

A survey of isoscalar monopole strengths and its fine structure on nuclei across the periodic table

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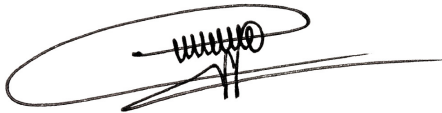
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Declaration

I, the undersigned, hereby declare that this thesis is my own unaided work. It is being submitted for the Degree of Doctor of Philosophy to the University of the Witwatersrand, Johannesburg. It has not been submitted previously for any degree or examination to any other university.

A handwritten signature in black ink, consisting of a large, sweeping loop on the left and a series of smaller, more intricate strokes on the right, ending in a sharp point.

Armand Bahini

June 17, 2021

Date

Abstract

Giant Resonances (GRs) are considered to be high frequency shape vibrations of the nucleus. This nuclear excitation mode exists in different forms, which can be classified according to the numerous modes that the protons and neutrons can oscillate, with or without specific orientation selections of their spin. The characteristic energies, strengths and widths of Giant Resonances are well known, with some GRs such as the Isovector Giant Dipole Resonance (IVGDR) first discovered as far back as before the World War II.

Since the new millennium it became apparent that the Isoscalar Giant Quadrupole Resonance (ISGQR) peak exhibits a fine structure that is independent of probe. Further it has since been established that the IVGDR also exhibits such fine structure. This fine structure as an additional GRs observable has been shown to be a useful tool to determine the damping mechanism of different shape vibration.

Until recently, there has not been experimental data available to comment on the possibility of the existence of fine structure in the Giant Monopole Resonance (GMR). This thesis is dedicated to the investigation of the Isoscalar Giant Monopole Resonance (ISGMR). The ISGMR was excited in nuclei across the periodic table by using inelastic α -particle scattering measurements acquired with a $E_\alpha = 200$ MeV beam at $\theta_{\text{Lab}} = 0^\circ$ and 4° . The high energy-resolution K600 magnetic spectrometer at iThemba LABS was used to detect the scattered alpha particles and an experimental energy-resolution of ~ 70 keV (FWHM) was achieved. This enabled the fine structure in the excitation energy region of the ISGMR to be investigated.

Alpha-particle has been proven to be the best probe to investigate the ISGMR

and as such, it has been widely used for nuclei across the periodic table to access the properties of the ISGMR as well as its strength distribution. For that, experiments are conducted at extreme forward angles including 0° and this is since the $L = 0$ Multipole peaks at that angle while all other have much lower yields. The multipole Decomposition Analysis (MDA) technique was employed in numerous studies to extract the $E0$ strength distribution in nuclei. However, due to the limitations in angular acceptance and resolution, the $E0$ strength distributions in the present project was determined via the Difference-of-Spectrum (DoS) method.

$E0$ strength distributions in ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg were determined and compared to previously published results from (low energy resolution) experiments at the Research Center for Nuclear Physics (RCNP) and Texas A&M University (TAMU). Overall, reasonable agreement was obtained. In order to gain insight into the ISGMR damping mechanism, the extraction of characteristic energy scales from the fine structure observed in the $E0$ strength distributions was performed using the technique of Continuous Wavelet Transform (CWT). The results were compared with theoretical predictions from the Phonon-Phonon Coupling (PPC) and Quasi-particle Random Phase Approximation (QRPA) models. In general, most of the extracted experimental scales were reproduced by the models. Hence, the present analysis suggests that the fine structure of ISGMR in medium to heavy mass nuclei across the periodic table arises from coupling to collective phonons and the non-harmonicity owing to interactions among phonons while it is explained by three factors in the studied light nuclei: deformation splitting of the levels, $E0/E2$ coupling, and Landau fragmentation.

Furthermore, the Fluctuation Analysis technique was employed to extract spin- and parity-dependent level densities of $J^\pi = 0^+$ in the ISGMR region. The extracted experimental level densities were compared with the phenomenological Back-Shifted Fermi Gas (BSFG) and the microscopic Hartree-Fock-Bogoliubov (HFB) and Hartree-Fock-BCS (HF-BCS) model predictions. This will be important for further studies combining level density data for $J^\pi = 0^+, 1^-, 1^+, 2^-$ and 2^+ states obtained from high resolution studies of other giant resonances, in order to test the spin and parity dependence of level densities.

Dedication

To Antigone Kpognisse and Pauline Comoulo.

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Chapter 1

Introduction

Giant Resonances (GRs) have been of interest in the field of nuclear physics for many years. They are defined as a small amplitude vibration around the ground state, at high frequency (high E_x), involving most of the nucleons (i.e. it is a collective mode) in the nucleus [Har01]. One of the reasons GRs are of interest is that they allow one to probe aspects of underlying nuclear structure and to test theoretical models [Dan66]. The oldest and best known giant resonance is the Isovector Giant Dipole Resonance (IVGDR) observed in 1937 by Bothe and Gentner [Bot37]. It was observed through photo-absorption measurements where various nuclei were bombarded with γ -radiation obtained from the ${}^7\text{Li}(p,\gamma)$ reaction. About three and a half decades later, Pitthan and Walcher [Pit71], Lewis and Bertrand [Lew72] discovered other giant resonances by means of inelastic proton scattering as well as electron scattering at high energy, viz. the Isoscalar Giant Quadrupole Resonance (ISGQR) and the Isovector Giant Quadrupole Resonance (IVGQR), as well as other magnetic resonances. These resonances are classified according to the quantum numbers, the orbital angular momentum L , the spin S and the isospin T , associated with their excitation.

Four decades after the first observation of the IVGDR, the Isoscalar Giant Monopole Resonance (ISGMR), which is the topic of this study, was observed by Harakeh *et al.* [Har77] through inelastic α -particle scattering at 120 MeV on ${}^{206,208}\text{Pb}$. Since then, much effort was put in to the theoretical and the microscopic descriptions of giant resonances. In the past decades, both alpha particles [You77, Har77, Har79, You97, You02, Pat14] and deuterons [Wil80,

[Pat14] as well as ^3He particles [Bue84] were used as probes to extensively study the ISGMR since its discovery. Recently, the ISGMR was investigated using a stored beams in inverse kinematics [Van14, Zam16]. However, the alpha particle has been established to be the best probe for ISGMR studies, explained by its zero spin and isospin, which enables the excitation of the electric isoscalar resonances predominantly. This is very important since in a microscopic view, giant resonances are described as a collective excitation of many particle-hole (p-h) states that arise from the residual p-h interaction. These interactions could be attractive or repulsive in nature defining isoscalar or isovector excitations, respectively [Har01], leading to an overlap of a number of giant resonances. This is shown in Fig. 1.1 where both the IVGDR and ISGMR peak almost at the same excitation energy.

The study of the ISGMR is important since knowledge of its excitation energy provides information relevant to the nuclear matter incompressibility coefficient which is crucial in the study of supernova collapse, neutron stars, etc. This is so because of the direct link of nuclear matter incompressibility and the curvature of the nuclear matter Equation Of State (EOS) [Bla80, Gar18, She20]. Further discussion is provided in Sect. 2.4.

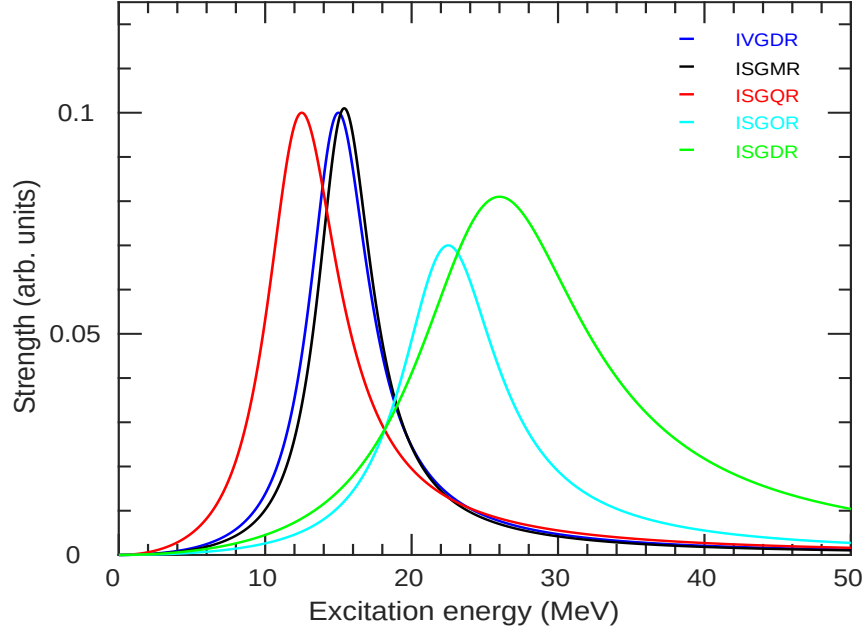


Figure 1.1: Various giant resonance strength distributions in ^{116}Sn showing an overlap in excitation energy (taken from [Pat15]).

High energy-resolution light-ion inelastic scattering experiments at iThemba LABS (South Africa) and the Research Center for Nuclear Physics (RCNP) at Osaka University (Japan) revealed that giant resonances carry fine structure [Olo20, Lat17, Don16, Jin14, Pol11, Usm09, She05]. Fine structure of giant resonances is understood to carry information on the underlying physical nature and the dominant decay mechanisms of the resonances. Through systematic studies at both iThemba LABS and RCNP, it has been established that the main mechanism responsible for fine structure differs for each resonance. It is due to coupling to low-lying surface vibrations in the case of the ISGQR [Kur18, Usm16, Usm11a, She09] and to Landau damping in the case of the IVGDR [Don20, Lat19, Neu19, Fea18]. Recently, exploration of the fine structure in the ISGMR region at iThemba LABS suggested that coupling to collective phonons and non-harmonicity due to interactions among phonons may explain the fine structure in doubly-magic calcium isotopes, namely ^{48}Ca and ^{40}Ca , while Landau damping is the source of fine structure seen in the singly magic isotopes, viz. ^{44}Ca and ^{42}Ca [Olo20]. For completeness, it is then of interest to know the mechanism leading to the fine structure in the case of the ISGMR across the periodic table. It is worth noting that in numerous investigations done on giant resonances in ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg , either through alpha, ^3He or deuteron inelastic scattering experiments, nothing was reported on the fine structure in the ISGMR region [Wil80, Bue84, Har01, Gar18].

Claims of significant nuclear structure contributions to the nuclear matter incompressibility have highlighted discrepancies between the data sets from Texas A&M University (TAMU) and RCNP groups. In fact, through inelastic α -particle scattering at 240 MeV and at small scattering angles, including 0° , Youngblood *et al.* [You13] investigated the ISGMR strength in $^{90,92}\text{Zr}$ and ^{92}Mo . They observed an unexpected variation in the extracted ISGMR strengths as well as its centroid. For example, the ISGMR centroid energy for ^{90}Zr was found to be 1.22 and 2.80 MeV lower than that of ^{92}Zr and ^{92}Mo , having significant impact on the extracted finite nucleus incompressibility K_A . These surprising results led to the conclusion that such variation may be due to nuclear shell structure effects [You13]. This conclusion contradicts the predictions from the hydrodynamic model which follows the $A^{-\frac{1}{3}}$ trend for the ISGMR energy. For further investigation, the group at RCNP investigated inelastic α -particle scattering at 386 MeV and

at small scattering angles, including 0° , in order to measure the ISGMR strengths in the same nuclei. Using both the Difference-of-spectra (DoS) and Multipole Decomposition Analysis (MDA) techniques, it was observed that the ISGMR strengths and energies in these three nuclei are rather similar, within experimental errors [Gup16a]. These findings contradict the previous claims from Ref. [You13] and the discrepancies were attributed to the phenomenological background subtraction method used by the TAMU group in their data analysis. The main difference between the two groups is that TAMU uses a phenomenological approach to subtract the background, which consists of a straight line at high excitation, joining onto a Fermi shape at low excitation to model particle threshold effects. The correctness of the background shape and magnitude is still under debate. On the other hand, measurements from RCNP are essentially free of all instrumental background due the ion-optics of their spectrometer which enables proper background subtraction during offline analysis at later stage [Gar18].

An independent survey of the monopole strength across the periodic table and extraction of incompressibility from data obtained at iThemba LABS could be helpful to understand the origin of these discrepancies. The K600 magnetic spectrometer was used in the present study. However, because of angular resolution limitations of the K600 0° Facility, the DoS method was chosen over the MDA which is the best and most reliable technique to obtain $E0$ strength distributions. As such, zero-degrees measurements ($\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$) and measurements around the minimum for the $L = 0$ angular distributions ($\theta_{\text{Lab}} = 2^\circ - 6^\circ$) were carried out. The $E0$ strength distribution can be accessed through the subtraction of the excitation energy spectrum for scattering angles around the $L = 0$ deep minimum where the sum of all other multipoles have a relatively flat angular distribution, from the 0° spectrum.

Further, this project seeks to investigate the fine structure of the ISGMR across the periodic table. This enables completion of previous studies carried out at iThemba LABS on the ISGQR [Kur18, Usm16, Usm11a], and the IVGDR [Don20, Lat19, Neu19, Fea18, Jin18]. In these works, characteristic energy scales are extracted through the Continuous Wavelet Transform (CWT) method followed by comparison with theoretical predictions from various models. The same technique is used in this work to extract the characteristic energy scales emerging from the fine structure in the region of

the ISGMR across the periodic table.

Finally, the $J^\pi = 0^+$ level densities can be extracted and added to the $J^\pi = 1^-, 2^+$ values from the other giant resonance studies at iThemba LABS, namely the Isovector Giant Dipole Resonance [Lat17, Jin14] and the Isoscalar Giant Quadrupole Resonance [Kur14, Pen14, Usm09]. Therefore, comparison with appropriate theoretical predictions such as spin distribution in level-density models can be done. More specifically the phenomenological Back Shifted Fermi Gas (BSFG) [Egi05, Rau97] and the microscopic Hartree-Fock-Bogoliubov (HFB) and Hartree-Fock-BCS (HF-BCS) [Hil01, Dem01] models can be tested.

This thesis is structured as follows:

- The theoretical considerations required in the study of giant resonances, more specifically the ISGMR, are presented in **Chapter 2**. Also discussed are the various theoretical models used in the microscopic description of the ISGMR strength function as well as those involved in the extraction of characteristic energy scales and nuclear level density.
- The experimental procedure followed for data extraction from both the zero- and four-degrees measurements are described in **Chapter 3**.
- The results of the analysed experimental data and the theoretical models are presented in **Chapter 4**. The procedure followed for accessing the $E0$ strength distribution in all the nuclei under investigation is presented as well as the extracted characteristic energy scales. The $J^\pi = 0^+$ spin-dependent level densities extracted from the experiments are discussed in conjunction with the model predictions.
- Conclusions and recommendations are presented in **Chapter 5**.

Chapter 2

Theoretical considerations

The theoretical considerations for the present study are discussed in this chapter. After a general description of nuclear giant resonances, with main focus on the Isoscalar Giant Monopole Resonance (ISGMR), the model predictions of giant resonance behaviour are further discussed with the illustration of unique techniques involved in the extraction of characteristic energy scales as well as level density in the region of the ISGMR.

2.1 Giant resonances

Giant resonances are defined as a small amplitude vibration around the ground state, at high frequency (high E_x), involving most of the nucleons (i.e. it is a collective mode) in the nucleus. That collective motion can also be seen in hydrodynamic terms as the oscillation of a liquid drop. The strength of a resonance is described by a transition amplitude which depends on the number of the nucleons participating in the response, the size of the system, and is limited by a sum rule [Har01].

2.1.1 Macroscopic picture

In order to provide a global understanding of Giant Resonance (GR) phenomena, both in stable and exotic nuclei, macroscopic and microscopic

approaches are required. In the macroscopic picture, the nucleus is described as a liquid drop of proton and neutron fluids where a GR is described as a small-amplitude collective oscillation about the nuclear equilibrium. As such, the surface of the nucleus is assumed to undergo small-amplitude oscillations about a mean spherical shape. The surface of the nucleus is expanded in terms of spherical harmonics described by [Har01]

$$R(\theta, \phi) = R_0 \left[1 + \sum_{\lambda\mu} \alpha_{\lambda\mu} Y_{\lambda\mu}^*(\theta, \phi) \right] \quad \lambda = 2, 3, \dots, \quad (2.1)$$

where $\lambda\mu$ are the angular momentum quantum numbers and $\alpha_{\lambda\mu}$ are the normal coordinates. The terms $R(\theta, \phi)$ and R_0 represent the surface coordinate and the equilibrium radius, respectively. The monopole and dipole resonances are not described by Eq. (2.1) since the former, $\lambda = 0$ and the latter, $\lambda = 1$ correspond to a compressional breathing mode without change of the shape and a displacement of the nucleus as a whole (which is forbidden), respectively.

GRs are observed in photonuclear reactions [Car71, Car74], reactions with incident projectiles such as electrons [Mül82], protons [Don18], alpha particles [You13], and sometimes in heavy-ion nuclear reactions. Most of the time they are due to the response of a nucleus to the electromagnetic field of the incident particle. The atomic nucleus is excited by the field, and therefore oscillates. External fields can be different in nature and thus have various multipolarities. They transfer to the nucleus the orbital angular momentum L , the spin S and the isospin T which characterise the transition between the ground state and the collective excited states. Thus, giant resonances are characterised by the quantum numbers L, S, T and are classified accordingly. Figure 2.1 shows a schematic representation of various giant resonance modes where each mode is identified by the angular momentum transfer ΔL , the spin excitation ΔS and the isospin excitation ΔT .

	Electric modes ($\Delta S = 0$)		Magnetic modes ($\Delta S = 1$)	
	Isoscalar ($\Delta T = 0$)	Isovector ($\Delta T = 1$)	Isoscalar ($\Delta T = 0$)	Isovector ($\Delta T = 1$)
$\Delta L = 0$				
$\Delta L = 1$	...			
$\Delta L = 2$				

Figure 2.1: Schematic representation of various collective modes with $\Delta L = 0$ (monopole), $\Delta L = 1$ (dipole) and $\Delta L = 2$ (quadrupole) (taken from [Har01]).

For giant resonances, there are two modes of excitations associated with the quantum numbers, viz. the electric modes and the magnetic modes, which correspond to nucleons with spin-up and spin-down oscillating in phase ($\Delta S = 0$) and out of phase ($\Delta S = 1$), respectively. Each spin mode is characterised by an isospin excitation. The isovector modes ($\Delta T = 1$) correspond to modes in which protons and neutrons oscillate out of phase with one another while isoscalar modes ($\Delta T = 0$) are modes in which protons and neutrons oscillate in phase. The first row of Fig. 2.1 is described as follows: for $\Delta S = 0$, $\Delta T = 0$ modes the nucleons move in phase, while for $\Delta S = 0$, $\Delta T = 1$ modes the protons and neutrons oscillate in opposite phase. Likewise for the magnetic mode columns, $\Delta S = 1$, $\Delta T = 0$ modes correspond to nucleons with spin up moving against nucleons with spin down, while $\Delta S = 1$, $\Delta T = 1$ modes correspond to protons with spin down moving against neutrons with spin up and *vice versa*.

2.1.2 Microscopic picture

Microscopically, giant resonances are well described by the coherent superposition of particle-hole (p-h) pair excitations which is induced by a transition operator $\mathcal{O}$ from the ground state $|\Psi_{\text{g.r.}}\rangle$ to the collective excited states $|\Psi_{\text{GR}}^{\lambda,\sigma,\tau}\rangle$ given by

$$|\Psi_{\text{GR}}^{\lambda,\sigma,\tau}\rangle = \mathcal{O}^{\lambda,\sigma,\tau} |\Psi_{\text{g.r.}}\rangle \quad (2.2)$$

with λ, σ, τ being the multipolarity, the spin and the isospin structure of the particle resonance, respectively [Har01].

For a resonance to be considered giant in nature, its observed strength should exhaust a significant fraction of the Energy Weighted Sum Rule (EWSR) sum rule. The total cross-section that can be expected for a mode with given L , T and S values is provided by the sum rule [Ber09]. Since giant resonances are transitions from the ground state to a collective excited state, their qualitative aspects can be understood using shell configuration as illustrated in Fig. 2.2. The major shells (energy levels) are denoted by N , $N + 1$, $N + 2$, $\dots$, and are separated by an energy $\Delta E \sim \Delta N \times 1\hbar\omega$ or $\sim 41A^{-1/3}$ MeV. The various transitions induced must be such that $\Delta N \leq L$ while, due to the conservation of parity, odd L transitions require $\Delta N = 1, 3, \dots$ and even L transitions $\Delta N = 0, 2, \dots$ [Har01]. It is worth noting that the ISGMR is a second-order effect; to first order, the transition operator $\mathcal{O}_{\mu}^{0,0,0}$ for ISGMR is a constant which cannot induce any transitions from the ground state to excited states [Pat15]. Further, not only the ISGMR but also the ISGDR is a second-order effect. First order of ISGDR would be a translational motion of the whole nucleus. This scheme is summarised in Table 2.1. For example, electric transitions between two major shells are considered to be $E1$ and transitions within a shell or across three major shells are considered to be $E2$, etc.

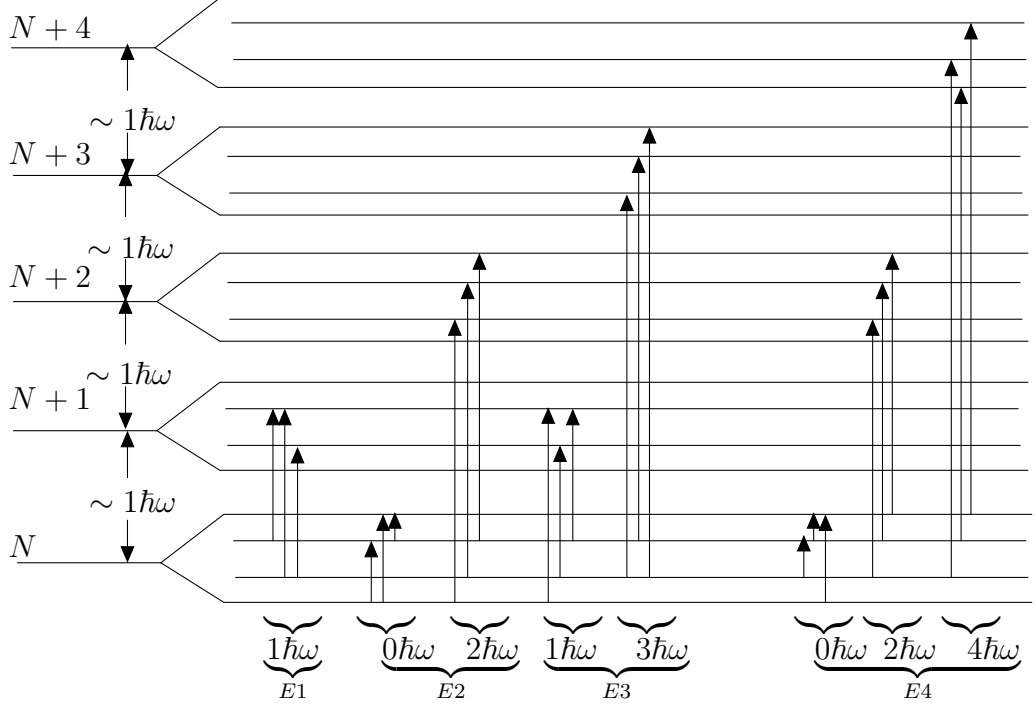


Figure 2.2: Schematic representation of electric multipole transitions between major shells of a hypothetical nucleus (taken from [Ber81]).

Table 2.1: Multipole excitations as particle-hole excitations across major shells. The brackets denote second-order effects transitions

Monopole	$\lambda = 0$	$(0\hbar\omega)$	$2\hbar\omega$	
Dipole	$\lambda = 1$	$1\hbar\omega$		
Quadrupole	$\lambda = 2$	$(0\hbar\omega)$	$2\hbar\omega$	
Octupole	$\lambda = 3$	$1\hbar\omega$	$3\hbar\omega$	
Hexadecapole	$\lambda = 4$	$(0\hbar\omega)$	$2\hbar\omega$	$4\hbar\omega$

In this model, a strong collective state is yielded following the interaction of the residual particle-hole (p-h). That strong collective state is a coherent superposition of all possible particle-hole interactions of a given multipolarity and parity [Har01]. In addition, for isoscalar transitions, the p-h residual interaction is attractive while it is repulsive for isovector excitations. These

together lead to the fact that various resonances overlap in excitation energy. As such, it is of utmost importance to choose the appropriate probe when studying a specific mode of vibration in order to attain selectivity in exciting the mode of interest. Further, the mixing of the simple collective 1p-1h state with more complicated many-particle many-hole of the same parity and spin implies resonances with large widths. The resonance width will be fully discussed in Sect. 2.2.

2.2 Damping of giant resonances

Giant resonances are defined with three parameters, namely the excitation energy, the strength and the width Γ_{GR} . The width is the result of the damping processes in the resonance and its value lies between 2 MeV and 5 MeV and can extend to 7 MeV in heavy deformed nuclei [Har01]. The Nuclear Shell-model can be used to calculate the width of giant resonances. But the various processes that contribute to the width make the calculations more complex with the size of the configuration space [Goe82]. The damping of resonances can be described by different components as follows

$$\Gamma_{\text{GR}} = \Delta\Gamma + \Gamma^{\uparrow} + \Gamma^{\downarrow} \quad (2.3)$$

with $\Delta\Gamma$ the Landau damping which describes the fragmentation of the initial one-particle one-hole (1p-1h) excitation, $\Gamma^{\uparrow}$ the escape width which corresponds to direct particle emission from (1p-1h) excitation, and $\Gamma^{\downarrow}$ the spreading width, which is the coupling to the more complex two-particle two-hole (2p-2h) to many-particle many-hole (np-nh) states. The pattern is clearly shown in Fig. 2.3.

Information on the dominant damping mechanisms of nuclear giant resonances can be found in the properties and characteristics of the fine structure [Usm16]. The spreading width $\Gamma^{\downarrow}$ leads to the pre-equilibrium and statistical decay observed in the compound nuclei. Its stochastic coupling mechanism is well described by the doorway model [Jin18, Neu19]. Fine structure is formed through the mixture of multiple scales of fluctuations which are induced by the decay of nuclear states as represented schematically

in Fig. 2.4.

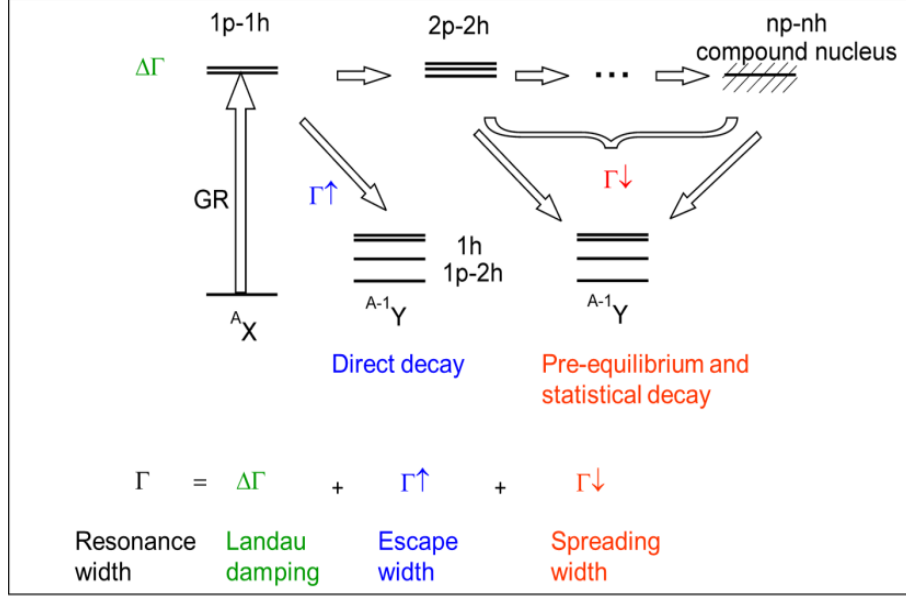


Figure 2.3: A schematic representation of the contributions to the width of a nuclear giant resonance (taken from [Neu19]).

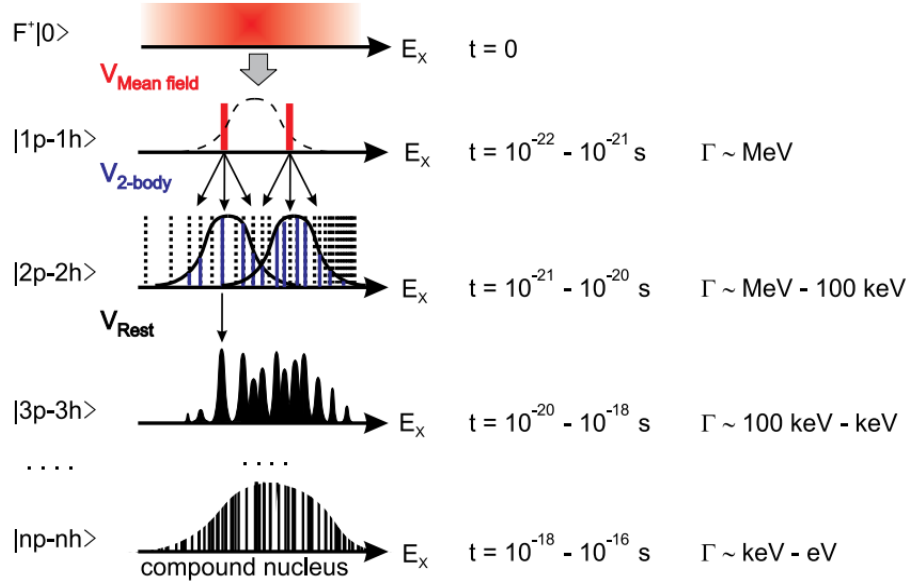


Figure 2.4: Different processes involved in the damping of giant resonances. The hierarchy of lifetimes and energy scales in the decay are indicated (taken from [She05]).

2.3 Microscopic models for the damping of giant resonances

Many microscopic models have been proposed to describe giant resonances. Such models include the Quasi-particle Phonon Model (QPM) [Sol92], the Random Phase Approximation (RPA), the Quasi-particle RPA (QRPA) [Paa07], the Second Random Phase Approximation (SRPA) [Dro90], the Quasi-particle Time Blocking Approximation (QTBA) [Lit08, Ego16], the Extended Theory of Finite Fermi Systems (ETFFS) [Kam04], the so-called Extended Time Dependent Hartree-Fock (ETDHF) method, the Self-consistent Separable Random Phase Approximation (SSRPA) [Nes06, Gia98] and the Phonon-Phonon Coupling (PPC) formalism [Ars17, Sev17].

These are self-consistent mean-field models with effective energy density functionals which are known as reliable tools for the description of nuclear structure and dynamics [Nes06]. GRs are described in these models by using only the energy density functional $E[\rho]$ which can be obtained in a non-relativistic framework (Skyrme, Gogny) or from a Lagrangian in the relativistic case [Kha10]. The action of the system can be written as

$$S = \int_{t_1}^{t_2} dt \int d\vec{r} (i\hbar\psi^*(\vec{r}, t)\partial_t\psi(\vec{r}, t) - E[\rho]) , \quad (2.4)$$

where $\psi(\vec{r}, t)$ denotes the state at a position $\vec{r}$ and time t . The particle density is given by

$$\rho(\vec{r}) = \sum_i \phi_i^*(\vec{r})\phi_i(\vec{r}) , \quad (2.5)$$

where the sum is over all nucleon states ϕ at a position $\vec{r}$. By applying the least action principle the Schrödinger (or Dirac in the relativistic case) equation is obtained. A nucleus is considered as many-body problem and as such the minimisation has to be performed on Slater determinant only. The action is rewritten as

$$S = i\hbar \sum_{i=1}^A \int_{t_1}^{t_2} dt \int d\vec{r} (\phi_i^*(\vec{r}, t) \partial_t \phi_i(\vec{r}, t) - E[\rho]) , \quad (2.6)$$

where ϕ are the single particles states. Thus, the least action principle yields the so-called Time Dependent Hartree Fock (TDHF) equation

$$i\hbar \partial_t \phi_i = \frac{\delta E[\rho]}{\delta \rho} \phi_i \hat{=} h \phi_i . \quad (2.7)$$

The mean field h is completely determined from the energy density functional. Considering the small amplitude harmonic perturbation of the ground state, the linear response is given by

$$\Pi = \Pi_0 + \Pi_0 \frac{\delta h}{\delta \rho} \Pi . \quad (2.8)$$

This perturbed response function Π yields the energies and transition probabilities (cross-sections) from the ground state to the excited states. The perturbed response is computed from the unperturbed one Π_0 which is built on the single-particle states obtained from the mean field description of the ground state [Kha10]. Hence, the excited energy spectrum is fully extracted. It should be noted that SSRPA is the most used model to describe giant resonances in deformed nuclei due to its high accuracy and a modest computational effort. The self consistency of this model makes it a powerful tool, with no additional parameter needed.

The features of each of the aforementioned microscopic models applicable to both ISGQR and IVGDR studies for nuclei across the periodic table are fully described elsewhere [Usm11a, Ars17, Sev17, Kur18, Don18, Lat19]. As such, only salient details will be given on RPA, QRPA and PPC models in the subsequent sections.

2.3.1 Random Phase Approximation (RPA) and Quasi-particle Random Phase Approximation (QRPA)

As stated earlier, the Random Phase Approximation (RPA) pioneered by Bohm and Pines [Boh51] forms part of the several microscopic models used to describe giant resonances. In that model, GRs are considered as a coherent superposition of 1p-1h excitation obtained from a correlated ground state. On that basis, its equation is given in [Tho61] and expressed as

$$\begin{pmatrix} A & B \\ -B^* & -A^* \end{pmatrix} \begin{pmatrix} X_{\text{ph}} \\ Y_{\text{ph}} \end{pmatrix} = E_{\text{ph}} \begin{pmatrix} X_{\text{ph}} \\ Y_{\text{ph}} \end{pmatrix} \quad (2.9)$$

with the matrices

$$A_{\text{php}'\text{h}'} = \varepsilon_{\text{ph}} \delta_{\text{pp}'} \delta_{\text{hh}'} + \left(\frac{\delta h_{\text{ph}}}{\delta \rho_{\text{p}'\text{h}'}} \right)_0 \quad \text{and} \quad B_{\text{php}'\text{h}'} = \left(\frac{\delta h_{\text{ph}}}{\delta \rho_{\text{p}'\text{h}'}} \right)_0. \quad (2.10)$$

The strength function $S_F(\omega)$ obtained following a perturbation from a hadronic external

$$\hat{F} = \sum_{\text{ph}} f_{\text{ph}} a_{\text{p}}^\dagger a_{\text{h}} \quad (2.11)$$

is expressed as

$$S_F(\omega) = -\pi^{-1} \sum_{\text{php}'\text{h}'} f_{\text{p}}^* R_{\text{php}'\text{h}'}(\omega) f_{\text{h}}, \quad (2.12)$$

where $a_{\text{p}}^\dagger$ and a_{h} denote the creation and annihilation operators. The matrix form of the ph propagator $R_{\text{php}'\text{h}'}$ is given by

$$R_{\text{php}'\text{h}'}(\omega) = \begin{pmatrix} \omega + i\eta - A & B \\ -B^* & -\omega + i\eta - A^* \end{pmatrix}^{-1}. \quad (2.13)$$

Hence, following the p-h states and the interaction involved, the RPA yields the collective phonons and can be used to describe both the centroid energies and Landau damping. Implemented in a continuum, the RPA can also describe the escape width but fails for the spreading width and the resulting fine structure

[Har01].

The coupling to the 2p-2h states yields the Second Random Phase Approximation (SRPA) [Dro86]. As such, Eq. (2.9) becomes

$$\begin{pmatrix} A & B & A_{12} & 0 \\ -B^* & -A^* & 0 & -A_{12}^* \\ A_{21} & 0 & A_{22} & 0 \\ 0 & -A_{21}^* & 0 & -A_{22}^* \end{pmatrix} \begin{pmatrix} X_{\text{ph}} \\ Y_{\text{ph}} \\ X_{2\text{p}2\text{h}} \\ Y_{2\text{p}2\text{h}} \end{pmatrix} = E_{2\text{p}2\text{h}} \begin{pmatrix} X_{\text{ph}} \\ Y_{\text{ph}} \\ X_{2\text{p}2\text{h}} \\ Y_{2\text{p}2\text{h}} \end{pmatrix}. \quad (2.14)$$

The matrices A_{12} and A_{22} define the coupling and interactions between 1p-1h and 2p-2h states whereas A and B are the RPA matrices. The SRPA model yields successful results in the description of damping processes [Yan86]. More details on the SRPA model can be found in Refs. [Pap14, Usm09].

Using the RPA as a starting point, the Quasi-particle Random Phase Approximation (QRPA) deals with the particle-particle (p-p), particle-hole (p-h) and hole-hole (h-h) in a similar way. Its equation is given by

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix} \begin{pmatrix} X_n \\ Y_n \end{pmatrix} = \omega_n \begin{pmatrix} X_n \\ -Y_n \end{pmatrix}, \quad (2.15)$$

where $\langle n \rangle$ denotes the excited state of energy ω_n . The QRPA model describes well both closed and open-shell nuclei. In this study, theoretical calculations were done on the light targets, ^{24}Mg and ^{28}Si using the QRPA model. For the aim of this project, a representative set of Skyrme forces is used, viz. the standard force SKM* [Bar82], SVbas, SV-mas [Klu09], SkT6 [Ton84] and SkP $^\delta$ [Dob95]. These forces were previously used in [Pea16, Gup16b, Kva16]. The results from the calculations are presented in the 0 – 30 MeV energy range.

2.3.2 Phonon-Phonon Coupling (PPC) method

The phonon-phonon coupling (PPC) formalism has been discussed in detail in Refs. [Gia98, Sev04, Sev08, Sev12] and as such only the different steps are briefly summarised here. For the present study, the anharmonicity of the low-energy vibrations is constrained by the coupling between one- and two-phonon

terms in the wave functions of excited states.

2.3.2.1 Implementation of QRPA

The starting point of the method is the Hartree–Fock (HF)-Bardeen–Cooper–Schrieffer (BCS) calculation [Rin80] of the ground state, where spherical symmetry is imposed on the quasi-particle wave functions. The continuous part of the single-particle spectrum is discretised by diagonalising the Skyrme HF Hamiltonian on a harmonic oscillator basis. As effective interactions, Skyrme interaction are used in the particle-hole (p-h) channel, and the pairing correlations are generated by a density-dependent zero-range force in the particle-particle (p-p) channel

$$V_{\text{pair}}(\mathbf{r}_1, \mathbf{r}_2) = V_0 \left[1 - \eta \left(\frac{\rho(r_1)}{\rho_0} \right)^\alpha \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) , \quad (2.16)$$

where $\rho(r_1)$ is the particle density in coordinate space; ρ_0 is equal to the nuclear saturation density; α , η and V_0 are model parameters. We use $\alpha = 1$; $\eta = 0$, $\eta = 0.5$, and $\eta = 1$ are the cases of a volume interaction, a mixed interaction and a surface-peaked interaction, respectively. The strength V_0 is fitted to reproduce the odd-even mass difference in the region of nuclei considered here. In order to limit the pairing single-particle space, we have used the smooth cutoff at 10 MeV above the Fermi energies [Sev08, Bon85, Kri90].

The residual interaction in the p-h channel $V_{\text{res}}^{\text{ph}}$ and in the p-p channel $V_{\text{res}}^{\text{pp}}$ can be obtained as the second derivative of the energy density functional with respect to the particle density and the pair density, accordingly. As proposed in Ref. [Gia98], we simplify $V_{\text{res}}^{\text{ph}}$ by approximating it by its Landau–Migdal form. All Landau parameters with $l > 1$ are equal to zero in the case of Skyrme interactions. We keep only the $l = 0$ terms in $V_{\text{res}}^{\text{ph}}$. In this work we study only normal parity states and one can neglect the spin-spin terms since they play a minor role [Sev02]. Also, the two-body Coulomb and spin-orbit residual interactions are dropped. Therefore, the residual interaction is written as

$$V_{\text{res}}^a(\mathbf{r}_1, \mathbf{r}_2) = N_0^{-1} \left[F_0^{(a)}(r_1) + F_0^{\prime(a)}(r_1) \tau_1 \cdot \tau_2 \right] \delta(\mathbf{r}_1 - \mathbf{r}_2) , \quad (2.17)$$

where a is the channel index $a = \{\text{ph}, \text{pp}\}$; τ_i are the isospin operators, and $N_0 = 2k_F m^* / \pi^2 \hbar^2$ with k_F and m^* standing for the Fermi momentum and nucleon effective mass.

The expressions for $F_0^{(\text{ph})}$, $F_0'^{(\text{ph})}$ and $F_0^{(\text{pp})}$, $F_0'^{(\text{pp})}$ can be found in Ref. [Gia81] and Ref. [Sev08], respectively. Since the definition of the pairing force (Eq. 2.16) involves the energy cutoff of the single-particle space to restrict the active pairing space within the mean-field approximation, this cutoff is still required to eliminate the p-p matrix elements of the residual interaction in the case of the subshells that are far from the Fermi energies as in Ref. [Sev08]. The p-h matrix elements and the antisymmetrised p-p matrix elements can be written as the separable form in the angular coordinates [Gia98, Sev08]. After integrating over the angular variables we use a N -point integration Gauss formula for the radial integrals. Thus, the residual interaction can be expressed as the sum of N terms in the finite rank separable approximation (FRSA) for the Skyrme residual interaction [Gia98, Sev08].

The phonon creation operators are introduced as [Sev08]

$$Q_{\lambda\mu}^+ = \frac{1}{2} \sum_{jj'} \left(X_{jj'}^{\lambda i} A^+(jj'; \lambda\mu) - (-1)^{\lambda-\mu} Y_{jj'}^{\lambda i} A(jj'; \lambda - \mu) \right), \quad (2.18)$$

$$A^+(jj'; \lambda\mu) = \sum_{mm'} \langle jm j' m' | \lambda\mu \rangle \alpha_{jm}^+ \alpha_{j'm'}^+, \quad (2.19)$$

where λ denotes the total angular momentum and μ is its z -projection in the laboratory system. The α_{jm}^+ (α_{jm}) is the quasiparticle creation (annihilation) operator and jm denote the quantum numbers $nljm$. One assumes that the ground state is the QRPA phonon vacuum $|0\rangle$ and the one-phonon excited states are $Q_{\lambda\mu}^+ |0\rangle$ with the normalisation condition

$$\langle 0 | Q_{\lambda\mu} Q_{\lambda'\mu'}^+ | 0 \rangle = \delta_{\lambda\lambda'} \delta_{\mu\mu'} \frac{1}{2} \sum_{jj'} \left(X_{jj'}^{\lambda i} X_{jj'}^{\lambda i'} - Y_{jj'}^{\lambda i} Y_{jj'}^{\lambda i'} \right) = \delta_{ii'}. \quad (2.20)$$

Making use of the linearised equation-of-motion approach one can get the QRPA equations [Rin80]. Solutions of this set of linear equations yield the one-phonon energies $\omega_{\lambda i}$ and the amplitudes $X_{jj'}^{\lambda i}$, $Y_{jj'}^{\lambda i}$ of the excited states. There are two types of QRPA matrix elements: the $A_{(j_1 j_1')(j_2 j_2')}^{(\lambda)}$ matrix related

to forward-going graphs and the $B_{(j_1 j'_1)(j_2 j'_2)}^{(\lambda)}$ matrix related to backward-going graphs, and the dimension of these matrices is the space size of the two-quasiparticle configurations. Using the FRSA for the residual interaction, the eigenvalues of the QRPA equations can be obtained as the roots of a relatively simple secular equation, where the matrix dimensions never exceed $6N \times 6N$ independently of the two-quasiparticle configuration space size [Gia98, Sev08]. The studies [Sev02] enable us to conclude that $N = 45$ for the rank of our separable approximation is enough for the electric excitations ($\lambda \leq 6$) in nuclei with $A \leq 208$.

2.3.2.2 Phonon-phonon coupling

Taking into account the basic ideas of the quasiparticle-phonon model (QPM) [Sol92], the Hamiltonian is then diagonalised in a space spanned by states composed of one- and two-QRPA phonons

$$\Psi_\nu(\lambda\mu) = \left(\sum_i R_i(\lambda\nu) Q_{\lambda\mu i}^+ + \sum_{\lambda_1 i_1 \lambda_2 i_2} P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu) [Q_{\lambda_1 \mu_1 i_1}^+ Q_{\lambda_2 \mu_2 i_2}^+]_{\lambda\mu} \right) |0\rangle, \quad (2.21)$$

with the normalisation condition

$$\sum_i R_i^2(\lambda\nu) + 2 \sum_{\lambda_1 i_1 \lambda_2 i_2} [P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu)]^2 = 1. \quad (2.22)$$

Because of the completeness and orthogonality conditions for the phonon operators, the bifermion operators $A^+(jj'; \lambda\mu)$ and $A(jj'; \lambda\mu)$ can be expressed by the phonon ones. The Hamiltonian can be rewritten in terms of quasiparticle and phonon operators [Sol92, Sev04, Sev12]. In order to let the two-phonon components of the wave functions obey the Pauli principle the exact commutation relations between the phonon operators should be taken into account. The Pauli principle corrections are dropped here since the effect on the giant resonances is small [Sol92].

The amplitudes $R_i(\lambda\nu)$ and $P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu)$ are determined from the variational principle

$$\delta(\langle \Psi_\nu(\lambda\mu) | H | \Psi_\nu(\lambda\mu) \rangle - E_\nu (\langle \Psi_\nu(\lambda\mu) | \Psi_\nu(\lambda\mu) \rangle - 1)) = 0, \quad (2.23)$$

which leads to a set of linear equations [Sev04, Sev12]

$$(\omega_{\lambda i} - E_\nu) R_i(\lambda\nu) + \sum_{\lambda_1 i_1 \lambda_2 i_2} U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i) P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu) = 0 , \quad (2.24)$$

$$2(\omega_{\lambda_1 i_1} + \omega_{\lambda_2 i_2} - E_\nu) P_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda\nu) + \sum_i U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i) R_i(\lambda\nu) = 0 . \quad (2.25)$$

The rank of the set of linear Eqs. (2.24) and (2.25) is equal to the number of one- and two-phonon configurations included in the wave function (Eq. 2.21). Its solution requires to compute the matrix elements coupling one- and two-phonon configurations

$$U_{\lambda_2 i_2}^{\lambda_1 i_1}(\lambda i) = \langle 0 | Q_{\lambda i} H [Q_{\lambda_1 i_1}^+ Q_{\lambda_2 i_2}^+]_\lambda | 0 \rangle . \quad (2.26)$$

The equations (2.24) and (2.25) have the same form as the QPM equations [Sol92], but the single-particle spectrum and the parameters of the residual interaction are calculated with the chosen Skyrme forces without any further adjustments.

The so-called FRSA was thus used to study the electric low-energy excitations and giant resonances within and beyond the Skyrme-QRPA [Sev04, Sev08, Sev12, Ars17, Sev17].

2.3.2.3 Details of calculations

In the presented calculations the single-particle continuum is discretised by diagonalising the HF Hamiltonian on a basis of 12 harmonic oscillator shells and cutting off the single-particle spectra at the energy of 100 MeV. This is sufficient to exhaust practically all the energy-weighted sum rule. Because of the large configurational space, we do not use effective charges.

For the investigation, we adopt two Skyrme interactions, namely, SLy4 [Cha98] (for ^{120}Sn , ^{208}Pb) and f^- [Les06] (for ^{58}Ni , ^{90}Zr). The parameters of the SLy4 interaction have been adjusted to reproduce nuclear matter properties, as well as nuclear charge radii, binding energies of doubly magic

nuclei (the effective mass value of 0.69 and a compression modulus of the symmetric nuclear matter of 230 MeV). We have also considered the force f^- which predicts an effective mass of 0.7 in symmetric matter, with negative isospin splitting of the effective mass in neutron-rich systems, $m_n^* < m_p^*$. The density of single-particle states near the Fermi level is related to the neutron and proton effective masses of the mean-field Hamiltonian. The strength V_0 is fitted to get a reasonable description of the experimental neutron pairing energies of ^{58}Ni , ^{90}Zr and ^{120}Sn

$$P_N = \frac{1}{2} [B(N, Z) + B(N+2, Z) - 2B(N+1, Z)] \quad (2.27)$$

for neutrons. Thus, the strength V_0 is taken equal to $-325 \text{ MeV}\cdot\text{fm}^3$ ($\eta = 0.25$) in the case of ^{58}Ni , ^{90}Zr (the f^- Skyrme set) and $-870 \text{ MeV}\cdot\text{fm}^3$ ($\eta = 1$) for ^{120}Sn (the SLy4 Skyrme set).

For the purposes of the present study, the calculation of the single spectrum and the parameters of the residual interaction are done with the Skyrme forces f^- (for ^{58}Ni and ^{90}Zr) and SLy4 (for ^{120}Sn and ^{208}Pb). The wave functions Eq. (2.21) of the 0^+ states are constructed by considering all one- and two-phonon configurations below 25 MeV built from the QRPA phonons with various multipoles $\lambda^\pi = 0^+, 1^-, 2^+, 3^-, 4^+, 5^-$ coupled to 0^+ . The theoretical results on ^{58}Ni , ^{90}Zr , ^{120}Sn and ^{208}Pb are presented in 7 – 25 MeV energy cutoff and there is an insignificant sensitivity of the calculation results near 25 MeV to the cutoff energy of the calculation space.

2.4 Isoscalar Giant Monopole Resonance and nuclear incompressibility

The Isoscalar Giant Monopole Resonance (ISGMR) is characterised by selection rules $\Delta L = 0$, $\Delta S = 0$ and $\Delta T = 0$ as illustrated in Fig. 2.1. It was observed for the first time in 1977 through inelastic scattering of 120 MeV alpha particles from ^{208}Pb [Har77, Har79]. This resonance has clearly been confirmed as such by Youngblood *et al.* [You77] with alpha inelastic-scattering measurements at very forward angles. Since then

extensive work was done to obtain a set of data in order to measure the centroid energy, the shape and the magnitude of the cross-section of the ISGMR [You97, You03, You13].

The ISGMR being the simplest electric vibrational resonance mode, is described as a collective oscillation of the surface of a nucleus in which protons and neutrons move in phase Fig. 2.1. It has been extensively studied during the last decades, see for example, Refs. [Col08, Avo13, Gar18] (and references therein) for recent reviews and discussions. This is because the ISGMR provides important information on the nuclear incompressibility K_A [Bla80, Uch04]. The centroid energy of the ISGMR as a function of nuclear mass was estimated by Bohr and Mottelson in their study of compressional modes of nuclei [Boh98] to be

$$E_{nL} = \frac{\hbar k_{nL}}{1.2} \sqrt{\frac{K_{NM}}{9m}} A^{-1/3} \text{ MeV} , \quad (2.28)$$

where k_{nL} are the eigenvalues of the motion for principal quantum number n and multipolarity L , K_{NM} represents the modulus of the nuclear matter incompressibility, and m the nucleon mass. From Eq. (2.28), the centroid energy of the ISGMR is shown to be directly related to the nuclear matter incompressibility modulus K_{NM} . This modulus belongs to the bulk properties used for the determination of the energy functional parameters for theoretical predictions [Bož15].

Furthermore, through alpha elastic/inelastic-scattering, it has been shown that the ISGMR excitation energy is proportional to the finite nucleus incompressibility K_A [Har01]. Very recently, U. Garg *et al.* [Gar18] in 2018 have successfully deduced the relationship between the excitation energy of the ISGMR and K_A , originally established in [Bla80], using a perturbative approach and the *Thouless theorem* [Tho61] expressed as

$$E_x^{\text{ISGMR}} = \sqrt{\frac{\hbar^2 A K_A}{m \langle r^2 \rangle_0}} , \quad (2.29)$$

where $\langle r^2 \rangle_0$ represents the ground state mean square radius. Under the condition that an accurate value of the nuclear matter radius can be obtained through elastic/inelastic scattering, Eq. (2.29) exhibits how one can obtain

the finite nucleus incompressibility from the ISGMR centroid energy.

The value of the modulus K_{NM} was estimated in Ref. [Boh98] to be 135 MeV. This implied the centroid energy of the ISGMR ($n = 1, L = 0$) to be

$$E_{\text{x}}^{\text{ISGMR}} \sim 65A^{-1/3} \text{ MeV}. \quad (2.30)$$

However, it has been found with the availability of experimental data, mostly from Texas A&M and RCNP groups, that the commonly accepted value for the nuclear matter incompressibility modulus is $K_{\text{NM}} = 230 - 240$ MeV, confirmed by relativistic as well as non-relativistic mean field models [Bož15]. This shifts the predicted centroid energy of the isoscalar monopole resonance. In Refs. [Har77, Har01], it is estimated as a function of nuclear mass A in medium-heavy nuclei ($A \geq 90$) to be

$$E_{\text{x}}^{\text{ISGMR}} \sim 80A^{-1/3} \text{ MeV}, \quad (2.31)$$

while around $A \sim 60$ mass region, the ISGMR might coincide with the ISGQR which leads to the centroid energy given by Eq. 2.30 [Har01]. The determination of the full $E0$ Energy Weighted Sum rule (EWSR) strength has been proven to be very difficult in light nuclei, with around only 30 – 40% of the $E0$ EWSR was found [Har77, Har01]. This implies the fragmentation of the strength distribution of the GRs. Nearby excitation of the ISGQR and other multipolarities completely overlap the ISGMR excitation energy. This yields difficulties in the determination of the K_{A} value in light nuclei.

The macroscopic and microscopic approaches to the infinite nuclear matter incompressibility coefficient have been fully described in detail elsewhere [Pat15, Gar18] (and references therein). As such, only salient details are provided in this section. For infinite nuclear matter ($N = Z$ and no Coulomb interaction), the incompressibility coefficient, K_{∞} is a very important physical quantity in the study of nuclei, supernova collapse, neutron stars, and heavy-ion collision, since it is directly related to the curvature of the Nuclear Matter (NM) Equation of State (EoS). It has been shown through the so-called leptodermous expansion Eq. (2.32) that there is a relationship

between the finite nucleus incompressibility and the infinite nuclear matter incompressibility [Bla80]

$$K_A = K_\infty + K_{\text{surf}}A^{-\frac{1}{3}} + K_\tau \left(\frac{N - Z}{A} \right)^2 + K_{\text{coul}}Z^2A^{-\frac{1}{3}}. \quad (2.32)$$

Here, N and Z are the number of neutrons and protons and $A = Z + N$. K_{surf} , K_τ and K_{coul} , represent the surface term, the asymmetry term and the Coulomb term in the nuclear incompressibility, respectively. The determination of the various parameters is done by fitting the empirical data. However, the various difficulties encountered during the extraction of the parameters rendered hopeless because they are correlated when fitted to data [Bla80, Col08, Pat14, Gar18]. Hence, the link between the compression of a finite nuclear system (the nucleus) and the infinite one (nuclear matter) is not straightforward. The connection between K_A and K_{NM} is made through the K_{NM} values of microscopic calculations capable to describe the experimental ISGMR energies over a wide mass range.

2.4.1 Experimental tools to study the ISGMR: α probe selectivity

During an excitation of a given giant resonance, other modes usually overlap in both excitation energies and scattering angles. This implies that one should use a probe which is as much as possible selective for the mode one wants to study [Har01]. The various tools used to study GRs are displayed in Fig. 2.5. Since the alpha particle is an $S = 0$, $T = 0$ probe, inelastic α -particle scattering will predominantly excite $\Delta S = 0$, $\Delta T = 0$ modes. Thus, for an even-even target nucleus with $J^\pi = 0^+$, the spin and parity of the excited states are given by ΔL and $(-1)^{\Delta L}$, respectively. Nevertheless, protons [Mar95] and deuterons [Wil80, Pat14] have also been used as probes to study isoscalar giant resonances. The deuteron is another example of an isoscalar probe because it has isospin of zero ($T = 0$). During inelastic scattering of the deuteron, spin-flip excitations can occur since the deuteron carries a spin of 1 unit in its ground state. However, the corresponding cross-sections are negligible compared to those for non-spin-flip excitations [Mor80]. Electromagnetic probes or charge-exchange reactions are

appropriate tools to study isovector resonances, in the sense that these probes can distinguish between protons and neutrons. However, for the extraction of nuclear incompressibility, which is a bulk property of nuclei, these modes remain inadequate. Furthermore, electrons are also used both as a source of isoscalar and isovector probes. However, both modes are strongly and equally excited, which makes the interpretation of such experiments quite difficult [Chen08].

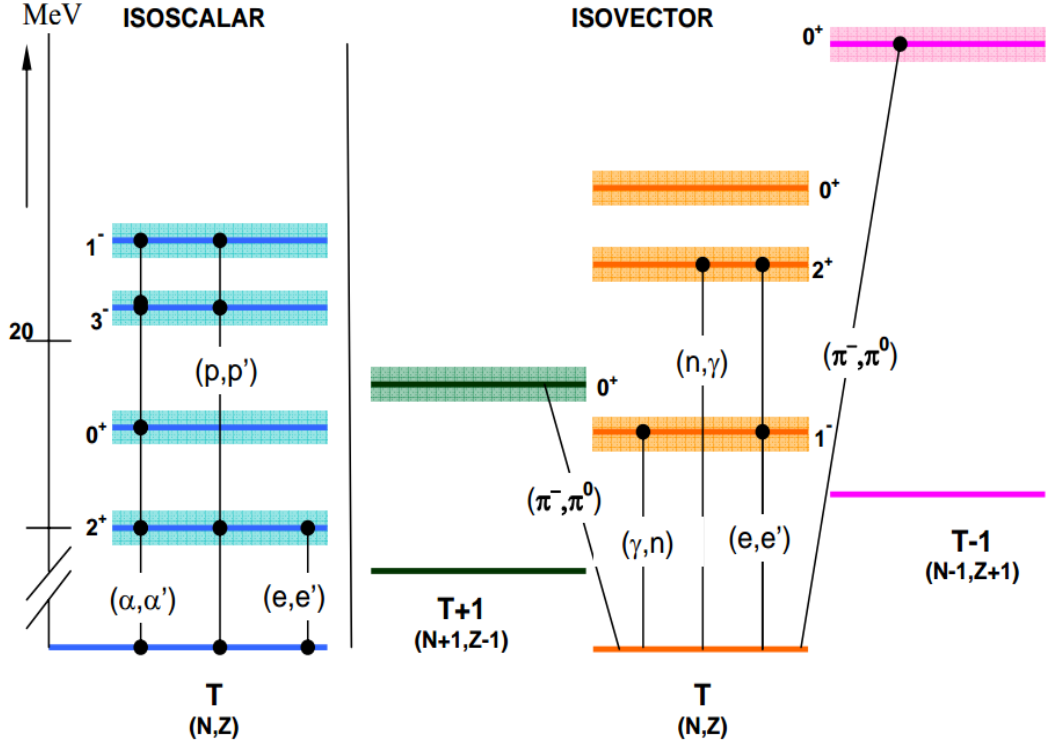


Figure 2.5: The main experimental tools used to study giant resonances (taken from [Chen08]).

2.5 Distorted-Wave Born Approximation (DWBA)

During an inelastic scattering direct reaction, it is well-established that the collective vibrational and rotational states are selectively excited [Har01]. The DWBA formalism remains an optimum tool used in the calculation of the differential cross-sections, which enables the extraction of the strength

distribution of the giant resonance. The DWBA formalism is described in [Sat83] and as such only a summary will be given in this section.

In a two-body scattering, $a + A \rightarrow b + B$, with an entrance channel for partition $a + A$ denoted by α and the exit channel by β for partition $b + B$, the total Hamiltonian, H is given by

$$H = H_\alpha + K_\alpha + V_\alpha , \quad (2.33)$$

where H_α , K_α and V_α represent the internal Hamiltonian for nucleus a and A , the kinetic energy of their relative motion and the mutual interaction potential, respectively. A complete set of internal states in channel β can be used to expand the total wavefunction ψ of the system. It is given by

$$\Psi_\alpha^+ = \sum_\beta \zeta_\beta(\vec{r}_\beta) \psi_\beta(x_\beta) , \quad (2.34)$$

where the (+) denotes the choice of an outgoing wave and α , the fact that it arises from an incident wave in the α channel. As such, the total wavefunction is composed of incoming plane waves only in the ground-state α channel while having outgoing spherical waves in all possible channels opened at the given incident energy. The estimation of the total wavefunction is through the DWBA formalism fully described in Ref. [Sat83]. Appropriate boundary conditions impose a new form of the expansion coefficients $\zeta_\beta(\vec{r}_\beta)$, where $f_{\beta\alpha}(\vec{r}_\beta, \vec{k}_\alpha)$ denotes the transition amplitude

$$\zeta_\beta(\vec{r}_\beta) \rightarrow e^{i\vec{k}_\alpha \cdot \vec{r}_\alpha} \delta_{\beta\alpha} + f_{\beta\alpha}(\vec{r}_\beta, \vec{k}_\alpha) \frac{1}{r_\beta} e^{ik_\beta r_\beta} . \quad (2.35)$$

It can be seen from the second term of Eq. (2.35) that the asymptotic direction of the relative momentum $\vec{k}_\beta$ is the same as $\vec{r}_\beta$, which implies a symmetric feature of the transition amplitude, $f_{\alpha\beta} = f_{\beta\alpha}$.

The differential cross-section for the transition from channel α to channel β is given by

$$\frac{d\sigma_{\beta\alpha}}{d\Omega} = \frac{|J_0^\beta| r_\beta^2}{|J_i^\alpha|} d\Omega , \quad (2.36)$$

where $|J_0^\beta| r_\beta^2$ is the outgoing flux per unit time going through the small area subtending a solid angle $d\Omega$ in channel β , and $|J_i^\alpha|$ the incident flux per unit time and unit area in channel α . Further, the differential cross-section can also be written in terms of the reduced masses in channels β and α and the transition amplitude

$$\frac{d\sigma_{\beta\alpha}}{d\Omega} = \frac{\frac{m_a M_A}{(m_a + M_A)} k_\beta}{\frac{m_b M_B}{(m_b + M_B)} k_\alpha} |f_{\beta\alpha}(\vec{k}_\beta, \vec{k}_\alpha)|^2 . \quad (2.37)$$

It can be seen from Eq. (2.37) that one needs to evaluate the transition amplitudes $f_{\beta\alpha}(\vec{k}_\beta, \vec{k}_\alpha)$ in order to calculate the differential cross-section. Knowing the interaction potential and the wavefunction, Eqs. (2.33) and (2.34), the normalised transition amplitude can be obtained exactly by solving the Schrödinger equation

$$H\Psi_\alpha^+ = (H_\beta + K_\beta + V_\beta)\Psi_\alpha^+ = E_\beta\Psi_\alpha^+ . \quad (2.38)$$

The transition amplitudes are thus obtained via the matrix elements

$$T_{\beta\alpha}(\vec{k}_\beta, \vec{k}_\alpha) = \langle e^{i\vec{k}_\beta \cdot \vec{r}_\beta} \psi_\beta | V_\beta | \Psi_\alpha^+(\vec{k}_\alpha) \rangle . \quad (2.39)$$

Using the DWBA formalism, one of the various ways for the determination of the wavefunction Ψ_α^+ is via the introduction of an auxiliary average potential $U_\beta(\vec{r}_\beta)$ in such a way that

$$V_\beta(\chi_\beta, \vec{r}_\beta) = U_\beta(\vec{r}_\beta) - W_\beta(\vec{r}_\beta) , \quad (2.40)$$

where the residual interaction $W_\beta(\vec{r}_\beta)$ is treated as a perturbation. As such, Eq. (2.38) is then modified as

$$[E_\beta - K_\beta - U_\beta(\vec{r}_\beta)]\zeta_\beta(\vec{r}_\beta) = \langle \psi_\beta | W_\beta(\vec{r}_\beta) | \Psi_\alpha^+ \rangle . \quad (2.41)$$

The solutions of the inhomogeneous Eq. (2.41) may be expressed in terms of the solutions of its homogeneous form

$$[E_\beta - K_\beta - U_\beta(\vec{r}_\beta)]\chi_\beta^{(+)}(\vec{k}_\beta, \vec{r}_\beta) = 0 , \quad (2.42)$$

where $\chi_\beta^{(+)}$ are the distorted waves and have the form of an incident plane wave plus outgoing (scattered) spherical waves. The (elastic) scattering of b on B is described by them. In the event of no Coulomb potential, these waves are given by

$$\chi_\beta^{(+)}(\vec{k}_\beta, \vec{r}_\beta) \rightarrow e^{i\vec{k}_\beta \cdot \vec{r}_\beta} + f_\beta^{(0)}(\theta) \frac{1}{r_\beta} e^{ik_\beta r_\beta} , \quad (2.43)$$

where $f_\beta^{(0)}(\theta)$ represents the scattering amplitude due to U_β alone. The potential U_β is obtained by fitting the experimental cross-sections. That procedure also leads to the distorted waves through a complex optical potential [Sat83].

