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Mechanisms of rockmass behaviour associated with slope instability

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DECLARATION

I declare that this dissertation is my own, unaided work. It is being submitted for the Degree of Master of Science in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

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at …….....

ABSTRACT

Many open pit mines are now being mined at very significant depths, often at depths far greater than was originally planned. Even deeper open pits are also being planned, to depths that would be considered deep by underground standards. Major rock slopes or high rock slopes result from deep open pits. The heights of the resulting rock slopes are usually beyond the current experience and knowledge base. Present understanding of the mechanisms of slope behaviour and failure for high slopes, and for slopes subjected to high in situ stress conditions, appears to be lacking. If the mechanisms of slope behaviour are not well understood, the validity of commonly applied methods of analysis of the stability of such slopes may be questionable.

The economic impact of excessively conservative design or of failures in these slopes can be very large. A major failure, apart from its immediate effect on production, could result in loss of ore reserves and can cause premature closure of the mine. Slope failures of any kind, if not properly managed, can represent a safety hazard for mining personnel. Large scale failures may also affect the surface surrounding the open excavation, which may involve structures and infrastructure. It is said that the optimally designed slope is one that fails the day that mining ceases. Slopes that do not experience some form of failure are said to be overdesigned.

This dissertation details the research that has been carried out into mechanisms of failure of slopes in discontinuous, hard rock masses. UDEC modelling code was used to model rock slopes in which variability occurs in the geometry of the geological planes of weakness. The aim was to determine how sensitive the mechanisms of failure of the slopes are to this variability, a variability that can typically occur in rock masses. The results of the analyses have important implications for the validity of methods of slope stability analysis commonly used for prediction and assessment of stability.

The range of numerical models analysed has shown that the behaviour can differ significantly from one model to the next. In particular, the variability in the

geometry of the discontinuities introduced into the models can have a very significant effect on the behaviour of the rock slopes. This is in spite of the fact that the rock masses being modelled can be considered to be statistically the same. In nature, the variability will probably be greater than that considered in the modelling. It is necessary to take this variability into account for realistic analysis and prediction of rock mass behaviour.

The analyses show that the rock slope deformations and failure did not involve a single failure surface, but are progressive, with deformation and local failure taking place throughout the slope height. In no case did failure involve displacement on a single failure plane. This places in question the conventional limit equilibrium approach to stability analysis, in which the stability of a failing mass above a defined or assumed failure surface is evaluated. No such surface could be defined for any of the models analysed. If such a failure surface was to be determined from the observations after failure, it would not be correct. The usual back-analysis approach to determine rock mass parameters is therefore also in doubt, since it will probably not take into account the actual rock slope failure mechanism. Therefore, although back analyses are considered to be important, the use of this approach could result in incorrect strength and deformation parameters for the rock mass and shear surfaces.

From the results of the analyses carried out, the following conclusions can be drawn regarding the deformation and failure of rock slopes in a systematically jointed rock mass:

- Deformation and failure will not be confined to specific failure surfaces, but will occur progressively throughout the mass. Multiple mechanisms of failure will occur, including shear and tensile failure on discontinuities, and shear, tensile and extensional failure of intact rock material. The accumulation of these types of localized failure will ultimately result in slope failure.

- Knowledge of the orientations and spacings of discontinuities in rock slopes does not allow prediction of a unique failure surface. Except in very specific situations, such a unique failure surface is unlikely to occur.
- Realistic prediction of real jointed rock slope behaviour is only likely to be possible using probabilistic approaches that take into account the variability in the rock mass.

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LIST OF SYMBOLS

2D	Two Dimensional
3D	Three Dimensional
3DEC	Distinct Element Code in Three Dimensions
c	Cohesion
CSIR	Commonwealth Scientific and Industrial Research
DDA	Discontinuum Deformation Analysis
DFN	Discrete Fracture Network
E	Static Young's modulus
ELFEN	A combined finite/discrete element program for the 2D and 3D modelling of jointed rock subjected to quasi-static or dynamic loading conditions.
FLAC	Fast Lagrangian Analysis of Continua
FLAC3D	Fast Lagrangian Analysis of Continua in Three Dimensions
FOSM	First-order second-moment
FS	Factor of safety
G	Shear modulus
GSI	Geological Strength Index
ISRM	International Society for Rock Mechanics
Ja	Joint alteration number
J_b	Block volume
JCS	Joint Wall Compressive Strength
Jn	Joint set number
Jr	Joint roughness number
JRC	Joint Roughness Coefficient
J_v	Volumetric joint count
Jw	Joint water reduction factor
K	Bulk modulus
kPa	Kilo Pascal
K_n	Joint normal stiffness
K_s	Joint shear stiffness

m	Material constant used with the Hoek-Brown failure criterion
MAP3D	3D rock stability analysis package using the Indirect Boundary Element Method
MINSIM	Suite of programs based on a three dimensional, linear, elastic, displacement discontinuity analysis of large scale mine layouts
MRMR	Mining Rock Mass Rating
PEM	Point estimate method
PFC	Particle Flow Code
PFC3D	Particle Flow Code in Three Dimensions
Q	Barton's tunnelling quality index rating system
RMR	Rock Mass Rating
ROCKPACK III	Package of programs for all phases of rock slope analysis and design where stability is controlled by the orientations and characteristics of rock mass discontinuities
ROCPLANE	Planar rock slope stability analysis tool
RQD	Rock Quality Designation
s	Material constant used with the Hoek-Brown failure criterion
SMR	Slope Mass Rating
SRM	Synthetic Rock Mass
SSPC	Slope Stability Probability Classification
SRF	Stress reduction factor
SWEDGE	Surface Wedge analysis tool
UDEC	Universal Distinct Element Code
VISAGE	Visage is a Finite Element system for geotechnics and rock mechanics.
τ	Peak shear strength
σ_n	Effective normal stress
ϕ_b	Basic friction angle
σ_{tj}	Joint tensile strength
τ	Shear stress
ϕ	Angle of internal friction

υ	Poisson's ratio
σ_1	Major principal stress
σ_2	Intermediate principal stress
σ_3	Minor principal stress
σ_c, UCS	Uniaxial Compressive Strength
σ_n	Effective normal stress
μ	Coefficient of friction

CHAPTER 1
Introduction

1.1 Background

Mining contributes considerably to the wealth of South Africa and the world as a whole, of which open pit mining has its fair share of contribution. Open pit mining is a mining method that involves design of rock slopes which should remain stable for the duration of the mining. Design of rock slopes involves the determination of optimal face inclination and height of benches, bench stacks and overall slopes, and the determination of optimal widths for spill berms and ramps. The economic impact of excessively conservative design or of failures in these slopes can be very large and every effort is required for an optimised design. The size of slopes means that they will almost always contain a number of significant structural features and a variety of geological materials (Hoek et al, 2000). These structural features have a huge impact on stability of rock slopes. Slopes need to be as steep as possible to minimize the amount of waste rock mined and hence to minimize mining cost, but the economic consequences of failure of slopes due to over-steepening can be disastrous. Factors taken into account in the design of rock slopes are the geological structure, ground water conditions, blasting practice, slope plan geometry and seismic activity of the area. It is often said that an optimally designed slope is one that fails the day that mining ceases (Department of Minerals and Energy Western Australia, 1999). Slopes that do not experience some form of failure are said to be overdesigned and such slopes are not optimized.

A major failure, apart from its immediate effect on production, could result in loss of ore reserves and can cause premature closure of the mine. Slope failures of any kind, if not properly managed, can represent a safety hazard for mining personnel, which in extreme circumstances could result in loss of life. Large scale failures may also affect the surface surrounding the open excavation, which may involve structures and infrastructure (Stacey, 2006).

Rock slope failures are events controlled by natural physical processes. Geological-geotechnical models that can be used to understand and to analyse these processes often include structural data as well as information on lithology, mineralization, alteration, weathering, hydrogeology and rock mass characteristics such as joint persistence and the condition of joints (Hoek et al, 2000).

Abramson (2002) reckons that slope stability evaluations are concerned with identifying critical geological, material, environmental, and economic parameters that will affect the project, as well as understanding the nature, magnitude, and frequency of potential slope problems. When dealing with slope stability analysis, previous geological and geotechnical experience in the area is valuable. The greater complexity of rock slope stability analyses is due largely to the presence of fractures and other geological discontinuities in the rock mass. These features produce discontinuities in the rock material and therefore affect the whole rock mass behaviour and they also usually control mechanisms of failure.

Stability analysis of slopes requires knowledge of the distribution, geometry, and engineering properties of the discontinuities within the rock mass. The quantity of the data collected, the methods employed, the quality of information obtained and its eventual usefulness will depend on many factors, notably the nature and seriousness of the problem, the accessibility of exposure, the time and cost justifiable for the task, and the experience and local knowledge of the investigator (Hencher, 1987**).** An investigation into the extent, orientation, and distribution of discontinuities for a particular slope will only be truly effective when geological nature of the structures is taken into account.

Slope design for open pit mining has traditionally been carried out using limit equilibrium and rock mass classification methods. Limit equilibrium methods deal with structurally controlled planar or wedge failures and circular or non-circular failure in homogeneous materials. An example of a limit equilibrium code is the 2D Slide program. Rock mass classification methods were introduced about 30 years ago

and are generally used in the preliminary design phase of the project when very limited rock mass data is available. Numerical methods have recently increased in their use for slope design especially for large scale rock slopes where horizontal stress effects are assured to play an important role. The 2D FLAC Slope, UDEC and Phase 2 are examples of numerical methods used for design of rock slopes. Traditionally designers have adopted methods to select appropriate and representative values for input parameters for slope design. In an attempt to overcome uncertainty and variability the probabilistic theory and statistical techniques have been applied to slope stability analysis.

1.2 Definition of Problem

Many open pit mines are now being mined at very significant depths, often at depths far greater than was originally planned. Even deeper open pits are also being planned, to depths that would be considered deep by underground standards. The heights of the resulting rock slopes are usually beyond the current experience and knowledge base. Present understanding of the mechanisms of slope behaviour and failure for high slopes, and for slopes subjected to high in situ stress conditions, appears to be lacking. If the mechanisms of slope behaviour are not well understood, the validity of commonly applied methods of analysis of the stability of such slopes may be questionable (Salim and Stacey, 2006).

Knowledge of failure behaviour/failure mechanism of rock slopes is important in that it allows proper analysis tools and techniques to be used. It is therefore important that tools and methods are developed for slope stability analysis and prediction of failure behaviour of major rock slopes as other failure mechanisms not previously taken into account in slope stability analysis have been observed.

Hoek et al (2000) accept the use of conventional limit equilibrium methods for analysis of structurally controlled and non-structurally controlled failures. Strength parameters used in such analysis are averaged. Hoek et al (2000) further accept that numerical modelling of slope deformation behaviour is adequately taken care of by numerical codes like FLAC and UDEC. The only shortcoming the authors indicate

around numerical modelling techniques is the effort required to acquire representative input parameters and effort in interpreting results of numerical analysis.

Hajiabdolmajid and Kaiser (2002) accept that limit equilibrium methods are reliable but lose their reliability wherever a number of deformability and strength zones exist as is the case with major rock slopes. They recommend numerical stress and deformation analysis techniques for slope stability analysis and prediction of failure behaviour of such slopes. They have also recommended numerical back analysis of slope failures and that modelled displacements should be compared with actual monitored displacements of rock slopes.

Dight (2006) has suggested techniques for prediction of failure behaviour of rock slopes based on three case studies of actual slope failures on "unknown" structures. These structures were unknown in the sense that they were only identified after failure of the rock slopes had occurred. Limit equilibrium methods could not be used as these structures were not known to exist previously. He observed that stress played a major role in producing new structures and his proposed methods take stress into account.

Sjoberg (2000) used numerical modelling to investigate failure mechanisms of generic slope geometries. He concluded that numerical modelling analysis is a useful tool for slope design. However, he does not recommend the use of limit equilibrium methods for design of major rock slopes unless the surface is predetermined. Even so, there is still a risk that if a failure surface is assumed a more critical mode of failure could be missed. Another weakness of limit equilibrium methods is that they assume rigid body movements. This assumption is questionable for major rock slopes. Comparing limit equilibrium methods and numerical analysis methods Sjoberg (2000) found that neither of the limit equilibrium methods could predict tensile failure while numerical methods did. He also found out that position of the failure surface in FLAC models differed from that in limit equilibrium analysis.

Terzaghi (1946) observed that joints are the most important causes of over break of rock in slopes. He therefore concluded that joints require cautious attention in slope stability analysis. Large variations in the properties and occurrence of the discontinuities intersecting the rock lead to a complicated composition and structure of the rock mass. The importance of any uncertainties in the input data can be quantified by carrying out a series of analyses in which each parameter is varied in turn within its probable range of values. Such a study is called sensitivity analysis (Hencher, 1987) and is particularly useful when assessing the cost effectiveness of different options for preventive or remedial works. Several authors have taken this type of analysis further and have expressed the likelihood of failure as a probability based on the statistical variation of the input parameters (McMahon, 1975; Piteau and Martin, 1977; Priest and Brown, 1983).

Saayman (1991) discussed the failure mechanism of a jointed rock mass as a stochastic process, i.e., different failure mechanisms occur depending on the particular combination of geometric and mechanical properties and loading conditions. Current slope stability analysis methods account for this in one of the two basic ways:

(i) Distribution of resisting and driving forces are obtained by utilizing distributions of geometric and strength properties, through simulation techniques, in basically deterministic failure mechanism and analysis methods. Probability of failure is determined from distributions of factor of safety.

(ii) The failure mechanisms themselves are first derived by simulating an appropriate number of variable geometry-strength property combinations. Distributions of resisting and driving forces are determined from summing the result of analysis of each path. Reliability is calculated from these.

The two approaches rely entirely upon proper definition of the geometric and mechanical properties of joints to overcome the problem of quantifying the

discontinuity persistence along a failure path. Persistence is a multi-variant function of joint orientations, joint length, joint spacing and shear strengths, as well as of the preferred mode of failure. All these parameters exhibit inherent variability. None of the probabilistic techniques studied incorporate all as variables. Generally, joint orientation is usually expressed deterministically. It has also to be assumed that in all cases sampling errors have been accounted for in advance.

It is very difficult to determine the position, length, orientation and strength of each discontinuity. This creates uncertainty by restricting the degree in which the model can be used to provide design data (e.g. expected displacement).

The distinguishing feature of slope stability problems in a rock mass is that the failure planes conform closely to pre-existing planes of weakness. The rock strength along a discontinuity is usually a small fraction of the strength of the intact material. Hence, the need to locate and establish the orientation and strength properties of critical discontinuities cannot be overemphasised (Patton and Deere, 1970).

1.3 Objectives

The objective of this research is to improve the understanding of mechanisms of failure of rock slopes in discontinuous, hard rock masses. Universal Distinct Element Code (UDEC) is used to model rock slopes in which variability occurs in the geometry of the geological planes of weakness. The aim is to determine sensitivity of mechanisms of failure of the slopes to this variability, a variability that typically occurs in rock masses. The results of the analyses will have important implications for the validity of methods of slope stability analysis commonly used for prediction and assessment of stability. It is hoped that once an understanding of mechanisms of failure of rock slopes in discontinuous, hard rock masses is attained it will then be possible to design optimum open pit slopes and be able to predict instabilities that can then be monitored and controlled.

1.4 Research Methodology

Literature survey was conducted to ascertain current methods available for the evaluation of rock slope stability. Numerous numerical models were run to evaluate the stability of rock slopes for different combinations of variability. The following variability and combinations of variability in rock mass parameters are assessed using UDEC models:

(i) Joint orientations;
(ii) Bedding plane dip;
(iii) Joint spacing;
(iv) Bedding plane spacing;
(v) Offset between joints;
(vi) Combinations of the above.

Rock mass strength parameters, joint normal and shear stiffness values were held constant. Observations of the process and mode of failure usually provide the feedback necessary to verify a new failure mechanism. It is therefore important to observe the failure process and failure modes of rocks at different stress levels. The effect of geometrical parameters on safety was examined by running a set of sensitivity studies. The value of a single parameter was changed by applying standard deviation from 25% to 75% while the other parameters were fixed to their "best estimate" values and variation in slope behaviour examined.

Also of prime importance in rock slope stability are parameters relating to water, seepage or external loads. These are however not addressed in this dissertation.

1.5 Content of the Dissertation

Chapter 2 details the background literature discussion on rock slope stability/instability as applied to open pit mining. Jointing in rock masses is discussed in Chapter 3 while Chapter 4 discusses previous work carried out using physical models and description of the numerical model together with input parameters. Results of numerical modelling are discussed in Chapter 5. The research conclusions and recommendations are given in Chapter 6.

CHAPTER 2

Rock Slope Stability

2.1 Mechanics of high rock slopes

Present understanding of the mechanisms of slope behaviour and failure for high slopes appears to be lacking. High slopes or major rock slopes result from mining of deep open pits. Stacey (1968) lists the following parameters as governing the stability of rock slopes:

(i) Geological structures,
(ii) Rock stresses,
(iii) Groundwater conditions,
(iv) Strength of discontinuities and intact rock,
(v) Pit geometry, including both slope angles and slope curvature,
(vi) Vibration from blasting or seismic events,
(vii) Climatic conditions, and,
(viii) Time.

This list demonstrates the many parameters required for assessment of rock slope stability. Rock slope stability analyses can be divided into two categories (Read, 2007); those directed at assessing the likelihood of structurally controlled, kinematically possible failures such as plane failures, wedge failures, and toppling failures; and those that attempt to assess the likelihood of a failure occurring through the rock mass. All of these analyses utilise the Mohr-Coulomb failure criterion. The Mohr-Coulomb criterion requires values of friction and cohesion for the rock mass. These parameters have been carried over into the more developed continuum and discontinuum numerical modelling tools for slope stability analysis.

Read (2007) identified two problems with the Mohr-Coulomb criterion:

(i) Mohr-Coulomb measures friction and cohesion at a point, which is then transferred to three-dimensional body of rock by assuming that the rock mass is isotropic which in a jointed rock mass is not the case.

(ii) Obtaining friction and cohesion values for a closely jointed rock mass is difficult, mostly because tri-axial testing of representative rock mass samples is itself made difficult by the complexity of performing tests on rock at a scale that is of the same order of magnitude as the rock mass. Sample disturbance and equipment size are the other major limitations. Consequently, the preferred method has been to derive empirical values of friction and cohesion from rock mass classification schemes, such as RMR, MRMR, and GSI that have been calibrated from experience. The classification schemes are discussed later in this chapter.

In order to gain overall understanding of rock slope behaviour rock mass structure, rock mass strength and slope failure mechanisms are described to explain important factors for high rock slopes behaviour. The four common failure mechanisms are plane, wedge, circular and toppling failure. Common rock mass classification schemes and their limitations are described and a review of numerical techniques for rock slope stability analyses are emphasised.

2.1.1 Rock mass structure

The knowledge of rock mass structure is an important task in mining and construction engineering since the behaviour of rock masses can be controlled by the presence of discontinuities (Hoek and Bray 1994, Hoek and Brown 1980). Piteau (1970) suggested that for purposes of analysing the behaviour of a rock mass certain fundamental properties of the rock mass must be appreciated. These are;

(i) That failure will tend to be confined to structural discontinuities, the strength of discontinuity being much less than the strength of intact rock,

(ii) The strength and deformational properties are directional, being dependent on the spatial distribution of the structural defects, i.e., the rock mass is anisotropic in nature,

(iii) The physical and lithological properties of the rock material are variable, i.e., the rock mass is heterogeneous in nature, and,

(iv) The rock mass is analogous to partitioned solid bodies made up of individual blocks, i.e., the rock mass is a discontinuous medium.

Characterisation of the rock mass is vital to the design of structures in rock. Figure 1 is a schematic illustration of the relationship between rock material, rock mass and discontinuities (Hoek et al, 1998a). A rock mass can be summarised as a discontinuous medium consisting of individual blocks that are heterogeneous in nature due to the variation of physical and lithological properties of the rock. The strength of the rock mass is mainly influenced by discontinuity features rather than the strength of intact rock. Intact rock is defined in engineering terms as rock containing no significant fractures. On the micro-scale intact rock is composed of grains, pore space and micro-fractures with the form of this micro-structure being governed by the basic rock forming processes. Discontinuities such as joints and faults may lead to structurally controlled instabilities whereby blocks form through the intersection of several joints, which are kinematically free to fall or slide from the excavation periphery as a result of gravity.

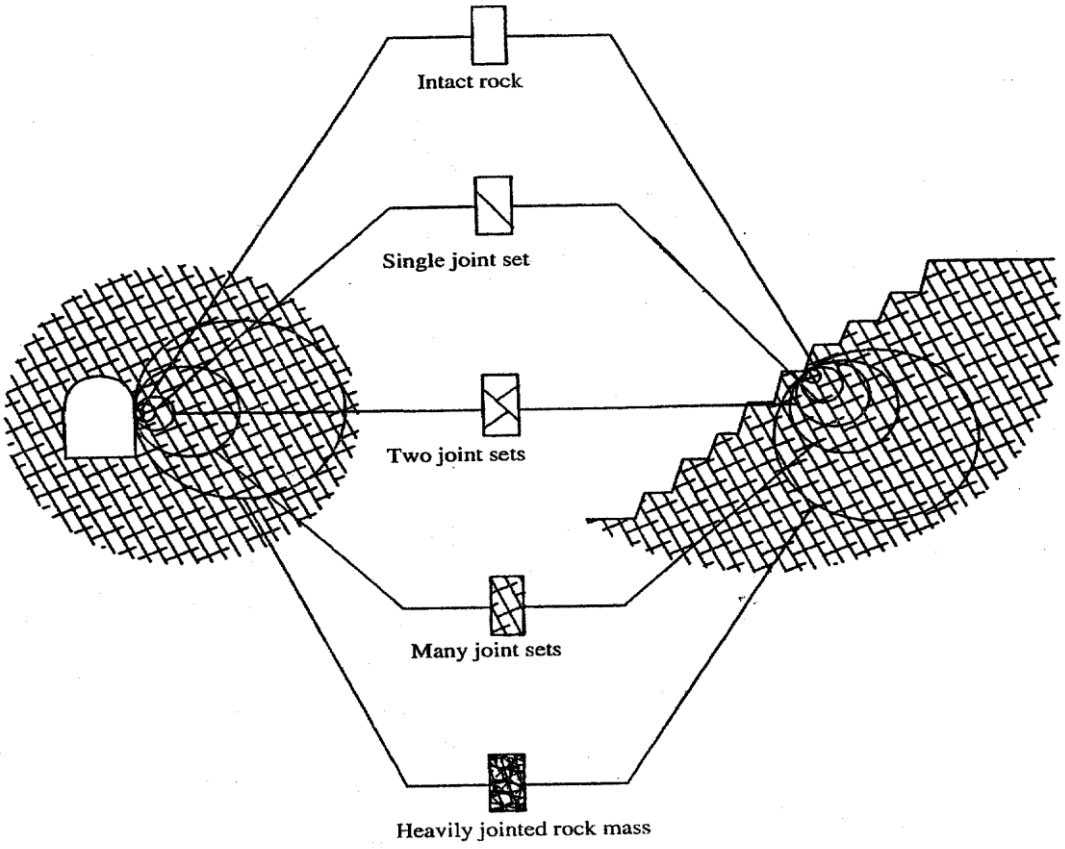


Figure 1: Relationship between rock material, rock mass and discontinuities (Hoek et al, 1998a)

Figure 2 and Figure 3 show two different geological conditions that are commonly encountered in the design analysis of rock slopes. These are typical examples of rock masses in which the strength of laboratory-size samples may differ significantly from the shear strength along the overall sliding surface. The figures show that in fractured rock the shape of the sliding surface are influenced by the orientation and length of the discontinuities (Wyllie and Mah, 2004). This is discussed in more detail in the following chapter.

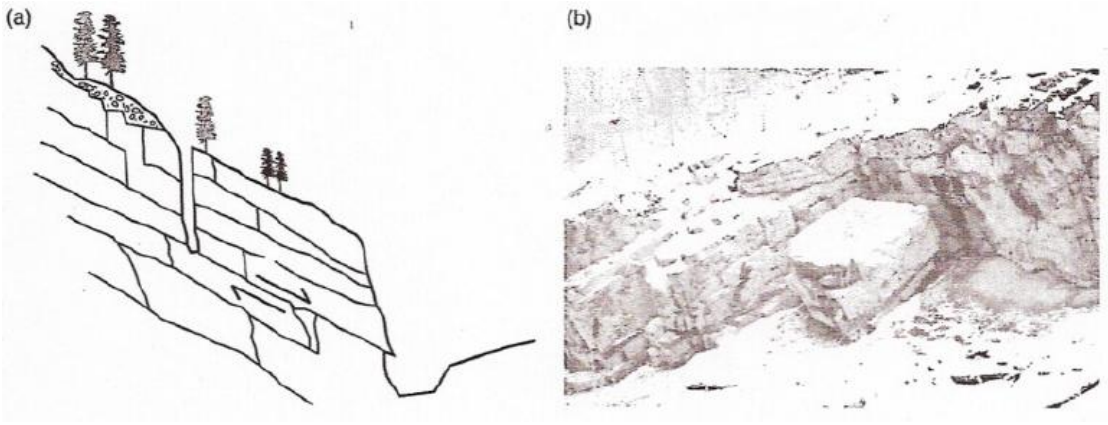


Figure 2: Plane failure on a continuous bedding plane dipping out of the slope (strong, blocky limestone, Crowsnet Pass, Alberta, Canada) (after Wyllie and Mah, 2004)

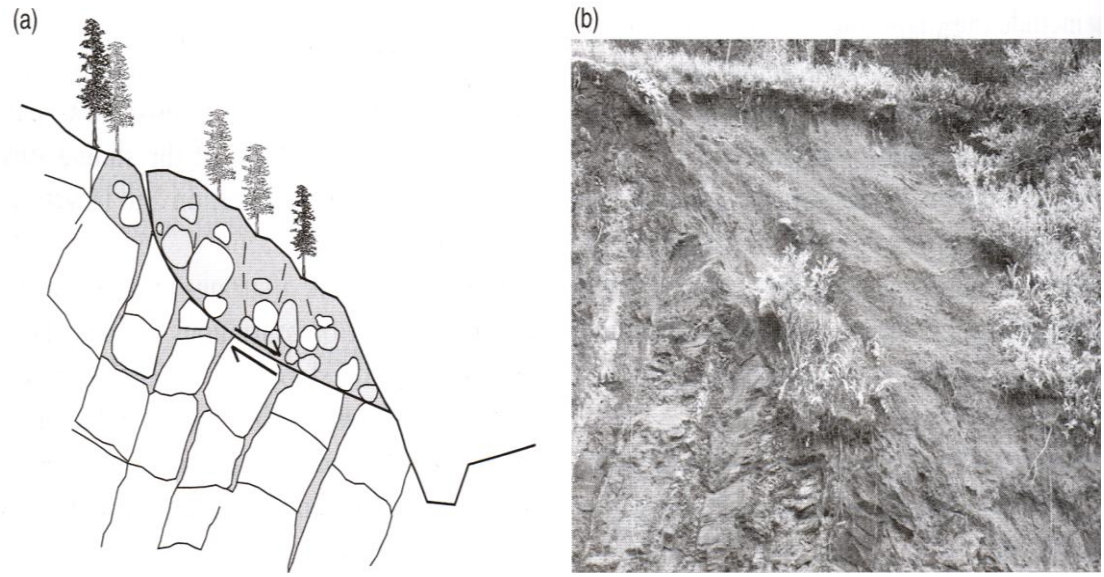


Figure 3: Circular failure in residual soil and weathered rock (weathered basalt, Island of Oahu, Hi) (after Wyllie and Mah, 2004)

Figure 2 shows a strong, massive limestone containing a set of continuous bedding surfaces that dip out of the slope face. Figure 3 shows a slope cut in a weathered rock in which the degree of weathering varies from residual soil in the upper part of the slope to slightly weathered rock at greater depth. Another geological condition that may be encountered is that of very weak but intact rock containing no discontinuities.

2.1.2 Rock mass strength

The strength of rock mass has not been investigated as frequently as the strengths of the intact rock and single discontinuities. However, it is generally accepted that strength is significantly reduced with increasing sample size. Analysis of failure mechanisms which involve one or more discontinuities are based on the concept that the strength of the rock mass is governed primarily by the strength of discontinuities and not by the strength of the intact rock itself. Due to the complexities in the behaviour of rock slopes, assessing the strength of the rock mass is difficult. Over the years there have been some successful attempts to determine the rock mass strength. Krauland et al (1989) acknowledged that there are four principal ways of determining the rock mass strength. These are:

(i) Mathematical modelling,
(ii) Rock Mass Classification,
(iii) Large scale testing, and,
(iv) Back-analysis of failures.

Empirically derived failure criteria for rock masses, often used in conjunction with rock mass classification can be added to this list. In mathematical models the strength of rock masses is described theoretically (Edoldro, 2003). The rock substance and the properties of the discontinuities are both modelled. A mathematical model requires determination of a large number of parameters and is often based on simplified assumptions.

Rock mass classification is often used in the primary stage of a project to predict the rock mass quality and excavation support design. The result is an estimate of the stability quantified in subjective terms. Additional information about the rock mass is obtained during the excavation phase and the classification system is continuously updated. The values obtained by some of the classification systems are used to estimate or calculate the rock mass strength using appropriate failure criteria.

Large-scale tests provide data on true strength of the rock mass at the actual scale of the construction, and, indirectly, a measure of the scale effect that most rocks exhibit. Large-scale tests are often neither practical nor economically feasible. For these reasons most researchers have studied the scale dependency of rock mass strength in a laboratory environment.

Back-analysis of previous failure is attractive as it permits more representative strength parameters to be determined. Failure must have occurred and the failure mode must be reasonably well established in order to carry out back analysis. Wyllie and Mah (2004) reckon that perhaps the most reliable method of determining the strength of a rock mass is to back analyze a failed or failing slope. This involves carrying out a stability analysis using available information on the position of the sliding surface, the ground water condition at the time of failure and any external forces such as foundation loads and earthquake motion. Some examples of back-calculated strength values for rock slopes are shown in Figure 4 (Hoek and Bray, 1994). These values are for slopes of various heights and in various rock types, ranging from very weak and weathered, to relatively hard and strong rocks.

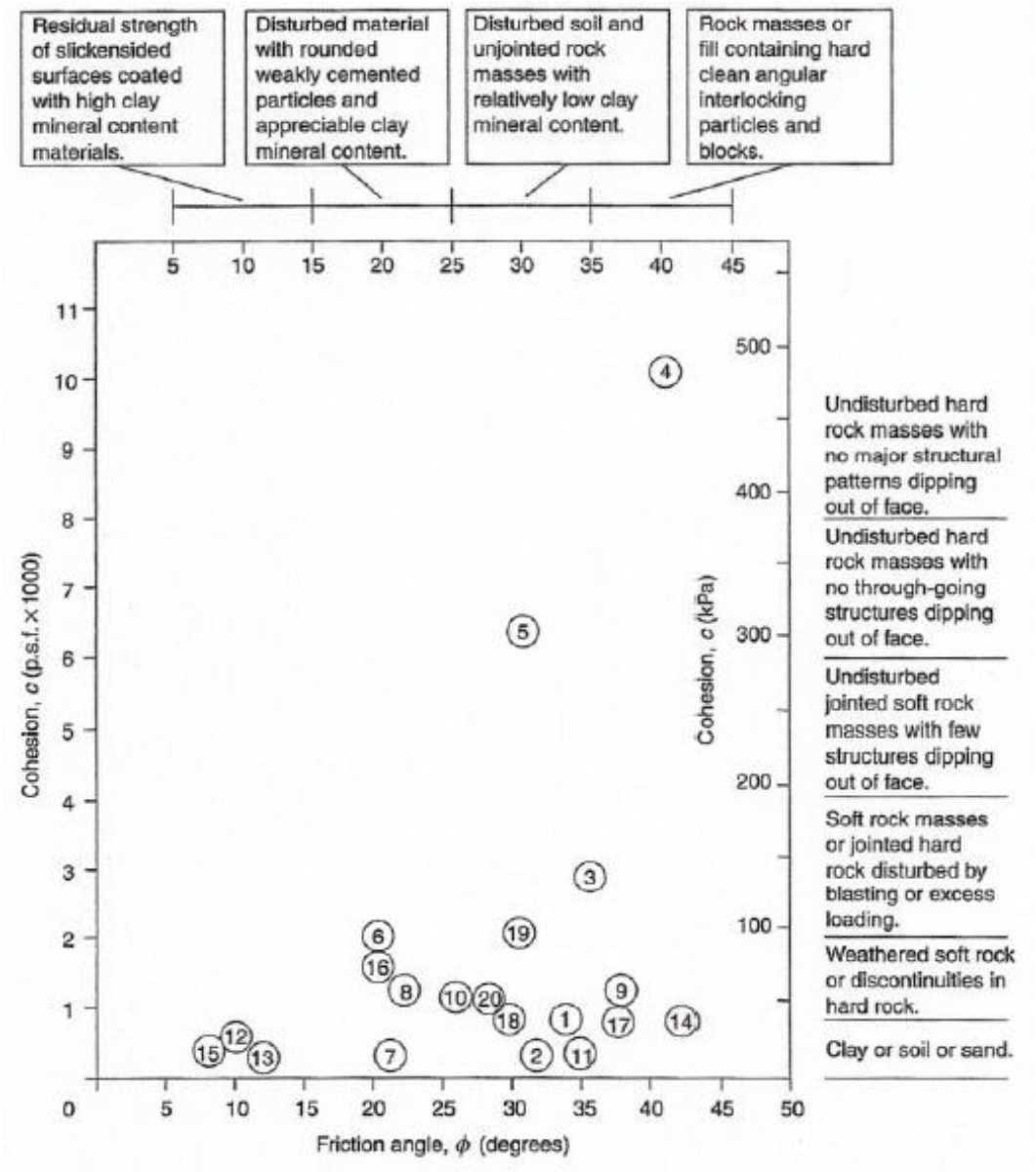


Figure 4: Relationship between friction angles and cohesive strength mobilized at failure for slopes (after Hoek and Bray, 1994)

While it is important for back analysis to be carried out in an attempt to improve understanding, the use of back analysis for determination of strength and deformation parameters for design and analysis purposes is considered to be problematic. A slope failure usually will involve deformations and local failures throughout the slope, with culmination of these smaller failures being the final failure or collapse, perhaps on a 'failure plane', and perhaps not. Often there are multiple mechanisms involved in the local and larger scale failures – shear, tension, extension,

crashing, rotation, bending, etc. Failure also may involve three dimensions instead of two. The questions that need to be asked are (Stacey, 2007a):

(i) In such cases, what should be back analysed?
(ii) Is there any validity in back analysed shear strength on the perceived 'failure plane'?
(iii) Is there any validity in back analysed rock mass cohesion, friction and deformability parameters and their use for 'calibration' of the slope or rock mass?

Most commonly used numerical methods utilise the Mohr-Coulomb failure criterion with its two basic parameters (friction angle and cohesion of rock mass) that are mostly derived from the back analysis. Questions raised by Stacey (2007a) are therefore crucial.

Sjoberg (1999) indicates that both the location and the orientation of each discontinuity must be quantified to be able to derive a criterion that describes the rock mass strength. There are very few examples of criteria that describe the strength of jointed rock masses in this manner. Ladanyi and Archambault (1970, 1980), Hoek and Bray (1994), suggested a modification of their criteria for single discontinuities, making the criteria applicable to rock masses. The failure criteria include terms for dilation rate, the ratio of the actual shear area to the complete surface area, the uniaxial compressive strength of the intact rock, and the degree of interlocking between the blocks in a rock mass. While this approach may seem attractive as it involves some consideration of the mechanics of block movement and intact rock failure, it is difficult to use in practise. As a result of the large number of input parameters and the difficulty in describing a large scale rock mass accurately, it usually takes considerable guesswork to come up with the input data.

