

3. for any $p \in S$ if $px \leq 0$ and $x \neq f(p)$, then $\xi(x) < \xi(f(p))$.

Let $\hat{\xi} = \max\{\xi(y) | y \leq x\}$, the function $x \mapsto \hat{\xi}$ is well defined and by condition (1) is continuous. By condition (2), if $x = (x_1, \dots, x_l)$, $y = (y_1, \dots, y_l)$ and $x_i > y_i$ for all i , then $\hat{\xi}(x) > \hat{\xi}(y)$. Moreover, it can be checked that for all $p \in S$, $px \leq 0$ and $x \neq f(p)$ implies that $\hat{\xi}(x) > \hat{\xi}(f(p))$. Hence the preference relation given by $x \succ y$ if and only if $\hat{\xi}(x) > \hat{\xi}(y)$ is continuous, monotone and generates f . $\square$

Thus, by construction $f^*(p) = f(p)$ for $p \in S_c$. If $p \in S \setminus S_c$ then

$$-f^*(p) = \begin{cases} \sum_{i=1}^l Dv_i(p)g_e(p) & \text{for } p \in S \setminus S_c \\ \eta(p) \left(\sum_{i=1}^l Dv_i(p)g_e(p) \right) + (1 - \eta(p))f(p)g_e(p) & \text{for } p \in S \setminus S_{c/2} \end{cases}$$

and the theorem is proved.

APPENDIX C. THE ROUTH-HURWITZ CONDITIONS FOR STABILITY

The Routh-Hurwitz conditions (as stated here and used in §8) provide sufficient conditions for the roots of a polynomial to have negative real parts. Since a necessary condition for a dynamical system to have a stable equilibrium point is that the Jacobian matrix have eigenvalues with negative real parts, these conditions provide a convenient method of testing the stability of an equilibrium without calculating the eigenvalues.

To state these conditions consider the following polynomial of degree n

$$(C.1) \quad a_0 x^n + a_1 x^{n-1} + \dots + a_n = 0$$

where we assume, without loss of generality, that $a_0 > 0$. Routh-Hurwitz then states that the roots of C.1 have negative real parts if and only if the first n of the following set of determinants

$$|a_1|; \begin{vmatrix} a_1 & a_3 \\ a_0 & a_2 \end{vmatrix}; \begin{vmatrix} a_1 & a_3 & a_5 \\ a_0 & a_2 & a_4 \\ 0 & a_1 & a_3 \end{vmatrix}; \begin{vmatrix} a_1 & a_3 & a_5 & a_7 \\ a_0 & a_2 & a_4 & a_6 \\ 0 & a_1 & a_3 & a_5 \\ 0 & a_0 & a_2 & a_4 \end{vmatrix}; \dots$$

are positive. (Note, here $|a_1| \equiv a_1$ and $a_m = 0$ for all $m > n$.)

The proof of this theorem can be found in Gantmacher [10] and will not be presented here. It makes use of some elementary yet technical arguments from complex analysis. The original proofs can be found in E.J. Routh *Stability of Given State of Motion* (London 1877), Adams Prize Essay; *Dynamics of a System of Rigid Bodies* (6th ed.; London 1930), part II, chap. vi; A. Hurwitz *Über die Bedingungen, unter welchen eine Gleichung nur Wurzeln mit negativen reellen Theilen besitzt* Mathematische Annalen XLVI (1895) 273-384.

