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CO–EVOLUTIONARY APPROACH TO THE MANPOWER PLANNING PROBLEM

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A dissertation submitted to the Faculty of Science, University of the Witwatersrand, in fulfilment of the requirements for the degree of Master of Science

Johannesburg, 2010

Abstract. This dissertation will investigate a realistically sized manpower planning problem. To perform the analysis, an appropriate model will be presented to solve a specific type of manpower problem. It allows for recruitment, dismissal, training of workers to different types, and multi–skilled workers. The training is enforced in the model through time delay. The model also includes consistency, demand and bound constraints that the solutions must satisfy. The solution of the model will then be investigated using a novel approach, the competitive co–evolutionary algorithm. The algorithm generates solutions in the same way a genetic algorithm does, and implements a predator–prey procedure to ensure satisfaction of the constraints. A numerical example will then be solved to illustrate the usefulness of the co–evolutionary algorithm when applied to this type of manpower problem.

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

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(Signature of candidate)

_____ day of _____ _____

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Notation

$\mathbb{Z}_+^N$	The set of all positive integers, including zero
$\text{card}(S)$	The cardinality of the set S

CHAPTER 1

Introduction

1. The manpower planning problem

Manpower planning is a problem faced by an organisation that requires an employment policy that ensures that its demands for the future are met [**1**]. These demands are often conflicting and solutions are difficult to find. These problems often consist of a large number of decisions that need to be made, making it necessary to model the problem mathematically.

The usefulness and scope of manpower planning has increased over the decades, given the increasing size and demands of organisations. Increased competition and a general need to minimise cost has spurred growth in the areas of research linked to manpower planning. Advances in various areas of mathematics have contributed to the ability to deal with the manpower planning problem more effectively. Large organisations, human resourcing departments and military departments consider manpower planning one of their central concerns and have invested large amounts of capital in dealing with the problem.

Manpower planning problems can be cast as optimisation problems [**2**]. These deal with the employment policy, the assignment of workers to tasks, predicting the future demand for manpower, and predicting the future supply for manpower. In some cases the supply and demand forecasting problems are treated as inputs to a more central problem: the employment policy and assignment of workers [**3**].

A paper by Edwards [**2**] surveyed the manpower planning models at the time. At this time several facets of the problem had already been identified as being crucial. These involve forecasting the demand for tasks in the future, forecasting the future supply of manpower and adjusting the organisation's policies to bridge the gap between these two facets.

Today, data collection is widespread and there are many methods available for accurately forecasting future demand. Simple methods for accomplishing this include

extrapolation and regression. The variables representing the demand for tasks can be stochastic or deterministic. A stochastic representation of these variables is more natural given the nature of forecasting demand for the future, with the uncertainty in demand increasing further into the future. Stochastic variables are usually associated with some distribution which is defined by some average value and an associated error. In the case of the normal distribution these are given by the mean and variance respectively. As a first approximation, these stochastic variables can be replaced by deterministic ones, with the demand values taking on some sort of average measure of the stochastic variables. The measure of error can then be used to simulate the true stochastic nature of the demand, as is done in Monte Carlo simulations.

Assuming that the forecasting problem is solved, the manpower planning problem can be split naturally into two remaining problems: the employment policy and assignment. The employment policy problem deals with ensuring that sufficient workers are available for assignment to the various tasks through the hiring, dismissal and training of workers while the cost of the process is minimised over a period of time. The assignment problem then deals with assigning the available workers to the various tasks.

The problems investigated are viewed at the macro level [**1, 3, 4**]. In this way they only track the total number of a specific worker type and not specific individuals. Models that track specific individuals would be examples of models that operate at the micro level. The tendency to use macro models is mainly to ensure the problem is still computationally tractable, but this approach has other benefits. Often the computational solutions to the manpower problem are not meant to be strict instructions of how an organisation should implement its employment policy, but are meant to act as guidelines. Macro models can assist in providing these guidelines by lumping workers into types, where these types are defined purely by a set of shared characteristics. This enforces an 'equality' amongst workers of the same type. Should an organisation require a micro view of its workers, the solution of a macro level based problem can be used to identify individuals and implement a preferred micro level solution.

Choosing an appropriate manpower planning model to represent the manpower planning problem defines the problem mathematically. The properties of the manpower problem suggest a certain natural structure. Quantities that are strictly integers pervade the problem, in particular the number of workers hired, trained,

and dismissed. The decision variables and by implication the state variables are therefore restricted to take on integer values.

The problem is generally formulated with a single objective function: the total cost of the process over a finite time horizon. The objective is then to minimise this cost, such that the constraints of the problem are met.

The manpower problem can involve optimising over one or more objective functions subject to certain constraints, that may be non–linear. The employment policy sub–problem involves recruitment, dismissal and training of workers and the assignment sub–problem involves assigning the available workers to specific tasks that must be performed. Both of these sub–problems must be addressed in such a way that the constraints of the problem are satisfied.

Models invariably have their drawbacks, and manpower models are no exception. Many models are formulated for extremely specific cases, with no possibility for extending the models to other types of problems. This is often illustrated by the assumptions the models make, such as homogeneous skill of the work force and lack of training mechanisms. These restrict the applicability of the models. More complex models rely on additional information: the ability to group workers and tasks into types, the ability to codify training programs, knowledge of demand for tasks in the future, and knowledge of restrictions on variables.

The size of the problem affects the solving of the model computationally. In reality, manpower problems are large due to the number of decision and state variables (which depend heavily on the planning horizon and number of workers in the problem). Some algorithms are developed exclusively for numerical examples which are small compared to the problems faced in reality.

Over the years several approaches to solving the manpower planning problem (or one of its sub–problems) have been introduced. Many use models that have some common properties while others deviate in drastic ways to solve very particular types of manpower planning problems.

Grinold [**5**] presents a manpower planning model that includes uncertain requirements for future demand. Finite Markov chains are used to represent the demand for manpower at each time step in the model. The objective functions chosen are based on minimising the expected value of a function of the difference between the supply and demand for manpower. The objective functions used in the problem are chosen to be quadratic for two reasons: they render the model solvable and

are also a reasonable measure of the systems actual performance versus its ideal performance. Time delayed training is modelled and continuation rates are used to represent the retention, promotion and retirement of workers in the system.

Poornachandra Rao [**6**] follows a dynamic programming approach to the manpower problem, seeking to minimise the total cost of the process. The model used is analogous to the Wagner-Whitten model in inventory management. He critically identifies recruitment, over–staffing, under–staffing, dismissal, recruitment and retention costs for use in the model. The model is limited in that it is isolated from the relevant organisations operating policies and the constraints that affect the manpower problem.

Cai and Li [**4**] make use of a genetic algorithm for solving the manpower problem. The model introduced allows for mixed skilled workers and does not allow for training of workers. Multiple objective functions are specified and integrated into the genetic algorithm to perform the various evolutionary operations. The primary objective is to minimise the total cost of the process, the secondary objective is to maximise the surplus of staff when assigning costs are approximately the same, and the tertiary objective is to reduce the variation of staff surplus over the time periods in the model. The genetic algorithm then uses all of these objectives to select breeders, and a multi–point crossover based on the Hamming distance between schedules is used to perform the crossover. A heuristic is used to reduce the infeasibility created during crossover. Cai and Li [**4**] noted that it is extremely important to find feasible regions before running the genetic algorithm and that large planning horizons would require alternative algorithms to solve the problem.

Zanakis and Maret [**1**] introduced a Markovian goal programming approach to the manpower problem, making use of a Markov type model based on historical probabilities. The model allows for hiring, dismissal, retirements and other stochastic processes. Guidelines specific to an instance of the problem are used to formulate a linear goal programming model. The objective function used is to minimise the overall cost of the process, and the model was solved for a single time step.

Brusco and Jacobs [**7**] used a simulated annealing approach to solve a scheduling problem. The problem is basically a shift assignment problem where the objective is to minimise the number of full time employees while satisfying the demand that is forecast. The model makes use of bounds in the problem, and does not consider constraints affecting the manpower problem.

Narasimhan [**8**] presented a custom algorithm to solve the assignment problem. The model proposed introduces various constraints on the system. A hierarchical model is used for workers to perform tasks, and the model does not allow for training of workers. While the algorithm is only applicable to the type of problem described, it has the desirable property where the computational time of the algorithm increases linearly with the problem size.

Lee, Cai and Teo [**3**] presented the manpower problem as a time delayed optimal control problem in discrete time. The model used allows for multi–skilled workers, time delayed training, hiring, dismissal and assignment of workers. The objective is to minimise the overall cost of the process. The problem initially consists of integer decision variables and is transformed into a discrete valued optimal control problem. Another transform is then used to reduce this problem to a standard optimisation problem involving continuous variables, allowing it to be solved by standard optimisation techniques.

Gass [**9**] gives an overview of military manpower planning models, highlighting Markov models, network–flow models and multi–year models. The network–flow models are powerful descriptive tools in that they do not require equations of state to display the transform of flows. These models however, break down when time dependent problems are considered. The multi–year models solve this shortcoming by transforming a previous time steps' workers.

The manpower model used in this paper is cast as a non–linear global optimisation problem with non–linear constraints and a single objective function. While very good global optimisation algorithms exist for particular problems, there are few that can handle constraints without additional assumptions. Those that do include penalty methods and heuristic methods. The penalty methods require other sub-problems be solved, and are not completely reliable [**10**]. The heuristic methods are used to alter infeasible solutions via some procedure, until they are feasible [**4**].

The problem of solving a manpower model is thus reduced to the task of finding an appropriate model, and a global optimisation algorithm capable of handling non–linear constraints. The dissertation will deal with the optimal employment policy and assignment of workers to tasks at a macro level. The problem will be cast as a non–linear integer programming problem with a single objective function, the overall cost of the process. The goal is to minimise the cost of the process and satisfy the constraints of the problem. All variables in the model are deterministic. The demand is assumed to be some maximum value of the expected demand, or an average value of the stochastic demand.

2. Co–evolutionary algorithms

There are a variety of ways in which one can apply global optimisation algorithms to handle constraints, yet the approaches yield mixed results depending on which problems they are applied to [**11**].

A common and easily implementable approach is to make use of exterior penalty methods. This approach introduces an auxiliary function with parameters that is added to the objective function, as shown by Luenberger in [**12**] and Soleimani-damaneh in [**13**]. The auxiliary function is chosen so as to penalise a solution heavily for not satisfying the constraints, while leaving the objective function unchanged for feasible solutions. There are several problems with this approach. The first is that the distortion of the objective function assumes some measure of feasibility of a solution, using this to make solutions gradually 'more feasible'. The second is that parameters introduced into the objective function must either be assumed or solved for as part of another optimisation problem. An example is given in [**10**], which describes a genetic algorithm that relies on penalty methods. This approach illustrates how the distortion of the objective function does not always yield satisfactory results.

Another approach to deal with the constraints is by using exterior point methods. These methods generate solutions that are not guaranteed to be feasible, and make use of the infeasible solutions that have been generated to generate solutions that are feasible. This process is often gradual, as the set of initial solutions must be combined to produce new solutions that are still not feasible. Some measure of feasibility is then used to gradually generate solutions that lie inside the feasible regions.

A naive way to implement a genetic algorithm to solve a problem with constraints is to reject the infeasible solutions at each iteration. This approach severely limits the size of the solution population in some problems. This approach may result in a solution population with an extremely low number of feasible candidates, if any at all. This is especially severe in problems where the ratio of feasible solutions to the total number of solutions in the problem domain is small. Problems of this nature tread a fine line between optimisation problems and satisfiability problems. Exterior point methods are more reliable in dealing with problems such as these, since they do not rely on solutions being feasible initially. In the context of the genetic algorithm, there is a need for diversity in the population of initial solutions, as well as subsequent generations. Solutions that are not initially feasible may

evolve to yield candidates that are more fit, and additional evolutionary operations may result in feasibility.

Co–evolutionary algorithms represent a class of new algorithms that have shown promise in global optimisation. They attempt to simulate the relationship between multiple species that interact with each other and adapt as a result of these interactions [**14**]. These include co–operative algorithms and competitive algorithms.

Co–operative co–evolutionary algorithms split the problem domain into sub–domains and breed species that form partial solutions to the problem. A merging of individuals from the various species then forms a complete solution to the problem. The separate evolution of these species can be linked to the varying of a single variable while all other variables are held constant. The separability of the variables in the objective function underlies this approach.

The competitive algorithms rely on evolving two types of populations, predators and prey. These populations evolve in such a manner that one is always trying to outdo the other. These dynamics can result in what is termed an arms race in the literature [**15**]. In this type of algorithm, the fitness of a species does not only depend on the individual's fitness, but also on the way it interacts with members of the other species [**16**].

The underlying mechanisms for co–evolutionary algorithms are not well understood. The beginnings of some progress in this area have been made by using a dynamical systems approach to study the effects of various processes on the ways in which the algorithms behave.

Westra and Paredis [**15**] used a co–evolutionary genetic algorithm to solve a path planning problem in two dimensional space with static obstacles. This made use of one population of goal seeking agents and another population of difficult starting points. It was found that the co–evolutionary genetic algorithm outperformed a normal genetic algorithm due to its ability to move out of local minima.

Potter and De Jong [**17**] introduced a general model for implementing the co–evolution of co–operating species. This was done by creating several sub–populations, each of which consisted of members that only represented a sub component of the complete solution. These sub–populations evolve separately and form fit, complete solutions. A critical finding was that the co–evolutionary algorithm performed worse on problems with interacting variables than a traditional genetic algorithm.

In [**18**], Popovici and De Jong investigated the cooperative co–evolutionary dynamics via fitness landscapes. A dynamical systems approach was used to analyse patterns that could lead to performance gains for the algorithm. The investigation found that the size of the population chosen and implementations of elitism affect the algorithm's performance in non–trivial ways.

Jansen and Wieg [**19**] provided evidence that the separability of the objective function in a problem is not the only property required to make a co–operative co–evolutionary algorithm beneficial. They noted that the explorative capabilities of such an algorithm are also an important factor. It was found that the expected frequency of mutation in a co–operative co–evolutionary algorithm is higher than a traditional evolutionary algorithm, and is the cause of this increased explorative capability.

Wiegand, Liles and De Jong [**20**] performed an empirical analysis of collaborative methods in cooperative co–evolutionary algorithms. The focus was on how the algorithm select collaborators for evaluation. It was concluded that for static objective functions, an optimistic approach on collaboration is best.

Iorio and Li [**21**] focused on the application of the cooperative co–evolutionary algorithm to multi–objective problems. A method of non–dominated sorting was used to reward successful collaborations. It was found that the algorithm was successful due to the fact that it acquired a large number of diverse, non–dominated solutions.

This dissertation will use a co–evolutionary algorithm to solve a manpower planning model. The competitive co–evolutionary algorithm will model the solutions as prey and the constraints of the problem as predators. These populations will then interact in the same way as they would in a traditional predator–prey model.

CHAPTER 2

Overview of the model

This chapter presents the essentials of the manpower planning model used. Details and consequences of the model are discussed in following chapters.

The manpower problem considered will consist of the the employment and assignment sub–problems.

We model the manpower planning problem at the macro level by introducing the concept of a worker type. We assume the problem consists of N worker types. All workers of a particular type are indistinguishable from one another.

For the employment problem, we need to ensure that the cost of the entire process is minimised while satisfying the constraints of the problem. We are required to make decisions such as the number of workers of a particular type to hire, dismiss and train at various points in time. The intervals at which these decisions are made can be assumed to be discrete, as there is normally a set interval in organisations to perform these types of actions. For example, workers are often hired or dismissed at the end of a month. Let t represent the current time step of the problem, where $t = 0, \ldots, T$ and T represents the final time up to which we want to solve the manpower problem. Our problem therefore consists of $T + 1$ time steps.

