INFORMATION COMMUNICATION TECHNOLOGY INTERVENTIONS IN TVET MATHEMATICS N4 INTEGRAL CALCULUS.

A Research Report submitted to the Faculty of Education, University of Witwatersrand, Johannesburg, in partial fulfilment of the requirements for the degree of Master of Education.

by

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DECLARATION

I, the undersigned, declare that this research project which I hereby submit for the degree of Master of Mathematics to the University of Witwatersrand contains my own, independent work and has not previously been submitted by anyone for any degree at this or any other university.

Signature ……………………………

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LIST OF ABBREVIATIONS AND ACRONYMS

NCS	National Curriculum Statement
SEFI	European Society of Engineering Education
ICT	Information Communication Technology
N4E1	N4 Electrical Group 1
N4M1	N4 Mechanical Group 1
ZPD	Zone of Proximal Development
3D	Three Dimensional View
CICDI	Confusion in Calculus Integral with Definite Integral
IKODRAMAG	Insufficient Knowledge of Drawing Mathematical Graphs
BITSHOSG	Basic Information Shown On Sketch Graph
CWASG	Calculations without the Accompanying Sketch Graph
IPI	Integration Performed Incorrectly
ZAM	Zero Area Magnitude
HOD	Head of Division
IPT	Information Processing Theory
UT	University of Technology
TVET	Technical and Vocational Training

CHAPTER ONE: BACKGROUND OF STUDY

1.1 Introduction and context of study

The study of teaching and learning of mathematics both internationally and locally has always been a topic for mathematics education research. Mathematics education research focuses on the teaching and learning of mathematics with the aim to improve how this subject can best be taught and learned with greater understanding. Many will agree with the notion that mathematics is a challenging subject and most students find it difficult to learn.

It is known that the importance of mathematics cannot be overemphasized. This study focuses on the teaching and learning of mathematics subject at Technical and Vocational Training (TVET) Colleges in South Africa. Mathematics forms the heart and the backbone of engineering courses both at TVET Colleges and universities of technology (UTs). For example, Penesis, Chin, Ranmuthugala and Fluck (2011) argue that one of the major factors that deter school pupils from selecting engineering courses is a dislike for mathematics. These researchers indicate that this trend can be reversed by changing the way mathematics is taught in schools.

Why is it necessary to study mathematics? People who studied mathematics know that this subject, amongst other things, is mainly about solving problems, producing mathematical conjectures and proving theorems. This means that mathematics is mainly about problem-solving and most of the time this requires a lot of abstract thinking. Courant, Robbins and Stewart (1996) also argue that mathematics connects abstract world of mental concepts to the real world of physical things. It cannot be discounted that abstract thinking is key in the learning of mathematics. Mulero, Segura and Sepulcre (2013) add that mathematics expresses itself everywhere in almost every facet of life - in nature all around us. Mulero et al. (2013) further indicate that mathematics is the language of engineering which helps us to describe our understanding of all that we observe. This is in line with Galileo Galilei that mathematics is the language with which God wrote the universe (Palmerino and Rita, 2006)

On the other hand, Mulero et al. (2013) agree with Aristotle who defined mathematics as the science of quantity, that is, the science of the things that can be counted and measured. It goes without saying that counting and measuring have vital roles in our daily life. Imagine how difficult life would be if human beings were unable to count and measure, e.g. days, months and years. It is true that mathematics cannot be divorced from our daily life and this is reason enough for most university degrees to require students to do mathematics as a subject. Mulero et al. (2013) argue that those students who did not take mathematics seriously in high school find several difficulties when they continue with mathematics at a tertiary level of study.

Mathematics as a subject has many branches of study. One of the distinct parts is the study of calculus. Naidoo (2007) indicates that first year mathematics courses at tertiary institutions consist of basic mathematics and calculus. In most cases the calculus component forms a major section of first year mathematics course. Much emphasis in calculus is placed on differentiation, integration and their applications. Tertiary engineering courses such as those of Integration, Laplace Transforms, Numerical Mathematics and Mathematics Modelling, have their roots in the understanding of elementary concepts in calculus. According to Naidoo (2007), South African universities of technology, mathematics is not required as a specialization, rather as an effective tool for studying engineering courses. It is likely that those who understand mathematics will also find it easy to deal with other subjects.

Let us check what researchers say about mathematics. Is mathematics understanding, crucial for engineering students? Bajracharya (2012) indicates that calculus provides logical paths to solve complex problems as used in a spectrum of disciplines. Standler (1990) indicates that the study of mathematics is important for artisans/engineers because the laws of nature are mathematical expressions. According to this researcher, mathematics equip students with technical analytic communication skills. Tall (1993) found that students had difficulties in understanding calculus when teachers used pen and paper method. Tall (1996) and McKnight (1987) found that teachers rely upon their own prepared materials as well as the student textbook and hardly make use of technology. Zachariades et al. (2007) agree that calculus concepts are abstract and complicated and therefore there is a need for more detailed investigations into student`s conceptualisations of calculus concepts.

These researchers paint the picture that one of the reasons why students find mathematics challenging is basically how teachers approach their work. Tall (1993) talks of pen and paper method of teaching mathematics where as Tall (1996) as well as McKnight (1987) indicate the use of textbook and no use of technology. Similarly, Naidoo (2007) argues that of all different ways of learning calculus, visualisation is amongst the best strategies to make students understand this topic better. This researcher believes that most students understand better when they see what the teacher or lecturer is talking about. This is where ICT tools come in handy.

1.2 What is calculus?

Rosenthal (1951) indicate that calculus is a mathematics study discovered independently by Sir Isaac Newton and Gottfried Leibniz in the 17th century. Calculus has two major branches in differential calculus as well as integral calculus. Differential calculus is the process of differentiation and determining derivatives. Differential calculus deals mainly with rates of change and slopes of graphs/curves. When we have a sketch of a function drawn, e.g. a cubic type graph, and we pick a point along this graph, we will have the slope of the tangent line at this point on the graph, equal the derivative of the function at this same point. Integral calculus concerns itself with accumulation of quantities and the area under and between graphs. The process of integration involves the determination of anti-derivatives or integrals.

The two branches are opposite operations. They undo each other. These two branches come to a common meeting point through the fundamental theorem of calculus. I therefore fully believe that calculus has widespread uses in engineering field of study and can solve many problems that elementary algebra alone cannot.

After having done a course on teaching and learning of algebra during first year of doing master`s course and also after reading Boyer (1970), it comes out clear that what algebra did to arithmetic is exactly what calculus is doing to algebra. Arithmetic deals with manipulation of numbers through the four basic operations. Algebra finds patterns between these numbers. Calculus finds patterns between equations. Research findings (e.g. Luneta and Makonye, 2010) indicate that both teachers and students register many challenges in the teaching and learning of calculus.

Orton (1983) discovered that students had little intuitive understanding and also had fundamental misconceptions about the concept of derivative in calculus. This means that students had superficial and incomplete understanding of many basic concepts in calculus. According to this author, this is attributed to the rote and manipulative learning that took place in earlier studies of calculus. Ferrini-Mundy (1994) indicates that students` understanding of calculus is done as a static collection of concepts and skills that need to be mastered one at a time. This should not be the case as mathematics concepts interconnect and built on to one another coherently.

Naidoo (2007) argues that the need for alternate methods of instruction to enhance teaching and understanding of calculus is essential. This researcher indicates that currently much of the focus in mathematics at most South African secondary schools is based on wanting to make mathematics simpler as far as possible because mathematics lessons in schools are algorithmic, focussing on standard methods, rules and procedures where little or no attention is given to genuine understanding of concepts.

1.3 Teaching and learning of calculus in South Africa

In the matric syllabus of South Africa, calculus forms about 12% of National Curriculum Statement (NCS) of 2011. To me, calculus is therefore studied at superficial level in South African secondary schools. Tall (1996), McKnight (1987) as well (1983) have indicated that teachers rely on textbook use, rote and manipulative learning for teaching calculus. These procedural approaches are responsible for students` superficial understanding of calculus. Learners' superficial understanding of calculus is carried over to tertiary institutions. Students continue to know less about calculus as teachers and lecturers continue to use old methods of teaching. Superficial understanding therefore becomes the source of errors and misconceptions developing in students.

Many lecturers who are employed at TVET Colleges are former secondary school mathematics teachers. Thus, their teaching style is in no way different to how they taught calculus in secondary schools. The situation at TVET College is that the first trimester students commit errors and misconceptions when learning calculus. In my opinion this is attributed to two factors. Firstly it is how they are taught and secondly their superficial knowledge on calculus is unable to carry them a little further in their understanding trajectory. The observation is that when students come to enrol at a TVET College, they have less calculus knowledge.

In a TVET College where I work, most lecturers use `pen and paper` method in the teaching and learning of Calculus (Tall, 1993). Lecturers rely heavily upon their own prepared materials as well as the student textbook and hardly make use of technology in teaching Calculus (Tall, 1996). It is also correct to stress that there is a need for more detailed investigations into student`s conceptualisations of calculus concepts (Zachariades et al., 2007).

I have come to believe that the errors as well as the misconceptions that students display in learning calculus, can be attributed to the types and methods of teaching used by lecturers. I work as a Head of Department (HOD) for Engineering Studies in a TVET College and I do class visits to many lecture rooms to observe how the teaching of mathematics is done. Visiting lecturers in their classrooms is one of my duties as the head of the division. I am also accountable for producing good results at the end of every trimester.

My observations are that lecturers are still using old methods of transmitting knowledge by standing in front of students and do the `talk and chalk`. Lecturers appear to be the final authorities for what they are saying to students. Their lessons are teacher - centred with little or no learner contributions. In almost all lecturer rooms there are no ICTs used except the scientific calculators which students use for finding answers when verifying the lecturer`s answer worked out on the board. In most cases our campus mathematics final results are not good as they are below college set target.

Apart from this research study being a requirement to be awarded a master degree by the University of Witwatersrand, this research serves to seek ways on how the engineering division at my work place can improve on the mathematics results and by so doing helping the college to reach its target. It is my duty at work to receive results from the Department of Higher Education, and analyse them and make presentations to the campus manager on how the engineering division performed every trimester.

1.4 Why chose calculus in TVET N4 Mathematics course?

The main reason I chose to do research on the teaching and learning of calculus is because it forms the lion`s share in the whole technical education curriculum (TVET mathematics curriculum). It forms about 80% of the N4 Mathematics syllabus and 100 % for both N5 Mathematics and N6 Mathematics. This means that if calculus is not mastered at this level, then students will not progress to N5 Mathematics and N6 Mathematics and their dreams of becoming artisans die off. Considering my observations that lecturers are same as high school teachers in terms of their teaching approach, and also that students come with background of superficial knowledge on calculus, it is likely that not many students understand calculus.

I agree with Shulman (1986) when he says that tertiary institutions are much like normal schools in carrying out their daily teaching operations. Ramsden (2003) argues that much university teaching is still based on the theory that students learn if lecturers transmit information to them in lecturer rooms in the in the same way it is done in our TVET college. This way of transmitting information to students by teachers, is one of the old methods of teaching. Du Preez, Steyn and Owen (2008) show that lectures in universities are presented in a traditional way just the way lesson are given in our campus.

Ramsden (2003) indicates that teaching at a higher institution of learning is not easy as it takes many years of practice to learn how to do it well, and even then, you will not have learned enough. According to Ramsden (2003), some lecturers do not know where to start improving it; at once overwhelmed by and unwilling to admit its complexity, they ask for a set of rules that will solve all their difficulties.

Gueudet (2008) looked at comparing the secondary school and university teachers' and found that even university teachers of all domains lack a clear objective for the teaching they deliver. According to this researcher, the students' personal work is more focused on learning to mimic strategies which lead to students aiming at learning to imitate the solutions more than at developing a real understanding

Sasman (2011) did a research in the National Senior Certificate (NSC) for the years 2008, 2009 and 2010 and found that in every examination candidates revealed deficient understanding of calculus. The researcher found that the errors students made indicate a lack of understanding and a very poor conceptual grasp. Sasman (2011) argues that many of the errors and misconceptions gathered from learner responses have their origins in poor understanding of the basics and foundational competencies taught in the earlier grades.

According to Sasman (2011), many educators are not confident in teaching some content in the NCS topics and some appear to be struggling with the depth of some topics which is a situation leading to familiar sections taught for long time and less familiar topics taught at a very superficial level. This results in educators struggling to teach the entire curriculum completely.

Jojo and Maharaj (2013) indicate that calculus is one of the topics introduced to matric learners at high school, yet a large number of them receive inadequate mathematics education and join the university mostly under-prepared for the study of differential calculus.

Du Preez, J., Steyn, T., & Owen, R. (2008) indicate that the pre-calculus mathematical knowledge of first-year engineering students is not at the expected level necessary for a study in calculus. According to Du Preez et al, the teaching of calculus at first-year tertiary level needs to equip engineering students with the fundamental mathematical skills required in engineering study.

N4 students` term assessments accessed from the campus archives display errors and evidence of misconceptions in the calculus section.

It makes sense to believe that if N4 Mathematics students are helped to improve their understanding of calculus, they will pass with good marks and progress to N5 as well as N6 Mathematics syllabi with no difficulty.

1.5 Research on teaching and learning of calculus.

Sarwadi and Shahrill (2014) define mathematics as a hierarchical build-up of concepts, skills and facts. According to these researchers, successful learning of this subject involves a systematic build-up of such hierarchy of concepts where ideas are to be understood and woven together for concepts to build on one another. It means, mathematics topics are connected into a series and should be taught as such as they build up into one meaningful idea. These researchers also see mathematics learning as cumulative where new knowledge gained is linked to the previous knowledge. Piaget (1972) has indicated that in the process of learning, knowledge is not constructed solely from experience but rather a mixture of experience and present knowledge structures. According to Piaget (1972), mental structures or schemata are constructed through interaction by processes called assimilation and accommodation. This means in Piagetian terms, once a schema or concept is formed, it is stable and resistant to change. A student's existing schema or concept will therefore determine what he or she learns from experience or instruction. What is more important is that if a student is unable to assimilate and accommodate, this creates a gap in the learning of the concept, and in turn, leads to mathematical errors or misconceptions.

Tall (1993) found that the teaching and learning of calculus, experiences challenges in how the message is given across by teachers to students. According to Tall (1993), teachers' messages may be framed in a language that evokes inappropriate ideas and may be presented in a sequence that is inappropriate for cognitive development. He further says that in learning calculus, the informal language used at the introductory stage can cause unforeseen difficulties. On the other side, Schwarzenberger (1980) argues that the mathematical difficulties in learning calculus arise as a result of not settling for simple explanations and result in underlying difficulties to haunt the students.

Muzangwa and Chifamba (2012) found that students plotted linear curves of the form: $y = mx + c$ instead of quadratic curves when given $x^2 - y + z = 4$. Students could not draw graphs correctly and therefore could not show the required region as asked in the question. Students failed to realise the limit values. Students could not integrate $\int e^t dt$. According to these authors, errors and misconceptions made by students in calculus, result from weak background of mathematics.

Pham (2012) argues that student performance should be assessed before, during, and after the instructional process to measure student's progress and to ensure if lesson objectives are achieved. Pham (2012) indicates that the adaptation of instruction requires teachers to involve careful assessment and planning for all students in the classroom, as well as the ability to select from a range of strategies to find the optimal match to the context. Teachers should also adapt instruction to meet the needs of students who either perform highly or lowly or those who will certainly make progress if they had received different types of presentation.

1.6 Errors and the Teaching of calculus

Tall (1993) indicates that in other countries like America, students usually meet calculus for the first time at college where most traditional calculus books include all the topics an instructor might wish to teach, with a large number of worked out examples and exercises to satisfy the most anxious student. This is similar with South Africa where procedural exercises produced high failure rates. This researcher further indicates that in Canada, school curriculum builders began investigating the use of computers in mathematics in general and later focused in calculus in particular. The reason was to improve understanding of calculus. According to Tall (1993), calculus is at the forefront of curriculum reform in mathematics with its vigour renewed by the advent of the computer. Mathematicians are discovering their zest for experiment and adventure and are passing on their enthusiasms to their students.

Luneta and Makonye (2010) indicate that the most crucial courses in higher education are calculus driven and further indicate that in South Africa where skills development is at the heart of producing artisans, it is observed that teachers and learners in schools register many challenges in dealing with this topic. In order that South Africa realizes her goal of producing artisans in TVET Colleges, students must understand Mathematics N4 as it is introductory and foundational into TVET mathematics programs.

In the process of studying mathematics as indicated in previous sections, TVET College students commit so many errors and display misconceptions in their attempts to learn calculus. Nesher (1987) points out those errors do not occur randomly but originate in a consistent conceptual frame-work based on earlier acquired knowledge. This researcher believes that any future instructional theory will have to change its perspective from condemning errors into one that seeks them. A good instructional program will have to predict types of errors and purposely allow them to occur. According to Nesher (1987), the road to a state of expertise is paved with errors and misconceptions where each error has the potential to become a significant milestone in the learning. Errors and misconceptions must be welcomed. Ryan and McCrae (2005) indicate that the knowledge of common mathematical errors and misconceptions of children can provide teachers with an insight into student thinking and a focus for teaching and learning of mathematics. A social constructivist view of learning suggests that errors and misconceptions are suitable for classroom consideration via discussion, justification, persuasion and even change of mind, so that it is the student who reorganises their own conceptions.

It is true that most tertiary institutions require a minimum level of school mathematics achievement in their admission procedures for applicants, however, such levels do not provide finer details about subject matter knowledge. TVET Colleges admit students who passed Mathematics and Physical Science with 40% or higher pass mark. These students did calculus at a shallow level in their matric lessons as per the curriculum. They enrol at a TVET College with the desire to become artisans. They find themselves doing a heavy calculus course in the introductory N4 Mathematics. N4 Mathematics is somehow a determining factor for students' success into realizing their dreams of becoming artisans. Most students are not copying with this subject. It is mentioned that lecturers use old and ineffective methods of teaching and students' lack of pre-requisite knowledge of calculus. Students at this level of study are experiencing integration for the first time. They did shallow differentiation which results in them struggling to transit from differentiation to integration. In their attempts to understand this calculus, they commit errors and misconceptions.

1.7 ICT tools in teaching and learning of calculus.

ICT stands for Information and Communications Technology or technologies. ICT tools enable modern computing in different types of devices, network components, applications, internet, digital media etc. to allow people to interact in the whole world. Noor-Ul-Amin (2013) explains ICT tools as the capacity to store and retrieve information using different devices.

All challenges reported above come into existence mainly from the manner in which teachers approach the teaching of calculus. Information Communication Technology (ICT) is the potential solution available to assist both teachers and students to understand calculus.

Horani and Daniel (2005) show that the use of ICT in teaching Mathematics makes teaching process more effective as well as enhance the students' capabilities in understanding basic concepts. Keong et al. (2005) agrees that ICT promotes greater collaboration among students and encourages communication and the sharing of knowledge. According to these authors, ICT gives rapid and accurate feedbacks to students and this contributes towards positive motivation which allows students to focus on strategies and interpretations of answers rather than spend time on tedious computational calculations.

Keong et al. (2005) further indicate that ICTs support constructivist pedagogy, where students use technology to explore and reach understanding of mathematical concepts. This approach promotes higher order thinking and better problem solving strategies. These authors indicate that teachers must be computer literate in order to use ICTs effectively to achieve good results.

ICT is used as an intervention strategy into the study of calculus as a way to eliminate the challenges TVET N4 Mathematics have. Keong et al. (2005) discovered that advances in technology have created many new opportunities in teaching mathematics and therefore there is a need for more research into the effect of technology in higher education.

This is exactly where the study fits in to determine how technology can best be used to address challenges TVET College N4 Mathematics students are experiencing when learning calculus. The findings by these researchers clearly indicate that ICTs will definitely change the situation for better. South Africa with its unique context of socio economic factors and disadvantaged communities may yield different results which may be helpful to TVET students.

1.8 Statement of the Problem

Most TVET N4 Mathematics students find the study of Mathematics topics too difficult. Students who enrol at a TVET College have the intentions of becoming artisans in future. The need for artisans is high in South Africa and TVET Colleges are mandated by government to train and produce artisans. Students need to know how to communicate their ideas as expected by the profession and also be equipped with analytical skills. All these can be achieved if students pass mathematics as one of the major subjects in the acquired course. If N4 Mathematics is well mastered by students, chances are that they will pass all the way to the exit level with good mathematics marks. The current situation is hopeless because these N4 Mathematics students fail their final examinations dismally. The technical mathematics involved here is mainly calculus driven. It is obvious that when students fail, it is because they are not answering questions correctly. Given the fact that the syllabus is mainly calculus, this makes me to conclude that on answering calculus questions, students commit errors. Students` low performance is attributed mainly to the gap that exists between the two level of study, i.e. matric calculus and college calculus. Matric calculus is shallow while the college calculus is deep. It is not easy for students to get their grip so quick taking into consideration the way they were taught at secondary schools by teachers and the way they are taught at college by lecturers. This is a huge problem and it affects almost everyone who studies mathematics/calculus on a higher level.

1.9 Purpose of Research

The study is pinned on constructivist theories and related frameworks to explore and find errors and misconceptions in TVET N4 Mathematics students learning about calculus and in particular the definite integral. This study agrees with Nesher (1987) that a good instructional program must predict the types of errors and purposely allow for such errors to happen. Part of the aim of the research study is to establish the basic pre-requisite calculus knowledge that TVET N4 Mathematics have, just before learning integral calculus. This study also focuses on the teaching methods lecturers apply when teaching calculus. The purpose of this study is to find types of errors and misconceptions TVET N4 Mathematics students do in learning about the integral calculus. This is where the basic pre-requisite knowledge about calculus will be established. The other aim is to determine how the Information Technological Communication (ICTs) can be used to improve teaching and understanding of calculus. This study wants to align itself with the idea of Nesher (1987) that teaching that has no intentions of detecting errors and misconceptions is like shooting in the dark where you cannot see your target. The ultimate goal of this research study is to determine the extent to which ICTs mediated teaching and learning advance mathematics learning for TVET students.

1.10 Research Questions

Considering the deliberations made above, the study wants to articulate the following research questions:

1. What prior knowledge of concepts do students bring when they enroll for a TVET N4 program?
2. How does ICTs mediation help to advance teaching integration?
3. How can students` understanding of the definite integral in TVET N4 calculus be improved by using ICTs.

1.11 Significance of the Study

It is true that in a situation where a student understands a particular mathematics topic, the level of confidence rises and this student is eager to participate in group discussions and therefore learn new things most correctly. Oliver (1989) indicates that errors and misconceptions form part of the learning process and teachers must create classroom atmosphere which is tolerant of errors and misconceptions in enhancing learning.

The study wants to highlight that students are making mistakes because they are making efforts to cope with mathematics taught in class. According to Olivier (1989), correct new learning depends on correct previous learning and similarly incorrect new learning sometimes depends on incorrect or incomplete previous learning. Teachers should therefore welcome errors and misconceptions as the normal way of students` effort to construct their own knowledge and these should not be avoided.

Students with less errors and misconceptions are likely to grow rich in conceptual knowledge which is rich in making connections, Hiebert and Lefevre (1986). Students with conceptual knowledge will therefore be able to make sense of mathematics in and outside the classroom situation. Luneta and Makonye (2010) argue that calculus errors and misconceptions at times result when learners fail to build procedures from conceptual knowledge.

It is significant to know what error types and misconceptions learners have in learning or performing a particular task. Luneta and Makonye (2010) further argue that errors and misconceptions emanate from prior knowledge as learners attempt to construct mathematical meanings. These authors believe that errors and misconceptions are a result of naive concept images that do not measure up to the concept definitions.

This study raises awareness on the importance of errors and misconceptions in a teaching and learning environment.

1.12 Conclusion

This chapter gives the context as well as the background of the study. It briefly explains mathematics and introduces calculus and further indicates how most researchers acknowledge the difficulty of this subject. I indicated that in other countries including South Africa, students study calculus for the first time in tertiary institutions. The traditional ways and methods of teaching and learning mathematics and calculus in particular, confuse students. This section also highlights the superficial level at which calculus is done at matric level in South Africa. I indicated in the TVET College where I work, lecturers do not use ICTs. Lecturers are fond of old traditional ways and methods of teaching mathematics. The introduction paints the picture of the current state of affairs in a TVET college where the research was conducted.

CHAPTER TWO: THEORETICAL FRAMEWORK

2.0 Introduction

This chapter outlines the main theories used in the teaching and learning of mathematics. The chapter starts by giving the two contradicting theories in the teaching and learning of mathematics which are behaviourism and constructivism. Brief explanations of the two mentioned theories are given but more emphasis is placed on constructivism because the research study is pinned on it. There are many theories in the teaching and learning of mathematics developed by researchers and relevant constructivists' theories and frameworks are highlighted and explained.

Fosnot and Perry (1996) describe behaviourism as a doctrine that explains learning as a system of behavioural responses to physical stimuli. Behaviourism theory concerns itself with the effects of reinforcement and external motivation and behaviours. Behavioural change is evidence that learning took place. Fosnot and Perry (1996) further indicate that behaviourists break a large area of study into smaller parts and pre-plan the parts into hierarchical order of simple to difficult areas of study. This theory considers students as passive participants who are in need of external motivation to learn. The main responsibility of teachers is to spend time in developing an orderly, well-structured curriculum. Teachers should as well determine how they will assess, motivate, reinforce, and evaluate the students. This teaching model is the same as splitting the whole object into parts and later assemble the whole parts to sum up to the initial object. In the same way a skill is divided into smaller skills which are given to students during teaching and learning process. At the end of the program, students are expected to have accumulated and mastered all smaller skills to add up the whole skill. Fosnot and Perry (1996) argue that behaviourism focuses on changing behaviour but offers no indication on the structural cognitive change in learners.

Unlike behaviourism, constructivism approaches teaching and learning of mathematics quiet differently. Fosnot and Perry (1996) indicate that the focus of constructivism is on the students' cognitive development and higher level of thinking and understanding. The key ideas in constructivism include: `schemata` which means the cognitive structures with pre- conceived ideas of the world. For example, a schema of my Sunday among others may be: waking up early, going to church, preparing a nice meal in the afternoon, preparing for work for the coming Monday, going to sleep early. Another person might have a completely different schema for a Sunday: waking up late, going to a restaurant for a breakfast, going to watch a movie until late, going home and sleeping early for going to work on a Monday. We therefore learn that there are different schemas for all kinds of activities.

The other constructivist key concepts are assimilation and accommodation. Assimilation is when students receive information which fits into their existing schemas and accommodation is when the information does not fit into the existing schemas (Piaget, 1964). These two learning processes, assimilation and accommodation, work together and lead to adaptation of our schemata to make accurate picture of the world. The last key concept is called equilibration, which is a process that strives to achieve cognitive balance between old/prior and new knowledge learnt. Other grand theories as well as related frameworks are explained in this chapter.

2.1. Piaget`s Theory

Piagetian constructivism concentrates on internal cognitive processes and does not consider the social aspect of the learner. The Piagetian equilibration theory focuses on two distinct learning processes of assimilation and accommodation. According to Piaget (1964) students are continually engaged in the process of adaptation of information in their schemata. This means that information is either assimilated or accommodated in the students` schemas. Piaget (1964) argues that if new information does not get assimilated nor accommodated, a cognitive conflict arises. It follows that the continuous organisation and re-organisation of information by assimilation and accommodation processes, lead to effective knowledge (cognitive development). Equilibration is like an engine that propels the construction of knowledge and by so doing creating opportunities for mathematics knowledge to take place. However there is a need to consider social aspects as Ernest (1996) argues that students are not passive recipients of knowledge but are actively constructing their own knowledge (Piaget, 1964).

Assimilation and accommodation processes as explained, make Piaget`s theory relevant to my research study. It is true that if students cannot do both processes then no learning may take place.

2.2 Types of constructivism

Researchers who read Piaget`s theory developed it further by considering the social aspects involved in teaching and learning of mathematics. Ernest (1996) distinguishes four different types of constructivism i.e. information processing theory, weak constructivism, radical constructivism and lastly the social constructivism. All constructivist types describe the process of understanding as the building up of mental structures. These types of constructivism are explained below.

2.2.1 Information Processing Theory

Ernest (1994) describes Information Processing Theory (IPT) as the simplest form of constructivism as it only accepts Von Glaserfeld's first principle of constructivism that knowledge is not passively received but actively built by the cognizing subject. Ernest (1994) indicates that IPT treats the mind as a computer which processes information and data, calling up various routines and procedures, organizing memorization and retrieving stored data.

According to Ernest (1994), the mind is not passive but functions like a complex electronic computer, Ernest (1994) also argues that IPT is the best learning theory that accounts for students' error patterns in mathematics. IPT is the study of how humans process information including acquisition, storage and retrieval of knowledge (Ernest, 194). In contrary to Ernest (1994), Mayer (1982) argues that IPT cannot be a form of constructivism because some knowledge learned by human beings is information admitted from outside and not constructed within, therefore this renders IPT not to be a form of constructivism. IPT`s ability of calling up routines and procedures is relevant to this study of calculus as it will improve procedural knowledge of students.

2.2.2 Weak constructivism

Ernest (1994) indicates that weak constructivism accepts Von Glaserfeld`s first principle of constructivism. Von Glaserfeld`s first principle states that knowledge is not passively received but actively built by the cognizing subject. The mind is seen as a soft computer (the brain) where data is processed and is self-constructed (Ernest, 1994). He further explains that, weak constructivism poses challenges in that if knowledge is individually and personally constructed, how can the truths of mathematics or facts about the world be known by any individual? This statement renders weak constructivism not to be relevant to my study. Constructed truths can only be known by means of information from the world. Weak constructivism is more local as opposed to global paradigm because it accounts for knowledge representations of individuals.

2.2.3 Radical constructivism

Radical constructivism embraces both first and second Von Glaserfeld `s principles of constructivism (Ernest, 1994). The second principle states that the function of cognition is adaptive and serves the organization of the experiential world. Radical constructivism takes the position that there is no ultimate true knowledge possible about the state of affairs in the world (fallibility). According to Ernest (1994), all constructed knowledge is radically constructed by individuals on the basis of cognitive process in dialogue with the experiential world. My research study supports the ‘fallibilist” position on the teaching and learning of mathematics. A fallibilist is someone who believes that mathematics knowledge is humanly constructed and as a result has some errors in it. Ernest (1995) indicates that mathematical knowledge fallibility and openness to revision, is in its proofs and concepts. Fallibilism rejects the absolutist view of mathematics. An absolutist is a person who believes that mathematics knowledge is beyond error and correction. According to Ernest (1995), absolutist -view supports teachers in giving routine mathematical tasks which promote the application of learnt procedures.

2.2.4 Social constructivism

Ernest (1994) indicates that social construction regards individual subjects and the realm of the social as individually interconnected as human subjects are formed through their interactions with each other. According to Ernest (1994), the mind is seen as part of a broader context of the social construction of meaning. It gives pride to place human beings and their language in the account of knowing. This researcher argues that social constructivism is about conventional knowledge which is lived and socially accepted. Language is at the centre of social construction. This type of constructivism is relevant to my study. Students need to learn together and help each other along the academic trajectory.

2.3 Concept image and concept definitions

Tall and Vinner (1981) introduce the two concepts “image and definition” in the teaching and learning of mathematics. These researchers explain the concept image as the total cognitive structures associated with a concept. The concepts `image and definitions` built over a period of time from different experiences. When an individual meets new stimuli, both concepts `image and definition` are affected individually and may be forced to change. Tall and Vinner (1981) indicate that the mental attributes linked to a particular concept may become a source for future conflict because the development of the concept image and concept definition is not coherent at all times. According to Tall and Vinner (1981), concept definition is words used to specify a concept and may be learnt by rote fashion or more meaningfully. Personal concept definition can differ from a formal concept definition. Students should always strive to give formal concept definitions by improving their personal concept images and concept definitions.

2.4 Vygotsky`s Social Cultural Theory

The study also takes a Vygotskian approach because of the belief that the world is socially constructed. Vygotsky (1930) indicates that human beings developed psychological tools to master their own behaviour. Karpov (2007) indicates that all human mental processes are mediated by these psychological tools. According to Vygotsky (1930), these tools are called signs and are used to aid thinking and behaviour. This researcher indicates that tools can only be fully understood through use. Vygotsky (1930) argues that it is possible to acquire a tool but fail to use it. It is indicated that speech is the most important sign system and it covers language used, writing covers reading and the number system is for quantifying objects. Vygotsky (1930) indicates that the mastery of reading and writing, the highest levels of thinking and abstract thinking or theoretical reasoning requires formal instruction. According to Vygotsky (1930), formal instruction makes students to gain awareness of the concepts and then put them to deliberate use. Daniels (2009) indicates that instruction is unnecessary if utilized for what has already matured in development. He argues that instruction should be the source of development.

Vygotsky (1930) argues that the student`s true potential can only be shown with the Zone of Proximal Development (ZPD) which means what students can do with little help. He therefore, argues that the ZPD gives an idea of those abilities that have matured and those that are in the process of maturing. Vygotsky (1930)`s ZPD means that by focusing on activities that students can accomplish with assistance, will reveal the abilities that are just beginning to develop. This means that the activities that students can do today with assistance, are the ones the students will be able to do successfully tomorrow without any assistance. A student is therefore said to have developed or learned something only when this student is able to do something new he/she could not do before. The teacher's most important role is to give assistance and support to the developing mind of the student.

Vygotsky (1930) distinguishes between scientific knowledge and spontaneous or everyday knowledge. Spontaneous knowledge refers to things that students learn on their own outside of school in their everyday lives. Scientific knowledge refers to knowledge attained through formal instruction at school. According to Davydov (1986) cited in Karpov (2007), spontaneous knowledge often leads to misconceptions. Karpov (2007) argues that traditional methods of teaching and learning, brought rote skills and verbal definitions rather than true scientific concepts. Mathematics teachers and lecturers need to provide support to students if they want to improve understanding of calculus on the part of students.

2.5 Dubinsky`s APOS Theory

Dubinsky and McDonald (2001) designed a powerful APOS theory with an attempt to understand Piaget`s development of logical thinking in learners. These researchers extended Piaget`s idea to deepen understanding of advanced mathematical concepts. APOS theory comes with four components arranged in a hierarchical order, i.e. A for Actions, P for processes, O for objects and S for schema. In some sense each conception in the list must be constructed before the next step if possible. However, in reality, when a student is developing her or his understanding of a concept, the constructions are not made in linear order.

