



UNIVERSITY OF THE
WITWATERSRAND,
JOHANNESBURG

**Application of sedimentological and gold assay data for
resource estimation: A case study of the Middelvlei Reef,
Witwatersrand Basin (South Africa)**

Disebo Letanta

Student Number 319194

School of Geosciences

University of the Witwatersrand

Johannesburg, South Africa.

Supervisor: Dr. Glen Nwaila¹

Co-supervisor: Prof. Christina Dohm²

External Co-supervisor: Mr. Mark Burnett³

¹ School of Geosciences, University of the Witwatersrand, Private Bag 3, Wits, 2050, South Africa

² School of Mining, University of the Witwatersrand, Private Bag 3, Wits, 2050, South Africa

³ AMC Consultants (Pty) Ltd., Nicholsons Walk Maidenhead, Berkshire, London SL6 1LD, United Kingdom.

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DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the degree of Master of Science in Geology at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination to any other University.

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(Signature of Candidate)

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day *month* *year*

ABSTRACT

Block modelling applied to point data is common practice in estimation of resources in the mining industry. This study compares the results of geostatistical block-modelling techniques applied to two different approaches to obtaining the point data. In the first instance, a novel approach to generating point grade data using a machine learning-algorithm to predict metal grades that are based on a support of assay, reef width and additional descriptive geological data is considered. This is in contrast to the conventional approach, which uses the assay data, reef widths only and interpreting geozones for estimation. The relevance and importance of this study centres on the need to efficiently use the vast amount of additional available historical data collected from the Witwatersrand Supergroup, at a time when most of the primary reef horizons are being depleted and information on secondary reef intersections is generally limited.

Several studies of the Middelvlei Reef have indicated a notable relationship between its sedimentological and depositional characteristics and the distribution of gold. The Middelvlei Reef is a medium- to large-pebbled oligomictic, auriferous conglomerate with interbedded quartzite and a thickness of two to four metres, along with pebble lags as thin as 10 cm. It is mined selectively, primarily for its gold content as a secondary reef at the study site.

This study compares ordinary and simple kriged block model estimates using the same omni-directional spherical variogram, based on observed assay data and enhanced quality grade data predicted by GS-Pred using the assay data and accompanying geological descriptions. The comparisons are conducted on the swath plots generated from the estimations on a more local scale. The swath plots indicate that the estimates for the data predicted by GS-pred produces estimates that are aligned to the sample points, particularly where the number of samples in the swaths is higher and where declustering has been carried out. The results indicate that data derived from GS-Pred can be incorporated into existing resource estimation procedures relevant to the operational mining environment. As GS-Pred is sensitive to the quality of the geological data collected, this study recommends that the limitations of the reliance on variable quality and often qualitative historical data be addressed to enhance GS-Pred performance and consistency. The recommendation also holds for other similar

machine learning-based approaches, as it pertains to data quality rather than algorithm formulation. For example, much of the geological descriptions, such as matrix colour and percentage mineralisation, are subjective and inconsistently interpreted within and between operations. Also, additional validation steps on assumptions and theory in the GS-Pred process should enhance this novel approach.

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Abbreviations and Acronyms

cmg/t	centimetre-grams per ton
CV	Coefficient of variation
g/t	grams per ton
IDW	Inverse Distance Weighting
ln	Natural logarithm
KNA	Kriging neighbourhood analysis
LOOCV	Leave One Out Cross-Validation
LRPCC	Linear regression Pearson correlation coefficient
MSE	Mean Squared Error
RMSE	Root Mean Square Error

1. Introduction

The Witwatersrand Basin is the world's largest gold province and has been exploited for its gold and uranium since its discovery in the late nineteenth century (Pretorius, 1963). From it, more than 54,000 tonnes of gold have been recovered (Frimmel, 2018). A remaining inferred resource of 30,000 tonnes of gold has been estimated for the basin (Tucker et al., 2016). Several estimation techniques have been used for resource estimation, the most basic of which involves polygonal, area-of influence or a nearest-neighbour approach, where an orebody intersection is assigned its area of influence, which is generally halfway to the next measured point (Armstrong, 1998). Weights can be applied to sampled grades, at known points, to estimate unknown points or blocks some distance away. In the case of the inverse distance interpolation methodology, the premise is that nearby points are more strongly correlated and, therefore, more similar than those further apart (Armstrong, 1998).

One of the more complex interpolators used in grade estimation is kriging, which is also based on the above-mentioned premise with an additional advantage of a model of the spatial correlation known as the variogram to determine optimum sample weights. Kriging uses both the spatial positions of samples and the spatial correlation between them to produce the best linear unbiased estimate, in the sense that a kriged estimate has the minimum estimation variance providing an indication of the confidence associated with the estimate. This methodology was named after Danie Krige, who originally developed it to avoid bias when estimating the average grade of mining panels in the 1950s. Krige did this by assigning a weight λ (which was determined by linear regression) to the average grade of the data in the panel and a complementary weight $1 - \lambda$ to the orebody's average grade (Krige, 1951). Georges Matheron, a French mining engineer and mathematician, invented and developed the mathematical theory for the practical methods of Krige several years later and named it Kriging in honour of fundamental work done by Krige (Matheron, 1963). The resource estimation techniques applied most in the Mineral Resource estimation of Witwatersrand gold deposits are based on some form of kriging. This estimation methodology produces estimates as well as a measure of the uncertainty associated with the estimates known as the kriging variance.

Conventionally, the gold grade of the Witwatersrand auriferous conglomerates (termed “reefs”) have been estimated using gold assay data and channel thickness measurements, with geological data, such as particle size, packing density, wt% pyrite being used qualitatively to assist in the definition of geozones for resource estimation. Geozones are individual domains that subdivide the geological model into distinct geologically homogeneous populations with distinguished spatial and statistical gold grade distributions (Dohm, 1995; Flitton and Peattie, 2016).

The distribution and concentration of gold in palaeoplacers, particularly those of Archean and Palaeoproterozoic age, often reflect the changes in the depositional environment and mineral source through space and time (Frimmel, 2018). Determining the sedimentological characteristics of palaeoplacer gold deposits is fundamental to understanding the nature of gold distribution and grade variability. A Microsoft Access based application (Seds Para) was recently developed at the School of Geosciences of the University of the Witwatersrand that captures detailed sedimentological data from the various conglomerate horizons present within the Witwatersrand goldfields (Nwaila et al., 2019). The availability of this application has created an opportunity for the integration of these sedimentological parameters into the mineral resource estimation process.

Machine learning, which is a field in artificial intelligence, is an application of computer science and statistical learning, in which patterns are identified in data by an algorithm and, from these patterns, predictions are made (Caté et al., 2017). The concept of it is that computer systems can learn from data, identify patterns and make decisions with minimal human involvement. There are three types of machine learning algorithms: supervised learning, unsupervised learning and reinforcement learning. A machine learning-based algorithm that uses sedimentological descriptions to build predictive metal grade models for the Witwatersrand goldfields on a per-target basis using spatially-localized data from a reference database was developed by Nwaila et al. (2019). These authors demonstrated that sedimentological descriptions provide sufficiently-quantitative data that allows for the determination of relationships between geology and gold grades, and the prediction of Witwatersrand gold grades for purposes such as resource estimation.

This research study will make use of combined supervised and unsupervised algorithms. Supervised learning refers to a target or outcome variable (dependent), which is predicted from a given set of predictors, which are independent variables called features (Zhang and Zhan, 2017). Unsupervised learning refers to algorithms that make use of unlabelled data, which has an unknown structure (Raschka, 2015). Machine learning has become an increasingly widely used tool in various Earth science fields, such as resource estimation, geophysics and facies modelling and classification (Caté et al., 2017; Zhang and Zhan, 2017, Nwaila et al., 2019). Through this application, a large amount of data can now be more efficiently used to make predictions and gain knowledge through self-learning algorithms (Raschka, 2015). A desirable outcome is to transform a knowledge-driven exploration industry towards a more data-driven approach (Caté et al., 2017).

South Africa was a world leader in gold production and its gold mining industry ranked first in the world in terms of production until 2009 (Neingo and Tholana, 2016). In 2018, it was ranked eighth (Dhiraj, 2018). South Africa still possesses a significant and globally competitive amount of gold. However, the industry is facing a number of challenges that impact production, including mounting production costs, labour unrest, gold price volatility, declining gold grades and mining of orebodies that have now reached mature stages, and that are becoming deeper and more challenging to access at depths approaching 4 km (Domínguez and Valero, 2013; Neingo and Tholana, 2016). These challenges are compounded by political, social, and environmental concerns, which act to dampen investor confidence (Neingo and Tholana, 2016). As a result, mines are developing and exploiting lower grade, more erratic deposits, requiring estimation techniques that will reduce the geological and estimation risk.

1.1. Problem statement

With the current technical, operational, and financial challenges facing the South African mining industry and the resultant decrease of funding available for capital and operational expenditures (Capex and Opex), there is a need for geological information obtained from mining and exploration to be used as efficiently as possible, specifically in areas where reef intersections are limited. In addition, large portions of the Witwatersrand gold deposits have been depleted, leaving relatively lower grade or

economically marginal horizons termed “secondary reefs”, requiring approaches to estimation that would potentially optimise the use of secondary data, and aid mine planning and decision-making processes at a time when there is reduced flexibility afforded to mine planning. A significant amount of geological data has been collected, but it does not get included in the estimation process, leading to data attrition. There is a potential for value addition and maximisation of existing geological information by utilising sedimentological descriptions and integrating them into the resource estimation process. This process can both lower estimation risks and enhance accuracy, and lower costs and increase productivity (Nwaila et al., 2019). This research may allow areas with sparse borehole information to be estimated with a lower estimation risk than that of current estimation methodologies based mainly on grade and reef width.

1.2. Aims and objectives

This research project aimed to derive and apply an integrated estimation approach by including sedimentological descriptions to the traditional estimation methodology of kriging gold grades and reef widths within predefined, geologically homogeneous zones through the application of GS-Pred. The objective was to compare the outputs of two estimation methods for a single area. It is envisaged that this methodology will be applicable during the various stages of mineral exploration, where sparse data are available. In this instance, reliance will be on limited and widely spaced borehole intersections located at distances far away from a mined-out area with dense sample data. For this research, the Middelvlei Reef of the Witwatersrand Supergroup at Sibanye Stillwater’s Driefontein Gold Mine was the focus of the study.

1.3. Research questions

With this endeavour, specific key questions will be answered, namely:

1. How can sedimentological descriptions be used to predict gold grades?
2. What is the most appropriate estimation technique to be applied using both sedimentological and assay data?
3. Which estimation method, namely a composite simple and ordinary kriged estimate, inverse distance weighting (IDW), will work well with GS-Pred-generated points in terms of regression slope and kriging efficiency?

1.4. The relevance of the current study

Any improvement in the quality of the grade and tonnage estimation of the Mineral Resource could potentially aid the mine planning and decision-making process and may assist in the optimal extraction of an orebody. This research may potentially produce an integrated resource estimation model that could support mine-planning and decision-making related to the mining and exploitation of the Middelvlei Reef at Driefontein Gold Mine. It is therefore expected that an improved Mineral Resource estimate, where there may be a paucity of grade information, will lead to a more reliable Mineral Reserve estimate. An improved estimate will be one where actual grades do not differ markedly from estimated grades, a problem which is common where there is limited information from the orebody. The financial and economic risks of development into mining areas that are later (during mining) discovered to be uneconomic can potentially be reduced by using more appropriate methodologies that will reduce risk and estimation uncertainty.

2. Literature review

2.1. Geological Setting

The Mesoarchean Witwatersrand Supergroup is located on the Kaapvaal Craton. It comprises two groups, the older West Rand and the younger Central Rand Group, the latter of which has nine major goldfields situated in it (Figure 2.1). The two groups are two independent stratigraphic sequences that developed in distinct basins and have a considerable interruption between their developments (Frimmel, 2014). The West Rand Group has a maximum age of 2985 ± 14 Ma and a minimum age of 2914 ± 8 Ma, while the Central Rand Group has maximum and minimum ages of 2902 ± 13 Ma and 2780 ± 3 Ma, respectively (Armstrong et al., 1991; Kositsin and Krapež, 2004). The West Rand Group comprises largely shales and was formed in a shallow marine to the distal fluvio-deltaic environment, while the Central Rand Group consists more of coarser quartzite and conglomerate units, which formed in a fluvial to fluvio-deltaic setting (Pretorius, 1974). It is the latter Central Rand Group which hosts the majority of its auriferous conglomerates, approximately eighty per cent of the mineralisation

(Therriault et al., 1997). The economic placer minerals were sourced from the granitic-greenstone basement and are likely to have been able to survive the environment due to low oxygen pressure, which would have prevailed at the time of transportation (Minter, 1977). The supergroup overlies the volcano-sedimentary Dominion Group or the Archaean granite-greenstone basement and is largely overlain by the metabasalts of the Ventersdorp Supergroup (Armstrong et al., 1991; Hall et al., 1997).

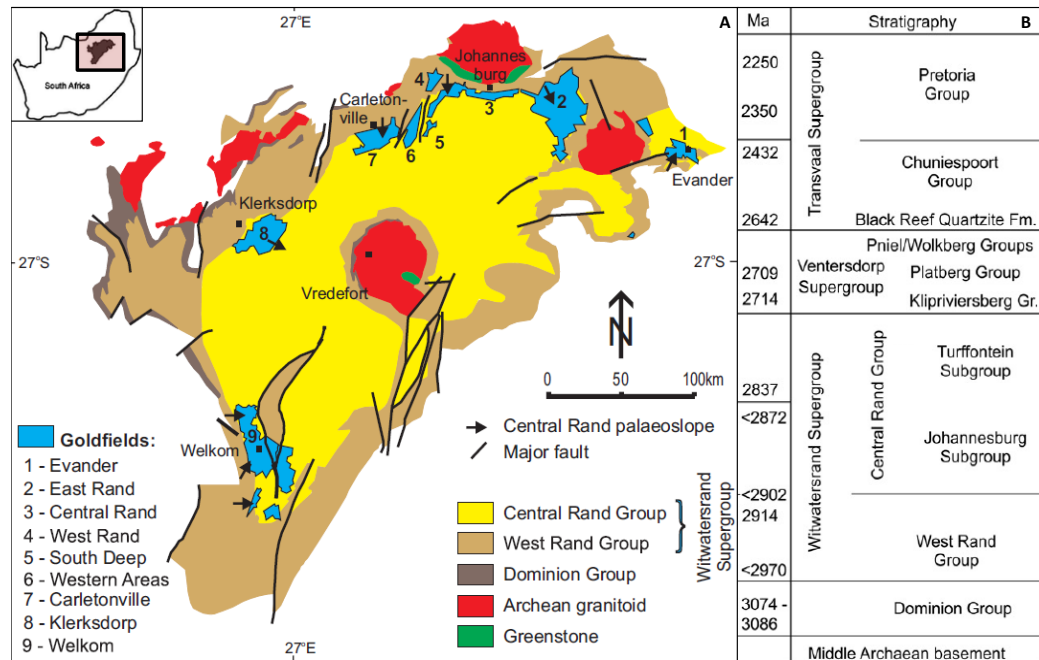


Figure 2.1: Location and geological map of the Witwatersrand Basin with producing goldfields (A) (adapted from Frimmel and Minter, 2002) and stratigraphy (B) (adapted from Koglin et al., 2010).

The Witwatersrand Basin has undergone low-grade greenschist facies metamorphism largely related to progressive burial and loading from the overlying Ventersdorp and Transvaal sequences, the intrusion of the Bushveld Igneous Complex at 2.054 Ga and fracturing from the Vredefort meteorite impact at 2.023 Ga (Frimmel, 2019). The gold occurrences within the Witwatersrand typically consist of mature pebbly quartz arenites, which may comprise several fluvial lithofacies, which vary from clast-supported oligomictic conglomerates to deflationary pebble lag surfaces (Frimmel and Minter, 2002). Significant gold mineralisation is restricted to fluvial deposits (Frimmel, 1997). Evidence for the detrital origin of the gold mineralisation is demonstrated in the toroidal-shaped gold particles extracted from a reef in the Welkom goldfield (Minter, 1999). The most commonly occurring indicator traits are unidirectional palaeo-currents

and bar-and-channel morphology (Els, 1998). Numerous palaeocurrent indicators sourced throughout the Central Rand Group were reviewed and integrated by Minter and Loen (1991) to reconstruct the placer dispersal patterns of the Supergroup.

The conglomerate units of the Witwatersrand Basin can be understood in terms of typical lithologies that would be encountered in a modern braided stream system (Robb and Meyer, 1995). The braided character is typically retained throughout the entire course of the fluvial system (Els, 1998). The economically significant placers were found by Minter (1977) to occur as regressive deposits on unconformities at sedimentary unit bases, transgressive deposits on angular unconformities at sedimentary unit bases or as terminal deposits on disconformities at the top of sedimentary units. Within the succession of the Central Rand Group, an upward decrease in the total gold yield and average grade is seen. The Carbon Leader and Main Leader Reefs represent the richest reefs at the bottom of the Central Rand Group. They have had average grades ranging between 20 and 40 grams per ton (g/t) while the Ventersdorp Contact Reef at the top of the Central Rand Group has had an average grade range of 5 to 12 g/t. The high gold grades in the older reefs are understood to be a consequence of a different mode of occurrence (Frimmel, 2018). The older reefs contain stratiform accumulations of hydrocarbon-rich material, and these carbon seams have been found to host a significant amount of the gold (Hallbauer and Joughin, 1973).

Braided streams consist of a succession of broad, shallow channels, bars and dry islands, and the elevated areas were active during flooding. Braided stream deposits have been outlined as consisting of up to three gravel facies, five sand facies and two fine-grained facies (Miall, 1977). A first-order control has been cited by Frimmel (2018) on Witwatersrand gold grade as being stratigraphy and sedimentary facies. He also noted that historically, this control has been successfully utilised in mining to delineate the orebody.

2.2. Economic reefs of the Driefontein gold mine

The Carbon Leader Reef is the principal orebody exploited on Driefontein gold mine, constituting approximately 62% of the mineral reserves (Sibanye Stillwater Limited, 2018). It is situated in the lower part of the Central Rand Group. Two sub-facies have been identified: a lenticular-shaped conglomerate and quartzite unit that resulted from

the active channelised flow and a single conglomerate sheet often with thin basal carbonaceous seams understood to be the result of sheet floods (Nami, 1983). Stream channels were interpreted to represent areas of high water velocity activity and erratic gold values and adjacent planes had consistently high gold values (Nami, 1983). The reef can be described as a placer that formed in response to fluvial degradation of alluvial fans developing towards the south into the Witwatersrand Basin (Buck and Minter, 1985). A basal hydrocarbon seam reaching a thickness of 1.2 cm occurs with the lower quartz-pebble conglomerate, and small nodules referred to as “fly-speck carbon” are also commonly observed locally (Fuchs et al., 2016).

The Middelvlei Reef, which is mined at Driefontein Gold Mine as a secondary reef, is part of the Central Rand Group of the Witwatersrand Supergroup (Figure 2.2). It is stratigraphically located approximately 50 m above the Carbon Leader Reef and was characterised by Jolly (1984) as being a middle to large pebble oligomictic, auriferous conglomerate, which contains interbedded quartzite and has a thickness of two to four metres, along with pebble lags as thin as 10 cm. This reef has a lower gold grade and a more erratic gold distribution than the Carbon Leader Reef (Jolly, 1984). The average mined grade of this reef is 5 g/t, and it has been mined selectively (Frimmel and Minter, 2002). Carbon, either as “fly-speck” or seams, is generally not prominent and has been noted in some thin reefs. The concentration mechanism for initial gold mineralisation was suggested to be mainly due to the confinement of heavy minerals in gravel, with a subsequent reworking of sediments responsible for additional concentration. The matrix is mostly quartz with pyrite as a widespread heavy mineral (Els, 1983). The most common gravel- and sand-fraction lithofacies were identified by Els (1987) as being massive, clast-supported conglomerates with white and smoky quartz clasts.

Group	Sub-group	Formation	Important Au-bearing conglomerates (reefs)
		Venterspost	Venterdorp Contact Reef (VCR)
Central Rand	Turffontein (~900 m)	Mondeor	Uitkyk EAs Bastard
		Elsburg	
		Kimberley	Beatrix, Denny's, VS5
	Johannesburg (~1300 m)	Booyens Shale	A, B, Crystallkop, Kimberley
		Krugersdorp	Bird, Basal, Vaal
		Luipaardsvlei	
		Randfontein	Livingstone
		Main	Middelvlei Carbon Leader, Main, North
		Blyvooruitzicht	Ada May, Beisa
West Rand			

Figure 2.2: Stratigraphic location of the Middelvlei Reef within the Central Rand Group of the Witwatersrand Supergroup (adapted from Jolley et al., 2004).

The palaeogeographic model of the Middelvlei Reef was described by Jolly (1984) as having two laterally neighbouring environments of deposition, which are a comprehensive active braided channel system with high energy conditions in the east and a floodplain with minor superimposed channels in the west. On the Doornfontein Consolidated Limited mining operation (Figure 2.3), Jolly (1984) used palaeocurrent indicators to determine a general unimodal palaeocurrent direction towards the south-south-east and due south in the eastern and western section of the mine, respectively. A general unimodal southerly palaeocurrent direction was also observed by Els (1987), with variations between south-east and south south-east in the eastern part of the mine and an easterly direction in the western part of the mine, indicating two populations of transported material (Els, 1983).