2.5.1 Sum rules

Quantum mechanically, GRs correspond to a transition between the ground state and a collective excited state. Known as a response of a nucleus to the electromagnetic field of the incident particle, that response can be seen as the sum of the various contributions of the multipole that compose the field. The EWSR remains an important tool for the analysis of GRs, especially for the isoscalar electric modes since it is obtained using only the ground state properties of the nucleus. The strength of a given resonance is measured through it. For a given multipolarity, the EWSR is defined as the sum of the product of the excitation energy with the transition probabilities from the ground state to the excited states

$$S(\mathcal{O}^{\lambda\tau}) \equiv \sum_n (E_n - E_0) | \langle n | \mathcal{O}^{\lambda\tau} | 0 \rangle |^2 . \quad (2.44)$$

Here, the transition operator $\mathcal{O}^{\lambda\mu}$ acts on the ground state and yields a set of collective excited states labelled by n . The multipolarity and isospin structure of the resonance are denoted by λ and τ , respectively.

Further, the transition operator can be expressed in terms of the spherical harmonics [Har01]

$$\mathcal{O}^{\lambda\tau} = \sum_{i=1}^A f(r_i) Y_{\tau}^{\lambda}(\Omega_i) , \quad (2.45)$$

where A denotes the mass term. The sum rule is then evaluated as

$$S(\mathcal{O}^{\lambda\tau}) = \frac{(2\lambda + 1)\hbar^2}{4\pi(2m)} A \left\langle \left(\frac{df}{dr} \right)^2 + \lambda(\lambda + 1) \left(\frac{f}{r} \right)^2 \right\rangle . \quad (2.46)$$

For the Isoscalar Giant Monopole Resonance ($\lambda = 0 = \tau$), the transition operation is given by

$$\mathcal{O}^{00} = \sum_{i=1}^A r_i^2 Y_0^0 , \quad (2.47)$$

and using Eq. (2.46) implies

$$S(\mathcal{O}^{00}) = \frac{\hbar^2 A}{2m} \langle r^2 \rangle . \quad (2.48)$$

Here, m , A and $\langle r^2 \rangle$ are the nucleon mass, the mass number, and the second moment of the ground state density, respectively.

2.6 Properties of the ISGMR

Since its discovery in 1977, the ISGMR has been extensively studied in past experiments and as such, its main features have been well established [Har77, You77, You97, Har01]. A brief overview of these features is provided in the following sections.

2.6.1 Resonance strength

Experimentally, the multipole strength distributions is determined in terms of fractions of 100% of the EWSR [Har81, Sat87] through a fitting of the linear combination of calculated angular distributions to the corresponding experimental cross-sections. An optimal fit is given by

$$\frac{d^2\sigma^{\text{exp}}}{d\Omega dE}(\theta_{\text{c.m.}}, E_x) = \sum_{L=0}^n a_L(E_x) \times \frac{d^2\sigma_L^{\text{DWBA}}}{d\Omega dE}(\theta_{\text{c.m.}}, E_x) , \quad (2.49)$$

where $d^2\sigma^{\text{exp}}/d\Omega dE$ is the extracted inelastic α -scattering cross-sections while $d^2\sigma_L^{\text{DWBA}}/d\Omega dE$ is the angular distribution of the calculated cross-sections for 100% EWSR. The coefficients $a_L(E_x)$ represents the fractional EWSR contribution. The strength is thus calculated from these coefficients. For the ISGMR, it is given by

$$S_0(E_x) = \frac{2\hbar^2 A \langle r^2 \rangle}{mE_x} a_0(E_x) , \quad (2.50)$$

where E_x is the excitation energy corresponding to a given state.

In heavy-mass nuclei $A \geq 90$, the strength distribution is concentrated and has a Lorentzian form. For some of these nuclei, the shape has a tail at the high-energy side [Har01]. The position of the centre of the peak is about $80A^{-1/3}$ MeV. As stated earlier on, the strength distribution is of significance for extracting the incompressibility. In recent work, almost 100% of the appropriate ESWR has been located in that nuclear mass region [You13, Gup16a]. In heavy deformed nuclei an additional low-energy bump is observed due to the coupling to the $K = 0$ component of the ISGQR, where K stands for the projection of the total angular momentum on the respective axis [Boh98]. On the other hand, the strength distribution is fragmented in sd-shell nuclei. Approximately, 50 - 100% of the appropriate sum rule has been located, whereas in lighter nuclei the strength distribution constitutes only a small part of the sum-rule strength [You99a, Har01]. This makes the identification of giant resonances in lighter nuclei ($A < 60$) somewhat equivocal.

2.6.2 Resonance width

As introduced in Sect. 2.2, the resonance width Γ_{GR} is crucial and needs the most difficult approach for its theoretical understanding. The resonance width of the ISGMR varies from about 2.5 to 4 MeV [Har01].

2.7 Extraction of characteristic energy scales

In order to gain insight into the characteristic energy scales of the fine structure, two different methods were initially employed, namely the entropy index method [Aib03] and a multifractal analysis [Lac00]. However, a promising method based on the Continuous Wavelet Transform (CWT) has been successfully investigated [She04]. The wavelet analysis which is a method based on Continuous Wavelet Transform is used in this study and it is discussed in the subsequent sections.

2.7.1 Wavelet analysis

It is widely accepted that many physical systems are structured into “levels” or “scales”. In order to efficiently extract the relevant degrees of freedom in such systems, a multi-resolution or scalable mathematical representation is of a great help. Wavelet analysis is a strong and effective tool used to analyse multiscale structures [Res12]. It is worth noting that Fourier analysis can also play the same role through the superposition of infinite sine and cosine functions to analyse periodic signals. The sinusoidal functions used to represent data are non-local and infinite, this then makes the Fourier analysis inappropriate in the case of the fine structure analysis of giant resonances. Wavelet analysis offers a great precision in the localisation in time of high-frequency signal aspects [Gra95]. In addition, wavelet analysis is not constrained to the usage of sinusoidal functions only. These together allow a study of the time evolution of the frequency pattern of a given signal with the best resolution [She05].

The choice of a wavelet plays an important role when performing wavelet analysis. The wavelet function is known as mother-wavelet and is given by [Mal98]

$$\Psi(x) = \frac{1}{\pi^{\frac{1}{2}} f_b} \exp \left(2\pi i f_c x - \frac{x^2}{f_b} \right) , \quad (2.51)$$

where f_b and f_c control the wavelet bandwidth and the centre frequency of the wavelet, respectively. This wavelet-function or *wavelets* must meet a set of requirements as follows:

- the function must be real, oscillating with mean value equals zero and
- the function must have finite length.

Mathematically, the above conditions mean

$$\int_{-\infty}^{\infty} \Psi^*(x) dx = 0 \quad (2.52)$$

and

$$K_{\Psi} = \int_{-\infty}^{\infty} | \Psi^2(x) | dx < \infty , \quad (2.53)$$

where $\Psi(x)$ is a real or complex function used as mother-wavelet with $\Psi^*(x)$ as its complex conjugate. Here, K_{Ψ} is the norm of the wavelet. The second condition defines the local feature of wavelets. This is of utmost importance since it makes wavelets an optimal tool for the description of the local behaviour of a given signal. Figure 2.6 shows some frequently used wavelet functions. Each of them is useful depending on the goal that needs to be reached. For example, the Haar wavelet is composed of successive Heaviside step functions and is suitable to analyse data with square wave. The other three wavelets, gradually, present better scale precision and localising properties than the Haar wavelet. More details on each of these wavelet functions can be found in Ref. [She05].

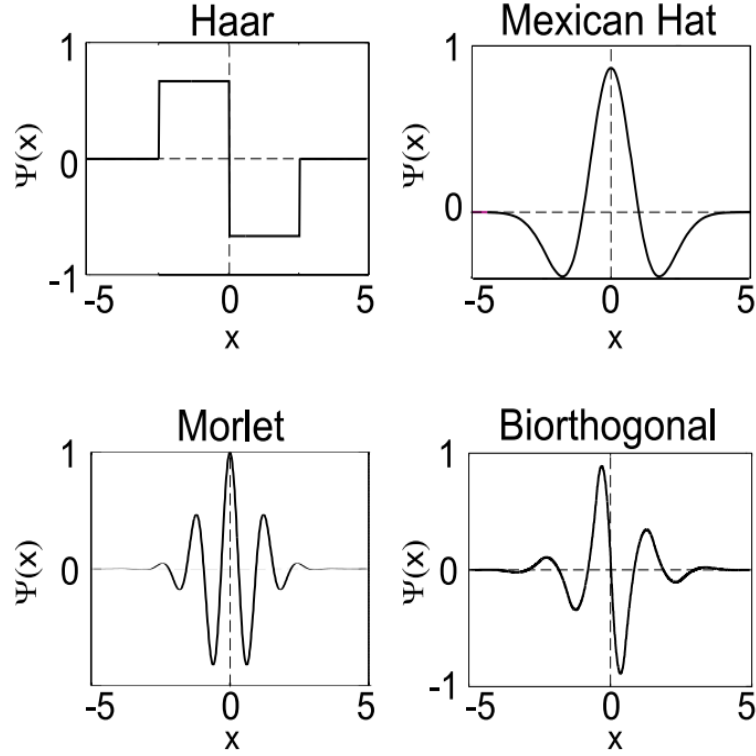


Figure 2.6: Schematic representation of some frequently used wavelet functions (taken from [She05]).

The Continuous Wavelet Transform (CWT) and the Discrete Wavelet Transform (DWT) are the two categories of wavelets transforms applied in the present work. Their main properties will be discussed in the subsequent sections.

2.7.1.1 Continuous Wavelet Transform (CWT) versus Discrete Wavelet Transform (DWT)

The convolution of a given signal $\sigma(E)$ with the wavelet function (generally complex-conjugated) yields the coefficients of the wavelet transform. This is explicitly given by [Mal98]

$$C(\delta E, E_x) = \frac{1}{\sqrt{\delta E}} \int \sigma(E) \Psi^* \left(\frac{E_x - E}{\delta E} \right) dE , \quad (2.54)$$

where $C(\delta E, E_x)$ are the coefficients of the wavelet transform, δE represents

the bin size and moreover a scaling factor. The parameter E_x shifts the wavelet position across the excitation energy range, hence allowing access to the scale localisation information.

The two parameters δE and E_x can either be varied continuously or in discrete steps defined by $\delta E = 2^j, E_{xk} = k \cdot \delta E, \quad k = 1, 2, \dots$. The former is known as the Continuous Wavelet Transform (CWT) while the later is the Discrete Wavelet Transform (DWT). The CWT yields high temporal and spectral resolution information, which provides certain easiness in the interpretation of the outputs. However, the CWT can become computationally demanding sometimes and does not provide a full reconstruction of the signal under question. On the other hand, the DWT is desirable in terms of timing and can reproduce considerable amounts of data without breaking the characteristics properties of the signal. Also, with its efficiency in the representation of the data, the DWT is used in filtering and compression. But the DWT lacks shift-invariance, i.e. the results are strongly dependent on the details of the chosen interval [She04, She08, She09, Kur18, Jin18]. This means that the DWT of a spectrum and its shifted version are then, not shifted versions of each other. Moreover, the DWT analysis can be used for a nearly model independent extraction of level densities.

It should also be noted that the values of the coefficients $C(\delta E, E_x)$ reflect on the similarity of the resulted wavelet with the original signal. If they are large, the resulted wavelet is very similar to the original spectrum while a strong difference between them results in small coefficients [Usm16].

Further, an important feature of wavelet transform is that it allows the reconstruction of the original spectrum, provided that the wavelets are orthogonal. This is done via the inverse wavelet transform given by [Mal98]

$$\sigma(E) = \frac{1}{K_\Psi} \int_{-\infty}^{\infty} \int_0^{\infty} C(\delta E, E_x) \frac{1}{\sqrt[5]{\delta E}} \Psi \left(\frac{E_x - E}{\delta E} \right) d(\delta E) dE_x, \quad (2.55)$$

$$\int_{-\infty}^{\infty} \Psi_{i,j}(E) \Psi_{k,l}^*(E) dE = \delta_{ik} \delta_{jl}. \quad (2.56)$$

It is worth noting that Eq. (2.56) is never satisfied in the CWT analysis. However, the DWT uses the choice of scales and energies based on powers of two, allowing reconstruction to be possible.

Another not less important feature of the wavelet analysis is the approximation of any background contribution in the experimental data, through the so-called *vanishing moments* of the wavelet function. This allows a nearly model independent extraction of nuclear level densities, since there is no background contribution to the wavelets coefficients. More details on the above features can be found in Ref. [She05]. It is now clear that wavelet transform is a powerful and more suitable tool in the study of the fine structure of nuclear resonances.

2.7.1.2 Wavelet power spectrum

The wavelet power spectrum is a projection of the squared wavelet coefficients onto the axis of the wavelet scale. Using a Fourier transform $\hat{f}(k)$ of a given function $f(x)$, the corresponding Fourier power spectrum $P_f(k)$ is given by:

$$P_f(k) = |\hat{f}(k)\hat{f}^*(k)|. \quad (2.57)$$

Analogously, the square root of the sum of the squares of the complex coefficients from the complex Morlet wavelet yields the power spectrum for a CWT case [Jin18].

2.8 Extraction of level densities

Nuclear level densities are of great importance in the field of nuclear physics and play a crucial role in astrophysical applications. They are defined as the number of levels in an energy unit (1 MeV) at a given excitation energy and remain an important ingredient in the calculation of reaction cross-sections [Rau97]. In addition, they are used in the prediction of reaction rates in

astrophysical reactions, via nuclear models. Nuclear level densities are extracted by counting levels of each spin and parity in experimental data. This requires a complete data set of known excited states in the low excitation energy region, just above the particle emission thresholds. Most information on level densities was retrieved from that region. However, recently, high energy-resolution inelastic scattering experiments allow the extraction of level densities by fluctuation analysis of data on the fine structure of electric and magnetic giant resonances [Kal06, Kal07, Usm11b, Usm12]. For the extraction of the $J^\pi = 0^+$ level densities and to get insight into the spin dependence of level densities, the same fluctuation analysis technique was applied in the present study. Details on the method are presented in Sect. 2.8.2.

2.8.1 Theoretical models for the extraction of nuclear level densities

Owing to its importance in nuclear reaction cross-sections, numerous approaches (models) are used to calculate level densities. These are the phenomenological Back-shifted Fermi Gas (BSFG) and the microscopic Hartree-Fock-Bogoliub (HFB) models. It is worth noting that Bethe in the early 1930's was the pioneer of the theoretical work that led to the calculation of level densities, through the Fermi Gas model and is mainly used to determine the cross-section of reactions occurring at a given energy. The density of states is defined as the number of excited states per unit energy [Bet37]

$$\rho(E) = \frac{1}{12a^{\frac{1}{4}}(E_x)^{\frac{5}{4}}} \exp\left(2\sqrt{a(E_x)}\right), \quad (2.58)$$

where E_x represents the excitation energy and a is the level density parameter. Despite its qualitative consistency with the experimental data, this model lacks for example, odd-even and collective effects.

2.8.1.1 Back-shifted Fermi Gas (BSFG) model

In order to take into account pairing and shell effect, a modification of the original Fermi Gas model, has led to the Back-shifted Fermi Gas model [Rau97, Egi05, Egi08]. The level density $\rho(E_x, J)$ in this model is expressed in terms of excitation energy E_x and spin distribution J . It is given by:

$$\rho(E_x, J) = \frac{1}{2} \rho(E_x) \mathbb{F}(J) , \quad (2.59)$$

where

$$\rho(E_x) = \frac{1}{\sqrt{2\sigma} 12a^{\frac{1}{4}}} \frac{\exp [4a(E_x - \Delta)]^{\frac{1}{2}}}{(E_x - \Delta)^{\frac{5}{4}}} \quad (2.60)$$

and

$$\mathbb{F}(J) = \frac{2J+1}{2\sigma^2} \exp \left[\frac{-J(J+1)}{2\sigma^2} \right] . \quad (2.61)$$

Here, Δ stands for the back-shift energy and $\sigma = 0.00888A^{\frac{2}{3}} [a(E_x - \Delta)]^{\frac{1}{2}}$ is the spin cut-off parameter, with A being the nuclear mass. More details on the calculation of the parameters involved in Eqs. (2.60) and (2.61) can be found in Refs. [Kur14, Jin14]. The BSFG model yields satisfactory results, mainly with light nuclei, up to 20 MeV excitation energy [Rau97, Egi05, Egi08].

Furthermore, the Constant-Temperature (CT) model [Eri60] also shows similar outputs at low excitation energy (particle separation energy) like the BSFG model [Egi88]. The level density in that model is given by:

$$\rho(E_x) = \frac{1}{T} \exp \left(\frac{E_x - E_0}{T} \right) \quad (2.62)$$

with T the nuclear temperature, E_x the excitation energy and E_0 the back-shifted energy.

2.8.1.2 Hartree-Fock-Bogoliubov (HFB) and Hartree-Fock-BCS (HF-BCS) models

The extraction of level density is performed well at low excitation energy via the use of the aforementioned phenomenological models. However, its extraction is quite difficult at high excitation energy. This is due to the high level density of nuclear states in that region. To accomplish such task, the HFB and HF-BCS models are widely employed [Hil01, Dem01, Gor08]. In addition to the parameters involved in the phenomenological models, the HFB and HF-BCS take into account the collective behaviour of nuclei and the level density is expressed in terms of parity π as well. The calculations in these models consider shell effects, pairing correlations, deformation effects and collective excitations over the range of nuclei. The outputs from the models show good agreement with experimental data.

2.8.2 Fluctuation analysis: assumptions and conditions of applicability

One of the key objectives of the present study is the extraction of the $J^\pi = 0^+$ level densities from the high energy-resolution alpha inelastic-scattering spectra. In order to achieve that goal, a fluctuation analysis method is employed. This method was first used in 1976 by Jonson *et al.* [Jon76] on β -delayed particle emission. Later on, it has been successfully applied on the analysis of electron scattering spectra [Mül82]. More recently, it has been used for the extraction of level densities from data on the fine structure of electric and magnetic giant resonances [Kal06, Kal07, Usm11b]. The autocorrelation function is one of the main features from the fluctuation analysis and it is employed in this study to extract information about the cross-section fluctuations with respect to a stationary mean value. This enables the extraction of level densities in the ISGMR region.

When performing fluctuation analysis, two conditions need to be considered depending on the comparability of the experimental energy-resolution ΔE and the mean levels spacing. In the situation where the average level width $\langle \Gamma \rangle$

is less than the average distance between levels $\langle D \rangle$ i.e. $\langle \Gamma \rangle \leq \langle D \rangle$, the high density of non-resolved states and their incoherent overlap lead to fluctuations. On the other hand, when $\langle \Gamma \rangle \geq \langle D \rangle$, the coherent overlapping states lead to fluctuations known as Ericson fluctuations [Eri60]. Because of the lack of extremely high statistics, we limit ourselves to the first condition for the present study.

In addition, the Random Matrix Theory (RMT) [Meh04] used in fluctuation analysis stays on two assumptions, namely on the probability for given spacing between adjacent states and the ground state decay widths. The former is assumed, in highly excited nuclei where levels of same spin and parity are mixed, to follow the Wigner distribution [Wig65]

$$P_W(D) = \frac{\pi D}{2 \langle D \rangle} \exp \left(-\frac{\pi D^2}{4 \langle D \rangle^2} \right), \quad (2.63)$$

while the latter obeys a Porter-Thomas distribution [Por56, Por65]

$$P_{PT}(\Gamma) = \sqrt{\frac{\langle \Gamma \rangle}{2\pi\Gamma}} \exp \left(-\frac{\Gamma}{2 \langle \Gamma \rangle} \right). \quad (2.64)$$

The assumptions above describe reasonably well nuclear excitations near the particle threshold [Haq82].

2.8.2.1 Methodology

In order to extract the $J^\pi = 0^+$ level densities from the (α, α') scattering data via the fluctuation analysis, several steps stated below need to be taken.

- The experimental (α, α') spectrum need to be cleared from any background, in order to make it free from contributions of other multipoles.
- In order to remove any fluctuation from counting statistics, the resulted spectrum obtained from the first bullet must be smoothed with a Gaussian function of width σ smaller than the experimental energy-resolution ΔE . The new spectrum obtained after this step is

labelled as $g(E_x)$.

- In a similar way, the resulting spectrum obtained from the first bullet must now be folded with a Gaussian function with a width $\sigma_>$ larger than the experimental energy-resolution. This is to ensure that one removes the gross structures in the resonance. The new spectrum obtained after this step is labelled as $g_>(E_x)$ and it defines the average about which the original points fluctuate.
- A stationary and dimensionless spectrum is obtained in the next step by calculating the ratio of $g(E_x)$ and $g_>(E_x)$. That new spectrum is given by Eq. (2.65) and its value fluctuates around an average intensity of one. This ratio is done in order to eliminate the long range correlations in the spectrum [Mat07],

$$\langle d(E_x) \rangle = \left\langle \frac{g(E_x)}{g_>(E_x)} \right\rangle, \quad (2.65)$$

where $\langle \rangle$ stands for the average done over the energy interval where the level spacing is extracted from. Furthermore, the fluctuations in the local intensity of $\langle d(E_x) \rangle$ may be expressed using an autocorrelation function and its variance given by Eqs. (2.66) and (2.67), respectively

$$C(\epsilon) = \frac{\langle d(E_x) \cdot d(E_x + \epsilon) \rangle}{\langle d(E_x) \rangle \cdot \langle d(E_x + \epsilon) \rangle} \quad (2.66)$$

and

$$C(\epsilon = 0) - 1 = \frac{\langle d^2(E_x) \rangle - \langle d(E_x) \rangle^2}{\langle d(E_x) \rangle^2}. \quad (2.67)$$

The parameter ϵ is the energy increment at a given excitation energy E_x .

In Ref. [Jon76], the autocorrelation function is approximated as follows:

$$C(\epsilon) - 1 = \frac{\alpha \langle D \rangle}{2\sigma\sqrt{\pi}} \times g(\epsilon, \sigma) \quad (2.68)$$

with

$$g(\epsilon, \sigma) = \exp\left(-\frac{\epsilon^2}{4\sigma^2}\right) + \frac{1}{\zeta} \exp\left(-\frac{\epsilon^2}{4\sigma^2\zeta^2}\right) - \left(\frac{8}{1+\zeta^2}\right)^{\frac{1}{2}} \exp\left(-\frac{\epsilon^2}{4\sigma^2(1+\zeta^2)}\right), \quad (2.69)$$

where $\zeta = \frac{\sigma_{\geq}}{\sigma}$ and $\alpha = \alpha_{\text{D}} + \alpha_{\text{I}}$ is the sum of the normalised variances of spacing and transition width distributions. The function $g(\epsilon, \sigma)$ is normalised such that $g(\epsilon = 0) = 1$.

It has been shown in Refs. [Kal04, Usm09, Pol11] that the most stable results may be obtained for $\sigma = \frac{1}{2}\Delta E$ and $\zeta = 2.5 - 3.5$. As such, the parameters α_{D} and α_{I} are assumed to follow Wigner and Porter-Thomas distributions, respectively,

$$\alpha = \alpha_{\text{W}} + \alpha_{\text{PT}} = 0.273 + 2.0. \quad (2.70)$$

Equation (2.68) shows a proportionality between the average level spacing $\langle D \rangle$ and the variance of $\langle d(E_{\text{x}}) \rangle$. The nuclear level densities are then extracted as

$$\rho(E) = \langle d(E_{\text{x}}) \rangle^{-1}. \quad (2.71)$$

Chapter 3

Experimental overview and data reduction

3.1 Overview

In this chapter the alpha inelastic-scattering experiments used to study the isoscalar giant monopole resonance in various nuclei (^{24}Mg , ^{28}Si , ^{58}Ni , ^{90}Zr , ^{120}Sn and ^{208}Pb) are discussed. In order to isolate the GMR, the excitation energy spectra associated with these nuclei were obtained at two scattering angles: 0° and 4° . The chapter is divided into two parts:

- the first part of the chapter provides a detailed description of the experimental equipment and the experiments that were conducted,
- the second part of the chapter, from Sect. 3.7 onwards, gives a detailed descriptions of the data reduction procedures crucial for obtaining background-free, accurately calibrated excitation energy spectra and experimental cross-sections.

3.2 Accelerator facility at iThemba LABS

The experiments for the investigation of the fine structure in the region of the ISGMR across the periodic table were performed at the iThemba Laboratory for Accelerator Based Sciences (iThemba LABS), situated near Cape Town in the Western Cape, South Africa. It is well known [Har01] that alpha inelastic-scattering is an excellent probe to excite the GMR. Furthermore, in order to be able to access the expected fine structure of the GMR, the detection system used in the alpha particle inelastic-scattering reaction should be a high resolution magnetic spectrometer. For this reason a 200 MeV dispersion-matched alpha particle beam delivered from the Separated-Sector Cyclotron (SSC) of iThemba LABS was used to induce inelastic alpha particle scattering, which was momentum-analysed in the K600 QDD magnetic spectrometer after having been incident upon various target nuclei.

3.2.1 Beam line at iThemba LABS

A schematic overview of the SSC facility at iThemba LABS is depicted in Fig. 3.1. Of great importance in all the processes undertaken by the lab, the Separated-Sector Cyclotron (SSC) produces beams ranging from light to heavy ions for nuclear physics research. It is worth noting that beams are also produced for other purposes such as beams for radiation biophysics research and for radioactive isotope production [Jin14]. Two injector cyclotrons, both with a K value of 8, provide low E beams for the SSC. These are the Solid Pole injector Cyclotron 1 (SPC1) and the Solid Pole injector Cyclotron 2 (SPC2). SPC2 has two external ion sources: the polarised proton ion source and the Electron Cyclotron Resonance (ECR) ion source. The alpha particle beam for the various experiments of this study was produced in the ECR ion source. The beam from the ion source is typically injected into SPC2 and accelerated to the specified energy of 200 MeV by the $K = 200$ Separated-Sector Cyclotron. The accelerated particles from the SSC are then delivered via the X, P1, P2 and S beamlines to the magnetic spectrometer [Nev11a]. The beamlines as well as associated hardware such as cleanup slits are indicated in Fig. 3.1.

The beam was set up to be dispersion matched, and some of the optimisation processes will be fully covered in Sect. 3.6.2.

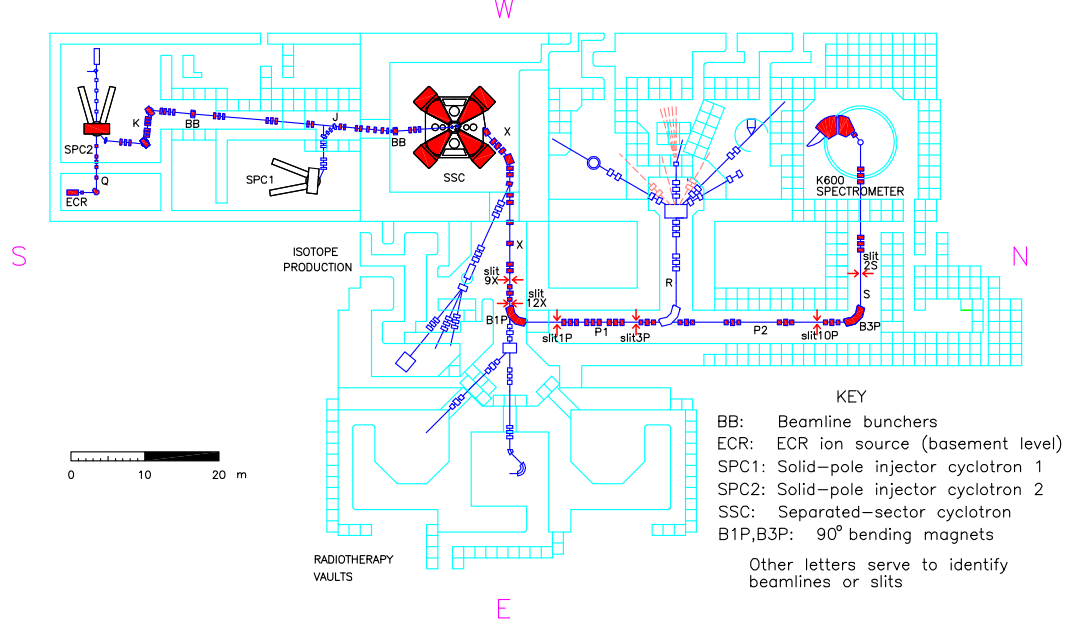


Figure 3.1: A schematic overview of the cyclotron facility at iThemba LABS.

In order to allow good energy-resolution measurements of scattered alpha particles detected by the focal plane detectors in the spectrometer, small energy-spread, small radial emittance and very little halo are the crucial conditions that the alpha particles beam delivered by the accelerator staff should satisfy. To obtain such a beam, a single turn extraction from SPC2 and the SSC must be achieved. Besides that, extra care must be taken in order to reduce potential sources of beam halo via the maximisation of the beam transmission. Furthermore, a small horizontal beam size at the object point must be ensured [Fuj02].

3.3 K600 magnetic spectrometer

After the beam interacted with the target, scattered charged particles are detected with the high resolution K600 magnetic spectrometer depicted in

Fig. 3.2. These charged particles are rigidity selected and vertically focussed by means of the five active elements of the K600 QDD magnetic spectrometer, namely a quadrupole magnet Q, two dipole magnets (D1 and D2) and two trim coils (K and H) [Nev11a]. The vertical focussing at the focal plane is ensured by the quadrupole magnet at the entrance of the spectrometer while the final horizontal focussing at the focal plane of particles leaving the residual nucleus in the same excited state is achieved through the use of the K and H trim coils. Also indicated in Fig. 3.2 are three distinct possible focal planes, viz. the low-, medium- and high-dispersion focal planes. These three focal planes result from the variation of the ratio of the fields of D1 and D2 which in turn allows the variation of the momentum dispersion between the nominal values of 6.2 cm/% and 10.9 cm/% [Don16].

Before the quadrupole magnet, the scattering chamber and collimator setup are situated. The Nationaal Instituut voor Kernfysica en Hoge-Energiefysica (NIKHEF) sliding seal scattering chamber used for the zero-degrees measurements (see Fig. 3.3) sits at the turning axis of the spectrometer and contains a turntable on which detectors or an internal beamstop (small Faraday cup) can be mounted as well as a target ladder with six available target positions [Jin14]. A collimator carousel, which holds six collimators, was used during these measurements. The Coincidence Array for K600 Experiments (CAKE) scattering chamber [Ads17b] (Fig. 3.4) was used in conjunction with a collimator ladder for the small-angle measurements. This scattering chamber was designed to contain up to six wedge-shaped Double-Sided Silicon Strip Detectors (DSSSDs) of the MMM design, placed in a lampshade configuration upstream from the target ladder. The large downstream opening of this scattering chamber makes it ideal to use in the small-angle measurements. The focal plane detector system will be fully covered in the next section.

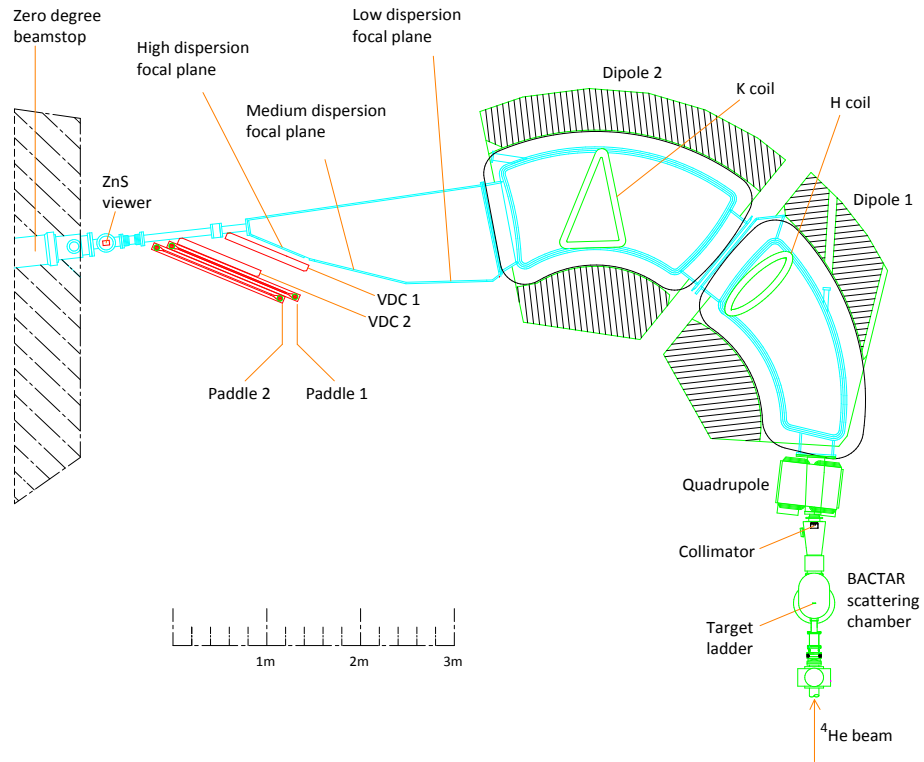


Figure 3.2: The layout of the K600 magnetic spectrometer.

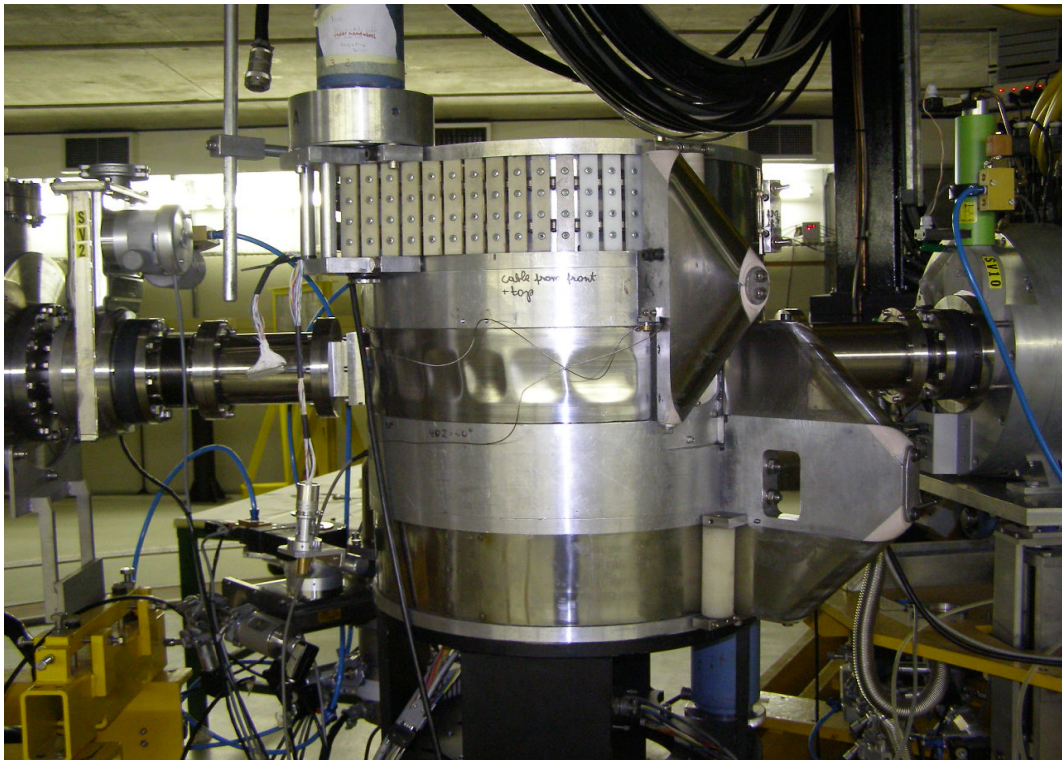


Figure 3.3: The NIKHEF sliding seal scattering chamber.

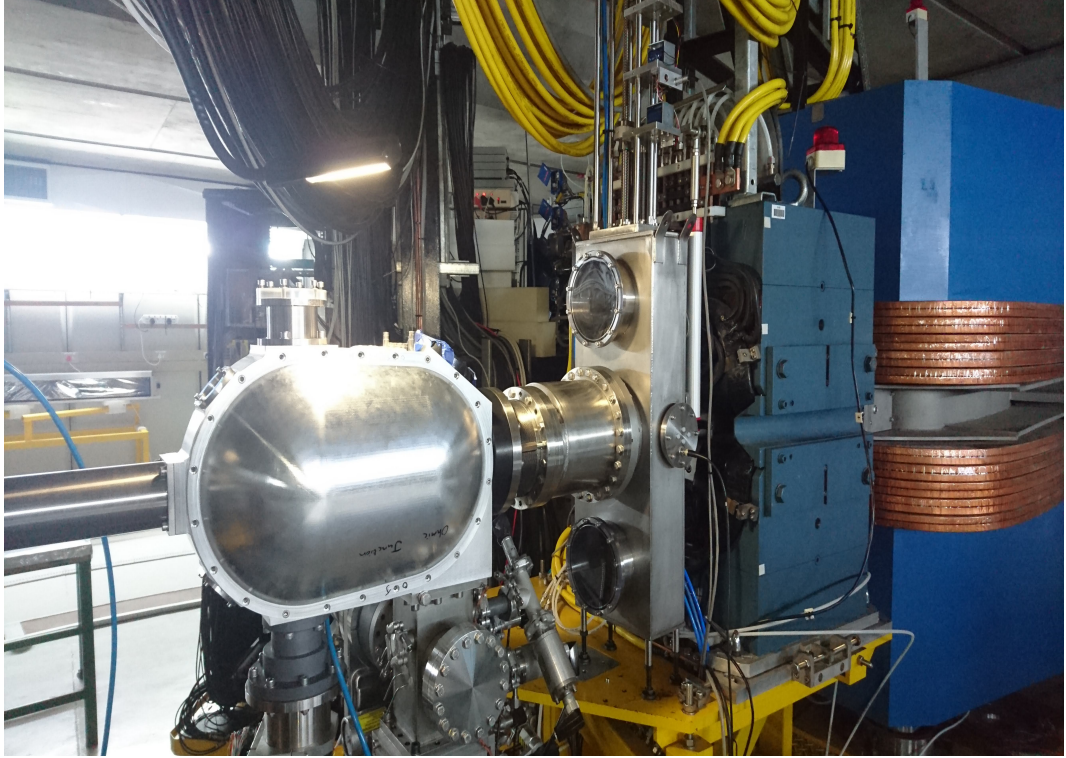


Figure 3.4: The CAKE scattering chamber.

3.3.1 Focal plane detector setup

A pair of plastic scintillator detectors located behind two Vertical Drift Chambers (VDCs) are the key components of the focal plane detector system. The technical specifications of these detectors are summarised in Table 3.1. The components of the focal plane detector setup has been described in detail elsewhere [Nev11a, Kur14, Jin14, Don16] and only the main points are summarised below.

Table 3.1: Technical specifications for the focal plane detector package.

X & U Vertical Drift Chambers	Specification
Wire configuration	vertical (X) and 50° (U)
Horizontal VDC acceptance	78 cm
vertical VDC acceptance	10 cm
Signal wire	20 μm gold plated tungsten
Guard wire	50 μm gold plated tungsten
Cathode planes	20 μm Al foil
Number of signal wires	198 (X) and 143 (U)
Number of guard wires	199 (X) and 144 (U)
Cathode-anode spacing	8 mm
Signal wire spacing	4 mm
Guard wire spacing	4 mm
Gas mixture	90% Ar and 10% CO ₂
Entrance and exit windows	25 μm mylar
Pre-amplifier	Technoland (13 units (X) and 9 units (U))
Typical guard wire voltage	~ -500 V
Typical cathode plane voltage	~ -3800 V
Scintillators	Specification
Scintillating material	Bicron BC-408
Scintillator dimensions	$1219.2 \times 101.6 \times 12.7$ (or 6.4) mm ³
Lightguide	Saint Gobain Crystals 90° adiabatic twisted pair
Photomultiplier tube (PMT)	Hamamatsu R329 – 02 PMT tube with a Hamamatsu E934 base
Typical PMT voltage	$\sim 1400 - 1800$ V for 0.5 inch
Maximum PMT voltage	2700 V

3.3.1.1 Plastic scintillator detectors

Also referred to as paddle detectors due to their geometry, the plastic scintillators, each with dimensions 122 cm $\times$ 10.2 cm, are located just behind the drift chambers and very close to the focal plane (indicated in Fig. 3.5).

They are 1/4" and 1/2" thick and are wrapped in sheets of aluminised mylar to avoid light penetration. Photomultiplier (PMT) tubes are connected vertically via 90° twisted adiabatic lightguides at both ends of each of the scintillators. This configuration enables close proximity to the 0° beampipe. In addition to the provision of event trigger signals, the scintillator detectors help identify scattered particles via $\Delta E - \Delta E$ particle identification spectra.

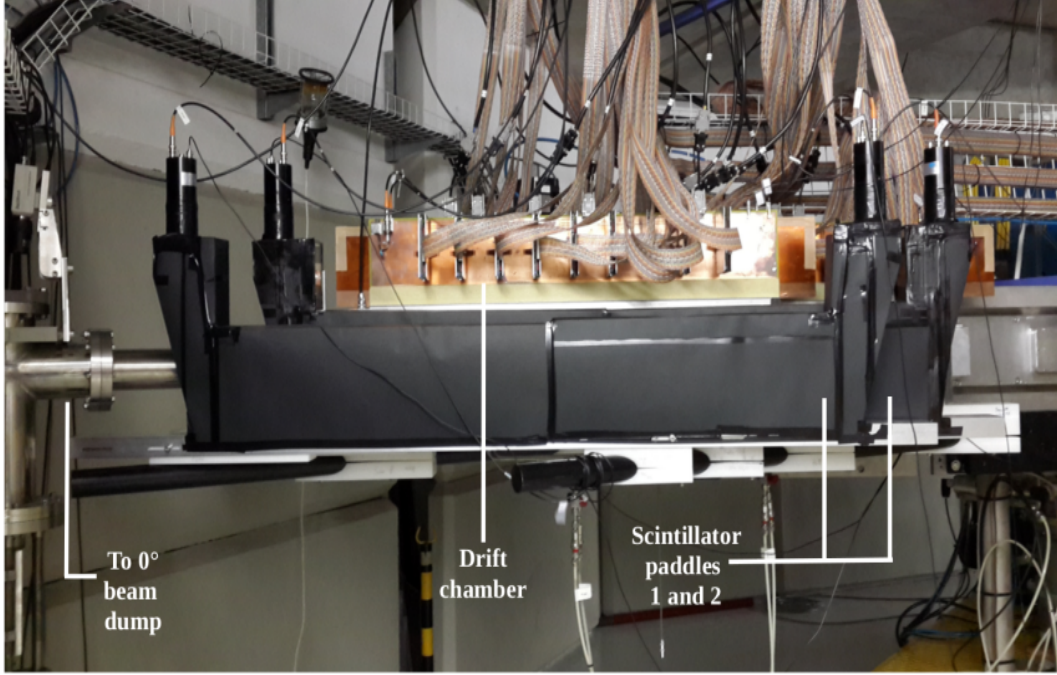


Figure 3.5: The K600 magnetic spectrometer focal plane detector package.

3.3.1.2 Vertical drift chambers

For an accurate determination of both the horizontal and vertical components of the particle tracks in the focal plane, the Vertical Drift Chambers (VDCs) are the essential tools to achieve such task. The accuracy in the position determination leads to good rigidity and scattering angle determination of scattered particles. As seen in Table 3.1, each of the VDCs possesses two wire-planes in an XU configuration with the X-wires perpendicular with respect to the scattering plane and the U-wires angled at 50°. Two mylar planes, 25 μm

thick serve to isolate the interior of the drift chambers from atmosphere. The detector interior is filled with gas mixture of 90% Ar and 10% CO₂.

3.3.2 K600 angle modes

Owing to its complex nature of the nuclear interaction, the experimental excitation spectra typically contain many contributions. However, states associated with low angular momentum transfer are favoured for reactions at 0°. This makes it a highly selective method to study excitations with low angular momentum transfer. The spectrometer can be operated at an angle of 0°. This is motivated by the fact that the first maximum of the ISGMR is observed at scattering angles of 0°. This is evidenced from the angular distribution calculations through the DWBA framework discussed in Sect. 4.2.1. However, it has been shown through systematics studies that intermediate energy alpha scattering at zero-degrees excites not only $E0$ but also isoscalar $E1$, $E2$ and, to lesser extent, $E3$ transitions [Tam11, Pol12]. The background contribution from other multipoles needs to be taken into account as it can impact the $J^\pi = 0^+$ strength and level density extraction. The way to take other multipoles into account is to first determine the strongest one of the multipoles, and then determine at which angle they are the strongest, corresponding to where the 0^+ state is the weakest.

As it happens a measurement at 4° is ideal, as the contribution from all other multipolarities have relatively flat distributions in this angular region while the ISGMR goes through a local minimum (see Figs. 4.7 and 4.8). Therefore, by subtracting the spectrum at the first minimum of the expected ISGMR angular distribution from the spectrum at 0°, one gets a fairly good idea of the 0^+ strength when looking at the difference spectrum. The inelastic scattering measurements for this study were performed at 0° and 4°.

3.3.2.1 Zero-degrees mode

In the zero-degrees scattering experiment, the circular spectrometer aperture covered $\pm 1.91^\circ$. In this mode, both the unscattered beam and the inelastic

scattered particles are transported through the K600 magnetic spectrometer. The primary beam is very close, approximately 10 cm, to the focal plane detectors positioned in the high-dispersion focal plane. For such measurements, the background contributions (discussed in Sect. 3.6.1) are potentially very high [Nev11b]. Past the high-momentum side of the focal plane, the beam then moves through the zero-degree beamline and is stopped by a Faraday cup situated within the wall of the experimental vault. This is illustrated in Fig. 3.2.

3.3.2.2 Four-degrees mode

In the small-angle scattering experiment, due to the limited acceptance of the spectrometer, the primary beam does not enter into it. The centre of the spectrometer aperture was set at an angle of 4° covering horizontal scattering angles from 2° to 6° . In this mode, a special Faraday cup adjacent to the K600 entrance aperture in front of the spectrometer quadrupole was used to stop the unscattered beam. The background conditions for finite angle measurements are better than for 0° measurements and one can operate the spectrometer in off-focus mode and also use the medium dispersion focal plane. In the off-focus mode the settings of the K600 quadrupole, normally set up to achieve vertical focusing of scattered particles in the focal plane, are changed to spread out the particles of interest vertically in the focal plane. This will allow for the determination of the vertical component of the scattering angle, which cannot be determined under focussed conditions as the vertical magnification matrix element of the K600 is small and results in a spread of vertical angles in the focal plane that are too small for the focal plane detectors to distinguish. As opposed to the high-dispersion focal plane, the medium dispersion focal plane covers a larger momentum range. This enables access to a broader excitation energy region [Nev11a].

3.4 Data acquisition system

The main components of the K600 data acquisition system are hardware modules using the Versa Module Europa (VME) standard. Signals from the

drift chambers are amplified by 16-channel electronic cards placed directly on the VDC. The signals are delivered to the CAEN V1190A Time-to-Digital Converters (TDCs), which has 100 picoseconds time resolution capacity, through 16-channel twisted-pair ribbon cables. The digitisation of the signals from the photomultiplier tubes attached to the scintillator detectors is achieved by a 12 bit current integrating CAEN V792 charge-to-digital converter (QDC). The trigger signals for the Maximum Integrated Data Acquisition System (MIDAS) [Mid07] which is used to save event-by-event data consist of the coincidence signal of the two scintillator detectors, considered as having 100% geometric efficiency for the detection of charged particles [Jin14, Don16]. For each scintillator detector the timing of the passage of a charged particle through the detector is established by means of a mean-timer (Lecroy Model 624). With its two inputs the individual timing signals from the PMTs at each end of the scintillator is established through the constant fraction technique using an Ortec 934 Constant Fraction Discriminator. If a coincidence event is established between the two scintillators within a time window of 72 ns a signal is sent to the QDC gate input and an event is registered, triggering the digitisation of PMT signals into the QDC as well as timing of any signals into the TDC within a time window of approximately $1\text{ }\mu\text{s}$ prior to as well after the arrival of a copy of the QDC gate signal provided to the TDC trigger inputs. During the digitisation phase of the QDC module, taking $6 - 7\text{ }\mu\text{s}$ per event, the scintillator coincidence unit is devoted to prevent the acceptance of another event. This is the main source of the deadtime of the DAQ.

3.5 Targets

In order to obtain the best possible energy-resolution that will allow for the survey of the monopole strength and the exploration of the fine structure of the ISGMR, typical self-supporting targets of areal densities ranging from 0.23 mg/cm^2 to 1.43 mg/cm^2 were used. The target thicknesses as well as their isotopic enrichment are summarised in Table 3.2. All the targets used during the experiments were prepared in the iThemba LABS target laboratory. Important criteria that targets must meet include good mechanical strength, stability under the beam irradiation and high isotopic enrichment [Sto14].

Among the factors that influence the target preparation techniques for nuclear structure experiments, the chemical and physical properties as well as the targets thickness remain of main importance. The material used in the target preparation process was supplied by the Oak Ridge National Laboratory and Isoflex USA in ingot form. The metallic targets e.g. ^{24}Mg , ^{58}Ni and ^{90}Zr were rolled. They were placed within a 1 mm thick polished stainless steel plates and then run through a set of rollers. The rolling process is fully described in Ref. [Sto14]. In order to avoid oxidation/water absorption, the ^{24}Mg material was rolled in an inert atmosphere.

The lead and tin targets were deposited on a glass slide that contains a thin film of a parting agent. This agent assists to release the evaporated material from the glass substrate. A potassium chloride salt was used as a parting agent and the process was conducted under vacuum. The released thin film of ^{208}Pb and ^{120}Sn were then mounted on the target frames that are used at the K600 spectrometer. Finally, the silicon target was made by etching silicon wafers using acid.

Table 3.2: Target specifications.

Target	Z	N	Areal density (mg/cm ²)	Isotopic enrichment (%)
^{28}Si	14	14	0.2321 ± 0.05	99.94%
^{58}Ni	28	30	0.7 ± 0.05	99.48%
^{90}Zr	40	50	0.9 ± 0.05	96.80%
^{120}Sn	50	70	1.08 ± 0.05	98.94%
^{208}Pb	82	126	1.43 ± 0.05	99.7%
Calibration targets				
^{24}Mg	12	12	0.23 ± 0.005	99.94%
^{197}Au	79	118	0.22 ± 0.005	99.7%

3.6 Experimental procedure

This section presents aspects of the experimental procedure followed for the experiments that form part of this study. During the experiments, the target nuclei were bombarded with an alpha beam at an incident beam of approximately 200 MeV over a period of four weekends with an average beam current value of 1 nA. The beam energy, experimentally obtained energy resolution, and the angular range covered for each experiment are summarised in Table 3.3. The second small-angle experiment performed to complete the data was on a different weekend from the 0° and the first small-angle experiments. The differences in the set-up of the dispersion-matched beam account for the difference in final energy resolution.

Table 3.3: Experimental details.

Experiment	Beam	Energy (MeV)	Energy res. (keV) FWHM	Inelastic angular range
PR236A	$^4\text{He}^{++}$	196	65	$0.0^\circ \rightarrow 1.91^\circ$
PR236A	$^4\text{He}^{++}$	196	65	$2.0^\circ \leq \theta_{\text{Lab}} \leq 6.0^\circ$
PR236B	$^4\text{He}^{++}$	196	70	$2.0^\circ \leq \theta_{\text{Lab}} \leq 6.0^\circ$

3.6.1 Background contributions

The background contributions to the position spectrum in the focal plane and their minimisation processes have been fully described in detail elsewhere [Nev11a, Kur14, Jin14, Don16] and only the main points are summarised here.

There are two main sources of background contribution to the position spectrum in the focal plane, viz. the beam-related background caused by beam halo, and the target-related background. Beam halo is composed of the beam particles whose coordinates in phase space differ from those of the core beam. Target-related background is a result of the elastic scattering off the

target itself and the re-scattering off any exposed hardware in the spectrometer [Nev11b].

The particles that make up the target-related background are detected in the focal plane with energies and angle coordinates that are similar to the inelastically scattered particles of interest. This makes the background minimisation quite difficult [Nev11b]. Fortunately, various conditions as described in Refs. [Nev11a, Jin14, Don16] have been achieved to ensure low target-related background. These depend on the ratios of the dipole fields in the K600 during zero-degrees experiments as well as the geometry of the collimator openings.

Procedures also exist to minimise the beam halo. The beam halo is mainly due to the interaction between the beam and beamline elements, especially at the beam extraction from the SSC as well as the beam size definition slit 9X, the emittance slit 12X and the energy spread definition slit 1P (see Fig. 3.1 for the location of these slits). A big component of this background can be removed by employing the beam halo slits 3P, 10P and 2S (see Fig. 3.1) that are located at horizontal and vertical focus points along the beamline.

During an experiment the minimisation of the beam halo conditions to the level of 50 Hz per nA is the first thing that is pursued once beam alignment on the scintillating beam viewer target is achieved. Images of the beam approximately 3 m upstream from the target, and at the target position, is shown in Figs. 3.6 and 3.7. This is then followed by optimising the dispersion matching conditions.

3.6.2 Dispersion matching

The dispersion matching process is well detailed in Refs. [Jin14, Don16] (and references therein) for further reading, and as such only summary will be provided here.

In order to obtain the best possible spatial definition in the focal plane of the K600 magnetic spectrometer as well as scattering angle definition, the matching of the ion optical characteristics of the beamline to the

spectrometer is of utmost importance. The concept of dispersion matching was first proposed by Cohen [Coh59]. This was later demonstrated by Blosser *et al.* [Blo71]. Cohen illustrated that if a proper ‘matching’ of the beamline and a spectrometer is achieved, the spectral resolution can be better than the beam resolution, up to the limit of the resolving power of the spectrometer. The dispersion matching process is composed of several procedures such as kinematic correction, focus matching, lateral dispersion matching and angular dispersion matching. In the work presented here, the conditions for dispersion matching [Fuj97] were realised using the faint beam method described in [Fuj02] for the zero-degrees measurements (and summarised below), and by optimising the resolution of the ground state of the $^{197}\text{Au}(\alpha, \alpha')$ reaction for the 4° measurements.

3.6.3 Faint beam technique

The faint beam method is a technique specifically used for both lateral and angular dispersion matching in a short time frame for zero-degrees experiments [Fuj02]. For small-angle measurements where the beam does not enter the K600, dispersion matching can be achieved using the ground state of a calibration target. In this study the ^{197}Au nucleus was used. This is, however, not applicable for 0° measurements due to the limitations of accessible low-lying excitation energies starting from 8 MeV approximately, which means that the ground state is not observed. The faint beam method relies on the fact that the matching conditions can be analysed by observing the beam properties in the focal plane, provided that a beam of far lower intensity, approximately 10^3 particles per second, is used [Fuj02]. At iThemba LABS, such intensity can be achieved by inserting three beam attenuation meshes between the ECR ion source and SPC2. These meshes reduce the number of alpha particles by a factor of 10^4 without affecting the beam profile and its ion-optical characteristics [Nev11a]. More details about the meshes are presented in Ref. [Don16]. Figure 3.8 illustrates the realisation of different matching conditions. The matching conditions are achieved via optimisation of last beamline quadrupole magnets. The quadrupole magnet Q5S is used to optimise the vertical focus and Q6S is used to optimise the horizontal focus and dispersion. Both of them together with the quadrupole magnet Q4S, with the beamline set up in the dispersive

mode, are used to optimise the resolution of the scattered particles by changing the current settings of these magnets. The result when all the three conditions are achieved is depicted in Fig. 3.8(a). It can be clearly observed that the position and angle spreads of the beam are greatly minimised.

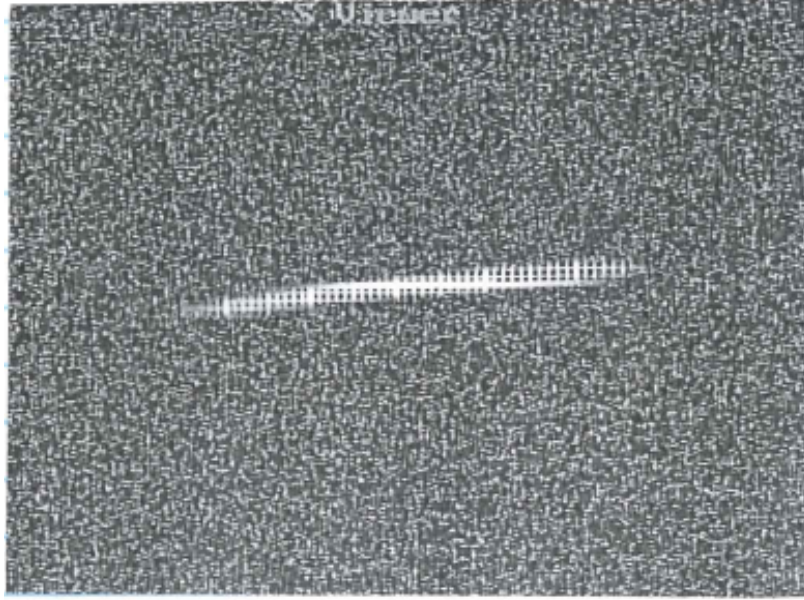


Figure 3.6: An image of the beam 3 m upstream from target position. The beam is approximately 50 mm wide at this point, and 3 mm high.

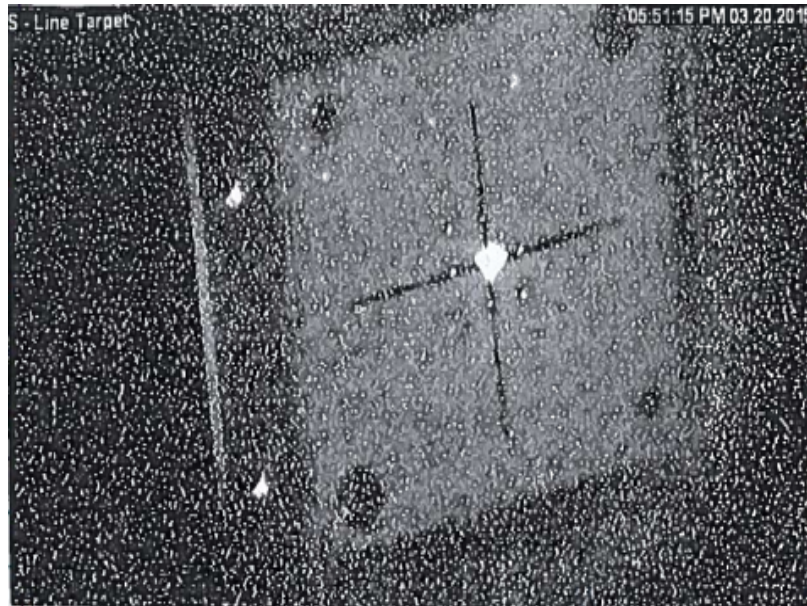


Figure 3.7: An image of the beam at the target position. The beamspot is approximately 3 mm $\times$ 3 mm big.

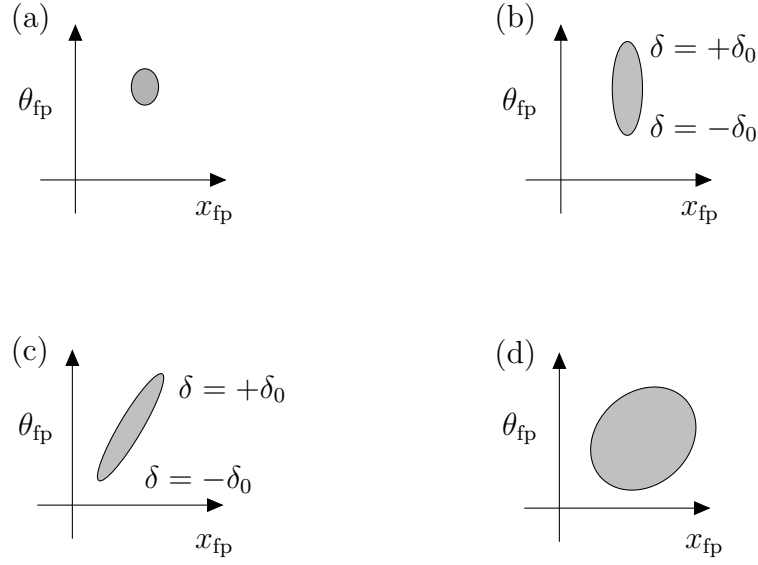


Figure 3.8: The θ_{fp} versus x_{fp} image of the beam in the focal plane. The beam ellipses are shown for the cases where (a) focus matching, lateral dispersion matching and angular dispersion matching are realised; (b) only focus matching and lateral dispersion matching are realised; (c) only focus matching is realised and (d) none of the focus, lateral or angular dispersion matching conditions are realised (taken from [Fuj02]).

3.7 Data reduction

During the experiment, data files were acquired with the MIDAS data acquisition system [Mid07]. They were analysed offline using ROOT software [Bru97] together with scripts created by users at iThemba LABS in C and C++ programming languages. Several steps are needed during the offline analysis process for the extraction of important information from the data files. After the particles of interest were properly identified, the gated spectra were further analysed for background reduction and energy calibration in order to reliably extract the corresponding cross-sections and, hence, the angular distributions.

3.7.1 Particle identification

In order to successfully interpret the information left behind as a particle passes through the focal plane, the particle identification is an important first step to select relevant particles for further analysis. The Time-Of-Flight (TOF) selection and the pulse-height information of the trigger signals from the scintillator counters were used in the particle identification procedure.

In order to successfully identify and group the detected particles, the Particle identification (PID) approach is used to interpret the information left behind as a particle passes through the focal plane detector system. The TOF selection is useful for PID since different types of particles may have similar rigidities, defined as

$$R = \rho B = \frac{p}{q} , \quad (3.1)$$

where B is the magnetic field, ρ is the radius of curvature, q , p and R are the particle's charge, momentum and its magnetic rigidity, respectively. This means that particles with the same momentum and charge end up at the same place in the focal plane. However, due to the difference in their respective masses, they will take different times to traverse the same distance between the target, the point of initial interaction, and the focal plane, where they are detected. The different TOF for different particles can be calculated knowing their momenta or kinetic energies, which will provide knowledge of their speed, and the distance to be travelled. The distance between the target and the center of the focal plane, for particles along the central ray of the K600, is approximately 9.1 meters. However, during an experiment, only a relative TOF value is determined, i.e. instead of measuring the time between the target and the focal plane, the time difference is taken between the beam RF signal and the time determined when the particles hit the scintillators.

The particle identification procedure also relies on the knowledge of the particle's energy loss in the plastic scintillators. When a charged particle passes through matter, it loses energy primarily by ionisation. The corresponding rate of energy loss is described by the Bethe-Bloch formula [Kno79] given by:

$$-\frac{dE}{dx} = 2\pi N_A r_e^2 m_e c^2 \rho \frac{Z}{A} \frac{z^2}{\beta^2} \left[\ln \left(\frac{2m_e v^2 \gamma^2 W_{\max}}{V^2} \right) - 2\beta^2 - \delta - 2\frac{C}{Z} \right] , \quad (3.2)$$

where $W_{\max}$ is the maximum energy that can be imparted to a free electron in a single collision, N_A is the Avogadro's number, r_e is the classical electron radius, m_e is the electron mass, ρ is the density of the medium, Z is the atomic number of the medium, A is the atomic mass number of the medium, z is the charge of the incoming particle, v is the speed of the incoming particle with $\beta = v/c$ and $\gamma = \frac{1}{\sqrt{1-\beta^2}}$, V is the mean excitation potential, δ is the density correction and C is the shell correction to the energy loss. In a scintillator this energy loss is converted to photons, and the energy loss is approximately proportional to the number of photons generated in the scintillators.

Light created within the scintillator is attenuated as it moves through the scintillator medium towards the photomultiplier (PMT). This is described as follows [Pat15]

$$I(x) = I_0 \exp \left(-\frac{x}{l} \right) , \quad (3.3)$$

where x denotes the distance travelled within the scintillator, I_0 is the initial photon number and l is the attenuation length of the scintillator material. The energy loss as the particles pass through the scintillator is successfully obtained through the geometric mean $\bar{I}$, as expressed in Eq. (3.4) of the number of photons from the two PMTs, where L represents the length of the scintillator [Pat15],

$$\bar{I} = \sqrt{I_L I_R} = \sqrt{I_0 \exp \left(-\frac{x}{l} \right) \times I_0 \exp \left(-\frac{L-x}{l} \right)} = I_0 \exp \left(-\frac{L}{2l} \right) . \quad (3.4)$$

The combination of energy loss and TOF information is used to identify different particle species in the focal plane. However, these parameters are also useful to minimise the contribution of instrumental background to the final spectra.

The problem with small-angle inelastic scattering measurements is not only identifying the particles of the correct type, but also differentiating between the inelastically scattered particles and particles originating from beam halo.

The particles, which are of the same species and energy range of the particles of interest, can be distinguished by a different TOF (for the same energy loss) and differences in measured scattering angles, since they originate from different scattering points.

Figure 3.9 shows the scintillator pulse height versus the TOF spectrum for Paddle 1 for the case where a ^{208}Pb target was used as well as for the case where no target was used. Figure 3.10 displays a two-dimensional histogram of TOF versus the horizontal focal plane position for the case where a ^{120}Sn target was used as well as for the case where no target was used. Similarly, Fig. 3.11 displays a two-dimensional histogram of TOF versus the horizontal scattering angle θ_{scat} for the case where a ^{28}Si target was used as well as for the case where no target was used. The software gate indicated (dashed contour lines) in each of these figures represent the gates used to select physics events for all targets, i.e. the inelastically scattered alpha particles of interest.