REFERENCES

- [1] A. Antoniou. *Applied Micro Economics*. Butterworths, 1992.
- [2] G. Debreu. *Theory of Value*. Yale University Press, 1959.
- [3] G. Debreu. Economies with a finite set of equilibria. *Econometrica*, 38:387-392, 1970.
- [4] G. Debreu. Smooth preferences. *Econometrica*, 40:603-616, 1972.
- [5] G. Debreu. Excess demand functions. *J. Math. Econom.*, 1:15-23, 1974.
- [6] E. Dierker. Two remarks on the number of equilibria of an economy. *Econometrica*, 40:951-955, 1972.
- [7] E. Dierker. *Topological Methods in Walrasian Economics*. Springer-Verlag, 1974.
- [8] E. Dierker. Regular economies. In K. J. Arrow and M. D. Intriligator, editors. *Handbook of Mathematical Economics*, vol. II, chapter 17. North-Holland Publishing Company, 1982.
- [9] C. Eaves and H. Scarf. The solution of systems of piecewise linear equations. *Mathematics of Operations Research*, 1:1-27, 1976.
- [10] F.R. Gantmacher. *The Theory of Matrices*. Chelsea Publishing Company, 1960.
- [11] J.D. Geanakoplos. Utility functions for debreu's excess demands. *mimeo. Harvard University*, 1978.
- [12] M. W. Hirsch. *Differential Topology*. Springer-Verlag, 1976.
- [13] M. W. Hirsch and S. Smale. On algorithms for solving $f(x) = 0$. *Comm. Pure Appl. Math.*, 32:281-312, 1979.
- [14] J.S. Jordan. Locally stable price mechanisms. *J. Math. Econom.*, 11:235-259, 1983.
- [15] A. Katok and B. Hasselblatt. *Introduction to the Modern Theory of Dynamical Systems*. Cambridge, 1995.
- [16] R. Mantel. On the characterization of aggregate excess demand. *J. Economic Theory*, 7:348-353, 1974.
- [17] A. Mas-Colell. On the equilibrium price set of an exchange economy. *J. Math. Econom.*, 4:117-126, 1977.
- [18] J. Milnor. *Topology from a Differentiable Viewpoint*. University press of Virginia, 1965.
- [19] J. Palis and W de Melo. *Geometric Theory of Dynamical Systems*. Springer-Verlag, 1982.
- [20] D. Saari. A method for constructing message systems for smooth performance functions. *J. Economic Theory*, 33:249-274, 1984.
- [21] D. Saari and C. Simon. Effective price mechanisms. *Econometrica*, 46:1097-1125, 1978.
- [22] D. Saari and S.R. Williams. On the local convergence of economic mechanisms. *J. Econ. Theory*, 40:152-167, 1986.
- [23] H. Scarf. Some examples of global instability of the competitive equilibrium. *International Econ. Review*, 1:157-172, 1960.
- [24] W. Schafer and H. Sonnenschein. Market demand and market excess demand functions. In K. J. Arrow and M. D. Intriligator, editors. *Handbook of Mathematical Economics*, vol. II, chapter 14. North-Holland Publishing Company, 1982.
- [25] S. Smale. Global analysis and economics iia. *J. Math. Econom.*, 1:1-14, 1974.
- [26] S. Smale. Global analysis and economics iv. *J. Math. Econom.*, 1:119-127, 1974.
- [27] S. Smale. Convergent processes of price adjustment and global newton methods. *J. Math. Econom.*, 3:107-120, 1976.
- [28] S. Smale. Dynamics in equilibrium theory. In *The Mathematics of Time*. Springer-Verlag, 1976.
- [29] S. Smale. Dynamics in general equilibrium theory. *American Economic Review*, 66:284-294, 1976.
- [30] S. Smale. *The Mathematics of Time*. Springer-Verlag, 1976.
- [31] S. Smale. Global analysis and economics. In K. J. Arrow and M. D. Intriligator, editors. *Handbook of Mathematical Economics*, vol. I, chapter 8. North-Holland Publishing Company, 1977.
- [32] S. Smale. The fundamental theorem of algebra and complexity theory. *Bull. Amer. Math. Soc. (N.S.)*, 4:1-36, 1981.
- [33] H. Sonnenschein. Do walras' identity and continuity characterize the class of community excess demand functions. *J. Economic Theory*, 6:345-554, 1973.
- [34] H. Sonnenschein. Market excess demand functions. *Econometrica*, 40:549-563, 1974.
- [35] N. Steenrod. *The Topology of Fibre Bundles*. Princeton University Press, 1951.