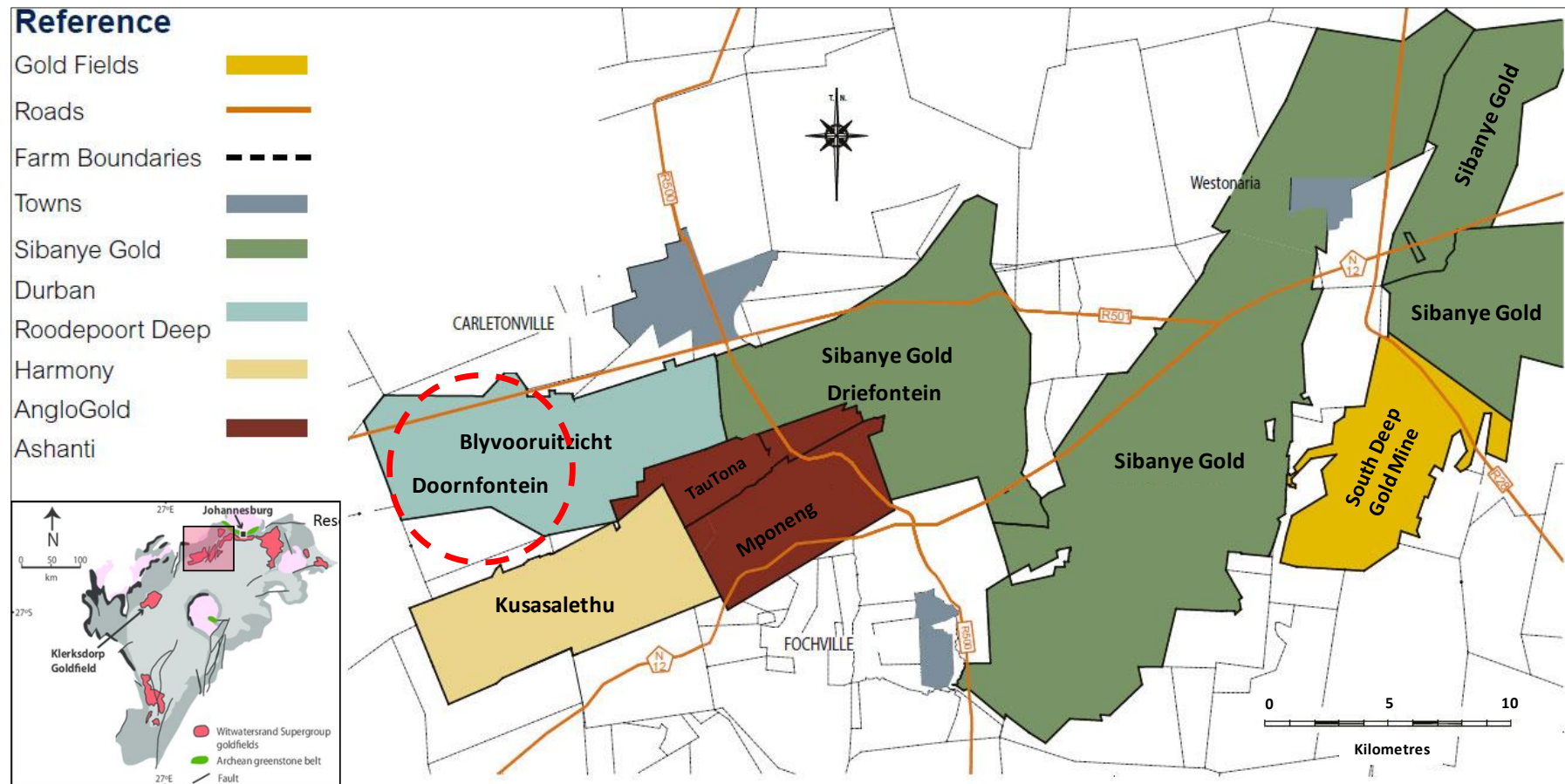


Figure 2.3: Location of Doornfontein Gold Mine (circled) within the Carletonville goldfield and the Witwatersrand Basin (Insert) (adapted from Smart et al. (2016) and Gold Fields Limited (2014)).

The Ventersdorp Contact Reef is a basal conglomerate of the Venterspost Conglomerate Formation, which is stratigraphically independent from the underlying Witwatersrand Supergroup and marks the base of the overlying Ventersdorp Supergroup (Hall et al., 1997). The Ventersdorp Contact placer, which comprises conglomerates and interbedded sandstones, overlies the Witwatersrand Supergroup with an angular unconformity. It is the only member of the Venterspost Formation in the Carletonville Goldfield (Krapez, 1985). Overlying basalt of the Ventersdorp Supergroup allowed for the preservation of its complex palaeogeomorphology (Krapez, 1985) and the reef has various undulating footwall lithologies. The degradation of Witwatersrand material and the development of a system of fluvial channels and terraces indicate that gold was concentrated by fluvial placer processes (Tucker et al., 2016). It was deposited unconformably across the Witwatersrand sequences and its gold mineralisation was noted by Jolley et al. (1999) to be hosted within steep- and shallow-dipping thrust related fractures associated with thrusting, hydrothermal fluid flow and alteration. The interaction of the thrusts with the architecture of the lithology in the footwall lithology and reef palaeo-topography control the mechanical linkage of the thrusts to the Ventersdorp Contact Reef (Jolley et al., 1999). The undulation of the reef plane follows topographic variations of the palaeosurface (Roberts and Schweitzer, 1999).

2.3. Sedimentological characteristics of palaeoplacer gold deposits

The goldfields of the Witwatersrand Basin were described by Pretorius (1974) as representing fluvial fans or delta fans, with the properties of the sediments making up the goldfield being transitional between those of an alluvial fan and a classic delta. In his study, Pretorius (1974) interpreted a goldfield as having formed at the interface between a fluvial system and shallow-water, lacustrine or inland-sea setting. Continued sediment reworking is conducive to placer formation and mineralisation in the form of heavy minerals is concentrated in this process (Buck and Minter, 1985). Multi-channel sequences of conglomerate and quartz arenite that were deposited by cyclical flood- and waning-stage flows resulted in thicker orebodies (Frimmel et al., 2005).

Similarities in sedimentological characteristics of auriferous palaeoplacer deposits exist between the Witwatersrand Basin and other deposits in the world. The auriferous

Matinenda Formation of the Huronian Supergroup in the Elliot Lake Area of Canada comprises cross-bedded sandstones and conglomerates, which represent a fluvial system of shallow braided channels (Fralick and Miall, 1989). The conglomerates are typical of gravel bars, and bedforms and the depositional model is the development of low-relief longitudinal bars (Fralick and Miall, 1989). The sedimentary features of the Mississagi Formation, also located in the Huronian Supergroup, were described by Whymark and Frimmel (2017) as characteristic of fluvial braided stream deposits. Two facies were described. The first is a distal facies comprising sandstone and thin, interbedded, well-sorted quartz and chert clast conglomerates and the second, a proximal facies, includes poorly sorted, clast-supported, polymictic boulder to cobble conglomerates that are interbedded with sandstone and mudstone.

A study was carried out by Burton and Fralick (2003) on gold and uranium palaeoplacer deposits to determine the processes responsible for the accumulation of heavy minerals. Among the requisite conditions cited in their study is a change in the regional slope, which results in gradient-parallel energy loss zones and infrequent major flood events. Stable bars and channels are more likely to host placer mineralisation due to their reworking by currents, which enhances prospects for heavy mineral entrapment by coarse bed materials (Smith and Minter, 1980). Another effect of reworking is that, within a braided system, regions of parallel and convergent flows would likely accumulate placer concentrations (Smith and Minter, 1980). Several common sedimentological parameters of palaeoplacer gold deposits have been highlighted by Minter (1991). Commonly, a lateral extent of mature quartz arenite on expansive unconformities, resulting from expansive erosion, is present. This is coupled with topographic relief allowing for deeply incised channels of gold-enriched sediments and their reworking. Lastly, intermittent tectonic rejuvenation of the basin margin, demonstrated by the stratigraphic stacking of sedimentary events indicating degradation/aggradation cycles, is encountered.

2.4. Methods for the evaluation of Witwatersrand-type gold ores and current challenges

Kriging was introduced to South African Witwatersrand gold ores when conditionally-biased estimates became insufficiently accurate for grade resource estimation, and improvement of mine valuation techniques was needed. A lognormal model was introduced by Herbert Sichel (1947) for gold values. In 1951, Danie Krige developed a statistical method that incorporated the spatial correlation of reef sampling data (Krige, 1951).

Kriging is a linear interpolation method that can be spatially applied, where the estimation at an unsampled position is given as the weighted sum of the surrounding sampled points based on the corresponding spatial variance (Sidler, 2003). It is referred to as the best linear unbiased estimator. Not only can kriging yield an optimal estimate for an unsampled area, but it also provides a measure of the precision of the estimate, known as the kriging variance, although other names are possible, such as the mean squared prediction error (Daya, 2015). The determination of a kriging variance has been cited as one of the advantages of geostatistics over traditional reserve assessment methods, which do not produce estimation error variances.

Whilst several types of kriging have been developed over time, for example, ordinary kriging, simple kriging, cokriging, indicator kriging, universal kriging and more (Sidler, 2003); this study focusses on ordinary kriging and simple kriging as these are most commonly applied methods in the resource estimation of the gold grades within the study area. Both techniques are discussed in some detail. Since ordinary kriging does not rely on a known mean of the region, it is the most applicable and common type of kriging (Sidler, 2003).

2.4.1 Variography

Variogram analysis is used for the quantitative definition of spatial continuity across various parameters, such as sample grades and thicknesses (Abzalov, 2014). The process entails calculating experimental variograms and subsequently fitting an appropriate model. The variogram model plays a fundamental role in any kriging estimation as it is used to optimise kriging weights of sample point used in the estimation to ensure a minimum estimation variance.

A variogram model of a key variable from a geospatial dataset exhibiting spatial correlations typically contains a sill, range, nugget effect representing the spatial correlation (Figure 2.4) of the variable. The sill, which is also the total variance, is the maximum variance that is reached by the variogram model and the range is the distance at which the sill is reached (Chilès and Delfiner, 2012).

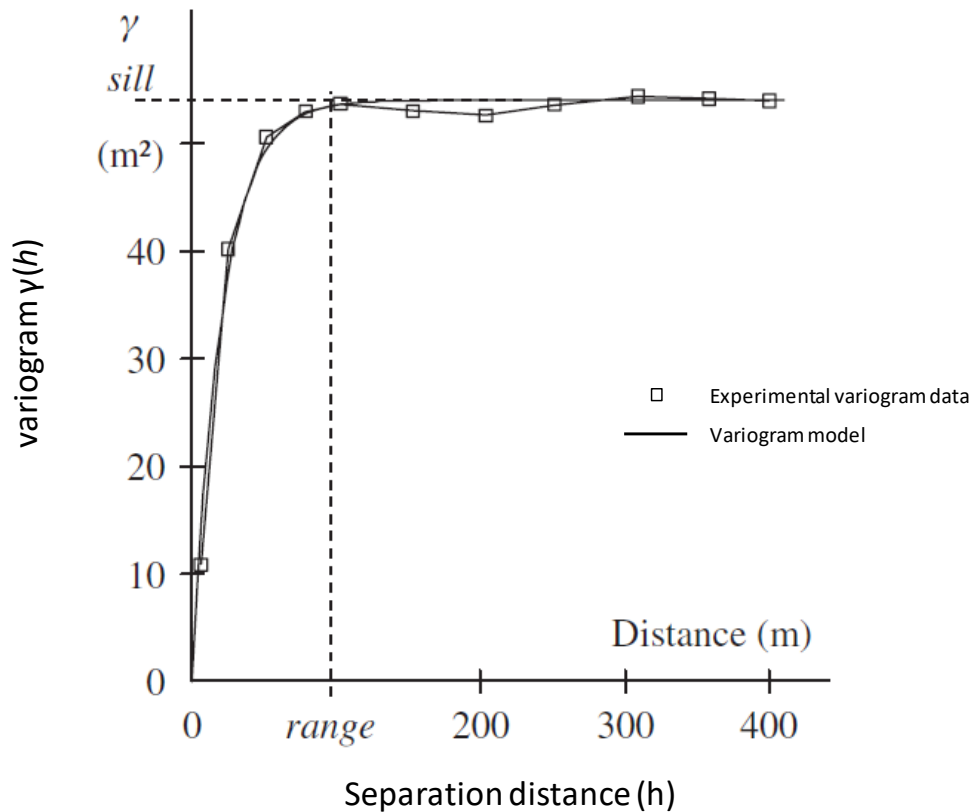


Figure 2.4: Variogram sill, range, nugget and spatial variance (adapted from (Chilès and Delfiner, 2012)).

2.4.2 Kriging Neighbourhood Analysis

The appropriateness of a kriging neighbourhood can be quantitatively determined with a variety of tests referred to as Quantitative Kriging Neighbourhood Analysis (QKNA). QKNA is a quantitative way of testing different estimation parameters and their effect on the quality of the estimate with the goal of selecting an optimal set of estimation parameters that minimise conditional bias (Vann et al., 2003). These parameters that are tested and optimised for the estimation are minimum and maximum number of samples, discretisation points and neighbourhood. This is done to ensure that

conditional bias is minimised. Kriging efficiency and slope of regression are the statistical measures of an optimal set of parameters (Vann et al., 2003).

2.4.3 Ordinary and Simple Kriging

The Ordinary Kriging estimate at a location $\vec{u}_0$ of a block $\hat{X}$, supported by n samples is given by:

$$\hat{X}(\vec{u}_0) = \sum_{i=1}^n \lambda_i X(\vec{u}_i).$$

Equation 2.1: Ordinary kriging estimate (Sidler, 2003).

Here, $X(\vec{u}_i)$ are the values of the surrounding samples at their locations $\vec{u}_i$ within some neighbourhood of the unknown block and λ_i are the kriging weights assigned to the values; with the weights determined by the spatial relationship of the data (Sidler, 2003), such as through a covariance matrix or a semi-variogram, and the location of the points supporting the estimation (Cressie, 1990).

The sum of the weights must be equal to one i.e. $\sum \lambda_i = 1$, a requirement that falls away in simple kriging as a weight is assigned to the mean (Rivoirard, 1987). The interpolation is based on the spatial correlation of random spatial variables, which can be mathematically modelled by several mathematical functions, such as the spherical, gaussian and exponential semi-variogram models (Matheron, 1963).

The method of Lagrange multipliers is used to minimise the mean squared prediction error, under the constraint that the sum of the weights is one (Isaaks and Srivastava, 1989, Rivoirard, 2005; Vann et al., 2003). The kriging variance σ_{OK}^2 for an ordinary kriged estimate at x_i is

$$\sigma_{OK}^2 = \sum \lambda_i \bar{\gamma}(x_i, V) - \bar{\gamma}(V, V) + \mu.$$

Equation 2.2: Ordinary kriging variance.

Here, $\bar{\gamma}(x_i, V)$ is the average of the variogram between the point x_i and the volume V , $\bar{\gamma}(V, V)$ is the average of the variogram between two points x and x' within the volume V , and μ is the Lagrange multiplier (Armstrong, 1998).

In ordinary kriging, although the mean value of the region is unknown a-priori, the data are assumed to conform to first-order stationarity, such that the mean could be estimated by the global mean of the sampling area (Sidler, 2003). In realistic applications, the mean often varies laterally at a desirable scale of observation and this variation can be accounted for in ordinary kriging, since no weight is assigned to the mean. Therefore, ordinary kriging has the advantage of not strictly requiring first-order stationarity and can be used to estimate the value at a point or the mean value over a block or domain (Sidler, 2003; Rivoirard, 2005).

This is not the case for simple kriging. There is no constraint on the sum of the weights as the remaining weight is allocated to the mean grade of the domain, commonly referred to as the weight of the mean. A “known mean” is used in simple kriging, assuming that such a global mean grade is known (Vann et al., 2003). The importance of the mean and its weight increases where there is less information in the kriging neighbourhood, indicating sparse data except when there are negative weights (Rivoirard, 1987).

Thus, simple kriging is commonly used at large mines where mined-out regions are used to estimate the kriged area means (Sidler, 2003). The use of the mean with its complementary weight compensates for localised sparseness of data, and allows the local estimates to tend towards the global mean in order to adhere to the assumption of stationarity (Rivoirard, 2005). As per Sidler (2003), the equations for a simple kriging estimate and its associated variance are

$$\hat{X}(\vec{u}_0) = m \left(1 - \sum_{i=1}^n \lambda_i \right) + \sum_{i=1}^n \lambda_i X(\vec{u}_i),$$

Equation 2.3: Simple kriging estimate.

where

$$m \left(1 - \sum_{i=1}^n \lambda_i \right) = \lambda_0$$

the weight of the regional mean (m) and

$$\sigma_{SK}^2 = \bar{C}(V, V) - \sum \lambda'_i \bar{C}(x_i, V),$$

Equation 2.4: Simple kriging variance.

where $\bar{C}(V, V)$ is the block variance or the dispersion variance of blocks and $\sum \lambda'_i \bar{C}(x_i, V)$ is the sum of the product of the simple kriging weights λ'_i and the average covariance between point x_i and the volume V (Armstrong, 1998).

The block variance BV is the dispersion variance of blocks in the deposit and is defined as

$$BV = \bar{C}(V, V) = \sigma^2 - \bar{\gamma}(V, V),$$

Equation 2.5: Block Variance.

which is the population variance (σ^2) minus the average semi-variogram value for block of volume V (Armstrong, 1998).

A measure of the efficiency of the kriging block estimates is referred to as kriging efficiency and is determined through the equation

$$KE(\%) = \frac{BV - KV}{BV}$$

Equation 2.6: Kriging efficiency.

Here BV is the variance of the blocks and KV is the kriging variance, and in this study, could be either σ_{SK}^2 or σ_{OK}^2 , depending on the estimation technique used (Krige, 1997). Ideally, KE should be high (close to one) as this will indicate a low kriging variance and it also indicates that the variance of the block estimates is approximately equal to the variance of the true block values (Krige, 1997).

The kriging slope of regression is obtained by cross-plotting the unknown true value of a random variable Z against the known estimate of the random variable Z^* , both with estimation volume V (Deutsch et al., 2014). The slope of regression (b) approximates the conditional bias of the kriging estimate in the equation

$$E\{Z_V \mid Z_V^* = z\} \cong a + bz$$

Equation 2.7: Conditional bias (Deutsch et al., 2014).

The slope of regression b is calculated by means of the estimation weights in the equation

$$b = \frac{Cov\{Z_V, Z_V^*\}}{\sigma_{Z_V^*}^2} = \frac{\sum_{i=1}^n \lambda_i \bar{C}(v_i, V)}{\sum_{i=1}^n \sum_{j=1}^n \lambda_i \lambda_j C(v_i, v_j)}$$

Equation 2.8: Kriging slope of regression (Deutsch et al., 2014).

Where λ_i are the kriging weights assigned to sample v_i , $\bar{C}(v_i, V)$ is the average covariance between the samples and the estimated volume V and $C(v_i, v_j)$ is the covariance between the sample v_i , and v_j , considered in the estimate. For simple kriging, the slope of regression will always be 1 as there is no lagrange multiplier necessary because the sum of the weights is not required to be equal to 1 as in the case of ordinary kriging (Deutsch and Deutsch, 2012).

The statistical distribution of Witwatersrand gold grades is lognormal, having a positive skewness with a tail of higher values (Rivoirard, 2005). This necessitates a log transformation and inverse-transformation of the block-grade estimate (Cressie, 1990). This transformation is carried out so that the data take on a more normal distribution, because the statistical methods used for estimation assume a Gaussian distribution. For this reason, lognormal estimators of the distribution's mean, which are obtained from the arithmetic mean and the variance of the log values, is a more reliable estimator of the mean than the arithmetic mean, of the untransformed values. The perpetuity of a lognormal distribution is assumed, from sample values in a panel or in a mine to panel values in a mine (Rivoirard, 2005). Deviation from lognormality can be treated with the incorporation of a third parameter, which entails the addition of a constant before taking logarithms (Krige, 1999). In some cases, the gold grades do not follow a three-parameter lognormal distribution and four-parameter compound lognormal (CLN) and

log-generalised inverse Gaussian (In-GIG) distributions are more applicable, albeit, mathematically more complex (Sichel et al., 1992).

The challenges that are encountered during resource estimation of Witwatersrand gold deposits are the significant depths of mining and sampling, and costly geological drilling, particularly when this drilling is carried out from surface and not from an underground development-end that has already been blasted. These challenges result in limited data availability for estimation in unmined locations (Minnitt et al., 2011; Flitton and Peattie, 2016).

2.5. Utilisation of sedimentological parameters in geostatistical modelling

A degree of similarity in the sedimentological trends between the Carbon Leader Reef and the Middelvlei Reef were observed when sedimentological and geostatistical studies were carried out by Reynolds and Stear (1987). This allowed sedimentological modelling and facies trends of the Carbon Leader Reef to set exploration targets for the Middelvlei Reef. The premise for this relation was that the Main Conglomerate Formation, in which both reefs are stratigraphically located, exhibits facies repetition and the reactivation of footwall structures that would result in a replication of major channel flow trends and channel distribution trends in both reefs. From this analysis, a facies model that links value distribution patterns to depositional trends was established for both reefs by Reynolds and Stear (1987).

