THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE USE OF GEOGEBRA IN ONE GAUTENG SCHOOL

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Abstract

This mixed-methods study drew from the social constructivist perspective and the Theory of Semiotic Mediation (TSM) (Vygotsky, 1978) to develop a pedagogical framework that supports the integration of ICTs in the teaching and learning of probability. The study investigated the impact of the use of GeoGebra on Grade 10 learners' understanding of probability. It sought to find ways in which GeoGebra can be used to overcome specific challenges Grade 10 learners have when solving probability problems. The research site was a school in Gauteng Province in Johannesburg, South Africa. Two groups of Grade 10 learners, the treatment group (n = 14) and the control group (n = 22) were taught in probability content for three weeks using standard instructional practice. No technology was used in this phase, but student-centred strategies which support active learning and interaction were adopted. Participants then wrote a pre-test covering probability concepts. Before writing the post-test, the treatment group received remedial intervention using GeoGebra, while the control group was taught without technology. The pre-test and post-test were quantitatively analysed using error analysis. Qualitative data were collected through participant observation and semi-structured individual interviews. The study found that the use of GeoGebra significantly reduced item difficulty levels and increased the incidence of use of correct methods in problem solving. Furthermore, it contributed to creating an active learning environment in which learner misconceptions were addressed. The findings highlight a teaching framework that can be used to leverage the potential of errors to support learner understanding of probability. We conclude that the semiotic potential of GeoGebra creates a learning environment where learner misconceptions in probability are resolved.

Key terms

Probability; Misconceptions; Models and Representations; ICT; Semiotic Mediation; Mathematics teaching; Mathematics learning

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Dedication

To my loving family.

Declaration

I, the undersigned, hereby declare that this work is my own original work. It is submitted for the Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before, in its entirety or in part, for any degree or examination at any other university.

Innocent Moyo Signature 

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List of acronyms

ATP	Annual Teaching Plan
CAPS	Curriculum and Assessment Policy Statement
CAS	Computer Algebra Systems
CCK	Common Content Knowledge
CK	Content Knowledge
DBE	Department of Basic Education
DGS	Dynamic Geometry Software
HCK	Horizontal Content Knowledge
ICT	Information Communication Technology
MDI	Mathematics Discourse in Instruction
MKT	Mathematical Knowledge for Teaching
NCTM	National Council of Teachers of Mathematics
NSC	National Senior Certificate
PCK	Pedagogical Content Knowledge
PSSM	Principles and Standards for School Mathematics
RTI	Response to Intervention
RME	Real Mathematics Education
SCK	Specialised Content Knowledge
SMK	Subject Matter Knowledge
TEAL	Technology-Enabled Active Learning
TPACK	Technological Pedagogical Content Knowledge
TK	Technological Knowledge
TSLN	Thinking Schools, Learning Nation
TSM	Theory of Semiotic Mediation
UTAUT	Unified Theory of Acceptance and Use of Technology
WSOE	Wits School of Education
ZPD	Zone of Proximal Development

CHAPTER ONE

INTRODUCTION TO THE STUDY

This study takes place at a time when the world faces a plethora of challenges that require technological solutions. The problems of climate change, economic recession, poverty, diversity, inequality and pandemics permeate across borders, causing setbacks of great proportion. A successful response to these challenges is not possible without a closer look at what education offers for sustainable development. Pedagogically, classroom practices need to shift in order to accommodate student-centred paradigms which support openness, critical thinking, sharing and collaboration. As a Mathematics educator, I feel challenged to investigate ways in which technology can be used in classrooms to support learning.

1.1 Background to the research

Teaching and learning practices around the globe have evolved since the beginning of the twentieth century to align with advances in technology and economic growth. These developments have shaped how teachers teach and why teachers teach. It can be assumed that in this fast-changing world, teachers who understand that learning is the goal of teaching will strive to integrate technology to make their classroom practices effective and relevant. This assumption can be premised on the reality of globalisation becoming the driving force behind the rapid transformation of the world economies. Education has been theoretically and empirically proven to shape economic growth and development (Nowak & Dahal, 2016). For example, Hanushek & Woessmann (2008) and Reyna & Brainerd (2007) posit that Mathematics knowledge is effective in human development and economic growth. Mathematics education practices around the world are shifting to benchmark themselves against the world standard. Many countries, including the United States of America (USA), Australia, Singapore and South Africa have undertaken curricula reforms in Mathematics education in response to global trends. Some of these reforms are highlighted below.

In the USA, the National Council of Teachers of Mathematics (NCTM, 2000) developed the Principles and Standards for School Mathematics (PSSM). The PSSM provides guidance for delivering student-centred Mathematics teaching that supports processes of problem solving, reasoning, communication, connections and representations (NCTM, 2000; Schoenfeld, 2002). In the Netherlands, the theory of Real Mathematics Education (RME) was developed in the 1970s to encourage the teaching of Mathematics from rich realistic situations (Van den Heuvel-Panhuizen & Drijvers, 2014). The RME concept supports learners' development of conceptual understanding (Ananiadou & Claro, 2009). In Singapore, the "Thinking Schools, Learning Nation" (TSLN) vision was introduced in the late 1990s to focus on opportunities for critical and creative skills, use of technology, lifelong learning and innovation (Ng, 2008; Mok, 2006). The TSLN reform also encourages self-regulated learning and cultivates a culture of academic rigour on the part of the learners.

Curriculum reform has not been neglected in South Africa either. Since the advent of democracy in 1994, a number of educational reforms have been undertaken to redress the ills from the Apartheid system. The current Curriculum and Assessment Policy Statement (CAPS) which was introduced in 2012, was preceded by other curricula reforms such as Outcomes Based Education (OBE) in 1997, National Curriculum Statement (NCS) in 2002 and the Revised National Curriculum Statement (RNCS) in 2009. The CAPS curriculum is a learner-centred curriculum reform which was developed to equip learners with critical and creative problem-solving skills (Department of Basic Education (DBE), 2011a, 2011b). The critical outcomes of the CAPS Mathematics curriculum include the correct use of the language of Mathematics, listening, communication, reasoning, application, investigation, analysis, representation, interpretation and problem solving (DBE, 2011a, 2011b; Cross, Mungadi & Rouhani, 2002). These specific skills are developed when students work through the following five learning outcomes:

- Numbers, operations and relationships
- Patterns, Functions and Algebra.
- Space and Shape (Geometry)
- Measurement
- Data handling (DBE, 2011a, 2011b).

It is envisaged that the teaching and learning of the above strands of Mathematics will develop necessary skills that are required for a successful interaction between democracy and economic development. South Africa is a new democracy emerging from a past that was marred by Apartheid injustices. This study is a response to the much-needed balance in the triad of democracy, economic growth and education. It focuses on the pedagogical aspect of Mathematics education.

1.2 Statement of the problem

There has been growing concern over students' performance in Mathematics examinations in South Africa over the past years (Sasman, 2011). Between 1995 and 2007 the percentage of learners who passed the Mathematics Higher Grade examination declined from 14.5% in 1995 to 7.2% in 2007 (Sasman, 2011). Between 2008 and 2010, after the Outcomes Based Education (OBE) curriculum was replaced with the Revised National Curriculum Statement (RNCS), the percentage pass rate improved, but was still below 50% (Sasman, 2011). Examination reports from 2012 to 2015 show that students' performance in the National Senior Certificate (NSC) Mathematics examination was consistently below 40% (DBE, 2014, 2015, 2016). A question-by-question analysis of Paper 1 shows that for a random sample of candidates, the average score in the probability question was below 39 % in 2014 and below 28% in 2015. This contrasted with other sections of the syllabus such as equations, inequalities, number patterns, finance and calculus where the average score was above 50% (DBE, 2014, 2015). The global picture also shows that performance levels in statistics and probability are low. For example, Kazemi, Shahmohammadi & Sharei (2013) observe that an "analysis of the learners' academic status at different levels of education shows that these learners have poor performance in probability and statistics subject" (p. 886). On the same subject, Chiesi & Primi (2010, p. 1) argue that "students find it difficult to grasp probability and statistical concepts."

Some of the factors cited for poor learner performance are centred around the teaching of Mathematics and the lack of learner motivation. An analysis of candidates' performance in the 2009 and 2010 NSC examinations revealed that intervention was needed to address "teaching strategies and methodology, content knowledge and understanding, planning to ensure curriculum completion, motivation and interest for both students and teachers" (Sasman, 2011, p. 12). This study is a response to challenges faced in the teaching and learning of Mathematics. The pedagogical value of Information and Communication Technologies (ICTs) as outlined in the policy documents is the focus of the study. Research shows that ICT integration still poses a huge challenge in South Africa (e.g. Padayachee, 2017; Dzansi & Amedzo, 2014; Graham, Stols & Kapp, 2020). For this reason, this study undertakes to address the problem of teaching and learning probability in South Africa. The use of ICTs in classroom practices requires an understanding of how learners create knowledge through the use of semiotic artefacts. I, therefore, investigate the pedagogical impact of ICT intervention on Grade 10 students' understanding of probability. I also seek to determine how GeoGebra can be used as a semiotic artefact to support probability learning and to overcome specific challenges related to probability understanding in a typical South African Grade 10 class.

1.3 Purpose of the Study

The purpose of this case study was to explore the impact of the use of GeoGebra intervention on Grade 10 learners' understanding of probability. In this research, GeoGebra intervention is defined as the use of GeoGebra as a semiotic artefact to mediate learning.

1.4 Research aim

The use of GeoGebra to overcome students' difficulties when solving probability problems has not been widely researched. The aim of this study was to explore how the use of GeoGebra as an alternative ICT mediation tool could impact on Grade 10 learners' understanding of probability.

1.5 Research objectives

The objectives of the study were:

- To identify the errors and misconceptions that Grade 10 learners make when modelling and solving probability problems.
- To explore how the use of GeoGebra can support the learning of probability concepts.
- To explore how GeoGebra can be used to address Grade 10 learners' errors and misconceptions in probability problem solving.

1.6 Rationale for the study

As already mentioned, learner performance in statistics and probability examinations is generally low. Also, there is evidence that teachers lack confidence in integrating technology in teaching, potentially compromising students' conceptual understanding (Lee and Hollebrands, 2008). This leads to low learner performance and places students' future at risk because statistical literacy and probability knowledge are required in real life. According to Garfield & Chance (2000), the goals of learning probability include to:

- understand the purpose and logic of statistical investigations
- understand the process of statistical investigations
- learn statistical skills
- understand probability and chance
- develop statistical literacy
- develop useful statistical dispositions
- develop statistical reasoning

(Garfield & Chance, 2000, p.100)

To appreciate the role of chance and randomness in real life, students need statistical knowledge. This knowledge is also needed for making decisions in the face of uncertainty. If the teaching and learning of probability is not addressed, students' success in Mathematics

Matric examinations can remain at risk, potentially affecting their career paths in fields that require statistical literacy. There is, therefore, a need for Mathematics teachers to be equipped with innovative pedagogical methods that address the learning needs of students. This study explored how GeoGebra intervention addresses challenges that are encountered in the teaching and learning of probability. Its findings provide a pedagogical framework that Mathematics teachers can adopt to enhance statistical and probabilistic understanding.

1.7 Rationale for choosing probability

Examiners' reports for the 2014 and 2015 Mathematics National Senior Certificate (NSC) examinations show an achievement of below 40% in the probability section (DBE, 2014, 2015, 2016). This is attributed in part to the challenges that South African Mathematics teachers face in teaching statistics and probability. These challenges include limited knowledge of statistics and probability content, lack of confidence and lack of competence to teach probability due to lack of exposure to probability learning and training. Some Mathematics teachers did not learn probability at school because it was not included in their Mathematics curriculum (DoE, 2011; Wessels & Nieuwoudt, 2011). Learners also face their own challenges when learning probability. According to Bennie (1998), learners can come to class with some preconceived intuitions which might not be well developed. This might, therefore, influence their probability thinking process.

Probability is included in Mathematics curricula around the world because of its application in real life situations. It plays an important role in people's personal and professional lives, in politics, health, insurance and business. Probability is also used in weather forecasting, sports strategising, insurance predictions, risk management, gaming and quality control, just to mention but a few. It also plays an important role in subjective decision making. Daily, people respond to situations subjectively and make smart decisions depending on their perception of the probability of an event happening or not happening. All these applications of probability make it necessary for teachers to equip themselves with the knowledge of teaching probability. This is important because teachers have a mandate to produce graduates that are statistically literate. Probability was, therefore, chosen for this study because the findings can potentially benefit Mathematics teachers in their quest for teaching methodologies that are innovative for the teaching of this new topic.

1.8 Rationale for choosing GeoGebra

It is presumed that the effective use of technology in Mathematics teaching and learning potentially enhances learners' understanding. The use of technology in teaching also provides learners with the opportunity to gain skills that are necessary for a seamless integration into the global and technological world. There are several dynamic Mathematics software that can be used to mediate learning. These include Autograph, 3DMath, Geometer's SketchPad and TinkerPlots. GeoGebra was preferred as a semiotic artefact for this study because of its accessibility, benefits, potentialities and affordances. GeoGebra is a free open-source software that can be downloaded and used offline on laptops, computers, tablets and cell phones without purchasing a license. It was specifically designed for Mathematics and can be used to create interactive and simulative activities that model real life situations. This ability affords learners to visualise situations that would otherwise be very hard to understand in their abstract form. As a Mathematics software with these features, GeoGebra can be an effective tool for enhancing learners' conceptual understanding of Mathematics.

1.9 Significance of the study

The teaching of Mathematics using ICT technologies is an area of interest. A wide range of research has been done on the teaching of Mathematics using technology. The findings of the current study can contribute to the already existing literature by providing an alternative pedagogical framework using GeoGebra. Moreover, the findings may inspire confidence in Mathematics teachers to try out GeoGebra in their pedagogical practices. This can ultimately benefit learners who need to improve their understanding of probability in order to realise improved results in school exiting assessments.

1.10 Research questions

The study seeks to find answers to the following research questions:

1.10.1 Primary research question

In what ways can GeoGebra be used to overcome errors and misconceptions that Grade 10 learners have when solving probability problems?

1.10.2 Research sub – questions

The following secondary questions will be answered:

1. What errors and misconceptions do Grade 10 learners make when modelling and solving probability problems?
2. How can the use of GeoGebra support the learning of various probability concepts? In particular, how can GeoGebra be used to address Grade 10 learners' errors and misconceptions in probability problem solving?

3. What pedagogical framework can be suggested for ICT mediations in resolving learner errors and misconceptions in probability?

1.11 Definition of key terms

1.11.1 Probability

Batanero, Chernoff, Engel, Lee, & Sanchez (2016) identify six different views that characterise the nature of probability: the classical, frequentist, propensity, logical, subjective and axiomatic views. The current study adopts the frequentist or frequential view of probability which defines probability as "the hypothetical number towards which the relative frequency tends when a random experiment is repeated infinitely many times" (Batanero, et al., 2016, p.4).This definition can be illustrated by an experiment which is repeated many times, such as tossing a coin 100 times and recording the frequencies of Heads (H) and Tails (T) that are obtained. The probability of obtaining a Head (H) will be the number of Hs obtained divided by 100. Thus, probability is a measure of the likelihood of an event occurring and can assume any numerical value from 0 to 1.

1.11.2 Misconceptions

Learners bring to class some prior knowledge or conception of Mathematics which might differ from the conventional explanation. Such knowledge and conception might act as a barrier to acquiring new knowledge (Gomez-Zwiep, 2008; Keeley, 2012). Misconceptions in the current study are defined as beliefs, conceptions or knowledge that learners bring to class and are inconsistent with accepted disciplinary interpretations. As suggested by Keeley (2012), misconceptions need not be considered as bad, but should be used as guidelines by teachers to plan their teaching and learning activities.

1.11.3 Models and representations

Probability can be a challenging concept for learners to grasp, partly because it is an abstract concept. In order to assist learners understand probability situations, the use of visual models, representations and symbols are suggested by Batanero and Diaz (2012), Batanero, et al,. (2016) and Hoffman (2006). Examples of representations are tree diagrams, Venn diagrams, contingency tables, two-way tables and outcome listings (Mutara & Makonye, 2016). Models and representations in this study are defined as visual representations of sample spaces which can take the form of tree diagrams, Venn diagrams, two-way tables and outcome listings.

1.11.4 ICT

Information and communication technologies (ICTs) are technologies that teachers and learners can use to access information and communication. ICTs in this study include computers, laptops, iPads, cell phones and Mathematics software and programmes that can be adopted for the learning of Mathematics. These are tools that teachers and learners can utilise to enhance learners' understanding of Mathematics concepts.

1.11.5 Semiotic mediation

According to Sáenz-Ludlow & Presmeg (2006), the teaching and learning of Mathematics are practices which support the use of symbols, instruments and signs to facilitate cognitive operations. Bussi & Boni (2003) cite a number of instruments that can be used in communicating mathematically, among which are technological objects (e.g. coins) and software that can be used to simplify mathematical situations. In this study, semiotic mediation is intervention that is given to learners with the aid of a technological tool in order to assist them develop a deeper conceptual understanding of the probability situations they are trying to solve.

The research process was guided by the theory of semiotic mediation (TSM) (Vygotsky, 1978) which is discussed later in Chapter Two. Key theoretical elements which informed the research methodology include constructivist principles, semiotic potential of an artefact and human mediation.

1.12 Methodology

This study followed a mixed methods design which employed both quantitative and qualitative data collection and analysis. Data were collected using a pre-test and post-test, interviews and participant observation. Students wrote a pre-test and a post-test on the probability section of the syllabus. Before the post-test was written, the treatment group received GeoGebra-assisted intervention lessons while the control group was taught using the standard instruction practice without any technology. During the intervention lessons, both classes went through the whole probability section, making sure that the errors that were made in the pre-test were thoroughly addressed.

Teaching before the pre-test: A purposive sample of Grade 10 learners at the researcher's school were taught probability content in the first quarter of the year (February/March). They were taught in their respective classes by their respective Mathematics subject teachers. The researcher taught the experimental group, and a colleague taught the control group. The colleague was a highly qualified Mathematics teacher. She was the head of the Mathematics department at the research site. In order to ensure that there was uniformity, both classes were taught following the same plan that the researcher designed and discussed with the colleague before and after each lesson. Student-centred approaches were adopted which allowed active learning as dictated by the academic policy of the school with regard to the standard of teaching. A total of three weeks were spent teaching this topic. None of the teachers used any technology to teach their classes, but formative assessments which took the form of classwork, homework and class tests were done to check for understanding as per school policy. Students from both groups, therefore, worked through the same activities and worksheets, and wrote the same formative assessments.

The Pre-test: The two groups of Grade 10 learners wrote a pre-test in the second quarter of the year in June. Two weeks before the pre-test, students in their respective classes went through a revision exercise of all the sections of the probability topic. The pre-test was marked by the researcher using a memorandum. Errors and misconceptions were recorded and analysed to establish trends.

GeoGebra Intervention and Participant Observation: After the pre-test was written and marked, both groups received intervention lessons to address the errors and the misconceptions that were picked in the pre-test. These interventions were given two weeks before the students wrote the post-test. The researcher taught the probability section to the treatment group using GeoGebra, while the control group was taught by their teacher using standard instruction practice where no technology was used. Students in the treatment group worked as a group and as individuals using GeoGebra to help them complete probability tasks.

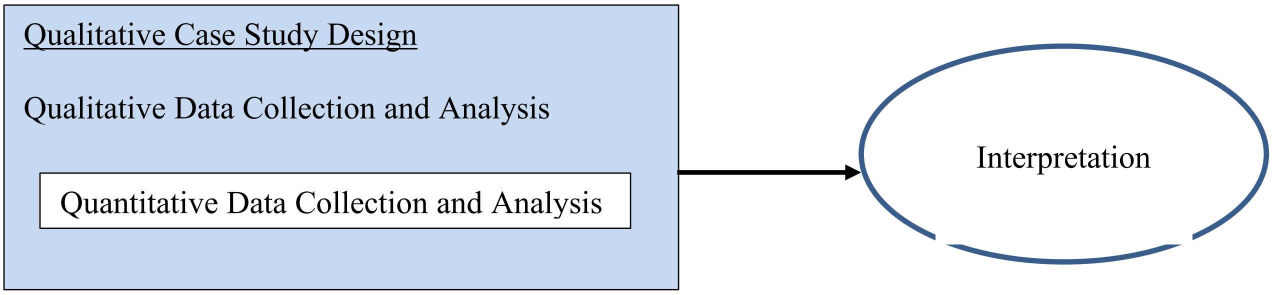
The Post-Test: A post-test was administered to both the treatment and the control groups two weeks after the interventions in the last quarter of the year in November. The test was marked by the researcher using a memorandum, and the errors that students made were recorded and analysed. The errors and frequency of errors in the pre-test and post-test were compared and tested for significance using statistical methods.

Semi-structured Interviews: From the group of students whose consent to participate in the study was obtained, seven of them agreed to participate in the interviews. The interviews were an attempt to get a more comprehensive understanding of their beliefs and thinking processes relative to the responses they gave in the pre-test. The interviews were also used to determine students' beliefs and perceptions about learning probability using GeoGebra. The seven students also agreed to be audio recorded. Transcripts of their responses were made and analysed for themes.

1.13 Research design and methodology

This study investigated the impact of GeoGebra on Grade 10 students' learning outcomes in their study of probability. The study fell within the mixed methods research paradigm because it used both qualitative and quantitative data collection and analysis approaches. Mixed methods research is defined as "the combination of qualitative and quantitative approaches in the methodology of a study" (Creswell & Plano Clark, 2011, p. 3). The mixed methods methodology was used for purposes of triangulation and complementarity (Creswell & Plano Clark, 2011, p.61). Triangulation is concerned with the corroboration and convergence of results, while complementarity "seeks elaboration, enhancement, illustration, and clarification of the results from the other method" (Creswell & Plano Clark, 2011, p. 62). The mixed methods design used in this study is the embedded design where the quantitative strand was added to the qualitative strand to enhance the design as suggested by Creswell & Plano Clark (2011). Figure 1.1 illustrates the research process in embedded mixed methods design.

Figure 1. 1 Research process for the embedded design (adapted from Creswell & Plano Clark, 2011, p.70)



The design was chosen for the study because it allows for an exploration of the impact of GeoGebra on students' learning outcomes using a variety of data sources such as written tests, observations, interviews, photographs and videos. According to Merriam (1985), data collected through observations and interviews are qualitative in nature and can be used to describe a case. Data collected from written tests in this study were quantitative in nature.

The unit of analysis for this study is the interaction between the class of Grade 10 learners and the GeoGebra artefact in the activity of learning probability. Baxter and Jack (2008) describe a unit of analysis or case of analysis as "a phenomenon of some sort occurring in a bounded context" (p. 545). A discussion of the boundaries of the unit of analysis for this study is presented in more detail in Chapter 3.

1.14 Division into chapters

The overall structure of my thesis consists of seven chapters, including this introductory chapter.

Chapter Two begins by discussing the purpose of conducting a literature review in research. This is followed by a discussion of learning theories and the theoretical framework which inform the study. A review of literature on key debates and issues related to my research problem is then presented. In the literature review I discuss the issue of Mathematics and probability teaching and learning. I also review literature on the use of ICTs in teaching.

In Chapter Three, I outline the research design and methodology to explain how I planned and conducted the study to explore the impact of GeoGebra on learners' understanding of probability. I start by considering the ontological, epistemological, axiological and methodological differences between three paradigms, viz: the positivist paradigm, the critical paradigm and the interpretivist paradigm. I then justify my interpretivist stand in the study in relation to data collection, data interpretation and data analysis. I then proceed to explain and justify my decision to adopt the quasi-experimental case study as a strategy for my research. I detail the actual procedure of sampling, data collection and data analysis. Finally, I examine issues of reliability, validity and ethics.

In Chapter Four, I present quantitative data analysis in the light of my first two research sub-questions, viz:

1) What errors and misconceptions do Grade 10 learners make when modelling and solving probability problems?

2) How can the use of GeoGebra support the learning of various probability concepts? In particular, how can GeoGebra be used to address Grade 1o learners' errors and misconceptions in probability problem solving?

I start by quantifying and explaining the types of errors that were identified in learners' work through error analysis procedures. I then analyse the effectiveness of GeoGebra in addressing specific learning outcomes using appropriate statistical procedures to test for significance in the observed differences.

In Chapter Five, guided by all my research sub-questions, I present qualitative data analysis to explicate how the use of GeoGebra potentially addresses learners' errors and misconceptions. I identify and interpret themes that emerged during interviews and observation to explain the effect of GeoGebra on specific learning outcomes.

In Chapter Six, I then merge quantitative and qualitative data analyses to give a consolidated discussion of the findings. In this chapter, I attempt to synthesise data, literature and my voice to answer, not only my first two research sub-questions, but also my third research sub-question, viz: What pedagogical framework can be suggested for ICT mediations in resolving learner errors and misconceptions in probability?

Finally, in Chapter Seven, the summary, conclusion and the recommendations of the study are presented.

1.15 Summary

In this chapter I introduced the study and gave a general overview of the research problem. I started by reviewing trends in Mathematics curricula in selected countries to provide a backdrop for the discussion of the nature of Mathematics teaching and learning. The purpose, aims and objectives of the study were then clarified. Also, the rationale for the study, and the justification for choosing probability and GeoGebra were outlined. Research questions that guide the researcher's selection and use of methodologies and methods throughout the study were also stated, followed by some highlights of the methodological stages and the methods of analysis that the study followed.

I now contextualise my study by identifying key issues and debates in probability teaching and learning in the literature review. I start by discussing the theoretical framework of the study.

CHAPTER TWO

THEORETICAL FRAMEWORK AND LITERATURE REVIEW

2.0 Introduction

This study explored the impact of GeoGebra on Grade 10 learners' understanding of probability. In this chapter, I discuss the theoretical framework that contributed to the understanding of the role of semiotic artefacts in Mathematics teaching and learning. This is followed by a review of related literature. The review explores issues and debates which relate to the research problem which was identified in the previous chapter. According to Nunan (1992, pp.216-217) the purpose of the literature review is to provide "background information on the research question, and to identify what others have said and/or discovered about the question. … [A literature review] extracts and synthesizes the main points, issues, findings, and research methods which emerge from a critical review of the readings." Nunan's explanation suggests that there is a relationship between the research problem and the literature review. On the same subject Ridley (2012), claims that literature review is used to identify a gap in previous research in relation to the research problem. Literature review, thus, contextualises research by situating it in already existing knowledge. Ridley (2012, p. 6) compares research to "a small piece in a complicated jigsaw puzzle." Her argument is that research does not exist on its own but relies on already existing knowledge. Another dimension to the function of the literature review is to identify theories and methodologies that influence research (Ridley, 2012; Webster & Watson, 2002).

The chapter is organised into three sections. In the first section, I focus on well-known theories of learning on which the theoretical framework of the study hinges. In the second section, I focus on the theoretical framework that underpins this study. I then move on to review literature in the third section. In the review, I cover litetature and previous research on teaching models and principles, with particular focus on Mathematics. The purpose of the discussion is to examine how different teaching and learning perspectives conceptualise mathematical understanding. The teaching and learning of probability is also discussed in this section. It covers errors and misconceptions learners make in probability problem solving situations. It also covers Mathematics teaching with special focus on students' errors and how these are analysed. This leads to a discussion of teaching interventions. Finally, ICT affordances and research on the use of ICTs are discussed in detail.

2.1 Theories of learning

Theories of learning explain how learners acquire, process and retain knowledge. For example, behaviorist theories advance that observable changes in learners' behaviour are evidence that learning has taken place (Semple, 2000). This implies that the role of the teacher in instruction is to drill the learner until the desired behaviour is observed. Thus, behaviorist theories can be criticised for promoting passive learning. On the contrary, cognitivism, which informs constructivism, argues for learning through active mental processes (Semple, 2000). Instruction is designed to allow learners to "express their thinking and organise their personal knowledge" (Semple, 2000, p. 1).

In this section, cognitive constructivism (Piaget, 1952) and social constructivism (Vygotsky, 1978) will be discussed, with the latter in more detail since it informs the theoretical framework that underpins this study. The origins of constructivism are linked back to the works of Socrates and Piaget (Amineh & Asl, 2015; Murphy, 1997). Although he was not a constructivist, some of Socrates' philosophical ideas supported constructivism. "Were he present today, he would likely show an interest in constructivism, no doubt recognizing in it some similarities with his own philosophy" (Murphy, 1997, p.2).

2.1.1 Cognitive constructivism

Piaget (1953) postulates that knowledge is constructed through assimilation, accommodation and equilibration (Kalina & Powell, 2009). Cognitive constructivism is critical for this study because it helps us to understand how participants engaged with the activities that were presented to them to acquire probability knowledge. First, the theory suggests that at any stage of growth, children apply their mental processes to make sense of the things around them (Piaget, 1953; Kalina & Powell, 2009). Second, when new information is introduced to learners, a state of mental disequilibrium occurs which causes the child to make some adjustments in his or her thinking to resolve the conflict. In this study, probability content which was introduced to the learners would be understood through the help of a GeoGebra artefact, thus aiding learners to deal with any disequilibration. Piaget argues that learning takes place when the child adjusts his or her thinking to resolve the conflict through the process of assimilation or accommodation. Assimilation "is when children bring in new knowledge to their own schemas, and accommodation is when children have to change their schemas to accommodate the new information or knowledge" (Kalina & Powell, 2009, p. 243). Thus, according to Piaget, individuals learn by cognitively constructing knowledge. Another aspect of cognitive constructivism which applies to this study is the notion of cognitive stages of development which determine what children can learn. This has important implications for teaching since learners' individual needs should be taken into account when planning instruction (Kalina & Powell, 2009). Furthermore, it allows individuals to learn at their pace. In essence, the learning activities and the methods of instruction should be selected carefully by the teacher to help students learn. However, this poses potential challenges for the teacher. First, developing learning activities and choosing methods of instruction which meet individual learners' needs are huge challenges on their own. Second, having successfully designed suitable activities for learners, the teacher should ensure that there is learning happening. Cognitive constructivism emphasises mental processes for acquiring knowledge. However, it does not say how learners deal with conceptual challenges during their learning. The deployment of a GeoGebra artefact in the learning environment was aimed at addressing this gap.

2.1.2 Social constructivism

Unlike cognitive constructivism, social constructivism incorporates collaboration and social interaction (Kalina & Powell, 2009; Amineh & Asl, 2015). It is based on assumptions about reality, knowledge and learning (Kim, 2001). According to Kim (2001), reality and knowledge are constructed by humans through human activity. Through coordination and collaboration with other people in a social context, humans acquire knowledge and understanding. In other words, humans "negotiate meaning until [they] come to believe that [they] all mean the same thing" Fosnot (2005, p. 5). Learning is, therefore, seen as a social process which "occurs when individuals are engaged in social activities" (Kim, 2001, p. 2). Vygotsky's (1978) sociocultural theory is developed on three interacting elements, viz: 1) social sources of individual thinking, 2) cultural tools and 3) the zone of proximal development. According to Vygotsky, learning takes place in sociocultural settings. The three elements are discussed below.

Social sources of individual thinking

Vygotsky (1978, p.57) posits that a child's cultural development occurs "first on the social level, and later, on the individual level; first, between people (interpsychological), and then inside the child (intrapsychological)." This means that what is there to be learnt by individuals exists first on the social level before the individual can acquire it. Vygotsky argues, therefore, that knowledge is constructed by learners through their interaction with other people using mediating cultural tools such as computers, and psychological tools such as signs, language and symbols (Woolfolk, 2010). This aspect guided the study during the development of probability activities that learners would interact with using a GeoGebra artefact. In particular, the nature of the learning activities that were selected for the class needed to match the cognitive growth of the learners and, at the same time, support the deployment of a GeoGebra artefact. It also provided guidelines on how to organise the class so that collaborative learning was supported.

