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EXPERIMENTAL HEAT TRANSFER COEFFICIENTS FOR THE COOLING OF OIL IN HORIZONTAL INTERNAL FORCED CONVECTIVE TRANSITIONAL FLOW

DOUGLAS GORDON ROGERS

A dissertation submitted to the Faculty of Engineering,
University of the Witwatersrand, Johannesburg
for the Degree of Master of Science in Chemical Engineering.

Johannesburg, 1981

DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the degree of Master of Science in Chemical Engineering at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other University.

SIGNED this 22 day of June in the year 1981.

D G ROGERS

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May 1981

SYNOPSIS

The heat transfer coefficients for the cooling of a Newtonian oil were determined for the ranges $550 < Re < 2800$ and $132 < Pr < 642$. The Reynolds number alone proved to be an insufficient criterion for the division of transfer regimes, in line with the view of Eckert. In each of three of the transfer regimes recognised heat transfer was correlated by a Hausen-type equation

$$Nu = A(Re^{0,8} - B)(1,8Pr^{1/3} - 0,8)[1 + (D/L)^{2/3}]\left[\frac{\eta}{\eta_w}\right]^{0,14}$$

The values found for A and B were 0,0184 and 213,9 for the mixed turbulent regime, 0,0277 and 272,5 for the upper transitional regime, 0,0176 and 106,1 for the middle transitional regime, while in the mixed laminar regime the correlating equation was $Nu = 0,0002\,Re^{1,42}Pr^{1/3}\left[\frac{\eta}{\eta_w}\right]^{0,14}$. In all these regimes the Stanton number increases with Reynolds number, while in true turbulent or laminar flow it decreases. In the lower transitional regime, characterised by low wall Reynolds numbers, the Stanton number fluctuated with Reynolds number and no good correlation was found.

KEYWORDS: Transfer, heat, transitional, flow, pipe, Nu, Re, Pr.

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1 INTRODUCTION

The flow of fluids is commonly divided into only two main flow patterns, the laminar flow and the turbulent flow regions. In most situations this subdivision is adequate in that the flow is either fully turbulent or fully laminar. However, an intermediate flow situation exists where the fluid flows in a so-called transitional fashion. In this transitional region a further distinction is made as to whether the flow is changing from laminar to turbulent or if the flow is changing from turbulent to laminar; the latter is called reversion, reverse transition or relaminarisation.

1.1 OCCURRENCE OF RELAMINARISATION

There are a number of instances where the relaminarisation of fluid flows may occur. These are[1]:

Reversion by dissipation. This is described as the relative increase in dissipation that occurs when the Reynolds number in a flow decreases. This can occur with the gradual enlargement of a pipe or channel if the angle of divergence is sufficiently small so that flow separation does not occur. It has also been experimentally observed that the decaying turbulent flow is strongly anisotropic under these conditions and that although the skin friction reaches the laminar value, the rest of the flow is still strongly turbulent. Other instances where this reversion may take place are when branching occurs or after a restriction in a pipe, for example an orifice, or in any flow where there is a Reynolds number decrease with time.

Stably stratified flows. This is the suppression of turbulence in the presence of a stabilising density gradient and can be observed to occur frequently in the atmosphere. In these instances it is the presence of a lighter fluid on top that causes the rising fluid to work against gravity and so turbulent energy may be converted into gravitational potential energy. This energy absorption leads to reversion of the flow.

Highly accelerated flows. Reversion has been observed to occur to a turbulent boundary layer subjected to a large favourable pressure gradient. This phenomenon is mostly limited to external flows.

Curved flows. Reversion has been observed during the radial Poiseuille flow between two parallel discs. However, the phenomenon does not appear to occur at a fixed Reynolds number and hence is more complex than may first be imagined.

Rotation. Reversion has been observed in a channel rotating about a spanwise axis. Under these circumstances it appears that the Coriolis force is providing the force for stabilising the flow.

Surface mass transfer. Experiments show that fully developed pipe flow exhibits some laminar features when a uniform circumferential injection is applied to the flow. It is probable that the injection of fluid is in some way affecting the eddies close to the wall and hence the entire flow.

Magnetohydrodynamic duct flows. Experiments have shown that with a sufficiently strong field the skin friction changes from the turbulent value to one characteristic of laminar magnetohydrodynamic flow. The experiments further show that under these circumstances the turbulent fluctuations do not disappear although they do not appear to contribute to momentum transport.

Thermal effects. There are two instances where thermal effects may cause reversion.

a. The heating of a gas in internal flow where with increasing temperature the gas density decreases and the velocity increases, therefore causing reversion due to acceleration. It is also possible that it is the increase in kinematic viscosity that is causing the reversion. This cause is seldom encountered in the process industry.

b. The cooling of a liquid in internal flow where it is likely that the increase in viscosity is causing the reversion, that is, reversion by dissipation. This case was selected for further study.

1.2 SPECIFIC PROBLEM SELECTED

The change in flow pattern necessarily changes the heat transfer rate which in turn affects the sizing of equipment to effect a given heat exchange duty. Since the heat transfer rate under laminar flow conditions is much less than the rate under turbulent conditions it is desirable to use turbulent flow conditions in equipment design. However, this cannot always be done, either due to limitations on available pumping power or because of the physical properties of the fluid. Situations may therefore exist for which equipment must be designed with transitional flow occuring.

Data and correlations for the heat transfer rate for laminar and turbulent conditions for internal fluid flow are numerous and acceptably accurate, however, a survey has shown[2] that there is a dearth of information concerning the transitional region.

The situation of turbulent to laminar transition is more often encountered in industry (cooling of liquids) than* the laminar to turbulent transition and it has therefore been decided to investigate the former situation, more specifically the effect of turbulent to laminar transition on the heat transfer of a cooling Newtonian liquid.

* Reference (2) forms an addendum to this report and serves as a survey of the pertinent literature. It is suggested that the addendum be read at this stage. The addendum starts on page 115.

The boundary conditions chosen for the experiments are:

i. Internal forced flow of a Newtonian fluid in a horizontal, circular tube

ii. A uniform temperature of the fluid at the start of the test section

iii. A uniform wall temperature on the outside of the tube

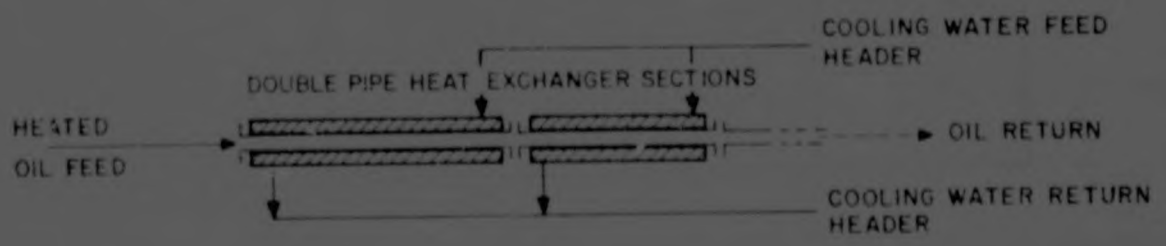
iv. A variable length test section to observe the effect of the length to diameter ratio.

v. Steady state condition.

These conditions have been chosen so as to simulate the situation occuring in a shell and tube exchanger where the tube side heat transfer coefficient limits the design.

2 EXPERIMENTAL AND PROCESSING OF RESULTS

The flow diagram of the experimental system is given in Figure 1 (Section 6.15) and may be summarised as follows:



There are two loops in the process, the one for the oil flowing on the inside of the test pipe and the other for the cooling water flowing on the outside or annulus. At no time are the fluids in direct contact. There are nine individual test sections, each identical, and fitted together to form one long heat exchanger. Detailed drawings of the test sections and the connecting pieces are given in Figures 2 to 15 (Section 6.15).

In the experiments Regal Oil B has been used as the inside fluid. A description of each process loop is given in Section 6.1.

The methods of flow measurement, temperature measurement and determination of physical properties are given in Sections 6.2, 6.3 and 6.4 respectively.

The quantities measured during the experimentation are the:

a. bulk oil inlet temperature

b. bulk oil outlet temperature

c. oil mass flow rate

d. cooling water temperature.

The range of dimensionless groups that the experiments cover is given in Table 1.

TABLE 1 Range of dimensionless groups covered by the data

Dimentionless group based on arithmetic average properties	Minimum	Maximum
Re	553	2 808
Pr	132,5	642,0
Gr	4 846	59 636
$\frac{\eta_w}{\eta}$	1,04	8,80
L/D	107	321

The experimental apparatus was designed with the aim of measuring the temperature profile with respect to length as the oil flowing in the jacketed pipe is cooled by the water in the jacket. The oil will enter in turbulent flow and as it cools will relaminarise and exit from the test section flowing laminarly. The temperature profile obtained will thus indicate how the heat transfer changes as the flow relaminarises.

However, it has proved impossible to measure the bulk temperature of the oil with a platinum resistance thermometer extending over the pipe diameter. The temperature profile in the oil causes the thermometer to measure some average value that is not the mean bulk value as is shown by the results in Section 6.9. It has also not been practicable to use a thermistor and traverse across the diameter to obtain a temperature and velocity profile and thus a bulk temperature, since the time required to do such measurements would require that a constant state condition exists for a long period of time; a requirement that is not usually achieved in relatively small apparatus of the nature used.

It has therefore been necessary to use a static mixer immediately before the temperature measurement point and since this disturbs the flow completely it has only been possible to measure the bulk temperature at the outlet from the test section. The temperature of the oil at the inlet is uniform because of the lagged section used and thus this temperature can be measured without a mixing device being used.

Using the experimental results in Section 6.12, the heat transfer coefficient has been calculated from

$$h_i = \frac{\frac{-\dot{m}_t \lambda}{r_i}\left\{b(\theta_{bo} - \theta_{bi}) + (C_o + b\theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\}}{2\Pi\lambda L - \dot{m}_t \ln\frac{r_o}{r_i}\left\{b(\theta_{bo} - \theta_{bi}) + (C_o + b\theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\}} \qquad (2.1)$$

as derived in Section 6.5. This calculation has been performed by computer using the programs described in Section 6.6.

2.1 ERROR ANALYSIS

Error analysis of both random and systematic errors for the experimentally determined heat transfer coefficient as given in Section 6.7, give an expected error of $E(h_i)/h_i \leq 6{,}4\%$. The systematic errors are a result of the assumptions that:

i. the outside wall temperature is equal to the cooling water temperature. This assumption is used since

   a. the heat flux in each section is unknown, and

   b. the flow rate of cooling water to each section is not known accurately.

   It is thus impossible to calculate the outside wall temperature accurately. (Correlations for the outside heat transfer coefficient are typically no more accurate than 10% to 20%.)

   The assumption introduces a systematic error into the experimental results which has been shown to be of the order of 2% in the calculation of the heat transfer coefficient and of the order of 5% in the calculation of the Grashof number (Section 6.7). Since the direction in which the error lies is always the same (underestimation of the heat transfer coefficient and Grashof number) it can be allowed for in correlating the data.

   If, however, this assumption is not made and the outside wall temperature is calculated using generally accepted correlations for the outside heat transfer coefficient and reasonable assumptions for the heat flux in each section, the error introduced will be random and of unknown magnitude. This is undesirable and hence working with a known magnitude systematic error has been chosen.

ii. the wall temperature is constant throughout the test section. Experimentation shows (Section 6.10) that there is a temperature rise of $\theta \approx 6$ °C of the cooling water flowing through each section. This will increase the mean bulk temperature by $\theta_b \approx 0,3$ °C which will cause a further systematic error of the order of 0,5% in the calculation of the heat transfer coefficient and of the order of 1% in the Grashof number. This error acts in the same direction as the other systematic error and increases the systematic error in the Nusselt group to $\frac{E(Nu)}{Nu} \leq 3\%$ and in the Grashof group to $\frac{E(Gr)}{Gr} \leq 6\%$.

3 DISCUSSION

3.1 COMPARISON OF EXPERIMENTAL RESULTS WITH CORRELATIONS FROM THE LITERATURE

Initially the data obtained from the experiments have been plotted using the equation of Hausen[3]:

$$Nu = 0,0235(Re^{0,8} - 230)(1,8Pr^{1/3} - 0,8)[1 + (D/L)^{2/3}](\eta/\eta_w)^{0,14} \quad (3.1)$$

with the result as plotted in Figure 21. From this it has been noted that it is not possible to correlate the data adequately using this form of correlation and relying on the Reynolds number alone to distinguish among the transfer regimes. The criteria established by Metais and Eckert[4] have then been applied to the data as indicated in Figure 22, to separate the data into various transfer regimes. This method effectively uses the Grashof, Prandtl and L/D ratios to characterise the effect of heat transfer on the transition mechanism. However, these subdivisions have proved inadequate and no reasonable correlation has been obtained for the data, neither with existing equations in the literature nor with modifications to such equations. Figure 23 shows the familiar Colburn form that is most often used as an example for correlations.

It has therefore been necessary to redefine the limits of the transfer regimes in such a manner that correlating equations can be found for the data in each region.

3.2 REDEFINITION OF TRANSFER REGIMES

Sieder and Tate (1936)[5] noted that the Stanton number decreases with increasing Reynolds number in both the laminar and the turbulent regimes. However, in the inbetween region the Stanton number appears to increas with increasing Reynolds number. This criterion has been applied to the data to subdivide the flow regions as given in Figure 24.

It has further been found that a Reynolds number effectively based on the wall viscosity as given by $Re\,(\frac{\eta_b}{\eta_{wl}})$ is more effective in correlating the regions than a bulk Reynolds number. This may intuitively be expected since the heat transfer is predominantly concerned with the region close to the wall and hence properties based on a wall temperature may be more effective in convection correlations.

The mixed turbulent and mixed laminar regions have been so termed because the Stanton number is still increasing with increasing Reynolds number in these regions, however, they are distinct from the transitional regions.

The transitional region further appears to divide into three distinct subregions the upper, middle and lower transitional regions. This indicates that the cooling rate is not adequately characterised by the $\frac{\eta}{\eta_w}$ ratio and the Grashof number, and that a more convenient length scale may exist on which to base dimensionless groups. It is possible that such a length scale may come either from a residence time concept from the view of allowing time and distance for free convection effects to set up, or from an entry length consideration.

3.3 CORRELATING EQUATIONS FOR EACH TRANSFER REGIME

With the transfer regions divided as given in Figure 24, correlating equations have been obtained for each regime.

3.3.1 Mixed turbulent regime

In this region the equation of Hausen has been found to be able to be conveniently modified to fit the data. As with correlations tried for all the regions the effects of the Prandtl group, the L/D ratio and the $\frac{\eta}{\eta_w}$ ratio have been tested without any conclusive evidence being found for cause to change the form presented by Hausen.

The modified equation that has been found to be most suited to the mixed turbulent data is given in Figure 25 as

$$Nu = 0{,}0184(Re^{0,8} - 213{,}9)(1{,}8Pr^{1/3} - 0{,}8)[1 + (D/L)^{2/3}](\frac{\eta}{\eta_w})^{0,14} \quad (3.2)$$

The exponent in the Reynolds group has been retained as 0,8 since this is generally accepted as being correct for turbulent flow. Equations tested with other values of the exponent did not prompt a change of the exponent from 0,8.

The correction factor of Gregorig[6], $\left[\frac{Pr_b}{Pr_w}\right]^p$, has been tested as a replacement of the $(\frac{\eta}{\eta_w})^{0,14}$ group but shows no marked improvement in the correlation. Because of the complexity of calculating the Gregorig factor it has been decided to retain the $(\frac{\eta}{\eta_w})^{0,14}$ factor.

Points on the boundaries of the region may be noted as correlating with subsequent correlations. The average data scatter for this correlation is of the order of 6%.

3.3.2 Upper transitional regime

As with the mixed turbulent region the Hausen form of the equation has been found to be the form most suitable for correlating the data. This is given in Figure 26 as

$$Nu = 0{,}0277(Re^{0,8} - 272{,}5)(1{,}8Pr^{1/3} - 0{,}8)[1 + (D/L)^{2/3}](\frac{\eta}{\eta_w})^{0,14} \quad (3.3)$$

The average data scatter for the correlation is of the order of 10% to 15%.

3.3.3 Middle transitional regime

As with the mixed turbulent and the upper transitional regions the Hausen form of the equation has been found to correlate the data most suitably. The equation as given in Figure 27 is

$$Nu = 0{,}0176(Re^{0,8} - 106{,}1)(1{,}8Pr^{1/3} - 0{,}8)[1 + (D/L)^{2/3}](\frac{\eta}{\eta_w})^{0,14} \quad (3.4)$$

The data point 59, has been assumed to be an experimental error since it will not fit in with any form of the equation nor in any other region.

3.3.4 Mixed laminar regime

The equation as given in Figure 28 has been found to be most suitable for correlating results in this region.

$$Nu = 0{,}0002Re^{1{,}42}Pr^{1/3}\left(\frac{\eta}{\eta_w}\right)^{0{,}14} \qquad (3.5)$$

The exponent of the Reynolds group, 1,42, raises doubt as to whether this region may in fact be termed laminar since it is characteristic of the laminar region when the exponent is $^1/_3$. It is thus possible that there are two regions rather than one in this area of Figure 24. However, the data set is too small to be able to separate the regions. It has thus been decided to accept the correlation.

3.3.5 Lower transitional regime

No suitable equation has been found to correlate the data in this region. The Stanton number appears to be random with respect to the Reynolds number as given in Figure 29 and it has thus been impossible even to use the data for scale-up. The only group that appears to correlate the data at all is

$$\left[\frac{Re\eta}{\eta_w}\right](L/D)^{0.2}Pr^{0.8} \qquad (3.6)$$

as given in Figure 30. It is doubtful whether this is in any way adequate, however.

Since the data in this region are predominantly for short length tubes it is possible that entry effects are causing the data scatter. These data are also characteristic of low mass flow rates used in order to obtain reasonable temperature drops during experimentation and it is possible that the nature of the flow is completely different to that of the other data.

Chronologically these data were obtained shortly before the temperature probe at the oil inlet was found to be faulty and thus experimental error could also be the cause of the data scatter.

3.4 COMPARISON OF EQUATIONS WITH THE HAUSEN EQUATION

In order to show the effect of subdividing the flow as given in Figure 24, a plot has been prepared of the data against the Hausen equation as given in Figure 31.

This shows the value of subdividing the flow into various regions in order to find an accurate correlating equation. However, it also indicates that the subdivisions as chosen here may

not be accurate since there is no definite trend in the equations obtained. It is therefore necessary to obtain additional data so that the regions may be defined more clearly.

4 CONCLUSIONS AND RECOMMENDATIONS

Conclusions resulting from the experimental programme and the subsequent processing of the experimental results are as follows:

1. *Correlations obtained for each transfer regime.*

   It has not been possible to determine the flow regime during internal pipe flow where heat transfer is taking place on the basis of the Reynolds group alone. Other factors influencing the transfer have been taken into account using the Grashof, Prandtl and L/D ratio groups.

   With the transfer divided into regimes as given in Figure 24, it has been possible to find equations that correlate each regime except for the lower transitional regime.

$$Nu = 0{,}184(Re^{0{,}8} - 213{,}9)(1{,}8Pr^{1/3} - 0{,}8)[1 + (D/L)^{2/3}]\left(\frac{\eta}{\eta_w}\right)^{0{,}14}$$

   Upper transitional: [Equation (3.3)]

$$Nu = 0{,}0277(Re^{0{,}8} - 213{,}9)(1{,}8Pr^{1/3} - 0{,}8)[1 + (D/L)^{2/3}]\left(\frac{\eta}{\eta_w}\right)^{0{,}14}$$

   Middle transitional: [Equation (3.4)]

$$Nu = 0{,}0176(Re^{0{,}8} - 106{,}1)(1{,}8Pr^{1/3} - 0{,}8)[1 + (D/L)^{2/3}]\left(\frac{\eta}{\eta_w}\right)^{0{,}14}$$

   Mixed laminar: [Equation (3.5)]

$$Nu = 0{,}0002Re^{1{,}42}\,Pr^{1/3}\left(\frac{\eta}{\eta_w}\right)^{0{,}14}$$

2. *Recommendations for further work.*

   Experience gained during this experimental programme has indicated that our understanding of transfer in the transitional region requires considerable additional data. A programme comprising the following conditions and equipment is proposed:

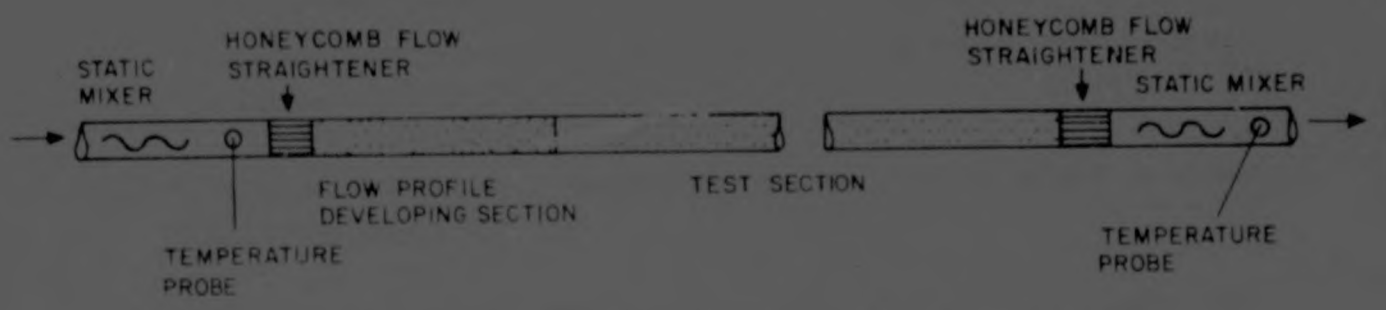
i. four different tube lengths ranging from 2 to 10 metres,

ii. straight lengths of tube without joints or internal protrusions to disturb the flow,

iii. three different tube diameters ranging from 19 to 38 millimetres,

iv horizontal, intermediate and vertical inclination of the test section,

v. a test section constructed as indicated below:



vi. that the cooling medium be water supplied to a multisectioned jacket around the test section,

vii. a turbine flow meter or other accurate type of flow meter in the test fluid loop,

viii. that the cooling water flow rate and bulk inlet and outlet temperatures are monitored to give an overall heat balance,

ix. a fully automated data logging with immediate evaluation of the data and graphical display; and using this facility to obtain a complete distribution of data points on a Metais Eckert type plot,

x. use of two or three different test fluids,

xi. steady state operation only,

xii. Reynolds numbers in the range $500 < Re < 10\ 000$,

xiii. turbulent to laminar transition initially.

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6 APPENDIX

6.1 EXPERIMENTAL

The two process loops as given in Figure 1 are as follows:

Oil side. Starting at the header tank the oil flows across electrical heaters to a rotary vane pump. This is a Variflo pump from Blackmer Pump Co. that is adjustable to obtain various flow rates. After the pump, the hot oil passes through a heavily insulated calming section. This section is to eliminate swirl introduced by the pump and to allow the fluid to reach thermal equilibrium.

The first temperature probe is at the end of the developing section, as detailed in Figures 9 to 15. These probes and connecting pieces have all been custom made to suit the dimensions of the pipe at each particular joint to prevent any projection or edge triggering turbulence in the flow. This is also why the temperature probes are retractable. To check the system for possible projections, water has been run down the inside pipe at a very high Reynolds number and the pressure drop has been monitored at each connecting piece using a multiple arm manometer as shown in Figure 16. Any surface irregularities are observed as apparent errors in the pressure drop and in this way each section has been corrected for alignment.

The first temperature probe is extended into the fluid at all times to measure the inlet oil temperature. The flat temperature profile at this point ensures that the temperature measured by the resistance thermometer is in fact the mean bulk temperature.

Undisturbed, the oil then flows for a variable number of sections with all the thermometers in the path withdrawn. The length of the test section flow has been varied during the experiments to obtain data on the effect of the length to diameter ratio on the heat transfer.

At the end of the test section the temperature of the fluid is measured after it has passed through a mixing device as detailed in Figure 17. This mixing device is necessary because of the strong temperature gradient in the fluid. Results without a mixing device (Section 6.9) show that the temperature interpreted by the thermometer can be up to 2 ^{o}C from the true bulk temperature.

After the final section of heat exchanger, the oil is routed through a rotameter and back to the holding tank.

Water side. The water is taken from a holding tank that is kept at constant temperature by continually introducing fresh water from the mains supply and allowing a continual overflow. The water is pumped by a Matheson and Bremner Multiflo pump into two headers, in one of which

6 APPENDIX

6.1 EXPERIMENTAL

The two process loops as given in Figure 1 are as follows:

Oil side. Starting at the header tank the oil flows across electrical heaters to a rotary vane pump. This is a Variflo pump from Blackmer Pump Co. that is adjustable to obtain various flow rates. After the pump, the hot oil passes through a heavily insulated calming section. This section is to eliminate swirl introduced by the pump and to allow the fluid to reach thermal equilibrium.

The first temperature probe is at the end of the developing section, as detailed in Figures 9 to 15. These probes and connecting pieces have all been custom made to suit the dimensions of the pipe at each particular joint to prevent any projection or edge triggering turbulence in the flow. This is also why the temperature probes are retractable. To check the system for possible projections, water has been run down the inside pipe at a very high Reynolds number and the pressure drop has been monitored at each connecting piece using a multiple arm manometer as shown in Figure 16. Any surface irregularities are observed as apparent errors in the pressure drop and in this way each section has been corrected for alignment.

The first temperature probe is extended into the fluid at all times to measure the inlet oil temperature. The flat temperature profile at this point ensures that the temperature measured by the resistance thermometer is in fact the mean bulk temperature.

Undisturbed, the oil then flows for a variable number of sections with all the thermometers in the path withdrawn. The length of the test section flow has been varied during the experiments to obtain data on the effect of the length to diameter ratio on the heat transfer.

At the end of the test section the temperature of the fluid is measured after it has passed through a mixing device as detailed in Figure 17. This mixing device is necessary because of the strong temperature gradient in the fluid. Results without a mixing device (Section 6.9) show that the temperature interpreted by the thermometer can be up to 2 ^{o}C from the true bulk temperature.

After the final section of heat exchanger, the oil is routed through a rotameter and back to the holding tank.

Water side. The water is taken from a holding tank that is kept at constant temperature by continually introducing fresh water from the mains supply and allowing a continual overflow. The water is pumped by a Matheson and Bremner Multiflo pump into two headers, in one of which

is a temperature probe as detailed in Figures 18 to 20. This probe gives the bulk temperature of the water supplied to each test section since the temperature of the water in each header is necessarily the same. The flow rate to each header has been determined using orifice plates with an orifice diameter of 20 mm (Section 6.8).

With the headers arranged in this way it is generally accepted that the flow will distribute evenly provided that each section is identical. The temperature of the water exiting from each section has been measured using platinum resistance thermometers and it has been found that the temperature rise is less than 1 ^{o}C (Section 6.10). The water is then returned to the holding tank.

6.2 FLOW MEASUREMENT

6.2.1 Flow rate of water in the annulus

The flow rate of the cooling water to each header has been measured on one occasion and thereafter conditions have been kept constant and the flow rate unaltered for all the experiments.

Orifice plates have been used in each of the lines and the pressure drop across each orifice has been measured using a differential pressure gauge (Section 6.8).

It has been found that the flow rate is essentially constant for small variations in the water temperature and that the flow appears to divide evenly among the nine test sections. The flow rate to each section has been taken as 0,98 kg/s which is the arithmetic average flow rate.

6.2.2 Flow rate of oil inside test section

The flow rate of the oil flowing on the inside of the test pipe determined using a calibrated Fischer and Porter precision bore rotameter. The rotameter has been calibrated with water at 15 ^{o}C and using a weighing tank system. The calibration points so obtained have been fitted using a linear least squares regression technique to give the equation for mass flow as

$$\dot{m}_1 = -0{,}015603 + 0{,}007762N_R \qquad (6.2.1)$$

with a regression coefficient r = 0,9989. N_R is the rotameter reading, and the equation is limited to $15 \leq N_R/\% \leq 100$ for an estimated error in the flow rate of $\frac{E(\dot{m}_1)}{\dot{m}_1} \leq 3\%$.

Correction has been made for different fluids using the equation recommended by Fischer and Porter

$$\dot{m}_2^2 = \frac{\dot{m}_1^2(\rho_t - \rho_2)\rho_2}{(\rho_t - \rho_1)\rho_1} \qquad (6.2.2)$$

No correction factor is necessary for changes in viscosity.

6.3 TEMPERATURE MEASUREMENT

Calibrated 100 ohm platinum resistance thermometers are used to measure temperature. This type of thermometer has a relatively fast response time τ where $\tau < 1$ second, a negligible offset drift with time, and is accepted as the International Practical Temperature Scale standard for temperatures $-180 < \theta/^{o}C < +600$ [c].

All the resistance thermometers used have been calibrated in an ice bath against a standard 100 ohm platinum resistance thermometer that has been calibrated by the Precise Physical Measurements Group of the CSIR. All the thermometers are thus referred to the same standard state and, therefore, temperature differences are known with great accuracy. The absolute temperatures are known only with respect to the standard thermometer used for the calibration and this is known to have an accuracy $E(\theta) \leq 0{,}10\ ^{o}C$.

The bridge used for determining the resistance of each thermometer is a Leeds and Northrup 8078 portable precision temperature bridge utilising a four lead, alternating current system of resolution $0{,}025\ ^{o}C$. The reading given by the bridge is in ohms and has to be subsequently processed to give temperatures. The various thermometers at the various points in the system have been selected for measurement using a simple manual switching box.

The temperature is determined from the resistance reading using the method proposed by Leeds and Northrup in the user manual for the bridge. This method is to first obtain a corrected value based on the calibrated value

$$R_2 = R_1 \frac{R^o}{R^o_o} \qquad (6.3.1)$$

where ^{o} refers to the calibration point.

Using the corrected value for the resistance, the temperature is then found by linear interpolation in the standard tables for 100 ohm platinum resistance thermometers based on the International Practical Temperature Scale of 1968. The expected error in the temperature is estimated as $E(\theta) \leq 0{,}05\ ^{o}C$.

$$\dot{m}_2^2 = \frac{\dot{m}_1^2(\rho_1 - \rho_2)\rho_2}{(\rho_1 - \rho_1)\rho_1} \qquad (6.2.2)$$

No correction factor is necessary for changes in viscosity.

6.3 TEMPERATURE MEASUREMENT

Calibrated 100 ohm platinum resistance thermometers are used to measure temperature. This type of thermometer has a relatively fast response time τ where $\tau < 1$ second, a negligible offset drift with time, and is accepted as the International Practical Temperature Scale standard for temperatures $-180 < \theta/^{o}C < +600$ [7].

All the resistance thermometers used have been calibrated in an ice bath against a standard 100 ohm platinum resistance thermometer that has been calibrated by the Precise Physical Measurements Group of the CSIR. All the thermometers are thus referred to the same standard state and, therefore, temperature differences are known with great accuracy. The absolute temperatures are known only with respect to the standard thermometer used for the calibration and this is known to have an accuracy $E(\theta) \leq 0{,}10\ ^{o}C$.

The bridge used for determining the resistance of each thermometer is a Leeds and Northrup 8078 portable precision temperature bridge utilising a four lead, alternating current system of resolution 0,025 ^{o}C. The reading given by the bridge is in ohms and has to be subsequently processed to give temperatures. The various thermometers at the various points in the system have been selected for measurement using a simple manual switching box.

The temperature is determined from the resistance reading using the method proposed by Leeds and Northrup in the user manual for the bridge. This method is to first obtain a corrected value based on the calibrated value

$$R_2 = R_1 \frac{R^{o}}{R_c^{o}} \qquad (6.3.1)$$

where ^{o} refers to the calibration point.

Using the corrected value for the resistance, the temperature is then found by linear interpolation in the standard tables for 100 ohm platinum resistance thermometers based on the International Practical Temperature Scale of 1968. The expected error in the temperature is estimated as $E(\theta) \leq 0{,}05\ ^{o}C$.

6.4 PHYSICAL PROPERTIES

No correction is included in the physical properties for effects of pressure since all the experiments have been conducted at ambient pressure using liquids only and the effect of pressure on liquid properties is negligible for small changes in pressure.

The physical properties are thus needed only for the liquid phase and only as a function of temperature. Values for the oil used (Regal oil B) and for water are tabulated in Section 6.14.

Since the physical properties are available only at discreet values of temperature and not in functional form, it is necessary to use interpolatory techniques to find values at arbitrary temperatures. The various interpolation techniques used are described below.

Viscosity. The liquid viscosity correlates fairly closely with the Andrade equation[8]

$$\eta = A_o \exp(B/T) \qquad (6.4.1)$$

A plot of $\ln(\eta)$ as a function of $\frac{1}{T}$ should, therefore, be linear. This has been noticed not to be quite accurate and a better fit has been obtained using a cubic spline fit[9] to $\ln(\eta)$ as a function of $\frac{1}{T}$.

The interpolated value is thus obtained from the cubic spline fitted to the data points in the immediate vicinity of the interpolation point.

Thermal conductivity. The liquid thermal conductivity follows the form[8]

$$\lambda = \lambda_o [1 - a(\theta - \theta_o)] \qquad (6.4.2)$$

Since θ_o is an arbitrary temperature, λ may be assumed to be linear with the centigrade temperature. This has been found to be true and a linear interpolation technique is adopted.

Specific heat. The specific heat is generally[10] assumed to be linear with temperature. It has been found that over reasonable variations in temperature this is the case and hence a linear interpolation technique is used.

Density. The form

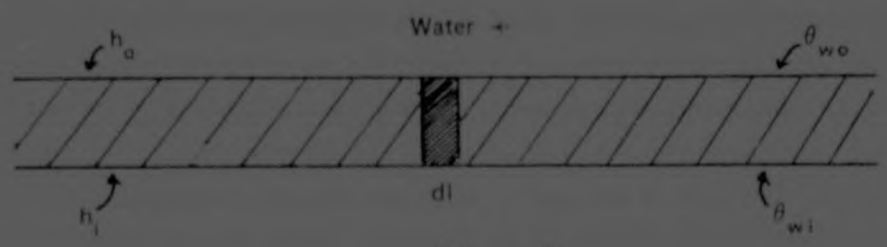
$$\frac{(\rho_\ell - \rho_v)_2}{(\rho_\ell - \rho_v)_1} = \left[\frac{T_c - T_2}{T_c - T_1}\right]^{1/3} \qquad (6.4.3)$$

is suggested in Perry[10]. This is not usable with the oil properties since the vapour properties and the critical temperature are unknown. The form of the equation suggests that $\ln(\rho)$ may be

linear; with $\ln(\theta)$, however, this is not so and the method of fitting cubic splines to ρ versus θ is used giving good results. This method is only acceptable for interpolation and not for extrapolation; however, all experiments have been conducted under conditions within the bounds of known ›

6.5 CALCULATION OF THE HEAT TRANSFER COEFFICIENT

The average heat transfer coefficient over the test section is calculated as follows and the geometry is given in the figure below:



A simple heat balance has been used to give

$$\theta - \theta_{wo} = dQ \left[\frac{1}{h_i 2\Pi r_i dl} + \frac{\ln \frac{r_o}{r_i}}{2\Pi \lambda_w dl} \right] \qquad (6.5.1)$$

where dQ may be written as

$$dQ = \dot{m}_i (C_o + b\theta) d\theta \qquad (6.5.2)$$

where $$C(\theta) = C_o + b\theta \qquad (6.5.3)$$

Equation (6.5.2) is substituted in Equation (6.5.1) and integrated from $\theta = \theta_{bi}$ to $\theta = \theta_{bo}$ and from $l = 0$ to $l = L$ assuming that:

i. the thermal conductivity of the wall (λ_w) is constant. This is reasonable for relatively small variations in temperature,

ii. the tube dimensions remain constant,

iii. the outside wall temperature θ_{wo} is constant.

Using separation of variables and solving for h_i gives:

$$h_i = \frac{\frac{-\dot{m}_i\lambda}{r_i}\left\{b(\theta_{bo} - \theta_{bi}) + (C_o + b\theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\}}{2\Pi\lambda L - \dot{m}_i \ln\frac{r_o}{r_i}\left\{b(\theta_{bo} - \theta_{bi}) + (C_o + b\theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\}} \qquad (6.5.4)$$

In this form h_i, the heat transfer coefficient, is an average value.

6.6 COMPUTER PROGRAMS

Data accumulated during experimentation is in the form of resistance measurements from the platinum resistance thermometers and of rotameter readings. From this raw data it is necessary to determine flow rates, temperatures, physical properties dimensionless members and the heat transfer coefficient. It is not practical to do this by hand for each experiment due to the amount of work necessary to interpolate physical properties. This task is therefore computerised as program 1 in FORTRAN on the CSIR's Control Data Corporation (CDC) Cyber 174. Program 1 reads in rotameter readings and resistances and gives out point values for physical properties and dimensionless groups and gives an average heat transfer coefficient and also arithmetic average dimensionless groups. This data is then in a reasonable format to be used in correlating other data.

In correlating the data a large number of variations need to be considered in the organisation of correlating equations. This is most conveniently done using computer program 2. All the data from program 1 is incorporated as a data input file to program 2. An interactive plotting routine and a Tektronix 4662 digital plotter are coupled to the program to obtain plots of the correlations tried.

Printouts of both programs are given in Section 6.13.

6.7 EXPECTED ERRORS

There are two types of errors that may be associated with experimental data, random errors and systematic errors, both forms of which are present in these experimental data. The random errors are associated with instrument accuracy and given in Section 6.7.1 and the systematic errors given in Section 6.7.2 are associated with the assumption of the outside wall temperature of the test sections being equal to the cooling water temperature. A summary of the expected errors is given in Table 2.

TABLE 2 Summary of expected errors

Variable	Random error	Systematic error	Maximum total error
θ_i/°C	0,05	0	0,05
θ_o/°C	0,05	0	0,05
θ_{wo}/°C	0,05	-1,3	1,35
$\dot{m}_i$/%	3	0	3
h_i/%	3,4	-3	6,4
Nu/%	3,4	-3	6,4
Re/%	3	0	3
Pr/%	0	0	0
Gr/%	0,5	-6	6,5

6.7.1 Random errors

The precision index of a general function, F(x), is defined as[11]

$$E(x)^2 = \sum_{i=1}^{N}\left[\frac{dF(x_i)}{dx_i}\right]^2 E(x_i)^2 \qquad (6.7.1)$$

Thus if the errors in the variables which make up the function F(x) are known then the error in F(x) is also known.

Heat transfer coefficient. The heat transfer coefficient is given by Equation (6.5.4). For practical purposes it is assumed that:

i. the error in the thermal conductivity is zero,

ii. the error in length measurement is zero,

iii. the errors in C_o and b are zero.

Let

$$A_1 = \frac{\dot{m}_i\lambda}{r_i}\left\{b(\theta_{bo} - \theta_{bi}) + (C_o + b\theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\} \qquad (6.7.2)$$

$$B_1 = 2\Pi\lambda L - \dot{m}_i\ln\frac{r_o}{r_i}\left\{b(\theta_{bo} - \theta_{bi}) + (C_o + b\theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\} \qquad (6.7.3)$$

Then it may easily be shown that

$$\frac{dh}{d\dot{m}_i} = \frac{2\Pi\lambda^2 bL}{B_1^2 r_i}\left\{(\theta_{bo} - \theta_{bi}) + (\frac{C_o}{b} + \theta_{wo})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\} \qquad (6.7.4)$$

$$\frac{dh}{d\theta_{bo}} = \frac{\lambda b\dot{m}_i}{B_1 r_i}\left\{1 + \left[\frac{\frac{C_o}{b} + \theta_{wo}}{\theta_{bo} - \theta_{wo}}\right]\right\} + \frac{A_1 b \ln(\frac{r_o}{r_i})\dot{m}_i}{B_1^2}\left[1 + \frac{\frac{C_o}{b} + \theta_{wo}}{\theta_{bo} - \theta_{wo}}\right] \qquad (6.7.5)$$

$$\frac{dh}{d\theta_{bi}} = \frac{(-1)\lambda b\dot{m}_i}{B_1 r_i}\left[1 + \left[\frac{\frac{C_o}{b} + B_1}{\theta_{bi} - \theta_{wo}}\right]\right] + \frac{A_1 b \ln(\frac{r_o}{r_i})\dot{m}_i}{B_1^2}\left[1 + \frac{\frac{C_o}{b} + \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right] \qquad (6.7.6)$$

$$\frac{dh}{d\theta_{wo}} = \frac{\lambda b\dot{m}_i}{B_1 r_i}\left[\frac{(\theta_{bo} - \theta_{bi})(\frac{C_o}{b} + \theta_{wo})}{(\theta_{bo} - \theta_{wo})(\theta_{bi} - \theta_{wo})} + \ln\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]$$

$$+ \frac{\dot{m}_i A_1}{B_1^2}\left\{\frac{[\ln(\frac{r_o}{r_i})C_o + \theta_{wo} b\ln(\frac{r_o}{r_i})](\theta_{bo} - \theta_{bi})}{(\theta_{bo} - \theta_{wo})(\theta_{bi} - \theta_{wo})} + b\ln(\frac{r_o}{r_i})\ln\left[\frac{\theta_{bo} - \theta_{wo}}{\theta_{bi} - \theta_{wo}}\right]\right\} \qquad (6.7.7)$$

The error in h is then

$$E(h)^2 = (\frac{dh}{d\dot{m}_i})^2 E(\dot{m}_i)^2 + (\frac{dh}{d\theta_{bo}})^2 E(\theta_{bo})^2 + (\frac{dh}{d\theta_{bi}})^2 E(\theta_{bi})^2 + (\frac{dh}{d\theta_{wo}})^2 E(\theta_{wo})^2 \qquad (6.7.8)$$

The expected errors in temperature and flow measurement are:

i. Temperature. The temperatures are measured using calibrated 100 ohm platinum resistance thermometers and a Leeds and Northrup 8078 portable precision temperature bridge. The makers claim a resolution of 0,025 °C under normal operating conditions. However, it is more realistic to double this value. The value for the precision index for temperature, $E(\theta)$, is thus 0,05 °C.

ii. Flow rate. The flow rate is measured using a Fischer and Porter FP–1–35–6–10/80 precision bore rotameter with a GNSVT66 stainless steel float. The rotameter has been calibrated with water at 16 °C using a weighing tank system. The expected error in the range $15 < N_R/\% < 100$ is $\frac{E(\dot{m}_1)}{\dot{m}_1} < 3\%$.

The correction for a different fluid in the rotameter is performed using the recommended equation of Fischer and Porter

$$\dot{m}_2^2 = \frac{\dot{m}_1^2 (\rho_1 - \rho_2)\rho_2}{(\rho_1 - \rho_1)\rho_1} \qquad (6.7.9)$$

This correction causes further degradation of accuracy in measurement.

In the experimental apparatus the rotameter is some distance from the last temperature measuring point. The effect of the error in temperature on the density correction factor has been determined as in Table 3.

TABLE 3 Error in flow rate due to temperature

Rotameter reading N_R/%	Fluid temperature T_1/ ^oC	Calculated mass flow rate $\dot{m}_2$/kgs^{-1}	Error due to temperature /%
88,5	54,16	0,6264	+ 0,09
88,5	56,76	0,6258	0,0
88,5	58,85	0,6254	- 0,06
88,5	59,03	0,6253	- 0,06

It is thus assumed that this error is negligible.

The overall accuracy of flow measurement is thus assumed to be $\frac{E(\dot{m}_i)}{\dot{m}_i} < 3\%$.

iii. The average expected error. An arbitrary experimental run selected is

θ_{bi} = 66,96 ^oC; θ_{bo} = 62,50 ^oC; θ_{wo} = 22,56 ^oC (run 1)

$\dot{m}_i$ = 0,5055 kg/s; L = 8,154 m; h = 169,9 W/m^{2}K

Fixed constants in the system are:

C	=	1804,32	J/kgK
b	=	3,56	J/kgK^2
λ	=	100	W/mK mean value for brass for $\theta/^oC$ = 0 ... 100
r_i	=	0,01257	m
r_o	=	0,01588	m

Using these values the following have been calculated:

$E(\dot{m}_i)$	=	0,015	kg/s
A_1	=	866 259,85	W^2/m^2
B_1	=	5097,83	WK

$$\frac{A_1}{B_1} = h = 169{,}927 \quad W/m^2K$$

$$\frac{dh}{d\dot{m}_i} = 337{,}837 \quad Ws/m^2K\ kg$$

$$\frac{dh}{d\theta_{bo}} = 36{,}478 \quad W/m^2K^2$$

$$\frac{dh}{d\theta_{bi}} = -39{,}836 \quad W/m^2K^2$$

$$\frac{dh}{d\theta_{wo}} = 4{,}6 \quad W/m^2K^2$$

Therefore

$$E(h)^2 = 25{,}68 + 3{,}327 + 3{,}967 + 0{,}041 = 33{,}051\ W^2/m^4K^2$$

and $\frac{E(h)}{h} = 3{,}4\%$.

Therefore the expected error in the heat transfer coefficient is $\frac{E(h)}{h} < 3{,}4\%$.

iv. Reynolds number. As previously it is assumed that:

a. there is no error in length measurement,

b. there is no error in physical properties.

Thus

$$\frac{E(Re)}{Re} = 3\% \quad \text{(the error in } \dot{m}_1\text{)}.$$

v. Nusselt number. With the previous assumption this reduces to:

$$\frac{E(Nu)}{Nu} = 3{,}4\%.$$

vi. Prandtl number. With the previous assumption, the Pr has no error.

vii. Grashof number. Using the previous assumptions and assuming no error in β, the only error is associated with the temperature.

Using the general form of the error equation:

$$\frac{dGr}{d\theta_1} = \frac{\beta g L^3 \rho^2}{\eta^2} \qquad (6.7.10)$$

$$\frac{dGr}{d\theta_2} = \frac{-\beta g L^3 \rho^2}{\eta^2} \qquad (6.7.11)$$

$$\left[\frac{E(Gr)}{Gr}\right]^2 = \frac{E(\theta_1)^2 + E(\theta_2)^2}{\theta_1 - \theta_2} \qquad (6.7.12)$$

and

$$\frac{E(Gr)}{Gr} = 0{,}5\%, \quad \text{with } \theta_1 = 25\ ^{o}C, \quad \theta_2 = 40\ ^{o}C \quad \text{(characteristic temperatures).}$$

6.7.2 Systematic errors

The only known systematic error that is encountered is the error in the outside wall temperature used in the calculation of the heat transfer coefficient (Nusselt number) and the Grashof number. This error is caused by assuming that the cooling water temperature is the same as the outside wall temperature and that the cooling water temperature is constant over the section. The reasons for these assumptions are given in Section 2. The error introduced has been estimated as follows:

Calculation of error in the outside wall temperature. The outside wall to fluid temperature difference is given by

$$\theta_{wo} - \theta_{wa} = \frac{\dot{q}}{h_o A} \qquad (6.7.13)$$

Typical experimental values for the right hand side of Equation (6.7.13) are:

$\dot{q} \approx 1{,}5$ KW/section

$A = 0{,}09\ m^2$/section

$h_o \approx 17\ 000\ W/m^2K$ (Section 6.11)

Using these values

$$\theta_{wo} - \theta_{wa} = 0{,}98\ ^{o}C.$$

Also the mean cooling water temperature is expected to be 0,3 ^{o}C higher than the inlet temperature (Section 6.10).

Therefore, effectively, using the outside inlet fluid temperature equal to the outside wall temperature introduces an error of $E(\theta_{wo}) \leq 1{,}3\ ^{o}C$.

Effect on the Nusselt number of a 1 ^{o}C error in the outside wall temperature. Using two arbitrary different wall temperatures to calculate the Nusselt number and using results from an arbitrary experiment gives for $\theta_{wo} = 22{,}56$, $Nu = 32{,}8$, and for $\theta_{wi} = 21{,}56$, $Nu = 32{,}1$.

That is, the Nusselt number is underestimated by 2% per degree Celsius of the outside wall temperature.

Effect on the Grashof number of a 1 ^{o}C error in the outside wall temperature. This is the only other group that was affected by the systematic error. The effect of an error in the outside wall temperature gives for θ_{wo} = 66,96, Gr = 17 347, and for θ_{wo} = 65,89, Gr = 16 434. Therefore the Grashof number is underestimated by approximately 5% per degree Celsius of the outside wall temperature.

6.8 ORIFICE PLATE RESULTS

These results have been obtained for the flow rate of water to the two inlet headers.

TABLE 4 Header with five sections

Temperature (θ)/ ^{o}C	$\frac{\dot{m}}{kg/s}$
25	4,793
28	4,816
30	4,817
34	4,812
38,5	4,797
43	4,777
44,8	4,777
Mean = 4,798/kgs^{-1} Standard deviation = 0,017/kgs^{-1}	

TABLE 5 Header with four sections

Temperature (θ)/ °C	$\dot{m}$ kg/s
27	4,0895
29	4,057
37	4,084
42,5	4,080
45	3,987
Mean = 4,0595/kgs^{-1}	
Standard deviation = 0,042/kgs^{-1}	

The mean flow to each section is thus 0,98 kg/s.

6.9 ERROR IN TEMPERATURE MEASUREMENT OF OIL WITHOUT MIXING DEVICE

The following measurements have been taken under normal operating conditions with

a. a plain thermometer, and

b. a mixing section and thermometer.

TABLE 6 Error in temperature measurement

Run	1	2	3	4	5
Plain θ_1/ °C	52,24	49,77	49,85	27,74	25,26
Mixed θ_2 / °C	50,31	47,12	47,20	26,55	24,52
Difference ($\theta_1 - \theta_2$) / °C	1,93	2,65	2,65	1,19	0,76

Run no. 5 has been done under adiabatic conditions and reflects the error in calibration of the thermometers. Taking this calibration error into effect, the average difference in the measured temperature to the true temperature is 1,35 ^{o}C.

6.10 TEMPERATURE RISE IN WATER ACROSS TEST SECTIONS

TABLE 7

	Run 1	Run 2	Run 3	Mean rise
Temperature of water ϑ_i /^{o}C	27,25	27,07	26,84	$\theta_o - \theta_i$
Temperature of water θ_o /^{o}C				
Section 1	28,26	28,08	27,82	1,01
2	27,97	27,77	27,54	0,71
3	27,82	27,64	27,41	0,57
4	27,77	27,59	27,36	0,52
5	28,03	27,87	27,61	0,78
6	27,82	27,64	27,41	0,57
7	27,77	27,61	27,38	0,53
8	27,77	27,59	27,38	0,53
9	27,74	27,56	27,36	0,50
				$\bar{m}$ = 0,64/^{o}C

6.11 HEAT TRANSFER COEFFICIENT IN THE ANNULUS

The heat transfer coefficient on the outside of the pipe is estimated using the equation of Monrad and Pelton[10] for annuli. The equation is

$$Nu = 0{,}020 Re^{0,8} Pr^{1/3} \left[\frac{D_2}{D_1}\right]^{0,53} \qquad (6.11.1)$$

with $Re = \frac{4\dot{m}}{\Pi\eta(D_1 + D_2)}$ and $Nu = \frac{h(D_2 - D_1)}{\lambda}$

In this case the dimensions of the annulus are:

Inside pipe ID = 24,94 mm OD = 31,75 mm

Outside pipe ID = 35,74 mm

$$\frac{D_2}{D_1} = 1{,}126.$$

The mass flow rate in each section is 0,98 kg/s as given in Section 6.8, and the fluid is water. From this it is possible to determine the heat transfer coefficient as in Table 8.

TABLE 8 Heat transfer coefficient in the annulus

$\theta_{w/a}$ / °C	Re	Pr	Nu	h_o/Wm^{-2}K^{-1}
15	16 274	7,99	99,561	14 846
20	18 451	6,95	105,085	15 881
25	20 773	6,09	110,566	16 931
30	22 197	5,39	120,444	17 960
35	25 749	4,80	121,285	18 998
40	28 399	4,3	126,454	20 029

6.12 EXPERIMENTAL RESULTS

θ_i °C	θ_o °C	θ_{we} °C	$\dot{m}$ kg/s	h_I W/m²K	Nu	St	Re	Pr	Gr_L	L m	
66.960	62.500	22.560	.5055	169.70	32.80	.8240E-04	1891.0	212.0	.5739E+12	8.154	
67.820	62.910	22.720	.4982	181.90	35.20	.8960E-04	1904.0	208.0	.6068E+12	8.154	
59.550	56.740	24.420	.5354	140.00	26.96	.6500E-04	1591.0	262.0	.2905E+12	8.154	
59.760	56.890	24.470	.5353	142.00	27.36	.6590E-04	1601.0	260.0	.2955E+12	8.154	
62.780	59.320	25.500	.6578	202.00	38.96	.7590E-04	2170.0	238.0	.3767E+12	8.154	
62.780	59.340	25.470	.6578	200.20	38.63	.7530E-04	2171.0	238.0	.3720E+12	8.154	
70.120	64.870	25.470	.5773	229.50	44.43	.9710E-04	2364.0	195.0	.6860E+12	8.154	
67.450	62.760	25.780	.7077	267.10	51.74	.9280E-04	2681.0	209.7	.5498E+12	8.154	
67.400	62.700	25.810	.7077	268.17	51.84	.9298E-04	2677.0	210.0	.5468E+12	8.154	
70.460	65.160	26.040	.6783	273.80	53.02	.9860E-04	2805.0	194.0	.6951E+12	8.154	10
70.480	65.180	26.040	.6783	273.60	52.99	.9850E-04	2808.0	193.0	.6967E+12	8.154	
71.820	66.280	22.480	.4833	183.70	35.60	.9260E-04	2079.0	186.0	.8368E+12	8.154	
71.690	66.250	22.460	.4833	180.40	34.96	.9096E-04	2073.0	187.0	.8312E+12	8.154	
78.520	71.900	22.380	.3277	132.90	25.85	.9770E-04	1695.0	158.0	.1367E+13	8.154	
69.100	64.660	22.100	.3464	109.50	21.19	.7730E-04	1390.0	199.0	.6988E+12	8.154	
68.840	64.320	22.100	.3537	114.40	22.15	.7920E-04	1406.0	200.0	.6815E+12	8.154	
65.370	60.880	22.050	.3612	125.00	24.14	.8520E-04	1281.0	223.0	.5035E+12	8.154	
65.130	60.620	22.030	.3613	126.40	24.41	.8620E-04	1270.0	224.0	.4920E+12	8.154	
53.070	50.650	21.970	.7831	196.20	37.60	.6298E-04	1828.0	329.0	.1603E+12	8.154	
54.600	52.520	21.970	.6811	139.50	26.80	.5130E-04	1700.0	308.0	.1930E+12	8.154	20
65.970	62.570	25.450	.4893	177.70	34.26	.8746E-04	1835.0	214.8	.2399E+12	6.341	
61.580	59.030	24.470	.6253	180.20	34.75	.7135E-04	2009.0	243.7	.1689E+12	6.341	
61.450	58.850	24.420	.6254	184.30	35.54	.7300E-04	1998.0	244.9	.1666E+12	6.341	
70.120	65.780	25.550	.3246	136.20	26.39	.1024E-03	1348.0	192.5	.3327E+12	6.341	
70.010	65.340	25.550	.3247	147.80	28.61	.1112E-03	1337.0	194.1	.3257E+12	6.341	
68.550	64.710	26.250	.5268	204.60	39.60	.9500E-04	2097.0	200.0	.2910E+12	6.341	
68.660	64.740	26.250	.5268	208.40	40.34	.9680E-04	2101.0	199.6	.2928E+12	6.341	
66.410	63.040	26.400	.6426	229.70	44.40	.8778E-04	2402.0	211.8	.2439E+12	6.341	
66.640	63.200	26.400	.6498	236.50	45.73	.8900E-04	2445.0	210.5	.2484E+12	6.341	
72.180	67.770	25.960	.3822	157.90	30.62	.1005E-03	1690.0	181.6	.3908E+12	6.341	30
72.000	67.770	25.960	.3822	151.60	29.40	.9650E-04	1685.0	182.0	.3870E+12	6.341	
62.650	59.710	25.680	.4264	143.50	27.69	.8320E-04	1413.0	236.9	.1780E+12	6.341	
62.420	59.260	25.650	.4265	154.90	29.88	.8980E-04	1396.0	239.6	.1726E+12	6.341	
63.640	60.150	25.580	.3324	129.90	25.08	.9650E-04	1129.0	231.6	.1916E+12	6.341	
65.990	60.670	25.650	.2095	120.60	23.30	.1418E-03	748.0	221.9	.2221E+12	6.341	
65.390	61.660	25.600	.2889	115.80	22.37	.9867E-04	1038.0	219.9	.2236E+12	6.341	
58.980	57.310	25.030	.6438	182.60	35.17	.7050E-04	1912.0	261.9	.4864E+11	4.530	
59.000	57.470	25.030	.4437	167.90	32.35	.6480E-04	1918.0	261.0	.4907E+11	4.530	
61.040	58.820	24.880	.4771	170.60	32.89	.8860E-04	1512.0	246.6	.5854E+11	4.530	
61.580	58.480	25.010	.3109	155.70	30.02	.1241E-03	982.0	246.0	.5939E+11	4.530	40
65.500	63.250	25.290	.5993	196.00	37.89	.8038E-04	2213.0	213.9	.8810E+11	4.530	
71.870	67.300	25.140	.3173	187.90	36.42	.1440E-03	1386.0	183.7	.1406E+12	4.530	
71.980	67.220	25.140	.3173	195.30	37.86	.1498E-03	1387.0	183.6	.1409E+12	4.530	
77.180	69.910	25.160	.1873	163.50	31.77	.2109E-03	924.0	165.2	.1968E+12	4.530	
76.950	69.490	25.160	.1946	175.10	34.01	.2176E-03	950.6	166.8	.1922E+12	4.530	
68.760	66.020	26.500	.6349	243.50	47.15	.9370E-04	2587.0	195.0	.1120E+12	4.530	

72.080	68.810	26.580	.5335	228.70	44.36	.1040E-03	2392.0	179.0	.1450E+12	4.530
72.860	69.570	26.610	.5262	224.00	43.46	.1030E-03	2415.0	175.0	.1545E+12	4.530
66.750	64.240	26.190	.4692	171.00	33.05	.8930E-04	1798.0	206.7	.9540E+11	4.530
62.180	60.150	25.910	.7082	231.30	44.63	.8070E-04	2344.0	236.7	.6410E+11	4.530
62.840	60.830	25.990	.6792	251.70	41.64	.7840E-04	2301.0	231.7	.6818E+11	4.530
65.550	62.970	24.620	.3971	147.60	28.53	.9138E-04	1461.0	214.7	.8880E+11	4.530
78.390	66.440	26.120	.1154	173.10	33.60	.3633E-03	553.0	172.6	.1865E+12	4.530
73.130	67.270	26.140	.2019	154.60	30.02	.1864E-03	899.8	180.8	.1459E+12	4.530
72.840	66.380	25.960	.2019	172.10	33.26	.2074E-03	884.0	184.1	.1403E+12	4.530
70.090	65.230	25.830	.2670	177.80	34.44	.1627E-03	1099.0	194.3	.1179E+12	4.530
67.530	64.900	26.040	.4041	151.80	29.38	.9202E-04	1586.0	202.3	.1021E+12	4.530
74.930	70.610	25.400	.2954	259.10	50.33	.2123E-03	1421.0	167.9	.3899E+11	2.7200
66.780	64.690	25.520	.6063	298.80	57.80	.1208E-03	2341.0	205.1	.2136E+11	2.7200
64.670	63.300	25.630	.6426	247.70	47.88	.9480E-04	2350.0	215.7	.1834E+11	2.7200
73.260	69.960	25.960	.1946	135.40	26.28	.1688E-03	903.0	173.2	.3495E+11	2.7200
79.880	76.530	26.010	.1870	117.10	22.82	.1501E-03	1051.0	145.5	.5794E+11	2.7200
82.370	74.360	25.500	.2159	318.50	62.05	.3534E-03	1222.0	145.8	.6052E+11	2.7200
81.740	77.230	25.230	.3813	275.40	53.69	.1727E-03	2274.0	140.0	.6523E+11	2.7200
80.850	74.380	25.390	.2519	302.60	58.82	.2380E-03	1395.0	148.2	.5709E+11	2.7200
80.220	72.340	25.160	.1980	295.50	57.50	.3590E-03	1057.0	153.8	.5255E+11	2.7200
80.750	73.050	25.160	.1900	259.80	50.58	.3467E-03	977.0	151.3	.5490E+11	2.7200
77.000	74.360	27.580	.1295	69.21	13.46	.1096E-03	677.0	155.2	.4639E+11	2.7200
77.110	74.020	27.610	.1944	121.50	23.64	.1500E-03	1013.0	155.7	.4600E+11	2.7200
83.320	79.310	27.640	.1869	136.90	26.71	.1750E-03	1143.0	134.7	.7039E+11	2.7200
82.400	81.430	27.510	.1796	31.51	6.15	.4178E-04	1115.0	132.5	.7306E+11	2.7200
63.383	61.714	37.663	.5223	143.73	27.75	.6721E-04	1830.6	226.4	.1363E+12	6.342
63.383	61.766	37.870	.5345	142.18	27.45	.6559E-04	1857.3	226.2	.1355E+12	6.342
54.267	53.253	34.068	.5795	119.45	22.96	.5165E-04	1457.3	306.0	.5724E+11	6.342
51.901	51.017	34.066	.5945	120.47	23.12	.5099E-04	1364.1	333.5	.4224E+11	6.342
72.499	69.856	36.317	.4959	155.01	30.08	.7587E-04	2272.0	175.2	.3299E+12	6.342
75.534	72.342	36.938	.4840	173.00	33.62	.8631E-04	2406.5	162.5	.4120E+12	6.342
77.472	73.990	37.120	.4752	178.05	34.64	.9019E-04	2490.2	155.0	.4762E+12	6.342
80.144	76.215	37.404	.4605	185.10	36.06	.9632E-04	2585.6	145.6	.5761E+12	6.342
82.427	78.180	37.818	.4531	189.60	36.98	.9992E-04	2696.2	138.1	.6730E+1.	6.342

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84.342 79.751 37.896  .4305 191.42  37.37  .1039E-03 2734.3  132.3  .7667E+12  6.342
85.576 81.613 48.705  .4239 202.63  39.59  .1135E-03 2751.9  127.4  .6544E+12  6.342
83.791 80.039 47.278  .4385 199.31  38.91  .1082E-03 2723.6  132.7  .5959E+12  6.342
83.633 79.961 47.045  .4370 193.83  37.83  .1056E-03 2706.2  133.0  .5940E+12  6.342
81.194 77.813 45.981  .4531 191.03  37.24  .1008E-03 2637.7  140.8  .5076E+12  6.342
79.280 76.137 45.566  .4605 187.69  36.56  .9774E-04 2551.3  147.3  .4416E+12  6.342
73.153 70.877 44.684  .4849 166.76  32.37  .8333E-04 2279.0  171.1  .2715E+12  6.342
68.081 66.227 44.139  .5050 166.54  32.24  .8063E-04 2041.9  196.6  .1701E+12  6.342
65.940 64.400 43.699  .5197 152.11  29.41  .7183E-04 1970.1  208.6  .1397E+12  6.342
56.999 55.802 35.023  .5645 127.13  24.47  .5616E-04 1571.2  278.2  .7583E+11  6.342  90
54.995 54.085 36.025  .5771 127.46  24.50  .5526E-04 1496.2  297.4  .5094E+11  6.342
53.565 52.785 38.517  .5846 124.40  23.90  .5337E-04 1436.8  312.6  .4071E+11  6.342
50.133 49.587 38.517  .6020 115.04  22.06  .4822E-04 1293.9  354.7  .2422E+11  6.342
47.097 46.707 38.362  .6245 112.55  21.55  .4572E-04 1184.6  399.1  .1428E+11  6.342
43.880 43.647 38.232  .6507 107.92  20.63  .4232E-04 1075.2  454.7  .7054E+10  6.342
40.096 40.044 39.293  .6370 165.87  31.64  .6690E-04  888.5  534.0  .7099E+09  6.342
36.136 35.929 31.227  .6670 111.42  21.21  .4325E-04  766.4  642.0  .3001E+10  6.342
42.144 41.858 35.463  .6163 105.73  20.19  .4377E-04  943.3  490.5  .7128E+10  6.342
46.733 46.240 36.680  .5883 116.85  22.37  .5043E-04 1096.1  405.9  .1583E+11  6.342
52.395 51.537 37.197  .5470 126.84  24.35  .5827E-04 1281.6  337.2  .3734E+11  6.342  100
53.669 52.785 37.609  .5433 123.63  23.75  .5706E-04 1235.1  312.1  .4335E+11  6.342
56.817 55.724 37.870  .5211 124.47  23.35  .5958E-04 1443.2  279.5  .6463E+11  6.342
60.149 58.847 38.025  .5061 124.06  23.91  .6079E-04 1578.6  250.0  .9524E+11  6.342
60.305 58.847 34.300  .4699 109.69  21.14  .5788E-04 1470.0  249.4  .1128E+12  6.342
63.748 61.948 34.455  .4550 117.35  22.66  .6226E-04 1596.1  224.3  .1587E+12  6.342
67.507 65.444 34.765  .4387 116.85  22.61  .6520E-04 1735.7  200.6  .2245E+12  6.342
70.693 68.236 35.049  .4074 119.64  23.19  .7150E-04 1771.9  183.9  .2937E+12  6.342
43.256 42.688 33.654  .4705 113.07  21.61  .6141E-04  786.3  470.3  .1109E+11  6.342
43.673 43.077 33.835  .4653 114.29  21.71  .6271E-04  755.7  462.3  .1177E+11  6.342
48.057 47.097 34.171  .4283 121.46  23.26  .7185E-04  836.4  388.4  .2375E+11  6.342  110
55.125 53.513 34.481  .3629 119.85  23.04  .8154E-04  945.8  299.9  .6033E+11  6.342
66.645 64.791 42.429  .3623 117.92  22.81  .7980E-04 1398.4  205.2  .1571E+12  6.342
 EOT ENCOUNTERED.
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Mixed turbulent

Run											
38	59.000	57.470	25.030	.4437	167.90	32.35	.6480E-04	1918.0	261.0	.4907E+11	4.530
51	62.840	60.830	25.990	.6792	251.70	41.64	.7840E-04	2301.0	231.7	.6818E+11	4.530
76	72.499	69.856	36.317	.4959	155.01	30.08	.7587E-04	2272.8	175.2	.3299E+12	6.342
77	75.534	72.342	36.938	.4840	173.00	33.62	.8631E-04	2408.5	162.5	.4120E+12	6.342
78	77.472	73.990	37.120	.4752	176.05	34.64	.9019E-04	2490.2	155.0	.4762E+12	6.342
79	80.144	76.215	37.404	.4605	185.10	36.06	.9632E-04	2585.6	145.6	.5761E+12	6.342
80	82.427	78.180	37.818	.4531	189.60	36.98	.9992E-04	2696.2	138.1	.6730E+12	6.342
81	84.342	79.751	37.896	.4385	191.42	37.37	.1039E-03	2734.3	132.3	.7667E+12	6.342
82	85.576	81.613	48.705	.4239	202.63	39.59	.1135E-03	2751.9	127.4	.8544E+12	6.342
83	83.791	80.039	47.278	.4385	199.31	38.91	.1082E-03	2723.6	132.7	.8959E+12	6.342
84	83.633	79.961	47.045	.4370	193.83	37.83	.1056E-03	2706.2	133.0	.8940E+12	6.342
85	81.194	77.813	45.981	.4531	191.03	37.24	.1006E-03	2637.7	140.8	.8076E+12	6.342
86	79.280	76.137	45.566	.4605	187.69	36.56	.9774E-04	2551.3	147.3	.7416E+12	6.342
87	73.153	70.877	44.684	.4849	166.76	32.37	.8333E-04	2279.0	171.1	.5715E+12	6.342
88	68.081	66.227	44.139	.5050	166.54	32.24	.8063E-04	2041.9	196.6	.4701E+12	6.342
89	65.940	64.400	43.699	.5197	152.11	29.41	.7183E-04	1970.1	208.6	.4397E+12	6.342

Mixed laminar

Run											
3	59.550	56.740	24.420	.5354	140.00	26.96	.6500E-04	1591.0	262.0	.2905E+12	8.154
4	59.760	56.890	24.470	.5353	142.00	27.36	.6590E-04	1601.0	260.0	.2955E+12	8.154
5	62.780	59.320	25.500	.6578	202.00	38.96	.7590E-04	2170.0	238.0	.3767E+12	8.154
6	62.780	59.340	25.470	.6578	200.20	38.63	.7530E-04	2171.0	238.0	.3770E+12	8.154
19	53.070	50.650	21.970	.7831	196.20	37.60	.6298E-04	1828.0	329.0	.1603E+12	8.154
20	54.600	52.520	21.970	.6811	139.50	26.80	.5130E-04	1700.0	308.0	.1934E+12	8.154
72	63.383	61.714	37.663	.5273	143.73	27.75	.6721E-04	1830.6	226.4	.1363E+12	6.342
73	63.383	61.766	37.870	.5345	142.18	27.45	.6559E-04	1857.3	226.2	.1355E+12	6.342
74	54.267	53.253	34.068	.5795	119.45	22.96	.5165E-04	1457.3	306.0	.5724E+11	6.342
75	51.901	51.017	34.068	.5945	120.47	23.12	.5099E-04	1364.1	333.5	.4224E+11	6.342
90	56.999	55.802	35.023	.5645	127.13	24.47	.5616E-04	1571.2	278.2	.7583E+11	6.342
91	54.995	54.085	38.025	.5771	127.46	24.50	.5526E-04	1496.2	297.4	.5094E+11	6.342
92	53.565	52.785	38.517	.5846	124.40	23.90	.5337E-04	1436.8	312.6	.4071E+11	6.342
93	50.133	49.587	38.517	.6020	115.04	22.06	.4822E-04	1293.9	354.7	.2422E+11	6.342
94	47.097	46.707	38.362	.6245	112.55	21.55	.4572E-04	1184.6	399.1	.1428E+11	6.342
95	43.880	43.647	38.232	.6507	107.92	20.63	.4232E-04	1075.2	454.7	.7054E+10	6.342
96	40.096	40.044	39.293	.6370	165.87	31.64	.6690E-04	888.5	534.0	.7099E+09	6.342
97	36.136	35.929	31.227	.6670	111.42	21.21	.4325E-04	766.4	642.0	.3001E+10	6.342
98	42.144	41.858	35.463	.6183	105.73	20.19	.4377E-04	943.3	490.5	.7128E+10	6.342
99	46.733	46.240	36.680	.5883	116.85	22.37	.5043E-04	1096.1	405.9	.1583E+11	6.342
100	52.395	51.537	37.197	.5472	126.84	24.35	.5827E-04	1281.6	327.2	.3734E+11	6.342
101	53.669	52.785	37.689	.5433	123.63	23.75	.5706E-04	1338.1	312.1	.4335E+11	6.342
102	56.817	55.724	37.870	.5211	124.47	23.95	.5958E-04	1443.2	279.5	.6463E+11	6.342
103	60.149	58.847	38.025	.5061	124.06	23.91	.6079E-04	1578.6	250.0	.9524E+11	6.342
104	60.305	58.847	34.300	.4699	109.69	21.14	.5788E-04	1470.0	249.4	.1128E+12	6.342
105	63.748	61.948	34.455	.4550	117.35	22.66	.6356E-04	1596.1	224.3	.1587E+12	6.342

Upper transitional

Run											
8	67.450	62.760	25.780	.7077	267.60	51.74	.9280E-04	2681.0	209.7	.5498E+12	8.154
9	67.400	62.700	25.810	.7077	268.10	51.84	.9298E-04	2677.0	210.0	.5468E+12	8.154
10	70.460	65.160	26.040	.6783	273.80	53.02	.9840E-04	2805.0	194.0	.6951E+12	8.154
11	70.480	65.180	26.040	.6783	273.60	52.99	.9850E-04	2808.0	193.0	.6967E+12	8.154
22	61.580	59.030	24.470	.6253	180.20	34.75	.7135E-04	2009.0	243.7	.1689E+12	6.341
23	61.450	58.850	24.420	.6254	184.30	35.54	.7300E-04	1998.0	244.9	.1666E+12	6.341
28	66.410	63.040	24.400	.6426	229.70	44.40	.8778E-04	2402.0	211.8	.2439E+12	6.341
29	66.640	63.200	24.400	.6498	236.50	45.73	.8900E-04	2445.0	210.5	.2484E+12	6.341
37	58.980	57.310	25.030	.6438	182.60	35.17	.7050E-04	1912.0	261.9	.4864E+11	4.530
41	65.500	63.250	25.290	.5993	196.00	37.89	.8038E-04	2213.0	213.9	.8810E+11	4.530
46	68.760	66.020	26.500	.6349	243.50	47.15	.9370E-04	2587.0	195.0	.1120E+12	4.530
50	62.180	60.150	25.910	.7082	231.30	44.63	.8070E-04	2344.0	236.7	.6410E+11	4.530
51	62.840	60.830	25.990	.6792	251.70	41.64	.7840E-04	2301.0	231.7	.6818E+11	4.530
72	63.383	61.714	37.663	.5273	143.73	27.75	.6721E-04	1830.6	226.4	.1363E+12	6.342
73	63.383	61.766	37.870	.5345	142.18	27.43	.6559E-04	1857.3	226.2	.1355E+12	6.342
76	72.499	69.856	36.317	.4959	155.01	30.08	.7587E-04	2272.8	175.2	.3299E+12	6.342
77	75.534	72.342	36.938	.4840	173.00	33.62	.8631E-04	2408.5	162.5	.4120E+12	6.342
88	68.081	66.227	44.139	.5050	166.54	32.24	.8063E-04	2041.9	196.6	.1701E+12	6.342
89	65.940	64.400	43.699	.5197	152.11	29.41	.7183E-04	1970.1	208.6	.1397E+12	6.342
96	40.096	40.044	39.293	.6370	165.87	31.64	.6690E-04	888.5	534.0	.7099E+09	6.342
105	63.748	61.948	34.455	.4550	117.35	22.66	.6356E-04	1596.1	224.3	.1587E+12	6.342
106	67.507	65.444	34.765	.4387	116.85	22.61	.6520E-04	1735.7	200.6	.2245E+12	6.342
107	70.693	68.238	35.049	.4074	119.64	23.19	.7150E-04	1771.8	183.9	.2937E+12	6.342

Lower transitional

Run											
1	66.960	62.500	22.560	.5055	169.70	32.80	.8240E-04	1891.0	212.0	.5739E+12	8.154
2	67.820	62.910	22.720	.4982	161.90	35.20	.8960E-04	1904.0	208.0	.6068E+12	8.154
7	70.120	64.870	25.470	.5273	229.50	44.43	.9710E-04	2364.0	195.0	.6860E+12	8.154
12	71.820	66.280	22.480	.4833	183.70	35.60	.9260E-04	2079.0	186.0	.8368E+12	8.154
13	71.690	66.250	22.460	.4833	180.40	34.96	.9096E-04	2073.0	187.0	.8312E+12	8.154
15	69.100	64.660	22.100	.3464	109.50	21.19	.7730E-04	1390.0	199.0	.6988E+12	8.154
16	68.840	64.320	22.100	.3537	114.40	22.15	.7920E-04	1406.0	200.0	.6815E+12	8.154
17	65.370	60.880	22.050	.3612	125.00	24.14	.8520E-04	1281.0	223.0	.5035E+12	8.154
18	65.130	60.620	22.050	.3613	126.40	24.41	.8620E-04	1270.0	224.0	.4920E+12	8.154
21	65.970	62.570	25.450	.4893	177.70	34.24	.8746E-04	1835.0	214.8	.2399E+12	6.341
26	68.550	64.710	26.250	.5268	204.60	39.60	.9500E-04	2097.0	200.0	.2910E+12	6.341
27	68.660	64.740	26.250	.5268	208.40	40.34	.9680E-04	2101.0	199.6	.2928E+12	6.341
30	72.180	67.770	25.960	.3822	157.90	30.62	.1005E-03	1690.0	181.6	.3908E+12	6.341
31	72.000	67.770	25.960	.3822	151.60	29.40	.9650E-04	1685.0	182.0	.3870E+12	6.341
32	62.650	59.710	25.680	.4264	143.50	27.69	.8320E-04	1413.0	236.9	.1780E+12	6.341
33	62.420	59.260	25.650	.4263	154.90	29.88	.8980E-04	1396.0	239.6	.1726E+12	6.341
38	59.000	57.470	25.030	.4437	167.90	32.35	.6480E-04	1918.0	261.0	.4907E+11	4.530
39	61.040	58.820	24.880	.4771	170.60	32.89	.8860E-04	1512.0	246.6	.5854E+11	4.530
47	72.080	68.810	26.580	.5335	228.70	44.36	.1040E-03	2392.0	179.0	.1450E+12	4.530
48	72.860	69.570	26.610	.5262	224.00	43.46	.1030E-03	2415.0	175.0	.1545E+12	4.530
49	66.750	64.240	26.190	.4692	171.00	33.05	.8930E-04	1798.0	206.7	.9540E+11	4.530
52	65.550	62.970	24.620	.3971	147.60	28.53	.9138E-04	1461.0	214.7	.8880E+11	4.530
57	67.530	64.900	26.040	.4041	151.80	29.38	.9202E-04	1586.0	202.3	.1021E+12	4.530
59	66.780	64.690	25.520	.6063	299.80	57.80	.1208E-03	2341.0	205.1	.2136E+11	2.7200
60	64.870	63.300	25.630	.6426	247.70	47.88	.9480E-04	2350.0	215.7	.1834E+11	2.7200
88	68.081	66.227	44.139	.5050	166.54	32.24	.8063E-04	2041.9	196.6	.1701E+12	6.342
89	65.940	64.400	43.699	.5197	152.11	29.41	.7183E-04	1970.1	208.6	.1397E+12	6.342

Mixed transitional

Run											
14	78.520	71.900	22.380	.3277	132.90	25.85	.9770E-04	1695.0	158.0	.1367E+13	8.154
24	76.120	65.780	25.550	.3246	136.20	26.39	.1024E-03	1348.0	192.5	.3327E+12	6.341
25	70.010	65.340	25.550	.3247	147.80	28.61	.1112E-03	1337.0	194.1	.3257E+12	6.341
34	63.640	60.150	25.580	.3324	129.90	25.08	.9650E-	1129.0	231.6	.1918E+12	6.341
35	65.990	60.670	25.650	.2095	120.60	23.30	.1418E-03	748.0	221.9	.2221E+12	6.341
36	65.390	61.660	25.600	.2889	115.80	22.37	.9867E-04	1038.0	219.9	.2236E+12	6.341
40	61.580	58.480	25.010	.3109	155.70	30.02	.1241E-03	982.0	246.0	.5939E+11	4.530
42	71.870	67.300	25.140	.3173	187.90	36.42	.1440E-03	1396.0	183.7	.1406E+12	4.530
43	71.980	67.220	25.140	.3173	195.30	37.86	.1498E-03	1307.0	183.6	.1409E+12	4.530
44	77.180	69.910	25.160	.1873	163.50	31.77	.2109E-03	924.0	165.2	.1968E+12	4.530
45	76.950	69.490	25.160	.1946	175.10	34.01	.2176E-03	950.6	166.8	.1922E+12	4.530
53	78.390	66.440	26.120	.1154	173.10	33.60	.3633E-03	553.0	172.6	.1865E+12	4.530
54	73.130	67.270	26.140	.2019	154.80	30.02	.1864E-03	899.8	180.8	.1459E+12	4.530
55	72.840	66.380	25.960	.2019	172.10	33.36	.2074E-03	884.0	184.1	.1403E+12	4.530
56	70.090	65.230	25.830	.2670	177.80	34.44	.1627E-03	1099.0	194.3	.1179E+12	4.530
58	74.930	70.610	25.420	.2954	259.10	50.33	.2123E-03	1421.0	167.9	.3899E+11	2.7200
59	66.780	64.690	25.520	.6063	298.80	57.80	.1208E-03	2341.0	205.1	.2136E+11	2.7200
61	73.260	69.960	25.960	.1946	135.40	26.23	.1688E-03	903.0	173.2	.3495E+11	2.7200
62	79.880	76.530	26.010	.1870	117.10	22.82	.1501E-03	1051.0	145.5	.5294E+11	2.7200
63	82.370	74.360	25.500	.2159	318.50	62.05	.3534E-03	1222.0	145.8	.6052E+11	2.7200
64	81.740	77.730	25.730	.3812	275.40	53.69	.1727E-03	2234.0	140.0	.6523E+11	2.7200
65	80.850	74.380	25.390	.2519	302.60	58.93	.2880E-03	1395.0	148.2	.5709E+11	2.7200
66	80.220	72.340	25.160	.1980	295.50	57.50	.3590E-03	1057.0	153.8	.5255E+11	2.7200
67	80.750	73.050	25.160	.1800	259.80	50.58	.3467E-03	877.0	151.3	.5490E+11	2.7200
68	77.000	74.360	27.560	.1295	69.21	13.46	.1286E-03	677.0	155.2	.4639E+11	2.7200
69	77.110	74.020	27.610	.1944	121.50	23.64	.1500E-03	1013.0	155.7	.4600E+11	2.7200
70	83.320	79.310	27.640	.1869	136.90	26.71	.1750E-03	1143.0	134.7	.7039E+11	2.7200
71	82.400	81.430	27.510	.1796	31.51	6.15	.4178E-04	1115.0	132.5	.7306E+11	2.7200