The difficulty associated with explicitly describing rock mass strength based on the actual mechanisms of failure has led to the development of strength criteria that treat the rock mass as an equivalent continuum (Sjoberg, 1999). Reviews of existing

empirical failure criteria were described by Helgstedt (1997) and Sheorey (1997), among others. The most well known and most frequently used of these criteria is the Hoek-Brown failure criteria which was first presented by Hoek and Brown (1980a, 1980b), in which shear strength is represented as a curved Mohr envelope, with subsequent revisions and updates by Priest and Brown (1983), Hoek and Brown (1988, 1997), Hoek et al (1998) and Hoek et al (2002). This strength criterion was derived from Griffith crack theory of fracture in brittle rock (Hoek, 1968), as well as from observations of the behaviour of rock in the laboratory and in the field (Marsal, 1967, 1973; Brown, 1970; Jaeger, 1970). The original criterion is mathematically written as follows:

$$\sigma_1 = \sigma_3 + \sqrt{m\sigma_3\sigma_c + s\sigma_c^2} \quad \text{.....} \quad \text{(i)}$$

Where σ_1 = Major Principal Stress

σ_3 = Minor Principal Stress

σ_c = Uniaxial Compressive Strength of the intact rock

m and s are material constants

Values for m and s can be determined from rock mass classification systems such as the CSIR RMR classification system by Beniawski (1976). The Geological Strength Index by Hoek et al (1992), Hoek (1994) and Hoek et al (1998a) was introduced after recognizing the fact that the RMR system was not adequate in characterising very weak rock masses. This index was subsequently extended for weak rock masses in a series of papers by Hoek et al. (1998b), Marinos and Hoek (2000) and Hoek and Marinos (2000).

From the above discussion it can be concluded that there is a high level of uncertainty in assessing the “total rock mass strength”. Since the initial assumptions are associated with rock slope design based on the rock mass strength understanding the effect of discontinuities and intact rock on rock mass strength is crucial.

2.1.3 Failure mechanisms

The most important requirement for rock slope stability evaluations is the determination of the correct failure mechanism(s). The four common mechanisms of slope failure that are in use in slope stability analysis and slope design are planar failure, wedge failure, shear failure on a circular or curved surface and toppling failure (Wyllie and Mah, 2004). These failure mechanisms are illustrated schematically in Figure 5.

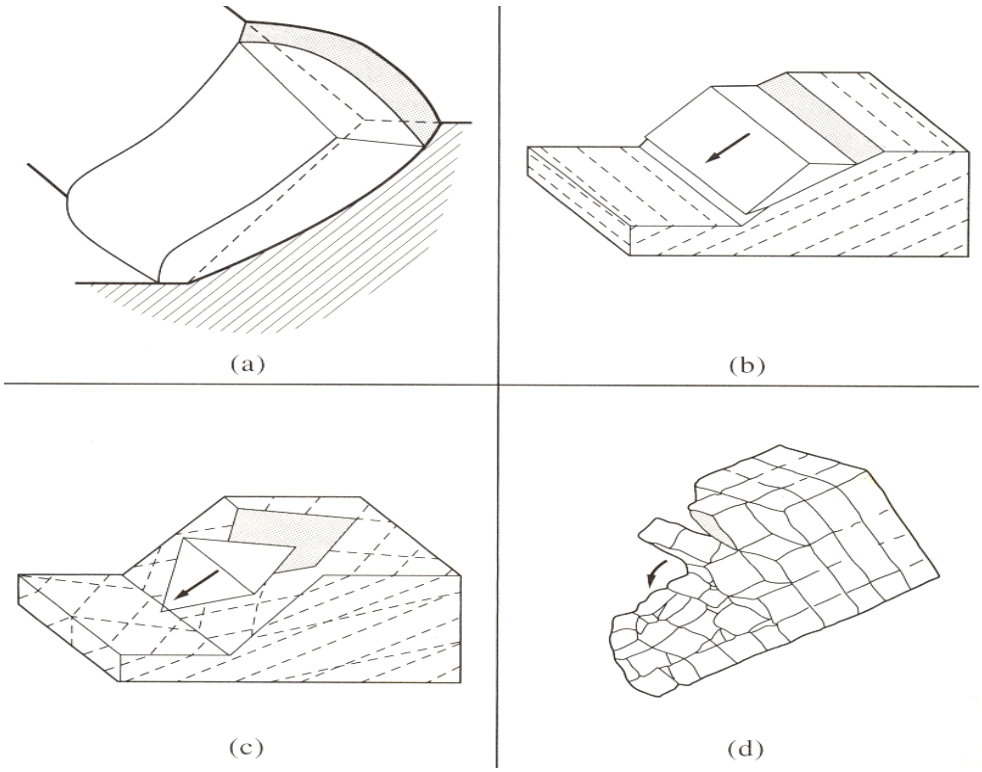


Figure 5: The four basic mechanisms of rock slope instability: (a) circular slip; (b) plane sliding; (c) wedge sliding; and (d) toppling [(a) after Hudson and Harrison, 1997, (b), (c), and (d) after Matheson, 1983]

So far the methods of analysis based on these mechanisms of failure have generally served the geotechnical engineering profession well. However, there is increasing evidence of slope behaviours that do not correspond with the common mechanisms, particularly when stress appears to be a factor (Stacey, 2007b). Despite the effort that has been put to slope stability and failure mechanisms in high rock slopes their understanding is generally poorly understood. This is particularly true for major hard-rock slopes, since there are few cases of large scale slope failures in these

slopes (Stacey, 2007b). Even for slopes in weaker rocks, which have experienced large-scale failure, several fundamental issues are still questionable (Stacey, 2007). These include, among others, the following:

(i) What are the conditions for the occurrence of different failure mechanisms?
(ii) How does failure initiate?
(iii) Does failure occur on a unique failure surface?
(iv) Do minor joints have any influence on stability of large slopes?
(v) How does progression of failure take place?

Consequently, there is need to study these aspects for commonly assumed failure mechanisms. Commonly applied failure mechanisms are discussed in the following sections. The objective is to provide a better understanding of the commonly used failure mechanisms and their shortfalls in explaining new failures observed in high slopes.

2.1.3.1 Plane failure

Plane failure involves sliding of a failure mass on a single surface as shown in Figure 6. The rock mass generally contains one or more sets of relatively uniformly distributed discontinuities with a relatively consistent length and spacing. The failure surface can generally be expected to form along a path of minimum resistance due to a combination of sliding and separation along discontinuities and failure through small amounts of intact rock (Piteau and Martin, 1981). It is rare to encounter all the geometric conditions required to produce such a failure in an actual rock slope. In order for this type of failure to occur, the following conditions must be satisfied (Wyllie and Mah, 2004):

(i) The plane on which sliding occurs must strike parallel or nearly parallel (within approximately ± 20°) to the slope face.

(ii) The sliding plane must "daylight" in the slope face. This means that the dip of the plane must be less than the dip of the slope face as shown in Figure 7a**,** that is, $\Psi^p < \Psi^f$.

(iii) The dip of the sliding plane must be greater than the angle of friction of this plane, that is, $\Psi^p > \Phi$.

(iv) The upper end of the sliding surface either intersects the upper slope, or terminates in a tension crack.

(v) Release surfaces that provide negligible resistance to sliding must be present in the rock mass to define the lateral boundaries of the slide. Alternatively, failure can occur on a sliding plane passing through the convex "nose" of a slope.



Figure 6: Planar failure on smooth, persistent bedding planes in shale (Interstate 40, near Newport, Tennessee), after Wyllie and Mah, 2004

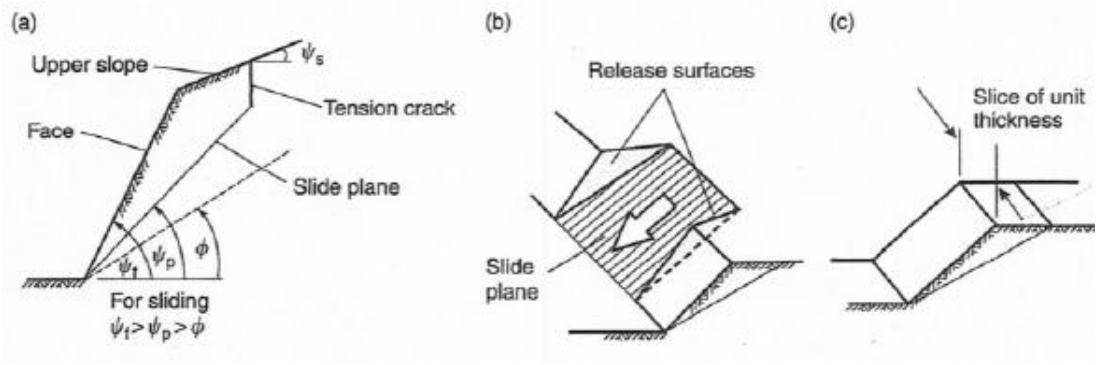


Figure 7: Geometry of slope exhibiting planar failure: (a) cross-section showing planes forming a planar failure; (b) release surfaces at ends of planar failure; (c) unit thickness slice used in stability analysis, after Wyllie and Mah, 2004

In the simplest case of pervasive, continuous joint plane that daylights the slope face, plane failure occurs if the friction angle is less than the dip of the joint plane. This is the kinematic criterion for slip. However, this is under the assumption that the rock mass is non-deformable (rigid body movements). The model tests by Sjoberg (2000) show that shear displacements are not uniformly distributed along a joint plane. In these models Sjoberg 9200) showed that the largest displacements occur near the slope face. For a slope with multiple joint planes or foliated slopes, slip did not occur on all joint planes that were kinematically free to move but was concentrated on joint planes close to the slope face.

Limit equilibrium analysis methods have been used extensively for assessing the deterministic factor of safety (FS) value for plane failure analysis. The factor of safety for plane failure is calculated by resolving all forces acting on the slope into components parallel and normal to the sliding plane. The vector sum of the shear forces, acting down the plane is termed the driving force. The vector sum of normal forces is termed the resisting force. The factor of safety of the sliding block is the ratio of the resisting forces to the driving forces, and is calculated as follows:

$$FS = \frac{\text{Resisting forces}}{\text{Driving forces}} \quad \text{...} \qquad \text{(ii)}$$

FS has been modified by taking into account the influence of ground water, roughness of surface plane, and seismic activities. For each design parameter single values are used that are assumed to be the average or best estimate values. In reality, due to variability, degree of uncertainty in measuring their values, and errors in the process of assessing their values each parameter has a range of values. The factor of safety may therefore be realistically expressed as a probability distribution, rather than a single value.

The results of a probabilistic stability analysis of the slope described by Wyllie and Mah (2004) shown in Figure 8 shows the range in the factor of safety values that are likely to exist in practice due to the variability in the slope parameters such as slope geometry. The following distributions were obtained in this example:

(i) Dip of the sliding plane, Ψ_p, has a normal distribution with a mean value of 20° and a standard deviation of 2.4°,

(ii) Cohesion has a c-skewed triangular distribution with most likely value of 80 kPa and maximum and minimum values of 130 kPa and 40 kPa, respectively,

(iii) Friction angle, Φ, has a normal distribution with a mean value of 20° and a standard deviation of 2.7°.

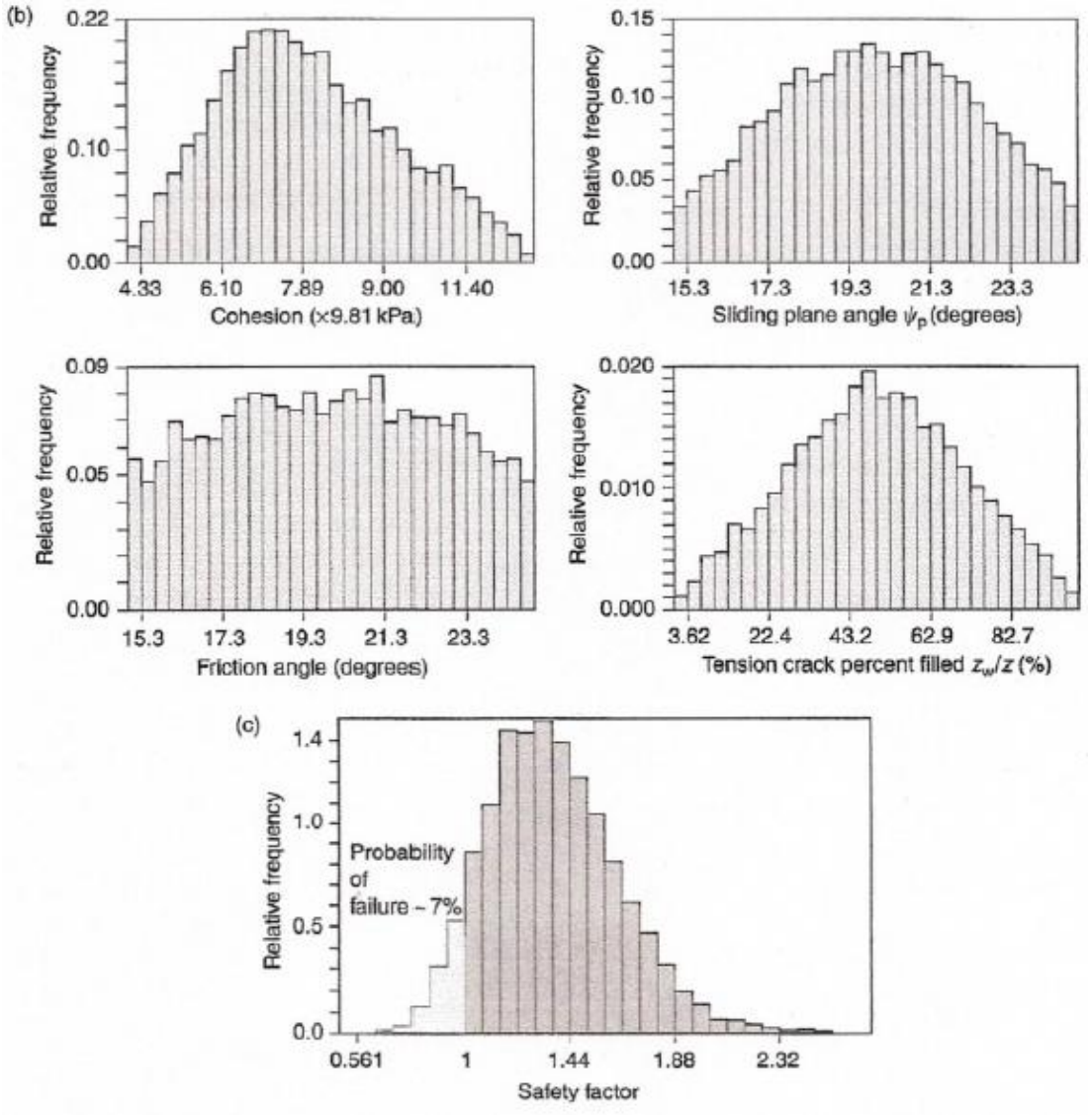


Figure 8: Probabilistic analysis of plane failure: (b) probability distribution of cohesion, friction angle, sliding plane angle and depth of water in tension cracks; (c) probability distribution of factor of safety showing 7% probability of failure, after Wyllie and Mah, 2004

The distribution shown in Figure 8(c) is a distribution of the factor of safety generated using Monte Carlo method. The method involved 10,000 iterations with values randomly selected from the input parameter distributions. The histogram shows that the mean, maximum and minimum factors of safety values are 1.36, 2.52 and 0.69 respectively. The factor of safety was less than 1.0 for 720 iterations so that the probability of failure is 7.2%. If the mean values of all the input parameters are used in the stability analysis, the deterministic factor of safety is 1.4. The sensitivity

analysis associated with these calculations shows that the factor of safety is most strongly influenced by the dip of the sliding plane. The analysis was performed using computer program ROCPLANE (Rocscience, 2003).

2.1.3.2 Wedge failure

Wedge failure occurs when two intersecting discontinuities form a tetrahedral failure block which could slide out of the slope along either one or the other of the discontinuities or along both discontinuities (Piteau and Martin, 1981). Wedge failure is probably the most common within the wide variety of mechanisms leading to failure of rock slopes (Hoek and Bray, 1981; Goodman and Kieffer, 2000). Wedge failure occurs over a much wider range of geological and geometrical conditions than plane failures. The wedge failure problem is extensively discussed in literature by Hoek and Bray (1981), Warburton (1981), Goodman (1989), Wittke (1990), Nathanail (1996), Low (1997), and Wang and Yin (2002).

Unstable wedges may be formed in rock slopes cut by at least two sets of discontinuities upon which sliding can occur (Hoek and Bray, 1981). Four different failure modes may be defined for a wedge (Goodman, 1989; Low, 1997):

(i) Sliding along the line of intersection of both planes forming the block,
(ii) Sliding along plane 1 only,
(iii) Sliding along plane 2 only,
(iv) A “floating” type of failure.

The geometry of the wedge for analysing the basic mechanics of sliding is illustrated in Figure 9**.** In order for the wedge failure to occur the following conditions must be satisfied (Hudson and Harrison, 1997);

(i) The dip of the slope must exceed the dip of the line of intersection of the two discontinuity planes associated with the potentially unstable wedge,
(ii) The line of intersection of the two discontinuity planes associated with the potentially unstable wedge must daylight on the slope plane,

(iii) The dip of the line of intersection of the two discontinuity planes associated with the potentially unstable wedge must be such that the strengths of the two planes are reached.

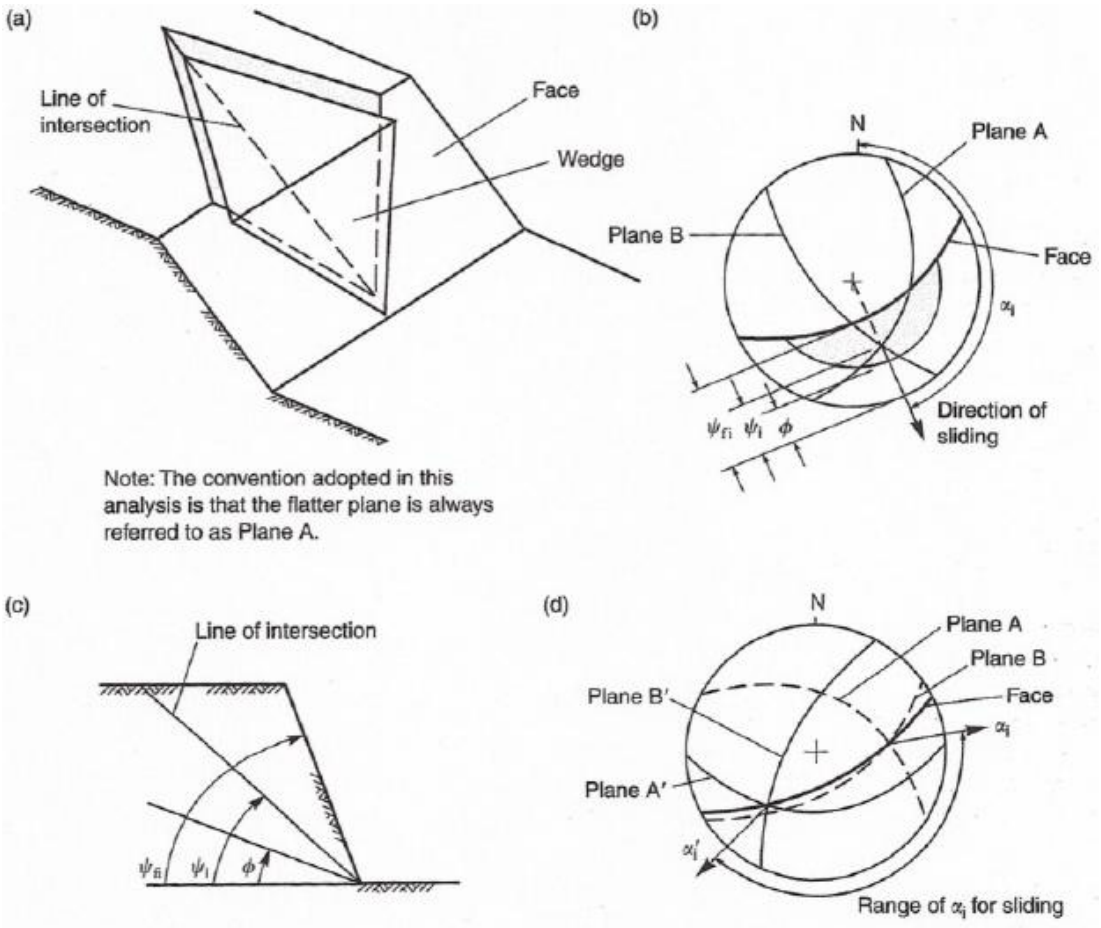


Figure 9: Geometric conditions for wedge failure: (a) pictorial view of wedge failure; (b) stereoplot showing the orientation of the line of intersection, and the range of the plunge of the line of intersection Ψ where failure is feasible; (c) view of slope at right angles to the line of intersection; (d) stereonet showing the range in the trend of line of intersection α_i where wedge failure is feasible (Wyllie and Mah, 2004)

Wedge failure mechanisms can also be described as active or passive (Nathanial, 1996). There are several geological scenarios in which active/passive wedge failure may be feasible. One surface may be a bedding plane while the other two may be persistent joints as shown in Figure 10. In the case of tightly folded strata, the fold limbs can form the outer boundaries of the wedges while crushed material in the core of the fold can form the inter-wedge surface as shown in Figure 11. An

active/passive wedge failure can also occur where a fault may form the outer boundaries of the wedges and the fault plane the inter-wedge surface as shown in Figure 12.

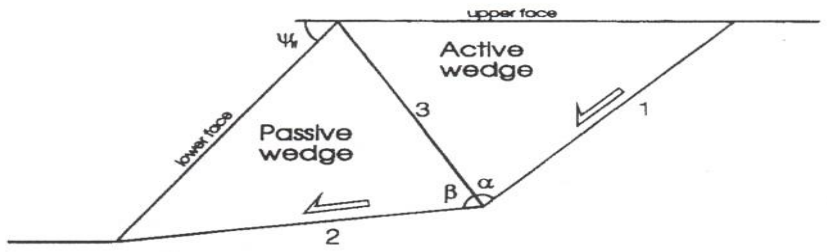


Figure 10: Geometry of active/passive wedge failure mechanism (Nathanail, 1996)

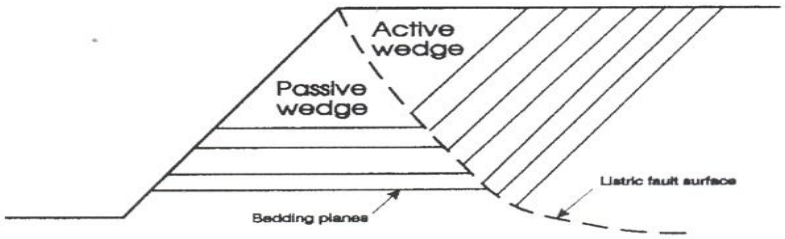


Figure 11: Geological scenario favourable for active/passive wedge failure in faulted strata (Nathanail, 1996)

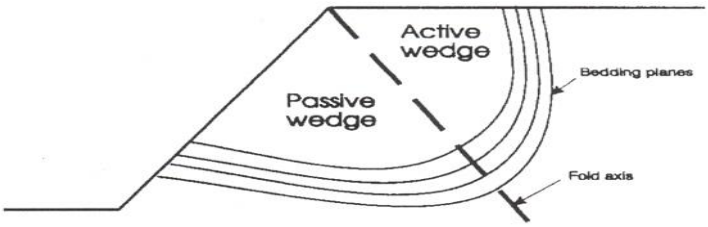


Figure 12: **Geological scenario favourable for active/passive wedge failure in folded strata (Nathanail, 1996)**

In the analysis of wedge failure, stereonets are used extensively to asses if wedge failure is kinematically feasible or not. In general, sliding may occur if the intersection point between the two great circles of the sliding planes lies within the shaded area in Figure 9. However, the actual factor of safety of the wedge cannot be determined from the stereonet. The factor of safety will depend on the geometry of

the wedge and the shear strength of each plane and water pressure (Wyllie and Mah, 2004). The limit equilibrium method is commonly used to find the factor of safety. The procedures are well documented in literature (Hoek and Bray, 1981; Wittke, 1970; Wyllie and Mah, 2004).

A detailed study of these procedures will realize that the problem is statically indeterminate. In the established force equilibrium equations, there are generally two unknown internal force vectors applied on the two failure surfaces, which involve a total of six components in the x, y, z co-ordinate system. The factor of safety to be evaluated adds one more unknown parameter. The number of available force equilibrium equations for the wedge block, normally expressed by the projection of forces on the co-ordinate axes, is three. Another two available equations can be provided by Mohr-Coulomb failure criterion that relates the magnitude of the normal and shear forces on the failure surfaces. Therefore, two assumptions must be made to allow the problem to be statically determinate (Chen, 2004). The ability to check the sensitivity of the factor of safety to variations in material properties is important because the values of these properties are difficult to define precisely.

A rapid check of the stability of a wedge can be done using friction-only stability charts. The charts and the process of utilising the charts to calculate the factor of safety is detailed in Hoek and Bray (1981) and Wyllie and Mah (2004). A slope with factor of safety, based on friction only, of less than 2.0 should be regarded as potentially unstable and require further detailed examination which will take into account wedge shape, dimensions, weight, water pressures, shear strengths, external forces, and bolting forces. Numerical modelling programs such as SWEDGE (Rocscience, 2001) and ROCKPACK III (Watts, 2001) are used as wedge stability analysis programs for comprehensive analysis.

2.1.3.3 Circular failure

Although this dissertation is concerned primarily with the stability of rock slopes containing well defined sets of discontinuities, it is also necessary to design cuts in weak materials such as highly weathered or closely fractured rock and rock fills. In such materials, failure occurs along a surface that approaches a circular shape as shown in Figure 13.

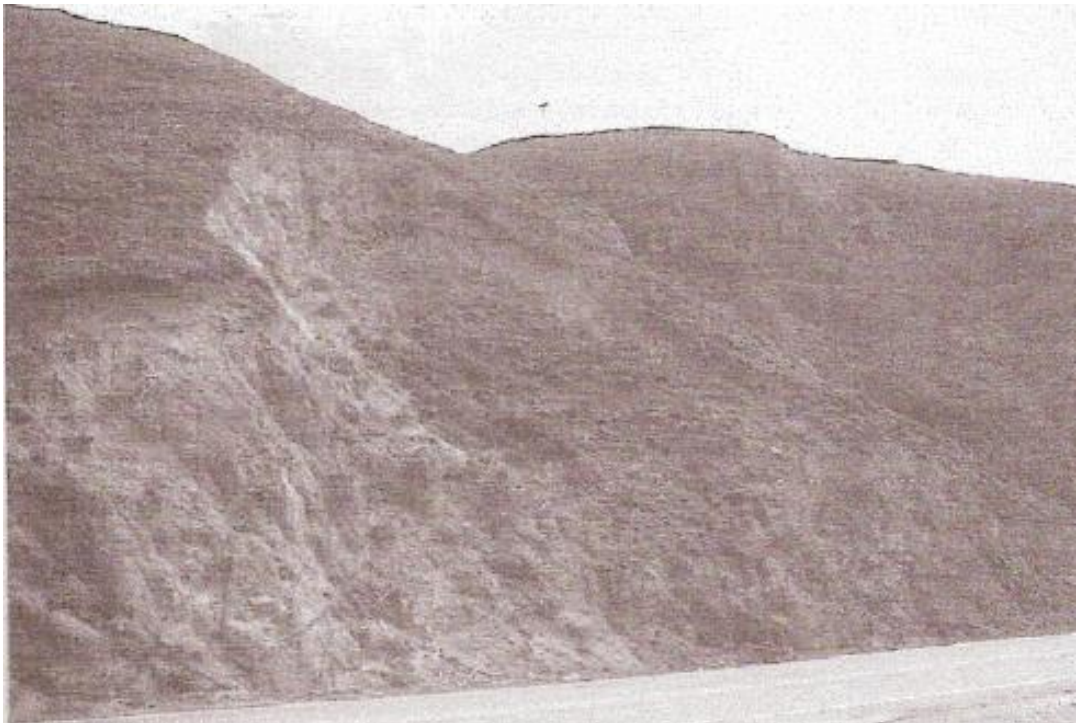


Figure 13: An illustration of circular slip

Circular shear failures are frequently observed in weak rocks (Hoek and Bray 1981) but also do occur in hard-rock slopes (Dahner-Lindqvist, 1992). Hudson and Harrison (1997) detailed the development of circular/curvilinear slips, see Figure 14. The slip surface is curved and usually terminates at a tension crack at the upper ground surface. The shape and location of the slip surface depends on the strength characteristics of the ground mass, which in turn depends on the rock mass structure. The actual failure mechanism probably involves slip along pre-existing discontinuities coupled with failure through intact rock bridges. Sjoberg (1999) admits that this will be difficult to simulate numerically, particularly for a large-scale rock slope. Rather, a pseudo-continuum approach with equivalent strength properties

for the rock mass as a whole is often taken. However, even with this simplified approach, elementary questions, such as where failure initiates and how it progresses, are left unanswered.

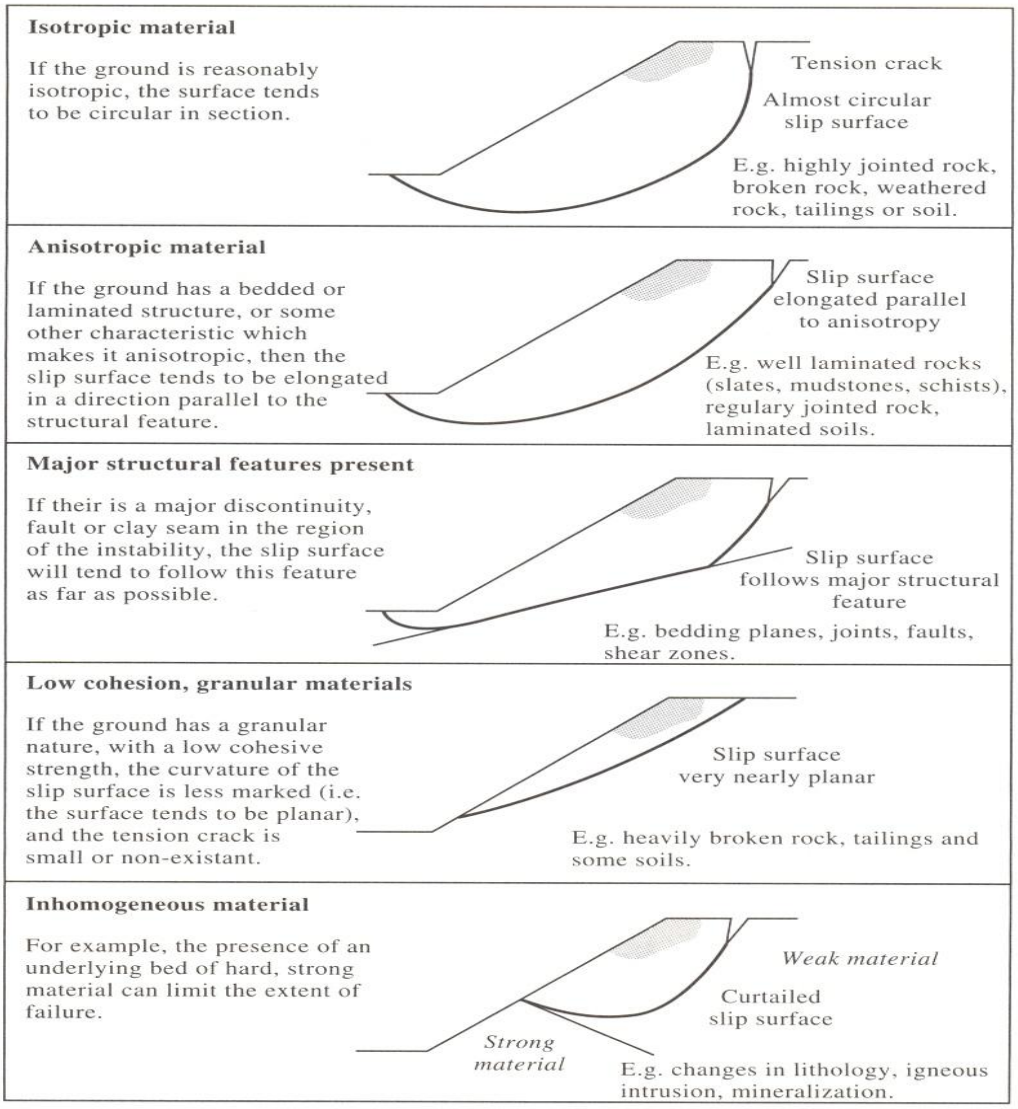


Figure 14: Development of curvilinear slips (Hudson and Harrison, 1997)

In an attempt to illustrate where failure initiates and how it progresses, Sjoberg (2000) introduced failure stages for circular rock mass shear failure in a slope. The different failure stages are summarized in Figure 15. Before failure, only elastic displacements result from the removal of rock. When mining to a new and critical slope height, yielding occurs, starting at the toe and spreads upwards. A band of

actively yielding elements forms followed by shear-strain accumulation at the toe progressing upwards and accompanied by slightly increasing displacements. For the cases studied, the displacements at this stage, i.e., before failure had developed fully were of the order of 0.2 m to 0.4 m. When the shear-strain accumulation has reached the crest, it can be said that a failure surface has formed, and larger displacements develop. Failure surface starts at the toe, followed by the middle and the crest of the slope. In the final stage, the failing mass can slide away from the slope.

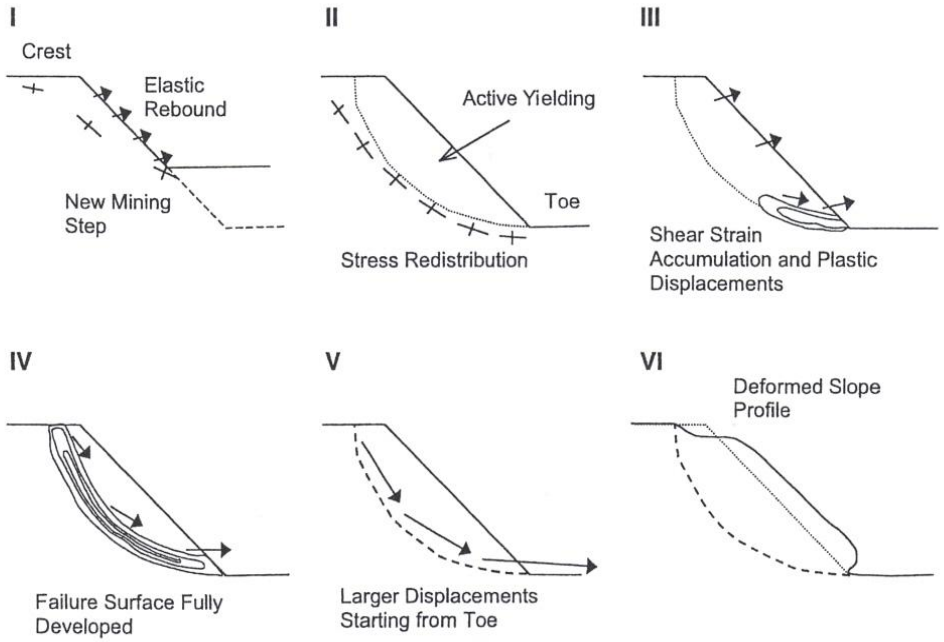


Figure 15: Failure stages for circular shear failure in a slope (Sjoberg, 2000)

2.1.3.4 Toppling failure

Toppling is defined as a mass-movement process characterized by the down-slope overturning, either through rotation or flexure, of interacting blocks of rock. Slopes with well-developed discontinuities or a pervasive foliation dipping steeply into the slope and trending parallel or sub-parallel to the slope crest are generally considered susceptible to toppling failure (Pritchard and Savingy, 1990).