In the petroleum industry, hydrocarbon resource modelling and evaluation is used to model parameters, such as porosity and permeability relationships (Cao et al., 2014). A reservoir characterisation technique has been outlined by Doligez et al. (2007) for modelling sedimentary structures from combined genetic and stochastic models and using the determined parameters in a geostatistical model. In the field of marine biology, multi-variate geostatistical modelling of sedimentological variables, such as grain size fractions and silt-clay percentage, can be carried out to predict species and habitat occurrences (Verfaillie et al., 2009). These applications indicate how geological parameters have been effectively quantitatively incorporated into other industries' kriging and resource modelling, despite kriging having strong origins in the gold mines.

2.6. The spatial relationship between gold and sedimentological parameters

Correlations between sedimentary characteristics and gold grade distribution in the Witwatersrand Basin have been established through various studies. A relationship was noted by Robb and Meyer (1995) between syn-sedimentary gold mineralisation and sedimentary features, including unconformities, fluvial channel ways, cross-bed foresets and deflations surfaces. Their study found that the highest concentration potential for heavy minerals, such as detrital gold, is in conglomerates, which have the best packing, the most resistant clasts and the highest abundance of foresets with pyrite.

Two Witwatersrand palaeoplacers were studied by Smith and Minter (1980), the Leader and the Elsburg Number 5 reefs located in the Welkom and Klerksdorp goldfields, respectively. This was done to understand the relationship between gold and uranium mineralisation and sedimentary textures and structures. Placer minerals were found to be concentrated on scouring surfaces and as a matrix in well-sorted conglomerates. The study also revealed that the orientations of some of the local high-grade trends are predictable, either from the related bar and channel orientations or with palaeocurrent directions, marking projections of continuity along with palaeoflow directions. A study that aimed to merge the geostatistical evaluations of determining ore-shoot directions with the approach of observing sedimentological features was carried out by Magri (1978). This study focused on palaeocurrent directions of the Vaal Reef on the Hartebeesfontein gold mine. A notable agreement between the geostatistical and sedimentological descriptions was observed, suggesting that the two fields could be of substantial mutual benefit.

Specific studies on the Middelvlei Reef include:

Low correlation coefficients between the gold grade of the Middelvlei Reef and the descriptions, such as percentage conglomerate (0.18) and average clast sizes (0.34) were calculated by Els (1987). An inverse relationship between gold grade and reef thickness was identified, with a correlation of 0.15. This suggested that higher gold concentrations are linked with thin, single units with larger pebble sizes. A similar observation linking thin pebble lags with notable gold concentration was made by Minter et al. (1992) on the Basal Reef.

The presence of a basal reef conglomerate, which is ubiquitous across the Doornfontein mine, was described by Jolly (1984) as a basal lag, and it was noted that the highest gold grades are encountered in the basal lag of the reef in the east of the mine and decrease as one moves westward. This basal lag decreases in thickness and maximum clast size from the east, which is linked to the active braided channel complex, to the west, which is associated with a floodplain environment, and the association of high grade with larger clast sizes was thought to be attributable to higher energy processes that contributed to a greater influx of gold (Jolly, 1984).

The lognormal distribution of Witwatersrand gold grades is also observed for the Middelvlei Reef. The reef has a series of zones of gold enrichment as well as paucity, and these are attributable to lateral sedimentological variations (Jolly, 1984). The variogram of the gold grades indicated the longest range of influence of 68 m in the 165°, suggesting the most considerable continuity of gold mineralisation of up to 68 m in this direction and for the area under study, which is similar to the palaeocurrent direction (Jolly, 1984).

3. Data and Methods

This study sought to analyse two sources of data in the form of assay values and sedimentological descriptions that had been obtained from underground chip sampling during mining and borehole intersections in unmined areas of the Middelvlei Reef and data that had been predicted using geological descriptions (Figure 3.1). A machine learning-based algorithm (GS-Pred, Nwaila et al., 2019) was used to predict sedimentological patterns of the study area and gold grades. The GS-Pred process consists of data pre-processing, linear regressions and cluster analysis and cross-validation of the prediction model. In-depth details of GS-Pred can be found in Nwaila et al. (2019), and are outlined in Chapter 4.

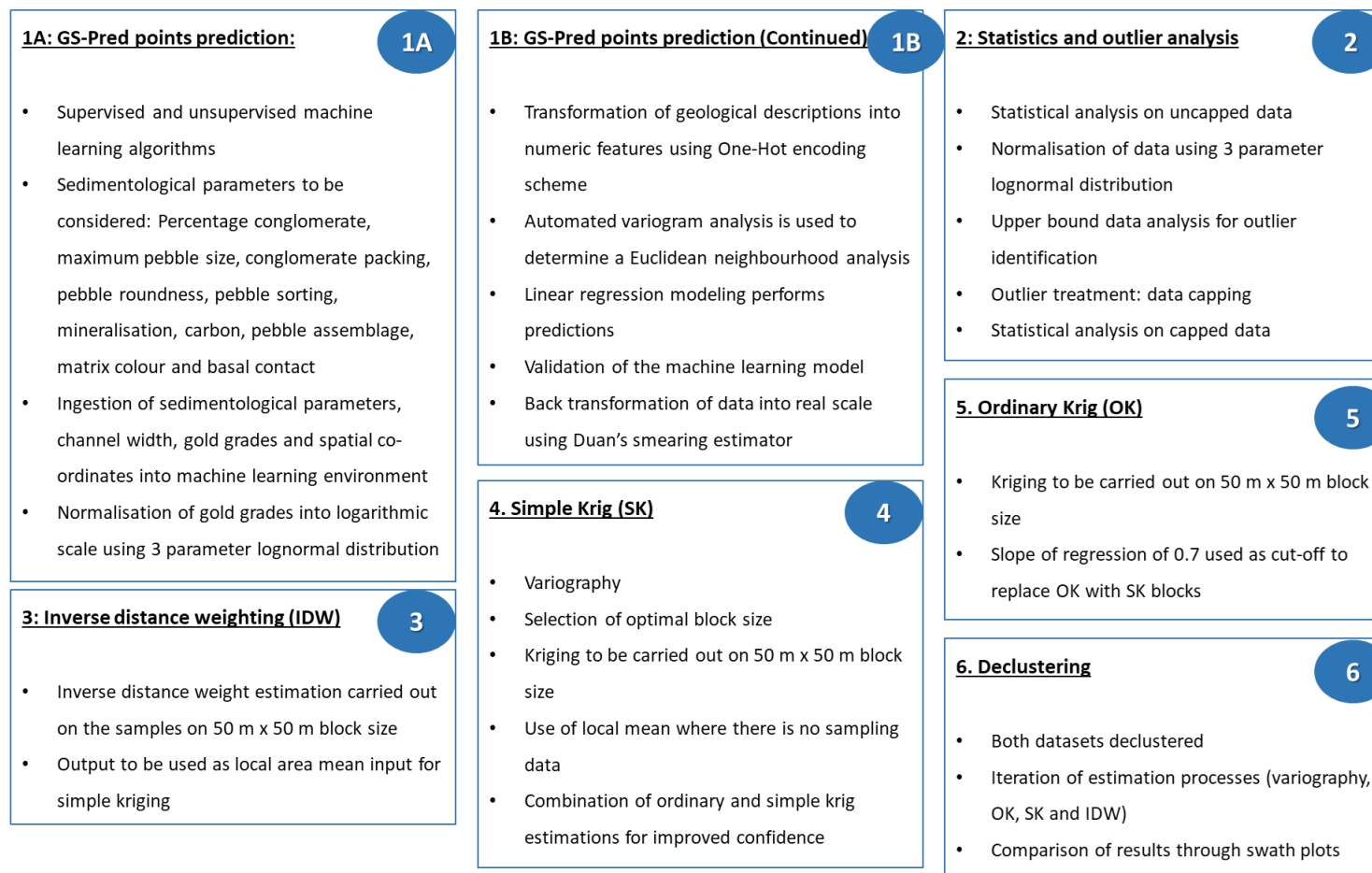


Figure 3.1: Steps followed in the estimation process.

3.1. Research locality

This study was carried out on the Driefontein Gold Mine located in the Carletonville goldfield of the Witwatersrand Supergroup and owned by Sibanye Stillwater Limited (Figure 2.4). An undisclosed area with dimensions of 1000 m by 1000 m of the mine was selected for the analysis. The Driefontein operation is located 70 to 80 km west of Johannesburg, near the town of Carletonville, and is made up of six shaft complexes that mine the Carbon Leader Reef, the Ventersdorp Contact Reef and the Middelvlei Reef. The reefs are accessed and exploited by scattered stoping and mini longwall stoping mining methods with closely spaced dip pillars, at depths of 600 to 3,420 m below surface (Sibanye Stillwater Limited, 2018).

3.2. Data analysis plan

The data analysis commenced with basic statistical analysis and outlier treatment and progressed to IDW, followed by ordinary and simple kriging. This method was carried out for both the assay data and the GS-Pred-generated data. The variable considered for the assay dataset was gold concentration (g/t), and for the predicted dataset, g/t, channel or reef width, percentage conglomerate, maximum pebble size, conglomerate packing, pebble roundness, pebble sorting, pyrite mineralisation, carbon, pebble assemblage, matrix colour and, basal contact. Due to the fact that the data of both datasets were not collected on a regular grid and were originally clustered in spacing, the two datasets were declustered and an iteration of the estimation process was also carried for both datasets on declustered data.

Manual data validation outside of GS-Pred consisted of identifying and removing any points that had zero (0 g/t) gold grade, which were below the detection limit; plotted outside of the study area and; spatial coordinate (x, y, z) duplicates, although GS-Pred is able to automatically remove such data through its data pre-processing (Nwaila et al., 2019). As it happens, no points were removed from either dataset through this validation. Grades in grams per ton (g/t) were considered in the handling of the gold assay data and not centimetre-grams per ton (cmg/t). The metal accumulation, or cmg/t, was not considered due to the incorporation of the channel width, or conglomerate thickness, as one of the input variables of the dataset.

4. GS-Pred algorithm prediction and descriptive statistics

4.1. GS-Pred Prediction

GS-Pred is a set of machine learning-based algorithms that combine sedimentological and assay data for the prediction of gold and potentially other metal grades, and the delineation of facies in conglomerate-hosted gold deposits (Nwaila et al., 2019). This type of machine learning uses a combination of supervised and unsupervised machine learning algorithms. The detailed procedures and algorithm descriptions employed by GS-Pred to predict grades used in this study have been summarised from Nwaila et al. (2019).

Essentially, GS-Pred sequentially carries out the following: it ingests a reference database consisting of geological descriptions of samples, their coordinates and known metal grades and channel widths, and a separate database of targets without metal grades; performs necessary data pre-processing for both the geostatistical and machine learning algorithms; performs geostatistical analysis of the reference database; serially builds prediction models for each target within the target database; makes predictions for each target based on its descriptive geology; optionally makes facies analyses using cluster analysis and depicts the database and prediction/clustering results. Below, the main components of GS-Pred are described.

Data pre-processing:

The reference and target databases both contain spatial coordinates, assay grades, channel widths and sedimentological descriptions for each sample, such as percentage conglomerate, maximum pebble size, conglomerate packing, pebble roundness, pebble sorting, pyrite mineralisation, carbon, pebble assemblage, matrix colour and basal contact. The reference database also contains known gold grades (in the case of gold grade predictions). The primary source database is a Microsoft SQL server which stores geological and assay data from several gold mines. From the SQL server, a subset of the data was retrieved and stored locally for use in Python.

The input data were clipped by gold grade (g/t). Two thresholds were used at the high and low percentiles to maximize prediction performance while retaining most of the data points. For academic purposes, the points are examined manually to understand

the nature of outliers. Network analysis-based outlier screening was performed by detecting anomalous relationships between the samples' geology and gold grade in a localized fashion. For the case study dataset, this yielded much of the same results as the aforementioned clipping and all points detected by clipping and network analysis-based outlier screening were removed. However, this provided a rationale for the removal of these data points without manual screening.

The clipped gold grades were subsequently normalised by adding a third parameter before the log transformation $[\ln(\text{Au} + \beta)]$ (Figure 4.1), to allow the geostatistical and machine learning algorithms to co-operate on the same gold grade data without compromising the validity of the results of the geostatistical algorithms. Here, Au is the gold grade in g/t and β is a small, positive constant. In GS-Pred β is ascertained by obtaining a value that reduces the D'Agostino's K-squared omnibus test result. The omnibus K-squared test relies on skewness and kurtosis, which are two statistical measures of a distribution's normality, to determine the distribution of a population (Date, 2019). For this dataset, β was 6e-06. No additional data manipulation was performed after this stage. Geological data were encoded into numeric labels and were rescaled before feeding into any machine learning algorithms.

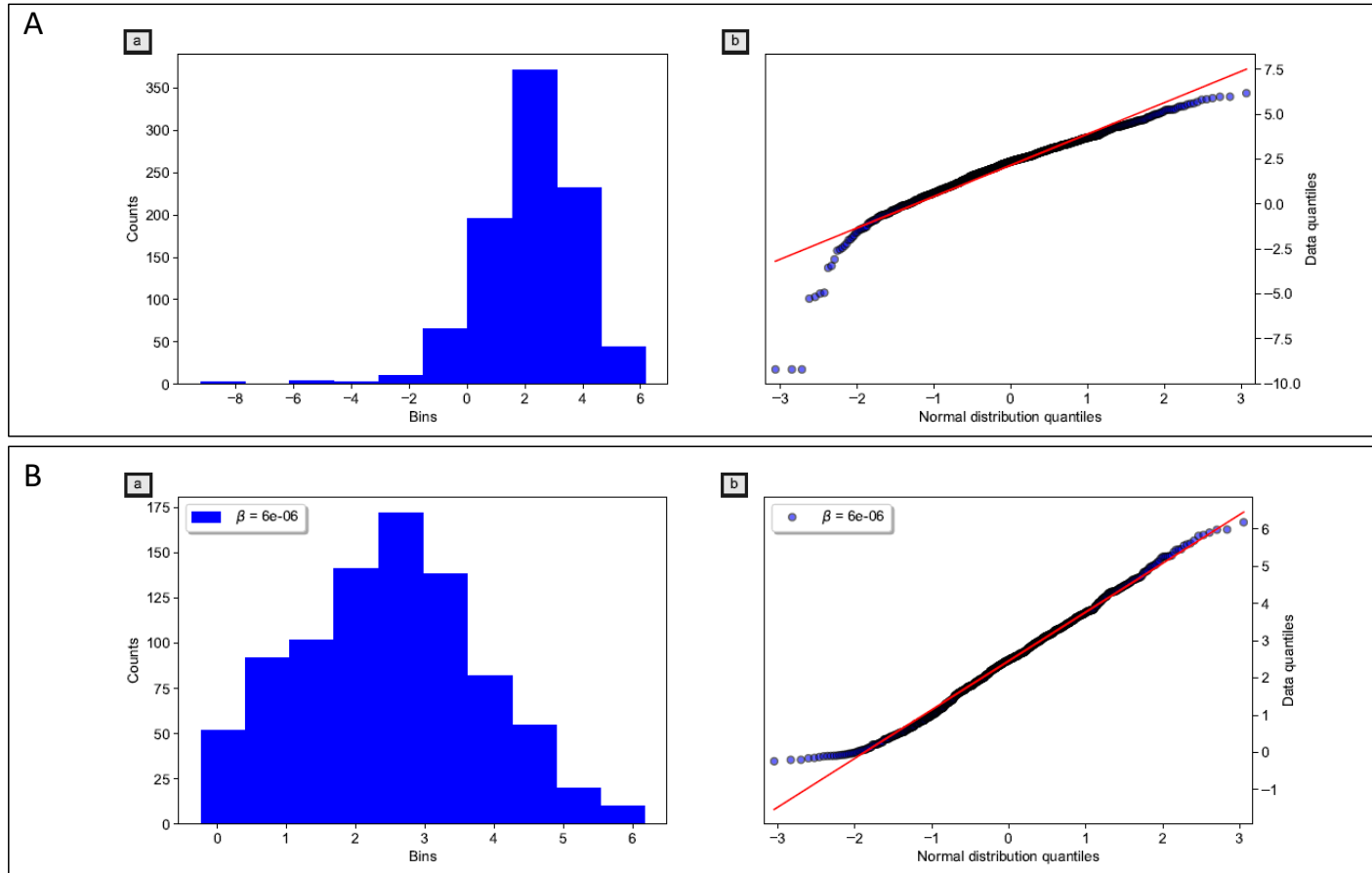


Figure 4.1: GS-Pred dataset from the Middelvrei Reef before (A) and after (B) the addition of a third parameter. The distribution is shown in histograms (a) and probability plots (b). The distribution is in natural logarithmic scale.

After the prediction has been completed, back-transformation is carried out. To re-transform, the model's predictions from the logarithmic scale into real scale the Duan's Smearing estimator is used. This estimator takes into account the fact that the expected value is a function of the exponent of the predicted value and an error term (Lesmeister and Chinnamgari, 2019). This smearing retransformation is used after fitting a linear regression model on a transformed scale after the natural logarithm of a variable has been estimated. For the back transformation of the gold grade, the Smearing factor can be expressed as

$$Au = \exp(\ln(Au)) \times \exp\left(\frac{1}{2} \sigma^2\right) - \beta$$

Equation 4.1: Duan's smearing estimator (Modified from Date (2020)).

GS-Pred makes use of an inverse transformation equation that relies on the mean squared error (MSE). The MSE, which is computed by the algorithm after the construction of the predictive model, measures the distance from the regression model to a set of points and squares them to remove negative signs. A smaller MSE indicates a closer fit to the data (Vernier, 2018). The square root of the MSE is the root mean square error (RMSE) and it can be interpreted as the standard deviation (from the true value) (Cimbala, 2011) to result in a modified transformation equation

$$Au = \exp\left[\ln(Au) + \frac{MSE}{2}\right] - \beta$$

Equation 4.2: Inverse transformation equation.

The sedimentological descriptions are nominal data and are transformed into numeric features that can be used in regression and clustering algorithms. Nominal data refers to variables that are labelled without any quantitative value, some examples in this study being matrix colour and clast roundness and sorting. Nominal data can also be referred to as categorical variables (Brownlee, 2017). This was carried out using a combination of encoding schemes, such as the One-Hot encoding scheme depending on the nature of the descriptions. As mentioned, in supervised learning when a network is being trained, labelled input is passed into the model and the model, in turn, produces a predicted output. The labels that are passed into the model are encoded so

that they can be suitable for the machine learning algorithms. The reason for this is that many machine learning algorithms are unable to operate directly on labelled data and require all input and output variables to be numeric (Brownlee, 2017). The encoding may be in the form of an integer or a vector of integers. One-Hot encoding transforms (encodes) categorical labels into binary vectors of zeros and ones. Each of these vectors has a length that is equal to the number of output categories. This type of encoding is so named because with each One-Hot encoded vector, every element in that vector will be a zero except for the element that corresponds to the actual category of the given input, which will be a one. A simple example of this would be the encoding of colour labels (Table 4.1).

Table 4.1: Example of One-Hot encoding carried out on colour labels. Each colour label is converted into a binary vector that has a length equal to the number of categories, in this case, four colours.

Orange	Black	Green	Red
1	0	0	0
0	1	0	0
0	0	1	0
0	0	0	1

Linear regressions and cluster analysis:

GS-Pred determines a spatial neighbourhood scale using a geostatistical approach. In particular, it uses an automated variogram analysis to fit a spherical model to the experimental variogram to determine the effective range and sill on the metal grades, in this instance, gold in g/t, within the reference database. Optionally, GS-Pred can refine the spherical model by maximizing the kriging performance of a subset or the entire reference database using a parameter-based grid search. The range is used as an estimate of the Euclidean neighbourhood radius, which sets the spatial boundary for the availability of training samples in automated regression modelling that predicts metal grades. Only samples contained within the neighbourhood are used on a per-target basis. Thereafter, and as needed by the operator, the algorithm iteratively infers all targets within the target database.

In machine learning, it is necessary to validate the stability of the machine learning model to ensure that the model can predict as competently for real data that it has not yet encountered as it can for a training dataset. This is known as cross-validation and it is done to ensure that the model is low on bias and variance (Gupta, 2017). One such method of cross-validation is “Leave One Out Cross Validation” (LOOCV). With this method, a single data point is left out of the training data, such that if there are n data points in the original dataset, $n-1$ samples are used to train the model (Shaikh, 2018). The remaining points are used to train the model and the process is iterated n times, each time omitting a single datapoint for all possible combinations. The omitted datapoint is then predicted and compared to the original, and an error is averaged for all of the iterations to give an average error (Gupta, 2017). A regression algorithm, known as linear regression Pearson correlation coefficient (LRPCC), forms a part of GS-Pred and performs a similar function of refining the machine learning model. Regression algorithms are utilised to determine relationships between a predicted quantity and its features. Cross-validation is used to ensure sound parameter selection for regression models. The output of this algorithm is the sum of the contribution of all the features used to make a prediction. Associated with this regression algorithm is a parameter, c , which can take on a value of 0 to 1, the range of the correlation coefficient of the linear fit of each model. The value for c can mute features that are not well correlated with the metal grade prediction, which is important as the relationship between gold grade and geological features is usually on a local scale. At a c value of 0, all features are used regardless of correlation, and at 1 only perfectly correlated features are used. At these extremes, either the prediction errors are at a maximum ($c = 0$) or very few points are predictable ($c = 1$). Through database-wide leave-one-out cross-validation, the parameter of c in LRPCC is set at 0.1. This value c was found by Nwaila, et al. (2019) to work well at 0.1 as the population of predictable targets and the reduction in average prediction error are optimised at this value. For GS-Pred, a target is considered predictable if it has a minimum of five training samples within its search neighbourhood, and its localised sum of the square of the Pearson correlation coefficients is greater than zero when using the LRPCC algorithm.