Luneta and Makonye (2010) explain the APOS theory as formation of concepts which first occurs as an activity, then a process, an object and lastly a schema. Luneta and Makonye (2010) indicate that in learning a mathematical concept, it first occurs as an activity to which learners attach little meaning, then the leaners view the concept as a process or procedure, thirdly the learners view the concept as an object internalised in the mind where lastly the object is incorporated in the learner`s schema. According to these researchers, doing mathematics should be dominated by activities and processes which should come before formation of mathematical concepts. Doing mathematical tasks involves conceptualising mathematical ideas which must be taught in context first as it helps learners to develop mathematical meanings before learning procedures. Luneta and Makonye (2010) further indicate that doing mathematics provides learners with physical and mental objects upon which mathematical meanings and concepts are imposed. These researchers indicate that once mathematical concepts are abstracted from reality through actions and processes, learners can hold them as generic mental objects which can apply to other practical and theoretical positions differently from concrete situation that bore them.

Tzur and Simon (2004) explains APOS theory`s A for action as any repeatable physical or mental manipulation that transforms objects. According to these researchers, an action requires a definite recipe, which the subject is aware of following. Tzur and Simon (2004) argue that the steps of the algorithm and the objects at all phases of the action must be present, either physically or in the imagination of the subject.

Tzur and Simon (2004) indicate that when the need for an explicit algorithm becomes less necessary and when the total action can take place entirely in the mind of the subject or can be imagined as happening, without following all of the specific steps, it means the action has been interiorized and it can now become a process.

These researchers further indicate that when it becomes possible for a process to be changed by action, it has been encapsulated to become an object. According to this theoretical view, there is only one way to make mathematical objects and that is by encapsulating a process. In many mathematical situations, it is important to go from object back to a process by de-encapsulating the object.

2.6 Sfard`s Theory

Sfard (1998, 1991) explains a learning theory that emerges from two sources of insight, i.e. observation in students' learning and investigating the historical developments of the concept. Sfard (1991) specified her object of study by using conception to refer to an individual's understanding and concept to refer to an agreed upon understanding that mature members of mathematical communities use. Sfard (1991) distinguished between two facets of mathematical understanding: a process (operational) and an object (structural).

Tzur and Simon (2004) points to Sfard`s theory as a reification theory distinguished into three phases, i.e. structuralization, interiorization and condensation. The structuralization is about stages in the development of a new concept, interiorization is when learners become familiar with a process where operations performed on lower-level mathematical objects will eventually give rise to a new concept. Lastly, condensation is when learners squeeze sequences of operations into more manageable units and thus become capable of thinking about a given process as a whole, including giving that condensed whole a name. Condensation is considered to be the birth of a new conception and lasts as long as the new entity is tightly connected to a certain process.

2.7 Ball`s Mathematical Content Knowledge (MCK) Framework

There will not be any teaching and learning if teachers do not have the knowledge of the subject they are supposed to teach. Ball (2008) indicates teachers must definitely know the subject they teach because if they do not know it well, they are unlikely to have the information they need to help students learn. This researcher further argues that just knowing the subject well is not good enough for teaching as teachers need to know mathematics in ways useful for making mathematical sense of students work and choosing powerful ways representing the subject so that it is understandable to students. It is further emphasised that content knowledge is immensely important to teaching and its improvement. According to Ball (2008), content knowledge is categorized into three specific dimensions: content knowledge, curricular knowledge and pedagogic content knowledge.

Content knowledge is the knowledge of the subject and its organizing structures which goes beyond just knowing facts and concepts. According to Ball (2008), teachers must understand the organizing principles and structures and the rules for establishing what is legitimate to do and say in a field. This also means that teachers need to understand that something is this way and that way and also know why it is this way and further know on what grounds its warrant can be asserted and furthermore know under what circumstances our belief in its influence can be weakened or denied.

Curricular knowledge refers to the representation of the full range of programs designed for the teaching of particular subjects and topic at a given level, the variety of instrumental materials available in relation to the programs and the set of characteristics that serve as both indications and contraindications for the use of particular curricular program materials in particular circumstances.

Pedagogic content knowledge refers to the worst useful forms of representation of ideas, powerful analogies, illustrations, examples, explanations and demonstrations in a word, the most useful ways of representing and formulating the subject that make it comprehensible to others.

2.8 Kilpatrick`s strands of Mathematical proficiency

Kilpatrick, Swafford and Findell (2001) came up with a comprehensive view of how students can learn mathematics successfully. Their view is aimed at promoting cognitive change in learners in order for them to be successful in learning mathematics. They argue that mathematics proficiency should consist of five components of conceptual understanding, procedural fluency, strategic competence, adaptive reasoning and productive disposition. Kilpatrick et al. (2001) indicate the following that:

Conceptual understanding refers to an integrated and functional grasp of ideas with no isolated facts or ideas and knowledge is a coherent whole. Students with conceptual understanding are able to verbalize connections amongst concepts and representations. It helps students not to make errors.

Procedural fluency is the knowledge of procedures, knowing when and how to use a particular rule in solving problems. It is about the skill of doing calculations accurately and efficiently.

Strategic competence is the ability to formulate mathematical problems, represent them and solve them.

Adaptive reasoning is the ability to think logically about the relationships amongst concepts and situations. This is seen as the glue that holds everything together and the lodestar that guides learning. It is a way of navigating through concepts, facts, solutions etc. to see if all make sense.

Productive disposition is seeing sense in doing Mathematics to the extent that one perceives it as both useful and worthwhile.

2.9 Conclusion

This section outlined the main theories and related frameworks in the teaching and learning of mathematics. The study is framed and guided by these theories and frameworks explained in this chapter. All theories and frameworks explained above, have one good thing in common and that is – they have a room to allow for learners to commit errors and misconceptions. The kind of teaching and learning atmosphere allowed by these theories is that learning should be learner- centred where the teacher should be aware that learners construct their learning in an active way.

We have seen how Tall and Vinner' s (1981) concepts, cognitive structures and mental pictures and the associated processes, are built over time with all kinds of experiences and change when students meet new stimuli. This change happens according to Piaget`s assimilation and accommodation processes. All constructivists acknowledge that learning does not happen passively on the part of the learner. Vygotsky (1930) indicates that language plays a significant role in the teaching and learning situations. His scientific knowledge is acquired through formal instruction and this instruction happens mainly through language. Dubinsky (2001) and Sfard (1991) theories indicate how learners come to develop mathematical concepts in their learning of mathematics.

Kilpatrick et al. (2001) strands of mathematical proficiency gave us something to think about if we want our students to be more proficient in learning mathematics. Finally, for teachers to achieve all these, they should have content knowledge, curricular knowledge lastly the pedagogical knowledge.

CHAPTER THREE: LITERATURE REVIEW

3.0 Introduction

The researchers of mathematics education have over the past decades grappled with the idea of trying to understand the errors and misconceptions done in the teaching and learning of Mathematics. Most of these researchers e.g. Luneta and Makonye (2010), Nesher (1987), Olivier (1989) etc. agree that errors and misconceptions are important, helpful and should not be ignored in the teaching and learning of mathematics.

3.1 Teaching and learning of calculus.

Selden, Selden and Mason (1994) conducted a research study on the teaching and learning of calculus with students from a calculus course with the aim of finding out whether A and B students (bright students) would perform better when confronted with non-routine/novel problems. The study also intended to find out on the level of routine calculus skills as well as the additional abilities in solving non-routine calculus problems encountered in mathematics. The study found that students committed many conceptual errors, they confused algebra with calculus, could not deal with differentiation, could not draw mathematical sketches correctly, and could not deal with equations and manipulations and finally displayed lots of misconceptions. Example of a routine test question asked in the research study:

a) $What\ is\ the\ slope\ of\ the\ line\ tangent\ to\ y = x^2\ at\ x = 1$?

b) $At\ what\ point\ does\ that\ tangent\ line\ touch\ the\ graph\ of\ y = x^2$.

As for the non-routing test questions, the following example was asked:

$Find\ values\ of\ `a`\ and\ `b`\ so\ that\ the\ line\ 2x + 3y = a\ \ is\ a\ tangent\ to\ the\ graph\ of$:

$f(x) = bx^2\ at\ a\ point\ x = 3$.

Selden et al. (1994, p.10) found that even good students from traditionally taught calculus classes cannot solve non-routine problems. The study revealed that many students preferred arithmetic and algebraic techniques in solving calculus problems and many made no use whatever of calculus. These researchers suggest that better teaching techniques should be developed to improve students` ability to solve calculus problems. The study by Selden et al. (1994) indicates that the traditional way of teaching calculus is in itself a problem and it is dominated by lots of errors and misconceptions. From this study it is clear that better methods of teaching calculus should be found.

Naidoo (2007) conducted a study on calculus and found that students can differentiate complicated functions analytically but cannot interpret differentiation graphically. This author suggests that students need to learn through discovery, visualisation and experimentation. On the other hand, Orton (1983) also conducted a study with the aim of investigating the understanding of differentiation by students at high school and training college students and found that students had little intuitive understanding as well as fundamental misconceptions about the derivative.

Students seem not to be interested in understanding what a derivative is but instead they are fond of mastering procedures on how to determine these derivatives. Students are keen of acquiring more of procedural knowledge in the expense of the conceptual knowledge.

White (1996) conducted a research study with students enrolled in a university calculus class and found that students have superficial and incomplete understanding of many basic concepts in calculus. According to this author, this situation is attributed to the rote and manipulative learning that takes place in an introductory course. Tall (1993) argues that whichever way the calculus is approached, there seem to be inherently difficult concepts which seem to cause problems no matter how they are taught. The limit concept creates a number of cognitive difficulties, including also difficulties embodied in the language. This author believes that when students meet difficulties, a dominant strategy for coping is to concentrate on the procedural aspects that are usually set in examinations.

Tall (1993) indicates that the reason why students find calculus very difficult is because students engage in calculations that are no longer performed by simple arithmetic and algebra.

According to Tall (1993), teachers often attempt to bypass the problems by using an informal approach method which have caused a general dissatisfaction with the calculus course in various countries round the world in the last decade.

Tall (1993) further argues that almost all possible ways of teaching calculus lead to inherently difficult concepts which seem to cause problems no matter how they are taught. The example of a limit concept is evident in creating a number of cognitive difficulties some of which are embodied in language, e.g. limit, tends to, approaches etc.

According to Tall (1993), the difficulties in understanding calculus are possibly caused by restricted mental images of functions, the Leibniz notation, which helps in addressing difficulties in translating real-world problems into calculus formulation. One of the crucial reason for students battling with calculus is that students prefer procedural methods rather than conceptual understanding. Lagrange (1999) also advocates for the graphic representations in the study of calculus. This author argues that diagrams can make it possible for teachers to guide students into novel experiences by drawing on students' prior knowledge and experience. Students interpreting mathematical meaning in these activities would form increasingly sophisticated mathematical conceptions.

Clement, Lochhead and Monk (1981) indicate that students have difficulty expressing relationships algebraically when learning about calculus problems as they cannot translate reliably between algebra and other symbol systems. According to these authors, students must learn to apply mathematics and translate a problem usually expressed in words into algebraic notation and retranslate a solution back into words

3.2 The use of ICT in the teaching of calculus

De Ting Wu (2004) advocates that computer technology is a powerful tool and a helpful aid in teaching and learning mathematics. According to this researcher, ICTs and its components of computation, visualization, and animation is helpful in developing new approaches to the teaching of calculus and ultimately to help students to overcome their difficulties in understanding this important concept. Computers enhance the study of mathematics thereby improving learners` interest and increasing motivation. When learners see videos and animations, they become so motivated to learn.

Tall (1993) indicates that various hypotheses have been put forward suggesting ways in which students' understanding of calculus might be improved. One of the ways in which understanding of calculus may be understood by students is by using computers. According to Tall (1993), a computer is a constructive activity that allows the student to learn by telling the computer how to carry out the required algorithms. In an activity conducted on checking understanding of calculus by Tall (1993), the results showed a significant improvement in the students using the computer over those who did not use the computer.

Noor-Ul-Amin (2013) believes that education is a socially oriented activity and quality education has traditionally been associated with strong teachers having high degrees of personal contact with learners. The use of ICT in education lends itself to more student-centred learning settings. But with the world moving rapidly into digital media and information, the role of ICT in education is becoming more and more important and this will continue to grow into the future. According to this researcher, ICTs do not only refer to computers and computer related activities but to the capacity to store and retrieve information with things like emails, internet service provision, telecommunication equipment and services, media and broadcasting, libraries and documentation centres, commercial information providers, network-based information services. This includes various kinds of ICT tools and products available and having relevance to education like teleconferencing, emails, audio conferencing, television lessons, radio broadcasts, interactive radio counselling, interactive voice response system, audiocassettes, CD ROMs, etc.

Noor-Ul-Amin (2013) argues that ICTs have the potential to innovate, accelerate, enrich, and deepen skills, to motivate and engage students, to help relate school experience to work practices, create economic viability for tomorrow's workers, as well as strengthening teaching and helping schools change. ICTs can be used to explore new opportunities for changing classroom practice. They are more facilitative in nature and they encourage and support independent learning because current learning theories are based on the notion that learning is an active process of constructing knowledge rather than acquiring knowledge and that instruction is the process by which this knowledge construction is supported rather than a process of knowledge transmission. In this domain, learning is viewed as the construction of meaning rather than as the memorisation of facts. Learning approaches using ICTs provide many opportunities for constructivist learning through their provision and support for resource-based, student- centred settings and by enabling learning to be related to context and to practice.

Noor-Ul-Amin (2013) enlightens us that ICTs play a major role on how the students should learn and has shifted the mode of curricula from teacher-centred forms of delivery to students-centred forms of delivery and by so doing addressing issues of motivation as students use videos, television and multimedia computer software that combine text, sound, and colourful moving images which can be used to provide challenging and authentic content that will engage the student in the learning process. It then follows that ICTs induce in students the interest to listen and become more involved in the lessons being delivered. Learning becomes more effective than the monotonous traditional classroom situations where the teacher just talks from a raised platform and the students just listen to the teacher. Noor-Ul-Amin (2013) indicates three main advantages of using ICTs in teaching. Firstly, images can be used to improve retentive memory in students, secondly ICTs make it possible for teachers to explain complex instructions and unsure students` comprehension and lastly teachers are able to create interactive classes and make lessons more enjoyable which then improves students' attendance and concentration.

3.3 ICT Mediation

The socio-cultural framework as mainly inspired by Vygotsky suggests the necessity within educational activities to interweave teaching and learning processes. Vygotsky (1978) argues that the cultural-historical framework is centred on educational activity in its entirety. This researcher highlights the crucial importance of tools and the role of the teacher. In Vygotsky`s terms, the tools are seen as mediators of the individual's actions and of the communication that takes place between participants in the activity where else on the other hand; the teacher is seen as a co-participant in the activity who gives assistance to the students' performance. Vygotsky (1978) distinguishes between two types of mediation tools for each human activity, i.e. the technical tools as well as the psychological tools. The technical tools, mediate the interaction of the individual with the outside world and brings about transformation in objects as it produces external effects on the object. On the other hand, the psychological tools are directed internally towards the mastery or control of one`s own processes or behaviour. Vygotsky (1978) indicates that the development of the higher psychological functions involved in all processes of comprehension and meaning construction, is the result of an integration of technical and psychological tools in the activity.

In taking the work of Vygotsky forward, Chiappini, Pedemonte and Robotti (2003) argue that technological tools assume a crucial role in supporting teaching and learning processes. These tools create activity settings where students can interact with mathematical knowledge and start to re-configure mathematical abstract objects in order to make them more concrete with the aim of using these objects in the student`s ZPD, where learning occurs and teaching is defined. Chiappini et al. (2003) strongly argue that a student`s ZPD is characterized by a strict connection between teaching and learning processes and further argue that learning happens with a student`s ZPD.

Chiappini et al. (2003) indicate that it is important to realize that the meaning resides not in the tools but in the aims for which the tool is used and the plans that are developed for using it. These plans are expressed in socially shared language, gradually realised in the course of the activity with the tool, constitute the meaning, in that they reflect the course of the activity mediated by the tool and orientated toward an aim. Chiappini et al. (2003) highlight that the key element in the construction of meaning is the type of activity setting that the teacher creates and manages, so as to maximise opportunities for technology-mediated co-participation and instructional conversation between teacher and student and among the students themselves mediated by technologies.

Aberšek and Aberšek (2010) found that ICTs create an information-rich learning environment in which students and teachers can explore the use of on-line knowledge bases, and information superhighways. These researchers believe that the school of 21st century will not have human teachers as they will be substituted by the use of ICTs.

3.4 Errors and misconceptions in the teaching and learning of Mathematics

Olivier (1989) defines errors as wrong answers due to planning which are systematic in that they are applied regularly in the same circumstances and considers misconceptions as underlying conceptual structures that give rise to errors. According to Olivier (1989), slips are wrong answers which happen carelessly and are quickly detected and corrected spontaneously. Slips are unavoidable and can be done by both novices and professors. Olivier (1989) indicates that errors are indicators of the existence of misconceptions. Olivier (1989) acknowledges that errors done in the course of teaching and learning mathematics are functions of cognitive variables, teachers, the curriculum, the social factors, affective factors, emotional factors, motivation, attitudes etc. Olivier (1989) argues that teachers cannot transmit knowledge ready and intact because errors and misconceptions are student`s efforts to intelligently construct their own knowledge. Misconceptions should therefore not be uprooted and avoided because they are part of the learning process and should be welcomed by both teachers and students.

Donaldson (1963) distinguishes between three types of errors in the teaching and learning of mathematics, i.e. Structural, Arbitrary and Executive errors. According to Donaldson (1963), structural errors arise from failure to grasp some principles or certain rule which is essential to finding solution, arbitrary errors are said to be those which behave arbitrarily and fail to take account of the constraints laid down in what is given e.g. Arbitrary errors could be caused by learners wanting questions to fit and suit what they know or are familiar with and lastly executive errors involve failure to carry out manipulations or procedures even though the required concepts have been understood.

Eisenhart, Borko, Underhill, Brown, Jones and Agard (1993) describe two types of errors which are procedural errors and conceptual errors. According to these researchers, procedural errors are associated with procedural knowledge and refers to mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms and conceptual errors are associated with conceptual knowledge and refers to knowledge of the underlying structure of mathematics which is about the relationships and interconnections of ideas that explain and give meaning to mathematical procedures.

Radatz (1979) classifies three error types. The first type of errors are as a result of lack of mastery of language used, the second are as a result of difficulties in obtaining visual information and lastly those that arise because of lack of prerequisite knowledge and skills.

The issue of language has also been raised by Mofolo-Mbokane (2013) who discovered that students do not possess a sufficient mathematics register, due to their poor mathematics background. Sarwadi and Shahrill (2014) indicate that if a learner is unable to assimilate and accommodate according to Piaget (1964), a gap is created in the learning of the concept and this in turn leads to errors and misconceptions. They advise that errors and misconceptions must not be seen as obstacles or 'dead ends', but must be regarded as an opportunity to reflect and learn. Teachers should therefore prescribe appropriate instructional strategies to be more diagnostically oriented in order to avoid any subsequent major conceptual problems.

It then follows that to know error types and misconceptions is important because it makes teachers know what they are dealing with.

Luneta and Makonye (2010) found that low learner achievement in calculus seems to be mainly due to knowledge gaps in algebra that forecloses substantial calculus epistemic access. Their argument is that algebra is the language of calculus. These authors indicate that difficulties encountered in the understanding of calculus are also due to learners' lack of knowledge in underlying calculus concepts, in which calculus techniques are instrumentally understood. These researchers also found that learners sometimes had good calculus concepts but weak mastery of algebra.

Luneta and Makonye (2010) further found that learners displayed insufficient knowledge of calculus terminology as well as difficulties due to non-conflicting but parallel calculus conceptual knowledge.

These researchers believe that for noted errors and misconceptions there is structure in the misconceptions learners have and that these misconceptions emanate from prior acknowledge as learners attempt to construct mathematical meanings. Luneta and Makonye (2010) further believe that learners' errors are therefore a result of naïve concept images that do not measure up to the concept definitions.

Muzangwa and Chifamba (2012) indicate that some learners had a weak background of mathematics such as a low pass at lower levels and yet most pre-calculus concepts such as algebra, limits, basic differentiation and integration have a strong link with high school mathematics. These authors found that misconceptions are a result of poor understanding of the basic themes of calculus like limits of functions and their representations. According to these researchers, some difficulties and misconceptions in calculus are as a result of teaching approach that emphasises to a larger extend procedural aspects of the calculus and neglects a solid grounding in the underpinnings of calculus.

Kiat (2005) found that students seemed to focus more on the procedural aspects of integration than on the conceptual aspects. According to Kiat (2005), students generally lack both conceptual and procedural understanding of integration.

With a study Kiat (2005) did on calculus, he found that the largest number of errors committed were technical errors which were primarily attributed to the students' lack of pre-calculus knowledge. This author further found that students had problem with the understanding of integration as the limit of a sum, and the relationship between a definite integral and areas under the curve.

According to Kiat (2005), many teachers have accepted the fact that integration cannot be made easy to be understood by students. Kiat (2005) indicates that the other difficulty in students understanding of integration of trigonometric functions which is found with a confusion of mixing up integration with differentiation. This researcher further found that students confused integration with differentiation and some could not recall trigonometric identities which they should introduce before doing integration.

This means that students either forgot the techniques of integrating trigonometric functions or lacked practice in this area. According to Kiat (2005), the lack of mathematical content knowledge in algebra affect the students` ability to integrate problems correctly. This means that students have problems with finding areas or generally in understanding the relationship between a definite integral and areas under the curve.

3.5 Teaching Intervention

Du Preez, Steyn and Owen (2008) conducted a research study in a South African university and found that first year calculus lecturers face a challenge of equipping engineering students with the necessary mathematical skills. It is true that without a solid mathematics foundation at this calculus level, students would find it too difficult to understand and apply knowledge involved in upper engineering classes. According to these researchers, it is of utmost importance to re-think the learning content and facilitation strategies in order to purposefully refocus the mathematical intervention to promote and encourage learning. Du Preez et al. (2008) strongly indicate that mathematics lecturers at this entry point into doing calculus at tertiary institutions should seriously take cognisance of what research findings tell on the importance of mathematical interventions. These researchers further indicate that the research study found that the mathematical experience in the first year has a definite effect on students' perseverance in engineering studies. These authors see this as a social and educational responsibility of the tertiary institution and the lecturing staff to make the most of the educational opportunity represented by a population of students who have the potential of succeeding in engineering.

Code, Piccolo, Kohler and MacLean (2014) indicate that the history of teaching calculus contains many efforts to improve students' conceptual understanding. These researchers found that most examinations focus predominantly on the procedural aspects of calculus in the expense of conceptual understanding. This has led to many challenges as students lacked conceptual knowledge. Their study was on related rates and linear approximation which are topics considered to be most difficult as seen in examination results. These researchers were aware of homework problems and past examination papers given to students. Related Rates, is a section about derivatives and linear approximation is more of graphical representation of the derivative. The intervention strategy was developed by these knowledgeable researchers whose point of departure was knowing the exact areas of difficulties, checking with what expects found and recommended to be done in such instances, knowing the content of the subject, probing questions, ability to link old and new information in the section of study, students given time to talk about their work, their understanding, minding the relationship between conceptual and procedural knowledge. Students were asked to explain their thinking to one another verbally and in writing.

Code et al. (2014) found that most students were able to draw sketches and give the correct related rate after the intervention strategy has been implemented. More students in the intervention section were successful in identifying the given rate and interpreting it as the \`time\` derivative. Few students could not derive an equation as they made mathematical errors somewhere in their work. This is to show that even with intervention, some students will still not improve their understanding, but at least the general picture is not as bad as before.

Tall (1996) indicates that the computer provides a new environment to explore the study of calculus as it significantly offer deeper insights different to a traditional approach. This researcher argues that students using paper and pencil drawings of graphs saw them as geometric shapes rather than a process of inputting x and outputting y. Tall (196) further argues that computer software can be used to represent function concepts graphically, sometimes with a facility to represent them in tabular form.

Ferrini-Mundy and Gaudard (1992) found that it is possible that procedural, technique-oriented secondary school courses in calculus may predispose students to attend more to the procedural aspects of the college or university course. The researchers argue that arriving at college and finding conceptual difficulties in the calculus may lead to students developing short-term techniques for survival. According to the authors, the short term techniques include amongst others what students have learned and invented for themselves (coping skills) so that they are able to pass.

Ferrini-Mundy and Graham (1991) show that the investigation of the student understanding of the concept of function is one of the most fundamental concepts underlying the calculus study. These researchers argue that despite the importance of functions, the students come to calculus with a primitive understanding of the concepts and hold deeply rooted and firm misconceptions. Ferrini-Mundy and Graham (1991) indicate that most entering students understand a function to be a formula; a graphical representation with no formula and meaning. According to the authors, students understand the concept of function as static.

Ferrini-Mundy and Graham (1991) further show that some of the difficulty in understanding calculus arises from an information translation of very sophisticated ideas and use of the examples of the definition of a sequence of real numbers tending to a limit. These authors also investigated the students` understanding of integration and found that students are able to apply basic techniques of integration but further probing indicated that they possessed fundamental misunderstandings about the underlying concepts. Further findings are that students do not understand the procedure of dissecting an area or volume, making use of the limit process, and the reasons why such a method works.

The study by these authors found that students` understanding of central calculus concepts are exceptionally primitive as students demonstrate virtually no intuition about the concepts and processes of calculus. It is found that students mimic examples and crank out homework problems that are predictably identical to the examples in the text which is a situation leading to misconceptions. Further research suggests that students have a strong commitment to these misconceptions which are resistant to change to direct instruction.

Cipra (1988) and White (1996) indicate in their studies that students enrolled in the traditional university calculus class have a very superficial and incomplete understanding of many of the basic concepts in calculus. They argue that this situation is as a result of rote and manipulative learning that takes place in an introductory course. On the other hand, Ferrini-Mundy (1998) describe students understanding of mathematics as a static collection of concepts and skills that need to be mastered one by one.

According to Ferrini-Mundy (1998), there is a need for alternate methods of instruction to enhance teaching and understanding of calculus. The author suggests that the syllabus and teaching methodology of the first year mathematics subjects had to include things like innovations based on information technology; opportunities for problem-based group work; opportunities for students to undertake self-learning of material deliberately not covered in lectures and finally a mixture of supervised and unsupervised learning activities. Similarly, Tall (1993) suggest various hypotheses that would improve students' understanding which looks at active learning, building up intuitions suitable for later formalizations, using computer graphics and computer programming and finally symbol manipulators.

Tall (1996) found that the most economically viable approaches that can make students understand calculus include the use of graphic calculators with graphic, numeric and subsequently symbolic facilities as well as having full facilities of a desk-top computer which has the advantage of portability, allowing the student to use them anywhere. Similarly, Ferrini-

Mundy and Graham (1991) indicate that the availability of new technology has an important influence on this new curriculum development effort in calculus. They argue that the most visible force for change in the mathematics curriculum is the computer, a mathematics-speaking device that has totally transformed science and society. The authors further argue that computers change both what is feasible and what is important in the mathematics curriculum by making it possible to do things in the classroom that could never be done before.

Du Preez, Steyn and Owen (2008) advise that mathematics educators should not start a calculus course at a knowledge level where they assume the students should be but rather meet the students where they are and get them actively involved. These authors believe that a thoughtful planning must go into deciding what, when and how to incorporate the review of pre-calculus 'must know' to support calculus topics so that students have a solid grounding in calculus. These researchers found that teachers of calculus at first-year tertiary level face the challenge to equip engineering students with the mathematical skills they need because without a solid foundation in mathematics at the calculus level, an engineering student will find difficulty in understanding and applying the knowledge involved in upper level engineering classes. According to Du Preez et al. (2008), there is a need to rethink the learning content and the learning facilitation strategies and to purposefully refocus the mathematical interventions so that the process of learning is promoted and encouraged.

Du Preez et al. (2008) strongly argues that basic concepts of calculus need to be revisited and reinforced at various times during the students' mathematics education. Their argument is that this will in turn bring stronger collaboration among secondary schools, colleges, and universities. They further argue that the role of technology can become useful in investigating the study of calculus where calculators and computers can serve as tools that aid such investigations. The other argument is that calculator does not only provide us with the opportunity of numerical approaches to calculus but also provide better understanding of the arithmetic that may lead to better understanding of the algebraically equivalent procedures.

3.6 Conclusion

The chapter outlined a number of ideas from literature perspective. Some of the main ideas are that research has found that ordinary and traditional ways of teaching, lead to students experiencing many difficulties in understanding mathematics and calculus in particular. Students taught using these methods could not solve both routine and non-routine calculus problems. Some researchers believe that calculus is too abstract and challenging for students and teaching it should include visual images and experimentation for students to have a concrete feel. It seems like most teachers over emphasize procedures over concepts and this leaves most students committing lots of errors and misconceptions. It is now a known fact that ICTs have the capacity to improve student`s understanding in learning mathematics as reported by most researchers. ICT tools and mediation in the teaching of mathematics is here to stay. There is whole lot of artifacts which can be used for different topics in the teaching and learning of mathematics. The chapter took us through a number of error types as classified by researchers. It is made clear that, mathematical errors and misconceptions are resistant and by ordinary means teachers may not have a way to deal with them if not by having a suitable intervention strategy.

CHAPTER FOUR: METHODOLOGY

4.0 Introduction

This chapter deals with issues of methodology and amongst other things, it explains the methods of collecting data. It is unfolding the outline of the research setting, the sampling, data analysis and further touches on issues of reliability and validity relevant to this study. Finally it highlights the ethical considerations about the research study and participants in the research study. This research study aims to explore and find types of errors and misconceptions TVET College Mathematics N4 students do in learning about the definite integral in calculating the magnitude of area under the curve.

4.1 Research Design

This research study is supported by a mixed- method design. A mixed - method design incorporates in it the quantitative as well as the qualitative designs. Quantitative research relies more on numerical data which is analyzed statistically. Muijs (2011) indicates that the disadvantage of using only the quantitative design is that it is not suitable for exploring a problem in depth and that it is shallow and does not help if one wants to get under the skin of phenomena. On the other hand, qualitative research is about establishing meaning of particular events or circumstances. Muijs (2002) defines qualitative research as an approach that encompasses a wide range of methods like interviews, case studies; ethnographic research and many others. This means qualitative research is subjective in nature as the researcher cannot detach himself or herself from the research. Bogdan and Biklen (1997) indicate that the qualitative research approach is interpretative in nature. Interpretation can only happen based on a particular theory of which theories stems from facts.

As articulated above, I chose to follow the mixed method approach because it widens the scope in terms of data analysis. I did not want to limit myself in this area of analysing data. Muijs (2002) explains mixed methods approach as a way of combining the quantitative and qualitative research designs which is a flexible approach. The approach is good in determining what the researcher wants to find out rather than being determined by a predetermined epistemological position. On the same understanding, Burke and Onwuegbuzie (2004) indicate that the goal of mixed methods research is not to replace either quantitative or qualitative approaches but rather to draw from their strengths and minimize the weaknesses of both in a research study. Mixed methods research is therefore helping to bridge the split between quantitative and qualitative research. Johnson and Onwuegbuzie (2004) argue that mixed method approach is the best research method as today's research world is becoming increasingly interdisciplinary, complex, and dynamic which suggests that researchers need to complement one method with another. These authors further argue that mixed methods allow researchers to mix and match design components that offer the best chance of answering their specific research questions. It is also seen as an attempt to use of multiple approaches in answering research questions, rather than restricting or constraining researchers' choices. Mixed methods approach is not a limiting form of research, rather an expansive and creative form of research. This gives the understanding that mixed methods enrich study results in ways that one form of data analysis does not allow.

This is a case study taking the form of an action research which is aiming at improving the existing classroom problems encountered in the study of calculus in the TVET College where I am based. Action research is mainly conducted to improve practices. It is an attempt to study the problem scientifically with an intention to guide, correct and evaluate the decisions and actions. Pernecky (1963) indicates that action research undertakes to resolve a problem than having to attempt to study facts. This means that the intention is to get depth and gaining insight toward the problem. This is about gathering relevant data of the problem and then come out clear on what to ask, what to test, how to test and with what to test, Pernecky (1963). According to Pernecky (1963), action research has the aim of studying an identified problem which is happening in a particular situation to a particular group of people.

Coghlan and Brannick (2014) define action research as a scientific approach used to study the resolution of important organizational problems together with those who experience these problems directly. They also agree that action research works through a cyclical four step process of consciously and deliberately: planning; taking action; evaluating the action; leading to further planning, and so on. Most importantly we learn from these researchers that the desired outcomes of the action research approach are not just solutions to the immediate problems but are important learning from outcomes both intended and unintended and a contribution to scientific knowledge and theory.

With this study the problem is the errors and misconceptions in the study of calculus by TVET college mathematics N4 students.

4.2 Research Site

The research study takes place in South Africa in a TVET College situated in the Eastern part of Gauteng Province. This TVET College has five campuses and one Artisan and Skills Developmental Centre, of which only two campuses offer N4 to N6 engineering courses. I shall name these campuses – Campus A and Campus B and further indicate that this study takes place at Campus A which offers N4-N6 electrical and mechanical engineering courses. N1 to N3 courses are offered at Campus C. Campus A has enrolled four groups of about forty students each doing Mathematics N4 program for the third trimester of the year 2017 with a split of two groups doing mechanical engineering course while other group is doing electrical engineering course. The research is conducted with two Mathematics N4 groups i.e. one Mechanical group (N4M1) and one Electrical group (N4E1).

4.3 Sampling

The sample of the research study is taken from the two groups of N4 Mathematics students doing an engineering qualification in a TVET college. The selected groups, each with about forty students, consist of both male and female students of different mathematical background enrolled at a TVET college. A full N4 TVET course, be it electrical or mechanical engineering, has four subjects building up a qualification. Admission requirements into the introductory N4 course, is a matric pass in both Mathematics and Physical science with 40% or higher. The whole cohort of students have passed their matric mathematics with 40% or higher. In their matric Mathematics, these students learned about calculus (differentiation in particular) on a superficial level in grade 12 Mathematics. None of them has done integration. These students learned a few rules of differentiation and are now expected to remember and reproduce some of these rules in order to understand deep integration. All these students should be at the same level of mathematics knowledge upon registration into N4 Electrical and N4 Mechanical engineering courses.

The campus from which the sample was taken has a capacity to take at the most four groups of forty students each. The study focused on the first two groups only, i.e. one mechanical and one electrical group. This was done to focus the study on a manageable size of participants of about eighty students in total.

The First Test Task was purposefully written by all students from the two groups with the aim of accumulating errors and misconceptions from their marked work. This task was given to check the effectiveness of the traditional methods of teaching calculus. The other intention was that of checking pre knowledge of calculus understanding done on previous studies. The First

Test Task was given to check the type of errors students would do in answering questions on calculus topic. The ultimate aim was to arrange the error types into categories according to error type as explained by researchers.

The selection for the interviews was done based on performance of students from the First Test Task. The idea was to get three sets of two students each who got the high marks, moderate marks and the lowest marks. The purpose of the interviews was to try and deepen the insight into students' errors and misconceptions about the calculus topic. The reason why the selection of students for interviews was varied, was to get error types on different levels of understanding of the topic.

