Galbraith and Haines (1998) have found that computer attitudes are more influential than attitudes to mathematics in influencing students' engagement with computer-related activities in mathematical learning. They identified six relevant factors: mathematics confidence, computer confidence, mathematics motivation, computer motivation, mathematics engagement, and computer-mathematics interaction, which as scales have been used widely and surprisingly reveal low correlations between attitudes to mathematics and attitudes to computers, regarding both confidence and motivation (Cretchley & Galbraith, 2002). Vale and Leder's (2004) research amongst high school students in Australia also found that attitudes to computers in mathematical learning were more strongly correlated to attitudes to computers than to mathematics. Interestingly, this relationship was stronger amongst boys than amongst girls.

Pierce and Stacey (2004) developed a framework that included the technical competencies and personal attributes (affect) as key aspects for the effective use of CAS in mathematical learning. Their research findings highlight the importance of both these aspects but suggest that "negative attitudes rather than technical difficulties can limit the effective use of CAS" (Pierce & Stacey,

2004, p.59). In more recent research, Pierce, Stacey and Barkatsas (2007) described the development of a new and simple scale called *Mathematics and Technology Attitudes Scale* (MTAS) for use by middle secondary school learners. This scale comprised five subscales, namely, affective engagement (AE), behavioural engagement (BE), confidence with technology (TC), mathematics confidence (MC) and attitude to learning mathematics with technology (MT). From a sample of 350 Grades 8 to 10 learners in six schools, findings include boys having notably higher scores than did girls on four scales excluding BE. The MT scale correlated positively with TC for boys and negatively with MC for girls. Generally, most learners in each school concurred that learning mathematics with technology was a better option.

Forgasz, Griffith and Tan (2006) carried out research on gender equity, beliefs and attitudes amongst teachers and students in secondary mathematics classrooms in Victoria, Australia. They collected data on teachers' and students' views on and perceptions of whether computers supported students' comprehension of mathematics. Approximately 60% of teachers believed that computers assisted students' mathematical understanding, with males being more positive than females. However, the students were not as convinced as teachers, with about 30% believing that computers assisted their understanding of mathematics, and again a greater percentage of male than female students supporting its effects. Their findings suggest a 'digital divide' exists between teachers' and students' beliefs of the use of technology in mathematical learning (Forgasz et al., 2006; Forgasz, Vale & Ursini, 2010).

Reflecting on a project, 'Teaching Mathematics with Technology' (EMAT), carried out in classrooms in Mexico, Ursini and Sacristán (2006, p.481) report that the use of computational tools impacted positively on children's attitudes towards mathematics:

> a study developed with 24 teachers and 1113 students (Ursini et al., 2004) [4] shows that there is a clear increase in their enthusiasm and motivation; and although the impact is different for girls and for boys, the behavioral changes observed seem to lead to more gender equity.

Many studies, some of which are mentioned above, suggest that there are some benefits to using CAS in learning mathematics. Ruthven and Hennessy (2002) elicited from teachers a model of the successful use of computers to support classroom teaching and learning in mathematics. In

[4] Unfortunately their research mentioned here is not in English, so the variables used to measure motivation are unknown to me, pending translation.

their studies, two themes amongst others emerged, namely ‘motivation improved’ (in generating student enjoyment and interest and building student confidence) and ‘engagement intensified’ (in securing the commitment and persistence of students in classroom activity).

Dunham and Hennessy (2008) carried out syntheses around issues related to equity and educational technology in mathematics, which include some reports on studies that involve attitudes, emotions and confidence with the use of educational technology in mathematics. I discuss some pertinent aspects of their syntheses below:

1. They report that affective factors, particularly attitude, motivation, and confidence level, influence participation as well as interaction with technology at school and home. However, they do not provide much detail of the studies in this regard.

2. Students enter university with different prior experiences and exposure to computers, which have an influence on their attitudes toward computer use and anxiety levels. The authors report on studies that show access to computers in the home improves students’ attitudes toward technology, particularly levels of anxiety and perceived usefulness. Even though computer experience is related to these, they state that “it is uncertain whether frequent use promotes a positive attitude or vice-versa” (p.383).

3. Differentials in experience could prevent some groups, such as those of lower socio-economic status (SES), and/or inner-city, rural, female students from benefiting from the full potential of educational technology.

4. The inequities in physical access at school are made worse by access in the home, which is mainly influenced by social factors, including family computer cultures (e.g. boys being more inclined than girls to having access to computers in the home) and parental support. The authors report on some studies in which parents’ encouragement were instrumental in motivating girls to use computers, both generally and at school.

5. The gender effect is evident with regard to attitudes toward computing technology but not towards portable computers and calculators, which appear to be suitable to girls. Several studies show that girls become more positive about using technology with experience and time but their confidence increases only slightly and remains less than that of boys.

6. Generally, inequities in the use of computers in mathematical learning can result from wide-ranging factors, such as the amount of use and access to technology; the type of use in the classroom (for example, programming versus drill, where drill and practice tend to be confined

to low-achieving and low-ability groups); the nature and quality of student interaction with computers; and the nature of group dynamics in the presence of technology.

7. "Research consistently demonstrates a relationship between the combination of computer experience (and competence) and attitudes toward technology (including anxiety) and a circular feedback effect of computer-related attitudes (especially enjoyment) upon performance." (p.390). Seen from an activity theory perspective, the imperatives of historical aspects (regarding prior experience) as well as affective aspects (mainly attitudes) are important aspects of research related to technology and learning.

In the above exposition I have examined many studies that have reported some positive results with respect to attitudes, increased enthusiasm and interest, motivation, class participation and group work. In the next section I discuss some of these findings and show how my study attempts to fill gaps in the literature.

2.6 MY RESEARCH AND THE CURRENT LITERATURE

In this section I show how my research relates to the current corpus of literature and why it should contribute to new knowledge in the field of mathematics education at tertiary level.

2.6.1 AFFECT AND MY STUDY

At this juncture, it is necessary to provide a succinct definition of the construct motive that I use in this study: I define motives to be the energising, personal reasons that prompt an individual to participate in a specific activity, in other words that have a directing function of the activity. The construct of motive is detailed in Chapters 3.5.7 and 3.6.

There is a paucity of research on students' motivations towards using CAS in mathematical learning. While Galbraith and Haines (1998) developed scales for quantitatively measuring students' computer-mathematics interaction and their relationship to mathematics motivation and computer motivation, they did not directly address motivation in mathematics mediated by CAS.

In the previous sub-section (2.5) I reviewed various studies that demonstrate the importance of research on affect in relation to CAS and mathematical learning. It is mostly accepted that "cognitive as well as affective factors – such as attitudes, beliefs, feelings and moods – must be explored if our understanding of the nature of mathematics learning is to be enhanced" (Leder & Forgasz, 2002, p.95). Research into the motives that students espouse about CAS in

mathematical learning is much needed to help educators understand what sustains their educational progress. Students could easily dismiss mathematics as difficult, irrelevant or boring, so: "We need engaging environments, in which the mathematics is actually *needed* for students to achieve goals that *they* find compelling, and *made visible* to students and *expressed in a language* with which they can connect" (Confrey et al., 2010, p.20, emphasis in original text). From an activity theory perspective it is these very motives and goals that are being investigated and which form the focal point of this study when students use CAS to learn mathematics.

Although research into some motivational factors with respect to CAS (in maths) is being carried out in the developed world, for example, Galbraith and Haines (1998), Forgasz et al. (2006) and Cretchley (2006, personal communication), these results may not directly translate to developing economies. For example, the great differences within the university sample with respect to access to and experience with computers between the privileged and less privileged students may result in different motivations that students have towards learning mathematics with computers. In addition, unlike the USA or Australia, where technology is used in the learning of mathematics at school level, technology in South Africa is only used with the privileged at school level.

In Australia, Stacey and her team lead the RITEMATHS project, the themes of which involve the use of real problems (R) and information technology (IT) to enhance (E) students' engagement with and achievement in mathematics (RITEMATHS, n.d). One of various research studies carried out within this project was by Pierce et al. (2007), on developing an attitudes scale. From Pierce and Stacey's (2006, p.214) research into secondary school teachers' reasons for using real world contexts in mathematical learning, it was found that teachers placed great importance on creating enjoyable and impressive lessons by choosing tasks situated in real world problems, so as "to promote students' affective engagement through simple pleasures". The teachers in the study were unanimous in stating that their choice of tasks was determined by their perceptions of whether they would "lead to increased student interest, engagement and improved attitude towards mathematics" (ibid., p.217). There may thus be a danger in teachers choosing tasks that appeal only to the sensory and affective dimension while not giving attention to the cognitive aspects of the tasks, and a possibility that the development of other vital mastery or learning goals might be ignored in this process (ibid.). A follow up to this research would be to investigate students' motivational aspects to determine reasons and motives for engagement in such tasks from their perspective.

Doerr and Pratt (2008) are amongst several researchers focusing on motivation, DEs and technology, but these are in the context of students modelling equations to 'real life' problems. In my study, students are given 'well-defined' engineering-related problems that simplify the complexity of the real situation. In doing so, important 'real' variables are excluded, which might send conflicting messages to students about whether unintended contextual factors should be disregarded.

Students' access to technology, both at school and at home, has increased significantly over the past few years, particularly in the USA, Australia and the UK (Dunham & Hennessy, 2008), however the situation in developing countries is still relatively poor. For example, the availability of computers in South African schools is 39.2%, with only 26.5% for teaching and learning (Department of Education, 2004, p.12). Of further concern, as Julie et al. (2010) write, is that the use of computers in mathematical learning at school is often aimed merely at improving achievement results and consequently there is a focus on drill-and-practice programmes. On the other hand, there seemed to be an early trend identified at a particular university in South Africa where students accessing computers at home was 71% in 2005, compared to 64% in 2004. In addition, 88% of students in 2005 claimed to be computer literate compared to 80% in 2004 (Berger, 2006). Although the latter statistics are encouraging, they indicate that more research is needed to assess whether the equity divide in accessing computers at home is narrowing for tertiary students across the country[5].

Thus far, the research on affect and computers in mathematical learning in South Africa has been mainly quantitative in nature, for example, Berger and Cretchley (2005) and Berger (2006). Elsewhere there are very few studies that involve analysing qualitative data, for example, Cretchley (2006, personal communication) collected data from students using open-ended questionnaires but, once more, the research lacked the rich theoretical frameworks necessary to understand the intricacies of learning. My study differs in its approach by using qualitative methods within an activity theory perspective.

2.6.2 THEORETICAL FRAMEWORKS AND MY STUDY

The use of socio-cultural frameworks in the context of affect and computers in mathematics learning is sparse. An example, in which socio-cultural frameworks, particularly activity theory

[5] This research involved the cohort of students studying Mathematics 1 major at this university (with sample size being approximately 230) and so was not representative of the larger tertiary sector or even that university.

as elaborated by Leontiev (1981, 1978), have been used, lies within statistics education. In Australia, Gordon (1998) used activity theory to explore the relationships between the goals of undergraduate students studying statistics for Psychology degrees, their conceptions of statistics, their approaches to learning statistics and the outcomes of their actions (performance in tests). A decade later, Stevenson (2008) produced a model of pedagogy which he developed from the activity theory framework and used to describe digital technologies as tool, tutor, environment or resource in pedagogical contexts. He also explored the metaphors of these descriptions, with the model taking into account relationships between teachers and learners that develop over time, motivated by the dynamics of the activity (using Leontiev's {1978} conceptions of activity, actions and operations) and the relationships that they create across time (using Engestom's {1987} conception of activity systems). He first produced an activity pentagon, the main use of which was to model the actions connected with an activity, with each of the five labels showing a particular facet of an action:

- *"Roles* that shape the relationships between teachers and learners…
- *Organisation* identifying how teachers and learners are grouped and managed…
- *Use* dealing with how a range of artefacts, including digital technologies, chalkboards, and materials designed for learning, are actually employed in practice…
- *Functionality* identifying the range of possible uses for artefacts as specified by their designers…
- *Operations* that represent the space of possibilities for an action provided by the technologies and the site being used…" (Stevenson, 2008, p. 840)

This activity pentagon model formed the slices of the main 'across time' model. For Stevenson (ibid., p.837), "learning takes place through the interplay of context, purpose, action sequences, and personal relationships". The model developed appears to concentrate on purposes as defined by curriculum statements and ICT development that are intended by the activity: "Activity theory is, after all, a description of a system built around mediated behaviour, which does not deal with people as individuals" (ibid., p.851). In Stevenson's model, attention is not given to learners' personal motives in an activity motivated by different objectives, such as students learning to get high marks or to please their teacher. Indeed, Stevenson's research was based on an activity system within which learning takes place. In contrast, the focus of my study is on individuals learning within a system, specifically on students' motives and goals for using CAS

in learning mathematics and with the individual in context. My ideas around activity theory are developed in the next chapter.

Recent research around the use of activity theory as a framework when researching CAS in first-year mathematics courses was conducted in Australia by Coupland (2004), using a combination of Leontiev's (1981, 1978) and Engeström's (1987) versions of activity theory. Amongst other areas of investigation, Coupland (2004) interrogated the introduction of CAS in a first-year university mathematics course, specifically examining how students in a specific environment responded to their initial experience with CAS and the relationships between their individual histories, their goals for learning mathematics and the variety of experiences they reported that related to using CAS for the first time. Mixed methods of qualitative and quantitative methods were used to garner data relevant to a case study group of around 100 students and two academics. The main findings of the study included:

> … the identification of the critical nature of *purpose*, or multiple motivating 'objects' in activity systems. Personal identity as a learner of mathematics is constructed through choosing to engage at surface or deep levels, alone or with others. Students with a low level of computing background who had a high level of engagement and sense of purpose in their mathematical learning reported that they appropriated the new tool for their own personal use. Students with a high level of computing experience who were unable to form goals congruent with the learning tasks were less likely to appropriate the tool. (Coupland, 2004, p.xi, emphasis in original text)

Whilst Coupland (2004) considered students' multiple motivating objects in activity systems, I distinguish between their multiple motives and the object of the activity (see Chapter 3 for a detailed discussion of these aspects). Coupland's (2004) study is significant in highlighting one of the ways in which the activity theory framework is used to gain an understanding of various experiences related to students learning mathematics with CAS. Moreover, she showed how activity theory offers a methodical way of describing and analysing CAS based interactions (in mathematics) within a cultural and social setting.

Other studies involving the use of activity theory and computers (not necessarily in mathematics learning) include Nardi (1996b), who presented various authors' views on how activity theory is used in a range of ways to create, model and appraise contexts that employ different digital technologies. To a large extent, they draw on Engeström's (1987) version of activity theory as opposed to Leontiev's (1978, 1981) earlier versions. In more recent work, Roth and Lee (2004)

have shown how activity theory can be used in science education, specifically the interpretation of graphs by scientists. I do not directly address the construct of instrumental genesis, which although a powerful framework is not directly related to my research questions. Rather, my study is concerned with the motivations or motives as postulated by Leontiev (1978, 1981) from the perspective of activity theory and goals related to actions, again defined using Leontiev's framework, that are espoused by students when they use CAS to learn mathematics. As mentioned above (see 2.3), for the most part instrumental genesis is concerned with the process of the artefact becoming a functional mathematical instrument in the hands of the user, and this involves the development of mental schemes.[6]

A traditional constructivist framework has been used to analyse the use of a tool in mathematical learning with focus more on cognitive aspects. The work carried out by Dubinsky and Tall (1991) (see 2.4) has been from the post-Piagetian standpoint, however the presence of computers in a laboratory and their dominating presence compared to the handheld graphic calculators inevitably encourage students to work together during the learning process, and may even favour the learning styles of some students using CAS. This lends itself to an interpretation involving the 'zone of proximal development (ZPD)', which with Vygotsky and the suitability of socio-cultural frameworks to my study is discussed further in Chapter 3. At this stage it is pertinent to note that socio-cultural frameworks are well-suited to being used as analytical lenses in learning involving computers, because, inter alia, tools are one of the most important features of these frameworks, and help in understanding learning as mediated by tools.

Despite a wealth of literature on motivation theory and the learning of mathematics, very few studies (reviewed above) have been conducted on motivations and CAS (in maths). There is also a considerable amount of literature on affect and technology (reviewed in 2.6.1), but it is beyond the scope of this thesis to provide a review of literature on motivation and mathematics as this in itself would constitute a thesis. I look briefly at some of the literature on general motivational theory (in Section B, 2.10) so as to make explicit the relationships between constructs used in achievement goal theory and intrinsic interest, achievement and approaches to learning.

[6] I suggest that for a future research project instrumental genesis could prove to be a useful framework if the research questions are modified slightly.

2.6.3 OTHER GAPS IN THE FIELD OF USING CAS (IN MATHEMATICS)

Berger's (2010) research, based on the theory of semiotics, produced crucial results related to how *degree* students use CAS in South Africa. I anticipate that some of these findings around the problems experienced by *degree* students may well resonate with problems experienced by *diploma* students when they use CAS in mathematics learning.

The research on affect and CAS in mathematics learning at tertiary level mostly involves students studying for degrees, whilst my study involves students studying towards formal career-focused qualifications. Research on affect with respect to the latter group is much needed to gain an understanding of what motivates students to use CAS in an environment wherein training them to pursue specific careers is the prime focus.

In the context of learning mathematics with CAS at tertiary level there are a large number of research findings involving students learning calculus at first-year level in academic programmes, but very few on learning to solve DEs (using CAS) by vocational students. This absence can be gauged from the most recent syntheses in the field, for example, see Blume and Heid (2008) and Heid and Blume (2008a). Moreover, the students in my study were studying the topic of differential equations during semesters three and four (see Chapter 4.3.1) and had already completed a first course in calculus. Hence, my study is much needed to shed light on this important stratum of tertiary students in South Africa, and arguably in the developing countries of the world.

Research has been carried out into mathematics and technology in the world of work, for example, Noss, Bakker, Hoyles, and Kent (2007), who researched workplace-based studies wherein the focus was on mathematical knowledge. In particular, they investigated various employees' use and knowledge of graphs in the manufacturing and service sectors in the United Kingdom (UK). This research was related to how people use mathematics and technology within their vocational careers and in the context of actual workplace scenarios. In contrast, my research focuses on how students learn mathematics with technology whilst they are in the formal training or learning stage of pursuing vocational qualifications at a higher education institution. The mathematics problems may include some engineering contexts but, as mentioned above, these problems are not the 'actual' ones that one would encounter in a workplace environment.

2.7. SUMMARY OF SECTION A

In this section of the literature review I have discussed various research studies that foreground both the opportunities and the challenges involved in the use of technology in mathematical learning. Computer environments may take on the burden of the cumbersome technical aspects of mathematical activity and encourage mathematical endeavours that promote conceptual understanding. These environments could also facilitate the co-development of mathematical techniques and theory, thereby providing potential avenues for learning to take place at more meaningful levels. There is general consensus that an assortment of technology may augment students' comprehension of mathematics (Zbiek & Hollebrands, 2008), nevertheless many studies point to a variety of complexities involved in the use of CAS as a tool in mathematical learning and teaching. Much of what is known derives from research studies where learning takes place in developed parts of the world. The potentialities of CAS as a tool in learning mathematics and the impediments that exist during its implementation in the learning process is certainly worthy of further research, especially in different contexts, such as learning mathematics in developing countries.

I have identified the following gaps in the literature around students using CAS in mathematics learning:

- Students' motivations and CAS in mathematics learning.
- Research in developing countries.
- The use of socio-cultural frameworks, particularly Leontiev's (1978, 1981) version of activity theory related to mathematical learning using CAS.
- CAS (in mathematics) and students studying towards vocational (non-degree) qualifications.

An important theoretical contribution of this study lies in the integration of motivation theory, viewed from a Western standpoint, and Leontiev's (1978, 1981) version of activity theory. This elaboration takes place in Chapter 3. The literature review on Western motivation is discussed in the next section.

I sum up this Section A literature review on the use of CAS in mathematical learning by concurring with Tall et al., that "a review of the research [*with calculus and technology*] does

not, and cannot, give definitive answers to whether reform courses using technology are “better” in general” (Tall et al., 2008, p.248, italicised words my addition).

SECTION B:

REVIEW OF MOTIVATIONAL THEORY

2.8 INTRODUCTION TO SECTION B

In this section of the literature review I discuss various constructs as put forward by Western motivational theorists, including achievement goal theory and expectancy-value theory. The construct of motivation I use for this study is developed in Chapter 3 from the perspective of activity theory. Within that chapter I discuss three significant categories of motives and show how the Western concept of motivation, namely some aspects of expectancy-value theory and achievement goal theory, fit into the three larger groupings as suggested by activity theorists.

I begin this section with discussing the need for researching the topic on motivation in learning contexts from a social and cultural perspective, then interrogate the construct of motivation as postulated by Western motivational theorists. I consider the specific theories of expectancy-value theory and achievement goal theory. A summary containing the theoretical constructs of motivation that are significant for this study completes this section of the literature review.

2.9 THE NEED TO RESEARCH MOTIVATION IN LEARNING CONTEXTS FROM A SOCIOCULTURAL STANDPOINT

In more current research on motivation in education there has been an interest in social and cultural factors which play a prominent role in influencing motivation theory and research, for example, Schunk, Pintrich and Meece (2008), and Brickman and Miller (2002).

Brickman and Miller (2002) developed a theoretical model which describes how individuals' future goals develop and guide their current actions. These future goals reveal culturally determined needs such as wanting to pursue an education, identifying a career and establishing a family. Various social and cultural factors, for example, family, peers and school, influence individuals' perceptions of what they want to pursue in life. These authors reported on three students from diverse backgrounds who held different beliefs as to how they would reach their future goals. Their histories and experiences had shaped the perceptions of themselves and their futures in significant ways. The authors argued that social and cultural factors influence the development of students' future goals and plans for education and these further influence academic motivation in their current educational setting. Their findings suggest that it is

important to approach the planning and development of educational instruction in ways that take cognisance of the influence of students' personal future goals on present academic motivation. The information I obtained from the participants relating to their future goals is useful in understanding the influence this has on their current academic achievement in mathematics and in using MATLAB to learn mathematics. Seen from an activity theory perspective, the individual's personal motives and goals are given attention in Brickman and Miller's (2002) research.

As Brickman and Miller (2002) argue, socioculturally-based theories, by their nature, may lead to better explorations of the intricacies surrounding students' motivation to learn. Vygotsky (1978) and Leontiev (1981) have posited that it is virtually impossible to separate students from the context in which they study. Theories that create more awareness of the range of sociocultural aspects influencing students' motivations "are likely to be of more value than theories (i.e. such as attribution theory, goal theory) that largely ignore these sociocultural elements" (Bempechat & Boulay, 2002, p.32).

Unfortunately, the methods used in achievement goal theory are mainly quantitative in nature, involving scales and the relationships between these scales. This type of research lacks the deep perspective that qualitative methods provide, with the exception of a few studies such as those of Bempechat and Boulay (2002) and Zan, Brown, Evans and Hannula (2006). One could speculate that as researchers who are steeped in motivational theory become increasingly aware of the social and cultural influences on learning there will be a corresponding increase in the number of goals that would with greater frequency be included in surveys, as is the case currently. From an activity theory perspective, consideration of individuals as 'beings' who constantly interact with the environment, context and social factors appears to resonate with the proposal made by some motivational theorists such as Bempechat and Boulay (2002) and Turner (2001).

Furthermore, Bempechat and Boulay (2002) have emphasised the urgent need to challenge the theoretical perspectives of goal theory that had guided the research thus far. These authors showed how students' own perspectives about learning had been neglected by theories and methods that paid limited attention to the social and cultural contexts in which achievement motivation develops. More specifically, they demonstrated how one can make sense of students' educational experiences and academic motivations by moving away from surveys and experiments and towards qualitative methods of inquiry.

For example, in my study, I found that, because of certain contextual factors, Tumi (a 98% scoring student in Mathematics 1) resorted to writing out entire MATLAB programmes by hand instead of programming directly on the computer. If I were to use the scales of achievement goal theory and quantitative methods to determine Tumi's goals, I would have possibly obtained a classification of mastery goals for her. Although she showed many signs of trying to understand the work and at times leaned towards deeper approaches to mathematical learning using CAS, there were many occasions in which she did not understand aspects of the work. Activity theory, with its inherent focus on the context of learning, provides a powerful framework to explain such instances.

My research project began as an inquiry appealing to traditional motivation theories. With this in mind I have realised that achievement goal theory as it currently appears in the literature is inadequate for examining the rich data that I collected. I argue that separating students' goals into the very few categories suggested by achievement goal theory loses sight of the social and cultural context that I am striving to maintain. Hence, I have retained achievement goal theory as a backdrop to this study, and have, where possible, made use of the terminology and definitions suggested by some of these models from achievement goal theory, bearing in mind that the work produced in the literature on achievement goals thus far has some merits and does provide a basis for expansion.

2.10 THE CONSTRUCT OF MOTIVATION

A major criticism of the research on motivation theory lies in the way researchers have defined similar motivational concepts differently. For researchers new to the field of motivation theory, examining this body of research is a daunting task. The way in which motivation constructs are defined determines the measures (usually quantitative) that are used to assess them and the consequent elucidation and interpretation of the results. Accordingly, there are several inconsistent results, possibly traceable to the differences in definitions and the measures used to assess the same construct (Schunk et al., 2008). Some of these conflicting results will be discussed in 2.10.2, under 'achievement goal theory').

Any standard text on motivation and learning reveals theories such as expectancy-value theory, attribution theory, social-cognitive theory, goals and goal-orientations, interest and affect theories, intrinsic and extrinsic motivation and, more recently, socio-cultural and teacher influences (see, for example, Schunk et al., 2008). This is sufficient to alert one to the complexity of the nature of motivation and motivational processes. The two main theories that

involve considerable amounts of research in the context of learning are achievement goal theory and expectancy-value theory. Goal-orientation theories were developed by educational and motivational psychologists to explain learning and achievement of academic tasks and in school settings (Schunk et al., 2008). Consequently, in this study I choose to focus on both these motivational theories.

At this stage it is necessary to give a broad definition of 'motivation' from the Western perspective (my definition for this study is developed in Chapter 3, using an activity theory construction). For Schunk et al. (2008, p.4, emphasis in original text) it is *"the process whereby goal-directed activity is instigated and sustained*". These authors point out that an important aspect of this definition lies in the word 'process' because motivation viewed as such implies that one infers motivations from the actions carried out by students and from their verbalisations. In this sense there are several indexes or behavioural indicators that relate directly to the component of sustaining in this definition of motivation (those relevant to this study are shown in Table 2.1, below). The index I had not selected for this study was the achievement index. I was not interested in the grades that students scored in their study of CAS in mathematics because these grades were not standalone ones appearing on result sheets, but contributed to the final symbol in the Mathematics 3 module. Consequently, studying task achievement meant that I had to study their overall achievement in Mathematics 3, which is not part of this study.

Table 2.1: Indexes of Motivation (Schunk et al., 2008, p.12, *italics* mine)

Index [*or indicators*]	**Relation to motivation**
Choice of tasks	Selection of a task under free-choice conditions indicates motivation to perform the task
Effort	High-effort - especially on difficult tasks – is indicative of motivation
Persistence	Working for a longer time - especially when one encounters obstacles – is associated with higher motivation

The indicator of *choice of tasks*, or interests relates directly to motivation in the sense that "when students have a choice, what they choose to do indicates where their motivation lies" (Schunk et al., 2008, p.11). In my study, students had little choice as to what they did in the labs because lecturers chose and solved problems as well as allocated tasks. What students chose to do when they learned mathematics mediated by CAS in their own time was to be useful in understanding where their motivation lay.

Effort is another important indicator of motivation in the context of learning, several studies having examined the correlation between motivational aspects and learning strategies or effort, for example, Pintrich and De Groot (1990). It is expected that students who are motivated to learn particular academic tasks will put in more mental effort, and "employ cognitive strategies they believe will promote learning: organising and rehearsing information, monitoring level of understanding, and relating new material to prior knowledge" (Schunk et al., 2008, p.12 citing the research by Pintrich & De Groot, 1990, & Pintrich, 2003). I believe that such strategies may result in deeper levels of learning (further discussion takes place in 2.10.3.4, and I make use of them in my analysis, see Chapter 5).

Regarding the indicator of *persistence* or time spent on a task, it is expected that students who are motivated to learn should persist in it, especially when faced with obstacles (Schunk et al., 2008). The determination and perseverance that one shows in particular activities or tasks is, according to Shunk et al. (2008), associated with higher motivation.

Schunk et al. (2008) have described indicators that one could use in a study on academic motivation, and I find the above indicators significant and relevant for this study. Indeed, I consider the actions related to how students regularly attend lectures, take notes, use resources, plan their studying, memorise, organise, make decisions, comprehend, solve problems, evaluate progress and form connections between what one has already learned to new tasks, as important sub-indicators of the three indicators mentioned above. I group some of these actions (or sub-actions) together so at to develop themes that connect with an activity theory perspective (see Chapter 5).

For Schunk et al. (2008, p.4, emphasis in original text), "Motivation involves **goals** that provide impetus for and direction to action". They argue that cognitive views of motivation stress the importance of goals, that is, individuals have some purpose in mind that they are endeavouring to realise or avoid. I interrogate these 'reasons' within an achievement goal-orientation

theoretical context in 2.10.3, after elaborating on expectancy-value theory and associated aspects important for this thesis.

2.10.1 EXPECTANCY-VALUE THEORY

Developed and elaborated by Eccles (1983) and Eccles and Wigfield (1995), expectancy-value theory reflects a social cognitive perspective that centres on "students' expectancies for academic success and their perceived value for academic tasks" (Schunk et al., 2008, p.50). This motivational theory has two components, namely expectancy and value. The former concerns students' beliefs and judgments about their capabilities of performing in upcoming areas in mathematics and succeeding at it (ibid.), hence there is a focus on future-oriented activities. My study is not concerned with this aspect but with the latter, the value component, which comprises different beliefs or perceptions that students have about the reasons they might engage in the task of using CAS in mathematical learning (ibid.). It looks at the extent to which students believe that the academic domains of mathematics and using CAS in mathematical learning are worth pursuing. Akin to goals, the values component is primarily concerned with the reasons students want to or do not want to use CAS to learn mathematics. The reasons for their beliefs could take the form of importance, intrinsic interest, extrinsic utility value and perceived costs (Eccles & Wigfield, 1995). Perceived costs are about students studying CAS (in maths) at the expense of pursuing other enjoyable activities, such as dancing (Scunk et al., 2008), though this is beyond the scope of the study as it involves investigating their hobbies and outside activities. Next I discuss three of the task value components that I use in this study:

1. *Importance* in this context relates to the value placed on doing well at a task (Schunk et al., 2008), that is, using CAS in learning mathematics. Statements such as '*It is important to me to do well in mathematics*'; '*It is important to me to get good grades in mathematics'* or '*It is important to me to learn the mathematics content*' are exemplars of this concept.

2. *Intrinsic interest* (or *intrinsic value*) refers to the inbuilt reasons students might have in their engagement with mathematics and/or using CAS (in maths). Such reasons could be the enjoyment and inherent challenge of studying mathematics using MATLAB (Wigfield & Eccles, 1992). Statements such as '*I find using MATLAB (in mathematics) very interesting*'; '*I am very interested in the mathematics content*' or '*I like the subject matter of the Mathematics 3 course (with CAS)*' are exemplars of this construct. Shunk et al. (2008) report that intrinsic interest is related to the joy and means of doing a task rather than to its ends (outcomes).

3. *Extrinsic utility value* is defined as how useful the task is for individuals in terms of their future aspirations, which include career ambitions. Extrinsic utility value is related more to the ends of a task (in this case studying CAS in mathematical learning) than to the means (Schunk et al., 2008). Statements such as '*I find the CAS (in maths) useful for what I want to do later (maybe in a follow up B.Tech course)*'; '*I think that I will be able to use what I learn in this MATLAB course in other Engineering courses*', or '*I think that the course material in this CAS (in maths) course is useful for me to learn*' are all exemplars of the utility value construct. For example, an Engineering student may not have much intrinsic interest in studying mathematics, but because he or she wishes to become an engineer, this course has a high utility value for him or her. In this sense, utility value is similar to extrinsic reasons (Schunk et al., 2008).

The above value components may have an influence on academic motivation in that they may lead to the student putting in more effort and persisting longer in learning the subject matter (Schunk et al., 2008). As Wigfield and Eccles (1992) assert, high intrinsic interest results in more engagement and more persistence since the student will be intrinsically motivated to work on the task.

2.10.2 ACHIEVEMENT GOAL THEORY

Research into achievement goal theory has been one of the most active areas of investigation of student motivation in academic situations (Pintrich, Conley & Kempler, 2003). Achievement goals are cognitively based (ibid.) and can be influenced by contextual factors rather than the more general construct of achievement motivation (Urdan & Giancarlo, 2002). Schunk et al. (2008, p.183) write that achievement "goal-orientation theories were developed specifically to explain achievement behaviour" on academic tasks. These theories focus on the goals of achievement tasks. For example, one can examine the particular achievement goals that students espouse related to the activity of using CAS in mathematical learning.

Achievement goal theory has been developed and detailed by theorists such as Ames (1992b), Dweck and Leggett (1988) and Wentzel (2000). Formerly, the dichotomous nature of students' goals was the main focus of achievement goal theory and learning (mastery goals versus performance goals), but over time the focus has changed to include some modifications, for instance, into mastery approach goals, performance approach goals, mastery avoidance goals, and performance avoidance goals. Eliott and McGregor (2001) used factor analysis to support the separation of the original dichotomous goal-orientations into this four-way split, however it

is surprising that these four goals were conceptualised without taking into account the social nature of individuals, which in an academic setting is a crucial criterion. In the activity theory framework, social components play a key role in learning, and Urdan and Maehr (1995) made an appeal for the inclusion of social goals into the framework of achievement goal theory. In this study I am concerned with the trichotomous framework (mastery goals, performance approach goals and performance avoidance goals) and the social goals, excluding mastery avoidance goals as none of the participants espoused them.

Pintrich (2003) defines 'goal-orientations' as the purposes or reasons for engaging in academic tasks, in other words *why* the student is trying to achieve academically in the task (see 3.6 where I locate this definition within activity theory). Achievement goal theory is thus concerned with an individual's perceived purposes for accomplishment, and as Urdan and Giancarlo (2002) state, it defines motivation in terms of quality. 'Quality' in this sense can be illustrated by the following scenario: two students using CAS in learning mathematics may have different reasons for wanting to achieve in the learning process yet they could both have the same amount of motivation to achieve (quantity). For example, Thembiso wants to achieve (in mathematics) so as to proceed to the last semester of the course whilst Tumi wants to achieve (in mathematics) because she finds the course inherently stimulating. Achievement goal theorists label these different perceived purposes of achievement as *goals*[7]. The reasons given by these two students for wanting to achieve may lead them to make use of different methods, styles and approaches when they use MATLAB in mathematical learning. Their underlying reasons could lead not only to different behaviours but also to different reactions to success and failure, as well as different notions about their abilities and the value attributed to mathematical tasks mediated by CAS (quality) (Urdan & Giancarlo, 2002). This definition, which includes quality, is particularly useful in this study as I am using qualitative methods to collect data and can examine the quality aspect using participants' own words.

The goals students pursue in a given situation, in this case the context of using CAS to learn mathematics, depends in part on the relatively "stable achievement goal orientations that they bring to the situation and in part on cues in the achievement context that make certain goals salient" (Urdan & Giancarlo, 2002, p.38). To address the first aspect, my study includes the reported goals (data obtained through interviews) that students have towards studying

[7] Urdan and Giancarlo (2002, p.38) note that "goals are described as overarching frameworks, or cognitive schemas through which individuals perceive academic tasks and achievement situations, interpret success and failure and think about and behave in the achievement situation".

mathematics. This may influence their goal-orientations towards using CAS in mathematical learning. The latter aspect relating to the cues in the achievement context could relate to the classroom milieu. Whether the classroom learning environment is conducive to promoting mastery or performance goals can be gauged from students' responses to the interview questions (the actual observations of the laboratory or classroom environment are beyond the scope of this study). Any other contextual influences on the goals that students espouse with respect to using CAS (in maths) have been analysed from interview data, screen recordings and observations of them whilst they solved problems in my presence.

Many researchers have examined how different achievement goal-orientations[8] relate to a multitude of outcomes, a few of which are achievement, motivation and cognitive aspects (such as deep or surface approaches to learning). I now discuss attributes of mastery and performance goal-orientations, followed by a discussion on the trichotomous goal-orientations (mastery, performance approach and performance avoidance goal-orientations). Within this discussion I consider how the goal-orientations interact with intrinsic motivation and achievement. This is followed by an examination of the relationships between achievement goal-orientations and deep or surface approaches and scrutiny of social goal-orientations.

2.10.2.1 DICHOTOMOUS: MASTERY AND PERFORMANCE GOAL-ORIENTATIONS

Various researchers have introduced very similar concepts but using different terms. For instance 'mastery and performance goal-orientations' were introduced by Ames (1992b) and Ames and Archer (1988), but are also known as 'learning and performance goal-orientations' by Dweck and Legett (1988), and Elliot and Dweck (1988). The terms 'task-involved' and 'ego-involved' were introduced by Nicholls (1984). Even though these constructs have been introduced with different underlying meanings, according to Schunk et al. (2008) there is enough conceptual overlap to justify treating them in a similar way. They illustrate their point by providing exemplars of similar items taken from the quantitative scales developed by these different goal achievement theorists.

Mastery goal-orientation is characterised by a focus on mastering the task; striving to improve oneself and accomplish something challenging from it; developing new skills; being

[8] Since activity theory uses the concept of goals as being related to actions, I reserve the word 'goals' for its specific use within the activity theory context. Henceforth, I use mastery goal-orientations and performance goal-orientations when I write about mastery goals and performance goals (from a non activity theory perspective) respectively.

competently engaged in it, either by developing or improving competence; and trying to gain understanding[9]. Performance goal-orientations are characterised by a focus on demonstrating ability, looking smart, demonstrating ability relative to others (for example, trying to exceed normal performance standards and outshining others in the task), and seeking public recognition (Ames, 1992a; Schunk et al., 2008; Urdan & Giancarlo, 2002).

2.10.2.2 TRICHOTOMY MASTERY, PERFORMANCE APPROACH, PERFORMANCE AVOIDANCE GOAL-ORIENTATIONS AND THEIR RELATIONSHIP TO INTRINSIC MOTIVATION AND ACHIEVEMENT

Midgley et al. (1998) found support for the trichotomous scales, clarified below:

Mastery goal-orientation

I like to learn new things. I like mathematics best when it really makes me think. I do my work in mathematics because I want to get better at it. I do my mathematics because I'm interested in it. I do my mathematics because I enjoy it.

Performance approach goal-orientation

It's important to me that the other students in my classes think that I am good at my mathematics. I want to do better than other students in my mathematics classes. I'd like to show my teachers that I'm smarter than the other students in my mathematics classes. Doing better than other students in mathematics classes is important to me.

Performance avoidance goal-orientation

It's very important to me that I don't look stupid in my mathematics classes. I do my mathematics so that I don't embarrass myself. The reason I do my mathematics is so my teachers don't think I know less than others. The reason I do my mathematics is so others won't think I'm dumb. One reason I would not participate in the mathematics class is to avoid looking stupid. One of my main goal-orientations is to avoid looking like I can't do my work.

Calling for the consideration of achievement goal-orientations into a trichotomous framework, Eliot and Harackiewizc (1996) found in an experimental study that participants holding mastery

[9] It is notable that this definition of mastery goal-orientation is compatible with that of mastery approach goal-orientation and consequently in this thesis I use both these terms interchangeably.

goal-orientations and those espousing performance approach goal-orientations had greater levels of task involvement and ensuing intrinsic motivation, whilst the performance avoidance goal-orientation participants had reduced task involvement (relative to the performance approach orientation) and consequently decreased levels of enjoyment of the activity. Thus, performance avoidance goal-orientation undermined intrinsic motivation relative to both a performance approach and mastery goal-orientation. Indeed, the performance avoidance goal-orientations have been shown to have deleterious consequences for interest and performance (Harackiewicz, Barron, Tauer, & Elliot, 2002). Participants within both the approach goal-orientations exhibited equivalent levels of intrinsic motivation.

In a recent longitudinal study by Harackiewicz, Durik, Barron, Linnenbrink-Garcia and Tauer (2008), the relationships between situational interest, individual initial interest, achievement goal-orientations and academic performance were explored amongst college students. These variables were also tracked and examined seven semesters later. Regarding the use of the phrase 'individual interest': students may enter the course with high levels of interest, possibly due to prior experiences with related content they found interesting. This individual interest may intensify if they find personal significance in the content. With respect to the phrase 'situational interest', students may enter the course with little prior knowledge and less initial interest but develop interest while interacting with the course materials, instructional designs and tasks. The first type of interest is termed 'individual interest', and the latter, which has significance for my study, 'situational interest'.

Participants in an introductory Psychology course were surveyed (ibid.) at various stages of the course and a set of data was obtained with respect to their initial interest in psychology, their achievement goal-orientations and situational interest. At the end of the semester, they were surveyed with respect to their grades and interest in the course. The last batch of data was collected seven semesters later and involved continued interest in psychology (average number of Psychology modules taken) and subsequent performance (grades). Findings indicated that interest and mastery goal-orientations are reciprocally related over time. Initial interest predicted mastery goal-orientation espousal, situational interest (for the duration of the course) and continued interest assessed a number of semesters later. Students entering the course with greater levels of initial interest in it might be inspired to learn more about it, that is, take on a mastery goal-orientation. The mastery goal-orientation in turn leads to the development of more interest as the students become task-oriented. The effects of initial interest on continued interest (beyond the introductory Psychology course) were partially mediated through mastery goal-

orientations, which can thus be viewed as a product of initial interest and as a predictor of interest.

A finding of their study revealed that situational interest during the course (not dependent on initial interest) predicted consequent course selections in Psychology. In my study, most students did not have prior exposure to using MATLAB in mathematical learning. Some participants might have had high levels of initial interest in wanting to learn to use MATLAB. This important result of Harackiewicz et al.'s (2008) research draws attention to the situational interest as a factor that could be used to predict continued interest in wanting to use MATLAB beyond this course. This is a significant factor as students in this study were registered within the vocational stream, being trained for the workplace environment. The question then arises: did the use of MATLAB arouse enough situational interest in the participants so that they may consider using this tool beyond the immediate classroom environment? Even though this last aspect does not form part of the study (students' goal-orientations), its influence on intrinsic interest and the situational interest is very relevant.

Harackiewicz et al.'s (2008) study suggests that both interest and performance are vital for maintaining continued interest in a domain. They also propose that both mastery and performance approach goal-orientations are important in predicting continued interest in an academic domain. In their study, they found more support for performance approach goal-orientations as a predictor of grades as compared with an indirect influence of mastery goal-orientations on grades.

Research carried out by Elliot, Shell, Henry and Maier (2005) found that performance avoidance goal-orientations undermined performance relative to performance approach and mastery goal-orientations. They also found that performance approach goal-orientations were just as positive for performance as mastery goal-orientations. Their research adds to the growing body of empirical work attesting to the claim that performance avoidance goal-orientations undermined performance relative to performance approach and mastery goal-orientations.

There is sufficient evidence in the literature to support the argument that mastery goal-orientations are positively linked to self-report of interest and enjoyment, however the results for performance approach goal-orientations are less consistent (Schunk et al., 2008). Some reports, such as those by Elliot and Harackiewizc (1996), show that performance approach goal-orientations can lead to interest in the task, task involvement and intrinsic motivation. Schunk et al. (2008) have noted that performance avoidance goal-orientations generally can have a

negative relation to interest, enjoyment and task value. With respect to performance avoidance goal-orientations and achievement, Elliot et al. (2005) claimed that there are sufficiently convincing findings in the measurement-based literature on achievement goal-orientations to deduce that performance avoidance goal-orientations are negatively related to performance attainment.

2.10.2.3 TRICHOTOMY: MASTERY, PERFORMANCE APPROACH, PERFORMANCE AVOIDANCE GOAL-ORIENTATIONS AND THEIR RELATIONSHIP TO DEEP AND SURFACE APPROACHES TO LEARNING

Schunk et al. (2008, p.193), in summarising various research studies on goal-orientations and their relationship to cognitive outcomes, report that students holding mastery goal-orientations also use "strategies that promote deeper processing of the material", for example, summarising, paraphrasing, organising, networking, as well as self-regulatory strategies such as planning, awareness and monitoring one's own learning and understanding. It is encouraging to note that this consensus exists, however the results for performance approach and avoidance goal-orientations and their association with cognitive outcomes are inconsistent: "taken together, the conflicting results suggest that approach performance goals do not have to be negatively related to cognitive self-regulatory activities in comparison to avoidance performance goals" (Schunk et al., 2008, p.195). As these authors argue, research in this area needs to go beyond the usual 'correlational' self-report studies as it seems that differences in the results might be due to the different measures used in evaluating similar constructs.

2.10.2.4 SOCIAL GOAL-ORIENTATION

Urdan and Maehr (1995) define social goal-orientation as the perceived social purposes for academic achievement. These goal-orientations are defined in addition to mastery and performance goal-orientations. Students may pursue achievement goal-orientations, such as being successful on academic tasks, getting good grades and doing better than others, and these could be related more to social goal-orientations than to mastery or performance goal-orientations.

Social goal-orientations can be portrayed as follows, albeit these characterisations are not mutually exclusive (Urdan & Maehr, 1995; Watkins, McInerney, Lee, Akande & Regmi, 2002; Wentzel, 2000):

Social approval goal-orientation: a student may pursue academic achievement (or underachievement) as a means of gaining approval from others.

Social responsibility goal-orientation: getting work done on time; cooperating with classmates.

Social interaction goal-orientation: trying to make or keep friends; trying to have fun with friends whilst learning.

Social affiliation goal-orientation: belonging to a group when doing academic work; the positive influence of friends while doing learning tasks.

Social compliance goal-orientation: making obvious that one is a good person.

Fortunately, social goal-orientations are defined in the same way as other achievement goal-orientations (such as mastery and performance goal-orientations), and since it is in line with the achievement goal theory as explicated by Ames (1992b) and Dweck and Leggett (1988) it will be used in this thesis.

As with most research in the socio-cultural paradigm, the context is imperative and plays a significant role in assisting with interpretation of the different goal-orientations that students hold. In this study the learning environment plays a critical role in determining the goal-orientations that students pursue (Ames, 1992b). For example, a student who believes her/his reason for getting a good grade in mathematics is to continue being part of the group of high achievers may find the social approval goal-orientation contributing to his putting more effort into studying. Meanwhile another student may have as his main reason for poor performance the gaining of approval from his peer group, who undervalue the study of mathematics (Urdan & Maehr, 1995). This student, who is mostly concerned with avoiding doing well in mathematics, is also in search of a social approval goal-orientation.

As Urdan and Maehr (1995) have argued, social approval goal-orientation is likely to have distinctly different effects on motivation, and this depends upon whose approval is being sought. There are positive and negative outcomes of achievement goal motivation, depending on the context in which the goal-orientation is pursued. Activity theory provides a framework to interrogate goal-orientations by giving consideration to diverse contexts.

The above examples suggest that one should not categorise social goal-orientations into a mutually exclusive class, as was the case initially with mastery and performance goal-orientations, but rather understand that social goal-orientations may overlap with each other and

with other goal-orientations, such as performance and mastery goal-orientations. Currently, many researchers are creating quantitative scales around the various constructs of social goal-orientations, for example, Horst, Finney and Barron (2007); however these interrelationships are not part of this study.

2.11 SUMMARY OF SECTION B

I have argued that the different goal-orientations discussed above contribute to the rich multiple goal-orientation nature of student motivation and are vital to facilitate understanding of the construct of motivation in learning activities. Of utmost importance is the realisation that students can espouse these multiple goal-orientations simultaneously. From an activity theory perspective, achievement goal-orientation theory can do well by paying attention to the individual's historical and cultural aspects of learning. Also, there should be a focus on the individual's distinctive context of learning.

It is encouraging to note that there is a considerable amount of recent literature using qualitative methods of inquiries on motivations and the learning of mathematics, for example, Zan et al. (2006); Evans, Morgan and Tsatsaroni (2006) and, DeBellis and Goldin (2006), though, again, a review of this literature is beyond the scope of this study. However, it is important to mention that the field of motivation with respect to mathematics and science education has undergone many changes in recent years as researchers increasingly have begun incorporating emotional factors into motivation, for example, Hannula (2006) and Roth (2007a). A review of the theoretical constructs on motivation that are important for this study is captured in Table 2.2:

Table 2.2: Summary: theoretical constructs of motivation

Constructs of motivation	**Brief description**
Indexes or indicators of motivation:	
• Choice of tasks	• Selection of tasks under free-choice indicates motivation to perform the task
• Effort	• High effort on difficult tasks is indicative of motivation
• Persistence	• Engaging with the task, especially when faced with obstacles, is an indicator of higher motivation
Task value components:	
• Importance	• Importance of task - importance of doing well or getting good grades in MATLAB (in maths) or important to learn the content of MATLAB (in maths)
• Intrinsic value or interest	• Intrinsic reasons students have for studying the activity – enjoyment, inherent challenge of studying CAS (in maths)
• Extrinsic utility value	• Usefulness – related to careers or other courses
Achievement goal theory:	
• Mastery goal-orientations	• Focus on mastering the task, striving to improve oneself in the task, striving to accomplish something challenging on the task, developing new skills, developing competence, improving and understanding and searching for meaning
• Performance approach goal-orientations	• Demonstrate ability and look smart, demonstrate ability relative to others, seek public recognition, seeking good grades, and be ego-involved
• Performance avoidance goal-orientations	• Avoid demonstrating lack of ability
• Social goal-orientations	Perceive social purposes for academic achievement, such as social approval, social responsibility, social interaction, social compliance, and social affiliation as goal-orientation

The motivation theory examined in this Section B and its link with activity theory will be theorised more clearly in the next chapter.

CHAPTER 3

THEORETICAL FRAMEWORK: AN EXPLICATION OF ACTIVITY THEORY

3.1 INTRODUCTION TO THE CHAPTER

Whilst I acknowledge the existence of various theories within the social cultural paradigm, such as situated learning, my primary concern is with the cultural, historical and social aspects of activity theory, in particular Leontiev's (1978, 1981) claims regarding activities, actions and operations. These constructs are invaluable for informing the empirical work and for the collection, analysis and interpretation of data. This chapter therefore examines the development of activity theory, from its roots in the work of Vygotsky, through the object of activity to the needs and relationship between motives and objects. I establish the links between motivation and motives, followed by learning at the level of action and operations. The notions of mediation, consciousness and context are discussed, culminating in presentation of the model that underpins this study.

To ensure ease of reading, pertinent activity theoretical constructs are briefly defined at this point. I provide an in-depth discussion of these constructs in 3.5, 3.6, 3.7 and 3.8. The outermost layer of Leontiev's three level activity structure, known as the activity level, is concerned with the global aspect of activity, and examines the context in which it takes place, for example, a learning situation. Associated with the activity level are the state of needs, motives and object of activity. Leontiev's (1978) description of needs is one that originates within a social and historical context, in which an individual has needs that demand to be satisfied. These needs include autonomy, competency and social belonging (Boekaerts, 1999). Regarding the definition of motives, I follow Leontiev's (1978) explanation that a motive takes on an arousing and directing function of the activity. Within the three broad categories of social (interrelations with others), self-related and cognitive motives, I situate the following specific motives of social approval goal-orientation: social responsibility goal-orientation; social interaction goal-orientation; social compliance goal-orientation; social affiliation goal-orientation; task value

components encompassing the importance of task (importance of doing well or getting good grades in MATLAB, or of learning the content of the MATLAB course); extrinsic utility value (future, usefulness – related to careers/ other courses); performance approach goal-orientations; task value components comprising intrinsic interest / intrinsic value; and mastery approach goal-orientations.

In this study the 'object' of activity is depicted as 'aim' and is associated with subjective qualities and imbued with individuals' motivations, motives and needs, as put forth by Leontiev (1978). The second layer of Leontiev's activity construction is actions, referring to conscious behaviour and actions that are driven by conscious goals. Whilst actions are conscious forms of behaviour, the means by which they are carried out are called *operations.* Operations refer to those actions that become automatic and non-conscious over time. Next I consider Vygotsky's theory.

3.2 SOCIO-CULTURAL THEORY AND VYGOTSKY'S THEORY

Activity theory was developed out of Vygotsky's socio-cultural[10] approach to learning by one of his students and collaborators, A.N Leontiev (Russell, 2002; Zincheko, 1995). Situated within socio-cultural theory, its purpose was "to understand how mental functioning is related to cultural, institutional, and historical context" (Wertsch, 1998, p.3). The Russian theorists, particularly Leontiev, and others viewed higher order mental processes as complex functional systems formed during the course of an individual's development whilst s/he interacts with others within a social environment or cultural system, and takes part in activity. These mental processes, in the form of ideas and knowledge development, occur in situations that are "functional in terms of the needs, expectations and goals of the individual or cultural group" (Crawford, 1996, p.133).

Vygotsky proposed that basic processes are transformed into higher cognitive states owing to ones involvement in culturally meaningful tools, such as language, whilst the individual interacts in a social environment (Hardman, 2005; Vygotsky, 1978). Vygotsky showed that language is used as the first interface between adult and child, and as a tool that enables communication with each other. Language progressively becomes internalised into a means for

[10] Although I am aware of the significant debates about the use of the terms 'cultural-historical' and 'socio-cultural', a discussion of them is beyond the scope of this thesis.

the child's thinking and control of activity. For Vygotsky (1978), the child's cultural development appears on two planes, firstly inter-psychological, as an interaction between people, and secondly intra-psychological.

Central to socio-cultural theory is Vygotsky's notion of *mediated action*, which he used to explain learning as individuals' construction of *meaning* while interacting with artefacts and social others in their environment (Yamagata-Lynch, 2003). This vibrant interaction plays a role in the social as well as cultural formation of the individual mind and influences individualistic knowledge construction (Wertsch, 1985).

According to Vygotsky (1978), development proceeds through the process of social interaction with more able others. His *zone of proximal development* (ZPD) delineated the distance between the level of actual development and the more advanced level of possible development, through the help of more capable participants. Every student's capacity can be extended when he or she is able to perform tasks with the assistance of more capable or experienced others, be it lecturers or peer students. Many students may thus participate in activities that are beyond their competence when acting alone, and it is within this 'zone of potentiality' that mediation as instruction provides the momentum for learning and consequently for change (Hardman, 2005).

Applied to this study, students use a computer as a tool that mediates the process of learning how to programme MATLAB software, with the specific purpose of solving DEs. I will argue that the MATLAB software, with its built-in error message functions, may operate within the students' ZPD. In addition, the laboratory set-up and after-hours opening may encourage students to work together in groups, and the completion of two MATLAB projects (as part of the Mathematics 3 course) by individual students may promote collaborative effort. In this context, one could therefore also study the ZPD of students.

From a Vygotskian perspective, learning could be understood as distributed; however, this does not situate it within a wider milieu of vibrant collective activities, nor provide a rich description of the energetic nature of activities (Lim & Chai, 2004). Vygotsky's theory may have implied the difference between individual actions and collective activity; but it is claimed that Leontiev articulated this distinction based on his famous example of the "primeval collective hunt" (to be elucidated in Chapter 3.3). Post-Vygotskian socio-cultural theorists have taken Vygotsky's mediated action as a basic component of their frameworks, and continue to develop and contribute to activity theory. For instance, Zincheko (1995) gives a clear exposition of the relationship between Vygotsky's and Leontiev's ideas, asserting that in Leontiev's theory of

activity all mental processes have an object-activity nature, which constitutes the merit of this strand. In Leontiev's psychological theory of activity the focus is on object-oriented and tool-mediated *actions*, whilst in Vygotsky's cultural-historical psychology the focus is on *meaning* (Zincheko, 1995). The cultural-historical school founded by Vygotsky emphasised the notion of mediation by language, while activity theory stresses mediation by tools and activities.

Working on the premise of Zincheko's (1995) proposal that the works of both the socio-culturists (Vygotsky and Leontiev) should be used in a mutually amplifying manner that enriches each other, I discuss (in Section 3.5) the framework of activity theory as elucidated by Leontiev, and show how this provides the principal analytical lens for this study. While Vygotsky perceived action mediated by signs as an essential system linking the external social world to the internal mental developments, Leontiev replaced these with activity as the essential scheme mediating between humans and their contexts (Wertsch, 1981a). Leontiev's approach to activity theory was criticised for appearing insensitive towards the role that culture and historical context play in the development of higher psychological functions as outlined by Vygotsky (Engeström, 1998), but Luria reinstated culture as an important contributor to activity theory. Recent approaches to activity theory, for example, by Gordon (1998) and Roth (2007a), have developed several conceptions of activity, integrating these two strands of Leontiev and Luria to varying degrees (Pietsch, 2005).

Although many cultural psychologists have described culture as basically a world of meanings which individuals confer on practices, for example, learning mathematics or learning to use computers, Ratner (1996) offers a conception of it from an activity theory perspective, describing the tenets of cultural psychology to be the formation of psychological functions as individuals involve themselves in functional social activities. These could take the form of possessing or manufacturing and distributing goods; creating families; educating; and exploring and comprehending the world. Within this perspective, one can see that psychological phenomena are steeped in practical activity. For Ratner, activity is organised in a particular tangible social system, hence psychological phenomena such as motivations, motives, attitudes, beliefs and emotions are developed within a particular economic, educational and scientific activity. As such, the individual does not exist separately from society, and in this study I see the use of tools (for example, MATLAB or computers) as an integral part of the above description of cultural practices.

3.3 THE DEVELOPMENT OF ACTIVITY THEORY

A few researchers, for example, Gordon (1998) have provided a detailed historical account of the development of activity theory; hence, I present an outline of its progression, with concepts pertinent to this study discussed in detail in the next section.

Three generations in the evolution of activity theory have been distinguished, the first of which centred on Vygotsky and mediation. The association between the individual or subject and the objects of the setting is mediated by cultural means and tools (Figure 3.1):

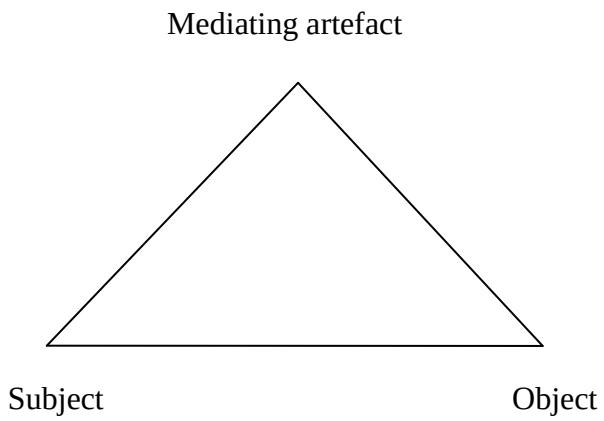


Figure 3.1: Vygotsky's mediation triangle

In the initial work of the cultural-historical school, this triangular relationship did not theoretically include mediation by others or social relations. Leontiev used the division of labour concept as a primary means to understand the evolution of mental functions, with Marx's concept of labour being his classic model of human object-oriented activity, hence contributing to the advancement of the above triangle. His distinction in conceptualising *activity* on the levels of *collective activity* and *individual* action is summarised by Engeström and Miettinen (1999, p.7):

> Mediation by other human beings and social relations was not theoretically integrated into the triangular model of action (proposed by Vygotsky, 1978). Such an integration required a breakthrough to the concept of activity by distinguishing between collective activity and individual actions. This step was achieved by Leontiev by means of reconstructing the emergence of division of labor.

The second generation of activity theory used Leontiev's three-level model of activity (activity, actions and operations levels) as its basis. Leontiev did not produce a diagrammatic model of a collective activity system, and Engeström proposed that Leontiev's model had failed to develop Vygotsky's model into one that creates an awareness of an activity system with its distinctive fundamental components. Consequently, Engeström (1987, p.36) produced the activity system as shown in the third generation activity theory model in Figure 3.2:

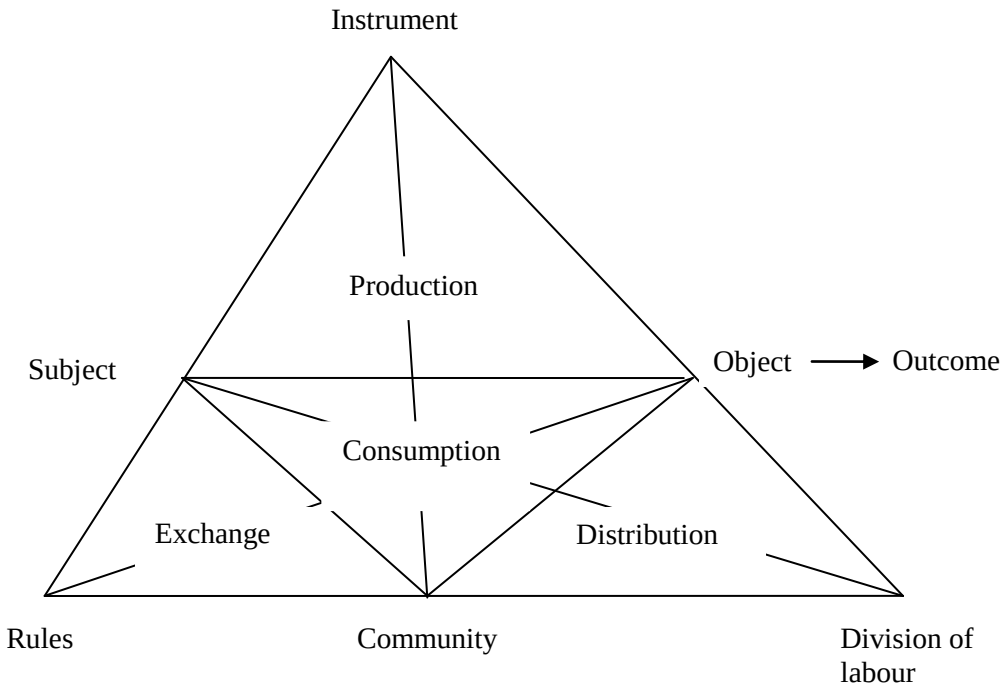


Figure 3.2: Engeström's activity system

The problem of levels in human functioning was posed theoretically by Leontiev (1981) in a hunting analogy, in which beaters share in the collective activity directed towards getting food, but have as the goal of their actions the frightening and driving of animals towards the hunters. The actions of an individual beater are therefore completed by other members of the group. In this activity, one interpretation could be accorded to Leontiev's distinction between the concepts of collective activity and individual action (Engeström & Miettinen, 1999). The actions of beating may not lead directly to the satisfaction of the beater's need for food, and the goal related to the actual process of frightening and chasing the animals towards the hunters did not coincide with the collective motive of the activity. Processes wherein the motive and goals do not coincide are called 'actions' (Leontiev 1981). Engeström and Miettinen (1999) add that one

could consider the beater's activity as the hunt, whilst the frightening of game could be thought of as the beater's action.

The individual's relationship with other group members could result in him or her receiving some of the meat of the hunted animal, as a consequence of combined effort within the activity context. Hence, this relationship is realised through the actions of other people within the context of activity. For Leontiev (1981), the connection between the motive of the activity and the goal corresponding to an action reveals objective social relations instead of natural ones. According to Leontiev (1978), the significance of human activity lies in the non-coincidence of motives and goals, with motives not always recognised by the individual. When individuals carry out different actions they may not be aware of the collective motives that evoke them, such is the nature of collective motives. It therefore becomes evident that the intermediate goals corresponding to the actions of individuals may or may not satisfy individual needs, although these could possibly be fulfilled through the realisation of collective motives (Pietsch, 2005). This idea of the existence of contradictions and tensions features prominently in Engeström's model of activity systems.

Kaptelinin (2005) argues that it is precisely the hunting activity articulated by Leontiev (1981) that has resulted in much confusion. This activity theorist points out that distinguishing the said activity on the basis of collective activities and individual actions does not seem to be consistent with the general framework developed by Leontiev. Kaptelinin (2005) maintains that Leontiev worked mainly within a psychological framework and primarily concentrated on the concept of activity as individually motivated, as will be explained below. Since there are many interpretations of the well-known hunting activity, the question arises as to which is most applicable. According to Kaptelinin (2005), this activity illustrates the distinction between individual activities and actions and correspondingly the difference between motives and goals. Further, "this example is used by Leontiev to illustrate that division of labor clearly induces a difference between what motivates a person (in this case, food) and to what person's actions are directed (in this case making animals run away)" (Kaptelinin, 2005, p.12). Hence, Kaptelinin (2005) draws attention to the piece of information that emphasises Leontiev's conceptualisation of the structure of individual activities rather than to activities that can only be collective. Comparable to this interpretation, I seek to understand what motivates an *individual* student to use MATLAB in mathematical learning, and what actions this student engages in to realise this activity, albeit not separating the student from the context of learning.

The hunting analogy demonstrates that the individual tool-mediated 'action' is insufficient as a unit of psychological analysis and Engeström (1987) proposes that if one were to take into account only an individual beater's actions, without deliberating on the collective activity and its motive, then the individual beater's actions would seem futile. As Engeström argues, analysing activity only on the basis of the individual's actions does seem pointless, but on closer inspection it is evident that he is proposing an examination of a collective activity system viewed from above, with its own collective motives. On the other hand, I postulate that according to Leontiev's theory one could look at the activity (as a unit of analysis) from the individual's perspective[11] and study what motives the individual espouses when participating in the particular activity[12] as opposed to a study of collective motives.

According to Engeström (1987, p.27, emphasis in original) "we may well speak of the activity *of the individual,* but never of *individual activity;* only actions are individual". In line with this, throughout the thesis, I refer to individuals as *individuals in context and within society,* shaped by rich historical, social and cultural factors, whilst interacting individually with activity in the course of their own development. In Leontiev's psychological theory, there is no separation between the individual and the external world. This relationship can be understood as the union of the subject and context, as the individual does not make sense of the external world through some type of screen, but rather there is a constitutive relationship between them (Pietsch, 2005). From an activity theory perspective, this relationship could be construed as the individual having an influence on the world through participation in activities, and conversely the activities having their own influence on the individual.

Activity theory provides a lens through which to analyse and illuminate how each individual negotiates his or her learning from the cultural-historical context, and it is within this rich context that my analysis takes place.

3.4 A VALIDATION OF LEONTIEV'S VERSION OF ACTIVITY THEORY

Activity is analysed at various levels and within two contemporary approaches, one developed by Leontiev (1978), the other by Engeström (1987). As Kaptelinin (2005) argues, these two

[11] I take note that one could also consider only individuals within Engeström's approach but the analysis of data involves an activity system as a unit of analysis rather than an activity.

[12] This aspect of Leontiev's theory is discussed in the next section.

approaches in activity theory do not compete with each other, but rather have different orientations that can be applied as analytical lenses to different situations. One could treat both as separate strands, with each serving its own purpose. Roth and Lee (2007) agree that they provide two different lenses through which to examine data, though this does not mean that one cannot use both together, as they are to be understood as complementary. Coupland (2004) has demonstrated how both may be used in a harmonious manner, having analysed data from different sources, namely students of mathematics and lecturers of mathematics, thus creating an activity system that demonstrates Engeström's approach.

Activity theory, as explicated by Engeström (1999), has been used widely in human-computer interaction, for example Nardi (1996a) and Christiansen (1996). He emphasises the use of activity systems as analytical frameworks to understand, amongst other activities, organisational set-ups, for example, a software team programming a system for a client, in which key players could be the team of software developers, project manager, subordinates and the client (Kuuti, 1996).

The pursuit of solutions to my research questions is central to my study, and although it will be interesting to look at Engeström's activity systems approach, I am concerned neither with organisational change nor collectively defined objects. Rather, it is the object-oriented activity of using MATLAB to solve DEs and the motives and motivations espoused by individual students as they take part in this activity that I seek to understand, along with their practice and actions related to this activity. I emphasise that the subject is not an isolated individual but that s/he acts in different social as well as societal networks, which situation, according to Lompscher (1999), is not to be ignored in the analysis of concrete activities using Leontiev's approach.

My understanding of Leontiev's (1981, 1978) theory of activity is that it has as its unit of analysis "*an activity",* whereas Engeström's is *"an activity system"*. This does not imply that Leontiev's concept of activity is not to be understood as an activity system, for although he explicitly focuses on activities in his theory his work does reveal implicitly the concept of activity systems. Reference to Leontiev's concept of activity systems is therefore to the system as encompassing "*activities",* which have at their core three levels, namely activity, actions and operations. I mainly examine *activities* (as elucidated by Leontiev) as a unit of analysis.