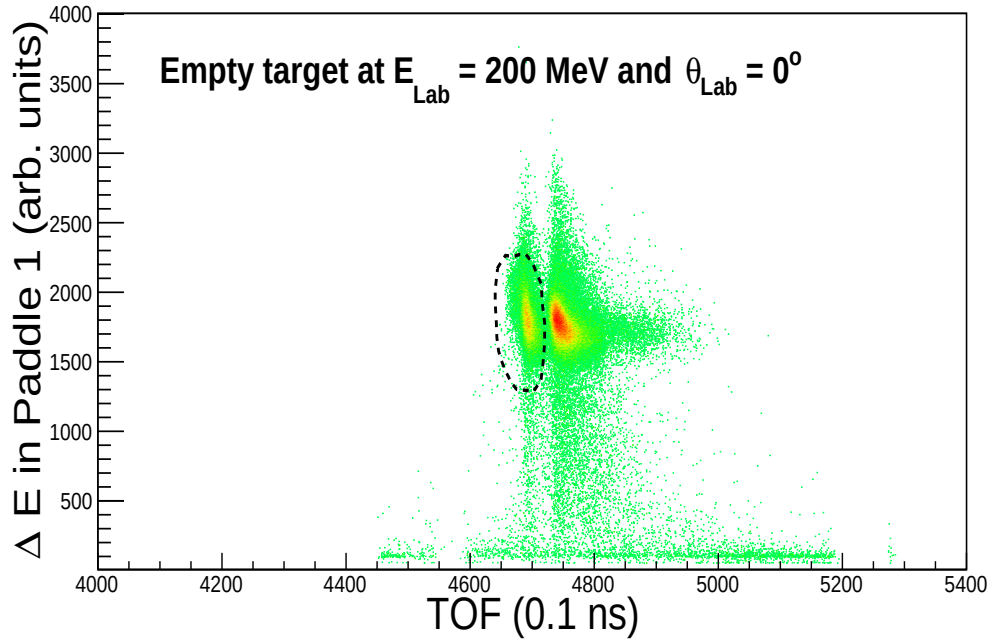
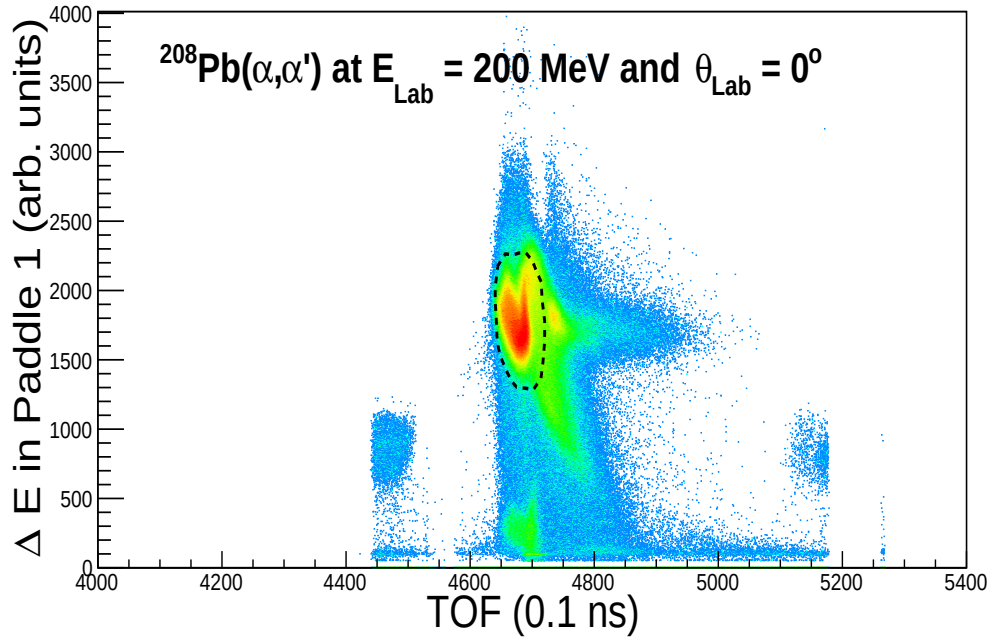


Figure 3.9: The pulse height in Paddle 1 plotted as a function of relative TOF values for the case where a ^{208}Pb target was used (top panel) and no target was used (bottom panel).

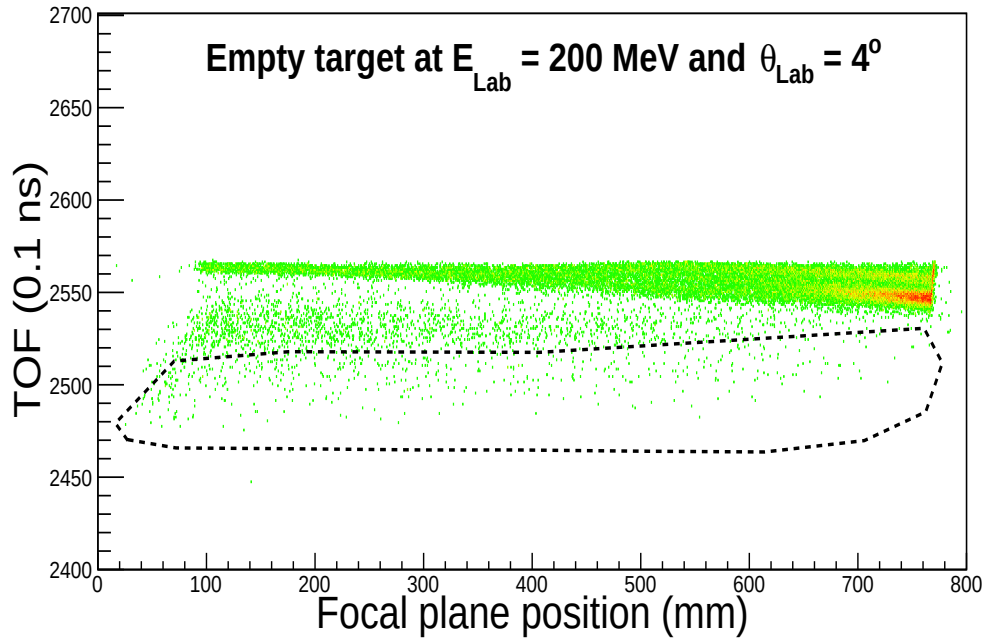
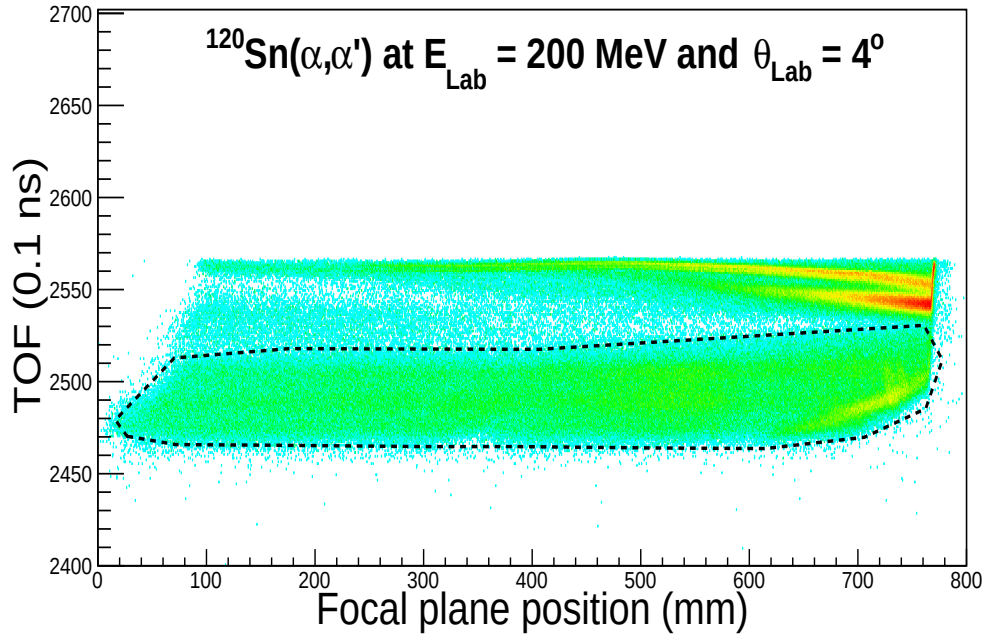


Figure 3.10: The relative TOF values plotted as a function of the focal plane position for the case where a ^{120}Sn target was used (top panel) and no target was used (bottom panel).

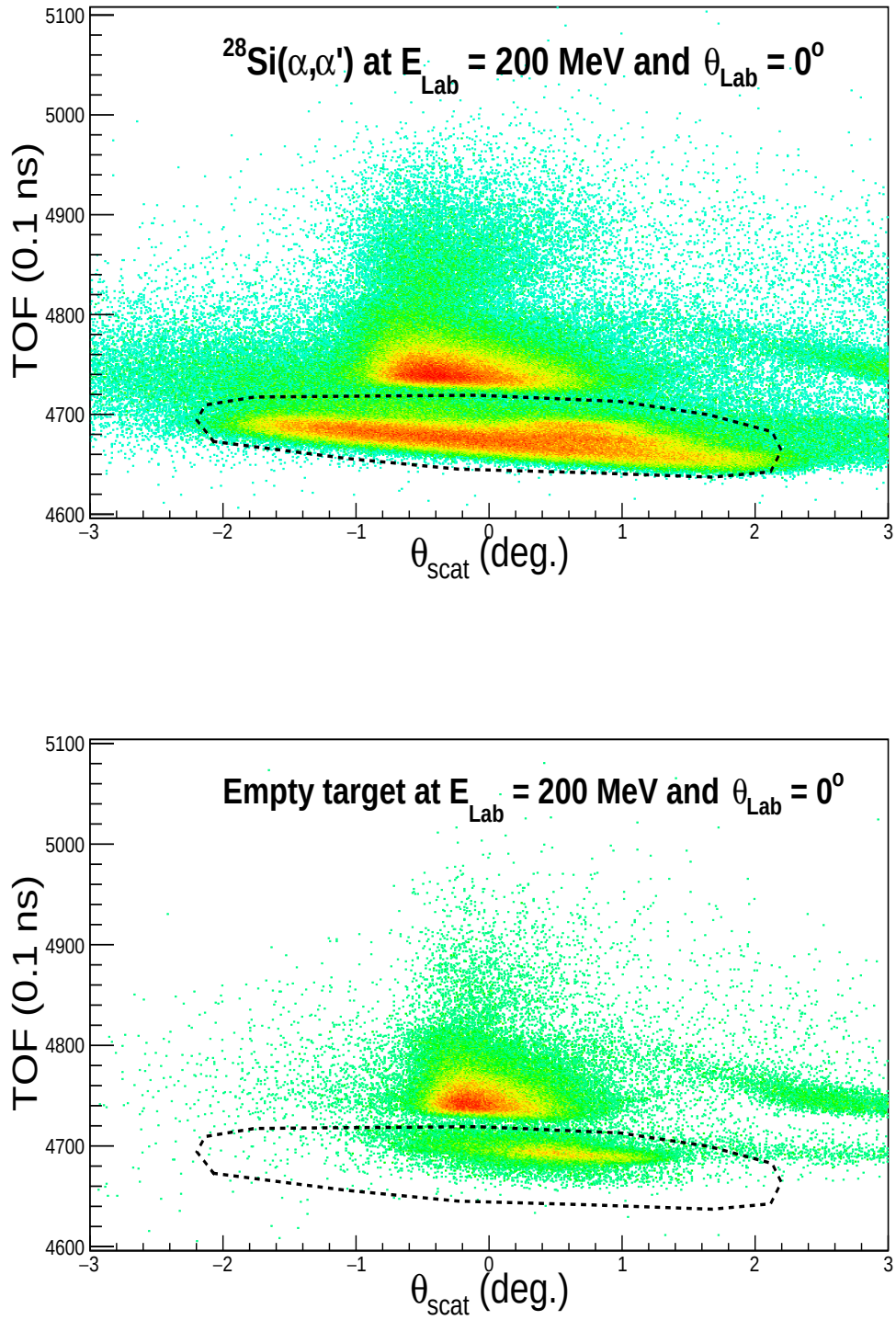


Figure 3.11: The relative TOF values plotted as a function of the horizontal scattering angle θ_{scat} for the case where a ^{28}Si target was used (top panel) and no target was used (bottom panel).

3.7.2 VDC data extraction

The various processes involved in order to obtain accurate focal plane position information are discussed in detail elsewhere [Nev01, Kur14, Jin14]. Only the main points are summarised below.

3.7.2.1 VDC functioning and focal plane position determination

The determination of both the horizontal and vertical components the focal plane coordinates, is possible through the three-dimensional particle track reconstruction which can be achieved using the signals from the multiwire drift chambers (MWDCs). This basic principle is depicted in Fig. 3.12. In order to ensure an accuracy in the coordinate measurement, the variations in the measured drift times must be corrected for. These variations are mostly due to the variations in ribbon cable lengths (MWDC pre-amplifier to TDC). The automation of the cable length correction process described in [Kur14] is done using a C++ code written by J.J. van Zyl [Zyl11]. This greatly helps since only minor manual adjustments are needed for the observed variations after the automated procedure is applied. Through the use of a Look-Up-Table (LUT), which may need offsets in some cases, the drift times are mapped to their associated drift distances.

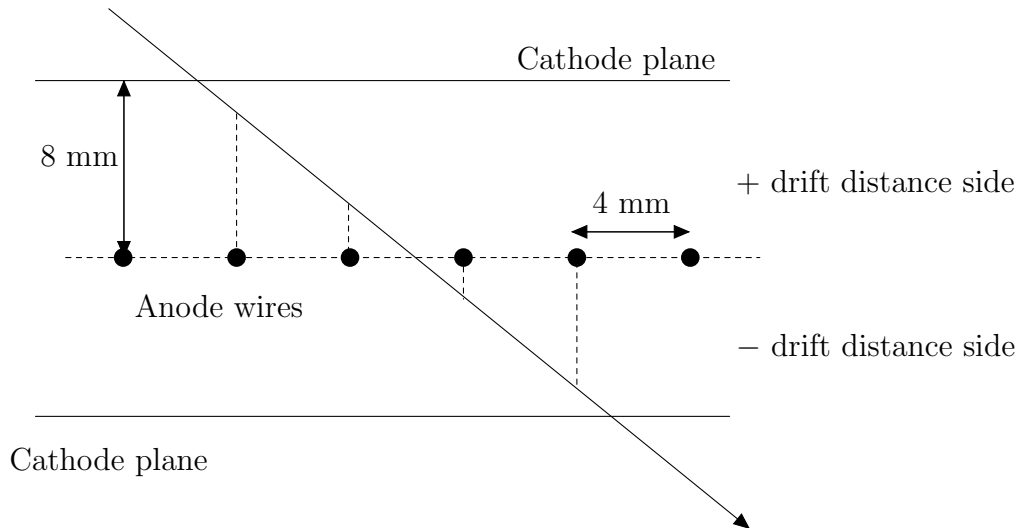


Figure 3.12: A schematic representation of the trajectory associated with a charged particle intersecting the VDC resulting in an event.

With reference to Fig. 3.12, the two-dimensional resolution (Res2D) spectrum Fig. 3.13 shows the relative position in the drift cell where the track crosses the wireplane. This spectrum indicates whether an offset inclusion is required or not from how the central regions of the spectra are aligned. Including a LUT offset allows for the alignment of the central regions and thus for the correct mapping from drift times to drift distances. Figure 3.13 displays the spectra before correction (upper panel) and the corrected two-dimensional spectra (lower panel).

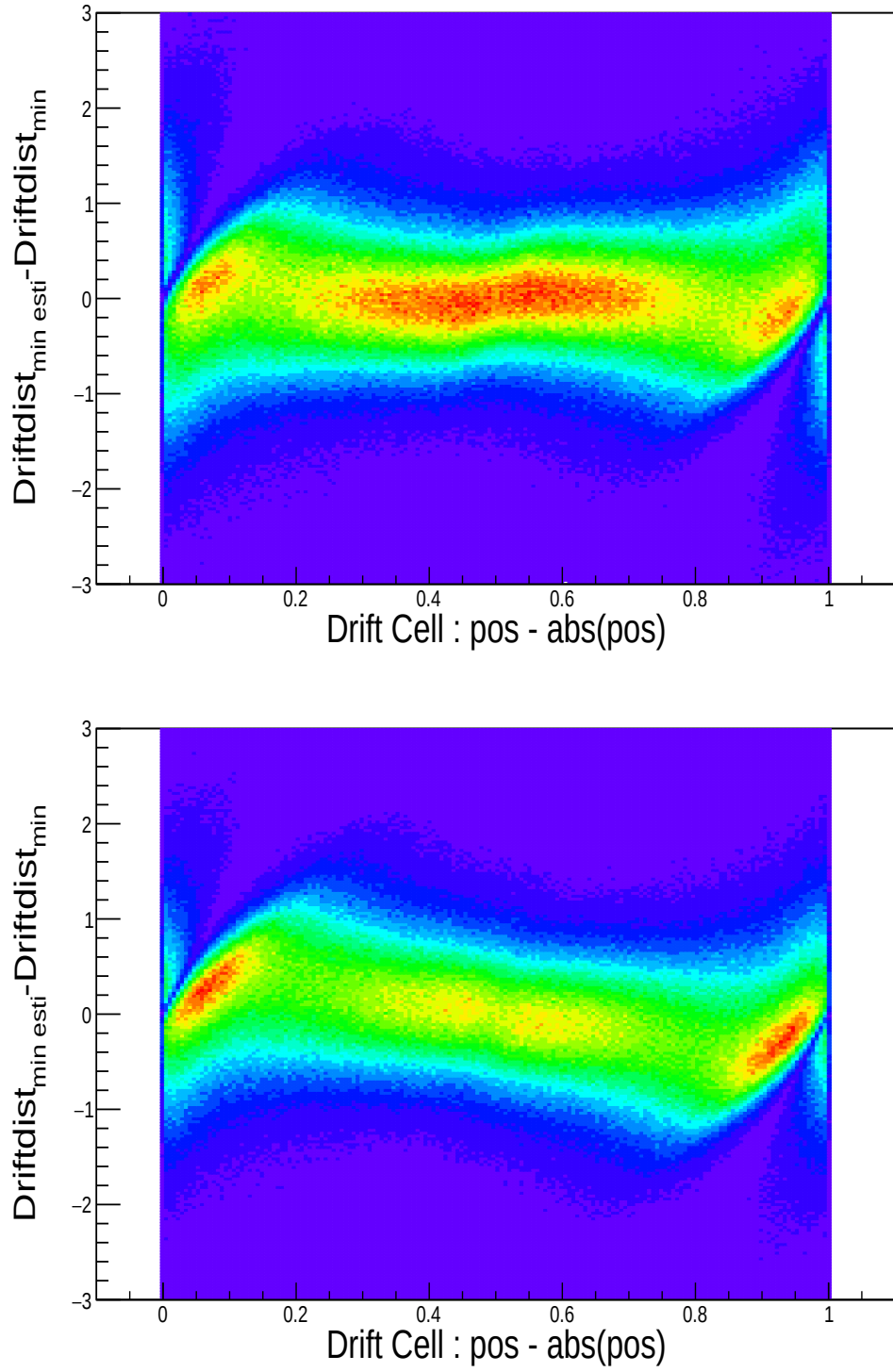


Figure 3.13: Resolution plot in need of a negative LUT offset correction (top panel) and the corrected spectrum (bottom panel).

3.7.2.2 Position resolution

The track angle on both sides of the wireplane should be more or less constant for a given n -wire VDC event, thus making the difference in slopes given in Eq. (3.5) equal to zero,

$$D = \frac{d_{t(1)} - d_{t(2)}}{\text{wire separation}} - \frac{d_{t(n-1)} - d_{t(n)}}{\text{wire separation}} , \quad (3.5)$$

where $d_{t(i)}$ is the drift distance as calculated from the drift time t of wire i . However, due to statistical fluctuations, a distribution of values around zero is obtained for the slopes from several events. The standard deviation σ_D of the difference in slopes is used in the calculation of the accuracy in the position determination σ_x of the drift chamber. The formula for such a calculation is derived in [Ber77] as

$$\sigma_x = \frac{\sigma_c}{\sqrt{n}} = \frac{\sigma_D}{2\sqrt{n}} , \quad (3.6)$$

where σ_c is the intrinsic cell accuracy and n is the number of wires used to determine the position. Knowing the accuracy in the position, one can calculate the intrinsic Full Width at Half Maximum (FWHM) position resolution of the drift chamber as follows

$$\text{FWHM}_{\text{pos}} = 2.35 \times \sigma_x = 2.35 \times \frac{\sigma_D}{2\sqrt{n}} . \quad (3.7)$$

Calculation of the position resolution for each of the wireplanes for a magnesium data file was done and the results are shown in Table 3.4.

Table 3.4: Position resolution of the four wireplanes for a given magnesium data file.

Wireplane	σ_D (mm)	FWHM _{pos}
X1	0.587	0.345
U1	0.459	0.269
X2	0.780	0.458
U2	0.607	0.357

3.7.2.3 Efficiency of the VDCs

The VDC efficiency per wireplane is given by the product

$$\varepsilon = \varepsilon_i \cdot \varepsilon_g , \quad (3.8)$$

where ε_i is the intrinsic efficiency and ε_g is the geometric efficiency. This number indicates the ability of the VDC to properly detect charged particles in the focal plane. Since the particles of a chosen rigidity were measured to be well-focussed in the vertical direction in the focal plane position, the geometric efficiency is assumed to be 100%. Thus one considers only the intrinsic efficiency which is given by

$$\varepsilon_i = \frac{\text{accepted No. of events (PID selected)}}{\text{total No. of events (PID selected)}} . \quad (3.9)$$

The following criteria are used to consider an event an accepted wireplane event

- the number of hit wires per wireplane should be between 3 and 6 and
- the drift times for each wire should fall within gated regions.

In order to calculate the total VDC efficiency ε_{tot} , one needs to take into account which wireplanes were used for the angle determination, described in

Sect. 3.7.2.4. In this study, X1, U1 and U2 wireplanes were used during the offline data analysis. For each measurement, PR236A and PR236B, the overall tracking efficiency was calculated by taking a product of the detection efficiencies of the three wireplanes,

$$\varepsilon_{\text{tot}} = \varepsilon_{\text{X1}} \times \varepsilon_{\text{U1}} \times \varepsilon_{\text{U2}} . \quad (3.10)$$

Here, ε_{X1} , ε_{U1} , and ε_{U2} are the efficiencies for the X1, U1, and U2 wireplanes, respectively. Each efficiency is calculated using Eq. (3.9). The overall efficiency ranges from 0.65 to 0.8 for this set of experiments.

3.7.2.4 Angle calibration

The measured angles of particle tracks in the focal plane can be translated to the angle at which a particle enters the spectrometer. This is a very important process as it is instrumental in determining angular distributions within the angular acceptance of the K600. Ideally, one requires a calibration of both the horizontal (θ_{scat}) and vertical (ϕ_{scat}) components of the scattering angle. Furthermore the horizontal component of the scattering angle plays an important role in the lineshape correction procedure (see Sect. 3.7.4). For all zero-degrees experiments the K600 was operated in vertical focussed mode to aid in the background subtraction routine. The full acceptance was, therefore, utilised to represent the cross-sections between 0° and 2° . The focussed mode was also employed for the 4 degrees measurements of the initial experiment (PR236A). Scattering angle selection was, however, made possible by the fact that influence of the vertical component of the scattering angle on the effective scattering angle is decreasing as the horizontal component of the scattering angle increases away from the zero-degrees. When it was realised that the background at 4° is much lower than at 0° , a “mild under-focus ion optical mode” was applied in the second part of the experiment PR236B, by decreasing the magnetic field of the quadrupole magnet Q by 5% relative to that used in PR236A. This was done to investigate if both the vertical and horizontal scattering angles can be accurately determined.

Angle calibration data were acquired at a finite angle of 4° using a multi-hole

(also called pepperpot) collimator as depicted in Fig. 3.14. A ^{197}Au target was used and only the elastic scattering peak was considered. By changing the magnetic field of the dipole magnet D1, the peak was moved across the focal plane, providing many calibration points.

Figure 3.15 shows plots of the horizontal focal plane angle (θ_{fp}) versus horizontal focal plane position (x_{fp}) for four different spectrometer fieldsets, obtained through the use of the pepperpot collimator. It can clearly be seen that the ground state shifts from high to low x_{fp} values as the magnetic field increases, i.e, from about 661.4 mm to 548.2 mm, to 386.4 mm and then to 184.2 mm as the field setting changes from 376.25 A to 381.25 A, to 388.75 A and then to 398.75 A, respectively. The current values indicated here represent the current in dipole D1 of the K600, relative to which the other K600 magnets are scaled.

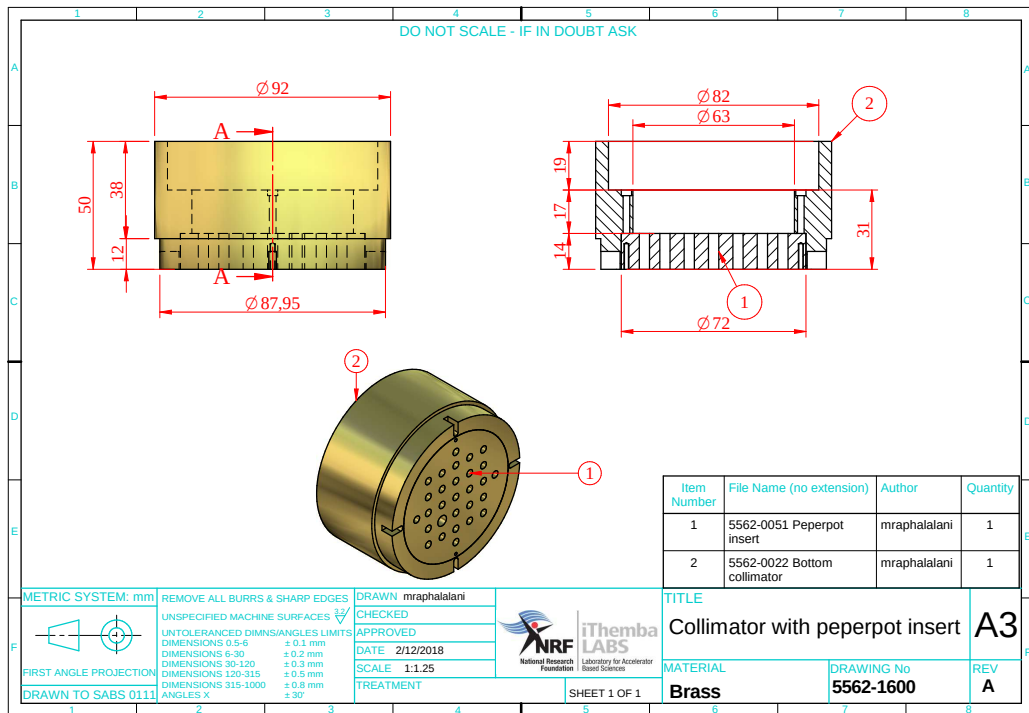


Figure 3.14: The pepperpot collimator used during experiments at iThemba LABS. The collimator is 15 mm thick in the central region with a 63 mm diameter. The first hole relative to the middle of the collimator on the x -axis/ y -axis is centred at 0.65° , the second at 1.3° and the third at 1.84° . These holes each span 4.77 mrad.

The horizontal component of the scattering angle of a scattered particle at

the target θ_{scat} was obtained by mapping the horizontal angle (θ_{fp}) of the particle at the detector to θ_{scat} using pepperpot collimator data. A series of peaks are observed in the (θ_{fp}) spectrum which represents the openings in the pepperpot collimator. The θ_{fp} to θ_{scat} conversion is obtained using the parameters from Gaussian fits to these peaks in conjunction with their associated focal plane positions, so that

$$\theta_{\text{scat}} = (a_1 x_{\text{fp}} + a_0) \theta_{\text{fp}} + (b_1 x_{\text{fp}} + b_0) , \quad (3.11)$$

where a_1, a_0, b_1 and b_0 are the fitting parameters to be determined. The observed peaks in θ_{fp} together with the associated Gaussian fits for one of the pepperpot measurements are shown in Fig. 3.16 (left). Further, the series of θ_{fp} to θ_{scat} conversions for the four different focal plane positions are displayed in Fig. 3.17. As shown in Fig. 3.18 (a), the x_{fp} dependence of the slope and offset of Eq. (3.11) can thus be determined.

During the offline analysis of the focus-mode data at 4° , an angle calibration based on only θ_{scat} information was done. This is mainly due to the fact that almost all vertical scattering information is lost in that vertical focussing mode. However, an angle calibration based on the combination of both the horizontal and vertical components of the scattering angle is possible when using the off-focus mode of the ion-optics. The vertical component of the scattering angle at the target (ϕ_{scat}) was determined mainly from the vertical position y_{fp} at the focal plane detector. The y_{fp} to ϕ_{scat} conversion is obtained using the parameters from Gaussian fits to the observed peaks in the (y_{fp}) spectrum Fig. 3.16 (right),

$$\phi_{\text{scat}} = a y_{\text{fp}} + b , \quad (3.12)$$

where a, b are the fitting parameters. However, the coefficients a and b are dependent on x_{fp} and θ_{scat} . This leads to the final calibration equation given by

$$\begin{aligned} \phi_{\text{scat}} = & \left[\left(\sum_{i=0}^3 a_i \theta_{\text{scat}}^i \right) + \left(\sum_{i=0}^3 b_i \theta_{\text{scat}}^i \right) x_{\text{fp}} \right] y_{\text{fp}} \\ & + \left[\left(\sum_{i=0}^3 c_i \theta_{\text{scat}}^i \right) + \left(\sum_{i=0}^3 d_i \theta_{\text{scat}}^i \right) x_{\text{fp}} \right] . \end{aligned} \quad (3.13)$$

The aim of the calibration procedure, therefore, was to determine the fitting parameters a_i, b_i, c_i and d_i .

It is important to note that the second and the fourth vertical rows of the pepperpot collimator were blanked out. This was done due to the weak off focus mode that was employed, which limits the vertical size of events in the focal plane. Under such conditions the focal plane detector resolving power only allows for three peaks to be discerned, as seen in Fig. 3.16.

For illustrative purposes, a visual representation of the differences between a proper θ_{scat} and ϕ_{scat} selection and selecting scattering angle only based on θ_{scat} is shown in Figs. 3.19 and 3.20. Figure 3.21 shows two-dimensional images of the pepperpot collimator data before (upper panel) and after (bottom panel) the reconstruction. The data are shown for one horizontal position at the detector, $x_{\text{fp}} = 661.4$ mm which corresponded to the magnetic field of the dipole magnet $D1 = 376.25$ A. Furthermore, a comparison of the angular distributions for the 9.30539 MeV 0^+ and 11.8649 MeV 1^- states in ^{24}Mg are shown in Figs. 3.22 and 3.23, respectively for angle cuts based on a combined θ_{scat} and ϕ_{scat} selection and selecting scattering angle only based on θ_{scat} . It can be seen that there is reasonable agreement between the results. Similarly, two of the target nuclei data after the reconstruction are displayed in the two-dimensional images, Fig. 3.24 in order to show the success of the calibration procedure. The red circle depicted on each plot corresponds to the angular acceptance ($\sim \pm 2^\circ$) of the circular entrance collimator to the $K = 600$ magnetic spectrometer of iThemba LABS.

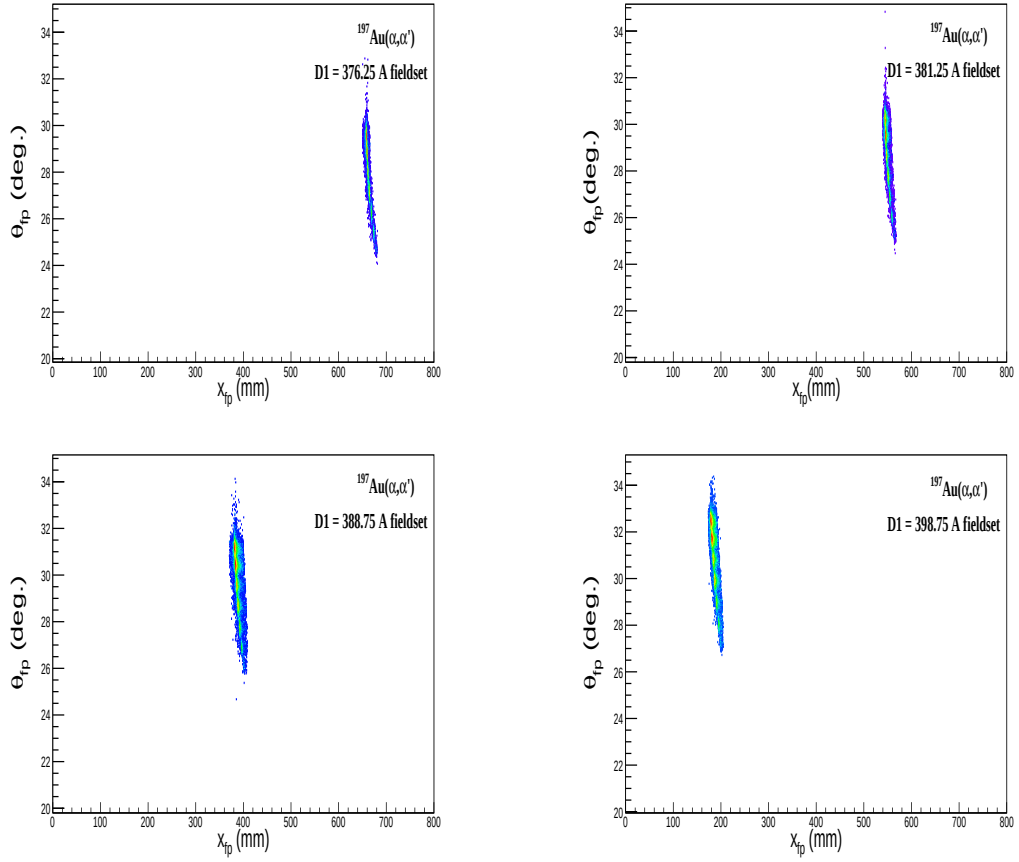


Figure 3.15: Histograms of the focal plane angle *versus* horizontal position, for four different spectrometer field settings. The prominent locus shown for each field represents the elastic peak.

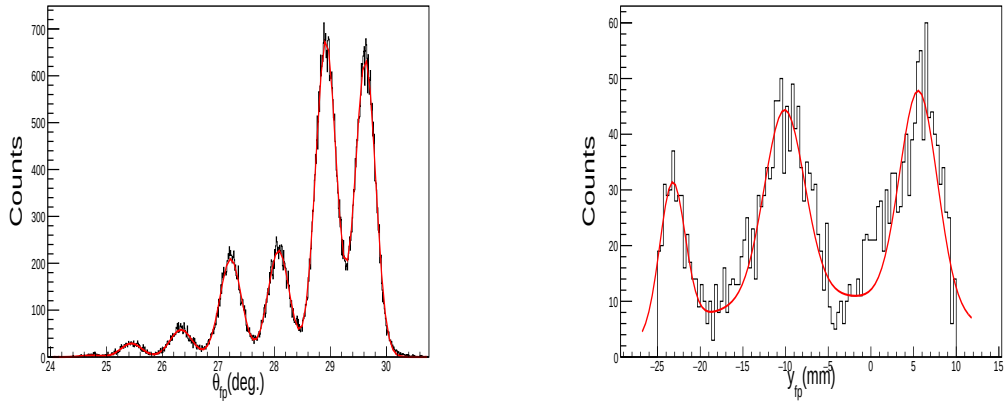


Figure 3.16: Left: a focal plane angle (θ_{fp}) pepperpot spectrum showing Gaussian fits to the observed peaks. Right: a vertical focal plane position (y_{fp}) pepperpot spectrum showing Gaussian fits to the observed peaks.

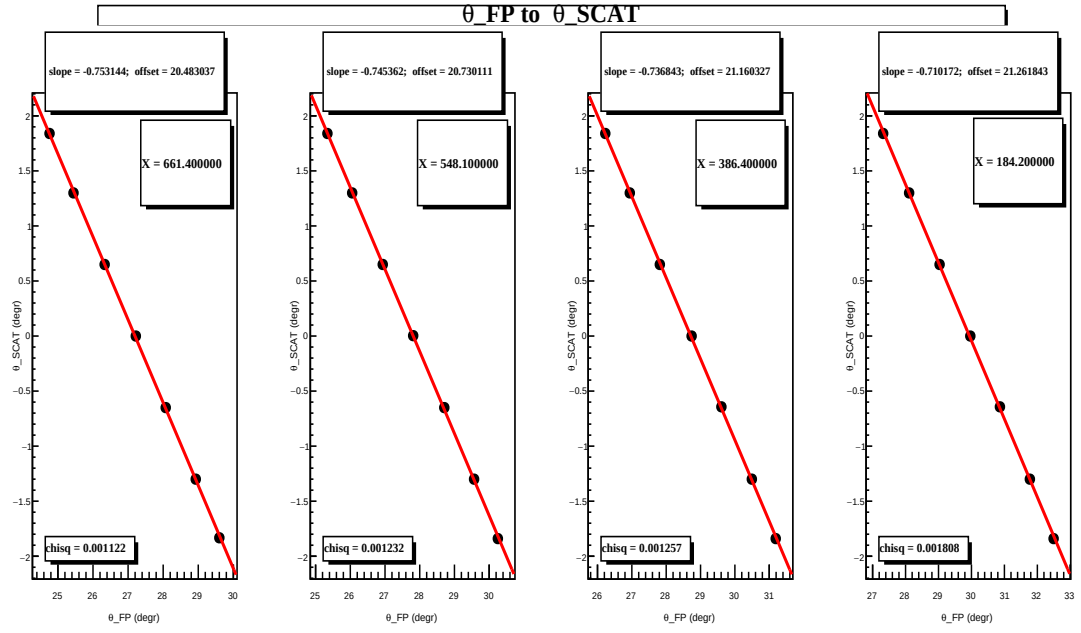


Figure 3.17: Graphs showing the conversion of focal plane angle (θ_{fp}) to scattering angle (θ_{scat}) for four pepperpot spectra with different focal plane positions.

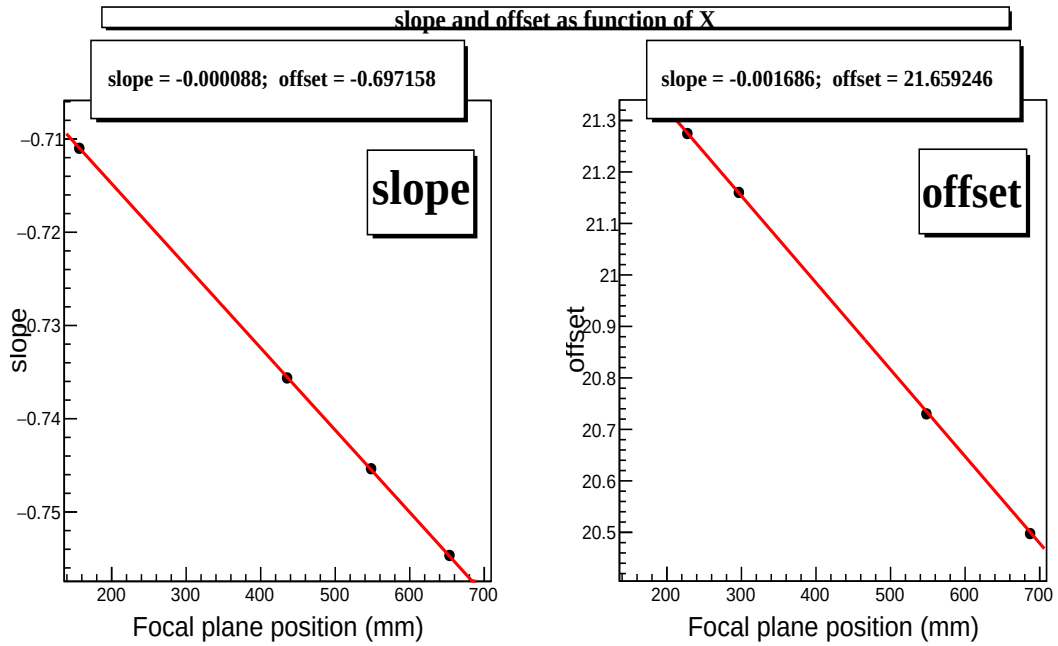


Figure 3.18: Graphs showing the determination of the x_{fp} dependence of the conversion of focal plane angle (θ_{fp}) to scattering angle (θ_{scat}), from the fits in Fig. 3.16.

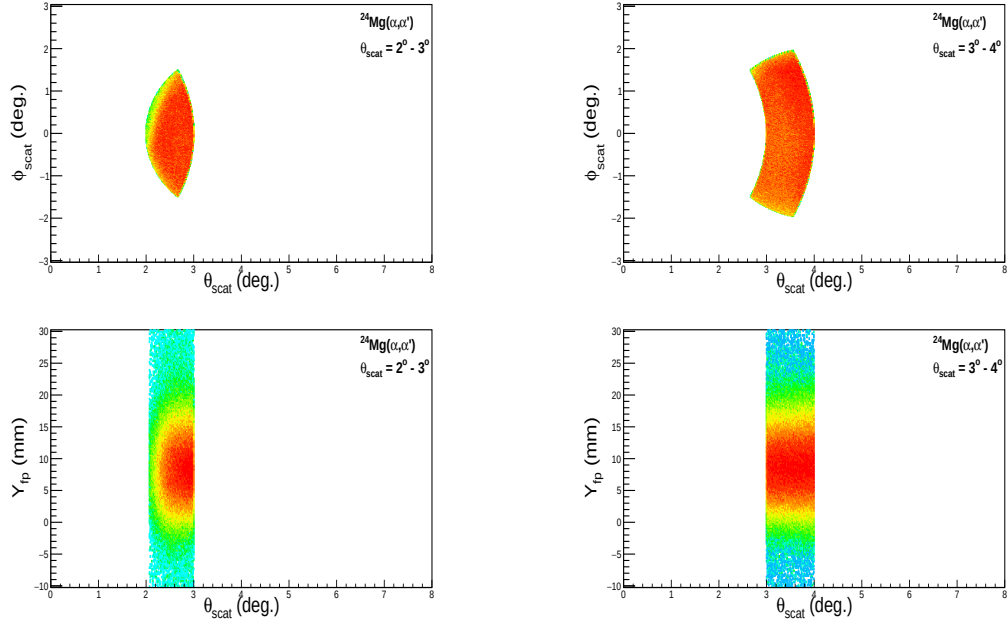


Figure 3.19: Different scattering angle selections, for data from the whole focal plane of ^{24}Mg . The top row represents scattering angle selection based on a combined θ_{scat} and ϕ_{scat} cut, for the ranges (from left to right) of $2^\circ - 3^\circ$ and $3^\circ - 4^\circ$. The bottom row represents scattering angle selection based only on θ_{scat} information, for the same ranges.

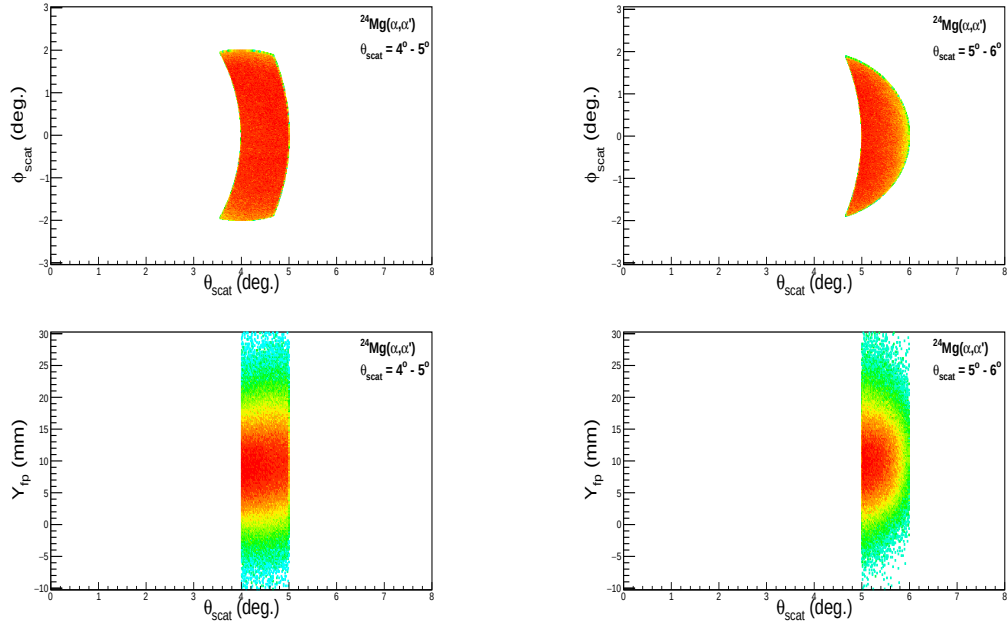


Figure 3.20: Same as in Fig. 3.19 but for the angle ranges $4^\circ - 5^\circ$ and $5^\circ - 6^\circ$.

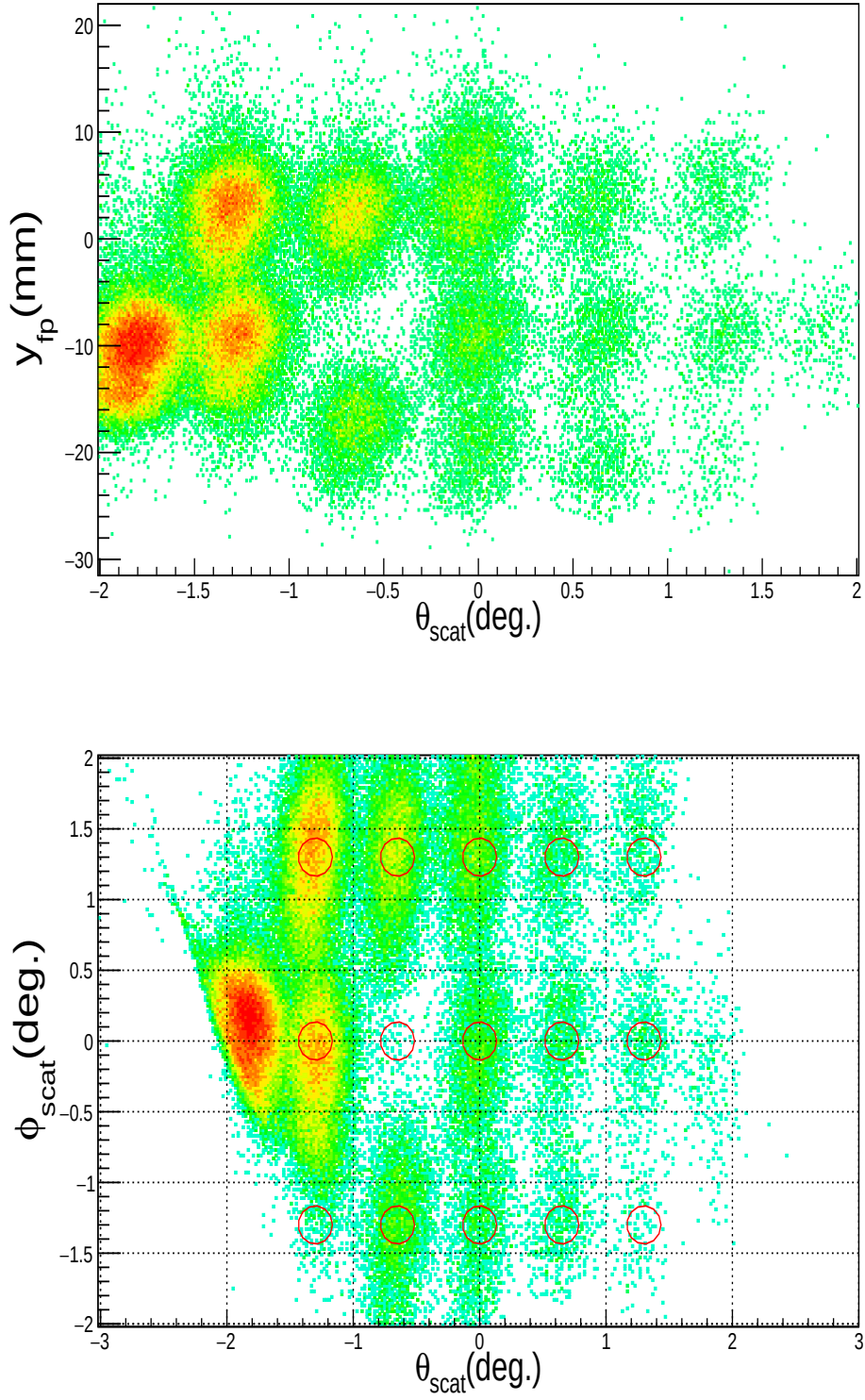


Figure 3.21: Reconstruction of the vertical (ϕ_{scat}) component of the scattering angle at the entrance of the spectrometer (bottom panel) from the information at the detector position (upper panel). Data are shown for one dipole magnet D1 settings (upper and lower panels). The red circles in the lower panel correspond to the hole positions of the pepperpot collimator.

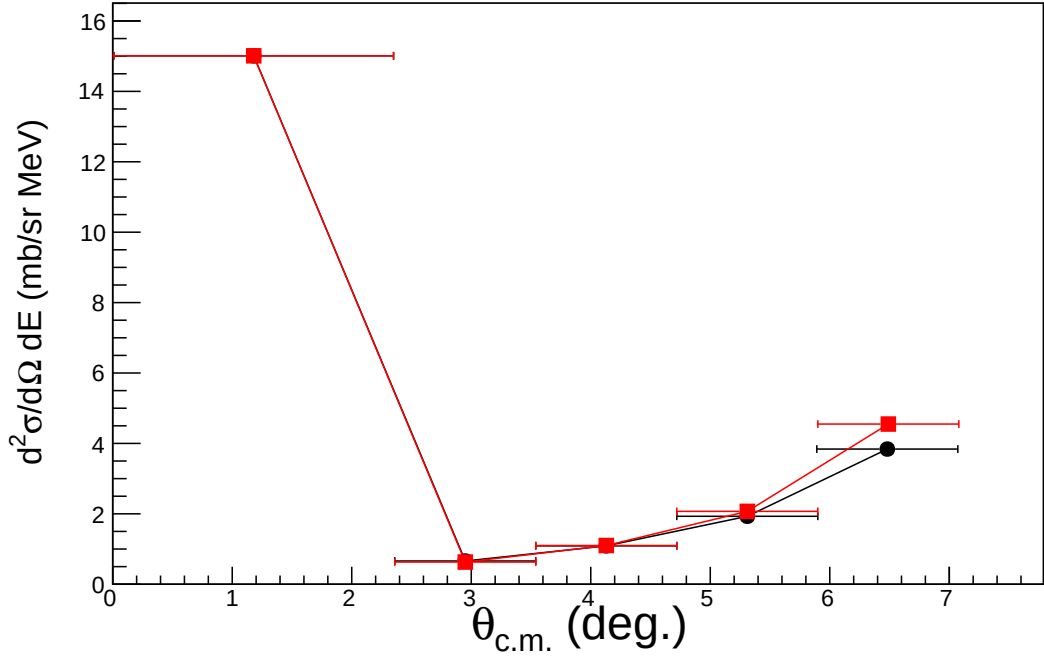


Figure 3.22: Differential cross-sections for the $J^\pi = 0^+$ state at 9.30539 MeV in ^{24}Mg . The data are represented by points with the horizontal error bar delineating the angular range covered. Black circles represent a scattering angle cut based on both θ_{scat} and ϕ_{scat} information. Red squares represent a scattering angle cut based only on θ_{scat} information. The vertical error bars represent statistical errors only (smaller than the symbol).

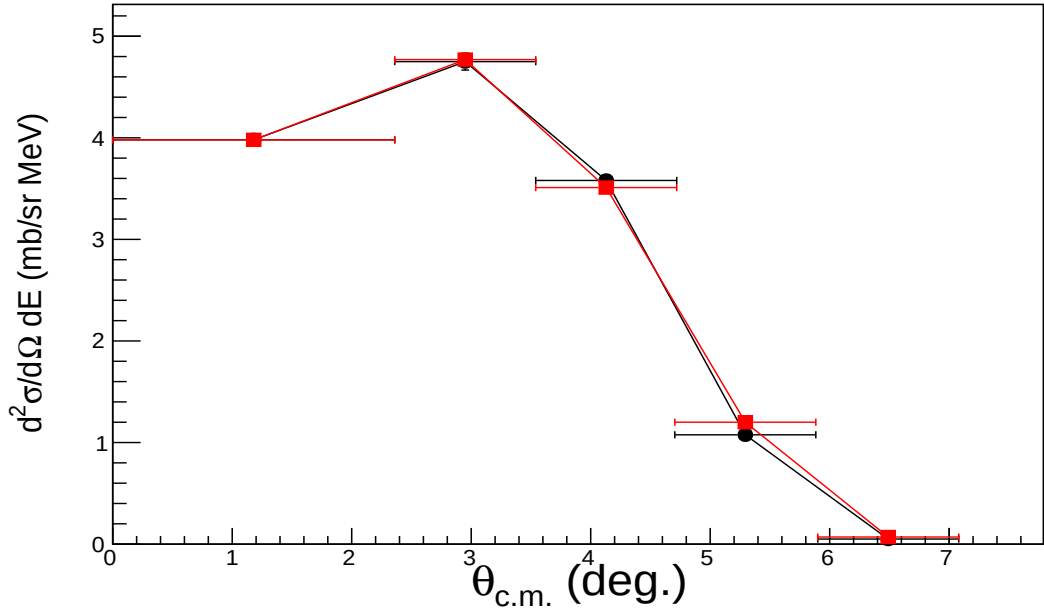


Figure 3.23: The same as in Fig. 3.22 but for the $J^\pi = 1^-$ state at 11.8649 MeV in ^{24}Mg .

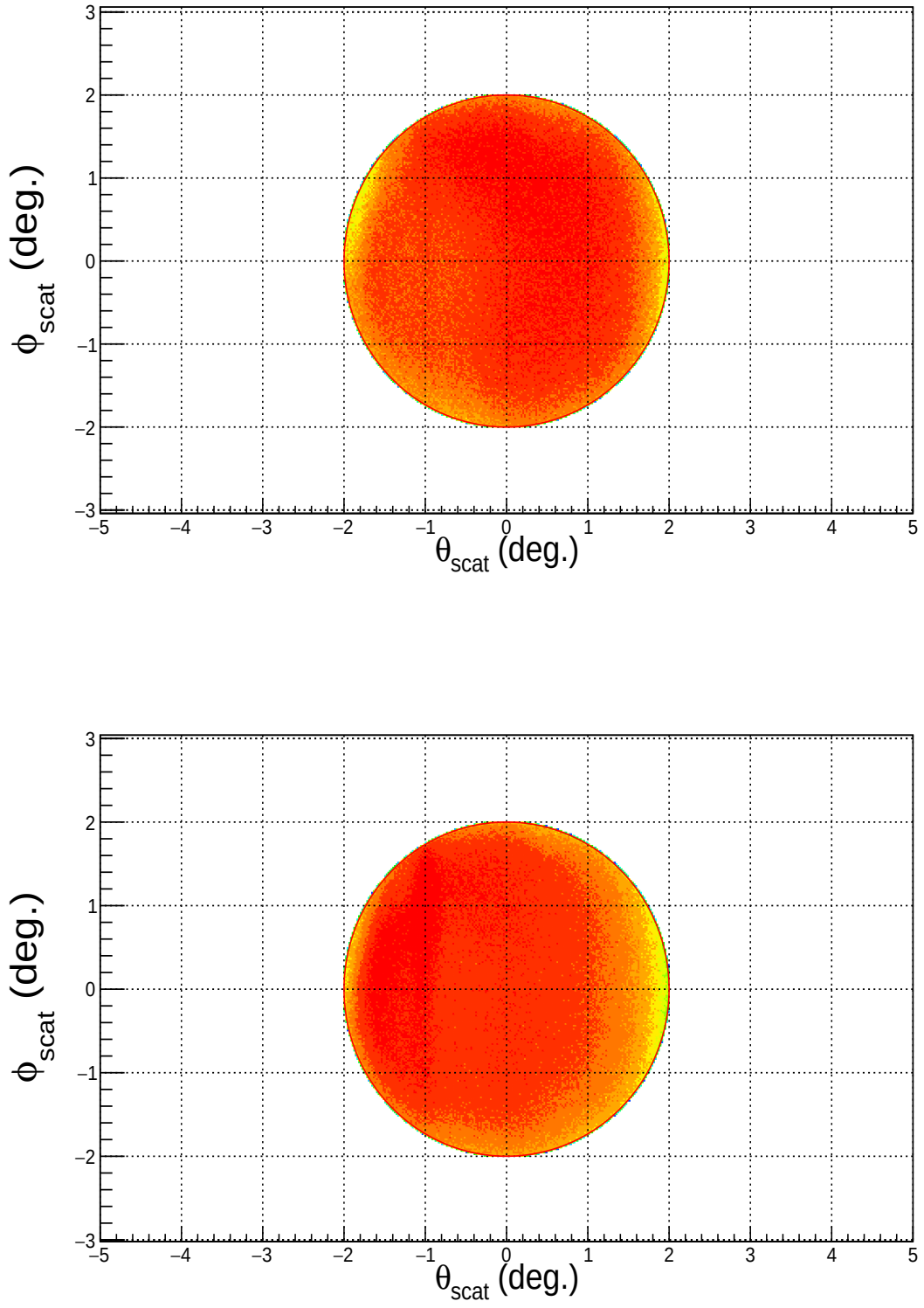


Figure 3.24: Reconstruction of the horizontal (θ_{scat}) and vertical (ϕ_{scat}) incidence angles at the entrance of the spectrometer. Data are shown for two different target nuclei, namely ^{28}Si (upper panel) and ^{120}Sn (lower panel).

3.7.3 Background subtraction

As mentioned in Sect. 3.6.1, instrumental background contributions are enhanced for 0° measurements even under the best experimental conditions. The distribution of the background events could be experimentally determined and be removed to yield a background-free spectrum. The method of background subtraction is described in [Jin14, Nev11a]. The spectrometer was operated in vertical focus mode for 0° experiments in order to enable effective background subtraction. As such, the events of interest are focused around $y_{\text{fp}} = 0$ while both background contributions (beam-related and target-related background) are expected to be equally distributed in the vertical direction. However, in the experiments reported on in this thesis, the optimum beam halo conditions were obtained with the central peak of the y_{fp} spectrum shifted slightly to a higher position. This implies that the vertical regions of background events above and below the area of interest are somewhat unequally sized. However the method described in [Jin14] is still valid for this study.

The distribution of the events in the y_{fp} plane is shown in Fig. 3.25. True events are concentrated and centred around $y_{\text{fp}} = 5$ mm while background (b.g.) events had approximately flat distributions. The bottom panel in Fig. 3.25 illustrates the focal-plane position spectrum for all selected events in the central region as well as the background that yields from the sum of the outer δy sections shown in green on the top panel of Fig. 3.25.

As stated earlier in Sect. 3.3.2.2, owing to the operating mode, the background contribution at finite angle measurements is less. However, careful analysis is still essential to properly handle background contributions for subtraction. In the four-degrees measurements offline analysis, a more sophisticated background subtraction method was applied. This was necessary since the background regions are not as big and rectangular as in the case of the zero-degrees measurements. The useful background regions have a trapezoid shape (Fig. 3.26). One can, therefore, not simply choose a rectangular region of background events and scale it to the size of the central region since that does not take the shape of the background region properly into account. In order to subtract the background regions (Y_{back}), each of them needs to be scaled up to the central region (Y_{central}) bin per bin taking

into account the changing size of the background region and the central region (Fig. 3.27 (a)). The results of the procedure are shown in Fig. 3.27 (b). The shape of the background represents the density of the background events per bin as well as the shape of the central region.

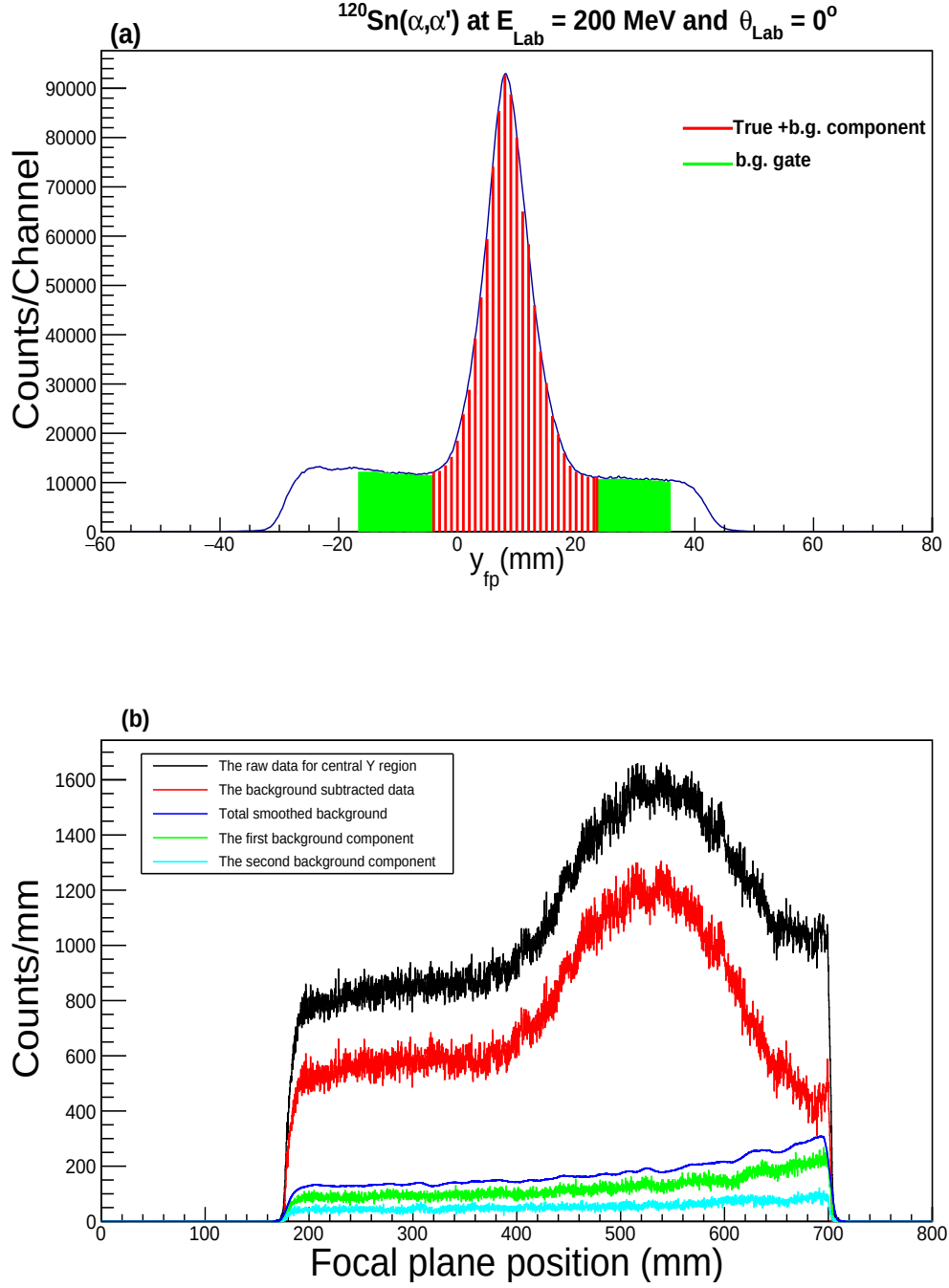


Figure 3.25: Histograms of the vertical and horizontal positions of scattered alpha particles in the focal plane for the $^{120}\text{Sn}(\alpha, \alpha')$ reaction at 0° are shown in panel (a) and (b), respectively. The events represented in the central red region in panel (a) were plotted by the thick black line in panel (b). The background components represented by the side green regions in panel (a) are shown in panel (b) as the thick cyan and green lines. These were averaged, smoothed and are plotted by the thin blue line in panel (b) after adding them together. The thick red line in panel (b) represents the background subtracted position spectrum.

