Chapter 3

Methodology and Research Design

We have briefly examined the nature and uses of theory - but in research should we begin with theory, or should theory itself result from the research?

John D. Dewey (1933)

Introduction

In this chapter I outline the design and methodologies that were used to conduct the research. A mixed model[17] design research approach was followed, combined with a sequential transformative methodology. The study consisted of three phases; (i) phase one which consisted of a document analysis, a set of video recorded lesson data, and a further document analysis, (ii) phase two a series of three intervention workshops aimed at introducing the notion of structure and early algebra to the teachers and (iii) phase three consisted of a second set of video recorded lesson data. A descriptive approach was followed whereby various inductive and deductive rounds of coding resulted in a set of coding matrices which were used to analyse and describe the data related to the document study. For the lesson data, a sequential transformative design was followed to provide answers related to the pedagogic changes that emerged after the series of intervention workshops. The two rounds of lesson data formed the basis for the sequential transformative design. The intervention workshops are described in detail in Chapter 5, and formed part of the design research process for the study. The analysis of the lesson sets used the mixed model approach whereby qualitative and quantitative data analysis was integrated throughout the study by using quantitative methods to support the qualitative discussion of the results. This chapter will give insight into the theoretical perspective, the research phases, the research methodology, the participants and the various coding matrices that were used to analyse the data in the three research phases.

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[17] Johnson & Onwuegbuzie (2004) suggest the term *mixed model*, rather than mixed methods, be used to differentiate research designs which *integrate* qualitative and quantitative data from those which merely employ both types of data.

The Theoretical Research Perspective: Mixed Model Design Research

A mixed model (Johnson and Onwuegbuzie, 2004; Mayring, 2001; Ercikan and Roth, 2006) approach to the research was selected as the most appropriate, as this study had a range of data sets that were analysed, using an integrated combination of qualitative and quantitative approaches. The term 'mixed model' refers to an emergent methodology of research that advances the systematic integration, or mixing, of quantitative and qualitative data within a single investigation or sustained program of inquiry (Whisdom and Creswell, 2013). The basic premise of this methodology is that such integration permits a more complete and synergistic utilization of data than does separate quantitative and qualitative data collection and analysis (Whisdom and Creswell, 2013). This includes the transformative designs that change one form of data into another, so that the data collected by mixed model designs can be merged (Caracelli and Green 1993; Mayring, 2001; Onwuegbuzie and Teddlie 2003; Srnka and Koeszegi, 2007; Ercikan and Roth, 2006).

In the last decade, mixed model procedures have been developed and refined to suit a wide variety of research questions (Creswell and Plano Clark, 2011). Some research studies have explored the effectiveness of enquiry that integrates qualitative and quantitative data (for example Adamson et al. 2004; Sandelowski 2000; Mayring, 2001; Weisner 2005; Erikan and Roth, 2009), but there remains a need for systematic information on how to operationalise such transformative analytic designs (Driscoll, et. al., 2007; Srnka and Koeszegi, 2007). The terms 'quantitising 'and 'qualitising' have been coined to describe such transformative designs (Tashakkori and Teddlie, 1998). The former describes the process of transforming coded qualitative data into quantitative data, and the latter converts quantitative data to qualitative data. Ercikan and Roth (2006) propose a continuum of generalisability that is based on the level of inference of knowledge derivation and the associated tendencies for knowledge characteristics. They maintain that polarising qualitative and quantitative processes is counter-productive, and outline how integration might occur by conceptualising a continuum instead of a dichotomy of generalisability.

This study investigates six Grade 2 teachers' teaching of sequencing and describes this in detail, observing their selection of examples and pedagogic approaches, in the context of EA. This study has three data sources: document analyses, lesson observations, and an intervention intended to advance EA practice. The research program is divided into three phases: (i) pre-intervention phase, which involves a document analysis, lesson observations and further document analysis, (ii) a series of three intervention workshops, and (iii) post-intervention lesson observations. Figure 3.1 illustrates the phases and their components.

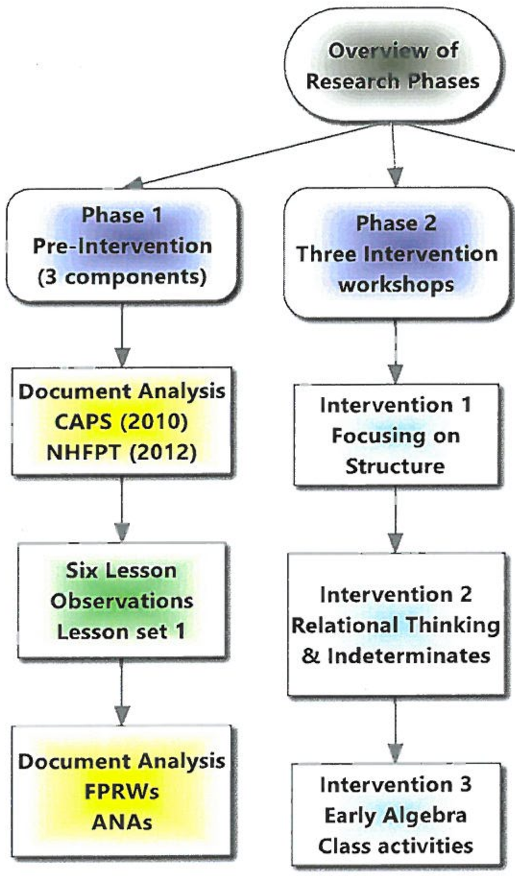

Figure 3.1: Flow chart of the three research phases showing the components in each phase.

The integrative approach to research is what underscores the methodological choices at the various phases of this research. A mixed model approach to the data was followed with primarily, but not exclusively, a sequential transformative strategy[18] (Creswell, 2003) for the data collection and interpretation. A mixed model provides greater flexibility and is more adaptable, as the quantitative part of the study could be used for generalisation of qualitative findings (Lobe, 2008). As Patton (1990) suggests: 'qualitative data can put flesh on the bones of quantitative results, bringing the results to life through in-depth elaboration' (Ibid, p.132).

[18] Collection and analysis of either quantitative or qualitative data first. The results are integrated in the interpretation phase, with the purpose to employ the methods that best serve the theoretical perspective.

The sequential transformative strategy is informed by the theoretical perspective and the advocacy for EA and the use of structure at FP when dealing with sequencing. The use of distinct phases in the research facilitates the implementation and the description of the results, with equal priority given to the qualitative and quantitative data. In phase 1 data was collected in three sequential stages: (i) document analysis of curriculum and support documents which informed (ii) the collection of the first set of lesson data which in turn necessitated (iii) a further document analysis of classroom resource texts. Phase 2 of the research was aimed at the implementation and development of EA and structure, and phase 3 comprised the collection of more lesson data to complete the iterative process of data collection in the sequential model.

A mixed model approach to this research study was chosen as it mirrors a natural way of information collection through integrating qualitative and quantitative methods (Ercikan and Roth, 2006) and has the potential to strengthen the rigour and enrich the analysis and findings (Srnka and Koeszegi, 2007). I report on the data through analysis, merging the qualitative and the quantitative, and by providing information on the occurrence of a specific phenomenon during phases one and three of the lesson data. For this purpose, I make use of co-ordinate pairs to quantatise the qualitative data, where the first entry indicates the percentage incidence of a concept in the first lesson set, and the second entry represents the percentage prevalence in the second lesson set. By doing this, I integrate the qualitative and quantitative nature of the data in the mixed model approach.

Design Research – Combining Developmental and Validation Approaches

According to Plomp (2013), 'design research is relevant for educational practice as it aims to develop research-based solutions for complex problems in educational practice or to develop or validate theories about processes of learning and teaching' (Ibid, p. 17). Plomp (2013) indicates that 'the research process incorporates systematic educational design processes', which are illustrated in Figure 3.2 on the next page. The design is cyclic in nature, and involves iterations when designing the intervention component until a balance between the ideals and the realisation has been achieved. In this study these iterations were followed at three levels in the design of the intervention, which will be described in the section that focuses on phase 2 of the research. The design research model that is followed in this study, which consists of

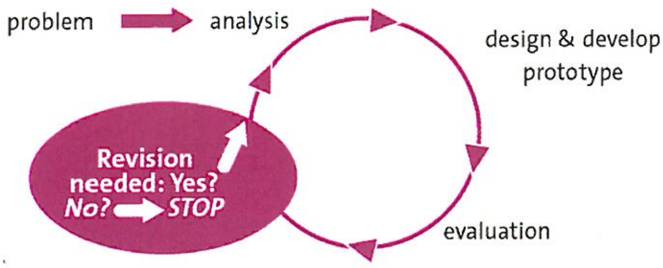

Figure 3.2: Iterations of systematic design cycles.

a preliminary round (phase one), an intervention (phase two) and a post-intervention (phase three), uses elements of a developmental study as well as a validation study (Plomp, 2013). The developmental component deals with designing a series of intervention workshops that will support teachers in developing their practice to enhance the notions of EA and structure. The validation component will explore whether this intervention enabled teachers to successfully deal relationally with sequencing at Grade 2 level. This model is appropriate as it allows for one research phase to be informed and shaped by previous phases and cycles through which the design was iterated.