6.13 COMPUTER PROGRAMS

6.13.1 Program 1

```
      PROGRAM MAIN (INPUT,OUTPUT,TAPE61=INPUT,TAPE62=OUTPUT,
     1TAPE7=OUTPUT,TAPE10,TAPE9,TAPE5=INPUT,TAPE6,TAPE63,TAPE64)
C
C     PROGRAM FOR USE WITH THE GREEN MAMBA
C
C
      DIMENSION RT(10),RE(10),PR(10),T(10),DEN(10),CND(10),
     A CP(10),VEL(10),VIS(10)
C       RT IS THE RESISTANCE THERMOMETER READING
C       RE IS THE CALCULATED REYNOLDS NUMBER
C        PR IS THE PRANDTL NUMBER
C        T IS THE LINEARLY INTERPOLATED TEMPERATURE
C       DEN IS THE CUBIC SPLINE INTERPOLATED DENSITY
C       CND IS THE LINEARLY INTERPOLATED THERMAL CONDUCTIVITY
C        CP IS THE LINEARLY INTERPOLATED SPECIFIC HEAT
C        VIS IS THE CUBIC SPLINE INTERPOLATED VISCOSITY
C       VEL IS THE VELOCITY AT EACH POINT
C      N IS THE NUMBER OF TEMPERATURE POINTS
       N=2
       WRITE(6,*)" GREEN MAMBA PROGRAM"
       WRITE(6,*)" WATER OR OIL (DEFAULT IS OIL)"
        IWATER=0
C       READ(5,232)KWATER
C      IF(EOF(5))71,88
88     CONTINUE
232    FORMAT(A3)
C      IF(KWATER.EQ.3HOIL)IWATER=0
C      IF(KWATER.EQ.3HWAT)IWATER=1
C
C  ------------------------------------------------------------------
C    OBTAIN THE EXPERIMENTAL DATA
C     READ TEMPERATURES DOWN THE EXCHANGER
C  DEV SECT 1   2   3   4   5   6   7   8   9   10
71      WRITE(6,400)
400    FORMAT(/" GIVE THE RESISTANCE THERMOMETER READINGS (I/O)")
       WRITE (7,*) "ENTER R(IN),R(OUT),ROTAMETER READING"
C
       READ(5,*)(RT(I),I=1,N)
       IF(EOF(5))20,10
10     CONTINUE
C      READ THE ROTAMETER READING ON THE OIL SIDE
       WRITE(6,401)
401     FORMAT(/" GIVE THE ROTAMETER READING IN PERCENT")
       READ(5,*)RR
       IF(EOF(5))20,11
11     CONTINUE
C  ------------------------------------------------------------------
```

```
C
C      FROM THIS DATA CALCULATE ALL THE TEMPERATURES
C       AND PHYSICAL PROPERTIES, AND PRANDTL AND REYNOLDS NUMBERS
       CALL TEMP(RT,T,N)
       IF(IWATER.EQ.1) GO TO 25
       CALL DENC(T,DEN,N)
       CALL COND(T,CND,N)
       CALL SPECIF(T,CP,N)
       CALL VISC(T,VIS,N)
       GO TO 27
25     CONTINUE
       CALL DENCW(T,DEN,N)
       CALL CONDW(T,CND,N)
       CALL SPECIFW(T,CP,N)
       CALL VISCW(T,VIS,N)
27     CALL ROTA(RR,VMASS,VEL,DEN,N)
       CALL REYN(DEN,0.02539,VIS,VMASS,VEL,RE,N)
       CALL PRANDTL(CP,VIS,CND,PR,N)
C
C       FROM THE ABOVE ALL THE REQUIRED DATA HAS BEEN CALCULATED AND STORED
C        IN THE RELEVANT ARRAYS OF DIMENSION 10 .THE DIAMETER OF THE TUBE IS
C        25.39 MM IN THE REYNOLDS CALC.
C ------------------------------------------------------------------------
C
C        THIS IS NOW PRINTED OUT
C
       CALL WRT(RR,VMASS,RT,T,DEN,CND,CP,VIS,RE,PR,VEL,N)
C ------------------------------------------------------------------------
C
C
C      CALCULATE THE HEAT BALANCE FROM THE DATA
C
       IF(IWATER.EQ.1)CALL HEATLW(T,VMASS,CP,RE,PR,CND,N)
       IF(IWATER.EQ.0)CALL HEATL(T,VMASS,CP,RE,PR,CND,DEN,VIS,N)
C
C ------------------------------------------------------------------------
C
C      REQUESTS FOR PLOTTER
C
       GOTO 71
17     WRITE(6,506)
506    FORMAT(/T5," DO YOU WANT TO PLOT    Y/N")
       READ(5,507)IPLOT
507    FORMAT(A10)
       IF(EOF(5))22,15
15      IF(IPLOT.EQ.1HN)GO TO 22
       WRITE(6,508)
508    FORMAT(T5,"WHICH TWO VARIABLES DO YOU WISH TO PLOT  X-Y"
      1,/,"      OR TYPE HELP TO LIST")
       READ(5,507)IPLOT
       IF(EOF(5))20,16
16     IF(IPLOT.EQ.1HH)WRITE(6,509)
       IF(IPLOT.EQ.1HH) GO TO 15
       IF(IPLOT.EQ."RES-TEMP")CALL RPLOT(RT,T,N)
       IF(IPLOT.EQ."RES-DEN")CALL RPLOT(RT,DEN,N)
       IF(IPLOT.EQ."RES-COND")CALL RPLOT(RT,CND,N)
       IF(IPLOT.EQ."RES-CP")CALL RPLOT(RT,CP,N)
```

```
      IF(IPLOT.EQ."RES-VIS")CALL RPLOT(RT,VIS,N)
      IF(IPLOT.EQ."RES-KEY")CALL RPLOT(RT,RE,N)
      IF(IPLOT.EQ."RES-PR")CALLRPLOT(RES,PR,N)
      IF(IPLOT.EQ."TEMP-RES")CALL RPLOT(T,RT,N)
      IF(IPLOT.EQ."TEMP-DEN")CALL RPLOT(T,DEN,N)
      IF(IPLOT.EQ."TEMP-COND")CALL RPLOT(T,CND,N)
      IF(IPLOT.EQ."TEMP-CP")CALL RPLOT(T,CP,N)
      IF(IPLOT.EQ."TEMP-VIS")CALL RPLOT(T,VIS,N)
      IF(IPLOT.EQ."TEMP-REY")CALL RPLOT(T,RE,N)
      IF(IPLOT.EQ."TEMP-PR")CALL RPLOT(T,PR,N)
      GO TO 17
509   FORMAT(/," RESISTANCE  RES,   TEMPERATURE  TEMP,  DENSITY  DEN"
     A,/" CONDUCTIVITY COND,  SPECIFIC HEAT  CP,  VISCOSITY VIS,"
     B,/," REYNOLDS  REY,    PRANDTL  PR  "
     C,/" E.G.   TEMP-DEN        TEMP-PR")
C
C ------------------------------------------------------------------
22      GO TO 71
20     STOP
       END
       SUBROUTINE DENC(T,DEN,N)
C  S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE   FOR
C     DENSITY FROM A GIVEN TEMPERATURE
C      USING CUBIC SPLINES
C      DATA IS FOR REGAL OIL B
C
C     T CONTAINS THE TEMPERATURE(S)
C     DEN  CONTAINS THE INTERPOLATED DENSITY(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      VTABLE  CONTAINS THE LOOK-UP TABLE
C
       DIMENSION T(N),DEN(N),DTABLE(9,2),C(4,9)
       DATA (DTABLE(I,2),I=1,9)/
      A 872.5,862.1,848.,837.8,825.,799.8,
      B 772.5,750.1,727.1/
       DATA (DTABLE(I,1),I=1,9)/
      A 20.,40.,60.,80.,100.,
      B 150.,200.,250.,300./
C ------------------------------------------------------------------1-------
C    CALCULATE THE CONSTANTS FOR THE SPLINE FIT
        CALL SPLICON(DTABLE,C,9)
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
        DO 1 I=2,9
        J=I
C                COUNTER VALUE AT EXIT
        IF(T(K).LT.DTABLE(1,1)) GO TO 3
        IF(T(K).LE.DTABLE(I,1)) GO TO 2
C IF NOT - LOOP
1       CONTINUE
```

```
C     ------------------------------------------------2----------
C     THE VALUE IS OUT OF THE TOP OF THE TABLE
      2     CONTINUE
C
C     THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C     PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
          WRITE(6,99) T(K)
99        FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2," WAS NOT FOUND"
         C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE DENSITY HAS BEEN"
         D,/,T' " LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C     CARRY ON INTO THE INTERPOLATION SECTION
C
C     INTERPOLATE LINEARLY FOR T
C
          DEN(K)=(T(K)-DTABLE(J-1,1))/(DTABLE(J,1)-DTABLE(J-1,1))
          DEN(K)=DEN(K)*(DTABLE(J,2)-DTABLE(J-1,2))+DTABLE(J-1,2)
C
C       THE DENSITY IS NOW INTERPOLATED
        CALCULATE THE NEXT VALUE
          GO TO 100
C     ------------------------------------------------2----------
C
          CONTINUE
C
C       THE DENSITY IS NOW INTERPOLATED USING A
C       CUBIC SPLINE ROUTINE
C       SUBROUTINE SPLICON CALCULATES THE INTERPOLATION CONSTANTS
C
          DEN(K)=(DTABLE(J,1)-T(K))*(C(1,J-1)*(DTABLE(J,1)-T(K))**2
         A  +C(3,J-1))
          DEN(K)=DEN(K)+(T(K)-DTABLE(J-1,1))*(C(2,J-1)
         A *(T(K)-DTABLE(J-1,1))**2
         A  +C(4,J-1))
C
C       THE DENSITY HAS BEEN INTERPOLATED BY SPLINES
C       CALCULATE THE NEXT VALUE
C
C
100       CONTINUE
C     ------------------------------------------------1----------
C
C       ALL FINISHED
C       RETURN WITH INTERPOLATED VALUES
C
          RETURN
          END
          SUBROUTINE SPLICON(TABLE,C,M)
C
C       SUBROUTINE TO CALCULATE THE CONSTANTS FOR THE
C       SPLINE FIT
C       REF: PENNINGTON ,R.H
C            INTRODUCTORY COMPUTER METHODS AND NUMERICAL
C              ANALYSIS.   MAC MILLAN 1965
C
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      DIMENSION TABLE(M,2),C(4,M),D(10),P(10),E(10),
     A A(10,3),B(10),Z(10),X(10),Y(10)
      DO 100 I=1,M
      X(I)=TABLE(I,1)
      Y(I)=TABLE(1,2)
100   CONTINUE
      MM=M-1
      DO 2 K=1,MM
      D(K)=X(K+1)-X(K)
      P(K)=D(K)/6.
2     E(K)=(Y(K+1)-Y(K))/D(K)
      DO 3 K=2,MM
3     B(K)=E(K)-E(K-1)
      A(1,2)=-1.-D(1)/D(2)
      A(1,3)=D(1)/D(2)
      A(2,3)=P(2)-P(1)*A(1,3)
      A(2,2)=2.*(P(1)+P(2))-P(1)*A(1,2)
      A(2,3)=A(2,3)/A(2,2)
      B(2)=B(2)/A(2,2)
      DO 4 K=3,MM
      A(K,2)=2.*(P(K-1)+P(K))-P(K-1)*A(K-1,3)
      B(K)=B(K)-P(K-1)*B(K-1)
      A(K,3)=P(K)/A(K,2)
4     B(K)=B(K)/A(K,2)
      Q=D(M-2)/D(M-1)
      A(M,1)=1.+Q+A(M-2,3)
      A(M,2)=-Q-A(M,1)*A(M-1,3)
      B(M)=B(M-2)-A(M,1)*B(M-1)
      Z(M)=B(M)/A(M,2)
      MN=M-2
      DO 6 I=1,MN
      K=M-I
6     Z(K)=B(K)-A(K,3)*Z(K+1)
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)
      DO 7 K=1,MM
      Q=1./(6.*D(K))
      C(1,K)=Z(K)*Q
      C(2,K)=Z(K+1)*Q
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)
7     C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
      RETURN
      END
      SUBROUTINE VISC(T,VIS,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE   FOR
C     VISCOSITY FROM A GIVEN TEMPERATURE
C     DATA IS FOR REGAL OIL B
C
C     T CONTAINS THE TEMPERATURE(S)
C     VIS  CONTAINS THE INTERPOLATED VISCOSITY(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      VTABLE  CONTAINS THE LOOK UP TABLE
C
```

```
      DIMENSION T(N),VIS(N),VTABLE(7,2),C(4,7)
      DATA (VTABLE(I,2),I=1,7)/
     A4.626,3.57,2.753,2.135,1.656,.802,.239/
      DATA (VTABLE(I,1),I=1,7)/
     A.00341,.00319,.003,.00283,.00268,
     B.00236,.00211/
C ------------------------------------------------1-------
C    CALCULATE THE CONSTANTS FOR THE SPLINE FIT
        CALL SPLICON(VTABLE,C,7)
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
       T(K)=1./(273.15+T(K))
        DO 1 I=2,9
        J=I
C                COUNTER VALUE AT EXIT
       IF(T(K).GT.VTABLE(1,1)) GO TO 3
       IF(T(K).GE.VTABLE(I,1)) GO TO 2
C IF NOT - LOOP
1       CONTINUE
C
C  ----------------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3       CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
       TK=1./T(K)-273.15
        WRITE(6,99) TK
99      FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE VISCOSITY HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C
C  INTERPOLATE LINEARLY FOR T
C
        VIS(K)=(T(K)-VTABLE(J-1,1))/(VTABLE(J,1)-VTABLE(J-1,1))
        VIS(K)=VIS(K)*(VTABLE(J,2)-VTABLE(J-1,2))+VTABLE(J-1,2)
       VIS(K)=EXP(VIS(K))*0.001
       T(K)=1./T(K)-273.15
C
C    THE VISCOSITY IS NOW INTERPOLATED
C    CALCULATE THE NEXT VALUE
       GO TO 100
C ------------------------------------------------2------
```

```
C
2       CONTINUE
C
C    THE VISCOSITY IS NOW INTERPOLATED USING A
C    CUBIC SPLINE ROUTINE
C    SUBROUTINE SPLICON CALCULATES THE INTERPOLATION CONSTANTS
C
        VIS(K)=(VTABLE(J,1)-T(K))*(C(1,J-1)*(VTABLE(J,1)-T(K))**2
       Z   +C(3,J-1))
        VIS(K)=VIS(K)+(T(K)-VTABLE(J-1,1))*(C(2,J-1)*(T(K)-VTABLE(J-1,1
       Z ))**2+C(4,J-1))
        VIS(K)=EXP(VIS(K))*0.001
        T(K)=1./T(K)-273.15
C
C    THE VISCOSITY HAS BEEN INTERPOLATED BY SPLINES
C    CALCULATE THE NEXT VALUE
C
C
100     CONTINUE
C-----------------------------------------------------1-------------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
        RETURN
        END
        SUBROUTINE SPECIF(T,CP,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE LINEARLY FOR
C     SPECIFIC HEAT FROM A GIVEN TEMPERATURE
C    DATA IS FOR REGAL OIL B
C
C     T CONTAINS THE TEMPERATURE(S)
C     CP  CONTAINS THE INTERPOLATED SPECIFIC HEAT(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      CPTABLE  CONTAINS THE LOOK-UP TABLE
C
        DIMENSION T(N),CP(N),CPTABLE(9,2)
        DATA (CPTABLE(I,2),I=1,9)/
       A 1871.5,1946.02,2018.04,2093.4,2160.39,
       B 2343.77,2522.55,2702.58,2890.99/
        DATA (CPTABLE(I,1),I=1,9)/
       A 20.,40.,60.,80.,100.,150.,200.,250.,300./
C-----------------------------------------------------1-------------
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
        DO 1 I=2,9
        J=I
```

```
C              COUNTER VALUE AT EXIT
      IF(T(K).LT.CPTABLE(1,1)) GO TO 3
      IF(T(K).LE.CPTABLE(1,1)) GO TO 2
C IF NOT - LOOP
1     CONTINUE
C
C  ---------------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3     CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
      WRITE(6,99) T(K)
99    FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2," WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE SPECIFIC HEAT HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C  ---------------------------------------------2----------
C
2     CONTINUE
C  INTERPOLATE LINEARLY FOR CP
C
      CP(K)=(T(K)-CPTABLE(J-1,1))/(CPTABLE(J,1)-CPTABLE(J-1,1))
      CP(K)=CP(K)*(CPTABLE(J,2)-CPTABLE(J-1,2))+CPTABLE(J-1,2)
C
C   THE SPECIFIC HEAT IS NOW INTERPOLATED
C   CALCULATE THE NEXT VALUE
100   CONTINUE
C-----------------------------------------------1----------
C
C   ALL FINISHED
C   RETURN WITH INTERPOLATED VALUES
C
      RETURN
      END
      SUBROUTINE COND(T,CND,N)
C   S.I. UNIST ARE USED
C
C    SUBROUTINE TO INTERPOLATE LINEARLY FOR
C    THERMAL CONDUCTIVITY FROM A GIVEN TEMPERATURE
C   DATA IS FOR REGAL OIL B
C
C    T CONTAINS THE TEMPERATURE(S)
C    CND  CONTAINS THE INTERPOLATED THERMAL CONDUCTIVITY(S)
C    N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C     CTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),CND(N),CTABLE(9,2)
      DATA (CTABLE(I,1),I=1,9)/
     A 20.,40.,60.,80.,100.,150.,200.,250.,300./
      DATA (CTABLE(I,2),I=1,9)/
     A  132.36,131.048,129.66,128.19,126.9,123.
     B ,119.42,115.19,112.36/
C ---------------------------------------------1----------
```

```
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
        DO 1 I=2,9
        J=I
C                COUNTER VALUE AT EXIT
        IF(T(K).LT.CTABLE(1,1)) GO TO 3
        IF(T(K).LE.CTABLE(1,1)) GO TO 2
C IF NOT - LOOP
1       CONTINUE
C
C  ----------------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3       CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
        WRITE(6,99) T(K)
99      FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE"
     A ". THE THERMAL CONDUCTIVITY HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C  ----------------------------------------------2----------
C
2       CONTINUE
C  INTERPOLATE LINEARLY FOR K
C
        CND(K)=(T(K)-CTABLE(J-1,1))/(CTABLE(J,1)-CTABLE(J-1,1))
        CND(K)=CND(K)*(CTABLE(J,2)-CTABLE(J-1,2))+CTABLE(J-1,2)
        CND(K)=CND(K)+0.001
C
C    THE THERMAL CONDUCTIVITY IS NOW INTERPOLATED
C    CALCULATE THE NEXT VALUE
100     CONTINUE
C---------------------------------------------1----------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
        RETURN
        END
        SUBROUTINE TEMP(RT,T,N)
C     S.I.  UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE LINEARLY FOR
C     TEMPERATURE FROM A GIVEN RESISTANCE THERMOMETER
C     RESISTANCE.
C     PRIMARILY INTENDED FOR USE WITH PRT'S.
C
```

```
C
C     RT CONTAINS THE RESISTANCE(S)
C     T  CONTAINS THE INTERPOLATED TEMPERATURE(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      RTTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION RT(N),T(N),RTTABLE(35,2)
      DATA (RTTABLE(I,1),I=1,35)/
     A 100.000,101.953,103.904,105.851,107.795,
     A109.737,111.675,113.611,115.543,117.472,
     B119.399,121.322,123.243,125.160,127.075,
     C128.986,130.895,132.801,134.703,136.603,
     D138.500,140.394,142.285,144.173,146.058,
     D147.941,149.820,151.696,
     E153.570,155.440,157.308,159.173,161.035,
     F162.894,164.750/
      DATA (RTTABLE(I,2),I=1,35)/
     Z0.,5.,10.,15.,20.,25.,30.,35.,40.,45.,50.,55.,
     E60.,65.,70.,75.,80.,85.,90.,95.,100.,
     F105.,110.,115.,120.,125.,130.,135.,
     G140.,145.,150.,155.,160.,165.,170./
C     ----------------------------------------------1---------
C
C     SET UP LOOP FOR NUMBER OF DATA POINTS
C
      DO 100 K=1,N
C
C     FIND THE POSITION OF RT IN THE LOOK-UP TABLE
C
      DO 1 I=2,35
      J=I
C                COUNTER VALUE AT EXIT
      IF(RT(K).LT.RTTABLE(1,1)) GO TO 3
      IF(RT(K).LE.RTTABLE(I,1)) GO TO 2
C IF NOT - LOOP
1     CONTINUE
C     ----------------------------------------------2---------
C     THE VALUE IS OUT OF THE TOP OF THE TABLE
3     CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C     PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
      WRITE(6,99) RT(K)
99    FORMAT(/,15,"THE RESISTANCE VALUE ",F10.2," WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE TEMPERATURE HAS BEEN"
     D,/,15," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C     CARRY ON INTO THE INTERPOLATION SECTION
C     ----------------------------------------------2---------
C
```

```
C
2     CONTINUE
C  INTERPOLATE LINEARLY FOR T
C
      T(K)=(RT(K)-RTTABLE(J-1,1))/(RTTABLE(J,1)-RTTABLE(J-1,1))
      T(K)=T(K)*(RTTABLE(J,2)-RITABLE(J-1,2))+RTTABLE(J-1,2)
C
C   THE TEMPERATURE IS NOW INTERPOLATED
C   CALCULATE THE NEXT VALUE
100    CONTINUE
C-------------------------------------------------1-------------
C
C   ALL FINISHED
C   RETURN WITH INTERPOLATED VALUES
C
      RETURN
      END
      SUBROUTINE PRANDTL(CP,VIS,CND,PR,N)
C
C   SUBROUTINE TO CALCULATE THE PRANDTL NUMBER
C   CP IS THE SPECIFIC HEAT
C     VIS IS THE VISCOSITY
C     CND IS THE THERMAL CONDUCTIVITY
C     PR IS THE CALCULATED PRANDTL NUMBER
C   N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C    ALL UNITS ARE IN S.I.
C
      DIMENSION CP(N),VIS(N),CND(N),PR(N)
CC ------------------------------------------------------10-------
C   SET UP LOOP FOR NUMBER OF DATA POINTS
      DO 1 I=1,N
C
C   CHECK THAT THERMAL CONDUCTIVITY IS NON ZERO
C
      IF(CND.LE.1.E-4) GO TO 2
C
C ------------------------------------------1-----------
C    CALCULATE THE PR
      PR(I)=CP(I)*VIS(I)/CND(I)
      GO TO 1
C ------------------------------------------1-----------
2     CONTINUE
C     PRINT A WARNING MESSAGE THAT THE PR IS NON FINITE
C
      WRITE(6,400) I
400   FORMAT(/,T5," THE PRANDTL NUMBER IS UNDEFINED AS THE"
     A" THERMAL CNDUCTIVITY IS ZERO",/,T5," THE PR(",I2,") HAS BEEN"
     B" SET TO 1.E+6")
      PR(I)=1.E+6
C
1     CONTINUE
      RETURN
      END
      SUBROUTINE REYN(DENS,DIA,VISCOS,VMASS,VEL,REYNOLD,N)
```

```
C
C      ROUTINE TO CALCULATE THE REYNOLDS NUMBER FROM EITHER
C      THE VELOCITY OR THE MASS FLOWRATE
C    DENS  IS THE DENSITY
C     DIA IS THE PIPE DIAMETER
C      VISCOS IS THE VISCOSITY
C     VMASS IS THE CALCULATED MASS FLOW
C      VEL IS THE CALCULATED VELOCITY
C     REYNOLD IS THE CALCULATED REYNOLDS FROM THE ABOVE
C    N IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C       .

C   IF THE MASS FLOW IS ZERO THEN THE INDIVIDUAL VELOCTIES ARE SPECIFIED
      DIMENSION DENS(N),VEL(N),VISCOS(N),REYNOLD(N)
      DO 2 I=1,N
      IF(VEL(I).LE.1.E-4) GO TO 1
C     ----------------------------------------1----------
C
      REYNOLD(I)=DENS(I)*VEL(I)*DIA/VISCOS(I)
      GO TO 2
C     ----------------------------------------1----------
1     CONTINUE
C   CHECK THAT THE MASS FLOW IS ALSO NOT ZERO
      IF(VMASS.LE.1.E-4) GO TO 3
C     ----------------------------------------2----------
      REYNOLD(I)=4.*VMASS/(3.14159*VISCOS(I)*DIA)
      GO TO 2
C     ----------------------------------------2----------
3     CONTINUE
C     WRITE WARNING
      WRITE(6,400)
400   FORMAT(/,T5," REYNOLDS CAN NOT BE CALCULATED AS BOTH"
     A" THE MASSFLOW AND VELOCITY ARE ZERO.",/,T5,
     B" THE RE IS SET TO ZERO.")
      DO 6 J=1,N
6     REYNOLD(J)=0.0
C
2     CONTINUE
        RETURN
       END
      SUBROUTINE ROTA(PERCEN,VMASS,VEL,DEN,N)
      DIMENSION VEL(N),DEN(N)
C
C   ROUTINE TO CALCULATE THE MASS FLOW RATE (VELOCITY) FROM THE ROTAMETER READ
C   THE VARIATION IN DENSITY OF THE FLUID IS INCLUDED IN THE CORRECTION
C    METHOD. THE VISCOSITY IS NOT INFLUENCING IF THE REYNOLDS NUMBER IS
C    GREATER THAN 1.E2
C
C    CHECK IF READING IS INSIDE CALIBRATION RANGE
      IF(PERCEN.GT.100.) GO TO 1
      IF(PERCEN.LE.15.) GO TO 1
C
C
```

```
C ------------------------------------------------------------1------------
3     CONTINUE
C   CALCULATE THE MASS FLOW FROM THE CORRELATING EQN.
C
      VMASS1=-0.015603+0.007762*PERCEN
C    CORRECT THIS FOR THE VARIATION IN DENSITY
C
      VMASS=VMASS1*SQRT((8020.-DEN(N))*DEN(N)/((8020.-999.)*999.))
C    DENSITY OF THE FLOAT IS 8020 KG/M3
C     DENSITY OF H20 AT 16 IS 999 KG/M3
C
C   CALCULATE THE VELOCITY IN THE PIPE AT EACH POINT
C      DIAMETER IS 25.39 MM
      DIA=.02539
      DO 7 JJ=1,N
7     VEL(JJ)=4.*VMASS/(DEN(JJ)*(3.14159*DIA*DIA))
C
      GO TO 2
C ------------------------------------------------------------1------------
1     CONTINUE
C  PRINT WARNING MESSAGE AND THEN CALCULATE FLOWRATE
C
      WRITE(6,400)
400   FORMAT(/,T5," THE ROTAMETER READING IS OUT OF RANGE ",
     B" OF THE CALIBRATION."
     A,/,"          THE MASS FLOW MUST BE CONSIDERED SUSPECT")
      GO TO 3
C  ==========================================================
2     CONTINUE
      RETURN
      END
      SUBROUTINE KPLOT(X,Y,N)
      DIMENSION X(12),Y(12,1),ICHAR(10),RANGE(4),ITITLE(144)
      DIMENSION NINCH(2),MASK(2000)
C READ THE TITLE AND AXIS CHARS
      WRITE(6,8)
 8    FORMAT(" TITLE?")
      READ(5,7) (ITITLE(I),I=1,72)
 7    FORMAT( 80A1)
      WRITE(6,9)
 9    FORMAT(" XAXIS?")
      READ(5,7) (ITITLE(I),I=73,108)
      WRITE(6,6)
 6    FORMAT(" YAXIS")
      READ(5,7) (ITITLE(I),I=109,144)
C FIND MAX VALUES VALUES OF X AND Y
      XMAX=-1.E10
      YMAX=-1.E10
      DO 1 I=1,N
      IF(X(I).LE.XMAX) GOTO 2
      XMAX=X(I)
 2    IF(Y(I).LE.YMAX) GOTO 1
      YMAX=Y(I)
 1    CONTINUE
```

```
C  FIND MIN VALUES OF X AND Y
      XMIN=1.0E10
      YMIN=1.0E10
      DO 10 I=1,N
      IF(X(I).GE.XMIN) G_TO 20
      XMIN=X(I)
 20   IF(Y(I).GE.YMIN) GOTO 10
      YMIN=Y(I)
 10   CONTINUE
      RANGE(1)=XMIN+0.1*(XMIN-XMAX)
      RANGE(2)=XMAX+0.1*(XMAX-XMIN)
      RANGE(3)=YMIN+0.1*(YMIN-YMAX)
      RANGE(4)=YMAX+0.1*(YMAX-YMIN)
      ICHAR(1)="+"
      IER=0
      NINCH(1)=20
      NINCH(2)=20
      CALL PLOTS(NINCH,"RICH",10)
      X(N+1)=RANGE(1)
      X(N+2)=(RANGE(2)-RANGE(1))/25
      Y(N+1)=RANGE(3)
      Y(N+2)=(RANGE(4)-RANGE(3))/15
      TITLE="MAMBA"
      NT=5
      TITLEX="XAXIS"
      NX=5
      TITLEY="YAXIS"
      NY=5
      WRITE(6,999)"N=",N
 999  FORMAT(A10,I5)
      WRITE(6,989)"X-VALUE",X
 989  FORMAT(A10,(4G16.7,3X))
      WRITE(6,989) "Y-VALUE",Y
      CALL GENPLT(X,Y,N,3,1,0.0,1,TITLE,NT,TITLEX,NX,TITLEY,NY,
     1            25.,15.,1,2.)
      CALL PLOT(0.,0.,999)
      RETURN
      END
      SUBROUTINE DENCW(T,DEN,N)
C  S.I. UNITS ARE USED
C
C    SUBROUTINE TO INTERPOLATE   FOR
C    DENSITY FROM A GIVEN TEMPERATURE
C     USING CUBIC SPLINES
C     DATA IS FOR WATER
C
C    T CONTAINS THE TEMPERATURE(S)
C    DEN  CONTAINS THE INTERPOLATED DENSITY(S)
C    N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C     VTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),DEN(N),DTABLE(9,2),C(4,9)
```

```
C  FIND MIN VALUES OF X AND Y
      XMIN=1.0E10
      YMIN=1.0E10
      DO 10 I=1,N
      IF(X(I).GE.XMIN) GOTO 20
      XMIN=X(I)
 20   IF(Y(I).GE.YMIN) GOTO 10
      YMIN=Y(I)
 10   CONTINUE
      RANGE(1)=XMIN+0.1*(XMIN-XMAX)
      RANGE(2)=XMAX+0.1*(XMAX-XMIN)
      RANGE(3)=YMIN+0.1*(YMIN-YMAX)
      RANGE(4)=YMAX+0.1*(YMAX-YMIN)
      ICHAR(1)="+"
      IER=0
      NINCH(1)=20
      NINCH(2)=20
      CALL PLOTS(NINCH,"RICH",10)
      X(N+1)=RANGE(1)
      X(N+2)=(RANGE(2)-RANGE(1))/25
      Y(N+1)=RANGE(3)
      Y(N+2)=(RANGE(4)-RANGE(3))/15
      TITLE="MAMBA"
      NT=5
      TITLEX="XAXIS"
      NX=5
      TITLEY="YAXIS"
      NY=5
      WRITE(6,999)"N=",N
 999  FORMAT(A10,I5)
      WRITE(6,989)"X-VALUE",X
 989  FORMAT(A10,(4G16.7,3X))
      WRITE(6,989) "Y-VALUE",Y
      CALL GENPLT(X,Y,N,3,1,0.0,1,TITLE,NT,TITLEX,NX,TITLEY,NY,
     1            25.,15.,1,2.)
      CALL PLOT(0.,0.,999)
      RETURN
      END
      SUBROUTINE DENCW(T,DEN,N)
C  S.I. UNITS ARE USED
C
C    SUBROUTINE TO INTERPOLATE   FOR
C    DENSITY FROM A GIVEN TEMPERATURE
C     USING CUBIC SPLINES
C     DATA IS FOR WATER
C
C    T CONTAINS THE TEMPERATURE(S)
C    DEN  CONTAINS THE INTERPOLATED DENSITY(S)
C    N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C     VTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),DEN(N),DTABLE(9,2),C(4,9)
```

```
      DATA (DTABLE(I,2),I=1,9)/
     A 998.203,992.162,983.284,971.817,957.854,916.59,
     B 864.304,799.361,712.251/
      DATA (DTABLE(1,1),I=1,9)/
     A 20.,40.,60.,80.,100.,
     B 150.,200.,250.,300./
C-----------------------------------------------------1-----
C
C    CALCULATE THE CONSTANTS FOR THE SPLINE FIT
      CALL SPLICON(DTABLE,C,9)
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
      DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
      DO 1 I=2,9
      J=I
C                COUNTER VALUE AT EXIT
      IF(T(K).LT.DTABLE(1,1)) GO TO 3
      IF(T(K).LE.DTABLE(1,I)) GO TO 2
1   IF NOT - LOOP
      CONTINUE
C-----------------------------------------------------2-----
C    THE VALUE IS OUT OF THE TOP OF THE TABLE
      CONTINUE
C
C    THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C    PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
      WRITE(6,99) T(K)
99    FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE DENSITY HAS BEEN"
     D./,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C    CARRY ON INTO THE INTERPOLATION SECTION
C
C    INTERPOLATE LINEARLY FOR I
C
      DEN(K)=(T(K)-DTABLE(J-1,1))/(DTABLE(J,1)-DTABLE(J-1,1))
      DEN(K)=DEN(K)*(DTABLE(J,2)-DTABLE(J-1,2))+DTABLE(J-1,2)
C
C    THE DENSITY IS NOW INTERPOLATED
C    CALCULATE THE NEXT VALUE
      GO TO 100
C-----------------------------------------------------2-----
C
3     CONTINUE
C
C    THE DENSITY IS NOW INTERPOLATED USING A
C    CUBIC SPLINE ROUTINE
C    SUBROUTINE SPLICON CALCULATES THE INTERPOLATION CONSTANTS
```

```
      DEN(K)=(DTABLE(J,1)-T(K))*(C(1,J-1)+(DTABLE(J,1)-T(K))**2
     A  +C(3,J-1))
      DEN(K)=DEN(K)+(T(K)-DTABLE(J-1,1))*(C(2,J-1)
     A *(T(K)-DTABLE(J-1,1))**2
     A  +C(4,J-1))
C
C    THE DENSITY HAS BEEN INTERPOLATED BY SPLINES
C    CALCULATE THE NEXT VALUE
C
C
100     CONTINUE                                        1
C-------------------------------------------------------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
        RETURN
        END
        SUBROUTINE VISCW(T,VIS,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE   FOR
C     VISCOSITY FROM A GIVEN TEMPERATURE
C     DATA IS FOR WATER
C
C     T CONTAINS THE TEMPERATURE(S)
C     VIS  CONTAINS THE INTERPOLATED VISCOSITY(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      VTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),VIS(N),VTABLE(7,2),C(4,7)
      DATA (VTABLE(I,2),I=1,7)/
     A 13.818,13.386,13.045,12.769,12.539,12.106,11.806/
      DATA (VTABLE(I,1),I=1,7)/
     A.00341,.00319,.003,.00283,.00268,
     B.00236,.00211/
C ------------------------------------------------------1-------
C    CALCULATE THE CONSTANTS FOR THE SPLINE FIT
        CALL SPLICON(VTABLE,C,7)
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
       T(K)=1./(273.15+T(K))
        DO 1 I=2,9
        J=I
C              COUNTER VALUE AT EXIT
C       IF(T(K).GT.VTABLE(I,1)) GO TO 3
        IF(T(K).GE.VTABLE(I,1)) GO TO 2
```

```
      DEN(K)=(DTABLE(J,1)-T(K))*(C(1,J-1)+(DTABLE(J,1)-T(K))**2
     A  +C(3,J-1))
      DEN(K)=DEN(K)+(T(K)-DTABLE(J-1,1))*(C(2,J-1)
     A *(T(K)-DTABLE(J-1,1))**2
     A  +C(4,J-1))
C
C    THE DENSITY HAS BEEN INTERPOLATED BY SPLINES
C    CALCULATE THE NEXT VALUE
C
C
100     CONTINUE                                        1
C-----------------------------------------------------------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
        RETURN
        END
        SUBROUTINE VISCW(T,VIS,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE   FOR
C     VISCOSITY FROM A GIVEN TEMPERATURE
C     DATA IS FOR WATER
C
C     T CONTAINS THE TEMPERATURE(S)
C     VIS  CONTAINS THE INTERPOLATED VISCOSITY(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      VTABLE  CONTAINS THE LOOK-UP TABLE
C
       DIMENSION T(N),VIS(N),VTABLE(7,2),C(4,7)
       DATA (VTABLE(I,2),I=1,7)/
      A 13.818,13.386,13.045,12.769,12.539,12.106,11.806/
       DATA (VTABLE(I,1),I=1,7)/
      A.00341,.00319,.003,.00283,.00268,
      B.00236,.00211/
C-------------------------------------------------------1--------
C    CALCULATE THE CONSTANTS FOR THE SPLINE FIT
        CALL SPLICON(VTABLE,C,7)
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
       T(K)=1./(273.15+T(K))
        DO 1 I=2,9
        J=I
C                COUNTER VALUE AT EXIT
C       IF(T(K).GT.VTABLE(1,1)) GO TO 3
        IF(T(K).GE.VTABLE(I,1)) GO TO 2
```

```
C IF NOT - LOOP
1       CONTINUE
C
C  ---------------------------------------------2-----------
C
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3       CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
        TK=1./T(K)-273.15
        WRITE(6,99) TK
99      FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2," WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE VISCOSITY HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C
C  INTERPOLATE LINEARLY FOR T
C
        VIS(K)=(T(K)-VTABLE(J-1,1))/(VTABLE(J,1)-VTABLE(J-1,1))
        VIS(K)=VIS(K)*(VTABLE(J,2)-VTABLE(J-1,2))+VTABLE(J-1,2)
       VIS(K)=EXP(VIS(K))*1.E-9
       T(K)=1./T(K)-273.15
C
C   THE VISCOSITY IS NOW INTERPOLATED
C   CALCULATE THE NEXT VALUE
        GO TO 100
C  ---------------------------------------------2-----------
C
C
2       CONTINUE
C
C   THE VISCOSITY IS NOW INTERPOLATED USING A
C   CUBIC SPLINE ROUTINE
C   SUBROUTINE SPLICON CALCULATES THE INTERPOLATION CONSTANTS
C
        VIS(K)=(VTABLE(J,1)-T(K))*(C(1,J-1)*(VTABLE(J,1)-T(K))**2
       Z  +C(3,J-1))
        VIS(K)=VIS(K)*(T(K)-VTABLE(J-1,1))*(C(2,J-1)*(T(K)-VTABLE(J-1,1
       Z ))**2+C(4,J-1))
        VIS(K)=EXP(VIS(K))*1.E-9
        T(K)=1./T(K)-273.15
C
C   THE VISCOSITY HAS BEEN INTERPOLATED BY SPLINES
C   CALCULATE THE NEXT VALUE
C
C
100     CONTINUE
C---------------------------------------------1-----------
C
C   ALL FINISHED
C   RETURN WITH INTERPOLATED VALUES
C
        RETURN
        END
```

```
      SUBROUTINE SPECIFW(T,CP,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE LINEARLY FOR
C     SPECIFIC HEAT FROM A GIVEN TEMPERATURE
C    DATA IS FOR WATER
C
C     T CONTAINS THE TEMPERATURE(S)
C     CP  CONTAINS THE INTERPOLATED SPECIFIC HEAT(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      CPTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),CP(N),CPTABLE(9,2)
      DATA (CPTABLE(I,2),I=1,9)/
     A 4183.,4179.,4185.,4198.,4219.,4320.,4510.,
     B 4870.,5650./
      DATA (CPTABLE(I,1),I=1,9)/
     A 20.,40.,60.,80.,100.,150.,200.,250.,300./
C ---------------------------------------------------1----------
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
        DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
        DO 1 I=2,9
        J=I
C                COUNTER VALUE AT EXIT
        IF(T(K).LT.CPTABLE(1,1)) GO TO 3
        IF(T(K).LE.CPTABLE(1,1)) GO TO 2
C IF NOT - LOOP
1       CONTINUE
C
C ---------------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3       CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
        WRITE(6,99) T(K)
99      FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE SPECIFIC HEAT HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C ---------------------------------------------2----------
C
2       CONTINUE
C  INTERPOLATE LINEARLY FOR CP
C
        CP(K)=(T(K)-CPTABLE(J-1,1))/(CPTABLE(J,1)-CPTABLE(J-1,1))
        CP(K)=CP(K)*(CPTABLE(J,2)-CPTABLE(J-1,2))+CPTABLE(J-1,2)
```

```
C
C    THE SPECIFIC HEAT IS NOW INTERPOLATED
C    CALCULATE THE NEXT VALUE
100     CONTINUE
C-----------------------------------------------------1-------------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
       RETURN
       END
       SUBROUTINE CONDW(T,CND,N)
C    S.I. UNIST ARE USED
C
C     SUBROUTINE TO INTERPOLATE LINEARLY FOR
C     THERMAL CONDUCTIVITY FROM A GIVEN TEMPERATURE
C    DATA IS FOR WATER
C
C     T CONTAINS THE TEMPERATURE(S)
C     CND  CONTAINS THE INTERPOLATED THERMAL CONDUCTIVITY(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      CTABLE  CONTAINS THE LOOK-UP TABLE
C
       DIMENSION T(N),CND(N),CTABLE(9,2)
       DATA (CTABLE(I,1),I=1,9)/
      A 20.,40.,60.,80.,100.,150.,200.,250.,300./
       DATA (CTABLE(I,2),I=1,9)/
      A .603,.632,.653,.670,.681,.687,.665,.616,.541/
C-----------------------------------------------------1-------------
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
       DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
       DO 1 I=2,9
       J=I
C                COUNTER VALUE AT EXIT
       IF(T(K).LT.CTABLE(1,1)) GO TO 3
       IF(T(K).LE.CTABLE(I,1)) GO TO 2
C IF NOT - LOOP
1      CONTINUE
C
C-----------------------------------------------------2-------------
C    THE VALUE IS OUT OF THE TOP OF THE TABLE
3      CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C    PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
       WRITE(6,99) T(K)
99     FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
      C " IN THE BOUNDS OF THE LOOK-UP TABLE"
```

```
     A ". THE THERMAL CONDUCTIVITY HAS BEEN"
     D,/,15," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C ---------------------------------------------2----------
C
C
2      CONTINUE
C   INTERPOLATE LINEARLY FOR K
C
        CND(K)=(T(K)-CTABLE(J-1,1))/(CTABLE(J,1)-CTABLE(J-1,1))
        CND(K)=CND(K)*(CTABLE(J,2)-CTABLE(J-1,2))+CTABLE(J-1,2)
C
C    THE THERMAL CONDUCTIVITY IS NOW INTERPOLATED
C    CALCULATE THE NEXT VALUE
100     CONTINUE
C---------------------------------------------1----------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
        RETURN
        END
        SUBROUTINE WRT(RR,VMASS,RT,T,DEN,CND,CP,VIS,RE,PR,VEL,N)
        DIMENSION RT(N),T(N),DEN(N),CND(N),CP(N),VIS(N),
       A RE(N),PR(N),VEL(N)
C
C     ROUTINE TO WRITE OUT RESULTS
C
        WRITE(6,403) RR,VMASS
403      FORMAT(1S," ROTAMETER READING IS ",F6.1,"  MASS FLOW IS ",
      AF10.4," KG/S")
        WRITE(6,404)
404    FORMAT(/,"   RESISTANCE      TEMPERATURE      DENSITY     ",
      A," CONDUCTIVITY"
      A,"     CP          VISCOSITY       REYNOLDS      PRANDTL"
      B,"      VELOCITY")
        WRITE(6,405)
405    FORMAT("       OHMS            CELSIUS          KG/M3          W/MK"
      A,"        J/KG K          KG/M S                                  ",
      A"     M/S")
        DO 5 I=1,N
        WRITE(6,402) RT(I),T(I),DEN(I),CND(I),CP(I),VIS(I),RE(I),PR(I)
      B,VEL(I)
5       CONTINUE
402    FORMAT(T2,F10.2,6X,F10.2,6X,F10.2,3X,F10.4,3X,F10.2,6X,
      AE10.4,3X,F10.2,3X,F10.2,3X,F10.2)
        RETURN
        END
        SUBROUTINE HEATLW(T,VMASS,CP,RE,PR,CND,N)
C
C      ROUTINE TO CALCULATE THE HEAT LOAD ETC.
C
C ---------------------------------------------
```

```
      DIMENSION RE(10),PR(10),CP(10),DT(10),TO(10),CPAVG(10),
     A DTL(10), REAV(10),PRAV(10), T(10),HEAT(10),AU(10)
     B ,ANU(10),ARU(10),AHU(10),CND(10),CNDAV(10)
     B ,RTO(10)
C
C ----------------------1----------------------------
C       SET LOOP TO N-1
        K=N-1
C
C       CALCULATE THE HEAT BALANCE
        DO 2 I=1,K
        CPAVG(I)=(CP(I+1)+CP(I))/2.
        DT(I)=T(I+1)-T(I)
        HEAT(I)=VMASS*CPAVG(I)*DT(I)
2       CONTINUE
C  ----------------------1----------------------------
C
C  ----------------------2----------------------
C
C   CALCULATE THE LOG MEAN TEMP DIFF
      WRITE(6,101)
101   FORMAT(//T2,"WHAT IS THE INLET HEADER RESISTANCE"
     A ," READING")
      READ(5,*)RTO(10)
      WRITE(6,104)
104   FORMAT(/" WHAT ARE THE OUTLET HEADER"
     A ," RESISTANCES (9)")
      READ(5,*)(RTO(I),I=1,9)
C     INLET OIL  1  2  3  4  5  6  7  8  9  OUTLET OIL
C
      CALL TEMP(RTO,TO,N)
      WRITE(6,106)
      WRITE(6,105)(TO(I),I=1,10)
106   FORMAT(/," THE TEMPERATURES ARE (1-9 AND INLET)")
105   FORMAT(/,10(F8.2))
      DO 3 I=1,K
      T1=TO(10)-T(I+1)
      T2=TO(I)-T(I)
      DTL(I)=(T1-T2)/(ALOG(T1/T2))
3     CONTINUE
C ----------------------2----------------------
C
C  ----------------------3----------------------
C     CALCULATE THE OVERALL HTC
      DO 4 I=1,K

C
C     THE INSIDE AREA IS 0.07116 ; RECIPROCAL = 14.0534
      AU(I)=HEAT(I)*14.0534/DTL(I)
4     CONTINUE
C  ----------------------3----------------------
C
C  ----------------------4----------------------
C     CALCULATE THE AVERAGE RE  ,PR AND CONDUCTIVITY
      DO 5 I=1,K
```

```
      REAV(I)=(RE(I+1)+RE(I))/2.
      PRAV(I)=(PR(I+1)+PR(I))/2.
      CNDAV(I)=(CND(I+1)+CND(I))/2.
5     CONTINUE
C     ----------4----------
C
C     ----------5----------
C  WRITE THIS ALL OUT
      WRITE(6,102)
102   FORMAT(//,T2,"   HEAT LOAD    DELTA T      DELTA T LM          U",
     A "          REAVG              PRAVG          AVG CONDUCTIVITY",
     B/,"           W                K                K                W/M2 K ",
     C ,"                                           W/M K")
      DO 6 I=1,K
      WRITE(6,103)HEAT(I),DT(I),DTL(I),AU(I),REAV(I),PRAV(I),CNDAV(I)
6     CONTINUE
103   FORMAT(3(F10.2,3X),3X,3(F10.2,3X),F10.4)
C     ----------5----------
C
C
C     CALCULATE THE INSIDE HTC FROM THE EQUATION OF EDE (1961)
      WRITE(6,998)
998   FORMAT(/" USING EQUATION OF EDE (1960)")
C
      DO 7 I=1,K
      ANU(I)=.026*(REAV(I)**.8)*(PRAV(I)**.4)
      AHU(I)=ANU(I)*CNDAV(I)/.025
C     THE TUBE DIAMETER IS 25 MM
      ARU(I)=1./(1./AU(I)-1./AHU(I))
7     CONTINUE
C
C     PRINT OUT THE RESULTS
      WRITE(6,109)
109   FORMAT(/,"      U                    NU          HI              "
     A ,"RESISTANCE OUTSIDE",
     B /,"   W/M2 K                                  W/M2 K          W/M2 K")
      DO 8 I=1,K
      WRITE(6,110)AU(I),ANU(I),AHU(I),ARU(I)
8      CONTINUE
110   FORMAT(4(F10.2,3X))
C
C     ----------------------------------------------------------------
C     CALCULATE THE INSIDE HTC FROM THE EQUATION OF HAUSEN (1974)
      WRITE(6,999)
999   FORMAT(/" USING EQUATION OF HAUSEN (1974)")
C
      XL=.906
C     THE TUBE LENGTH IS 906 MM
      DO 76 I=1,K
      ANU(I)=.0235*(REAV(I)**.8-230.)*(1.8*PRAV(I)**.3-.8)*
     A (1.+(.025/XL)**.666)
      XL=XL+.906
      AHU(I)=ANU(I)*CNDAV(I)/.025
      ARU(I)=1./(1./AU(I)-1./AHU(I))
76    CONTINUE
```

```
C     PRINT OUT THE RESULTS
      WRITE(6,199)
199   FORMAT(/,"       U                    NU          HI                "
     A ,"RESISTANCE OUTSIDE",
     B /,"    W/M2 K                          W/M2 K              W/M2 K")
      DO 88 I=1,K
      WRITE(6,220)AU(I),ANU(I),AHU(I),ARU(I)
88     CONTINUE
C
220   FORMAT(F10.2,3X,E10.4,3X,2(F10.2,3X))
      RETURN
      END
      SUBROUTINE HEATL(T,VMASS,CP,RE,PR,CND,DEN,VIS,N)
C
C   ROUTINE TO CALCULATE THE H.T.C.
C
      DIMENSION RE(N),PR(N),CND(N),CP(N),DEN(N),VIS(N),T(N)
C
C     ------------------------------------------------------------
C     READ THE TUBE LENGTH AND THE WALL TEMPERATURE
      WRITE(6,100)
100   FORMAT(/," GIVE THE WALL RESISTANCE THERM. READING")
      WRITE (7,*) "ENTER T(WATER),LENGTH"
      READ(5,*)RTO
      WRITE(6,101)
101   FORMAT(/," GIVE THE TUBE LENGTH (M)")
      READ(5,*)XL
      CALL TEMP(RTO,TO,1)
      WRITE(6,102)TO
102   FORMAT(/," THE WALL TEMPERATURE IS    ",F6.2)
C     ------------------------------------------------------------
C
C CALCULATE THE AVERAGE RESULTS
C
C
      CPAVG=(CP(1)+CP(2))/2.
      CNDAV=(CND(1)+CND(2))/2.
      REAV=(RE(1)+RE(2))/2.
      PRAV=(PR(1)+PR(2))/2.
C
C   THE THERMAL CONDUCTIVITY OF BRASS IS TAKEN AT 50 DEG C
C     AS 100.34 W/MK
C     THE TBE I.D. IS 12.47 MM
C              O.D.     15.88
C     ------------------------------------------------------------
C     CALCULATE THE H.T.C. ASSUMING CONSTANT CP
      X1=ALOG((TO-T(2))/(TO-T(1)))
      HI1=100.34*VMASS*CPAVG/.01247*(X1/(2.*3.14159*
     A 100.34*XL-VMASS*CPAVG*ALOG(15.88/12.47)*X1))
C     CALCULATE THE NU AND ST NUMBERS
      AST1=HI1/CPAVG/(VMASS/0.0005)
      ANU1=HI1*0.025/CNDAV
```

```
C
C  ------------------------------------------------------------------
C
C     CALCULATE THE GRASHOF NUMBER
C     BETA IS 636 E-6
C
      GR1=(9.8*636.E-6*(XL**3)*(DEN(1)**2)*(T(1)-T0))/(VIS(1)**2)
      GR2=(9.8*636.E-6*(XL**3)*(DEN(2)**2)*(T(2)-T0))/(VIS(2)**2)
      GRAV=(GR1+GR2)/2.
C
C  ------------------------------------------------------------------
C
C     WRITE THIS OUT
      WRITE(6,103)
103   FORMAT(//,"    HI              ST        NU              RE        PR        GR")
      WRITE(6,104)HI1,AST1,ANU1,REAV,PRAV,GRAV
104   FORMAT(6(E10.4))
C
C  ------------------------------------------------------------------
C
C      CALCULATE THE H.T.C ASSUMING CP(T)
C      CP 0  =1804.52     B= 3.56
      X2=(1804.52+3.56*T0)*ALOG((T(2)-T0)/(T(1)-T0))
      X3=3.56*(T(2)-T(1))+X2
      HI2=(-1.)*(VMASS*100.34*X3/0.01247)/(2.*3.14159*100.34*
     A XL-VMASS*ALOG(15.88/12.47)*X3)
      AST2=HI2/CPAVG/(VMASS/0.0005)
      ANU2=HI2*0.025/CNDAV
C
C  ------------------------------------------------------------------
C
      WRITE(6,103)
      WRITE(6,104)HI2,AST2,ANU2,REAV,PRAV,GRAV
C ***************************************************************
C WRITE RESULTS IN SEPERATE FILE FOR PGM RDGR
C***************************************************************
C105   READ (64,*)
C     IF (EOF(64)) 106,105,106
106   WRITE(64,107) T(1),T(2),T0,VMASS,HI2,ANU2,AST2,REAV,PRAV,GRAV,
     1   XL
 107  FORMAT (3(F6.3,1X),F6.4,1X,F6.2,1X,F6.2,1X,E10.4,1X,F6.1,
     1 1X,F6.1,1X,E10.4,2X,F5.3)
C  ------------------------------------------------------------------
      RETURN
      END
```

6.13.2 Program 2

```
      PROGRAM DGR(INPUT,IN,OUT,OUTPUT,TAPE5=IN,TAPE6=OUT,TAPE7
     A ,TAPE63,TAPE9,TAPE10,TAPE61=INPUT,TAPE62=OUTPUT,TAPE3)
C
C     PROGRAM TO PLOT AND CORRELATE THE DATA TAKEN FROM THE GREEN MAMBA
C     THE EXPERIMENTAL DATA IS ON TAPE7
C
      DIMENSION TIN(200),TOUT(200),TW(200),VMASS(200),HI(200),
     A  ANU(200),AST(200),RE(200),PR(200),GR(200),XL(200),VRAT(200)
      DIMENSION ANUC(200),GZ(200),PRAT(400),X(200,9),Y(200,9),NR(9)
     A ,GRL(150)
      COMMON  X,Y,NR
C
C ------------------------------------------------------------------
C     SET THE FLAGS FOR THE VARIOUS CASES
C
C  REENTRY POINT
7132   CONTINUE
       ICHI=0
C                IF ICHI=1 AND IAVG=1 THEN THE CHI2 TEST IS DONE
       IXTRA=0
C              IF IXTRA=1 THEN THE DATA OF SIEDER/TATE ETC IS INCLUDED
       IHAUSEN=0
C              IF IHAUSEN=1 THEN THE HAUSEN CORRELATION IS USED
       ICOLB=0
C               IF ICOLB=1 THEN THE COLBURN TYPE PLOT IS DONE
       IST=0
C       IF IST=1  THEN THE SIEDER TATE TYPE PLOT IS DONE
       IDEPEW=0
C             IF IDEPEW=1 THE A DEPEW/AUGUST TYPE PLOT IS DONE
       IDOUG=0
C             IF IDOUG=1 THEN MY CORRELATIONS ARE DONE
       IAVG=0
C            IF IAVG=1 THEN THE AVERAGE PERCT. ERROR IS CALCULATED
       IMET=0
C          IF IMET=1 THEN THE METAIS ECKERT PLOT IS DONE
       ILENGTH=12
C         IF ILENGTH NOT = 0 THEN NUMBERED PLOT IS DONE FOR
C           THE SPECIFIED EXCHANGER LENGTH ---
C     SEE STATEMENTS 810-890 FOR CODES (1-9)
C
C      IF ILENGTH = 12 THEN A PLOT OF ALL THE DATA IS MADE
C
C
       IKUZ=0
C            IF IKUZ = 1 THEN THE KUZNETSOVA PLOT IS DONE
C
       INR=0
C         IF INR=1 THEN A NU - RE PLOT IS DONE
C
       ING=0
C         IF ING=1 THEN A NU - GR PLOT IS DONE
C
```

```
      IPE=0
          IF IPE=1 THEN A ST - PE PLOT IS DONE
C
C
      IRG=0
          IF IRG=1 THEN A ST - RE GR PLOT IS DONE
C
C
      IRE=0
          IF IRE=1 THEN A ST - RE PLOT IS DONE
C
C
      ISG=0
          IF ISG=1 THEN A ST - GR  PLOT IS DONE
C
C
      INP=0
          IF INP=1 THEN THE NU - PE PLOT IS DONE
C
C
      IGR=0
          IF IGR=1 THEN THE NU - RE GR  PLOT IS DONE
C
C
C
C
C
C
C
C
C   READ IN SELECTION
C
752   READ(5,732)IM
732   FORMAT(A3)
      IF(IM.EQ.3HSTO)STOP
      IF(IM.EQ.3HCON)GO TO 742
      IF(IM.EQ.3HICH)ICHI=1
      IF(IM.EQ.3HIXT)IXTRA=1
      IF(IM.EQ.3HIHA)IHAUSEN=1
      IF(IM.EQ.3HICO)ICOLB=1
      IF(IM.EQ.3HIST)IST=1
      IF(IM.EQ.3HIDE)IDEFEW=1
      IF(IM.EQ.3HIDO)IDOUG=1
      IF(IM.EQ.3HIAV)IAVG=1
      IF(IM.EQ.3HIME)IMET=1
      IF(IM.EQ.3HIKU)IKUZ=1
      IF(IM.EQ.3HILE)READ(5,*)ILENGTH
      IF(IM.EQ.3HINR)INR=1
      IF(IM.EQ.3HING)ING=1
      IF(IM.EQ.3HIPE)IPE=1
      IF(IM.EQ.3HIRG)IRG=1
      IF(IM.EQ.3HIRE)IRE=1
      IF(IM.EQ.3HISG)ISG=1
      IF(IM.EQ.3HINP)INP=1
      IF(IM.EQ.3HIGR)IGR=1
      GO TO 752
C
742   CONTINUE
C
C
C-----------------------------------------------------------------
C   READ IN THE DATA
      REWIND 7
```

```
      DO 699 I=1,9
699   NR(I)=0
      N=0
      DO 1 I=1,400
      N=N+1
      READ(7,*)TIN(I),TOUT(I),TW(I),VMASS(I),HI(I),ANU(I),AST(I),RE(I),
     A  PR(I),GR(I),XL(I)
      IF(EOF(7))2,1
1     CONTINUE
2     CONTINUE
C-----------------------------------------------------------------
      N=N-1
      KKK=N
C     IF IXTRA IS 1 READ IN THE DATA
C    OF SIEDER/TATE  NORRIS/SIMMS
      IF(IXTRA.EQ.0) GO TO 89
      KK=N+1
      KKK=N+20
      DO 88 I=KK,KKK
      READ(5,*) VRAT(I),ANU(I),RE(I),PR(I),XL(I)
      VRAT(I)=1./VRAT(I)
88    CONTINUE
89    CONTINUE
C
C-----------------------------------------------------------
C
C
C     DETERMINE THE VISCOSITY RATIO
C
      CALL VISRAT(VRAT,TIN,TOUT,VMASS,XL,HI,N)
C     VRAT IS V W/V B
C
C    DETERMINE THE GREGORIG CORRECTION IF EXTRA DATA IS NOT PRESENT
      IF(IXTRA.EQ.0)CALL GRLS(PRAT,TIN,TOUT,VMASS,XL,HI,RE,N)
C
C-----------------------------------------------------------------
C     CALCULATE THE GRASHOF AND GRAETZ NO. FOR FURTHER USE
C     THE GRASHOF NUMBER BASED ON LENGTH IS GRL
C
      DO 115 I=1,KKK
      GZ(I)=.785*RE(I)*PR(I)*0.0254/XL(I)
      GRL(I)=GR(I)
      IF(IXTRA.EQ.0)GR(I)=GR(I)*(1.639E-5/(XL(I)**3))
115   CONTINUE
C
C    WRITE THESE RESULTS TO THE OUTPUT FILE
C
C
C
C      BRANCH ROUND THE FOLLOWING IF AVERAGE ERROR IS TO BE FOUND
      IF(IAVG.EQ.1)GO TO 789
C
C     BRANCH ROUND IF A NU - RE PLOT IS TO BE DONE
      IF(INR.EQ.1)GO TO 789
```

```
      DO 77 I=1,KKK
C
C  ---------------------------------------------------
C  ROUTINE TO DO CORELLATION
C
      IF(IHAUSEN.EQ.1)
     A RE(I)= 0.0184*(RE(I)**0.8-213.7)*
     B (1.8*PR(I)**0.3-0.8)*
     C (1.+(0.0254/XL(I))**0.66)*
     D (VRAT(I)**(-0.14))
      IF(IDEPEW.EQ.1)
     A RE(I)=0.023*(RE(I)**0.8)
     B *(PR(I)**0.33)*(0.3*VRAT(I)**0.3+0.45)
      IF(ICOLB.EQ.1)ANU(I)=ALOG(ANU(I)*(VRAT(I)**0.14)*
     A (PR(I)**(-0.33)))
      IF(ICOLB.EQ.1)RE(I)=ALOG(RE(I))
      IF(IST.EQ.1)RE(I)=1.488*(RE(I)**0.33)*
     A (PR(I)**0.33)*((0.0254/XL(I))**0.33)*
     B (VRAT(I)**(-0.14))*(1.+0.015*(GR(I)**0.33))
      IF(IDOUG.EQ.1)
     A RE(I)=0.0002*(RE(I)**1.42)*(PR(I)**0.3)/(VRAT(I)**0.14)
      IF(IMET.EQ.1)ANU(I)=ALOG(RE(I)/VRAT(I))
      IF(IMET.EQ.1)RE(I)=ALOG(GR(I)*PR(I)*(0.0254/XL(I)))
      IF(IKUZ.EQ.1)ANU(I)=ALOG(ANU(I)*(PR(I)**(-.4))*(VRAT(I)**.11))
      IF(IKUZ.EQ.1)RE(I)=ALOG(RE(I))
      IF(ING.EQ.1)RE(I)=((RE(I)/VRAT(I))**2.)*
     A ((XL(I)/0.0254)**0.2)*(PR(I)**0.33)
      IF(IPE.EQ.1)ANU(I)=AST(I)
      IF(IPE.EQ.1)RE(I)=RE(I)*PR(I)
      IF(IRG.EQ.1)ANU(I)=AST(I)
      IF(IRG.EQ.1)RE(I)=RE(I)*GR(I)
      IF(IRE.EQ.1)ANU(I)=AST(I)
      IF(ISG.EQ.1)ANU(I)=AST(I)
      IF(ISG.EQ.1)RE(I)=GR(I)
      IF(INP.EQ.1)RE(I)=RE(I)*  (I)
      IF(IGR.EQ.1)RE(I)=RE(I)*GR(I)
77    CONTINUE
789     CONTINUE
C
C-----------------------------------------------------------
C          DETERMINE THE AVERAGE PERCENTAGE ERROR
C          BETWEEN THE EQUATION AND THE DATA
C
      IF(IAVG.EQ.0)GO TO 799
      E2=0.
      NN=0
C                        E2 IS THE AVERAGE PERCENTAGE ERROR
      DO 999 I=1,KKK
      IF(IHAUSEN.EQ.1)ANUC(I)=0.0235*((RE(I)**0.8)-230.)*(1.8*(PR(I)
     A **0.3)-0.8)*(1.+(0.0254/XL(I))**.66)*(VRAT(I)**(-0.14))
      IF(ICHI.EQ.1)GO TO 89
```

```
      E1=100.*(ANUC(I)-ANU(I))/ANU(I)
      IF(RE(I).LE.1600..AND.XL(I).GE.1.)E2=E2+ABS(E1)
      IF(RE(I).LE.1600..AND.XL(I).GE.1.)NN=NN+1
      GO TO 999
C
891   CONTINUE
      E1=((ANU(I)-ANUC(I))**2)/ANUC(I)
      IF(RE(I).GE.1600..AND.XL(I).GE.3.)E2=E2+E1
      IF(RE(I).GE.1600..AND.XL(I).GE.3.)NN=NN+1
C
999   CONTINUE
      IF(ICHI.EQ.1)GO TO 892
      E2=E2/NN
C       THE AVERAGE PERCENTAGE ERROR IS FINISHED
      STOP
C
892   CONTINUE
      WRITE(6,*)"CHI2",E2,"POINTS",NN
       STOP
C
799   CONTINUE
C  ------------------------------------------------------------------------
      IF(ILENGTH.EQ.12)GO TO 66
      DO 10 I=1,N
810   IF (XL(I).EQ.0.906) CALL SETSUB(RE,ANU,1,I)
820   IF (XL(I).EQ.1.812) CALL SETSUB(RE,ANU,2,I)
830   IF (XL(I).EQ.2.718) CALL SETSUB(RE,ANU,3,I)
840   IF (XL(I).EQ.3.624) CALL SETSUB(RE,ANU,4,I)
850   IF (XL(I).EQ.4.530) CALL SETSUB(RE,ANU,5,I)
860   IF (XL(I).EQ.5.436) CALL SETSUB(RE,ANU,6,I)
870   IF (XL(I).EQ.6.342) CALL SETSUB(RE,ANU,7,I)
880   IF (XL(I).EQ.7.248) CALL SETSUB(RE,ANU,8,I)
890   IF (XL(I).EQ.8.154) CALL SETSUB(RE,ANU,9,I)
10    CONTINUE
66    CONTINUE
C  ========================================================================
C   NOW DO THE PLOTTING OF THE DATA
C
      CALL RPLOT(KKK,ILENGTH,RE,ANU)
      GO TO 7132
      END
      SUBROUTINE SETSUB(RE,ANU,L,I)
C     SUBROUTINE TO SET UP 2-DIMENSIONAL ARRAY CONTAINING ALL DATA
C     FOR PLOTTING SUBROUTINE, SORTED ACCORDING TO L/D RATIO.
      DIMENSION NR(9),X(200,9),Y(200,9),RE(400),ANU(400)
      COMMON  X,Y,NR
      NR(L)=NR(L)+1
      X(NR(L),L)=RE(I)
      Y(NR(L),L)=ANU(I)
      RETURN
      END
      SUBROUTINE RPLOT (N,ILENGTH,RE,ANU)
C
C   ROUTINE TO PLOT THE DATA
```

```
      COMMON X,Y,NR
      DIMENSION X(200,9),Y(200,9),ICHAR(10),RANGE(4),NINCH(2),
     1  X1(200),X2(200),X3(200),X4(200),X5(200),X6(200),X7(200),
     1  X8(200),X9(200),Y1(200),Y2(200),Y3(200),Y4(200),Y5(200),
     1  Y6(200),Y7(200),Y8(200),Y9(200),XX(3),YY(3),NR(9),
     1   RE(400),ANU(400),A(25),B(25)
      EQUIVALENCE (X(1,1),X1(1)),(X(1,2),X2(1)),(X(1,3),X3(1)),
     1           (X(1,4),X4(1)),(X(1,5),X5(1)),(X(1,6),X6(1)),
     1           (X(1,7),X7(1)),(X(1,8),X8(1)),(X(1,9),X9(1)),
     1           (Y(1,1),Y1(1)),(Y(1,2),Y2(1)),(Y(1,3),Y3(1)),
     1           (Y(1,4),Y4(1)),(Y(1,5),Y5(1)),(Y(1,6),Y6(1)),
     1           (Y(1,7),Y7(1)),(Y(1,8),Y8(1)),(Y(1,9),Y9(1))
C ---------------------------------------------------------------
C READ THE TITLE AND AXIS CHARS
      TITLE="    "
      TITLEX="       "
      TITLEY="        "
C ---------------------------------------------------------------
C     SCALE AXES FOR ALL PLOTS
C
      CALL SCALE(RE,15.,N,1)
      CALL SCALE(ANU,10.,N,1)
      XX(1)=XX(2)=RE(N+1)
      XX(3)=RE(N+2)
      YY(1)=YY(2)=ANU(N+1)
      YY(3)=ANU(N+2)
C   SET UP THE CALLING ROUTINES
      NT=10
      NX=10
      NY=10
      X1(NR(1)+1)=X2(NR(2)+1)=X3(NR(3)+1)=X4(NR(4)+1)=X5(NR(5)+1)=XX(1)
      X6(NR(6)+1)=X7(NR(7)+1)=X8(NR(8)+1)=X9(NR(9)+1)=XX(1)
      X1(NR(1)+2)=X2(NR(2)+2)=X3(NR(3)+2)=X4(NR(4)+2)=X5(NR(5)+2)=XX(2)
      X6(NR(6)+2)=X7(NR(7)+2)=X8(NR(8)+2)=X9(NR(9)+2)=XX(2)
      Y1(NR(1)+1)=Y2(NR(2)+1)=Y3(NR(3)+1)=Y4(NR(4)+1)=Y5(NR(5)+1)=YY(1)
      Y6(NR(6)+1)=Y7(NR(7)+1)=Y8(NR(8)+1)=Y9(NR(9)+1)=YY(1)
      Y1(NR(1)+2)=Y2(NR(2)+2)=Y3(NR(3)+2)=Y4(NR(4)+2)=Y5(NR(5)+2)=YY(2)
      Y6(NR(6)+2)=Y7(NR(7)+2)=Y8(NR(8)+2)=Y9(NR(9)+2)=YY(2)
C ---------------------------------------------------------------
C
      CALL PLOTS(NINCH,"RICH",10)
      IF (ILENGTH.NE.0) 50,60
50    IF(ILENGTH.NE.12) CALL NUMPLT(ILENGTH,XX,YY,RE,ANU,N)
       IF(ILENGTH.EQ.12)  CALL NUMPLT(ILENGTH,XX,YY,RE,ANU,N)
      GOTO 99
60    CONTINUE
      CALL GENPLT(XX,YY,1,3,-1,0.0,0,TITLE,NT,TITLEX,NX,TITLEY,NY,
     1           15.,10.,1,0.)
      IF (NR(1).NE.0) CALL GENPTT(X1,Y1,NR(1),1,-1,0.0,0)
      IF (NR(2).NE.0) CAL       TT(X2,Y2,NR(2),2,-1,0.0,0)
      IF (NR(3).NE.0) CALL        T(X3,Y3,NR(3),3,-1,0.0,0)
      IF (NR(4).NE.0) CALL GENPTT(X4,Y4,NR(4),4,-1,0.0,0)
      IF (NR(5).NE.0) CALL GENPTT(X5,Y5,NR(5),5,-1,0.0,0)
      IF (NR(6).NE.0) CALL GENPTT(X6,Y6,NR(6),6,-1,0.0,0)
```

```
      IF (NR(7).NE.0) CALL GENPTT(X7,Y7,NR(7),7,-1,0.0,0)
      IF (NR(8).NE.0) CALL GENPTT(X8,Y8,NR(8),8,-1,0.0,0)
      IF (NR(9).NE.0) CALL GENPTT(X9,Y9,NR(9),9,-1,0.0,0)
      GOTO 99
 99   CALL PLOT(0.,0.,999)
C-----------------------------------------------------------------------
      RETURN
      END
      SUBROUTINE NUMPLT (IL,XX,YY,RE,ANU,N)
C-----------------------------------------------------------------------
C     THIS SUBROUTINE PLOTS THE DATAPOINTS OF A DATASET WITH SPECIFIED L/D
C     RATIO USING NUMBERS TO IDENTIFY THE POINTS.
C     THE DATA IS PRINTED IN THE SAME SEQUENCE IN FILE "OUT".
C     NOTE: SYMBOLS ARE NOT SELF CENTERED !!!!!!
C-----------------------------------------------------------------------
      DIMENSION X(200,7),Y(200,9),XX(3),YY(3),NR(9)
     C ,ANU(400),RE(400)
      COMMON X,Y,NR
 20   WRITE (6,30) IL
 30   FORMAT (//,T9,"**** NUMPLOT FOR L/D RATIO CODE ",I2," ***",//,
     1T7,"POINT NO   SYMBOL       X-VALUE         Y-VALUE"/)
      PNUM=0.
      M=NR(IL)
      CALL PLOT(1.0,1.0,-3)
      CALL FACTOR(.6)
      CALL AXIS(0.0,0.0,10H              ,-10,15.,0.,XX(2),XX(3))
      CALL AXIS(0.0,0.0,6H       ,6,10.,90.,YY(2),YY(3))
      IF(IL.EQ.12)GO TO 100
      GO TO 101
100   DO 60 K=1,N
      XX(1)=(RE(K)-XX(2))/XX(3)
      YY(1)=(ANU(K)-YY(2))/YY(3)
      PNUM=PNUM+1.
      CALL NUMBER(XX(1),YY(1),.14,PNUM,0.,-1)
 60   CONTINUE
      GO TO 99
101   CONTINUE
      DO 50 L=1,M
      XX(1)=(X(L,IL)-XX(2))/XX(3)
      YY(1)=(Y(L,IL)-YY(2))/YY(3)
      PNUM=PNUM+1.
      WRITE (6,40) L,L,X(L,IL),Y(L,IL)
 40   FORMAT (T10,I3,T19,I3,T28,E10.5,T41,E10.5)
      CALL NUMBER(XX(1),YY(1),.14,PNUM,0.,-1)
 50   CONTINUE
 99   RETURN
      END
      SUBROUTINE VISC(T,VIS,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE   FOR
C     VISCOSITY FROM A GIVEN TEMPERATURE
C     DATA IS FOR REGAL OIL B
C
```

```
C     T CONTAINS THE TEMPERATURE(S)
C     VIS  CONTAINS THE INTERPOLATED VISCOSITY(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      VTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),VIS(N),VTABLE(7,2),C(4,7)
      DATA (VTABLE(I,2),I=1,7)/
     A4.626,3.57,2.753,2.135,1.656,.802,.239/
      DATA (VTABLE(I,1),I=1,7)/
     A.00341,.00319,.003,.00283,.00268,
     B.00236,.00211/
C ---------------------------------------------------1---------
C
C   CALCULATE THE CONSTANTS FOR THE SPLINE FIT
      CALL SPLICON(VTABLE,C,7)
C
C
C   SET UP LOOP FOR NUMBER OF DATA POINTS
C
      DO 100 K=1,N
C
C   FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
      T(K)=1./(273.15+T(K))
      DO 1 I=2,9
      J=I
C              COUNTER VALUE AT EXIT
      IF(T(K).GT.VTABLE(1,1)) GO TO 3
      IF(T(K).GE.VTABLE(1,1)) GO TO 2
C IF NOT - LOOP
1     CONTINUE
C
C  ---------------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3     CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
      TK=1./T(K)-273.15
      WRITE(6,99) TK
99    FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2," WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE. THE VISCOSITY HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C
C  INTERPOLATE LINEARLY FOR T
C
      VIS(K)=(T(K)-VTABLE(J-1,1))/(VTABLE(J,1)-VTABLE(J-1,1))
      VIS(K)=VIS(K)*(VTABLE(J,2)-VTABLE(J-1,2))+VTABLE(J-1,2)
      VIS(K)=EXP(VIS(K))*0.001
      T(K)=1./T(K)-273.15
C
C   THE VISCOSITY IS NOW INTERPOLATED
C   CALCULATE THE NEXT VALUE
      GO TO 100
```

```
C ------------------------------------------------------2-----
C
2       CONTINUE
C
C    THE VISCOSITY IS NOW INTERPOLATED USING A
C    CUBIC SPLINE ROUTINE
C    SUBROUTINE SPLICON CALCULATES THE INTERPOLATION CONSTANTS
C
       VIS(K)=(VTABLE(J,1)-T(K))*(C(1,J-1)*(VTABLE(J,1)-T(K))**2
      Z   +C(3,J-1))
       VIS(K)=VIS(K)+(T(K)-VTABLE(J-1,1))*(C(2,J-1)*(T(K)-VTABLE(J-1,1
      Z ))**2+C(4,J-1))
       VIS(K)=EXP(VIS(K))*0.001
       T(K)=1./T(K)-273.15
C
C    THE VISCOSITY HAS BEEN INTERPOLATED BY SPLINES
C    CALCULATE THE NEXT VALUE
C
C
100     CONTINUE
C-------------------------------------------------1-------------
C
C    ALL FINISHED
C    RETURN WITH INTERPOLATED VALUES
C
       RETURN
       END
       SUBROUTINE SPECIF(T,CP,N)
C    S.I. UNITS ARE USED
C
C     SUBROUTINE TO INTERPOLATE LINEARLY FOR
C     SPECIFIC HEAT FROM A GIVEN TEMPERATURE
C    DATA IS FOR REGAL OIL B
C
C     T CONTAINS THE TEMPERATURE(S)
C     CP  CONTAINS THE INTERPOLATED SPECIFIC HEAT(S)
C     N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C      CPTABLE  CONTAINS THE LOOK-UP TABLE
C
       DIMENSION T(N),CP(N),CPTABLE(9,2)
       DATA (CPTABLE(I,2),I=1,9)/
      A 1871.5,1946.02,2018.04,2093.4,2160.39,
      B 2343.77,2522.55,2702.58,2890.99/
       DATA (CPTABLE(I,1),I=1,9)/
      A 20.,40.,60.,80.,100.,150.,200.,250.,300./
C ------------------------------------------------------1-------
C
C    SET UP LOOP FOR NUMBER OF DATA POINTS
C
       DO 100 K=1,N
C
C    FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
       DO 1 I=2,9
       J=I
```

```
C              COUNTER VALUE AT EXIT
       IF(T(K).LT.CPTABLE(1,1)) GO TO 3
       IF(T(K).LE.CPTABLE(I,1)) GO TO 2
C IF NOT - LOOP
1      CONTINUE
C
C  ---------------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3      CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
       WRITE(6,99) T(K)
99     FORMAT(/T5,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
     C " IN THE BOUND  OF THE LOOK-UP TABLE. THE SPECIFIC HEAT HAS BEEN"
     D,/,T5," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C  ---------------------------------------------2----------
C
2      CONTINUE
C  INTERPOLATE LINEARLY FOR CP
C
       CP(K)=(T(K)-CPTABLE(J-1,1))/(CPTABLE(J,1)-CPTABLE(J-1,1))
       CP(K)=CP(K)*(CPTABLE(J,2)-CPTABLE(J-1,2))+CPTABLE(J-1,2)
C
C   THE SPECIFIC HEAT IS NOW INTERPOLATED
C   CALCULATE THE NEXT VALUE
100    CONTINUE
C-------------------------------------------------1----------
C
C   ALL FINISHED
C   RETURN WITH INTERPOLATED VALUES
C
       RETURN
       END
       SUBROUTINE VISRAT(VRAT,TIN,TOUT,VMASS,XL,HI,N)
C
C      SUBROUTINE TO CALCULATE THE VISCOSITY RATIO
C     DEFINED AS VIS WALL/ VIS BULK
C     WHERE THE BULK VISCOSITY IS ARITHMETC MEAN OF
C     THE VISCOSITY AT THE INLET AND OUTLET.
C
       DIMENSION VRAT(N),TIN(N),TOUT(N),VMASS(N),XL(N)
      A ,HI(N),T(3),V(3)
       DO 1 I=1,N
C
C     FIND THE AVERAGE SPECIFIC HEAT OF THE OIL
       T(1)=TIN(I)
       T(2)=TOUT(I)
       CALL SPECIF(T,V,2)
       CPAV=(V(1)+V(2))/2.
C
```

```
C     CALCULATE THE INSIDE WALL TEMPERATURE
C
      TWI=VMASS(I)*CPAV*(TOUT(I)-TIN(I))/(3.14*0.0254*
     A XL(I)*HI(I))+(TOUT(I)+TIN(I))/2.
C
C     CALCULATE THE VISCOSITIES
C
      T(3)=TWI
      CALL VISC(T,V,3)
      VB=(V(1)+V(2))/2.
      VRAT(I)=V(3)/VB
1     CONTINUE
C
C     ALL CALCULATIONS DONE
      RETURN
      END
       SUBROUTINE SPLICON(TABLE,C,M)
C
C     SUBROUTINE TO CALCULATE THE CONSTANTS FOR THE
C     SPLINE FIT
C     REF: PENNINGTON ,R.H
C          INTRODUCTORY COMPUTER METHODS AND NUMERICAL
C            ANALYSIS.   MAC MILLAN 1965
C
      DIMENSION TABLE(M,2),C(4,M),D(10),F(10),E(10),
     A A(10,3),B(10),Z(10),X(10),Y(10)
      DO 100 I=1,M
      X(I)=TABLE(I,1)
      Y(I)=TABLE(I,2)
100   CONTINUE
      MM=M-1
      DO 2 K=1,MM
      D(K)=X(K+1)-X(K)
      F(K)=D(K)/6.
2     E(K)=(Y(K+1)-Y(K))/D(K)
      DO 3 K=2,MM
3     B(K)=E(K)-E(K-1)
      A(1,2)=-1.-D(1)/D(2)
      A(1,3)=D(1)/D(2)
      A(2,3)=F(2)-F(1)*A(1,3)
      A(2,2)=2.*(F(1)+F(2))-F(1)*A(1,2)
      A(2,3)=A(2,3)/A(2,2)
      B(2)=B(2)/A(2,2)
      DO 4 K=3,MM
      A(K,2)=2.*(F(K-1)+F(K))-F(K-1)*A(K-1,3)
      B(K)=B(K)-F(K-1)*B(K-1)
      A(K,3)=F(K)/A(K,2)
4     B(K)=B(K)/A(K,2)
      Q=D(M-2)/D(M-1)
      A(M,1)=1.+Q+A(M-2,3)
      A(M,2)=-Q-A(M,1)*A(M-1,3)
      B(M)=B(M-2)-A(M,1)*B(M-1)
      Z(M)=B(M)/A(M,2)
      MN=M-2
```

```
      DO 6 I=1,MN
      K=M-I
6     Z(K)=B(K)-A(K,3)*Z(K+1)
      Z(1)=-A(1,2)*Z(2)-A(1,3)*Z(3)
      DO 7 K=1,MM
      Q=1./(6.*D(K))
      C(1,K)=Z(K)*Q
      C(2,K)=Z(K+1)*Q
      C(3,K)=Y(K)/D(K)-Z(K)*P(K)
7     C(4,K)=Y(K+1)/D(K)-Z(K+1)*P(K)
      RETURN
      END
      SUBROUTINE COND(T,CND,N)
C   S.I. UNIST ARE USED
C
C    SUBROUTINE TO INTERPOLATE LINEARLY FOR
C    THERMAL CONDUCTIVITY FROM A GIVEN TEMPERATURE
C   DATA IS FOR REGAL OIL B
C
C    T CONTAINS THE TEMPERATURE(S)
C    CND  CONTAINS THE INTERPOLATED THERMAL CONDUCTIVITY(S)
C    N  IS THE NUMBER OF DATA POINTS TO BE PROCESSED
C     CTABLE  CONTAINS THE LOOK-UP TABLE
C
      DIMENSION T(N),CND(N),CTABLE(9,2)
      DATA (CTABLE(I,1),I=1,9)/
     A 20.,-0.,60.,80.,100.,150.,200.,250.,300./
      DATA (CTABLE(I,2),I=1,9)/
     A  132.36,131.048,129.66,128.19,126.9,123.
     B ,119.42,115.19,112.36/
C ---------------------------------------------1--------
C
C   SET UP LOOP FOR NUMBER OF DATA POINTS
C
      DO 100 K=1,N
C
C   FIND THE POSITION OF T IN THE LOOK-UP TABLE
C
      DO 1 I=2,9
      J=I
C             COUNTER VALUE AT EXIT
      IF(T(K).LT.CTABLE(1,1)) GO TO 3
      IF(T(K).LE.CTABLE(I,1)) GO TO 2
C IF NOT - LOOP
1     CONTINUE
C
C  ---------------------------------------2----------
C  THE VALUE IS OUT OF THE TOP OF THE TABLE
3     CONTINUE
C
C THE VALUE IS NOT FOUND WITHIN THE BOUNDS OF THE TABLE
C
C  PRINT WARNING AND INTERPOLATE USING LAST 2 VALUES IN THE TABLE
```

```
      WRITE(6,99) T(K)
99    FORMAT(/15,"THE TEMPERATURE VALUE ",F10.2,"  WAS NOT FOUND"
     C " IN THE BOUNDS OF THE LOOK-UP TABLE"
     A ". THE THERMAL CONDUCTIVITY HAS BEEN"
     D,/,15," LINEARLY EXTRAPOLATED FROM EXISTING VALUES")
C
C  CARRY ON INTO THE INTERPOLATION SECTION
C ---------------------------------------------2---------
C
2     CONTINUE
C  INTERPOLATE LINEARLY FOR K
C
      CND(K)=(T(K)-CTABLE(J-1,1))/(CTABLE(J,1)-CTABLE(J-1,1))
      CND(K)=CND(K)*(CTABLE(J,2)-CTABLE(J-1,2))+CTABLE(J-1,2)
      CND(K)=CND(K)*0.001
C
C   THE THERMAL CONDUCTIVITY IS NOW INTERPOLATED
C   CALCULATE THE NEXT VALUE
100   CONTINUE
C---------------------------------------------1---------
C
C   ALL FINISHED
C   RETURN WITH INTERPOLATED VALUES
C
      RETURN
      END
      SUBROUTINE GREG(PRAT,TIN,TOUT,VMASS,XL,HI,RE,N)
C
C    ROUTINE TO CALCULATE THE CORRECTION FACTOR OF GREGORIG
C
      DIMENSION RE(N),PRAT(N),TIN(N),TOUT(N),VMASS(N),XL(N)
     A ,HI(N),T(3),CP(3),AK(3),V(3)
      DO 1 I=1,N
C
C    FIND THE AVERAGE SPECIFIC HEAT OF THE OIL
      T(1)=TIN(I)
      T(2)=TOUT(I)
      CALL SPECIF(T,V,2)
      CPAV=(V(1)+V(2))/2.
C
C    CALCULATE THE INSIDE WALL TEMPERATURE
C
      TWI=VMASS(I)*CPAV*(TOUT(I)-TIN(I))/(3.14*0.0254*
     A XL(I)*HI(I))+(TOUT(I)+TIN(I))/2.
C
C
      T(3)=TWI
      T(1)=(TIN(I)+TOUT(I))/2.
      T(2)=(T(1)+T(3))/2.
      CALL SPECIF(T,CP,3)
      CALL VISC(T,V,3)
      CALL COND(T,AK,3)
      PRF=CP(2)*V(2)/AK(2)
      PRW=CP(3)*V(3)/AK(3)
```

```
      PRB=CP(I)*V(I)/AK(I)
      XIPR=(PRF-PRW)/(PRB-PRW)-0.5
      DENM=(PRB**0.05)*(RE(I)**0.02)*((0.5-XIPR)**0.01)
      XNUM=((0.5-XIPR)**0.2)*0.264+0.0757
      XP=XNUM/DENM
      PRAT(I)=(PRB/PRW)**XP
1     CONTINUE
C
C   ALL CALCULATIONS DONE
      RETURN
      END
      SUBROUTINE REG(X,Y,N)
      DIMENSION X(120),Y(120)
      XBAR=0.
      YBAR=0.
      XX=0.
      YY=0.
      XY=0.
      DO 1 I=1,N
      XX=XX+X(I)*X(I)
      YY=YY+Y(I)*Y(I)
      XY=XY+X(I)*Y(I)
      XBAR=XBAR+X(I)
      YBAR=YBAR+Y(I)
1     CONTINUE
      WRITE(6,*)N," XX",XX,"YY",YY
      WRITE(6,*)"XBAR",XBAR,"YBAR",YBAR
      C=FLOAT(N)
      XBAR=XBAR/C
      YBAR=YBAR/C
      SX=XX-C*XBAR*XBAR
      SY=YY-C*YBAR*YBAR
      SXY=XY-C*XBAR*YBAR
      WRITE(6,*)"SX",SX,"SY",SY,"SXY",SXY
      R=SXY/SQRT(SX*SY)
      A=SXY/SX
      B=YBAR-A*XBAR
      WRITE(6,*)"  R  ",R,"  A  ",A,"  B  ",B
      STOP
      END
```

```
      PRB=CP(1)*V(1)/AK(1)
      XIPR=(PRF-PRW)/(PRB-PRW)-0.5
      DENM=(PRB**0.03)*(RE(I)**0.02)*((0.5-XIPR)**0.01)
      XNUM=((0.5-XIPR)**0.2)*0.264+0.0757
      XP=XNUM/DENM
      PRAT(I)=(PRB/PRW)**XP
1     CONTINUE
C
C   ALL CALCULATIONS DONE
      RETURN
      END
      SUBROUTINE REG(X,Y,N)
      DIMENSION X(120),Y(120)
      XBAR=0.
      YBAR=0.
      XX=0.
      YY=0.
      XY=0.
      DO 1 I=1,N
      XX=XX+X(I)*X(I)
      YY=YY+Y(I)*Y(I)
      XY=XY+X(I)*Y(I)
      XBAR=XBAR+X(I)
      YBAR=YBAR+Y(I)
1     CONTINUE
      WRITE(6,*)N," XX",XX,"YY",YY
      WRITE(6,*)"XBAR",XBAR,"YBAR",YBAR
      C=FLOAT(N)
      XBAR=XBAR/C
      YBAR=YBAR/C
      SX=XX-C*XBAR*XBAR
      SY=YY-C*YBAR*YBAR
      SXY=XY-C*XBAR*YBAR
      WRITE(6,*)"SX",SX,"SY",SY,"SXY",SXY
      R=SXY/SQRT(SX*SY)
      A=SXY/SX
      B=YBAR-A*XBAR
      WRITE(6,*)"  R  ",R,"  A  ",A,"  B  ",B
      STOP
      END
```