The whole classroom discussions purposefully focused on both active and passive students to manifest their problem areas.

Finally, the Follow-up Test Task was given to measure the effectiveness of the intervention strategy which was used. This task was written by all students from both groups. The idea was to check how whether there was any improvement achieved as a result of the intervention strategy.

4.4 Data Collection Methods

Data collection was done in four distinct stages taken in order, i.e. First Test Task, Interviews, whole class discussions and lastly Follow-up Test Task. This is in line with Pham (2012) who advises that student's performance should be assessed before, during, and after the instructional process to measure student's progress and to ensure if lesson objectives are achieved. The researcher started by engaging with students and giving them lessons on calculus as arranged with group lecturers. Five lessons were given to groups. The lessons were given with the purpose of preparing students for assessment on the section of calculus.

The first stage happened after a series of lessons on the topic of calculus were given to both groups by the researcher. The lessons covered the pre-requisite calculus knowledge, introduction to integration, indefinite integrals as well as definite integrals. Students were made to see the relationship between differentiation and integration using simple examples found in their prescribed textbooks. A table showing standard derivatives as well as integrals was used to make clear this relationship between differentiation and integration. (See Table 7 in the appendix section). There were about five detailed lessons given to students. The lessons pulled from integration, showing the relationship with differentiation and checking on the pre-requisite knowledge of students. The group lecturers have already completed differentiation section with students. The teaching of integration section was intended to result into giving students an assessment that was used to collect the first set of data.

4.4.1 First Test Task

The First Test Task was administered to collect data which showed areas where students were battling. This was given just after the traditional teaching of the topic. The idea behind giving this task was to collect types of errors students might have and further realize the misconceptions formed in the process of them trying to understand the topic. The other intention of giving this First Test Task was to try and evaluate how effectiveness of the traditional method of teaching calculus done earlier. This was the only way to collect data after the first lessons that took place to see the types of errors students would commit and further the type of misconceptions formed in students. Well thought questions were chosen to fulfil this idea.

4.4.2 Interview sessions

The interview of a few selected students was done based on the performance from the First Test Task. The target was to have two students per group of students who got very low marks, followed by those who got average marks and finally those who got high marks. The intention was to get ideas and explanations from different levels of understanding from these groups. These ranges of responses helped in mapping forward the intervention strategy.

4.4.3 Whole Class Discussions

Whole class discussions was done to share and extend the type of errors found during marking of First Test Task as well during interview sessions to every member of the class. One would have loved to interview all students but due to time constraints it was not possible. The whole class discussions afforded an opportunity to all students to have a say in the feedback session and indicate on personal struggles in understanding the topic. In other words, the whole class exercise was done to involve all students in an active way in looking at problem areas done in the First Test Task.

4.4.5 Follow-up Test Task

The follow-up Test Task was purposefully given to mainly check whether the intervention strategy improved all problem areas encountered during all past stages of collecting data. It would serve no purpose of identifying problem areas and working on them if there was not going to be a mechanism to test if the situation improved. This Follow-up Test Task was given to check if all gaps identified earlier, were closed. The Follow-up Test task was precisely given to determine the effectiveness of the intervention strategy used after the problem areas due to First Test Task errors and misconceptions, feedback sessions, interviews and the whole class discussions were done.

4.5 Data Collection Instruments

The two instruments i.e. First Test Task and Follow-up Test Task questions, were carefully selected from previous national examinations on N4 TVET mathematics. These questions meet the relevant examination standard of the N4 TVET mathematics course. The questions have been tried and tested to be very good for the testing of knowledge relevant to the N4 mathematics level. The questions have been used repeatedly at the same level of national examinations with the same results.

The interview questions were discussed amongst myself the researcher and the two mathematics lecturers who were teaching the two identified groups in the study. This was done with the idea to try and strike a level of validity and reliability in the interview questions. The other person who joined us was a mathematics senior lecturer who volunteered to be part of the team. He brought his lot of experience into the exercise and benefited us a lot. He helped phrase some of the interview questions.

There were about five highly qualified English lecturers who checked all language issues in the instruments used to collect data. One of them was a senior lecturer in the department of English studies and she has many years of experience in the teaching and marking of English subject. The other was an English remedial lecturer with university English studies in her qualifications. These lecturers worked on all chapters of the research and corrected language issues in these chapters. The senior lecturer asked for a digital copy of the research so that she could indicate the corrected areas in colour. The corrected copy was then given to the remedial lecturer who then read and gave her input to the document. The remedial lecturer did not find much to change because already there were no or little mistakes left in the document.

The remedial lecturer was also a CAMI champion. CAMI is a kind of computer software installed in the campus computers to assist students who are weak in mathematics and English. This remedial lecturer has extensive knowledge on the teaching and learning using ICT resources in the campus. Most of her remedial work is done through computers which are installed in her remedial workshop.

In terms of the lay-out of the instruments used to collect data, there is a standard lay-out prescribed by the college. Every assessment should take this particular format. Deviations are not accepted. The instruments used to collect data were done exactly same way the college would want staff design their assessments. Once the assessments are done, they are sent for copies. The photocopier operator will not make copies if the standard is not adhered to. The photocopier relies on the signature of the head of division approving to make copies.

4.6 Data collection process

The processes of collecting data were well thought of in terms of how they unfolded. The First Test Task was conducted immediately after the teaching sessions which focused on the topic from which the task questions were selected. All students from both groups were brought together in the campus auditorium to write the First Test Task. The task was treated as normal test in which all conditions for a test applied. Students used exam pads which were checked for cribbing notes.

The Follow-up Test Task was administered after the feedback sessions, whole class discussions and the interventions strategy was implemented. This task was administered in the same way as the First Test Task in terms of how and where is was written.

4.7 Calculus lessons given by the Researcher before assessment.

The researcher started by teaching calculus to the N4 mathematics students. It is good practice to teach the topic first before assessing students. The plan was to first teach the topic and later assess students on it.

The researcher used ordinary and traditional methods of teaching as most lecturers did. This was done to see how effective these methods of teaching would be in making students understand calculus. The teaching was dominated to a greater extent by the researcher.

Teaching by telling was the order of the day. Students were passive participants. Teaching pulled off from establishing whether students had pre-requisite knowledge on calculus. The researcher did not get much information on this point because most students were quiet and did not show signs of willingness to talk to the researcher. Students were used to keeping quiet during their lessons.

4.8 First Test Task and Interviews.

This test was given immediately after the completion of teaching calculus section to the two groups. It consisted of five carefully selected questions on calculus. The first two questions intended to check for basic pre-requisite knowledge and involved indefinite integral while the remaining three were on definite integral asking for the application of integration in calculating area magnitude under graphs. The first two questions were five marks each, while the remaining questions were given six marks each. Marks were given only to key steps. Sixty four students in total wrote this test. The duration of the test was one and a quarter hours. The test was marked and feedback given to students. Only 16 students managed to pass and the rest of 48 students failed dismally. The students with worst case scenarios and those with interesting cases were interviewed in order to deepen the insight of the errors and misconceptions made. All findings were recorded, filed and kept safe. All errors and misconceptions discovered were noted and brought to the attention of the whole class for further discussions. The intention was to deepen insight into the student's errors and misconceptions done in this test.

Interviews were done to deepen the awareness of the existence of other error types as well as underlying misconceptions which could not be captured during the First Test Task. A total of six students forming three groups of two members each were interviewed. The first group was for students who performed poorly, the second group scored average marks and the last group was for bright students who scored high marks. The interviews took the approach of probing questions in order to persuade students to manifest their errors and misconceptions about the test performance. The interviews were helpful as they broadened the area of errors and misconceptions. This means that errors which could not be detected as appearing in the answer sheets of students were now realized.

4.9 Intervention Strategy

Intervention should take place only after identifying errors and misconceptions. To me it does not mean teaching the topic again as if it was never taught. It should be about hammering on the identified errors and misconceptions. It is good practice that teachers and lecturers should assess students only after they shall have taught the topic of the assessment. Pham (2012) argues that student performance should be assessed before, during, and after the instructional process to measure student's progress and to ensure if lesson objectives are achieved.

The lessons given by researcher from which the First Test Task was prepared, were mainly teacher centred. This is how mathematics lecturers give their lessons in this campus. As the head of division, one of my responsibilities is to support lecturers in almost everything they do. It came out clear from the class visits that lecturers are still using traditional methods of teaching mathematics. This led me present my lessons same as how group lecturers do.

In giving my first lessons prior to the First Test Task, my teaching style resembled those of group lecturers with students remaining mostly passive as I was teaching. Out of this exercise, the test was administered, then marked and feedback given to students. After having identified error types from students' responses and after the students' manifestation of their errors and misconceptions in the interview sessions and whole class discussions, an intervention strategy was rolled out to students. This was about teaching calculus using ICT tools.

An intervention strategy adopted a new approach in teaching and learning calculus. The traditional methods of teaching were thrown away and got replaced by the use of ICTs. The intervention strategy took the form of re-teaching the topic but mainly hammering on the identified errors and misconceptions by a way of using ICTs. Graph sketching was one of the most common source of errors committed.

Most researchers as discussed in this study agree that sketches and diagrams should be used when teaching definite integral and ICTs are perfect tools to do just that. Five more lessons were prepared using ICTs and presented to students. All lessons were dominated by video watching with a process of pausing and explaining some parts and aspects of the lesson. These lessons were no longer teacher – centred, instead students became active participants showing lot more interest in doing calculus. This strategy was mainly focusing teaching and correcting the identified errors and misconceptions seen in test results. A number of `You Tube` videos were shown to students.

The first video was watched from: https://youtu.be/Vv1BUCkgsr8. This video took students through differentiation and this was done to link previous knowledge on differentiation to new knowledge on integration. The other reason why this video was shown is that students needed to realize that the two sections cannot be divorced from each other. It’s like two sides of a coin. The other reason was to provide a bridge for safety passage from differentiation to integration.

The second video was from: https://youtu.be/49FfXVOIalE: Basic Integration. This video shows all the integration basics necessary for beginners to know about integration. It shows basic functions and how they are integrated.

The third video shown was from: https://www.youtube.com/watch?v=LkdodHMcBuc: This was about the definite integral. This video deals only with the definition and does not touch on calculations. It makes students to close most gaps left by the First Test Task and interviews. Lastly all following videos were watched:

1. https://www.youtube.com/watch?v=0ddipzqkgyk : introduction to the definite integral – you tube
2. https://www.youtube.com/watch?v=ppholu5-e_0: definite integrals - calculus – you tube
3. https://www.youtube.com/watch?v=gx8bwiigfkq: definite integrals – you tube
4. https://www.youtube.com/watch?v=odwktt0rmdg: the definite integral – you tube

These videos are about integration. These videos are prepared and presented by mathematics subject expects. They show clear graphs and offer free professional lectures which were recorded in simple English for students to understand and follow. Sketches drawn in these videos show representative strips, names of graphs, areas shaded and Cartesian planes suitably scaled. Integration has been done correctly at a pace well thought for beginners. The videos come with one excellent feature of making it possible to `pause` every time the teacher would want to emphasize something and every time students would want something to be explained. This happened frequently with these students as there were many areas they needed much clarity.

4.10 A Follow-up Test Task

A Follow-up Test Task was given to both groups to check whether the ICT strategy addressed all areas of errors and misconceptions. This Follow-up Test Task was given to see if there was any improvement in the understanding of calculus by students. Four carefully selected questions were prepared on definite integral and each asked for six marks. Students, who were weak and poor in responding to earlier questions on First Test Task, were now followed to check their performance in comparison to Follow-up Test Task. This was mainly to check the extent to which ITCs were effective in diminishing the errors and misconceptions made earlier. The scripts were marked and checked against all errors and misconceptions spotted earlier to see the extent to which the intervention strategy managed in helping students improve on errors and misconceptions made earlier.

4.11 Data Analysis

4.11.1 Inductive approach

An inductive approach was used in the study. The process started by the researcher collecting data relevant to topic. The idea was to collect substantial amount of data and then stepping back to get a bird's eye view of the data collected. The initial focus was on specific situations which ultimately led to the general situation with the goal of looking for patterns in the data and working to develop a theory that could explain those patterns.

Michalski (1983 explains inductive approach as a process of acquiring knowledge by drawing inductive inferences from teacher or environment provided facts which involves operations of generalizing, transforming, correcting and refining knowledge representations. This researcher indicates that inductive approach provides the ability to make accurate generalizations from scattered facts and to discover patterns in collections of observations. According to Michalski (1983), inductive approach further provides improvement of current techniques and a basis for developing alternate knowledge acquisition methods. Michalski (1983) argues that inductive approach is used to detect and rectify inconsistencies, remove redundancies and to cover gaps in the topic of study. This means that inductive approach is used to establish new concepts or theories that characterize given facts by presenting inference rules for generalization concept descriptions and methodology.

On the other hand Thomas (2006) explains to inductive approach as primarily using detailed readings of raw data to derive concepts, themes, or a model through interpretations made from this raw data by researcher. In other words it means that the researcher begins with an area of study and allows the theory to emerge from the data. According to Thomas (2006), the primary purpose of the inductive approach is to allow research findings to emerge from the frequent, dominant, or significant themes inherent in raw data, without the restraints imposed by structured methodologies. Thomas (2006) argues that the two purposes underlying the development of the general inductive analysis approach are: 1. to condense extensive and varied raw text data into a brief, summary format and 2. to establish clear links between the research objectives and the summary findings derived from the raw data.

Jebreen (2012) explains the inductive approach as referring to an approach that primarily uses detailed reading of raw data to derive concepts, themes, and models, through the researcher's interpretations of the raw data by beginning from the area of study and create a theory from the collected data. This researcher further says that the purpose of this approach is to allow the result to emerge from the frequent, significant themes discovered in the raw data without applying any structured methodology. According to Jebreen (2012), describing the data gives insight about phenomena rather than supporting the result with the data. Jebreen (2012) argues that inductive analysis approach is used to combine varied raw data into a brief summary; to create clear links between the objectives of the research and the results from the raw data and make those links clear to others and how those links fulfil the research objectives and to develop a theory based on the experiences and processes revealed by the text data.

This study has adopted an inductive analysis in its approach. Raw data was collected by the researcher in active engagement with both the N4 mathematics lecturers as well as the students identified for the study.

Data analysis is guided mainly by the research design which the research study adopted. Caracelli and Greene (1993) indicate that qualitative data is numerically coded and quantitative data is statistical analysis and narratives. These authors indicate that the mixed method highlights the integrative potential of both qualitative and quantitative approach. This approach is used in analysing data collected for this study.

This section foreground the analytic tool used to analyse the collected data. Data analysis has its influence in chapters two and three on conceptual framework and literature review respectively. Chapter two on conceptual framework highlights a number of major theories and frameworks used in the teaching and learning of mathematics. Amongst these theories and frameworks, a few is used in this study. Kilpatrick`s five strands of proficiency is one such theory which has been used to analyse data collected. This was incorporated with Dubinsky`s APOS theory, Tall and Vinner`s concept image and definition and Sfard`s object theory. These three theories were used to check students 'errors and misconceptions committed in the tasks given. This is not to say that ideas from other theories were not used. The other part of the analysis considers pedagogic content knowledge by the lecturer. It is because I believe that some errors and misconceptions result from the teacher`s content knowledge. Therefore Ball`s content knowledge framework is used to analyse data from this position.

The data analysis is divided into two distinct areas. The first area is explained above which focuses at students` errors and misconceptions from the theorists' point of view. The second part is considering the literature review by researchers. Section 2.4 explains errors and misconceptions committed in the teaching and learning of mathematics. This study mainly considers the three error classifications of Donaldson (1963), Eisenhart et al. (1993) and Radatz (1979) in analysing data. Donaldson (1963) talks about three types of errors in structural, arbitrary and executive errors, Eisenhart et al. (1993) explains two types of errors in conceptual and procedural errors and lastly Radatz (1979) classifies error types into lack of pre-requisite knowledge, lack of language and difficulties in obtaining visual information. These are explained in chapter two earlier.

4.11.2 Analytical Tool

This is the data analysis tool used in the study which has its roots in chapters two and three. Chapter two dwelled more on grand theories as well as frameworks in the teaching and learning of mathematics. Chapter three gave much details on the types of errors and misconceptions other researchers found in the teaching and learning of calculus. The analytic tool in this study pinned on two main sections, i.e. the first part as well as the second part.

First Part of analytic tool

Focus and attention was given to the three outstanding researchers mentioned in this study in chapter three. Data collected was analysed according to three theorists in Donaldson (1963), Eisenhart et al. (1993) and Radatz (1979).

Firstly, errors and misconceptions were checked if they were Donaldson (1963) `s structural, arbitrary and executive. Secondly errors and misconceptions were checked if they were Eisenhart et al (1963)`s conceptual or procedural in nature and finally if errors and misconceptions were as a lack of prior knowledge or lack of obtaining visual information.

This component of the analytic tool was found by the researcher to be enough and detailing but not complete in terms of analysing the collected data.

Second Part of analytic tool

This part was meant to complete the data analysis together with the first part mentioned above for the information collected. This part focused on major theories as well as frameworks dealt with in chapter two of this study. Amongst other things, this section focused on analysing data from a perspective of Ball`s teacher content knowledge, Kilpatrick`s five strands of mathematical proficiency, Piaget`s assimilation and accommodation laws, Dubinsky`s APOS theory and Tall and Vinner (1981) `s Concept image and concept definitions. This section was not limited to the above theories but tried to touch on all possible ideas emerging from chapter two on conceptual framework.

4.12 Reliability and Validity

Golafshani (2003) explains reliability as the extent to which results are consistent over time. According to this researcher, if the results of a study can be reproduced under a similar methodology, then the research instrument is considered to be reliable.

Brown & Dowling (2001) say that reliability involves the consistency, dependability or stability of the results whereby if the test is repeated or used many times by a different researcher; the same results are achieved. According to Cohen et al. (2000), reliability is concerned with precision and accuracy of data interpretation.

The research study considered these guidelines as proposed by these researchers mentioned above. In an attempt to address issues of reliability, clear instructions and as well the solutions for both tests were given. The marking guidelines (solutions to questions), were done in a way that any another knowledgeable teacher can be able to use and mark the tests.

To improve on reliability of the responses, same students were used for both tasks. Caution was taken in designing both tests in order to accurately measure students' performance on the definite integral. The method of collecting data was credible and it was done through the First-Test Task, interview sessions of six selected students, corrections session, whole class discussions, and Follow up-Test Task. Lesson vignettes (pictures of students' actual responses to task questions) are included to show this. I promise to be free from bias as a researcher as much as possible. As a developing researcher I am aware that this area can always improve with lot of research writing.

Kimberlin and Winterstein (2008) explains validity as the extent to which an instrument measures what it intended to measure. These researchers indicate that validity requires an instrument to be reliable. According to these researchers, validity is not a property of the test itself but instead, it is the extent to which, the interpretations of the results of a test are warranted, which depends on the test's intended use. On the other hand, another researcher, Golafshani (2003) indicates that validity determines whether the research truly measures that which it was intended to measure. Mason and Bramble (1989) explain validity as the degree to which the collected data measures accurately what it is supposed to measure. Another researcher, Denscombe (2002) says that validity concerns the accuracy of the questions asked, data collected and the explanations offered to these questions.

In this research study, the instruments used for collecting data were carefully designed by taking into consideration the intentions of the study. The study aimed at finding out errors and misconceptions TVET N4 Mathematics do in answering questions on definite integral by first trying to find out on the baseline calculus prerequisite students have before learning about deep calculus. It was anticipated that the questions designed for the students were surely going to enable the researcher to get valid data through students` responses. The students realized that taking part in the research study would benefit them in many ways since they would be able to improve their understanding of the definite integral with the ultimate goal of scoring higher on this section in final their examinations. This means that students would make less or no errors in their final examinations. Same students were allowed to sit for both tasks as a means to control issues of validity and reliability. Questions were varied in both tasks to avoid familiarity of responses. The two tasks had almost the same instructions' written over same length of time and given under the same conditions.

The data analysis of both tasks was done the same way. If any expert or any other knowledgeable person is allowed to work on the students written responses, surely the outcomes will resemble mine.

A team comprised of myself the researcher, two mathematics N4 lecturers as well as a senior lecturer teaching higher levels of N5 and N6 mathematics, helped with data analysis process. I produced the error categories to the team and they found it friendly to follow. The senior lecturer got interested in students who could not draw sketch graphs correctly. He indicated that students needed to know graphs in order for them to cope with mathematics N5 and N6. The other two lecturers checked the integration techniques done by students. They helped categorize errors into identified codes. The senior lecturer shared with us (myself the researcher and two other lecturers) that a strong foundation should be laid in mathematics N4 and that was the reason he was joining us. The team understood the error categories so well and they looked for such errors and recorded them. They found the exercise interesting and indicated that they were going to do it for their term tests.

4.13 Ethical Considerations

Ethical considerations are like laying ground rules between the researcher and the participants of the research study. Bassey (2003) and Cohen et al. (2000) indicate that ethics relate to the rights and the interests of the participants in the research. This exercise helps to prevent students against the threat of marginalization and exploitation.

Students were given information sheet packages with consent forms for willingness to participate in the study. Lecturer information sheet and consent forms were issued and signed. The parents' information sheets were not issued since all students were over eighteen years of age. All findings were treated with confidentiality without mentioning real names of students and their institution`s name. Students were named: Student 1, 2, 3 etc. with no real names used.

A detailed application to the TVET college principal requesting permission to do research in the college was sent. In allowing me to do research in the college, the department of higher education and training (DHET) had to be contacted. Another application had to be submitted to DHET offices in Pretoria. Permission was granted and the principal had to inform the campus manager where the research took place.

The information sheet contained important information to all participants indicating clearly that no one will be paid for taking part in the research study. It indicated clearly that participation was voluntarily and this ruled out the possibility that the research would be biased in terms of responses that the students would give compared to if they were paid. The participants were assured of their safety that they would not be harmed during their participation, and that the research was solely based on improving their knowledge on definite integral.

The Department of higher education and training as well as the college where the study was conducted gave me permission to carry out the research. All participants gave their written consent to take part in the study. All students were above the age of eighteen years old. All participants received information sheets which detailed the conditions for the research study. Students gave consent for both written and interview exercises. The information sheets made it very clear that the tasks were meant only for research purposes and that no marks will be used as term test marks. They were also told that nobody was forcing them to take part in the study and further that there were no incentives to be given at the end of the study. It was made clear to students that they had the right to withdraw from study without fear of any penalty that might arise. There was no need to get parents' consent as all were above age. Lecturers were also served with information sheets. They gave their written consent. No lecturer`s name was mentioned in the study.

Further ethical considerations have been given much thought in the study. Issues of anonymity and confidentiality have been given much attention during data collection processes. During the writing of the First Test Task, students were told to use their student numbers on the answer scripts they were using and therefore did not use their real names and surnames. Their student numbers were generated by an `Adapt It` system used by the college during registration of students. Each student has a unique and different student number. It is highly unlikely to memorize one`s student number as it is a long digit number. Students used the same student numbers when writing the Follow-up Test Task.

During the interview sessions, students were referred to and named as student 1, student 2 up to the last student 6. No real names were used. The same thing happened with the whole classroom discussions where students were named first student, second student and third student and so on. This was done to avoid using real student`s names.

4.14 Conclusions

This chapter gave ideas on what the teaching methods were used during the teaching and learning of calculus. It also shared light on the analytic tool used to analyse data collected. Finally, it explained reliability and validity relevant for the study. The ethical considerations which are relevant to this research study have been explained in this chapter.

CHAPTER FIVE: DATA ANALYSIS

5.0 Introduction

In this chapter 5, data analysis is done. This chapter reflects back and assembles ideas from all sections and analyses the collected data. Data has been collected in four main stages of First Test Task, Interviews, whole class discussions and Follow-up Task. The interviews were conducted, feedback given and whole class discussions were held. Chapter Five is arranged in the following manner: It pulls off from analyzing students First Test Task performance and proceed to interviews which are followed by whole class discussions and then lastly a Follow-up Task. It compares the students` performances from all points of data collection. It attaches more meaning on the data collected in terms of what mathematics researchers say. This section finally consolidates all data according to the analytical strategies set out in chapter four on methodology. It closes with an overall analysis of all data collected.

5.1 Data collected from the First-Test Task.

The marking of this Task revealed many errors and misconceptions done by students. It is found that students are battling with the learning and understanding of calculus. Students lack baseline calculus knowledge as well as integration skills. Students lack knowledge in sketching mathematical graphs. The intended definite integral was not understood. This subsection focuses on the types of errors and misconceptions committed during this Task. It further puts categories on the errors and misconceptions made by students. Each error is evidenced by some images of the student`s actual work as captured. These images are called lesson vignettes. The First Test Task was administered with the aim of collecting data which revealed the error types made by students. Most students were found battling with the mastery of mathematical sketch graphs. Students were unable to draw mathematical sketches and in some instances some students did not draw these graphs at all because they lacked the necessary knowledge.

Some students had problems with confusing integration and differentiation. Kiat (2005) found that students had problems in confusing integration with differentiation. Most students did not show understanding between definite integral and the definite integral. The integration skills remained a big challenge as most students could not integrate correctly different function types. In one of the most worry some situations, some students were able to draw the correct sketches and showed the area but surprisingly went on to calculate the magnitude of such area as zero. This showed lack of understanding of the concept. Some students decided to leave questions unattended mainly because they did not know how to go about answering the calculus questions

The following categories were formed and used for data analysis. The first category was for those students who confused ordinary integration with definite integrals. This also considered those who also confused differentiation with integration. The second category looked at students who showed insufficient knowledge of drawing sketch graphs. The Cartesian plane as well as the suitable scale used, formed part of the category. The third was the information that one would show on the drawn graph. Students who did not know what information to show on sketch graphs, were categorized under this group of error type. The next category focused much on students who went straight to calculations without first having to draw the sketches. Their calculations were not accompanied by sketches. The other category looked deeper into the knowledge of performing integration in different functions. The other category considered students who drew the sketches correctly and indicated the area by either shading or dotting but went on to calculate the area as zero magnitude. This indicated that the concept of area under graphs was not well understood. The other category considered students who left questions unanswered simply because they did not have an idea of how to go about attempting those questions.

5.1.1 Students' confusion of ordinary integration with definite integral. (SCOIDI)

Figure 1 shows students adding constant when finding definite integral whereas others do not add the constant when determining the ordinary integration. Students confuse the addition of the constant term \`C\` and do not know exactly when to add and not to add it. Figures 1, a to c, below, show students adding a \`C\` to the definite integral. On the other side, figures 1, d to e, show no \`C\` addition to ordinary integration. Many students committed this error. The \`C\` addition of fig 1, d to e, is done by \`the marker\` for purposes of giving feedback.

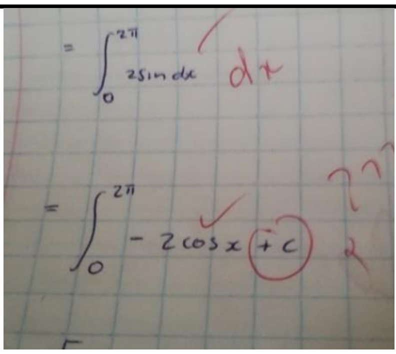


fig 1.a

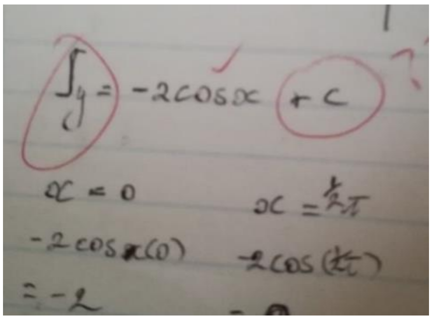


fig 1.b

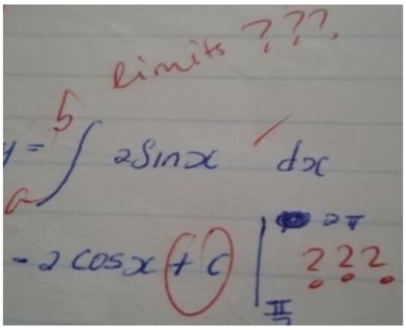


fig 1.c

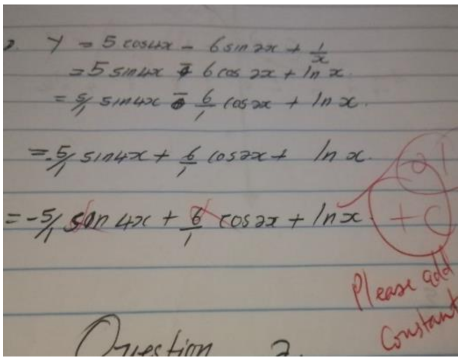


Fig 1. d

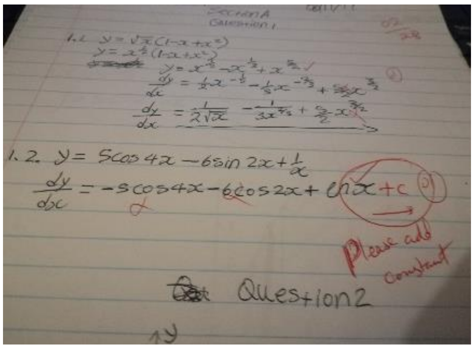


figure 1.e

Figure 1: Confusion of integral calculus with definite integral

5.1. 2 Insufficient knowledge of drawing Mathematical graphs (INKDMAG)

Figure 2 shows students who could not draw mathematical graphs correctly. They do not know the basic shapes of most mathematical graphs. The trigonometric graphs (Cosine and Sine graphs) are a challenge since most students do not understand the concept of the period and the amplitude. The following figures show these incorrectly drawn sketches. Figure 2, a and d, should be exponential graphs that appear far from being correct. Figure 2.b should be the graph of rectangular hyperbola but looks like an exponential graph. Figure 2.c looks like a parabola and yet it should be a sine graph.

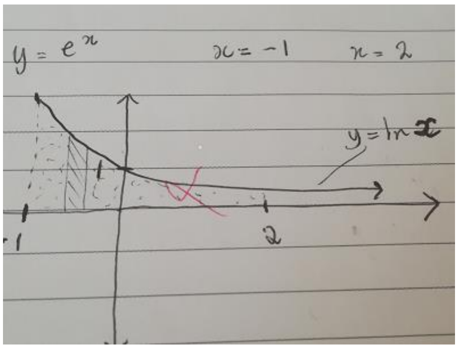


Figure 2. a

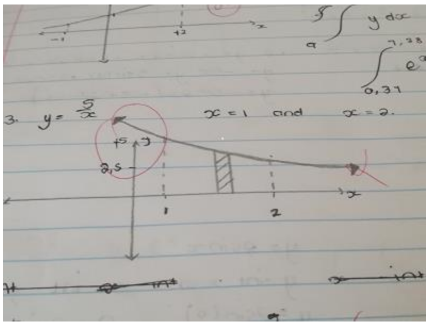


figure 2.

b

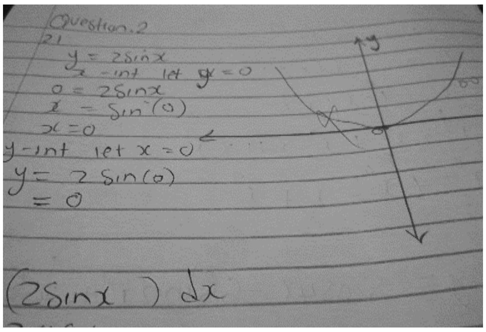


Figure 2. c

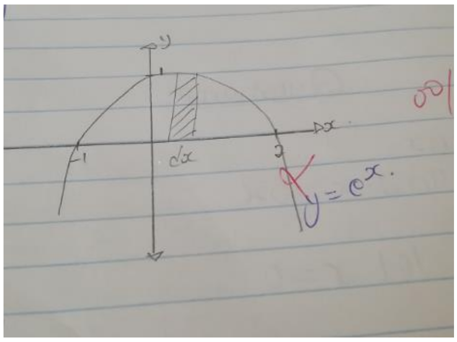
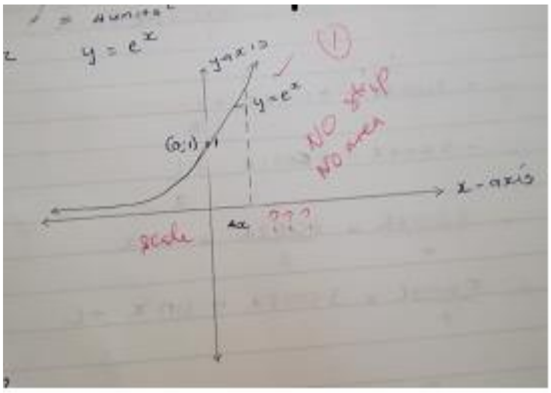


figure 2.d

Figure 2: Insufficient knowledge of drawing Mathematical graphs

5.1.3 Basic information to be shown on sketch graphs. (BITSHOSG)

Figure 3.a. does not give any idea of the area to be calculated as it does not show the described area, the representative strip to be used to do this calculation and lastly no scaling of the Cartesian plane on both the x-axis and the y-axis. Figures 3, b to d, do not show either area or representative strips.



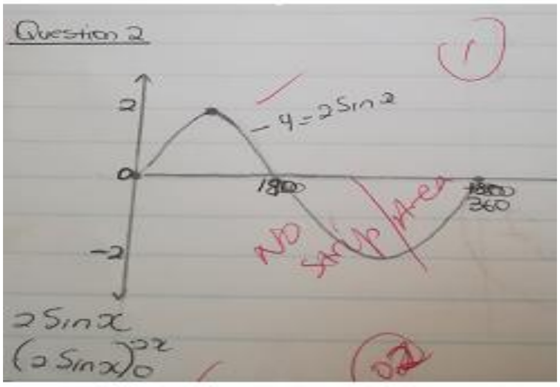
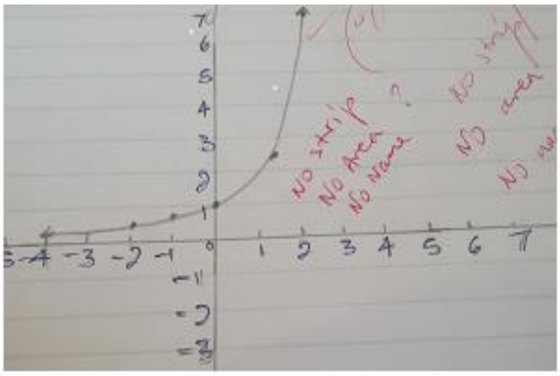


Fig 3.a

Fig 3. b



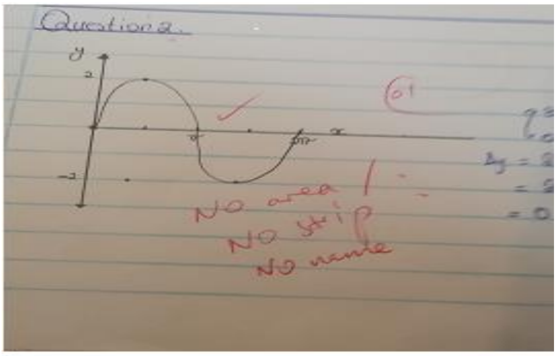


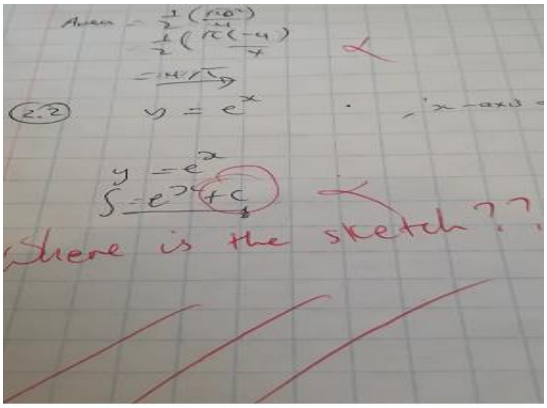
Figure 3.c

Figure 3.d

Figure 3: Basic Information to be shown on Sketch Graphs

5.1.4 Calculations without the accompanying sketch graphs (CWASG)

Figure 4 images bellow, show calculations only and no sketch graphs are drawn. Figure 4.d shows an attempt to use the table method of drawing graphs. Table method of drawing graphs is for beginners.