Johnson and Onwuegbuzie (2004) posit that researchers should plan 'to develop a design that answers their own research questions within the constraints and boundaries of the study context' (Ibid, p.20). Furthermore Ercikan and Roth (2006) argue that 'research questions should drive the choice of research methods' (Ibid, p21) and the design and methods used to analyse and report data. This research employs a variety of research methods in order to address the research questions that it seeks to answer.

As outlined in chapter one, the three research questions that this study seeks to answers are:

1. To what extent and in what ways do the curriculum documents, the supporting materials and the materials used for teaching encourage and support teachers in their development of structural thinking and algebraic reasoning when working with sequencing?
2. How do teachers work with sequencing at a Grade 2 level?
3. How are teachers able to change or supplement their teaching after a series of workshops on algebraic reasoning and structural thinking?

Each of the research questions seeks to describe and understand 'what is' and discover the 'nature of things' related to (i) curriculum and supporting documents, insofar as the notion of structure and algebraic reasoning is concerned, (ii) how teachers deal with sequencing in Grade 2, and (iii) how teachers adjust their pedagogy after an intervention aimed at developing their understanding of structure and algebraic reasoning. A mixed model approach is appropriate as it provides answers to the research questions from both a qualitative and a quantitative perspective and works hand over hand to interpret and analyse the data by providing the type and frequency of a phenomena in the data sets.

In the sections that follow I will elaborate on the rationale for the methods that will be used to analyse and report on the data.

Research Methodology: Data Collection and Coding

According to Merriam (1998) and Yin (2003), the most important characteristic of qualitative research is that the primary instrument in the collection and the analysis of the data is the researcher. Qualitative data consists of 'direct quotations from people about their practice and experiences, opinions, feelings and knowledge through detailed descriptions of people's activities, behaviours, actions recorded in observations or extracted from various types of documents' (Patton, 1990, p10). Merriam (1998) and Yin (2003) also point out that the data collections that can be used in qualitative research are document study, direct observation, participant observation and physical artefacts. This study will include document analyses, direct observation and participant observation.

Document Analysis – Policy and Resources

Zhang and Wildemuth (2009) caution that the documents selected for analysis must be chosen appropriately so as to enrich the research process. The documents under review were selected for analysis because they determine policy in terms of CA2 and strongly influence classroom practice through providing teachers with activities to use in the classroom. The document analysis component of the research deployed a mixed model design to analyse the data and report the findings in order to address the first research question.

Content analysis as a research method is a systematic and objective means of describing and quantifying phenomena. It can also be deployed as a method for analysing documents (Elo & Kyngas, 2007). I used both qualifying and quantifying approaches in the document analysis as part of the mixed model approach to the document analysis. Zhang and Wildemuth (2009) argue that, when engaging in qualitative content analysis, a set of systematic and transparent procedures for processing data should be followed in order to support valid and reliable inferences. This involves the three stages of preparation, organising and reporting processes and results.

In the preparation stage, I extracted the sections that deal with CA2 in all of the documents under review. These sections were pasted into a document and organised according to the order in which they appear in each of the documents under review. These sections, which were selected because of their focus on CA2 and the broader notion of sequencing in the FP, formed the unit of analysis[19] for this component of the study. Once the sections of documents were prepared in the manner describe above, I continued to develop a coding scheme to arrange the data and organise overall themes that emerged during the initial analysis.

I firstly used open coding (inductive approach) to develop a list of codes based on criteria that emerged from the initial exploration of the data. The codes, which were developed from the inductive approach applied to the CAPS (2010) and the NHFPT (2012), were verified by using the constant comparative method (Glaser & Strauss, 2006) which entailed:

> (i) the systematic comparison of each text assigned to a category with each of those already assigned to that category, in order to fully understand the theoretical properties of the category; and
> (ii) integrating categories and their properties through the development of interpretive memos which *was parameterised through developing descriptors in the code memo*.
>
> (p.101: Italics added by author)

In the second round of coding, I deployed a deductive approach (Hsieh and Shannon, 2005, and Zhang and Wildemuth, 2009) to identify the components of the triadic notion (patterns, structure and algebraic reasoning). During the deductive coding phase my analysis of the relevant theory and research findings in the field guided my coding and descriptors for my individual codes. Since my analytical framework is based on the triad, I focused on these aspects in the documentation that was identified for analysis. I explored both the *manifest content* and the *latent content* within these documents (Elo and Kyngas, 2007, p109). The manifest content is that which is visible and on the surface – it refers to the obvious visible

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[19] The unit of analysis is the major entity that is being analysed in a study - it is the *what* or *who* that is being studied.

content. The latent content of a document refers to the meaning that underlies what is said or shown by the manifest content (Fraenkel, Wallen and Hyub, 2012).

After completing the coding analysis of the CAPS (2010) and the NHFPT (2012), I applied the same approaches to the remaining sets of documents, namely the ANA guidelines, exemplar items and assessment items as well as the FPRWs, since these texts formed the classroom resources that were used by the teachers in this study. In my analysis, I sought to determine whether the triad is visible in the surface content of the curriculum and supporting documents, and also whether the underlying meaning of this visible content facilitates the implementation of the triad in class.

The categories that were identified in the initial document analysis were: types of patterns, pedagogical components of patterns, objects and actions used in patterns, micro descriptions of patterns, instructions that accompany the activities on pattern and the cognitive demand of pattern activities. This analysis was done by hand and Microsoft Excel was used to record various codes that were assigned to the data. The reason this coding was done by hand was that the data relating to CA2, in all the documents under review, was spread over sections in the documents, and needed to be extracted from the various sections in order to combine these for coding. The method of combining the documents also made it easier to cross reference and compare codes across the different documents when applying the constant comparative method (Glaser & Strauss, 2006).

The final stage involved tabulation of categories and themes, and finally the provision of a description and analysis of the matrix and the findings. It is standard practice to use quotations to justify conclusions (Schilling, 2006). These quotations were taken from the documents under review and discussed in the findings and conclusions. I have, however, also made use of matrices, graphs and charts (Miles & Huberman, 1994) to display data.

Observation through Digital Video Recordings

A major source of my data collection is digital video recordings. Video recordings provide a rich source of visual data which can be viewed and reviewed as often as is necessary for analysis. Opie (2004) warns that people may consciously or unconsciously change the way they behave when they realise they are being observed. For this reason, the video equipment was discretely placed at the back of each classroom. The teachers' initial lessons on pattern, the intervention workshops, as well as the second series of six lessons, were digitally video recorded. The first round of recordings was spread out over a period of two months. This was followed by three intervention workshops which were recorded over a three-month period and a final round of lesson observations which were recorded over a six-week period. For the intervention workshops with the teachers, the recording equipment was focused on both the researcher and the participating teachers. Two sets of lessons were observed, and thus a total of twelve lessons, of an average length of eighty-five minutes each, were digitally recorded (See Fig 3.3 for details). The researcher's role was to record each of the lessons and make field notes where necessary. The aim of recording each of the lessons was to have a record of how the teachers taught pattern which could be accessed for further scrutiny and transcription. These videos were transcribed, coded and analysed. These processes will be described in detail in the sections that follow.

Teacher	Lesson 1	Lesson 2
1	73 min	69 min
2	86 min	108 min
3	78 min	73 min
4	104 min	95 min
5	75 min	68 min
6	79 min	112min

Figure 3.3: The duration (in minutes) of each of the digitally video recorded lessons.