6.14 PHYSICAL PROPERTIES

6.14.1 Regal oil B (R and O)

TABLE 9 Physical properties

θ °C	C J/kgK	λ W/mK x 10^3	η kg/ms x 10^3	ρ kg/m^3	Pr
20	1871,5	132,36	102,1	872,5	1443,64
40	1946,02	131,048	35,53	862,1	527,61
60	2018,04	129,66	15,69	848,0	244,2
80	2093,40	128,19	8,46	837,8	138,13
100	2160,39	126,90	5,24	825,0	89,21
150	2343,77	123,00	2,23	799,8	42,49
200	2522,55	119,42	1,27	772,5	26,83
250	2702,58	115,91		750,1	
300	2890,99	112,36		727,1	

C_o = 1804,52 J/kgK, b = 3,56 J/kgK^2

$$\beta = \frac{1}{V}\left[\frac{\partial V}{\partial T}\right]_P = 636 \times 10^{-6}/\ ^oC$$

6.14.2 Water

Physical properties for water were taken from "Thermodynamic and Transport Properties of Fluids", Y R Mayhew and G F C Rogers, Oxford, (1973).

6.15 FIGURES

Figure 1 Schematic layout of the experimental rig
Figure 2 Assembled section of the heat exchanger
Figure 3 Item 2 of Figure 2
Figure 4 Item 3 of Figure 2
Figure 5 Item 4 of Figure 2
Figure 6 Item 5 of Figure 2
Figure 7 Item 6 of Figure 2
Figure 8 Item 7 of Figure 2
Figure 9 Connecting piece
Figure 10 Item 1 of Figure 9
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Figure 16 Manometer
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Figure 29 Lower transitional data as a function of the Stanton and Reynolds groups
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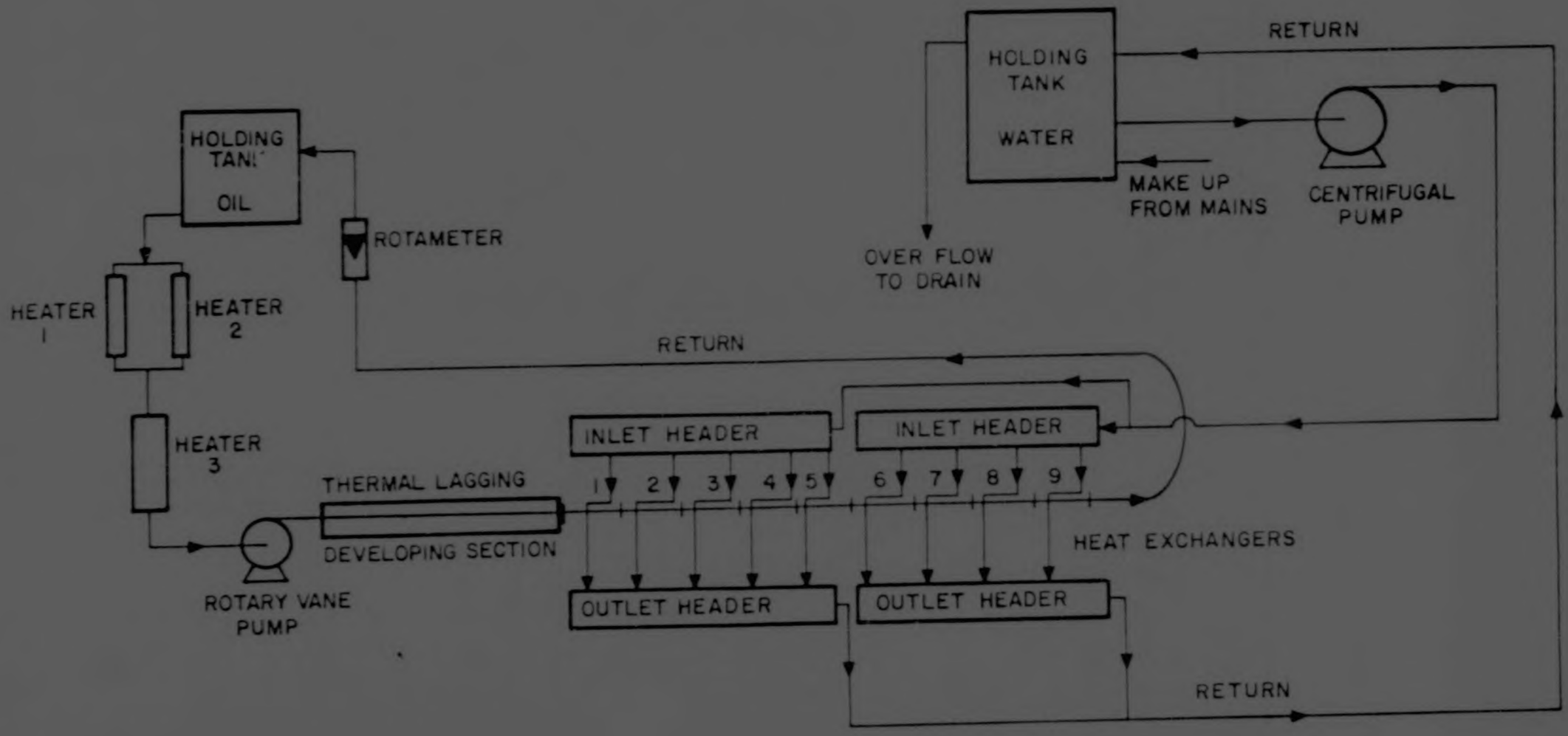


FIGURE 1 Schematic layout of the experimental rig

FIGURE 2 Assembled section of the heat exchanger

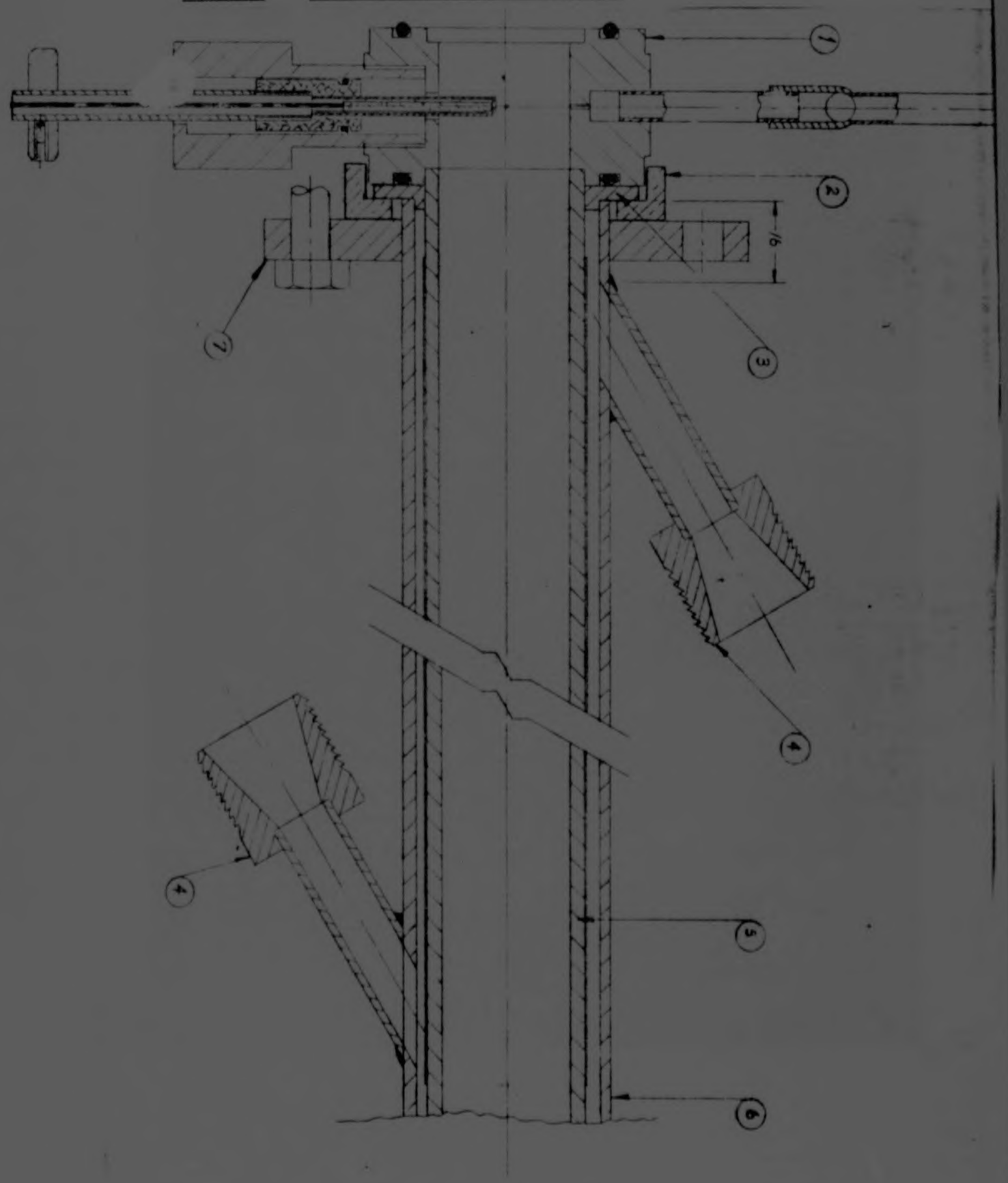


FIGURE 3 Item 2 of Figure 2

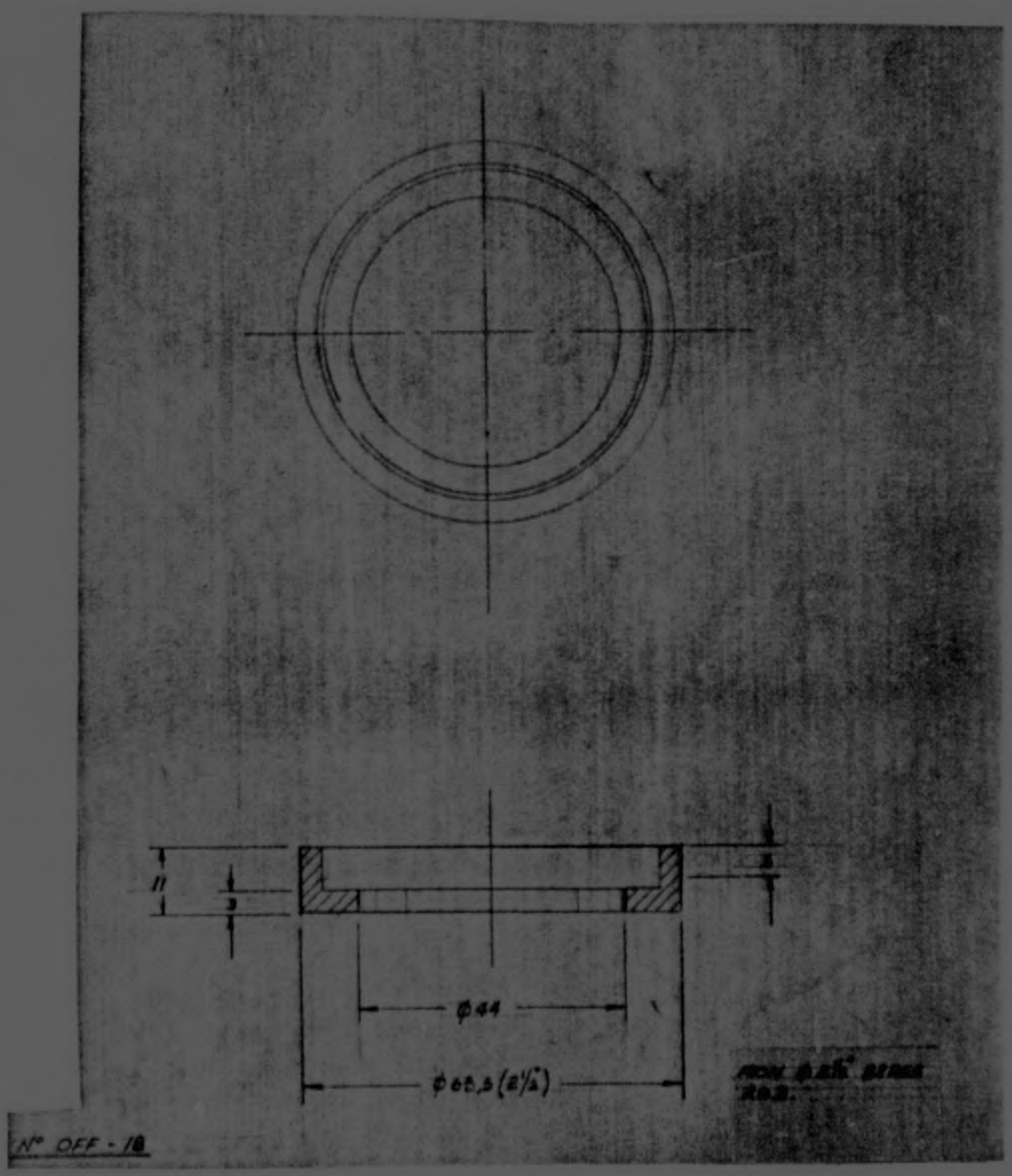


FIGURE 4 Item 3 of Figure 2

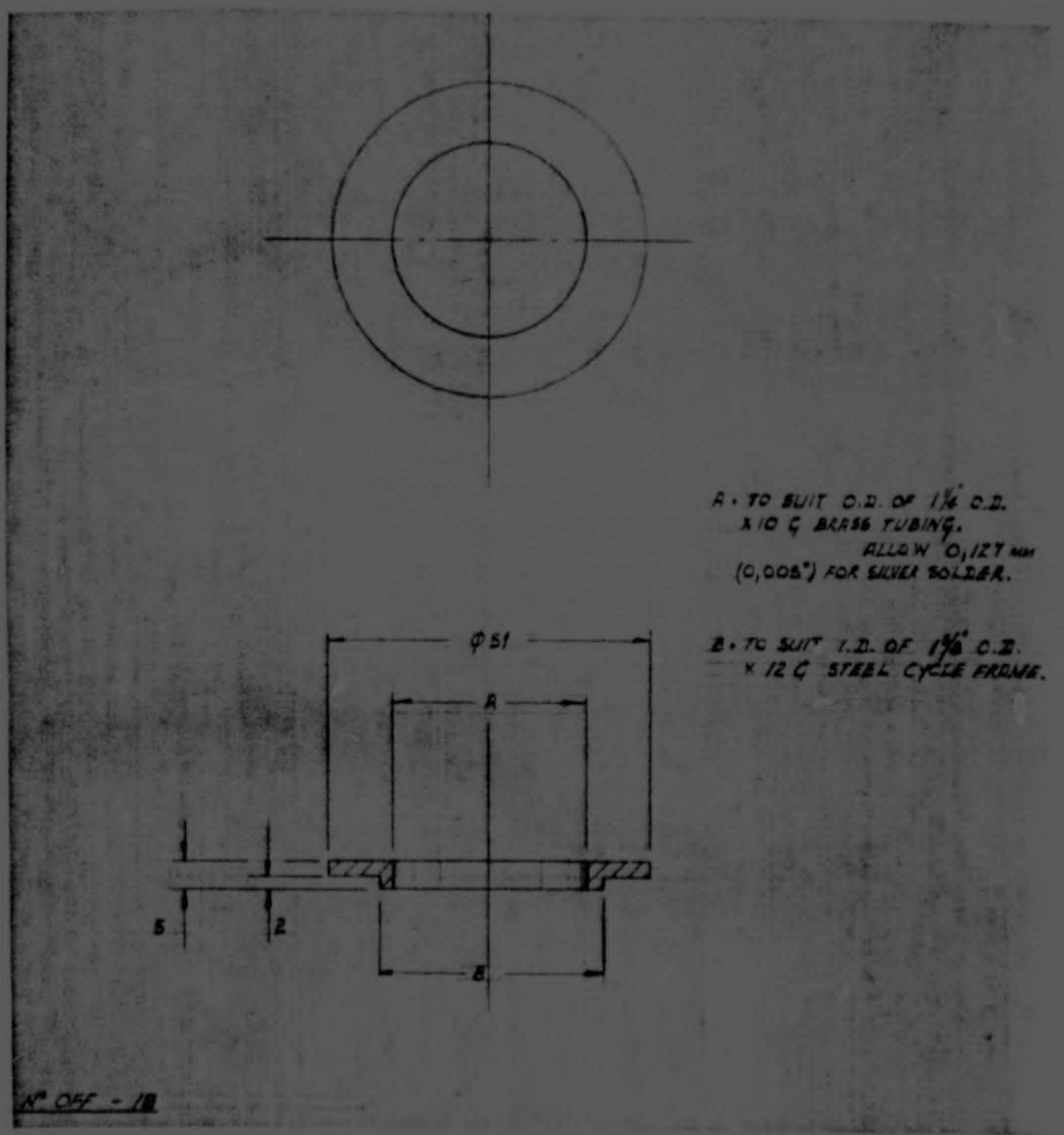
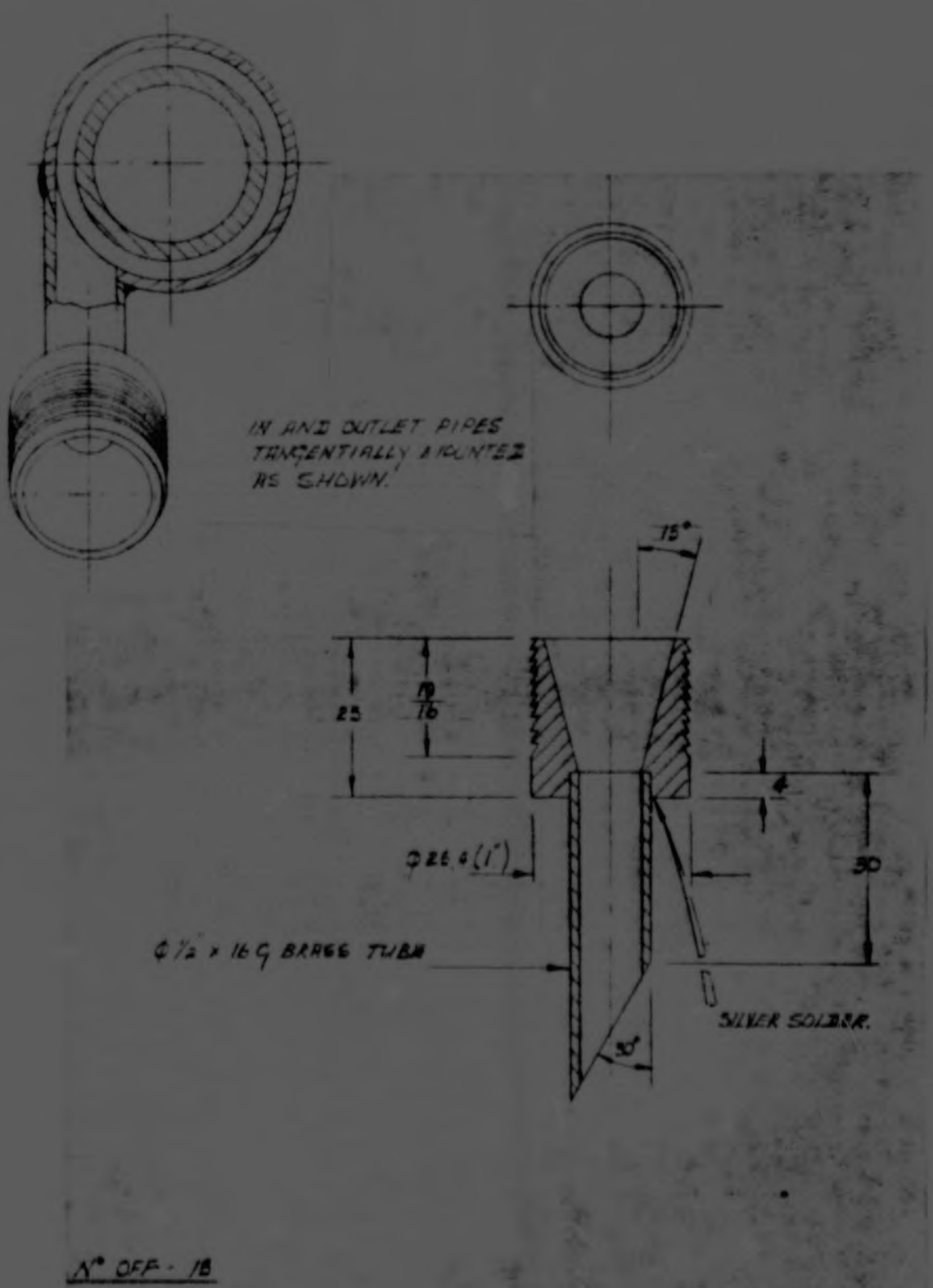


FIGURE 5 Item 4 of Figure 2



4° OFF - 18

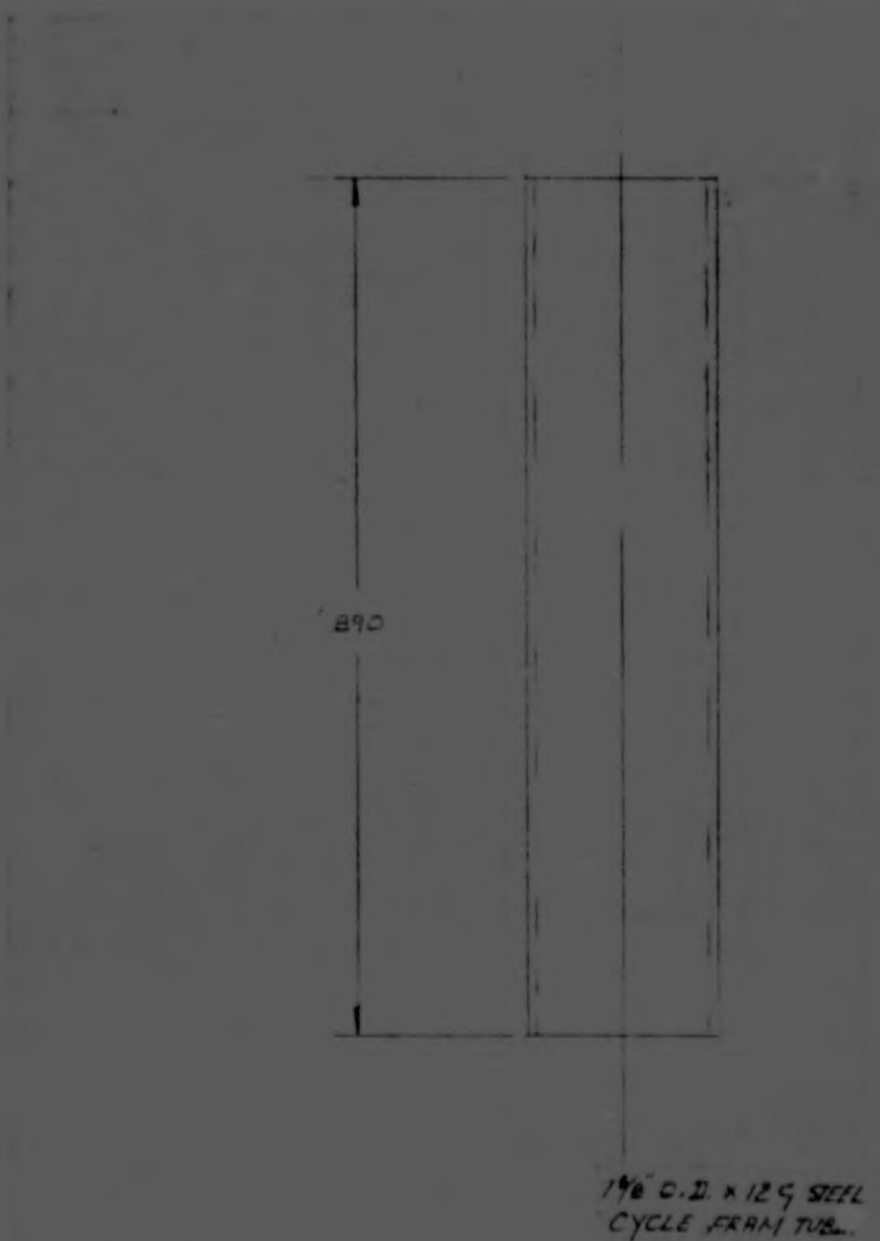
FIGURE 6 Item 5 of Figure 2



1¼" O.D. x 10G
BRASS TUBING

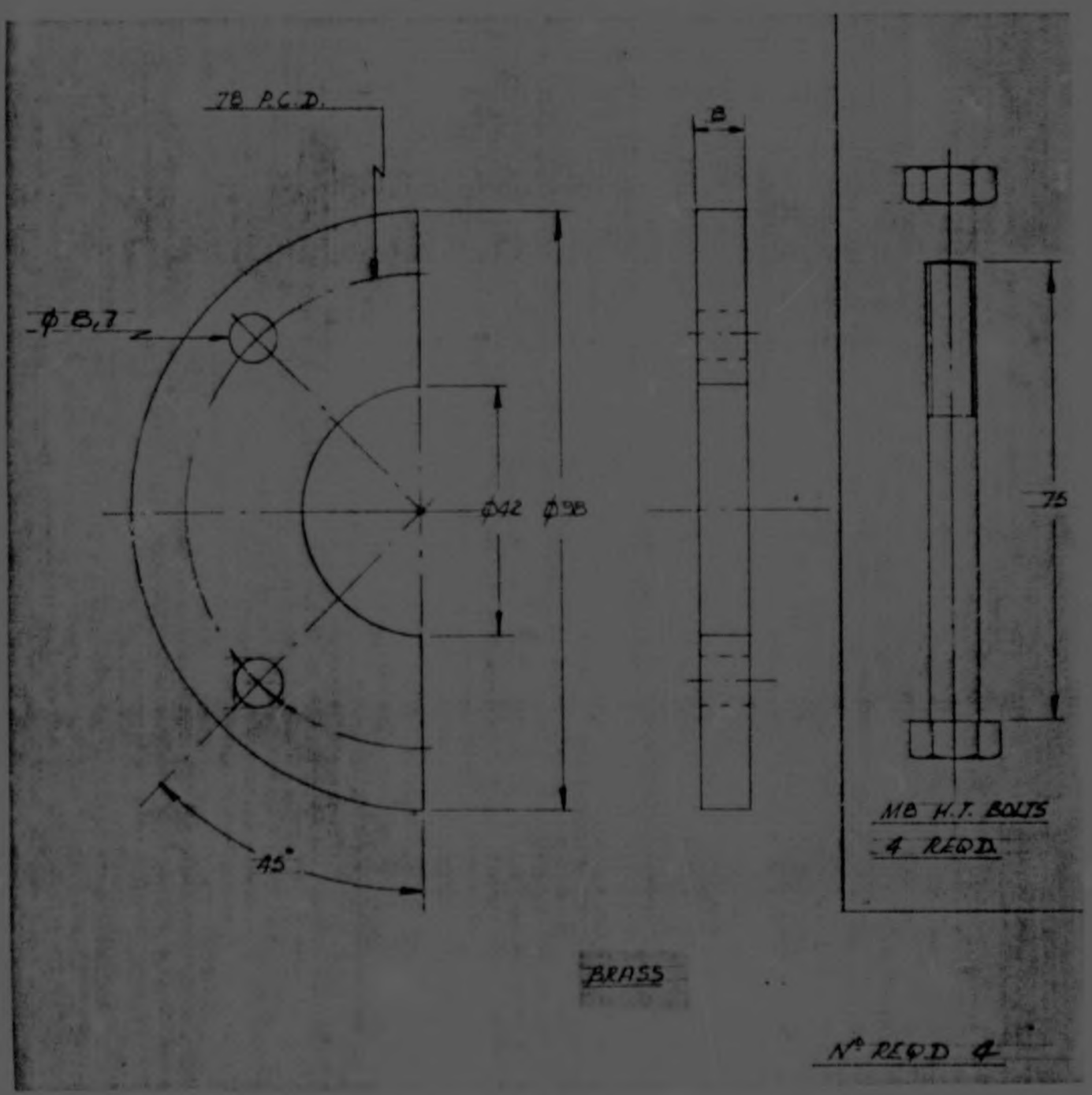
N° OFF - 9

FIGURE 7 Item 6 of Figure 2



Nº OFF : 2

FIGURE 8 Item 7 of Figure 2



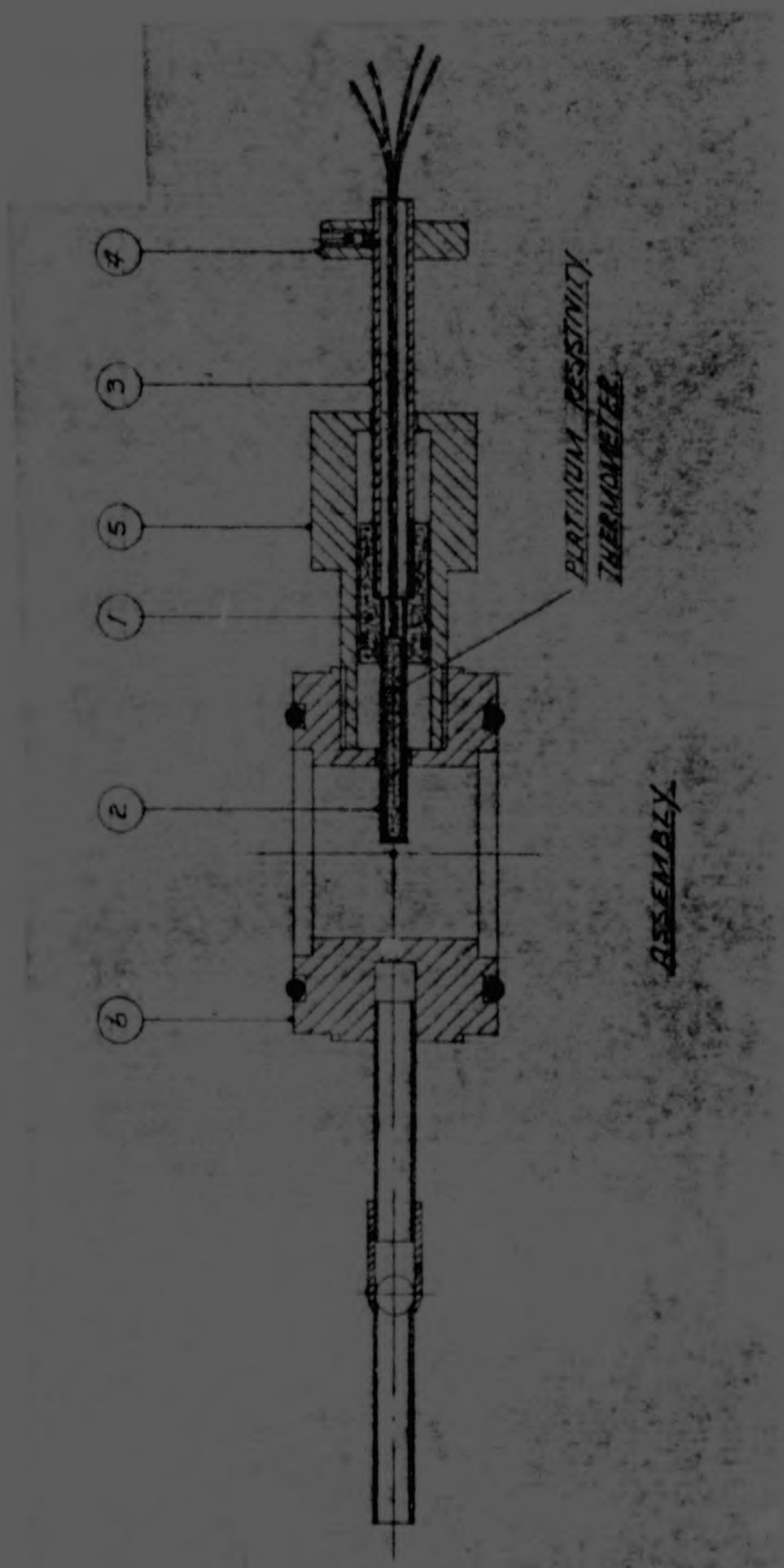
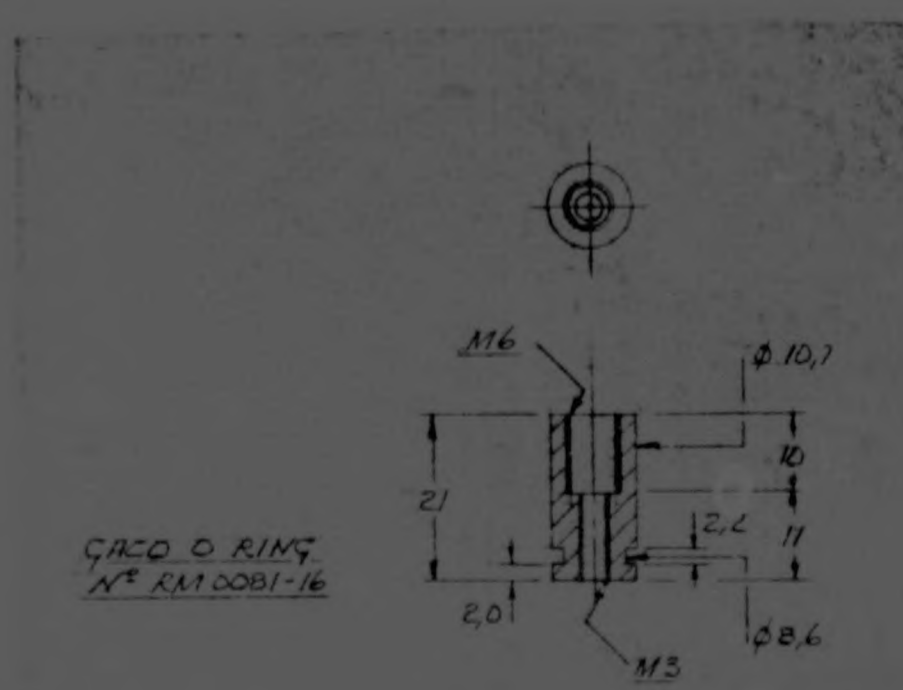


FIGURE 9 Connecting piece

FIGURE 10 Item 1 of Figure 9



GACO O RING
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PAPER BASE.

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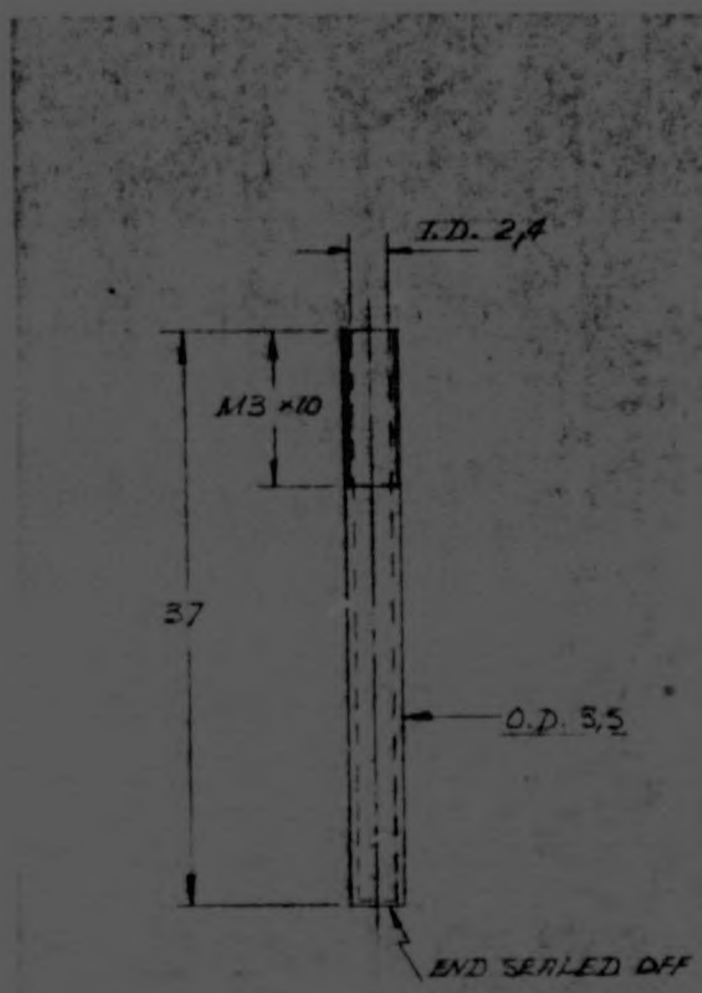
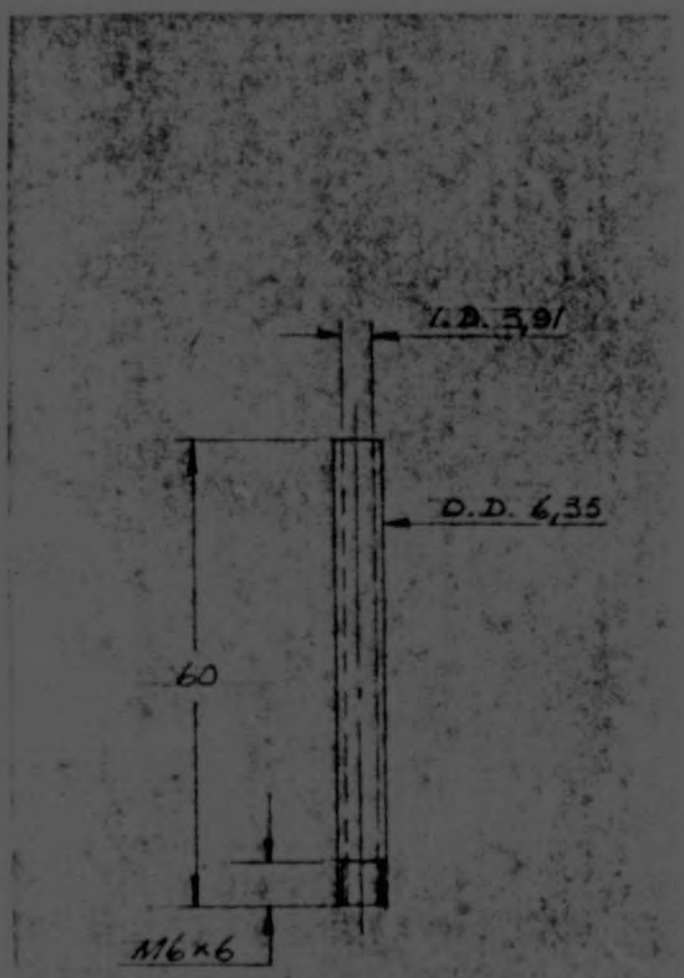
FIGURE 11 Item 2 of Figure 9



PART N° 2 OF ITEM 1
MAT. COPPER
SCALE 2:1

FIGURE 12 Item 3 of Figure 9



PART Nº 3 OF ITEM 1
MAT. BRASS
SCALE 1:1

FIGURE 13 Item 4 of Figure 9

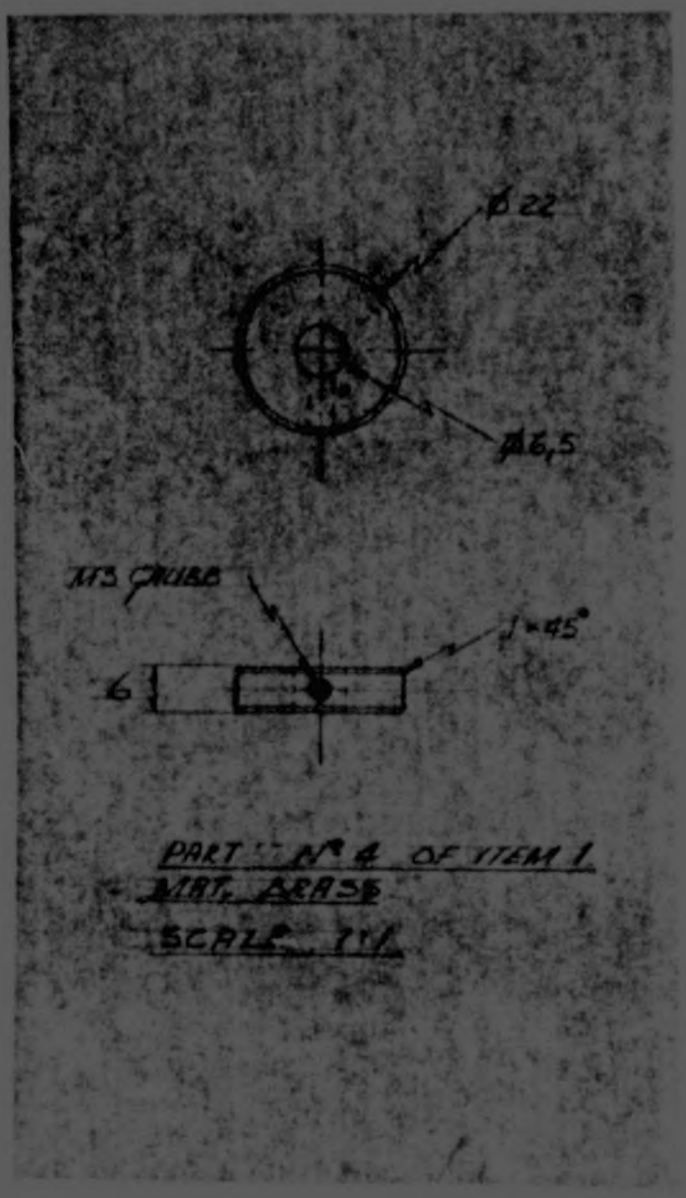
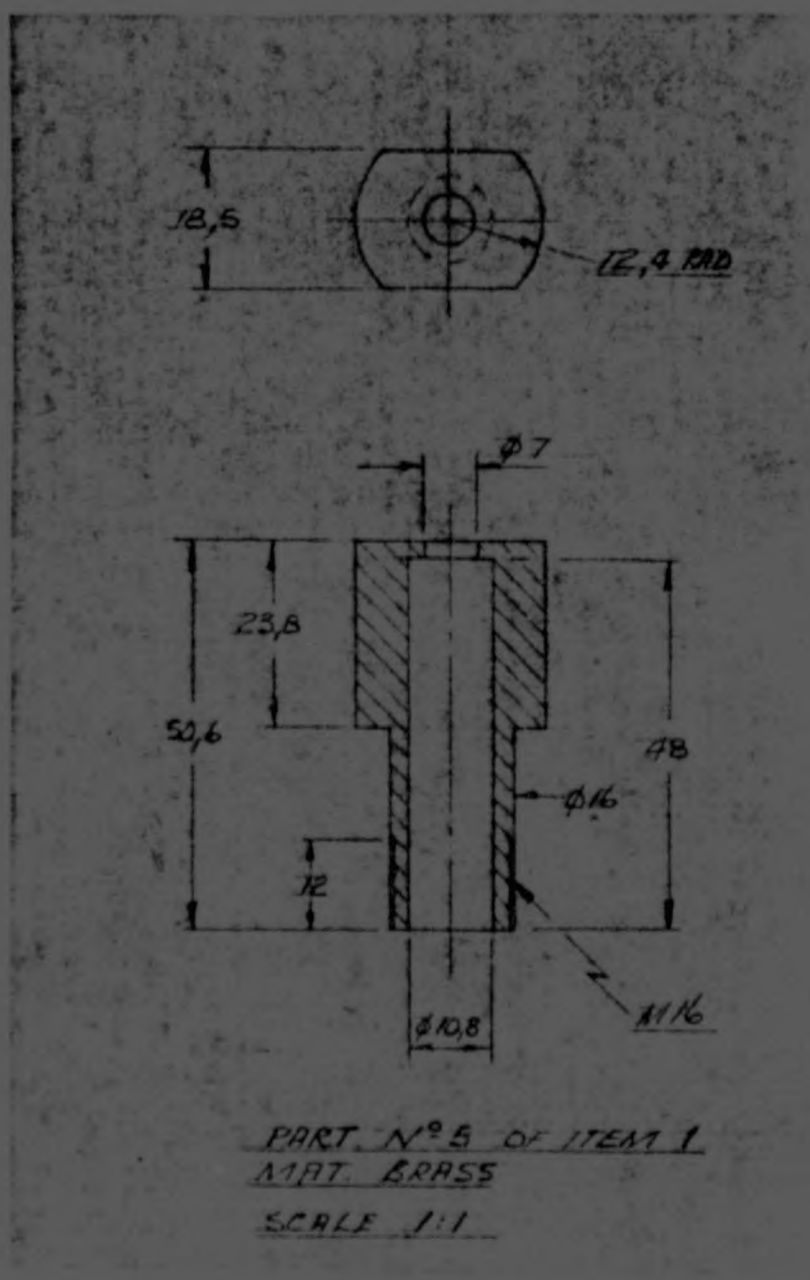


FIGURE 14 Item 5 of Figure 9



PART Nº 5 OF ITEM 1
MAT. BRASS
SCALE 1:1

FIGURE 15 Item 6 of Figure 9

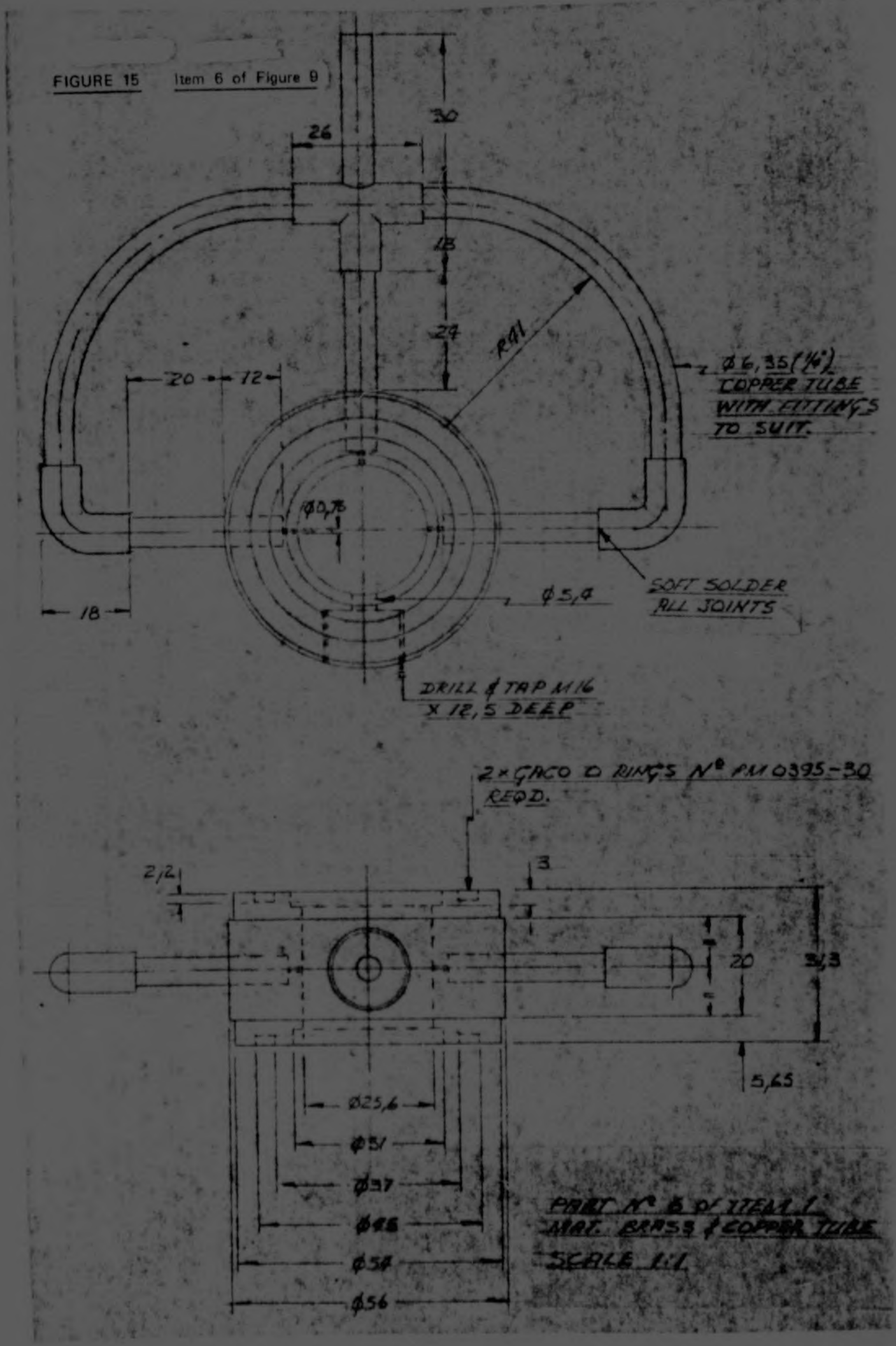


FIGURE 16a Manometer

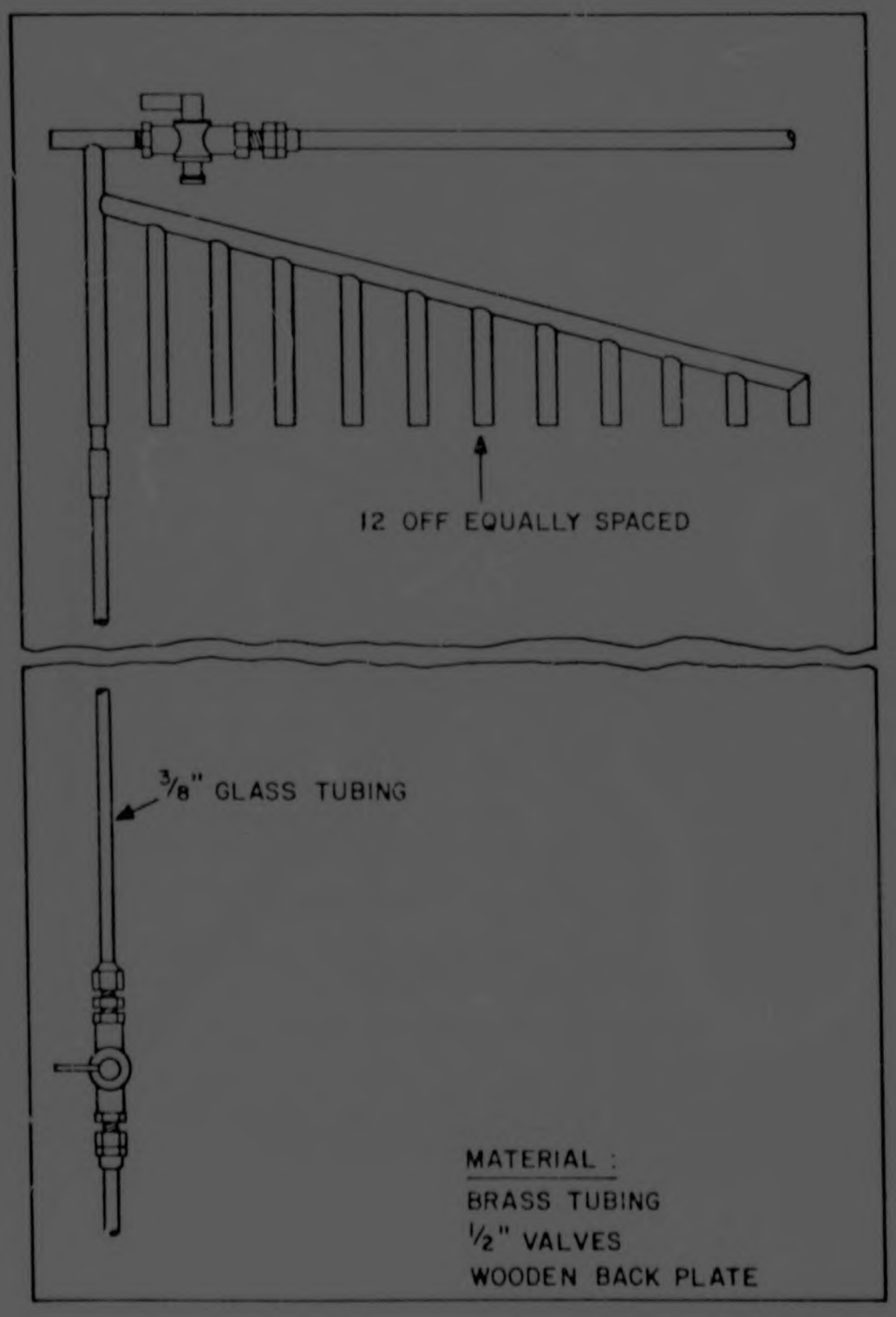


FIGURE 16b Manometer

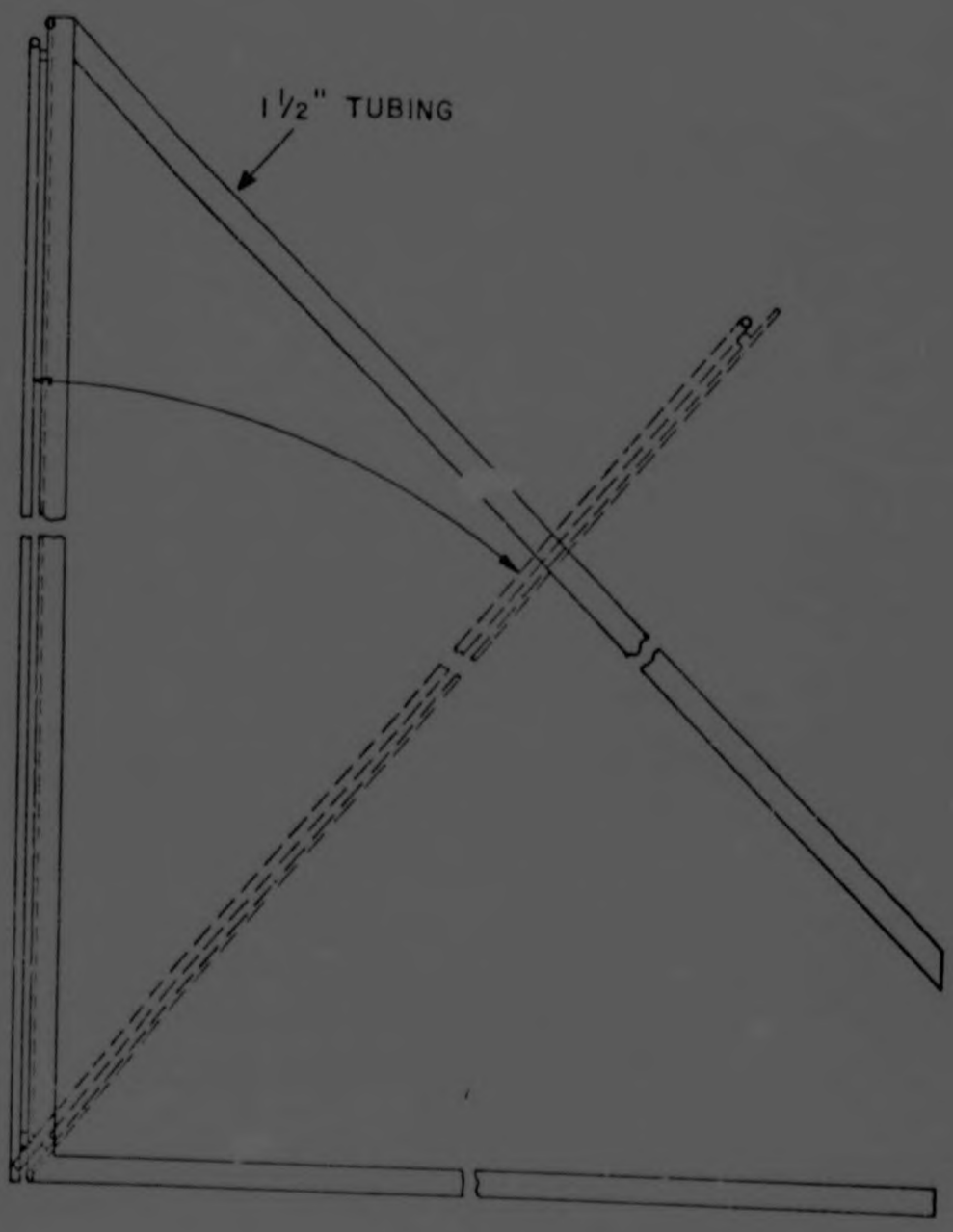
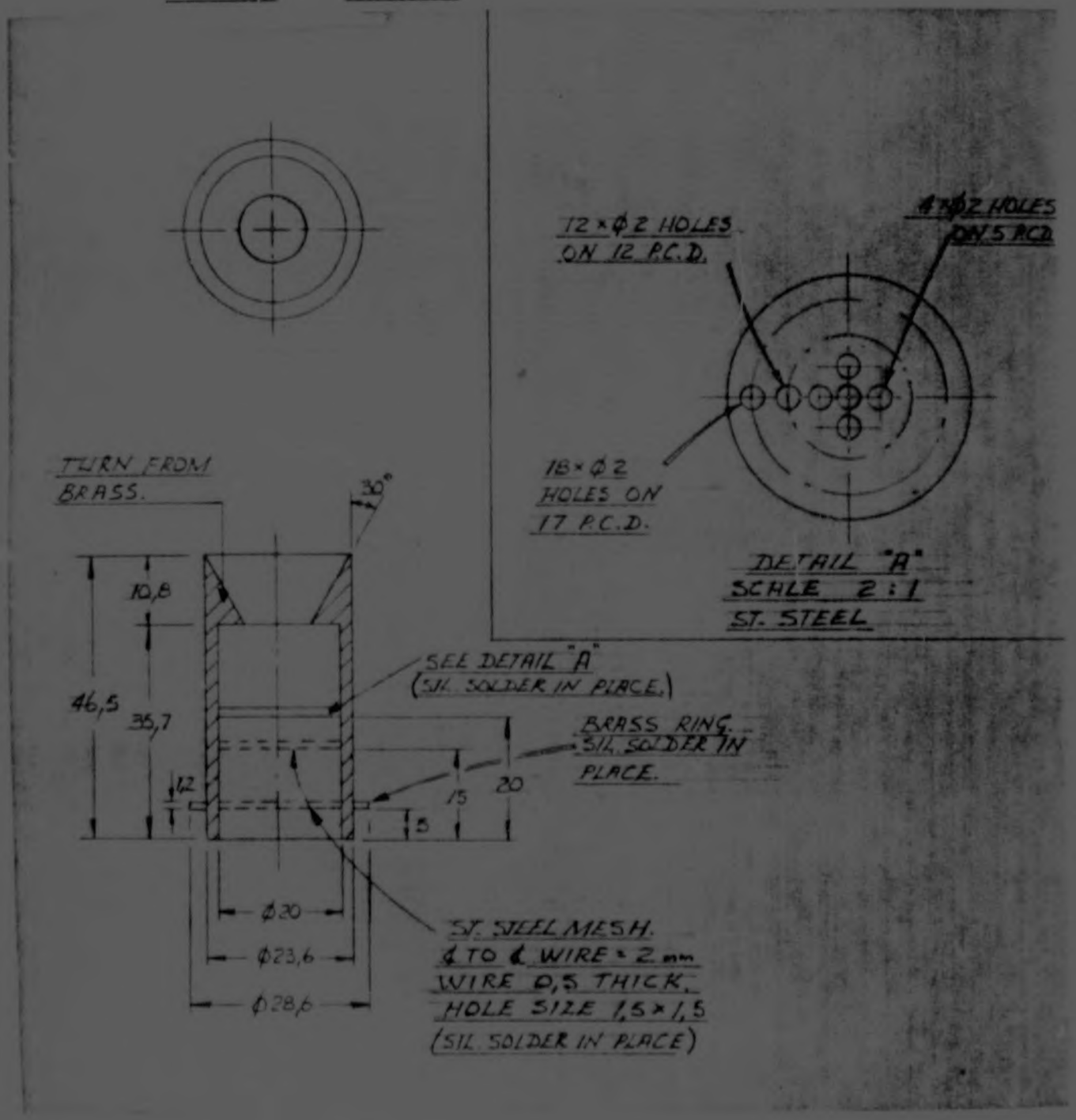
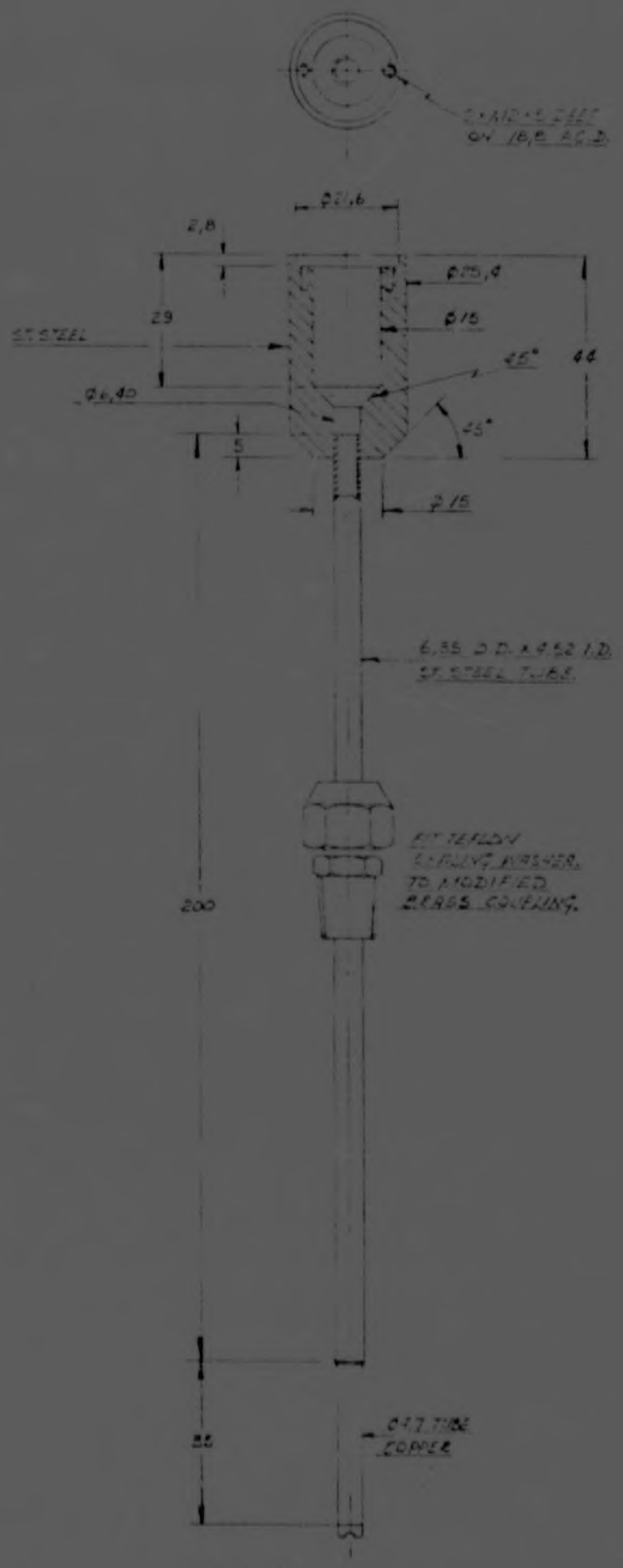


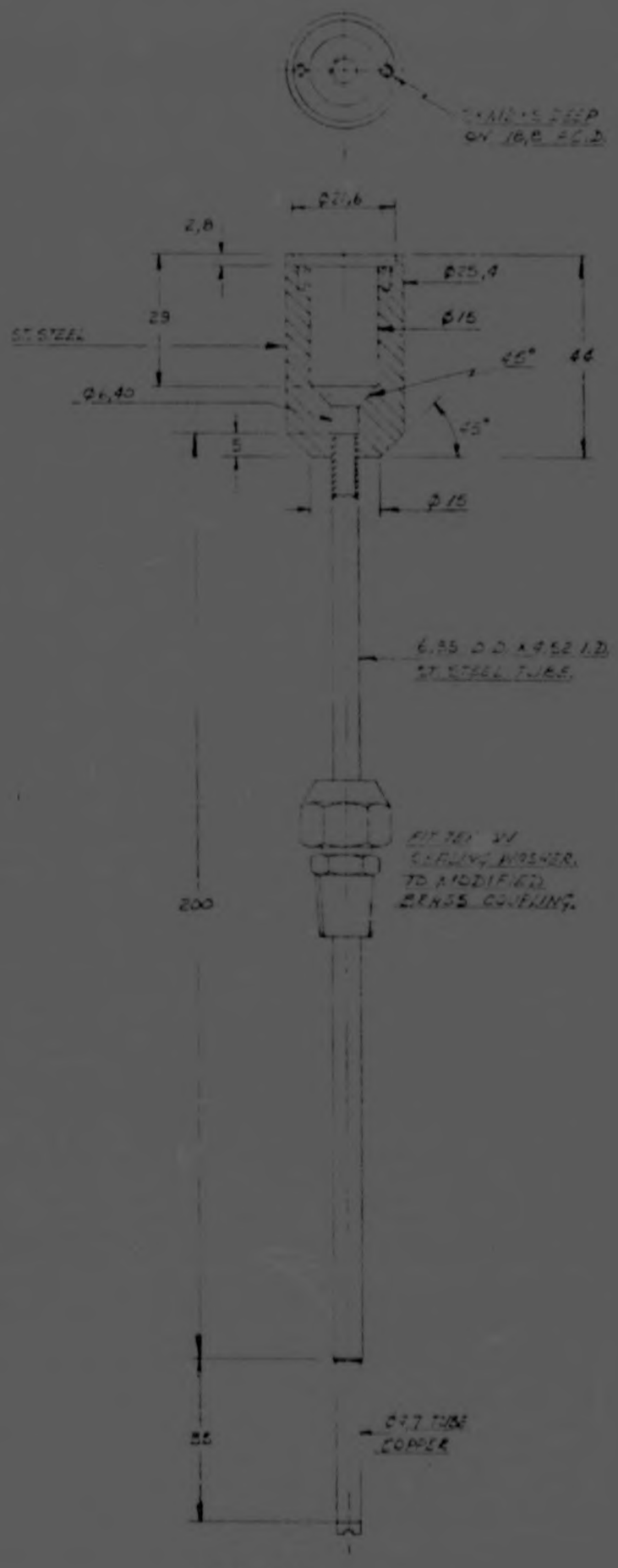
FIGURE 17 Static mixer





SCALE 1:1

FIGURE 18 Temperature probe for water temperature in the inlet header



SCALE 1:1

FIGURE 18 Temperature probe for water temperature in the inlet header

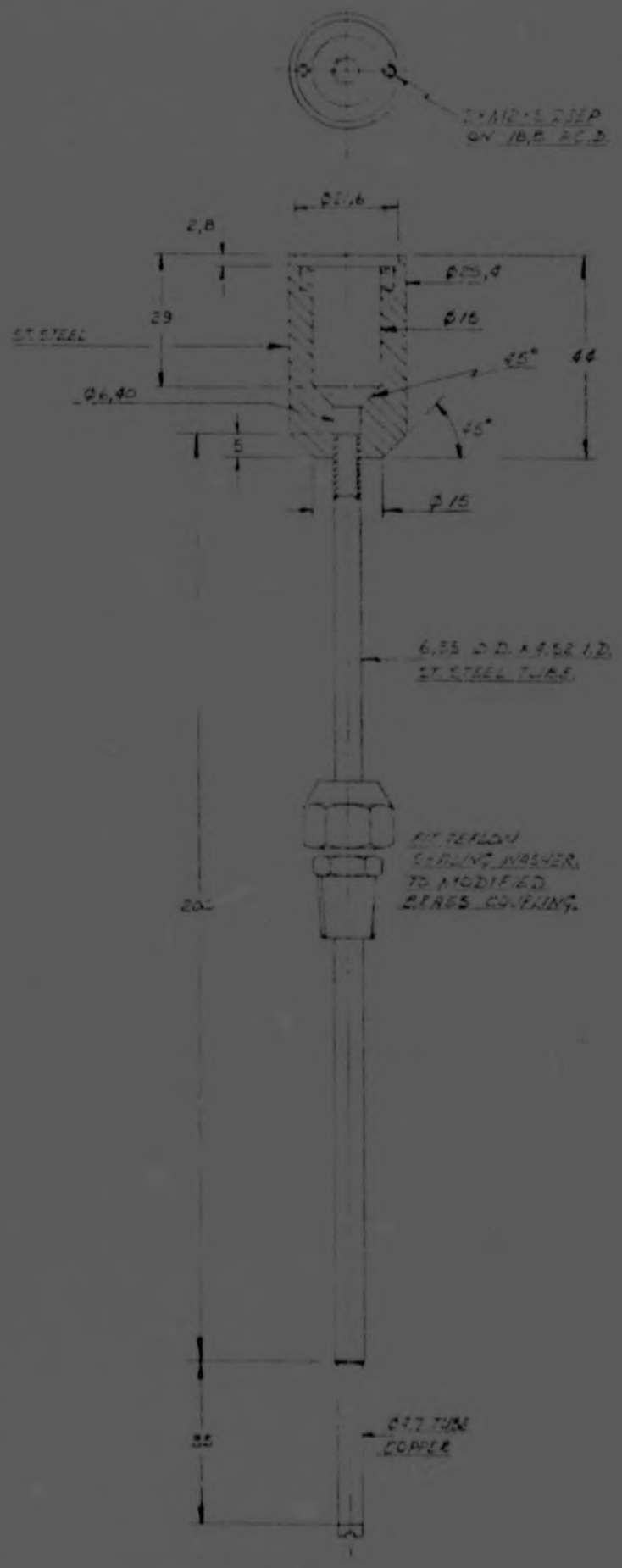
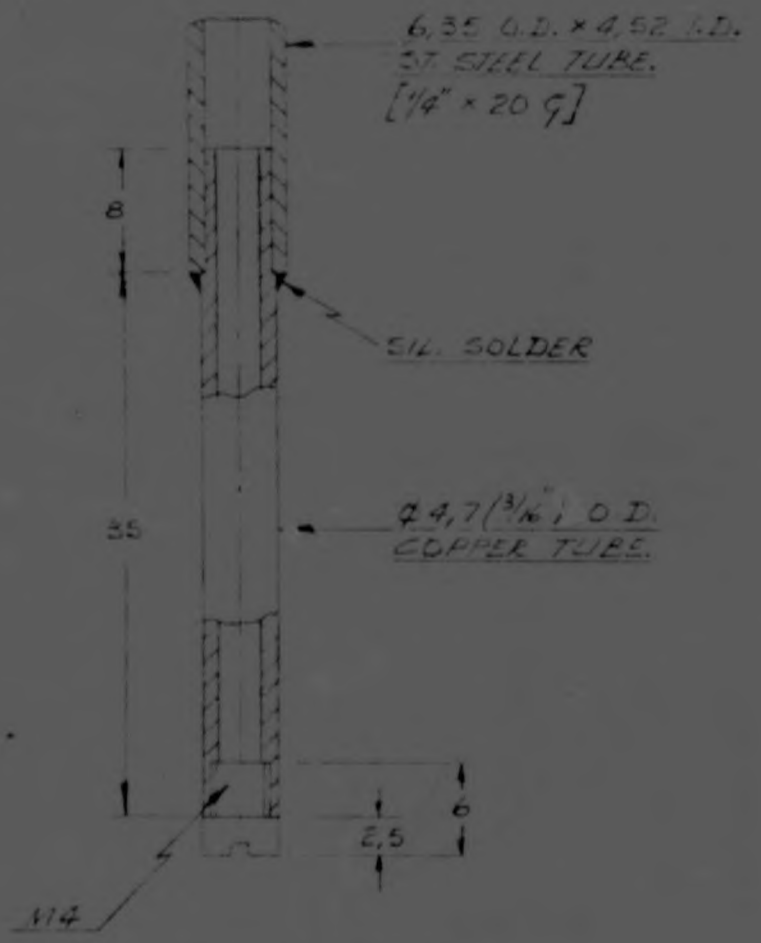


FIGURE 18 Temperature probe for water temperature in the inlet header

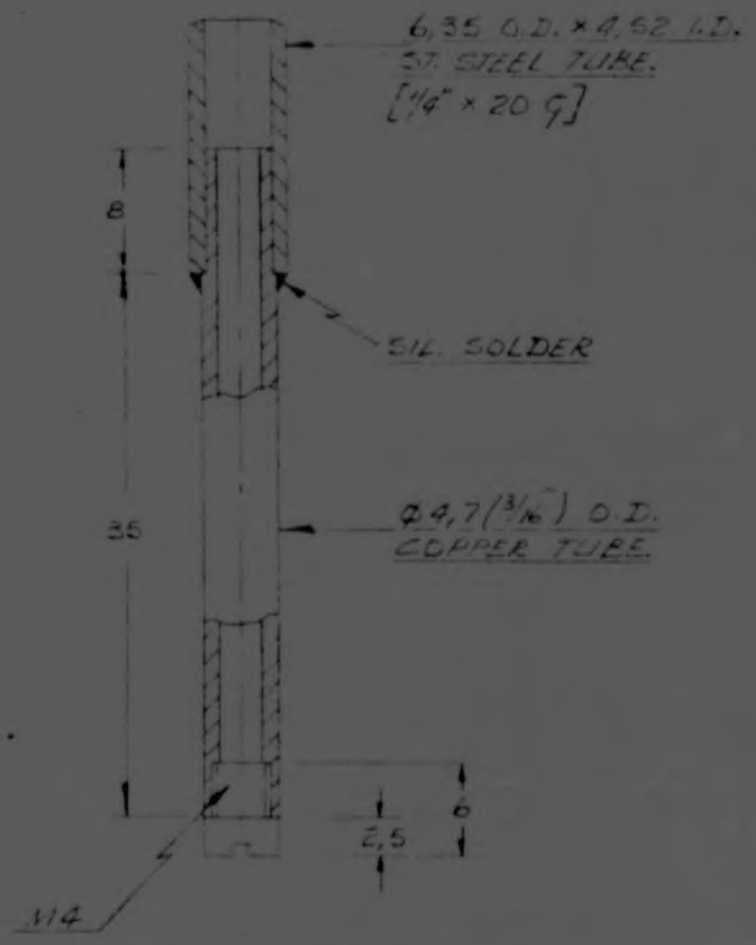
FIGURE 19 Detail of Figure 18



THERMO POCKET
DETAIL.