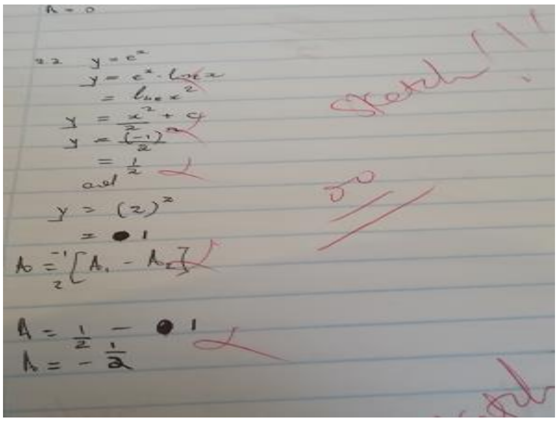
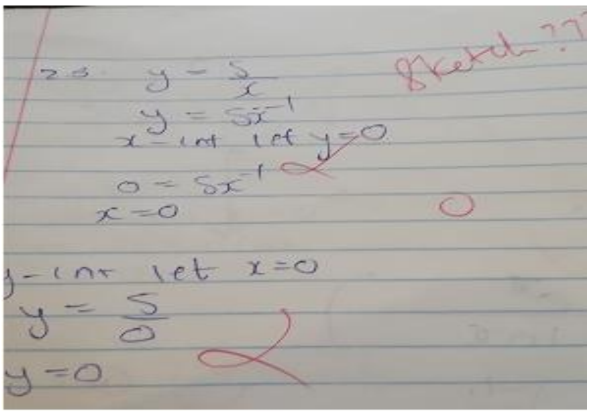


Fig 4. a

Fig 4. b



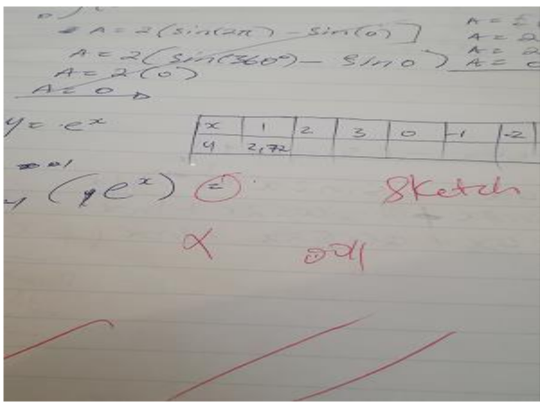
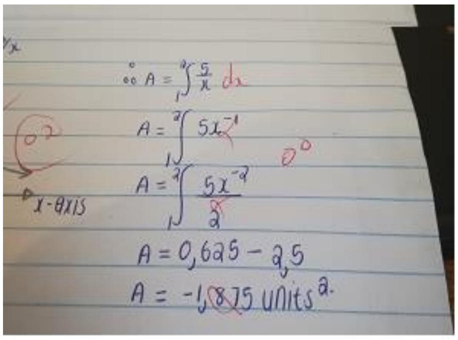


Fig 4.c **Fig 4. d**

Figure 4: Calculations without the accompanying sketch graphs

5.1.5 Integration Performed Incorrectly (IPI)

Figures 5, a to f, show incorrect integration of different functions. Serious errors are committed as shown by all these vignettes below. Students are far from doing the correct integration. Figure 5.c shows no integration done by the student.



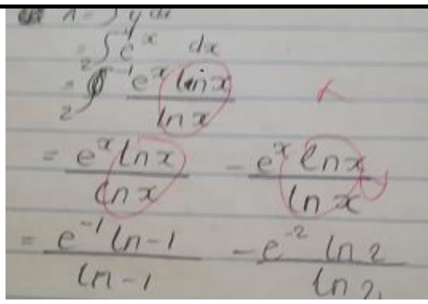
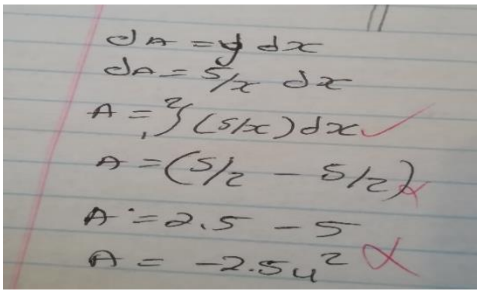


Fig 5.a

Fig 5.b



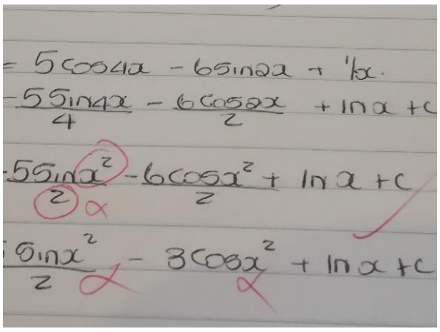
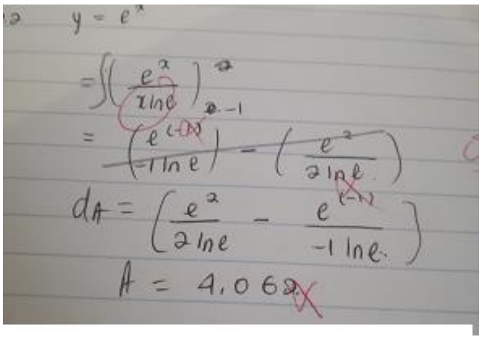


Fig 5.c

Fig 5.d



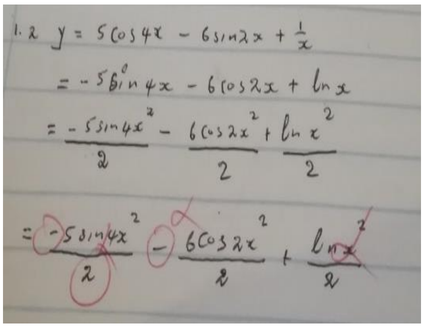


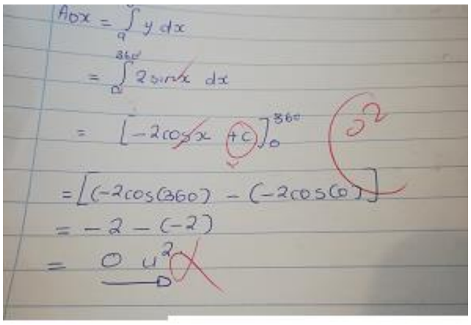
Fig 5.e

Fig 5.f

Figure 5: Integration performed incorrectly

5.1.6 Zero area magnitude (ZAM)

Figure 6 shows students who gave a zero area magnitude. Figure 6.a shows calculations that yielded the zero area magnitude while the other figure 6.b shows a shaded area for zero area magnitude.



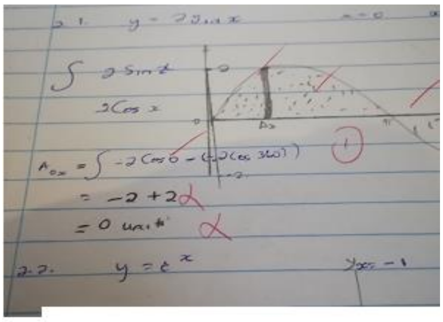


Fig 6.a **Fig 6. b**

Figure 6: Zero Area Magnitude

In all these Figures, 1 to 6, shown above, we see different lesson vignettes capturing different errors. These figures are given to facilitate the analysis for different types of errors committed in students` attempts to respond to the questions. All shown figure above, reveal the weakness and lots of misunderstanding in students.

5.2 Summary of errors committed in First Test Task.

This sub-section gives the summary of all errors committed during this stage of data collection. This sub-section gives codes and their explanations in a table form which is further used to analyze the errors committed during this First Test Task. The codes are as follow:

No	Error explanation/Category	Code
1	**Students confusing ordinary integration with definite integral.** When students do not know when to add and not to add constant \`C\`.	**SCOIDI**
2	**Insufficient knowledge of drawing mathematical graphs.** Students lack basic knowledge in drawing sketch graphs. Students do not know the basic shape of the sketch graphs. Trigonometric graphs (y=asinbx and y=acosbx) come with period and amplitude	**INKDMAG**
3	**Basic information to be shown on sketch graphs.** Cartesian plane should be well scaled and named, give the name of the graph, show limiting values, shade or dot the area, show representative strip.	**BITSHOSG**
4	**Calculations without the accompanying sketch graphs.** Calculations should be preceded by a sketch.	**CWASG**
5	**Integration performed incorrectly.** Students should master all integrating techniques. Trigonometric functions are no exceptions.	**IPI**
6	**Zero area magnitude** For a graph drawn and area shaded, the magnitude cannot be zero.	**ZAM**
7	**Questions left unattended** Students left blank spaces.	**QLU**

Table 1: First Test Task error categories

5.2.1 Frequency Table on errors committed during the First Test Task.

The majority of the errors committed are in accordance with Selden et al. (1994) that students lack knowledge in drawing mathematical sketches. Most students confused differentiation with integration. It was found out that some students had superficial and incomplete understanding of basic calculus concepts, (White, 1996). It was also evident that students had little intuitive understanding as well as fundamental misconceptions about calculus. Orton (1983) indicated that students liked procedures more than understanding the concepts. This was evident where students showed no understanding of the solution but showed little move procedurally. Most of the errors identified fitted well with what researchers found with other studies done elsewhere in the world.

Almost 71% of all errors committed were in line with Selden et al. (1994) where students could not draw mathematical graphs correctly. This included those who could draw sketches but not show things like representative strips, limiting values etc. Their overall knowledge of mathematical graph sketching was totally incomplete and lacked mastery.

The remaining 29% of errors committed were in line with both Orton (1983) and Selden et al. (1994) in that students could not integrate functions correctly. Some students confused differentiation and integration in their executions. Most showed little if no understanding of integration techniques.

Table 2 below shows the number of students who committed errors in the questions asked in First Test Task. The last row gives the numbers for each error category indicated on top.

SCOIDI Students confusing ordinary integration with	**INKDMAG** Insufficient knowledge shown on graphs	**BITSHOSG** Graphs drawn but students did not show the following basic information on sketch graphs	**CWASG** No sketches drawn	**IPI** Integration performed incorrectly	**ZAM** Zero area magnitude	**QLU** Questions left not unanswered

definite integration.						
Constant `C` added to definite integral	Insufficient knowledge on sketching graphs	The following not shown: ✓ Limiting values ✓ Representative strip ✓ Shading of area ✓ Naming of graph				
33	**36**	**115**	**11**	**48**	**27**	**8**

Table 2: First Test Task error frequencies

Total Errors committed by students = 278 errors.

Table 2 above displays the error frequencies in the First-Test Task given to students. Two hundred and seventy-eight errors in total were committed. The most observed error was one of integration performed incorrectly, which contributed to 17% or 48 errors out of 278 total errors.

Almost 15% or 42 errors out of 278 total errors were committed on representative strips not drawn. Almost 12%, 06% and 08% of errors were committed on no naming of sketch graphs, no shading of area and limiting values not shown respectively. About 10% of errors were done on giving zero area magnitude. 13% of errors were committed on insufficient knowledge of drawing sketch graphs. About 03% of errors were found on leaving questions unattended. Almost 12 % errors of were committed on confusing the addition of the constant value when integrating functions.

Almost 30% of errors committed, are on integration. This means that students have difficulties in dealing with integration of different functions. One of the things I have done as a researcher in choosing questions was that of varying questions. This was done with the view to bring up different sketch graphs for different function types which students had to integrate. The integration techniques required and expected for the solution of the functions asked, where mostly covered in this task.

The remaining 70% of errors are committed on sketch graphs. Students do not know how to draw their mathematical sketches. They do not know the basic shapes of these graphs. They confuse these graphs. It is also seen that the trigonometric graphs are the most difficult sketches to draw.

These two situations of not being able to draw sketch graphs correctly and not being able to integrate different functions correctly, are main areas of concern from the First Test Task. All errors committed are from these two identified areas.

5.3 Interview sessions and data collected

The interview questions included the why and how type questions. Students who did not draw sketches were asked why they not drew those sketches. Students answered that they did not know it was necessary. Those who drew sketches correctly and did not show limiting values and other relevant information on the sketches were asked why they did not show such information on drawn sketches. They responded that they did not know what to show on drawn sketches. Students who did not draw sketches were asked how they were going to see the area they were calculating and responded they did not know what to do.

The how type of questions as well as the why type questions were not used in depth to capture the understanding of students.

The sub-section shows the interview responses from all six students. These students were interviewed after feedback from First Test Task was given where students have performed badly. The first two students obtained low marks, followed by another two students who obtained average marks. The last two students did well and obtained good marks. The transcripts below show only the important aspects of the interviews.

Researcher and Student 1

The student obtained low marks in the First Test Task. This student did not draw sketches at all when answering the questions on definite integral.

Researcher: *The Test questions asked you to draw the sketches. Do you think this is necessary in calculating the area magnitude?*

Student: *No.*

Researcher: *Is that the reason why you did not draw graphs in answering these questions?*

Student: *Yes.*

Researcher: *ok, I understand but tell me one thing please: Can you draw those graphs asked in the Test?*

Student: *No, I don't know how to draw them.*

Researcher: *Good and thank you for opening up.*

***Researcher**: The sketch helps you have an idea of the area you are calculating. If you do not draw sketches, how are you going to see the area you are calculating?*

Student: *I don't know. I just calculate.*

***Researcher**: It`s ok.*

***Researcher**: You did not show or use limiting values in your calculations. Why this? What do you think is the purpose of the limits?*

Student: *I don't know. What are the limits?*

***Researcher**: They are boundaries. In Question 2.2 the question indicated that x=-1 and x=2. These are limits. You use them when calculating the area magnitude. They indicate that in calculating the area magnitude, one must start summing up all strips from -1 to 2 on the graph.*

Student: *Ok, Thank You Sir.*

***Researcher:** My pleasure. Let me ask you something. Do you think we can have a negative area magnitude?*

Student*: No. Our Engineering Science teacher told us that we can never have negative area.*

***Researcher:** That's good. It`s true.*

***Researcher**: You performed poorly in this Test. Why is this so?*

Student*: I don't understand this section.*

***Researcher:** What exactly do you not understand?*

Student*: Drawing the graphs and integrating.*

***Researcher**: Ok, I see. We will do corrections and please tell me where you not following.*

Student*: Ok, I will.*

***Researcher**: Thanks. We are done.*

Student 1 is totally lost and does not understand the concept of definite integral. This student cannot do both mathematical graph sketching and integration of functions. The student confirmed that he does not know how to draw graphs and also how to integrate. He does not understand the concept of calculus. He lacks both conceptual knowledge as well as procedural knowledge about calculus studied at this level. His concept image as well as concept definition is almost empty. There is a bit of incorrect procedures involved in the attempt to answer questions but lacked depth. There is nothing much this student is able to attach and make connections to the concept of calculus.

Researcher and student 2

The student obtained low marks in the First Test Task. This student drew all sketches correctly but did calculations incorrectly.

***Researcher**: In the Test you were asked to draw sketches. Do you think this is necessary in calculating the area magnitude?*

***Student**: Yes.*

***Researcher**: Ok, I see you have drawn sketches correctly but did calculations wrongly. Did you get lost on what to do?*

***Student**: Yes. I can draw but I cannot calculate the area correctly.*

***Researcher:** Ok, I see. I understand but tell me one thing please: Can you integrate?*

***Student**: Yes, I can but with simple functions like in question 1.*

***Researcher**: Good.*

***Researcher**: I hope you know that the sketch helps you have an idea of the area you calculating. If you drew them you should know what and how to go about calculating the area magnitude.*

***Student**: I don't know. I can draw but cannot calculate.*

***Researcher:** It`s ok. What do you think is your problem?*

***Student**: Doing the calculations to find the total area.*

***Researcher**: On your sketches you only drew the sketches but did not show all other features like the area as well as the limiting values. Do you know what the limits are for in doing these calculations? What is their purpose?*

***Student:** I don't know. What purpose do they serve?*

Researcher: *They are boundaries, just like in Question 2.2 the question indicated that x=-1 and x=2. These are limits. You use them when calculating the area magnitude.*

Student: *Ao!!! I did not know this.*

Researcher: *Good, now you know neh?. Let me find out on something. Do you think is possible to have a negative area magnitude?*

Student: *No. We can`t. Area cannot be negative.*

Researcher*: That’s wonderful. It`s true, We cannot have a negative area..*

Researcher: *You performed badly in this Test. Why did you do this?*

Student: *I can only draw graphs but calculations…, no. I don’t know where to start.*

Researcher*: What exactly is the problem?*

Student*: Doing integration after drawing those sketches.*

Researcher*: It`s Ok, I am planning to do corrections with you all in class and please tell me where you not following.*

Student*: Ok Sir.*

Researcher*: Thanks. We have come to the end of this interview.*

Student 2 is able to draw graphs correctly. This student`s pre-requisite knowledge on sketch graphs is at an advanced level. This student can integrate simple functions correctly.

The fact that this student demonstrated full understanding of the pre-requisite knowledge, it means the challenge might be with how the lecturer introduced and taught calculus at this level of study. This student could not show limiting values on correct drawn graphs and could not integrate functions seeing for the first time in N4 mathematics. This is an ideal candidate to learn N4 calculus. This student does not show convincing understanding on the calculus concept but displays full potential to do so. The concept image as well as the concept definition are likely to take shape as soon as intervention is put in place and done.

Researcher and student 3

The student obtained average marks in the First Test Task. This student drew sketches and done calculations for all questions.

Researcher*: The Test you wrote, asked for sketches. Do you think this is necessary in calculating the area magnitude?*

Student*: Yes, it is Sir.*

Researcher*: Why do you think so?*

Student: *You draw sketches to see the area.*

Researcher: *Ok, I see. I agree. But you drew one graph the incorrect way, which is the hyperbola graph. Can you integrate?*

Student*: Yes, I know because I did not know how the graph looks like. I can integrate but not all questions.*

Researcher*: Good. Which questions did you not integrate correctly?*

Student*: The y=2sinx function. I missed out on the minus sign. I confuse the sine and the cos.*

Researcher*: Ok my student. Surely you know that the sketch helps you have an idea of the area you calculating, like you indicated. What do you think is the importance of the limits?*

Student: *I don't know but I know we use them.*

Researcher*: It`s ok. I will show you later when we doing corrections with all students in class.*

Student*: Doing corrections? That will be helpful. I want to see where I messed up.*

Researcher*: Good. All of your answers are positive values. Is it possible to have a negative area?*

Student: *Err Err, No. It is not possible.*

Researcher*: That is correct my student.*

Researcher: *Tell me where you battling with this Test?*

Student: *I want some few tips on how to go about solving these calculations correctly.*

Researcher*: The session we will be having on corrections will help address that concern of yours.*

Student: *That will then make me happy.*

Researcher*: It`s Ok, We will have the session soon.*

Researcher: *Thanks you too. We are done.*

Student 3 understand the concept little better but like any other student has few problem areas which need attention. The rectangular hyperbola graph can be a challenge to most students when considering the asymptotes of the graph. The integration of trigonometric functions is also a big problem for most students. This student has a better concept image as well as the concept definition. He is able to talk about the concept in a way he understands and believes it should be done.

Researcher and student 4

The student obtained average marks in the First Test Task. This student drew sketches and did calculations for all questions.

Researcher*: This Test you wrote, asked for sketches. Do you think this is necessary in calculating the area magnitude?*

Student*: Yes, I think so.*

Researcher*: Why is that so?*

Student: *You can see the area you are calculating after drawing the sketches.*

Researcher*: Sure I agree. I see you managed to draw graphs correctly but calculated wrongly the area magnitude. Why did you get the zero area?*

Student*: Yaah!!! That question I want to see the answer. I got zero because two minus two gives zero. Is it wrong?*

Researcher*: Good. It is true that two minus two is zero. In this case it is not what is happening. That area cannot be zero. You will see by the time we will be doing corrections in class.*

Student: *I cannot wait to have that session on corrections. I really want to see how it is done because I thought I got it right.*

Researcher*: What do you think is the importance of the limits?*

Student: *We use them when we substitute after integration. Am I wrong?*

Researcher: *No, You are right.*

Student*: Thank You Sir.*

Researcher*: Tell me my student, did you find a negative answer area in the past when practising for this Test?*

Student*: Yes I did.*

Researcher*: Please explain to me how you got it.*

Student*: If limiting values are say -1 and 2, then I put -1 first and then put 2 then I end up with a negative answer*

Researcher*: That is true. I want you to know that you are doing the solution the wrong way. You should always substitute the bigger limiting value first followed by the smaller limiting value. Got it?*

Student: *Ooh Is that so? I did not know it.*

Researcher*: It`s Ok, you are here to learn.*

Student: *Thank you so much Sir...*

Researcher*: Thank you. We are done for now. See you when we will be doing corrections in class.*

Student 4 like student 3 understands the concept a little better but has some few misunderstanding areas with the calculations. This student settled for a zero area magnitude even when the graph is drawn and shaded between limiting values. The other problem is that this student did not know how to arrange the limiting values when substituting to get the final answer. This student can integrate different functions with the exception of the trigonometric ratios. Both their conceptual knowledge and procedural knowledge are at a promising stage which can be improved by an intervention.

Researcher and student 5

The student obtained good marks in the First Test Task. This student drew sketches and done calculations for all questions.

Researcher*: The Test you wrote, asked you to draw sketches. Why do you think this is necessary in calculating the area magnitude?*

Student: *The sketch shows the area nicely. You can see what you are calculating. You can also see whether the area is positive or negative.*

Researcher*: is that so? So we can have a negative area?*

Student*: Yes, Err, mara…*

Researcher: *Please tell me what you mean.*

Student*: I mean, with the y=2sinx question, there are two parts that are equal. One above x-axis and one below y-axis.*

Researcher: *...so, does it mean that one is positive and the other is negative?*

Student: *Yaa, mara I don't know.*

Researcher*: so how do you go about calculating the final total area of those two equal parts?*

Student: *I must add them together. Therefore I will not have a zero area. I will have two times the one magnitude to avoid adding minus and a plus to get zero.*

Researcher*: Now, You are talking my dear. Thank you for such a brilliant idea.*

Student: Thanks Sir.

Researcher*: Tell me now, what if you find a negative answer area?*

Student*: No it is not possible because area magnitude cannot be negative.*

Researcher*: That`s my student. Thank you so much.*

Student*: My pleasure Sir.*

Researcher*: Let us talk about boundary values.*

Student: *What about them Sir?*

Researcher*: What are limiting values for?*

Student: *These are x-values that tells us where to start and stop when integrating.*

Researcher: *Thank you. Tell me more about this.*

Student: *We integrate first and then put the bigger one first and then the smaller one and then get the answer.*

Researcher: *In doing corrections, what will you want me to mostly touch on?*

Student*: Sketch graphs like the one in question 2.2 with two parts equal, one on top and one below.*

Researcher*: Thank you, I will definitely touch on something like that my dear.*

Student: *I can`t wait.*

Researcher*: Thank you, Keep well.*

Student 5 understands the concept well. The procedural knowledge as well as the conceptual knowledge are developed correctly. This student is able to express his understanding of the concept in his own words. He is a good student. He should be asked to explain to students who do not understand calculus.

Researcher and student 6

The student obtained good marks in the First Test Task. This student drew sketches and done calculations for all questions.

Researcher: *In the Test you wrote, the question instruction is that you draw sketches. Why do you think this is necessary in calculating the area magnitude?*

Student*: The sketch is used to indicate the area we are calculating.*

Researcher*: How so, my student?*

Student*: On the sketch one can see the area clearly, the limiting values which we use to substitute for upper limit value and the lower limit value to get the correct area. You also dot or shade the area to indicate the area clearly.*

Researcher*: Ok, that is good.*

Researcher*: A negative area magnitude, can we have such?*

Student*: NO, not at all.*

Researcher*: I can see you did so well in this test. Do you think you understand everything in this section?*

Student: *No, I cannot say I understand everything here.*

Researcher*: What do you want to know more about this section?*

Student: *I want more challenging problems to practice myself to prepare for examinations.*

Researcher*: Please help me in doing the corrections and let us help your fellow students together.*

Student: Ok, I don't have any problem with that.

Researcher: Thank you, see you then.

Student 6 like student 5 understands the concept well. Both the conceptual knowledge as well as procedural knowledge are well developed. This student committed little slips with no or little misconceptions. He understands the procedures on calculating the area under graph. His lay-out of the solutions is well executed which shows understanding of the procedural approach. Graphs have been drawn correctly. His understanding of the calculus topic is at a satisfactory level. He has better concept image as well as concept definition.

5.4 Whole class discussions combined with feedback and re-teaching of topic.

The interviews were followed by a session on corrections of the First Task Test. This was combined with giving feedback. The researcher made and copied solutions and gave each student a copy of the solutions. The researcher started probing students on some of their responses and questions concerning the errors and misconceptions done on their answer scripts. The researcher indicated that anyone can talk about anything concerning the Test. Courageous students stood up and manifested their errors and misconceptions to the whole class.

The researcher took one question and solution at a time and then opened up a platform for the class to talk about the solutions. The transcripts show only the important aspects of the discussions.

This came out of the whole class discussions:

First Student*: Good Morning Sir. I want to talk about this question of the y=2sinx.*

Researcher*: Good, Let us talk about it.*

First Student*: Yaa, there is too much here. Firstly I confused this y=2sinx with y=sin2x, but now I see, Ooh Yes. These two halves are equal. Am I correct?*

The student said this words pointing to the one part of the graph above the x-axis and the other below the x-axis.

Student continues*: I f we say this is positive and this is negative then they will add to zero and yet we have the area.*

Researcher*: That`s good. I am impressed. You have explained it far better my good student. Let me add something to this: In trying to avoid getting zero, you can also think this way:*

$$Total\ Area = \int_0^{2\pi} 2sinxdx = 2\int_0^{\pi} 2sinxdx = 2\int_{\pi}^{2\pi} 2sinxdx.$$

I say 2 times the one part since the two parts are equal as agreed.

Second Student*: This is perfect, I am ok now.*

Researcher*: Perfect? What do you mean my dear? In what way? Please share with us all.*

Second Student*: I mean that it is making perfect sense now. Thank you for this.*

***Researcher:** I am happy.*

Researcher to whole class*: Let us continue. What else is not clear?*

Third Student*: Sir tell me if I am wrong. 5 over x, is it not same as 5 times x to the exponent minus one?*

Researcher*: It is my dear.*

Third Student*: ....but when we integrate we add one to the exponent and then divide by the sum we get, am I correct?*

Researcher*: Yes you are, but..., that situation you talking about is a different one.*

Third Student*: How so?*

***Researcher**: $\int \frac{5}{x} dx$ is a standard integral. Remember the $\int \frac{k}{x} dx$ story we talked about when introducing the table of standard derivatives and integrals to you. This must be $5lnx$+C.*

***Third Student**: Eishh, I remember now. Thank You.*

***Researcher**: Thank you too for remembering and also understanding as well.*

***Researcher**: Please allow me talk about the limiting values and how you should show them on the integral sign when calculating the area magnitude. This is how you will show limits on integral sign: $Total\ area = \int_a^b f(x)dx$, where b >a. Thank You.*

***Fourth Student**: This section is difficult. How do I know all graphs?*

***Fifth Student responding to Fourth Student**: My dear, remember we did a module on Sketch Graphs and I think we must revisit that module.*

***Fourth Student**: Do you think so?*

***Fifth Student responding to Fourth Student**: Yes, because all graphs asked here in this Test, are there in that module. I checked quickly.*

***Fourth Student**: Ok Thanks. I will go check too.*

A few more Students agreeing to go and check the module. We will also do that.

***Researcher**: Good, It is good when we learning together.*

***Researcher**: Another thing is that when we calculate the magnitude of the area under graphs, which is the definite integral, we do not add the constant value `C`. The constant value `C` is added only, when we determine the calculus integral from simple functions.*

***Sixth Student**: but this integration…, do we get the correct answer or are we estimating the answer?*

***Researcher**: We get the exact answer.*

***Sixth Student**: I don't believe it. The shape is not regular.*

***Researcher**: It is true we get the exact answer. Draw the sketch on the board and let`s prove it. The best example is when we have an area bounded by straight line graph. The researcher quickly came up with this question: Calculate the area bounded by the graph of $x + y = 1$, the x-axis and the y-axis. The researcher asked the student to draw this sketch on the board for all students to see.*

Sixth Student: I don't know how to draw this graph. Sorry Sir.

Researcher: It is ok, I know you are not alone. This we can prove by using the triangle area formula of half base multiply be the perpendicular height. (See section 6.2 under discussions)

The whole class discussions were more helpful in that they revealed further errors and misconceptions. Students confessed their confusion over trigonometric graphs of cosine and sine graphs regarding amplitude and period. It came out clear that student could not show the correct arrangement of limits when executing the solutions. The zero area magnitude came out of the discussions and was explained clearly. The accuracy of integration in calculating the area of the irregular shape led to students gaining confidence over this topic of calculus. This was achieved by using ordinary method compared to integration method. (see section 6.2)

5.5 Data collected from Follow-up Test Task

The marking of this Task revealed that most students were able to perform better as compared to the First Test Task. There are areas of improvement seen in the performance of students` work. Most students were able to rectify and resolve most of the errors and misconceptions they did during the First-Test Task. The following section shows the errors and misconceptions committed during Follow-up Task which are classified according to categories used for First Test Task errors and misconceptions.

5.5.1 Students confusing ordinary integration with definite integral. (SCOIDI).

In figure 7 below, this time around, the constant `C` is dealt with correctly. Most students are giving their magnitude as a squared unit value. This is good practice. This shows improved understanding as compared to First Test Task responses. There is no longer the `C` confusion and instead, students learned that area is measured in squared unit value. Figures 7, a and b, shows great improvement as compared to what students did during the First Test Task.

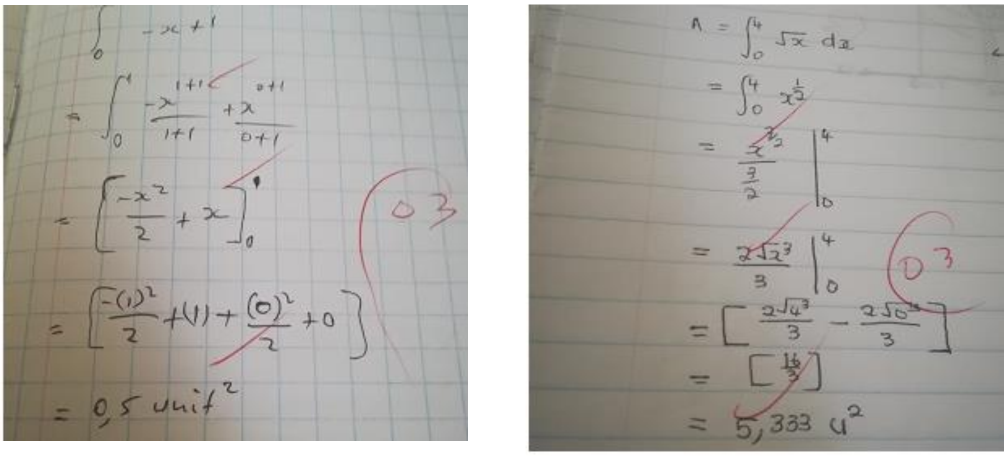


Fig 7.a

Fig 7.b

Figure 7: Confusion of integral calculus with definite integral is being cleared

5.5.2 Insufficient knowledge of drawing Mathematical graphs (INKDMAG)

Figure 8 below shows students` correctly drawn graphs. Challenging graphs were asked but students managed to draw them correctly. Figure 8.a is a difficult graph to remember and draw for most students. Asking this parabola graph of $y = \sqrt{x}$ instead of $x = y^2$ had intentions to test the graphs sketching knowledge and skills. For students to realize that these graphs are same, simply shows improved understanding. Figures 8, b to d, show correct drawn sketches. Drawing sketches correctly goes with showing important features on these graphs.

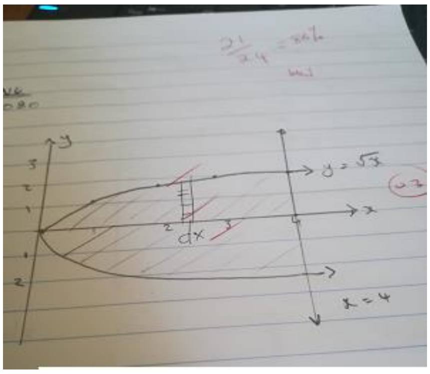


Fig 8.a

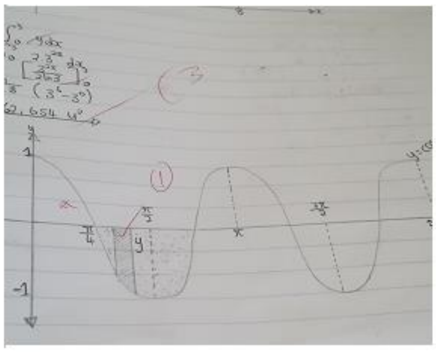


Fig 8.b

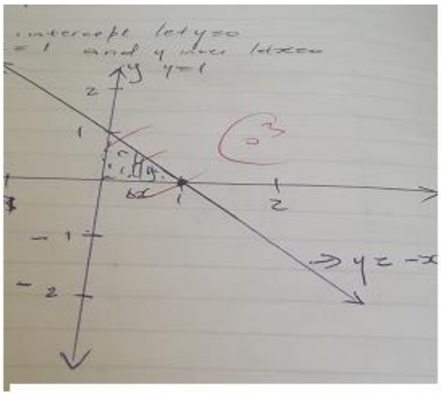


Fig 8.c

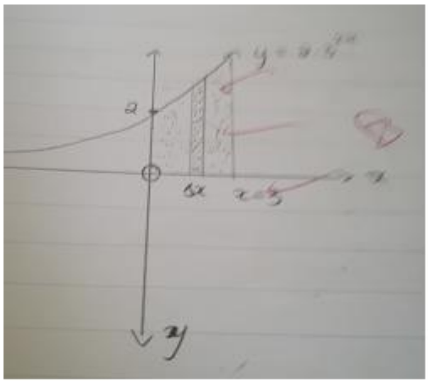


Fig 8.d

Figure 8: Improved knowledge in drawing sketch graphs.