Descriptive Approach to Data

In my study I subscribed to the descriptive approach by observing how teachers teach pattern before and after the intervention workshops. I described the activities and the pedagogy that they use to make sense of sequencing in general. The intervention component of my study is also described in detail and is grounded in current theoretical perspectives in the field related to the triad of pattern, structure and algebraic reasoning. The framework for each part of the intervention was informed and shaped by the needs of the teachers who participated in the study, and the intentional learning that was planned by the researcher regarding the triad. Thus, whereas the description of the first lesson set focuses primarily on the teachers' general approach to pattern, the description of the second lesson set concentrates on improvements to the initial pedagogy and particularly the pedagogical choices that were made to enhance the notion of structure in FP sequencing activities.

The Coding-Analysis-Coding Process

The application and re-application of codes to the data results in codification – a process that permits data to be 'segregated, grouped, regrouped and relinked in order to consolidate meaning and explanation' (Grbich, 2007, p. 21). Bernard (2006) states that analysis 'is the search for patterns in data and for ideas that help explain why those patterns are there in the first place' (p. 452). Coding is thus used to analyse and group similarly coded data items into categories of code families, and from there the process of identifying patterns and larger emergent themes in the analysis process continues. For the coding of the lesson data, I started with the initial coding using an inductive approach in which the data was labelled and coded according to general trends that emerged. During this initial round of coding, the data was broken down into first level categories and concepts that formed the master headings and sub-headings, which expanded during further coding and analysis.

Corbin & Strauss (2008, p. 55) contend that 'pre-established theories can inform, if not drive, the coding process itself'. Guided by the research focus of pattern, structure and algebraic reasoning, I embarked on a second round of focused coding in which I re-examined the initial codes and delved deeper into the data to expand on coding categories and develop further sub-categories and extensions of the initial open coding and code descriptors. This involved an in-depth search for the triad in the lesson data through open (data driven) and focused (theory driven) coding. A third round of coding was carried out to confirm concepts and categories, to ensure accuracy and to explore links and relationships that emerged between concepts and categories that were defined. During the fourth round of micro-analysis, influences and consequences of broader overlapping associations between categories in the lesson data were examined, and themes that emerged were refined. I used ATLAS.ti software to code and analyse the lesson data. Figure 3.4 (on the next page) shows the cycles of coding that were followed in the analysis of lesson data. Each round of coding was recorded and included in the code families that were generated using ATLAS.ti and these form the collective bank of codes that were assigned to the data during the analysis process.

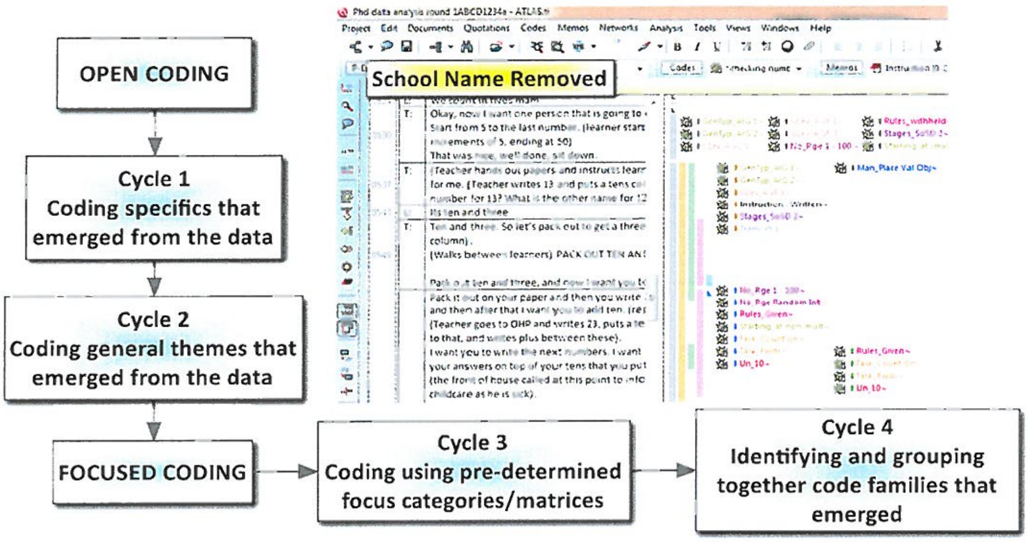

Figure 3.4: The cycles of coding that were followed to code the lesson data and the document analysis.

The Research Participants

The lesson data used in this study was collected while observing six Grade 2 teachers teaching pattern in three different schools. Each teacher taught two lessons on the topic of pattern, one during the initial round of data collection and a second lesson after the series of intervention workshops. The six teachers involved in this study all teach at former Model C[20] schools in the South African public school system and were selected using a purposive convenience sampling approach, based on their geographic proximity and demographic similarities to ensure homogeneity and obviate social and economic variables that may skew the findings. The researcher has, through previous contact, built good relationships with the schools and it made sense to continue working with them. All three schools are within a range of three to six kilometres from Tembisa, a township[21]. Of the schools' total learner populations, 68% come from Tembisa and travel to school by taxi or arranged community transport, or they walk to school. The remaining 22% of learners are from nearby communities or reside within the areas in which the schools are located. Collectively black learners comprise 98% of the schools' populations. The schools are public schools and are mainly funded by the government. They do, however, finance additional needs by collecting supplementary fees from parents, and are all classified as either quintile 4 and 5 schools. The

[20] Model C school are schools that served white learners only during apartheid and are now racially integrated. They are better resourced that township schools for black learners.
[21] The word 'township' is used to denote the residential areas that were created under the segregationist policies of apartheid to house black people.

schools are generally well resourced, but do not currently receive sufficient funding from government to maintain infrastructure and expand resources. Despite this, the schools are well maintained and learning resources are readily available and in good condition.

The teachers in this sample collectively had eighty years of teaching experience, ranging from three years to twenty-three years of experience, at the time of conducting the study (see Table 3.1 below). There are six teachers in the sample, two of whom have diplomas in teaching FP learners at colleges of education[22] in South Africa, one has a teaching degree from the University of Zimbabwe, and three have degrees obtained recently from different universities in South Africa. The average age of the sample of teachers is thirty-five years and six months, ranging from twenty-five to forty-five years of age. All the teachers in the sample are female, of whom three are white and three are black. Only four of the teachers matriculated with mathematics as one of their subject choices, while the other two qualified up to Grade 9 with school mathematics.

	Age	Race	Tertiary Qualification	Institution	Teaching Experience (years)	Home Language	Position	Level of school maths
T1	32	Black	Degree	University of Pretoria	9 (Gr 2/3)	Sepedi	Teacher	Grade 12
T2	28	White	Degree	University of Johannesburg	6 (Gr 2/3)	Afrikaans	Teacher	Grade 12
T3	25	White	Degree	North-West University	3 (Gr 2)	Afrikaans	Teacher	Grade 12
T4	40	Black	Diploma	Soweto College of Education	17 (Gr 2/4)	isiZulu	HoD	Grade 9
T5	43	White	Diploma	Johannesburg College of Education	22 (Gr 1/2/3)	English & Portuguese	Teacher	Grade 12
T6	45	Black	Degree	University of Zimbabwe	23 (Gr 2/3/4)	SiNdebele	Teacher	Grade 9

Table 3.1: A summary of the demographics of the sample of teachers participating in the study.

The teachers in the sample were selected based on their replies to invitations that were sent to six different schools in the area. To minimize the differences across participating teachers, only teachers who were academically qualified with either a diploma in teaching FP or a Bachelor of Education (B-Ed) degree with FP specialization, were selected. The participating teachers are all from the same school district, Ekurhuleni North, and are located in the same cluster of schools. The schools are administered by the same district office and officials. The study included teachers with varying levels of experience at teaching FP mathematics,

[22] The colleges of education have been incorporated into the universities and no longer exist as independent colleges.

especially at Grade 2 level. Two of the participating teachers also have experience in teaching up to Grade 4 mathematics. Only one of the teachers has a management position (head of department) in the school, and at the time was applying for a deputy principal position.

Data Analysis: The Three Research Phases

The aim of my study was to understand and describe how teachers taught and worked with sequencing at FP Grade 2. I investigated whether and how a structural approach to arithmetic in the domain of pattern (CA2) might enhance teaching and help teachers improve their instructional practices. There were three phases in the research study, which applied a mixed model approach to the collection and analysis of the data across the phases (Figure 3.1 on p.52). The phase components consisted of an initial document analyses, an initial set of six teaching observations, a further document analysis of teaching resources (phase 1), a series of three intervention workshops (phase 2) and a final set of six post-intervention lesson observations (phase 3). I will now proceed to describe the process of analysis in each of the phases in the research. I will give detailed information regarding the research methods that were used in the particular phases, the research question that was addressed by the research phase, the description of the aims in the particular phase, as well as the final reporting phase, where the mixed model approach was followed to interpret and report the findings.