As data has been collected to inform the research questions, my observations of how students use CAS in mathematical learning, as well as the screen recordings that form an integral part of

my data collection, make the *activity, actions and operations* concept of activity theory suitable as a lens through which to analyse data. For example, when analysing the screen dumps I look for certain aspects that can be classified as an action (conscious doings), or as an operation (non-conscious aspects). Further, I am able to ascertain whether some of these actions have become operations as students proceed with their learning (see Chapter 5 for more details and exemplars).

3.5 CONSTRUCTS ASSOCIATED WITH LEONTIEV'S ACTIVITY THEORY

Leontiev proposed three levels for understanding activity, providing three vantage points from which I examine the data in this study. These are defined according to their *functionality* rather than their intrinsic properties (Wertsch, 1981a). According to Leontiev (1978, 1981), activities are undertaken in order to fulfil motives as well as needs or desires. At the uppermost level, collective activity is correspondent with motives towards obtaining a particular object or outcome. Put in a slightly different way, collective activity is driven by object-related motives. Kaptelinen (2005) and Foot (2002) pose significant questions about whose motives these are. I discuss Kaptelinin's (2005) views on the object of activity (in Section 3.5.4) and draw mainly on his knowledgeable contribution to the 'object of activity' and the corresponding motives that are linked to the object.

Leontiev's three-tier structure of activity and its associated concepts is illustrated as follows:

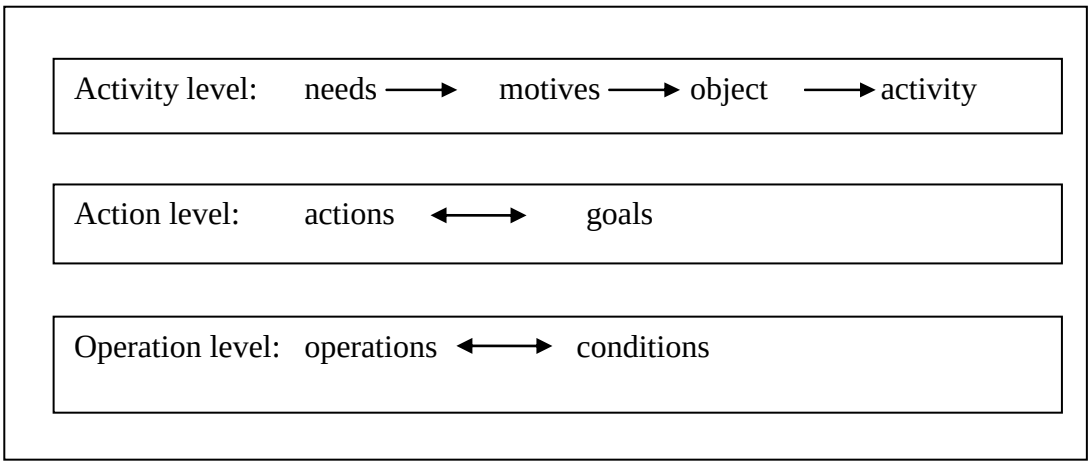


Figure 3.3: Leontiev's activity levels

3.5.1 CONSTRUCT OF ACTIVITY

Lompscher (1999, p.11, emphasis in original) wrote that an essential facet of human life is that "individuals ensure their existence and development by *activity*". Based on their interactions with the world, individuals change and shape their societal life and the formation and process of activity are determined by the object(s) of the activity as well as the conditions under which the activity occurs. Individuals use available means or tools in a relatively acceptable manner to translate aspects of the world into personally devised objects. This occurs "under concrete cultural-historical, societal and natural, social and individual *conditions*" (Lompscher, 1999, p.12, emphasis in original).

The outermost level is concerned with the global aspect of activity, which Leontiev (1978, p.50), defines as a "unit of life mediated by psychic reflection, the real function of which is that it orients the subject in the objective world". It is a structure delineating the context in which activity takes place, for example, a school-going activity, work or hobby. Activity therefore orients the student in the world in which s/he lives, and changes that world in a mutual way.

Consistent with this definition, I consider the activity associated with my analysis of data to be: *students learning to use MATLAB in mathematical learning*[13]. I take note that there might have been different activities that students pursue whilst they are in the computer laboratory and supposedly learning how to use MATLAB to solve DEs, but I have not collected data within that setting. My data has been collected purposively when students use MATLAB to solve DEs whilst I observe them doing so. In this context, I was particularly interested in their motivations towards this activity and their practice (actions and operations) as they went about learning to use this software. To gain a better insight into students' motivations towards CAS in mathematical learning and their practice in this context, I collected interview data on how they went about learning mathematics and their motivations towards it. No observations took place for this kind of data; therefore, I consider the associated activity to be *students' reflections on the activity of learning mathematics.*

In arguing that socio-cultural research must formulate its position in relation to the individual-society standpoint, Wertsch (1995) points out a need to formulate the task of socio-cultural research in terms of the relationship between mental functioning and socio-cultural setting. In

[13] Although the activity is *students learning to use MATLAB in mathematical learning*, this mathematical learning primarily involves, but is not limited to, the solving of DEs.

my study, the unit of analysis is the activity of *students using CAS in a mathematical learning environment*, which articulation also emphasises the unity of individual and context. Thus, activity could be viewed as a system that has its own formation with internal transformations and its own development: "Activity is a molar, not an additive unit of the life of the physical, material subject" (Leontiev, 1978, p.50).

Activity is the functional unit which mediates between people and their social and physical environments (Wertsch, 1981a), and is not simply derived through a series of reactions and actions. For Engeström (1987, p.28), "human practice is not just a series or sum of actions", and students participating in the activity of learning to use MATLAB in solving DEs bring to this learning context, amongst other things, their own unique histories which influence the ways they go about this learning. Accordingly, there is a rich historical, social and cultural context imbuing this activity, as opposed to simply a series of actions.

Gordon (1998, p.6) provides an exposition of *activity,* based on Leontiev's work, that "individuals act in accordance with their purposes and needs which are shaped by and reflect histories and resources, both personal and cultural". The individual, the social, and the cultural environments interact mutually and are inherent in this construct. I use this explanation of activities to position the development of those emanating from my data, and from an analytic perspective ask the questions: *Why does the activity exist? What motives does the individual have?* and *Which needs of the individual is the pursuit of this activity striving to fulfil?* Lompscher (1999, p.12, emphasis in original) wrote that activity is "*psychologically regulated* and characterised by different degrees of goal-directedness, consciousness and other aspects", thus the structure of activity could be described as hierarchical, suggesting that activity consists of and is realised by actions and in turn by automated operations.

3.5.2 CONSTRUCT OF ACTIONS

The middle level of individual (or group) action is driven by conscious goals, by which individual actions can be distinguished at the intermediate level within the activity system, and which are immediate in nature. Making up and realising the activity, for Leontiev (1981), *actions* are always directed towards a goal, while an activity is developed through its goal-directed actions, subordinated to conscious purposes (Engeström, 1987). For students using MATLAB in mathematical learning, such actions may include writing out entire MATLAB codes before typing them onto the computer, or even attending each laboratory session with the compulsion of making copious notes. Significant here is that the overall activity is focussed

towards certain motives and objects whilst individual actions may be directed towards more transitional goals. Actions are conscious behaviour with the intermediate goals determined "with regard to the social relations in which they are set" (Gordon, 1998, p.41).

Content, arrangement, inter-relations and hierarchy of actions depend on the physical activity of which they are components (Lompscher, 1999). The corresponding goals could be on different levels and some may be more significant than others, largely depending on the respective motivation, motives and needs that impel the activity. From an analytic perspective, it is at this action level of analysis that the question arises: how is the task of using MATLAB to learn mathematics carried out by the individual? This reflective mode may also correspond to the level of actions where occurs the planning, decision-making, comparison of techniques, review of concepts, change and re-evaluation (Norman, 1993).

3.5.3 CONSTRUCT OF OPERATIONS

The lowest level of Leontiev's activity system corresponds to routine operations and is influenced by the conditions and tools of the action at hand (Lompscher, 1999). Actions are conscious behaviours, while the means by which they are carried out are called *operations*, and depend on the conditions under which a tangible goal is realised. It is on this level that the conditions, particularly the tools, which demarcate the precise mechanisms for carrying out the action, are present (Gordon, 1998). For example, tools could refer to CAS. Operations may start out as conscious actions, but over time become automatic and non-conscious. For Popov (1998), they represent the technical side of carrying out actions, so can constitute a technique or skill, and require adroitness. The important characteristic of an operation is that it is carried out without conscious deliberation or attention to the process of execution, referred to as 'automatic process'. Operations allow people to act without thinking consciously about each distinct step, and represent the routine aspects of carrying out a task (Popov, 1998). This tendency to automation led Leontiev (1981) to see the general fate of operations as assuming the functionality of a machine. A contextual example of an operation would be a student becoming conscious of moving between different screens when using CAS for the first few times. This action is subordinated to a goal, that is, the ability to generate graphs when introduced to MATLAB. With experience, the act of moving between screens may be performed non-consciously as a means to some other action, finding and displaying the numerical and analytical solutions of a DE. This is then classified as an operation.

3.5.4 CONSTRUCT: OBJECT OF ACTIVITY

Leontiev (1978, p.52) draws attention to one of the most important constructs, known as the *object of activity*, stating that the term "objectless activity" is meaningless. Even though activity may appear to exist without an object, it is imperative that "scientific investigation of activity necessarily requires discovering its object". He also points out the dual nature of object of activity:

> …first, in its independent existence as subordinating to itself and transforming the activity of the subject; second, as an image of the object, as a product of its property of psychological reflection that is realised as an activity of the subject and cannot exist otherwise.

In other words the object is transformed into its subjective form (or image) and the activity is converted into objective results (Leontiev, 1981).

Involvement in activity forms the basis on which individuals transform parts or features of the world into an object, which they act upon (in accordance with the available means and possibilities) so as to fulfil their needs (Lompscher, 1999). In trying to comprehend the construct of 'object of activity', it becomes necessary to consider firstly the origin of this concept, and secondly the specific manner in which it is used within the two approaches of Leontiev and Engeström. Finally, the definition of the construct of object I use for this study has to be clarified and suitable exemplars given.

Activity theory is in its developmental phase, with the journal *Mind, Culture and Activity*, as recently as 2005, dedicating an entire volume to discussion around the construct *object of activity*. This, as one of its most basic concepts, also proves one of the most elusive, due firstly to the difficulties experienced in the translation of ideas from Russian to English, as well as the concept of object having originated in German philosophy, and secondly to the different interpretations attributed to the same concept within two approaches in activity theory, namely those of Leontiev and Engeström. The Russian words *objekt* and *predmet* have similar meanings, both translated into English as 'object', but there are subtle differences between them that ultimately influence their use in particular ways and contexts. For instance, Kaptelinin (2005) criticised Leontiev's book *Problems of the Development of Mind* (1959/1981) for not distinguishing clearly between the underlying meanings of the two words, however, in Leontiev's later book (1975/1978), *Activity, Consciousness and Personality*, his elaboration of the construct of 'object of activity' was presented in a way that did encompass their distinct

meanings (Kaptelinin, 2005). The term *predmet* denotes the object as having more *subjective* qualities, while in *objekt* they are more *objective* and represent material reality. *Objekt* "was used to describe a pole of the 'subject-object' opposition, through which opposition the notion of activity as a process of mutual transformations between subject and object was defined" (ibid., p.7).

It is apposite that the translations of the terms to their English equivalence of 'object' meant that their original meanings are at times still not given due prominence. It thus becomes a challenge, when engaging with the literature, to identify the orientation of object as having subjective or objective qualities. Kaptelinin (2005, p.8) proposes a few simple guidelines to assist in identifying the intended meaning of object[14]:

- Importance should be placed on the context in which the word 'object' is used.
- 'Object' is likely to have the meaning of *predmet* if there is prominence given to "intentional, social, meaningful and integrated qualities".
- In the expression of 'the object of activity' and related uses, object has the meaning of *predmet*.
- When one considers Leontiev's (1981, 1978) formulation of activity as a mediated 'subject-object' interaction, then in this context and related uses, object has the meaning of *objekt.*

The above clarification of object results in its particular use within the two perspectives of Leontiev (object used in the sense of *predmet*) and Engeström (object used in the sense of *objekt*). Table 3.1 (below) outlines the two perspectives on the object of activity, and is followed by a description of each.

[14] These rules are given in the context of readers engaging with the English translations of Leontiev's (1978) work.

Table 3.1: The two perspectives of object of activity (Source: Kaptelinin, 2005, p.11)

Facets of Activity	**Leontiev**	**Engeström**
Activities are carried out by	Individuals (predominantly)	Communities
Activities are performed	Both individually and collectively	Collectively
The object of activity is related to	Motivation, need ("the true motive")	Production (what is being transformed into the outcome)
Application domain	Psychology	Organisational change

For Leontiev (1978), the object (*predmet*, having subjective connotations) of activity is considered an object of activities carried out mainly by individuals (either collectively or individually) and is related to motivation[15] (Kaptelinin, 2005). The individual is the centre of attention in Leontiev's theory because he developed the concept of activity theory within the psychological domain of analysing individuals' participation in activity:

> Human psychology is concerned with the activity of concrete individuals that takes place either in conditions of open association, in the midst of people, eye to eye with the surrounding object world – before the potter's wheel or behind the writing desk. Under whatever kind of conditions and forms human activity takes place, whatever kind of structure it assumes, it must not be considered as isolated from the social relations, from the life of society. (Leontiev, 1978, p.51)

Hence, for Leontiev, all activities are social, even those that appear to be carried out in seclusion. From this perspective it is important to regard activity as embedded in societal and social contexts. Nevertheless, the awareness is primarily of 'concrete individuals' engaged in individual activities and taking into account the individual's motivation that provides the impetus for his or her participation in activities.

In this study I use the formulation of 'object' in the sense of Leontiev's *predmet*, depicted as 'aim', associated with subjective qualities and imbued with individuals' motivations, motives

[15] I explain this relationship and provide examples when discussing motives and needs in Sections 3.5.6, and 3.5.7.

and needs. Primarily concerned with their motives towards and practice (actions and operations) in the activity of learning to use MATLAB to solve DEs, object is likely to be associated with their intentions and the meaning they ascribe to this activity.

Nardi (2005) provides an illustration which helps to differentiate between the two senses of object. The first is linked to *objekt*, and denotes that which is to be realised, such as a cure for cancer; the second is linked to *predmet*, as a scheme of *motive-object*, such as contributing to making the world a better place, and is linked as a motive to the object of curing cancer. Finally, Foot (2002) draws attention to the term 'object' having the dual meaning in English of conceptual or material entity and aim, where sense is determined by the context. To me, the word 'object' as described by Leontiev's *predmet* has its sense embodied in the word 'aim', and henceforth I use the terms 'object' and 'aim' interchangeably.

3.5.5 THE RELATIONSHIP BETWEEN MOTIVES AND OBJECTS

According to Kaptelinin (2005, p.9), Leontiev considered the entirety of life processes of an individual as "an overarching context for activities (including actions and operations)" …

> Thus in the total flow of activity that forms human life, in its higher manifestations mediated by psychic reflection, analysis isolates separate (specific) activities in the first place according to the criterion of motives that elicit them. Thus actions are isolated – processes that are subordinated to conscious goals, finally, operations that directly depend on the conditions of attaining concrete goals. (Leontiev, 1978, p.66)

For Leontiev, activities are distinguished by motives that inspire them, and the tenet of object-oriented or object-related activity is the cornerstone of the theoretical framework of activity theory. This view of the object of activity incorporates the concept of motive, and, as mentioned above, the standard definition of object relates to the object-fulfilling motives through its goal-directed actions:

> According to the terminology I have proposed, the object of an activity is its true motive. It is understood that the motive may be either material or ideal, either present in perception or existing only in the imagination or in thought. Leontiev (1978, p.62)

While Gordon (1998) and Coupland (2004) do not explicitly distinguish between object and motive, there are two viewpoints of the statement "the object of an activity is its true motive". The first is in agreement, exemplified by Lompscher (1999) as objects which a student

represents cognitively, and which satisfy a need, becoming emotionally significant. As such they become the *real motive* of an object-oriented activity, and "In this sense, motives represent a unity of cognition and emotion" (Lompscher, 1999, p.12). The second position is one of disagreement, as argued by Kaptelinin (2005), that the definition poses several conflicting views. Firstly, "if the object of an activity is its true motive" then this means there is no difference between the constructs of 'object of activity' and 'motive of activity'. Hence, it appears that two powerful words such as 'object' and 'motive' are used to represent the same thing. Secondly, the concept does not tell apart true motives from untrue ones, leaving the distinction (if any) between them unworkable (Kaptelinin, 2005). Thirdly, how does one incorporate the multi-motive idea that was clearly introduced by Leontiev into the definition?

In his defence, Leontiev (1978, p.62) points out that "an activity does not exist without a motive", but can have several motives: "activity necessarily becomes multi-motivational, that is, it responds simultaneously to two or more motives" (ibid., p.123). While he does not provide adequate illustrations of it[16], this leaves an opening that could be filled in different ways, a flexibility in the way the theory can be used that my study aims to exploit.

Kaptelinin (2005), supported by Nardi (2005), proposes that the definition of the object of activity as "its true motive" be revisited, and calls for the separation of the object of activity from its motive, through introducing valuable models:

[16] Leontiev's examples are premised on an assumption that there is a 1:1:1:1 relationship between activities, needs, motives and objects (Kaptelinin, 2005).

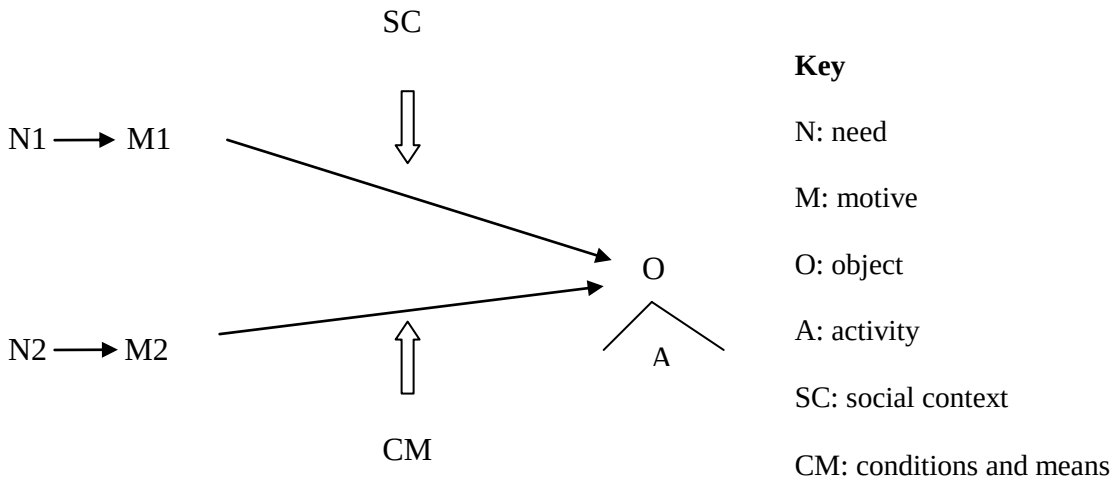


Figure 3.4: A model encompassing number of motives – one object - one activity (Kaptelinin, 2005, p.16).

I now discuss the connection between needs, motives, objects and activity in relation to the above model, in which there may well be one or more motives of the individual that influence the activity; furthermore these motives correspond to the needs of the individual. If there are several conflicting needs then these could be interpreted as different aspects of the same activity. The object is different from any effective motives and is jointly defined by the entire set of motives the individual makes every effort to realise, in and through participation in activity. Nevertheless, even if these motives are very powerful, "the activity does not have a direction and does not really start until the object of activity is defined" (Kaptelinin, 2005, p.16).

This idea is illustrated by Kaptelinin (2005) using another hunting activity, different from Leontiev's (1978) 'primeval collective hunt' example discussed above. Taking a hunting activity that is shaped by a few motives, for example food and self-preservation, he posits that if both are strong enough and the prey is dangerous then the hunter is faced with a dilemma, of either starving to death or assaulting the animal and run the risk of being killed. More probably, the hunter's activity will be focussed towards a desired result that will make attaining both motives possible. The hunter could chase the animal until it gets tired and stops being a danger, in which case both food and self-preservation are the motives. The object associated with this hunting activity is safely to obtain food by chasing the animal until it gets tired, which is what

provides the structure and focus for the activity. Although this example does not explicitly point out the needs of the individual, one can postulate that it is survival.

In summary, the features of the model (Kaptelinin, 2005) used in this study are:

- There may well be many motives but there is only one object of the activity.
- The object of activity is jointly determined by all effective motives, and is dynamically constructed by taking into account the various constraints that exist when individuals interact with and participate in it. These may comprise the needs of the individual that the activity is striving to fulfil, accessible or available means, other likely connected activities and other individuals involved (each with their personal motives and objects).
- The object of activity is both motivating and directing the activity.

This model makes possible addressing my concern with poly-motivated activities.

3.5.6 CONSTRUCT OF NEEDS

There is a profound and close relationship between the concepts of object-oriented activity, need and motivation in Leontiev's activity theory framework, and I now seek to clarify his notion of 'need', particularly in relation to motivation. Arguing against the established convention in psychology which considers the organic needs of an individual as the basis of human motivation, he criticised the concepts and terms used in traditional psychology: "There is no need to investigate all of these confused concepts and terms that characterise the present condition of the problem of motives" (Leontiev, 1978, p.116), by which he meant desire, interest, impulse, emotion and instinct.

Leontiev (ibid., p.62) introduced his well-known phrase "the object of an activity is its true motive" with the purpose of addressing this problem of mystifying concepts related to motivation (Miettinen, 2005). A need becomes a motive capable of directing the actions only when the need finds its object, and this takes the form of cultural and historical object. Human needs could also be described as having a historical materialistic sense (Leontiev, 1978), such that they "are disconnected from their biological foundations and connected to the development of the material culture … [and, citing Marx, 1993] … a historically created need has taken the place of the natural one" (Miettinen, 2005, p.54). Hence, this notion of needs evolves from an individual's human practice and interaction with activity, culture and society as opposed to

needs that arise from a biological basis. This forms one of the main ideologies of Leontiev's activity theory framework.

In providing a coherent consideration of the nature and sources of distinctively human needs within an activity theory perspective, Miettinen (2005) discusses the idea of the surfacing of higher object-related needs in the context of collective work activities. This approach is steeped in the belief that needs are social in origin. He proposes that *artefact-mediated desire for recognition* is a resource for making sense of the formation of individual motives within collective work activities, and points out that the concept of *desire for recognition* is based on the position advocated by Hegel (1983). Miettinen (2005, p.55) notes that in this conception, a "…constitutive human social need, a desire for approval and recognition is taken as a starting point. This social need can become attached to any cultural object that comes to symbolise it". It is relatively uncomplicated to consider the notion that desire for recognition is both a social and a cultural need, but how does this compare with Leontiev's development of needs in his activity theory framework? Leontiev (1978, p.59) declares the social nature of needs in an activity: "…that is indigenously social, that is, develops only under conditions of cooperation and sharing by people". Hence, there is comparability between the notions of needs as advocated by Hegel and Leontiev's (1978) theory of needs (Miettinen, 2005). It is important to note that needs developed by the latter theorist are within an object-oriented activity context[17].

An individual's participation in activity not only satisfies his or her needs but also produces new needs of material and cultural origin: "Through activity people not only modify objects, means and conditions, they change themselves as well and become conscious of this activity" (Lompscher, 1999, p.12). Further, Lompscher (1999) proposes that needs (organic as well as cultural) have to be satisfied by appropriate objects, initiated and discovered through the individual's human-world interaction, that is, through the process of activity. Needs could be viewed either as in starting points and presuppositions of activity, or as "results or products of activity in the sense that they are assigned a meaning by concrete objects" (Lompscher, 1999, p.12). For Leontiev (1978), needs emerge initially only as a condition and prerequisite for activity, but once the individual begins to act on them, a transformation instantly takes place and they stop being what they were, as needs in themselves. However, as the activity proceeds the more the needs are converted into their result. The most important change that involves the

[17] Miettinen (2005) points to an elaboration of object-oriented and social nature of human needs by citing Holzkamp-Osterkamp's theory of needs, wherein she makes a distinction between biological needs and productive needs. The latter involve primary needs, such as controlling one's own social and cultural environment by contributing to the upholding and advancement of community life.

transition of needs to the psychological level "consists in the beginning of the active connection of needs with the objects that satisfy them" (ibid., p.116).

I emphasise Leontiev's (1978) description of needs as one that originates within a social and historical context, where an individual has needs that necessitate the satisfaction thereof, so that s/he can maintain her/his physical existence in society. Within this perspective I draw on categories of psychological needs that are frequently underscored in educational situations, including autonomy, competency and social belonging (Boekaerts, 1999). These are not incompatible with Leontiev's description of needs, as he alludes to the formation of special types which he refers to as "objective-functional", such as work and artistic creation.

In delineating the categories of needs for this study, I now give exemplars. Social belonging could include a desire for recognition; for example, recognition as someone 'clever' enough to use computers for learning mathematics. Some students are eager to learn to use MATLAB in mathematical learning, not only because they envisage its usefulness when they join industry as graduate engineers but also because they wish to satisfy needs of becoming competent users of the software. An individual may also realise a need to be competent by choosing to solve mathematical tasks confidently, or a need to have a good understanding of a topic in mathematics mediated by CAS. Some students choose to study mathematics and computers in mathematical learning because ultimately they desire to become professionals, and in this way satisfy their need to function as independent and self-sufficient members of society. For Foot (2002), 'need states' include the intrinsic love of learning, the development of one's academic career or the attainment of income to provide for material necessities of life. These need states are all well-suited to Leontiev's activity theory framework, as some students choose to study CAS in mathematical learning because of their intrinsic love for the learning of mathematics (see Chapters 5 and 6).

3.5.7 THE ACTIVITY THEORETICAL CONCEPTION OF MOTIVATION AND MOTIVES

Motives are to be understood as psychological components of activity. Leontiev's (1978, p.116) explanation of motive is that it takes on an arousing and directing function of the activity:

> The fact is that in the subject's needy condition itself the object that is capable of satisfying the need is not sharply delineated. Up to the time of its first satisfaction the need "does not know" its object, it must still be disclosed. Only as a result of such disclosure does need

acquire its objectivity and the perceived (represented, imagined) object, its arousing and directing function of activity; that is, it becomes a motive.

Further, Leontiev (1978) points out that subjective experiences and wishes do not make up motives because in themselves they are incapable of producing directed activity. Instead, the psychological problem is for one to realise what constitutes the object of the given desire or wish.

Lompscher (1999) provides an explanation of the concept of motivation regarding learning activities as an individual proceeds through life from the stages of a pre-scholar. Based on the work of Leontiev (1994), he notes that there are three main levels of general motives for learning as an individual proceeds through schooling years:

1. The level of motives that lie in learning itself (when the individual is at pre-school stage s/he takes on a new role as a pupil and her/his motive is to partake in a learning activity that is valued by society).

2. The level of motives that lie in school life and relationships within class and to the school collective community (when learning becomes the usual everyday activity, new learning motives are formed, for example, pursuing good grades or position in class).

3. The level of motives that lie in the world, such as future career and life's opportunities (towards the last few years of schooling learning motives take the form of preparing for future life).

From an activity theory perspective, Lompscher (1999) proposes three broad groups of motives, namely, those that reflect a *social nature*, those connected with *self-related* reasons, and those that correlate with *cognitive* motives.

Social nature motives take the form of identifying with teachers or others in the peer group; communication between peers and teachers; co-operation with teachers and other learners; interrelations with peers, teachers and other members of the educational society; learning as a means to help and support others; and feelings of duty to learn. These in a way depend on the social and societal conditions of life and the existing conditions and state of affairs in school and classroom.

Self-related motives are more intimately connected with the individual learner, "he himself, his own personal development and well-being, his achievement and self-perfection, his success and

position compared with others" (Lompscher, 1999, p.17). This category emphasises the way the student considers him/herself in an individual situation, and how s/he positions her/himself in relation to others[18].

Different levels of **cognitive motives** coincide with the developmental levels of an individual as s/he interacts with the learning activity. A lower level of cognitive activity is motivated by learning motives related to the student learning isolated facts, singular relations and similar things, and the student being most interested in getting a result (namely surface approaches to learning). With development of thinking, cognitive motives reach a higher level and students become interested in the methods and modes essential to arrive at a result. The use of cognitive tools features prominently when students use deep approaches to engage in subject matter. Those who have learned that there is more to learning than a surface approach begin to gain interest in these methods and find ways that reveal deeper approaches and understanding. This, according to Lompscher (1999), is a powerful learning motive because it is not limited to singular observable facts or results, but rather encompasses good understanding of the material. Such learning is dominated by intrinsic interest in the learning object itself and could result in learning over longer periods. The lower level of cognitive motives directed towards superficial learning is not able to inspire long-lasting cognitive activity because it can be easily satisfied and hence quickly lose its motivational possibilities (Lompscher, 1999).

In the above categorisation of motives it may appear that self-related and cognitive motives are not social in nature. Seen from an activity theory perspective these latter groups of motives are undeniably social in nature, even the most individual action or motive being characterised as social by Leontiev (1978). I remind the reader that Leontiev's (1978) conception of the social disposition of activities and actions that appear as individualistic in nature are imbued with social practices and relations:

> Human psychology is concerned with the activity of concrete individuals that takes place either in conditions of open association, in the midst of people, eye to eye with the surrounding object world – before the potter's wheel or behind the writing desk. Under whatever kind of conditions and forms human activity takes place, whatever kind of structure it assumes, it must not be considered as isolated from the social relations, from the life of society. (Leontiev, 1978, p.51)

[18] I discuss how self-related motives are associated with mastery and performance approach goal-orientations in Chapter 3.6.

Furthermore, I draw attention to Lompscher's (1999) characterisation of social motives, which underscores the interrelations of an individual with others, for example, teachers or peer group members. In his conception of social motives he considers the learner as part of the educational activity system and as such this category focuses on the interactive nature of learning with others, namely, learning as a means to help and support others; and feelings of duty to learn. These in a way emphasise the broader societal conditions of life and the existing conditions in the school and classroom. Notwithstanding, in this thesis I use these categories as analytic ones to effect a consistent analysis of the various forms of motives and to draw comparisons with the Western categories of motivation, to be discussed in 3.6. However, I argue throughout that the individual is a 'being' in context.

Individual motives are established in the course of activity, not merely given (Lompscher, 1999). Motives are distinctive attributes of human beings and cannot be explained by the biological formation of an individual. Miettinen (2005, p.53) asserts that motives emerge in the appropriation and "development of objects and artefacts in collective human activities". In the course of activity, new motives regularly surface or change their arrangement and significance, while some also lose their strength and disappear gradually. Lompscher (1999) draws attention to an important consideration in the study of motives of an activity, namely that each structural component of human activity, whether it is the object, social relations, means and conditions or cause and consequences, could in this process become learning motives of further activity.

The hierarchical structure associated with Leontiev's conception of the levels of activity is primarily concerned with motivation (Lompscher, 1999). At the activity level, human activity is inspired by various motives which differ in significance and position in the motivational structure. These may also come into disagreement with each other or appear on an equal standing. Motives that give the activity its personal sense are described as 'sense-forming' (Leontiev, 1978), and determine the personal meaning of an activity for an individual, as well as the nature of the activity and its location in his or her life and activity system (Lompscher, 1999).

The activity structure emphasises the relationship between motives and goals, a possible strength of this conceptualisation lying in the manner in which an activity theory perspective emphasises how goals develop, what they depend on and how they operate (Lompscher, 1999). This is in contrast to other motivational theories which view goals as given. An important aspect of goals in the activity theoretical framework is that they derive from motives. As Lompscher (1999, p.13) writes: "motives generate goals for actions necessary for reaching the desired

result". He points out that one of the functions of goals lies in anticipating results and hence they establish the path towards the results, directing the action, which in turn constitutes part of a motivated activity. Goals are subordinate in relation to motives. Leontiev (1978) stresses the difference between motives and goals in the psychological analysis of activities, while the personal assessment of reaching a goal depends primarily on the relationship between it and the motives leading the whole activity (Lompscher, 1999).

It is apposite to examine how the above elaboration of the groups of motives fits in with Leontiev's concept of motivation. In this study, I follow Leontiev (1978) and define motives as the energising, personal reasons that prompt an individual to participate in a specific activity, that is, have a directing function. This conceptualisation implies that the abovementioned three groups of motives are directly linked to the concept of motivation as put forth by Leontiev. The important aspect is that motivation in this framework is considered "in its substance as object-related and object-determined as is activity itself" (Lompscher, 1999, p.12). Hence, throughout this study I deem motivation to be intimately linked to the object-oriented activity (of learning mathematics using CAS) as opposed to a description of it in its generality as a broad psychological construct. The concept of motives viewed in this activity theory framework involves how an individual acts with regard to a certain collectiveness of relationships, that is, toward society and toward the person himself (Leontiev, 1978). An example could be work activity, which is socially motivated and directed towards material reward (Leontiev, 1978). The latter motive fulfils a stimulating feature of the activity.

Research in the activity theory paradigm is continuing to evolve rapidly, with important constructs such as motivation, emotions and identity having only recently been studied within the framework of cultural-historical activity theory (CHAT), for example, Roth et al. (2004) and Roth (2007a). These concepts have been deliberated upon in ways that bring to light the important contributions of culture, social relationships and personal histories towards understanding the identities and motivations of an 'individual'[19] within the activities of learning scenarios and workplace environment (in mathematics and science education contexts).

Roth's (2007a) significant contribution to third generation CHAT was made by putting forward a way in which emotion and its associated dimensions of identity and motivation can be incorporated into activity theory. He developed a model to illustrate how emotions, motivations

[19] The term 'individual' is used in the same way as conceptualised throughout this thesis: individual in union with the context.

and identity are incorporated into an approach grounded in CHAT, in which *practical action* plays a key role (e.g. as analytic starting point), based on the integral relation between actions and emotions.

The theory of motives may involve a self-indulgent nature for individuals, if "…All activity of man is in some way subordinated to the principle of maximising positive and minimising negative emotions" (Leontiev, 1978, p.120). An individual may wish to experience satisfaction and be free from suffering, and these comprise underlying motives that drive him or her to make choices as to which activity to become involved in. "Emotions are not subordinated to activity but appear to be its result…" (Leontiev, 1978, p.120).

Roth (2007a) conceptualises emotional valence and its associated payoffs as individuals making choices and deciding on which activities to participate in, so that there is some reward either on a short- or long-term basis. Their framing of goals that have a higher possibility of success implies a higher emotional valence (Roth, 2007a), and from this perspective one has to analyse practical actions and practical reasons to gain an understanding of emotions. Motivation builds on the motive, goals and corresponding emotional valences available in practical actions; hence it interrelates with conscious or non-conscious contemplation of emotional rewards that result from the action. Some examples of reward are satisfaction, expansion of action possibilities, sense of success and expansion of control over life conditions (Roth, 2007a). Even though I do not examine the aspect of emotions in this study, I draw upon Roth's (2007a) rich description of payoffs.

3.6 ESTABLISHING THE LINK BETWEEN MOTIVATIONS AND MOTIVES

In Chapter 2, I outlined the theory of motivation as expounded by Western motivational theorists. Accordingly, the definition of motivation is given as "motivation *is the process whereby goal-directed activity is instigated and sustained*" (Schunk et al., 2008, p.4, emphasis in original). Pintrich (2003) defines 'goal-orientations' or 'achievement goals' as the purposes or reasons for engaging in academic tasks. In Chapter 3.5.7, I drew on Leontiev's definition of motive as taking on an arousing and directing function of the activity. In the context of this study I define motives to be the energising, personal reasons that prompt an individual to participate in a specific activity, in other words that have a directing function of the activity. Motives give the activity its direction and content.

The theory of achievement goals as outlined in Chapter 2 is not within a cultural, historical framework, mainly due to the excessive use of surveys argued therein. However, achievement goals do provide elaboration of motives that can fit into the activity theory framework if they take into account the cultural, historical and social aspects. Hence, I propose to use the elaboration of achievement goals by considering these goal-orientations as subsets of the broad theory of motives proposed by Lompscher (1999). In this way, when I use achievement goals I also ensure that these goal-orientations have a directing function of the activity and are imbued with cultural and historical aspects.

Lompscher's (1999) three categories of motives (social - interrelations with others, self-related and cognitive, 3.5.7), that form the broad categories for this study, provide a comprehensive outline; however one criticism of Lompscher is that he does not adequately operationalise them. Hence, I also draw on the work of Western motivational theorists to provide an in-depth explanation of some of the motives (see Figures 3.5 and 3.7, below). I now discuss how achievement goals and expectancy-value theory can be viewed as subsets of the categories of motives.

Recently, achievement goal theory has placed emphasis on social goal-orientations (Chapter 2), and these goals are compatible with the category of social motives as proposed by Lompscher (1999). In Chapter 2, I discussed social goal-orientations as comprising social approval goal-orientations, social responsibility goal-orientations, social status goal-orientations and social affiliation goal-orientations. Social learning motives as proposed by Lompscher (1999) take the form of identifying with teachers or others in the peer group; communication between peers and teachers; co-operation with teachers and other learners; interrelations with peers, teachers and other members of the educational society; learning as a means to help and support others; and feelings of duty to learn.

Performance approach and performance avoidance goal-orientations find their source in cultural motives (such as motivated by status). Those who hold performance approach and performance avoidance goal-orientations consider themselves in relation to others and these goal-orientations could easily be considered as subsets of the category of self-related motives.

Three components of expectancy-value theory, namely, students placing a value on the task component, how they value the importance of the task, and their extrinsic utility value, all relate to social and cultural value components. For example, the cultural setting determines how students value the importance of learning to use CAS in mathematical learning. Extrinsic utility

value has sources in students' needs to graduate as professional engineers and hold jobs in this field. The abovementioned value components could be considered as subsets of the broad category of self-related motives. The fourth component of expectancy-value theory, namely intrinsic interest, is considered as a sub-category of cognitive motives.

Cognitive motives, according to Lompscher (1999), point to students being interested in the methods and modes essential to arrive at a result. Such students are interested in deeper approaches to learning and learn with understanding. Mastery goal-orientations fall into the category of cognitive motives as students who hold such are motivated to have a deep understanding of the learning task and are inherently interested in learning the subject matter. Viewed in this way, students with mastery goal-orientations will display actions that are related to using deep approaches in learning. The value component of intrinsic interest or value can be viewed as part of this cognitive motives category.

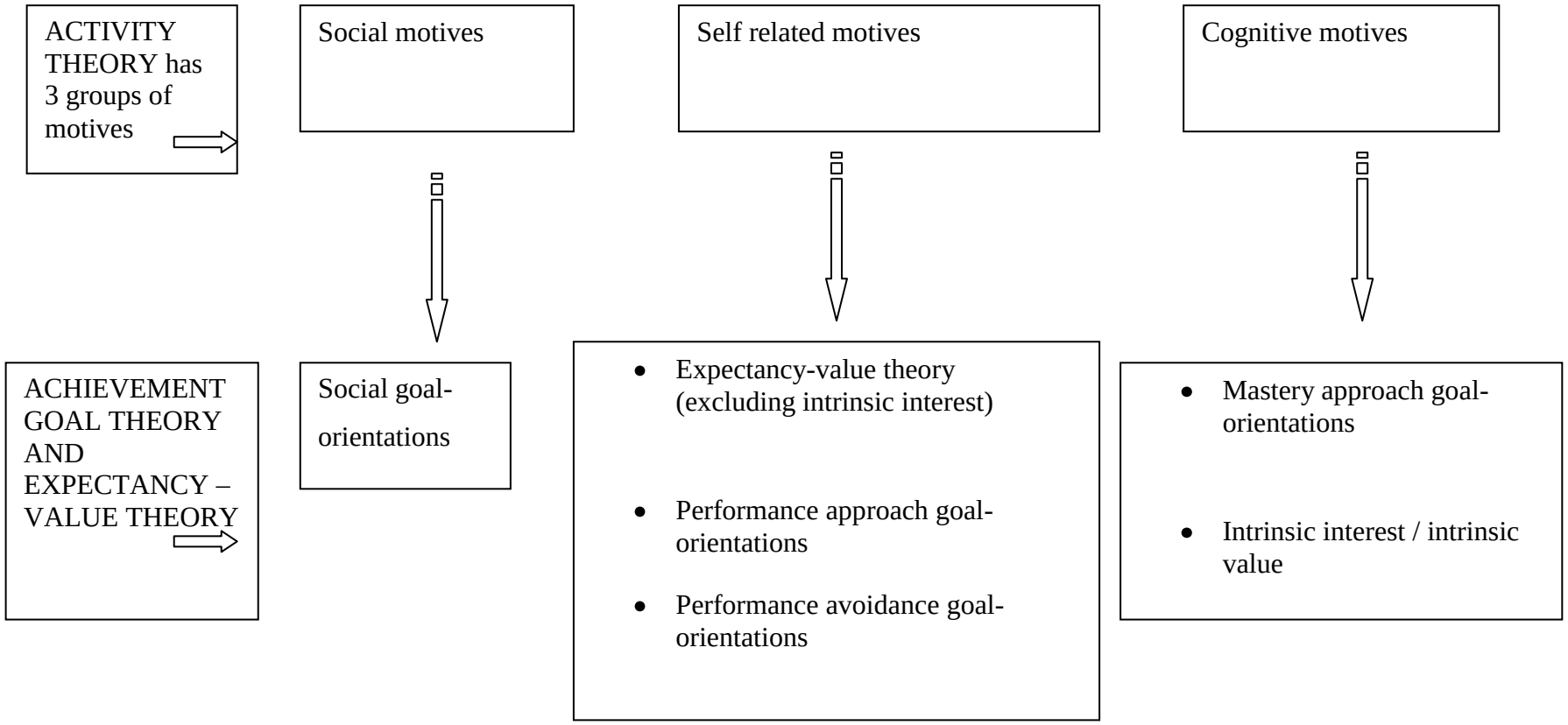


Figure 3.5: Link between motives (activity theory, achievement goal theory, and expectancy-value theory)

3.7 LEARNING AT THE LEVELS OF ACTIONS AND OF OPERATIONS

The action and operation levels of an activity provide distinct stages at which to analyse a learning activity. Conscious actions are associated with the act of planning and reflection, while non-conscious operations are related to the acquisition of tacit or understood knowledge (Popov, 1998). Learning at the level of operations requires students to know automatically the learning matter, whether it is easy or difficult. This could involve a student mastering the work either through understanding or through swotting and cramming. The alternatives feature at the level of actions, and analyses at both levels are vital for insight into students learning mathematics using CAS.

In relation to learning with technology, Norman (1993) posits two modes of cognition, *experiential* and *reflective*. The former correlates with the level of operations, seen by Popov (1998) as expert behaviour and skilful performance, where the individual acts effortlessly and without a need for planning or contemplation. For Norman (1993), this mode primarily involves reactive cognition, in other words the automatic or spontaneous responses of an individual, for example, a student studying mathematics at university not pausing to think about the answer to 1+1. The latter, reflective mode, correlates with the level of actions, and involves contemplation, reflection, decision-making, planning, comparing and contrasting concepts, and problem-solving methods. It leads to new ideas, novel or original responses, and involves conceptually driven cognition (Norman, 1993). Learning at the level of actions implies that the student is an active participant in the meaning-making process. When reflective tasks become routine they are carried out in experiential or operation mode, for example, the skilful act of knowing complex procedures in a mathematical problem-solving scenario. Students internalise a process that previously required conceptual or other tools. In this study, an important aspect to consider is that actions mediated by tools could change to operations, so that the tool recedes from conscious attention (Leontiev, 1978; Roth, 2003).

In providing a historical overview of the school-going activity, Engeström (1987) puts it that text (including arithmetic algorithms) takes the role of an object instead of an instrument or tool in the learning process:

> This object is moulded by pupils in a curious manner: the outcome of their activity is above all the same text reproduced and modified orally or in written form (summarised,

classified, organised, recombined, and applied in a strictly predetermined manner to solve well-structured, 'closed' problems). (Engeström, 1987, p.52)

The text in the activity systems, where it is created and used, is commonly referred to as 'dead text', that is, it "becomes a closed world, a dead object cut off from its living context" (Engeström, 1987, p.53). Problem-solving in mathematics for engineering courses within the vocational stream at tertiary level, related to the mathematics in which the participants in this study engage, is not about solving real problems from the real world but about solving problems in simulated engineering contexts often devoid of salient constraints and variables. Learning 'dead text' does have rewards, whether in the form of gaining marks or grades and entrance to higher studies or obtaining scholarships.

3.8 THE NOTIONS OF MEDIATION, CONSCIOUSNESS AND CONTEXT

Throughout this chapter, I have considered aspects of mediation, consciousness and context. These constructs are dominant in activity theory and warrant further attention.

Tools mediate thought and behaviour during the interaction between the subject and the context of the learning activity (Engeström, 1987; Nardi, 1996c). They may be material tools, such as pen and paper techniques, computer, MATLAB and textbooks; psychological tools, such as symbolic systems (algorithms, syntax, multiple representations); or knowledge of concepts and rules, learning strategies and techniques (Lompscher, 1999). Tools can mirror cognitive functions, for example a process to find the numerical solutions of DEs, where values are repeatedly or mechanically substituted into formulae in order to produce the solution.

From an activity theory perspective, the mediating artefacts usually refer to the tools that the subject uses to act on the object space. In my study one could consider the material tools that the subject uses to solve DEs as being the computer, CAS (MATLAB), pen and paper and 'textbook'. Students engage with the MATLAB software and its particular syntax in this course so that they can solve DEs by programming MATLAB. Consequently, they are taught programming skills in MATLAB with this specific purpose in mind.

Leontiev (1978) argues that meanings mediate thought but in themselves cannot give rise to it. He conveys the notion of personal meaning-making as:

> an investigation of the formation of mental processes and meanings (ideas) may express only one part of the total movement of activity, but this may be a very important part: the assimilation by the individual of methods of thought worked out by humanity. But this does not cover only cognitive activity, its formation, or its function. Psychological thought (and individual consciousness as a whole) is wider than those logical operations and those meanings in whose structures they are encased. (Leontiev, 1978, p.60)

The individual's involvement in activity, which has come about because of cultural historical development, results in ascribing meaning to various aspects of the activity, including processes at the actions and operations levels, and these meanings impact consciousness in larger ways, often beyond involvement in the activity.

In the school of CHAT, Vygotsky and Leontiev have attempted to explain how the individual mind or consciousness develops. This is also reflected in the title of Leontiev's (1978) book: *Activity, Consciousness and Personality,* which explicitly refers to activities as the mediator between individual and reality:

> to the proposition that internal psychological activities originate from practical activity, historically accumulated as a result of the education of man based on work in society, and that in separate individuals of every new generation they are formed in the course of ontogenetic development is attached yet one more very important proposition. It consists of this that simultaneously there takes place a change in the very form of the psychological reflection of reality: Consciousness appears as a reflection by the subject of reality, his own activity, and himself. (Leontiev, 1978, p.59)

It becomes evident that, within this activity conception, consciousness is a reflection of some process. It is by means of one's various actions, including mental ones such as reflection, that one transmits images from participation in activities and the external world to the internal cognitive sphere. Activity thus mediates between external environments surrounding humans and their internal domain (Gordon, 1998).

Roth (2007b, p.657) points out that when Leontiev and Vygotsky write about mediation, "it always pertains to conscious activity and its reflection in mind". Hence, "the activity of the subject, external and internal, is mediated and regulated by a psychic reflection of reality" (Leontiev, 1978, p.75). It can be deduced from this that from Leontiev's theory of activity, mediation should be considered at the levels of activity (which is driven by conscious motives) and actions (which are driven by conscious goals). Operations, which can be viewed as

conditioned events, emerge from "a dialectic of structures of both *material* and *psychological respectively*" (Roth, 2007b, p.658, emphasis in original), and that "all human experience is shaped by the tools and sign systems we use", with mediators connecting people personally to the world in which they live (Nardi, 1996a, p.9).

In the state of operations, Roth (2007b) finds the tool withdrawing from conscious activity altogether and not standing between the subject and object of consciousness. Due to the immediate nature of operations and non-conscious state, operations are therefore non-mediated performances. For example, initially students are very conscious about whether to use the MATLAB editor or an M-file to type in the syntax and code, but after a few practice sessions this conscious choice fades and joins the level of non-conscious operations, allowing them to direct their consciousness to other matters. External material entities can thus be seen as transforming into processes that occur at the mental level of consciousness (Coupland, 2004). For Nardi (1996a, p.7), "the object of activity theory is to understand the unity of consciousness and activity. Activity theory incorporates strong notions of intentionality, history, mediation, collaboration and development in constructing awareness". This activity theorist argues that consciousness is not a collection of isolated cognitive actions, such as decision-making and remembering, but rather is located in one's everyday practice. As Leontiev argues, "…consciousness is a product of society, it is produced… consciousness is *co-knowing*, but only in that sense that individual consciousness may exist only in the presence of social consciousness and of language that is its real substrate" (1978, pp. 57,60, emphasis in original). It is during the process of material production that language is produced and is a carrier of the socially developed meanings contained in it.

Of significance to this study, when considered in human-computer interaction, is that consciousness carries with it the notion of access to cognitive resources. For example, someone skilled at solving DEs has an "associated mental ease and access to certain cognitive resources peculiar to experts who have become very good at something" (Nardi, 1996a, p.11). Learners or trainees, on the other hand, spend time on, and consciously struggle with those actions that would eventually become automatic operations necessitating slight or no conscious awareness. Consequently, they deliberate on task actions whilst working with fewer cognitive resources, and, with time and experience, more of these deliberations reach the level of operations and increase their cognitive resources (Nardi, 1996a). This has a direct implication for my study as I analyse data pertaining to students or trainees and their corresponding actions and operations as they participate in the activity of mathematics (with CAS).

It is important also to elucidate the terms 'subject' and 'object of activity', to enable a thorough analysis of activity. Roth positions this as follows: "an activity cannot be reduced to the subject of activity or the object of activity, because these terms are mutually exclusive yet constitutive of each other", but although innately different, they are "expressions of the *same* consciousness" (Roth, 2007b, p.660, emphasis in original). In the context of students participating in the activity of learning how to use MATLAB to solve DEs, "The unit cannot be reduced to them although they constitute the unit" (Roth, 2007b, p.660). Choosing to study MATLAB in solving DEs implies that someone is making this choice and that something (MATLAB) is being studied. In my study, I endeavour to illuminate how this learning activity develops, how it shapes the individual and how it is also shaped by the context and setting of which the individual is a constituent part. For Leontiev (1981, p.47) "with all its varied forms, the human individual's activity is a system in the system of social relations. It does not exist without these relations".

Nardi (1996c) argues that one is motivated to study 'context' because this helps to understand relations amongst individuals, tools and social others. In Chapter 3.5.1, I argued from an individual-society standpoint that the unit of analysis is the activity of *students using CAS in a mathematical learning environment*. This then compels me to "focus on the unfolding of real activity in a real setting" (Nardi, 1996c, p.71), with the context as an important shaper of activity. However, for Leontiev (1978), what mattered was that examining actions and operations in individual experiments became significant only if they could be understood in the wider context of the study of the unity of the subject and object, and of the social, cultural and historical nature of the relations between man and the objective world. Consequently the role of the institution in the social constitution of learning is given attention throughout this thesis.

The construct of context in activity theory may be interpreted in different ways, one of which is that the activity itself is the context, in the sense that the actions, operations, motives, and objects *are* the context (Nardi, 1996c). Individuals empowered with their own decision-making abilities "consciously and deliberately generate contexts (activities) in part through their own objects…" (ibid., p.76). Context is entwined with individuals, activity, artefacts and other individuals influencing the activity, so context could be viewed as both internal to individuals, involving their goals and motives, and at the same time external, involving artefacts, the role of other people and specific settings. For Nardi (ibid., p.76): "in activity theory, external and internal are fused, unified". Throughout this study, I emphasise the indivisibility of individual and context.

The importance of the activity theory framework to this investigation is twofold. As Gordon (2004, p.42) puts it, "firstly the framework centres our attention on student's actions, including mental actions and their goals – what students do and why they so act. Secondly, it brings the context of learning to the foreground". When students learn numerical methods with CAS they interpret it within their own contexts. The aforementioned notions of context are illustrated in Chapter 6.

3.9 DEVELOPMENT OF MODEL

Modifying the illustration of activity levels that Roth (2007a) put together, I use Figure 3.4 (taken from Kaptelinin, 2005) to develop my model of the activity levels for learning mathematics using CAS (Figure 3.6, below). My analysis of students' activity of learning mathematics mediated by CAS is carried out using this model as an analytic means. Linking my two main research questions with this model will show how they can be answered or elaborated through using it:

1. What are the motives of students towards using CAS as a tool in mathematical learning?

2. How do students use CAS in mathematical learning (i.e. what are their actions and operations from an activity theory perspective?)

To answer question 1, I draw on Leontiev's elaboration of the activity level and action level. The former is associated with the dimensions of individual needs, motives and objects. For ease of reference, I provide an overview of the constructs of needs and motivation in Figure 3.7 (below), which also illustrates the origins of the motives. Motivation can also be surmised from students' actions, such as choice of tasks, effort and persistence (Chapter 2), hence I infer motivation from these, either reported to me in interviews or determined from my observations of them using CAS in mathematical learning). To answer question 2, I draw on Leontiev's notion of the constructs of actions and operations, by which students' goals as well as the means needed to carry out the activity are accentuated.

As mentioned above, the inclusion of CAS in the mathematical learning process may evoke feelings, needs and motivations, which will influence, for example, the length of time students persevere with learning tasks and the kind of learning approaches and strategies they adopt. My investigation seeks to understand how and why they use CAS in the manner they do from the activity theory perspective. Throughout this investigation, the emphasis is on the individual

acting in his or her social and cultural world. In this framework, motivations of students are inseparable from their thinking and actions.

The activity theoretical framework is valuable for examining the intricacies involved in students learning mathematics with CAS, and their motivations. This framework is suitable as it relates students' *actions* and their *motivations* to the context of learning. In other words, activity theory provides a way of interpreting and analysing students' actions, operations and motivations when using CAS in learning mathematics.

3.10 SUMMARY

In this chapter, I laid the foundation for the use of Leontiev's theory of activity for analysing and interpreting data, as well as discussing findings. I clarified and deliberated upon a considerable number of constructs that I use in this thesis. To this end, I examined notions of activity, actions, operations, object of activity, the relationship between motives and objects and needs. The activity theoretical conception of motivation and motives was investigated and I largely used research by Lompscher (1999) to establish a link between these two constructs. I considered what it means to use the framework to investigate learning at the levels of actions and operations. The broad ideas of mediation, consciousness and context were reviewed. Most importantly, the development of the model that underpins this study was produced, by drawing on Kaptelinin's (2005) and Roth's (2007a) prototypes.

CHAT can provide an impressive methodological framework, a valuable theoretical structure and pertinent terminology and expressions for analysis of learning activities in the field of mathematics, science and computer education. The theory as explicated by Leontiev (1978, 1981) illuminates the strong links between personal sense which is related to the reality of the individual's life (objects, motives and needs) and the dominant culture of learning in society and real life. The framework provides a way of analysing tool- or computer-mediated learning activities that are central to the development of society.

In the next chapter I make clear the methodological aspects of this study.

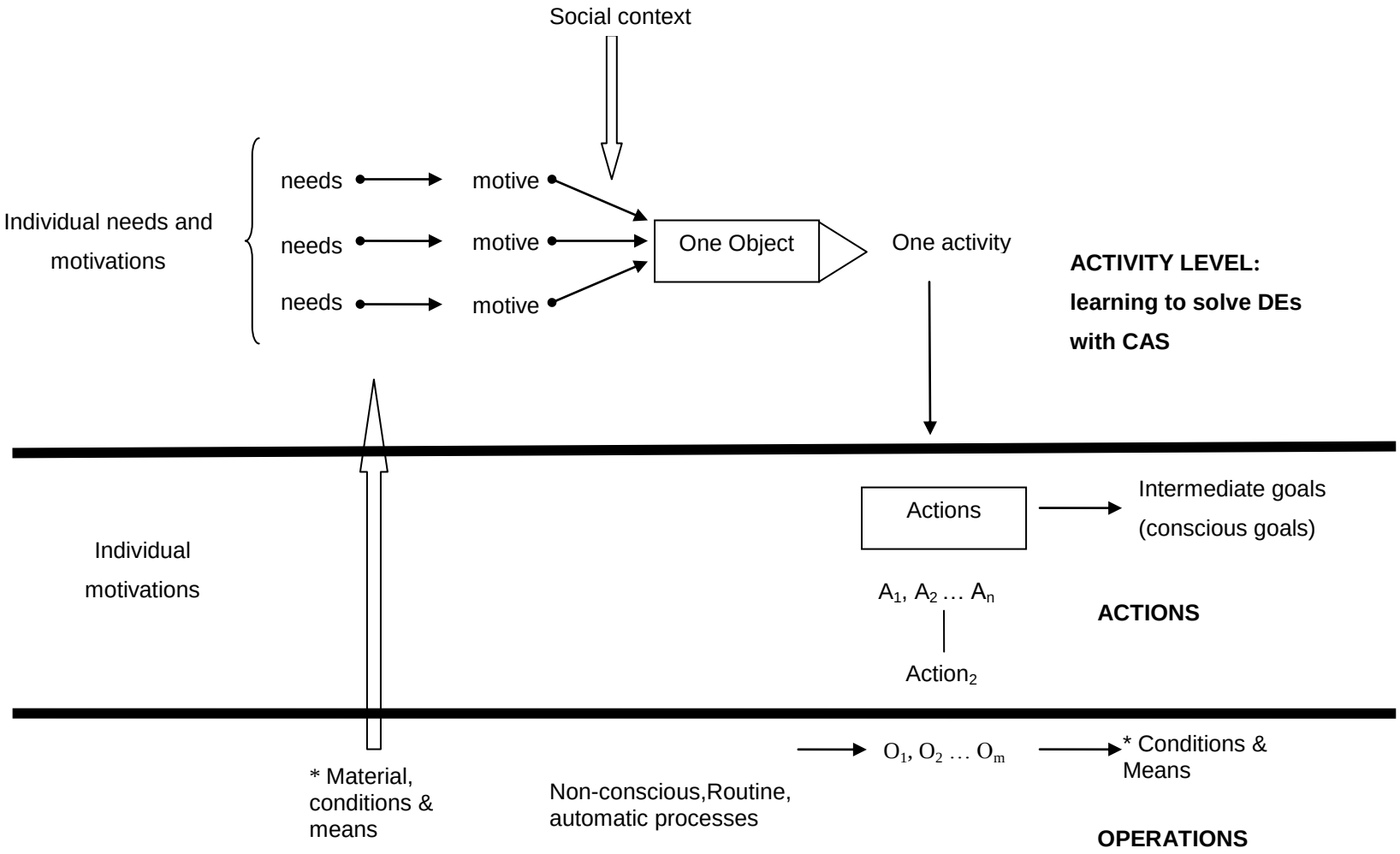


Figure 3.6: Model of the activity levels for learning mathematics using CAS

origin Lompscher's theory; *italicised*: source from Western motivation theory.

Needs	**Motives**		
	Social motives (interrelations)	**Self-related motives**	**Cognitive motives**
Need for recognition or approval. Social belonging (could include need for recognition). Autonomy Competency Need to develop ones career Need to obtain income to provide for material necessities of life	Identify with peers/ teachers. Communication with peers/others. Co-operation with teachers, other learners. Inter-relation with peers, teachers, others. Learning as means to help & support others. Feelings of duty to learn. *Social approval goal-orientation* *Social responsibility goal-orientation* *Social interaction goal-orientation* *Social compliance goal-orientation* *Social affiliation goal-orientation*	Motives that involve "he himself, his own personal development and well-being, his achievement and self-perfection, his success and position compared with others". *Task value components - Importance of task (importance of doing well or getting good grades in MATLAB or important to learn the content of the MATLAB course).* *Extrinsic utility value (future, usefulness – related to careers/ other courses).* *Performance approach goal-orientations (demonstrating ability & looking smart, demonstrating ability relative to others, seeking public recognition, grades, ego-involved).* *Performance avoidance goal-orientations (avoid demonstrating lack of ability).*	Interested in methods and modes which are essential to arrive at a result. Interested in methods and ways that reveal deeper approaches and understanding *Task value components - Intrinsic interest / intrinsic value (refers to intrinsic reasons students have in studying the activity – enjoyment, inherent challenge of studying CAS in maths).* *Mastery approach goal-orientations (focus on mastering the task, striving to improve oneself in the task, striving to accomplish something challenging on the task, developing new skills, developing competence, improving & understanding, searching for meaning).*

Figure 3.7: Outline of some constructs used in the study

CHAPTER 4

METHODOLOGY AND METHODS

4.1 INTRODUCTION

In this chapter I describe how I conducted my study, and the organisation, planning and techniques used to answer the research questions. I explain my choice of methods and how I planned to analyse data using coding methods (Neuman, 2003), which together with their rationale and depictions constitute the methodology. Furthermore, I show how I have satisfied criteria related to validity, reliability and issues of ethics, all of which are crucial to judging the quality and integrity of qualitative research.

According to Opie (2004, p.16), methodology:

> refers to the theory of getting knowledge, to the consideration of the best ways, methods or procedures, by which data that will provide the evidence basis for the construction of knowledge about whatever it is that is being researched, is obtained … [and procedures or methods refer to the] specific research techniques that are used in order to collect and then analyse data.

However, before discussing this methodology I make explicit the positions that underline my methods of research, namely the nature of 'the world out there' (the ontological question), and how I can get to know it (the epistemological question). Particularly, I am interested in how I can gain access to the selected students' motives and their actions when they learn mathematics using CAS.

As mentioned above, I regard individuals as 'beings' in a social context and within society, in line with Crotty's (2003, p.55) argument that, "all meaningful reality is socially constructed", that is, individuals interpret it within a social context. He also argues that a socially constructed view of reality does not preclude a belief that the world is real, nor imply that it is not real.

Thus, the individual, be it researched or researcher, is able to experience reality as being structured and in turn as structuring reality.

With regards to epistemology, I assume that meaning is not discovered but constructed by the individual as s/he participates in activities. As Crotty (2003, p.9) contends, people construct meaning as they engage with the world that they are constantly interpreting: "in this understanding of knowledge, it is clear that different people may construct meaning in different ways, even in relation to the same phenomenon". Thus, in order for me to access aspects of knowledge relating to the research topic I interviewed case study students, which allowed me to elaborate an activity theory framework in which to interpret students' motivations and their actions regarding learning mathematics mediated by CAS. My use of interviews provided access to the experiences, both motivational and action-driven, that students had undergone with respect to using MATLAB in mathematical learning. Use of computer screen recordings of problem-solving sessions provided valuable access to students' actions, and my constructionist stance in epistemology is compatible with realism in ontology (Crotty, 2003).

4.2 METHODOLOGY

This research is primarily located within an interpretive paradigm, drawing on qualitative methods, because of its special value for investigating complex issues and gaining a deep understanding around the phenomenon of learning with CAS.[20] It employs the case study methodology, "the distinctive need" for which, Yin (2009, p.4) argues, "arises out of the desire to understand complex social phenomena [and to] "investigate a contemporary phenomenon in depth and within its real-life context" (p.18). The case study is appropriate for reaching the goal of understanding the motives of students towards using CAS in mathematical learning and how they go about this activity, all set within a well-defined socio-cultural framework and context. With research questions that "seek to explain some present circumstance", in this case how students use MATLAB in mathematical learning, Yin (2009, p.4) proposes that case study as a methodology is appropriate. Moreover, in the first research question, 'what are the motives of students…?', the use of 'what' is indicative of an exploratory study, hence case study is again relevant as a methodology.

[20] I have in Chapters 2 and 3 provided an extensive elaboration upon and characterisation of the theoretical fields and relevant constructs. In chapters 5 and 6, I engage in discussions of participants' results. Consequently, in order to understand the empirical domain using constructs from the theoretical field I have to define and demarcate the empirical domain. This is done in Section 4.3, wherein I discuss the empirical field and empirical setting.

Since the case study methodology involves multiple sources of evidence (Yin, 2009), I have diversified my data collection sources, namely interviews, observations, problem-solving sessions (or computer screen analyses) and reflective interviews. This diversification was useful in validating the data and made triangulation possible. I chose the case study students purposefully, as will be discussed in 4.4.2. The case study inquiry also benefits from an elaborated theoretical framework to guide data collection and analysis (Yin, 2009), as explicated in Chapters 2 and 3. A case may be an individual, while case study research includes both single and multiple cases (Yin, 2009). I discuss results pertaining to two case study students in depth (Chapter 5), and carry out a cross-case analysis of three case study students in Chapter 6. Hence, my study could be classified as involving multiple cases in which each case is an individual *student using CAS in a mathematical learning environment*.

4.3 EMPIRICAL DOMAIN

The empirical domain generally consists of the empirical field, empirical setting and the findings (Brown & Dowling, 1998).

4.3.1 EMPIRICAL FIELD

The empirical field is described as "the broad range of practices and experiences to which the research relates" (Brown & Dowling, 1998, p.141), which for my research is undergraduate students enrolled in the Mathematics 3 semester course of Electrical, Mechanical, Computer Systems and Industrial Engineering diplomas at the University of Johannesburg (UJ), one of the two comprehensive universities in South Africa that was constituted as a result of the mergers between universities and technikons[21] in 2005. The course had a diverse student population with the majority from previously disadvantaged groups[22] and thus deprived of good quality education, and basic necessities such as water, electricity and housing. The minority were from advantaged groups in the sense of economics and education. The students entered university having studied at a range of schools[23] with an assortment of mathematics qualifications, varying from an 'A' to 'D' in Higher Grade matriculation Mathematics, or at least a 'C' in Standard Grade. In the first semester of 2007 there were 217 registered students for the Mathematics 3

[21] Technikons are in some parts of the world referred to as polytechnics. They have recently been amalgamated with universities or changed their names to 'universities of technology'.

[22] During the apartheid era the population was grouped as follows: Whites, Blacks, Indians and Coloureds – with the last three groups generally comprising the disadvantaged.

[23] I will discuss this variety in 4.4.2.1.

semester course (Mathematics 3 is studied in the students' third semester at university, which is typically in the second year of their diploma study), and 203 for the same course run in the second semester of that year. It was not compulsory for certain groups of students to choose to study the Mathematics 3 course; indeed the Mechanical Engineering students had a choice as to whether they wished to study Mathematics 3. The students were allocated to different groups for theory lectures, mainly according to the respective diplomas they were studying. Each group had four 40-minute theory lectures per week, and were further sub-grouped for MATLAB practicals with each group allocated three 40-minute lab sessions. Some of the MATLAB sessions were run concurrently in two labs with two different lecturers because the labs seated a maximum of 40 students. Apart from the lecturer of the MATLAB component, there was one tutor in the lab to assist students if required. Upon completion of the formal lecture times, students were expected to practice on their own, using either their personal computers or, after hours, the labs. The labs were open from 16h30 till 19h00 Mondays to Fridays, but during these times they could be used by any student wishing to access any of the available software (such as *Excel*, *Word* or MATLAB).

I collected data in the second semester of 2007, at the beginning of which I handed out questionnaires to 24 Electrical and Mechanical Engineering students, a group I refer to as the *critical group* (details in 4.4.2.1). From this group I selected ten students, a core group I refer to as my *critical cases* (details in 4.4.2.2). I was not lecturing any of the Mathematics 3 theory or MATLAB sessions during this data collection phase.

Kotze (2007) was prescribed as the Mathematics 3 course 'textbook', but a crucial point is that although students and lecturers referred to this as such it was actually a set of notes. Overall, its presentation was relatively bland, unlike the standard calculus textbooks that made use of colourful graphs and tables. Moreover, all A4 pages had been compressed to A5 and ring-bound to produce a flimsy document with unusually small layout for a prescribed textbook. The MATLAB component was written in a way that merely gave examples and exercises, and, as I argue below, this encouraged students merely to copy the code onto the MATLAB editor. The 'textbook' lacked explanations and elaborate discussions necessary to encourage comprehension and knowledge construction. The book was written by the MATLAB course coordinator, who

was a lecturer on both the theory and the MATLAB components[24]. Henceforth I refer to this set of notes as the 'textbook'.

