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ESSAYS ON SPECULATIVE BUBBLES IN FINANCIAL MARKETS

by

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A Thesis Submitted to the Faculty of Commerce, Law and Management of the University of the Witwatersrand, in the School of Economic and Business Sciences

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Doctor of Philosophy

in

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SUPERVISOR: Professor Christopher Malikane

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Abstract

Essays on Speculative Bubbles in Financial Markets

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The first essay formulates a dynamic rational contagion model in order to analyse the evolution of speculative bubbles. The model consists of two laws of motion: the speculative bubble and the probability of the bubble. The first essay shows that the model has two stable equilibria and one unstable equilibrium. The dynamics of both the nonlinear speculative bubbles and the probability interact to form two stable equilibria and one unstable equilibrium which lead to ballooning and busting of the speculative bubbles. These features of speculative bubbles are driven by the speculators's herd behaviour, the bubbles size, the speed of change, the strength of infection, and the effects of both the bubbles and the short-term interest rate on the transition probability.

The second essay extracts speculative bubbles from two financial markets: the foreign exchange and the stock markets for South Africa between 1995Q2 and 2008Q4. The second essay uses the no-arbitrage models for the exchange rate and the stock price. By invoking the rational bubbles theory and using the residuals, we compute the asset price bubbles using the expectational restriction for rational bubbles theory. Three robustness checks on the computed bubbles confirm that speculative bubbles are present in the stock price and the exchange rate. By using

graphs of speculative bubbles, we show that the speculative bubbles are consistent with the existence of bubble episodes as documented in the literature.

The third essay formulates a macro-model of a small-open economy in order to investigate the relative performance of optimal monetary policy rules that respond to speculative bubbles and those that do not. The model consists of two nonlinear speculative bubbles: the stock price and the exchange rate bubbles. These speculative bubbles interact with the IS curve, the Phillips curve and the asset prices. The findings show that policy rules that respond to speculative bubbles dominate rules that do not.

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Lastly, I offer my best regards and God's blessings to all those who supported me in any respect during the course of my PhD programme.

Declaration

The work submitted in this Thesis is the result of my own investigation, except where otherwise stated. It has not already been accepted for any degree, and is also not being concurrently submitted for any other degree.

Signed by Candidate:.....

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Date: October 18, 2011

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Introduction

1.1 Brief Outline of the Topic

The thesis title constitutes three essays on speculative bubbles in the South African stock and foreign currency markets during the period of 1995Q2 through 2008Q4. Specifically, the thesis first formulates a rational contagion analytical framework in order to explain how speculative bubbles rise and bust in financial markets. Secondly, the thesis extracts speculative bubbles from the overall stock market index and a bilateral exchange rate in order to study time-series properties and understand their nature. Thirdly, the thesis formulates a macro-model of a small-open economy in order to investigate the relative performance of optimal monetary policy rules that respond to speculative bubbles and those that do not.

This thesis identifies three gaps in the literature. The first gap is that the existing rational contagion frameworks that explain why speculative bubbles rise and bust in financial markets do not use speculative bubbles and endogenous probability functions. The second gap is that the existing literature on extraction of speculative bubbles for South Africa do not use the expectational restriction under rational bubbles theory. The third gap is that the existing literature for South Africa does not test whether optimal monetary policy rules that respond to speculative bubbles dominate those that do not.

The thesis addresses the three gaps in three essays. The first essay addresses the first gap by formulating a contagion analytical framework. The essay extends Orlean's (1989) rational bubbles theoretical framework by using Lux's (1995) pure contagion model. The

essay then extends Lux's (1995) model by using speculative bubbles and the endogenous probability function from Semmler and Zhang (2007). Filardo (2004) forms the basis for justifying the use of the speculative bubble as an indicator of investors's responses to asset price changes. The probability function is used to incorporate transitions from pessimism to optimism and vice versa into the contagion model (Lux (1995)). The model has two stable equilibria and one unstable equilibrium. The dynamics of both the non-linear speculative bubbles and the probability interact to form two stable equilibria and one unstable equilibrium which lead to ballooning and busting of speculative bubbles. These dynamics are driven by speculators's herd behaviour, the bubbles size, the speed of change, the strength of infection, and the effects of both the bubbles and the short term interest rate on transition probability.

The second essay addresses the second gap by extracting speculative bubbles from two financial markets: - the stock and the foreign exchange markets for South Africa between 1995Q2 and 2008Q4. The second essay uses the no-arbitrage models for the exchange rate and the stock price (Gordon (1959, 1962), Isard (2006), Blanchard (2009)). The essay invokes the rational bubbles theory (Blanchard and Watson (1982)) in order to use the expectational restriction for computing speculative bubbles. The unit root tests and the Granger causality tests show that the extracted bubbles are extraneously determined. The graphical analysis of the speculative bubbles indicate consistency with the existence of bubbles episodes as documented in the literature. Thus, the presence of speculative bubbles is an indication that asset prices are disconnected from their fundamental values. Addition-

ally, autoregression analysis indicates the existence of weak herd behaviour in the financial markets for South Africa.

The third essay handles the third gap. The essay formulates a macro-model of a small-open economy in order to investigate the relative performance of optimal monetary policy rules that respond to speculative bubbles against those that do not. The model consists of two nonlinear speculative bubbles: the stock price and the exchange rate bubbles (Semmler and Zhang (2007)). The speculative bubbles interact with the IS curve, the Phillips curve (Ball (1998); Svensson (1998)) and the asset prices (Smets (1997); Svensson (1998); Wu and Xiao (2008)). The essay finds that optimal rules that respond to speculative bubbles dominate those that do not.

1.2 Motivation of the Study

The study uses the three gaps in the literature as motivation. Firstly, from the theoretical view, the literature uses contagion models to explain why speculative bubbles balloon and bust, but it is important to examine their dynamics when these models explicitly use them as endogenous variables and where the transition probability are a function of both interest rate and bubbles. Secondly, from the empirics perspective, it is valuable to evaluate speculative bubbles which are generated by the expectational restriction in the South African context. Thirdly, from both the theoretical and empirical perspectives, optimal policy rules for South Africa have concentrated on evaluating responses to overall asset prices, but it is worthy examining responses to speculative bubbles.

Speculative bubbles can bust at any time and therefore they need considerable theoretical and empirical analyses, required for policy implications. The three questions that the study seeks to answer include: Firstly, how are speculative bubbles formed and how do they balloon and bust? Secondly, how can speculative bubbles be identified and extracted and what are their properties? Thirdly, does responding to speculative bubbles by the central bank improve macroeconomic stability or not?

1.3 Objectives of the Study

The study has three objectives, which address the above three questions.

1. To explain why speculative bubbles balloon and bust. This objective addresses the first question. The aim here is to formally define speculative bubbles and then derive factors that drive speculative bubbles in steady state. To do this thesis use the rational contagion analytical framework.

2. To extract speculative bubbles from the stock price and the bilateral nominal exchange rate between the United States dollar and the South African rand and study their nature and properties. This objective addresses the second question. The thesis uses the expectational restriction under rational bubbles theory for extraction.

3. To investigate whether optimal rules that respond to speculative bubbles dominate those that do not, in macroeconomic stabilization. The third objective addresses the third question. The thesis calculates the optimal monetary policy rules and estimate the variances for real output gap and inflation in order to determine which rule

stabilizes the economy better. The thesis also compares the estimates of the welfare losses for optimal rules that respond to speculative bubbles and those that do not.

1.4 Contributions of the Study

There are three contributions. The first essay derives endogenous speculative bubbles and includes the interest rate in an endogenous transition probability of bubbles in a rational contagion framework. The second essay extracts speculative bubbles in the South African stock and foreign exchange markets by using the expectational restriction. In the third essay, the thesis uses the extracted speculative bubbles in a derived optimal monetary policy rule and examines whether responding to speculative bubbles dominate those optimal rules that do not, using South African data.

1.5 Literature Review

Chapter 1 constitutes the first essay. There are three controversies in interpreting the behavioural view to the existence of speculative bubbles. Orlean (1989), Lux (1995), and Calvo and Mendoza (1997) provide the first interpretation which favours using the case of multiple equilibria, which implies the presence of contagious behaviour among investors . In the first interpretation, speculative bubbles are a manifestation of differences in valuations by optimists and pessimists. Within this group are those who favour using the assumption of perfectly rational behaviour (Orlean (1989), Dupuy (1989), Banerjee (1992) and those who prefer the irrational behavioural assumption (Lux (1995), Shiller (2005)). Calvo and

Mendoza (1997), and Bernanke and Gertler (1999), among others, favour a second interpretation which posits that regulatory changes or economic liberalization lead to behavioural change among investors. This second interpretation indicates that speculative bubbles are a clear indication of monetary induced booms such as reductions in interest rates. French (2009) gives the third interpretation that contagion is transmitted internally by actions of investors within a market due to increases in money supply and the Austrian malinvestment theory. In the third interpretation, speculative bubbles are an ultimate manifestation of increases in money supply and malinvestments. This study favours a mixture of the three interpretations of contagion.

Chapter 2 forms the second essay. The South African literature posits that there are two main approaches to extracting speculative bubbles. There are those who favour using the rational bubbles theory on one hand and those who favour using econophysics theory. Zhou and Sornette (2009) favour the econophysics approach. Those who favour using the rational bubbles are divided. Yang (2006) uses the value frontier framework to measure speculative bubbles. Ahmed, Rosser, and Uppal (2010) use a mixture of rational bubbles and fractal analysis to test for speculative bubbles. There is no evidence that the expectational restriction has been used for South African data. Global literature that have applied the expectational restriction include Semmler and Zhang (2007).

Chapter 3, which represents the third essay surveys the global and local literature. Three distinct notions exist on whether central banks should respond to speculative bubbles in their conduct of monetary policy or not. Focussing on South African literature, Aron and Muellbauer (2000), and Malikane and Semmler (2008) favour the first notion that central

banks should actively respond to asset prices. On the other hand, Parusel and Viegi (2009) favour the second notion of a reactive monetary policy response to asset prices. The third notion that favours an active monetary policy response to speculative bubbles is a new area for studies on South Africa. However, the key proponents of the third notion include Kent and Lowe (1997), Filardo (2001, 2004), Genberg (2001), Rudebusch (2005), Kontonikas and Montagnoli (2006) and Semmler and Zhang (2007). Chapter 3 favours the third notion.

1.6 Analytical Framework/Methodology

Chapter 1 uses the behavioural view in formulating the rational contagion model in order to explain why speculative bubbles balloon and bust in financial markets. Chapter 3 employs the no-arbitrage theoretical framework in formulating a rational bubbles model for extracting speculative bubbles from the stock price and the exchange rate. Chapter 3 uses a macro-model of a small-open economy in order to formulate an optimal monetary policy rule that responds to speculative bubbles. The speculative bubbles, at the Thesis's centre stage, form the basis for each analytical framework.

Chapter 1 has one theoretical hypothesis which asserts that the stability of speculative bubbles is determined by herd behaviour, the bubble size, the interest rate, and the probability of bubbles. Thus, based on the hypothesis, the theoretical method is based on the application of the pure contagion model to the stability analysis of speculative bubbles. The first step is to endogenize the probability function so as to redefine the time paths of both the nonlinear speculative bubbles and the probability function. The study adopts Semmler

and Zhang (2007) probability function in order to introduce regulatory practice effects on the time paths of bubbles and their probabilities.

Chapter 2 tests the empirical hypothesis that speculative bubbles do not exist in both the stock price and the exchange rate in South Africa. The empirical methodology to the extraction of speculative bubbles uses residual based cointegration to estimate the residuals. The estimated residuals are used in the expectational restriction to compute the speculative bubbles. Semi-parametric residuals based tests are compared to parametric tests before extracting bubbles. The reason for this approach is that speculative bubbles are unobserved and therefore are part of the error term in a fundamentals asset price model. Speculative bubbles exist, if cointegration is rejected and vice versa. The standard unit root tests, the autoregressive order process (Flood and Garber (1980)) and the Granger causality tests (Granger (1969)) are employed as robustness tests on the extracted speculative bubbles. Speculative bubbles should be non-stationary in order to pass the first test of a speculative bubble. The second check implies that speculative bubbles should have an autoregressive structure. The third test is that the asset prices should Granger cause their market fundamentals, in order for speculative bubbles to exist.

The empirical hypothesis for Chapter 3 asserts that the optimal monetary policy rule that responds to speculative bubbles dominates the one that does not. The estimation method begins with determining the parameters of the structural model: one with speculative bubbles and the other without. The second step computes the parameters in the optimal monetary policy. Thirdly, the study generates the variances of inflation and real

output gap and the loss function value. Lastly, the study compares which optimal rule dominates the other.

1.7 Findings of the Study

Chapter 1 shows that in steady state, speculative bubbles can be stable and unstable, driven by the herd behaviour; bubbles size; the transition probability; the interest rate; the speed of change and the bubbles sign. The equilibria conditions for speculative bubbles are unsustainable leading to balloons and busts.

Chapter 2 finds evidence for the existence of speculative bubbles in the two asset prices and these results are consistent with the literature (Yang (2006), Zhou and Sornette (2009), Bond (2010) and Ahmed et al. (2010)). The chapter identifies six currency busts around: 1996Q4, 1998Q4, 2002Q2, 2004Q1, 2006Q1 and 2008Q3 and stock market price bubble busts around 1996Q4, 1997Q4, 1998Q4, 2001Q3, 2002Q1, 2003Q2 and 2008Q4.

Chapter 3 establishes that, first, monetary policy should respond to speculative bubbles in an aggressive manner. The most responsive of all target variables in the optimal rule is real output gap followed by inflation and real exchange rate. The reaction to speculative bubbles gives a lower Taylor curve than not responding to them. Therefore, this study is in favour of those who argue that central banks should respond to speculative bubbles.

1.8 Limitations of the Study

This study defines speculative bubbles using the expectational restriction, identifies and extracts speculative bubbles using South African data only. This study can be extended by considering a large number of small-open economies. The empirical analysis in Chapters 2 and 3 can be extended to other trading partner countries and changing the analytical method to panel data analysis. The idea would be to see which country stabilizes inflation and real output gap variance the most.

The thesis is restricted to the relationship between South Africa and the United States of America. However, the foreign exchange market can be extended to other trading partner countries.

1.9 Outline of the Thesis

The thesis consists of the Introduction, three main essays, the Conclusion and Policy Implications. The Introduction presents a summary of the entire proposed study. Chapter 1 presents the first essay of the Thesis entitled "Formation of Speculative Bubbles in Financial Markets". Chapter 2 is the second essay entitled "Extraction of Speculative Bubbles in Financial Markets". Chapter 3 presents the third essay entitled "Monetary Policy and Speculative Bubbles ". Chapter 4 gives the Conclusion and Policy Implications. Each of the three essays has its own Reference section and Appendices where applicable.

References

AHMED, E.; ROSSER JR., J.B., AND UPPAL, J.Y. (2010). "Emerging Markets and Stock Market Bubbles: Nonlinear Speculation?", *Emerging Markets Finance and Trade*, Vol. 46, No. 4, July-August, pp. 23-40.

ARON, J. AND MUELLBAUER, J. (2000). *Estimating Monetary Policy Rules for South Africa,* Central Bank of Chile Working Papers, December, No. 89, pp. 1-48.

BALL, L. (1998). "Policy Rules for Open Economies", *NBER Working Paper Series*, Working paper no. 6760, October, pp.1-23.

BANERJEE, A.V. (1992). "A Simple Model of Herd Behaviour", *Quarterly Journal of Economics*, August 1992, pp. 797-817.

BERNANKE, B., AND GERTLER, M. (1999). " Monetary Policy and Asset Price Volatility", *Economic Review*, fourth quarter, 1999, Federal Reserve Bank of Kansas City, pp. 17-51.

BLANCHARD, O. (2009). *Macroeconomics*, Pearson Prentice Hall, United States of America, pp. 412-

BLANCHARD, O.J. AND WATSON, M.W., (1982). "Bubbles, Rational Expectations, and Financial Markets", *Crises in the Economic and Financial Structure,* Wachtel , P. (Ed), Lexington, Mass: Lexington Books.

BOND, P. (2010). "South Africa's Bubble Meets Boiling Urban Social Protest", *Monthly Review*, Vol., 62, No. 2.

CALVO, G.A. AND MENDOZA, E.G. (1997). Rational Herd Behaviour and the Globalization of Securities Markets. *Institute for Empirical Macroeconomics*, Discussion Paper no. 120.

DUPUY, J.P. (1989). Common Knowledge, Common Sense. *Theory and Decision,* 27, pp. 37-62.

FILARDO, A.J. (2001). "Should Monetary Policy respond to asset price bubbles? Some Experimental results", In: Kaufman, G. (ed) Asset Price bubbles: implications for monetary and regulatory policies, vol. 13, *Elsevier Science,* Amsterdam, pp. 99-123.

FILARDO, A. J. (2004).” Monetary Policy and Asset Price Bubbles: Calibrating the Monetary Policy tradeoffs”, *BIS Working Paper* No. 15.

FLOOD, R., AND GARBER, P. (1980). “Market Fundamentals vs. Price Bubbles: The First Tests”, *Journal of Political Economy*, 88, pp. 745-770.

FRENCH, D.E. (2009). *Early Speculative Bubbles and Increases in the Supply of Money*, Mises Institute, Alabama, second edition. Chapter 8.

GENBERG, H. (2001). “Asset prices, monetary policy and macroeconomic stability”, *De Economist*, 149, pp.433-53.

GORDON, M.J.(1959). "Dividends, Earnings and Stock Prices", *Review of Economics and Statistics*, 41(2), pp. 99-105.

GORDON, M.J.(1962). *The Investment, Financing, and Valuation of the Corporation*. Homewood Press, IL., R.D. Irwin.

GRANGER, C.W.J.(1969). "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods", *Econometrica*, 37, pp. 424-438.

ISARD, P. (2006). "Uncovered Interest Rate Parity", *IMF Working Paper*, wp/06/96

KENT, C. AND LOWE, P. (1997). “Asset Price Bubbles and Monetary Policy”, *Reserve Bank of Australia Research Discussion Paper*, no. 9709.

KONTONIKAS, A. AND MONTAGNOLI, A. (2006). “Optimal monetary policy and asset price misalignments”, *Scottish Journal of Political Economy*, vol. 53, no. 5, November. Pp.636-654.

LUX, T. (1995). "Herd Behaviour, Bubbles and Crashes", *The Economic Journal,* Vol. 105, No. 431, pp. 881-896.

MALIKANE, C. AND SEMMLER, W. (2008). “Asset prices, Output and Monetary Policy in a small open economy”, *Metroeconomica,* 59 (4), pp. 666-686.

ORLEAN, A. (1989). "Mimetic Contagion and Speculative Bubbles", *Theory and Decision*, 27, pp. 63-92.

PARUSEL, M. & VIEGI, N. (2009). *Economic Policy in Turbulent Times*, TIPS Discussion Papers Series. TIPS 2009, pp. 1-27.

RUDEBUSCH, G.D. (2005). “Monetary Policy and Asset Price Bubbles”, Federal Reserve Bank of San Francisco (FRBSF), *Economic Letter*, vol. 18., no. 2005, August.,pp. 1-3.

SEMMLER, W. AND ZHANG, W. (2007). “Asset price volatility and monetary policy rules: a dynamic model and empirical evidence”, *Economic Modelling,* 24, pp. 411-30.

SHILLER, R.J. (2005). *Irrational Exuberance*, Princeton University Press, Princeton, second edition, Pp. 226-

SMETS, F. (1997). “Financial asset prices and monetary policy: Theory and evidence”, *Bank for International Settlements Working Paper* no. 47, Sept.

SVENSSON, L.E.O. (1998). "Open Economy Inflation Targeting", *NBER Working Paper Series*, Working paper 6545, pp. 1-50.

WU, GUOJUN AND XIAO, Z. (2008). “Are there Speculative Bubbles in Stock Markets? Evidence from an Alternative Approach”. *Statistics and its Interface*. Volume 1, pp.307-320.

YANG, Q. (2006). *Stock Bubbles: Theory and Estimation*. PhD Thesis, Brunel University, Business School, December.

ZHOU, W. AND SORNETTE, D. (2009). "A Case Study of Speculative Financial Bubbles in the South African Stock Market 2003-2006", *Physica,* A (388), pp. 869-88.

Chapter 1
Formation of Speculative Bubbles in Financial Markets

1.1 Introduction

The essay formulates a dynamic rational contagion framework in order to explain how speculative bubbles rise and bust in financial markets. The rational contagion framework is built around a self-regulating process of infection among rational speculative investors. The transition probabilities for transiting from optimism to pessimism and vice versa, are a function of speculative bubbles and the interest rate. Positive speculative bubbles are indicators of optimistic sentiments while negative speculative bubbles are an indicator of pessimistic sentiments in the financial markets. The instability in the speculative bubble is brought about by excessive herd behaviour. However, when the herd behaviour is very low, speculative bubbles evolve in a stable manner.

One way of explaining contagion is through the behavioural idea, which posits that speculative bubbles spread like an infection from one investor or a group of investors to another. However, those who favour this idea are divided. There are three interpretations of investors' behaviour. Firstly, there are those who believe that contagion can be transmitted when rational investors take actions which lead to excessive co-movements in prices, independent of fundamentals. In this case, contagion is transmitted internally by actions of investors within a market due to increases in money supply and the Austrian malinvestment theory (French (2009)). The Austrian theory posits that increases in money supply lower interest rates below the natural rate and this leads to the malinvestment problem and speculative bubbles.

Secondly, there are those who favour using the case of multiple equilibria, which implies contagious behaviour among investors (Orlean (1989), Lux (1995), Calvo and Mendoza (1997)). Within this group are those who favour using the perfectly rational behaviour

assumption (Orlean (1989); Dupuy (1989); Banerjee (1992)) and those who favour the irrational behaviour assumption (Lux (1995), Shiller (2005)).

Thirdly, there are those who believe that changes in regulatory practices can induce investors to alter their behaviour after a crisis (Calvo and Mendoza (1997), Bernanke and Gertler (1999)). In this case, speculative bubbles are the ultimate manifestation of monetary induced booms such as reduction in interest rates. This study favours a mixture of the second and the third interpretations. However, from the third interpretation, this study follows the assumption of perfect rationality.

There are two main steps in formulating the analytical framework. The first step employs Orlean (1989) to formalize the definition for speculative bubbles as asset price movements, independent of market fundamentals. The rational bubbles theory, which is founded on the assumption of perfect rationality is used. The rational bubbles theory posits that there are two solutions to an asset price namely; the fundamental and the non-fundamental solutions. The non-fundamental solution is the one which defines the law of motion of speculative bubbles. An important feature of the non-fundamental solution is that expected speculative bubbles obey the expectational restriction, which states that the expected speculative bubbles grow and bust because of the effect of the probability of bubbles emerging, the growth rate of speculative bubbles, the size of the speculative bubbles and the residuals from the fundamental solution of the asset pricing model.

The second step extends Orlean (1989) by employing Lux's (1995) pure contagion model, based on a probabilistic approach, which postulates that probabilities should depend on the actual distribution of speculative bubbles. According to the pure contagion model, the transition probability is a function of the speculative bubble, which is an indicator of rational herd behaviour. The first extension to the Lux's (1995) model is to change the indicator of investors' behaviour by favouring the speculative bubbles. This treatment of speculative bubbles has support from Minsky (1982), Kindleberger (2000), and Filardo (2004), who posit that speculative bubbles are an indicator of the behaviour of speculative investors. The second extension to Lux's (1995) probability postulate is by extending the probability function to include the interest rate. This adjustment is consistent with Kent and Lowe (1997) and Semmler and Zhang (2007). Thus, the study uses an endogenized

probability function, which is a function of speculative bubbles and the interest rate. The endogenous probability allows macroeconomic factors, such as interest rate, to influence the process of opinion formation and therefore have an effect on the dynamics of speculative bubbles.

This study demonstrates that perceived changes in the speculative bubbles to bust can generate boom-bust cycles and thereby producing asset price movements that can be periodically stable and unstable. The structure of the probability model explains market sentiments, whereby speculative bubbles are generated by the interaction of pessimistic and optimistic speculative investors, indicating their differences in the evaluations of assets. The analytical framework shows that the law of motion of speculative bubbles is grounded in the properties of the probability distributions, interactions among speculative investors who act as infection, which leads to bubbles deviating from their stability paths and causing macroeconomic instability and poor financial intermediation. Therefore, asset prices are influenced by the strength of infection, the speculative bubbles size, the growth rate and decrease rate of speculative bubbles, the effects of both the bubbles and the interest rate on transition probabilities and the sign function of the speculative bubbles.

The rest of this essay is structured as follows. Section 2 presents the contagion analytical framework. Section 3 provides the stability analysis. Section 4 concludes.

1.2 Contagion Analytical Framework

This section considers an hypothesis that the ballooning and busting of speculative bubbles is caused by investors' behaviour. This chapter begins with developing a specification for the speculative bubbles and then link it to a pure contagion analytical framework. The rational bubbles model provides a formal definition for speculative bubbles for both the stock price and the exchange rate. We employ Orlean (1989) to explain factors underlying speculative bubbles. The relationship between speculation and liquidity, as first discussed in Keynes (1936), is the starting point to explaining speculative bubbles formation. Speculation and liquidity are two sides of the same coin.

Speculative markets exist since speculative investors can buy an asset and sell it at a later date and be able to make a capital gain or loss. As more liquidity is pumped into the financial market, speculation becomes stronger. We assume that there are two financial markets for the stocks and the exchange rate. The economy is described by two no-arbitrage models: the Gordon model (Gordon (1959, 1962)) as applied in Wu and Xiao (2008) and the uncovered interest parity as used by Isard (2006). The fundamental value of the stock price (d_t) is the present value (PV) of the future streams of dividends (d_{t+n}) over its investment horizon. And the fundamental value of the exchange rate is the interest rate differential $\left(i_t - i_t^f\right)$, which is equal to the difference between the domestic interest rate (i_t) and foreign interest rate $\left(i_t^f\right)$. These fundamental values are updated every time information becomes available up to infinity (Tirole (1985)).

The financial market model is described by the interaction between the stock price and the exchange rate, as presented below:

$$s_t = \frac{s_{t+T}}{(1+R_{t+T})^T} + \sum_{n=0}^{\infty}\left(\frac{d_{t+n}}{(1+R_{t+n})^n}\right) \tag{1.1}$$

$$e_t = e_{t+T} + \left(i_t - i_t^f\right) \tag{1.2}$$

where, s_t denotes nominal stock price at time t, e_t denotes the nominal exchange rate at time t, R_{t+T} and R_{t+n} denotes time-varying discount rate for equity and T denotes the terminal time or maturity time, where $T > 0$. Eqns. (1.1) and (1.2) are evaluations of the stock price and the exchange rate by speculative investors, respectively. An important question the thesis raises is how the fundamental values in eqns. (1.1) and (1.2) can be equal to the price levels. This is possible for the stock price, if the PV of the expected terminal price is equal to zero and the exchange rate equilibrium is possible, if the forward premium is equal to the interest rate differential. In these two cases, no speculative bubbles will exist.

Given that agents are independent and doing their own evaluations of asset prices, it will not be possible for everyone to arrive at the same evaluations for fundamentals and asset prices. Arguably, speculative investors prefer to invest in stocks when the PV of the terminal price is greater than zero, which is in line with the concept of speculation.

Additionally, speculative investors would prefer to invest in foreign currency when the forward premium is greater than the interest rate differential. There is no speculation if either the PV of the terminal stock price is equal to zero or the forward premium is equal to the interest rate differential. If we assume speculation exists, the stock price will deviate from its fundamental value and the forward premium will not be equal to the interest rate differential. This is the well known long-standing difference between the asset price and its fundamental value which has been traditionally called a "speculative bubble" (Orlean (1989)). The key question to deal with is whether it is the fundamental solution which exists or it is the speculative bubble solutions to eqns. (1.1) and (1.2) which exist.

In a financial market with the interaction of speculative investors, the difference between the two evaluations in eqns. (1.1) and (1.2) is the nature of information used in calculating the following two types of expectations:- self-reference and specular expectations. Self-reference is a process of trying to foresee which average opinion corresponds to a particular formal structure. Specularity is an essential cause of the disconnection of prices from objective information. Thus, a link between self-reference and specular expectations exists and therefore the focus is on the nature of the rational behaviour of speculative investors and economic constraints. The main constraints include; transaction costs, the degree of agent's confidence in their individual valuations of the fundamental value and prices, their willingness to take chances, and the amount of liquidity that they have on hand for dealing with an unforeseen need to make payments. However, because of human nature and the presence of constraints, speculative investors are obliged to shift from a behaviour of enterprise to the one of speculation (Orlean (1989)).