One of the first qualitative papers describing field examples of toppling failures was by de Frietas and Waters (1973). This paper was pivotal in persuading geotechnical engineers and geomorphologists to accept toppling as a significant and distinct mass-movement process and mode of failure. Goodman and Bray (1976) summarized the

state of knowledge of toppling at the time, defined and discussed the basic types of toppling failure, and described the limit equilibrium method of analysis for toppling failure.

Toppling can occur at all scales in all rock types (de Frietas and Watters, 1973). The process commonly affects back slopes of highway and railway cuts (Piteau et al, 1979; Brown et al, 1980; Piteau and Martin, 1981; Ishiada et al, 1987), mine bench or mine slopes (Wyllie 1980; Piteau and Martin, 1981; Piteau et al, 1981), and excavations for other engineered structures (Woodward 1988).

Sjoberg (2000) explained the failure stages of toppling failure mechanisms. These are summarized in Figure 16. Following the elastic rebound, failure starts in the form of slip along the steeply dipping joints in the slope. Joint slip starts at the toe and progresses toward the crest, with accompanying stress redistribution around this region. The depth to which slip along the joints develops is determined by slope angle, the friction angle of joints, and the stress state. Rock columns are compressed, which creates the necessary space for a slight rotation of the columns, starting at the toe. For a high slope, even the elastic deformation of the rock mass can be enough to allow a small rotation. This is followed by tensile bending failure at the base of rotating column, which subsequently progress toward the crest. Finally a base failure surface has developed along which the failed material can slide.

The conditions necessary for toppling failure to occur are:

(i) The joints must dip relatively steeply into the slope and they must be able to slip relative to each other,
(ii) The rock mass must be able to deform substantially for toppling to have “room” to develop, and,
(iii) The rock-mass tensile strength must be low to allow tensile bending failure at the base of toppling columns.

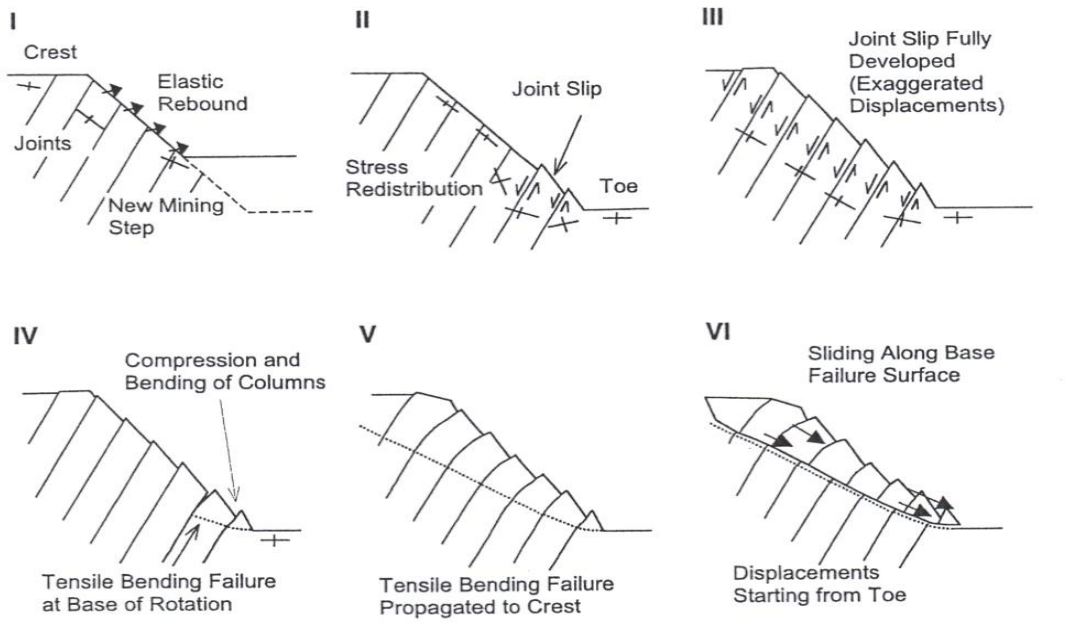


Figure 16: Failure stages for large-scale toppling failure in a slope (Sjoberg, 2000)

Four principal types of toppling failure have been defined as block, flexural, block-flexural and secondary toppling (Goodman and Bray, 1976; Evans, 1981).

Block toppling is the consequence of more widely spaced steep joints, combined with flatter, often roughly orthogonal cross-joints, which divide a stronger rock mass into blocks of finite height, see Figure 17. The cross-joints provide release surfaces for rotation of the blocks. The blocks rotate forward out of the slope driven by own weight, and stability depends on the location of the centre of gravity of the blocks relative to their bases. Such a system tends to fail catastrophically. Once blocks begin to tip, the stabilizing forces acting on the bases decrease. The dominant direction of the major principal stress in block toppling is affected by distressing in the direction parallel with the slope as individual columns move forward away from those situated further upslope (Nichol, 2002).

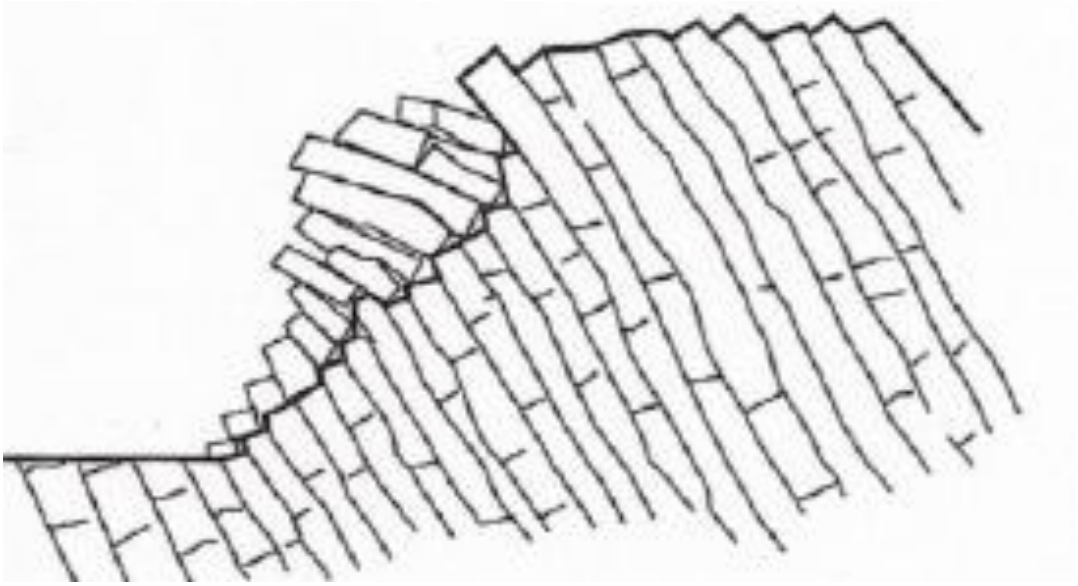


Figure 17: A schematic of block toppling failure (Goodman and Bray, 1976)

Block toppling occurs in stronger rock containing both a steep joint set and well-developed pre-existing cross joints. It is a brittle process, leading potentially to large, extremely rapid slope failures.

Flexural toppling is a ductile, self-stabilizing process. It occurs in weak rock masses dominated by a single closely spaced discontinuity set and relatively free of cross joints (Nichol, 2002). Flexural toppling is a distinctive mechanism of failure of slopes excavated in a rock mass intersected by a set of parallel discontinuities (i.e. bedding planes, foliation etc.) dipping steeply inside the slope as shown in Figure 18. When a slope is excavated in such a rock mass, the intact rock layers will tend to bend into the excavation. This will essentially involve mutual sliding, bending and subsequent fracturing of the rock layers once the local bending stress attain the layer tensile strength (Adhikary and Dyskin, 2007). Such a discontinuity system produces a rock mass composed of a stack of rock columns, which can be visualised as an array of interacting cantilever beams fixed at a certain depth, and free to bend into the excavation. In such cases, the rock columns bend forward under their own weight and transfer load to the underlying columns, thus giving rise to tensile and compressive bending stresses. Failure is initiated when tensile (bending) stress in the toe of the column exceeds the tensile strength of the rock (Adhikary et al, 1997).

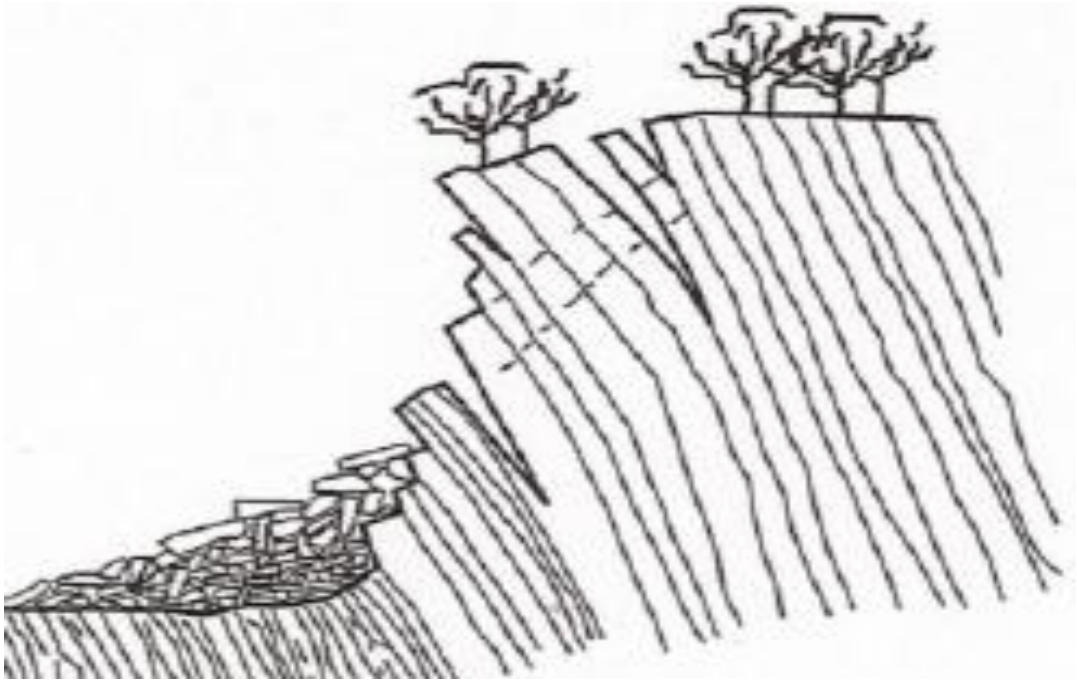


Figure 18: A schematic of flexural toppling failure (Goodman and Bray, 1976)

Block-flexure toppling is characterized by pseudo-continuous flexure along columns that are divided by numerous cross joints. Instead of the flexural failure of continuous columns resulting in flexural toppling, toppling of columns in this case results from accumulated displacement on the cross-joints. As a result of the large number of small movements in this type of topple, there are fewer tension cracks than in flexural toppling, and fewer edge-to-face contacts and voids than in block toppling (Wyllie and Mah, 2004).

Secondary toppling is divided into four categories;

(i) toppling at head of slide;
(ii) toppling at toe of slide with shear movement of upper slope;
(iii) toppling of columns in strong upper material due to weathering of underlying weak material;
(iv) toppling at pit crest resulting in circular failure of upper slope;

According to Wyllie and Mah (2004), these failures are initiated by some undercutting of the toe of the slope, either by natural agencies such as scour or weathering or by human activities. In all cases, the primary failure mode involves sliding or physical breakdown of the rock, and toppling is induced in the upper part of the slope as a result of this primary failure.

A further example of toppling mechanism was described by Sjoberg (2000). For the configuration which flexural toppling cannot develop for the case of joints dipping parallel to, or steeper than, the slope, a different type of behaviour was observed in the models, termed "underdip" toppling failure, which was first proposed by Cruden (1989). Shearing along the steeply inclined joints was followed by bending of rock columns and heaving of the slope toe.

The most popular analytical technique to predict whether a slope will topple is the limit equilibrium method developed by Goodman and Bray (1976). The technique is discussed in detail later in the dissertation. There are several limiting assumptions to limit equilibrium method for toppling analysis;

(i) No blocks can be both toppling and sliding.
(ii) The method only applies to block toppling of continuous columns. Columns may be jointed, but slip on joints or overturning of individual blocks defined by joints within a column is not allowed.
(iii) The columns are rigid. The technique cannot accurately analyze toppling with internal column deformations.
(iv) The location and inclination of the stepped failure plane must be assumed.
(v) The slope geometry is currently restricted to a uniform step and block width.
(vi) The analysis is by definition a static balance of forces and cannot incorporate variations in joint strength due to joint mobilization. (Pritchard and Savingy, 1990)

From the late 1970's, in an attempt to overcome the limitations of the limit equilibrium method, finite element method and distinct element method have been used to evaluate toppling failure mechanisms. Although finite element methods overcome many of the limitations of the limit equilibrium method, the finite element method has a limited ability to model a toppling rock mass because of its continuum formulation but on the other hand, due to discontinuous formulation, in which the rock mass is represented by an assemblage of blocks and the discontinuities dividing the blocks act as boundary interactions with a prescribed joint behaviour, distinct element modelling has an advantage of evaluating toppling failure.

2.2 Design methods

2.2.1 Introduction

An elementary feature of all design methods is that shear takes place along either a discrete sliding surface, or within a zone, behind the face. If the shear force is greater than the shear strength of the rock on this surface the slope will be unstable. Instability could take the form of displacement that may or may not be tolerable, or slope may collapse either suddenly or progressively (Wyllie and Mah (2004). There are many methods available for this type of analyses, including rock mass classification systems, limit equilibrium and numerical modelling.

Among the available design methods for assessing rock slope stability are the deterministic and probabilistic methods. A deterministic analysis, because of its simplicity, is commonly used to evaluate the stability of a slope system, based on fixed values of the necessary parameters, most importantly discontinuity parameters. The difficulty with using deterministic design approaches lies in constructing a model which is representative of the actual slope behaviour.

In probabilistic approach all the factors governing slope stability exhibit inherent variations. Representative point estimates are very difficult, if not impossible, to obtain. The stochastic nature of the input parameters is included. Parameters are described as distribution of values rather than single absolute values. By combining

the probabilities of each parameter value, the probability of slope failure can be calculated (Sage, 1976; Coates, 1977)

The following sections describe both the deterministic and probabilistic methods.

2.2.2 Limit equilibrium methods

The failure surface is assumed in limit equilibrium analyses. The shear strength of material is normally described by the Mohr-Coulomb or Hoek-Brown failure criteria. None of the basic equations of continuum mechanics regarding equilibrium is satisfied except for forces (Chen, 1975). In the simplest form of limit equilibrium analysis, only the equilibrium of forces is satisfied. The sum of forces acting to induce sliding is compared with the sum of forces available to resist failure to obtain a safety factor. The safety factor can also be formulated as a ratio between the actual cohesion or friction angle of the slope and the cohesion or friction required for the slope to be stable (Stacey, 1968), or in terms of resisting and driving moments, which is useful for the analysis of circular shear failure.

A factor of safety of less than 1.0 indicates the possibility of failure occurring. If there are several potential failure modes or different failure surfaces that have a calculated factor of safety less than 1.0, all these can fail. The condition of limit equilibrium strictly means that the only admissible factor of safety is 1.0 (Sjoberg, 1999).

The assumptions implied by the factor of safety equation are, however more far reaching. It is assumed in all limit equilibrium analysis that the shear strength is fully mobilized along an entire failure surface at the time of failure. This is not always true – with the possible exception of simple planar shear or wedge failures (Sjoberg, 1999). In other cases, and particularly for complex large scale failures of progressive nature, the shear resistance could differ significantly from point to point on the failure surface. This is a function of both varying shear strength and stress conditions along the failure surface. It is also assumed that the material in the failure zone can be subjected to unlimited deformations without loss of strength and that the

displacements within the sliding body are small compared with the displacements in the failure zone (Bernander and Olofsson, 1983). This assumption of rigid body movement is acceptable when failure occurs in the form of massive sliding along pre-existing discontinuities and the rock mass moves more-or-less as a coherent mass. It is less acceptable when failure is more progressive in nature and no clearly pre-defined failure surface exists (Stacey, 1973). Pritchard and Savingy (1990) suggest that limitations restrict the method to the analysis of small-scale toppling where the process operates across a planar failure surface, and failure is facilitated by joint shear and separation.

Although these inherent assumptions cannot be neglected, the factor of safety approach is still a very useful concept for engineering design. For slope design, Jennings and Stefan (1967) proposed that a factor of safety equal to unity should be used. The maximum achievable slope angle can be calculated following which the slope angle should be made slightly less steep to create a safety margin for the pit slope.

Eberthard et al (2002) noted that *"Limit equilibrium analysis only provides a snapshot of the conditions at the moment of failure, and as such they provide a simplified answer as to why the slope failed, but not within the context of time as to why now"*

2.2.3 Rock mass classification methods

Rock mass classification methods were introduced about 30 years ago and have been used in civil and mining engineering as the preliminary approach to assess the engineering behaviour of rock masses. During the feasibility and preliminary design stages of a project, when very limited detailed information on the rock mass, in situ stress state and hydrologic characteristics is available, the use of a rock mass classification scheme can be of considerable benefit (Hoek, 2007).

Common rock mass classification systems are discussed in this section. These are the Rock Mass Rating (RMR), Mining Rock Mass Rating, (MRMR), Rock Tunnelling Index (Q), Geological Strength Index (GSI) and Slope Mass Rating (SMR) systems.

The Rock Mass Rating or Geo-mechanics classification system was developed by Bieniawski in 1976. Bieniawski made significant changes to the ratings assigned to different parameters as more case records became available (Bieniawski, 1989). The following five parameters are used to classify a rock mass using RMR.

(i) Rock Quality Designation (RQD)
(ii) Rock material strength (UCS),
(iii) Spacing of discontinuities,
(iv) Condition of discontinuities,
(v) Groundwater condition.

In addition to these a sixth parameter, discontinuity orientation, in relation to direction of excavation advance is applied.

Rock Quality Designation (RQD) is an index of rock quality. It is defined as the percentage of the sum of core lengths greater than 100mm to the total sum of the core run. The rock material strength is the Uniaxial compressive strength of intact rock usually obtained from laboratory testing. Spacing of discontinuities describes the frequency of jointing. Condition of discontinuities describes the surface conditions and infilling of the discontinuities. Groundwater condition gives an estimate of the conditions that are likely to be encountered during the mining/excavation phase. RQD, spacing and condition of discontinuities are obtained from core logging or surface mapping of structures.

The rock mass rating value is obtained by summing the five parameter values and adjusting this total by taking the joint orientations into account. In applying this classification system the rock mass is divided into a number of structural regions and each region is classified separately. The boundaries of the structural regions usually

coincide with a major structural feature such as fault or with a change in rock type. In some cases, significant changes in discontinuity spacing or characteristics, within the same rock type, may necessitate the division of the rock mass into a number of small structural regions (Hoek, 2007). Different extents of weathering may also necessitate the division of the rock mass into geotechnical zones.

Rock mass rating values range from 0 to 100. Lower values indicate poor quality rock while higher values indicate good quality rock. The values indicate rock conditions such as fair rock or good rock, etc., and appropriate action, such as support are selected for each rock condition. The adjustment takes into account the significance of the different joint orientations and applies to that joint set that is of most significance. For example, for a tunnel, it will be the set that strikes parallel to the tunnel axis. Where no one joint set has particular significance the averages of the rating values for each set are taken into account in determining the RMR value.

Laubscher realised the RMR system was somewhat conservative as it was originally based on civil engineering case histories. Several modifications have been proposed by Laubscher (1977, 1984), Laubsher and Taylor (1976) and Laubsher and Page (1990) in order to make the classification more relevant to mining applications. After those modifications, the Mining Rock Mass Rating (MRMR) system is now a completely independent rock mass classification system. MRMR takes into account the same parameters as the RMR system, as defined by Bieniawski and adjustments are applied to the RMR value to take account of weathering of the rock mass, joint orientation relative to excavation, stress effects and effects of blasting.

Laubscher's MRMR system was originally based on case histories drawn from caving operations in Africa. Case histories from around the world have been added to the database. Relationships have been found between RMR and MRMR. The MRMR value is usually about 5 points lower than the RMR value. The MRMR classification system is better suited to real stability assessment since it is also concerned with cavability (Stacey, 2007a).

Barton et al. (1974) of the Norwegian Geotechnical Institute proposed a Tunneling Quality Index (Q), drawn from an evaluation of a large number of case histories of underground excavations, for the determination of rock mass characteristics and tunnel support requirements. The numerical value of Q is defined by;

$$Q=\frac{RQD}{Jn}x\frac{Jr}{Ja}x\frac{Jw}{SRF} \quad \text{..... (iii)}$$

where:

RQD is the rock quality designation

Jn is the joint set number

Jr is the joint roughness number

Ja is the joint alteration number

Jw is the joint water reduction factor

SRF is the stress reduction factor.

SRF/Jn represents rock block size, Jr/Ja represents inter-block shear strength and Jw/SRF represents the confining stress.

During the 30 years since the Q-system was introduced it has received much attention worldwide. Through numerous papers, several improvements and/or adjustments of the system have been published, most of them by its originators/ researchers at the Norwegian Geotechnical Institute. Literature on the developments of the Q-system and related papers can be found in Palmstrom and Broch (2006).

The Q system does not take the rock material strength into account explicitly, although it is implicitly included in arriving at the SRF assessment. The orientation of joints is also not taken into account since it is considered that the number of joint sets, and hence the potential freedom of movement for rock blocks is more important. A recent critique to the Q system by Palmstrom and Broch (2006) points out that, the ratio *(RQD/Jn)* does not provide a meaningful measure of relative block

size, and the ratio *(Jw/SRF)* is not a meaningful measure of the stress acting on the rock mass to be supported.

Palmstrom and Broch (2006) consider that the classification systems provide good checklists for collecting rock mass data, and may be of use in planning stage studies "for tunnels in hard and jointed rock masses without overstressing". Palmstrom and Broch (2006) do not support the use of these systems for final design.

Since the mentioned rock mass classification systems are complicated and the required parameter values are hard to determine, Hoek (1994) and Hoek et al (1998a) introduced the Geological Strength Index (GSI) which provides a simple visual method of quantifying the strength of rock mass for different geological conditions. Values of GSI are related to both the degree of fracturing and the condition of fracture surfaces as shown in Figure 19 and Figure 20. This index was subsequently extended for rock masses in a series of papers by Hoek et al (1998b), Marinos and Hoek (2000), and Hoek and Marinos (2000). As indicated in the Figure 20, it is recommended that a range of GSI values, rather than a single value, should be adopted.

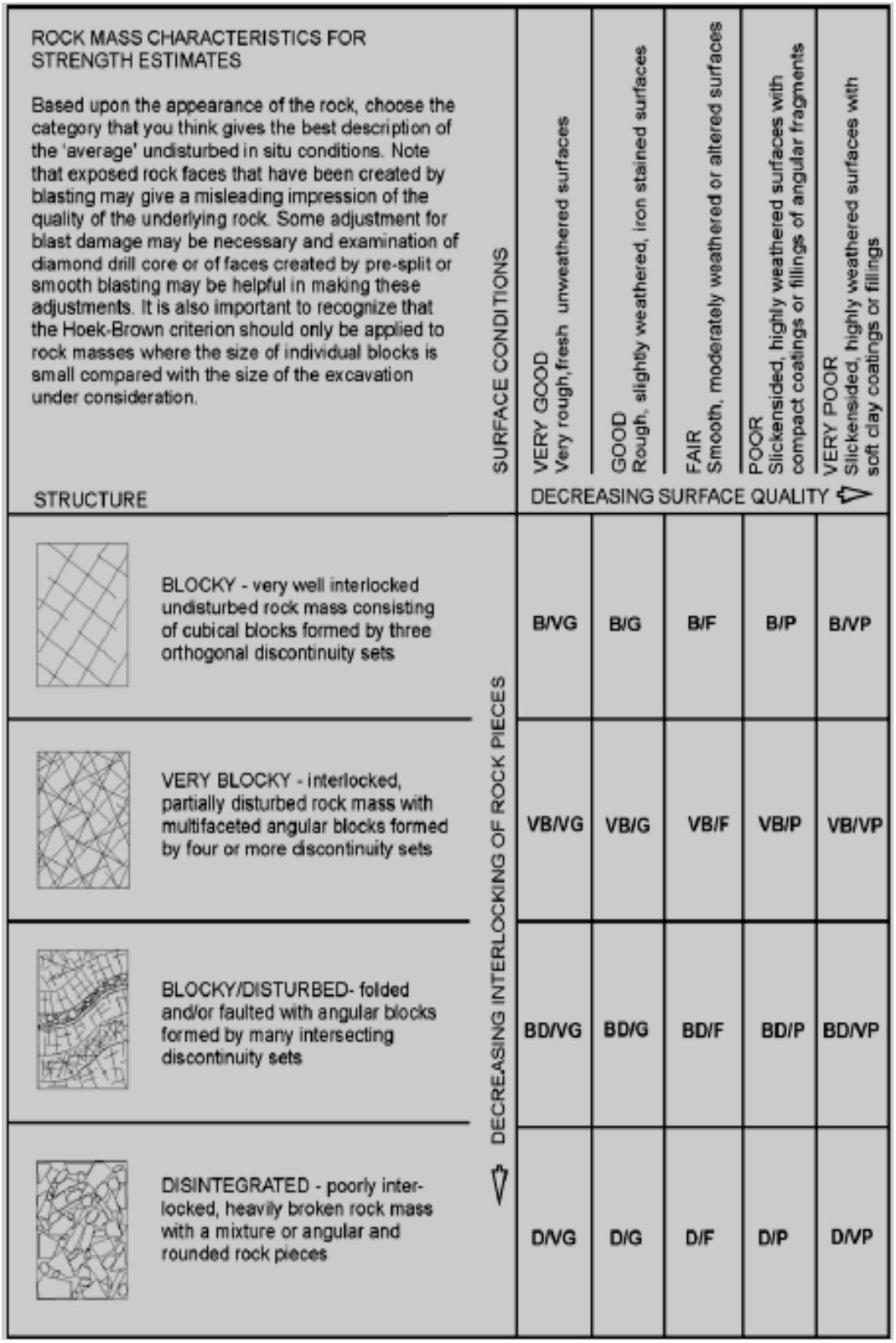


Figure 19: Characterisation of rock masses on the basis of interlocking and joint alteration (Hoek and Brown, 1997)

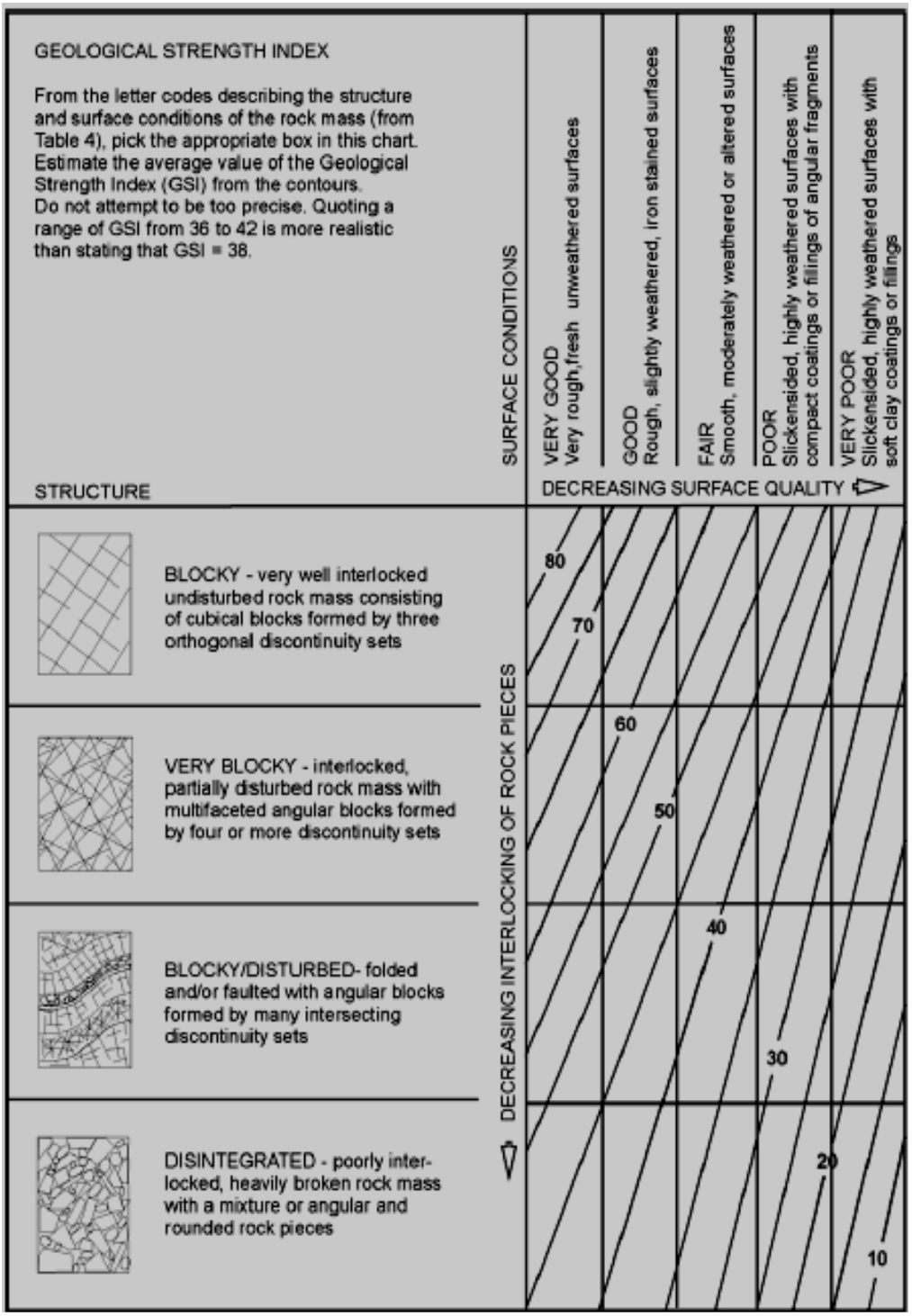


Figure 20: Estimate of Geological Strength Index GSI based on geological descriptions (Hoek and Brown, 1997)

Introduced in 1994 (Hoek et al., 1998a) to replace Bieniawski's RMR in the generalised Hoek-Brown criterion, the concept of the 'Geological Strength Index' (GSI) recognised the difficulties encountered by the criterion when the value of RMR was less than 25. Principal benefits claimed for the GSI concept were that it was based more heavily on fundamental geological observation and less on 'numbers' as featured by RMR. By sidestepping RMR it was said to avoid double counting of joint spacing, which features within the expression for both RQD and RMR, and avoid double counting of UCS, which features within expressions for both RQD and the generalised Hoek-Brown criterion.

The benefits claimed are however, somewhat cosmetic. Fundamental geological observation is accentuated, but the charted GSI values (Hoek et al., 1998a), are still those of Bieniawski RMR (1976). It follows that joint spacing remains double counted, UCS remains double counted and uncertainties of RQD as a parameter for determining rock mass strength (Read, 2007) have not been avoided.

The RMR system has also been modified into a classification system specifically aimed at rock slopes which was developed by Romana (1985) and several changes have been proposed by Romana (1991, 1993, and 1997). The system is called the Slope Mass Rating (SMR). The RMR system is applied, and four adjustment factors, namely accounting for joint and slope geometries, and the excavation method, are then added. The resulting SMR rating is grouped into one of five stability classes, which are then used to determine the overall stability for the slope and also for selection of the suggested support. SMR was also suggested by Bieniawski (1989) for use in slope stability analysis.

Tsiamboas and Telli (1991) compared the RMR and SMR system for limestone slopes. They found that the RMR system leads to an underestimation of the stability conditions of limestone cuts whilst the SMR was a better predictor. It should be noted that the only stability problems the slopes had were rock falls that were structurally controlled.

A similar but less comprehensive approach to slope stability classification was proposed by Haines and Tebrugge (1991) who based their classification on the MRMR system. They correlated slope design curves with rock mechanics on the basis of slope angle versus slope height. A simplified classification system was also developed by Hawley et al. (1994) for use in the preliminary slope design of large scale pits in South America and Canada. There have also been some attempts to develop a single design formula for slopes, similar to the formulas often used in pillar design. Sah et al. (1994) formulated an equation for applying regression analysis to a number of case studies. This was purely empirical, and different failure modes were not differentiated. The reliability of such a formula for general application must therefore be questioned. Another recent approach to rock slope stability called Slope Stability Probability Classification (SSPC) was developed by Hack (1998). The system is based on a three-step approach and on the probabilistic assessment of independently different failure mechanisms in a slope. According to Hack et al. (2003) the scheme classifies rock mass parameters in one or more exposures and allowances are made for weathering and excavation disturbance. This gives values for the parameters of importance to the mechanical behaviour of a slope in an imaginary un-weathered and undisturbed 'reference' rock mass. Assessment of the stability of the existing slope or any new slope in the reference rock mass taking into account both methods of excavation and future weathering is then made.

Due to simplicity of rock mass classifications with which rock masses can be classified, and with which a number can be put to their quality, it has been used extensively by civil and mining engineers around the world. Engineers tend to make their assessments on design and mechanism of failure according to the determined number from any rock mass classification system used. If the requirement is to predict the development of failure, or to determine whether the excavation will stabilise naturally the analysis is completely inadequate. This is because we cannot predict the failure zone geometries accurately and cannot take into account the correct mechanism of failure. However, from a design point of view, in which stability needs to be preserved and failure contained, such an analysis will be quite

adequate. It will determine the necessary potential volume of failure and the necessary length of anchors.

Consequently, in civil engineering applications, in which there is conservatism and usually a large factor of safety, any shortcomings in the use of rock mass classification approaches are masked. However, in mining, prediction is usually required and there is necessarily a lesser margin of stability. The following are some aspects to beware of when using rock mass classification methods, based on lecture notes produced by Stacey (2007a);

(i) Owing to the availability of a rock mass quality number, the "feel" for rock mass and the understanding of the likely rock mass behaviour may be lost by the inexperienced user;

(ii) General rock mass classification is not applicable to a wide range of rock masses, and this can be overlooked with the availability of a rock mass quality number. Many rock engineers nowadays appear to expect the rock mass to behave according to a number rather than according to the real in situ rock mass characteristics;

(iii) The availability of correlation between the rock mass quality number and rock mass deformation and strength parameters has facilitated sophisticated non-linear numerical stress analysis for design of support. These analyses are often carried out without the necessary understanding of the mechanisms of deformation and failure of the rock mass. This can result in completely incorrect assessment of stability, and support requirements;

(iv) The prediction of stability, or instability, requires a thorough understanding of the potential failure behaviour of the rock mass and the appropriate failure mechanisms. It is probably necessary to consider a range of potential failure mechanisms before making a stability prediction;

(v) Variability in input parameters for rock mass classification needs to be considered. The risk involved will not be taken into account with the use of a single number for the rock mass quality.