SCALE 2:1

FIGURE 19 Detail of Figure 18



THERMO POCKET
DETAIL.

SCALE 2:1

FIGURE 20 Detail of Figure 18

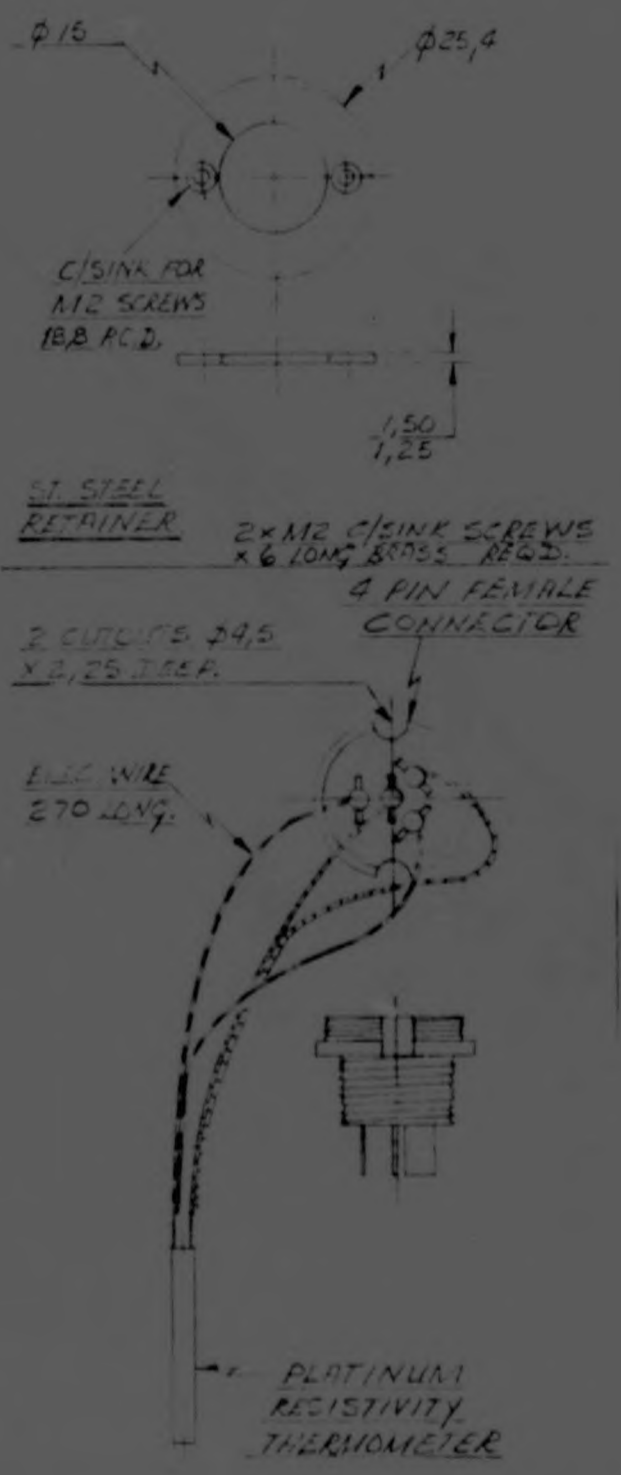


FIGURE 21 Data plotted against the Hausen equation

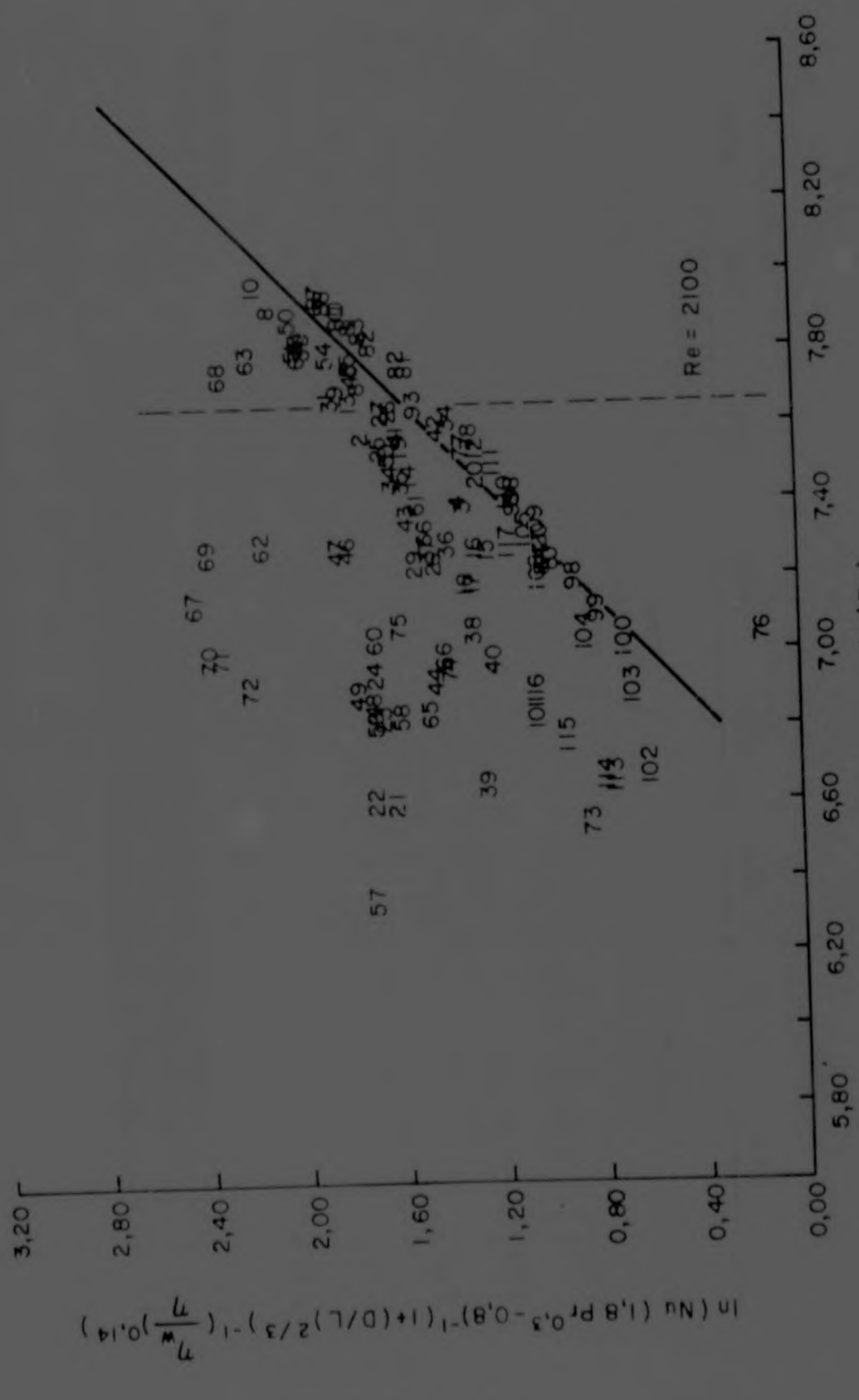


FIGURE 22 Metais and Eckert plot of the data

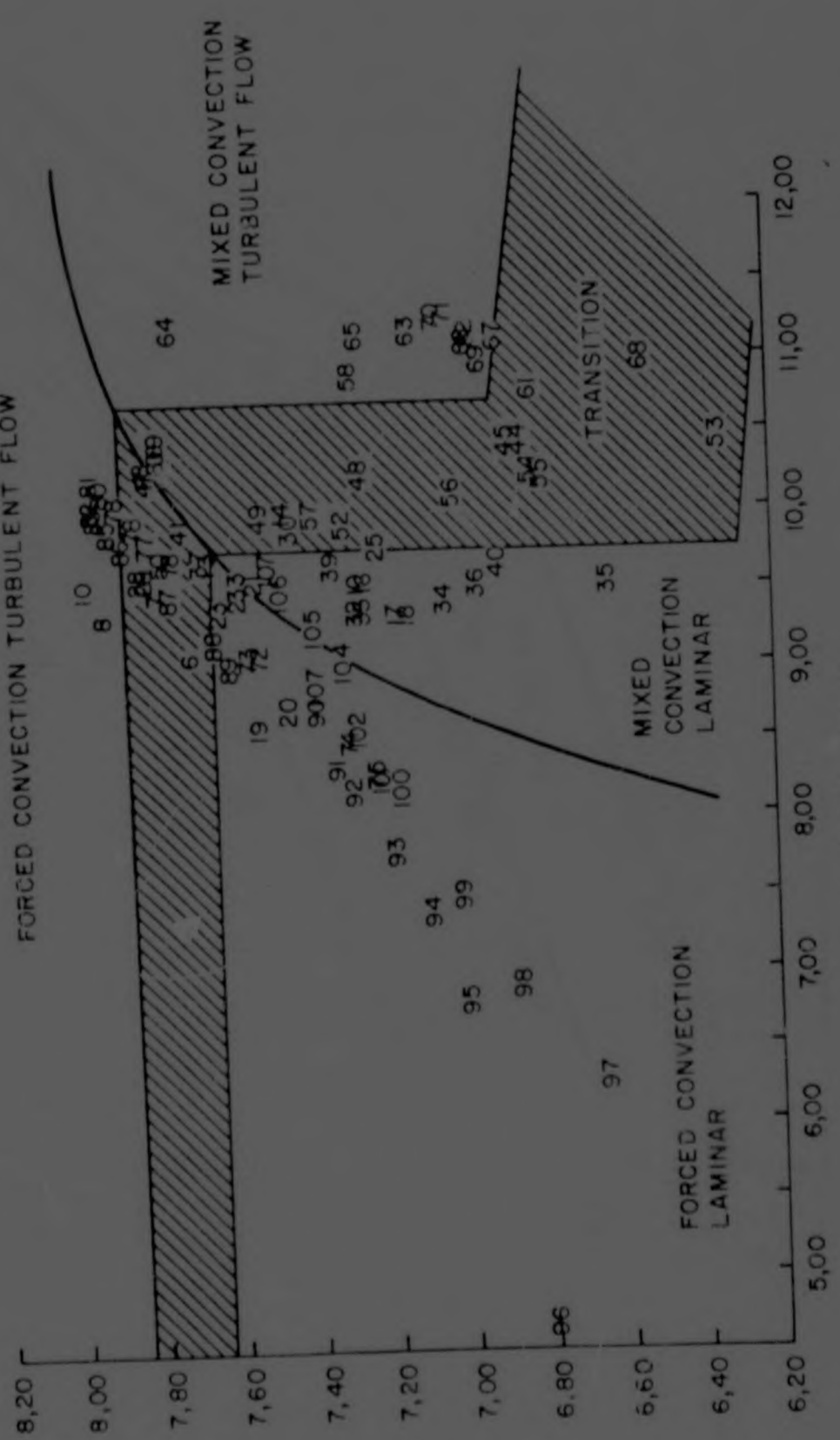
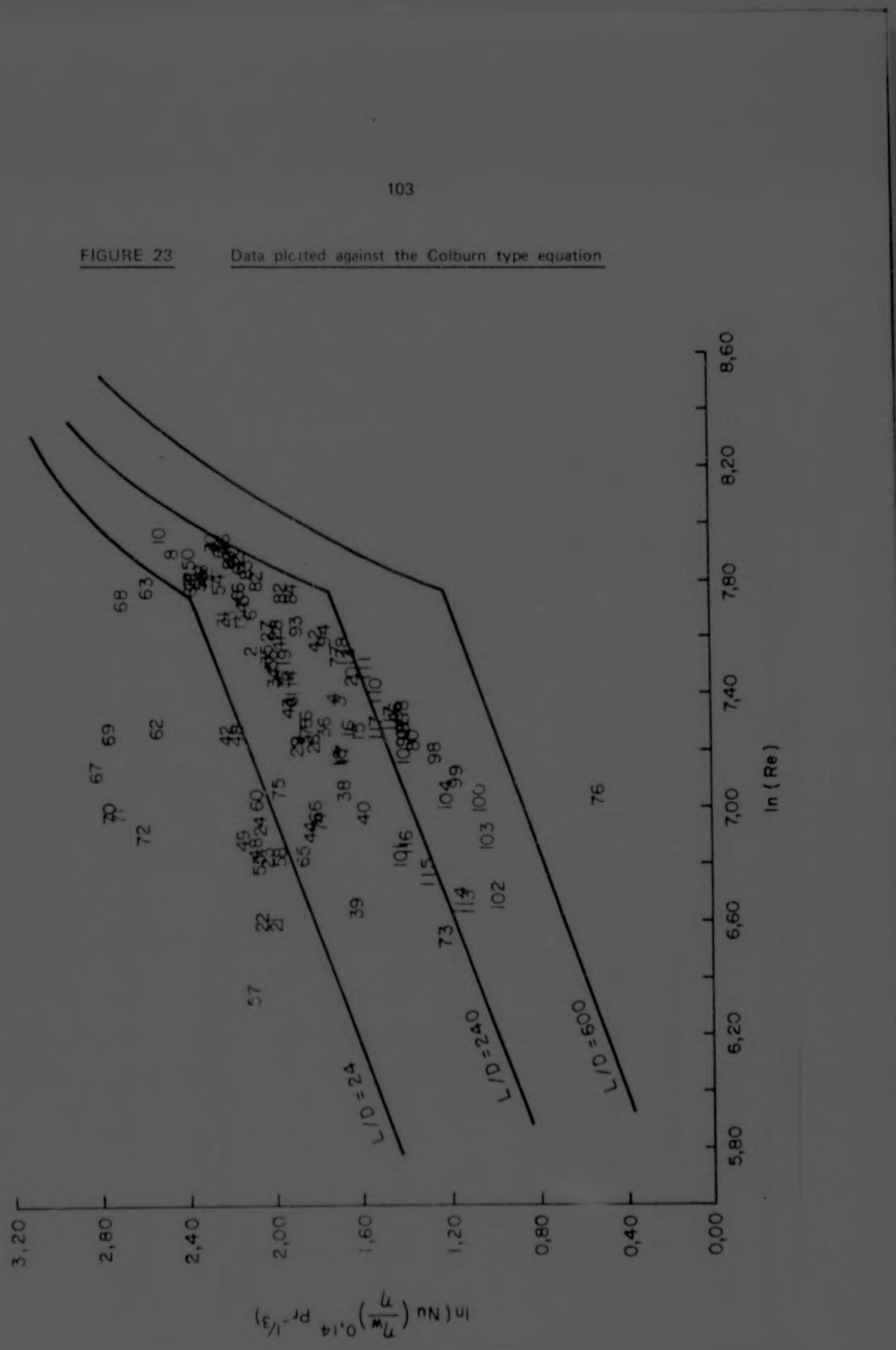


FIGURE 23 Data plotted against the Colburn type equation



Author Rogers D G
Name of thesis Experimental heat transfer coefficients for the cooling of oil in horizontal internal forced convective Transitional flow 1981

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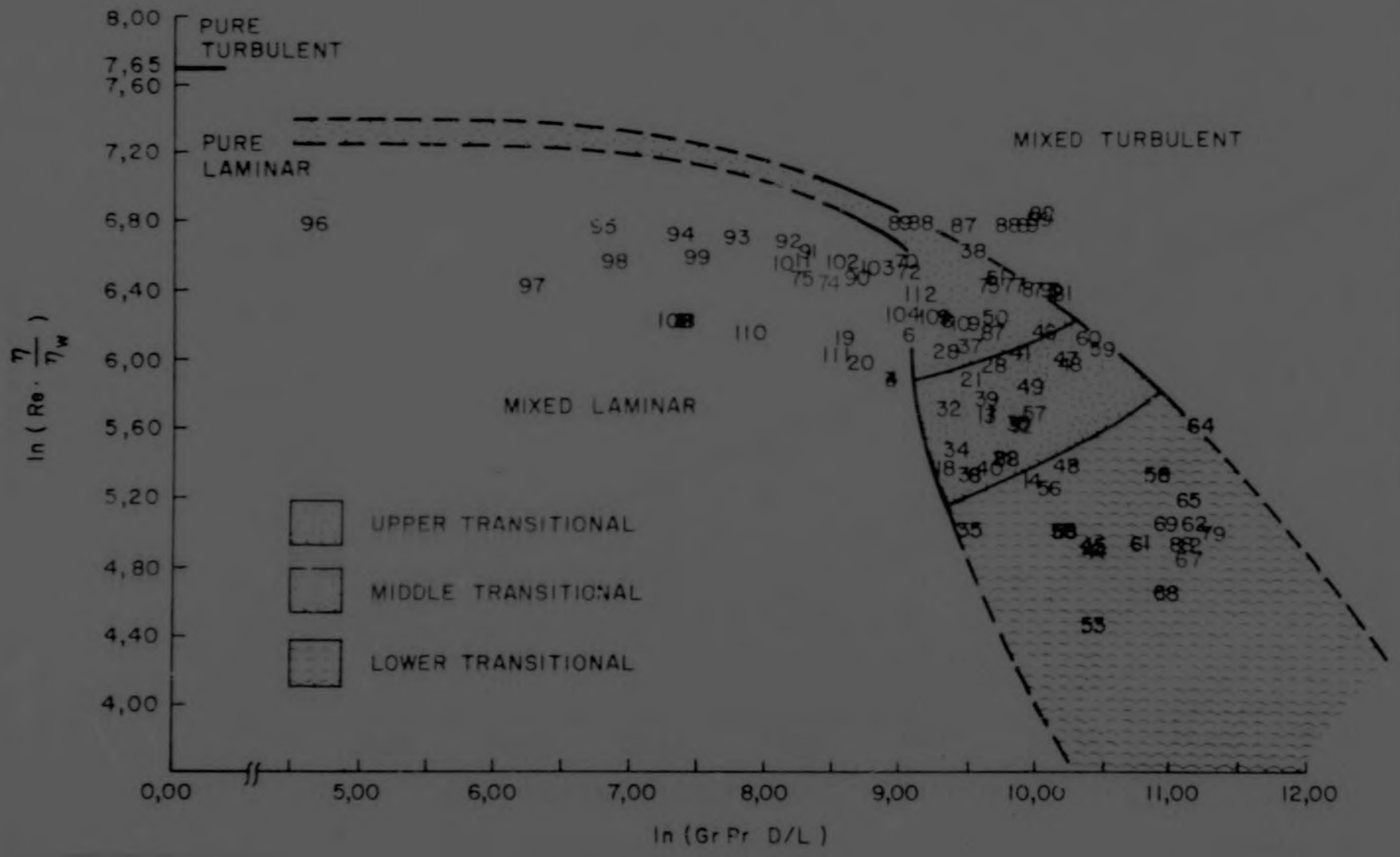
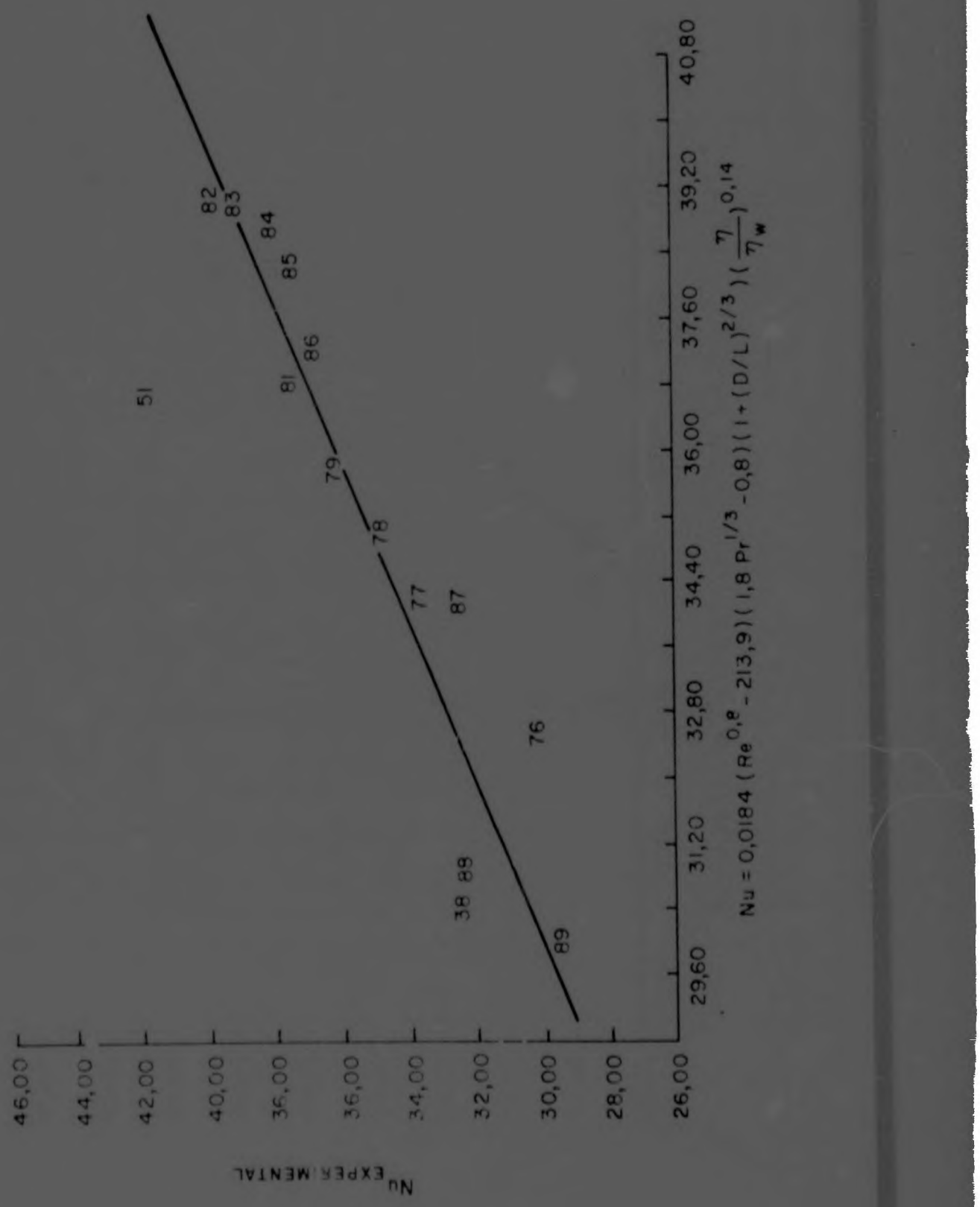


FIGURE 24 Modified Metais and Eckert plot

FIGURE 25 Mixed turbulent equation



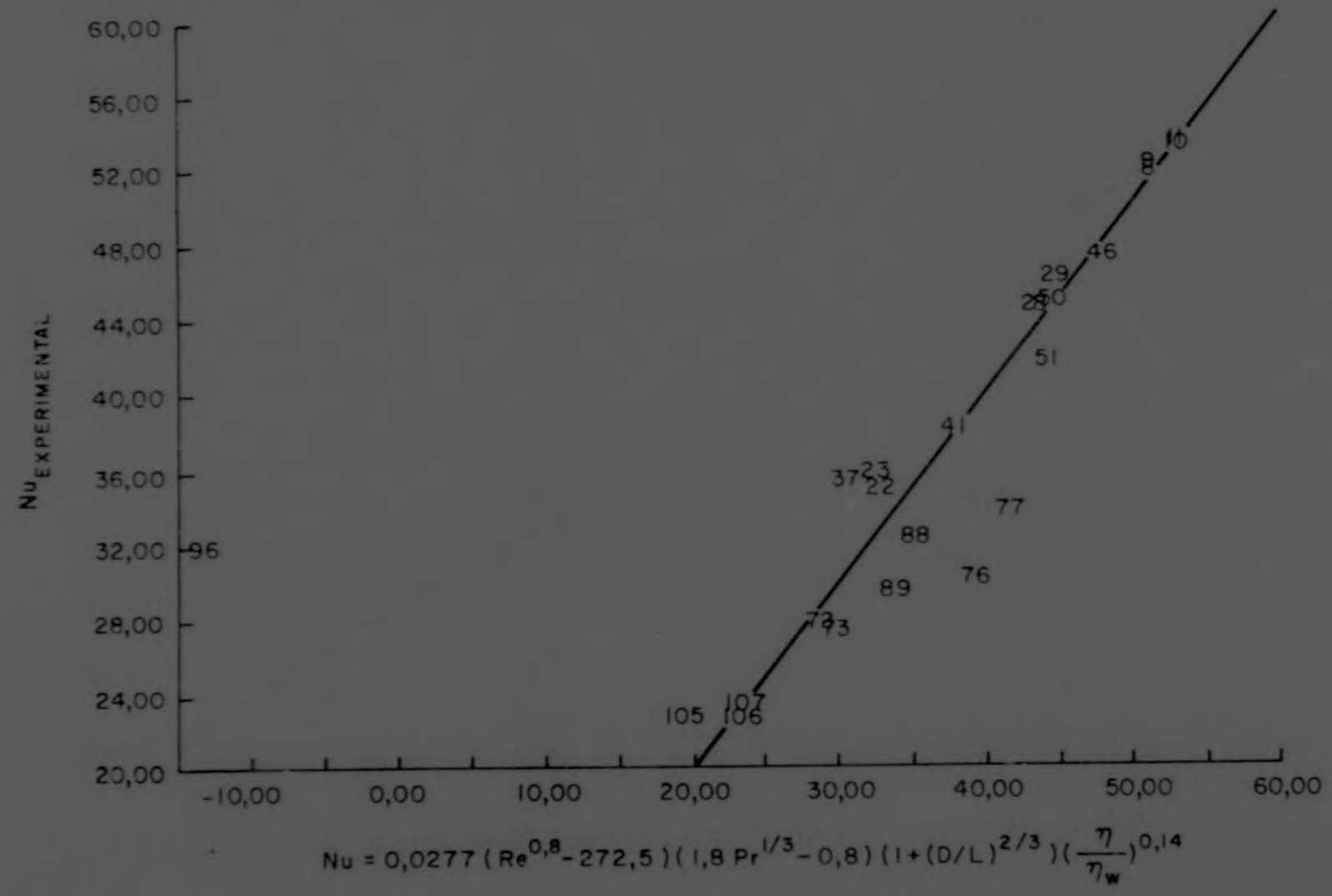


FIGURE 26 Upper transitional equation

FIGURE 26 Upper transitional equation

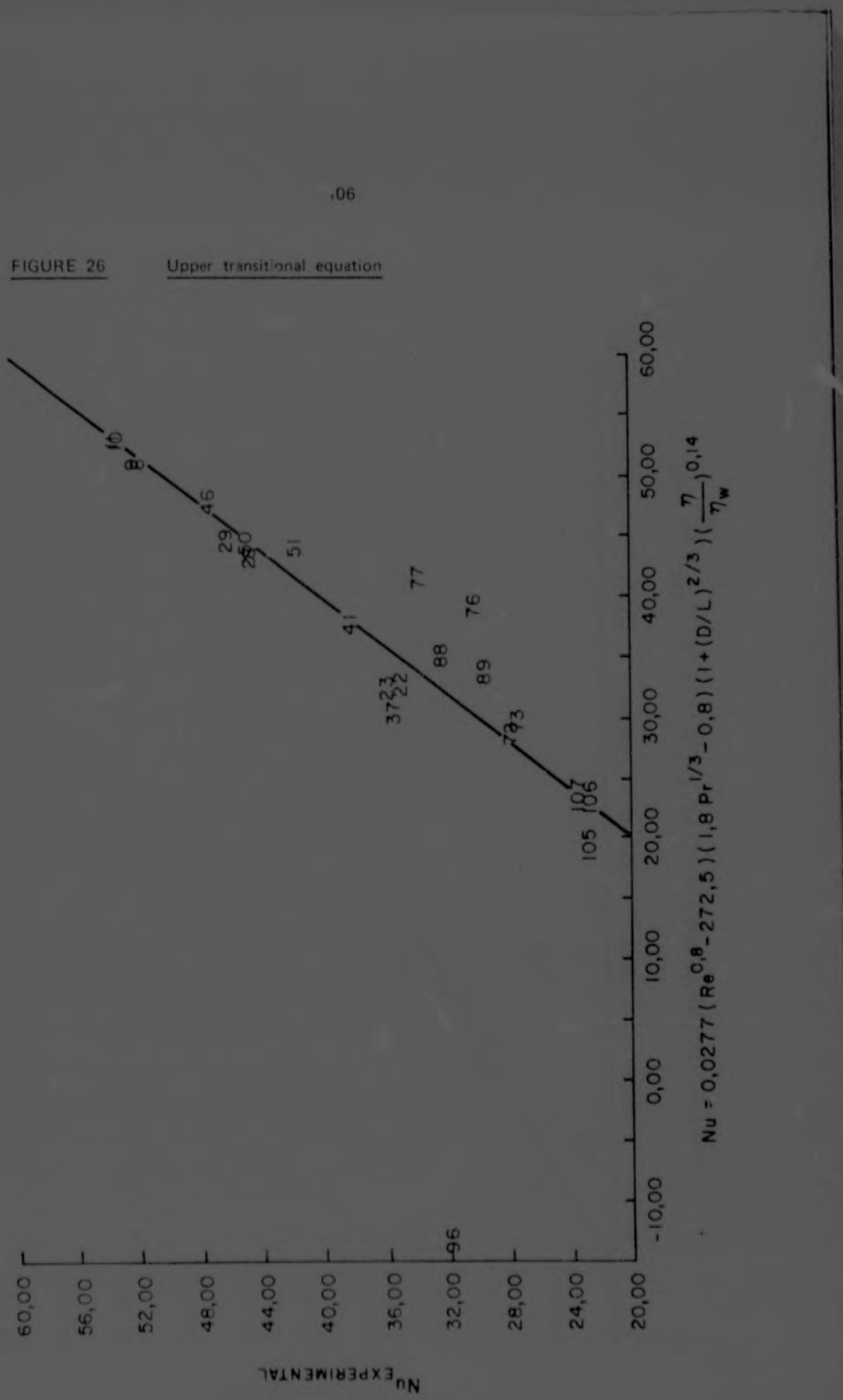
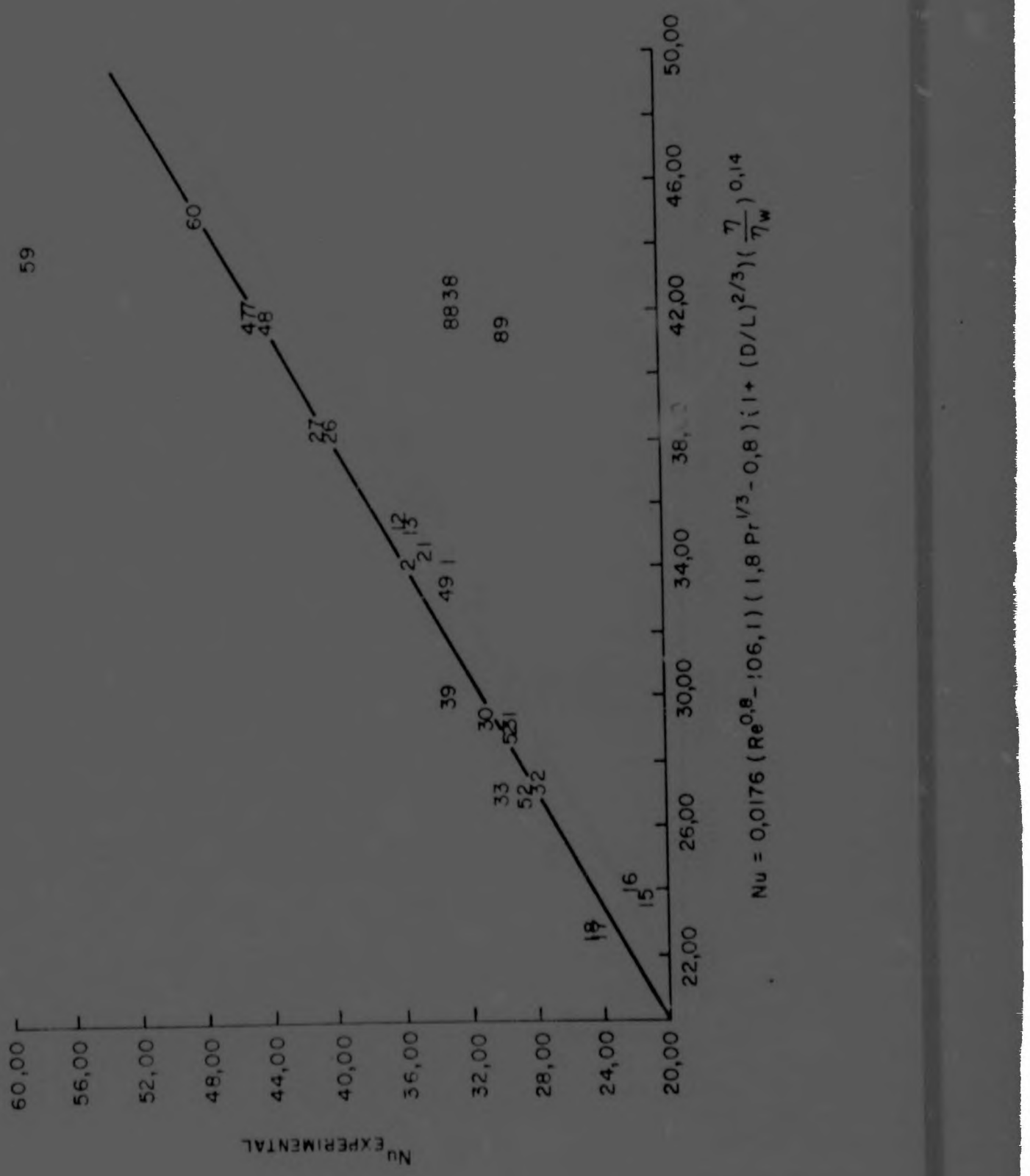


FIGURE 27 Middle transitional equation



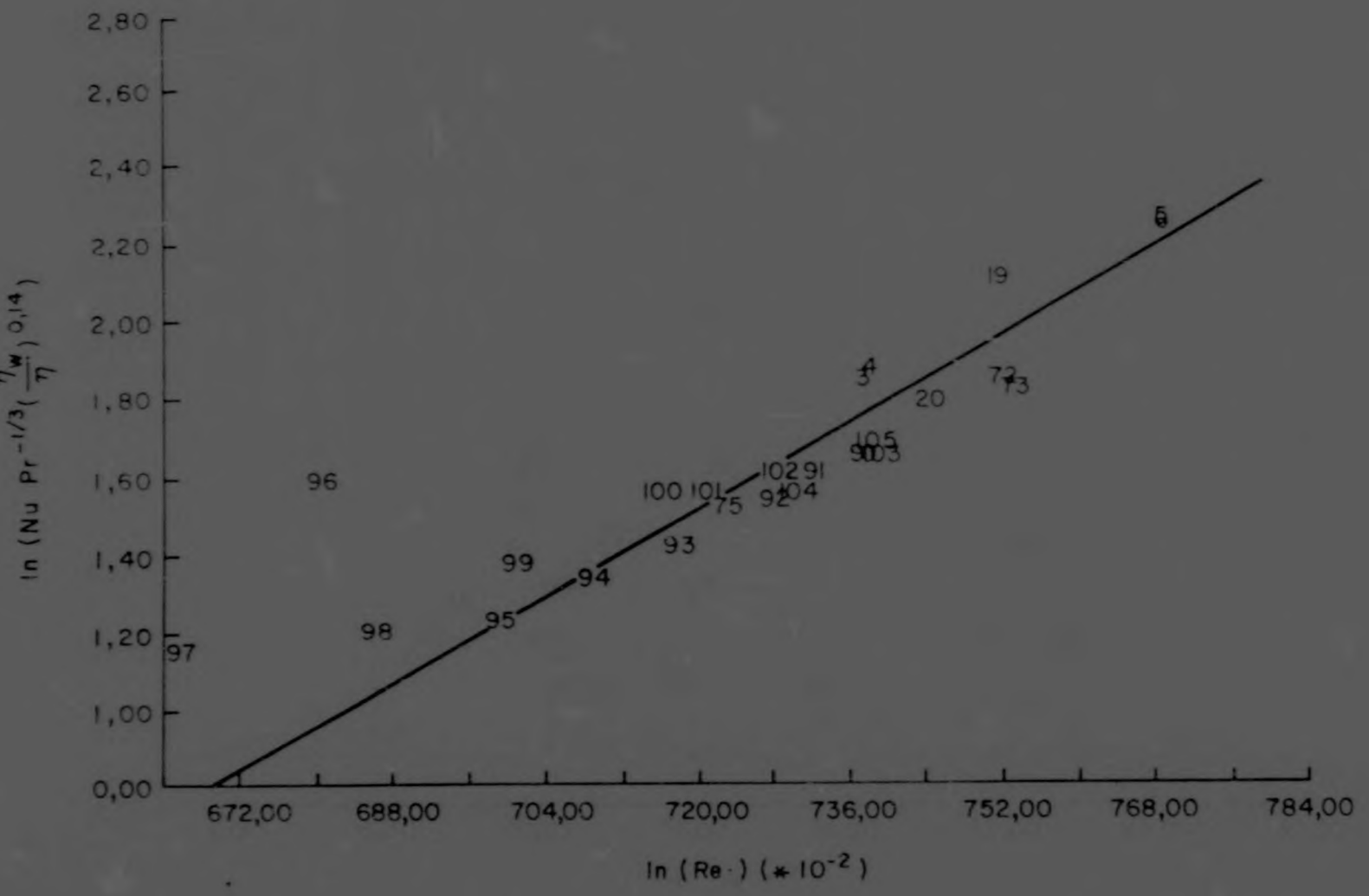


FIGURE 8 Mixed laminar equation

FIGURE 29 Lower transitional data as a function of the Stanton and Reynolds groups

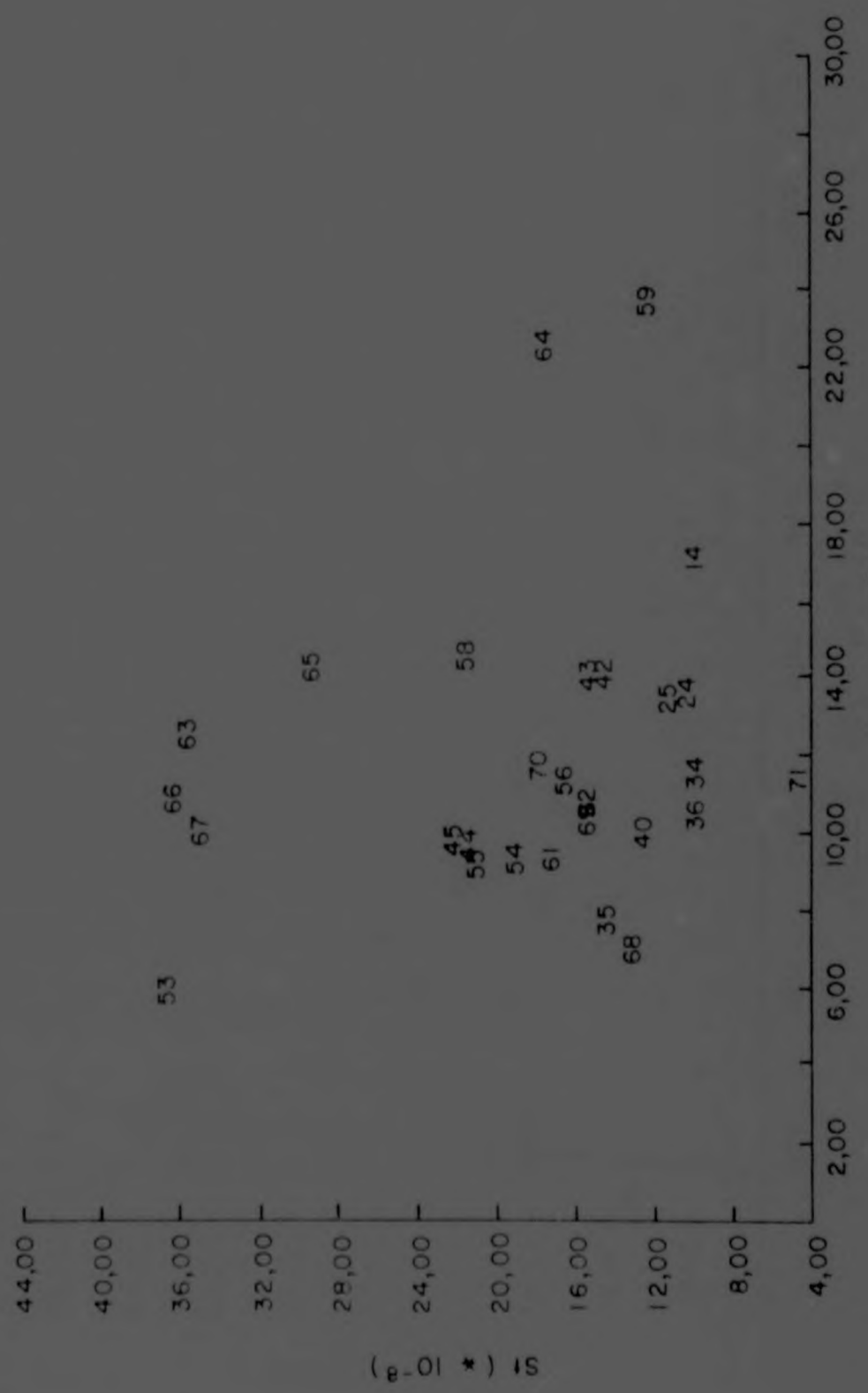


FIGURE 29 Lower transitional data as a function of the Stanton and Reynolds groups

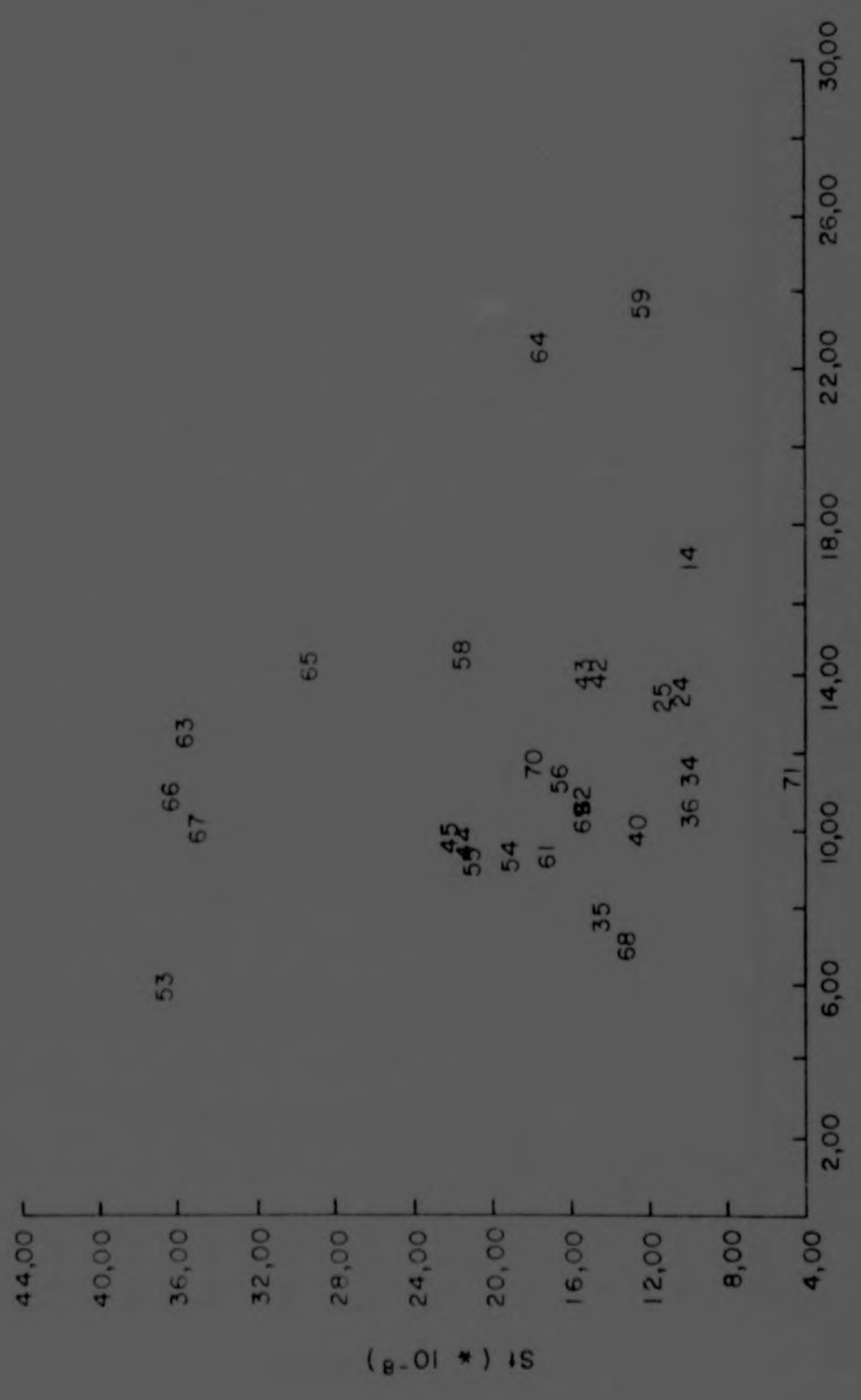
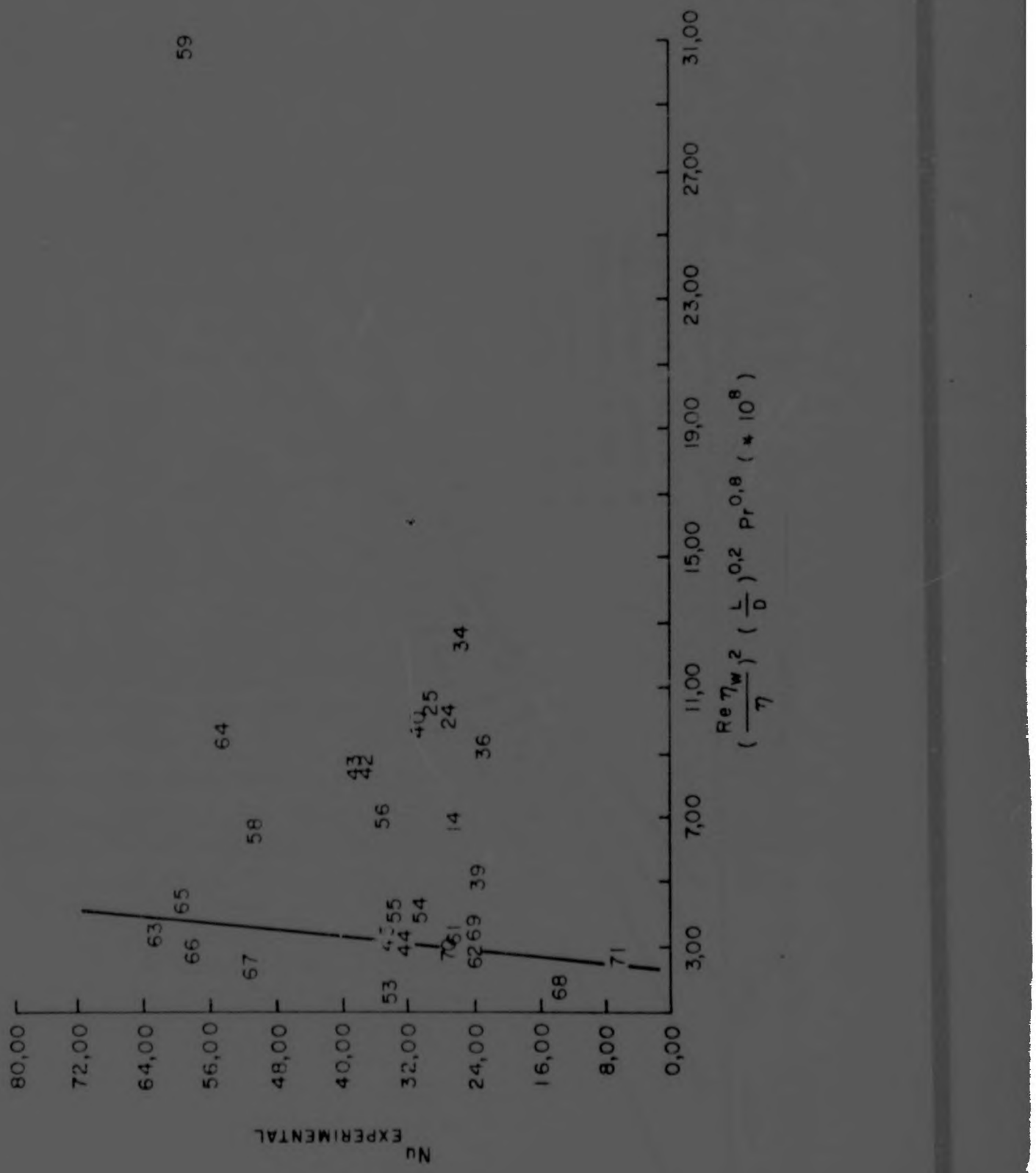


FIGURE 30 Lower transitional equation



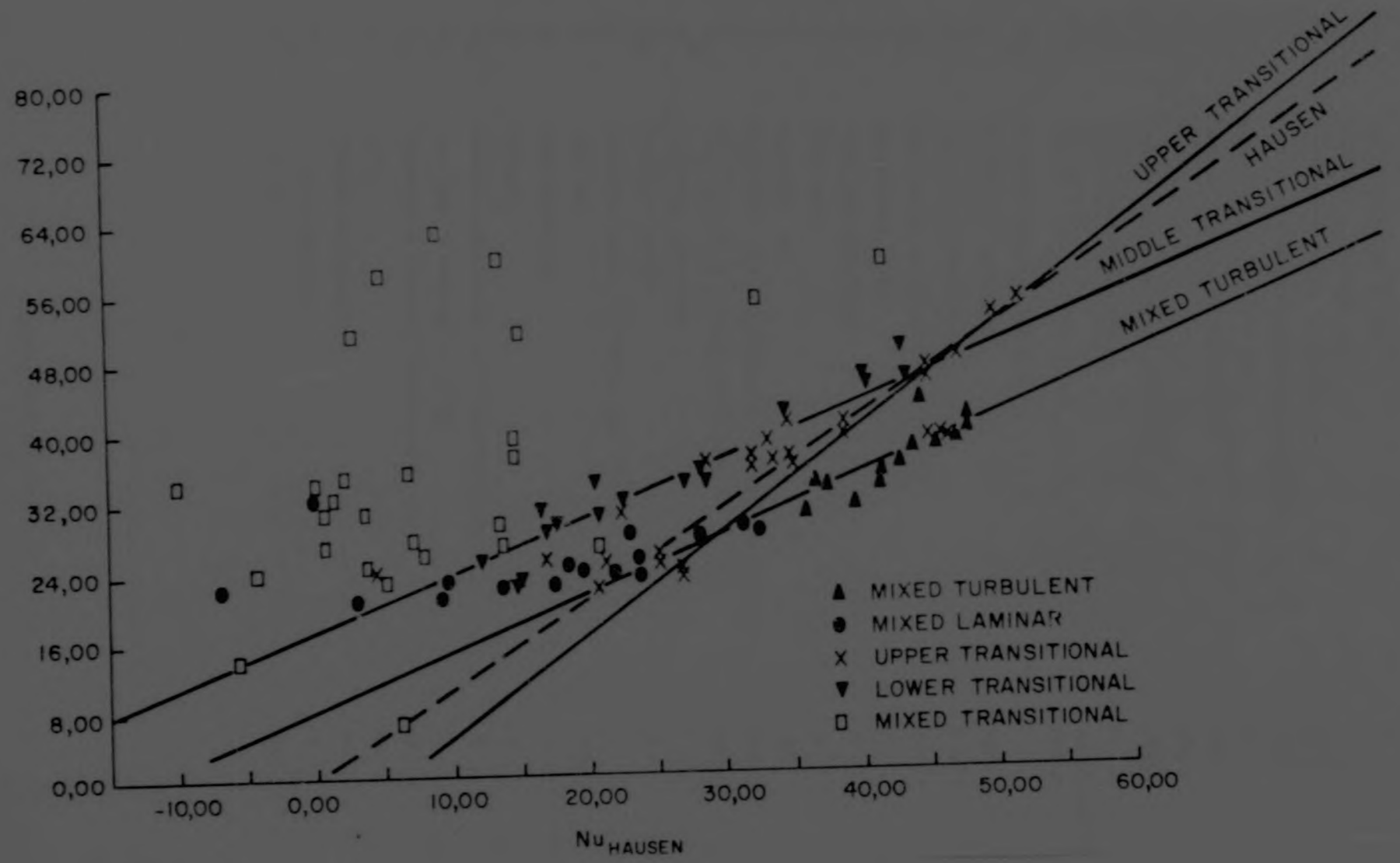


FIGURE 31 Hausen equation compared with derived equations

7 NOMENCLATURE

A	Surface area used in 6.7.2	m^2
A_1	Coefficient as defined in 6.7.1	W^2/m^2
A_0	Coefficient in Andrade equation	kg/ms
a	Coefficient in thermal conductivity equation	W/mK
B_1	Coefficient as defined in 6.7.1	WK
b	Coefficient in specific heat equation	J/kg
C	Specific heat	J/kgK
C_0	Specific heat at reference temperature	J/kgK
D	Inside diameter of tube	m
E(x)	The error in x	dimensions are that of x
$F(x_i)$	A general function used in 6.7	
g	Gravitational acceleration	m/s^2
h	Heat transfer coefficient	W/m^2K
h_0	Heat transfer coefficient in the annulus	W/m^2K
ℓ	Characteristic length	m
dl	Incremental tube length	m
L	Tube length	m
$\dot{m}$	Mass flow rate	kg/s
$\dot{m}_1$	Mass flow of oil as calculated from calibration equation	kg/s
$\dot{m}_2$	Mass flow of oil corrected for density	kg/s
$\bar{m}$	Arithmetic mean value	
N_R	Rotameter reading	%
p	Correction factor of Gregorig used in 3.4.1	
$\dot{q}$	Heat flux	W/m^2
$\dot{Q}$	Heat transfer rate	W
R	Resistance	Ω
R_0	Resistance of standard resistance thermometer	Ω
R_1	Resistance as measured by bridge	Ω
R_2	Resistance corrected to standard tables	Ω
r	Radius	m
T	Absolute temperature	K
T_c	Critical temperature	K
w	Velocity	m/s

Greek symbols

β	Coefficient of volume expansivity	1/K
η	Dynamic viscosity	kg/ms
η/η_w	Ratio of bulk to wall viscosity	
θ	Temperature	^oC
θ_0	Datum temperature in thermal conductivity equation	^oC
θ_{wa}	Temperature of the water in the cooling jacket	^oC
λ	Thermal conductivity	W/mK
λ_0	Thermal conductivity at datum temperature	W/mK
ρ	Density	kg/m^3
ρ_f	Density of rotameter float	kg/m^3
τ	Response time of resistance thermometer	s

Subscripts used in general equations

(Specific instances are included previously)

i	At inlet conditions
o	At outlet conditions
wi	At inside wall conditions
wo	At outside wall conditions
ℓ	Pertaining to liquid
v	Pertaining to vapour
b	At bulk conditions
bi	At bulk inlet conditions
bo	At bulk outlet conditions

Dimensionless groups

(Properties are evaluated at the bulk temperature)

Grashof number $Gr = \beta g L^3 \rho^2 \Delta\theta / \eta^2$

$Gr_D = \beta g D^3 \rho^2 \Delta\theta / \eta^2$

$Gr_L = \beta g L^3 \rho^2 \Delta\theta / \eta^2$

Nusselt number $Nu = \frac{hD}{\lambda}$

Prandtl number	Pr	=	$\frac{C\eta}{\lambda}$
Reynolds number	Re	=	$\frac{\rho wD}{\eta} = \frac{4\dot{m}}{\Pi\eta D}$
Stanton number	St	=	$\frac{h}{\rho wC} = \frac{Nu}{RePr}$

FORCED CONVECTIVE HEAT TRANSFER IN SINGLE PHASE FLOW OF A NEWTONIAN FLUID IN A CIRCULAR PIPE

An annotated summary of empirical correlations

DOUGLAS GORDON ROGERS

SYNOPSIS

An extensive bibliography of empirical correlations for the Nusselt group for internal Newtonian pipe flow has been compiled to facilitate the design of heat transfer equipment. An index is provided for locating experimental heat transfer coefficients for particular fluids and flow conditions. The area of transitional flow is lacking in experimental data and more data must be collected in this region before reliable predictions of heat transfer coefficients may be made.

KEYWORDS: Heat transfer coefficient, turbulent, laminar, transition, review internal flows, empirical correlations, fluid flow.

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1 INTRODUCTION

Order and Simplification
are the first steps towards
the mastery of a subject.

Thomas Mann

Reliable heat transfer coefficients for designing heat exchange equipment are often difficult to obtain and the design engineer may on occasion question the applicability of a chosen coefficient. This doubt may result in over conservative design methods being used and more expensive units than necessary being designed.

This bibliography was compiled as an aid to the designer to find an accurate heat transfer coefficient for the conditions for which he is designing, and to clarify the state of experimental data and correlations as a first step to improving the design process.

The heat transfer coefficient for the transfer of heat to or from a non-porous wall to a fluid is defined by the equation

$$q = h(\theta_{wall} - \theta_{fluid}) = -k_{fluid} \frac{\partial \theta}{\partial y} \Bigg|_{y = wall}$$

The magnitude of the heat transfer coefficient, h, has been determined to depend substantially on the velocity field and it is thus convenient to subdivide heat transfer coefficients according to the flow pattern. The dependence of h on other factors such as the Prandtl number, Grashoff number and Peclet number does not enable a convenient subdivision to be made.

1.1 FLOW REGIMES

Internal forced flow is characterised by the three regions, laminar, turbulent and transitional flow, the latter occurring in the intermediate region between laminar and turbulent flow. The transitional flow may be further subdivided into laminar to turbulent transitional and turbulent to laminar transitional (also called reverse transition), depending on the history of the flow. The three fundamental regions are traditionally characterised by the Reynolds number as

Laminar $Re < 2\,100$
Transitional $2\,000 < Re < 10\,000$
Turbulent $Re > 5\,000$

These regions are loosely defined since the geometry and the heat transfer rate affect the transition process.

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These regions are loosely defined since the geometry and the heat transfer rate affect the transition process.

For internal pipe flow Metais and Eckert (1964) recognised the effect of free convection on the flow pattern and incorporated the $GrPr$ product to further subdivide the flow into the regions

Forced convection turbulent
Free convection turbulent

Forced convection laminar
Free convection laminar

Mixed convection turbulent
Mixed convection laminar.

In this report only the three fundamental regions of laminar, turbulent and transitional flows and only experimental results and correlations are considered. For reviews on the theoretical models the texts of Shah and London (1978) for laminar flow models and Reynolds and Cebeci (1976) and Launder and Spalding (1972) for turbulent flow models are recommended. There is no specific text for transitional flows and this region is usually included in turbulent flow modelling.

1.2 LOCATING INFORMATION

Fluid and Equation indexes were prepared for locating original experimental data for particular fluids and flow conditions. For example, if a heat transfer coefficient is required for heating molasses in turbulent flow in a horizontal pipe, the Fluid index (Section 4.2) indicates that Friend and Metzner (1958) obtained experimental data; cross-referring to 1958 in the Equation index (Section 4.3) for the article by Friend and Metzner will give the data and an accurate heat transfer coefficient.

Alternatively the Equation index may be used for evaluating a given correlation for the Nusselt number (or other heat transfer group). For example, if the equation of Malina and Sparrow (1964) was in question, the entry in the Equation index will indicate how the equation fits given experimental data.

The Equation index also contains entries for which there is no cross reference in the Fluid index. These entries are largely analytical solutions, or correlations based on other researchers' data.

For literature other than English, the VDI–Wärmeatlas, Berechnungsblätter für den Wärmeübergang, is recommended as an interesting summary.

2 CHRONOLOGICAL SUMMARY OF EXPERIMENTAL HEAT TRANSFER COEFFICIENT CORRELATIONS

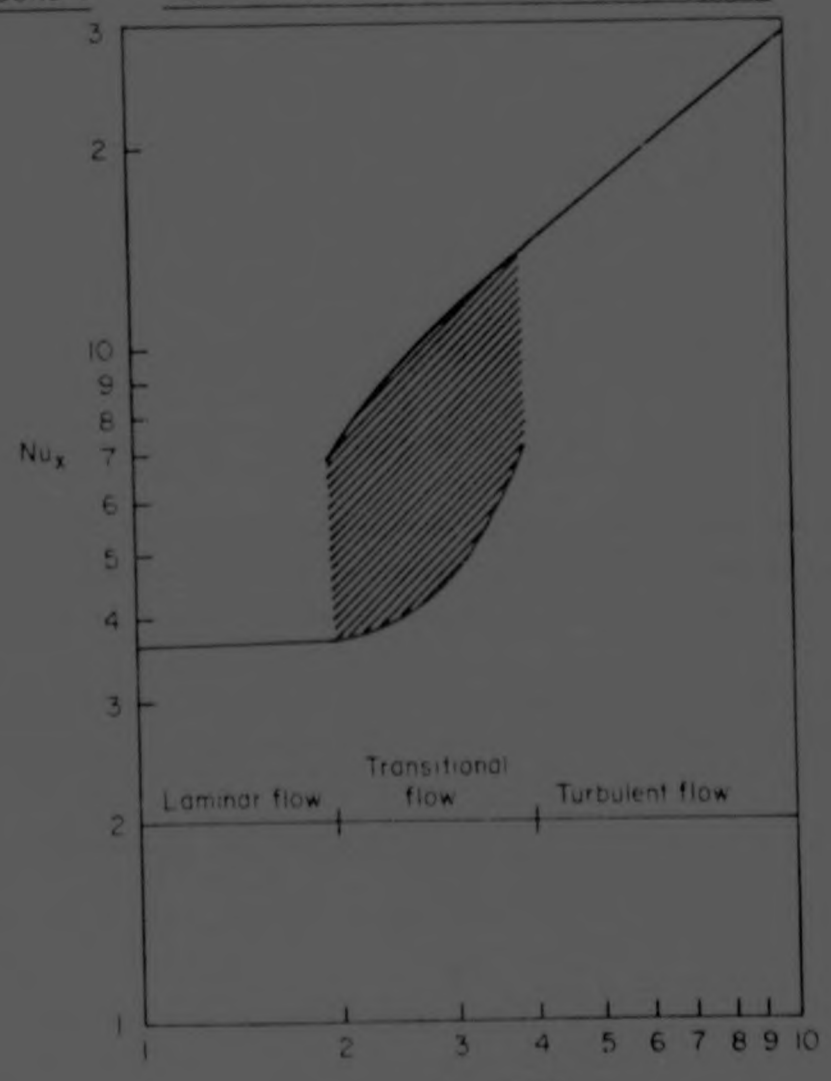
Osborne Reynolds (1874, 1884) was one of the first researchers to recognise and quantify the modes in which fluids flow in pipes. This he did as follows:

> *"... In the first place, it has shown that the property of viscosity or treacliness, possessed more or less by all fluids, is the general influence conclusive to steadiness, while on the other hand, space and velocity are the counter influence..."*

Reynolds therefore, divided fluid flow into two regions which have since been termed *laminar* and *turbulent* flow, which in isothermal conditions are distinguished by the Reynolds group, $\frac{\rho v d}{\mu}$.

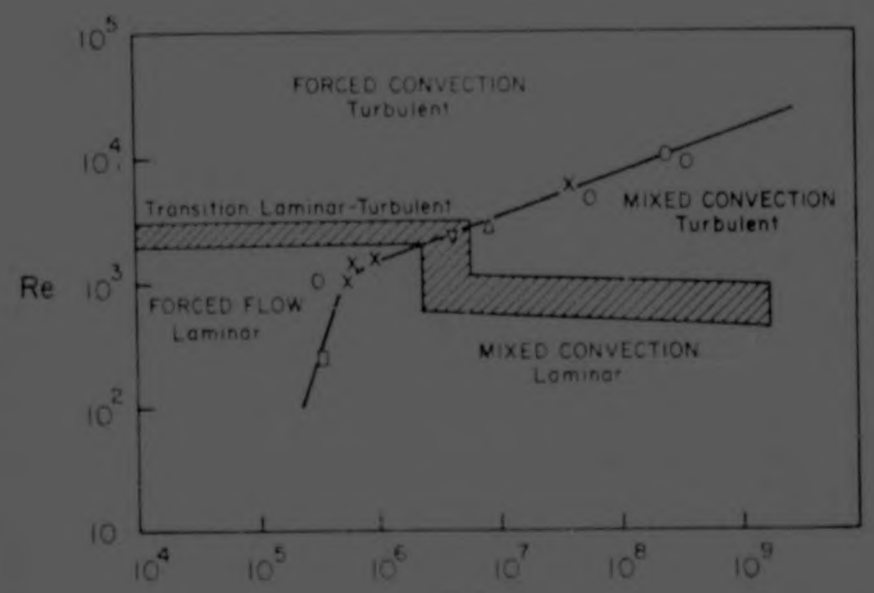
It was not until circa 1940 that an intermediate region of fluid flow which had a marked effect on the heat transfer, was discerned. This region was termed *transitional flow* and correlations of the form of Figure 1 were accepted; noteworthy is the lack of indication in the figure of how to select a Nusselt number in the transition region.

FIGURE 1 Nusselt numbers for transitional gas flow (Pr = 0,7)



Circa 1954 Eckert and Diaguila and later Metais and Eckert (1964) identified the effect of free convection on the extent and location of the three flow regions and presented results as in Figure 2.

FIGURE 2 Regimes of free, forced, and mixed convection for flow through horizontal tubes



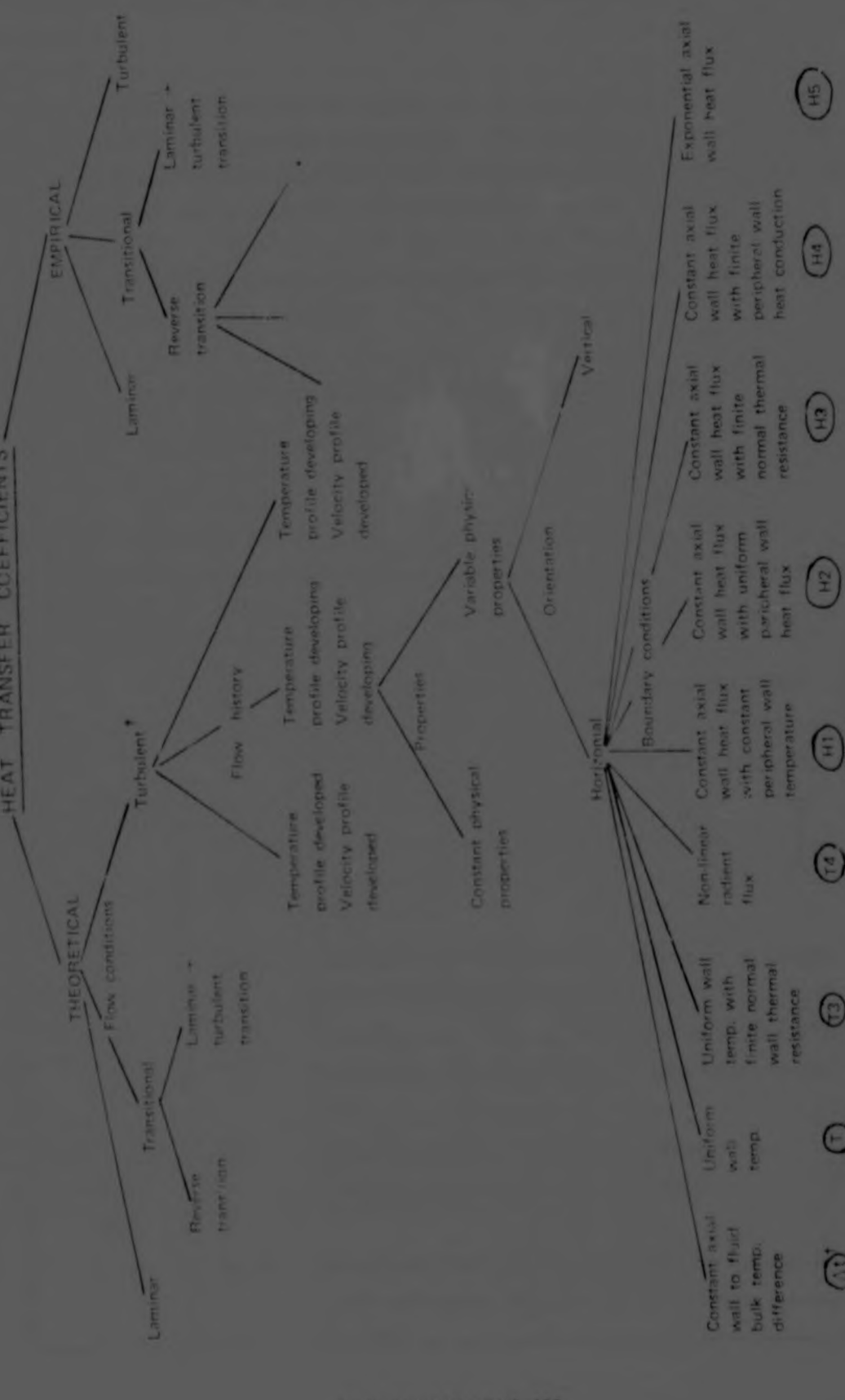
($10^{-2} < Pr\frac{d}{L} < 1$) Metais and Eckert (1964)

(The free convection limit was not established through lack of data).

However, correlations for determining the heat transfer in the transition region were still inadequate and circa 1970 Bankston further subdivided the region into the separate cases of *laminar to turbulent transition* and turbulent to laminar transition or *reverse transition*.

With these subdivisions of the flow conditions, Figure 3 has been constructed. It is envisaged that this Figure will be enlarged in the future as a more complete understanding is reached.

FIGURE 3 Schematic representation of flow division and boundary conditions for heat transfer in pipes



† Boundary conditions after Shah and London (1978)

‡ Asymptotes: Re → ∞, Pr → ∞; Re → ∞, Pr → 0 [Churchill (1977)] ; RePr → 0 [Lyon (1951)]

• All sublayers may be developed in the same manner as has been done for the turbulent, theoretical, heat transfer coefficients

2.1 TURBULENT FLOW HEAT TRANSFER CORRELATIONS

This section summarises the most important points that may be extracted from the extended survey in section 3.

Since the available literature on heat transfer extends over a relatively long time span, it is often difficult to visualise the progression of the science. To facilitate the visualisation of the state of experimental turbulent flow heat transfer coefficient correlations Table 1 has been constructed. From this it is relatively easy to grasp the chain of thought through the time span.

In the other sections on laminar and transition flow heat transfer correlations, similar tables have been drawn up to facilitate visualisation.

TABLE 1 A chronological summary of turbulent flow heat transfer correlation methods

UWT = const. wall temp.
UHF = uniform heat flux

BOUNDARY CONDITIONS	DATE	
	1905 1909	Nusselt and Boussinesque from dimensional analysis suggest $Nu = \phi(Re)\,\phi(Pr)$
UWT $Pr = 0,7$	1917	Nusselt suggests $Nu = \phi(RePr)(d/L)^n$ for a developing velocity profile.
	1916 – 1919	Taylor proposes $\frac{1}{C_H} = \frac{2}{C_f}\left[1 + \frac{U_s}{U}(Pr - 1)\right]$ linking friction and heat transfer
	1922	McAdams and Frost take the viscosity at an average film temperature to align data.
$L/D > 35$	1924	McAdams and Frost suggest $Nu = \phi(Re)\,(1 + a\,d/L)^n$ eliminating the effect of infinite tube length on the Nu.
UWT $Pr = 2,48$ $7,35$	1924	Rice incorporates a temperature difference term, possibly to account for free convection.
	1929	Keevil and McAdams note the effect of the wall to bulk temperature difference to give different velocity profiles

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	1929	Keevil and McAdams note the effect of the wall to bulk temperature difference to give different velocity profiles

BOUNDARY CONDITIONS	DATE	
UWT L/D 59 - 224	1931	Lawrence and Sherwood find that d/L has only an effect for viscous liquids.
	1931	Drew, Hogen and McAdams suggest using the Gz number ($\equiv a RePr\, d/L$).
UWT	1933	Colburn suggests using film temperatures to align the data and suggests that for large Gr the group $(1 + a\,Gr^{1/3})$ should be included.
UHF L/D = 48 Pr 2 - 5	1945	Bernardo and Eian note that the St correlated results better in the lower turbulent region.
	1950	Deissler concludes that the effect of fluid properties across the tube can be eliminated by evaluating the properties at a temperature close to the average of the wall and bulk temperatures.
	1951	Lyon notes that there is an expected minimum of the Nu as Pr → 0.
	1954	Deissler notes that increasing Pr eliminates the entrance effect and that the effect of variable viscosity can be eliminated by evaluating the viscosity at temperatures which are a function of the Pr.
UWT L/D = 5 Pr = 0,7	1954	Eckert and Dianuila note the effect of GrPr on determining the limits of the flow regions.
UHF	1955	Hartnett defines the thermal entrance length and notes that at high Re the Pr has little effect.
UWT Pr = 0,7	1961	Jackson, Spurlock and Purdy experimenting with developing velocity profiles note an effect of L/d but not of free convection. In their experiments the boundary layer did not fill the tube.
	1963	Petukhov and Popov suggest the equation $Nu = \dfrac{f/8\ RePr}{a + b\sqrt{f/8}\ (Pr^{2/3} - 1)}$

BOUNDARY CONDITIONS	DATE	
UHF L/D = 30 Pr = 7 - 8	1964	Allen and Eckert use the wall to fluid temperature difference to correlate the local Nu.
UWT L/D = 31 Pr = 0,71 - 5,52	1965	Kolář uses a turbulent $Re_t = \frac{u_m d}{\sqrt{\frac{1}{B}}}$ to correlate data.
UHF L/D = 21 Pr = 0,7 - 14,3	1967	Gowen and Smith show that the universal temperature profile is dependent on Pr and Re.
UWT L/D = 80 Pr = 2 - 8	1971	Herbert and Sterns for vertical tubes note that the GrPr has little effect at high Re but increases as Re decreases.
	1972	Gross and Thomas note that inclusion of the $\frac{dP}{dx}$ term improves a theoretical model.
	1974	Mori, Sakakibara and Tanimoto find that for Gz = 50 the ratio of the wall thermal conductivity to that of the fluid and the wall thickness may be significant.