In the Runge-Kutta II and IV portions of the 'textbook' pertaining to solving first order DEs there were worked out examples given with MATLAB codes, but without graphs representing the solutions. Only one example in the mentioned portions had a table of numerical solutions, thus it appeared that students were expected to copy the MATLAB code from the 'textbook' examples and produce numerical solutions, some of which they would not find out were correct or not. This approach does not encourage students first to work through the examples on their own and then compare their solutions to those of the examples in the text, because not all examples had numerical solutions or graphs displayed. In summary, the book provided examples that focused only on the MATLAB code and were devoid of all-inclusive solutions in the Runge-Kutta II and IV portions. The 'textbook' was obviously not geared towards encouraging a multiple representation approach since not all examples consisted of tables of solutions or corresponding graphs to the solutions of DEs.

A stark comparison with previous textbooks[25] was that in the older ones the numerical solutions to DEs were worked out using pen and paper techniques and substitution. The MATLAB book was sold as a separate booklet comprising MATLAB examples and exercises. Students could, if they wished, make reference to the main textbook to produce one or two numerical solutions to DEs using substitution methods, and this could act as a check as to whether they had produced the correct numerical solutions when using MATLAB.

Sections covered using MATLAB were not discussed in the theory classes, but rather during MATLAB lectures, and in labs where each student had access to a computer. During the 120 minutes allocated (I also call this 'MATLAB practical times'), the lecturer taught students using overhead transparencies or *PowerPoint* slides to develop MATLAB programmes using relevant examples. These slides also appeared on the students' computers, and the remaining lab time was used by students either to redo these examples or to practise exercises from the 'textbook'. The students were allowed to interact with each other and, if they chose, to solve problems in pairs or small groups. To a large extent the formal part of the MATLAB lectures still used

[24] There were two lecturers who taught the full-time theory groups and three who taught the MATLAB practical sections.

[25] These texts were again written by lecturers from the Mathematics department.

traditional teaching methods, with the lecturer as the source of authority and conveyor of information.

This Mathematics 3 course had many assessments, with the weighting of theory to MATLAB practical being in the ratio of 70:30. In the MATLAB component, students handed in two projects (project 1 totalled 15 and project 2 totalled 20), and did a MATLAB test (totalling 30). Finally, they sat a MATLAB examination towards the end of the semester. All students did the same MATLAB projects, tests and exam.

4.3.2 EMPIRICAL SETTING

My empirical field can be localised to an empirical setting, the latter essentially being a process of making choices with respect to the research design and data collection techniques (Brown & Dowling, 1998, p.141). I interviewed and observed ten students (chosen from the critical group), in four interviews, two long and two short. Each interview was audio-recorded for later transcription (see 4.4). In the first long interview (Interview 1), at the beginning of the semester, I gathered information from students on their background, motives towards learning mathematics, and actions that they carried out whilst doing so. In the second long interview (Interview 2), towards the end of the semester, I did the same (excluding the background issue), but this time I questioned them on their motives and actions related to both the Mathematics 3 and MATLAB components. During the semester I observed students solve DEs using MATLAB in my office. There were two such problem-solving sessions, audio-taped and screen recorded using *Bulent* screen recording software. The two shorter interviews took place immediately after these problem-solving sessions, which I term ‘reflective interviews’, and in which I questioned students about the task(s) they had just solved. Students generally participated in pairs during these problem-solving sessions, though at times I could not schedule a pair to attend at the same time due to their other commitments, in which case they solved problems alone in my presence. Throughout the sessions they were encouraged to talk and seek assistance when required.

Table 4.1 (below) summarises data collected in the empirical setting, presented in the first column and sequenced in the order in which I carried out the study.

Table 4.1: Summary of methods used to collect data

Type of data	Nature of instrument	Description of data	Participants' involvement
Interview 1	Semi-structured interviews, approximately 45 min. Audio-taped and transcribed.	Background, motives towards learning mathematics and actions that they carried out whilst learning mathematics.	Individual.
Problem-solving Session 1	Screen dump of students using MATLAB to find numerical solutions of first order DEs (Euler methods); transcribed by recording all movements of cursor and typing observed on the screen. Observations written. Discussion between student-student and interviewer-student, audio-taped and transcribed.	Students' actions whilst they solved the tasks; some motives and goals may also have been inferred through actions undertaken.	Mainly in pairs, on occasion individually.
Reflective Interview 1 (during or after Problem-solving Session 1)	Semi-structured interviews about tasks in Problem-solving Session 1. They were also about actions reported whilst students studied on their own. Approximately 20 minutes long. Audio-taped and transcribed.	Students' explanations on aspects of the task(s) solved in Problem-solving Session 1. Students reports on how they solved MATLAB problems in their own time.	Mainly in pairs, on occasion individually.
Problem-solving Session 2	Screen dump of students using MATLAB to find numerical solutions of first order DEs (Runge-Kutta II and IV methods); transcribed by recording all movements of cursor and typing observed on the screen. Observations written. Discussion between student-student and interviewer-student, audio-taped and transcribed.	Students' actions whilst they solved the tasks; some motives and goals may also have been inferred through actions undertaken.	Mainly in pairs, on occasion individually.
Reflective Interview 2 (during or after Problem-solving Session 2)	Semi-structured interviews about tasks in Problem-solving Session 2. They were also about actions reported whilst students studied on their own. Approximately 20 min long. Audio-taped and transcribed.	Students' explanations on aspects of the task(s) solved in Problem-solving Session 2. Students reports on how they solved MATLAB problems in their own time.	Mainly in pairs, on occasion individually.
Interview 2	Semi-structured interviews, approximately 1 hour. Audio-taped and transcribed.	Motives towards learning Mathematics 3 and actions that they carried out whilst learning Mathematics 3. Motives towards learning MATLAB (in maths) and actions that they carried out whilst learning MATLAB (in maths).	Individual.

I used data from all these sources to provide an in-depth and complete discussion, having collected it from two major sources, namely, the problem-solving sessions with DEs and the interviews in which students spoke more broadly about their actions and motives towards the use of CAS in mathematics.

4.4 METHODS

As indicated above, a number of different methods were employed in the data collection. These are dealt with in turn here.

4.4.1 THE ATTITUDES TO COMPUTERS SURVEY

In the first week of the second academic semester (June 2007), I conducted a small survey amongst Mathematics 3 Mechanical and Electrical Engineering diploma students (empirical field). I selected these two (out of a possible five) groups out of convenience, and invited all students from these two classes to fill out a short closed-ended questionnaire on their attitudes towards computers[26]. The computer attitudes questionnaire was from Berger and Cretchley's (2005) research, in turn adapted from the computer attitudes questionnaire of Cretchley et al. (2000) (Appendix A). I further requested that they provide me with the symbol of the results they had obtained in the previous Mathematics 2 course, as well as indicate whether the high school attended was in a township (see 4.4.2.1). I contend that if I could obtain information on these three factors then I would be in a position to put together a matrix from which I could purposefully choose students for the critical cases[27].

I guaranteed anonymity and informed students in writing as well as verbally that the marks would be used only to help me make an informed selection of possible students to be invited to participate in the qualitative research project. I also requested that all those who were willing to be video-taped, interviewed, observed, and have their computer work recorded indicate their willingness on the *attitude to computers* survey sheet). Students in these classes were given an information sheet containing the purpose of my research, the methods I planned to use to obtain data and my contact details, as well as those of my supervisor. Since this was the beginning of

[26] Attitude to computers are correlated with attitudes to using computers in mathematical learning (Galbraith & Haines, 1998).

[27] I also wished to make sure that students from township schools and those who did not attend township schools were duly represented.

the semester, when students were still registering or writing supplementary exams, the class attendance was low.

4.4.2 SELECTION OF PARTICIPANTS

Participants were selected in several groups.

4.4.2.1 CRITICAL GROUP

The *attitudes to computers* survey was completed by 24 students, all Black, with 23 from the previously disadvantaged groups and one a foreign student from North Africa. Sixteen of these students were from the Mechanical Engineering group and the remaining eight from the Electrical Engineering group. Below is a summary of characteristics of the 24 students in this 'critical group', sorted into four categories: academic status; attitudes to computers; type of school attended; and whether they studied Mechanical or Electrical Engineering. Academic status refers to the students' previous Mathematics 2 grade and comprises Grades A (75% +); B (70% - 74%); C (60 - 69%) and D (50% - 59%). I further sub-divided the attitudes to computer results into four sub-categories, namely, students who obtained a total score of:

- 52+ out of 65 - very strong attitude to computers
- 46 to 51 – strong attitude
- 39 to 45 – good attitude
- 33 to 38 – poor attitude.

Regarding the type of school attended, 'T/S' refers to township[28], while 'not T/S' refers to those who had not attended township schools. Township schools generally lack resources in terms of facilities, labs, computers, teachers and books. One student mentioned rural schools, another foreign schools, and I captured the information as such.

[28] Township schools are situated in areas designated for separate development during the apartheid period. Townships were mainly situated outside the major cities, for example, in the apartheid era the name 'Soweto' was an abbreviation of a collection of *SO*uth *WE*st *TO*wnships of Johannesburg (Adler, 2001:162).

Table 4.2: Numbers of students doing Mathematics 3 surveyed for critical group by academic status, attitudes to computers, type of school and diploma registered.

Academic status	Total number of students	Attitudes to computers	Students by type of school	Electrical or mechanical engineering
A	2	2 Very strong	Not T/S 1 Foreign 1	Mechanical 1 Electrical 1
B	3	1 very strong 2 strong	T/S 2 Not T/S 1	Electrical 2 Mechanical 1
C	6	3 very strong 3 strong	T/S 4 Not T/S 1 Rural 1	Electrical 3 Mechanical 3
D	12	2 very strong 8 strong 2 good	T/S 7 Not T/S 5	Electrical 2 Mechanical 10
Passed supplementary	1	1 good	Not T/S 1	Mechanical 1

In percentage terms, 33,3% of the students had very strong attitudes to computers, 54,2% had strong attitudes and 12,5% had good attitudes. Moreover 54,1% were from township schools and 37,5% were not, whilst 4,2% were from a rural school and 4,2% a foreign school.

4.4.2.2 CRITICAL CASES

Having grouped the students by academic status, types of schools and attitudes to computers scale, I decided on the number of students to select in each of these categories for the research project. However, firstly I returned to the surveys and noted the number of times students chose 'neutral' from items on the *attitudes to computers* survey. I then eliminated some who had chosen the option of 'neutral' more times than the rest (in this way a large number of students

obtaining ‘D’ symbols and two obtaining ‘C’ symbols were not selected) [29]. Those students who made selections of more than one choice or scratched what they had originally chosen were also eliminated as they displayed a sense of uncertainty.

My selection was not proportionate to numbers represented in each category. I purposely chose more students with ‘A’ to ‘C’ academic status because it was my belief that I would be able to obtain rich data with students from these categories. However, I made certain that 50% were selected from those who attended township schools as these represented an important category of students attending the University. I chose the student who attended the rural school and the one who was a foreigner. Again, the latter is important because there are an increasing number of foreign nationals studying at this institution. I was biased in choosing more Electrical Engineering than Mechanical Engineering students, consequent upon my experience that working with the former revealed that they generally conversed better in English than the latter[30].

In summary, my cross-selection of students comprised six with very strong attitudes to computers, three with strong attitudes and one good. In Table 4.3 (below), I present a summary of the number of students selected by sub-category.

[29] I believed that those who chose ‘neutral’ often may have had difficulty expressing their views and experiences. This would be an important factor to take into account when conducting interviews.

[30] In support of my view and experience, I note that the minimum admission requirements for the Electrical Engineering students was English ‘E’ on Higher Grade or ‘C’ Standard Grade and that the Mechanical students could be admitted based on a pass in English.

Table 4.3: Numbers of students selected from critical group as critical cases (for interviewing and observations) by academic status, type of school, attitudes to computers and diploma registered for

Academic status	Students by type of school	Attitudes to computers	Electrical or Mechanical Engineering	Number of students selected as critical cases
A	Not T/S 1	1 very strong	1 Mechanical	2
	Foreign 1	1 very strong	1 Electrical	
B	Not T/S 1	1 very strong	1 Electrical	3
	T/S 2	2 strong	1 Electrical; 1 Mechanical	
C	Not T/S 1	1 very strong	1 Mechanical	4
	T/S 1	1 very strong	1 Electrical	
	T/S 1	1 strong	1 Electrical	
	Rural 1	1 very strong	1 Mechanical	
D	T/S 1	1 good	1 Electrical	1

The above selection of ten students were then invited to be the critical case study students for this research project, with all ten agreeing to participate, attending and completing every aspect.

4.4.2.3 CASES SELECTED FOR DISCUSSION

It was an important yet complex decision to select which of the ten participants' data to discuss. I discuss results related to three in Chapter 6, represented as follows: one student who had attended a township school (Tumi); one who had attended a rural school (Thembiso); and one foreign student who had attended a school in a North African country (Abraham)[31]. Most

[31] Pseudonyms are used here and elsewhere to preserve anonymity.

importantly, these three were purposively selected for discussion because their motives illuminated my three categories. Although all three held multiple motives, with each there seemed to be certain motives that predominated over others. For example, Tumi had motives prevalent from the category of cognitive motives, Thembiso had more self-related motives, and Abraham had motives from the social (interrelations with others) motives category. I therefore chose one case study student to represent each of the three categories of motives. Yin (2009, pp.26, 47) also points out that one should choose the cases that are most likely to illuminate one's research questions for discussion, including the choice of unique cases.

In Chapter 5, I present the case studies of Tumi and Thembiso in detail, but I present only the table of results pertaining to Abraham's data. This choice was made because Tumi's actions were in many ways also representative of another student's actions, but Tumi spoke clearly of her motives and actions. I discuss Thembiso in detail because he articulated his self-related motives very well. In particular, he spoke fondly of his extrinsic motives. Thembiso was also chosen because his extrinsic motives were similar to two other participants but his articulations were clear and expressive. Abraham espoused motives that were both social (interrelations with others) and cognitive, and he was chosen because of the unique way in which he contributed to others' learning in terms of his social motives. I did not discuss Abraham in detail, due to space constraints, but I discuss and compare his motives and actions in the discussion chapter (6).

In the above elucidation I have argued why the three case study students were chosen for discussion. Of the remaining students not chosen for discussion, one had difficulty expressing herself in English and most often repeated similar answers to different questions. Another solved the MATLAB problems with ease, making very few errors but speaking softly during the interviews, rendering gaps in the transcriptions of interview data. The difficulties experienced by two students in using MATLAB are similar to those captured in this thesis, and their data would not have added any new insights.

4.4.3 THE INTERVIEWS

Kvale (1996, p.5-6) describes the purpose of a research interview as being "to obtain descriptions of the life world of the interviewee with respect to interpreting the meaning of described phenomena". Interviews seemed an appropriate way to search out the students'

reflections on their needs, motives and some actions[32] associated with the activity of using MATLAB in mathematical learning. Considering I also wanted information related to how they had studied for previous mathematics courses, and why they chose to study mathematics using CAS, I decided to hold two interviews, namely Interview 1 at the beginning of the semester and Interview 2 towards the end. To support my use of interviews as a tool to collect data on motivations, I follow Lompscher's (1999, p.19) advice that in a study on students' motivations and motives one could ask students about the choices of methods used or the types of task they usually chose to solve. One would also ask them to explain why they chose those tasks.

There were various questions that I asked before, during and after problem-solving sessions. As noted above (Table 4.1), the formal questioning took the form of Interview 1, Interview 2, Reflective Interview 1 and Reflective Interview 2. The interviews were semi-structured in that my responses and further probing depended to a large extent on how interviewees responded to the questions. At the beginning of the semester, I interviewed each of the ten participants, individually, probing to obtain information on their backgrounds, mostly related to the type of school attended, their knowledge of computers and mathematics, motives for learning mathematics and actions in which they generally engaged when learning mathematics. These questions covered a wide spectrum in that students were asked to reflect on the time when they chose to study high school mathematics and the reasons for this choice (if they could remember). I also prodded to find out their immediate plans on completing this diploma. This helped to identify underlying needs.

After a few weeks, and timed in a way that students would have covered the section on Euler's solutions to DEs during formal lectures, I arranged for the first problem-solving session to take place. After this I conducted Reflective Interview 1, usually carried out with the pair of students or alone if they solved the problems individually. In Reflective Interview 1, I probed to see if students could answer questions about the tasks they had just completed. After a few more weeks, and synchronised with the coverage of Runge-Kutta methods covered in formal lectures, I arranged for Problem-solving Session 2. As with Problem-solving Session 1, I conducted Reflective Interview 2, primarily asking questions about the tasks they had solved in this problem-solving session.

Finally, towards the end of the semester and soon after Problem-solving Session 2, I conducted Interview 2 with each participant and probed to obtain details of why they studied MATLAB in

[32] To observe their actions undertaken, I arranged the problem-solving sessions.

mathematical learning and how they went about engaging in this activity. I also asked similar questions with respect to their activity of learning Mathematics 3, some based on what other activity theorists have probed (Coupland, 2004; Gordon, 1998), for example, *How do you usually go about learning how to use MATLAB to solve DEs? Regarding your answer to the previous question, why did you choose those methods? What do you think is needed to be successful at learning how to use MATLAB to solve DEs?* Coupland (2004, p.105) raised similar questions, expecting to attain multiple motivating objects that students espoused.

I piloted some of the interview questions from Interview 1 prior to data collection, using a sample of one engineering student, who incidentally had not used MATLAB in mathematical learning. Additionally, I made a hasty transcription of the first participant's Interview 1 data and discussed this with my supervisor so as to have an idea of whether I was getting answers to my research questions using the current line of questioning.

Throughout this report I emphasise that Interviews 1 and 2 comprise data that is obtained from students' *reporting* their motives and actions to me; these motives and actions were not *observed* by me. However, large portions of the reflective interviews and student-student interactions (if they worked in pairs) or lecturer-student interactions constitute data collected during the actual problem-solving sessions, and I observed these actions in which they were engaged.

4.4.4 PROBLEM-SOLVING SESSIONS

A rationale for use of problem-solving sessions is followed here by details of the structure of both the sessions.

4.4.4.1 Overview

I requested each participant to choose a partner, either one they already knew from the research project or one I selected for them. In this way participants were invited to attend the problem-solving sessions in pairs. However, some solved the problems alone because of other last-minute commitments. Every problem-solving session (with a maximum of two participants) took place in my office, where each student had access to either a desktop or laptop computer. These sessions were audio-recorded with the computer screens recorded using *Bulent* screen recording software. To a certain degree, the problem-solving sessions were designed so that I could observe how students engaged in the activity of using MATLAB to solve DEs. The actual

tasks were designed with both research and pedagogic intentions, however the tasks were to a large extent similar in nature to what students might encounter in MATLAB tests and/or exams.

The problem-solving sessions were structured along selected lines and semi-structured along others. They were structured in that all participants were given the same set of tasks in Problem-solving Session 1, and some participants (four out of ten) were given the same task in Problem-solving Session 2, with the remaining six solving a set of similar tasks to the first four, but with slight modifications (see 4.4.4.2 and 4.4.4.3). It was semi-structured in the sense that my prompts, questioning and responses differed with respect to the actions of the participants and my relationship with that particular student. These problem-solving sessions were not in any way simulations of the MATLAB laboratory sessions, although I may have taken on a teaching role in a few aspects, for instance explaining to students how they should type and present all numerical methods studied (Euler, Runge-Kutta II and IV), in one m-file or how to substitute into functions. My responses as interviewer during the recorded (audio) part of these sessions did indeed constitute a research role.

There are major differences between the problem-solving sessions and other sites where a student may engage in using MATLAB to solve DEs. I focussed more on requesting students to produce their own solutions, and although they were encouraged to ask their partners or me for help, or engage in discussions, they had eventually to construct their answers using MATLAB. This also meant that I left them to struggle and only intervened when called upon to provide assistance, or when I thought it was necessary to do so, for example when they could not go further without my help. My presence, unlike on occasions when they solved problems on their own, must have affected the students and perhaps their actions.

It is necessary to take cognisance of the possibility that the problem-solving sessions may have some common characteristics with a typical MATLAB learning situation. Nevertheless, I contend that the problem-solving sessions in this study certainly exposed a large number of actions in which participants had engaged, and perhaps concealed others.

There were certain guidelines that I implicitly followed to support my research requirements. In essence I wished to see what actions or operations each student had adopted when solving the DE using MATLAB, for instance: *Was s/he able to work through error messages, if so how? Could s/he interpret graphs and determine whether solutions were correct or not? How did s/he use the 'textbook' in this process of learning? Whom or what did s/he resource when in*

difficulty? Did s/he understand the use of MATLAB commands - if not, did s/he seek clarification?

The guidelines, briefly, were as follows:

- Firstly, the student was given a mathematical task which s/he was expected to do on her/his own either by following the 'textbook' or requesting help from her/his partner or me. At this initial stage of the session I limited my help and only provided assistance, for example to open a new m-file. If s/he was unable to proceed s/he had to ask her/his partner or use the 'textbook'. The 'textbook' contained similar examples to the ones given in the task and the task was similar to what was covered in the MATLAB formal classes.

- When students requested help and it happened to be well into the problem-solving sessions I gave hints based on when and where I thought it was necessary. Usually after the first hour of the sessions I may have given more assistance (perhaps I felt that the student had struggled enough or that I had sufficient data on certain aspects of her/his MATLAB solution).

- There were two tasks given for each problem-solving session. Part 1 was compulsory and Part 2 was not. However, students were urged to complete Part 2 (all students attempted Part 2; most completed the solutions but not all got them right). Of significance here is that I allowed students at any stage of the session to modify a previous part if they so wished (these going back to previous solutions may have indicated certain behaviour in that they discovered some aspects were wrong only when they solved Part 2, or some went back to change a correct Part 1 solution, which is indicative of some uncertainty experienced).

In a way, the above plan guided me to leave some students to struggle with syntax until I felt it was necessary to intervene.

4.4.4.2 Structure of Problem-solving Session 1

As stated above, ten students solved the tasks allocated in Problem-solving Session 1, either individually or in pairs. Once students had completed the section on Euler's solutions to DEs using MATLAB during formal lectures, and where possible had practiced on a few of these exercises, they could participate in the problem-solving session. I did have participants who only practiced during formal MATLAB class time and not after hours, but they still participated in the problem-solving session. It was not my intention to get students to attempt the problem-

solving sessions after having practiced solving a great number of problems, but rather I was interested in capturing actions or operations as they went about their *learning* to solve DEs using MATLAB. The maximum duration of the problem-solving session was about two hours, although not strictly applied. Consequently, not all students completed both tasks I had designed for the session.

At the commencement of the problem-solving session, I repeated to students that this was not a test and that they could treat it as a practice session in which they were learning how to solve DEs using Euler's method. I provided them with the prescribed 'textbook' (Kotze, 2007), writing material and pens. They were told they could use any other material, such as class notes or other problems they had already solved. I also conveyed to them that I was conducting research into how they went about solving the problem using MATLAB, and that it did not matter if they were not yet experts at doing this. I gave a few examples, such as my interest in how they could handle or find out about what the error messages pointed to.

The students controlled the pace of the problem-solving session, with some managing to finish early, whilst others took their time. If one student produced the correct solution then the other would naturally look at it and try to pinpoint his/her own mistakes, or aim towards this achievement. Some students did not produce the correct solutions, and I provided more assistance, even after the formal data collection process.

The tasks in Problem-solving Session 1

The different steps in part 1 were designed to enable students to reach the stage of constructing both Euler and analytical solutions to the problem. I chose a task comprising sub-problems that allowed for the use of such methods as comparison, reflection, manipulation and interpretation. This task was taken from Kotze (2007, p.225) with some steps modified. This variety of actions also formed the core of my questions in Reflective Interview 1. In Part II, I created the task that concentrated more on how students would handle the input into MATLAB of more complicated-looking DEs.

Part I

Consider the following differential equation with the given initial condition

$$\frac{dy}{dx} = e^{x} - \frac{y}{x} \text{ with } y(1) = 1$$

1. Write a MATLAB code for an Euler solution of this differential equation over a domain [1, 2] with step size $h = 0.1$. Copy and save this code to a new M-file (call this file your name).

2. Work with 15 significant figures and display a table of values for x and y. Copy this table to your M-file.

3. Solve this DE analytically (on the page provided), use the linear method.

4. Display a table of values containing x and both the Euler and analytic solution.

5. Plot the two solutions, make use of a legend to distinguish between the two graphs, clearly label the axes and give your graph the title of your name.

Part II

Congratulations, now that, you have come this far, copy the MATLAB code that you have from Part 1 above and paste this to a new M-file, save this M-file as your surname. Edit the code so that you can now find the Euler solution for this differential equation:

$$\frac{dy}{dx} = \frac{2}{3}e^x - \frac{y \sin x}{2x^2}$$

with the same initial condition and step-wise increments as above.

Display a table of values containing x and the Euler solution and save this to your M-file.

I present the solutions to these tasks in Appendix B. Euler's method involves the following:
We can approximate a first order DE of the form $dy/dx = f(x,y)$ over an interval $a \leq x \leq b$ if the initial condition $y(x_1)=y_1$ is given. The Euler formula is $y_{i+1}=y_i+h.f(x_i, y_i)$.

4.4.4.3 Structure of Problem-solving Session 2

Problem-solving Session 2 took place in the midst of a student strike on campus, during which the labs were closed. Consequently, not all participants had been able to practice in the labs on how to solve DEs using Runge-Kutta II and IV methods, but they had completed these sections in the formal MATLAB lectures. Some were also able to complete a few exercises using these

methods during the formal MATLAB class time. Most were concerned because they had to prepare for an upcoming MATLAB test, so they welcomed the problem-solving session, perhaps using it as an opportunity to practise.

As before, ten students solved the tasks allocated in Problem-solving Session 2, either individually or in pairs. The maximum duration allocated was again two hours, although not applied stringently. Not all completed both the tasks I had designed for the session. I structured the Problem-solving Session 2 similarly to Session 1, in that students were encouraged to consult each other and me, as well as use class notes, the prescribed 'textbook' and other resources. Once more, students had full control over the pace of the session.

The tasks in Problem-solving Session 2

Along the same lines as the creation of tasks in Problem-solving Session 1, the different steps in Parts I and II of Problem-solving Session 2 were designed so that students could construct both Runge-Kutta II and IV solutions to the problem. This task also enabled the possibility of association, deliberation, strategy, understanding and making use of different representations (symbolic, graphical, and numerical). I deliberately used variables *t* and *v* instead of *x* and *y* to determine if students would have difficulty identifying independent and dependent variables, as substitution of these variables into the auxiliary equations of the Runge-Kutta methods was important in order to obtain solutions. Part 1 of the task was modified from an assignment given in Kotze's (2007, p.221) 'textbook'.

Part I

Consider the following differential equation with the given initial condition

$$\frac{dv}{dt} = 1 + vt \quad with\, v(0) = 2$$

1. Write a MATLAB code for a Runge-Kutta II solution of this differential equation with $0 \le t \le 1$ with step size h = 0.1. Copy and save this code to a new M-file (call this file 'your name second' e.g. Jeeva second).

2. Work with 15 significant figures and display a table of values for *t* and *v*. Copy this table to your M-file.

Type in the following code to simultaneously find the Euler solution in the same M-file

```
y1(1)=2;
for i=1:m
    y1(i+1)=y1(i)+h*(1+t(i)*y1(i));
end
disp ([t' v' y1'])
```

Part II

Congratulations, now find the Runge-Kutta order IV solution for the same differential equation.

1. Display a table of values containing *t,* RK II and RK IV solutions.

2. Plot the two solutions, make use of a legend to distinguish between the two graphs, clearly label the axes and give your graph the title of 'your name 23'.

As noted above, there were four students who completed the above tasks in Problem-solving Session 2 (of whom Abraham was one), and the remaining six (including Tumi and Thembiso) attempted a modified problem wherein I changed the previous task to include a square root:

$$\frac{dv}{dt} = 1 + v\sqrt{t} \quad with\ v(0) = 2$$

Correspondingly, the Euler solution which I gave them in Part 1 also changed as follows.
Type in the following code to simultaneously find the Euler solution in the same M-file

```
y1(1)=2;
for i=1:m
        y1(i+1)=y1(i)+h*(1+sqrt(t(i))*y1(i));
end
disp ([t' v' y1'])
```

After having observed the first four participants solve the problem (they had some difficulties but managed to obtain the solutions), I felt that the modified problem would provoke a better challenge than the original one and this happened in the problem-solving sessions. However, Abraham made similar mistakes to Tumi and Thembiso, although to a lesser extent. It can be argued that he may have solved a slightly easier problem, but in my analysis, having transcribed all mistakes and captured the actual mistakes using codes and encapsulating them into larger actions, the extent of the mistakes was not really the crux. Rather, the point is that similar mistakes were indeed made (see the summary of codes for each of the three participants in Chapter 5). Overall, I suggest that these different tasks did not necessarily complicate the study, as the actions that participants displayed in solving them were indeed comparable, as can be gauged from Chapter 6.

Because I did not want students to waste time in working out the Euler solution (this was tested in Problem-solving Session 1), I gave them the corresponding code so that they could merely type in this solution and concentrate on interpreting or comparing solutions. A Runge-Kutta II and IV solution involves substituting into two and four auxiliary equations respectively. Below I briefly outline these methods.

We can approximate a first order DE of the form $dy/dx = f(x,y)$ over an interval $a \leq x \leq b$ if the initial condition $y(x_1)=y_1$ is given.

The Runge-Kutta II formula encompasses two auxiliary equations k_1 and k_2:

$k_1 = h.f(x_i , y_i)$

$k_2 = h.f(x_i+h , y_i+k_1)$

$y_{i+1}=y_i+ ½ (k_1 + k_2)$

The Runge-Kutta IV formula encompasses four auxiliary equations k_1, k_2, k_3 and k_4:

$k_1 = h.f(x_i , y_i)$

$k_2 = h.f(x_i+½h , y_i+½k_1)$

$k_3 = h.f(x_i+½h , y_i+½k_2)$

$k_4 = h.f(x_i+h , y_i+k_3)$

$y_{i+1}=y_i+ 1/6(k_1 + 2k_2+2k_3+k_4)$

I present the solutions to the tasks of Problem-solving Session 2 in Appendix C.

4.5. RESEARCH RIGOUR

The concepts of validity and reliability are originally located within a presumably objective reality (Cohen, Manion & Morrison, 2001), and are designed for experimental studies that involve testing hypotheses, measuring variation and testing the significance of findings, hence their use in establishing trustworthiness in quantitative research is recognised. Several researchers have argued for acceptable alternative criteria of validity and reliability (Hitchcock & Hughes, 1995; Lincoln & Guba, 1985) for qualitative research, to be applied as follows.

4.5.1 RELIABILITY

Reliability is associated with the replicability of studies, that is, if carried out on a similar group of respondents in a similar context would similar results be found? There must be something tangible and unchanging 'out there' that can serve as a benchmark if replication is to make sense (Lincoln & Guba, 1985). I studied students' motivations and actions, but these were neither static nor stable across different groups of respondents or even within the same group. Clearly, the above definition of reliability becomes problematic to studies set within the interpretive context. The notion of reliability in qualitative research is taken to be *dependability* and *consistency* (Lincoln & Guba, 1985), but if the data was[33] to be collected by someone else using the same instrument, would the findings be the same?

According to Hitchcock and Huges (1995), "Reliability concerns the extent to which a particular technique will produce the same kinds of results, however, whenever and by whomever it is carried out" (p.107), and to this end some of the various forms of dependability are member checks (respondent validation), triangulation, prolonged engagement, persistent observations in the field and audit trails (Lincoln & Guba, 1985).

Triangulation: I asked other researchers to verify my codes and analytic categories. In this context, they were given samples of my interview transcripts and checked that I had analysed these appropriately. This process has helped me to determine whether my codes and categories are credible. I write about triangulation with respect to data collection methods within the validity part of this section.

Audit trails: I ensured that all raw data, including video recordings, screen recordings, written field notes and transcriptions, were safely stored. I have kept all evidence, including how the structure of categories was arrived at, my methodological notes on procedures and strategies, as well as my notes on trustworthiness relating to credibility, dependability and confirmability. Further, these audit trails proved helpful in organising, relating, cross-referencing and attaching priorities to data that may otherwise have remained undifferentiated until the writing task was undertaken (Lincoln & Guba, 1985).

Persistent observation and prolonged engagement: The technique of persistent observation, that is identifying those characteristics that are most relevant to the problem under scrutiny, and

[33] Although 'data' is a Latin plural of datum it may also be treated as an uncountable entity, thus acting grammatically as a singular noun, as is the case in this paper.

prolonged engagement, in which the researcher is involved with a site sufficiently long, helped to take care of any distortions in the data that might have arisen (Lincoln & Guba, 1985). As a previous lecturer of the same course (students using MATLAB in the study of numerical solutions for DEs), I am not new to the site and I was still involved with it for the full duration of the semester in which the data was collected. I also observed a different part-time group of students whilst they were learning how to solve DEs using MATLAB in the laboratory.

4.5.2 VALIDITY

Validity is concerned with the extent to which the researcher truly and accurately captures events as they had originally occurred (Hitchcock & Hughes, 1995). In other words, does the researcher provide an accurate and true description, interpretation and reconstruction of the events? (Lincoln & Guba, 1985). In this context, Maxwell (2002) provides an elucidation of descriptive validity, interpretive validity and generalisability, defining it as what the researcher reports having seen or heard. Meanwhile, interpretive validity refers to inferences the researcher makes from the words and actions of participants in the situations studied, with the interpretive accounts in their own language.

Descriptive validity: In order to make certain that the description of events, statements and actions related to what participants did and said are valid I firstly ensured that all problem-solving sessions and interviews were audio-recorded. Secondly I also video-recorded the case study students whilst they solved the problems in my office. Although the video-recordings were not analysed, they serve as evidence should a reader be interested in questioning aspects of the study. Moreover I have ensured that all interview transcripts have been carefully transcribed, by a professional transcriber, who made the transcription of interview data. I rechecked the transcription indicating special features of speech, such as pauses or hesitations. I carried out the transcription of interactions between participants and myself in the problem-solving sessions and reflective interviews. I transcribed the screen dumps by meticulously recording all on-screen activity, paying attention to every stroke of the keyboard and noting the exact time, to the second, to which the entries pointed.

Interpretive validity: I interpreted the events, statements and behaviour using the participants' own language and concepts from the theoretical field. To this end I gave explicit exemplars of empirical data relating to concepts from the theoretical field. Moreover, participants themselves gave detailed explanations because of my cross-questioning, and these assisted me in reaching interpretive validity.

Generalisability: referring to the extent to which one can extend the version of a particular situation to other persons or settings (Maxwell, 2002), generalisability is an important consideration of any research project. Lincoln and Guba (1985) suggest that *transferability* should be used as a substitute for generalisability within the interpretive paradigm. I draw attention to the unique context of this study and the fact that it is grounded in the social framework of this particular institution, and how students learn to solve DEs within a defined teaching and learning environment. To this end, I provide a detailed description of the research context, as well as data collection methods, interpretations and the methods that transformed the text into findings. Thus, a potential user of my research project will be in a better position to decide the extent to which transfer to other groups, situations or disciplines could be possible (Cohen et al., 2001).

Triangulation of methods: The credibility of a study is enhanced by the use of different data collection methods. I used interviews, reflective interviews, observations and two problem-solving sessions. For instance, having read students' reports in interviews that they wrote MATLAB code using pen and paper techniques before typing onto the computer, I backed up the claims through observing these actions during the problem-solving session(s).

In this study I addressed the issue of validity in a pragmatic way, keeping the description close to the data and using my theoretical framework to effect the analysis. My results were presented to researchers in various forums.

4.5.3 ETHICAL CONSIDERATIONS

Ethics generally pertain to beliefs about what is correct or incorrect, proper or inappropriate, and good or bad (McMillan & Schumacher, 2006). It is my responsibility as a researcher to deliberate these considerations and make the best judgment possible. I made certain that the participants were not subjected to loss of self-esteem or human dignity. It is also my duty to protect the rights and welfare of the participants and I have done so to the best of my ability. For instance, I obtained written permission from the Dean of the Science Faculty at the University in which I teach, who allowed me to proceed with my intended research. I obtained ethical clearance from the University of Witwatersrand, School of Education, in which I am registered for a PhD degree (ethics protocol number: 2007ECE15). I explained the aim and purpose of my research to the Dean of the Science Faculty and to the Head as well as Deputy Head of Mathematics at my University. I also informed lecturers on the MATLAB course of my intention and objectives in carrying out this research project.

I made available written feedback of the results of the research project so that all participants and key stakeholders (notably the Dean, Head of Mathematics Department, MATLAB lecturers and students) have access to it. Throughout the research, when I was unsure of any matters relating to ethics, I approached my supervisor as well as the ethics division of the University at which I am registered, for advice and assistance.

Worthiness of the research project: I intend to contribute to the field of mathematics education in several ways. Firstly, the amplification of activity theory from Leontiev's perspective, and merging more recent work from activity theorists such as Kaptelinin and Lompscher, allowed me to elaborate a more systematic framework for the study. It is envisaged that the combination of modern (Western) theory on motivations and motives from the activity theory perspective, and how they converge or not, will shed new light on this massive theory of motivations. These results can be useful for those struggling to find explanations beyond the usual quantitative studies of Western motivational theory. Moreover, the results on students' motives towards solving DEs by using MATLAB and their actions in this respect should be valuable for lecturers not only in South Africa but also in other countries.

Informed consent: The students from the critical group who filled in the initial survey signed informed consent forms. All participants were informed of the aims of the study and its methodology. They volunteered to participate in the study and signed consent forms agreeing to be audio-recorded and video-recorded. Participants were informed in writing that they had the right to withdraw from the research project at any stage, and this would not be held against them in any way. Nobody withdrew.

Privacy, confidentiality and anonymity: At all stages of the research project, for example at the beginning of each interview and problem-solving session, I informed participants that they would be guaranteed anonymity and that no real names would be used in the write-up and presentations of results, rather pseudonyms would be used. The students' responses were treated with the utmost confidentiality. When they signed the consent forms I also signed the form, assuring them of complete confidentiality. The transcriber, being in the field of higher education, was aware of the requirement to keep all information about participants confidential, and could be trusted in this respect. Next I consider the ethical issues that I had encountered in the field.

In a thesis of this magnitude there are ethical issues that inevitably arise during the various stages of research. I remind the reader that the larger Mathematics 3 group comprised 203

students and I selected a convenient sample of ten critical case study students from a smaller critical group of 24. Although Abraham was the only foreign student amongst the ten critical case study students there were a number of other foreign students in the main group of 203 students. Consequently, he is not easily identifiable by teachers or students who knew the class, with the exception of his partner and friend who accompanied him to the MATLAB problem-solving sessions.

Further discussion of ethical issues in the field: When I became aware that Tumi's learning suffered due to computer access problems (to be discussed in Chapters 5 and 6), I experienced feelings of guilt in that I could only afford her one extra practice session during which she could solve any number of MATLAB problems using the computer. There were other students in a similar predicament to Tumi and this worried me too. At that point my queries with the lab management did not result in any immediate resolution to the problem as new labs were a future project of the recently merged institution.

The issue of researching students' learning whilst they were actively engaged with the course material challenged my thinking as I could not obtain much data on Leontiev's operations level of learning. Notwithstanding, I was able to investigate in detail students' actions and conscious goals, which would not have emerged so clearly had data been collected perhaps a semester later, when students would have completed the MATLAB course and, as I speculated, more operations would surface. During the writing up I faced the ethical uncertainty of whether I would do justice to Leontiev's three levels, but with the amount of analysis corresponding to the first two levels of his framework I was reassured that the thesis would make a valuable contribution to students learning.

In summary, I have ensured that participants were not harmed, that they were informed of the purpose of the research and the methods used to obtain data. They consented to be part of it, and their privacy has been and will be respected.

4.6 DATA ANALYSIS PROCEDURES: OPEN-, AXIAL- AND SELECTIVE- CODES

Having dealt with much of the process of transcribing in 4.5.2., I now explain the format that I made use of.

4.6.1 INTRODUCTION

The interviews were transcribed verbatim, including repetitions and hesitations. I reference the interviews as follows -

TH-int1(239 – 240): Thembiso's Interview 1 with line numbers 239 to 240
TH-int2(239 – 240): Thembiso's Interview 2 with line numbers 239 to 240
TH-ref-int1(239 – 240): Thembiso's Reflective Interview 1 with line numbers 239 to 240
TH-ref-int2(239 – 240): Thembiso's Reflective Interview 2 with line numbers 239 to 240.

For Tumi's and Abraham's transcripts, I use the same style and instead of TH, I use TU and AB respectively.

Moreover I have used the following convention:
.. for a pause or hesitation.

I use *JP* to refer to myself as the interviewer in these transcripts. Below I elucidate the process of coding. It is vital to note that the dialogue between student-student and student-lecturer (me) during the Problem-solving Session 1 was transcribed in the same document as Reflective Interview 1 and similarly for Problem-solving Session 2.

Transcripts of the screen recording of both problem-solving sessions are captured numerically, for example as follows:

*16a 01.18.56 p(i+1)=2/3*exp(x(i))-(y(i).*sin(x(i))./2*x(i) ^2;*
16b 01.19.50 Evaluates the code and looks at graph, closes graph and goes back to m-file.
16c 01.20. 13 removes dot from . 01.20.31 adds brackets*
*p(i+1)=2/3*exp(x(i))-(y(i)*sin(x(i))./(2*x(i) ^2);*

The numbering in Problem-solving Session 2 is a continuation of the numbering from Problem-solving Session 1.

Several authors suggest that codes are efficient and help to organise data through its labelling and retrieving access (Miles & Huberman, 1994; Neuman, 2003). One of the well-known proposals in analysing qualitative data is for the researcher to identify and locate patterns and associations within the participants' words and actions (Neuman, 2003), but at the same time remain in close contact with the construction of reality as seen from the participants' perspective. My choice of analysing data using the three levels of analysis as proposed by Neuman (2003), namely, open-, axial- and selective- coding, made this a possibility.

All of the transcripts of the interviews and of screen-recorded problems were analysed in an inductive way. I began analysing data by using open codes, which refer to the formation of labels corresponding to particular pieces of the textual data. In a first pass through data I assigned initial labels (or codes) to each sentence, concentrating on key words and phrases articulated by the participants. This I did twice, reading and re-reading transcripts because a particular sentence could be implying different depictions, which I could capture using various labels or open codes. Although this detailed open coding analysis was at a low level of abstraction, it enabled me to underscore critical terms and key events from deep inside the data (Neuman, 2003). Consequently, meaning was given even to the smallest component of data. The manner in which I coded is comparable to the descriptive level of coding as proposed by Gibbs (2007). An advantage of creating codes at this level was that it forced me to pay close attention to the participants' utterances and to construct codes that reflected their experiences rather than mine (Gibbs, 2007). During this initial assigning of codes phase I consulted with my analytic memos made during the data collection phase. In some cases these served to ratify my initial labelling or open coding of data (Neuman, 2003).

Neuman (2003) proposes that the next level of coding makes 'axial codes', which requires a second pass through data. At this level I began making connections between the codes identified in the previous open coding process. I concentrated on the initial codes and grouped them whilst attempting to organise them into categories. I examined causes and consequences, strategies, interactions and processes (Neuman, 2003). I suggest that both the open coding and axial coding levels enabled me to saturate the data as I had to re-read the interview transcripts to ensure that all relevant utterances were considered for coding and analysis.

The final level of coding is selective coding and involves a last pass through grouped data. After having identified the major axial codes I scanned the data and previous codes and looked selectively for cases illustrating themes. In this process I made comparisons and contrasts to

identify main themes (Neuman, 2003). It is important to note that the categories developed are neither mutually exclusive nor exhaustive, but rather overlapping.

4.6.2 EXAMPLES OF DATA ANALYSIS WITH RESPECT TO INTERVIEW DATA

At this juncture, it is apposite to illustrate my coding process. I use an example from Tumi's transcript:

TU-int2(129 –130). JP: What are some of the things that you found difficult about using MATLAB specifically to solve differential equations? You mentioned one or two now, but..

Tumi: I would say like.. making mistakes, most of the time when you're typing in, because like sometimes you find that you have a lot of brackets, you miss one, and you get weird solutions and yes..

At the open coding level, I assigned a code as 'issues with brackets'. There were other codes assigned to this excerpt but for purposes of this illustration I limit my discussion to this code. At this level of analysis I remained at the descriptive level. At the axial coding level of analysis I grouped this code into a larger code called 'problems with syntax'. Brackets are just one aspect of the larger scheme of syntax when using MATLAB to programme a solution to a DE. Consequently, I grouped this code into an action called 'strives to get right the syntax'. Although Tumi does not talk about putting in much effort to get right the syntax in this excerpt, this is implied from what she says. Moreover, I looked for other exemplars in Tumi's interview transcript as well as in her actual problem-solving sessions. In Problem-solving Session 1, Tumi goes to great lengths to correct her programme code when it comes to brackets usage. This piece of information, linked with what she articulates above, suggests that 'strives to get right the syntax' is an apt code to use for this action.

During the selective coding process I decided upon selecting the axial code of 'strives to get right the syntax' and constituting a major theme which, together with other codes, I identified as 'processes followed when solving a differential equation'. The themes that were formed were neither mutually exclusive nor exhaustive. Indeed, certain actions could belong to more than one theme.

A great advantage of using the above coding process was that when I dealt with data that depicted actions, I was able to map the axial codes directly onto my activity theory framework, so 'strives to get right the syntax' was mapped as an action in Leontiev's hierarchy of activity levels. In this mapping process I had first to illuminate the goals that correspond to different actions so that I could work within the elucidation of Leontiev's concepts.

I draw attention to an important aspect in naming most of my axial codes. If these were to be linked with actions then I primarily used 'action' verbs to name these codes. From an activity theory perspective this allowed me to depict actions within Leontiev's activity theory framework. Some examples of these 'action' verbs include *tries to, compares, writes, redoes, solves,* and *studies*. This way of giving names in the form of 'action' verbs to sub-actions or actions is not new, see for example, Lautenbach (2005). The codes that were linked to motives of activity theory obviously did not follow this pattern of naming my labels.

One could say that this open coding process began in a grounded way. This process, however, continued with a more concept-driven coding approach in mind. I chose the above approach to code data because it not only made possible analysis of the content of the text but led to the following important points as suggested by Gibbs (2007):

- It enabled me to group together text that was similarly coded (i.e. related to similar action). Hence I could get to the essence of the code.

- I could also examine how, within a particular theme, the thought or scheme changed as the participant proceeded with his/her learning during the course of the semester.

- I could explore how these themes contrasted and compared with those of other participants.

At this juncture I list the major themes regarding actions that emanated from the analysis of both interview data and problem-solving sessions of all three participants:

- **Theme: Actions relating to optimising the environment**. This theme includes actions such as use of resources, independent studies, studies in a group, and attending lectures.

- **Theme: Actions relating to personal power** - this theme relates to making decisions with regard to what one studies and when one studies. It includes actions such as last minute studying of MATLAB (in maths), studying throughout the duration of the mathematics course, and choosing exercises to solve that include challenging ones. The decisions related to problem choice relate closely to motivational theory (for example,

students who have cognitive motives will display actions that include selecting challenging problems to solve).

- **Theme: Actions that could lead to learning MATLAB (in maths) operationally**. This theme includes actions related to solving problems repeatedly, so that they can be done at an operational level. Some examples are actions of practice; actions of students redoing the class examples and memorising MATLAB statements.

- **Theme: Actions relating to comprehension of maths (with CAS)**. This theme includes actions related to wanting to develop meaning, such as applying theory, assessing progress, studying theory and checking or verifying solutions. Note that this theme relates closely to the cognitive motives of learning.

Coupland (2004) used phenomenological methods to construct categories of students' engagement in mathematical learning. In her evaluation of students' experiences within an activity theory context she developed different categories. One category related to the goal of students seeking understanding and another to students learning maths at the level of operations. These two categories are similar to the ones that I have developed.

The themes that emanated from students' problem-solving sessions using MATLAB to solve DEs are:

- **Theme: Actions related to general computer use**. This theme includes actions such as using copy and paste techniques, putting effort into basic computing skills, opening of files and saving of files.

- **Theme: Actions related to the use of commands in MATLAB**. This theme includes actions such as the use of plot, display and legend commands in MATLAB.

- **Theme: Actions related to processes followed when solving a DE**. This theme includes actions such as writing out code before typing, striving to get the syntax right, making sense of error messages, rectifying minor errors after perusal of some of the error messages, paying attention to the sequence of steps in MATLAB and interpreting graphs.

- **Theme: Actions related to overcoming difficulties associated with maths whilst using CAS to solve DEs**. This theme includes actions such as working hard with independent and dependent variables and deliberating with functions and substitution.

A brief discussion of how these themes relate to each other and to the theoretical framework ensues. I refer the reader to Chapter 6.3, wherein a detailed argument of how each theme relates to the theoretical framework is undertaken.

There is a strong link between the institutional context of learning and the two themes of 'actions relating to optimising the environment' and 'actions relating to personal power'. The institution plays a key role in the social constitution of learning, with the three participants sharing common experiences in their diploma study and within the actual institutional milieu.

The two themes of actions that could lead to learning MATLAB (in maths) operationally and actions relating to comprehension of maths (with CAS) illuminate a range of learning experiences within the activity theoretical conception of learning. Specifically, Leontiev's (1978) notion of learning at the level of operations and learning encompassing consciousness are dealt with in these themes. Furthermore, the theme of actions relating to comprehension of maths (with CAS) relates closely to the cognitive motives of learning.

The two themes of actions related to general computer use and actions related to overcoming difficulties associated with mathematics whilst using CAS to solve DEs relate to Leontiev's proposal of personal histories. In the former theme consideration is given to the history of tool or computer usage, and in the latter aspects of student's history of mathematics are dealt with. The theme of actions related to the use of commands in MATLAB underscores the conditions under which operations arise.

Psychological tools as well as physical or material tools are imperatives of the activity theory framework and attention to these is given in the theme of actions related to processes followed when solving a DE.

Motives were coded using the framework outlined in Figure 3.7 (Chapter 3), where I engaged in a detailed discussion on producing the theoretical categories of motives from the perspectives of activity theory[34] and the Western approaches to motivation theory (from Chapter 2). Data related to motives was analysed from participants' interview transcripts into the three categories of cognitive motives, self-related motives and social (interrelations with others) motives.

[34] In particular Lompscher (1999).

4.6.3 EXAMPLES OF DATA ANALYSIS WITH RESPECT TO PROBLEM-SOLVING SESSIONS

Below I illustrate an example of coding from Tumi's Problem-solving Session 1.

2d goes to end of h=0.1; enters a line and types in m=(b-a); deletes ;
2e puts in) to get: m=(b-a)) deletes this bracket: m=(b-a)
2f m=(b-a)? deletes ?
2g m=(b-a)/m deletes m
17.10 2h m=(b-a)/h;
3a goes to line after x=1:h:2; types in y(1)=1;
3b 17.50 i=1:m;
3c 18.20 goes back and types in 'for' to get:
for i=1:m;
3d 18.49 y(i=1 deletes 1
3e 18.54 y(i=+ deletes + deletes =
3f 19.34 y(i+)=1= deletes =
3g 19.49 y(i+)=1+ deletes + deletes 1

Open codes for episodes 2f, 2g: keyboard use of / and ? keys; 3d-3g: keyboard use of + and = keys.
Axial code: struggles with the use of symbols that are represented by the same key on the keyboard (in other words misuse of the shift function). I grouped this to produce the axial code or action as 'Tumi puts effort into basic computing skills (this includes typing: struggle with the keys of the keyboard)'. The formation of the theme that incorporates this code is called 'actions related to general computer use'.

4.6.4 SELECTING ASPECTS OF DATA ANALYSIS FOR DISCUSSION

As detailed above, I analysed all interview transcripts, made a thorough analysis of students' reflections upon their learning of mathematics at high school level and levels 1, 2 and 3 of the diploma programme they were pursuing. In this respect I was able to analyse data into 'motives towards mathematics' and 'actions followed with respect to the activity of learning mathematics'. I could not however include all these aspects in the discussion chapter (my study deals specifically with students using CAS in mathematical learning). I thus chose to discuss only the needs, motives and object of activity related to how they reflected upon learning

mathematics. I felt that in order to understand students' motives towards the activity of learning mathematics with CAS I first needed to understand their motives towards the activity of learning mathematics; hence my discussion of these aspects.

4.7 SUMMARY

In this chapter I have clarified my methodology and methods. I provided information on how I went about selecting participants for the study, and on how I selected participants for discussion in the next few chapters. I discussed important tools used to gather data, particularly interviews and problem-solving sessions. In addition, I discussed how I achieved validity, reliability and how I handled concerns of ethics within this interpretive paradigm. Finally, I made clear my procedures used in analysing data. To this end I supplied examples related to the analysis of interview transcripts and screen recording transcripts. In the next chapter I present the results of this data analysis for Thembiso and Tumi.

Chapter 5

A structured analysis: exposing students' needs, motives, object of activity, actions and operations

5.1 INTRODUCTION

The aim of this chapter is to explicate the needs, motives, object of activity, actions and operations that emerged from analysis of data from Thembiso and Tumi. As noted in Chapter 4, only a tabular summary is presented of the analysis of Abraham. In order to understand better the motivations for using CAS (in maths) it is necessary first to develop a solid background to students' motives and needs with respect to learning mathematics without CAS, which together with their reflections provide an indispensable context. I therefore analyse two activities, namely their reflections on the activity of learning mathematics and the activity of learning mathematics with CAS. In the latter, students' reflections on this activity as well as actions carried out in the problem-solving sessions are discussed. To this end I present the analysis of the interviews with Thembiso and the two problem-solving sessions. I elaborate upon and discuss the activity theory model as it applies to him, beginning with his background, followed by discussions of his needs and motives with respect to his reflections on the activity of learning mathematics. I elucidate his needs, motives and object of activity for learning mathematics using CAS. The actions he carries out when learning mathematics using CAS are considered, and then I examine operations before closing the discussion with a summary, wherein I show the link between the activity theory model and empirical data. Similarly, I analyse interviews with Tumi and problem-solving sessions and this presentation follows the outline of my analysis of Thembiso. Throughout these discussions I provide evidential support from the transcripts of interviews and the two MATLAB problem-solving sessions.

5.2 THEMBISO'S INTERVIEWS AND PROBLEM-SOLVING SESSIONS

Portrayal of Thembiso's background will help in understanding and appreciating his motivations and actions with respect to the activity of learning mathematics using CAS, followed by a discussion of the needs and motives related to his reflections on the activity of learning mathematics. This provides the backdrop to his needs and motives when learning mathematics using CAS, together with the object of activity, actions and operations. In distinctive activity theory style I show how his personal histories play a significant role in the motives and actions adopted in current learning situations.

5.2.1 THEMBISO'S BACKGROUND

Thembiso presents himself as a self-assured individual who likes to impress others with his knowledge (use of technical language) and grades. He enjoys popularity, as can be inferred from his utterances in Interview 1:

> TH-int1(296). I enjoy. I enjoy the spotlight.

His mother tongue is Sepedi but he also speaks English clearly, loudly and confidently. He attended an agricultural school situated in the midst of farms but states that it was well-resourced with modern facilities that included a swimming pool, sports facilities and computer laboratory. Prior to enrolling for the current diploma programme in Mechanical Engineering, Thembiso studied the first year of a degree in Civil Engineering. He found that he was not prepared for the challenges associated with studying a degree, claiming the work was difficult and that he was not sufficiently mature to dedicate time to studying.

An important aspect of Thembiso's previous learning styles is his reported lack of effort towards studying. There are many episodes of his student life in which he reflects that he put in less effort than he would have liked to. When questioned why he found Grade 11 mathematics difficult he said:

> TH-int1(150). I didn't give it my all because at that stage I lived alone. I lived with my little sister. My mum moved to Polokwane, about 100kms, so she would come back on weekends. So that.. I didn't have that **drive** or the responsibility to, ok, **now** I have to study, now I have to.. yes, so I never gave it that much effort [words in bold his emphasis].

> TH-int1(264). … Part of the reason I didn't do so good in RAU [this was the university at which he studied the Civil Engineering degree] was immaturity. So here my diploma has taught me, ok fine, you need to give your priority basically. Which one comes first and.. so yes, to an extent it has equipped me but not fully? I don't think it has fully equipped me to make it at varsity yet.

Thembiso is of the opinion that a hard worker is someone who gives 90% of his time to his studies and is prepared to study on Fridays. He states that he is not yet a hard worker. I construe that he views effort as the key to success, yet he still displays confidence and enthusiasm in wanting to pursue his dreams. These virtues are sustained by the influence of a strong presence of family seniors, for instance an aunt who is his role model, as well as a supportive mother who believes in his abilities. They have a positive impact on Thembiso's career choice:

> TH-int1(340). … I have an aunty who has also done mechanical engineering. So she's, she's basically the pillar. She's basically pushing me. And my mum - because my mum knows nothing about engineering. All she knows is that it's difficult. But then I, I admire her.. mm.. is it resilience? I'm not sure [of] the word. Because coming from varsity I had very bad grades and then she still had that, ok, you can make it, it's all in you. Some parents would have lost hope and my son, engineering's not for you. So yes, I think she will be pleased, but most my aunt, my aunt, yes [this response was in the context of who would be pleased when he graduates].

The failure in the first year degree courses prompted him to study the current Mechanical Engineering diploma, for which he expressed exhilaration:

> TH-int1(24). …But then eventually later on, especially my course mechanical… I loved cars and the mechanical aspect, how they worked, so I find it very conducive to my hobbies.
>
> So coming here, changing from theory to practical, it was the catalyst for me to love my subject.

In the interviews, Thembiso conveys a strong desire towards obtaining a degree in Mechanical Engineering upon completion of the Mechanical Engineering diploma. He believes that by studying for the current diploma he will acquire the necessary skills related to commitment and dedication that will be needed when he engages in studying towards a degree. Obtaining this diploma is viewed as a springboard to pursuing a degree, though he would first like to work for

two years to gain practical experience. He expressed a sense of urgency in wanting to complete this degree:

> TH-int1(176-180). …because I'm serious about S3[35]. I'm quite serious.
>
> …Because right now I can see the finish line that, I er.. once after S4[36] I'm done, I have P1 and P2 [P1 and P2 are abbreviations for Practical-training 1 and Practical-training 2 respectively[37]]. So I'm motivated, I do not want another semester.
>
> I want to get through this, yes, as quick as possible. Hence I'm doing seven subjects.

In terms of the theory outlined in Chapter 3, I describe Thembiso's several needs that sustain his engagement in learning as follows. His current and future aims reveal culturally determined needs such as wanting to pursue an education and identifying a career. His immediate needs are to complete the diploma and obtain a degree. He also wishes to earn an income to provide for material necessities of life:

> TH-int1(22). …So eventually as.. I thought there was money; I was basically headed for the money [in engineering].

Thembiso's needs may be encapsulated as follows: he would like eventually to become an autonomous member of society. Even though he has not explicitly stated this need, it is implied in his articulations.

Thembiso learned to use computers whilst still in school, and although he exudes confidence when speaking about it he compares himself with computer 'geeks', whose level he claims he has not yet attained. Thembiso has access to computers at home but is dissatisfied with online access on his campus, which has only one dedicated computer laboratory for use of the Internet, and an insufficient number of computers for a growing student population. When asked if he uses the Internet often, his reply was:

> TH-int1(280-282). I don't know.. I only use it when I er.. because the one here at school [at the university campus], it has de-motivated me basically because it's slow and I hate

[35] S3 stands for semester three and he is currently studying semester three courses.

[36] S4 stands for semester four, which is the last semester of the diploma theory programme.

[37] In order to obtain the diploma, students need to pass the four theory semesters and serve two semesters in practical experiential training.

queues. So I hate going to the Internet lately. But then.. so I only go there for assignments and.. yes, not there.. just because I have extra time.

I can use my phone at home - at residence I use my phone as a modem. But only when I am er.. for example, an academic record is required for my CV, that's when I have to take it, download it from school. But I never go to.. just passing time no; too expensive..

In Interview 1, he reflected the immensely positive affect of working with computers:

TH-int1(288). I love computers - yes I do, I do, so I'm enjoying…

When questioned about what it is around computers that he loves, he responded:

TH-int1(290). The fact that I'm the master, it's the slave, I get to.. it gets to do whatever I want…

Certainly, these are his perceptions of using computers prior to learning about programming to solve DEs using MATLAB. After learning to use MATLAB to find solutions to DEs, Thembiso does not feel the same way about him being the master in this specific relationship to computer use:

TH-int2(98). Initially I loathed MATLAB, I didn't really.. because it was complicated, it was complicated. In terms of we were exposed to other softwares, of which they were straight to the point, but then MATLAB required.. um, even the, er.. what's the other part, there's the editing part and the what's that M part, not the M-file, the other one.. command window. The command window, mm.. when it gives you errors you don't understand them…

5.2.2 THEMBISO'S NEEDS AND MOTIVES WHEN HE REFLECTS UPON LEARNING MATHEMATICS

In this section I discuss the needs Thembiso has with mathematics specifically, then elaborate on motives espoused when he learns mathematics at the Mathematics 3 level.

5.2.2.1 NEEDS

During Thembiso's high school days he took pride in identifying himself with those scholars who studied mathematics (Interview 1). He uses labels such as 'smart' and 'coward' to differentiate between individuals who study mathematics and those who do not. From a cultural

perspective the scholars at the school he attended appeared to promote the study of mathematics, and he was part of this peer group. When questioned in Interview 1 as to why he chose to study mathematics in Grade 10 at high school he responded as follows:

> TH-int1(140). Ok, that, I was, I was very immature then so I didn't have a goal at where or what I wanted to do. But then the fact that ok my sister did maths, so it played a role in the fact, and we considered people who didn't take maths as cowards basically. Yes, so that was the main reason actually I took maths that my sister would help me if I struggled and I also don't want to be, don't want to be a coward.

His response when asked who the 'we' is when he said, "we considered people…" was:

> TH-int1(142). The rest of us who did maths. Actually the whole school. It's like mm.. no I wouldn't give an example about here at school, it's not fair. But for example if you didn't take maths, you would take travel and tourism. Something that was the choices we had. So those who went for travel and tourism, they were more into, no, I've never been good at maths. Yes so I was like, I do not want to be part of that group. We want to be part of the smart ones.

Behind a motive is a need, of which the individual may however not be aware (Leontiev, 1981), and seeking recognition is a human social need. Thembiso is aware of his needs and makes decisions with this awareness in mind. A crucial one could be recognition as part of the group of 'smart' students studying mathematics. Labels such as 'smart' have become part of his identity, and he does not wish to be characterised as 'coward' or 'dumb' by choosing not to study mathematics. In his opinion he has to do mathematics to show others that he has the courage to do so, and he wishes to live up to the very expression that he and his peer group had coined during high school days. Years later one can notice the influence of these personal histories on his decision-making abilities, i.e. he made the choice to study Mathematics 3 when it was not a compulsory option in the Mechanical Engineering diploma. Although there were other motives that played a role in his choice to study Mathematics 3 (to be discussed below), the main underlying need remained a need to be recognised as 'smart' by studying it:

> TH-int1(134). …Added to the fact that I always thought of maths as a compulsory subject from high school, from high school as if you're not doing maths you're dumb. I've always been one of that group who'd say that. So and now.. it's sort of **hardwired** in me that "do maths!" yes, do maths… [his emphasis in bold].

Various social and cultural factors, such as family, peers and school, influenced his perceptions of what he wishes to pursue in life. His histories and experiences have shaped many perceptions of himself and his future in significant ways.

5.2.2.2 MOTIVES

In the interviews Thembiso gives the impression that he is not inherently interested in studying mathematics for the love of it. In the reported motives to study Mathematics 3 he alludes to motives that I classify as 'self-related', while those I categorise as cognitive in nature are absent from his discourse. Even though he states that he finds certain sections of mathematics, for example geometry, interesting, these do not have a directing and energising function on the activity of learning at the levels of Mathematics 1, 2 and 3[38]. He expresses no interest in learning mathematics related to the methods and modes of problem solving, which are essential to arrive at results.

He has motives that I classify as social[39] in nature, but these are not directly related to pursuing the study of mathematics:

> TH-int1(270). No, group work doesn't work for me. It doesn't. I just can't. I tried group work in a different subject.. last week Monday, was it Monday; no I think it was on Monday. It didn't quite work out. Because I talk a lot. Because we end up talking and being off subject. So I prefer studying on my own.

Thembiso's social[40] motives will be elaborated on in the section on motives espoused when he learns mathematics mediated by CAS (5.2.3.2).

Self-related motives

Within the assemblage of self-related motives I explain how Thembiso clearly has different motives to learn mathematics. At the beginning of the Mathematics 3 course he eagerly

[38] Mathematics 1, 2 and 3 refer to the semester levels studied for this diploma, and none include geometry.

[39] Note that social in this context refers to the interrelations of an individual with others

[40] interrelations with others

expressed a motive to obtain a good grade in this course. When questioned as to why this was important he replied:

> TH-int1(320). It's important because for companies, for companies, yes in terms of bursaries and internships, yes I think it would make a difference. Compared to Cs, at least an 'A' there yes, it would add **colour** [his emphasis in bold].

> TH-int1(306). …I've set myself a target of at least a distinction per semester. So Mathematics 3 should be part of it.

Thembiso believes that getting an 'A' symbol in Mathematics 3 would make a difference by opening up the possibility of more opportunities for scholarships and internships. Furthermore, this performance approach goal-orientation of seeking good grades is not accompanied by a rationale of putting in more effort into studies. He wants to get a good symbol with less effort:

> TH-int1(120). Looking forward to it (studying Mathematics 3), well er.. the reason is I have a positive.. I don't have the right word for it.. I could say, approach. Because the last semester 75% had distinctions. A hundred percent pass rate. So, I'm guessing, not only will this subject improve my academic history, it will also be less of a hassle if I had to for example, there was a 30% pass rate, so I know I have to give it **more** work to make sure I'm not part of the 70% [his emphasis in bold]. Yes, so in this instance I know it's better and from other students they're also saying it's quite ok. [He gives an example that if there was a 30% pass rate this would imply that he would have to work harder in the subject compared to when there is a pass rate of 100%].

Taking into account the context in which this information on pass rates was passed onto Thembiso, it appears that his lecturer had told the lecture group at the beginning of the semester that the previous Mathematics 3 course had 100% of the students passing Mathematics 3 and 75% obtaining distinctions. Consequently, Thembiso had a motive towards obtaining a good symbol, albeit with a more relaxed notion of the effort required. Even though he stated that he plans to get this 'A' in Mathematics 3 by giving it time, he ultimately does not allocate much time for studying Mathematics 3, as will be determined from the actions undertaken.

In the interviews, Thembiso states that he also chose to study Mathematics 3 to avoid taking a more difficult module. He had a choice between an Electrical Engineering module and Mathematics 3, but according to hearsay from other students, who said that the electrical module was difficult, he decided to select Mathematics 3 and do the Electrical Engineering

module part-time, whilst aiming to study for a B.Tech qualification[41] in the future. The reason he was concerned about choosing the challenging module during this semester was that he had a workload of seven subjects for which to obtain credits during this short semester. It is evident that trying to complete a large number of modules for an engineering qualification has had an influence on his decisions.

Two main motives in opting to study Mathematics 3 are related to reasons that involve an extrinsic utility value. The first is that Mathematics 3 is required for a future qualification. Thembiso claims to have chosen to study Mathematics 3 because:

> TH-int1(134). … I also saw at B.Tech level there's Maths 4 or something too that (I) need to take maths. So obviously I'm going to need Maths 3.

Thembiso would like to pursue a higher qualification upon completion of the diploma, and has identified the B.Sc Engineering degree or B.Tech qualification as possibilities. This information relating to his future aims is useful in understanding the influence this has on his current decision to study mathematics.

The second extrinsic utility value motive is that studying Mathematics 3 is useful because of its application to industry and to engineering scenarios. When questioned (Interview 2) about some of the things he liked in the Mathematics 3 course, he said:

> TH-int2(6). Mm, the application questions, the application questions. It gives me er.. a feeling that, like, I'm doing engineering and then it's not just Maths 3 where I have to calculate *X*. Now I have to find springs, shock absorbers of cars, the forces in them for example, using maths.

5.2.3 THEMBISO'S NEEDS, MOTIVES AND OBJECT OF ACTIVITY WHEN LEARNING MATHEMATICS WITH CAS

The particular needs, motives and object of activity in learning mathematics with CAS gleaned from interviews with Thembiso are now discussed in turn.

---

[41] B.Tech is a Bachelor of Technology programme and a more advanced diploma than the current Mechanical Engineering one. At times it is considered equivalent to a B.Sc honours degree although there is ongoing debate on this. The orientation is still practical, not theoretical like the typical honours degrees.