Rational bubbles theory provides a means to explaining the number of particular solutions which exist in the price evaluation process in eqns. (1.1) and (1.2). The theory uses rational expectations on the PV of the expected terminal prices as:

$$s_{t+T} = E_{T-1}\left(s_{t+T} \mid I_{t+T-1}\right) \tag{1.3}$$

$$e_{t+T} = E_{t+T-1}\left(e_{t+T} \mid A_{t+T-1}\right) \tag{1.4}$$

where, E denotes the mathematical expectation and I and A are the information sets for the stock and foreign exchange markets, respectively. Eqns. (1.3) and (1.4) indicate that

the expected prices are on average equal to the prices that are effectively realized and a self-fulfilling condition is by implication recognized in these equilibrium conditions. The self-fulfilling prophecy about expected prices does cause speculative bubbles to emerge even when agents are rational. The differences in the processing of information concerning future prices leads to differences in evaluations. This leads to a disconnection between asset prices and their objective information. The disconnection is called speculative bubbles. It is standard in the literature that speculative bubbles satisfy the following expectational restriction:

$$E_t b_{s,t+1} = \delta^{-1} b_{st}, \text{ with } \delta > 0 \tag{1.5}$$

$$E_t b_{e,t+1} = \delta^{-1} b_{et}, \text{ with } \delta > 0 \tag{1.6}$$

where, $\delta^{-1} = (1 + R_t)$ denotes the growth factors and specifically $R_t =$ required market return and it can be positive or negative. This implies that many financial episodes where bubbles were observed and interpreted as resulting from irrational behaviour (Shiller (1981, 2005) now can also result from rational behaviour, under rational bubbles theory (see Blanchard and Watson (1982)). Eqns. (1.5) and (1.6) show that speculative bubbles are independent of the fundamentals and there are possibilities that they can have different forms. Furthermore, eqns. (1.5) and eqn. (1.6) imply that speculative bubbles generally grow without any connection to fundamentals and this type of speculative bubble is known as a deterministic bubble (West (1987)).

Most importantly, one of the motivations for holding a speculative asset experiencing a price bubble is the expectation that the price will continue to rise. In the case where an independently and identically distributed (IID) error term also determines the evolution of the bubble, the bubble would also be driven stochastically by an extraneous factor unrelated to economic fundamentals. Borrowing from Blanchard and Watson (1982) and Orlean (1989), one of the different forms for speculative bubbles is:

$$b_{j,t+1} = \begin{cases} \delta^{-1} b_{j,t} + \varepsilon_{t+1}, & \text{with probability } p_j \\ \varepsilon_{t+1}, & \text{with probability } (1 - p_j) \end{cases} \tag{1.7}$$

where, j denotes either s, or e; p is the probability of a speculative bubble increasing and $(1 - p)$ is the probability of the speculative bubble crashing and ε is an IID noise term with

mean zero and constant variance. These bubbles seem more realistic because they can start, crash and start up again over and over (Blanchard and Watson(1982)). The rational bubble in eqn. (1.7) obeys the expectational restrictions defined by eqns.(1.5) and (1.6). Eqn. (1.7) illustrates stochastic speculative bubbles.

The expectational restriction condition shows that speculative bubbles can perfectly arise as a result of the operators having highly rational attitudes. The most important implication of the rational bubbles theory is that speculation may not be considered as something that is necessarily desirable from the economic system's efficiency point of view, but that speculation can lead to a disconnection between asset prices and fundamental values. This disconnection in turn may lead to a faulty overall allocation of factors of production, what the Austrian business cycle theory refers to as malinvestment (French (2009)). The existence of eqns. (1.5) and (1.6) formalizes the disconnection between asset prices and fundamental values. Eqns. (1.5), (1.6) and (1.7) describe a process of pure self-validation taking place independently of fundamentals. This expectational restriction reinforces Keynes's (1936) long established view which has since dispelled the notion of incompatibility between rationality and speculative bubbles. Therefore, it is the very rationality of speculative investors, in their desire to make the best possible use of the market's constraints, that leads mechanically to the emergence of speculative bubbles (Orlean (1989)).

This thesis has shown that eqn. (1.7) is the mathematical definition of speculative bubbles. We now formalize eqn. (1.7) into a contagion analytical framework. An economy is described by two types of rational speculative investors:- the optimists are those with bullish sentiments and pessimists are those with bearish sentiments about the market price. Under the assumption of perfect rationality, speculative bubbles are known by speculative investors. The speculative bubbles (b) are given in eqn. (1.7). It follows that $b = 0$ corresponds to a situation where the number of optimists is equal to the pessimists. This is the case of no contagion. Therefore, situations of $b \gtrless 0$ exhibit more or less predominant optimism or pessimism. This is the moderate case of contagion. In extreme cases, $b = 1$ or $b = -1$. In these cases, all speculative investors would have the same opinion and either everyone is selling or is buying. In this case, contagion is very strong. This description implies that speculative bubbles are bounded between 1 and -1. The bounding of the speculative bubble, is supported by Lux (1995).

Contagion is the infection of attitudes, with a high portion of optimistic rational speculators changing the attitudes of the smaller number of pessimistic speculators, and vice versa. Contagion theory postulates that there exists two types of transition probabilities: The transition probability from optimism to pessimism $\left(\rho_{-+}\right)$ and the transition probability from pessimism to optimism $\left(\rho_{+-}\right)$. In the presence of contagion, both transition probabilities should depend on the actual level of speculative bubbles;

$$\rho_{-+} = \rho_{-+}(b), \quad \rho_{+-} = \rho_{+-}(b) \tag{1.8}$$

where, ρ_{+-} implies that speculators are upgrading asset values and ρ_{-+} implies that speculators are downgrading asset values. Thus, ρ_{+-} is the probability that optimists are expecting bubbles to rise and ρ_{-+} is the probability that the optimists are expecting bubbles to fall. Rational speculative bubbles can rise and burst and therefore they can be positive or negative (b_+, b_-).

Under rational bubbles theory, the evolution of the speculative bubble in period t+1 is determined by the current period bubble, the growth rate of the bubbles and the level of shocks, given the probability of the bubble increasing (Blanchard and Watson (1982)). This expectational restriction is shown in eqn. (1.7). By adding the positive and negative speculative bubble states, gives:

$$E_t b_{t+1} = \delta^{-1} \rho\, b_t = (1 + R_t)\, \rho\,(b_t) \tag{1.9}$$

Eqn. (1.9) is the mean value equation for the original speculative bubble based on the expectational restriction. This thesis confines the analysis to mean values in order to neglect intrinsic dynamics of variances and higher moments. Doing so is sufficient in order to determine the most probable development from any initial state (Lux (1995)). Suppressing the time subscripts in eqn. (1.9) transforms the mean-value equation into a dynamic one for the change in time of the speculative bubble as:

$$\frac{db}{dt} = (1 + R)\left(\rho_{+-}(b) - \rho_{-+}(b)\right) b \tag{1.10}$$

where, $\rho = \left(\rho_{+-}(b) - \rho_{-+}(b)\right)$. Eqn. (1.10) is consistent with Lux (1995). However, the dynamics of the speculative bubbles in eqn.(1.10) are determined by their growth rates or decrease rates (R), the difference between the transition probabilities and the speculative

bubble size. Thus, all transition probabilities have to be positive by definition; and to grasp the rational contagion idea, the probability for a transition from pessimistic to optimistic attitude is assumed to be larger than in the opposite direction if the prevailing disposition about the asset price is already optimistic. It seems reasonable to assume that the relative change in the probability to switch from one state to another is determined by the change in bubbles and the strength of infection (Lux (1995)). Algebraically, this is written as:

$$\frac{d\rho_{+\text{-}}}{\rho_{+\text{-}}} = a\ db; \quad \frac{d\rho_{-+}}{\rho_{-+}} = -a\ db \tag{1.11}$$

where, eqn.(1.11) implies states that the relative change in the probability to switch from pessimism to optimism increases linearly with changes in the rational speculative bubble db, and vice versa; and a is the measure of strength of infection or rational herding behaviour (Lux (1995)). These assumptions suggest that the probability switching is determined by the speed of change (v), strength of infection (a) and the level of bubble (b):

$$\rho_{+\text{-}}(b) = v(\exp\{ab\}), \quad \rho_{-+}(b) = v(\exp\{-ab\}) \tag{1.12}$$

Eqn. (1.12) shows the probability function for optimists and pessimists, respectively. Eqn.(1.12) allows for switches even in the presence of market efficiency $(b = 0)$ since; $\rho_{+\text{-}} = \rho_{-+} = v > 0$. This implies that changes in the speculative bubble occurs due to the differences in information possessed by speculators. Thus, the time development of the mean-value of the speculative bubbles defined in eqn.(1.10) becomes;

$$\frac{db}{dt} = bv(1+R)(\exp\{ab\}) - bv(\exp\{-ab\}) \tag{1.13}$$

Using the De Moivre's theorem, the Euler's eqn. (1.13) is solved for sinh and cosh function as (Weidlich and Haag (1983)) (see Appendix A.A for derivation);

$$\dot{b} = v\left[2+R\right](b\sinh ab - b\cosh ab) \tag{1.14}$$

Eqn. (1.14) uses the standard sine-cosine formula used by Lux (1995) which implies that the evolution of the time path of the speculative bubbles follows the hyperbolic sine and cosine. Thus, the time path of the speculative bubbles in eqn.(1.14) can be expressed using $tanh$ as (Weidlich and Haag (1983)):

$$\dot{b} = v\,[2 + R]\,(b\tanh(ab) - b)\cosh(ab) \tag{1.15}$$

Eqn.(1.15) is now the description of a pure rational contagion dynamics (see Appendix A.A). The outcome of this differential equation is well known from other applications in Weidlich and Haag (1983) and (Lux, 1995) of the same ideas. Eqn. (1.15) implies that the time path of the asset bubble depends on the speed of change (v), the bubbles size (b), strength of infection (a) and the discount rate (R), which is the key determinant of the growth rate of the speculative bubble. This thesis illustrates the description of pure rational contagion, in eqn. (1.15) by using Figure 2.1, which gives a basic framework for the analysis of the effects of rational herd behaviour on transition probabilities, contagion and stability conditions of speculative bubbles.

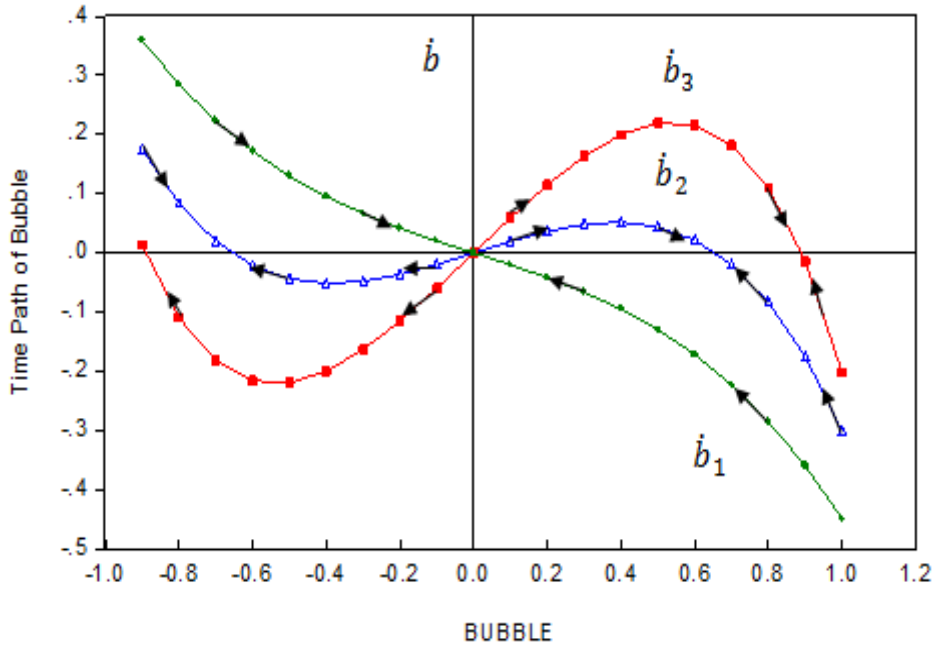

Fig. 2.1- Pure Contagion and Stability Conditions

Note: a=0.8$\left(\equiv \dot{b}_1\right)$; a=1.2$\left(\equiv \dot{b}_2\right)$; a=1.6$\left(\equiv \dot{b}_3\right)$

Source:Author

Figure 2.1 gives a basic framework for the analysis of rational herd behaviour and it is based on the equilibrium condition Tanh (ab)= b. To simplify the analysis the literature sets,$v = R = 1$ (Lux (1995)). On the vertical axis is the time path bubbles and on the horizontal axis is the size of the bubble. The stable path is when the herd effect is relatively

weak, as represented by the $\dot{b}_1$graph for $a = 0.8$. The implication for this is that since the contagion effect is weak, all deflections into one direction will die out in the course of events and the system will revert to a state of stability after some disturbance. However, if $a > 1$, the graphs are depicted by $\dot{b}_2$ (for $a = 1.2$) and $\dot{b}_3$ (for $a = 1.6$), which indicate that any small deviations from the balanced state is capable of turning a majority of speculators bullish or bearish through mutual infection. This makes their equilibrium point, $(b = 0)$ to be unstable. Any deflection from it results into a snowball-like cumulative infection process. These dynamics lead to two stable equilibria, at $b < 0$ and $b > 0$. At these positions the majority of rational speculators will either be in bullish or bearish mood, respectively, as long as the level of bubble does not increase or fall further. Therefore, these stable points are not sustainable.

1.3 Stability Analysis

We now extend the above analytical framework by assuming an endogenous probability function for the speculative bubbles. This thesis defines the behaviour of speculators by using the probability that speculators put on speculative bubbles. Thus, knowledge about the law of motion of probability functions helps in understanding the dynamics of speculative bubbles. Since the concern is with speculative bubbles which are created by the interaction of optimists and pessimists with asymmetric information, we formalize the steady-state equilibrium paths for the interaction between the probability function and the speculative bubbles.

Assuming that the agents' average expected probability of a positive speculative bubble is 0.5, this thesis extends the model by endogenizing the probability function, as suggested by Semmler and Zhang (2007). Because probabilities should be bounded, nonlinear and asymmetric, we use the bounded probability function: bounded between 0 and 1, nonlinear and asymmetric around 0. Using the primitive function concept and employing the probability function for optimistic and pessimistic investors in eqn.(1.8), we obtain the following expressions:

$$\rho_{+-}(b,r)_{t+1} = \frac{1}{2}\left(1 - \tanh\left(\omega\left(b_t, r_t\right)\right)\right) \tag{1.16}$$

and

$$\rho_{-+}(b,r)_{t+1} = \frac{1}{2}\left(1 - \tanh\left(\vartheta\left(b_t, r_t\right)\right)\right) \tag{1.17}$$

where, b_t is the level of bubble, r_t is a the interest rate. Eqns. (1.16) and (1.17) indicate the effects of the level of bubble and the interest rate on optimistic and pessimistic sentiments, respectively. This analytical framework is representative of the mixture of three theoretical sources of speculative bubbles, namely; rational psychology of speculative investors, represented by the sentiments measures; the perfect rationality of speculative investors represented by rational expectations; and the regulatory instrument represented by interest rates. The importance of this specification is that now the endogenous probability function includes a mixture of all three sources of speculative bubbles. The probability function represents either optimistic or pessimistic sentiments and it is bounded between 0 and 1 (see Appendices A.B and A.C for details).

The time path of the level of the bubble can be expressed as the difference between the asset valuations of the optimists and pessimists. Using eqn.(1.10), the time path of the level of the bubble is given as:

$$\frac{db}{dt} = (1+R)\rho_{+-}(b_{t+1}, r_{t+1}) - \rho_{-+}(b_{t+1}, r_{t+1}) \tag{1.18}$$

Eqn. (1.18) shows the time path of the level of the bubble, which is still within the pure contagion analytical framework. Using eqn.(1.15), and substituting for the speculative bubble term b , which is obtained by dividing the bubble in eqn. (1.7) by the transition probability to obtain the law of motion for the current measure of the level of the bubble as:

$$b_{t+1} = \frac{\delta}{\rho_{t+1}} b_t + \varepsilon_{t+1} - \varepsilon_{t+1} \rightarrow b_t = \frac{\rho_{t+1} b_{t+1}}{\delta}. \tag{1.19}$$

And ignoring time scripts for the time path equations and substituting b only for the $tanh$ and $cosh$ terms gives (see Appendix A.D for details);

$$\dot{b} = \left[\begin{pmatrix} \tanh(\cosh\left(a\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b,r\right)\right)\right)-\left(1-\tanh\left(\vartheta\left(b,r\right)\right)\right)\right)^2\right)) \\ -\cosh\left(\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b,r\right)\right)\right)-\left(1-\tanh\left(\vartheta\left(b,r\right)\right)\right)\right)^2\right) \end{pmatrix}\right] (2v+Rv)\left(ab^2\delta^{-2}R\right) \tag{1.20}$$

Eqn. (1.20) is the steady state of the speculative bubble and follows the definition of the hyperbolic sine and cosine function. Eqn.(1.20) defines state-dependent speculative bubbles in steady state.

Assuming that the relative change in the probability to switch from one state to another is determined by the change in speculative bubbles and the strength of infection or herd behaviour described by (eqn.(1.11)), the transition probability is rewritten as:

$$\rho_{+-} = \frac{1}{a}\frac{d\rho_{+-}}{db}; \qquad \rho_{-+} = -\frac{1}{a}\frac{d\rho_{+-}}{db} \tag{1.21}$$

thus, using eqn.(1.21) the time path of the combined transition probabilities for optimists and pessimists as the difference between their valuations is:

$$\dot{\rho} = \rho_{+-}adb - \rho_{-+}adb \tag{1.22}$$

and the financial market's time path for the transition probability from optimism to pessimism is solved in Appendix A.E as:

$$\dot{\rho} = \frac{1}{2}\left(\tanh\left(\phi_3 f_p(b) - \phi_1 f_o(b) + \left(\phi_4 - \phi_2\right) sign(b)r\right)\right) adb \tag{1.23}$$

where, $f_o(b)$ and $f_p(b)$ are the linex functions for optimists and pessimists, respectively, defined as:

$$f_o(b) = v\left[\exp\{ab\} - ab - 1\right],\ v > 0, a \neq 0 \tag{1.24}$$

$$f_p(b) = v\left[\exp\{-ab\} - ab - 1\right],\ v > 0, a \neq 0 \tag{1.25}$$

where, v scales the function, a determines the asymmetry of the function (see Appendix A.C, eqn. (A.20)). Eqn.(1.23) defines the steady state probabilities for the financial market, while eqns. (1.24) and (1.25) are consistent with the probability switching eqn. (1.12).

The speculative bubbles are treated as stochastic and can be positive or zero or negative. In order to accurately capture the influence of price changes on opinion formation, we employ the probability of bubbles. An increase in the probability of the bubbles is an indication that asset prices are expected to decrease and a decrease in the probability indicates that asset prices are expected to increase, for positive bubbles. The opposite is the case for negative bubbles. The probability function therefore guides the herd behaviour of speculative investors. The steady state price stability equilibrium is determined by (see Appendix A.E for our solution):

$$\rho = \frac{\delta}{2ab}\left(\begin{array}{c} 2\ln a + 2\ln\left(\rho + b - \delta\right) + 4\ln v + \ln\phi_1 - \ln\phi_2 - \ln\phi_3 + \ln\phi_4 \\ +2\ln sign(b)r \end{array} \right) \quad (1.26)$$

Eqn. (1.26) indicates that in steady state, asset prices can be influenced by strength of infection (a), speed of change (v), the bubble size (b), the discount factor (δ), the effects of bubbles on the probability switching (ϕ_1, ϕ_3), effects of the interest rates on probability switching (ϕ_2, ϕ_4) and the sign of bubbles $(sign\,(b))$.

Therefore, eqns.(1.20) and (1.23) are plotted in Figures 2.2 and 2.3 using simulated data, with the $\dot{\rho}_{+-} = 0$ curve sloping upwards and the $\dot{b} = 0$ curve being nonlinear. The vertical axis measures the probability of bubbles while the horizontal axis measure the bubble sizes. This model is robust in explaining the unstable nature of both deterministic and collapsible speculative bubbles. Figures 2.2 and 2.3 show the intertemporal equilibria at three points of intersection. For example, Figure 2.2 shows the stable bubble path and the stable probability for optimists.

The $\dot{b}$ graph, which describes eqn.(1.20), shows the steady state evolution of the speculative bubbles when herd behaviour is high, at $a = 2$. The $\dot{\rho}$-graph, which follows eqn.(1.23), depicts the steady state evolution of the probability of bubbles increasing as bubble sizes increase. This behaviour is depicted by deterministic bubbles. It should be noted that the condition for the existence of speculative bubbles equilibria is the same as in the pure rational contagion case and therefore the stability conditions remain the same. However, in Figure 2. 2 the unstable equilibrium, where $b = 0$ is very strong, implying the presence of strong instability.

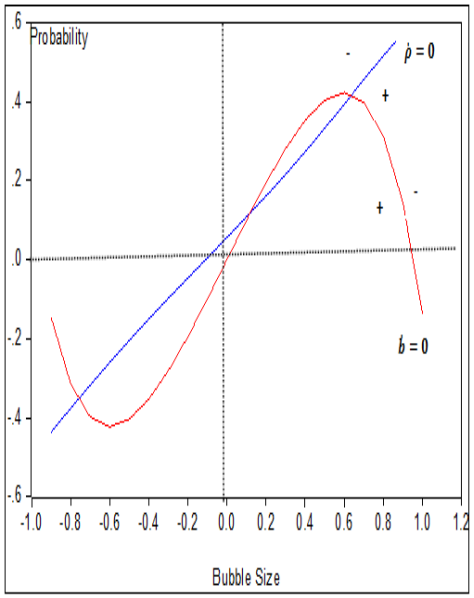

Fig.2.2- Phase Diagram I

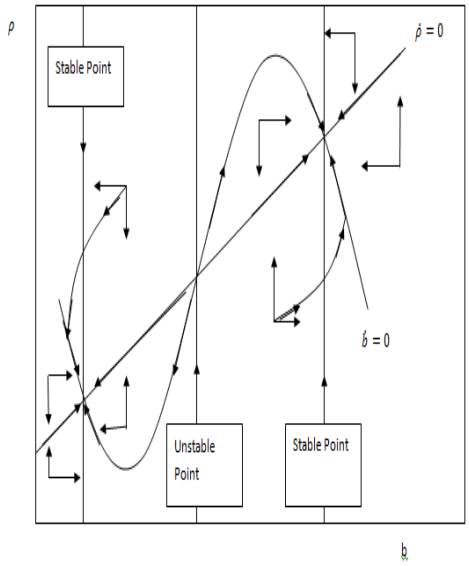

Fig. 2.3- Phase Diagram II

Source:Author

Figure 2.2 is the phase diagram indicating the steady states for the speculative bubbles and the transition probability and assuming $v = R = 1$. The figure indicates that speculative bubbles are very unstable and give rise to a possibility of multiple equilibria in contagion models. The figure reports two stable equilibria and one unstable equilibrium as shown in Figure 2.3. The vertical axis measures the probability function ρ. The stable equilibria points are at (-0.78, -0.38) and at (0.62, 0.42) while the unstable point is at (0.10, 0.11). The instabilities in the bubbles are coming from (i) the measure of strength of infection or rational herd behaviour a, (ii) the effects of speculative bubbles and interest rate on the probability function, (iii) the size of the speculative bubble and (iv) the discount rate R,or the assets' required rates of return. Thus, if the speculative bubble is between -0.78 and 0.10, it implies that the bubble will adjust towards -0.78 and if on the other hand the bubble is between 0.10 and 0.62, it will adjust towards the 0.62 stable point. Figure 2.3 shows the arrows, which indicate direction of contagion effects, with implications on the stability of asset prices. In this analysis, we postulate that the time path of the probability is a log linear trend and has support from Kent and Lowe (1997).

1.4 Conclusion

Firstly, the study develops an analytical framework for the dynamics in speculative bubbles, which have a probability of crashing and regenerating, assuming rational expectations. The rational contagion analytical framework that is developed in this essay employs a combination of features from Orlean (1989); Lux (1995); Blanchard and Watson (1982) and Semmler and Zhang (2007). Therefore, the analytical framework shows that speculative bubbles are unstable and a permanent feature of asset prices. The instabilities in the speculative bubbles positively depend on the strength of herd behaviour and whether speculative bubbles are positive or negative. The asset price equilibria when speculative bubbles exist are not sustainable.

Secondly, by including the interest rate in the probability function, the contagion model predicts that the effect of monetary policy on herd behaviour depends on the size and sign of the speculative bubble. For instance, on the one hand, if the bubble is positive, an increase in the interest rate lowers the probability that the speculative bubble increases in the next period. On the other hand, if the bubble is negative, an increase in the interest rate increases the probability that the speculative bubble balloons in the next period.

Lastly, an increase in the strength of infection generates unstable bubbles and therefore they can either balloon or collapse rationally in order to adjust the market price. Thus, this study analyses the rationality of the contagion of opinions in explaining the dynamic structure of speculative bubbles. The contagion model has all the three features of the rational behavioural view:- investors' practices, regulatory practices and multiple equilibria and these modify the Lux (1995) pure contagion model.

References

BANERJEE, A.V. (1992). "A Simple Model of Herd Behaviour", *Quarterly Journal of Economics*, August 1992, pp. 797-817.

BERNANKE, B., AND GERTLER, M. (1999). " Monetary Policy and Asset Price Volatility", *Economic Review*, fourth quarter, 1999, Federal Reserve Bank of Kansas City, pp. 17-51.

BLANCHARD, O.J. AND WATSON, M.W., (1982). "Bubbles, Rational Expectations, and Financial Markets", *Crises in the Economic and Financial Structure,* Wachtel , P. (Ed), Lexington, Mass: Lexington Books.

CALVO, G.A. AND MENDOZA, E.G. (1997). Rational Herd Behaviour and the Globalization of Securities Markets. *Institute for Empirical Macroeconomics*, Discussion Paper no. 120.

DUPUY, J.P. (1989). Common Knowledge, Common Sense. *Theory and Decision,* 27, pp. 37-62.

FILARDO, A. J. (2004)." Monetary Policy and Asset Price Bubbles: Calibrating the Monetary Policy tradeoffs", *BIS Working Paper* No. 15.

FRENCH, D.E. (2009). *Early Speculative Bubbles and Increases in the Supply of Money*, Mises Institute, Alabama, second edition. Chapter 8.

GORDON, M.J.(1959). "Dividends, Earnings and Stock Prices", *Review of Economics and Statistics*, 41(2), pp. 99-105.

GORDON, M.J.(1962). *The Investment, Financing, and Valuation of the Corporation*. Homewood Press, IL., R.D. Irwin.

HAKEN, H. (1993). *Synergetics: An introduction,* 3rd edition, Berlin: Springer.

ISARD, P. (2006). "Uncovered Interest Rate Parity", *IMF Working Paper*, wp/06/96.

KENT, C. AND LOWE, P. (1997). "Asset Price Bubbles and Monetary Policy", *Reserve Bank of Australia Research Discussion Paper no. 9709.*

KEYNES, J.M. (1936). *The General Theory of Employment, Interest and Money,* (Harcourt Brace, London).

KINDLEBERGER, C.P. (2000). *Manias, Panics and Crashes: A history of financial Crises*, 4th Edition, 16.

LUX, T. (1995). "Herd Behaviour, Bubbles and Crashes", *The Economic Journal,* Vol. 105, No. 431, pp. 881-896.

MINSKY, H. (1982). *Can it Happen Again? Essays on Instability and Finance*, Armonk: M.E. Sharpe.

NOBAY, A.R. AND PEEL, D.A. (2003)."Optimal Discretionary Monetary Policy in a Model of Asymmetric Central Bank Preferences", *Economic Journal,* 113, 489, pp. 657-665.

ORLEAN, A. (1989). "Mimetic Contagion and Speculative Bubbles", *Theory and Decision*, 27, pp. 63-92.

SEMMLER, W. AND ZHANG, W. (2007). “Asset Price Volatility and Monetary Policy Rules: A Dynamic Model and Empirical Evidence”, *Economic Modelling*, 24, pp. 411-30.

SHILLER, R.J. (1981). "Do Stock Prices Move Too Much to be Justified by Subsequent Changes in Dividends?" *The American Economic Review*, vol. 71, No. 3, pp. 421-436.

SHILLER, R.J. (2005). *Irrational Exuberance*, Princeton University Press, Princeton, second edition, Pp. 226-

TIROLE, J. (1985). “Asset Bubbles and Overlapping Generations”, *Econometrica,* Vol. 53, No. 6, pp. 1499-1528.