The characterisation above allows us to introduce employment variables for the problem. Let $h_i(t)$ be the number of type i workers hired at time t, $f_i(t)$ be the number of type i workers dismissed at time t and $u_{ij}(t)$ be the number of type i workers to be trained to type j at time t. These decision variables provide a basis for expressing the decisions we make regarding our employment policy. With these variables defined, some basic costs can be considered. Let $c_i^h(t)$ be the cost of hiring a new worker of type i at time step t. Then the total hiring costs for worker type i at that time step are

$$h_i(t)c_i^h(t).$$

Similarly, we can define the cost for dismissing a type i worker at time t as $c_i^f(t)$. The total dismissal costs for type i workers at that time step are

$$f_i(t)c_i^f(t).$$

To implement training in the model, each worker type i has associated with it a set of worker types $L(i)$. Given $i \neq j$, the interpretation of this set is that a worker of type i may be trained to a worker of type j if $j \in L(i)$.

Let us define the cost for sending a type i worker for training to a type j at time t as $c_{ij}^u(t)$. The total cost for training worker type i at time t must be summed over all worker types that worker type i can be trained to. The total training costs for worker type i at that time step are then

$$\sum_{j \in L(i)} u_{ij}(t)c_{ij}^u(t).$$

These costs are a direct result of the decisions that are made. However, recurring costs such as salaries and the maintenance of workers in training also need to be taken into account. To account for these costs we introduce the following state variables. Let $x_i(t)$ be the number of workers of type i at time t and let $y_{ij}(t)$ be the number of workers in training from worker type i to worker type j at time t. These state variables allow us to quantify the costs associated with salaries and maintaining workers in training. At each time step workers not in training are paid their salaries, and workers in training are paid the salaries they were earning before they entered training.

Let $s_i(t)$ be the salary of a type i worker at time t. The total cost of salaries for type i workers at time t is then

$$x_i(t)s_i(t).$$

The total cost for maintaining a type i worker undergoing training to another worker type at time t must be summed over all worker types that worker type i can be trained to. The total training maintenance costs for all type i workers at that time step are then

$$\sum_{j \in L(i)} y_{ij}(t)s_i(t).$$

To calculate the cost of the entire process, we now need to sum the above costs over all worker types, and over each time period. This leads to the following total cost function

$$C = \sum_{t=0}^{T} C(t),$$

where

$$C(t) = \sum_{i=1}^{N} \left[h_i(t)c_i^h(t) + f_i(t)c_i^f(t) + x_i(t)s_i(t) + \sum_{j \in L(i)} u_{ij}(t)c_{ij}^u(t) + \sum_{j \in L(i)} y_{ij}(t)s_i(t) \right].$$

Given our introduction of the state variables $x_i(t)$ and $y_{ij}(t)$, we need to find expressions for these in terms of our problems parameters and decision variables. We will model the training in the model by removing the workers in training from the pool of assignable workers and returning them to the pool after a period of time equal to the duration of training. Let τ_{ij} be the duration of training required to train a type i worker to a type j worker. We impose the following restrictions on the values of τ_{ij}, given by

$$\tau_{ij} \geq 1, \quad \tau_{ij} \in \mathbb{Z}_+.$$

The state variables can be defined by specifying the flow of workers of a given type from one time step to the next and specifying initial conditions. The number of workers of worker type i at the next time step is affected by the current numbers of workers $x_i(t)$, workers hired $h_i(t)$, dismissed $f_i(t)$, workers exiting for training $\sum_{j \in L(i)} u_{ij}(t)$, and workers returning from training after the appropriate time delay $\sum_{j|i \in L(j)} u_{ji}(t - \tau_{ji})$. The set indicated by $j|i \in L(j)$ should be interpreted as the set of all worker types j that can be trained to type i. The number of workers at time step $t + 1$ is given by

$$x_i(t+1) = x_i(t) + h_i(t) - f_i(t) - \sum_{j \in L(i)} u_{ij}(t) + \sum_{j|i \in L(j)} u_{ji}(t - \tau_{ji}).$$

Initially ($t = 0$), we assume there are a given number of workers of each type, given by

$$x_i(0) = \xi_i,\ i = 1, \ldots, N.$$

The number of workers in training at the next time step is affected by the current numbers of workers undergoing training $y_{ij}(t)$, workers entering training $u_{ij}(t)$, and workers exiting training after the appropriate delay $u_{ij}(t - \tau_{ij})$. This is given by

$$y_{ij}(t + 1) = y_{ij}(t) + u_{ij}(t) - u_{ij}(t - \tau_{ij}).$$

Initially ($t = 0$), we assume there are a certain number of workers in training,

$$y_{ij}(0) = \chi_{ij},\ i = 1, \ldots, N,\ j \in L(i).$$

Natural bounds for the hiring, dismissing, and training of workers are known. These are either derived from business policy or physical limitations in the employment environment. Bounds on hiring exist as a limitation on a maximum number of a particular worker type that the organisation can afford. Let $b_i^h(t)$ be the maximum number of workers of type i that can be hired at time t. The hiring bound constraint then takes the form

$$h_i(t) \leq b_i^h(t),\ i = 1, \ldots, N.$$

Similarly, labour regulations may limit the number of workers that can be dismissed, and limitations on training facilities may restrict the number of workers that can be trained. These can similarly be cast in the form of bound constraints as

$$\begin{aligned} f_i(t) &\leq b_i^f(t),\ i = 1, \ldots, N, \\ u_{ij}(t) &\leq b_{ij}^u(t),\ i = 1, \ldots, N,\ j \in L(i). \end{aligned}$$

We now consider the assignment sub–problem. Each worker type i is capable of performing a set of tasks $M(i)$. The tasks form the basis for which demand must be satisfied. Thus by assigning a sufficient number of workers to a task, the demand

for a task can be met. Let the demand for a task m at time t be given by $d_m(t)$. The assignment decisions we are then required to make can be expressed by defining the variables $v_{im}(t)$, the number of type i workers assigned to task m at time t. The satisfaction of the demand requirement can then be expressed by the constraint

$$\sum_{i|m\in M(i)} v_{im}(t) \geq d_m(t),\ m = 1, \ldots, M.$$

This ensures that demand is met or exceeded at every time step.

The parameters $d_m(t)$ are assumed to be inputs to the problem. This is effectively a problem to forecast future demand for tasks, for which methods already exist.

A crucial link between the employment and assignment sub–problems is a consistency requirement. At any particular time step, we are not permitted to assign more workers to tasks than the number of workers the organisation actually possesses at that time. We thus need to ensure that at each time step, and for each type of worker, the number workers assigned to all possible tasks does not exceed the number of workers available. This translates into the following consistency constraints

$$\sum_{m\in M(i)} v_{im}(t) \leq x_i(t),\ i = 1, \ldots, N.$$

Since any employment decisions that are made at the final time step can only increase the cost of the process without satisfying any constraints, we impose the condition that we may not hire, dismiss or train any workers at the final time step T, as given by

$$h_i(T) = f_i(T) = u_{ij}(T) = 0,\ i = 1, \ldots, N,\ j \in L(i).$$

1. The objective function

We cast the manpower planning problem as an optimisation problem. We seek to minimise the overall cost of the process, that is given by

$$\min C = \sum_{t=0}^{T} C(t) \tag{1.1}$$

where

$$\begin{aligned} C(t) = \sum_{i=1}^{N} [h_i(t)c_i^h(t) + f_i(t)c_i^f(t) + x_i(t)s_i(t) \\ + \sum_{j\in L(i)} u_{ij}(t)c_{ij}^u(t) + \sum_{j\in L(i)} y_{ij}(t)s_i(t)]. \end{aligned} \tag{1.2}$$

2. Model dynamics

The evolution of the system is governed by the balance equations

$$x_i(t+1) = x_i(t) + h_i(t) - f_i(t) - \sum_{j\in L(i)} u_{ij}(t) + \sum_{j|i\in L(j)} u_{ji}(t-\tau_{ji}), \tag{2.1}$$

$$y_{ij}(t+1) = y_{ij}(t) + u_{ij}(t) - u_{ij}(t-\tau_{ij}). \tag{2.2}$$

Supplemented by the initial conditions

$$x_i(0) = \xi_i,\ i = 1,\dots,N, \tag{2.3}$$

$$y_{ij}(0) = \chi_{ij},\ i = 1,\dots,N,\ j \in L(i). \tag{2.4}$$

As well as the enforcement of the final conditions

$$h_i(T) = f_i(T) = u_{ij}(T) = 0,\ i = 1,\dots,N,\ j \in L(i). \tag{2.5}$$

3. Model constraints

The constraints for the system are of three types: demand, consistency and bound constraints.

The demand constraints are given by

$$\sum_{i|m\in M(i)} v_{im}(t) \geq d_m(t),\ m = 1,\dots,M.$$

The worker assignment consistency constraints are given by

$$\sum_{m \in M(i)} v_{im}(t) \leq x_i(t),\ i = 1, \ldots, N.$$

The bound constraints for decision variables $h_i(t)$, $f_i(t)$, $u_{ij}(t)$ are given by

$$\begin{aligned} h_i(t) &\leq b_i^h(t),\ i = 1, \ldots, N, \\ f_i(t) &\leq b_i^f(t),\ i = 1, \ldots, N, \\ u_{ij}(t) &\leq b_{ij}^u(t),\ i = 1, \ldots, N,\ j = 1 \in L(i). \end{aligned}$$

The control and state variables are constrained to take on positive integer values, specifically

$$\begin{aligned} h_i(t) &\in \mathbb{Z}_+^N,\ i = 1, \ldots, N,\ t = 0, \ldots, T-1, \\ f_i(t) &\in \mathbb{Z}_+^N,\ i = 1, \ldots, N,\ t = 0, \ldots, T-1, \\ u_{ij}(t) &\in \mathbb{Z}_+^N,\ i = 1, \ldots, N,\ j \in L(i),\ t = 0, \ldots, T-1, \\ v_{im}(t) &\in \mathbb{Z}_+^N,\ i = 1, \ldots, N,\ m \in M(i),\ t = 0, \ldots, T, \\ x_i(t) &\in \mathbb{Z}_+^N,\ i = 1, \ldots, N,\ t = 1, \ldots, T, \\ y_{ij}(t) &\in \mathbb{Z}_+^N,\ i = 1, \ldots, N,\ j \in L(i),\ t = 1, \ldots, T. \end{aligned}$$

CHAPTER 3

Analysis of the model

This chapter analyses the model presented in more detail. The objective function, balance equations and constraints are all studied and some of their properties discussed.

1. The optimisation problem

We cast the manpower planning problem as an optimisation problem, minimising the overall cost, C, given by (1.1).

2. The objective function

The objective function of the model is defined to be the total cost of the process. The previous chapter showed how various costs, such as hiring, dismissal, training, and maintenance costs make up the total cost, as given by (1.2)

The total cost of the process consists of costs from each time step, over the entire planning horizon.

The objective function contains the decision variables $h_i(t)$, $f_i(t)$, $u_{ij}(t)$ *and* the state variables $x_i(t)$, $y_{ij}(t)$. It should be noted that the objective function is linear in the decision and state variables. The linearity of the objective function is thus determined by whether the state variables are linear functions of the decision variables. In this model, the state functions are non–linear functions of the decision variables due to time delay terms. This results in a non–linear objective function for the problem.

3. Model dynamics

The model dynamics are given by the equations (2.1) and (2.2), the initial conditions are given by the equations (2.3) and (2.3), and the final conditions are given by equation (2.5).

The time delay term in the argument for the training is critical to tracking the number of workers currently in training. The structure of the balance equations ensures that the workers in training are not in the pool of assignable workers. The time delay arguments also ensure that workers re–enter the pool of assignable workers after the correct duration of training time. The state variables $y_{ij}(t)$ are thus only used as a mechanism for tracking the number of workers in training. The time delay arguments introduce an element of non–linearity which, while making the model more realistic, introduces complexities that are not easily to analyse. The usual assumptions of linearity associated with the objective function and constraints which involve the state variables are thus no longer valid.

The initial conditions set the number of workers of type i at time $t = 0$ equal to ξ_i, and set the number of workers undergoing training from type i to type j at time $t = 0$ to χ_{ij}. For a case where $\chi_{ij} \neq 0$, additional information is required for the problem to be well defined. The section *Initial conditions for training* in the appendix provides a more detailed description.

The final conditions are imposed since the effect of employment variables at the final time step T has no impact on the satisfaction on the demand constraints at the final time step. This is since $x_i(T)$ and $y_{ij}(T)$ are functions of control variables of all previous time steps only.

The decision variables in the problem are $h_i(t)$, $f_i(t)$, $u_{ij}(t)$ for all $0 \leq t < T$ and $v_{im}(t)$ for all $0 \leq t \leq T$. Together with the initial conditions, these determine the values of the variables $x_i(t)$ and $y_{ij}(t)$ for all t, and thus the value of the objective function. The satisfaction of the constraints is also determined by these variables.

4. Model constraints

The demand constraints

$$\sum_{i|m\in M(i)} v_{im}(t) \geq d_m(t),\ m = 1, \ldots, M$$

require that a sufficient number of workers be assigned to perform a particular task at a specific time step. Evaluation of the demand constraints only requires knowledge of the demand and the required control variables $v_{im}(t)$. The evaluation of this constraint is thus computationally cheap, since it is the comparison of a sum of decision variables and a constant value.

The demand constraints are also used to set up the upper bounds for each of the decision variables $v_{im}(t)$. This is done by setting the upper bound of each $v_{im}(t)$ to the relevant demand constant, plus some percentage.

The worker assignment consistency constraints

$$\sum_{m\in M(i)} v_{im}(t) \leq x_i(t),\ i = 1, \ldots, N$$

require that a sufficient number of workers to perform a task are available for assignment. For a particular worker type i, the total number of workers assigned over all tasks m, $\sum_{m\in M(i)} v_{im}(t)$ cannot exceed the number of workers available at that time, $x_i(t)$. Were this constraints not present, it would be possible to assign workers that the organisation does not possess in sufficient numbers. These constraints are functions of the assignment control variables *and* the state variables $x_i(t)$. The constraints are thus non–linear and require computation of $x_i(t)$, which is far more expensive to compute than the other constraints in the problem. The state variables $x_i(t)$ become progressively more convoluted (with the contribution of time delay terms) and more expensive to calculate as t increases. The constraints for $t = 0$ are in fact linear, since $x_i(0) = \xi_i$ is a constant.

These constraints form the link between the assignment and employment aspects of the problem. Due to the constraints being a function of the assignment and employment variables, they are also the most difficult to satisfy.

The set indicated by $i|m \in M(i)$ should be interpreted as the set of all worker types i that are capable of performing task m.

The bound constraints for decision variables $h_i(t)$, $f_i(t)$, $u_{ij}(t)$, given by

$$h_i(t) \leq b_i^h(t),\ i = 1, \ldots, N,$$
$$f_i(t) \leq b_i^f(t),\ i = 1, \ldots, N,$$

and

$$u_{ji}(t) \leq b_{ij}^u(t),\ i = 1, \ldots, N,\ j = 1 \in L(i)$$

impose upper limits on the number of workers that may be hired, dismissed or trained at time t, for each type of worker. As simple functions of the bounds and control variables, these constraints are computationally cheap to compute.

Since the bound constraints are all linear and form bounding regions for the variables, they are used to specify the domain for each the employment variables. Due to this fact, they are also trivially satisfied. They only need to be considered since genetic operations might increase the value of a particular employment variable outside of the range specified by these constraints.

The decision and state variables are constrained to take on positive integer values. These restrictions are implicitly satisfied by using a particular integer parametrisation for the decision variables in the algorithm. The fact that the state variables are functions of the decision variables, along with the consistency constraints ensures that the state variables satisfy the integer constraints.

5. Problem size

Let card(S) represent the cardinality of the set S and let r represent the number of tasks in the problem.