4.2. Initial statistical analysis and data treatment processes

GS-Pred generated gold grade data was compared through statistical analysis with the assay data that was used as a reference database for the GS-Pred output. The GS-Pred dataset had fewer data points than the original assay dataset. From GS-Pred, samples are predicted over a range of search radii, for example, from 6 m to 100 m; and a condition is set that a minimum number of sedimentological features and points are required to make predictions. The true versus predicted points and distribution of residuals are then plotted. Subsequently, cross-validation techniques are used to only export points with higher accuracy for the purpose of kriging. If the objective is to get the most likely accurate value, the search will be limited, perhaps to 30 m rather than 100 m and this will yield a lower number of samples compared to the actual assays, which is the case in this study. Therefore, the lower number of samples has to do with strict neighbourhood search routine in GS-pred that is biased to higher accuracy predictions. One could relax the search and get more points out but this will compromise the quality of the predictions.

All descriptive statistics analyses in this chapter and the next were carried out in Microsoft Excel. Descriptive statistics were determined for the two datasets and histograms plots were constructed (Figures 4.2 and 4.3).

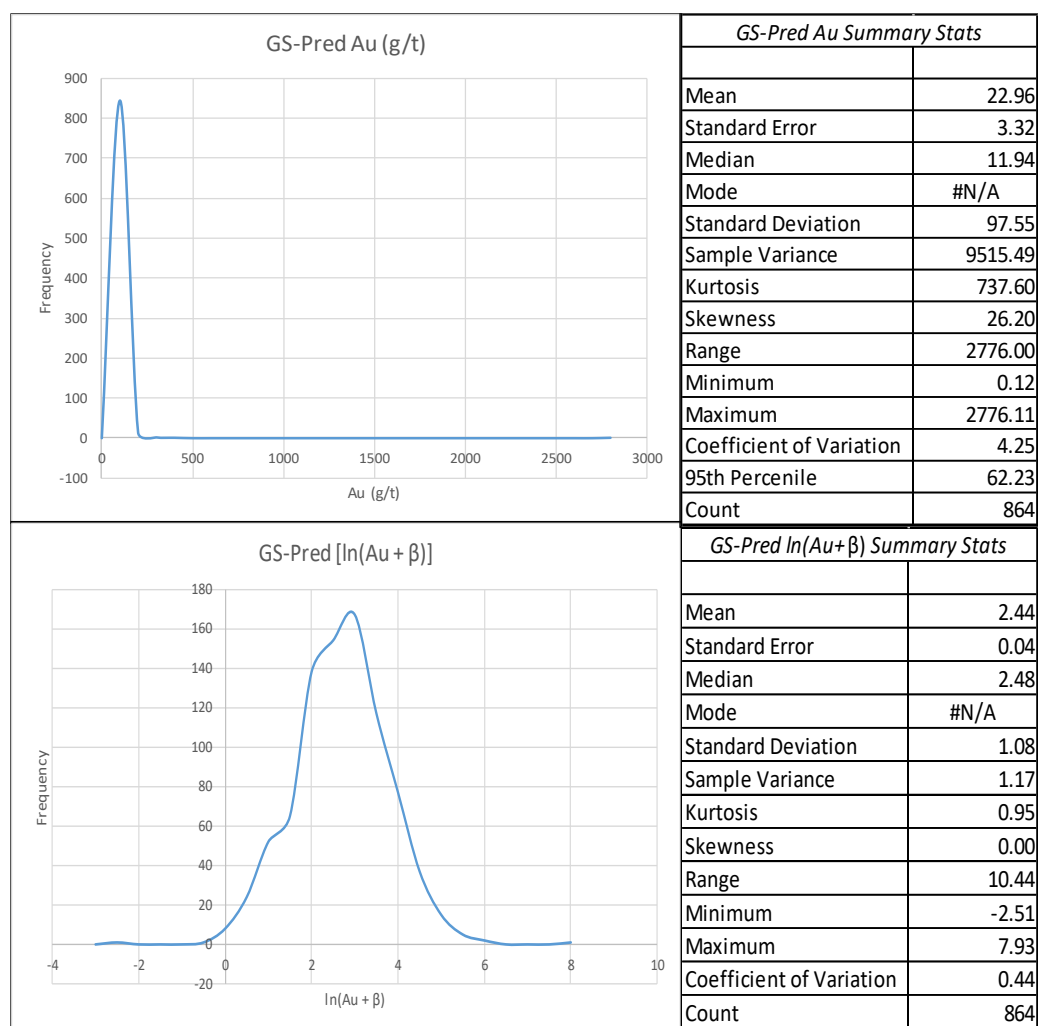


Figure 4.2: Raw GS-Pred data summary statistics and histograms.

A coefficient of variation (CV) greater than 1 (the standard deviation is higher than the mean) implies that there is a tail of high values and these will have a notable influence on the estimation process. If the CV is greater than 2, a decision on how to treat extreme values ought to be considered, as these will unduly dominate the estimation (Isaaks and Srivastava, 1989). The CV for the GS-Pred-predicted output of 4.25 was high, which indicates a long tail of high values in the histogram of the dataset. In addition, the maximum observation of 2776.11 g/t was far higher than the observation at the 95th percentile of 62.23 g/t.

The datasets were normalised with the addition of a third parameter β , by making use of the Goal Seek function in Microsoft Excel. The target value was determined to minimize the skewness of the datasets to zero by changing the β value. The β value was

determined individually for each dataset and was 1.99 for the GS-Pred dataset and 1.68 for the assay dataset.

The CV for the assay dataset was lower than that of the GS-Pred dataset at 1.84 (Figure 4.3). However, the wide difference between the 95th percentile and maximum observation was noted as an indication of potential outliers.

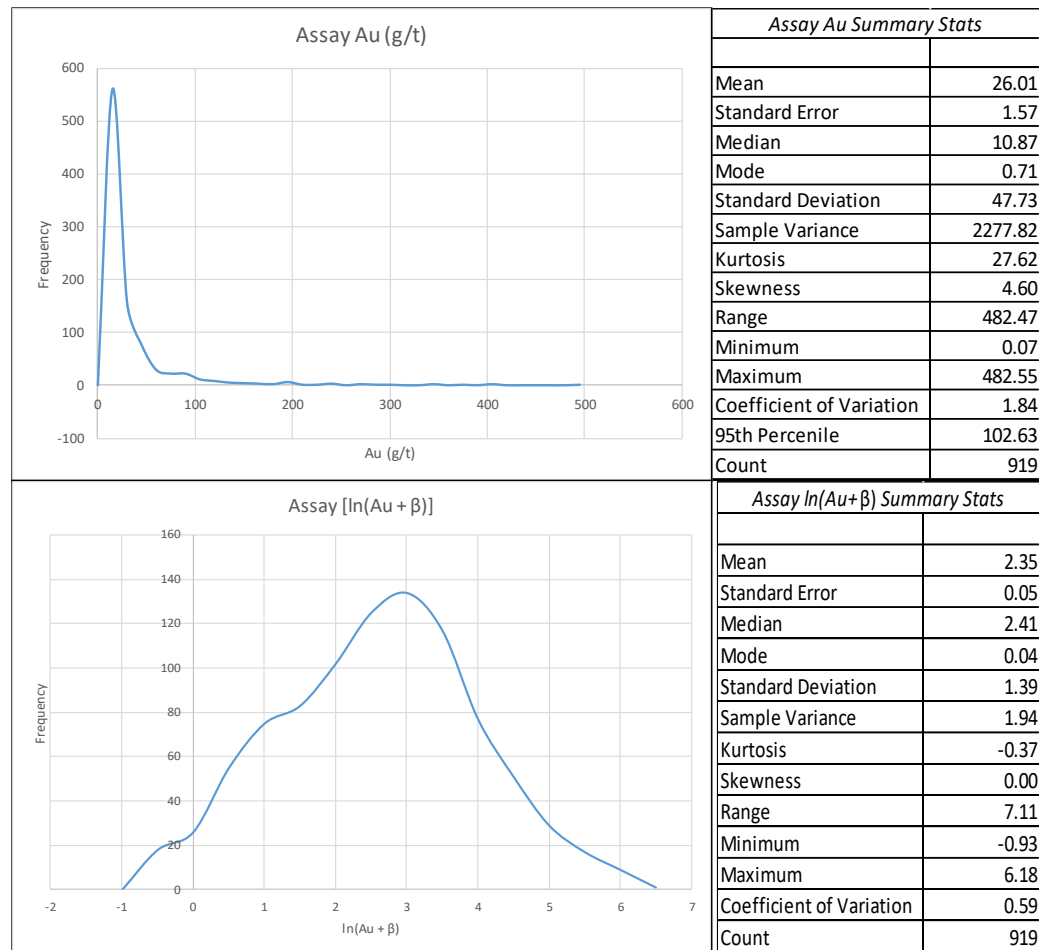


Figure 4.3: Raw assay data summary statistics and histograms.

4.3 Treatment of outliers

A potential risk associated with extremely high grades is the overestimation of the deposit or the overestimation of block grades in the areas in the neighbourhood of the extremely high-grade data points. Several steps have been outlined by Srivastava (2001) on how to evaluate and address outliers. Some of these include utilising probability and scatter plots and data posting to identify outliers, and to evaluate them

visually in their spatial context. Although one of the measures listed by Strivatstava (2001) is to discard the outlier, industry practice is to cap it at an acceptable grade (Leuangthong and Nowak, 2015). For both datasets, outliers were observed and were taken as data points with values beyond the upper bound of each dataset. For both datasets, the outliers were spatially located throughout the study area (Figure 4.4).

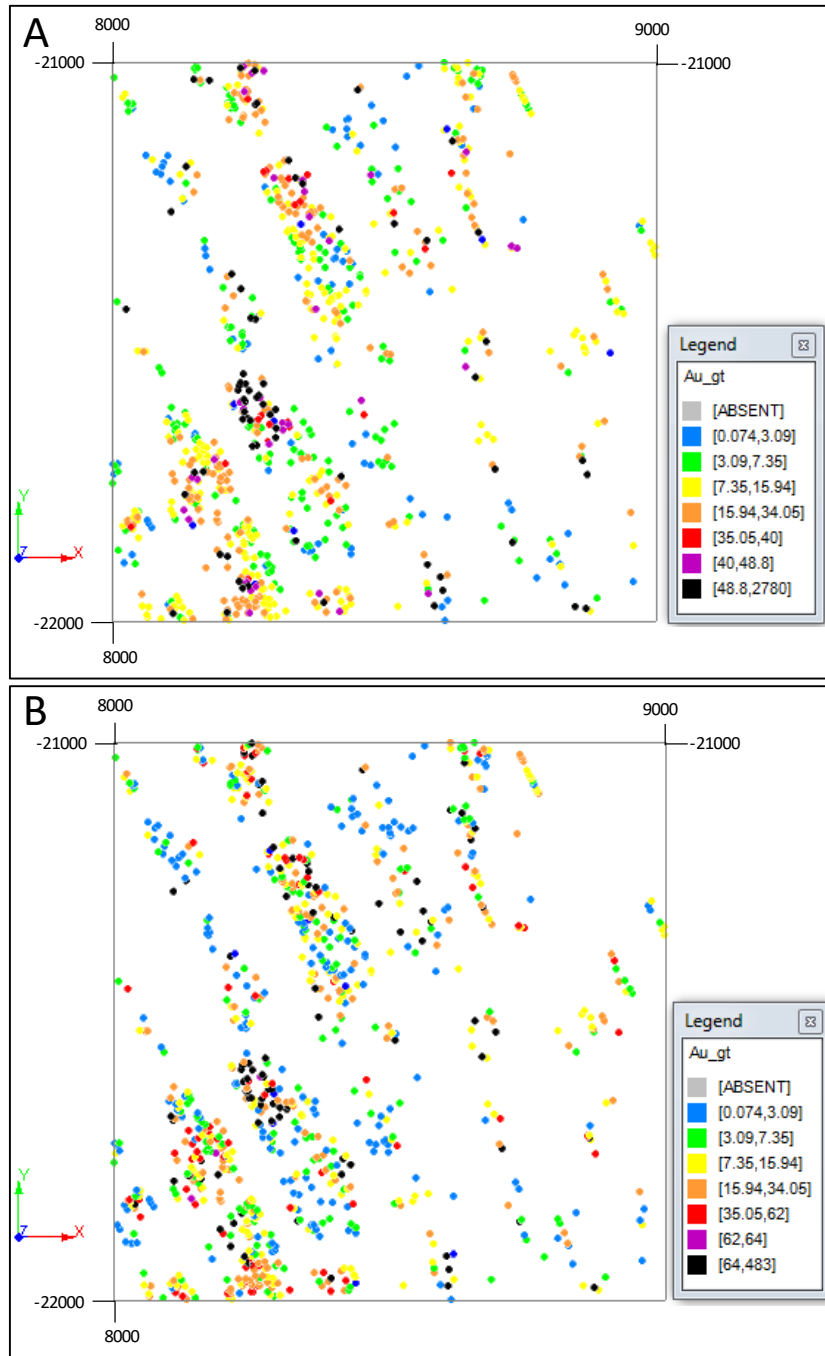


Figure 4.4: The location of outliers throughout the study area for GS-Pred (A) and assay data (B) represented by black on the values legend. The colour scheme is based on the gold grade (Au_gt) in g/t.

Grade capping entails reducing grades that have been determined to be extremely high, or outliers, to a lower, capped grade for resource estimation. This process allows for such data points to be retained in the database and used throughout the estimation process (Leuangthong and Nowak, 2015). Due to the presence of extremely high values

indicated by CV greater than 1, and the fact that the grades in the 95th percentiles of the datasets are far lower than the maximum values of the respective zones, a decision was made to cap the data to the upper bound of each dataset. This resulted in the capping of 69 data points in the GS-pred dataset to 48.83 g/t, approximately 8% of the data. The consequence of capping data and adding a third parameter β to it is the distortion that the distribution undergoes when only the histogram is considered. A high-frequency tail is observed due to the number of data points that have been capped to a single value. The result of the mode being higher than the mean and median is a direct consequence of the capping. A plot of these datasets on a probability plot indicates that the departure from normality is not significant as the majority of the data points lie on the distribution fit line (Figure 4.5).

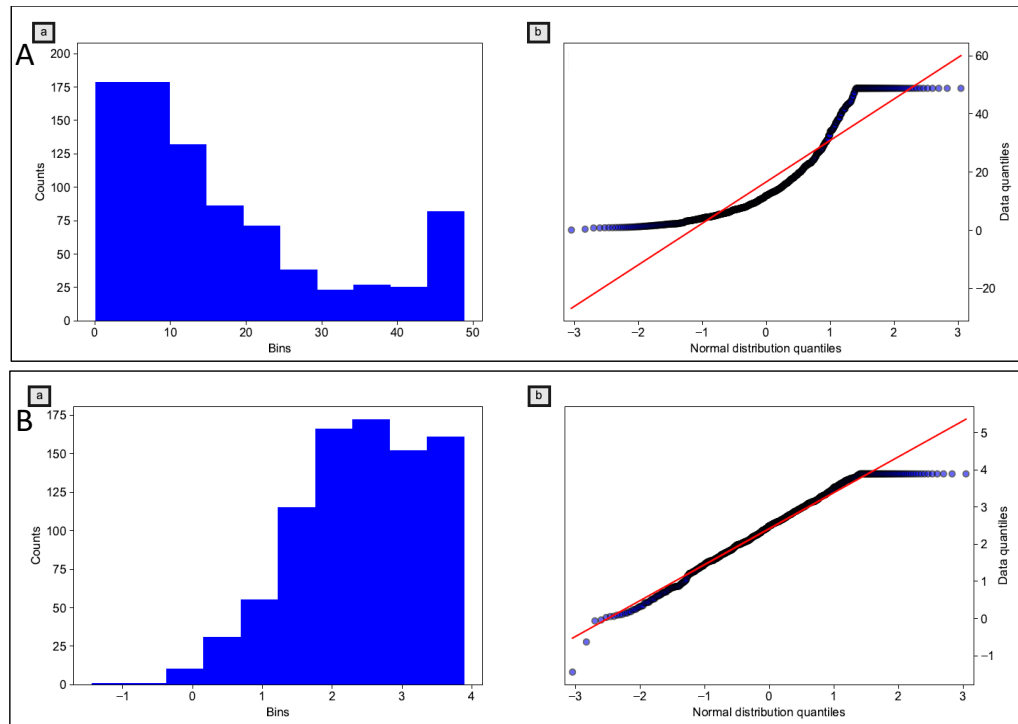


Figure 4.5: Capped GS-Pred data and before (A) and after the addition of a third parameter, β (B) plotted as a histogram (a) and a QQ-plot (B).

In the assay dataset, 93 data points (approximately 10% of the data) were capped to 63 g/t (Table 4.2). The reduction in the CV of the capped dataset is smaller than that of the GS-Pred dataset, however, it is now closer to 1, meaning that the tail of high values will not have as much influence on the estimation process as might have been the case for the uncapped data.

Table 4.2. Capped assay data summary statistics

Assay Summary Statistics	<i>Au</i>	<i>ln(Au+6)</i>
Mean	18.63	2.52
Standard Error	0.65	0.03
Median	10.87	2.53
Mode	63.01	4.17
Standard Deviation	19.73	1.04
Sample Variance	389.37	1.08
Kurtosis	0.25	-1.07
Range	62.93	3.60
Skewness	1.22	0.00
Minimum	0.07	0.57
Maximum	63.01	4.17
Coefficient of Variation	1.06	0.41
95th Percentile	63.01	4.17
Count	919	919

Box and whisker plots for the datasets demonstrate a notable difference between the raw and capped data (Figure 4.6). Capping reduced the CV in each dataset and ensured that the maximum observations were closer to the 95th percentile than in the raw datasets. The standard error and variances were also significantly reduced. The upper bounds, or whiskers, of the GS-Pred and assay datasets at 48 g/t and 62 g/t are largely in agreement with the extent of capping carried out, as values beyond these two grades would be considered outlier observations. The means of the datasets plot close to the third quantiles and this is a result of the positively skewed distribution.

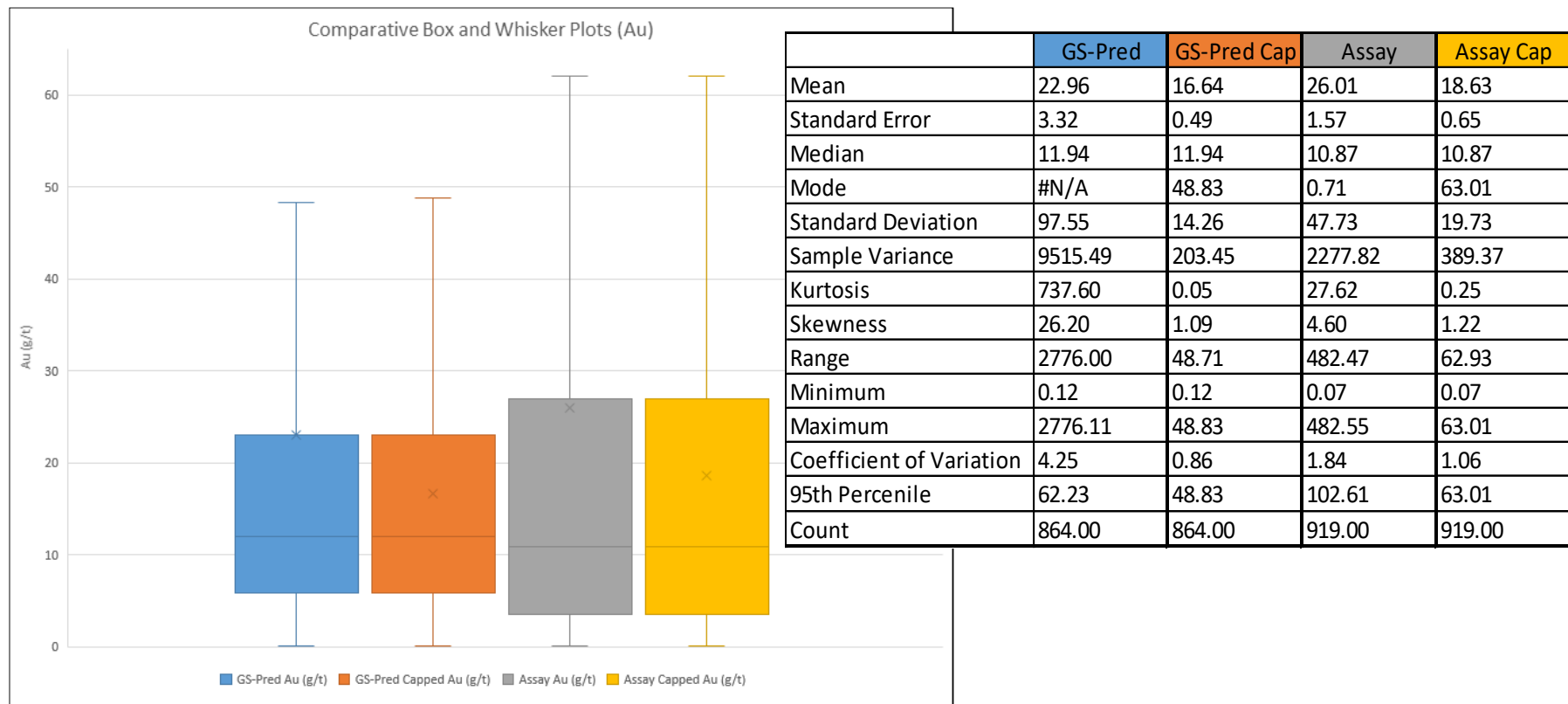


Figure 4.6: Box and whisker plots of raw and capped datasets from GS-Pred and assays before 3 parameter natural log transformation.