5.5.3 Basic information to be shown on sketch graphs. (BITSHOSG)

Figure 8 sketches above, have been drawn correctly with basic information shown on them as required by good practice. In these figures above, most of the basic information needed to be shown on sketch graphs, have been shown. The basic information refers to the limiting values, area indicated, name of the graph, the representative strip shown and the suitable scale used for the Cartesian plane. Figures 9, b and d, show almost all basic information needed to be shown on sketch graphs. There is an improvement of this error committed earlier. All figures 9, a to d, show important features needed to be shown on sketch graphs.

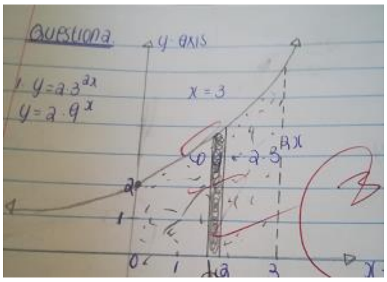


Fig 9.a

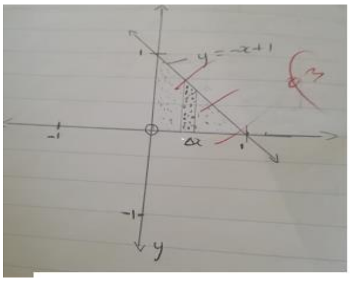


Fig 9.b

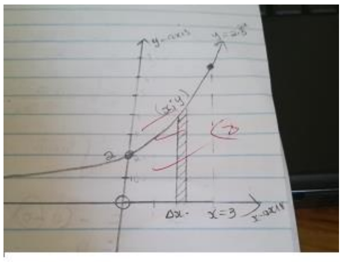


Fig 9.e

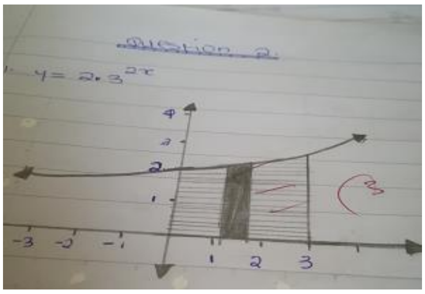


Fig 9.c

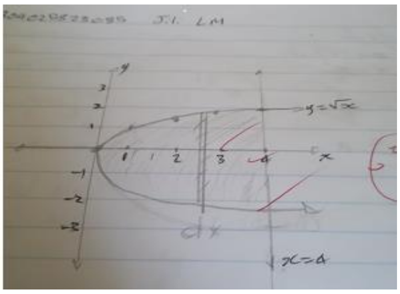


Fig 9.d

Figure 9: Basic information shown on sketch graphs

5.5.4 Calculations without the accompanying sketch graphs (CWASG)

All figures 10, a to d, show sketch graphs showing clearly the area shaded or dotted, the name of the graph and the representative strip accompanying calculations when determining the area under the graph. These figures 10, a to d, all show improved understanding. Correct sketches are accompanying correct calculations in the sketches below.

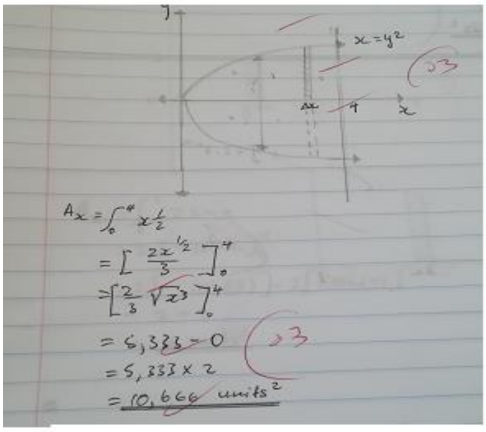


Fig10.a

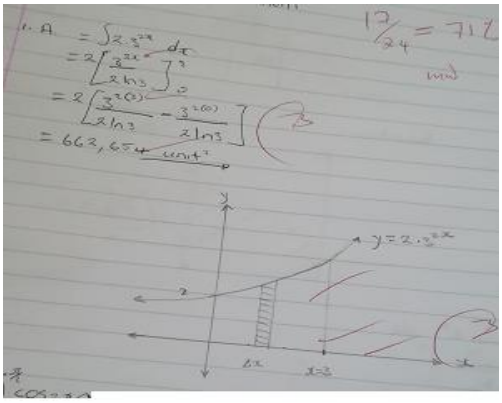


Fig 10.b

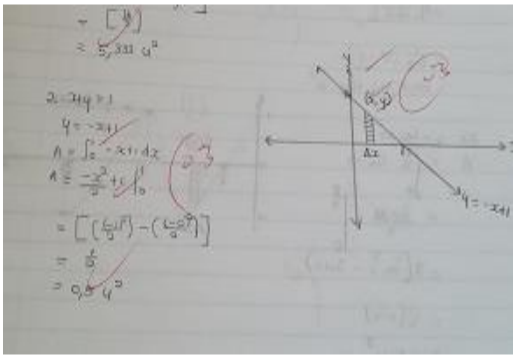


Fig 10.c

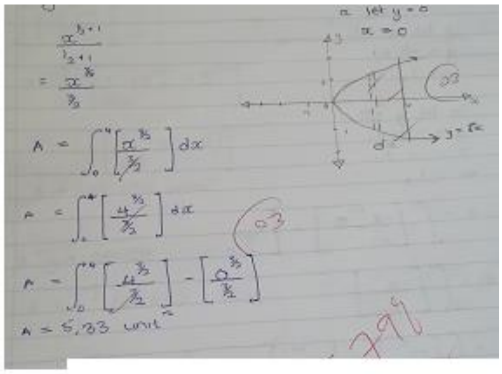
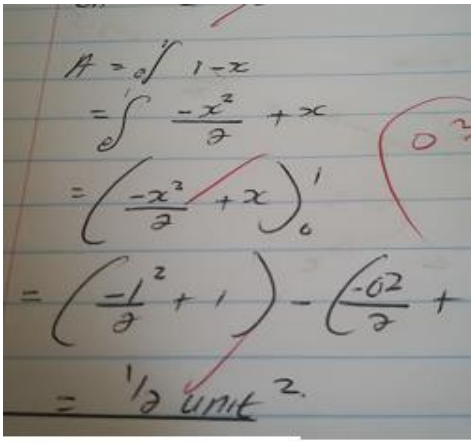


Fig 10.d

Figure 10: Sketch graphs accompanying the calculations.

5.5.5 Integration Performed Incorrectly (IPI)

Figures 11, a to d, show integration performed correctly by students. Students show the ability to integrate different functions correctly. This is a great improvement as compared to First Test Task errors and misconceptions committed earlier. These figures below show great understanding on how to integrate different functions correctly.



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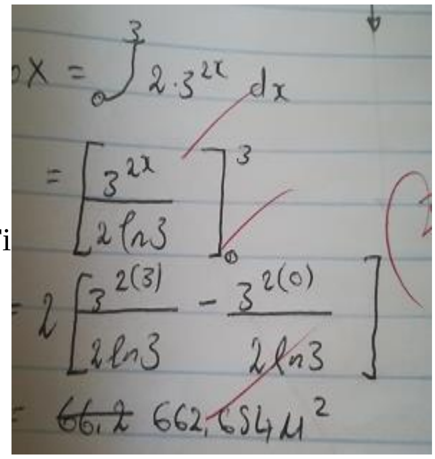
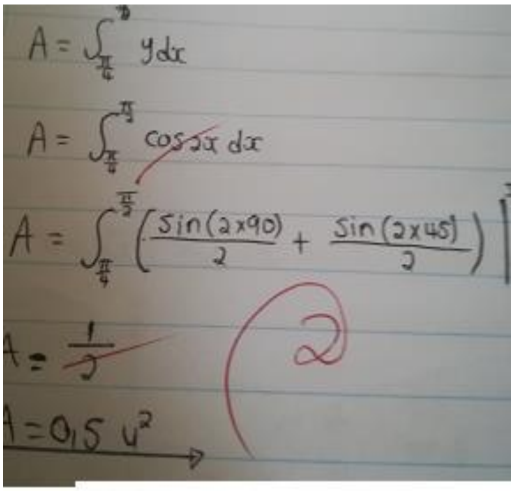


Fig 11.a

Fig 11.b



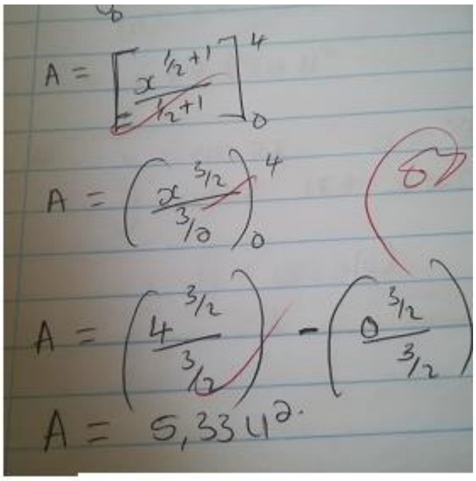


Fig 11.c

Fig 11.d

Figure 11: Integration performed correctly

5.5.6 Zero area magnitude (ZAM)

Figure 12 show calculations or the graph of y=cos2x for intervals of $0 \; and \; \frac{\pi}{2} radians$ created the same scenario as that of y=2sinx between x=0 and x=2π radians. The question on trigonometric graph was handled better this time as most students correctly calculated the answer. Few still gave their area magnitude as zero.

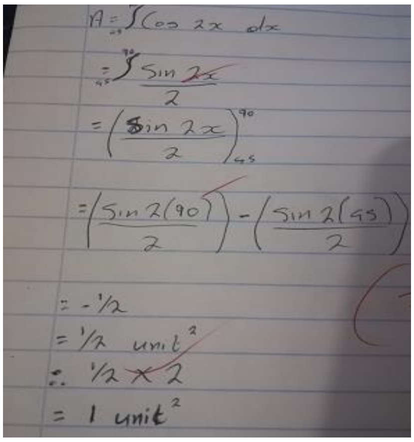


Fig 12.a

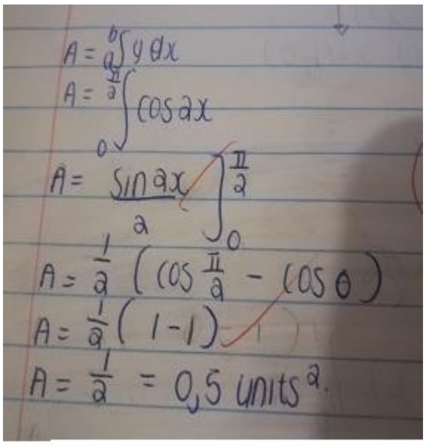


Fig 12.b

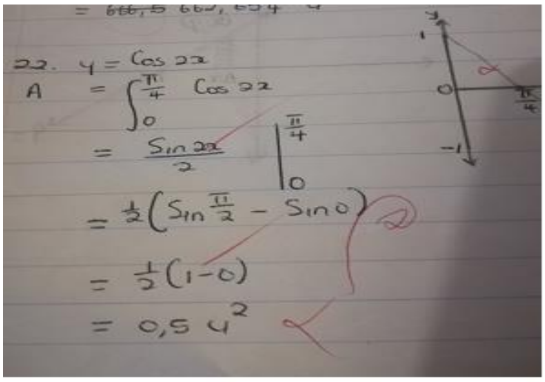


Fig 12.c

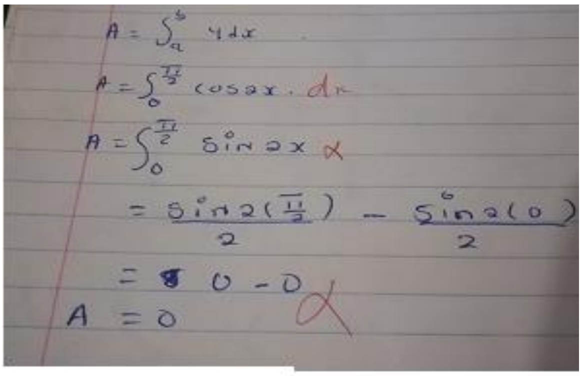


Fig 12.d

Figure 12: Zero area magnitude

The other method of analyzing data is by giving the data in a table form as seen for First Test Task. The following table shows the error frequencies for Follow-up Test Task. It basically indicates the number of students who committed errors as classified in Table 1.

SCOIDI	INKDMAG	BITSHOSG	CWASG	IPI	ZAM	QLU
Students confusing ordinary integration with definite integration.	Insufficient knowledge shown on graphs	Graphs drawn but students did not show the following basic information on sketch graphs	No sketches drawn	Integration performed incorrectly	Zero area magnitude	Questions left not unanswered
Constant \`C\` added to definite integral	Insufficient knowledge on sketching graphs	The following not shown: ✓ Limiting values ✓ Representative strip ✓ Shading of area ✓ Naming of graph				
2	**4**	**22**	**2**	**8**	**3**	**0**

Table 3: Follow-Up Test Task error frequencies

Total number of errors committed = 41.

Table 4 above displays the error frequencies in the Follow-up Test Task given to students. Only 41 errors were committed. The most resistant error was that of 20% on integration performed incorrectly. Only 05% added \`C\` to definite integral as well as not drawing sketches for the calculations. 10% still show insufficient knowledge on drawing graphs. 17% did not shade or dot the area. 15% did not name their graphs. 12% did not show the representative strip. 10% did not show the limiting values. 07% were committed on zero area magnitude. These figures are far less as compared to those obtained from First Test Task.

The table below shows the comparison between the First-Test errors and the Follow Up-Test errors committed by students. The errors committed during the First-Test Task are more than those committed during the Follow up Test Task. Fewer mistakes were made during Follow up-Test Task. Comparison of the First Test Task errors with Follow up Test Task errors.

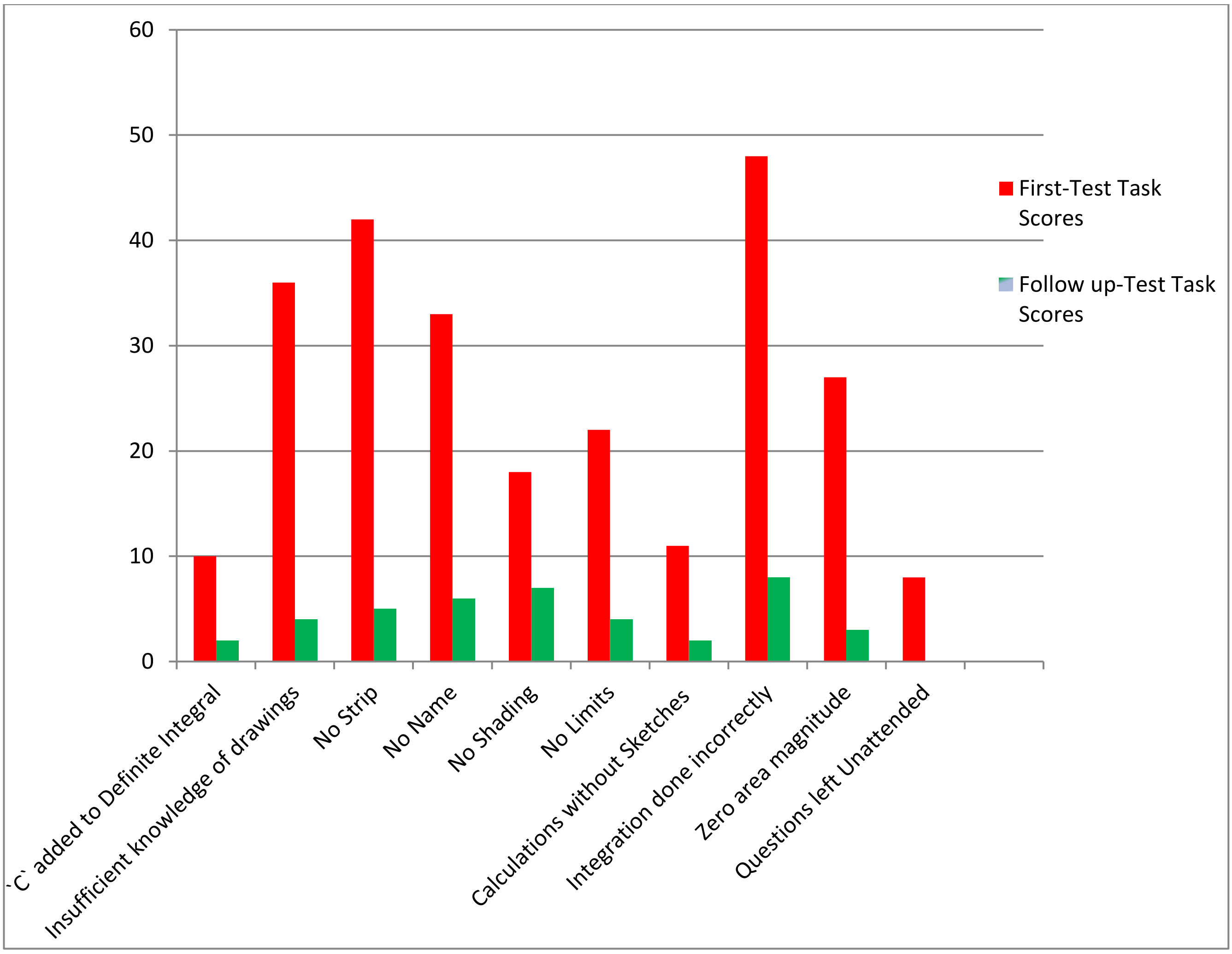


Table 4: First Test Task errors against Follow-Up Task errors

Table 5 below compares the pass mark as determined by the Department of Higher Education in the TVET sector as 40% in all Technical subjects offered at TVET colleges. The table below shows the pass rate comparisons between the First-Test Task and the Follow up-Test Task. The pass rate for the First Test Task percentage is sitting at 25 % while it is sitting at 100% for the Follow-up Test Task. This shows great improvement as well as understanding of this calculus topic.

Task Given	**Students Performance**	**Comment**
First-Test Task % pass	Total Wrote = 64 Number passed = 16 Number failed = 48 Percentage pass = 25%	Students experienced many challenges with this section on definite integral. Too many errors were committed during this Test.
Follow up-Test Task % pass	Total Wrote = 36 Number passed = 36 Number failed = 0 Percentage pass = 100	Students committed errors but were lesser as compared with the First Test Task. There was a great improvement.

Table 5: Pass rate comparison of First Test and Follow-Up Test Tasks

5.6 Concepts needed to answer calculus questions in both tasks given to students

The tasks intended to check on the understanding of certain concepts embedded in the area of calculus study. This is given in Table 6 below.

Item	Concepts and Procedures needed to answer Test Questions
1	**Determine the integral of:** $y = \sqrt{x}(1 - x + x^2)$ Simplification by multiplying out the brackets. Think of differentiation and integration relationship. Notation sign integration. Standard integral of function form: y $= ax^n$.
2	**Determine the integral of :** $y = 5cos4x - 6sin2x + \frac{1}{x}$ Knowledge of trigonometric functions. Standard derivatives on trigonometric functions, Standard integral of function form: $y = ax^n$ where $n = -1$.
3	**Calculate area bounded by: The curve of** $y = 2sinx$ and the $x - axis$ for $x = 0\ to\ x = 2\pi.$ Graph sketching and skills, trigonometric graphs, amplitude, period, limiting values, shading the graph, identifying the area, naming the graph, Cartesian plane scaling,
4	**Calculate the area bounded by: The curve** $y = e^x$, the $x - axis, and\ the\ lines\ x = -1\ and\ x = 2.$ Graph sketching knowledge and skills, exponential graphs, positioning boundaries, integrating functions of type: $y = Ka^{mx}$. limiting values, shading the graph, identifying the area, naming the graph, Cartesian plane scaling,
5	**Calculate the area bounded by:** $y = \frac{5}{x}$, the $x - axis$ and the lines $x = 1\ and\ x = 2$ Graph sketching knowledge and skills, recognising type of hyperbola graph, positioning boundaries, integrating functions of type: $y = ax^n$ where $n = -1$, limiting values, shading the graph, identifying the area, naming the graph, Cartesian plane scaling.

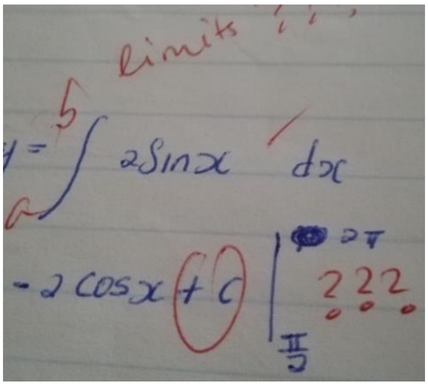
Table 6: Concepts and procedures needed to answer questions

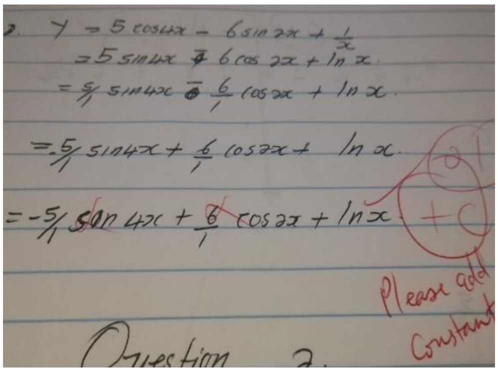
The following section considers item by item analysis indicating the types of errors and misconceptions committed during the First Test Task.

5.7 Item by item analysis (Conceptual and Literature point of view)

This section considers the items of the Test Task. An item will be made out of two or three vignettes. The aim is to capture a substantive amount of error types and misconceptions that might arise in these actual lesson vignettes.

5.7.1 Item 1 from Task 1

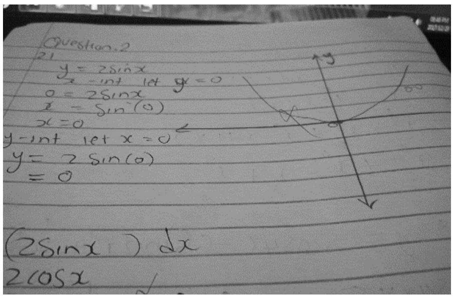




Item 1 from Task 1: First error types

Students commit conceptual errors as they do not know when to add and not to add a constant when integrating. There is also arbitrary error occurred when the learner ignore to show the limiting values as given in the question. There is also executive error type because even though the student is aware of what to do after understanding the concept in solving this problem, they still fail to carry out procedures required to solve the question.

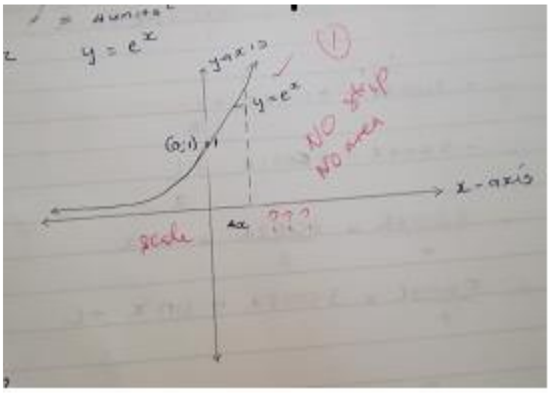
5.7.2 Item 2 from Task 1

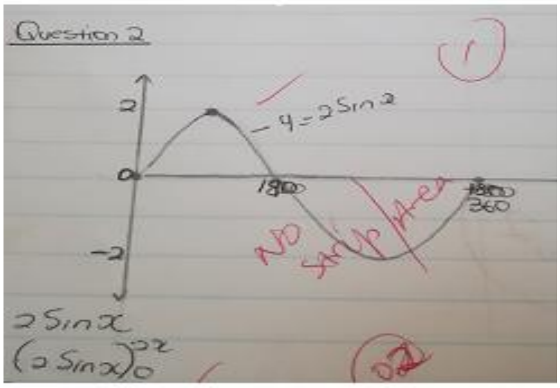


Item 2 from Task 1: Second error types

This is arbitrary error as students chose to draw the graph they knew. Students did not have an idea of the visual information on how the basic shapes should be like. The error is due to lack of pre-requisite knowledge students needed to have in drawing these graphs.

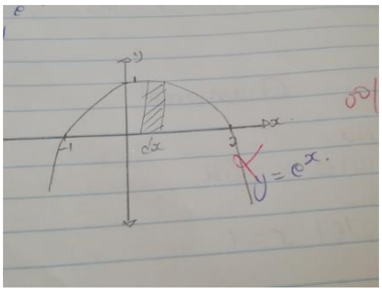
5.7.3 Item 3 from Task 1





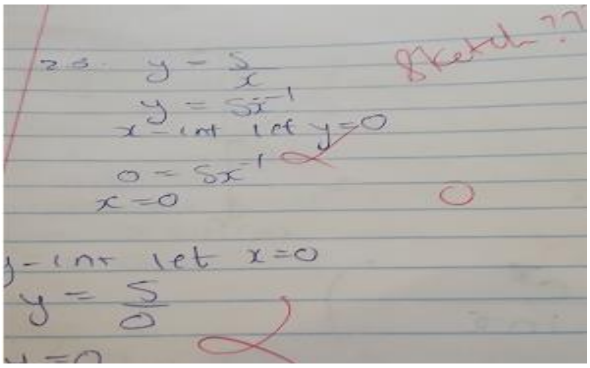
Item 3 from Task 1: Third error types

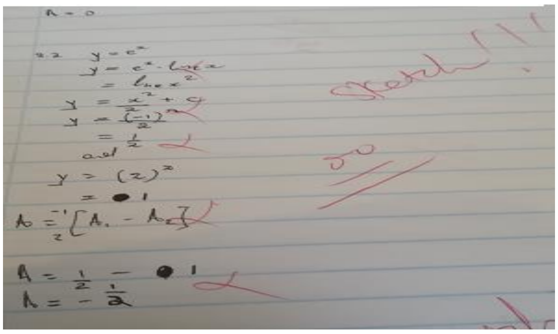
Errors committed here are arbitrary in nature as students fail to show limiting values given in the question. There is structural error committed since students could not grasp the basic final form of how the graphs should appear in order for area to be calculated. Executive error occurred due to students failing to come up correct graph, which indicates the area to be calculated.



Even though students understand the concept, they fail to have what is needed to help them do the solution.

5.7.4 Item 4 from Task

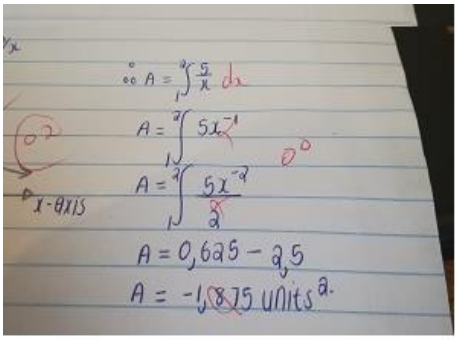


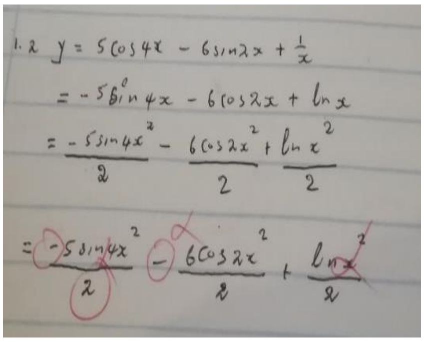


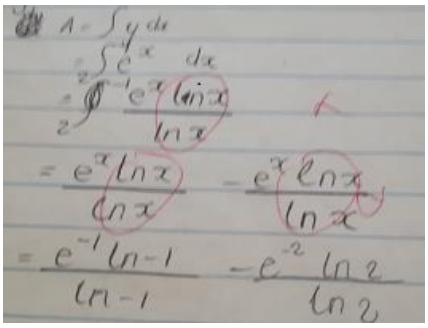
Item 4 from Task 1: Fourth error types

Structural errors occurred as a result of students failing to use the rule of integrating different functions. Arbitrary error occurred when students chose to ignore to use what is given in the question in coming up with their limits when these values are given. Conceptual error occurred when students could not realize the importance of graphs for the calculations that are following. The relationship thereof is not realized. The error can also be as a result of students failing to have a visual picture of how the graph should look. There are also errors as a result of lack of basic knowledge in sketching graphs. Procedural error occurred as a result of failing to carry out the correct procedures in solving the question.

5.7.5 Item 5 from Task 1



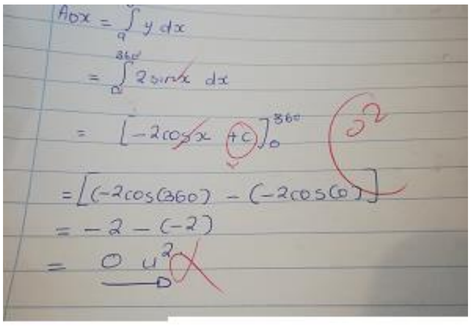


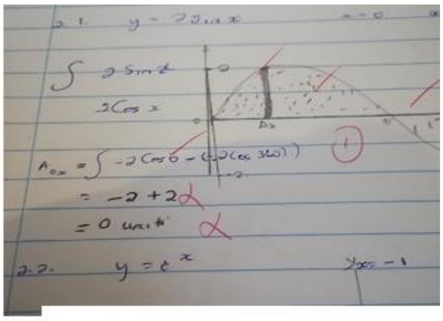


Item 5 from Task 1: Fifth error types

The error types in these vignettes are structural in that students failed to apply rules involved in carrying out integration of different functions. Executive error occurred due to failure to carry out procedures on integration even after understanding the concept. Procedural errors in failing to apply algorithm involved.

5.7.6 Item 6 from Task 1





Item 6 from Task 1: Sixth error types

Conceptual error occurred when students fail to understand the concept of area. For area dotted or shaded, how is it then zero? They fail to understand the meaning of area in this context and fail to see the underlying structure as well as meaning in this mathematics question.

5.8 General analysis of item by item analysis for errors and misconceptions

5.8.1 Determine the integral of: $y = \sqrt{x}(1 - x + x^2)$

The question wanted students to quickly come to their senses that the brackets should first be multiplied out as a means of simplification. Students needed to know that for this type of function, which is in the form: $y = ax^n$, the procedure required to solve this problem is when one is added to the exponent, get the sum, and divide the term by the sum. They are expected to add the constant at the end of the solution. Students needed to be able to flip from differentiation quickly to integration as the two concepts are working in reverse to each other.

Students lacked basic mathematical skills of multiplying the brackets. Some integrated the root of x outside the bracket as a stand-alone term and jumped into the brackets and integrated each term as if there was no bracket. Most students did not add the constant to their integrals.

5.8.2 Determine the integral of: $y = 5cos4x - 6sin2x + \frac{1}{x}$

This question tested the ability to integrate the trigonometric ratios with the last term testing on the previous knowledge when the value of \`n\` is negative. A formula sheet provided could be used to show how trigonometric functions are integrated. The question wanted to test on the sign change involved in determining the integrals of trigonometric functions. The last term wanted to test if students are aware of it as a standard integral and not following the ordinary technique of adding one to exponents' procedure as explained in 5.8.1 above.

5.8.3 **Calculate area bounded** by: The curve of: $y = 2sinx$ and the $x - axis$ for $x = 0\ to\ x = 2\pi.$

This question expected students to be able to draw the trigonometric graph with limiting values given. It intended to check on the knowledge of period and amplitude because these have an influence on the area magnitude. The question was chosen wisely as it intended to check the understanding when two equal parts of area are lying on positive side and negative side of Cartesian plane added to give the total magnitude. This is deceitful as it appears as if areas cancel out to give zero area magnitude.

Students added the two equal magnitudes and yielded a zero area magnitude. Some students confused amplitude and period concepts.

5.8.4 **Calculate area bounded by**: The curve

$y = e^x$, the $x - axis, and\ the\ lines\ x = -1\ and\ x = 2.$

The question wanted to check on the mathematical graph sketching skills and whether students know basic shapes of graphs as well as key features of these graphs. Students could not draw this graph. The limits were put to check if students know their Cartesian plane well.

Students committed arbitrary error as they ignored the negative sign and use the positive signs for both limits.

5.8.5 Calculate area bounded by: $y = \frac{5}{x}$, the $x - axis$ and the lines $x = 1\ and\ x = 2$

This question wanted to check the knowledge of graphs. Some students confused this graph with a straight line graph and by so doing committing arbitrary error. It also intended to check with integration skills of the function type shown in the question.

Students still failed to see it as a standard integral of `klnx + c` form.

5.9 Conclusion

Data gathered and accumulated from all points of collection reveal many weaknesses on the part of students who took part in the research study. The summary of errors and misconceptions committed in both tasks given, are a worrying factor even though the second assessment reveal improved performance. The concepts shown on table 6 further tells that the gap between where students are in terms of calculus knowledge compared to what they were supposed to do, is so wide. The item by item analysis reveals so many errors committed.

CHAPTER 6: DISCUSSIONS

6.0 Introduction

This chapter focuses on the discussions about the error types and misconceptions done in a TVET College by mathematics students who enrolled for an introductory N4 program. It is the turning point to a lengthy research study undertaken to understand the types of errors and misconceptions confronting the TVET college N4 mathematics students. Chapter six discusses all sets of data collected from all different points. This chapter is guided by the theoretical, the conceptual and the literature review outlined in earlier chapters. It is aiming to inform the findings, reflections and recommendations which are following in the next chapter.

Luneta and Makonye (2010) indicate that South Africa has an inefficient education system which produces poor mathematics results compared to other countries of the world. A research conducted by Luneta and Makonye (2010), found that some students’ errors and misconceptions caused by South Africa’s Education System in mathematics could not be classified with the systems used by other countries. According to Luneta and Makonye (2010), error analysis is a dynamic process as errors are not mutually independent but closely associated and interwoven with each other.

6.1 Linking data and Literature

This sub-section considers the analytical framework outlined earlier. It gives the type of errors and misconceptions by researchers against what students committed in both Tasks given and interactions done. Olivier (1989) indicates that errors are indicators of the existence of misconceptions and happen as a result of many factors like teachers, motivation, social factors, attitudes and many other factors. From constructivists` point of view, errors are intelligent constructs of knowledge by students. Sarwadi and Shahrill (2014) hold high the Piagetian idea that when students fail to assimilate or accommodate, a gap is created in the learning of the concept and this lead to the birth of errors and misconceptions coming into existence.