The number of decision variables n_d in the problem is

$$\begin{aligned} n_d &= (\#\text{ Hiring variables}) + (\#\text{ Dismissal variables}) + (\#\text{ Training variables}) \\ &\quad + (\#\text{ Assignment variables}) \\ &= (TN) + (TN) + \left(T \sum_{i=1}^{N} \operatorname{card} L(i) \right) + (T+1) \left(\sum_{i=1}^{N} \operatorname{card} M(i) \right) \\ &= (T) \left(2N + \sum_{i=1}^{N} \operatorname{card} L(i) \right) + (T+1) \left(\sum_{i=1}^{N} \operatorname{card} M(i) \right). \end{aligned}$$

The number of state variables n_s in the problem is

$$\begin{aligned} n_s &= (\#\text{ Workers of a type variables}) + (\#\text{ Workers in training variables}) \\ &= (TN) + \left(T \sum_{i=1}^{N} \operatorname{card} L(i) \right) \\ &= (T) \left(N + \sum_{i=1}^{N} \operatorname{card} L(i) \right). \end{aligned}$$

The number of constraints n_c in the problem is

$$\begin{aligned} n_c &= (\#\text{ Hiring constraints}) + (\#\text{ Dismissal constraints}) + (\#\text{ Training constraints}) \\ &\quad + (\#\text{ Demand constraints}) + (\#\text{ Consistency constraints}) \\ &= (TN) + (TN) + \left(T \sum_{i=1}^{N} \operatorname{card} L(i) \right) + ((T+1)r) + ((T+1)N) \\ &= (T+1)(N+r) + (T) \left(2N + \sum_{i=1}^{N} \operatorname{card} L(i) \right). \end{aligned}$$

For example, a problem that consists of 10 worker types ($N = 10$), 15 tasks ($r = 15$), taken over 12 time ($T = 11$) steps with $\sum_{i=1}^{N} \operatorname{card} L(i) = 10$ and $\sum_{i=1}^{N} \operatorname{card} M(i) = 15$ gives

$$\begin{aligned} n_d &= 510, \\ n_s &= 220, \\ n_c &= 630. \end{aligned}$$

CHAPTER 4

Analysis of the problem

This chapter gives brief overviews of topics that have an impact on the problem under consideration. These include the exponential growth faced by problems of this nature, the discrete nature of the problem, the complexity of the problem from the view of computer science, and various methods of solving the problem.

1. Exponential growth

Exponential growth is a well known problem that results from the exponential increase in size of a solution space when there is a linear increase in the number of variables. Brute force algorithms check every solution in a solution space to find the global optima, and are thus susceptible to this problem. Solving problems with small numbers of variables using a brute force algorithm is tractable, but as soon as the number of variables in the problem increase beyond a certain point, brute force algorithms become an impractical method of solution.

Many algorithms that are widely in use exhibit exponential complexity in the worst case. However, these algorithms are widely used because they perform better than the worst case for many classes of problems they are used to solve. Even the well known simplex method, renowned for its ability to solve large–scale, realistic problems in less than exponential time, is known to have worst case exponential complexity.

To illustrate the exponential growth of a problem, let l_i^b be the inclusive minimum bound for variable i and let u_i^b be the inclusive maximum bound for variable i. The total number of solutions n_s in a search space of N variables is then given by

$$n_s = \prod_{i=1}^{N} (u_i^b - l_i^b + 1).$$

As an example, consider a problem with 10 variables, each of which can take on 10 possible values. The total number of solutions in the search space is given by

$$\begin{aligned} n_s &= \prod_{i=1}^{N}(u_i^b - l_i^b + 1) \\ &= \prod_{i=1}^{10}(10) \\ &= 10^{10}. \end{aligned}$$

Compare this with the number of variables in a problem which has only 10 more variables than the original problem. The total number of solutions in this new search space is given by

$$\begin{aligned} n_s' &= \prod_{i=1}^{N}(u_i^b - l_i^b + 1) \\ &= \prod_{i=1}^{20}(10) \\ &= 10^{20}. \end{aligned}$$

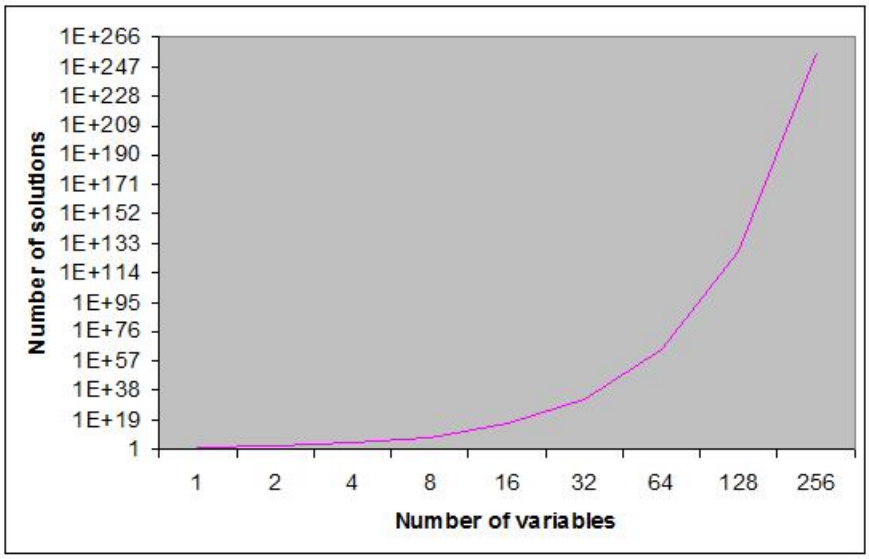
The result shows there is an increase of ten orders of magnitude between the sizes of each search space, that was only brought on by doubling the number of variables in the problem. Since even small problems generate a large solution space that needs to be searched, larger problems result in search spaces which are practically impossible to check using brute force algorithms. Heuristic algorithms are used to sample the search space and use certain rules to improve on the initial solutions.

Figure 4.1 and Table 4.1 illustrate the relationship between the number of variables and number of solutions when each of the variables added to the problem has 10 possible values.

TABLE 4.1. Number of variables versus number of solutions

Number of variables	Number of solutions
1	10
2	100
4	10000
8	1×10^8
16	1×10^{16}
32	1×10^{32}
64	1×10^{64}
128	1×10^{128}
256	1×10^{256}

FIGURE 4.1. Number of variables versus number of solutions



This example illustrates a fundamental problem with algorithms that exhibit worst case exponential behaviour. Given the ability to compute the original problem, we would need to have 10^{10} times more computing resources to solve a problem that only has double the number of variables. Methods to circumvent this problem have been and remain a major focus of modern computer science.

2. Optima in continuous and discrete systems

Discrete systems deal with variables whose values can take on, at most, a countably infinite number of values. The countable nature of the values is a result of some indivisible unit that is a characteristic of the set of numbers used to represent these

values. Together with an appropriate set of bounds on the variables and a particular set of numbers, a discrete system can consist of variables that only take on a finite number of values. This finite number of values may result in a solution space that is impractical to check completely.

Continuous systems deal with variables whose values take on an uncountable range of values. The uncountable nature of the range of values is a result of the inability to order the set of values.

The concept of a local minimum in a continuous system can be defined. x' is a local minimum of f if

$$\begin{aligned} \exists \quad \epsilon > 0 \quad \text{st} \quad \forall x \quad \text{satisfying} \quad d(x, x') \leq \epsilon \\ \Rightarrow f(x') \leq f(x), \end{aligned}$$

where $d(x, x')$ is a metric on the space.

If we further assume that f is strictly convex and there is a unique point x^* such that $f'(x^*) = 0$, then the following two conditions are sufficient for a global optimum, given a unique point x^*:

$$\begin{aligned} f'(x^*) &= 0 \\ f''(x^*) &> 0. \end{aligned}$$

The calculus of continuous systems thus allows one to find the global minimum using a straightforward procedure. Discrete systems hold no such advantage, as the concept of convexity is not well defined. The only way to find a global minimum is to check the objective function value of every candidate solution, and return the optimal solution.

Algorithms to optimise an objective function typically generate an initial solution, or a set of initial solutions. These are used in an iterative manner with a set of rules to generate new solutions. The set of rules used fall into two major categories: deterministic and heuristic.

Convex optimisation methods typically make use of the calculus of Newton and Leibnitz, utilising gradient information about the problem. This can be used in various ways by the algorithms to decide on how to move towards better solutions.

This forms the basis for the convex optimisation methods such as the steepest descent and conjugate gradient algorithms.

By contrast, algorithms that deal with discrete problems can assume no knowledge of a gradient for the problem, since it is not well defined. The algorithm used still requires some way to generate better solutions. Instead of using a deterministic rule to calculate a path to move along, a heuristic rule is used. A process that makes use of a solution's objective function values is applied in conjunction with the generation of random numbers to produce a set of decisions that should, on average, generate better solutions.

3. Constraint satisfaction problems

Constraint satisfaction problems deal with a set of candidate solutions that must satisfy a set of constraints. These problems deal with large solution spaces where it is intractable to use brute force methods to solve the problem.

Consider the manpower model presented with a rigorous set of constraints. In a practical sense, the problem moves away from being a pure optimisation problem to a constraint satisfaction problem. The primary objective is then to find *a* solution that satisfies the constraints of the problem, assuming such a solution exists.

Several methods are used to solve constraint satisfaction problems, one of which is backtracking. Due to the large solution spaces involved, the methods must have mechanisms to discard large areas of the search space (such as the pruning mechanism used by branch and bound) while using a reasonable amount of resources.

The primary factor in deciding whether the manpower problem should be considered as an optimisation problem or a constraint satisfaction problem is the ratio of the total number of feasible solutions s to the total number of solutions n. The total number of solutions is generally easy to compute given the range of each of the variables in the problem, but calculating s would in general require evaluating every constraint in the problem for each possible solution. This brute force method would result in the solution to the problem, but is practically impossible given all but the most trivial problems.

We thus consider a uniformly random sample of the solution space, of size n_z. We can then compute the number of feasible solutions in the sample, s_z, and use this to form an approximation of the ratio of feasible solutions to total solutions

$$\frac{s}{n} \approx \frac{s_z}{n_z}$$
$$s \approx \frac{ns_z}{n_z}.$$

In the limit of the sample size approaching infinity, the approximation becomes exact

$$\lim_{n_z \to \infty} \frac{s_z}{n_z} = \frac{s}{n}.$$

This is an important step in deciding how to approach the problem, since the methods for dealing with optimisation problems and constraint satisfaction problems have completely different goals.

4. Computational complexity

The study of complexity in computational terms is concerned with the amount of resources required to solve a problem, assuming it is solvable. The resources that are normally considered are storage and the time required to solve the problem.

The resources needed to solve a problem are dependent on the inputs of a problem. Given that the amount of resources used to solve the problem is expressed as a function of the inputs, the required resources for the problem can be calculated. The primary resources considered are the time required to find a solution to a problem (associated with the time complexity of a problem) and the memory required (associated with the space complexity of a problem). Thus, as the number of inputs to a problem are changed, the resources required to solve the problem are affected as well.

5. Complexity classes

A major area of research in Computer Science is to classify problems into complexity classes. These classes represent problems with common characteristics in terms of the ability we possess to test a candidate solution, and how many resources are consumed in finding solutions to the problem.

A complexity class is a set of problems that are solvable by a model with certain resources. These resources are traditionally expressed as functions of the inputs of the problem.

The discussion of complexity classes and the solutions of problems using algorithms are often described using a mathematical abstraction of a computer called a Turing machine. This mathematical construct consists of a tape of infinite length, as well as a 'head' that can read a bit from the current section of the tape, and write a bit to the current section of the tape. The Turing machine also has a set of rules that specify what data should be written and how the head should move (a unit forwards or backwards), given the current state and data on the current section of the tape.

Two useful types of Turing machines are used in complexity theory: deterministic and non–deterministic. The deterministic Turing machine follows the definition given above, while the definition for a non–deterministic Turing machine is modified somewhat. A non–deterministic Turing machine calculates all possible transitions from the current state in parallel, until a solution is reached. One interpretation of this definition is that at every possible branch a new instance of a deterministic Turing machine is spawned to compute the result of that specific branch.

The complexity class P is the set of all decision problems that are solvable in polynomial time by a deterministic Turing machine. Thus, if x is the number of inputs to the problem, a P class problem requires the following amount of resources to solve

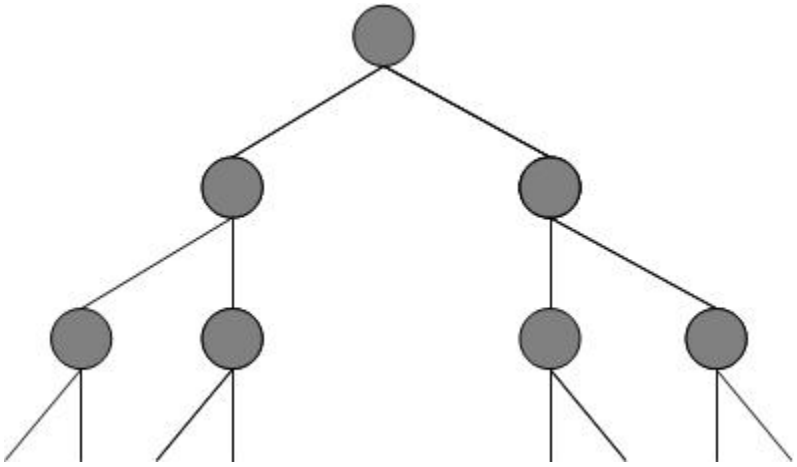
$$x^a$$

for some constant a.

In practice, the constant a cannot be a large number for the problem to be practically solved.

The class NP is the set of decision problems that are solvable in non–deterministic polynomial time by a non–deterministic Turing machine. A NP problem also has the characteristic whereby a candidate solution can be checked to be a solution to the problem in polynomial time. The Travelling Salesman problem and Boolean Satisfiability problem are two examples of NP problems.

FIGURE 4.2. Branch and bound tree



It is well known that the non–linear integer program is a NP–Hard problem. This class of problem is at least as hard as the hardest problem in NP. There is currently no algorithm that is known to solve NP–Hard problems in polynomial time.

6. Non–linear integer programming problem

In general the Manpower problem can be expressed as

$$\min \quad C(A, B),$$

such that

$$f_i = 0 \quad i = 1, \ldots, p,$$
$$g_j \geq 0 \quad j = p+1, \ldots, q,$$

where A and B are vectors of the decision and state variables respectively. The functions f_i and g_j are functions of the decision and state variables.

This is the form of a non–linear integer programming problem. This general formulation captures the familiar optimisation problems of constrained linear optimisation, unconstrained optimisation and other cases. This more general problem can be solved by various methods, some of which are outlined below.

7. Branch and Bound

Branch and Bound is a global optimisation technique that is widely used. It abstracts the problem into an implicit tree structure, minimising the need for storage. The tree represents the solution space and the algorithm systematically searches through this tree to find the optimal solution. A solution is represented by a traversal from a node at the top of the tree to the bottom of the tree. Once a parametrisation is chosen for how to map a solution into the tree structure, the algorithm can use various rules to navigate the search space. Figure 4.2 shows a special form of a tree structure, the binary tree, commonly used to represent problems which can be parametrised by a binary structure.

Branch and bound can eliminate large areas of the solution space during a single step if an appropriate parametrisation is chosen. It does so by selecting which part of the tree to traverse next (branching), and then calculating upper and lower bounds for the solutions that form a subset of the current node in the tree (bounding). The bounds are then used to decide whether to proceed branching down the current node, or whether the current section of the tree can be pruned. Pruning is a process whereby all nodes below a particular node are removed from consideration, since they are known to not contain the optimal solution. This process continues until the entire tree has been pruned or one solution remains.

In the worst case, the algorithm will have to traverse the entire tree to find the optimal solution, resulting in exponential time complexity. An appropriate parametrisation is required to try and avoid this. The other critical requirement is having an efficient way of computing the bounds on the objective function for subsets of solutions.

The manpower problem can utilise a modified branch and bound algorithm. The bounding step can be modified so that in addition (or precluding) the objective function bounds being calculated, the constraints in the problem are used to check whether the tree can be pruned. Given the varying degree of constraints affecting solutions in the manpower problem, the weaker constraints will be satisfied by most solutions while the more restrictive constraints will result in large areas of the search space being removed from consideration.