- [36] W. Szlenk. *An Introduction to the Theory of Smooth Dynamical Systems*. Polish Scientific Publishers/Wiley, Warszawa/Chichester, 1984.
- [37] H. Varian. A remark on the boundary restrictions in the global newton method. *J. Math. Econom.*, 4:127-130, 1977.
- [38] von Geldrop. Extension of a theorem of smale on equilibria for pure exchange economies. *J. Math. Econom.*, 5:245-253, 1978.
- [39] F. Warner. *Foundations of Differentiable Manifolds and Lie Groups*. Scott, Foresman and Company, 1971.
- [40] G.W. Whitehead. Homotopy properties of the real orthogonal groups. *Annals of Mathematics*, 43:132-146, 1942.
- [41] S.R. Williams. Implementing a generic smooth function. *J. Math. Econom.*, 13:273-288, 1984.
- [42] S.R. Williams. Necessary and sufficient conditions for the existence of a locally stable message process. *J. Economic Theory*, 35:127-154, 1985.

INDEX

- Algebraic normal bundle, 32
- Baire Space, 10
- Boundary, 8
- Boundary operator, 35
- Budget Plane, 4
- Budget Set, 4
- Chain, 35
- Characteristics, 71
- Chart, 8
 - adapted, 6
 - local, 6
 - submanifold, 7
- Codimension, 7
- Commodity
 - bundle, 3
 - space, 37
 - free, 3
 - noxious, 3
 - scarce, 3
- Computational sequence, 72
- Critical point, 13
- Cycles, 35
- Degree, 32, 34
- Demand Function, 4
- Economy, 3, 38
 - regular, 39, 41
- Effective price mechanism, 56
- Embedding, 6
- Equilibria
 - classical, 38
 - extended, 37, 38
 - stable, 44
- $E_{xx}(r, u)$, 38
- Essential, 31
- Excess demand
 - aggregate
 - strongly regular, 71
- Excess Demand Function
 - aggregate, 5, 56
 - individual, 5
- Fibre map, 28
- Generic, 13
- Halfspace, 8
 - proper, 8
- Homology group, 35
- Homotopic, 29, 58
- Homotopy equivalent, 62
- Immersion, 6
- Indifference Surface, 4
- Inessential, 31
- Initial Endowment, *see* Resources 4
- Irreducible, 31
- Isotopy group, 23
- Jets, 10
- Lie
 - algebra, 20
 - bracket, 7, 20
 - group, 20
 - group homomorphism, 21
 - subgroup, 21
- Locally Compact, 6
- Manifold, 8
 - differentiable, 6
 - homogenous, 21
 - submanifold, 7
 - neat, 8
 - triangulation, 34
- Marginal rate of substitution, 39
- Market mechanism, 66
- Matrix
 - skew symmetric, 23
- Newton method
 - generalised, 67
 - global, 53
 - orthogonal, 61
- Orientation, 31
- Oriented vector space, 31
- Paracompact, 6
- Polar decomposition, 26
- Preferences, 4
- Price, 3
 - system, 3
- Price mechanism, 56
 - hyperbolically locally stable, 61
 - at unique equilibria, 65
 - locally stable, 57
 - at unique equilibria, 65
- Privacy preserving, 71
- Private information, 71
- Reducible, 31
- Regular value, 13
- Representation, 21
- Residual, 13
- Resources, 4
- Simplex, 34
- Simplicial complex, 34