2.2.4 Numerical modelling

Limit equilibrium analysis, accompanied by stereographic techniques, remain an essential first stage method of analysis in rock slope design. With the rapid advancement in computer technology and the availability of relatively inexpensive commercial modelling codes, numerical modelling methods are now standard in rock slope investigations.

Many rock slope stability problems involve complexities relating to acquiring input parameters, geometry, material anisotropy, non-linear behaviour, in situ stresses, and the presence of several coupled processes, e.g., pore pressures, seismic loading, etc. Numerical methods of analyses used for rock slope stability may be conveniently divided into three approaches: continuum, discontinuum, and hybrid modelling. Figure 21 provides a summary of commonly used numerical techniques.

Analysis method	Critical input parameters	Advantages	Limitations
Continuum Modelling (e.g. finite-element, finite-difference)	Representative slope geometry; constitutive criteria (e.g. elastic, elasto-plastic, creep etc.); groundwater characteristics; shear strength of surfaces; *in situ* stress state.	Allows for material deformation and failure. Can model complex behaviour and mechanisms. Capability of 3-D modelling. Can model effects of groundwater and pore pressures. Able to assess effects of parameter variations on instability. Recent advances in computing hardware allow complex models to be solved on PC's with reasonable run times. Can incorporate creep deformation. Can incorporate dynamic analysis.	Users must be well trained, experienced and observe good modelling practice. Need to be aware of model/software limitations (e.g. boundary effects, mesh aspect ratios, symmetry, hardware memory restrictions). Availability of input data generally poor. Required input parameters not routinely measured. Inability to model effects of highly jointed rock. Can be difficult to perform sensitivity analysis due to run time constraints.
Discontinuum Modelling (e.g. distinct-element, discrete-element)	Representative slope and discontinuity geometry; intact constitutive criteria; discontinuity stiffness and shear strength; groundwater characteristics; *in situ* stress state.	Allows for block deformation and movement of blocks relative to each other. Can model complex behaviour and mechanisms (combined material and discontinuity behaviour coupled with hydro-mechanical and dynamic analysis). Able to assess effects of parameter variations on instability.	As above, experienced user required to observe good modelling practice. General limitations similar to those listed above. Need to be aware of scale effects. Need to simulate representative discontinuity geometry (spacing, persistence, etc.). Limited data on joint properties available (e.g. jk_n, jk_s).
Hybrid/Coupled Modelling	Combination of input parameters listed above for stand-alone models.	Coupled finite-element/distinct-element models able to simulate intact fracture propagation and fragmentation of jointed and bedded media.	Complex problems require high memory capacity. Comparatively little practical experience in use. Requires ongoing calibration and constraints.

Figure 21: Numerical method of analysis (after Coggan et al, 1998)

Examples of commercially used continuum modelling codes are MAP3D, MINSIM, and an example of a discontinuum code is UDEC.

Continuum modelling is best suited for the analysis of slopes that are comprised of massive rock, intact rock, weak rocks, and soil-like material or heavily fractured rock masses. Most continuum codes incorporate a facility for including discrete fractures such as faults and bedding planes but are inappropriate for the analysis of blocky media (Eberhardt et al., 2002). The continuum approaches used in rock slope stability include the finite-difference and finite-element methods.

Finite Element method (FEM) is popular for slope stability analysis in situations where the failure mechanism is not controlled completely by discrete geological

structures. Although FEM overcame most of the limitations of limit equilibrium methods, the finite element method has a limited ability to model rock mass behaviour due to its continuum formulation. Duncan (1996) and Griffiths and Lane (1999) summarized the results of a survey on slope analysis using FEM and provided a number of valuable lessons concerning the advantages and limitations of the FEM methods for use in practical slope engineering problems. Nevertheless, the currently widely accepted FEMs are based on hypothetical stress–strain constitutive models for intact rocks and have difficulties in simulating multiple joint sets involved in a large-scale rock mass. Wang et al (2003) concluded that these factors result in inaccuracy in capturing the true mechanical behaviour of a rock mass and therefore bring about problems in slope stability analysis and slope design, such as establishing slope failure development and the final failure surface.

In recent years the vast majority of published continuum rock slope analyses have used 2-D finite difference code, FLAC (Itasca, 2001). This code allows a wide choice of constitutive models to characterize the rock mass and incorporates time dependent behaviour, coupled hydro-mechanical and dynamic modelling.

Two dimensional continuum codes assume plane strain conditions, which are frequently not valid in inhomogeneous rock slopes with varying structure, lithology and topography. 3-D continuum codes such as FLAC3D (Itasca, 2001) and VISAGE (Vips, 1999) enable the engineer to undertake 3-D stability analyses of rock slopes.

Since a rock mass is not a continuum, its behaviour is dominated by discontinuities such as faults, joints, and bedding planes. In some cases, the failure surface can occur in intact rock portions if the rock is very weak. The behaviour of these features plays a critical part in stability evaluation (Bhasin et al., 2004). Discontinuum methods were developed to take into account the effect of discontinuities on the behaviour of the rock masses.

Discontinuum methods (e.g. discrete-element, distinct-element) treat the problem domain as an assemblage of distinct, interacting bodies or blocks which are subjected

to external loads and are expected to undergo significant motions with time. Algorithms generally use a force-displacement law to specify the interaction between deformable intact rock blocks and a law of motion, which determines displacements induced in the blocks by out-of-balance forces. Joints are viewed as interfaces between the blocks and are treated as a boundary condition rather than as a special element in the model. Block deformability is introduced through the discretization of the blocks into internal constant strain elements (Eberhardt et al., 2004). Discontinuum modelling constitutes the most commonly applied numerical approach to rock slope analysis, the most popular method being the distinct element method (Hart, 1993). Distinct-element codes such as UDEC (Itasca, 1995) use a force displacement law specifying interaction between the deformable joint bounded blocks and Newton's second law of motion, providing displacements induced within the rock slope. The dual nature of these discontinuum codes make them particularly well suited to rock slope instability problems. They are capable of simulating large displacements due to slip or opening up of discontinuities. Discontinuum codes are capable of modelling the deformation and material yielding of the joint-bounded intact rock blocks, which is particularly relevant for high slopes in weak rock and for complex modes of rock slope failure (Stead and Eberhardt, 1997). Simulation must always be verified with field observations and wherever possible instrumented slope data. This becomes even more relevant with the development of 3-D discontinuum codes such as 3DEC (Itasca, 2001).

According to Eberhardt et al. (2004), one limiting assumption required by the distinct-element method is the inclusion of fully persistent, interconnected discontinuities. This condition would be most applicable at the stage in the progressive failure process where a significant portion of the shear surface has already developed.

The discrete element method (DEM) introduced by Cundall and Strack (1979) describes the mechanical behaviour of assemblages of discs (2D) and spheres (3D). The method was developed for deformation and stability analysis of multiple jointed rock masses, for instance around underground excavations and for slope stability

analysis Cundall and Hart (1993) and Cundall (1980). The method is based on the use of an explicit numerical scheme in which the interaction of the particles is monitored by contact and the motion of the particles modelled particle by particle. The basic difference between DEM and continuum based methods is that the contact patterns between components of the system are continuously changing with the deformation process for the former, but are fixed for the latter (Jing 2003). An important recent development in discontinuum codes is the application of distinct-element methodologies and particle flow codes, e.g., PFC (Itasca, 2001). This code allows the rock mass to be represented as a series of spherical particles that interact through frictional sliding contacts. Clusters of particles may be bonded together through specified bond strengths in order to simulate joint bounded blocks. This code is capable of simulating fracture of intact rock blocks through the stress induced breaking of bonds between the particles. This is a significant development as it allows the influence of internal slope deformation to be investigated resulting from yield and intact rock fracture of jointed rock (Stead et al., 2006). Wang et al. (2003) demonstrated the application of PFC in the analysis of heavily jointed rock slopes, see Figure 22.

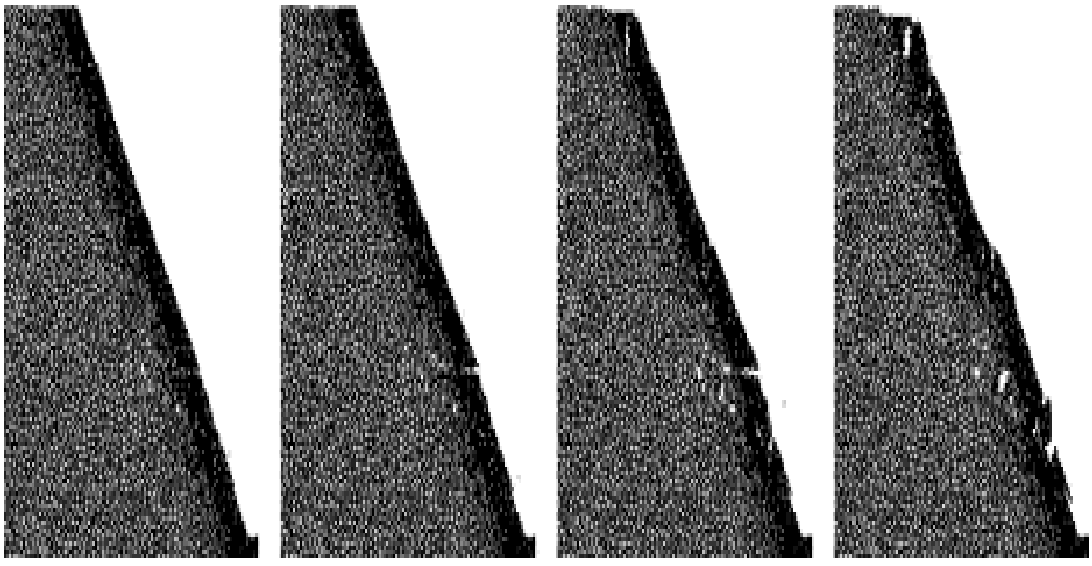


Figure 22: Simulation of a rock slope failure development for a rock mass using PFC (after Wang et al., 2003)

Another discontinuum method, the Discontinuum Deformation Analysis (DDA), developed by Shi and Goodman (1989) has been used with considerable success in the modelling of discontinuous rock masses, both in terms of rock slides (Sitar and MacLaughlin, 1997) and rock falls (Chen and Ohnishi, 1999).

By coupling individual numerical methods, the strengths of each method can be preserved while its weakness can be minimised or eliminated. The combination of individual methods and their associated models can create a model that best describes the specific problem. Hybrid numerical models have been used for a considerable time in underground rock engineering including coupled boundary-/finite-element and coupled boundary-/distinct-element solutions. Recent advances include coupled particle flow and finite-difference analyses using FLAC3D and PFC3D. These hybrid techniques already show significant potential in the investigation of such phenomena as piping slope failures, and the influence of high groundwater pressures on the failure of weak rock slopes (Stead et al, 2001). One of the latest developments in hybrid numerical technique is the combined finite-/discrete-element code; ELFEN (2001), incorporating fracture propagation and adaptive re-meshing capabilities. ELFEN can be used in the analysis of varied rock slope failure mechanisms. Models commence with a continuum representation using finite elements of the rock slopes. Discrete fractures may also be represented. Stead and Coggan (2006) explained that, in ELFEN, progressive fracturing is allowed to occur according to the selected fracturing criterion, in the process forming discrete elements with the newly formed fracture-bounded blocks being composed of deformable finite elements. The code is hence able to simulate the "total rock slope failure process" from initiation, through transport and comminution to deposition.

For most rock slope stability problems, the volume of rocks under surveillance contains many thousand, if not millions, of fractures. It is not possible to explicitly map each and every fracture. It is necessary, therefore, to utilise models which can maximise the use of field data and can then be incorporated into a numerical code. An important recent development introduced by Pine et al (2006) and Pine et al (2007) may overcome these difficulties. Pine (2007) argued that in order to capture

the essentially discontinuous nature of fractures, appropriate discontinuous kinematic mechanisms must be introduced. Continuum, distinct element or particle methods alone cannot accommodate the key phenomena so most rational approach to the modelling of multi fracturing materials is offered by a combined continuum/ discontinuum approach. The key development in the new approach is the creation of the link between the codes so that stochastic geometric models of fracture systems generated in FracMan can be processed as geo-mechanic models into the continuum/discontinuum method ELFEN.

FracMan is an interactive discrete feature analysis and geometric modelling software package developed to model the geometry of discrete features and perform data analysis, including faults, fractures, geologic modelling, spatial analysis, visualization, flow and transport, and geomechanics. Stochastic modelling of fractures in rock masses, notably the use of the FracMan suite of codes (Dershowitz et al., 1998), is well developed in some important areas. Typical applications include the modelling of fluid flow through rock fracture systems for the purposes of analysing the potential for radionuclide transport in the vicinity of proposed waste repositories (Dershowitz, 1992) and the exploitation of fracture-dominated hydrocarbon (oil and gas) reservoirs. Use of rock fracture data (orientation, continuity, spacing, surface condition) can be maximised for purposes of creating the most complete description of the rock mass as it exists, in situ, prior to further loading (Pine et al., 2007).

Cundall and Damjanac (2009) used the synthetic rock mass (SRM) numerical approach to model failure of rock slopes. In this model, a “lattice” of point masses is attached by springs, and fracture is allowed by breakage of springs and joint slip. This approach has been programmed/ coded at Itasca as Slope Model. This model accepts a discrete fracture network (DFN). Cundall and Damjanac (2009) points out that the main application of Slope Model, which allows new fracturing through intact rock, is used in the assessment of large slopes in jointed rock masses where induced stresses may be sufficient to cause new fracturing. Cundall and Damjanac (2009) verified Slope Model with numerous examples, one being that of bridging fracture.

This illustrated deep-seated failure of a large rock slope through fracturing of intact rock bridges. The fractures formed in the gaps between the joints and that allowed the overall slope to fail. This cannot be simulated by currently available programs like Slide, for example.

Failure of large rock slopes may involve the combination of several different failure mechanisms. The advantage of using numerical models over the limit equilibrium models described earlier is that they can be used to model progressive failure and displacement as opposed to a simple factor of safety. It can be shown that failure occurs in several distinguishable phases and that significant displacements occur before the failure surfaces have developed fully. Modelling can also help to distinguish the types of failures that can occur in different geomechanical environments. Two-dimensional analysis is still considered sufficient in most cases, given the current limited knowledge of rock structure and failure mechanisms in three dimensions.

2.2.5 Probabilistic analysis

Natural materials comprising most rock slopes possess an innate variability that is difficult to predict or calculate. Uncertainty also arises from insufficient information concerning the site condition and incomplete understanding or simplification of a failure mechanism. Therefore uncertainty and variability in geologic conditions and geotechnical parameters are distinctive characteristics of engineering geology (Einstein and Baecher, 1982). In rock slope stability analysis, the uncertainty and variability may be in the form of a large scatter in discontinuity attitude data and the geometry of jointing or laboratory test results. It is difficult to obtain a great number of field specimens with uniform properties. Consequently, one of the most important challenges in rock slope engineering is the selection of a representative value for stability analysis from an array of widely scattered data.

Engineers and researchers, attempting to limit and quantify variation and uncertainty in their data, have adopted various methods to select appropriate, representative values for discontinuity parameters. In an attempt to overcome uncertainty and

variability, the probabilistic theory and statistical techniques have been applied to slope stability analysis.

In the probabilistic analysis the factor of safety is considered as a random variable and can be replaced by the probability of failure to measure the level of slope stability. The probability of failure is simply defined as the probability of having FS≤1 given as a percentage that is equal to the area that FS≤1 under the probability density function (PDF) for factor of safety.

Several authors (Carter and Lajtai, 1992; Muralha and Trunk, 1993; Trunk, 1993; Leung and Quek, 1995; Wolff, 1996; Feng and Lajtai, 1998, Duzgun et al., 2003; Park et al, 2005) have developed different probabilistic analysis methods for rock slope stability. Basically these approaches use three different reliability analysis methods, such as Monte Carlo simulation, FOSM (first-order second-moment) and PEM (Point estimate method). The Monte Carlo simulation method is commonly used to evaluate reliability of the slope system when direct integration of the system function is not practical, but the PDF of each component variable is completely prescribed. In this procedure, values of each component are generated randomly by its respective PDFs and then these values are used to evaluate the factors of safety. By repeating this calculation, the probability of failure can be estimated by the proportion of results in which the safety factor is less than one. This estimation is reasonably accurate only if the number of simulations is very large.

The advantage of this method is that the complete probability distribution for the FS is obtained if the PDF of input parameters is assessed precisely and the correlation between the input parameters is estimated. The disadvantage is that large numbers of simulations are required when the failure probability is relatively small. In addition, this simulation may cause errors if the probability of failure is very low. When the PDFs of the component variables are not available, but their mean and coefficient of variance (cov) are known, the first-order second-moment method (FOSM) can be used to calculate approximately the probability of failure. This method is based on the truncated Taylor series expansion of the FS beyond the first-order term. It yields

a good approximation if the uncertainties of the variables are small. Inputs and outputs are expressed as expected values and standard deviations.

Advantages of this method are:

(i) Calculation is simple

(ii) Only information of moments (that is, mean and variance as first moment and second moment) are needed rather than a complete distribution function.

The disadvantage of the method is that mathematical calculations are difficult when the number of component variables is large. An alternative method for calculating the moments for the FS is to use the point estimate method (PEM). The FS is determined for all possible combinations of one low and one high value (point estimate) of each component variable and then the combinations are weighted by the product of their associated probabilities (Harr, 1987; Wolff, 1996). This method also requires only moment information for the variables. An advantage of this approach is that correlation of random variables can be easily considered in the calculation of moments for FS. However, both FOSM and PEM provide only estimates of mean and standard deviation for the FS and these methods do not provide information regarding the distribution of FS (Park and West, 2001).

2.3 Conclusions

The approach to rock slope analysis is continuously evolving to accommodate the necessity to quantify risks associated with failure. Although conventional methods are available to calculate the probability of a rock slope failure, all too frequently, the complexity of the rock failure mechanism casts considerable doubt on the calculated risk. The four common mechanisms of slope failure that have been used as the basis for slope stability analysis and slope design for many years do not always explain some of the failure mechanisms which have been encountered recently.

Sjoberg (2000) concluded that “There are, without a doubt, many other possible mechanisms, and there is also the possibility that these currently unknown (or poorly investigated) mechanisms are crucial for higher and steeper slopes than those presently existing.”

Stacey (2007b) quoted several unexpected slope failures from several publications. He concluded that “from the observations and interpretations it may be concluded that slope behaviour is possibly always much more complex than the common slope failure mechanisms may indicate, and that validity of methods of analysis based on these common mechanisms may often be in doubt”.

There are still lots of uncertainties in determining the behaviour of rock slopes and, therefore, slope failure mechanisms. The question is what needs to be done to minimize uncertainties in slope stability analysis so that rock slope behaviour can be better predicted. The author believes that problems that face rock slope engineers in successfully determining the behaviour of rock slopes can be overcome by utilising probabilistic based methods while bearing in mind the effects of the variability in the parameters towards slope behaviour. The results of probabilistic analyses should be seen as guidance without ignoring the engineering judgement following careful field observations.

CHAPTER 3

Jointing in a rock mass

3.1 Introduction

The engineering properties of a rock mass depends often far more on the system of joints within the rock mass than the strength of the rock itself. From an engineering point of view knowledge of the type, frequency and properties of the joints are often more important than the types of rocks involved. In assessing the potential failure mechanisms of a particular slope, consideration is given to the classification and characteristics of discontinuities. It is desirable to describe and classify the joints in such a manner that their influence can be assessed qualitatively and, if possible, quantitatively.

3.2 Rock joints

Joints are a particular type of geological discontinuity but the term tends to be used generically in rock mechanics and usually covers all types of structural weaknesses with the exception of faults (Hoek, 1984). Geometric and mechanical characterization of rock joints is the basis for most of the work of engineering geologist, civil and mining engineers when dealing with rock masses. Characterization of discontinuities is an important part of the engineering characterization of rock masses. Complete description of joints is difficult because of their three-dimensional nature, their limited exposure in outcrops, borings or tunnels, and their stochastic character.

Joints tend to form at practically all ages in the history of rocks. Different hypotheses have been suggested to account for the development of joints (Leith, 1923; Price, 1966; Hobbs et al, 1976; and Suppe, 1985). Gumede (2005) summarized that, in general, joints are a result of rock reaction to:

(i) Earth stresses responding to the mantle / crustal movements.
(ii) Different expansion and contraction rates within a rock material or rock mass in response to temperature changes, mineral alterations and variations in moisture content.
(iii) Rock material or rock mass response to stress changes due to loading and unloading of the overburden through denudation processes.

Although it is generally agreed that joints form mainly due to tensile stresses, they can, however, also form under compressive stresses (Price, 1966).

There is a general agreement (ISRM, 1978; Herget, 1977) regarding the scope of description required to characterize the nature of discontinuities and main attributes are listed below and illustrated schematically in Figure 23;

(i) orientation (mean dip and dip direction);
(ii) spacing
(iii) persistence
(iv) roughness and waviness
(v) wall strength
(vi) aperture
(vii) filling
(viii) seepage
(ix) number of sets
(x) block size

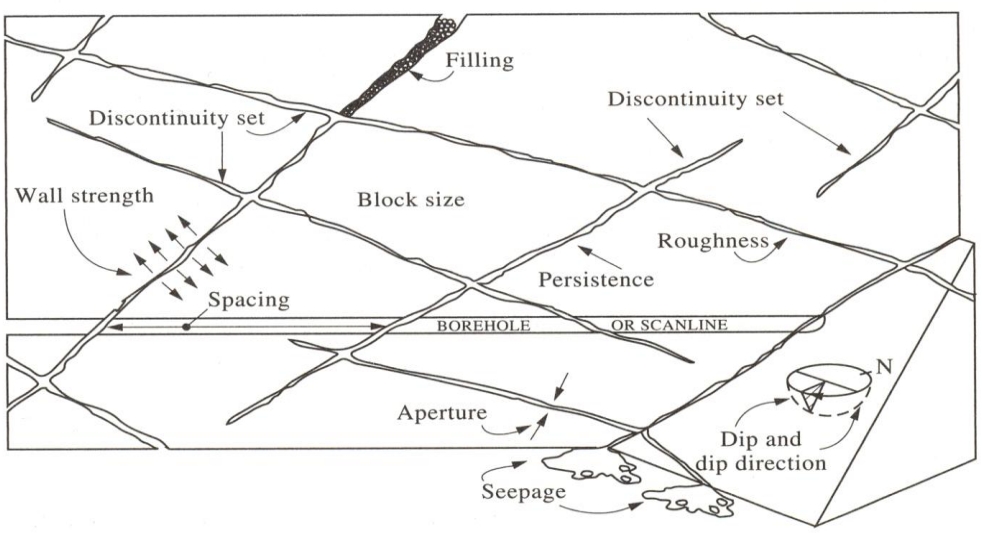


Figure 23: Schematic of the primary geometrical properties of discontinuities in rock (after Hudson, 1989)

Three main methods used for obtaining geometric properties of discontinuities are;

(i) Photographic interpretation and measurement;
(ii) Mapping exposed faces;
(iii) Drilling.

The various fracture data collection methods were summarized by Call (1992) as follows:

(i) Fracture-Set Mapping. Fracture sets are visually identified during the course of regular geologic mapping, and the fracture set orientation, length, and spacing are recorded.
(ii) Scan-Line Mapping (Detail-Line mapping). This is a systematic spot sampling method in which a measuring tape is stretched along the bench face or outcrop to be measured. For all the fractures along the tape, the point of intersection with the tape, orientation, length, roughness, filling type, and thickness are recorded.
(iii) Cell Mapping. The bench face or outcrop is divided into cells. Normally, the width of cell is equal to one to two times the height of the cell. Within

each cell, the fracture sets are visually identified, and the orientation, length, and spacing characteristics are recorded along a line that is oriented in any direction.

(iv) Oriented Core. Oriented core provides fracture orientation and spacing data, but length cannot be determined with this technique.

Cell mapping (Call et al, 1976 and Call, 1992), and fracture-set mapping (Call et al, 1976; LaPointe and Hudson, 1985; Warburton, 1980), are the favoured methods as more data can be collected over a bigger area to better define the limits of structural domains and the variability of the joint characteristics with structural domain. The choice of the technique depends on the type and number of staffing available. The scan-line method requires little or no judgement in data collection; cell-mapping and fracture-set mapping require geological judgements to be made. The scan-line represents detailed information at one location equivalent to one or two cells. It would take three to seven times longer to map enough scan lines to cover the same area using the cell or fracture-set methods. The normal scan-line is horizontal and has the inherent problem of mapping those joint sets that do not intersect that horizontal line, such as flat-dipping joint sets or sets that strike parallel to the wall orientation. The only way to map those sets is to map a vertical scan-line or a face perpendicular to the wall. Fracture-set and cell-mapping permit mapping all sets in all directions. The scan-line method can be used when confirming the distribution of the structures and also when individuals collecting the data lack geological training (Nicholas and Sims, 2000).

Discontinuities within the rock mass can be sampled only by drilling or excavation. In rock, core drilling is generally carried out using either double or preferably triple-tube barrels to reduce disturbance (Hencher, 1987). Oriented coring is used either to collect data where surface data are limited or to determine whether the structural domains mapped at the surface extend behind the pit walls. Oriented core does not provide length data. Additionally, the oriented data is more scattered than is the surface mapping data because the oriented core represents only 7 cm^2 to 15 cm^2 of the fracture plane. It does not, therefore, represent an average orientation. Also, the

oriented core has a definite blind zone, which must be considered when analyzing the data (Nicholas and Sims, 2000)

For locations where there is good rock exposure and the structure uniform Wyllie and Mah (2004) suggest that as few as 20 measurement should provide information on the orientation of the sets, with a further 50 to 100 measurement required to define typical properties such as persistence, spacing, and infilling of joints.

3.3 Classification of discontinuities

Field studies have shown that rocks are invariably jointed in preferential directions and joints occur in parallel groups which are called joint sets (Piteau, 1970) that are either sub-parallel or have similar geomechanical properties. Single minor joints have limited influence on overall slope stability, dependent of course on the size of the slope under question. Due to their repeated appearance with similar systematic orientations they can influence slope stability as a joint set. These joint sets intersect to form joint systems. When failure in a rock mass takes place, one or more joint sets are generally involved, hence the importance of joint characterisation. Detailed joint characterisation usually gives insight into the state or manner of rock mass deformation or the structural region concerned (Leith, 1923). Depending upon their mode of origin, the characteristics of the joint sets can vary greatly. Not only can the average spacing between joints vary within wide limits, but the nature and degree of joint infilling materials, physical characteristics of their planes, and their degree of development can be vastly different. Due to variations of these properties one joint set can have very different effects on shear characteristics than another joint set. For each of the joint sets the various properties of each set must be considered individually.

Geological investigations usually categorize discontinuities according to the manner in which they were formed. This is useful for geotechnical engineering because discontinuities within each category usually have similar properties as regards both dimensions and shear strength properties that can be used in the initial review of

stability conditions. The following are standard definition of the most common types of discontinuities (Wyllie and Mah, 2004):

Fault – Discontinuity along which there has been observable amount of displacement. Faults are rarely single planar units; normally they occur as parallel or sub-parallel sets of discontinuities along which movement has taken place to some extent.

Bedding – This is a surface parallel to the surface of disposition, which may or may not have a physical expression. The original attitude of the bedding plane is not necessarily horizontal.

Foliation – Parallel orientation of platy minerals or mineral banding in metamorphic rocks.

Joint – Discontinuity in which there has been no observable relative movement

Cleavage – Parallel discontinuities formed in incompetent layers in a series of beds of varying degrees of competency are known as cleavages. In general, the term implies that the cleavage planes are not controlled by mineral particles in parallel orientation.

Schistosity - Foliation in schist or other coarse grained crystalline rock due to the parallel arrangement of mineral grains of the platy or prismatic type such as mica.

3.4 Joint properties

The following sub-sections describe joint properties which include orientation, length, spacing and shear strength.

3.4.1 Joint orientation

Joint orientation describes the attitude of a discontinuity in space. Orientation is the most important joint property since joints that are favourably orientated with regard to stability, relative to a free face, effectively neutralise the effects of other joint properties. Joint orientations are usually stochastic and probability distributions are used to describe them. Not surprisingly, it is for this reason that statistical techniques were introduced at an early stage to sample and evaluate joint orientation distributions (Schmidt, 1932). Joint orientations are uniquely described by the dip and dip direction angles which are defined below (Wyllie and Mah, 2004) and shown schematically in Figure 24;

Dip is the maximum inclination of a discontinuity to the horizontal (angle ψ).

Dip direction or *dip azimuth* is the direction of the horizontal trace of the line of dip, measured clockwise from north (angle α).

Strike, which is an alternative means of defining the orientation of a plane, is the trace of the intersection of an inclined plane with a horizontal reference plane.

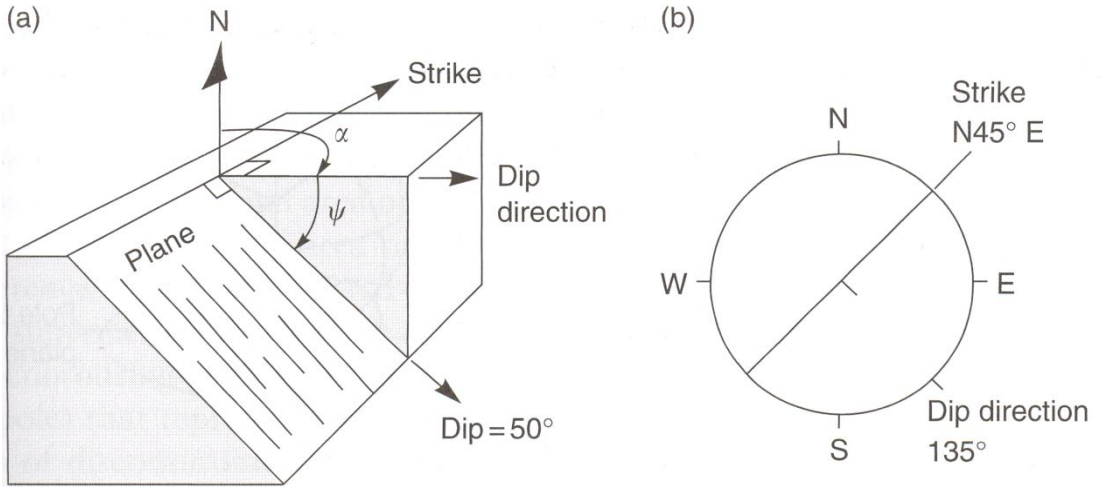


Figure 24: Terminology defining discontinuity orientation: (a) isometric view plane (dip and dip direction); (b) plan view of plane (after Wyllie and Mah, 2004)

One of the most important aspects of rock slope analysis is the systematic collection and presentation of geological data in such a way that it can easily be evaluated and incorporated into stability analysis. Spherical projections are probably the most common graphical methods in use today to represent orientation data. Since structural geological features occur in three dimensions with a degree of natural scatter, the spherical projections have been used to represent and analyze three dimensional orientation data in two dimensions. Saymaan (1991) explained that this method of modelling distributions of joint orientations has some shortcomings. Graphical methods frequency polygons and rose diagrams all represent dip and dip direction separately in two dimensions, despite the fact that they are correlated and distributed in three dimensions. The joint probability distribution of dip and dip direction is thus impossible to evaluate from such representations.

Given an ideal scenario where the stress propagation during joint development is constant and the rock material and rockmass are homogeneous, isotropic and elastic, joints will be found in single sets with insignificant dispersions. The relationship between a changing stress field acting on an inhomogeneous and inelastic rock material and rockmass is the reason for joint orientation dispersion (Saayman 1991). Methods of determining orientation dispersion and from this the standard deviation can be found in Goodman (1980), McMahon (1982), and Morriss (1984).

In sampling for joint orientations three main types of errors need to be considered. These are:

(i) Sampling error or bias;
(ii) Estimation or statistical error;
(iii) Measurement inaccuracy, which can again be sub-divided into:
- Random (measurement) errors; and
- Systematic errors

Sampling errors is caused by non-representative sampling plans. Estimation error is the result of fluctuations in statistics from one sample to the next (taken from the same population). Measurement errors are caused by imprecise readings and inaccuracies in the instruments with which individual elements are measured (Saayman, 1991).

3.4.2 Joint length

The length of a joint describes the potential failure plane. It indicates the extent to which joints and rock material individually affect the engineering properties of a rock mass. The measurable part of a joint length is referred to as the trace length. Zhang and Einstein (1998) explained that measured trace lengths can be obtained from three types of sampling on an exposure (including natural outcrops, rock cuts, and tunnel walls). These are:

(i) Sampling the traces that intersect a line drawn on exposure, which is known as scanline sampling, see Figure 25;

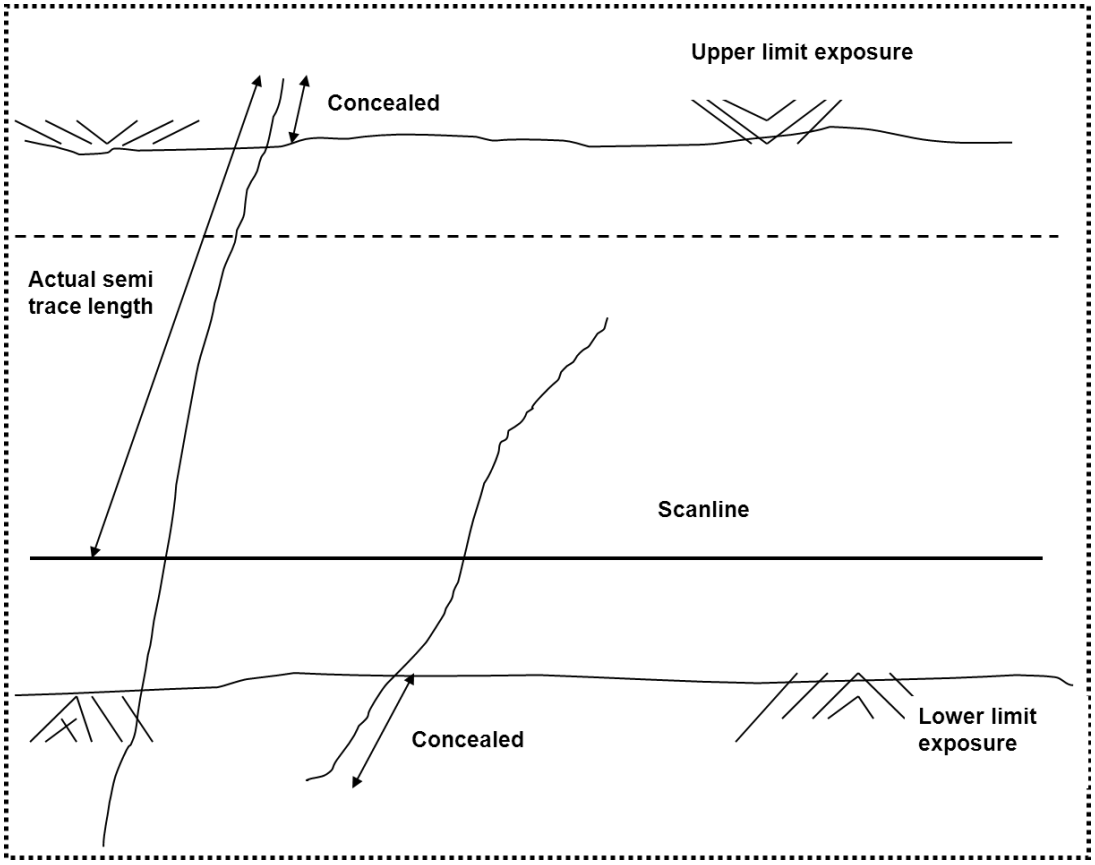


Figure 25: Trace lengths mapping in a scan-line survey (after Priest and Hudson, 1981)

(ii) Sampling the traces that intersect a circle drawn on the exposure, which is known as circle sampling;

(iii) Sampling the traces within a finite size area (usually rectangular or circular shape) on the exposure, which is known as area (or window) sampling. This is shown in Figure 26.