5.2.3.1 NEEDS

In addition to having the privilege of being exposed to computers during his high school days, Thembiso had, during this data collection phase, access to a computer in his home environment. His wish to be recognised as part of the cluster of 'smart' students is not only satisfied by his association with the subject of mathematics and those who study it, but also with his involvement with computers and the associated technical jargon:

> TH-int1(290). …And I sort of become part of this elite groups of… ok, I'm not necessarily a geek yet. But then that group, I get, I know my way around it I can say. So I'm enjoying it.
> TH-int1(294). …It also gives people this belief, you know when you start speaking all this technical terms of computers that this person is intelligent, so that kind of thing, yes.

By knowing about computers and the accompanying technical terminology, Thembiso wishes to be an embodiment of intelligence, showing others that he is intelligent.

5.2.3.2 MOTIVES

Motives identified were categorised as either social or self-related.

Social motives (interrelations with others)

I infer from both the interview data and observation of problem-solving sessions that Thembiso enjoys communicating with others whilst learning mathematics with CAS. Firstly, I discuss his reported (Interview 2) inter-relationship with another student in a more superior position and studying towards a B.Tech qualification. During his study of MATLAB in mathematical learning, and two days before the MATLAB test, Thembiso explained the work covered in MATLAB laboratory sessions to this friend who had not attended any of the laboratory sessions during the semester. Whilst doing so he claims that he was going through most of the material for the first time since attending lectures. Thembiso reflected on this association as providing assistance to a friend, but also acknowledges a beneficial involvement is arising:

> TH-int2(182). It, um.. the fact, ok I was helping him but I was gaining a lot, I was gaining a lot. … So when he came there it was more of a revision for me, for me. Because I have done, these are the things that I've done in class. So we were just going through the examples and everything and I was quite happy with it.

Moreover, I got an impression of Thembiso's self-esteem in this relationship:

> TH-int2(180). …So explaining also from chapter one to.. it also helped me. Yes, surprisingly. It helped me to understand everything, yes. And he's in B.Tech, mind you.

From the above discussion, I assume that Thembiso would also like to impress this group of friends. He came across as an individual who enjoys establishing inter-relationships with peers, and explained in detail the solution of a DE using analytical methods to Manto (his partner during the MATLAB problem-solving sessions). This will be described further in Section 5.2.4.

Self-related motives

Performance approach goal-orientations are an important aspect of self-related motives. Thembiso admits that obtaining a good grade in the MATLAB examination is important to him. He is content with MATLAB grades contributing towards the examination marks of Mathematics 3:

> TH-int2(274). I've set myself goals. …And part of my goals was.. ok, per semester, I have to have a distinction. … But maths I kind of slipped a bit. But then I can um.. I went down to just getting a B, a 'B', which is at least 70. For me it's enough.

> *TH-int2(248). No, no. I would have got a worse symbol* [when questioned if he would have got a better symbol in Mathematics 3 if MATLAB was not counting towards the examination mark].

Students studying towards a vocational qualification such as engineering find the application of MATLAB interesting (Periasamy, 2008a). Thembiso expresses positive affect towards any type of application as it sustains his interest in learning to use MATLAB, which he finds meaningful and enjoyable to apply. Unlike the Electrical Engineering students, who are usually able to see relevant application of numerical solutions to DEs within their field of study, Thembiso, a student of Mechanical Engineering, reported in interviews that he sees the application outside his field:

> TH-int2(128). I enjoy er.. the fact that, for example, also application questions, there's some part about bacteria and er.. we did the same things in Maths 2, er.. yes, so right now we're doing them in.. I enjoy application questions because it gives me a feeling that ok what I'm doing is not pointless, it's not pointless. I can apply this somehow. So I, I, I enjoy the application questions of MATLAB.

After spending an entire semester learning to use MATLAB in mathematics, he perceives no direct relevance to his field of study of using it specifically when solving DEs:

> TH-int2(140). Um.. ow, ow.. I'm not really sure, I'm not really sure. Because DEs, mm.. besides the application questions of DEs, I wouldn't say I see the purpose of it, I wouldn't say.

Nevertheless, he is convinced that companies are using MATLAB in this era of technology so he believes that it is required in the workplace environment:

> TH-int2(138). Um.. so far.. I could say.. um, ok most, most companies in industry today they're reverting to MATLAB compared to hand. So it would.. having both of them at my disposal it would be ok, it would be ok…

I assert that the self-related motives that Thembiso alluded to above are extrinsic utility value motives. He finds learning to use MATLAB useful because of its envisaged application in the workplace as well as other areas of life.

Students learn how to solve DEs using analytical methods in the previous Mathematics 2 course. Currently they are exposed to solving the same DEs as well as finding numerical solutions to other DEs using MATLAB[42]. Thembiso is in a good position to spell out his motives with respect to MATLAB use:

> TH-int2(288). I would prefer, er.. the MATLAB because number one it's, it's accurate, it's accurate. And unlike, compared to calculation, if I make a mistake in calculation, I might still find an answer. If I make a mistake in MATLAB I might not find an answer, but then if I do find an answer, er.. most probably you could see, you could see based on er. whatever er. like the MATLAB will tell me, the graph will tell me - you could see this is completely out of hand, you could see. Another thing, MATLAB it's shorter, it's shorter compared to calculations. It requires; it takes less time. So I would take MATLAB compared to other er. methods.

> TH-int2(284). …I'm, I'm, I'm glad it's er. compulsory to take it. I am glad because it helps a lot. And it's very simple.

[42] Although some lecturers might have shown students how to find the numerical solutions to DEs by pen and paper substitution techniques, most would have directed this time to teaching them how to use MATLAB to solve them.

I deduce from the above that he is motivated to use MATLAB because of its potential to produce accurate solutions (compared to pen and paper solutions). He claims that MATLAB is timesaving, helpful, and easy to use. Some of his motives seem incongruous because he spends much time in trying to produce the solutions in both problem-solving sessions. I assume that the problem-solving sessions were the learning phase, and at the end of the semester he is convinced that MATLAB is a timesaving device. The creation of graphs with MATLAB that corresponds to the table of numerical solutions is important for him to realise if he has correctly solved the problem. These different representations are useful in his learning process.

5.2.3.3 OBJECT OF ACTIVITY

Thembiso's object of the activity is to get a good symbol in MATLAB assessments by studying at the last minute, redoing class examples and similar exercises and memorising MATLAB statements. I argue that his aim is to show others that he is intelligent and by choosing to participate in the activity of learning mathematics and its accompanying association with CAS, he strives to achieve this aim, which seems to manifest itself in his social motives. He is studying MATLAB because he perceives it as useful for the future, and finds using it to be easy, accurate and timesaving.

5.2.4 ACTIONS CARRIED OUT BY THEMBISO WHEN LEARNING MATHEMATICS WITH CAS

I combined actions related to learning mathematics with CAS into the following themes.

THEME: ACTIONS RELATING TO OPTIMISING THE ENVIRONMENT

There are many factors that may enhance the learning process, for example, attending lab sessions, questioning the lecturer, making use of resources other than the prescribed 'textbook'[43] and discussing with others. Below I expand on how Thembiso makes optimal use of factors associated with the learning situation.

[43] As previously explained (Chapter 4.3.1), the prescribed 'textbook' comprises a series of notes compiled by a senior lecturer of the course and it lacks a thorough explanation on the section of solving DEs using MATLAB.

Action: dutifully attends MATLAB laboratory sessions

In the mathematical learning process of the Mathematics 2 and Mathematics 3 (theory) courses, Thembiso reported in Interview 1 that he placed great emphasis on attending lectures. He felt that this was important because it enabled him to understand the work covered in class. However, he made some statements from which it could be determined that his understanding of the work covered in class was not at a deep or meaningful level. For instance, he said that by attending lectures he had some hint of the work covered:

> TH-int2(92). …Because um, it's not so difficult that you will get stuck every chapter. You can complete it in say a week, the entire book; provided you attended.. you have a clue of what's going on.

He stated that he dutifully attends the laboratory sessions. From the above line of reasoning and the quote below it is evident that the goal corresponding to the action of dutifully attending MATLAB lectures is 'to have some clue' of the work covered in the laboratory. I argue that he attends MATLAB laboratory sessions so that he can obtain the work carried out in class and solve a few problems:

> TH-int2(150)…Also the same way as the theory. I get, I go to classes. I make sure I attend all my classes. Especially since its Mondays, I attend all my classes, I get examples, I get um.. there are class questions that we are required to do, and then we save them…

Action: uses resources

Thembiso's use of resources to learn MATLAB in mathematical learning is very limited. Other than the prescribed 'textbook' he does not consult any books or the Internet. When questioned about his use of such resources he responded:

> Th-int2(176). No, I haven't, I haven't done that, I haven't done that. I am quite satisfied with the material that was given to us.

In addition, MATLAB has a help-file that should provide valuable assistance, albeit not according to Thembiso:

> TH-int2(208). No. I tried, I tried um.. my sister just installed it. So um.. she, she told me it, it helps a lot. So er.. when I went.. it is very complicated, I don't know how.. Because once you open a window it gives you a lot of, a lot of things, so I decided no, no.

Having attempted to use the help-file, he found it complex and consequently abandoned it.

The resources that he makes use of during the MATLAB mathematical learning process are classroom-solved examples. He claims that he copies the examples that the lecturer solves in the MATLAB class and studies them again two days before a test or examination[44]. He reflects that he also solves exercises during the MATLAB lecture time and uses them again for studying before a test or examination. He makes use of the prescribed 'textbook', a constant reliance upon which I observed during Problem-solving Session 1, during which he resourced his partner Manto and me:

> 5a Thembiso compares his table values with Manto's table.
> 6a : Discussion with Manto: you see anything different there?
> TH-ref-int1(9-12). Manto: I see the equations are different; – *h*.
> Thembiso: oh I used *x*.
> Manto: No *h* supposed to be there all the time.
> Thembiso: I didn't put in that *h*.

It appears that Thembiso leaves the learning of MATLAB (in maths) to the last minute[45]; and consequently is unable to resource others, even if he requires assistance.

> TH-int2(162). …I just couldn't understand the questions, er.. that was required. So I, I leave it. Because it's too late to consult the likes of Peter and other students.

However he is not averse to seeking assistance:

> TH-int2(206). I would ask others provided there's still time, there's still time. I would ask er.. yes because Peter lives around, yes, close to me. I would go asking him, if he doesn't understand; if there's still time I would come to a lecturer.

Thembiso makes use of resources so that they could assist the learning process, which is the goal corresponding to the action of 'uses resources'.

---

[44] This aspect of studying two days before a test or examination will be discussed in detail in the next theme on personal power.

[45] This aspect will be discussed in the section on action: cram or study mathematics (with CAS) last minute before a test.

Action: questions about subscripts in MATLAB

In Problem-solving Session 1, Thembiso questions how subscripts should be represented in MATLAB:

> TH-ref-int1(1). Can I just ask a question regarding this *y(1)?* I have noticed in that you can't use *0* but then it's *y(0)=1* and *y(1)=1*.

The goal for this action is to understand subscripts in MATLAB.

Action: tries to get access to MATLAB software

The most important means when learning to use MATLAB in solving DEs are the computer and MATLAB software. I assume that the goal is for Thembiso to gain access to the software so as to practise and learn mathematics. When questioned at the beginning of the MATLAB course if there might be any obstacles in his path that would prevent him from obtaining a distinction, Thembiso stated:

> TH-int1(314-316). No, I don't think so. I don't see anything that.. unless.. no, I don't, I don't.
> JP: Unless? Thembiso: I would say unless my computer fails but there are still computers here at school for me to practise and.. no, I don't see anything else, in terms of the MATLAB…

His computer did not function for most of the semester and he had not made use of the campus computers laboratories for practicing. After Problem-solving Session 1, he admitted to not having done any of the MATLAB homework problems:

> TH-ref-int1(100-103). I have never done them.
> … The thing is I have MATLAB on my computer but the computer isn't working. At residence, it doesn't work so I've been planning on once I fix it then I can start practising.
> … today, today I was going to fix it. That is why I was running around but I haven't made any progress.

By Problem-solving Session 2, Thembiso had still not repaired his computer:

> TH-ref-int2(80). I was planning on installing MATLAB but then.. computer problems.. I am working on it. I am working on it before the test.

He reported that he had managed to get his computer to work before the second test, which was just prior to the MATLAB examination. In summary, he had dutifully attended lectures, copied classroom-solved examples and completed the solutions to some exercises in class. Apart from these he had not been able to engage with any of the exercises or examples on his own, due to computer access problems[46].

> TH-ref-int1(100-102). I have never done them.
> … The thing is I have MATLAB on my computer but the computer isn't working. At residence, it doesn't work so I've been planning on once I fix it then I can start practising.

Note that his style of learning the Mathematics 2 and Mathematics 3 theory points to:

> TH-int1(172). (I) usually would not do it, yes.. for as long as I understood what happened in class that day, yes, I wouldn't do it. I would er.. make a note of what I had to do. But I would not do it.
> TH-int1(95-96). JP: One week before the exam? Thembiso: Not exams, tests, tests. I would go through everything, try to understand.

Thus in Thembiso's case I argue that the lack of access to a working computer (with MATLAB installed) did not influence the way he approached the learning of mathematics with CAS. Nevertheless, he left the learning until two days before the MATLAB major test, similar to his reported previous approaches to learning mathematics without CAS:

> TH-int2(158). No, no, it was a thing I was waiting for my computer to get, to get fixed. So right now I, I wouldn't do them immediately, I wouldn't. Even if I knew the sixth floor, they're open on.. I never went there, I never went there. But right now I'm just, I've done them, I would dedicate maybe er.. on a weekend, I would give Saturday, Saturday, for example last week we were writing on Monday, I gave Saturday to, to MATLAB until I understood everything…

Action: explains MATLAB to other students

Thembiso reports in the interview that he prefers to study Mathematics 2 and Mathematics 3 (theory) courses independently. Although his reflection on group study shows that he has motives that I categorise as social (talking too much and then not about the topic of study during

[46] Except for two MATLAB projects or assignments that were mandatory to complete and he claimed to have used his friends' computers.

one of the attempted group work sessions), he feels that group members generally tend to decrease his pace:

> TH-int2(78). It's a thing, mm.. they tend to slow me down. I cannot study when I'm with someone. I usually mm.. get relaxed, talk a lot, or maybe he's busy with a part that he doesn't understand but I understand. So it gets to slow me down because I have to explain that part to him. So I prefer doing it on my own.

This conveys a sense of superiority, giving the impression that he has to help others and that he is not the recipient of help from the group members.

Thembiso assumes an active role in explaining MATLAB to others (see Section 5.2.3.2). The goal corresponding to the action of 'explains MATLAB to others' is that Thembiso wishes to satisfy motives that I classify as social. He enjoys inter-relationships with others and claims that he was professional in the manner in which he explained MATLAB to the senior student:

> TH-int2(200). No, no, no. I'm guessing it's a thing of, he's a much older person, so I was more professional. But if it was someone my age, we would play and talk a lot and yes, eventually we wouldn't go anywhere…

THEME: ACTIONS RELATING TO PERSONAL POWER

In discussing this theme I focus on how Thembiso gained control over his student life, particularly as a student of MATLAB whilst learning mathematics. I give attention to his choices concerning what and when he studied MATLAB in learning mathematics.

Action: cram or study mathematics (with CAS) last minute before a test

Thembiso claims to have crammed or studied MATLAB (in maths) at the last minute, prior to MATLAB tests or examination. As reported, he attended MATLAB lectures, had 'some clue' of how the examples were solved, solved exercises during the lecture time and did not engage with the content until an assessment drew near:

> TH-int2(180). I dedicate I dedicated a day. I, I wrote last week. I gave Saturday to MATLAB. It's a thing of setting a schedule for yourself, I did that before, er.. during the week, that on Saturday I'll be busy with MATLAB until I can see I'm satisfied and I understand. So I did that, er.. just go about the whole day, the whole day, using the same

technique I use, I started from chapter one even though I knew I understood, from where we are required to plot graphs and stuff. And then another thing that helped me was this, this student who has not attended at all. So it was a thing I, I have to explain to him how we go about it. So explaining also from chapter one to.. it also helped me…

With respect to the action of cramming MATLAB in mathematics learning at the last minute, I infer that Thembiso wished to prepare just for the MATLAB assessment. When questioned if he would have studied had there been no test, he stated:

TH-int2(186-188). I don't think I would have done it, I would have done it.
JP: And why not? Thembiso: The reason is six other subjects are demanding, are demanding.

It is notable here that there appears to be a recurring pattern with respect to his approach to learning mathematics (theory) as well as mathematics mediated by MATLAB, namely, leaving the bulk of the learning to two days before an assessment. Even though Thembiso began the Mathematics 3 course with renewed vigour, where (he reports that) for the first time in his mathematics study for the diploma qualification he caught up with work that he had missed due to his absence from the first mathematics lecture, this energy was not sustained[47]. The question arises as to why Thembiso left the learning of MATLAB (in maths) to the last minute. Of the few possible explanations, the first relates to the issue of having a large workload due to the number of courses he was studying (see previous quote), which left him with little time to spend on practicing MATLAB. The second relates to his learning style, namely, by which he forces himself to study MATLAB in learning mathematics because of the existence of assessments (both tests and examinations). The third relates to his motivational profile. He holds performance approach goal-orientations in which he mainly wishes to achieve good grades and consistent with the literature on students holding such goal-orientations he wishes to score well with minimal effort, and is concerned about the image that he portrays. One could conjecture that if he leaves the studying of MATLAB (in maths) to the last minute, and it happens that he does not do well in the assessment, then he could always defend his results by claiming that he did not put consistent or adequate effort into practicing MATLAB in learning mathematics.

[47] Despite the fact that he was driven because he was nearing the end of his studies and wanted to obtain the diploma as soon as possible.

Action: redoes class examples

During this last minute study, Thembiso alludes to revising work that had been covered previously in class. He reported that he redoes the examples solved in the MATLAB lectures by writing them down on paper before typing them onto MATLAB:

> TH-int2(150-152). …So once I'm practising um, er.. differential equations, I would go through the examples which we did in class on paper, on paper.. I usually put them on paper, and then afterwards I would do them on the computer.
> … Redo the examples, yes, on the computer and check them.

The goal is to check whether he gets the same answers as did the lecturer when it was done in class.

Action: chooses which tasks to solve and Action: attempts to solve more difficult problems

In Interview 2 he described going through all class examples but not all of the exercises:

> TH-int2(158). …I gave Saturday to, to MATLAB until I understood everything. So once there was that. Yes but then I did everything, I did everything.

When questioned about the exercises, he stated:

> TH-int2(160). Not all of them, but then I covered everything.

In this context 'did everything' and 'covered everything' refer to revising the work completed during lectures.

> TH-int2(154). When I'm done with the exercises, I mean the examples I would go to the exercises - the ones in the book that we haven't done.

Thembiso seeks confirmation of methods by first doing known class examples and then attempting to solve exercises from the 'textbook'. The goal associated with the action of 'chooses which tasks to solve' is to become familiar with worked out class examples.

Most importantly (as discussed in Chapter 2) the kinds of exercises that students select to study of their free choice are an indicator of motivation. Thembiso claimed that he attempts to solve more difficult problems:

> TH-int2(204). Um, I, I would usually mark it; er.. go to the rest, go to the rest that I understand. And if, if I'm, I'm done and I have to come back to this one, then the thing was I couldn't understand what was required. I didn't understand the question. So I battled but eventually I couldn't, I just leave it, I just left it.

From the following I deduce that he does not persist in solving problems that are challenging in nature:

> TH-int2(162). I did have, I did have challenging ones that I couldn't do. I just couldn't understand the questions, er.. that was required. So I, I leave it. Because it's too late to consult the likes of Peter and other students.

I surmise that the goal corresponding to the action of 'attempts to solve more difficult problems' is to prepare for an assessment.

Action: chooses Euler over Runge-Kutta II and Runge-Kutta IV methods

Thembiso wants to learn as few of the numerical methods as possible. When questioned about which of the three methods (Euler, Runge-Kutta II and IV) is more accurate he chose Euler because he assumed that the first method taught to him was the most important one:

> TH-ref-int2(63). I would usually prefer the Euler. It could be a mindset thing because it's the first thing I learned. So it was programmed in me that it's the 'more' better assumption.

I subsequently informed him that the Euler method lacks accuracy; he realised that this method will not be used widely in industry:

> TH-ref-int2(66). So why do they have to teach us the Euler and Runge-Kutta II if they could? Yes it's into detail but then we never going to use the Euler say in industry or anywhere.

Notably, his motive to learn a method is with a view as to where it would be useful as opposed to learning for the love and interest in it.

A scenario was posed to Thembiso in which he was asked to imagine he worked for an engineering company and carrying out some modelling of equations. He arrived at a DE that needed to be solved and the company had MATLAB available. Would he choose to solve the DE using Runge-Kutta II or an Euler solution? Thembiso's response to this hypothetical question was:

TH-ref-int2(69). From today I would go for the Runge-Kutta IV but then if it's complex I would for an assumption depending on how much into detail the company wants it, then I would for the Euler.

Having learned that the Runge-Kutta IV is the more accurate numerical approximation to use, Thembiso consents that in real life application there is a possibility of him choosing less complex methods, such as the Euler method, unless the company demands more accurate solutions. The goal corresponding to the action of choosing Euler methods over Runge-Kutta is to be successful in finding a solution, even if it is not the most accurate one.

Action: completes MATLAB projects that are similar to class examples

His approach to learning mathematics mediated by MATLAB is an assessment-driven approach. Apart from the practise that he gets during the MATLAB lectures he only uses MATLAB on his own because of the obligatory MATLAB projects (or assessments):

TH-int2(212). …I could go to them for.. but not to practise, to do the assignment, to do the assignment. Because it's not very um.. different from the exercises we did in class. So using the exercises and stuff I just refer and do the project.

The marks obtained in the MATLAB projects contribute to the term mark for each student. The action is to complete the MATLAB projects with the goal being to obtain a mark. The MATLAB projects are similar to class examples and class exercises, thus there is an indication that the projects are neither very demanding nor designed in a manner that elicits investigation or exploration. Thembiso completes the projects by resourcing worked out class examples.

THEME: ACTIONS THAT COULD LEAD TO LEARNING MATHEMATICS (WITH CAS) OPERATIONALLY

As detailed in Chapter 3, an important aspect of learning from the perspective of the activity theory framework is the notion that conscious deliberate actions should, over time, become non-conscious and automatic operations. Next, I elaborate on how Thembiso could be engaging in a few actions that I classify as operationalising aspects of MATLAB in learning mathematics, albeit without comprehension.

Action: memorises statements or methods

To ascertain whether Thembiso would memorise aspects of mathematics, I suggested another scenario:

> TH-int2(193-198). JP: Let's suppose that you are writing a test next week and you got all the answers - … you got all the memorandums to the Mathematics 3 problems. You are confused as to how some of the answers were obtained. Let's say the theory first. What will you do?
> Thembiso: …So it's a thing of, I would try to, to memorise it. To memorise the, the steps of how they went about solving it.
>
> JP: Would you do the same with the MATLAB stuff that you don't understand and you got all the answers to?
> Thembiso: Um.. The thing is, ok, they have given us memorandums, questions with the memorandums, as well, yes. Um.. I think I would, I would but then I haven't, I'm not exactly sure what I would memorise in MATLAB. But provided there was er.. a few steps that you go about it. For example I would say, when we solve um.. when I was explaining to this B.Tech er.. student, in terms of $M = B - A$ over H. It's a thing that er.. I know it has something to do with it's the number of times, something like that, but then I couldn't exactly explain to him. But then what I'm saying you have to put it this way in every.. so I told him, just, just memorise this line, just just know it like that, yes. So it's a thing, if you don't understand um.. say and it comes, it keeps repeating, memorise it then.

Certain statements of MATLAB, such as the division of the domain into equal sub-intervals, appear in every numerical solution of DEs. Thembiso has memorised this statement and does not know why it is required in the solution, nor could he explain the existence of such a statement to the student he was helping. He does not attempt to find out why this statement is necessary. I contend that he does not persist in wanting to engage with MATLAB (in maths) in a deep way, as this is evidence of a surface approach to learning MATLAB in mathematics. The goal related to the action of 'memorising statements or methods' is to know the solutions or methods just prior to a test.

Action: redoes class examples by first writing them out

As previously elaborated, Thembiso claims to make use of methods such as redoing MATLAB class examples by first writing them out and then typing them into MATLAB to verify the

solutions. These methods can be described as actions that are in the process of becoming operations.

> TH-int2(150-152). It's just a matter of practicing. Also the same way as the theory. … I attend all my classes, I get examples, I get um.., there are class questions that we are required to do, and then we save them. So once I'm practicing um, er.. differential equations, I would go through the examples which we did in class on paper, on paper.. I usually put them on paper, and then afterwards I would do them on the computer.
> JP: Redo the examples? Thembiso: Redo the examples, yes, on the computer and check them.

He reflected that he closely follows the techniques used by the lecturer, but does not examine any other methods that are used in the examples of the 'textbook':

> TH-int2(164). This method it's er very effective for me. Because mm, I, I get to go through using not the 'textbook', but the examples that were done in the 'textbook' using the lecturer's technique. So this helps me in terms of, if I get stuck somewhere, er.. say in the exercises that I'm doing, and then using the lecturer's technique, I can go back to using the example from the 'textbook'. But that usually doesn't help because the lecturer will give us the simplest and user-friendly technique available. So um, this method it's effective because I get to practise er the examples, understand how I should go about it – solving and then that's when I go to the exercises.

It consequently becomes evident that the goal associated with the action of 'redoes class examples' is to understand techniques or procedures.

Thembiso's style of learning is firstly to solve problems that are similar to worked out examples covered in class. If he completes this process and has time he then attempts the challenging exercises in the 'textbook', provided they have answers. If he cannot solve these then he tends to ignore them. Because he leaves his studying to the last minute, when faced with difficult problems he does not seek assistance. He organises his learning schedule in such a way that he struggles to fit into the agenda the task of solving challenging problems, leaving no time for experimentation or investigation, both of which are crucial to the learning of mathematics using MATLAB. Consequently, it appears that the MATLAB component has not stimulated his curiosity sufficiently to influence him towards treating the learning of MATLAB (in maths) differently from the learning of mathematics theory.

THEME: ACTIONS RELATING TO COMPREHENSION OF MATHEMATICS (WITH CAS)

In Thembiso's case, there are a few actions that relate to the comprehension of mathematics (with CAS), discussed here.

Action: interprets graphs and Action: checks the y values in the output

Considering the discussion thus far, I argue that Thembiso puts very little effort into seeking meaning in the learning of mathematics using CAS. However, an exception is that he reports interpreting graphs after having produced the numerical solution to DEs:

> TH-int2(240). Usually. ok before, before I think the second last session I had.. before the previous one I think.. It was a thing I would go there; the main thing was to battle to get a graph portrayed. So once er.. I, I, it would display - it will plot a graph, I was happy, I was happy that um.. But then I learned a few things. Number one was graphs should not cross. So I know once when they cross there's er.. a mistake somewhere. Usually instead of the dy/dx, the independent and dependent variable you would swap them. So I learned that…

Thembiso, like most of the other students, would only relax once he obtained a graph, irrespective of whether it was correct. Usually, producing a graph meant that he had managed to overcome syntactical problems as well as interpret error messages and therefore he was glad to produce a graph plot. This was the case when he first learned to programme using MATLAB:

> TH-ref-int1(105). JP: Do you check to see that you have got the correct graph?
> Thembiso: Not really, not really. Usually the **problem** [his emphasis] is having a graph. So once there is a graph, usually we just assume it is correct.
> TH-ref-int1(106). JP: But when you plot the two graphs together and they appear totally different?
> Thembiso: Then it's a problem. JP: Then do you check? Thembiso: Then I check yes. JP: But if you got one single graph are you happy with that? Thembiso: I am happy. I am usually happy.

As the learning proceeded and the solutions to DEs required more than one method, namely Euler, Runge-Kutta II and Runge-Kutta IV, Thembiso claimed to have subsequently learned ways to interpret these graphs. In this context he stated that the graphs should not cross each other, and he also performed the act of comparing the particular graphs of the solutions to the

DE. This comparison usually assisted in determining whether he has produced correct solutions. Consequently the goal for the action of 'interprets graphs' is to determine if his solution to the DE is correct.

Based on observations and conversations during the problem-solving sessions, I conclude that Thembiso actively interprets graphs. In Problem-solving Session 1, he interprets the graphs of Euler and the analytical solution, and in Problem-solving Session 2, he interprets the graphs of Euler and Runge-Kutta II as well as the graphs of Runge-Kutta II and IV.

Problem-solving Session 1:

9a evaluates the code and produces 2 graphs
9a TH-ref-int1(32-34). Thembiso: This plot is different.
JP: So what does it tell you?
Thembiso: Something we did wrong.

Initially he has some idea of how the graphs of both the numerical and analytical solutions should look. Upon producing a table of numerical solutions in MATLAB, he immediately plots the graphs to determine if his solutions are correct:

13a. 1.00.31 types in plot *(x,y,x,z)*
13a TH-ref-int1(51). Thembiso: I will just see, I am happy. JP: Why are you happy?
13b TH-ref-int1(52). Thembiso: Basically compared to the first one- yes they do vary slightly. I guess considering what I saw in the book it's relatively close enough.
TH-ref-int1(53). Thembiso: It's actually identical to this one.

The graphs corresponding to the Euler and analytical solutions or the Euler and Runge-Kutta II and IV solutions differ only marginally. In other words, students realise that they have produced the correct solutions to the DE by checking to see that these graphs are indeed close to each other. Thembiso learns by closely and constantly comparing with the 'textbook', even comparing his graphs (produced by MATLAB) with the 'textbook' ones, although the latter are different graphs corresponding to different problems. He examines how close the graphs in the 'textbook' are to each other and whether the graphs that he had produced in the problem-solving session are just as close:

TH-ref-int1(53). It's actually identical to this one [pointing to 'textbook'].

Problem-solving Session 2:

When solving the DE using Euler and Runge-kutta II methods, Thembiso uses techniques of examining the closeness of graphs by zooming into the figure. He states that he would like to have an analytical solution to assist him in deciding if the solution is correct. Nevertheless, he realises that the table of numerical values look very different, and he has identified that the graphs differ right from where they begin. From examples given to students (in the 'textbook'), graphs ought to be close enough to each other from the beginning of the interval and usually deviate towards the end of the interval:

> 35g,h,i TH-ref-int2(35). Thembiso: We still get relatively the same graph. JP: But it is not crossing over. It's an improvement of your earlier one.
> TH-ref-int2(36). Thembiso: I am not too certain. Not really because the table is still very different. Graph is not crossing.
> JP: What do you think? How do you feel?
> TH-ref-int2(37). Thembiso: I can't exactly say because if there was, there was an analytical I could basically get a rough idea using the hand solution you taught us. But then now.. they deviate immediately from start.

When solving the second task, Thembiso correctly associates Runge-Kutta II being close to Runge-Kutta IV – note that even though these graphs are very close they are still incorrect (he has used the same dependent variable *v* for both Runge-Kutta II and Runge-Kutta IV solutions):

> 40a 25.46 zooms into the figure containing two graphs, observes two graphs being close to each other and away from the third graph, then closes this figure screen
> 40b 26.05 goes to plot, enters a space between *v* and *t* to get:
> plot(*t,v, t,y,t,Y),* he then deletes this space.
> TH-ref-int2(95). I plotted but I didn't quite check, yes I could see Runge-Kutta II and Runge-Kutta IV.. almost make one.. yes, yes, I am happy.

Action: explains MATLAB to other students

In mathematics education literature, discussing with others is often construed as indicative of a deep approach to learning. Even though Thembiso appears to be describing procedures, for example the division of the domain into intervals, I assume that when he explains MATLAB to others this explanation elicits fruitful discussions and engagement with the learning material. Consequently, I consider this action as part of the theme: actions relating to comprehension of mathematics (with CAS).

TH-int2(198). …For example I would say, when we solve um.. when I was explaining to this B.Tech er.. student, in terms of $M = B - A$ over H. It's a thing that er.. I know it has something to do with it's the number of times, something like that, but then I couldn't exactly explain to him. But then what I'm saying you have to put it this way in every.. so I told him, just, just memorise this line…

During Problem-solving Session 1 he again explains to Manto without thorough knowledge of the content:

TH-ref-int1(17). I am not 100% sure but I think, I think this the one with the *y* goes to the other side; *e* to the *x* stays.

THEME: ACTIONS RELATED TO GENERAL COMPUTER USE

Thembiso maintains that his prior experience of working with computers has proved useful for this MATLAB course, especially familiarity with the keyboard. He draws on these prior skills to facilitate the learning of MATLAB (in maths). I assert that ultimately the use of the keyboard should become an operation so that the focus is not on the typing but rather on the MATLAB code (and mathematics). Prior experience mediates the use of MATLAB in that he can now type faster.

TH-int2(289-292). JP: Do you think that your previous experience with working with computers has helped you out in the MATLAB course?
Thembiso: It has helped a lot, it has helped a lot and again judging by other students, um.. I've noticed students who have computers, they don't struggle with MATLAB, and then students who don't [have computers], they do struggle.

JP: In what way do you think it has helped you out?
Thembiso: I cannot say there's any clear-cut advantage, for example, learning *Word* and stuff that will help you in MATLAB. I guess it's just a thing, eventually, it er.. Computer, the keyboard becomes a part of you. I think this is kind of deep. But then you sort of get used to it, used to a computer. Whereas when you don't use a computer at all, and suddenly you get.. because number one they're very slow. You get very slow when it comes to typing for example um.. everything. So unlike, unlike er.. when you're used to a computer, there's no advantage that I can point out for example, learning Word and other computer

programs that you can use for MATLAB. There isn't. But it just feels natural, it just feels natural.

Action: opens a new m-file

In Problem-solving Session 1, Thembiso appears to be confused with opening a new m-file:

1c, d opens up a new m-file.
JP assists him to open an m-file. Confusion between whether he should use the command window or m-file to type the code: click on command window (it comes into view), goes back to click on the m-file.

When one double clicks on the MATLAB icon, the command window comes into immediate view. This command window could be used to type in MATLAB statements but then the statements are immediately compiled by MATLAB as opposed to typing them into an m-file and the compilation takes place after all the code has been typed in[48]. Initially students are taught to use the command window of MATLAB to enter statements to produce simple sketches, and subsequently they are taught to use the m-file for programming to solve DEs. He opens the command window and I help him to open a new m-file but he is puzzled about where to type the MATLAB code. The goal corresponding to the action of opening an m-file is that he aims to solve the MATLAB problems. At the start of the second task (in Problem-solving Session 1) he questions me on the new m-file's appearance:

14b TH-ref-int1(75). Thembiso: The white one isn't it also a new m-file?
JP: The white is also a new one.

This affirms that he had difficulty distinguishing between a new m-file and the command window.

Action: puts effort into typing initial lines

In Problem-solving Session 2, Thembiso puts effort into typing in the easy initial line of *a=0:*

27b typed in clear all
27c 18.16 typed *a= 0* with a space before *0*

[48] When students type in the code in an m-file, they then highlight the code and evaluate it; subsequently MATLAB shows the error messages if, for example, any of the syntax is wrong or MATLAB produces the solution.

a= 0 had a red line under *a=*

27d 18.40 took space out

27e 18.45 deleted *a=0*

27f types *a=0*;

I deduce that Thembiso puts effort into typing in *a=0*, but in Problem-solving Session 1 the typing of these statements appear to be operations. Perhaps he has now made a typing error. In addition, after deleting the space before *'0'* he is unsure whether he has taken care of the spaces and consequently deletes and retypes the statement. I assume that he wants to be certain that the statement is now correct. This act of making typing mistakes whilst entering statements in MATLAB does not feature often in Thembiso's interaction with MATLAB. The goal of this action is to solve the DE (without error messages).

Action: carries out certain procedures with ease

Thembiso displays confidence in the manner in which he uses basic computer techniques such as: 'copy and paste', 'highlight code' and 'zooms in' to examine graphs closely. The goal associated with 'copy and paste' is to make typing into MATLAB easier or to reproduce the table of solutions from command window into m-file. Copy and paste techniques are important because they assist in speeding the process of programming. An example is the similarity of the auxiliary equations in Runge-Kutta IV solutions:

K2=h(1+(sqrt(t(i)+0.5*h)*(v(i)+0.5*K1)))*

K3=h(1+(sqrt(t(i)+0.5*h)*(v(i)+0.5*K2)))*

The goal for the sub-action of 'zooms into figure' is to examine the graphs closely so that Thembiso could ascertain if he has produced the correct graphs and/or solution. Since the graphs of any two of the numerical solutions, for example Runge-Kutta II and Runge-Kutta IV, should be close to each other, it becomes difficult to see if they cross or they deviate much from each other, more so because the interval of the horizontal axis is usually small [0,1]. He uses the zoom function to observe these graphs more closely.

35h highlights, evaluates code, studies two graphs, zooms in and out

32b 42.58 highlights values in table, right clicks, copies and pastes into the m-file

38a 17.25 types in *K3=h*(1+sqrt*

38b 17.51 deletes *h*(1+sqrt* from *K3*, copies right hand side of the equation of *K2* and pastes into the right hand side of *K3*

Action: questions about one of the basic keys of the keyboard (insert key)

Even though he skilfully uses basic techniques related to computers, there are aspects of certain keys with which he is still unfamiliar. For instance, he questions me as follows:

> TH-ref-int2(21). Thembiso: I have a problem - this cursor it writes over.
> JP: You have the insert key on.

The goal of this action is that he wants to return the cursor to its original mode.

THEME: ACTIONS RELATED TO THE USE OF COMMANDS IN MATLAB

I consider Thembiso's use of various MATLAB commands.

Action: types in the plot command with ease to produce graphs when the dependent variables are y and z and the independent variable is x

In Problem-solving Session 1, he uses the plot command with ease. In this context the plot command is related to the dependent variables being *y* and *z* and the independent variable being *x*. However, in Problem-solving Session 2, he experiences difficulty when using the plot command related to the independent variable *t* and dependent variables *y* and *v*. He has difficulty applying the syntax if the variables are different to *x*, *y* and *z*. I construe that he uses the plot command with little understanding of its precise syntax.

He relies greatly on producing graphs to determine if he has the correct solution. Even if there are obvious differences in the solutions to an analytical and Euler solution he does not examine the table of solutions, but rather plots the graphs and examines their proximity, as evidenced in Problem-solving Session 1:

> 8b Evaluates code, produces a table of values
> 8b these values are incorrect; some are positive whilst others are negative. Does not take any notice of this, instead moves on to immediately plot graphs.
> 8c goes back to m-file and types in Plot *(x,y,x,z)*

Noticeable is the uncertainty with the plot command when the variables are changed (in Problem-solving Session 2):

34b 48.51 plot *(t,v,t,y)*

TH-ref-int2(23). I can try. I can try.

34b TH-ref-int2(24). like this.. I can't..

TH-ref-int2(25). JP: *t* with the *v; t* with the *y* - correct.

In Runge-Kutta IV method he again doubts his use of the plot command:

39f 24.50 types in plot *(t,v,t,y,t,Y)*

40b 26.05 goes to plot, enters a space between *v* and *t* to get:

plot*(t,v, t,y,t,Y),* then he deletes this space.

Action: makes use of MATLAB commands

Although Thembiso makes use of MATLAB commands he does not fully comprehend the meaning or syntax of these commands, as evidenced below:

MATLAB command 'format long':

I questioned him about the use of the 'format long' command:

TH-ref-int1(138).

JP: Is it using the 15 significant figures when it does the calculations?

Thembiso: I think so. I think so because if you go to format short, probably that tables would be rounded off, which would be the graphs would deviate more. So for accuracy.

JP: Maybe you should try that out and see – (use) format long and format short, see what the difference is between the two…

I conclude that he is unable to explain that 'format long' and 'format short' are about the display of the solutions. He assumes that the calculations of the solution depend on the choice of format type. I assume that he follows closely with worked out class (or 'textbook') examples because he merely uses the command without being able to explain why.

MATLAB command 'hold on':

Similarly, he uses the 'hold on' command without understanding its use (Problem-solving Session 1):

23a TH-ref-int1(90). Thembiso: *y(i) + h* times… I just started the whole thing.

TH-ref-int1(91). I don't know – do you see like, ok no, I am not sure about this but I just tried like 'hold on' and then it uses the same [variable values as the first Runge-Kutta II or Euler values].

Manto: What I did was I just edited everything..

Thembiso is perplexed about producing solutions to Euler and Runge-Kutta II in the same m-file. He is of the opinion that by using 'hold on' the dependent variables for the second solution, namely those related to Runge-Kutta II, will not assume values from the first solution (he was using the same name for dependent variables for both the Euler and Runge-Kutta II solutions, which was incorrect).

Also in Problem-solving Session 2, he questioned why 'hold on' is used:

37a Thembiso: this 'hold on' it's not being used in this instance.

TH-ref-int2(92). JP: Hold on comes with the plotting. If you feel that you want to use it you can.

TH-ref-int2(93). Thembiso: But it's not necessary?

MATLAB command 'disp':

Thembiso expresses doubt when using the display command to display multiple graphs. In Problem-solving Session 2, he questions the syntax of the display function:

39a 21.15 highlights disp and plot from the Euler code; right clicks; does not make any selection. [I suppose he had intentions of copying these two lines]

39b 21.57 types in disp *([*

39c 23.54 disp *([t' v'*

39d 24.16 goes back to *Y(1)=2*, deletes *(1),* then types it in again

39e 24.45 continues typing in disp*([t' v' y' Y'])*

TH-ref-int2(94). Thembiso: the difference Runge-Kutta II and Runge-Kutta IV.. how do you put them together it's *(t, v)* for Runge-Kutta II. I mean here the display. *Y1* it's for Runge-Kutta IV and then *y* is for Runge-Kutta IV and then *t, v* – is it supposed to be there?

MATLAB command 'legend':

Thembiso uses the 'legend' command without knowing its syntax (N.B. commas should have been used in the syntax to separate the names of graphs):

40c 27.00 types in legend (‘RkII then he deletes kII and types in ‘ ‘RK2’ to produce the statement: legend(‘RK2’ ‘euler’ ‘RK3’)

Then, without questioning to find out what the syntax of this legend function should be, he:

41d 31.37 deletes legend statement from m-file.

I construe that his action of deleting the legend statement and not replacing it implies that he is not enthusiastic about learning the correct way of producing this command. Perhaps his immediate goal is first to produce the solution (and graphs) to the DE and later concentrate on easy commands such as ‘legend’, which are there to identify the graphs.

Based on the above evidence related to not knowing how or why MATLAB commands (plot, format long, hold on, display and legend) are used, I contend that Thembiso uses commands of MATLAB at a superficial level, and that in some instances perseveres neither to find out what mistakes he has made in the syntax of the command, nor why the command is used. The goal for making use of MATLAB commands is to produce a solution or graph.

Action: carries out certain procedures with ease (e.g. puts cursor over red squiggle which appears below any errors

When statements in MATLAB are syntactically incorrect, MATLAB shows a red squiggle below the error in that statement. The user has to place the cursor over this squiggle and a pop-up appears containing hints related the error. This ongoing feedback mechanism gives the user an opportunity to change the syntax before compiling the programme[49]. After being alerted to this feedback, Thembiso places the cursor over the squiggle and reads the corresponding message in the pop-up:

37g reads the pop-up message that appears when cursor is on red squiggle below ‘for’: apparently an END is missing, possible match ‘for’.

[49] I liken this ongoing feedback to typing in a *Word* document where if the spelling is unrecognisable then there appears a red squiggle below the misspelled word.

THEME: ACTIONS RELATED TO PROCESSES FOLLOWED WHEN SOLVING A DE

Various actions, such as those related to writing MATLAB code, making sense of error messages, rectifying errors and putting effort into getting right the syntax make up this theme.

Action: writes out parts of the MATLAB code before typing onto computer and Action: types statements from those given in the instruction sheet

Thembiso reports in the interview that he writes out part of the MATLAB code before typing onto the computer. It will be recalled that within the theme of actions related to general computer use, Thembiso states that it feels natural for him to use a computer. At this juncture he claims that he has to write out the entire MATLAB code because it does not feel natural to think and type the MATLAB code. Consequently, his statements in this context could be interpreted as: it feels natural to be in front of the computer typing. At the same time he feels that typing the MATLAB syntax directly onto the computer (without writing the code on paper) is not a natural process, begging the question as to whether these feelings are influenced by the atypical syntax of MATLAB.

> TH-int2(109-110). JP: The first stuff, *a = 0*.
> Thembiso: Those are easy. And um, once you get to, you get to um, the equations after 'for' loops (the auxiliary equations), once you get to those, as long as you have them on paper, it's very simple, it's just putting in, er.. figures.
> TH-int2(111-114). JP: So you write your 'for' loops out. You prefer to do that?
> Thembiso: Yes, I prefer writing it down first.
> JP: Why?
> Thembiso: Because um, using a computer it's, it doesn't come naturally. It's better thinking and applying to a pen rather than applying to a keyboard. So it doesn't, it doesn't feel natural when I have to think of *K*.. even if I have the formula book and I have *Y* plus *i* of *1*. I prefer writing it down first, putting the star in, in in the replacement to um.. times, the multiplication sign, and the dot where it's needed, yes. And the square root, putting in that arrow thing.. and to, I mean SQRT for the square root. I prefer writing them down like that, and then afterwards I just copy what I wrote down.

The important aspect here is that Thembiso *believes* that putting the auxiliary equations on paper will make it easy for him to copy from and type onto the computer. In both problem-solving sessions, he writes out the entire code before typing onto the computer:

1e (Problem-solving Session 1)

Writes out entire code - starting from:

clear all a=1 b = 2 m = $\frac{b-a}{h}$ *y(1)=1 for i=1:m y(i+1)=y^i +h * exp (x^i) - y^i / x^i*

end format long disp([x' y'])

There is some incongruity in his actions. Considering the action of 'writes out parts of the MATLAB code before typing onto computer' and the action of 'types statements from those given in the instruction sheet', in the latter he types the statements incorrectly onto the MATLAB editor. It thus becomes a challenge to understand why he writes out parts of the MATLAB code because he cannot copy directly onto the MATLAB editor, as evidenced below.

In 32e he omits the subscript *i* that should accompany the *y* variable on the right hand side. N.B., this equation is given in the instruction sheet.

32e 43.54 for *i=1:m*

y(i+1)=y+h(1+sqrt(t(i))*y(i),* pauses for a while, then puts in another *)* at the end.

In 28e, when typing the statement of *m,* he realises that *t=(b-a)/h* is incorrect, so he corrects the statement. This also indicates that he is not typing according to what he has written. I propose that the reason Thembiso writes out the MATLAB code (or parts of it) is to satisfy his *belief* that writing assists in getting right the syntax, even though he may not be typing according to what has been written (or given).

28c 19.44 *t=(b-a)/h;*

In 28c there is an incorrect use of the independent variable *t*, which he uses to divide the interval into sub-intervals instead of choosing a new (dummy) variable.

28d 20.41 *v(1)=2*

28e 20.55 *m=*

28f goes back to 28c and changes to *t=a:H:b;*

The goal associated with the action of 'types statements from those given in the instruction sheet' is to copy the given solution onto the MATLAB editor.

Action: makes sense of MATLAB error messages

Thembiso reflects in the interviews on a vital aspect of attempting to make sense of error messages whilst using MATLAB to solve DEs:

> TH-int2(237-238). JP: Is there anything about the MATLAB itself, the software that you would like to change if you had a choice?
> Thembiso: Er.. The command window, the command window. For example, the errors. The errors are just annoying. The fact that they get to use complicated language. Maybe people eh..eh..who, say for example, are mechanical [engineering], they don't er.. are not required to know those terms. Parentheses and matrix dimensions. If they could um.. try to make it as general as possible. For example, the brackets are not, you know balanced. It would be very, very aiding, helpful. So other than that, that's the only annoying part of MATLAB that I have a problem with the error messages.

Thembiso expresses negative affect towards MATLAB error messages and according to him they are listed using complicated language. I argue that initially they appear to defy their purpose of helping him correct wrong syntax. However, with some familiarity and over time he could understand the meaning conveyed by some of the common error messages:

> TH-int2(210). Because the help-file I was going er.. to it so it can help me with understanding the command window errors, the errors. But then, I couldn't get there. I just couldn't get there, so I decided no. Because I, I already understood a lot via practise.. no not practise, um.. experience, experience. Once, it gives you something with matrix dimensions, it's either the dot or bracket or something, you know, you know um.. So the experience helps. Once you've seen this before, you know probably there's a bracket missing. But then I haven't utilised the help function at all.

When questioned at the end of the course if he has an improved understanding of the error messages, he states:

> TH-int2(124). Er.. I sort of have an idea, I cannot, I cannot say, because probably it's going to be something new that I haven't seen before, but I'm happy, I'm happy with them.

There is still some apprehension that there may be new error messages that he might not be able to comprehend. The goal for the action of 'makes sense of MATLAB error messages' is to correct errors related to syntax.

Action: rectifies minor errors after perusal of some of the error messages

As discussed in his personal reported goals, Thembiso expresses a desire to produce a solution or graph. To this end he rectifies minor errors after perusal of some of the error messages. In Problem-solving Session 1, he easily interprets the error message and corrects the errors as follows:

error message is *???m=(b-a)h;*
Error: unexpected MATLAB expression.
4b types in the / in the statement of *m=(b-a)/h*
4c error message is *??? y(i=1)=y(i)+exp(x(i))-y(i)/x(i);*
Error: The expression to the left of the equals sign is not a valid target for an assignment.
4d goes back to m-file, changes the *y(i=1) to y(i+1).*

Action: strives to get right the syntax

I assert that the goal for this action is to produce a solution or graph to the problem. Thembiso does not understand some of the error messages that point to syntactical problems. In his endeavours to produce correct statements and eventually a compiled MATLAB programme, he tries to alter the syntax in many different ways, often repeating previous steps, for example, putting brackets in a certain position, removing them when he gets the error message, and later putting them back in the same position. He also uses trial and error methods with syntax. He may type in mathematically equivalent statements in the hope that he will get right the syntax. This is because he does not really understand the meaning of some of the error messages. I give only a few exemplars from Problem-solving Session 1 below. It is noticeable that in Problem-solving Session 2 he hardly made any syntactical errors.

Task 1 in Problem-solving Session 1

Issues with ;

2b goes back to put in the ; after *a=1 b=2 and h=0.1*

Nor did Thembiso put the semi-colon in his written code, even though he had earlier stated that he writes out everything.

Types in mathematically equivalent statements:

10b Thembiso types *z=exp(x)-(exp(x)/x)+1/x;*

In 10b there is an act of typing in an equivalent expression to 8a: observe that *(exp(x)+1)/x* is equivalent to *(exp(x)/x)+1/x.* By typing in equivalent expressions, Thembiso is expecting to get no errors in MATLAB. I argue that he experiences difficulty with the syntax.

10d 51.11 types *z=exp(x)-exp(x)/x+1/x*

In 10d he removes brackets from the second term of the 10b statement (cf 10h, below).

Indiscriminate use of brackets:

10e 51.12 ???error using = = > mrdivide
Matrix dimensions must agree
10f 51.43 removes plot *(x,y,x,z)*
10h 53.25 types in *z=exp(x)-(exp(x)/x)+1/x*

In 10h Thembiso puts the brackets in once more at the same position as before (cf. 10b and 10d). He does not understand error messages and does unexplained performances such as removing and inserting brackets in the same position.

10i 54. 20 evaluates, gets same error message
10j 55.28 puts brackets in then removes them.

In summary, he performed the following steps:

z=exp(x)-(exp(x)+1)/x;
z=exp(x)-(exp(x)/x)+1/x; [types but does not evaluate this code]
types *z=exp(x)-exp(x)/x+1/x;* [removes brackets, evaluates code]
types *z=exp(x)-(exp(x)/x)+1/x* [puts brackets in again, evaluates code]

Uses methods of trial and error with syntax:

11a TH-ref-int1(37). I am having problems putting it. I am not quite sure - MATLAB is telling me …[here he is trying to make sense of MATLAB error messages]
TH-ref-int1(38). JP: Error using MRDIVIDE. Remember the dot – when you divide two variables, what do we follow the divide by?…this is something unexpected MATLAB operator.

12a 55.07 types in the dots*: z=exp(x)-exp.(x) /. x+1/x*

12b 56.14 highlights so as to evaluate code but does not evaluate code

12c 56.29 instead goes back to add another dot before / sign: *z=exp(x)-exp(x). /. x+1/x* and removes the dot after exp

The above depicts an act of learning by trial and error methods. Thembiso is not sure where the dot goes, so he puts it in two places, perhaps taking advantage of the fact that MATLAB will point out the error if the operator is unrecognisable.

Does not understand error messages:

12e 56.40 evaluates code and gets error message:

??? z=exp(x)-exp(x). /. x+1/x

Error: unexpected MATLAB operator

12f 56.59 removes dot after / sign: *z=exp(x)-exp (x). / x+1/x*

12g 57.05 highlights code, evaluates, error message

??? error using = = > mrdivide

matrix dimensions must agree

12g 59.23 puts in brackets *z=exp(x)-(exp (x). / x)+1/x,* evaluates, gets error message

59.29 ??? error using = = > mrdivide

matrix dimensions must agree

12h 59.42 goes to m-file, checks the brackets by clicking on the left one

N.B., there is a misunderstanding of error messages: he assumes that matrix dimensions refer to brackets and thus puts in another set of brackets in 12g.

Thembiso is conscious of putting in brackets even when they are making no difference, in other words he still gets the same error messages:

12h TH-ref-int1(45-50). JP: What's happening Thembiso?

Thembiso: Having a problem with these dots.

JP: You got two divisions there

Thembiso: This and for this? JP: Yes.

Thembiso: And this brackets, can I remove them? JP: It is up to you?

Thembiso: Ok let me look close enough.

12g 59.58 puts in dot just before the second /

z=exp(x)-(exp (x). / x)+1./x

Task 2 in Problem-solving Session 1.

Puts in dots with / when not required:

Thembiso struggles with syntax, specifically the dot, even when the dot is not required in this problem:

> 14e TH-ref-int1(77). Thembiso: The dots are not necessary? Manto: That's what I think.
> TH-ref-int1(78). Manto: Where did you have your dot on?
> TH-ref-int1(79). Thembiso: No the last one there are variables there, somewhere there has to be dots.
> 14e 01.13.11 thereafter types
> *p(i+1)=2*exp(x(i))./3-(y(i).*sin(x(i))./2*x^2;*
> 14d plot *(x,y,x,z,x,p)*

Does not understand error messages:

Thembiso struggles to make sense of MATLAB error messages.

> 15a 01.16.35 Evaluates code produces two graphs and obtains error message:
>
> *???p(i+1)=2*exp(x(i))./3-(y(i).*sin(x(i))./2*x^2*
> |
>
> Error: expression or statement is incorrect - - possibly unbalanced *(, {, or [.*
> 15a TH-ref-int1(80).Thembiso: Are you not supposed to suppress this? Because it's telling me…

He speaks about suppressing the statement. N.B., in MATLAB, suppressing a statement is when a ; (semi-colon) is put at the end of the statement so that the statement is suppressed in the output of the problem. Thembiso interprets this error message as if there is a ; missing when the error message obviously points to an unbalanced *(or { or [*.

> TH-ref-int1(81). JP: Expression is taken as incorrect – there is something that should come in the end. It's unbalanced with the brackets and there it shows you where the thing is unbalanced.

Action: pays attention to the sequence of steps in MATLAB even when the order does not matter

All MATLAB codes are evaluated together and the position of statements in the code is not important. However, Thembiso gives constant attention to the order of statements in the code. He has not taken advantage of the opportunity to use MATLAB and test to observe if there would be a difference if he were to type MATLAB statements in any order or in any sequence. He does not appear to be investigative in his approach to learning mathematics using MATLAB. I assert that the goal to get right the sequence of steps in MATLAB is to satisfy his belief that this may reduce obtaining MATLAB errors so that he could produce a solution or graph.

2d Before the statement of *m =...,* he enters a line, types in *y(1)=1;*
3d goes back: enters a line before *m = (b-a)h*; types in *x=a;h;b,*

THEME: ACTIONS RELATED TO OVERCOMING DIFFICULTIES ASSOCIATED WITH MATHEMATICS WHILST USING CAS TO SOLVE DES

In Chapter 4.4.4.3, I listed the auxiliary equations for the Runge-Kutta methods[50]. In order to produce the correct auxiliary equations of *k1, k2* and so on, the student should first determine which variables are independent and which are dependent from the given DE[51]. Secondly, they should be able to substitute into the formulae corresponding to the auxiliary equations. In this theme, the mathematical notions of identifying independent and dependent variables as well as

---

[50] The Runge-Kutta II formula encompasses two auxiliary equations k_1 and k_2:

$k_1 = h.f(x_i , y_i)$ $\quad$ $k_2 = h.f(x_i+h , y_i+k_1)$

$y_{i+1}=y_i+ ½ (k_1 + k_2)$

The Runge-Kutta IV formula encompasses four auxiliary equations k_1, k_2, k_3 and k_4:

$k_1 = h.f(x_i , y_i)$ $\quad$ $k_2 = h.f(x_i+½h , y_i+½k_1)$ $\quad$ $k_3 = h.f(x_i+½h , y_i+½k_2)$ $\quad$ $k_4 = h.f(x_i+h , y_i+k_3)$

$y_{i+1}=y_i+ (1/6)(k_1 + 2k_2+2k_3+k_4)$

[51] Recall that the DE that was given in Problem-solving Session 2 is: $\frac{dv}{dt} = 1 + v\sqrt{t}$ *with* $v(0) = 2$.

Note that

$f(t,v) = \frac{dv}{dt}$

substitution and functions are under review. A discussion of both these interconnected actions follows.

Action: works hard with independent and dependent variables and Action: deliberates with functions and substitution

In Problem-solving Session 2, Thembiso begins typing onto the MATLAB editor by using the independent variable *t* to divide the interval into sub-intervals instead of choosing to use a new (dummy) variable.

28c 19.44 *t=(b-a)/h;*
28d 20.41 *v(1)=2*
28e 20.55 *m=*

Regarding 28e I construe that when typing the statement for *m* (which usually represents the dummy variable that divides the interval into sub-intervals) he realises that *t=(b-a)/h* is incorrect, consequently changing the statement to:

28f goes back to 28c and changes to *t=a:H:b;*

The above indicates that Thembiso had initially not worked out that *t* is the independent variable.

Thembiso produces the correct auxiliary equation for *k1* (Note that producing the *k1* equation is easy as it merely requires the given DE to be multiplied by *h* even though he has not identified the independent and dependent variables).

29a 21.55 for *i=1:m*
k1=h(1+v(i)*sqrt(t(i)))*

Thembiso has not distinguished between independent and dependent variables and his substitution into the *k2* auxiliary equation is incorrect, as evidenced in:

29d 24.15 *k2=h*(1+v(i)+H,* deletes and puts in *h*
k2=h(1+v(i)+h)*(sqrt(t(i))+k1);*

The errors with independent and dependent variables can be noticed again when he uses *t* as the dependent variable:

30a 26.46 *t(i+1)=t(i)+0.5*(k1+k2)*

Alerted to his erroneous use of variables he becomes attentive to his mistakes:

TH-ref-int2(2). So it's almost identical to this example 4 (pointing to the 'textbook' where example 4 contains the solution to the differential equation *dy/dx = 1+x*y*). The Euler solution is it the same as this one? JP: You see after *k1, k2* you put in *y(i+1)*- is it this one? JP: No it's like this..

TH-ref-int2(6). ... JP: I am going to explain this - you got to replace the *x* with *x+h* and *y* with *y+k1*.

TH-ref-int2(7). Thembiso: Oh *k1*? That's my mistake.

TH-ref-int2(9). Thembiso: I think I've replaced the wrong thing with the wrong thing. Instead of *v*, I put in *t*.

30d 28.42 goes back to 30a and changes the *t's* to *v's*

v(i+1)=v(i)+0.5(k1+k2)*

Although he corrects the statement of 30a (and does nothing with *k2*, see 29d above) his struggle with independent and dependent variables can be deduced from the erroneous display function:

31b 29.03 disp*([v' t*

31c 29.46 deletes *v' t*

types in *t'* and *v* to get disp*([t' v])*

Thembiso becomes concerned when he produces graphs that cross each other. Recall that the given DE is *dv/dt = 1+v*sqrt(t).* The original *k1* that he typed in was *k1=h*(1+v(i)*sqrt(t(i))).* In his attempt to rectify the wrong graphs he modifies the *k1* equation to produce a mathematically equivalent one:

35a 1.09.00 changes equation *k1* to

k1=h(1+sqrt(t(i))*v(i));*

From the above I infer that he is struggling with substituting into functions.

Thembiso could not resolve the issue of producing graphs that cross each other:

TH-ref-int2(29). Thembiso: But the problem is where? So I am just going to do it again and see.

I explain to Thembiso that he has used the independent and dependent variables incorrectly and that *t* represents the independent variable and *v* the dependent one. He reacts by merely swapping *v(i)* with *sqrt(t(i))* and vice versa. He modifies the previously typed *k2* equation *(k2=h*(1+v(i)+h)*(sqrt(t(i)+k1))* to produce:

> 35b 1.09.48 enters line after *k1* and types in
> *k2=h*(1+sqrt(t(i))+h)*v(i)+k1));*

An important aspect in the above modified *k2* equation relates to function substitution. N.B. *h* should have been included in the square root function but he is unaware of this substitution:

> 36a TH-ref-int2(38). JP: Let me see what values I got [looking at my notes].
> JP: Thembiso in Runge-Kutta II the problem here is that you are multiplying the square root of *t* with the *v* but in that multiplication your *h* is not included in your square root.
> TH-ref-int2(39). Thembiso: Oh it was supposed to be included in the square root?
> TH-ref-int2(40). JP: You want $t+h$ to be under the square root sign..
> 36a 01.23.51 goes back to *k2* in the statement of 35b, removes bracket after *t(i)* to produce:
> *k2=h*(1+sqrt(t(i)+h)*v(i)+k1));*

In addition, he articulates his difficulty with functions and substitution:

> TH-ref-int2(72). Thembiso: Basically you get where you went wrong but then ok.. I would like e.g. The *dy/dx* (he means *dv/dt*) I had no idea. I went through that but I couldn't see anything. So I was basically stranded there.

In the above actions I propose that the goal is to find the solution to the DE.

Action: uses the same dependent variables when doing two or more numerical methods (e.g. Euler, Runge-Kutta II and Runge-Kutta IV) in one m-file.

During the MATLAB solutions of Runge-Kutta I explain to the participants that different dependent variables must be used to represent the different numerical methods of Euler, Runge-Kutta II and Runge-Kutta IV. Thembiso used the variable *v* for Runge-kutta II; he used *y1* for the Euler solution and *Y* for the Runge-Kutta IV solution, but he did not apply this consistently, as evidenced in the Runge-Kutta IV solution:

> 37d continues typing:
> *K1=h*(1+sqrt(t(i))*v(i));*

K2=h(1+(sqrt*

37e 14.14. enters a line before *K1* and types in for *i=1:m*

comes back to *K2* and continues typing

K2=h(1+(sqrt(t(i)+0.5*h)*(v(i)+0.5*K1))*

In 37d and e above these equations still contain the variable *v(i)* that was already used as a dependent variable in the Runge-Kutta II code (Thembiso should have used *Y*). He is unaware that using the same dependent variable across the different numerical methods will produce wrong solutions (in spite of me explaining this aspect earlier in the session):

TH-ref-int2(96-98). JP to Manto: You using *v's* again, did we not use *v's* before. Manto: We did.

JP: It confuses this *v* with that *v*.

Thembiso: It's not the same *v*?

JP: No, what did you use?

Thembiso: I used *v* as well.

Thembiso changes some of the *v(i)* to the variable *Y(i)*. He is confused with the use of *v(i)* (from the previous Runge-Kutta II solution) and *Y(i)* (the new dependent variable introduced for the Runge-Kutta IV solution). This is evident in the way he goes about changing *v(i)* to *Y(i)* in the *K4* equation and then changes it back to *v(i):*

41b 30.30 modifies *K2* equation - changes *v(i)* to *Y(i).*

does the same for *K3* and *K4* equations. He thus produces:

K2=h(1+(sqrt(t(i)+0.5*h)*(Y(i)+0.5*K1)));*

K3=h(1+(sqrt(t(i)+0.5*h)*(Y(i)+0.5*K2)));*

K4=h(1+(sqrt(t(i)+h)*(Y(i)+K3)));*

31.11 changes *Y(i)* in *K4* back to *v(i)* to get:

K4=h(1+(sqrt(t(i)+h)*(v(i)+K3)));*

Eventually he does correct the above usage of dependent variables. The goal associated with the action of 'uses the same dependent variables when doing two or more numerical methods in one m-file' is to produce a solution to each of the three numerical methods.

5.2.5 OPERATIONS RELATED TO THEMBISO'S ACTIVITY OF LEARNING MATHEMATICS WITH CAS

Thembiso effortlessly opens the MATLAB programme and the help-file, yet throughout Problem-solving Session 1 he does not use this help-file. At this initial stage of his learning to use MATLAB he claims that he does not know what the help-file is about:

1a easily opens up MATLAB with a view of the command window
1b easily opens up help-file
TH-ref-int1(132). JP: What about the help files? TH-ref-int1(133).
Thembiso: Never exploited that part.
JP: Do you think it might help? TH-ref-int1(135).
Thembiso: I don't know what it's about.

He easily types the initial lines of the MATLAB code in Problem-solving Session 1:

2 a starts typing in clear all;

a=1

b=2

h=0.1

The opening of the MATLAB programme and the command window, as well as the typing of the initial codes, all appear to be done in an automatic way; hence classified as operations. It will be recalled that he did not know whether to use the command window or m-file to type in the code in Problem-solving Session 1 but he easily used the m-file in Problem-solving Session 2. This choice between command windows and m-file has now joined the level of operations:

27a 11.41 easily opens a new m-file

Once all the MATLAB code has been typed into an m-file, Thembiso easily selects all code (either by highlighting or using Ctrl A) and evaluates the code by selecting evaluate from the file menu; in other words he produces the compiled form of the MATLAB code. This he does many times in both the problem-solving sessions:

4a selects the entire code (it highlights), evaluates the code.

This process of selecting and evaluating code appear as operations.

5.2.6 SUMMARY OF THEMBISO'S ACTIVITIES

The needs, motives, actions and operations from the above discussions are summarised in Figures 5.1 to 5.3 (below). I use the model developed in Figure 3.6 as the basis for the explication of the links between the conceptual variables of my study (e.g. needs, motives) and the empirical data that originated from the analysis of Thembiso's undertakings. These ways of using tables to present the association between conceptual variables and themes (or codes) illustrate the essence of their relationships and provide a succinct summary of the needs, motives, actions and operations of each participant. I have separated the actions into those that were derived from reported interviews (or participants' reflections) and those that I had captured during the 'actual' problem-solving sessions. This justifies my listing (or repetition of) the same actions but within the two different contexts (reported interviews vs. actual problem-solving sessions).

A summary of Thembiso's reflections on the activity of learning mathematics

Needs

Mathematics 1 and 2
Need to be recognised as part of the group of 'smart' students by studying mathematics.

Mathematics 3
Need to be recognised as part of the group of 'smart' students by choosing to study Mathematics 3.

Motives

Mathematics 3

Self-related motives:

- Performance approach: seeking to obtain 'A' symbol in Mathematics 3.
- Chooses to study Mathematics 3 so as to avoid taking a more difficult course.
- Extrinsic utility value – opting to study Mathematics 3 because it is useful for future studies.
- Extrinsic utility value – Mathematics 3 is useful because of its application to industry and to engineering scenarios.