The natural log-transformed data with the addition of a third parameter is less variable than the untransformed data, as indicated by the lower CV, standard error and sample variances across both datasets (Figure 4.7). The values of kurtosis and skewness are also consistent with a normally-distributed dataset after the transformation.



Figure 4.7: Box and whisker plots of raw and capped datasets from GS-Pred and assays after three-parameter natural log-transformation (outlier datapoints not shown).

The final datasets were used for estimation (Figure 4.8). All variography and estimation processes were carried out in Studio RM Datamine.

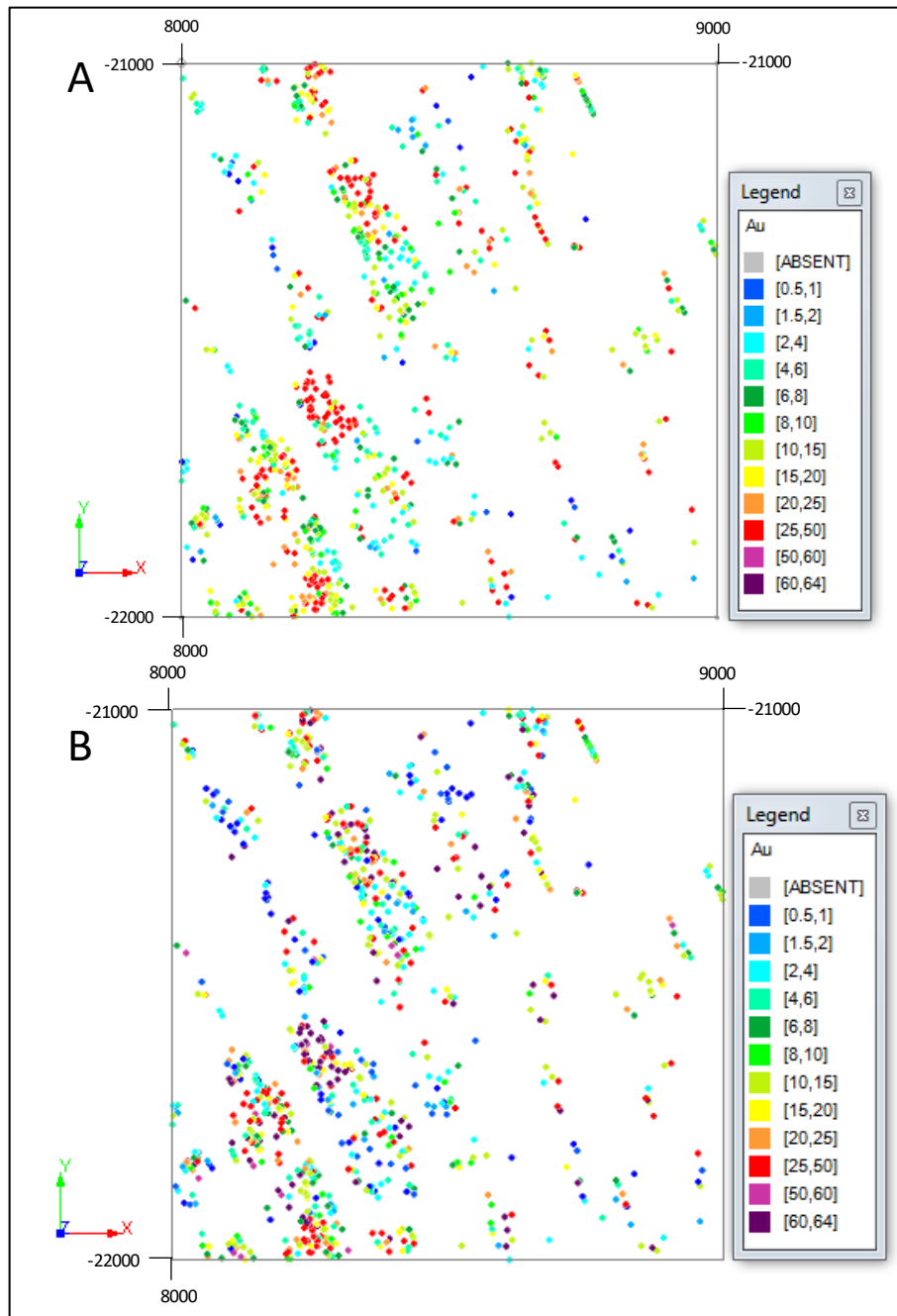


Figure 4.8: Plot of GS-Pred data (A) and assay data (B) to be used in estimation and block modelling. The colour scheme is based on the gold grade (Au_gt) in g/t.

5. Block Modelling

5.1. Variogram modelling

Due to the spatial and statistical similarity of the two datasets, a single variogram was used for the assay data and the data generated through GS-Pred for the resource modelling (Figure 5.1). A three-parameter natural log transformation was carried out on the assay data and this transformed data was used to model the variogram with three spherical structures. The variogram was normalised to unity and modelled in the 115.5° direction.

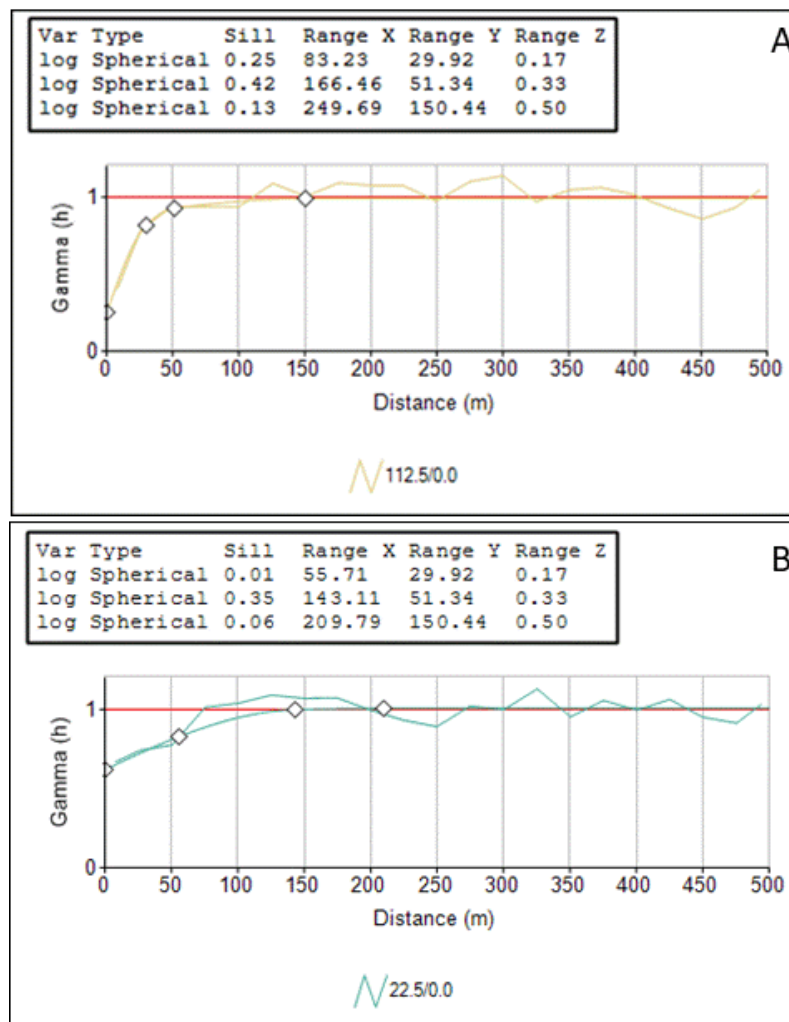


Figure 5.1: Variogram model for log-transformed grades for the estimation of the GS-Pred and assay data in the directions of maximum (A) and minimum, or perpendicular, (B) continuity.

5.2. Quantitative Kriging Neighbourhood Analysis (QKNA)

Kriging by nature produces smoothed estimates resulting in conditional bias. Kriging efficiency and slope of regression are the statistical measures that are used to select an optimal set of set of the kriging neighbourhood parameters to reduce the impact of the conditional bias (Vann et al., 2003).

As previously outlined from Krige (1997), conditional bias occurs when higher-grade blocks are over-valued and lower-grade blocks are under-valued. Ordinary kriging tends to produce conditionally biased estimates since the slope of regression is usually less than 1 while simple kriging produces smoothed estimates since the slope of regression is equal to unity. Causes of conditional bias range from sub-optimal kriging parameters and spatial structure (isotropic and anisotropic considerations), block size and incorrect block discretization. The KNA carried out in Chapter 5 allowed for the optimal kriging parameters, which increased the slope of regression and lowered the kriging variance, to be selected. In this study, an isotropic semivariogram was used and therefore the semivariogram does not capture directional or zonal anisotropy. Isotropic models, which assume that the variogram reaches the sill or range at the same point in all directions, are commonly used in geostatistical modelling due to them being easier to build.

From the variogram modelling, optimal kriging parameters were determined through a Kriging Neighbourhood Analysis (KNA). The parameters considered were minimum and maximum number of samples, discretisation and neighbourhood extent. In all of the estimations, the minimum number of samples was 5. This ensures that the number of samples is always greater than the number of degrees of freedom that are lost in a dataset, which are the mean, variance, standard deviation and error of the mean. The KNA indicated that the optimum maximum number of samples for the estimation was 19. For the datasets, a maximum number of samples of 19 samples results in the kriging slope of regression approaching unity and the estimation variance being at its lowest (Figure 5.2).

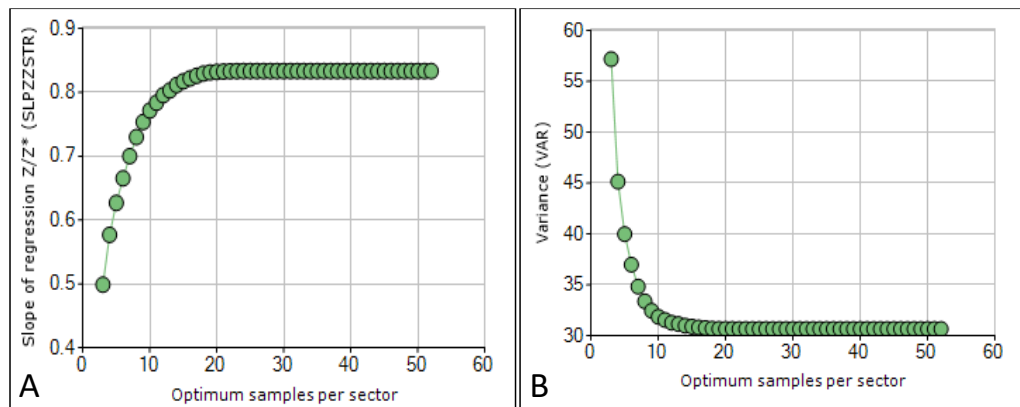


Figure 5.2: Determination of optimum maximum samples by KNA using the slope of regression (A) and kriging variance (B).

A block size kriging neighbourhood analysis was carried out in Studio RM Datamine on the kriging block size using 10 and 30 as the minimum and maximum number of samples, respectively, and a 50 m x 50 m block size was selected (Figure 5.3). On average, the closest samples had a 5 m spacing. Although a smaller block size would have resulted in a higher kriging efficiency and kriging slope of regression, it would have resulted in fewer samples per block.

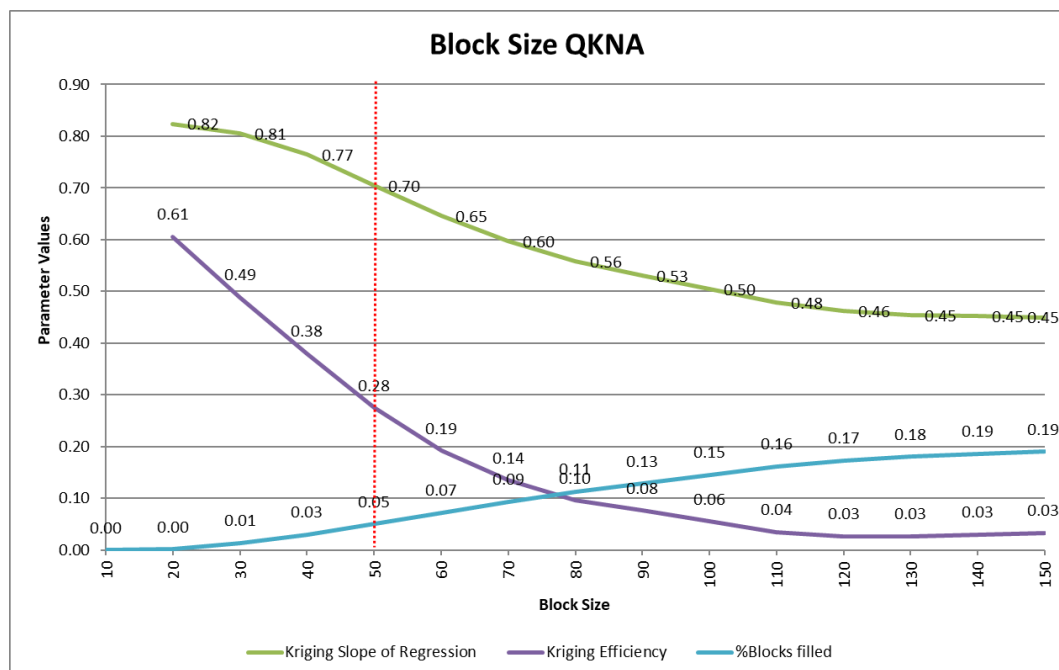


Figure 5.3: Block Size QKNA for both datasets

5.3. Ordinary kriging block modelling

A 50 m by 50 m ordinary kriging block model was created (Figure 5.4) and the kriging slopes of regression were also determined (Figure 5.5). Ordinary kriging produced block estimates for most of the study area except for the north-eastern portion due to a lack of data in this region. Simple kriging would be able to improve the confidence of the project area's estimate where there are no samples by relying on an area mean. The importance of the area mean is noted in this instance, where there is less information in the kriging neighbourhood.

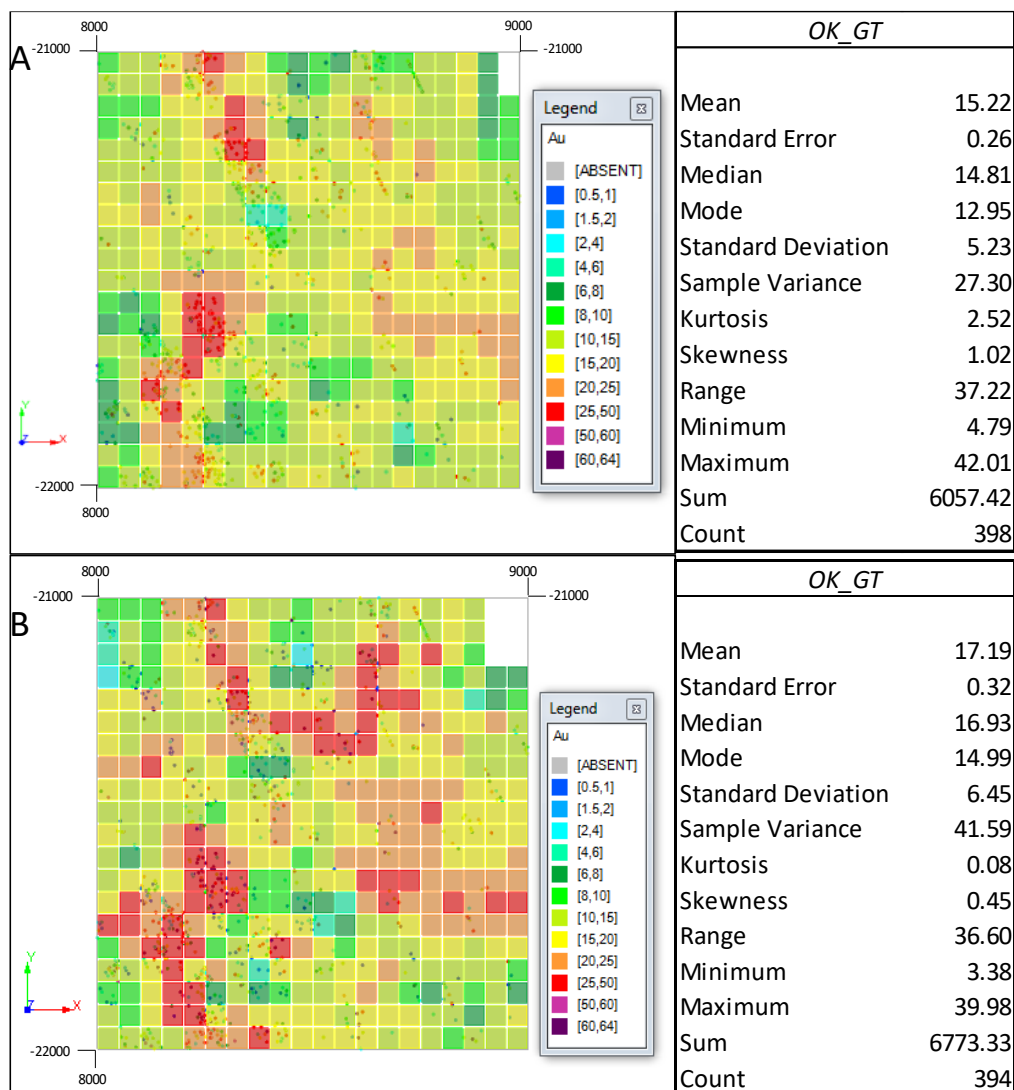


Figure 5.4: Ordinary kriged estimate estimates for GS-Pred data (A) and assay data (B) with corresponding gold estimate statistics.

The kriging slope of regression for both datasets was greater than 0.7. This indicates that the final, composite ordinary and simple kriged estimate models would be dominated by ordinary kriging estimates as the cut-off for the composite models was 0.7 and greater.

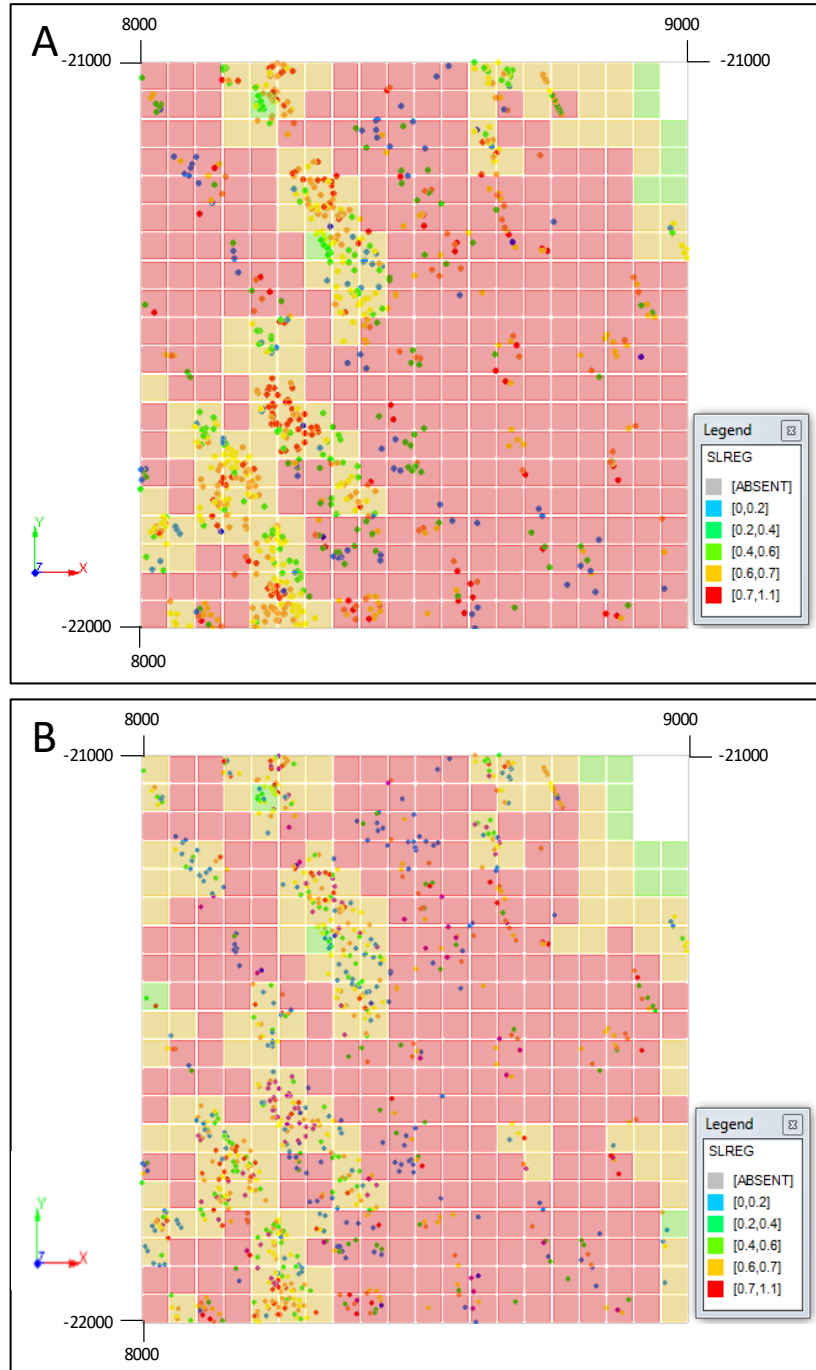


Figure 5.5: Kriging slope of regression for GS-Pred data (A) and assay data (B) ordinary kriged estimate estimates.