2.1.1 Conclusions

For heat transfer to a fluid with constant properties, a fully developed velocity profile and without free convection effects, the Nusselt number correlation is of the form $Nu = \phi(Re).\psi(Pr)$.

For free convection effects the term $(1 + a\,Gr^{1/3})$ may be included as a multiplier. [The effect of using the group as a multiplier is discussed by Brown and Thomas (1965)].

For a developing velocity profile the term $(L/D)^n$ or inclusion of a friction term $\sqrt{f/8}$ may be included as a multiplier. $(L/D)^n$ may be criticised as predicting an infinite Nu for an infinite tube length and $[1 + (L/D)^n]$ is sometimes used to eliminate this incongruity. The alternative use of the Graetz number is not to include entrance effects but to define the ratio of the rate of heat transfer by convection to the rate of heat transfer by conduction.

Variable fluid properties are accounted for by the ratio Nu/Nu_o as explained in section 2.4.

The most often cited equation is that of Petukhov and Popov (1963), with the variable fluid property correction term of Hufschmidt, Burck and Riebold (1966) which is

$$Nu = \frac{f/8 \; RePr}{1{,}07 + 12{,}7\sqrt{f/8}\,(Pr^{2/3} - 1)} \cdot \left[\frac{Pr}{Pr_w}\right]^{0{,}11}$$

Alternatively the equations recommended by Metais and Eckert (1964) may be used.

2.2 LAMINAR FLOW HEAT TRANSFER CORRELATIONS

As in section 2.1, Table 2 has been constructed to facilitate visualisation.

TABLE 2 A chronological summary of laminar flow heat transfer correlations

BOUNDARY CONDITIONS	DATE	
UWT UHF	1885 1910	Graetz and Nusselt formulate an analytical solution for fully developed velocity and temperature profiles.
UWT UHF	1928	Lévêque extends the result to developing velocity profile giving $Nu = a(RePr\,d/L)^{1/3}$
	1930	Dittus and Boelter include the term ΔT_{LM} to include variable physical properties.
UWT $Pr = 2 - 8$	1930	Colburn and Hougen find that $Nu = aPr^{1/3}\,Gr^{1/3}$ with properties based on a film temperature, independent of the Re.
UHF $L/D = 150$	1931	Kirkbride and McCabe propose the form $Nu = a + \frac{b}{RePr\,d/L} + \frac{c}{(RePr\,d/L)^n}$
	1933	Colburn includes the term $1 + a\,Gr^{1/3}$ to account for free convection and bases properties on a film temperature.
$L/D = 90$	1936	Sieder and Tate suggest the property correction $Nu = a(RePr\,d/L)^{1/3}\left[\frac{\mu b}{\mu w}\right]^{0{,}14}$

BOUNDARY CONDITIONS	DATE	
UWT Pr = 43 L/D = 20 - 602	1942	Martinelli et al criticise the use of $1 + aGr^{1/3}$ as a multiplier as this would indicate an increasing effect of free convection with increasing Re, contrary to their observed results for vertical tubes. (See Brown and Thomas, 1965).
UWT Pr = 60 - 1000 L/D = 40 - 193	1943	Kern and Othmer suggest $\frac{1 + aGr^{1/3}}{\log Re}$
	1959	Stephan suggests $Nu = f_n(L/d Pe)$ for constant properties and $Nu = f_n(L/d Re,Pr)$ for variable properties.
UHF Pr = 0,7 - 8	1961	Ede suggests $Nu = a + bGr^{1/3}$
L/D = 72	1962	Oliver criticises $Gr^{d}L$ as producing a term in D^4 and suggests Gr^{L}/d.
L/D = 36 - 72	1965	Brown and Thomas find that for horizontal tubes the free convection increases with increasing Re.
UHF L/D = 127 Pr = 0,7	1966	Mori et al find that free convection effect starts at $ReRa = 10^3$ and that the critical Re depends on the intensity of the secondary flow.
UHF Pr = 0,7 L/D = 80	1966	McComas and Eckert notice that the free convection effect increases as the ratio of Gr to Re increases.
	1967	Iqbal and Stachiewicz analytically find that the tube inclination has little effect on the Nu.
UHF Pr = 2 - 8 L/D = 700	1968	Shannon and Depew notice that for $\frac{(GrPr)^{1/4}}{Nu_o} < 2$ the natural convection is negligible.
L/D = 28	1971	Depew and August suggest using the group $Gz\ Gr^{n}\ Pr^{m}$

2.2.1 Conclusion:

For a fully developed parabolic velocity profile, constant physical properties and no free convection effects, the Nusselt number expected from theory is for:

uniform wall temperature $Nu_\infty = 3{,}657$

and for uniform heat flux $Nu_\infty = \frac{48}{11} = 4{,}364$

For a developing velocity profile the term $(L/D)^n$ or $[1 + (L/D)^n]$ may be included as a multiplier. This latter form eliminates the problem of infinite tube lengths giving infinite heat transfer coefficients.

Free convection effects are not easily accounted for, however, the forms $1 + aGr^n$ and $\frac{1 + aGr^n}{Pg\ Re}$ are accepted as reliable. In some instances the Grashoff group alone is included as a multiplier, and in the case of Ede (1961) the Nusselt group is correlated as a function of the Grashoff number only.

The extent or intensity of free convection may be expressed as a function of the ReRa product or as Ra^n/Nu_o.

Variable physical properties are included by using the $\frac{Nu}{Nu_o}$ form of correlation as given in section 2.4.

No single equation is recommended for all fluids and conditions, however, the most recent equation by Depew and August (1971)

$$Nu = 1{,}75[Gz + 0{,}12(GzGr^{1/3}\ Pr^{0{,}36})^{0{,}86}]^{1/3} \left[\frac{\mu b}{\mu w}\right]^{0{,}14}$$

may be reliable for most conditions.

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$$Nu = 1{,}75[Gz + 0{,}12(GzGr^{1/3} Pr^{0{,}36})^{0{,}88}]^{1/3} \left[\frac{\mu b}{\mu w}\right]^{0{,}14}$$

may be reliable for most conditions.

2.3 TRANSITIONAL FLOW HEAT TRANSFER CORRELATIONS

As in the previous sections, Table 3 has been constructed to facilitate visualisation.

TABLE 3 A chronological summary of transitional flow heat transfer correlations.

BOUNDARY CONDITIONS	DATE	
	1933	Colburn constructs graphical correlations using the ratio $\frac{\mu_{average}}{\mu_{film}}$ as a distinguishing criterion.
	1936	Sieder and Tate in an analagous form use $\frac{\mu_{average}}{\mu_{wall}}$
	1940	Norris and Streid notice that there is a transition region which may extend from Re = 2 100 to Re = 10 000.
Pr = 35 → 140 L/D = 234	1942	Norris and Sims experimentally determine the relation $\frac{h}{CpG} = 0{,}0067\, Pr^{-0{,}8} \left[\frac{\mu}{\mu_w}\right]^{0{,}14} = St$
	1966	Petersen and Christiansen found the relation $St = St_{10\,000} \left[\frac{St_{cr}}{St_{10\,000}}\right]^{\phi Re} \left[\frac{Re}{10\,000}\right]^{3} \left[\frac{10\,000}{2\,100}\right]^{3\phi Re}$ and found no effect of L/d.
	1970	McEligot, Coon and Perkins note that the acceleration parameter may be used for predicting laminarisation.
	1970	Bankston likens the relaminarisation to the reverse transition in external flows.
	1970	Coon and Perkins find criteria for predicting the transition with gases.
	1970	Dyban and Epik suggest that more than one exponent for the Re is necessary to correlate the heat transfer.
UWT L/D = 77 → 231	1975	Pechenegov finds $Nu_x = CRe_x^n$, where C and n are functions of L/d.
	1977	Churchill proposes a comprehensive correlating equation for the total flow region; however, the equation is very cumbersome and is made up of various individual equations.

2.3.1 Conclusions

The extent of the transition region is not clearly definable and depends on the conditions of the system.

The most promising correlations for the heat transfer coefficient appear as a function of the Stanton number, rather than the Nusselt number.

The effects of free convection, variable physical properties and $^{L}/_{D}$ ratio on the heat transfer have not been determined extensively.

The only available data are:

for air	Pechenegov 1975
and for liquids	Norris and Sims 1942

and these are therefore recommended.

2.4 CORRECTION METHODS FOR VARIABLE PHYSICAL PROPERTIES

It is most often convenient to interpret experimental results as a deviation from some specific datum. In heat transfer to a fluid it is convenient to represent the datum as the limiting case of zero heat flux, under which restriction physical properties will be effectively constant due to the uniform temperature fields.

The heat transfer under finite heat flux is then some function of the limiting case of zero heat flux. This is expressed as

$$\frac{Nu}{Nu_o}$$

where Nu_o is the limiting case of zero heat flux. Table 4 has been constructed from correlations in the literature to summarise available correction methods.

TABLE 4 Correction factors for variable physical properties

FLUID	AUTHOR	YEAR	CORRECTION FACTOR
GASES	Il'in	1951	$Nu = CRe^{0,8}\left[\frac{T_w}{T_b}\right]^n$ Air $\left[\frac{T_w}{T_b}\right]$ 0,5 – 0,9 0,9 – 1,2 1,2 – 2,3 C 0,0218 0,0212 0,0223 n 0 −0,27 −0,58
	Humble, Lowdermilk and Desmon	1951	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^n$ Air $n = 0$ at $\frac{T_w}{T_b} < 1$ $= -0,55$ at $\frac{T_w}{T_b} > 1$
	Kays	1955	$\frac{h}{h_{isothermal}} = \left[\frac{T_{wall}}{T_{mixed\ mean}}\right]^{-n}$ $n = 0,25$ heating $n = 0,08$ cooling
	Bialokoz and Saunders	1956	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,5}$ Air $\frac{T_w}{T_b} = 1,1 - 1,73$
	Wright and Walters	1959	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,575}$ Hydrogen $\frac{T_w}{T_b} = 1 - 4$
	McCarthy and Wolf	1960	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,3}$ Hydrogen $\frac{T_w}{T_b} = 1,5 - 2,8$
	Taylor and Kirchgessner	1960	$\frac{Nu}{Nu_o} = 1$ Helium $\frac{T_w}{T_b} = 1,6 - 3,9$
	McCarthy and Wolf	1960	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,7}$ Hydrogen Helium $\frac{T_w}{T_b} = 1,5 - 9,9$

FLUID	AUTHOR	YEAR	CORRECTION FACTOR	
GASES	Wieland	1962	$\frac{Nu}{Nu_o} = 1$	Helium and Hydrogen
	Taylor	1963	$\frac{T_w}{T_b} = < 2,8$ $1,5 - 5,6$	
	Petukhov and Popov	1963	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_f}\right]^{-0,47}$	
	Kutateladze (through Petukhov)	1963	$\frac{Nu_f}{Nu_o} = \left[\frac{2}{\sqrt{\frac{T_w}{T_f}} + 1}\right]^2$	
	Kirillov and Malugin	1963	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,5}$ $\frac{T_w}{T_b} = 1,1 - 2,3$	Nitrogen
	McEligot, Magee and Leppert	1965	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,5}$	Air Carbon di-oxide Argon
	Lelchuk, Elphimov and Fedotov	1965	$\frac{T_w}{T_b} = 1,1 - 2,5$ $1,1 - 2,7$	Nitrogen
	Perkins and Worsoe-Schmidt	1965	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,7}$ $\frac{T_w}{T_b} = 1,3 - 7,5$	Nitrogen
	Volkov and Ivanov	1966	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,55}$ $\frac{T_w}{T_b} = 1,1 - 2,1$	Air
	Petukhov Kirillov and Maidonic	1966	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{n}$ $n = -(0,9 \log \left[\frac{T_w}{T_b}\right] + 0,205)$ $\frac{T_w}{T_b} = 1 - 6$	Nitrogen

FLUID	AUTHOR	YEAR	CORRECTION FACTOR
GASES	Kutateladze (through Gregorig 1970)	–	$\frac{Nu}{Nu_o} = 1,27 - 0,27 \left[\frac{T_w}{T}\right]$ cooling $= \left[\frac{T}{T_w}\right]^{0,55}$ heating
	McEligot, Ormand and Perkins	1966	$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,5}$
	Zucchetto, and Thorsen	1973	$\frac{Nu}{Nu_o} = \left[\frac{T_s}{T_b}\right]^{-0,17}$
LIQUIDS & GASES	Kaye and Furnas	1934	$h_{cooling} = h_{heating} \left[\frac{\mu_{heating}}{\mu_{cooling}}\right]^r$ $r = 0,5$ liquids $= 1,0$ gases
LIQUIDS	Dittus and Boelter	1930	$Nu = aRe^{0,8}Pr^n$ $n = 0,3$ cooling $= 0,4$ heating
	Colburn	1933	$\frac{Nu}{Nu_o} = \left[\frac{\mu_{avg}}{\mu_{film}}\right]^{1/3}$
	Kraussold	1933	$Nu = aRe^{0,8}\left[d/L\right]^{0,054} Pr^n$ $n = 0,37$ heating $= 0,3$ cooling
	Sieder and Tate	1936	$\frac{Nu}{Nu_o} = \left[\frac{\mu_{bulk}}{\mu_{wall}}\right]^{0,14}$
	Michejev and Zukauskas (through Gregorig 1970)	1952	$\frac{Nu}{Nu_o} = \left[\frac{Pr}{Pr_w}\right]^{0,25}$

FLUID	AUTHOR	YEAR	CORRECTION FACTOR
LIQUIDS	Jakovlev (through Gregorig 1970)	1957	$\frac{Nu}{Nu_o} = \left[\frac{Pr}{Pr_w}\right]^{0,11}$
	Kutateladze (through Gregorig 1970)	1968	$\frac{Nu}{Nu_o} = \left[\frac{Pr}{Pr_w}\right]^{n}$ $n = 0,25$ cooling $= 0,06$ heating
	Malina and Sparrow	1964	$\frac{Nu}{Nu_o} = \left[\frac{\mu_b}{\mu_w}\right]^{0,05}$
	Hufschmidt, Burck and Riebold	1966	$\frac{Nu}{Nu_o} = \left[\frac{Pr}{Pr_w}\right]^{0,11}$
	Hackl and Gröll	1969	$\frac{Nu}{Nu_o} = 0,645\left[\frac{\mu_w}{\mu_b}\right] + 0,355$
	Shannon and Depew	1969	$\frac{Nu}{Nu_o} = \left[\frac{\mu_w}{\mu_b}\right]^{m}$ $m =$ fn(Gz) in graphical form.
	Gregorig	1970	$\frac{Nu}{Nu_o} = \frac{\lambda_k \kappa_{Pr}}{\kappa_\mu}$
	Kuznetsova	1972	$\frac{Nu}{Nu_o} = \left[\frac{\mu}{\mu_w}\right]^{0,11}$
	Gregorig	1976	$\frac{Nu}{Nu_o} = \left[\frac{Pr_b}{Pr_w}\right]^{P}$ $P = \frac{0,264(\frac{1}{2} - xPr)^{0,20} + 0,0757}{Pr_b^{0,05} Re_b^{0,02}(\frac{1}{2} - xPr)^{0,01}}$ $xPr = \frac{Pr_t - Pr_w}{Pr_b - Pr_w} - \frac{1}{2}$

From this Table it may be concluded that the correlations for gases and liquids are of different form.

For gases the form

$$\frac{Nu}{Nu_o} = f_n(T,T_{wall})$$

appears to be the only form of correction.

For liquids there is a large variety of correction forms; however, it is physically most likely that use of the Prandtl group may be the most reliable form. The correction term of Gregorig (1976) is thus possibly the most favourable correction method.

2.5 CONCLUSIONS

Data and correlations for the heat transfer coefficient for laminar and turbulent flow of fluids with variable physical properties, developing velocity profiles and free convection effects are extensive. It is therefore not a problem for the design engineer to select a reliable coefficient for most circumstances and for unusual circumstances reliable prediction methods are available.

The area of transition flow has not been extensively examined and methods for determining the extent of this region and obtaining heat transfer coefficients may be unreliable.

More data must be collected for the transition region before reliable predictions of heat transfer coefficients may be made.

3 EXTENDED CHRONOLOGICAL SURVEY

3.1 TURBULENT FLOW CORRELATIONS

Although turbulence as such was not observed until circa 1884, Boussinesque in 1877 proposed a theory of *eddy diffusion* to account for the larger measured pressure gradient in pipe flow than that predicted by the theory of Hagen (1839) and Poiseuille (1841, 1846). He effectively introduced a mixing coefficient A_T for the Reynolds stress in turbulent flow by defining

$$\tau_t = -\rho\overline{u'v'} = A_T \frac{d\bar{u}}{dy}$$

This has the disadvantage that A_T is not a property of the fluid but is dependent on the mean velocity.

Later Boussinesque (1905) from dimensional analysis derived the functional form

$$h = \frac{k}{d} \phi \left[\frac{dup}{\mu}\right] \psi \left[\frac{Cp\mu}{k}\right]$$

where ϕ and ψ were functions to be derived from experimental results. He did not, however, verify this experimentally.

Around the same time Nusselt (1909) also using dimensional analysis, suggested that

$$h = \frac{bk_{wall}}{d^{1-n}} \left[\frac{uCp}{k}\right]^n$$

and experimentally determined $n = 0{,}786$ and $b = 15{,}90$. The group $\frac{hd}{k}$ was later to be termed the *Nusselt number*.

In 1917 Nusselt extended his previous results to include a developing velocity profile and determined that for air

$$Nu = 0{,}03622 \left[d/L\right]^{0{,}54} \left[\frac{\rho duCp}{k}\right]^{0{,}786}$$

was found to hold. The d/L term allowed for the developing profile.

G I Taylor (1916 – 1919) extended Reynolds' analogy to include two regions of flow inside the pipe; a laminar region where ε (the *"coefficient of turbulent exchange"*) is negligible and a turbulent region where the viscosity is negligible. From this he deduced the relation

$$\frac{Cp\rho u(\theta_o - \theta)}{q} = \frac{\rho u^2}{\tau} \left[1 + \frac{u_b}{u}(Pr - 1)\right]$$

where u_δ is the velocity at the laminar – turbulent layer.

Gröber (1921) proposed the provisional equation for both liquids and gases

$$Nu = \frac{a}{r^{0,05}} \left(\frac{d\nu\rho C_p}{k}\right)^{0,79}$$

possibly as an extension of Nusselt's results. This was later criticised by McAdams and Frost (1924) as failing to allow satisfactorily for variations in diameter and velocity.

McAdams and Frost (1922) noted that a *critical velocity* that defined the boundary between laminar and turbulent flow could be expressed as

$$u_c = \frac{16\mu}{\rho d f_c}$$

and also noted that in certain cases the transition could be delayed far in excess of this velocity. Using the results from experiments with light oils and water they suggested a simplified form of the Boussinesque equation as

$$Nu = aRe^b$$

where the viscosity was taken at the average film temperature in an attempt to correlate the data satisfactorily.

Two years later McAdams and Frost (1924), after experimenting with the heating of water put forward the equation

$$Nu = 3\left[1 + \frac{50d}{L}\right] Re^{0,8}$$

after noticing a pronounced effect of the L/d ratio.

Rice (1924), using collected data and basing physical properties on the film temperature proposed an equation of the form

$$Nu = a\Delta\theta\, Pr^{0,5}\, Re^{5/6}$$

for use with gases and liquids in flow well above the critical value. The use of the $\Delta\theta$ suggests the influence of free convection.

In 1925 Prandtl developed his mixing length hypothesis:

$$\tau_t = \rho l^2 \left|\frac{d\bar{u}}{dy}\right| \frac{d\bar{u}}{dy} \qquad (l \text{ is the } \textit{Prandtl mixing length}).$$

This theory has subsequently been used extensively and still finds numerous applications.

Cox (1928), using semi-theoretical considerations derived the equation

$$Nu = aRe^{5/6} Pr^{1/3}$$

and based the physical properties on the film temperature as had McAdams and Frost (1922).

Morris and Whitman (1928) experimented extensively with three petroleum oils and using the form

$$Nu = a\phi(Re)\,\psi(Pr)$$

presented the results in graphical form, differentiating between heating and cooling. The Nu was defined as a point value rather than length averaged value and the use of average fluid properties rather than film properties gave more consistent results. Figures 4 and 5 are the plots obtained and it is interesting to note that there is more scatter in the data for cooling, suggesting an additional mechanism not allowed for in the equation used.

FIGURE 4 The heat transfer coefficient as a function of a dimensional Reynolds number for heating of oils. (Morris and Whitman 1928)

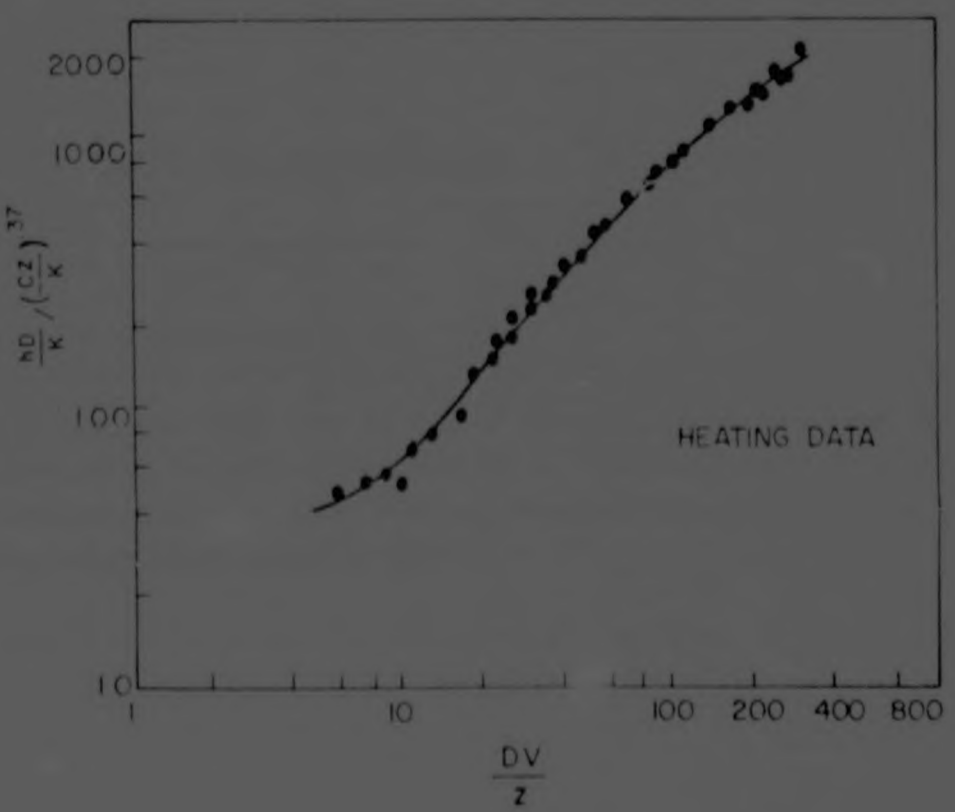
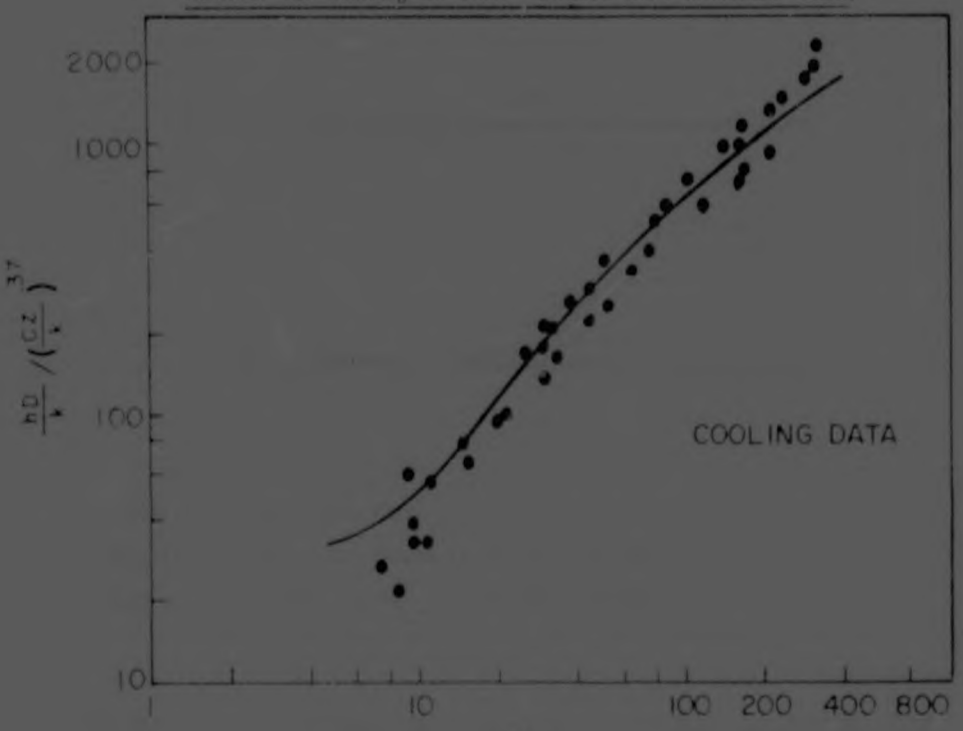


FIGURE 5 The heat transfer coefficient as a function of a dimensional Reynolds number for cooling of oils. (Morris and Whitman 1928)



Prandtl (1928) further developed the results of Taylor (1916 – 1919) by observing that for smooth pipes

$$\frac{v_s}{v}$$

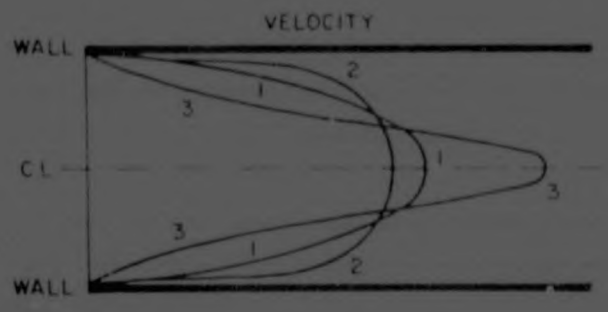
was proportional to $Re^{-1/8}$. This led to the equation:

$$C_H = \frac{C_f}{2} \frac{1}{1 + \beta Re^{-1/8} (Pr - 1)}$$

which was later found to hold for small Pr only.

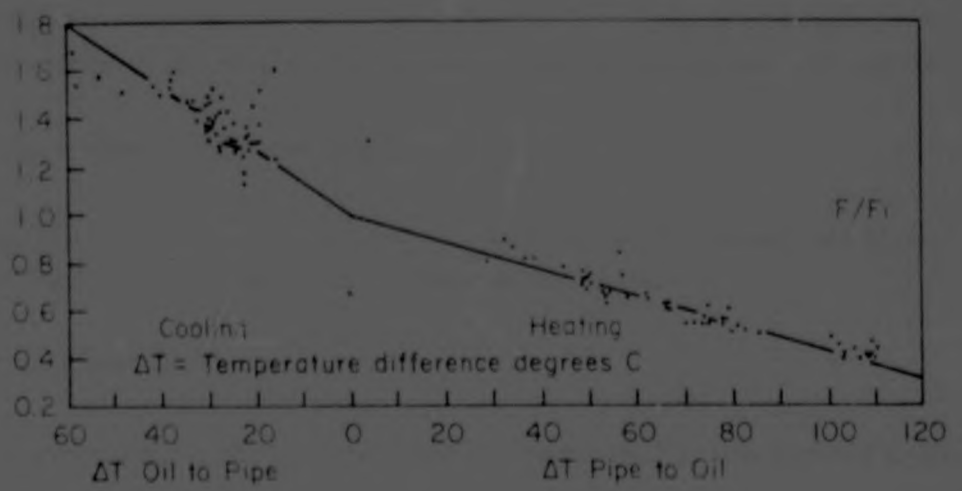
Keevil and McAdams (1929) noted the effect of the direction of the heat transmission on the velocity profile and on the friction factor. Physically the situation was represented as in Figure 6, and using the experimental results of two oils they quantified the effect in the form of the wall to bulk temperature difference as in Figure 7.

FIGURE 6 Effect of heat transmission on velocity distribution in viscous motion. (Keevil and McAdams 1929)



Curve 1 — isothermal flow.
Curve 2 — heating of liquid or cooling of gas.
Curve 3 — cooling of liquid or heating of gas.

FIGURE 7 The effect of ΔT on the friction factor in laminar flow (Keevil and McAdams 1929)



As with the results of Morris and Whitman (1928) the data for cooling were more scattered indicating an unaccounted for mechanism.

Eagle and Ferguson (1930), after an exhaustive survey of the published data, concluded that it was impossible to deduce any general rule by which heat transfer coefficients could be predicted under any given conditions.

They also proposed that in addition to a film and core region there was an intermediate layer, which led to the equation

$$\frac{au_m}{k_o} = A + B\,(Pr - 1) + C(Pr - 1)^2$$

where A, B and C are functions of the Re. Experimenting with water, they noted that there was agreement between the equation and the experimental data. They also postulated that the scatter in experimental results could be caused by a free convection effect.

Dittus and Boelter (1930) in a now classic paper experimentally determined correlations for the heating and cooling of oils. As with previous investigators they could not reconcile the heating and cooling data and suggested the equations

$Nu = aRe^{0,8}Pr^{0,4}$ for heating, and

$Nu = bRe^{0,8}Pr^{0,3}$ for cooling.

The effect of an entrance region was noted but was not correlated due to insufficient data. An equation for laminar flow was also suggested in which a $^{d}/_{L}$ term was included for entrance effects and the term ΔT_{LM} to account for free convection effects.

Lawrence and Sherwood (1931) investigated the effect of the tube length using water. Stender* in 1930 had noted that the use of the factor $\left[{}^{d}/_{L}\right]^{p}$ used as a multiplier in the Nusselt equation indicated zero heat transfer for infinite pipes, and had proposed the equation

$$Nu = a(RePr)^{0,75} + bRePr({}^{d}/_{L}).$$

However, using experimental data on flow without a developing section Lawrence and Sherwood concluded that the $^{d}/_{L}$ ratio had no effect and proposed the equation

$$Nu = aRe^{0,7}Pr^{0,5}$$

They noted, however, that for oils in the semi turbulent region the $^{d}/_{L}$ ratio appeared to have a pronounced effect but did not have sufficient data to correlate the results.

Drew, Hogan and McAdams (1931) reviewed the available equations and using previously published data concluded that no models were adequate to describe the true situation, and suggested

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*Stender, Wiss. Veröffentlich. Siemens Konzern, 9, 88 (1930), as referred to by Lawrence and Sherwood (1931).

the use of the group $\frac{WCp}{kL}$ (Graetz number) to correlate the data. It is very interesting to note that the $Gz = aRePr^{d}/L$ which ties in with previous correlations.

Nusselt (1931), using the data of Burbach* and of Eagle and Ferguson (1930) substantiated his postulate that the equation was of the form

$$Nu = aRe^{0,764}\,Pr^{0,355}\left[d/L\right]^{0,0552}$$

Sherwood and Petrie (1932) experimented extensively with the heating of several liquids of Pr from 1 to 20 and successfully correlated the results using an equation of the Dittus and Boelter (1930) form as suggested in 1931. They further noted that the results were inaccurate for lower Re.

Murphree (1932) criticised the Prandtl model as not showing the effect of large Pr effectively and, assuming the model of eddy currents whose value was zero at the wall and increased to a constant value in the bulk of the fluid, derived the formula

$$Nu = \frac{1 - \xi}{1 + \xi} \cdot \frac{1}{3\left[Pr\,\frac{\varepsilon}{\nu}\right]^{1/3}} \cdot \psi\left[Pr.\,\varepsilon/\nu\right]$$

where ψ is a complicated function. When applied to experimental results the model was found to be reasonable.

Colburn (1933) introduced the j_H factor which he defined as

$$j_H = \frac{h}{CpG}\,Pr^{2/3}$$

and postulated a direct link between this and the friction factor $f/2 = \frac{R}{\rho v^2}$

Using others' experimental results and basing the fluid properties on a film temperature defined as

$\theta_f = \theta_{aug} + \frac{1}{2}(\theta_{wall} - \theta_{aug})$ for turbulent flow and

$\theta_f = \theta_{aug} + \frac{1}{4}(\theta_{wall} - \theta_{aug})$ for laminar flow,

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*Burbach, Th and Hermann, R. "Strömingswiederstand und Wärmeübergang in Rohren", Liepzig 1930, p 45, as referred to by Nusselt (1931).

the use of the group $\frac{WCp}{kL}$ (Graetz number) to correlate the data. It is very interesting to note that the $Gz = aRePr^{d}/L$ which ties in with previous correlations.

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$\theta_f = \theta_{aug} + \frac{1}{2}(\theta_{wall} - \theta_{aug})$ for turbulent flow and

$\theta_f = \theta_{aug} + \frac{1}{4}(\theta_{wall} - \theta_{aug})$ for laminar flow,

---

*Burbach, Th and Hermann, R. "Strömingswiederstand und Wärmeübergang in Rohren", Liepzig 1930, p 45, as referred to by Nusselt (1931).

he derived the equations

$$j_H = a + b\left[\frac{dG}{\mu}\right]^{-0,32} \qquad \text{for turbulent flow and}$$

$$j_H = a\left[\frac{dG}{\mu}\right]^{-2/3}\left[\frac{L}{d}\right]^{-1/3} \qquad \text{for laminar flow}$$

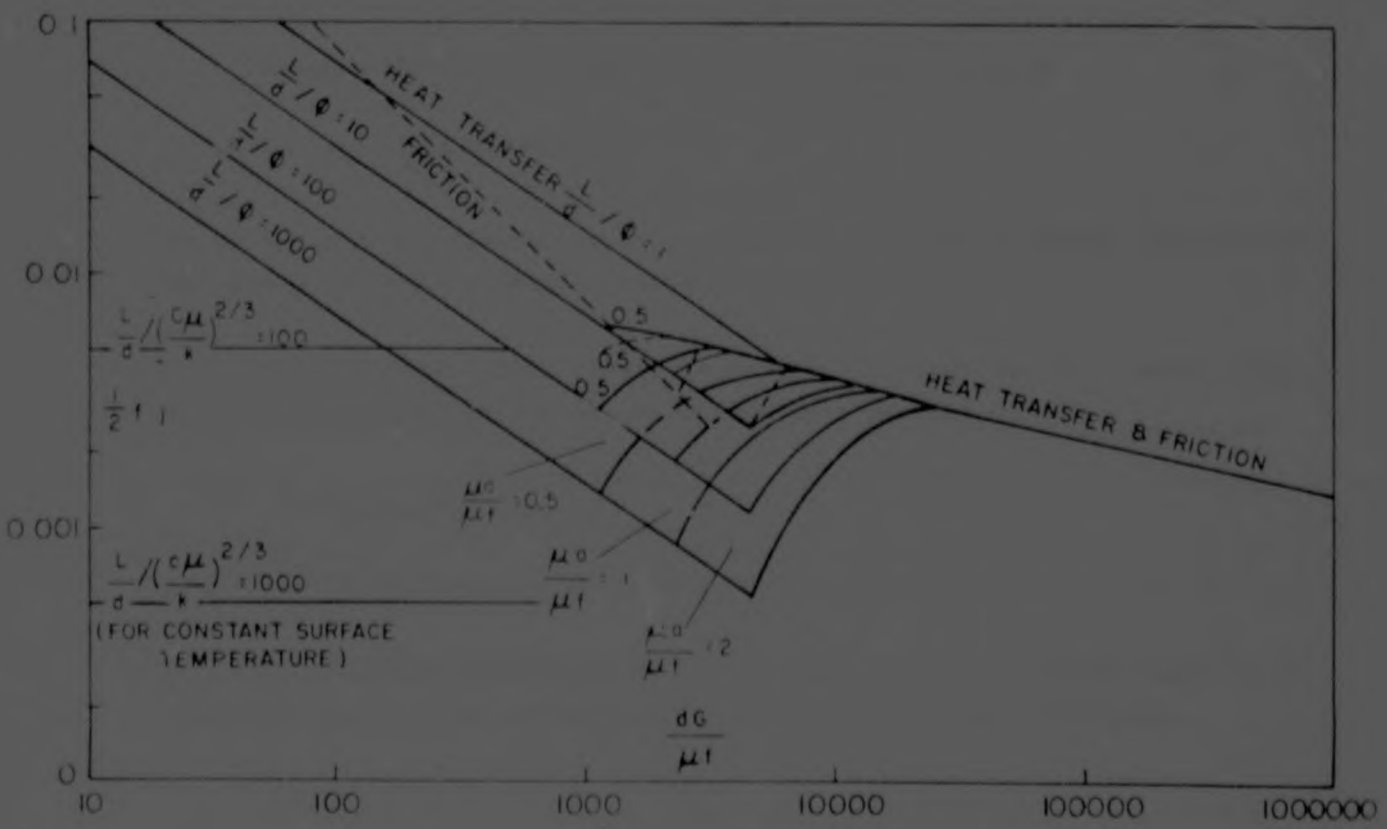
He suggested the use of the parameter

$$\left[\frac{\mu_{avg}}{\mu_{film}}\right]^{1/3}$$

to bring the heating and cooling data into line and noted that for large Gr, inclusion of the term $(1 + 0,015\,Gr^{1/3})$ was necessary.

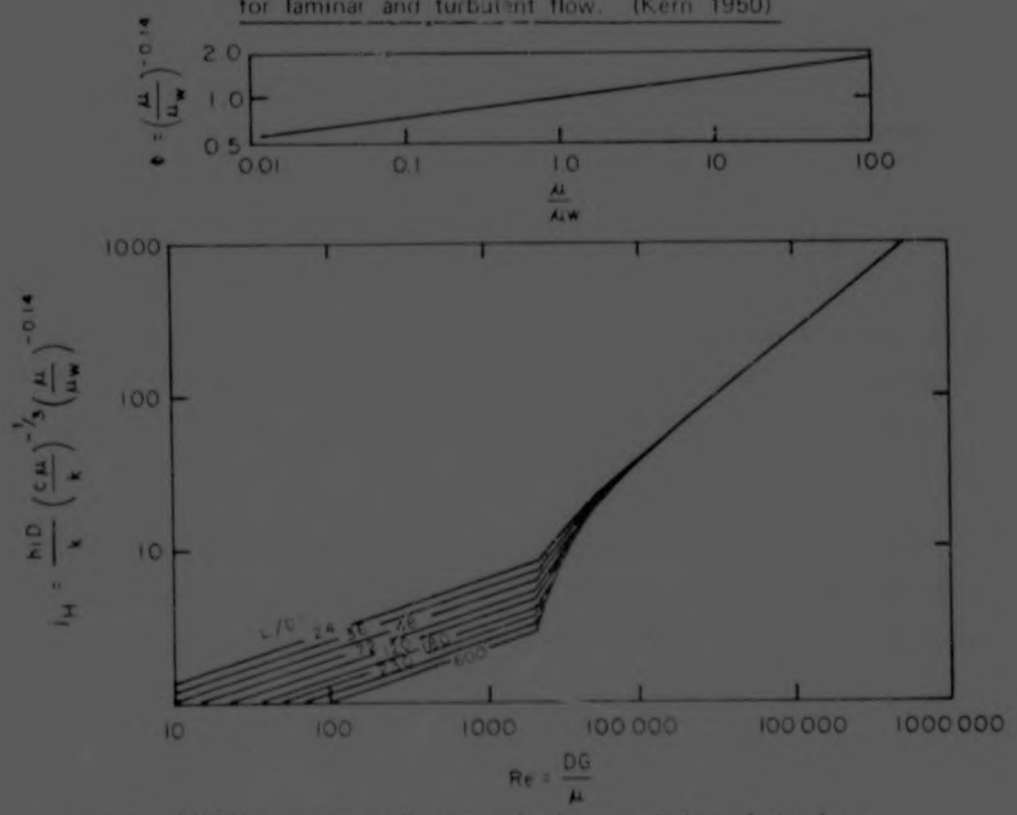
The j_H and $f/_2$ factors did not correlate in the laminar region but were in good agreement for large Re. In accordance with friction data Colburn presumed an analogous form for the heat transfer coefficient and hence constructed Figure 8, which extends from laminar to turbulent flow. The form of the curve in the transition region is, however, arbitrary and not based on sound experimental data.

FIGURE 8 The functions j and j_H as a function of the Reynolds number for laminar and turbulent flow. (Colburn 1933)



This work of Colburn's was a major advance and graphs of the form of Figure 8 are still used, as for example, in Kern's book "Process Heat Transfer" (Figure 9)

FIGURE 9 The functions j and j_H as a function of the Reynolds number for laminar and turbulent flow. (Kern 1950)



Kraussold (1933) experimentally determined a correlation of the form

$$Nu = aRe^{0.8}Pr^{0.37}\left[d/L\right]^{0.054}$$

for the heating of fluids and changed the exponent of the Pr to 0,3 for cooling. This indicated that there was an entrance effect, contrary to previous results.

Kaye and Furnas (1934) advanced the theory of a stationary film at the wall to explain the difference in heating and cooling data and suggested a correction factor

$$h_{cooling} = h_{heating}\left[\frac{\mu_{heating}}{\mu_{cooling}}\right]^{r}$$

where r was 0,5 for liquids and 1,0 for gases.

Sieder and Tate (1936) in a now renowned paper, presented experimental results for the heating and cooling of oils, and based on the results, suggested the use of the factor

$\frac{\mu_{bulk}}{\mu_{wall}}$ to bring the heating and cooling data into agreement. They suggested a value of 0,14 for the exponent of this correction factor and noting that the ratio L/d was not necessary in turbulent flow produced a correlation as in Figure 6. This form is still widely accepted although the scatter of the original data is fairly large.

Von Kármán (1939) introduced the concept of a buffer layer between the laminar sublayer and the core in pipe flow. Using this theory and the Reynolds analogy he obtained, using Nikuradse's velocity profiles, (1932), the relation

$$\frac{1}{C_H} = \frac{2}{C_f} + 5\left[\frac{2}{C_f}\right]^{1/2}\left\{Pr - 1 + \ln\left[1 + 5/6\,(Pr - 1)\right]\right\}$$

which for $Pr = 1$ reduces to the Reynolds analogy and for $Pr - 1$ small reduces to a form similar to that of Taylor (1916 – 1919). Comparison of this equation with the results of Dittus and Boelter (1930) was good for $Pr < 25$. This effect of the Pr may be expected when using a Reynolds analogy.

Boelter, Martinelli and Jonassen (1941) extended Von Kármán's model by including variable viscosity in the laminar sublayer and constant viscosity outside. This constant viscosity was an average viscosity, not a viscosity at an average temperature. They derived an equation by adding the three resistances

$$q/A = -k\frac{d\theta}{dy} \qquad \text{laminar layer}$$

$$\frac{q}{A\rho Cp} = -\left[\frac{\nu}{Pr} + \varepsilon\right]\frac{d\theta}{dy} \qquad \text{buffer layer}$$

$$\frac{q}{A\rho Cp} = -\varepsilon\frac{d\theta}{dy} \qquad \text{turbulent core}$$

which were in agreement when compared with the data of Morris and Whitman (1928).

Hausen (1943) used the results of Sieder and Tate (1936) and others with special attention to the effect of L/d and arrived at the function

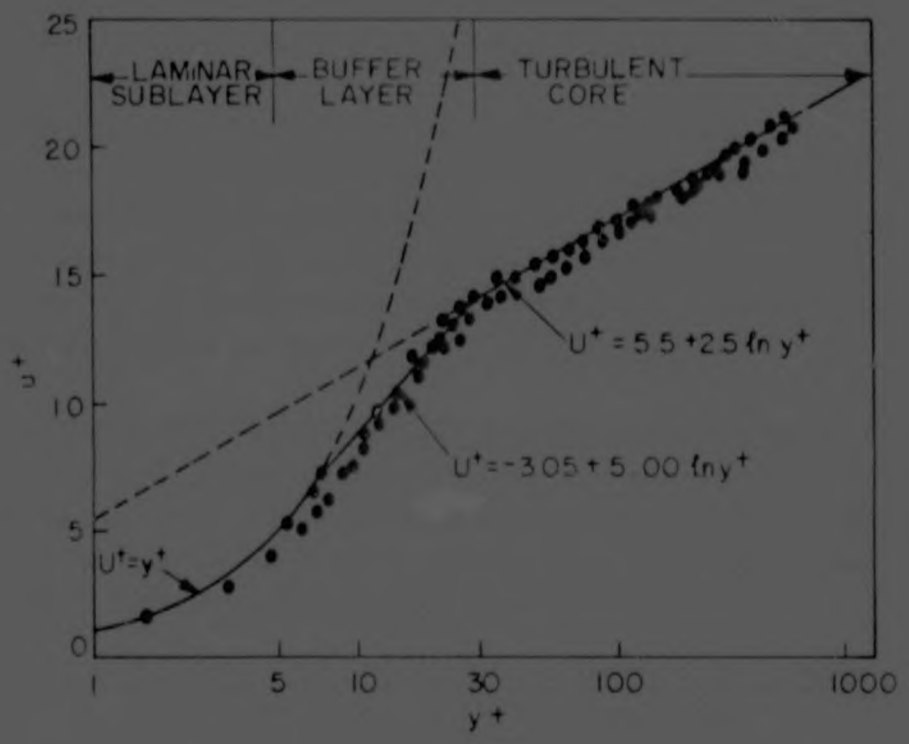
$$Nu = a(Re^{2/3} - 125)Pr^{1/3}\left[1 + (d/L)^{2/3}\right]\left[\frac{\mu}{\mu_w}\right]^{0,14}$$

This equation does not show the anomaly of zero heat transfer for infinite tube length.

Bernardo and Eian (1945) experimented with ethylene glycol and found that the Dittus and Boelter (1930) form of the equation correlated the results satisfactorily. They also noted that the Stanton number $\frac{h}{CpG}$ correlated the results better in the lower turbulent region.

Martinelli (1947) extended the model proposed by Von Kármán to include low Prandtl numbers by considering molecular conduction in the turbulent core. He also constructed a generalised velocity distribution using the results of Nikuradse (1932) and Reichardt (1943) as given in Figure 10.

FIGURE 10 Generalised velocity distribution for turbulent flow in tubes (Martinelli 1947)



Using dimensional analysis Deissler (1950) determined that the eddy conductivity could be expressed as $\varepsilon = n^2 uy$

where n^2 is experimentally determined. From this he developed velocity profiles that agreed with experimental results. Subsequently he extended this analysis (1950) to include variations in physical properties and concluded that the effect of fluid properties across the tube could be eliminated by evaluating the properties at a temperature close to the average of the wall and bulk temperatures.

Two years later Deissler and Eian (1952) extended this work to include Pr < unity and found that the predicted results agreed with experimental results for air.

Danckwerts (1951) while considering liquid-film coefficients in gas absorption proposed the *surface renewal* model. The idea of a stagnant film near the wall was abandoned and the idea of eddies continually exposing fresh sections of the surface was considered.

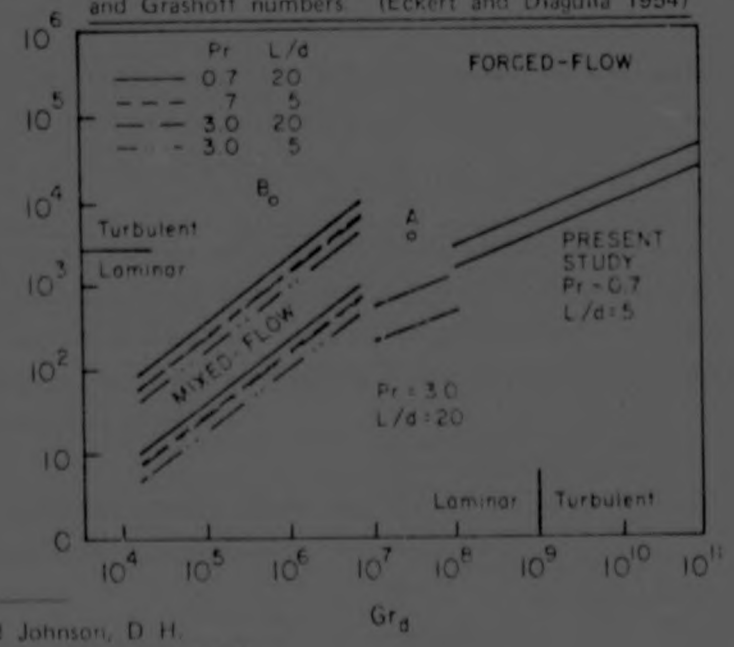
Lyon (1951), using the assumptions of fully developed velocity and temperature profiles, uniform heat flux and $\epsilon_h = \epsilon_m/Pr_t$ arrived at an integral form for the Nusselt number which was shown to be in reasonable agreement with experimental results. *He further noted from this integral form that there was an expected minimum for the Nu as Pr → 0 in turbulent flow, which was around 7.* This was later used by Churchill (1977) in his method of using asymptotes to obtain interpolatory equations.

Lin, Moulton and Putnam (1953) assumed an eddy viscosity in the region close to the wall and derived an equation that fitted experimental velocity profiles very well. This then eliminated the concept of a laminar sublayer as in the surface renewal model.

Deissler (1954) pointed out that the inadequacy of his previous results was due to the expression used for the eddy diffusivity close to the wall. A new expression, modified to account for the effect of kinematic viscosity in reducing turbulence close to the wall, gave improved results and indicated that except at low Re the entrance effect decreased with increasing Pr. His analysis also indicated that the effect of variable viscosity could be eliminated by evaluating the viscosity at a temperature which was a function of the Pr.

Eckert and Diaguila (1954) experimented with air flowing in vertical tubes and found that with properties based on a film temperature Hausen's equation underestimated the heat transfer coefficient at high Re. Using the results of Martinelli and Boelter and Watzinger and Johnson (1939)* they divided the fluid flow into the three regions of *forced, mixed* and *free* flow using the criterion $Re_x = a(PrGr_x)^{0.33}$ and plotted the results as in Figure 11.

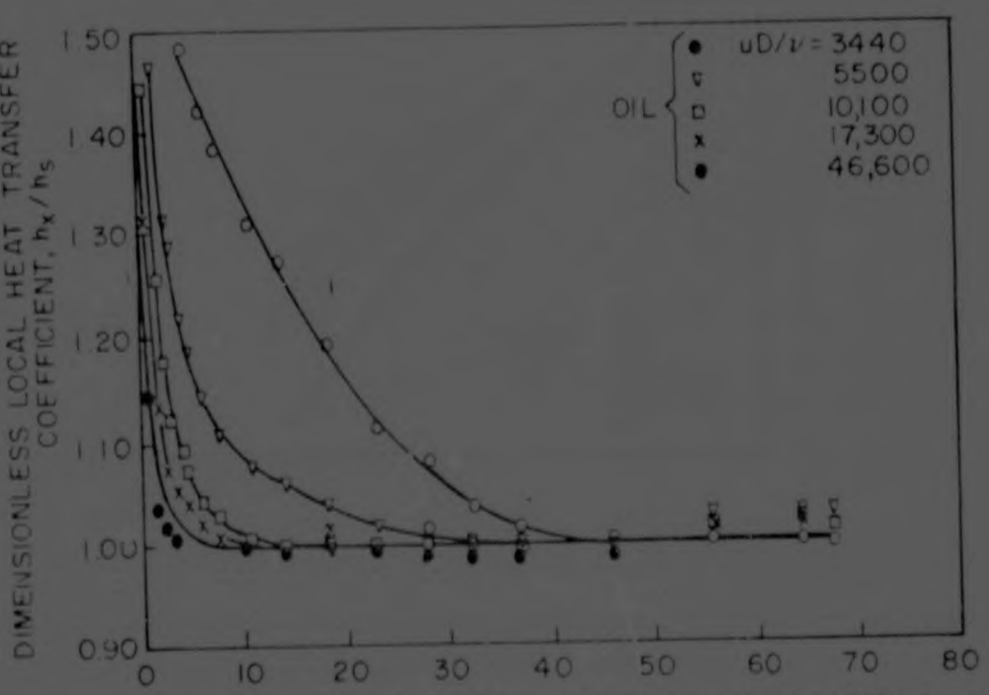
FIGURE 11 Subdivision of the flow regions as a function of the Reynolds and Grashoff numbers (Eckert and Diaguila 1954)



* Watzinger, A. and Johnson, D. H.
Forsh. ad. Geb. Ingen. 10, 182 (1939)
as referred to by Eckert and Diaguila (1954).

Hartnett (1955) experimented with oil and water with a fully developed velocity profile to determine the *thermal entrance length*, defined as the length needed for the heat transfer coefficient to reach a constant value. He concluded that at high Re the Pr had little effect on the entry length. Figure 12 is a typical result.

FIGURE 12 Thermal-Entry-Length results for oil flow in the transition region (Hartnett 1955)



Deissler (1955) analytically derived expressions for the thermal entrance length for uniform heat flux and uniform wall temperature and obtained results which agreed well with experimental results.

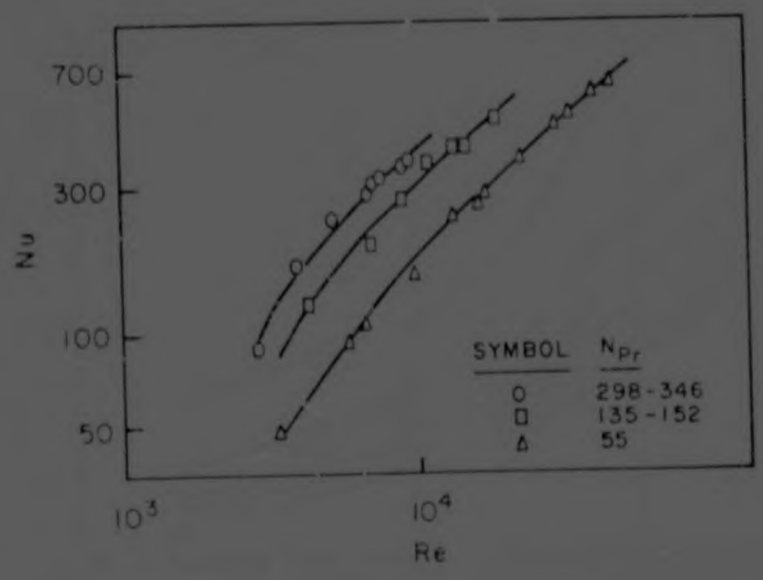
Sparrow, Hallman and Siegel (1958) analytically derived an expression for the thermal entrance length assuming a uniform temperature and a fully developed velocity profile. A similar approach to that of Graetz was used and yielded results that agreed with those of Deissler (1955). They noted that the effect of increasing Pr was to decrease the thermal entrance length, which is expected from the physics.

Friend and Metzner (1958) modified and simplified Reichardt's (1943) analysis for the case of moderate Pr to the form

$$St = \frac{1/2}{1{,}2 + (Pr - 1)b(Pr)\sqrt{1/2}}$$

where $\phi(Pr)$ was experimentally determined to be 11,8 $Pr^{-1/3}$. Interesting to note is the consistency of the results through the transition region in Figure 13.

FIGURE 13 The Nusselt number as a function of the Reynolds number for heating of molasses. (Friend and Metzner 1958)



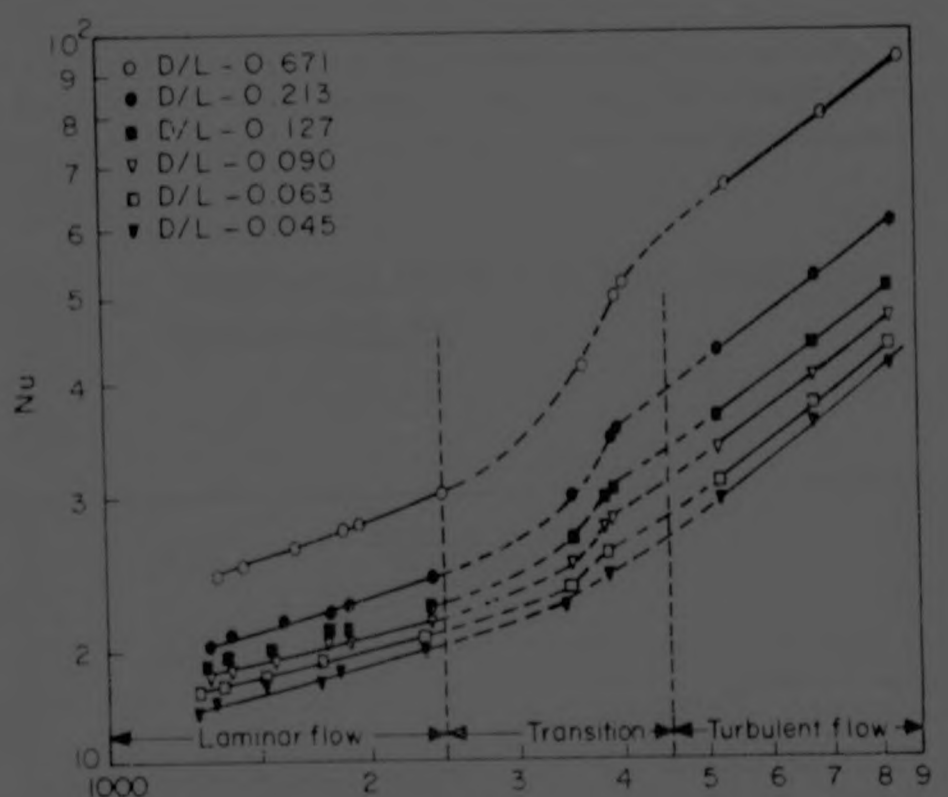
Azer and Chao (1960) modified Prandtl's mixing length hypothesis to include a continuous change of momentum and energy during the flight of an eddy. This translates to

$$\frac{\varepsilon_H}{\varepsilon_M} = \text{variable.}$$

Results from this model agreed with experimental results.

Jackson, Spurlock and Purdy (1961) experimented with air with a developing velocity profile and found an effect of tube length but not of free convection. The tubes used in the experiments were not long enough to allow the boundary layer to completely fill the tube. Their results for the lower turbulent – transitional region were consistent. See Figure 14.

FIGURE 14 Nusselt number as a function of the Reynolds Prandtl group for air. (Jackson, Spurlock and Purdy 1961)



Petukhov and Popov (1963) used a numerical procedure to solve the heat and momentum equations assuming variable physical properties, and found a more generalised form of the Lyon (1951) integral. From experimental results for constant physical properties they derived the interpolation formula.

$$Nu = \frac{f/8\ RePr}{1{,}07 + 12{,}7\sqrt{f/8}\ (Pr^{2/3} - 1)}$$

which is similar to the form of Friend and Metzner (1958). For variable physical properties Petukhov and Popov (1963) suggested the use of

$$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_f}\right]^n$$

at high temperature loads. They also mentioned the factor

$$\frac{Nu_f}{Nu_o} = \left[\frac{2}{\sqrt{\frac{T_w}{T_f}} + 1}\right]^2$$

of Kutateladze (1962) in the Russian literature.

Dipprey and Sabersky (1963) looked at the effect of surface roughness and proposed the *cavity vortex* theory. This assumes that the wall consists of small cavities and that the flow in and about these cavities consists of one or more standing vortices.

Metais and Eckert (1964) summarised the correlations for horizontal and vertical flow and presented the results in graphical form. Available experimental data did not permit the establishment of a limit between free and mixed convection in horizontal tubes. Their results are reproduced in Figures 15 and 16.

FIGURE 15 The extent of the flow regions for flow in horizontal tubes (Metais and Eckert 1964)

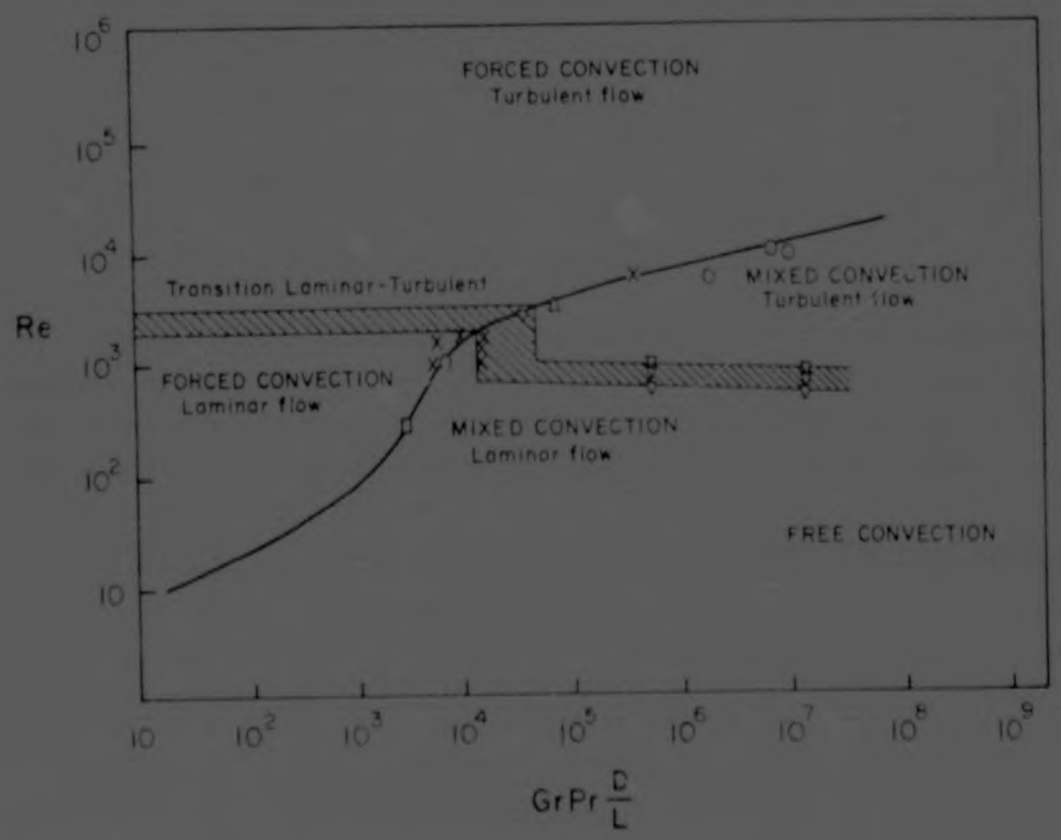
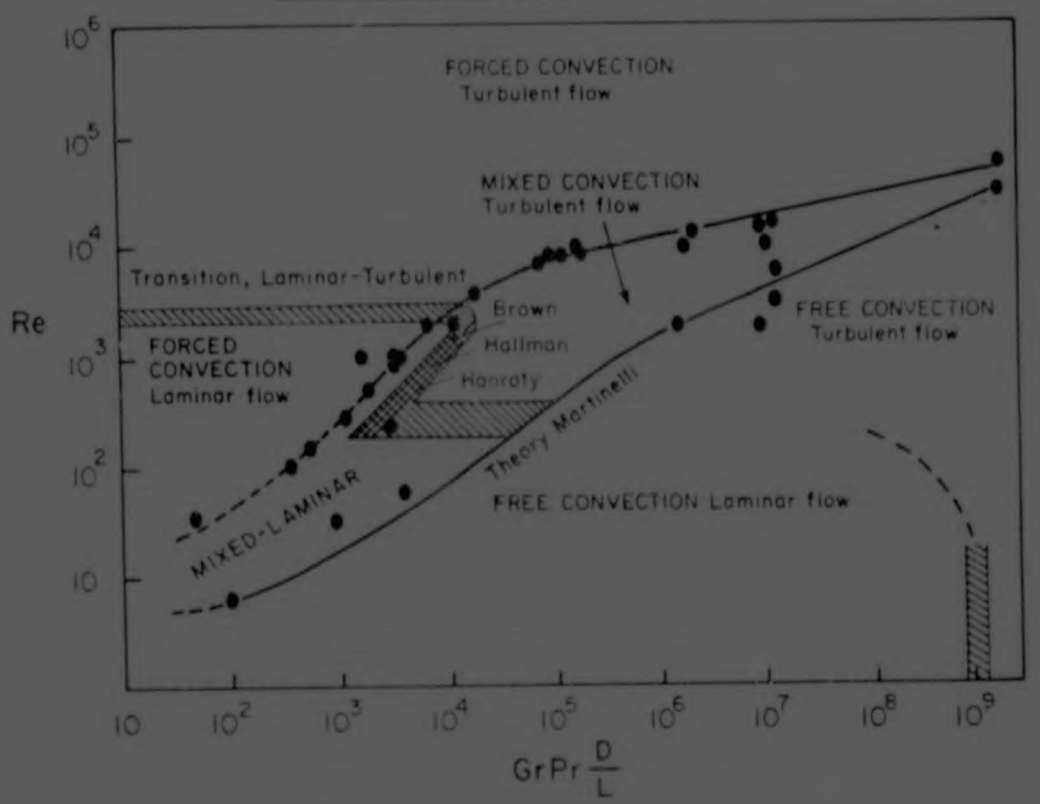
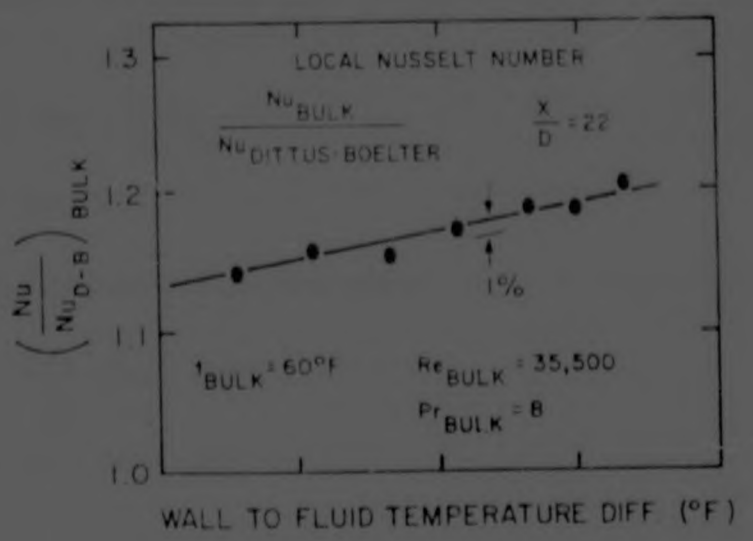


FIGURE 16 The extent of the flow regions for flow in vertical tubes (Metais and Eckert 1964)



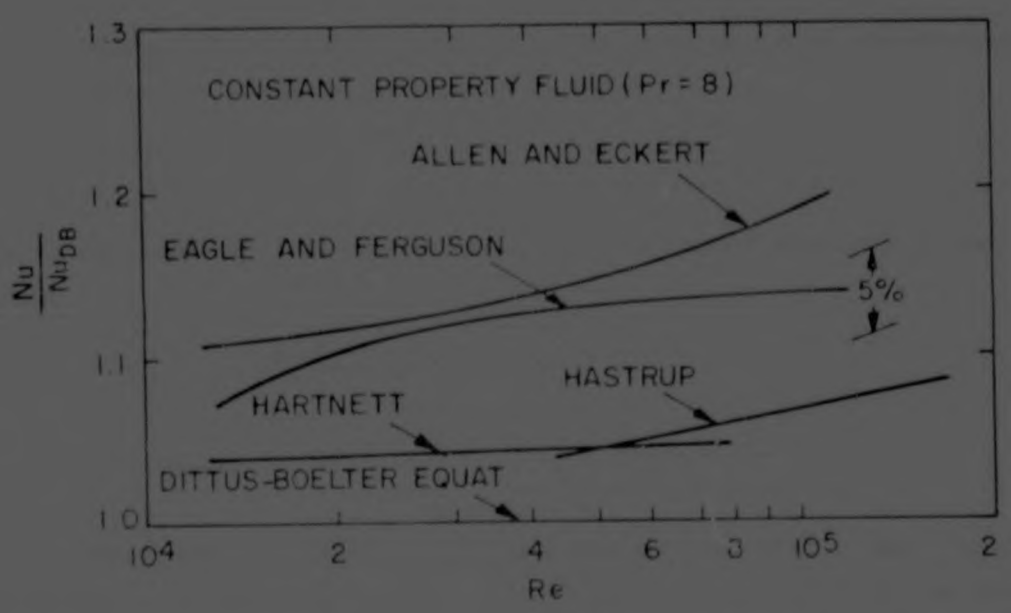
Allen and Eckert (1964) experimented with water and used the wall to bulk fluid temperature difference, to show the effect of variable physical properties, to correlate the results. Their results, compared with the equation of Dittus and Boelter (1930) are shown in Figure 17. They noted that the entrance effects were very similar to those of Hartnett (1955).

FIGURE 17 Comparison of results of Allen and Eckert with Dittus and Boelter (Allen and Eckert 1964)



They also noted that the correction factor of Sieder and Tate (1936) was not satisfactory but did not suggest an alternative. Their data are compared with those of others in Figure 18.

FIGURE 18 Comparison of heat transfer coefficients with those of other investigators. Thermally developed, constant property case: Pr 8. Uniform wall heat flux. (Allen and Eckert 1964)



Malina and Sparrow (1964) extended the work of Allen and Eckert (1964) to include oils and suggested the correction factor

$$\left[\frac{\mu_b}{\mu_w}\right]^{0.05}$$

be used for variable physical properties.

Subbotin, Ibragimov and Nomofilov (1965), after experiments over a wide range of Re and Pr, suggested that

$$Nu = 7{,}24 - \frac{9{,}5}{\log Re}$$

be used for constant properties.

Kolář (1965) derived the semi-theoretical relation

$$Nu = a\left[\frac{u_m d}{\nu\sqrt{f/8}}\right]^b Pr^{0.5}$$

where the group was called the turbulent Re_t.

Hufschmidt, Burck and Riebold (1966) suggested, after experiments with water, that the group

$$\left[\frac{Pr}{Pr_w}\right]^{0,11}$$

be used for temperature dependent properties and used with the equation of Petukhov and Popov (1963).

McEligot, Ormand and Perkins (1966) found that at $Re > 4\ 000$ the Dittus and Boelter (1920) relation was satisfactory and suggested the use of

$$\frac{Nu}{Nu_o} = \left[\frac{T_w}{T_b}\right]^{-0,5}$$

for variable properties. They used the ratio of T_w to T_b plotted against the modified Reynolds group

$$Re = \frac{4\dot{m}\,T_b}{\pi\,\mu\,d\,T_w}$$

to distinguish between the flow regions.

Gowen and Smith (1967) experimentally investigated the effect of the Pr based on a film temperature on the temperature profile. Assuming equal eddy diffusivities for heat and momentum they derived a universal temperature profile of the form

$$t^+ = A_s \ln y^+ + B$$

where A_s was constant and B_s a function of the Pr. Their experiments showed t^+ to have a dependence on the Re.

Polyakov (1968), after experimentation, concluded from the inadequacy of the Petukhov and Popov (1963) equation to correlate results at large Gr, that free convection must still be influencing heat transfer in the turbulent region.

In 1968 the Engineering Sciences Data Unit published the first of their summaries on heat transfer in pipes. These summaries are updated at regular intervals.

Lawn (1969), from a theoretical discussion, concluded that the turbulent $Pr_t = \frac{\epsilon_H}{\epsilon_m}$ must be dependent on the Re which severely limits its usefulness in practical calculations and which questions its physical significance.

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Hackl and Gröll (1969) and Hausen (1969) suggested the correction factor

$$Nu/Nu_o = 0{,}645 \left[\frac{\mu_w}{\mu_b} \right] + 0{,}355$$

be used in preference to the Sieder and Tate (1936) form. This correction was obtained after numerous experiments with oils.

McEligot, Smith and Bankston (1970), in an excellent paper, reviewed the theoretical models for turbulent flow, and using a finite difference scheme solved the flow equations assuming temperature dependent physical properties and found a substantial improvement on previous models. They concluded that mixing length models are better than eddy diffusivity models for predicting the heat transfer and suggested the Van Driest mixing length model as the best. A summary of the models is given in the Appendix 4.4.

Gregorig (1970), from theoretical considerations, arrived at a complicated correction factor for physical properties. This factor was of the form

$$K = \frac{K_\lambda K_{Pr}}{K_\mu}$$

where each K_λ is a function of the temperature and the L/d ratio.

Herbert and Sterns (1971) experimented with water flowing in vertical tubes and noted that at high Re the product GrPr had little effect, but had a strong influence at low Re.

Gross and Thomas (1972) introduced the term $\frac{dP}{dx}$ into the eddy penetration model and found that this improved the correlation. This suggests that it may be necessary to include the $\frac{dP}{dx}$ term in empirical correlations.

Launder and Spalding (1972) summarised the models of turbulence in a very readable form and this is recommended for a study of the theoretical models.

Pennell, Sparrow and Eckert (1972) and Kudva and Sesonske (1972) found from experiments that in the region near the wall and in the region near the centre line, the ratio of the turbulence intensity to the friction velocity is independent of the Re.

Zucchetto and Thorsen (1973) noted that for air the variable physical properties could be corrected by

$$\frac{Nu}{Nu_o} = \left[\frac{T_s}{T_b} \right]^{-0{,}17}$$

Mori, Sakakibara and Tanimoto (1974) analytically found that at Gz = 50, the ratio of the thermal conductivity of the wall to that of the fluid, and the thickness of the tube wall, may become significant factors on the heat transfer.

Sleicher and Rouse (1975) reviewed a few empirical equations and recommended Petukhov (1970) as the best. Using others' experimental data they developed an equation similar in form to that of Petukhov but in which the exponents of the Re and Pr were functions of the Pr.

Hrycak and Andrushkiw (1974) used Meissner's entropy principle to develop a method of predicting the critical Re and found the method to be fairly accurate.

Gregorig (1976) considered the case where the Pr number is a non-linear function of temperature, and using similarity considerations arrived at a correction factor

$$\frac{Nu}{Nu_o} = \left[\frac{Pr_b}{Pr_w}\right]^P$$

where P is a complicated function of the Re and Pr.

Hanna and Sandall (1978) developed an analytical approximation for

$$\frac{St/\sqrt{f/2}}{\left[St/\sqrt{f/2}\right]_o}$$

and found it to be in reasonable agreement with Petukhov's (1970) experimental results.

Polley (1979) using data on air, determined that the Engineering Sciences Data Unit correlations were significantly better than the Dittus and Boelter (1930) or Sieder and Tate (1936) equations.

No further papers have been traced since this.

3.2 LAMINAR FLOW CORRELATIONS

Graetz in 1885 and later Nusselt in 1910 used the following assumptions to obtain a solution for laminar pipe flow:

i) fully developed parabolic velocity profile

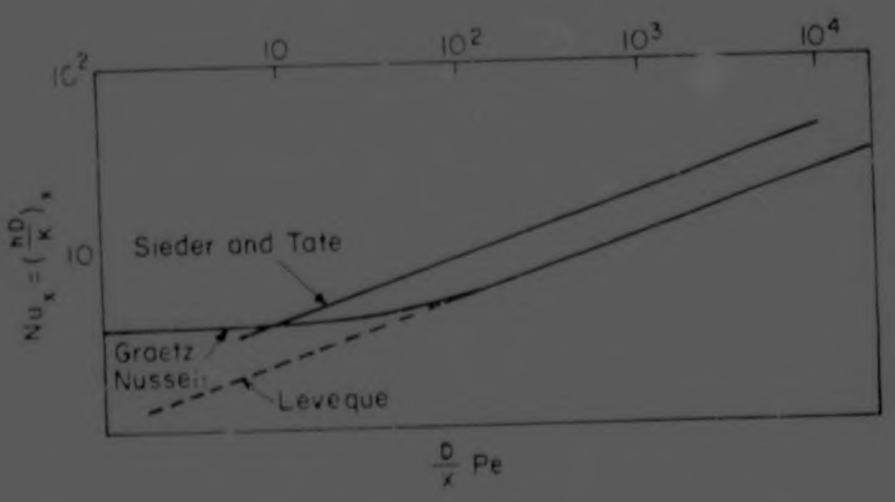
ii) constant wall temperature boundary condition

iii) negligible axial conduction

iv) constant fluid properties.

The resulting partial differential equation was solved to give a series solution for the Nusselt number as a function of the Peclet number which gives the result in Figure 19.

FIGURE 19 Local Nusselt number for laminar flow (HSU 1963)



The results of Sieder and Tate, and Leveque's extension of the Graetz result, are included in the plot for comparison.

Leveque (1928) extended this result to include a developing velocity profile, giving

$$Nu = 1.607 \left[{}^{d}/_{L} \; Re \, Pr \right]^{1/3} \qquad \text{(Figure 1)}$$

Dittus and Boelter (1930), using experimental data on the heating and cooling of oils added the term ΔT_{LM} to include variations in physical properties.

Colburn and Hougen (1930) experimented with water in vertical flow and concluded that the heat transfer was independent of the velocity and proposed an equation of the form

$$Nu = a\, Pr^{1/3}\, Gr^{1/3}$$

with properties evaluated at the film temperature.

Kirkbride and McCabe (1931) criticised the use of a parabolic velocity profile since this was applicable to isothermal cases only. From experimental results on water and oil they derived the equation

$$Nu = a + \frac{b}{RePr\,{}^{L}/_{d}} + \frac{c}{\left[RePr\,{}^{L}/_{d}\right]^{0,454}}$$

Colburn (1933) introduced the j_H factor and suggested using the term $1 + 0{,}015\,Gr^{1/3}$ to correct for bouyancy effects. Properties were based on a film temperature defined as

$$\theta_f = \theta_{avg} + {}^{1}/_{4}\left[\theta_w - \theta_{avg}\right]$$

Sieder and Tate (1936) experimented with oils and recommended the use of Colburn's (1933) correction factor for free convection. The factor

$$\frac{\mu_{bulk}}{\mu_{wall}}$$

was suggested as a correction for variable physical properties and they proposed

$$Nu = a\left[RePr\,{}^{d}/_{L}\right]^{1/3}\left[\frac{\mu_b}{\mu_w}\right]^{0,14}$$

Martinelli et al (1942) experimented with vertical flow and found an equation developed by Martinelli and Boelter

$$Nu = 1{,}75\,F_1\left[Gz + 0{,}0722F_2\left[\frac{GrPrd}{L}\right]^{0,75}_{w}\right]^{1/3}$$

was in reasonable agreement with the experimental results. They criticised the use of $1 + 0{,}015\,Gr^{1/3}$ as a multiplier as this would imply that the effect of free convection would increase with increasing Gz which was contrary to what was physically expected.

Hausen (1943) paid special attention to the effect of the ${}^{L}/_{d}$ ratio and used Sieder and Tate's and others' results to give the equation

$$Nu = a + \frac{b\,RePr\,{}^{d}/_{L}}{1 + c\left[RePr\,{}^{d}/_{L}\right]^{2/3}}$$

Kern and Othmer (1943) experimented with oils and from the results suggested the term

$$2{,}25\ (1 + 0{,}01\,Gr^{1/3})/\log Re$$

as a correction for free convection effects.