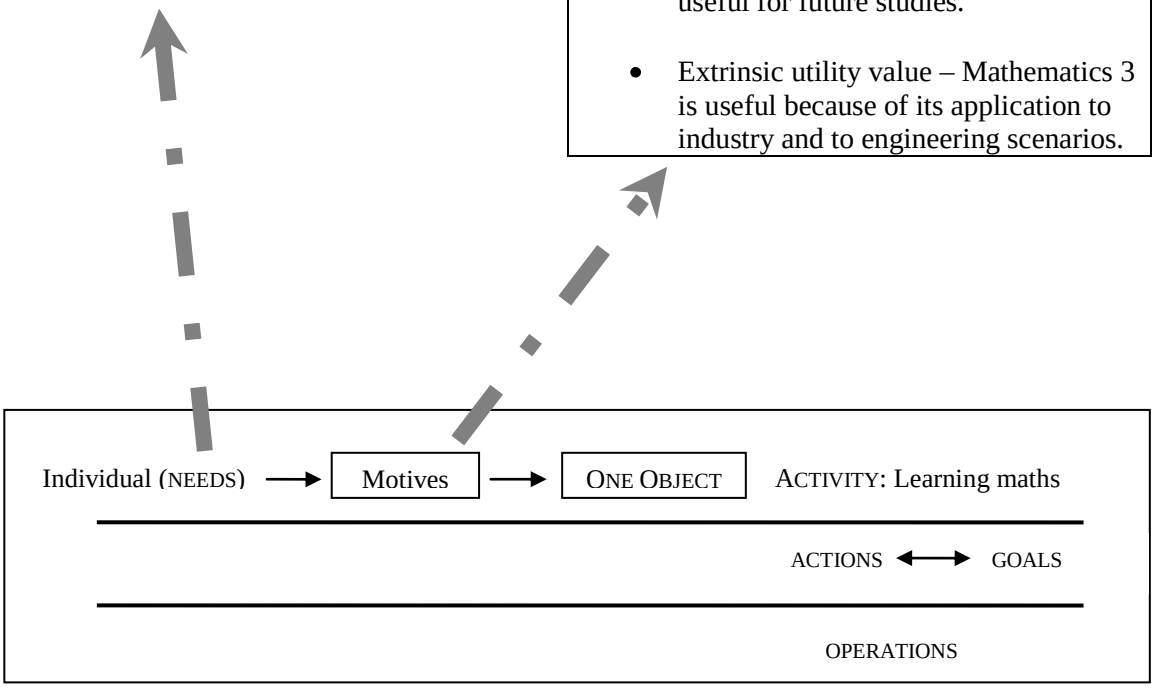


Figure 5.1: Needs and motives from the analysis of Thembiso's interviews - reflections on the learning mathematics activity

A summary of Thembiso's activity of learning mathematics with CAS

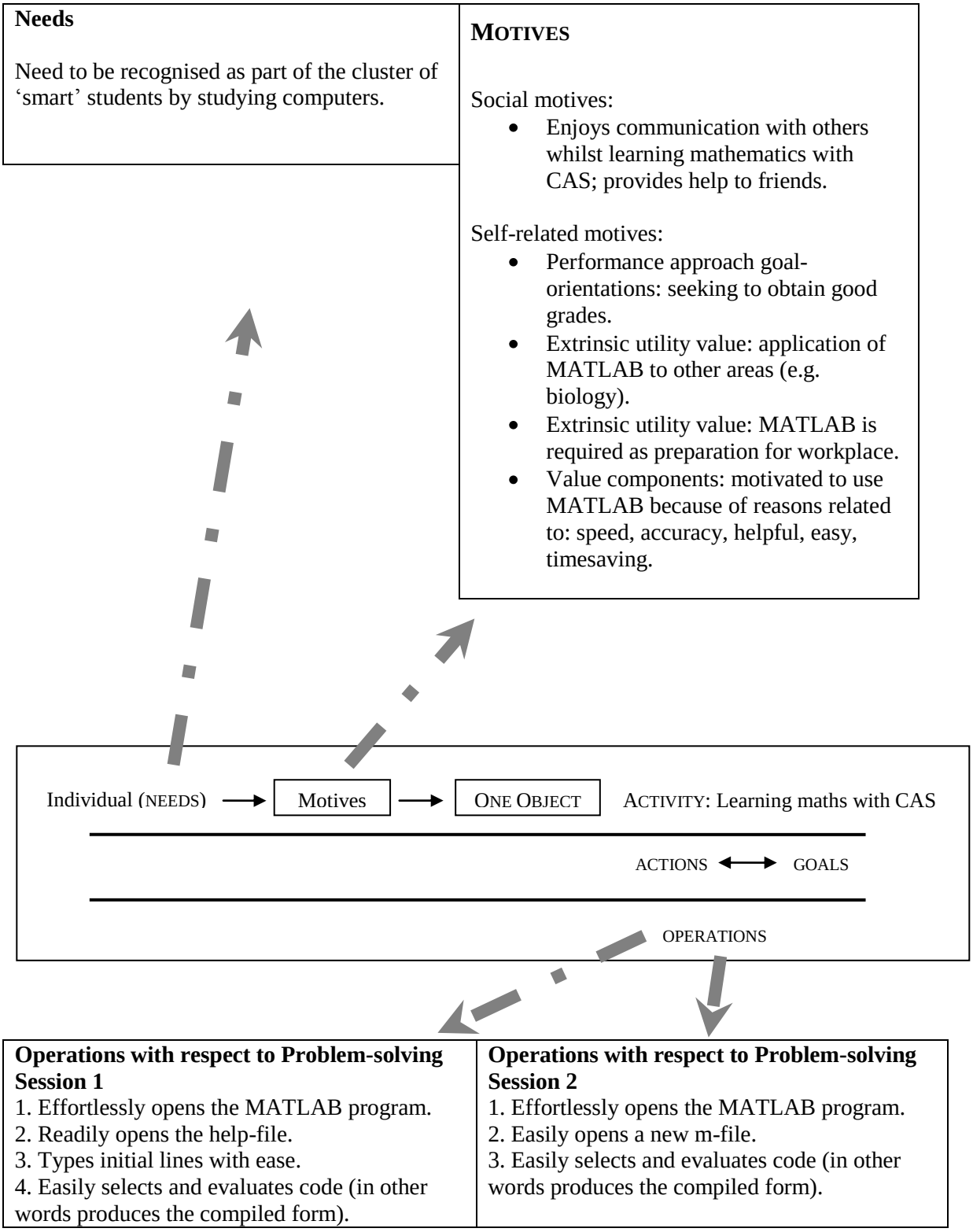


Figure 5.2: Needs, motives and operations from the analysis of Thembiso's learning mathematics with CAS activity

Theme: actions relating to comprehension of mathematics with CAS

Actions reported in learning mathematics with MATLAB

1. Interprets graphs. Goal: to determine if his solution to the DE is correct or not.
2. Explains MATLAB to other students. Goal informed by social motives – enjoys inter-relationships with other (also more advanced) contemporaries.

Actions carried out in problem solving session 1

1. Interprets graphs of Euler and analytical solution. Goal: to determine if his solution to the DE is correct or not.
2. Explains MATLAB/analytical solution to Manto. Goal: Social motives – enjoys inter-relationship with others.

Actions carried out in problem solving session 2

1. Interprets graphs of:
   - Euler and Runge-Kutta II
   - Runge-Kutta II and Runge-Kutta IV.

Goal: to determine if his solution to the DE is correct or not.
2. Checks the *y* values in the output. Goal: to determine if his solution to the DE is correct or not.

Theme: actions relating to personal power

Actions reported in learning mathematics with MATLAB

1. Cram/study mathematics (with CAS) last minute before a test. Goal: wants to prepare just for the assessment.
2. Redoes class examples. Goal: to check whether he gets the same answers as the lecturer did when it was done in class.
3. Chooses which tasks to solve. Goal: to become familiar with worked out class examples.
4. Attempts to solve more difficult problems. Goal: to prepare for an assessment.
5. Chooses Euler methods over Runge-Kutta II and Runge-Kutta IV methods. Goal: to be successful in finding a solution even if it is not accurate.
6. Completes MATLAB projects that are similar to class examples. Goal: to score marks.

Theme: actions that could lead to learning mathematics (with CAS) operationally.

Actions reported in learning mathematics with MATLAB

1. Memorises statements or methods. Goal: to know the solutions or methods just prior to a test.

2. Practises class examples by first writing them out. Goal: to understand techniques or procedures.

Theme: actions relating to optimising the environment

Actions reported in learning mathematics with MATLAB

1. Dutifully attends MATLAB laboratory sessions. Goal: 'to have a clue of what's going on in class'.
2. Uses resources. Goal: to assist the learning of solving DEs using MATLAB.
3. Tries to get access to MATLAB software. Goal: to gain access to the software so as to practise and learn mathematics.
4. Explains MATLAB to other students. Goal: Social motives – enjoys inter-relationships with other (also more advanced) contemporaries.

Actions carried out in problem solving session 1

1. Uses resources when in difficulty. Goal: to assist the learning of MATLAB.
2. Questions about subscripts in MATLAB. Goal: to understand subscripts in MATLAB.
3. Explains MATLAB/analytical solution to Manto. Goal: Social motives – enjoys inter-relationship with others.

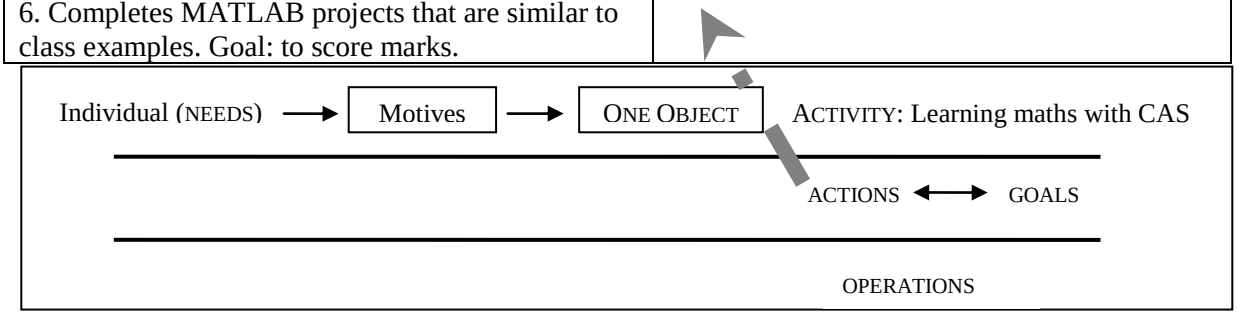


Figure 5.3: Themes and actions from Thembiso's learning mathematics with CAS activity

Theme: actions related to overcoming difficulties associated with mathematics whilst using CAS to solve DEs **Actions carried out in Problem-solving Session 2** 1. Works hard with independent and dependent variables. Goal: aims to find a solution to the DE problem. 2. Uses the same dependent variables when doing two or more numerical methods (e.g. Euler, Runge-Kutta II and Runge-Kutta IV) in one m-file. Goal: aims to produce a solution to each of the three numerical methods. 3. Deliberates with functions and substitution. Goal: aims to solve the DE.	**Theme: actions related to general computer use** **Actions carried out in Problem-solving Session 1** Opens a new m-file. Goal: aims to solve the MATLAB problems. **Actions carried out in Problem-solving Session 2** 1. Puts effort into typing initial lines (previously this was an operation in Problem-solving Session 1). Goal: to solve the DE. 2. Carries out certain procedures with ease, for example: • copy and paste. Goal: to make typing into MATLAB easier or to reproduce a table of solutions from command window into m-file. • zooms into figures. Goal: to examine graphs closely so that he could ascertain if he has produced the correct graphs and solution. 3. Questions about one of the basic keys of the keyboard (insert key). Goal: to return the cursor to its original mode.
Theme: actions related to the use of commands in MATLAB. **Actions carried out in Problem-solving Session 1** 1. Types in the plot command with ease to produce graphs when the dependent variables are *y* and *z* and the independent variable is *x*. Goal: to produce graph(s). 2. Makes use of MATLAB commands. Goal: to produce a solution or graph. **Actions carried out in Problem-solving Session 2** 1. Makes use of MATLAB commands such as hold on, display, plot, and legend. Goal: to produce a solution/ graph. 2. Carries out certain procedures with ease (e.g. puts cursor over red squiggle which appears below any errors). Goal: to read the error message in the corresponding pop-up.	

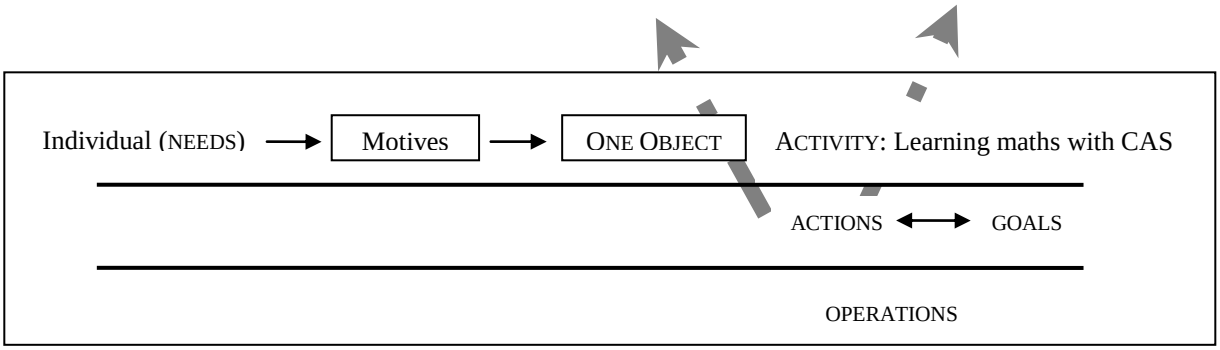


Figure 5.3: Themes and actions from the analysis of Thembiso's learning mathematics with CAS activity (continued)

Theme: actions related to processes followed when solving a DE

Actions reported in learning maths with MATLAB
1. Writes out parts of the MATLAB code before typing onto computer. Goal: to satisfy his belief that writing out assists in getting right the syntax.
2. Makes sense of MATLAB error messages. Goal: to correct errors related to syntax.

Actions carried out in problem-solving session 1
1. Writes out entire MATLAB code before typing onto computer. Goal: to satisfy his belief that writing out assists in getting right the syntax.
2. Rectifies minor errors after perusal of some of the error messages. Goal: wants to produce a solution or graph.
3. Strives to get right the syntax. Goal: to produce a solution or graph.
4. Pays attention to the sequence of steps in MATLAB even when the order does not matter. Goal: to satisfy his belief that this may reduce obtaining MATLAB errors so that he could produce a solution or graph.
5. Interprets graphs of Euler and analytical solution. Goal: wants to determine if his solution to the DE is correct or not.

Actions carried out in problem-solving session 2
1. Writes out entire MATLAB code before typing onto computer. Goal: to satisfy his belief that writing out assists in getting right the syntax.
2. Strives to get right the syntax. Goal: to obtain a solution or graph.
3. Pays attention to the sequence of steps in MATLAB. Goal: to satisfy his belief that this may reduce obtaining MATLAB errors so that he could produce a solution/graph.
4. Interprets graphs of: Euler and Runge-Kutta II
Runge-Kutta II and Runge-Kutta IV
Goal: wants to determine if his solution to the DE is correct or not.
5. Types statements from those given in the instruction sheet. Goal: to copy the given solution onto the MATLAB editor.

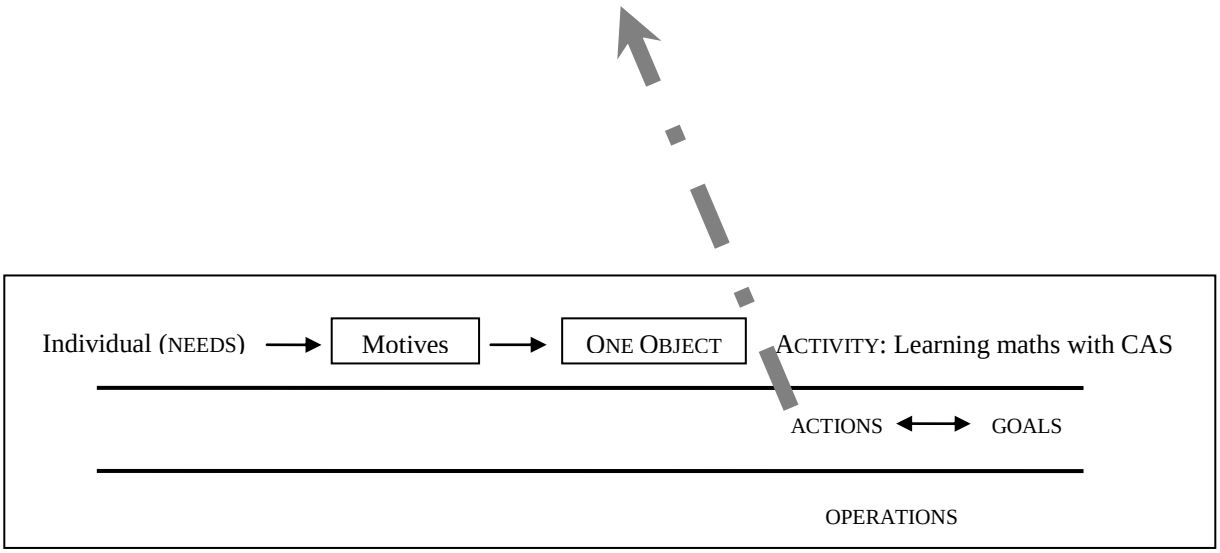


Figure 5.3: Themes and actions from the analysis of Thembiso's learning mathematics with CAS activity (continued)

5.3 ANALYSIS OF TUMI'S INTERVIEWS AND PROBLEM-SOLVING SESSIONS USING THE ACTIVITY THEORY MODEL

I begin with an account of Tumi's background, followed by the needs and motives obtained from her reflections on the activity of learning mathematics. More importantly for the activity of using CAS in mathematics learning, I concentrate on elucidating her needs, motives, object of activity, actions and operations as per the activity theory model shown in Figure 3.6.

5.3.1 TUMI'S BACKGROUND

Tumi presents herself as a student who exudes confidence and enjoyment in studying mathematics both at high school and at university:

> TU-int1(258). … it happened like ok most of the time when I am writing, let's say if I write a test. I feel like I **do do** well before I get before I actually get the results [her emphasis].

Mathematics is described as one of Tumi's comforting subjects:

> TU-int1(80). … actually like maths was er like one of my consolation subjects, yes.

Her home language is Sepedi and she speaks clearly in English. Tumi attended a school in a township in Mpumalanga which she describes as rural, probably the reason that her exposure to computers only began once she attended university.

> TU-int1(231). No the thing is that I got here like at school we didn't have any computers you know. I didn't even know how to switch on a computer.

Tumi is studying towards an Electrical Engineering (heavy current) diploma. She claims that this career choice was her decision, and describes the involvement of her parents in her career plans as follows:

> TU-int1(24). Yes, I was like at home like in a way it is like parents are not really involved with the school things you know. They don't understand this whole thing you know. Like they always say you must become a teacher you know.

Tumi is self-motivated to study, but obtaining this qualification would also make her mother happy. She is also perhaps setting an example for siblings to follow:

> TU-int1(40-42). …like ok in a way ok I am pleasing myself but I know that my mum would be very happy because I will be the first child to graduate at home you know from her. … there are eight children I am third one all the others haven't graduated.

Tumi claims to be an achiever in mathematics, nonetheless her mother, who is a teacher of mathematics, appears to have contributed to her development in the subject:

> TU-int1(57-60). Ok. like my mum she is a teacher. She teaches maths and biology so she always like even when I was young she always giving me numbers to work out and stuff you know. I tend to like maths very much. …And for my matric. I actually got B higher grade.

> TU-int1(74). Ya my mum actually had a part, a big role to play in all that in geometry. Actually before like, before I started grade 12, like in Grade 11 we didn't do the geometry stuff and that. I didn't really understand but during the December holidays my mum was like teaching me the geometry stuff like at school she is a teacher like when she gives a thing, like when she gives a test to her students, she will give me the question paper to write her a memo and then I used to do that.

Tumi was led to believe by her community that obtaining a 'B' higher grade symbol in mathematics at matric level was an accomplishment, until she saw others at university with 'A' symbols:

> TU-int1(62). And you know I got here, I was like ok most of the time, like when I am at home 'B' higher grade is a big thing. When I get here and I am standing in the queue people have A's I am like oh my gosh!

Tumi has ambition; she wants to obtain the diploma so that she could become employable and obtain essential material necessities of life. She also aspires to study further and develop her career:

> TU-int1(44-48). Tumi: Haaa, Ok, guess I just want to get a job and start working. …Yes and buy a car.
> JP: Any other kind of plans after that.
> Tumi: Just buy a car, a house and you know do my B.Tech.

I interpret the above (Interview 1 data) to be: Tumi expresses a need for autonomy. She chooses to study this Engineering diploma as it affords her the opportunity to satisfy her need to function

as an independent and self-sufficient individual in society. In terms of the theory outlined in Chapter 3, I describe Tumi's needs that sustain her commitment to learning as follows. She has a need to develop her career in the sense of satisfying a more immediate need, to obtain this diploma. Moreover, an equally important need to fulfil is her desire to obtain a B.Tech qualification in the future.

As discussed in Chapter 2, effort is a vital index of motivation and this is noticeable in Tumi's reported description of learning events in mathematics, as can be deduced from the excerpts below. Even though she claims that Mathematics 1 was challenging and she knew that she had to work hard, she achieved a very high percentage in Mathematics 1:

> TU-int1(97). Ok. Like, in school it [mathematics] was like it was interesting in a way because like most of the time like you know; how can I put this - it wasn't that hard you know but when you get here you know it's challenging you know you really need to study like, yes yes.

> TU-int1(98-99). JP: Do you enjoy the challenges of studying maths at University? Tumi: Yes actually I do because most of the time when I am studying theory I like can't really get this on my head and every time you know I just want to do maths something like do a calculation or something like that. I am like ok, you know I think that I tend to concentrate a lot on maths than I do on other subjects.

> TU-int1(134-137). JP: Were there anything in Maths I and 2 that you did not like? Tumi: No actually I got an 'A' for my Maths 1, I think. Yes I got 98% for my final Maths 1. Yeah I think Maths 1 was very easy.

From an activity theory perspective it is important to examine the history related to Tumi's computer usage as this may well have an influence on some actions taken to learn MATLAB in mathematical learning. Tumi began her encounter with computers once she entered university to study for the Electrical Engineering diploma, and claims to have become familiar with some software such as *Word*, *Excel* and *PowerPoint*. When questioned if she used computers often, her answer was:

> TU-int1(209). No not really no.

She stated that she initially found working with computers to be complex but now claims that it is manageable, describing her encounters with computers as follows:

TU-int1(229 –231). Tumi: No it is not that bad at all. Even though at first I.. it was very difficult for me but right now it is not that difficult at all.
JP: So what makes it easier you said it difficult now it is better what makes it easier?
Tumi: No the thing is that I got here like at school we didn't have any computers you know. I didn't even know how to switch on a computer. Yeeah, so I used to get very frustrated like when you get here, like when you doing computer course, I mean the computer system they expect you to know the basics you know like where you save, where you do those things, you know it was very confusing for me at first, but yes.

5.3.2 TUMI'S NEEDS AND MOTIVES WHEN SHE REFLECTS UPON LEARNING MATHEMATICS

In this section I begin the discussion on Tumi's description of needs as made apparent when she reflected on how she studied Mathematics 1, 2 and 3 (whilst registered for the diploma). This is followed by an elaboration of motives espoused when she learns mathematics at university level.

5.3.2.1 NEEDS

Tumi reports in Interview 2 that she engages in a process of consistently solving mathematics problems. This technique enables her to solve problems more easily in tests or examinations as well as tutorials. Her preparation includes solving problems that are similar in nature. By persisting in her habit of solving exercises from the 'textbook', she claims to develop the competency needed to solve tutorial problems (which mainly comprise past year's examination questions):

TU-int2(91-92). JP: You also said you practise a lot as a method. Why would you choose a method like that?
Tumi: Because I know that if I do these exercises - like most of the time, like, I don't forget them easily, like, even if I see a problem like that one, I'm sure that it helps me; it becomes easier for me to solve problems like that one.
TU-int2(93-94). JP: So you say, even if you see one. Where would you see one like that?
Tumi: See for example in an exam. Even when I'm doing the tutorials; it's easier for me to do them. They say if you didn't practise as often as you could, like when you get problems

like that in the tutorial you have to go back to the 'textbook' and refer back. But if you did them then, like, ok, I've seen this before.
TU-int2(95-96).JP: It makes your tutorial..?
Tumi: Easier.

Tumi claims that university mathematics equips her with the capacity to develop problem-solving skills or to become adept at this.

TU-int1(85-87). JP: Do you see any purpose in the maths that you learn at university?
Tumi: Yes, I think that er it gives you an ability to solve problems easier, you know.

Tumi's experience of university mathematics is one that involves challenges. In addition, she desires to spend time studying mathematics and expresses a longing to be continually active in solving mathematics problems. She also wants to be able to solve problems successfully in mathematics and persists in doing so, as can be deduced from Interview 1. She claims to be deliberating more on mathematics than on her other subjects and intersperses her studying of other theory courses with solving mathematics problems or carrying out calculations:

TU-int1(98-99). JP: Do you enjoy the challenges of studying maths at university?
Tumi: Yes actually I do because most of the time when I am studying theory I like can't really get this on my head and every time you know I just want to do maths something like do a calculation or something like that. I am like ok, you know I think that I tend to concentrate a lot on maths than I do on other subjects.
TU-int1(105). Tumi: …Yes, I do because like most of the time I get frustrated if I can't solve something you know.

Tumi is also quick to add that time constraints limit the amount of time that she has to spend on studying mathematics. I contend that she associates her affection for mathematics with her need for competency:

TU-int1(107-109). Yes but, yeah but I don't really have enough time to study maths all the time but ya I do enjoy studying maths, I do. … Actually when I understand what's going on.

I conclude that Tumi wants to solve mathematics problems with confidence, which reveals a need for competency in mathematics.

5.3.2.2 MOTIVES

From the interviews, Tumi's motives in learning mathematics may be categorised as both cognitive and self-related.

Cognitive motives

In the interviews, Tumi speaks positively about mathematics and expresses the same affection for Mathematics 3 as articulated for her previous mathematics courses. Her interest in studying mathematics was made obvious:

> TU-int1(128-133). JP: Do you remember much of the maths content that was taught to you in semester two maths [this is Mathematics 2]?
> Tumi: Yes I do actually. JP: What were some of the things that you remember or stands out?
> Tumi: Ok like when we were doing partial fractions yes I liked it very much.
> JP: Integration by parts?
> Tumi: Integration by parts yeah partial fraction, yes it was very interesting to me, yes.
> TU-int2(371-372). JP: Even though it's compulsory, do you have any regrets about studying Maths 3?
> Tumi: No. I think maths it's one of those subjects that I like so much. So I don't have regrets.

Moreover, important verbalisations such as: 'I just want to do maths' are indicators of motives that are mastery approach goal-orientations:

> TU-int1(99). …and every time you know I just want to do maths something like do a calculation or something like that. I am like ok, you know I think that I tend to concentrate a lot on maths than I do on other subjects.

Tumi reports that she independently selects problems from the 'textbook' to work through. She claims she does not spend much time in doing problems of a similar nature but rather focuses on solving a variety of problems, including those that are challenging. What students choose to do on their own are indicators of where their motivation lies (Schunk et al., 2008), and Tumi's choice to solve problems of a demanding nature is evidence of motives related to mastery approach goal-orientations.

> TU-int2(53-58). JP: Ok, so you do select your own problems?

Tumi: Yes.

JP: And more or less how do you go about selecting your problems?

Tumi: Ok like normally I would start with the examples. I do the examples, after that I look at the exercises and I try to do.. ok the ones which are similar - I don't really like doing. I'll do maybe one or two of them and then go to the next one, the one that I think that is different from the ones that I did.

JP: Challenging ones? Do you choose the challenging ones?

Tumi: Yes, I do.

Self-related motives

Even though Tumi's motives in studying mathematics are related to intrinsic interest and the desire to develop competence in mathematics, she also displays other motives. In the context of Mathematics 3, she affirms that she would like to get a good grade and I contend that this is a performance approach goal-orientation. She reports in Interview 1 that she plans to work consistently in mathematics to achieve a good grade:

TU-int1(238-241). JP: What grades do you expect to get at the end of this Maths 3 course?

Tumi: Hopefully I'm I'm hoping to get a distinction.

JP: How do you plan to go about getting your distinction?

Tumi: Ok, like every time when I have done something in class, I must go back and make sure that I study that section because, yes you can never go wrong. Yes as long as you study, yes, I think that I'll be fine.

There is evidence of extrinsic utility value, which I deduce from the Interview 1 transcript: Tumi is studying mathematics so as to do well and so improve her career opportunities. These motives form part of the energising personal reasons that direct her activity of learning mathematics. Behind these motives lies her need for autonomy as discussed above. She strives to achieve and hopes to be rewarded in terms of a sponsorship:

TU-int1(254-256). JP: Can I ask you - why is it important to get this distinction? Tumi: Because I think that I will have better chances of getting a job. Yes yes and maybe even a sponsor.

JP: Are you currently on a …

Tumi: NSFAS loan, yes. [The National Student Financial Aid Scheme]

Tumi reflects on the importance of studying Mathematics 3, which she finds crucial because of its application to engineering scenarios. This is not surprising since most students studying towards a vocational qualification place importance on the extrinsic utility value of mathematics. I argue that she values the application of Mathematics 3 and its applicability to other modules in the engineering diploma.

> TU-int2(3-4). Some things that I like about Maths 3. um..What do I like? Ok, it helps, it helps me.. like understanding Maths 3 helps me to solve other problems other than maths, like with control systems we use Laplace to solve um. If it wasn't for Maths 3 it was going to be difficult for me to understand what I was doing with those other problems in control systems.
>
> TU-int2(19-20). JP: Do you see any purpose in studying the Maths 3 course?
>
> Tumi: Yes, I do, because it helps me to solve other problems. Not only related to maths but control systems, things like that.

5.3.3 TUMI'S NEEDS, MOTIVES AND OBJECT OF ACTIVITY WHEN LEARNING MATHEMATICS USING CAS

In this section I briefly discuss the needs that Tumi has when studying computers in mathematical learning. I focus on her motives when she learns mathematics using CAS, followed by an elucidation of object of activity, and most importantly her actions and operations.

5.3.3.1 NEEDS

I have provided an explanation on Tumi's 'need for competency' when it comes to her participation in mathematics, however due to the context in which she learns mathematics using CAS (Chapter 5.3.4, wherein I give details on the institutional context) there are no explicit needs that Tumi displays or articulates related to the activity of studying CAS in mathematics learning. Nevertheless, I suggest that I could reasonably infer her needs. Tumi's need for autonomy and her need to develop her career still underlie her drive for involvement in learning activities.

5.3.3.2 MOTIVES

Tumis's motives when learning mathematics using CAS are both cognitive and self-related.

Cognitive

I argue that Tumi's reports of her inherent interest in studying mathematics continue when she learns mathematics with CAS. Reflecting on MATLAB, she has found it to be very interesting. In addition, she values the MATLAB learning experience:

> TU-int2(350). Yes, I think it's.. a good experience for me. I take MATLAB as like, I value it the same as theory. The stuff that we're doing in theory class.
> TU-int2(307-308). JP: Yes. Generally about MATLAB and the whole course, what would you suggest?
> Tumi: I think it's fine to work with. It's very interesting.
> TU-int2(382). Tumi: Not really except that MATLAB is very interesting, it was very interesting for me.

Tumi claims that the many mistakes she has made whilst working with MATLAB proved to be a fruitful learning experience for her and she appreciates making them. I argue, based on Interview 2, that she is striving to improve herself in the activity of learning mathematics with CAS. She aspires to develop competency in the task of using MATLAB to learn mathematics:

> TU-int2(359-360). JP: Is getting a good grade in the MATLAB exam important to you?
> Tumi: I would say yes, but then like so far with MATLAB.. ok, I did both good and bad, but I still don't regret it because.. like I know my mistakes now. And I'm glad that I made mistakes because by now if I just go the correct stuff I wouldn't know like what is it that I'm doing wrong, what is it that I need to do, what is it that I need to solve.

I maintain that the motives Tumi espouses are mastery approach goal-orientations, whereby she endeavours to seek understanding and indulge in self-improvement. By learning to use MATLAB in mathematical learning she states that she has developed new skills. Specifically, she claims that the MATLAB experience has endowed her with an enhanced alertness and has helped her to become a discerning individual – to be aware of right from wrong. Moreover, I deduce from her articulations in Interview 2 that she believes that MATLAB had played a vital role in her improved cognitive abilities. There appears to be an indication of cognitive development:

> TU-int2(353-356). JP: Now if MATLAB was not a compulsory requirement of the maths course, would you study MATLAB?
> Tumi: I think I would. I think it's very interesting.

JP: What would your reasons be?

Tumi: It's just a good experience, like you know when you're doing the graphs and then you're doing a lot of mistakes and you have to go back. I think like it helps me to become very sharp, to know when I'm doing something that this is wrong. Like most of the time I want to write something, you just know that this is wrong here. I get that feeling sometimes.

TU-int2(361-362). JP: Is getting a good grade in the MATLAB exam, is that important to you?

Tumi: Ok, I'lI say ya, yes. It would be great to get a good grade, as long as I understood what I did.

Tumi also expresses an interest in MATLAB because of the challenges presented by MATLAB:

TU-int2(141-144).

JP: Was MATLAB interesting?

Tumi: It is.

JP: What aspects of it are interesting, why would you say it's interesting?

Tumi: I'd say it's.. it's challenging. It gives you something to do.. sometimes you think that you know something but then you realise that you don't really know. Like when you're trying to solve an equation in class.. or maybe writing a test, you keep on writing [typing], you don't even know what you're doing is right or wrong. You just draw it and then yes.. you think that it's right but then maybe you're totally off.

Self-related

I elucidate the multi-motive facet of Tumi's self-related motives. I begin with the personal motive of Tumi wanting to obtain good grades in MATLAB (in mathematical learning and thereby achieve a good symbol in Mathematics 3 – this I classify as a performance approach goal-orientation). Tumi finds it important to get a good grade provided she comprehends what she has learned:

TU-int2(361-364). JP: Is getting a good grade in the MATLAB exam, is that important to you?

Tumi: Ok, I'lI say ya, yes. It would be great to get a good grade, as long as I understood what I did.

JP: So why is it important to get a good grade?

Tumi: Ok the grades contribute to what my final mark in maths.. It would be nice to have a good symbol in maths.

Tumi reports in Interview 2 that she finds it useful to learn to use MATLAB (in maths) although currently using MATLAB to solve DEs does not feature as applications in any of her other courses in Electrical Engineering[52]. However, Tumi speculates that using MATLAB specifically to solve DEs will be useful for her future if she decides to continue with a career in electrical systems. The motive to learn MATLAB is classified as extrinsic utility value when MATLAB is perceived as useful to her future career specialisations:

TU-int2(181-182). JP: So why is it important to learn how to use MATLAB to solve differential equations?

Tumi: Ok like for example, in the future maybe.. let's say you take.. because when we get to S4[53] I think that you're going to.. like after completing S4 you're going to choose whether you want to be.. let's say if you're trying to.. you're going to work with systems. I think that could be an advantage for you in order to, if you understand the MATLAB.

TU-int2(192). …ok, like going to the previous question, like I said.. yes, I think knowing about MATLAB and stuff, I think that it's very important because they say for example, when you're doing Electrical Engineering and then maybe you become a protection engineer and stuff, they say you're trying to design a machine.. they say you're trying to find out the lifespan of that machine, you're going to need these equations to calculate how long it's going to work perfectly and then it's going to depend on the time that you're going to…how frequently you use that machine, and then the environment where you put it, the temperatures and stuff. I'm sure that you're going to find an equation like for that thing that.. the temperature, the time, you're going to solve everything.

For Tumi, MATLAB's importance is related to its use as a verification of the solutions produced using pen and paper methods. Comparing solutions are viewed as deeper approaches to learning and will be discussed further under the theme of 'actions related to comprehension of mathematics (with CAS)'.

TU-int2(171-172). JP: Do you think it's important to learn how to use MATLAB?

Tumi: I would say yes, because like I said I don't think that, ok, like.. just to see if the solutions that you're getting are correct.

---

[52] The issue of relevance of the topic of DEs to the Electrical Engineering curriculum needs attention.

[53] Recall S4 is Semester 4 and refers to the last theory semester of the diploma course.

Tumi is motivated to use MATLAB because it is timesaving, easier to use than pen and paper methods, produces neat graphs, produces results easily, and makes it easier to solve problems because of its facility to produce two or three representations to the same problem. Even though Tumi experiences much difficulty and spends time in trying to produce solutions to DEs in Problem-solving Session 1, she still reports that MATLAB is a timesaving tool, provided that one has a good knowledge of the work:

> TU-int2(105-106). JP: …What are some of the things that you like about MATLAB and why?
> Tumi: Like, I'll say.. ok if you know your work it won't take you a lot of time to solve a problem. TU-int2(108). To.. I'll say differential equations [she talks about problem solving in the DE context].
> TU-int2(379-380). JP: We can approach problems in numerical methods in different ways [I remind the reader that numerical methods are used to obtain the solution to DEs usually when you cannot find the analytical solution]. You've learned about 3 of them: Runge-Kuttas and Euler. There are many different ways, number one is mental calculations. …Then there is pen and paper. Then it's scientific calculators. Then it's MATLAB. Which do you prefer and why?
> Tumi: I would say.. doing MATLAB. I would say it's much better because you could use two or three different methods to see if you get the same solutions. I think it's easier to compare with that. And then when you're using a calculator you could make a mistake, press one instead of two, or.. sometimes you need to divide then subtract again, put the brackets.. we have to go back and put the bracket. I think it's much better to use the MATLAB.

Tumi believes that producing the numerical solutions to DEs by hand will lead to mistakes in the numerous substitutions that one has to carry out, and that the process takes much longer to produce solutions compared to using MATLAB:

> TU-int2(118-122). Tumi: I'll say that okay MATLAB generally saves time. It's supposed to save time like when solving problems.
> JP: And is it happening?
> Tumi: Yes, that's what I like. Rather than let's say if you're trying to plot the graph, you have to substitute the values, and get a graph paper and yes.. I think it gives you.. the solutions are better, you're just trying everything, it gives you the results.
> JP: You say the solutions are better, what do you mean by the solutions are better?

> Tumi: I mean you get the results easier than when you're doing them like.. writing it by hand.

I deduce that Tumi is pleased about MATLAB's facility of error messages that appear to assist in correcting errors and thereby hasten the process of producing a solution, as is evidenced:

> TU-int2(148-152). Tumi: …They say like, like most of the time if you were only like.. I think it's easier to get correct solutions than when you're just doing it by hand.
> JP: Can you give me an example of that perhaps?
> Tumi: I think it's much better to use the MATLAB because let's say if you're solving a problem, you won't really know what.. like ok, maybe you do know what you're doing, but it's easier to make mistakes than when we're doing MATLAB. Because most of the time if you make a mistake it tells you that there's a mistake. It gives you an error message, so you can quickly go back and try to change it.

5.3.3.3 OBJECT OF ACTIVITY

I argue that for Tumi, the object is to develop competence, skill and increased cognitive ability in learning mathematics with CAS soon after the work is covered in class by rewriting MATLAB code and solving exercises. She aims to obtain a good symbol in the MATLAB component and prepare herself to apply MATLAB (in maths) to future engineering contexts.

5.3.4 ACTIONS CARRIED OUT BY TUMI WHEN LEARNING MATHEMATICS WITH CAS

I elaborate on Tumi's actions when she learns mathematics with CAS, which include those derived from the analysis of interview data and from actual problem-solving sessions.

THEME: ACTIONS RELATING TO OPTIMISING THE ENVIRONMENT

The social context of learning mathematics with CAS provides a dynamic setting to examine how Tumi makes optimal use of the environment.

Action: studies independently

In Interview 2, Tumi reports that she studies Mathematics 3 according to her own schedule, including coming to terms with the work covered in class, usually at the end of the same day[54]. Studying with others does not work for her as she claims that group work is time-consuming. In the interviews, she speaks often about a studying pace and gives the impression that other students have a slower one than her, perhaps because she works consistently and well ahead of any assessment:

> TU-int2(291-294). JP: Do you study Maths 3 together with other students perhaps in a group?
> Tumi: Most of the time I do it by myself.
> JP: Why?
> Tumi: For me.. ok like, for me, like ok most of the time,there's this other girl at where I'm staying, like it's very hard for me to do it like.. I like using my own timetable. I don't like.. let's say most of the time, like you do exercises like, you know the previous test that we wrote. She wanted to do the exercises in the books and stuff, and it's like, that's when like she's starting with that, and for me I did those things a long time ago and I don't think it's necessary for me to do that. I only work through the tutorial problems.
> TU-int2(295-298). JP: Do you find yourself explaining maths to her?
> Tumi: Yes, sometimes but.. ok for me like when I'm studying in a group, I feel like those people are pulling me behind. I think it takes a lot of time when you are studying with other people. Most of the time I do it by myself, if I get a problem I'll go ask somebody. Same, I don't mind if somebody comes and starts asking me, but studying together..
> TU-int2(297). JP: How about MATLAB in a group?
> Tumi: No.

Based on the above information I assert that the goal that corresponds to Tumi studying independently is to maintain her own studying pace in MATLAB. Although she explains that she studies Mathematics 3 independently, her studying of MATLAB in mathematical learning follows a similar pattern:

> TU-int2(218-228). Tu: Ok, most of the time I go back home, I take the 'textbook', and then I look at what we did in class, try to do those examples that we did in class on my own…

[54] This will be further discussed under the theme of actions relating to personal power.

… On my own first. Most of the time in class, we do them in class, the lecturer explains what's happening there. Ok, but then I need to do let's say if I understand what she said in class, I start by doing those examples, the ones that we did in class, do them again…

Further there is no competition when it comes to scoring grades in the MATLAB projects:

TU-int2(368). No. That's why most of the time I like studying on my own. Because I don't really like comparing stuff with other people because I know they have their studying pace, I have my studying pace. So we're very different.

Action: accesses laboratory only a small number of times

A critical aspect pertaining to the use of MATLAB (in maths) relates to having access to a computer and MATLAB software. Tumi claims that she did not have access to these tools at home but relied primarily on the ones in campus. This meant having to use the laboratory after 16h30 on weekdays in order to practise. The campus on which she was studying is situated in an unsafe area in Johannesburg, consequently her learning to use MATLAB (in maths) was affected in dire ways:

TU-ref-int2(76-77). Tumi: I haven't done it. Because practise makes perfect – we all know that and I am not practising. … Labs opening at 4h30 and sometimes it's not even opened till 5 pm.
TU-ref-int2(78-79). JP: Do you feel that if the labs were opened during the day, you would have been doing more practise.
Tumi: Yes yes
TU-ref-int2(81). Tumi: Sometimes we have free periods cos' on Wed we have free periods from 9h30 – 13h30 we have to stay around here.
TU-ref-int2(83). Tumi: Yes if the labs were opened, I would go there during that time.
TU-ref-int2(85). JP: You write a test this week and an exam in two weeks? Weekends: is the lab opened?
TU-ref-int2(87-90). Tumi: No.
JP: would that make a difference?
Tumi: Yes it would.
JP: but you both stay close by?
Jane: It's very dangerous, you can't walk late.
Tumi: even during the day. Ok I have a sister here they took her phone just around 13h00-13h30.

TU-ref-int2(91). Tumi: Especially late whew.

Tumi claims that she has hardly practised on the computer except during MATLAB class time and she tries her best to go to the labs to practise, just a day before the MATLAB test:

TU-int2(229-236). JP: Now you said sometimes if you get a chance you go to the lab. Can you talk more about that?

Tumi: Yes, most of the time I don't really have time to go to the lab because of.. because like sometimes they open the labs late. Like today it's Saturday, the labs are closed. So most of the time, when I get a chance I go to the labs and try to do..

JP: Have you been getting the chance often or not?

Tumi: Not but then ok, this week I went there once.

JP: To write the test. Over and above that?

Tumi: No, not to write the test. Just to do the problems.

JP: Did you manage to do a lot?

Tumi: Ok there was this other.. I think it was two of them. Because I went there, I had the test the following day, I was writing on Thursday, I went there on Wednesday afternoon. I tried to do two examples.

TU-int2(251-252). JP: And then you left the lab quite late alone?

Tumi: Yes, it was quarter to six. I say whew.

Tumi reports that she writes out the MATLAB code of solutions to DEs. This does not afford her a valuable learning experience as she is unable to determine immediately whether what she has written is correct or not, compared to using MATLAB wherein one gets immediate feedback when compiling programmes. If the opportunity arises she attends the laboratory to access the computers and type the code after she has written it. Consequently she is unhappy with the manner in which she has been practising to use MATLAB to solve DEs:

TU-int2(237-240). JP: Overall are you satisfied with the way you are studying, going to the lab and working on MATLAB?

Tumi: Not necessarily. Sometimes I wish I had a computer. I think that would make my life easier if I had a computer accessible next to me.

JP: You think you would have practised differently had you had a computer with MATLAB installed on it?

Tumi: Yes, because most of the time when you're doing like things.. if you're doing the problem by hand [writing the MATLAB code], you're not really sure if it's correct or not. You can't even see the graphs.

TU-int2(269-270). JP: If you don't get a chance to get to the lab before a test, what happens then?
Tumi: I just write by hand.

The action of accessing the laboratory only a small number of times has a goal: Tumi wants to practise solving MATLAB exercises using a computer. A further consequence of attending labs infrequently is as follows:

TU-int2(246). I'm in the lab.. I think that um.. most of the time let's say that there are these mistakes that we keep on doing without realising it, but when you're actually in the labs, you can see the mistakes. Like, oh, I've been doing that mistake a lot. I've been doing that most of the time without noticing [she has been making the mistakes whilst writing out the MATLAB solutions to the DEs].

Even when Tumi accesses the lab she cannot dedicate the occasion just to using MATLAB in mathematical learning. She reports of lab work in other subjects that demands her attention and time:

TU-int2(241-244). JP: …Do you know how many times you went to the lab to practise?
Tumi: It's not a lot of times..
JP: About?
Tumi: About.. let's say, like most of the time I would go to the lab not only to practise MATLAB because I have protection we also use um the computer, even control systems we use the computers to practise. It's not a lot. Because they open up late, that's the problem.
TU-int2(335-336). JP: If the labs were open during the day would that make a difference?
Tumi: It would, because most of the time we are free and then like, most of time, say I finish around about lunch time, and then I think that I could use that time to go to the labs. Because like right now, there's a lot of stuff that you're using that needs computers, like control systems, protection, MATLAB. Because most of the time like when you get a chance to go to the labs it's only for a short period of time and you have so much to do. It's a lot.

Action: uses resources

The goal for this action is to gain knowledge from the 'textbook' or others. Apart from the 'textbook' Tumi has not consulted other material such as relevant books or Internet resources

related to MATLAB. She has tried using the help-files but is not happy with this resource and finds it inaccessible and confusing. Her reflections on the use of resources in Interview 2 are:

> TU-int2(285-286). JP: With MATLAB have you ever consulted any books, other books, internet resources?
>
> Tumi: No. I'll say that most of the time I don't come across a lot of problems. Maybe it's because I'm using my pen and paper, ya. So I haven't done that. Like the only one that I was wondering about was the one that I was asking you, because that was the first time I see..
>
> TU-int2(302). Ok, like for me the help-file is confusing. Because it has a lot of things and I don't really know where to go.

Tumi asserts in the interviews that she primarily resources the prescribed 'textbook':

> TU-int2(218). Ok, most of the time I go back home, I take the 'textbook', and then I look at what we did in class, try to do those examples that we did in class on my own.
>
> TU-int2(222). Tumi: …Write them out. And then find problems and then write them down.. like if I get a chance to go and use a computer, I start with those examples first and then write [type] them.. like say, put it on the screen and try to run them and see if it's the same.. because sometimes when they give you an example in the text book, they also have the graph. See if the graph is correct and look at the tables if their values correspond with the ones in the 'textbook'.
>
> TU-int2(299-300). JP: Suppose you come across a differential equation using MATLAB and you experience difficulties with that, what do you do? You're in the lab and you've got this difficulty?
>
> Tumi: I'll take a 'textbook', look if there's something that's similar to that. If I can't get it right still, I'll ask somebody.

Tumi, in Problem-solving Session 2 encounters a new error message involving the word 'integer' and she is anxious to know its meaning and how to resolve this error. She expresses a desire to consult some books but unfortunately the library is closed on Sundays and on most Saturday afternoons.

> TU-int2(287-290).
>
> JP: The integer? Tumi: That was the first time I saw it so I was like, ok, yes…
>
> JP: We'll have a look at that now.

Tumi: Because right now I'm anxious. I'm thinking ok, what am I going to do? Am I going to.. I'm thinking about getting another 'textbook' if we can't find it? But the library is closed tomorrow.

Tumi accedes to asking others for assistance if she cannot find answers in the 'textbook' – consistent with this articulation she requested help during problem- solving session 1.

8c. TU-ref-int1(24). Tumi: Can you please help me out here.
JP: Yes, ask you partner for help.
Tumi: Would you please help me here, I have no idea what it is that I am doing wrong. *X* of *i* here again.
Jane: ok that's what I did, I am not sure if it's going to come right.
Tumi: Wow ok.
JP: Got it?
Tumi: Yes.
Jane: Yes.

In addition, during Problem-solving Session 1, Tumi typed statements into the m-file by making use of her class notes - referring to the initial statements of *a*=... *b*=...

I argue that Tumi mainly resources her class notes and 'textbook'. It is imperative to note that her actions of using resources during the problem-solving sessions are similar to what she has reflected upon during interviews.

THEME: ACTIONS RELATING TO PERSONAL POWER

I give attention to how Tumi managed the learning process of MATLAB in mathematical learning in spite of the many obstacles that she faced. In illuminating the choices made by her regarding what and when she studied MATLAB in learning mathematics, I begin by discussing two inter-related actions:

Action: chooses to solve problems quickly after lectures and Action: Writes out solutions to MATLAB homework problems

I postulate that the goal for the first action is to become familiar with already solved examples and to attempt solving exercises:

Tu-int2(43-46). JP: Imagine that you start a new topic in class today. Let's say that your topic is Laplace transforms. How do you go about studying the topic?
Tumi: Ok, it's introduced in class. When I get home I'll try to look at the examples, and then try to do one or two exercises.
JP: You try to do it the same day or before the next lecture or after that or before the test?
Tumi: Ok, like same day most of the time…

The goal corresponding to the second action is for Tumi to have the solutions ready so that when she gains access to the laboratory, she types it into MATLAB as evidenced by:

TU-int2(260). Most of the time I do them by hand. And then when I get a chance I go to the lab.

The students' histories are an important aspect of the activity theory framework. Students bring to the learning process their previous learning methods. Tumi reported in the interviews that she usually solves the homework problems on the same day that it is given out in the mathematics class. She continues in this tradition when she learns mathematics mediated by MATLAB. Since access to computers was an issue she resorts to writing out the solution to MATLAB problems on the same day they are given out in class. Circumstances have forced her into this mode of writing MATLAB code[55] as opposed to typing directly onto MATLAB software. Tumi's enthusiasm to complete homework problems on the same day that the sections are covered in class prevails, in spite of the computer access problem.

TU-int2(259-264). JP: And then the rest of the problems? Whatever's in the book?
Tumi: Most of the time I do them by hand. And then when I get a chance I go to the lab.
JP: You do them immediately, the same day, you do them after that, how long..?
Tumi: You mean the..?
JP: By hand.
Tumi: The same day.

The Electrical Engineering students have a compulsory test week wherein all lectures cease and they write tests in each subject. The influence of this context on Tumi's normal learning style is immense. She is unable to carry out her usual practice of wanting to solve problems soon after

[55] I will discuss later how she in any way prefers to write out parts of the MATLAB syntax first before typing onto the computer.

they are given out in class, as she has to pay attention to the tests being written during this test week:

> TU-ref-int2(63-65). JP: Have you been practising to write by hand?
> Tumi: Ya especially the ones with the Euler method.
> JP: Is it bec of not able to come in the evenings.
> Tumi: yes, yes
> TU-ref-int2(67-68). JP: Have you practised the Runge-Kutta by hand?
> Tumi: Not really no because we only did it a week before last but then we had this test week - we can't do anything. Ya and I just thought that MATLAB is easy so it can't really give me a problem.

Action: redoes class examples followed by solving exercises

As Tumi mentioned, her style of learning mathematics (subsequent to attending lectures) involves redoing classroom examples. Selecting and solving exercises from the 'textbook' (including challenging ones) follows this. In the context of learning mathematics with MATLAB, she pursues the same tradition. She would like to establish whether she has understood what the lecturer did in class and consequently works through already solved classroom MATLAB examples. This approach involves writing out the entire code to solutions of MATLAB examples and exercises. The goal corresponding to the action of 'redoes class examples followed by solving exercises' is for Tumi to determine if she has understood the examples solved by the lecturer in class. Her goal is also to apply what she has learned so as to solve other problems (exercises) from the text:

> TU-int2(256-258). Tumi: Ok, like most of the time they will give us, let's say, like most of the time, like she does an example, and then after doing that example she gave us a chance to do that example. Then [she] gives us a problem to do. So normally I would do that example and then do the problem again. That problem that we were given.
> JP: Do you have enough time to do that? Tumi: Yes.
> TU-int2(218-228). Tumi: Ok, most of the time I go back home, I take the 'textbook', and then I look at what we did in class, try to do those examples that we did in class on my own..
> JP: Without a computer?
> Tumi: Without a computer.
> JP: So you write them out?

Tumi: You write them out. And then find problems and then write them down.. like if I get a chance to go and use a computer, I start with those examples first and then write [in this context she means type] them.. like say, put it on the screen and try to run them and see if it's the same.. because sometimes when they give you an example in the 'textbook', they also have the graph. See if the graph is correct and look at the tables if their values correspond with the ones in the 'textbook'..

JP: So you redo the examples first?

Tumi: Yes, I do.

JP: You redo the examples, writing them out?

Tumi: Yes.

JP: On your own or looking at the answers in the book, or what do you do?

Tumi: On my own first. Most of the time in class, we do them in class, the lecturer explains what's happening there. Ok, but then I need to do let's say if I understand what she said in class, I start by doing those examples, the ones that we did in class, do them again.. I do that.

Action: chooses Runge-Kutta methods over Euler methods

In contrast to Thembiso, who questioned why the Euler method was taught when it is the least accurate one, Tumi relishes the experience associated with learning Euler methods. Her envisaged choice of methods in a real-life (workplace) context is to choose the most accurate method, namely Runge-Kutta IV, in spite of it being the longest (and perhaps the most difficult) of the three approximation methods learned. I propose that the goal is to choose the most accurate method to solve DEs. Moreover, Tumi asserts that she would take on the added accountability of finding the analytical solution (is this exists), then compare this to the Runge-Kutta II and IV solutions. Such reported actions are indicative of deep approaches to applying mathematics using MATLAB:

TU-ref-int2(124-125). JP: Which of all the approximations that you learned is most accurate?

Tumi: It's Runge-Kutta IV.

TU-ref-int2(126-127). JP: Let's say you have a DE that you produced in some circuit (workplace scenario). You got MATLAB available. Which of the methods would you choose and why?

Tumi: I would most definitely use Runge-Kutta IV cos' I'm sure, I know that it is much closer to the real one.

TU-ref-int2(128-129). Tumi: Yes, I would check analytical solution. If I could see the Runge-Kutta II closer to the analytical I'll get worried because I expect the Runge-Kutta IV to be closer.

JP: So will you also do all the methods?

TU-ref-int2(130-132). Tumi: But I won't do Euler.

JP: Do you think learning about Euler was a waste of time?

Tumi: Not really, no. like before I knew the Runge-Kutta methods, I thought it [Euler] was great. It's simple, it's short and ya..

THEME: ACTIONS THAT COULD LEAD TO LEARNING MATHEMATICS (WITH CAS) OPERATIONALLY

Having discussed how Tumi redoes class examples after attending lectures, I consider how this action could be construed as one that leads to learning mathematics (with CAS) operationally.

Action: redoes class examples followed by solving exercises

Tumi's action of redoing class examples is an attempt to become skilful at solving problems in MATLAB (in mathematics learning). She affirms that even if she understands the content taught in class she nevertheless begins her independent study by redoing the solutions of known classroom solved examples and by meticulously writing the MATLAB code:

TU-int2(228). On my own first. Most of the time in class, we do them in class, the lecturer explains what's happening there. Ok, but then I need to do let's say if I understand what she said in class, I start by doing those examples, the ones that we did in class, do them again.. I do that.

TU-int2(218-224). Ok, most of the time I go back home, I take the 'textbook', and then I look at what we did in class, try to do those examples that we did in class on my own..

JP: Without a computer?

Tumi: Without a computer.

JP: So you write them out?

Tumi: You write them out. And then find problems and then write them down.. like if I get a chance to go and use a computer…

JP: So you redo the examples first?

Tumi: Yes, I do.

Tumi alludes to practicing Mathematics 3 material so that she could recall what she has previously studied and understood. In learning Mathematics 3, she reports that she uses practise as a method:

> TU-int2(90-92). Tumi: I think like doing the exercises as often as I can. I think yes and then if you get stuck somewhere ask somebody. That really helps.
> JP: You also said that you practise a lot as a method. Why would you choose a method like that?
> Tumi: Because I know that if I do these exercises.. like most of the time, like, I don't forget them easily, like, even if I see a problem like that one, I'm sure that it helps me.. it becomes easier for me to solve problems like that one.

Tumi claims that she prepares for Mathematics 3 tests by revising and practising familiar problems.

> TU-int2(68). You know like when I'm about to write a test I just go through the stuff that I did, it's more like revision, and I try to pick up problems, like exercises that I've already done and try to do it and see if I could remember them. Because sometimes you do these exercises and then you do others, you do a lot of work after that, so I just go back and try to refresh my memory.

In contrast, Tumi prepares for a MATLAB test by making an effort and struggling to access the labs so that she could type her written MATLAB solutions onto the computer[56]. Although practise is a means that Tumi alludes to in learning mathematics, she was not afforded this prospect in learning mathematics with CAS because of the distinctive context of learning using CAS (without proper access to computers). Instead her practise took the form of writing code:

> TU-ref-int2(60-62). JP: So how are you going to prepare for the test and the exam that's coming up?
> Tumi: I will also come on Thursday because I am writing tomorrow. Difficult. But even if we write by hand ok, can get most of them right but then you will never know for sure.
> TU-int2(267-270). JP: How do you go about preparing for the MATLAB test then?

[56] I scheduled the last interview on a Saturday so that I created a chance for Tumi to use my computer for most part of that day. My observations: she typed in her written codes as well as attempted to solve challenging exercises from the 'textbook' using the MATLAB software. This was in preparation for the major test and upcoming MATLAB (in maths) examination.

Tumi: Ok, like, most of the time I look at the stuff that we did previously. I look at them, and then like most of the time I would go to the lab if I have a chance. I'd go there and then try to do some of the examples on my own…

JP: If you don't get a chance to get to the lab before a test, what happens then?

Tumi: I just write by hand.

THEME: ACTIONS RELATING TO COMPREHENSION OF MATHEMATICS (WITH CAS)

First I elaborate on two interrelated actions:

Action: tries to solve analytically first and Action: compares MATLAB solutions with the values that she produced by substitution (pen and paper)

Tumi uses manual methods to check the numerical solution produced by MATLAB; she works out the solution to Euler's numerical solution by manually substituting values into the formula. Verification of MATLAB solutions produced by the computer is an indicator of Tumi's diligence in her learning. Below is her report on this action:

TU-int2(97-98). JP: Let's consider the MATLAB software that you used mainly to solve differential equations: Euler, Runge-Kutta 1, Runge-Kutta 2. How did you feel about using it?

Tumi: I think it makes life easier. But then sometimes you get these messages.. like ok, even if you get.. what I've realised now is that most of the time you get the results but they're not correct. And you have to be really careful. Especially with the Euler. It's easier to see that you did a mistake with Euler, go back and just do the substitution manually and then yes..

TU-int2(163-170). JP: Because I remember you were the only student at the beginning [of Problem-solving Session 1] with a pen and paper, you were calculating table values before you did the Euler.

Tumi: Yes.

JP: So are you going to talk about that?

Tumi: Because I was trying to see like.. when you get a table, especially with an Euler, I was just trying to find out if the values are correct. Like using a pen and paper. I think it's an easier way to find out if you made a mistake.

JP: But it can take long to produce a table of ten values.

Tumi: It can.

JP: But you're not doing all ten?

Tumi: No. Just maybe two or three.

Based on the above I propose that the goal of the first action is to check the solution obtained from substitution (pen and paper techniques) against that obtained from numerical methods in MATLAB, or, as in the case of Problem-solving Session 1, the goal is to answer the question in the given task. The goal of the second action is to ratify the solution produced from using MATLAB.

Tumi's approach to learning encompasses a multi-method perspective. She is dissatisfied with using MATLAB to produce the numerical solutions but relies on her substitution techniques by hand. Most of the problems requiring numerical solutions cannot be solved analytically, hence she finds it difficult to plot or produce solutions using analytical means:

TU-int2(99-100). JP: Describe some of your experiences of using MATLAB to solve differential equations.

Tumi: What I normally did is if I get a problem, I'll try to solve it analytically and do the Euler methods or the Runge-Kutta II and IV. But then what I found is that most of the time it's very difficult to plot or find a table of the analytical solutions. Sometimes difficult to do that.

Tumi's analytical solutions provide her with a verification system for MATLAB numerical solutions, and these reported actions are consistent with those observed in Problem-solving Session 1:

Tumi worked on the analytical solution first. Consulted with her notebook for the formula of integration by parts. Tumi had solved the DE analytically but made a mistake just in the last step of the solution. She then made use of the Euler formula to substitute and obtain correct value for y_2.

8d Tumi compares the MATLAB table of values with the y_2 value that she had worked out by hand for the Euler approximation.

Although Tumi produces solutions to Euler methods using pen and paper, she is not aware that she can use such techniques to find solutions involving Runge-Kutta methods. She finds that there is a great deal of uncertainty with these and does not know how to validate the solutions produced by MATLAB. She had, in Problem-solving Session 2, produced MATLAB solutions to Runge-Kutta II and Runge-Kutta IV methods but was unable to tell if these were correct (she thought they were when they were not). In the previous editions of the 'textbook', the methods

involving manual substitution were given in detail but recently have been omitted. The current chapter on MATLAB merely gives the MATLAB procedures involved in finding the numerical solutions of DEs. Tumi made a plea for alternate methods to verify the solutions of Runge-Kutta methods obtained from using MATLAB:

> TU-int2(116). What I don't like about MATLAB, especially with the Runge-Kutta's, it's very difficult to tell that you've made a mistake. You can't really do.. there's no way of knowing that you made a mistake.
> TU-int2(303-304). JP: If you were to change anything about the MATLAB course, what would that be?
> Tumi: I'd say that ok, you know what I wish that there was a way in which one could see.. especially with the Runge-Kutta.. to see if we're doing something wrong or we're doing something right, there was a way in which you could check let's say manually like, same as like when you do Euler solutions, it's easier to check by hand.

The demanding work of finding then comparing solutions produced by substitution with numerical solutions produces a sense of gratification for Tumi and boosts her self-esteem:

> TU-int2(109-114). JP: What are some of the things that you liked about using MATLAB to solve differential equations?
> Tumi: You know like after writing (writing in this context means typing) everything and then you just evaluate the stuff that you did and you get your results, I think you feel good about yourself. Especially if you can see that the values are correct, the solutions are correct. If you feel like they are correct..

Action: interprets graphs

One of the vital aspects involving numerical solutions of DEs using MATLAB is for students to generate and interpret corresponding graphs. I draw attention to the stages of learning that Tumi goes through with respect to producing graphs for the solution of DEs in MATLAB. At first she is content with being able to produce a graph, perhaps because she found difficulty in interpreting and correcting the error messages:

> TU-int2(132-134). Tumi: Initially I thought that it was very easy, but now ok, I think it's challenging. Because like at first I didn't really look.. because I was only happy if I got a solution and a graph. I didn't really try to see if the graph is correct, if the values are

correct. I didn't look at that. I thought it was very easy. I thought it was all about finding a graph and the values. But now I realise that it's a lot more than that.

JP: What would that lot more be?

Tumi: Like checking if the graphs are correct, checking if the values are closer together.

TU-ref-int2(118-121). Tumi: Ok , most of the time I was just happy to get an output.

JP: You normally think if it gives you an output..

Tumi: You normally think if it gives you an output - you think it's right.. most of the time I get frustrated when I get the error messages, you know. And the moment I get an output, I am like oh thank God!

JP: You never think it's wrong?

Tumi: You just assume it's right.

In Problem-solving Session 1 Tumi gets diverse solutions for the same DE using the analytic method and numerical approximation of Euler techniques. She studies both the graphs but does not realise they are incorrect.

8e studies both the graphs.

When alerted by me to her having produced incorrect solutions[57], Tumi begins to check the graphs and solutions that she subsequently produces. As she progresses with her learning she becomes focussed on interpreting graphs. She has comprehended that the Runge-Kutta IV graph should be much closer to the analytical graph, as evidenced by her responses in Interview 2:

TU-ref-int2(99). JP: How do you know if you are getting the correct Runge-Kutta table values?

TU-ref-int2(101). Tumi: Ya because for me like, most of the time if I did, let's say ..ok normally what I like doing: like I would draw an analytical graph with the Euler, Runge-Kutta II, Runge-Kutta IV and then I'll see that most of the time Runge-Kutta IV is much closer to the analytical graph than the other one.

TU-int2(341-342). JP: Many students said that when they do a MATLAB problem to solve differential equations, they said to me that they don't even know what the graph should look like and it makes them feel uncertain. How do you feel?

[57] She thought that her solutions were correct because she compared the first solution value of her pen and paper substitution with the MATLAB one and they concurred. However, the rest of the MATLAB solutions were incorrect and she had not worked out the corresponding ones manually.

Tumi: Most of the time when I start doing the problem I'm not sure how the graph should look like. But then as soon as I get.. like when I do these numerical methods, like I think that they should look similar, more or less the same.

Tumi reports in the interview about the closeness of the graphs:

TU-ref-int2(4). 05:09 Tumi: Because they almost alike Runge-Kutta II and Euler.
TU-ref-int2(7-9). JP: Are you happy with your solutions?
Tumi: Ok, like I've seen that ok, like, they are not really that close but I think that's to be expected because I'm sure that the Runge-Kutta IV.. ok I am expecting the Runge-Kutta IV to be closer to the Euler solution than..
JP: You got right idea with the wrong ones in it. Runge-Kutta IV should be closer to Runge-Kutta II or to the analytical.
Tumi: Interrupts or acknowledges Runge-Kutta IV closer to the analytical.

Moreover she easily uses functions such as zoom to get a closer look at the graphs:

22d 01.21.14 Zooms in: observes graphs – both graphs are very close to each other.

The goal corresponding to the above action of interprets graphs is to determine if her solution to the DE is correct.

THEME: ACTIONS RELATED TO GENERAL COMPUTER USE

Given that Tumi entered university with no knowledge of how to use computers, and she does not have a personal computer, it is not surprising that she experiences problems with basic computing skills in both the problem-solving sessions.

Action: puts effort into basic computing skills

I suggest that the goal for this action is to produce solutions to DEs. I focus on the attempts she puts into typing, the struggle she has with the keyboard, alluding to the use of the shift function, and trials with highlighting text. These struggles are prominent in both problem-solving sessions and are similar; hence my examples come from both of the sessions.

Attempts with typing:

Tumi types in capital letters, for instance *V* and *H*, when lower case should be used:

47.36 20a for *i=1:m*

k1= deletes *k*

types in *K:* *K1=H** deletes *H*

types in *h: K1=h*(1=* deletes =

K1=h(1+V(i)*sqrt (* 52.17 deletes *V(i)*

K1=h(1+sqrt(t(i))*V(i));*

Tumi uses (alphabet) *o* instead of (number) *0*:

20b *K2=h*(1+sqrt(t(i))+o.5* deletes the (alphabet *o*)

20c *K2=h*(1+(sqrt(t(i))+0.5*h)*(V(i)+0.5*K1));*

Tumi struggles with typing:

20f 59.56 types in *k* deletes *k*

types in *y* deletes *y*

types *V(i+1)=v* deletes *v*

V(i+1)= (deletes *(*

01.00.50 *V(i+1)=V(i) =* deletes =

V(i+1)=V(i)+(10 deletes *0*

V(i+1)=V(i)+(1) deletes *)*

V(i+1)=V(i)+(1? deletes *?*

V(i+1)=V(i)+(1/6)(K1+2*K2+*

Struggle with use of the shift function:

Tumi types in: *) instead of 0* *? and /* *+ and =* *(and 9.*

Since all these symbols share one key on the keyboard, I assume that she has difficulties using the shift key or function.

2d goes to end of *h=0.1;* enters a line and types in *m=(b-a);* deletes ;

2e puts in *)* to get: *m=(b-a))* deletes this bracket *m=(b-a)*

2f *m=(b-a)?* deletes *?*

2g *m=(b-a)/m* deletes *m*

17.10 2h *m=(b-a)/h;*

3d 18.49 *y(i=1* deletes *1*

3e 18.54 *y(i=+* deletes + deletes =

3f 19.34 *y(i+)=1=* deletes =

Trials with highlighting text:

Highlighting of MATLAB code appears to be problematic. Although Tumi manages to highlight the code and then copy and paste them, the very act of highlighting does not appear to be effortless.