VARIAN, H.R. (1975). "A Bayesian Approach to Real Estate Assessment". In: Fienberg, S.E., Zellner, Anold (eds.), *Studies in Bayesian Econometrics and Statistics in Honour of Leonard J. Savage.* Amsterdam, North Holland, pp. 195-208.

WEIDLICH, W. AND HAAG, G. (1983). *Concepts and Models of a Quantitative Sociology*, Berlin, Springer.

WEST, K. (1987). "A Speculation Test for Speculative Bubbles", *Quarterly Journal of Economics*, 102, 553-80.

WU, GUOJUN AND XIAO, Z. (2008). “Are there Speculative Bubbles in Stock Markets? Evidence from an Alternative Approach”. *Statistics and its Interface*. Volume 1, pp.307-320.

Chapter 2
Extraction of Speculative Bubbles

2.1 Introduction

This essay extracts speculative bubbles from South Africa's overall stock market price index and a bilateral exchange rate in order to study time-series properties and understand their nature. Two asset pricing models for the stock price and the exchange rate are estimated and residual-based cointegration tests are rejected thereby indicating the presence of speculative bubbles. This thesis computes speculative bubbles by using the expectational restriction-based model (see Semmler and Zhang (2007)). The unit root tests, autoregressive process check and the Granger causality test indicate that the computed asset price bubbles are extraneously determined. The presence of speculative bubbles in financial asset prices implies that asset prices are disconnected from their fundamental values. When prices are disconnected from their fundamentals, the perceived changes in speculative bubbles to bust can generate boom-bust cycles. The busting of speculative bubbles is normally followed by a credit crunch and a deterioration of households' and firms' balance sheets leading to a decline in social welfare. Knowledge about the properties of speculative bubbles can help in controlling them and therefore providing a soft landing.

There are several views on how to determine the existence of speculative bubbles in the literature and more specifically for South Africa. On the one hand, there are those who favour using the rational bubbles theory and on the other hand those who favour using the econophysics theory (see Bouchand and Potters (2003)). The econophysics approach posits that searching for fundamentals is irrelevant because of fundamental uncertainty and it uses high frequency data for prices (Zhou and Sornette (2009)). Those who favour using the rational bubbles theory employ the fundamental in the asset pricing model, but are divided. There are those who use the value frontier framework to measure speculative bubbles (Yang (2006)). Some use rational bubbles theory with fractal analysis (Ahmed et al. (2010)); such as the Hurst (1951) exponent method (Peters (1994)); the Hamilton (1989)

regime switching methods and the Brock, Dechert and Scheinkman (1997) independence tests. Others use the expectational restrictions for speculative bubbles (Semmler and Zhang (2007)). This study follows the expectational restriction for measuring speculative bubbles because it has a rich theoretical background (Blanchard and Watson (1982)).

This study employs two empirical models based on the solutions to no-arbitrage conditions for asset pricing under rational bubbles theory (Blackburn and Sola (1996)). The no-arbitrage condition for the stock pricing model is the Gordon model (Gordon (1959, 1962); Wu and Xiao (2008)) while that for the exchange rate is the uncovered interest parity condition (Isard (2006); Blanchard (2009)). There are two particular solutions to the no-arbitrage conditions:-the fundamental solution, which exists if bubbles do not exist; and the non-fundamental solutions. The non-fundamental components are called "speculative bubbles", which follow the expectational restriction. The expectational restriction implies that speculative bubbles are extraneously determined. Therefore, any extraction method that uses the expectational restriction generates speculative bubbles.

The study uses the residual estimates of the fundamental models for both the stock price and the exchange rate using South African data to compute the speculative bubbles. A residual based cointegration method detects for the presence of speculative bubbles in the residuals. The study uses the residual based cointegration method in order to extract speculative bubbles from the residuals, since bubbles are unobservable in the two empirical models. The study computes the speculative bubbles for the stock price and the bilateral exchange rate using the expectational restriction. The expectational restriction is capable of accurately tracking speculative bubbles episodes using South African data.

The findings suggest that speculative bubbles exist in the stock price and the bilateral exchange rate between 1995Q2 and 2008Q4. The residual based cointegration test results indicate that there is no cointegration, which implies the presence of speculative bubbles. The robustness test for the residual-based cointegration tests also suggest the presence of speculative bubbles. The results for the robustness of the extracted speculative bubbles using the unit root tests confirm that the extracted bubbles have a unit root and therefore are speculative bubbles. The autoregressive order process results also confirm the results of the unit roots tests on extracted bubbles by indicating that the extracted bubbles have an autoregressive process of order one (AR(1)). The study uses Granger

causality test results between the asset price and its fundamental, asset price and the speculative bubble, and the fundamental and the speculative bubble to show that the asset prices are exogenous in the asset pricing model. The Granger causality tests indicate that asset prices Granger cause their respective fundamentals, indicating that the extracted bubbles are speculative bubbles. By plotting graphs for the speculative bubbles, there is evidence of consistency with the bubbles episodes in the literature. For example, this study identifies six currency busts around: 1996Q4, 1998Q4, 2002Q2, 2004Q1, 2006Q1, and 2008Q3 and slightly more episodes of stock market bubble busts around: 1996Q4, 1997Q4, 1998Q4, 2001Q3, 2002Q1, 2003Q2 and 2008Q4. These financial crisis dates mimic both the domestic and international financial crisis periods, which have been documented in the literature.

The rest of this essay is structured as follows: Section 2 provides the empirical literature. Section 3 gives an outline of the model. Section 4 gives the methodology and empirical results. Section 5 concludes.

2.2 Empirical Literature

There are several studies which have identified the presence of speculative bubbles in asset prices and can be categorized according to three distinctive views. Ahmed, Rosser and Uppal (2010) provide empirical evidence of the presence of speculative bubbles in emerging markets's stock markets, which also include results on South Africa. The estimates cover the period between early 1990s through 2006 and based on daily returns. They study residuals of Vector Autoregressive-based fundamentals by using the Hamilton regime switching model (Hamilton (1989)), the rescaled range analysis of Hurst (1951) and the Brock, Dechert and Scheinkman (1997) test. This methodology is based on the standard approach to identifying a bubble which is known as the rational bubbles model (Blanchard and Watson (1982)). The empirical method in Ahmed, Rosser and Uppal (2010) uses the stochastic rational bubbles model which posits that speculative bubbles are present in the residuals. This standard approach is a method which identifies the speculative bubble as an asset price straying from the fundamental value and the estimated residuals. However, there are problems with this approach. The first problem with the standard approach is the identification

of the fundamental value. The second problem with the standard approach is that the fundamental itself may be changing over time and in some complicated fashion (Flood and Garber (1980)).

Yang (2006) provides empirical evidence about the existence of speculative bubbles in 37 heterogenous countries, which include South Africa. Yang (2006) develops a value frontier framework, which is based on the same concept as the standard approach. The value frontier framework is a method of detecting bubbles based on the "value frontier" concept. The extraction of bubbles first start with the identification of all known fundamental determinants of asset prices and then determining the fundamental valuation method. The regressions of asset prices on the fundamentals picks up the fundamental value with the least inflated asset price as the value frontier. This value frontier is then used as a benchmark to measure the deviations of the asset prices relative to the value frontier.

Zhou and Sornette (2009) test 45 indices and common stocks in South Africa and found evidence for the presence of speculative bubbles. The methodology that they use posits that it is not possible to know the fundamental value because of the fundamental uncertainty. The argument is that searching for fundamentals is irrelevant because all that matters are short-term dynamics at high frequencies. This view is held by developers of the econophysics approach (Bouchand and Potters (2003)). Zhou and Sornette (2009) define a bubble as a faster-than-exponential acceleration with significant log-periodic oscillations. They establish that the mini-crash that occurred around mid-June 2006 was only a partial correction, which resumed into a renewed bubbly acceleration bound which was predicted to end in 2007. They use a rational expectations model in the presence of noisy imitative traders. They do not use fundamentals in their regression analysis.

Many studies from developed countries which have provided empirical evidence on the presence of speculative bubbles in financial markets exist. The relevant ones to this study are those which stick to the rational bubbles theory by using the expectational restriction which is capable of integrating regulatory practice, perfect rationality and rational herd behaviour (Semmler and Zhang (2007)). Semmler and Zhang (2007) specify extracted speculative bubbles using a modified expectational restriction of the rational bubbles theory. The modification allows them to specify a speculative bubble as either positive or negative. The residuals based test is also used to establish the presence of bubbles and

to extract them. To our knowledge, no one has applied this approach on the South African stock and foreign exchange market prices data.

2.3 Outline of the Model

A financial market is described by two asset pricing models: a stock pricing model and an exchange rate determination model. I adopt two no-arbitrage asset pricing models: the Gordon model (Gordon (1959, 1962); Wu and Xiao (2008)) and the uncovered interest rate parity model (Svensson (1998); Isard (2006); Blanchard (2009)) as shown below;

$$S_t = \frac{S_T}{(1+R_t)^T} + \sum_{n=1}^{\infty}\left(\frac{d_{t+n}}{(1+R_t)^n}\right) + d_t \tag{2.27}$$

$$e_t = e_{t+T} + \left(i_t - i_t^f\right) \tag{2.28}$$

where, S_t is the stock price at time t, S_T is the expected terminal stock price at maturity time T, d_{t+n} is the future stream of dividend payouts for an infinitely lived stock, R_t is an appropriate discount rate at time t, e_t is the nominal exchange rate at time t (a higher e implies an appreciation), e_{t+T} is the expected exchange rate at maturity date T, i_t is the current short-term domestic interest rate and i_t^f is the current short-term foreign interest rate. Eqns. (2.27) and (2.28) describe that the asset pricing is driven by the no-arbitrage conditions for the stock market and the foreign exchange markets. Eqn. (2.27) is the Gordon model and eqn. (2.28) is the uncovered interest parity condition. The expected terms in eqns. (2.27) and (2.28) are uncertain conditional terms and they generate speculative behaviour which leads to the emergence of speculative bubbles.

Under the rational bubbles theoretical framework, which is developed in Blanchard and Watson (1982), speculative bubbles are the non-fundamental solutions of the asset pricing models. By invoking the rational bubbles theory, the general solutions to eqns. (2.27) and 2.28) follow solution methods by Wu and Xiao (2008) and Meese (1986), respectively. The structures of the general solutions are:

$$S_t = d_t + b_{st} \tag{2.29}$$

$$e_t = \left(i_t - i_t^f\right) + b_{et} \tag{2.30}$$

where, b_{st}, b_{et} are the speculative bubbles for the stock price and the exchange rate, respectively. Eqns. (2.29) and (2.30) are known as rational bubbles models for the stock price and the exchange rate, respectively. Eqn. (2.29) is the general solution based on the present value principle of the Gordon model for stock price valuation. However, eqn. (2.30) is the general solution for the exchange rate determination, which is based on the uncovered interest parity condition.

Following Blanchard and Watson (1982) and Meese (1986), the speculative bubbles are generated by extraneous events or rumours and satisfy the following expectational restrictions:

$$E_t b_{s,t+1} = \beta^{-1} b_{st} \tag{2.31}$$

$$E_t b_{e,t+1} = \psi^{-1} b_{et} \tag{2.32}$$

where, E_t is the expectations operator, β^{-1} and ψ^{-1} are appropriate growth factors for the stock price bubble and the exchange rate bubble, respectively. Eqns. (2.31) and (2.32) are the expectational restriction conditions for stock price bubbles and exchange rate bubbles, respectively. Eqns. (2.31) and (2.32) also imply that speculative bubbles can take on different forms satisfying expectational restrictions such as the West's (1987) collapsing bubbles, the Evans's (1991) periodically collapsing and regenerating bubbles the Diba and Grossman's (1988) shrinking bubbles and the Semmler and Zhang's (2007) positive and negative bubbles. The Semmler and Zhang (2007) asset price bubble evolves in the following manner:

$$b_{s,t+1} = \begin{cases} \beta_1^{-1} b_{s,t} + u_{t+1}, \text{ with probability } \rho_s \\ \beta_2^{-1} b_{s,t} + u_{t+1}, \text{ with probability } 1 - \rho_s \end{cases} \tag{2.33}$$

$$b_{e,t+1} = \begin{cases} \psi_1^{-1} b_{e,t} + v_{t+1}, \text{ with probability } \rho_e \\ \psi_2^{-1} b_{e,t} + v_{t+1}, \text{ with probability } 1 - \rho_e \end{cases} \tag{2.34}$$

where, $\beta_1^{-1} = (1 + R_{1,st})$ denotes a time varying growth factor for stock bubbles, $\psi_1^{-1} = (1 + R_{1,et})$ denotes a time varying growth factor for exchange rate bubbles, $\beta_2^{-1} = (1 - R_{2,st})$

denotes a time-varying decrease factor for stock bubbles, $\psi_2^{-1} = (1 - R_{2,e})$ denotes a time-varying decrease factor for exchange rate bubbles. R is the gross market return on the asset and employed as a key determinant of the speculative bubbles growth or decrease rate (Poterba and Summers (1986)) and Cunado et al. (2005)). ρ denotes the probability for the bubble to increase and $1 - \rho$ denotes the probability for the bubble to fall. u and v are the stochastic error terms which drive the speculative bubbles and are assumed to be driven stochastically by extraneous factors unrelated to the fundamentals. These error terms are independently and identically distributed noise terms.

The speculative bubbles in eqns.(2.33) and (2.34) seem very realistic since a positive bubble would imply an overvaluation of the asset price and a negative bubble would imply an undervaluation. Thus, the presence of speculative bubbles imply the presence of asset price misalignments.

One of the most important motivations for holding speculative assets that are experiencing price bubbles is the expectation that the price will continue to rise. The rational bubbles theory has shown that bubbles can arise as a result of the investors having rational attitudes (Blanchard and Watson (1982)). The most important contribution of the rational bubbles theory is that speculation can no longer be considered as something that is necessarily desirable from the economic efficiency point of view, but that speculation can lead to severing of connections between asset prices and fundamental values. This disconnect, which is formalized in eqns. (2.31) and (2.32), in turn leads to a faulty overall allocation of factors of production.

Since rational agents know the probability function of a bubble emerging, state-dependent speculative bubbles can be reformulated from eqns.(2.33) and (2.34) following Semmler and Zhang (2007). This is done by multiplying the respective probabilities to the current speculative bubble and summing the two equations. The simplified models for each asset price bubble is given as:

$$b_{s,t+1} = \frac{(\beta_1 (1 - \rho_s) + \rho_s \beta_2)}{\beta_1 \beta_2} b_{s,t} + u_{t+1} \tag{2.35}$$

and

$$b_{e,t+1} = \frac{(\psi_1 (1 - \rho_e) + \rho_e \psi_2)}{\psi_1 \psi_2} b_{e,t} + v_{t+1} \tag{2.36}$$

Thus, the evolution of the bubble is determined by the growth factor of the bubble, the probability of collapse, the decrease factor, the probability of the bubble emerging, the current bubble and the stochastic disturbance term, which captures all other factors. This study employs these bubbles because they can interchange their states from being positive to being negative.

To test for speculative bubbles, the study employs empirical models based on Wu and Xiao (2008) and Isard (2006) stock price and exchange rate models, respectively, and using the following regressions:

$$S_t = \kappa_1 + \kappa_2 d_t + u_t \tag{2.37}$$

$$e_t = \mho_1 + \mho_2 \left(i_t - i_t^f \right) + v_t, \tag{2.38}$$

where, κ_1 and κ_2 are parameters for the stock price regression model and $\mho_1$and $\mho_2$ are the parameters for the exchange rate regression model. u_t and v_t denote normal error terms and they include speculative bubbles and risk premia.

Eqn. (2.37) regresses the stock price on the dividends and this implies that the extraction of speculative bubbles from the stock price uses eqn. (2.35). Eqn. (2.38) is the regression of the nominal exchange rate on the interest rate differential. This thesis extracts speculative bubbles from the exchange rate by using eqn. (2.36).

2.4 Methodology and Empirical Results

Given the above empirical models, this study tests the null hypothesis that speculative bubbles do not exist in both the stock price and the exchange rate. This thesis employs the bilateral exchange rate between the United States dollar and the South African Rand, to capture the effects of the dollar denominated global trade on the Rand. In this case, the United States dollar is used a convertable vehicle currency for world trade. An increase in the exchange rate implies an appreciation and a decrease is a depreciation.

Equations (2.37) and (2.38) are the empirical models for this study. Firstly, this thesis tests for the presence of speculative bubbles in both the foreign exchange and the stock mar-

kets using residual based cointegration approach. The study uses the Phillips and Ouliaris (1991) and the Engle and Granger (1987) cointegration tests. These types of tests have less chance of rejecting cointegration when there is one and accepting cointegration when there is none. These tests consider the null hypothesis of no cointegration. By treating the bubble as a unit root process, they test the null hypothesis that the asset price contains a unit root. The existence of bubbles is done by testing for unit roots on the residuals from the cointegrating regression. If there are unit roots, then the speculative bubbles exist and if there are no unit roots then the speculative bubbles do not exist. The justification for this method is that, since bubbles are unobservable, they are captured in the residuals which tend to drift apart persistently. This type of bubbles are non-stationary. Cointegration analysis helps establish whether bubbles are stable or are unstable. Unstable bubbles are generated when the cointegration test fails and stable bubbles are present when a cointegrating relationship exists.

We check for robustness of these results by using the traditional approach, such as the Johansen (1991) cointegration test. Secondly, we use the expectational restriction condition to compute the speculative bubbles. This rational bubbles theory property admits time varying asset returns as discount rates. The time varying discount rates can be positive or negative and therefore the speculative bubbles resemble that specified by Semmler and Zhang (2007). Thirdly, we carry out three robustness tests on the extracted bubbles as further justifications for maintaining that the extracted bubbles are speculative bubbles. The three robustness tests are the "standard " unit root tests, the autoregressive order process (AR(p)), and the Granger Causality test. Fourthly, we graph the extracted speculative bubbles to identify bubble episodes which are consistent with the literature (Reinhart and Rogoff (2009); Zhou and Sornette (2009); Bond (2010)).

This thesis conducts unit roots test to determine the stationarity of the stock price index, the dividends, the bilateral nominal exchange rate, and discount rates for South Africa and United States using the standard Augmented Dickey and Fuller (1979) test, the Phillips and Perron (1988) test and the Kwiatkowski, Phillips, Schmidt and Shin (1992) test.

We simulate the speculative bubbles using eqns. (2.35) and (2.36) using the probability of the bubble of 0.5, also used by Semmler and Zhang (2007). The justification is

that because bubbles are linked to residuals, which can either be positive or negative, the probability of the residual being either positive or negative is 0.5. Assuming that the speculative bubble starts at zero, then the expected speculative bubble in the next period is the outcome of the residual corresponding to that period and it can be either positive or negative. Since little is known about the sign of the noise in the asset pricing model, speculative investors then may expect it to be positive or negative with an equal probability of 0.5.

This study's empirical methodology is designed to overcome the identification problem for bubble phenomena, which is related to the problem of distinguishing any type of bubble from an expected future change in market fundamentals and the detection problem associated detecting a periodically collapsing bubble when the residuals of the fundamentals regression are integrated (Blackburn and Sola (1996), Wu and Xiao (2008)).

2.4.1 Data Description

This thesis estimates asset pricing models using the quarterly data for South Africa and the United States of America between 1995Q2 and 2008Q4 sourced from the International Monetary Fund's International Financial Statistics data base, International Monetary Fund Web Site at Wits University Library and from the McGregor BFA data buffet web site at the University of Cape Town Library. The main variables for the data collection are the gross domestic product; Johannesburg Stock Exchange all share index; dividends, domestic short-term interest rate, the United States of America short-term interest rate, and the bilateral nominal exchange rate of the United States of America dollar per South African Rand.

2.4.2 Discussion of Results

This paper employs two formal tests for the stationarity of key variables. The two types of tests test the null hypothesis that the series have a unit root. Thus, rejecting the null hypothesis of a unit root would mean that the series does not have a unit root (i.e. a series is stationary).

Table 2.1. Unit Root Tests: 1995Q2-2008Q4

Variables	**ADF Test**			**PP Test**		**Decision:**
	Level	1st Diff	2nd Diff	Level	1st Diff	Integration
Interest rate-SA	-1.48	-5.86**		-1.48	-5.80**	I(1)
Nominal exchange rate	-0.15	-6.28**		-2.34	-6.33**	I(1)
Stock price index	-2.31	-5.31**		-0.73	-5.41**	I(1)
Stocks dividends	6.79	0.17	-6.49**	6.74	-3.77**	I(2)
Interest rate-US	-1.51	-2.37	-7.66**	-1.47	-2.41**	I(2)

Note: 1%(**), 5%(*) and at10%(***) levels of significance.
$Source$:$Author$

Table 2.2. Residual-Based Co-integration Tests

Residuals Tests	Exchange rate		Stock Price	
	Value	Prob.*	Value	Prob.*
Phillips-Ouliaris tau-statistic	-2.70	0.40	-2.33	0.37
Phillips-Ouliaris z-statistic	-11.81	0.45	-12.46	0.20
Engle-Granger tau-statistic	-2.73	0.38	-2.23	0.42
Engle-Granger z-statistic	-12.21	0.42	-11.59	0.24
*Mackinnon (1996) p-values				

$Source$:$Author$

Table 2.1 shows that the exchange rate and the stock price are integrated of order one, implying that they have a unit root. This result justifies the use of a cointegration test (Engel and Granger (1987)). The United States short-term interest rate is integrated of order two (I (2)) while the South African short-term rate is integrated of order one (I (1)). The dividends per share is integrated of order two (I(2)). The I (1) and I (2) variables enter the cointegrating regression in levels and first difference, respectively. All the unit root tests are rejected at 1% level of significance.

This thesis provides residual-based cointegration results in Table 2.2. Table 2.2 indicates the Phillips and Ouliaris (1991) and the Engle and Granger (1987) cointegration tests results for the exchange rate model. The null hypothesis of no cointegration is not rejected. This implies that the speculative bubbles exist in the bilateral exchange rate of the US dollar per South African rand. Table 2.2 also indicates the Phillips-Ouliaris and the Engle-Granger cointegration test for the stock price model with only the dividends as a fundamental. The null hypothesis for the cointegration test cannot be rejected. The rejection of cointegration implies the presence of stock price bubbles.

Table 2.3. Johansen Co-integration Tests: Asset Pricing Models

Trace Statistics				
1). Exchange Rate				
Hypothesized		Trace	Critical Values	
No. of CE(s)	Eigenvalue	Statistic	5%	1%
None	0.16	13.28	15.41	20.04
At most 1	0.08	4.08	3.76	6.65
2). Stock Price				
None	0.14	8.12	15.41	20.04
At most 1	0.0007	0.04	3.76	6.65
Maximum Eigenvalue Statistics				
3). Exchange Rate				
Hypothesized		Max-Eigen	Critical Values	
No. of CE(s)	Eigenvalue	Statistic	5%	1%
None	0.16	9.20	14.07	18.63
At most 1	0.08	4.08	3.76	6.65
4). Stock Price				
None	0.14	8.08	14.07	18.63
At most 1	0.0007	0.04	3.76	6.65

Source:Author

Traditional cointegration tests results are given in Table 2.3 as a robustness check on the residual based cointegration tests. Table 2.3 indicates two blocks for the Johansen (1988, 1991) cointegration test for both the exchange rate and the stock price models (Johansen and Juselius (1990)). The first block shows trace statistics as measures for the number of cointegrating vectors and the second block shows the maximum eigen value statistics. Both the trace and the maximum eigen value statistics indicate that there is no cointegrating relationship between the bilateral nominal exchange rate and the interest rate differential, on one hand, and between the stock price and the dividends on the other hand. These results are consistent with the residual based cointegration tests, which show that there is no long-run relationship between asset prices and their fundamentals. Therefore, there is evidence that speculative bubbles exist.

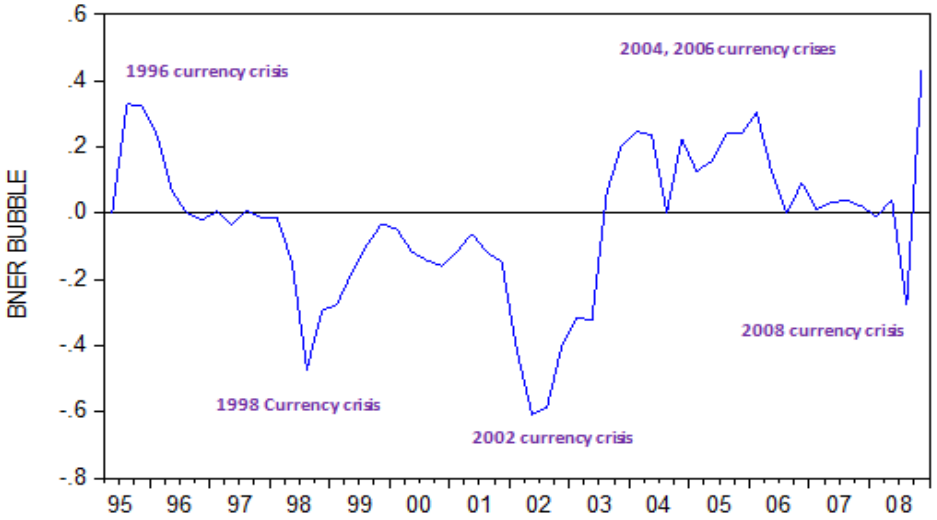

Fig.3.1- Foreign Exchange Market Bubbles

Source:Author

Figures 3.1 and 3.2 show the extracted exchange rate bubble and stock price bubble indices over time, respectively. Figure 3.1 identifies six bubble bust episodes, which coincide with the currency crises periods which have been reported in the literature (Reinhart and Rogoff (2009); Bond (2010)). The figure shows that the exchange rate bubble is consistent with the following domestic currency crises: 1996, 1998, 2002, 2004, 2006, and 2008 (Bond (2010)). Specifically, the figure indicates an undervaluation episode following the 1998 Asian financial crisis and the 2002 South African currency crisis. This is followed by an episode of positive bubbles between 2004 and 2008Q1. The model shows that the 2006 currency crisis in South Africa led to exchange rate readjustment to equilibrium but with a bubble bust towards end of first quarter of 2008 and a reversal by the end of 2008 into a positive bubble episode.

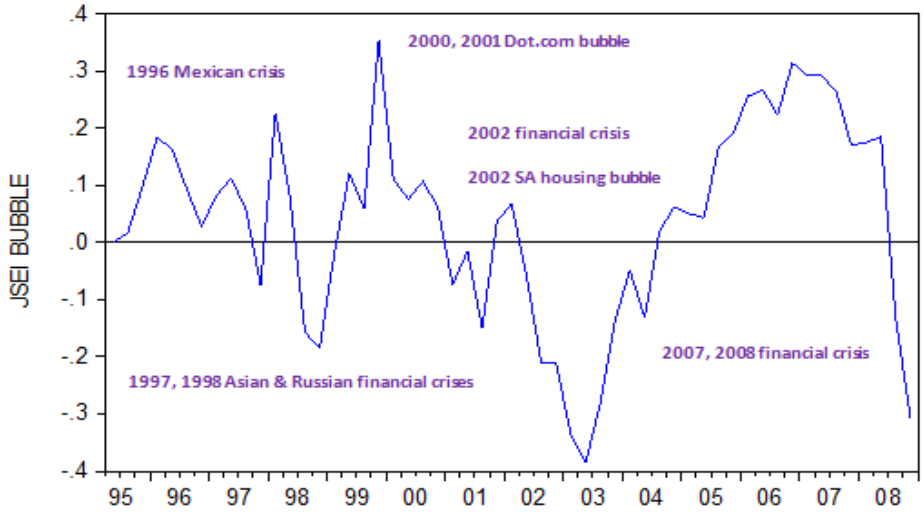

Fig. 3.2- Stock Market Index Bubbles

Source:Author

Fig. 3.2 identifies eight (8) stock bubble burst episodes, which coincide with both domestic and world financial crisis periods which have been reported in the literature (Reinhart and Rogoff (2009); Zhou and Sornette (2009); Bond (2010)). The figure shows that the stock price bubble is consistent with the following stock bubble crises: 1996, 1997, 1998, 2000, 2001, 2002, 2007, and 2008. These episodes coincide with the 1996 Mexican economic crisis, the 1997-98 Asian and Russian financial crisis, the 2000- 2001 dot com bubble, the 2002 South African housing bubble, the 2006-2007 commodity price booms, the 2007-2008 financial crisis. The international financial crises suggest the existence of contagion or spill over effects to the South African stock market. This has support in Forbes and Rigobon (2002) who define markets contagion as significant increases in cross-market co-movements.