The zone of proximal development

The Vygotskian perspective maintains that what is learnt should be matched with the child's developmental level. Children experience two levels of development in learning, the actual developmental level and the level of potential development. The former is the stage where the child is independently and mentally able to solve assigned tasks without the help of another person. Vygotsky (1978) describes this as "the level of development of a child's mental functions that has been established as a result of certain already completed developmental cycles" (p. 85). On the other hand, the level of potential development is the stage where the child can only solve problems through adult guidance or from a more knowledgeable peer. The gap between the actual developmental level and the potential level is called the zone of proximal development (ZPD). It is "a zone where learning occurs when a child is helped in learning a concept in the classroom" (Kalina & Powell, 2009, p. 244). The ZPD notion suggests that participants in the study should be grouped in such a way that they can learn from each other through interaction. Furthermore, in developing probability test items and activities for such a study, an awareness of the zone of proximal development (ZPD) is critical. Some questions which are considered to be very difficult require scaffolding in the ZPD to facilitate learning.

The role of cultural tools and signs

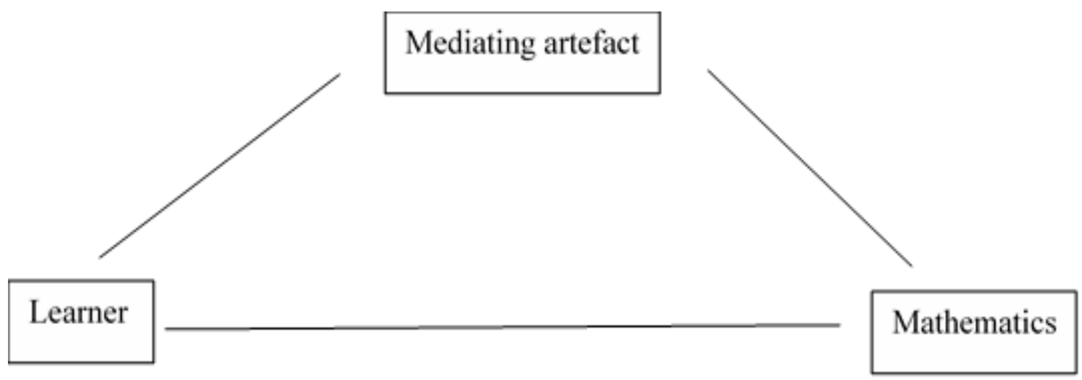
According to Amineh & Asl (2015), language is an important cultural tool through which humans construct reality. On the same subject, Kalina & Powell (2009, p. 245) argue that "language enhances learning and … precedes knowledge or thinking." As children use language as a cultural tool to accomplish a task, the way they think about the activity is shaped. Signs develop when a child is accomplishing a task using a tool. These signs are meanings that develop when the child grapples with the task using a tool. Vygotsky (1978) refers to the process of constructing meanings using tools and signs as internalisation. The role of cultural tools and signs was a critical dimension in the process of selecting learning activities, allocating teacher roles, allocating learner roles and also in the organisation of the class for effective learning. It shaped the role of the mediating artefact in the whole process of probability knowledge creation and allowed individual learners to come up with their own interpretations of the object of learning. In the next section, I discuss the theory of semiotic mediation (TSM) (Vygotsky, 1978) as the main contributor on the role of semiotic artefacts in learning

Mathematics.

2.2 The Theoretical Framework: Theory of Semiotic Mediation (TSM)

The framework that underpins this study is the Theory of Semiotic Mediation (TSM) (Vygotsky, 1978). TSM is based on constructivist theories of learning and it explains how mathematical knowledge is created using a semiotic artefact. It postulates that learning can be mediated using artefacts to enhance internalisation of mathematical concepts. The notion of mediation is "used in relation to the support that a specific tool gives one in the accomplishment of a task through its use, [and is] also related to the potentiality of fostering learning processes with respect to a specific piece of knowledge, for instance mathematical knowledge" (Mariotti & Maffia, 2018, p. 51). Thus, TSM specifically addresses the relationship between the artefact, what is learned (mathematical content) and human mediation in the teaching-learning process (Mariotti & Maffia, 2018). It models the teaching-learning process around the semiotic potential of an artefact and explains how the use of a specific tool can evoke a specific mathematical knowledge. For example, the use of an abacus can evoke polynomial notation of numbers (Mariotti & Maffia, 2018). As learners use an artefact, not only do they become aware of the artefact they are using, but they also become aware of the procedure of using the artefact to accomplish the task. This evokes mathematical knowledge. Figure 2.1 models the process of mediation using an artefact.

Figure 2. 1 A model of the process of mediation by an artefact (Drijvers, Kieran & Mariotti, 2010)



TSM models the teaching-learning process around a didactical cycle. A didactical cycle is a teaching sequence that is followed to make knowledge accessible to learners. According to Mariotti & Maffia (2018, p.50), the potential of an artefact depends a lot on the orchestrating role of the teacher. They argue that the teacher is responsible for designing learning activities that exploit the semiotic potential of an artefact. Thus, in the context of TSM, the didactical cycle addresses the teacher's crucial role to design effective intervention. The cycle starts when the teacher assigns students activities to carry out using the artefact. The activities are designed in such a way as to provoke a production of individual psychological signs which can be interpreted as mathematical meanings. Students complete the given activities using the artefact (Bussi and Mariotti, 2008; Mariotti & Maffia, 2018). As they interact with the task, the artefact and with each other, signs such as words or gestures are produced, which may be interpreted as mathematical. The teacher's role, therefore, is to focus on the production of these signs and on their transformation into mathematical meanings.

The second set of activities in a didactical cycle involves assigning students activities that promote individual production of signs (Bussi and Mariotti, 2008; Mariotti & Maffia, 2018). This could take the form of students writing individual reflections on their experience of using the artefact or writing their own conclusions from a group activity they were involved in.

In the final stage, students are given activities that promote a collective production of signs. These activities involve the whole class, with the teacher giving guidance through a discussion of specific mathematical aspects. Such an activity is likely to result in students sharing their personal psychological signs to yield collective signs. Figure 2.2 illustrates a didactical cycle.

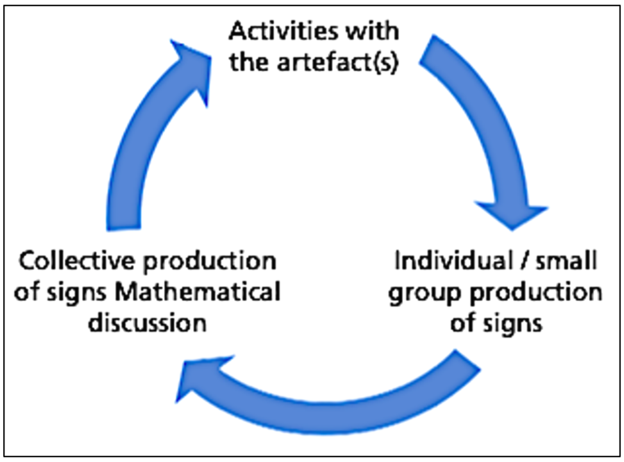


Figure 2. 2 The didactical cycle (Mariotti & Maffia, 2018, p. 53).

As already stated, GeoGebra was used in this study as a semiotic artefact to acquire probability knowledge. Learners were assigned problems to solve using the artefact. Any signs that developed as learners worked with the artefact were observed and used to explain the impact of GeoGebra.

The role of semiotic artefacts in the production of Artefact signs, Mathematics signs and Pivot signs.

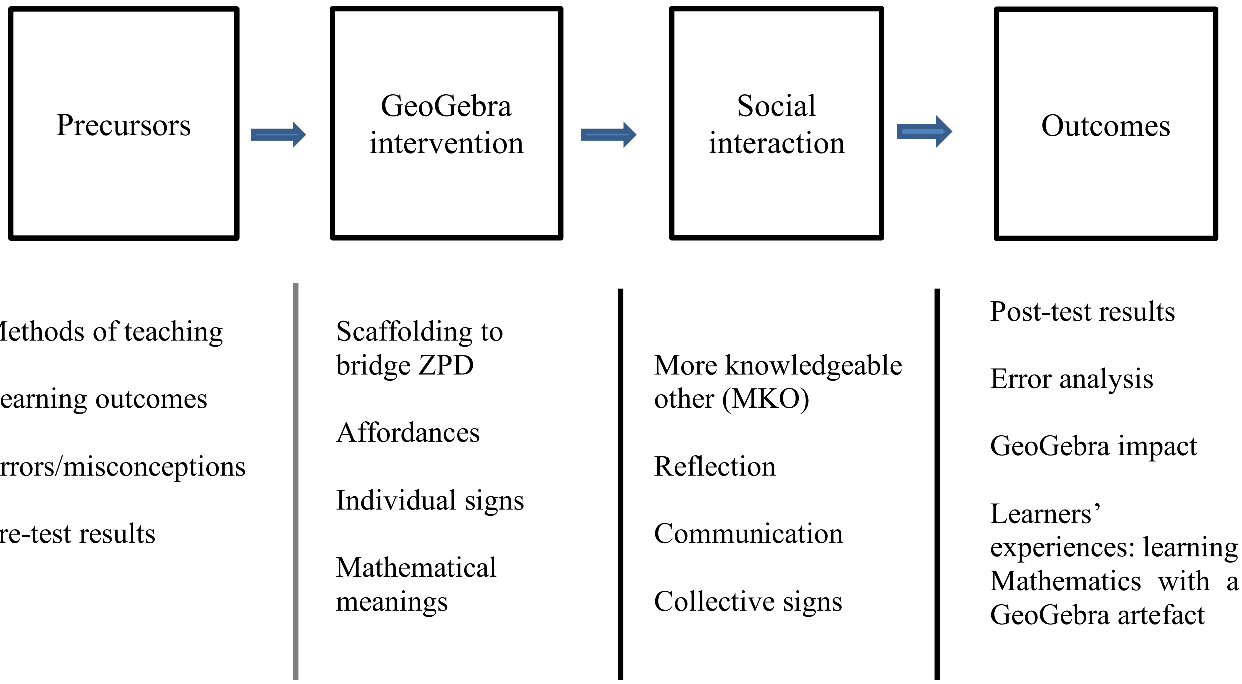
When students use an artefact to accomplish a task, signs emerge which relate to the task and to their collaborating partners (Bussi & Mariotti, 2008). The sociocultural perspective (Vygotsky, 1978) claims that artefacts help learners to accomplish what would otherwise be impossible to accomplish without them. Through the guidance of the teacher, the signs that emerge when students use the artefact in mathematical activities potentially evolve into recognisable mathematical meanings. Bussi & Mariotti (2008) identify three categories of signs: artefact signs, Mathematics signs and pivot signs.

Artefact signs emerge when students use the artefact to accomplish an activity. As students work with the artefact, they construct ideas based on the context in which they are working. According to Bussi & Mariotti (2008), these signs have personal meanings since they are related to how the student is experiencing the learning situation. When students work in groups using the artefact, there is a potential need to communicate. This communication will likely rely on the signs that are produced when using the artefact. Examples of artefact signs include gestures and drawings (Bussi & Mariotti, 2008; Turgut, 2015). Mathematics signs are mathematical meanings which can take the form of a definition or a proof (Bussi & Mariotti, 2008). The teacher can use his/her expert ability to link artefact signs to Mathematics signs at the level that his students will understand. Pivot signs are described as "hybrid terms in natural language" (Turgut, 2015; p. 2423). These signs are useful in translating artefact signs to Mathematics signs.

The conceptual framework

This study explored the impact of the use of GeoGebra on learners' understanding of probability. The following conceptual framework demonstrates how the aims of this study were expected to be accomplished.

Figure 2. 3 The conceptual framework for studying the impact of GeoGebra on students' learning of probability.



2.3 Mathematics teaching in the 21st century

Kilbane & Milman (2013) argue that the success of any educator depends on his/her ability to design effective instruction for his/her learners. Global trends (e.g. in technology) suggest that teachers can no longer ignore technology when designing instruction for their learners. Today's learners have unique individual needs. They are growing up in a digital era where access to information is instant. This suggests that the 21st century teacher should strive to remain technologically relevant. As already argued in Chapter One, low achievement in examinations can be linked to the state of Mathematics teaching. Epistemological themes on how Mathematics is learnt are important when addressing students' achievements in Mathematics (Schoenfeld, 2016). In this section, I discuss examples of teaching models that have been researched over the past decades. The principles that drive these models could help educators to link their teaching to existing theories of learning in their quest for effective learning programmes.

2.3.1 Mathematics teaching models

Teaching models are frameworks for implementing instruction. They are designed to "promote specific learning outcomes related to required standards in the academic disciplines through the use of a specially orchestrated set of activities" (Kilbane & Milman, 2013, pp.18-19). The set of activities are tasks that learners engage in through steps that support specific learning goals. According to Maheshwari & Maheshwari (2013), teaching models have six attributes, viz:

- focus (objectives of teaching and learning environment)
- syntax (sequence of steps and activities that are followed)
- principles of reaction (the planned teacher's reaction to students' responses)
- the social system (teacher and learner activities)
- the support system (additional requirements), and
- application (use of learned material in other situations)

Thus, teaching models will differ from each other according to their attributes, and according to the theory of learning that informs them. Joyce & Weil (2003, p.7) point out that "how teaching is conducted has a large impact on students' abilities to educate themselves." They emphasise the importance of engaging students in robust cognitive and social tasks and argue that outstanding teachers teach students to learn. According to Joyce & Weil (2003), effective teaching models are measured not only by their ability to achieve their objectives, but also "by how well they increase the ability to learn" (p. 7). As Kilbane & Milman (2013) reminds us, "models of teaching or models of learning, are instructional tools that should be in every teacher's tool kit (p. 54).

Numerous research-based teaching models have been developed over the past decades. Four families of teaching models have been identified as information processing models, personal models, social interaction models and behaviour modification models (Maheshwari & Maheshwari, 2013). Information processing models are concerned with mastery of inquiry skills, academic concepts and logical reasoning and thinking skills. Personal models, on the other hand, are concerned with increasing learners' competence, creativity and openness to new experience (Maheshwari & Maheshwari, 2013). Social interaction models are aimed at enhancing collaboration and solving problems in a social context. Finally, behaviour modification models emphasise change in behaviour. Thus, each model focuses on the purpose for which it was developed. I briefly discuss five research-based teaching models below. These are the path-smoothing teaching model (Wigley, 1992), the challenging model (Wigley, 1992), the Mathematics discourse in instruction (MDI) (Adler & Ronda, 2015) and Lesh's translation model (Suh, 2007; Johnson, 2018), and the three-phase model (Martin & Speer, 2009).

The path-smoothing teaching model

The path-smoothing teaching model aims to smooth the path for learners (Wigley,1992). It follows the following pattern:

- the teacher states the kind of problem on which the class will be working,
- learners are led through a method for tackling the problem,
- learners work on exercises to practise the methods given, and
- the class revisits similar subject matter through revision (Wigley, 1992, p.4).

The path-smoothing model of teaching is, however, criticised for its inability to support insightful learning. This is because learners are taught through repetition, and teachers offer explanations instead of provoking debate to clarify meaning. "Inevitably, pupils' perceptions remain unexamined if they passively agree to the arguments in order that work can proceed" (Wigley, 1992, p.5). This view is shared by Foster (2013), who argues that chalk-and-talk pedagogy "severely restricts students' opportunities to engage in authentic mathematical thinking and deprives them of the enjoyment of solving richer, more worthwhile problems, which would forge connections across diverse areas of the subject" (p. 563). Foster attributes such pedagogy "partly to an assessment-dominated curriculum and partly to a systemic de-professionalisation of teachers through a performance accountability culture in which they are constantly required to prove to non-specialist managers that they are effective" (Foster, 2013, p.563). According to Truxaw & DeFranco (2008), these transmission methods offer little opportunity for students to create meaning for themselves. This is caused by the fact that students are drilled to reproduce what they were taught procedurally. This leads to passive learning and encourages rote learning of procedures rather than inductive learning.

The challenging model

Wigley (1992) advocates for a model for classroom practice which challenges and engages the learner by fostering a conjecturing atmosphere. He proposes what he calls a challenging model. The main features of the model include the following paths. First, the teacher presents a challenging context or problem, and learners are given time to work on it and make conjectures about methods or results. Second, a variety of ways to deal with the situation are established from learners' work and shared by the teacher with the class. Third, strategies which evolve from learners' work are applied to a variety of problems. Finally, learners review their work and identify what they have learnt and how it relates to other knowledge. This model requires a lot of time since learners work through the problem with minimum to no assistance from the teacher. The teacher should necessarily be careful not to oversimplify the problem because he or she must challenge the learners. To a certain extent, this model potentially supports critical learning. According to Jarworski (2006), critical inquiry amounts to learning through practice. Learning follows the process of inquiry where learners critically acquire knowledge, are allowed "to wonder, to ask questions, and to seek to understand by collaborating with others" (Jarworski,2006, p.200). However, this model does not mention the role of social contexts when learning.

The Mathematics Discourse in Instruction (MDI)

Adler & Ronda (2015) developed the Mathematics Discourse in Instruction (MDI), a teaching model that supports active learning in a sociocultural setting. MDI works through four interacting variables:

- the object of learning (the lesson goal),
- exemplification (examples and related tasks),
- explanatory talk (talk about what counts as Mathematics in a lesson), and
- learner participation (learner-learner and learner-teacher interaction).

It advances that teaching and learning starts with an awareness of the object of learning. This could be the concept or content that should be learned. In order to assist learners grasp the concept, some form of mediation in the form of exemplification and explanatory talk by the teacher are needed. Exemplification and explanatory talk can be achieved using available sociocultural tools. The framework, therefore, supports the teaching and learning of Mathematics using available technological tools in a sociocultural appropriate context.

Lesh's translation model

Lesh's translation model (Suh, 2007; Johnson, 2018) adds the dimension of representation to the teaching of Mathematics. It argues that mathematical ideas can be represented in various different modes to help learners grasp the concepts. According to this model, ideas can be represented as real-life situations, manipulatives, pictures, written symbols or verbal symbols. The various modes of representation should be taken from the sociocultural context of the learners. Johnson (2018) suggests that the translation model can be extended to include moving pictures using technology.

The knowledge-based model

Martin & Speer (2009) propose a knowledge-based model of teaching practice where teaching is a cycle comprising three phases, viz: knowledge, implementation and analysis. In this model, the teacher draws from his/her knowledge of the subject and of the students. The second phase, which is the actual implementation of the lesson includes class activities. This phase is also influenced by teacher knowledge. In the analysis phase, the teacher reflects on the performance of his/her students and of his/her own teaching. What happens in the implementation and analysis phases in turn influences teacher knowledge of the subject and the students. Thus, the model is cyclic. The model suggests that learners' experiences of learning Mathematics, their attitudes towards Mathematics and their beliefs about Mathematics can be influenced by the teacher. In a study by Domino (2009) to determine Mathematics teachers' influences on students' attitude towards Mathematics, it was found that teachers influenced students' understanding of Mathematics and students' attitude towards Mathematics. The way teachers taught Mathematics was cited as a contributor to learners' positive experiences of learning the subject. This included the way teachers made the lessons interesting or fun for learners, the way student activity was promoted during the lessons, the way Mathematics content was related to real life, the pacing of the lessons and the extent to which students were assisted and cared for. On the contrary, students cited lessons in which they were not afforded the opportunity to be actively engaged as contributors to their negative learning experiences. They were also not happy with lessons in which attention was not given to them or their learning needs.

Some of the teaching models discussed in the foregoing paragraphs were designed a long time ago (e.g. the challenging model). A close look at these models shows that they can be adopted with some modification for the 21st century classroom. According to Kilbane & Milman (2013), the teacher should possess a clear understanding of the characteristics of the 21st century learner in order to design an effective teaching model. Moreover, the teacher should be aware of the global trends that influence education. Kilbane & Milman (2014, p. 5) argue that "technology functions as both the fuel and the tool for educational change" and posit that technologies are capable of transforming teaching models to make them effective. Johnson's (2018) research supports this view. In her study, Johnson (2018) used Lesh's translation model to show that mathematical representation can be enhanced by utilizing technology. The 21st century classroom is diverse in terms of learners' needs. The teacher's challenge becomes complex as a result since meeting the unique needs of all the learners with varying learning and cultural backgrounds is not an easy task. Kilbane & Milman (2013) address this problem well when they suggest that today's learners need flexible teachers who can design effective instructional programmes. These are "teachers who know when and how to use standard, modified, and customized materials to help them achieve their highest potential (Kilbane &

Milman, 2013, p. 19).

There is a strong relationship between teaching models and theories of learning upon which they are developed. Moreover, teaching models are strongly influenced by specific educational principles that drive learning. The teaching principles encapsulated in the National Council of Teachers of Mathematics (NCTM) are one such case.

2.3.2 NCTM teaching principles and strategies

National Council of Teachers of Mathematics (NCTM) suggests six research-based principles for effective Mathematics teaching. These principles, which are regarded as necessary components of an effective Mathematics program, are centred around teaching and learning, access and equity, curriculum, tools and technology, assessment and professionalism (Huinker, Leeinwand & Brahier, 2014). A brief description of these principles is given below.

- *Teaching and learning*: According to the NCTM guidelines, the teaching of Mathematics should result in students reasoning in a mathematical way. Mathematics teachers should adopt and employ strategies that support effective learning. Such teaching strategies allow students to engage individually and collaboratively, helping them to begin to make sense out of the mathematical ideas that are there. Engaged learners have a greater chance to reason in a mathematical way (Huinker et al., 2014) than learners who are not.
- *Access and equity*: This means that students' access to effective teaching and learning should be prioritised. Students should have access to a high-quality curriculum and learning resources.
- *Curriculum*: Mathematics teaching should develop connections between Mathematics and the real world.
- *Tools and technology*: Mathematics teaching should allow for the integration of technology to support students' learning.
- *Assessment:* Assessment in Mathematics teaching should be part of instruction and should yield useful feedback for decision making.
- *Professionalism*: Mathematics teachers must be professionals who make the success of their students a top priority.

These principles require Mathematics teachers to adopt specific classroom practices which support effective teaching and learning. Huinker et al.,(2014) further identify eight

Mathematics teaching practices that characterise effective teaching and learning, viz:

- Establish Mathematics goals to focus learning. Goals help in guiding instructional decisions in the lesson.
- Implement tasks that promote reasoning and problem solving. Students must be actively engaged in order for their problem-solving strategies to develop.
- Use and connect mathematical representations. Students should be assisted to see connections between mathematical representations. This will help them to understand the concepts that are involved.
- Facilitate meaningful mathematical discourse. Students should be encouraged to share their mathematical ideas with each other.
- Pose purposeful questions. Questioning is an important component of a lesson because it helps in assessing students' reasoning and sense making.
- Build procedural fluency from conceptual understanding. Students' use of procedures should be built on conceptual understanding. Students should not be applying procedures without the necessary conceptual understanding of the situation.
- Support productive struggle in learning Mathematics. Students should be supported to productively grapple with Mathematics.
- Elicit and use evidence of student thinking. Students' thinking should be used as evidence of their mathematical understanding.

(Huinker et al., 2014, p. 535)

In developing an effective teaching model for Mathematics, these practices should be used as guiding principles. Probability was the main focus of this study. In the next section, the nature of probability and the challenges associated with acquiring probability knowledge are discussed.

2.4 The nature of probability

Probability is a concept whose interpretation varies depending on the view one holds about it. According to Liu & Thompson (2007) probability is either frequency-based or belief-based. Frequency-based probability uses stochastic laws to process measures of chance. On the other hand, belief-based probability relies on the degree of belief one holds about the situation under consideration. Batanero et al.(2016) argue that students bring to class some intuitive ideas about probability which are influenced by their beliefs. Thus, even young children can use such terms as "probable" and "likely" to express their degree of belief. These intuitive ideas can be used as a starting point in developing a more formal understanding of probability..

Another perspective about probability is the classical perspective. It uses a sample space of equally likely outcomes to measure the likelihood of an event occurring (Batanero et al., 2016; Batanero and Diaz, 2012). As an example, when a six-sided fair cube numbered 1 to 6 is rolled, the sample space, S, can be represented as S = {1;2;3;4;5;6} where each number is equally likely to be rolled. The probability of rolling an even number is calculated by dividing the number of even numbers in the sample space, by 6, the number of possible outcomes. This definition is consistent with the high school probability formula, $\mathrm{P(A)} = \frac{\mathrm{n(A)}}{\mathrm{n(S)}}$.

Frequentists view probability as the relative frequency of an event when a random experiment is repeated many times under identical conditions (Batanero and Diaz, 2012). The assumption is that if an experiment is carried out a large number of times, the relative frequency of the desired events stabilise and estimate the probability of those events occurring. This view is relevant for this study, particularly for Grade 10 learners who have to carry out such experiments using tools like coins, dice and playing cards.

An awareness of the various perspectives of probability can shed some light on how probability knowledge should be acquired. I now turn to the teaching and learning of probability and maintain that the teaching models which were discussed earlier apply here as well.

2.5 Teaching and learning statistics and probability: some challenges

Statistical literacy is the ability to interpret and communicate statistical information. It enables individuals to make choices in various real-life contexts such as workplaces. Gal (2004) argues that statistical literacy is an interaction between the learners' knowledge of statistics and their beliefs and attitudes.

Learning statistics involves acquiring and developing reasoning, thinking and literacy skills through interaction with data (Ben-Zvi & Garfield, 2004). Garfield (2002) notes that there is no consensus on how statistical reasoning is taught and argues that even though statistical procedures and concepts are taught in the classroom, the guarantee that statistical reasoning is successfully developed in the process is not there. This highlights the importance of teaching statistics for conceptual understanding. Students learn about qualitative data, quantitative data, continuous data and discrete data, but sometimes fail to understand why these forms of data land themselves to graphs, tables and statistical analysis. Effective statistics and probability instruction should, therefore, assist students to conceptualise relationships between different forms of data and their different forms of representations.

Learning statistics also involves making judgments. In probability, these judgements are made under uncertainty. According to Garfield & Ben-Zvi (2007), such judgments are sometimes influenced by faulty intuitions, posing a challenge because the biases and faulty beliefs that learners bring to the learning environment can hinder new knowledge acquisition. There are, therefore, several challenges that are associated with the teaching and learning of statistics and probability. Some sources of these challenges include the abstract and complex nature of concepts, inadequate knowledge base, misuse of representations, learner's intuitive ideas, irreversibility of probability experiments and the representativeness heuristic. These concepts are briefly explained below.

The abstract and complex nature of statistics and probability: Some challenges in statistics and probability teaching and learning are caused by the abstract and complex concepts. For example, the mean of the numbers 12; 13; 15; 17 and 20 is 15.4. However, the answer 15.4 does not appear among the original data set. Learners may, therefore, find it difficult to conceptualise mean in this example because it is abstract. To overcome such difficulties, Garfield (2002) suggests that statistical reasoning should be developed through activities using visual simulations. In the same vein, Konold, Harradine & Kazak (2007) suggest that learning probability through modelling can bring a deeper conceptual understanding of the real phenomenon that is being studied. Modelling allows learners to construct representations of the phenomenon under study, helping them to view the situation in a less abstract way.

Inadequate knowledge base: Challenges also arise when students lack some understanding of basic Mathematics which is required to learn statistics. For example, students need computational knowledge of percentages, fractions, decimals and proportions to carry out successful data analysis using summary statistics such as mean. When this knowledge is missing, difficulties are likely to be encountered (Gal, 2004).

Misrepresentations: Other difficulties that are encountered in probability learning arise from the misuse of representations. There is evidence that learners face challenges when creating representations in probability problem-solving (e.g. Mutara & Makonye, 2016). Representations, such as tree diagrams, Venn diagrams, possibility spaces and contingency tables help students to understand probability situations. However, when these are incorrectly created or interpreted, difficulties arise. In their analysis of students' representations of a conditional probability problem, Nascimento, Morais & Martins (2016) found that only 47% of the subjects correctly used a table or a tree diagram to solve problems.

Learner's intuitive ideas: Learners' preconceived ideas can interfere with the acquisition of new concepts. According to Batanero & Diaz (2012), there is evidence that some of the informal ideas learners hold are actually misconceptions that can be difficult to get rid of. Modelling through simulations can assist in simplifying such problems.

Irreversibility of probability experiments: The irreversible nature of probability activities add to the list of challenges faced. Other topics in Mathematics use operations that can be reversed. For example, in computational arithmetic, the operation of adding 2 to 3 can be reversed using the reverse operation of subtraction to go back to the original value. Learners are able to see that if $2 + 3 = 5$, then $5 - 3 = 2$ and $5 - 2 = 3$. However, in a random probability experiment like flipping a coin, the samples obtained are always different and the experiment cannot be reversed. This can be a source of learning difficulties which requires adequate pedagogical training to correctly explain the related concepts (Batanero, 2014).

The representativeness heuristic: Tversky and Kahneman (1974) argue that people judge certain types of probability situations using a representativeness heuristic which is based on the evaluation of the degree to which one event resembles the other. This can lead to misconceptions of chance as illustrated in the following example. When a coin is tossed many times, and the outcomes are {Head, Head, Head, Head, Head, Head, Head,}, people using the representativeness heuristic may expect a tail (T) on the next toss because they think it is about time a tail showed up, or a head (H) because of the history of heads. Mistakes of this nature are referred to as negative and positive recency mistakes respectively (Bryant & Nunes, 2012; Tversky and Kahneman, 1974).

There are many other reported difficulties that are faced by learners in probability learning. Some of them include the confusion between mutually exclusive events and independent events, as well the incorrect use of notation such as $n(A)$ instead of $P(A)$ (DBE, 2015, 2016).

Sarwadi and Shahrill (2014) argue that the learning of Mathematics involves a systematic buildup of concepts which calls for the learner and the teacher to handle in such a way as to allow for the concepts to hierarchically build on each other. Learners will always face difficulties when learning probability unless the way probability is taught is reformed. In particular, as long as teaching and learning probability is only focused on learning rules and procedures, these challenges will persist. Sarwadi and Shahrill (2014) warn that "teaching students only procedural skills will impair learning in the classroom and will not equip students well with the necessary skills mathematically for the future"(p.2). Probability learning and teaching should rather emphasise conceptual understanding. The difficulties learners experience in Mathematics can be understood through errors analysis. In the next section, students' errors and misconceptions when learning Mathematics are highlighted.

2.6 Errors students make in Mathematics

Students' difficulties in Mathematics sometimes manifest in the form of errors in their written work. An error analysis of students' written work can help identify these difficulties and reveal knowledge gaps. Newman (1977) identifies five categories of student errors in problem solving. These are errors of reading, comprehension, transformation, process skills and encoding (Fitriani, Turmudi, & Prabawanto, 2018). Reading errors involve situations where students fail to recognise words or symbols in a context-based problem. This leads to failure to use the given information to answer the posed question. Errors of comprehension include instances where students do not understand what the problem means despite reading the problem accurately. Transformation errors occur when students fail to translate a mathematical word problem into mathematical symbols or language such as graphs or equations. Process skill errors occur when students fail to use the correct procedure in solving the problem. They include the wrong choice and wrong use of rules in answering a problem. They also include computational errors that occur in the process of applying a procedure to solve the problem. Encoding errors have to do with students' mistakes in relating a solution to the problem. They include incorrectly writing an answer to a problem or failure to justify the result that was obtained.