Kays (1955) numerically solved the energy equations using the velocity profiles of Langhaar and neglecting the radial velocity end axial conduction terms. Agreement with experimental results for gases was good and the correction factor

$$\frac{h}{h_{isothermal}} = \left[\frac{T_{wall}}{T_{mixed\ mean}}\right]^{-m}$$

where m = 0,25 for the heating and 0,08 for the cooling of gases, for temperature dependent properties, was advanced.

Stephan (1959) argued that for constant properties the function was of the form

$$Nu = f_1\left[\frac{L}{D}\ Pe\right]$$

and that for variable properties

$$Nu = f_2\left[\frac{L}{D}\ Re,Pr\right]$$

From semi-theoretical considerations he arrived at the interesting form

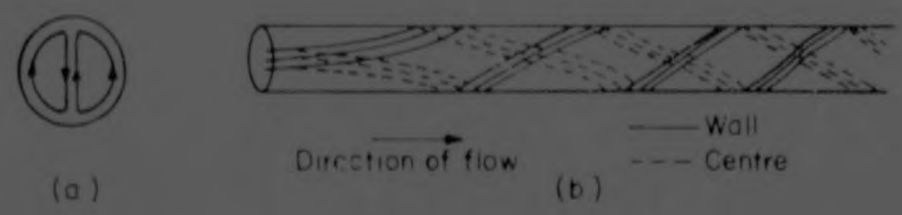
$$\overline{Nu} = 3{,}66 + \frac{0{,}0677(PrRe_D{}^{d}/L)^{1,33}}{1 + 0{,}1Pr(Re_D{}^{d}/L)^{0,83}}$$

Ede (1961) experimented with air and water and found that the data could be correlated using the Grashoff number only in the form

$$Nu = 4{,}36(1 + 0{,}06Gr^{0,3})$$

Oliver (1962) criticised the use of the term Gr^{d}/L since this produces a term in D^4 which may have a strong influence not physically expected and suggested the use of $GrPr^{L}/d$. He noted that the effect of the tube length on the free convection effects should physically be different for horizontal and vertical tubes and postulated that for horizontal tubes the flow pattern would be as in Figure 20.

FIGURE 20 Probable flow pattern in horizontal tube with heated wall, (a) end elevation and (b) horizontal elevation. (Oliver 1962)



Brown and Thomas (1965) experimented with water in horizontal tubes and found that contrary to Martinelli's work on vertical tubes, it appeared that free convection effects increased with the flow rate. This was reconciled by postulating that at higher flow rates the temperature gradient at the wall would be higher than at low flow rates and hence the free convection effect would be greater.

Mori et al (1966) found from experiments with air at high heat flux that the free convection had a pronounced effect which started at about $ReRa = 10^3$. They also noted that the critical Reynolds appeared dependent on the intensity of the secondary flow and suggested

$$Re_{cr} = 128\ (ReRa)^{1/4}$$

McComas and Eckert (1966) noticed that the effect of free convection in air flows increased as the ratio of Gr to Re increased and suggested using Oliver's (1962) equation.

Iqbal and Stachiewicz (1967) looked analytically at the effect of the tube inclination on combined free and forced convection and concluded that this had little affect on the Nu but did affect the friction factor. They noted that a temperature dependent density affected mainly the velocity field and not the temperature field.

Mori and Futagami (1967) extended the work of Mori et al (1966) and experimentally determined that the vortices in the secondary flow pattern moved closer to the wall with increasing ReRa (intensity of the secondary flow). The results are reproduced in Figure 21.

FIGURE 20 Probable flow pattern in horizontal tube with heated wall, (a) end elevation and (b) horizontal elevation. (Oliver 1962)

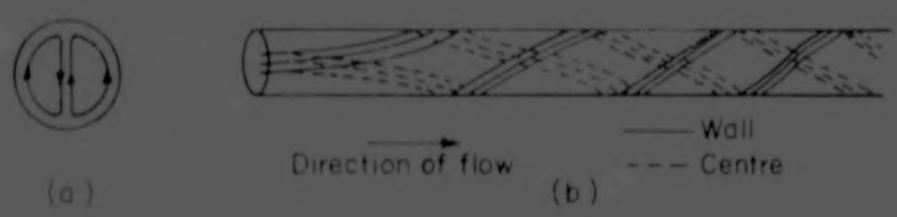


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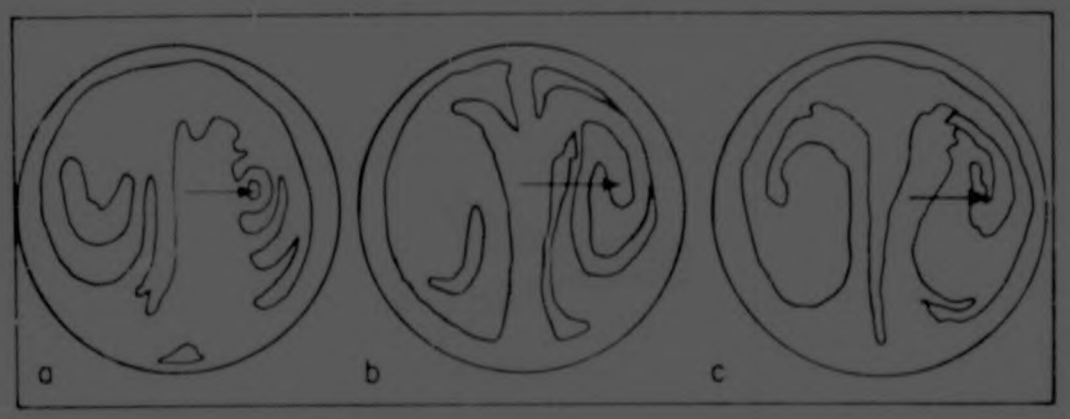
FIGURE 21 Secondary flow pattern. (Mori and Futagami 1967)

(a) ReRa = 2×10^4 (b) ReRa = 9×10^4 (c) ReRa = $1{,}6 \times 10^5$

Centres of vortices move closer to wall for increasing ReRa.



Shannon and Depew (1968) experimented with the heating of water at low Reynold-number and noticed a pronounced effect of the L/d ratio and from this concluded that for

$$\frac{(GrPr)^{1/4}}{Nu_o} < 2$$

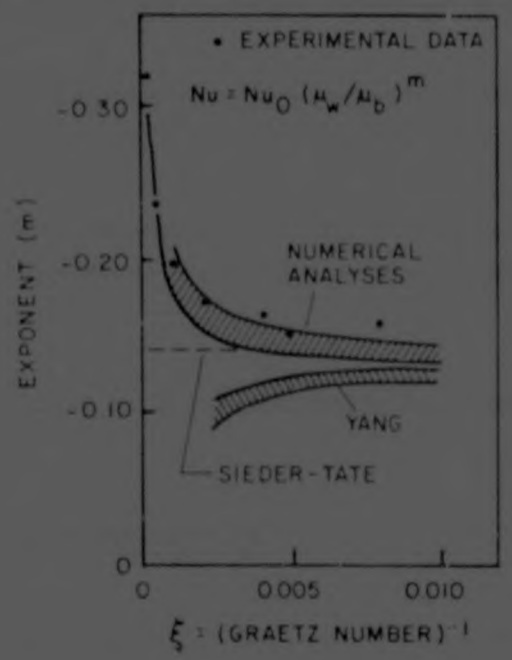
the natural convection influence was negligible.

A year later Shannon and Depew (1969) concluded from a theoretical consideration that for an experimental dependence of μ on T the temperature distribution in the fluid is a function of the viscosity ratio and the Gz only. From experiments with ethylene glycol they found the relation

$$\frac{Nu}{Nu_o} = \left[\frac{\mu_w}{\mu_b}\right]^m$$

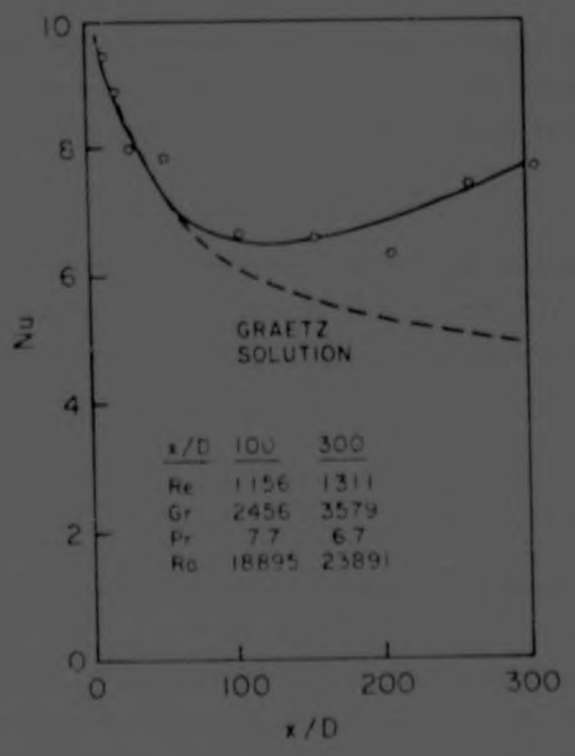
where m is specified in Figure 22.

FIGURE 22 Exponent m versus ξ. (Shannon and Depew (1969)



Kupper, Hauptmann and Iqbal (1969) experimented with uniform heat flux on water and although unable to correlate the data to show the effect of free convection, found a strong correlation with $^{L}/_{d}$ ratio as in Figure 23.

FIGURE 23 Variation of Nusselt number along tube (Kupper et al 1969)



Shah and London (1971) summarised the analytical solutions for laminar flow in a large variety of geometries. A summary of their findings is reproduced in the Appendix 4.5.

Porter (1971) summarised (without the knowledge of Shah and London) the laminar flow of Newtonian and Non-Newtonian fluids.

Depew and August (1971) experimentally determined that the effect of free convection was not correlated adequately by $GrPr\,{}^{L}/_{d}$ for ${}^{L}/_{d} < 50$ and suggested using $GzGr^{n}Pr^{m}$ since ${}^{L}/_{d}$ ceased to have significance in this range.

There appears to have been no further experimental investigation after this date.

3.3 TRANSITIONAL FLOW CORRELATIONS

Colburn (1933) and Sieder and Tate (1936) were the first to suggest heat transfer correlations in the transition region. These correlations, derived from the analagous plots for fluid friction, give a discontinuity where the extended turbulent range meets the laminar range. Colburn used the ratio

$$\frac{\mu_{average}}{\mu_{film}}$$

to correlate the results and Sieder and Tate used $\frac{\mu_{average}}{\mu_{wall}}$.

Norris and Streid (1940) noted that there may be a distinct region where the flow *"may be neither laminar nor turbulent, but of an intermediate nature"*. They noted that the transition region began at approximately Re = 2 100 and extended as far as Re = 10 000 for large values of ${}^{L}/_{d}$.

Norris and Sims (1942) experimented with the cooling of oils of high Pr in vertically downward flow in the semi-turbulent region of $1\,500 < Re \leq 11\,000$. The velocity profile was undeveloped and ${}^{L}/_{d}$ was fixed in the experiments. Using physical properties based on an average inlet and outlet temperature they derived the equation

$$\frac{h}{C_p G} = 0{,}0067 Pr^{-0{,}8} \left[\frac{\mu}{\mu_{wall}} \right]^{0{,}14}$$

which correlated the results better than the methods of Colburn or of Sieder and Tate.

Senecal and Rothfus (1953) investigated laminar to turbulent transition in isothermally flowing air and noted that transition was limited to a narrow range of Re 2100 to 2800 and marked by extreme changes in the velocity profile. They found that friction data were independent of the

direction of approach to the transition region but did not extend their experiments to non-isothermal flow.

Ede (1961) experimented with air and water in the transition region but could not find any satisfactory method of correlating the data.

Petersen and Christiansen (1966) looked specifically at the transition region for Newtonian and Non-Newtonian fluids and empirically derived the relation

$$St = St_{Re = 10\,000} \left[\frac{St_{\text{critical Re}}}{St_{Re = 10\,000}}\right]^{\phi(Re)} \left[\frac{Re}{10\,000}\right]^{\beta} \left[\frac{10\,000}{2\,100}\right]^{\beta\,\phi(Re)}$$

where β is the slope of the heat transfer correlation curve chosen for the turbulent region and

$$\phi(Re) = 1{,}635 \log\left[\frac{1}{1{,}133}\ \frac{(Re - 710)}{(Re - 1\,800)}\right]$$

They found that $\phi(Re)$ effectively eliminated the effect of L/d ratio.

McEligot, Ormand and Perkins (1966) experimented in the transition region but the results could not be correlated.

Bankston (1970) experimented with hydrogen and helium undergoing turbulent to laminar transition at high heat loads and found that transition occurred at a bulk Re far in excess of the minimum turbulent Re for adiabatic flows. He also likened the process to the reverse transition of boundary layers in external flows.

McEligot, Coon and Perkins (1970) summarised the research on laminarisation in tubes and pointed out that the acceleration parameter used in external flows could be used for internal flows for predicting laminarisation.

Coon and Perkins (1970) looked specifically at the reverse transition of air and concluded that if

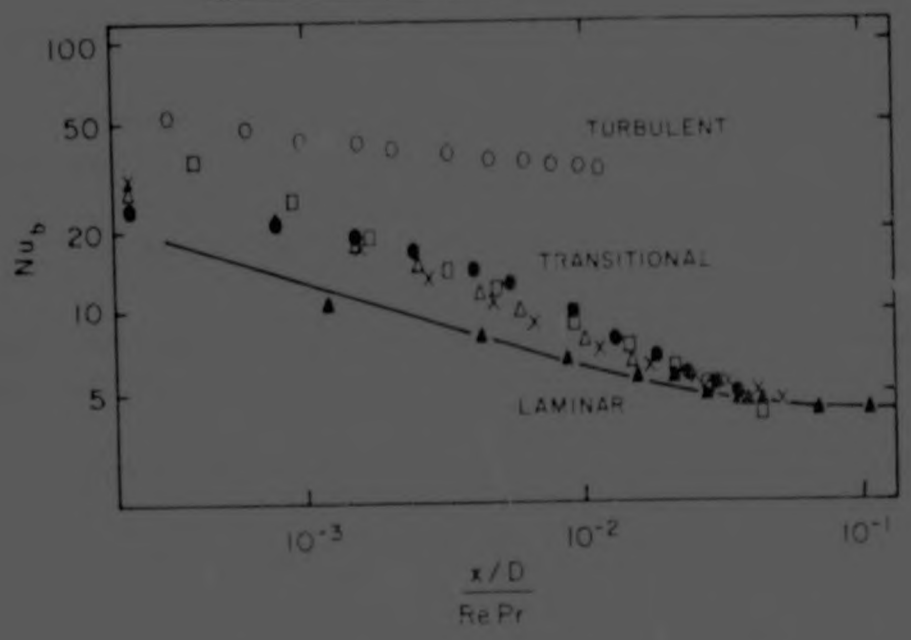
$$\frac{4q}{G^2 dT\, C_p} \approx 1{,}5 \times 10^{-6}$$

based on bulk properties was exceeded for an initially turbulent flow then the turbulent flow correlations did not predict the heat transfer coefficient acceptably. They further noted that downstream of the point

$$\left[\frac{x}{D}\right] \approx (2 \times 10^{-5})\,(T_{inlet})\,(Re_{b\ inlet})^2 \left[\frac{T_w}{T_b}\right]_{max}$$

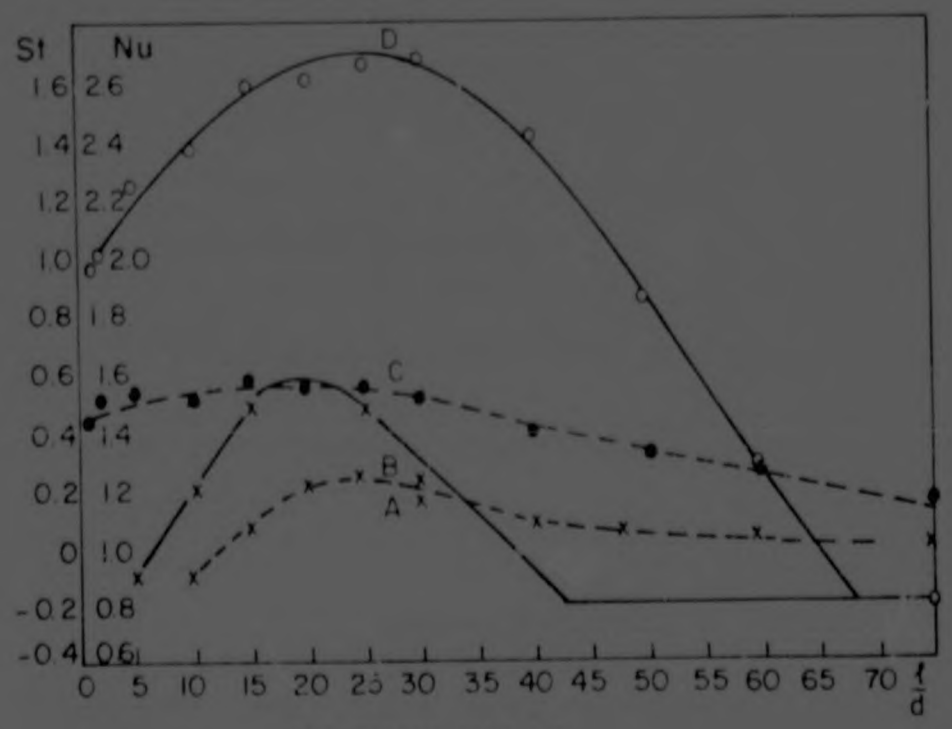
the laminar flow equations predicted the heat transfer coefficient. Their results in the transition region do not appear to be haphazard. See Figure 24.

FIGURE 24 Bulk Nusselt number data compared to Graetz Solution length: laminar flow, turbulent flow and flow with reverse transition. (Coon and Perkins 1970)



Dyban and Epik (1971) looked at reverse transition using air flow in nozzles and concluded that the local Nu could not be expressed in terms of the Re with only one exponent for the Re. However, they did not suggest a correlation. Figure 25 is a plot of the results obtained.

FIGURE 25 Change in the Re exponent for transient flow regimes (solid lines – local heat transfer; dashed lines – average heat transfer): A, B – conical convergent nozzle; C, D – Vitoshinskiy nozzle. (Dyban and Epik 1971)



Kuznetsova (1972) looked at the transitional flow of oil (without saying whether this was reverse transitional or laminar to turbulent) and from the results recommended that the equation

$$\overline{Nu} = 0{,}013\, Re^{0.8}\, Pr^{0.4} \left[\frac{\mu}{\mu_w}\right]^{0.11}$$

be used. The results were well correlated by this. However, much of the original text was lost in translation.

Pechenegov (1975) experimented with air undergoing reverse transition and found that the results could be correlated using the equation

$$Nu_x = cRe_x^{\,n}$$

where c and n are functions of L/d. This was in accordance with the expectations of Dybon and Epik (1971) that no single exponent could be used for the Re.

Churchill (1977) proposed a comprehensive correlating equation for the total flow region

$$Nu = \left\{ Nu_{laminar}^{d} + \left\{ Nu_{transitional}^{c} + \left[\left(Nu_{Pe \to 0}^{Pr \to 0} \right)^{b} + \left\{ \left(Nu_{Re \to \infty}^{Pr \to 0} \right)^{a} + \left(Nu_{Re \to \infty}^{Pr \to \infty} \right)^{a} \right\}^{b/a} \right]^{c/b} \right\}^{d/c} \right\}^{1/d}$$

The individual expressions for the Nu's were found from expressions in the literature. In particular, in the transition region, Churchill proposed using the asymptote

$$Nu_{transitional} = Nu_{laminar,Re=2\,100} \left[e^{(Re - 2\,200)/730} \right]$$

and a value of the exponent $c = -2$. For the entire turbulent and transition region the function

$$Nu_T = \left[\frac{1}{Nu_{turbulent}^{2}} + \frac{1}{Nu_{transitional}^{2}} \right]^{-1/2}$$

was proposed.

Since this time no literature has appeared.

4 APPENDIX

4.1 REFERENCES

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4.2 FLUID INDEX

This index is for tracing original experimental data on the heat transfer to fluids in pipes. The heat transfer conditions are limited to the internal flow of Newtonian fluids in laminar, transitional or turbulent flow in horizontal or vertical circular pipes.

Cross-referencing with the EQUATION INDEX is intended and actual data may be obtained from that index or the original literature.

The work of *Porter (1971)* is recommended for references for Non-Newtonian fluids in laminar flow and for Newtonian fluids in different geometries.

FLUID	RANGE		REFERENCE
	Reynolds	Orientation	
Air	Unknown	Horizontal	Nusselt (1917)
	300 – 100 000	Horizontal	Ede (1961)
	Laminar	Horizontal	Mori and Futagami (1967)
	100 – 900	Horizontal	McComas and Eckert (1966)
	100 – 13 000	Horizontal	Mori et al (1966)
	Laminar and turbulent	Horizontal	Jackson et al (1961)
	Turbulent		Subbotin et al (1965)
	4 500 – 140 000	Horizontal	Kolář (1965)
	Turbulent	Vertical	Gowen and Smith (1967)
	Turbulent	Horizontal	Pechenegov (1975)
	Turbulent	Horizontal	Zucchetto and Thorsen (1973)
	Turbulent		Rice (1924)
	Turbulent	Horizontal	Deissler and Eian (1952)
	Turbulent	Vertical	Eckert and Diaguila (1954)
Askarel	3 500 – 11 000	Vertical	Norris and Sims (1942)
Acetone	1 000 – 100 000	Horizontal	Sherwood and Petrie (1932)
Benzene	1 000 – 100 000	Horizontal	Sherwood and Petrie (1932)
n-Butanol	1 000 – 100 000	Horizontal	Sherwood and Petrie (1932)
Butanol	5 000 – 300 000	Horizontal	Bernado and Eian (1945)
Corn syrup	Turbulent	Horizontal	Friend and Metzner (1958)
Ethanol	Laminar	Horizontal	Depew and August (1971)
	Laminar	Horizontal	Oliver (1962)
Ethylene glycol	6 – 300	Horizontal	Shannon and Depew (1969)
	5 000 – 300 000	Horizontal	Bernado and Eian (1945)
	Turbulent	Vertical	Gowen and Smith (1967)

FLUID	RANGE		REFERENCE
	Reynolds	Orientation	
Glycerol	Laminar	Horizontal	Sieder and Tate (1936)
	Laminar	Horizontal	Depew and August (1971)
	Laminar	Horizontal	Oliver (1962)
Helium	1 450 – 45 000	Vertical	McEligot et al (1966)
Kerosene	1 000 – 100 000	Horizontal	Sherwood and Petrie (1932)
Mercury	Turbulent		Subbotin et al (1965)
Molasses	Turbulent	Horizontal	FrienJ and Metzner (1958)
Nitrogen	1 450 – 4 500	Vertical	McEligot et al (1966)
Oil	Laminar	Horizontal	Sieder and Tate (1936)
	Laminar	Vertical	Martinelli et al (1942)
	Laminar and turbulent	Horizontal	Dittus and Boelter (1930)
	Laminar	Horizontal	Kern and Othmer (1943)
	Laminar	Vertical	Test (1968)
	2 300 – 48 000	Vertical	Hartnett (1955)
	3 500 – 11 000	Vertical	Norris and Sims (1942)
	Turbulent	Horizontal	Malina and Sparrow (1964)
	4 000 – 11 000	Horizontal	Hackl and Grõll (1969)
	Turbulent		McAdams and Frost (1922)
Gas oil	1 862 – 43 000	Horizontal	Morris and Whitman (1928)
Straw oil	1 862 – 43 000	Horizontal	Morris and Whitman (1928)
Transformer Oil		Horizontal	Kuznetsova (1972)
Fuel oil		Horizontal	Kuznetsova (1972)
Light motor oil	1 862 – 43 000	Horizontal	Morris and Whitman (1928)
Velocite oil	Laminar and turbulent	Horizontal	Keevil and McAdams (1929)

FLUID	RANGE		REFERENCE
	Reynolds	Orientation	
Spindle oil	Laminar and turbulent	Horizontal	Keevil and McAdams (1929)
Light oil	Laminar	Horizontal	Kirkbride and McCabe (1931)
Rabbeth oil	Laminar and turbulent	Horizontal	Keevil and McAdams (1929)
Heavy fuel oil	Laminar	Horizontal	Kirkbride and McCabe (1931)
Sodium	Turbulent		Subbotin et al (1965)
Sugar solution	Turbulent	Horizontal	Friend and Metzner (1958)
Water	1 862 – 43 000	Horizontal	Morris and Whitman (1928)
	Laminar	Vertical	Colburn and Hougen (1930)
	Laminar	Horizontal	Kirkbride and McCabe (1931)
	1 000 – 100 000	Horizontal	Sherwood and Petrie (1932)
	Laminar	Horizontal	Sieder and Tate (1936)
	Laminar	Vertical	Martinelli et al (1942)
	Laminar	Horizontal	Depew and August (1971)
	100 – 2 000	Horizontal	Kupper et al (1969)
	300 – 100 000	Horizontal	Ede (1961)
	2 300 – 48 000	Vertical	Hartnett (1955)
	Laminar	Horizontal	Oliver (1962)
	Laminar	Horizontal	Brown and Thomas (1965)
	120 – 2 300	Horizontal	Shannon and Depew (1968)
	5 800 – 71 000	Vertical	Herbert and Sterns (1971)
	14 000 – 50 000		Walker and Bott (1973)
	5 000 – 300 000	Horizontal	Bernado and Eian (1945)
	14 000 – 500 000	Vertical	Dipprey and Sabersky (1963)
	13 000 – 111 000	Horizontal	Allen and Eckert (1964)
	Turbulent	Horizontal	Malina and Sparrow (1964)
	4 500 – 140 000	Horizontal	Kolář (1965)
	Turbulent		Subbotin et al (1965)
	20 000 – 640 000	Horizontal	Hufschmidt et al (1966)
	Turbulent		McAdams and Frost (1922)
	Turbulent		Rice (1924)

FLUID	RANGE		REFERENCE
	Reynolds	Orientation	
Water	Turbulent		McAdams and Frost (1924)
	Turbulent	Horizontal	Eagle and Ferguson (1930)
	Turbulent and laminar	Horizontal	Dittus and Boelter (1930)
	Turbulent	Horizontal	Lawrence and Sherwood (1931)
	Turbulent	Horizontal	Nusselt (1931)

4.3 EQUATION INDEX

This index is used in conjunction with the FLUID INDEX for tracing equations or direct experimental results. Likewise the heat transfer conditions are limited to the internal flow of Newtonian fluids in laminar, transitional or turbulent flow in horizontal or vertical circular pipes.

Both theoretically based and empirical equations are incorporated.

For a more complete review of the theoretical equations the works of *Shah and London (1978)* for laminar flow and *Launder and Spalding (1972), Petukhov (1970), McEligot et al (1970) and Reynolds and Cebeci (1976)* for turbulent flow are recommended.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, CO_2

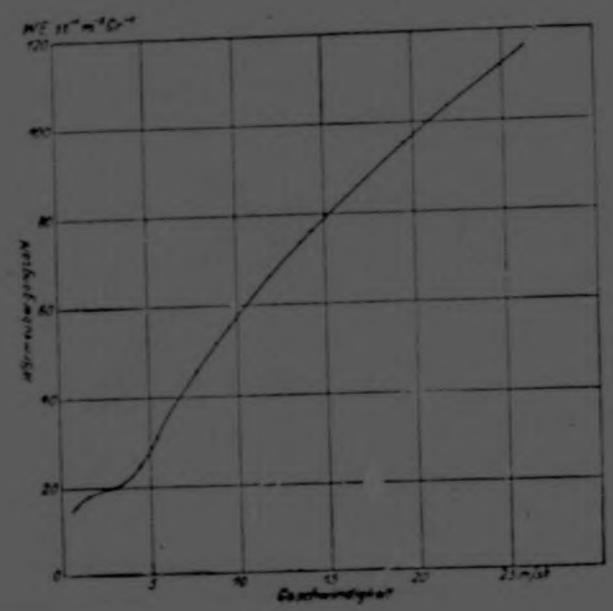
METHOD: Steam heating in double pipe exchanger.

AUTHOR(S): Nusselt, W.

SOURCE: Z. VDI, Band 53, Nr 44, 1808 (1909)

EQUATION PROPOSED:

$$h = 15{,}90 \frac{k_{\text{wall temp}}}{d^{0.214}} \left[\frac{uCp}{k_{\text{average temp}}} \right]^{0.786}$$



REYNOLDS: Turbulent

PRANDTL: ~0,7

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: Various

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Constant wall temperature (condensing steam)

AUTHOR(S): Nusselt, Wilhelm Von

SOURCE: Z. VDI, Band 61, Nr 33, 685 (1917)

EQUATION PROPOSED:

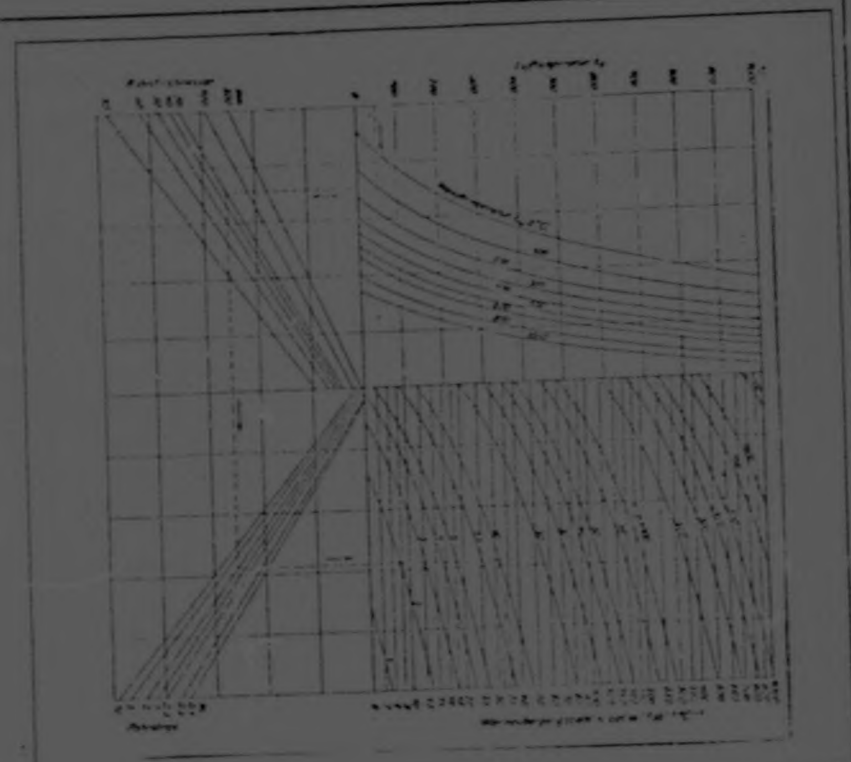
$$\frac{hmd}{k_m} = 0{,}03622 \left[d/L\right]^{0{,}054} \left[\frac{du\rho_m Cp_m}{k_m}\right]^{0{,}786}$$

$$\boxed{Nu = 0{,}03622 \left[d/L\right]^{0{,}054} \left[RePr\right]^{0{,}786}}$$

$$\lambda_m = \frac{1}{T_w - T_o} \int_{T_o}^{T_w} \lambda dT$$

T_o = gas temperature

T_w = wall temperature



REYNOLDS: Unknown

PRANDTL: Unknown

GRASHOF: Unknown

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: Various

AUXILIARY INFORMATION Paper in German and the layout is confusing.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, Oil

METHOD: Data of other workers

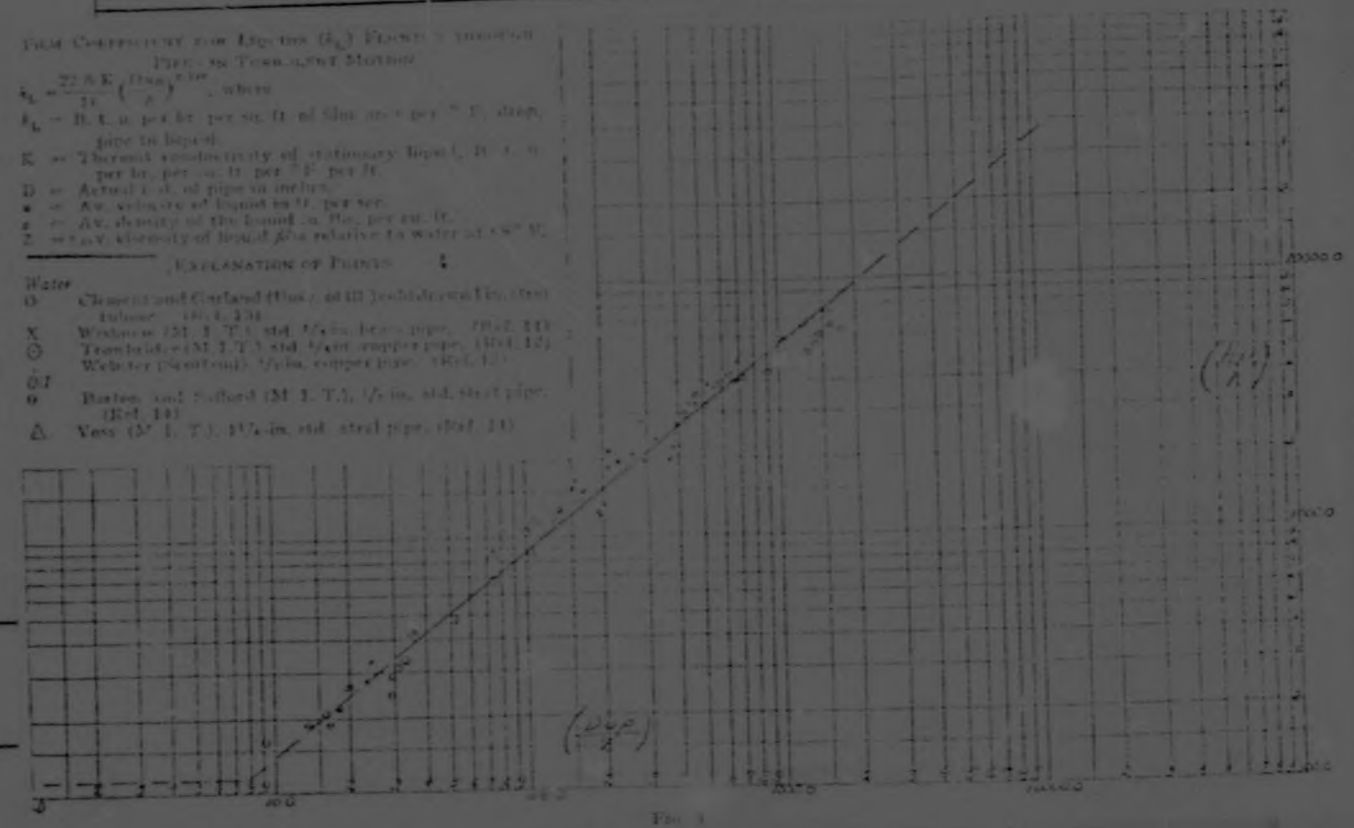
AUTHOR(S): McAdams, W. H.
Frost, T. H.

SOURCE: Ind. Eng. Chem., 14, 1101 (1922)

EQUATION PROPOSED:

$$h = \frac{22,6}{D}\left[\frac{Duo}{z}\right]^{0,796}$$

z = the viscosity at the average film temperature referred to water at 68°F.



REYNOLDS: Turbulent **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, Water

METHOD: Not given but assumed to be constant wall temperature.

AUTHOR(S): Rice, C W.

SOURCE: Ind. Eng. Chem., 16, (5), 460 (1924).

EQUATION PROPOSED:

$$W_c = \left[\frac{Ak\Delta t}{K_o D}\right] \cdot \left[\frac{\mu Cp}{k}\right]^{\beta} \cdot \left[\frac{\rho Dv}{\mu}\right]^{\alpha} \quad \text{(watts)}$$

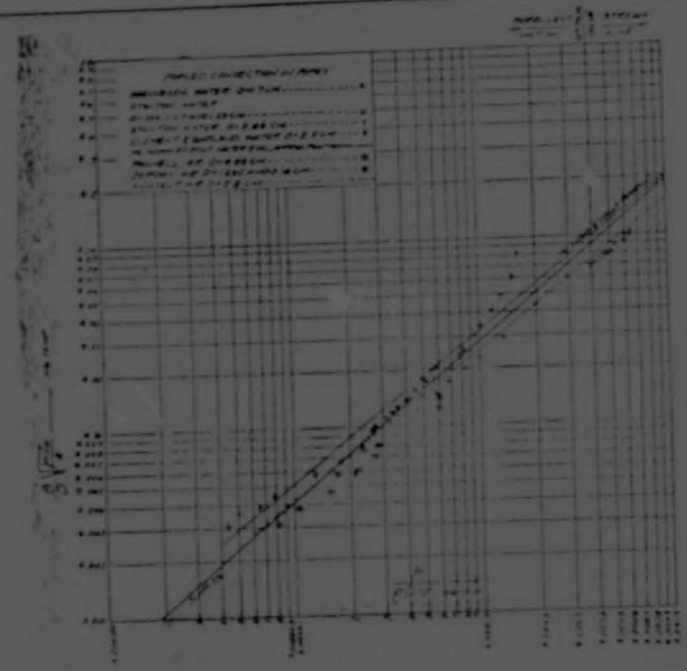
For smooth pipes : $K_o = 60$, $\beta = \frac{1}{3}$, $\alpha = \frac{5}{6}$

A = area of surface

Δt = temperature difference

If R = mechanical resistance, dynes/cm^2, then

$$W_c = \left[\frac{Ak\Delta tR}{v\mu}\right] \cdot \left[\frac{\mu Cp}{k}\right]^{\beta} \quad \text{(watts)}$$



Where $B = K_o D \left[\frac{k}{\mu Cp}\right]^{\beta} \left[\frac{\mu}{\rho Dv}\right]^{\alpha}$

REYNOLDS: Turbulent

PRANDTL: 2,48 - 7,35

GRASHOF: Unknown

HORIZONTAL/VERTICAL: Not given

LENGTH/DIAMETER: Not given

AUXILIARY INFORMATION Fluid variables taken at the average film temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Using the data of Stanton, Webster and Bray and Sayler.

AUTHOR(S): McAdams, W H
Frost, T H

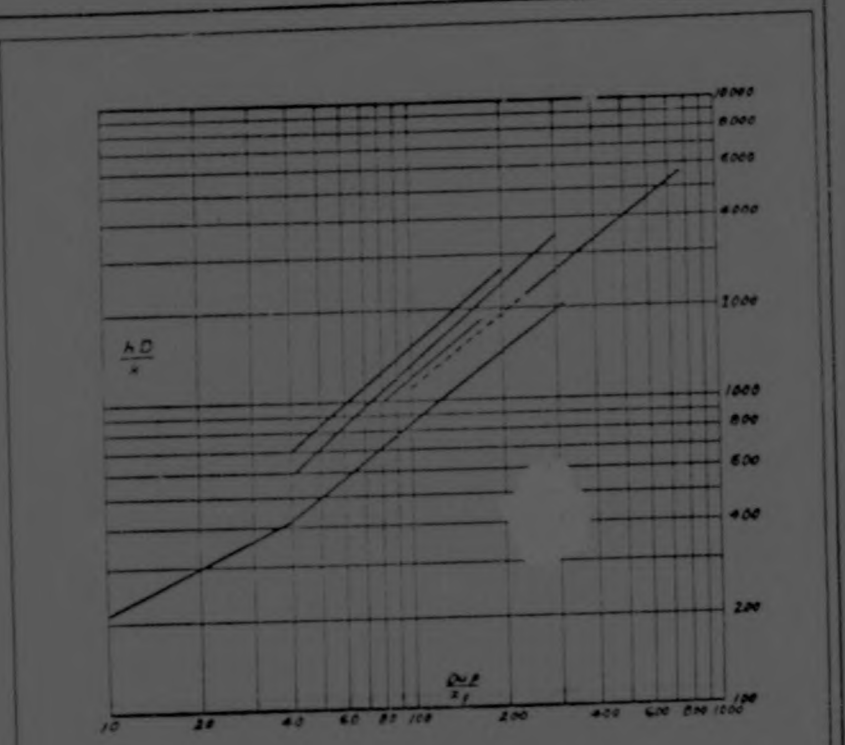
SOURCE: Refrig. Eng., 10, (9), 323 (1924).

EQUATION PROPOSED:

$$h = \frac{138}{D^{0,2}} \left[\frac{u}{z_t}\right]^{0,8} \left[1 + \frac{50D}{L}\right] \left[\frac{BTU}{hrft^2\ ^oF}\right]$$

where z is the viscosity related to the viscosity at 68^oF.
k is taken as constant 0,329.BTU /$hrft^2\ ^oF$

The graph is a plot of the various data showing the spread of results.



REYNOLDS: Turbulent
PRANDTL:
GRASHOF:
HORIZONTAL/VERTICAL: Unknown
LENGTH/DIAMETER: > 35

AUXILIARY INFORMATION L/D must be greater than 35

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Gas oil, Straw oil, Light motor oil, Water.

METHOD: Steam heating, water cooling of double pipe heat exchanger.

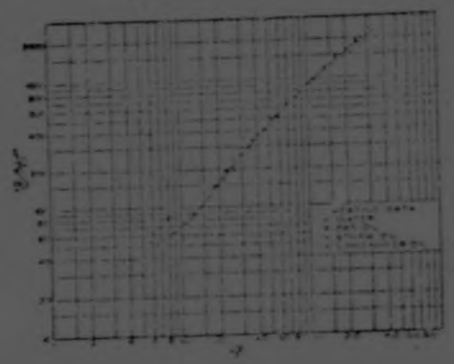
AUTHOR(S): Morris, F H
Whitman, W G

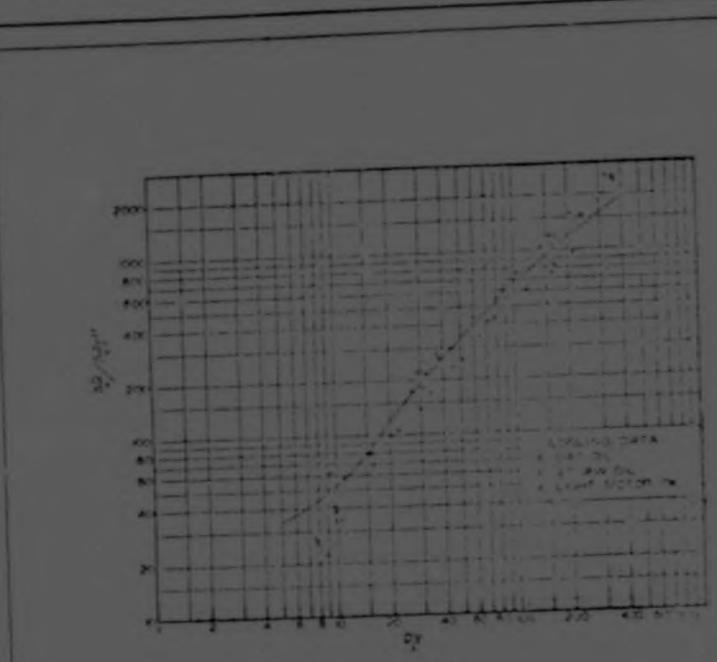
SOURCE: Ind. Eng. Chem., 20, (3), 234 (1928).

EQUATION PROPOSED: To non-dimensionalise:

$$Nu = \frac{Nu_{mw}}{12} \qquad Re = 124{,}13\ Re_{mw} \qquad Pr = 2{,}42\ Pr_{mw}$$

Use is made of the graphs rather than an equation. The parameters given are dimensioned and must be adjusted as above.





REYNOLDS: 1 862 → 43 445

PRANDTL: 2,83 → 750

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 196

AUXILIARY INFORMATION Physical properties at main bulk temperature
12 inch entrance region.

HEAT TRANSFER COEFFICIENTS IN TUBES

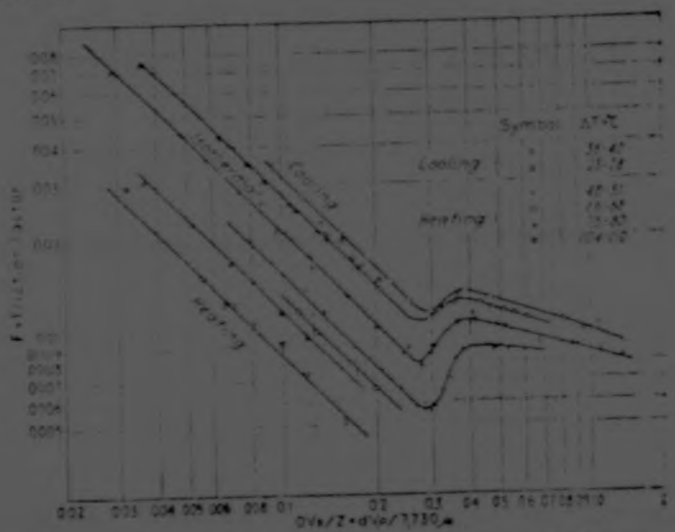
FLUID(S): Velocite oil, Spindle oil, Rabbeth oil.

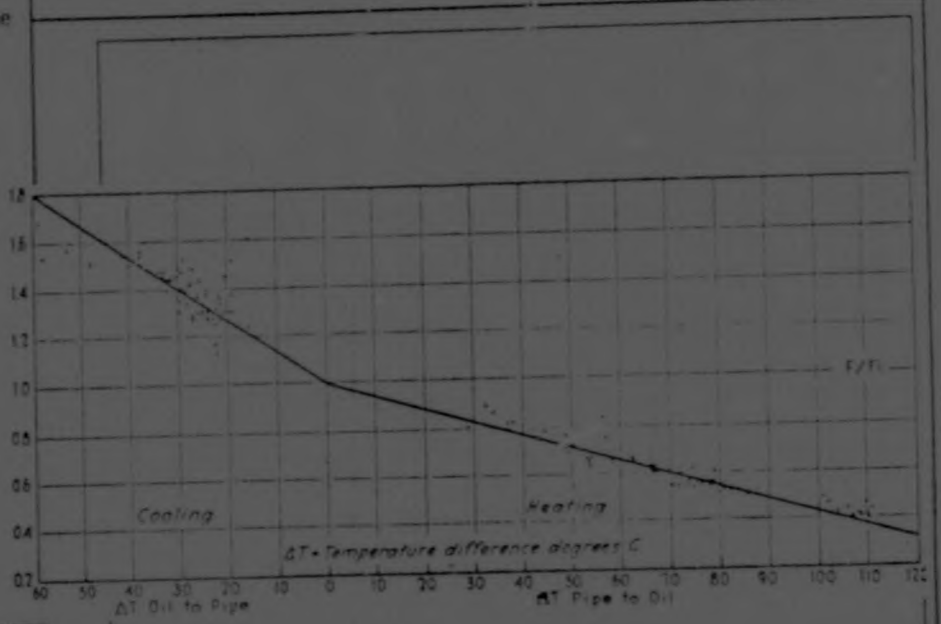
METHOD: Steam heating and water cooling of a jacketed pipe.

AUTHOR(S): Keevil, C S
McAdams, W H

SOURCE: Chem. and Met. Engr. 36, (8), 464 (1929)

EQUATION PROPOSED: Graphs are presented showing the variation of friction factor (fanning) with the difference between pipe and fluid average temperature.





REYNOLDS: Viscous and turbulent

HORIZONTAL/VERTICAL: Horizontal

PRANDTL:

GRASHOF:

LENGTH/DIAMETER: 105, 110

AUXILIARY INFORMATION Entrance region of 100D. Properties at main bulk temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Electrical heating using alternating current

AUTHOR(S): Eagle, A
Ferguson, R M

SOURCE: Proc. Inst. Mech. Engrs. (London). (1930).

EQUATION PROPOSED:

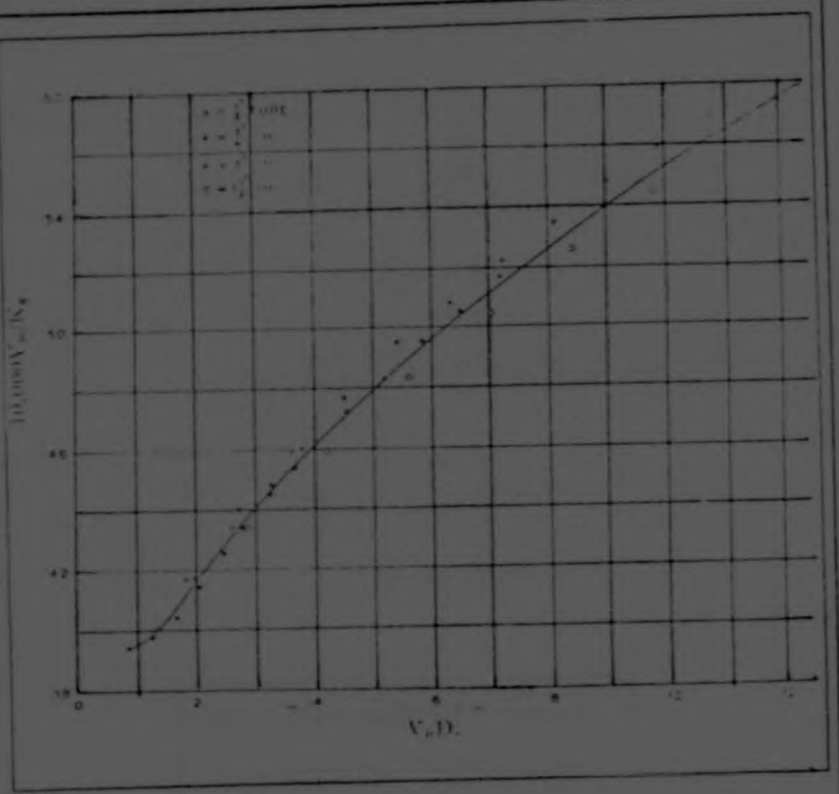
$$h_o - h = \left[\frac{\beta Pr}{\alpha + \beta(Pr - 1)}\right]^2 \left[\frac{-1}{2Pr}\ \frac{dPr}{dt}\right] H$$

H = heat flux in BTU/ft^{2}hr

h_o = coefficient at $H = 0$ in BTU/hr ft^2 ^oF and is referred to the tube wall temperature.

V_o = mass velocity

The graph shows experimental results at a temperature of 60^oF.



REYNOLDS: Turbulent
HORIZONTAL/VERTICAL: Horizontal
PRANDTL: 2,99 → 10,6
GRASHOF:
LENGTH/DIAMETER: 48 → 144

AUXILIARY INFORMATION α and β are determined by experiment such that for water $\frac{\beta Pr}{\alpha + \beta(Pr - 1)} = \frac{Pr}{2{,}45 + Pr}$

HEAT TRANSFER COEFFICIENTS IN TUBES

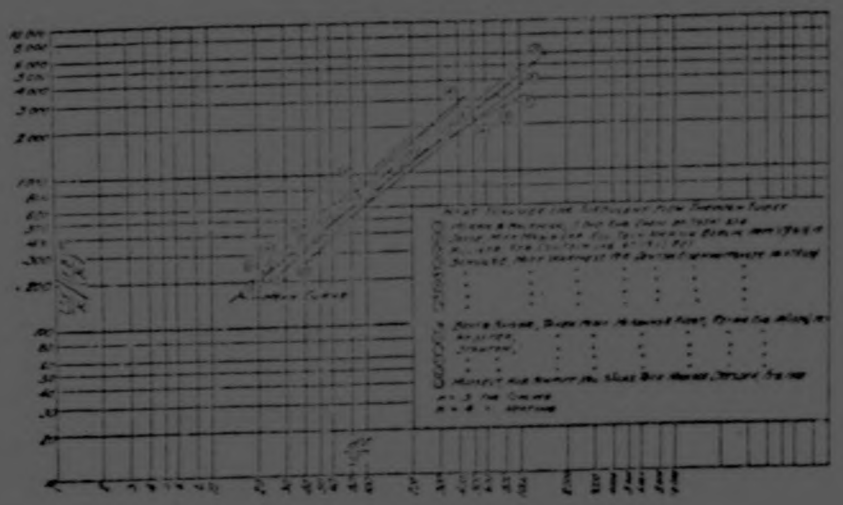
FLUID(S): Oils, water

METHOD: Results of others used.

AUTHOR(S): Dittus, F W
Boelter, L M K

SOURCE: Univ. Calif. Pubs. in Eng. 2, (13) 443 (1930).

EQUATION PROPOSED:



$$\frac{hd}{k} = 19{,}5\left[\frac{dv}{z}\right]\left[\frac{Cpz}{k}\right]^{n} \quad \text{Turbulent flow}$$

$n = 0{,}3$ cooling

$= 0{,}4$ heating

$$h = 20\left[\frac{k\Delta T_{LM}}{d}\right]^{1/3} S^{2/3} Cp\left[\frac{uds}{\mu}\right]^{1/12}\left[1 + \frac{50}{r}\right] \quad [\text{BTU/hr ft}^2\ {}^{o}\text{F}]$$

in viscous flow

$Nu = 0{,}024\ Re^{0,8} p_r^{0,4}$ heating

$Nu = 0{,}026\ Re^{0,8} p_r^{0,3}$ cooling

REYNOLDS: Turbulent & Laminar **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION z is the viscosity. Properties are taken at the mean steam temperature. S is the specific gravity.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Steam heating or water cooling. Upward and downward flow.

AUTHOR(S): Colburn, A P
Hougen, O A

SOURCE: Ind. Eng. Chem. 22, 522 (1930)

EQUATION PROPOSED:

$$h = 0{,}128\ k_f \left[\frac{Cp\,\mu_f}{k_f}\right]^{1/3} \left[\frac{D^2 \rho_f^2 \beta_f \Delta t g}{\mu^2_f}\right]^{1/3}$$

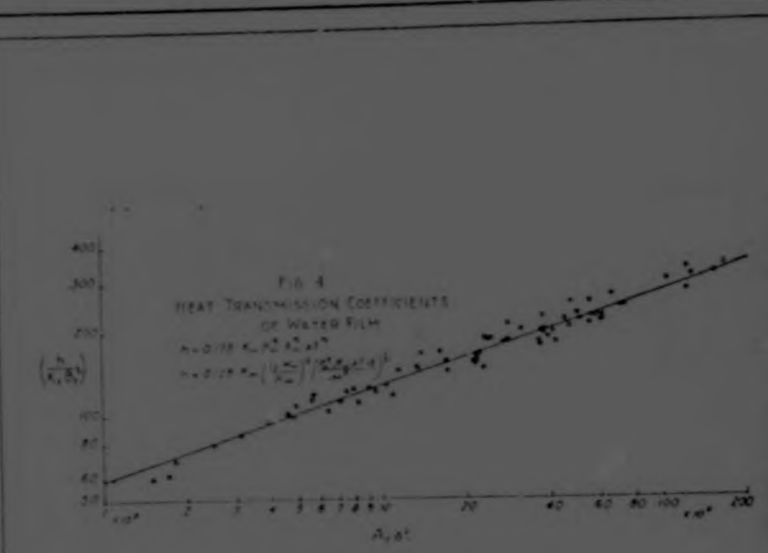
[BTU/hr ft^2 ^{0}F]

$h_w = 0{,}42 t_w^3 \sqrt{\Delta t}$

$= 0{,}37 t_f^3 \sqrt{\Delta t}$ for upward flow [BTU/hr ft^2 ^{0}F]

$h_w = 0{,}44 t_f^3 \sqrt{\Delta t}$ for downward flow

Δt is the temperature drop across film.



REYNOLDS: Laminar
PRANDTL:
GRASHOF:
HORIZONTAL/VERTICAL: Vertical
LENGTH/DIAMETER:

AUXILIARY INFORMATION
t_f = average temperature of water film
t_w = temperature of main water stream

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Gases

METHOD:

AUTHOR(S): Stender

Wiss Veröffentlich Siemens–Kazern 9, 88 (1930)

SOURCE: Lawrence & Sherwood (1931)

EQUATION PROPOSED:

$$\frac{hd}{k} = C_1\left[\frac{du\rho Cp}{k}\right]^{0.75} + C_2\left[\frac{du\rho Cp}{k}\right]\left[\frac{d}{L}\right]$$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Condensing steam in a surrounding jacket.

AUTHOR(S): Lawrence, A E
Sherwood, T K

SOURCE: Ind. Eng. Chem. 23, 301 (1931)

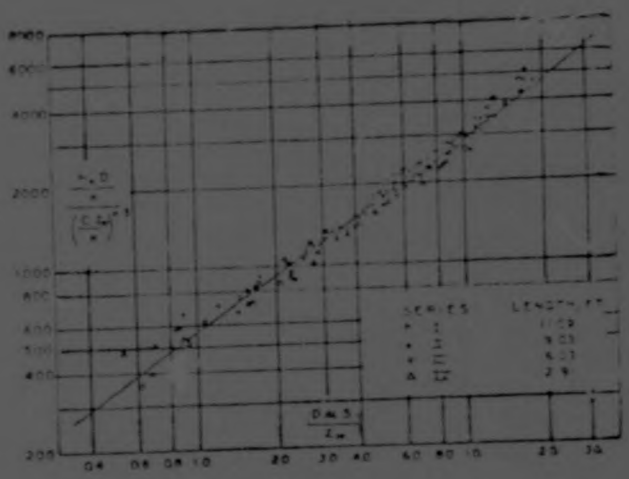
EQUATION PROPOSED:

$$\frac{h_o D}{k} = 450 \left[\frac{D u \Delta}{zm}\right]^{0,7} \left[\frac{Cpzm}{k}\right]^{0,5}$$

zm is the viscosity in centipoises taken at arithmetic mean film temperature

Δ is the specific gravity.

$$Nu = 0,056\ Re^{0,7} Pr^{0,5}$$



REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 59 → 224

AUXILIARY INFORMATION The experiments were conducted particularly to look at tube length effect. Arithmetic mean temperature for properties.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, light oil, heavy fuel oil.

METHOD: Electric heating.

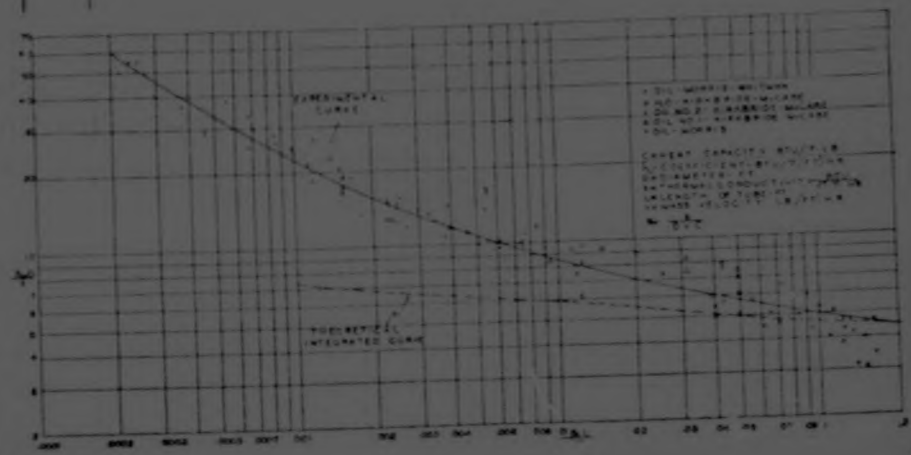
AUTHOR(S): Kirkbride, C G
McCabe, W L

SOURCE: Ind. Eng. Chem., 23, 625 (1931)

EQUATION PROPOSED:

$$\frac{h_{av} D}{k} = 3{,}65 + \frac{0{,}0065}{(Pe\ L/D)} + \frac{0{,}513}{(Pe\ L/D)^{0{,}454}}$$

Properties taken at arithmetic mean fluid temperature.



REYNOLDS: Viscous flow

HORIZONTAL/VERTICAL: Horizontal

PRANDTL:

GRASHOF:

LENGTH/DIAMETER: 150

AUXILIARY INFORMATION 5 ft calming section

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, light oil, heavy fuel oil.

METHOD: Electric heating.

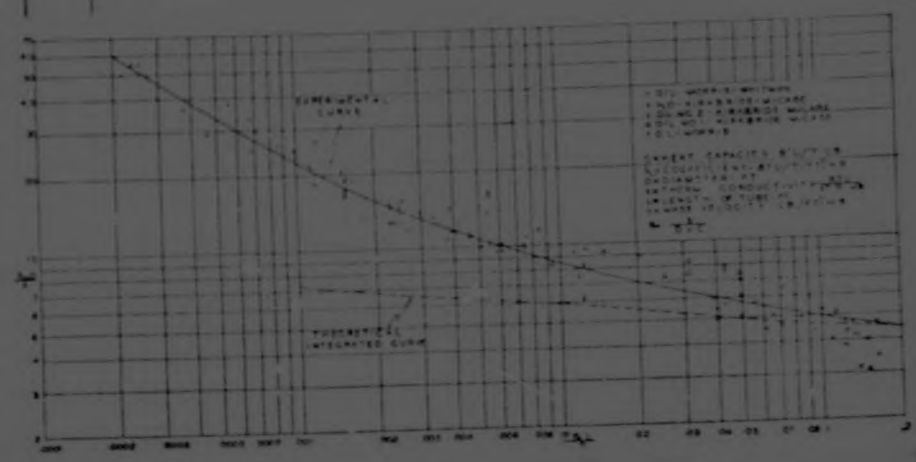
AUTHOR(S): Kirkbride, C G
McCabe, W L

SOURCE: Ind. Eng. Chem., 23, 625 (1931)

EQUATION PROPOSED:

$$\frac{h_{av} D}{k} = 3{,}65 + \frac{0{,}0065}{(Pe\ ^{L}/_{D})} + \frac{0{,}513}{(Pe\ ^{L}/_{D})^{0{,}454}}$$

Properties taken at arithmetic mean fluid temperature.



REYNOLDS: Viscous flow

HORIZONTAL/VERTICAL: Horizontal

PRANDTL:

GRASHOF:

LENGTH/DIAMETER: 150

AUXILIARY INFORMATION 5 ft calming section

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Using data of Burbach, Akad. Leipzig 1930

AUTHOR(S): Nusselt, W

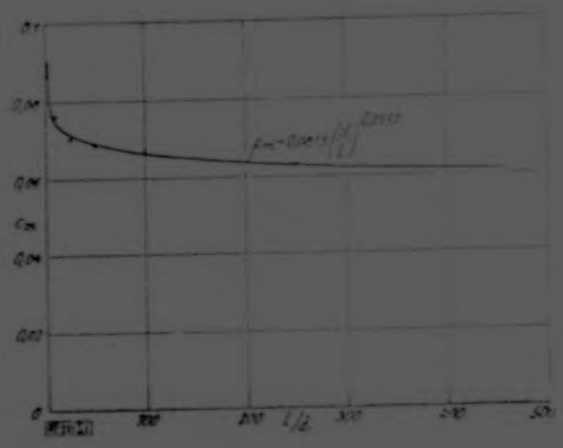
SOURCE: Forsch Gebiete Ingw. Band 2, Nr 3, 309 (1931)

EQUATION PROPOSED:

$$\frac{hd}{k_m} = 0{,}0855 \left[d/L\right]^{0{,}0552} \left[\frac{\rho u d}{\mu}\right]^{0{,}764} \left[\frac{C_p \mu}{k}\right]^{0{,}355}$$

Nusselt suggested that this formula would apply to laminar flow as well.

Using the data of Eagle and Ferguson the exponents found were 0,819 and 0,365.



The effect of tube length

REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 10 → 400

AUXILIARY INFORMATION The article is in German.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, Acetone, Benzene, Kerosene, n-butyl alcohol.

METHOD: Jacketed pipe, steam and hot water heating.

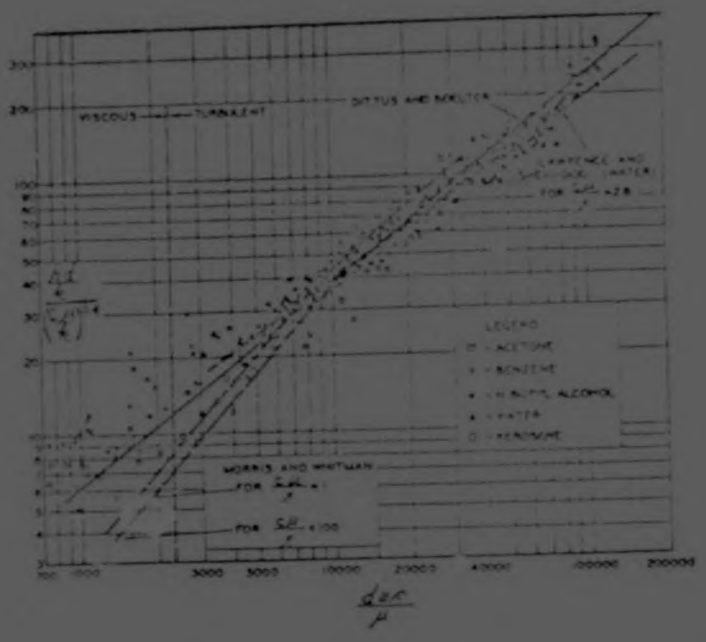
AUTHOR(S): Sherwood, T K
Petrie, J M

SOURCE: Ind. Eng. Chem., 24, 736 (1932)

EQUATION PROPOSED:

$$\frac{hd}{k} = 0{,}024 \left[\frac{du\rho}{\mu}\right]^{0{,}80} \left[\frac{Cp\mu}{k}\right]^{0{,}40}$$ turbulent flow.

Properties taken at the arithmetic mean temperature.



REYNOLDS: 1 000 → 100 000 **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 97

AUXILIARY INFORMATION 55 inch developing section.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): All fluids

METHOD: Combined data

AUTHOR(S): Colburn, A P

SOURCE: Trans. AIChE. 29, 175 (1933)

EQUATION PROPOSED:

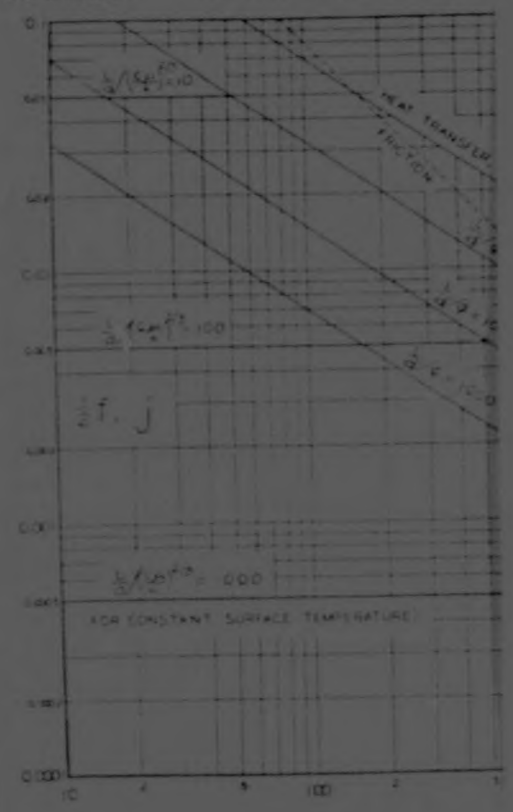
$$j = \left[\frac{h}{CpG}\right]\left[\frac{Cp\ \mu_f}{k}\right]^{2/3} = 0{,}0007 + 0{,}065\left[\frac{dG}{\mu_f}\right]^{-0{,}32}$$

Properties at film temperature. Turbulent flow.

$$j = \left[\frac{h_a}{CpG}\right]\left[\frac{Cp\mu}{k}\right]^{2/3} = 1{,}5\left[\frac{dG}{\mu}\right]^{-2/3}\left[\frac{L}{d}\right]^{-1/3}$$

Properties at arithmetic mean. Laminar flow.

Multiply by $(1 + 0{,}015\ Gr^{1/3})\left[\frac{\mu_a}{\mu_f}\right]^{-1/3}$ for large Gr.



REYNOLDS: All **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION $t_f = t_a + \frac{1}{2}(t_w - t_a)$ in turbulent flow, and $t_f = t_a + \frac{1}{4}(t_w - t_a)$ in laminar flow.

$$\phi = \left[\frac{\mu_a}{\mu_f}\right]^{-1/3}(1 + 0{,}015\ Gr^{1/3})$$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): All fluids

METHOD: Combined data

AUTHOR(S): Colburn, A P

SOURCE: Trans. AIChE. 29. 175 (1933)

EQUATION PROPOSED:

$$j = \left[\frac{h}{CpG}\right]\left[\frac{Cp\ \mu_f}{k}\right]^{2/3} = 0{,}0007 + 0{,}065\left[\frac{dG}{\mu_f}\right]^{-0{,}32}$$

Properties at film temperature. Turbulent flow.

$$j = \left[\frac{h_a}{CpG}\right]\left[\frac{Cp\mu}{k}\right]^{2/3} = 1{,}5\left[\frac{dG}{\mu}\right]^{-2/3}\left[\frac{L}{d}\right]^{-1/3}$$

Properties at arithmetic mean. Laminar flow.

Multiply by $(1 + 0{,}015\ Gr^{1/3})\left[\frac{\mu_a}{\mu_f}\right]^{-1/3}$ for large Gr.

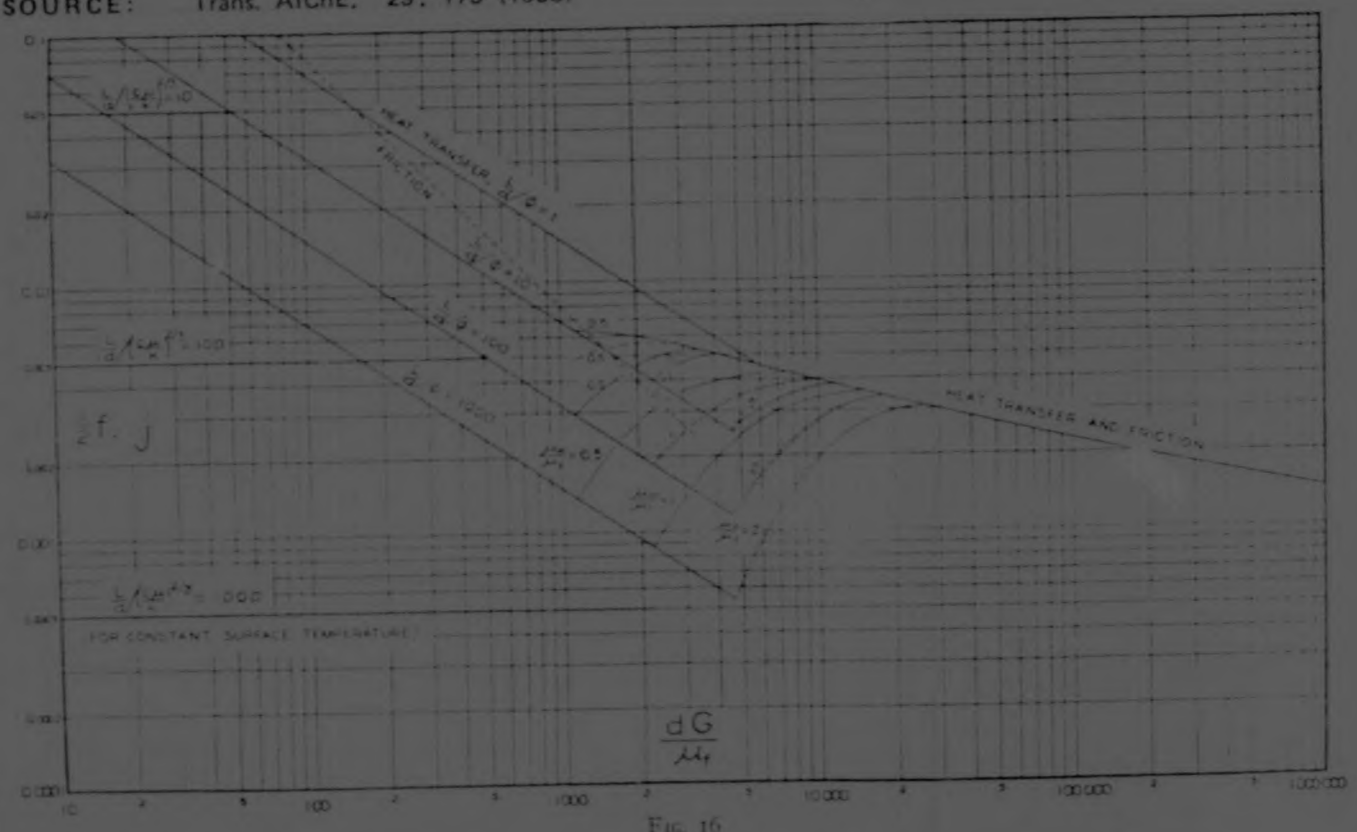


Fig. 16

REYNOLDS: All **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION $t_f = t_a + \frac{1}{2}(t_w - t_a)$ in turbulent flow, and $t_f = t_a + \frac{1}{4}(t_w - t_a)$ in laminar flow.

FLUID(S):

METHOD:

AUTHOR(S): Kraussold, H

Forsch Ing. Wes. Bd 4, 39 (1933)

SOURCE: Hausen, H. (1943)

EQUATION PROPOSED:

$$Nu = 0{,}032\ Re^{0.8}\ Pr^{0.37}\left[\frac{d}{L}\right]^{0.054} \quad \text{heating}$$

$$Nu = 0{,}032\ Re^{0.8}\ Pr^{0.3}\left[\frac{d}{L}\right]^{0.054} \quad \text{cooling}$$

REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil, glycerol, water

METHOD: Water heating and cooling

AUTHOR(S): Sieder, E N
Tate, G E

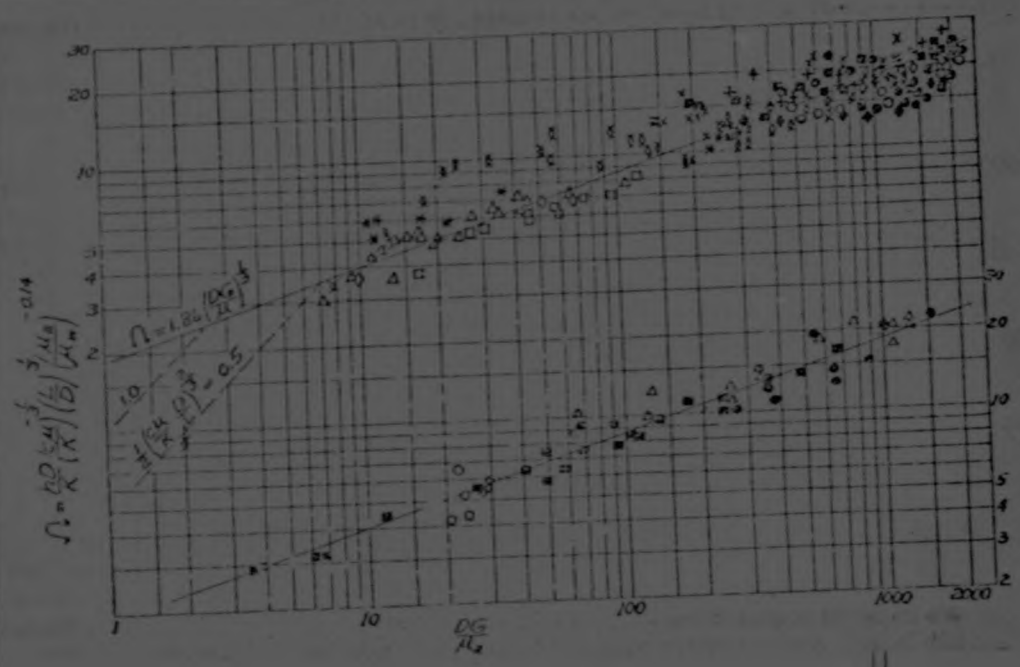
SOURCE: Ind. Eng. Chem. 28, 1429 (1936)

EQUATION PROPOSED:

$$Nu = 1{,}86\ Re^{1/3}\ Pr^{1/3}\left[\frac{L}{D}\right]^{-1/3}\left[\frac{\mu_a}{\mu_w}\right]^{0{,}14}$$

Multiply by $0{,}8(1 + 0{,}015\ Gr^{1/3})$ for $Gr > 25\,000$

The equation was developed for viscous flow and in form was found to apply in the turbulent region as well.



REYNOLDS: Viscous
PRANDTL:
GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 90

AUXILIARY INFORMATION 2 ft calming section. Properties evaluated at main bulk temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Heating and cooling of fluids without change in state.

METHOD:

AUTHOR(S): Drew, T B
Hottel, H C
McAdams, W H

SOURCE: Trans. AIChE 32, 271 (1936)

EQUATION PROPOSED:

Turbulent flow

$$\left[\frac{h}{CpG}\right]\cdot\left[\frac{Cp\mu}{k}\right]^{n} = 0{,}023\left[\frac{DG}{\mu}\right]^{-0{,}2}$$

$n = 0{,}6$ heating

$n = 0{,}7$ cooling

Streamline flow

$$\left[\frac{h_{am}D}{k}\right] = 1{,}65\left[\frac{WCp}{kN}\right]^{1/3}\left[\frac{\mu}{\mu_s}\right]^{1/3}\left[1 + 0{,}015\,Gr^{1/3}\right]$$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION Results are presented with no basis for choice.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil, water

METHOD: Steam heating

AUTHOR(S): Martinelli, R C; Southwell, C J; Alves, G; Craig, H L; Weinberg, E B; Lansing, N F; Boelter, L M K

SOURCE: Trans. AIChE, 493 (1942)

EQUATION PROPOSED:

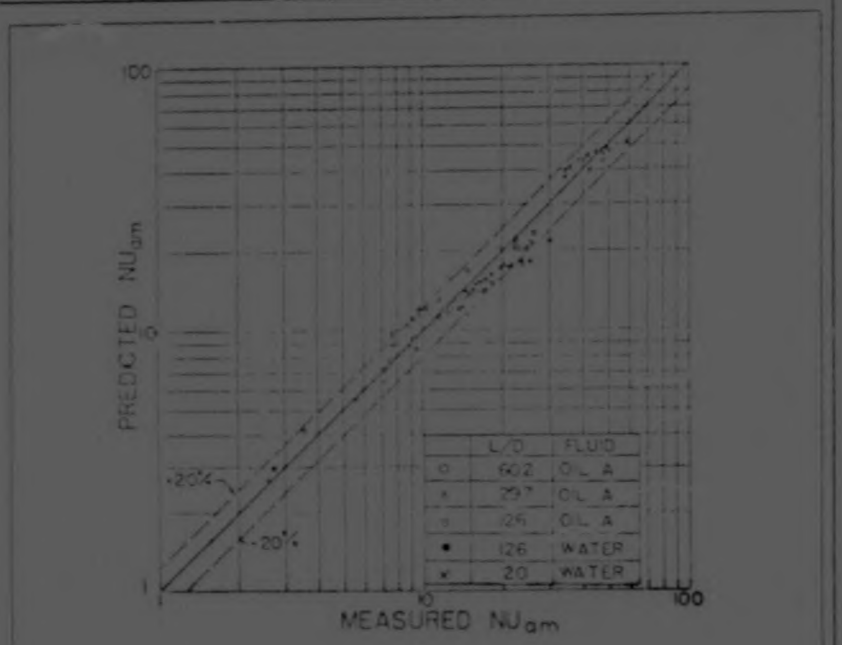
$$Nu_{am} = 1{,}75\, F_1 \sqrt[3]{Gz_m} + 0{,}0722\, F_2 \left[\frac{GrPrD}{L}\right]_w^{0{,}75}$$

$$F_1 = \frac{\left[\frac{\Pi\, Nu_{am}}{Gz_m}\right]}{\ln\left[\frac{2 \times \frac{\Pi\, Nu_{am}}{Gz_m}}{2 - \frac{\Pi\, Nu_{am}}{Gz_m}}\right]}$$

correction factor for using arithmetic mean temperature.

$$F_2 = \frac{5}{4}\int_0^L \left[\frac{x}{L}\right]^{1/4} e^{-3/4} \cdot \frac{\Pi\, Nu_{am}}{F_1 Gz_m} \cdot \left[\frac{x}{L}\right]^{2/3} \frac{dx}{L}$$

The use of the exponent 0,84 in the Gr group is suggested as being superior to 0,75.