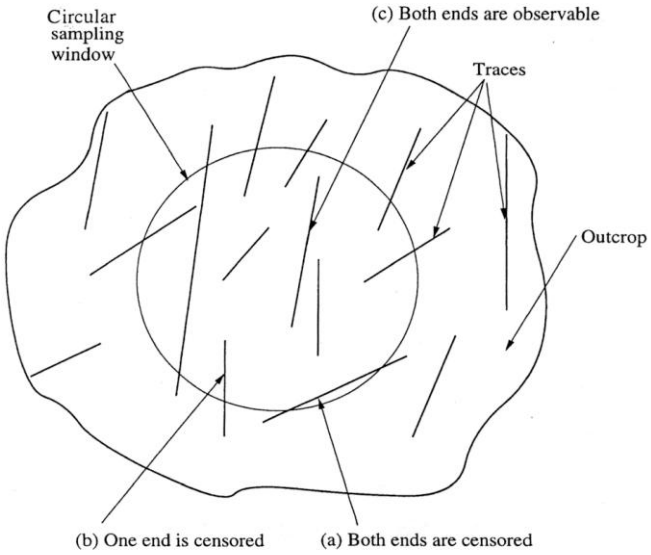


Figure 26: Discontinuities intersect a circular sampling window (Weiss, 2008)

It is not easy to measure absolute length of a joint since the entire joint surface cannot easily be seen. Hence joint trace lengths are measured in "exposures" and the overall length is statistically predicted thereafter. Accurate measurement of joint lengths is more challenging than the measurement of other joint properties mainly because of the following (Beacher and Lanney, 1978; Einstein et al., 1979; Priest and Hudson, 1981; Einstein et al, 1983; Kulatilake and Wu, 1984):

(i) Size bias due to the likelihood of measuring larger joints only, as they have a higher probability of being sampled than smaller joints. This bias affects the results in two ways. A larger discontinuity is more likely to appear in an outcrop than smaller ones and a longer trace is more likely to appear in a sampling area than a shorter one.

(ii) Censoring bias due to long joint traces which tend to extend into or out of the visible excavation face resulting in the impossibility of measuring the actual full lengths of the joints concerned. It is therefore common in joint length mapping not to measure joint lengths that are above a pre-defined length (censoring). Censoring bias consequently has the effect of underestimating the actual joint length.

(iii) Truncation bias due to trace lengths below some known cut-off length that cannot be measured practically. In practice, very small trace lengths are difficult or sometimes impossible to measure. Thus in field mapping, it is common to measure joints from a given minimum length (truncation). Joints below that given length are not measured; hence truncation errors result in the overestimation of joint lengths. The extent of this bias depends on the truncated length in relation to true joint lengths.

(iv) Orientation bias due to the probability of a discontinuity appearing in an outcrop depends on the relative orientation between the outcrop and the discontinuity. Introducing an orientation error during the measuring stage effectively means that the joint(s) will be assigned to the incorrect joint set.

Saayman (1991) concluded, after carrying out an extensive sampling experiment, that in determining joint length, the benefits of analytical techniques to correct biases will be limited and dip lengths of critical joints dipping towards a slope cannot yet be determined with sufficient reliability. It has been argued (Einstein et al, 1983) that the average length of the joints in samples can be up to twice as large as the true unbiased population average.

Given the stochastic character of joint trace lengths, evaluation of extensive data has shown that the frequency distribution of joint lengths may be described by a number of distributions: exponential (e.g Robertson, 1977; Call et al., 1976; Barton, 1976), lognormal (e.g. MacMahon 1971; Bridges, 1976; Baecher et al, 1977), hyperbolic (Segall and Pollard, 1983), and Gamma-1 (Dershowitz, 1984). In most cases these authors collected data and then determined the best fitting distribution. Different mechanical processes lead to different distributions, e.g. uniform processes to exponential distributions, multiplicatory process as they occur is breakage to lognormal distributions and the continuity of the process from smallest to largest sizes to hyperbolic distributions, (Dershowitz and Einstein, 1988).

3.4.3 Joint spacing

Joint spacing is a measure of joint intensity in a rock mass, i.e. the number of joints per unit distance normal to the dip direction of the set. It is taken as the perpendicular distance between adjacent joints. A highly jointed rock mass (closely spaced joints) is generally of poor quality than a sparsely jointed rock mass (widely spaced joints). Though orientation is considered a more important joint property than spacing, it should be noted that a widely spaced joint set, though critically oriented relative to the excavation, may have an insignificant effect on the stability of the excavation. Spacing is utilized in several ways for slope stability analysis:

(i) Spacing is used as a qualitative measure of the relative importance of joint sets in terms of their population densities.

(ii) Joint spacing has been used in the various rock mass classification systems, where it is used in terms of either joint frequency or rock quality designation (RQD).

(iii) Spacing is used quantitatively as an input parameter into wedge and stepped path type failure analysis.

Joint spacing is applied as one of six input parameters in the rock mass rating (RMR) system. ‘‘*It is widely accepted that spacing of joints is of great importance in appraising a rock mass structure. The very presence of joints reduces the strength of a rock mass and their spacing governs the degree of such a reduction*’’, (Bieniawski, 1973). The RMR applies ratings of joint spacing according to the classification system by Deere (1968). When one distinct joint set occurs, it is easy to measure the spacing. But when more than one joint set occurs, or for more complicated jointing pattern, Bieniawski (1973) did not indicate how to calculate the spacing (Palmstrom, 2001). According to Edelbro (2003) the lowest rating should be considered if there is more than one joint set and the spacing of joints vary.

For a linear survey the discontinuity intersection points may be evenly spaced, clustered, random or a combination of these. Priest and Hudson (1976) studied the results of comprehensive scan line mapping from three different tunnels in the UK.

The rock types encountered were chalk, limestone, sandstone and mudstone and it was found that, unless there is a large predominance of evenly spaced discontinuities, any combination of evenly spaced, clustered or randomly positioned discontinuities will produce a negative exponential distribution of spacing as shown in Figure 27. Priest and Hudson's findings have been supported by others (Call et al, 1976; Wallis and King, 1980). However, Sen and Kazi (1984) recommend the use of a log normal distribution for the analysis of discontinuity spacing estimates and reported that it provides greater flexibility because both the average discontinuity spacing and the variance of the discontinuity spacings are considered.

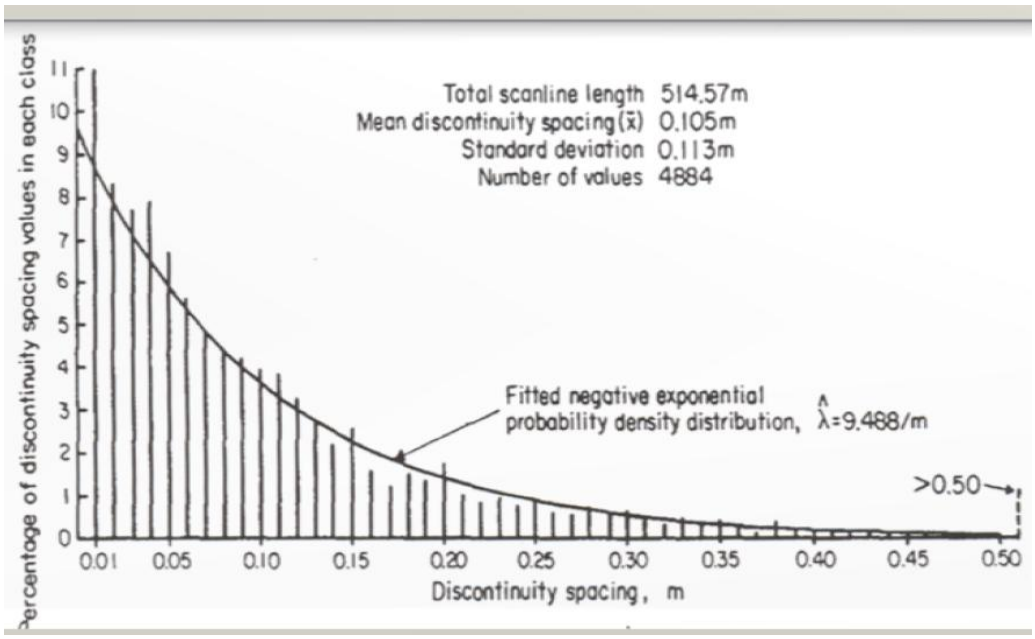


Figure 27: Example of a negative exponential distribution of discontinuity spacings (after Priest and Hudson, 1976)

Joints delineate blocks. The block sizes and shapes are related to the degree of jointing, since the discontinuities that cut the rock mass in various directions create blocks between each other. The block size is an extremely important parameter in rock mass behaviour. Where relatively regular jointing exists, it may be possible to give adequate characterization of the jointing pattern according to the system presented by Dearman (1991) in Figure 28. However, in most cases, there is no regular jointing pattern; a rough characterization of the blocks, which is shown in Figure 29, is more practical. The block shape influences the correlation between the

block volume (Vb) and the volumetric joint count (Jv) (Palmstrom, 2001). Maximum and minimum block volume and volumetric joint count can only be directly determined if the joint spacing is known. The block shape depends mainly on the differences between the joint set spacing.

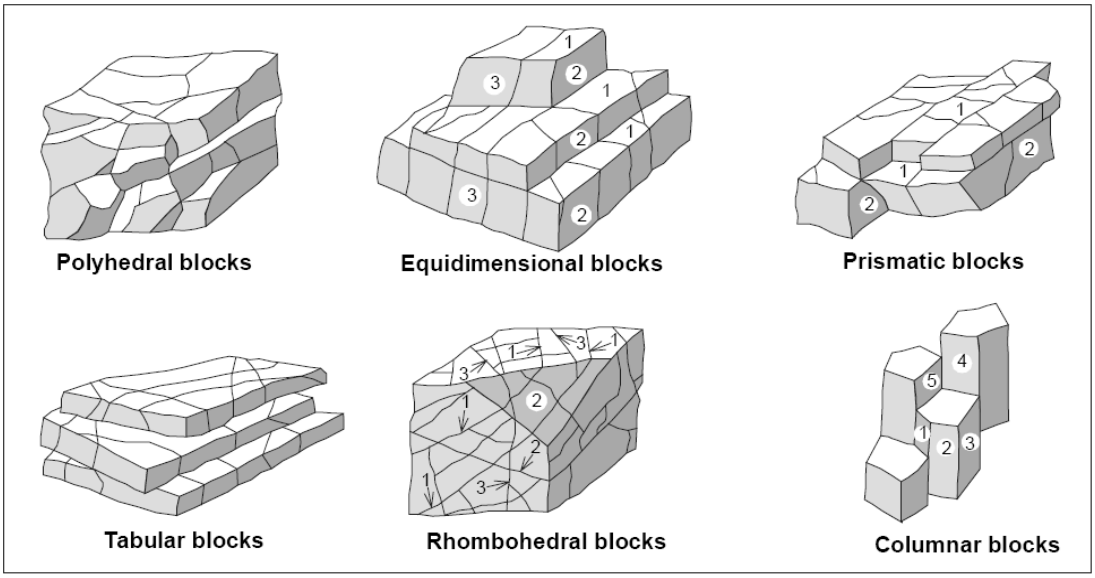


Figure 28: Examples of jointing pattern (after Dearman, 1991)

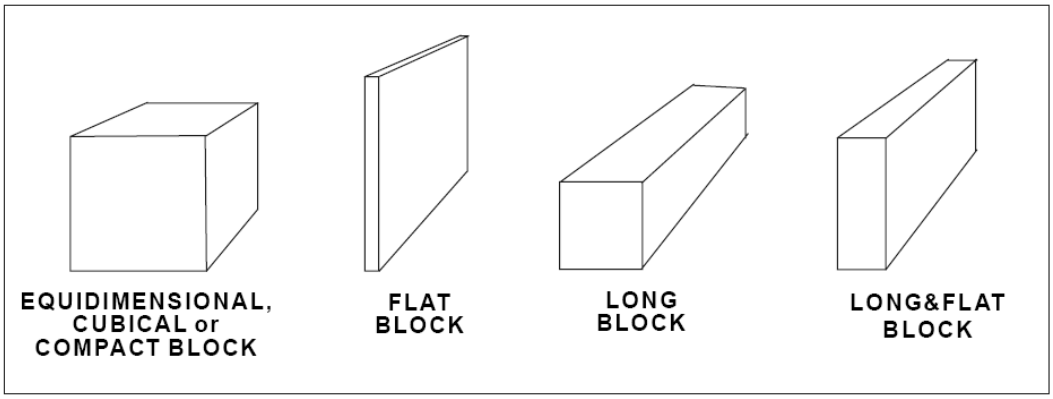


Figure 29: Main types of blocks (from Palmstrom, 1995)

Depending on its position in space a discontinuity can either lead to or detract from stability of the slope. Only those joints, for example, that have a spatial distribution within the region of the slope where failure is physically possible are of concern. It is these factors that will determine both the failure mechanism and the configuration of

the ultimate path of failure. The spatial distribution of discontinuities should be known for stability conditions to be predicted (Piteau, 1970).

Salim and Stacey (2006) also demonstrated the effect of joint offsets and argued that if joints are not offset, it is possible for opening up of the rock mass to take place very easily across these effectively continuous joints. This is due to the fact that there is not sufficient contact length on the bedding planes either side of the joints to generate shear strength, which will in turn generate effective tensile strength across the joints.

Priest (1993) described two separate criteria by which the reliability of mean discontinuity spacing estimates can be assessed:

(i) Inaccuracy is caused by the tendency of the mean estimate to be biased by some persistent factor that causes the result to be consistently in error. Short sampling lines may produce inaccuracy where discontinuity sets with average spacing values greater than the length of the sampling line may not be encountered by that line.

(ii) Imprecision is caused by the tendency of small samples to produce mean values that exhibit inconsistent random deviations from the true population mean. The precision of mean discontinuity spacing estimates will improve with an increase in the number of measured and calculated discontinuity spacing.

The percentage error recommended in practice lies in the range of 5-7% (Sen and Kazi, 1984). From literature survey the mean discontinuity spacing percentage deviation for each of the boreholes and scan lines is significantly greater than this recommended range at the 99% confidence level. It has also been observed that the percentage deviation decreases as the sample size increases, indicating that a great number of spacing measurement will provide a more reliable mean set spacing estimates.

3.5 Joint shear strength

At shallow depth, where stresses are low, failure of the intact rock material is minimal and the behaviour of the rock mass is controlled by sliding on discontinuities. In order to analyse the stability of this system of individual rock blocks, it is necessary to understand the factors that control the shear strength of the discontinuities which separate the blocks (Hoek, 2007). The term shear strength is used to describe the resistance against shearing developed along a surface and the main contributing factors for persistent joints.

Many slope failures occur through closely jointed soil and rock where the joints contribute to a general weakening effect of the rock mass (Skempton and La Rochelle, 1965; Koo, 1982). Determination of reliable shear strength values is crucial, as will be shown later; relatively small changes in shear strength can result in significant changes in the slope behaviour. Major advances have been made over the last three decades in the understanding of the factors controlling the shear strength of joints and some accepted methods for measuring or estimating strength have been established. The shear strength of discontinuities in rock has been extensively discussed by a number of authors such as Patton (1966); Goodman (1970); Ladanyi and Archambault (1970); Barton (1971, 1973, 1974); Barton and Choubey (1977).

The first known shear-strength criterion was proposed by Coulomb in the Eighteenth Century. Studying friction between two flat surfaces, he concluded that the relationship between normal and shear loads may be expressed as:

$$\tau = \mu.\sigma_n \quad \text{..... (iv)}$$

where τ is the shear strength; $_n$ is the normal stress; and μ is the coefficient of friction, which is a material property and can be expressed as:

$$\mu = \tan \phi_b \quad \text{..... (v)}$$

where ϕ_b is the basic friction angle.

Patton (1966) was the first researcher in rock mechanics to relate the shear behaviour of joints to normal load and roughness. His work is based on an idealized model of a joint in which roughness is represented by a series of constant-angle triangles or saw-teeth. Patton observed that at low normal loads the shear strength of joints may be expressed as:

$$\tau = \sigma_n \tan(\phi_b + i) \quad \text{..... (vi)}$$

where τ = Peak shear strength

σ_n = Effective normal stress

i = angle of inclination of the failure surfaces with respect to the direction of application of the shearing force.

At high normal loads, when the tips of most asperities are sheared off, he found that the shear strength can be expressed in Mohr-coulomb criterion as:

$$\tau = c + \sigma_n \tan\phi_r \quad \text{..... (vii)}$$

where c = Cohesion

ϕ_r = residual friction angle

Combining the two failure criteria together, Patton (1966) obtained a bilinear envelope that describes fairly well the shear strength of plane surfaces containing a number of regularly spaced teeth of equal dimensions as shown in Figure 30. He postulated that the two lines represented different modes of failure. At low normal stresses, i.e. line OA in the diagram, the peak shear strength is governed by sliding, while at high normal stresses (line AB) the shear strength is governed by shearing of rock surface asperities. He concluded that changes in the slope of the failure envelope reflects changes in the mode of failure and changes in the mode of failure are related to physical properties of irregularities along the failure surface.

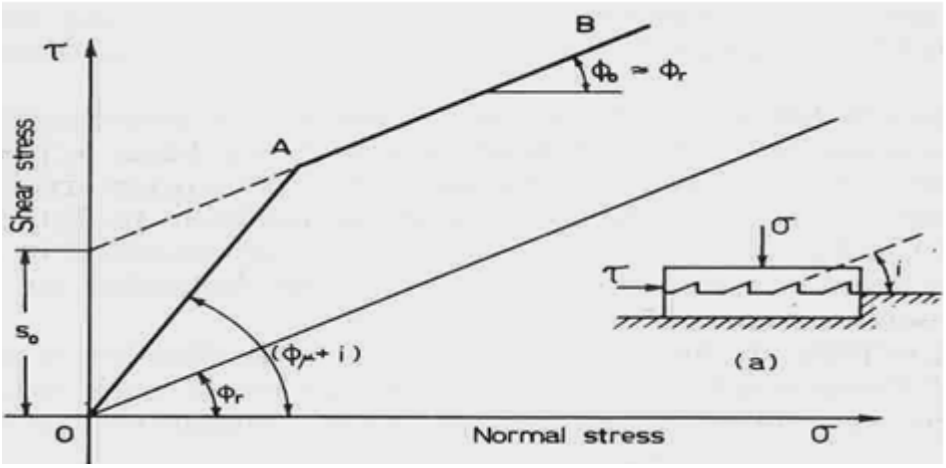


Figure 30: Bilinear failure envelope for multiple inclined surfaces (Patton, 1966)

Patton's model was extended to natural profiles by Maksimovic (1992) to take into account dilatation. To describe the variation of dilatation of rough joints as a function of normal load he proposed the following equation to estimate the peak-shear strength of a joint:

$$\tau = \sigma_n \tan\left\{\phi_b + \frac{\Delta\phi}{1+\frac{\phi_n}{p_n}}\right\} \quad \text{..... (viii)}$$

where $\Delta\phi$ = inclination of the steepest asperities

p_n = the median angle pressure

A technical problem in trying to use this method is that it is imperative to perform at least three shear tests on the same surface to calculate the parameter p_n (Maksimovic 1996). The transition from dilatancy to shearing was studied theoretically and experimentally by Ladanyi and Archambault (1970) who approached the problem of joint-shear strength by identifying the areas on the joint surface where sliding and breaking of asperities are most likely to occur.

While Patton's approach has the merit of being very simple, it does not reflect the reality that changes in shear strength with increasing normal stress are gradual rather than abrupt. Barton and co-workers (1973; 1976, 1977, 1990) studied the behaviour

of natural rock joints and proposed the widely used empirical equation for analysis and prediction of shear strength of rock joints. The equation can be re-written as:

$$\tau = \sigma_n \tan\left[JRC \log_{10}\left(\frac{JCS}{\sigma_n} \right) + \phi_b \right] \quad \text{..... (ix)}$$

where $\tau =$ Peak shear strength

$\sigma_n =$ Effective normal stress

$JRC =$ Joint Roughness Coefficient

$JCS =$ Joint Wall Compressive Strength

$\phi_b =$ Basic friction angle

It is clear that, by comparing Barton's original equation to that proposed by Patton, the difference between the two expressions is that the roughness angle *i* of Patton's equation has been replaced by a term dependent on normal stress that contains JRC.

For the study of the behaviour of discontinuities for a range of normal stresses and different rock types normally encountered in mining excavations, equation (ix) can be considered to be adequate (Hoek, 2007). The main parameters that must therefore be calculated to determine shear strength of discontinuity surfaces are JRC, JCS, ϕ_b .

Barton proposed estimating JRC either by back-analyzing shear tests that have been performed, or by field estimates. It can visually be estimated by observing the surface and matching it with the roughness profiles shown in Figure 31. The International Society for Rock Mechanics later adopted these standard profiles in their suggested procedure for measuring the roughness of discontinuities (ISRM, 1978). This method is valid for small-scale laboratory specimen. However, in actual field conditions where the length of the surface is large, JRC must be estimated for the full-scale surface using profilometer as shown in Figure 32. For a certain profile length of the surface, the maximum asperity amplitude is measured in millimetre using the profilometer and the value of JRC is noted from graphs shown in Figure 32.

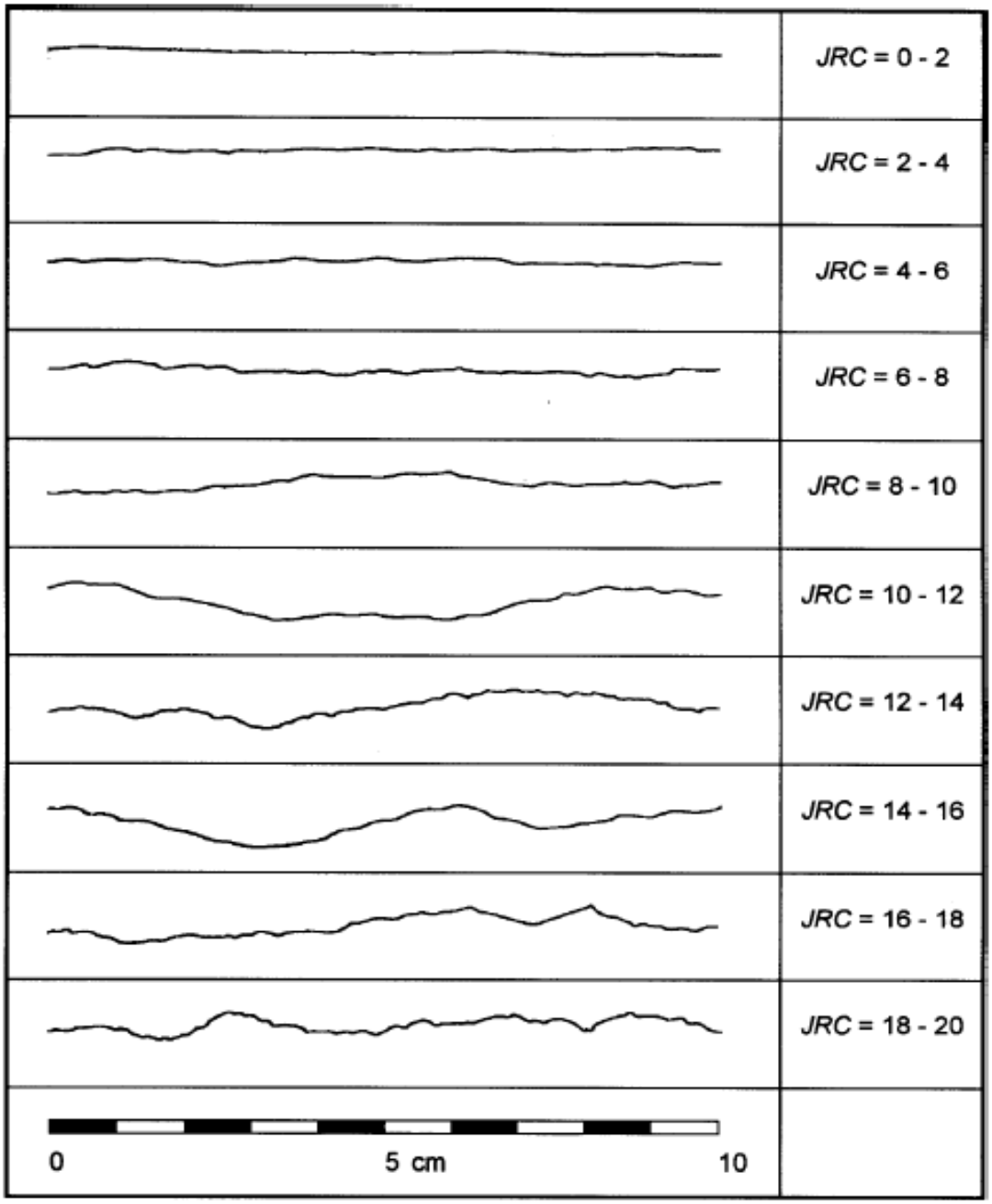


Figure 31: Roughness profiles and corresponding JRC values (after Barton and Choubey, 1977)

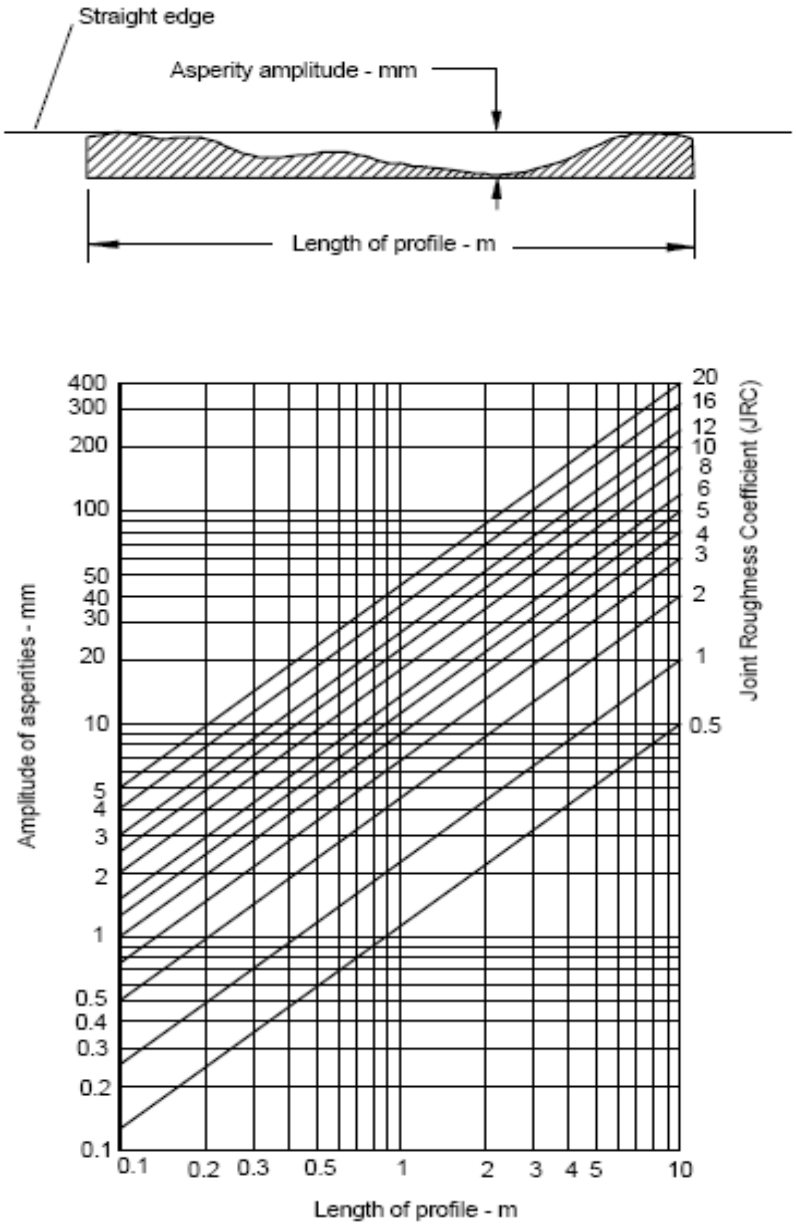


Figure 32: Method for estimating JRC from measurements of surface roughness amplitude from a straight edge (from Barton and Bandis, 1982)

This visual-comparison method for estimating JRC has been judged to be subjective and unreliable by several investigators (e.g. Hsiung et al. 1993, Maerz et al. 1990). Therefore many researchers have studied alternative ways to calculate JRC, and consequently, many parameters have been proposed in the literature (Grasselli, 2001). Many researchers have investigated the correlation between statistical parameters (Tse and Cruden 1979, Reeves 1985) or fractal dimensions (Lee et al.

1990) of the profiles and JRC values. However, the JRC values themselves include a few problems. For example, while shear strength of a joint depends on the direction of shearing (Huang and Doong 1990, Jing et al. 1992), the statistical parameters and fractal dimensions give no directional information (Grasselli, 2001).

Where the state of weathering of both the rock material and the joint walls is similar, samples of rock material tested in uniaxial compression can be used to estimate JCS (Wines and Lilly 2003). Where joint walls are weathered to a limited depth, methods of point load testing (Hoek and Bray, 1977) and Schmidt hammer techniques may be appropriate. Where no direct measurements are available, a ratio of JCS/σ_c equal to 1/4 may be used (Barton, 1973) or field index testing may be used. The suggested methods for estimating joint wall compressive strength were published by the ISRM (1978). The use of the Schmidt rebound hammer for estimating joint wall compressive strength was proposed by Deere and Miller (1996), as shown in Figure 33.

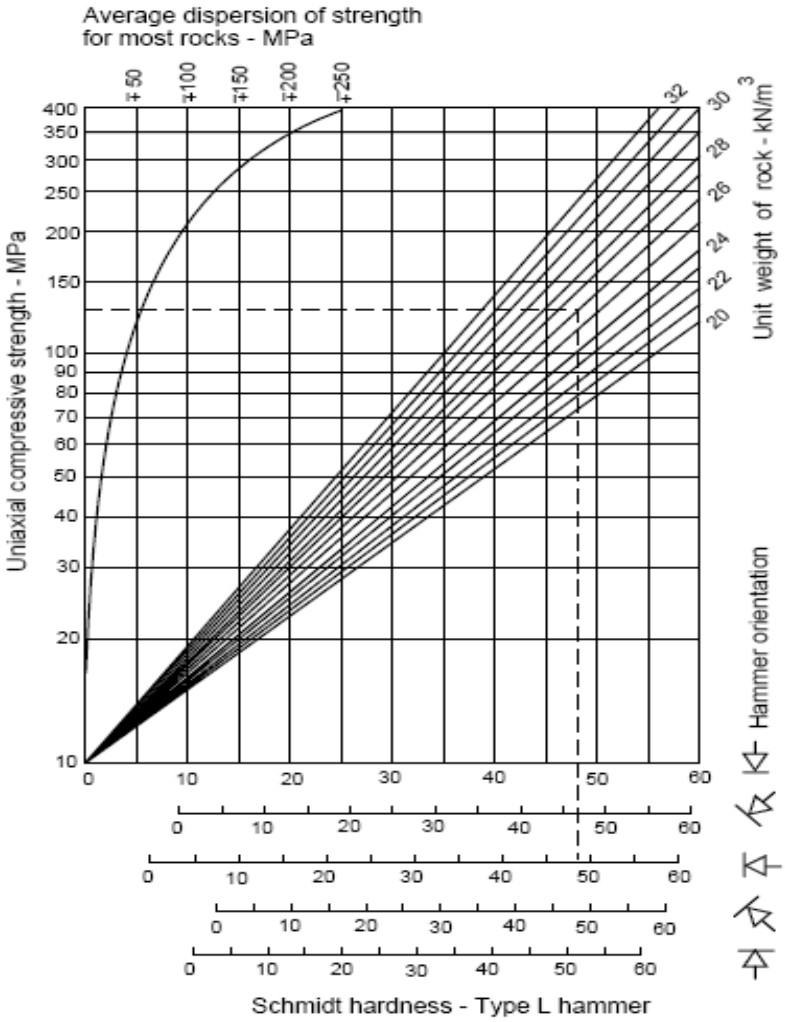


Figure 33: Estimate of joint wall compressive strength from Schmidt hardness

If JRC and JCS are determined from laboratory-scale samples it may be necessary to incorporate scale corrections in order to get the corresponding values for *in situ* block size. Barton and Choubey (1977) noted that as the joint length increases, joint wall contact is transferred to the larger and less steeply inclined asperities as the peak shear strength is approached, resulting in larger individual contact areas being less steeply inclined in relation to the mean plane of the joint when compared to the small, steep asperities, resulting in reduced JRC values. Consequently, the reduction in JRC and JCS values will result in a reduction in shear strength as the discontinuity length is increased and small-scale roughness of a surface becomes less significant compared to the dimensions of the discontinuity, and eventually large-scale

undulation has more significance than the roughness (Barton and Bandis, 1982; Bandis, 1993).

The basic friction angle can be estimated from direct shear test on smooth rock surfaces that have been prepared by means of a smooth, clean diamond saw cut (Hoek and Bray, 1977). Barton and Choubey (1977) reported that the basic friction angle for most smooth unweathered rock surfaces lies between 25° and 35°. Stimpson (1981) suggested the use of tilt testing of diamond core samples for the estimation of the basic friction angle. He observed the core surfaces produced by typical core drilling procedures are pre-cut and smooth and therefore not dissimilar to saw cut rock surfaces.

The discussion presented above has dealt with the shear strength of discontinuities in which rock wall contact occurs over the entire length of the surface under consideration. This shear strength can be reduced drastically when part or the entire surface is not in intimate contact, but covered by soft filling material such as clay gouge (Hoek, 2007). If the discontinuity contains an infilling, the shear strength properties of the fracture are often modified, with both the cohesion and friction angle of the surface being influenced by the thickness and properties of the infilling. The presence of infillings along discontinuity surfaces can have a significant effect on stability. It is important that infillings be identified in the investigation program, and that appropriate strength parameters be used in design (Wyllie and Mah, 2004). A comprehensive review of the shear strength of filled discontinuities can be found in Barton (1974).

Many of the analyses used to calculate factors of safety against sliding are expressed in terms of the Mohr-Coulomb cohesion (*c)* and friction angle (φ), which are the main parameters in numerical modelling of jointed rock. The Mohr-Coulomb equation is non-linear and involves the use of c and φ. It becomes necessary to estimate these values. The friction angle of a discontinuity surface can be determined in the laboratory using direct shear test. Direct shear tests on small samples of rock joints can only be expected to provide the following data:

(i) The basic frictional resistance for groups of like joints once the effects of sample variability have been removed.

(ii) Information on the mechanics of shearing of the natural joint that can aid in the selection of a roughness coefficient for the joint in situ (Hencher, 1987)

It is also possible to measure normal stiffness of the discontinuity infilling during direct shear tests (Wyllie, 1999).