Students also make careless errors or slips in Mathematics. Herholdt & Sapire (2014, p. 43) describe such errors as "random errors in declarative or procedural knowledge, which do not indicate systematic misconceptions or conceptual problems." Another category of errors identified by Herholdt & Sapire (2014) are bugs, also known as pervasive errors. These errors are classified as systematic errors, and occur when students' reasoning is incorrect, leading to faulty conceptual or procedural understanding of the work. Brodie (2014) and Herholdt & Sapire (2014) concur that students who make such errors believe that they are correct. The reason for this is because according to the learner, these misconceptions are truth and cannot be easily removed or replaced.

Sarwadi & Shahrill (2014) hold the view that students' errors are products of students' misconceptions. They further argue that errors are "unique and they reflect their [students] understanding of a concept, problem or a procedure" (Sarwadi & Shahrill, 2014, p. 1). Thus, students' errors are systematic since they are not brought about by chance. According to

Sarwadi & Shahrill (2014), students' errors take three dimensions. First, students make errors when they fail to see a connection between what they are doing and what they already know. Second, errors occur when students fail to connect new knowledge with pre-conceived knowledge. Third, errors arise from students' overgeneralisation of rules. These types of errors tend to persist if proper intervention is not planned.

Lai (2012) categorises student errors in Mathematics as factual, procedural and conceptual. These errors occur when students lack necessary knowledge to solve a problem, when they fail to pay attention or when they are careless. According to Lai (2012) factual errors are slips because they do not necessarily occur due to some lack of understanding. Instead, they occur when learners fail to recall basic facts that they need to solve a mathematical problem. Procedural errors, on the other hand, arise when students fail to follow correct procedures when answering a question (Lai, 2012). Conceptual errors are bugs or pervasive errors which arise because of lack of understanding of a concept.

An understanding of why these errors occur can help teachers to evaluate their instructional methods. The following section discusses the sources of student errors in Mathematics.

2.6.1 Sources of errors and misconceptions in Mathematics

Constructivists view learning as a process where students are actively involved in constructing knowledge. Knowledge is constructed when learners blend prior knowledge with present knowledge. According to Sarwadi and Shahrill (2014), learners' previous experiences are critical in knowledge acquisition. Through assimilation and accommodation, "a students' existing schema or concept will … determine what he or she learns from experience or instruction" (Sarwadi & Shahrill, 2014, p.2). Students bring to class previously learned knowledge from their past experiences. Thus any new information that is presented to them is assimilated into what they already know. The learner's existing knowledge may require some restructuring in order to accommodate the new knowledge. Sarwadi and Shahrill (2014) argue that when a child fails to assimilate and accommodate, a gap is created "in the learning of the concept, which in turn leads to mathematical errors and misconceptions" (p.2). They argue that some of the sources of learners' errors are the faulty problem-solving strategies that were learnt from previous experience as well as students' failure to grasp the teaching technique that the teacher employs.

Other errors learners make are systematic. Brodie (2014) and Sarwadi & Shahrill (2014) concur that systematic errors are pervasive and persistent. These errors result from students' misconceptions and may require some pedagogical intervention to address them. Makonye & Fakude (2016) studied Grade 8 learners' errors and misconceptions in the learning of addition and subtraction of directed numbers. They concluded that the sources of errors "seemed to be lack of reference to mediating artifacts such as number lines or other real contextual situations" (Makonye & Fakude, 2016, p. 1). Thus, mediating artefacts are applauded for their semiotic potential to build conceptual understanding (Mariotti & Maffia, 2018). Makonye & Fakude (2016), therefore, recommend that conceptual understanding should be built "through use of multirepresentations and other contexts meaningful to learners" (p. 1). This is in agreement with Lesh's translation model which argues that conceptual understanding is developed when mathematical ideas are represented in various modes (Suh, 2007).

Student errors are also systemic since they affect every learner across the curriculum (Brodie, 2014; Sarwadi & Shahrill, 2014). According to Sarwadi & Shahrill (2014, p. 3), systemic errors reflect learners' "understanding of a concept, problem or a procedure." Students sometimes fail to successfully develop correct conceptual understanding of mathematical contexts, leading to misconceptions. For this reason, misconceptions are, therefore, conceptual structures that students form in their minds when they are learning. According to Brodie (2014) they are evidence of learner cognition and needs. An analysis of these learner errors can give the teacher some insight into how learners think and what instructional interventions and strategies are appropriate to address them. Instead of viewing errors and misconceptions as obstacles to learning, it is important to see them as opportunities for learning and lenses that can be used to understand students' thinking.

2.6.2 Mathematics instruction with focus on errors and misconceptions

This study focused on how GeoGebra can be used to address learners' errors and misconceptions when solving probability problems. Research shows that Mathematics teachers sometimes do not communicate students' errors openly to avoid embarrassing or confusing the learners (Bray, 2013). However, an open analysis of students' errors can promote conceptual understanding (Bray, 2013). Research shows that discussing learner errors can help in overcoming some of the errors they make. For example, Makonye & Khanyile (2015) investigated the effect of probing Grade 10 learners on their errors in a written test. Participants wrote a pre-test covering content in algebraic fractions. After the test was analysed for errors, interviews in which learners were probed on the errors they made in the pre-test were conducted. The post-test results showed that the participating students overcame 98.6 percent of the errors they made in the pre-test. This is evidence that learner errors can be discussed in a classroom context without causing humiliation to the learners. Makonye & Khanyile (2015) recommend that teachers should probe learners on errors in their work in order to overcome the errors. There should, therefore, come a time in the classroom when the teacher openly discusses the type of errors students exhibit in their work.

The use of a semiotic artefact in a didactical cycle to accomplish a task can enhance conceptual understanding. A didactical cycle is a sequence of activities that the teacher plans for the learners to complete using a semiotic artefact (Bussi & Mariotti, 2008) (see section 2.2). For this study, the didactical activities were planned by the researcher and assigned to learners. These activities helped to locate learners' errors and misconceptions. As suggested by Bray (2013) and Makonye & Khanyile (2015), the study incorporated a focus on students' errors and misconceptions into instruction in order to enhance probability understanding. The study assumed that an awareness of students' errors and misconceptions can lead to important teacher and learner decisions which support probability understanding. This is particularly true in the light of Bray's (2013) framework for "designing and implementing lessons to leverage the instructional potential of errors" (p. 426). In this study, the pre-test played the important role of locating the errors and misconceptions Grade 10 learners have when solving probability problems.

2.6.3 Framework for Mathematics instruction with focus on errors and misconceptions

According to Bray (2013), a public discussion of students' errors can enhance mathematical understanding. Bray (2013) suggests a framework that can be followed to teach Mathematics with a focus on misconceptions. The framework has four phases: 1) selecting mathematical tasks, 2) planning for instruction, 3) students working on tasks and 4) public discussion. These phases are explained below.

Selecting mathematical tasks: The first phase involves selecting mathematical tasks for learners. The role of the teacher is to select mathematical tasks that have the potential to bring out the misconceptions that students have. Examples of such tasks are problem-based tasks that require students to give reasons and to justify their thinking (Bray, 2013). The Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011a) states that mathematical modelling should be the focus of the Mathematics curriculum. It points out that "contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible" (p. 8). This emphasises the importance of designing tasks that are context based and have the potential to challenge the students to the extent that any underlying misconceptions are brought to the surface.

Planning for instruction: In the second phase, after selecting mathematical tasks, the teacher then plans for instruction. Bray (2013) argues that as part of planning, teachers should anticipate the type of errors students will make as they work on the tasks. This anticipation will assist the teacher to plan in advance how they are going to leverage the potential of the errors. *Students working on tasks*: In the next phase, students get to actively work on tasks. When students are working, the teacher should observe how they grapple with the tasks and the errors they make.

Public discussion: The final phase is an open or public discussion of solutions and the errors that were made. The whole class is involved in analysing the errors that were made. Table 2.1 summarises Bray's (2013) framework.

Table 2. 1 Framework for mathematics instruction with focus on errors (Bray, 2013, p. 430)

Phase	**Strategies**
Selecting mathematical tasks	• identify the mathematical focus and related misconceptions. • Use problem-based tasks that emphasize the mathematical focus. • tweak task features (contexts, numbers) to provoke misconceptions.
Planning for instruction	• anticipate students' errors and identify underlying misconceptions. • Consider how errors can be used instructionally to illuminate mathematical ideas. • Plan a way to find out how students are approaching tasks. • Make provision for students to analyze and publicly discuss flawed solutions.
Students work on tasks	• Promote a culture of intellectual risk taking. • Find out how students are approaching tasks and the errors they are making. • Decide on a plan for public discussion (solutions to include, order, purpose).
Public discussion of errors	• engage students as a community of learners to analyze flawed solutions. • emphasize the conceptual lessons of errors. à Honor error makers and learning from mistakes.

2.6.4 Framework for analysing errors that students make in Mathematics

Errors in Mathematics are caused by lack of conceptual and procedural understanding. According to Herholdt & Sapire (2014), an error pattern analysis can help Mathematics teachers in developing effective pedagogical practices. Herholdt & Sapire (2014) propose a five-step framework that can be used to analyse students' work for errors. The five steps are: 1) determine the level of difficulty of each test item, 2) determine students' use of correct methodologies, 3) identify instances where correct methodology was used but led to an incorrect result, 4) identify items that were not attempted, and 5) identify the most commonly occurring errors (Herholdt & Sapire, 2014p.49).

Difficulty levels: The difficulty levels of test items indicate the items in a test which were difficult, moderate or easy for the test takers. Herholdt & Sapire (2014) state that the difficulty indices indicate the "phases of mathematical conceptual development through which learners pass" (p.47). Thus, to a large extent, difficulty indices help determine the conceptual levels of understanding that the test takers have acquired.

Students' use of correct methodologies: The second step in error analysis involves recognising what is working well in terms of students' choice of methods to answer the posed questions. It is important to have a clear idea of the concepts and skills that have been grasped by learners, hence the importance of determining the extent to which students used correct methods which led to correct results in answering the questions.

Correct methodology leading to incorrect results: In the third step of error analysis, the extent to which correct methods were used, but incorrect results were obtained is determined. According to Herholdt & Sapire (2014), this information is helpful because it sheds some light on where intervention should be directed.

Items that were not attempted: The next step takes into consideration the questions that students did not attempt. This step is undertaken in order to determine students' areas of weakness and their knowledge gaps. These cases would then need to be attended to during intervention lessons.

Most commonly occurring errors: The final error analysis step is to identify the most common errors that students made for each test item. The purpose of this step is to provide the teacher with a clear "overview of learner errors, the underdeveloped concepts on which they are based (where relevant) and ideas for how to address these errors" (Herholdt & Sapire, 2014, p. 49).

This study investigated the impact of GeoGebra intervention in addressing learner errors and misconceptions. An error analysis of both the pre-test and the post-test were conducted and the results were used to determine the impact of GeoGebra intervention on learners' understanding of probability.

2.7 Planning for intervention

The purpose of error analysis is to get some insight into possible explanations for errors learners make in their work. After analysing learners' errors, appropriate intervention should be planned and given. Planning for intervention involves thinking about remediating strategies that effectively address those errors (Herholdt & Sapire, 2014). In the current study, a GeoGebra intervention programme was planned to address learning gaps in probability understanding. This section focuses on planning learning interventions to improve learners' mathematical understanding. I start by discussing some indicators for mathematical understanding.

2.7.1 Indicators of mathematical understanding

According to Pirie & Schwarzenberger (1988), mathematical understanding involves "comprehension of concepts, the relationships between these concepts and ordinary language or physical objects" (p. 461). A child who has acquired deep mathematical understanding should be able to see relationships between concepts and justify why and how certain mathematical conclusions are reached. Niemi (1996) suggests four indicators that can be used to explain students' levels of mathematical understanding, viz: representational fluency or knowledge, problem solving, justification and explanation measures.

Representational fluency as an indicator of mathematical understanding

Representational fluency is the ability by learners to create, use and interpret pictorial, graphic and symbolic representations to develop deeper mathematical understanding. The notion of representational fluency is congruent to Lesh's translation model which was discussed earlier. According to Suh & Moyer (2007), representational fluency can be measured by analysing the frequency at which students used representations in answering questions in a written test. In the context of probability, these representations can take the form of Venn diagrams, tree diagrams, outcome tables and contingency tables. The current study required learners to adapt some of these representations to answer probability questions in the pre-test and post-test. Also, these representations were used in some questions that were posed. Learners were, therefore, expected to interpret probability contexts which were presented in various forms, including Venn diagrams. Table 2.2 shows the criteria that can be used to analyse students' representational fluency.

Table 2. 2 Translating probability problems into other representations (adapted from Suh & Moyer, 2007)

Solution strategies	No. of students	%
Used primarily pictorial representations		
Used primarily symbolic representations		
Used pictorial and symbolic representations		
Used primarily algorithms		
No strategy shown		

Problem solving and justification as an indicator of mathematical understanding

Another indicator of students' mathematical understanding is their ability to solve problems and to justify their procedures (Niemi, 1996). This also includes using representations to justify why a given solution is correct. For example, in the study of probability, students can be given probability situations which require them to draw or use Venn diagrams to justify an answer. For instance, given the sample space S and its subsets A and B defined as: $S = \{1; 2; 3; \dots ; 10\}$, $A = \{2; 3; 5 ; 7\}$ and $B = \{1; 2; 5; 10\}$, students can draw a Venn diagram to justify that $P(A \cap B) = 0{,}2$. In the current study, respondents were given similar probability problems to solve using Venn diagrams and probability rules.

Rosli, Goldsby & Capraro (2013) argue that traditional tests do not accurately measure students' problem-solving abilities since they "only focus on students' mathematical skills and procedures" (p. 54). They argue that problem solving and problem posing are cognitive activities which can be measured using a performance rubric. Performance rubrics are based on the requirement for teachers to assign students mathematical tasks and learning activities which are based on real life situations and engage students in active learning using various learning approaches (NCTM, 2000). A performance rubric is capable of assessing "students' mathematical concepts, procedures, processes, and disposition toward mathematics … at a specified level of performance in a rubric based on what they know or what they can do" (Rosli et al., 2013, p.56). An anaholistic rubric is an example of a performance rubric which combines analytic and holistic rubrics to assess different levels of students' conceptual and procedural understanding in problem solving and problem posing. The four stages of problem solving, namely, understanding the problem, planning a strategy, carrying out the solution, and looking back at the solution (Rosli et al. 2013, NCTM, 2000) can be used as criteria for assessing students' understanding. Table 2.3 shows a performance rubric for assessing conceptual and procedural understanding in problem solving.

Table 2. 3 Performance rubric for assessing conceptual and procedural understanding in problem solving (adapted from Rosli et al., 2013)

	Marks			Total
Criteria	0	1	2	
Understanding the problem	Complete misunderstanding of the problem	Part of the problem misunderstood or misinterpreted	Complete understanding of the problem	
Planning a solution	No attempt, or totally inappropriate plan	Partially correct plan based on part of the problem being interpreted correctly	Plan could have led to a correct solution if implemented correctly	
Getting an answer	No answer, or wrong answer based on an inappropriate plan	Copying error; computational error; partial answer for a problem with multiple answers	Correct answer	

When people are solving problems, they formulate new questions which help them develop a deeper understanding of the posed problem. This is described by Rosli et al (2013) as problem posing and it shows the level of understanding of the learner. Relevant performance rubrics can be used to assess students' problem posing. An example of a problem posing rubric suggested by Rosli et al (2013, p. 57) uses four criteria, viz: understanding the concept, solution of the problem, creativity of the problem and solution of partner's problem. Students are asked to formulate questions for each other based on the mathematical concept they are studying. They then solve each other's problems. Table 2.4 shows an example of a performance rubric for assessing mathematical understanding in problem posing.

Table 2. 4 Performance rubric for assessing mathematical understanding in problem posing (Adapted from Rosli et al., 2013)

		Marks		Total
Criteria	1	2	4	
understanding the concept	Poor understanding	Some understanding	Complete understanding	
solution of the problem	Attempted to solve	Partially correct	All correct	
creativity of the problem	Comparable to types in text	Somewhat different from text	Completely different from text	
solution of partner's problem	Attempted to solve	Partially correct	All correct	

An awareness of the students' levels of mathematical understanding provides a good starting point for teachers who want to plan for learning intervention for their learners. In the next section, I discuss the Response To Intervention (RTI) model to illustrate how Mathematics teachers can go about developing an intervention programme that addresses learners' specific challenges.

2.7.2 A response to intervention (RTI) model

The Curriculum and Assessment Policy Statement (CAPS) gives guidelines to teachers about the aims of teaching Mathematics, the skills that are required to teach and learn Mathematics, the content areas that should be focused on in Mathematics, as well as the weightings of content areas in Mathematics (DBE, 2011b, p.8). This is in addition to the other guidelines and policies that govern the teaching and learning of Mathematics in South Africa. The policy statement, therefore, is detailed and clearly specifies the progression of content. It also gives guidelines to clarify how content should be taught and how time should be allocated to each section. However, as Huinker, et al., (2014) correctly state, standards or policies do not teach; teachers do. This means that focus should be placed on the specific actions that teachers need to enact in order to ensure that effective teaching takes place and all students learn Mathematics.

Learning intervention programmes are designed to improve learners' abilities in their learning areas, as well as to fill knowledge gaps that exist. Some of the learning difficulties result from knowledge gaps that exist as a result of unfinished learning from previous grades, but others are difficulties students have in understanding a particular learning area. According to Bryant, Bryant, Gersten, Scammacca & Chavez (2008), children's learning problems should be identified early so that intervention is sought to overcome them. Fuchs and Fuchs (2001) discuss the response to intervention (RTI) model which can be adopted to address students' learning difficulties. The response to intervention (RTI) model consists of three levels, viz: primary, secondary and tertiary intervention. Primary intervention (also referred to as Tier 1) is the basic general education level where quality instruction is given to all students in the classroom. Secondary intervention (Tier 2) is focused on those students that have been identified as struggling in Tier 1 with specific content areas and have not managed to meet the acceptable standard. Individual attention is then planned for them so that they grasp the content. Tertiary intervention (Tier 3) focuses on students who are struggling with the content to the extent that they need extra lessons, extra strategies or additional resources to help them grasp. These interventions are planned to address learning difficulties that students might have in Mathematics.

Different approaches can be adopted for intervention using the TRI model. According to Bryant et al.(2008, p. 22), some of these approaches include "peer assisted tutors", "verbalisation of cognitive strategies" and "physical (concrete) and visual (pictorial) representations." This study drew from the response to intervention (RTI) model in a number of ways. First, all students who participated in the study received appropriate quality instruction in probability in the classroom. This was in line with Klingner & Edwards' (2006, p.109) argument that children should receive "culturally responsive, appropriate, quality instruction that is evidence based." Thus, every effort was made to ensure that participating learners received quality instruction at all the levels of intervention. Second, those students who did not meet the set standards in the first level of intervention were identified and given small group instruction to try and improve their understanding of the content. Those students who still had difficulties after intervention in Tier 2 were further assisted through intensive intervention strategies in preparation for their final examinations. Studies show that teaching interventions can be effective in addressing specific learning challenges. Some examples of such studies are presented in the next section.

2.7.3 Factors and variables for the effectiveness of teaching interventions

There are several factors which affect the effectiveness of teaching interventions. Many studies have been carried out which seek to evaluate learning effectiveness of various interventions (e.g. Al-rahmi, Othman & Yusuf, 2015; Lee, Kim, , Park, Kim & Jeong ,2012). Al-rahmi, et al.(2015) investigated the effectiveness of e-Learning in Malaysian higher education using a survey questionnaire. They investigated the factors that affect e-learning effectiveness through a survey in which 268 university undergraduate students participated. Their study showed that there was a correlation between certain specific factors and e-learning effectiveness. In particular, their study found that "self-efficacy, interface, community, usefulness, students' satisfaction and intention to use e-learning" were important indicators of the effectiveness of e-learning (Al-rahmi, et al., 2015, p. 625).

Other studies (e.g. Lee et al.,2012), identified attitude, satisfaction and participation as indicators for the effectiveness of their intervention. For example, Lee et al.(2012) identified the following factors as indicators of learner satisfaction: learners' performance, learners' views on the ease of use of Digital Textbooks, learners' views on how well content was organised in the Digital Textbooks, and learners' views on how Digital Textbooks supported interaction with other subjects. They collected data from 197 students to analyse the effectiveness of u-Learning environment and Digital Textbooks. Their focus was on the learners' levels of satisfaction with Digital Textbooks. The study found that the level of student satisfaction in Digital Textbooks declined. However, when 2 226 students were asked about learning performance, Lee et al. (2012) found that students preferred Digital Textbooks to paper textbooks.

Outhwaite, Faulder, Gulliford & Pitchford (2018) and Outhwaite, Gulliford & Pitchford (2020) also carried out studies to determine the effectiveness of a Mathematics app intervention on a group of students in early education in the United States. They employed a pupil-level randomised control trial (RCT) where a group of students were taught Mathematics using an interactive Mathematics app for a certain period of time. Another group was taught the same content, but did not use the app. The impact of the Mathematics app intervention was measured after twelve lessons by considering the effectiveness of the implementation of the app in the classroom setting as compared to the standard instructional practice (Outhwaite, et al. 2018 & Outhwaite et al., 2020). The results showed that there was "significantly greater math learning gains for both forms of app implementation compared with standard math practice" (Outhwaite, et al. 2018, p. 284).

Thus, research evidence shows that teaching interventions can be planned to focus on specific factors. The extent to which an intervention is effective can then be known by measuring how those factors or variables changed as a result of the intervention. This study is interested in ICT intervention in learning probability. In the next section, ICT affordances are discussed in detail.

2.8 ICT integration in Mathematics teaching and learning

Mathematics curricula around the globe advocate for learning experiences where students are given the opportunity to use ICTs to acquire knowledge. One of the aims of the current CAPS Mathematics curriculum in South Africa is "to promote knowledge in local contexts, while being sensitive to global imperatives" (DBE, 2011, p. 4). The use of ICTs to mediate students' learning experiences is encouraged and recognised as an effective way of delivering learner-centred education. This study investigated the impact of ICTs in probability teaching. Some of the benefits of using ICTs in pedagogical practices are highlighted below.

2.8.1 Affordances of ICTs in education

The emphasis on integrating ICTs in education is necessitated by the perceived benefits of ICTs in maximising learning. ICT affordances focus on the opportunities which technology provides to support learning activities (Hammond, 2010). According to Salomon (1993) cited in Conole & Dyke (2004, p. 115), "affordance refers to the perceived and actual properties of a thing, primarily those functional properties that determine just how the thing could possibly be used." They argue that a clear understanding of these affordances enable us to use the technologies effectively to support learning. Conole & Dyke (2004) propose a taxonomy of ICT affordances which consists of accessibility, speed of change, diversity, communication and collaboration, reflection, multimodal and non-linear, risk, fragility and uncertainty, immediacy, monopolization and surveillance. Some of these ICT affordances which apply to this study are outlined below.

Accessibility

Access to information has become relatively easy due to the available range of ICT (Conole & Dyke, 2004). This means that teachers and learners can use the available technology for learning. In this study, GeoGebra was used because of its accessibility for both teachers and learners.

Diversity

ICTs offer access to a wide range of knowledge since the whole world is easily accessible. As Conole & Dyke (2004, p.117) argue, “exposure to the experience of others is a key ingredient to effective learning and a potential affordance of ICT.” In this study, learners had the opportunity to learn from each other in a sociocultural context.

Communication and collaboration

Technology opens up possibilities for communication and collaboration as the need to engage increases (Conole & Dyke, 2004). Students worked together in this study, and shared ideas which were influenced by the use of GeoGebra software.

Reflection

Conole & Dyke (2004, p. 118) state that “ICT has the potential to enable reflection and criticality to be enhanced. It presents new opportunities for knowledge claims to be considered and subjected to the critical gaze of much wider and more diverse communities of practice.” In this study, learners brought to class some preconceived knowledge about some aspects of probability. Through the use of GeoGebra, an opportunity was created for them to reflect on their ideas and make necessary adjustments. In the same vein Podworny (2016) argue that technology supports creation of simulations that can serve as tools for reasoning. This is supported by Savard, Freiman, Theis & Larose (2013) who advance that technology assists in deepening conceptual understanding of concepts that learners sometimes find difficult to understand. It has also been reported that the use of technology in instruction yields high levels of focus, attention and motivation in learners (Beck and Huse, 2007; Bester & Brand, 2013).

Immediacy

The use of ICTs has the potential to speed up information processing to make it available almost immediately. The use of GeoGebra in this study allowed students to access results of their activities immediately. This is discussed in detail under GeoGebra affordances. According to Batanero et al.(2016), data collection, data storage, data representation and data organisation can be performed relatively fast and accurately using technology.

However, despite the known benefits of integrating ICTs into teaching, literature shows that the use of technology in classrooms worldwide is limited (Ertmer and Ottenbreit-Leftwich, 2010; Buabeng-Andoh, 2012). The following section discusses factors that influence ICT integration in learning.

2.8.2 Factors influencing teachers' use of ICTs in learning

According to Czerniewicz, Ravjee & Mlitwa (2006, p.7) South Africa has "no specific technology policies in higher education explicitly steering practices." Even in secondary education, despite the mention of ICTs in national policy documents such as the White Paper 7 (DoE, 2004) and CAPS (DBE, 2011a, 2011 b), there are no practical actions or guidelines that are given which direct the integration of ICTs in learning. School authorities, for example, are not adequately informed about the integration of ICTs in education (Bialobrzeska & Cohen, 2005) and teachers lack training in technology use. Several reasons are cited why teachers may not use technology in their classrooms.

One of the major reasons why teachers may not use ICT in their teaching is lack of necessary ICT skills and technological pedagogical training (Bell & Glen, 2008). In order to teach using technology, Koehler & Mishra (2009) argue that teachers need technological pedagogical content knowledge (TPACK). TPACK framework combines the teacher's content knowledge (CK), pedagogical content knowledge (PCK) and technological content knowledge (TCK). These three interact and culminate in TPACK. Content knowledge (CK) depicts knowledge of the subject matter that should be learned or taught (Koehler & Mishra, 2009). It also includes the teacher's knowledge of mathematical concepts and the alternative representations of these concepts. On the other hand, pedagogical content knowledge (PCK) is the teacher's specialised knowledge of a variety of aspects that are related to his/her teaching. These include the teacher's knowledge of the cognitive demands of the Mathematics tasks on learners (Ding, He and Leung, 2014) as well as the learning difficulties and misconceptions of students. PCK also depicts the teacher's understanding of the methods and strategies of presenting mathematical knowledge. According to Mishra & Koehler (2006, p. 1027), "a teacher with deep pedagogical knowledge understands how students construct knowledge, acquire skills, and develop habits of mind and positive dispositions toward learning." Besides CK and PCK, technological content knowledge (TCK) is also required to teach mathematical content using technology (Mishra & Koehler, 2006). TCK is knowledge about how technology integration can transform the subject matter for better understanding. Therefore, to use an artefact such as GeoGebra to teach probability requires the teacher's knowledge of how GeoGebra can be used to achieve such a task. Thus, teachers who lack TCK are not likely to use technology in their teaching. The TPACK framework, therefore, suggests that teachers' propensity to use technology depends on a number of factors, including teachers' understanding of how subject content can be represented using technology, teachers' knowledge of how technology can be integrated into pedagogy, and teachers' knowledge of how technology can simplify mathematical concepts.

Another reason why teachers may not integrate technology in their teaching is the lack of adequate resources. These include financial constraints and limited ICT resources. For example, according to Lundall & Howell (2000), out of 23 518 schools in South Africa, only 13 % of them had computers. Not much has changed in this regard to date.

According to Umugiraneza, Bansilal & North (2018), even though 80% of the Mathematics teachers they surveyed had a positive view about the usefulness of technology in enhancing mathematical understanding, these teachers did not use technology in their classrooms, save the calculator. The use of technology to teach is influenced by the teachers' perceived need to use it. Technologies are introduced in organisations to improve productivity. Whether those technologies are used or not depends on the perceptions that are held about them. Several models have been proposed to determine user acceptance of new technologies. The Unified Theory of Acceptance and Use of Technology (UTAUT) model (Venkatesh, Morris, Davis & Davis, 2003; Venkatesh, Thong & Xu, 2012) is one such example. The UTAUT model describes four constructs that determine or influence user acceptance of technology. These are performance expectancy, effort expectancy, social influence and facilitating conditions (Venkatesh et al., 2003, p.447). Figure 2.4 illustrates the constructs of the UTAUT model and how they interact with other factors. These constructs are explained below.

Performance expectancy

According to Venkatesh et al. (2003), performance expectancy is the degree to which technology users believe that using the technology will result in increased job performance. It is characterised by five predictors of intention, which are perceived usefulness, extrinsic motivation, job-fit, relative advantage and outcome expectations (Venkatesh, et al., 2003; Venkatesh, et al., 2012). Perceived usefulness is the belief that the use of technology will increase performance. In this study, perceived usefulness is viewed as the belief teachers and learners have that the use of technology in teaching and learning will improve understanding and performance. Extrinsic motivation is what motivates the user to use technology. In Mathematics teaching and learning, the motivating factor for the use of technology could be the anticipated improved conceptual understanding or performance as a result of the technology. The job-fit predictor is viewed in this study as the potential benefit that technologies are capable of yielding. Technology users, therefore, take into account the affordances of the system to determine their intention to use it. GeoGebra affordances will be discussed later in the chapter. Relative advantage describes the extent to which the use of a particular technology is perceived to be better than another. In this study, the relative advantage of GeoGebra simulation over rolling dice or tossing coins can be understood in the light of immediacy.

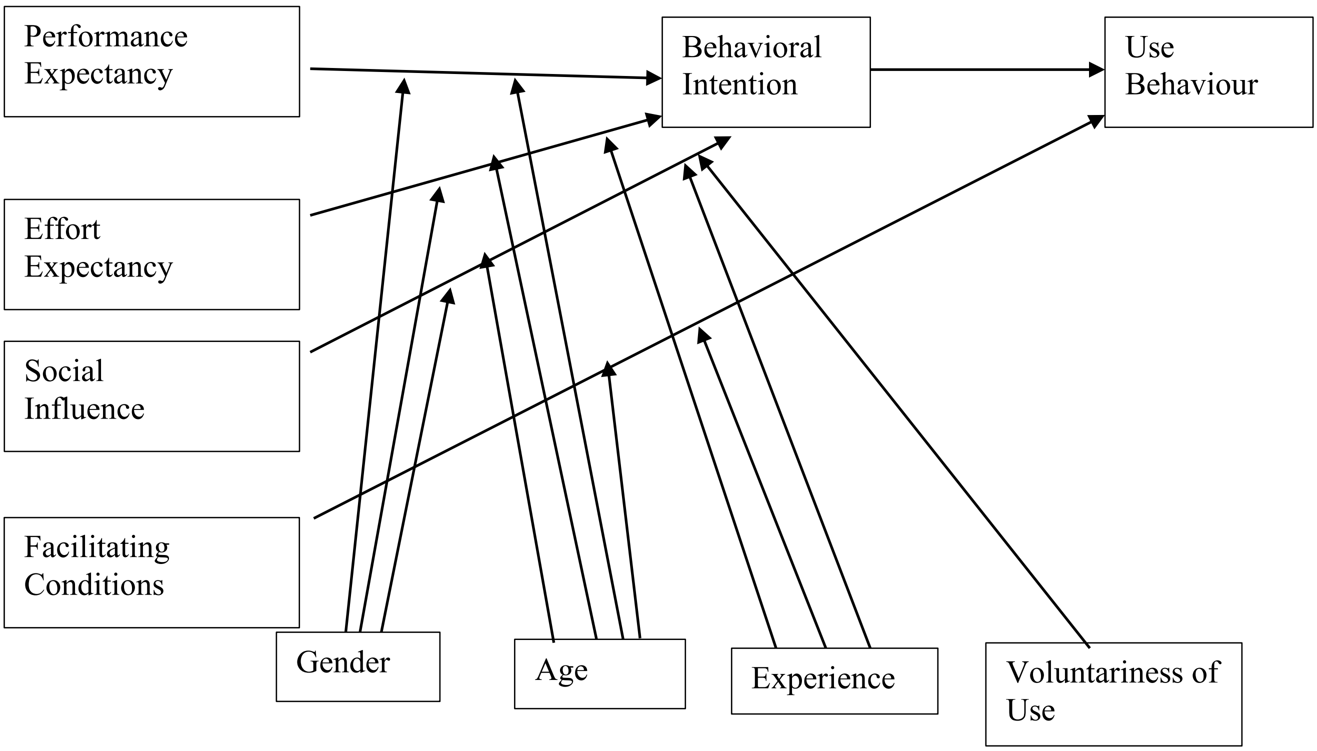


Figure 2. 4 Unified theory of acceptance and use of technology (UTAUT) (Venkatesh et al., 2003)

Effort expectancy

Venkatesh et al. (2003) define effort expectancy as “the degree of ease associated with the use of the system.” (p. 450). There are three predictors that are associated with effort expectancy, viz: perceived ease of use, complexity and ease of use. Perceived ease of use refers to user beliefs that using technology will be effortless. Complexity is the extent to which technology is difficult to use. Users may not be willing to use technology if they perceive it to be complicated for them to use.