8. Dynamic programming

A method that effectively makes use of splitting a problem into sub–problems is dynamic programming. It is based on the assumption that if a problem can be broken up into sub–problems, the optimal solutions of the sub–problems form part of the optimal solution to the complete problem.

The general solution for a dynamic programming problem is a solution to the Bellman equation. The model described can be cast into the recursive form of the Bellman equation, and solved using the appropriate methods.

In the manpower planning problem, the following initial and final conditions fix the start and end points of the problem

$$\begin{aligned} x_i(0) &= \xi_i, \\ y_{ij}(0) &= \chi_{ij}, \\ h_i(T) = f_i(T) &= u_{ij}(T) = 0. \end{aligned}$$

Dynamic programming can operate in one of two ways. We can either split the main problem in sub–problems, find the optimal solutions for the sub–problems and recombine these to form the optimal solution to the main problem. The alternative approach is to solve a subset of sub–problems and then use these to generate larger problems that are then solved.

When using the first approach, at each step we would consider all possibilities from the previous system state, eliminating steps that would result in the problem not being solved in an optimal manner.

The problem with a dynamic programming approach is that the set of previous states at each step are numerous, resulting in a large number of sub–problems. Attempting to solve the problem in such a way would require traversing large sections of the search space, making this method of solution inefficient.

9. Optimal control approach

The manpower problem presented can be considered as a discrete time–delayed optimal control problem. The objective in this approach is to find a discrete–valued control (subject to the restrictions of the problem), such that the overall cost of the process is minimised.

This approach is used in [**3**]. The problem described above is then transformed into a discrete–valued optimal control problem, followed by a Control Parametrisation Enhancement Transform. This transforms the problem into a continuous optimisation problem. An appropriate algorithm for solving the resulting problem can then be used to solve the original problem.

10. Modified genetic algorithm approach

This section highlights the approach of a slightly modified genetic algorithm to solve the manpower planning model.

Genetic algorithms are widely used in global optimisation problems due to their ease of use and the straightforward manner in which they can be programmed. However, they are ultimately used because they are successful due to their ability to solve a wide range of problems, while assuming very little about the details of a particular problem.

Genetic algorithms simulate biological operations to generate solutions in an iterative manner, making use of heuristics to avoid local optima. They begin by generating an initial population that is used to progressively reach more optimal solutions. This algorithm is typically used to solve unconstrained, non–linear optimisation problems.

The genetic algorithm begins by taking a domain for the problem and the objective function to optimise as inputs. The basic building blocks of a genetic algorithm are chromosomes, that are used to describe a particular parametrisation of solutions. Thus, given some candidate solution s, applying the parametrisation Γ results in a chromosome c. The parametrisation is chosen such that parametrisation Γ is invertible and well defined, so

$$\Gamma(s) = c$$

and

$$\Gamma^{-1}(c) = s.$$

Let A be a vector of decision variables and let B be a vector of state variables. The objective function $C(A, B)$ is then used to construct a fitness function F. In general, $F(C(A, B))$ and the transition from the objective function value to the fitness function value is cheaply computed. The fitness function is constructed

in such a way that solutions that are more optimal have higher fitness values. The exact way in which F is constructed depends on whether the problem is a minimisation or maximisation problem.

The two major genetic operations used in genetic algorithms are breeding and mutation. Breeding is the operation that is used to generate more fit chromosomes, on average, from existing chromosomes. The mutation operation is used to modify an existing chromosome, affecting its underlying structure in a random way. This is used to perturb solutions that have clustered around local minima.

The breeding operation in a genetic algorithm is used to gradually generate chromosomes that are more fit than their predecessors. A central concept that is assumed is that, on average, when using the breeding operations on sets of the fitter chromosomes in a population, more fit solutions will be generated. Mathematically, given a set of initial chromosomes P and a non–deterministic breeding operation B, the offspring of the breeding operation would be a population P'. Let N be the Nth breeding operation, $\overline{f}$ be the fitness of the initial population, and $\overline{f^*}$ be the average fitness of the population after the $N'th$ breeding operation. After each breeding operation the set P is updated so that

$$P = P \bigcup P'$$

and

$$\lim_{N \to \infty} B(P)$$

results in

$$\overline{f^*} \geq \overline{f}.$$

When facing a problem that includes constraints, a mechanism is needed so that the constraints can be included into the genetic algorithm. A simple approach is to only consider solutions for breeding when they are feasible. This results in an interior point method, since we only generate solutions from feasible chromosomes.

Formally, the steps for the algorithm are as follows:

```
Input  : Problem data
Output: List of fittest, feasible solutions
Initialise initial chromosome population, constraints;
while Stop conditions not reached do
    breeders = SelectFeasibleSolutions(population, numberOfOffspring)
    foreach breedingPair in breeders do
        offspringPair = Breed(breedingPair)
        offspringPair = Mutate(offspringPair)
        population = population ⋃ offspringPair
    end
end
```

Algorithm 1: Modified genetic algorithm

The requirement that only solutions that are feasible are available for selection at each iteration places stringent requirements on the chromosomes that are selected for breeding. All infeasible points that are generated are worthless since they are not eligible for selection. Thus problems with non–rigorous constraints will cope fairly well, while problems with low numbers of feasible solutions that are clustered in separate regions will be extremely difficult to solve with this algorithm.

An obvious problem arises with the generation of an initial population for problems with stringent constraints. The number of feasible solutions generated might be low, or even zero. In the case where there are no feasible solutions the algorithm cannot continue and must re–populate the initial chromosome population in an attempt to generate feasible solutions. In the case of a low number of feasible solutions, the algorithm will, at best, rely on its breeding rule and the region defined by the feasible solutions to generate more fit, feasible solutions.

To illustrate the point, several runs to generate an initial population were performed using the main problem (described in chapter 8). A summary of the relevant variables and the runs is given below.

The results show the sensitivity of this algorithm to the low number of feasible solutions in the model. With no feasible solutions available in the above samples,

TABLE 4.2. Problem summary

Problem summary	
Number of decision variables	103
Number of constraints	122
Total number of candidate solutions	$\approx 1.725 \times 10^{69}$

TABLE 4.3. Modified genetic algorithm initial solution feasibility

Results		
Run	Initial chromosomes	Feasible solutions
1	10^4	0
2	10^5	0
3	10^6	0
4	2×10^6	0
5	3×10^6	0
6	4×10^6	0
7	5×10^6	0
8	6×10^6	0
9	7×10^6	0
10	8×10^6	0

the algorithm is unable to continue and perform genetic operations in an attempt to improve on the available feasible solutions.

Even if several solutions were available after the initial population generation, the algorithm would only be able to breed a limited number of new solutions. It is clear that the constraints in the model place rigorous restrictions on the solutions. Of the large number of solutions, only very few are feasible. Uniformly generating solutions with the sample sizes indicated is a method that is unable to generate a single feasible solution.

A less direct approach will be needed to generate feasible solutions and deal with the rigorous constraints in the model.

CHAPTER 5

Sensitivity analysis

1. Overview

The manpower planning model developed has been cast as a non–linear optimisation problem with constraints. More realistic manpower planning problems have progressively larger number of variables. This results in the generation of a larger number of constraints. As non–trivial constraints are added to a system, the number of feasible solutions decrease.

By contrast, the size of the candidate solution space increases exponentially with respect to a linear increase in the number of decision variables. Assuming that solutions are generated in a uniformly random way, the probability of initially generating feasible solutions is significantly reduced.

These two factors result in the ratio of the number of feasible solutions to the total number of solutions decreasing rapidly as variables are added to the manpower planning model.

A simple analysis of the sensitivity of a particular solution would be to analyse small changes in the values of a particular cost, and note how this affects the overall cost of the process. The analysis becomes straightforward if we consider the costs to be continuous functions $c(t)$. This is true when a particular solution is considered, since the decision and state variables take on constant values. The total cost of the process can then be viewed as a function C of all costs, c_i.

Perturbation theory can then be applied to the resulting continuous function to analyse changes to the overall cost as a result of perturbations in the cost functions that make it up. This can be done using the partial derivatives with respect to the cost functions. A perturbation in the cost c_p is then given by

$$\frac{\partial C}{\partial c_p}.$$

In any optimisation problem that contains constraints, sensitivity analysis of the solutions generated is an important aspect. Consideration of a solution's insensitivity to perturbations in the constraints is a concern that can override the need to optimise the objective function. The manpower planning model presented is one of these problems. While the decision variables are directly controlled, the demand parameters, $d_m(t)$, are estimates of demand in the future, and are thus inherently uncertain. By considering the feasibility of a solution when certain constraints are perturbed, we can gain insight into the robustness of a set of decisions in the face of demands that differ from the initial estimates.

Optimal solutions of problems involving meaningful constraints reside on the edge of a feasible region, and there exists some perturbation of the constraints that will result in the solution no longer being feasible. In practical terms, the strategy of paying slightly more to mitigate risk is common practice. This suggests that robust solutions that have a slightly higher cost than the optimal solution might be more valuable.

2. Sensitivity of the bound constraints

The constraints provide direct upper bounds for the employment decision variables. Along with the lower bound enforced by the whole number requirement on the variables, the domain for the employment variables is specified.

The bound constraints for $h_i(t)$, $f_i(t)$ and $u_{ij}(t)$ are easily satisfied since the constraints that they must satisfy are directly used to generate their respective solutions spaces. Only genetic mutations might result in solutions that do not satisfy these constraints.

3. Sensitivity of the fulfilment constraints

The sensitivity of the fulfilment constraints

$$\sum_{i|m\in M(i)} v_{im}(t) \geq d_m(t),\ m = 1, \ldots, M,$$

require a combination of assignment variables to exceed a given parameter, the future demand for a task. This makes the fulfilment constraints non–trivial to satisfy.

The lower bounds on the assignment variables are defined by the whole number requirement, and the upper bound can be set by adding some constant to each $d_m(t)$.

4. Sensitivity of the assignment consistency constraints

The sensitivity of the assignment consistency constraints

$$\sum_{m \in M(i)} v_{im}(t) \leq x_i(t),\ i = 1, \ldots, N$$

are difficult to analyse due to the mixing of employment and assignment variables. It is clear that a combination of assignment variables having to be less than or equal to a complicated dependency of decision variables will be difficult to satisfy, as shown by

$$\sum_{m \in M(i)} v_{im}(t) \leq x_i(t-1) + h_i(t-1) - f_i(t-1) - \sum_{j \in L(i)} u_{ij}(t-1) + \sum_{j | i \in L(j)} u_{ji}(t - \tau_{ji} - 1).$$

The assignment consistency constraints to not specify any information on the bounds of any decision variables in the model.

5. The perturbation of a constraint

With the goal of testing how a robust a solution is, we need to define the perturbation of a constraint. In this case, we restrict the result of perturbing a constraint to a new constraint that is at least as restrictive as the original constraint. If a solution s satisfies a constraint $g(s')$, it will satisfy any constraint that is less restrictive than $g(s')$. In the case of sensitivity analysis, this provides no information at all on the sensitivity of a solution. It is thus only dependent on constraints that are more restrictive than the original constraint.

Assume a solution s and an original constraint $g_1(s')$ that s satisfies. Let $g_2(s')$ be a constraint that is the result of a perturbation of $g_1(s')$. The sensitivity of s is analysed by considering

$$g_2(s).$$

For a set of constraints $G(s')$, we may consider the number of new perturbed constraints, $G'(s')$, where each element of $G'(s')$ is the result of a perturbation of each respective member of $G(s')$.

Any constraint $g(x)$ in the manpower planning model can be cast into the form

$$g(s) \geq 0.$$

This allows us to define a perturbation of the constraint which we know is more restrictive by some unit α. In this case

$$g(s) \geq \alpha,$$

where α is some small natural measure in the space of the problem. Since the parameters occurring in each of the constraints are limited to whole numbers, the smallest possible unit of measure is $\alpha = 1$.

The demand parameters in the model are the parameters that are most likely to change. From the above argument, the raw perturbation of a fulfilment constraint takes the following form

$$\sum_{i|m\in M(i)} v_{im}(t) \geq d_m(t) + \alpha,\ m = 1, \ldots, M,$$
$$\sum_{i|m\in M(i)} v_{im}(t) \geq d_m(t) + 1.$$

By setting $d_m(t) + 1 = d'_m(t)$ the we note that the constraint has been transformed into a more restrictive constraint with a new demand $d'_m(t)$.

6. Uncertainty of the demand parameters

The uncertainty of the parameters $d_m(t)$ in the fulfilment constraints can be modelled using a probability distribution that accurately models the uncertainty of the parameters at each point in time. We assume that there is some cut–off demand, after which higher values of the demand parameters have zero probability of occurring. Demand parameters take on a discrete set of values, and we use this to define the distribution for them as

$$D_m(t) = d_m(t) + X,$$

where $d_m(t)$ is the deterministic demand, and X is some discrete random variable.

A scenario-based approach can be used to produce a number of possible scenarios that result from the possible values of the demands generated. Solutions can then be tested against each of these scenarios to study their feasibility in each of the scenarios.

7. Probability distribution for the demand parameters

$D_m(t)$ can thus take on the following values

$$d_m(t), d_m(t)+1, \ldots, [d_m(t)]_{\max},$$

where $[d_m(t)]_{\max}$ is the cut–off demand.

Since X is a discrete random variable, it must have a countable number of values it can take on, each with non–zero probability. By the definition of a distribution

$$\sum_{d=0}^{\infty} [P(D_m(t) = d)] = 1.$$

8. Defining the measure of feasibility

The measure of the feasibility of a solution is defined by its feasibility in the scenarios that are produced. It is also prudent to consider the probability of a given scenario occurring to use as a weight for the feasibility of a solution. This will give us a measure of a solutions feasibility with a perspective on how likely a given scenario is to occur. Solutions that are are feasible in a larger number of scenarios must have feasibility ratings that are strictly higher than solutions are that are feasible in fewer scenarios.

Since the total number of scenarios may be very large, we need to consider a sample of the possible scenarios. Given a sample of N_Q scenarios I, each scenario i has associated with it a probability of occurrence that is calculated from the probability distributions used and the actual values for the perturbed parameters in the

scenario. For a scenario with a set of demand parameters δ, the probability of the scenario is given by

$$P_i^{\delta} = \prod_{t=0}^{T} P(D(t) = \delta(t)).$$

Let ϕ_i be an indicator function for scenario i that acts on a candidate solution s, and is defined as

$$\phi_i(s) = \begin{cases} 1 & \text{if } s \text{ is feasible in scenario } i \\ 0 & \text{if } s \text{ in not feasible in scenario } i, \end{cases}$$

If we let P_i represent the probability of occurrence of scenario i, let $F(s)$ represent the feasibility of a solution s, and let Z be the maximum number of scenarios, then we can then precisely define the measure of feasibility of a solution s in the following way

$$F(s) = \sum_{i}^{Z} P_i \phi_i(s).$$

This measure has some useful properties. The first is that there are lower and upper bounds on the measure. If a solution is feasible in no scenarios, it takes the lowest value of the measure, 0. If a solution satisfies all of the scenarios, it will take the maximum value, 1.

The second useful property is that due to the behaviour of the indicator function ϕ_i, the measure coincides with the probability of any of the scenarios occurring for which s is feasible. Higher measures of feasibility are thus directly linked to solutions that are more likely to satisfy a randomly generated scenario.

Typically, the number of scenarios is quite large, with many scenarios having a very small probability of occurrence. It is thus more tractable to generate a sample of the set of all possible scenarios (without replacement).We still retain all of the properties described above if we normalise the measure with the total probability of occurrence of all the scenarios in the sample. If we let Q be the set of sample scenarios, and n_Q be the cardinality of the set Q, then we can define the normalisation constant

λ as

$$\lambda = \left(\sum_{i=0}^{n_Q} P_i \right)^{-1}.$$

The measure of feasibility is then generalised to

$$F(s) = \lambda \sum_{i=0}^{n_Q} P_i \phi_i(s).$$

This expression gives a normalised measure of feasibility in the sample space defined by Q.