The ratios of the estimated speculative bubbles to their actual asset prices, in percentage, are plotted in Figure 3.3, from which we make several observations. Firstly, it is evident that the stochastic speculative bubbles for the exchange rate fluctuate greatly and account for a substantial portion of the actual bilateral exchange rate in the sample, especially during the currency crisis periods. Secondly, compared to their mean estimates in Figure 4, the exchange rate bubbles are found to be significantly negative at 1% level in 1998, 2002 and 2008Q3 when the Rand depreciated greatly. The results suggest that be-

tween 20% and 60% of the actual exchange rate during these periods might have been caused by speculative bubbles. The study finds that the composition of exchange rate bubbles in the exchange rate is in line with those of Wu (1995).

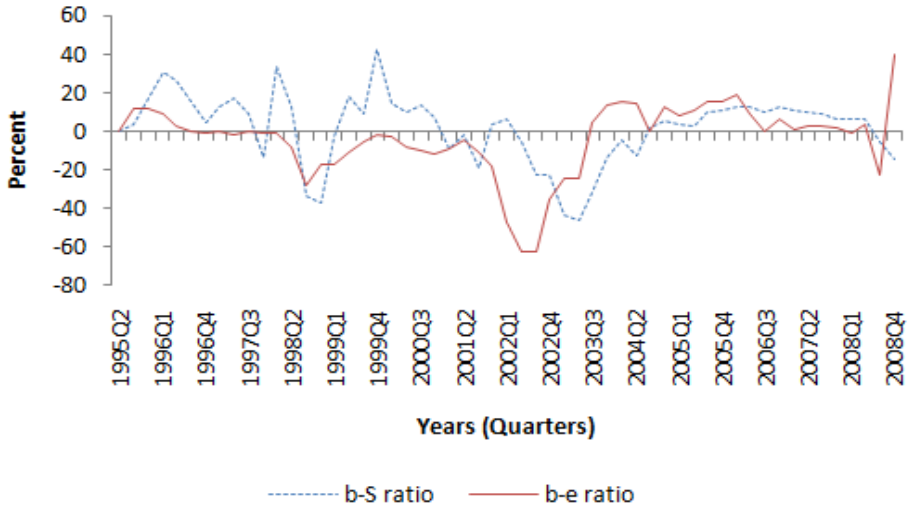

Fig. 3.3- Bubbles to Asset Price Ratios

(Note: b-S ratio denotes stock bubble to price ratio; b-e ratio denotes exchange rate bubble to exchange rate ratio)

Source:*Author*

Thirdly, it is also evident that the stochastic speculative bubbles for the stock price fluctuate greatly and account for a substantial portion of the actual stock price in the sample, especially during the 1996 Mexican crisis, the 1998 Asian and Russian financial crises, the 2000-01 dot.com financial bubble, and the recent 2008 financial crisis as shown in Figure 3.2. Secondly, compared to their mean estimates in Figure 3.5, the stock price bubbles are found to be significantly positive at 5% level over the financial boom-bust periods of the sample period. The results suggest that between 20% and 40% of the actual stock price during the boom-bust periods might have been caused by speculative bubbles. The composition of stock price bubbles in the stock price is also in line with those of Wu (1997).

Figures 3.4 and 3.5 show the statistics for both the exchange rate bubbles and the stock price bubbles, respectively. The statistics are statistically significant as shown in the two figures. This thesis concludes that the extracted speculative bubbles are non-normal and with fat tails in their empirical distributions as indicated by the kurtosis.

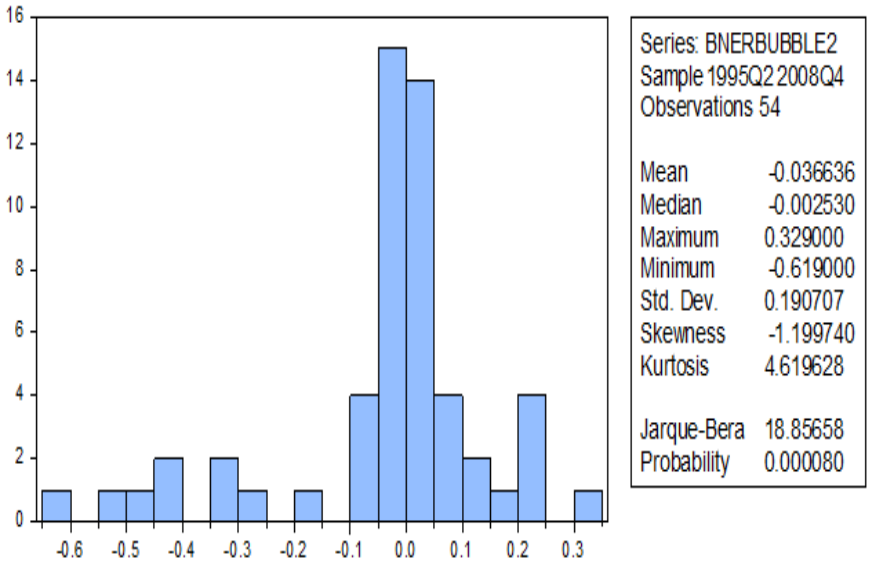

Fig 3.4- Exchange Rate Bubbles

Source:Author

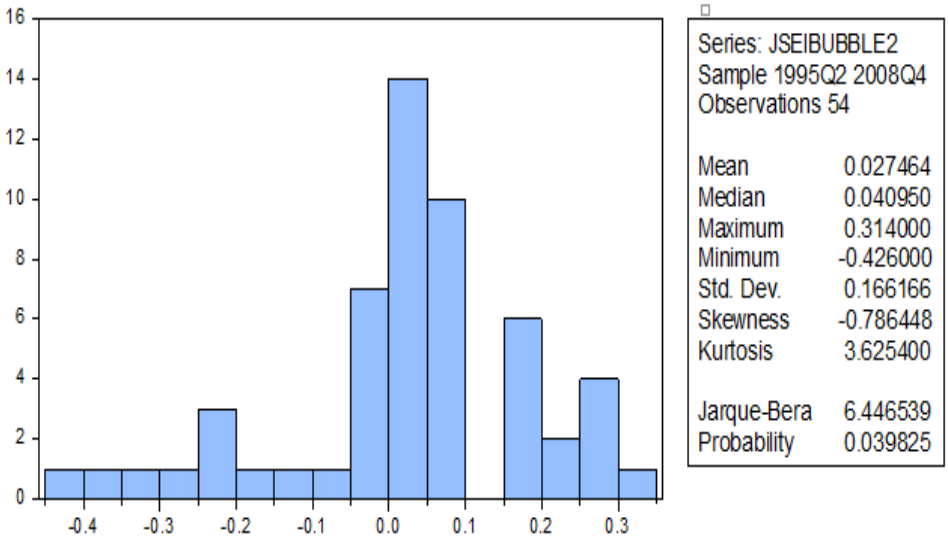

Fig 3.5- Stock Price Bubbles

Source:Author

A check on the autoregressive order p (AR(p)) of the speculative bubbles in levels employs eqns. (2.35) and (2.36). The AR model results, in Table 2.4, indicate that both stock price and exchange rate bubbles have a coefficient of less than one indicating that the herd behaviour is weak. Applying pure contagion equilibrium analysis in Chapter 1,

weak herd behaviour implies the stability condition holds. The study determines this result by regressing the expected speculative bubbles on its current value. However, we first construct correlograms for speculative bubbles as shown in Figures 3.6 and 3.7. The optimal lag length is associated to the partial correlation function, before it decays within the limit bounds. The Q-statistic and the probability indicate the order of the autoregressive process for each bubble (Enders (2004)). The dotted lines in Figures 3.6 and 3.7, are approximate two standard error bounds. Thus, autocorrelation outside the bounds implies that it is significantly different from zero at 5 % level of significance. Furthermore, the partial autocorrelation (PAC) of a pure autoregressive process of order 1, AR (1), cuts off at lag 1. Therefore, Figures 3.6 and 3.7 show that the speculative bubbles are AR (1).

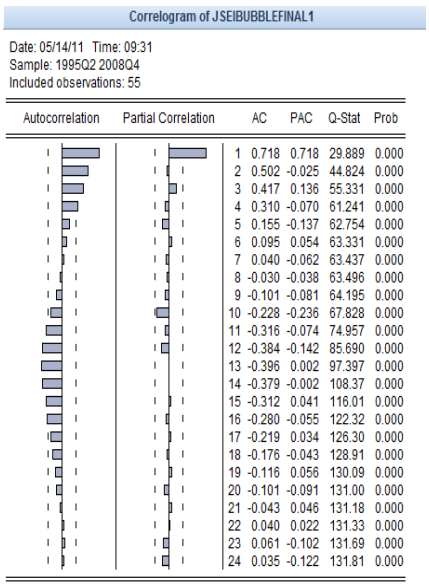

Fig. 3.6-Stock Bubbles

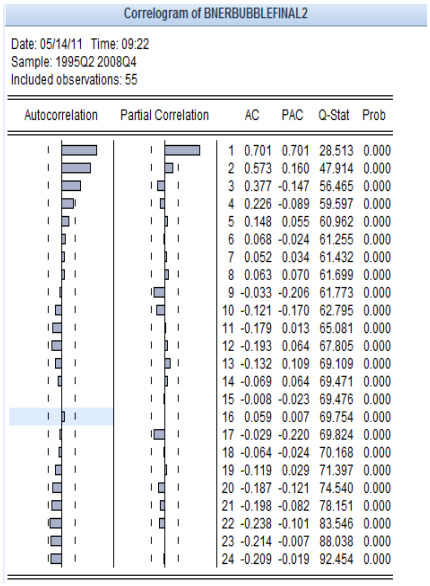

Fig. 3.7- Exchange Rate Bubbles

Source:*Author*

The AR(1) process for each of the speculative bubbles in eqns (2.35) and (2.36) are estimated using the ordinary least squares method and the regression results are shown in Table 2.4. These results confirm the existence of weak herding in both the stock and the foreign exchange markets for South Africa between 1995Q2 and 2008Q4.

Figures 3.8 and 3.9 indicate serial correlation tests. The serial correlation tests are insignificant for residuals indicating that the moving average process does not exist. Thus, the autoregressive process results are consistent the coefficients in Flood and Garber (1980).

Table 2.4. Results for Autoregressive processes for Bubbles: 1995Q2-2008Q4

AR Model	Coefficient	se	t-stat	ρ	R^2	log L
$b_{s,t+1} = \alpha_{bs} b_{s,t} + u_{t+1}$	0.78	0.09	8.55	0.00	0.56	39.9
$b_{e,t+1} = \alpha_{be} b_{e,t} + v_{t+1}$	0.76	0.10	7.89	0.00	0.53	23.8

$Note$: se= standard error; t-stat= t-statistic, ρ= probability, logL=log likelihood statistic
$Source$:$Author$

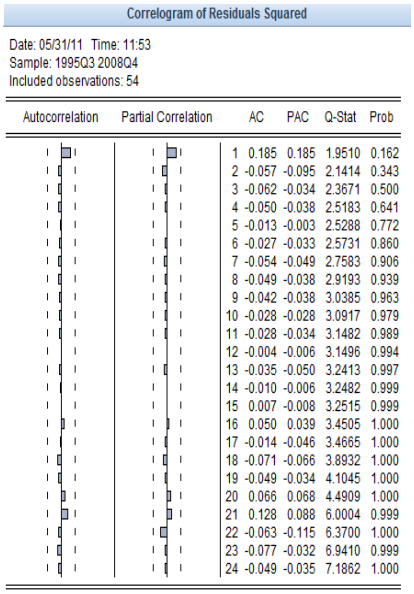

Fig.3.8-Residuals for b_{et+1}

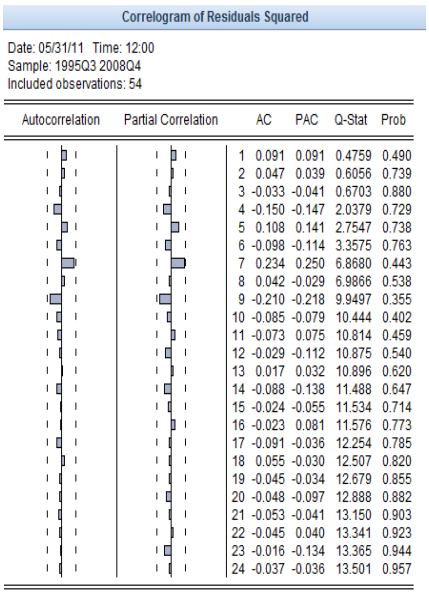

Fig.3.9- Residuals for b_{st+1}

$Source$:$Author$

We carry out further robustness tests on the speculative bubbles. These tests include unit root tests on the speculative bubbles and Granger causality tests between the asset prices and fundamentals; asset prices and bubbles; and fundamentals and bubbles. Employing rational bubbles theory, the generated speculative bubbles should have a unit root. And employing Engle and Granger (1987), the asset prices should be exogenous to their market fundamentals for the generated asset price bubbles to be known as speculative bubbles. Therefore, the asset price should Granger cause the fundamental.

The validity of the estimated speculative bubbles can be checked by employing the above stated robustness checks. The question that may be asked is whether the estimated speculative bubbles contain any information about the behaviour of investors in both the stock and the foreign currency markets.

Table 2.5. Speculative Bubbles Properties: 1995Q2-2008Q4

Variables	**ADF Test**		**PP Test**		**Decision:**
	Level	1st Diff	Level	1st Diff	Integration
Exchange rate Bubble	-2.44	-7.11**	-2.41	-7.11**	I(1)
Stock price bubble	-2.34	-7.48**	-2.40	-7.50**	I(1)

Note: 1%(**), 5%(*) and at10%(***) levels of significance.
$Source$:$Author$

Table 2.6. Granger Causality Tests for the Rational Bubbles Models: 1995Q2-2008Q4

	lags=2		**lags=4**
Null Hypothesis	F-Statistic		F-Statistic
1. Exchange Rate Rational Bubbles Model			
BNER does not Granger Cause Interest differential	4.12*	L	1.99
Interest differential does not Granger Cause BNER	1.42	L	1.16
Bubble does not Granger Cause interest differential	0.93	L	0.46
Interest differential does not Granger Cause Bubble	1.00	L	0.49
Log BNER does not Granger Cause Bubble	5.37**	D	2.72*
Bubble does not Granger Cause log BNER	0.15	D	1.12
2. Stock Price Rational Bubbles Model			
Stock price does not Granger Cause Dividends	4.14*	L	3.61**
Dividends does not Granger Cause Stock price	0.83	L	1.26
Stock Bubble does not Granger Cause Dividends	5.12**	L	3.19*
Dividends does not Granger Cause Stock bubbles	0.71	L	1.07
Stock bubble does not Granger Cause stock price	1.52	L	2.61*
Stock price does not Granger Cause stock bubble	2.92***	L	2.81*

Note: 1%(**), 5%(*) and at10%(***) levels of significance. Decision: If F-Statistic is significant, Reject Ho.
$Source$:$Author$

Table 2.5 indicates that the exchange rate bubble and the stock price bubble have unit roots. The null hypothesis of a unit root is not rejected in levels for both the speculative bubbles. Therefore, the study concludes that the extracted asset price bubbles are indeed speculative bubbles.

Table 2.6 indicates the Granger causality tests (Granger (1969)) for both the foreign currency market and the overall stock market index for South Africa. The F-statistics are computed by varying the lag structure. The results indicate that the bilateral nominal exchange rate provide statistically significant information about the forecasted values of both the interest rate differential and the exchange rate bubbles, given the lag length of two. That is, the pairwise tests indicate that the bilateral nominal exchange rate Granger causes both the interest rate differential and the exchange rate bubbles. This implies that the bilateral nominal exchange rate takes precedence over the interest rate differential in the cointegrat-

ing regression. The other results for the exchange rate model imply that neither the bilateral nominal exchange rate bubble nor the interest rate differential Granger causes each other. This means that neither exchange rate speculative bubbles nor the interest rate differential includes information about each other. Thus, neither of the two can be used to forecast the other. The results for the exchange rate model imply that the generated nominal exchange rate bubbles are indeed extraneously determined and are called speculative bubbles.

Considering the stock pricing model, Table 2.6 also indicates that the stock price Granger causes the dividends. This implies that the stock price is useful in forecasting dividends and not the other way round. The results also show that the Granger causality between the stock price and the stock price bubbles can not be determined. However, the stock price bubbles Granger causes the dividends. This implies that the stock price bubbles are used to forecast dividend payouts. The conclusion is that the extracted stock price bubbles are speculative bubbles.

2.5 Conclusion

This study establishes the presence of speculative bubbles in both the stock price and the bilateral nominal exchange rate in South Africa between 1995Q2 and 2008Q4. The extracted speculative bubbles have fat tails in their distributions, and these properties are similar to their asset prices. The study employs the standard no-arbitrage models to extract speculative bubbles. The stock price model is the standard present value formula, while the exchange rate model is the uncovered interest parity condition. By employing a robust fully modified least squares method, the study shows that the evolution of speculative bubbles follows a three state process. That is, they can either be negative or positive or even zero. The tests for the existence of speculative bubbles are done by using two different cointegration techniques, namely: Engle and Granger (1987) and Phillips and Ouliaris (1991) cointegration tests. The post-estimation and post extraction phase uses the unit roots and the Granger causality tests to justify the that the extracted asset price bubbles are indeed speculative bubbles. The basic idea is that if fundamentals Granger cause asset prices, then the extracted bubbles would not be speculative bubbles. However, speculative bubbles

can Granger cause fundamental values, as is the case for the stock pricing model. The autoregressive order process indicates weak herd behaviour in both the stock and foreign exchange markets, implying stability in speculative bubbles.

The implications of the presence of speculative bubbles is that when bubbles are in a zero state it implies that the asset price is correctly priced in the market while positive bubbles means the existence of overvalued asset prices and busts of asset bubbles are a possibility. But if the asset price bubble is negative, the implication is that financial assets are undervalued. The empirical evidence indicates that speculative bubbles exist in South African financial markets.

References

AHMED, E.; ROSSER JR., J.B., AND UPPAL, J.Y. (2010). "Emerging Markets and Stock Market Bubbles: Nonlinear Speculation?", *Emerging Markets Finance and Trade*, Vol. 46, No. 4, July-August, pp. 23-40.

BLACKBURN, K. AND SOLA, M. (1996). 'The Market Fundamentals versus Speculative Bubbles: A New Test Applied to German Hyperinflation', *International Journal of Finance and Economics*, 1, 303-16.

BLANCHARD, O. (2009). *Macroeconomics*, Pearson Prentice Hall, United States of America, pp. 412-

BLANCHARD, O.J. AND WATSON, M.W., (1982). "Bubbles, Rational Expectations, and Financial Markets", *Crises in the Economic and Financial Structure,* Wachtel , P. (Ed), Lexington, Mass: Lexington Books.

BOND, P. (2010). "South Africa's Bubble Meets Boiling Urban Social Protest", *Monthly Review*, Vol., 62, No. 2.

BOUCHAUD, J.P., AND POTTERS, M. (2003). *Theory of Financial Risk and Derivative Pricing: From Statistical Physics to Risk Management*. Cambridge, UK: Cambridge University Press.

BROCK W. A., DECHERT, W.D., AND SCHEINKMAN, J.A. (1997). "A Test for Independence Based on the Correlation Dimension." *Econometric Reviews,* 15, pp. 197-235.

CUNADO, J., GIL-ALANA, L.A, PEREZ DE GRACIA, F. (2005), "A test for rational bubbles in The NASDAQ stock index: a fractionally integrated approach", *Journal of Banking and Finance*, 29, 2633 - 2654.

DIBA, B.T., AND GROSSMAN, H.I. (1988). "The Theory of Rational Bubbles in Stock Prices", *The Economic Journal,* Vol., 98, No. 392, pp. 746-754.

DICKEY, D.A. AND FULLER, W.A. (1979). " Distribution of Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association*, 74, 427-431.

ENDERS, W. (2004). *Applied Econometric Time Series*, John Wiley & Sons, Inc, 2nd Edition, Chapter 2.

ENGEL, R., AND GRANGER, C. (1987). "Cointegration and Error Correction: Representation, Estimation and Testing ". *Econometrica*. 55, 251-276.

EVANS, W. G. (1991). ‘ Pitfalls in Testing for Explosive Bubbles in Asset Prices’, *The American Economic Review*, vol. 81, pp.922-930.

FLOOD, R., AND GARBER, P. (1980). “Market Fundamentals vs. Price Bubbles: The First Tests”, *Journal of Political Economy*, 88, pp. 745-770.

FORBES, K. AND RIGOBON, R. (2002). "No Contagion, only Interdependence: Measuring Stock Market Co-movements.", *Journal of Finance,* 5, pp. 2223-2261.

GORDON, M.J.(1959). "Dividends, Earnings and Stock Prices", *Review of Economics and Statistics*, 41(2), pp. 99-105.

GORDON, M.J.(1962). *The Investment, Financing, and Valuation of the Corporation*. Homewood Press, IL., R.D. Irwin.

GRANGER, C.W.J.(1969). "Investigating Causal Relations by Econometric Models and Cross-Spectral Methods", *Econometrica*, 37, pp. 424-438.

HAMILTON J. D. (1989). “A New Approach to the Economic Analysis of Nonstationary Time Series and the Business Cycle.” *Econometrica* 57, pp. 357-384.

HURST H. E.(1951). “Long Term Storage Capacity of Reservoirs.” *Transactions of the American Society of Civil Engineers,* 116, pp. 770-799.

INTERNATIONAL MONETARY FUND (2009). *International Financial Statistics*, Wits University Library website.

ISARD, P. (2006). "Uncovered Interest Rate Parity", *IMF Working Paper*, wp/06/96

JOHANSEN, S. (1988). "Statistical Analysis of Cointegration Vectors", *Journal of Economic Dynamics and Control,* 12, pp. 231-54

JOHANSEN, S., AND JUSELIUS, K. (1990). “Maximum Likelihood Estimation and Inference on Cointegration with Application to the Demand for Money”. *Oxford Bulletin of Economics and Statistics*.52, 169-210.

JOHANSEN, S. (1991). "Estimation and Hypothesis Testing of Cointegration Vectors in Gaussian Vector Autoregressive Models", *Econometrica*, 59, pp. 1551-1580.

KWIATKOWSKI, D.; PHILLIPS, P.C.B.; SCHMIDT, P.; AND SHIN, Y. (1992). "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root. How sure are we that economic time series have a unit root? *Journal of Econometrics* 54, pp.159-178.

MACKINNON, J.G. (1996). "Numerical Distribution Functions for Unit Root and Cointegration Tests", *Journal of Applied Econometrics*, 11, pp. 601-618.

MEESE, RICHARD, A. (1986). "Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates?" *The Journal of Political Economy*, vol. 94, no. 2, pp. 345-373.

PETERS, E.E. (1994). *Fractal Market Analysis*. John Wiley & Sons, Inc

PHILLIPS, P.C.B. AND OULIARIS, S. (1991). "Asymptotic Properties of Residual Based Tests for Cointegration ". *Econometrica*, 58, 165-194.

PHILLIPS, P.C.B. AND PERRON, P. (1988). " Testing for Unit Roots in Time Series Regression", *Biometrika*, 75, 335-346.

POTERBA, J.M. AND L.H. SUMMERS (1986). "The Persistence of Volatility and Stock Market Fluctuations", *The American Economic Review*, Vol. 76, No. 5, pp. 1142-1151.

REINHART, C.M. AND ROGOFF, K.S. (2009). *This Time is Different: Eight Centuries of Financial Folly*, Princeton University Press, Princeton and Oxford.

SEMMLER, W. AND ZHANG, W. (2007). "Asset price volatility and monetary policy rules: a dynamic model and empirical evidence", *Economic Modelling,* 24, pp. 411-30.

SVENSSON, L.E.O. (1998). "Open Economy Inflation Targeting", *NBER Working Paper Series*, Working paper 6545, pp. 1-50.

WEST, K. (1987). "A Speculation Test for Speculative Bubbles", *Quarterly Journal of Economics*, 102, 553-80.

WU, Y. (1995). "Are there Rational Bubbles in Foreign Exchange Markets? Evidence from an Alternative Test", *Journal of International Money and Finance*, Vol. 14, No. 1, pp. 27-46.

WU, Y. (1997). "Rational Bubbles in the Stock Market: Accounting for the U.S. Stock - Price Volatility", *Economic Inquiry,* Vol. XXXV, (April), pp. 309-319.

WU, G. AND XIAO, Z. (2008). "Are there Speculative Bubbles in Stock Markets? Evidence from an Alternative Approach". *Statistics and its Interface*. Volume 1, pp.307-320.

YANG, Q. (2006). *Stock Bubbles: Theory and Estimation*. PhD Thesis, Brunel University, Business School, December.

ZHOU, W. AND SORNETTE, D. (2009). "A Case Study of Speculative Financial Bubbles in the South African Stock Market 2003-2006", *Physica,* A (388), pp. 869-88.

Chapter 3
Monetary Policy and Speculative Bubbles

3.1 Introduction

Evidence exists which suggest that speculative bubbles grow from time to time and that such bubbles may cause economic distortions such as financial and real economic instabilities. Specifically, such economic distortions for South Africa are largely attributed to volatilities in the interest rates, the overall stock market index and the exchange rates, especially after the liberalisation of financial markets in 1995. South Africa's relatively advanced financial markets and having moved towards inflation targeting in 2000, have attracted scholars in establishing how monetary policy rules, which include asset prices, perform in stabilizing the economy (Aron and Muellbauer (2000), Malikane and Semmler (2008)). However, these studies do not separate asset prices into fundamental and non-fundamental components and therefore the optimal policy response implies reacting to the overall asset prices, as opposed to reacting separately to each component.

This essay formulates a macro-model of a small-open economy in order to investigate the relative performance of optimal monetary policy rules that respond to speculative bubbles and those that do not. The study posits that an expected change in the speculative bubble reflects a change in the current asset prices and therefore a change in the expected real output gap and inflation. Stabilization measures for expected inflation require the consideration of expected speculative bubbles because they account for changes in the current asset prices. Since speculative bubbles are asset price movements, independent of fundamentals, as long as they can disturbilize current asset prices, they remain a threat to real economic and consumer price stability. Therefore, they should be incorporated in the central bank's reaction function (Kent and Lowe (1997), Filardo (2001, 2004)). However, economists are divided on whether a central bank should respond to speculative bubbles in its conduct of monetary policy or not. Three distinct notions exist.

The first notion favours the central banks' active response to asset prices. The proponents of this notion include; Smets (1997), Svensson (1998, 2003), Blanchard (2000), Cecchetti et al. (2000), Filardo (2000), Aron and Muellbauer (2000), Borio and Lowe (2002), and Malikane and Semmler (2008). For instance, Borio and Lowe (2002) justify their notion by arguing that booms and busts in asset prices should be considered as part of a broader set of symptoms that typically include a build up of debt and high rate of capital accumulation. Rising asset prices and debt accumulation lead to stretched households and corporate balance sheets, vulnerable to sharp corrections.

The second notion favours a reactive monetary policy response to asset prices and proponents include; Bernanke and Gertler (1999, 2001), Mishkin (2001), Gilchrist and Leahy (2002), Parusel and Viegi (2009), among others. They posit that a central bank dedicated to price stability should pay no attention to asset prices *per se*, except insofar as they signal changes to expected inflation. The main argument by Bernanke and Gertler (2001) is that since asset prices tend to exhibit large swings, a policy rule that responds to them would tend to exhibit large swings in the interest rate. This tends to generate high volatility in output and inflation. Bernanke and Gertler (1999) argues that pricking the perceived speculative bubbles runs the risk of generating financial panics.

The third notion favours the central bank's reaction to speculative bubbles. Those in favour include; Kent and Lowe (1997), Filardo (2001, 2004), Genberg (2001), Rudebusch (2005), Kontonikas and Montagnoli (2006), and Semmler and Zhang (2007), among others. Their main argument is that central banks should prevent speculative bubbles from busting by using the preemptive power of short-term nominal interest rate. Under this notion, the asset price is decomposed into its market fundamental and speculative bubbles and therefore recognizes the imperfect nature of financial markets. However, there is a research gap in South Africa regarding the third notion.

The estimation method has the following steps. Firstly, the paper estimates two structural systems, one with speculative bubbles and the other without bubbles. The structural system consists of an IS curve, a Phillips curve, real stock price and bilateral real exchange rate models. The current real asset prices are determined by current fundamentals and endogenous expected speculative bubbles. Secondly, the parameters for the optimal monetary policy rule are computed by using the estimated parameters of the structural model.

Thirdly, the variances for inflation and real output gap and the loss function for each of the two optimal monetary policy rules are generated.