Phase One: The Pre-Intervention Phase

The investigation started with a document analysis focusing on the curriculum statement CAPS (2010) and the supporting document NHFPT (2012) to determine what these two documents propose for the teaching of sequencing at FP level in CA2. During the first set of lesson observations, all six teachers used two classroom resources, namely the Foundation Phase Rainbow Workbooks (FPRWs) and the Annual National Assessment (ANA) workbooks, exemplar items as well as past assessment items. The ANA resources listed above are made available by the Department of Basic Education (DBE) to teachers to assist in the preparation of learners for the annual assessment that takes place once a year. These materials are comprised of a set of small booklets that contain exemplar items and guidelines to prepare learners for the assessment. The FPRWs are textbooks that are provided, free of charge, by the DBE and form part of the DBE's range of interventions aimed at improving the performance of South African learners in the first six grades. The FPRWs are intended to supplement textbooks (DBE, 2010) by providing additional exercises for learners. These materials formed the basic resources from which the activities that the six teachers engaged in the class when teaching sequencing were selected. It was therefore essential to understand what the policy documents, the supporting materials, as well as the textbook materials, propose in relation to CA2.

Creating inductive and deductive codes

Data coding in research methodology is a preliminary step to analysing data, which makes the process of data analysis easier. Applying codes inductively and deductively leads to a complementary approach to the coding of data, and often one approach leads to the other (Schutt, 2006). The document analysis used both inductive and deductive approaches, which were completed through four rounds of data coding described earlier in this chapter[23]. The term[24] by term activities and descriptions in the CAPS (2010) were coded, resulting in a total of fifty-seven different data- driven codes, later grouped into five coding families which described the data. These are shown in detail in Table 3.2.

	CODE FAMILIES				
	Types of Pattern	**Pattern Resource**	**Actions / Instructions of Pattern Activity**	**Pattern Pedagogical Resources**	**Micro Detail of Patterns**
INDIVIDUAL CODE ITEMS PER FAMILY	• Geometric Patterns • Number Patterns • Symmetry Patterns • Auditory Patterns • Noise Patterns • Game Patterns	• Shapes • Colour • Percussion • Repetition • Words • Pictures • Rhythm • Counters • Coins • Peg Boards • Numbers	• Describing • Copy & Extend • Copy, Extend & describe • Create own • Complete the given diagram • Fill in the missing numbers • Fill in the number cards • Complete the number chains • Fill in the number line	• Kinaesthetic • Concrete objects • Semi concrete objects • Count objects • Daily counting • Counting in groups of... • Tables • Input/output diagrams	• At least up to 100 (C) • At least up to 200 (C) • At least up to 1000 (C) • Forwards (C) • Backwards (C) • Number sequences • See the logic of the pattern • See the part that is repeated • Repeating groups where objects the same, position different • Repeating groups where objects are different, but repeat in the same way • Size of shape / numbers change in a predictable way • Groups where same size objects / shapes increase / decrease in same way • In 10's from any multiple of 10 (C) • In 5's from any multiple of 5 (C) • In 2's from any multiple of 2 (C) • In 1's from any number (C) • In 2's from any multiple of 2 (C) • In 3's from any multiple of 3 (C) • In 4's from any multiple of 4 (C) • In 100's from any multiple of 100 (C) • In 50's from any multiple of 50 (C) • In 20's from any multiple of 20 (C) • In 25's from any multiple of 25 (C) ***(C)** *related to counting activity*

Table 3.2: Coding items for the CAPS (2010) curriculum specifications and term descriptions in CA2.

[23] This was done on p.58

[24] 'Term' indicates a quarter of the academic year in a school. One year will have four terms.

The five coding families that were identified during the first round of inductive coding were: (i) the types of patterns as described in the CAPS document, (ii) the resources that can be used to illustrate patterns, (iii) the intended action based on the instruction accompanying the activity, (iv) the pedagogical resources suggested for patterning activities and (v) the micro details of pattern activities. I examined the CAPS document to identify the relations that was established between patterns, structural thinking and algebraic reasoning in CA2. I looked for instances where the notions of structure and the development of algebraic reasoning skills, through a focus on expressing generalities, were explicated and connected. This analysis was done to address part of research question 1, concerned with the CAPS (2010) National Curriculum Statement, and what this document makes apparent for the teaching of sequencing in CA2.

The NHFPT (2012) was analysed in order to investigate what this document describes in extension and support of what the CAPS national curriculum has outlined for CA2. The NHFPT document is an extension of the descriptors in the CAPS document, and is intended to provide guidelines and further discussion of what is already made available in the CAPS curriculum. For this reason, the extensions that the NHPFT provide were discussed along with their implications for the meaning it creates for the implementation of CA2. This discussion is descriptive in nature and is based on inductive and deductive analytical procedures. The analysis of the CAPS and the NHFPT provided me with evidence that the concepts of structure and algebraic reasoning are included in both of the documents.

The first lesson set data analysis informed a decision to analyse the FPRWs and the ANAs, since it was established that these resources shape practice. This decision was based upon the realisation that these classroom resources shape practice in the FP mathematics class. Particular attention was paid to the tasks that are made available for learning in CA2 in the materials and texts with which participant teachers worked. All of these classroom texts are important in order to understand how teachers teach pattern, why they teach pattern in a particular way and what supports, shapes or informs their choices. As for the initial document analysis, the focus in all of the documents under review was CA2. The analysis of the classroom resource texts was undertaken to address research question 1. The inductive codes are represented in Table 3.3 (on p.65) which includes the code families as well as all the individual codes that were used to code the data.

CODE FAMILIES CREATED DURING TEXT ANALYSIS

	Pattern Resource	Type of Pattern	Pattern Detail	Cognitive demand	Instructions accompanying the activity
INDIVIDUAL CODE ITEMS PER FAMILY	• Number • Shape -pattern using shapes -tessellation / transformation of a unit of repeat • Alphabet • Drawing • Colours • Rhythmic Claps • Pictures of Objects • Money coins • Symmetry (reflective / Rotational) • Time	• Different Pattern • Number pattern • Repeating pattern • Repeating pattern with growth	• Forwards • Backwards • With the grain • Across the grain • Growth number	• **Difficulty 1** Questions have three consecutive numbers, or the rule is given. • **Difficulty 2** Questions do not have three consecutive numbers, and the rule is withheld. • **Difficulty 3** First and last number in sequence given with 3 or more spaces left open – rule withheld. • **Question unclear**	• What comes next • Fill in the missing numbers • Fill in the missing EVEN numbers • Fill in the missing ODD numbers • Complete the patterns • Complete each row using the pattern • Copy the patterns into the spaces • Extend the pattern • Colour the beads as you count in... • Clap the pattern • Describe each pattern in words • Colour the pattern that comes next • Draw the next pattern • Draw your own pattern • Complete the following.... (condition) • Create your own colour patterns... • Count in... • Draw or paste pictures of things that came in .... • Use number line to write a pattern • Match the pattern • Complete these number lines • Trace the pattern and then colour it • In what are we counting? • Write the (fives) .... From ... To ... • Tick all the .... • Write the numbers that are both in ... And ... • Colour in the pattern...what do you notice • Change hrs to minutes and vice versa • Count on the number line • Where do they belong? • Count from ... To ... • What's the jump? • What is the pattern • What's the difference between the numbers? • Do you notice the pattern? • Describe it (the pattern) • Write down the pattern • What pattern do you see?

Table 3.3: An illustration taken from the initial open coding cycle of the classroom text resources used by the teachers.

The code families that were generated from the inductive coding process were based on the resources that are recommended for the teaching of patterns, the types of patterns that are mentioned in the texts, the details of the patterns that are discussed, the cognitive demand of the pattern activities that are made available and finally the instructions that accompany each pattern activity. A total of 62 codes were created which were grouped into five code families.