> 53.00 11b highlights code first she highlights all except last line, then highlights all code, then highlights a few lines. This is followed by highlighting all code from part 1 and right clicking, copy and paste into same m-file but below the first table values.

I have discussed how Tumi easily zooms in to examine graphs (refer to theme of actions relating to comprehension of mathematics with CAS). Consequently, she does carry out certain procedures with ease. The goal is to examine graphs closely so that she could ascertain if she has produced the correct solution to the DE.

THEME: ACTIONS RELATED TO THE USE OF COMMANDS IN MATLAB

Tumi puts effort into producing the correct syntax with respect to MATLAB commands.

Action: makes use of MATLAB commands

Next I consider how Tumi made use of various commands, such as 'clear all', 'disp', 'plot' and 'legend' in Problem-solving Session 1, along with 'ylabel' and 'legend' in Problem-solving Session 2.

MATLAB command 'Clear all':

Tumi does not use certain MATLAB commands like 'clear all' because she does not understand why this command is there:

> TU-ref-int1(16). Tumi: Is it necessary to put in 'clear all' before everything.
> JP it's always good, you know why… [drilling noise].

MATLAB command 'disp':

There is uncertainty with the display command and she types in pisp instead of disp:

23.18 4e types in pisp then goes back to end enters a line before pisp and types in format long

4f pisp*()*

4g pisp*([])*

4h pisp*([x',y'])* (types in *y* then deletes *y*, types in *y* again)

29.22 5j puts the pisp right: disp*([x',y',y1])*

MATLAB command 'title':

Tumi types in title and then deletes it:

33.08 6a title*(*

33.39 6b deletes title*(*

MATLAB command 'plot' and 'legend':

There are issues with the plot command. Here Tumi tries to instruct MATLAB to use colours when sketching ('b' is blue and 'r' is red). She does not get this right therefore in 12a she deletes the colours and leaves it simply as *plot(x,y).* She also struggles with the syntax of the legend command.

29.31 5k plot*(x,y,'b',x,y1,'r')*

30.25 5l legend (numerical ; deletes ;

5m legend (numerical, analytical)

32.08 5n legend ('numerica'l, analytical) puts cursor over l and reads message 'invalid syntax at T, possibly a *), } or]* is missing

32.27 5o deletes ' before l and types in ' after numerical; also puts in the ' before the word analytical and after the word analytical:

legend('numerical', 'analytical')

01.05.50: 12a removes *r* from plot command, then deletes *y1, x and b*: plot*(x,y)*

Tumi obtained an error message related to legend. Instead of seeking help, she chose to delete the legend statement, possibly wishing first to produce the solution to the DE then later deal with these less important commands. Tumi does not question greatly the use of the commands or their syntax when she is in doubt, nor seek to find the correct way to use these commands from the 'textbook'. Her use of MATLAB commands is at a superficial level, not increasing her understanding of most commands:

01.06.38 12c produces a table of values with a warning

1.0000 1.0000 …

warning: ignoring extra legend entries

> in legend at 292

12g 01.10.32 selects from part 2 the statements of plot, xlabel, ylabel and legend and deletes them.

MATLAB command 'ylabel':

In Problem-solving Session 2 there are still issues with some commands. Tumi correctly types in *20 0 ylabel('v')* but when there is an error message pointing to

21a 01.08.58 ???undefined function or variable *"V"*

Tumi acts as follows:

21c deletes ylabel*('v')*

21d highlights evaluates selection

???undefined function or variable *"V"*

21e types in ylabel*(* deletes *(*

ylabel*('v')*

The above act of deleting *ylabel('v')* and retyping it indicates that Tumi is not confident with the syntax of the *ylabel* command.

MATLAB command: 'legend'

I would have expected Tumi to have mastered the legend command by Problem-solving Session 2 but there is still a struggle to get the command right. The legend statement in *20p: legend('RKII' 'RKIV')* just needs a comma ' ,' to separate RKII and RKIV. Instead she removes the quotes ' ' which are needed and puts in the comma.

21 l …Also there is an error message: ???legend('RKIV' 'RKII')

Error: Unexpected MATLAB expression

22b 01.19.50 alters legend of RKIV to read as:

legend(RKII, RKIV)

22c 01.20.05 highlights, evaluates selection …

error message: ??? Undefined function or variable 'RKII'.

23a 01.32.27 legend ('RKII', 'RKIV')

Action: carries out certain procedures with ease (e.g. puts cursor over red squiggle which appears below any errors)

There are occasions in which Tumi reads the error message associated with the cursor placed over the red squiggle. The goal is to know what the errors are before compiling the programme (see 5n above).

THEME: ACTIONS RELATED TO PROCESSES FOLLOWED WHEN SOLVING A DE

Within this theme, consideration is given to a range of actions as performed by Tumi, such as those related to writing MATLAB code, making sense of error messages, rectifying errors, striving to get right the syntax and paying attention to the sequence of MATLAB code even when the order did not matter.

Action: writes MATLAB code for the auxiliary equations before typing onto computer

Tumi claims that she finds the method of writing MATLAB code first and then typing to be easier than just typing directly onto the computer. Her concern is with the particular syntax of MATLAB and its associated issues with brackets. Tumi believes that she finds it easier to read and identify variables like *X* and *Y* on paper than on the computer. The goal of writing parts of the MATLAB code before typing onto the computer is to satisfy her belief that writing assists in getting right the syntax.

TU-int2(201-208).
JP: How do you usually go about learning how to use MATLAB to solve differential equations?
Tumi: Like, most of the time if I get an equation, a problem, I will write it out. I'll write down first like, the stuff that I need to do, the equation first.. the problem first, and then try to do it manually first before I could do it on the MATLAB, on the computer.
JP: You're using pen and paper?
Tumi: Yes, using pen and paper. And then after that just try the equation because it's just substitution.. just copy everything from where you have written.
JP: Why do you choose to use those methods?
Tumi: Because I think that when you're using pen and paper, it's easier to find.. because it's easier to find, um.. to find. It's easier to work out the solution than when you're doing it on a computer because you might miss the bracket, and then most of the time like

looking at the problem it's easier to see that ok, this is *Y* and it's not *X* like rather than when you're doing it on the computer you write, like say you, you write *X* instead of *Y* or something. I think it's easier to do it on pen and paper.

JP: For you it's easier?

Tumi: Because you write it down and then after that you just copy everything on there.

In Problem-solving Session 2:

Tumi started to write just a few lines of the code: *a=0, b=1 and h=0.1,* then began typing these lines as well as *m=(b-a)/h, t=a:h:b and v(1)=2.* She wrote out the auxiliary equations *k1, k2 and v(i+1)* for Runge-Kutta II solution, then typed these into MATLAB.

Action: types statements from those given in the instruction sheet

Although Tumi feels that it is much easier to transfer what she has written on paper to the MATLAB editor, she does not always copy accurately. In Problem-solving Session 2, the Euler solution was given in the instruction sheet to students and Tumi types in a different and incorrect statement, compared to what was given. The goal for this action is to copy the solution from the instruction sheet onto the MATLAB editor. This is what she produced:

19a 31.57 *y1(1)=2;*
for *i=1:m*
first types in: *y1=y(1)*
then modifies this statement to: *y1(i+1)=y1(i+1)+h*(1+sqrt(t(i))*y1(i));*
19b end (she ignores the *y1* which has a red squiggle below it).

She later corrected the *y1(i+1)* to *y1(i)* in the first term on the right hand side of this equation. A general comparison of what she has written and its transfer to the MATLAB editor reveals that most often Tumi types according to what she has written. She even returns to the paper, changes some aspects and then types these changes onto the computer. Tumi had originally written the code represented in *17k* but then altered this on paper to reflect that of the code in *18b*.

18b puts in *(* and a *)* for *sqrt(t)+K1)*
altogether:
k2=h(1+v(i)+h)*(sqrt(t)+K1));*
Compare 18b *k2=h*(1+v(i)+h)*(sqrt(t)+K1));* with
17k *k2=h*(1+v(i)+h)*sqrt(t+K1));*

Action: makes sense of error messages

Initially, Tumi found that locating errors in the MATLAB code was an arduous task as the error messages did not indicate the exact position of errors in the code.

> TU-ref-int1(137-138). JP: Yeah, Ok do you understand what it says when the matrix dimension agree and don't agree?
> Tumi: Sometime I really don't, I really don't.
> JP: You don't understand sometimes?
> TU-ref-int1(141). Tumi: I think it will be more nicer if it indicated the line which you made a mistake so that you could just go there and adjust.
> JP: Yeah.
> TU-ref-int1(147). Tumi: Me I get frustrated if I had to look at one thing all over again you know.
> JP: So you prefer to have the line numbered?
> TU-ref-int1(148). Tumi: Yes.
> TU-ref-int2(119). Tumi: you normally think if it gives you an output- you think it's right.. most of the time I get frustrated when I get the error messages, you know. And the moment I get an output, I am like oh thank God!

Tumi states that she compiles each part (either an Euler or Runge-Kutta II or Runge-Kutta IV) separately so that she is able to find errors related to the coding of that specific part. In this way she tries to resolve the errors guided by the error messages. The goal for the action of 'makes sense of error messages is that Tumi wants to resolve errors so as to produce solutions and results. Tumi's comments on error messages made a few days before the final examination are as follows:

> TU-int2(135-138). JP: When you use MATLAB specifically to solve differential equations, was there any part of it that was boring?
> Tumi: Not really. Most of the time it was very challenging. Like let's say when I try to run a programme, and then I get these error messages and then I had to go back and look for it.
> JP: How did you feel about the error messages?
> Tumi: I think like most of the time they are very helpful because like, most of the time if I get an error message I will start with the first.. let's say I'm solving one equation using three methods, I'll start with the first method and see if I get an error, then highlight that until I see what that error is.

I argue that Tumi appears to have moved from the phase of challenging initial encounters with error messages to subsequent skill in locating the errors. This move is mediated by experience, practise and knowledge:

> TU-int2(139-140). JP: How about understanding the error messages?
> fmi: Yes, I think I understand it a lot better now. At first, I just saw this red thing; I was like, what the hell is that? I'm trying to.. because I used to look at the whole problem, like go back, step-by-step, look at it. Ok now I think it's much better because I just go, I try to look where the error is and try to fix that.

Moreover, Tumi classifies comprehending error messages and locating the position of errors as one of the main tools needed successfully to solve DEs using MATLAB:

> TU-int2(213-214). JP: What do you think is needed to be successful at learning how to use MATLAB to solve differential equations?
> Tumi: Ok first of all, I think you should know.. you should understand the question, I mean the problem that you need to solve. And then you should understand the error messages when you find them, and then you should know where to go.. like I think the error messages, you should know where it's directed to.

Action: rectifies minor errors after perusal of some of the error messages

In Problem-solving Session 2, I observe that Tumi corrects a few minor errors after inspecting the error messages. The goal is to produce the solution or graph(s).

> 18f 20.27 highlights all code, evaluates selection, gets an error message which is displayed at the end of all the code:
> 21.44 ??? undefined function or variable *'K1'*.
> 18g 22.47 changes the *K1* in right hand side of the *k2* statement to lowercase *k1*
> *k2=h*(1+v(i)+h)*(sqrt(t)+k1));*
> 18f Rectifies errors after perusal of some of the error messages. Yet she ignores the upper case *K1* and *K2* in the very next line, that is, in the expression for *v(i+1).*
> 18h highlights the statement clear all
> then highlights the rest, right clicks and evaluates selection
> 22.56 gets an error message which is displayed at the end of all the code: ??? undefined function or variable *'K1'*.
> 18i Goes back to the *v(i+1)* statement and changes the capital *K1* and *K2* into lower case *k1* and *k2* resp:

v(i+1) = v(i)+0.5(k1+k2);*

Action: strives to get right the syntax

The goal with this action is to produce a solution or graph to the problem. She draws attention to the manner in which she could misuse brackets, and this might result in incorrect solutions:

> TU-int2(129-130). JP: What are some of the things that you found difficult about using MATLAB specifically to solve differential equations? You mentioned one or two now, but…
> Tumi: I would say like.. making mistakes, most of the time when you're typing in, because like sometimes you find that you have a lot of brackets, you miss one, and you get weird solutions and yes..

Tumi tries to rework the syntax in many different ways. One example is that in response to an error message she will remove brackets from a certain position, then replace them in the same position and evaluate the code, only to get the identical error message. A few examples from both problem-solving sessions follow.

Issues with ' ; '

Tumi erroneously puts in ; at the end of the 'for' statement:

> 3c 18.20 goes back and types in 'for' to get:
> for *i=1:m;*

Concern with brackets:

Tumi is usually concerned with the use of brackets and she types both the open and closed brackets before entering anything into them:

> 4f pisp*()*
> 4g pisp*([])*

Uncertainty with syntax of ./

Regarding the error message in 33.50, Tumi is aware that there are issues with './' but she erroneously puts this in the *y(i+1)* expression which does not require './'. I deduce that she does not understand why or when to use the dot with a division sign. Note that './' should be used only for the analytic solution *y1*.

33.46 6c highlights code, right clicks and chooses evaluates selection.

33.50 error message appears after legend statement

???error using ==> mrdivide

matrix dimensions must agree

6d back to m-file – looks around the *y(i+1)* statement in the for loop.

35.59 6e highlights *(exp(x)-(y/x)* in the *y(i+1)= y(i)+h*(exp(x)-(y/x));*

6f puts in . before /:

y(i+1)= y(i)+h(exp(x)-(y./x));* and in

y1=exp(x)-(exp(x)+1)./x;

Does not understand error messages:

Tumi tries many modifications but still gets the same error message:

38.44 error message after legend statement

???in an assignment *A(I)=B,* the number of elements in *B* and *I* must be the same

39.47 7c puts in the *i* that accompanies *y* on right hand side: *y(i+1)= y(i)+h*(exp(x)-(y(i)./x));*

40.03 7d ???in an assignment *A(I)=B,* the number of elements in *B* and *I* must be the same

7e goes to m-file, removes ; from the statement of *i=1:m;* in the for loop.

40.50 7f ???in an assignment *A(I)=B,* the number of elements in *B* and *I* must be the same

Indiscriminate use of brackets:

TU-ref-int1(49). 23.04. JP: you don't think it's right?

Tumi: no.

JP: no?

TU-ref-int1(51). JP: Do you think your error lies with the 'for' loop again? Tumi: yes.

TU-ref-int1(52). 26.00 JP: what makes you think that Tumi?

Tumi: I think I made a mistake here because there are so many brackets.

JP: Ok… Tumi: Ya.

7a 36.54 highlights evaluates code

error msg after legend statement

???in an assignment *A(I)=B,* the number of elements in *B* and *I* must be the same

7b back to m-file moves around the *y1* statement

inserts a set of brackets:

y1=exp(x)-((exp(x)+1)./x); compare with 6f:

6f: *y1=exp(x)-(exp(x)+1)./x;*

11f *y(i+1)= y(i)+h*((2/3)*exp(x(i)-(y(i)*sin(x(i)/(2*(x(i)^(2)));*

(whilst doing this, Tumi types in *0*, then deletes *0* and puts in *)* instead):

y(i+1)= y(i)+h((2/3)*exp(x(i)-(y(i)*sin(x(i)/(2*(x(i)^(2))));*

11g 01.04.57 not sure of brackets, checks brackets and adds *)*

y(i+1)= y(i)+h((2/3)*exp(x(i)-(y(i)*sin(x(i)/(2*(x(i)^(2)))));*

The act of deleting *)* and replacing it results in no change to the statement, yet Tumi continues to evaluate the code - expecting to make the error message disappear:

23b 01.33.22 highlights, evaluates code:

??? k2=h(1+(sqrt(t(i)+h)*(v(i)+k1));*

Error: unbalanced or unexpected parenthesis or bracket

Uses methods of trial and error:

I assume that Tumi does not understand the error message in 21a; consequently, she uses techniques wherein she deletes 'ylabel', evaluates the code, obtains the same error message and then retypes 'ylabel':

21a 01.08.58 highlights, evaluates selection, error message at end of code:

???undefined function or variable *"V"*

21b goes to 'for' loop of RK4, moves cursor around *K1, K2, K3, K4*, deletes *V* from right hand side of: *V(i+1)=V(i)+(1/6)*(K1+2*K2+2*K3+K4);*

types in *V* again.

21c deletes ylabel*('v')*

21d highlights evaluates selection

???undefined function or variable *"V"*

21e types in ylabel*(* deletes *(*

ylabel*('v')*

21f 01.11.14 goes to *V(i+1)* statement of RK IV 'for' loop

21g 01.11.48 goes to *K1: K1=h*(1+sqrt(t(i))*V(i));* deletes *)* on extreme right end and puts it back.

21h 01.12.27 moves cursor over *K3* and *K4*. Returns to command window to look at error message.

Action: pays attention to the sequence of steps in MATLAB even when the order does not matter

Tumi pays attention to the arrangement of the MATLAB code and where in this sequence she has to type in certain MATLAB statements. I purport that the goal to get right the sequence of steps in MATLAB is perhaps to satisfy her belief that this may lessen MATLAB error messages, enabling her to produce a solution or graph. She does not experiment with MATLAB to determine whether there would be any difference were she to type MATLAB statements in any sequence:

1b on line 2, types in *x=1:0.1:2;*
1c line3 types in *h=0.1* then erased this
1d back to line1 entered a line and in line 2 typed *h=0.1;*
13.12 2a entered a line before *h=0.1*; types in *a=*

THEME: ACTIONS RELATED TO OVERCOMING DIFFICULTIES ASSOCIATED WITH MATHEMATICS WHILST USING CAS TO SOLVE DES

In this theme the mathematical procedure of substitution and functions are considered.

Action: deliberates with functions and substitution

In Task 1 of Problem-solving Session 2 Tumi types the auxiliary equation as:

17k *k2=h*(1+v(i)+h)*sqrt(t+k1));*

Her utterances indicate that she knows that *t* is the independent variable and *v* the dependent one but the actual substitution in 17k is however incorrect – note that the correct *k2* equation is:

k2=h(1+v(i)+k1)*sqrt(t(i)+h));*
TU-ref-int2(12) Tumi: One that is independent is *t* and *v* is dependent.

In 17k Tumi includes the constant *k1* within the square root (in other words she substitutes into the square root) even though she uses the incorrect constant.

She proceeds to alter 17k to produce a completely different statement. Here she removes *k1* from within the square root:

18b *k2=h*(1+v(i)+h)*(sqrt(t)+K1));*

This modification is indicative of Tumi's uncertainty with functions and substitution as she alters a 'correct'[58] statement to produce an incorrect one. Later in the Problem-solving Session she states that she had correctly included the constant within the square root but did not know why she changed it:

TU-ref-int2(48). …JP: Lets go back to the formula, I'll help you with the RK II.
JP: Explains *dv/dt*, independent, dependent. *K1* is okay, *k2* says *h* times $f(x_i+h).(y_i+k1)$.
JP explains the substitution - replace the *t* with $t_i + h$..
Tumi: I did that at first but don't know why I changed it.
22h 01.30.50: removes *)* after *t(i)* from *k2=h*(1+(sqrt(t(i))+h)*(v(i)+k1));* to produce
k2=h(1+(sqrt(t(i)+h)*(v(i)+k1));*

Regarding Task 2 of Problem-solving Session 2, Tumi produces the *k2* auxiliary equation as:

20c *K2=h*(1+(sqrt(t(i))+0.5*h)*(V(i)+0.5*K1));*

She substitutes the constants *h* and *k1* in the correct way but does not include *0.5*h* within the square root sign. I alert Tumi to the incorrect substitution and inform her that there is a problem with the Runge-Kutta II solution:

TU-ref-int2(15-17). Tumi: The first one was fine [here she means the Runge-Kutta II solution].
Tumi: Yes I see it now.
JP: the first one is fine?
TU-ref-int2(18-20).Tumi: I did it like.. No, I think that it doesn't really make that much of a difference because like here, like multiplication sign [she refers to the auxiliary equation *k1* of Runge-Kutta II].
JP: In the *k1* it is correct, in the *k2*?
Tumi: Ok, ok I see it.
JP: It will make a difference in your *k2*?
22a 01.19.20 Goes to *k2* in RK II *code k2=h*(1+v(i)+h)*(sqrt(t)+k1));*
Changes *h* to *k1* and *k1* to *h* to produce:
k2=h(1+v(i)+k1)*(sqrt(t)+h));*

[58] Correct in the sense of including the constant within the square root

In 22a, Tumi swaps *h* and *k1* but still does not check to see that her substitution into the function is correct in the sense of including *h* within the square root sign, including the subscript *i* with *t* as well as multiplying the *(v(i)+k1)* with *(sqrt(t)+h)*.

In her utterances (see *TU-ref-int2(18-20)* above), Tumi is adamant that the *k1* is correct but she still proceeds to produce a mathematically equivalent statement to her previous *k1*:

> 22e 01.24.01 goes to *k1* of RK II: changes position of *v(i)* from *k1=h*(1+(v(i)*sqrt(t(i))));* to
> *k1=h*(1+(sqrt(t(i))*(v(i)));*

She changes the *k2* auxiliary equation of Runge-Kutta II but has still not included *h* within the square root sign:

> 22f 01.25.45 goes to *k2* of RK II: changes products from:
> *k2=h*(1+v(i)+h)*(sqrt(t(i))+k1));* to
> *k2=h*(1+(sqrt(t(i))+h)*(v(i)+k1));*

Tumi evaluates the code and produces solutions to the DE using Runge-Kutta methods. These are incorrect but very close to the correct ones. The graphs of Runge-Kutta II and Runge-Kutta IV are also deceivingly close to each other. I assert that the closeness of the graphs leads her to assume that she has produced the correct solutions. Moreover, it takes her some time to realise that *h* should be included within the square root as is evidenced below. In her quest to produce the correct solutions she questions me on carrying out impractical steps, such as interchanging the *k* and *h* instead of checking her substitution into the formulae of the auxiliary equations. I conclude that this is indicative of the difficulty that Tumi experiences with functions and substitution whilst solving the DE using MATLAB.

> TU-ref-int2(33-43). JP: Did you correct the *t's* and *v's* in the Runge-Kutta II?
> Tumi: No I didn't. Do you think that could be the problem?
> JP: I think that could be the problem. You don't think that could be a problem?
> Tumi: No because it's multiplication sign.
> JP: But then remember the formula for Runge-Kutta II. What's the formula for Runge-Kutta II? It's *x+h* and *y+k1* – in your *k2*? …
> Tumi: Oh I see, it will make a difference. Is it ok if I just change the *k* and the *h* and leave it like that?
> JP: Is that what the formula says?

Tumi: It says $1+V_i+k1$.

JP: Let's see the graph, identical. Are you happy with your graphs?

Tumi: yes

JP: Are you happy with your table of values?

Tumi: Yes, I am. [I showed Tumi the correct solutions].

TU-ref-int2(33-43). Tu: yeah.

TU-ref-int2(48). …JP: Lets go back to the formula, I'll help you with the RK II.

JP: Explains *dv/dt*, independent, dependent. *K1* is okay, *k2* says *h* times $f(x_i+h).(y_i+k1)$.

JP explains the substitution - replace the *t* with $t_i + h$..

Tumi: I did that at first but don't know why I changed it.

22h 01.30.50: removes *)* after *t(i)* from *k2=h*(1+(sqrt(t(i))+h)*(v(i)+k1));* to produce *k2=h*(1+(sqrt(t(i)+h)*(v(i)+k1));*

In 22h Tumi finally produces the correct *k2* auxiliary equation. I postulate that the goal corresponding to the action of 'deliberates with functions and substitution' is to solve the DE correctly.

5.3.5 OPERATIONS RELATED TO TUMI'S ACTIVITY OF LEARNING MATHEMATICS WITH CAS

In both problem-solving sessions, Tumi is aware of working in an m-file and not in the command window[59]. Since she effortlessly opens the MATLAB programme in Problem-solving Session 1, I assume that this is an operation:

1a easily opens up MATLAB with view of command window and a new m-file

In Problem-solving Session 2:

17a 5.20 Tumi easily opens up command window. However Tumi struggles with opening an m-file.

[59] The command window is not usually used to produce the programme code for the solution to DEs. One could use the command window to check if the syntax is correct as it gives immediate feedback but no participant in this research study did that.

5.3.6 SUMMARY OF TUMI'S ACTIVITIES

Comparable to the summary of Thembiso's analysis I use the model developed in Figure 3.6 to indicate the links between the conceptual variables and the empirical data that were derived from Tumi's analysis. I summarise needs, motives, actions and operations from the above discussions in Figures 5.4 to 5.6:

A summary of Tumi's reflections on the activity of learning mathematics

Needs	**Motives**
Mathematics 1, 2 Need for competency in mathematics. **Mathematics 3** Need for competency in mathematics.	**Mathematics 1, 2** • Cognitive motives- intrinsic interest in studying mathematics. **Mathematics 3** Self-related motives: • Performance approach: seeking to obtain 'A' symbol in Mathematics 3 to increase the probability of finding a job. • Mathematics 3 is important because of its application to engineering scenarios. Cognitive motives: • Intrinsic interest in Mathematics 3. • Mastery approach goal-orientations: seeking understanding.

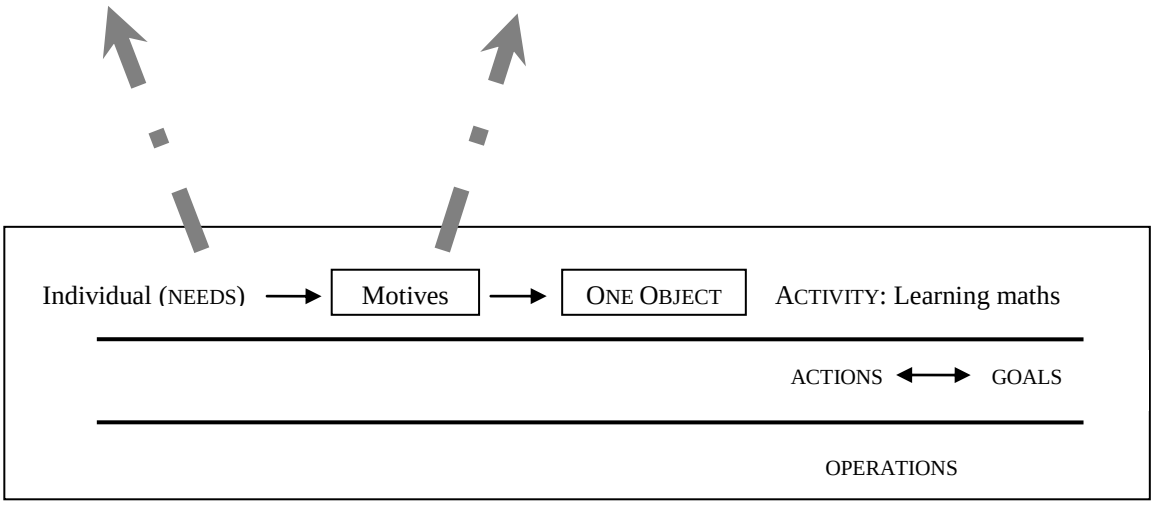
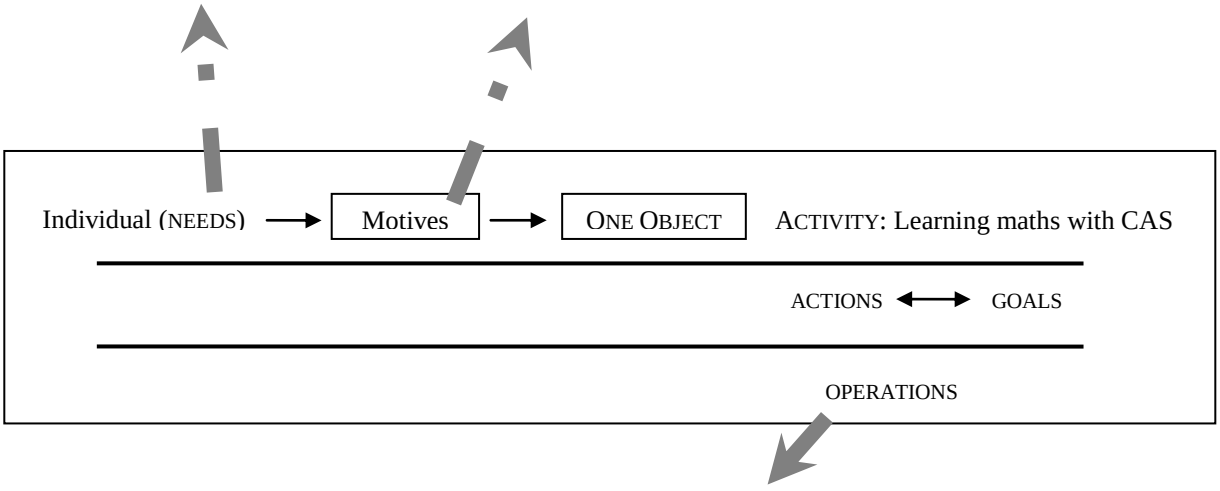


Figure 5.4: Needs and motives from the analysis of Tumi's interviews - reflections on the learning mathematics activity

A summary of Tumi's activity of learning mathematics with CAS

Needs	**Motives**
No explicit needs but I can infer Tumi's needs as follows: there is a need for autonomy and the need to develop her career - these appear to be the underlying needs that drive her learning activities.	Self-related motives: • Extrinsic utility value – MATLAB is perceived useful for future career specialisations. • Performance approach: would like to obtain good grades in MATLAB. • Value components -motivated to use MATLAB because it is timesaving, easier than hand methods, produces neat graphs, produces results easily. MATLAB makes it easier to solve problems. Cognitive motives: • Finds intrinsic value in studying MATLAB. • Mastery approach goal-orientations: wanting to develop competence and develop new skills. • Believes that MATLAB played a vital role in improved cognitive abilities.



Operations with respect to Problem-solving Session 1	**Operations with respect to Problem-solving Session 2**
Effortlessly opens the MATLAB program.	Effortlessly opens the MATLAB program.

Figure 5.5: Needs, motives and operations from the analysis of Tumi's learning mathematics with CAS activity

Theme: actions relating to comprehension of mathematics with CAS

Actions reported in learning mathematics with MATLAB

1. Tries to solve analytically first. Goal: to check her analytic solution against the solution obtained from numerical methods in MATLAB.
2. Compares MATLAB solution with the values that she produced by substitution (pen and paper) Goal: to ratify the solution produced from using MATLAB.

Actions carried out in Problem-solving Session 1

1. Tries to solve analytically first. Goal: to answer the question in the given task; also to check her substitution (into Euler formula) against the solution obtained from numerical methods in MATLAB.
2. Compares MATLAB solution with the values that she produced by substitution. Goal: to ratify the solution produced from using MATLAB.
3. Interprets graphs. Goal: to determine if her solution to the DE is correct or not.

Actions carried out in Problem-solving Session 2

Interprets solutions/ graphs. Goal: to determine if her solution to the DE is correct or not.

Theme: actions that could lead to learning mathematics (with CAS) operationally

Actions reported in learning mathematics with MATLAB

Redoes class examples followed by solving exercises. Goal: to determine if she has understood the examples solved by the lecturer in class. Goal: to apply what she has learned to other problems (exercises) from the text.

Theme: actions relating to personal power

Actions reported in learning mathematics with MATLAB

1. Chooses to solve problems quickly after lectures. Goal: to become familiar with already solved examples and to attempt solving exercises.
2. Writes out solutions to MATLAB homework problems. Goal: to have it ready so that when she gains access to the laboratory, she types it into MATLAB.
3. Redoes class examples followed by solving exercises. Goal: to determine if she has understood the examples solved by the lecturer in class. Goal: to apply what she has learned to other problems (exercises) from the text.
4. Chooses Runge-Kutta IV methods over Euler methods. Goal: to choose the most accurate method to solve DEs.

Theme: actions relating to optimising the environment

Actions reported in learning mathematics with MATLAB

1. Studies independently. Goal: to maintain her own studying pace in MATLAB.
2. Accesses laboratory only a small number of times. Goal: to practise MATLAB exercises.
3. Uses resources. Goal: To gain knowledge from the 'textbook' or others.

Actions carried out in Problem-solving Session 1

Resources text, class notes, JP/Jane. Goal: To gain knowledge from the 'textbook' or others.

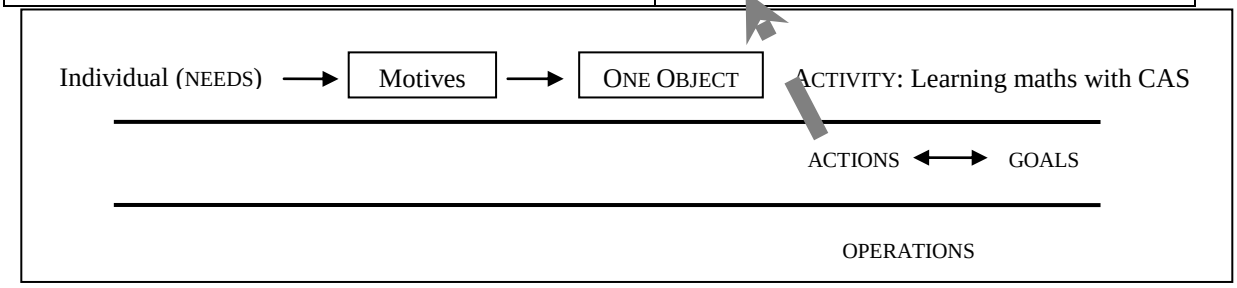


Figure 5.6: Themes and actions from the analysis of Tumi's learning mathematics with CAS activity

Theme: actions related to general computer use

Actions carried out in Problem-solving Session 1
Tumi puts effort into basic computing skills (such as typing: struggle with the keys of the keyboard, highlighting text). Goal: to produce solutions to DEs.

Actions carried out in Problem-solving Session 2
1. Puts effort into basic computing skills (such as typing: struggle with the keys of the keyboard, highlighting text). Goal: to produce solutions to DEs.

2. Carries out certain procedures with ease (zooms in to examine graphs closely). Goal: to examine graphs closely so that she could ascertain if she has produced the correct solution to the DE.

Theme: actions related to the use of commands in MATLAB

Actions carried out in Problem-solving Session 1
Makes use of MATLAB commands. Goal: to produce a solution/graph.

Actions carried out in Problem-solving Session 2
1. Makes use of MATLAB commands. Goal: to produce a solution/graph.
2. Carries out certain procedures with ease (e.g. puts cursor over red squiggle below any errors. Goal: to read the error message in the corresponding pop-up).

Theme: actions related to overcoming difficulties associated with mathematics whilst using CAS to solve DEs

Actions carried out in Problem-solving Session 2
Deliberates with functions and substitution. Goal: to solve the DE correctly.

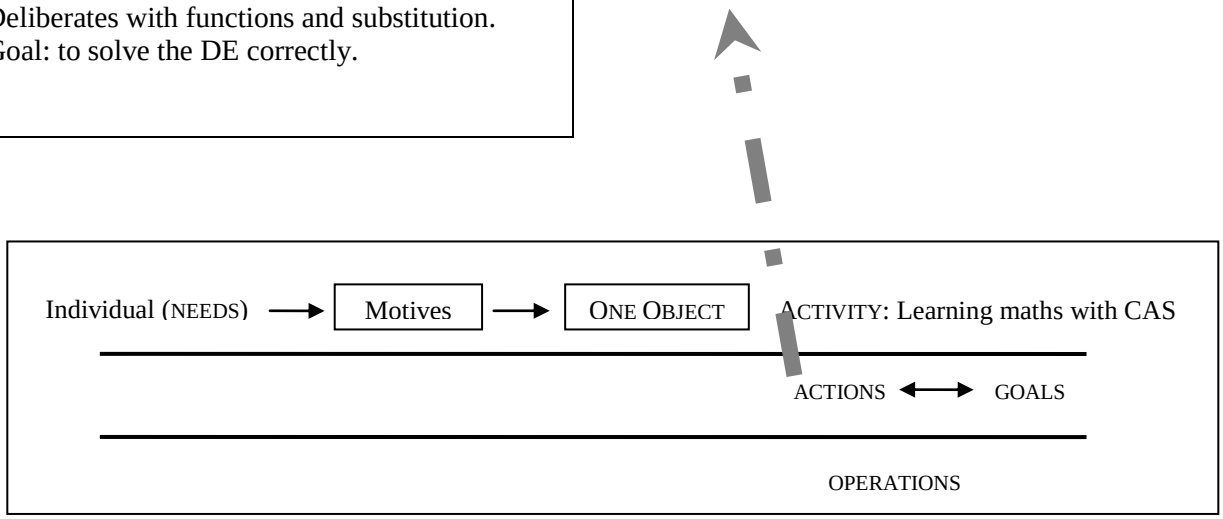


Figure 5.6: Themes and actions from the analysis of Tumi's learning mathematics with CAS activity (continued)

Theme: actions related to processes followed when solving a DE

Actions reported in learning mathematics with MATLAB

1. Writes out solutions to MATLAB homework problems. Goal: to have it ready so that when she gains access to the laboratory, she types it into MATLAB.
2. Writes MATLAB code before typing onto the computer. Goal: to satisfy her belief that writing code assists in getting right the syntax.
3. Makes sense of error messages. Goal: wants to resolve errors so as produce solutions and results.
4. Strives to get right the syntax. Goal: Tumi just wants to produce a solution/graph.
5. Interprets graphs obtained when plotting the solution to Euler, Runge-Kutta II and Runge-Kutta IV methods. Goal: to determine if she has obtained the correct solution.

Actions carried out in Problem-solving Session 1

1. Strives to get right the syntax. Goal: to obtain a solution/graph.
2. Pays attention to the sequence of steps in MATLAB even when the order does not matter. Goal: to reduce obtaining MATLAB errors so as to produce a solution/graph.
3. Interprets graphs. Goal: to determine if her solution to the DE is correct or not.
4. Compares MATLAB solution with the values that she produced by substitution. Goal: to ratify the solution produced from using MATLAB.

Actions carried out in Problem-solving Session 2

1. Writes MATLAB code for the auxiliary equations before typing onto computer. Goal: to satisfy her belief that writing code assists in getting right the syntax.
2. Rectifies minor errors after perusal of some of the error messages. Goal: to produce the solution.
3. Strives to get right the syntax. Goal: to produce a solution/graph.
4. Types statements from those given in the instruction sheet. Goal: to copy the solution from the instruction sheet onto the MATLAB editor.
5. Interprets solutions/ graphs. Goal: to determine if her solution to the DE is correct or not.

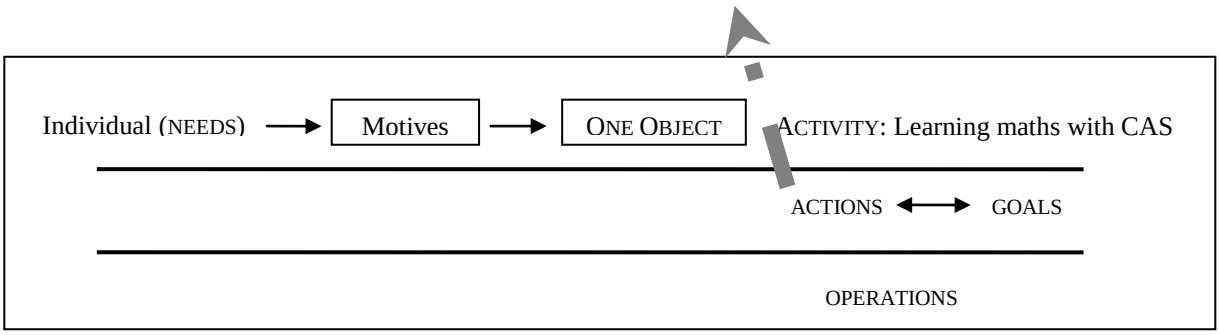


Figure 5.6: Themes and actions from the analysis of Tumi's learning mathematics with CAS activity (continued)

5.4 ANALYSIS OF ABRAHAM'S INTERVIEWS AND PROBLEM-SOLVING SESSIONS USING THE ACTIVITY THEORY MODEL

I present only a summary of Abraham's data analysis due to space constraints. Abraham clearly espoused motives that I classify as cognitive and social (interrelations with others), but I include him in the study mainly because of his deep-seated social (interrelations with others) motives. His motives are different from those held by the other two participants and necessitate discussion. Even though I do not make known the analysis of Abraham's data, I include his actions, operations, needs, motives and goals in the discussion in Chapter 6. The motives of the three case students illuminate the three main categories of motives, namely, cognitive, self-related and social motives; resulting in a comprehensive study.

Comparable to the summary of the other two participants I use the model in Figure 3.6 to indicate the links between the conceptual variables and the empirical data that originated from Abraham's analysis. I summarise needs, motives, actions and operations from the analysis of Abraham's data in Figure 5.7 to 5.9 (below).

A summary of Abraham's reflections on the activity of learning mathematics

Needs	Motives
Mathematics 1 and 2 Need for competency in mathematics. **Mathematics 3** Need for developing competency in Mathematics 3.	**Mathematics 1 and 2** Cognitive motives: • Interested in being challenged by tasks in mathematics. • Mastery approach goal-orientations: mastering the task of learning mathematics and solving problems, striving to improve oneself in the task of learning mathematics. Self-related motives: • Extrinsic utility value – Mathematics 1, 2 is useful because of its relations to electrical engineering courses and examples. • Regards studying mathematics as important. Social motives: • Communication with peers, learning as a means to help and support others, identify with peers. **Mathematics 3** Cognitive motives: • Mastery approach goal-orientations: developing understanding. • Intrinsic interest in mathematics. Self-related motives: • Important to study Mathematics 3 as it is needed for other engineering modules. • Values mathematical knowledge and the Mathematics 3 course. • Extrinsic utility value- Mathematics 3 is useful for application problems in engineering scenarios. • Performance approach: seeking to obtain 'A' symbol in Mathematics 3. Social motives: • Communication with peers, learning as a means to help and support others.

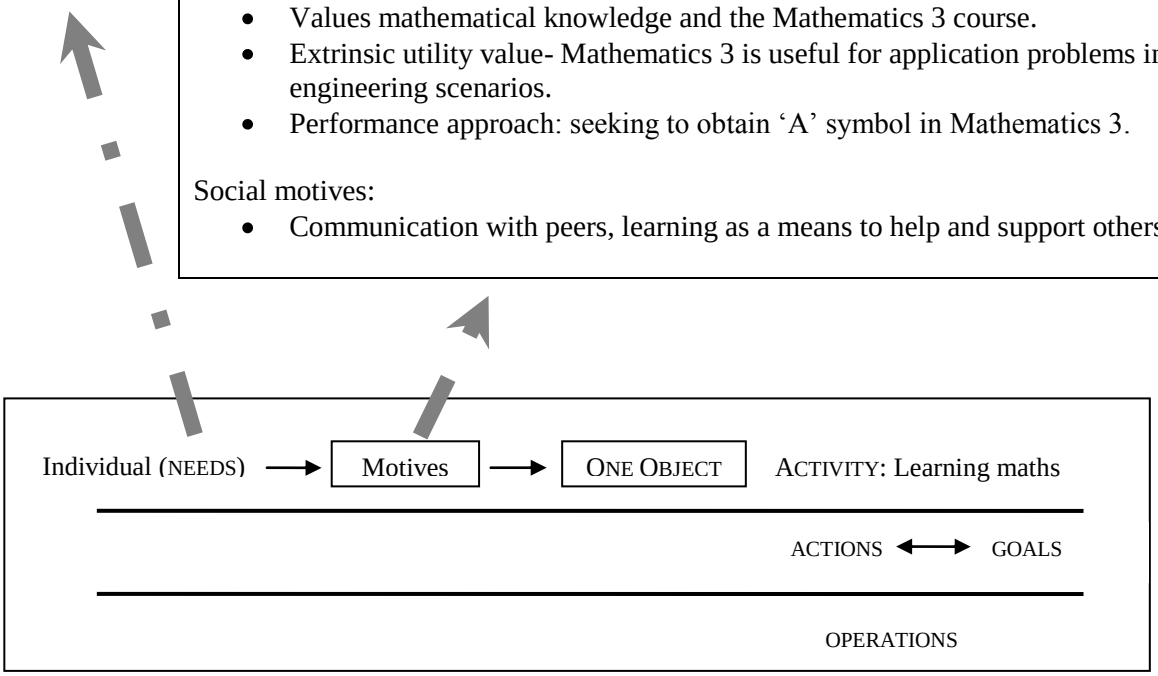


Figure 5.7: Needs and motives from the analysis of Abraham's interviews - reflections on the learning mathematics activity

A summary of Abraham's activity of learning mathematics with CAS

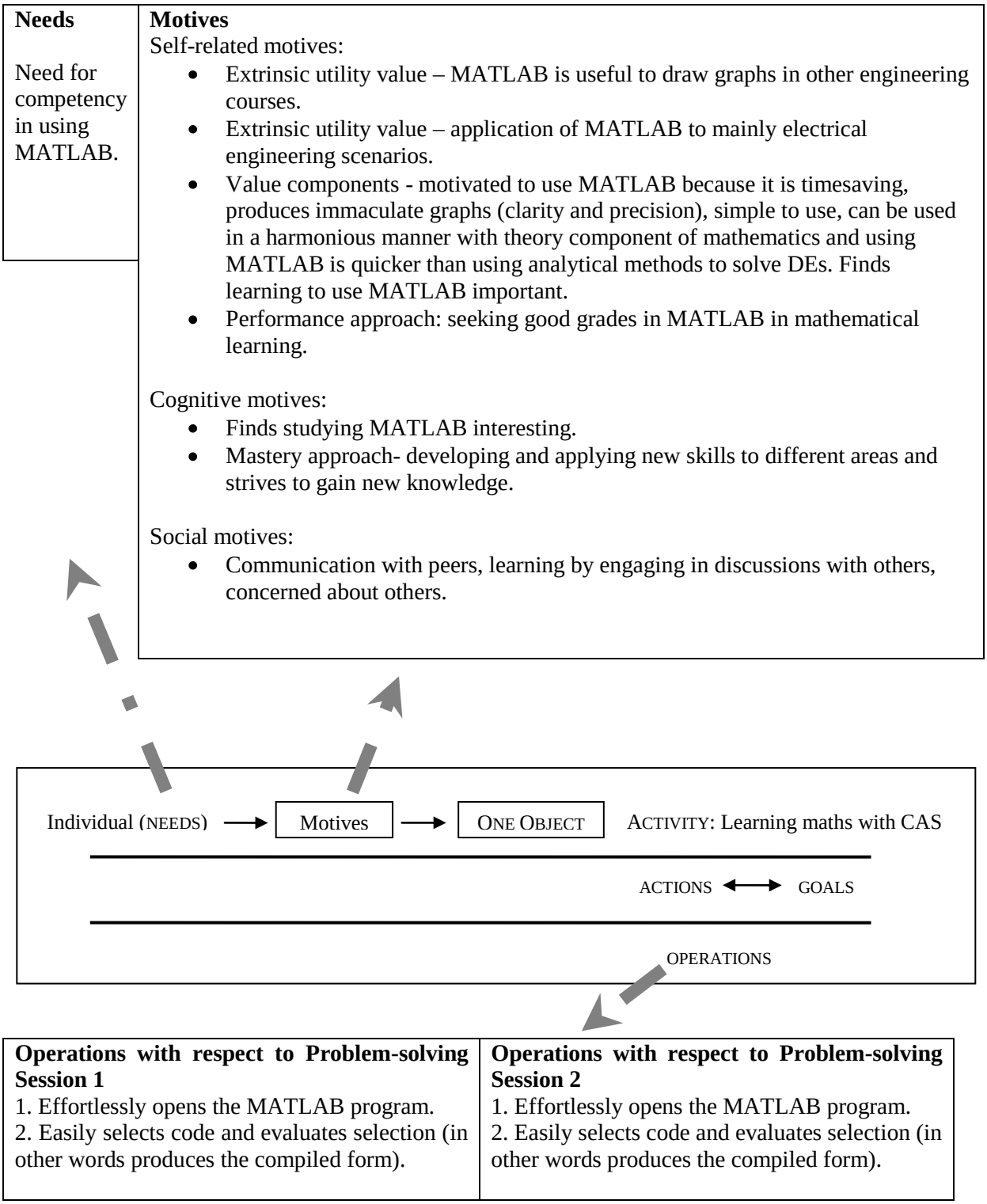


Figure 5.8: Needs, motives and operations from the analysis of Abraham's learning mathematics with CAS activity

Theme: actions relating to comprehension of mathematics with CAS

Actions reported in learning mathematics with MATLAB
1. Discusses and debates with group members. Goal: to understand and know mathematics (with CAS).
2. Tries to solve analytically first. Goal: to challenge himself.
3. Interprets graphs – Euler, Runge-Kutta II, Runge-Kutta IV. Goal: to determine if his solution to the DE is correct or not.

Actions carried out in Problem-solving Session 1
1. Tries to solve analytically first. Goal for part 1 of the given task: to answer the question asked in the task; goal for part 2: to challenge himself.
2. Interprets graphs of Euler and analytical solution. Goal: to determine if his solution to the DE is correct or not.
3. Checks the *x* values in the output. Goal: to determine if he has produced correct solutions.

Actions carried out in Problem-solving Session 2
1. Checks the *t* values in the output. Goal: to determine if he has produced correct solution for the independent variable.
2. Interprets graphs of: Runge-Kutta II and Runge-Kutta IV. Goal: to determine if his solution to the DE is correct or not.

Theme: actions relating to personal power

Actions reported in learning mathematics with MATLAB
1. Selects MATLAB exercises to solve mainly related to electrical engineering and chooses to solve challenging exercises. Goal: to practise and learn how to use MATLAB to solve engineering problems.
2. Chooses to solve problems soon after lectures. Goal: to understand each piece of work soon after it is covered in class.
3. Applies MATLAB to engineering scenarios. Goal: to fulfil self-related motives of wanting to see the relevance to the engineering field.

Actions carried out in Problem-solving Session 1
Types in the 'textbook' example onto the MATLAB editor. Goal: to solve the task allocated for the Problem-solving Session.

Theme: actions that could lead to learning mathematics (with CAS) operationally

Actions reported in learning mathematics with MATLAB
Uses methods of practise. Goal: to know techniques and procedures.

Actions carried out in Problem-solving Session 1
Types in the 'textbook' example onto the MATLAB editor. Goal: to solve the task allocated for the Problem-solving Session.

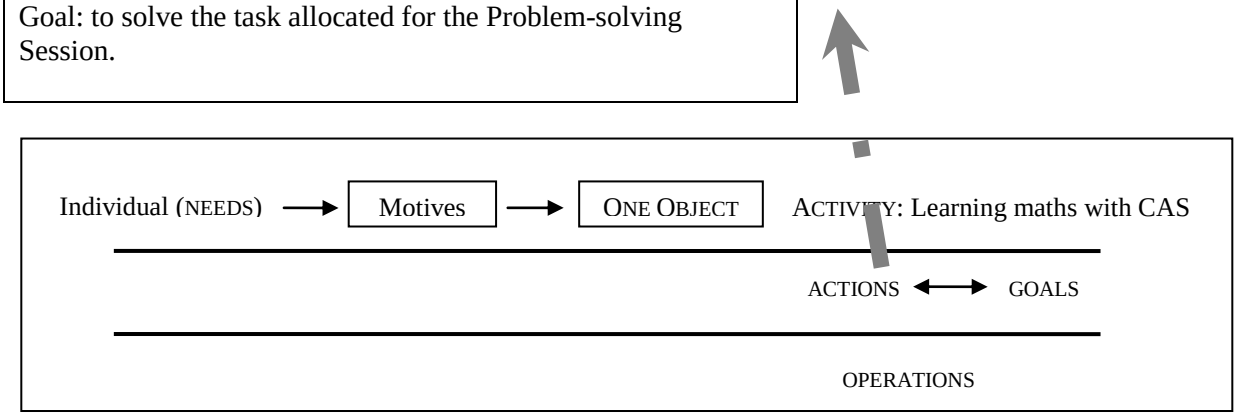


Figure 5.9: Themes and actions from the analysis of Abraham's learning mathematics with CAS activity

Theme: actions related to general computer use	**Theme: actions related to the use of commands in MATLAB**
Actions reported in learning mathematics with MATLAB Uses copy and paste techniques. Goal: to make typing into MATLAB easier or to reproduce a table of solutions from command window into m-file or to save time. **Actions carried out in Problem-solving Session 1** Carries out certain procedures with ease. (e.g. resizing MATLAB command window and editor or m-file, uses undo button, copy and paste techniques, saves file). Goal: to see the command window and m-file next to each other; to undo cursor movements; to save time; to save the files. **Actions carried out in Problem-solving Session 2** 1. Carries out certain procedures with ease. (e.g. resizing MATLAB command window and editor or m-file, uses undo button, copy and paste techniques, saves file). Goal: as above. 2. Puts effort into the use of a decimal for Task 1. Goal: to produce the correct solution.	**Actions carried out in Problem-solving Session 1** Makes use of MATLAB commands such as xlabel, display, plot, and legend. Goal: to produce a solution/ graph. **Actions carried out in Problem-solving Session 1** 1. Makes use of MATLAB commands such as display, plot, and legend. Goal: to produce a solution/ graph. 2. Carries out certain procedures with ease (e.g. puts cursor over red squiggle which appears below any errors). Goal: to read the error message in the corresponding pop-up.
	Theme: actions related to overcoming difficulties associated with mathematics whilst using CAS to solve DEs **Actions carried out in Problem-solving Session 1** Endeavours to substitute. Goal: to find a solution to the DE. **Actions carried out in Problem-solving Session 2** Deliberates with functions and substitution. Goal: aims to solve the DE.

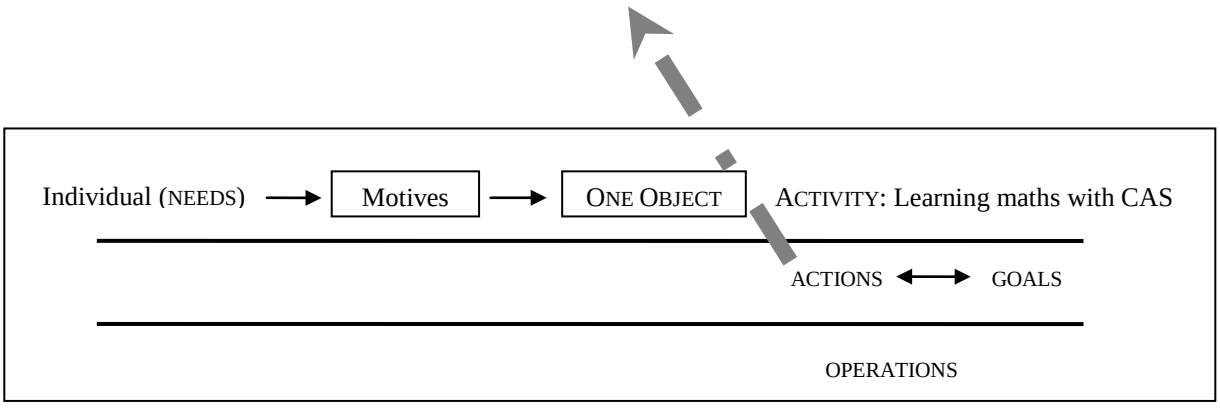


Figure 5.9: Themes and actions from the analysis of Abraham's learning mathematics with CAS activity (continued)

Theme: actions related to processes followed when solving a DE

Actions reported in learning mathematics with MATLAB
1. Strives to get right the syntax. Goal: to obtain a solution/graph.
2. Makes sense of error messages. Goal: to resolve errors so as produce a solution/ graph.
3. Interprets graphs – Euler, Runge-Kutta II, Runge-Kutta IV. Goal: to determine if he has obtained the correct solution.

Actions carried out in Problem-solving Session 1
1. Strives to get right the syntax. Goal: to obtain a solution/graph.
2. Pays attention to the sequence of steps in MATLAB even when the order does not matter. Goal: to reduce obtaining MATLAB errors so as to produce a solution/graph.
3. Interprets graphs of Euler and analytical solution. Goal: to determine if his solution to the DE is correct or not.
4. Uses methods of checking the *x* values in the output. Goal: to determine if he has produced correct solution.
5. Types in the 'textbook' example onto the MATLAB editor. Goal: to solve the task given in the Problem-solving Session.
6. Rectifies minor errors after perusal of some of the error messages. Goal: to produce a solution/graph

Actions carried out in Problem-solving Session 2
1. Writes out some code before typing onto computer. Goal: possibly to get right the syntax.
2. Strives to get right the syntax. Goal: to produce a solution/graph.
3. Uses methods of checking the *t* values in the output. Goal: to determine if he has produced correct solution.
4. Pays attention to the sequence of steps in MATLAB. Goal: to reduce obtaining MATLAB errors so as to produce a solution/graph.
5. Rectifies minor errors after perusal of some of the error messages or after re-checking the code. Goal: to produce a solution/graph.
6. Interprets graphs of: Runge-Kutta II and Runge- Kutta IV. Goal: to determine if his solution to the DE is correct or not.
7. Types statements from those given in the instruction sheet. Goal: to copy my solution.

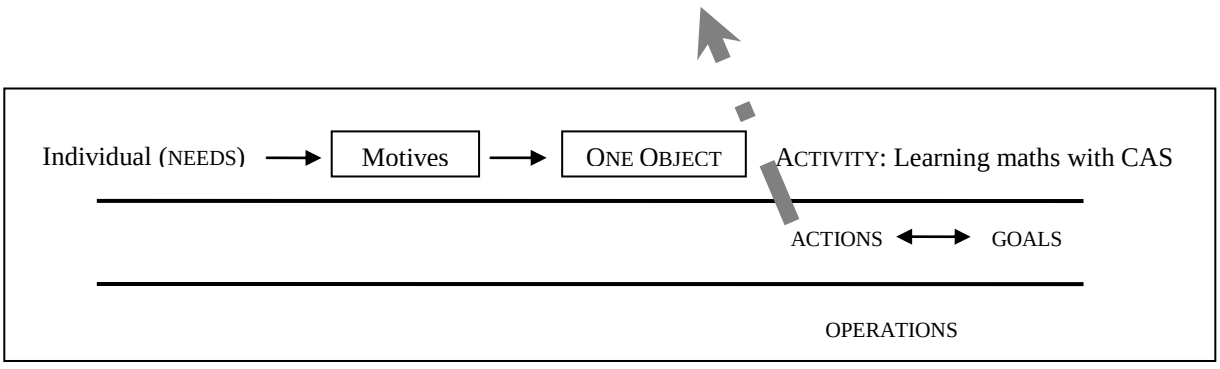


Figure 5.9: Themes and actions from the analysis of Abraham's learning mathematics with CAS activity (continued)

Theme: actions relating to optimising the environment
Actions reported in learning mathematics with MATLAB
1. Studies independently. Goal: to produce solutions on his own first.
2. Accesses MATLAB software. Goal: to practise using MATLAB (in maths).
3. Uses resources. Goal: to gain knowledge from the 'textbook' and others.
4. Studies in a group – solves problems with classmates. Goal: to fulfil social motives.

Actions carried out in Problem-solving Session 1
1. Resources text or JP when in difficulty. Goal: To gain knowledge from the 'textbook' or others.
2. Advises his partner Zonke. Goal: to assist others.

Actions carried out in Problem-solving Session 2
1. Resources text or JP when in difficulty. Goal: to gain knowledge from the 'textbook' or others.
2. Advises Zonke. Goal: to assist others.

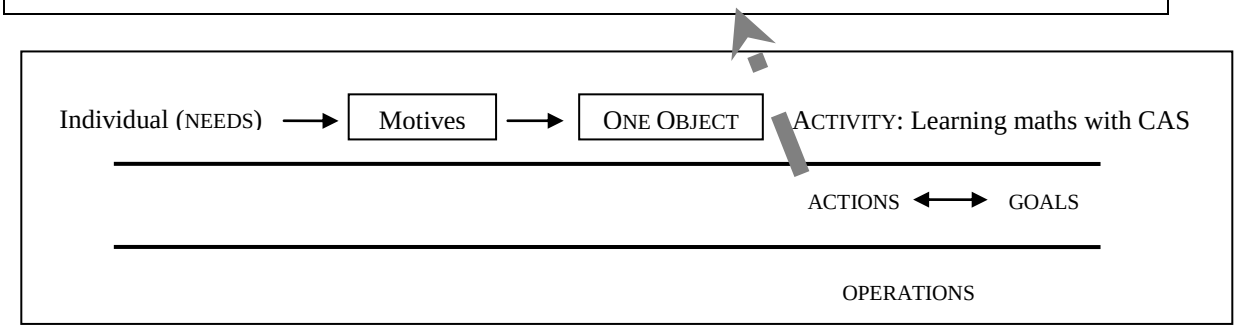


Figure 5.9: Themes and actions from the analysis of Abraham's learning mathematics with CAS activity (continued)

5.5 SUMMARY

I used the theoretical constructs of Leontiev's activity theory model (namely, needs, motives, object of activity, actions and operations) to analyse and structure the empirical data. I gave exemplars of empirical data from two participants (Thembiso and Tumi), and for the third participant, Abraham, I gave a tabular summary. My elucidations were largely related to the activity of using CAS in mathematical learning and where appropriate I drew on details from the students' reflections on the activity of learning mathematics. Particularly I analysed students' needs and motives pertaining to their reflections on the activity of learning mathematics. Chapter 6 resumes the discussion and amplification of findings interconnected to all three participants' results, paying attention to explicit comparison and contrast of results. Most importantly, I expand, clarify and deepen the link between empirical data referred to in Chapter 5 and the theory developed in Chapters 2 and 3.

Chapter 6

DISCUSSION AND FINDINGS

6.1 INTRODUCTION

In this chapter I expand, clarify and deepen the link between empirical data referred to in Chapter 5 and the theory developed in Chapters 2 and 3. I discuss how the analytic accounts described in Chapter 5 answer my research questions. I deepen this explication by comparing the findings related to the three participants' data.

The research questions are restated for ease of reference[60]:

1. What are the motives of students towards using CAS as a tool in mathematical learning?

2. How do students use CAS in mathematical learning (i.e. what are their actions and operations from the activity theory perspective?)

3. What relationships are evident between the first two questions and how might these be explained?

As described in Chapter 3, Leontiev's (1978, 1981) activity theory framework clarifies three hierarchical levels. On the activity level there are compelling links between the individual's object of activity, motives and needs. The second level provides for awareness of the individual's contemplative nature, in other words his/her actions, which are directed towards goals. These goals, which are informed by the individual's motives, are constructed as he or she engages with real activities. Operations, which make up the third level, are performed without

[60] In chapter 4.3.2 I pointed out that although I use the phrase 'mathematical learning' I am (to a large extent) examining how students use CAS in solving DEs because I have observed their actions within this specific setting. However, in the interviews the students gave answers not only on how they use CAS in solving DEs but also on how they use MATLAB generally in mathematical learning.

conscious awareness. In this thesis I have formulated my first two research questions around the aforementioned Leontiev's three levels.

I use and discuss the main elements of activity theory as possible explanations for my empirical findings. These elements include but are not limited to socio-cultural history, needs, multiple personal motives of individuals, the object of activity, the hierarchical structure of activity, consciousness, the mediating role of tools and others in learning, as well as individual development and the context of learning.

Using these theoretical constructs I provide answers for the three research questions. Research Question 3 is answered in 6.5: I connect and show the link between motives and actions. In order to do this I first discuss motives in 6.2 (which answers Research Question 1) then elaborate on actions in 6.3, followed by operations in 6.4 (these answer Research Question 2).

6.2 A COMPARISON OF STUDENTS' OBJECT OF ACTIVITY AND MOTIVES

I used Lompscher (1999) to group motives into three larger categories: self-related, cognitive and social (interrelations with others), as elaborated upon in Chapter 3 (refer to Figure 3.7). Leontiev's activity theory framework provides a powerful system with which to view students' motives: he proposes that in response to various needs which are defined within a social and cultural context, students have multiple, subjective motives and an object related to the activity concerned. These inform and guide the students' in deciding how and where they should place their effort and spend their time with respect to the task concerned. Moreover, students' goals that correspond to actions carried out in the task are inter-related to the motives that they espouse. Below I evaluate and discuss students' motives, by comparing and contrasting them across the three categories of motives developed.

6.2.1 SELF-RELATED MOTIVES

There follows a discussion on performance approach goal-orientations, extrinsic utility value and task value components.

PERFORMANCE APPROACH GOAL-ORIENTATIONS

All the participants were striving towards the attainment of good grades; hence they espoused performance approach goal-orientations. Tumi reported that she was planning to study consistently and expected to obtain a distinction, in the hope that this would help her secure a sponsorship. I draw attention to the power of societal context and societal needs and their influence on individual motive formulation, which in Tumi's case are exemplified by society offering rewards in the form of scholarships linked to good grades. Students therefore formulate a few motives that are related to what society deems important. Human motives cannot be explained by the biological foundation of an individual but "they emerge in the appropriation, use, and development of objects and artefacts in collective human activities" (Mietttinen, 2005, p.53), as illustrated by Tumi's motive.

An important aspect of my activity theory framework is that personal motives arise out of a need to subscribe to the values espoused by society members. Thembiso stated that he was aiming to obtain an 'A' in Mathematics 3, believing this would augment his academic record, and studying Mathematics 3 would fulfil his need to be recognised as clever by members of society.

Abraham was of the opinion that grades were unimportant to him but that the knowledge gained through the studies was significant. He however acknowledged that the presence of assessments did compel one to pursue good grades and to maximise one's efforts in one's studies. Consequently, the pursuit of good grades was definitely an important motive for Abraham, and this motive I classify as a performance approach goal-orientation. He had a strong need for competency in using MATLAB, as will be explained in 6.2.2, even though he espoused a performance approach goal-orientation. In his hierarchical structure of motives I assume that of wishing to get good grades was less intense than the cognitive one. Motives could change their position and significance, and some even lose their power (Lompscher, 1999).

I note that the impetus to perform is different in all three case study students, yet they displayed the motive of seeking to obtain good grades in this activity of learning mathematics using MATLAB.

EXTRINSIC UTILITY VALUE

All three participants stressed the usefulness of MATLAB to their intended profession, to be expected since their study of Mathematics 3 was part of a vocational qualification in

engineering. Thembiso and Tumi perceived MATLAB as potentially useful, whereas Abraham found utility value in the present, that is, applying it immediately to electrical engineering scenarios[61]. For example, he claimed to have used MATLAB to draw graphs in other engineering courses whilst he was studying the then MATLAB component in mathematical learning. These actions may have been enabled by his having more ready access to the software and computers than did Thembiso or Tumi. Abraham also felt that he had more knowledge of electrical engineering than the younger students as this had been one of his previous careers. Perhaps this may have led to him being more investigative in his approaches to learning engineering content. Using MATLAB to explore mathematical solutions or producing graphs in other subjects involves investigating and transferring the knowledge gained from one domain to another. I deduce that Abraham achieved the personal appropriation of the CAS tool for his own use. In a situation where much can be learned about a new tool through its use in various circumstances, those students who are able to do this are at an advantage.

There are possibly flaws in the context of solving DEs using MATLAB that could affect the way students construct personal motives and goals. Such a situation is illustrated by the case study of Thembiso, a student of Mechanical Engineering, who could not find immediate application of MATLAB to his field of study but who was motivated by its application to other areas (for example, biology) and its potential application in his future career. The question arises as to whether such a student could form personal motives and goals leading to constructive engagement with CAS in mathematical learning in a context in which he does not perceive immediate use to his field of study (Coupland, 2004). Considering examples involving DEs in mechanical engineering could be one way to amplify such students' engagement and interest in the course. It is evident that Thembiso was motivated by a perceived understanding that MATLAB could be used considerably in industry and he felt that he was going to need it in his future workplace or in the study of a B.Tech qualification in the future.

My activity theory framework supports the notion of the influence of cultural dynamics. The usefulness of mathematics to a future qualification or to industry could be viewed as a factor that is culturally framed. Qualified engineers[62] have contributed to production of a syllabus in

[61] Abraham reported that he only solved MATLAB problems from the 'textbook' pertaining to electrical engineering situations

[62] The Engineering Council of South Africa (ECSA) audits and accredits the engineering diplomas. The council comprises senior engineers (who have a close link with engineering companies) as well as engineering academics. They also contribute to the formation of syllabus and the nature of practical training.

which the sections on mathematics are purposively chosen, which relate to the engineering field, and/or which are stepping-stones to the pursuit of further studies in the field. The qualified engineers' recommendations thus affect curriculum planning.