CHAPTER 6

Refining the model

This chapter covers refinements to the model to make it more tractable. These include the relaxation of a particular class of constraints, the separability of the model, and the effect of simulating multiple populations in the process of solving the model.

1. Constraint relaxation

As noted in a previous chapter, a modified genetic algorithm performed poorly when attempting to generate solutions in a uniformly random way. A co–evolutionary algorithm will require at least some feasible solutions in its initial population in order to generate fitter, feasible solutions.

The low number of feasible solutions indicate that the constraints place restrictions on the system that limit the number of feasible solutions drastically. As a first step, we can examine the model and check whether there are any classes of constraints that can be relaxed in such a way that the optimal solution will remain the same.

For the optimal solution to remain the same, we must guarantee that all new solutions have a greater objective function value than the original solutions. Consider the objective function given by (1.1).

All costs in the problem are positive numbers, and all of the decision and state variables that multiply these costs are restricted to take on non–negative values.

The size of the feasible set of solutions can be increased by relaxing any of the bounds on $h_i(t)$, $f_i(t)$ and $u_{ij}(t)$. This however, may reduce the value of the optimal solution. One approach to deal with this is to set up and solve a sequence of problems where the bounds on $h_i(t)$, $f_i(t)$ and $u_{ij}(t)$ are decreased in increments, finally reaching their original values.

Consider the functions that define the dynamics of the state variables (2.1) and (2.2).

The state equations show that only the hiring variables, $h_i(t)$, have a consistent effect of increasing the state variables $x_i(t+1)$. Since $h_i(t)$ and $x_i(t+1)$ only occur with positive costs in the objective function, increasing $h_i(t)$ will result in an increase of the objective function.

In an analogous way, decreasing $f_i(t)$ will have a consistent effect of increasing the state variables $x_i(t+1)$. However, reducing the bounds on the variables $f_i(t)$ introduces the possibility that the optimal solution to the original problem is not feasible in the new problem.

Thus $h_i(t)$ is the only decision variable whose bounds we can increase while guaranteeing that the optimal solution to the original problem will remain feasible in the new problem.

The only equation that directly controls the upper bounds on the variables $h_i(t)$ are the hiring bound constraints,

$$h_i(t) \leq b_i^h(t),\ i = 1, \ldots, N.$$

By increasing the value of each $b_i^h(t)$ by some number, we can relax the hiring bound constraints and allow for the generation of feasible solutions in the initial population. The new bound constraints are then

$$h_i(t) \leq b_i^h(t) + \beta_i^h(t),\ i = 1, \ldots, N.$$

The bound constant can be factored so that

$$h_i(t) \leq b_i'^h(t),\ i = 1, \ldots, N,$$

where $b_i'^h(t)$ is a new bound and $b_i'^h(t) = b_i^h(t) + \beta_i^h(t)$.

In terms of the manpower problem, this seems to be a logical choice. If the guidelines given by the hiring bounds are too restrictive to formulate a feasible employment policy, then the guidelines should be relaxed. Hiring more workers to fulfil the demand for tasks in the future is the most direct (yet most expensive way) to ensure that demand is met.

2. Separability of the employment and assignment aspects

Another strategy that is commonly used throughout mathematics is the splitting of problems into sub–problems. While this is not always possible, the strategy can provide several advantages when it is applicable.

The manpower planning model considered is a combination of two distinct problems, an employment policy facet and an assignment facet. This is evident from the lack of assignment decision variables in the objective function. The objective function and bound constraints are clearly components relating to the employment facet. The satisfaction constraints relate to the assignment facet. The link between the two facets is provided by the consistency constraints, which contain both employment and assignment decision variables.

When both facets are considered simultaneously, generating uniformly random candidate solutions is insufficient. It results in too few feasible solutions, or none at all. To increase the probability of generating feasible solutions, consider the following two models:

Model 1

$$\min C = \sum_{t=0}^{T} C(t)$$

subject to

$$C(t) = \sum_{i=1}^{N} \left[h_i(t)c_i^h(t) + f_i(t)c_i^f(t) + x_i(t)s_i(t) + \sum_{j \in L(i)} u_{ij}(t)c_{ij}^u(t) + \sum_{j \in L(i)} y_{ij}(t)s_i(t) \right]$$

with the constraints

$$h_i(t) \leq b_i^{'h}(t),\ i = 1, \ldots, N$$
$$f_i(t) \leq b_i^f(t),\ i = 1, \ldots, N$$
$$u_{ij}(t) \leq b_{ij}^u(t),\ i = 1, \ldots, N,\ j \in L(i)$$

and

Model 2

$$\min \sum_{i=1}^{N} \sum_{m \in M(i)} v_{im}$$

with the constraints

$$\sum_{i|m \in M(i)} v_{im}(t) \geq d_m(t),\ m = 1, \ldots, M$$
$$\sum_{m \in M(i)} v_{im}(0) \leq x_i(0),\ i = 1, \ldots, N$$

Model 1 is the employment aspect of the original model, and Model 2 is the assignment aspect of the original model with an additional objective function specified. The lack of assignment control variables in the original model's objective function forces us to specify an objective function if we wish to consider Model 2 as an optimisation problem. The objective function is specified in such a way that it is diametrically opposed to the demand constraints.

Model 1 has the trivial solution where all decision variables take on zero values, so there is no need to solve the model. The initial population that is generated in a uniformly random way should be passed directly to the original model.

Mathematically, the demand constraints only place a lower bound on combination of assignment variables and do not specify the maximum values the variables can take in this space. Without an objective function that is diametrically opposed to the constraints, Model 2 becomes trivial to solve, with solutions taking on the maximum values possible. The practical interpretation of the objective functions is that assigning the minimum number of workers required would free up workers so that they could be assigned to other tasks.

Most consistency constraints are conspicuously absent from either model for two specific reasons. Firstly, the consistency constraints mix assignment and employment control variables (through the state variables $x_i(t)$ and decision variables $v_{im}(t)$). In the context of these models, they cannot be specified as being a part of

either, since this mixes the solutions of both models. Secondly, the consistency constraints are the most restrictive constraints on the system. By solving these models first, we are more likely to generate solutions that are feasible in the original model.

Solving both models would not necessarily give the optimal solution to the problem, as they are only portions of the original problem. However, a solution in the original model, when separated, is certainly a solution in both of these models. By considering the two facets in isolation we decrease the complexity significantly and allow for a greater possibility of generating feasible solutions.

By using a co–evolutionary algorithm to solve each model, we will be supplied with candidate employment solutions and candidate assignment solutions. These solutions can then be combined and used as members of an initial population in the original model.

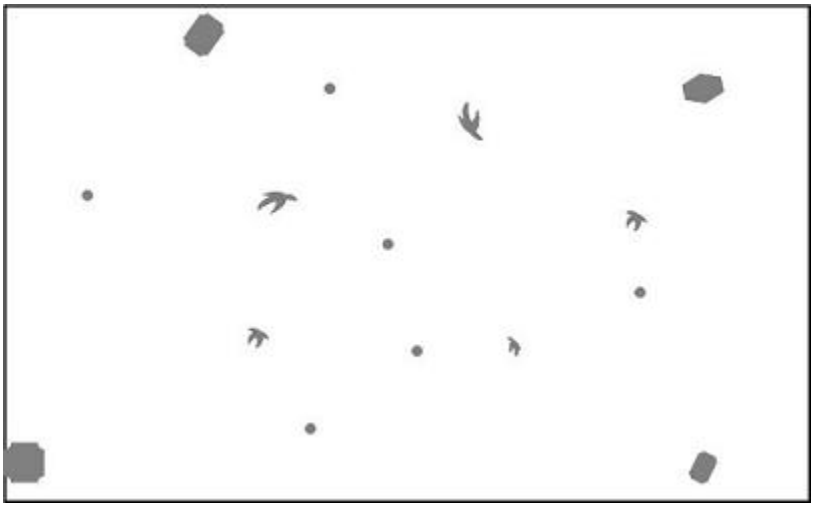
3. The power of multiple populations

Co–evolutionary algorithms differ from evolutionary algorithms in one major sense: co–evolutionary algorithms simulate multiple interacting populations. As a result of this, the interactions between the populations must be modelled. The overall fitness of an individual is thus not only dependent on its usual fitness, but also on how it interacts with other populations.

Co–operative co–evolutionary algorithms are a type of algorithm that use multiple populations co–operating to reach solutions for a problem. An obvious application of this is the separability of certain aspects of the problem. As an example, consider two variables in an objective function that are not coupled. Adjusting one variable while holding the other constant will give an indication of what effect the variable has on the value of the objective function. Populations simulating each of these variables independently would then allow an optimal solution to be found by combining the results of both populations. So separately, the populations only model some aspect of the original model, but when the populations cooperate they form a complete solution to the model.

The co–operative approach can be used to model the employment and assignment aspects of the manpower planning model. The solutions can then be combined and used to provide candidate solutions for the complete model.

FIGURE 6.1. Overall solution space



Competitive co–evolutionary algorithms are a type of algorithm that simulate the interactions of multiple populations that compete with one another. The overall effect is still to improve the solutions to a problem, but the effect on individual members of each population might be detrimental.

The competitive approach can be utilised by considering a predator–prey model. The solutions are modelled as a prey population and the set of constraints are modelled as a predator population. The predators and prey populations interact in some way, with the predators eliminating some of the prey. Thus, solutions that fulfil the constraints survive, while those that do not are eliminated from the prey population. In this way, solutions that satisfy the constraints have a greater probability of being chosen for breeding operations. When the algorithm terminated, a larger set of solutions that satisfy the constraints are available.

Multiple populations can also be used when separating the employment and assignment aspects of the manpower model. The original model has a large number of rigorous constraints and variables. Consider the qualitative Figure 6.1 representing the solution space of the original model.

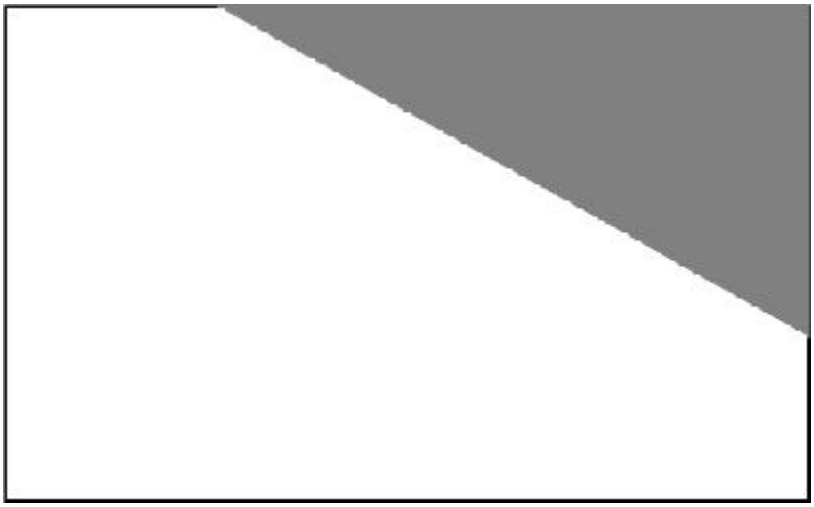
The small pockets of feasible solutions are difficult to find using standard methods for generating an initial population.

By splitting the original model into Model 1 and Model 2, we are left with two solution spaces to consider, but each of which has less constraints and variables

FIGURE 6.2. Employment solution space



FIGURE 6.3. Assignment solution space



than the original problem. In addition to this, the effects of the constraints for each of the models is less restrictive. This results in the qualitative representation given by Figure 6.2 and Figure 6.3.

A method of generating uniformly random solutions is now much more likely to result in feasible solutions.

The solutions from these two models can then be combined in such a way that they produce solutions in the original model. Solutions in the original model are

produced by the product of the solution populations in Model 1 and Model 2. Thus, 1000 employment solutions combined with 1000 assignment solutions result in 1 000 000 solutions in the original problem. We can thus cover a far larger part of the original solution space and focus on areas that are more likely to contain feasible solutions.

4. Objective function minimisation versus feasibility, resource usage

Given the nature of the manpower planning model, it is necessary to investigate what is expected of the outputs. Given that it has been cast as an optimisation problem, minimisation of the objective function appears to be the overriding concern. Yet the previous sections have highlighted the need for solutions that are feasible and insensitive to small changes in certain constraint parameters. Solutions of this nature should also be produced in a reasonable amount of time, with a realistic amount of resources.

The discussion of the manpower planning problem given in [**22**] considers the manpower planning requirements of the US armed forces. The need for this entity to meet its obligations while minimising costs is paramount. It is expected to be able to successfully carry out missions relating to several large scale conflicts simultaneously. This is no simple problem, as the complex organisation structure (including ranks, experience, age, and specialisation) lead to a large number of worker types and complex training programs. The demands to be met for engaging in conflicts is inherently uncertain, and the planning horizon can extend to the order of several years. Stochastic attrition must also be taken into account to realistically model the system.

Solutions to this kind of problem cannot be sensitive to slight changes in the values of uncertain parameters in the constraints. Even if the cost of such solutions is considerably lower than others, they lose their usefulness as soon as they become infeasible. When combat readiness of a unit depends on such solutions, they must be feasible for a range of changes to the uncertain parameters.

These arguments clearly show that the minimisation of the objective function is not the primary goal. The overriding goal is to provide feasible solutions that are insensitive to changes in uncertain parameters. The solutions with the minimum cost from this result set can then be chosen and implemented. The trade off between higher costs for increased insensitivity to uncertain parameters is one that should be made by the organisation. The manpower planning model should provide the

organisation with the relevant information to decide on the degree of trade–off they desire.

CHAPTER 7

The algorithm

1. Overview of the algorithm

To solve the manpower planning model, a mixed cooperative and competitive co–evolutionary algorithm is used.

The first step is a to consider the co–evolutionary algorithms associated with Model 1 and Model 2. Each of these are run to produce employment and assignment solutions. These are then combined to produce initial populations for a co–evolutionary algorithm associated with the original model. This allows the majority of the constraints to be checked before the original model is even considered. This mechanism to generate an initial population for the original model is thus more effective that a method of generating uniformly random solutions.

Each of the two initial co–evolutionary algorithms solve a model that is less restrictive than the original model. The key idea is that a feasible solution of the original model, when separated, must be a feasible solution in Model 1 and Model 2. Let m_1 and m_2 be maps from a feasible solution s_0 in the original model to solutions s_1 and s_2 in each of the other models respectively. Then

$$s_1 = m_1(s_0)$$
$$s_2 = m_2(s_0).$$

are both feasible solutions in the spaces of the respective model by virtue of the fact that they only contain a subset of the constraints from the original model.

Each of the co–evolutionary algorithms consists of one prey and one predator population. The predators perform predation operations on the prey, affecting their chances of passing into the next generation. The remaining prey then breed, producing new solutions, and the process continues until some stop condition is reached.

Prey represent solutions of the control variables, thus representing a complete solution. The state variables can be computed from the values of the decision variables, and the objective function can then be evaluated. Predators represent the constraints for a particular model, with a measure of their effectiveness against solutions.

In this implementation of the algorithm, we allow the prey populations to evolve so that fitter solutions can be generated. The predator populations, by contrast, are kept static and do not evolve. The reason for this is straightforward: we want our solutions to satisfy the constraints we specify initially, not some other constraints that are the result of genetic operations.

When an interaction takes place between a solution and a constraint, there are two possibilities. If the solution satisfies the constraint, it survives and continues breeding. If it does not satisfy the constraint, it is eliminated and the constraint's fitness is increased. After several generations, this gives us an excellent measure of which constraints in the problem are trivially satisfied and which constraints are extremely difficult to satisfy.

