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Dynamical systems in Walrusian general equilibrium

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3. for any $p \in S$ if $px \leq 0$ and $x \neq f(p)$, then $\xi(x) < \xi(f(p))$.

Let $\hat{\xi} = \max\{\xi(y) | y \leq x\}$, the function $x \mapsto \hat{\xi}$ is well defined and by condition (1) is continuous. By condition (2), if $x = (x_1, \ldots, x_l), y = (y_1, \ldots, y_l)$ and $x_i > y_i$ for all i, then $\hat{\xi}(x) > \hat{\xi}(y)$. Moreover, it can be checked that for all $p \in S, px \leq 0$ and $x \neq f(p)$ implies that $\hat{\xi}(x) > \hat{\xi}(f(p))$. Hence the preference relation given by $x \succcurlyeq y$ if and only if $\hat{\xi}(x) > \hat{\xi}(y)$ is continuous, monotone and generates f. □

Thus, by construction $f^*(p) = f(p)$ for $p \in S_\epsilon$. If $p \in S \backslash S_\epsilon$ then

$$-f^*(p) = \begin{cases} \sum_{i=1}^{l} Dv_i(p) g_\epsilon(p) & \text{for } p \in S \backslash S_\epsilon \\ \eta(p) \left(\sum_{i=1}^{l} Dv_i(p) g_\epsilon(p) \right) + (1 - \eta(p)) f(p) g_\epsilon(p) & \text{for } p \in S \backslash S_{\epsilon/2} \end{cases}$$

and the theorem is proved.

Appendix C. The Routh-Hurwitz Conditions for Stability

The Routh-Hurwitz conditions (as stated here and used in §8) provide sufficient conditions for the roots of a polynomial to have negative real parts. Since a necessary condition for a dynamical system to have a stable equilibrium point is that the Jacobian matrix have eigenvalues with negative real parts, these conditions provide a convenient method of testing the stability of an equilibrium without calculating the eigenvalues.

To state these conditions consider the following polynomial of degree n

$$a_0 x^n + a_1 x^{n-1} + \ldots + a_n = 0 \tag{C.1}$$

where we assume, without loss of generality, that $a_0 > 0$. Routh-Hurwitz then states that the roots of C.1 have negative real parts if and only if the first n of the following set of determinants

$$|a_1|;\quad \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix};\quad \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix};\quad \begin{vmatrix} a_1 & a_3 & a_5 & a_7 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix};\quad \ldots$$

are positive. (Note, here $|a_1| \equiv a_1$ and $a_m = 0$ for all $m > n$.)

The proof of this theorem can be found in Gantmacher [10] and will not be presented here. It makes use of some elementary yet technical arguments from complex analysis. The original proofs can be found in E.J. Routh *Stability of Given State of Motion* (London 1877), Adams Prize Essay; *Dynamics of a System of Rigid Bodies* (6th ed.; London 1930), part II, chap. vi; A. Hurwitz *Uber die Bedinungen, unter welchen eine Gleichung nur wurzen mit negativen reellen Theilen beszit* Mathematische Annalen XLVI (1895) 273-384.

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