0,84 was used for $\left[\frac{GrPrD}{L}\right]_w$

REYNOLDS: Viscous flow

PRANDTL: 49

GRASHOF: $10^5 \rightarrow 10^6$

HORIZONTAL/VERTICAL: Vertical

LENGTH/DIAMETER: 20; 126; 297; 602

AUXILIARY INFORMATION Arithmetic mean temperature is used for properties of Gz and Nu. w refers to wall temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil, Askarel (a chlorinated hydrocarbon)

METHOD: Water cooling in double pipe exchanger. Flow vertically downwards.

AUTHOR(S): Norris, R H
Sims, M W

SOURCE: Trans. AIChE, 469 (1942)

EQUATION PROPOSED:

$$\frac{h}{CpG} = 0{,}0067 \left[\frac{Cp\mu}{k}\right]^{-0{,}8} \left[\frac{\mu_a}{\mu_w}\right]^{0{,}14}$$

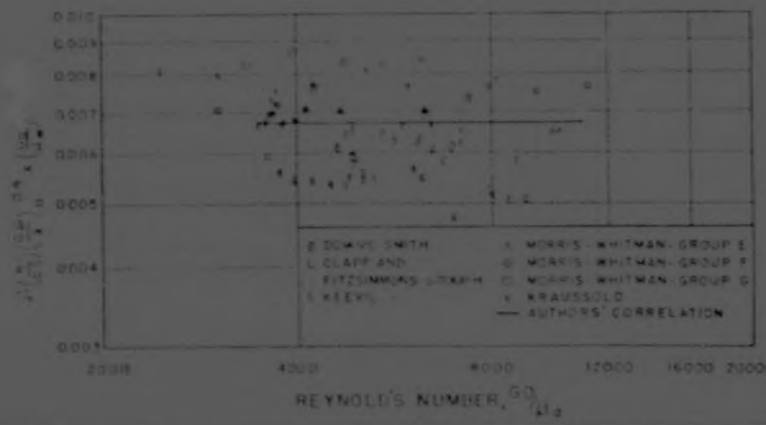
agreement of data is to ± 5%.

or $$Nu = 0{,}0067\ Re Pr^{0{,}-} \left[\frac{\mu_a}{\mu_w}\right]^{0{,}14}$$

$$Re = 3\,500 \rightarrow 11\,000 \qquad \frac{\mu_a}{\mu_w} = 0{,}32 \rightarrow 0{,}76$$



Author's data.



REYNOLDS: Transitional **HORIZONTAL/VERTICAL:** Vertical

PRANDTL: 35 → 140

GRASHOF: **LENGTH/DIAMETER:** 234

AUXILIARY INFORMATION Properties based on average liquid temperature

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil

METHOD: Steam heating.

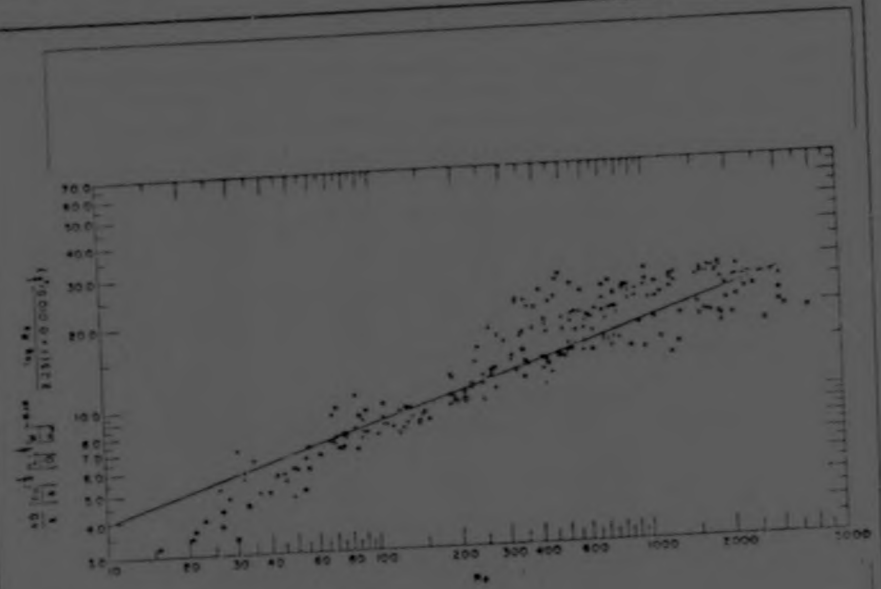
AUTHOR(S): Kern, D Q
Othmer, L F

SOURCE: Trans. AIChE 517 (1343)

EQUATION PROPOSED:

$$Nu = 1{,}86 \left[RePr \frac{d}{L}\right]^{1/3} \cdot \left[\frac{\mu}{\mu_w}\right]^{0{,}14} \cdot \frac{2{,}25\,(1 + 0{,}010 Gr_e^{1/3})}{\log Re}$$

which is a corrected form of the Sieder and Tate equation.



REYNOLDS: Laminar
PRANDTL: 60 → 1 000
GRASHOF: 300 → 22,75 x 10^6

HORIZONTAL/VERTICAL: Horizontal
LENGTH/DIAMETER: 48; 100; 193

AUXILIARY INFORMATION 3 ft entrance section. Properties at bulk average temperatures.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil

METHOD: Steam heating.

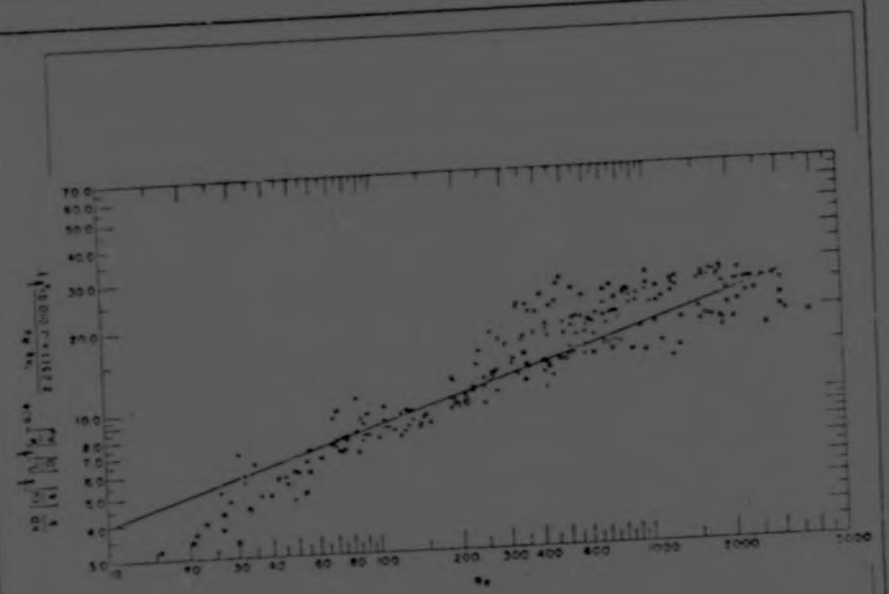
AUTHOR(S): Kern, D Q
Othmer, D F

SOURCE: Trans. AIChE 517 (1943)

EQUATION PROPOSED:

$$Nu = 1{,}86 \left[RePr \frac{d}{L}\right]^{1/3} \left[\frac{\mu}{\mu_w}\right]^{0{,}14} \cdot \frac{2{,}25\,(1 + 0{,}010 Gr_e^{1/3})}{\log Re}$$

which is a corrected form of the Sieder and Tate equation.



REYNOLDS: Laminar

PRANDTL: 60 → 1 000

GRASHOF: 300 → 22,75 × 10^6

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 48; 100; 193

AUXILIARY INFORMATION 3 ft entrance section. Properties at bulk average temperatures.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): All

METHOD: From others' results.

AUTHOR(S): Hausen, H.

SOURCE: Z. VDI Beiheft Verfahrenstechnik, (4), 91 (1943)

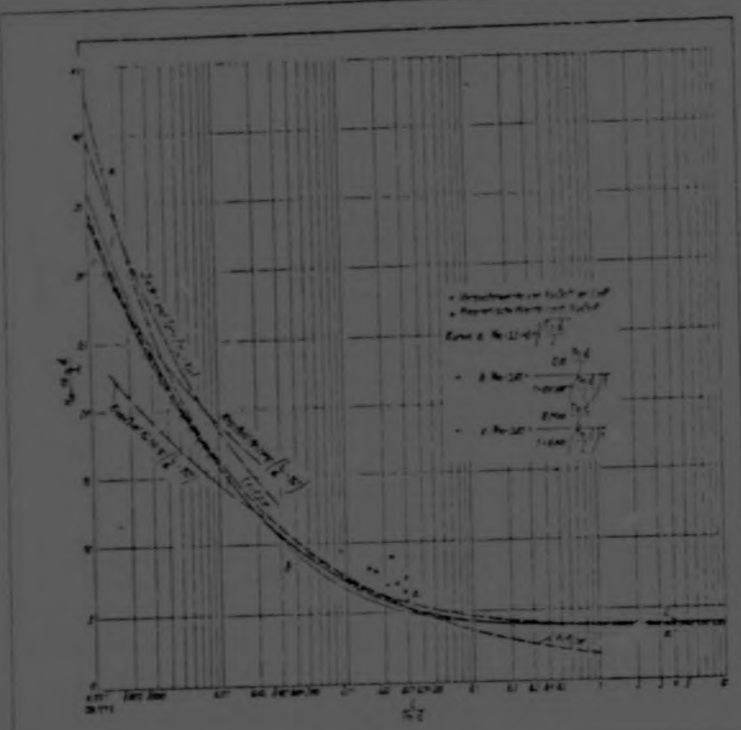
EQUATION PROPOSED:

$$Nu = 0{,}116\left[Re^{2/3} - 125\right] Pr^{1/3}\left[1 + \left[\frac{d}{L}\right]^{2/3}\right]\cdot\left[\frac{\mu}{\mu_w}\right]^{0.14}$$

in turbulent flow.

$$Nu = 3{,}65 + \left[\frac{0{,}0668\left[Pe\,\frac{d}{L}\right]}{1 + 0{,}045\left[\frac{Pe\,d}{L}\right]^{2/3}}\right]$$

in laminar flow.



REYNOLDS: Turbulent and viscous

HORIZONTAL/VERTICAL: Horizontal

PRANDTL:

GRASHOF:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Ethylene Glycol, water, butanol

METHOD: Electrically heated tube.

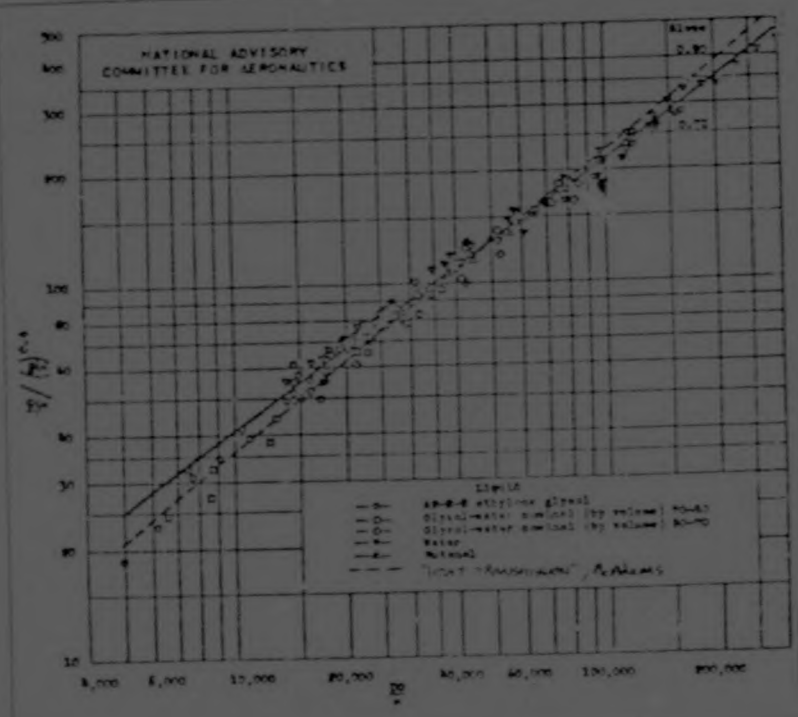
AUTHOR(S): Bernado, E.
Eian, C. S.

SOURCE: NACA Arr. No. E5F07 E136 (1945)

EQUATION PROPOSED:

$$Nu = 0{,}048\ Pr^{0.4}\ Re^{0.73}$$

average scatter of ± 10%



REYNOLDS: 5000 → 300 000

PRANDTL: 2 → 5

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 48

AUXILIARY INFORMATION Properties are taken at the mean bulk temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD:

AUTHOR(S): Il'in, L N.
Kotloturbostroenie No. 1 (1951)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = C Re_b^{0,8} \left[\frac{T_w}{T_b}\right]^n$$

	$\frac{T_w}{T_b}$ 0,5 ÷ 0,9:	0,9 ÷ 1,2:	1,2 ÷ 2,3
C	0,0218	0,0212	0,0223
n	0	−0,27	−0,58

REYNOLDS: $7 - 60 \times 10^3$ HORIZONTAL/VERTICAL:

PRANDTL:

GRASHOF: LENGTH/DIAMETER: 59; 62

AUXILIARY INFORMATION

$\frac{T_w}{T_b}$ = 0,56 ÷ 2,3

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD:

AUTHOR(S): Humble, L V
Lowdermilk, W H
Desmon, L G

NACA Reb. 1020 (1951)

SOURCE: Through Petukhov (1970).

EQUATION PROPOSED:

$$Nu_b = 0{,}023\, Re_b^{0,8}\, Pr_b^{0,4}\, \frac{T_w}{T_b} \qquad (\frac{x}{d} > 60)$$

$$n = 0 \quad \text{at} \quad \frac{T_w}{T_b} < 1$$

$$n = -0{,}55 \quad \text{at} \quad \frac{T_w}{T_b} > 1$$

REYNOLDS: $7 \cdot 300 \times 10^3$

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER: 30 - 120

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 0{,}46 \rightarrow 3{,}5$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Electrical heating.

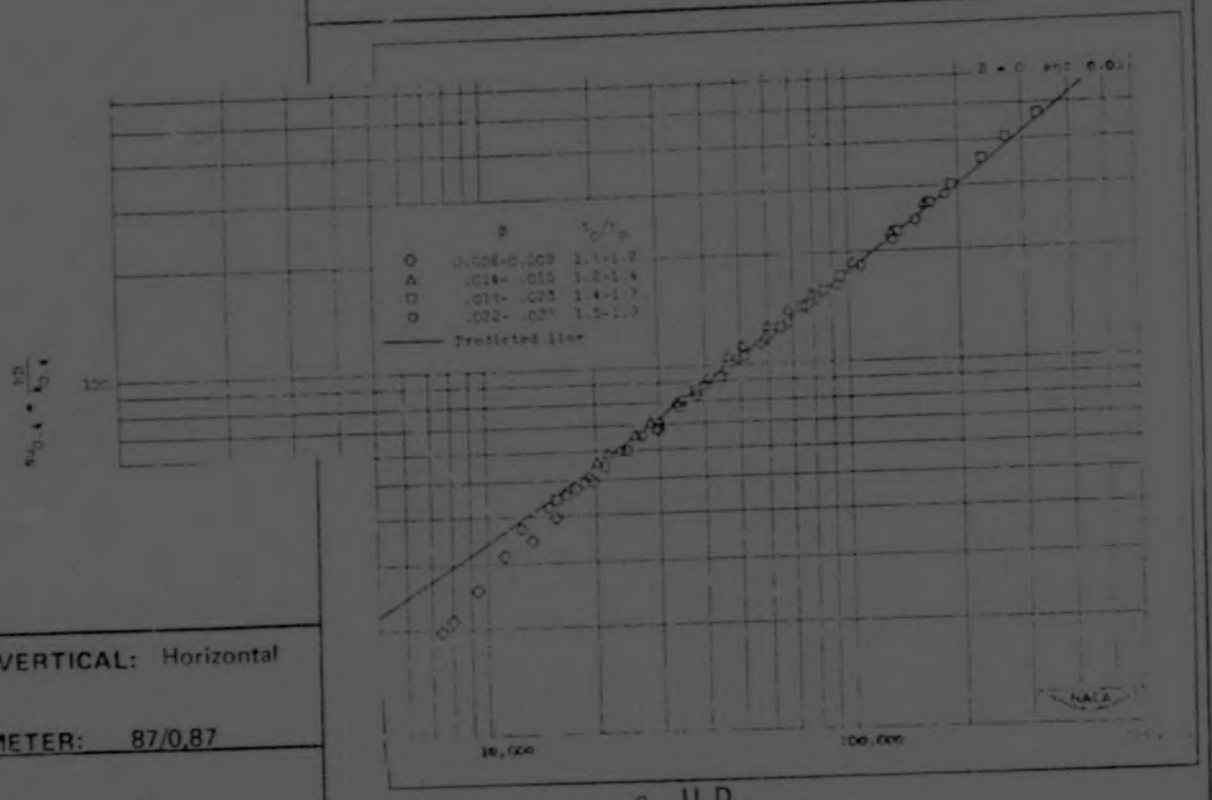
AUTHOR(S): Deissler, R G
Eian, C S

SOURCE: NACA TN 2629 (1952)

EQUATION PROPOSED:

$$U^+ = \int_0^{y^+} \frac{dy^+}{(1 + \beta t^+)^d + \frac{n^2 U^+ y^+}{1 - \beta t^+}}$$

$$t^+ = \int_0^{y^+} \frac{dy^+}{\frac{(1 + \beta t^+)^d}{Pr} + \frac{n^2 U^+ y^+}{1 - \beta t^+}}$$



REYNOLDS: Turbulent

PRANDTL: 0,73

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 87/0,87

AUXILIARY INFORMATION

$t_{0,4} = 0{,}4(t_o - t_b) + t_b$ o refers to wall

$$Re_{0,4} = \frac{\rho_{0,4} U_b D}{\mu_{0,4}}$$

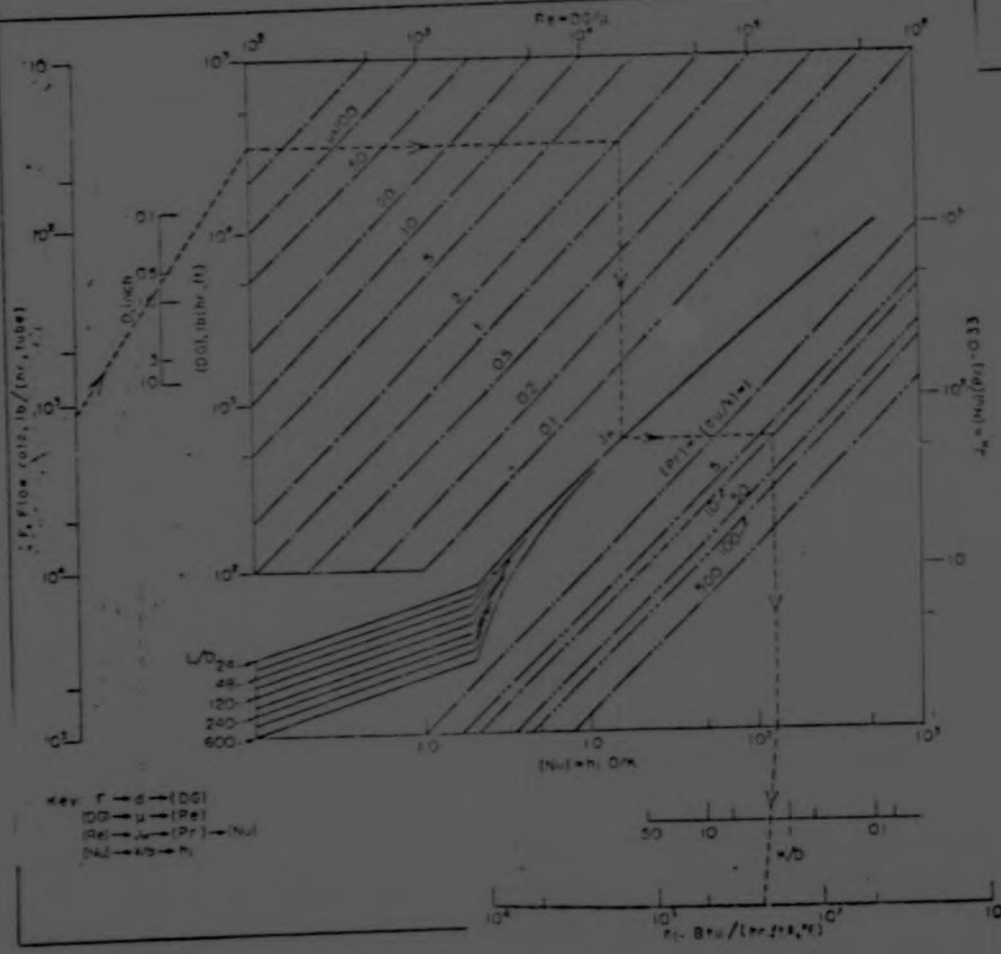
HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Tang, Y S

SOURCE: Chem. Eng. 174, Dec. 1953.



This is a nomograph of the equations given by McAdams in "Heat Transmission" (1942).

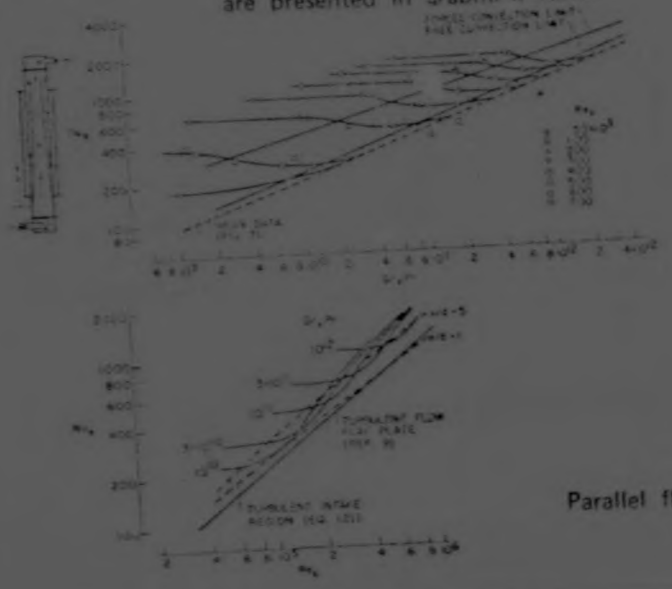
HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Steam heating

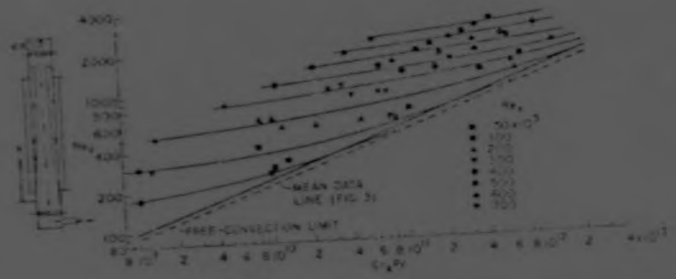
AUTHOR(S): Eckert, E R G
Diaguila, A J

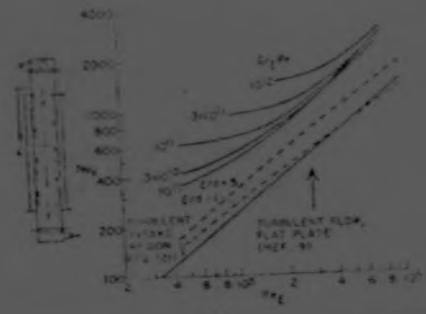
EQUATION PROPOSED: No equation is proposed and the results are presented in graphical form.



Parallel flow

SOURCE: Trans. ASME 497 (1954)





Counter flow

REYNOLDS: Turbulent

PRANDTL: 0,7

GRASHOF:

HORIZONTAL/VERTICAL: Vertical

LENGTH/DIAMETER: 5

AUXILIARY INFORMATION: The equation of Martinelli and Boelter is examined.

Author Rogers D G
Name of thesis Experimental heat transfer coefficients for the cooling of oil in horizontal internal forced convective Transitional flow 1981

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HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil, water

METHOD: Electrical heating for the determination of thermal entry lengths

AUTHOR(S): Hartnett, J P

SOURCE: Trans. ASME 1211, (Nov. 1955)

EQUATION PROPOSED: No equation is proposed. Heat transfer coefficients are presented graphically as a function of the tube length.

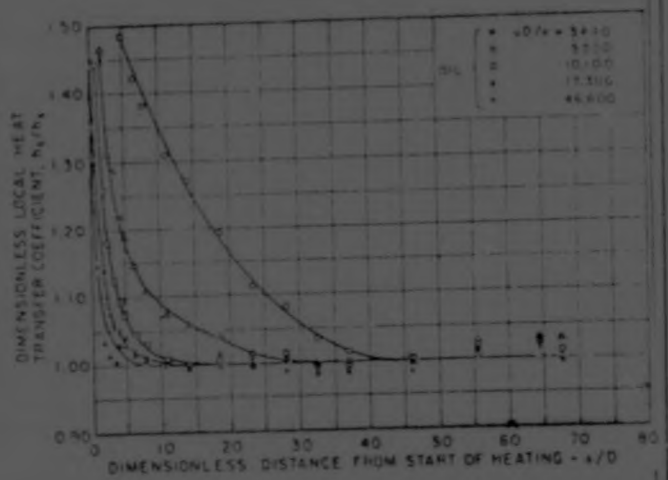
THERMAL-ENTRY-LENGTH RESULTS—WATER

$uD/\nu \times 10^{-3}$	x_{fd}/L	L_e/D	$(L_e/D)_{5\%}$
	Before oil tests		
16.9	6.8	10.7	5.2
21.3	6.7	7.7	3.7
22.7	6.5	7.7	3.2
28.1	7.2	7.7	3.2
30.3	7.0	7.7	2.6
33.6	7.3	7.7	2.9
44.0	7.0	9.2	2.6
44.3	7.3	9.2	2.6
45.3	7.4	9.2	2.8
45.4	7.3	9.2	2.5
45.4	7.0	9.2	2.5
46.8	7.3	10.7	2.5
46.8	7.0	10.7	2.5
50.5	7.4	13.8	2.6
50.8	7.4	9.2	2.2
50.8	7.4	10.7	2.2
51.2	7.5	10.7	2.8
57.4	7.6	12.3	3.4
59.6	7.4	10.7	3.4
62.7	7.3	11.7	3.0
73.9	7.5	13.8	3.4
81.9	7.9	12.8	3.4
86.2	7.8	15.0	3.1
89.2	7.5		
	After oil tests		
19.3	6.5	10.7	3.1
58.7	6.6	9.2	1.7
55.2	6.9	10.7	2.5
54.0	7.0	14.0	3.1

THERMAL-ENTRY-LENGTH RESULTS—OIL

$uD/\nu \times 10^{-3}$	x_{fd}/L	L_e/D	$(L_e/D)_{5\%}$
1.58	459	74*	59
1.94	480	71*	58
2.37	379	74*	62
2.97	41		31
3.41	350	41	28
4.29	346	41	28
5.20	223	29	19
5.83	250	13.8	9.2
10.1	206	13.8	9.2
13.3	178	16.9	11.5
14.8	137	13.8	9.2
16.2	90	10.7	7.7
16.3	135	7.4	4.6
17.2	138	13.8	9.2
17.3	87	13.8	9.2
22.8	98	7.7	4.6
24.6	107	10.7	7.7
24.6	107	15.3	10.7
25.8	88	7.8	4.6
34.9	63	13.8	9.2
40.4	63	6.0	3.1
46.0	51	9.4	6.1

* Thermal-entry length actually greater than tube length.



Transition region.

REYNOLDS: 2300 – 17000 – 48000

HORIZONTAL/VERTICAL: Vertical

PRANDTL:

GRASHOF:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD:

AUTHOR(S): Bialokoz, J
Saunders, O
Combustion and Boilerhouse Engineering, Nov. (1956)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}022\, Re_b^{0,8} Pr_b^{0,4} \left[\frac{T_w}{T_b}\right]^{-0,5}$$

REYNOLDS: $124 \rightarrow 435 \times 10^3$ HORIZONTAL/VERTICAL:

PRANDTL:

GRASHOF: LENGTH/DIAMETER: $29 \rightarrow 72$

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}1 \rightarrow 1{,}73$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Corn syrup, molasses, sugar solution

METHOD: Steam jacket heating

AUTHOR(S): Friend, W L
Metzner, A B

SOURCE: AIChE J. 4, 393 (1958)

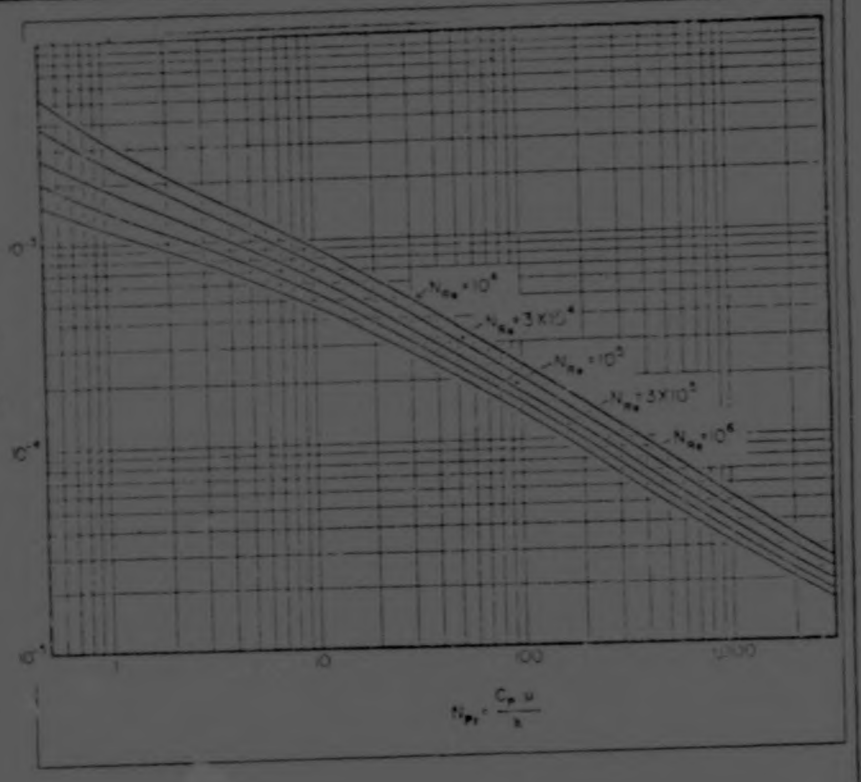
EQUATION PROPOSED:

$$St = \frac{h}{CpG} = \frac{\frac{f}{2}}{1{,}20 + 11{,}8\sqrt{\frac{f}{2}}\,(Pr - 1)\,(Pr)^{-1/3}}$$

f is the fanning factor

The standard deviation from the experimental results is 9,4%.

The use of $\left[\frac{\mu_b}{\mu_w}\right]^{0,14}$ is also suggested.



REYNOLDS: Turbulent
PRANDTL: 50 → 600
GRASHOF:
HORIZONTAL/VERTICAL: Horizontal
LENGTH/DIAMETER: 158

AUXILIARY INFORMATION 10 ft calming section.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Petukhov, B S
Kirillov

Teploenergetica, 4, 63 (1958)

SOURCE: Sleicher, C A. Int. Heat Mass, (1975)

EQUATION PROPOSED:

$$Nu_o = \frac{RePr\frac{f}{f_s}}{K + 12{,}7(Pr^{2/3} - 1)\sqrt{\frac{f}{8}}}$$

$$K = 1{,}07 + \frac{900}{Re} - \frac{0{,}63}{1 + 10Pr}$$

REYNOLDS:

PRANDTL: 0,5 ÷ 2 000

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): All

METHOD: Using the results of others.

AUTHOR(S): Stephan, K

SOURCE: Chem. Ing. Tech. 31, 773 (1959)

EQUATION PROPOSED:

$$Nu = 3{,}66 + \frac{0{,}0677(PrRe\,\frac{d}{L})^{1{,}33}}{1 + 0{,}1\,Pr(Re\,\frac{d}{L})^{0{,}83}}$$

REYNOLDS: Laminar **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION Paper is in German.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD: Summary

AUTHOR(S): Coates, J
Pressburg, B S

SOURCE: Chem. Eng. 66, (28) 67 (1959)

EQUATION PROPOSED:

	Conditions	General Form	
No Phase Change: Fluid Inside Conduit			
Turbulent flow Eq. (A)	$N_{Re} > 2,000$ $0.7 < (C_p\mu/k) < 120$	$\frac{hD}{k} = a\left(\frac{DG}{\mu}\right)^{b}\left(\frac{C_p\mu}{k}\right)^{c}$	$\frac{hD}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.33}$
	$L/D > 50$ $\mu_{wall} < 2$	$\frac{h}{CG} = a\left(\frac{DG}{\mu}\right)^{b}\left(\frac{C_p\mu}{k}\right)^{c}$	$\frac{h}{CG} = 0.023\left(\frac{DG}{\mu}\right)^{-0.2}\left(\frac{C_p\mu}{k}\right)^{0.67}$
Eq. (B)	$N_{Re} > 10,000$ No restriction on viscosity. Otherwise same as Eq. (A)	$\frac{hD}{k} = a\left(\frac{DG}{\mu}\right)^{b}\left(\frac{C_p\mu}{k}\right)^{c}\left(\frac{\mu}{\mu_w}\right)^{d}$	$\frac{hD}{k} = 0.023\left(\frac{DG}{\mu}\right)^{0.8}\left(\frac{C_p\mu}{k}\right)^{0.33}\left(\frac{\mu}{\mu_w}\right)^{0.14}$
Eq. (C)	Liquid metals only	$\frac{hD}{k} = a + b(N_{Re}N_{Pr})^{c}$	$\frac{hD}{k} = 7.0 + 0.025\left(\frac{DGC}{k}\right)^{0.8}$
Stream line flow Eq. (D)	$N_{Re} < 2,000$	$\frac{hD}{k} = a\left(\frac{wC}{kL}\right)^{b}$	$\frac{hD}{k} = 2.0\left(\frac{wC}{kL}\right)^{0.8}\left(\frac{\mu}{\mu_w}\right)^{0.14}$
Eq. (E)	Limiting case of Eq. (D) when $T_{wall} = T_{wall}$		$\frac{hD}{k} = \frac{2w}{\pi kL}$
Streamline flow and natural convection Eq. (F)	$N_{Re} < 2,000$ Low viscosity or high ΔT or both.	$\frac{hD}{k} = a\left[\frac{wC}{kL}(1 + bZ^{n})\right]^{m}$	$\frac{hD}{k} = 1.75\left(\frac{\mu}{\mu_w}\right)^{0.14}\left[\frac{wC}{kL}(1 + 0.015Z^{1/3})\right]^{0.33}$
	Coefficient at a varying axial tube orientation and whether fluid is heated or cooled.	$Z = \frac{D}{L}\left(\frac{\rho^2\beta g}{\mu^2}\right)\left(\beta\Delta T\right)\left(\frac{C_p\mu}{k}\right)$	

REYNOLDS:

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD: Others' results.

AUTHOR(S): Hausen, H.

SOURCE: Allgem. Wärmetech. 9, 75 (1959)

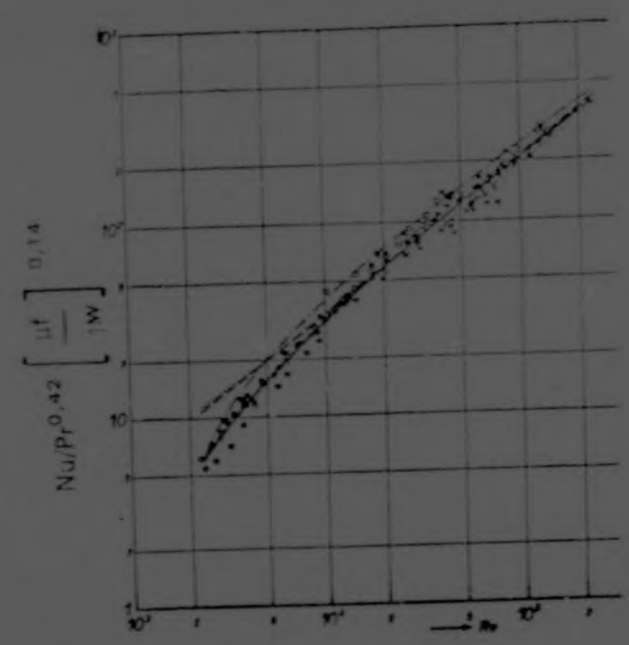
EQUATION PROPOSED:

$$Nu = 0{,}037\,(Re^{0,75} - 180)\,Pr^{0,42}\left[1 + \left[\frac{d}{L}\right]^{2/3}\right]\left[\frac{\mu_f}{\mu_w}\right]^{0,14}$$

in turbulent flow, and

$$Nu = 3{,}65 + \frac{0{,}19\left[Pe\,\frac{d}{L}\right]^{0,8}}{1 + 0{,}117\left[Pe\,\frac{d}{L}\right]^{0,467}}$$

in laminar flow.



REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Hydrogen

METHOD:

AUTHOR(S): Wright, F C.
Walters, H.

WADC Tech. Rep. 59 – 423, Aug. (1959).

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}021\ Re_b^{0,8}\ Pr_b^{1/3} \left[\frac{T_w}{T_b}\right]^{-0{,}575}$$

at large $\frac{\ell}{D}$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1 \rightarrow 4$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Petukhov, B S
Nolde, L D

Teploenergetica, 6, 72 (1959)

EQUATION PROPOSED:

$$Nu = 0{,}21 \left[RePr \ \frac{d}{L} \right]^{0{,}8} \left[GrPr \ \frac{d}{L} \right]^{0{,}07}$$

SOURCE: Herbert (1971)

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Vertical

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Hydrogen, Helium

METHOD:

AUTHOR(S): McCarthy, J R
Wolf, H

Rep. No. RR-60-12 Rocketdyne, Canoga Park, California (1960)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}045\ Re_b^{0,8}\ Pr_b^{0,4} \left[\frac{\ell}{D}\right]^{-0,15} \left[\frac{T_w}{T_b}\right]^{-0,7}$$

REYNOLDS: $5 \rightarrow 1500 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 26 – 67

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}5 \rightarrow 9{,}9$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Hydrogen

METHOD:

AUTHOR(S): McCarthy, J R
Wolf, H.
Am. Rocket Soc. J. 30, No. 4 (1960)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}023\, Re_b^{0,8}\, Pr_b^{0,4} \left[\frac{T_w}{T_b}\right]^{-0,3}$$

REYNOLDS: $7 \rightarrow 1500 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 43,67

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}5 \rightarrow 2{,}8$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Helium

METHOD:

AUTHOR(S): Taylor, M F.
Kirchgessner, T A.

Am. Rocket Soc. J. 30, No. 4 (1960)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_t = 0{,}021\ Re_t^{0,8}\ Pr_t^{0,4}$$

at great $\frac{\ell}{D}$

REYNOLDS: 3,2 → 60 x 10^3 **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 60,92

AUXILIARY INFORMATION

$\frac{T_w}{T_b}$ = 1,6 → 3,9

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Steam heating

AUTHOR(S): Jackson, T W
Spurlock, J M
Purdy, K R

SOURCE: AIChE J. 7, 38 (1961)

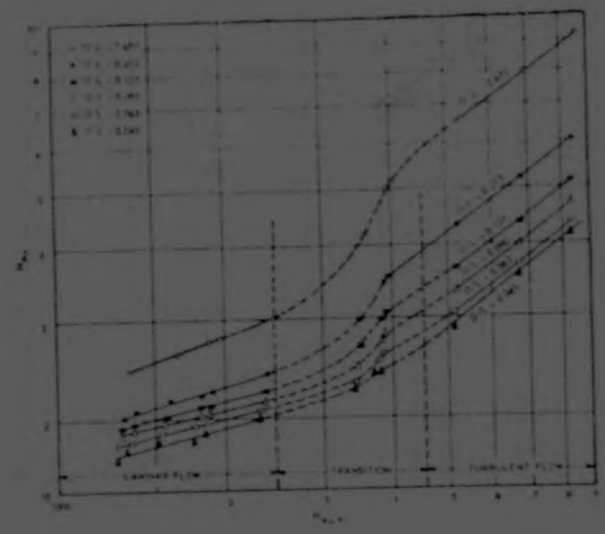
EQUATION PROPOSED:

$$Nu = 2{,}67\,Gz^{1/3}\left[1 + \frac{0{,}0087^2}{Gz^{1/2}}\left[\frac{GrPr}{\Pi\,Nu}\left[1 - e^{\frac{-\Pi\,Nu}{Gz}}\right]\right]^{1{,}5}\right]^{1/6}$$

for lamina flow $60 < Gz < 1300$

$$Nu = 0{,}023\,Re^{0.8}\,Pr^{1/3}\left[1 + 3\,\frac{d}{L}\right] \quad \text{for } Re > 7000$$

Grashof is based on the log mean temperature difference.



REYNOLDS: Turbulent and laminar

HORIZONTAL/VERTICAL: Horizontal

PRANDTL: 0,7

GRASHOF:

LENGTH/DIAMETER:

AUXILIARY INFORMATION Properties at bulk average temperatures.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, air

METHOD: Electrical heating

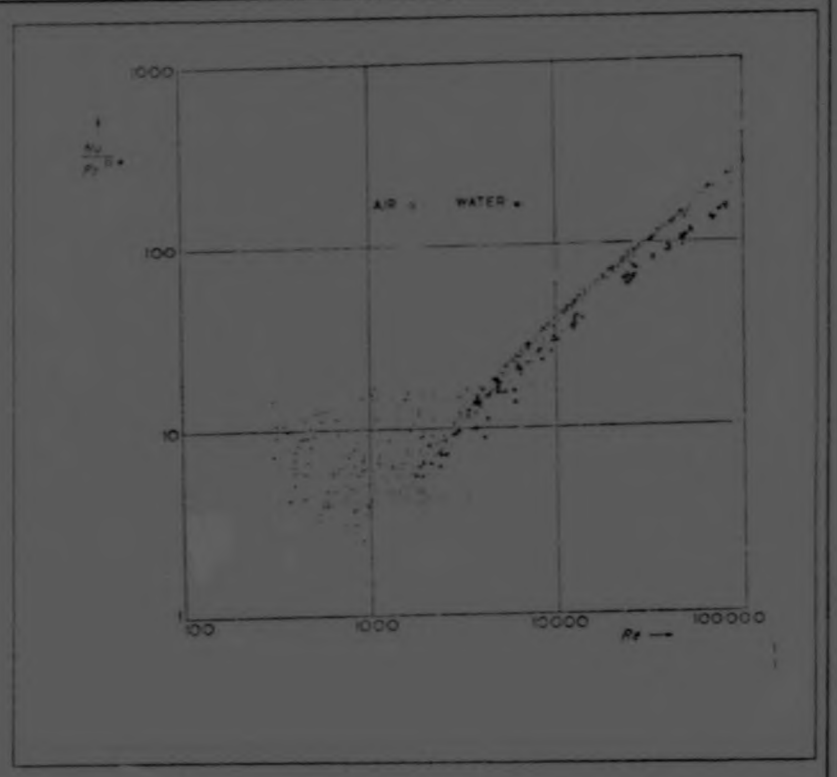
AUTHOR(S): Ede, A J

SOURCE: Int. J. Heat Mass, 4, 105 (1961)

EQUATION PROPOSED:

$Nu = 0{,}023\ Re^{0.8}\ Pr^{0.4}$ turbulent flow

$Nu = 4{,}36\ (1 + 0{,}06\ Gr^{0.3})$ laminar flow



REYNOLDS: 300 – 100000

HORIZONTAL/VERTICAL: Horizontal

PRANDTL:

GRASHOF:

LENGTH/DIAMETER:

AUXILIARY INFORMATION Physical properties calculated at the bulk average temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Glycerol, ethanol, water

METHOD: Water cooling and heating

AUTHOR(S): Oliver, D R

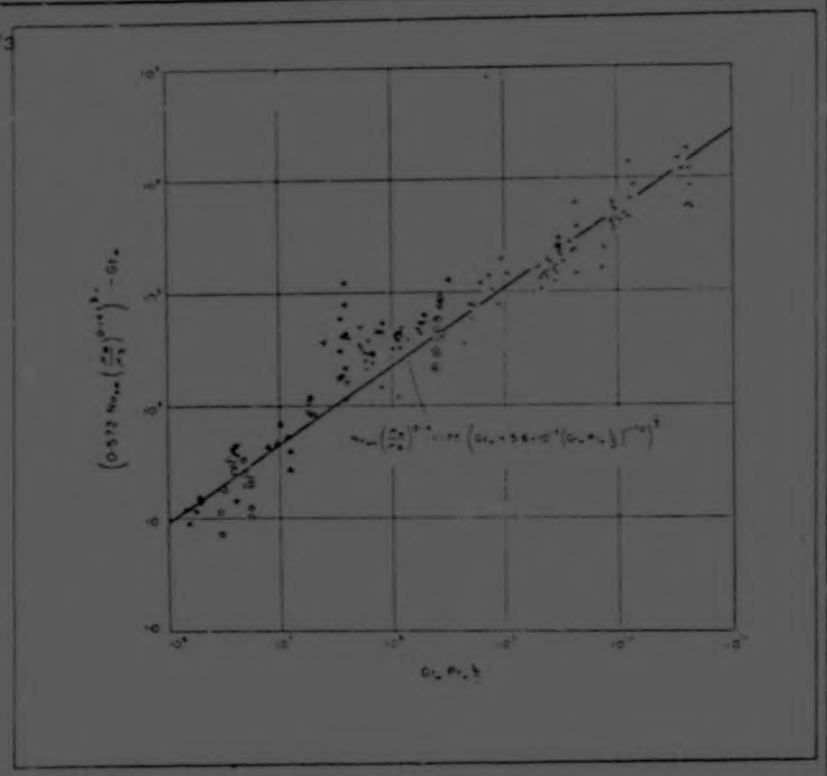
SOURCE: Chem. Eng. Sci., 17, 335 (1962)

EQUATION PROPOSED:

$$Nu_{am}\left[\frac{w}{\mu_b}\right]^{0,14} = 1,75\left[Gz_m + 5,6 \times 10^{-4}\left[Gr_m Pr_m \frac{L}{d}\right]^{0,70}\right]^{1/3}$$

for $Gz > \Pi Nu_{am}$

errors do not exceed $\pm$ 20%



REYNOLDS: Laminar

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 72

AUXILIARY INFORMATION 1,5 it calming section. Mean fluid temperatures used for properties.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Helium, hydrogen

METHOD:

AUTHOR(S): Wieland, W F

AIChE Symp. Nuc. Eng. Heat Transfer, Chicago, (1962)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_f = 0{,}021\ Re_f^{0,8}\ Pr_f^{0,4}$$

far from the entry.

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER: 250

AUXILIARY INFORMATION $\frac{T_w}{T_b} < 2{,}8$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Kutateladze

EQUATION PROPOSED:

$$\frac{Nu_f}{Nu_{of}} = \left[\frac{2}{\sqrt{\frac{T_w}{T_f}} + 1}\right]^2$$

Through Petukhov and Popov (1963)

SOURCE: The Element of heat exchange (Russian) 1962

REYNOLDS: **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Hydrogen, helium

METHOD:

AUTHOR(S): Taylor, M F

("Heat Transfer and Fluid Mechanics Institute") p 51, Stanford University Press. California (1963).

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_f = 0{,}021\, Re_f^{0,8}\, Pr_f^{0,4}$$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER: 77

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}5 \rightarrow 5{,}6$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Krasnoshchekov, E A
Sukomel, A S
(1963)

SOURCE: Kuznetsova (1970)

EQUATION PROPOSED:

$$Nu = 0{,}35 \left[Re\ Pr\ \frac{d}{L} \right]^{0{,}6} \left[Gr\ Pr\ \frac{d}{L} \right]^{0{,}1}$$

REYNOLDS: HORIZONTAL/VERTICAL:

PRANDTL:

GRASHOF: LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Krasnoshchekov, E A
Sukomel, A S
(1963)

SOURCE: Kuznetsova (1976)

EQUATION PROPOSED:

$$Nu = 0{,}35 \left[Re\ Pr \frac{d}{L} \right]^{0{,}6} \left[Gr\ Pr \frac{d}{L} \right]^{0{,}1}$$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Nitrogen

METHOD:

AUTHOR(S): Kirillov, U V
Malugin, Yu, S

High Temp. Heat Physics, 1, No. 2, (1963)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}021\ Re_b^{0,8}\ Pr_b^{0,4} \left[\frac{T_w}{T_b}\right]^{-0,5}$$

at $\frac{l}{D} > 30$

REYNOLDS: $7 \rightarrow 160 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 1,38

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}1 \rightarrow 2{,}3$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Electrically heated.

AUTHOR(S): Dipprey, D F
Sabersky, R H

SOURCE: Int. J. Heat Mass, 6, 329 (1963)

EQUATION PROPOSED:

$$\frac{2C_H}{C_f} = Pr^{-2/3}$$

where

$$C_H = \frac{q_o}{\rho U_m Cp(T_w - T_L)}$$

$$C_f = \frac{2\tau_o}{\rho U_m^2}$$

T_L = mean temperature

REYNOLDS: 14 000 - 500 000 **HORIZONTAL/VERTICAL:** Vertical

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 90

AUXILIARY INFORMATION 45 D entrance region.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Incompressible fluid

METHOD: Numerically

AUTHOR(S): Petukhov, B S
Popov, V N

SOURCE: High Temperature, 1, 69 (1963)

EQUATION PROPOSED:

$$Nu = \frac{\frac{f}{8}\,RePr}{1{\cdot}07 + 12{\cdot}7\sqrt{\frac{f}{8}}\,(Pr^{2/3} - 1)}$$

ant physical properties.

For high temperature loads

$$\frac{Nu_f}{Nu_{of}} = \left[\frac{T_w}{T_f}\right]^n \qquad n = -0{\cdot}47$$

The equation was proposed by Petukhov and Kirillov in 1958. Teploenergetica No. 4.

REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

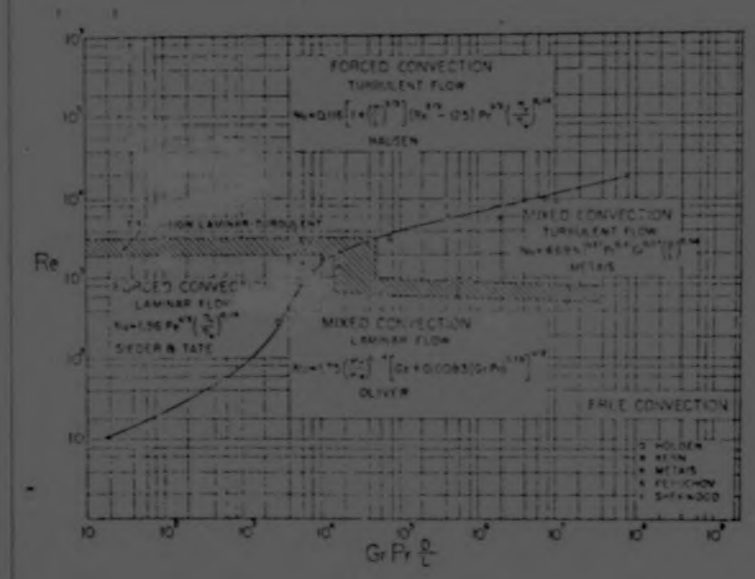
METHOD: Uniform wall temperature.

AUTHOR(S): Metais, B
University of Minnesota (1963)

EQUATION PROPOSED:

$$Nu = 4{,}69\ Re^{0.27}\ Pr^{0.21}\ Gr^{0.07} \left[\frac{d}{L}\right]^{0.36}$$

SOURCE: Eckert and Metais (1964)



REYNOLDS: Turbulent **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

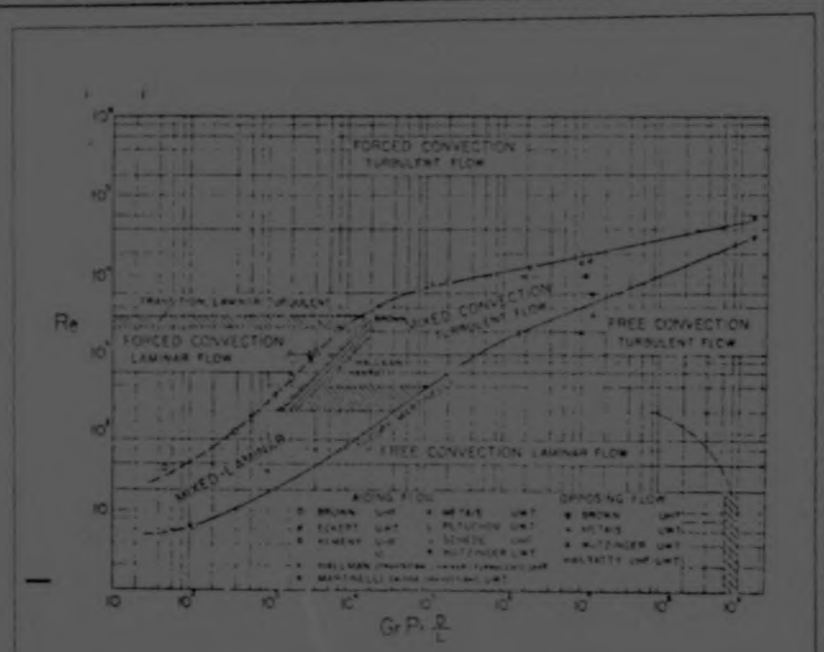
METHOD:

AUTHOR(S): Metais, B
Eckart, E R G

EQUATION PROPOSED:

Read in conjunction with Metais (1963). Valid for uniform wall temperature and uniform heat flux.

SOURCE: J. of Heat Transfer, 295 (1964)



REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Vertical

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Uniform wall heat flux.

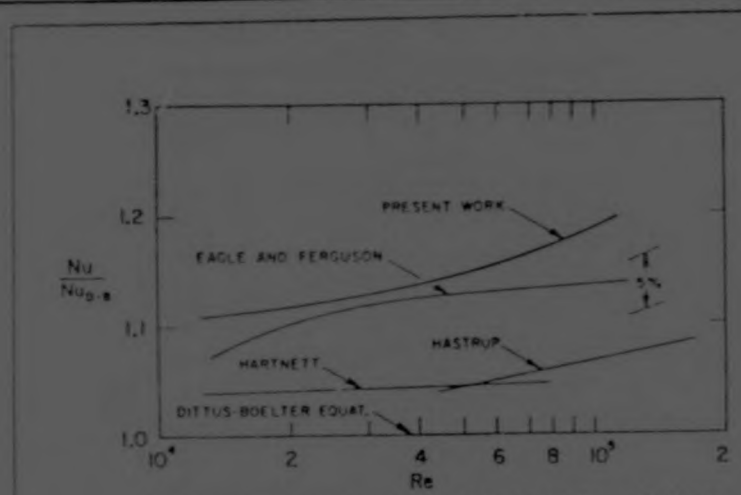
AUTHOR(S): Allen, R W
Eckert, E R G

SOURCE: Trans. ASME, 86, 301 (1964)

EQUATION PROPOSED:

$$St = 0{,}000370 + 0{,}000926\ Re^{-0.269}$$

Correlation is to be better than 1%.



Comparison of heat-transfer coefficients with those of other investigators. Thermally developed, constant property case: Pr = 8. Uniform wall heat flux.

REYNOLDS: 15000, 111000 Turbulent

PRANDTL: 7 → 8

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 30 D

AUXILIARY INFORMATION Properties taken at bulk temperature

96 D entrance region.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, oil

METHOD: Electrical heating

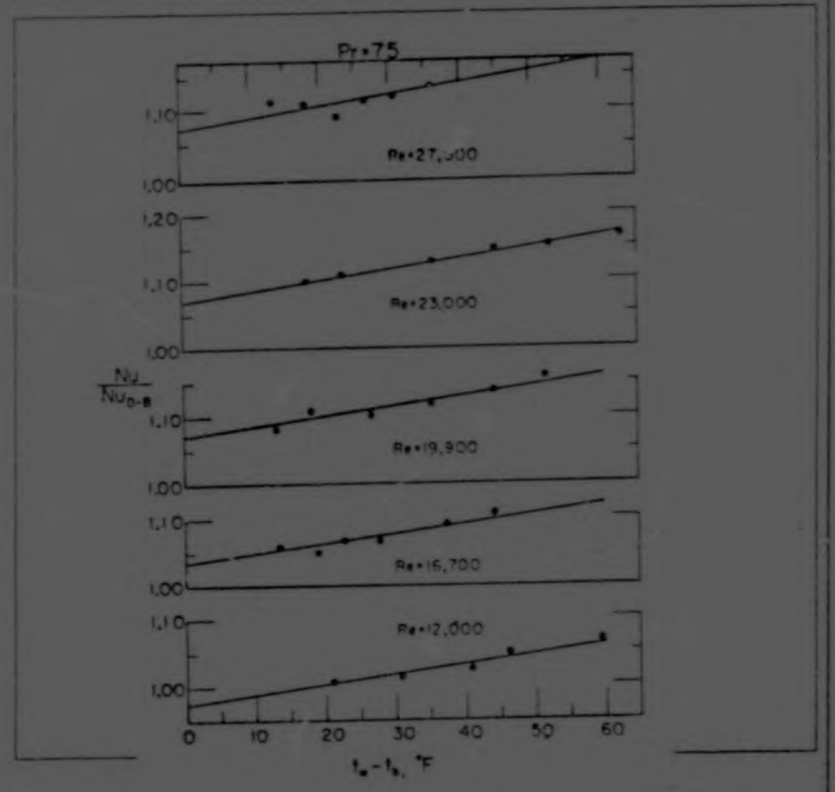
AUTHOR(S): Malina, J A
Sparrow, E M

EQUATION PROPOSED:

$$Nu = 0{,}023\, Re^{4/5}\, Pr^{1/3} \left[\frac{\mu_b}{\mu_w}\right]^{0{,}05}$$

i.e. a correction to the Dittus and Boelter equation.

SOURCE: Chem. Eng. Sci. 19, 953 (1964)



REYNOLDS: Turbulent
PRANDTL: 3 → 75
GRASHOF:

HORIZONTAL/VERTICAL: Horizontal
LENGTH/DIAMETER: 30

AUXILIARY INFORMATION 96 D developing section.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Water cooling and heating

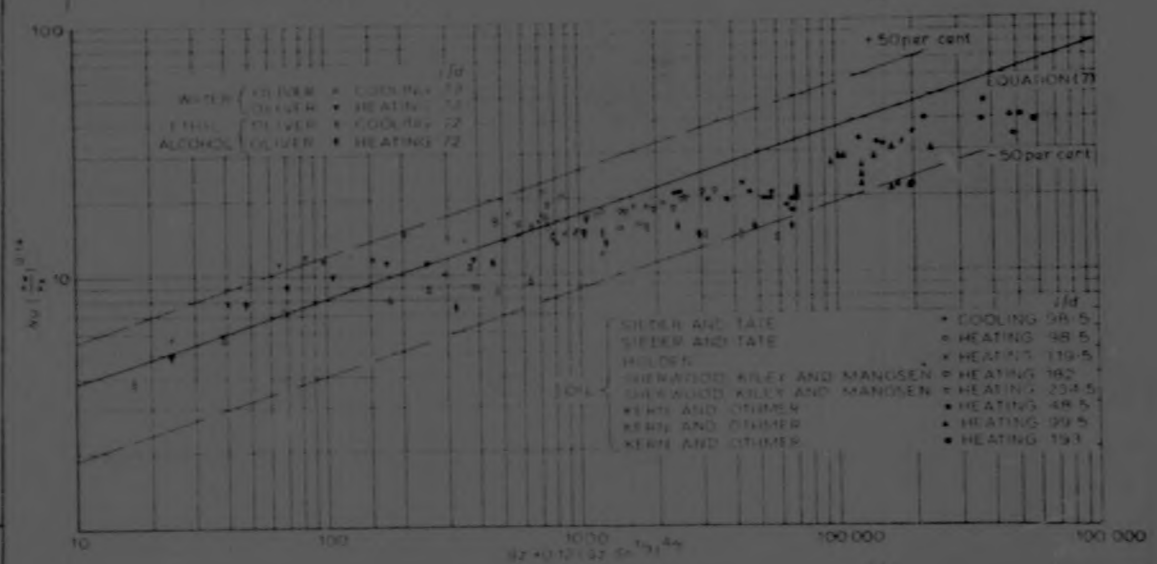
AUTHOR(S): Brown, A R
Thomas, M A

EQUATION PROPOSED:

$$Nu = 1{,}75\left[Gz + 0{,}012\left[GzGr^{1/3}\right]^{4/3}\right]^{1/3}\left[\frac{\mu_b}{\mu_w}\right]^{0.14}$$

Correlates the majority of data to 8% and the majority of published data to 50%.

SOURCE: J. Mech. Eng. Sci., 7, (4), 440 (1965)



REYNOLDS: Laminar **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: $4 \times 10^4 \rightarrow 480 \times 10^4$ **LENGTH/DIAMETER:** 72; 108; 36

AUXILIARY INFORMATION All properties are evaluated at the bulk average temperature. Calming section was included.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Water cooling and heating

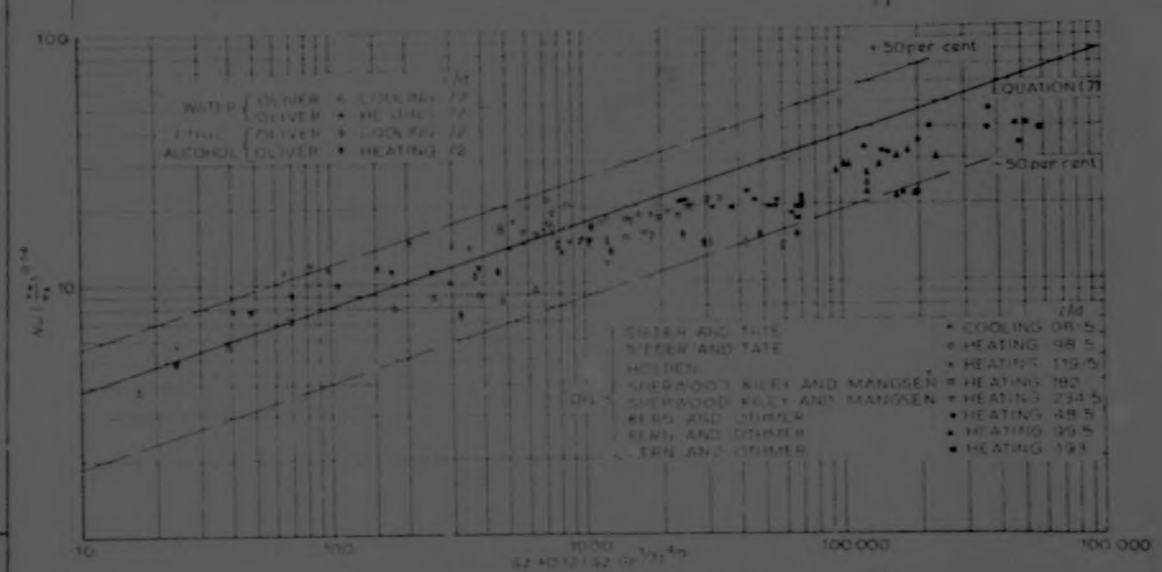
AUTHOR(S): Brown, A R
Thomas, M A

EQUATION PROPOSED:

$$Nu = 1{,}75\left[Gz + 0{,}012\left[GzGr^{1/3}\right]^{4/3}\right]^{1/3}\left[\frac{\mu_b}{\mu_w}\right]^{0{,}14}$$

Correlates the majority of data to 8% and the majority of published data to 50%.

SOURCE: J. Mech. Eng. Sci., 7, (4), 440 (1965)



REYNOLDS: Laminar **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: $4 \times 10^4 \rightarrow 480 \times 10^4$ **LENGTH/DIAMETER:** 72; 108; 36

AUXILIARY INFORMATION All properties are evaluated at the bulk average temperature. Calming section was included.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, water

METHOD: Condensing steam, rough pipes.

AUTHOR(S): Kolář, V

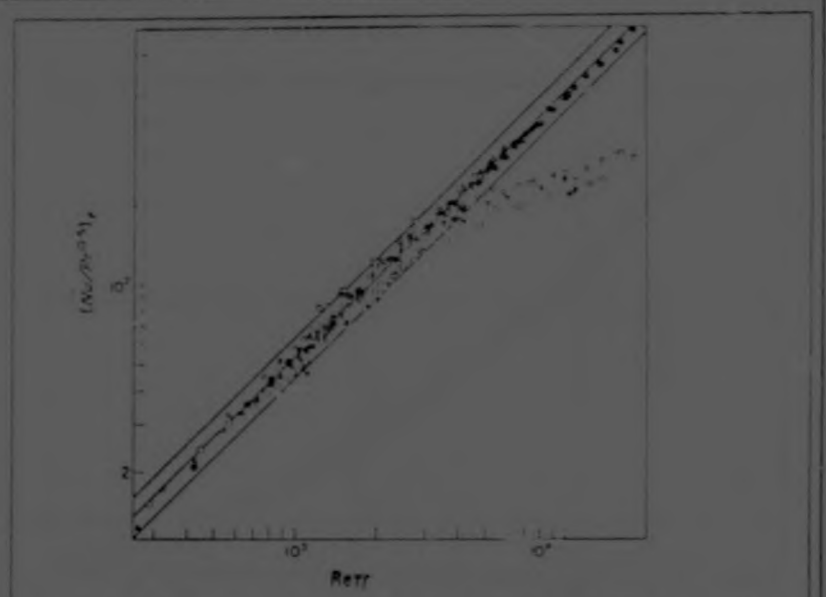
SOURCE: Int. J. Heat Mass, 8, 639 (1965)

EQUATION PROPOSED:

$$Nu_f = 0{,}05814\, Re_{Tf}^{0{,}806}\, Pr^{0{,}5}$$

Re_{Tf} is the Re_T based on the film temperature.

$$Re_T = \frac{u_m \sqrt{\frac{f}{8}}\, d\rho_m}{\mu_m}$$



Plot is not of given equation but demonstrates the effect of using Re_{Tf}

REYNOLDS: 4500 → 140000 **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL: 0,71 → 5,52

GRASHOF: **LENGTH/DIAMETER:** 31

AUXILIARY INFORMATION $t_f = \frac{t_s + t}{2}$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Mercury, air, sodium, water

METHOD:

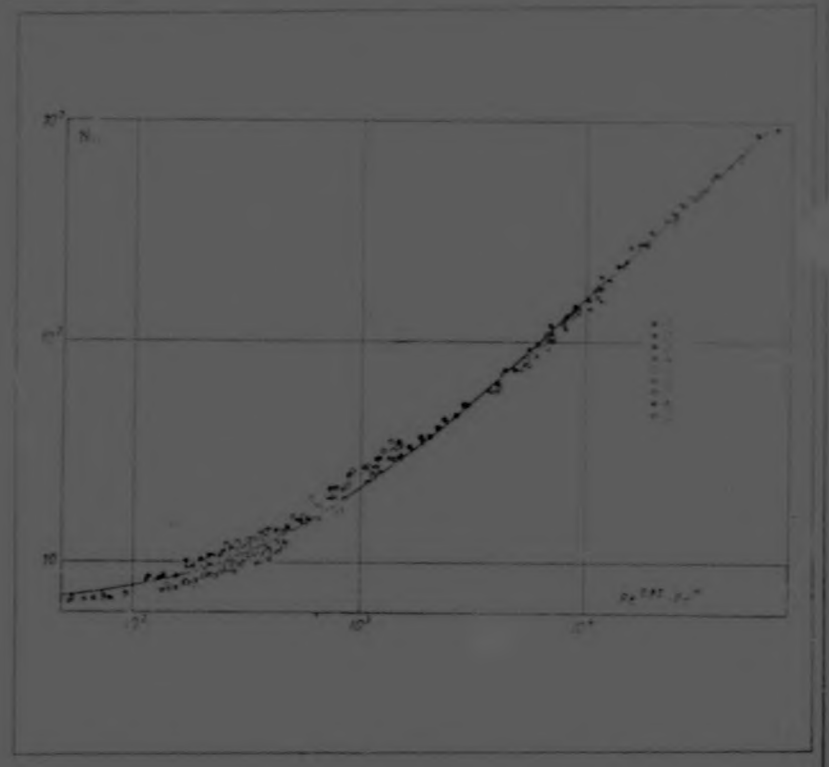
AUTHOR(S): Subbotin, V. I.
Ibragimov, M. Kh.
Nomofilov, E. V.

SOURCE High Temp. 3, 380 (1966)

EQUATION PROPOSED:

$$Nu = Nu_0 + aRe^m Pr^n$$

$$= 7{,}24 - \frac{9{,}5}{\log Re} + 0{,}0155\, Re^{0{,}82}\, Pr^{0{,}58} - 0{,}18\,th(0{,}8 \log Pr)$$



REYNOLDS: Turbulent
PRANDTL: $0 < Pr < 5$
GRASHOF:

HORIZONTAL/VERTICAL:
LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Nitrogen

METHOD

AUTHOR(S): Perkins, H C
Worsoe-Schmidt, P

Int. J. Heat Mass Trans. 8, 7, 1011 (1965)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}024\ Re_b^{0,8}\ Pr_b^{0,4} \left[\frac{T_w}{T_b}\right]^{-0,7}$$

$$\frac{l}{D} > 40$$

REYNOLDS: $18 - 280 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 160

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}3 - 7{,}5$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, carbon dioxide, argon

METHOD:

AUTHOR(S): Lelchuk, V L
Elphimov, G I
Fedotov, Yu P

Heat and Mass Transfer, Minsk VI, (1965)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}021\ Re_b^{0,8}\ Pr_b^{0,4} \left[\frac{T_w}{T_b}\right]^{-0,5}$$

at $\frac{\ell}{D} > 50$

REYNOLDS: $14 \rightarrow 600 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** $77 \rightarrow 206$

AUXILIARY INFORMATION

$\frac{T_w}{T_b} = 1{,}1 \rightarrow 2{,}7$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Nitrogen

METHOD:

AUTHOR(S): Petukhov, B S
Kirillov, V V
Maidonic, V N

Int. Heat Trans. Conf. 3, Chicago, p 28, (1966)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}021\, Re_b^{0.8}\, Pr_b^{0.4} \left[\frac{T_w}{T_b}\right]^n$$

at $\frac{\ell}{D} > 80$

$$n = -\left[0{,}9 \log\left[\frac{T_w}{T_b}\right] + 0{,}205\right]$$

REYNOLDS: $13 \rightarrow 300 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** $80 \rightarrow 100$

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1 \rightarrow 6$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, helium, nitrogen

METHOD:

AUTHOR(S): McEligot, D M
McGee, P M
Leppert, G

J. Heat Transfer, 87, No. 1, (1965)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}021\, Re_b^{0,8}\, Pr_b^{0,4} \left[\frac{T_w}{T_b}\right]^{-0,5}$$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER: 160

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}1 \rightarrow 2{,}5$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD:

AUTHOR(S): Volkov, P M
Ivanov, A V

Heat Transfer and Hydrodynamics in elements of power equipment.
Tr, TskTI 73, (1966)

SOURCE: Through Petukhov (1970)

EQUATION PROPOSED:

$$Nu_b = 0{,}0193\, Re_b^{0{,}8}\, Pr_b^{0{,}4} \left[\frac{T_w}{T_b}\right]^{-0{,}55}$$

$$\frac{\ell}{D} > 100$$

REYNOLDS: $14 \rightarrow 400 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 48 → 370

AUXILIARY INFORMATION $\frac{T_w}{T_b} = 1{,}1 \rightarrow 2{,}1$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Electrical heating

AUTHOR(S): Mori, Y
Futagami, K
Tokuda, S
Nakamura, M

SOURCE: Int. J. Heat Mass, 9, 453, (1966)

EQUATION PROPOSED:

$$Nu_x = 0{,}61\,(ReRa)^{1/5}\left[1 + \frac{1{,}8}{(ReRa)^{1/5}}\right]$$

for laminar flow.

In turbulent flow at Pr = 0,72

$Nu = 0{,}0204\,Re^{4/5}$ in agreement with Colburn.

REYNOLDS: 100 → 13000 **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 127

AUXILIARY INFORMATION 7 m entrance section.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

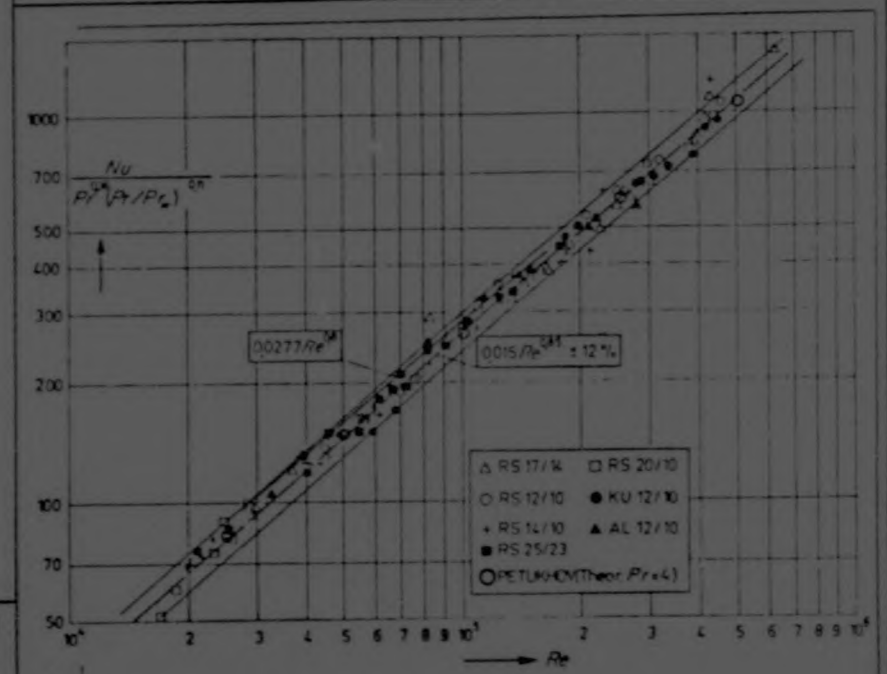
METHOD: Constant heat flux.

AUTHOR(S): Yakovlev, V V
Atonmaya Energiya 8, (3) 250 (1960)
and Kernenergie, 3, 1098 (1960)

EQUATION PROPOSED:

$$\frac{Nu}{Pr^{0,36}\left[\frac{Pr}{Pr_w}\right]^{0,11}} = 0,0277\ Re^{0,8}$$

SOURCE: Hufschmidt et al (1966)



REYNOLDS: Turbulent **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Electrical heating. Results of others.

AUTHOR(S): Hufschmidt, W
Burck, E
Riebold, W

SOURCE: Int. J. Heat Mass, 9, 539 (1966)

EQUATION PROPOSED:

Using Petukhov and Popov equation with a Pr correction factor.

$$Nu = \frac{\sqrt{\frac{f}{8}}\ RePr}{1{,}07 + 12{,}7\sqrt{\frac{f}{8}}\left[Pr^{2/3} - 1\right]}\left[\frac{Pr}{Pr_w}\right]^{0.11}$$

The use of the Pr group was first suggested by Yakovlev (1960)

REYNOLDS: 20000 - 640000 **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL: 2 - 5,5

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION A good review of others' contributions is given.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, nitrogen, helium.

METHOD: Electrical heating.

AUTHOR(S): McEligot, D M
Ormand, L W
Perkins, H C

SOURCE: Trans. ASME C, 239 (1966)

EQUATION PROPOSED:

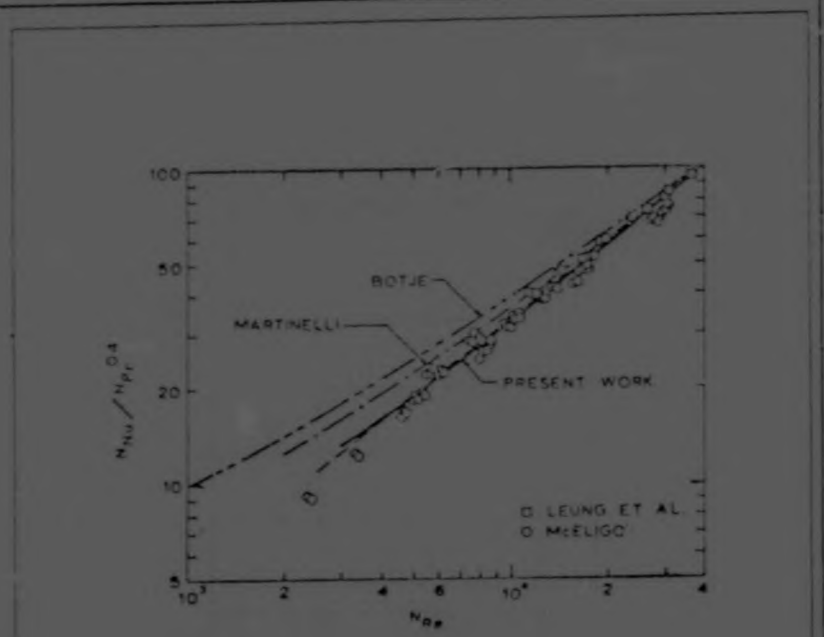
$$Nu = 0{,}21\, Re^{0,8}\, Pr^{0,4} \exp(-90\, q^{+}) \qquad \text{for } Re > 15\,000$$

$$\text{where } q^{+} = \frac{\text{heat flux}}{\text{enthalpy} \times \text{mass flow rate}} \quad \text{(heat flux parameter)}$$

(initial, no flow)

$$0 < q^{+} < 0{,}004$$

$$Nu = Nu_{\text{const. prop.}} \left[\frac{T_w}{T_b}\right]^{-0,5}$$



REYNOLDS: 1450 → 45000 **HORIZONTAL/VERTICAL:** Vertical

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** > 30

AUXILIARY INFORMATION Dittus and Boelter equation fits their data to ± 5%.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Electrical heating

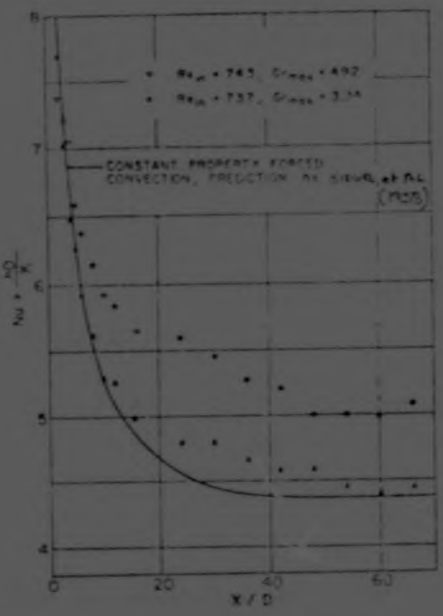
AUTHOR(S): McComas, S T
Eckert, E R G

EQUATION PROPOSED:

$$Nu = A\left[Gz + B\left[GrPr\,\frac{L}{d}\right]^{3/4}\right]^{1/3}$$

A and B are constants.