There is a cyclical effect in the production of knowledge, with practising engineers having found use for CAS in their professional work and this knowledge being reproduced in institutions of higher learning. Once students reproduce it they in turn qualify and join industry as practising engineers, and it in turn is produced and reproduced in the workplace. This consumption of knowledge by society influences and changes society as a whole, and consequently activity orients the student in the world in which s/he lives and that s/he changes in a mutually constitutive way. The idea of producing and reproducing knowledge in institutions of learning and its consumption in society is a common theme in activity theory (based on Marxist theories) (refer to Chapter 3.7). The aforementioned production and reproduction of knowledge illustrates that activities are oriented toward collective (societal) motives, for example, learning to programme computer software as a guarantee of the trainee engineer's future survival, which have arisen in the course of cultural-historical development (Roth, 2007a).

According to Leontiev (1978, p.50), activity is defined as a "unit of life mediated by psychic reflection, the real function of which is that it orients the subject in the objective world". The extrinsic utility value motives espoused by participants exemplify Leontiev's conviction about the psychic reflection of reality because participants, while actively engaging in the learning activity (an objective reality), also regularly reflect on and think about its utility value in the future.

In vocational studies, mathematics is learned or taught primarily with the purpose of application. Most engineering students do not aim to become mathematicians but rather they learn mathematics because of its significant utility value. The motives espoused by the case study students evidently epitomise the specific group of extrinsic utility values.

TASK VALUE COMPONENTS

Cultural tools such as CAS were developed for use by professional engineers, so when engineering students learn how to use them they have to find out what is useful and valuable, especially when exposed to CAS for the first time, as in the case of the participants. All three participants found value in the use of MATLAB in mathematical learning. Two task value components common to the three students relate to MATLAB being easy to use and a

timesaving device. Tumi was the only participant to use pen and paper, and when she did she worked out a few numerical solutions to the DE, and her comment was that MATLAB made it easier than 'hand methods' to find solutions. Abraham also stated that using MATLAB was quicker than using analytical methods to solve DEs.

Tumi and Abraham valued the neat graphs that MATLAB produced, with the latter speaking passionately about their clarity and precision, which may well have inspired him to use MATLAB to draw them in other engineering courses. On the other hand, Thembiso was motivated to use MATLAB because of the accuracy and speed of the software, which naturally is only as accurate as the programming code input of the student[63]. Abraham, with his prior experience in engineering and mathematics, was well poised to talk about the harmonious integration of MATLAB with the theory component of mathematics.

These students not only saw the potential for using MATLAB (in maths), as deliberated upon under 'extrinsic utility value' (above), but they also realised other potentialities and values of CAS whilst involving themselves in the CAS activity. This indicates that students form their own consequential subjective motives whilst they endeavour to find meaning in the use of the new tool CAS. An important consideration is that this group of task value motives succinctly makes apparent how motives are established in the participants' course and process of activity, as alluded to in Chapter 3.5.7.

6.2.2 COGNITIVE MOTIVES

For ease of reading I remind the reader that Leontiev's (1978) description of needs is one that originates within a social and historical context, where an individual has needs that demand satisfaction. The different needs that I consider in this thesis are social belonging (perhaps including a need for recognition or approval), autonomy, competency and the need to develop one's career. Some students choose to study mathematics and computers in mathematical learning because ultimately they desire to become professionals, and in this way satisfy their need to function as independent and self-sufficient members of society.

Regarding the definition of motives, it would be recalled that I follow Leontiev's (1978) explanation that a motive takes on an arousing and directing function of the activity. Considering the three broad categories of social (interrelations with others), self-related and

[63] Note that in 6.3 I discuss the difficulties experienced by the students regarding this.

cognitive motives, together with the following specific motives of social approval goal-orientation, social responsibility goal-orientation, social interaction goal-orientation, social compliance goal-orientation, social affiliation goal-orientation, task value components comprising the importance of task, extrinsic utility value (underscoring the usefulness to ones careers/ other courses), performance approach goal-orientations, task value components (mainly consisting of intrinsic interest / intrinsic value) and mastery approach goal-orientations.

I argue that Tumi and Abraham found intrinsic value in studying MATLAB, espousing as they did their mastery approach goal-orientations. In spite of the lack of access to computers, Tumi tried to gain proficiency in MATLAB, which she persisted in learning to use by writing out MATLAB codes to solve DEs as opposed to actually using MATLAB to solve DEs. She seemed to believe that writing code, even without a computer, would improve her proficiency. Tumi valued the development of new skills and believed that her engagement with MATLAB had played a vital role in her improved cognitive abilities. Abraham came across as being appreciative of the way he developed and applied new skills to different areas. He too was striving to gain new knowledge and cherished the learning of MATLAB in mathematics. I argue that Abraham was striving to fulfil a need for competency in mathematics using MATLAB by creating a forum for discussions and arguments (this is elaborated in 6.2.3 under social motives). In contrast, Thembiso expressed little interest in studying MATLAB as a tool in its own right. I assert that his motives for studying mathematics (with CAS) related more to performance approach goal-orientations, extrinsic utility value, task value components and social (interrelations with others) motives.

Miettinen (2005)[64] notes that many theories on the starting point of the personality connect needs to the notion of competence or capability. This activity theorist proposes that "when a need becomes attached to an object [*such as mathematics*], it also assumes the form of the skill to manipulate and use these objects [*mathematics*]" (Miettinen, 2005, p.56, my insertions and emphases). Applied to the context of this study, both Abraham and Tumi reported that in their earlier experiences of learning mathematics they had been skilful problem-solvers, and very interested in that activity. I assume that their intrinsic interest and skill in problem-solving became attached to the subject of mathematics. This earlier association with mathematics may have given rise to current cognitive motives for seeking competency in mathematics so that they would wish to study any mathematics course competently and in meaningful ways, thereby

[64] N.B. Miettinen based this statement on the work of a few researchers.

sustaining the original intrinsic interest associated with it. Hence, it seems that the two participants' historical components influenced their formulation of current cognitive motives with respect to learning mathematics with CAS. As Roth (2007a, p.43) argues, "being good at something feeds back, makes it interesting and enjoyable to engage in, thereby producing and reproducing emotion, enjoyment, and motivation".

In summary, Tumi and Abraham espoused motives that I describe as cognitive in nature, illustrating Kaptelinin's (1996, p.62) notion that "…Activity theory states that tools not only change the task but often empower the individual, even if the external tool is no longer used".

6.2.3 SOCIAL MOTIVES

I argue that Thembiso and Abraham espoused social (interrelations with others) motives in learning mathematics with CAS although their underlying rationale differed widely. Thembiso enjoyed communication with peers, coming across as being pleased to demonstrate his ability and in the process showing others that he was clever. He reported that whilst he was studying for a MATLAB (in maths) assessment he provided assistance to a friend who had no knowledge of the MATLAB component[65]. Although Thembiso articulated that he preferred not to study mathematics with others in a group, I noticed (refer to 5.2.3.3) a great disparity when it came to his study of mathematics with CAS. Perhaps the CAS reflected the potential for being a catalyst in bringing about discussions and explanation.

On the other hand, Abraham enjoyed sharing knowledge[66], engaging other students in discussions and communicating with peers. His concern for other students took the form of him being the leader of a group of students, and he reported that he motivated his group members to study and provided a forum for learning by discussion. He stated that he encouraged the group members to question and probe and claimed that in this way his progress was also assessed. If he could not answer their questions then he had to revisit the theory in mathematics. I argue that the social motives (interrelations with others) that Abraham formulated were steps towards answering his critical need, namely for competency in learning MATLAB (in maths). This need

[65] It will be recalled that this friend was studying towards a higher qualification, namely, B.Tech in Engineering.

[66] This I infer from his articulations in the interviews and my observations of the problem-solving sessions.

evolves from his human practice and interaction with activity, culture and society, as opposed to one that arises from biological basis (discussed in Chapter 3.5.6) – one of the main philosophies of Leontiev's activity theory framework.

Abraham's prior knowledge[67] of mathematics and engineering facilitated his learning process, as he claimed to recognise the use and relevance of mathematics in engineering scenarios. He reported that he used his knowledge and experience to motivate other students so that they might realise the importance of what they were learning. For him, learning was not just about getting better grades but also about finding use for what he had learned in his intended career as principal of a technical high school.

According to Abraham, the group members compared MATLAB solutions and if theirs differed then this initiated dialogue. If someone in the group was unable to display the solution to a DE then the group members assisted that individual. Abraham claimed mainly to solve MATLAB (in maths) problems on his own; he subsequently assisted group members or sought their assistance when confronted with any difficulties. For Abraham, I argue that the social motives that he held regarding Mathematics 3 and MATLAB (in maths) were social responsibility goal-orientations, that is, engaging in mutual assistance and communicating amongst peers. In addition, Abraham's discussion of MATLAB problems with classmates was indicative of deeper approaches to learning (refer to 6.3.4 for further discussion).

Abraham was studying for the diploma to pursue, in due course, an M.Tech. degree in engineering, which would enable him to become the principal of a technical school in North Africa. In Abraham's case there were particular social motives for learning which I describe as a means to help and support others. In this regard, it is also instructive to note that he was a priest and may still have been motivated by pastoral concerns or historically rooted motives to assist others in their learning endeavours. Another perspective on Abraham's social motives is that of cognitive engagement, which I discuss in 6.3.4.

Activities are socially constructed and as a consequence of the above discussion on students' subjective motives I pose the question as to whether they converge with any of the collective course outcomes. Amongst other objectives, the course designers convey some main outcomes, two of which are that the student should "work effectively as a member of a team or group" and the course will provide the student with a basic introductory knowledge of DEs, "which can be

[67] Termed 'historical aspects' in activity theory expressions.

applied in practical techniques in his/her chosen field of study or workplace" (Kirchener, 2008, p.3). I argue that there is some alignment between individual participant's motives and goals and those of the institutional curriculum planners (the manifestation of such alignment in various motives was discussed above). For instance, Abraham and Thembiso held motives that I describe as social (interrelations with others) in nature, and these could well be in harmony with course objectives and outcomes.

In summary, participation in activity articulates the individual's active relationship with the world. The individual integrates culturally structured ways of dealing with the demands of the educational learning environment by being dynamic in his or her association with other human beings and with the physical setting. Activity mediates between the individual and the educational environment and changes in activity guide changes in the way the individual reflects on the activity and corresponding actions. As discussed, Thembiso and Abraham held motives regarding the activity of learning mathematics with CAS that I interpret as social (interrelations with others), though their underlying needs were very different. In this regard, the construct of activity embraces the interaction and relations between subjects acting upon and producing a number of different motives, with the intention of satisfying various human needs (Lompscher, 1999).

6.2.4 OBJECT OF ACTIVITY

The image of the object of activity exists twice: first in its objective form in the world and second as a reflection in the individual, which is in its subjective form (Leontiev, 1981). The objects of activity for each of the three participants demonstrate this notion of image. For Thembiso, I argue that his object of activity is to get a good symbol in MATLAB assessments by studying at the last minute, redoing class examples and similar exercises, and memorising MATLAB statements. Thembiso's aim was to show others that he was 'smart' and by choosing to participate in the activity of learning mathematics and its accompanying association with CAS, he was striving to achieve that aim. His involvement in the CAS activity was the objective reality, while the influence the activity had on his enjoyment of the portrayal of intelligence could represent one aspect of the subjective form.

For Tumi, I assert that her object was to develop competence, skill and increased cognitive ability in learning mathematics with CAS soon after the work was covered in class by rewriting MATLAB code and solving exercises. Here too the object of activity was penetrating the

subject, altering her consciousness and transforming her (Kuuti, 1996). I contend that Abraham's object of activity was to develop and learn new skills so as to apply them to different areas. His aim was also to gain new knowledge and to pass it on to others (as an intended principal of a technical college). He claimed that he involved peers in his studies so that they might act as a stimulant to his own learning by challenging him. However, at the same time he could represent a change in their lives by being a mentor and expressing concern for their learning. The object of activity was transforming Abraham as an individual and demonstrates how "the subjective image of the external world is the product of the activity of the subject in that world" (Leontiev, 1978, p.39).

6.2.5 SUMMARY AND FINDINGS OF STUDENTS' MOTIVES

In this section I have explored a range of motives made manifest by the participants. This study is about the multiple motives of each individual participant as they carry out their activity of learning mathematics with CAS. It is apposite to remind the reader that motives are set within the individuals' unique personal, social, cultural and historical contexts. Inevitably, their learning was shaped by many motives and goals but overall it remained the case that there was an examination at the end of the course and I posit the view that participants would have been aware of this (evidenced by the finding that they all held performance approach goal-orientations). Hence, any motives expressed by the participants must be understood in the context of the presence of an examination obligation.

One major finding of this study is that students hold multiple motives regarding their learning of CAS (in maths). In activity theory terms the three students made apparent their personal energising reasons that underlie their participation in the activity concerned. The answers to Research Question 1 on students' motives about using CAS in mathematics learning are summarised below.

Within the category of **self-related motives** the students espoused motives that I classify as performance approach goal-orientations (directed towards obtaining good grades), extrinsic utility value and task value components. They all advocated extrinsic utility value motives because they found purpose in wanting to study MATLAB (in maths) for reasons relating to future career or applications of MATLAB to other relevant engineering contexts. All three participants found value in the task of learning to use MATLAB (in maths). Two task value components that were held by the three students relate to MATLAB being easy to use and a

timesaving device. For example, Tumi and Abraham believed that MATLAB made it easier to solve problems compared to hand methods. They also valued the neat graphs that MATLAB produced. Thembiso found that he was motivated to use MATLAB because of the accuracy and speed of the software.

Regarding the category of **cognitive** motives, Tumi and Abraham found intrinsic value in studying MATLAB. In addition, both possessed motives that I describe as cognitive in nature, though this was not the case with Thembiso, who was striving to fulfil a need to be recognised as part of the cluster of 'smart' students by studying computers. Tumi believed that her participation in the MATLAB activity had improved her cognitive abilities and she appreciated the development of new skills. Abraham too was appreciative of participating in the MATLAB activity, and applied the new skills learned from using MATLAB to other areas of his engineering courses. He too was striving to gain knowledge in the use of the new tool MATLAB (in maths).

Regarding the study of mathematics (with CAS), Abraham and Thembiso espoused motives that I classify as **social** (interrelations with others)**,** yet their underlying rationale was dissimilar. For Abraham I argue that his motives were informed by social responsibility goal-orientations, for instance, assisting peers and encouraging communication amongst group members as well as learning as a means to help and support others. Thembiso enjoys communication with peers, is content to demonstrate his ability and in the process show others that he is clever. In such a context, where a good deal can be learned about a new tool by asking others, Tumi, who did not espouse such motives, was clearly at a disadvantage.

As discussed in Chapter 3, the Russian theorists, particularly Leontiev, conjecture that higher order mental processes are formed during the course of an individual's personal development whilst s/he interacts with others. In Abraham's case I argue that his claim of working with others who also challenge him in educationally useful ways can only encourage learning at meaningful levels. For him, CAS became a tool for investigating on his own and pondering with others, all set within a socially supportive and encouraging environment.

Activity theory considers activity as a mediator between individuals and the world. Initially there is an interaction between the user and the CAS, then this focus shifts "to a larger context of interaction of human beings with their environment, that is, transcending the user interface to reality" and beyond the individual-CAS interaction (Kaptelinin, 1996, p.47). In this study I note that CAS may have been used by individuals to support meaningful motives, such as social,

self-related and cognitive motives, which usually exist beyond the situation of the individual human-computer interaction. Kaptelinin (1996, p.49) succinctly puts it as "human beings usually use computers not because they want to interact with them but because they want to reach their goals beyond the situation of the 'dialogue' with the computer".

6.3 A COMPARISON OF STUDENT'S ACTIONS

The action and operation levels of an activity unquestionably provide distinct stages through which to analyse a learning activity. Since conscious actions are associated with the act of planning and reflection, as argued above, actions have associated with them goals that are conscious and intermediate in nature. In this case, actions reveal how the activity of using MATLAB (in maths) unfolds for each participant. In a study on students' motives with respect to studying mathematics mediated by CAS, one could consider the various actions related to how students plan, commit to memory, organise, make decisions, understand, solve problems and assess their own progress (Schunk et al., 2008). From an activity theory perspective there is an added opportunity to include those actions that are in the process of becoming operations.

Leontiev (1978, p.63) writes that, "correspondingly, actions are not special 'units' that are included in the structure of activity. Human activity does not exist except in the form of action or a chain of actions". My Research Question 2, on how students use CAS (in maths), is illuminated by a wide range of actions. The individual actions carried out with associated goals formed are related to the motives and needs that the students espouse. These are all elements of the personal, social and cultural context of learning mathematics (with CAS). The crucial choices and decisions that students make are discussed in this section. Specifically, the actions of the three participants will be compared and contrasted within the grouping of eight themes.

6.3.1 THEME: ACTIONS RELATING TO OPTIMISING THE ENVIRONMENT

An important tenet of activity theory is emphasis on the relations between context (and environment) and the students' activities of learning. The participants with their motives and goals, the tool with its syntax and error messages, and the situation in which each is embedded, are brought to the fore in the ensuing discussion. There is no separation of the individual from

the context in which the learning takes place. Indeed "actions are always situated into a context, and they are impossible to understand without that context…" (Kuuti, 1996, p.26). Kaptelinin (1996, p.49) points out that one "should include the meaningful context of the user's goals, environment, available tools, and interactions with other people". In this study I emphasise various aspects of this keystone 'context' of activity theory by considering the theme of 'optimising the environment'.

One of the characteristic features of an undergraduate's life is that learning depends on available material resources, such as the Internet, textbook, lecture notes, help-files of MATLAB, computer, software and, most importantly, social interactions with others (lecturer, other students, and tutors). These are only a few of the tools that mediate thought and behaviour whilst the participant interacts with the learning activity (Engeström, 1987; Nardi, 1996c). The student's own thoughts and experiences of his or her immediate environment are a web of complex relations, and this adds to the possibilities he or she can or cannot exploit.

To begin with, one of the cornerstones of learning to use CAS (in maths) relates to accessing fundamental tools. Trying to gain access to MATLAB or computers was an important action for each participant. As mentioned in Chapter 5.3.4, Tumi reported that she accessed the labs (to practice on her own) on only a few occasions, such as a day before the MATLAB tests. To compensate for not having consistent access to the CAS tool Tumi regularly resorted to writing out the solution to MATLAB exercises. This was her method of learning mathematics with CAS enforced by the circumstances in which she found herself within this institution of learning. Tumi's actions in this rare context indicate her persistence in learning in the face of adversity. It is noticeable how the greater context of living in an unsafe society had a direct impact on her actions in learning to use MATLAB (in maths).

Abraham also reported that during the first two weeks of the semester, when he did not have the software, he could not access the labs after 16h30 because the area was unsafe. His usual style of learning, namely, solving problems soon after the sections had been covered in class, was compromised by the circumstances of studying in an unprotected neighbourhood. This is similar to the issues that Tumi faced. From an activity theory perspective, I cannot neglect the role that the institution and society play in shaping participants' learning activity.

As discussed in Chapter 5.2.4., Thembiso claimed that he had his own computer with the MATLAB software installed but for most part of the semester it was not functional and he did not go to the labs to practice. The computer was repaired just prior to the MATLAB

examination. For Thembiso, his context of learning in previous mathematics courses was in any case to leave the learning to just before an assessment. I contend that he adopted a similar (historical) style of learning at the last minute when it came to his study of mathematics with CAS.

It will be recalled that the MATLAB lecture time consisted of one-hour lectures and one-hour of practice time. During the latter, students were supposed to work with the MATLAB software, though not all of them did. However, in this study, all three participants reported that they made good use of this one-hour practice time to work on exercises and examples. In Tumi's case this largely formed the only 'real' practice sessions that she had during the semester. Such constructive utilisation of class time is an indicator of the participants' keenness and motivation to engage with the learning material. On the one hand, Thembiso's and Tumi's learning was constrained by a lack of access to CAS outside the classroom, but on the other hand the context of the scheduled practice time during lectures enabled them to gain familiarity with the CAS tool as they could engage in anything using MATLAB. By appropriating certain features, such as proper utilisation of lab practice time, I infer that the students open up endless possibilities to transform their own learning process.

Coupland (2004, p.70) posits that "activity theory provides an explanation for the development of consciousness in the individual, by emphasising the role of social interactions and the mediation of signs and sign systems". As discussed in 6.2.3, the social relations formed by participants as they involve themselves in the activity of using CAS (in maths) play a major role in their learning. There are many educational advantages, with one of the spin-offs being the development and improvement of communication skills. This is a vital aspect of learning, especially when English is either a second or third language for the participants and they are primarily being prepared for a world of work in which English is the dominant language. It is significant that Abraham came from a Francophone country in Africa and his communication with local students (of his group) was in English. As outlined in Chapter 3.2, the cultural-historical school founded by Vygotsky emphasises the notion of mediation by language, and two of the participants stated that their communication and forming of interactions with others made a positive contribution to their own learning of mathematics with CAS.

Nardi (1996c) puts it that the role that cultural values play in shaping activity should form part of the context. Furthermore, Triandis (2002) proposes that studying in a group context helps students appreciate and respect the cultures of other group members. In a multi-cultural society, these kinds of interactions during learning can only be encouraged. Education that focuses on

improving group and cooperative relationships may result in creating the possibilities of "productive, constructive, and smooth relationships, both inside and outside the classroom" (ibid., p.12). This overall development of the individual is an important aspect of learning viewed from an activity theory perspective, and echoes the unison of individual and context that is a common theme throughout this thesis.

The construct of mediation is a central theme in activity theory (refer to the discussion in Chapter 3.8). The 'textbook'[68], class notes and solved examples amongst other material tools mediate the learning for all participants. All three students reported in the interviews that they accessed classroom-solved examples, but their main resource was the prescribed 'textbook' in their learning of MATLAB (in maths). Thembiso and Tumi claimed that they attempted to use MATLAB's help-file but found it difficult and complicated, while Abraham did not try using the help-file. The goal formation of wanting to produce solutions to DEs is mediated by the participants' various use of means, such as 'textbook', help-file and solved examples, and because actions realise goals all these means have mediated their goal (Roth, 2007b).

It is vital to note that within this theme of actions relating to optimising the environment I concentrated largely on the institutional context. Apart from each student's personal histories and current experiences, they share a myriad of common experiences brought about by being part of the same diploma study in a specific institutional context. The institutional context, with its own conventions and specific constraints such as curriculum, tests, examinations textbook, lecture notes, computer software, computer labs, MATLAB lectures incorporating practice sessions and, social interactions with others (lecturer, other students, and tutors), influences students' motives and actions and is underscored throughout this chapter. I remind the reader that this institutional context is not specific only to this theme.

6.3.2 THEME: ACTIONS RELATING TO PERSONAL POWER

As individuals, people are in a position to exercise choice and to know the consequences of their decisions. In this theme I examine how the participants achieved control of their study of MATLAB whilst learning mathematics, in particular the question: What choices did they make concerning what and when they studied MATLAB in learning mathematics? Effort is another

[68] Note that the 'textbook' and its related issues were discussed in Chapter 4.

index that is pertinent to learning. Motivated students will be expected to make an effort to succeed in learning. As indicated in Chapter 2.10.1, the indexes of motivation are choice of tasks, effort and persistence (Schunk et al., 2008). I also contrast exemplars of choice of tasks and study-time for the three participants.

Abraham and Tumi reported that they engaged in solving MATLAB problems throughout the semester, while Thembiso crammed or studied at the last minute, just before a major assessment (and close to the MATLAB exam). Both Tumi and Abraham claimed that they chose to solve MATLAB problems soon after the content had been covered in lectures[69]. They worked in a consistent manner when they learned mathematics with CAS. However, Thembiso claimed that if there were no assessments he would not spend time on his study of MATLAB (in maths) as his six other academic subjects were demanding of his time. I assert that his learning was driven mainly by the presence of assessments and grading. It is notable that all the participants gave accounts of how they went about studying previous mathematics courses. They appeared to study mathematics with CAS once lectures had finished, using approaches and learning styles similar to those adopted in their previous mathematics learning (Periasamy, 2008b). Indeed, the personal histories of engagement with similar courses will encourage individuals to take up certain positions, and the participants appear to have adopted comparable learning styles.

A vital aspect of my motivational framework is that the type of exercise that students select to study according to their free choice is an indicator of motivation. When students have a choice, what they prefer to do indicates where their motivation or enthusiasm lies (Schunk et al., 2008). Abraham and Tumi reported that they selected challenging exercises to work out, while Thembiso attempted to solve difficult problems. However, Thembiso's endeavours were unsuccessful because he left his studying to the last minute and did not have time to work on challenging problems or to ask others for help in solving them. Remembering that motives give rise to goals, one explanation for Abraham's and Tumi's actions of choosing to solve challenging problems could be that they wished to satisfy their cognitive motives in learning mathematics (with CAS). A second explanation could be that they were preparing for MATLAB assessments and espoused performance approach goal-orientations that had to be fulfilled.

How did the students make use of the examples presented by the lecturer during the formal MATLAB class? Two of the participants, Thembiso and Tumi, reflected that they redid

[69] As previously mentioned all three participants claimed that they used the lab time effectively but here the focus of my discussion is on what happens subsequent to the lab sessions.

classroom-solved examples by writing out the entire MATLAB code. Thembiso's actions in this regard give the impression that he was putting in a great deal of effort, but his goals reflect that he just wished to get the same solutions as had the lecturer when they were done in class. In contrast, Tumi had two goals for this action, the first being to determine if she had understood the examples solved by the lecturer, and the second to apply what she had learned to the solution of other problems (exercises) from the 'textbook'. Although Abraham did not comment on redoing already solved examples, he typed out an example from the 'textbook' onto the MATLAB editor during Problem-solving Session 1. Correspondingly, his goal was to solve the allocated DE problem, which nevertheless was different from the 'textbook' example. I assume that at this early stage of learning to solve DEs, Abraham found it convenient to copy the 'textbook' code.

Although the students were involved in actions that were comparable (in the above elucidation), it is instructive to note that their reasons for doing so vary. The goals are constructed by the individuals and are informed by their motives as they go about unravelling the requirements of the activity of learning mathematics with CAS. Each participant has unique requirements, for instance, Tumi's goals were formed to satisfy her cognitive motives when learning mathematics with CAS and were distinctively different from those reported by Thembiso.

Thembiso reflected that there was a possibility that he would choose to use the easier Euler methods over the more challenging Runge-Kutta methods to solve problems in the workplace environment, even though he was aware of greater accuracy using the latter methods. On the other hand, Tumi reported that she would choose Runge-Kutta methods over Euler because the former are accurate. Abraham was not asked the question relating to method choice in the interview. It is important to note the choices made by participants who hold chiefly performance approach goal-orientations (Thembiso) and mastery approach goal-orientations (Tumi). Students who are performance goal-oriented are motivated by extrinsic forces, such as grades and norm-referenced standards, and usually choose goals that are very easy or very difficult (Meece, 1994).

6.3.3 THEME: ACTIONS THAT COULD LEAD TO LEARNING MATHEMATICS (WITH CAS) OPERATIONALLY

As explained in Chapter 3.5.3, activity theory in the context of learning posits that it is vital for several actions to become operations, and I contend that all participants were engaged in some form of learning mathematics with CAS so that it could be done at an operational level. This is

an important aspect of the learning process and relates to the participants' skilful use of MATLAB (in maths).

What kinds of actions lead to automated ones? I assert that when students engage in a practice of redoing already solved class examples, they do so because they want to learn MATLAB steps, procedures or methods so that a few of these can become operations. For example, by writing the code of already solved examples, one becomes familiar with the syntax of MATLAB and these can turn into operations. As discussed in 6.3.2, Tumi and Thembiso reflected that they redid class examples by first writing them out. Although their reported goals were different for this action, this action could also be interpreted as one that leads to learning mathematics (with CAS) in an operational manner.

Usually when students practise many questions they do so to become familiar with the methods so that assessments can be made at an operational level (Coupland, 2004). All three participants described some form of practice when it came to their learning. Abraham wrote that if "one practises very well and has a good knowledge of application then one can achieve". I maintain that what starts out as conscious actions for the participants may result in some of the actions or some sub-actions becoming operations over time, especially if they practise and become familiar with the learning material and MATLAB software. This I argue from the activity theoretical perspective of how actions are realised in and through operations involving learning.

Memorising statements of MATLAB is also an action that results in learning at the level of operations. The issue here is whether one commits to memory after having gained understanding or whether one memorises without any understanding. Thembiso claimed that he did memorise MATLAB statements when solving DEs. For example, the statement associated with dividing the interval into sub-intervals appears in every solution of DEs using MATLAB. He had memorised this statement without being able to explain how it was arrived at. Thembiso's goal for this action was to know the solutions or methods just prior to a test. I claim that this goal is formed in response to the underlying motive of wishing to get a good grade - a performance approach goal-orientation.

In summary, the development of my model of the activity levels in Chapter 3.5 makes apt provision for explaining learning activities. Conscious actions of repetitively practising the learning activities, as well as memorising features of the learning task, ought to find expression as non-conscious operations over time. I propose that this is the way that learning has happened for these students in mathematics, viz., from high school days they probably learned

mathematics by copying worked out examples from either the text or figures of authority, and solved similar exercises before proceeding to challenging ones. Tumi and Thembiso indicated this with respect to their then current mathematics study and Abraham copied the 'textbook' solution of a DE directly onto the MATLAB editor. This does not imply that they did not seek to make personal meaning, but generally this as how learning new sections in school mathematics occurred for them. This is also evidenced by their being mindful of 'practising' in learning activities. The aforementioned is in line with Engeström's (1987) discussion on 'dead text' (refer to Chapter 3.7).

6.3.4 THEME: ACTIONS RELATING TO COMPREHENSION OF MATHEMATICS WITH CAS

Educators are generally interested in encouraging students to seek personal meaning and comprehension in any learning activity undertaken. This search is revealed through various actions carried out by the participants as the activity of learning mathematics (using CAS) unfolds. As reviewed in Chapter 3, the individual's higher cognitive functions develop in and through participation in purposive activity. Wertsch (1998) writes that these higher order mental processes develop whilst the individual is in a social and cultural network with others, and involves him or herself in activity, whilst for Leontiev (1981, p.58), "under social conditions that allow for the full development of human beings, intellectual activity is not isolated from practical activity".

An activity is developed through its goal-directed actions, "carried out in variable concrete circumstances" and subordinated to conscious intentions (Engeström, 1987, p.28). Consequently, I discuss and review the following relevant actions associated with the theme of making personal meaning: interprets graphs; verifies MATLAB solutions; tries to solve analytically first; compares MATLAB solutions with the values that is produced by substitution (pen and paper); and discusses and debates with others or group members.

Initially, Thembiso was the only participant who paid attention to interpreting graphs, yet he was performance goal-orientated and did not engage with the material consistently (apart from attending MATLAB lectures). Each time he produced a table of solution values to the DE he immediately plotted the graphs to examine their proximity. On the other hand, Tumi and Abraham were not aware that they should be interpreting graphs in order to determine if they

had correct solutions. They subsequently became aware of the importance of graphical examination and interpretation through my mediation.

It will be recalled that Abraham typed out an example from the 'textbook'; this solution together with his own analytical solution resulted in two graphs that were far apart. He claimed that he had completed the solution to the task and I subsequently questioned him about the graphs produced. I contend that Abraham thought he had completed the task because he was able to produce graphs even if they were not the correct ones and had wide gaps between them. He stated that he did not know what to expect with graphs[70]. Meanwhile, Thembiso was very much aware of checking the proximity of graphs. However, when producing the graph to just one of the methods (for example, Euler) he indicated that he would only relax once he had produced a graph – accurate or inaccurate as it might be. In Problem-solving Session 1, Tumi thought it involved getting her MATLAB programme to compile without errors and produce any graph, and she reflected that she did not even check to see if the graphs were correct. Their statements about being at ease so long as they produced graphs (even if the graphs were wrong) is indicative of the difficulties experienced with the syntax of MATLAB, and interpreting and correcting error messages. The context of paying attention to the actual process of programming (e.g. syntax and resolving error messages) initially took away their concentration on learning mathematics using CAS. The students seem to have regarded the task as complete once they had realised their purpose of producing any graphical representation in MATLAB.

In line with Leontiev's (1978) proposal on consciousness (Chapter 3.8), the situation of learning to programme MATLAB (in maths) had all the participants fully absorbed in the actions of getting right the syntax and interpreting and resolving error messages. As pointed out, Tumi and Abraham were not even aware of examining graphical outputs during the initial stages of learning MATLAB. However, as time progressed their knowledge through experience and mediation expanded to the extent that the moments of reflection concerning the particular structure of syntax recedes from consciousness, thereby allowing the students to direct their consciousness to other matters. In other words, once the programme is compiled without errors and a solution is produced, the student no longer has to think about getting right the syntax or correcting error messages. This leaves him or her with time to concentrate on other issues, such as examining and exploring the graphs produced.

[70] Note that in Abraham's case this problem-solving session took place after just one formal lecture in Euler's methods and Abraham had only solved one exercise in class on his own.

Roth (2007b) writes that in the state of operations the tool does not stand *between* the subject and object of consciousness but rather recedes from conscious activity altogether. There are instances of deliberate actions disappearing into non-consciousness when a participant's concentration shifts to the graphical output. Indeed, the notion of consciousness provides an explanation for their evolution of learning and accounts for the significant moments of higher engagement in the activity of solving DEs with MATLAB. As discussed (Chapter 3.8), activity theory is concerned with the development of human consciousness and the above example illustrates how this is located in relation to everyday practice.

However, the use of MATLAB in this particular way is such that even if one pays attention and gives thought to checking the graphs, one cannot always know that one has the correct ones. For example, Tumi and Thembiso could associate Runge-Kutta II as being close to Runge-Kutta IV, and at times these graphs were deceptively close to each other, yet they were incorrect. In the problem-solving sessions involving Runge-Kutta II and IV methods I observed that these two students relied mainly on examining graphs and they believed that their solutions were correct when they were not. It is a necessary condition that graphs should be close but not sufficient. The goal with respect to the action of interpreting graphs was to determine if the solution to the DE was correct, and I conclude that they could not know if they have ultimately achieved this important goal.

Given the above situation, it is encouraging to note that two participants went about verifying MATLAB solutions using methods other than graphical ones. Both Tumi and Abraham adopted different ways of checking to see if they had produced correct solutions. Abraham relied on examining the values of the independent variables in the output to determine if he had produced correct solutions, and thus ensured that the differences between the values were indeed constant. However, this is not always a precise method as it does not give any indication of whether the solutions to the DE or *y* values are correct. Nevertheless, it was Abraham's way of determining if he was on the correct path and his way of making personal meaning out of the use of the new tool in the particular context of solving DEs.

It is notable that to find the numerical solutions of DEs, students could solve the DE using CAS and could also use pen and paper techniques by substituting values in the auxiliary equations. Tumi worked out a few values for the DE (using pen and paper) by substituting in the auxiliary equation of the Euler method. In her case, the goal formation of wanting to produce solutions to DEs is mediated by her reflection of using both the means (pen and paper as well as CAS), and because actions realise goals both the means have mediated her goal (Roth, 2007b).

Tumi then compared the values obtained through actual substitution with those produced by MATLAB. This gave her an added advantage of comparing and checking to see if she had produced the correct solutions using MATLAB. None of the other students in this research project compared MATLAB solutions in this manner. Verifying solutions produced by the computer takes time and effort, and is an indicator of persistence. Furthermore, this is indicative of some deep approaches to studying mathematics with computers and is linked to a mastery approach goal-orientation. As noted above, such motives are classified as cognitive and the corresponding actions of 'compares MATLAB solution with the values produced by substitution' are indicative of Tumi's attempts at personal meaning-making when using CAS.

A few of the DEs in the MATLAB course were previously solved using analytical methods during an earlier semester. Were the participants enthusiastic about solving the DEs analytically whilst producing the MATLAB solutions? Thembiso was not keen to do so, but Tumi and Abraham believed it important to produce analytical solutions even when these were not a requirement of the MATLAB task. Abraham's reasons were that he first tried to find analytical solutions and his goal was to challenge himself. He believes that analytical techniques require more cognitive effort compared to MATLAB programming, and that programming MATLAB to solve DEs does not involve integrating much previous knowledge of mathematics or theory. I assert that he attempted to find analytical solutions to satisfy his need for competency. He was oriented to a mastery approach and interested in ways and methods of integrating theory into the problem-solving process, whereas Tumi's reasons were based on her beliefs that analytical solutions provide her with a verification system for MATLAB numerical solutions (discussed in Chapter 5.3.4).

Learning with the aim of making personal meaning is also about encompassing a multi-method perspective. Students who are motivated to learn are expected to put more mental effort into their learning process. They should employ cognitive strategies that they consider will advance their learning. Such procedures include "organising and rehearsing information, monitoring understanding and relating new material to prior knowledge" (Schunk et al., 2008, p.12). I argue that both Tumi and Abraham were passionate about putting greater mental effort into their learning of mathematics with CAS. They were keen to link previous methods learned (i.e. analytical solutions) with new and current methods of solving DEs (i.e. producing numerical solutions using MATLAB). Thembiso, on the other hand, did not express enthusiasm for the use of previously learned methods.

Next I detail how interactions with others are viewed as an important aspect of comprehending mathematics (with CAS).

Abraham's professed acts of encouraging debates, arguments and discussions, and assessing his own progress, demonstrate the idea of comprehending mathematics (with CAS). I assert that he was seeking to fulfil a need in respect of becoming competent in the learning activity. His stated actions are suggestive of learning using multifaceted and meaningful approaches. As discussed (Chapter 5.2.4), Thembiso explained MATLAB to a B.Tech student but I argue that he appeared to be describing procedures, for example the division of the domain into intervals. In addition, during the problem-solving sessions, Thembiso explained to his partner even though he did not have thorough knowledge of the subject matter. Nonetheless, I contend that Thembiso's interaction with other students made him develop an awareness of himself, even though he appeared to be describing procedures. Thembiso believed that his explanations to others helped him develop an enhanced understanding of the learning material.

For each participant, their engagement in the activity of learning mathematics with CAS was motivated by their underlying needs. Such needs give rise to motives and "one motive may obviously find expression in various goals and actions" (Engeström, 1987, p.28). It will be recalled that Abraham had a need for competency in using MATLAB (in maths), while Tumi had a need for autonomy and to develop her career. These needs gave rise to various motives. For Tumi I argue that her cognitive motives were to develop competence and learn and develop new skills, while for Abraham, his cognitive motive was to gain new knowledge. These motives gave rise to goals associated with the actions related to comprehension of mathematics with CAS. In Tumi's case, one goal was to check her analytic solution against the solution obtained from numerical methods in MATLAB, while Abraham's main goal was to challenge himself. Through Tumi's and Abraham's actions and endeavours, I notice that they were seeking to satisfy personal cognitive motives, particularly mastery approach goal-orientations.

Abraham acted on his need to gain competency in using MATLAB (in maths), which together with his social (interrelations with others) motives gave rise to his actions of discussing and debating with group members. His goal was to understand and know mathematics (with CAS). Thembiso too developed his personal meaning with respect to using CAS (in maths) but not to the same extent or depth as Tumi or Abraham. Thembiso's need was primarily concerned with being recognised as 'smart', as well as impressing others with his knowledge and explanations. Through his conscious actions of explaining MATLAB to others he was striving to satisfy them.

In summary, the participants were engaged in developing their understanding and making sense of the CAS tool (in maths) in diverse ways. In this theme, the interplay between needs and motives and how they give rise to goals for actions that could result in personal meaning-making with the CAS tool have been examined. In Chapter 3.5.6 I argued that, through activity, people amend objects, means and conditions and, most importantly, transform themselves. This transformation was evident in all three of the participants' actions of making meaning with CAS, as described above. Such actions and transformations have to be initiated and discovered through the individual's human-world interaction, in other words through the process of activity (Lompscher, 1999).

6.3.5 THEME: ACTIONS RELATED TO GENERAL COMPUTER USE

In activity theory, emphasis is given to the contexts of learning and personal historical background. Participants bring to the learning process their histories of computer use, their experiences and their beliefs. Computer experience is about personal socio-cultural history, and insights into participants' histories may well inform their current levels of engagement with the activity of learning mathematics using CAS. In the ensuing discussion, I consider the students' reported prior computer knowledge and determine whether this has any influence on their general computer use when using MATLAB to solve DEs.

Thembiso and Abraham claimed familiarity with the use of computers, while Tumi learned to use them just two semesters prior to studying this MATLAB component. Tumi's struggle with basic computing skills was evident throughout both problem-solving sessions. Overall, in the problem-solving sessions, Abraham and Thembiso displayed confidence in their use of computers, although both also had a small number of negligible problems with basic computer use. In other words, they expressed a good amount of skill in their general computer usage, possibly mediated by their prior experience.

As detailed in Chapter 5.3.4, Tumi would spend time and effort painstakingly typing and deleting just to produce one statement. She also struggled with highlighting text, and thus did not make as much use of copy and paste techniques as did the other two participants.

I conclude that there is undoubtedly an association between the students' reported prior ability and history of computer use, and their current levels of proficiency and competence with respect to general computer use whilst using MATLAB (in maths). In activity theory terms, knowledge

of the personal histories of individuals within a social and cultural framework is vital if one is to gain insight into how the individual makes current use of the tool. Though it is important to take into account the personal historicity I agree with Coupland (2004, p.137) that personal history does not determine activity: "Someone's computing background, for example, would be expected to influence but not fully determine all future successes with new learning tools". This brings to mind Berger and Cretchley's (2005) research (discussed in Chapter 2), which found that students studying at tertiary levels may be able to take full advantage of technological resources in spite of not having been exposed to them at school level. In this study it is evident that although Tumi lacked the necessary experience and skills with computers, she still strove to make personal meaning of the mathematics mediated by CAS (as discussed in Chapter 6.3.4).

6.3.6 THEME: ACTIONS RELATED TO THE USE OF COMMANDS IN MATLAB

An important aspect of the activity theory framework is the emphasis on operations, which are determined by the conditions under which they are carried out. Many commands of MATLAB have strict syntax associated with them. This may cause students to experience difficulty in turning various actions into operations. One such category of actions is the use of MATLAB commands. I assert that the specific syntax played a role in constraining the students in making their actions become operations.

Abraham easily made use of MATLAB commands, such as 'xlabel', 'display', 'plot' and 'legend'. Most often he would copy these commands, that he had typed in for the solution of Task 1, paste them into the solution of Task 2 and modify them without difficulty to produce the correct MATLAB command statements. Tumi and Thembiso had struggled with the syntax of various commands, nor could they explain why certain commands were being used. I would have expected that by Problem-solving Session 2 the use of MATLAB commands should have become operations in Leontiev's terms, but this did not happen for Tumi and Thembiso.

Thembiso used commands of 'format long', 'hold on', 'disp' and 'legend', but he either did not know the syntax of these commands or did not know why a few commands were being used. Likewise, Tumi either had difficulties with the syntax of certain commands or she did not know why they were used: for example, 'clear all', 'display', 'title', 'plot', 'legend' and 'ylabel'. In Problem-solving Session 2, neither of these participants could get right the syntax for the

'legend' command. Consequently, they deleted this command. Tumi also deleted the use of commands such as 'title' and 'ylabel', because of syntactically related errors. Probably they worked mainly towards their goal of producing the solution to DEs; their aim was to get the programme to run free of errors and produce graphs.

In activity theory terms, they consciously deleted commands when they realised that they were not getting right the syntax of commands so that they could deliberate on producing other aspects of the code that were useful in solving the DE. It is important to note that the commands they deleted are not integral to the solution of the DE. These examples also exemplify how external acts such as deleting commands becomes internalised as consciousness, and students act with this consciousness in mind.

Regarding the activity theory model in Figure 3.6, which underpins this study, practical action follows upon action. In Tumi's and Thembiso's quest to solve the given tasks, their actions encompass typing commands into the MATLAB editor and, when the compiler rejects the code, they follow this action with other actions of deleting the code. In this way, their acts exemplify an important aspect of my model that actions give rise to further actions. I contend that students learn through their actions in context.

There are "ranges of actions in most situations and the acting subject must select" what to do (Roth, 2007a, p.59). This researcher contends that acting concretely realises the choice of one plausible option amongst many and the choices are invariably leaning towards higher emotional valence. Those actions that participants choose are associated with emotional valence and its associated pay-offs (refer to the discussion in Chapter 3.5.7).

The choice of actions made by Tumi[71] and Thembiso[72] exemplify these pay-offs, possibly in the sense of the satisfaction enjoyed when producing a solution to the DE or its corresponding graph. Put differently, the sense of success associated with producing a compiled (error-free)

---

[71] TU-ref-int2(118-121). Tumi: Ok , most of the time I was just happy to get an output.

JP: You normally think if it gives you an output..

Tumi: You normally think if it gives you an output - you think it's right… most of the time I get frustrated when I get the error messages, you know. And the moment I get an output, I am like 'oh thank God'!

[72] TH-int2(240). Usually. Ok before, before I think the second last session I had… before the previous one I think… It was a thing I would go there; the main thing was to battle to get a graph portrayed. So once er… I, I, it would display - it will plot a graph, I was happy, I was happy that um…

MATLAB programme (this is one of their many goals) compels them actually to delete less important commands without forsaking the actual solution.

My interpretation of learning within the activity theory framework is that learning to programme MATLAB is a human activity that takes place in its particular historical, social and cultural context, imbued with the individual's motives, actions and goals. In this theme I expand on how students make choices whilst they are engaged in the activity of solving DEs using MATLAB. These choices exemplify how actions follow actions and, more importantly, how the participants act in relation to emotional-payoffs, which on this occasion are related to their immediate goals. From an activity theory perspective, participants have a "practical understanding of the progress toward reaching the intended goals and motives; they exhibit this understanding to others as the activity unfolds" (Roth, 2007a, p.49).

6.3.7 THEME: ACTIONS RELATED TO OVERCOMING DIFFICULTIES ASSOCIATED WITH MATHEMATICS WHILST USING CAS TO SOLVE DEs

The complexity of relating traditional mathematics notation to MATLAB notation posed a problem to the participants. All three experienced difficulty associated with mathematics whilst using CAS to solve DEs. Abraham is a student who scored straight A's (100% in Mathematics 1 and 2 in the diploma study) and he also taught mathematics to learners. In an ordinary learning mathematics context (without CAS) I am confident that both Abraham and Tumi (who scored 98% in Mathematics 1) would have been able to substitute correctly. However, in the context of learning mathematics with CAS, neither could produce correct function substitutions. I assert that the unique but also demanding context of learning to use a new tool in mathematics had an influence on the way they went about pursuing known mathematical endeavours.

My first explanation relates to the conditions of including the subscript *i* as part of the argument of the exponential function. In Problem-solving Session 2, the participants found that substituting in functions was a complex task. Even in Problem-solving Session 1, Abraham struggled to substitute in the Euler formula. He omitted the subscript '*i'* that should accompany *x* in *exp(x(i))*. When he could not resolve the associated error message he requested my assistance and I alerted him to the manner in which he had substituted. He corrected the error and stated that he had not thought about including the '*i'*. As discussed in Chapter 3.5.3,

automatic operations are driven by the conditions and tools of the action at hand. I argue that the conditions relating to including a subscript as part of the argument of the function (which students have not previously encountered) may have caused known operations to become questionable.

It seems that even those students who were proficient, competent and confident in the prior mathematics learned, as in the case of Tumi and Abraham, experienced trouble and deliberated with functions and substitution under the specific conditions of the action. Activity theory asserts that operations could briefly lose their position as operations under different conditions of the action being carried out, as evidenced in Tumi's and Abraham's substitution techniques.

When I alerted Tumi and Thembiso to their incorrect substitutions, both engaged in swapping around variables without checking their substitutions in the formulae of the auxiliary equations. Consequently, constants that should have been included within the square root sign were not. In Tumi's case, she initially included a constant (a wrong one) within the square root function but shortly thereafter she changed this. Abraham used copy and paste techniques to produce the auxiliary equations for Runge-Kutta IV methods but did not check the substitution in the formulae of each auxiliary equation. One plausible explanation could be that they were focussed on learning how to programme MATLAB, and it will be recalled (Chapter 5) that they were absorbed in their goals of producing solutions to the DEs without any error messages when compiled. Consequently, they had decreased levels of attention and concentration on the mathematics of substitution. In activity theory terms, the consciousness of each participant was on the CAS tool and not on the mathematics of substitution (in spite of my mediation) during their engagement in the activity of programming MATLAB.

In Problem-solving Session 2, the independent variable was t and dependent variable v. Thembiso did not identify which variable was independent and which dependent, while Abraham and Tumi correctly identified these. I suggest three explanations related to Thembiso's struggle with independent and dependent variables. The first is that the notion of independent and dependent variables has not been operationalised during Thembiso's previous mathematics learning. These aspects also relate directly to Thembiso's history of learning mathematics and point to the current gaps in his mathematical knowledge.

The second possible explanation is that Thembiso had operationalised the notion of independent and dependent variables but because of the unique context of learning to use the CAS tool these operations, which are conditioned by the context, became actions again. Another possible

reason that Thembiso displayed difficulty with identifying independent and dependent variables is that he was neither keen on nor enjoyed accessing previously learned mathematics knowledge. It was mainly through my mediation that Thembiso ultimately realised the goal of finding a solution to the specific DE.

In this theme I have explained how automatic operations depend on the conditions and tools of actions. I discussed examples that demonstrate this dependence by examining mathematical aspects of substitution in functions with respect to the particular conditions of including MATLAB subscripts as part of the argument of functions. I also discuss student's consciousness being directed to the CAS tool rather than to the mathematics of substitution.

6.3.8 THEME: ACTIONS RELATED TO PROCESSES FOLLOWED WHEN SOLVING A DE

Psychological tools as well as physical or material tools are important aspects of the activity theory framework. As discussed (Chapter 3.8), tools mediate thought during the interaction between the subject and the context of the learning activity (Engeström, 1987; Nardi, 1996c). The psychological tools could encompass beliefs, symbolic systems such as algorithms, syntax, knowledge of concepts and rules, learning strategies and techniques (Lompscher, 1999). I argued that the different depictions related to the activity levels, goal-directedness of actions and other conscious as well as non-conscious aspects contribute to the claim that activity is psychologically regulated (Lompscher, 1999).

In this theme I consider some of the actual processes that students carried out so as to achieve their primary goal of finding solutions to DEs using MATLAB. In particular, I discuss actions of how the participants went about writing out parts of the MATLAB code before typing onto the computer; how they made sense of error messages; the ways in which they endeavoured and struggled to get right the syntax; and how they paid attention to the sequence of steps in MATLAB, even when the order was not important.

Thembiso and Tumi believed that writing out MATLAB code before typing onto the computer assisted them in getting right the syntax. Thembiso wrote out the entire MATLAB code in both problem-solving sessions, while Tumi wrote out the code only for the auxiliary equations in Problem-solving Session 2. Abraham did not indicate anything regarding the writing out of MATLAB code, however he did write out some code in Problem-solving Session 2.

Nonetheless, all three students attempted typing the Euler solution that was given in the instruction sheet onto the MATLAB editor, and all copied the statements incorrectly. This leads me to conclude that Thembiso's and Tumi's statements that writing out helps to get right the syntax is an unjustified belief as they could not even copy the given statements directly onto the MATLAB editor.

I assert that Tumi's and Thembiso's activity of learning to solve DEs with MATLAB was mediated by material resources of pen and paper use, as well as psychological beliefs that writing code helps them to produce correct auxiliary equations. In this context, Abraham's activity was mediated by pen and paper but I cannot make a reasoned analysis of his beliefs because he did not articulate reasons for writing out code.

As stated above, language is a fundamental component of activity theory. When learning to programme MATLAB there is also the issue of learning a new technical language, as in interpreting error messages. The participants struggled to make sense of the technical language used in MATLAB's error messages, therefore they worked hard to put right their syntax because often they could not interpret or correct error messages. One example was Thembiso's failure to understand terms such as 'parenthesis'.

The difficulties with correcting errors could be attributed either to their not knowing where in the code the errors were located, or to their not knowing what the errors meant. They found that trying to resolve error messages was a confusing undertaking, and consequently engaged in various sub-actions to correct the errors. Some of these included changing correct mathematical statements into incorrect ones; changing mathematical statements to equivalent ones and not realising that they were the same; indiscriminate use of brackets; and using methods of trial and error. For instance, in response to an error message, they would remove brackets from a certain position, replace them in the same position and evaluate the code only to get the same error message. It is notable that actions resulted in various further actions and these were not always approached in a logical manner. Their consciousness was directed at trying to get rid of the error message and this happened at the expense of rational thought and action. I argue that the relationship with the use of the computer (and software) is very complex, but while this is an aspect of instrumental genesis (discussed in Chapter 2.3) it is not the immediate focus of my research.

One of the collective objects or motives of the module of learning Mathematics 3 with CAS is that students should "communicate effectively using everyday language and mathematical

language and symbols to describe processes and to solve problems" (Kirchener, 2008, p.3). The question arises as to whether the difficulty that the participants experienced in the use of syntax and interpreting error messages is in agreement with the course motive of communicating effectively using mathematical language and symbols. I assert that the participants' immediate goal of wanting to produce solutions to DEs did come into conflict with institutional or departmental collective motives when they could not always achieve it. Leontiev (1981) distinguishes between processes wherein the collective, socially constructed motive and individual goals do not coincide, and calls these processes 'actions'. I have labelled the participants' analogous actions as 'strives to get right the syntax'.

Thembiso lucidly stated that the complicated language used in the error messages was a challenge to his powers of comprehension. This defies the purpose of error messages, namely to help the user to correct the syntax. Abraham said that with the syntax one has to be precise, for example, the brackets should be in place as well as the semi-colon. Furthermore, they experienced problems with the use of the technical language. Tumi said that after reading the error messages she could not even locate where in the MATLAB code the actual errors were. She was of the opinion that the main skills that are needed to solve DEs using MATLAB successfully are to interpret error messages and to locate the exact position of the errors in the MATLAB code. By the end of the semester, the CAS software, through its constant feedback via error messages, mediated the participants' goal to generate solutions to the DE.

Also towards the end of the semester, the three participants concurred that they were in a better position to handle error messages – due to experience and familiarity now mediating the way they approached resolving error messages and associated syntactical problems. It is evident that they were learning a technical language through use, confirming Wertsch's claim that "our knowledge of the world is mediated by our interaction with it" (1981b, p.38).

It is important to note that this learning process with programming in MATLAB is a two-way process. MATLAB gives feedback (via error messages) respecting obvious syntactical misuse, and the user adjusts code to conform to this proposition. In this way the user is learning the correct use of syntax and the software scaffolds his/her knowledge. Examined from an activity theory perspective, this development exemplifies the notion that participation in activity changes the subject in reciprocal ways, the first having just been described.

The second way could be that, due to students' involvement in the activity and their cumulative experiencing of difficulty with error messages, the developers of the MATLAB software could

change it and make it more user-friendly. Hence, the subjects' involvement in activity could possibly transform the activity in the future. The activity of CAS (in maths) that the student is participating in has itself arisen historically and culturally. The methodology for programming CAS, for instance, has been expanded upon and enhanced by present-day programmers and continues to evolve rapidly in line with modern advancement in technology.

All the participants assumed, incorrectly, that the order of MATLAB statements was significant; hence they were frequently aware of sequencing their steps in the MATLAB code. Often they would type a statement and realise that this should appear after some other statement. They would subsequently move the statement to a new position in the MATLAB code. No one tested to see what happens if they did not follow the 'textbook' example with respect to sequencing steps in MATLAB, which evaluates the entire code together and as such following a rigid sequence of statements is not required. One possible justification could be that the participants were learning by closely following texts or figures of authority. As discussed above (Chapter 3.7), the idea of producing and reproducing knowledge is an important aspect in activity theory. Students produce and reproduce knowledge and it may well be that all knowledge has not been explored or fully understood. CAS provided the opportunity for experimentation, but this was not taken up by the participants in this situation.

In summary, I have examined how psychological beliefs, material resources and MATLAB's constant feedback (via error messages) mediate the participants' goal of finding solutions to DEs. The issue of learning a technical language through use was addressed. I discussed how actions gave rise to further actions and argued that the latter was not always rationally performed, because the consciousness of each participant was on eradicating error messages and correcting associated syntax, as opposed to being attentive to the mathematics. Finally, a vital aspect of participating in activity relates to how the activity and the subject have the potential to change in mutual ways, as suggested in the above discussion.

6.3.9 SUMMARY AND FINDINGS OF STUDENTS' ACTIONS

In Section 6.3, the actions, which are intertwined with the cultural, historical and social matrix of the participants, were considered and deliberated upon. Students, trainees or novices usually start at a technical level in their learning and gradually move onto the conceptual levels. To this end I considered certain themes related to actions about the use of technical MATLAB commands and general computer use, as well as other themes about making personal meaning

with CAS and actions that are in the process of becoming operations or joining the level of cognitive resources. In this way I became aware of a range of actions covering a wide spectrum of students' involvement in the activity of learning mathematics (with CAS).

In answering Research Question 2, on how students use CAS in mathematical learning, I have provided explanations for participants' actions within the eight themes developed from the analysis of data. Each theme was discussed, elucidated and most importantly justified by drawing on concepts and notions of activity theory, as well as the literature review in Chapter 2. CHAT (Leontiev, 1978) provides a language with which to articulate the actions and operations associated with the three participants' activity of learning to solve DEs using MATLAB. Indeed, it offers ways of examining learning by linking motivated human activities, goal-driven actions and tacit operations. Next I summarise the findings within each theme.

Theme: actions relating to optimising the environment. One of the findings of this research supports the notion that learning is shaped by context and available resources. My activity theory framework brings to the fore the context of learning. The lack of access to computers and MATLAB impacted on how two participants approached their learning activity. Tumi resorted to writing out the solutions to MATLAB exercises whilst Thembiso left the learning of mathematics with CAS to the last minute, just prior to a major assessment. Nonetheless, the tradition of having one-hour practice times at the end of formal lectures afforded all the participants an opportunity to practise and their immediate engagement with the learning material during this designated practice time is an indicator of their motivation to learn. Such actions demonstrate how participants make optimal use of the formal lecture situation.

There were various resources, such as 'textbook', lecture notes, classroom solved examples, computer, software and interactions with others, that mediated the learning of mathematics (with CAS) for the participants. However, Thembiso and Tumi, who attempted using the help-files of MATLAB, soon abandoned them due to their complexity.

Mediation emerged as a central idea in this thesis, and I considered how participants made use of various mediating tools to realise intermediate goals. I discussed how forming social relations has educational spin-offs, such as improving communication skills and respecting the cultures of others. All of these are vital to the overall development of individuals and society and my activity theory framework supports the notion of this growth.

Theme: actions relating to personal power. The critical indicators of motivation include actions related to participants' effort, persistence, selection of tasks (Schunk et al., 2008) and

choice of study time. Salient findings are that Tumi and Abraham, with key cognitive motives, selected challenging tasks to solve and also reported that they solved MATLAB problems consistently throughout the semester. Thembiso, with prominent performance approach goal-orientation, studied mathematics with CAS at the last minute, just prior to an assessment, and did not persevere at solving challenging problems.

The participants adopted learning styles which were similar to the ones reported when they learned mathematics without CAS, and this relates to their histories of personal engagement with mathematics. Apart from histories, the view that activity theory is goal-driven is fundamental to Leontiev's (1978, 1981) theory, and is considered next.

A prominent result is that the participants learned by redoing already solved examples, but they had different goals for doing so. For Tumi, a student with major cognitive motives, her goal was to determine whether she understood the solved examples and to apply what she had learned to the solution of other problems. For Thembiso, meanwhile, a student with significant performance approach goal-orientation, his goal for such an action was to get the same answer as the one obtained by the lecturer when the example was completed in class. These goals that are formulated by participants are informed by their motives, and in so doing illustrate an important notion of activity theory that needs give rise to motives, which in turn could inform goal formation (Leontiev, 1978).

Theme: actions that could lead to learning mathematics (with CAS) operationally. The most important finding in this theme relate to the participants' reported involvement in some form of *practising* to solve problems. I argue that participants engaged in such actions because they ultimately wished to solve problems at an operational level.

Thembiso, Tumi and Abraham learned mathematics (with CAS) by redoing worked out examples from either the text or those solved in class, and they reported solving similar exercises[73]. Such actions are in line with Engeström's (1987) discussion of 'dead text'.

Thembiso, who espoused mainly performance approach goal-orientations, reported that he memorised MATLAB statements, devoid of understanding. I assert that such actions demonstrate a surface approach to learning and he did this so that he could recall such statements at an operational level. These learning experiences beg the question as to why

[73] N.B., Tumi and Abraham with cognitive motives reported that they also solved problems that were challenging in nature.

students resort to using surface approaches in learning mathematics using CAS. One would expect that the CAS should serve as a tool to promote deeper levels of understanding and learning.

Theme: actions relating to comprehension of mathematics with CAS. This theme relates to Leontiev's theory of meaning-making. The activity theory notion of human consciousness being located in everyday practice was exemplified in this theme when participants' awareness shifted from syntactical problems and resolving error messages to embracing an examination of the graphical output of CAS.

Tumi and Abraham, who held dominant cognitive motives, were keen on linking previous methods (analytical solutions of DEs) studied with new ones (numerical solutions of DEs), thus demonstrating a desire for personal meaning-making. They verified solutions to MATLAB using varied approaches, such as examining the table of independent values or comparing substituted values of numerical solutions produced by hand with those of MATLAB. In addition, Abraham talked about linking the theory to practical. He reported that he applied MATLAB to other learning contexts in his field of study and I assert that he believed that he was appropriating the tool for his own use. The finding that participants who hold prime cognitive motives use deeper approaches to learning mathematics (with CAS) was explicated within this theme.

Nonetheless, Thembiso, who espoused mainly performance approach goal-orientation, also performed actions that were related to comprehending mathematics (with CAS) when he interpreted graphs. Unlike Tumi and Abraham, he did not use any other methods of verification. I note that although he had surface approaches to learning, the aforementioned actions exemplify his attempts to understand mathematics with CAS.

Thembiso and Abraham, who espoused social (interrelations with others) motives regarding their learning of CAS (in maths), believed that their learning had benefited through either providing assistance to or receiving help from others. In addition, Abraham reported that he encouraged his group members to challenge him in the mathematical learning process (involving CAS). Such actions of forming a study group (as in the case of Abraham) and teaching others to use CAS (as in the case of both participants) could be interpreted as actions that are associated with the comprehension of mathematics with CAS.

Theme: actions related to general computer use. A significant finding is that there is a relationship between participants' reported history of tool use and their then levels of

proficiency in general computer use[74]. Thembiso and Abraham, with good levels of prior computer skills did not struggle with the general use of computers (such as highlighting text, typing of symbols and saving files) when using MATLAB to learn mathematics. Tumi, with low levels of computer experience, struggled with general computer use whilst programming MATLAB to solve problems.

Theme: actions related to the use of commands in MATLAB. I focussed on the constraints and conditions related to the syntax of MATLAB commands. There are many ways to achieve a goal and these could be determined by the individual whilst interacting with the general structure of actions. The actual actions of Thembiso and Tumi, such as deleting MATLAB commands, are in themselves irrational yet they make sense when understood as part of the larger system of the activity.

Thembiso and Tumi struggled with the syntax of various MATLAB commands, such as 'format long', 'hold on', 'display', 'legend', 'clear all', 'title', 'plot', and 'ylabel'. Nor could they explain why these commands were used. In activity theory terms, they consciously deleted a command when they realised that they were not getting right its syntax, so that they could deliberate on producing other aspects of the code that were crucial to solving the DE. Put differently, the findings of this theme suggest that the two participants made a choice to pursue their goal of producing a solution to the DE and thereby seeking positive pay-offs, as opposed to spending time correcting insignificant MATLAB commands. The sense of success associated with producing a compiled (error-free) MATLAB programme (this is one of their many goals) compelled them to delete less important commands without compromising the actual solution.

Such actions also exemplify the notion of how external acts such as deleting commands become internalised as consciousness. Students act with this consciousness in mind.

Theme: actions related to overcoming difficulties associated with mathematics whilst using CAS to solve DEs. I examined the complex nature of relating traditional mathematics notation to MATLAB notation with respect to substitution. The finding that became apparent is that the participants' concentration was on the CAS tool (its associated syntax and error messages) and not on the mathematics of function substitution. All the participants found it difficult to relate traditional mathematics notation to MATLAB notation as far as functions and substitution were concerned. Tumi and Abraham had very high levels of prior mathematics knowledge yet they

[74] The latter was inferred from data of the problem-solving sessions.

had problems and deliberated with functions and substitution under the specific conditions of using CAS in mathematics learning. In their case these conditions might have caused known operations (of substitution) to resurface as actions.

Theme: actions related to processes followed when solving a DE. I scrutinised some of the psychological tools that mediate learning. Tumi and Thembiso believed that writing all or parts of MATLAB code before typing into the software would be beneficial. These beliefs together with MATLAB's constant feedback via error messages mediated the participants' goal of finding solutions to DEs.

This research has highlighted the difficulties experienced by the participants regarding the use of the syntax of MATLAB and learning a new technical language as in interpreting error messages. The findings on the complexities of syntax related to CAS concur with several other studies; for example, Berger (2006); Drijvers (2000) and Pierce and Stacey (2004).

Actions gave rise to sub-actions and the latter were not always judiciously executed because the participants' consciousness was directed at clearing error messages and correcting related syntax. Some of the sub-actions included participants changing correct mathematical statements into incorrect ones, changing mathematical statements to equivalent ones and not realising that they were the same, indiscriminate use of brackets, and using methods of trial and error.

Towards the end of the semester, the three participants agreed that they were in a better position to manage error messages. I argue that this is due to experience having mediated their approach to understanding error messages and resolving errors.

6.4 A COMPARISON OF STUDENTS' OPERATIONS

Operations are the third level in Leontiev's hierarchy of activity levels. They represent the skilful, automatic and non-conscious doings of individuals. Learning at the level of operations requires students to know automatically the learning matter, whether it is straightforward or complicated. However, developing skill in using MATLAB to solve DEs is dependent on the conditions and means of the actions being carried out. Operations are constrained by the conditions and context, many of which, with the constraints and means, were discussed above (6.3), as they formed part of observable actions or conscious reports made by the participants. The MATLAB syntax, with its strict rules and the language used in error messages, limits some

actions from becoming operations. Observing operations is not an easy task; consequently there were very few operations that emerged from participants' data.

The students were taught to type basic MATLAB statements in the command window and use the m-files to programme MATLAB to solve DEs. Initially, Thembiso did not know whether he should use the command window or the m-file to type in the programming code. The choice between command window and m-file were conscious actions for Thembiso, but by Problem-solving Session 2 he could easily open and use an m-file. Here again I assume that his easy access to the m-file has become an operation and was not in conscious thought compared to the struggle he displayed in Problem-solving Session 1. In contrast, the other two participants could easily open the MATLAB programme and use m-files.

Thembiso and Abraham were able to select and evaluate code in order to produce the compiled form of the programme without difficulty. This they did many times during both the problem-solving sessions and I argue that this process was done at the operation level. Thembiso typed the initial lines, such as *a=0,* with ease in Problem-solving Session 1, and I assume that these were operations.

In this study it appears that the operations mainly relate to basic computing functions and not to the actual use of MATLAB code or syntax. It will also be recalled that I discussed how certain conscious doings temporarily join the level of operations, enabling the participant to direct their consciousness to other matters (6.3.4). In summary, I note that many of the actions had not become operations by Problem-solving Session 2, for example, the use of MATLAB commands.

It will also be recalled that Research Question 2 is related to how students use CAS (in maths) and although in this study there were only a few operations that emerged, the learning largely took the form of conscious, reflected actions and associated goals. I refer the reader to Chapter 7.3 wherein I acknowledge that in a future study involving Leontiev's framework I would use think aloud protocols in anticipation of obtaining more data related to the operations level of this framework.

6.5 ANSWERING RESEARCH QUESTION 3

Bearing in mind that answers to Research Question 3 were noticeable in the discussion and deliberation in 6.3, for the sake of completeness I recapitulate salient motives and actions corresponding to each case study student. In this way the link between motives and actions may be affirmed.

Students hold a myriad of motives, which coupled with their own personal histories and the current context of their learning may influence the goals (and actions) they formulate as they engage in the activity of learning mathematics with CAS. This web of goals relating to the actions concerned and motives relating to this activity are interconnected and logical. Throughout this study I have considered students' motives, their needs and their actions as being intertwined with the social and cultural context of learning mathematics (with CAS).