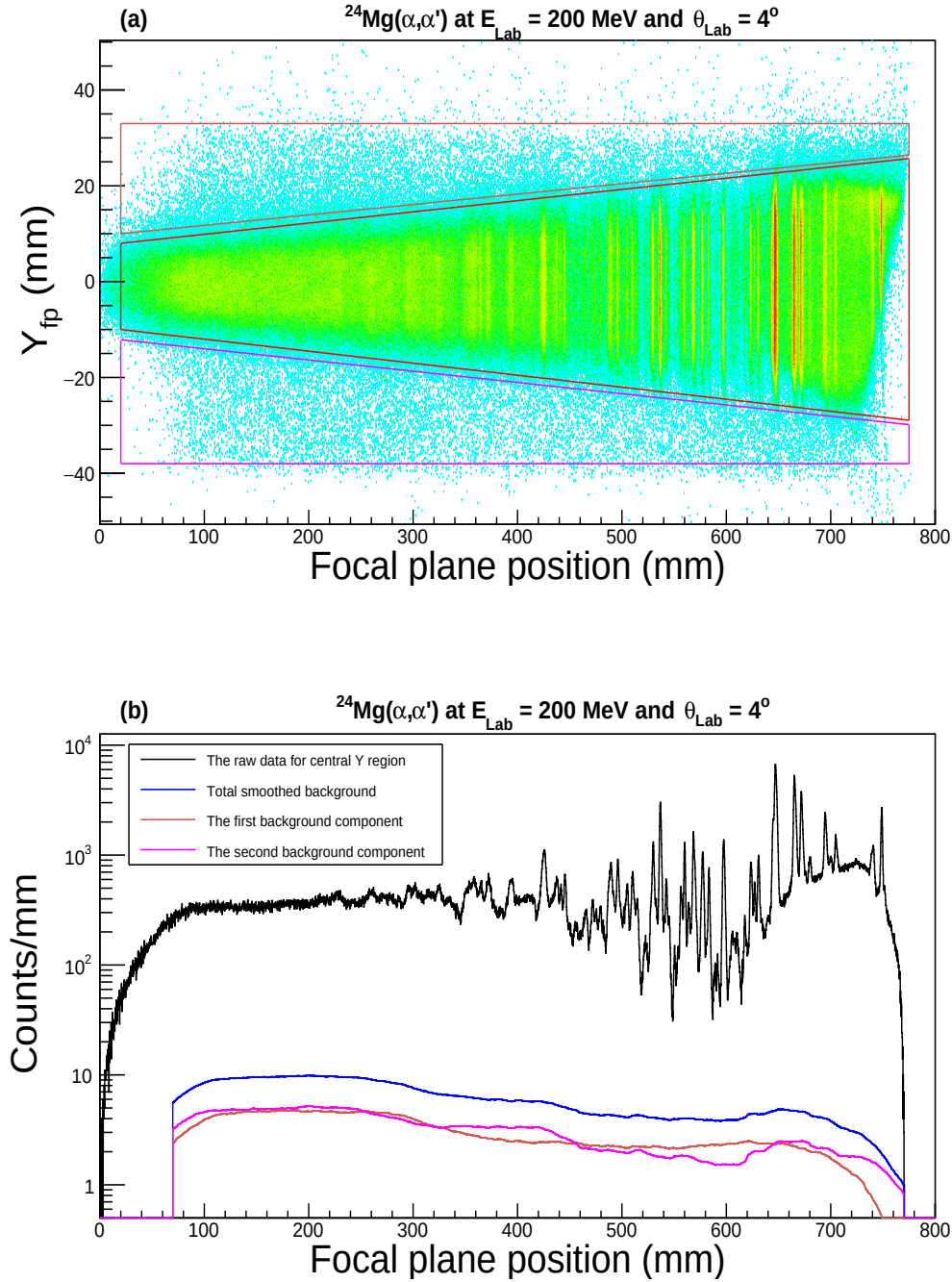


Figure 3.26: Histograms of the vertical versus the horizontal positions of scattered alpha particles in the focal plane for the $^{24}\text{Mg}(\alpha, \alpha')$ reaction at four degrees are shown in panel (a), while panel (b) represents only the horizontal positions. The events in the background regions selected in panel (a) (magenta and brown trapezoid contours) were smoothed and are plotted by the thin magenta and brown lines in panel (b). Also shown are the data representing the central data region (thick black line) and the averaged background component (dark blue).

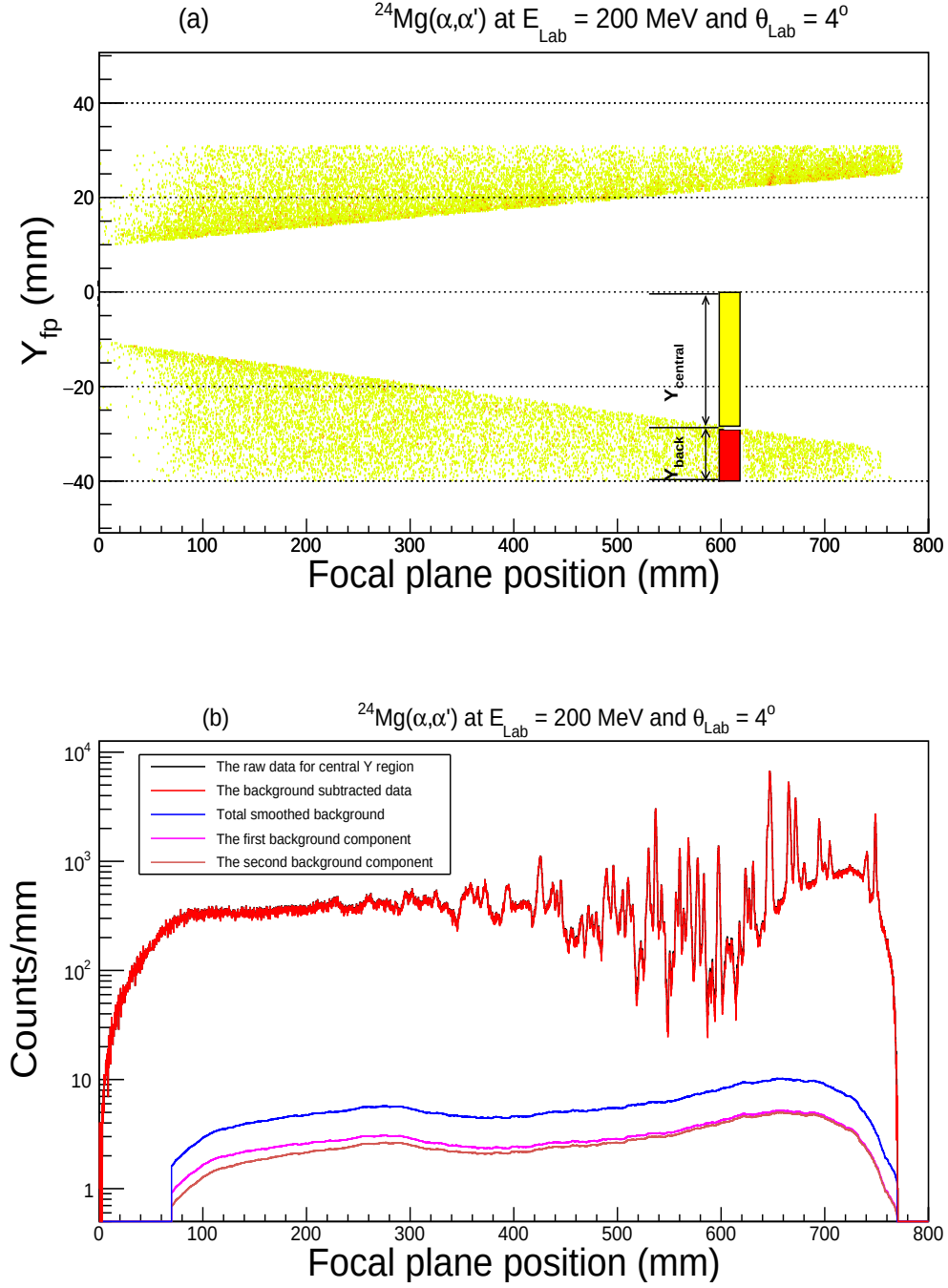


Figure 3.27: A histogram of the vertical versus the horizontal positions of background events is shown in panel (a). To represent the background in the central region the number of events in the background region must be scaled according to the ratio $Y_{\text{central}}/Y_{\text{back}}$ as shown in panel (a). The horizontal position spectra in panel (b) shows the background data shown in Fig. 3.26 that were corrected for the shape of the background regions. The final smoothed background that is to be subtracted from the data represented in the central cut is shown as the blue line.

3.7.4 Lineshape correction

The optical aberrations of the K600 together with kinematic broadening effects can considerably affect the energy resolution if suitable corrections are not made. These effects can be seen in the angular dependence of the focal plane position for a given nuclear state. It is worth noting that the K- and H-coils are used to optimise energy resolution during the experiment. However, these correction coils cannot fully correct the observed structures, and lineshape correction is a necessary step to be taken in order to obtain the best possible energy resolution. In order to have an idea of how much the lineshape of a given state plotted with the coordinates of scattering angle θ_{scat} and focal plane position x_0 need to be corrected, one extracts (x_0) coordinates at regular intervals along that state to attempt reproducing its overall shape. A polynomial of fifth order given by Eq. (3.14) is used to fit these coordinates plotted so that

$$x_{\text{state}} = f_0 + \sum_{i=1}^5 f_i \theta_{\text{scat}}^i . \quad (3.14)$$

Therefore, the lineshape of the state in the focal plane position spectrum is corrected as follows:

$$X_C = x_{\text{fp}} - \left(f_0 + \sum_{i=1}^5 f_i \theta_{\text{scat}}^i \right) , \quad (3.15)$$

where X_C and x_{fp} represent the corrected focal plane position and the non-corrected one, respectively.

Additionally, it is observed that the focal plane position is weakly dependent on the vertical focal plane position y_{fp} . As stated in Sect. 3.7.3, the data were not centred around the vertical zero and as such a dependence on y_{fp} is modified to include these offsets. Further to this, a dependence of the θ_{scat} correction factors on x_{fp} should be implemented. This is due to a small change in the ion optical character across the focal plane, and is necessary to ensure that states on either end of the focal plane appears vertically straight. The lineshape correction equation then becomes

$$X_C = x_{\text{fp}} - \left[f_0 + \sum_{i=1}^5 f_i \theta_{\text{scat}}^i + g(y_{\text{fp}} + y_{\text{offset}}) + h(x_{\text{fp}} + x_0) \theta_{\text{scat}}^2 \right] . \quad (3.16)$$

The shape of the line of a prominent state together with some surrounding states before the correction and after the correction is displayed in Fig. 3.28 where the upper panel displays θ_{scat} versus x_{fp} for the case where no lineshape correction has been applied and the lower panel displays the resultant spectrum following a full lineshape correction of the form given in Eq. (3.16). Figure 3.29 shows an example of the overall effect of the lineshape correction on the position spectrum.

It is worth noting that the kinematics differences (affecting kinematic broadening of states) between the $\text{Mg}(\alpha, \alpha')$ and $\text{Si, Ni, Zr, Sn, Pb}(\alpha, \alpha')$ reactions are very small at and close to 0° . As an example for confirmation, relativistic kinematic calculations were performed to establish the energy of the detected particle in the focal plane for the $^{24}\text{Mg}(\alpha, \alpha')$ and $^{208}\text{Pb}(\alpha, \alpha')$ reactions at 0° and 1.91° with $E_\alpha = 200$ MeV. At an angle of 0° , the values obtained for an excitation energy of 10 MeV were 189.967 MeV and 189.987 MeV for the $^{24}\text{Mg}(\alpha, \alpha')$ and $^{208}\text{Pb}(\alpha, \alpha')$ reactions, respectively. At an angle of 1.91° , the values obtained for an excitation energy of 10 MeV were 189.937 MeV and 189.983 MeV for ^{24}Mg and ^{208}Pb , respectively. The kinetic energy difference between 0° and 1.91° were thus calculated to be 30 keV and 4 keV for the $^{24}\text{Mg}(\alpha, \alpha')$ and $^{208}\text{Pb}(\alpha, \alpha')$ reactions, respectively. This is much smaller than the energy resolution of the measurements. Small kinematic differences were not only seen at the given excitation energy used in the calculation but over the whole range of the excitation energy values. Lineshape correction parameters established through the $^{24}\text{Mg}(\alpha, \alpha')$ reaction is, therefore, applied to all targets for 0° measurements. However, the kinematics differences between the $\text{Mg}(\alpha, \alpha')$ and $\text{Si, Ni, Zr, Sn, Pb}(\alpha, \alpha')$ reactions at 4° are not negligible. As an example to confirm this, at an angle of 4° , the values of the energy of the detected particle in the focal plane, obtained for an excitation energy of 10 MeV were 189.804 MeV and 189.969 MeV for $^{24}\text{Mg}(\alpha, \alpha')$ and $^{208}\text{Pb}(\alpha, \alpha')$ reactions, respectively.

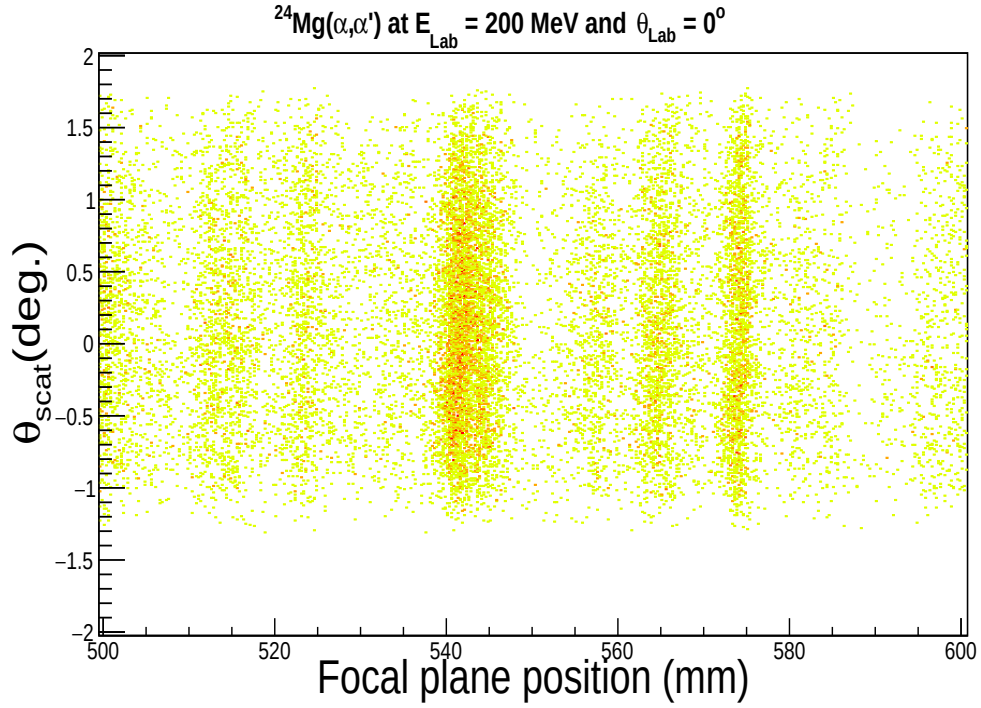
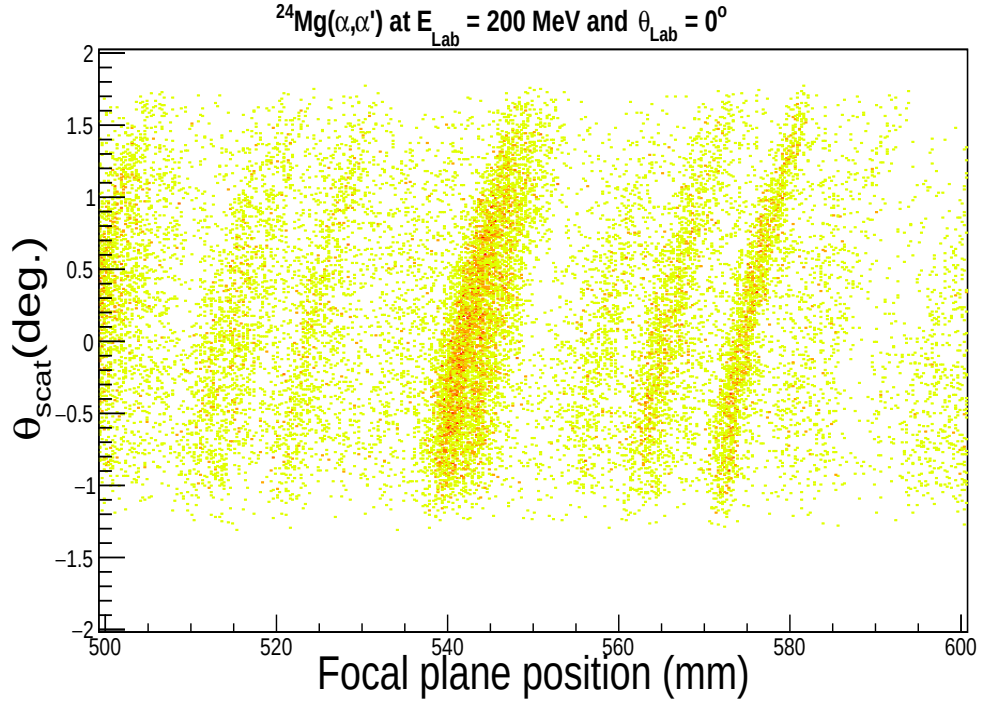


Figure 3.28: The effect of lineshape correction. Top panel: θ_{scat} versus x_{fp} with no lineshape correction applied. Bottom panel: θ_{scat} versus x_{fp} , following a full lineshape correction.

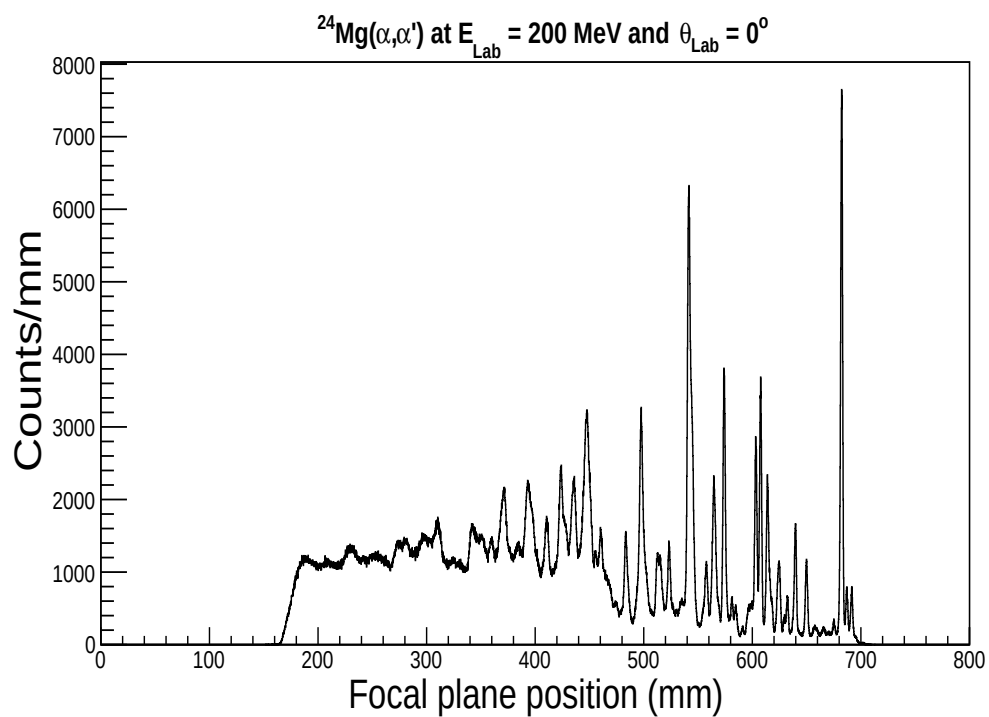
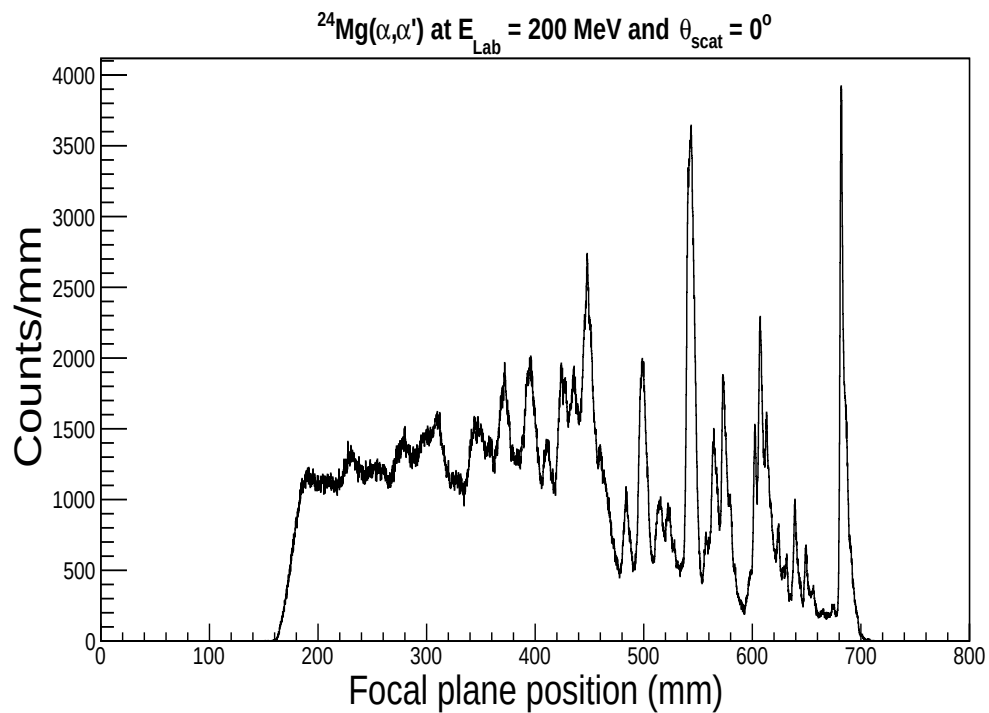


Figure 3.29: The effect of lineshape correction. Top panel: the position spectrum with no lineshape correction applied. Bottom panel: the focal plane position spectrum following a full lineshape correction.

It was, therefore, not possible to blindly apply the ^{24}Mg lineshape correction factors to all targets at 4° . Some targets had suitably strong individual states that allowed for target-specific correction factors to be established. In the case of the lead and tin targets, there are no strong states on which to perform a lineshape correction. Therefore, the correction had to be made using the information obtained from the correction on the magnesium spectrum. In the case of ^{24}Mg , the lineshape correction takes into account the kinematic broadening of the $^{24}\text{Mg}(\alpha, \alpha')$ reaction as well as the correction of aberrations. In order to correct the lineshapes of Pb and Sn, additional kinematic broadening was calculated and added to the Mg correction parameters. The theoretical calculations that lead to the linear term used to update the lineshape correction equation (3.16) is detailed in Appendix. A.

3.7.5 Data offsets and chained data

The day/night temperature changes and its subsequent effect on SSC magnetic fields together with frequent RF changes performed by beam operators during beam optimisations result in small variations in the beam energy over time. This implies small shifts in the focal plane positions of the detected particles. Before combining the usable data files, one needs to take into account these shifts in the focal plane. As a result, the focal-plane position resolution is improved considerably by including the position offsets in the chaining procedure as can be seen in Fig. 3.30. In some cases big position shifts could even result in additional false peaks in the focal plane spectrum.

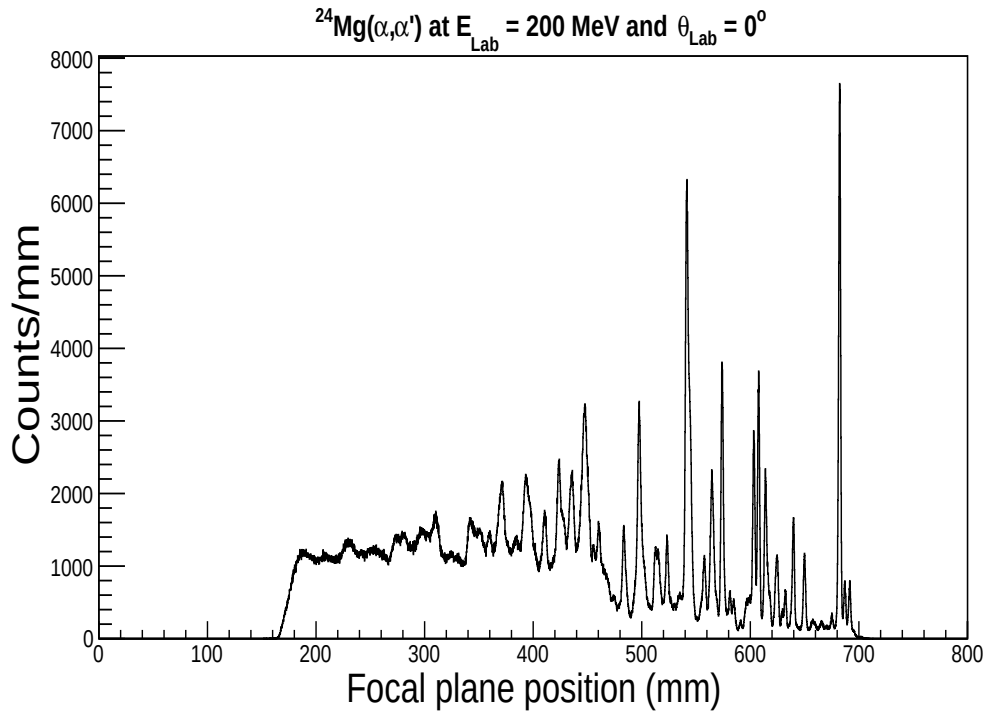
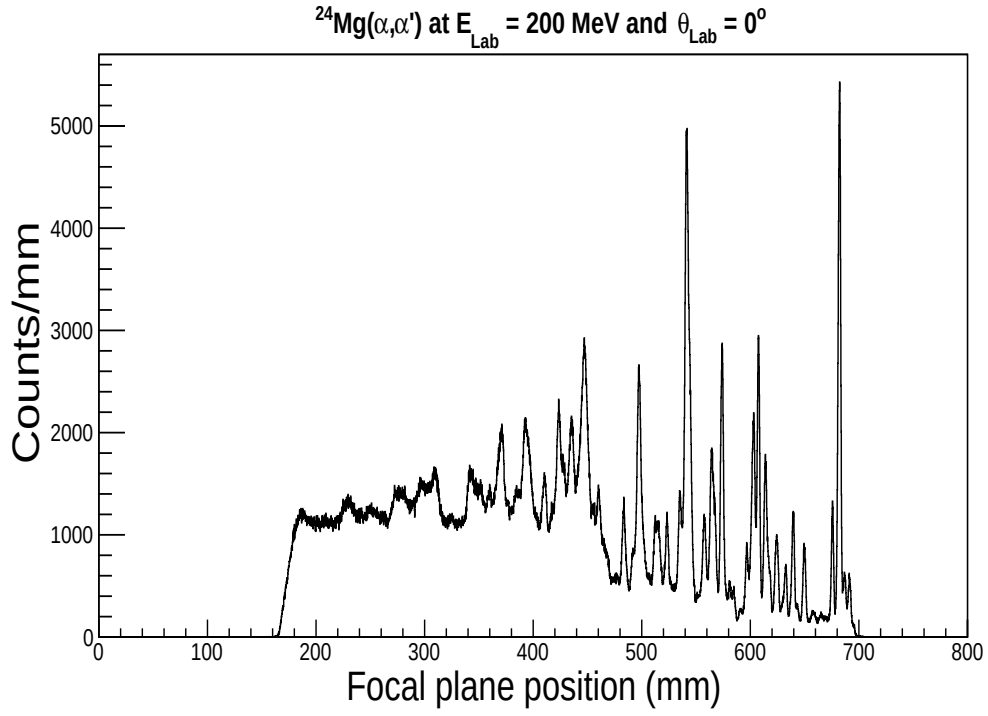


Figure 3.30: The effect of position offsets on the resolution of the chained spectrum where (top panel) illustrates the case where the offsets have been neglected and (bottom panel) illustrates the effect of offset inclusion.

3.7.6 Energy calibration

The position of particles in the focal plane of the magnetic spectrometer depends on the rigidity of the particle. Since we are only concerned with the detection of one type of particle, this is akin to momentum selection. Following a focal plane position to momentum calibration, the conversion from momentum to excitation energy is achieved using a kinematics routine implemented in the analysis code. The ^{24}Mg nucleus with its many identifiable peaks was used as calibration target for this study.

The basic geometry of nuclear reactions in the reaction plane is shown in Figure 3.31. The projectile, target, ejectile and recoil nuclei are labelled 0, 1, 2 and 3. Conservation of linear momentum along and perpendicular to the beam direction leads to

$$\begin{cases} p_0 &= p_2 \cos \theta + p_3 \cos \beta \\ 0 &= p_2 \sin \theta - p_3 \sin \beta \end{cases} \quad (3.17)$$

and conservation of the total energy gives

$$E_0 + m_1 + Q = E_2 + E_3 + E_x , \quad (3.18)$$

where $E = T + m$ and $E^2 = p^2 + m^2$. The kinetic energy is represented by T and E is the total energy (kinetic + rest-mass energy). The Q -value of the reaction is given by $Q = m_0 + m_1 - m_2 - m_3$. In the case of inelastic scattering it is equal to 0. Squaring and adding Eqs. (3.17) yields

$$p_0^2 + p_2^2 - 2p_0p_2 \cos \theta = p_3^2 . \quad (3.19)$$

The excitation energy is derived from two-body kinematics as described in [Whe14]

$$\begin{aligned}
E_x &= E_0 + m_1 - E_2 - E_3 \\
&= E_0 + m_1 - \sqrt{p_2^2 + m_2^2} - \sqrt{p_3^2 + m_3^2} \\
&= E_0 + m_1 - \sqrt{p_2^2 + m_2^2} - (p_0^2 + p_2^2 - 2p_0p_2 \cos \theta + m_3^2)^{\frac{1}{2}} . \quad (3.20)
\end{aligned}$$

Here, θ is the scattering angle of the particle in the laboratory frame. The ejectile momentum p_2 is obtained from the position-momentum calibration procedure described as follows: after background subtraction, lineshape correction and the position offsets being included, the identification of the known excited states in the position spectrum was done. A relativistic kinematics code was used to calculate the momentum (p) values of the identified states. The focal plane positions x_{fp} were plotted against the calculated momenta p as shown in Fig. 3.32. A second-order polynomial was used to fit the data and as such the x_{fp} to p calibration yields

$$p = c_1 x_{\text{fp}}^2 + c_2 x_{\text{fp}} + c_3 . \quad (3.21)$$

The resulting parameters c_1, c_2 and c_3 are provided in Fig. 3.32. With knowledge of the observed momentum p_2 and the known masses, K600 angle and beam energy, Eq. (3.20) was used to calculate E_x .

The calibrated ^{24}Mg excitation energy spectra are shown on Fig. 3.33 for both experiments (PR236A and PR236B). A comparison of the energy values for the excited states of ^{24}Mg obtained in this work and those given by the National Nuclear Data Center (NNDC) [NNDC] as well as the ones published by K. van der Borg *et al.* [Bor81] are displayed in Fig. 3.34. The states indicated are the identified states used in the calibration.

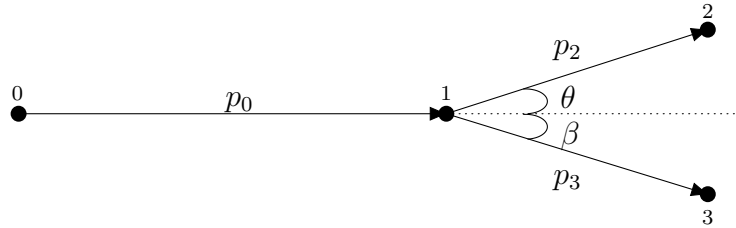


Figure 3.31: Basic geometry of nuclear reactions in the reaction plane.

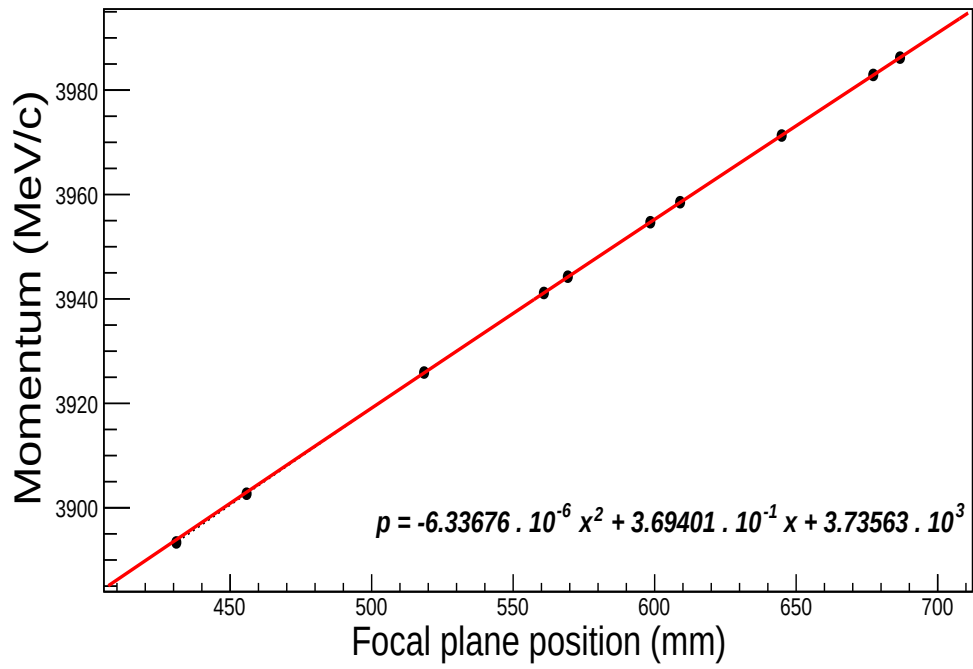
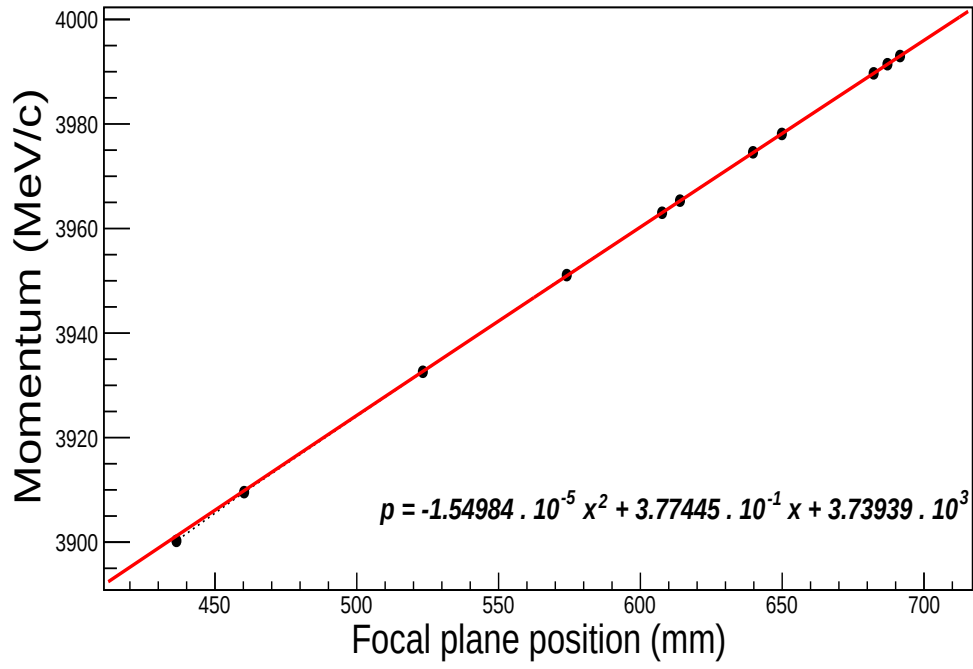


Figure 3.32: The momentum calibration curves obtained for the 0° measurements (top panel) and for the 4° measurements (bottom panel).

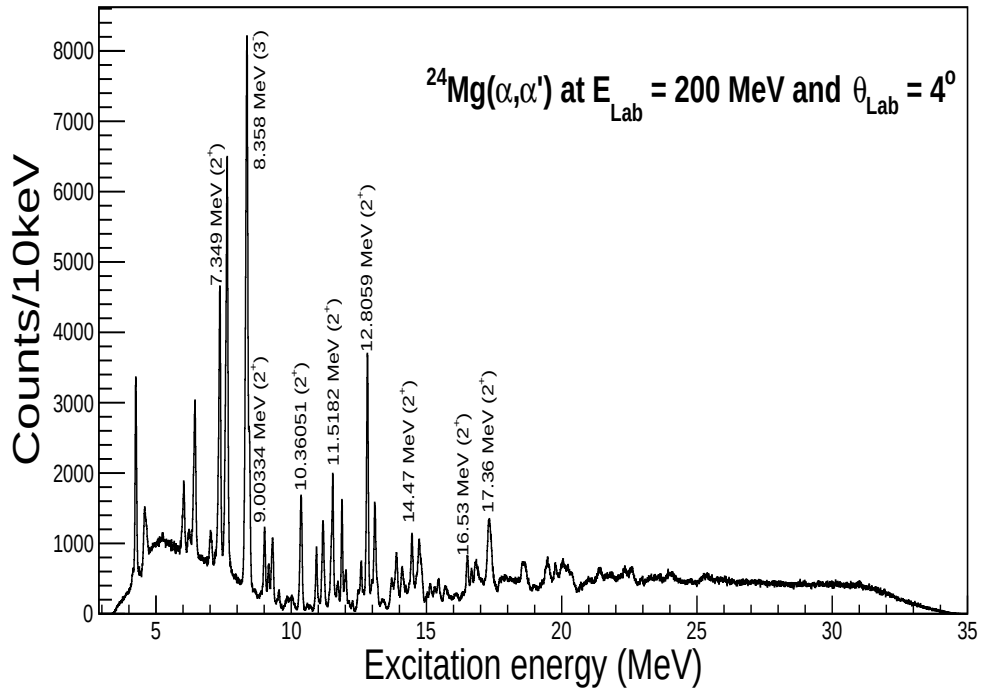
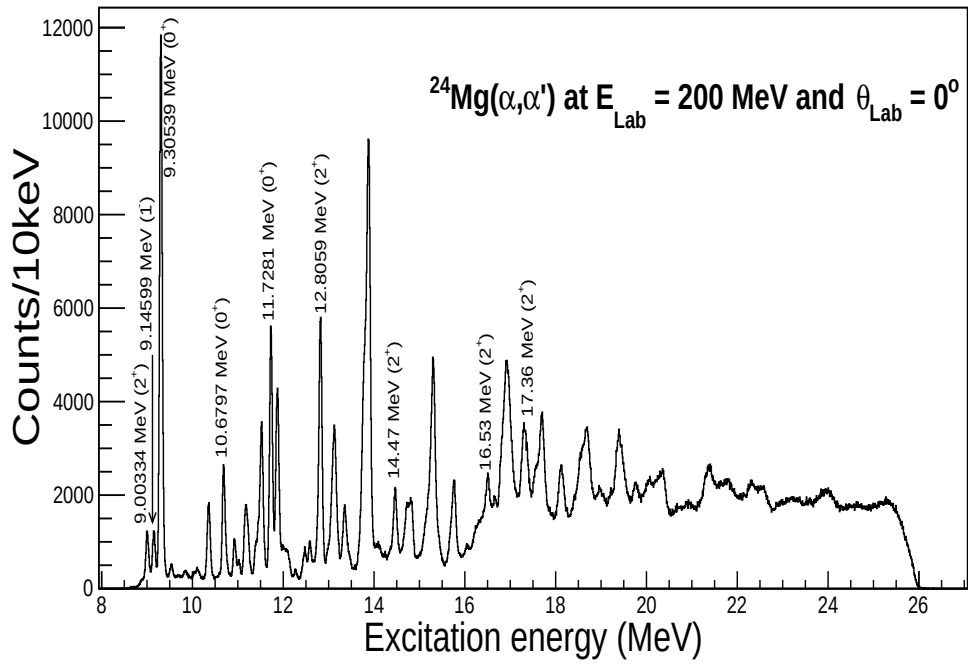


Figure 3.33: Calibrated excitation-energy spectra obtained for the $^{24}\text{Mg}(\alpha, \alpha')$ reaction at iThemba LABS for the 0° measurements (top panel) and for the 4° measurements (bottom panel). Some of the prominent states chosen for calibration are shown.

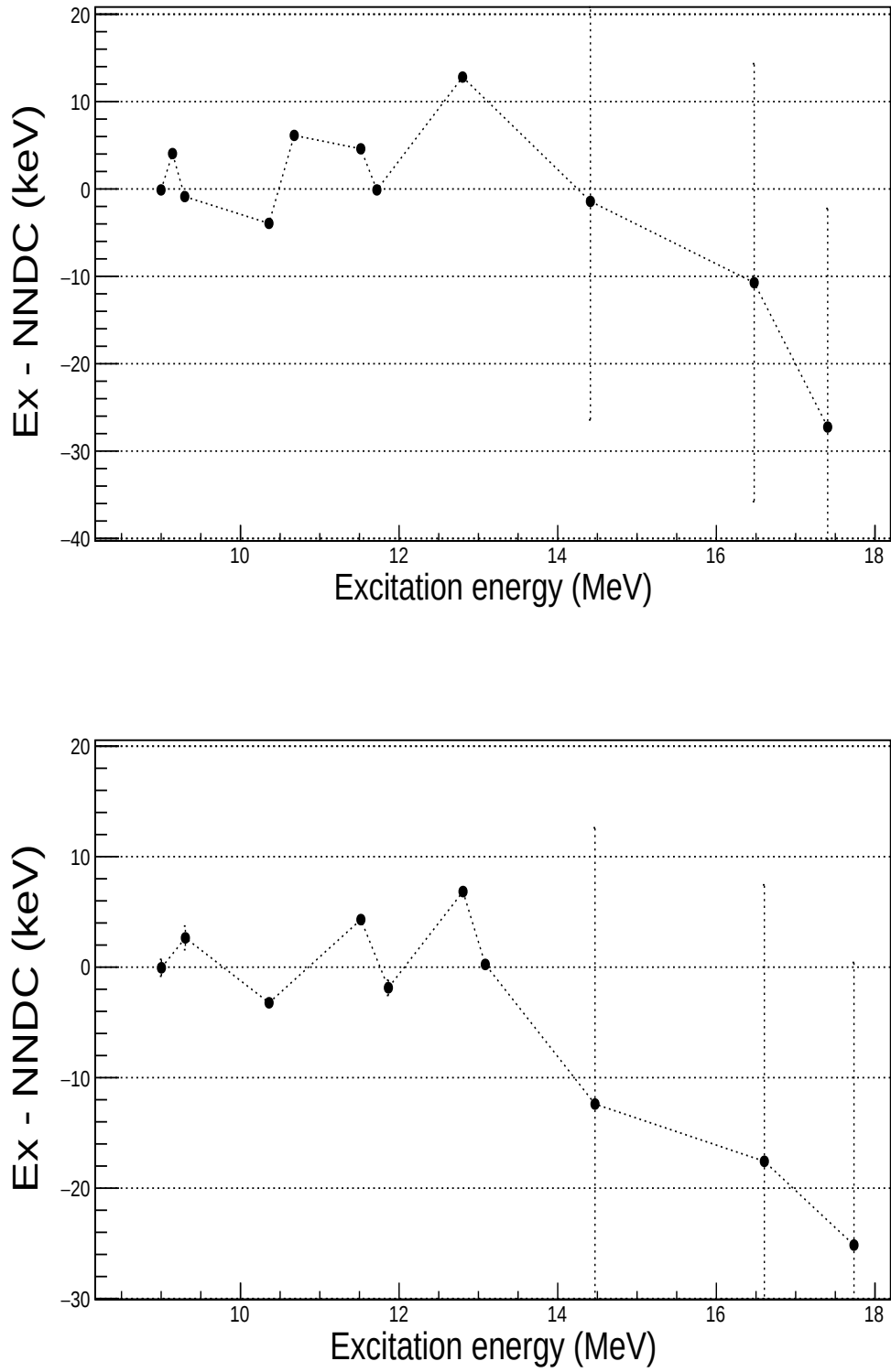


Figure 3.34: A comparison of the energy values for the excited states of ^{24}Mg obtained in this work and those provided by the NNDC [NNDC], for the 0° measurements (top panel) and for the 4° measurements (bottom panel). The three last states are as published by K. van der Borg *et al.* [Bor81]

3.7.7 Double differential cross-sections

The double differential cross-sections are calculated using the instrumental background free spectrum of the target of interest. With units of $\text{mb} \cdot \text{sr}^{-1} \cdot \text{MeV}^{-1}$, the double differential cross-sections is defined as

$$\frac{d^2\sigma}{d\Omega dE} = \frac{10^{27} \cdot N_c}{N_0 \cdot \rho \cdot D \cdot \Delta\Omega \cdot \Delta E \cdot \varepsilon_{\text{tot}}} , \quad (3.22)$$

where N_c is the corrected number of counts in a bin representing inelastically scattered α -particles, N_0 is the total number of incident alpha particles on the target, ρ is the number of target nuclei per unit area (in mb^{-1}), and D is the electronic dead-time correction factor, which ranges from 0.9937 to 0.9987 for this set of experiments. It is the ratio between the inhibited and uninhibited pulser for the scalar read-out of the current integrator (CI) value ($D = CII/CIU$). $\Delta\Omega$ is the solid angle of the K600 magnetic spectrometer in sr and has a value of 3.48 sr and 3.84 sr, for the zero- and four-degrees measurements, respectively. ΔE is the energy bin size (in MeV) and ε_{tot} is the total VDC efficiency of the K600 defined in Sect. 3.7.2.3.

The number of incident alpha particles, N_0 is calculated as

$$N_0 = \frac{CI \cdot R \cdot 10^{-12}}{2e} , \quad (3.23)$$

where R is the selected range (in nA) of the current integrator, and represents 1000 counts per second for a full scale current read-out. The parameter $2e$ is the alpha particle charge (in coulombs) where e is the elementary charge.

Further, the number of target nuclei per unit area ρ is given by:

$$\rho = \frac{\lambda \cdot N_A}{A} , \quad (3.24)$$

where λ is the target thickness (in g/cm^2), N_A for Avogadro's number and A is the mass number of the considered nucleus.

The double differential cross-sections were extracted for the ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg nuclei. The statistical errors associated with the

cross-sections are deduced as follows:

$$\Delta \frac{d^2\sigma}{d\Omega dE} \Big|_{\text{stat}} = \frac{1}{\sqrt{N_{\text{counts}}}} \frac{d^2\sigma}{d\Omega dE} , \quad (3.25)$$

where N_{counts} is the number of counts per energy bin.

The systematic uncertainties have been estimated from the measured double differential cross-sections by considering various sources, namely

- (i) *Target thickness*: using the alpha particle transmission method, a 4% systematical error in the target thickness has been considered.
- (ii) *Current integrator*: considering the description from its manual and the accuracy of the charge collection on the different beamstops, one can contemplate a maximum uncertainty on the integrated current of 1%.
- (iii) *Background subtraction error*: due to different selection criteria of software gates defining the background, it is estimated that there is a yield uncertainty of approximately 1% to 2% from light medium mass to heavy mass nuclei, respectively.
- (iv) *Solid angle error*: this accounts for propagating measurements uncertainties of the collimator opening and the distance between the target and collimator. A safe upper limit for this systematical error is considered to be 0.3%.
- (v) *Software angle cut*: this uncertainty is due to the spread in angles of the beam on the target. It amounts to approximately 2% uncertainty in the yield.

The overall systematic uncertainties on the double differential cross-sections amount to approximately 5% using the quadratic sum given by

$$\Delta \frac{d^2\sigma}{d\Omega dE} \Big|_{\text{syst}} = \sqrt{\sum_k \left(\frac{\Delta X_k}{X_k} \right)^2} \frac{d^2\sigma}{d\Omega dE} , \quad (3.26)$$

where X_k are all uncertainty-afflicted experimental sources mentioned above.

Chapter 4

Results, analysis and discussion

The experimental results are compared to the theoretical predictions in this chapter. First of all, the calibrated and background subtracted spectra for the zero- and four-degrees measurements are presented. Discrete peaks from $^{16}\text{O}(\alpha, \alpha')$ are identified in the $^{208}\text{Pb}(\alpha, \alpha')$ spectra and using a $^{16}\text{O}(\alpha, \alpha')$ spectrum, the contaminant was removed to obtain oxygen-contamination-free spectra. The procedure followed to achieve such a task is explained in Sect. 4.1.1, whereafter the experimental cross-sections are presented in Sect. 4.1.2. In order to extract the strength of the ISGMR, the Difference-of-Spectra (DoS) method is performed as a proof of principle study using data obtained at 0° and 4° . This procedure is outlined in Sect. 4.2. Due to the overlap mainly with the ISGQR and ISGDR, one needs to determine the appropriate scattering angle cut from the 4° data, where the ISGMR has a deep minimum, that can successfully lead to the determination and extraction of the ISGMR. For that, DWBA calculations were employed and the results are detailed in Sect. 4.2.1. Fraction $E0$ EWSR and $E0$ strength results for all nuclei investigated in this study are presented in Sect 4.2.3 together with previously published results from the Texas A&M University (TAMU) and the Research Center for Nuclear Physics (RCNP) groups.

It should be noted that the DoS method has some shortcomings, such as the fact that the subtraction of the small-angle cut spectrum from the 0° spectrum does not ultimately remove the IVGDR contribution which, like the ISGMR, also peaks at 0° and which becomes important due to the Coulomb interaction which is stronger for heavier nuclei. A modified DoS procedure that requires

knowledge of the fractional strengths of the different multipolarities is discussed in Sect. 4.3.1 followed by the comparison of the findings with published results from TAMU and RCNP in Sect. 4.3.2.

The extraction of characteristic energy scales of the fine structure of the ISGMR from both experimental data and theoretical predictions using the Continuous Wavelet Transform (CWT) technique are presented in Sect. 4.4. The extraction of the spin-dependent level densities of 0^+ states ends the chapter.

4.1 Overview of the measured experimental alpha inelastic scattering data: zero-degrees and four-degrees measurements

Figures 4.1 and 4.2 show the excitation energy spectra for ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg at $\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$ and $\theta_{\text{Lab}} = 2^\circ - 6^\circ$, respectively. The prominent 12.049 MeV $0^+ \text{ }^{16}\text{O}$ peak in the ^{208}Pb 0° spectrum indicates the presence of some oxygen contamination in the ^{208}Pb target. The 11.520 MeV 2^+ state is also visible in the 4° dataset for ^{208}Pb . The procedure followed for the removal of the oxygen contaminants is detailed in the following section.

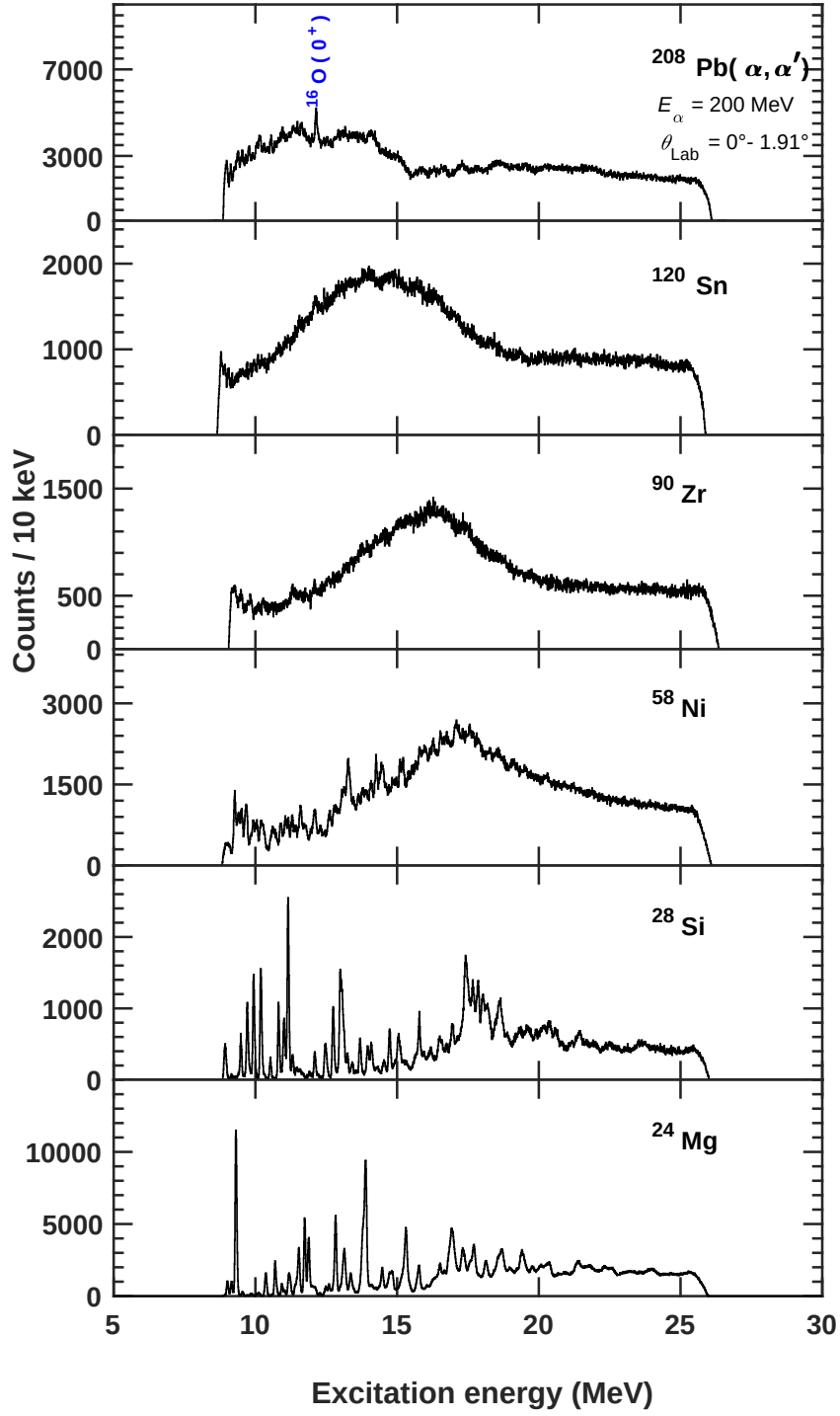


Figure 4.1: Excitation energy spectra for $^{208}\text{Pb}(\alpha, \alpha')$, $^{120}\text{Sn}(\alpha, \alpha')$, $^{90}\text{Zr}(\alpha, \alpha')$, $^{58}\text{Ni}(\alpha, \alpha')$, $^{28}\text{Si}(\alpha, \alpha')$ and $^{24}\text{Mg}(\alpha, \alpha')$ at $E_\alpha = 200$ MeV and $\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$.

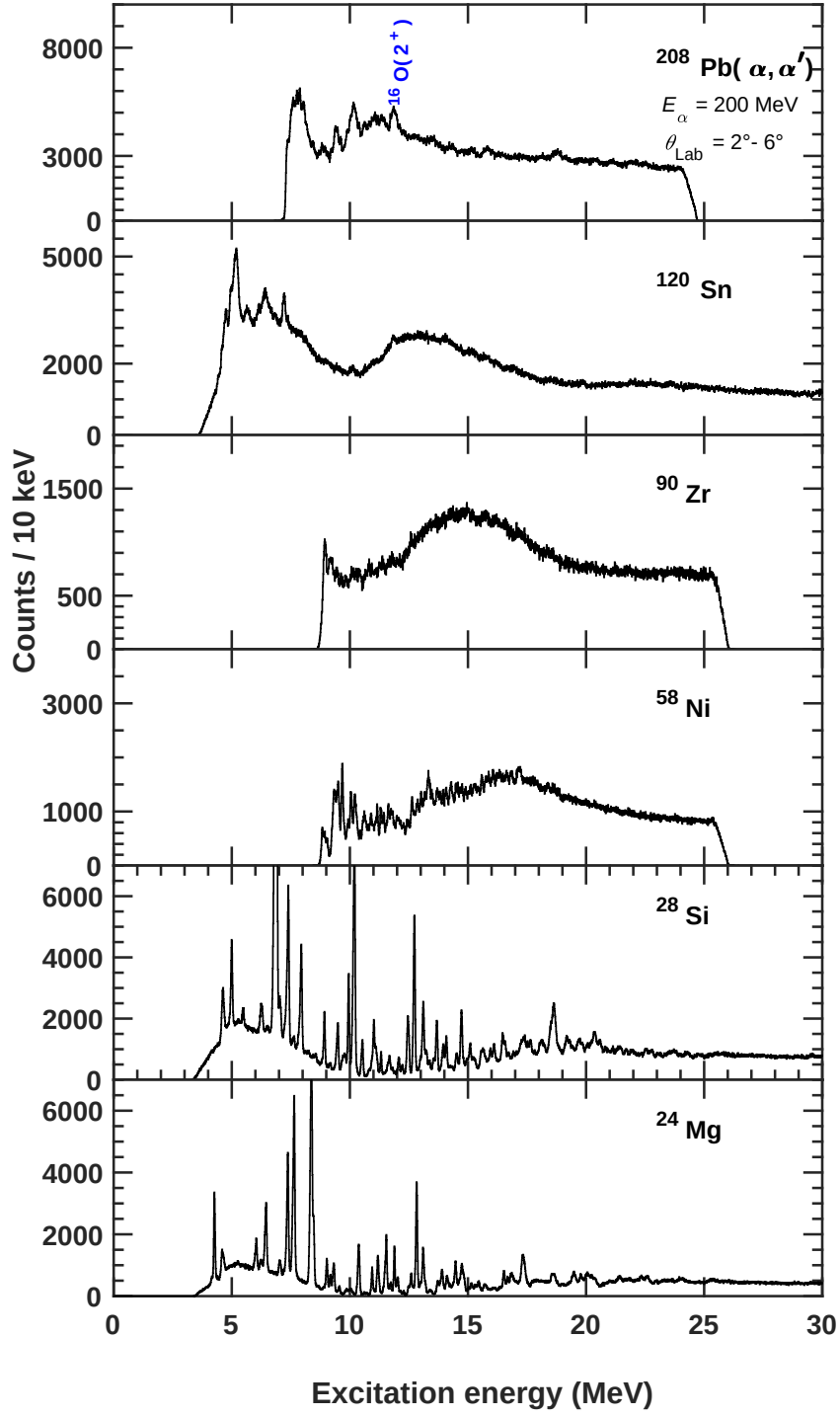


Figure 4.2: Excitation energy spectra for $^{208}\text{Pb}(\alpha, \alpha')$, $^{120}\text{Sn}(\alpha, \alpha')$, $^{90}\text{Zr}(\alpha, \alpha')$, $^{58}\text{Ni}(\alpha, \alpha')$, $^{28}\text{Si}(\alpha, \alpha')$ and $^{24}\text{Mg}(\alpha, \alpha')$ at $E_\alpha = 200$ MeV and $\theta_{\text{Lab}} = 2^\circ - 6^\circ$.

4.1.1 Removal of oxygen (^{16}O) contamination

The 12.049 MeV 0^+ and 11.520 MeV 2^+ states in ^{16}O observed in the ^{208}Pb spectra at 0° and 4° , respectively, originate from the water contamination on the target nucleus during the manufacturing process. It should be noted that the excitation energies of these states are slightly shifted from their normal positions due to the differing masses of ^{16}O and ^{208}Pb since the rigidity calibration was performed using the Pb target mass. This is seen in the top panels of Figs. 4.1 and 4.2. While the identifiable peaks of ^{16}O sit at the lower energy side of the excitation energy spectrum, ^{16}O also contributes to the background underneath the ISGMR region. It is then essential to get an accurate $^{16}\text{O}(\alpha, \alpha')$ spectrum in order to successfully remove this background from the Pb spectrum. In order to achieve this, mylar $\text{C}_{10}\text{H}_8\text{O}_4(\alpha, \alpha')$ and carbon $^{12}\text{C}(\alpha, \alpha')$ measurements were performed at both 0° and 4° . After normalising¹ the spectrum from the ^{12}C nucleus to the strength of the ^{12}C 9.641 MeV peak in the mylar spectrum, the ^{12}C spectrum was removed from the mylar spectrum through direct subtraction. The contribution of hydrogen to the 0° spectrum can be ignored as it affects lower excitation energies than observed in the spectrum. At 4° , the hydrogen can be seen below 12 MeV. However, it is absent in the $2^\circ - 2.9^\circ$ angle cut spectrum that will be used in the DoS process as discussed in Sect. 4.2.2. The resulting subtracted spectrum can be considered as the $^{16}\text{O}(\alpha, \alpha')$ spectrum. In a similar manner, the $^{16}\text{O}(\alpha, \alpha')$ spectrum was normalised to the strength of the ^{16}O 12.049 MeV and 11.520 MeV peaks in the ^{208}Pb spectra and then removed from each of spectrum at both 0° and $2^\circ - 2.9^\circ$ ². The results obtained for both 0° and $2^\circ - 2.9^\circ$ are shown in the panels (a), (b) and (c) of Figs. 4.3 and 4.4.

¹This is done based on the number of counts in the 9.641 MeV peak in both mylar and carbon spectra by fitting the peak using a Gaussian plus a polynomial background. The area under the peak is then obtained and this leads to an appropriate scaling.

²It is worth noting that due to the DoS method to be applied at later stage, the small-angle cut $2^\circ - 2.9^\circ$ for the $^{16}\text{O}(\alpha, \alpha')$ spectrum was used in the process of oxygen contaminants removal from the 4° data.

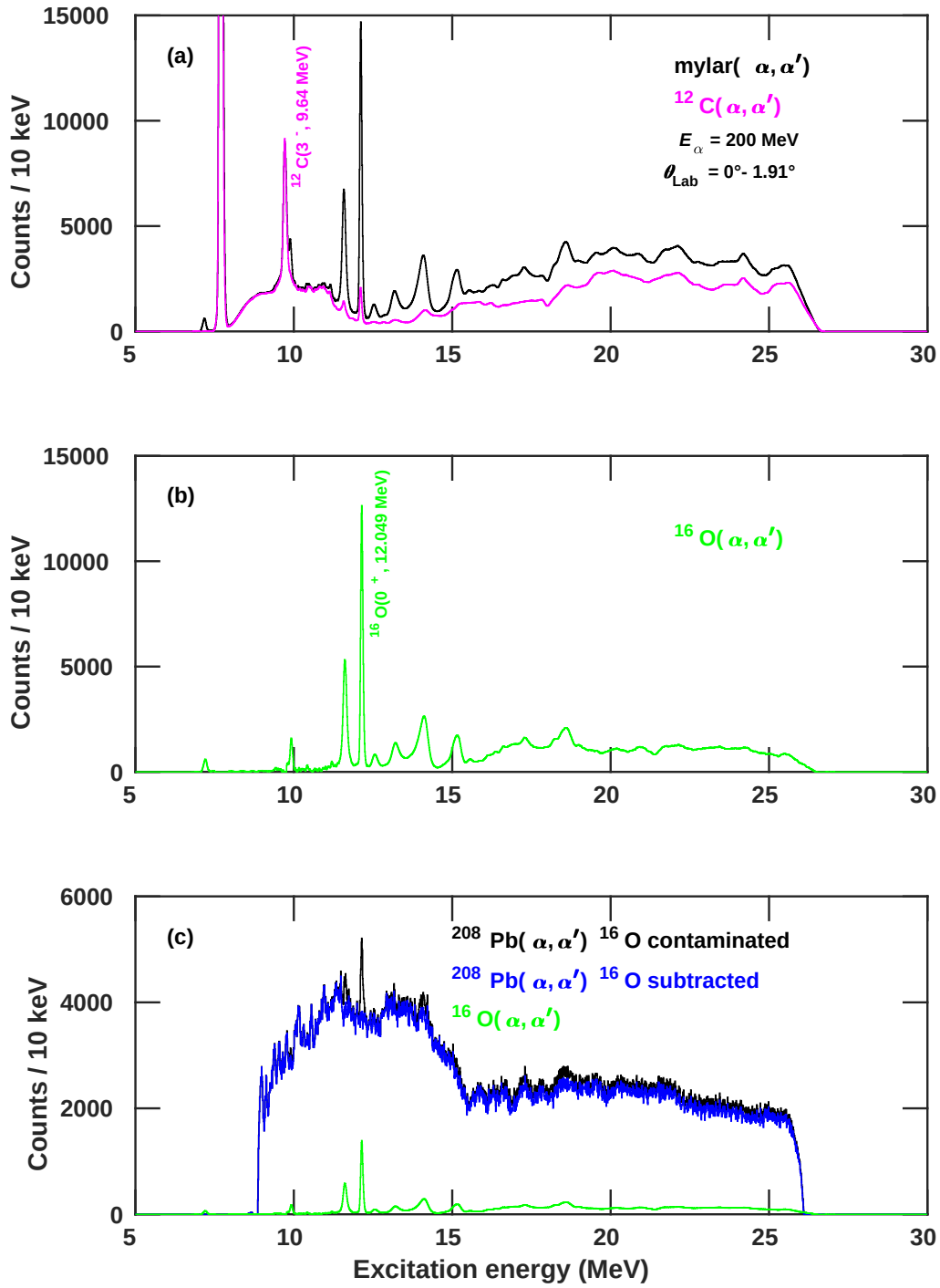


Figure 4.3: (a) Excitation energy spectrum of mylar overlaid with ^{12}C for $0^\circ - 1.91^\circ$. Extracted ^{16}O is shown in panel (b). The spectra of oxygen-contaminated ^{208}Pb , oxygen-free ^{208}Pb and normalised ^{16}O spectrum before subtraction are shown in panel (c).