Donaldson (1963) classified errors into three categories explained earlier in chapter three (section 3.4). Most students in the First Test Task committed many of these types of errors classified by Donaldson (1963). Item 1 from Task 1 shows the arbitrary error students committed by choosing to ignore the limiting values which must be shown on the integral sign when calculating the area magnitude. Showing limits on integral sign is good practice. This item 1 from Task 1 also shows the executive error as students fail to carry out the procedure even after displaying strong evidence that they understand the concept. Procedurally, these students are wrong but understand what must be done to answer the question.

Item 2 from Task 1 shows the evidence of arbitrary error when students decided to draw sketch graphs which were not asked by the question. They chose to ignore the graphs asked by the question and drew what they know or what they preferred. This may be argued that they drew these graphs believing that they are correct.

In item 3 from Task 1 we see arbitrary error when students decided to ignore the limiting values similar to item 1 from Task 1. There is also structural error when students drew wrong graphs showing little details which make it difficult to have an overview of how the area should look like. It is a rule to draw the graph correctly as this gives an idea of how the area looks. It is amazing to expect a correct calculated area answer when the area is drawn incorrectly or drawn with little details. When students fail to come up with the final graph drawn correctly, this to me is executive error because they understand the concept but fail to carry out the procedure of calculating the area magnitude. The procedure is that students should correctly draw the graph, show all details and information on it, shade or dot the area, put limiting values and then continue to do calculations. Students understand that they must do this, but fail to execute it.

Item 4 from Task 1 shows structural error when students fail to use the rule for integrating different functions. They seem to approach different functions the same way and this is wrong. Different function types are integrated differently. Arbitrary error is also seen here when students chose to ignore to use given limits and are calculating their own limits. There is structural error when students fail to integrate: $\int \frac{5}{x} dx \ and \ \int e^x dx.$ Instead of recognizing these functions as standard integrals, students failed to do so. Students failed to carry out procedures on integration even after having understood the concept and these lead to executive errors. .

On the other side Eisenhart et al. (1993) made two error classifications in conceptual errors as well as procedural errors. Item 1 from Task 1 shows evidence of conceptual error as students do not know when to add and not to add a constant when integrating. The concept of area calculation is not understood when students add a constant value `C` to the magnitude. Students will also not add a constant to ordinary integrals after determining these integrals. In item 4 from Task 1, students commit conceptual error by not realizing the importance of graphs for the calculations that are doing. The relationship of sketches and calculations is not realized. Students failed to carry out the correct procedures in solving the question and by so doing committing procedural error.

Item 5 from Task 1 shows the procedural error when students failed to apply the correct algorithm in calculating the area magnitude. The procedure used is totally wrong and will not yield the expected correct answer.

Item 6 from Task 1 shows conceptual error in students failing to understand the concept of area. For area dotted or shaded, it will definitely not have a zero magnitude because it is there. They fail to understand the meaning of area in this context and fail to see the underlying structure as well as meaning in this mathematics question.

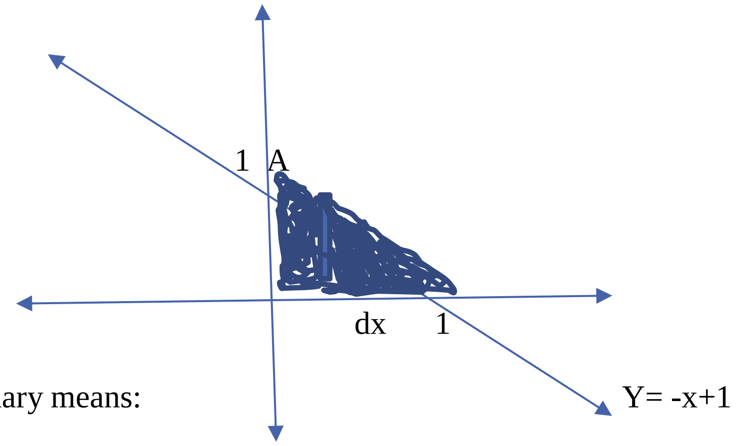
Lastly Radatz (1979) has three error categories as seen earlier in the study. Chances for students to commit errors as a result of language issues are zero. The questions for both tasks used students' friendly language of: `determine the integral of function of ….. and also ` Calculate the magnitude of the area bound by the graph of ….`. Yet there are many chances that errors can emerge as a result of difficulties in obtaining the visual information. This is quite relevant in the study of calculus where students are expected to draw sketch graphs and check how the area distribution looks like. Other errors can develop as a result of lack of pre-requisite knowledge. Item 1 from Task 1 shows errors as a result of pre-requisite knowledge. Students showed lack of pre-requisite knowledge in graph sketching. Item 2 from Task 1 shows lack of prior knowledge on drawing mathematical graphs. Students could not draw simple graphs of the form: $y = e^x \ and \ y = 2sinx.$ Some of the sketches needed students to have visual information before they could draw these graphs. According to Radatz (1979), errors were committed as a result of lack of this visual information.

6.2 Further General Discussions considering outlined analytical strategies

Most students who wrote the First Test Task committed lot of errors and misconceptions in answering the questions asked. The error types range from drawing sketch graphs to doing calculations. When students fail to draw the sketch graphs correctly, it becomes difficult for them to see how the area they are calculating looks like. They do not have an idea of how this area is spread and therefore they struggle to realize the limiting values. Mofolo-Mbokane (2013) believes that the visual imagery or visual representation makes the understanding of Mathematics a little easier. This is true. Radatz (1979) also indicates that difficulties in obtaining visual information lead to students committing many errors. There is a lot of evidence from First Test Task on these error types. During the feedback, correction session as well as whole class discussions, students did not have faith in the method of using integration to get the total area. An example was quickly designed and the question put on the board. The student was asked to draw this sketch on the board but he indicated that he does not know how the graph looks.

Example: Calculate the area bounded by the graph of: x+y=1, the x-axis and the y-axis.

Solution:



Solution by ordinary means:

The area of Triangle ABC = $\frac{1}{2} \times base \ \times perpendicular\ height.$

$= \frac{1}{2} \times 1\ \times 1.$

$= \frac{1}{2}\ unit\ squared.$

By means of integration:

$$Total\ area = \int_0^1 (-x+1)dx$$

$$= \left(\frac{-x^2}{2} + x\right) \text{ (for limits x=1 and x=0)}$$

$$= \left(\frac{-1^2}{2} + 1\right) - \left(\frac{-0^2}{2} + 0\right)$$

$$= \left(\frac{-1}{2} + 1\right) - 0$$

$$= \left(\frac{1}{2}\ unit\ squared.\right)$$

This means that sketches help students in understanding the question better. According to Mofolo-Mbokane (2013), the correctly drawn sketch can be used to address a number of issues, for an example, in a situation where the area is enclosed between two graphs, students can easily read off the limiting values and points of intersection without having to calculate them, for example: when the graphs pass through the origin it is easy to tell that one of the point of intersection is a (0, 0) point. Some students do not know what basic information to show on the sketch graphs. Figures 3, a to d, are good examples. Students drew graphs only and did not show the area described, no naming of graphs and no representative strips shown. It is good practice that the graph should be named, the area shaded or dotted and the representative strip be shown.

Graph sketching remains a big challenge to most students. Students draw mathematical graphs incorrectly. They also do not know the basic shapes of these graphs. Figures 2, a to d, clearly show how students confuse graphs. In Figure 2,d, an exponential graph is confused with a parabola graph. It is surprising because one of the TVET N4 mathematics module is `Sketch Graphs`. This Module introduces students to all possible graphs and it precedes the module on Differentiation and Integration. This should serve as pre-requisite knowledge needed for students to master graph drawing.

A hand full of students did not draw graphs and they went straight to calculations. This is bad practice. Figure 4.d shows a table drawn with the intention of using it to draw the graph. At this level of study, students should recognize graphs and know the basic shapes of graphs without resorting to the table method. Table method of drawing simple mathematical graphs is for beginners, that is N1 to N3 students.

Students could not integrate some trigonometric functions and some basic functions which is information made available for them on Mathematics N4 Formula Sheet (same as Table 7). Students confuse the minus sign and the division by derivative of the angle 2x when having to integrate $\int Sin2x\ dx$. Some could not recognize the $\int \frac{k}{x} dx\ to\ be\ integrated\ as\ klnx + C$. It is amazing to see that other students found the area magnitude to equal zero even after having drawn the sketch and shaded the area.

The First Test Task reveals a number of errors and misconceptions that students do in calculating the area under graphs by means of integration. The errors are both procedural and conceptual in nature. Eisenhart, Borko, Underhill, Brown, Jones and Agard (1993) indicate that procedural errors refers to mastery of computational skills and knowledge of procedures for identifying mathematical components, algorithms, and definitions. The following is a good example of students lacking procedural knowledge. Some students are totally lost and they do not know how to go about carrying out the calculations.

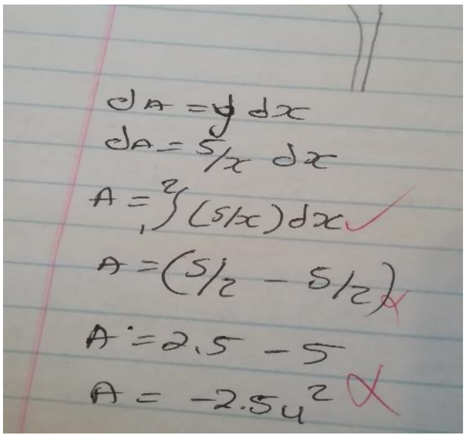


Figure 5.c: Example of lack of knowledge of procedures.

This student lacked knowledge of procedures and did not know what to do as he /she did not integrate at all. Eisenhart et al. (1993) indicate that conceptual errors refer to knowledge of the underlying structure of mathematics which is about the relationships and interconnections of ideas that explain and give meaning to mathematical procedures. Looking at figure 5.c, it is clear to deduce that this particular student does not understand the concept at all. Much needs to be done to help such students in order to make them understand the concept of the definite area.

This high number of errors and misconceptions committed during this First Test Task can be attributed to the pre-requisite knowledge needed before doing deep calculus and definite integral. Radatz (1979) indicates that some errors arise as a result of lack of prerequisite knowledge and skills. There is clear evidence to this effect as most students could not draw simple graphs. Those who were able to draw, could not show basic information on these graphs and further could not perform simple integration.

6.3 Discussing Interviews and Feedback Sessions

The first student (student1) who was interviewed could neither draw nor integrate. He neither drew sketches nor integrated correctly. He disclosed that he does not know how to draw sketch graphs and how to integrate. He lacks both conceptual and procedural knowledge. The only little thing this student knows is that area can never be negative.

Student 2 drew graphs correctly but could not perform integration correctly. This student is better off as compared with the first student. This student is seen to be on the verge of breaking into full understanding if carefully helped. He/she understands how and why the sketch graph is drawn but does not know what information to show on the sketches. There is something positive to build on in helping this student. The situation with this student is not totally hopeless.

The third student (student 3) could do both drawings and integration but certain sketch graphs and certain functions posed challenges. Little help is needed to advance this student.

The fourth student is similar because he/she can integrate and draw graphs but has slight problems with conceptual understanding of the area bound by trigonometric graphs. Simple explanations made this student to fully understand his/her mistake.

The last set of students interviewed are what most teachers would like to have in his/her classroom. They do little mistakes and are too quick to understand where they went wrong.

The teacher does not have to dig so deep to make them understand. A little effort is needed to help these students out. These students understand why sketches must be drawn and what basic information should be shown on the sketches. They know the importance of limiting values and how to arrange them on integral sign. Like any other student, they have few grey areas where they need some help from the teacher to make them understand fully. They seem to have the necessary conceptual and procedural knowledge of how to go about solving the definite integral problems. The interviews provided deeper insight into students' thinking. Some students were totally lost and could not follow what was taking place. Most of them lacked both graph sketching skills as well as integration skills.

The whole class discussions which took the form of feedback session revealed yet other errors and misconceptions which could not be identified during the First-Test Task and the interview sessions. Two important things came to the fore during the feedback session. Firstly it is the confusion of y=2sinx with y=sin2x graphs and secondly the arrangement of limiting values on the integral sign. The amplitude and period concepts were explained clearly to the full satisfaction of the student during the intervention. The feedback session made student to believe in the integration as a means to calculate the area magnitude of irregular shapes.

6.4 Discussion of the Intervention Strategy used

ICT tools were used as an intervention strategy to improve the situation of errors and misconceptions. The intervention strategy of ICTs brought light into many areas that were not understood by students. All identified errors and misconceptions were brought forward for all student to see. The researcher organized all ICT tools needed to confront the errors and misconceptions shown in the First Test Task. Calculators were readily available as all students had these tools for rechecking the answers. A video on introductory and basic differentiation which served as pre-requisite for integral calculus was shown with great success. This video clip closed so many gaps of misunderstanding created during the normal lessons. Various videos on basic integration and introduction to definite integral were shown. These videos showed sketch graphs in all possible viewing angles. On the sketch graphs students could clearly see the area described by the question, representative strips shown, limiting values correctly shown and the naming of the sketches drawn. In one example, the tutor starts explaining how integration is used to calculate the total area of irregular figures by showing the method of squares with little parts of graphs left out in the estimation of the area.

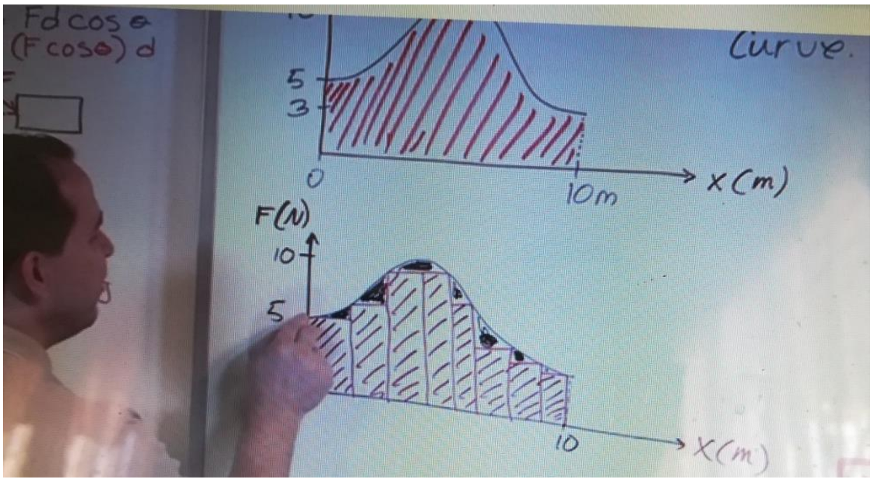


Figure 13: Introduction Explanations to Definite Integrals

This picture was taken from a You-Tube videos by stopping it and capturing this picture using a powerful cell phone and then the picture got uploaded here. This picture made a big difference in making students understand how integration is so powerful in calculating the area of irregular shapes. This picture was used to explain that the darker spots on top of rectangular blocks, are excluded in the first estimation of the total area. It was explained that by further dividing into finer and smaller rectangles, these darker spots disappear as they now form part of the rectangles and therefore the total area can be correctly calculated by means of integration where it still means the area of rectangle is length multiplied by breadth. This video was paused many times by students to get this explanation clear. At the end it helped a great deal.

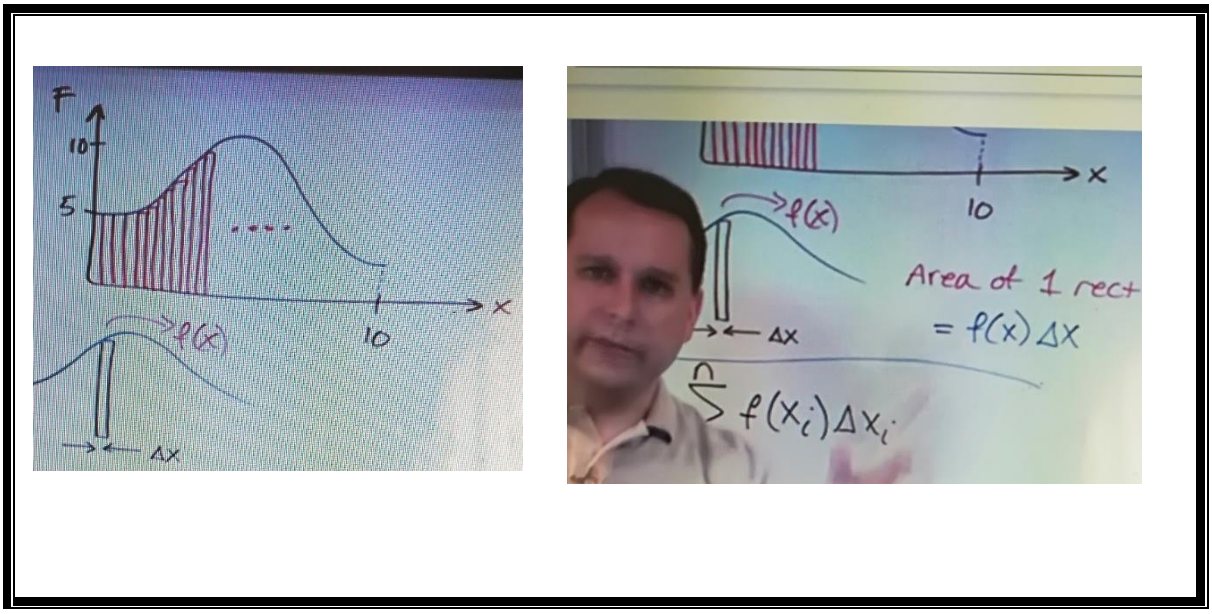


Figure 14: Dividing into finer and smaller rectangles

This picture was captured in the same way as the one above in figure 13. It was used to explain that the finer rectangles now cover the total irregular shape. The area of rectangles equal length times breadth where the length is the f(x) with dx as the breadth.

These videos gave good examples on the definite integrals with calculations done step by step to help students follow and see how it is done. Videos with different examples of different functions and graphs were viewed by students. These examples shown included those of exponential graphs, straight line graphs, parabola graphs, hyperbola graphs and many more of the prescribed graphs in N4 Mathematics syllabus. Enough time on successive days was given to students to view these videos. The good thing was that students would play these videos and listen to the tutors and where they needed clarity they paused for the researcher`s attention. Students also discussed amongst themselves with greater success.

The video watching, class discussions and the individual questions, made a big improvement on the learning of definite integral. Students were then ready to write the Follow up Test Task to check if the intervention strategy had effects on their understanding of definite integral.

6.5 Discussing the Follow-up Test Task

The Follow up Test Task revealed that there was definitely an improvement on many areas of definite integral learning. Students were able to draw correct graphs. They could now show the basic information on the graphs. They indicated the representative strips, named their graphs, shaded their areas and did many of the things they saw on videos.

Students were able to perform integration correctly. They were able to arrange the limiting values correctly. They got correct answers with the correct unit of area as seen in videos. All students passed the Follow up Test Task because they understood better after watching videos. This is compared to 75% fail obtained in the First Test Task. There is a great improvement as a result of the ICT intervention strategy. First Test Task yielded 25% pass and Follow-up Task gave 100% pass.

It is fair to conclude that the teaching of this section through the ICT tools has advanced students` understanding of the concept of definite integral to a greater extent. Keong, Horani and Daniel (2005) indicate that the use of ICT in teaching Mathematics can make the teaching process more effective as well as enhance the students’ capabilities in understanding basic concepts. These researchers believe that ICT promotes greater collaboration among students and encourages communication and the sharing of knowledge. Keong et al (2005) indicate that ICT tools can be used as a graphical visualising tool. As a class, we what Keong et al. (2005) say by browsing search engines on relevant `You Tube` videos. Videos were able to make students understand better on integration of different functions and drawing of different graphs. ICT tools used managed to make abstract concepts sound concrete through the use of graphical visualising tools. The use of ICTs in using `ready drawn sketch graphs` is more sensible than to draw from scratch with pencil and paper. It saves time and addresses issues of accuracy.

It is true that as seen by Ruthven et al (2004) that ICT tools make lessons more interesting to students as they arouse their interest because things are done differently. Students find computers exciting and they become happy when they are given a chance to be taught using a computer. According to Ruthven et al (2004), the use of ICT tools make mathematical activities more interesting, exciting and fun and exciting. This is exactly what happened in the lecture room where I was doing the intervention. ICTs usage promotes less hand writing which is often something that the low achievers are happy about as it overcomes difficulties in drawing accurate graphs (Ruthven et al., 2004).

6.6 Overall Discussions

Students at this level of study are introduced to integration for the first time and it is therefore likely that they will battle with integration and commit many errors and misconceptions in their attempt to understand the topic of calculus. Students could not perform integration of simple functions. Table 7 shows standard derivatives and integrals for simple functions. Students seem not to have mastered this table. This table 7 shows the pre-requisite knowledge that students should have acquired before learning the definite integral. Students show superficial knowledge of integration when faced with different functions e.g. in trying to integrate: $\int \frac{5}{x} dx$, students gave the answer as: $\frac{-5x^{-2}}{-2} + C$. This shows little knowledge of integration. The other situation is of the: $\int e^x dx$ which has been integrated as: $\frac{e^x}{xlne} + C$. This is a hopeless situation in as far as integration is concerned.

The Follow-up Test Task revealed few errors and misconceptions committed as compared to the First Test Task. There has been a lot of improvement in almost all error and misconception areas identified earlier. There were only 41 errors committed with the Follow-up Test Task as compared to 278 errors committed during First-Test Task. Students were able to draw sketch graphs correctly with basic information shown on the sketches. Integration was perfected, and all students passed this Follow up Test Task.

The ICT intervention proved to have improved the students ‘understanding of the definite integral. Errors committed during the Follow up Test Task were categorized in the same way as those of First Test Task. It is seen that less errors were committed during the Follow up Test Task compared to First Test Task. The confusion about the addition of the constant value `C` was cleared and only a few students continued to add `C` to the definite integral. Figures 7, a and b, show correct calculations of the definite integral with the magnitude of area given as a squared unit value. This is good practice. Students continued to improve their understanding of drawing sketch graphs.

Figures 8, a to d, show clear understanding of drawing sketch graph and showing on graphs basic information necessary for area calculations. For students to recognize that $y = \sqrt{x}$ is a parabola, they deserve some praise. There is great improvement in drawing sketches as compared to First Test Task graphs.

Figures 9. a to e, show improved understanding on what basic information should be shown on graphs. The representative strips are shown, the graphs are named and the Cartesian planes are suitably scaled. The piece of area to be calculated is clearly shown by a way of dotting or shading. Students improved on drawing sketches before doing calculations. Calculations are accompanied by correctly drawn sketches. This is good practice indeed.

Figures 10. a to d, show perfect way of approaching questions on definite integral. Sketches are drawn correctly with correct calculations done alongside or underneath. There is great improvement on this part of calculations accompanied by correct sketches.

The next figures 11, a to d, show integration well executed. The procedural steps are done correctly. This shows good understanding of the definite integral.

A zero area magnitude is still posing some challenges as few students still calculate their final answers as zero magnitude. The general picture of students who got zero area magnitude with First Test Task and Follow up Test Task reduced from 27 to just 2 students respectively. This is a great improvement.

The improved understanding shown with Follow up Test Task is as a result of re-teaching the topic of definite integral using ICTs. The fact that there were less errors committed during Follow up-Test Task is an indicator that re-teaching of the topic was effective to a greater extent. The ICT intervention strategy took the form of re-teaching the topic.

Constructivists believe that teachers do not go to class and teach errors and misconceptions to students but instead students construct their own understanding from what is taught by the teachers. According to Nesher (1987), teachers must welcome errors and mistakes students do. This researcher says that errors and misconceptions in students originate from previous lessons taught. This then makes us understand that errors and misconceptions happen just when students try to make sense of what the teacher is saying. Again, constructivists make us understand that students do not understand everything they are taught by their teachers simply because knowledge cannot be transmitted intact form the teacher to the students. Students listen and make sense of the lesson in their own way. This is true with this study because even after the re-teaching of topic by using ICT intervention strategy, students did not get 100% marks. This means that there are still gaps left open. It is also true that some students realized their errors and misconceptions after the re-teaching of the topic and this made them rectify and resolve some of their errors and misconceptions committed earlier.

The Follow up Test Task yielded an improvement in areas of procedural and conceptual knowledge. Researchers, Hiebert and Levefre (1986) argue that Mathematics is mainly about procedural and conceptual knowledge. They argue that students may not understand mathematics if these two concepts remain two separate entities and also if any is deficient.

Mofolo-Mbokane (2013) who conducted a research in TVET Colleges found that most challenges experienced by students emanates from lack of the mathematics register and the superficial knowledge of mathematics. He indicated that the students ` mathematics content knowledge is too low. This research study found that most students did not battle so much with the mathematics register but instead with the superficial knowledge of the calculus topic as well as the low mathematics content. Students could not draw sketch graphs correctly. They do not know what basic information to show on drawn sketches. They do not give names to their graphs. They do not show the representative strips. They neither shaded nor dotted areas. They do not understand integration at all because they lack integration techniques. Students experienced lot of challenges in answering questions in calculating area under graphs by using integration. This is an indicator that the pre-requisite knowledge into integral calculus is little and also weak. The arrangement of Modules in the TVET N4 Mathematics textbook is that the differentiation module precedes the integration module. This is because the differentiation should lay a foundation for integration. Understanding differentiation leads to understanding integration because these two concepts coincide.

6.7 Mathematical proficiency

Students have committed many errors and misconceptions in this study. Students' lack of pre-requisite knowledge in calculus led to them committing errors and misconceptions in this study. Students` low level of understanding calculus is a problem.

Thinking of this study from Kilpatrick et al. (2001) point of view, it is clear that students lack the five strands of mathematical proficiency. The errors and misconceptions reveal that students do not have conceptual understanding. They don't understand the concept of calculus.

According to Kilpatrick et al. (2001) it is true that students did not have a grasp of ideas about the intended lesson outcomes. Students could hardly draw correct graphs and do correct integration. Students show evidence of low levels of procedural fluency. They do not know when and how to use a particular rule for a solution. A typical example was when a student took down the function as it was and substituted the limiting values without first integrating the function in order to get the solution.

These first two strands of Kilpatrick et al. (2001) resonate with Hiebert and Levefre (1986) who argue that understanding mathematics is about procedural and conceptual knowledge. According to these researchers, students will find it difficult to understand mathematics if one of the two is deficient.

The fact that students could not solve these calculus problems, it means that they do not have the strategic competence. They also do not have adaptive reasoning because they could not navigate through the concept to check if their answers made sense or not. The zero area magnitude is a good example where students drew the graph, shaded the area and finally calculated the magnitude of this area to equal zero.

Students did not have both insight as well as grounded knowledge on historical development of the concept (pre-requisite knowledge of calculus). Students found this section difficult as a result of these situations (Sfard, 1991).

6.8 Conclusions

The analysis of data collected reveals so many errors and misconceptions committed by students. Data was analyzed taking into consideration the conceptual framework as well as the literature review. It came out clear that each stage of collecting data yielded more results. The First Test Task led to students committing many errors and misconceptions which were addressed during corrections and whole class discussions. The interview sessions as well as the whole class discussions, added more errors and misconceptions which were addressed during the intervention exercise. Finally, the Follow-up Test Task showed an improvement in the errors and misconceptions committed earlier.

CHAPTER SEVEN: FINDINGS, REFLECTIONS AND RECOMMENDATIONS

7.0 Introduction

The chapter gives the findings as well as the reflections and limitations of the study. The last section of this chapter is the recommendations. The research study investigated errors and misconceptions TVET N4 mathematics students do in learning about calculus. The study aimed at finding out about pre-requisite calculus knowledge in TVET College N4 mathematics students before learning deep calculus. The study further aimed at determining how ICT mediation would help advancing teaching of calculus. Lastly, the study wanted to discover the extent ICT mediation would advance teaching and learning of mathematics for TVET students.

7.1 Findings

In responding to the research question on pre-requisite knowledge about calculus, it is discovered that students lack this knowledge. Most students showed insufficient pre-requisite knowledge on calculus. Students could not perform simple integration and this is an indicator that they did not understand well the simple differentiation. Understanding differentiation makes the transition to understanding integration possible. Mathematical graphs sketching is a nightmare for most students as they could not draw most graphs correctly. One student appeared blank as he could not draw even the simplest straight line graph during the whole class discussions. The N4 Mathematics syllabus has a module on `Sketch Graphs` which is done before differentiation and integration. It is proven and can also be concluded that students did not understand and master the type of graphs dealt with in the module on `Sketch Graphs`. The differentiation, integration and their applications deal with graphs from this module. It suffices to conclude that students' pre-requisite knowledge about definite integral is insufficient. The research study found two key areas in students` lack of knowledge on: how to draw mathematical sketch graphs as well as the inability to integrate different functions correctly. Most students could not draw trigonometric graphs correctly. There is a serious confusion in identifying and drawing the graphs of y=2sinx and y=sin2x. The situation of the amplitude and the period with the graphs of y=asinbx and y=acosbx is confusing to students.

In drawing sketch graphs, students do not know what basic information to show on the graphs. The areas to be calculated were not shaded or dotted and the representative strips were not shown and drawn. Graphs were not named. In some instances, students left empty spaces wherein it is believed they intended to draw graphs but lacked the necessary knowledge to execute the solution.

As if the failure to draw mathematical sketch graphs was not enough, most students had problems with integration and therefore could not calculate the final area magnitudes correctly. These students seem not to be aware that different function types are integrated differently. There is no blanket approach in as far as integration is concerned.

On a little positive note, one of the findings that gives hope is the issue of language usage which appeared not to be a problem with most students. The students` mathematics register seemed to be appropriate for the level of study they are in. Students` general language use is excellent. No student complained about the use of language during all the phases of data collection. The language used in assessments was acceptable and suitable for both the students and the level of study.

Discussions above revolve around the students. Students cannot take the whole blame for errors and misconceptions in their attempt to learn and understand calculus. Some of the findings emanated from the lecturers side. The lecturers` old teaching methods also contributed to students committing errors and misconceptions in their learning of calculus. During my lecturer class room visits, I discovered that they were still using the old `talk and chalk` method of teaching. I call it the `talk and chalk` method even though they are using whiteboards because nothing is different in terms of the two scenarios. Lessons are lecturer-centred with students as passive participants. I also found that few active students were engaging with the lecturer but the majority were passive. The other thing I noticed is that the lecturers are relying on the prescribed textbook in their teaching of the subject. The textbook is the only available resource to use to plan for lessons. We know that textbooks are not perfect.

The other situation that seems to be true is the fact that some lecturers are not confident with teaching calculus. Luneta and Makonye (2010) indicate that in South Africa where skills development is at the heart of producing artisans, it is observed that teachers and learners in schools register many challenges in dealing with the topic of calculus. I concur with this statement as it is also one of my findings. The lecturer got extremely excited when I taught calculus to her students. She joined a few classes and indicated that she learned a few techniques on how to teach calculus.

The other research question is on the use of ICTs and how these ICTs advance teaching and learning of TVET N4 mathematics. It was discovered that ICT tools sparked great interest in learning calculus and definite integral. Students confessed their enjoyment in watching `You Tube` videos, which brought some kind of entertainment and fun, lots of meaningful learning and more of improved understanding on learning calculus. Students enjoyed pausing the video clips whenever they felt like discussing something seen or said by the tutor and also where they needed more explanations from the researcher. Surely ICTs must find a place within the teaching and learning of mathematics at TVET Colleges. My finding in using ICT tools is that students looked and behaved totally different to the ones taught by lecturers in old fashioned style. They displayed evidence of enjoying their lessons. They became active. They could discuss ideas and concepts. They were no longer passive as before. They got involved. The ICT tools came out tops and managed to curb and diminish errors and misconceptions committed earlier. A bigger percentage of errors and misconceptions were diminished and by so doing improving understanding on the part of students. The ICT tools raised bars in both procedural knowledge as well as the conceptual knowledge because students could afterwards draw and integrate correctly.

The greatest finding is that nothing can take the place of ICT tools in the teaching and learning of calculus. This is not to say that ICT tools should replace the lecturers. Lecturers must make full use of ICT tools to advance their students ‘understanding of mathematics topics. It is about time that teachers and lecturers use ICT tools predominantly in their teaching of calculus. For those students who are seeing integration for the first time in their schooling years, it is likely for them to struggle with the understanding of this topic, ICT tools have the solution. The superficial level at which calculus is done at matric level is not enough foundation to lay anti-derivatives (integration) to students, ICT tools have the solution. Most researchers like Ferrini-Mundy and Lauten (1994), Kedibone (2011), Orton (1983) and Radatz (1979) indicated that visual information is quite necessary in the study of calculus since it is involving graphs and to this challenge, ICT tools offer just the solution these researchers are talking about. It is because ICT tools come with visual representations and detailed sketches with full step by step explanations and calculations done by subject experts. The `pencil and paper` method used, is a technique prone to making students commit errors and misconceptions, which is by far inferior to ICT strategies of teaching.

7.2 Reflections and Limitations of Study

This research study was done at the level of master at one of the best universities in the country and also in the world. The researcher`s developing experience of conducting research is in itself a challenge capable of bringing some limitations on how to execute the task effectively and quite appropriately.

The research took place in a TVET College with a calendar of about eleven weeks per trimester. It is know that there are three trimesters in every academic year. The effective eleven weeks of a trimester are broken into registrations, teaching and examination times. The time for effective teaching and learning is about only seven weeks. The first and last two weeks of a trimester are reserved for registrations and examinations respectively. This situation was already a challenge in terms of collecting data by giving the First-Test Task, doing the interviews with six students, doing corrections and giving feedback with further re-teaching of the topic and lastly administering the Follow-up Test Task. There was hardly enough time to do all these exercises in this short space of time. This posed a serious challenge when the Head of Division had to authorize the time for the researcher and the students to meet and agree about the processes of research study. This time constraints necessitated for students to avail themselves during their free keys and lunch times.

The other serious challenge was when the researcher had to be granted permission to conduct research at this particular campus. The application forwarded to the office of the principal, for some reasons took too long to be resolved. The permission was granted just when students were about to prepare for their final examinations. This was during the time students had already built on their term marks and they were taking their little time left before they sit for examinations to relax a bit. The study was then inserted into that slot where students were little relaxed. The researcher had asked the register lecturer to leave out the section on integral calculus on agreement that he (the researcher) will teach it. The teaching of this topic happened as per the researcher`s plan for collecting data. There was indeed little time to carry out the research study.

The other factor is that the sample for this study is small and therefore may not be generalized to all TVET N4 Mathematics students in the country. The college whose students are sampled for the study is where the researcher is working. There can be issues of biasness in some processes of the study. The other factor was that of not probing students too deep during the interview sessions. Time constraints was a reason.