Using South African quarterly time series data between 1995Q2 and 2008Q4, the study finds that the volatilities of inflation and real output gap are lower for an optimal monetary policy rule that includes speculative bubbles than the one that does not include them. This implies that the optimal response to both fundamentals and speculative bubbles dominates the optimal response to fundamentals. This study also finds that the real stock market price and the bilateral real exchange rate have a smaller impact on real output gap than the real interest rate. Furthermore, the real output gap accounts for a larger impact on inflation than the change in the real exchange rate. The findings suggest that monetary authorities should respond to both speculative bubbles and fundamentals aggressively. These findings are consistent with Rotemberg and Woodford (1997), Woodford (1999), Rudebusch and Svensson (1999), Semmler and Zhang (2007), among others.

The rest of the essay is structured as follows. Section 2 presents an outline of the model. Section 3 presents the model estimation. Section 4 presents the empirical results. Section 5 concludes. References and Appendices thereafter.

3.2 Outline of the Model

A small-open economy is described by three sectors of an economy namely, the IS curve; the Phillips curve; the financial market represented by the capital market and the foreign exchange market. This model is consistent with Svensson (1997, 1998, 1999) and Semmler and Zhang (2007). There are two asset prices: the real stock price and the bilateral real exchange rate. The central bank controls the economy through the short-term interest rate as a policy instrument and follows the Taylor type interest rate rule. The following equations summarize the structure of our model:

$$y_t = \theta_{yy}(L)\, y_t - \theta_{yr} r_{t-1} - \theta_{yq} q_{t-1} + \theta_{ys} s_{t-1} + \varepsilon_t \tag{3.39}$$

$$\pi_t = \theta_{\pi\pi}(L)\, \pi_t + \theta_{\pi y} y_{t-1} - \theta_{\pi q} \Delta q_{t-1} + \eta_t; \text{ such that } \sum \theta_{\pi\pi}(L) = 1 \tag{3.40}$$

$$s_t = -\theta_{sr} r_t + \theta_{sy} y_t + \theta_{sb} b_{s,t+1} \tag{3.41}$$

$$q_t = \theta_{qr}(r_t - r_t^f) + \theta_{qb} b_{e,t+1} \tag{3.42}$$

$$b_{e,t+1} = \left(\frac{1}{2} \left(1 - \tanh\left(\omega\left(b_{e,t}, r_t\right)\right)\right)\left(R_{e1} + R_{e2}\right) - R_{e2} + 1 \right) b_{e,t} + v_{t+1} \tag{3.43}$$

$$b_{s,t+1} = \left(\frac{1}{2} \left(1 - \tanh\left(\vartheta\left(b_{s,t}, r_t\right)\right)\right)\left(R_{s1} + R_{s2}\right) - R_{s2} + 1 \right) b_{s,t} + u_{t+1} \tag{3.44}$$

where, y_t is the real output gap, r_t is the real short-term interest rate (i.e. the nominal short-term rate minus the domestic inflation), r_t^f is the real short-term foreign interest rate, q_t is the log of the bilateral real exchange rate defined as the United States dollar per unit of the South African rand (a higher q means appreciation), s_t is the log real stock price level, d_t is the log real stock market dividends, π is inflation rate, $b_{st,}$ $b_{s,t+1}$ are the current and future nominal stock price bubbles; b_{et}, $b_{e,t+1}$ are the current and future bilateral nominal exchange rate bubbles, θ_{ij} are positive coefficients; R_{s1} and R_{e1} are the growth rates of the stock price bubble and exchange rate bubble, respectively; R_{s2} and R_{e2} are the decrease rates of the stock price bubble and the exchange rate bubble, respectively. $\tanh\left(\omega\left(b_{e,t}, r_t\right)\right)$ is is the hyperbolic tangent of some probability function ω (see Appendix B.A, eqn. (B.11)). $\tanh\left(\vartheta\left(b_{s,t}, r_t\right)\right)$ is also the hyperbolic tangent of some probability function ϑ (see Appendix B.A, eqn. (B.12). These probability functions have been used in Kent and Lowe (1997) and modified by Semmler and Zhang (2007). The $\theta_{ii}\left(L\right)$ functions are standard multiple lag operator functions. The terms ε, η, u, and v are white-noise shocks in their respective models.

Eqn.(3.39) describes a small-open economy's IS-Curve. Real output gap is determined by lagged real output gap, lagged real interest rate, lagged real exchange rate, and lagged real stock price and a demand shock, which represents changes in consumer and business confidence. This model extends the Svensson (1998) open economy IS curve by employing the real stock price. The real interest rate has a negative effect on real output gap. There is a positive relationship between real output gap and its lags. The real stock price positively impacts real output via quantity of wealth and investment balance-sheet ef-

fects (International Monetary Fund (2003)). Real output gap is negatively affected by the real exchange rate, through a current account channel. Both the real stock price and the real exchange rate are driven by market fundamentals and non-fundamental components. The non-fundamental components are known as speculative bubbles, which are asset price changes, driven independently of their fundamental determinants and can collapse or rise at any time without any warning.

Eqn.(3.40) is a small-open economy Phillips curve, which follows Ball (1998) and Svensson (1998:16). Inflation is measured by the growth in consumer price index. Thus, the inflation model, which is the aggregate supply curve, is an accelerationist version of the Phillips curve that has been augmented to positively depend on the influence of lagged real output, negatively depends on lagged real exchange rate appreciation/depreciation, and positively depends on the lagged inflation values. The random disturbance term captures the cost-push shocks to inflation.

Eqns.(3.41) and (3.42) represent the dynamic evolution of asset prices;- the real stock price (s_t) and the real exchange rate (q_t), respectively. These two asset pricing models extend the Kontonikas and Montagnoli's (2006) asset price model by assigning a hyperbolic tangent probabilistic structure to the non-fundamental variable, the speculative bubbles. Both the real stock price and the real exchange rate models depict the actual financial market behaviour where we assume a partial adjustment mechanism, which allows observed asset prices to be misaligned and therefore prices can move independently of their market fundamentals. We allow for the asset prices to be determined by the fundamentals and the speculative bubbles.

From eqn. (3.41) the fundamentals determinants of the real stock price are the real interest rate (Wu and Xiao (2008)) and the real output gap (Smets (1997)). We use real output gap instead of the real dividends because according to Smets (1997) the dividends are proportional to output. The real interest rate has a negative impact on the real stock price, while the real output gap has a positive impact. The non-fundamental determinant of the real stock price are the speculative bubbles, which are a solution to the nominal stock pricing model under rational bubbles theory (Blanchard and Watson (1982)). The speculative bubbles obey the expectational restriction of the non-fundamental solution and are extraneously determined by rational bubbles theory.

The fundamental for the real exchange rate in eqn. (3.42) is the real interest rate differential, which is determined from the uncovered real interest rate parity condition. The real interest rate differential has a positive effect on the real exchange rate. This thesis augments the uncovered real interest rate parity condition with speculative bubbles in the exchange rate to illustrate the presence of deviations from interest parity. This model specification is broader than just augmenting the interest parity with risk premium to indicate possible deviations from parity, as in Svensson (1998). In this model, the risk premium is captured in the shocks, which are a part of the speculative bubbles. Thus, by specifying the asset pricing models with both fundamentals and speculative bubbles, we allow for the effects of having multiple prices in financial markets on both real output gap and inflation.

Eqns. (3.43) and (3.44) describe the dynamic evolutions of the speculative bubbles in the nominal stock price and nominal exchange rate, respectively. These models follow Semmler and Zhang (2007). These speculative bubbles can be positive or negative driven by the objective probabilities of them rising or busting. This specification implies that speculative bubbles can occur again and again after it busts. First, Bernanke and Gertler (1999) and Kent and Lowe (1997) use a bubble which will not occur again after the bust and second, Blanchard and Watson (1982) employ a bubble which will never be negative. We use non-stationary bubbles and ones which do not need to increase before exploding. By implication, asset prices are also non-stationary and highly volatile.

Furthermore, eqns. (3.43) and 3.44) are solutions from some expectational restriction-conditions of the rational bubbles theory (see Appendix B.A). The bubbles models augment the speculative bubbles with hyperbolic tangents for some probability function in order to have a probability that is bounded between 0 and 1 (Semmler and Zhang (2007)). The growth rates for speculative bubbles can either be negative or positive and therefore we use the market returns as proxies for the discount rates (Cunado et al. (2005)). Anything that is not included in the speculative bubbles models but can affect speculative bubbles are captured by the noise shocks. The difference between eqns. (3.43) and (3.44) and the Semmler and Zhang (2007) is that this thesis endogenizes the probability function that is augmented in the speculative bubbles models and we include them in the optimization process. By doing so this thesis allows the current size of the speculative bubble and its growth rate to

be influenced by the current interest rate. This makes speculative bubbles to be consistent with rational expectations (Kent and Lowe (1997)).

The structural model for this study describes the interactions of economic variables for a small-open economy and can be used to derive an optimal monetary policy rule. In an economy described by the IS curve, the Phillips curve, and two asset prices augmented with speculative bubbles, we assume that the monetary authority sets monetary policy by adjusting the short-term real interest rate. However, in practice, central banks use nominal interest rates as their policy tool. In this case, given the inflation rate, the central bank is essentially choosing real interest rate when it picks its nominal interest rate. Woodford (2001) provides explanations for the type of assumptions that would justify using short-term real interest rate as a sufficient variable to link monetary policy to the macroeconomic environment via the equilibrium relationship between the IS curve and the Phillips curve and financial markets.

The structure of the model implies that at time t, the social planner chooses the interest rate, r_t, which affects concurrent real exchange rates, next period's output growth, which in turn affects stock prices, while contemporaneous inflation and real output growth are predetermined by past decisions, past nominal speculative bubbles and current exogenous shocks. The structure of the model is summarized in eqns.(3.39) through (3.44). This thesis solves the structural model more compactly in two simultaneous equations represented by the IS and Phillips curves (See Appendix B.A for the solution):

$$y_{t+1} = \varphi_t + z_{t+1} \tag{3.45}$$

$$\pi_{t+1} = k_t + \eta_{t+1} \tag{3.46}$$

where, $\varphi_t = \tau_1 y_t - \tau_2 r_t - \tau_3 b_{e,t} + \tau_4 b_{s,t} + \tau_5 r_t^f$ is the control variable of the central bank as π_t and y_t are predetermined and r_t is chosen, τ_i are the parameters and $k_t = \theta_{\pi\pi}(L)\,\pi_t + \theta_{\pi y} y_t - \theta_{\pi q}\Delta q_t$ is the state variable at time t.

Assuming that the central bank's intertemporal quadratic loss function L, penalizes both real output gap and inflation volatility, its job is to minimize the loss function:

$$\min \frac{1}{2} E_t \sum_{t=1}^{\infty} \lambda^t L, \quad \lambda \in (0,1) \tag{3.47}$$

where,

$$L = \pi_t^2 + \mu_y y_t^2 = \left(\left[k_t + \eta_{t+1} \right]^2 + \mu_y \left[\varphi_t + z_{t+1} \right]^2 \right) \tag{3.48}$$

Subject to

$$k_{t+1} = \pi_{t+1} + \theta_{\pi y} y_{t+1} - \theta_{\pi q} \Delta q_{t+1} \tag{3.49}$$

$$\Leftrightarrow k_{t+1} = k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \tag{3.50}$$

where, the inflation coefficient is equal to one, $\theta_{\pi\pi}(L) = 1$; $\xi_{t+1} = \eta_{t+1} + \theta_{\pi y} z_{t+1}$ is a Gaussian process. y^2 is the variance of real output gap, π_t^2 is the variance of inflation, $\mu_y \geq 0$ is the penalty on output gap stabilization. $0 < \lambda < 1$ is a discount factor. Eqn. (3.50) indicates the law of motion of the state variable. The implication for having eqns.(3.45) and 3.46) is that as both the interest rate and consequently output φ_t, are chosen, the only state variable is inflation k_t. Thus, the value function, $V(k_t)$, is the expected value of the policymaker's loss function if φ_{t+i} is set optimally. The value function is defined in terms of the state variable, k_t. We use the Bellman's dynamic programming principle as an optimization technique. This thesis first specifies the value function by using the two constraints, eqns.(3.45) and (3.46), to obtain eqn. (3.51) (see Appendix B.D for details).

$$V(k_t) = \min_{\varphi t} E_t \{ \frac{1}{2} \left(\left[k_t + \eta_{t+1} \right]^2 + \mu_y \left[\varphi_t + z_{t+1} \right]^2 \right) + \lambda V(k_{t+1}) \} \tag{3.51}$$

The first-order condition that yields the optimal response is given in eqn.(3.52) and its derivation is shown in Appendix B.D:

$$\mu_y \varphi_t + \theta_{\pi y} \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) = 0 \tag{3.52}$$

The first order condition and the envelop theorem allow us to derive the optimal path for the control variable (see Appendix B.D for details):

$$\varphi_t = -\left(\frac{\theta_{\pi y}\lambda}{\mu_y + (\theta_{\pi y})^2 \lambda}\right) k_t + \left(\frac{\lambda \mu_y}{\mu_y + (\theta_{\pi y})^2 \lambda}\right) E_t \varphi_{t+1} \tag{3.53}$$

Given the linear-quadratic structure of the model, the solution is of the form $\varphi_t = ck_t$. The interest rate, which is set by policy makers, is derived by using the definitions for φ_t, k_t,and c to obtain the optimal real interest rate rule (see Appendix B.D):

$$r_t = \alpha_\pi \pi_t + \alpha_y y_t + \alpha_{be} b_{e,t} + \alpha_{bs} b_{s,t} + \alpha_{rf} r_t^f + \alpha_q \Delta q_t \tag{3.54}$$

where, the $\alpha_j' s$ in eqn.(3.54) are calculated parameters indicating interest rate adjustments, where, $a_\pi = \left(\frac{c_2}{\Omega_1}\right)$ denotes the adjustment in interest rate due to inflation; $a_y = \left(\frac{(c_2\theta_{\pi y} - \tau_1)}{\Omega_1}\right)$ denotes the adjustment in interest rate due to real output gap; $a_{be} = \left(\frac{\Omega_2}{\Omega_1}\right)$ denotes the adjustment in interest rate due to speculative bubbles in the nominal exchange rate; $a_{bs} = \left(\frac{\Omega_3}{\Omega_1}\right)$ denotes the adjustment in interest rate due to speculative bubbles in the nominal stock price; $a_{rf} = \left(\frac{\tau_5}{\Omega_1}\right)$ denotes the adjustment in interest rate due to foreign real interest rate; and $a_q = \left(\frac{c_2\theta_{\pi q}}{\Omega_1}\right)$ denotes the adjustment in interest rate due to the change in the bilateral real exchange rate (see Appendices B.C and B.D for more on definitions). Eqn.(3.54) is the reaction function of the central bank and it is known as an extended Taylor rule, augmented to include the central bank's reactions from speculative bubbles, foreign interest rates and changes in the exchange rate.

The Taylor principle implies that the inflation coefficient should exceed the value of 1, to ensure a real interest rate response that will lead to lower inflation (Kontonikas and Montagnoli (2006)). The inflation coefficient in the interest rate reaction function a_π should be greater than 1 in order to satisfy the stability condition that the real interest rate increases in response to inflation, with higher values implying a more aggressive responses. The stability condition states that the optimized policy adjustment factor, c_2, must be greater than Ω_1.This implies that the countercyclical monetary policy response can be effective by ensuring that the real interest rate increases in response to higher real output gap, lower stock price bubbles and higher real exchange rate bubbles and vice versa.

The optimal monetary policy rule is highly robust to mispecification because it incorporates the probability features of speculative bubbles (see Appendix B.A). Thus, the

optimality of a monetary policy action depends on the subjective assessments about the probability of a bubble emerging or busting in the next period (Bordo and Jeanne (2002)).

For example, using the definition for the adjustment in interest rate due to speculative bubbles in both the nominal stock price and the exchange rate, the effect of the probability on monetary policy rule is positive. Thus, the effects of the change in probabilities on coefficients for speculative bubbles in both the stock price and exchange rate depend on whether the bubble is positive or negative. If the bubble is positive a larger probability leads to a higher coefficient and as a result , a higher real interest rate. This is consistent with the intuition that in order to eliminate a positive bubble which is likely to continue to increase, it is necessary to raise the real interest rate to reduce the real stock price. However, raising the interest rate also causes the real exchange rate to appreciate, making exports less competitive. To every policy reaction, both the stock price and the real exchange rate respond differently, but depends on whether the speculative bubbles is negative or positive (see Appendix B.B). This is so because it is usually argued that there exists a negative relationship between the stock price and the interest rate and a positive relationship between the exchange rate and the interest rate.

3.3 Model Estimation

Based on the structural model, this study tests the null hypothesis that the optimal rule that includes speculative bubbles does not stabilize an economy better than the one that does not. The first step is to conduct unit root tests on all variables in the structural model using the standard Augmented Dickey and Fuller (1979) test, the Phillips and Perron (1988) test, and the Kwiatkowski, Phillips, Schmidt and Shin (1992) test. We test for unit roots in data in order not to mispecify the structural model by including variables which do not guarantee convergence to equilibrium during estimation.

The last column of Table 3.7 gives the order of integration for all the variables. For some series when the ADF tests and the PP tests differ, I use the Kwiatkowski-Phillips-Schmidt-Shin (1992) test. The Kwiatkowski-Phillips-Schmidt-Shin tests the null hypothesis that the series are stationary. This is done for real interest rate for the United States.

Table 3.7. Unit Root Tests for the Monetary Model: 1995Q2-2008Q4

Variables	**ADF Test**		**PP Test**		IO
	Level	1st Diff	Level	1st Diff	
y	-3.26*	-3.49**	-2.47	-3.51**	I(1)
r	-3.38*	-7.33**	-3.49*	-8.28**	I(1)
π	-3.68**		-3.77**		I(0)
q	-2.47	-5.34**	-2.38	-5.37**	I(1)
s	-0.43	-6.64**	-0.59	-6.69**	I(1)
r^f	-2.24	-4.15**	-4.05**		I(1)
e	-2.38	-6.28**	-2.34	-6.33**	I(1)
b_e	-4.10**		-4.21**		I(0)
b_s	-2.92*	-8.3**	-2.9*	-8.91**	I(1)

Note: ** =1%; *=5% and ***=10% levels of significance.
$Source$:$Author$

The second step is to estimate the structural model represented in eqns. (3.39) through (3.44) as a systems regression method. We estimate two structural systems, one with speculative bubbles and the other without bubbles. The third step is to calculate the parameters for the optimal monetary policy rule in eqn. (3.54) using estimated parameters from the structural model. The fourth step is to simulate the variances for inflation and real output gap and compute the loss function.

A simultaneous equation system approach allows for the parameters of various models in a system to be contemporaneously estimated. Since we are studying a single country, a preferred econometric method for systems equations which include asset price bubbles would be ones that account for heteroskedasticity and contemporaneous correlation in the errors across equations. The Seemingly Unrelated Regression (SUR) method is ideal for a single country regression analysis, although for multiple countries, a generalized method of moments (GMM) can be used. The SUR is also known as the multivariate regression, or Zellner's method, and has been employed to estimate the parameters of a system in which the estimators of the cross-covariance matrix are based upon estimators of the unweighted system.

There are other systems estimation techniques such as the full-information maximum likelihood (FIML). The FIML method, which according to Meese (1986) does poorly when the speculative bubbles terms are included in an asset pricing model is not adopted. The poor performance of maximum likelihood methods has also been highlighted in Filardo

(2001), among others who also use asset price bubbles in the estimation of the interest rate rule.

This study's econometric framework follows the Granger representation theorem (Engel and Granger (1987)) and it accommodates the SUR method for estimating a short-run structural model. The Granger representation theorem posits that the moving average autoregressive and error correction representation are connected for cointegrated systems. Cointegration implies that deviations from equilibrium are stationary, with finite variances, even though the series themselves are nonstationary and have infinite variances.

After estimating the structural model, we calculate the unconditional variances of inflation and real output gap using the same structural model. This thesis assumes three values for the penalty on the variance of real output gap, μ_y; as: 0.01, 0.5 and 1.5. We construct graphs depicting Taylor curves by using simulated variances for inflation and real output gap. The idea for doing so is to determine which optimal rule gives the lowest Taylor curve. The Taylor curve is drawn with variance of inflation , π^2, on the horizontal axis and variance of real output gap, y^2, on the vertical axis. The Taylor curve can be used to resolve the decision problem on which monetary policy rule should be adopted:-either strict inflation targeting, or output targeting, or flexible inflation targeting.

3.4 Empirical Results

3.4.1 Data Description

The study uses quarterly data between 1995Q2 and 2008Q4. There are two key data sources: The International Monetary Fund's (2009) International Financial Statistics, Quarterly Issues and from the McGregor BFA data buffet website. The study period covers a single political regime under the African National Congress rule and covers the financial crises periods between 1996Q1 and 2008Q4. Key variables for the model are the real gross domestic product for South Africa; the real short-term interest rates for South Africa (discount rate); Consumer price index for South Africa; the nominal speculative bubbles for the Johannesburg Stock Exchange all share index (JSEI); nominal speculative bubbles for

the bilateral nominal exchange rate; real short term interest rate for the United States (Discount Rate) differential, and the bilateral real exchange rate for the United States dollar per unit of the South African Rand.

3.4.2 Calculating State-dependent Speculative Bubbles

The study uses the following expectational restriction equation under rational bubbles theory to extract speculative bubbles:

$$b_{t+1} = u_{t+1} + \rho\left(1 + R_t\right) b_t \tag{3.55}$$

where, ρ is initially assumed to be 0.5. Since the speculative bubble begins at zero, eqn. (3.55) implies that the probability of the bubble emerging in the next period depends on the probability of having a negative or positive residual. The probability of having a positive or negative residual is always equal to 0.5. Thus, the speculative bubble in the first period is the corresponding residual in that period (Semmler and Zhang (2007)). The proxy for the discount factor is the market return. Figures 4.1 and 4.2 indicate the extracted speculative bubbles for the bilateral nominal exchange rate and the stock price, respectively.

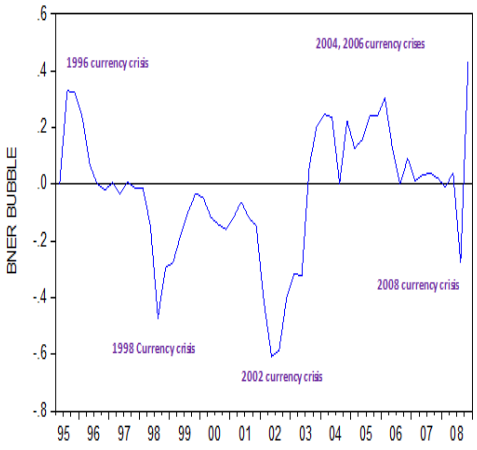

Fig. 4.1- Exchange Rate Bubbles

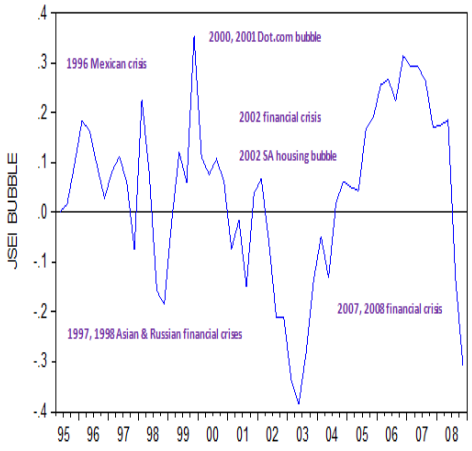

Fig. 4.2- JSE Index Bubbles

Source:*Author*

The extracted speculative bubbles closely follow the major financial crisis periods over the sample period and which is in agreement with Reinhart and Rogoff (2009) and Bond (2010). After extraction we then employ Semmler and Zhang (2007) endogenous

Table 3.8. Parameters for the Estimation of Probability of Bubbles

Parameters	Result	SZ (2007)
ϕ_1	0.31	na
ϕ_2	0.08	na
ϕ_3	0.34	0.4
ϕ_4	0.12	0.8
a	15	10
v	1	1
R_{e1}, R_{e2}	time-varying	$g_1 = 0.01$, $g_2 = 3.0$
R_{s1}, R_{s2}	time-varying	

Note: SZ (2007) denotes Semmler and Zhang (2007).
$Source: Author$

probability model (see Appendix B.A, eqns. (B.9) and (B.10)). The parameters for the calculation of the probability function are presented in Table 3.8.

Table 3.8 shows the empirical results for the probability functions for each of the two speculative bubbles, for the whole period. The parameter estimates for the variables used in simulating probabilities for bubbles measure the effects of bubbles and the interest rate on the magnitude of probability switching. The size of probability switching, as determined from the linex function (see Appendix B.A, eqn. (B.14)), is a dependent variable in an ordinary least squares (OLS) regression equation to determine the parameter values for the effects of speculative bubbles size and the interest rate on the probability function of the speculative bubbles. The standard full information maximum likelihood systems regression method is employed to determine the parameter estimates. The estimated parameters in Table 3.8 are comparable to those in Semmler and Zhang (2007) and Kent and Lowe (1997) for industrialized countries.

3.4.3 Parameter Estimation: Structural Model Results

This study calculates the coefficients for the optimal monetary policy rule and investigates its macroeconomic performance by simulating variances of inflation and real output gap. The study compares an optimal rule with bubbles to the one without bubbles and then decides which one achieves the best stabilization of real output gap and inflation volatility.

Table 3.9 shows that the real interest rate has a larger impact on real output gap than both the real exchange rate and the real stock price. However, lagged real output gap

Table 3.9. Structural Model Results (standard errors in parenthesis)

1). Model With Bubbles	
Real Output Gap	Bilateral Real Exchange Rate
$\theta_{yr} = -0.04^{*}(0.01)$	$\theta_{qr} = 0.05^{***}(0.31)$
$\theta_{yy} = 0.97^{**}(0.13)$	$\theta_{qb} = 0.08^{***}(0.06)$
$\theta_{yq} = -0.008^{*}(0.004)$	$\theta_{qr^f} = -0.56^{***}(0.32)$
$\theta_{ys} = 0.007^{*}(0.003)$	
Inflation	Real Stock Price
$\theta_{\pi\pi} = 0.63^{**}(0.11)$	$\theta_{sr} = 0.22^{***}(0.33)$
$\theta_{\pi y} = 0.48^{*}(0.21)$	$\theta_{sy} = 0.65^{**}(0.18)$
$\theta_{\pi q} = -0.03^{**}(0.009)$	$\theta_{sb} = 0.08^{***}(0.09)$
2). Model Without Bubbles	
Real Output	Bilateral Real Exchange Rate
$\theta_{yr} = -0.03^{*}(0.12)$	$\theta_{qr} = 0.08^{***}(0.31)$
$\theta_{yy} = 0.96^{**}(0.03)$	$\theta_{qr^f} = -0.73^{*}(0.31)$
$\theta_{yq} = -0.008^{*}(0.004)$	
$\theta_{ys} = 0.007^{*}(0.0004)$	
Inflation	Real Stock Price
$\theta_{\pi\pi} = 0.62^{**}(0.08)$	$\theta_{sr} = 0.17^{***}(0.32)$
$\theta_{\pi y} = 0.60^{**}(0.23)$	$\theta_{sy} = 0.68^{**}\ (0.17)$
$\theta_{\pi q} = -0.04^{**}(0.009)$	

Note: **, *, and *** denote 1%, 5%and10% levels, respectively.
$Source$:$Author$

accounts for the largest variation in real output gap. The comparison of the effects of both real interest rate and the real exchange rate justifies why the interest rate is preferred as an instrument. Knedlik (2006) gets the similar result from using the monetary conditions index that the interest rate has a larger effect on real output gap than the real exchange rate. The study uses the estimated coefficients to determine the adjustment parameters of all the variables in the optimal monetary policy rule.

Table 3.9 reports the estimated parameters of two types: parameters from a model with speculative bubbles and without them. Both the real exchange rate and the real stock price maintain the same coefficients as before, but the impact of the real interest rate falls when speculative bubbles are excluded. The parameters which determine the Phillips curve change by increasing the responsiveness of inflation to excess demand. The responsiveness of inflation to the exchange rate improves but remains smallest. The real exchange rate model also shows an increased impact of both the South Africa and United States short term interest rates. The real stock price model results are mixed with an increased impact

of real output gap and a decreased impact of real interest rate. The results confirm that the Blanchard-Tobin effect is a real problem with the real interest rate. The deleterious income effects of the real exchange rate is also real and is referred to as the Dornbusch effect. On the other hand, the indirect Mundell effect is a real threat and is represented by the indirect deleterious effect of inflation on real output growth.

3.4.4 Evaluation of Monetary Policy Rules

The study evaluates the optimal monetary policy rules across three sets of central bank preference choices of weights on the variance of real output gap. Table 3.10 shows the coefficients for the optimal monetary policy rules and Figures 4.3 and 4.4 and Table 3.11 gives an evaluation of the performance of the optimal rules.