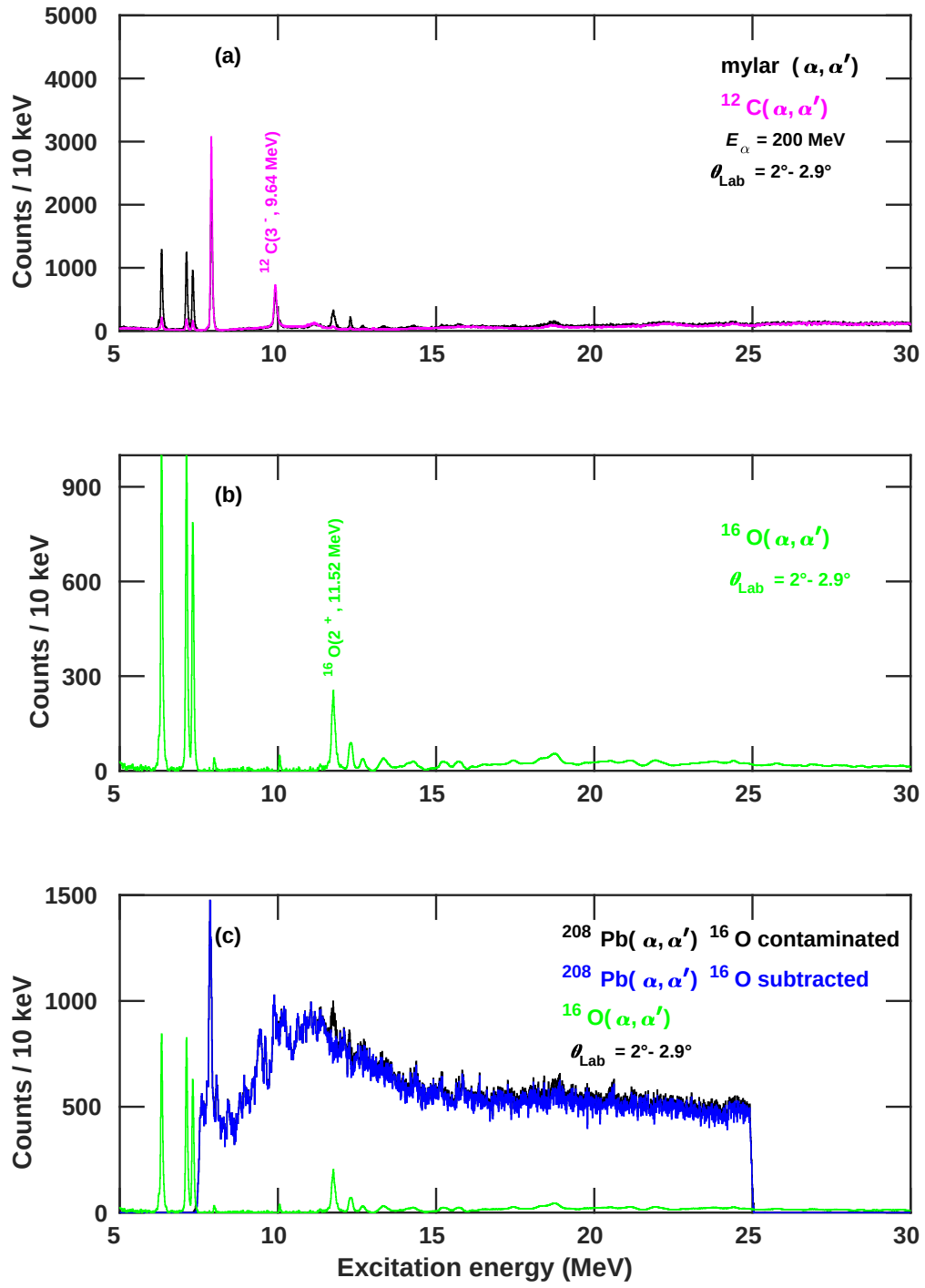


Figure 4.4: Same as Fig. 4.3 but for $2^\circ - 2.9^\circ$.

4.1.2 Experimental cross-sections for ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg

Following the removal of oxygen contaminants from the $^{208}\text{Pb}(\alpha, \alpha')$ spectra, the double differential cross-sections for all the nuclei under investigation are evaluated using Eq. (3.22) and are shown in Figs. 4.5 and 4.6 for both 0° and 4° data³⁴. For the ^{208}Pb nucleus, the finite angle spectrum represents only the $2^\circ - 2.9^\circ$ angle selected region.

³Technically, the cross-sections should be referred to as triple differential cross-sections $d^3\sigma$ as three independent variables, θ , ϕ and E were involved in the calculations.

⁴The yield in cross-sections observed at lower excitation energy region are non-physical effects due to acceptance issue with the spectrometer.

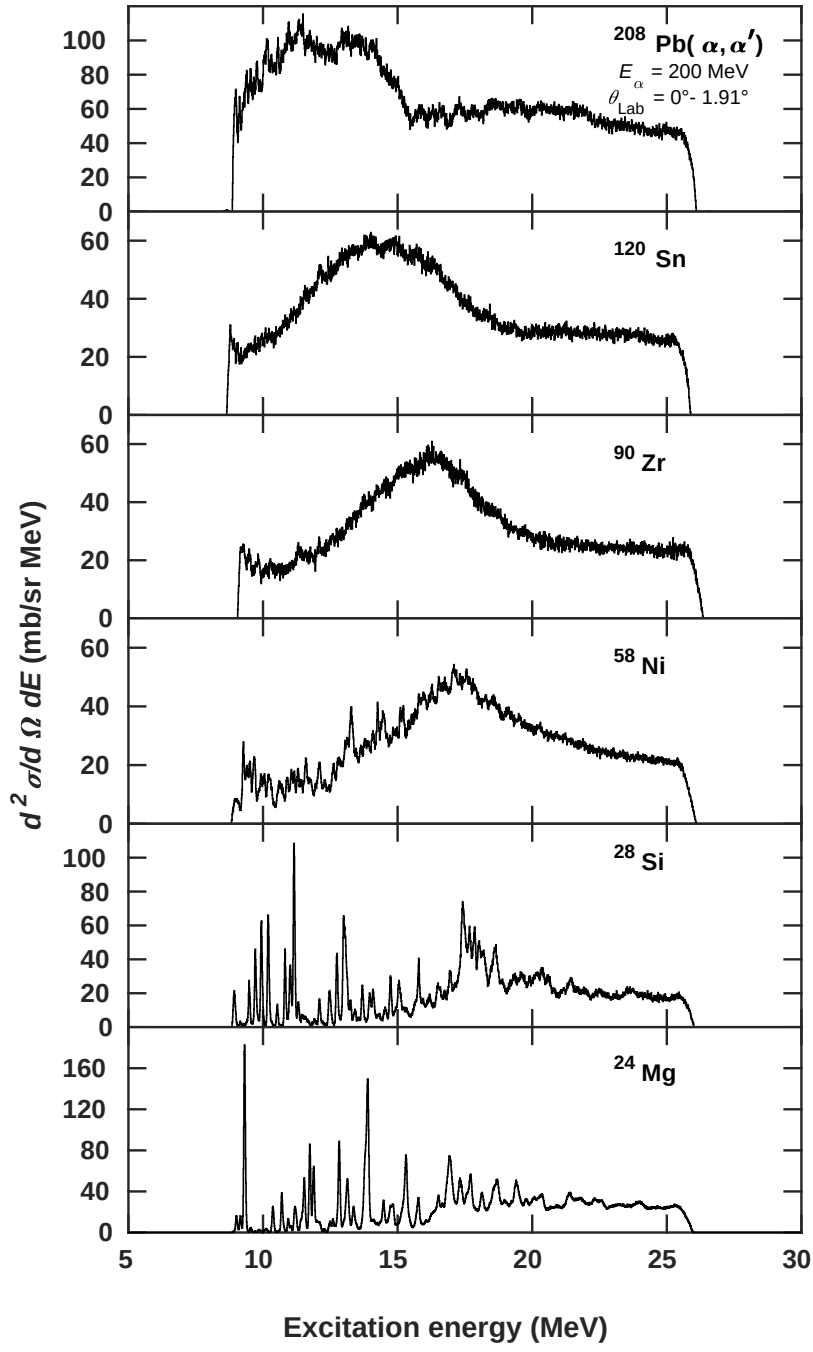


Figure 4.5: Double differential cross-sections obtained for $^{208}\text{Pb}(\alpha, \alpha')$, $^{120}\text{Sn}(\alpha, \alpha')$, $^{90}\text{Zr}(\alpha, \alpha')$, $^{58}\text{Ni}(\alpha, \alpha')$, $^{28}\text{Si}(\alpha, \alpha')$ and $^{24}\text{Mg}(\alpha, \alpha')$ for the angle range $0^\circ - 1.91^\circ$.

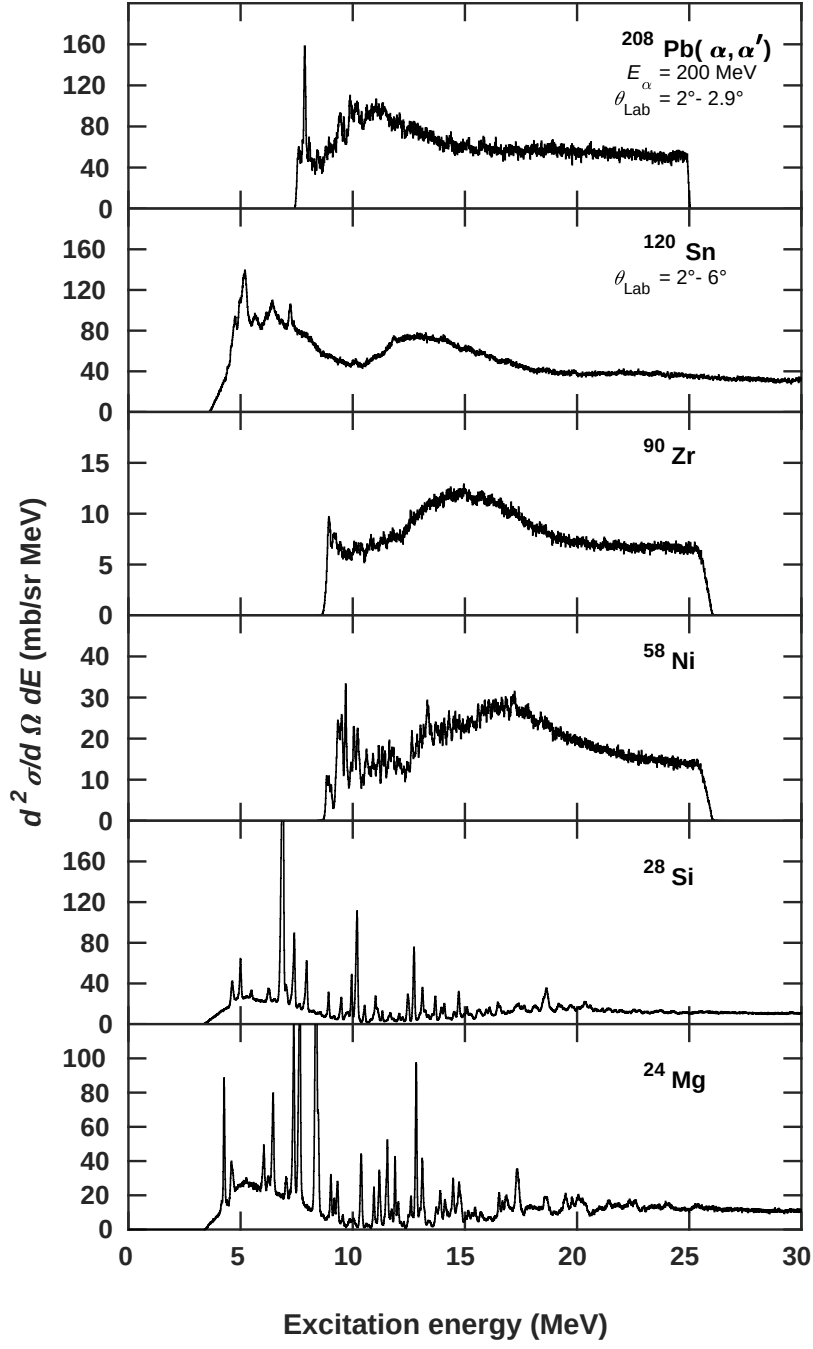


Figure 4.6: Double differential cross-sections obtained for $^{208}\text{Pb}(\alpha, \alpha')$, $^{120}\text{Sn}(\alpha, \alpha')$, $^{90}\text{Zr}(\alpha, \alpha')$, $^{58}\text{Ni}(\alpha, \alpha')$, $^{28}\text{Si}(\alpha, \alpha')$ and $^{24}\text{Mg}(\alpha, \alpha')$ for the angle range $2^\circ - 6^\circ$. The top panel indicates the double differential cross-sections obtained from oxygen-free $^{208}\text{Pb}(\alpha, \alpha')$ for the angle range $2^\circ - 2.9^\circ$.

4.2 Extraction of $E0$ strength in the studied nuclei

The Multipole Decomposition Analysis (MDA) approach remains a better tool to extract the $E0$ strength from (α, α') experimental data. For that, the accepted norm is that one needs to obtain data with angular distribution $0^\circ \leq \theta_{\text{Lab}} \lesssim 10^\circ$ in steps of about $\sim 0.5^\circ$. This requires a magnetic spectrometer with good angular resolution and capable of providing data within that angular range (see Refs. [You97, Gup16a]). Since the investigation of the fine structure of the ISGMR stands as the prime objective of this project, the dataset does not have information beyond 6° to allow for the MDA method to be performed.

As such, the $E0$ strength in the present data has been extracted through the DoS method. The method consists of subtracting a spectrum obtained from an angle cut where the angular distributions for the other multipolarities are nearly flat, and that of GMR is at a minimum, from the spectrum obtained with data taken at 0° . This method, therefore, requires the determination of the suitable angle cut from the measurement at $\theta_{\text{Lab}} = 2^\circ - 6^\circ$, which is obtained from DWBA calculations detailed in Sect. 4.2.1.

4.2.1 Angular distributions using DWBA in the studied nuclei

In the present project, the DWBA calculations were carried out according to the method described in [Sat97] and discussed in Sect. 2.5. A density-dependent single-folding model for the real part of the potential $U(r)$, obtained with a Gaussian α -nucleon potential, and a phenomenological Woods-Saxon potential for the imaginary term of $U(r)$ were used, so that the α -nucleus potential can be written as

$$U(r) = V_{\text{fold}}(r) + i \frac{W}{\{1 + \exp[(r - R_I)/a_I]\}}. \quad (4.1)$$

The real single-folded potential $V_{\text{fold}}(r)$ is obtained by folding the

ground-state density with the density-dependent α -nucleon interaction

$$V_{\text{fold}}(r) = -V \int d^3r' \rho(r') [1 - \beta \rho(r')^{2/3}] \exp(-z^2/t^2) , \quad (4.2)$$

where $z = |r - r'|$ is the distance between the centre of mass of the α particle and a target nucleon, $\rho(r')$ is the ground-state density of the target nucleus at the position r' of the target nucleon. The parameters $\beta = 1.9 \text{ fm}^2$ and range $t = 1.88 \text{ fm}$ were taken from [Sat97]. Here, V is the real potential depth and W is the depth of the Woods-Saxon type imaginary part of the potential, with radius $R_I = r_I(A_t^{1/3} + A_p^{1/3})$ and diffuseness a_I . The subscripts p and t refer to projectile and target, respectively. The ground-state density $\rho(r)$ of the target nucleus at the position r is given by

$$\rho(r) = \frac{\rho_0}{1 + \exp\left(\frac{r-c}{a}\right)} , \quad (4.3)$$

where the Fermi-distribution parameters c and a are the adjusted half-density radius and the diffuseness, respectively.

DWBA calculations were carried out on the nuclei under investigation, namely ^{24}Mg , ^{28}Si , ^{58}Ni , ^{90}Zr , ^{120}Sn and ^{208}Pb using the computer code PTOLEMY [Rho80a, Rho80b]. The Optical Model Parameters (OMP) used in the DWBA calculations, viz. r_I, a_I, V and W for each nucleus were obtained by fitting elastic scattering cross-sections. The procedure is fully described in [Nol87]. Due to the lack of elastic scattering data in our experiments, the parameters were taken from publications of the TAMU group for each of the nuclei under investigation. This is justified by the closeness of their beam energy (240 MeV) to this study. In order to illustrate that the PTOLEMY calculations are properly performed using the published fractional strengths for the various multipoles of an excitation energy representing the maximum of the GMR, the results published by the TAMU group in [You99b], [You02], [Lui06], [Lui15] and [You04] for ^{24}Mg , ^{28}Si , ^{58}Ni , ^{90}Zr , ^{116}Sn and ^{208}Pb were reproduced and are shown in the left side panels of Figs. 4.7 and 4.8. Here, the ^{116}Sn nucleus is considered because no result has been published by the TAMU group on ^{120}Sn . As such, the OMP obtained for ^{116}Sn were used for the ^{120}Sn nucleus in the iThemba LABS dataset. By employing the same fractional strengths, calculations of angular distributions were done at $E_\alpha = 200 \text{ MeV}$ and the

results are shown on the right side panels of Figs. 4.7 and 4.8.

The EWSR serves as input to PTOLEMY by adjusting the coupling strengths between the ground and excited states. The idea is that a standardised amount of the EWSR is assumed, and the corresponding deformation parameters/reduced transition probabilities can be calculated with that value, which then yields the DWBA outputs. Those DWBA outputs can then be used to fit the experimental distributions and, therefore, give the percentage of that standardised amount of the EWSR which is found in the energy bin for various multipolarities. This is elucidated in [Har81, Sat87, Har01]. Tables 4.1 and 4.2 show the OMP and the Fermi distribution parameters, respectively, used in the DWBA calculations with references listed.

Table 4.1: Optical model parameters taken from the TAMU group publications.

Nucleus	V (MeV)	W (MeV)	r_I (fm)	a_I (fm)	Refs.
^{24}Mg	41.02	35.39	0.934	0.614	[You99b]
^{28}Si	44	32.5	0.9306	0.687	[You02]
^{58}Ni	41.19	40.39	0.821	0.974	[Lui06]
^{90}Zr	40.2	40.9	0.786	1.242	[Lui15]
$^{116/120}\text{Sn}$	36.7	23.94	0.998	1.047	[You04]
^{208}Pb	43.3	61.4	1.032	0.567	[You04]

Table 4.2: Fermi-distribution parameters taken from [Fri95].

Nucleus	^{24}Mg	^{28}Si	^{58}Ni	^{90}Zr	$^{116/120}\text{Sn}$	^{208}Pb
c (fm)	2.979	3.155	4.08	4.909	5.433	6.670
a (fm)	0.523	0.623	0.515	0.515	0.515	0.545

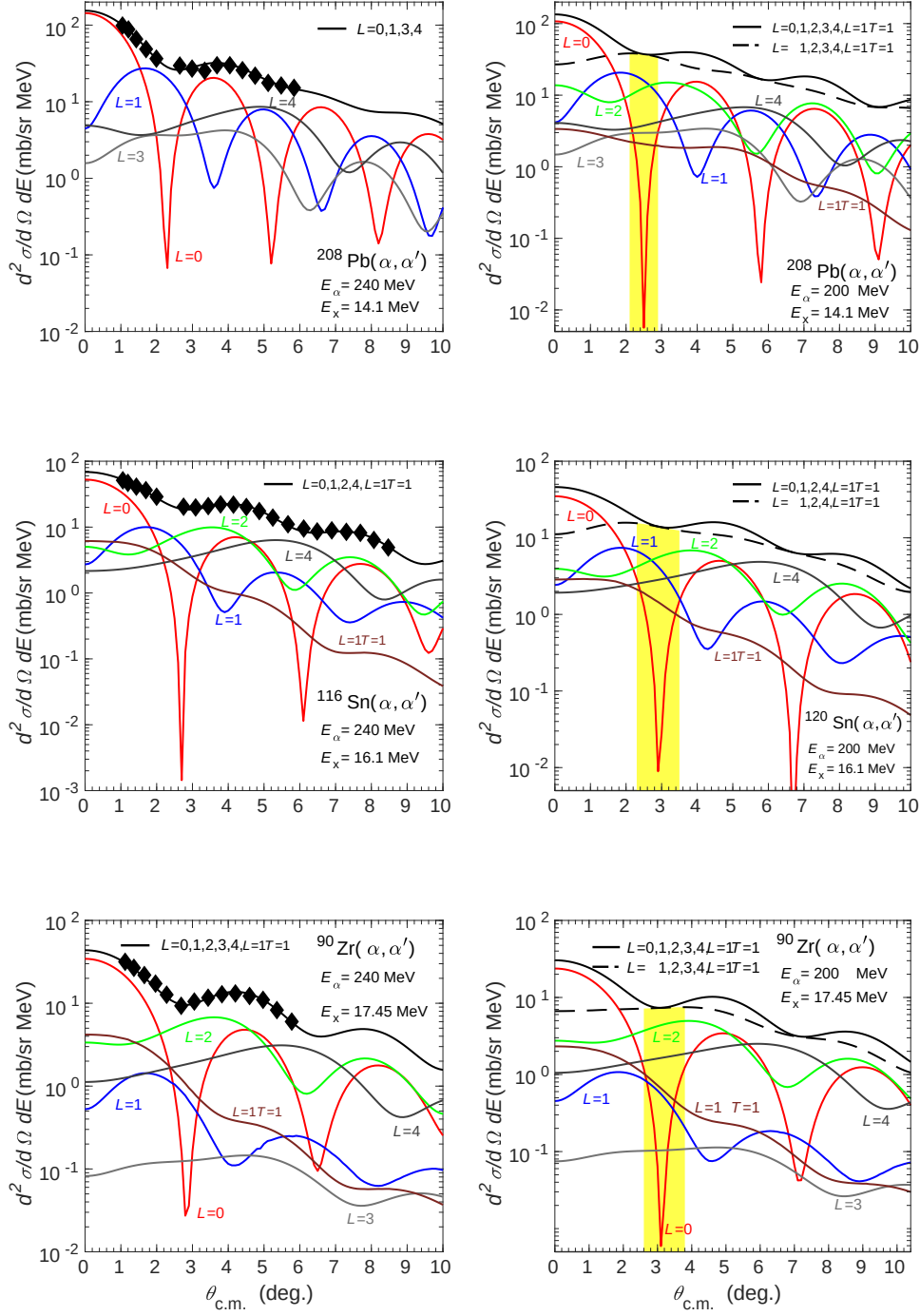


Figure 4.7: Angular distributions of the differential cross-sections for inelastic alpha particle scattering on ^{208}Pb , $^{116/120}\text{Sn}$ and ^{90}Zr at $E_x = 14.1$ MeV, 16.1 MeV and 17.45 MeV, respectively. The left side panels represent the TAMU group results from their MDA analysis at beam energy $E_\alpha = 240$ MeV while the right panels are the DWBA results obtained at beam energy $E_\alpha = 200$ MeV. The solid black line denotes the sum of all the multipoles indicated on each panel. The black dashed line represents the sum of all multipoles except $L = 0$. The data shown on the left panels are taken from [You04] and [Lui15].

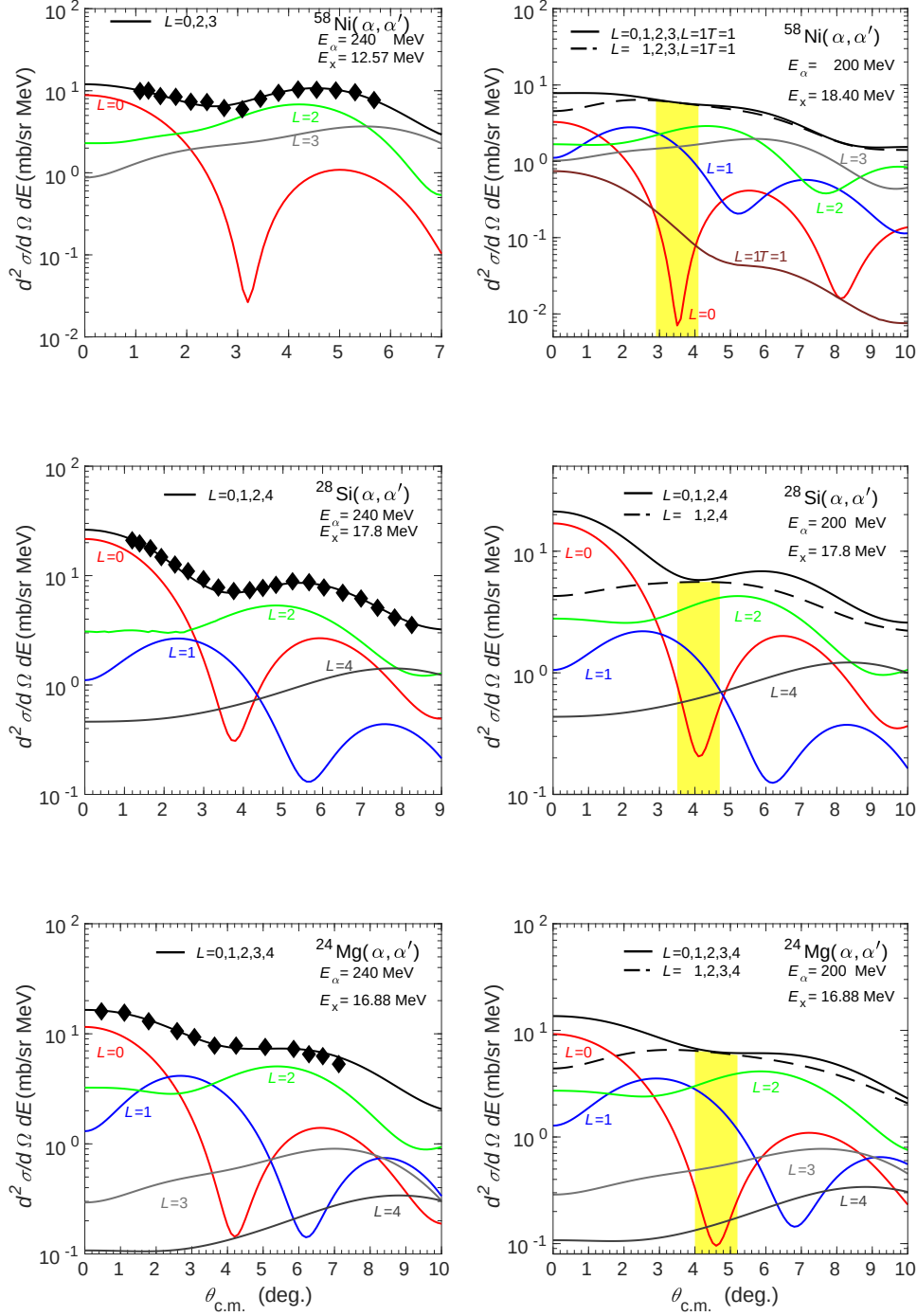


Figure 4.8: Angular distributions of the differential cross-sections for inelastic alpha particle scattering on ^{58}Ni , ^{28}Si and ^{24}Mg at $E_x = 12.57(18.4)$ MeV, 17.8 MeV and 16.88 MeV, respectively. The left side panels represent the TAMU group results from their MDA analysis at beam energy $E_\alpha = 240$ MeV while the right panels are the DWBA results obtained at beam energy $E_\alpha = 200$ MeV. The solid black line denotes the sum of all the multipoles indicated on each panel. The black dashed line represents the sum of all multipoles except $L = 0$. The data shown on the left panels are taken from [Lui06], [You02] and [You99b].

4.2.2 Difference-of-Spectra (DoS) results

As stated earlier, the $E0$ strength for all the nuclei under investigation is extracted using the DoS method. The technique relies on the subtraction of all other multipoles from the 0° spectrum using a subset of the angle range from the 4° data where the $L = 0$ contribution is going through its first minimum. Ideally, the sum of angular distributions of other multipoles (all except $L = 0$) is nearly flat within the region $0^\circ \leq \theta_{\text{c.m.}} \leq 4.5^\circ$. This is shown by the black dashed lines in the DWBA calculations shown in Figs. 4.7 and 4.8. Hence, the subtraction of the inelastic spectrum of the flat angular distribution where $L = 0$ is at a minimum from the 0° spectrum represents essentially the $E0$ component excited in α -inelastic scattering close to 0° (see Refs. [You98], [You99a], [Gup16a] and [Gar18] for further readings).

The direct subtraction of the angle cut spectrum from the spectrum at $0^\circ \leq \theta_{\text{Lab}} \leq 1.91^\circ$ yields the difference spectrum shown by the black spectra in Figs. 4.9 - 4.11 for each of the nuclei under investigation. The angle cut for each nucleus is indicated by the highlighted (yellow) area in the right side panels of Figs. 4.7 and 4.8. Table 4.3 lists the angle cuts both in the laboratory and centre-of-mass (c.m.) frames. The angle corresponding to the minimum of $L = 0$ is also indicated.

Table 4.3: Angle cuts implemented in the 4° dataset to define the angular region around the first $L = 0$ minimum.

Nucleus	^{24}Mg	^{28}Si	^{58}Ni	^{90}Zr	^{120}Sn	^{208}Pb
Angle cut (θ_{Lab})	$3.39^\circ - 4.4^\circ$	$3.03^\circ - 4.07^\circ$	$2.7^\circ - 3.81^\circ$	$2.48^\circ - 3.62^\circ$	$2.22^\circ - 3.38^\circ$	$2.0^\circ - 2.9^\circ$
Angle cut ($\theta_{\text{c.m.}}$)	$4^\circ - 5.2^\circ$	$3.5^\circ - 4.7^\circ$	$2.9^\circ - 4.1^\circ$	$2.6^\circ - 3.8^\circ$	$2.3^\circ - 3.5^\circ$	$2.05^\circ - 2.95^\circ$
$L = 0$ minimum ($\theta_{\text{c.m.}}$)	4.6°	4.1°	3.5°	3.2°	2.9°	2.5°

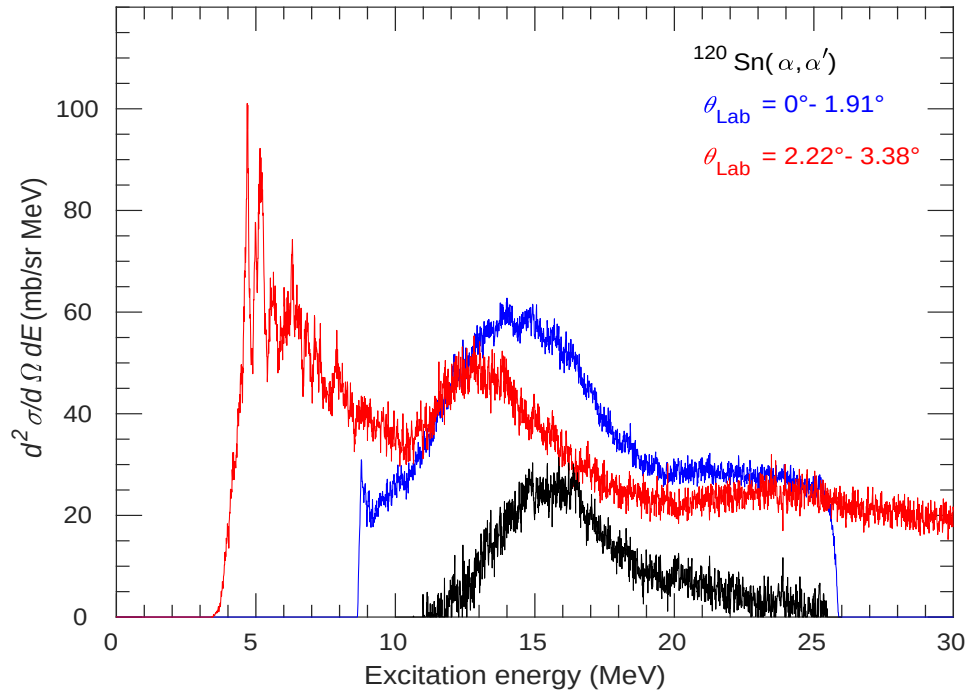
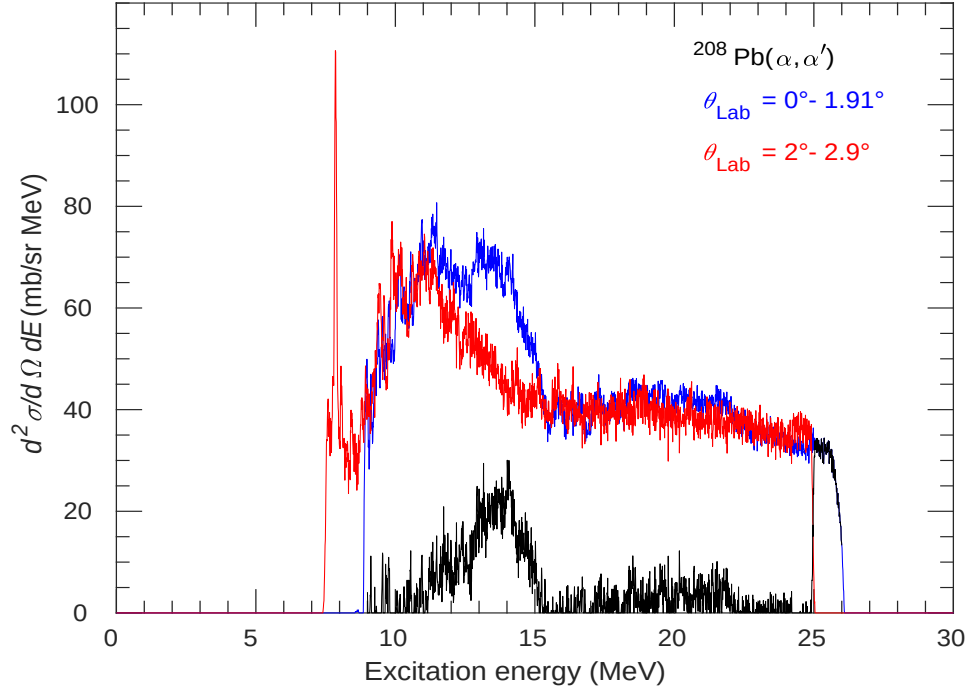


Figure 4.9: Double differential cross-sections for $^{208}\text{Pb}(\alpha, \alpha')$ (top panel) and $^{120}\text{Sn}(\alpha, \alpha')$ (bottom panel) at $E_\alpha = 200$ MeV. The blue spectrum represents the data acquired at $0^\circ \leq \theta_{\text{Lab}} \leq 1.91^\circ$, the red represents the angle cut spectrum at $2^\circ \leq \theta_{\text{Lab}} \leq 2.9^\circ$ for ^{208}Pb and at $2.22^\circ \leq \theta_{\text{Lab}} \leq 3.38^\circ$ for ^{120}Sn , respectively. The black spectrum represents the difference spectrum for each nucleus.

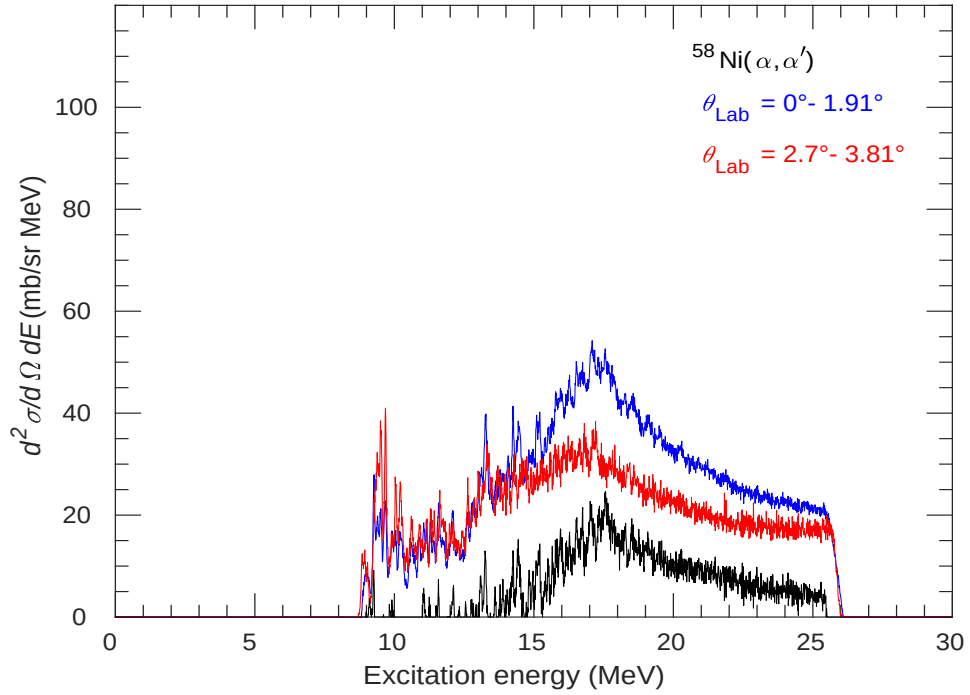
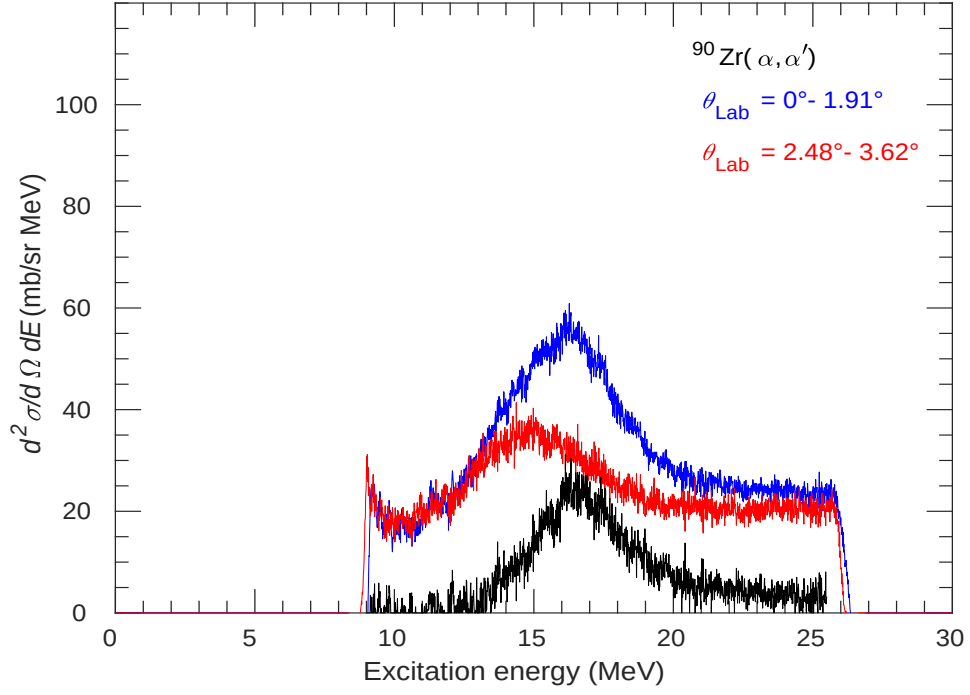


Figure 4.10: Double differential cross-sections for $^{90}\text{Zr}(\alpha, \alpha')$ (top panel) and $^{58}\text{Ni}(\alpha, \alpha')$ (bottom panel) at $E_\alpha = 200$ MeV. The blue spectrum represents the data acquired at $0^\circ \leq \theta_{\text{Lab}} \leq 1.91^\circ$, the red represents the angle cut spectrum at $2.48^\circ \leq \theta_{\text{Lab}} \leq 3.62^\circ$ for ^{90}Zr and at $2.7^\circ \leq \theta_{\text{Lab}} \leq 3.81^\circ$ for ^{58}Ni , respectively. The black spectrum represents the difference spectrum for each nucleus.

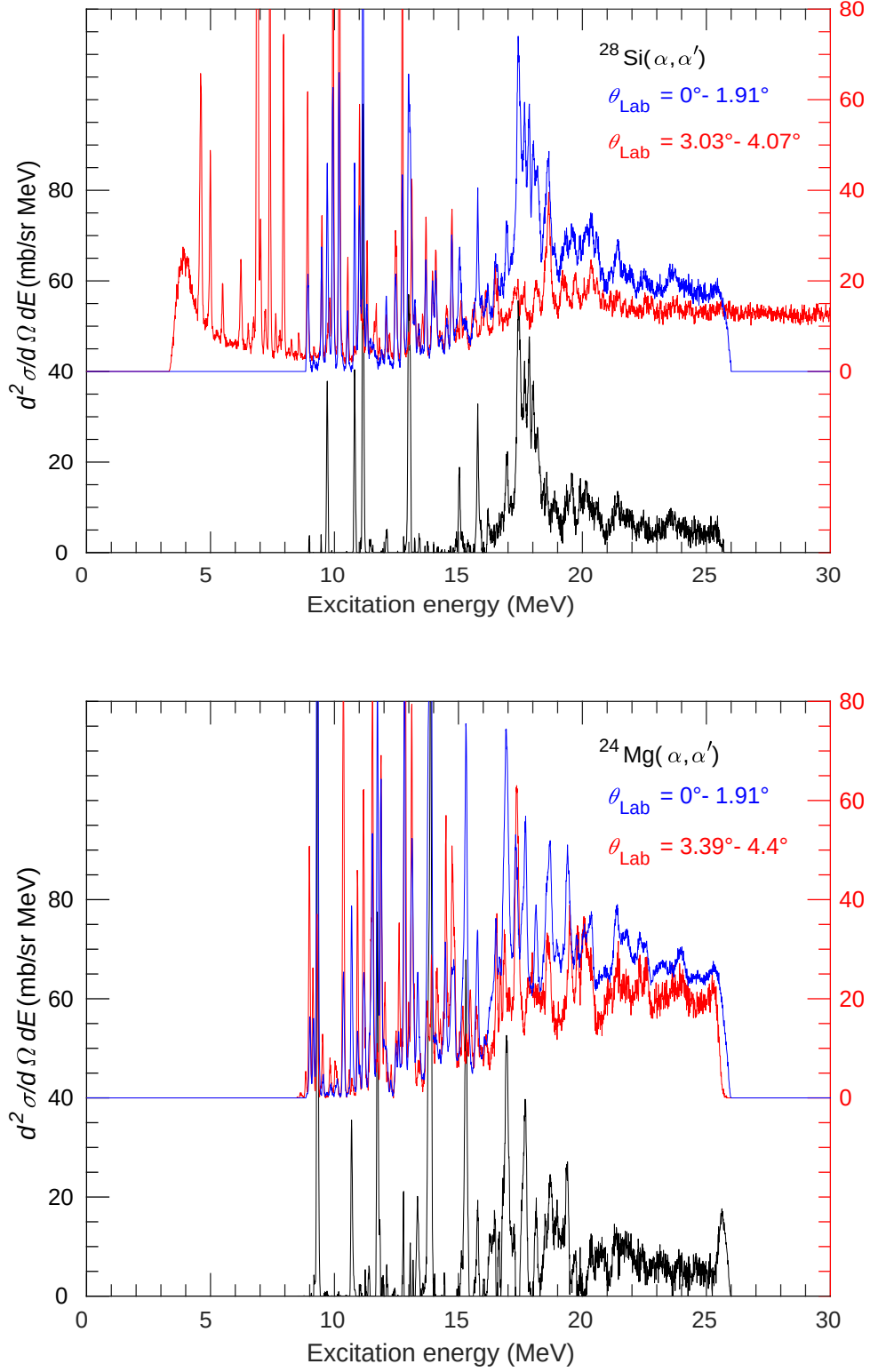


Figure 4.11: Double differential cross-sections for $^{28}\text{Si}(\alpha, \alpha')$ (top panel) and $^{24}\text{Mg}(\alpha, \alpha')$ (bottom panel) at $E_\alpha = 200$ MeV. The blue spectrum represents the data acquired at $0^\circ \leq \theta_{\text{Lab}} \leq 1.91^\circ$, the red represents the angle cut spectrum at $3.03^\circ \leq \theta_{\text{Lab}} \leq 4.07^\circ$ for ^{28}Si and at $3.39^\circ \leq \theta_{\text{Lab}} \leq 4.4^\circ$ for ^{24}Mg , respectively. For clarity purpose, these spectra are shifted by 40 mb/sr MeV. The black spectrum represents the difference spectrum for each nucleus.

4.2.3 Determination of the Fraction $E0$ EWSR and the $E0$ strength distributions

In order to obtain the Fraction $E0$ EWSR and $E0$ strength from the difference spectrum shown as the black spectrum in each of the Figs. 4.9 - 4.11, several steps were required as listed below. Considering the ^{208}Pb as an example case:

- Starting from the difference spectrum shown in Fig. 4.12(a), the spectrum is rebinned from 10 keV to 1 MeV bins shown in Fig. 4.12(b)⁵. This is to enable comparison with published MDA results from both RCNP and TAMU groups on similar nuclei at beam energy $E_\alpha = 386$ MeV and $E_\alpha = 240$ MeV, respectively.
- The rebinned spectrum is converted from the laboratory to the centre-of-mass (c.m.) frame using the corresponding factor for each nucleus obtained via kinematics calculations. This is shown in panel (c) of Fig. 4.12.
- The theoretical DWBA cross-sections at $\theta_{\text{c.m.}}^{\text{av.}} = 1.041^\circ$ ⁶ ($\theta_{\text{Lab}}^{\text{av.}} = 1.02^\circ$) corresponding to the average cross-sections between $\theta_{\text{c.m.}} = 0^\circ - 2^\circ$ and for 100% of the EWSR as a function of excitation energy are calculated using the code PTOLEMY [Rho80a, Rho80b] and the OMP shown in Table 4.1. This is shown in panel (d) of Fig. 4.12.
- The Fraction $E0$ EWSR per MeV (see Eq. (2.49)) is obtained by dividing the extracted experimental $E0$ component at $\theta_{\text{c.m.}}^{\text{av.}} = 1.041^\circ$ by the corresponding $L = 0$ 100% EWSR calculation at the same angle. The result is shown in panel (e) of Fig. 4.12. For comparison purposes, the Fraction $E0$ reported by the TAMU and/or RCNP groups are also shown. The RCNP group results are published as $E0$ strength only. As such, the corresponding Fraction $E0$ EWSR are calculated using Eq. (2.50).

⁵The rebinning may differ from nucleus to nucleus due to comparison with published MDA results from both RCNP and TAMU groups on similar nuclei. See panels (b) of Figs. 4.12 - 4.17.

⁶This angle differs from nucleus to nucleus as indicated in the panel (c) of Figs. 4.12 - 4.17.

- and using Eq. (2.50) in conjunction with the parameters in Table 4.4, the Fraction $E0$ EWSR/MeV is converted to $E0$ strength. The result is shown in panel (f) of Fig. 4.12. For comparison purposes, the $E0$ strength reported by the TAMU and/or RCNP are also shown. The TAMU group results are published as the Fraction $E0$ EWSR only. As such, the corresponding $E0$ strength are calculated using Eq. (2.50).

In order to estimate the errors on the calculated Fraction $E0$ EWSR and $E0$ strength, the total error (sum of statistical and systematical errors) given by Eqs. 3.25 and 3.26, for each E_x bin, is propagated to the difference spectrum using a quadratic sum of the errors on both the zero- and four-degrees measurements cross-sections. This error is then also propagated through the whole process to extract the Fraction $E0$ EWSR and $E0$ strength.

The same procedure above was followed for ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg . The results are shown in Figs. 4.13 - 4.17.

Table 4.4: Moments of the ground state density for the nuclei under investigation taken from [Fri95].

Nucleus	A	$\langle r^2 \rangle$ (fm ²)
^{24}Mg	24	9.345
^{28}Si	28	9.753
^{58}Ni	58	14.258
^{90}Zr	90	18.233
^{120}Sn	120	21.669
^{208}Pb	208	30.294

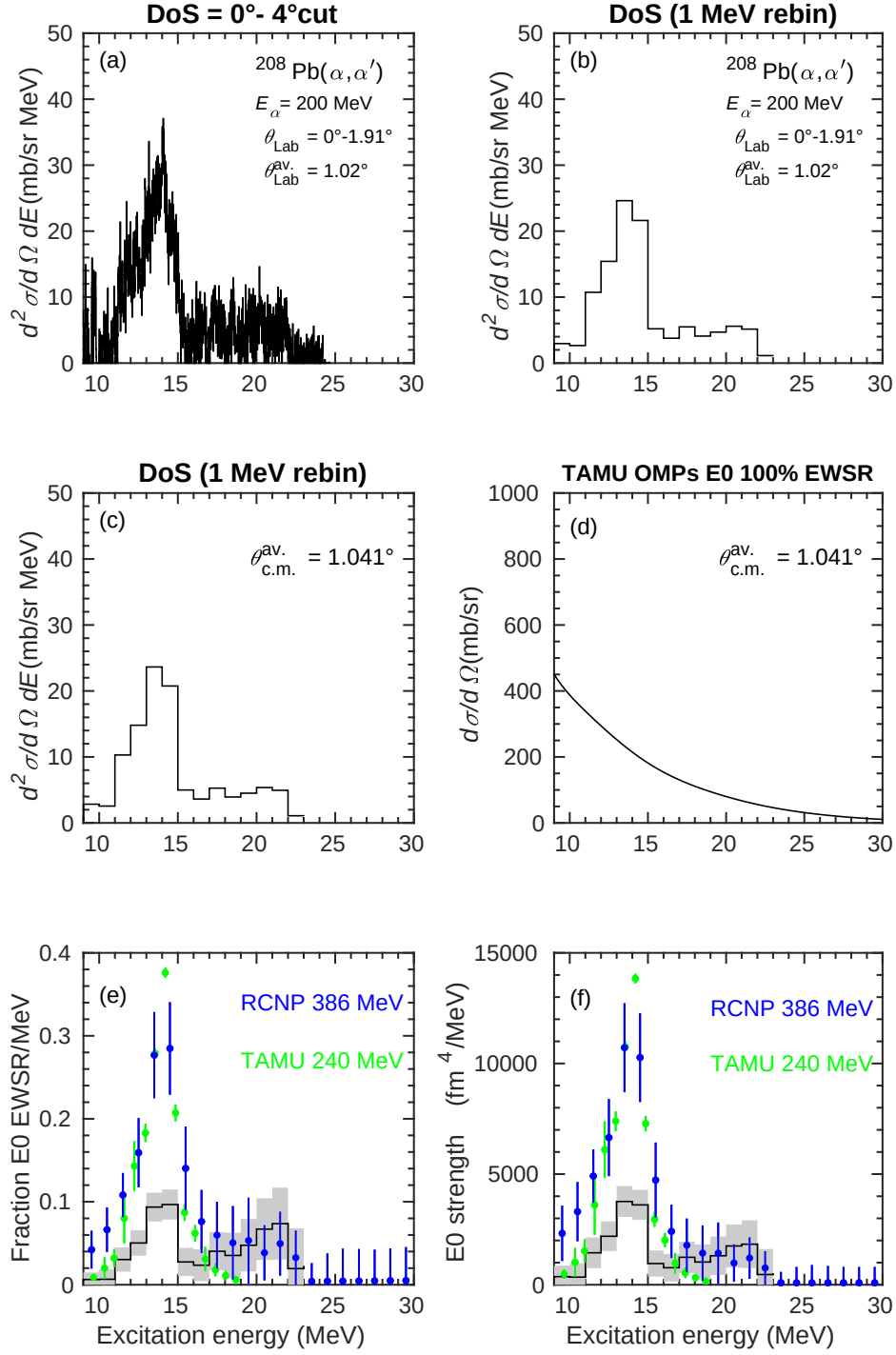


Figure 4.12: An outline of the conversion process from the DoS cross-sections of $^{208}\text{Pb}(\alpha, \alpha')$ to Fraction $E0$ EWSR per MeV and then to $E0$ strength distribution (explanation given in text). Error bars are displayed as grey bands. Data points shown as blue and green filled circles with vertical error bars are taken from [Pat13] and [You04].

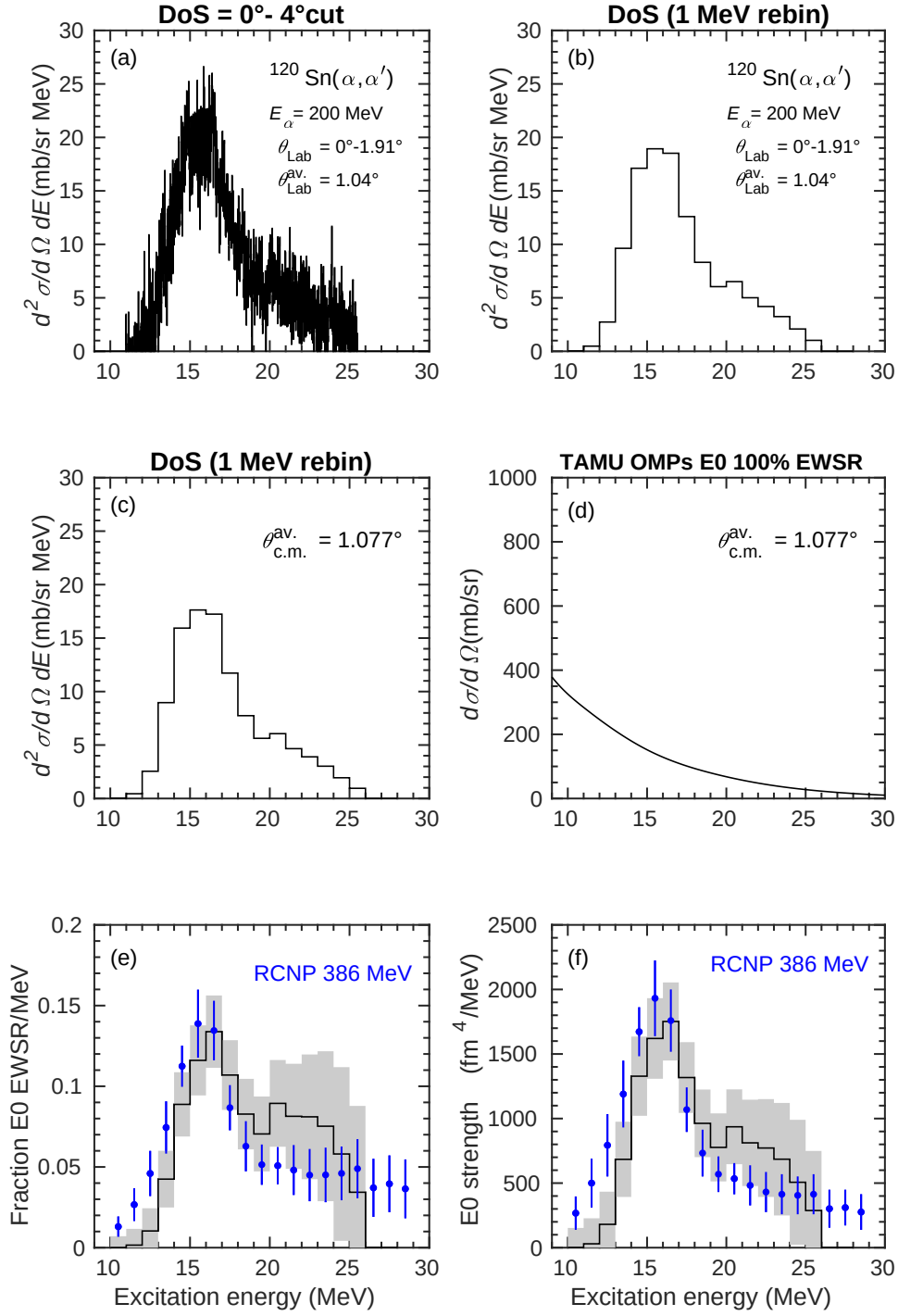


Figure 4.13: An outline of the conversion process from the DoS cross-sections of $^{120}\text{Sn}(\alpha, \alpha')$ to Fraction $E0$ EWSR per MeV and then to $E0$ strength distribution (explanation given in text). Error bars are displayed as grey bands. Data points shown as blue filled circles with vertical error bars are taken from [Li10].

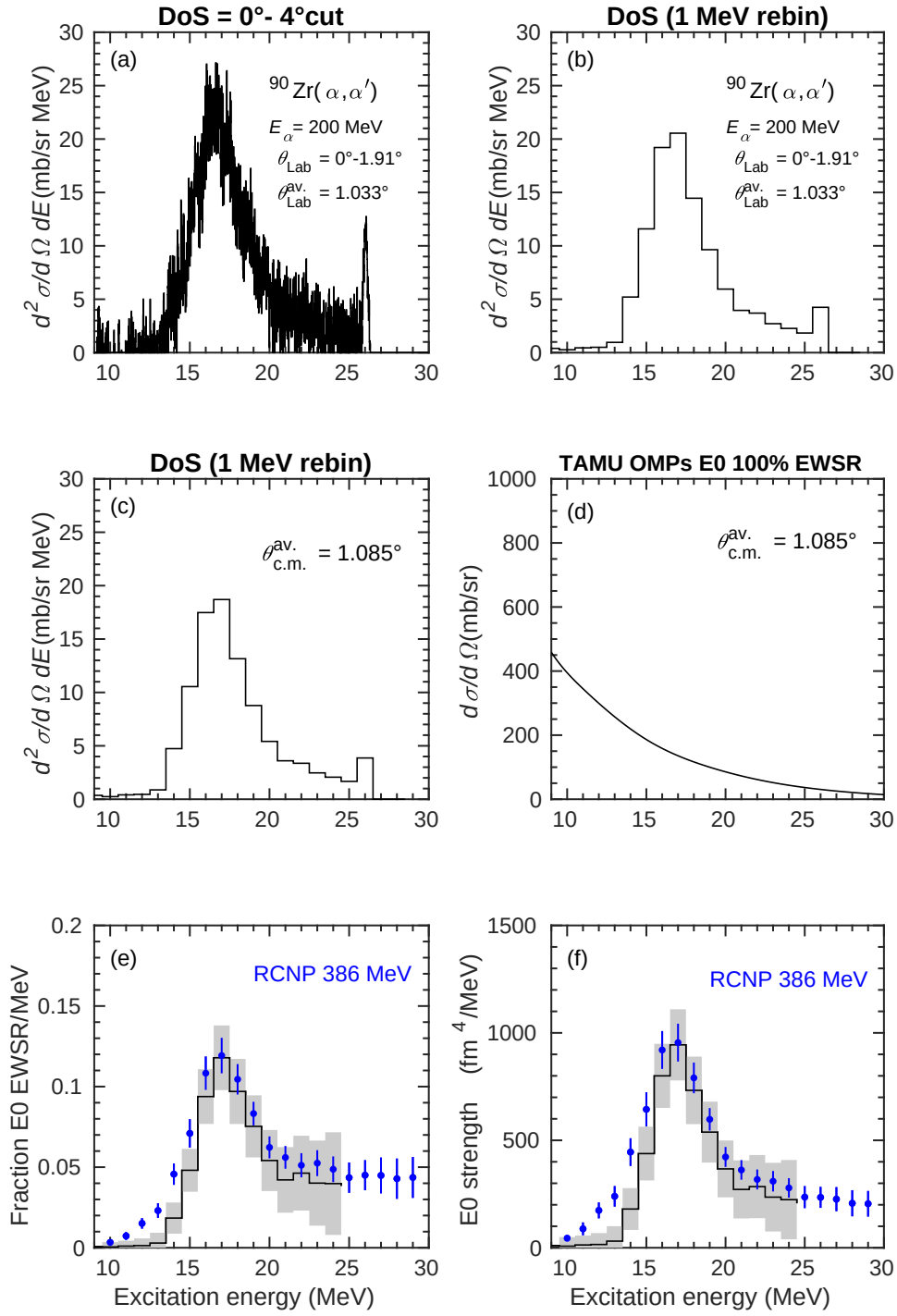


Figure 4.14: An outline of the conversion process from the DoS cross-sections of $^{90}\text{Zr}(\alpha, \alpha')$ to Fraction E0 EWSR per MeV and then to E0 strength distribution (explanation given in text). Error bars are displayed as grey bands. Data points shown as blue filled circles with vertical error bars are taken from [Gup18].

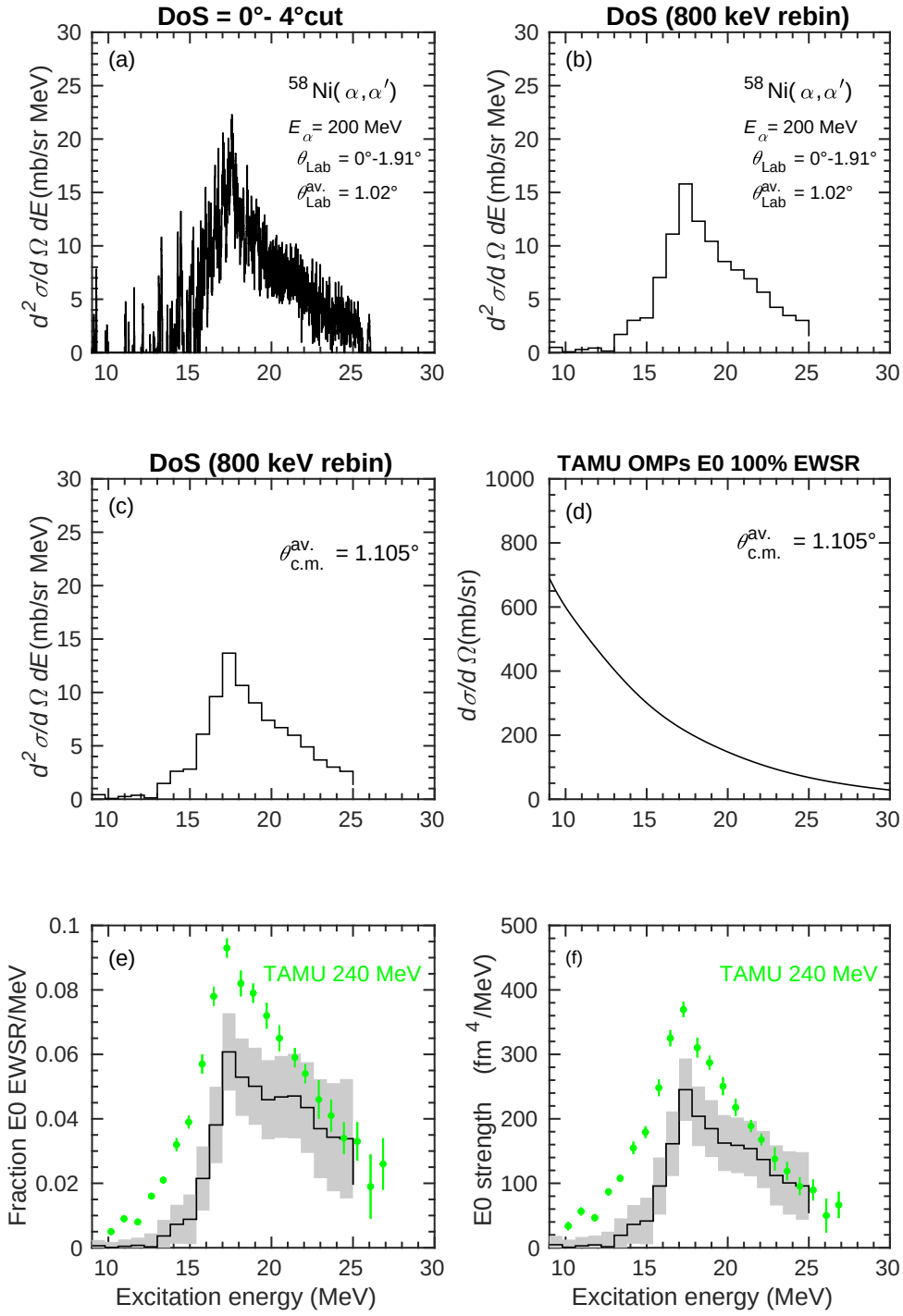


Figure 4.15: An outline of the conversion process from the DoS cross-sections of $^{58}\text{Ni}(\alpha, \alpha')$ to Fraction E0 EWSR per MeV and then to E0 strength distribution (explanation given in text). Error bars are displayed as grey bands. Data points shown as green filled circles with vertical error bars are taken from [Lui06].