An important aspect of this study is that I examine students' motives from a multi-motive perspective. Although the three students espoused a mixture of motives, in each case study student there were particular motives that are striking compared to the rest. For example, in Thembiso's case I argue that he exuded performance approach goal-orientations with strong conviction when learning mathematics with CAS. His motive of seeking to obtain good grades because it 'adds colour' to his academic record must be interpreted against his need to be recognised as part of the group of 'smart' students. I argue that his social (interrelations with others) motives too are interpreted against this background. Thembiso expressed little interest in studying mathematics (with CAS) because he found it inherently interesting or enjoyed the associated challenges. Next, I review the salient actions that such a student engages in when learning mathematics with CAS.

Thembiso stated that he crammed mathematics (with CAS) at the last minute before an assessment. To this end, he reported that he redid class examples by writing them out so as to ascertain whether he could get the same answer as did the lecturer. He claimed to work mainly through already solved class examples but not all exercises. During his last minute study he described having attempted to solve more difficult problems but eventually had to leave them out because he did not understand how to do them, and there was no time to seek assistance. In the interviews he stated that he would choose to solve a DE using easier (Euler) methods even though he was aware that this method was not the most accurate one. Thembiso memorised statements of MATLAB code without knowledge of why they are used.

I argue that his approach to learning mathematics (with CAS) is at a surface level and demonstrates a shallow understanding. In the problem-solving sessions I observed that Thembiso did not know most of the MATLAB commands or the reasons why some of them were used. In spite of the approach to learning mathematics (with CAS) described above, he carried out some actions, such as 'interprets graphs' and 'interacts with others', which are usually classified as actions leading to the comprehension of mathematics (with CAS).

From the above description I perceive that there is a relationship between Thembiso's performance approach goal-orientations and his surface approach to learning mathematics (with CAS). This could be explained in terms of the motivation theory outlined in Chapter 2, by which students with primarily performance approach goal-orientations usually involve the ego and are often committed to perform and demonstrate high ability with little effort as reported in the syntheses of Schunk et al. (2008). These students are likely to define success in relation to others. Thembiso's motives, goals and actions exemplify this particular relationship.

I now discuss the relationships between Tumi's motives and salient actions. Although she held multiple motives I contend that she espoused predominantly cognitive motives towards studying mathematics (with CAS). I assert that Tumi found intrinsic value in studying MATLAB (in maths) and she held mastery approach goal-orientations in wanting to develop competence and skill with respect to using MATLAB (in maths). Tumi believed that her participation in the activity of learning to use MATLAB (in maths) had played a vital role in improving her cognitive abilities. Nonetheless, in the problem-solving sessions I observed that she did not know most of the MATLAB commands or why they were used.

What were Tumi's salient actions when learning mathematics with CAS? Tumi first tried to solve any DE using analytical methods. She then substituted in a numerical formula to find a few solutions to the DE before attempting the solution using MATLAB. Once she produced solutions to the DE using MATLAB she compared these with the ones she had produced using pen and paper techniques. Tumi claimed that her approach to learning was to solve problems soon after formal lectures. She claimed that she redid classroom-solved problems because she wished to understand the examples. In addition she said that she prepared to apply what she has learned to solve other exercises and problems. The context of learning in an unsafe environment played a major role in Tumi's actions with respect to learning mathematics (with CAS). She recounted that she wrote out solutions to MATLAB exercises so that she could type them onto the MATLAB editor when or if she gained access to a computer. In this way she persisted in her study of the MATLAB component. Consequently, there is a relationship between Tumi's

mastery approach goal-orientations and the effort that she put into her studying. Tumi's actions exemplify the discussion (refer to Chapter 2) on how students who are motivated to learn persist at the task, especially when faced with obstacles.

It will be recalled that Abraham described significant motives which I place in two major categories, namely *social* (interrelations with others) and *cognitive*. He professed to finding the study of MATLAB (in maths) interesting. I assert that he espoused mastery approach goal-orientations because he also articulated developing and applying new skills to different areas, as well as striving to gain new knowledge. Abraham's social motives encompassed learning as a means to help and support others.

Abraham's actions consisted of trying to solve DEs analytically, even though the analytical methods were not always required. He did this because he wanted to challenge himself. Abraham had a few ways of determining whether he had produced correct solutions to DEs, for example, he also checked the values of the independent variables in the output. He said he had formed a study group and encouraged debates and discussions with group members, which I therefore describe as a deep approach to learning. I contend that such actions helped him fulfil his need for competency in learning mathematics (with CAS) as well as satisfy his social motives. In the problem-solving sessions Abraham's concern about his partner's learning was especially evident.

As discussed in Chapter 2, the selection of challenging tasks under free choice indicates motivation to perform the task and Abraham's reported corresponding actions in the learning of mathematics (with CAS) alludes to this. Similar to Tumi's, his actions entailed solving problems soon after formal lectures were concluded. Abraham reflected that he applied his knowledge of MATLAB to solve problems in other engineering courses.

I conclude that there is indeed a relationship between Abraham's motives of social concern, cognitive motives and his actions. In summary, both Tumi and Abraham held motives that I describe as cognitive in nature, and they particularly espoused mastery approach goal-orientations to learning MATLAB (in maths). Their actions to a large extent encompassed studying consistently throughout the semester, verifying solutions to MATLAB other than using graphical interpretation and attempting to solve analytically, even when this was not required. Both put in great effort and persistence towards studying this MATLAB (in maths) component.

It has been argued that mastery approach goal-orientation is characterised by a focus on mastering the task; striving to improve one's self in the task; striving to accomplish something

challenging on the task; developing new skills; being competently engaged in the task either by developing or improving competence; and trying to gain understanding (Ames, 1992b; Schunk et al., 2008; Urdan & Giancarlo, 2002). Mastery goal-orientations reflect an emphasis on improvement and understanding, even if it means making mistakes (Midgley et al., 1998). Students with this goal-orientation search for meaning, show strong intrinsic motivation and often get lost in their work. In Abraham's and Tumi's case I find a correlation between the cognitive motives espoused[75] and acts involving high effort or persistence in learning, an important finding of the study.

6.6 SUMMARY OF CHAPTER 6

Leontiev posits that, at the activity level, purposive intentional activity driven by needs, personal motives and object offers a valuable level to analyse activity. Activity theory focuses awareness and importance of the motives of an activity, and within this framework I have established and explained the multiplicity of motives through the three categories proposed by Lompscher (1999).

In this chapter, I linked the themes that emerged from the empirical data to the theory and compared and contrasted the three participants' motives, actions and operations when they learned mathematics using MATLAB. I provided summaries for each of these sub-sections, during which I showed how my three research questions were answered with respect to the three students' data. Activity theory contains many features, such as, tools, mediation, historicity, context, consciousness, personal meaning-making, conditions and others, and these were considered essential in my discussion. I have used activity theory as a "powerful and clarifying descriptive tool rather than a strong predictive theory" (Nardi, 1996a, p.7). In the next chapter I draw a conclusion to this thesis.

[75] Note that mastery approach goal-orientations is a subset of cognitive motives.

Chapter 7

Conclusion

7.1 INTRODUCTION

This final chapter reflects on the study as a whole. It begins with the contribution of the thesis to new knowledge. I reflect upon the limitations of this study, suggest possible directions for future research and make recommendations for specific educational practice. My personal reflections bring this chapter to a close.

7.2 CONTRIBUTION OF THESIS TO NEW KNOWLEDGE

A significant endeavour of this thesis is to interpret, apply and develop Leontiev's activity theory. My detailed elaboration of constructs of activity theory attempted to define and make sense of a number of ill-defined concepts and has in the process operationalised certain critical variables such as motives and object of activity. A major contribution of this thesis lies in its contribution to educational theory development.

The model that I had developed in Figure 3.6, by merging Kaptelinin's (2005) and Roth's (2007a) prototypes, is an important contribution to the discipline of activity theory, underscoring as it does the distinctive nature of individuals and their motivations viewed together with the three levels of Leontiev's (1978, 1981) activity scheme. The model includes salient imperatives associated with motivation, such as individuals' needs, motives, object and goals, whilst also emphasising the social context and material conditions and means.

From a theoretical point of view this study attempts to build a bridge between two theoretical frameworks: motivational theory, as portrayed by Western motivational theorists, and Leontiev's (1978, 1981) activity theory framework, comprising the three levels of activity, actions and operations. It presents a new framework by expanding the former into larger, more

general and flexible categories of motives as postulated by Lompscher (1999) from an activity theoretical position. This new framework enabled me to include important motives related to students' use of CAS (in maths) that may otherwise have been undifferentiated. Nonetheless, the work of Western motivational theorists provided a starting point for my understanding of motivational theory in this thesis, and it enabled me to elaborate and explain motives more clearly from an activity theory stance. However, activity theory, with its focus on social, historical and cultural contexts, provides a valuable framework for a study involving motivations in a learning milieu.

Another main contribution of this thesis lies in its empirical contribution and its significance in the field of education. It will be noted that it is through the analysis that the strength of the theoretical modifications can be seen. The detailed analysis provides greater insight into how students learn. I use these three students' data to explain how learning happens for them, but the new insights gained in our understanding of the processes of learning is just not limited to them or to students in diploma courses in Higher Education using CAS but paves the way to understand how learning happens in general. Indeed, this thesis underscores important ways of examining learning by linking motivated human activities, goal-driven actions and tacit operations.

This study has attempted to illuminate students' motivations and actions whilst they learn mathematics mediated by CAS within a tertiary engineering programme, and in the process has addressed the many gaps that were identified in Chapters 1 and 2. It will be recalled that there is a paucity of research on students' motivation and the learning of mathematics using CAS, one salient study being the research carried out by Galbraith and Haines (1998), who developed scales for quantitatively measuring students' computer-mathematics interaction and their relationship to mathematics motivation and computer motivation. However, they did not directly address motivation in mathematics mediated by CAS. The finding in my study that students espouse multiple, subjective motives, informed by various needs, in addition to the motives being inter-related to the goals that correspond to conscious actions of learning mathematics with CAS, attempts to narrow the aforementioned gap.

The gap is further narrowed when I consider some important findings involving the relationship between students' motivations and their actions when learning mathematics (with CAS). Specifically, there is a relationship between students espousing performance approach goal-orientations and their surface approaches to learning mathematics (with CAS). Such findings could be explained in terms of the motivation theory outlined in Chapter 2, by which students

with primarily performance approach goal-orientations usually involve the ego and are often committed to demonstrate high ability with little effort, as reported in the syntheses of Schunk et al. (2008). These students are likely to define success by comparing it to others. This study also highlights the relationship between students' mastery approach goal-orientations and the corresponding effort that they put into their study of mathematics (with CAS). The study underscores how students who are motivated to learn persist with the task, especially when there are difficulties in their path.

The second gap identified in Chapters 1 and 2 that this study intends to fill relates to students' motivation and mathematical learning with CAS taking place in a developing country as compared to the greater part of research on affect which is in developed countries. Inevitably, the issues of access and inequity arise. In developed countries, Dunham and Hennessy (2008) found that the inequities in the use of computers in mathematical learning can result from the amount of use and access to technology, indeed a finding that resonates with my study. They also found that differentials in experience could prevent certain groups, such as those of lower socio-economic status and/or female students from acquiring the full potential of educational technology in mathematics. This is comparable to the findings of this study in which there is an association between students' reported prior ability or history of computer use, and their then levels of proficiency and competence with respect to general computer use whilst using the MATLAB software in mathematical learning.

Furthermore, some of the difficulties with syntax and interpreting graphs reported by students in this study resonate with those experienced by students in the developed world (see Drijvers, 2000).

This study has drawn on socio-cultural frameworks, particularly Leontiev's (1978, 1981) version of activity theory, thus contrasting with an exclusive focus on quantitative methods as is usual in research into motivations, and addresses the third gap that was identified in Chapters 1 and 2.

The fourth gap concerns the lack of research on motivations and CAS in mathematical learning within the vocational stream, as compared to academic degree programmes. Notwithstanding, I contrast some of the findings related to students' *actions* in this study with results found when degree students learn mathematics (with CAS). In a survey involving 108 questionnaires administered on the technical affordances of CAS to first year mathematics degree students, Berger (2006) found that many students experienced technical difficulties using CAS, for

example, problems with syntax and interpreting output. The finding that became apparent in my study is that the participants' concentration was on the CAS tool (its associated syntax and error messages) and not on the mathematics of function substitution. This research has highlighted the problems encountered by the participants regarding the use of the syntax of MATLAB and learning a new technical language, as in interpreting error messages. The findings on the complexities of syntax related to CAS concur with several other studies; for example, Berger (2006); Drijvers (2000) and Pierce and Stacey (2004).

7.3 LIMITATIONS OF THIS STUDY

I realise that a great part of the journey in the study was to unravel the ways in which the theoretical framework could be used and applied. I experienced lengthy periods of frustration investigating the approaches of Leontiev (1978, 1981) and Engeström (1987) and the suitability or unsuitability of the latter approach to my research. I took these different theoretical paths but later realised that Leontiev's activity theory construction provided a more illuminating framework for a study on motivations and actions. It was through the avenue of secondary sources, such as Roth (2007a), Lompscher (1999), Gordon (1998) and Coupland (2004), that I gained the confidence to apply Leontiev's activity theory to the study. At a later stage I returned to his primary work to gain a deeper understanding of his ideas and approaches.

Trying to understand the concepts of activity theory was certainly time-consuming, and was exacerbated by ambiguous translations from the Russian language into English and the corresponding interpretations or misinterpretations. For example, constructs such as 'object of activity' were interpreted in different ways by activity theorists themselves. As pointed out in Chapter 3, as recently as 2005 esteemed activity theorists were still debating this construct.

The activity theory framework, with its powerful yet flexible nature, encouraged me to pursue its use as a theoretical lens for this study. Once I delineated the terminology of the framework, the analysis, and most importantly the discussion of results, benefited from the language and expressions. My engagement and perseverance in trying to understand the framework contributed to my evolvement and development as a researcher. I was grateful for having had the opportunity to use the framework. In a future study involving Leontiev's framework I would use methods encompassing think aloud protocols, as this may provide enhanced insights into participants' goals and operations.

Originally I had planned to collect qualitative data during the actual MATLAB lecture time, especially the last one hour of the session wherein students practised on their own. However, the design had to change because the lecturers on this course, at the last moment, denied me access to their classrooms.

This thesis has attempted to present the motives and actions of case study students when they engage in the activity of learning mathematics (with CAS). The results of the research may not be generalisable to other settings, especially if the importance of the context is to be regarded (Neuman 2003; Miles & Huberman, 1994). Every student has his or her own unique circumstances, historical aspects and cultural upbringing. Despite this study not perhaps being generalisable, it does illuminate critical issues that may resonate with other findings or be taken up for future research. For example, the after-hours lab access within the precincts of a dangerous suburb of Johannesburg needs revisiting, as it deprives the female (and male) students of the chance to engage in much needed practice.

Due to the lack of research on motivations and CAS in mathematical learning it becomes difficult to evaluate the empirical findings related to Research Questions 1 (based on motives) and 3 (based on the relationship between motives and actions) of this study with prior comparable studies. The findings that related to Research Question 2 were compared with other studies (refer to 7.2.) but the absence of a body of research on solving DEs using CAS within the vocational stream again makes an assessment of this study not easy. However it is envisaged that my study would stimulate research into the intricate areas of student motivation and mathematical learning using CAS, not disregarding research into actions and CAS (in mathematics) within a vocational stream.

It will be recalled (Chapter 4.5.3) that this study exposes only a few operations with respect to students learning mathematics (with CAS). I argue that whilst researching students' learning when they were simultaneously engaged with the course material may have contributed to the few operations. I refer the reader to 7.4 wherein I discuss a possible resolution.

7.4 SUGGESTIONS FOR FUTURE RESEARCH

Developing a survey around motives that are classified using Lompscher's (1999) three categories would be an interesting future research project and may provide illuminating results if data were to be gathered using the setting of students learning MATLAB (in maths).

Exploring students' motivation using other recent methods, such as investigating emotions and voice pitches, is recommended for future research projects.

Bulent Screen Recorder was a valuable data-capturing tool as it caught every keystroke, pause, deletion, and copy and paste technique. Further research is needed to ascertain its utility value in research.

Although I am very pleased with the motivational framework (see Figure 3.7) which I developed using both Western motivational theory and Leontiev's activity theory, it could be further refined and built upon by more research.

I have collected considerable data related to students' reported needs, motives and actions when they had learned Mathematics I and II (excluding CAS). There appears to be some relationship between these characteristics and the needs, motives and actions when they learn mathematics with CAS (see Periasamy, 2010). An expansion of this exploration could prove to be a worthwhile prospect.

I did not interview the course designer, though she would have had her own objects, needs, motives and actions in mind when she introduced the MATLAB component into the Mathematics 3 module. Determining such objects, needs, motives and actions could prove to be a useful future project. Now that this study has established a range of motives and actions of participants with respect to learning mathematics using CAS, an interesting future study would be to examine the tensions and contradictions in an activity system, perhaps using Engeström's (1987) framework, with the *subjects* comprising students and their motives, and the *community* consisting of lecturers, institution and the Engineering Council of South Africa.

7.5 RECOMMENDATIONS FOR SPECIFIC EDUCATIONAL PRACTICES

Although CAS was used in a specific way to teach students to programme MATLAB to solve DEs, I recommend that attention be given to the design of tasks that promote mathematical thinking and deeper levels of engagement as opposed to encouraging learning by means of replicating known exemplars. CAS could also be incorporated into the mainstream teaching as opposed to only two sections of the mathematics syllabus, as it could also serve as a means to

demonstrate its use as a tool to check answers. More exemplars on mechanical engineering should be included in the tasks.

Students should be encouraged to use library books and other recommended books on the subject content of MATLAB programming (in maths), as this would engender a deeper understanding of programming. I propose that more attention be given to the use of help-files in MATLAB as this could serve as a valuable resource, especially regarding the learning of syntax and trying to make sense of complicated error messages. From my experience I argue that the key to gaining competence in the use of any software is through navigating its help-files, clearly a skill that needs to be developed in students.

In the course design I recommend an avenue for students' reflections which should serve as a feedback component to the design of the course as well as to the lecturers. In these ways students' voices would be heard more often. In the past there were dedicated tutorials for the MATLAB (in maths) component, supervised by lecturers and tutors. I suggest that the tutorials be reinstated because they provide students with much needed practice time during the day and the support of knowledgeable others to scaffold their learning. The participants in this study agreed that the MATLAB tutorials should be restored.

The safety of the campus wherein this study was conducted was alluded to in Chapter 6. In this regard my recommendations are that students should gain access to the laboratory facilities on the other three campuses, which may be relatively safer in the evenings. Furthermore, since this campus does not allow students to access the computer labs during the day for practice, as compared to the other campuses, the students could benefit from daytime access at different sites. Daytime laboratory access at weekends should be considered at this campus so as to enhance ease of access.

It will be recalled that the participant (Tumi) with low levels of computer experience struggled with general computer use whilst programming MATLAB to solve problems (Chapter 6.3.9). This issue raises two questions: Firstly, why do students leave school being computer illiterate? and secondly, are there insufficient levels of computer skills development during the first two semesters of the diploma study? At the time of writing, the participants have one semester remaining to complete the theoretical component of their studies, so does this imply that a few students would graduate as engineers having low levels of computer skills, especially when the task of a vocational programme is to produce graduates who are competent for the workplace? I suggest that further research be carried out in determining students' general computer skills

prior to graduation and the necessary steps be taken to ensure that the would-be engineers have adequate computer skills.

7.6 PERSONAL REFLECTIONS

My experiences during this study have been varied and ongoing. I entered it with a state of mind steeped in quantitative practice brought on by my own personal history as a postgraduate in pure mathematics. Simultaneously I was thrilled by the prospect of working with progressive and exciting new ideas in students' motivations and their actions using CAS to learn mathematics. The possibility of gathering data using screen recordings and interviews seemed a novel and new approach to me. As the study progressed, the richness of data that emerged from the use of qualitative methods attracted me towards this research methodology and this change in mindset has seen an immense transformation in my thinking. This journey has also made me realise the extent of the complexities that exist in the world of qualitative research.

During the research process I came to discover that "in a word, society produces the activity of the individuals it forms" (Leontiev, 1981, p.48); that activities and tools are social and cultural constructions; and that students learn how to use tools and engage in activities so as to ensure their own existence in society. In other words, activities are oriented towards collective motives and have arisen in the course of historical-cultural development (Roth, 2007a). Indeed, what a learning experience the theory of activity has provided for me.

I conclude the study with a quotation that clearly captures the essence of the research:

> What the subject sees in the object world are motives and goals, and conditions of his activity must be received by him in one way or another, presented, understood, retained, and reproduced in his memory; this applies also to processes of his activity and to the subject himself - to his condition, characteristics, and idiosyncracies. Leontiev (1978, p. 75)

APPENDIX A: ATTITUDE TO COMPUTERS SURVEY

All data will be used anonymously.
If you do not agree to the data being used (anonymously) for educational research purposes, do not fill in the survey.

Indicate your views on items below by circling the number

For each item there is a row of numbers (1 – 5) corresponding to a five point scale. Choose only **one** response by **circling** only **one** of the numbers. The numbers stand for the following responses:

Strongly disagree	Disagree	Neutral	Agree	Strongly agree
1	2	3	4	5

For example:

If you **disagree strongly** with the statement *"using computers is easy."* Then circle the number 1 as follows

(1) 2 3 4 5

	Strongly Disagree	Disagree	Neutral	Agree	Strongly agree
1 I have less trouble learning how to use a computer than I do learning other things.	1	2	3	4	5
2 When I have difficulties using a computer, I know I can handle them.	1	2	3	4	5
3 I am not what I would call a computer person.	1	2	3	4	5
4 It takes me longer to understand computers than the average person.	1	2	3	4	5
5 I have never felt myself able to learn how to use computers.	1	2	3	4	5
6 I enjoy trying new things on a computer.	1	2	3	4	5
7 I find having to use computers frightening.	1	2	3	4	5
8 I find many aspects of using computers interesting and challenging.	1	2	3	4	5
9 I don't understand how some people seem to enjoy spending so much time at a computer.	1	2	3	4	5

10 I have never been very excited about computers.	1	2	3	4	5
11 I find using computers confusing.	1	2	3	4	5
12 The technical language puts me off using computers.	1	2	3	4	5
13 I'm nervous that I'm not good enough with computers to be able to use them to learn.	1	2	3	4	5

By signing below, it means that I am willing to take part in the research project. I do not mind being video-taped (once), interviewed (3 times), observed (during Problem-solving Sessions) and have my computer work recorded (2 times). This will all take place during the second semester of 2007.

If you are **willing to take part in the research project**, then please fill in form below; otherwise leave the space below blank.

Surname and initials:

Signature:

Telephone number:

APPENDIX B (SOLUTIONS TO MATLAB TASKS IN PROBLEM –SOLVING SESSION 1)

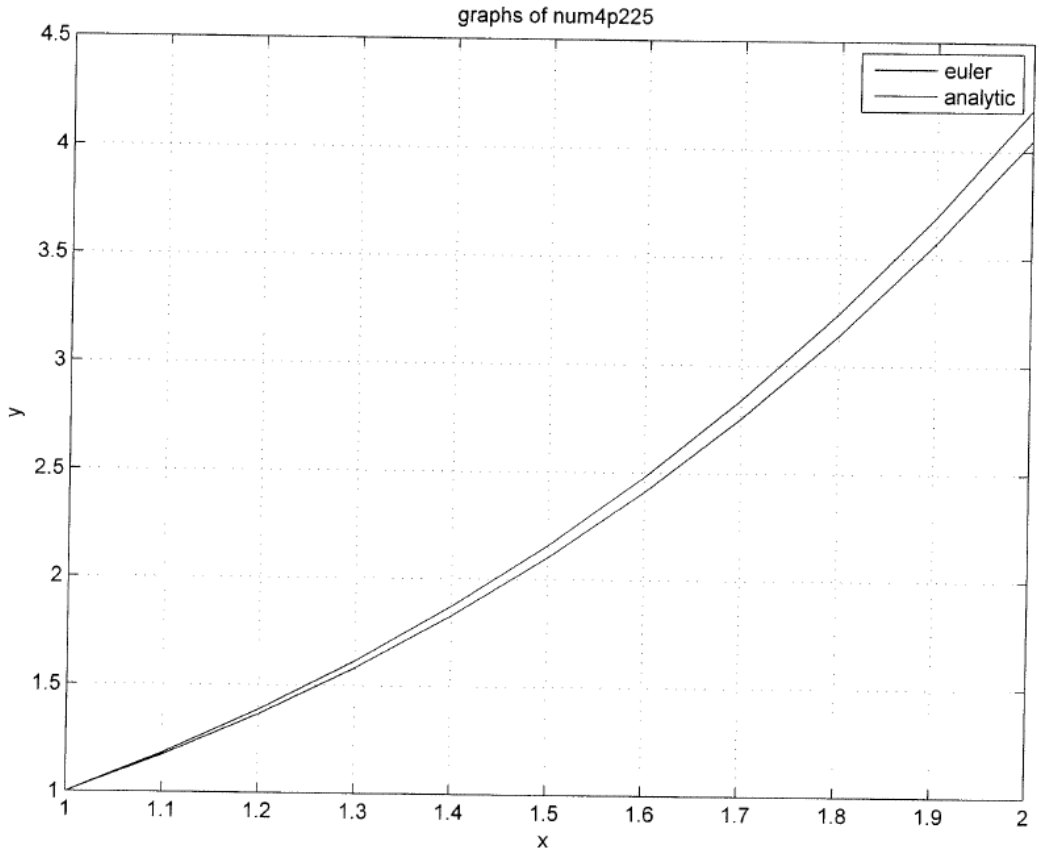
PROBLEM-SOLVING SESSION 1 PART 1

07/08/23 02:22 E:\matlab num4p225\num4p225.m 1 of

```
clear all
a=1;
b=2;
h=0.1;
x=a:h:b;
y(1)=1;
m=(b-a)/h;
for i=1:m
      y(i+1)=y(i)+h*(exp(x(i))-y(i)/x(i));
   end
format long
disp([x' y'])
   1.000000000000000   1.000000000000000
   1.100000000000000   1.171828182845905
   1.200000000000000   1.365714950436375
   1.300000000000000   1.583917063506998
   1.400000000000000   1.829006956153000
   1.500000000000000   2.103883598826539
   1.600000000000000   2.411793599271909
   1.700000000000000   2.756359741756926
   1.800000000000000   3.141615672591004
   1.900000000000000   3.572046214999465
   2.000000000000000   4.052633226858998

z=exp(x)-(exp(x)./x)+1./x;
plot(x,y,x,z)
grid
xlabel('x')
ylabel('y')
title('graphs of num4p225')
legend('euler','analytic')

disp([x' y' z'])
   1.000000000000000   1.000000000000000   1.000000000000000
   1.100000000000000   1.171828182845905   1.182196911267858
   1.200000000000000   1.365714950436375   1.386686153789424
   1.300000000000000   1.583917063506998   1.615991538681364
   1.400000000000000   1.829006956153000   1.872914276241335
   1.500000000000000   2.103883598826539   2.160563023446021
   1.600000000000000   2.411793599271909   2.482387159148168
   1.700000000000000   2.756359741756926   2.842213631887670
   1.800000000000000   3.141615672591004   3.244287761961310
   1.900000000000000   3.572046214999465   3.693318420027022
   2.000000000000000   4.052633226858998   4.194528049465325
```

analytical:

$$y = e^{x}\left(-\frac{e^{x}}{x}\right) + \frac{1}{x}$$


graphs of num4p225
euler
analytic
y
x
4.5
4
3.5
3
2.5
2
1.5
1
1
1.1
1.2
1.3
1.4
1.5
1.6
1.7
1.8
1.9
2

APPENDIX C

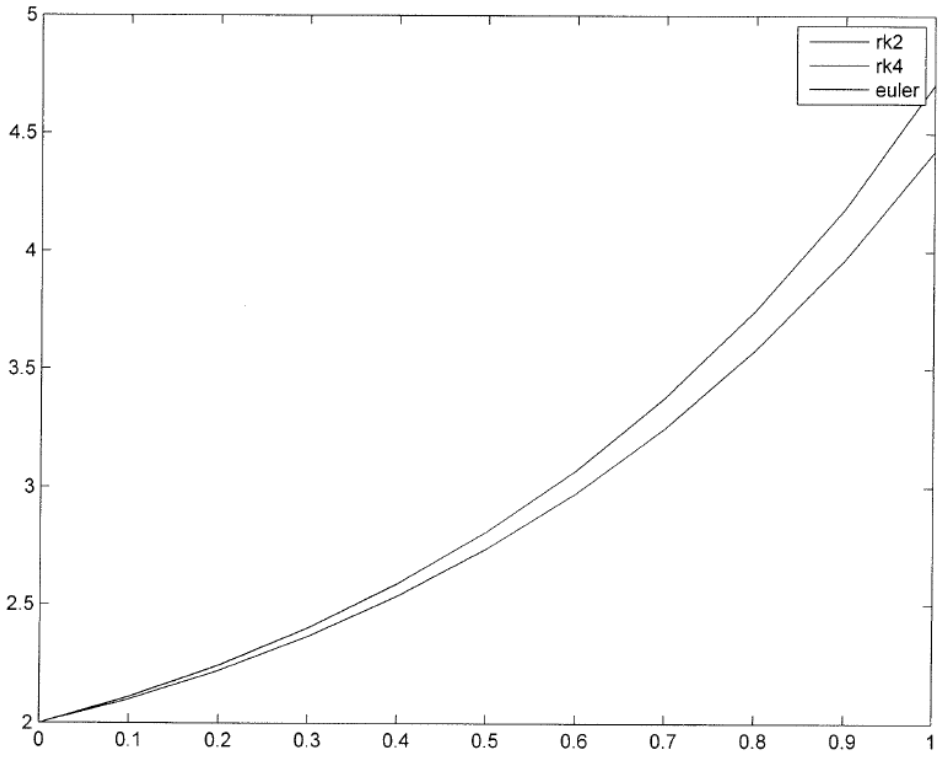
SOLUTIONS TO MATLAB TASKS (IN PROBLEM-SOLVING SESSION 2 – ORIGINAL PROBLEM)

It is notable that Abraham solved the following task of: *dv/dt = 1+vt, v(0) = 2*

```
%PART 1: dv/dt = 1+vt, v(0) = 2

%RK II

clear all
a=0;
b=1;
h=0.1;
v(1)=2;
t=a:h:b;
m=(b-a)/h;
for i=1:m
   k1=h*(1+v(i)*t(i));
   k2=h*(1+(t(i)+h)*(v(i)+k1));
   v(i+1)=v(i)+0.5*(k1+k2);
end
format long
disp([t' v'])

%PART 2: RK IV

%change the variable for the dependent variable v becomes y
%change the names for k1, k2 etc...could use rk1, rk2
y(1)=2;
for i=1:m
   rk1=h*(1+y(i)*t(i));
   rk2=h*(1+(t(i)+0.5*h)*(y(i)+0.5*rk1));
   rk3=h*(1+(t(i)+0.5*h)*(y(i)+0.5*rk2));
   rk4=h*(1+(t(i)+h)*(y(i)+rk3));
   y(i+1)=y(i)+(1/6)*(rk1+2*rk2+2*rk3+rk4);
end
disp([t' v' y'])
plot(t,v,t,y)
legend('rk2','rk4')

%   SOLUTIONS:
%    t                          RK II
     0                  2.000000000000000
```

0.100000000000000 2.110500000000000
0.200000000000000 2.243368550000000
0.300000000000000 2.401625774315000
0.400000000000000 2.589123651880614
0.500000000000000 2.810723339867122
0.600000000000000 3.072529208569614
0.700000000000000 3.382195918464635
0.800000000000000 3.749330760921184
0.900000000000000 4.186021466338801
1.000000000000000 4.707530602239512

%	*t*	*RK II*	*RK IV*
	0	*2.000000000000000*	*2.000000000000000*
	0.100000000000000	*2.110500000000000*	*2.110359001041667*
	0.200000000000000	*2.243368550000000*	*2.243090719562103*
	0.300000000000000	*2.401625774315000*	*2.401219699623060*
	0.400000000000000	*2.589123651880614*	*2.588605854466363*
	0.500000000000000	*2.810723339867122*	*2.810123212831983*
	0.600000000000000	*3.072529208569614*	*3.071896083766579*
	0.700000000000000	*3.382195918464635*	*3.381609274840198*
	0.800000000000000	*3.749330760921184*	*3.748915037636345*
	0.900000000000000	*4.186021466338801*	*4.185967615298488*
	1.000000000000000	*4.707530602239512*	*4.708127510580721*



SOLUTION TO MATLAB TASKS (IN PROBLEM-SOLVING SESSION 2 –MODIFIED)

It is notable that Thembiso and Tumi solved the modified DE of *dv/dt=1+v*sqrt(t), v(0)=2.*

07/10/05 11:00 E:\matlab num4p225\rk2rk4eulermodified.m

```
%dv/dt = 1+v*sqrt(t), v(0) = 2
%RKII

clear all
a=0;
b=1;
h=0.1;
v(1)=2;
t=a:h:b;
m=(b-a)/h;
for i=1:m
    k1=h*(1+v(i)*sqrt(t(i)));
    k2=h*(1+sqrt((t(i)+h))*(v(i)+k1));
    v(i+1)=v(i)+0.5*(k1+k2);
end
format long
disp([t' v'])
%RK IV
%change the variable for the dependent variable v becomes y
%change the names for k1, k2 etc...could use rk1, rk2
y(1)=2;
for i=1:m
    rk1=h*(1+y(i)*sqrt(t(i)));
    rk2=h*(1+(sqrt(t(i)+0.5*h))*(y(i)+0.5*rk1));
    rk3=h*(1+sqrt((t(i)+0.5*h))*(y(i)+0.5*rk2));
    rk4=h*(1+(sqrt(t(i)+h))*(y(i)+rk3));
    y(i+1)=y(i)+(1/6)*(rk1+2*rk2+2*rk3+rk4);
end
disp([t' v' y'])
plot(t,v,t,y)
legend('rk2','rk4')
```

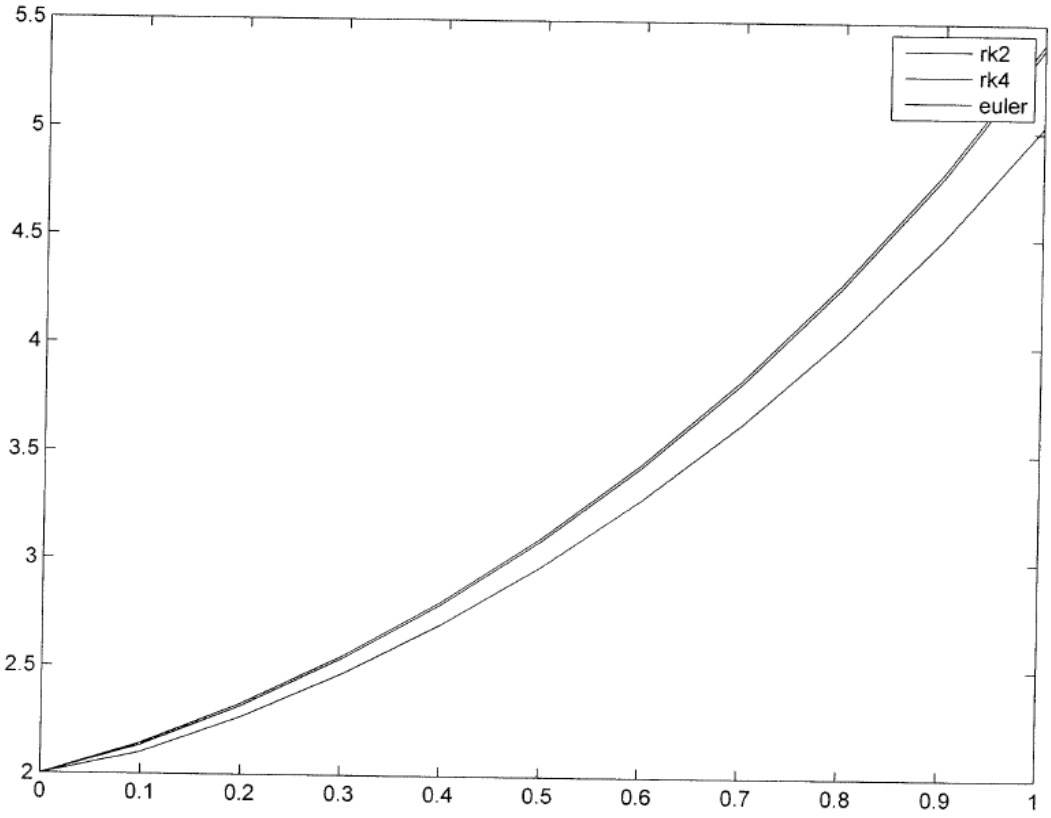
```
%euler
y1(1)=2;
for i=1:m
    y1(i+1)=y1(i)+h*(1+sqrt(t(i))*y1(i));
end
disp([t' v' y' y1'])
plot(t,v,t,y,t,y1)
legend('rk2','rk4','euler')
```

```
%       t                       RK2
 0   2.000000000000000
    0.100000000000000   2.133203915431768
    0.200000000000000   2.318377191443033
    0.300000000000000   2.539287089009187
    0.400000000000000   2.796688090197399
    0.500000000000000   3.093794107157891
    0.600000000000000   3.435344064782421
    0.700000000000000   3.827420208295601
    0.800000000000000   4.277493149280848
    0.900000000000000   4.794579056759158
    1.000000000000000   5.389477548575277
```

```
%          t                       RK2                       RK4
 0    2.000000000000000   2.000000000000000
```

```
0.100000000000000   2.133203915431768   2.142042696765493
0.200000000000000   2.318377191443033   2.328280703616346
0.300000000000000   2.539287089009187   2.550071381216330
0.400000000000000   2.796688090197399   2.808437967256090
0.500000000000000   3.093794107157891   3.106685251298265
0.600000000000000   3.435344064782421   3.449614556568876
0.700000000000000   3.827420208295601   3.843370007220830
0.800000000000000   4.277493149280848   4.295493303198608
0.900000000000000   4.794579056759158   4.815086737499714
1.000000000000000   5.389477548575277   5.413056356644777
```

```
%       t                RKII                RK IV                Euler
0   2.000000000000000   2.000000000000000   2.000000000000000
0.100000000000000   2.133203915431768   2.142042696765493   2.100000000000000
0.200000000000000   2.318377191443033   2.328280703616346   2.266407830863536
0.300000000000000   2.539287089009187   2.550071381216330   2.467764670354510
0.400000000000000   2.796688090197399   2.808437967256090   2.702929708011257
0.500000000000000   3.093794107157891   3.106685251298265   2.973877992664246
0.600000000000000   3.435344064782421   3.449614556568876   3.284162922167679
0.700000000000000   3.827420208295601   3.843370007220830   3.638553088243425
0.800000000000000   4.277493149280848   4.295493303198608   4.042976280578963
0.900000000000000   4.794579056759158   4.815086737499714   4.504591072370716
1.000000000000000   5.389477548575277   5.413056356644777   5.031934103861271
```


rk2
rk4
euler
5.5
5
4.5
4
3.5
3
2.5
2
0
0.1
0.2
0.3
0.4
0.5
0.6
0.7
0.8
0.9
1

REFERENCES

Adler, J. (2001). *Teaching mathematics in multilingual classrooms*. Dordrecht: Kluwer Academic Publishers.

Aldis, G.K., Sidhu, H.S., & Joiner, K.F. (1999). Trial of calculus and Maple with heterogeneous student groups at the Australian Defence Force Academy. *The International Journal of Computer Algebra in Mathematics Education, 6*(3), 167-189.

Ames, C. (1992a). Classrooms: Goals, structures and student motivation. *Journal of Educational Psychology, 84*(3), 261 – 271.

Ames, C. (1992b). Motivation: What teachers need to know? *Teachers College Record, 91*(3), 409 – 421.

Ames, C., & Archer, J. (1988). Achievement goals in the classroom: Students' learning strategies and motivation processes. *Journal of Educational Psychology, 80*(3), 260-267.

Artigue, M. (2002). Learning mathematics in a CAS environment: The genesis of a reflection about instrumentation and the dialectics between technical and conceptual work. *International Journal of Computers for Mathematical Learning, 7*(3), 245 – 274.

Bempechat, J., & Boulay, B.A. (2002). Beyond dichotomous characterisations of student learning: New directions in achievement motivation research. In D.M. McInerney & S. Van Etten (Eds.), *Research on sociocultural influences on motivation and learning* (Vol 1., pp. 17 – 36). Information Age Publishing.

Berger, M. (2006). Mathematical affordances of CAS in an undergraduate mathematics course. In D. Hughes-Hallett, I. Vakalis, & H. Arikan (Eds.), *Proceedings of the 3rd International Conference on the Teaching of Mathematics at the Undergraduate Level*, Istanbul. CD Rom.

Berger, M. (2010). Using CAS to solve a mathematics task: A deconstruction. *Computers & Education, 55*(1), 320-332.

Berger, M., & Cretchley, P. (2005). Technology and mathematics learning at university level: A South African perspective. *African Journal of Research in Mathematics, Science and Technology Education, 9*(2), 97 – 108.

Blume, G. W., & Heid, M. K. (2008). *Research on technology in the teaching and learning of mathematics: Cases and perspectives.* (Vol 2.). Charlotte, NC: Information Age.

Boekaerts, M. (1999). Self-regulated learning: Where we are today? *International Journal of Educational Research, 31*(6), 445-457.

Brickman, S., & Miller, R. (2002). The impact of sociocultural context on future goals and self-regulation. In D.M. McInerney & S. Van Etten (Eds.), *Research on sociocultural influences on motivation and learning* (Vol 1., pp. 119 – 138). Information Age Publishing.

Brown, A., & Dowling, P. (1998). *Doing research/reading research: A mode of interrogation for education.* London: The Falmer Press.

Bulent, B. (2007). Bulent's Screen Recorder, version 4.1.5. Available online, retrieved April 17, 2007 from http://www.thesilver.net/download.asp?id=1

Christiansen, E. (1996). Tamed by a rose: Computers as tools in human activity. In B.A. Nardi (Ed.), *Activity theory and human-computer interaction* (pp. 175 – 198). Cambridge, MA: Massachusetts Institute of Technology.

Cohen, L., Manion, L., & Morrison, K. (2001). *Research methods in education*. London: RoutledgeFalmer.

Confrey, J., Hoyles, C., Jones, D., Kahn, K., Maloney, A.P., Nguyen, K.H., Noss, R., & Pratt, D. (2010). Designing software for mathematical engagement through modeling. In C. Hoyles and J-B. Lagrange (Eds.), *Mathematics education and technology – rethinking the terrain: The 17th ICMI Study* (pp. 19 – 45). Springer: New York.

Coupland, M. (2004). *Learning with new tools*. Unpublished doctoral thesis, The University of Wollongong, Australia. Available online, Australian Digital Theses: http://adt.caul.edu.au/

Cretchley, P., & Galbraith, P. (2002). Mathematics or computers? Confidence or motivation? How do these relate to achievement? *Proceedings of the 2nd International Conference on the Teaching of Mathematics (undergraduate)*. CD, Crete: Wiley, or online, http://www.math.uch.gr/~ictm2/Proceedings/pap318.pdf

Cretchley, P., Harman, C., Ellerton, N., & Fogarty, G. (2000). MATLAB in early undergraduate mathematics: An investigation into the effects of scientific software on learning. *Mathematics Education Research Journal, 12*(3), 219-233.

Crotty, M. (2003). *The foundations of social research.* London: Sage Publications.

DeBellis, V. A., & Goldin, G. A. (2006). Affect and meta-affect in mathematical problem solving: A representational perspective. *Educational Studies in Mathematics, 63*(2), 131–147.

Department of Education. (2004, August 26). *White Paper on e-Education: Transforming learning and teaching through information and communication technologies (ICTs).* Pretoria, South Africa: Department of Education. (Government Gazette, Vol. No. 26734).

Doerr, M., & Pratt, D. (2008). The learning of mathematics and mathematical modelling. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 259 - 285). Charlotte, NC: Information Age.

Drijvers, P. (2000). Students encountering obstacles using CAS. *International Journal of Computers for Mathematical Learning, 5*(3), 189-209.

Drijvers, P., & Gravemeijer, K. (2005). Computer algebra as an instrument: Examples of algebraic schemes. In D. Guin, K. Ruthven & L. Trouche (Eds.), *The didactical challenge of symbolic calculators: Turning a computational device into a mathematical instrument* (pp.163 – 196). New York: Springer.

Drijvers, P., & Trouche, L. (2008). From artifacts to instruments: A theoretical framework

behind the orchestra metaphor. In G. W. Blume & M. K. Heid (Eds.), *Research on technology in the teaching and learning of mathematics: Cases and perspectives* (Vol. 2, pp.363 – 391). Charlotte, NC: Information Age.

Dubinsky, E (2010). Writing computer programs to learn mathematics. In V. Mudaly, R. Mudaly, & M. Van Wyk (Eds.), *Proceedings of the 18th Annual meeting of the Southern African Association for Research in Mathematics, Science and Technology Education in South Africa.* Kwa-Zulu Natal: SAARMSTE. CD Rom.

Dubinsky, E., & Tall, D. (1991). Advanced mathematical thinking and the computer. In D. Tall (Ed.), *Advanced mathematical thinking* (pp. 231-243). Dordrecht, Netherlands: Kluwer Academic Publishers.

Dunham, P., & Hennessy, S. (2008). Equity and use of educational technology in mathematics. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 345-418). Charlotte, NC: Information Age.

Dweck, C. S., & Leggett, E. L. (1988). A social-cognitive approach to motivation and personality. *Psychological Review, 95*(2), 256–273.

Eccles, J. (1983). Expectancies, values and academic behaviours. In J.T. Spence (Ed.), *Achievement and achievement motives* (pp. 75 – 146). San Francisco: Freeman.

Eccles, J. S., & Wigfield, A. (1995). In the mind of the actor: The structure of adolescents' achievement task values and expectancy-related beliefs. *Personality and Social Psychology Bulletin, 21*(3), 215–225.

Elliot, A. J., & McGregor, H. A. (2001). A 2 x 2 achievement goal framework. *Journal of Personality and Social Psychology, 80*(3), 501–519.

Elliot, A.J., & Harackiewicz, J. M. (1996). Approach and avoidance achievement goals and intrinsic motivation: A mediational analysis. *Journal of Personality and Social Psychology, 70*(3), 461 – 475.

Elliot, A.J., Shell, M.M., Henry, K.B., & Maier, M.A. (2005). Achievement goals, performance contingencies, and performance attainment: An experimental test. *Journal of Educational Psychology, 97*(4), 630-640.

Elliot, E. S., & Dweck, C. S. (1988).Goals: An approach to motivation and achievement. *Journal of Personality and Social Psychology, 54*(1), 5–12.

Engeström, Y. (1987). The emergence of learning activity as a historical form of human learning. In Y. Engeström (Ed.), *Learning by expanding: An activity-theoretical approach to developmental research* (Chapter 2, pp. 1-79) Helsinki: Orienta-Konsultit. Available online, retrieved March 16, 2011 from http://lchc.ucsd.edu/mca/Paper/Engeström/expanding/toc.htm

Engeström, Y. (1998). *Cultural-historical activity theory.* Retrieved May 30, 2007 from University of Helsinki, Center for Activity Theory and Developmental Work Research Website: http://www.edu.helsinki.fi/activity/6.htm

Engeström, Y. (1999). Activity theory and individual and social transformation. In Y. Engeström, R. Miettinen, & R-L. Punamaki (Eds.), *Perspectives on activity theory* (pp. 18–38). Cambridge, UK: Cambridge University Press.

Engeström, Y., & Miettinen, R. (1999). Introduction. In Y.Engeström, R. Miettinen, & R-L. Punamaki (Eds.), *Perspectives on activity theory* (pp.1-15). Cambridge, UK: Cambridge University Press.

Evans, J., Morgan, C., & Tsatsaroni, A. (2006). Discursive positioning and emotion in school mathematics practices. *Educational Studies in Mathematics, 63*(2), 209–226.

Foot, K. (2002). Catching an evolving object. *Mind, Culture, and Activity, 9*(2), 132–149.

Forgasz, H.J., Griffith, S., & Tan, H. (2006). Gender, equity, teachers, students and technology use in secondary mathematics classrooms. In C. Hoyles, J. Lagrange, L.H. Son, & N. Sinclair (Eds.), *Proceedings of the 17th ICMI Study Conference: Technology Revisited,* Hanoi University of Technology, 3rd – 8th Dec.

Forgasz, H.J., Vale, C., & Ursini, S. (2010). Technology for mathematics education: Equity, access and agency. In C. Hoyles & J-B. Lagrange (Eds.), *Mathematics education and technology – rethinking the terrain: The 17th ICMI Study* (pp.385-403). Springer: New York.

Galbraith, P. & Haines, C. (1998). Disentangling the nexus: Attitudes to mathematics and technology in a computer learning environment. *Educational Studies in Mathematics*, *36*(3), 275-290.

Gibbs, G.R. (2007). *Analysing Qualitative Data.* Singapore: Sage Publications.

Gordon, S. (1998). *Understanding students learning statistics: An activity theory approach*. Unpublished doctoral thesis, The University of Sydney, Sydney. Available online, Australian Digital Theses: http://adt.caul.edu.au/

Gordon, S. (2004). Understanding students' experiences of statistics in a service course. *Statistics Education Research Journal, 3*(1), 40 – 59.

Guin, D. and Trouche, L. (1999). The complex process of converting tools into mathematical instruments: The case of calculators. *International Journal of Computers for Mathematical Learning, 3*(3), 195 – 227.

Hannula, M.S. (2006). Motivation in mathematics: Goals reflected in emotions. *Educational Studies in Mathematics, 63*(2), 165 – 178.

Harackiewicz, J. M., Barron, K. E., Tauer, J. M., & Elliot, A. J. (2002). Predicting success in college: A longitudinal study of achievement goals and ability measures as predictors of interest and performance from freshman year through graduation. *Journal of Educational Psychology, 94*(3), 562–575.

Harackiewicz, J.M., Durik, A.M., Barron, K.E., Linnenbrink-Garcia, L., & Tauer, J.M. (2008). The role of achievement goals in the development of interest: Reciprocal relations between achievement goals, interest, and performance. *Journal of Educational Psychology, 100*(1), 105–122.

Hardman, J. (2005). An exploratory case study of computer use in a primary school mathematics classroom: New technology, new pedagogy? *Perspectives in Education, 23*(4), 1-13.

Hegel, G.W.F. (1983). *Jena lectures on the philosophy of the spirit. In Hegel and the human spirit: A translation of the Jena lectures of the philosophy of the spirit (1805-6)* (L.Rauch, Ed. & trans.). Detroit, MI: Wayne University Press.

Heid, M. K., & Blume, G. W. (Eds.). (2008a). *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1). Charlotte, NC: Information Age.

Heid, M. K. (1988). Resequencing skills and concepts in applied calculus using the computer as a tool. *Journal for Research in Mathematics Education, 19*(1), 3-25.

Heid, M. K., & Blume, G. W. (2008b). Technology and the development of algebraic understanding. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 55–108). Charlotte, NC: Information Age.

Hitchcock, G. & Hughes, D. (1995). *Research and the teacher.* London: Routledge.

Hitt, F., & Kieran, C. (2009). Constructing knowledge via a peer interaction in a CAS environment with tasks designed from a task-technique, theory perspective. *International Journal of Computers for Mathematical Learning, 14*(2), 121-152.

Hollebrands, K., Laborde, C., & Straber, R. (2008). Technology and the learning of geometry at the secondary level. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 155 - 205). Charlotte, NC: Information Age.

Horst, S.J., Finney, S.J., & Barron, K.E. (2007). Moving beyond academic achievement goal measures: A study of social achievement goals. *Contemporary Educational Psychology, 32*(4), 667–698.

Johnson, D.C. (2000). Algorithmics and programming in the school mathematics curriculum:

Support is waning – Is there still a case to be made? *Education and Information Technologies, 5*(3), 201-214.

Johnston, P.H., & Nicholls, J.G. (1995). Voices we want to hear and voices we don't. *Theory into practice, 34*(2), 94-100.

Julie, C., Leung, A. Thanh, N.C., Posadas, L.S., Sacristan, A.I. & Semenov, A. (2010). Some regional developments in access to and implementation of digital technologies and ICT. In C. Hoyles & J-B. Lagrange (Eds.), *Mathematics education and technology – rethinking the terrain: The 17th ICMI Study* (pp. 361 – 383). Springer: New York.

Kaptelinin, V. (1996). Computer-mediated activity: Functional organs in social and developmental contexts. In B.A. Nardi (Ed.), *Activity theory and human-computer interaction* (pp.45-68). Cambridge, MA: Massachusetts Institute of Technology.

Kaptelinin, V. (2005). The object of activity: Making sense of the sense-maker. *Mind, Culture and Activity, 12*(1), 4 - 18.

Kieran, C., & Saldanha, L. (2008). Designing tasks for the codevelopment of conceptual and technical knowledge in CAS activity: An example from factoring. In G. W. Blume & M. K. Heid (Eds.), *Research on technology in the teaching and learning of mathematics: Cases and perspectives* (Vol. 2, pp.393 – 414). Charlotte, NC: Information Age.

Kieran, C., Drijvers, P., Boileau, A., Hitt, F., Tanguay, D., Saldanha, L., et al. (2006). The co-emergence of machine techniques, paper-and-pencil techniques, and theoretical reflection: A study of CAS use in secondary school algebra. *International Journal of Computers for Mathematical Learning, 11*(2), 205 - 263.

Kirchner, E. (2008). *Learning guide Mathematics III MAT2BW2.* Johannesburg: Faculty of Science, University of Johannesburg.

Kotze, H. (2007). *Differential equations with applications.* (unpublished notes, University of Johannesburg).

Kuuti, K. (1996). Activity theory as a potential framework for human-computer interaction

research. In B.A. Nardi (Ed.), *Activity theory and human-computer interaction* (pp. 17 – 44). Cambridge, MA: Massachusetts Institute of Technology.

Kvale, S. (1996). *Interviews: An introduction to qualitative research interviewing*. London: Sage Publications.

Lagrange, J-B. (1999). Complex calculators in the classroom: Theoretical and practical reflections on teaching pre-calculus. *International Journal of Computers for Mathematical Learning, 4*(1), 51- 81.

Lautenbach, G.V. (2005). *Lecturers' changing epistemologies and pedagogies during engagement with information and communication technology in an education faculty*. Unpublished doctoral thesis, University of Johannesburg, Johannesburg.

Leder, G., & Forgasz, H. (2002). Measuring mathematical beliefs. In G. Leder, E. Pehkonen & G. Törner (Eds.), *Beliefs: A hidden variable in mathematics education?* (pp. 95-114). Dordrecht: Kluwer.

Leontiev, A.N. (1978). *Activity, consciousness and personality*. Engelwood Cliffs: Prentice-Hall.

Leontiev, A.N. (1981). The problem of activity in psychology. In J.V. Wertsh (Ed.), *The concept of activity in Soviet psychology* (pp. 37 – 71). New York: M.E. Sharpe.

Leontiev, A.N. (1994). The development of the learning motive in children. *Multidisciplinary Newsletter for Activity Theory, 13*(14), 24-29.

Lim, C.P., & Chai, C.S. (2004). An activity theoretical approach to research of ICT integration in Singapore schools: Orienting activities and learner autonomy. *Computers and Education, 43*(1), 215-236.

Lincoln, Y.S., & Guba, E.G. (1985). *Naturalistic inquiry*. Thousand Oaks, California: Sage Publications.

Lompscher, J. (1999). Motivation and activity. *European Journal of Psychology of Education,*

Vol. 14(1), 11-22.

Machin, M.C., & Rivero, R.D. (2002). Students' attitudes towards mathematics and computer when using Derive in the learning of calculus concepts. *The International Journal of Computer Algebra in Mathematics Education, 9*(4), 259 – 283. Academic Research Library.

MATLAB & SIMULINK, Release 2007a, The Mathworks, www.mathworks.com/patents.

Marx, K. (1993). *Grundrisse.* London: Penguin Books.

Maxwell, J.A. (2002). Understanding and validity in qualitative research. In B.M. Miles & A.M. Huberman (Eds.), *The qualitative researcher's companion.* London: Sage Publications.

McMillan, J.H., & Schumacher, S. (2006). *Research in education: Evidence-based inquiry.* (6th ed.). Boston: Pearson Education, Inc.

Meece, J. L. (1994). The role of motivation in self-regulated learning. In D. H. Schunk & B. J. Zimmerman (Eds.), *Self-regulation of learning and performance: Issues and educational applications*. (pp. 25 – 44). Hillsdale, NJ: Lawrence Erlbaum Associates, Publishers.

Midgley, C., Kaplan, A., Middleton, M., Maehr, M.L., Urdan, T., Anderman, L.H,, et al. (1998). The development and validation of scales assessing students' achievement goal orientations. *Contemporary Educational Psychology, 23*(2), 113–131.

Miettinen, R. (2005). Object of Activity and Individual Motivation. *Mind, Culture and Activity, 12*(1), 52-69.

Miles, B.M., & Huberman, A.M. (1994). *Qualitative data analysis: An expanded sourcebooks*. (2nd ed.). London: Sage Publication.

Monaghan, J. (2005), Computer Algebra, Instrumentation and the Anthropological Approach. Paper presented at the 4th CAME conference. Retrieved March 24, 2007 from http://www.lonklab.ac.uk/came/events/CAME4/index.html

Nardi, B.A. (1996a). Activity theory and human computer-interaction. In B.A. Nardi (Ed.), *Activity theory and human-computer interaction* (pp. 7 – 16). Cambridge, MA: Massachusetts Institute of Technology.

Nardi, B.A. (1996b). *Context and consciousness: Activity theory and human-computer interaction.* Cambridge, MA: Massachusetts Institute of Technology.

Nardi, B.A. (1996c). Studying context: A comparison of activity theory, situated action models, and distributed cognition. In B.A. Nardi (Ed.), *Activity theory and human-computer interaction* (pp. 69 – 102). Cambridge, MA: Massachusetts Institute of Technology.

Nardi, B.A. (2005). Objects of desire: Power and passion in collaborative activity. *Mind, Culture and Activity, 12*(1), 37 - 51.

Neuman, L.W. (2003). *Social research methods: Qualitative and quantitative approaches*. (5th ed.). Boston: Pearson Education Company.

Nicholls, J. G. (1984). Achievement motivation: Conceptions of ability, subjective experience, task choice, and performance. *Psychological Review, 91*(3), 328–346.

Norman, D.A. (1993). *Things that make us smart: Defending human attributes in the age of the machine.* Reading, Massachusetts: Addison-Wesley Publishing Company.

Noss, R., Bakker, A., Hoyles, C., & Kent, P. (2007). Situating graphs as workplace knowledge. *Educational Studies in Mathematics*, *65*(3), 367-384.

Opie, C. (2004). *Doing educational research*. London: Sage Publications.

Periasamy, J. (2008a). An exploration of second year university maths students' motivations towards the use of technology in solving differential equations. In. M.V. Polaki, T. Mokuku & T. Nyabanyaba (Eds.). *Proceedings of the 16th Annual Conference of the Southern African Association for Research in Maths, Science and Technology Education, 14th–17th Jan, 2008.* Maseru, Lesotho: SAARMSTE.

Periasamy, J. (2008b). An exploration of students' experiences in the learning of mathematics mediated by computer algebra systems. In W-C Yang, M. Majewski, T. Alwis, & K. Khairiree (Eds.), *Proceedings of the 13th Asian Technology Conference in Mathematics*, Suan Sunandha Rajabhat University, Thailand: ATCM.

Periasamy, J. (2010). A student's actions when he learns mathematics using MATLAB. In V. Mudaly, R. Mudaly, & M. Van Wyk (Eds.), *Proceedings of the 18th Annual meeting of the Southern African Association for Research in Mathematics, Science and Technology Education in South Africa.* Kwa-Zulu Natal: SAARMSTE. CD Rom.

Pierce, R. & Stacey, K. (2006). Enhancing the image of mathematics by association with simple pleasures from real world contexts. *Zentralbrlatt fur der Mathematick (ZDM) Electronic-Only Publication*, 38(3), 214-225.

Pierce, R., & Stacey, K. (2004). A framework for monitoring progress and planning teaching towards the effective use of computer algebra systems. *International Journal of Computers for Mathematical Learning, 9*(1), 59 – 93.

Pierce, R., Stacey, K., & Barkatsas, A. (2007). A scale for monitoring students' attitudes to learning mathematics with technology. *Computers & Education*, *48*(2), 285-300.

Pietsch, J. (2005). *Collaborative learning in mathematics: Realising reform principles in traditional classrooms.* Unpublished doctoral thesis, The University of Sydney, Sydney. Available online, Australian Digital Theses: http://adt.caul.edu.au/

Pintrich, P. R., & De Groot, E. (1990). Motivational and self-regulated learning components of classroom academic performance. *Journal of Educational Psychology, 82*(1), 33–40.

Pintrich, P. R., Conley, A.M. & Kempler, T.M. (2003). Current issues in achievement goal theory and research. *International Journal of Educational Research, 39*(4-5), 319–337.

Pintrich, P.R. (2003). A motivational science perspective on the role of student motivation in learning and teaching contexts. *Journal of Educational Psychology, 95*(4), 667-686.

Popov, O. (1998). *Some educational implications of the cultural-historical activity theory.*

Retrieved, November 22, 2006, from http://www.educ.umu.se/~popov/chat.htm

Ratner, C. (1996). Activity as a key concept for cultural psychology. *Cultural Psychology, 2*(4), 407-434.

RITEMATHS. (n.d.) RITEMATHS project web site. Retrieved August 10, 2009, from http://extranet.edfac.unimelb.edu.au/DSME/RITEMATHS/

Roth, W-M. (2003). Competent workplace mathematics: How signs become transparent in use. *International Journal of Computers for Mathematical Learning, 8*(3), 161-189.

Roth, W-M. (2007a). Emotion at Work: A Contribution to third-generation cultural-historical activity theory. *Mind, Culture and Activity*, *14*(1-2), 40–63.

Roth, W-M. (2007b). On mediation: Toward a cultural-historical understanding. *Theory & Psychology, 17*(5), 655-680.

Roth, W-M., & Lee, Y.J. (2004). Interpreting unfamiliar graphs: A generative, activity theoretic model. *Educational Studies in Mathematics, 57*(2), 265-290.

Roth, W-M., & Lee, Y.-J. (2007). "Vygotsky's neglected legacy": Cultural-historical activity theory. *Review of Educational Research, 77*(2), 186 -232.

Roth, W-M., Tobin, K., Elmesky, R., Carambo, C., McKnight, Y-M. & Beers, J. (2004). Re/making identities in the praxis of urban schooling: A cultural historical perspective. *Mind, Culture and Activity, 11*(1), 48-69.

Russell, D.R. (2002). Looking beyond the interface: Activity theory and distributed learning. In M.R. Lea & K. Nicholl (Eds.), *Distributed learning*: *Social and cultural approaches to practice* (pp. 64–82). New York: RoutledgeFalmer.

Ruthven, K. (2002). Instrumenting mathematical activity: Reflections on key studies of the educational use of computer algebra systems. *International Journal of Computers for Mathematical Learning, 7*(3), 275 - 291.

Ruthven, K., & Hennessy, S. (2002). A practioner model of the use of computer-based tools and resources to support mathematics teaching and learning. *Educational Studies in Mathematics, 49*(1), 47-88.

Schunk, D.H., Pintrich, P.R., & Meece, J.L. (2008). *Motivation in education – theory research and applications.* (3rd ed.). Upper Saddle River, NJ: Merrill-Prentice Hall.

Stevenson, I. (2008). Tool, tutor, environment or resource: Exploring metaphors for digital technology and pedagogy using activity theory. *Computers & Education, 51*(1), 836-853.

Tall, D., Smith, D., & Piez, C. (2008). Technology and calculus. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 207–258). Charlotte, NC: Information Age Publishing.

Triandis, H.C. (2002). Modern education needs cross-cultural psychology. In D.M. McInerney & S. Van Etten (Eds.), *Research on sociocultural influences on motivation and learning* (Vol 1., pp. 1 – 13). Information Age Publishing.

Trouche, L. (2004). Managing the complexity of human/machine interactions in computerized learning environments: Guiding students' command process through instrumental orchestrations. *International Journal of Computers for Mathematical Learning, 9*(3), 281 - 307.

Turner, J.C. (2001). Using context to enrich and challenge our understanding of motivational theory. In S. Volet & S. Jarvela (Eds.), *Motivation in learning contexts: theoretical and methodological implications* (pp.85 – 104). Amsterdam: Pergamon.

Urdan, T., & Giancarlo, C. (2002). A comparison of motivational and critical thinking orientations across ethnic groups. In D.M. McInerney & S. Van Etten (Eds.), *Research on sociocultural influences on motivation and learning* (Vol 1., pp. 37 – 60). Information Age Publishing.

Urdan, T., & Maehr, M. (1995). Beyond a two-goal theory of motivation and achievement: A case for social goals. *Review of Educational Research, 65*(3), 213 – 243.

Ursini, S., & Sacristán, A.I. (2006). On the role and aim of digital technologies for mathematical learning: Experiences and reflections derived from the implementation of computational technologies in Mexican mathematics classrooms. In C. Hoyles, J. Lagrange, L.H. Son, & N. Sinclair (Eds.), *Proceedings of the 17th ICMI Study Conference: Technology Revisited,* Hanoi University of Technology, 3rd – 8th Dec, 2006.

Vale, C., & Leder, G. (2004). Student views of computer based mathematics in the middle years: Does gender make a difference? *Educational Studies in Mathematics, 56*(3), 287-312.

Vygotsky, L.S. (1978). *Mind in society: The development of higher psychological processes.* Cambridge, MA: Harvard University Press.

Watkins, G., McInerney, D., Lee, C., Akande, A. & Regmi, M. (2002). Motivation and learning strategies: A cross-cultural perspective. In D.M. McInerney & S. Van Etten (Eds.), *Research on sociocultural influences on motivation and learning* (Vol 2., pp. 329 – 343). Information Age Publishing.

Wentzel, K. (2000). What is it that I am trying to achieve? Classroom goals from a content perspective. *Contemporary Educational Psychology, 25*(1), *105* – 115.

Wertsch, J. V. (1981a). The concept of activity in Soviet psychology: An introduction. In J.V. Wertsch (Ed.), *The concept of activity in Soviet psychology*, (pp. 3-35). New York: M. E. Sharpe, Inc.

Wertsch, J. V. (1985). *Vygotsky and the social formation of mind.* Cambridge, Massachusetts: Harvard University Press.

Wertsch, J. V. (1995). The need for action in sociocultural research. In J. V. Wertsch, P. del Río & A. Alvarez (Eds.), *Sociocultural studies of mind* (pp. 1-34). New York: Cambridge University Press.

Wertsch, J.V. (1981b). The problem of activity in psychology: Editor's introduction. In J.V. Wertsh (Ed.), *The concept of activity in Soviet psychology* (pp. 37 – 40). New York: M.E. Sharpe.

Wertsch, J.V. (1998). *Mind as action.* New York, Oxford: Oxford University Press.

Wigfield, A., & Eccles, J. S. (1992). The development of achievement task values: A theoretical analysis. *Developmental Review, 12*(3), 265–310.

Yamagata-Lynch, L.C. (2003). Using activity theory as an analytic lens for examining technology professional development in schools. *Mind, Culture and Activity, 10*(2), 100 – 119.

Yerushalmy, M. (1991). Student perceptions of aspects of algebraic function using multiple representation software. *Journal of Computer Assisted Learning, 7*(1), 42-57.

Yin, R.K. (2009). *Case study research: Design and methods.* (4th ed.). London: Sage Publications.

Zan, R., Brown, L. C., Evans, J., & Hannula, M. S. (Guests Eds.) (2006). Affect in mathematics education: An introduction. [PME special issue]. *Educational Studies in Mathematics*, 63(2), 113-121.

Zbiek, R.M., & Hollebrands, K. (2008). A research-informed view of the process of incorporating mathematics technology into classroom practice by inservice and prospective teachers. In M. K. Heid & G. W. Blume (Eds.), *Research on technology and the teaching and learning of mathematics: Research syntheses* (Vol. 1, pp. 287 - 344). Charlotte, NC: Information Age.

Zinchenko, V. P. (1995). Cultural-historical psychology and the psychological theory of activity: Retrospect and prospect. In J. V. Wertsch, P. del Rio & A. Alvarez (Eds.), *Sociocultural studies of mind* (pp. 37-55). New York: Cambridge University Press.