Transcription and coding of pre-intervention lesson data

Transcripts possess 'temporal-historical dimensions' (Duranti, 2007, p.302) and it is essential to prepare these as accurately and as descriptively as possible, before starting the process of analysis. Transcripts are used for analysis, but are also evidence of practice and of procedures followed in that analysis (Duranti, 2007) and further provide evidence of the researcher's analytic claims (Ashmore & Reed, in Davidson, 2009). The series of data emerging from the initial six lessons was digitally recorded and transcribed, and subjected to four rounds of rigorous inductive and deductive coding. The transcriptions sought, as far as possible, to maintain morphological naturalness[25] (McLellan, MacQueen, and Neidig, 2003). The transcripts were prepared by the researcher using the basic principle of transliteration into text of the events, actions, activities, artefacts, writing, discourse, pedagogy and the semiotic resources employed by the teachers and the learners. The transcripts also included screenshots that illustrated aspects that were difficult to describe through transcription, such as samples of learner work or teachers' instructional materials written on the board or signalled to the class. Each transcribed file was checked again by the researcher once the initial transcription was concluded to ensure the accuracy and the completeness of each transcript. Once the transcripts had been finalised, they were prepared for coding using ATLAS.ti software.

During the initial analysis an inductive method of open coding was followed to identify themes across the six lessons in an attempt to analyse practice as accurately and extensively as possible. Figure 3.5 shows (in blue) an example of one such a problem where data-driven open codes were assigned to the data during analysis. This process was undertaken to aid in finding answers to research questions two and three. A further three secondary rounds of coding deployed a deductive approach of focused coding in the analysis of the teaching of

[25] Morphological naturalness means preserving all possible contextual information about the observed lessons that goes beyond the literal discourse between the teachers and learners. This implies that word form, commentary form, and punctuation should strive to capture the original speech presentation as closely as possible, to include the unspoken nuances and actions during the lessons.

pattern. In the analysis stages I organised the codes into categories and interpreted the data according to emerging themes that recur and show relevance to the study.

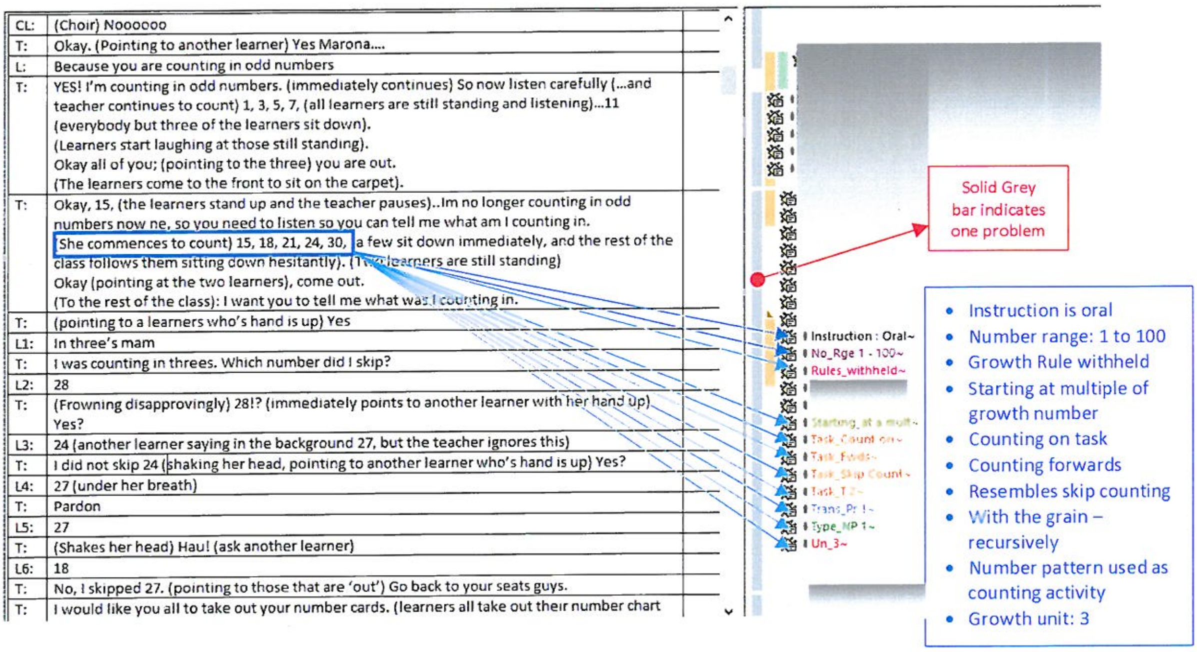

Figure 3.5: An illustration taken from the initial open coding cycle of lesson set 1 showing ten codes assigned to the problem posed by the teacher.

Phase Two: The Intervention Workshops

The intervention phase of the research was conceptualised to follow a design research approach as explained earlier. The intervention took teachers through a series of discussions and activities related to structural and relational thinking in order to demonstrate how a structural approach can enhance algebraic reasoning skills in young learners. The workshops were activity-based and theoretically grounded in the three habits of mind as proposed by Driscoll (1999) (see Chapter 2, p.26 for details). In all the activities, the focus was an investigation of relationships that foreground structural extensions of arithmetic. The series of workshops were thus designed to further the goals of early algebra through introducing teachers to the concept of relational thinking and structural extension when working with sequencing in CA2 in Grade 2 Mathematics. To accomplish this, teacher knowledge about working relationally and structurally was extended in the first two workshops and in part of the third workshop. The intervention workshops thus had a dual purpose: to extend teacher knowledge about structure and structural extension by providing relationally rich mathematical experiences, and secondly to extend and enrich classroom pedagogy by reexamining CA2 activities for Grade 2. Teachers were afforded the opportunity to revisit the types of activities that were made available to learners from texts used in class during lesson set 1. Teachers worked on adjusting and supplementing these activities to promote algebraic reasoning as a fundamental characteristic of sequencing problems. These re-worked activities formed part of the design cycles and processes that were followed to improve classroom practice and texts, and were used by the teachers in the second lesson set. During the workshops the teachers also revisited their pedagogical approaches to teaching patterns. The workshops were designed around the teachers' needs and the intended learning. Re-design to accommodate emergent issues in particular workshops also took place.

Cycles in planning the workshop design

The iterations, and the cycle of design and development, were followed over a period of time, as well as between each of the workshops. The dearth in working relationally in CA2, which was identified in the first lesson set data, as well as the document analysis that focused on the textbook resources, was the key component that the series of workshops was designed to address. There were two levels of design that took place in phase two of the research, the initial workshop and the following two intervention workshops and the activities that were used in class. The initial two workshops that formed part of this study were informed by previous workshops with a similar focus. The prior workshops were designed for FP teachers and presented at the Association for Mathematics Educators of South Africa (AMESA) regional conferences over a period ranging from 2009 to 2013: With cycles of the design, the workshops were improved and adjusted to work at an increasingly deeper conceptual level with FP teachers. The design of the first two workshops in this study was an extension of the designs of the previous AMESA workshops and included a theoretical component based on current research in the field. I also considered the analysis of the first lesson set and included a focus on working with structure in the FP classroom. From the initial lesson analysis, it was evident that a more pronounced focus on structure was required to support the aims of the study. The first two workshops have, therefore, undergone three iterations of development to take the form that they have in this study.

As the aim of the workshops was to develop collectively the notion of structure with FP teachers in CA2, there was also a cycle in the design across the workshop sequence which was largely informed by the responses and needs of the teachers as these became evident during the preceding workshop. The second workshop was planned to develop the notion of working relationally with an unknown quantity, based on the experience of introducing an indeterminate in the first workshop. During workshops one and two, teachers often referred to the (in)ability of their learners to work at the level that was suggested by some of the activities in the workshops. The third workshop gave the teachers an opportunity to reflect on the activities that they used in the first set of lessons and in the previous two workshops. These activities were discussed and adapted to focus on the triad of pattern, structure and early algebra and some of the activities were used in the second, post-intervention, round of lessons. In this sense, the cycle that the teachers were progressing through was one of new learning, revisiting pedagogical approaches to teaching pattern, looking at activities that are made available and refining and adjusting these to enhance structural and relational thinking. In order to create a clear horizon view of the landscape of pattern in CA2, the workshops were planned to enhance the understanding of the teachers with regards to the potential that a relational and structural approach to pattern offers for early algebra and a relational understanding of mathematics at FP level.