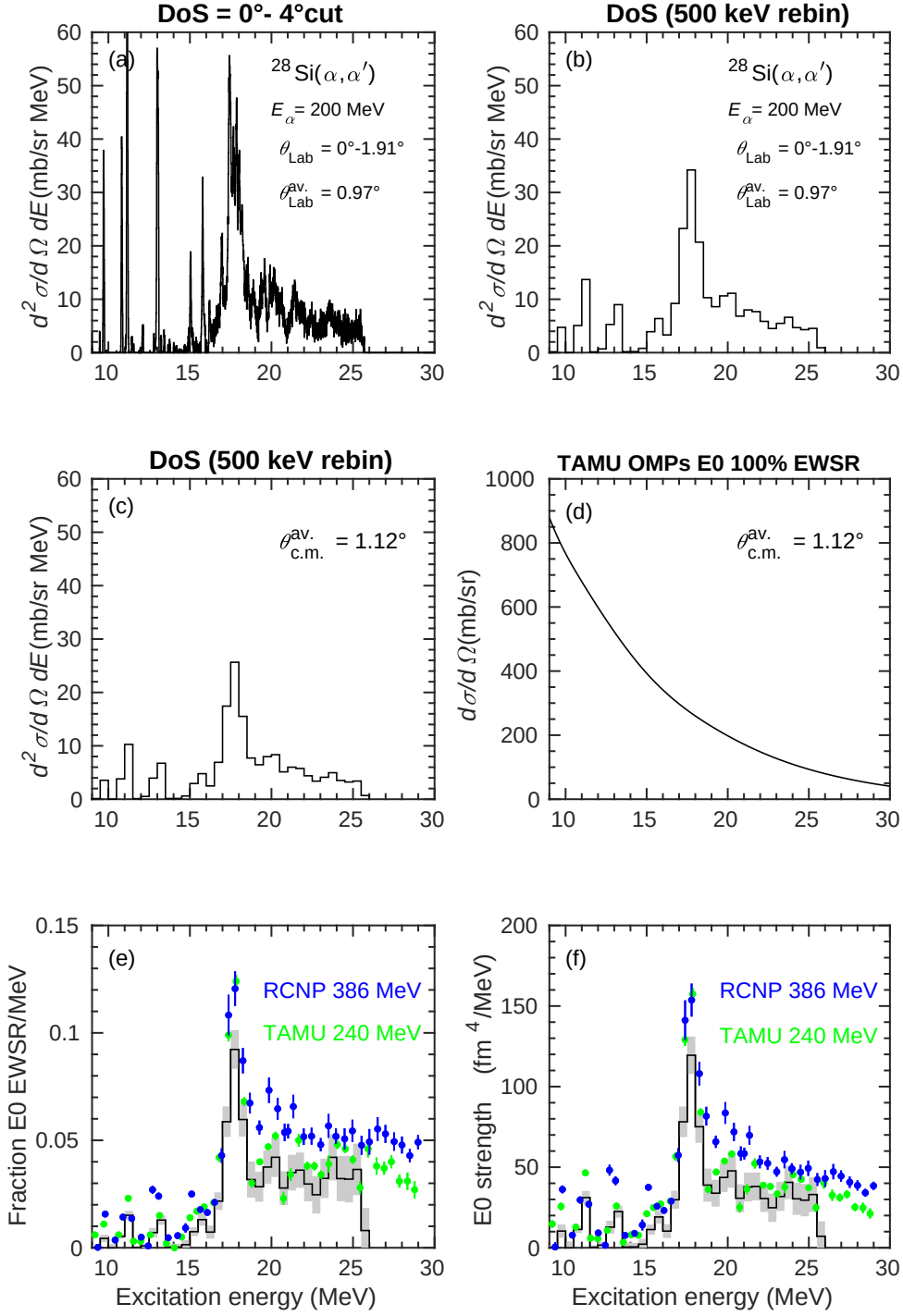


Figure 4.16: An outline of the conversion process from the DoS cross-sections of $^{28}\text{Si}(\alpha, \alpha')$ to Fraction $E0$ EWSR per MeV and then to $E0$ strength distribution (explanation given in text). Error bars are displayed as grey bands. Data points shown as blue and green filled circles with vertical error bars are taken from [Pea16] and [You07].

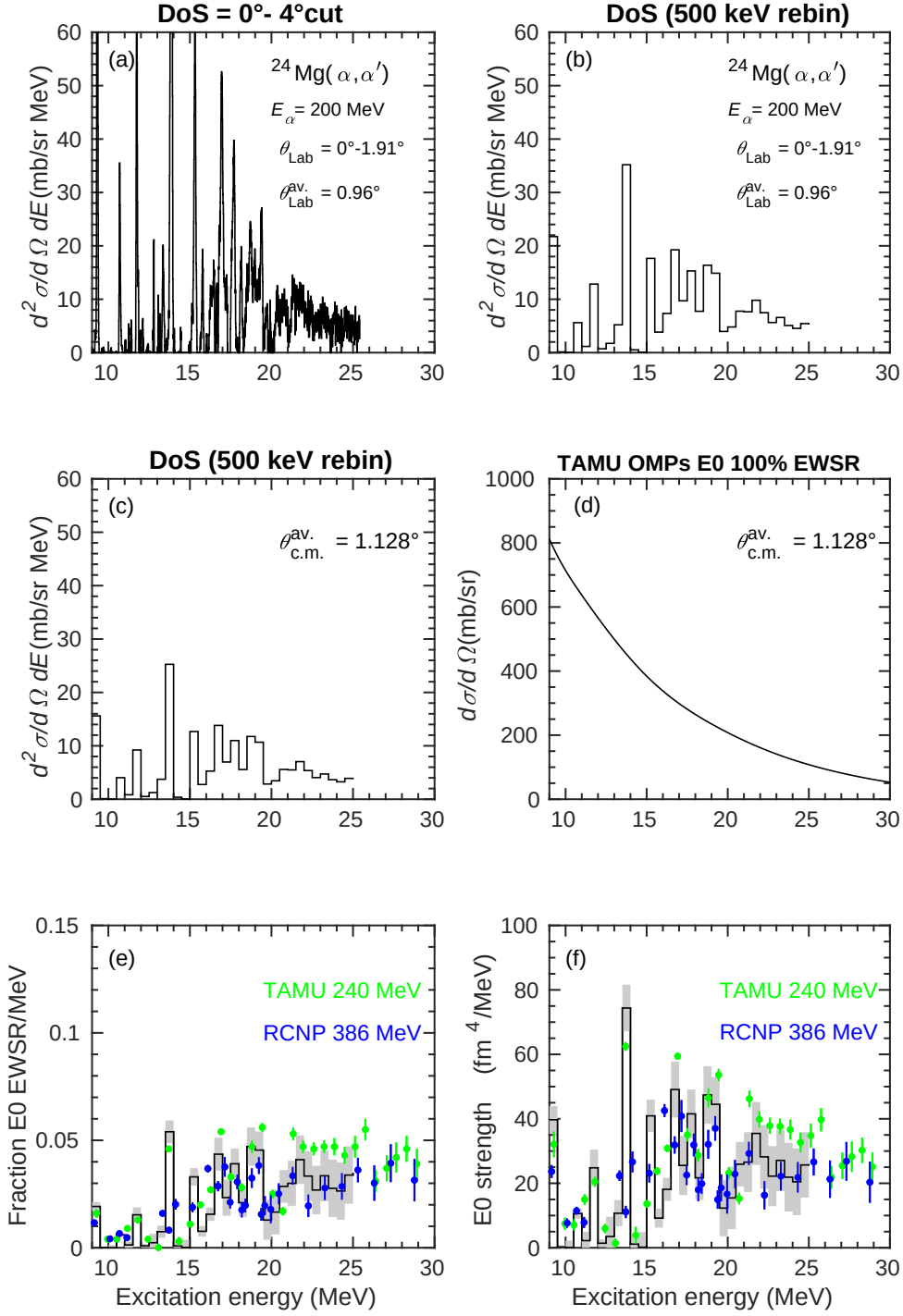


Figure 4.17: An outline of the conversion process from the DoS cross-sections of $^{24}\text{Mg}(\alpha, \alpha')$ to Fraction $E0$ EWSR per MeV and then to $E0$ strength distribution (explanation given in text). Error bars are displayed as grey bands. Data points shown as blue and green filled circles with vertical error bars are taken from [Gup16b] and [You09].

4.2.4 Comparison of iTL ISGMR strengths with TAMU and RCNP results

Either or both of the TAMU and RCNP groups reported Fraction $E0$ EWSR or $E0$ distributions, respectively, in all the nuclei under investigation in this study. They used the MDA approach on data acquired from inelastic α scattering at 240 MeV and 386 MeV, respectively. Considering the panels (e) and (f) of Figs. 4.12 - 4.17 in our results, the following conclusions can be drawn on each nucleus:

- **^{208}Pb**

Considering the panels (e) and (f) of Fig. 4.12, there seems to be a big discrepancy between the strength extracted from this work compared to the results from the RCNP and TAMU groups. Both the Fraction $E0$ EWSR and $E0$ strength are consistently lower than both the RCNP and TAMU results, from the low excitation energy region up to the GMR region. The excess strength observed in results from this study from 17 MeV and above is considered to be artificial and due to the background conditions that are much more severe for heavy nuclei such as Pb. This is explained by the target-related background (see Sect. 3.6.1) and elastic cross-sections/Rutherford scattering cross-sections scales with Z^2 .

- **^{120}Sn**

There is reasonable agreement with the RCNP Fraction $E0$ EWSR and $E0$ strength. However, at excitation energies below 16 MeV the iThemba LABS strength is consistently lower than the results from RCNP. In addition, at higher excitation energies the present results agree with the RCNP results within the grey uncertainty bands.

- **^{90}Zr**

Overall there is, within error bars, an excellent agreement with RCNP Fraction $E0$ EWSR and $E0$ strength. But similar to ^{120}Sn , the present results yield weaker strength below 14 MeV.

- **^{58}Ni**

Our results peak at the same energy as the TAMU group results, even though the value extracted for the $E0$ strength at the maximum is lower

than extracted by [Lui06]. As seen in other nuclei, the results reported by the TAMU group have a sharply downward trend for their strength above the peak. This is a known behaviour which might be related to their phenomenological background subtraction procedure. On the low excitation energy region, our results yield a weaker strength than the TAMU group.

- **^{28}Si**

The present results are for the most part consistent with previous findings by the TAMU group, except for a lower value of the $E0$ strength at the peak of the distribution. The results from RCNP, on the other hand, exhibits a much stronger $E0$ strength above 19 MeV than both the iThemba LABS and TAMU results. It is worth noting that some of the strength at lower excitation energy values are known not to belong to the GMR. It has been confirmed recently that the 0^+ states at 9.71 MeV, 10.81 MeV, 11.14 MeV and 12.99 MeV can be considered to be potential bandheads for superdeformed bands in ^{28}Si corresponding to its exotic structures [Ads17a]. The spin parity assignments shown in NUDAT [NNDC] have not been updated to show these newly established 0^+ spin parity assignments, and still show 5^- , 2^+ , 2^+ and 0^+ , respectively.

- **^{24}Mg**

For the most part the present results agrees well within error bars with both results from the RCNP and TAMU groups. However, above 17 MeV the Fraction $E0$ EWSR and $E0$ strength extracted by the TAMU group is consistently higher than both the iThemba LABS and RCNP results.

While the agreement between results from this study with results from work done at RCNP and TAMU seems to range from fair to good for most nuclei, the big difference observed for the ^{208}Pb results required a closer look. One notices from panel (d) of Fig. 4.12 that the 100% EWSR $E0$ cross-sections obtained from DWBA calculations using the OMP provided by the TAMU group are higher than one would expect from naively comparing with the trend of the cross-sections as mass increases. Similarly, when looking at the parameters listed in Table 4.1, it can be noticed that the value of diffuseness

parameter a_1 seems small for ^{208}Pb when compared to the trend observed for the other nuclei. This called for an investigation into the sensitivity of the results based on the choice of the OMP. For this purpose the global OMP recently proposed by Su Xin-Wu and Han Yin-Lu [Xin15] were employed to determine the theoretical DWBA cross-sections for 100% EWSR calculations. Similar to the work described in [Sat97], the optical model parameters used in their study are the standard form of Woods-Saxon Eqs. (4.1) and (4.2). In addition, the real V_R and imaginary W_V potential depths are energy dependent and taken as polynomial form given by

$$V_R(E) = V_0 + V_1E + V_2E^2 + V_3\frac{(N - Z)}{A} + V_4ZA^{-\frac{1}{3}} , \quad (4.4)$$

and

$$W_V(E) = \max \{0, U_0 + U_1E\} , \quad (4.5)$$

where A , Z and N are the mass, proton and neutron number of the target nucleus, respectively. The incident alpha energy in MeV is denoted by E . The parameters that appeared in Eqs. (4.4) and (4.5) are listed in Table 4.5.

Using a beam energy of $E = 200$ MeV, the OMP obtained for the iThemba LABS data are listed in Table 4.6. Following the steps listed in Sect. 4.2.3, the Fraction $E0$ EWSR and $E0$ strength were extracted using the OMP listed in Table 4.6. The results obtained are shown in Figs. 4.18 - 4.23. It can be seen that for all the nuclei under investigation except ^{208}Pb , there is a little difference in $E0$ strength depending on which OMP are used. This again reveals that there are unresolved issues with the ^{208}Pb extracted Fraction $E0$ EWSR and $E0$ strength using the OMP from Ref. [You04].

Calculations for both folded OMP from Ref. [You04] and the global OMP [Xin15] were performed in order to compare with the angular distribution data for elastic scattering and the low-lying 3^- state $E_x = 2.61$ MeV in ^{208}Pb at 240 MeV, available from Ref. [Cla01]. Panels (a) and (b) of Fig. 4.24 indicate that DWBA calculations using the folded OMP are clearly superior to the global OMP calculations (see Fig. 4.25). It is, therefore, suggested that in future an attempt should be made to extract a different set of parameters

for the ^{208}Pb folded OMP, starting with a more realistic value for a_I and then varying the depth parameters V together with the imaginary depth parameter W to achieve a good fit for the elastic and low lying states in ^{208}Pb .

Table 4.5: Global optical model parameters taken from [Xin15].

Parameter	Value	Unit
V_0	175.0881	MeV
V_1	-0.6236	
V_2	0.0006	MeV ⁻¹
V_3	30.0000	MeV
V_4	-0.2360	MeV
U_0	-4.0174	MeV
U_1	0.1409	
r_V	1.4259	fm
a_V	0.5587	fm

Table 4.6: Optical Model Parameters obtained from using Eqs. (4.4) and (4.5) and parameters listed in Table 4.5.

Nucleus	V_R (MeV)	W_V (MeV)	r_V (fm)	a_V (fm)
^{24}Mg	73.3863	24.1626	1.4259	0.5587
^{28}Si	73.2800	24.1626	1.4259	0.5587
^{58}Ni	73.6960	24.1626	1.4259	0.5587
^{90}Zr	75.5950	24.1626	1.4259	0.5587
^{120}Sn	76.9768	24.1626	1.4259	0.5587
^{208}Pb	77.4481	24.1626	1.4259	0.5587

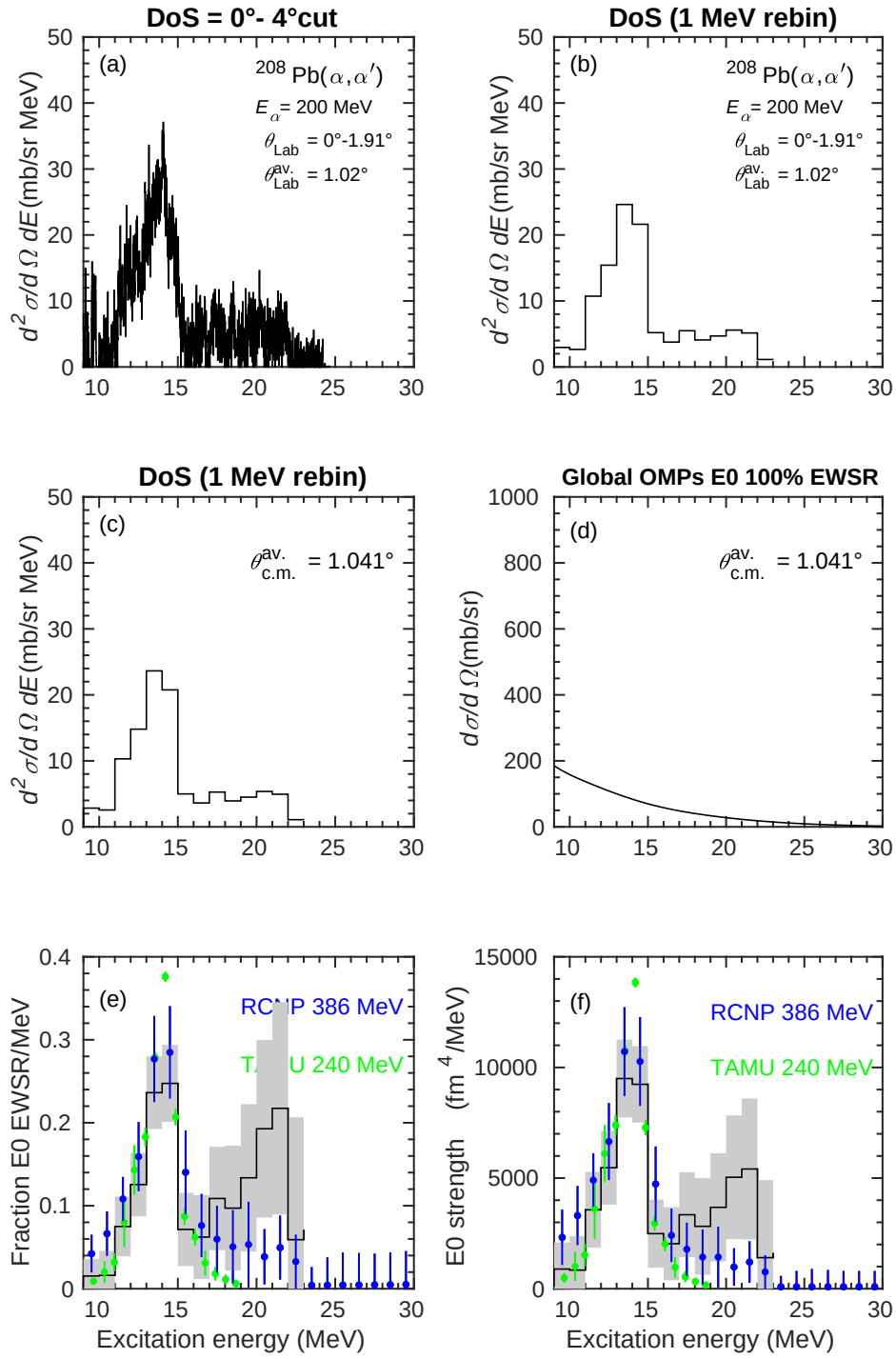


Figure 4.18: Same as Fig. 4.12 except for the use of global OMP instead of TAMU OMP.

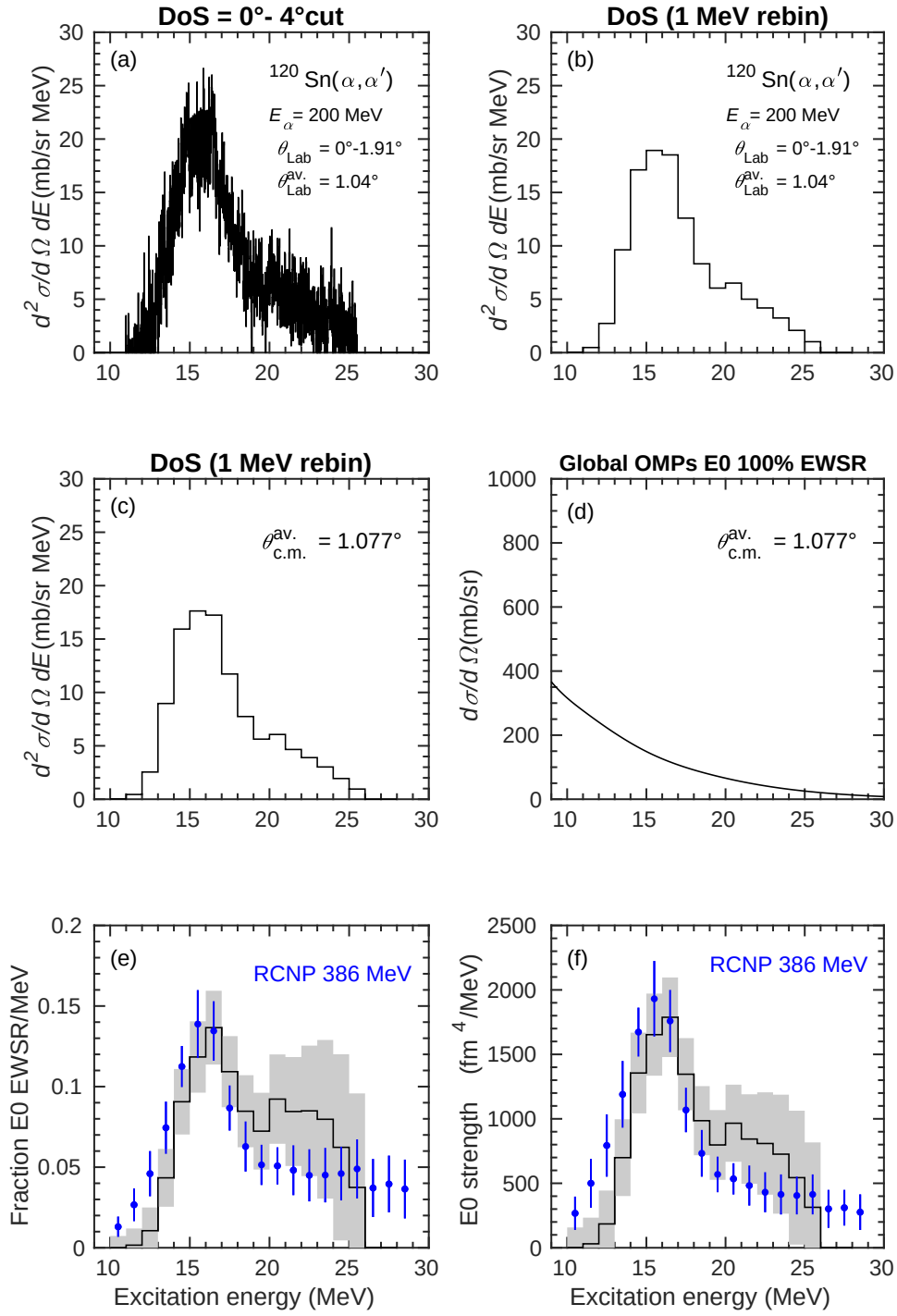


Figure 4.19: Same as Fig. 4.13 except for the use of global OMP instead of TAMU OMP.

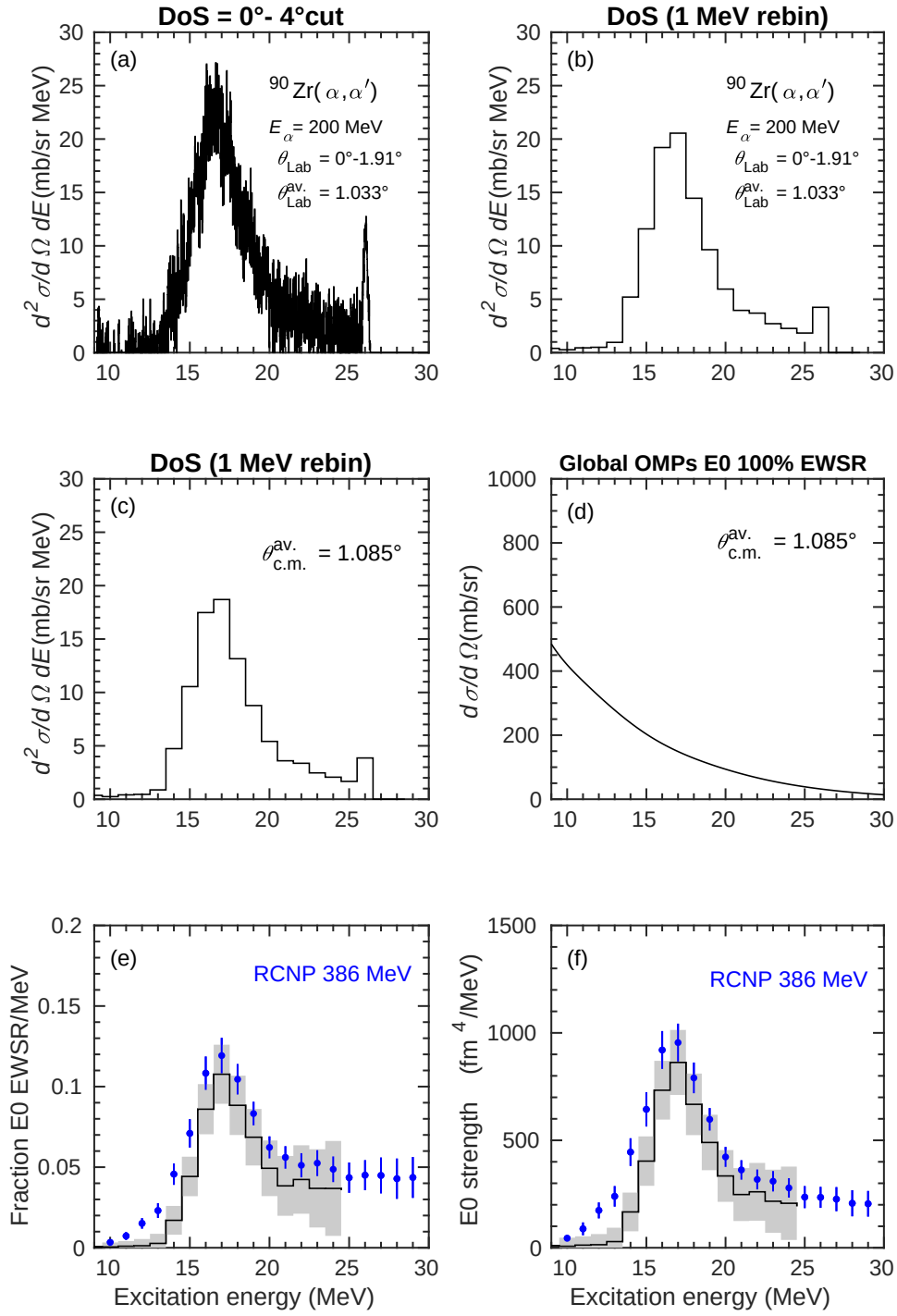


Figure 4.20: Same as Fig. 4.14 except for the use of global OMP instead of TAMU OMP.

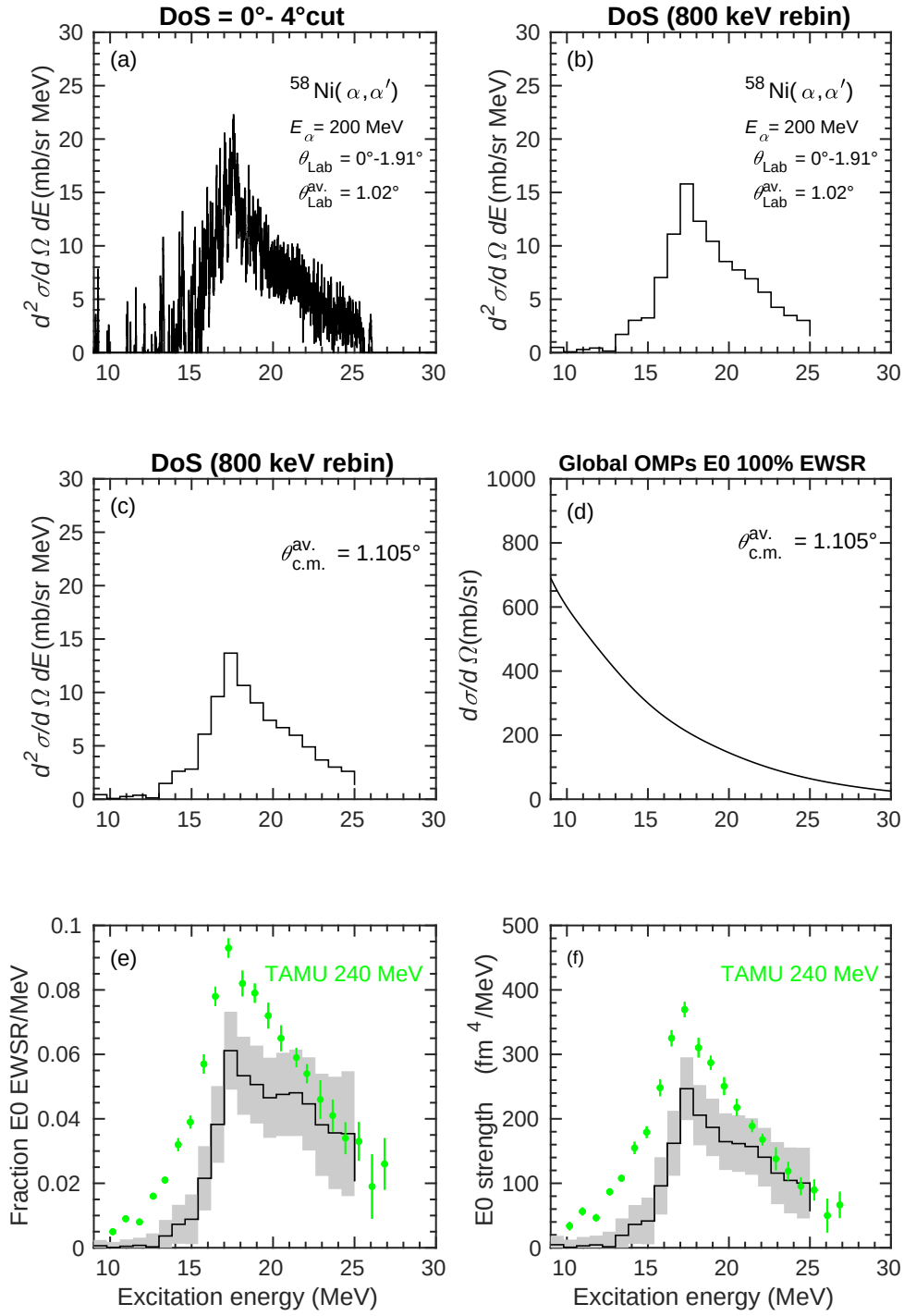


Figure 4.21: Same as Fig. 4.15 except for the use of global OMP instead of TAMU OMP.

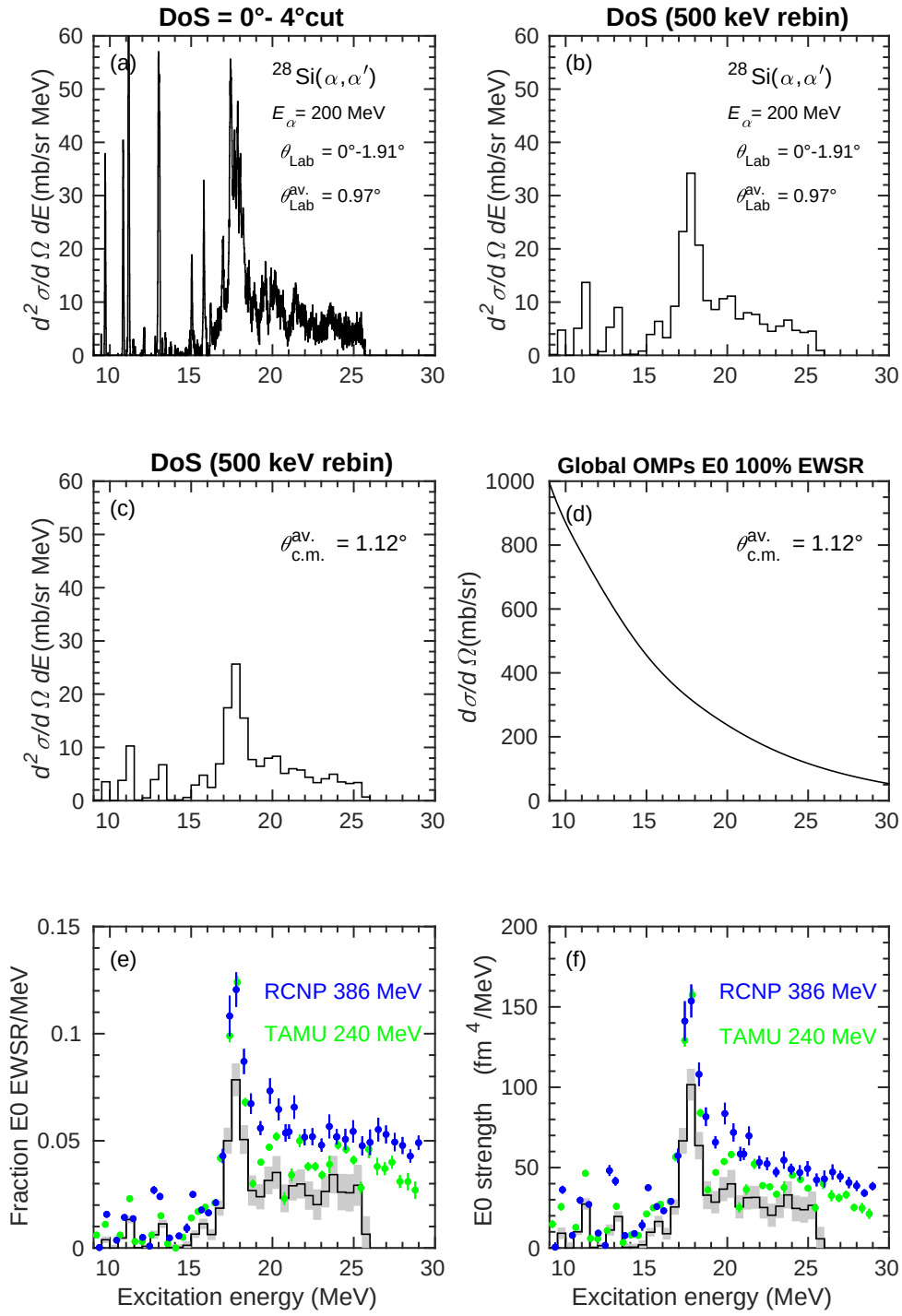


Figure 4.22: Same as Fig. 4.16 except for the use of global OMP instead of TAMU OMP.

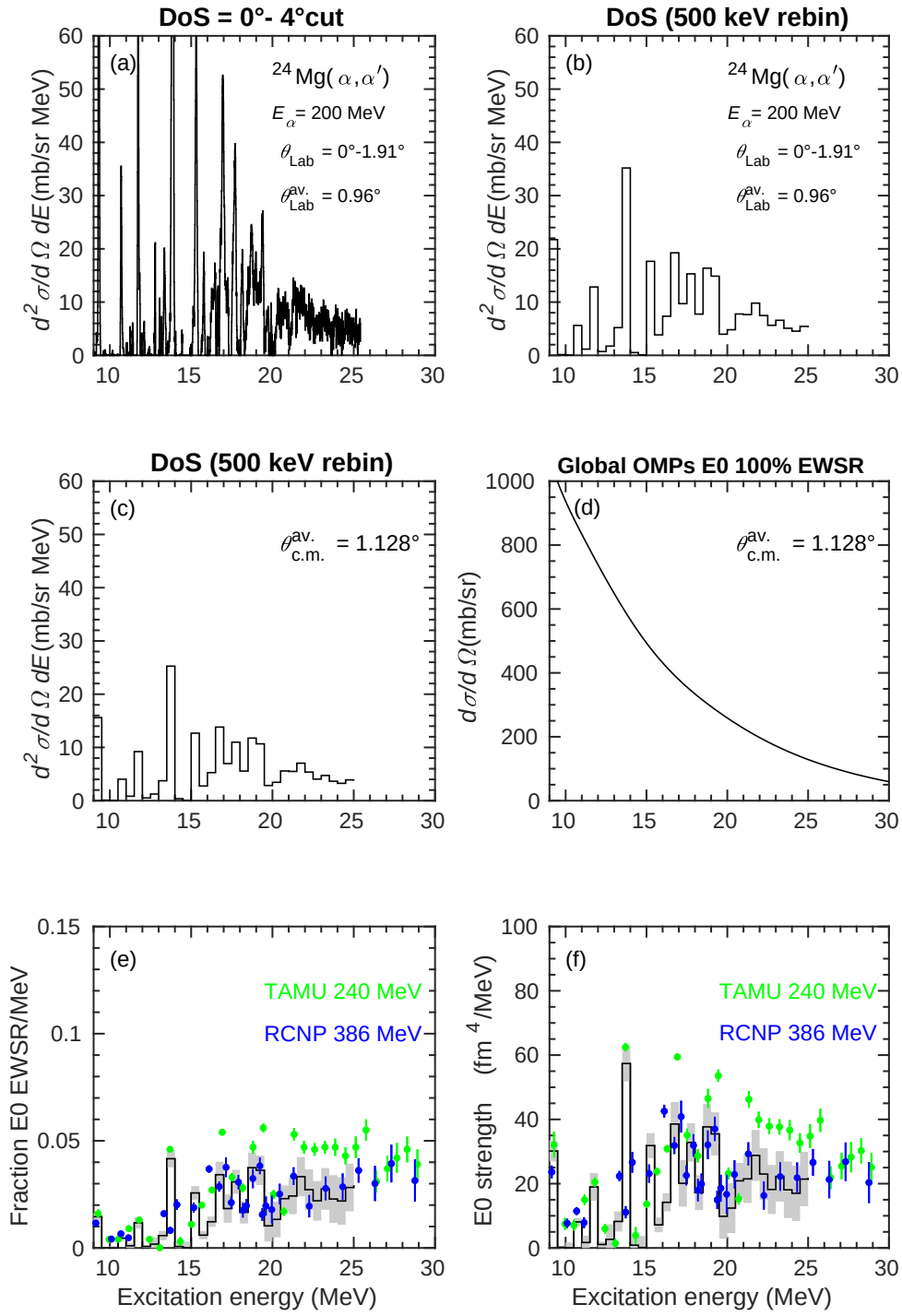


Figure 4.23: Same as Fig. 4.17 except for the use of global OMP instead of TAMU OMP.

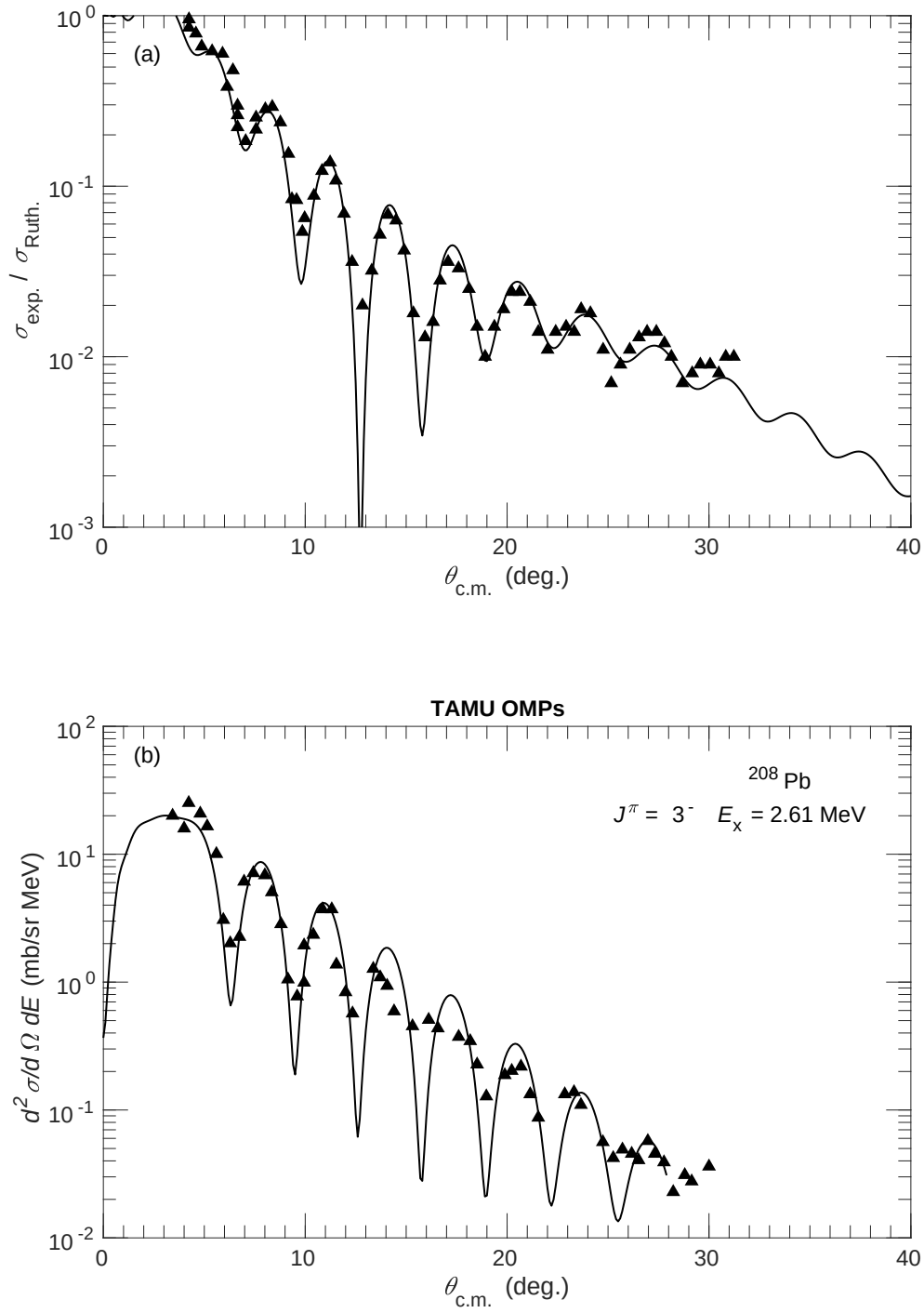


Figure 4.24: Angular distribution of the ratio of α elastic scattering cross-sections to Rutherford scattering (a) and double differential cross-sections of inelastic scattering of 3^- state (b) at $E_\alpha = 240 \text{ MeV}$ on ^{208}Pb (using TAMU OMP). The data shown are taken from [Cla01].

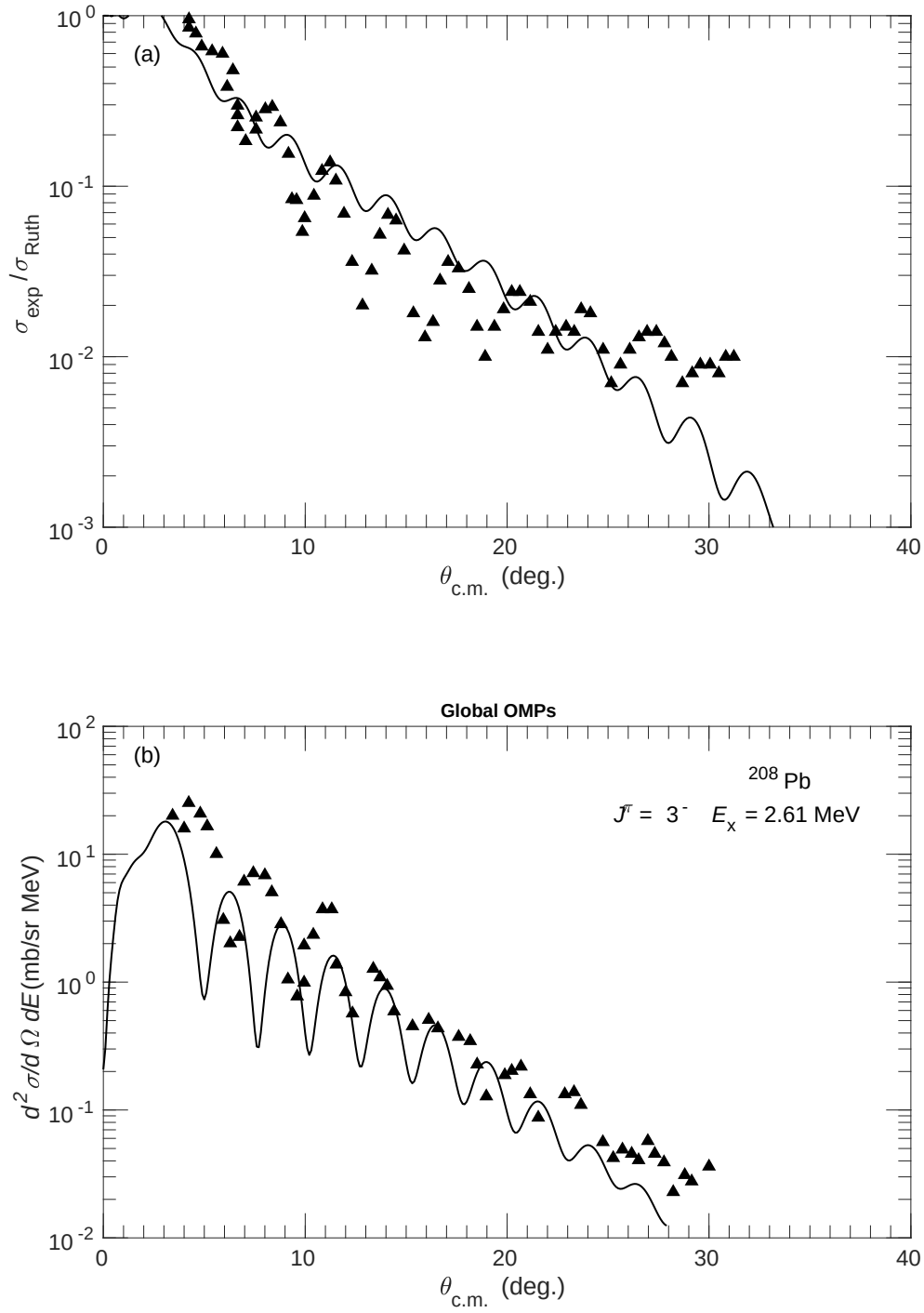


Figure 4.25: Same as Fig. 4.24 except for the use of global OMP instead of TAMU OMP.

In summary, considering the differences in beam energies, methods applied (MDA and DoS), experimental conditions, it can be seen that there is a

reasonable agreement in both peak positions and amplitudes of the $E0$ strength extracted in this work with those reported by the TAMU and RCNP groups. This then leads to the conclusion that the structure observed in the DoS spectrum arises essentially from the $E0$ strength.

4.3 Overview of iTL ISGMR strength distributions in the studied nuclei

In this section, an overview of the strength distribution in the nuclei under investigation is presented. A modified DoS procedure based on the subtraction of IVGDR contribution is presented in Sect. 4.3.1 for ^{90}Zr as a test case. The properties (centroid and width) of the ISGMR are presented in Sect. 4.3.2 followed by the comparison with TAMU and RCNP groups results from their MDA analysis. The experimental ISGMR are compared with the theoretical calculations in Sect. 4.3.3.

4.3.1 Discussion of modified DoS: case of ^{90}Zr

As stated earlier, the application of the DoS technique does not ultimately remove the IVGDR contaminant which becomes important for medium and heavy mass nuclei due to large Coulomb contribution to the cross-sections. The reason for this statement is as follows: the DoS method relies on subtraction of all other multipoles from the zero-degrees spectrum using a finite angle cut where the $L = 0$ contribution is essentially not present. Because the IVGDR contribution is also forward peaked (although not as strongly as the GMR), only approximately 20% of its strength is present in the finite angle cut. Combined with the fact that the IVGDR strength overlaps in excitation energy with the GMR strength, while the isoscalar $L = 1, 2, 3$ contributions are found at slightly lower or higher excitation energies than the ISGMR (see Fig. 1.1), the full IVGDR contribution can, therefore, not be subtracted from the zero-degrees spectrum by using the standard DoS method.

Usually, the IVGDR is subtracted out of the experimental data prior to the

MDA analysis. This is achieved by using photonuclear data in conjunction with DWBA calculations based on the Goldhaber-Teller model [Gup18, Gup16a, You04]. In principle, photonuclear data are available for ^{208}Pb and ^{120}Sn , but not yet attempted to use these data, hence, further analysis will be carried out on heavy nuclei as an ongoing project on this topic. The following method was used to account for the IVGDR in our data which consisted of reproducing the results from MDA analysis in [Gup18] on ^{90}Zr at 15 MeV, 20 MeV and 25 MeV where multipolarities up to $L = 6$ including the IVGDR distribution as a function of scattering angle were considered. Further, using the same fractional strength for all the multipolarities including the IVGDR and lowering the beam energy from 386 MeV to 200 MeV, DWBA calculations were performed and an estimation of the amount of IVGDR contribution present in the iThemba LABS data could be retrieved (see Fig. 4.26). It can be seen that the IVGDR contribution is reduced by a factor of 5.3 just by reducing the incident alpha-particle energy.

Therefore, noting that the IVGDR contribution from the results in [Gup18] amounts to approximately 17.3% of the measured cross-sections at the peak of the GMR, the IVGDR contaminant reduces to $\simeq 5.5\%$ of the measured cross-sections at 0° in the iThemba LABS data. This is the strength of the IVGDR contribution estimated in the zero-degrees spectrum. In addition, present results at 200 MeV seem to indicate that all other multipoles except $L = 0$ (not including the IVGDR) lead to a dip of about 20% in the 0° measured cross-sections spectrum angle region in comparison to that in the angle cut from the 4° measured cross-sections. By inspection, this is due to the multipole angular distributions being “pulled out” from 0° at 200 MeV in comparison to 386 MeV, which leads to a corresponding drop in strength in the measured angle region $\theta_{\text{Lab}} = 0^\circ$ to 1.91° . The double differential cross-sections of the IVGDR as a function of excitation energy was estimated using the non-standard Lorentzian equation [Car74, Car71] given by

$$\sigma(E_x) = \frac{\sigma_0}{1 + \left[(E_x^2 - E_0^2)^2 / E_x \Gamma^2 \right]}, \quad (4.6)$$

where σ_0 , E_0 and Γ are the peak, the centroid and the width of the resonance, respectively. The excitation energy range considered here is denoted by E_x . The centroid and width are taken from the Lorentzian parameters given in [Gup18]. The strength σ_0 of the IVGDR is estimated as explained above.

Hence, by taking into account the dip observed and the fact that 20% of the IVGDR was already removed through the subtraction of the 4° angle cut spectrum from the 0° spectrum, both the IVGDR and the 4° angle cut spectra were scaled down accordingly prior to their subtraction from the 0° measured cross-sections. The results of such process are shown in Fig. 4.27.

Following the steps described in Sect. 4.2.3, the Fraction $E0$ EWSR and $E0$ strength were obtained from the modified DoS spectrum. The results are shown in Fig. 4.28. The errors bars displayed as grey bands include 15% of the cross-sections of the estimated IVGDR contributions in addition to the uncertainties listed previously in Sect. 3.7.7.

By comparing Figs. 4.14 and 4.28, it can be seen that at and below 15 MeV the discrepancies between the results from this work and the RCNP group results have decreased somewhat when using the method of IVGDR subtraction and scaling of the finite angle data. It should be emphasised though that in order to scale the finite degree angle cut, one requires knowledge of the fractional strengths of the different multipolarities. It, therefore, has the potential effect of biasing the present extraction of $E0$ strengths towards the results of the study from which the fractional strengths originate. Furthermore, the required scaling parameter clearly needs to be excitation energy sensitive, which it is not at present.

Overall, however, the results confirm the conclusion in [Shl87] where it was shown that the IVGDR cross-sections may be safely neglected for beam energy less than 200 MeV in extracting the $E0$ strength from the ISGMR.

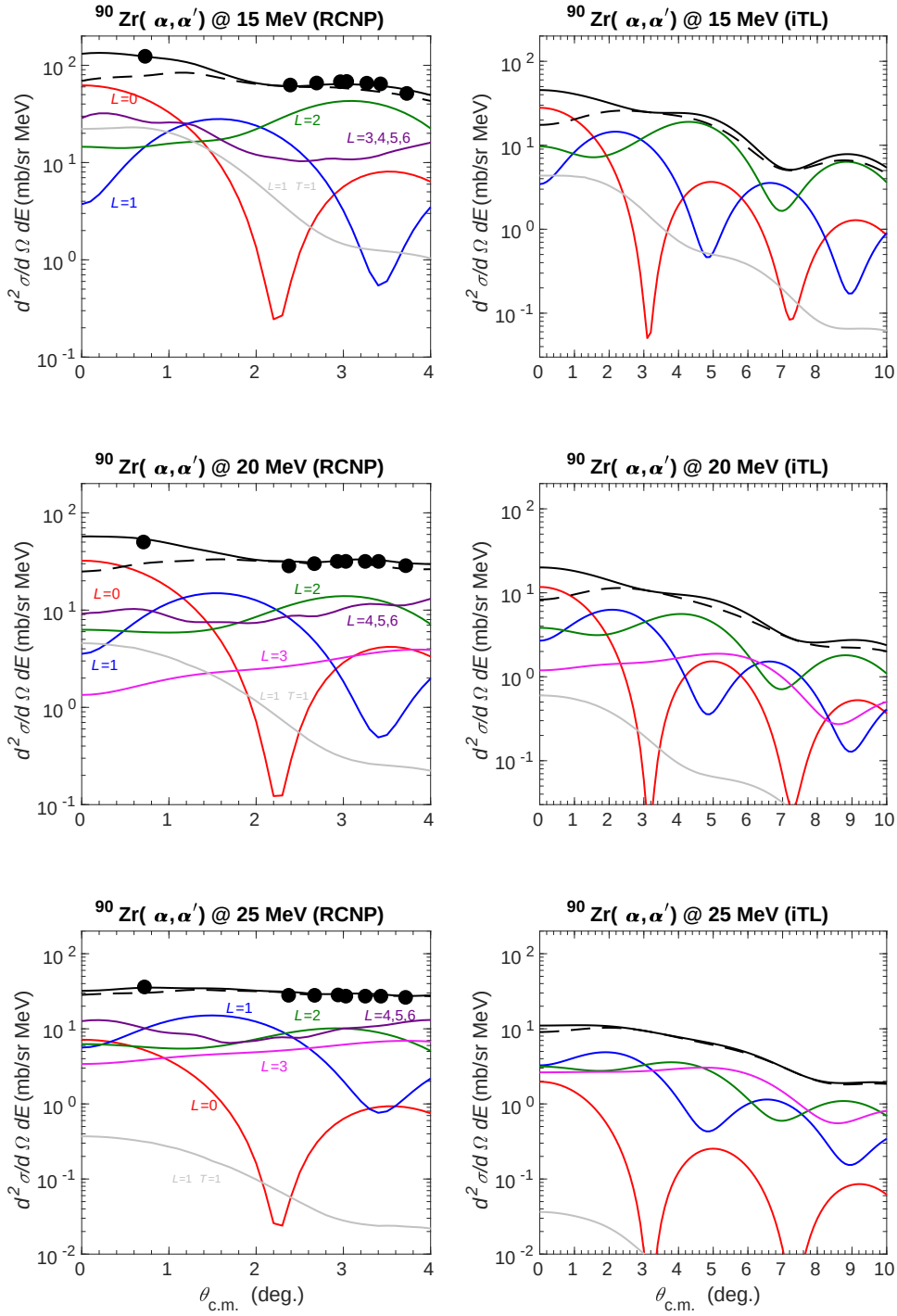


Figure 4.26: MDA analysis results for ^{90}Zr (left panels) and DWBA calculations at $E_{\text{Lab}} = 200$ MeV (right panels). The black dashed line represents the sum of all multipoles except $L = 0$ and the IVGDR. The data shown on the left panels are taken from [Gup18].

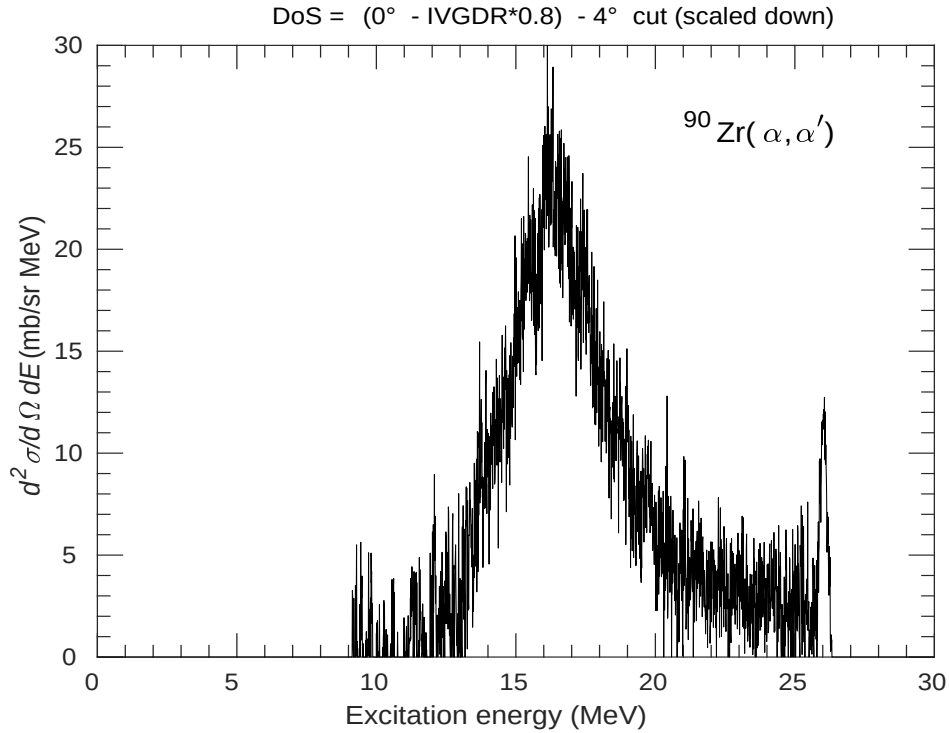
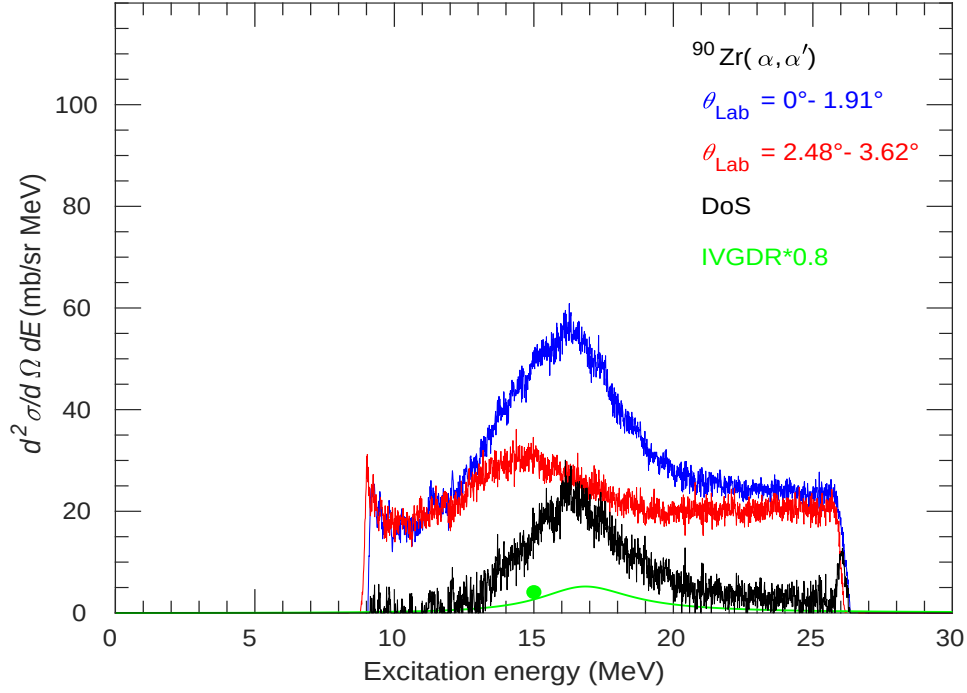


Figure 4.27: Double differential cross-section for $^{90}\text{Zr}(\alpha, \alpha')$. The blue spectrum represents the data acquired at $0^\circ \leq \theta_{\text{Lab}} \leq 1.91^\circ$, the red represents the angle cut spectrum at $2.48^\circ \leq \theta_{\text{Lab}} \leq 3.62^\circ$ (scaled down) and the green represents the estimated IVGDR contribution. The black spectrum represents the difference spectrum (top and bottom panels). The green dot indicates the position of the peak of the IVGDR as taken from the RCNP MDA analysis results.

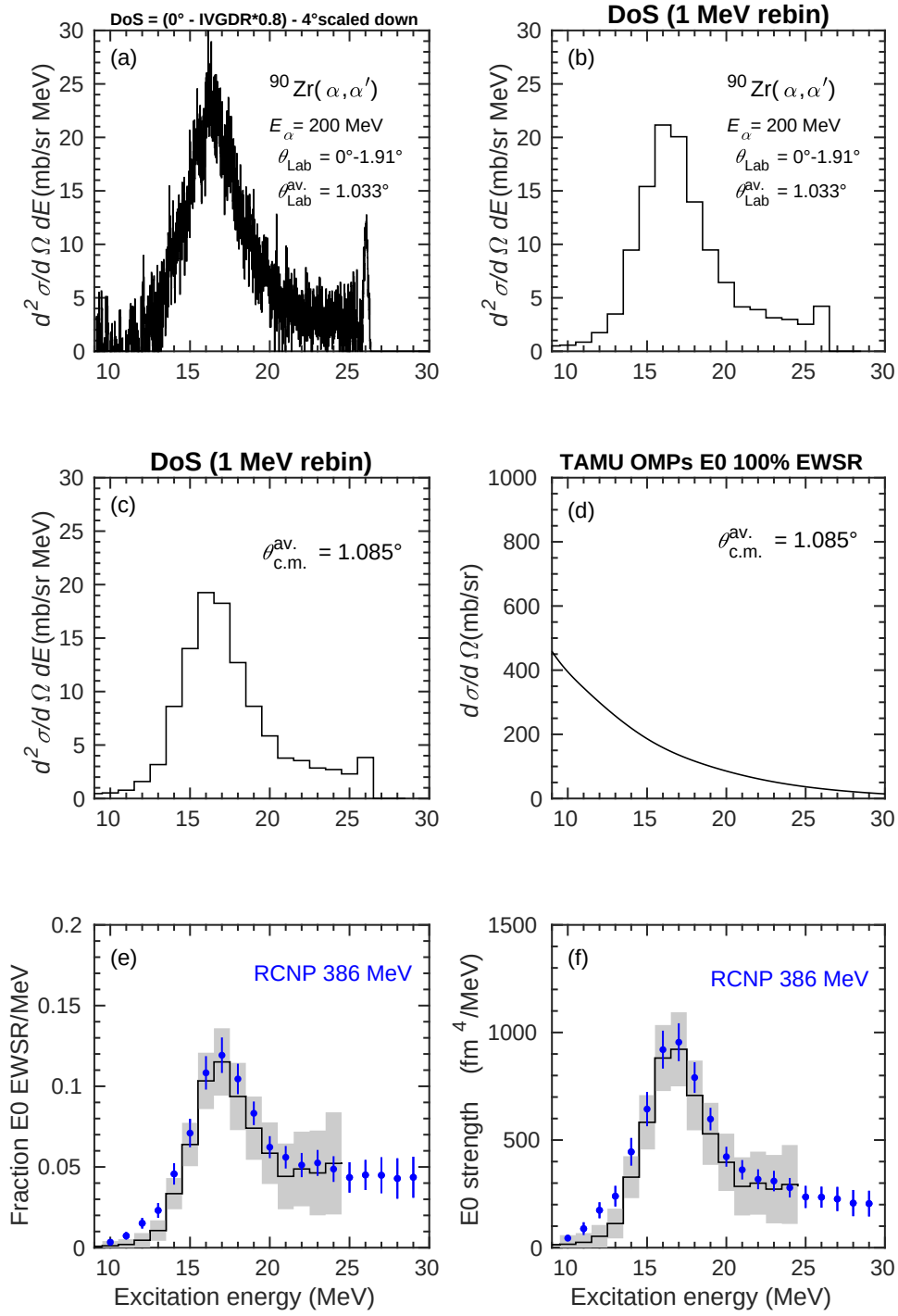


Figure 4.28: Same as Fig. 4.14 but the IVGDR-free difference spectrum is used here. Error bars are displayed as grey bands. Data points shown as blue filled circles with vertical error bars are taken from [Gup18].

4.3.2 Energy and width of the ISGMR in the studied nuclei

In the last two decades, extensive studies were made using the MDA method to determine the isoscalar giant monopole $L = 0$ ($E0$ strength distribution) and other higher multipoles, namely $L = 1, 2, 3, \dots$ across the periodic table and via (α, α') scattering [Gup18, Pea16, Gup16a, Pat13, Li10, Lui06, You04]. In the present work, the $E0$ strength distribution was accessed using the DoS technique. This method enables the almost complete removal from the 0° spectrum where $L = 0$ dominates, of the underlying background from all other multipoles with $L \geq 1$. The extracted $E0$ strength distribution in all the nuclei under investigation is shown in Fig. 4.29. It can be seen that the ISGMR in the medium to heavy mass nuclei consists of a Lorentzian bump with a maximum located at an excitation energy value following the trend of $A^{-\frac{1}{3}}$ dependence. This is depicted by the Lorentzian fit shown as the blue solid line in the four top panels in Fig. 4.29.

In addition, for all the nuclei under investigation, it can be seen that a lot of fine structure is present across the excitation energy range. This then reveals that the ISGMR is not just a Lorentzian structure but it also consists of noticeable fine structure for nuclei across the periodic table. For the centroid and width of the ISGMR to be extracted from the $E0$ strength distribution in ^{208}Pb , ^{120}Sn , ^{90}Zr and ^{58}Ni , the spectra displayed in Fig. 4.29 were rebinned to 1 MeV⁷ so that the fine structure is smoothed out. As such, a constrained Lorentzian⁸ fit was employed (see Figs. 4.30 and 4.31) to extract the centroid and width values with their corresponding errors which represent the uncertainty due to the fitting of the $E0$ strength. The values obtained in this work are comparable and agree within errors with the previously published results from both TAMU and RCNP groups from their MDA analysis. These are listed in Table 4.7 where other nuclei including those also studied at iThemba LABS⁹ in recent years have been added.

⁷For internal consistency, 800 keV bin size was used for the ^{58}Ni nucleus as in Figs. 4.15 and 4.21.

⁸The low excitation energy region is fitted well while only few data points in the region above the peak of the ISGMR were included.

⁹These are ISGMR experimental data taken at iThemba LABS with the results presented in [Olo20] and [Moo19] from the DoS technique on $^{40,42,44,48}\text{Ca}$ and Discrete Wavelet Transform (DWT) background-subtraction technique on ^{58}Ni and ^{90}Zr , respectively.