We compute the model's parameters in eqn. (3.54) using the coefficients in Table 3.9. The structure of the optimal monetary policy rule for calculating these parameters is derived in Appendix B.D. The monetary policy adjustment coefficient is c_2. This thesis assumes three sets of central bank preferences, which are consistent with Rudebusch and Svensson (1999). The sets of preferences are represented by $\{\mu_y, c_2\} = \{0.01, -1.78\}$; $\{0.05, -1.85\}$; $\{1.5, -1.88\}$), as illustrated in Table 3.10. Additional information at the bottom of Table 3.10 shows the sample period averages for probability features. The average growth rate of the stock price bubble over the sample period is 180 percent and its average decrease rate is -70 percent. The average increase rate of the exchange rate bubble is 1.6 percent while its average decrease rate is -88 percent.

Table 3.10 shows computed coefficients for the optimal monetary policy rule, in basis points. The coefficients indicate an aggressive monetary policy. Monetary policy remains aggressive whether speculative bubbles are included or excluded from the structural model. However, the central bank responds more aggressively when speculative bubbles are included in the optimal monetary policy rule. Although Woodford's (1999:76) result does not include all the target variables being considered here, his results are consistent with his inflation coefficient of 46.1 in the optimal rule. The conclusion is that targeting real output gap gives a much more aggressive reaction than targeting inflation. Malikane and Semmler (2008) also conclude that more aggressive response is coming from real output gap.

Table 3.10. Optimal Rule over Different Preferences

Optimal rule- with bubbles				Optimal rule -no bubbles		
μ_y	0.01	0.5	1.5	0.01	0.5	1.5
c_2	-1.78	-1.85	-1.88	-1.78	-1.85	-1.88
coefficients	{in basis points}			{in basis points}		
α_π	47.05	49.04	49.65	30.13	31.40	31.80
α_y	48.11	49.06	49.35	34.25	35.02	35.25
α_{bs}	-0.79	-0.79	-0.79	-	-	-
α_{be}	0.06	0.06	0.06	-	-	-
α_f	0.42	0.42	0.42	0.27	0.27	0.27
α_q	-1.41	-1.47	-1.49	-1.21	-1.26	-1.27
Estimated information						
ρ_s	0.22	ρ_e	0.26			
R_{s1}	1.798	R_{e1}	0.016			
R_{s2}	-0.696	R_{e2}	-0.879			

Source:Author

As implied by this study's Phillips curve, monetary policy must be aggressive and preemptive in order to head off inflation ahead of time. The reason being that inflation expectations are slow to change. The results show that preemptive monetary policy rule is a possibility and has support from Rudebusch and Svensson (1999), Ball (1998), Kontonikas and Montagnoli (2006). This study allows the expected probability of the speculative bubbles to be estimated in order to influence the coefficient of the speculative bubbles. The knowledge about the empirical probability gives power to central banks on when to influence the evolution of the speculative bubbles. The empirical probability gives information on the likelihood of a speculative bubble.

Figures 4.3 and 4.4 depict the efficiency of the optimal monetary policy, by comparing the one with speculative bubbles and another without. The study uses the variances of inflation and real output gap as measures of macroeconomic stability. The Taylor curve shows that the optimal monetary policy with speculative bubbles is more efficient than the one without. The optimal rule that excludes speculative bubbles is above that which includes them. The variances of inflation and real output gap are higher for an optimal rule that excludes speculative bubbles than the one that includes them. Therefore, the central bank should respond to speculative bubbles. The Taylor curves further indicate that the macroeconomic effects of responding to the speculative bubbles have stabilizing effects.

The graphs are drawn as a way of comparing the stabilization effects of the central bank's reaction function on the variances of inflation and real output gap. Each figure compares an optimal monetary policy rule with speculative bubbles to the one without them.

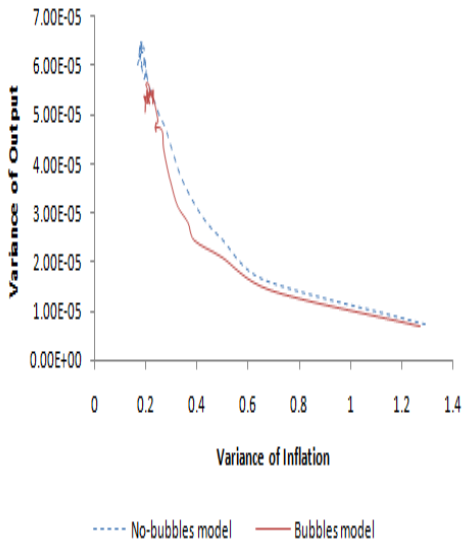

Fig. 4.3- Performance of the Optimal rule, $\mu_y = 0.5$

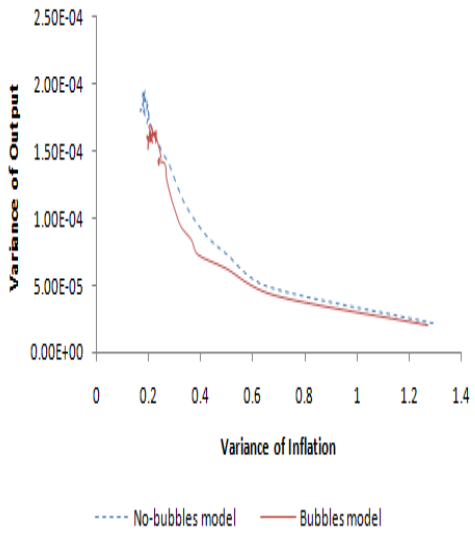

Fig. 4.4- Performance of the Optimal rule, $\mu_y = 1.5$

Source:Author

Figure 4.3 shows the result for the central bank's preference of a 0.5 weight on the variance of real output gap. In this case, the optimal rule with the speculative bubbles gives the lower Taylor curve implying that it has a better performance, especially in the middle portions of the figure. This result shows that the flexible rule would give the lowest variances for inflation and real output gap. Figure 4.4 shows that if the central bank increases its punishment for volatility of real output gap, the optimal rule with speculative bubbles out-performs the one without. Basically, the results depicted in Figures 4.3 and 4.4 indicate the same pattern of results that the optimal rule with speculative bubbles will tend to be superior to the one without bubbles.

The optimal rule with bubbles has a larger stabilization effect than the rule without bubbles, but Table 3.11 shows that stabilization is achieved at greater loss to welfare. The results for the objective function of the central bank indicates higher losses for the bubbles rule across all the three central bank's preferences. Table 3.11 indicates that the statistics

Table 3.11. Performance of the Optimal Rule

	Preferences	Standard Deviation		Objective Function
Rules	Implicit μ_y	π	$\bar{y}$	$Loss$
Bubbles	0.01	0.26	0.001	1.27
No-bubbles	0.01	0.49	0.001	0.24
Bubbles	0.5	0.51	0.01	0.26
No-bubbles	0.5	0.49	0.01	0.24
Bubbles	1.5	0.51	0.01	0.26
No-bubbles	1.5	0.49	0.01	0.24
RW (1997)		(0.39)	(11.3)	(0.93)

Note: RW (1997) denotes Rotemberg and Woodford (1997)
$Source$:$Author$

for the standard deviations and the objective function are for the whole sample period, as period averages.

The results in Table 3.11 have empirical support from the literature (Rotemberg and Woodford (1997)). Rotemberg and Woodford (1997) report values for standard deviations of inflation and real output gap that are less than 1 for inflation and their reported value for the loss function is also less than 1.

This thesis shows that responding to speculative bubbles is better than not responding to them. This implies that the optimal rule that includes speculative bubbles stabilizes inflation and real output gap better than without them. But the study also shows that stabilization of inflation and real output gap variances can be achieved at a greater welfare loss when responding to speculative bubbles. This is comparable to Semmler and Zhang (2007) who conclude that responding to speculative bubbles with a zero bound interest rate is desirable but that there are increasing welfare loss implications associated with it.

3.5 Conclusions

This study develops a dynamic optimal monetary policy rule, which has a role for speculative bubbles. The objective of the paper has been to determine the effects of the central bank's response to speculative bubbles on the macroeconomic performance for South Africa between 1995Q2 -2008Q4. By estimating an endogenized probability function for

each of the asset price bubbles, the study incorporates the non-linear effects of economic booms and recessions in the optimal monetary policy rule, since the endogenous probability that enters the model is a non-linear function of interest rate and speculative bubbles.

The model shows that a non-linear combination of the optimal interest rate rule and speculative bubbles is a possibility with both highly and lowly induced bubbles. Thus, as the probability of the bubble falls, the speculative bubbles will increase, allowing negative bubbles to become less and less negative. The possibility of nonlinearity implies the possible existence of multiple equilibria in financial asset price models and the existence of non-linear rules.

The significance of speculative bubbles in the interest rate rule indicates that the South African Reserve bank may to some extent have taken into account the financial markets imbalances in the past decade. Simulations indicate that the overheating of the financial markets can turn out to be good for targeting speculative bubbles because of the inflation and real output gap variance minimization effects. But targeting asset price bubbles may not be as loss minimizing as the non- bubble targeting rules. The results suggest that the monetary policy rule is aggressive and that speculative bubbles should be included in the optimal monetary policy rule for macroeconomic stabilization reasons. Thus, penalizing real output gap is beneficial to an economic planner and policy actions that aim at targeting inflation should not ignore imbalances in the financial markets.

References

ARON, J. AND MUELLBAUER, J. (2000). *Estimating Monetary Policy Rules for South Africa,* Central Bank of Chile Working Papers, December, No. 89, pp. 1-48.

BALL, L. (1998). "Policy Rules for Open Economies", *NBER Working Paper Series*, Working paper no. 6760, October, pp.1-23.

BERNANKE, B. AND GERTLER, M. (1999). " Monetary Policy and Asset Price Volatility", In *New Challenges for Monetary Policy*, A symposium sponsored by the Federal Reserve Bank of Kansas City, Jackson Hole, Wyoming, 26-29 August.

BERNANKE, B. AND GERTLER, M. (2001). " Should Central Banks Respond to Movements in Asset Prices?, *American Economic Review,* May.

BLANCHARD, O. (2000). "Bubbles, Liquidity Traps, and Monetary Policy," in A. Posen and R. Mikitani (eds), *Japan's Financial Crisis*, Washington D.C.: International Institute of Economics.

BLANCHARD, O.J. AND WATSON, M.W., (1982). "Bubbles, Rational Expectations, and Financial Markets", *Crises in the Economic and Financial Structure,* Wachtel , P. (Ed), Lexington, Mass: Lexington Books.

BOND, P. (2010). "South Africa's Bubble Meets Boiling Urban Social Protest", *Monthly Review*, Vol., 62, No. 2.

BORDO, M. AND JEANNE, O. (2002). "Boom-Busts in Asset Prices, Economic Instability, and Monetary Policy," *NBER Working Paper*, no. 8966, May.

BORIO, C. AND LOWE, P. (2002). *Asset Prices, Financial and Monetary Stability: Exploring the Nexus*, Paper Presented at the BIS Conference on "Changes in Risk through Time: Measurement and Policy Options", BIS Working Papers, No. 114, July.

CECCHETTI , S.G.; GENBERG, H.; LIPSKY, J.; AND WADHWANI, S. (2000). "Asset prices and Central Bank policy", *Geneva Reports on the World economy* No.2 , International Centre for Monetary and Banking Studies and Centre for Economic Policy Research, Geneva.

CUNADO, J., GIL-ALANA, L.A, PEREZ DE GRACIA, F. (2005), "A test for rational bubbles in The NASDAQ stock index: a fractionally integrated approach", *Journal of Banking and Finance*, 29, 2633 - 2654.

DICKEY, D.A. AND FULLER, W.A. (1979). " Distribution of Estimators for Autoregressive Time Series with a Unit Root", *Journal of the American Statistical Association,* 74, 427-431.

ENGEL, R., AND GRANGER, C. (1987). "Cointegration and Error Correction: Representation, Estimation and Testing ". *Econometrica*. 55, 251-276.

FILARDO, A. J. (2000). " Asset Prices and Monetary Policy", *Federal Reserve Bank of Kansas City Economic Review*, third quarter.

FILARDO, A.J. (2001). "Should Monetary Policy respond to asset price bubbles? Some Experimental results", In: Kaufman, G. (ed) Asset Price bubbles: implications for monetary and regulatory policies, vol. 13, *Elsevier Science,* Amsterdam, pp. 99-123.

FILARDO, A. J. (2004)." Monetary Policy and Asset Price Bubbles: Calibrating the Monetary Policy tradeoffs", *BIS Working Paper* No. 15.

GENBERG, H. (2001). "Asset prices, monetary policy and macroeconomic stability", *De Economist*, 149, pp.433-53.

GILCHRIST, S. AND LEAHY, J. V. (2002). "Monetary policy and asset prices", *Journal of Monetary Economics*, 49, pp. 75-97.

INTERNATIONAL MONETARY FUND (2003). "When Bubbles Burst", *World Economic Outlook*, Chapter 2, April, pp. 61-94.

INTERNATIONAL MONETARY FUND (2009). *International Financial Statistics*, Quarterly, IMF website- Wits University Library.

KENT, C. AND LOWE, P. (1997). "Asset Price Bubbles and Monetary Policy", *Reserve Bank of Australia Research Discussion Paper*, no. 9709.

KNEDLIK, T. (2006). "Estimating Monetary Policy Rules for South Africa", *South African Journal of Economics*, 74(4), pp. 629-642.

KONTONIKAS, A. AND MONTAGNOLI, A. (2006). "Optimal monetary policy and asset price misalignments", *Scottish Journal of Political Economy*, vol. 53, no. 5, November. Pp.636-654.

KWIATKOWSKI, D.; PHILLIPS, P.C.B.; SCHMIDT, P.; AND SHIN, Y. (1992). "Testing the Null Hypothesis of Stationarity against the Alternative of a Unit Root. How sure are we that economic time series have a unit root? *Journal of Econometrics* 54, pp.159-178.

LUX, T. (1995). "Herd Behaviour, Bubbles and Crashes", *The Economic Journal,* Vol. 105, No. 431, pp. 881-896.

MALIKANE, C. AND SEMMLER, W. (2008). “Asset prices, Output and Monetary Policy in a small open economy”, *Metroeconomica,* 59 (4), pp. 666-686.

MEESE, RICHARD, A. (1986). “Testing for Bubbles in Exchange Markets: A Case of Sparkling Rates?” *The Journal of Political Economy*, vol. 94, no. 2, pp. 345-373.

MISHKIN, F.S. (2001). “The transmission mechanism and the Role of asset prices”, NBER Working paper No. 8617.

MUSSA, M. (1976). “The Exchange Rate, the Balance of Payments, and Monetary and Fiscal Policy under a Regime of Controlled Floating” , *Scandinavian Journal of Economics*, 78, 229-48.

NICHOLLS, D. AND QUINN, B. (1982). *Random Coefficient Autoregressive Models: An Introduction*. Springer-Verlag.

NOBAY, A, R. AND D.A. PEEL(2003). "Optimal Discretionary Monetary Policy in a Model of Asymmetric Central Bank Preferences", *Economic Journal*, Royal Economic Society, 113 (489), pp. 657-665.

PARUSEL, M. & VIEGI, N. (2009). *Economic Policy in Turbulent Times*, TIPS Discussion Papers Series. TIPS 2009, pp. 1-27

PHILLIPS, P.C.B. AND PERRON, P. (1988). “ Testing for Unit Roots in Time Series Regression”, *Biometrika,* 75, 335-346.

POTERBA, J.M. AND L.H. SUMMERS (1986). "The Persistence of Volatility and Stock Market Fluctuations", *The American Economic Review*, Vol. 76, No. 5, pp. 1142-1151.

ROTEMBERG, J.J. AND WOODFORD, M. (1997). " An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy", *NBER Macroeconomics Annual*, Vol. 12, pp. 297-346.

REINHART, C.M. AND ROGOFF, K.S. (2009). *This Time is Different: Eight Centuries of Financial Folly*, Princeton University Press, Princeton and Oxford

RUDEBUSCH, G.D. (2005). “Monetary Policy and Asset Price Bubbles”, Federal Reserve Bank of San Francisco (FRBSF), *Economic Letter*, vol. 18., no. 2005, August.,pp. 1-3.

RUDEBUSCH, G. D. AND SVENSSON, L.E.O. (1999). “Policy Rules for Inflation Targeting”, in John B. Taylor (ed), *Monetary Policy Rules*. Chicago, The University of Chicago Press.

SEMMLER, W. AND ZHANG, W. (2007). “Asset price volatility and monetary policy rules: a dynamic model and empirical evidence”, *Economic Modelling,* 24, pp. 411-30.

SMETS, F. (1997). “Financial asset prices and monetary policy: Theory and evidence”, *Bank for International Settlements Working Paper* no. 47, Sept.

SVENSSON, L.E.O. (1997). "Inflation Forecasting Targeting: Implementing and Monitoring Inflation Targets". *European Economic Review.* 41, pp. 1111-1146.

SVENSSON, L.E.O. (1998). "Open Economy Inflation Targeting", *NBER Working Paper Series*, Working paper 6545, pp. 1-50.

SVENSSON, L.E.O. (1999). "Inflation Targeting: Some Extensions". *Scandinavian Journal of Economics.* 101 (3), pp. 337-361.

SVENSSON, L.E.O. (2003). " Monetary Policy and Real Stabilisation", *National Bureau of Economic Research*, *Working Paper 9486*, February.

VARIAN, H.R. (1975). A Bayesian Approach to Real Estate Assessment. In: Fienberg, S.E., Zellner, Amold (eds.), *Studies in Bayesian Econometrics and Statistics in Honour of Leonard J. Savage.* Amsterdam, North-Holland, pp. 195-208.

WOODFORD, M. (1999).Optimal Monetary Policy Inertia, *NBER Working Papers,* no. 7261,

WOODFORD, M. (2001). “The Taylor Rule and Optimal Monetary Policy”, *American Economic Review*, Vol. 91, May, pp. 232-37.

WU, GUOJUN AND XIAO, Z. (2008). “Are there Speculative Bubbles in Stock Markets? Evidence from an Alternative Approach”. *Statistics and its Interface*. Volume 1, pp.307-320.

Chapter 4
Conclusions and Policy Implications

4.1 Conclusions

Chapter 1 formulates the analytical framework for the dynamics in speculative bubbles, which have a probability of crashing and regenerating, assuming rational expectations. The rational contagion analytical framework which is developed in this essay employs a combination of features from Orlean (1989); Lux (1995); Blanchard and Watson (1982) and Semmler and Zhang (2007). The findings indicates that speculative bubbles are unstable and a permanent feature of asset prices. The instabilities in the speculative bubbles positively depend on the strength of herd behaviour and whether speculative bubbles are positive or negative. By implication, the asset price equilibria when speculative bubbles exist are not sustainable.

The inclusion of the interest rate in the probability function is meant to incorporate in the contagion model the effect of monetary policy on herd behaviour. The first essay analyses the rationality of the contagion of opinions in explaining the dynamic structure of speculative bubbles. The contagion model has all the three features of the rational behavioural view:- investors' practices, regulatory practices and multiple equilibria.

Chapter 2 measures speculative bubbles in both the stock price and the bilateral nominal exchange rate in South Africa between 1995Q2 and 2008Q4. The computed speculative bubbles have fat tails in their distributions, and these properties are similar to their asset prices. The study's theoretical basis is grounded in the standard no-arbitrage models. The stock price model is the standard present value formula, while the exchange rate model is the uncovered interest parity condition. The essay also shows that the evolution of speculative bubbles follows a three state process. That is, they can either be negative or positive or even zero. The tests for the existence of speculative bubbles are done by using two different cointegration techniques, namely: Engle and Granger (1987) and Phillips and Ouliaris (1991) cointegration tests. The post-estimation and post extraction phase uses the

unit roots and the Granger causality tests to justify that the extracted asset price bubbles are indeed speculative bubbles. The basic idea is that if fundamentals Granger cause asset prices, then the extracted bubbles would not be speculative bubbles. However, speculative bubbles can Granger cause fundamental values, as is the case for the stock pricing model.

Chapter 3 derives a dynamic optimal monetary policy rule, which responds to speculative bubbles and fundamentals. The objective of the essay is to compare the performances of optimal rules which respond to speculative bubbles and those which do not. The essay incorporates the non-linear effects of economic booms and recessions in the optimal monetary policy rule by using an endogenous probability which is a non-linear function of interest rate and speculative bubbles.

The analytical framework shows that a non-linear combination of the optimal interest rate rule and speculative bubbles is a possibility with both highly and lowly induced bubbles. Therefore, as the probability of the bubble falls, the speculative bubbles will increase, allowing negative bubbles to become less and less negative. The possibility of nonlinearity implies the possible existence of multiple equilibria in financial asset price models and the existence of non-linear rules.

The significance of speculative bubbles in the interest rate rule indicates that the South African Reserve bank may to some extent have taken into account the financial markets imbalances in the past decade. The results suggest that the monetary policy rule is aggressive and that speculative bubbles should be included in the optimal monetary policy rule for macroeconomic stabilization reasons.

4.2 Policy Implications

The study posits that there is a strong relationship between the interest rate, bubble size and the transition probability. For example, as the interest rate increases, the transition probability for the expected stock price bubbles falls sharply for positive bubbles and increases less sharply for negative stock bubbles. However, at the same time, an increase in interest rate causes the transition probability for the expected exchange rate bubbles to increase sharply for positive bubbles and to fall less sharply for negative bubbles. A decrease in

interest rate tends to give opposite effects on the transition probability for expected speculative bubbles. Estimating the transition probability is useful to the central bank in order to know when to react to expected speculative bubbles. The change in the transition probability due to changes in interest rates posses a major policy challenge. For example, a raise in the interest rate appreciates the exchange rate and a fall in the transition probability. This happens because if speculative bubbles are positive, lower interest rates balloons speculative bubbles in stocks. However, an interest rate hike leads to a reduction in the stock prices and to a buying frenzy.

The use of optimal policy rules, which incorporate both speculative bubbles, does not discriminate the policy dilemma of choosing between stock price and exchange rate targeting, since the central bank only has one policy objective of improving macroeconomic stability. An optimal monetary policy rule that responds to speculative bubbles can achieve such a policy objective.

The central bank can best achieve both inflation targeting and the best compromise between inflation and real output gap stability by engaging in proactive monetary policy forecasting activities, where the central bank selects the feasible combination of inflation and real output gap projections which minimize the loss function and the corresponding interest rate and sets the short-term nominal rate accordingly.

In these activities, asset price developments and potential asset price bubbles are taken on board and responded to the extent that they affect the projections of the target variables, such as inflation and real output gap. Situations can arise when asset price developments are deemed unsustainable and hence speculative bubbles, and when a future collapse of speculative bubbles is imminent. If the probability of such a future collapse is deemed to impact on inflation or real output gap projections, the central bank may want to adjust policy to moderate asset price developments and reduce the probability of future busts, thereby achieving more preferable inflation and real output gap projections.

Appendix A
Appendix to Chapter 1

A.A Derivation of the Pure Contagion Model

The time path of the speculative bubble is expressed and conforms to the Master equation approach (Weidlich and Haag (1983), Lux (1995)):

$$\frac{db}{dt} = bv(1+R)(\exp\{ab\}) - bv(\exp\{-ab\}) \tag{A.1}$$

let $\frac{db}{dt} = x$,

$$x = bv(1+R)(\exp\{ab\}) - bv(\exp\{-ab\}) \tag{A.2}$$

Expanding eqn. (A.2) gives

$$x = (1+R)\, bve^{ab} - b\frac{v}{e^{ab}} \tag{A.3}$$

Using De Moivre's theorem, the Euler's eqn. (A.3) is solved for sinh and cosh function as (Weidlich and Haag (1983));

$$\begin{aligned} x &= bv \sinh ab \cosh ab^2 v + bv\left(\cosh ab + \sinh ab\right)(1+R) \\ &= bv\left(\cosh ab + \sinh ab\right)(1+R) - b\frac{v}{\cosh ab + \sinh ab} \end{aligned} \tag{A.4}$$

Let $x = \dot{b}$, eqn. (A.4) becomes

$$\dot{b} = bv\left(R+1\right)\left(\sinh ab + \cosh ab\right) - bv\left(\cosh ab + \sinh ab\right)^{-1} \tag{A.5}$$

Applying the De Moivre's rule that:

$$\left(\cos x + i \sin x\right)^n = \cos\left(nx\right) + i \sin\left(nx\right) \tag{A.6}$$

Eqn. (A.5) becomes:

$$\dot{b} = bv\left(1+R\right)\left(\sinh ab + \cosh ab\right) - bv\left(-\cosh ab - \sinh ab\right) \tag{A.7}$$

$$\dot{b} = bv\,(1+R)\,(\sinh ab + \cosh ab) + bv\,(\sinh ab + \cosh ab) \tag{A.8}$$

The mean value for the speculative bubbles follow the standard approach:

$$\dot{b} = v\,[2+R]\,(b\sinh(ab) - b\cosh(ab)) \tag{A.9}$$

Eqn. (A.9) uses the standard sine-cosine formula which implies that the evolution of the time path of the speculative bubbles follows the hyperbolic sine and cosine standard approach. The final mean value solution is derived using the following standard explicit mean value formula:

$$\frac{dx}{dt} = \Upsilon\,[\tanh(u) - x]\cosh(u) \tag{A.10}$$

where, $u = ab$, $x = b$, $\Upsilon = v\,[2+R]$. Thus, the time path of the speculative bubbles in eqn.(A.9) can be expressed using $tanh$ as:

$$\dot{b} = v\,[2+R]\,(b\tanh(ab) - b)\cosh(ab) \tag{A.11}$$

This solution transformation has support from Weidlich and Haag (1983) and Lux (1995).

A.B Proof of Boundedness of the Endogenous Probability Function

The boundedness property of eqn.(1.16) can be proved by first taking the derivative of the probability function, $\rho(b)$ with respect to bubble size, b and then taking limits as bubble size, b , approaches positive infinity and negative infinity;

$$\begin{aligned}\frac{d\rho_{+-}(b)}{d(b)} &= \frac{d}{db} - \frac{1}{2}\tanh(b) = \frac{1}{2}\tanh^2(b) - \frac{1}{2} \\ &= \frac{1}{2\cos^2 i(b)}\left(\cos^2 i(b) - 1\right) - \frac{1}{2} = -\frac{1}{2\cosh^2(b)} < 0\end{aligned} \tag{A.12}$$

Eqn.(A.12) indicates the opposite effects between the probability and the bubbles. We prove that the probability function is bounded between 0 and 1 by taking the limits on the probability function and solving as;

$$\lim_{(b)\to\infty} \frac{1}{2}\left(1-\tanh(b)\right)=0 \tag{A.13}$$

and

$$\lim_{(b)\to-\infty} \frac{1}{2}\left(1-\tanh(b)\right)=1 \tag{A.14}$$

This concludes the proof.