The researcher works as Head of Division (HOD) at the campus where the study was conducted. This may somehow have an influence on the part of students in terms of giving their consent thinking that by not giving it, they may then somehow suffer the consequences in their study of the subject. Some may give concerned just to impress the HOD and may appear as cooperative students where else they are not interested in taking part in the study.

The other challenge was that of having to give this study full attention when at the same time HOD responsibilities had to be attended to. At times students were left to discuss amongst themselves with the brighter students put in charge of the discussions and surprisingly the sessions turns out to be more educative. This was an eye opener that students can help each other learn more meaningfully. The HOD`s situation had its toll on the researcher when he had to find time to mark both pieces of Tasks. There was no way the researcher could have asked for assistance in marking the Tasks as this could have reduced the chances of spotting and seeing errors and misconceptions committed by students.

7.3 Recommendations

The research study made it clear that the researcher intended to help students improve their understanding of TVET calculus in Mathematics N4. In order to achieve this great idea, the exact errors and misconceptions must first be identified. According to Olivier (1989), errors and misconceptions are student`s efforts to intelligently construct their own knowledge and errors and misconceptions should therefore not be uprooted and avoided as they are part of the learning process. Instead, they should be welcomed by both teachers and students. Inappropriate and radical uprooting of students' errors and misconceptions may shake the student`s confidence and cause confusion (Olivier, 1989).

This research study welcomed the idea that locating student`s errors and misconceptions on a particular topic of study (calculus) and further probing, help students realize their problems and difficulties. It makes sense to believe that hammering and attending to the identified errors and misconceptions in a particular area of study, improves the students` understanding of the topic. Nesher (1987) strongly argues that teachers should welcome and tolerate errors and misconceptions and most importantly use them as feedback mechanism for effective learning. She argues further that the type of teaching that has no intention to detect errors and misconceptions is like shooting in the dark. It is easy to miss the target in the dark.

In light of these statements by researchers, this research study wants to explicitly give the following recommendations:

- ✓ Further research should be done on TVET N4 Mathematics students' background knowledge of calculus before learning deep and integral calculus. This area is not given much attention in this study since it is not the main area of focus. Students are weak in this area and therefore cannot proceed smoothly into deep and integral calculus. Student's knowledge of differentiation as well as integration indicated not to have a strong grip. Differentiation is the reverse of integration and vice versa and students cannot disjoint the two in their understanding.
- ✓ The ICT tools should be used in the teaching and learning of deep calculus in TVET Colleges. It is because these tools bring with them the advantage of visual representations of sketch graphs drawn by experts, introductory explanations to both differentiation and integration, definite integral examples done and solved step by step procedures, integration done correctly by experts etc. The ICT brings meaningful learning that take some form of entertainment and fun into the classroom and by so doing motivating students to learn. ICTs is used to stimulate interests and spark class discussions about the topic of study.

The teaching and learning of mathematics should pay special attention to student`s errors and misconceptions where an intervention strategy to help the students resolve these errors and misconceptions are always made the order of the day.

The intervention strategy should take the form of re-teaching or revisiting the topic or any form that may be found to be effective. This means that these errors and misconceptions must be identified and acknowledged and then be analysed in order to find an appropriate intervention strategy. When errors and misconceptions are not addressed this keep on affecting new learning in a negative and bad way. Further damage is continuing to deepen the misunderstanding of topics.

The teachers of TVET N4 Mathematics should make sure that before they teach definite integral, they diagnose students' background knowledge. This will help them to know what they are dealing with unlike if they were to realize at the end of the current topic that students are less knowledgeable. The one focus should be on the sketch graphs. Students should know how to draw all graphs prescribed for N4 Mathematics. They should be able to recognize basic shapes of graphs by analysing the standard formulae of graphs. For example, $y = 3x-5$, should quickly be seen and understood as a straight line graph with a positive gradient. They need not rely on tables for sketching graphs.

Teachers of mathematics and calculus in particular, must make it a habit of giving enough exercise questions on integrating techniques. Students should be made aware that different functions are integrated differently and there is no blanket approach. In this way, calculus will be best understood by students.

7.4 Reflections

This chapter concludes all processes involved in the research study from chapter one to chapter seven. Chapter one gives the background and the context of the study followed by chapter two on conceptual framework which considered some of the relevant grand theories and frameworks in the teaching and learning of mathematics. These grand theories and frameworks serve as the backbone of the research study. Literature review is explained in chapter three. This literature serves as the bloodstream of the research study as it is found in almost every part of the study. Literature review is followed by chapter four on methodology. Methodology chapter outlines the types of methods used in data collection. This data is analyzed in chapter five and discussed in chapter six. Chapter seven gives the discussions on findings, the reflections and limitations of the study and lastly it is the recommendations.

This study has been an eye opener in many ways. Recommendations given in this study should be a 'must implement' if mathematics teaching and learning is to be improved in TVET colleges.

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9: APPENDICES

Appendix 1: Ethics clearance from the Witwatersrand University

Wits School of Education

WITS UNIVERSITY

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

10 October 2017

Student Number: 445465

Protocol Number: 2017ECE038M

Dear Mmachuene William Hlako

Application for Ethics Clearance: Master of Science

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate, has considered your application for ethics clearance for your proposal entitled:

ICT interventions in TVET Mathematics N4 integral calculus.

The committee recently met and I am pleased to inform you that **clearance was granted**.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

MMabeta

Wits School of Education

011 717-3416

cc Supervisor - Dr Judah Makonye

Appendix 2: Letter to College Principal requesting permission to carry out research.

2449 Masinga Street
Tsakane
1550

The Principal
Ekurhuleni East TVET College
Sam Ngema Drive
Kwa-Thema
1575
26 June 2017

REQUEST FOR PERMISSION TO CONDUCT RESEARCH IN EKURHULENI EAST (TVET) COLLEGE DAVEYTON CAMPUS WITH REPORT 191 MATHEMATICS N4 STUDENTS.

Dear: Mrs. H Sibande

My name is Hlako M William. I am a second year student of Masters at the University Of Witwatersrand, School Of Education. This letter serves as a request to conduct research in your college at one of the campuses (Daveyton Campus).
I am doing research on Errors and Misconceptions that TVET Mathematics N4 students make and have in learning about integral calculus when calculating magnitudes of areas under graphs. The application of definite integral is an official topic for TVET college Mathematics N4 syllabus. It constitutes a bigger percentage of the passing mark for this level. The research aims at finding the type of errors and misconceptions students make and have in solving definite integral problems. The study is to be carried out on the Mathematics N4 students simply because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and throughout this time I have seen so many students committing lots of errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

The research will involve:

1. Two groups of Mathematics N4 students writing a test (First Test) of five carefully selected questions about the baseline and prerequisite integral calculus knowledge for 1h15 minutes. There will be eighty students at the most since each group has forty students. In writing this test, students will be instructed to use their student numbers as a way of identification on their answering scripts. No names will be allowed. These groups are the Mechanical and Electrical Engineering students whose official time table shows that they are attended to by their lecturer

during the first and third periods respectively. The task will be written during the students `normal periods as reflected on the official campus time table. After the scripts shall have been marked, close attention will be paid to errors and mistakes done and these errors will then be categorized and classified for analysis.

2. The First Test will be followed by interviews with selected students. These students will be informed about the confidentiality of the information they will be giving that it will be documented and stored in lockable cabinet in my office at work. About six students with special and interesting cases based on the types of errors made, will be interviewed so as to deepen the understanding of the type of errors and misconceptions they have on the indicated section. This exercise is done to probe and elicit their thinking on the errors they shall have made in the task. All errors identified will be brought to class for discussions with all students. Teaching, which is based on these errors and misconceptions, will then follow in order to address the identified errors and misconceptions. ICT technologies will be used to aid understanding in students.
3. The Follow up Test with about five questions on the same topic will be given to the same students to check if the intervention strategy has been effective in addressing identified errors and misconceptions. It will be written over a period of one and quarter hours by the same students. The scripts will be marked and taking particular note to the errors made earlier will be taken in order to see if they still exist. The extent to which the teaching strategy has been effective will then be determined by using quantitative measures. This test is anonymous and students will be expected to use their student numbers and no real name will be allowed.

The First and Follow up Test marks will not be used as term test marks. Students will be reassured that they can withdraw their permission at any time during this project without any penalty should they feel to withdraw. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. However, the participants will in turn benefit from the study since this is an official topic in the syllabus. This will have positive impact on the Mathematics N4 pass rate. The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing from the study. Your individual privacy as the college principal will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

May I please know if your office will require any further information?

I look forward to your positive response as soon as it is convenient to you.

Yours faithfully

Hlako MW

Phone: 073 1551 535

Signature: ------------------------------ Date: 26/06/2017 ------------------------------

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: **Judah.Makonye@wits.ac.za**

Appendix 3: Letter of Request to do Research in a TVET College sent to Department of Higher Education.

higher education
& training
Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

DHET 004: APPENDIX 1:

APPLICATION FORM FOR STUDENTS TO CONDUCT RESEARCH IN PUBLIC COLLEGES

1. APPLICANT INFORMATION

1.1.	Title (Dr /Mr /Mrs /Ms)	Mr	
1.2	Name and surname	Mmachuene William Hlako	
1.3	Postal address	2449 Masinga Street Tsakane Brakpan	
1.4	Contact details	Tel	0117321094
		Cell	0731551535
		Fax	
		Email	velihlako@gmail.com
1.5	Name of institution where enrolled	Witwatersrand University	
1.6	Field of study	Humanities/Education	
1.7	Qualification registered for	*Please mark relevant option:*	
		Doctoral Degree (PhD)	
		Master's Degree	X
		Other (please specify)	

2. DETAILS OF THE STUDY

2.1	Title of the Study

ICT interventions in TVET Mathematics N4 integral calculus.

2.2 Purpose of Study

P
o
s

The research intends to find errors and misconceptions Technical and Vocational Education and Training (TVET) College Mathematics N4 Students do and have in integral calculus when answering questions on the definite integral. The research will be taking place at Ekurhuleni East TVET college in Daveyton Campus. The participants will be students from two N4 engineering groups i.e. Electrical group (N4E1) and Mechanical group (N4M1). Each group has 40 students. Calculus constitute 40 percent of the final N4 Mathematics examination. Calculus done at this level is introductory as well as foundational. The next two levels i.e. N5 and N6 Mathematics are 100 % calculus. If the students can minimise on the errors and misconceptions they are doing in N4

Mathematics, they will start mastering Calculus and then score good marks in their examinations. It is highly likely that they will find their next level N5 and N6 Mathematics easier. The type of errors and misconceptions students do and have in this section, will make me to devise and implement a teaching technique to address these learner difficulties. I intend using Information Communication Technology (ICT) in intervening on the errors and misconceptions students do in this definite integral. This research will be guided mainly by the three questions: 1. What baseline calculus prerequisites TVET College N4 students have before learning integral calculus? 2. How ICT mediation may help to advance teaching integration? 3. To what extend does ICT's mediated teaching and learning advance mathematics learning in a TVET College? Students will first have a lesson on Calculus. The intention of the lesson is to find out if students have prerequisite knowledge about integration. Students will thereafter be given a test to collect this data. This test will ask questions on calculus prerequisites and definite integral questions on calculations of area under graphs. This test will then be marked and feedback given to students. Interviews with interesting cases will be arranged. The whole class group discussion will be arranged to deepen insight into errors and misconceptions students have. ICT interventions will be done. You Tube videos showing graphs with area calculations will be used to address errors and misconceptions on definite integral. Follow up test will be given to see if ICT intervention strategy was effective. The findings will be documented and kept. Strategies found to be effective will be shared with fellow lecturers who are teaching Mathematics. The intention is to improve the results of Mathematics N4 as a subject in the TVET Colleges. If given a chance, I will address conference about my findings with further intentions of publishing in the journal.

DHET 004: APPENDIX 1: APPLICATION FORM FOR STUDENTS TO CONDUCT RESEARCH IN PUBLIC COLLEGES

3. PARTCIPANTS AND TYPE/S OF ACTIVITIES TO BE UNDERTAKEN IN THE COLLEGE

Please indicate the types of research activities you are planning to undertake in the College, as well as the categories of persons who are expected to participate in your study (for example, lecturers, students, College Principals, Deputy Principals, Campus Heads, Support Staff, Heads of Departments), including the number of participants for each activity.			
3.1	Complete questionnaires	**Expected participants (e.g. students, lecturers, College Principal)**	**Number of participants**
		a) Students	80
		b)	
		c)	
		d)	
		e)	
3.2	Participate in individual interviews	**Expected participants**	**Number of participants**
		a) Students	10
		b)	
		c)	
		d)	
		e)	
3.3	Participate in focus group discussions/ workshops	**Expected participants**	**Number of participants**

		a) Students	80
		b)	
		c)	
		d)	
		e)	
3.4	**Complete standardised tests (e.g. Psychometric Tests)**	**Expected participants**	**Number of participants**
		a) Students	80
		b)	
		c)	
		d)	
		e)	
3.5	**Undertake observations** *Please specify*	Not applicable	
3.6	**Other** *Please specify*		

4. SUPPORT NEEDED FROM THE COLLEGE

Please indicate the type of support required from the College (Please tick relevant option/s)			
Type of support		*Yes*	*No*
4.1	The College will be required to identify participants and provide their contact details to the researcher.	X	
4.2	The College will be required to distribute questionnaires/instruments to participants on behalf of the researcher.		X

4.3	The College will be required to provide official documents. ***Please specify the documents required below***	X	
	Registers, mark-sheets for tests records.		
4.4	The College will be required to provide data *(only if this data is not available from the DHET).* ***Please specify the data fields required, below***	X	
	Students age range, sex i.e. whether males or females, academic history of students, contact details.		
4.5	***Other, please specify below***		

5. DOCUMENTS TO BE ATTACHED TO THE APPLICATION

The following 2 (two) documents must be attached as a prerequisite for approval to undertake research in the College	
5.1	Ethics Clearance Certificate issued by a University Ethics Committee Yes
5.2	Research proposal approved by a University Yes

6. DECLARATION BY THE APPLICANT

I undertake to use the information that I acquire through my research, in a balanced and a responsible manner. I furthermore take note of, and agree to adhere to the following conditions:

a) I will schedule my research activities in consultation with the said College/s and participants in order not to interrupt the programme of the said College/s.

b) I agree that involvement by participants in my research study is voluntary, and that participants have a right to decline to participate in my research study.

c) I will obtain signed consent forms from participants prior to any engagement with them.

d) I will obtain written parental consent of students under 18 years of age, if they are expected to participate in my research.

e) I will inform participants about the use of recording devices such as tape-recorders and cameras, and participants will be free to reject them if they wish.

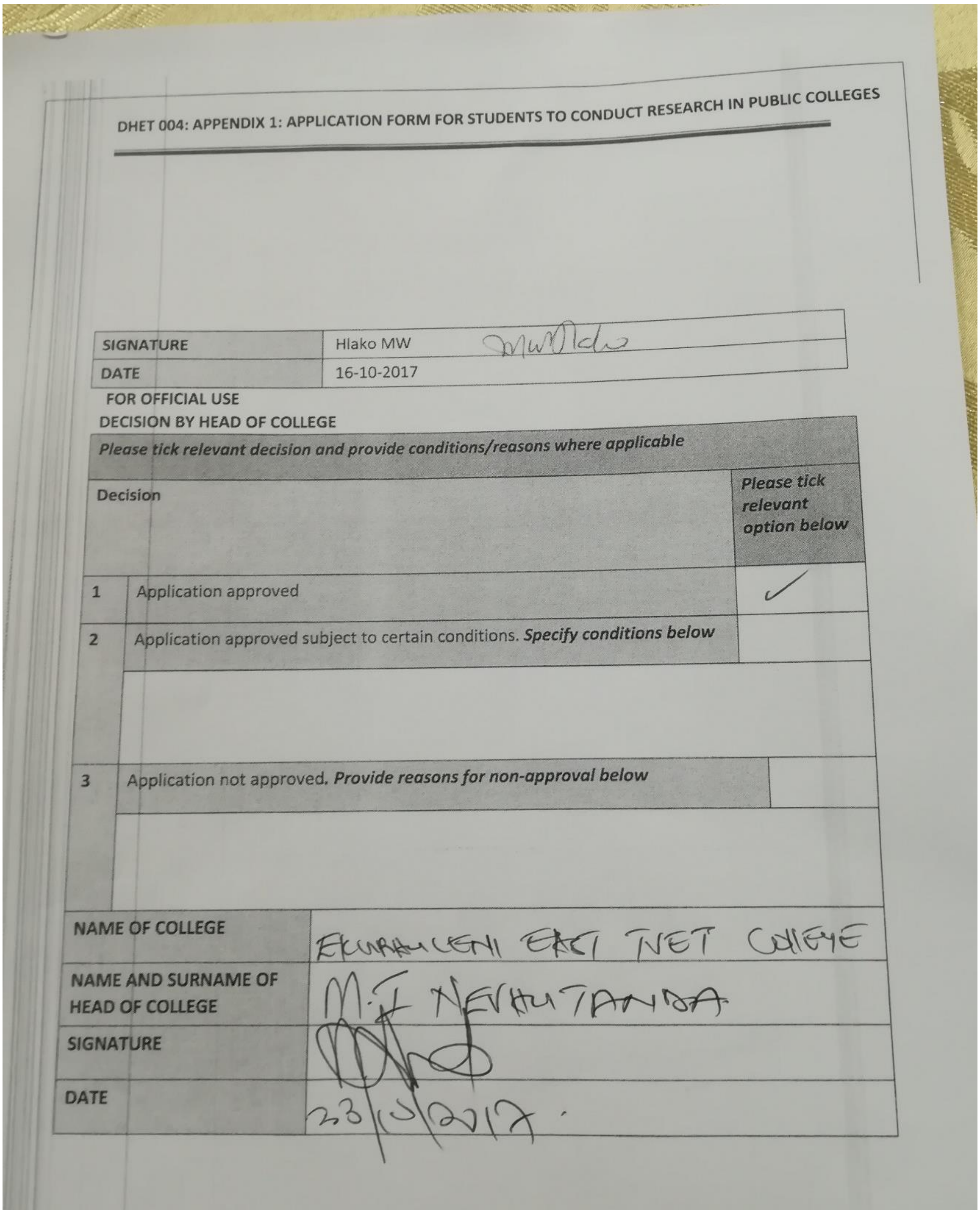
f) I will honour the right of participants to privacy, anonymity, confidentiality and respect for human dignity at all times. Participants will not be identifiable in any way from the results of my research, unless written consent is obtained otherwise.

g) I will not include the names of the said College/s or research participants in my research report, without the written consent of each of the said individuals and/or College/s.

h) I will send the draft research report to research participants before finalisation, in order to validate the accuracy of the information in the report.

i) I will not use the resources of the said College/s in which I am conducting research (such as stationery, photocopies, faxes, and telephones), for my research study.

j) Should I require data for this study, I will first request data directly from the Department of Higher Education and Training. I will request data from the College/s only if the DHET does not have the required data.



DHET 004: APPENDIX 1: APPLICATION FORM FOR STUDENTS TO CONDUCT RESEARCH IN PUBLIC COLLEGES

SIGNATURE	Hlako MW	MWHlako
DATE	16-10-2017	

FOR OFFICIAL USE

DECISION BY HEAD OF COLLEGE

Please tick relevant decision and provide conditions/reasons where applicable		
Decision		***Please tick relevant option below***
1	Application approved	✓
2	Application approved subject to certain conditions. ***Specify conditions below***	
3	Application not approved. ***Provide reasons for non-approval below***	

NAME OF COLLEGE	EKURHULENI EAST TVET COLLEGE
NAME AND SURNAME OF HEAD OF COLLEGE	M.F NEVHUTANDA
SIGNATURE	
DATE	23/10/2017.

Appendix 4: Permission letter by college principal

higher education & training
Department: Higher Education and Training
REPUBLIC OF SOUTH AFRICA

EKURHULENI EAST TVET COLLEGE
"Committed to Excellence"

HEAD OFFICE
Sam Ngema Road
Kwa-Thema, Springs
Tel: (011) 730 6600
Fax: (011) 736 9909
Email: info@eec.edu.za
Private Bag X52
SPRINGS
1560

Date: 23 October 2017 2017

PERMISSION LETTER BY THE PRINCIPAL OF THE COLLEGE

Permission from the Ekurhuleni East TVET College (EEC) to carry out the proposed research study in Daveyton Campus

Title of the Research: Information Communication Technology (ICT) interventions in TVET College N4 Mathematics in learning integral calculus.

I, Mbengeni Itumeleng Nevhutanda, the Deputy Principal Academic Affairs of

Ekurhuleni East TVET College, have read and understood the contents of the letter seeking permission for doing research on Mathematics N4 students on the errors and misconceptions they have in learning about definite integral.

I therefore permit Mr MW Hlako to do the above proposed research study at Daveyton Campus of EEC.

Signature:

Signed at Kwa-Thema Central Office on this day of 23 October 2017

E-mail: info@eec.edu.za Website: www.eec.edu.za

ARTISAN AND SKILLS DEVELOPMENT CENTRE
10 Argon Street
Fulcrum, Springs
Tel: (011) 730 6600
Fax: (011) 736 9909
Email: info@eec.edu.za
Private Bag X52
SPRINGS
1560

BENONI CAMPUS
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Fax: (011) 425 3439
Email: info@eec.edu.za
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BENONI
1500

DAVEYTON CAMPUS
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Private Bag X01
DAVEYTON
1507

BRAKPAN CAMPUS
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Brakpan
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Fax: (011) 740 9188
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BRAKPAN
1540

KWA-THEMA CAMPUS
Sam Ngema Rd.
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Tel: (011) 730 6600
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Email: info@eec.edu.za
Private Bag X79
SPRINGS
1560

SPRINGS CAMPUS
Plantation Rd.
Springs
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Fax: (011) 362 6182
Email: info@eec.edu.za
Private Bag X21
SPRINGS
1560

Appendix 5: Signed information sheet for students

INFORMATION SHEET FOR STUDENTS DATE: 23 October 2017

Dear Student

My name is Hlako MW and I am a second year Masters student at the University of the Witwatersrand. I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students simply because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

My investigation involves establishing baseline and prerequisite knowledge about integral calculus by firstly teaching this topic and later giving a test to check the understanding. A test of five questions will be given. This test will be marked with all errors recorded. Selected students will be invited for interviews about their work. I will be the only interviewer to the students.

The errors will be brought to the whole class for discussions. Teaching strategy will be devised to diminish these errors and misconceptions. ICT will be the centre of focus in addressing the errors made. Finally a follow up test will be given to check the final understanding of the topic and whether the strategy has been effective. Feedback will be given to students in all cases.

Remember, these are not replacing term tests, they are not for marks and it is voluntary, which means that you don't have to do it if you feel like. Also, if you decide halfway through that you prefer to stop, this is completely your choice and will not affect you negatively in any way. I will not be using your own name but I will use pseudonyms so no one can identify you. All information about you will be kept confidential in all my writing about the study. Also, all collected information will be stored safely and destroyed between 3-5 years after I have completed my project.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Hlako MW

Phone: 073 1551 535

Info – sheet.

Signature: ------------------------------------ Date: 26/10/2017 ----------------------------

Student Full Names and Surname: THANDOKAZI PORTIA NGUNDU ------------------------

Student Number: --

Student Contact Details/Cell phone number: 063 323 1946 --------------------------

Student Signature: TP --

Date of signature: 2017/11/05 --

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

INFORMATION SHEET FOR STUDENTS DATE: 23 October 2017

Dear Student

My name is Hlako MW and I am a second year Masters student at the University of the Witwatersrand. I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students simply because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

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I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Hlako MW

Phone: 073 1551 535

Signature: ----------------------------- Date: 26/6/2018

Student Full Names and Surname: TSHILIDZI NEMAMILWE

Student Number: 9712176331081

Student Contact Details/Cell phone number: 0720692130

Student Signature: -----------------------------

Date of signature: 06/11/2017

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

INFORMATION SHEET FOR STUDENTS DATE: 23 October 2017

Dear Student

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I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Hlako MW

Phone: 073 1551 535

Signature: -- Date: 26/10/2017 ----------------------

Student Full Names and Surname: THANGAVHUELELO MUGANO

Student Number: ~~0610582015~~ 9407290438082

Student Contact Details/Cell phone number: 0610682015

Student Signature: --

Date of signature: 06/11/2017

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

INFORMATION SHEET FOR STUDENTS DATE: 23 October 2017

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I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

Hlako MW

Phone: 073 1551 535

Signature: .... Date: 16/10/2017

Student Full Names and Surname: THOMAS SILUCIA

Student Number: 9207025553084

Student Contact Details/Cell phone number: 0748417969

Student Signature: ....

Date of signature: 06/11/2017

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: judah.Makonye@wits.ac.za

Appendix 6: Information sheets for the teachers

INFORMATION SHEET TEACHERS

Date: 26 June 2017

Dear Mrs. Ditsego VH

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

The research will involve:

- Two groups of Mathematics N4 students writing a test (First Test) of five carefully selected questions about the baseline and prerequisite integral calculus knowledge for one and quarter hours. In writing this test, students will be instructed to use their student numbers as a way of identification on their answering scripts. No names will be allowed. These groups are the Mechanical and Electrical Engineering students whose official time table shows that they are attended to by their lecturer during the first and third periods respectively. The task will be written during the students 'normal periods as reflected on the official campus time table. After the scripts shall have been marked, close attention will be paid to errors and mistakes done and these errors will then be categorized and classified for analysis.
- The First Test will be followed by interviews with selected students. About six students with special and interesting cases based on the types of errors made, will be interviewed so as to deepen the understanding of the type of errors and misconceptions they have on the indicated section. This exercise is done to probe and elicit their thinking on the errors they shall have shown in the task. All errors found and identified will be brought to class for discussions with all students. Teaching, which is based on these errors and misconceptions, will then follow in

order to address the identified errors and misconceptions. ICT technologies will be used to aid understanding in students. These students will be informed about the confidentiality of the information they will be giving that it will be documented and stored in lockable cabinet in my office at work.

The Follow up Test with about five questions on the same topic will be given to same students to check if the intervention strategy worked. It will be written over a period of one and quarter hours by the same students. The scripts will be marked and again taking particular note to the errors made earlier in order to see if they still exist. The extent to which the teaching strategy has been effective will then be determined by using quantitative measures. This test is anonymous and students will be expected to use their student numbers and no real name used.

The research participants which in this case are students will not be advantaged or disadvantaged in any way. The First and Follow up Test marks will not be used as term test marks. Students will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The participants will in turn benefit from the study since this is an official topic in the syllabus.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing from the study. Your individual privacy as a campus manager will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: ....   Date: 26/06/2017

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

Appendix 7: Teachers' consent form

TEACHER`S CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to be a participant in the research study I am doing with the University of Witwatersrand.

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

TEACHER`S INFORMED CONSENT:

I, (full names of teacher) ---VIVIENNE H. DITSEGO-----------------------

Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do. Yes/No
- I have received, read and understand the information sheet about the study. Yes/No
- I agree to be observed when teaching the baseline as well as the prerequisite integral calculus knowledge. Yes/No
- I do not have to answer every question and can withdraw from the study at any time. Yes/No
- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study. yes/No
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project. Yes/No

Teacher Signature ________ Date 26/06/17

Teacher Contact Details/Cell phone Number: 083 770 8186

INFORMATION SHEET TEACHERS

Date: 26 June 2017

Dear Mrs. Nadason S

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

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order to address the identified errors and misconceptions. ICT technologies will be used to aid understanding in students. These students will be informed about the confidentiality of the information they will be giving that it will be documented and stored in lockable cabinet in my office at work.

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The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing from the study. Your individual privacy as a campus manager will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: .... Date: 26 June 2017

Research Supervisor:

Dr Judah Makonye

Office: 011 717 4506

TEACHER`S CONSENT FORM

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I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

TEACHER`S INFORMED CONSENT:

I, (full names of teacher) ---Salome Nadason---

Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do. Yes/No
- I have received, read and understand the information sheet about the study. Yes/No
- I agree to be observed when teaching the baseline as well as the prerequisite integral calculus knowledge. Yes/No
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- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study. yes/No
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project. Yes/No

Teacher Signature ____________ Date 26 June 2017

Teacher Contact Details/Cell phone Number: 072 408 3430

INFORMATION SHEET TEACHER/S

Date: 19 October 2017

DIYASE BALLENG OTHELIA

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

The research will involve:

4. Two groups of Mathematics N4 students writing a test (First Test) of five carefully selected questions about the baseline and prerequisite integral calculus knowledge for one and quarter hours. In writing this test, students will be instructed to use their student numbers as a way of identification on their answering scripts. No names will be allowed. These groups are the Mechanical and Electrical Engineering students whose official time table shows that they are attended to by their lecturer during the first and third periods respectively. The task will be written during the students `normal periods as reflected on the official campus time table. After the scripts shall have been marked, close attention will be paid to errors and mistakes done and these errors will then be categorized and classified for analysis.
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Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: ---------------------------------- Date: 08/10/2018

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

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TEACHER`S INFORMED CONSENT:

I, (full names of teacher) DIJASE BALLENG OTHELIA

Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do. Yes/No
- I have received, read and understand the information sheet about the study. Yes/No
- I agree to be observed when teaching the baseline as well as the prerequisite integral calculus knowledge. Yes/No
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- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study. yes/No
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project. Yes/No

Teacher Signature Date 08/10/18

Teacher Contact Details/Cell phone Number: 0846691393

INFORMATION SHEET TEACHER/S

Date: 19 October 2017

CONFidence Kgomotso Madzwigi

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

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Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: ---------------------------------- Date: ---08/10/2018----------------

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

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I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

TEACHER`S INFORMED CONSENT:

I, (full names of teacher) CONFIdence Kgomotso Madzwigi

Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do. Yes/No
- I have received, read and understand the information sheet about the study. Yes/No
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- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study. Yes/No
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project. Yes/No

Teacher Signature________ Date 08/10/18

Teacher Contact Details/Cell phone Number: 076 6575 748

INFORMATION SHEET TEACHER/S

Date: 19 October 2017

S'bongile Olphie Masuku

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

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The research participants which in this case are students will not be advantaged or disadvantaged in any way. The First and Follow up Test marks will not be used as term test marks. Students will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The participants will in turn benefit from the study since this is an official topic in the syllabus.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing from the study. Your individual privacy as a campus manager will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: ----------------------------------- Date: 08/10/2018

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

TEACHER'S CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to be a participant in the research study I am doing research with the University of Witwatersrand.

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

TEACHER'S INFORMED CONSENT:

I, (full names of teacher) ----- S.O. Masuku -----

Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do.
  Yes/No
- I have received, read and understand the information sheet about the study.
  Yes/No
- I agree to be observed when teaching the baseline as well as the prerequisite integral calculus knowledge.
  Yes/No
- I do not have to answer every question and can withdraw from the study at any time.
  Yes/No
- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study.
  yes/No
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project.
  Yes/No

Teacher Signature ____________ Date 08/10/2018

Teacher Contact Details/Cell phone Number: 0722556787

INFORMATION SHEET TEACHER/S

Date: 19 October 2017

Theressa Busisiwe Tsoka

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

The research will involve:

4. Two groups of Mathematics N4 students writing a test (First Test) of five carefully selected questions about the baseline and prerequisite integral calculus knowledge for one and quarter hours. In writing this test, students will be instructed to use their student numbers as a way of identification on their answering scripts. No names will be allowed. These groups are the Mechanical and Electrical Engineering students whose official time table shows that they are attended to by their lecturer during the first and third periods respectively. The task will be written during the students `normal periods as reflected on the official campus time table. After the scripts shall have been marked, close attention will be paid to errors and mistakes done and these errors will then be categorized and classified for analysis.
5. The First Test will be followed by interviews with selected students. About six students with special and interesting cases based on the types of errors made, will be interviewed so as to deepen the understanding of the type of errors and misconceptions they have on the indicated section. This exercise is done to probe and elicit their thinking on the errors they shall have shown in the task. All errors found and identified will be brought to class for discussions with all students. Teaching, which is based on these errors and misconceptions, will then follow in

order to address the identified errors and misconceptions. ICT technologies will be used to aid understanding in students. These students will be informed about the confidentiality of the information they will be giving that it will be documented and stored in lockable cabinet in my office at work.

The Follow up Test with about five questions on the same topic will be given to same students to check if the intervention strategy worked. It will be written over a period of one and quarter hours by the same students. The scripts will be marked and again taking particular note to the errors made earlier in order to see if they still exist. The extent to which the teaching strategy has been effective will then be determined by using quantitative measures. This test is anonymous and students will be expected to use their student numbers and no real name used.

The research participants which in this case are students will not be advantaged or disadvantaged in any way. The First and Follow up Test marks will not be used as term test marks. Students will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The participants will in turn benefit from the study since this is an official topic in the syllabus.

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Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: ---------------------------------- Date: 08/10/2018

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

TEACHER`S CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to be a participant in the research study I am doing research with the University of Witwatersrand.

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

TEACHER`S INFORMED CONSENT:

I, (full names of teacher) Theressa Busisiwe Tsoka

Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do. Yes/No
- I have received, read and understand the information sheet about the study. Yes/No
- I agree to be observed when teaching the baseline as well as the prerequisite integral calculus knowledge. Yes/No
- I do not have to answer every question and can withdraw from the study at any time. Yes/No
- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study. yes/No
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project. Yes/No

Teacher Signature ________ Date 08/10/2018

Teacher Contact Details/Cell phone Number: 072 231 9936

INFORMATION SHEET TEACHER/S

Date: 19 October 2017

MOSHIA ANNAH CHO~~F~~KOE

My name is Hlako M William. I am a second year Masters student in the School of Education at the University of the Witwatersrand.

I am doing research on errors and misconceptions TVET Mathematics N4 students have in learning about integral calculus when calculating magnitudes of areas under graphs. The research aims to find the type of errors and misconceptions students have in solving these definite integral problems. The study is to be carried out on the Mathematics N4 students because I realized that these students do not have a good grasp on this calculus topic. I have taught TVET N4 Mathematics for over thirty trimesters and over this time I have seen so many students committing so many errors and displaying misconceptions when learning calculus. I strongly believe that diagnosing these students' errors and misconceptions about solutions of the definite integral can help in devising best strategies to teach this concept better.

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The research participants which in this case are students will not be advantaged or disadvantaged in any way. The First and Follow up Test marks will not be used as term test marks. Students will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks in participating in this study. The participants will not be paid for this study. The participants will in turn benefit from the study since this is an official topic in the syllabus.

The names of the research participants and identity of the school will be kept confidential at all times and in all academic writing from the study. Your individual privacy as a campus manager will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely

Hlako MW

Phone: 073 1551 535

Signature: amuNlal ------------------------------ Date: 08/10/2018

Research Supervisor:

Dr Judah Makonye
Office: 011 717 4506
Mobile: 078 689 4572
Email: Judah.Makonye@wits.ac.za

TEACHER`S CONSENT FORM

Please fill in and return the reply slip below indicating your willingness to be a participant in the research study I am doing research with the University of Witwatersrand.