The most reliable values are obtained if a sample with a smooth, planar surface is used because it is found that, with an irregular surface, the effect of surface roughness can make the test result difficult to interpret. It can be difficult to measure the cohesion of a surface in a direct shear test because, if the cohesion is very low, it may not be possible to obtain an undisturbed sample. If the cohesion is high and the sample is intact, the material holding the sample in the test equipment will be stronger than the infilling if the sample is to shear. Where it is important that the cohesion of a weak infilling be measured, an *in situ* test of the undisturbed material may be required (Wyllie and Mah, 2004). A true cohesion will result from impersistence of joints and several authors (Jennings, 1970 and Einstein et al., 1983) have addressed the problem theoretically but at present there may not be any better methods available than to use engineering judgement following careful field description. Field values for cohesion might be calculated by back-analysis of failed slopes.

Joint stiffness parameters are also fundamental input in numerical modelling of jointed rock masses and describe the stress-deformation characteristics of joints (Wines and Lilly, 2003). Barton (1972) described the joint shear stiffness (K_s) as the average gradient of the shear stress-shear displacement curve for the section of the curve below peak strength. Shear stiffness can be estimated from direct shear testing results, and its values will depend on the size of sample tested and will generally increase with an increase in normal stress (Bandis et. al. 1983)

The UDEC user's manual (2000) states that the shear stiffness for rock joints with clay-infilling can range from roughly 10-100 MPa/m while that for tight joints in granite and basalt can exceed 100 GPA/m. Hamman and Coulthard (2007) present typical ranges for joint shear stiffness including values of 250-450 GPa/m for jointing in basalt (Wines and Lilly, 2004).

Normal joint stiffness is a measure of additional displacement that occurs in a specimen with a joint compared to the displacement that would be measured for an identical specimen without a joint (Hopkins, 2000). Normal joint stiffness introduced by Goodman et al (1968) as one of three parameters that the authors suggest could be used to describe potential behaviour of a joint. Barton (1972) described joint normal stiffness (K_n) as the normal stress per unit closure of joint. Bandis et al. (1983) proposed that joint normal stiffness is influenced by:

(i) The initial actual contact area;
(ii) The joint wall roughness;
(iii) The strength and deformability of the asperities; and the thickness, type and physical properties of any infill material.

Joint normal stiffness can be estimated from laboratory testing. The UDEC user's manual (2000) states that the normal stiffness for rock joints with clay-infilling can range from roughly 10-100 MPa/m while that for tight joints in granite and basalt can exceed 100 GPa/m. Hamman and Coulthard (2007) present typical ranges for joint normal stiffness including values of 300-550 GPa/m for basalt.

3.6 Uncertainties and error in joint measurement

Einstein and Baecher (1982, 1983) have defined the main sources for uncertainties and errors in engineering geology and rock mechanics. These are:

(i) Spatial variability.
(ii) Measurement errors (sample disturbance, random errors, bias errors, statistical fluctuation).
(iii) Model uncertainty.
(iv) Omissions.

The ideal characterization of jointing would involve the specific description of each joint in the rock mass. Exact definition of its position and geometric and mechanical properties are imperative. This is not always possible for a number of reasons (Palmstrom, 2001):

(i) The visible parts of joints are limited and thus prevent complete observation;
(ii) Joints at a distance from the exposed surfaces cannot be directly observed;
(iii) Direct (visual or contact measurement) and indirect (geophysical) observations have limited accuracy.

Robertson (1977) noted that the measured strike may vary as much as ±20°. For attitude measurements of planar features, Friedman (1964) estimates accuracy of ±1° for dips greater than 70° and ±3° for inclinations of 30-70°. In reality, the variability could probably be greater than that considered by the authors.

Uncertainties in jointing measurements can be illustrated with an example of the Coulomb model which describes the dependence of shear resistance on normal stress by a linear relationship. The friction and cohesion parameters of this model are subject to the previously mentioned spatial variability and measurement errors. In

addition the model itself is actually an idealized representation of data points which might be more accurately related by a curve (Mohr model) as indicated in Figure 34.This is model uncertainty (Einstein, 2003). Its existence has been recognized since the beginning of the period under consideration (Baecher, 1972) but it is only relatively recently that detailed investigations of this uncertainty have been conducted (Lacasse and Nadim, 1996). Einstein and Karam (2001) have shown that for instance, in landslide analyses, there are up to 20% differences in safety factors depending on the chosen model.

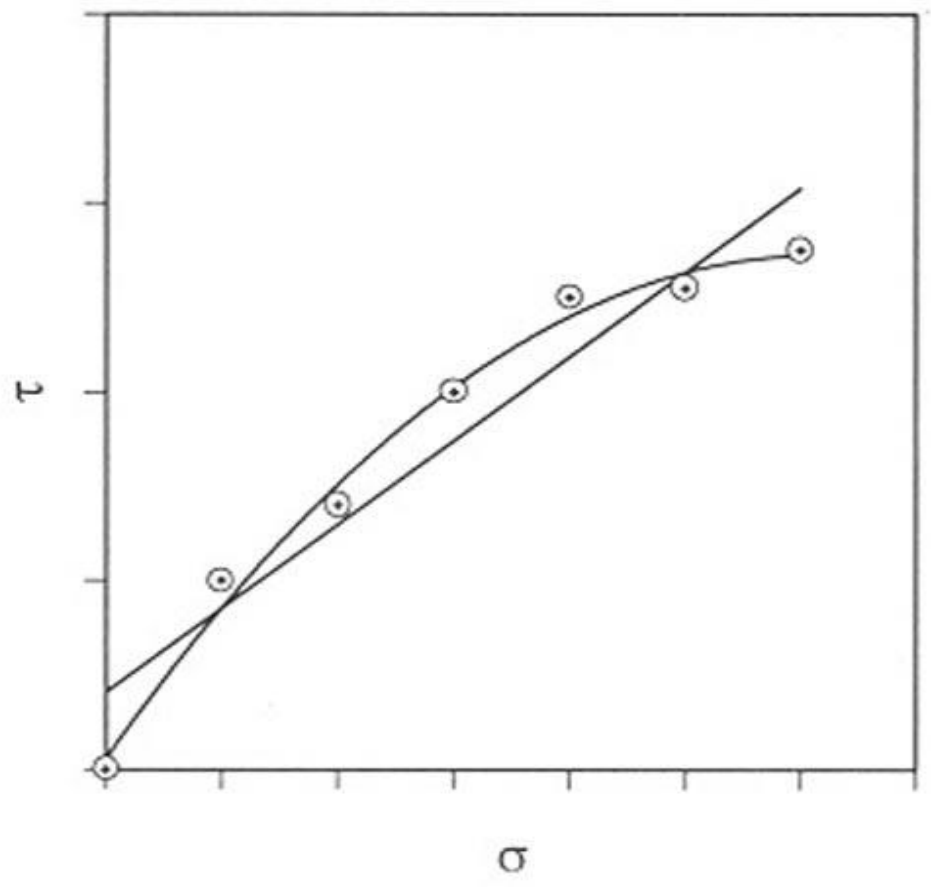


Figure 34: Model Uncertainty of the Coulomb versus Mohr τ – σ Model (after Einstein, 2003)

The following features may cause uncertainties and errors in determining joint parameters:

(i) The spatial occurrence, variations and large volume of the material (i.e. rock mass) involved in a rock construction (Dershowitz and Einstein, 1988);

(ii) The methodology followed in carrying out the investigations;

(iii) Uncertainties connected to the joints measured, as only a portion of the joints may be exposed, which are considered to be representative joints within the entire rock mass;

(iv) Effect of microscopic discontinuities;

(v) Outcrops or surfaces may not be representative as a result of weathering;

(vi) In excavated surfaces and in drill cores it may be difficult to distinguish between natural and artificially induced discontinuities (Palmstrom, 2001);

(vii) Limitations in drill core logging: artificial breaks are included, and information relating to the waviness and continuity of joints is minimal as shown in Figure 35. In addition soft gouge is often lost during core recovery (Palmstrom, 2001).

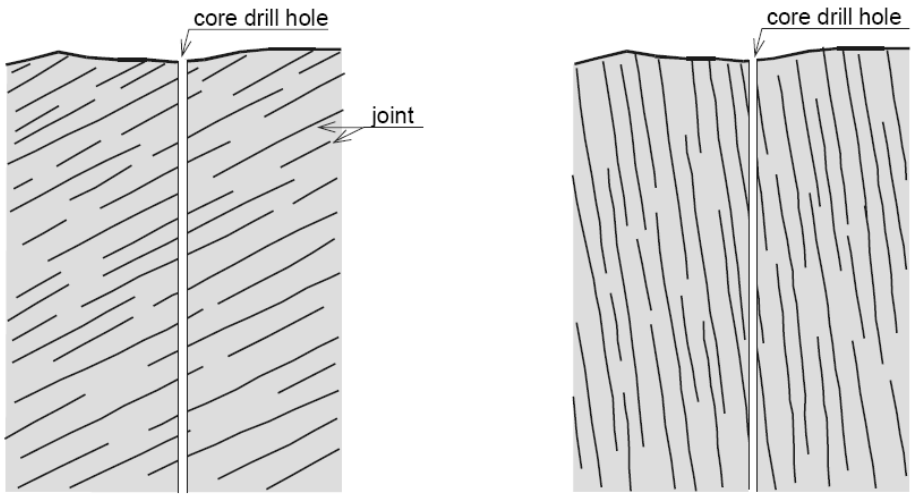


Figure 35: The angle between the joints and the drill core may strongly influence on the length of the core pieces (after Palmstrom, 2001)

(viii) The way the description is performed or the quality of the characterization made of the various parameters in rock masses. As most of the input parameters in rock engineering and rock mechanics are found from observations, additional errors may be introduced from poorly defined descriptions.

3.7 Conclusions

Joints show great variation in both geometric and mechanical properties and some of the most significant errors for joint characterization are:

(i) Small joints are often disregarded;
(ii) Very large fracture surfaces may be measured more than once;
(iii) Joints almost parallel to the foliation or bedding may be overlooked.

Despite the amount of research that has been conducted in the determination of both geometric and mechanical properties in rock joints, fundamental issues, in the context of slope reliability, regarding the distribution of joint orientation, spacing and trace length without associating them to particular locations, reliabilities of empirical methods and laboratory tests are not satisfactory. It is almost impossible to measure absolute length of a joint since the entire joint surface cannot easily be seen. Part of joint traces can be measured and the overall continuity for the joint set can be predicted in a statistical sense which causes uncertainties and errors which must be considered when analyzing the data. Incorporating uncertainties in performance analysis is still the major problem in slope stability analysis.

The degree of work that also has been devoted to providing techniques for measurement, data reduction and presentation associated with each of those main aspects of geometrical and strength parameters of joints have been highly variable. There is no standardized, or correct, method of measuring and characterizing rock joint parameters, because the accuracy of measuring the separate parameters depends on the engineering objective/judgement.

An ideal characterization of jointing would involve the specific description of each joint in the rock mass, exactly defining its geometric and mechanical properties. It is not always possible to determine exact values for each joint. Joints in a rock mass are, therefore, usually described as an assemblage rather than individually. The assemblage has stochastic character in that joint characteristics vary in space (Dershowitz and Einstein, 1988). Such variation may be minute as in the case of the orientation of a joint set approximately parallel or they may be large if a particular property has substantial variability.

One of the reasons for the widespread use of the Mohr-Coulomb constitutive law in rock engineering is its simplicity in application. It is, however, not a particularly satisfactory peak strength criterion for rock material (Brady and Brown, 1985). The reasons are:

(i) It implies that a major shear fracture exists at peak strength; observations show that this is not always the case;
(ii) It implies a direction of shear failure which does not always agree with experimental observation, and;
(iii) Experimental peak strength envelopes are generally non-linear; they can be considered linear only over limited ranges of normal or confining stress.

Mohr coulomb measures friction and cohesion at a point. These values are transferred to the three-dimensional body of rock mass by assuming that the rock mass is isotropic, which in a jointed rock mass is not the case.

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CHAPTER 4
Physical and numerical modelling

4.1 Introduction

Two main model types are demonstration model (physical model) and simulation models. An example of a demonstration model is the base friction angle determination model which involves placing the material whose base friction angle is required on an inclined plane. The angle of inclination is increased, relative to the horizontal surface, until the material starts to slide down the inclined plane. The angle from the horizontal to the inclined plane is the base friction angle of the material. Examples of simulation models include numerical models. The weakness of physical models for slope stability analysis is that they have some limitations, e. g., they cannot take effects of stress into account. Numerical models, on the other hand, represent real dimensions and other real conditions.

4.2 Physical modelling

A physical model is a smaller or larger physical copy of an object (Wikipedia, 2010). The object being modelled may be small (for example, an atom) or large (for example, the Solar System). In the context of the dissertation, the object being modelled is the open pit which is large compared to the physical model. The geometry of the model and the object it represents are often similar in the sense that one is a rescaling of the other; in such cases the scale is an important characteristic. However, in many cases the similarity is only approximate or even intentionally distorted. Physical models allow visualization, from examining the model, of information about the object the model represents.

4.3 Results of physical modelling and the model geometry

The physical model and its results are described in detail by Stacey (2006). The purpose of the physical model was to illustrate the mechanism(s) of slope behaviour. The weakness of the physical model is that it could not take in situ stress into consideration. The results of the physical models are summarized in the following paragraphs. The typical geometry of the two dimensional physical models is shown in Figure 36, representing a rock mass containing bedding planes and cross joints.

Although the discontinuity orientations were the same in the range of two-dimensional models tested, different ratios of joint plane spacings to bedding plane spacings (S) were used. Four such ratios were tested, as summarized in Table 4.1.

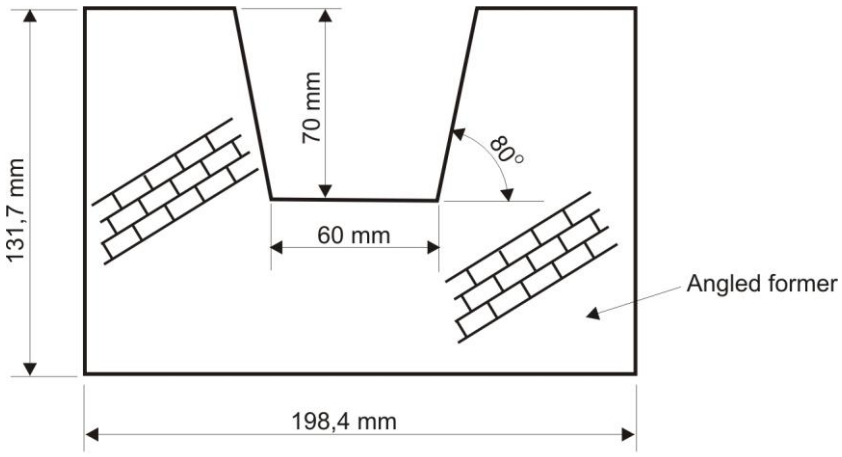


Figure 36: Configuration of two-dimensional models (Stacey, 2006)

Table 1: Spacing of joints and bedding planes in the models (Stacey, 2006)

Bedding plane Spacing (mm)	Joints spacing (mm)	Spacing Ratio S	Number of Models
3.4	12.7	3.74	4
4.6	12.7	2.76	5
3.4	6.35	1.87	4
6.35	6.35	1.0	5

Figure 37, Figure 38, Figure 39 and Figure 40 show examples of typical sequences of failure of the two-dimensional models. The number of models tested is given in Table 1. It is immediately apparent from these figures that failure takes place by progressive sliding on the bedding planes, generally throughout the height of the slope, with tensile opening of the cross joints. Rotation of blocks was also observed, particularly in the case of the 1:1 joint to bedding plane spacing. Failures are therefore combinations of different mechanisms. Stepped "surfaces" were formed that had average angles of inclination that were much steeper than the dip of the bedding planes.

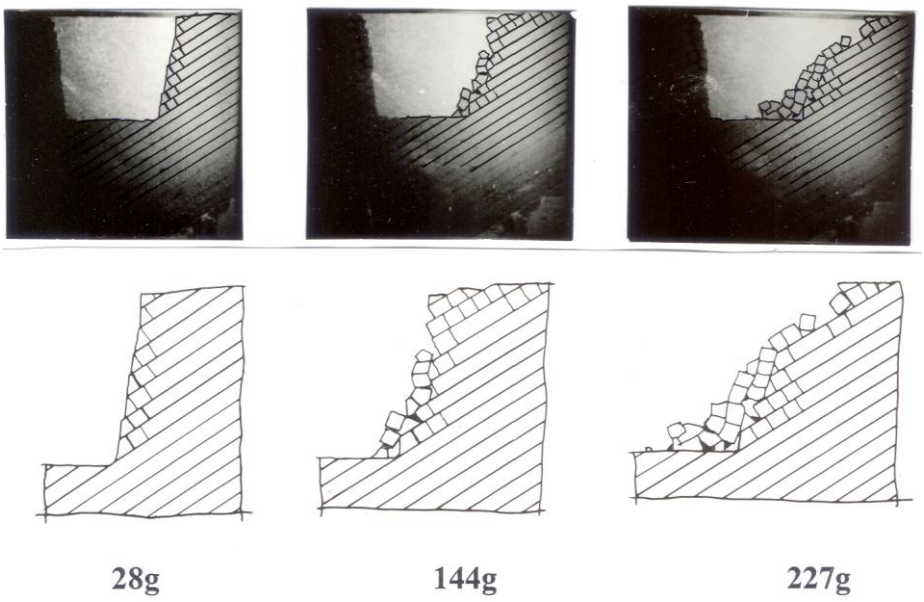


Figure 37: Two-dimensional model (S = 1.0) (Stacey, 2006)

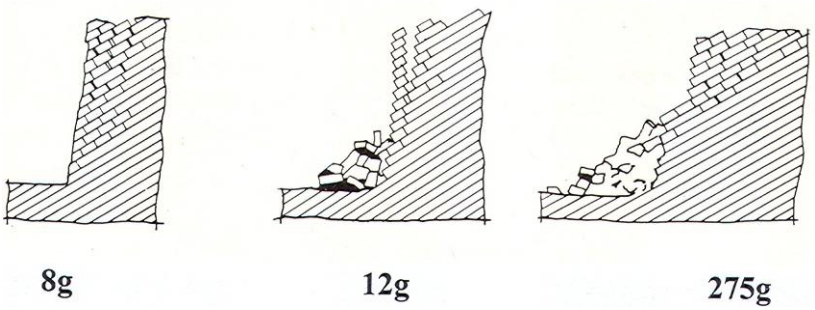


Figure 38: Two-dimensional model (S = 1.87) (Stacey, 2006)

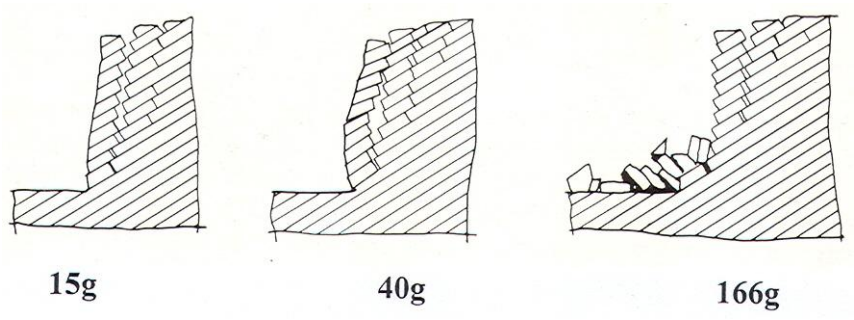


Figure 39: Two-dimensional model (S = 2.76) (Stacey, 2006)

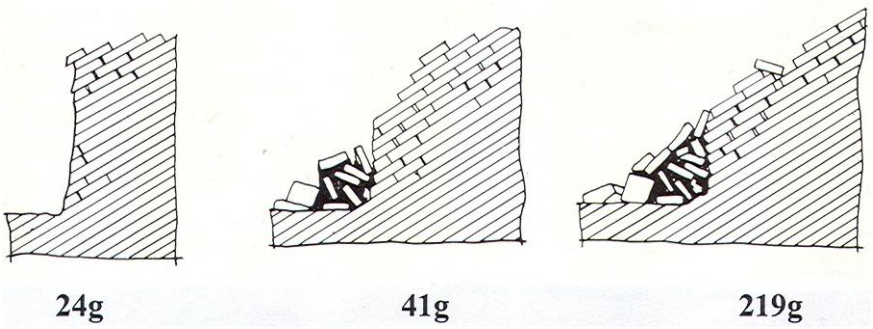


Figure 40: Two-dimensional model (S = 3.74) (Stacey, 2006)

A slight bulging of the slope face just above the toe was noticed before failure in some of the models. This was apparently due to sliding on the bedding planes in this region. After collapse, the presence of the collapsed debris at the base of the slope generally had a stabilizing effect on the remaining slope.

The mode of failure of the model slopes appeared to be controlled entirely by the ratio of the joint spacing to the bedding plane spacing. This in turn controls the capacity of the configuration to transmit tensile stress and also the amount of sliding which can take place in the "coherent" mass before complete separation of adjacent pieces occurs resulting in collapse.

The mode of failure of all model slopes tested was progressive. Failure of the right hand slope was by progressive sliding down along bedding planes. In no case did

collapse of the entire slope occur without warning. There was no clearly defined single surface of failure, and these slopes would therefore not be amenable to conventional slope stability analysis by limit equilibrium methods. Failure of the left hand slope only occurred with S = 1.0.

Although the behaviour of models with the same S value was similar, there was some variation in the centrifugal acceleration at which corresponding deformations, and collapse, occurred for models with this S value. It is believed that the main reason for this was the variation in the relative locations of the pieces of model material. This inconsistency regarding the gravitational acceleration at which failure initiated and collapse ultimately occurred was one of the motivations for the numerical analyses described below.

4.4 Numerical modelling

Numerical models represent real dimensions and other real conditions of rock slopes unlike physical models. In situ stress effects are taken into account in numerical models. Numerical models will therefore illustrate real situations better than the physical models. Numerical models were used in this research to understand the mechanisms of slope behaviour in real situations.

There are many numerical modelling methods available today, each with their own strengths and weaknesses. Besides the general prerequisites, such as the ability to handle different slope geometries, it is essential that the modelling tool allows simulation of rock mass failure.

Numerical modelling of the mechanical behaviour of jointed rock mass is a difficult task as the discontinuities not only require special modelling consideration, but also require different treatments depending upon the scale of the problem involved. Considering the jointed rock mass, the stability of the slopes can be governed by characteristics of joints behaviour rather than by the physical properties of the rock mass itself. Distinct element simulation is effective in analysing the stability of

jointed rock masses. The Universal Distinct (UDEC), a distinct element simulation code, is used in this research for the analysis of jointed rock masses. Given qualitative or uncertain data on the properties and distributions of joints, and in situ conditions, sensitivity studies of a system can be undertaken using UDEC.

The idea that certain key parameters of the un-deformed rock mass may influence failure behaviour in a quantifiable way was examined through a parametric study of a large rock slope using UDEC. The complete range of options available in UDEC far exceeds the scope of this dissertation. However, it is considered appropriate to summarise the program.

UDEC (Itasca, 1995) allows finite displacements and rotations of discrete deformable blocks, including complete detachments, and automatically recognizes new contacts. The blocks are separated by joints, which are considered to be interfaces (i.e., the discontinuity is treated as a boundary condition). A soft-contact approach is used to treat the relative normal displacements at the block contacts. A finite normal stiffness is used to represent the measurable stiffness that exists at a contact or joint. Realistic representation of crushing of the corners of the blocks, which would occur as a result of stress concentration, is achieved by rounding the corners so that blocks can smoothly slide past each other when two opposing corners interact.

UDEC (Itasca, 1995) analysis includes sliding along and opening/closure of rock discontinuities as controlled by the joint properties (normal and shear stiffness, cohesion, friction, etc.). The dual nature of these discontinuum codes make them particularly well suited to rock slope instability problems; they are capable of simulating large displacements due to slip, or opening, along discontinuities; and they are capable of modelling the deformation and material yielding of the joint-bounded intact rock blocks, which is particularly relevant for high slopes in weak rock and for complex unexplained modes of rock slope failure.

In addition, the program (Itasca, 1995) can:

(i) Simulate the effect of rock support mechanics, via various structural elements that interact with the rock blocks.
(ii) Perform effective stress analyses, based on assumed distributions of water pressure within the rock material and/or in joints.
(iii) Have additional features added by the user, using the in-built programming language, FISH.

Hamman and Coulthard (2007) summarized some of the issues that need to be kept in mind when using UDEC. The program is designed for nonlinear stress analyses, with plastic yield within rock material (in shear and in tension), slip and separation on joints, and large strains and displacements. The final state of any nonlinear system depends strongly on the stress path. It is therefore critical that analyses using UDEC are set up so that the numerical model is subjected to a stress path that reflects as closely as possible that for the real system.

The explicit finite difference method allows fully dynamic analyses to be performed for systems with time varying loads, but quasi-static analyses can also be performed when loads (or unloadings, for the case of excavations) vary slowly with time. The latter are still based on integration of the equations of motion, but with artificial damping applied as part of the numerical solution process to make the system respond in a quasi-static manner. In such cases, the sudden changes in applied loads within the model when an excavation is created or extended will induce transient stress waves. As these are not 'real', the modeller must ensure that they do not cause physically inappropriate yield to occur anywhere within system. This is particularly important with jointed systems, where transient tensile stress waves crossing a joint may make it open and close almost instantaneously, thereby 'losing' its shear stress and perhaps dropping it to residual strength.

Each step of the analysis must be ‘stepped’ long enough for the system to come to equilibrium, if indeed that is the computed final state. Alternatively, if a localised or extensive failure is predicted to occur, the user must ensure that numerical solution proceeds long enough to demonstrate the mechanism and extent of any failure, but not continue unnecessarily with the analysis beyond that point.

Geomechanics invariably deals with effectively semi-infinite rock masses. A numerical model must be truncated at some point, and boundary conditions applied to simulate the far-field rock response. The user must ensure that these artificial boundaries are sufficiently far from the region of interest that they do not significantly influence the computed response.

UDEC (Itasca, 1995) is a 2 Dimensional code. An engineer performing the modelling must determine whether this might unduly influence the results, and then decide whether a 3 Dimensional analysis might be necessary for a full understanding of the system’s behaviour.

For a relatively long open pit, a two dimensional model is justified, except at the pit corners and, of course, at the lateral boundaries of slope failures. It is believed that many fundamental issues regarding failure mechanisms can be studied without entering the complexities of three-dimensional modelling. However, a 2D model very often represents a worst-case approximation of a real slope, because the beneficial horizontal ‘arching’ around the periphery of the pit is neglected. Unless the real mechanism of failure involves 3D wedges that are not simulated in 2D model, it is likely that a 3D model would predict that the pit is more stable than 2D models indicate.

Another limiting assumption required by the distinct-element method is the inclusion of fully persistent, interconnected discontinuities. When discontinuities do not intersect to form blocks, the UDEC program may become impractical.

Interpretation of results is another important part of the numerical modelling analysis. Unlike conventional, implicit, finite-element programs, which produce a solution after the calculation phase, the results from UDEC must be interpreted by the user to assess whether the system is stable, unstable, or in steady-state plastic flow. There are four different indicators that can be used for this (Itasca, 1995):

(i) Maximum unbalanced force;
(ii) Block/grid point velocities;
(iii) Plastic indicators, and;
(iv) Histories.

During time stepping, the unbalanced force is determined for the model; this indicates whether blocks in the model are moving or not, and is continuously updated on the screen. The unbalanced force is important in assessing the state of the model for static analysis. A model is considered stable when the maximum unbalanced force is almost zero, or if the unbalanced forces decrease by 3-4 orders of magnitude, then the model is indicating that the problem is moving towards a stable equilibrium. If the unbalanced force increases or remains the same, then the model suggests that blocks are moving or failing (Itasca, 1995).

The velocities of deformable blocks may be assessed by plotting the whole field of velocities. It is likely that steady-state conditions are indicated in the model if grid point velocities have converged to near-zero values. If the velocities show high non-zero values, then either the block is failing, or steady plastic flow is occurring within the block (Itasca, 1995).

If there are certain variables that are of particular interest (e.g displacements and stresses), the history command should be employed to track these variables in the regions of interest. After some timestepping has taken place, the plots of these histories often provide the way to find out how the system is behaving.

Plasticity indicators are used on the UDEC models to reveal those zones in which the stresses satisfy the yield criterion (Itasca, 1995). A failure mechanism is indicated if there is a contiguous line of active plastic zones that join two surfaces. Initial plastic flow often occurs at the beginning of a simulation, but subsequent stress redistribution unloads the yielding elements so that the stresses no longer satisfy the yield criterion ("yielded in the past"). Only the actively yielding elements ("at yield surface" and "tensile failure") are important for the detection of a failure mechanism (Eberhardt, 2004).

The system can also be unstable, meaning that it is heading for ultimate failure or collapse. An unstable model is usually characterized by a non-zero, often fluctuating; maximum unbalanced force, as well as increasing velocities and displacements. The model can also collapse due to displacements becoming very large, thus distorting individual elements badly and prohibiting further timestepping.

In this study, a simulated model was run to investigate the influence of various parameters. This increases understanding of rock slope behaviour as well as the effects of variation of joint parameters in failure mechanism. It is important to point out that the word "model" implies a simplification of the real world, as the real world often is too complex for us to understand (Starfield and Cundall, 1988). The results were examined for each stage of the analysis, to determine whether any failure mechanisms were developing.

Each stage in the analysis actually consists of two sub-stages:

(i) The system is stepped close to equilibrium, with all rock and joint strengths increased, to avoid physically spurious yield of the rock mass or joints during the numerical damping of stress waves that inevitably arise in the model when loading is suddenly changed.

(ii) The excavation is made after the system has reached the equilibrium stage and strengths are returned to 'real' values. The system is again stepped to equilibrium or until a failure mechanism clearly develops.

These can be seen via plots of plastic state (which show rock regions of active shear or tensile yield, as well as the regions where yield has occurred in the past) and joint slip/separation. A failure mechanism can develop by active shear or tensile yield through intact rock material and/or by slip/separation on joints or shears.

4.5 The numerical model , boundary conditions and in-situ stress field

For accurate analysis of pit slopes using plastic models, the size of the model should be at least three times the pit width and three times the final pit depth, see Figure 41 (Sjoberg 2000).

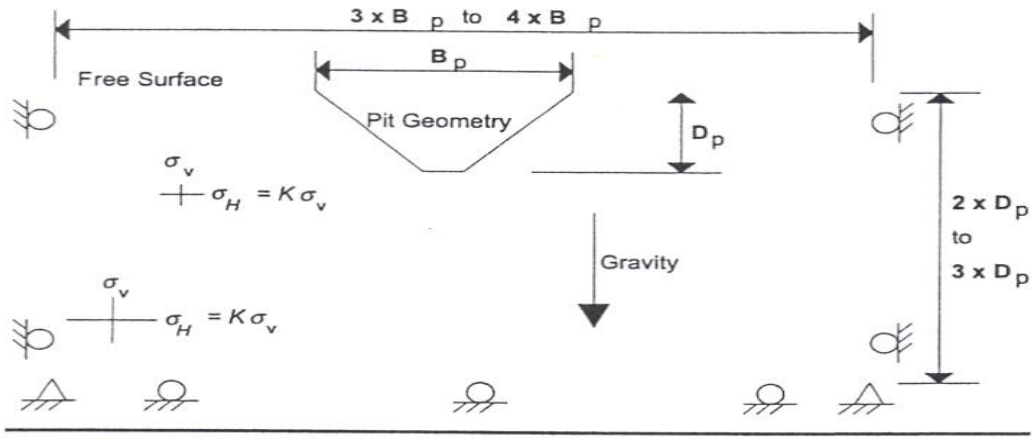


Figure 41: Model set up (size, boundary conditions and virgin stresses) to simulate slope failures (Sjoberg, 2000)

The in situ state of stress is a major factor in many rock mechanics problems. The tectonic history of the rock mass may lead to in situ stresses substantially different from a stress state created by gravitational loading. The in situ conditions must therefore be specified for the present type of the non-linear analyses. Results reported in Lemos (1987) show that changes in the horizontal to vertical in situ stress ratio may lead to slip on different joint sets.

The pit configuration of the two dimensional model used with the UDEC code is an in situ version of the physical model, see Figure 42. The dip of the bedding planes is 33° with 14 m spacing and the dip of the cross joints is 57° with 40 m spacing (Stacey, 2006).

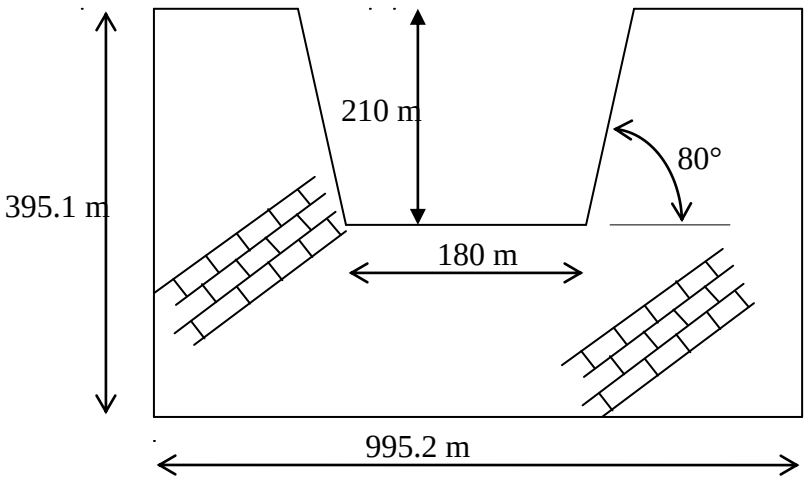


Figure 42: Configuration of two-dimensional models (Stacey, 2006)

4.6 Material model used in the analysis

UDEC allows a large number of constitutive models to be employed. Some of these are specifically suited for modelling of soils, while others are more generally applicable for rock materials. In this study Mohr-Coulomb plasticity constitutive model is used for deformable-block material.

In UDEC all plastic models are based on fundamental plasticity theory. The model is characterized by its yield function, hardening/softening functions, and flow rule. The yield function defines the stress state at which plastic flow takes place and is represented by a yield surface. All points inside the yield surface behave elastically, whereas a point on the yield surface is said to be at yield. The yield function is denoted as f with the condition that $f=0$ at yield. The material is elastic if $f<0$. The softening/hardening function determines whether the material is perfectly plastic or if its strength increases or decreases with increased straining. Finally, the flow rule

specifies the direction of the plastic strain increment vector as that normal to the potential surface defined by the potential function (Sjoberg, 1999).

The coulomb slip criterion is also used as a constitutive model for joints in UDEC. In this model failure occurs if the shear stress acting along the joint plane reaches the shear strength. Input parameters for this model are joint cohesion, joint friction angle, and joint tensile strength. Furthermore, the elastic deformation of the joint is assumed to follow a linear relation both for normal and shear deformations. Input parameters for this are the joint normal (k_n) and joint shear (k_s) stifnesses. Using this constitutive model in UDEC, deformation can occur over the joints both in the shear and normal directions. In addition, it is possible to simulate peak and residual strength of joints (with instantaneous strength loss at failure). Tensile failure is modelled using a simple tensile strength criterion, which states that tensile failure occurs when the minor principal stress reaches the joint tensile strength, σ_{tj}. The joint tensile strength is set to zero (instantaneous softening) when tensile failure occurs (Sjoberg, 1999).

4.7 Mechanical properties of rocks

Table 2 shows laboratory scale strength properties while Table 3 shows laboratory scale elastic constants used in the models.