Social influence

According to Venkatesh et al. (2003), social influence is the perception by a user about the extent to which other significant people expect him or her to use technology. There are three predictors for social influence, namely the subjective norm, social factors and image. Subjective norm is the perceived pressure other people expect one to be able to use, or not to use, technology. Social factors involve the interpersonal support that a user receives from his/her team. In a classroom setting, this involves learners helping each other or the teacher helping the learners. Image refers to the perceived status that comes with using technology. The CAPS curriculum makes it clear that technology should be integrated into the teaching and learning of Mathematics. The use of technology in this study was, therefore, a response to current educational policy expectation.

Facilitating conditions

Facilitating conditions is the extent to which a user believes that adequate support exists for him to use technology (Venkatesh et al., 2003). Three predictors for facilitating conditions include perceived behavioral control, facilitating conditions and compatibility. Perceived behavioral control refers to how easy or difficult it is to perform an action. In the context of this study, this refers to the availability of resources, knowledge, opportunities and the compatibility of the GeoGebra software in probability knowledge acquisition. Stols, Ferreira, Pelser, Olivier, Van der Merwe, De Villiers, & Venter (2015) define compatibility as "the extent to which new technology is perceived as complying with prospective users' current values, needs and previous experiences" (p.4). For the current study, compatibility is viewed as the extent to which the researcher's and students' preferred working styles were supported by the GeoGebra technology. Stols et al. (2015) studied Mathematics teachers' perceptions about integrating technology in the classroom using the Unified Theory of Acceptance and Use of Technology (UTAUT) framework. Their findings showed that with regard to effort expectancy, participating teachers found technology overwhelming. However, teachers indicated that they had adequate access to necessary technology. Due to their perceived limited technological skills, participating teachers did not use technology in their teaching. The UTAUT model, therefore, offers a framework for analysing not only teachers', but also students' perceptions about using technology in the classroom.

I now shift to research findings on technology intervention in the teaching of Mathematics. This review further helps in identifying research areas that previous studies have covered and, therefore, reveals gaps in research that require attention.

2.9 Research on the use of technology in teaching Mathematics

Many studies (for example, Jaffer, Ng'ambi, & Czerniewicz, 2007) show that technology-enhanced instruction has a positive impact on knowledge retention, learning outcomes, achievement and motivation. However, there seems to be limited research on the use of GeoGebra in probability learning. The current study addresses this gap by suggesting GeoGebra as an alternative resource for enhancing probability understanding.

A study by Jaffer, et al. (2007) shows that integration of ICTs into education increases learner motivation and enhances knowledge production, information sharing, communication and conceptual understanding. Although these studies report a positive impact of ICTs on learning outcomes, they do not explain how the use of ICTs actually leads to those new outcomes. This gap is also identified by Tolani-Brown, McCormac & Zimmermann's (2010) in their analysis of research on the impact of ICTs on learning outcomes in developing countries. Mathematical understanding is the objective of teaching and learning Mathematics. The use of a technological tool in learning Mathematics is an intervention that is aimed at enhancing mathematical understanding. According to Jaffer, et al. (2007) and Frith, Jaftha & Prince (2004), the design of educational technology interventions should be driven by the educational needs of teachers and learners. The following paragraphs report on previous studies which examine the impact of technology on specific dependent variables such as learner competence, learner motivation and learning experiences.

There is research evidence that integrating technology in class influences change in students' conceptions and competencies. Istenic Starčič, Cotic, Solomonides, and Volk (2016) explored the experiences of 150 preservice student teachers when ICT supported learning was introduced in their educational technology course. The participants had no prior experiences of digital storytelling or multimodal design when the study began. During their course, the students participated in instructional and learning activities that required them to use digital resources and ICT software and hardware. Their competencies and the impact of ICT integration in the course were then evaluated by identifying a set of learning outcomes. Pre-test and post-test scores were used to generate quantitative data, while qualitative data was collected through reflections of the participants. The study findings showed that participants' conceptions changed during the course from passive participants to active producers of media content. The findings also revealed that students' technical and pedagogical competency in using ICT technology and designing digital artefacts improved as a result of this intervention.

Star, Chen, Taylor, Durkin, Dede & Chao (2014) studied the impact of technology-based strategies for enhancing motivation in Mathematics. In order to understand the potential impact of technology-based learning on students' motivation in Mathematics, Star et al.(2014) investigated the relationship between technology-based activities, students' engagement and learning of Algebra. Three technology-based activities of algebraic nature were assigned to a large group of students and their teachers in order to investigate the relationship between these technology-based activities and students' engagement and competence. The researchers sought to determine the impact of technology-based intervention on students' motivation and the extent to which that impact was a result of the technology that was used. The findings on the impact of technology-based intervention on motivation were mixed. No gains were observed in some criteria for motivation, for example, self-efficacy. However, modest improvements in Mathematics learning were observed in the students' scores. The study highlighted the need to match technology-based motivational learning activities to the developmental stage of the learners.

It is generally accepted that integrating technology into teaching and learning enhances the learning experience. However, the way teachers use technology in the classroom heavily influences the effects and impact of that technology on learning outcomes (Drijvers, Doorman, Boon Reed & Gravemeijer (2010). Teachers who do not see the value of integrating technology into teaching and learning will most likely avoid using it to achieve their educational goals. Drijvers et al.(2010) also argue that teachers who find it difficult to adapt their teaching strategies to fit to situations where technology is integrated create another problem. This problem is caused by the unfamiliar atmosphere and the demands that the technology brings into the learning situation. Drijvers et al.(2010) investigated the type of orchestrations that teachers develop when using technology and how these are related to the teachers' views on the role of technology in Mathematics teaching and learning. Their study revealed that teachers' preferences of a particular orchestration were related to their views on technology-based instruction.

Carr (2012) examined the effects of iPad use on 5th-grade students' Mathematics achievement. The experimental group used iPads for a certain period of time in the school term while the control group did not. A pre-test and a post-test were administered before and after the iPad intervention respectively. The findings were analysed quantitatively and showed that the difference in achievement between the two groups was not significant. However, each group achieved higher scores in the post-test than in the pre-test. Carr (2012) recommends that increasing the intervention duration time and the number of participants could yield significant effects. Carr (2012) also recommends that collecting qualitative data in future research could yield more significant effects. The current study employed both quantitative data collection and qualitative data collection but recruited 36 participants compared to 104 participants in Carr's (2012) study.

Shieh (2012) examined the impact of Technology-Enabled Active Learning (TEAL) on students' performance and teacher's teaching of physics. They used pre-tests and post-tests, interviews and class observations as their data sources. Technology-Enabled Active Learning (TEAL) is a "pedagogical innovation established in a technology-enhanced multimedia studio, emphasizing constructivist-oriented teaching and learning … and features media-rich software for simulation and visualization to facilitate students' learning" (Shieh, 2012, p. 206). Two groups of students, the treatment group and the control group, wrote the pre-test and the post-test. Before the post-test, TEAL intervention was given to the treatment group in the TEAL studio, while the control group received their intervention using the traditional lecture methods in the classroom. The findings revealed that there were gains that were evident as a result of students' exposure to the TEAL intervention. The gains included increased test results, increased interest in physics classes and increased active participation by students during lessons. On the part of teachers, the results revealed that they became more enthusiastic about helping their students. Due to the TEAL intervention, students' achievements increased and this, together with their improved positive responses to the teachers' instruction, in turn motivated the teachers to become more dedicated to their students. Though there was no significant difference between the treatment and control groups' prior knowledge as revealed by the pre-test results, the findings revealed that the treatment group recorded a positive gain in the tests, while the control group achieved negative gains. Thus, the treatment group improved significantly in conceptual understanding.

The following section discusses the affordances of GeoGebra in teaching and learning Mathematics. GeoGebra was used in this study to mediate learning of probability to a group of Grade 10 learners.

2.10 Semiotic affordances and constraints of GeoGebra

Artefacts that can be used in a social context to communicate personal and mathematical meanings are regarded as semiotic resources. Van Leeuwen (2005) describes semiotic resources as artefacts that have a set of affordances based on their past and possible uses. GeoGebra is considered in this study as constituting a semiotic resource for the construction of probability knowledge. This suggests that probability concepts can be developed through the use of GeoGebra. Mariotti (2012) and Turgut (2015) suggest that the semiotic potential of an artefact can be analysed using its epistemological, cognitive, historic and didactic perspectives. Turgut (2005) states in particular that "the process of outlining the semiotic potential of an artefact needs a deep analysis encompassing an epistemological and didactic-cognitive perspective" (p. 2421). This section presents what can be done with GeoGebra in supporting the teaching of probability.

GeoGebra is a dynamic software whose affordances lie in its ability to combine functionalities of both Computer Algebra Systems (CAS) and Dynamic Geometry Software (DGS) to handle algebraic functions and geometric relationships (Hohenwarter and Jones, 2007). GeoGebra tools can be used to analyse statistical data. For instance, it is possible, using GeoGebra, to "create mathematical objects and explore them visually and dynamically" (Aizikovitsh-Udi & Radakovoc, 2011, p.4944). The use of GeoGebra as a dynamic tool provides a rich learning experience for students (Prodromou, 2014), and enhances conceptual understanding by providing opportunities for students to discover relationships. It also allows the students to interact with the software and to "construct solutions in a responsive milieu that provides feedback on their actions" (Olsson, 2017, p. 14).

GeoGebra is useful, not only for data management and data analysis, but also for providing students with opportunities to explore probability models. It can be used to engage learners in reasoning, modelling and exploration, thus improving students' problem-solving abilities (Prodromou, 2014; Bu, Mumba & Alghazo, 2011; Hohenwarter & Jones, 2007). It allows learners to use representations and models (Prodromou, 2014). For example, statistical data can be represented as graphs, bar charts, pie charts and probability functions.

The GeoGebra software is capable of showing multiple windows on the same screen, with one window displaying algebraic functions and the other window showing a visualisation of the function (see appendices D1, D2, D3 and D4). This allows the user to input a function and see a visualisation of the function immediately. GeoGebra shows the various representations as a graph or a table, and this provides immediate visual feedback to students, helping them create mathematical relationships (Kllogjeri & Shyti, 2010).

The ability for GeoGebra to construct many graphs and many diagrams using drag-and-drop tools gives students adequate time to interact with the multiple representations and observe patterns which lead to generalisations (Olsson, 2017). In statistics and probability, data collection and representation can be time consuming, but the use of technology can overcome that and leave the students with adequate time to interact with the data and explore relationships and generalisations (Batanero et al., 2016).

Sliders (see appendix D3) enable users to adjust parameters and visualise the dynamic effect it has on the shapes. The graphics can also be animated to enhance understanding of the relationship between parameters.

Learning probability can be enhanced when opportunities are created for learners to actively participate in activities that promote construction of mental and conceptual models of the situation being studied (Bu, Spector & Haciomeroglu, 2011). Through representations such as diagrams and tables, learners can improve their understanding of mathematical concepts and relationships.

GeoGebra can be used to simulate situations where a random experiment is repeatedly carried out. As an example, a random experiment involving flipping a coin a large number of times can be simulated using GeoGebra functionalities. Appendix D4 illustrates a simulation of flipping a coin 236 times, with heads (H) showing 130 times. The bars visualise the number of heads (H) and tails (T) obtained, and their heights will keep changing as the number of flips increase.

Studies by Olsson (2017) show that GeoGebra supports collaborative learning. Students working in groups were able to share ideas as evidenced by the conversations they held in the GeoGebra environment that they were working in. Also, students showed creative reasoning when they gave feedback in respect of the tasks they were working on. Another study by López-Pérez, Pérez-López, & Rodríguez-Ariza (2011) also shows that the use of ICTs in learning reduces the dropout rate and improves examination results.

GeoGebra, like any other technological tool, has associated constraints. According to Olsson (2017), the computer-generated feedback that students receive after entering specific commands in GeoGebra require the ability of the student to interpret the feedback. Unlike feedback that is given by a person, which is "explicitly formulated to help the student solve a specific task", computer feedback is "an automatically generated response to an action" (Olsson, 2017, p.21). Olsson (2017) further explains that computer-generated feedback is a direct response to the input data and must therefore, be interpreted "with respect to the purpose of the input" (p.21). Students, therefore, should be able to interpret computer feedback, which can be a problem for them. The ability to input the correct command does not guarantee that students will interpret the feedback accurately, thus the teacher's assistance will always be needed.

The other constraint lies in the students' ability to solve problems using GeoGebra. Olsson (2017) argues for a didactic approach of enhancing learning, where students are afforded the opportunity to construct a solution for an intellectually challenging problem through collaboration with others using a dynamic software (p.24). In a study, Olsson (2017) concluded that GeoGebra supports problem solving and reasoning. However, if students are guided in solving those problems, then the use of GeoGebra potentialities becomes limited. The study also showed that "only students whose reasoning was characterized as creative used feedback elaborately and reached a solution to the task" (p.31). This suggests that feedback that students get from GeoGebra require creative reasoning in order to be used conclusively. This goes back to the fact that computer-generated feedback still needs to be interpreted, and some students may find that challenging. After creating several representations using GeoGebra, it is, therefore, still possible that wrong interpretations of those graphs can be given, especially if the students do not understand the purpose of the data that was entered. There is no automatic guarantee that students will always see relationships in the various visualisations that they have created using GeoGebra.

Moreover, like any other ICT technological tool, GeoGebra may not always improve students' learning outcomes or performance. Neither may it always make teachers deliver effective lessons. Ertmer et al.(2010) argue that "knowing how to use technology hardware (e.g., digital camera, science probe) and software (e.g., presentation tool, social networking site) is not enough to enable teachers to use the technology effectively in the classroom" (p.260). They argue that the effective use of ICTs requires teachers' knowledge of a lot more other variables such as planning to teach with ICTs, selecting software that addresses learners' educational needs, and evaluating outcomes that are supported by the use of technology. As stated earlier, the teacher's knowledge of mathematical content and of the affordances of the technological tool is critical in teaching Mathematics using technology. This includes the knowledge of the relationship between the skills and concepts that they want to develop in learners.

It is also uncertain that learners will automatically benefit from merely using GeoGebra. López-Pérez et al.(2011) argue that integration of technology into teaching is not the only variable required for learners' outcomes to improve, but it should be combined with face-to-face interaction and learner motivation in a complementary way. This implies that the use of technology on its own does not necessarily produce the desired outcomes.

2.11 Summary

This literature review focused on issues that influence the teaching and learning of Mathematics and probability. The nature of probability and its teaching should be matched with the demands of the 21st century to produce graduates who can apply acquired skills in real life. Literature reveals that the successful teaching of Mathematics and probability using technology requires an awareness of the affordances of ICTs to address challenges that learners encounter when acquiring knowledge. Further research on probability teaching is required in South Africa to address the challenges that teachers and learners face in probability teaching and learning. In particular, research on Mathematics teaching should focus on teaching interventions using ICTs. This review reveals that there are gaps in research regarding the impact of GeoGebra on addressing specific challenges that learners face when solving probability problems. Whereas there is adequate literature which shows that learners face difficulties in understanding probability, there are not enough studies which link GeoGebra to the difficulty levels of probability questions in learners' written assessments. Furthermore, although there are several studies which investigate errors learners make in Mathematics, there seems to be little research on how GeoGebra intervention impacts on specific learning outcomes which are the object of error analysis. This study, therefore, sought to highlight the impact of GeoGebra on specific challenges that error analysis exposes. These challenges include among others, learners' perceived difficulty of questions in tests, use of strategies in solving problems, learners' errors and misconceptions and the prevalence of these errors. In the next chapter, the research design and methodology of the study are explained.

CHAPTER THREE

RESEARCH DESIGN AND METHODOLOGY

3.0 Introduction

Research methodology is a systematic way that research inquiry follows to solve the research problem (Wahyuni, 2012; Kothari, 2004). It is a plan or strategy that the researcher follows to answer research questions. According to Scotland (2012, p. 9), methodology is concerned with answering "why, what, from where, when and how data is collected and analysed" (Scotland, 2012. p.9). Hathaway, Urubuto, Henry, Byiringiro & Cartledge (2018, p. 16) argue that the methodology chapter provides sufficient information to allow the readers to "evaluate the persuasiveness of the study for themselves and replicate the study if needed." In this chapter, the research design and data collection methods used to collect and analyse data are discussed and justified. The justification links the research with research methodology literature. In addition, ethical considerations for the study and the participants are outlined. Finally, the limitations experienced during data collection, and issues of validity and reliability are discussed. I first discuss the research paradigm of the study.

3.1 Research Paradigms: Interpretivism and positivism

A research paradigm is viewed as a philosophical assumption and belief about how the world is perceived (Wahyuni, 2012). It is concerned about the worldview that the researcher holds and "perceives to be truth, reality and knowledge" (Ryan, 2018, p. 2). A research paradigm, therefore, shapes the way the researcher thinks, and provides a framework that guides inquiry. There are several paradigms that influence social research, among which are the critical paradigm, the positivist/postpositivist paradigm and the interpretivist/constructivist paradigm. For the purpose of this research, more emphasis will be placed on the positivist paradigm and the interpretivist paradigm since they influenced the research methods of this study in specific ways. In my research I refer to both methodologies because a mixed-methods research approach was used. Each paradigm can be distinguished by its ontological, epistemological, methodological and axiological stance. I will first define these terms and then move on to show how the various research paradigms answer these philosophical questions.

Ontology is concerned with the nature and structure of existence (Crotty, 1998). Ryan (2018, p.2) defines ontology as "the values a researcher holds about what can be known as real and what someone believes to be factual." Ontological considerations, therefore, seek to answer the question, “what is the form and nature of reality?” (Aliyu, Singhry, Adamu & AbuBakar, 2015, p.3). Thus, ontology deals with reality and studies existence in the universe.

Epistemology, on the other hand, is concerned about how people come to know the world (Ryan, 2018). It involves knowledge and is concerned about “what is entailed in knowing, that is, how we know what we know” (Crotty, 1998, p. 3). In particular, epistemology answers the question, “what is the basic belief about knowledge?” (Aliyu et al., 2015, p.3). Epistemology, therefore, deals with “the nature of knowledge, its possibility, scope and general basis” (Hamlyn, 1995, cited in Crotty, 1998, p. 16).

Methodological issues address how the researcher goes about finding out that which can be known (Aliyu et al., 2015). Thus, methodology involves strategies or plans of action that govern the choice of methods.

Axiology has to do with values and ethics. Tomar (2014, p. 1) defines axiology as the “branch of philosophy concerned with the general problem of values, that is, the nature, origin, and permanence of values.” Axiology answers the question, ‘what ought to be?’ The ethical considerations of this research which will be discussed later in this chapter seek to answer the axiological question. I will now move on to discuss the research paradigms in relationship to their ontological, epistemological, methodological and axiological questions. I will also show how the positivist and interpretivist paradigms specifically informed my research.

3.1.1 Critical paradigm

According to Ryan (2018, p. 10), critical paradigm is a theory that “seeks to challenge world views and the underlying power structures that create them.” The ontology of the critical theory argues that reality is that which has been “shaped by social, political, cultural, economic, ethnic, and gender values” (Scotland, 2012, p.13). This then implies that beings can “reconstruct their own world through action and critical reflection” (Aliyu, 2015, p. 4). Epistemologically, the critical theory assumes that we come to know through power and cultural structures. For this reason, critical theorists believe that change to improve humanity can be achieved through raising awareness about oppression (Ryan, 2018). The methodologies that critical theory employs are aimed at “interrogating values and assumptions, exposing hegemony and injustice, challenging conventional social structures and engaging in social action” (Scotland, 2012, p. 13). A critical researcher, therefore, employs action research methods and data sources such as open-ended interviews, focus groups, open-ended observations and journals (Scotland, 2012). The researcher’s role is that of a facilitator, who influences the research to a great extent to promote consciousness.

3.1.2 Positivism

Ryan (2018, p. 4) argues that "positivism is considered a form of, or a progression of empiricism. [It] believes knowledge should be objective and free from any bias stemming from the researcher's values and beliefs." Positivists believe that reality is the same for every person. For this reason, the ontology of a positivist researcher is realism (Scotland, 2012), a view that objects or facts exist whether humans know about them or not and can be proven. Thus, reality does not exist in the mind of the knower, but is discoverable through hypothesis and experiment (Ryan, 2018; Scotland, 2012).

The epistemological view underpinning positivism is objectivism (Crotty, 1998). This view argues that "things exist as meaningful entities independently of consciousness and experience, that they have truth and meaning residing in them as objects (objective truth and meaning), and that careful scientific research can attain that objective truth and meaning" (Crotty, 1998, p.6). This implies that knowledge is viewed as objective and discoverable through quantitative research methodologies. Positivist researchers, therefore, tend to adopt quantitative methodology and experimental methods which involve control groups, treatment groups, pre-tests and post-tests. The role of a positivist researcher is, therefore, to manipulate the variables in the study to eliminate bias and investigate phenomena.

With respect to axiology, a positivist researcher is bound to be unethical in data collection because of the desire to establish the truth. This is so because a positivist researcher goes about investigating phenomena in an objective and independent manner, often controlling the investigated (Aliyu, et al. 2015). Positivist research is also characterised by empirical methods which involve experimentation, quantification, manipulation of variables and statistical analysis. Although, the current study falls under interpretivism, some processes of research methods that were adopted are informed by positivism. Positivism employs quantitative research methods, and this study adopts quantitative methods to answer some research questions.

As already mentioned, the study followed a mixed-methods research design. The quantitative strand of the study borrowed from the positivist paradigm to collect and analyse data through quasi-experimental approaches. Quantitative data were collected using a pre-test and post-test. Data were analysed and interpreted quantitatively using inferential statistics. It was also important to identify and control specific variables to ensure that the observed outcomes were a result of the GeoGebra intervention and not any other factor. These variables will be discussed later in this chapter. The main research paradigm for this research is interpretivism. I now discuss interpretivism and its related interpretivist methodologies in the next sub-sections.

3.1.3 Interpretivism

Interpretivism has a relativist ontological perspective of reality (Ryan, 2018; Scotland, 2012). It posits that reality is relatively and subjectively knowable only through socially constructed meanings (Ryan, 2018). Different individuals may, therefore, view meaning differently since reality is mediated through individual people's senses, perceptions, experiences and feelings (Ryan,2018). Interpretivism believes that knowledge is situated in the social and cultural context of the knower and is discoverable through collaboration. As humans interact, they create knowledge and meaning about their world.

An interpretivist researcher gains understanding of a phenomenon under study when he or she participates in the social and cultural context in which it occurs. Thus, the researcher seeks understanding from the participants' perspectives (Scotland, 2012). In line with the interpretivist paradigm, this study employed open-ended interviews and observations to gain insight into participants' perspectives. Collected data were analysed through interpretation. The research methods used in this study will be discussed later in this chapter. In the foregoing sections, I showed how positivist and interpretivist paradigms relate to my research. The positivist paradigm helped me to decide on quantitative data collection and analysis methods while interpretivism contributed towards qualitative data collection and analysis. I now turn to some interpretive methodologies which relate to my research.

3.1.4 Interpretive methodologies

This research adopted the interpretivist paradigm. It focused on the challenges that are faced in the teaching and learning of probability at Grade 10 level. In particular, the study sought to identify the value of GeoGebra in addressing Grade 10 students' specific challenges in learning probability. The theory of semiotic mediation (TSM) (Vygotsky, 1978) was adopted as the framework for the study. The TSM framework explains how mathematical knowledge is created using an artefact. It claims that mathematical knowledge is created when learners interact in a sociocultural setting (Bussi & Boni, 2003; Mariotti, 2012).

The interpretive paradigm is not a single paradigm, but a family of paradigms that share common principles. This study drew from three of these, viz: hermeneutics, phenomenology and grounded theory. Some principles of hermeneutics, phenomenology and grounded theory informed the methodology and methods of this research. These principles are discussed below to justify how they influenced this study.

According to Ryan (2018), hermeneutics has a lot to do with the interpretation and understanding of texts and documents. Webb & Pollard (2006, p.31), citing Lee (1994), state that the concept of text has been extended "to include not just documentary artefacts that human subjects create, but also their individual actions, group behaviours, and even social institutions, all of which, as text analogues, have meanings that can be read and interpreted." Although this study is not primarily hermeneutic, an analysis and interpretation of the participants' actions, behaviours and written work were undertaken. In order to get a deeper understanding of students' conception of probability, data in the form of text were gathered. In addition, students' behaviours and actions were observed. These needed to be analysed and interpreted. It was, therefore, fitting to adopt the interpretivist paradigm in order to support the interpretation of students' written responses, their actions and their behaviours during GeoGebra lessons.

Some phenomenological principles also informed this research. According to Ryan (2018), phenomenology focuses on the interpretation of people's experiences. McMillan & Schumacher (2010, p.24) state that phenomenology is concerned with the meanings of a lived experience. In other words, the researcher makes sense out of the information and perceptions gathered from the participants by employing inductive research methods such as interviews and participant observation. This research focused on students' experiences of using GeoGebra to learn probability. The interpretive approach was fitting since a deep understanding of the impact of GeoGebra on students' learning outcomes required the researcher to make sense out of students' lived experiences and actions during class activities. In particular, students were observed using the artefact to learn probability. They were further interviewed in order to get an understanding of their experiences. Each student had his or her own perspective after their interaction with the GeoGebra artefact. Their different views and experiences were interpreted to determine the impact of the artefact on students' understanding of probability.

Another paradigm which influenced this research was Grounded theory. Charmaz (2006, p.2) states that the methods of Grounded theory "consist of systematic, yet flexible guidelines for collecting and analyzing qualitative data to construct theories 'grounded' in the data themselves." Qualitative data were collected from participants using observations, interviews and written tests. Initial coding and focused coding were then used to identify themes for understanding students' experiences in learning probability with a GeoGebra artefact. Initial coding or open coding involved taking a close look at the participants' responses and actions in order to identify themes that emerged. I closely read through the interview transcripts, watched the video clips and viewed photographs of the lessons to get the initial idea of what was happening. Charmaz (2006) suggests that each word, line or phrase in transcripts should be read with an open mind in order to discover possible codes as they emerge. This was necessary in this research since the themes that are reported in the study were not pre-determined but emerged from the data during the analysis process. This was in line with Engward (2013, p. 37) who states that in grounded theory, "the researcher should not predetermine a priori about what he or she will find, and what and how social phenomena should be viewed." Therefore, in line with Grounded theory principles, I avoided making assumptions about the effect of GeoGebra intervention and instead, adopted participants' perspectives. Focused coding was also applied by revisiting the codes that were obtained in open coding and adjusting them as more understanding of the data emerged.

The interpretive approach was fitting for this study because it allowed for the interpretation of qualitative data to arrive at findings. It also allowed the researcher to seek answers to the research problem through the experiences of participants in a social setting (Thanh & Thanh , 2015). Unlike positivism which views reality as absolute and discoverable only through rigid methods, interpretivism allowed for flexibility in data interpretation and analysis. The notion of flexibility was important because the researcher was able to find answers to the research questions by considering different viewpoints of the participants. This provided an in-depth insightful understanding of the situation under study. The participants in this study were learners from different backgrounds with different learning abilities. To a large extent their different views influenced by their different backgrounds. The interpretive approach was also fitting because the interpretation of participants' views was informed by constructivism. The constructivist principles supported collaboration among participants. Students were able to interact with the GeoGebra artefact to construct knowledge in accordance with the guidelines outlined in the theoretical framework of semiotic mediation and Vygotsky's sociocultural perspective. The researcher was also able to interact with the participants in order to gain a deeper understanding of their views and experiences of learning probability using GeoGebra. The researcher aimed to gather some data through observation methods. This required the researcher to participate in the study as a participant observer.

The preceding sections discussed research paradigms and methodologies that informed this study. This discussion was important to explain the philosophical perspectives that underpinned this research. The research design that was followed in the study is discussed next.

3.2 Research design: quasi-experimental case study within a mixed-methods design

A research design is a blueprint that deals with plans and procedures for intervention and data collection (McMillan & Schumacher, 2010). It specifies what data will be obtained for the study, as well as the conditions under which this will be done. It also specifies from whom data will be obtained, when it will be obtained and how it will be analysed (Yin, 2017). Creswell, Hanson, Clark Plano & Morales (2007, p. 237) describe research design as approaches to research "that encompass formulating research questions and procedures for collecting, analyzing, and reporting findings." The choice of a research design for this study took into account the need to interpret students' experiences of using technology to learn probability. A group of Grade 10 learners from one school participated. The study investigated the impact of the use of GeoGebra intervention on students' probability learning. It, therefore, needed a design that would establish a cause-and-effect relationship between the use of GeoGebra and specific learning outcomes to measure the impact of the intervention. For that reason, this study adopted a quasi-experimental case study. The use of GeoGebra by a particular group of Grade 10 students is a case that helps to shed some light on the impact of ICTs on students' understanding of probability.

Case study

According to McMillan & Schumacher (2010, p. 544), a case study is "an in-depth exploration of a bounded system." It entails studying a "single unit for the purpose of understanding a larger class of (similar) units" (Gerring, 2004, p. 342). According to Baxter & Jack (2008) such a study should provide tools for researchers to study complex phenomena within their contexts. A case study design was adopted because of its attributes that fit this research. I discuss these attributes next.

A case study allows the researcher to explore a bounded system in order to obtain an in-depth understanding of a phenomenon. This research explored ways in which GeoGebra can be used to overcome specific challenges that Grade 10 learners have when solving probability problems. Creswell et al. (2007) state that the focus in case study research is "on the issue with the individual case selected to understand the issue"(p. 245). Thus, this research aimed at building a deep contextual understanding of the case of GeoGebra and Grade 10 learners. It focused on reaching a deep understanding of participants' lived experiences of learning probability using a GeoGebra artefact.

Second, a case study is specific regarding the case or unit of analysis that is being studied. Miles & Huberman (1994, p. 25) define a case or a unit of analysis as "a phenomenon of some sort occurring in a bounded context." Therefore, a case is that which the researcher wants to analyse. It could be an individual or a group of individuals or a program, just to mention a few. In this study, the researcher wanted an in-depth understanding of the impact of GeoGebra on Grade 10 learners' understanding of probability. Thus, the unit of analysis was the interaction between the class of Grade 10 learners and the GeoGebra artefact in the activity of learning probability. The study sought to understand Grade 10 learners' errors and misconceptions in probability problem solving and to compare students' learning outcomes pre GeoGebra intervention with outcomes post GeoGebra intervention.