A.C Properties of the Endogenous Probability Function

Semmler and Zhang (2007) assumes that the probability that the speculative bubble will increase or decrease in the next period can be influenced by the interest rate, r, and the size of the speculative bubble, b. Thus, assuming a time varying probability function, the next period probability function for the speculative bubbles, $\rho(b,r)_{t+1}$, follows two paths, which are defined using the endogenous probability function as:

$$\rho_{+-}(b,r)_{t+1}=\frac{1}{2}\left(1-\tanh\left(\omega\left(b_t,r_t\right)\right)\right) \tag{A.15}$$

and

$$\rho_{-+}(b,r)_{t+1}=\frac{1}{2}\left(1-\tanh\left(\vartheta\left(b_t,r_t\right)\right)\right) \tag{A.16}$$

where,

$$\omega\left(b_t,r_t\right)=\phi_1 f_o(b)_t+\phi_2 sign(b_t)r_t,\ \ \phi_i>0 \tag{A.17}$$

and

$$\vartheta\left(b_t,r_t\right)=\phi_3 f_p(b)_t+\phi_4 sign(b_t)r_t,\ \ \phi_i<0 \tag{A.18}$$

where, $\rho_{+-}(b)_{t+1}$ and $\rho_{-+}(b)_{t+1}$, are period t+1 probability functions for the optimists and the pessimists, respectively, and $sign(b)$ is the sign function for bubbles which is specified as a three state Markov chain process:

$$sign(b)_t=\begin{array}{c}1,\text{if }(b)_t>0\\0,\ \text{if }(b)_t=0\\-1,\ \text{if }(b)_t<0\end{array} \tag{A.19}$$

where, $f_o(b)$ in eqn.(A.17) is the linex function for optimists and $f_p(b)$ in eqn. (A.18) is the linex function for pessimists. The linex function is nonnegative and asymmetric around

0. Borrowing from Semmler and Zhang (2007), the following is the specification for the LINEX function as it is found in Varian (1975) and Nobay and Peel (2003):

$$f(b) = v\left[\exp\{ab\} - ab - 1\right],\ v > 0, a \neq 0 \tag{A.20}$$

where, v scales the function $f(b)$, and a determines the asymmetry of the function $f(b)$. For analytical purposes, Semmler and Zhang's (2007) specifications for $v = 1$ and $a > 0$ are adopted. Applying the standard probability structure for comparative purposes, eqn.$(A.20)$ gives shape to the probability function for each of the asset price bubbles. The trajectory of the probability function will be more flatter when the bubble is negative than when the bubble is positive, as long as a is not equal to zero. Furthermore, eqns.$(A.15)$ and $(A.16)$ indicate the compactness of the probability function (that is, it is bounded between 0 and 1). Using the general eqn.$(A.12)$, the following important properties of the probability functions for each of the two asset price bubbles are deduced:

$$\frac{d\rho_{+\,-}(b,r)_{t+1}}{d(b)_t} = -\frac{\phi_1\alpha\left(\exp\{\alpha b_t - 1\}\right)}{2\cosh^2(\omega\left(b_t, r_t\right))} < 0\ ,\forall\ (b)_t > 0 \tag{A.21}$$

$$and\ \ \frac{d\rho_{-\,+}(b,r)_{t+1}}{d(b)_t} = -\frac{\phi_3\alpha\left(\exp\{\alpha b_t - 1\}\right)}{2\cosh^2(\vartheta\left(b_t, r_t\right))} > 0\ ,\forall\ (b)_t < 0 \tag{A.22}$$

Eqns. $(A.21)$ and (A.22) indicate that the slope or the rate of change of the probability function for optimists and pessimists is negative if the bubble is positive and is positive if the bubble is negative. The probability functions that are defined in eqns. (A.21) and (A.22) are asymmetric around $b = 0$. Furthermore, eqns. (A.15) and (A.16) indicate that the effects of the current bubbles b_t on $\rho_{+\,-}(b,r)_{t+1}$ and b_t on $\rho_{-\,+}(b,r)_{t+1}$, depend on the sign function, eqn.(A.19) for each bubble (i.e. whether the bubble is optimistically or pessimistically generated or zero-bubble). This is consistent with both Semmler and Zhang (2007) and Kent and Lowe (1997). As more traders realize that the bubbles exist, they become more reluctant to buy the assets. This points in the direction of the effects of bubbles on the fundamentals. The effect of the bubble on the fundamental is always negative. Thus, rational traders know that if the bubble is very large, the asset value or fundamental value will be very low and will be unwilling to buy the asset at that very high

price and everyone will be stuck with the asset, unless the bubble bursts. The price can either fall sharply or gradually depending on the nature of the probability function.

A.D Steady State Solution for the Speculative Bubbles

The definition of both the optimistically generated bubbles and the pessimistically generated bubbles in eqn. (1.19),is included in time varying probability to determine the stability implications of the growth paths of bubbles $\left(\dot{b}\right)$ as:

$$\dot{b} = v\,(2+R)\,(\tanh(ab) - b)\,R\cosh(ab) \tag{A.23}$$

Using eqn.(1.3), the law of motion of the social response index is:

$$b_{t+1} = \frac{\delta}{\rho_{t+1}} b_t + \varepsilon_{t+1} - \varepsilon_{t+1} \tag{A.24}$$

$\rightarrow$

$$b_t = \frac{\rho_{t+1}}{\delta} b_{t+1} \tag{A.25}$$

Substituting eqn. (A.25) into eqn. (A.23) and dropping time subscripts gives;

$$\dot{b} = v\,(2+R)\left(\tanh(a\left(\frac{\rho}{\delta}b\right)) - \left(\frac{\rho}{\delta}b\right)\right)R\cosh\left(a\left(\frac{\rho}{\delta}b\right)\right) \tag{A.26}$$

Probability switching is determined by $\phi_1, \phi_2, \phi_3, \phi_4$,strength of infection (a), bubble size (b), bubble sign (sign (b)), and interest rate (r), and speed of adjustment (v) and are all factors affecting the social response or contagion in the financial market. These factors are implied in eqn. (A.26) (see Appendix A.C). Grouping like terms gives;

$$\dot{b} = (2v + Rv)\left(\tanh(R\cosh\left(a^2\frac{\rho^2}{\delta^2}b^2\right)) - R\cosh\left(a\frac{\rho^2}{\delta^2}b^2\right)\right) \tag{A.27}$$

$$\dot{b} = (2v + Rv)\left(\tanh(\cosh\left(a^2\rho^2b^2\delta^{-2}\right)R) - \cosh\left(a\rho^2b^2\delta^{-2}\right)R\right) \tag{A.28}$$

Substituting for the difference of the two probabilities for optimists and pessimists (see Appendix A.C)

$$\dot{b} = \left[\begin{pmatrix} \tanh(\cosh\left(a\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b,r\right)\right)\right)-\left(1-\tanh\left(\vartheta\left(b,r\right)\right)\right)\right)^2\right)) \\ -\cosh\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b,r\right)\right)\right)-\left(1-\tanh\left(\vartheta\left(b,r\right)\right)\right)^2\right) \end{pmatrix}\right] (2v+Rv)\left(ab^2\delta^{-2}R\right) \tag{A.29}$$

$$\dot{b} = \{\tanh(\cosh\left(a\left(\frac{1}{2}\begin{pmatrix} (1-\tanh\left((\phi_1 f_o(b)+\phi_2 sign(b)r\right))) \\ -(1-\tanh\left(\phi_3 f_p(b)+\phi_4 sign(b)r\right)) \end{pmatrix}\right)^2\right) - \cosh\left(\frac{1}{2}\begin{pmatrix} (1-\tanh\left(\phi_1 f_o(b)+\phi_2 sign(b)r\right)) \\ -(1-\tanh\left(\phi_3 f_p(b)+\phi_4 sign(b)r\right)) \end{pmatrix}\right)^2\} (2v+Rv)\left(ab^2\delta^{-2}R\right) \tag{A.30}$$

$$\dot{b} = \{\tanh(\cosh a\left(\frac{1}{2}\begin{pmatrix} \left(1-\tanh\left(\begin{pmatrix} \phi_1\left(v\left[\exp\{\alpha b\}-\alpha b-1\right]\right) \\ +\phi_2 sign(b)r \end{pmatrix}\right)\right) \\ -\left(1-\tanh\begin{pmatrix} \phi_3\left(v\left[\exp\{-\alpha b\}-\alpha b-1\right]\right) \\ +\phi_4 sign(b)r \end{pmatrix}\right) \end{pmatrix}\right)^2) - \cosh\frac{1}{2}\begin{pmatrix} (1-\tanh\left(\phi_1\left(v\left[\exp\{\alpha b\}-\alpha b-1\right]\right)+\phi_2 sign(b)r\right)) \\ -(1-\tanh\left(\phi_3\left(v\left[\exp\{-\alpha b\}-\alpha b-1\right]\right)+\phi_4 sign(b)r\right)) \end{pmatrix}^2\} (2v+Rv)\left(ab^2\delta^{-2}R\right) \tag{A.31}$$

$$\dot{b} = \{\tanh(\cosh a\left(\frac{1}{2}\begin{pmatrix} \left(1-\tanh\left(\begin{pmatrix} \phi_1\left(v\left[\exp\{\alpha\left(\frac{\rho}{\delta}b\right)\}-\alpha\left(\frac{\rho}{\delta}b\right)-1\right]\right) \\ +\phi_2 sign(b)r \end{pmatrix}\right)\right) \\ -\left(1-\tanh\begin{pmatrix} \phi_3\left(v\left[\exp\{-\alpha\left(\frac{\rho}{\delta}b\right)\}-\alpha\left(\frac{\rho}{\delta}b\right)-1\right]\right) \\ +\phi_4 sign(b)r \end{pmatrix}\right) \end{pmatrix}\right)^2) - \cosh\left(\frac{1}{2}\begin{pmatrix} \left(1-\tanh\left(\phi_1\left(v\left[\exp\{\alpha\left(\frac{\rho}{\delta}b\right)\}-\alpha\left(\frac{\rho}{\delta}b\right)-1\right]\right)+\phi_2 sign(b)r\right)\right) \\ -\left(1-\tanh\left(\phi_3\left(v\left[\exp\{-\alpha\left(\frac{\rho}{\delta}b\right)\}-\alpha\left(\frac{\rho}{\delta}b\right)-1\right]\right)+\phi_4 sign(b)r\right)\right) \end{pmatrix}\right)^2\} (2v+Rv)\left(ab^2\delta^{-2}R\right) \tag{A.32}$$

A.E Steady State Solution for the Probability

The time path of the probability function for the combined optimists and pessimists is given as:

$$\dot{\rho}_{+-}\left(b,r\right)_t = \rho_{+-}\alpha db - \rho_{-+}\alpha db \tag{A.33}$$

$$\dot{\rho}(b,r) = \frac{1}{2}\left((1-\tanh\left(\omega\left(b,r\right)\right)) - (1-\tanh\left(\vartheta\left(b,r\right)\right))\right)\alpha db \tag{A.34}$$

Substituting eqns. (A.17) and (A.18) into eqn. (A.34) gives:

$$\dot{\rho} = \frac{1}{2}\left(\begin{array}{c}(1-\tanh\left((\phi_1 f_o(b) + \phi_2 sign(b)r\right))\right) \\ -\left(1-\tanh\left(\phi_3 f_p(b) + \phi_4 sign(b)r\right)\right)\end{array}\right)\alpha db \tag{A.35}$$

$$\begin{aligned}\dot{\rho} &= \frac{1}{2}\alpha db - \frac{1}{2}\tanh\phi_1 f_o(b)\alpha db - \frac{1}{2}\tanh\phi_2 sign(b)r\alpha db \\ &\quad -\frac{1}{2}\alpha db + \frac{1}{2}\tanh\phi_3 f_p(b)\alpha db + \frac{1}{2}\tanh\phi_4 sign(b)r\alpha db\end{aligned} \tag{A.36}$$

$$\begin{aligned}\dot{\rho} &= -\frac{1}{2}\tanh\phi_1 f_o(b)\alpha db - \frac{1}{2}\tanh\phi_2 sign(b)r\alpha db \\ &\quad +\frac{1}{2}\tanh\phi_3 f_p(b)\alpha db + \frac{1}{2}\tanh\phi_4 sign(b)r\alpha db\end{aligned} \tag{A.37}$$

$$\dot{\rho} = \frac{1}{2}\left(\tanh\left(\begin{array}{c}\phi_3 f_p(b) + \phi_4 sign(b)r - \phi_1 f_b(b) \\ -\phi_2 sign(b)r\end{array}\right)\right)\alpha db \tag{A.38}$$

$$\dot{\rho} = \frac{1}{2}\left(\tanh\left(\phi_3 f_p(b) - \phi_1 f_b(b) + \left(\phi_4 - \phi_2\right) sign(b)r\right)\right)\alpha db \tag{A.39}$$

Let $\dot{\rho} = 0$, and simplifying gives;

$$\begin{aligned}0 &= -\frac{1}{2}\phi_1 f_o(b)\alpha db - \frac{1}{2}\phi_2 sign(b)r\alpha db + \frac{1}{2}\phi_3 f_p(b)\alpha db \\ &\quad +\frac{1}{2}\phi_4 sign(b)r\alpha db\end{aligned} \tag{A.40}$$

$$0 = -\phi_1 f_o(b) - \phi_2 sign(b)r + \phi_3 f_p(b) + \phi_4 sign(b)r \tag{A.41}$$

$$\begin{aligned}0 &= -\phi_1\left(v\left[\exp\{\alpha b\} - \alpha b - 1\right]\right) + \phi_3\left(v\left[\exp\{-\alpha b\} - \alpha b - 1\right]\right) \\ &\quad +\phi_4 sign(b)r - \phi_2 sign(b)r\end{aligned} \tag{A.42}$$

$$\begin{aligned}0 &= -\phi_1 v\exp\{\alpha b\} + \phi_1\alpha b v + \phi_1 v + \phi_3 v\exp\{-\alpha b\} - \phi_3\alpha b v - \phi_3 v \\ &\quad +\phi_4 sign(b)r - \phi_2 sign(b)r\end{aligned} \tag{A.43}$$

$$\begin{aligned}0 &= \phi_3 v\exp\{-\alpha b\} - \phi_1 v\exp\{\alpha b\} + \phi_1\alpha b v + \phi_1 v - \phi_3\alpha b v - \phi_3 v \\ &\quad +\phi_4 sign(b)r - \phi_2 sign(b)r\end{aligned} \tag{A.44}$$

Taking logs, we get

$$\begin{aligned} 0 &= \ln\phi_3 + \ln v - \alpha b - (\ln\phi_1 + \ln v + \alpha b) + \ln\phi_1 \\ &\quad + \ln(\alpha + b + v + \phi_1 v - \phi_3 + \alpha + b + v - \phi_3 + v) \\ &\quad + \ln\phi_4 + \ln sign(b)r - \ln\phi_2 + \ln sign(b)r \end{aligned} \tag{A.45}$$

$$\begin{aligned} 0 &= -\ln\phi_3 - 2\alpha b + 2\ln a + 2\ln b + 4\ln v + \ln\phi_1 \\ &\quad + \ln\phi_4 + \ln sign(b)r - \ln\phi_2 + \ln sign(b)r \end{aligned} \tag{A.46}$$

Simplifying and substituting for b

$$2\alpha\left(\frac{\rho}{\delta}b\right) = 2\ln a + 2\ln\left(\frac{\rho}{\delta}b\right) + 4\ln v + \ln\phi_1 - \ln\phi_2 - \ln\phi_3 + \ln\phi_4 + 2\ln sign(b)r \tag{A.47}$$

Eliminating logs and solving for ρ

$$\rho = \frac{\delta}{2\alpha b}\left(\begin{array}{c} 2\ln a + 2\ln\left(\frac{\rho}{\delta}b\right) + 4\ln v + \ln\phi_1 - \ln\phi_2 - \ln\phi_3 + \ln\phi_4 \\ +2\ln sign(b)r \end{array}\right) \tag{A.48}$$

$$\rho = \frac{\delta}{2\alpha b}\left(\begin{array}{c} 2\ln a + 2\ln(\rho + b - \delta) + 4\ln v + \ln\phi_1 - \ln\phi_2 - \ln\phi_3 \\ +\ln\phi_4 + 2\ln sign(b)r \end{array}\right) \tag{A.49}$$

Appendix B
Appendix to Chapter 3

B.A Evolution of Speculative Bubbles

We specify the law of motion of the evolution of speculative bubbles in the stock price and the exchange rate using Semmler and Zhang (2007) as follows:

$$b_{s,t+1} = \begin{cases} \beta_1^{-1} b_{s,t} + u_{t+1}, \text{ with probability } \rho_s \\ \beta_2^{-1} b_{s,t}, \text{ with probability } 1 - \rho_s \end{cases} \tag{B.1}$$

$$b_{e,t+1} = \begin{cases} \psi_1^{-1} b_{e,t} + v_{t+1}, \text{ with probability } \rho_e \\ \psi_2^{-1} b_{e,t}, \text{ with probability } 1 - \rho_e \end{cases} \tag{B.2}$$

where, $\beta_1^{-1} = (1 + R_{1,st})$ denotes a vector of time varying growth factor for stock bubbles, $\psi_1^{-1} = (1 + R_{1,et})$ denotes a vector of time varying growth factor for exchange rate bubbles, $\beta_2^{-1} = (1 - R_{2,st})$ denotes a vector of time-varying decrease factors for stock bubbles, $\psi_2^{-1} = (1 - R_{2,e})$ denotes a vector of time-varying decrease factors for exchange rate bubbles. R is the market return on the asset and employed as a key determinant of the speculative bubbles growth or decrease rate (Poterba and Summers (1986:1143) and Cunado et al. (2005)). ρ denotes the probability for the bubble to increase and $1 - \rho$ denotes the probability for the bubble to fall. u, v are the stochastic error terms which drive the speculative bubbles and are assumed to be driven stochastically by extraneous factors unrelated to the fundamentals. These error terms are independently and identically distributed noise terms. The speculative bubbles in eqns.(B.1) and (B.2) seem very realistic since a positive bubble would imply an overvaluation of the asset price and a negative bubble would imply an undervaluation. Thus, the presence of speculative bubbles imply the presence of asset price misalignments.

One of the most important motivations for holding speculative assets experiencing price bubbles is the expectation that the price will continue to rise. The discovery of rational bubbles has shown that bubbles can perfectly arise as a result of the investors's having highly rational attitudes. The most important contribution of the discovery of rational bub-

bles is that speculation can no longer be considered as something that is necessarily desirable from the economic system's efficiency point of view, but that speculation can lead to a complete severing of connections between asset prices and fundamental values.

Since rational agents know the probability function of a bubble emerging, state-dependent speculative bubbles can be reformulated from eqns.(B.1) and (B.2) following Semmler and Zhang. This is done by multiplying the respective probabilities to the current speculative bubble and summing the two equations. The simplified models for each asset price bubble is given as:

$$b_{s,t+1} = \rho_{st+1}\left(1 + R_{1,st}\right) b_{s,t} + u_{t+1} + \left(1 - \rho_{st+1}\right)\left(1 - R_{2,st}\right) b_{s,t} \tag{B.3}$$

where

$$\left(1 - \rho_{st+1}\right)\left(1 - R_{2st}\right) b_{st} = b_{st} - b_{st}\rho_{st+1} - R_{2st}b_{st} + R_{2st}b_{st}\rho_{st+1} \tag{B.4}$$

and

$$\rho_{st+1}\left(1 + R_{1st}\right) b_{st} = b_{st}\rho_{st+1} + R_{st}b_{st}\rho_{st+1} \tag{B.5}$$

$$b_{s,t+1} = R_{st}b_{st}\rho_{st+1} + u_{t+1} + b_{st} - R_{2st}b_{st} + R_{2st}b_{st}\rho_{st+1} \tag{B.6}$$

$$b_{s,t+1} = \left(\rho_{st+1}\left(R_{1st} + R_{2st}\right) - R_{2st} + 1\right) b_{st} + u_{t+1} \tag{B.7}$$

The the law of motion of the speculative bubble for the exchange rate can also be specified by applying symmetric rule to give;

$$b_{e,t+1} = \left(\rho_{et+1}\left(R_{1et} + R_{2et}\right) - R_{2et} + 1\right) b_{et} + v_{t+1} \tag{B.8}$$

Eqn.(B.8) defines the evolution of the speculative bubbles in the exchange rate in period t+1 and eqn.(B.7) defines the evolution of the speculative bubbles in the stock price in period t+1. Thus, the evolution of the speculative bubble is determined by the growth and decrease factors of the speculative bubbles, the probabilities of speculative bubble growth and bust, the current size of the speculative bubble and the stochastic disturbance term, which captures all other factors that affect speculative bubbles. This study employs these bubbles because they can interchange their states from being positive to being negative.

There are two ways of endogenizing the probability function: a linear and a non-linear approach. Kent and Lowe (1997) takes a linear approach while Semmler and Zhang (2007) take a nonlinear approach.

We introduce into the optimization problem a nonlinear probability function which is bounded between 0 and 1, and which indicates the effects of the size of the bubble and the level of the interest rate on the probability function. This implies that the probability function can be measured using the following specification as suggested by Semmler and Zhang:

$$\rho_{et+1} = \frac{1}{2}\left(1 - \tanh\left(\omega\left(b_{e,t}, r_t\right)\right)\right) \tag{B.9}$$

$$\rho_{st+1} = \frac{1}{2}\left(1 - \tanh\left(\vartheta\left(b_{s,t}, r_t\right)\right)\right) \tag{B.10}$$

with

$$\omega\left(b_{e,t}, r_t\right) = \phi_1 f_{be}(b_{e,t}) + \phi_2 sign(b_{e,t}) r_t, \;\; \phi_i > 0 \tag{B.11}$$

and

$$\vartheta\left(b_{st}, r_t\right) = \phi_3 f_{bs}(b_{s,t}) + \phi_4 sign(b_{s,t}) r_t, \;\; \phi_i > 0 \tag{B.12}$$

where, $\rho_{e,t+1}$ and $\rho_{s,t+1}$, are the expected probability functions for the exchange rate bubble and the stock price bubble, respectively; ϕ_1 and ϕ_3 denote the effects of bubbles on the probability and ϕ_2 and ϕ_4 denote the effects of the interest rate on the probability; and $sign(b_{it})$ is the sign function of speculative bubbles and it follows three states as stated below:

$$sign(b_{et}) = \begin{array}{c} 1, \text{if } b_{et} > 0 \\ 0, \text{ if } b_{et} = 0 \\ -1, \text{ if } b_{et} < 0 \end{array} \quad and \;\; sign(b_{st}) = \begin{array}{c} 1, \text{if } b_{st} > 0 \\ 0, \text{ if } b_{st} = 0 \\ -1, \text{ if } b_{st} < 0 \end{array} \tag{B.13}$$

where, $f_j(b_{jt})$ is the so-called linex function which is nonnegative and asymmetric around 0. Employing Semmler and Zhang, the linex function can be specified by following the specification in Varian (1975) and Nobay and Peel (2003) as:

$$f(b) = v\left[\exp\{ab\} - ab - 1\right], \; v > 0, a \neq 0 \tag{B.14}$$

where, v denotes the speed of infection and scales the linex function $f(b)$, and α denotes herd behaviour and determines the asymmetry of the function $f(b)$.For analytical purposes, we adopt Semmler and Zhang's specifications for $v = 1$ and $a > 1$. By applying the standard probability structure for comparative purposes, eqn. (B.14) gives shape to the probability function for each of the asset price bubbles. The probability function trajectory will be more flatter when the bubble is negative than when the bubble is positive, as long as a is not equal to zero. Therefore, eqn. (B.14) measures the probability switching from pessimistic market sentiments to optimistic ones (Lux (1995)).

Substituting eqns. (B.11), B.12), B.13), and (B.14) into eqns. (B.9) and B.10) gives:

$$\rho_{et+1} = \frac{1}{2}\left(1 - \tanh\left(\phi_1\left(v\left[\exp\{ab\} - ab - 1\right]\right) + \phi_2 sign(b_{e,t})r_t\right)\right) \tag{B.15}$$

$$\rho_{st+1} = \frac{1}{2}\left(1 - \tanh\left(\phi_3\left(v\left[\exp\{ab\} - ab - 1\right]\right) + \phi_4 sign(b_{s,t})r_t\right)\right) \tag{B.16}$$

Eqns. (B.15) and (B.16) are the expected probabilities of bubbles increasing in the future period and depend on the herd behaviour (a), bubble size (b), sign of the speculative bubble, the interest rate (r), the speed of infection (v), the effects of bubbles on probabilities (ϕ_1, ϕ_3), and the effects of interest rate on probabilities (ϕ_2, ϕ_4).

The final solutions describing the law of motion for the speculative bubbles in both the stock price and the exchange rate is solved by substituting eqns. (B.16) and (B.15) into eqns. (B.7) and (B.8), respectively, to give:

$$b_{e,t+1} = \begin{pmatrix} \frac{1}{2}\left(1 - \tanh\left(\phi_1\left(v\left[\exp\{ab\} - ab - 1\right]\right) + \phi_2 sign(b_{e,t})r_t\right)\right) \\ (R_{1et} + R_{2et}) - R_{2et} + 1 \end{pmatrix} b_{et} + v_{t+1} \tag{B.17}$$

$$b_{s,t+1} = \begin{pmatrix} \frac{1}{2}\left(1 - \tanh\left(\phi_3\left(v\left[\exp\{ab\} - ab - 1\right]\right) + \phi_4 sign(b_{s,t})r_t\right)\right) \\ (R_{1st} + R_{2st}) - R_{2st} + 1 \end{pmatrix} b_{st} + u_{t+1} \tag{B.18}$$

B.B Properties of the Probability Function

The probability function has two properties: the effect of speculative bubbles and the interest rate on the probability function depends on whether the bubble is negative or positive. The theoretical properties of the probability functions for speculative bubbles can be derived in the following manner. By differentiating eqns.(B.9) and (B.10) with respect to the speculative bubble, the following defines the first property of the probability function for each of the two speculative bubbles:

$$\frac{d\rho^{e}_{t+1}}{db_{et}} = -\frac{\phi_1 a\left(\exp\{ab_{et}-1\}\right)}{2\cosh^2(\omega\left(b_{et},r_t\right))}\begin{Bmatrix}<0\text{ , if } b_{et}>0\\ >0\text{ , if } b_{et}<0\end{Bmatrix} \tag{B.19}$$

$$and\ \frac{d\rho^{s}_{t+1}}{db_{st}} = -\frac{\phi_3 a\left(\exp\{ab_{st}-1\}\right)}{2\cosh^2(\vartheta\left(b_{st},r_t\right))}\begin{Bmatrix}<0\text{ , if } b_{st}>0\\ >0\text{ , if } b_{st}<0\end{Bmatrix} \tag{B.20}$$

Eqn. (B.19) indicates that the slope or the rate of change of the probability function for the exchange rate bubble is negative if the bubble is positive and is positive if the bubble is negative. The same interpretation applies for the probability function for the stock price bubble in eqn.(B.20). The probability function that is defined in eqns. (B.9) and (B.10) is asymmetric around $b_{it} = 0$. Furthermore, eqns. (B.19) and (B.20) indicate that the effects of the current speculative bubbles on their respective probabilities, depend on the sign function. This is consistent with both Semmler and Zhang and Kent and Lowe: As more traders realize that the bubbles exist, they become more reluctant to buy the assets. Thus, rational traders know that if the bubble is very large, the asset value or fundamental value will be very low and will be unwilling to buy the asset at that very high price and everyone will be stuck with the asset, unless the bubble bursts. This idea is in line with Semmler and Zhang who argue that the price can either fall sharply or gradually depending on the nature of the probability function or the strength of herd behaviour α.

For example, if the exchange rate bubble is positive, it implies that the probability function of the bubble will decrease in the next period. Therefore, eqn. (B.19) shows that the larger the exchange rate bubble the lower the probability that the positive bubble will increase in the next period. The negative exchange rate bubble is interpreted in the opposite manner. If the exchange rate bubble is negative, an increase in the bubble implies that the probability of the bubble will increase in the next period. Thus, as the exchange

rate depreciates continuously, it gets closer and closer to its lowest point and therefore, it is more and more likely to appreciate in the future. But we assume that the current negative exchange rate bubble does not influence the probability in period one as much as the positive one because practically, traders might usually be more pessimistic in bear markets than optimistic in bull markets. Additionally, it is more difficult to activate a foreign exchange market when it is in recession than hold it down during booms. To a larger extent, this is what is explained by the linex function in eqn. (B.14). The probability function is flatter when the speculative bubble is negative than when its is positive (Semmler and Zhang (2007)). A similar analysis can be done for the probability function for the stock price bubble. Note that the definition for an asset price appreciation is the positive change and a depreciation is a negative change.

The second property is the effect of the current interest rate on the next period's probability and can also be studied by taking the partial derivatives of the probability functions in eqns. (B.9) and (B.10), with respect to the interest rate r_t as follows:

$$\frac{d\rho_{e,t+1}}{dr_t} = -\frac{\phi_2 sign(b_{et})}{2\cosh^2(\omega\left(b_{et}, r_t\right))}\begin{Bmatrix} > 0 \text{ , if } b_{et} > 0 \\ < 0 \text{ , if } b_{et} < 0 \end{Bmatrix} \tag{B.21}$$

$$\frac{d\rho_{s,t+1}}{dr_t} = -\frac{\phi_4 sign(b_{st})}{2\cosh^2(\vartheta\left(b_{st}, r_t\right))}\begin{Bmatrix} < 0 \text{ , if } b_{st} > 0 \\ > 0 \text{ , if } b_{st} < 0 \end{Bmatrix} \tag{B.22}$$

The first partial derivative in eqn. (B.21) shows that if the exchange rate bubble is positive (that is, over-valuation), an increase in the current interest rate, r_t , will increase the probability that the exchange rate bubble will increase in the next period. However, if the bubble is negative (that is, undervaluation), an increase in the current interest rate, r_t, will lower the probability that the exchange rate bubble will decrease in the next period. In the case of the stock price bubble, if the bubble is positive, an increase in the interest rate will lower the probability that the bubble will increase in the next period. If the stock price bubble is negative, an increase in the current interest rate will increase the probability that the bubble will decrease in the next period. Thus, the same policy action will lead to different effects on the probability of the asset price bubble increasing. This is the dilemma policy makers face. If there are positive asset price bubbles in both markets, either an increase or a decrease in the interest rate triggers different reactions in the financial market: the exchange

rate bubble is likely to increase (with positive bubbles) while the stock price bubble is likely to reduce. An ideal financial market situation is to maintain an undervalued exchange rate and to have a bubbly stock price, before increasing interest rates. Alternatively, if the current exchange rate is overvalued and the stock price is undervalued, then the reduction in the current interest rate will have similar positive effects in the financial markets.