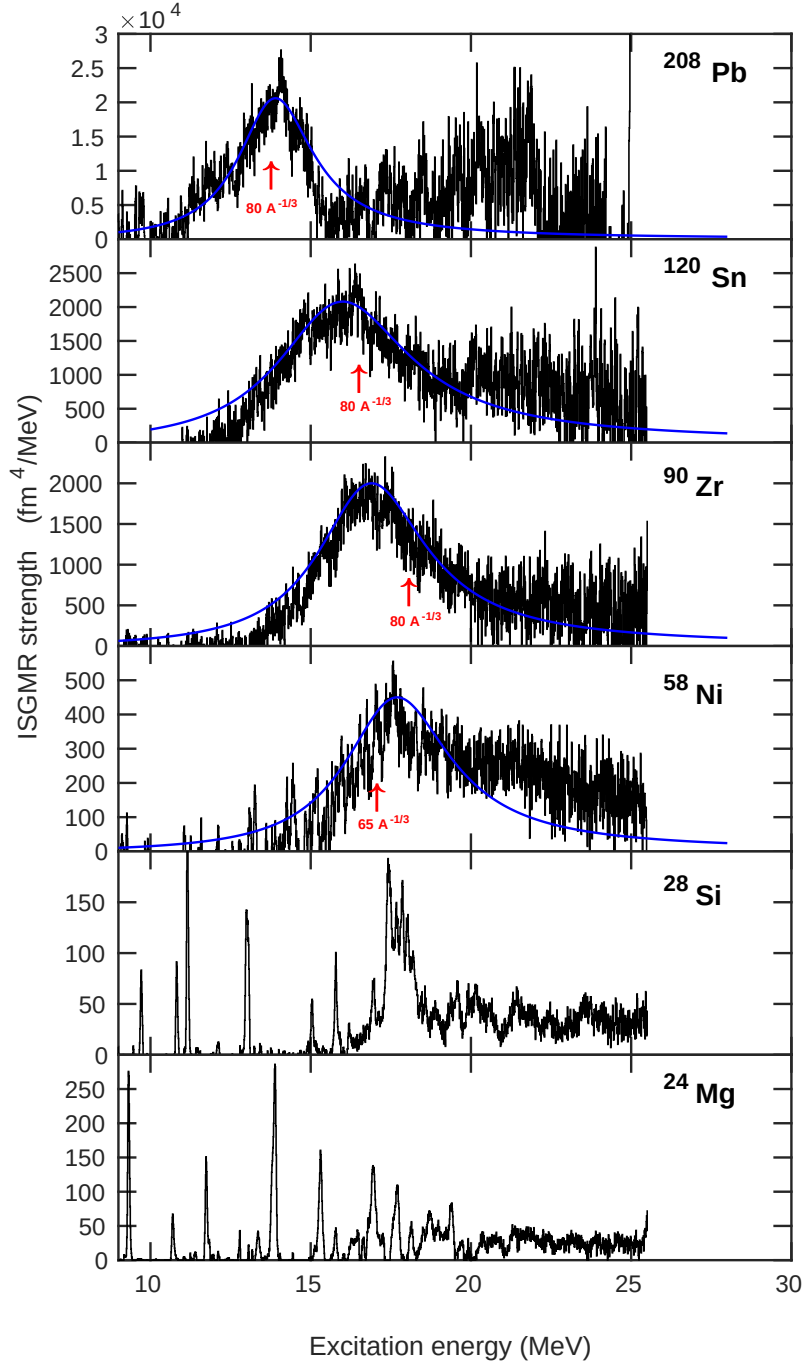


Figure 4.29: ISGMR strength distribution for $^{208}\text{Pb}(\alpha, \alpha')$, $^{120}\text{Sn}(\alpha, \alpha')$, $^{90}\text{Zr}(\alpha, \alpha')$, $^{58}\text{Ni}(\alpha, \alpha')$, $^{28}\text{Si}(\alpha, \alpha')$ and $^{24}\text{Mg}(\alpha, \alpha')$. The blue solid line represents a Lorentzian fit to the experimental data. The red arrows indicate the phenomenological $A^{-\frac{1}{3}}$ expression of the centroid.

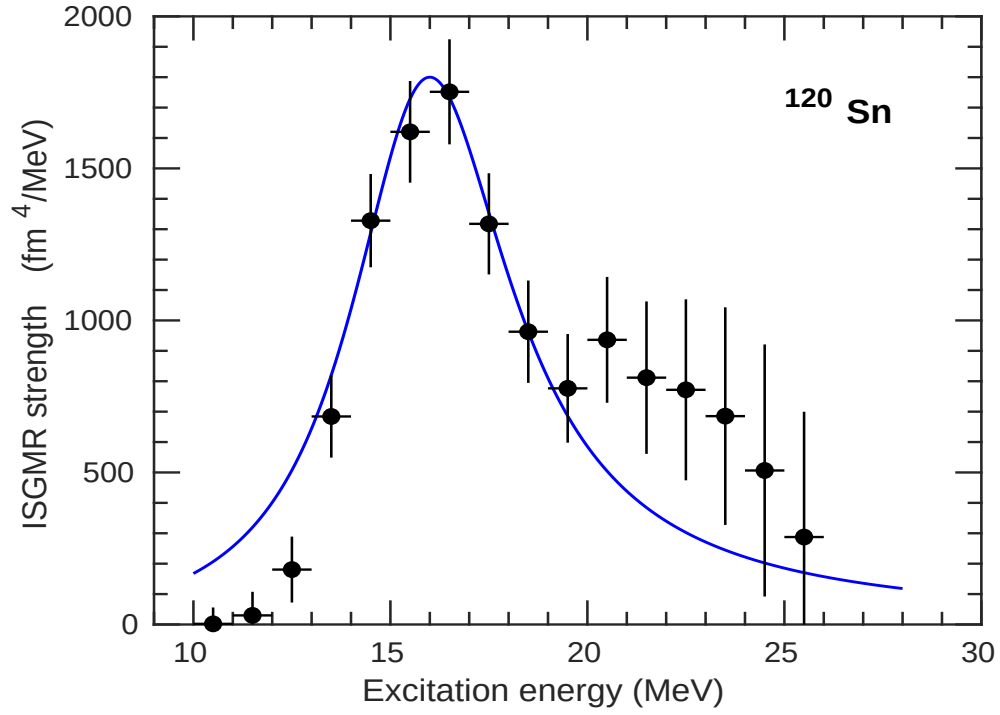
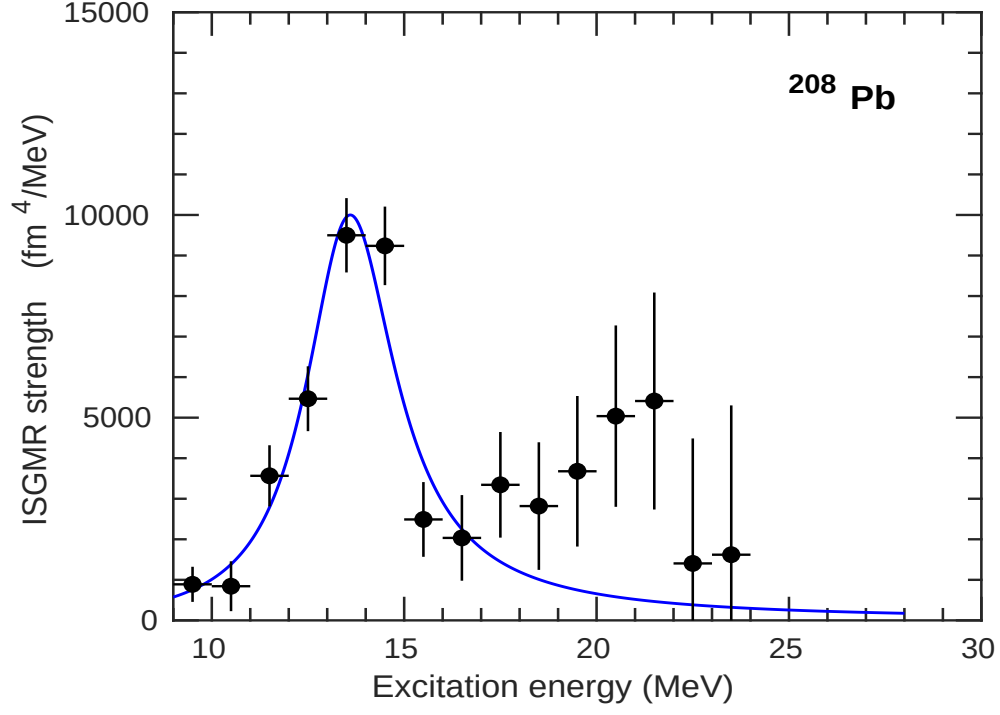


Figure 4.30: ISGMR strength distribution for $^{208}\text{Pb}(\alpha, \alpha')$ (top panel) and $^{120}\text{Sn}(\alpha, \alpha')$ (bottom panel). The blue solid line represents a Lorentzian fit to the experimental data. Vertical error bars are calculated as discussed in the last paragraph of Sect. 4.2.3 and horizontal error bars are half of an energy bin width.

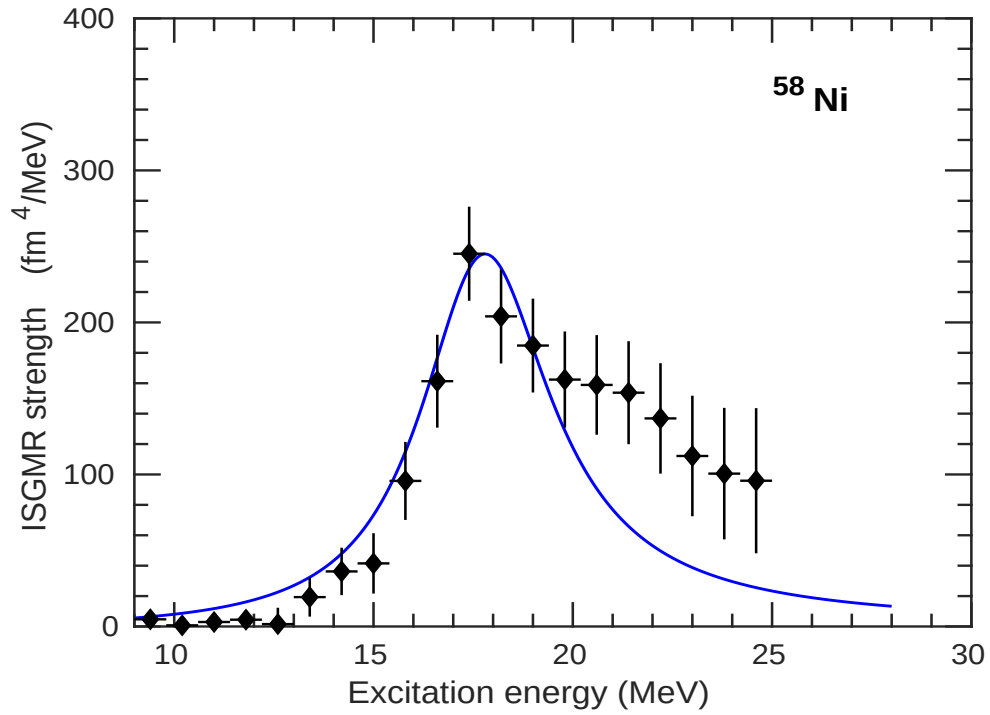
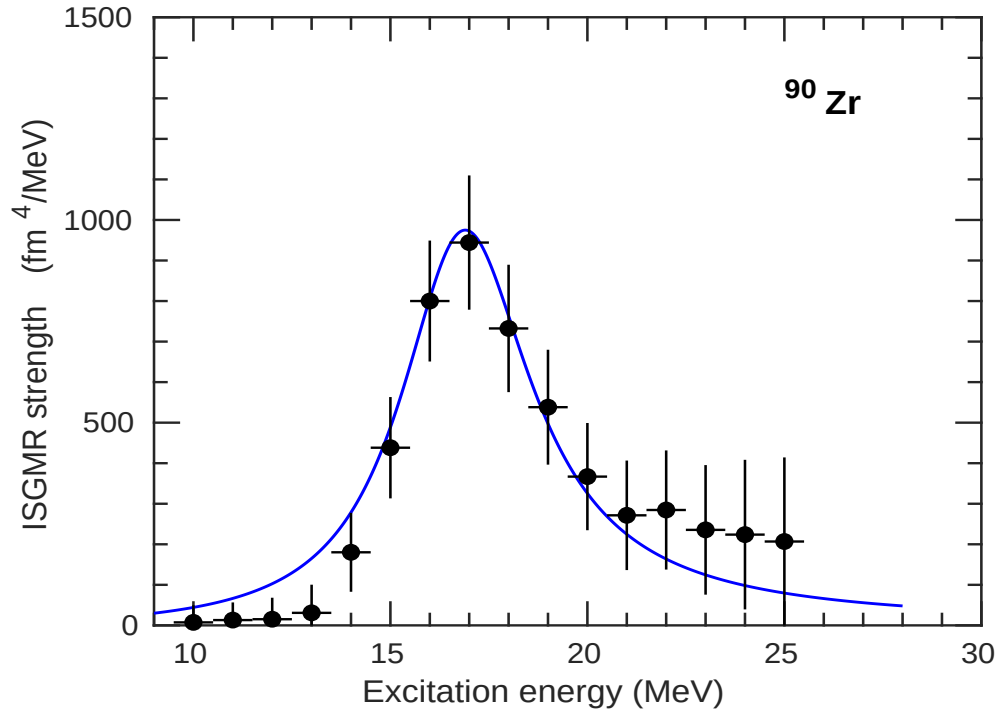


Figure 4.31: Same as Fig. 4.30 but for ⁹⁰Zr and ⁵⁸Ni.

Table 4.7: Lorentzian-fit parameters for the ISGMR as well as the $E0$ energy weighted sum rule (EWSR) strength located in nuclei across the periodic table. Results from this study (fits as in Figs 4.30 and 4.31) are listed. Results from selected previous (α, α') measurements at iThemba LABS, RCNP and TAMU are also presented for comparison.

Nucleus	Centroid (MeV)	Width (MeV)	EWSR (%)	Reference
^{208}Pb	13.60 ± 0.20	2.85 ± 0.20	85 ± 25	Present work
	13.60 ± 0.10	3.10 ± 0.40	147 ± 18	[Pat14]
	13.70 ± 0.10	3.30 ± 0.20	149 ± 18	[Pat13]
	13.96 ± 0.20	2.88 ± 0.20	99 ± 15	[You04]
^{124}Sn	14.80 ± 0.20	4.50 ± 0.50	105 ± 5	[Li10]
	15.34 ± 0.13	5.00 ± 0.53	106 ± 20	[Lui04]
^{120}Sn	15.70 ± 0.40	5.00 ± 0.40	107 ± 10	Present work
	15.40 ± 0.20	4.90 ± 0.50	108 ± 7	[Li10]
^{116}Sn	15.80 ± 0.10	4.10 ± 0.30	99 ± 5	[Li10]
	15.85 ± 0.20	5.27 ± 0.25	112 ± 15	[You04]
^{92}Mo	16.85 ± 0.10	4.44 ± 0.26	64.2 ± 6	[Gup18]
	16.80	4.00	107 ± 13	[You13]
^{92}Zr	16.25 ± 0.10	5.33 ± 0.12	78.9 ± 7	[Gup18]
	16.60	4.40	103 ± 12	[You13]
^{90}Zr	16.90 ± 0.20	4.05 ± 0.20	82 ± 13	Present work
	15.90 ± 0.20	4.30 ± 0.80	—	[Moo19]
	16.76 ± 0.12	4.96 ± 0.32	74.7 ± 9	[Gup18]
	16.55 ± 0.08	4.20 ± 0.30	95 ± 6	[Gup16a]
	17.10	4.4	106 ± 12	[Lui15]
^{58}Ni	17.80 ± 0.50	4.00 ± 0.60	65 ± 17	Present work
	16.94 ± 0.20	4.00 ± 0.80	—	[Moo19]
	18.14 ± 0.14	7.40 ± 0.13	85 ± 13	[Lui06]
^{48}Ca	17.90 ± 0.10	5.80 ± 0.20	—	[Olo20]
^{44}Ca	17.20 ± 0.10	6.00 ± 0.20	—	[Olo20]
^{42}Ca	17.30 ± 0.10	6.00 ± 0.20	—	[Olo20]
^{40}Ca	18.00 ± 0.10	6.70 ± 0.30	—	[Olo20]

4.3.3 Comparison of experimental ISGMR and theoretical calculations in the studied nuclei

Theoretical calculations of the $E0$ strength functions were performed using the PPC model discussed in Sect. 2.3.2 for $^{208}\text{Pb}(\alpha, \alpha')$, $^{120}\text{Sn}(\alpha, \alpha')$, $^{90}\text{Zr}(\alpha, \alpha')$ and $^{58}\text{Ni}(\alpha, \alpha')$. In order to allow a direct comparison between theory and experiment, the calculated $E0$ strength distributions were smoothed with a Gaussian function of Full Width Half Maximum (FWHM) of 70 keV comparable to the energy resolution obtained during experiments. The results are shown in the bottom panels of Figs. 4.32 - 4.35.

The PPC calculations reveal noticeable fine structure and the position of the main $E0$ strength seems to agree more or less with the experimental data. There is a shift of the centroid to the higher excitation energy region for ^{208}Pb and ^{90}Zr where extra $E0$ strength is calculated above the ISGMR peak for the latter. For the ^{58}Ni nucleus, there is a good agreement between the position of the centroid while for the ^{120}Sn , the theoretical predictions yield much broader ISGMR than the experimental results. The parametrisation SLy4 has a large incompressibility parameter $K = 229.9$ MeV. This is one of the reasons why PPC sometimes overestimates the ISGMR energy [Cha98]. These are shown in the top panels of Figs. 4.32 - 4.35 where 2 MeV wide Lorentzian folded PPC result overlays the experimental data. The experimental data for ISGMR in the prolate ^{24}Mg and oblate ^{28}Si are best described by the Skyrme force SkP $^\delta$ with the low compressibility $K = 202$ MeV. The narrow ISGMR peak at 13.5 MeV in ^{24}Mg and 17.5 MeV in ^{28}Si as well as the integral ISGMR strengths are well reproduced. Hence, the present study suggests low incompressibility for light deformed nuclei. Further the ISGMR strength is separated into a narrow peak at 13 – 18 MeV and broad structure at 19 – 26 MeV. Comparison of the ISGMR and $K = 0$ branch of ISGQR strength distributions justifies that the narrow peak appears from the deformation induced coupling between the monopole and quadrupole modes [Gup18, Kva16]. The broad structure is the main part of the ISGMR and SkP $^\delta$ [Dob95] calculations describe both parts of the ISGMR well. These results are shown in Figs. 4.36 and 4.37.

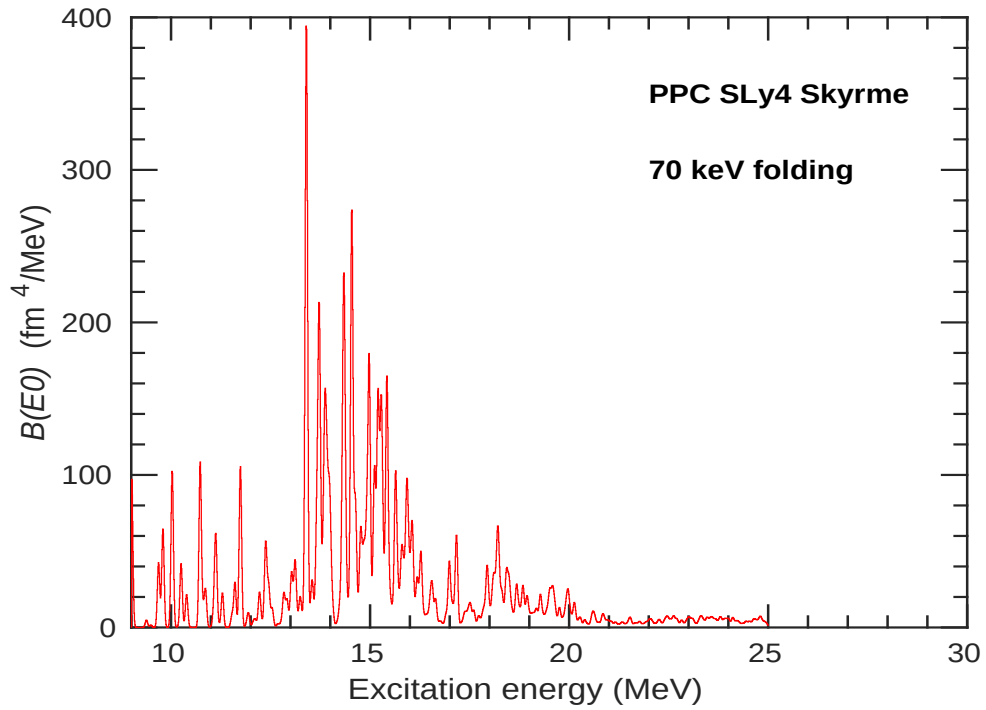
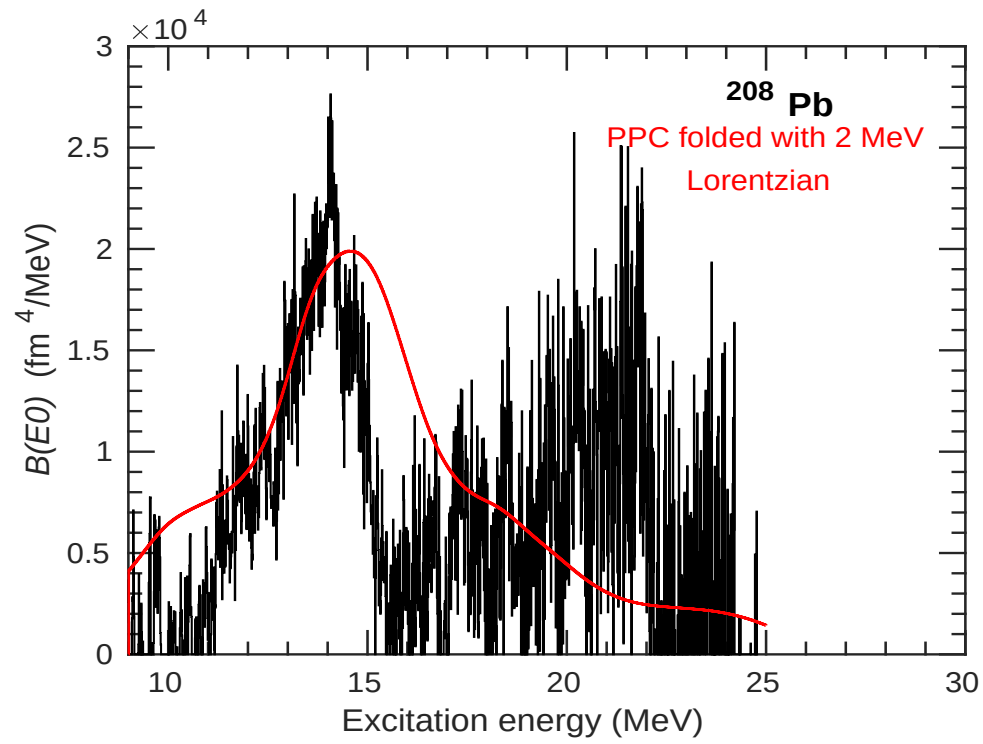


Figure 4.32: ISGMR strength distribution for ^{208}Pb (see Fig. 4.12) overlaid with PPC result (in red) folded with 2 MeV Lorentzian (top panel). Strength functions obtained via the PPC model (bottom panel).

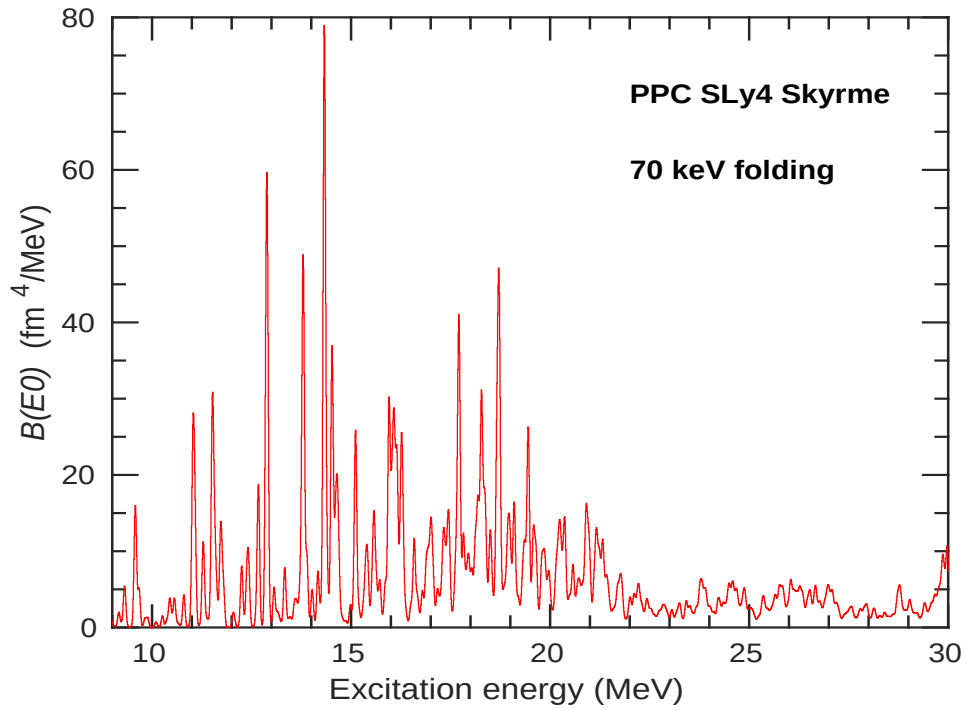
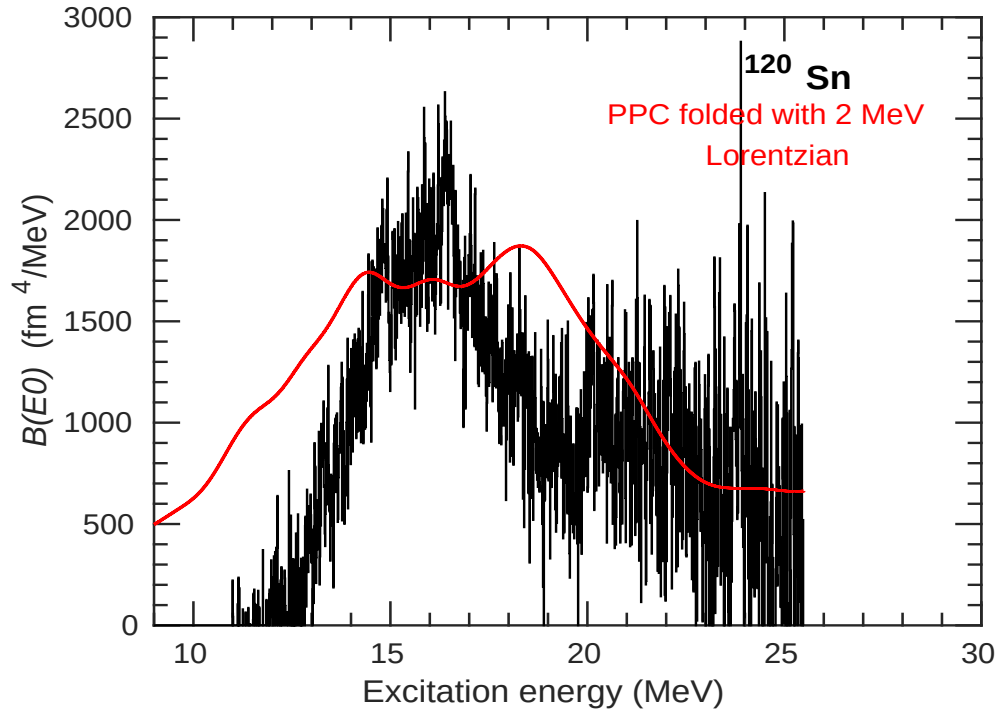


Figure 4.33: ISGMR strength distribution for ^{120}Sn (see Fig. 4.13) overlaid with PPC result (in red) folded with 2 MeV Lorentzian (top panel). Strength functions obtained via the PPC model (bottom panel).

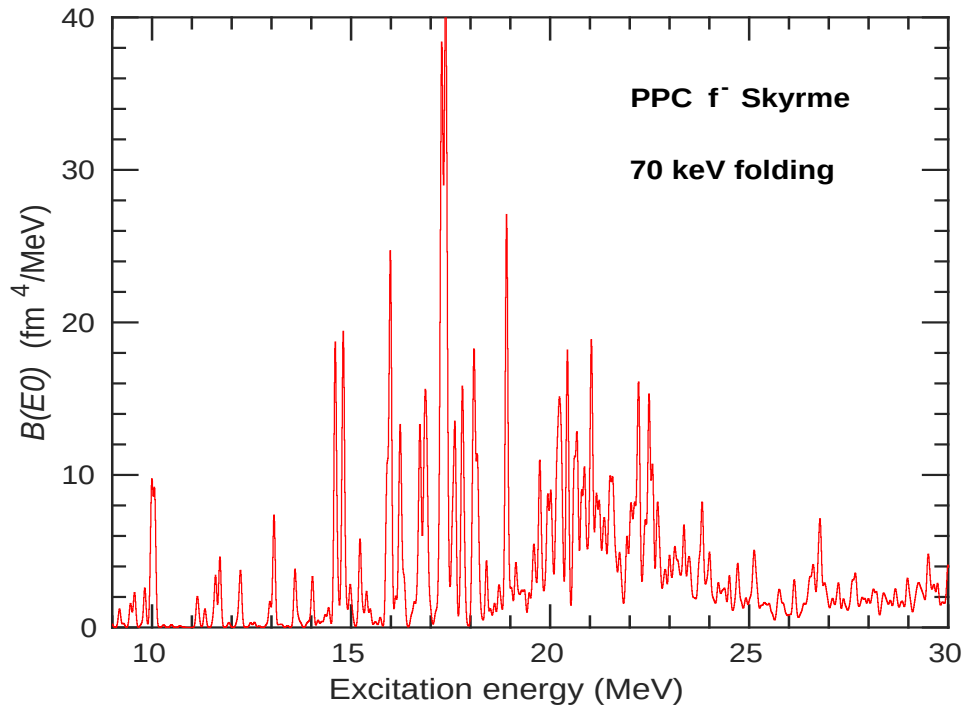
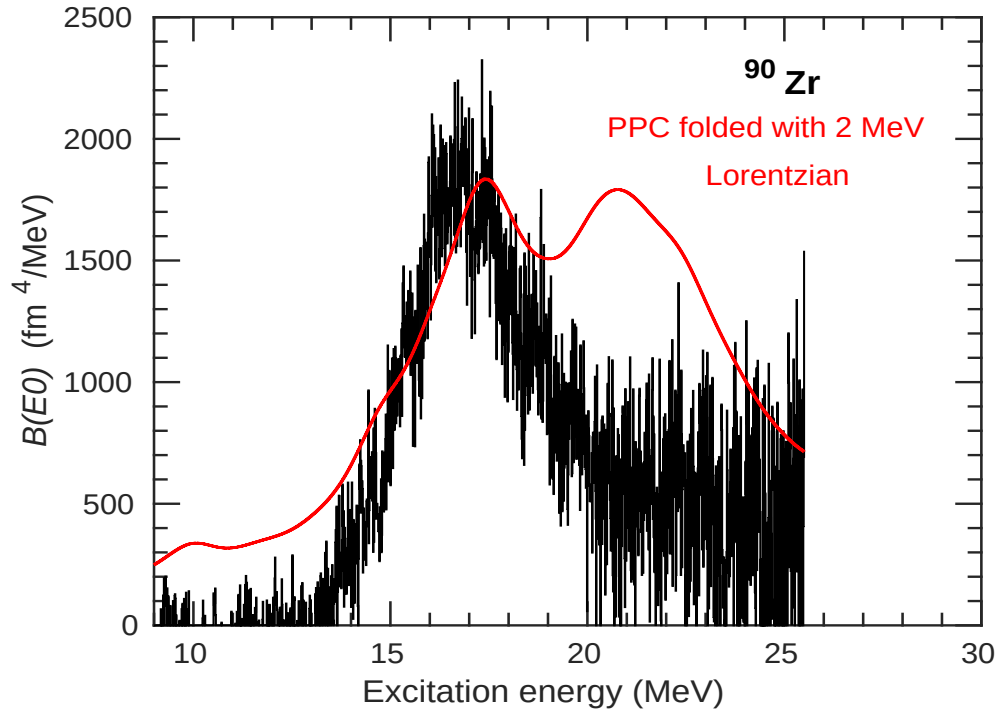


Figure 4.34: ISGMR strength distribution for ^{90}Zr (see Fig. 4.14) overlaid with PPC result (in red) folded with 2 MeV Lorentzian (top panel). Strength functions obtained via the PPC model (bottom panel).

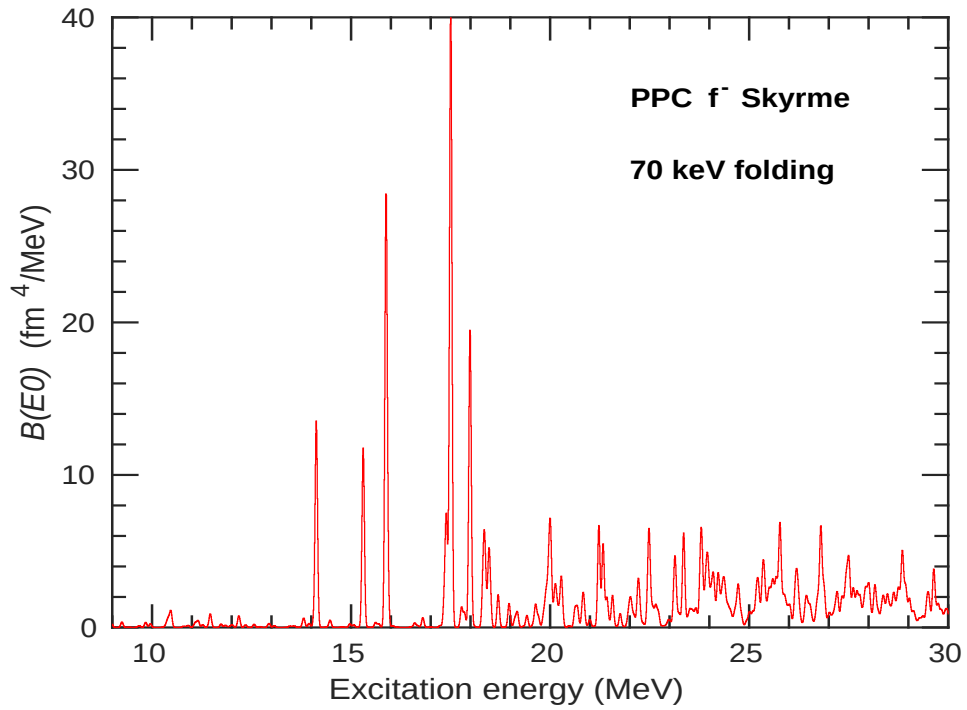
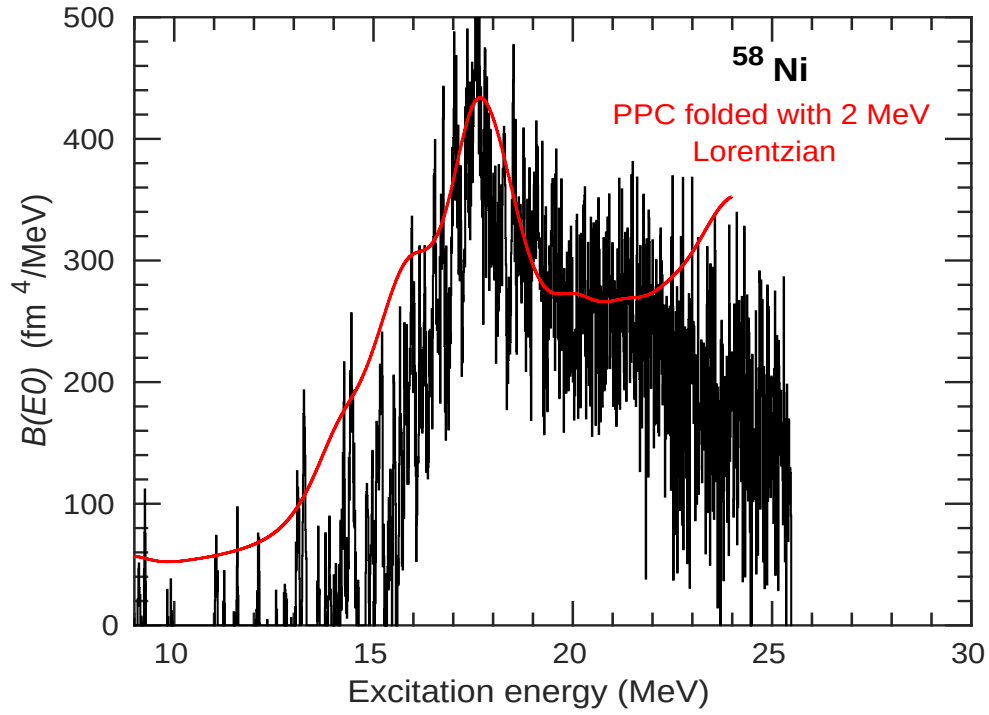


Figure 4.35: ISGMR strength distribution for ^{58}Ni (see Fig. 4.15) overlaid with PPC result (in red) folded with 2 MeV Lorentzian (top panel). Strength functions obtained via the PPC model (bottom panel).

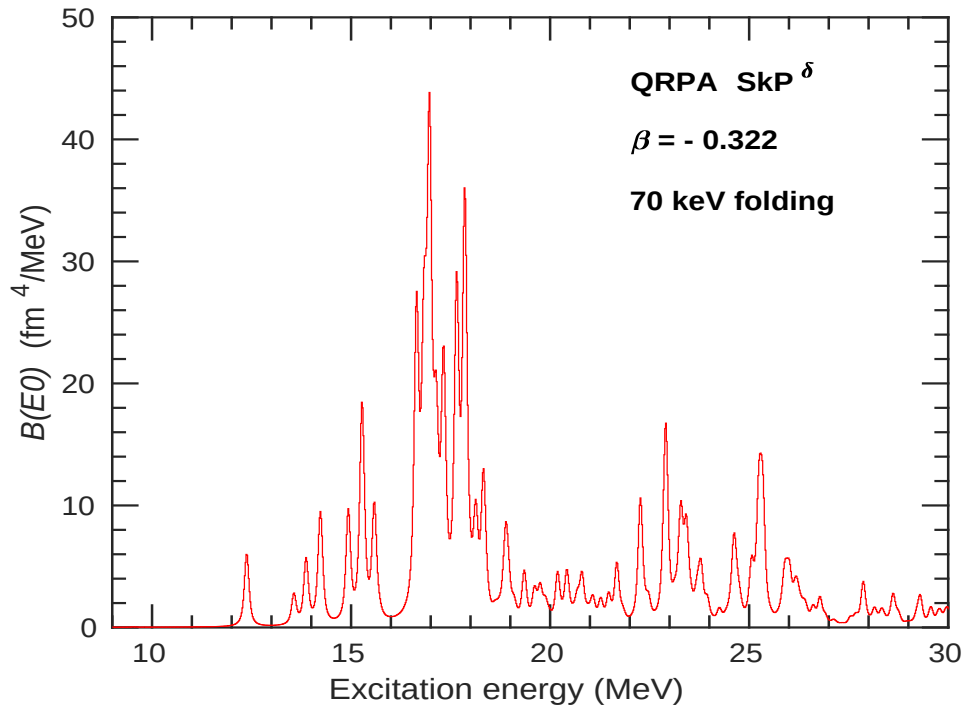
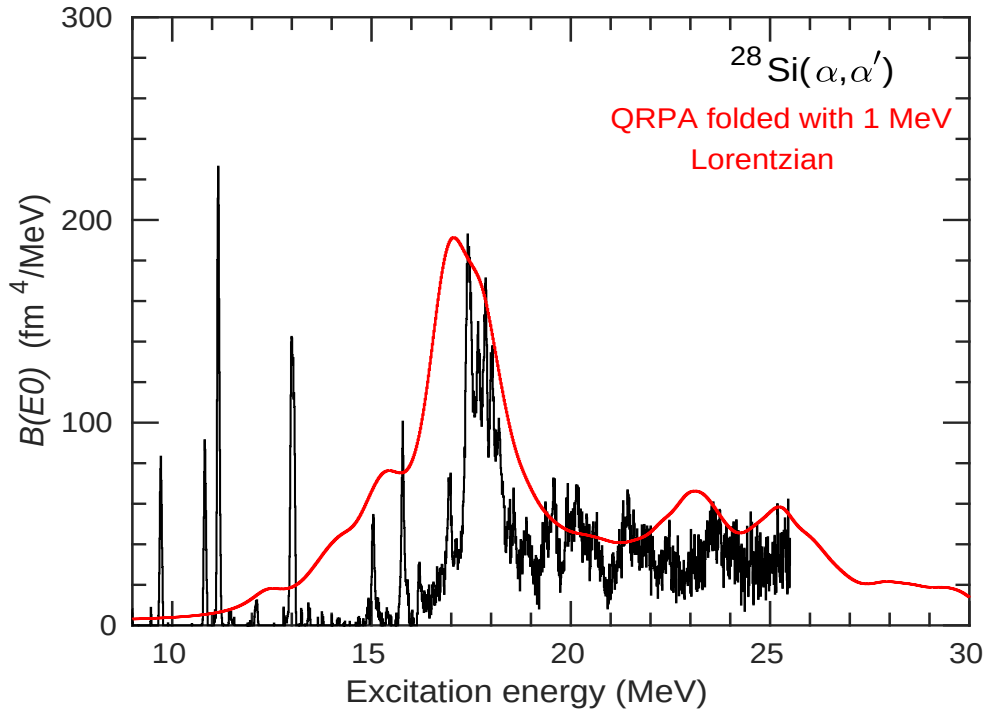


Figure 4.36: ISGMR strength distribution for ^{28}Si (see Fig. 4.16) overlaid with QRPA result (in red) folded with 1 MeV Lorentzian (top panel). Strength functions obtained via the QRPA model (bottom panel).

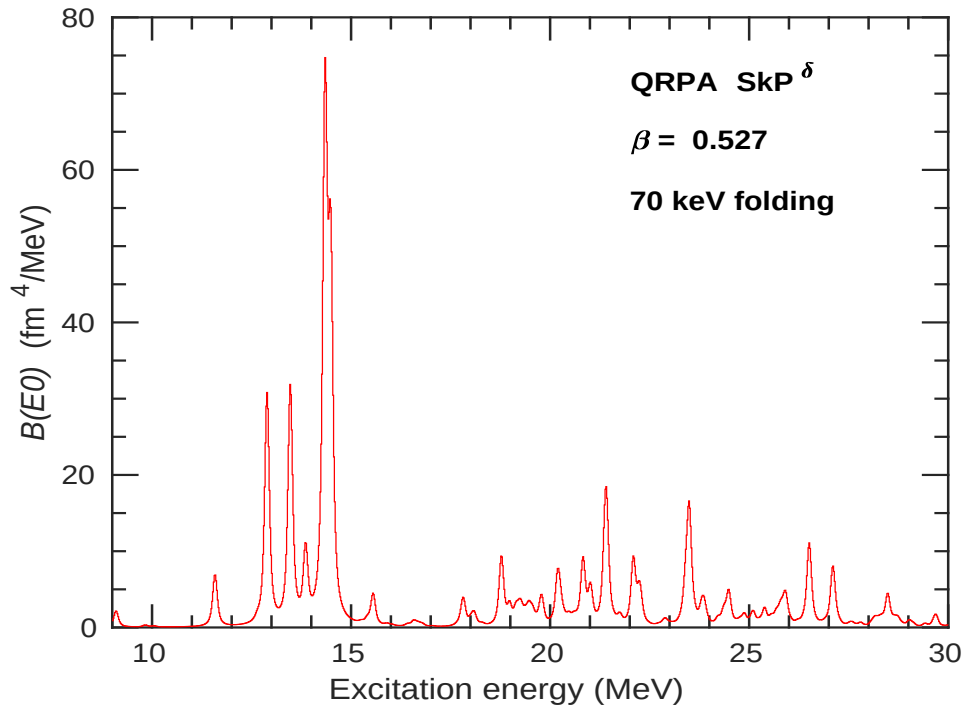
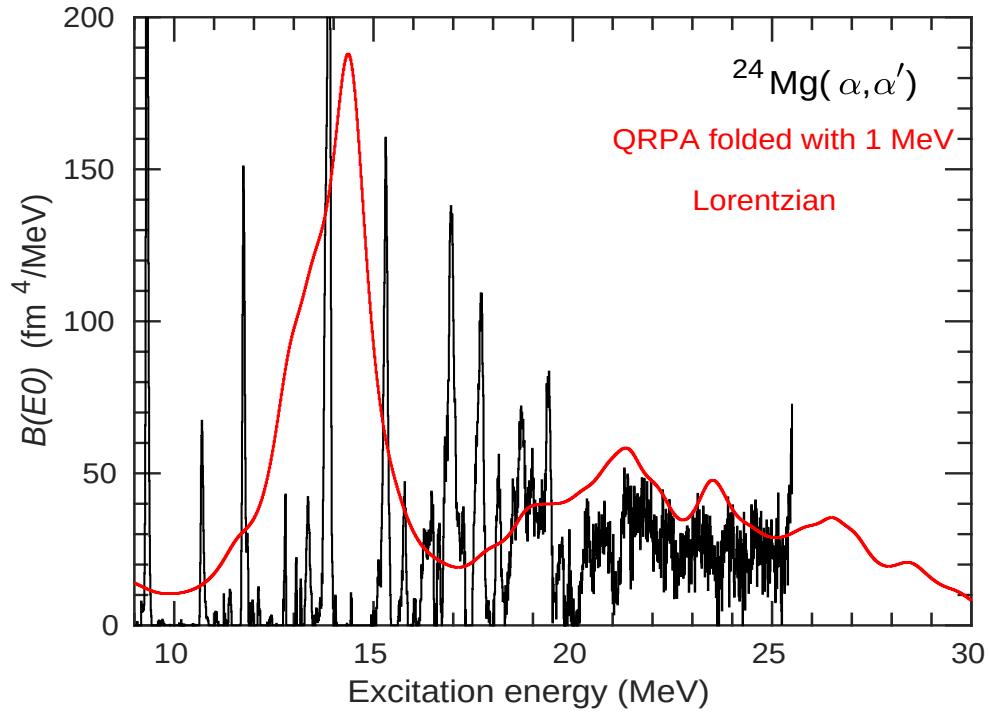


Figure 4.37: ISGMR strength distribution for ^{24}Mg (see Fig. 4.17) overlaid with QRPA result (in red) folded with 1 MeV Lorentzian (top panel). Strength functions obtained via the QRPA model (bottom panel).

4.4 Fine structure analysis of ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg

As introduced in Sect. 2.7, extraction of characteristic wavelet length-like energy scales from measured excitation energy spectra using wavelet analysis will allow comparison with the theoretical predictions of the ISGMR from the PPC model. The difference spectrum obtained via the subtraction of the small angle cut spectrum from the 0° spectrum represents the extracted $E0$ strength used in the process. Sect. 4.4.1 presents the results of the wavelet analysis for the nuclei under investigation which are discussed in Sect. 4.4.2. For readability all the figures and tables are shown at the end of this Section.

4.4.1 Extraction of characteristic energy scales in the studied nuclei

The wavelet analysis was performed following the techniques outlined in Sect. 2.7. A Continuous Wavelet Transform (CWT) was used to generate the wavelet coefficients (Eq. 2.54) as a function of excitation energy, for each of the spectra of the nuclei under investigation. It has been shown in [Olo20] that the DoS technique has no adverse effect on the extracted wavelet energy scales since the power spectra show identical scales with relative strength either using the difference spectrum or the zero-degrees spectrum. This is confirmed in Figs. 4.38 and 4.39 for the ^{90}Zr nucleus as a test case and for the two energy regions chosen for the investigation. The results are shown by the two-dimensional plots in the second and fourth panels on the right hand side of Figs. 4.38 and 4.39 in which blue (red) regions indicate negative (positive) values. This is explained by the oscillatory structure of the mother wavelet (Eq. 2.51) used in the analysis. Further, the extracted wavelet coefficients have been projected onto the scale axis to generate the power spectrum shown in the second and fourth panels on the left hand side of Figs. 4.38 and 4.39. This spectrum displays the distribution of the scales in the excitation region chosen for the analysis and the presence of typical scales is indicated by the peaks and points of inflection in the power spectrum. As

such, the value of a particular scale is given by the difference between consecutive minima (or maxima). This is known as “length-like scale”. For a better display of the scales, the power spectrum is plotted as square root of power or power/scale, depending on the nucleus under investigation.

The results of the wavelet analysis on the measured spectra and the theoretical predictions are shown in Figs. 4.40 - 4.46. For each nucleus, different excitation energy regions have been considered for the analysis depending on the location of the maximum of the ISGMR. These regions are indicated by the vertical dashed lines shown in panels on the right-side of Figs. 4.40 - 4.46. As an example, the oscillatory structure displayed in Fig. 4.40 by the two-dimensional plot for the ^{208}Pb experimental double differential cross-section shows a distinct peak in the coefficient plot at $E_x \approx 14$ MeV with an energy scale value of ≈ 2700 keV. This corresponds to the main $E0$ strength in the panel above it. The standard deviation of the peak is calculated by dividing the FWHM (obtained here as the half of the 3000 keV wavelet energy scale and known as “width-like scale”) by the usual factor of 2.355.

4.4.2 Comparison and discussion of experimental and theoretical extracted scales in the studied nuclei

In this Section, the extracted scales both from the measured experimental cross-sections and the theoretical predictions are discussed. The power spectra were obtained by summing the wavelet coefficients within the excitation energy regions delimited by the black dashed lines in Figs. 4.40 - 4.46. The scales are displayed as filled circles and the associated error is given by one standard deviation of the corresponding width-like scale. For comparison purposes and in order to facilitate the determination of similar scales in the corresponding power spectra for the calculations of the PPC and QRPA models, the results obtained from experiments are also displayed as vertical grey bars in all left-side panels of Figs. 4.40 - 4.46. In order to group the scales, three different energy regions are considered, namely less than 300 keV, 300 to 1000 keV and above 1000 keV. The extracted energy scales both experimental and theoretical are listed in Tables 4.8 - 4.14. When two scales agree within error bars, they are

placed in the same column. These scales are known as equivalent characteristic energy scales i.e. they are present in both experimental data and the PPC and QRPA model predictions.

From Tables 4.8 - 4.14, it can be seen for the three different energy regions considered, that scales are present in both the experimental data and the theoretical calculations. As such, for the nuclei under investigation, the PPC and QRPA models reproduce well the experimental scales for the medium to heavy mass nuclei and light mass nuclei, respectively. For the doubly-magic nucleus ^{208}Pb , the scales in the first and second energy regions agree within error bars but the model fails to reproduce the scale in the third energy region. The scales of second and third classes are reproduced by the PPC model calculations in the singly-magic nucleus ^{120}Sn while the scale predicted by the model in the first energy region is higher than the experimental scale. Excellent agreement is seen between the model predictions and the experimental scales extracted from the singly-magic nucleus ^{90}Zr for the two excitation energy regions chosen for the analysis (comprised of the main ISGMR and above) and for all the three classes. The same applies to the third singly-magic nucleus considered in this study, namely ^{58}Ni . As for the light nuclei, namely ^{28}Si and ^{24}Mg , there is good agreement between the scales extracted experimentally and from the theoretical calculations using the SkP^δ Skyrme force parametrisations within the QRPA framework, although for ^{28}Si more scales are seen from the theoretical calculations, and for ^{24}Mg not all experimental scales are observed in the QRPA calculations.

In summary, both the PPC and QRPA models reproduce quite well the characteristic energy scales extracted from the experimental data. Hence, the present analysis suggests that the fine structure of ISGMR in medium to heavy mass nuclei across the periodic table arises from coupling to collective phonons and the non-harmonicity owing to interactions among phonons. Unlike ISGMR in close to spherical nuclei ^{208}Pb , ^{120}Sn , ^{90}Zr and ^{58}Ni , the fine structure in well-deformed nuclei ^{28}Si and ^{24}Mg is mainly provided by three factors: deformation splitting of the levels, $E0/E2$ coupling, and Landau fragmentation. The PPC effect, which is important in close to spherical ^{208}Pb , ^{120}Sn , ^{90}Zr and ^{58}Ni , is of a minor importance in highly deformed nuclei like ^{28}Si and ^{24}Mg . However, these conclusions stem from knowledge of the ingredients of the theoretical calculations used in the

comparison. Other models could provide more insights where the PPC and QRPA failed to reproduce the experimental scales.

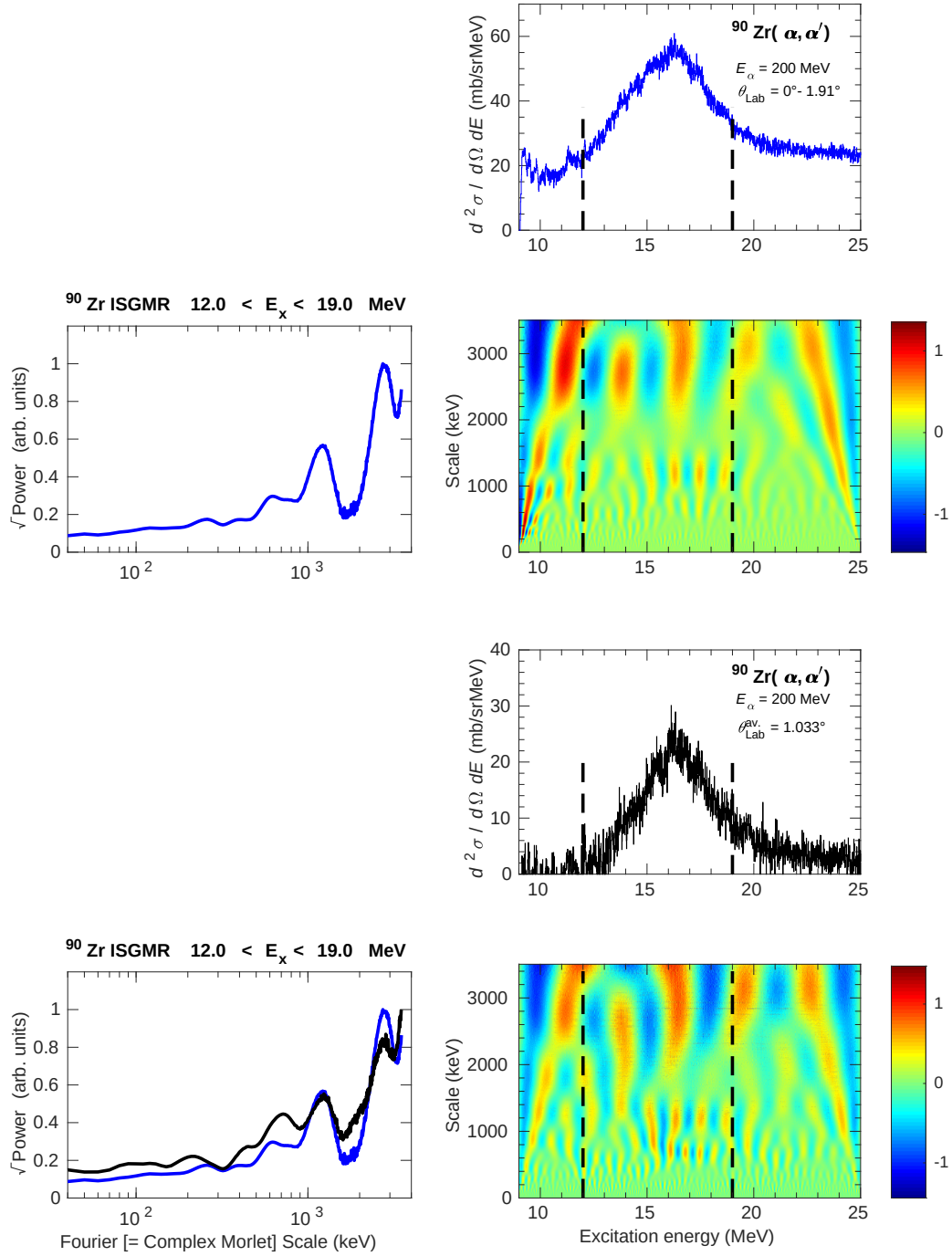


Figure 4.38: Top set (right column): Experimental double differential cross-section for $^{90}\text{Zr}(\alpha, \alpha')$ measured at $\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$. Density plot of the real part of the CWT coefficients of the data (top set lower right). Top set (left column): The corresponding power spectrum for the excitation energy region indicated by the vertical dashed lines ($12 \text{ MeV} \leq E_x \leq 19 \text{ MeV}$). Bottom set: Same as the top set but for the difference spectrum, respectively. The corresponding power spectrum in black superimposed on power spectrum from $\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$.

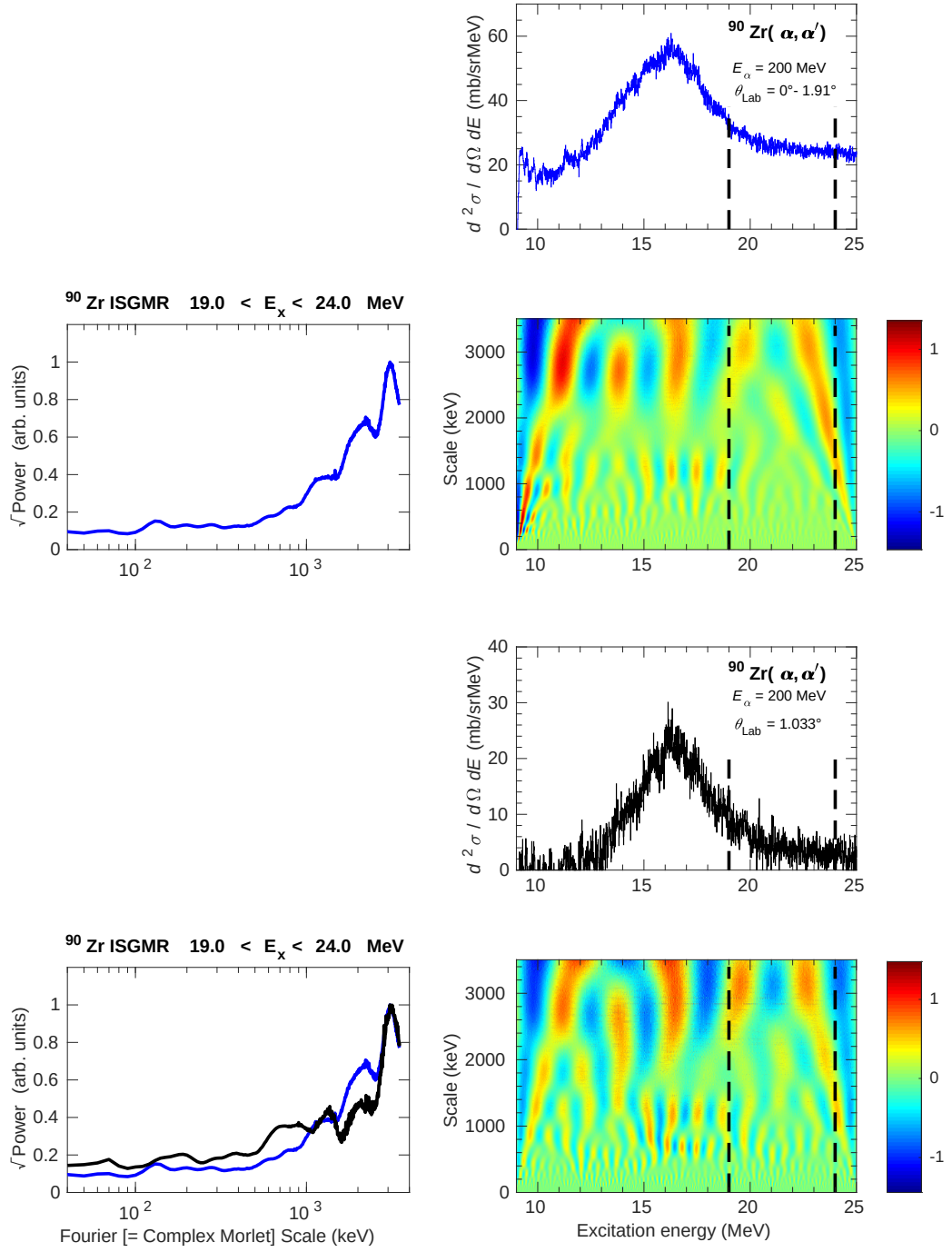


Figure 4.39: Same as Fig. 4.38 but for the excitation energy region 19 – 24 MeV.

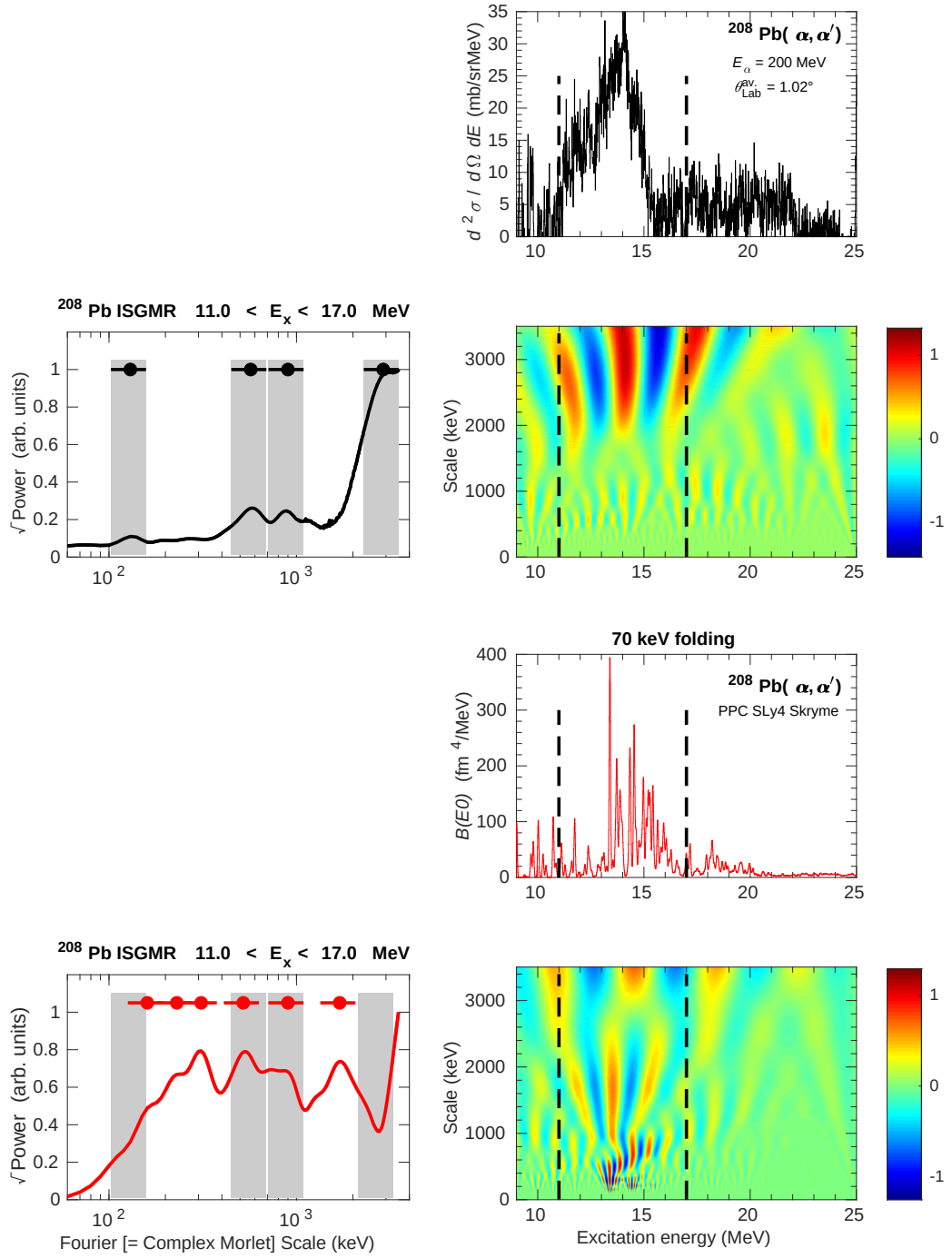


Figure 4.40: Top set (right column): Difference spectrum for $^{208}\text{Pb}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av.}} = 1.02^\circ$. Density plot of the real part of the CWT coefficients of the data (top set lower right). Top set (left column): The corresponding power spectrum for the excitation energy region indicated by the vertical dashed lines ($11 \text{ MeV} \leq E_x \leq 17 \text{ MeV}$). Bottom set: Same as the top set but for the PPC model calculations. The scales are displayed as filled circles and the associated error is given by one standard deviation of the corresponding width-like scale. The results obtained from experiments are displayed as vertical grey bars.