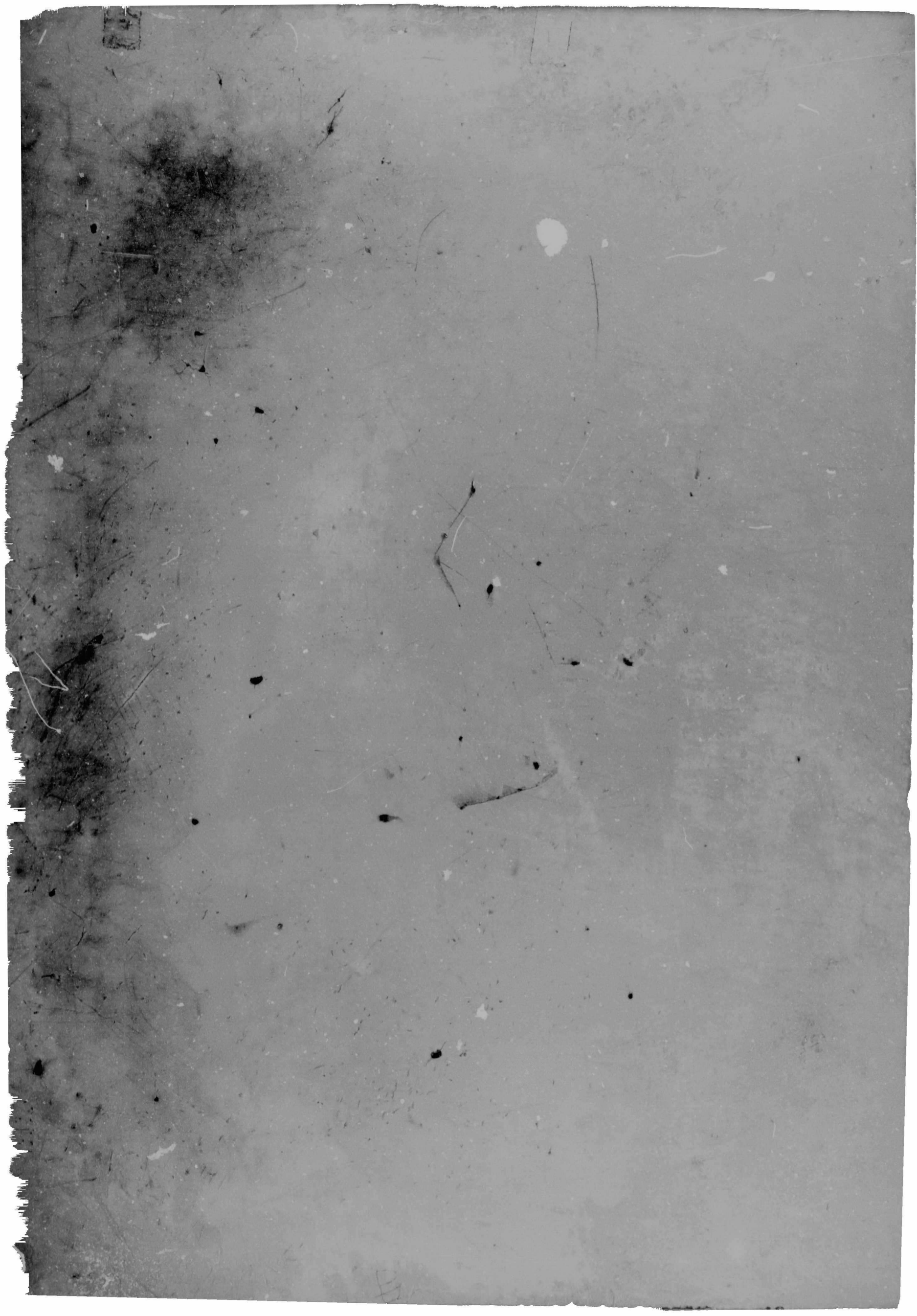
SMD Theorem, 48
Stability
 asymptotic, 57
 Lyapunov, 57
Stability sequence, 73
State, 38
 attainable, 38
Static analysis, 1
 comparative static analysis, 1

Tangent
 space, 7
 vector, 7
Topology
 strong, 9, 12
 weak, 9, 12
Transversal, 39
Transverse, 13
Trivialisation, 27

Utility Function, 4
Utility function, 37

Vector bundle, 27
 atlas, 27
 chart, 27
 map, 28
Vector field, 7
 left invariant, 20

Walras' law, 5
Wealth, 4



Author Davey S J

Name of thesis Dynamical systems in Walrasian General Equilibrium 1997

PUBLISHER:

University of the Witwatersrand, Johannesburg

©2013

LEGAL NOTICES:

Copyright Notice: All materials on the University of the Witwatersrand, Johannesburg Library website are protected by South African copyright law and may not be distributed, transmitted, displayed, or otherwise published in any format, without the prior written permission of the copyright owner.

Disclaimer and Terms of Use: Provided that you maintain all copyright and other notices contained therein, you may download material (one machine readable copy and one print copy per page) for your personal and/or educational non-commercial use only.

The University of the Witwatersrand, Johannesburg, is not responsible for any errors or omissions and excludes any and all liability for any errors in or omissions from the information on the Library website.