5.4. Simple kriging block modelling

Simple kriging was preceded by IDW with a power of 1, meaning that the averaging of samples within each block was determined. The IDW aimed to determine the local area mean, which is required for the simple kriged estimate. This IDW model served as an input proto-model for the simple kriged estimate model (Figure 5.6). There is a difference in the variance of the simple kriged estimates, which is higher than those of the ordinary kriged estimates. The IDW block model is the same as that of the simple kriged estimate.

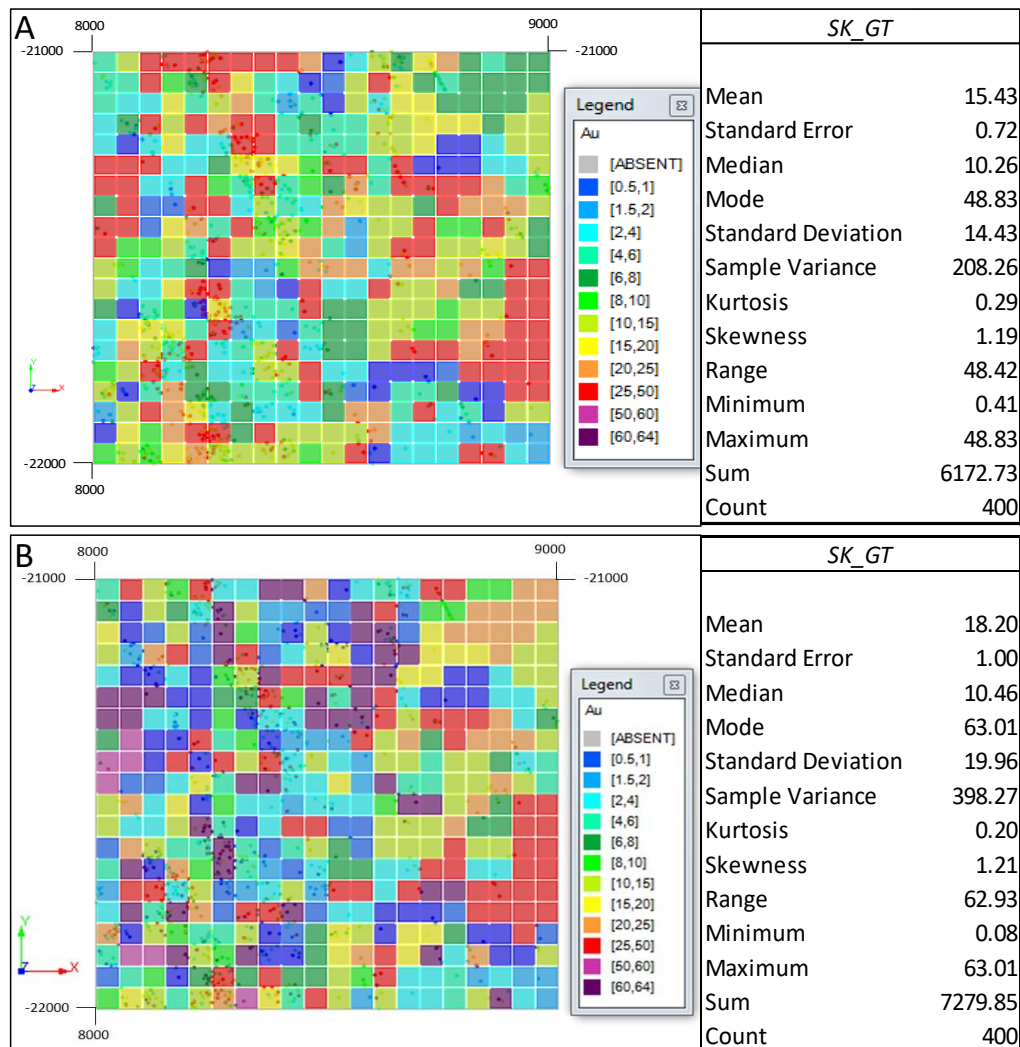


Figure 5.6: Simple kriged estimate block model for GS-Pred data (A) and assay data (B) with corresponding gold estimate statistics.

5.5. Composite ordinary and simple kriging model and validation

The blocks from the ordinary kriging that had a kriging slope of regression of less than 0.7 were removed and replaced with the estimates of the simple-kriged estimates and a composite block model was produced (Figure 5.7). The aim of combining the two estimations was to produce a block model with higher confidence and to have estimated gold values where there were no samples. In the latter instance, the local area mean will have a higher weighting for the estimate in such areas. The effect of combining the two estimates was a reduction in the variance of the estimates, compared to the estimates produced by the simple kriging alone. Compositing on this basis is justified as slope of regression is an indicator of conditional bias and should ideally be as close to 1 as possible.

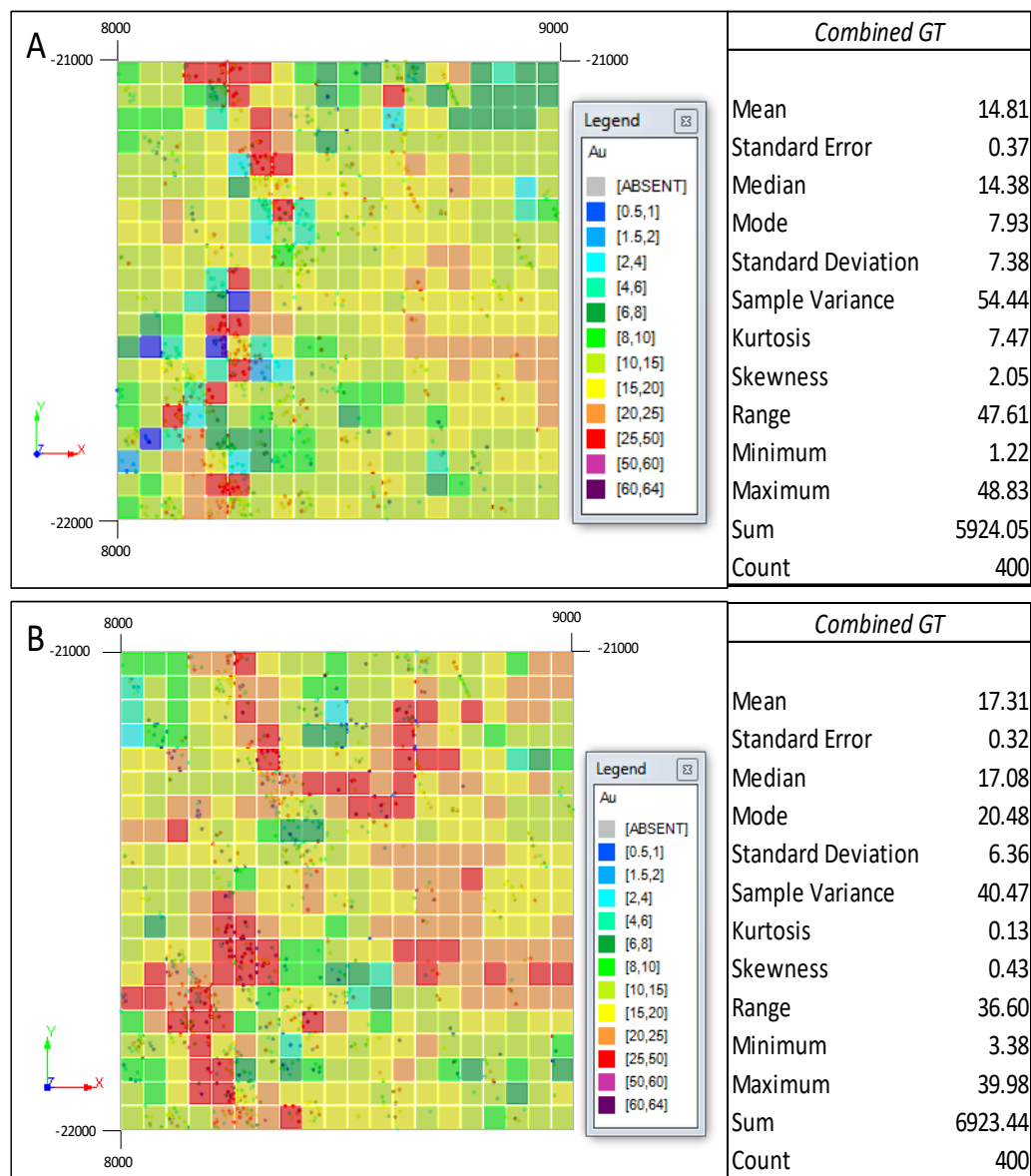


Figure 5.7: Composite ordinary and simple kriged estimate block models for GS-Pred data (A) and assay data (B) with corresponding gold estimate statistics.

The kriging efficiency was calculated and a visual observation indicates that it was, overall, higher for the GS-Pred data than it was for the assay data (Figure 5.8).

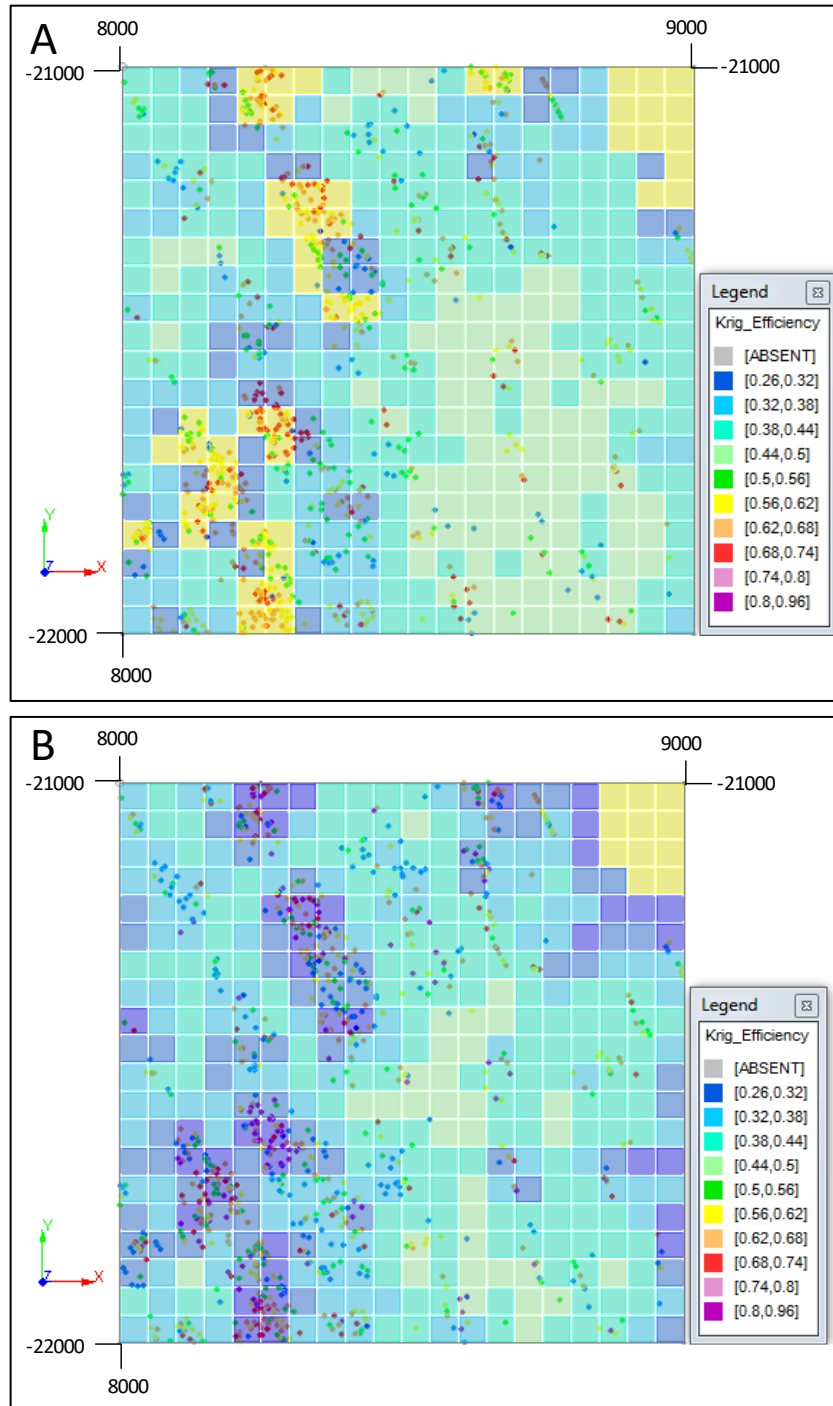


Figure 5.8: Kriging efficiency for composite ordinary and simple kriged estimate block models for GS-Pred data (A) and assay data (B).

The comparison of the kriging slopes of regression and kriging efficiencies are best summarised in a box and whisker plot (Figure 5.9). These two measures and their comparisons were compared on a global scale.

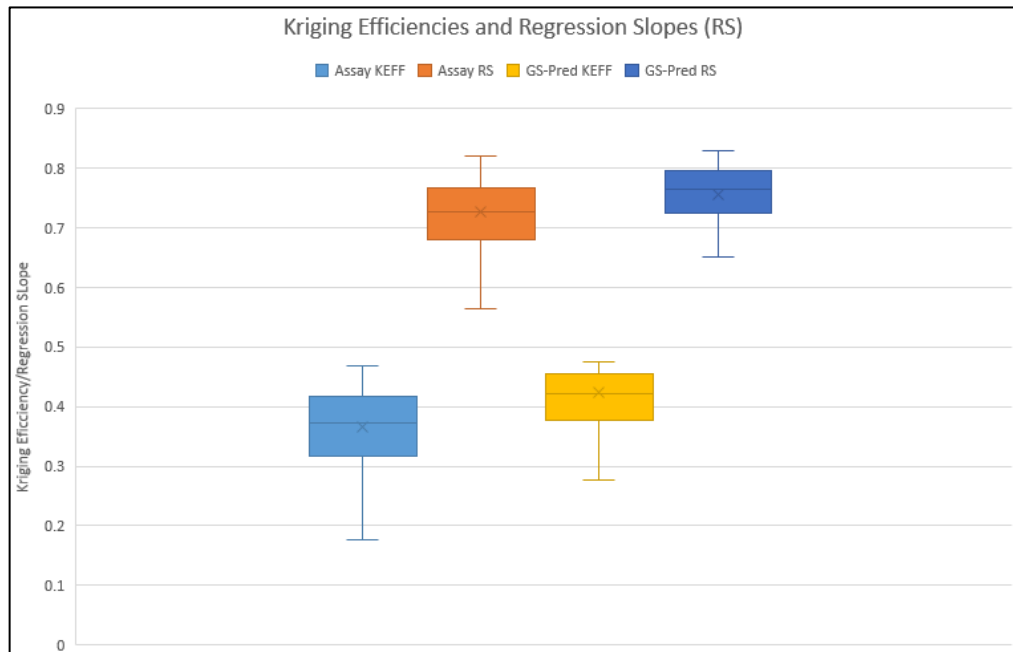


Figure 5.9: Box and whisker summary of the kriging efficiencies and kriging slope of regression of the estimations of GS-Pred generated and assay data.

A swath plot was produced for this final composite model for the purpose of validation, carried out along the X- and Y-Axis on a 50 m by 50 m corridor (Figure 5.10). A swath plot is a visual validation tool used to compare two sets of data in a particular direction. In this instance, the average estimated grades were compared to the average sample grades in the X and Y directions at a 100 m interval. This validation is carried out to determine how well the estimation fits the sampling data, or rather, to identify if there is any bias or smoothing in the estimates. The swath plots for both datasets indicate that there is under-estimation (bias) between the estimated blocks and the sample grades, particularly where the number of samples is lower. This under-estimation is more marked in the X-direction than in the Y-direction for GS-Pred and less in the case of the assay data. A similar observation that indicates more bias for GS-Pred than for The assay data in the X-direction than in the Y-direction is made for the assay data. A possible reason for this might lie in the orientation of the point data which is more aligned to the Y direction than the X direction.

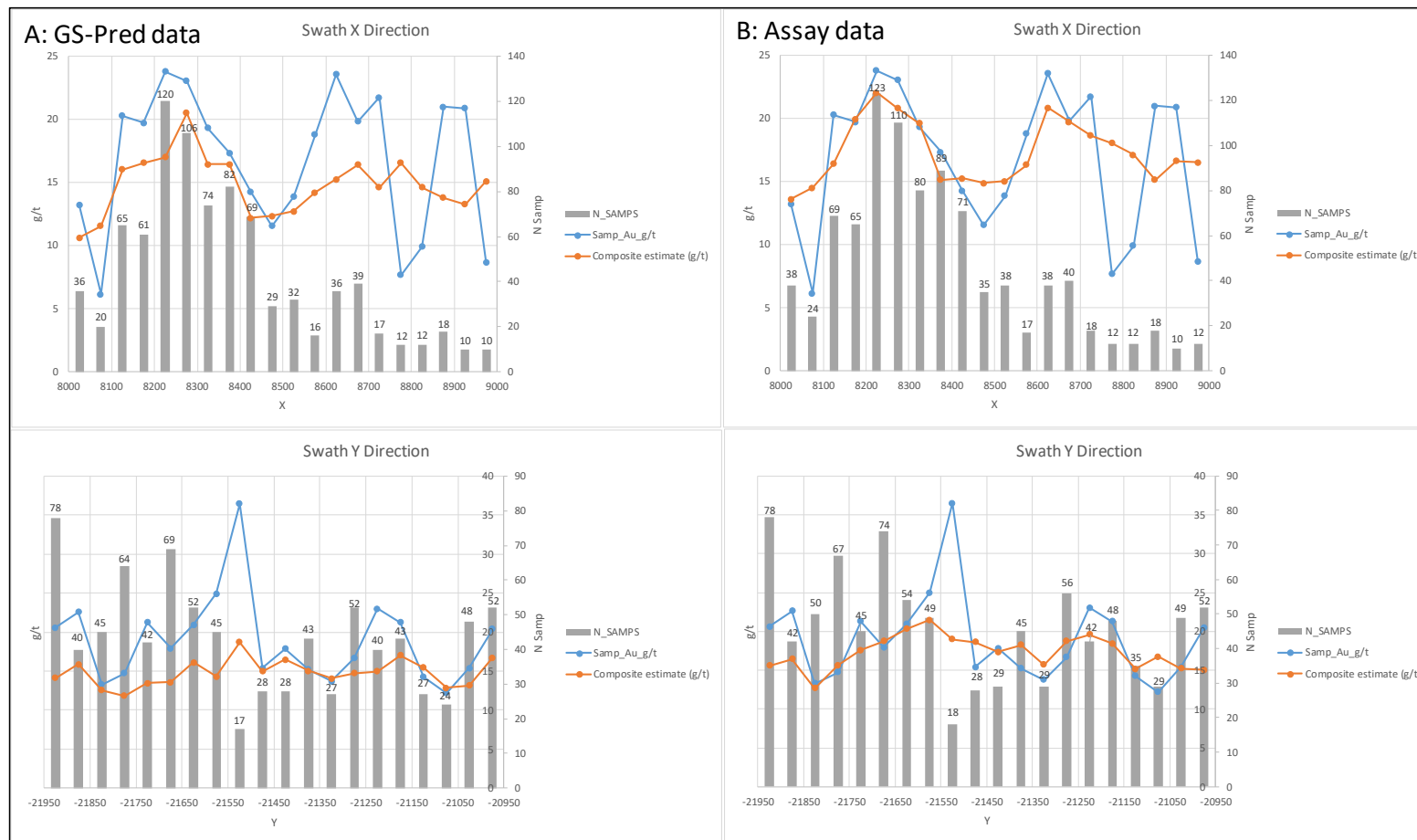


Figure 5.10: Swath plot for the composite ordinary and simple kriged estimate block models and the sample data for GS-Pred data (A) and assay data (B).