Third, a case study emphasises the boundedness of the phenomenon under study. Baxter & Jack (2008) state that a case or unit of analysis should be bounded in order to avoid the problem of answering questions that ought not to be answered. Boundaries can be set on the unit of analysis in a number of ways. Baxter & Jack (2008) suggest three ways by which cases can be bounded, viz: binding by time and space, binding by time and activity, and binding by definition and context. For this study boundedness was achieved by focusing on the impact of GeoGebra on one particular group of learners. The participants were a class of Grade 10 learners who were taught probability at two stages, first in class without technology and again in class using GeoGebra technology. These interventions were guided by the prescribed Grade 10 syllabus. Also, participants were students from a single school in Gauteng, South Africa. The cases were, thus, bounded by time since data were collected from the students in one particular year.

Fourth, a case study highlights the use of multiple data collection methods to study a phenomenon. McMillan & Schumacher (2010) argues that the in-depth exploration of a phenomenon in a case study is based on extensive data collection. This study employed multiple data collection methods. Pre-test and post-test data were collected in order to identify errors and misconceptions in students' work. These data were mainly quantitative and were also used to evaluate the effectiveness of GeoGebra intervention on students' learning outcomes. Qualitative data were collected through participant observation and semi-structured interviews.

Fifth, a case study is undertaken on a single unit in order to gain an understanding of a larger population of similar units (Gerring, 2004). This means that the findings from a case study can be replicated. This was achieved in this study by taking into account issues of triangulation in order to validate the methods and the results. These methods are discussed in detail later in this chapter.

A case study design was fitting for this inquiry because of the nature of questions that the inquiry sought to answer. Baskarada (2014) states that a case study is appropriate when the study seeks to answer "how" and "why" research questions. This study sought to determine how GeoGebra intervention affected learning outcomes.

Quasi experiment

This study adopted the quasi-experimental design. According to McMillan & Schumacher (2010), quasi-experimental designs can be used if participants are not randomly assigned into treatment and control groups. For this study, the treatment group received intervention through GeoGebra technology, while the control group received theirs through standard practice methods with no technology. Thus, the control group was used to capture what would have been the outcomes if the GeoGebra intervention was not given (White & Sabarwal, 2014). The intervention was then tested to measure the extent to which it achieved its objectives. For this study, the control and treatment groups were a group of Grade 10 learners at the same research site. These will be discussed in detail later in this chapter.

3.3 Sampling

Sampling involves the selection of the research site and the research participants (Creswell & Plano Clark, 2011). Before I explain how these were selected, I will discuss the sampling procedures that were used in this research.

Sampling procedures

Convenience sampling was used in this study to select the research site and the research participants. According to Etikan, Musa & Alkassim (2016, p. 2), convenience sampling is "where members of the target population that meet certain practical criteria, such as easy accessibility, geographical proximity, availability at a given time, or the willingness to participate are included for the purpose of the study." Purposive sampling was also used to select participants. Neuman & Robson (2014) argue that purposive sampling can be used when the researcher wants to select cases which are informative or wants cases that allow for in-depth investigation to gain deeper understanding of a phenomenon. On the same topic, Etikan et al., (2016, p. 2) state that purposive sampling is "the deliberate choice of a participant due to the qualities the participant possesses. Simply put, the researcher decides what needs to be known and sets out to find people who can and are willing to provide the information by virtue of knowledge or experience." A researcher can use purposive sampling to pick cases that possess particular characteristics that are of interest to him or her (Cohen, Manion & Morrison, 2013). Thus, qualitative researchers recruit participants "who have experienced the central phenomenon or the key concept being explored in the study" (Creswell and Plano Clark, 2011, p. 173). The selection of the research site and the participants is discussed in the following paragraphs. Reasons for using convenience and/or purposive sampling for the selection of these are also discussed.

3.3.1 Selection of the research site

The research site was a school located in one of the districts in Johannesburg, in the Gauteng province of South Africa. The school was conveniently selected because of its proximity and accessibility to the researcher who was an educator at the site. This made obtaining permissions to carry out the study easy. Using nonrandomised convenience sampling to choose the site was also economic, both financially and in terms of time. Any other randomly chosen site was going to be expensive for the researcher because it was going to involve a lot of travelling.

The school was also selected because of its potential to provide adequate data to answer the research questions. Creswell & Plano Clark (2011) state that a qualitative researcher can identify a small number of sites and participants as long as these can provide in-depth information about what is being studied. The other convenience for choosing this particular site was ease of academic access. Since I worked with the teachers, our working relationship was good, allowing me to get their support to conduct my research. For instance, after explaining my study to the teachers and the head of Mathematics, I got permission to revise the Grade 10 Mathematics year planner to accommodate the study. The probability topic was going to be taught as the last topic in October, but I was allowed to move the teaching of the topic to February/March. This allowed me adequate time to do my research. It would not have been possible to make these changes in other schools.

Background information about the research site

The site was a co-educational, independent day school catering for the educational needs of learners from Grade 0 to 12. The grades were split into three phases, viz: the Pre-school phase (Grade 0), the Preparatory school phase (Grades 1 – 7) and the High school phase (Grades 8 – 12). Each phase had its own learning facilities in its designated area within the whole college. However, they shared some facilities such as the auditorium, the swimming pool and the sporting fields. The school had a well-developed infrastructure with state-of-the-art sporting, cultural and academic facilities. It also had well-equipped classrooms, computer laboratories and a well-equipped media centre. The school was technology-driven. Every classroom was equipped with a data projector, a sound system, a laptop and iPad for every teacher. Wi-fi connectivity was available, allowing every teacher, student and staff unlimited access to it via all their devices.

The majority of the learners resided in wealthy estates within a walking distance from the school. Some drove cars or golf carts to and from school. However, a small number of learners resided more than seven kilometres from the school. Some of the learners in this category were less privileged and were learning on sponsorships, bursaries or scholarships. They travelled to school by public transport.

The high school phase of the college enrolled around 400 students every year from diverse ethnical, racial and religious backgrounds. About 20% of the students were international students mainly from Asia and countries around Africa. The majority of the learners at the school were predominantly white, representing around 90% of the school population. The teaching staff consisted of around 55 predominantly white teachers.

The school offered a wide curriculum. It offered the Independent Examination Board (IEB) examinations and the Cambridge Assessment International Education (CAIE).

3.3.2 Selection of research participants

Two Grade 10 classes at the research site were conveniently recruited to participate in the study. The choice of the participants was convenient because the researcher taught a Grade 10 class at the site. The other class was taught by a colleague. The Grade 10 Mathematics lessons were timetabled at the same time. The researcher planned all the lessons and discussed them with the colleague to help her adhere to some minimum expectations. I also set the probability questions for the cycle tests and examinations throughout the year.

One class was assigned as a control group and the other as the treatment group. My rapport with both groups was good, and for that reason, many students were willing to participate in the study. My choice of the Grade 10 class was based on my personal conviction that the study of probability should be thoroughly addressed in the Further Education and Training (FET) phase. This is partly because the FET phase is an exit point into the real world for high school learners. I also chose the Grade 10 class because of their cognitive levels of development compared to lower grades. The probability concepts that I wanted to explore using GeoGebra are introduced in Grade 10 and consolidated in Grade 11 and Grade 12.Thus, the Grade 10 class was judged by the researcher to possess the knowledge and the experience that were useful for the study.

As already stated, the sample consisted of two Grade 10 classes which were purposively and conveniently drawn from the participating school. There were 55 Grade 10 learners in the school in total, 39 of whom did Mathematics, and 16 did Mathematical Literacy. The students taking Mathematics were grouped into two classes. Initially, one class (the treatment group) had 17 students and the other class (the control group) had 22 students. Three of the participants in the treatment group withdrew from the study when they moved schools. The data collected from them in the pre-test were, therefore, not included in this study. This group also comprised students of mixed mathematical abilities and diverse socio-economic home backgrounds. The students were not grouped according to their mathematical ability, and therefore, were of mixed ability. The control group (n = 22) was taught by a colleague, who was the head of the Mathematics department. She was highly experienced and qualified to teach at this level. Her class also consisted of mixed ability learners of diverse socio-economic home backgrounds.

On average, the performance of these learners in written tests and examinations was good. All students taking Mathematics were expected to achieve a minimum mark of 40% to stay in the Mathematics class. Students that were struggling to get the minimum mark were advised to consider moving to Mathematical Literacy, but not forced to. The demographics of the participants are summarised in Table 3.1 table below.

Table 3. 1 Demographical data of participants

Control Group (n = 22)				Treatment Group (n = 14)			
	Male	Female	Total		Male	Female	Total
White	9	8	17	White	6	3	9
Black	0	5	5	Black	3	2	5
Total	9	13	22	Total	9	5	14

Assigning participants to control and treatment groups

As already stated, participants were assigned to the control group (n = 22) and the treatment group (n = 14). Both groups wrote a pre-test and a post-test. Before the pre-test, both groups received quality teaching where standard teaching methods were used. No ICT mediating tool was used during these lessons. The classes were taught again after the pre-test results were analysed. To determine the impact of GeoGebra, the treatment group received GeoGebra-assisted lessons while the control group received quality teaching using standard teaching methods. No ICT intervention was used in teaching the control group.

The independent variables that were manipulated in this research were the GeoGebra semiotic artefact and the GeoGebra-supported probability lessons. Besides these two, all the other variables were kept constant. In particular, all the probability lessons before and after intervention were conducted at the same time. Both groups followed a common learning timetable and assessment timetable. The probability activities that they used were all planned by the researcher to maintain uniformity. Both classes were taught by highly qualified teachers who had years of experience teaching the subject. The two groups also wrote the same tests which were set by the researcher and moderated by the colleague who taught the control group. Any observable changes were, therefore, attributable to the GeoGebra artefact and the GeoGebra-supported lessons.

Since this was a nonprobability quasi-experimental study, no randomisation was used to assign the participants. The researcher decided to assign the class that he taught as the treatment group for convenience. The two groups were not aware that they were participating as treatment group or control group. This was blinded to them in order to avoid a situation where they changed their behaviours in order to artificially fit into their role.

As stated earlier, the two classes consisted of mixed ability learners. Their performance in written tests were also comparable. Thus, the possibility that the observed changes were a result of students' abilities was minimized, if not eliminated.

Every effort was, therefore, made to ensure that any observed changes were a result of the independent variable.

3.4 Mixed methods

Although this study is predominantly qualitative, some research questions needed quantitative data to answer them. The mixed-methods approach was adopted to address this aspect. According to Johnson, Onwuegbuzie & Turner (2007, p.123), mixed methods research is the type of research "in which a researcher or team of researchers combines elements of qualitative and quantitative research approaches for the broad purposes of breadth and depth of understanding and corroboration." Creswell & Plano Clark (2011) discuss several research problems that may necessitate the use of mixed methods. They argue that mixing can be done in any of the following circumstances: when one data source is insufficient, when initial results need to be explained, when exploratory findings need to be generalised, and when the study needs to be enhanced using a second method. The mixed-methods approach was used in this study because the qualitative and quantitative data sources were insufficient on their own to fully answer all the research questions. This research needed qualitative data to fully understand the reasoning behind students' errors and misconceptions when solving probability problems. Qualitative data were also needed to understand the experiences of the participants in learning probability using GeoGebra. To obtain an understanding of the cause-and-effect relationship between GeoGebra and students' specific learning outcomes, quantitative data were needed.

The mixed methods approach, therefore, was able to offset the weaknesses of qualitative and quantitative methods.

Mixing was also useful for triangulation and complementarity. Triangulation corroborates and converges the results of the same phenomenon from different methods (Johnson et al.,2007; Creswell & Plano Clark, 2011). Complementarity, on the other hand, elaborates, enhances, illustrates or clarifies overlapping, but different results of a phenomenon from one method with results from the other method (Johnson et al. 2007). For this study, the embedded mixed methods approach was used. In an embedded mixed methods, the second data set can be collected and analysed before, during or after the collection and analysis of the larger design has been done (Creswell & Plano Clark, 2011). In this research, the quantitative strand was embedded within the qualitative strand. For this reason, the study is denoted by QUAL (quan).

I now focus on qualitative and quantitative research methods to discuss how these were used in this study. The research methods were grounded within the interpretive paradigm to "yield insight and understandings of behaviour and explain actions from the participant's perspective" (Scotland, 2012, p.12). I discuss first, the qualitative research methods.

3.4.1 Qualitative research methods

According to Ochieng (2009, p. 14), "qualitative researchers are interested in meaning, how people make sense of their lives, experiences and their structures of the world." Ochieng (2009) argues that meaning is discovered by interpreting participants' experiences in a setting. The qualitative phase of this study was interested in gaining understanding of students' misconceptions in probability, and ways in which GeoGebra can be used to address them. Data were gathered from participants in a social natural context of a classroom setting as suggested by the sociocultural constructivist perspectives (Vygotsky, 1978). The impact of GeoGebra was not predetermined by the researcher but was discovered through an inductive interpretation of students' experiences before, during and after GeoGebra intervention. The results were used to develop a pedagogical framework for teaching and learning probability using GeoGebra. The research instruments that were adopted to generate qualitative data included semi-structured interviews and participant observation. These are discussed separately below.

Semi-structured interviews

Semi-structured interviews were conducted to try and understand the reasoning behind students' errors and misconceptions. According to Wahyuni (2012), semi-structured interviews are methods of data collection where participants share their "perspectives, stories and experiences regarding a particular social phenomenon being observed by the interviewer" (p.

73). A semi-structured interview also allows participants to share their views, not only on predefined questions, but also on any other perspective that might arise during the interview. Flexibility was allowed in this research to accommodate any information that might unexpectedly emerge. Wahyuni (2012) refers to this as responsive interviewing. An interview protocol showing a set of questions that would be asked was developed, but flexibility was allowed to accommodate any emerging ideas. According to Creswell & Plano Clark (2011) an interview protocol assists in ensuring that the interview is organised and the questions asked help to answer the research questions. On the same subject, Castillo-Montoya (2016) argues for an interview protocol refinement framework that can enhance the reliability of interview protocols. The framework describes a process that follows four phases, which are:

- ensuring interview questions align with research questions,
- constructing an inquiry-based conversation,
- receiving feedback on interview protocols, and
- piloting the interview protocol Castillo-Montoya (2016, p. 811)

The interview questions for the study were aligned with the research questions as suggested by Castillo-Montoya (2016). Appendix C.1 shows examples of interview questions that were asked, and Appendix C.2 shows how the interview questions align with the research questions. The interview questions were based on the participants' experiences of learning probability in class, as well as on their responses in the pre-test. These questions sought to find a deeper understanding of the misconceptions Grade 10 learners held when solving probability problems, as well as students' beliefs, experiences, attitudes and perceptions about their learning of probability using GeoGebra. The interviews were audio-recorded with the permission of the participants.

The interviews took place in the researcher's classroom during tea break or lunch break or after school when students had no lessons. Seven students were interviewed separately on separate days. Interviewees were assured of confidentiality before the interviews started. They were assured that their names were not going to be used in the report, but only their ideas or views. Newcomer, Hatry & Wholey (2015) state that in confidential interviews, respondents are more likely to give honest responses to the interviewer. I needed honest responses from the interviewees, hence the importance of assuring interviewees about the confidentiality of the interviews.

To make the interviewees feel free, I spoke to each one of them before the start of the interview and assured them that they did not have to answer all the questions I would ask them. I also let them know that if they were not comfortable answering a particular question, they should just say so. The interviewees were also made aware that their responses were not any form of assessment, and that I was just asking them in order to get their views about the subject matter. They were assured that they were free to answer the questions in any language of their choice and to ask questions if they did not understand.

I had a good teacher-learner rapport with all my students, and that made the interview sessions smooth and productive. The interviewees were free to talk to me, and that allowed them to share as much of their views as possible. All the interviewees gave me permission to audio record the interviews. We negotiated a sitting arrangement where the recordings would be audible. The researcher's cellphone and iPad were used to record the interviews.

Advantages and disadvantages of semi-structured interviews

Semi-structured interviews were preferred because they allowed face-to-face conversations with the participants. Opdenakker (2006) states that "face-to-face interviews are characterised by synchronous communication in time and place" (p. 3). This allows the researcher to take note of cues such as voice, intonation and body language (Opdenakker, 2006) which can go a long way in providing the researcher with extra information. During the interviews, I was able to capture the facial expressions and the body language of the interviewees at the same time that I captured their verbal responses.

Another advantage of face-to-face interviews is what Opdenakker (2006) terms time delay between question and answer. Because we were in one place, interviewees responded to questions almost immediately. According to Opdenakker (2006), this ensures that the answers are spontaneous and genuine because there is no overthinking on the part of the interviewee.

A third advantage is the fact that I could easily record the interviews using an audio recorder. This allowed me to listen to the audios several times, and to transcribe the audios into text.

As already alluded to above, interviewees can answer questions dishonestly or untruthfully. To ensure that interviewees' responses were candid, I assured them before the start of each interview of the confidentiality of the interview as suggested by Newcomer et al.(2015).

Newcomer et al.(2015) states that semi-structured interviews can be time consuming. Since break times were used, a maximum of 30 minutes was possible for each interview that was conducted during break, which meant that some questions were not asked for some interviewees since these interviews took the form of a natural conversation between the interviewer and the interviewee. The interviews that were conducted after school were also controlled in terms of time because the interviewees needed to be released early for their extra-curricular activities. After the timetabled lessons, students had forty (40) minutes before they went to their various extra-curricular activities. That time was used for consolidation and consultation, where learners were expected to go to their teachers for further assistance in the subject areas where they struggled. Arrangements were made with some participants to have their interviews during that period. The maximum time that was possible for these after-lesson interviews was, therefore, thirty (30) minutes as well. The average length of the interviews was twenty (20) minutes. However, this worked to an advantage in the sense that interviewees did not get tired or restless, since the time spent per interview in this study was not lengthy. Since the interviews were face-to-face and individually conducted, getting time slots for all seven interviews, however, took long.

Participant observation

Participant observation took place in the researcher's classroom where the treatment group attended their Mathematics classes every day. The learning space was, therefore, familiar space to the learners. This provided several advantages, which included maximisation of learning time since learners knew where to find everything they needed for the lessons. Learners were allowed to sit in their usual places in order to avoid any form of anxiety related to sitting preferences. I also wanted to maintain the same atmosphere that always prevailed in our other lessons. I preferred to use my classroom instead of the computer laboratory because I wanted the class to use the same set of data at any given time, and I thought it would be beneficial if we worked from one device in class instead of everyone using their own device. The laptop in the classroom and the data projector were effective in all the intervention lessons that were planned. Learners were assigned tasks to complete as homework, and they used their personal devices to access the lessons at home. The classroom was spacious and well ventilated, with adequate fans to control the temperature.

Participant observation was used to collect qualitative data from participants in the treatment group. The GeoGebra-mediated intervention lessons were taught by the researcher. Probability worksheets, each representing a lesson lasting approximately 40 minutes, were planned and delivered. The lessons focused on probability concepts such as mutually exclusive events, inclusive events, complementary probability events, modelling relative frequency of an event in an experiment carried out a large number of times, use of Venn diagrams to model and solve probability problems. GeoGebra activities were used to model these events and experiments. Data related to how the participants used GeoGebra and how they collaborated with each other to complete the probability activities were collected through participant observation to help answer the research questions.

Observation was guided by the epistemological and ontological principles of social constructivism. Field notes were taken to record what the participants were doing and how they were using GeoGebra to interact with each other and with probability activities. The flexible and adaptive nature of the qualitative design also allowed the researcher to take note of anything else that was relevant, including any informal conversations among the researcher and the participants. The assistance of a colleague was sought to video-record some learning activities.

Handling challenges associated with participant observation

There are several challenges that are associated with participant observation. One of them is the Hawthorne effect (Oswald, Sherratt & Smith , 2014). According to Oswald et al., (2014), "the Hawthorne effect is when there is a change in the subject's normal behaviour, attributed to the knowledge that their behaviour is being watched or studied" (p. 53). To overcome this effect, Oswald et al.(2014), suggest that the researcher can establish a good working and trusting relationship between himself and the participants. In this study, a colleague was invited to my class to help take pictures and video clips of some lessons. It is possible, therefore, that some of the learners might have changed their normal behaviours because of her presence. To minimise this effect, the colleague visited the class for only two lessons. However, there was no significant deviation from normal behaviour that was observed when the other teacher was in class.

3.4.2 Quantitative research methods

Quantitative research is an empirical research paradigm which collects numerical or quantified data. Ingham-Broomfield (2014, p. 33) states that it is "a means for testing objective theories by examining the relationship among variables." Embedded in this definition is the philosophical notion that quantitative research falls under positivism. In quantitative research, the researcher controls the variables in such a way as to eliminate bias, investigate phenomena and generalise the findings.

This study used quantitative methods to collect and analyse data to identify students' errors and misconceptions in probability problem solving. Data were collected through a pre-test and post-test (appendix G) and analysed using error analysis. The observed learning outcomes before and after GeoGebra intervention were analysed using statistical methods to establish whether the changes were statistically significant or not. The results were then used to determine the impact of GeoGebra intervention on students' specific learning outcomes. The quantitative phase of the study helped to determine the cause-and-effect relationship between GeoGebra and students' learning outcomes. The research instruments used for this phase were pre- and post-tests. I will now discuss each of these research instruments below.

Pre-test

In the first quantitative data collection phase, during the second quarter of the year (July), participants wrote a pen-and-paper pre-test. Berry (2008) defines a pre-test as an assessment that is used at the beginning of a course for the purpose of determining "a subject knowledge baseline" (p.19). The purpose of the pre-test was to determine the errors and the misconceptions that students make when solving probability problems. The pre-test consisted of probability questions covering content that had previously been quality-taught by the researcher and a colleague during the first quarter of the year (February/March). The test was marked using a memorandum, and errors were quantitatively analysed to examine the misconceptions that learners had when solving probability problems.

Post-test

In the second quantitative data collection phase, during the fourth quarter of the year (November), both groups of learners wrote a post-test. The post-test was written after the treatment group had received GeoGebra-assisted intervention lessons in probability and the control group had also received intervention from their teacher. Intervention for the control group did not use GeoGebra or any other ICT tool, but was conducted through standard instruction methods. The purpose of the post-test was to collect further quantitative data on the errors and misconceptions that students made when solving probability problems.

Some considerations for using pre-post-test design

The merits and demerits of using a pre-post design were taken into account. Marsden & Torgerson (2012) state that "interventions that rely on the single group "pre-experimental" research design (also known as 'before and after' or 'pre- and post-test' design) may be threatened by a number of biases" (p. 584). They argue that the observed outcomes when learners are pre-tested and post-tested following an intervention may be a result of the intervention, but there is a possibility that the outcomes are a result of other variables such as "history, maturation, test effect and regression to the mean (RTM)" (p. 584). To increase validity in this study, two separate groups of learners were recruited. GeoGebra intervention was given to one group (the treatment group) and not to the other group (the control group). The pre-test and post-test were, therefore, not written by a single group, but two in order to increase the chances of linking the observed outcomes to the intervention.

Initially, the treatment group had seventeen (17) participants, three (3) of whom withdrew from the study after transferring to other schools. All three had written the pre-test but had not written the post-test or participated in the interviews. Their pre-test results were, therefore, excluded from the study. Only data from the remaining fourteen (14) participants were analysed. The sample size of (n = 14) was large enough to generate both quantitative data and qualitative data.

Another consideration was the time gap between the pre-test and post-test. There was a four-month gap between the pre-test and the post-test. Yu & Ohlund (2010) and Marsden & Torgerson (2012) argue that the gap between the two tests can jeopardise validity, a phenomenon they refer to as history. This implies that the outcomes in the post-test could be influenced by other events that took place between the pre-test and the post-test. I addressed this factor in my research by using the difference-in-differences (DiD) estimate to ensure that the observed difference was a result of the intervention and no other factors.

Maturation is another factor that I took into account. If data collection takes a long time to complete, participants may mature and perform well in the post-test regardless of the intervention (Yu & Ohlund, 2010). As already stated, there was a gap of four months between the pre-test and the post-test. However, during that period students were on school holidays for one month and were busy with other topics when they returned. The majority of them did not get a chance, therefore, to go over the probability topic on their own. Some of them had even forgotten most probability concepts when intervention was given and needed to be reminded of them. The school programme and calendar, thus worked in favour of this study in the sense that the effect of history caused by the gap between the pre-test and post-test was minimised, if not completely eliminated.

Statistical regression to mean (RTM) was avoided in this study because the two classes were composed of mixed ability learners. This was made possible since the classes were not grouped according to their mathematical abilities, but rather according to their subject combinations. Statistical regression occurs when participants are selected in such a way that those with similar characteristics or those whose scores are extremely high or extremely low are grouped together (Yu & Ohlund, 2010). High scoring students are likely to improve their scores after any intervention. This effect was neutralised by the fact that both the treatment and control groups were composed of students of mixed mathematical abilities.

The pre-test and post-test were used because of their ability to immediately pinpoint the errors that learners make. The tests were marked using a memorandum and error analysis was conducted after each test was written and marked.

In the preceding section, I discussed the mixed-methods design and justified the use of qualitative and quantitative research methods. Each research instrument was discussed, making sure that their merits and limitations are clarified. This was important because it validated the data collection methods. I will now turn to the classroom activities that were planned to allow GeoGebra intervention.

3.5 The design of the GeoGebra classroom activities

This study focused on probability learning. Learning is a mental function which can be mediated by the use of tools. Hasan (2005), citing Vygotsky, points out that tools are artificial stimuli which humans use to enhance their mental activities. The design of GeoGebra classroom activities drew from the Vygotskian notion of semiotic mediation. Hasan (2005) describes four elements that are necessary for a tool to be used for mediation. First, there must be an intention to mediate learning using a tool. For this study, probability activities were planned to allow learners to use GeoGebra simulations. The researcher also provided guidance on the use of GeoGebra to acquire probability knowledge. This was achieved through clear instructions to students. Second, Hasan (2005) states that a tool is said to mediate if there is a content of mediation. For this study, probability and its related concepts were the content of mediation. The syllabus and the assessment guidelines as stipulated in the CAPS document were consulted in order to cover the topic. Third, Hasan (2005) points out that learning using a tool should be an interactive process. This implies that the process of semiotic mediation necessarily requires a mediatee. The classroom activities were, therefore, planned with students in mind. The content was at their level, and their timetabled lessons were used to eliminate inconveniences. Fourth, the circumstances for mediation were taken into account as well (Hasan, 2005). The learning environment, including the sitting arrangement, was designed in such a way as to support interaction among students.

Before the pre-test was written, both the treatment and control groups received quality teaching on the probability topic in the first quarter of the year. Standard teaching methods were used without any ICT tool. The pre-test was then written in the second quarter in July. Two weeks before the test, both groups revised the topic to refresh their memories. In the fourth quarter of the year in October, both classes received further teaching. The treatment group was taught using GeoGebra while the control group was taught using standard teaching methods without any ICT tools. The post-test was then written in November. In this section, the GeoGebra intervention is discussed in detail.

The GeoGebra artefact and GeoGebra-supported lessons were used as independent variables to establish a cause-and-effect relationship between GeoGebra and learning outcomes. The Grade 10 syllabus and assessment guidelines as stipulated in the Curriculum and Statement

Policy Statement (CAPS) were used to plan the lessons. This ensured that the content was at the right level for the learners. Since the topic had been taught before, it was possible to cover the topic in four GeoGebra intervention lessons. The activities were designed by the researcher in the form of four worksheets. Each worksheet was a lesson lasting 30 to 40 minutes. The concepts covered in each worksheet are described below.

Worksheet 1 : Probability concepts

The aim of this lesson was to give the learners an opportunity to revise and familiarise themselves with probability concepts. The concepts that were covered in the lesson included understanding of the meaning of the following aspects: experiment, possibility, fair, bias, trial, outcome, random, event, sample space, probability, exclusive and inclusive. The class was given the following YouTube link to a video by Khan Academy to introduce some of these concepts: https://www.youtube.com/embed/uzkc-qNVoOk. Students were asked to watch the video and then answer the questions on the worksheet (Appendix F.1).

The researcher guided the learners in this lesson and discussed these concepts in context. Students were asked to give examples of their own to show that they understood the concepts.

Worksheet 2: Probability models to understand theoretical probability and experimental probability using a coin simulation

This worksheet (Appendix F.2) was aimed at comparing theoretical probability with experimental probability using a coin simulation. The link https://ggbm.at/LZbwMZtJ was given to allow students to toss a coin a large number of times in order for them to recognise a trend in the outcomes. They recorded the outcomes during the GeoGebra simulation activity and the researcher posed relevant questions to probe them to think. In particular, students were asked to give their opinions on the relationship between the relative frequencies of heads and the theoretical probability of heads.

Worksheet 3: Probability models to understand theoretical probability and experimental probability using a die simulation

In this lesson (Appendix F.3), theoretical probability and experimental probability were compared and discussed using a GeoGebra simulation of rolling dice. The GeoGebra link https://ggbm.at/UsoH4eNl was given to students to allow them to complete the tasks. Using a simulation of rolling two dice, students were asked to define X as “the sum of the numbers rolled.” They recorded the outcomes in an outcomes table and discussed the relationship between the relative frequencies and the theoretical probabilities of the outcomes.

Worksheet 4: Outcomes tables and Venn diagrams

In this lesson (Appendix F.4), two dice were rolled and events A and B were defined as:

A : Sum of the numbers appearing on the dice

B: At least one of the dice shows a 2.

Using a simulation of rolling dice in the link, https://ggbm.at/J9NyWRa5, students drew outcomes tables and used the obtained data to draw Venn diagrams.

During these intervention lessons, field notes were taken to capture students' responses to the activities. Students' levels of engagement, interaction and participation were also observed. The researcher also observed any behavior and challenges that were observable in order to gather as much qualitative data as possible.

The next section will focus on the challenges that were encountered during data collection. These challenges included teaching plans, quality instruction, scheduling of intervention lessons and student participation in the study.

3.6 Limitations experienced during data collection

There were several challenges regarding the implementation of the planned GeoGebra intervention lessons. These challenges included scheduling of the probability topic in the year planner, ensuring high-quality instruction of the probability topic, scheduling of the GeoGebra intervention lessons and students' participation. These are discussed in detail below.

Scheduling of the probability topic in the Mathematics teaching year planner.

The first challenge was related to the Annual Teaching Plans (ATPs). According to the school's Grade 10 Mathematics planner, the probability topic was scheduled to be taught as the last topic towards the end of the year. For all the grades, from Grade 9 to Grade 12, the school Annual Teaching Plans (ATPs) scheduled the probability topic to be taught at the end as the last topic. The guidelines that are provided in the Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011b) regarding content to be taught, sequencing of topics per term and pacing of topics throughout the year list probability as the last topic. Textbook authors also leave the topic till the final chapter. In Paper 1 Mathematics examinations, probability questions generally appear somewhere towards the end of the paper. Although the CAPS document states that the order of topics is not prescriptive, teachers tend to follow the order given in the ATPs prescriptively. For this research, the school year planner did not provide adequate time for the study to be conducted. The teaching of probability as the last topic, two to three weeks before end of year examinations would not adequately accommodate GeoGebra integration because of examination anxiety and pressure. It was necessary for the researcher to, therefore, request that the Grade 10 probability topic be moved and be among the first three topics to be taught in the first term of the year. Permission was granted by the Mathematics head of subject and the fellow Grade 10 Mathematics teachers to move the teaching of the probability topic to the first quarter of the year in February/March.