Formally, the steps for the algorithm are as follows:

```
Input  : Problem data
Output: List of fittest, feasible solutions
Initialise initial prey population, predator population;
while Stop conditions not reached do
    for offspring ← numberOfOffspring do
        breeders = SelectBreeders(preyPopulation, numberOfOffspring);
        foreach breedingPair in breeders do
            offspringPair = Breed(breedingPair)
            offspringPair = Mutate(offspringPair)
            preyPopulation = preyPopulation ⋃ offspringPair
        end
    end
    for attacks = 1 to maximumAttacks do
        predator = SelectPredator(predatorPopulation);
        prey = SelectRandomPrey(preyPopulation);
        if predator Kills Prey then
            preyPopulation = preyPopulation \{prey};
            predator.fitness = predator.fitness + 1;
        end
    end
end
```

Algorithm 2: Co–evolutionary algorithm

2. Implementation

The algorithm is implemented in C#, a strongly typed, object–oriented language based on Microsoft's .NET platform. The language provides a structured environment to model the manpower planning model and the co–evolutionary algorithm on an appropriate level of abstraction. The object-oriented nature of the language allows concepts such as inheritance and polymorphism to assist in the abstraction process.

The various test cases and the main problem were run on an AMD Athlon(tm) 64 X2 dual core processor 5400+, with 3.25 GB of RAM, running Windows XP Professional and the required .NET 3.5 Framework.

A code profiling tool, NProf, was used to profile the code and target specific pieces of the code that required optimisation to improve performance. FXCop, a tool that checks conformance with Microsoft's .NET Framework Design Guidelines was used

to ensure the guidelines were reasonably followed. Inheritance diagrams and code documentation were generated using Graphviz and Doxygen.

3. Programming concepts

The principles underlying object oriented programming are expressed in a way that allows one to model complex systems in a manner that is natural to the problem. The ability to create user defined types (classes) allows one to capture the essential aspects of the system that must be modelled, while maintaining a level of complexity that is manageable.

In the case of the manpower planning problem, there are several different layers that are modelled. The first is the actual manpower model, expressed in terms of workers, tasks, costs, and so forth. Another level is the modelling of the algorithm, its operations (such as mutation and breeding), and the members of the populations (predators and prey). Finally, there is a mathematical layer that represents the objective function, the balance equations and so on.

Object oriented programming revolves around three central principles: Encapsulation, inheritance and polymorphism.

Encapsulation is the principle that is concerned with separating objects and hiding the implementation of an objects behaviour from other objects. The objects should clearly specify what operations are allowed to be performed on them by other objects, and which operations they are allowed to perform by themselves. The actual implementation of methods in objects is hidden so that other objects behaviour is not dependent on changes to these. This creates a distinct zone of demarcation for objects (the clients) that access the methods of another object (the server). The server exposes a set of interfaces, while hiding the implementation of these interfaces. This eliminates the dependency of the clients on the server's implementation, and they are only dependant on the interfaces. As an example, consider the task information class. This class exposes certain information on which workers can perform a particular task. The public interface defined by this class is one that allows a client to call the CanBePerformedBy method, passing in a task. The client can then expect a list of workers that can perform the task to be returned. The internal implementation is done via a lookup in a dictionary, but this is hidden from the client.

```
public class TaskInfo
```

```
{
private Dictionary<Task, List<WorkerType>> _canBePerformedBy;

public List<WorkerType> CanBePerformedBy(Task task)
    {
         ...
    }
}
```

Inheritance allows for levels of abstraction by defining the shared properties of groups of objects. The class defining the common behaviour of the other classes is referred to as the base class, and classes inheriting from this are termed derived classes. All classes that inherit from this class are guaranteed to satisfy the behaviour set out by the base class. This reduces the amount of code that would otherwise be needed to model a problem. Inheritance also results in generalisations of types, as the base class represents a generalisation of the derived classes. As an example, a simple genetic algorithm manager type is defined to inherit from a co–evolutionary algorithm type, and the base classes Run method is overridden.

```
public class SimpleGAManager : CoevolutionaryAlgorithm
{
     ...
}

public override void Run()
{
     ...
}
```

Figure 7.1 shows the detailed inheritance diagram for the Chromosome class. This shows the classes that inherit from the chromosome class, as well as the various member methods and variables applicable to each class. Figure 7.3 shows the simplified inheritance diagram for the same class.

Figure 7.2 shows the collaboration diagram for the co–evolutionary algorithm class. This shows the member variables of the co–evolutionary algorithm class, as well as the various member methods and variables applicable to each object.

FIGURE 7.1. Detailed chromosome inheritance

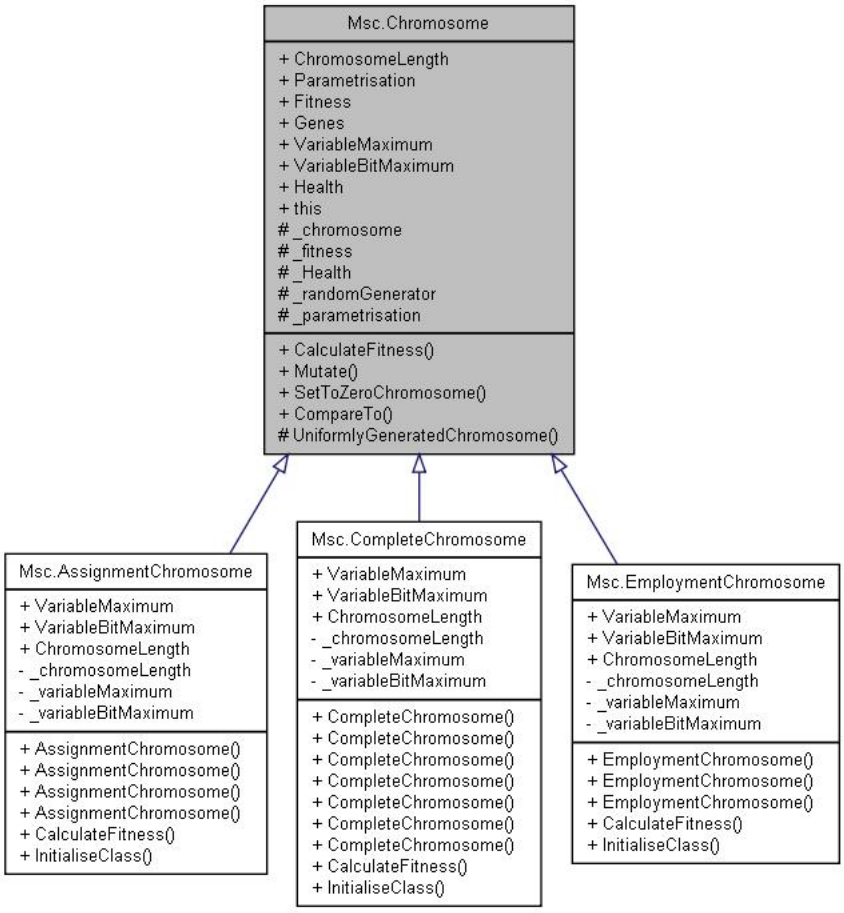


Figure 7.4 shows the inheritance diagram for the Constraint class. This shows the various classes that inherit from the Constraint class.

Polymorphism allows a disparate collection of objects that share the same base class to be treated as a collection of the same objects. In this way, operations may be performed on a collection of these objects without actually knowing the underlying types of the objects. As an example, consider the abstract class Chromosome

```
abstract public class Chromosome : IComparable<Chromosome>
{
     ...
}
```

FIGURE 7.2. Co–evolutionary algorithm collaboration graph

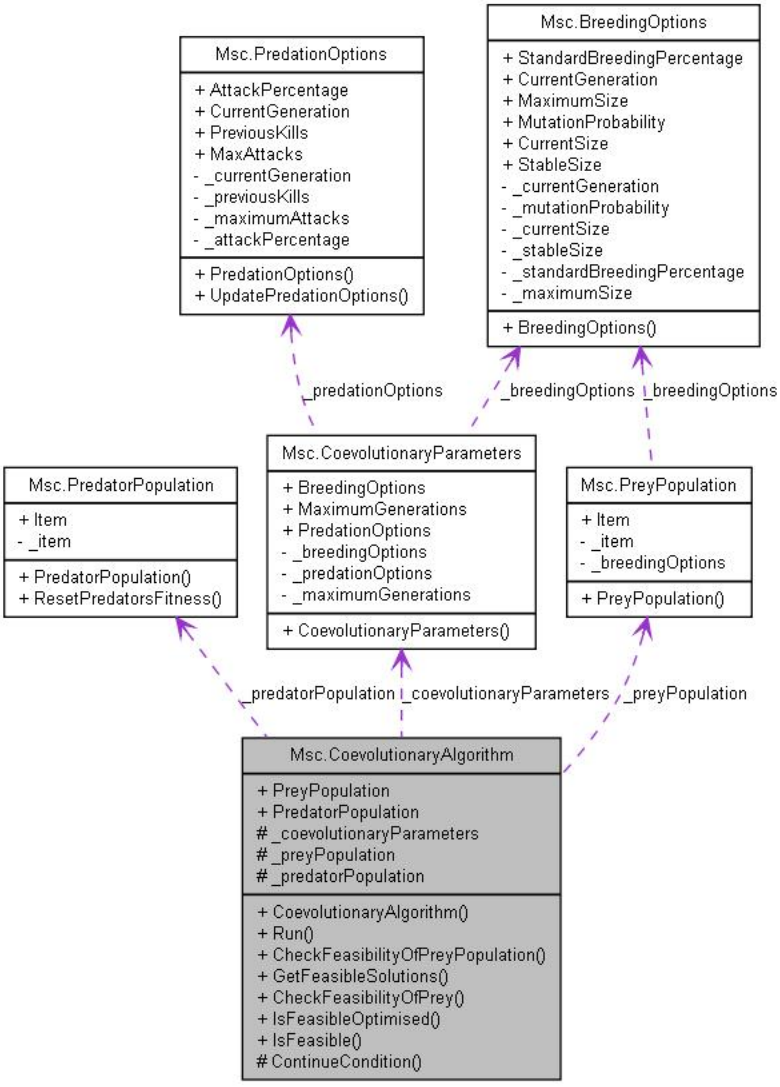


FIGURE 7.3. Chromosome inheritance

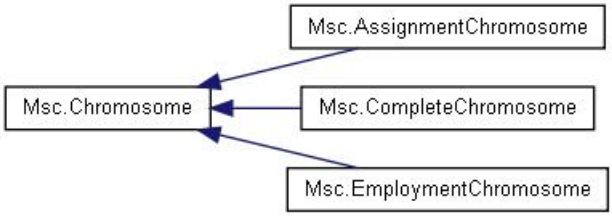
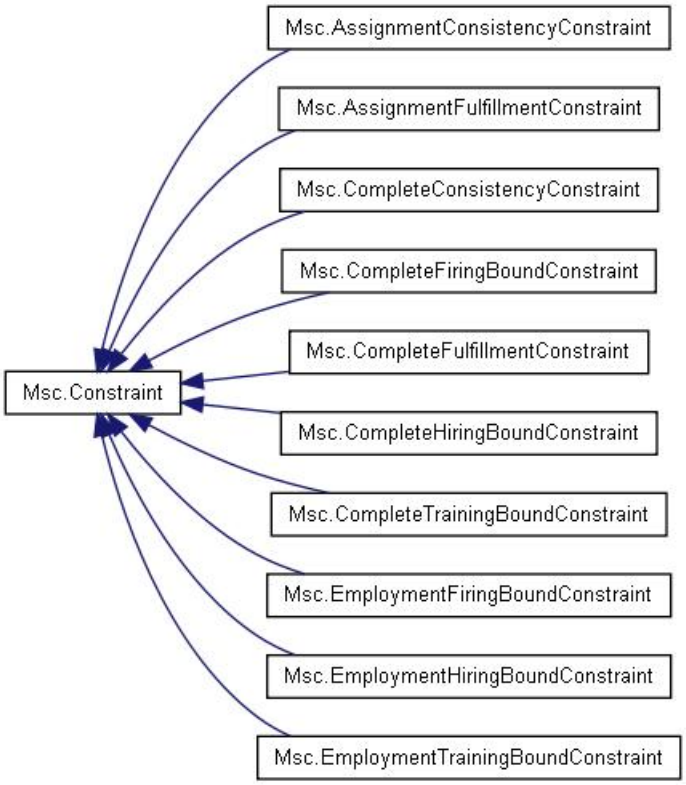


FIGURE 7.4. Constraint inheritance



By definition, an abstract class cannot be instantiated. Even so, polymorphism can be used to loop over a list of items that all inherit from the Chromosome class

```
foreach (Chromosome chromosome in preyPopulation.Item)
{
     ...
}
```

Another powerful concept is that of reflection. This allows a class to inspect its own properties and possibly modify its behaviour. As an example, consider the abstract Chromosome class. Since it is abstract, we cannot construct an instance of it. At a point in the code where polymorphism is used to loop over a collection of Chromosomes, information about the underlying class is needed. In the code below, reflection is used to query the constructor information for the class underlying the Chromosome object. This is then used to instantiate another instance of the type.

```
ConstructorInfo constructorInfo =
```

```
    preyPopulation.Item[preyIndex].GetType().GetConstructor(
        new Type[] { preyPopulation.Item[preyIndex].GetType() });

tempObject = constructorInfo.Invoke(
    new object[] { preyPopulation.Item[preyIndex] });
```

CHAPTER 8

Test cases

This chapter considers three test cases, each of which considers a case of the manpower planning model with certain characteristics. Each of the cases is small enough to be checked using a brute force algorithm, allowing comparison of the optimal solution with the solutions generated by the co–evolutionary algorithm.

1. Test case 1

Test case 1 considers a manpower planning model in which there is only a single worker type, a single task, and no training. This is the most straightforward scenario, where a single worker is only responsible for performing a single task.

The following information defines the test case

TABLE 8.1. Test case 1 - General information

General parameters	
Number of constraints	14
Number of decision variables	10
Number of state variables	3

TABLE 8.2. Test case 1 - General parameters

General parameters	
T	3
N	1
m	1

TABLE 8.3. Test case 1 - Task information

Tasks	
$M(1)$	$\{1\}$

TABLE 8.4. Test case 1 - Hiring bounds

Hiring bounds	
$b_1^h(t)\ \forall\ t$	2

TABLE 8.5. Test case 1 - Dismissal bounds

Dismissal bounds	
$b_1^f(t)\ \forall\ t$	1

TABLE 8.6. Test case 1 - Demand

$d_m(t)$				
	$t=0$	$t=1$	$t=2$	$t=3$
$m=1$	1	3	2	2

TABLE 8.7. Test case 1 - Hiring costs

Hiring costs	
$c_1^h(t)\ \forall\ t$	1

TABLE 8.8. Test case 1 - Dismissal costs

Dismissal costs	
$c_1^f(t)\ \forall\ t$	1.5

TABLE 8.9. Test case 1 - Salaries

Salaries	
$s_1(t)\ \forall\ t$	0.7

TABLE 8.10. Test case 1 - Initial workers

Initial workers	
$x_1(0)$	2

2. Test case 1 - Brute force algorithm results

The optimal solution given by the brute force algorithm returned a feasible solution with an objective function value of 6.6. The values of the decision and state variables are given in Table 8.11.

TABLE 8.11. Test case 1 - Optimal solution

Optimal solution	
$h_1(0)$	1
$h_1(1)$	0
$h_1(2)$	0
$f_1(0)$	0
$f_1(1)$	0
$f_1(2)$	0
$v_{1,1}(0)$	2
$v_{1,1}(1)$	3
$v_{1,1}(2)$	2
$v_{1,1}(3)$	3
$x_1(0)$	2
$x_1(1)$	3
$x_1(2)$	3
$x_1(3)$	3

3. Test case 1 - Co–evolutionary algorithm results

The optimal solution given by the co–evolutionary algorithm returned a feasible solution with an objective function value of 6.6. The values of the decision and state variables are given in Table 8.12.