The content of the intervention workshops

As mentioned, the final workshops were designed after analysis of the first set of lesson data. The design was in response to the practices of teaching pattern with the specific purpose of providing an experience to teachers in structural thinking and the pedagogical shifts that would enhance structure. With these foci in mind, workshop one was planned to extend teacher knowledge of structure and structural extension. The workshop dealt with a conceptually rich activity that uses a three by three calendar square to explore structurally and focus on contextual considerations to deepen structural knowledge in arithmetic. In this workshop the notions of surface and systemic structure were introduced and teachers were afforded the opportunity to generalise through a process of objectification[26], as described by Radford (2010). The second workshop featured three activities to introduce participants to the notion of working relationally with unknown quantities. Various pedagogical tools were used to ground the notion of input-output relations and to build on developing functional thinking. Teachers were introduced to the notion of doing and undoing and the mathematical practices that operate structurally to enhance processes that function to reverse certain operations. This workshop was also designed to further teacher knowledge in the area of structural extension and early algebra, and to provide teachers with sample activities that enhance the habits of

[26] The workshop activities are discussed in detail in Chapter 5.

mind as proposed by Driscoll (1999). The third and final intervention workshop foregrounded regularity in Grade 2 pattern activities. In this workshop internet-based activities were used to introduce teachers to sample problems that enhance algebraic reasoning skills at Grade 2 level. Magic squares were used to broaden structural extension of activities where magic squares are used in class, and finally generalised objects of knowledge were introduced for teacher knowledge extension. The final focus in workshop three was to revisit activities and pedagogies that the teachers used in lesson set one, and to provide a pedagogical approach that enhances the notions of structural extension and generalisation when engaging with these activities.

Phase Three: Post Intervention Lesson Observation

The post-intervention stage consisted of a further six lesson observations to determine the extent to which, and the ways in which, teachers changed their teaching of pattern after the series of workshops described earlier. These lessons were also digitally video recorded, transcribed and coded in the manner described for the pre-intervention lessons. The recording took place over a six-week period. In order to address research questions two and three, a clear categorisation was necessary to bring the theory and practice of teaching together. During the analysis, I looked for similar patterns between what the teachers did in a lesson and across the various lessons, and also what they did that was unusual, particularly regarding the teaching of sequencing in general. This also entailed comparisons across lesson set one and two. I was chiefly interested in looking at the opportunities that teachers utilised to work with structural thinking to enhance algebraic reasoning skills in the domain of pattern. Therefore, I identified themes and sub-themes, built and applied coding schemes, described trends and phenomena that became apparent and compared trends and data items. The framework that I used was developed and refined by repeating and revising the analysis and coding, which is described in the sections that follow.

Extended Inductive and Deductive Coding Matrices

In the sections that follow, I will describe the deductive coding matrices that were used in this study. These matrices were used for the document analysis and/or the lesson data in phase one and phase three of the data collection and analysis. The coding matrices that were designed for this study dealt with patterns, pattern tasks, structural and relational thinking, visual representation of patterns and levels of generality. The deductive coding themes were developed from theory in the field, and then revisited with the inductive coding to merge themes and develop the final coding matrices that were created to address the research questions. The coded items in the qualitative data were later quantitised and merged in the interpretation process (Onwuegbuzie and Teddlie, 2003) to indicate the frequency with which a particular concept was present in the data.

Coding matrix for pattern

The coding matrix for patterns consisted of two defining categories which placed activities into the two categories suggested by Liljedahl (2004): repeating pattern (illustrated in Table 3.4(a)), for the patterns with cyclic structure; and number pattern (illustrated in Table 3.4(b)), for those patterns that had embedded growth structure[27].

The patterns matrix initially consisted of six types of number pattern entries and three types of repeating pattern entries (codes NP 1 – 6 and RP 1 – 3). After the first lesson set was analysed, there were a few categories that emerged in addition to what had already been included on the matrix. As a result, the pattern matrix was re-worked and extended by adding components to the number pattern section of the description and codes (NP 7 – 11). Thus, in total, there were fourteen codes that described the repeating patterns and the number patterns that were used in class and that were also located in the resource documents under review. Tables 3.4 (a) and (b) illustrates the details of the coding described above.

REPEATING PATTERNS		
Literature / Definition	**Code**	**Description**
Patterns with a discernible unit of repeat. Contains a cyclic structure. Generated by the repeated application of a smaller portion (core) of the pattern, which is never interrupted. Different patterns can have the same form.	**RP 1**	Uses repeating patterns with a discernible unit of repeat.
	RP 2	Varies repeating pattern using shapes, symbols, numbers etc.
	RP 3	Makes the cyclic structure available.

Table 3.4(a): Coding matrix for repeating patterns used in the analysis of the documents and lesson data.

[27] This is discussed in detail in Chapter 2, p 32-34.

NUMBER PATTERN		
Literature / Definition	**Code**	**Description**
Patterns constructed solely on numbers. This type of pattern cannot be transferred to a non-numeric pattern without the loss of some crucial property of the pattern. The pattern consists of growth where the growth is either multiplicative or additive, positive or negative.	NP 1	Number patterns are used as counting activities.
	NP 2	Growth patterns are used with first difference constant.
	NP 3	Patterns of the type a, b, c, ... The first three consecutive numbers are given and learners are required to continue the pattern.
	NP 4	Patterns with intermediate terms missing: a, _, _, b, c, d,_
	NP 5	a , _ , _ , _, b (beginning and end given, with a given number of spaces in between)
	NP 6	_, _ , a, b, _, _ (A consecutive pair is given with terms to both sides asked)
	NP 7	Growth pattern, multiplicative growth used.
	NP 8	a, __ , b: Requiring the calculation of the mean number.
	NP 9	Teacher gives the rule and random numbers, asks for surrounding numbers.
	NP 10	One number given, learners create own pattern and rule.
	NP 11	Rule given, learners required to create own pattern.

Table 3.4(b): Coding matrix for number patterns used in the analysis of the resource documents and lesson data.

Coding matrix for types of number pattern tasks

Initially there were only three types of tasks that were identified (T 1 – 3), but soon this list grew to include ten descriptors of the tasks that teachers were using in class, and also tasks made available for learning in the classroom text resources. These additions were based on describing the micro-elements that tasks comprised. These emerged through the extension of the analysis of the resource documents to include counting and ordering activities which did not form part of the focus on sequencing in CA2. Working with positive and negative growth in number sequences was added to the types of sequences, even where these included activities that resembled counting forwards and backwards in the pattern class. Table 3.5 shows the details of the matrix that was used to code and analyse the types of tasks that were used in the lessons and also those that were included and described in the documents under review.

	Literature / Definition	Code	Description
Task focus	Types of tasks that are made available: identify, copy, extend and create own patterns. Tasks work recursively (with the grain) or using covariance (across the grain)	**T 1**	Types of tasks that are made available: identify, copy, extend and create patterns.
		T 2	With the grain (working recursively with growth) to extend pattern.
		T 3	Across the grain (towards generalization by focusing on covariance / functional thinking)
Growth particulars	Working with a sequence of numbers where the numbers become smaller when reading from left to right / Counting backwards	**Tsk_Bckw**	Counting Backwards / Sequenced numbers get smaller.
	Working with a sequence of numbers where the numbers become bigger when reading from left to right / Counting forwards	**Tsk_ Fwds**	Counting Forwards / Sequenced numbers get bigger.
Counting	Counting is required, starting at a random point and counting on.	**Tsk_Count on**	Counting from a random starting point and applying the regularity from this point onwards.
	Skip counting is used as a number pattern activity	**Tsk_Skip Count**	Counting in twos, threes, fours etc.
	Repeated addition is used to work with number pattern with constant growth structure	**Tsk_Rep Add/Subtr**	Pattern is taught as repeated addition or subtraction.
Random Foci	Numbers are ordered according to size (big to small)	**Tsk_Order act**	A sequence of three numbers that need ordering using a regularity that is noticed and explicated.
	Teacher makes up word problems in an attempt to provide a quasi-contextual problem	**Tsk_Word Prob**	Impromptu word problems made up and presented to learners.

Table 3.5: Coding matrix for number pattern tasks used to analyse resource documents and lesson data.

Coding matrix for structural and relational thinking in lesson data

When considering the notion of structural development, I looked particularly for instructions or evidence that a relational view of numbers was necessary and possible. Structure is also explicated by using objects that communicate regularity. This could be problematic in itself, as the structure will remain at the indiscernible level if not used and made visible by the teacher.