Ensuring high-quality instruction of the probability topic

Another challenge which is related to the delay in teaching probability was time constraint. The researcher observed that in Grade 9 the teaching of probability was rushed. This was caused by time constraints and other factors such as pressure from end of year examinations. For this study, it was important for students to receive high-quality probability teaching as advanced in the response to intervention (RTI) model (Klinger & Edwards , 2006). It was, therefore, important to minimise problems associated with time constraints by teaching the topic a bit early in the year. Studies show that teachers' pedagogical decisions can be influenced by time constraints. For example, Teig, Scherer & Nilsen (2019) investigated the relationship between teacher self-efficacy in teaching and perceived teaching challenges that are related to time constraints. They found that teachers who perceived time constraints as obstacles did not frequently use inquiry-based methods in their teaching.

The Curriculum and Assessment Policy Statement (CAPS) encourages the use of teaching strategies that promote the development of problem-solving and cognitive skills. These strategies enable students to "identify, investigate and solve problems creatively and critically" (DBE, 2011b, p. 9). This aim cannot be achieved if there is no adequate time allocated to the teaching of Mathematics. There are many reasons that are cited which contribute towards the loss of teaching time in South African schools. Among these is the unrealistic expectation that the lengthy Mathematics curriculum can be completed within the limited allocated teaching time using teaching strategies that CAPS advocates. In reality, the time allocated to the teaching of Mathematics ends up used for other purposes. For instance, teaching time is lost to other school programmes such as school camps, sports meetings and school assemblies. Also, the CAPS curriculum places a lot of emphasis on assessment, which takes up a lot of teaching time. The mid-year examinations and the end-of-year examinations alone both use up almost two months during the year. This is over and above the many hours that students spend writing cycle tests and class tests. Investigative approaches to learning Mathematics are necessary for the development of cognitive skills and for understanding what is learnt. This helps learners to apply what has been learnt. However, if time is not adequately allocated, the use of such strategies gets compromised. Teig et al. (2019) suggest that a discussion about allocating more teaching time in science is necessary. In order to engage learners in inquiry-based probability learning, more than nine hours may be required. The Annual Teaching Plans (ATPs) allocate two weeks for probability teaching. This is equivalent to nine hours for probability, which is not enough if innovative strategies are to be used.

In order to address the perceived lack of investigative teaching of probability, the researcher planned the teaching of the probability topic himself by sequencing the concepts into lessons and suggesting a variety of classroom activities that could be used. The researcher also planned formative assessments for the involved classes in order to ensure uniformity. The initial teaching of the topic was done in February/March and the pre-test was written in July, a fortnight before students broke away for the school holiday in August. The aim was to ensure that enough time was allocated to the teaching and learning of probability before the pre-test and the post-test were administered.

Scheduling of the GeoGebra intervention lessons

GeoGebra intervention lessons were planned for October the same year and the post-test was taken in November. It was found that scheduling GeoGebra intervention lessons outside the timetabled times after school was going to be difficult due to students' commitment to other after-school activities such as sports and clubs. To address this problem, the intervention was given in October during timetabled lessons. One consideration about this arrangement was how the intervention was going to be scheduled within the timetable without disrupting the teaching of other topics. Fortunately, the syllabus was completed in time and there was a week available in October to revise probability as part of preparation for the forthcoming end-of-year examinations. The timing worked in favour of the students.

Students' participation

This research started with 21 students in the treatment group, who all wrote the pre-test. However, seven of these pulled out of the study because they either opted to study Mathematical Literacy or they moved school. Their pre-test results were, therefore, not included in this study. Only 14 students in the treatment group participated in the study from the beginning till the end. Out of the 14 participants in the treatment group, only 5 agreed to participate in the interviews. All the 22 students in the control group participated in the study from the pre-test stage to the post-test stage, although only 2 of them agreed to participate in the interviews.

GeoGebra-assisted lessons were conducted in the researcher's classroom except for some homework tasks that students completed on their devices at home. In class, the GeoGebra software was used to simulate probability experiments such as rolling dice or tossing coins. Students were able to participate in these lessons because the commands that were required to use the software were simple. Each simulation was accessed by clicking on a link that was given. To carry out the appropriate GeoGebra action, all that was required was to select the right number of coins or dice and then to click on the start, stop, pause, reset or any relevant key.

The simulations were pre-programmed in the software. By clicking the correct option, it was possible to generate data. At the same time, the software produced graphical visualisations that accompanied the data that were generated. Students were able to see these visualisations because they were data-projected on the screen that was fixed in front of the class. The experiments could be paused, stopped or resumed at any point. This allowed the class to engage in discussions. In order to get students to experience the GeoGebra effect from the same source, some experiments were carried out in class using one laptop instead of letting each student carry out their own experiment on their own devices. This allowed the class to engage in discussions and to focus on the same aspect of the lesson. At the same time, this gave the researcher a chance to interact with the class as a collective during the experiment.

In the preceding section, challenges related to the actual data collection process were highlighted. Both qualitative and quantitative data were collected for this research. The following section is dedicated to the data analysis techniques that were employed in this research.

3.7 Data analysis techniques

To investigate the impact of ICT mediation on Grade 10 learners' understanding of probability, qualitative data were collected through interviews, observation, video clips and photographs while quantitative data were collected through a pre-test and post-test. The use of multiple data sources necessitated the use of multiple data analysis techniques. According to Leech & Onwuegbuzie (2007), utilising multiple data analysis types enhances data analysis triangulation which can lead to a better understanding of the phenomenon being studied. In this section, I discuss data analysis techniques that were employed. These techniques include coding and error analysis.

Table 3.2 shows a summary of the data collection methods and the data analysis methods for each of the research sub-questions.

Table 3. 2 Matrix of research questions, data collection methods and data analysis methods

Primary Research Question: *In what ways can GeoGebra be used to overcome errors and misconceptions that Grade 10 learners have when solving probability problems?*

Research sub question	Data collection method	Data analysis method
What errors and misconceptions do Grade 10 learners make when modelling and solving probability problems?	Pre-test (QUANTITATIVE - qualitative) Post-test (QUANTITATIVE – qualitative)	Error analysis Difference in Differences
What errors and misconceptions do Grade 10 learners make when modelling and solving probability problems?	Semi structured interviews (QUALITATIVE) • Audio recordings • Interview transcripts	Coding/constant comparison analysis Framework approach
How can the use of GeoGebra support the learning of various probability concepts? In particular, how can GeoGebra be used to address Grade 10 learners' errors and misconceptions in probability problem solving?	Participant Observation (QUALITATIVE) • Photos • Videos • Student scripts • Participant observation • Observation/Field notes	Coding/ constant comparison analysis

3.7.1 Qualitative data analysis techniques: coding

As earlier stated, qualitative data were collected from multiple sources, including semi-structured individual interviews, participant observation, photographs and videos. The reason for using multiple data sources was to generate adequate information about participants' experiences of using GeoGebra to learn probability. I also needed a platform where participants'' experiences and views could be verified by looking at them from different sources.

Qualitative data were analysed using coding, also known as constant comparison analysis. The aim was to identify underlying themes in the dataset (Leech & Onwuegbuzie, 2007). According to Saldaña (2009), coding is an interpretive act which requires the researcher's ability to summarise what the participant says or what the researcher observes using descriptive words. Through the process of coding, underlying themes that emerge from data are identified. Saldaña (2009, p.3) defines a code as "a word or short phrase that symbolically assigns a summative, salient, essence-capturing, and/or evocative attribute for a portion of language-based or visual data." Therefore, codes are "tags or labels for assigning units of meaning to the descriptive or inferential information compiled during a study" (Miles & Huberman, 1994,p. 56). According to Leech & Onwuegbuzie (2007), coding can be undertaken deductively, inductively or abductively. In deductive coding, codes are determined before the analysis is done, and then looked for in the data. In inductive coding, codes are obtained from the data. In abductive analysis codes emerge iteratively. In this study, codes were inductively obtained using processes of framework approach (discussed below) and some principles of grounded theory (discussed below and also earlier in section 3.1.4). Inductive coding was necessary because the pedagogical framework for learning probability using GeoGebra was not pre-determined but emerged from the analysis of the data that were collected.

Leech & Onwuegbuzie (2007) and Baskarada (2014) suggest coding steps that a researcher can follow to analyse qualitative data. These steps were followed to analyse interview and observation data to generate relevant themes for the phenomenon under study. The first step that I undertook was to read the transcripts. Leech & Onwuegbuzie (2007) and Baskarada (2014) suggest that coding should start with reading the research memos. Research memos are conceptual write-ups or ideas which can be in the form of a sentence, a paragraph or a few pages (Miles & Huberman, 1994). After reading through the transcripts, the data were then chunked into smaller but meaningful parts which were labelled using the same code.

Some principles of Framework approach, Grounded theory, phenomenology and hermeneutics were adopted to analyse qualitative data. I briefly discuss how these paradigms guided my analysis in the following paragraphs.

The Framework approach influence

Hackett & Strickland (2018) describe the Framework approach as a systematic qualitative data analysis method that allows for the identification and analysis of themes in text data. On the same subject, Smith & Firth (2011) argue that the framework approach is effective for "making the data analysis transparent and illustrating the linkage between the stages of the analysis" (p. 4). The approach has five stages which are connected to explain how data analysis is done from the start to finish. These stages are: 1) familiarisation, 2) thematic framework, 3) indexing and sorting, 4) data summary and display, and 5) mapping and interpretation (Hackett & Strickland, 2018).

Familiarisation analysis stage is where the researcher familiarises himself/herself with the participants' responses by immersing himself/herself in the data. For this study, interviews with each participant were audio recorded. The audio data were transcribed and read through carefully in order to understand students' responses. In the pre-test, post-test and observation activities, students responded to some questions by way of explanation. This generated data in text form which required analysis. Such text was read carefully to familiarise with its contents. Video recordings and photographs that were taken during intervention lessons also provided data which the researcher viewed carefully to familiarise with and make sense of.

Using the ideas that were identified in the first stage and taking into account the aims of the study, the data were coded to develop a thematic framework. Each text data was read carefully and codes or labels were used to describe what the researcher interpreted to be the meaning of the text. The codes that were identified included the behaviours of the participants, the beliefs of the participants in relation to a particular probability concept, and the reactions of the participants while solving probability problems using GeoGebra. The codes were put into categories and used to develop the analytical framework that was applied to the qualitative data. Qualitative interview data analysis, therefore, involved grouping together into themes and subthemes ideas which were similar and relevant to the aims of the study.

Grounded theory influence

In Grounded theory, Charmaz (2006, p. 46) views coding as “the pivotal link between collecting data and developing an emergent theory to explain these data.” On the same subject, Engward (2013) argues that Grounded theory methodology provides a framework for data analysis which can be adopted to inductively identify codes in qualitative research. In this study I used some of the Grounded theory coding strategies to analyse qualitative data. In particular, the strategies of initial coding and focused coding (Charmaz, 2006) were used to identify themes. These strategies were described in section 3.1.4. Both initial coding and focused coding were used to identify themes which assisted in developing a pedagogical framework for teaching probability using GeoGebra.

Phenomenological influence

Individual interviews and participant observation generated qualitative data which were useful in describing participants’ own experiences of using GeoGebra as a semiotic artefact for learning probability. In generating the findings from interview data and observation data, I interpreted what the participants actually said, or were seen doing. This was to ensure that the findings came from the data, and not from the researcher’s prior conceptualisation (Roulston, 2014). According to Roulston (2014), it is important for the researcher “to remain open to what is in the data (p. 302). Roulston (2014) further argues that the researcher should spend time reflecting on data and what they mean. I read through the transcripts several times and used only data that related to the phenomenon under study. The findings of this research were, therefore, influenced by principles of phenomenology.

Hermeneutical influence

Hermeneutics deals with the interpretation of texts and documents (Ryan, 2018; Wernet, 2014). Wernet (2014) advances that text can take the form of an utterance which must be interpreted to obtain its objective meaning. He proposes that the interpretation of a text should be guided by four principles of interpretation, viz: "to exclude the context, to take the literal meaning of text seriously, sequentiality and extensivity" (Wernet, 2014, p. 239). Excluding the context before interpreting a text does not mean that the context in which the utterance was made should not be considered per se. As Wernet (2014) argues, the meaning of a text should first be understood as such before its context is considered. "The contextualization follows the context-free interpretation of a text (Wernet, 2014, p. 239). This is important because it gives the researcher the opportunity to "contrast a latent and a manifest level of meaning in every text sequence" (Wernet, 2014, p. 239). The second principle to be considered when interpreting utterances is that the literal meaning of a text should be taken seriously. This means that the researcher should concentrate on the actual text that the participants said when interpreting for meaning. The third principle of interpretation, that is sequentiality, specifies that the text must be analysed line by line (Wernet, 2014). Finally, the principle of extensivity suggests that interpretation should be done in depth (Wernet, 2014). Thus, all the text fragments should be included in the analysis so that chances of distorting the meaning of a text are minimized (Wernet, 2014). These principles were taken into account when interpreting the actual words that participants uttered in the interviews and during observation.

Qualitative data in this research were also collected in the form of visual and video data. I discuss these in the next paragraph.

Transcription and coding of visual and video data

During GeoGebra intervention, clips of video recordings and photographs were taken in order to capture the class layout and atmosphere, as well as participants' gestures, expressions, actions, body language and conversations. Screen recordings of some lessons were also made using the Open Broadcaster Software (OBS). Participants were also observed using GeoGebra to solve probability problems. Their strategies and use of representations to simplify the problems were also noted. The photographs were viewed carefully, and the videos watched repeatedly to familiarise with data. According to Bailey (2008) transcribing visual and video data into written form is an interpretive procedure which requires the researcher to make judgments about what detail to include or exclude. On the same subject, Saldaña (2009) states that coding of data requires the researcher's ability to summarise what the participant says and what he observes using descriptive words. Using codes, emerging patterns in visual data (photographs) and in video recordings were observed and categorised into themes. These observations were analysed and used to answer the research questions. According to Saldaña (2009), the identified codes, categories and themes can then develop into a theory. These observations were integrated into a pedagogical framework for learning probability using GeoGebra.

3.7.2 Quantitative data analysis techniques: error analysis

Learners' errors, slips and misconceptions in the pre-test and post-test were used to investigate the impact of GeoGebra on specific learning outcomes. Brodie (2014) argues that errors should be viewed as "evidence of learner thinking and learner needs."(p.222). Thus, teachers can draw from learners' errors by learning how to handle them and how to use them to support the development of mathematical concepts in learners. Brodie (2014) further explains that the occurrence and persistence of learner errors can be understood through the lenses of constructivism. The errors that learners make in Mathematics arise when they apply their misconceptions. Misconceptions are, therefore, conceptual structures that learners individually and subjectively construct. They are not easy to remove because according to the learners' current knowledge these are meaningful conceptual structures and truths. However, learners end up committing errors because what they consider to be "truth" may not be recognised as mathematically correct knowledge by a more knowledgeable other (MKO). An analysis of learners' errors was undertaken, not only to determine the value of GeoGebra pedagogical intervention, but also to identify patterns and processes that should be considered when developing a pedagogical framework for probability teaching.

According to Shalem, Sapire & Sorto (2014) and Herholdt & Sapire (2014), special categories of teacher knowledge can assist to explain the type of errors learners make in Mathematics. Pedagogical content knowledge (PCK), in particular, is essential to identify and classify the type of errors students make when solving mathematical problems. In this study, the errors students made were identified and classified as procedural errors, conceptual errors and factual errors (Lai, 2012). According to Lai (2012), procedural errors occur when students do not follow correct steps to solve a problem. Conceptual errors, on the other hand, occur when a student does not have a full understanding of a specific Mathematics concept. Factual errors are mistakes that occur when students have not mastered, or fail to recall, a basic fact that is necessary to find a solution to the problem. The process of error analysis is described in the following paragraphs.

Error analysis

Error analysis processes advanced by Lai (2012) and Herholdt & Sapire (2014) were combined to identify and analyse errors. Herholdt & Sapire (2014) suggest that error analysis should follow five steps which are: 1) identification of level of difficulty of the question, 2) identification of cases where correct methods are used, 3) identification of cases where correct methods are used but the results are incorrect, 4) identification of cases where questions are not attempted and 5) identification of cases where the most common errors occur. Lai (2012) also specifies that when doing error analysis, students' responses should be analysed, and that observed error patterns should be described.

In this study, the prevalence of each type of error that students made in the pre-test was compared to the prevalence of errors made in the post-test to help determine the impact of GeoGebra intervention. Inferential test statistics and difference in difference (DiD) analyses were done to explain the impact that GeoGebra intervention had on the prevalence of errors made.

The analysis started by calculating difficulty indices and discrimination indices of test items as suggested by Herholdt & Sapire (2014). According to Koçdar, Karadag & Sahin (2016) item difficulty "is the percentage of learners who answered an item correctly and ranges from 0.0 to 1.0. The closer the difficulty of an item approaches to zero, the more difficult that item is" (p. 16). This means that higher difficulty values indicate easier questions, while lower values indicate difficult questions. On the other hand, the discrimination index of an item is "the ability to distinguish high and low scoring learners. The closer this value is to 1, the better the item distinguishes the learners who get a high score from those who get a low score" (Koçdar et al., 2016, p. 16). A positive discrimination index is considered as a desirable result since it indicates that top scoring students got the question correct more than did low scoring students. A negative discrimination index, on the other hand, indicates that students in the bottom half got the question correct more than did the students in the top half. Good question items are the ones that discriminate between high achievers and low achievers. To calculate the levels of difficulty and discrimination indices, the following procedure was used. The total scores (see Appendix E1 and Appendix E2) obtained by learners in the tests were ranked in numerical order from the highest to the lowest. The scores were then grouped into the top and bottom halves, the top half being the top high scores and the bottom half being the bottom low scores. The difficulty index of each item was then calculated using the formula:

$$\text{difficulty level} = \frac{\text{number of correct answers}}{\text{number of students}}$$

The following scale was used to classify items as either difficult, moderate or easy:

- Difficult items: An item is difficult if the difficulty index $\leq 0{,}5$
- Moderate items: An item is moderate if $0{,}51 \leq$ difficulty index $\leq 0{,}84$
- Easy items: if difficulty index $\geq 0{,}85$.

The discrimination index (r) of each item was calculated using the formula:

$$\text{Discrimination Index} = \frac{N_{top\ half} - N_{bottom\ half}}{N_{each\ half}}$$

were

- $N_{top\ half}$ = number of students in the top half who got correct answers,
- $N_{bottom\ half}$ = number of students in the bottom half who got correct answers and
- $N_{each\ half}$ = number of students in each half.

The following scale was used to classify the discrimination index of a test item as good, fair or poor:

- an item has a good discrimination if the index is above 0.30,
- an item has a fair discrimination if the index is between 0.10 and 0.30, and
- an item has a poor discrimination if the index is below 0.10.

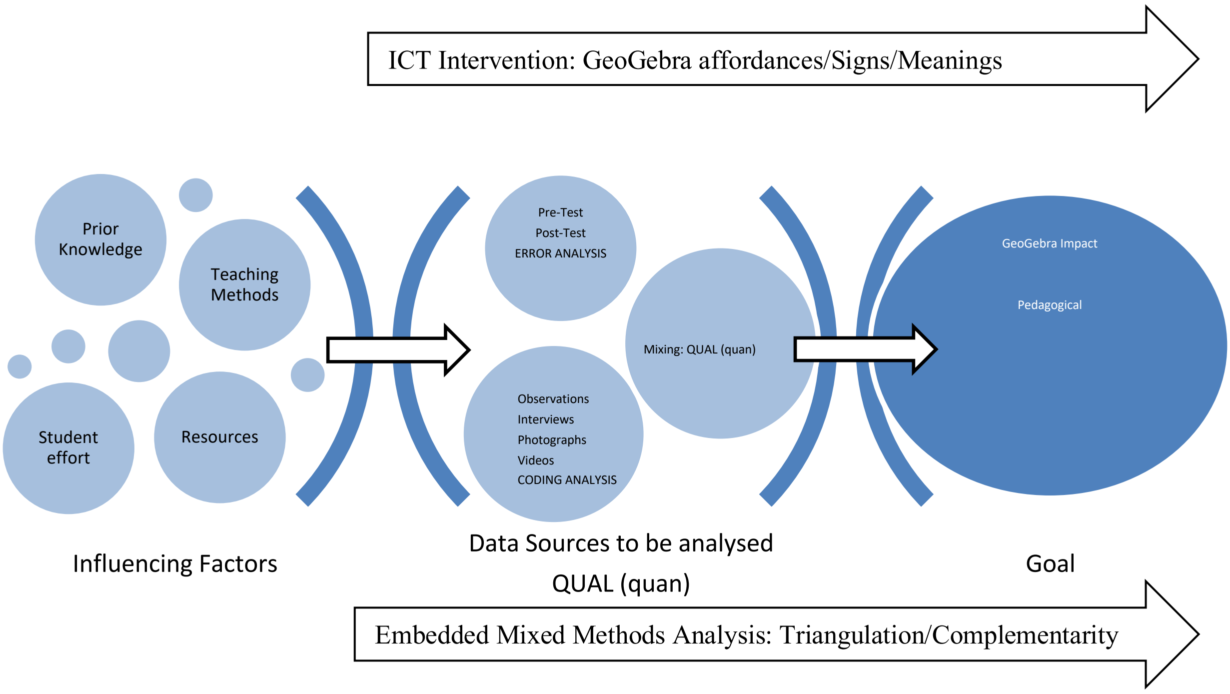
3.7.3 The analytical framework

The analytical framework presented below was developed by taking into account the research questions and the learning outcomes that are associated with probability. According to Keshavarz (2011), learning outcomes are measurable knowledge, skills and attitudes teachers expect students to demonstrate after learning a course.

Learning outcomes can be assessed in many different ways, including direct methods such as standardised tests to measure students' knowledge, skills and abilities. They can also be assessed using indirect assessment tools such as survey forms, and the success rate at which students graduate or progress to study at higher levels (Keshavarz, 2011).

In this study, after receiving quality teaching and intervention in the probability topic, students wrote a pre-test and a post-test at the respective phases of data collection. It was assumed in developing the analytical framework that the learning outcomes measured in the pre-test and post-test were influenced by a number of elements such as prior knowledge, student effort, student attention, teaching methods, resources and time. The learning outcomes from the pre-test and post-test were analysed quantitatively using error analysis. Interview and observation data were qualitatively analysed separately and combined with quantitative data findings to make explicit the value of GeoGebra in learning probability. Figure 3.1 shows the analytical framework this study followed.

Figure 3. 1 Analytical Framework



3.8 Validity and Reliability

The criteria of research trustworthiness includes credibility, transferability, dependability and confirmability (Wahyuni, 2012). Measures were taken in this study to ensure validity and reliability of results.

This being a quasi-experimental case study that employed mixed methods strengthens the validity of its findings. According to Bärnighausen, Tugwell, Røttingen, Shemilt, Rockers, Geldsetzer, ... & Atun, 2017, p. 21) , quasi-experiments can "generate causal evidence with a high degree of external validity" and "avoid the threats to internal validity that arise when participants in nonblinded experiments change their behavior in response to the experimental assignment to either intervention or control arm." In other words, quasi experiments effectively address issues of internal and external validity.

Rogers & Révész (2019, p.3) describe validity as "the soundness of a study, the degree to which the results of a study accurately answer the question that it set out to answer." According to Wahyuni (2012), careful selection of a case is crucial to achieve validity. The probability topic that the study is focusing on was selected from the Grade 10 students' Mathematics syllabus, and the activities that were assigned in class were in line with the subject assessment guidelines (SAGs) for the grade. In order to achieve validity, data were collected using multiple sources which included tests, audio recorded interviews, participant observation field notes, video recording and photographs.

External validity refers to "the ability of a study's conclusions to be transferred to other studies" (Fusch et al., 2018, p. 21) irrespective of population or setting. It is the extent that the study findings are transferable to other settings (Wahyuni, 2012). To achieve transferability, a detailed description of the research site, and a detailed physical description of the learning space where the GeoGebra activities took place were provided (section 3.3).

Construct validity is the ability of a study's inferences to relate to the conceptual framework of the study (Fusch et al., 2018). This was achieved in this study through triangulation by using multiple data sources and research instruments.

Validity in data analysis was achieved by using different analysis methods. Hussein (2009) argues that whenever both qualitative and quantitative data are used in a study, "then more than two methods are needed in the analysis towards attaining data validation within the single paradigm: and further extending the analysis between the two paradigms for completeness purposes" (p. 4). The quantitative findings of the study were reached through error analysis using Herholdt & Sapire's (2014) error analysis framework, while qualitative findings were arrived at through coding. The interpretive research paradigm was adopted to yield insight and understanding of the participants' behaviours, actions and interview responses. To maintain validity of analysis, my interpretation of the participants' views was guided by the interpretive paradigm and the theoretical framework that were adopted for the study. Further, I based my interpretation on the conceptual framework and analytical framework.

Measures were also taken to eliminate the influence of external factors on the outcomes of the study. This was to ensure that the results were only attributable to the influence of the independent variables, GeoGebra intervention in this study. As already stated, this quasi-experimental design followed a pre-post test design. The treatment group received GeoGebra intervention while the control group did not. The learning outcomes between the two groups were compared before and after intervention. In order to ensure validity and reliability of the results, specific variables had to be managed. This included eliminating history bias, selection bias and lack of blinding.

First, history bias which could result from the length of time between the pre-test and post-test was a possible threat to validity. The pre-test was given in July, while the post-test was written four months later in November, two weeks after intervention. The time length between the tests was carefully considered to ensure that it did not affect the study in a negative way. Soon after the pre-test, students went on a month-long school holiday. They, therefore, did not receive any additional teaching until they returned to class in September. Upon returning to class, both groups were busy learning other topics and finishing up the syllabus. The only time they received instruction in probability again was during the intervention week. The outcomes that were recorded for both classes were, therefore, attributable to the intervention they received.

Second, selection bias which could be caused by the different characteristics between the treatment and control groups was minimized. The treatment and control groups were selected from the same school and were of mixed ability. Their performance in Mathematics assessments were comparable. Therefore, the outcomes that were observed before and after intervention cannot be attributed to ability, but to the independent variable.

Third, bias that could have resulted from lack of blinding was eliminated because the researcher was a teacher at the research site and taught the grade. During observation, no significant deviation from normal behaviour was observed. An observation guide was used to direct the researcher's attention to behaviours of interest such as how learners used the artefact, how learners used data that were obtained from GeoGebra use and the level of engagement of learners. I had taught the same students the previous two years in Grade 8 and Grade 9. The learners were used to me and my teaching style and were comfortable in my class. For this reason, my presence was not obtrusive.

Rogers & Révész (2019) argue that although reliability cannot guarantee validity, it is a prerequisite for validity. They describe reliability as "the extent to which a measurement or an experimental procedure elicits consistent interpretations about the construct that it sets out to measure" (Rogers & Révész, 2019, p. 3). A study's reliability may suffer when data collection and data analysis procedures and instruments are not accurate. These procedures and data collection instruments were discussed in detail in section 3.4 in order to ensure reliability

3.9 Data saturation and triangulation

Data collection procedures for this research were discussed earlier in this chapter. I was aware of the importance of using data collection methods that would eventually result in data saturation. According to Fusch & Ness (2015), it is important for researchers to reach data saturation because failure to do so might hamper content validity of the research. Fusch & Ness (2015), argue that "data saturation is reached when there is enough information to replicate the study" (p. 1408). In their study, Guest, Bunce & Johnson (2006), found that after interviewing twelve out of sixty participants, there was no new information that was emerging. They discovered that the basic elements of "metathemes were present as early as six interviews" (Guest, et al., 2006, p. 59). For this study, a total of seven learners were interviewed. Two of them were interviewed after they had written the pre-test, and five were interviewed after the GeoGebra intervention lessons after the post-test was written. The number of students interviewed was the same number of students who had given consent to be interviewed. The interview questions that were asked were structured and because of limited time, it was not possible for each interviewee to respond to all the questions. As a result of the flexibility of the study as explicated in the theoretical framework, some questions were skipped for some participants. The responses from the interviewees indicated that acceptable data saturation levels were reached because the respondents did not offer very diverse views about the phenomenon under study. Their responses were comparative and fitted well under the themes that were identified through coding analysis. Engward (2013) stresses the importance of considering the experiences of the participants in a study when exploring a social phenomenon since these guide research. The findings in research are merely reflections of patterns in the experiences of the participants. After the interviews, the themes that were identified were a reflection of the experiences of the learners that participated in the study.

The quantitative data collection methods that were used in the study were the pre-test and post-test. The tests were administered to 1) identify the errors and misconceptions that Grade 10 learners made when solving probability problems, and 2) to investigate the impact of GeoGebra on students' errors and misconceptions. The study identified three common types of errors that students made, viz: factual errors, procedural errors and conceptual errors. The reasoning which led to students' errors was similar, suggesting that data saturation was well achieved in this respect. All the errors that were made were systemic and common for all the students who happened to make them.

In order to enhance chances of reaching data saturation, Fusch et al., (2018) suggest that triangulation, the use of multiple data sources, can be used to add depth to the data that are collected . Triangulation further helps in ensuring reliability and validity of the collected data and the results (Fusch et al., 2018). In particular, the external validity and construct validity of a study's findings can be enhanced through triangulation. In the context of this study, a pre-test was used to collect quantitative data to identify errors students made when solving probability problems. This was followed by interviews with some of the students to gain a deeper understanding of their misconceptions. With these results in mind, an intervention was developed to address students' errors and misconceptions. The intervention for the treatment group was in the form of GeoGebra-assisted lessons. At the intervention phase of data collection, the researcher, as a participant observer, observed and recorded student behaviours and any verbal communication that was deemed relevant to the research question and sub-questions. More interviews after the GeoGebra intervention were conducted with some students to try and understand their experiences about the use of GeoGebra in learning probability. Qualitative data were, therefore, obtained from the interviews and observations.

Triangulation is used in research to increase the validity and reliability of research findings (Hussein, 2009; Noble & Heale, 2019). There are several types of triangulation that are used in social research: data triangulation, theory triangulation, investigator triangulation, analysis triangulation and methodological triangulation (Noble & Heale, 2019; Hussein, 2009). Both data triangulation and methodological triangulation were used in this study to increase the reliability and validity of the results. Data triangulation was employed to reach data saturation (Fusch & Ness, 2015) as earlier discussed. This was achieved by employing several data collection methods such as pre-test, post-test, semi-structured interviews, observation, video clips and photographs to collect information to study the impact of GeoGebra on Grade 10 learners' understanding of probability.

Methodological triangulation involves the use of several data collection methods to study the same phenomenon to ensure that the data obtained is rich (Hussein, 2009; Fusch & Ness, 2015). For this study, both qualitative and quantitative data collection methods were used to collect and analyse data. Data analysis methods included error analysis, statistical significance testing and coding. Fusch & Ness (2015) argue that the use of multiple sources of data enhance the validity of multiple data analysis methods.

3.10 Dependability

The research design and the steps detailing the process of carrying out the research served the purpose of dependability. The documents used in the study will be kept as confirmation that the findings are not my own preferences.

3.11 Piloting the research instrument

To test the research instruments and to identify any possible problems in the study, the pre-test was administered to a group of Grade 11 learners at the same site. This group of learners had studied probability before in Grade 10, and they were examined on the topic in the end-of-year examinations. This class was not taught probability using technology, and they would cover the Grade 11 probability content in four months' time. The only knowledge they had on probability was, therefore, the Grade 10 probability content that they learnt in Grade 10. In order to refresh their minds, they were given up to a week to revise the probability content that they covered in Grade 10. They were given a Grade 10 probability worksheet to assist them revise. The Grade 11 pilot group wrote the Grade 10 pretest which was marked using a memo. Their responses and feedback gave an idea of the appropriateness and effectiveness of the pretest and the memo. The pilot did not indicate any serious problems with the pre-test.