5.6. Declustering

The datasets used for these estimations consist of points that are spatially clustered in places and are not on a regular grid, leaving considerable areas containing little to no samples. This creates a challenge for resource estimation, as areas with no samples have no data from which to estimate. Declustering is a common practice to resolve this issue in drill hole or sample data, which is often not collected on a regular grid. This irregular sampling results in areas where there are higher sampling densities and other areas, which are relatively sparsely sampled. In such instances, a biased histogram and data statistics result (Rondon and Trueman, 2014). During declustering, a grid is placed over the stationary domain with a cell size that is commonly the approximate spacing of the data, where sampling is sparse. A count of the number of occupied cells is obtained and these occupied cells receive equal weight. Closely positioned data receive reduced weighting and distantly spaced data receive increased weight. Data in the cell then equally share the weight assigned to the cell and each cell is subsequently assigned a single value (Deutsch, 2015). It is important to remember that the weighting changes the influence of each sample and not the sample value (Pyrz and Deutsch, 2007). The two datasets were declustered using a grid size of 50 m x 50 m x 1 m in Studio RM Datamine (Figure 5.11).

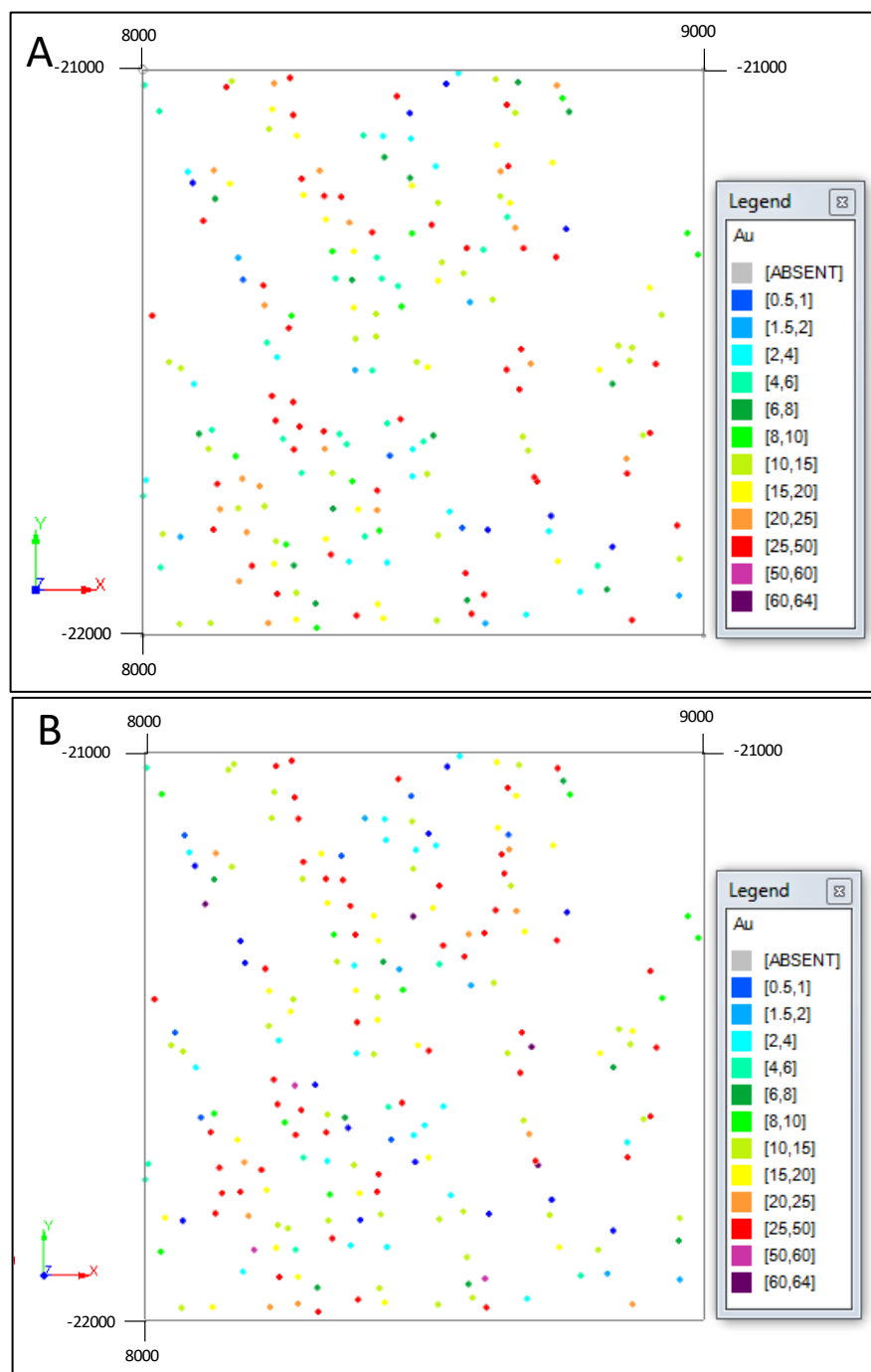


Figure 5.11: Declustered datasets from GS-Pred data (A) and assay data (B).

The effect of the declustering process is given for the two datasets (Table 5.1). In the table, it can be seen that the declustering has resulted in several cells (200 and 206 for GS-Pred and the assay data, respectively), in which each sample is assigned an equal weight based on their count in each cell. Where there are fewer samples in a cell, the weighting is higher, and where there are more samples in a cell, the weighting is

decreased. The number of samples (864 and 919 for GS-Pred and the assay data, respectively) remains unchanged through the declustering process.

Table 5.1: Summary of the declustering of GS-Pred and assay Data.

<u>GS-Pred</u>				<u>Assay</u>			
NSAMPLES	DCWEIGHT	NCELLS	COUNT	NSAMPLES	DCWEIGHT	NCELLS	COUNT
1	4.32	55	55	1	4.46	52	52
2	2.16	32	64	2	2.23	37	74
3	1.44	29	87	3	1.49	27	81
4	1.08	15	60	4	1.12	13	52
5	0.86	13	65	5	0.89	13	65
6	0.72	14	84	6	0.74	17	102
7	0.62	9	63	7	0.64	12	84
8	0.54	3	24	8	0.56	3	24
9	0.48	3	27	9	0.50	3	27
10	0.43	10	100	10	0.45	11	110
11	0.39	2	22	11	0.41	2	22
12	0.36	4	48	12	0.37	5	60
13	0.33	4	52	13	0.34	3	39
14	0.31	2	28	14	0.32	3	42
15	0.29	1	15	15	0.30	1	15
16	0.27	1	16	16	0.28	1	16
17	0.25	1	17	17	0.26	1	17
18	0.24	1	18	18	0.25	1	18
19	0.23	1	19	19	0.23	1	19
Sum		200	864	Sum		206	919

In addition, the summary statistics of the weighted samples after declustering were obtained and are summarised below (Table 5.2). For both of the datasets, the variance and CV were reduced, indicating reduced variability.

Table 5.2: Summary of the statistics of the datasets before and after declustering.

	GS Pred Data (g/t)	Declustered GS Pred Data (g/t)	Assay Data (g/t)	Declustered Assay Data (g/t)
Mean	16.64	17.14	18.63	15.41
Skewness	1.09	1.10	1.22	0.82
Kurtosis	0.05	0.97	0.25	-0.07
Variance	203.45	201.90	389.37	134.90
STDev	14.26	14.21	19.73	11.61
Standard error	0.49	0.99	0.65	0.82
Min	0.12	0.17	0.07	0.84
Max	48.83	63.01	63.01	48.83
Range	48.71	62.84	62.93	47.99
Coefficient of variation	0.86	0.83	1.06	0.75

Subsequently, an experimental variogram was calculated for the declustered datasets and, again, only a single variogram was deemed necessary for both datasets (Figure 5.12). The variogram was calculated and modelled from the log-transformed assay data after declustering. Three spherical structures were used to model the variogram, which was isotropic for ease of use, with a maximum range of 130 m. The variogram was normalised to unity.

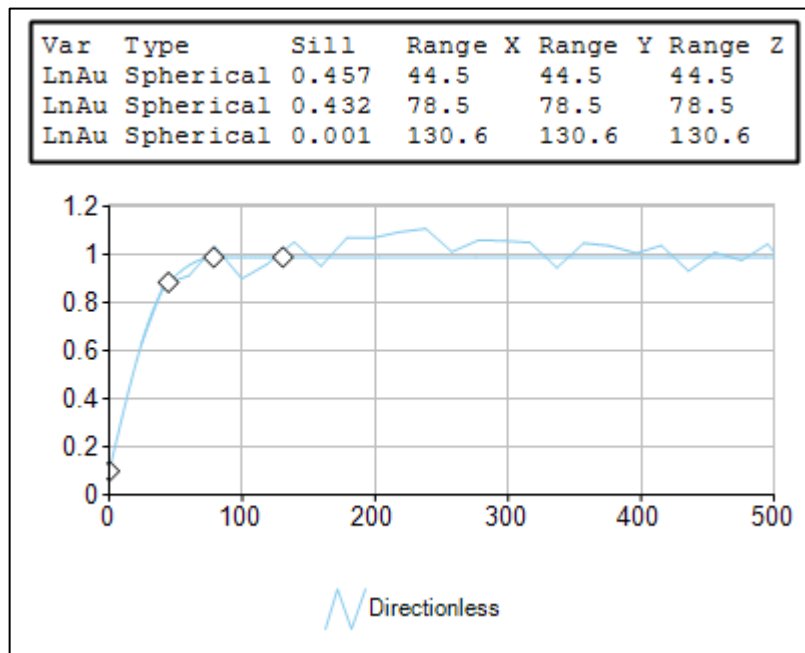


Figure 5.12: Isotropic variogram for log-transformed grades for declustered estimation.

5.7. Declustered ordinary kriging block modelling

The modelling for the declustered datasets was carried out with the estimation and search parameters used for the non-declustered data, so there was no need to determine optimal kriging parameters. Again, ordinary kriging was carried out on the datasets and produced a block model with lower variability than that of the original dataset (Figure 5.13).

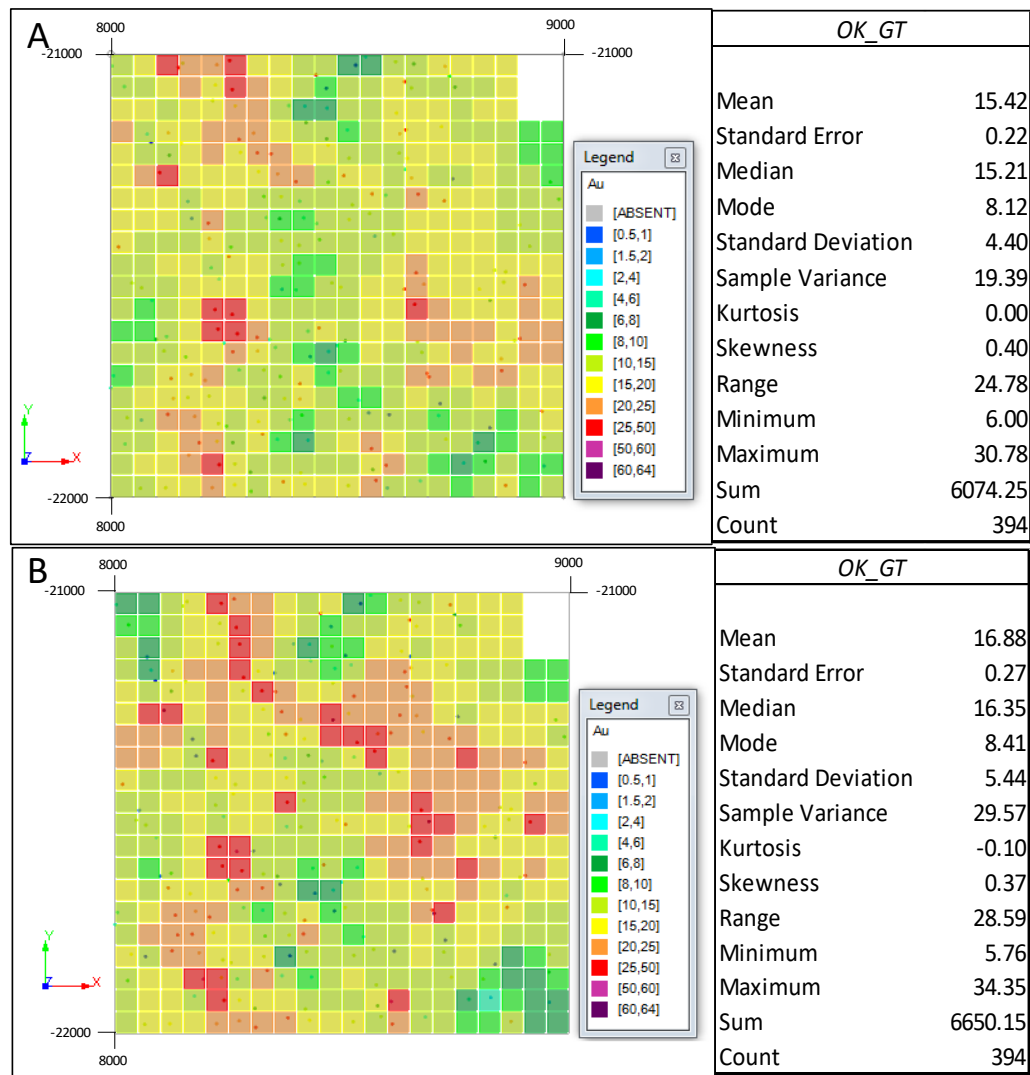


Figure 5.13: Ordinary-kriged estimates for declustered GS-Pred data (A) and assay data (B) with corresponding gold statistics.

For the declustered ordinary kriging, the kriging slope of regression was greater than 0.7 over a significantly greater proportion of the study area for both datasets (Figure 5.14). This indicated that almost all of the blocks in the composite estimation models would be contributed to by ordinary kriging.

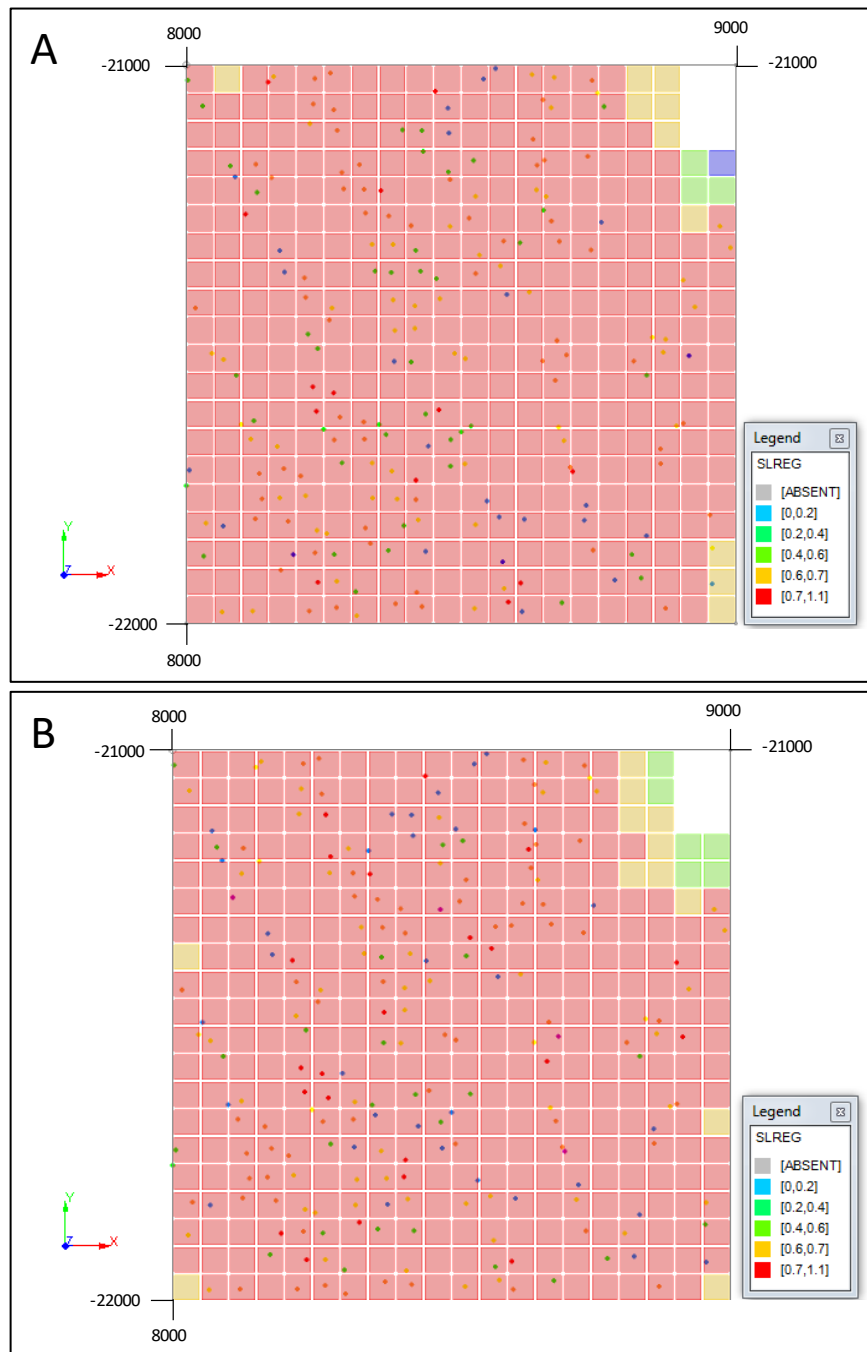


Figure 5.14: Kriging slope of regression for declustered GS-Pred data (A) and assay data (B) ordinary kriged estimate estimates.

5.8. Declustered simple kriging block modelling

An IDW block model was carried out using a power of 1 and the local means would be used as an input for the simple kriging block model. The simple kriging produced a block model that was substantially more variable in estimated values than the model

produced by the ordinary kriging (Figure 5.15). Most blocks reflected the value of samples they contained.

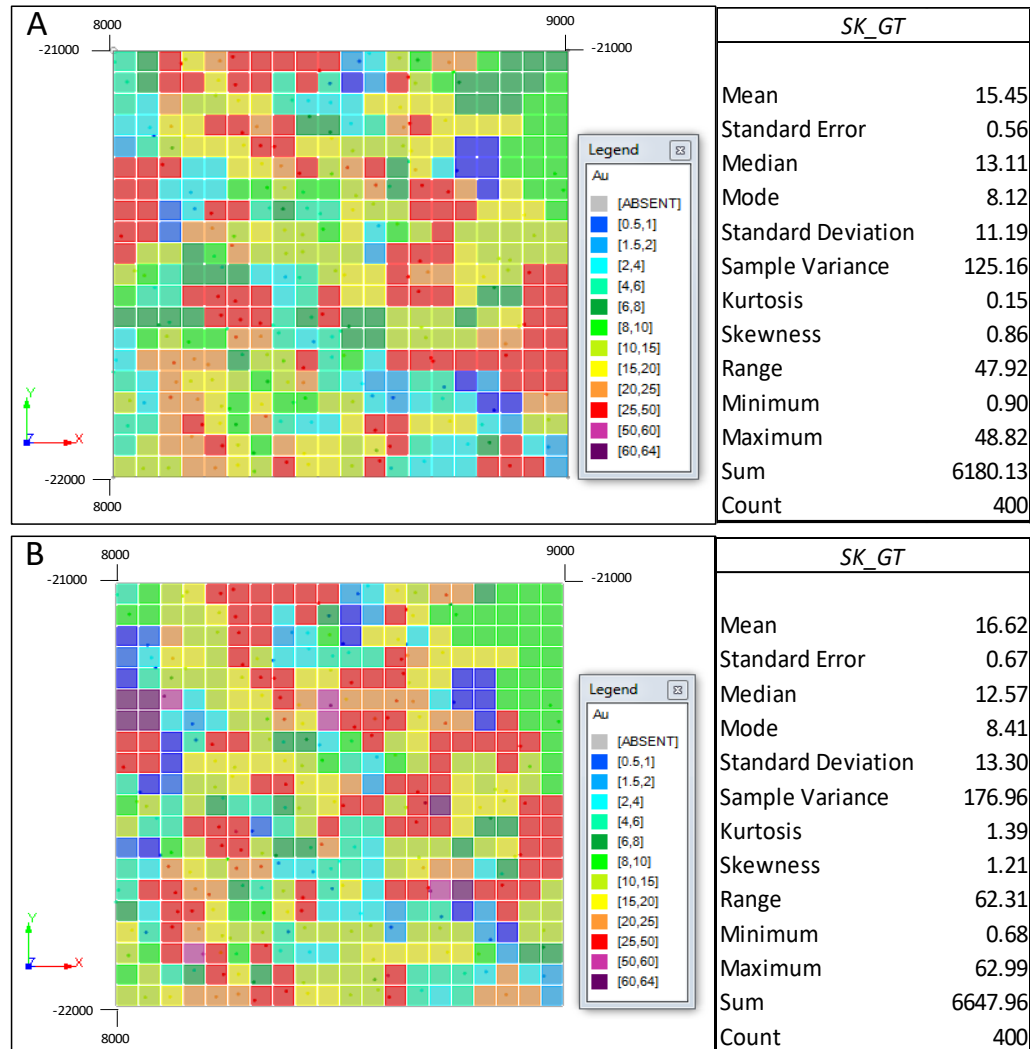


Figure 5.15: Simple kriged estimate block model for declustered GS-Pred data (A) and assay data (B) with corresponding gold statistics.