This is similar to the idea of unit construction in shape and space, where the pre-determined units that are often given in activities mask the structural aspects which learners must discover through a process of guided re-invention (Freudenthal, 1990). Through carefully selected activities, which create the need for some formal regularity to be used in a relational way, structure and structural extension become the vehicles that shape mathematical activity into more sophisticated forms. A focus on structure gives learners access to more advanced mathematical thinking. It requires the teacher to emphasise process, rather than product. The codes in Table 3.6 were applied to the lesson data sets in this study.

		Literature / Definition	Code	Description
Arithmetic to Algebraic thinking	Process driven	Instruction foregrounds process when working with arithmetic expressions. Focus is on the process that is engaged when solving problems. (How / Why...)	PR 1	Structure is used as procedures that are performed and foregrounded when working with arithmetic expressions.
	Product Driven	The final solution that is produced at the end of the process. The focus of instruction is getting the correct answer. (What...)	PR 2	Arithmetic instruction is focused on and driven by the need to arrive at a final solution that is correct.
	Relational Thinking	Structure is used on a level that does more than simply recognise the elements and properties of a relationship. Mathematical properties are used, explained and connected. Focus is on different ways in which to perform a calculation	Rel 1	Structure is used on a level that extends understanding beyond the surface features.
			Rel 2	Properties are used, explained and connected.
			Rel 3	Focus on different ways to perform a calculation / operation.
Stages of Structural Thinking	Pre-Structural	Responses and representations lack any evidence of mathematical or spatial structure	SoSD1	Responses do not show any evidence of structure being used.
	Emergent Structural	Responses and representations contain some elements of structure. (Inventive semiotic stage)	SoSD2	Responses and representations contain some elements of structure.
	Partial-Structural	Demonstrate some aspects of mathematical notation or symbolism. Make use of spatial features such as grids or arrays in order to communicate structural and mathematical thinking	SoSD3	Make use of spatial features (grids or arrays) in order to communicate structural thinking.
			SoSD4	Demonstrate some aspects of mathematical notation or symbolism.
	Stage of structural development	Ability to integrate the mathematical and spatial features which will allow for generalisation of patterns algebraically. Possesses extensive knowledge about mathematical objects, the relationship between these objects as well as the properties of the objects. Uses forms of arithmetic to identify structure	SoSD5	Integrate mathematical and spatial features which will allow for generalisation of patterns algebraically.
			SoSD6	Possess extensive knowledge of mathematical objects, the relationships between objects and the properties of objects.
			SoSD7	Uses forms of arithmetic to identify structure.

Table 3.6: Coding matrix focusing on structural features of activities made available in the lesson data.

Since the primary focus of this study is the use of structure in the Grade 2 number pattern classroom, the codes in this table were assigned by using a variety of theoretical perspectives drawn from recent academic studies on structure. The stages of structural development mirror the work done by Joanne Mulligan and her colleagues in the PASMAP over the years.

Coding matrix for visual representation of pattern in lesson data

Teachers used different ways to signify important aspects related to the process of sequencing. The visual representations, which emerged when analysing the data items, can be divided into four broad categories: number charts, structure, manipulatives and semiotic resources as shown in Table 3.7. A popular tool that was used was the number chart, as it was used in counting activities and also to work with structural aspects of counting. There are two codes (Vis1 and Vis2) for the use of the number chart which excludes any reference to structure whilst engaging the chart in class activity. Where the chart was used structurally, it was coded using the appropriate codes in the structure and relational thinking category.

	Literature / Definition	Code	Description
Number Chart	Using the number chart can blur the lines between growth and repeating pattern. Patterns appear as arrays on the chart which removes focus from growth to repeat.	Vis1	Using the number chart (100) to count with.
		Vis2	Using the number chart (100) to identify patterns as arrays on the chart.
Structure	Giving first three consecutive terms in a number sequence and giving the growth rule, can resemble counting activity and blur the lines between counting and more advanced notions used when dealing with sequencing numbers.	Vis3	Number sequences with a, b, c and rule given - counting on activities not explicating structure.
		Vis4	Number sequences with a, b, c and rule given - counting on activities explicating structure.
	Number lines have embedded structural aspects that can be used to develop both single variant and covariant change in sequencing activities.	Vis5	Using a number line.
Manipulatives	Manipulatives are used in ways that do not foreground sequencing notions and distort the intended cognitive activity. They may also be used inappropriately and mask the development of advanced mathematical ideas.	Vis6	Using an inappropriate manipulative, or using a manipulative inappropriately.
Semiotic Resources	Structural characteristics are foregrounded using various semiotic resources. Growth number is foregrounded as a key part of the developing sequence.	Vis7	Consecutive numbers or spaces are given on board and hoops or 'plus numbers' are used to explicate structural aspects.

Table 3.7: The coding system that describes the visual representations used by teachers in the sequencing class.

Only one of the teachers used the number chart to explicate structural aspects. This is discussed in detail in chapter six. Teachers also worked with sequences where consecutive items, as well as the growth rule, were given, but then continued working with these sequences in two different ways. The first way involved either using the growth number to indicate explicitly and to discuss the recursive growth in the sequenced items, or secondly by failing to draw attention to these aspects and merely focusing on the correctness of answers provided by the learners. Table 3.7 shows the details of the coding system.

Teachers also used number lines extensively. I coded any use of the number line and combined this code with codes that indicate relational and structural use of the number line. The use of manipulatives ranged from number cards, place value objects, the abacus, Dienes blocks, magnets, counters, digital clocks, toy cars and animals, necklaces grouped in tens, rulers and balls. These manipulatives were used extensively and mainly in lesson set one. The code that was assigned indicated only the ineffective use of a manipulative. It was also necessary to note when teachers used symbols, words or gestures to explicate structural aspects on the board, as these semiotic resources were used to explicate structural aspects related to growth sequences. Such actions were mainly focused on advancing a recursive view on sequencing numbers.

<u>Deductive coding matrix for levels and types of generality in lesson data</u>

Generalisation lies at the heart of mathematical activity (Liljedahl, 2004). The triad of pattern, structure and algebraic reasoning forms the means that develops the habits of mind that learners need to seek for the general in the specific (Mason 1996). I used the work of Luis Radford to develop the coding system that was used for levels and types of generality. The analysis took on a deductive form as Radford's theoretical perspectives and contributions in the field of generalised mathematical thinking were used to describe and analyse aspects of the general that occurred in the lesson data. The types of generality used were: naïve induction, arithmetical and algebraic generalisations. When I was coding the lesson data, it soon became evident that there were different forms of naïve induction that emerged during the classroom discussions.

Learners would provide an answer and change it immediately if the teacher did not acknowledge it as correct, or when the class responded negatively to the answer. Learners would also provide a correct answer to a question from the teacher, but then fail to provide an

		Literature / Definition	Code	Description
Types of generality	Naïve Induction	No attempt at noticing a commonality between the terms. Learners resort to guessing to create a rule to express the pattern. These rules are merely hypotheses. Naïve induction does not lead to algebraic generalisations. (Trial and Error / Guessing)	NI 1	Learners guess and change their answers immediately when the teacher does not confirm or the class's answers do not concur.
			NI 2	Learners guess the correct rule but cannot provide a mathematical reason for their answers when asked to do so.
			NI 3	Learners guess the correct answer but apply the rule by using their number chart.
	Arithmetic	Operates on or with number. Noticing a commonality in given particular terms, and forming a genus. Fails to provide a direct rule that generalises the pattern.	ArG1	Operates on or with number
			ArG2	Noticing a commonality and forming a genus
	Algebraic	Algebraic generalisations operate on or with the unknown. This genus must crystalise into a schema to be algebraic. A direct rule that can be used as an expression to calculate any general term of the pattern, is provided. The alphanumeric system is used to express generality.	AG 1	Genus crystalises into a schema
			AG 2	Provide a direct rule that can be used as an expression to calculate any general term of the pattern.
Levels of generality	Factual	Operates at level of numbers and facts. Indeterminate remains unnamed. Generalisations rest on actions performed on numbers. Actions made up of words, gestures, specific perceptual activities.	AGF1	Operates at the level of numbers and facts
			AGF2	Generalisations rest on actions performed on numbers
			AGF3	Performing actions on numbers using words, gestures and perceptual activities.
	Contextual	Indeterminate is named. Goes beyond the particular and generalisations deal with a new object: a general which is named through an embodied and situated description of it. Noticed commonalities in the perceptual, formed a general concept (a genus) which applies beyond the perceptual.	AGC1	Indeterminate is named. General object named and described. (Goes beyond particular figures and generalisations deal with a new object, which is a general figure).
			AGC2	Noticing commonalities using the structure of the object.
			AGC3	General objects are named through an embodied and situated description of them.
			AGC4	Formed a genus and can apply it to all the terms in the sequence.
	Symbolic	Indeterminate is named and expressed through alphanumeric symbols. Formal algebraic representation (formula) representing all terms in the sequence. Generalised object crystallised into a schema.	AGS1	Letter symbol is used as placeholder for an unknown and also as a generalised representative of a range of values (alphanumeric system).
			AGS2	Generalised object crystallised into a schema.