SOURCE: Trans. ASME C, 88, 147, (1966)



REYNOLDS: 100 → 900 **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: 1 → 1000 **LENGTH/DIAMETER:** ~ 80

AUXILIARY INFORMATION Properties evaluated at local average temperature

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD: Others' data

AUTHOR(S): Petersen, A W
Christiansen, E B

SOURCE: AIChE J. 12, 2, 221 (1966)

EQUATION PROPOSED:

$$St = St_{Re=1000} \left[\frac{St_{critical\ Re}}{St_{Re=10000}}\right]^{\phi(Re)} \left[\frac{Re}{10000}\right]^{\beta} \left[\frac{10000}{2100}\right]^{\beta\,\phi(Re)}$$

β is the slope of the j-factor vs Re curve in the turbulent region on a log log plot.

$$\phi(Re) = 1{,}635 \log\left[\frac{1}{1{,}133} \frac{(Re - 710)}{(Re - 1800)}\right]$$

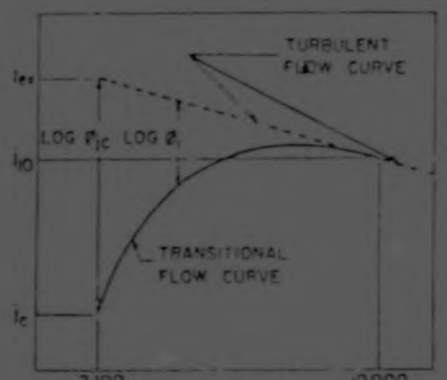


Illustration of the deviation of the transitional flow heat transfer curve from the extrapolated turbulent flow line on a log log plot and a graphical representation of $\log \phi_i$ and $\log \phi_{ic}$.

ϕ is the ratio at a given flow rate in the transition region, of the actual heat transfer rate to the heat transfer rate that would be obtained if fully turbulent flow heat transfer mechanism existed.

REYNOLDS: Transitional

PRANDTL: > 2

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER: 118 → 197

AUXILIARY INFORMATION Data for air are not in good agreement.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air, aqueous ethylene glycol

METHOD: Electrical heating.

AUTHOR(S): Gowen, R A
Smith, J W

SOURCE: Chem. Eng. Sci., 22, 1701 (1967)

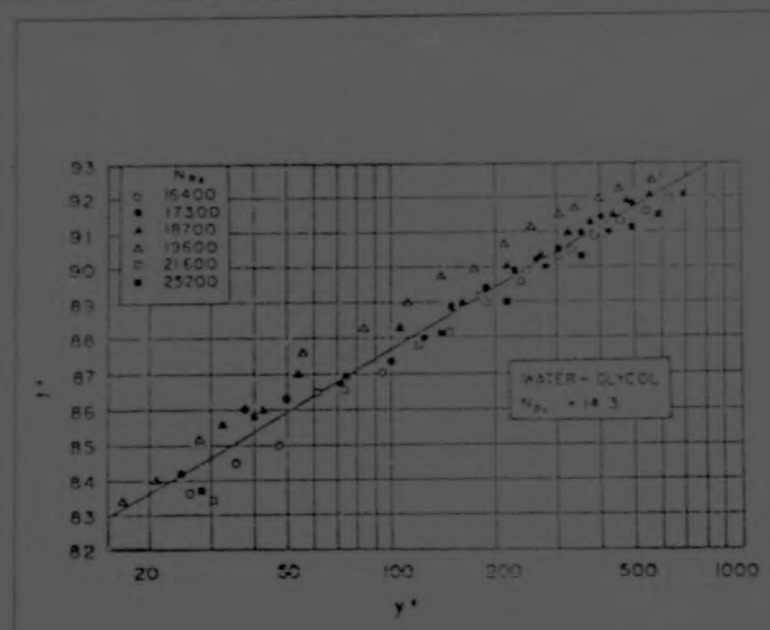
EQUATION PROPOSED:

$$St = \frac{\sqrt{\frac{f}{2}}}{B_s + \left[\sqrt{\frac{2}{f}} - 5{,}5\right]}$$

$$B_s = 5{,}5 \ln\left[\frac{5Pr + 1}{30}\right] + 8{,}55 + 5Pr$$

B_s is the constant in the universal temperature profile.

$$t_B^+ = \frac{\sqrt{\frac{f}{2}}}{St}$$



REYNOLDS: 10 000 → 50 000 **HORIZONTAL/VERTICAL:** Vertical

PRANDTL: 0,7 → 14,3

GRASHOF: **LENGTH/DIAMETER:** 21

AUXILIARY INFORMATION Film viscosity used for Pr.

$$T_f = \frac{T_B + T_w}{2}$$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Uniform heating

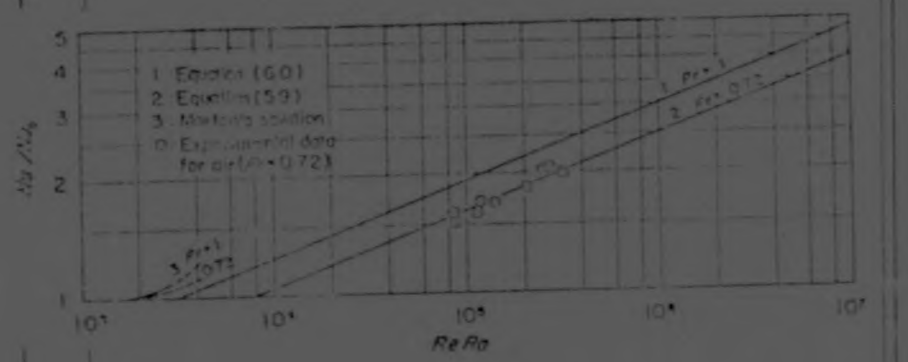
AUTHOR(S): Mori, Y
Futagami, K

SOURCE: Int. J. Heat Mass, 10, 1801 (1967)

EQUATION PROPOSED:

$$\frac{Nu}{Nu_o} = 1 + (0.1036 - 0.0007\,Pr + 0.3334\,Pr^2)\left[\frac{ReRa}{4608}\right]^2$$

using the results of their previous work



REYNOLDS: Laminar **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil

METHOD: Water or steam heating

AUTHOR(S): Test, F L

SOURCE: Trans. ASME C, 385 (1968)

EQUATION PROPOSED:

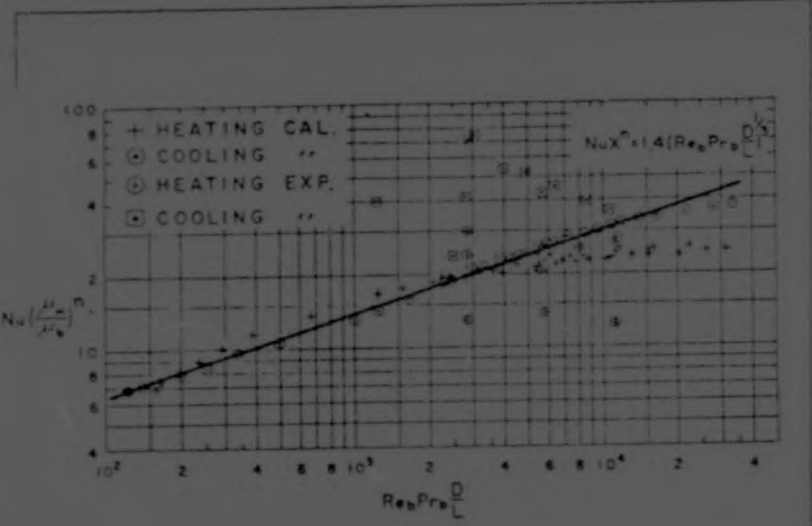
$$Nu_b \left[\frac{\mu_w}{\mu_b}\right]^n = C_1 \left[Re_b\, Pr_b\, \frac{d}{L}\right]^m$$

$n = 0{,}05$ heating

$n = 1/3$ cooling

$C_1 = 1{,}4$

$m = 1/3$



REYNOLDS: Laminar

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Vertical

LENGTH/DIAMETER: 240

AUXILIARY INFORMATION Properties based on bulk average temperature.

HEAT TRANSFER COEFFICIENTS IN TUBES

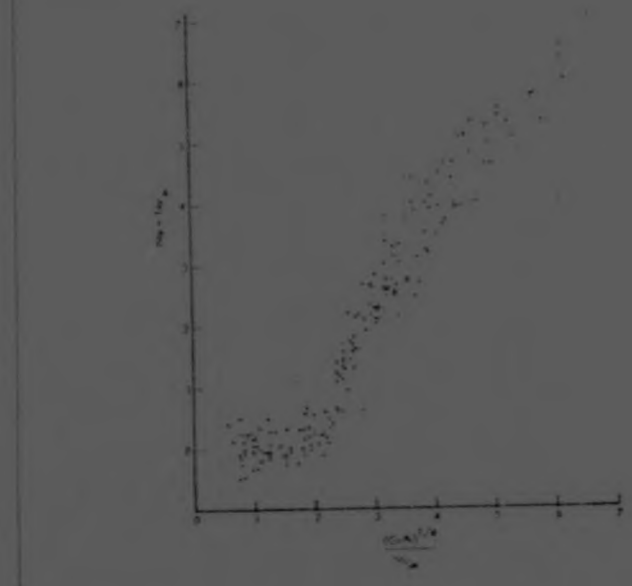
FLUID(S): Water

METHOD: Electrical heating

AUTHOR(S): Shannon, R L
Depew, C A

SOURCE: Trans. ASME C, 353 (1968)

EQUATION PROPOSED: In graphical form. It is suggested that natural convection is unimportant for $(GrPr)^{1/4}/Nu_x < 2$, where Nu_x is the solution from Siegel et al (1958).



REYNOLDS: 120 → 2300

PRANDTL:

GRASHOF: $2{,}5 \times 10^5$

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: → 700

AUXILIARY INFORMATION Average properties were used.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Everett, M

SOURCE: Chem. Engr. CE159, Sept. (1969)

EQUATION PROPOSED:

$$Nu = 0{,}0225 Re^{0.795} Pr^{\gamma} \left[\frac{\mu}{\mu_w}\right]^n$$

$$\gamma = 0{,}495 - 0{,}0225 \ln Pr$$

$$n = \left[\frac{Re}{870\,000}\right]^{0{,}84} \qquad Re < 62\,500$$

$$n = 0{,}11 \qquad Re > 62\,500$$

These are modified forms of the ESDU (1967)

REYNOLDS: $> 4 \times 10^3$ **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION Correlation is to within 10,2%

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Oil

METHOD:

AUTHOR(S): Ha[illegible], A
Gröll, W

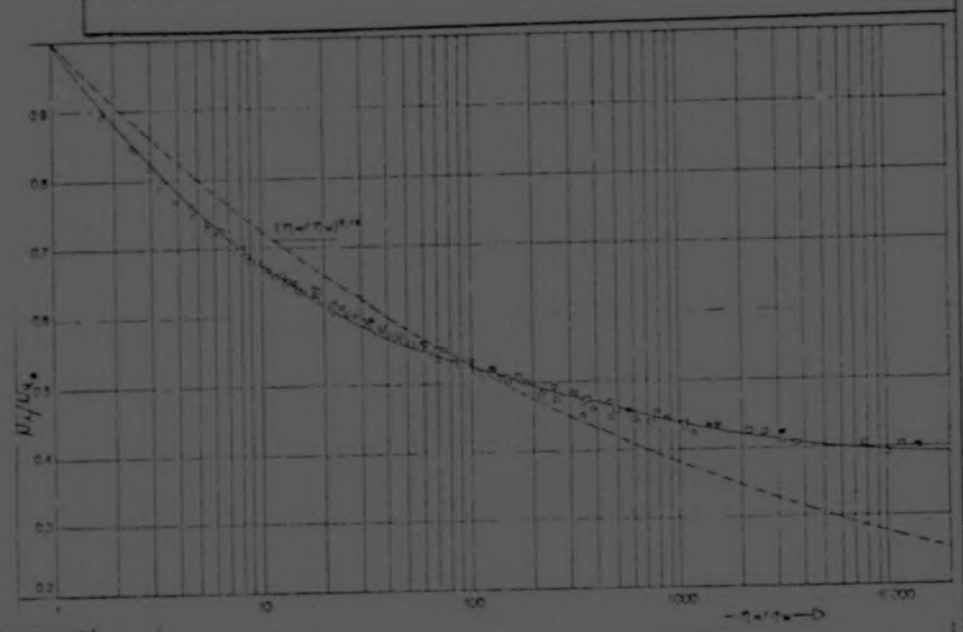
SOURCE: Verfahrenstechnik 3(4) 141 (1969)

EQUATION PROPOSED: Suggested a new correction factor for viscosity and presented this graphically.

Hausen, H [Verf. 3, (8) 355 (1969) and p 480]
showed that the equation

$$\frac{Nu}{Nu_o} = 0.645\left[\frac{\mu_w}{\mu_b}\right]^{-0,3} + 0,355$$

fitted the data.



REYNOLDS: 4000 → 11000 **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Ethylene glycol

METHOD: Electrical heating

AUTHOR(S): Shannon, R L
Depew, C A

SOURCE: Trans. ASME C, 91, 251 (1969)

EQUATION PROPOSED:

$$Nu = Nu_o \left[\frac{\mu_w}{\mu_b} \right]^m$$

REYNOLDS: 6 → 300
PRANDTL: 26 → 500
GRASHOF: → 2800

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER:

AUXILIARY INFORMATION

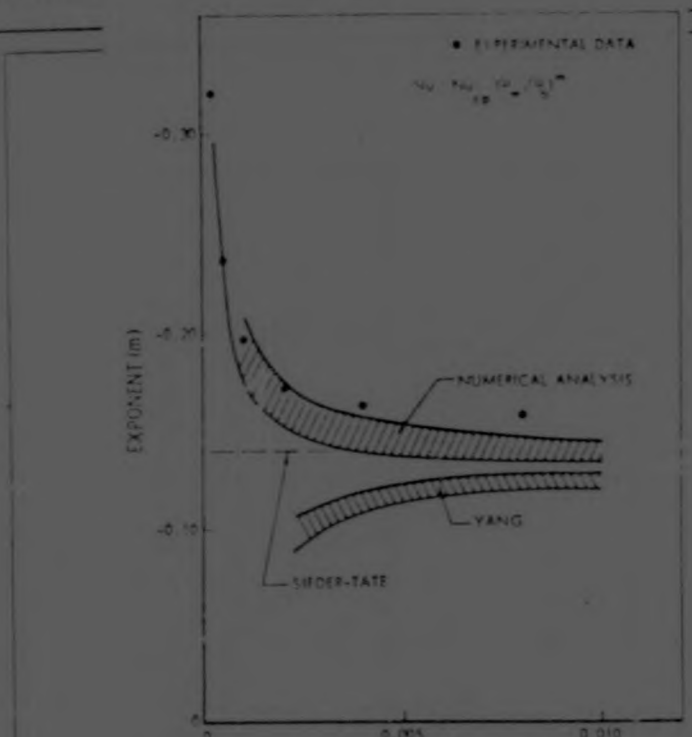


Fig. 7 Exponent m versus ξ

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Electrical heating

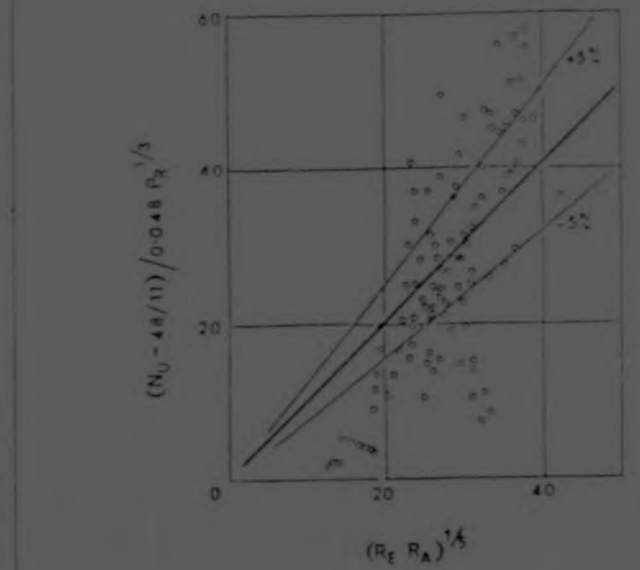
AUTHOR(S): Kupper, A
Hauptmann, E G
Iqbal, M

SOURCE: Solar Energy, 12, 439 (1969)

EQUATION PROPOSED:

$$Nu = \frac{48}{11} + 0.048\,Pr^{1/3}\,(ReRa)^{1/5}$$

correlates 53% of the data to 10%.



REYNOLDS: 100 → 2000 **HORIZONTAL/VERTICAL:** Horizontal
PRANDTL: 4 → 9
GRASHOF: 300 → 30000 **LENGTH/DIAMETER:** 310

AUXILIARY INFORMATION 1 ft entrance section. Bulk temperatures used for properties.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Krasnoschekov
Protopopov

High Temp. 4, (3) (1963)
9, (6) (1971)

SOURCE: Petukhov, Design and Theory Source book, 1974

EQUATION PROPOSED:

$$Nu_b = Nu_o \left[\frac{\rho_w}{\rho_b}\right]^m \left[\frac{\overline{Cp}}{Cp_b}\right]^n$$

REYNOLDS: HORIZONTAL/VERTICAL:

PRANDTL:

GRASHOF: LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Temperature dependent physical properties

METHOD: Using others' results and theoretical study.

AUTHOR(S): Gregorig, R

SOURCE: Wärme und Stoffübertragung, 3, 26 (1970)

ECUATION PROPOSED:

$$K_a = \frac{K_k K_{Pr}}{K_\nu}$$

$$K_k = \left[\frac{k_w}{k_m}\right]^{A(1 + 2x_k)}$$

$$K_{Pr} = \left[\frac{Pr_w}{Pr_m}\right]^{0,72A(1 + 2x_{Pr})}$$

$$K_\nu = \left[\frac{\nu_w}{\nu_m}\right]^{0,94A(1 + 2x_\nu)}$$

$$x_\lambda = \frac{\lambda_B - \lambda_w}{\lambda_m - \lambda_w} - \tfrac{1}{2}$$

$$A = 1 - \exp\left[-0,185\left[\frac{L}{D}\right]^{1/3}\right]$$

REYNOLDS: **HORIZONTAL/VERTICAL:**

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION w = wall temperature
m = mean temperature. B = at $T_B = (T_m + T_w)/2$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water, ethanol, glycerol / water

METHOD: Cooling with refrigerant

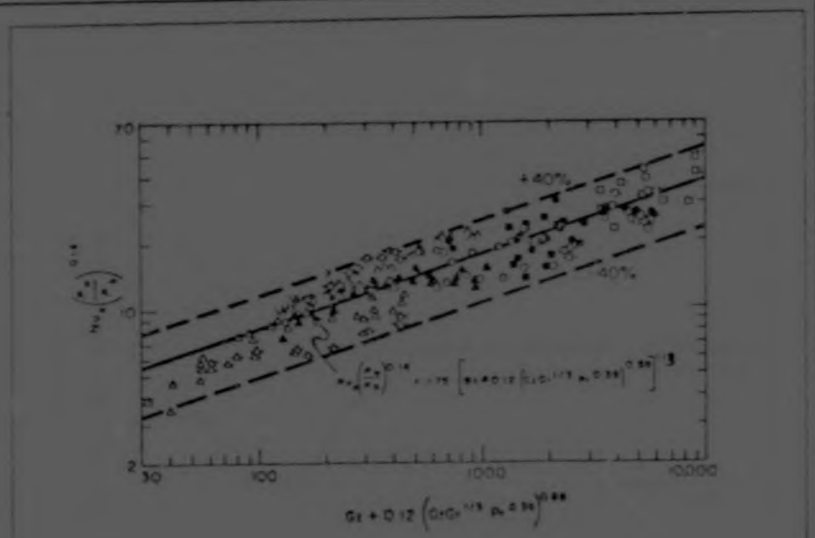
AUTHOR'S): Depew, C A
August, S E

SOURCE: Trans. ASME C, 380 (1971)

EQUATION PROPOSED:

$$Nu = 1{,}75\left[Gz + 0{,}12\left[GzGr^{1/3}\,Pr^{0{,}36}\right]^{0{,}88}\right]^{1/3}\left[\frac{\mu_b}{\mu_w}\right]^{0{,}14}$$

correlates most data to within 40%



REYNOLDS: Laminar **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 28,4

AUXILIARY INFORMATION Properties at bulk average temperatures.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD: Steam heating

AUTHOR(S): Herbert, L S
Sterns, U J

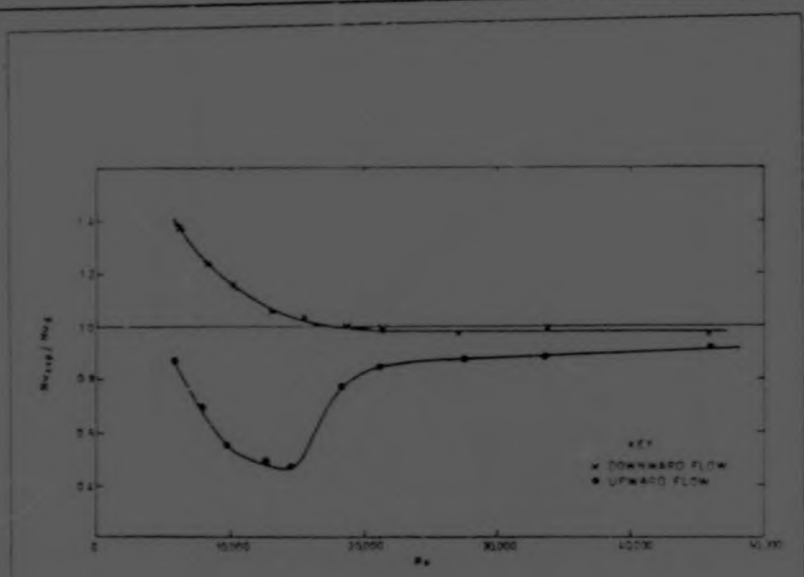
SOURCE: Chem. Eng. J. 4, 46 (1972)

EQUATION PROPOSED:

$Nu = 0{,}56\ Re^{0.47}\ Pr^{0.4}$ downward flow $Re < 15000$

$Nu = 8{,}5 \times 10^{-2}(GrPr)^{1/3}$ upward flow
Re 4500 → 15000
Gr $3 \times 10^6 - 30 \times 10^6$

or $Nu = 0{,}0225\ Re^{0.795}\ Pr^{(0.495 - 0.0225\ \ln Pr)}$ ..... (a)



Experimental results compared with equation (a)

REYNOLDS: 5600 → 71000

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Vertical

LENGTH/DIAMETER: 80

AUXILIARY INFORMATION Agreement with equation (a) was not good

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Data of Lawn (1969)

METHOD: Inclusion of the term $\frac{dP}{dx}$ in the eddy penetration model.

AUTHOR(S): Gross, R. J.
Thomas, L. C.

SOURCE: Trans. ASME C 494 (1972)

EQUATION PROPOSED:

$$Nu = \frac{T_o - T_i}{T_o - T_b} \frac{U_b}{U_i} \frac{f}{2} Re\sqrt{Pr} \left[\frac{1 + \sqrt{1 - \left[\frac{16}{X}\right]}}{2} \right]$$

$$X = \left[Re\sqrt{\frac{f}{2}} \right] \left[U_b \frac{\sqrt{\frac{f}{2}}}{U_i} \right]$$

U_i = eddy velocity at the first instant of renewal

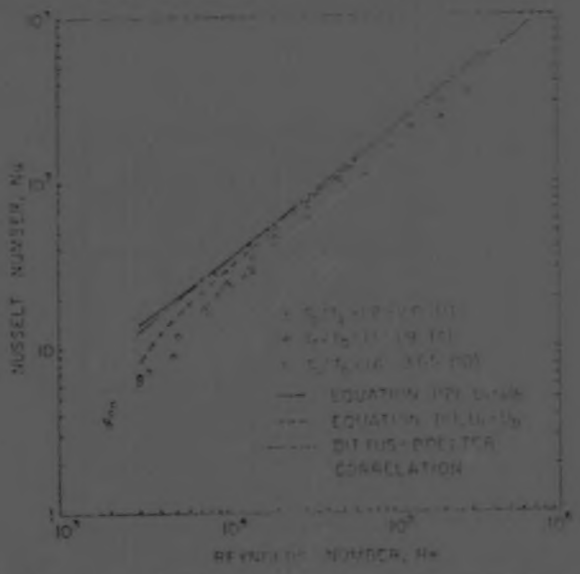


Fig. 1 Comparison of predictions for Nusselt number with experimental data, after Lawn

REYNOLDS: Turbulent — Transitional

HORIZONTAL/VERTICAL:

PRANDTL:

GRASHOF:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Transformer oil, fuel oil

METHOD: Electrical heating

AUTHOR(S): Kuznetsova, V V

SOURCE: Teploenergetika, 19, (5) 84 (1972)

EQUATION PROPOSED:

$$Nu = 1{,}23\left[Pe\,\frac{d}{L}\right]^{0.4}\left[\frac{\mu}{\mu_w}\right]^{-1/6}$$

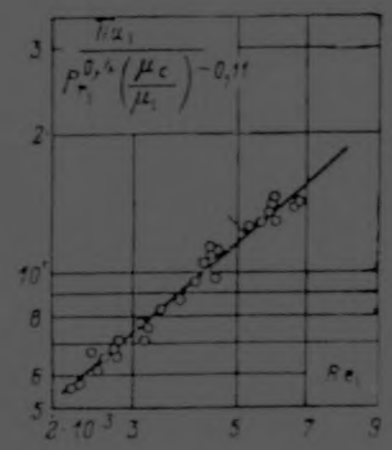
Re 400 - 1900

Pr 170 - 640

$$Nu = 0{,}013\,Re^{0.8}\,p_r^{0.4}\left[\frac{\mu}{\mu_w}\right]^{-0.11}$$

Re 2000 - 8000

Pr 70 - 200



REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER:

AUXILIARY INFORMATION Properties at bulk temperatures.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

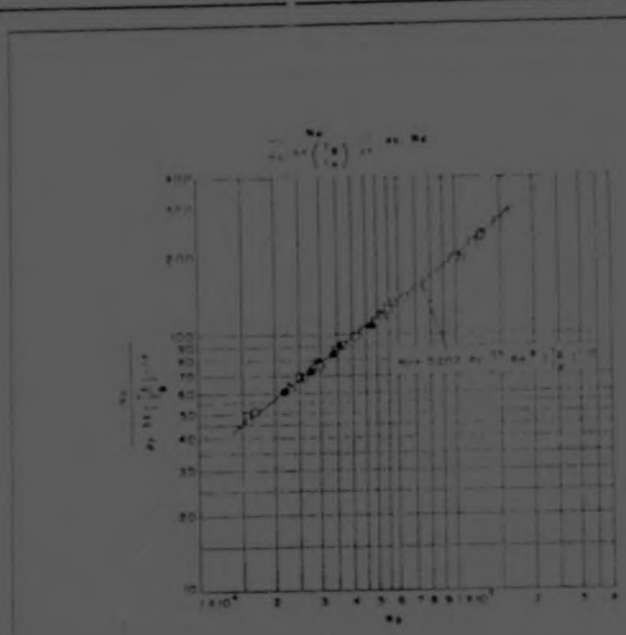
METHOD: Uniform surface temperature.

AUTHOR(S): Zucchetto, J
Thorsen, R S

SOURCE: Trans. ASME C, 134 (1973)

EQUATION PROPOSED:

$$Nu = 0{,}0207\, Pr^{0.33}\, Re^{0.8} \left[\frac{T_s}{T_b}\right]^{-0{,}17}$$



REYNOLDS: Turbulent **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:**

AUXILIARY INFORMATION Properties evaluated at $T_b = T_s + \Delta T_{LM}$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Water

METHOD:

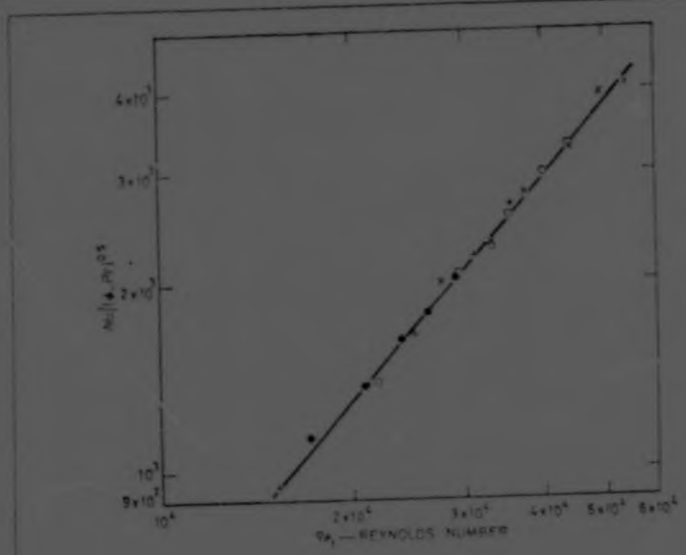
AUTHOR(S): Walker, R A
Bott, T R

SOURCE: Chem. Engr 151 (1973)

EQUATION PROPOSED:

$$Nu = 0{,}0114\sqrt{\frac{f}{2}}\ Re^{1{,}174}\ p_r^{0{,}5}$$

Properties at average temperature.



× : $Pr \sim 2.8$
⊙ : $Pr \sim 3.4$
+ : $Pr \sim 4.0$
○ : $Pr \sim 4.7$
Slope = 1.17

REYNOLDS: 14000 → 50000 HORIZONTAL/VERTICAL:

PRANDTL: 2,0 → 5,5

GRASHOF: LENGTH/DIAMETER:

AUXILIARY INFORMATION The authors were looking at the effect of sand grain roughness.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD:

AUTHOR(S): Hausen, H

SOURCE: Wärme und Stoffübertragung, 7, 222 (1974)

EQUATION PROPOSED:

$$Nu = 0{,}0235 \left[Re^{0.8} - 230 \right] \left[1{,}8\, Pr^{0.3} - 0{,}8 \right] \left[1 + \left[\frac{d}{L} \right]^{2/3} \right] \left[\frac{\mu}{\mu_w} \right]^{0{,}14}$$

REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION Altered form of the equation presented in 1959.

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD: Others' results

AUTHOR(S): Gnielinski, V

SOURCE: Forsch. Ing.–Wes, 41, (1) 8 (1975)

EQUATION PROPOSED:

$$Nu = 0{,}0214\left[Re^{0{,}8} - 100\right]Pr^{0{,}4}\left[1 + \left[\frac{d}{L}\right]^{2/3}\right]\left[\frac{T}{T_w}\right]^{0{,}45}$$

$$0{,}6 < Pr < 1{,}5$$

$$2300 < Re < 10^6$$

$$Nu = 0{,}012\left[Re^{0{,}87} - 280\right]Pr^{0{,}4}\left[1 + \left[\frac{d}{L}\right]^{2/3}\right]\left[\frac{Pr}{Pr_w}\right]^{0{,}11}$$

$$1{,}5 < Pr < 500$$

REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD: Using others' results.

AUTHOR(S): Sleicher, C A
Rouse, M W

SOURCE: Int. J. Heat Mass, 18, 677 (1975)

EQUATION PROPOSED:

$$Nu_b = 5 + 0{,}015\, Re_b^{a}\, Pr_w^{b} \qquad 0{,}1 < Pr < 10^5 \qquad \text{.....(a)}$$

$$10^4 < Re < 10^6$$

$$a = \frac{0{,}88 - 0{,}24}{(4 + Pr_w)}$$

$$b = {}^{1}/_{3} + 0{,}5\, e^{-0{,}6 Pr_w}$$

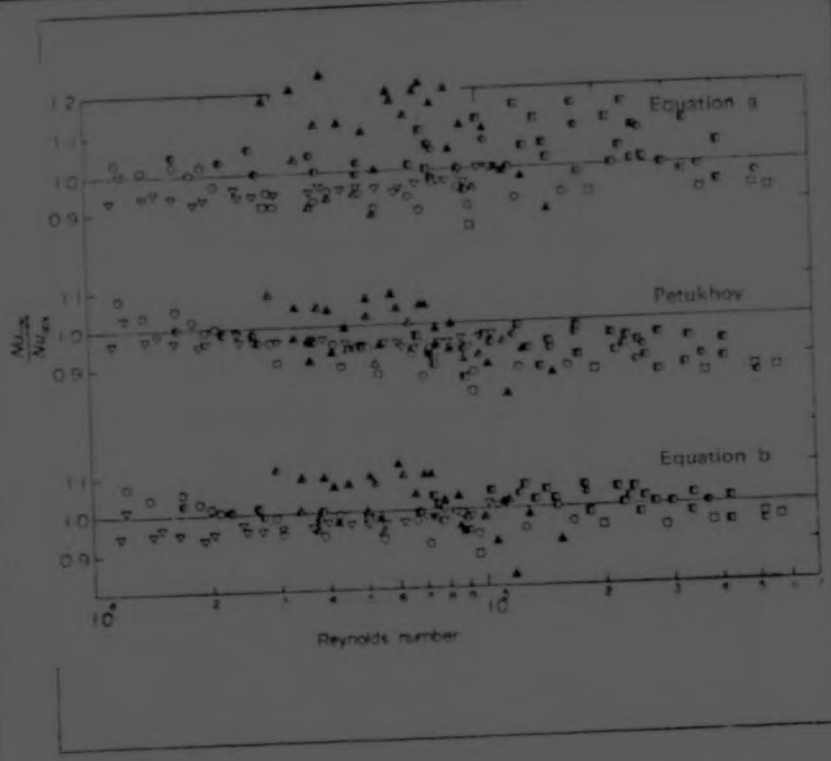
or if $\frac{T_w}{T_B} > 2$

$$Nu_b = 5 + 0{,}012\, Re_b^{0{,}83} \left[Pr_b + 0{,}29\right]\left[\frac{T_w}{T_b}\right]^{n} \qquad \text{.....(b)}$$

$$n = -\log_{10}\left[\frac{T_w}{T_b}\right]^{1/4} + 0{,}3$$

$$0{,}6 < Pr < 0{,}9 \qquad \frac{x}{d} > 40$$

$$10^4 < Re < 10^6$$



REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

AUXILIARY INFORMATION

Petukhov, Adv. Heat Trans. 6, 503 (1970)

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Constant wall temperature.

AUTHOR(S): Pechenegov, Yu. Ya

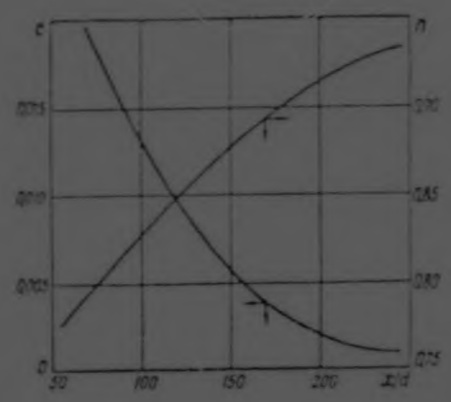
SOURCE: Heat Trans. Sov. Res., 7, (6) 60 (1975)

EQUATION PROPOSED:

$$Nu_{wall,x} = C\,Re^{n}_{wall,x}$$

where C and n are given in the graph.

Correllation is to within 3%.



c and n in formula as functions of x/D.

REYNOLDS: Lower turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 77; 154; 231

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Constant wall temperature.

AUTHOR(S): Pechenegov, Yu. Ya

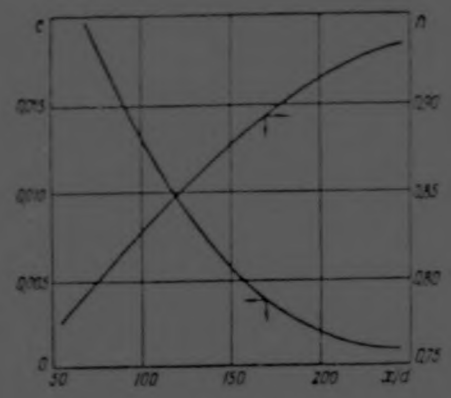
SOURCE: Heat Trans. Sov. Res., 7, (6) 60 (1975)

EQUATION PROPOSED:

$$Nu_{wall,x} = C\,Re^{n}_{wall,x}$$

where C and n are given in the graph.

Correllation is to within 3%.



c and n in formula as functions of x/D.

REYNOLDS: Lower turbulent **HORIZONTAL/VERTICAL:** Horizontal

PRANDTL:

GRASHOF: **LENGTH/DIAMETER:** 77; 154; 231

AUXILIARY INFORMATION

CSIR REPORT CENG 327

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Air

METHOD: Constant wall temperature.

AUTHOR(S): Pechenegov, Yu. Ya

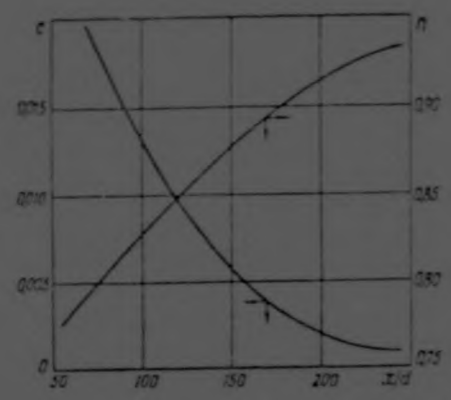
SOURCE: Heat Trans. Sov. Res., 7, (6) 60 (1975)

EQUATION PROPOSED:

$$Nu_{wall,x} = C\,Re^{n}_{wall,x}$$

where C and n are given in the graph.

Correllation is to within 3%.



c and n in formula as functions of x/D.

REYNOLDS: Lower turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL: Horizontal

LENGTH/DIAMETER: 77; 154; 231

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

METHOD: Others' results.

AUTHOR(S): Gregorig, R

SOURCE: Wärme und Stoffübertragung, 9, 61 (1976)

EQUATION PROPOSED:

$\frac{L}{d} > 70$, cooling of liquids only.

$$\frac{Nu_b}{Nu_o} = \left[\frac{Pr_b}{Pr_w}\right]^P$$

$$P = \frac{0{,}264(½ - x_{Pr})^{0{,}20} + 0{,}0757}{Pr_b^{0{,}05}\, Re_b^{0{,}02}\, (½ - x_{Pr})^{0{,}01}}$$

$$x_{Pr} = \frac{Pr_f - Pr_w}{Pr_b - Pr_w} - ½$$

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION $t_f = \frac{[t_b + t_w]}{2}$

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): All

METHOD: Generalised correlation.

AUTHOR(S): Churchill, S W

SOURCE: Ind. Eng. Chem. Fund. 16, (1) (109) (1977)

EQUATION PROPOSED:

$$\left[Nu\right]^{10} = \left[Nu_{\ell}\right]^{10} + \left[\frac{e^{\frac{(2200 - Re)}{365}}}{\left[Nu_{\ell c}\right]^{2}} + \left[\frac{1}{Nu_{o}^{o} + \frac{0{,}079\, Re\sqrt{f}\, Pr}{(1 + Pr^{4/5})^{5/6}}}\right]^{2}\right]^{-5}$$

$Nu_{\ell c}$ = > laminar regime at Re = 2100

Nu_{o}^{o} = > as Pr → 0 and Re → 2100

Nu_{ℓ} = > laminar regime

REYNOLDS:

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S): Fluids with temperature dependent viscosity

METHOD: Others' results

AUTHOR(S): Hanna, O T
Sandall, O C

SOURCE: Trans. ASME C, 100, 224 (1978)

EQUATION PROPOSED:

$$\frac{\frac{St}{\sqrt{\frac{f}{2}}}}{\left[\frac{St}{\sqrt{\frac{f}{2}}}\right]_o} = \left[1 - \frac{1}{A}\right] + \frac{H}{A(T_c - T_w)}$$

$$H = \int_{T_w}^{T_c} \frac{\rho_c}{\rho} \, dT$$

$T_c \geq$ bulk conditions

For heating A = 2,4; For cooling A = 1,05

REYNOLDS: Turbulent

HORIZONTAL/VERTICAL:

PRANDTL:

GRASHOF:

LENGTH/DIAMETER:

AUXILIARY INFORMATION

HEAT TRANSFER COEFFICIENTS IN TUBES

FLUID(S):

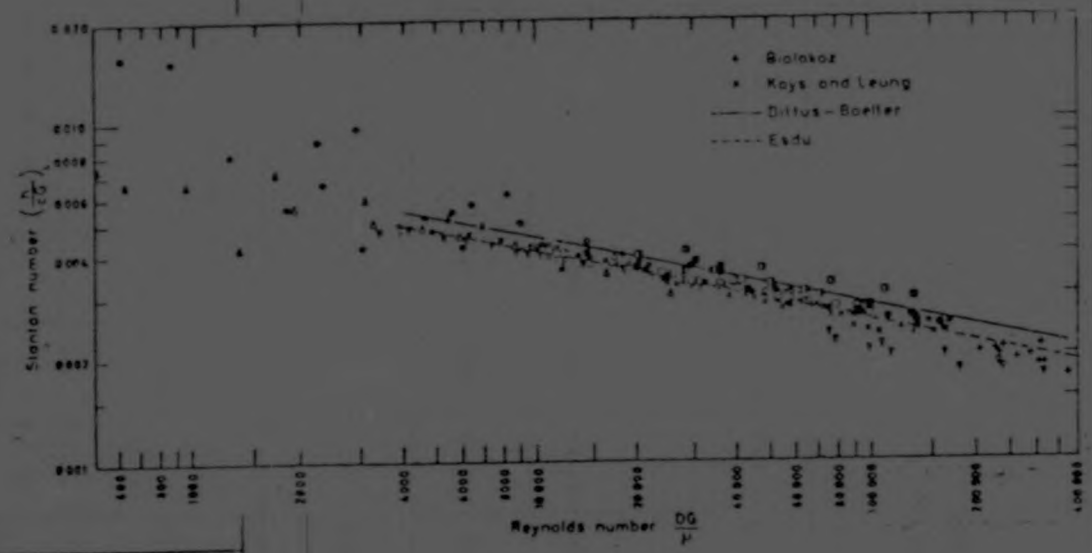
METHOD:

AUTHOR(S): Polley, G T

EQUATION PROPOSED:

Suggests the use of the ESDU correlation (1967).

SOURCE: Chem. Engr, 233, (April 1979)



REYNOLDS: Turbulent

PRANDTL:

GRASHOF:

HORIZONTAL/VERTICAL:

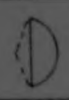
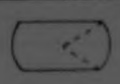
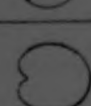
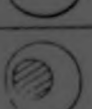
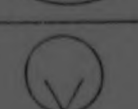
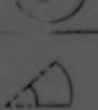
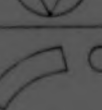
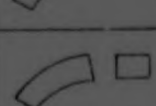
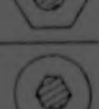
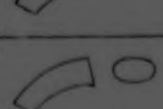
LENGTH/DIAMETER:

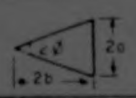
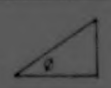
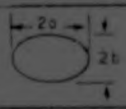
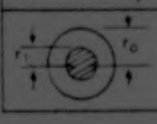
AUXILIARY INFORMATION

4.4 SUMMARY OF ANALYTICAL SOLUTIONS FOR TURBULENT FLOW FROM McELIGOT, SMITH AND BANKSTON (1970)

"Title"/Reference	Basic Representation	x	y^+_ℓ	Range	PREDICTION FOR CONSTANT PROPERTIES, Re = 10^5 Nu/Nu_{DB}	f/f_{DKM}
Reichardt "local"	$\frac{\varepsilon}{\nu_y} = \frac{\kappa}{6}\left[y_y^+ - y_\ell^+ \tanh\frac{y_y^+}{y_\ell^+}\right]\left[2-\frac{y}{r_w}\right]\left[1+2\left[1-\frac{y}{r_w}\right]^2\right]$	0,4225	11,0	Wall → C_ℓ	0,945	0,989
Reichardt modified "wall"	Same except replaced y_y^+ by y_w^+ in argument of tanh.	Same	Same	Same	Same	Same
Reichardt "wall"	$\frac{\varepsilon}{\nu_w} = \frac{\kappa}{6}\left[y_w^+ - y_\ell^+ \tanh\frac{y_w^+}{y_\ell^+}\right]\left[2-\frac{y}{r_w}\right]\left[1+2\left[1-\frac{y}{r_w}\right]^2\right]$	Same	Same	Same	Same	Same
Three layer modified Martinelli	Smith, S B. MSE Report, Univ. Arizona, 1967.	0,4	–	–	0,964	0,993
Sparrow, Hallman and Siegel "local"	$\varepsilon = n^2 u y\left[1-e^{\frac{-n^2 uy}{\nu^2}}\right]$; $n = 0,124$ $\frac{\varepsilon}{\nu_y} = \kappa\, y_y^+\left[1-\frac{y}{r_w}\right]-1$	0,36	26,0	$y_y^+ < y_\ell^+$ $y_y^+ > y_\ell^+$	0,943	0,984
Sparrow, Hallman and Siegel "wall"	Viscous sublayer same (except range); Core: $\frac{\varepsilon}{\nu_w} = \kappa\, y_w^+\left[1-\frac{y}{r_w}\right]-1$	0,36	26,0	$y_w^+ < y_\ell^+$ $y_w^+ > y_\ell^+$	Same	Same
Van Driest "local"	$\ell = \kappa y\left[1-\exp\left[-y_y^+/y_\ell^+\right]\right]$	0,4	26,0	Wall → C_ℓ	1,03	1,06
Van Driest "wall"	Same except replaced y_y^+ by y_w^+ in argument of exp.	Same	Same	Same	Same	Same
Kendall et al. "local"	$\frac{d\ell_y^+}{dy_y^+} = \frac{\kappa y_y^+ - \ell_y^+}{y_\ell^+}\sqrt{\frac{\tau_y}{\tau_w}}$	0,42	11,83	Wall → C_ℓ	1,03	1,06
Kendall et al. "wall"	$\frac{d\ell_y^+}{dy_y^+} = \frac{\kappa y_y^+ - \ell_y^+}{y_\ell^+}\sqrt{\frac{\tau_y/\rho_y}{\tau_w/\rho_w}}$	Same	Same	Same	Same	Same
Kendall w/ Clauser "wall"	Wall region same form as Kendall et al. "wall"; Core (or wake): $\varepsilon + \nu = 0,018\, u_{c_\ell}^*$	0,44	11,83	$\varepsilon+\nu \leq$ 0,018 $u_{c_\ell}^*$	0,939	1,00

4.5 SUMMARY OF ANALYTICAL SOLUTIONS FOR LAMINAR FLOW IN VARIOUS GEOMETRIES [SHAH AND LONDON (1971)]

GEOMETRY		GEOMETRY	
	Equilateral triangular duct with rounded corners		Longitudinal flow between cylinders, square array
	Sine ducts		Pascal's limacon
	Circular sector ducts		Curvilinear polygonal ducts
	Circular segment ducts		Ovaloid ducts
	Flat sided circular duct		Confocal elliptical ducts
	n-sided cusped ducts		Circular duct with rounded corner square cores
	Moon shaped ducts		Circular duct with elliptical cores
	Cardioid duct		Elliptical ducts with circular cores
	Eccentric annular ducts		Internally finned tube
	Annular sector ducts		Curved circular ducts
	Regular polygonal ducts with central circular cores		Curved rectangular ducts
	Circular duct with central regular polygonal cores		Curved elliptical ducts
	Longitudinal flow between cylinders, triangular array		Curved concentric annular ducts

GEOMETRY		^1Re	Nu_{H1}	Nu_T
	Straight circular duct	16	48/11	3,657
	Parallel plates	24	140/17	7,541
	Rectangular ducts $2b/2a \sim 0 - 1$			
	Isosceles triangular ducts $2\phi \sim 0 - 180^o$			
	Right triangular duct $\phi \sim 0 - 180^o$			
	n-sided regular polygonal ducts $n \sim 3 - \infty$			
	Elliptical ducts $2b/2a \sim 0 - 1$			
	Concentric annular ducts $r_i/r_o \sim 0 - 1$			

4.6 NOMENCLATURE

A — Transfer area

a, b, $c_{\text{subscripted}}$ — Empirical coefficient

Cp — Specific heat (J/kgK)

C_f — Coefficient of friction

$\tau / \rho u^2$

C_H — Coefficient of heat transfer

$\frac{q}{Cp\rho u\theta}$

d, D — Inside diameter of tube (m)

f — (Fanning) friction factor

$\frac{2R}{\rho u^2}$

g — Gravitational acceleration (m/s^2)

G — Mass velocity $\frac{4\dot{m}}{\pi d^2}$ (kg/sm^2)

Gr — Grashof number

$\frac{\beta g \ell^3 \rho^2 \Delta\theta}{\mu^2}$

Gz — Graetz number

$\frac{\dot{m}Cp}{k\ell} = \frac{\pi}{4} \frac{Re_d Prd}{\ell}$

h — Heat transfer coefficient (W/m^2K)

j — Colburn's factor

j_H

k — Thermal conductivity (W/mK)

ℓ — Characteristic length (m)

L — Tube length (m)

ℓ — Prandtl's mixing length

$\dot{m}$ — Mass flow rate (kg/s) $\frac{\pi d^2}{4} \rho u$

Nu — Nusselt number

$\frac{hd}{k}$

p — Pressure (N/m^2)

Pe — Peclet number

$\frac{\rho u d Cp}{k} = Re_d Pr$

Pr — Prandtl number

$\frac{Cp\mu}{k}$

q — Heat flux (W/m^2)

Q — Heat flow rate (W)

Ra — Raleigh number GrPr

Re — Reynolds number

$\frac{\rho u \ell}{\mu}$

r — Radius (m)

R — Resistance to flow

S — Cross sectional area (m^2)

St — Stanton number

$\frac{h}{\rho u Cp} = \frac{Nu}{Re_d Pr}$

T — Temperature (K)

t — Time (s)

u — Velocity x (m/s)

U — Overall heat transfer coefficient (W/m^2K)

Velocity y (m/s)

Velocity z (m/s)

Direction along axis of pipe (m)

Direction along radius (m)

Ratio of viscosity to the viscosity of water at 68 ^{o}F

Thermal diffusivity

$\frac{k}{\rho Cp}$ (m^2/s)

Coefficient of volume expansion

$(\frac{1}{K})$

Dynamic viscosity

$\left[\frac{kg}{ms}\right]$

Kinematic viscosity

$\frac{\mu}{\rho}$ (m^2/s)

Temperature (K)

Density (kg/m^3)

Shear stress (N/m^2)

Eddy diffusivity (m^2/s)

Eddy diffusivity of heat (m^2/s)

Eddy diffusivity of momentum (m^2/s)

Layer thickness (m)

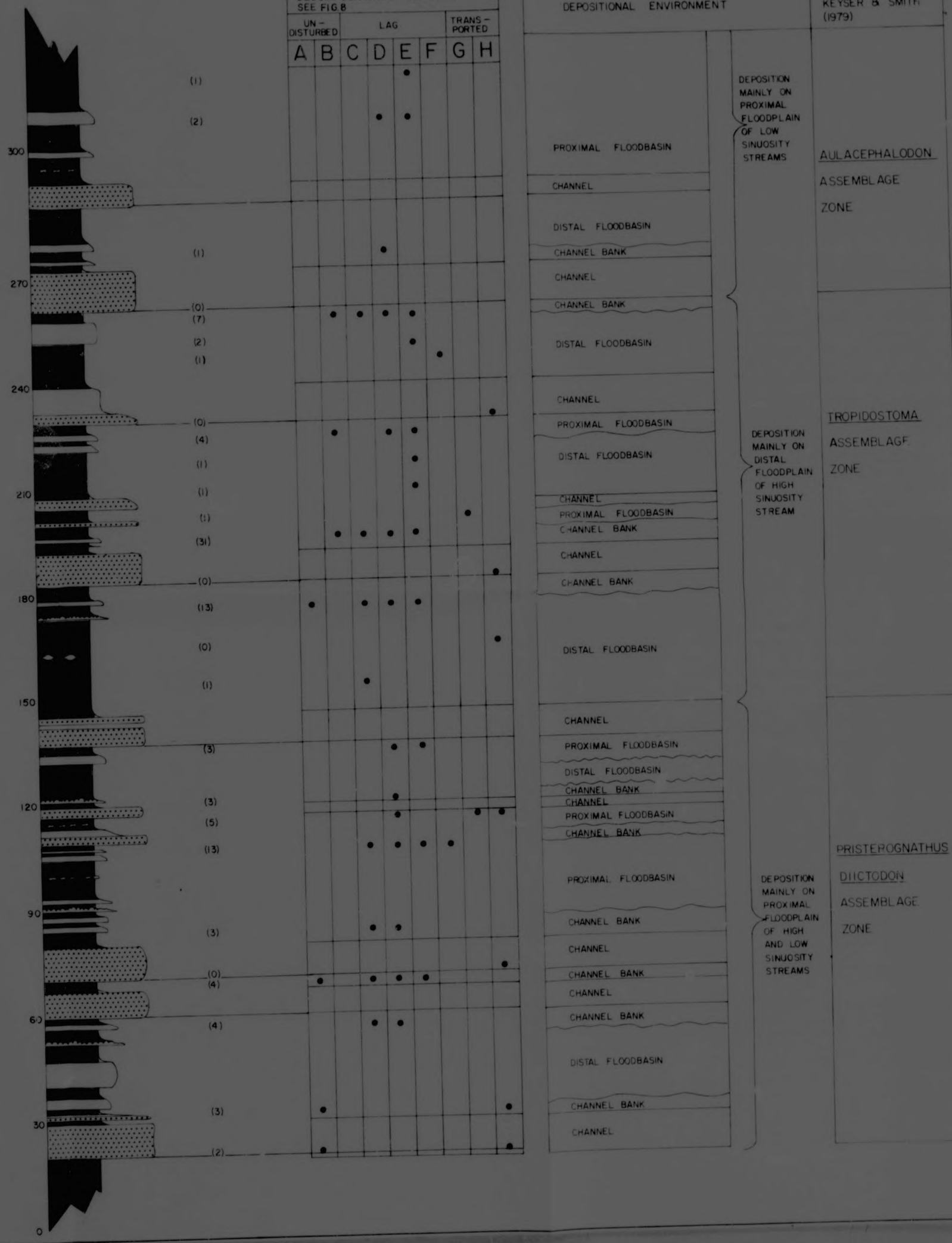
Subscripts

b	Bulk
am	Arithmetic mean
avg	Average
$\mathcal{C}_L$	Centreline
w	Wall
i	Inlet, intermediate
o	Outlet at zero heat flux;
o	Constant properties
lm	Log mean
d	Based on diameter
x	Based on x distance
L	Based on tube length
f	Film
1,2, etc.	Arbitrary basis
∞	As infinite conditions are reached.

APPENDIX IV

COMPOSITE SECTION OF TRAVERSES A, B AND C (SEE MAP I) SHOWING LITHOLOGY AND DEPOSITIONAL ENVIRONMENTS AND THE TAPHONOMY OF DIICTODON FELICEPS FOSSILS

REFER TO FIG. 8 FOR EXPLANATION OF TAPHONOMIC CLASSES



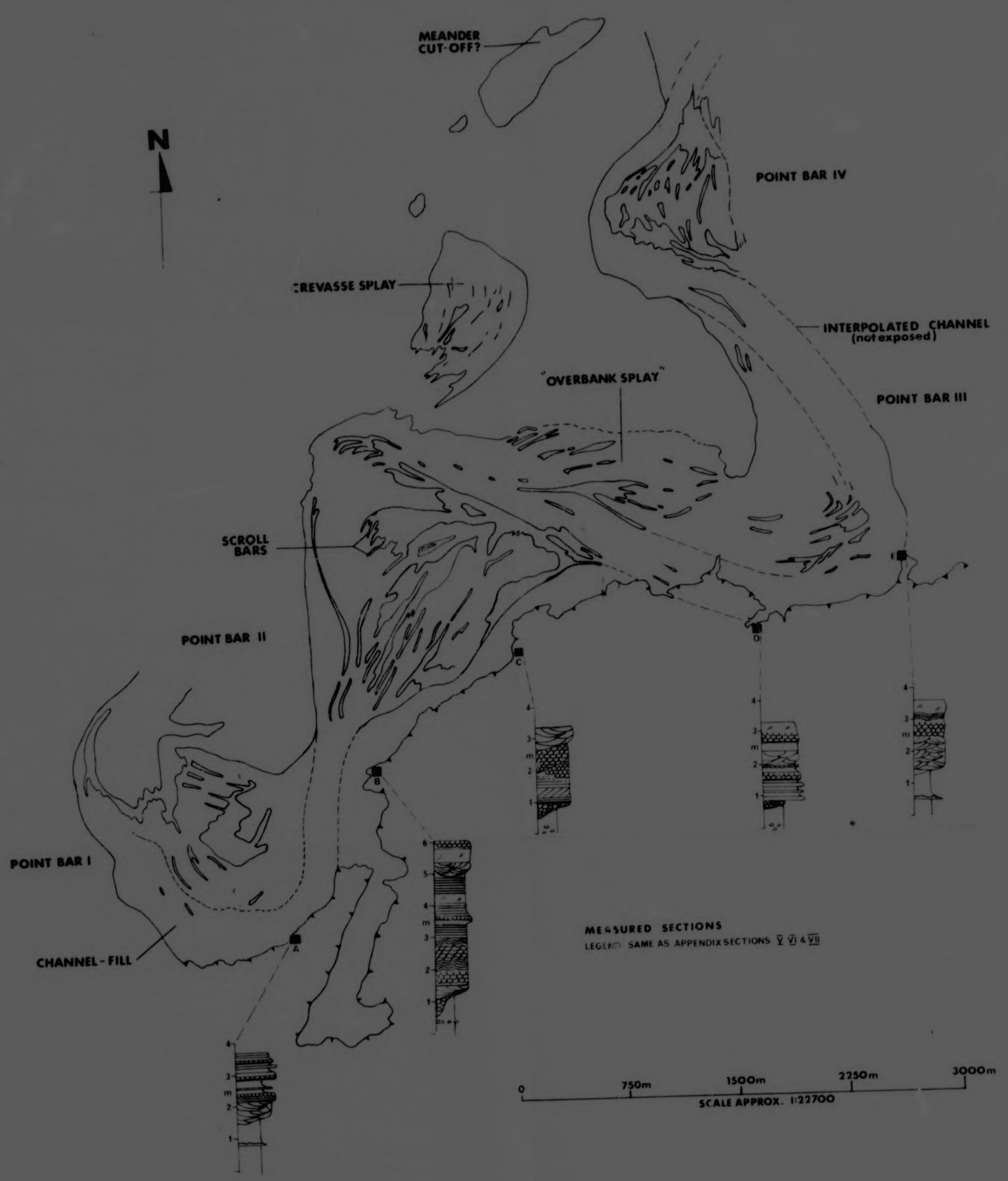
LEGEND

- SANDSTONE
- SILTSTONE
- MUDSTONE
- CALCAREOUS CONCRETIONARY LAYER
- "CHERT"
- CALCAREOUS NODULES
- FOSSIL LOCALITY
- (3) NO. OF DIICTODON FELICEPS
- (0) OBSERVED FOSSIL (NOT COLLECTED)

SCALE IN METRES

APPENDIX III

TYPE A BEDS : PLAN OF REIERSVLEI SANDSTONE

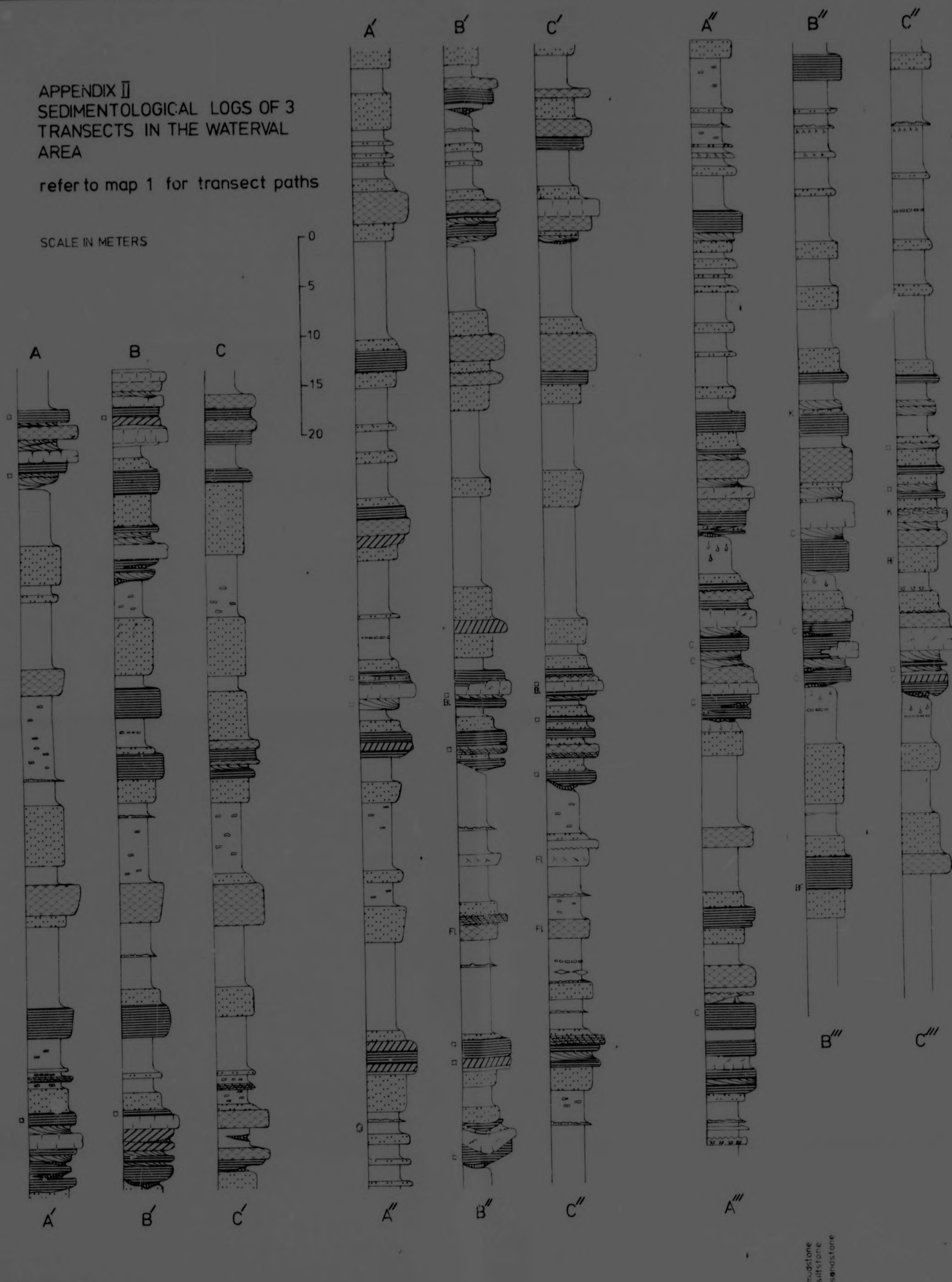


APPENDIX II
SEDIMENTOLOGICAL LOGS OF 3 TRANSECTS IN THE WATERVAL AREA

refer to map 1 for transect paths

SCALE IN METERS

0
5
10
15
20





MAP 3: BIOZONE MAP OF THE WATERVAL - BERGVALLEI AREA

LEGEND

- AULACEPHALODON / CISTECEPHALUS ASSEMBLAGE ZONE
- TROPIDOSTOMA / ENDOTHIODON ASSEMBLAGE ZONE
- UPPER / LOWER PRISTEROGNATHUS/DICTODON ASSEMBLAGE ZONE
- FOSSIL LOCALITY (SEE APPENDIX I)
- DOLERITE
- SANDSTONE OUTCROP
- DYKE

0 750 1500 2250 3000 metres

Map 2 DETAILED BIOZONATION OF THE REGION SURROUNDING THE STUDY AREA

SCALE: 1:100 000



MAP I: THE GEOLOGY OF THE WATERVAL AREA

0 750 1500 2250 3000 metres

LEGEND

- MAJOR SANDSTONE 1
- MAJOR SANDSTONE 2
- EXTENSIVE SANDSTONE OUTCROPS
- MINOR SANDSTONE OUTCROPS
- DOLERITE
- MUDROCK AND ALLUVIUM
- LINE OF TRANSECT
- DYKE

ROODBERG

EKLIPSBERG

BERGVALLEI

WATERVAL

LIEFONTEIN

REIERSVLEI SANDSTONE

25°30'

32°00'

01 02 03 04 05 06 07 08 09 10 32°11'

DEPARTEMENT VAN MYNWESE
DEPARTMENT OF MINES

GEOLOGIESE OPNAME
GEOLOGICAL SURVEY

AFDELING/DIVISION
GEOLOGY

KOPIEREG

MAP I: THE GEOLOGY OF THE WATERVAL AREA

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SAMGESTEL DEUR / COMPILED BY: R.M.H. Smith

GETEKEN DEUR / DRAWN BY: F. Haale

VERWYSINGS / REFERENCES

TEGNIESE KONTROLE / TECHNICAL CONTROL: R.M. Kayne

DATUM / DATE: August 1979

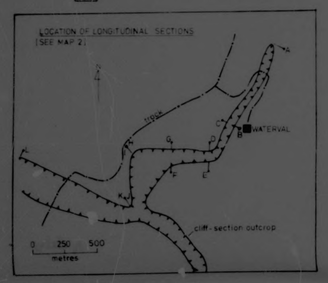
KLASSIFIKASIE/CLASSIFICATION

NR. / NO.

LEGEND TO LONGITUDINAL SECTIONS

- SANDSTONE UNIT I
- SANDSTONE UNIT II
- SANDSTONE UNIT III
- EROSION AND REACTIVATION SURFACES

LOCATION OF LONGITUDINAL SECTIONS (SEE MAP 2)

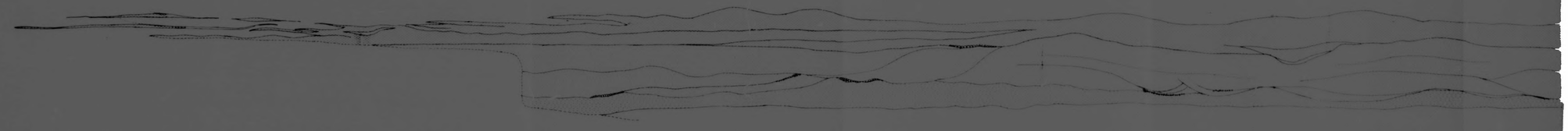


LEGEND TO COLUMNAR LOGS

- MUDSTONE
- SILTSTONE
- RIPPLE CROSS-LAMINATION
- CLIMBING RIPPLE LAMINATION
- INCIPIENT RIPPLE LAMINATION
- WAVY OR PARALLEL LAMINATION
- HORIZONTAL LAMINATION
- MASSIVE BEDDING
- FLAT BEDDING
- TROUGH CROSS-BEDDING
- PLANAR CROSS-BEDDING
- MUDSTONE PEBBLE CONGLOMERATE
- CALCAREOUS NODULES AND NODULAR LAYERS
- PALAEOSURFACE
- SOLE STRUCTURES
- B BONE FRAGMENTS
- SLUMPED MUDSTONE
- P PLANOLITES

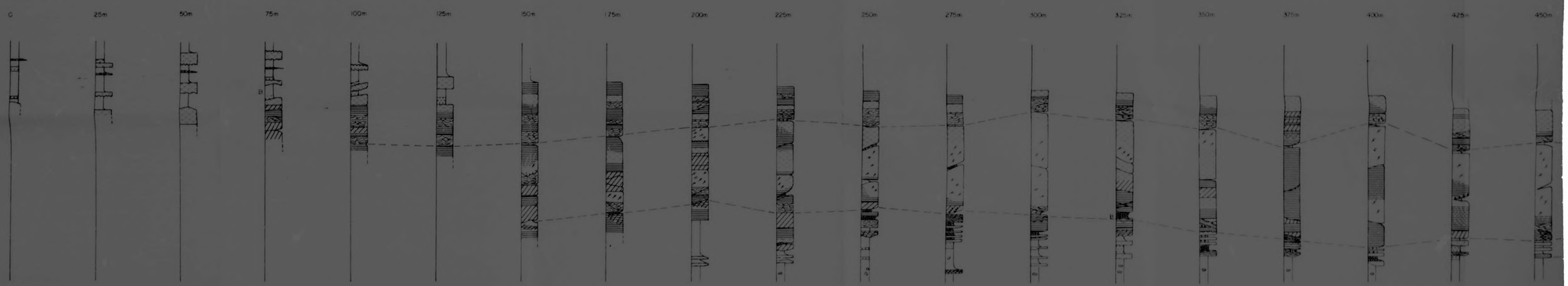
SCALE IN METRES 1:500

A



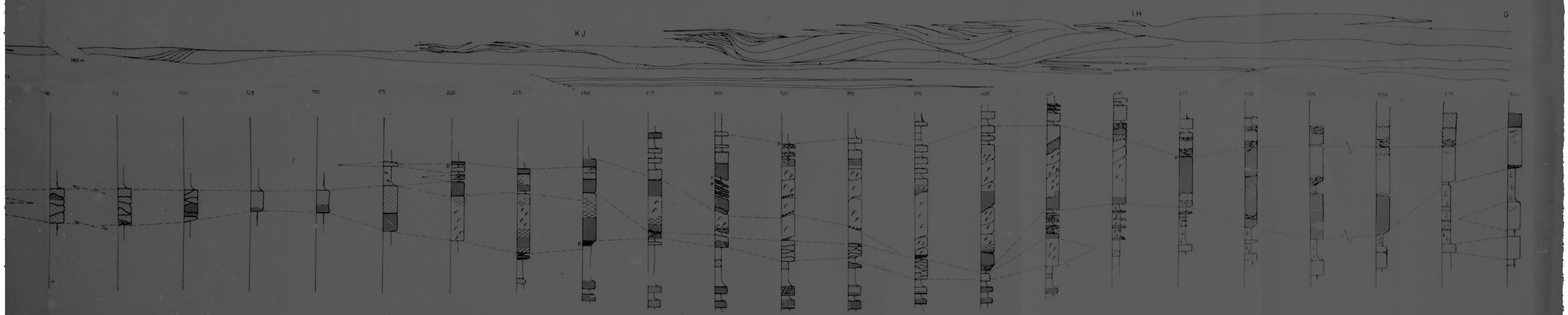
HORIZONTAL SCALE IN METRES (1:500)

0 25m 50m 75m 100m 125m 150m 175m 200m 225m 250m 275m 300m 325m 350m 375m 400m 425m 450m



SCALE IN METRES 1:100

LONGITUDINAL AND COLUMNAR SECTIONS SHOWING

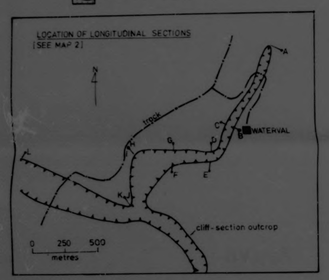


LONGITUDINAL AND COLUMNAR SECTIONS SHOWING SEDIMENTARY STRUCTURES OF A CLIFF EXPOSURE ON WATERVAL : SECTIONS G-H, I-J, K-L

LEGEND TO LONGITUDINAL SECTIONS

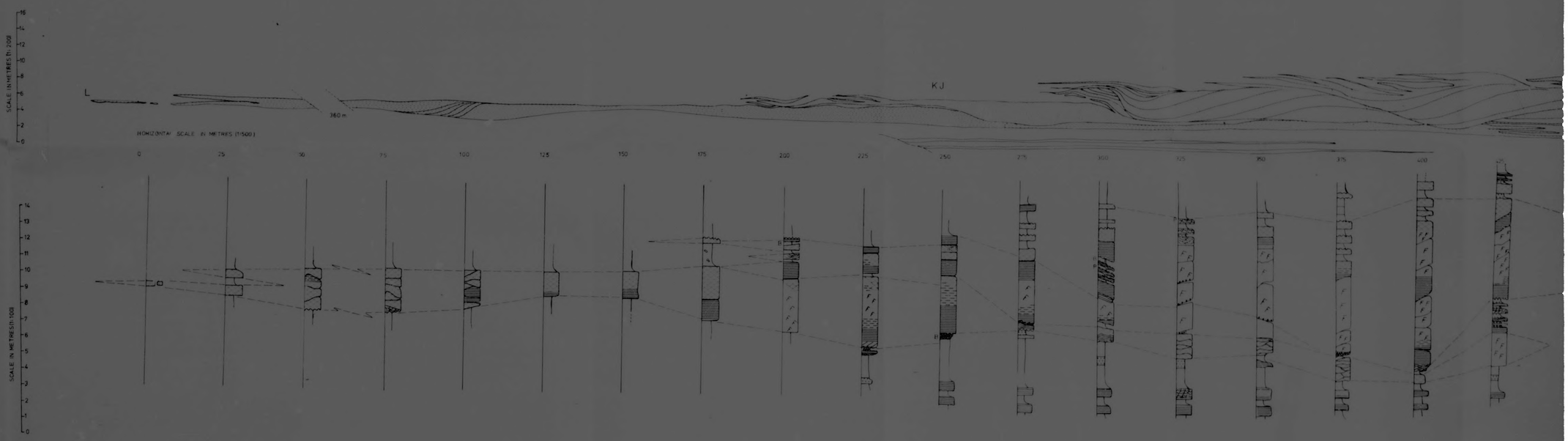
- SANDSTONE UNIT I
- SANDSTONE UNIT II
- SANDSTONE UNIT III
- EROSION AND REACTIVATION SURFACES

LOCATION OF LONGITUDINAL SECTIONS
[SEE MAP 2]



LEGEND TO COLUMNAR LOGS

- MUDSTONE
- SILTSTONE
- RIPPLE CROSS-LAMINATION
- CLIMBING RIPPLE LAMINATION
- INCIPIENT RIPPLE LAMINATION
- WAVY OR PARALLEL LAMINATION
- HORIZONTAL LAMINATION
- MASSIVE BEDDING
- FLAT BEDDING
- TROUGH CROSS-BEDDING
- PLANAR CROSS-BEDDING
- MUDSTONE PEBBLE CONGLOMERATE
- CALCAREOUS NODULES AND NODULAR LAYERS
- PALAEOSURFACE
- SOLE STRUCTURES
- B BONE FRAGMENTS
- SLUMPED MUDSTONE
- P PLANOLITES



LONGITUDINAL AND COLUMNAR SECTIONS SHOWING SEDIMENTARY STRUCTURES OF A CLIFF EXPOSURE ON WATERVAL : SECTIONS G-H, H-J, K-L

Author Rogers D G
Name of thesis Experimental heat transfer coefficients for the cooling of oil in horizontal internal forced convective Transitional flow 1981

PUBLISHER:
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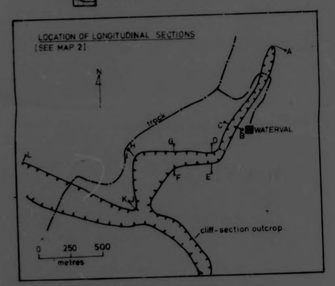
LOCATION OF LONGITUDINAL SECTIONS (SEE MAP 2)



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- PALAEOSURFACE
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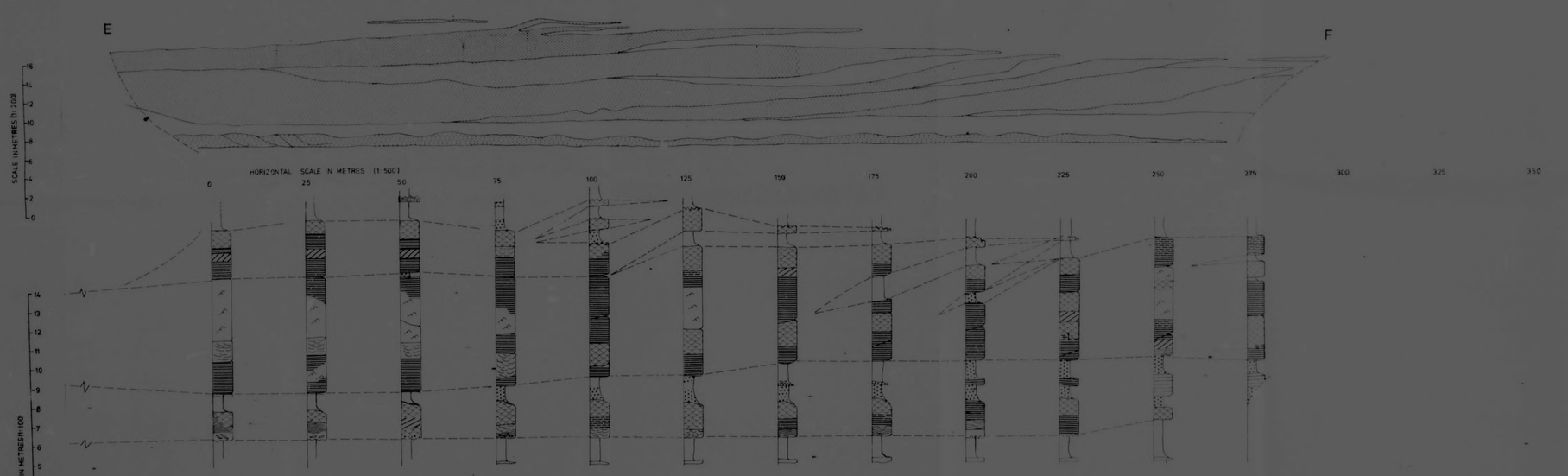
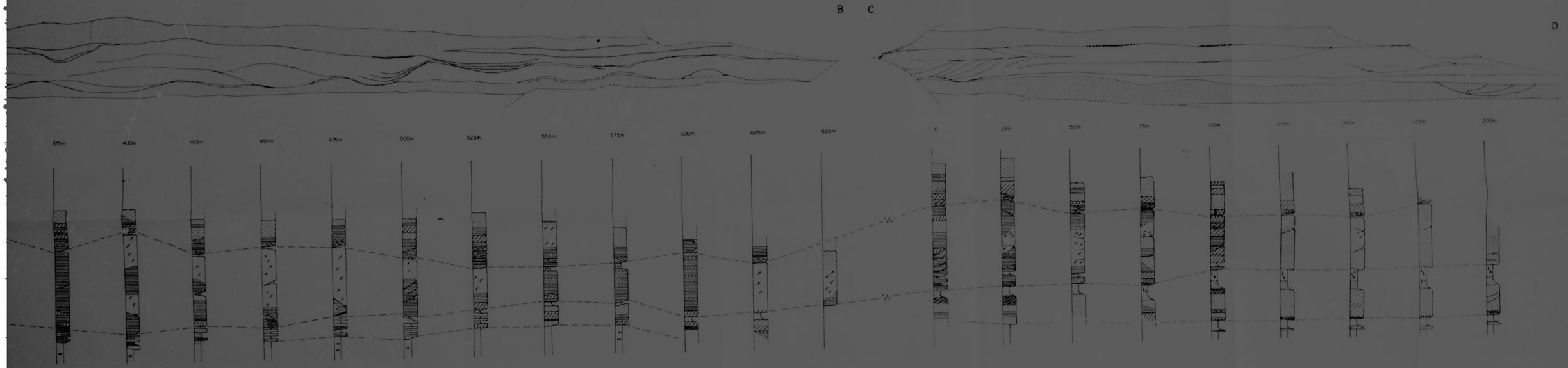


LONGITUDINAL AND COLUMNAR SECTIONS SHOWING SEDIMENTARY STRUCTURES OF A CLIFF SECTION ON THE FARM WATERVAL : SECTION E-F



DINAL AND COLUMNAR SECTIONS SHOWING SEDIMENTARY STRUCTURES OF A CLIFF EXPOSURE ON THE FARM WATERVAL: SECTION ABCD

Author Rogers D G
Name of thesis Experimental heat transfer coefficients for the cooling of oil in horizontal internal forced convective Transitional flow 1981

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