Table 2: Selected strength properties (laboratory-scale) for rocks (adopted from Goodman, 1980)

Rock Formation	Friction Angle (degrees)	Cohesion (MPa)	Tensile strength (MPa)
Berea sandstone	20	27.2	1.17

Table 3: Selected elastic constants (laboratory-scale) for rocks (adopted from Goodman, 1980)

Rock Formation	E (GPa)	υ	K (GPa)	G (GPa)
Berea sandstone	19.3	0.38	26.8	7.0

E = Young's modulus

υ = Poisson's ratio

K = Bulk modulus

G = Shear modulus

4.8 Mechanical properties of joints

Selected strength properties used in the analyses are shown in Table 4.

Table 4: Selected strength properties for rock joints (adopted from discussion with T.R. Stacey)

	Friction angle (degrees)	Cohesion (MPa)
Rock joints	25	0.5

The normal and shear stiffness of discontinuities are often subject to debate. It is often argued that they are difficult to determine, which is true, and that they have significant influence on the results, which is not necessarily correct. The limited amount of published data on joint stiffness indicates that shear stiffness is lower than the normal stiffness, but that the difference between the two becomes smaller for higher normal stress (Bandis et al, 1983). However, for the modelling approach used in this work, exact values of joint stiffness are not necessary. The joint stiffness only affects displacements before joint slip, which are very small in most cases and not very interesting, particularly when extensive joint slip is expected (Sjoberg, 1999). Furthermore, it has been shown that the elastic stress distribution is relatively insensitive to the choice of joint stiffness (Kulatilake et al., 1992). Finally, a very high joint stiffness will result in a smaller time step and longer calculation times in UDEC. The values used in this work were chosen in accordance with these criteria,

not based on laboratory tests. Modelling results also confirmed the relative insensitivity regarding choice of joint stiffness parameters.

CHAPTER 5

Results of numerical modelling

5.1 Introduction

Different combinations of variations of joint dip angle, joint spacing, bedding dip angle, and bedding spacing were used for the study to evaluate their effects on the behaviour of the excavated slope in the rock mass when the joint strength parameters are continuous. UDEC modelling was carried out to investigate the following:

(i) Variability in joint dip;
(ii) Variability in bedding plane dip;
(iii) Variability in offsets between joints;
(iv) Variability in joint spacing;
(v) Variability in bedding plane spacing;
(vi) Combinations of the variability in (i) to (v).

The result from the basic model which does not include any of the variabilities in (i) to (v) above is shown in Figure 43. It can be seen in this model that deformation of the right hand slope has taken place by sliding on bedding planes, leaving two observable "subsidence cracks", and there is joint separation in several locations. There are clear deformations a small height above the toe of the slope. Minor failure of very small blocks, by sliding or toppling or a combination of both, has also occurred from the left hand slope.

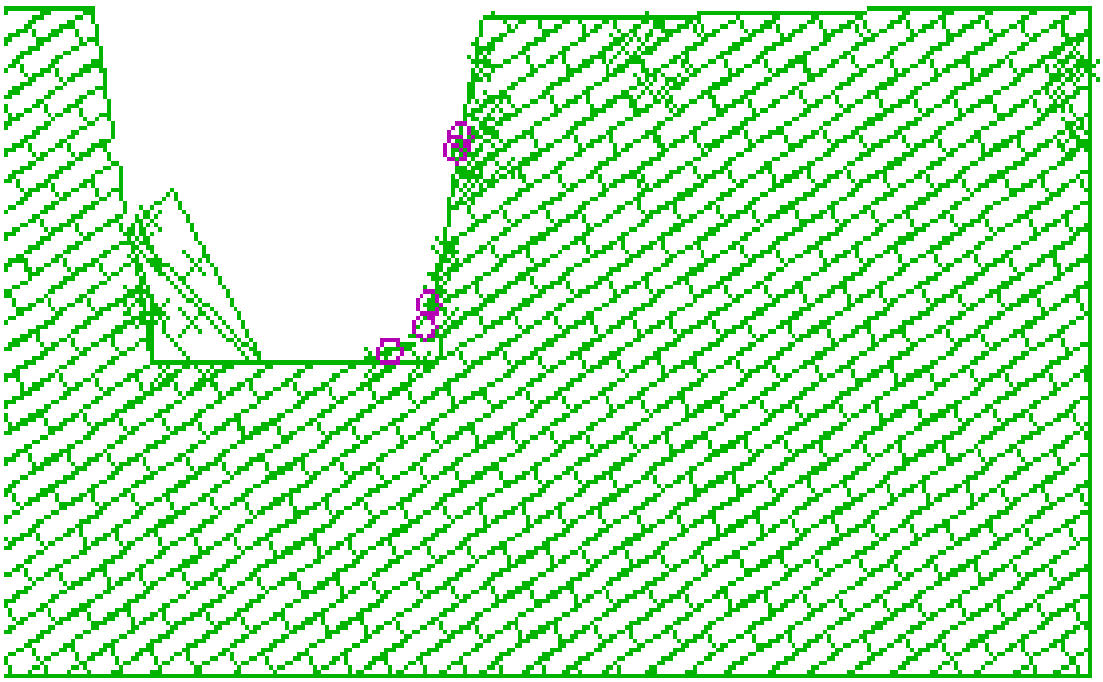


Figure 43: Basic model with no variability

The base case is the model in which there was no variability in the joint dip, bedding plane dip, joint spacing and bedding plane spacing. The results for 22 cases analyzed using UDEC are summarized in the following paragraphs. By maintaining the same time-base of each output sequence for each site, direct comparison of the results becomes possible.

Plasticity indicators were used to reveal those zones in which the stresses satisfy the yield criterion. As discussed earlier, a failure mechanism is indicated if there is a contiguous line of active plastic zones that join two surfaces. Initial plastic flow often occurs at the beginning of a simulation, but subsequent stress redistribution unloads the yielding elements so that their stresses no longer satisfy the yield criterion ("yielded in the past"). Only the actively yielding elements ("at yield surface" and "tensile failure") are important for the detection of a failure mechanism (Eberhardt, 2004)

5.2 Effects of joint orientation

The effects of joint orientation on stability of slopes have been evaluated by examining the slope behaviour in which variability occurs in the dip angle of joints. 25%, 50%, and 75% standard deviation of joint dip angles were considered:

5.2.1 Standard deviation of joint dip angle of ±25% of mean values

Small deformations are observable along the bedding planes of the right hand side of the slope, see Figure 44. Minor block fall out can also be seen from the left hand slope possibly due to sliding or toppling. Opening up of a couple of joints along the bedding planes occurred, which indicates that bedding planes slid through to the pit. The results do not vary widely from those observed for the base case model.

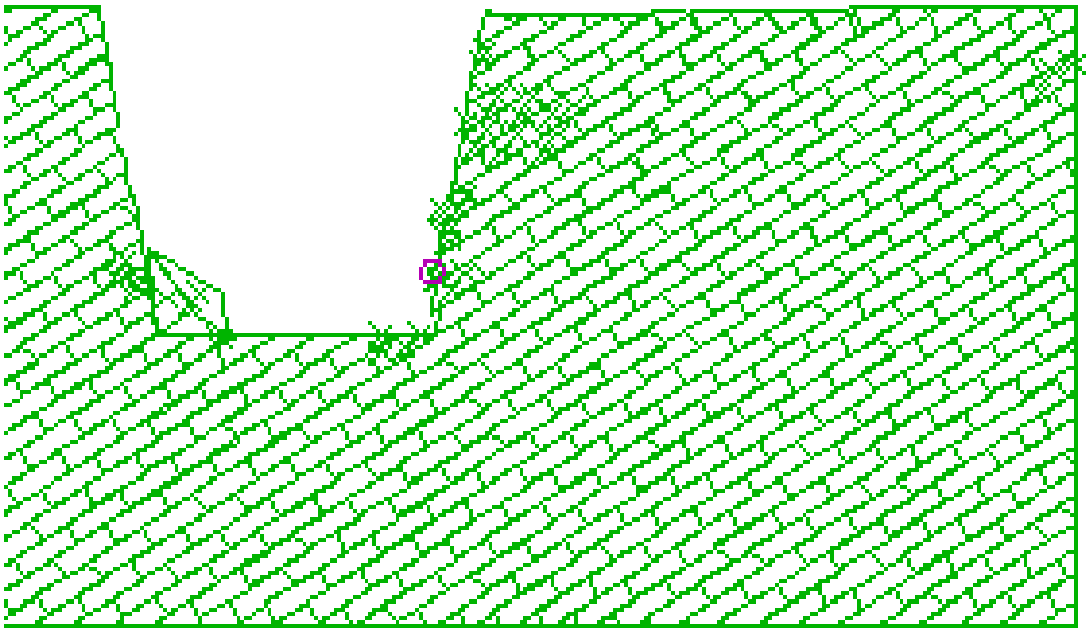


Figure 44: 25% standard deviation of joint dip angle

5.2.2 Standard deviation of joint dip angle of ±50% of mean values

The results for a 50% standard deviation of joint dip angle are shown in Figure 45. It can be seen that block fall out has occurred near the middle of the left hand slope. No block fall out is observable from the right hand slope. The behaviour of the right

hand side of the slopes is characterized by opening up of joints close to the face of the slope and there are small deformations.

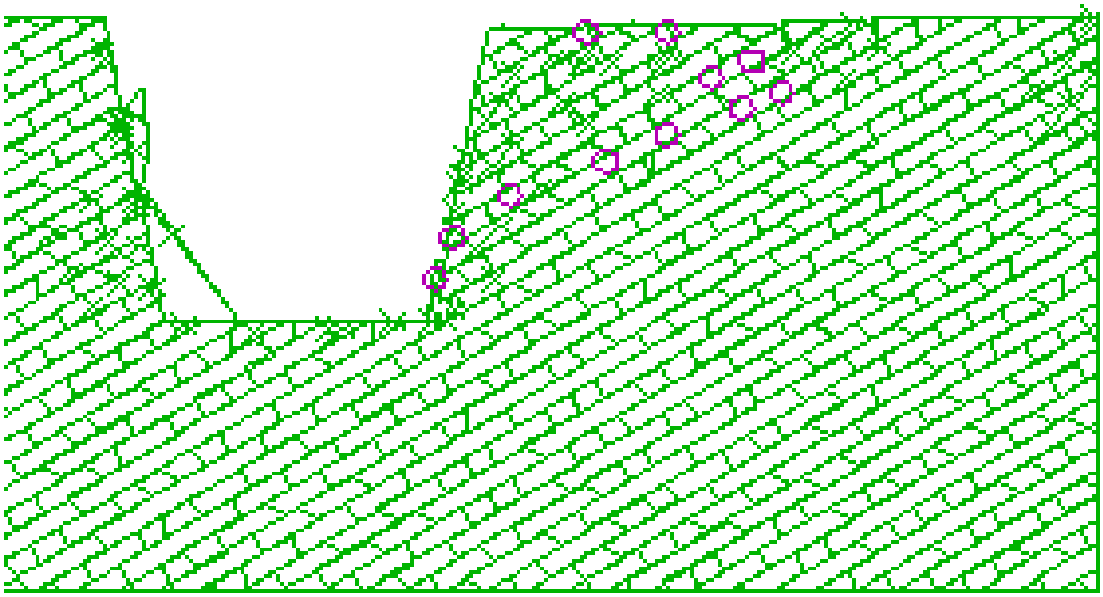


Figure 45: 50% standard deviation of joint dip angle

5.2.3 Standard deviation of joint dip angle of ±75% of mean values

The results for a 75% standard deviation of joint dip angle are shown in Figure 46. Deformations down dip are greater, and it can be seen that block fall out has occurred near the toe of the right hand slope. Minor block fall out can also be seen from the left hand slope.

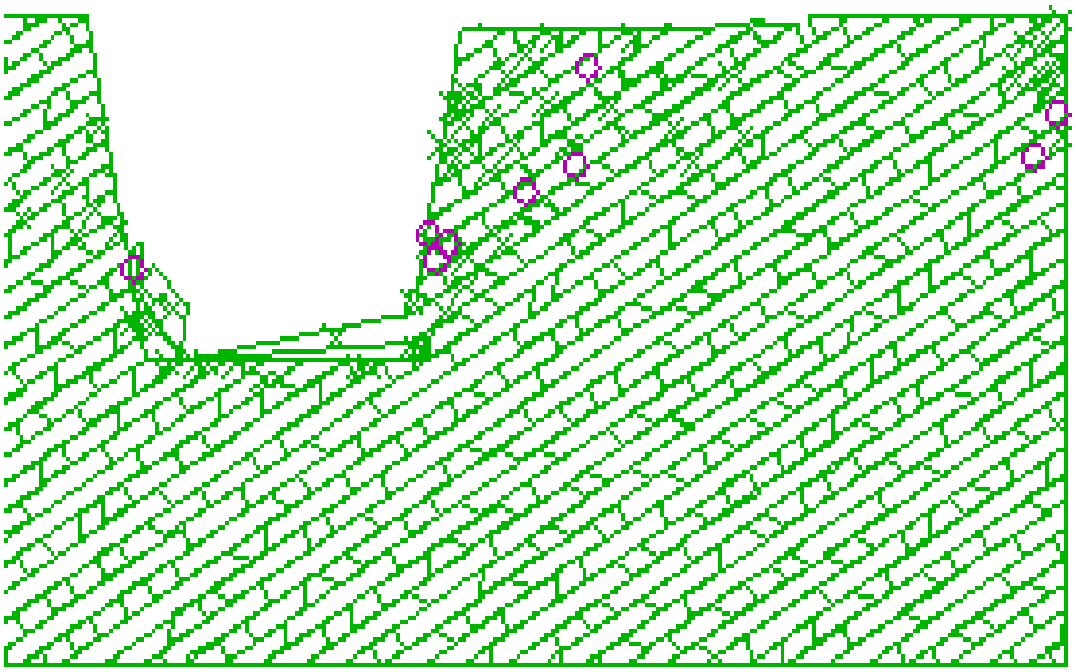


Figure 46: 75% standard deviation of joint dip angle

5.3 Effects of bedding orientation

The effects of bedding orientation on stability of slope have been evaluated by examining the slope behaviour in which variability occurs in the dip angle of bedding planes. For this exercise 1°, 2°, 3° standard deviations of bedding dip angles were considered.

5.3.1 Standard deviation of bedding plane of ±1° of mean values

Figure 47 shows the rock mass generated, and the resulting slope failure for a 1^{o} standard deviation in the bedding dip. On the right hand side of the slope joints have opened up causing small deformations along these bedding planes. On the left hand side of the slope large blocks have fallen out for a height equal to half the slope height, from the middle of the slope to the toe of the slope.

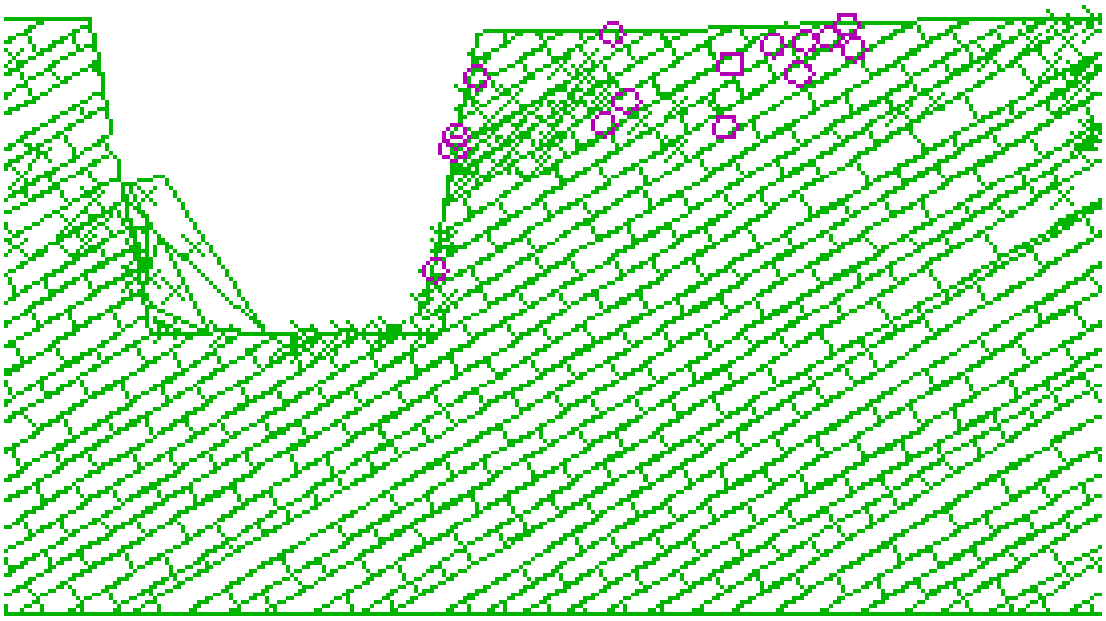


Figure 47: 1° standard deviation of bedding dip angle

5.3.2 Standard deviation of bedding plane of ±2° of mean values

The observed failure is quite similar to the one observed for the 1° standard deviation of bedding dip angle. There are observable large deformations of the right hand slope. Block failure as a result of sliding or toppling or both occurs on the left hand slope. Bed separation takes place on the right hand slope. The results are shown in Figure 48.

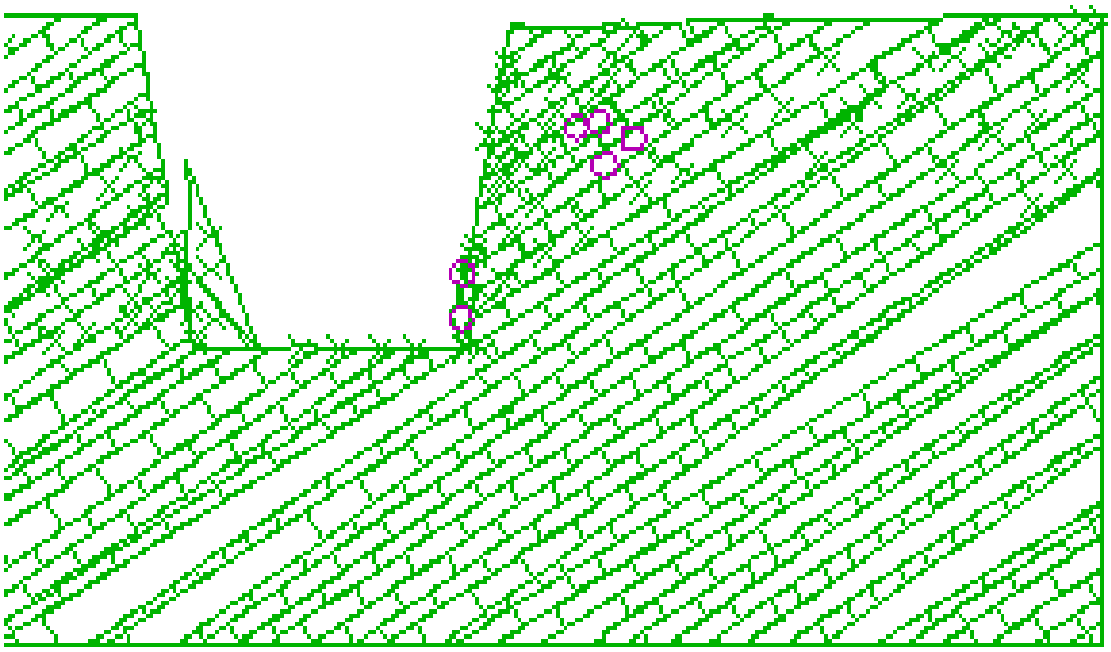


Figure 48: 2° standard deviation of bedding plane

5.3.3 Standard deviation of bedding plane of ±3° of mean values

Owing to the length of the bedding plane surfaces, a small variation in their dip angle can have a significant influence on the appearance of the rock mass. Some beds are thinner and others thicker than the average. Figure 49 shows the rock mass generated, and the resulting slope failure for a 3^{o} standard deviation in the bedding dip. It can be seen that substantial deformation of the right hand slope has taken place, with associated block fall out. It is clear that deformation has taken place throughout the height of the right hand slope. Some fall out has also occurred from the left hand slope.

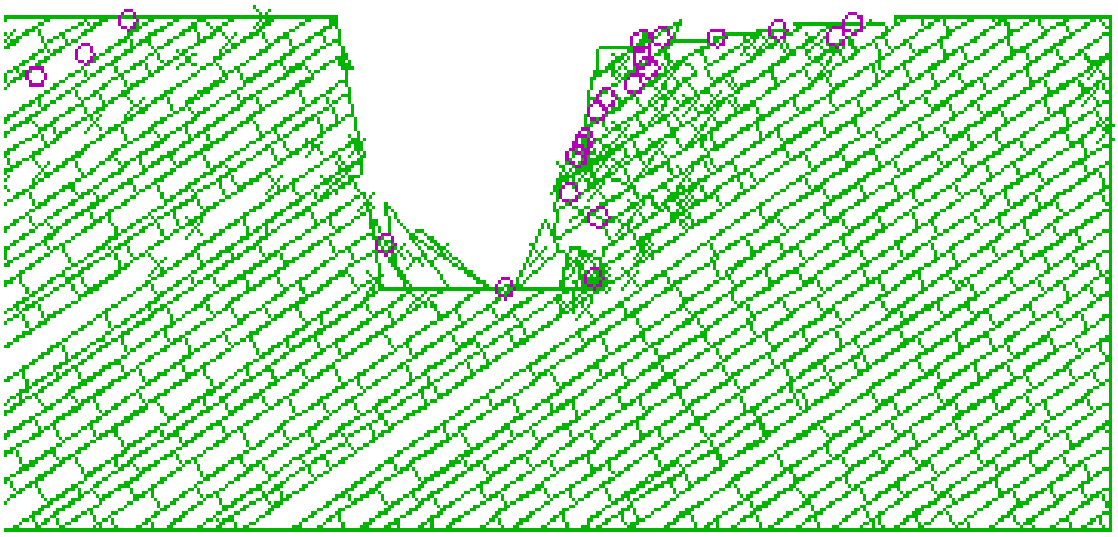


Figure 49: 3° standard deviation of bedding plane

5.4 Effects of combinations of bedding and joint orientation

The effects of bedding and joint orientation on stability of slopes have been evaluated by examining the slope behaviour in which variability occurs in both dip angle of bedding planes and dip angle of joints. For this exercise 1°-25%, 1°-50%, 1°-75%, 2°-25%, 2°-50%, and 2°-75% standard deviation of bedding dip angles and joint dip angles were evaluated. The results are discussed in the following paragraphs.

5.4.1 Standard deviation of bedding plane of ±1° and standard deviation of joint dip angle of ±25% of mean values

The results show minor failure at the toe of the left hand slope. There are slightly larger failures on the right hand side slope towards the middle height of the slope. Results are shown in Figure 50.

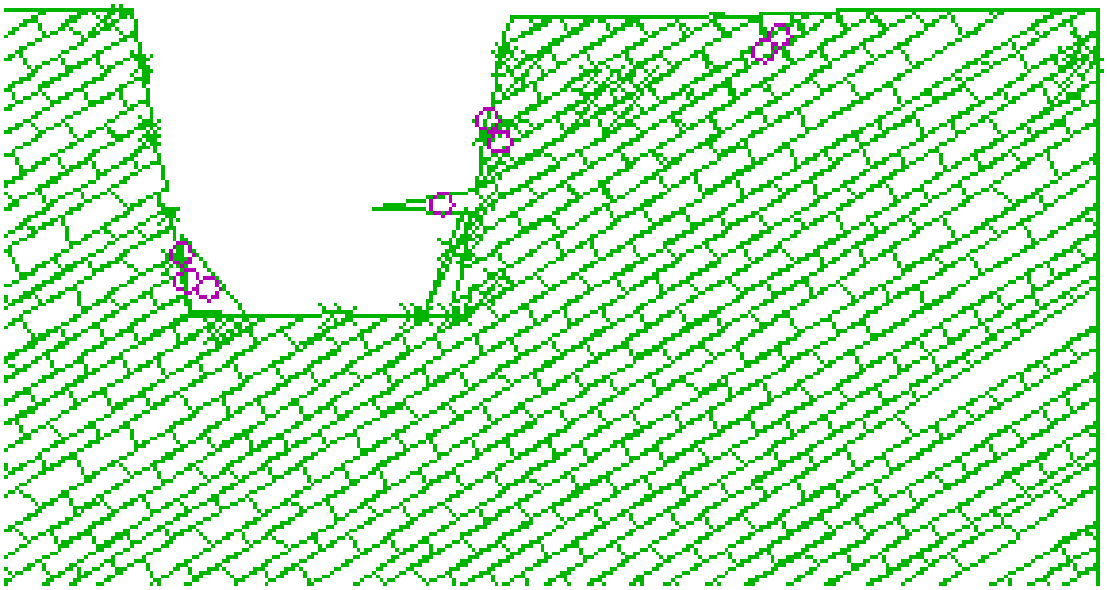


Figure 50: 1° SD of bedding plane-25% SD of joint dip angle

5.4.2 Standard deviation of bedding plane of ±1° and standard deviation of joint dip angle of ±50% of mean values

The deformation throughout the slope, and the significant relative displacement on one of the bedding planes, are evident. Block failure is observable on the right hand slope towards the middle height of the slope. Results are shown in Figure 51.

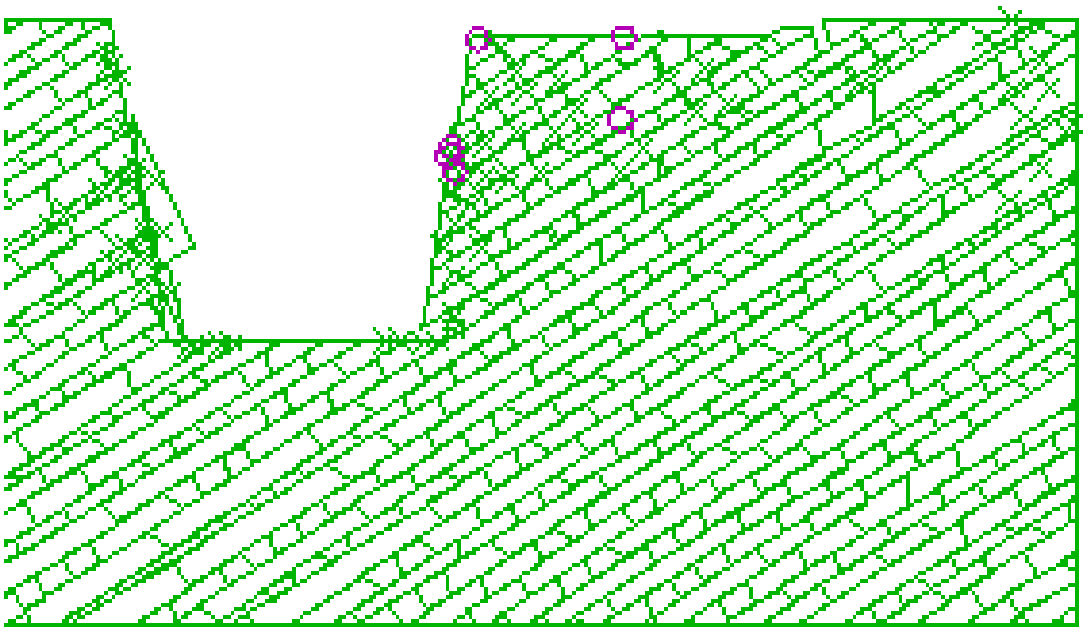


Figure 51: 1° SD of bedding plane-50% SD of joint dip angle

5.4.3 Standard deviation of bedding plane of ±1° and standard deviation of joint dip angle of ±75% of mean values

For this analysis the results show that block fall-out takes place on both the right hand slope and the left hand slope. The sliding mechanism dominates the central part of the rock mass on the right hand slope. Rock slope failure taking place on the left hand slope is possibly due to sliding or toppling or both. Results are shown in Figure 52.

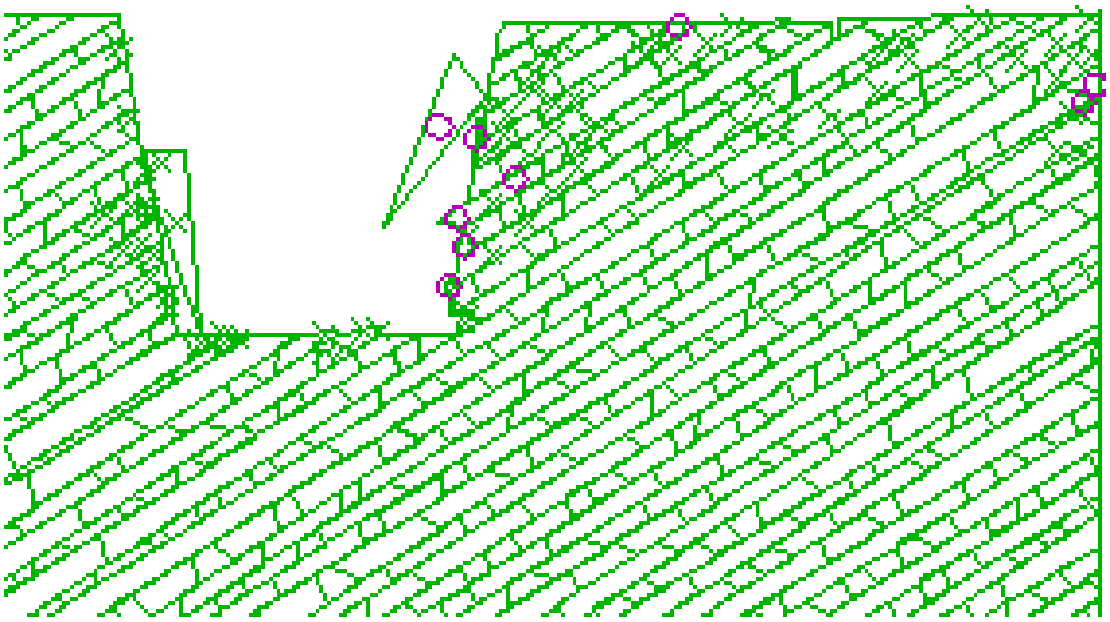


Figure 52: 1° SD of bedding plane-75% SD of joint dip angle

Comparing the three results for 1 degree standard deviation of bedding and 25%, 50% and 75% standard deviation of joint dip angle it is evident that the higher the standard deviation of joint dip the larger the failure surface and deformations.

5.4.4 Standard deviation of bedding plane of ±2° and standard deviation of joint dip angle of ±25% of mean values

Deformations took place along the bedding planes on the right hand side slope as a result of opening up of joints. The face on the right hand side of the slope is shown to bulge into the pit as a result of the large deformations taking place. Block failure took place on the left hand slope towards the toe of the slope possibly due to sliding or toppling, or both. Results are shown in Figure 53.

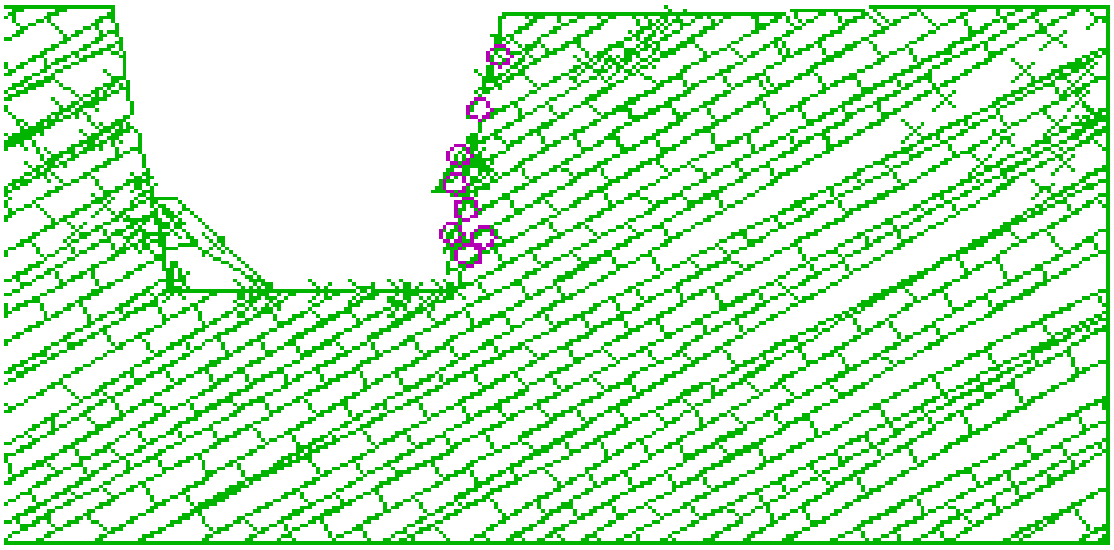


Figure 53: 2° SD of bedding plane-25% SD of joint dip angle

5.4.5 Standard deviation of bedding plane of ±2° and standard deviation of joint dip angle of ±50% of mean values

There are large deformations along the bedding planes on the right hand slope. Block failure also took place at the toe of the right hand slope. The pit bottom experienced large block failure. Neither block failure nor deformations are indicated on the left hand slope. Results are shown in Figure 54.

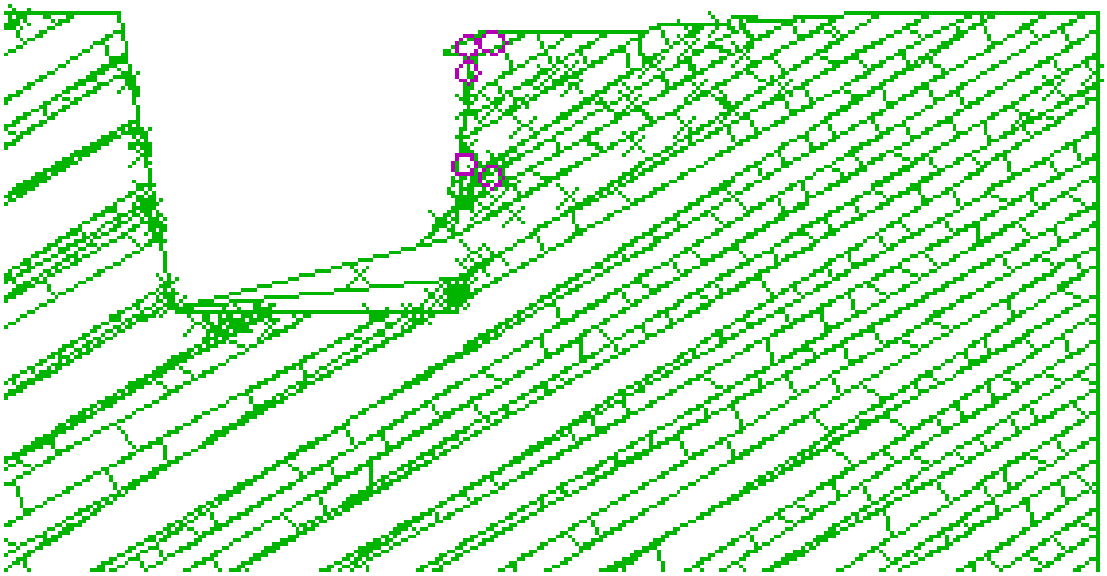


Figure 54: 2° SD of bedding plane-50% SD of joint dip angle

5.4.6 Standard deviation of bedding plane of ±2° and standard deviation of joint dip angle of ±75% of mean values

The deformation throughout the slope is evident, and the bulging of the face is comparable with that shown in Figure 39 for a physical model. Neither block failure nor deformations took place on the left hand slope and pit bottom. Results are shown in Figure 55.