TABLE 8.12. Test case 1 - Co–evolutionary Optimal solution

Optimal solution	
$h_1(0)$	1
$h_1(1)$	0
$h_1(2)$	0
$f_1(0)$	0
$f_1(1)$	0
$f_1(2)$	0
$v_{1,1}(0)$	1
$v_{1,1}(1)$	3
$v_{1,1}(2)$	2
$v_{1,1}(3)$	3
$x_1(0)$	2
$x_1(1)$	3
$x_1(2)$	3
$x_1(3)$	3

The objective function value is the same as the value of the optimal solution found by the brute force algorithm. Only assignment decision variable values differ slightly. This is expected as the assignment decision variables do not affect values of the objective function.

The sensitivity analysis sample was almost complete, with a coverage of 0.999. This revealed that the optimal solution found had a feasibility measure of 0.594, and the solution found by the co–evolutionary algorithm had a feasibility measure of 0.495.

The co–evolutionary algorithm found solutions with higher feasibility at a higher cost. This is expected, as higher costs often involve hiring more workers, making them available for increases in demand in the future.

TABLE 8.13. Test case 2 - General information

General parameters	
Number of constraints	28
Number of decision variables	20
Number of state variables	6

TABLE 8.14. Test case 2 - General parameters

General parameters	
T	3
N	2
m	2

TABLE 8.15. Test case 2 - Task information

Tasks	
$M(1)$	$\{1\}$
$M(2)$	$\{2\}$

TABLE 8.16. Test case 2 - Hiring bounds

Hiring bounds	
$b_1^h(t)\ \forall\ t$	2
$b_2^h(t)\ \forall\ t$	2

4. Test case 2

Test case 2 considers a manpower planning model in which there are multiple worker types, multiple tasks, and no training. This scenario is used to highlight the aspect of multi–skilled workers.

The following information defines the test case

TABLE 8.17. Test case 2 - Dismissal bounds

Dismissal bounds	
$b_1^f(t)\ \forall\ t$	1
$b_2^f(t)\ \forall\ t$	1

TABLE 8.18. Test case 2 - Demand

$d_m(t)$				
	$t=0$	$t=1$	$t=2$	$t=3$
$m=1$	1	2	1	1
$m=2$	0	1	2	2

TABLE 8.19. Test case 2 - Hiring costs

Hiring costs	
$c_1^h(t)\ \forall\ t$	1.2
$c_2^h(t)\ \forall\ t$	1.5

TABLE 8.20. Test case 2 - Dismissal costs

Dismissal costs	
$c_1^f(t)\ \forall\ t$	2
$c_2^f(t)\ \forall\ t$	2.4

TABLE 8.21. Test case 2 - Salaries

Salaries	
$s_1(t)\ \forall\ t$	1
$s_2(t)\ \forall\ t$	1.2

TABLE 8.22. Test case 2 - Initial workers

Initial workers	
$x_1(0)$	2
$x_2(0)$	1

5. Test case 2 - Brute force algorithm results

The optimal solution given by the brute force algorithm returned a feasible solution with an objective function value of 12.3. The values of the decision and state variables are given in Table 8.23.

TABLE 8.23. Test case 2 - Optimal solution

Optimal solution	
$h_1(0)$	0
$h_2(0)$	0
$h_1(1)$	0
$h_2(1)$	1
$h_1(2)$	0
$h_2(2)$	0
$f_1(0)$	0
$f_2(0)$	0
$f_1(1)$	0
$f_2(1)$	0
$f_1(2)$	0
$f_2(2)$	0
$v_{1,2}(0)$	2
$v_{2,1}(0)$	0
$v_{1,2}(1)$	2
$v_{2,1}(1)$	1
$v_{1,2}(2)$	2
$v_{2,1}(2)$	2
$v_{1,2}(3)$	2
$v_{2,1}(3)$	2
$x_1(0)$	2
$x_2(0)$	1
$x_1(1)$	2
$x_2(1)$	1
$x_1(2)$	2
$x_2(2)$	2
$x_1(3)$	2
$x_2(3)$	2

6. Test case 2 - Co–evolutionary algorithm results

The solution given by the co–evolutionary algorithm returned a feasible solution with an objective function value of 13.5. The values of the decision and state variables are given in Table 8.24.

TABLE 8.24. Test case 2 - Co–evolutionary Optimal solution

Optimal solution	
$h_1(0)$	0
$h_2(0)$	1
$h_1(1)$	0
$h_2(1)$	0
$h_1(2)$	0
$h_2(2)$	0
$f_1(0)$	0
$f_2(0)$	0
$f_1(1)$	0
$f_2(1)$	0
$f_1(2)$	0
$f_2(2)$	0
$v_{1,2}(0)$	2
$v_{2,1}(0)$	1
$v_{1,2}(1)$	2
$v_{2,1}(1)$	2
$v_{1,2}(2)$	1
$v_{2,1}(2)$	2
$v_{1,2}(3)$	2
$v_{2,1}(3)$	2
$x_1(0)$	2
$x_2(0)$	1
$x_1(1)$	2
$x_2(1)$	2
$x_1(2)$	2
$x_2(2)$	2
$x_1(3)$	2
$x_2(3)$	2

The objective function value of the solution found by the co–evolutionary algorithm is only slightly higher than the value of optimal solution found by the brute force algorithm.

The sensitivity analysis sample was almost complete (a coverage of 0.969). This revealed that the optimal solution found by the brute force algorithm had a normalised feasibility measure of 0.303, while the solution found by the co–evolutionary algorithm had a normalised feasibility measure of 0.253.

The co–evolutionary algorithm found solutions with higher feasibility at a higher cost. This is expected, as higher costs often involve hiring more workers, making them available for increases in demand in the future.

TABLE 8.25. Test case 3 - General information

General parameters	
Number of constraints	22
Number of decision variables	19
Number of state variables	6

TABLE 8.26. Test case 3 - General parameters

General parameters	
T	2
N	2
m	2

TABLE 8.27. Test case 3 - Tasks

Task information	
$M(1)$	$\{1\}$
$M(2)$	$\{1, 2\}$

TABLE 8.28. Test case 3 - Training information

Training	
$L(1)$	$\{2\}$

7. Test case 3

Test case 3 considers a manpower planning model in which there are multiple worker types, multiple tasks, and training programs for a subset of the worker types. This is the most general scenario, where workers are responsible for various tasks and are able to be trained to other worker types.

The following information defines the test case

TABLE 8.29. Test case 3 - Hiring bounds

Hiring bounds	
$b_1^h(t)\ \forall\ t$	2
$b_2^h(t)\ \forall\ t$	2

TABLE 8.30. Test case 3 - Dismissal bounds

Dismissal bounds	
$b_1^f(t)\ \forall\ t$	1
$b_2^f(t)\ \forall\ t$	1

TABLE 8.31. Test case 3 - Training bounds

Training bounds	
$b_{12}^u(t)\ \forall\ t$	1

TABLE 8.32. Test case 3 - Training times

Training times	
τ_{12}	1

TABLE 8.33. Test case 3 - Demand

$d_m(t)$			
	$t=0$	$t=1$	$t=2$
$m=1$	1	2	1
$m=2$	0	1	2

TABLE 8.34. Test case 3 - Hiring costs

Hiring costs	
$c_1^h(t)\ \forall\ t$	1.2
$c_2^h(t)\ \forall\ t$	1.5

TABLE 8.35. Test case 3 - Dismissal costs

Dismissal costs	
$c_1^f(t)\ \forall\ t$	2
$c_2^f(t)\ \forall\ t$	2.4

TABLE 8.36. Test case 3 - Training costs

Training costs	
$c_{12}^u(t)\ \forall\ t$	0.5

TABLE 8.37. Test case 3 - Salaries

Salaries	
$s_1(t)\ \forall\ t$	1
$s_2(t)\ \forall\ t$	1.2

TABLE 8.38. Test case 3 - Initial workers

Initial workers	
$x_1(0)$	2
$x_2(0)$	1

TABLE 8.39. Test case 3 - Initial workers in training

Initial workers in training	
$y_{12}(0)$	0

8. Test case 3 - Brute force algorithm results

The optimal solution given by the brute force algorithm returned a feasible solution with an objective function value of 7.9. The values of the decision and state variables are given in Table 8.40.

TABLE 8.40. Test case 3 - Optimal solution

Optimal solution	
$h_1(0)$	0
$h_2(0)$	0
$h_1(1)$	0
$h_2(1)$	1
$f_1(0)$	0
$f_2(0)$	0
$f_1(1)$	0
$f_2(1)$	0
$u_{1,2}(0)$	0
$u_{1,2}(1)$	0
$v_{1,2}(0)$	2
$v_{2,2}(0)$	0
$v_{2,1}(0)$	0
$v_{1,2}(1)$	2
$v_{2,2}(1)$	0
$v_{2,1}(1)$	1
$v_{1,2}(2)$	2
$v_{2,2}(2)$	0
$v_{2,1}(2)$	2
$x_1(0)$	2
$x_2(0)$	1
$x_1(1)$	2
$x_2(1)$	1
$x_1(2)$	2
$x_2(2)$	2
$y_{1,2}(0)$	0
$y_{1,2}(1)$	0
$y_{1,2}(2)$	0

9. Test case 3 - Co–evolutionary algorithm results

The optimal solution given by the co–evolutionary algorithm returned a feasible solution with an objective function value of 9.4. The values of the decision and state variables are given in Table 8.41.

TABLE 8.41. Test case 3 - Co–evolutionary Optimal solution

Optimal solution	
$h_1(0)$	0
$h_2(0)$	0
$h_1(1)$	0
$h_2(1)$	2
$f_1(0)$	0
$f_2(0)$	0
$f_1(1)$	0
$f_2(1)$	0
$u_{1,2}(0)$	0
$u_{1,2}(1)$	0
$v_{1,2}(0)$	2
$v_{2,2}(0)$	0
$v_{2,1}(0)$	1
$v_{1,2}(1)$	2
$v_{2,2}(1)$	0
$v_{2,1}(1)$	1
$v_{1,2}(2)$	1
$v_{2,2}(2)$	1
$v_{2,1}(2)$	2
$x_1(0)$	2
$x_2(0)$	1
$x_1(1)$	2
$x_2(1)$	1
$x_1(2)$	2
$x_2(2)$	3
$y_{1,2}(0)$	0
$y_{1,2}(1)$	0
$y_{1,2}(2)$	0

The objective function value of the solution found by the co–evolutionary algorithm is only slightly higher than the value of optimal solution found by the brute force algorithm.

The sensitivity analysis sample was almost complete (a coverage of 0.999). This revealed that the optimal solution found by the brute force algorithm has a normalised

feasibility measure of 0.457, while the best solution found by the co–evolutionary algorithm had a normalised feasibility measure of 0.381.

The co–evoltionary algorithm found solutions with higher feasibility at a higher cost. This is expected, as higher costs often involve hiring more workers, making them available for increases in demand in the future.

CHAPTER 9

Main problem

The following chapter considers the main problem. This problem is adapted from the sample problem in [**3**], but the problems share the same optimal solution.

1. Main problem

The following information defines the main problem, which is adapted from [**3**], where an optimal solution was found. It is a realistically sized manpower planning problem, with multi–skilled workers, training programs, and a restrictive set of demand parameters. The co–evolutionary algorithm will be used to solve the problem, and its solutions compared with the solution found in [**3**]. A feasibility analysis will then be done on the solutions.

TABLE 9.1. Main problem - General information

General parameters	
Number of constraints	122
Number of decision variables	103
Number of state variables	45

TABLE 9.2. Main problem - General parameters

General parameters	
T	9
N	3
m	2

TABLE 9.3. Main problem - Task information

Tasks	
$M(1)$	$\{1\}$
$M(2)$	$\{2\}$
$M(3)$	$\{1, 2\}$

TABLE 9.4. Main problem - Training information

Training	
$L(1)$	$\{3\}$
$L(2)$	$\{3\}$
$L(3)$	$\{\}$

TABLE 9.5. Main problem - Hiring bounds

Hiring bounds	
$b_1^h(t)\ \forall\ t$	4
$b_2^h(t)\ \forall\ t$	3
$b_3^h(t)\ \forall\ t$	2

TABLE 9.6. Main problem - Dismissal bounds

Dismissal bounds	
$b_1^f(t)\ \forall\ t$	2
$b_2^f(t)\ \forall\ t$	2
$b_3^f(t)\ \forall\ t$	2

TABLE 9.7. Main problem - Training bounds

Training bounds	
$b_{13}^u(t)\ \forall\ t$	2
$b_{23}^u(t)\ \forall\ t$	2

TABLE 9.8. Main problem - Demand

$d_m(t)$										
	$t=0$	$t=1$	$t=2$	$t=3$	$t=4$	$t=5$	$t=6$	$t=7$	$t=8$	$t=9$
$m=1$	2	2	3	2	1	2	6	2	2	6
$m=2$	2	3	1	2	7	2	2	8	5	1

TABLE 9.9. Main problem - Hiring costs

Hiring costs	
$c_1^h(t)\ \forall\ t$	1.2
$c_2^h(t)\ \forall\ t$	1.5
$c_3^h(t)\ \forall\ t$	3.0

TABLE 9.10. Main problem - Dismissal costs

Dismissal costs	
$c_1^f(t)\ \forall\ t$	2.0
$c_2^f(t)\ \forall\ t$	2.4
$c_3^f(t)\ \forall\ t$	4.0

TABLE 9.11. Main problem - Salaries

Salaries	
$s_1(t)\ \forall\ t$	1.0
$s_2(t)\ \forall\ t$	1.2
$s_3(t)\ \forall\ t$	2.0

TABLE 9.12. Main problem - Training costs

Training costs	
$c^u_{23}(t)\ \forall\ t$	0.3
$c^u_{13}(t)\ \forall\ t$	0.5

TABLE 9.13. Main problem - Training times

Training times	
τ_{13}	2
τ_{23}	1

TABLE 9.14. Main problem - Initial workers

Initial workers	
$x_1(0)$	2
$x_2(0)$	2
$x_3(0)$	0

TABLE 9.15. Main problem - Initial workers in training

Initial workers in training	
y_{13}	0
y_{23}	0

TABLE 9.16. Main problem optimal solution - hiring variables

Optimal solution - hiring variables	
$h_1(0)$	0
$h_2(0)$	0
$h_3(0)$	1
$h_1(1)$	0
$h_2(1)$	0
$h_3(1)$	0
$h_1(2)$	0
$h_2(2)$	2
$h_3(2)$	0
$h_1(3)$	0
$h_2(3)$	2
$h_3(3)$	0
$h_1(4)$	0
$h_2(4)$	0
$h_3(4)$	0
$h_1(5)$	1
$h_2(5)$	0
$h_3(5)$	0
$h_1(6)$	0
$h_2(6)$	0
$h_3(6)$	0
$h_1(7)$	0
$h_2(7)$	0
$h_3(7)$	0
$h_1(8)$	0
$h_2(8)$	0
$h_3(8)$	0

2. Optimal solution results

TABLE 9.17. Optimal solution - dismissal variables

Optimal solution - dismissal variables	
$f_1(0)$	0
$f_2(0)$	0
$f_3(0)$	0
$f_1(1)$	0
$f_2(1)$	0
$f_3(1)$	0
$f_1(2)$	0
$f_2(2)$	0
$f_3(2)$	0
$f_1(3)$	0
$f_2(3)$	0
$f_3(3)$	0
$f_1(4)$	0
$f_2(4)$	0
$f_3(4)$	0
$f_1(5)$	0
$f_2(5)$	0
$f_3(5)$	0
$f_1(6)$	0
$f_2(6)$	0
$f_3(6)$	0
$f_1(7)$	0
$f_2(7)$	2
$f_3(7)$	0
$f_1(8)$	0
$f_2(8)$	0
$f_3(8)$	0