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

TEACHER`S INFORMED CONSENT:

I, (full names of teacher) MOSHIA ANNA CHOKOE

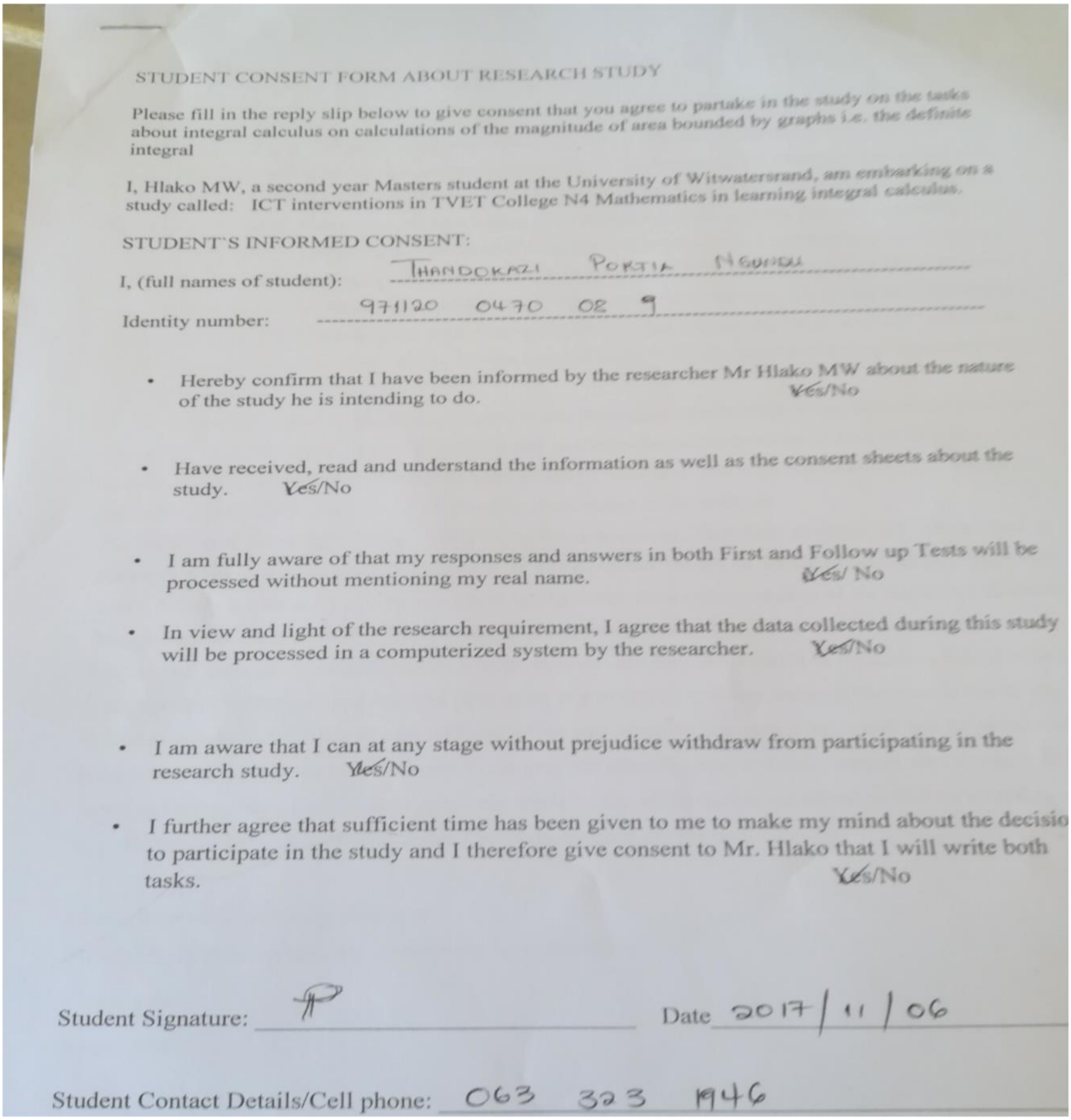
Give my consent for the following:

- I confirm that Mr. Hlako MW have explained to me about the nature of the study he is intending to do. **Yes/No**
- I have received, read and understand the information sheet about the study. **Yes/No**
- I agree to be observed when teaching the baseline as well as the prerequisite integral calculus knowledge. **Yes/No**
- I do not have to answer every question and can withdraw from the study at any time. **Yes/No**
- I agree that I will allow Mr. Hlako MW to teach, give tests to the groups identified for the study. **yes/No**
- Have been made aware that all the data collected during this study will be destroyed within 3-5 years after completion of the project. **Yes/No**

Teacher Signature Moshi'a A-C. Date 08/10/2018

Teacher Contact Details/Cell phone Number: 079 667 3804 / 073 1477523

Appendix 8: Signed students consent forms



STUDENT CONSENT FORM ABOUT RESEARCH STUDY

Please fill in the reply slip below to give consent that you agree to partake in the study on the tasks about integral calculus on calculations of the magnitude of area bounded by graphs i.e. the definite integral

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

STUDENT`S INFORMED CONSENT:

I, (full names of student): THANDOKAZI PORTIA NGUNDU

Identity number: 971120 0470 08 9

- Hereby confirm that I have been informed by the researcher Mr Hlako MW about the nature of the study he is intending to do. Yes/No
- Have received, read and understand the information as well as the consent sheets about the study. Yes/No
- I am fully aware of that my responses and answers in both First and Follow up Tests will be processed without mentioning my real name. Yes/ No
- In view and light of the research requirement, I agree that the data collected during this study will be processed in a computerized system by the researcher. Yes/No
- I am aware that I can at any stage without prejudice withdraw from participating in the research study. Yes/No
- I further agree that sufficient time has been given to me to make my mind about the decisio to participate in the study and I therefore give consent to Mr. Hlako that I will write both tasks. Yes/No

Student Signature: ______________ Date 2017/11/06

Student Contact Details/Cell phone: 063 323 1946

STUDENT CONSENT FORM ABOUT RESEARCH STUDY

Please fill in the reply slip below to give consent that you agree to partake in the study on the tasks about integral calculus on calculations of the magnitude of area bounded by graphs i.e. the definite integral

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

STUDENT`S INFORMED CONSENT:

I, (full names of student): THANGAVHUELELO . MUANO

Identity number: 9407290438082

- Hereby confirm that I have been informed by the researcher Mr Hlako MW about the nature of the study he is intending to do. Yes/No
- Have received, read and understand the information as well as the consent sheets about the study. Yes/No
- I am fully aware of that my responses and answers in both First and Follow up Tests will be processed without mentioning my real name. Yes/ No
- In view and light of the research requirement, I agree that the data collected during this study will be processed in a computerized system by the researcher. Yes/No
- I am aware that I can at any stage without prejudice withdraw from participating in the research study. Yes/No
- I further agree that sufficient time has been given to me to make my mind about the decision to participate in the study and I therefore give consent to Mr. Hlako that I will write both tasks. Yes/No

Student Signature: ____________ Date 06/11/2017

Student Contact Details/Cell phone: 0610682015

STUDENT CONSENT FORM ABOUT RESEARCH STUDY

Please fill in the reply slip below to give consent that you agree to partake in the study on the tasks about integral calculus on calculations of the magnitude of area bounded by graphs i.e. the definite integral

- I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

STUDENT`S INFORMED CONSENT:

I, (full names of student): Wandile Wiseman

Identity number: 980925 6385 084

- Hereby confirm that I have been informed by the researcher Mr Hlako MW about the nature of the study he is intending to do. Yes/No
- Have received, read and understand the information as well as the consent sheets about the study. Yes/No
- I am fully aware of that my responses and answers in both First and Follow up Tests will be processed without mentioning my real name. Yes/ No
- In view and light of the research requirement, I agree that the data collected during this study will be processed in a computerized system by the researcher. Yes/No
- I am aware that I can at any stage without prejudice withdraw from participating in the research study. Yes/No
- I further agree that sufficient time has been given to me to make my mind about the decision to participate in the study and I therefore give consent to Mr. Hlako that I will write both tasks. Yes/No

Student Signature: ______________ Date 06 / 1 / 2017

Student Contact Details/Cell phone: 078 734 1754

STUDENT CONSENT FORM ABOUT RESEARCH STUDY

Please fill in the reply slip below to give consent that you agree to partake in the study on the tasks about integral calculus on calculations of the magnitude of area bounded by graphs i.e. the definite integral

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

STUDENT`S INFORMED CONSENT:

I, (full names of student): THOMAS SIWELA

Identity number: 9207025553084

- Hereby confirm that I have been informed by the researcher Mr Hlako MW about the nature of the study he is intending to do. Yes/No
- Have received, read and understand the information as well as the consent sheets about the study. Yes/No
- I am fully aware of that my responses and answers in both First and Follow up Tests will be processed without mentioning my real name. Yes/ No
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- I am aware that I can at any stage without prejudice withdraw from participating in the research study. Yes/No
- I further agree that sufficient time has been given to me to make my mind about the decision to participate in the study and I therefore give consent to Mr. Hlako that I will write both tasks. Yes/No

Student Signature: ____________ Date 06/11/2017

Student Contact Details/Cell phone: 0748417969

STUDENT CONSENT FORM ABOUT RESEARCH STUDY

Please fill in the reply slip below to give consent that you agree to partake in the study on the tasks about integral calculus on calculations of the magnitude of area bounded by graphs i.e. the definite integral

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

STUDENT'S INFORMED CONSENT:

I, (full names of student): TSHILIDZI NEMAMILWE

Identity number: 9712176531081

- Hereby confirm that I have been informed by the researcher Mr Hlako MW about the nature of the study he is intending to do. Yes/No

- Have received, read and understand the information as well as the consent sheets about the study. Yes/No

- I am fully aware of that my responses and answers in both First and Follow up Tests will be processed without mentioning my real name. Yes/ No

- In view and light of the research requirement, I agree that the data collected during this study will be processed in a computerized system by the researcher. Yes/No

- I am aware that I can at any stage without prejudice withdraw from participating in the research study. Yes/No

- I further agree that sufficient time has been given to me to make my mind about the decision to participate in the study and I therefore give consent to Mr. Hlako that I will write both tasks. Yes/No

Student Signature: ______ Date 06 / 11 /2017

Student Contact Details/Cell phone: 072 069 2130

STUDENT CONSENT FORM ABOUT RESEARCH STUDY

Please fill in the reply slip below to give consent that you agree to partake in the study on the tasks about integral calculus on calculations of the magnitude of area bounded by graphs i.e. the definite integral

I, Hlako MW, a second year Masters student at the University of Witwatersrand, am embarking on a study called: ICT interventions in TVET College N4 Mathematics in learning integral calculus.

STUDENT'S INFORMED CONSENT:

I, (full names of student): Amos MKhonto

Identity number: 8606226194088

- Hereby confirm that I have been informed by the researcher Mr Hlako MW about the nature of the study he is intending to do.
  Yes/No ✓
- Have received, read and understand the information as well as the consent sheets about the study. Yes/No
- I am fully aware of that my responses and answers in both First and Follow up Tests will be processed without mentioning my real name.
  Yes/ No ✓
- In view and light of the research requirement, I agree that the data collected during this study will be processed in a computerized system by the researcher. Yes/No ✓
- I am aware that I can at any stage without prejudice withdraw from participating in the research study. Yes/No ✓
- I further agree that sufficient time has been given to me to make my mind about the decision to participate in the study and I therefore give consent to Mr. Hlako that I will write both tasks.
  Yes/No ✓

Student Signature: Amos Date 06/11/2017

Student Contact Details/Cell phone: 073 654 1745

Appendix 9: First Test Task

higher education & training
Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

EKURHULENI EAST TVET COLLEGE
"Committed to Excellence"

RESEARCH STUDY: MATHEMATICS N4 FIRST TEST TASK

DATE: 13 July 2017 DURATION: 75 MINUTES

INSTRUCTIONS

- Answer all questions asked in this question paper.
- The Question Paper has two sections i.e. Section A and Section B.
- Section A is on prerequisite knowledge about definite integral. Section B is about the definite integral.

SECTION A

Question 1

Determine the integrals of the following functions:

1.1 $y = \sqrt{x}(1 - x + x^2)$ 05

1.2 $y = 5cos4x - 6sin2x + \frac{1}{x}$ 05

SECTION B

Question 2

In each of the following cases use integration and a sketch to calculate the area bounded by:

2.1 The curve $y = e^x$ and the $x - axis$ for $x = 0\ to\ x = 2\pi$. 06

2.2 The curve $y = e^x$, the $x - axis, and\ the\ lines\ x = -1\ and\ x = 2$. 06

2.3 The curve ,y= the $x - axis$ and the lines $x = 1\ and\ x = 2$. 06

TOTAL MARKS = 28

Appendix 10: First Test Task memorandum/marking guideline

MATHEMATICS N4 — MARKING GUIDELINE

FIRST TEST TASK — by Hlako M

1.

1.1. $\int \sqrt{x}(1-x+x^2)\,dx$

$= \int x^{\frac{1}{2}}(1-x+x^2)\,dx$

$= \int (x^{\frac{1}{2}} - x^{\frac{3}{2}} + x^{\frac{5}{2}})\,dx$ ✓

$= \dfrac{x^{\frac{3}{2}}}{\frac{3}{2}} - \dfrac{x^{\frac{5}{2}}}{\frac{5}{2}} + \dfrac{x^{\frac{7}{2}}}{\frac{7}{2}}$ ✓ $+ c$

$= \frac{2}{3}x^{\frac{3}{2}} - \frac{2}{5}x^{\frac{5}{2}} + \frac{2}{7}x^{\frac{7}{2}} + c$ ✓ ✓ ✓

05

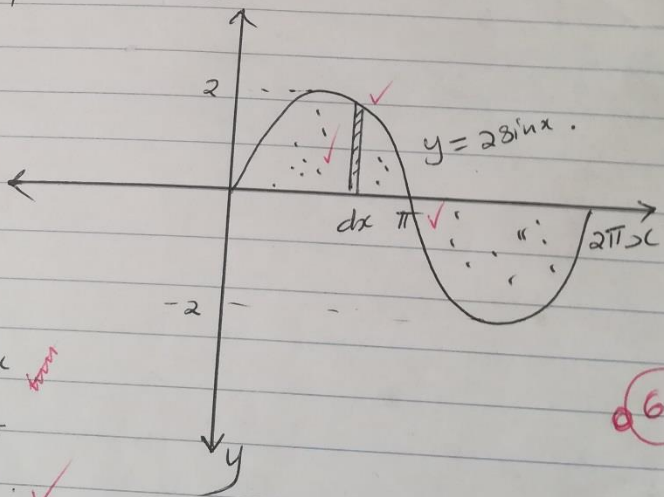
1.2. $\int \left(5\cos 4x - 6\sin 2x + \frac{1}{x}\right)dx$

$= \dfrac{5\sin 4x}{4} + \dfrac{6\cos 2x}{2} + \ln x + c$ ✓✓ ✓✓ ✓

5

2.

2.1.



$dA = y\,dx$

$dA = 2\sin x\,dx$ ✓

$A = 2\int_0^{\pi} 2\sin x\,dx$

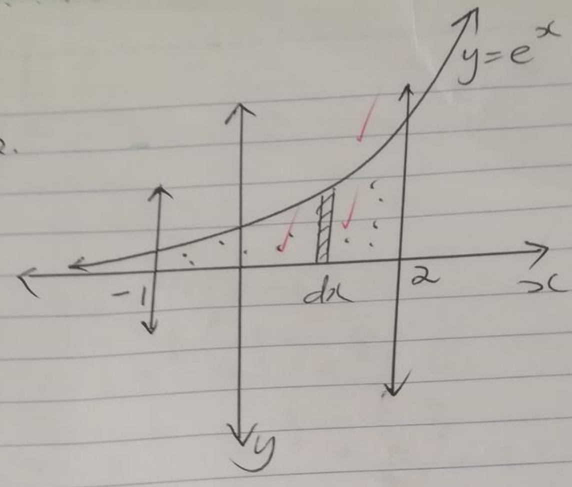
$A = 2(-2\cos x)_0^{\pi}$ ✓

$A = 2\left[(-2\cos\pi) - (-2\cos 0)\right]$

$A = 2\left((-2(-1)) - (-2\times 1)\right] = 2(4) = 8u^2$ ✓

06

2.2.



$dA = y\,dx$

$dA = e^x\,dx$ ✓

$A = \int_{-1}^{2} e^x\,dx$

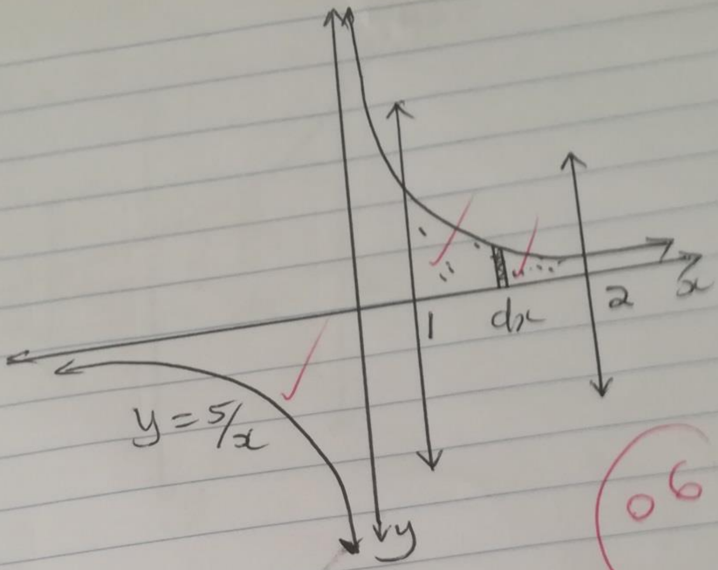
06

$A = \left(e^x\right)_{-1}^{2}$ ✓

$A = e^2 - e^{-1}$

$A = 7{,}021\ u^2$ ✓

2.3.



06

$dA = y\,dx$

$dA = \frac{5}{x}\,dx$ ✓

$A = \int_1^2 \frac{5}{x}\,dx$

$A = \left(5\ln x\right)_1^2$ ✓

$A = 5\ln 2 - 5\ln 1$ ✓

$A = 3,466\,u^2$ ✓

MATHEMATICS N4 MARKING GU

FIRST TEST TASK by Hlako M

1.

1.1. $\int \sqrt{x}(1-x+x^2)\,dx$.

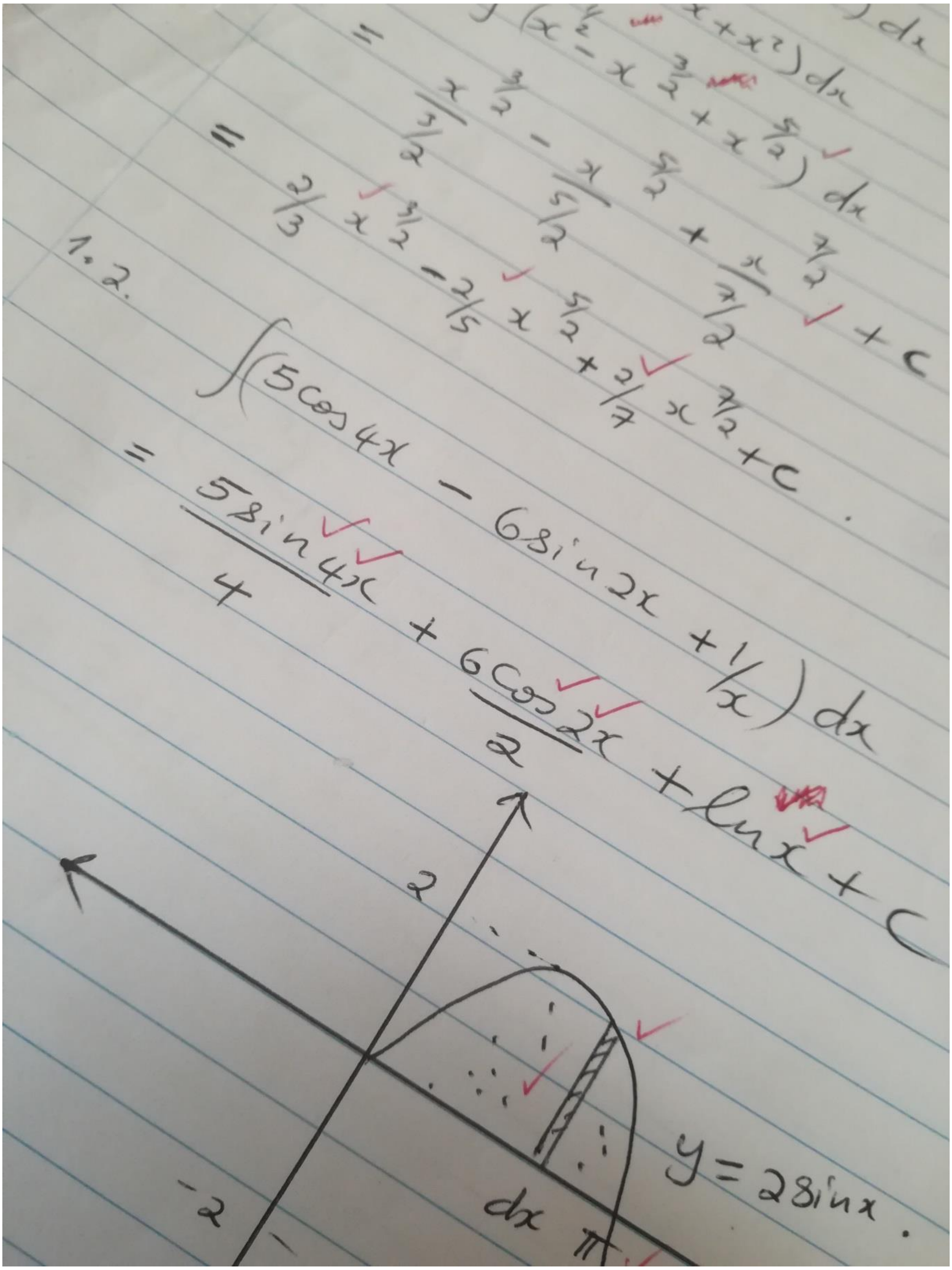
$= \int x^{1/2}(1-x+x^2)\,dx$

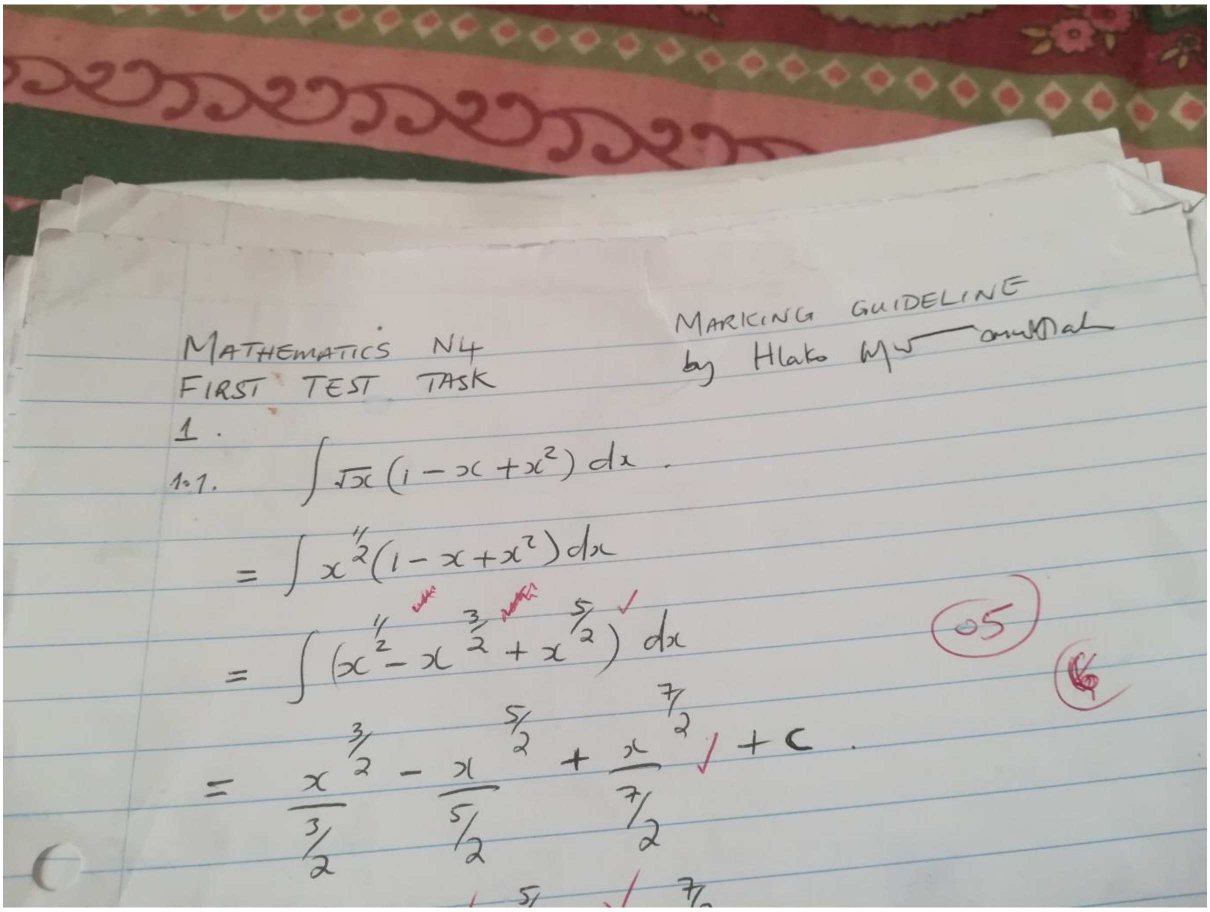
$= \int (x^{1/2} - x^{3/2} + x^{5/2})\,dx$

$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + \frac{x^{7/2}}{7/2} + C$.

$= \frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} + \frac{2}{7}x^{7/2} + C$.

. $\int\left(5\cos 4x - 6\sin 2x + \frac{1}{x}\right)$


1.2.
∫(5cos4x − 6sin2x + 1/x) dx
= 5sin4x/4 + 6cos2x/2 + ln x + C
y = 2sin x.
2
-2
dx
π


MATHEMATICS N4
FIRST TEST TASK
MARKING GUIDELINE
by Hlako
1.
1.1. $\int \sqrt{x}(1-x+x^2)\,dx$.
$= \int x^{1/2}(1-x+x^2)\,dx$
$= \int (x^{1/2} - x^{3/2} + x^{5/2})\,dx$
$= \frac{x^{3/2}}{3/2} - \frac{x^{5/2}}{5/2} + \frac{x^{7/2}}{7/2} + C$.
05

Appendix 11: Follow Up Test Task

higher education & training

Department:
Higher Education and Training
REPUBLIC OF SOUTH AFRICA

EKURHULENI EAST TVET COLLEGE
"Committed to Excellence"

RESEARCH STUDY: MATHEMATICS N4 FOLLOW UP TEST TASK

DATE: 27 July 2017 DURATION: 75 MINUTES

INSTRUCTIONS

- Answer all questions asked in this question paper.

Question 1

In each of the following cases use integration and the sketch to calculate the area bounded by:

1. The curve $y = \sqrt{x}$ *and the line* $x = 4$. 06

2. The line $x + y = 1$, *the* $x -$ *axis and the* $y -$ *axis.* 06

Question 2

1. Determine the area between the curve $y = 2.3^{2x}$, *thex* $-$ *axis, the* $y -$ *axis and the line* $x = 3$. 06

2. Calculate the area bounded by the curve $y = cos2x$, *the* $x -$ *axis and the orinates* $x = \frac{\pi}{2}$.

06

TOTAL = 24

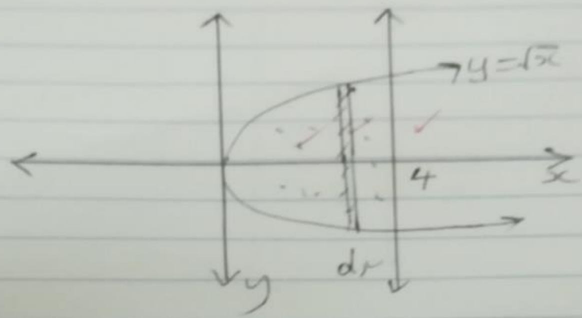
Appendix 12: Follow Up Test Task Memorandum/Marking Guideline

FOLLOW – UP TEST TASK.

MATHEMATICS N4 MARKING GUIDELINE

Q1.

1.1 $y = \sqrt{x}$ and the line $x = 4$.



3

$dA = y\,dx$

$dA = \sqrt{x}\,dx$

$A = \int_a^b x^{\frac{1}{2}}\,dx$

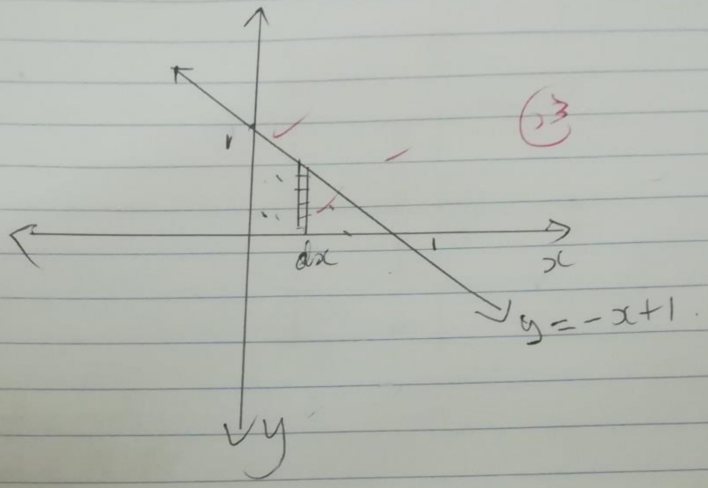
$A = \left(\frac{2}{3}x^{\frac{3}{2}}\right)_0^4$

$A = \frac{2}{3}(4)^{\frac{3}{2}} - \frac{2}{3}(0)^{\frac{3}{2}}$ 3

$A = \frac{2}{3} \times 2^3 = \frac{16}{3} = 5{,}333u^2$

Please accept $2 \times 5{,}333 = 10{,}667\ u^2$.

2. $x + y = 1$; x - axis and y - axis



03

$dA = y\,dx$

$dA = (-x+1)\,dx$

$$A = \int_0^1 (-x+1)\,dx$$

$$A = \left(\frac{-x^2}{2} + x\right)\Big|_0^1$$

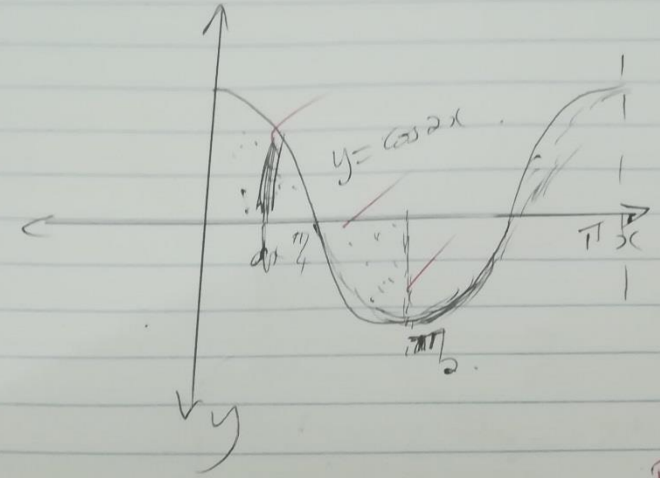
$$A = \left[\left(\frac{-1^2}{2} + 1\right) - \left(\frac{-0^2}{2} + 0\right)\right]$$

$$A = -\frac{1}{2} + 1$$

$$A = \frac{1}{2}\,u^2$$

03

(2) $y = \cos 2x$, x-axis and $x = \pi/2$.



(S3)

Period $= \frac{360°}{2} = 180°$

$dA = y\,dx$

$dA = \cos 2x\,dx$

$A = 2\int_0^{\pi/4} \cos 2x\,dx$

$A = 2\left(\frac{\sin 2x}{2}\right)_0^{\pi/4}$

(S2)

$A = (\sin 2x)_0^{\pi/4}$

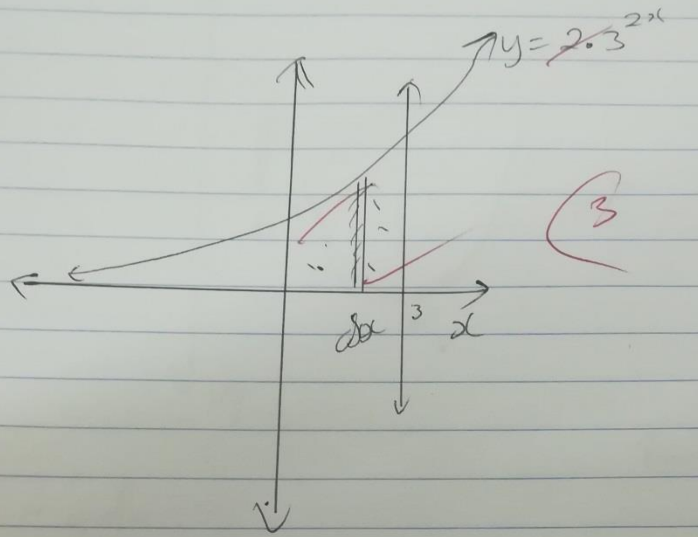
$A = (\sin 2\cdot\pi/4 - \sin 2\cdot 0)$.

$A = 2(\sin \pi/2 - \sin 0)$

$A = (1)\,u^2. = 1\,u^2.$

①

$y = 2.3^{2x}$; x-axis and $x = 3$.



$dA = y\,dx$

$dA = 2.3^{2x}\,dx$

$A = \int_0^3 2.3^{2x}\,dx$

$A = \left(\frac{2.3^{2x}}{2\ln 3}\right)_0^3$ 3

$A = \left[\left(\frac{3^6}{\ln 3}\right) - \left(\frac{3^0}{\ln 3}\right)\right]$. $= 663{,}564 - 0{,}9102$

$A = 662{,}654\ u^2$.

Appendix 13: Table 8: Table of Standard Derivatives and Integrals.

TABLE 7: TABLE OF STANDARD DERIVATIVES AND INTEGRALS

Differentiation $\longrightarrow$	
$\longleftarrow$	Integration
$f(x)$	$\int dx$
Kax^n	$\frac{Kax^{n+1}}{n+1}$
$Klnx$	$\frac{K}{x}$
$sinx$	$cosx$
$cosx$	$-sinx$
$tanx$	sec^2x
$cotx$	$-cosec^2x$
$secx$	$secxtanx$
$cosecx$	$-cotxcosecx$

- $\int tanx\ dx = lnsecx + c$

- $\int cotx\ dx = lnsinx + c$

THE END OF RESEACH STUDY