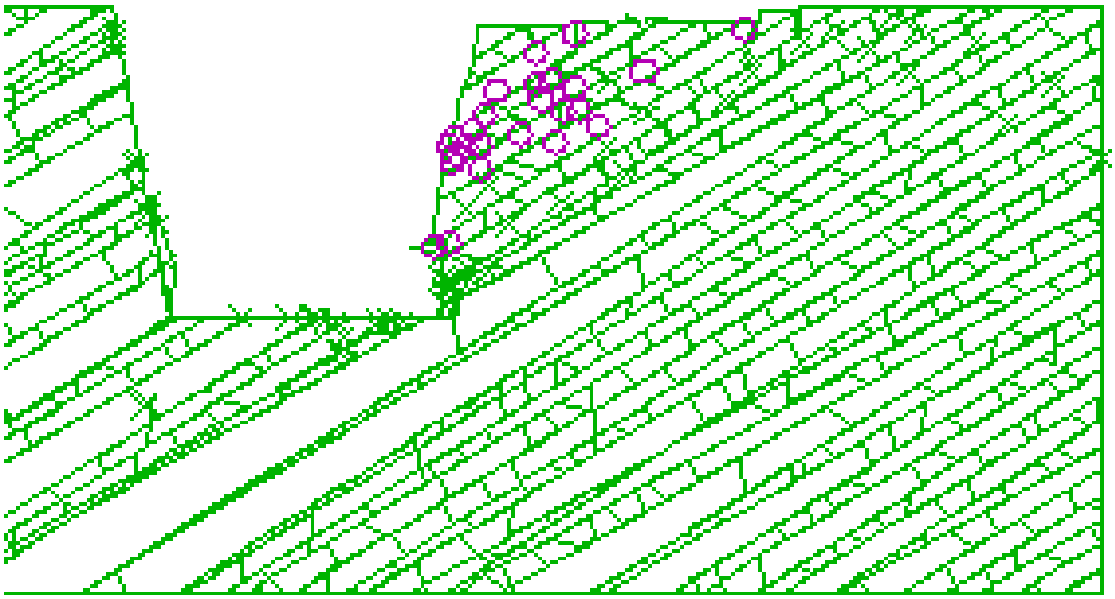


Figure 55: 2° SD of bedding plane-75% SD of joint dip angle

Comparison of the results for 2 degree standard deviation of bedding plane and 25%, 50% and 75% standard deviation of joint dip angle shows failure of the right hand side slope.

5.5 Effects of joint spacing

The effects of joint spacing on the stability of slopes have been evaluated by examining the slope behaviour in which variability occurs in the spacing of joints. For this exercise 25%, 50%, and 75% standard deviation of joint spacing were considered.

5.5.1 Standard deviation of joint spacing of ±25% of mean spacings

Minor block failures took place on both slopes. Block failure occurs at mid-height of the left hand slope and at the toe of the right hand slope. Results are indicated in Figure 56.

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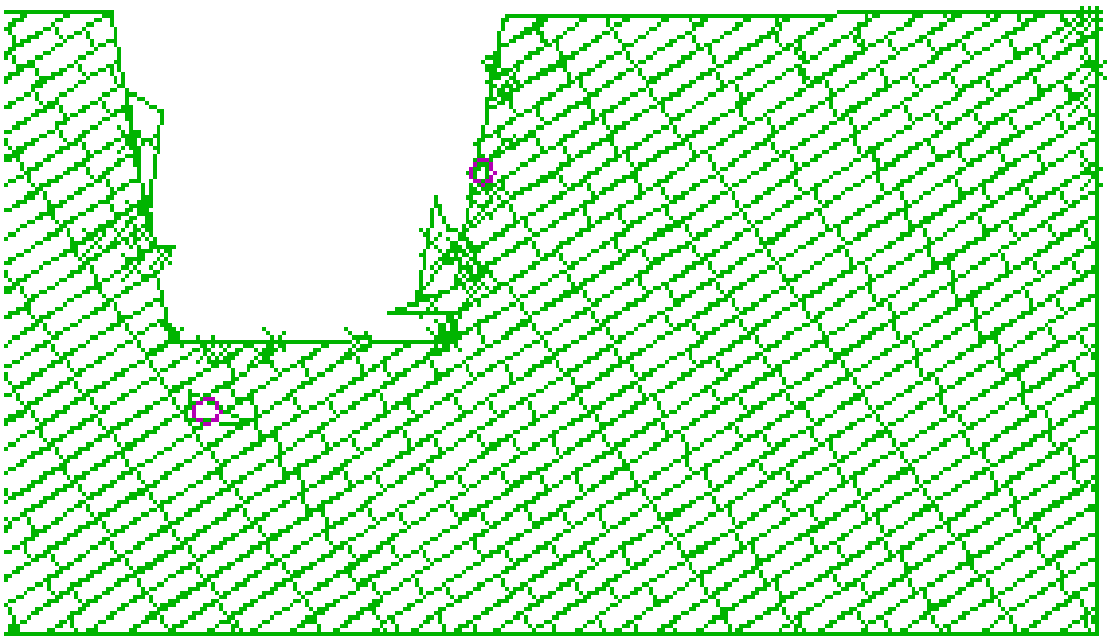


Figure 56: 25% standard deviation of joint spacing

5.5.2 Standard deviation of joint spacing of ±50% of mean spacings

Minor block failure occurred at the toe of the right hand slope while there was no failure on the left hand slope. There are small deformations along the bedding planes on the right hand side of the slope, see Figure 57.

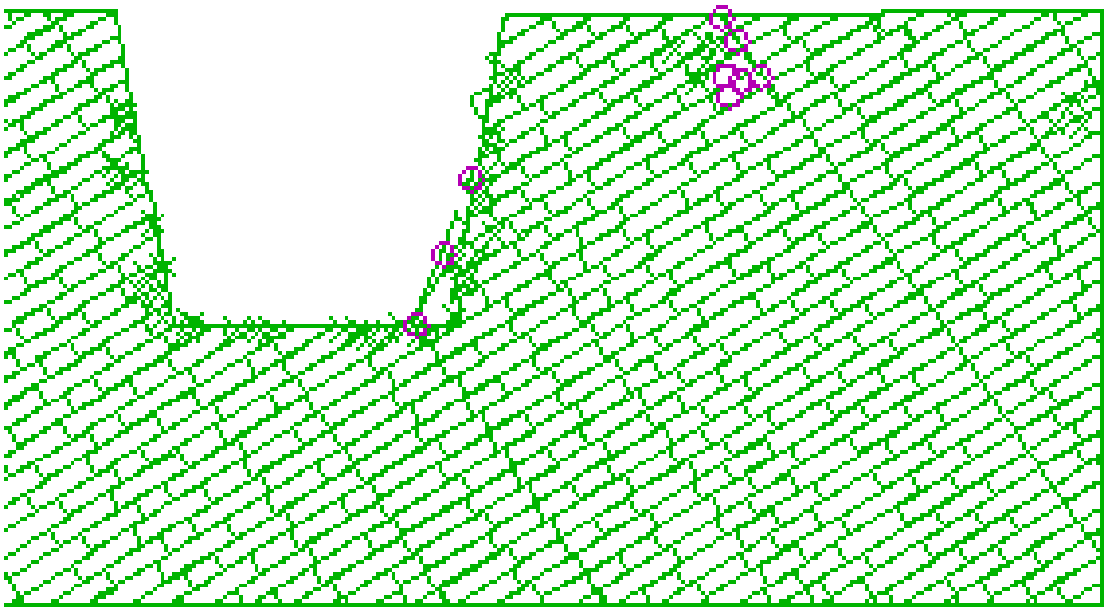


Figure 57: 50% standard deviation of joint spacing

5.5.3 Standard deviation of joint spacing of ±75% of mean spacings

There are very minor deformations along the bedding planes on the right hand side slope. Block failure is indicated on both sides of the slopes, see Figure 58.

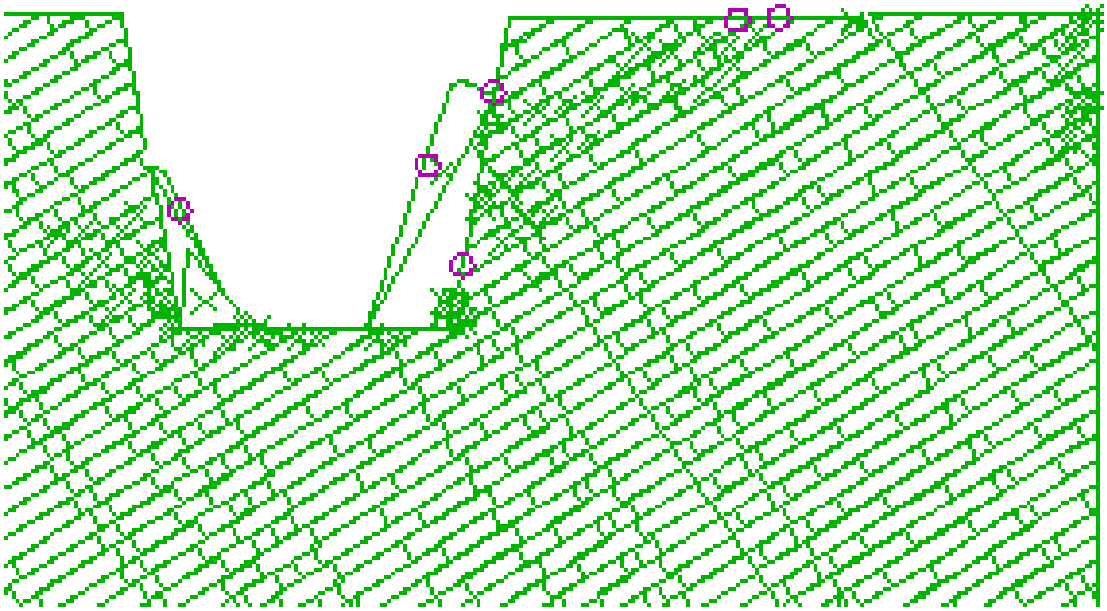


Figure 58: 75% standard deviation of joint spacing

Variation in the joint spacing alone has a relatively minor effect on slope behaviour. The reason for this could be that joints generally remain offset, and hence they have some effective tensile strength. Results show that, except for some minor block fall out, there is no major deformation of the slope. The same variability in the bedding plane spacing has a much more dramatic effect on the deformation of the slope as described below.

5.6 Effects of bedding spacing

The effects of bedding plane spacing on the stability of slopes have been evaluated by examining the slope behaviour in which variability occurs in the spacing of bedding. For this exercise 25%, 50%, and 75% of standard deviations of bedding spacing is used.

5.6.1 Standard deviation of bedding spacing of ±25% of mean spacings

Very minor block failure is indicated on the left hand slope. There are very small deformations along the bedding planes on the right hand slope. Sliding into the pit took place in two bedding planes which are close to the bottom level of the pit, see Figure 59.

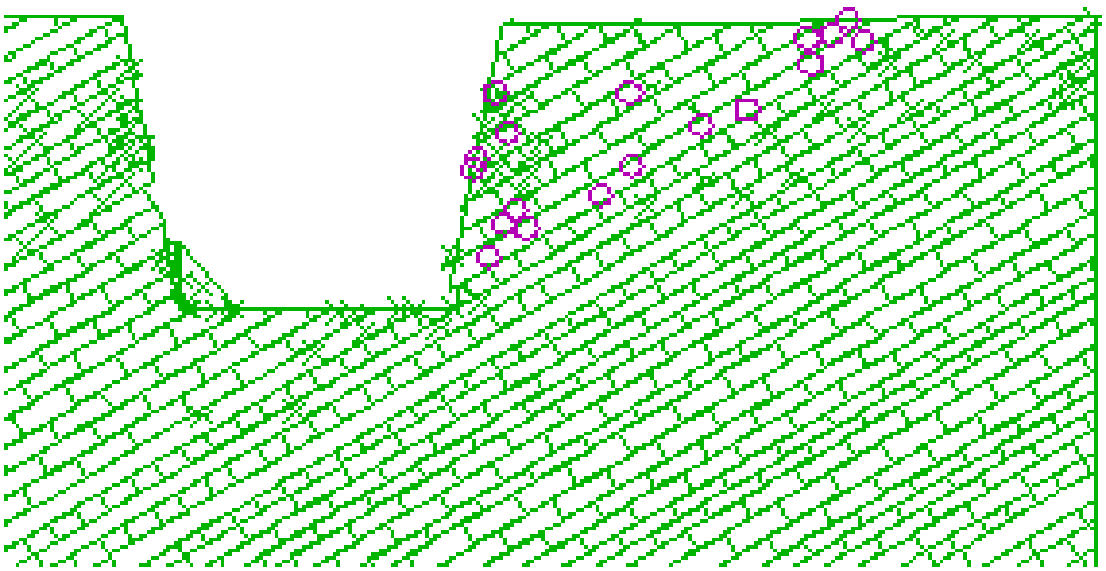


Figure 59: 25% standard deviation of bedding spacing

5.6.2 Standard deviation of bedding spacing of ± 50% of mean spacings

Block failure is larger than for the previous model and also occurs at the toe of the left hand slope. There are small deformations along the bedding planes on the right hand side of the slope. Results are shown in Figure 60.

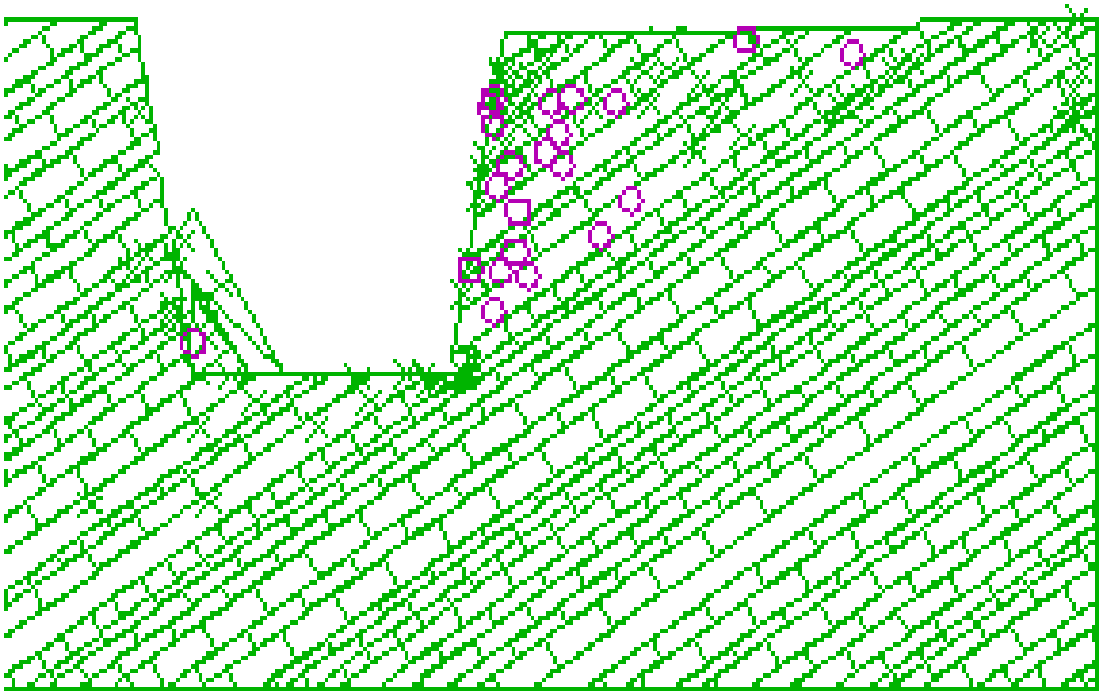


Figure 60: 50% standard deviation of bedding spacing

5.6.3 Standard deviation of bedding spacing of ±75% of mean spacings

There is significant deformation causing slope failure of the right hand slope for the maximum variability, as shown in Figure 61. With lesser variability, deformations were much smaller. Minor block failure occurred on the left hand slope. Minor slope overhangs are evident. Slope overhangs were also observed in some of the physical models.

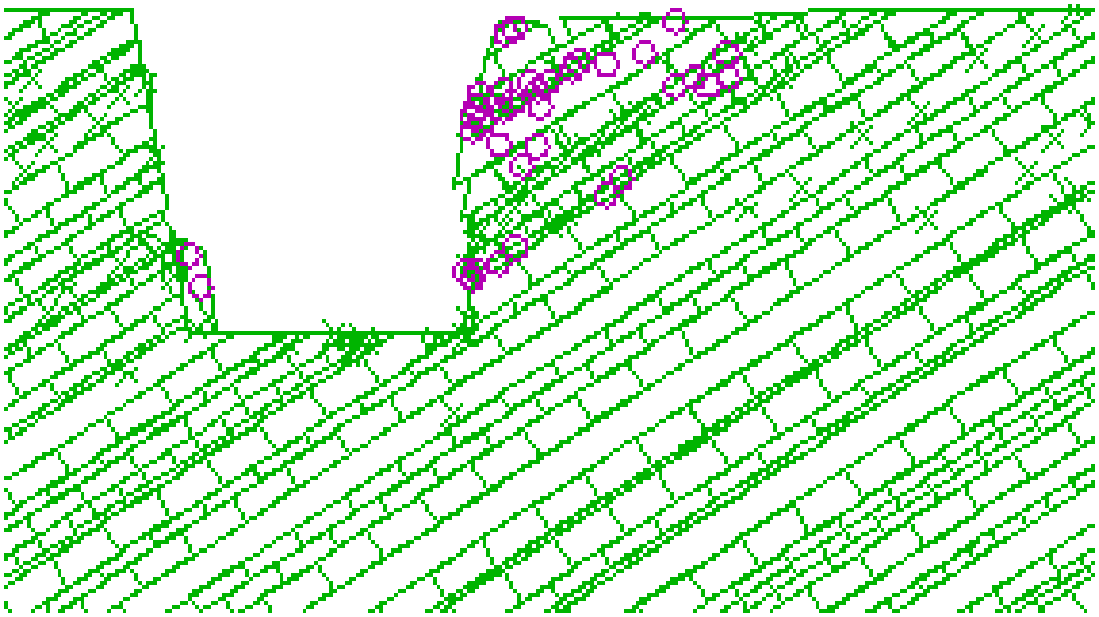


Figure 61: 75% standard deviation of bedding spacing

5.7 Effects of combinations of bedding and joint spacing

The effects of bedding spacing and joint spacing on the stability of slopes have been evaluated by examining the slope behaviour in which variability occurs in the spacing of bedding and joints. For this exercise 25%-25%, 50%-50%, and 75%-75% standard deviation of bedding spacing and joint spacing were considered.

5.7.1 Standard deviations in bedding plane and joint spacings of ±25% of mean spacings

Very minor deformations took place along the bedding planes of the right hand slope, see Figure 62. There is nothing notable happening on the left hand slope.

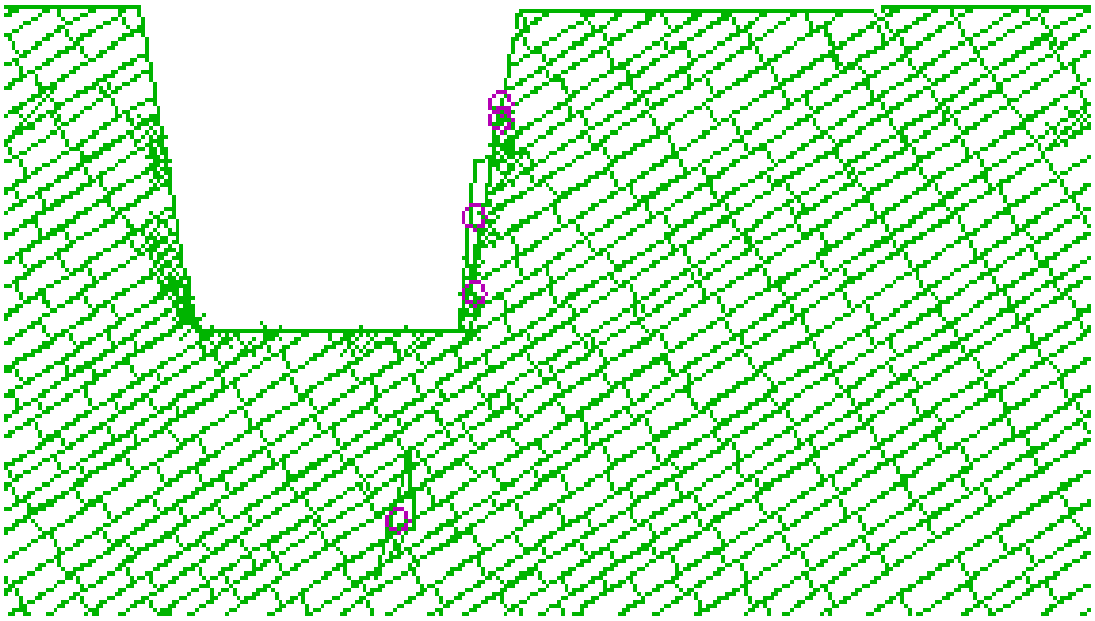


Figure 62: Standard deviations in bedding plane and joint spacings of ±25% of mean spacings

5.7.2 Standard deviations in bedding plane and joint spacings of ±50% of mean spacings

Block movement on a plane has been observed along the bedding planes of the right hand slope. Block failure occurred on the left hand slope. Results are shown in Figure 63.

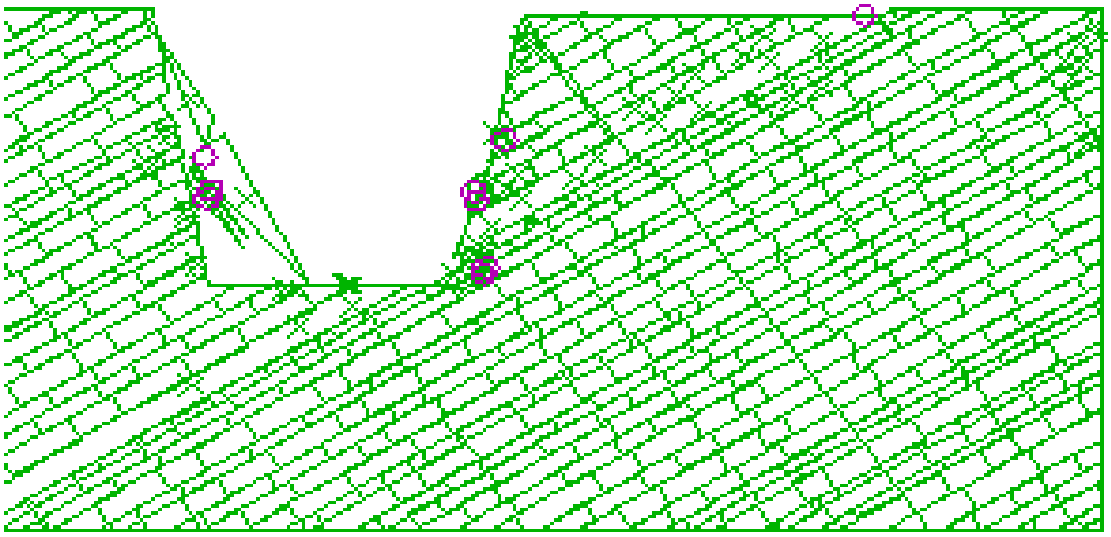


Figure 63: Standard deviations in bedding plane and joint spacings of ±50% of mean spacings

5.7.3 Standard deviations in bedding plane and joint spacings of ±75% of mean spacings

There is significant deformation of the right hand slope for the maximum variability, as shown in Figure 64. Slope overhangs as indicated were also observed in some of the physical models. With lesser variability, deformations were much smaller.

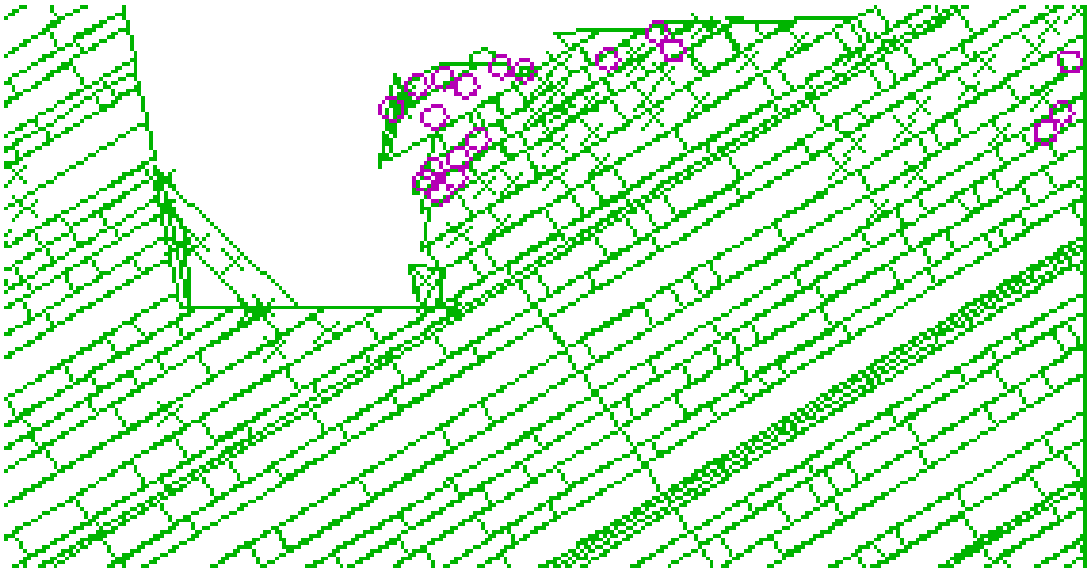


Figure 64: Standard deviations in bedding plane and joint spacings of ±75% of mean spacings

The three models clearly illustrate the effect of the increase in standard deviations in bedding plane and joint spacing at the same time on the behaviour of rock slopes.

5.8 Effects of joint offsets

If joints are not offset, it is possible for opening up of the rock mass to take place very easily across these effectively continuous joints. This is due to the fact that there is not sufficient contact length on the bedding planes either side of the joints to generate shear strength, which will in turn generate effective tensile strength across the joints. The effects of joint offsets on the stability of slopes can be evaluated by examining the slope behaviour in which variability occurs in the offsets of joints. For

this exercise variation of joint offsets in two different UDEC models were considered.

5.8.1 Relative joint location 50% SD of joint spacing

There are large deformations along the bedding planes of the right hand slope. Block failure occurred on both slopes. Results are shown in Figure 65.

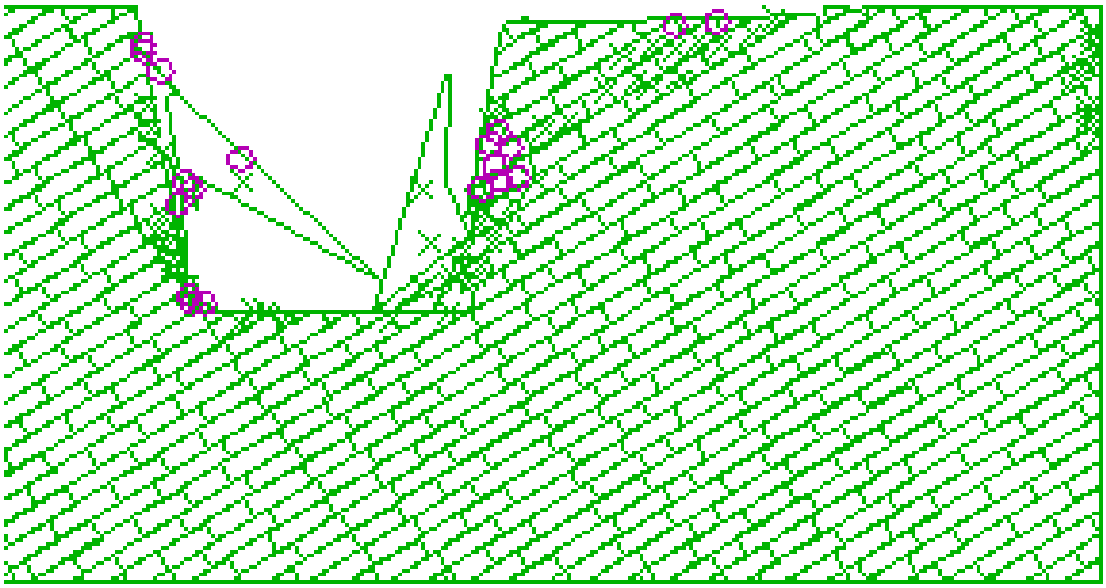


Figure 65: Relative joint location 50% SD of joint spacing

5.8.2 Relative joint location 50% SD of bedding spacing

Complete failure of the right hand slope occurred while no failure of the left hand slope took place. Easy joint opening and consequently large deformations of the rock mass caused complete failure of the right hand slope, see Figure 66.

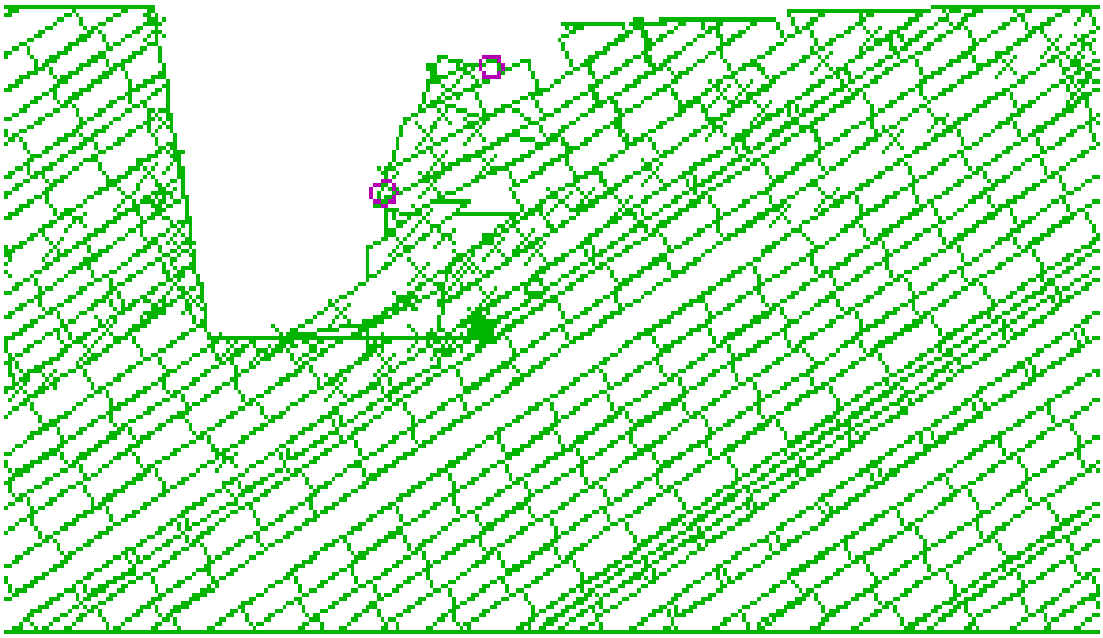


Figure 66: Relative joint location 50% SD of bedding spacing

5.9 Interpretation of the results of the numerical analyses

The results of the numerical analyses have confirmed that the behaviour of the physical models were realistic in spite of the limitations of the small scale of the models. The numerical model behaviour mirrored the physical model behaviour in many cases.

The range of models analysed has shown that the behaviour can differ significantly from one model to the next. In particular, the variability in the geometry of the discontinuities introduced into the models can have a very significant effect on the behaviour of the rock slopes. This is in spite of the fact that the rock masses being modelled can be considered to be statistically the same. In nature, the variability will probably be greater than that considered in the modelling. It is necessary to take this variability into account for realistic analysis and prediction of rock mass behaviour, and an interesting approach in this regard is described by Pine et al (2006). This involves the generation of joint traces in a three dimensional rock mass model from the statistical parameters that define the joints, and the modelling of this rock mass

using a numerical modelling package that takes into account several mechanisms of failure.

The analyses show that the rock slope deformations and failure did not involve a single failure surface, but are progressive, with deformation and local failure taking place throughout the slope height. In no case did failure involve displacement on a single failure plane. This places in question the conventional limit equilibrium approach to stability analysis, in which the stability of a failing mass above a defined or assumed failure surface is evaluated. No such surface could be defined for any of the models analysed. If such a failure surface was to be determined from the observations after failure, it would not be correct. The usual back-analysis approach to determine rock mass parameters is therefore also in doubt, since it will probably not take into account the actual rock slope failure mechanism. Therefore, although back analyses are considered to be important, the use of this approach could result in incorrect strength and deformation parameters for the rock mass and shear surfaces.

CHAPTER 6

Conclusions and recommendations

6.1 Conclusions

The characterization of rock masses for engineering applications is subject to uncertainties due to limited data that are typically available during site characterization, and due to inherent variability of properties within the rock mass. Accordingly, the problem of decision-making under uncertainty has become a topic of increasing interest for the rock engineers.

A proper description or geotechnical calculation to determine the behaviour of a rock mass should theoretically include all properties in a rock mass including all spatial variation of the properties. In practice this would be unrealistic and is also not possible without disassembling the rock mass. The smaller the allowed variability of the properties of rock mass the more accurate the geotechnical calculations can be. Smaller variability of the properties involves, however, collecting more data and is thus more costly, and the whole analysis also becomes more complex.

The aim of this research was to improve the understanding of mechanisms of failure of rock slopes in discontinuous, hard rock masses. UDEC was used to model rock slopes in which variability occurs in the geometry of the geological planes of weaknesses. More than 250 UDEC models were run in the process, of which 22 have been discussed in this research. A basic UDEC model, which did not include any variability in parameters discussed, was built, run and analysed. Further UDEC models which included variability and combinations of variability in geometric properties of joints were built and run. Results were analysed and comparisons made between the different models. Results of numerical models were also compared with those of physical models previously obtained by Stacey (2006).

The results of the numerical analyses have confirmed that the behaviour of the physical models were realistic in spite of the limitations of the small scale of the models.

From the results of the analyses carried out, the following conclusions can be drawn regarding the deformation and failure of rock slopes in a systematically jointed rock mass:

(i) Deformation and failure will not be confined to specific failure surfaces, but will occur progressively throughout the mass. Multiple mechanisms of failure will occur, including shear and tensile failure on discontinuities, and shear, tensile and extensional failure of intact rock material. The accumulation of these types of localized failure will ultimately result in slope failure.

(ii) Knowledge of the orientations and spacing of discontinuities in rock slopes does not allow prediction of a unique failure surface. Except in very specific situations, such a unique failure surface is unlikely to occur.

(iii) Realistic prediction of real jointed rock slope behaviour is only likely to be possible using probabilistic approaches that take into account the variability in the rock mass.

(iv) The degree of rock instability varies in the different models as shown by different areas (volumes) of unstable rock.

(v) Slope failures have been shown to be progressive. Slope monitoring is therefore an important part of slope design.

(vi) Use of Slope Model, which utilises discrete fracture network, for stability analysis of major rock slopes has shown great potential as the model has added capability of allowing fracturing through intact rock.

6.2 Recommendations

The following are the recommendations from this research:

(i) The probabilistic sensitivity analysis clearly shows that any future data gathering effort should concentrate on reduction of uncertainties associated with joint parameters.

(ii) Only UDEC code has been used in this research. It is recommended that a code like ELFEN should be used for comparison purposes.

(iii) Stress was maintained as a constant parameter in the numerical models. It is recommended that models should also be run for different stress levels to check the effect of this parameter on the behaviour of major rock slopes.

(iv) Joint geometry has shown to have a huge influence on slope behaviour. Variation of joint strength parameters cohesion and friction should be researched as these would have a huge impact on slope stability. A code like ELFEN should be used for this purpose. UDEC does not have the capability to simulate this. The question of the validity of back analysed rock mass cohesion, friction and deformability parameters and their use for 'calibration' of the slope or rock mass could then be answered.

(v) In addition to the results of slope stability analysis, engineering judgement is recommended for an optimal slope design.

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