3.12 Ethical considerations

Stevens (2013) defines ethics as a study of "good conduct and the grounds for making judgements about what is good conduct" (p. 3). The British Educational Research Association (2011) gives guidelines regarding voluntary informed consent of participants, guarantee of anonymity, openness and disclosure, right to withdraw and many other issues. These guidelines were followed to ensure that the research met all the standards of ethics.

After obtaining an ethical clearance from the Research Ethics Committee from the University of the Witwatersrand, permission was sought from the research site to carry out the study. The purpose of my research was discussed with the school authorities and the participants, assuring them that participation in the study would not compromise the quality of their learning. This helped in ensuring that participants made informed decisions.

The researcher also endeavoured to respect participants by asking for permission from their parents to participate in the research. Participation was voluntary and they were assured that any data collected from them would be kept anonymous and confidential. Participants were also assured that they could withdraw from the research at any stage. I also assured the participants that their safety and comfort during data collection was guaranteed, and that they would be protected from harm. As an insider researcher, my knowledge of the school and its safety rules made it easy for me to ensure that participants were free from harm.

3.13 Summary

In this chapter, I provided an outline of how the study was conducted. The research paradigm, the research design and data collection and analysis methods were also discussed. I also provided justification for the choice of these methods in order to enhance the validity of the study results. The research site and the sample were discussed in more detail, and issues of validity and reliability in data collection and data analysis were highlighted. A pre-test was administered to a group of Grade 10 students to determine the errors and misconceptions they have when solving probability problems. This was followed by interviews to try and get a deeper understanding of the students' responses. Interview data and observation data were analysed using qualitative data analysis methods. Students were then taught using GeoGebra as an intervention tool. In the final phase of the study, students wrote a post-test in order to assess the impact of GeoGebra intervention. Data were analysed using both quantitative and qualitative methods. Quantitative data included the frequencies of various errors that students made in the pre-test and in the post-test. Qualitative and quantitative results were mixed using mixed methods research methodology for purposes of triangulation.

The next chapter presents findings from the quantitative data.

CHAPTER FOUR

QUANTITATIVE DATA ANALYSIS OF ERRORS IN THE PRE-TEST AND POST-TEST

4.0 Introduction

Chapter Three discussed the research paradigms that informed this research. This discussion helped to shape the methodology that was needed to appropriately answer the research questions of the study. This was a mixed-methods study which necessitated both the use of qualitative and quantitative research methods. The quantitative data collection phase identified learners' errors and misconceptions in probability problem solving. These data were collected using a pre-test and post-test. The questions (items) in these tests were designed to elicit underlying misconceptions in learners' responses. In order to maintain an acceptable curriculum standard, the CAPS assessment guidelines in the Grade 10 assessment policy (DBE, 2011) were consulted when compiling the tests. In this chapter, quantitative analysis results of learners' errors and misconceptions are presented. The purpose of the analysis was to answer the research question which sought to identify learners' errors and misconceptions. The analysis was also useful for finding ways in which GeoGebra can be used to overcome specific challenges learners have when solving probability problems. To help me answer the research questions, I computed descriptive statistics, including mean and percentages. I then carried out significance tests to determine whether the observed changes were significant or not. A difference or change in the dependent variable was considered significant based on the probability level obtained from the test. In this study, the probability level $p < 0.05$ was used to claim significance. In Chapter Six, the discussion of these results will then be provided, together with the qualitative results. I start by providing a summary of the flow of participants at the different stages of my research.

4.1 Participant flow

At the beginning of the study, a total of 55 Grade 10 learners were enrolled at the research site. Of this number, 16 studied Mathematical Literacy, and so they were not recruited to participate in the study. The remaining 39 students were recruited. The participants were assigned to the treatment group ($n = 17$) and the control group ($n = 22$). All the 39 participants wrote the pre-test. However, three (3) participants from the treatment group pulled out of the study before the second stage of data collection after they had written the pre-test. One of them changed to Mathematical Literacy, while two moved to different schools. The pre-test data collected from the three participants were, therefore, not used in this study. The number of participants in the treatment and control groups who participated at all the different stages of this study is summarised in Table 4.1.

Table 4. 1 Number of participants at each stage of data collection

	Wrote the pre-test	Participated in the interview	Attended GeoGebra lessons	Wrote the post-test
Treatment group	n = 17	n = 5	n = 14	n = 14
Control group	n = 22	n = 2	n = 22	n = 22

Participants in this study were Grade 10 learners aged between 15 and 16 years. Of the 36 participating students, 18 were boys and 18 were girls (see Table 3.1 for demographics).

In the following section, the findings from error analysis of the pre-test and post-test are presented.

4.2 Specific assessment objectives that error analysis focused on

The stages of error analysis as proposed by Herholdt & Sapire's (2014) were used to analyse quantitative data. Four of these stages include: 1) difficulty levels and discrimination indices of test items, 2) students' use of methods in the tests, 3) incidence of questions not attempted, and 4) incidence of most common errors. Pre-test items and post-test items which addressed the same specific assessment objective were analysed under the four stages of Herholdt & Sapire's (2014) error analysis. Test items were classified under the same specific assessment objective if they satisfied the following criteria: a) the items (questions) require the use of the same cognitive mathematical processes to answer them, b) the items (questions) can be answered using the same strategies, c) the items require the same skills to answer the question. The specific assessment objectives for the analysis are stated below.

Specific Assessment Objective 1: Learners demonstrate knowledge and understanding of probability concepts. Learners who demonstrate knowledge and understanding of probability concepts were recognised by their ability to recall and apply various probability concepts, probability terminology and probability notation. Appendix G.3 shows the pre-test questions (item 1) and post-test questions (item 5) under this assessment objective.

Specific Assessment Objective 2: Learners demonstrate the ability to interpret probability models to compare the relative frequency of events with the theoretical probability. The notion of relative frequency as an approximation to probability measure was developed in class by performing a very large number of trials. The test questions in this category, therefore, required learners to draw from similar situations by interpreting the results in the context of a given problem. The pre-test questions (item 2) and post-test questions (item 1 & item 2) for this assessment objective are shown in Appendix G.4.

Specific Assessment Objective 3: Learners demonstrate the ability to analyse a problem and select a suitable strategy to solve it. Test items that address this objective required learners to use Venn diagrams, or the following probability rules for events A and B in a sample space S to solve probability problems: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$; A and B are mutually exclusive if $P(A \text{ and } B) = 0$; A and B are complementary if they are mutually exclusive and $P(A) + P(B) = 1$. Pre-test questions (item 3) and post-test questions (item 3) were paired together for purposes of analysis, while pre-test questions (item 4) was paired together with post-test questions (item 4). Appendix G.5 shows these test items. The results are reported in the next section.

4.3 Analysis of learners' challenges in understanding probability concepts

Quantitative data analysis began with item analysis to determine the level of difficulty and the discrimination index of each question in the pre-test and post-test. The levels of difficulty of each test item and the discrimination indices were calculated using learners' actual responses in the tests. The following criteria were used to categorise a question as difficult, moderate or easy:

Difficult questions	$\text{difficulty index } \leq 0{,}5$
Moderate questions	$0{,}51 \leq \text{difficulty index} \leq 0{,}84$
Easy questions	$\text{difficulty index} \geq 0{,}85$

A question had a good, fair or poor discrimination index if it fell in the following categories:

Questions with good discrimination	$\text{discriminaion index} > 0{,}3$
Questions with fair discrimination	$0.1 < \text{discrimination index } \leq 0{,}3$
Questions with poor discrimination	$\text{discrimination index } \leq 0{,}1$

4.3.1 Learners' challenges in understanding probability concepts

Pre-test Item # 1 and post-test item # 5 (see below) were designed to solicit for students' understanding of probability concepts and probability events (Appendix G1).

- [From pre-test item 1]: Events X and Y are defined as follows: X: Boys in Grade 10 who play soccer; Y: Girls in Grade 10 who play soccer. State whether or not events X and Y are complementary.
- [From post-test item 5]: During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows: 28 learners said they preferred B, "Boerewors rolls and koeksisters", 48 said they preferred K, "Kota with atchar, chips and Russian sausages", x said they preferred both B and K, 20 said they preferred neither. State, with a reason, whether B and K are mutually exclusive, complementary or inclusive.

The questions were designed to check for learners' understanding of the following probability concepts: bias, outcomes, mutually exclusive, complementary, inclusive. A context was described and learners answered the accompanying question (s). The way learners answered these questions indicated whether they understood the covered concepts or not. The actual scores obtained by each learner were used to calculate the level of difficulty of these questions. Appendix E1 and Appendix E2 show pre-test and post-test raw marks for both groups respectively. The difficulty and discrimination indices of these questions by the treatment group and control group are summarised in Table 4.2 below.

Table 4. 2 Difficulty and discrimination indices for paired test items: Learners' understanding of probability concepts and events

		Number of learners with correct answers	**% of learners who got incorrect answers**	**Difficulty index**	**Discrimination index**
Pre-test (Item 1)	Treatment (n = 14)	1	92.9	0,07	0,14
	Control (n = 22)	0	100	0,00	0,00
Post-test (Item 5)	Treatment (n = 14)	7	50	0,50	0,14
	Control (n = 22)	6	72.7	0,27	0,18

Pre-test Item # 1 turned out to be difficult for the majority of the students (92.9 percent from the treatment group; 100 percent from the control group). Despite an average percentage score of 60 percent for both groups on this item, no student managed to get full marks for it. The item had a difficulty index of 0.07 and 0.00 for the treatment group and the control group respectively, indicating that the item was quite difficult for students in both groups. The discrimination ability of pre-test Item #, however, was poor for both the treatment group and the control group (discrimination index = .14 for the treatment group; discrimination index = .00 for the control group).

Item #5 in the post-test also solicited for students' understanding of probability concepts and events. This time, a reason to justify the answer was required. Events were described and students were asked to state whether they were mutually exclusive, complementary or inclusive, and to give a supporting reason. This question also proved to be difficult for both groups (50 percent from the treatment group; 72.7 percept from the control group). This was despite the mean percentage score of 61 percent and 39 percent for the treatment group and control group respectively. The difficulty index for the treatment group was 0.50, while the index for the control group was 0.27. The majority of the students (64 percent of the two groups combined), did not get the answers correct for this item. Either they gave a correct classification of the event with a wrong reason, or they gave a wrong classification with or without a reason. The difficulty seemed to lie in the fact that students had not conceptually grasped the full meanings of the types of different probability events to the extent of explaining them comprehensively.

There was also evidence that students used guess work in the post-test to answer the first part of the question which required them to state whether the described events were mutually exclusive, inclusive or complementary. The second part of the question required a reason to be given to support their choice, but the majority of the students were not able to give correct reasons, indicating that they had partial knowledge of these concepts. Hence, most of them simply guessed answers. When students know that they lose nothing for taking a wrong guess, they can randomly pick an answer from the given list of alternatives and hope that they get it right. When they are asked to support their choices, misconceptions then come to light. Guessing answers in Mathematics assessments is not only regarded as bad behaviour but is also viewed as a sign of lack of comprehension. Vanderoost, Janssen, Eggermont, Callens and De Laet (2018) suggest that guessing can be discouraged by adopting scoring methods that measure partial knowledge such as negative marking and elimination testing. In negative marking, particularly in multiple choice assessments, students are penalised when they give wrong answers in order to discourage them from guessing. Students who are aware that they will lose marks for every wrong answer may not want to take the risk of guessing out the answer but prefer instead to leave that question blank. In elimination testing, students are rewarded for eliminating distractors, but penalised for eliminating the correct answer. This scoring method measures and rewards partial knowledge while still penalising misconception and guessing. Vanderoost et al. (2018) found that elimination testing with adapted scoring is an effective scoring method for discouraging guessing. This question (post-test Item #5) was prone to guessing, but the requirement to support the answer with a reason might have helped to reduce guessed answers.

These results show that students do not have a complete understanding of the various probability concepts. There is need for these concepts to be learnt thoroughly since they become part of the probability language probability problem solving.

4.4 Analysis of learners' challenges in understanding single and compound events

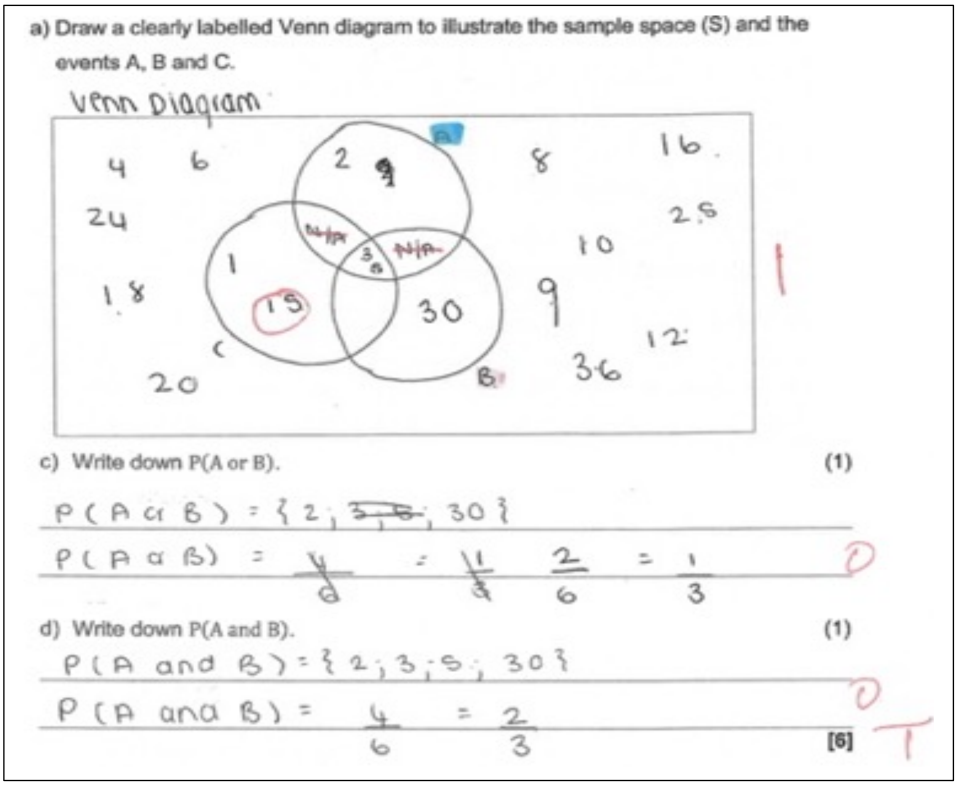
Pre-test Item # 2 (see Appendix G.1) examined students' understanding of experimental probability involving Venn diagrams in single and combined inclusive events. The question modelled an experiment where two dice were rolled several times and the outcomes were multiplied to generate the sample space, S = {1; 2; 3; 4; 5; 6; 8; 9; 10; 12; 15; 16; 18; 20; 24; 25; 30; 36}. Learners were then asked to draw a Venn diagram showing the following subsets of S: A = {prime numbers}, B = {multiples of 15} and C = {factors of 15}. Learners then used the Venn diagram to calculate the following probabilities:

- the probability of outcomes that are prime numbers, P(A),
- the probability of outcomes that are prime numbers or multiples of 15, $P(A \cup B)$
- the probability of outcomes that are both prime numbers and multiples of 15, $P(A \cap B)$

In this question A, B and C were considered as single events, while $A \cap B$ and $A \cup B$ were considered as compound events. After drawing a Venn diagram, learners were expected to identify these events on the diagram and calculate their probabilities. Both groups struggled with this item. The main difficulty that learners encountered was to identify the section of the Venn diagram which represented the events so that they could correctly calculate the required probabilities. The underlying misconception that led to such errors was language related. The meaning of *OR* and that of *AND* in the context of probability were misunderstood, leading to errors in learners' answers. Some learners understood *(A or B)* to mean *(A only) combined with (B only)*. Thus, for these learners, $A \cup B = (A \cap B') \cup (B \cap A')$, which incorrectly excludes the outcomes in $A \cap B$ and leads to the wrong conclusion that $P(A \cup B) = P(A \cap B') +$

$P(B \cap A')$ even when A and B are mutually inclusive. Some learners also misinterpreted the meaning of *(A and B)* to mean *all outcomes in A combined with all outcomes in B*. This led to a wrong conclusion that $A \cup B = A \cap B$ as seen in Figure 4.1. For this question, the treatment group achieved a success rate of 43 percent, while the control group achieved 27 percent. The respective item difficulty levels were calculated and found to be 0.21 for the treatment group and 0.18 for the control group.

Figure 4. 1 An extract of a learner's answer showing lack of understanding of events A∪B and A∩B.



The percentage number of students who got wrong answers was 78.6 percent for the treatment group and 81.8 percent for the control group. Table 4.3 shows the difficulty levels of this item for each group.

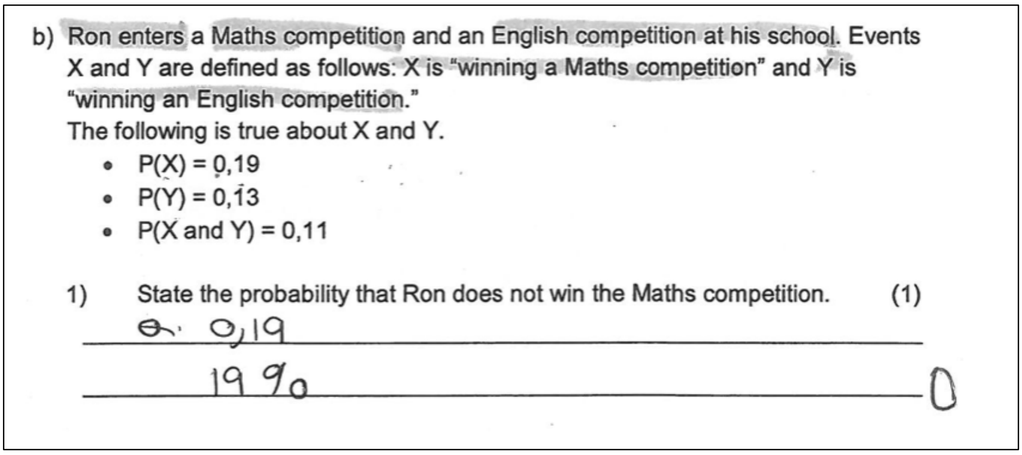
Table 4. 3 Difficulty and discrimination indices for paired test items: Experimental probability involving probability of single events and combined inclusive events using a Venn diagram or probability rules

Paired items		Number of learners with correct answers	% of learners who got wrong answers	Difficulty index	Discrimination index
Pre-test (Item 2)	Treatment (n = 14)	3	78.6	0.21	0,43
	Control (n = 22)	4	81.8	0.18	0,36
Post-test (Item 1)	Treatment (n = 14)	4	71.4	0,29	0,29
	Control (n = 22)	2	90.9	0,09	0,00
Post-test (Item 2)	Treatment (n = 14)	3	78.6	0,21	0,43
	Control (n = 22)	0	100	0,00	0,00

In the post-test (see Appendix G.2), a similar question was asked. This is shown as post-test Item # 1 and post-test Item # 2 in table 4.3. In post-test Item # 1, a mathematical context modelling an experiment of rolling a die was given. Students were then asked to calculate the probability of obtaining the event, "an even number or a factor of 12" when a die is rolled. No particular method was prescribed to answer the question, and so students could use a Venn diagram or use the probability rule, P(A or B) = P(A) + P(B) – P(A and B).

In post-test Item # 2, learners worked with the single events X and Y where, X = {winning a Maths competition} and Y = {winning an English competition}. They were given that P(X) = 0.19, P(Y) = 0.13 and P(X and Y) = 0.11. Students were asked to calculate the probability of not winning the Maths competition and the probability of winning either of the competitions. Since no particular method was prescribed, some students used a Venn diagram, while others used the probability rules. Students made various errors which showed that their understanding of single and compound events was not fully developed. For example, some learners were not able to differentiate between an event and a complementary event (see Figure 4.2).

Figure 4. 2 An extract of a learner's answer showing lack of understanding of an event and its complementary event


b) Ron enters a Maths competition and an English competition at his school. Events X and Y are defined as follows: X is "winning a Maths competition" and Y is "winning an English competition."
The following is true about X and Y.

- P(X) = 0,19
- P(Y) = 0,13
- P(X and Y) = 0,11

1) State the probability that Ron does not win the Maths competition. (1)

0,19

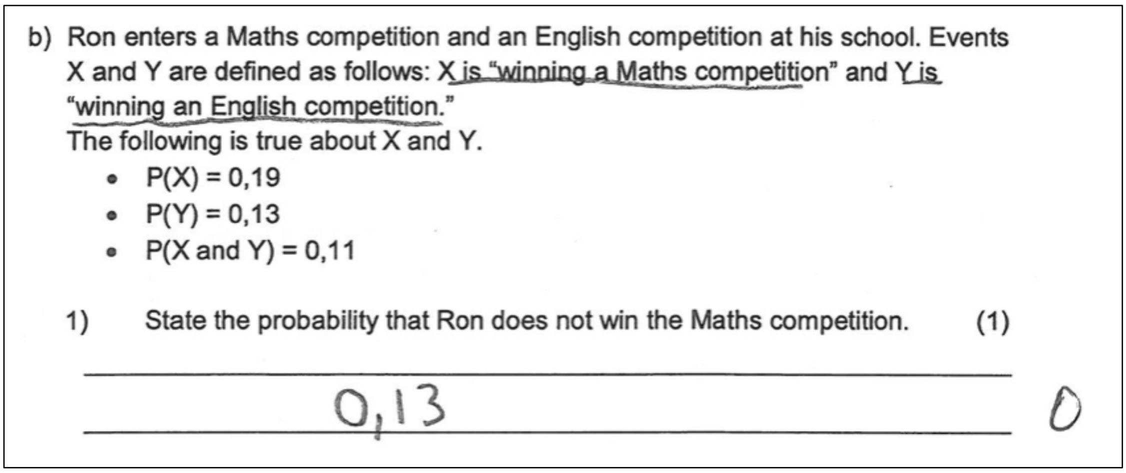
19 %

0

Other learners misinterpreted the question. For example, the statement "probability that Ron does not win the Maths competition" was interpreted to mean "probability that Ron wins the English competition". Figure 4.3 shows an example of an answer which was interpreted using this thinking. The learner did not understand that there were other alternative outcomes in the question. Instead, he concluded that if Ron does not win the Maths competition, then he must win the English competition. Also, the learner did not understand that the events X and Y were not exhaustive.

This question proved to be a very difficult question for both groups (treatment group difficulty levels = 0.29 and 0.21; control group difficulty levels = 0.09 and 0.00). The treatment group achieved success rates of 29 percent for Item # 1 and 36 percent for Item # 2, while the control group achieved 9 percent and 18 percent respectively.

Figure 4. 3 An extract of a learner's answer showing a wrong interpretation of an event


b) Ron enters a Maths competition and an English competition at his school. Events X and Y are defined as follows: X is "winning a Maths competition" and Y is "winning an English competition."
The following is true about X and Y.

- P(X) = 0,19
- P(Y) = 0,13
- P(X and Y) = 0,11

1) State the probability that Ron does not win the Maths competition. (1)

0,13

0

4.5 Analysis of learners' challenges when using representations in probability problem solving

The problem-solving question in the pre-test (Item # 3 and Item # 4) (see Appendix G1) was scaffolded to assist students. The question asked respondents to determine the probability that a learner selected from a group is left-handed or plays soccer given the following information:

- number of learners = 40
- number of learners who are left-handed = 7
- number of learners who play soccer = 18
- number of learners who are soccer players who are left-handed = 4
- all learners are either right-handed or left-handed, but not both.

The expectation was for the question to be answered with the aid of a Venn diagram. Students were thus expected to draw a Venn diagram before answering the follow-up questions. The pre-test questions proved to be difficult for the treatment group (difficulty index for item # 3 = 0.21; difficulty index for item # 4 = 0.14) and moderate for the control group (difficulty index for item # 3 = 0.73; difficulty index for item # 4 = 0.81). Table 4.4 shows the difficulty and discrimination indices for item # 3 while Table 4.5 shows the difficulty and discrimination indices for item # 4.

Table 4. 4 Difficulty and discrimination indices for paired test items: Experimental probability involving single events, combined events and combined inclusive events using a Venn diagram or probability rules

Tested content: Experimental probability involving probability of single events, probability of combined events and probability of combined inclusive events using a Venn diagram or probability rules

Paired items		Number of learners with correct answers	% of learners who got wrong answers	Difficulty index	Discrimination index
Pre-test (Item 3)	Treatment (n = 14)	3	78.6	0,21	0,43
	Control (n = 22)	16	27.3	0,73	0,85
Post-test (Item 3)	Treatment (n = 14)	9	35.7	0,64	0,43
	Control (n = 22)	15	31.8	0,68	0,45

Table 4. 5 Difficulty and discrimination indices for paired test items: Interpretation of Venn diagrams to determine probability of combined events

Paired items		Number of learners with correct answers	% of learners who got wrong answers	Difficulty index	Discrimination index
Pre-test (Item 4)	Treatment (n = 14)	2	85.7	0,14	0,29
	Control (n = 22)	18	18.2	0,81	-0,09
Post-test (Item 4)	Treatment (n = 14)	3	78.6	0,21	0,43
	Control (n = 22)	8	63.6	0,36	0,36

In the post-test, the problem-solving question (Item # 3 and Item # 4) also required students to use a Venn diagram. A mathematical context was given showing food preferences of a group of 80 pupils where :

- 28 of them preferred B,
- 48 preferred K,
- an unknown number preferred both B and K
- 20 preferred neither.

Item # 3 required students to draw a Venn diagram and Item 4 asked learners to use the Venn diagram to calculate P(B or K) and P(neither B nor K). There was an improved performance in the post-test compared to the pre-test for item 3 for the treatment group (pre-test mean = 43 percent, difficulty index = 0.21; post-test mean = 71 percent, difficulty index = 0.64). The improvement in performance seemed to lie on the context that was used in the posed problem. In the pre-test question, learners struggled to figure out the number of people in a group of 40 who were right-handed and did not play soccer given that:

- 7 were left-handed
- 18 played soccer
- 4 played soccer and were left-handed, and
- 40 were either right-handed or left-handed, but not both.

The majority of the students (78.6 percent from the treatment group; 27.3 percent from the control group) who failed to draw a correct Venn diagram were not able to determine the number of learners who were right-handed and did not play soccer. They failed to interpret the context. The question in the post-test, on the other hand, did not create much problems for students. Students were asked to draw a Venn diagram to illustrate the given information. The majority of the respondents (64.3 percent of the treatment group; 68.2 percent of the control group) entered the numbers in the appropriate regions of the Venn diagram correctly.

4.6 Analysis of students' use of methods in the tests

There were questions in the pre-test and post-test which lended themselves to the use of Venn diagram strategies. Pre-test Item # 2 and pre-test Item # 3 (see Appendix G1) were examples of such questions. In the post test, Item #1 and Item #2 (see Appendix G 2) did not specify a method or strategy, and students could use Venn diagram strategies or probability rules to answer them. In this section, an analysis of the use of methods in the pre-test and post-test by both the treatment and control groups is reported. Correct methods included correct use of Venn diagrams. For a Venn diagram method to be correct, the actual outcomes, number of outcomes or probability of outcomes needed to be placed in their correct subsets of the Venn diagram. Students could also use probability rules to solve the problems. A correct use of probability rules included choosing the correct rule for the given situation and substituting in it to reach an answer. Four possible outcomes were observed in students' work. I categorised them as A, B, C and D. These categories are illustrated below.

Category A: Students drew correct Venn diagrams and used correct follow-through steps to reach the correct result. This was labelled as category A in the analysis and it refers to cases where a correct Venn diagram was drawn and a correct answer was reached. These responses exemplify cases where learners fully answered the posed questions correctly. To illustrate category A, the following question from the post-test will be used:

During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows:

- 28 learners said they preferred B, "Boerewors rolls and koeksisters"
- 48 said they preferred K, "Kota with atchar, chips and Russian sausages"
- x said they preferred both B and K
- 20 said they preferred neither.

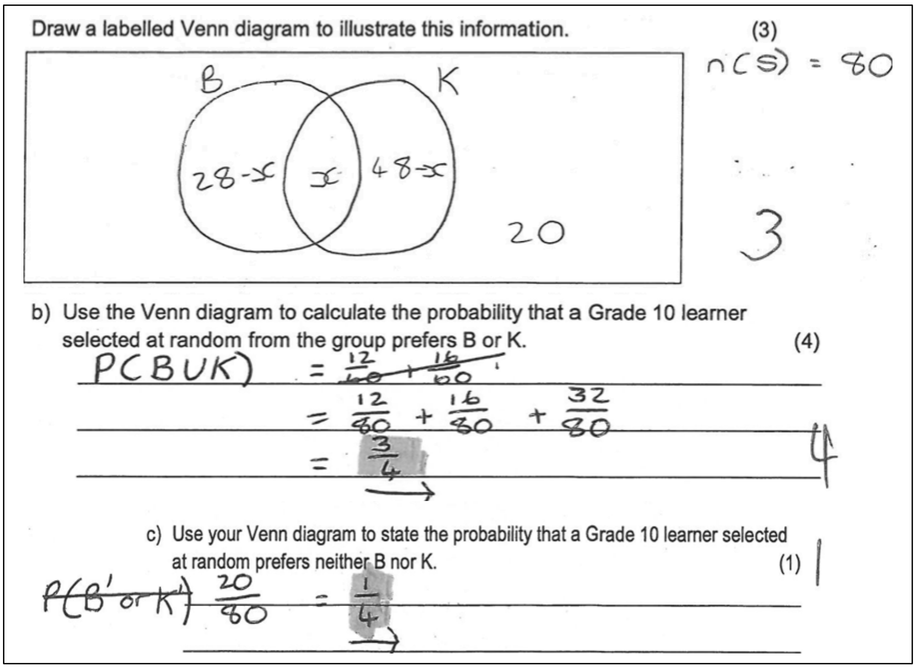
a) Draw a labelled Venn diagram to illustrate this information.

b) Use your Venn diagram to calculate the probability that a Grade 10 learner selected at random prefers B or K.

c) Use your Venn diagram to calculate the probability that a Grade 10 learner selected at random prefers neither B nor K.

Figure 4.4 illustrates category A learners' responses. It shows that a correct Venn diagram was drawn and a correct answer was reached.

Figure 4. 4 Learners' use of correct Venn diagrams followed by correct answers (Category A)



4.6.1 Analysis of students' use of correct methods followed by correct results (Category A)

Two questions in the pre-test required students to show working. They either used Venn diagrams or probability rules to answer the questions. A small percentage of respondents from the treatment group (21 percent for Item # 2 ; 14 percent for Item # 3) used correct methods. The percentage use of methods in the pre-test is summarised in Table 4.6. A small percentage of respondents in the control group (9 percent for Item 2 ; 27 percent for Item 3) used correct methods to answer the questions.

There was an improvement in the use of correct methods by both groups in the post-test (Table 4.7). However, this increase represented a small percentage of respondents who used correct methods. The percentage of respondents in the treatment group who used correct methods and got the correct answer was 36 percent for Item # 1 and 36 percent for Item # 2. On the other hand, the corresponding percentage of respondents in the control group who used correct methods and got correct answers was 14 percent in Item # 1 and 27 percent in Item # 2.