5.9. Composite declustered ordinary and simple kriged estimate model and validation

Due to the high kriging slope of regression across almost the entire study area, the composite kriging had more blocks derived from ordinary kriging and, thus, relatively lower variability between the block values compared to blocks estimated by simple kriging (Figure 5.16).

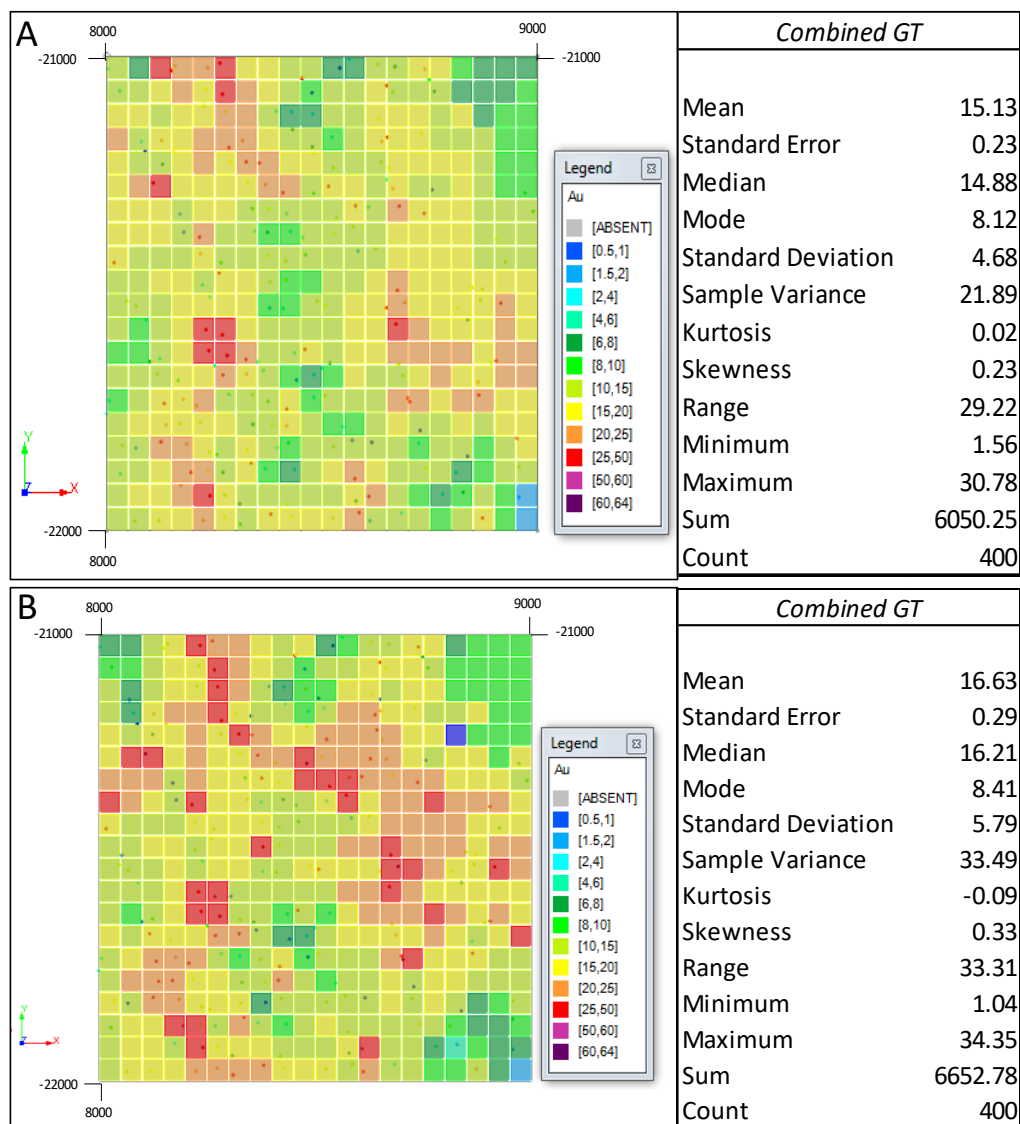


Figure 5.16: Composite ordinary and simple kriged estimate block models for declustered GS-Pred data (A) and assay data (B) with corresponding Au statistics.

The kriging efficiency improved overall for both datasets after the declustering (Figure 5.17). The kriging slope of regression also improved.

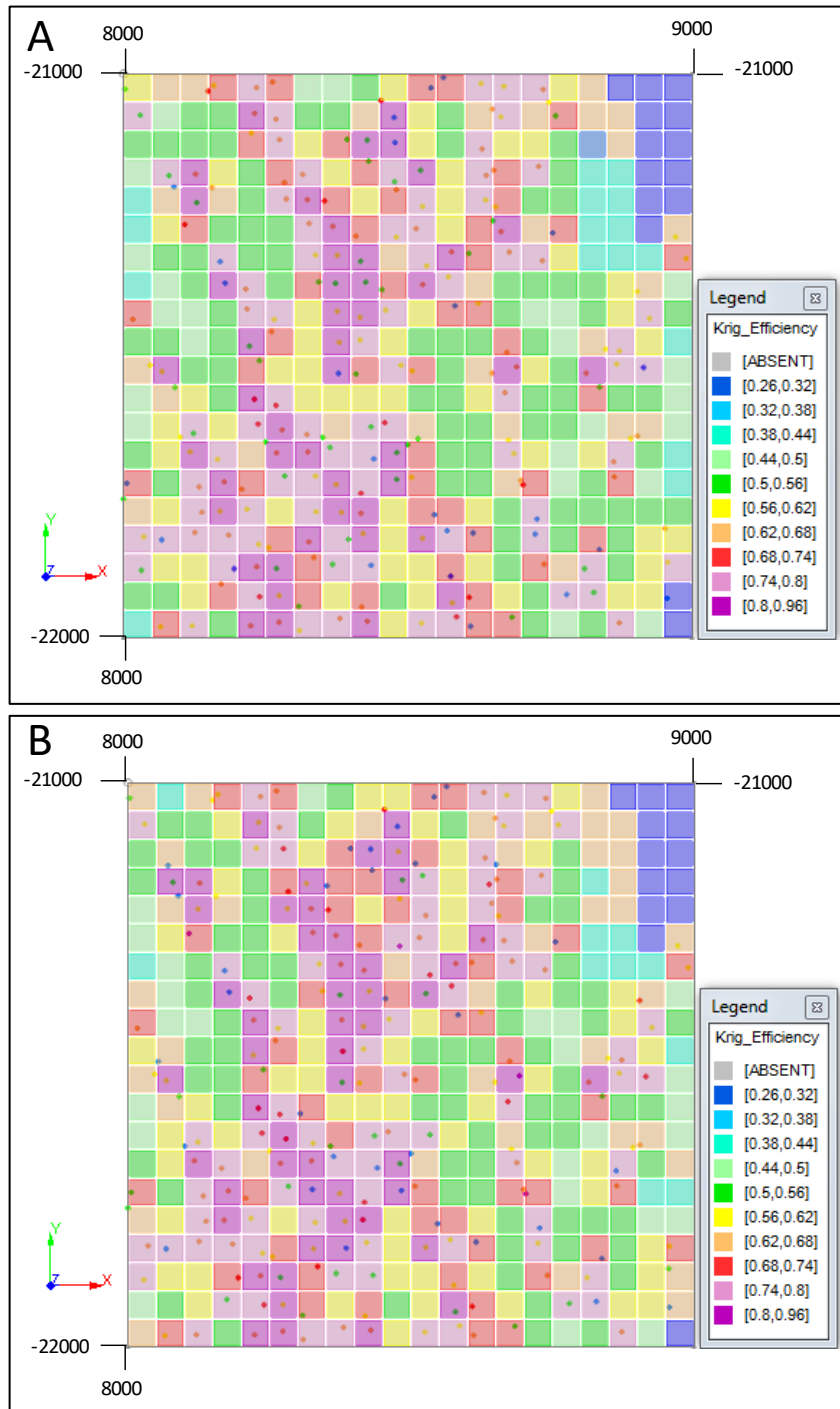


Figure 5.17: Kriging efficiency for composite ordinary and simple kriged estimate block models for declustered GS-Pred data (A) and assay data (B).

The comparison of the kriging slope of regression and kriging efficiencies on a global scale were summarised in a box and whisker plot (Figure 5.18).



Figure 5.18: Box and whisker summary of the kriging efficiencies and kriging slopes of regression of the estimations of declustered GS-Pred generated and assay data.

Swath plots comparing the different datasets were produced (Figure 5.19). The fit of the estimates to the average samples improved for GS-Pred on the declustered data, with the bias reduced. The smoothing effect of kriging is observed in the estimates of both the GS-pred data as well as the Assay data estimates. The assay data estimates do, however, show less bias than the GS-Pred estimates.

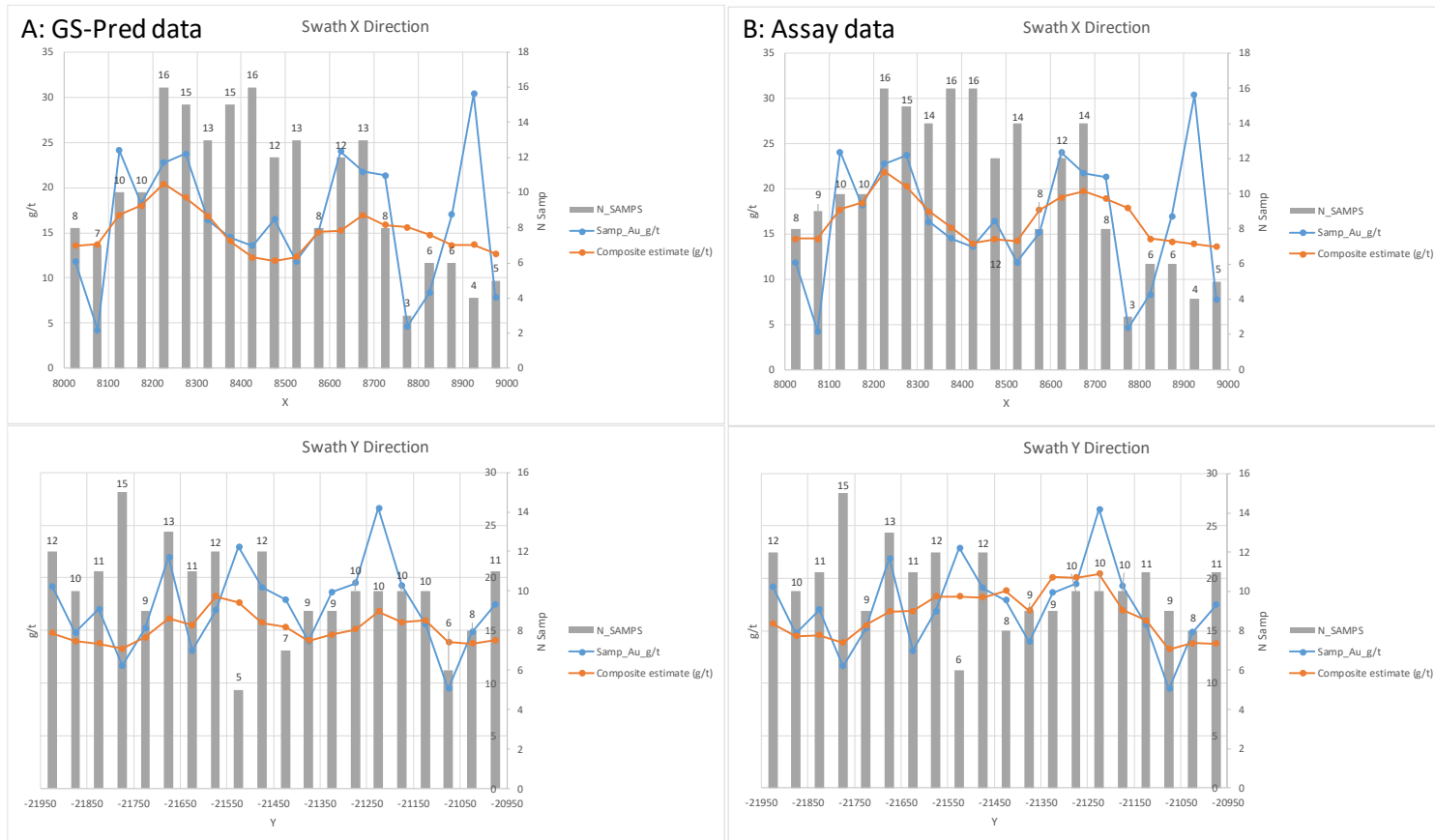


Figure 5.19: Swath plot for the composite ordinary and simple kriged estimate block models and the sample data for declustered GS-Pred data (A) and assay data (B).

5.10. Summary of various estimation methods

Overall, the effect of declustering decreased the estimation variance (Figure 5.20). The declustered estimates of the GS-Pred data indicated a lower variance than that of the assay data, as well as a lower standard error. This observation is expected as the declustered data had a lower variability.

	50m x50m Assay IDW (g/t)	50m x50m Assay OK-SK Combined (g/t)	50m x50m GS Pred IDW (g/t)	50m x50m GS Pred OK-SK Combined (g/t)	50m x50m Declustered Assay Pred IDW (g/t)	50m x50m Declustered Assay Combined OK-SK (g/t)	50m x50m Declustered GS Pred IDW (g/t)	50m x50m Declustered GS Pred OK-SK Combined(g/t)
Mean	16.72	17.31	15.43	14.81	16.72	16.63	15.50	15.13
Skewness	1.21	0.43	1.18	2.05	1.21	0.32	0.85	0.23
Kurtosis	1.20	0.11	0.27	7.36	1.20	-0.10	-0.01	0.01
Variance	212.60	40.37	207.70	54.30	212.60	33.41	145.20	21.84
STDev	14.58	6.35	14.41	7.37	14.58	5.78	12.05	4.67
Standard error	0.73	0.32	0.72	0.37	0.73	0.29	0.60	0.23
Min	0.17	3.38	0.41	1.22	0.17	1.04	0.84	1.56
Max	63.01	39.98	48.83	48.83	63.01	34.35	48.83	30.78
Range	62.84	36.60	48.42	47.61	62.84	33.31	47.99	29.22
Coefficient of variation	0.87	0.37	0.93	0.50	0.87	0.35	0.78	0.31

Figure 5.20: Statistical summary of the various estimations performed.

6. Discussion

The variography carried out in this study indicated the highest continuity in the direction 112.5° (east-south-east), which differs from the 165° (south-south-east) observed by Jolly. This may be due to localised changes or a function of new information resulting in a change in the interpretation of geology. From the observations and comparisons made in Chapter 5, the GS-Pred-generated dataset produced estimates that were comparable to those of the assay results for the purpose of block modelling.

Bias was observed, mostly for the data estimations that took place before declustering, as seen on the swath plots. If the GS-Pred data represent the true deposit characteristics, there will be slight bias towards certain directions. Additionally, the reason for the increased bias in the X-direction seen in both datasets compared to the Y-direction is due to there being more sample data in the latter than in the former and the sample alignment to the y direction.

The GS-Pred data are produced based on sedimentological observations and the reference dataset, meaning that each datum had been derived from higher dimensional observations. Declustering of both datasets enhanced the quality of the estimates further, as evidenced by the improved kriging slopes of regression and kriging efficiencies and the reduction in the bias observed in the swath plots. For both the declustered and original data, the GS-Pred dataset produced estimates that had more blocks with a kriging slope of regression of 0.7 and higher. A limitation of this comparison is the fact that anisotropy was not taken into account and the lower kriging efficiencies and kriging slope of regression for the assay data may be an artefact of this limitation. However, anisotropy could also be taken into account in an expanded study for the datasets and a thorough comparison should be performed on an equal basis.

Since reference data containing sedimentology was used to generate the GS-Pred dataset, the results do not capture gold remobilized and/or re-concentrated by post-depositional events. However, if data capturing the effects of the post-depositional events were available, GS-Pred is capable of incorporating quality enriched data containing, for example, petrography and geochemistry, and automatically discard and/or attenuate unusable dimensions of through cross-validation on a per-point basis. This flexibility is a huge advantage of machine learning algorithms. In the instance of

the Middelvlei Reef, where there has been a substantial amount of evidence to suggest a strong influence of sedimentological parameters on gold grade, this observation may be less significant than on an orebody that may have been considerably altered and a consequent re-mobilisation of gold has taken place. It would be interesting to apply the algorithm on other gold-bearing conglomerates having taken alteration effects into account and possibly incorporating them into the data.

Another topic of contention is the recording of mostly qualitative data. The recording of certain parameters, such as matrix colour and clast roundness may be subject to great variability as its observations are largely subjective and open to deliberation, having an impact on the quality of the sedimentological data being incorporated into the estimation. It is, therefore, paramount that rigorous metrology of geology be established in the environment in which such an integrated estimation approach is carried out. In conventional estimation, where the only variable considered is assay values, there is an established emphasis on ensuring the quality of the data that is used through quality assurance and quality control (QAQC). An integrated method requires a new metrological framework and potentially more meticulousness because of the increased number of variables that are introduced into the estimation process, each one requiring acceptable quality for a reliable estimation to be achieved.

A limitation to which sedimentological data are captured exists, in that descriptions of parameters that exist in three dimensions, such as clast roundness and packing, are often described and represented in two dimensions. Therefore, the extent of these limitations and their impact on the estimation process need to be fully understood. Another facet of the limitations of recording such data is that there are often vertical variations in the appearance of these parameters. Mineralisation might be higher towards the bottom of a reef horizon compared to the top, likewise with clast size and packing. The manner in which these observations are made and recorded needs to be consistent, across underground and borehole sampling, and across the personnel responsible for the capturing of this data. This is essential if the integrity of the sedimentological database is to be preserved and used for machine learning-type of applications. Rigorous QA-QC measures exist for the collection of assay data and an adjacent framework for sedimentological data needs to be established to facilitate the adoption of new technology in the mining industry. By extension, if GS-Pred or other

machine learning algorithms were to generalise knowledge across physically segregated locations, such as two isolated mines, then an interoperable data standard, including metadata standards must be established to ensure prediction validity and software compatibility across various locales.

The advantages and disadvantages of using either dataset have been summarised (Table 6.1). The decision to use either will depend on these comparisons.

Table 6.1: Comparative advantages and disadvantages of using assay data and GS-Pred data in the resource estimation process

GS-Pred Advantages	GS-Pred Disadvantages	Assay Advantages	Assay Disadvantages
Can predict into unknown areas using historical data	Quality of input data requires higher scrutiny	Only a single set of attributes/ assay data needs be considered. Lower potential for input and quality errors	Limited dimensionality of using only assay values, missed opportunity of using acquired sedimentological data
Higher dimensionality to data points	Application of algorithms, without a dedicated graphical software front-end may be too specialised for operation personnel, lack of knowledge of the theory	Simpler to apply in estimation processes in the operational environment	Estimation is based on assay data only, ignores potentially associated geology in a geozone because geozones are identified largely on thickness and grades only, and, in some cases, on the presence of structural discontinuities

		Can estimate into unknown areas using historical data	
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7. Conclusion and recommendations

This study compared the results of block-modelling techniques on point data that are generated by two different approaches. The block modelling and point data are commonly used for resource estimation in the mining environment. One approach, pioneered by Nwaila et al. (2019) is GS-Pred, which uses a machine learning-based algorithm to predict metal grades. The other conventional approach uses mostly only assay data, reef widths and geozones. The data employed in this study covers an undisclosed mine area of $\sim 1 \text{ km}^2$ from the Driefontein Gold Mine, located in the Carletonville goldfield of the Witwatersrand Supergroup and owned by Sibanye Stillwater Limited. Contained within the data are assay values, geospatial coordinates of the samples and geological descriptions of the samples, such as basal contact and percentage conglomerate. This data was used to produce block models and predicted gold grade data via GS-Pred, which produced predicted point- data that are also used to produce block models for resource estimation. Swath plots for the GS-Pred data, especially where the number of samples is relatively higher, indicate that the predicted points can be used for estimation. The swath plots for the predicted points can be expected to improve where there is more consistent and uniform geological data.

A premise of predicting gold grades from sedimentological descriptions is that historical data can be used to deduce grade distribution in areas that are under-sampled, leveraging the strong relationship between sedimentology and gold grade. The application of machine learning facilitates this process. As this study is exploratory and novel, it highlighted issues related to experimentally applying a methodology without prior verification of the assumptions. However, the results show that, for the case study using the provided dataset, GS-Pred generates reasonable quality data compared to the reference assay-based approach that can be used in traditional resource estimation-types of block modelling that are common in the mining industry.

On the operational level, there are often standards that outline the methodology of the recording of geological descriptions. This study recommends that the adherence to existing standards, and perhaps to enhanced standards in light of the adoption of machine learning, be safeguarded for the collection of qualitative data, in order to ensure consistency and, consequently, a more robust geological data framework. Such

a framework is a foundational pillar for the adoption of advanced analytics, including machine learning in the industry and academia. However, there is a need for increased and perhaps manual validation to confirm the validity of assumptions through the data and analytics pipeline, and thereby enhance the quality of output from GS-Pred. Additionally, it would be of interest to record and incorporate post-depositional descriptive data, such as alteration. In locations with a strong alteration influence, the delineation of facies and prediction of point-wise gold grades should be greatly enhanced.

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