Table 3.8: Coding matrix focusing on types and levels of generality.

explanation for how they had arrived at the answer. Such correct answers usually did not lead to any further exploration and were often arrived at merely by following arrays on the number chart. These arrays had been coloured in when learners had used the chart during counting activities. For a code to be an algebraic generalisation, symbolic forms of expressing regularity had to be present in the work. The codes that I used most frequently were the arithmetic generalisations, as most of the generalisations happened in the domain of number. The full list of codes and descriptions is indicated in Table 3.8.

In items coded as arithmetic generalisations, learners and teachers expressed some form of generality that existed between the terms in the sequence, using mostly the recursive relationship that existed in the sequence of numbers. The generalisation happened at the level of 'language' and did not include any symbolic notation. The formation of a genus was evident where the generality was expressed in words (saying). This generality can also be expressed mathematically as counting in twos becoming adding two to one number in order to obtain the next number. Here learners and teachers were required to operate with the observed regularity through carrying out processes and performing actions with the sequenced items. Learners also performed operations to identify commonalities between terms. Such actions were either coded as factual or contextual, depending on the level of sophistication of the generalised object.

Further open coding matrices for lesson set data

As indicated earlier in this chapter, various rounds of inductive and deductive coding were carried out. During the inductive coding processes, a variety of noteworthy components to lessons in the two lesson sets were identified. On the basis of these observations trends that required further follow up were noted. For this purpose, codes were assigned during the inductive coding process. These codes signalled the starting points and the range over which sequences were created as well as the growth unit size, the use of the rule underpinning the sequence structure, manipulatives that were used to build sequences and also general instructional attributes. These codes are included in the matrix illustrated in Table 3.9 and were used to interrogate the pedagogical attributes that are discussed in Chapter 6 in the following illustrations:

- Figure 6.1, p.191 and discussion on p.193 – Written / Oral Instruction
- Figure 6.3, p.204 – General attributes of sequencing
- Table 6.3, p.212 – Most frequently occurring attributes for growth sequences

- Table 6.4, p.214 – Starting ranges of sequencing for each teacher
- Figure 6.6, p.215 – Illustrating the starting intervals per teacher
- Figure 6.7, p.219 – Combining common attributes in growth sequences per lesson set.

Starting Point of Sequence	
Code	**Descriptor**
Starts at 0	Sequence starts at 0.
Starts at 1	Sequence starts at 1.
Starts at multiple	Sequence starts at a multiple of the growth number.
Starts at non-multiple	Sequence starts at a non-multiple of the growth number.
Starts at smallest multiple	Sequence starts at smallest multiple of the growth number.

Number Range of Sequence	
Code	**Descriptor**
No range 0-100	Numbers used that lie between 0 to 100
No range 1-100	Numbers used that lie between 1 to 100
No range 1-200	Numbers used that lie between 1 to 200
No range 100-200	Numbers used that lie between 100 to 200
No range Random interval	Numbers used that do not fit or cross over ranges above

Growth Number Size	
Code	**Descriptor**
Un 1	Growth Number is 1
Un 2	Growth Number is 2
Un 3	Growth Number is 3
Un 4	Growth Number is 4
Un 5	Growth Number is 5
Un 6	Growth Number is 6
Un 7	Growth Number is 7
Un 10	Growth Number is 10
Un 11	Growth Number is 11
Un 100	Growth Number is 100
Un 1000	Growth Number is 1000
Un Dec	Growth Number is decimal number

Rules and Sequencing	
Code	**Descriptor**
Rule asked then given	Teacher gives the rule and then ask for the rule.
Rule given then asked	Teacher instructs learners to count in *'a'* then requires learners to identify the rule as add *'a'*.
Rule asked	Teacher asks learners to look for the rule and to verbalise it
Rule given	Teacher gives rule and then ask for a sequence to be made using the rule
Rule withheld	No rule is given by the teacher and learners expected to find the growth rule.

Manipulatives	
Code	**Descriptor**
Man-everyday objects	Teachers used everyday objects such as analogue clock, calendar, toy balls.
Man-number objects	Teachers used number objects such as toy animals with four legs, or toy cars with four wheels
Man-ordered number objects	Teachers used objects with an ordered number range such as a number chart, a number lines or tables.
Man-place value objects	Teachers used objects with a place value structure such as the Abacus, Denes blocks, Place value number cards, unit counters, etc.

General Instruction	
Code	**Descriptor**
Instr - Oral	Teacher uses only oral instructions to teach – nothing is written on the board to explain sequencing features.
Instr -Writ	Teacher uses both oral and written instruction to explain relational features in sequencing.
Instr - LQ	Teacher makes use of learners' contributions to extend understanding in class – both correct and incorrect contributions are explored.
Instr - Focus	The lesson is started by naming what the focus will be for the day. Either by writing on the board, or by explaining orally what the focus will be.

Table 3.9: Inductive codes generated from the lesson data.

Coding to Percentages

Part of the process of analysis, involved converting codes into percentages to use in the discussions of the findings of the study. These percentages are used in chapters 4, 6 and 7 to 'quantitise' aspects of the lessons that were analysed. These percentages derived from analysis of the different characteristics that were grouped into code families to describe the attribute(s) of lessons and tasks upon which this study focused. The totals that were used to convert the tallies (from coding) to percentages are mentioned in each chapter. For example, in Chapter 4 the key focus is on the enumeration and classification of activities in the different resources, so the totals differ depending on the focus. The total number of activities in the FPRWs was 533 of which 47 were classified as different patterns, 395 as growth patterns, 83 as repeating patterns and 8 as patterns that shows growth using shapes. The total number of activities in the ANAs was 196 of which 3 were classified as different patterns, 152 as growth patterns, 30 as repeating patterns and 11 as patterns that show growth using shapes. The latter category in both texts included discrete parts of a shape pattern growing spatially in quantity, as well as a continuous shape showing growth using a size that increases[28]. These categories are discussed later in Chapter 4 when comparing categories across the texts, so the totals shift to the number of activities in each category of pattern activity. Across the chapters the totals that were used to convert to tallies to percentages are mentioned in each case.

Summary

The research followed a mixed model design research approach which advocates the systematic integration of quantitative and qualitative data. The three phases of the research worked with a variety of research methods to address the research questions. The coding throughout the study was both open (inductive) coding, which was data driven, and focused (deductive) coding, where the research in the field informed the coding of the data. After the data was coded, certain coding families were identified from themes that emerged in the data. Table 3.10 shows a summary of the research phases (RP) that were undertaken, the research questions (RQ) that were addressed in each of the phases as well as the methods of analysis that were used in each of the phases.

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[28] These two categories are discussed and illustrated in Chapter 4, p.99, Fig 4.12.

	RQ 1	RQ 2	RQ3	Transcription	Open Coding of data items	Focused Coding of data items	Manual coding using Excel	Coding using Atlas.ti	Document analysis	Lesson observations	Intervention workshops
RP1											
RP2											
RP3											

Table 3.10: The research phases, research questions and methods of analysis.

The chapters that discuss the data analysis, chapters four, five, six and seven will make use of a method that quantifies the occurrence of certain phenomenon, whilst working with qualitative data. This method was chosen to integrate both types of data (qualitative and quantitative) and provides the reader with an indication of whether there was a decline or an incline in particular phenomenon that shaped the lessons. I will cross reference the coding matrices where they informed the findings that are discussed and illustrated in the Chapters 4, 5, 6 and 7.

In the four chapters that follow, I discuss the document analysis (Chapter 4), the intervention workshops (Chapter 5), how teachers initially taught sequencing before the intervention workshops (Chapter 6) where I also make use of the lessons post the intervention workshops to compare increase / decrease in incidence of key factors in the lessons. Lastly I look at the use of structure in the FP classroom (Chapter 7) post intervention workshop series.