The empirical validity of the relationship between the probability function and the speculative bubbles for the bilateral exchange rate bubbles and the stock price bubbles is given in Figures 4B1 through 4B4. This enables us check how the estimated probability functions for speculative bubbles are related to bubble sizes, as predicted by probability theory. Figures 4B1 and 4B3 confirm the law of motion of the empirical probability function for the speculative bubbles in the stock price and the exchange rate. The results are supported by Semmler and Zhang (2007).

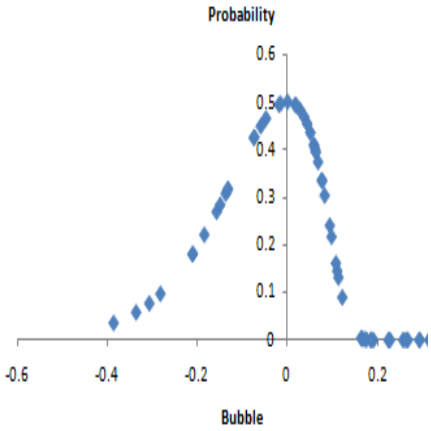

Fig. 4B1- b_{st} & $\rho_{t+1}; r = 0$

$Source$: $Author$

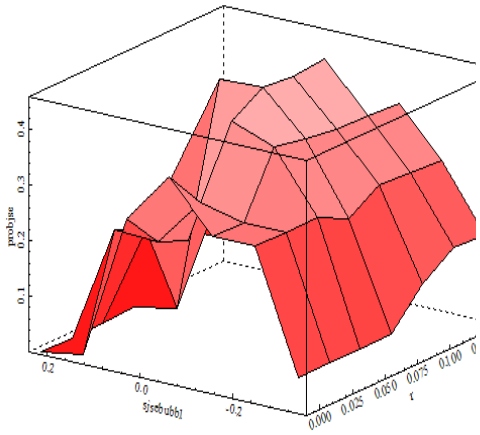

Fig. 4B2- r_t, b_{st} & ρ_{t+1}

$Source$: $Author$

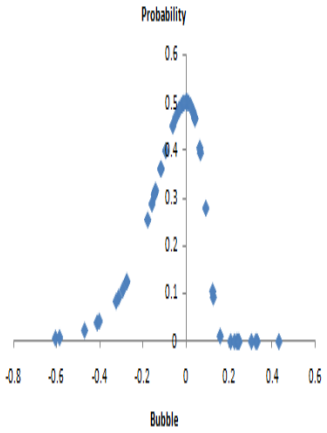

Fig. 4B3- b_{et} & $\rho_{e,t+1}; r = 0$

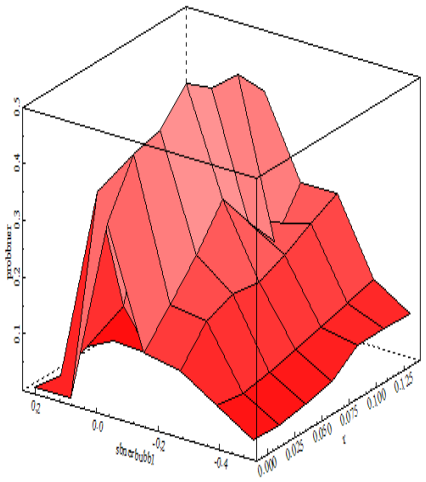

Fig. 4B4-$r_t, b_{et}, \& \rho_{t+1}$

Source:Author

The empirical probability function is asymmetric and weakly bounded between 0 and 0.5 across speculative bubbles. The theoretical reason for this is grounded in the argument that since little is known about the signs on the residual process of the asset pricing model (i.e. the bubble process), economic agents might expect the noise in the bubble process to be either positive or negative with an equal probability of 0.5. Figures 4B2 and 4B4 are constructed by allowing the interest rate to vary over time.

The above analysis shows that the interest rate drives the probability function of bubbles in the next period and therefore stimulates changes in both the speculative bubbles and the behaviour of speculative investors. Thus, interest rates can make speculative bubbles to increase or can be used to bust speculative bubbles.

B.C Compact Form of the Structural Model

Substituting eqns. (3.43) and eqn. (3.44) into eqns. (3.41) and eq. (3.42), respectively, gives

$$s_t = -\theta_{sr} r_t + \theta_{sy} y_t + \theta_{sb} \left(\left(\rho_{s,t+1} \left(R_{s1} + R_{s2} \right) - R_{s2} + 1 \right) b_{s,t} + u_{t+1} \right) \tag{B.23}$$

$$q_t = \theta_{qr}(r_t - r_t^f) + \theta_{qb}\left(\left(\rho_{e,t+1}\left(R_{e1} + R_{e2}\right) - R_{e2} + 1\right) b_{e,t} + v_{t+1}\right) \tag{B.24}$$

given that,

$$\rho_{st+1} = \frac{1}{2}\left(1 - \tanh\left(\phi_3\left(v\left[\exp\{ab\} - ab - 1\right]\right) + \phi_4 sign(b_{s,t})r_t\right)\right)$$

and

$$\rho_{et+1} = \frac{1}{2}\left(1 - \tanh\left(\phi_1\left(v\left[\exp\{ab\} - ab - 1\right]\right) + \phi_2 sign(b_{e,t})r_t\right)\right)$$

Substituting eqns. (B.23) and eqn. (B.24) into eqn. (3.39) and the forward looking IS curve becomes;

$$y_{t+1} = \frac{\gamma_1}{1-\gamma_2}y_t - \frac{\gamma_3}{1-\gamma_2}r_t + \frac{\gamma_4}{1-\gamma_2}r_t^f - \frac{\gamma_5}{1-\gamma_2}b_{e,t} + \frac{\gamma_6}{1-\gamma_2}b_{s,t} + z_{t+1} \tag{B.25}$$

where

$$z_{t+1} = \frac{1}{1-\gamma_2}\left(\varepsilon_{t+1} - \theta_{yq}v_{t+1} + \theta_{ys}u_{t+1}\right) \tag{B.26}$$

$$\begin{aligned}
\gamma_1 &= \theta_{yy};\ \gamma_2 = \left[\theta_{ys}\theta_{sy}\right];\ \gamma_3 = \left(\theta_{yr} + \theta_{yq}\theta_{qr} + \theta_{ys}\theta_{sr}\right);\ \gamma_4 = \theta_{yq} \\
\gamma_5 &= \theta_{yq}\theta_{qb}\left(\rho_{e,t+1}\left(R_{e1} + R_{e2}\right) - R_{e2} + 1\right); \qquad \text{(B.27)} \\
\gamma_6 &= \left[\theta_{ys}\theta_{sb}\left(\rho_{s,t+1}\left(R_{s1} + R_{s2}\right) - R_{s2} + 1\right)\right]
\end{aligned}$$

where, γ_5 ,γ_6 indicate the effects of the speculative bubbles' endogenous probability functions (see Appendix B.A) on the coefficients of the exchange rate bubble and the stock price bubble, respectively. Substituting for the endogenous probability functions, the coefficients on the two speculative bubbles takes the form;

$$\gamma_5 = \theta_{yq}\theta_{qb}\left(\frac{1}{2}\left(1 - \tanh\left(\omega\left(b_{e,t}, r_t\right)\right)\right)\left(R_{e1} + R_{e2}\right) - R_{e2} + 1\right) \tag{B.28}$$

$$\gamma_6 = \theta_{ys}\theta_{sb}\left(\frac{1}{2}\left(1 - \tanh\left(\vartheta\left(b_{s,t}, r_t\right)\right)\right)\left(R_{s1} + R_{s2}\right) - R_{s2} + 1\right) \tag{B.29}$$

Taking expectations on eqn. (B.25), yields

$$E_t y_{t+1} = w_1 y_t - w_2 r_t + w_3 r_t^f - w_4 b_{e,t} + w_5 b_{s,t} \tag{B.30}$$

where,

$$\begin{aligned} w_1 &= \frac{\theta_{yy}}{(1-\theta_{ys}\theta_{sy})}; w_2 = \frac{(\theta_{yr}+\theta_{yq}\theta_{qr}+\theta_{ys}\theta_{sr})}{(1-\theta_{ys}\theta_{sy})}; w_3 = \frac{\theta_{yq}}{(1-\theta_{ys}\theta_{sy})}; \\ w_4 &= \frac{\theta_{yq}\theta_{qb}\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b_{e,t},r_t\right)\right)\right)\left(R_{e1}+R_{e2}\right)-R_{e2}+1\right)}{(1-\theta_{ys}\theta_{sy})}; \\ w_5 &= \frac{\theta_{ys}\theta_{sb}\left(\frac{1}{2}\left(1-\tanh\left(\vartheta\left(b_{s,t},r_t\right)\right)\right)\left(R_{s1}+R_{s2}\right)-R_{s2}+1\right)}{(1-\theta_{ys}\theta_{sy})} \end{aligned} \tag{B.31}$$

Substituting eqn. (B.30) into eqn. (B.25) eliminates the expectations in eqn. (B.25) giving;

$$y_{t+1} = \tau_1 y_t - \tau_2 r_t - \tau_3 b_{e,t} + \tau_4 b_{s,t} + \tau_5 r_t^f + z_{t+1} \tag{B.32}$$

where,

$$\begin{aligned} \tau_1 &= (\gamma_1+\gamma_2 w_1) = \left(\theta_{yy}+\theta_{ys}\theta_{sy}\left(\frac{\theta_{yy}}{(1-\theta_{ys}\theta_{sy})}\right)\right); \\ \tau_2 &= (w_2+\gamma_3) = \left(\frac{(\theta_{yr}+\theta_{yq}\theta_{qr}+\theta_{ys}\theta_{sr})}{(1-\theta_{ys}\theta_{sy})}+(\theta_{yr}+\theta_{yq}\theta_{qr}+\theta_{ys}\theta_{sr})\right); \\ \tau_3 &= (w_4+\gamma_5) = \frac{\theta_{yq}\theta_{qb}\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b_{e,t},r_t\right)\right)\right)\left(R_{e1}+R_{e2}\right)-R_{e2}+1\right)}{(1-\theta_{ys}\theta_{sy})} \\ &\quad +\theta_{yq}\theta_{qb}\left(\frac{1}{2}\left(1-\tanh\left(\omega\left(b_{e,t},r_t\right)\right)\right)\left(R_{e1}+R_{e2}\right)-R_{e2}+1\right); \\ \tau_4 &= (w_5+\gamma_6) = \frac{\theta_{ys}\theta_{sb}\left(\frac{1}{2}\left(1-\tanh\left(\vartheta\left(b_{s,t},r_t\right)\right)\right)\left(R_{s1}+R_{s2}\right)-R_{s2}+1\right)}{(1-\theta_{ys}\theta_{sy})} \\ &\quad +\theta_{ys}\theta_{sb}\left(\frac{1}{2}\left(1-\tanh\left(\vartheta\left(b_{s,t},r_t\right)\right)\right)\left(R_{s1}+R_{s2}\right)-R_{s2}+1\right); \\ \tau_5 &= (w_3+\gamma_4) = \left(\frac{\theta_{yq}}{(1-\theta_{ys}\theta_{sy})}+\theta_{yq}\right) \end{aligned} \tag{B.33}$$

We define φ_t of the control variable of the central bank as π_t and y_t are predetermined and r_t is chosen

$$\varphi_t = \tau_1 y_t - \tau_2 r_t - \tau_3 b_{e,t} + \tau_4 b_{s,t} + \tau_5 r_t^f \tag{B.34}$$

Therefore, the original structural model eqns. (3.39)-(3.44) can be compactly written as

$$y_{t+1} = \varphi_t + z_{t+1} \tag{B.35}$$

$$\pi_{t+1} = k_t + \eta_{t+1} \tag{B.36}$$

where,

$$k_t = \theta_{\pi\pi}(L)\,\pi_t + \theta_{\pi y} y_t - \theta_{\pi q}\Delta q_t \tag{B.37}$$

Eqn. (B.37) is the state variable at time t.

B.D Derivation of the Optimal Rule

We employ the optimization method in Kontonikas and Montagnoli (2006). Assuming that the central bank's intertemporal quadratic loss function L, penalizes both real output gap and inflation volatility, its job is to minimize the loss function:

$$\min \frac{1}{2} E_t \sum_{t=1}^{\infty} \lambda^t L, \qquad \lambda \in (0,1) \tag{B.38}$$

where,

$$L = \pi_t^2 + \mu_y y_t^2 = \left(\left[k_t + \eta_{t+1} \right]^2 + \mu_y \left[\varphi_t + z_{t+1} \right]^2 \right) \tag{B.39}$$

Subject to

$$\begin{aligned} k_{t+1} &= \pi_{t+1} + \theta_{\pi y} y_{t+1} - \theta_{\pi q}\Delta q_{t+1} \\ \Leftrightarrow k_{t+1} &= k_t + \theta_{\pi y}\varphi_t - \theta_{\pi q}\Delta q_{t+1} + \xi_{t+1} \end{aligned} \tag{B.40}$$

where, the inflation coefficient is equal to one, $\theta_{\pi\pi}(L) = 1$; $\xi_{t+1} = \eta_{t+1} + \theta_{\pi y} z_{t+1}$ is a Gaussian process. y^2 is the variance of real output gap, π_t^2 is the variance of inflation, $\mu_y \geq 0$ is the penalty on output gap stabilization. $0 < \lambda < 1$ is a discount factor. Eqn. (B.40) indicates the law of motion of the state variable. The implication for having eqns.(3.45) and 3.46) is that as both the interest rate and consequently output φ_t, are chosen, the only state variable is inflation k_t. Thus, the value function, $V(k_t)$, is the expected value of the policymaker's loss function if φ_{t+i} is set optimally. The value function is defined in terms of the state variable, k_t.

We use the Bellman's dynamic programming principle. We substitute the two constraints, eqns.(3.45) and (3.46) into the value function to obtain;

$$V(k_t) = \min_{\varphi t} E_t \{ \frac{1}{2} \left(\left[k_t + \eta_{t+1} \right]^2 + \mu_y \left[\varphi_t + z_{t+1} \right]^2 \right) + \lambda V(k_{t+1}) \} \tag{B.41}$$

The first-order condition that yields the optimal response is given as:

$$\frac{\partial V(k_t)}{\partial \varphi_t} = 0 \Leftrightarrow \mu_y \varphi_t + \theta_{\pi y} \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) = 0 \tag{B.42}$$

In order to derive an expression for $E_t V'(k_{t+1})$ in eqn.(B.42), we use the envelop theorem:

$$\begin{aligned} dV(k_t) &= E_t \frac{\partial}{\partial k_t} \{ \frac{1}{2} \left[k_t + \eta_{t+1} \right]^2 + \mu_y \frac{1}{2} \left[\varphi_t + z_{t+1} \right]^2 \\ &\quad + \lambda V \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) \} dk_t \end{aligned} \tag{B.43}$$

$$V'(k_t) dk_t = E_t \left[k_t + \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) \right] dk_t \tag{B.44}$$

$$V'(k_t) = k_t + \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) \tag{B.45}$$

Multiplying eqn.(B.45) by $\theta_{\pi y}$ we obtain;

$$\theta_{\pi y} V'(k_t) = \theta_{\pi y} k_t + \theta_{\pi y} \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) \tag{B.46}$$

Adding eqn.(B.46) to eqn.(B.42) gives;

$$\begin{aligned} 0 &= \mu_y \varphi_t + \theta_{\pi y} \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) + \theta_{\pi y} V'(k_t) \\ &\quad - \theta_{\pi y} k_t - \theta_{\pi y} \lambda E_t V' \left(k_t + \theta_{\pi y} \varphi_t - \theta_{\pi q} \Delta q_{t+1} + \xi_{t+1} \right) \end{aligned} \tag{B.47}$$

Simplifying eqn. (B.47) gives eqns. (B.48) and (B.49):

$$\mu_y \varphi_t + \theta_{\pi y} V'(k_t) = \theta_{\pi y} k_t \tag{B.48}$$

$$V'(k_t) = k_t - \left(\frac{\mu_y}{\theta_{\pi y}} \right) \varphi_t \tag{B.49}$$

Eqn.(B.49) implies that

$$E_t V'(k_{t+1}) = E_t \left[k_{t+1} \right] - \left(\frac{\mu_y}{\theta_{\pi y}} \right) E_t \varphi_{t+1} \tag{B.50}$$

Substituting for $E_t \left[k_{t+1} \right]$ from eqn.(B.40) into eqn.(B.50) yields;

$$E_t V'(k_{t+1}) = k_t + \theta_{\pi y}\varphi_t - \left(\frac{\mu_y}{\theta_{\pi y}}\right) E_t\varphi_{t+1} \tag{B.51}$$

Substituting eqn.(B.51) back into the first order condition yields;

$$\mu_y\varphi_t + \theta_{\pi y}\lambda\left[k_t + \theta_{\pi y}\varphi_t - \left(\frac{\mu_y}{\theta_{\pi y}}\right) E_t\varphi_{t+1}\right] = 0 \tag{B.52}$$

or

$$\varphi_t = -\left(\frac{\theta_{\pi y}\lambda}{\mu_y + (\theta_{\pi y})^2\lambda}\right) k_t + \left(\frac{\lambda\mu_y}{\mu_y + (\theta_{\pi y})^2\lambda}\right) E_t\varphi_{t+1} \tag{B.53}$$

When policy is set at time t, k_t summarizes the state, so the optimal policy, given the linear-quadratic structure, will be of the form;

$$\Leftrightarrow \varphi_t = ck_t \tag{B.54}$$

which implies that

$$E_t\left[\varphi_{t+1}\right] = cE_t\left[k_{t+1}\right] = c\left(\left(1 + \theta_{\pi y}c\right)k_t\right) \tag{B.55}$$

Using the optimal policy condition eqn.(B.54), eqn. (B.53) becomes

$$ck_t = -\left(\frac{\theta_{\pi y}\lambda}{\mu_y + (\theta_{\pi y})^2\lambda}\right) k_t + \left(\frac{\lambda\mu_y}{\mu_y + (\theta_{\pi y})^2\lambda}\right) c\left(1 + \theta_{\pi y}c\right)k_t \tag{B.56}$$

where, eqn.(B.56) shows what the central bank's policy rule implies concerning interest rates. A quadratic equation for c can be derived from eqn.(B.56);

$$\theta_{\pi y}\lambda\mu_y c^2 - \left(\mu_y - \lambda\mu_y + \theta_{\pi y}^2\lambda\right)c - \theta_{\pi y}\lambda = 0 \tag{B.57}$$

Eqn.(B.57) is consistent with Kontonikas and Montagnoli (2006). The solutions to eqn.(B.57) are:

$$c_1 = \frac{1}{2}\left(\frac{\theta_{\pi y}^2(\lambda - 1)}{\theta_{\pi y}\lambda\mu_y} + \frac{\theta_{\pi y}\lambda}{\theta_{\pi y}\lambda\mu_y}\sqrt{4\mu_y}\right) \tag{B.58}$$

$$c_2 = \frac{1}{2}\left(\frac{\theta_{\pi y}^2(\lambda - 1)}{\theta_{\pi y}\lambda\mu_y} - \frac{\theta_{\pi y}\lambda}{\theta_{\pi y}\lambda\mu_y}\sqrt{4\mu_y}\right) \tag{B.59}$$

To determine which of these solutions to accept, note that

$$k_{t+1} = k_t + \theta_{\pi y}\varphi_t = (1 + \theta_{\pi y}c)k_t \tag{B.60}$$

so that k_{t+1} is a stable process if and only if $c < 0$ so that $1 + \theta_{\pi y}c < 1$. We are looking for a negative solution for c. First, by multiplying c_1c_2,we find a negative solution and we conclude that one of the equations is negative. Since the requirement for optimal solutions is a negative value for c, we find that c_2, is the negative value. The standard optimal policy rule can be written as:

$$\varphi_t = c_2 k_t \tag{B.61}$$

$$\varphi_t = \left[\frac{1}{2}\left(\frac{\theta_{\pi y}^2(\lambda - 1)}{\theta_{\pi y}\lambda\mu_y} - \frac{\theta_{\pi y}\lambda}{\theta_{\pi y}\lambda\mu_y}\sqrt{4\mu_y}\right)\right] k_t \tag{B.62}$$

Eqn.(B.62) determines the optimal policy rule. Therefore, the statement that $\varphi_t$equals c_2k_t is used to solve for the optimal interest rate rule:

$$\begin{aligned} & \tau_1 y_t - \tau_2 r_t - \tau_3 b_{e,t} + \tau_4 b_{s,t} + \tau_5 r_t^f \\ = \;& -c_2\left(\pi_t + \theta_{\pi y}y_t - \theta_{\pi q}\Delta q_t\right) \end{aligned} \tag{B.63}$$

Collecting like terms together, gives

$$-\tau_2 r_t - \tau_3 b_{e,t} + \tau_4 b_{s,t} = -\tau_5 r_t^f - \tau_1 y_t - c_2\left(\pi_t + \theta_{\pi y}y_t - \theta_{\pi q}\Delta q_t\right) \tag{B.64}$$

The left-hand side becomes

$$\begin{aligned} & -\tau_2 r_t - \left(\frac{1}{2}\left(1 - \tanh\left(\omega\left(b_{e,t}, r_t\right)\right)\right)\left(R_{e1} + R_{e2}\right) - R_{e2} + 1\right) \\ & \left(\frac{1}{(1 - \theta_{ys}\theta_{sy})} + 1\right)\left[\theta_{yq}\theta_{qb}\right] b_{e,t} \\ & + \left(\frac{1}{2}\left(1 - \tanh\left(\vartheta\left(b_{s,t}, r_t\right)\right)\right)\left(R_{s1} + R_{s2}\right) - R_{s2} + 1\right) \\ & \left(\frac{1}{(1 - \theta_{ys}\theta_{sy})} + 1\right)\left[\theta_{ys}\theta_{sb}\right] b_{s,t} \end{aligned} \tag{B.65}$$

Using definitions from Appendix B.A for $\left(\omega\left(b_{e,t}, r_t\right)\right)$ and $\left(\vartheta\left(b_{s,t}, r_t\right)\right)$ and substituting back into eqn. (B.65) gives

$$
\begin{aligned}
&-\tau_2 r_t - \left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{yq}\theta_{qb}]\, b_{e,t}\,(R_{e1}+R_{e2})\,\frac{1}{2}\tanh\phi_2 sign(b_{e,t}) r_t \\
&-\left((R_{e1}+R_{e2})\left(\frac{1}{2}-\frac{1}{2}\tanh\phi_1 f_{be}(b_{e,t})\right)-R_{e2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{yq}\theta_{qb}]\, b_{e,t} \\
&+\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{ys}\theta_{sb}]\, b_{s,t}\,(R_{s1}+R_{s2})\,\frac{1}{2}\tanh\phi_4 sign(b_{s,t}) r_t \\
&+\left((R_{s1}+R_{s2})\left(\frac{1}{2}-\frac{1}{2}\tanh\phi_3 f_{bs}(b_{s,t})\right)-R_{s2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{ys}\theta_{sb}]\, b_{s,t}
\end{aligned}
\tag{B.66}
$$

Collecting like terms gives;

$$
\begin{aligned}
&-\left(\begin{array}{c}\tau_2+\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{yq}\theta_{qb}]\, b_{e,t}\,(R_{e1}+R_{e2})\,\frac{1}{2}\tanh\phi_2 sign(b_{e,t}) \\ -\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{ys}\theta_{sb}]\, b_{s,t}\,(R_{s1}+R_{s2})\,\frac{1}{2}\tanh\phi_4 sign(b_{s,t})\end{array}\right) r_t \\
&-\left((R_{e1}+R_{e2})\,\frac{1}{2}\,(1-\tanh\phi_1 f_{be}(b_{e,t}))-R_{e2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{yq}\theta_{qb}]\, b_{e,t} \\
&+\left((R_{s1}+R_{s2})\,\frac{1}{2}\,(1-\tanh\phi_3 f_{bs}(b_{s,t}))-R_{s2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{ys}\theta_{sb}]\, b_{s,t}
\end{aligned}
\tag{B.67}
$$

This simplifies to:

$$
\begin{aligned}
&-\left(\tau_2+\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)\frac{1}{2}\left(\left(\begin{array}{c}[\theta_{yq}\theta_{qb}]\, b_{e,t}\,(R_{e1}+R_{e2})\tanh\phi_2 sign(b_{e,t}) \\ -[\theta_{ys}\theta_{sb}]\, b_{s,t}\,(R_{s1}+R_{s2})\tanh\phi_4 sign(b_{s,t})\end{array}\right)\right)\right) r_t \\
&-\left(\left((R_{e1}+R_{e2})\,\frac{1}{2}\,(1-\tanh\phi_1 f_{be}(b_{e,t}))-R_{e2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{yq}\theta_{qb}]\right) b_{e,t} \\
&+\left(\left((R_{s1}+R_{s2})\,\frac{1}{2}\,(1-\tanh\phi_3 f_{bs}(b_{s,t}))-R_{s2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{ys}\theta_{sb}]\right) b_{s,t}
\end{aligned}
\tag{B.68}
$$

Let

$$
\begin{aligned}
\Omega_1 &= \left(\tau_2+\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)\frac{1}{2}\left(\begin{array}{c}[\theta_{yq}\theta_{qb}]\, b_{e,t}\,(R_{e1}+R_{e2})\tanh\phi_2 sign(b_{e,t}) \\ -[\theta_{ys}\theta_{sb}]\, b_{s,t}\,(R_{s1}+R_{s2})\tanh\phi_4 sign(b_{s,t})\end{array}\right)\right) \\
\Omega_2 &= \left(\left((R_{e1}+R_{e2})\,\frac{1}{2}\,(1-\tanh\phi_1 f_{be}(b_{e,t}))-R_{e2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{yq}\theta_{qb}]\right) \\
\Omega_3 &= \left(\left((R_{s1}+R_{s2})\,\frac{1}{2}\,(1-\tanh\phi_3 f_{bs}(b_{s,t}))-R_{s2}+1\right)\left(\frac{1}{(1-\theta_{ys}\theta_{sy})}+1\right)[\theta_{ys}\theta_{sb}]\right)
\end{aligned}
\tag{B.69}
$$

Then, the left hand side solution is given as;

$$-\Omega_1 r_t - \Omega_2 b_{et} + \Omega_3 b_{st} \tag{B.70}$$

Substituting eqn. (B.70) into the left hand side of eqn. (B.64) gives;

$$-\Omega_1 r_t - \Omega_2 b_{et} + \Omega_3 b_{st} = -\tau_5 r_t^f - \tau_1 y_t - c_2 \pi_t + c_2 \theta_{\pi y} y_t - c_2 \theta_{\pi q} \Delta q_t \tag{B.71}$$

The solution for the optimal rule is obtained from eqn. (B.71) as:

$$r_t = \left(\frac{c_2}{\Omega_1}\right) \pi_t - \left(\frac{(c_2 \theta_{\pi y} - \tau_1)}{\Omega_1}\right) y_t - \left(\frac{\Omega_2}{\Omega_1}\right) b_{et} + \left(\frac{\Omega_3}{\Omega_1}\right) b_{st} + \left(\frac{c_2 \theta_{\pi q}}{\Omega_1}\right) \Delta q_t + \left(\frac{\tau_5}{\Omega_1}\right) r_t^f \tag{B.72}$$

In shorter notation, eqn. (B.72) becomes:

$$r_t = \alpha_\pi \pi_t - \alpha_y y_t + \alpha_{be} b_{e,t} - \alpha_{bs} b_{s,t} + \alpha_{rf} r_t^f + \alpha_q \Delta q_t \tag{B.73}$$

where, $\Delta q_t = q_t - q_{t-1}$ denotes change in the real exchange rate, where, the alphas in eqn.(B.73) are indicators of interest rate weights on optimal policy rule drivers, where, $a_\pi = \left(\frac{c_2}{\Omega_1}\right)$ denotes interest rate weight on inflation; $a_y = \left(\frac{(c_2 \theta_{\pi y} - \tau_1)}{\Omega_1}\right)$ denotes interest rate weight on real output gap,; $a_{be} = \left(\frac{\Omega_2}{\Omega_1}\right)$ denotes interest rate weight on nominal exchange rate bubbles; $a_{bs} = \left(\frac{\Omega_3}{\Omega_1}\right)$ denotes interest weight on nominal stock price bubbles; $a_{rf} = \left(\frac{\tau_5}{\Omega_1}\right)$ denotes interest weight on foreign real interest rate; and $a_q = \left(\frac{c_2 \theta_{\pi q}}{\Omega_1}\right)$ denotes interest rate weight on the change in the bilateral real exchange rate.