TABLE 9.18. Optimal solution - training variables

Optimal solution - training variables	
$u_{1,3}(0)$	0
$u_{2,3}(0)$	0
$u_{1,3}(1)$	0
$u_{2,3}(1)$	0
$u_{1,3}(2)$	0
$u_{2,3}(2)$	0
$u_{1,3}(3)$	1
$u_{2,3}(3)$	0
$u_{1,3}(4)$	0
$u_{2,3}(4)$	2
$u_{1,3}(5)$	0
$u_{2,3}(5)$	0
$u_{1,3}(6)$	0
$u_{2,3}(6)$	0
$u_{1,3}(7)$	0
$u_{2,3}(7)$	0
$u_{1,3}(8)$	0
$u_{2,3}(8)$	0

TABLE 9.19. Optimal solution - assignment variables

Optimal solution - assignment variables	
$v_{1,1}(0)$	2
$v_{2,2}(0)$	2
$v_{3,1}(0)$	0
$v_{3,2}(0)$	0
$v_{1,1}(1)$	2
$v_{2,2}(1)$	2
$v_{3,1}(1)$	0
$v_{3,2}(1)$	1
$v_{1,1}(2)$	2
$v_{2,2}(2)$	2
$v_{3,1}(2)$	1
$v_{3,2}(2)$	0
$v_{1,1}(3)$	2
$v_{2,2}(3)$	4
$v_{3,1}(3)$	1
$v_{3,2}(3)$	0
$v_{1,1}(4)$	1
$v_{2,2}(4)$	6
$v_{3,1}(4)$	0
$v_{3,2}(4)$	1
$v_{1,1}(5)$	1
$v_{2,2}(5)$	4
$v_{3,1}(5)$	1
$v_{3,2}(5)$	0
$v_{1,1}(6)$	2
$v_{2,2}(6)$	4
$v_{3,1}(6)$	4
$v_{3,2}(6)$	0
$v_{1,1}(7)$	2
$v_{2,2}(7)$	4
$v_{3,1}(7)$	0
$v_{3,2}(7)$	4
$v_{1,1}(8)$	2
$v_{2,2}(8)$	2
$v_{3,1}(8)$	1
$v_{3,2}(8)$	3
$v_{1,1}(9)$	2
$v_{2,2}(9)$	2
$v_{3,1}(9)$	4
$v_{3,2}(9)$	0

TABLE 9.20. Main problem co–evolutionary solution - hiring variables

Co–evolutionary solution - hiring variables	
$h_1(0)$	1
$h_2(0)$	2
$h_3(0)$	2
$h_1(1)$	1
$h_2(1)$	0
$h_3(1)$	3
$h_1(2)$	3
$h_2(2)$	1
$h_3(2)$	3
$h_1(3)$	0
$h_2(3)$	0
$h_3(3)$	1
$h_1(4)$	0
$h_2(4)$	0
$h_3(4)$	0
$h_1(5)$	0
$h_2(5)$	0
$h_3(5)$	0
$h_1(6)$	0
$h_2(6)$	0
$h_3(6)$	0
$h_1(7)$	1
$h_2(7)$	1
$h_3(7)$	0
$h_1(8)$	1
$h_2(8)$	3
$h_3(8)$	0

3. Co–evolutionary algorithm results

The objective function value of the solution found by the co–evolutionary algorithm is 283.8, compared to the optimal solution which has an objective function value of 119.1.

The sensitivity analysis sample had a coverage of 0.277. The actual optimal value was found to have a normalised feasibility measure of 0.152, while the best solution found by the co–evolutionary algorithm had a normalised feasibility measure of 0.524. This shows that while the cost of the solution found by the co–evolutionary algorithm is higher, it is far less sensitive to changes in the demand parameters.

TABLE 9.21. Main problem co–evolutionary solution - dismissal variables

Co–evolutionary solution - dismissal variables	
$f_1(0)$	0
$f_2(0)$	0
$f_3(0)$	0
$f_1(1)$	0
$f_2(1)$	0
$f_3(1)$	0
$f_1(2)$	0
$f_2(2)$	0
$f_3(2)$	0
$f_1(3)$	0
$f_2(3)$	0
$f_3(3)$	0
$f_1(4)$	0
$f_2(4)$	0
$f_3(4)$	0
$f_1(5)$	0
$f_2(5)$	0
$f_3(5)$	0
$f_1(6)$	0
$f_2(6)$	0
$f_3(6)$	0
$f_1(7)$	0
$f_2(7)$	0
$f_3(7)$	0
$f_1(8)$	0
$f_2(8)$	0
$f_3(8)$	0

TABLE 9.22. Main problem co–evolutionary solution - training variables

Co–evolutionary solution - training variables	
$u_{1,3}(0)$	2
$u_{2,3}(0)$	0
$u_{1,3}(1)$	0
$u_{2,3}(1)$	1
$u_{1,3}(2)$	0
$u_{2,3}(2)$	0
$u_{1,3}(3)$	2
$u_{2,3}(3)$	2
$u_{1,3}(4)$	0
$u_{2,3}(4)$	0
$u_{1,3}(5)$	0
$u_{2,3}(5)$	0
$u_{1,3}(6)$	0
$u_{2,3}(6)$	0
$u_{1,3}(7)$	1
$u_{2,3}(7)$	0
$u_{1,3}(8)$	2
$u_{2,3}(8)$	1

TABLE 9.23. Main problem co–evolutionary solution - assignment variables

Co–evolutionary solution - assignment variables	
$v_{1,1}(0)$	2
$v_{2,2}(0)$	2
$v_{3,1}(0)$	0
$v_{3,2}(0)$	0
$v_{1,1}(1)$	2
$v_{2,2}(1)$	4
$v_{3,1}(1)$	0
$v_{3,2}(1)$	2
$v_{1,1}(2)$	3
$v_{2,2}(2)$	0
$v_{3,1}(2)$	4
$v_{3,2}(2)$	1
$v_{1,1}(3)$	2
$v_{2,2}(3)$	3
$v_{3,1}(3)$	0
$v_{3,2}(3)$	0
$v_{1,1}(4)$	2
$v_{2,2}(4)$	3
$v_{3,1}(4)$	1
$v_{3,2}(4)$	6
$v_{1,1}(5)$	3
$v_{2,2}(5)$	3
$v_{3,1}(5)$	1
$v_{3,2}(5)$	2
$v_{1,1}(6)$	3
$v_{2,2}(6)$	3
$v_{3,1}(6)$	8
$v_{3,2}(6)$	1
$v_{1,1}(7)$	3
$v_{2,2}(7)$	4
$v_{3,1}(7)$	0
$v_{3,2}(7)$	8
$v_{1,1}(8)$	3
$v_{2,2}(8)$	6
$v_{3,1}(8)$	3
$v_{3,2}(8)$	2
$v_{1,1}(9)$	5
$v_{2,2}(9)$	2
$v_{3,1}(9)$	2
$v_{3,2}(9)$	0

TABLE 9.24. Main problem co–evolutionary solution - available workers

Co–evolutionary solution - available workers	
$x_1(0)$	2
$x_2(0)$	2
$x_3(0)$	0
$x_1(1)$	3
$x_2(1)$	4
$x_3(1)$	2
$x_1(2)$	4
$x_2(2)$	4
$x_3(2)$	5
$x_1(3)$	7
$x_2(3)$	5
$x_3(3)$	8
$x_1(4)$	7
$x_2(4)$	5
$x_3(4)$	9
$x_1(5)$	7
$x_2(5)$	5
$x_3(5)$	9
$x_1(6)$	7
$x_2(6)$	5
$x_3(6)$	9
$x_1(7)$	7
$x_2(7)$	5
$x_3(7)$	9
$x_1(8)$	8
$x_2(8)$	6
$x_3(8)$	9
$x_1(9)$	9
$x_2(9)$	9
$x_3(9)$	9

TABLE 9.25. Main problem co–evolutionary solution - available workers

Co–evolutionary solution - workers in training	
$y_{1,3}(0)$	0
$y_{2,3}(0)$	0
$y_{1,3}(1)$	2
$y_{2,3}(1)$	0
$y_{1,3}(2)$	2
$y_{2,3}(2)$	1
$y_{1,3}(3)$	0
$y_{2,3}(3)$	0
$y_{1,3}(4)$	2
$y_{2,3}(4)$	2
$y_{1,3}(5)$	2
$y_{2,3}(5)$	0
$y_{1,3}(6)$	0
$y_{2,3}(6)$	0
$y_{1,3}(7)$	0
$y_{2,3}(7)$	0
$y_{1,3}(8)$	1
$y_{2,3}(8)$	0
$y_{1,3}(9)$	3
$y_{2,3}(9)$	1

CHAPTER 10

Conclusion

In this dissertation we reviewed the manpower planning problem and its role in managing the resources of an organisation to meet its demands for the future.

The manpower planning problem was cast as a mathematical model. The model allows for various classes of constraints to model the restrictions on the system, mechanisms for hiring, dismissing, training and assigning workers to tasks. Balance equations were introduced to track the number of workers, as well as the number of workers in training. The balance equations, objective function and constraints were analysed. It was noted that while the objective function is linear in the decision and state variables, the state variables are non–linear. The non–linearity is a result of the time delay terms in the state equations. The overall effect is that the objective function and several classes of constraints are non–linear.

Various alternatives for modelling the problem were highlighted, and their strengths and weaknesses discussed. The exponential problem faced when solving problems with gradually larger numbers of variables was discussed, and the effect on the manpower planning model was considered. It was found that the restrictive constraints in the manpower planning model, along with the exponential problem, result in a rapidly decreasing ratio of feasible solutions to candidate solutions as the size of the problem increases.

The discrete nature of the problem was analysed and compared to methods used to solve continuous problems. A modified genetic algorithm was used to attempt to solve a medium sized manpower planning problem, and it was found that the algorithm could not progress due to a lack of feasible solutions in the initial generation of candidate solutions.

The sensitivity analysis of solutions was then considered as integral to analysing solutions to the manpower planning model. While optimisation problems focus primarily on optimising solutions with respect to the objective function, it may

be more important to also minimise the sensitivity of solutions to changes in parameters within constraints. In the case of the manpower planning model, these parameters are the demands for tasks in the future. Given the inherent uncertainty of the demand parameters, it is prudent to consider the feasibility of the solutions when the parameters are altered. A measure of feasibility was defined that coincided with the probability of a solution being feasible across a range of scenarios. This definition was extended to allow for a only a sample of the total number of the scenarios to be considered.

Methods for refining the model and making the model more tractable to solve were then introduced. The hiring bound constraints were highlighted as a class of constraints that could be relaxed to introduce a larger number of initial, feasible solutions. The employment and assignment aspects were then separated from the original model into two separate models. These new models were then shown to encapsulate some aspects of the original model, and methods for utilising these models to solve the original problem were discussed. The use of multiple populations was discussed and it was found that these could be applied to the two separated models to produce a higher number of initial, feasible solutions in the manpower planning model.

The co–evolutionary algorithm was then introduced and its method of operation defined. The algorithm uses a predator–prey model to simulate an optimisation problem with candidate solutions and constraints. The solutions are modelled as a prey population that breed new solutions to the problem, and a predator population was used to model the constraints in the system. Interactions between the predators and prey result in solutions that do not satisfy the constraints being eliminated gradually.

Three small test cases that could be checked with a brute force algorithm were then considered. It was shown that in some cases the co–evolutionary algorithm found the optimal solutions. In cases where it did not, the solutions objective function values differed only slightly. The feasibility of the solutions was also analysed.

A realistically sized manpower problem was then considered. The optimal solution found in [**3**] was compared to the best solution found by the co–evolutionary algorithm. The co–evolutionary algorithm produced a result with a higher objective function value, but had a much higher feasibility rating than the optimal solution. This is preferable as the uncertainty in the demand constraints' parameters require a set of decisions that are reasonably insensitive to changes in these parameters.

APPENDIX A

Calculations

1. Initial conditions for training

This section considers the special case where $\chi_{ij} \neq 0$.

A set of initial conditions $y_{ij}(0) = \chi_{ij}$ is not sufficient to determine a unique past for the problem. There are many states of the problem for times $t < 0$ that will lead to the initial conditions being satisfied, with differing effects on the state variables for $t > 0$. Thus, while we know how many workers are in training from one type to another at time $t = 0$, it is unknown how long each of the workers has been in training. This prevents us from knowing when workers should re–enter the pool of assignable workers. A unique past is guaranteed in the special case where $\chi_{ij} = 0$ $\forall \quad \chi_{ij}$.

This can be illustrated by considering the balance equations which govern the dynamics of the system,

$$x_i(t+1) = x_i(t) + h_i(t) - f_i(t) - \sum_{j \in L(i)} u_{ij}(t) + \sum_{j|i \in L(j)} u_{ji}(t - \tau_{ji})$$
$$y_{ij}(t+1) = y_{ij}(t) + u_{ij}(t) - u_{ij}(t - \tau_{ij}).$$

Setting $t = 0$ gives

$$x_i(1) = x_i(0) + h_i(0) - f_i(0) - \sum_{j \in L(i)} u_{ij}(0) + \sum_{j|i \in L(j)} u_{ji}(-\tau_{ji})$$
$$y_{ij}(1) = y_{ij}(0) + u_{ij}(0) - u_{ij}(-\tau_{ij}).$$

All variables are determined for the problems with the exception of the terms $u_{ij}(-\tau_{ij})$. These terms represent the control variables for training decisions that were made in the past ($t < 0$). By setting $t = 1, \ldots, T$, we note that we require the

information for some $u_{ij}(t)$ where $t < 0$. Specifically, we require information for all

$$u_{ij}(t), \quad t = -1, \ldots, -\tau_{ij},\ i = 1, \ldots, N,\ j \in L(i).$$

To generalise the initial conditions, this information needs to be added to the model. Thus, the knowledge of control variables in the previous time steps

$$u_{ij}(t), \quad i = 1, \ldots, N,\ j \in L(i),\ t = -1, \ldots, -\tau_{ij},$$

and the initial conditions

$$y_{ij} = \chi_{ij}$$

specify the problem uniquely when there are workers in training at $t = 0$.

2. Maximum cardinality of the sets L(i) and M(i)

Given a problem with N worker types and r tasks, it is useful to have an upper bound on the number of training variables (u_{ij}) and task variables (v_{im}) as simple functions of N and r. This can be done by making some assumptions about the structure of the problem.

To establish an upper bound, some basic assumptions must be made. Let the workforce exhibit a hierarchical structure, where workers who are higher in the hierarchy are capable of performing more tasks than workers lower in the hierarchy. We assume the tasks exhibit the same structure. A worker lower down in the hierarchy can be trained to any higher level in the hierarchy. Training is not circular, so for a given worker type i, there exists no sequence of training programs that will result in the worker being trained back to type i. The worker type at the top of the hierarchy is capable of performing all tasks. If worker type i can perform n number of tasks, worker type $i-1$ can perform $n-1$ number of tasks. This results in each a worker type at each lower level in the hierarchy being able to perform one less task than a worker at the higher level. We further assume that the number of tasks is equal to, or exceeds the number of worker types.

The above assumptions result in worker type 1 having training programs to all worker types, and worker type N having no training programs. In general

$$\operatorname{card} L(i) = N - i$$

and

$$\begin{aligned}
&\sum_{i=1}^{N} \operatorname{card} L(i) \\
&= \sum_{i=1}^{N} (N-i) \\
&= \sum_{i=1}^{N} N - \sum_{i=1}^{N} i \\
&= N(N) - \frac{N(N+1)}{2} \\
&= \frac{N}{2}(N-1).
\end{aligned}$$

The above assumptions also give the following expression for the cardinality of $M(i)$

$$\operatorname{card} M(i) = r - (N-i)$$

and

$$\begin{aligned}
&\sum_{i=1}^{N} \operatorname{card} M(i) \\
&= \sum_{i=1}^{N} (r - (N-i)) \\
&= \sum_{i=1}^{N} r - \sum_{i=1}^{N} N + \sum_{i=1}^{N} i \\
&= N(r) - N^2 + \frac{N(N+1)}{2} \\
&= N(r - \frac{1}{2}(N-1)) \\
&= Nr - \sum_{i=1}^{N} \operatorname{card} L(i).
\end{aligned}$$

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