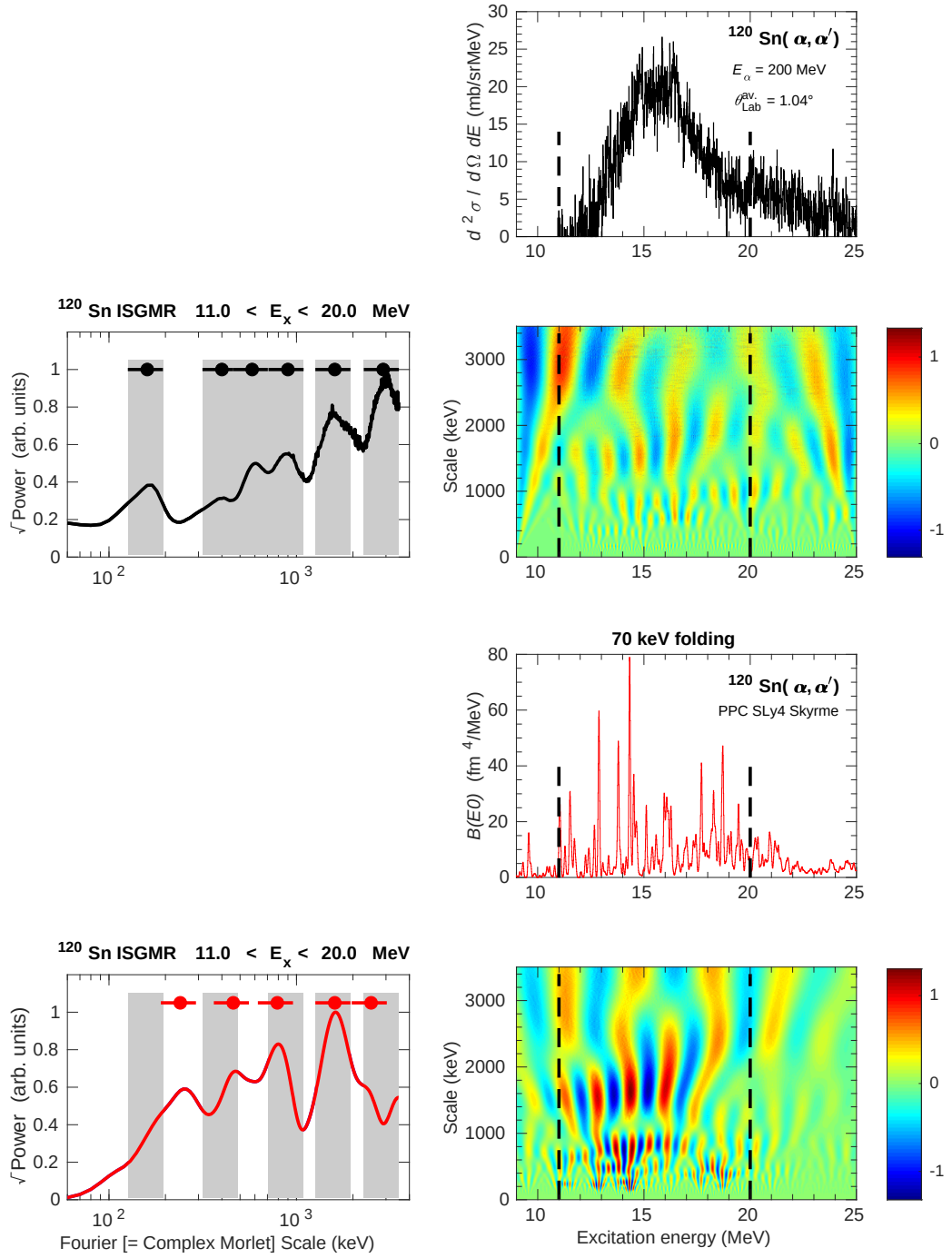


Figure 4.41: Same as Fig. 4.40 but for $^{120}\text{Sn}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av}} = 1.04^\circ$ and the excitation energy region $11 \text{ MeV} \leq E_x \leq 20 \text{ MeV}$.

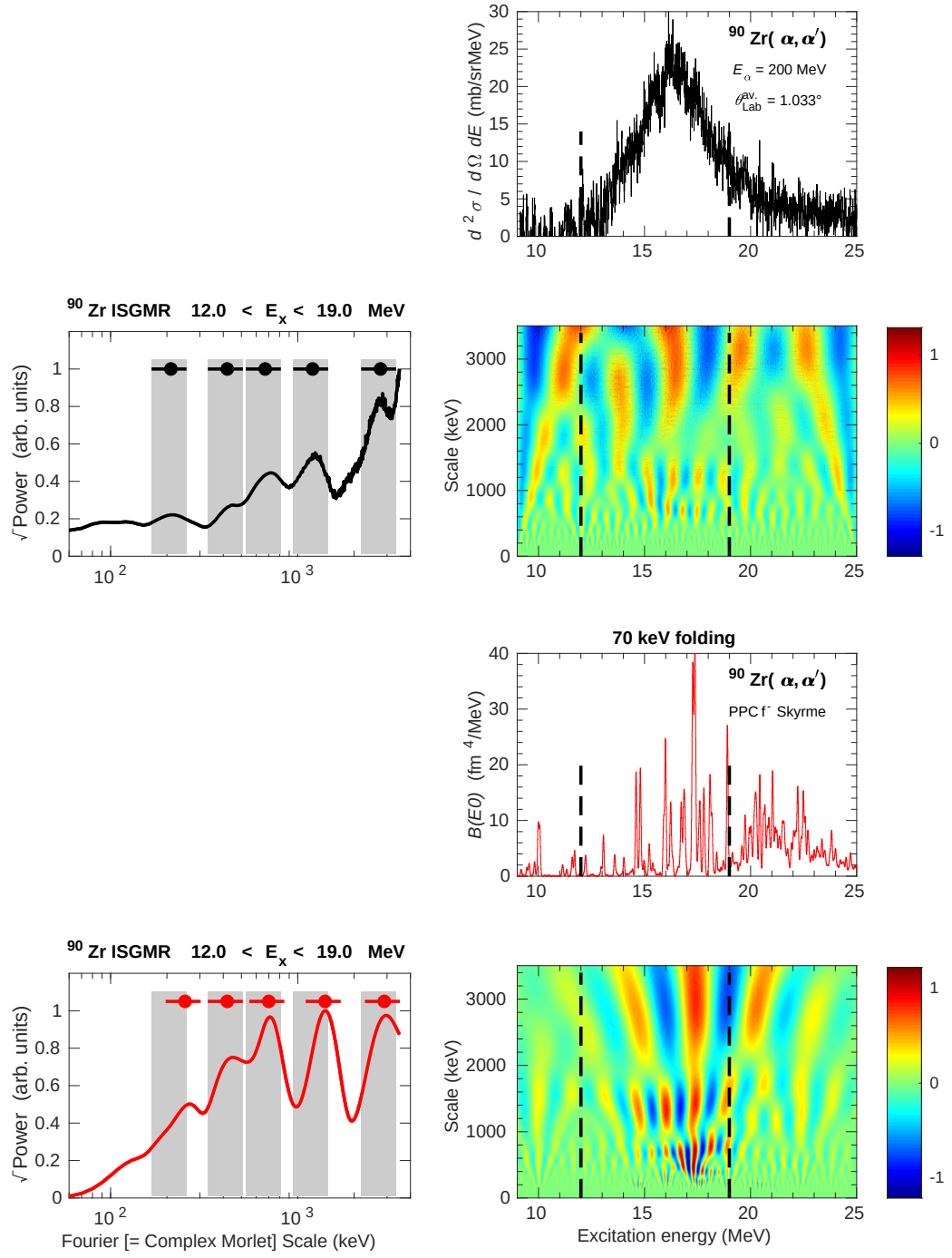


Figure 4.42: Same as Fig. 4.40 but for $^{90}\text{Zr}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av}} = 1.033^\circ$ and the excitation energy region $12 \text{ MeV} \leq E_x \leq 19 \text{ MeV}$.

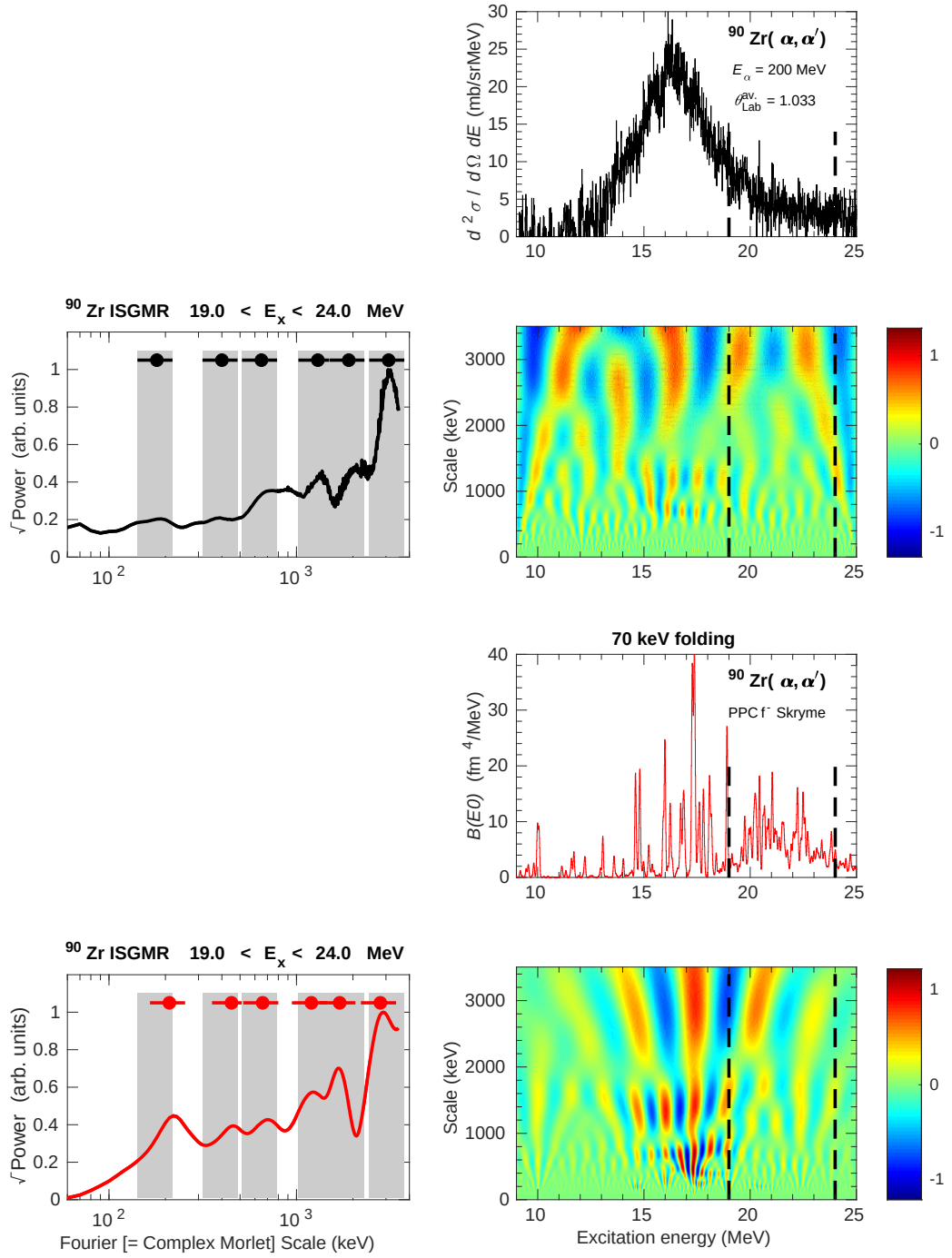


Figure 4.43: Same as Fig. 4.40 but for $^{90}\text{Zr}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av}} = 1.033^\circ$ and the excitation energy region $19 \text{ MeV} \leq E_x \leq 24 \text{ MeV}$.

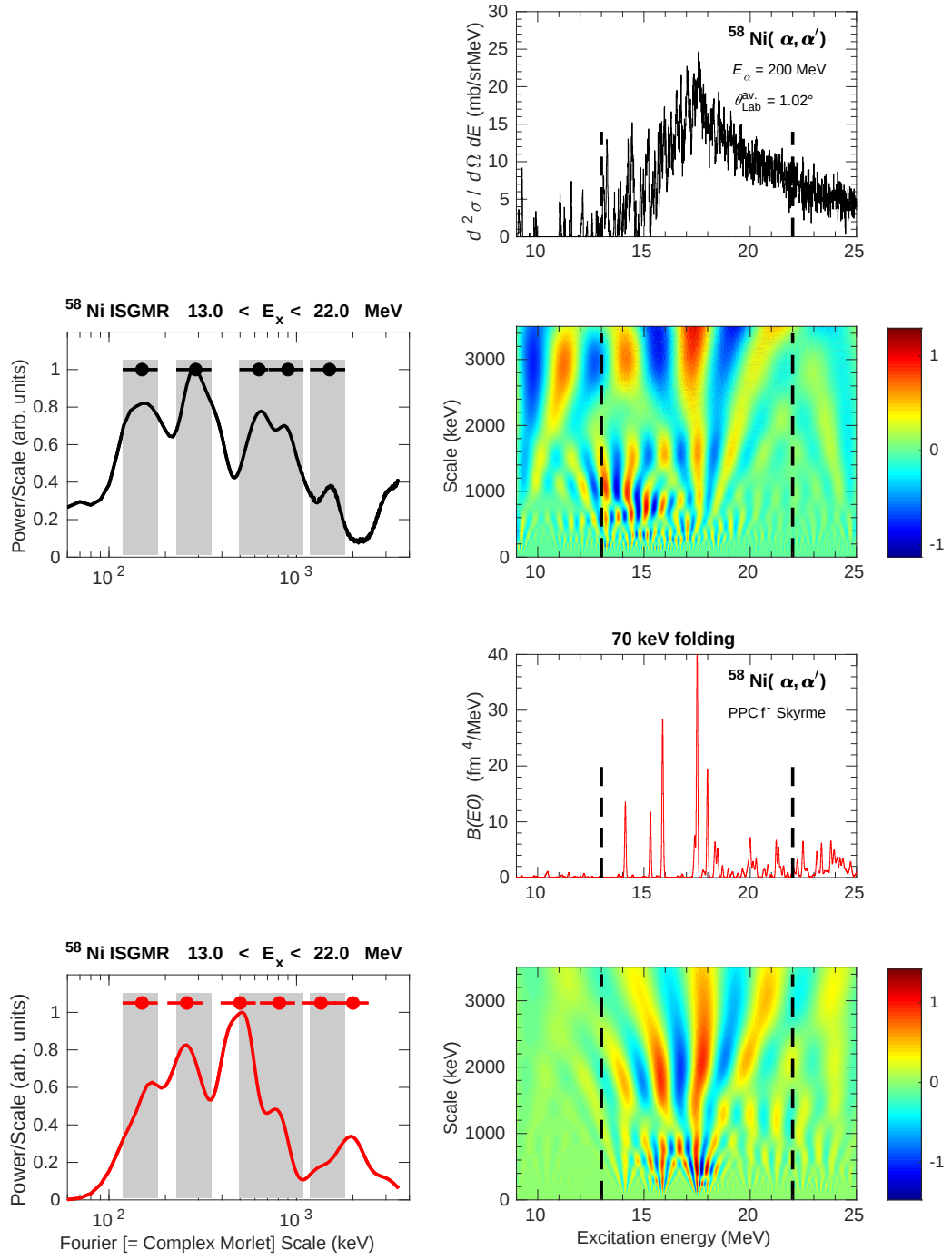


Figure 4.44: Same as Fig. 4.40 but for $^{58}\text{Ni}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av}} = 1.02^\circ$ and the excitation energy region $13 \text{ MeV} \leq E_x \leq 22 \text{ MeV}$.

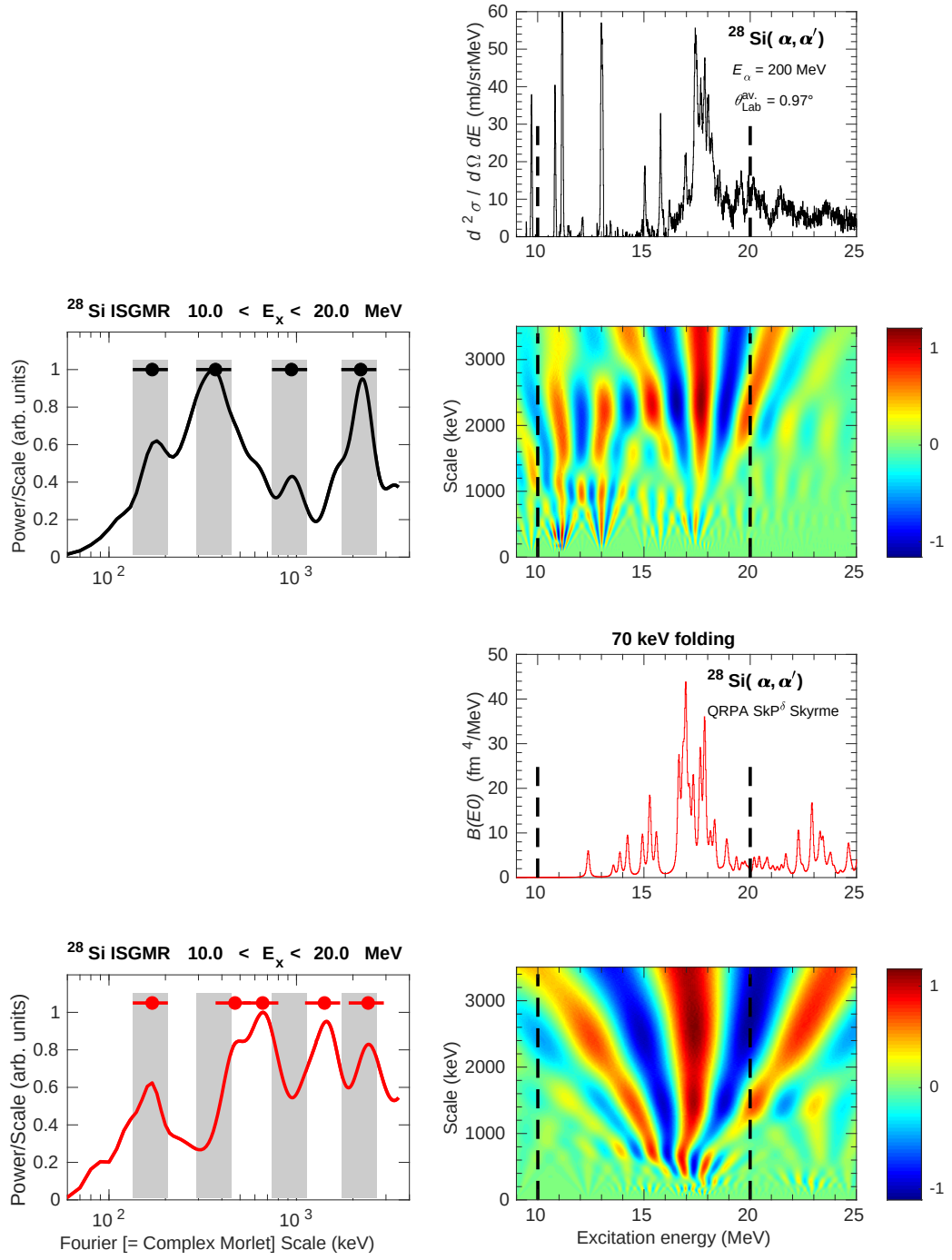


Figure 4.45: Same as Fig. 4.40 but for $^{28}\text{Si}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av}} = 0.97^\circ$ and the excitation energy region $10 \text{ MeV} \leq E_x \leq 20 \text{ MeV}$.

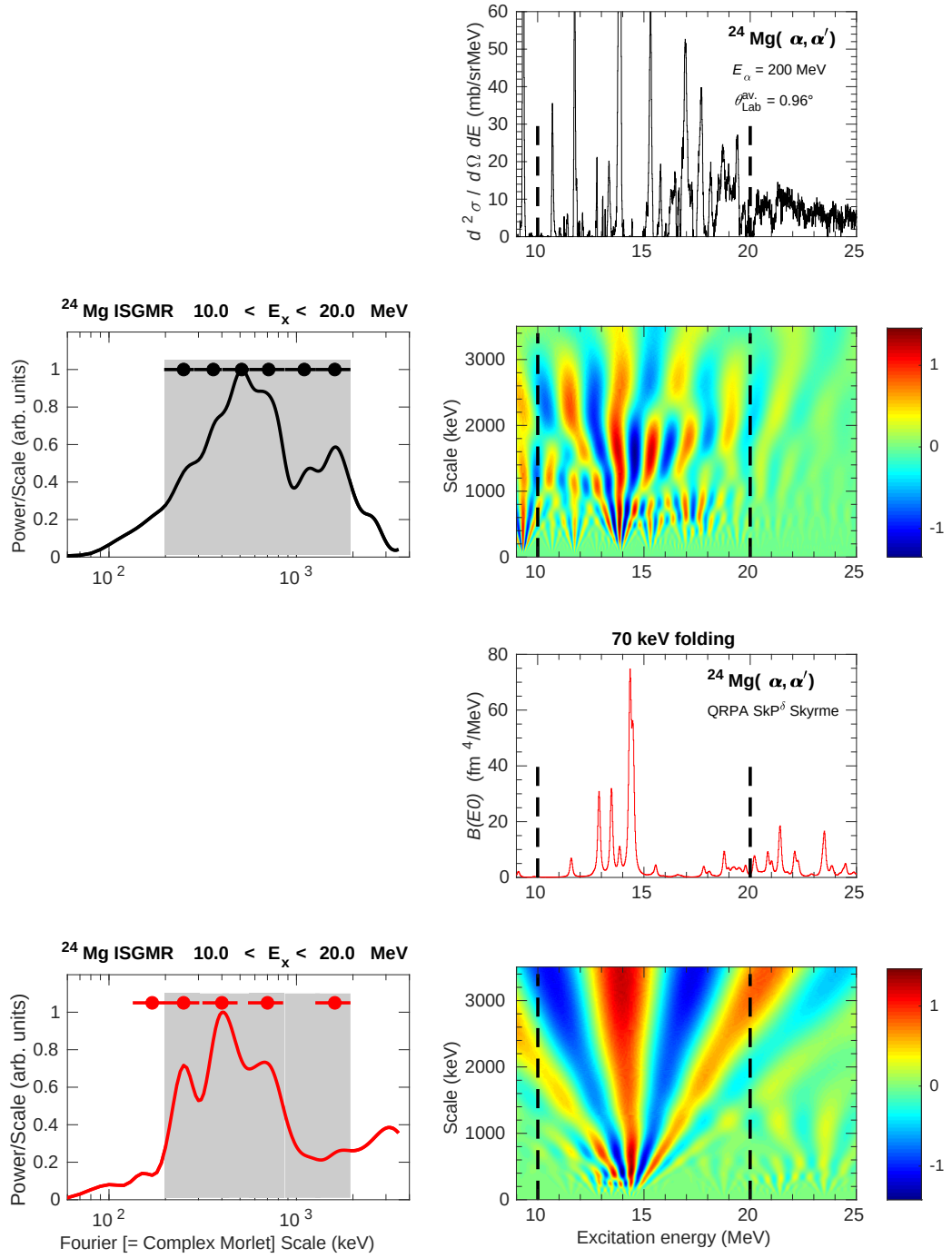


Figure 4.46: Same as Fig. 4.40 but for $^{24}\text{Mg}(\alpha, \alpha')$ at $\theta_{\text{Lab}}^{\text{av}} = 0.96^\circ$ and the excitation energy region $10 \text{ MeV} \leq E_x \leq 20 \text{ MeV}$.

Table 4.8: Energy scales extracted for ^{208}Pb : $11 \text{ MeV} \leq E_x \leq 17 \text{ MeV}$.
Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	$\Delta E \lesssim 300 \text{ keV}$		Scales			$\Delta E \geq 1000 \text{ keV}$
		$300 \leq \Delta E \leq 1000 \text{ keV}$					
^{208}Pb	Experiment	130		570	900		2700
	PPC	160	230	310	520	900	1700

Table 4.9: Energy scales extracted for ^{120}Sn : $11 \text{ MeV} \leq E_x \leq 20 \text{ MeV}$.
Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	$\Delta E \lesssim 300 \text{ keV}$	Scales			$\Delta E \geq 1000 \text{ keV}$	
			$300 \leq \Delta E \leq 1000 \text{ keV}$				
^{120}Sn	Experiment	160	400	580	900	1600	2900
	PPC	240	460		790	1600	2500

Table 4.10: Energy scales extracted for ^{90}Zr : $12 \text{ MeV} \leq E_x \leq 19 \text{ MeV}$.
Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	$\Delta E \lesssim 300 \text{ keV}$	Scales		$\Delta E \geq 1000 \text{ keV}$
			$300 \leq \Delta E \leq 1000 \text{ keV}$		
^{90}Zr	Experiment	210	420	670	1200 2770
	PPC	250	420	700	1400 2900

Table 4.11: Energy scales extracted for ^{90}Zr : $19 \text{ MeV} \leq E_x \leq 24 \text{ MeV}$.
Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	Scales		
		$\Delta E \lesssim 300 \text{ keV}$	$300 \leq \Delta E \leq 1000 \text{ keV}$	$\Delta E \geq 1000 \text{ keV}$
^{90}Zr	Experiment	180	400 650	1300 1900 3100
	PPC	210	450 660	1200 1700 2800

Table 4.12: Energy scales extracted for ^{58}Ni : $13 \text{ MeV} \leq E_x \leq 22 \text{ MeV}$.
Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	Scales		
		$\Delta E \lesssim 300 \text{ keV}$	$300 \leq \Delta E \leq 1000 \text{ keV}$	$\Delta E \geq 1000 \text{ keV}$
^{58}Ni	Experiment	150 290	630 900	1500
	PPC	150 260	500 810	1350 2000

Table 4.13: Energy scales extracted for ^{28}Si : $10 \text{ MeV} \leq E_x \leq 20 \text{ MeV}$.
Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	Scales		
		$\Delta E \lesssim 300 \text{ keV}$	$300 \leq \Delta E \leq 1000 \text{ keV}$	$\Delta E \geq 1000 \text{ keV}$
^{28}Si	Experiment	170	370	2200
	QRPA	170	470 660	1410 2410

Table 4.14: Energy scales extracted for ^{24}Mg : $10 \text{ MeV} \leq E_x \leq 20 \text{ MeV}$. Equivalent characteristic energy scale values are vertically aligned.

Nucleus	Dataset	Scales					
		$\Delta E \lesssim 300 \text{ keV}$		$300 \leq \Delta E \leq 1000 \text{ keV}$			$\Delta E \geq 1000 \text{ keV}$
^{24}Mg	Experiment	250		360	510	710	1100 1600
	QRPA	170	250	400 700		1600	

4.5 Level density analysis in the ISGMR region

In this section, the extraction of the level density of $J^\pi = 0^+$ states in ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg using Fluctuation Analysis as described in Sect. 2.8.2 is presented. The analysis was performed within an excitation energy range of 10 to 24 MeV and the extracted level densities are further compared with microscopic and phenomenological models.

4.5.1 Fluctuation analysis

As stated earlier on, the level densities in this study are extracted via the fluctuation analysis discussed in Sect. 2.8.2. The experimental autocorrelation function $C(\epsilon = 0) - 1$ is proportional to the mean level spacing $\langle D \rangle$. This is shown by Eq. (2.68) and as such the level densities can be obtained once the mean level spacing is retrieved from knowing the autocorrelation function. The determination of background contributions and their complete removal from the alpha inelastic-scattering spectra in this study is of utmost importance prior to the extraction of level spacing from the autocorrelation function values. The various background contributions were already largely removed before the

level densities extraction step. This is explained by the removal of instrumental background through the data reduction process detailed in Sect. 3.7. The physical background contributions arising from other multipoles excited in the vicinity of $L = 0$ are also removed via the DoS technique detailed in Sect. 4.2.2.

There is a subtle, but important, difference in the way the DoS spectra used for the calculation of level densities were extracted compared to the DoS method described in Sect. 4.2.2. There, the cross-sections obtained from the finite angle cut representing the angular region around the minimum of the $L = 0$ distribution was subtracted from the cross-sections obtained in the zero-degrees measurements, which spans the laboratory angle range $\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$. This difference was then divided by a DWBA calculation for 100% EWSR calculated at the average angle spanned by the $\theta_{\text{Lab}} = 0^\circ - 1.91^\circ$ measurements to provide the fractional strength results. Alternatively, one could scale up the cross-sections measured in the zero-degrees measurements in order to represent the cross-sections at exactly 0° , subtract from that the cross-sections for the finite angle cut, and then divide this difference spectrum by a DWBA calculation for 100% EWSR at 0° in order to provide a fractional strength result. For such a scaling up exercise, one needs to have full knowledge of the fractional strengths of all multipolarities in order to accurately predict the shape of the total cross-sections as a function of angle.

For the simplistic assumption inherent in the DoS method these two approaches should yield similar results over most of the excitation energy region. However, at lower excitation energies the $L = 0$ strength is not as dominant as at higher excitation energies. The first method described above can, therefore, result in a negative DoS cross-sections where a weak $L = 0$ cross-section exists. Therefore, in order to be sensitive to $J^\pi = 0^+$ level densities below the peak of the ISGMR, the second approach was used to generate the DoS cross-sections spectra used in this section.

4.5.2 Extraction of $J^\pi = 0^+$ level density in ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg

The background-free difference spectra were used in the fluctuation analysis following the steps listed in Sect. 2.8.2.1. The experimental energy resolution in this study is approximately 70 keV (FWHM) and as such, the spectra were smoothed and folded with a Gaussian of width $\sigma = 35$ keV and $\sigma_{>} = 105$ keV to yield the spectra $g(E_x)$ and $g_{>}(E_x)$, respectively. For illustrative purposes, Fig. 4.47 depicts the fluctuation analysis procedure for ^{90}Zr where the spectra $g(E_x)$ and $g_{>}(E_x)$ overlaying each other are shown in the second panel followed by the dimensionless stationary spectrum $d(E_x)$ shown in the third panel. In the fourth lower panel, the autocorrelation functions for both theory and experiments are displayed.

Considering the aforementioned values of σ , $\sigma_{>}$ and taking $\alpha = 2.273$, the mean level spacing $\langle D \rangle$ can be obtained as follows [Mat07]

$$\langle D \rangle = [C(\epsilon = 0) - 1] \times k \quad (4.7)$$

with

$$k = \frac{2 \times 0.035 \times \sqrt{\pi}}{2.273 \times \left[1 + \frac{1}{\zeta} - \frac{2\sqrt{2}}{\sqrt{1+\zeta^2}} \right]} = 0.124366, \quad (4.8)$$

where $\zeta = \frac{\sigma_{>}}{\sigma} = 3$. Hence, the level density per energy bin is calculated using Eq. (2.71).

The results obtained from the extraction of $J^\pi = 0^+$ level densities in ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg are shown in Figs. 4.48 - 4.50. The results from both the phenomenological (BSFG) [Rau97], [Egi05] and microscopic models (HFB and HF-BCS) [Hil01], [Dem01] are also displayed for comparison purposes.

Considering the results for each of the nuclei under investigation, the following conclusions can be drawn:

- **^{208}Pb**

The extracted experimental level densities in the excitation energy region, $E_x = 10$ to 14 MeV agree well with BSFG model [Rau97] both in terms of magnitude and energy dependence (top panel of Fig. 4.48). Only the extracted level density at 15 MeV agrees with both the HFB and HF-BCS parametrisations. However, from 16 MeV and above, there is a noticeable discrepancy between the extracted experimental level densities and all the theoretical predictions. In fact, the extracted level densities with the experimental energy resolution in this study are limited by a maximum value of 10^4 MeV^{-1} . This then explains the failure in reproducing any theoretical predicted level density value beyond that limit.

- **^{120}Sn**

Here, none of the extracted level densities from experiment agrees with either the phenomenological or microscopic models (bottom panel of Fig. 4.48). There is a pronounced disagreement between the results and can, once again, be explained by the lowest value of level density (10^4 MeV^{-1}) at $E_x = 10$ MeV obtained using the parametrisations from all the models considered in this study. In addition, the insufficiency of statistics in the experimental data could also explain the discrepancies.

- **^{90}Zr**

There is an excellent agreement between the extracted experimental level densities and the microscopic models in the excitation energy region of 10 to 16 MeV (top panel of Fig. 4.49). From there, discrepancies are observed between the models predictions and the extracted experimental level densities. This is still explained by the limited maximum value of level densities extracted with the experimental energy resolution in this study.

- **^{58}Ni**

The extracted level densities in the excitation energy region, $E_x = 12$ to 22 MeV agree reasonably well with the theoretical predictions (bottom panel of Fig. 4.49). From 12 to 15 MeV, the calculated level densities from the BSFG model [Egi05] are very comparable to the extracted level densities from experiment. The microscopic models and the BSFG

model [Rau97] reproduce well the experimental level densities within the excitation energy region of 17 to 22 MeV.

- **^{28}Si and ^{24}Mg**

In the excitation energy region considered $E_x = 12$ to 23 MeV, the extracted level density from the experimental data for the light nuclei are well reproduced within errors by the parametrisations of the microscopic model HFB (with considerable fluctuations) (Fig. 4.50). However, it must be noted that the extracted level densities in the excitation energy region $E_x = 20$ to 25 MeV are influenced by α -clustering and may belong to the ISGDR. This is explained by the deformation-induced splitting of ISGMR in ^{28}Si and ^{24}Mg into two humps: a narrow peak emerging from the coupling between ISGMR and ISGQR and wide structure at $20 - 25$ MeV representing the main ISGDR. This is discussed in [Pea16] for ^{28}Si and [Gup16b] for ^{24}Mg . All the approximations from the phenomenological models fall below the experimental level densities across the excitation energy range.

The theoretical predictions using the HFB parametrisation describe qualitatively quite well the experimental level densities extracted for ^{24}Mg , ^{28}Si and ^{58}Ni . For ^{90}Zr and ^{208}Pb the agreement is good only up to $E_x = 16$ and 15 MeV, respectively. All models predictions fail to reproduce level densities for ^{120}Sn . In terms of magnitude, the extracted experimental values of the level densities are within errors limits comparable with the models calculations for ^{24}Mg , ^{28}Si , ^{58}Ni , ^{90}Zr and ^{208}Pb (for the latter, only up to $E_x = 16$ and 15 MeV, respectively). However, none of the level densities go beyond 10^4 and this is in particular the case for heavier systems at larger excitation energies. Here, the errors on the experimental level density arise mainly from the smoothing parameters varied by 10% and the range of the excitation energy interval chosen for the analysis (0.5 and 2 MeV).

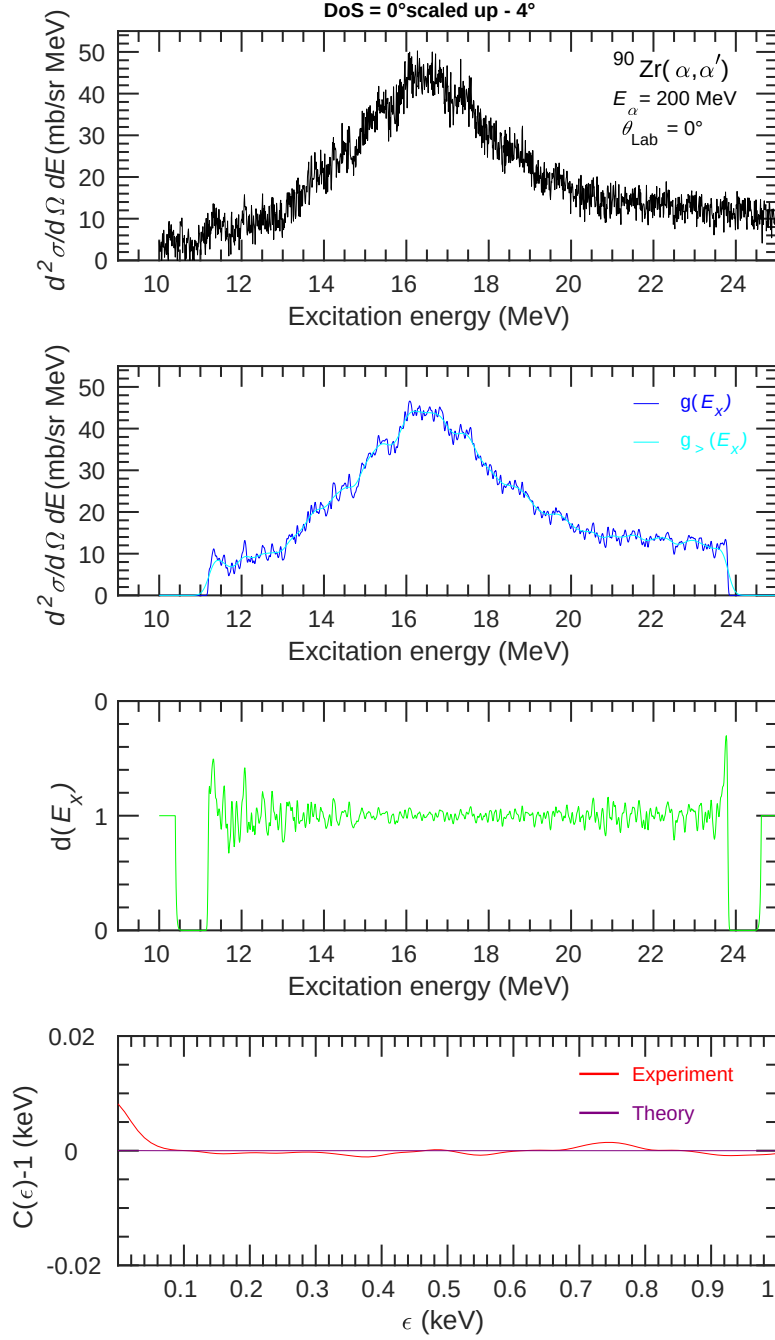


Figure 4.47: Experimental double differential cross-section for $^{90}\text{Zr}(\alpha, \alpha')$ at $\theta_{\text{Lab}} = 0^\circ$ (first top panel). Smoothed spectra $g(E_x)$ and $g_>(E_x)$ overlaying each other (second top panel) followed by the resulting stationary spectrum (first bottom panel). The autocorrelation functions both experimental (red line) and theoretical (purple line) are displayed in the second bottom panel.

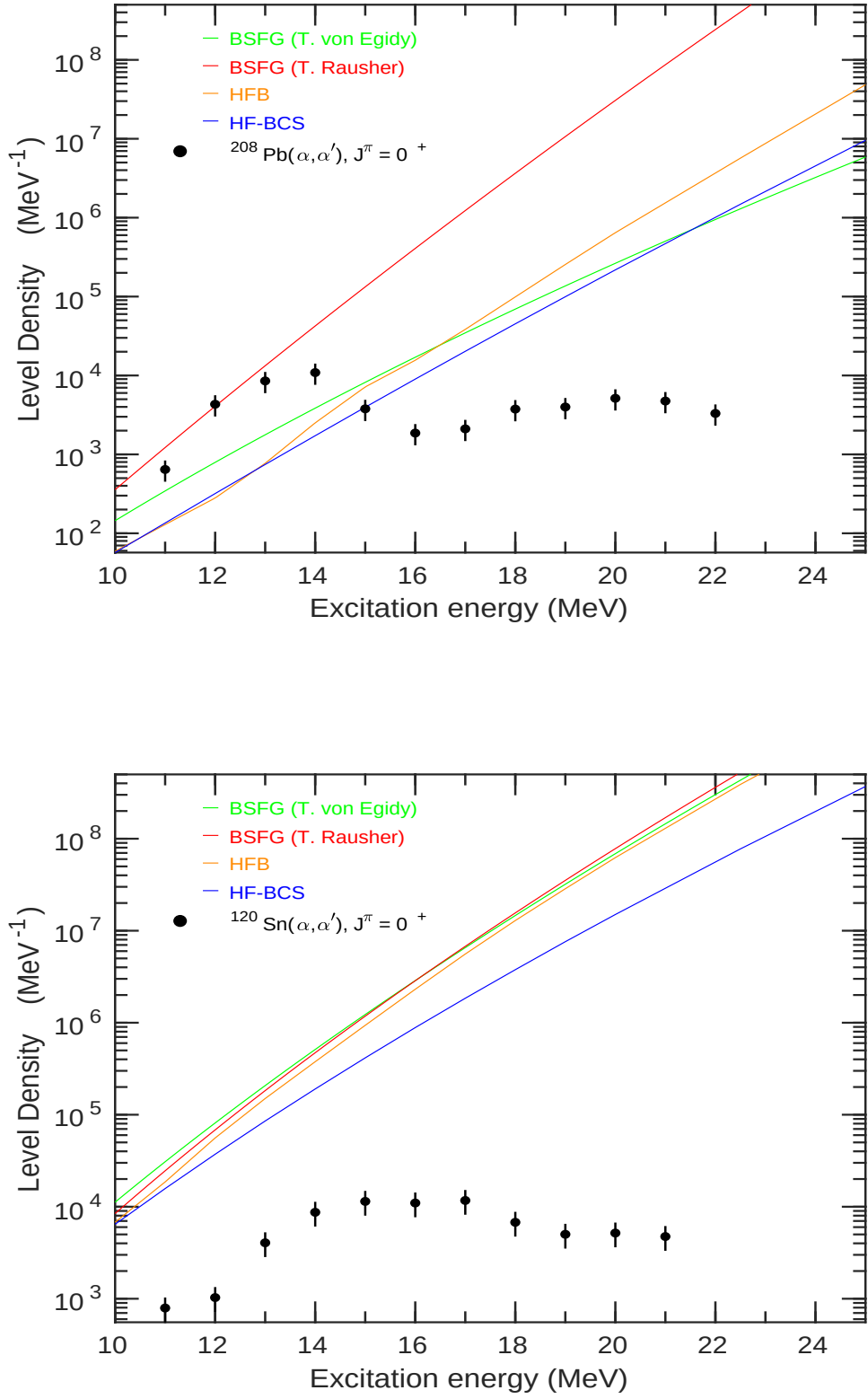


Figure 4.48: $J^\pi = 0^+$ level densities extracted for $^{208}\text{Pb}(\alpha, \alpha')$ (top panel) and $^{120}\text{Sn}(\alpha, \alpha')$ (bottom panel) at $\theta_{\text{Lab}} = 0^\circ$. The theoretical predictions from both the phenomenological and microscopic models are also shown.

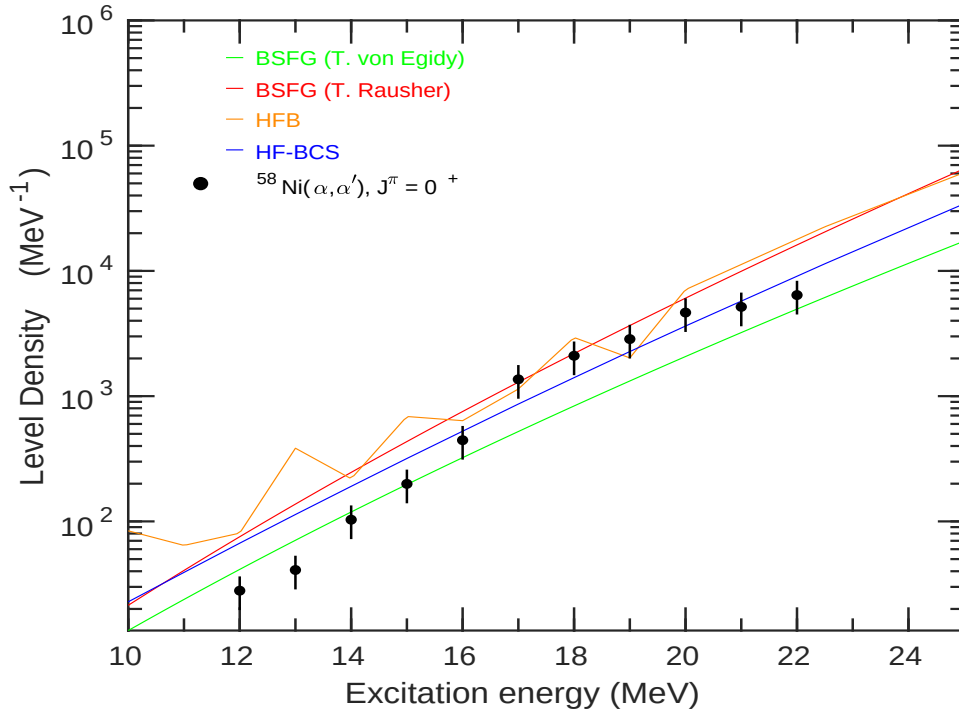
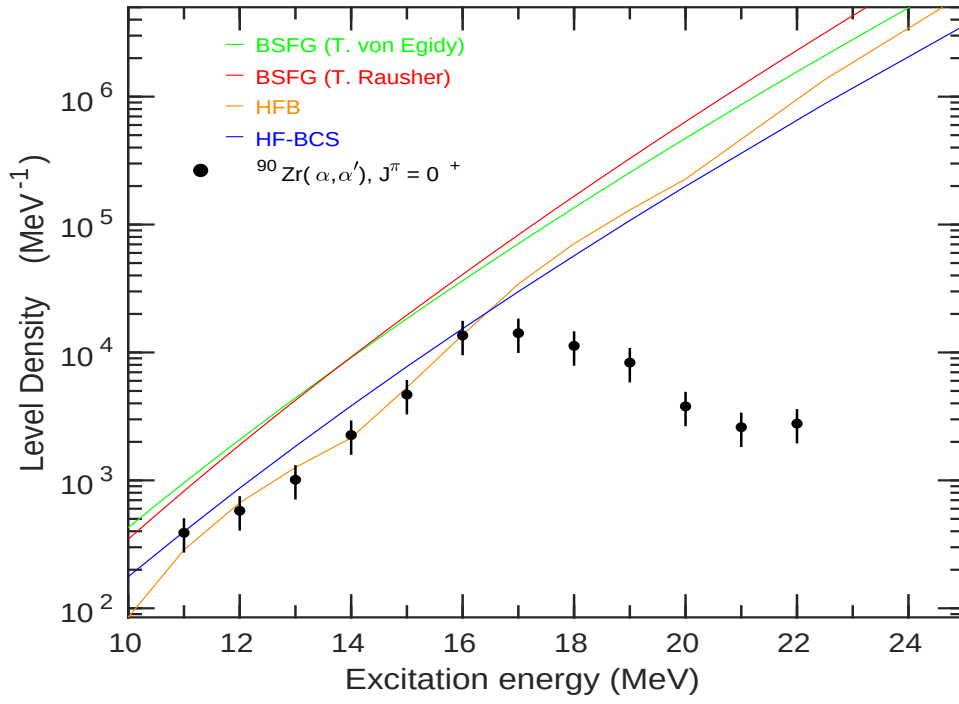


Figure 4.49: Same as Fig. 4.48 but for ^{90}Zr and ^{58}Ni .

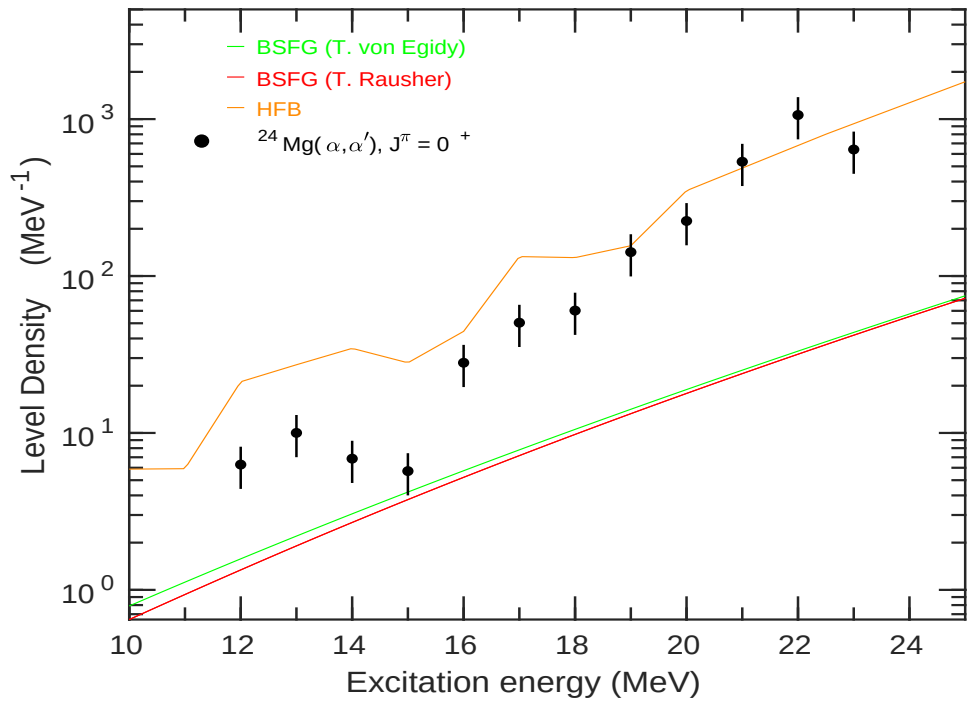
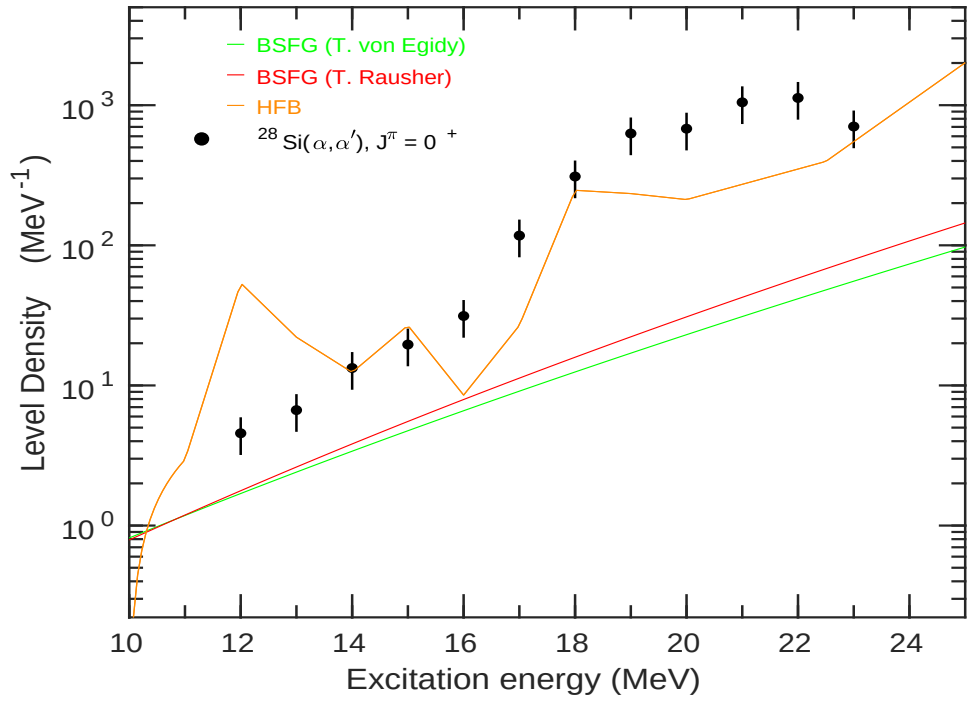


Figure 4.50: Same as Fig. 4.48 but for ^{28}Si and ^{24}Mg .

Chapter 5

Conclusions

The Isoscalar Giant Monopole Resonance (ISGMR) excited in nuclei across the periodic table ranging from heavy to light mass has been investigated using inelastic α -particle scattering measurements acquired with a $E_\alpha = 200$ MeV beam at $\theta_{\text{Lab}} = 0^\circ$ and 4° . The K600 magnetic spectrometer and the Separated Sector Cyclotron (SSC) at iThemba LABS were used to carry out the experiments needed to achieve such a task. For all the targets under investigation, substantial fine structure was observed in the excitation energy region of the ISGMR. Prior to the DoS technique being applied in order to extract the $E0$ strength distribution, the oxygen contaminant peaks identified in the ^{208}Pb experimental spectra were removed through direct subtraction using the information obtained from separate measurements.

For the application of the DoS technique, DWBA calculations were used to determine the angle cut for the measurements at 4° . The difference spectrum obtained for each nucleus was converted to the Fraction $E0$ EWSR and $E0$ strength which were compared with MDA analysis results obtained by the TAMU and RCNP groups. Overall, the strength functions obtained in this project have shown reasonable agreement, in centroids and widths, with results from both groups for all the nuclei under investigation. However, the phenomenological background subtraction method employed by the TAMU group before their MDA analysis may lead to unreliable results as well as questionable OMP values such as the one used for ^{208}Pb in [You04] and shown in this work to be problematic. On the other hand, the results from the RCNP group are potentially overestimated due to a lack of subtraction of

any continuum prior to the MDA analysis. Further, it has been shown in this work that the DoS technique has some shortcomings since the IVGDR contribution is not fully removed and needs to be treated separately using photonuclear data.

Further, characteristic energy scales of the ISGMR in ^{208}Pb , ^{120}Sn , ^{90}Zr , ^{58}Ni , ^{28}Si and ^{24}Mg were extracted using the Continuous Wavelet Transform (CWT) technique and compared with theoretical predictions from the Phonon-Phonon Coupling (PPC) and QRPA models. In general, the extracted experimental energy scales are well reproduced by the PPC and QRPA models. It is, therefore, suggested that the fine structure of ISGMR in medium to heavy mass nuclei across the periodic table arises from coupling to collective phonons and the non-harmonicity owing to interactions among phonons. The fine structure in well-deformed nuclei ^{28}Si and ^{24}Mg is mainly provided by three factors: deformation splitting of the levels, $E0/E2$ coupling, and Landau fragmentation. Better insights could be obtained following theoretical results from other models.

Using the fluctuation analysis, spin- and parity-dependent level densities of $J^\pi = 0^+$ states were extracted in the ISGMR region and compared with those from theoretical predictions using the phenomenological BSFG and microscopic HFB and HF-BCS models. In general, reasonable agreements could be achieved for ^{24}Mg , ^{28}Si , and ^{58}Ni throughout the excitation energy range considered, except for ^{90}Zr and ^{208}Pb which deviated beyond 15 MeV, as well as overall lower values obtained for ^{120}Sn , with predominantly very high level densities up to 10^8 MeV^{-1} in all the theoretical predictions. This discrepancy can be attributed to maximum obtainable value of 10^4 MeV^{-1} limitations in the applied code used for the extraction of level densities.

With regard to further investigation, the present study could be extended by attempting a MDA analysis with the few angular datapoints for more insights into the ISGMR strength distribution across the periodic table. A series of measurements of elastic scattering data could also be taken in order to retrieve the Optical Model Parameters (OMP) at 200 MeV needed in the DWBA calculations performed in this project. In addition, available photonuclear cross-sections data could be used to ensure that for all nuclei studied the IVGDR contribution is removed from the excitation energy

spectra before applying the DoS method.

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Appendix A

Lineshape correction at 4°

As introduced in Sect. 3.7.4, due to the non negligible kinematics differences of the scattered particles from inelastic-scattering of light and heavy targets at small angles, the lineshape correction parameters obtained from the light nucleus with strong identifiable states cannot be directly used to correct the lineshapes of states in the heavy nuclei. One needs to obtain a correction factor to be used in order to update the lineshape correction Eq. 3.16, for heavy nuclei that does not have strong individual states. As an example, results of kinematics calculations for the reactions $^{24}\text{Mg}(\alpha, \alpha')$ and $^{208}\text{Pb}(\alpha, \alpha')$ are provided in Table A.1.

Table A.1: Kinematics calculations for inelastically scattered α -particles from different targets with the same rigidity at 4° .

	θ_{K600}	E_α (MeV)	$QB\rho$ (e.kG.cm)
^{24}Mg $E_x = 10 \text{ MeV}$	2°	189.927	4019.85
	4°	189.803	4018.52
	6°	189.601	4016.31
^{208}Pb $E_x = 10.165 \text{ MeV}$	2°	189.818	4018.67
	4°	189.804	4018.52
	6°	189.781	4018.27

Let us consider for 4° measurements scattered α particles at a particular

rigidity value at the center of the K600 angle acceptance, i.e. 4° . This translates to different values of excitation energies in targets of different mass. For a specific excitation energy E_x in any nucleus, Table A.1 shows that the energy difference between the energy of the scattered particles E_α is higher between 4° and 6° than between 2° and 4° . These can be seen in Fig. A.1.

The lineshapes in Fig. A.1 are a consequence of the curvature of E_α as a function of scattering angle, resulting from the kinematics of inelastic scattering. This is depicted in Fig. A.2.

To understand the procedure necessary to correct the lineshapes for heavy nuclei, consider the following: a rigidity calibration was performed for the ^{24}Mg target after the lineshapes of states in the θ_{scat} versus E_x plot were corrected so that all states are represented by straight lines centred around the K600 central angle, in this case 4° . This lineshape correction had the effect of providing an E_x spectrum relevant for the kinematics conditions at 4° , providing a rigidity calibration that is valid for all targets at 4° . This has the effect of transforming the lineshapes in Fig. A.1 to look like those in Fig. A.3, where it is clear that the lineshape of the heavy nucleus is subjected to the same change as that of the light calibration target.

In order to correct the lineshape of the heavy target, one needs to correct its shape in the θ_{scat} vs. E_x plot. That shape can to first order be approximated with a straight line that is independent of the value of E_x , since the function of the energy difference (E_x at θ minus E_x at 4°) does not vary much over the focal plane, as shown in Fig. A.4. The additional lineshape correction required for a heavy nucleus, assuming that the basic lineshape correction for the ^{24}Mg case is already affected, can then be achieved by adding a value ΔE_x

$$\Delta E_x = -1 \times \frac{|E_\alpha^{6^\circ} - E_\alpha^{2^\circ}|}{6^\circ - 2^\circ} \quad (\text{A.1})$$

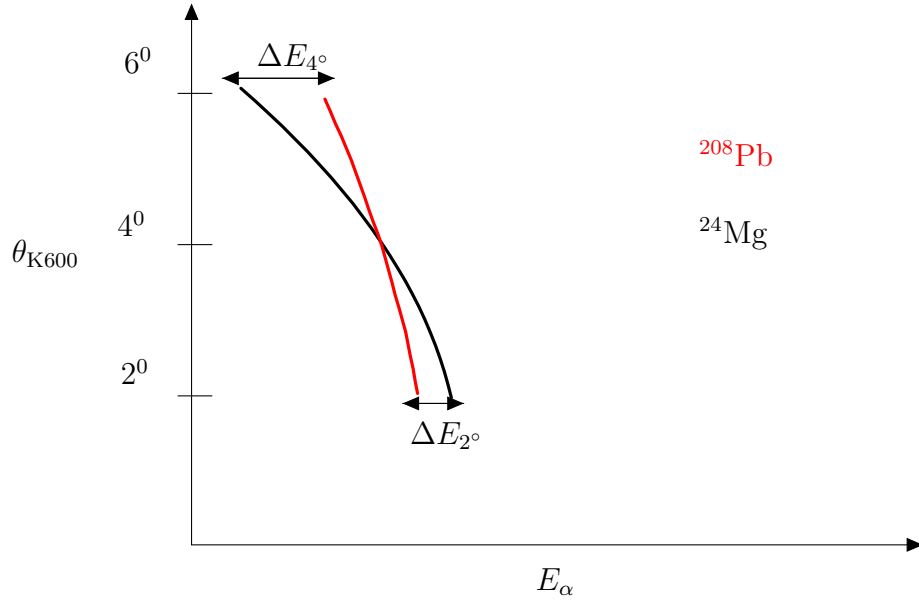


Figure A.1: Scattering angle *versus* energy of the scattered particles for the light and heavy nuclei. Different excitation energy E_x in the two nuclei are chosen to have the same rigidity at 4° .

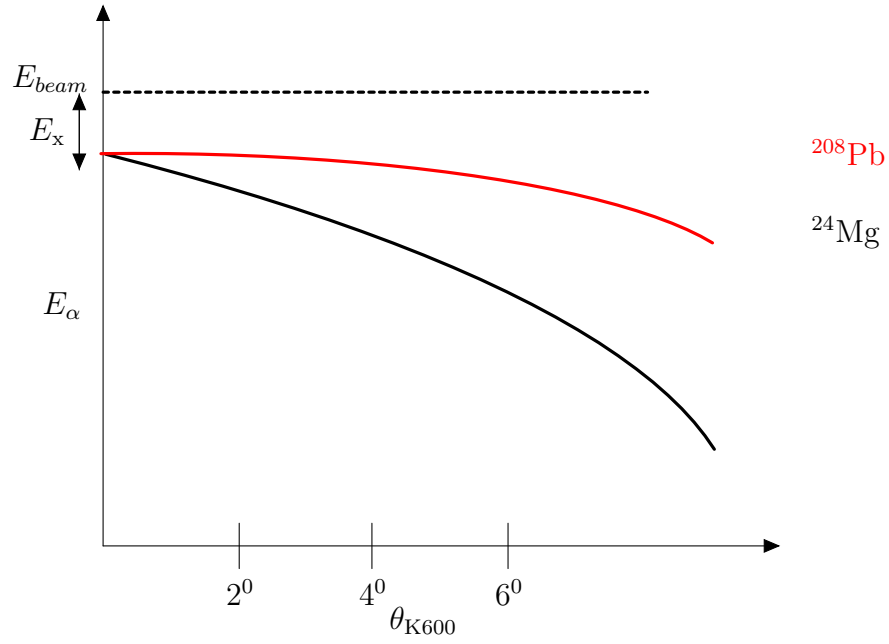


Figure A.2: Experimental curvature followed by scattered particles from light target (black curve) *versus* heavy target (red curve).

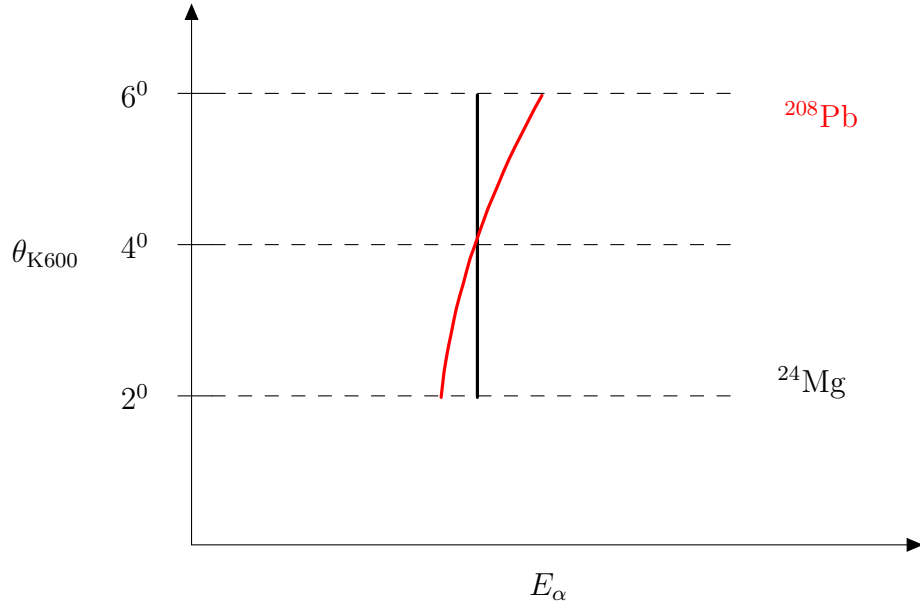


Figure A.3: Scattering angle *versus* energy of the scattered particles for the light and heavy nuclei after lineshape correction on the light nucleus being implemented.

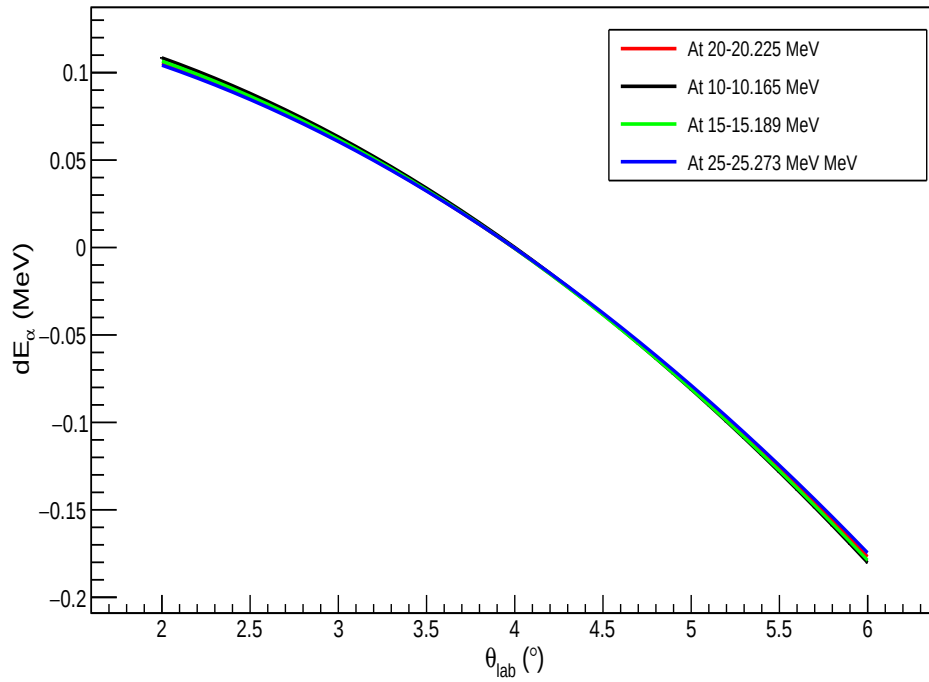


Figure A.4: Difference between E_α of an inelastically scattered alpha particle at angle θ from the kinetic energy E_α of an inelastically scattered α -particle at 4° (both with a ^{208}Pb target) for various excitation energies.