Table 4. 6 Students' use of methods in the pre-test (Item 2 & Item 3)

Correct Venn diagram, correct follow through method = A Correct Venn diagram, incorrect follow through method = B
Incorrect Venn diagram, correct follow through method = C Incorrect Venn diagram, incorrect follow though method = D

	Pre-test Item 2 Percentage of respondents under each category				Pre-test Item 3 Percentage of respondents under each category			
	A %	**B %**	**C %**	**D %**	**A %**	**B %**	**C %**	**D %**
Pre-test (Treatment group)	21	36	0	43	14	29	21	36
Pre-test (Control group)	9	23	4	64	27	9	14	50
Average %	**15**	**29.5**	**2**	**53.5**	**20.5**	**19**	**17.5**	**43**

Table 4. 7 Students' use of methods in the post-test (Item 1 & Item 2)

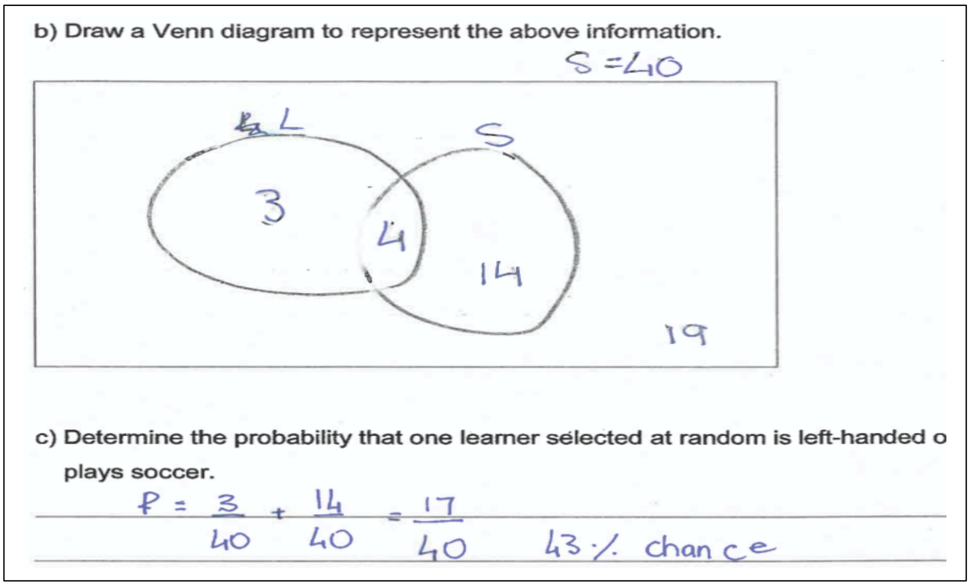
Correct Venn diagram, correct follow through method = A Correct Venn diagram, incorrect follow through method = B
Incorrect Venn diagram, correct follow through method = C Incorrect Venn diagram, incorrect follow though method = D

	Post-test Item 1 Percentage of respondents under each category				Post-test Item 2 Percentage of respondents under each category			
	A %	**B %**	**C %**	**D %**	**A %**	**B %**	**C %**	**D %**
Post-test (Treatment group)	36	0	0	64	36	28	0	36
Post-test (Control group)	14	0	0	86	27	45	5	23
Average %	**25**	**0**	**0**	**75**	**31.5**	**36.5**	**2.5**	**29.5**

Category B: In category B, students drew correct Venn diagrams, but used an incorrect follow through procedure to reach their answer. Figure 4.5 illustrates an example of learners' responses in category B. The posed question stated that in a group of 40 learners , 7 are left-handed, 18 play soccer, 4 play soccer and are left-handed, and all are either left-handed or right-handed, but not both. Learners were asked to represent the information on a Venn diagram, and then use the diagram to answer the questions that followed.

The response shows that the respondent was able to draw a correct Venn diagram for the question but failed to interpret the Venn diagram to correctly answer the question that followed. The Venn diagram correctly shows the number of actual outcomes entered in their proper subsets. For that part, the respondent got full marks. The next question required the respondent to determine the probability that the learner selected at random is left-handed (L) or plays soccer (S).

Figure 4. 5 An extract of an answer by learner C14 showing a correct Venn diagram followed by an incorrect interpretation (Category B).



Lack of understanding of the meaning of "or" for inclusive events, led this learner to think that n(L or S) excludes the 4 which is in the common region. Thus, a wrong procedure was used to answer the question. The student thought that P(L or S) = P(L only) + P(S only). This resulted in the incorrect answer of 43%. This error pattern occurred when respondents failed to identify event "L or S" using a Venn diagram. The error also indicates that the respondents had limited understanding of different types of events, in particular mutually exclusive and inclusive events. The answer of 43% suggests that the respondents thought that "L or S" meant L and S are mutually exclusive.

4.6.2 Students' use of correct methods followed by incorrect results (Category B)

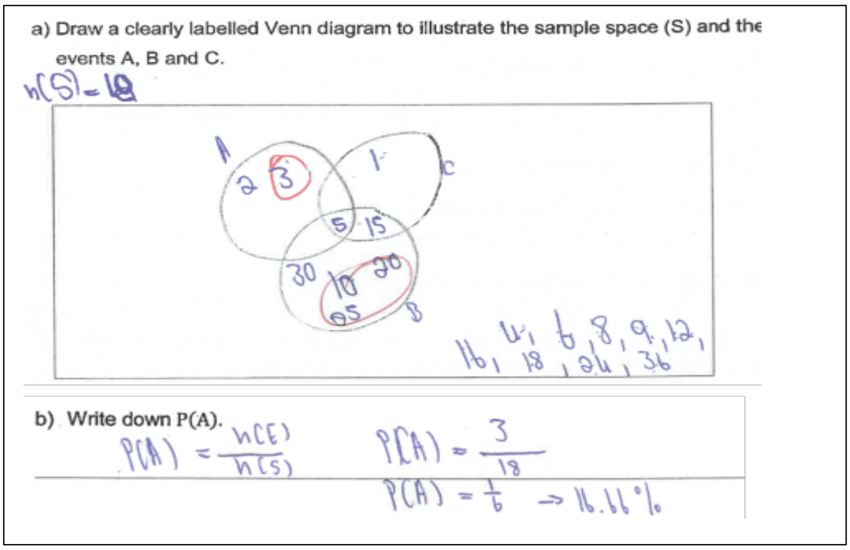
Students' responses indicated that there were cases where correct methods were used in answering questions. Respondents either drew a correct Venn diagram or picked the correct probability rule to answer questions. However, in some of these cases, the majority of the respondents were not able to follow through their methods to reach the correct result. These findings are reported below for each group and test.

A small percentage of respondents in the treatment group (36 percent in pre-test Item # 2; 29 percent in pre-test Item # 3) used correct methods in the pre-test items but were not able to follow through their methods to get the correct answers. Also, the percentage of respondents in the control group who used correct methods but failed to follow them through to get correct answers was small (23 percent in pre-test Item # 2; 9 percent in pre-test Item # 3).

In the post-test, there was no respondent who used a correct method and failed to follow it through to get the correct answer for Item # 1. However, seeing that only a small percentage of responses fell under category A, this means that the majority of them fell under either category C or D. For post-test Item # 2, 28 percent of the respondents in the treatment group started with the correct method but failed to use it to get the answer. The corresponding percentage of respondents in the control group for this category was 45 percent.

Category C: Category C responses were cases of students' answers where incorrect Venn diagrams were drawn, but a correct follow-through method was used to answer the question(s). These responses did not receive full scores. Figure 4.6 illustrates an example of learners' responses in category C. The Venn diagram was incorrect because the number of elements in some regions of the diagram were incorrect. However, the learner applied correct follow through interpretation to calculate P(A).

Figure 4. 6 An extract of an answer by learner T1 showing an incorrect Venn diagram followed by a correct interpretation (Category C)



In this question the sample space, S = {1;2;3;4;5;6;8;9;10;12;15;16;18;20;24;25;30;36}, was obtained by multiplying the outcomes when two dice are rolled. Events A, B and C were defined as: A = {prime numbers, B ={ multiples of 15} and C = {factors of 15}. The outcome 3 was correctly identified as belonging to A, but the learner failed to identify it as a factor of 15. As a result, 3 was not written inside event C. The learner placed other outcomes in wrong positions as well, including {5; 10; 20; 25} which were wrongly placed in B when they are not multiples of 15. These errors made the Venn diagram incorrect. However, the incorrect Venn diagram was correctly interpreted to answer the question that followed, which required the value of P(A). The learner was able to see that there are three outcomes in event A and he used that information correctly to answer the question. This case fits the category C.

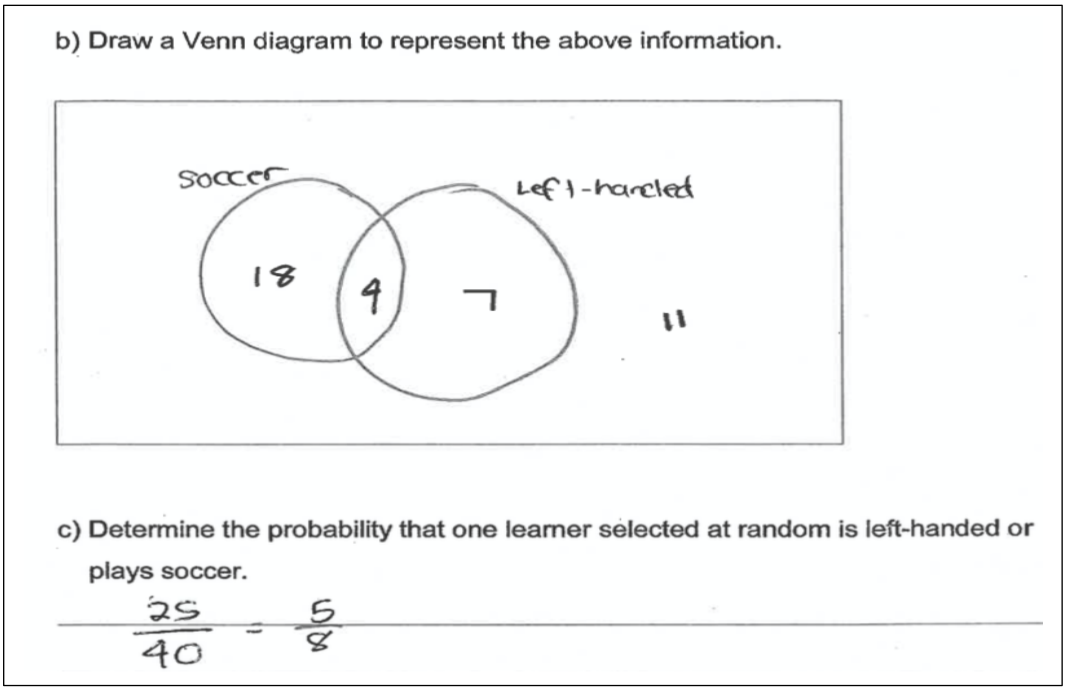
4.6.3 Students' use of incorrect Venn diagrams followed by correct answers (Category C)

There were cases where students drew incorrect Venn diagrams but interpreted them correctly to answer questions. In such cases, students' follow-through methods and answers were awarded marks because correct reasoning was applied. In the pre-test, the percentage of respondents in the treatment group who fit this category was 0 percent for pre-test Item # 2, and 21 percent for pre-test Item # 3. The corresponding percentages for the control group were 4 percent for pre-test Item # 2, and 14 percent for pre-test Item # 3.

In the post-test, the percentages of respondents who fit this category were low compared to the percentages in the pre-test (0 percent of the respondents in the treatment group in post-test Item # 1; 0 percent of the respondents in the treatment group in post-test Item # 2; 0 percent of the respondents in the control group in post-test Item # 1; 5 percent of the respondents in the control group in post-test Item # 2).

Category D: In category D, students drew incorrect Venn diagrams and applied an incorrect follow-through method to try and answer the question(s). An example of category D responses is illustrated in Figure 4.7 below. The response in Figure 4.7 shows that the learner was not able to use the given information to answer the question. It was given in the question that n(S) = 18, n(L) = 7 and n(S and L) = 4. First, the learner entered wrong values in the subsets, making his Venn diagram incorrect. The question following the Venn diagram was supposed to be answered by correctly interpreting the Venn diagram (even if the Venn diagram was wrong). However, the student failed to interpret the Venn diagram when he excluded 4, the number of outcomes in the common region, to get n(L or S).

Figure 4. 7 An extract of an answer by learner C5 showing an incorrect Venn diagram followed by an incorrect interpretation (Category D)



4.6.4 Students' use of incorrect Venn diagrams followed by incorrect answers (Category D)

In the majority of cases in the pre-test, students who were not able to draw correct Venn diagrams were also not able to correctly interpret them to arrive at the answer. From the treatment group, 43 percent of the respondents drew incorrect Venn diagrams and failed to interpret them in pre-test Item # 2. In the pre-test Item # 3, the percentage of respondents who drew incorrect Venn diagrams and failed to interpret them correctly was 36 percent. The corresponding percentage of respondents in the respective items was 64 percent for Item # 2 and 50 percent for Item # 5.

In the post-test, the percentage of respondents who drew incorrect Venn diagrams and failed to interpret them was 64 percent for post-test Item # 1 and 36 percent for post-test Item # 2. The control group corresponding percentages were 86 percent in post-test Item # 1 and 23 percent in post-test Item # 2. These figures also represent the percentages of respondents whose solutions were incorrect and, therefore, scored no marks. The reason for this was because a wrong Venn diagram was not allocated marks. A wrong interpretation of the wrong Venn diagram also got no marks since it equated to wrong procedures for answering the question.

4.7 Analysis of the incidence of questions that were not attempted

There were no significant incidences of questions that were not attempted in the pre-test. From the treatment group, there was only one student who did not attempt Item 3 and Item 4, most probably because the student ran out of time. From the control group, three students did not attempt one or two items in the pre-test. All items were attempted in the post-test by both the treatment and the control groups. Table 4.8 shows a summary of items that were not attempted.

Table 4. 8 Incidence of questions that were not attempted

	Pre-test: Number of students who did not attempt this item				Post-test: Number of students who did not attempt this item			
	Item 1	**Item 2**	**Item 3**	**Item 4**	**Item 1**	**Item 2**	**Item 3**	**Item 4**
Treatment group	0	0	1	1	0	0	0	0
Control group	1	1	1	1	0	0	0	0
Total	**1**	**1**	**2**	**2**	**0**	**0**	**0**	**0**

4.8 Analysis of incidence of most frequent errors

According to Herholdt & Sapire (2014), an analysis of most frequent errors in students' work can yield important information. This includes the errors that learners make, the concepts that are not fully developed in the learners, and the methods that can be adopted to address these errors. The errors that learners make can also show how learners were thinking, and how they ought to be assessed (Herholdt & Sapire, 2014). Literature shows that errors in probability problem solving arise when learners hold certain probability misconceptions (Khazanov & Prado, 2010; Garfield & Ahlgren, 1988; Jun & Pereira-Mendoza, 2002; Shaughnessy, 1977). Examples of such misconceptions include equiprobability, outcome approach, compound approach (Jun & Pereira-Mendoza, 2002), representativeness bias and outcome orientation (Garfield & Ahlgren, 1988; Khazanov & Prado, 2010; Shaughnessy, 1977).

In this study, misconceptions that produced patterns of errors included overgeneralisation, representativeness bias, positive recency effect and negative recency effect (Hokor, Apawu, Owusu-Ansah, & Agormor, 2022). These will be explained later in the sections below. Lai (2012) posits that errors learners make in Mathematics can be classified as factual errors, procedural errors and conceptual errors. In this study, each item was considered, and the errors made by students were identified. The errors were coded into three broad categories as factual errors (F), procedural errors (P) and conceptual errors (C). These types of errors were discussed in Chapter Two.

4.8.1 Factual errors (F) made by learners

Factual errors occurred when learners failed to recall basic facts about properties of certain types of numbers or when they overgeneralised over properties of certain types of numbers. For example, there was evidence that a learner who knew what prime numbers are would still exclude some prime numbers from the set of prime numbers. In some cases, non-prime numbers were included in the set of prime numbers. This suggests that the meaning of a prime number was extended to include numbers that are not prime. In order to draw Venn diagrams, some questions required learners to first identify outcomes of the described events. Some of these outcomes included multiples of numbers, factors of numbers and prime numbers. There were incidences where these outcomes were either excluded from the lists to which they belonged or included in the lists to which they did not belong. According to Gal (2004), lack of conceptual knowledge and understanding of other areas of Mathematics can impact negatively on the successful acquisition of probability knowledge. In this study, learners committed a variety of factual errors which were categorised as F1 – F6 using the following criteria:

- F1 errors: learners included numbers that are not multiples of a given number in the set of multiples.
- F2 errors: learners excluded numbers that are multiples of a given number from the set of multiples
- F3 errors: learners included numbers that are not factors of a given number in the set of factors
- F4 errors: learners excluded numbers that are factors of a given number from the set of factors
- F5 errors: learners included numbers that are not prime in the set of prime numbers
- F6 errors: learners excluded prime numbers of a given number from the set of prime numbers.

These errors resulted in learners giving incorrect solutions or drawing incorrect Venn diagrams. For example, in pre-test Item # 2, it was given that

- S = {1;2;3;4;5;6;8;9;10;15;16;18;20;24;25;30;36}
- A = {outcomes that are prime numbers}
- B = {outcomes that are multiples of 15}
- C = {outcomes that are factors of 15}.

In order to draw a correct Ven diagram from this context, learners were required to identify the outcomes of A, B and C so that they could write these in their correct regions in the Venn diagram. Table 4.9 shows the prevalence of factual errors that learners made in the pre-test and post-test by the treatment group and control group.

Table 4.8 shows that the most frequent of these errors was the inclusion by learners of outcomes which are not prime numbers in the set of prime numbers. This error accounted for 45.1 percent of all the errors (on average) from both the treatment and control groups. At Grade 10 level, students still confuse prime numbers. This is largely because they would have forgotten everything about the properties of prime numbers which they study in great depth in lower grades. In lower grades, students deal with many questions that ask them to prime factorise numbers. However, in Grade 10, factorisation is mainly focused on algebraic expressions. Not much attention is given to the properties of prime numbers because these are assumed knowledge.

Table 4. 9 Incidence of factual errors by treatment and control groups in the pre-test and post-test

Factual errors (F) – Treatment group

		Pre-test		Post-test	
Type	Error description	Freq	%F	Freq	%F
F1	Includes wrong multiples	1	7.7	0	0.0
F2	Excludes correct multiples	4	30.8	0	0.0
F3	Includes wrong factors	0	0	0	0.0
F4	Excludes correct factors	1	7.7	0	0.0
F5	Includes wrong prime outcomes	7	53.8	0	0.0
F6	Excludes correct prime outcomes	0	0	0	0.0
	Total	13	22*	0	0*

Total No. of errors by this group [Pre-test = 58; Post-test = 44]

Factual errors (F) – Control group

		Pre-test		Post-test	
Type	Error description	Freq	%F	Freq	%F
F1	Includes wrong multiples	7	31.8	0	0.0
F2	Excludes correct multiples	4	18.2	0	0.0
F3	Includes wrong factors	0	0.0	0	0.0
F4	Excludes correct factors	3	13.6	5	100
F5	Includes wrong prime outcomes	8	36.4	0	0.0
F6	Excludes correct prime outcomes	0	0.0	0	0.0
	Total	22	21*	5	6*

Total No. of errors by this group [Pre-test = 107; Post-test = 87]

Total number of errors as a percentage of all the errors for the particular test by group

4.8.2 Procedural errors (P) made by learners

Participants made procedural errors when they failed to follow the correct steps in answering the questions. According to Hokor, et al. (2022, p. 3), learners face procedural difficulty when they fail "to carry out manipulations or algorithms despite having understood the concepts behind the probability problem." In this study, some learners were able to identify the correct rule to use, but failed to pay attention to the necessary conditions for that rule to work. For example, to use the rule for $P(A\ or\ B)$, learners needed to understand whether A and B were inclusive or exclusive in order to correctly manipulate $P(A\ and\ B)$. Some procedural errors resulted from learners' lack of full conceptual understanding of some aspects of the posed questions. The errors were coded and categorised as follows:

- P1 errors: misunderstanding of the probability of union of two events
- P2 errors: misunderstanding of probability of single events
- P3 errors: misunderstanding of probability of intersection of two events
- P4 errors: misunderstanding of probability of inclusive events
- P5 errors: misunderstanding of probability of complementary events.

The various sub-categories of procedural errors are illustrated next.

4.8.2.1 Learners' procedural errors involving union of inclusive events (P1 errors)

Some procedural errors occurred when students attempted to calculate the probability of the union of two inclusive events using probability rules. This error was coded as P1 The rule, $P(A\ or\ B) = P(A) + P(B) - P(\ A\ and\ B)$ was taught in class and students needed to learn it. This rule could be used in questions where the sample space and two or three subsets with common outcomes were given. As an example, Item # 2 in the pre-test required the following events and their outcomes to be used:

S = {1;2;3;4;5;6;8;9;10;12;15;16;18;20;24;25;30;36},

A = {2;3;5}

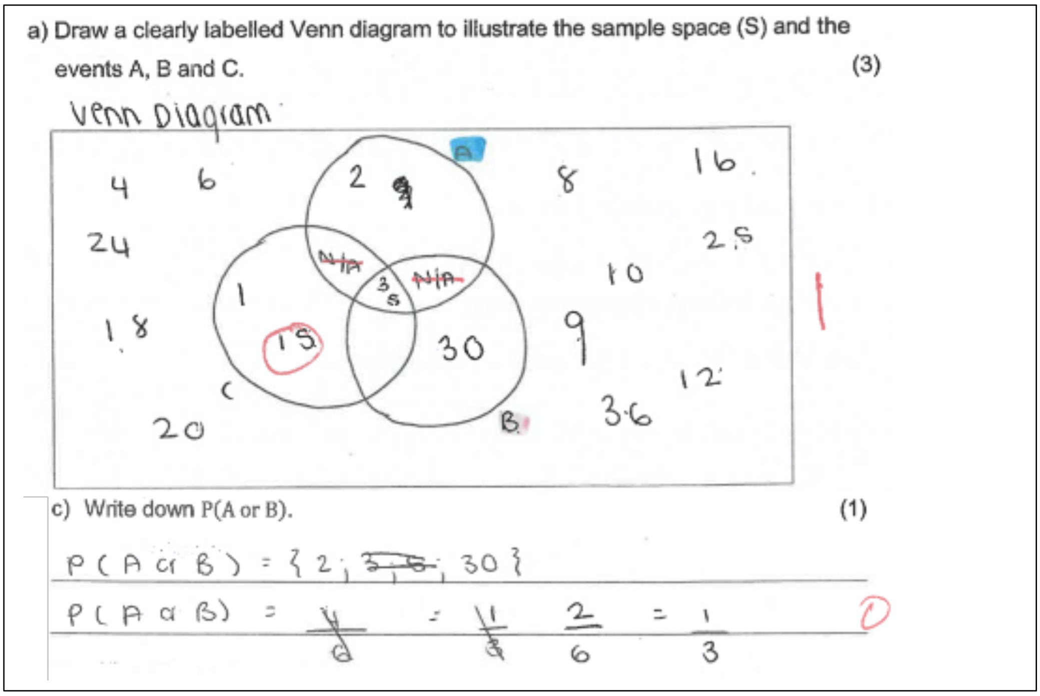
B = {15;30}

C = {1;3;5;15}.

Using the above information, students were asked to draw a Venn diagram to illustrate the outcomes of the events S, A, B and C, and then write down $P(A\ or\ B)$. To answer this question correctly, students needed to understand the meaning of $(A\ or\ B)$. Students who answered the question incorrectly excluded the outcomes which were in the intersection of A and B, suggesting a misconception in the meaning of $(A\ or\ B)$. In the pre-test, the number of P1 errors (sum = 9) as a percentage of all the procedural errors (P) (sum = 27) for the treatment group (n

= 14) was 33.3 percent. For the same group, in the post-test, the number of P1 errors (sum = 20) as a percentage of all procedural errors (P) (sum = 31) was 64.5 percent). Table 4.9 shows these results. The prevalence of P1 errors for the control group in the pre-test (sum = 15) as a percentage of all procedural errors (sum = 42) from the group was 35.7 percent. In the post-test, the number of P1 errors was 35 out of the number of all procedural errors (sum = 53). This translated to the percentage frequency of 66.0 percent. Table 4.10 shows these results. This indicates that P1 errors were most prevalent in both groups. The findings suggest that students do not understand (A or B) to mean either A or B or both. Figure 4.8 shows an example of a procedural error involving learners' misunderstanding of the union of two events and their related probability (error P1).

Figure 4. 8 An extract from learner C22 showing procedural errors involving union of inclusive events

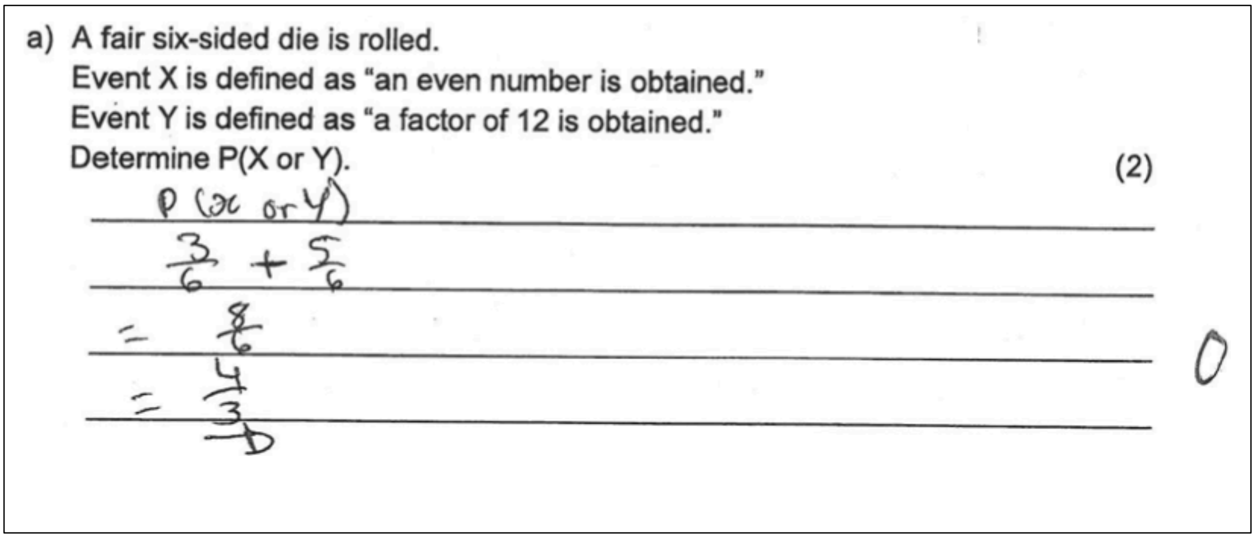


In Figure 4.8 the student thought that there were only two outcomes in (A or B) and wrote that $P(\text{ A or B}) = \frac{2}{6}$. This error is caused by the students' lack of understanding of the meaning of the probability terminology (A or B) and used a wrong method because of the confusion between (A or B) and (A and B). This was just one example of students' lack of complete understanding of the meaning of the word OR as used in probability. The idea that the event (A and B*)* is a subset of (A or B) is not fully developed in students' minds. It needs to be taught thoroughly in Grade 10.

A further procedural error that the student in Figure 4.8 made was to divide by 6 instead of 18. Students tend to restrict their answers to the subsets of the sample space and disregard those outcomes that are outside the subsets. In this example, all the outcomes of the event $(A \cup B \cup C)^{/}$ were disregarded in answering this question, leading to a procedural error. Thus some procedural errors occur when students fail to identify the correct number of outcomes in the sample space. This leads to the use of incorrect steps in answering questions that involve the probability of the union of inclusive events.

These errors were more pronounced in questions where a Venn diagram was not a requirement. In Item 1 in the post-test, it was given that after rolling a six-sided die, the desired outcomes were X = {even numbers} and Y = {factors of 12}. Most students knew that the outcomes in X and Y are {2;4;6} and {1;2;3;4;6} respectively. However, errors occurred when students interpreted P(X or Y) to be equal to P(X) + P(Y) although X and Y were not mutually exclusive. Students who approached the question this way ended up getting a probability value of $\frac{4}{3}$. It did not seem to occur to these students that probability measure cannot exceed 1. Figure 4.9 shows an example of this error.

Figure 4. 9 Extract of work by learner T13 showing a procedural error involving union of events



4.8.2.2 Learners' procedural errors when solving single event problems (P2 errors)

There were incidences where procedural errors resulted from learners' failure to identify all outcomes of a single event in a Venn diagram. This error was categorised as P2 (Table 4.9, Table 4.10). The percentage of P2 errors by the treatment group in the pre-test was 3.7 percent. In the post-test, it was much higher at 19.4 percent. The corresponding results for the control group were 4.8 percent and 22.6 percent in the pre-test and post-test respectively. In Item # 2 in the pre-test, for example, event A = {prime numbers} and C = {factors of 15} have some outcomes in common, viz: (A and C) = {3; 5}. Respondents drew a Venn diagram to assist them answer this question. However, errors occurred when the outcomes {3;5} were excluded from event A, apparently because they were in C. Thus, learners did not understand that outcomes in A can also be in C. Figure 4.10 illustrates this error.

Figure 4. 10 Extract of work by learner C1 showing a procedural error involving single events

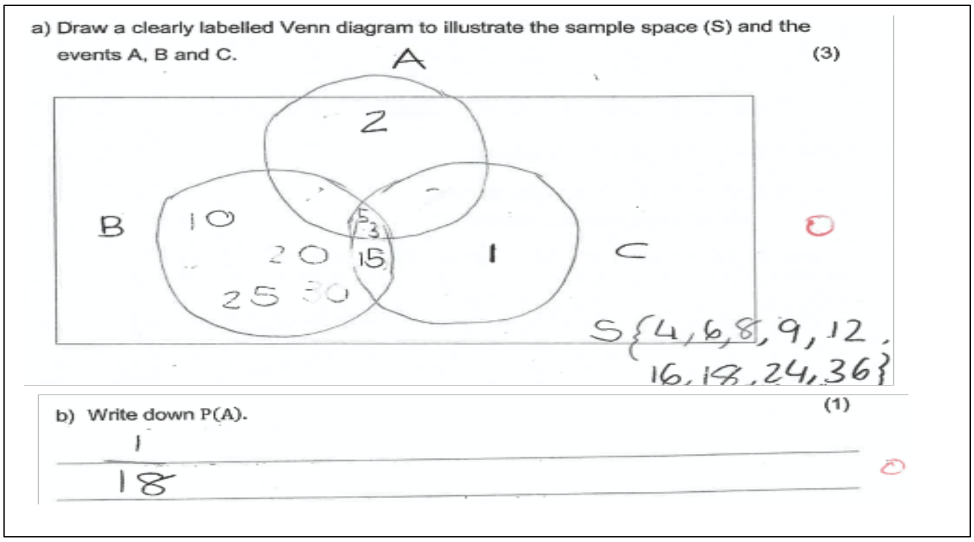


Figure 4.10 shows that to calculate P(A), the learner did not use those outcomes of A which belonged to other events. The learner failed to recognise that {3;5} was also an event within the event A. In the mind of this student, event A should only be restricted to A, and should not include other events. This suggests that learners do not understand the concept of combined events. In particular, students do not understand that events can be combined to form another event or a subset of another event. In order to address the challenges that Grade 10 learners encounter when dealing with the probability of combined events, adequate emphasis in Grade 10 probability teaching must be given to compound stochastic events. The difficulty that students had with combined events in this study confirms Iversen & Nilsson's (2019) study about students' reasoning about compound stochastic events. Iversen and Nilsson (2019) found that students in lower secondary school struggle to apply certain types of reasoning to probability tasks, including well-grounded combinatorial reasoning and multiplicative reasoning.

In another example of P2 error type, Item # 3 in the pre-test asked students to determine the probability of not winning a Maths competition, given the following information:

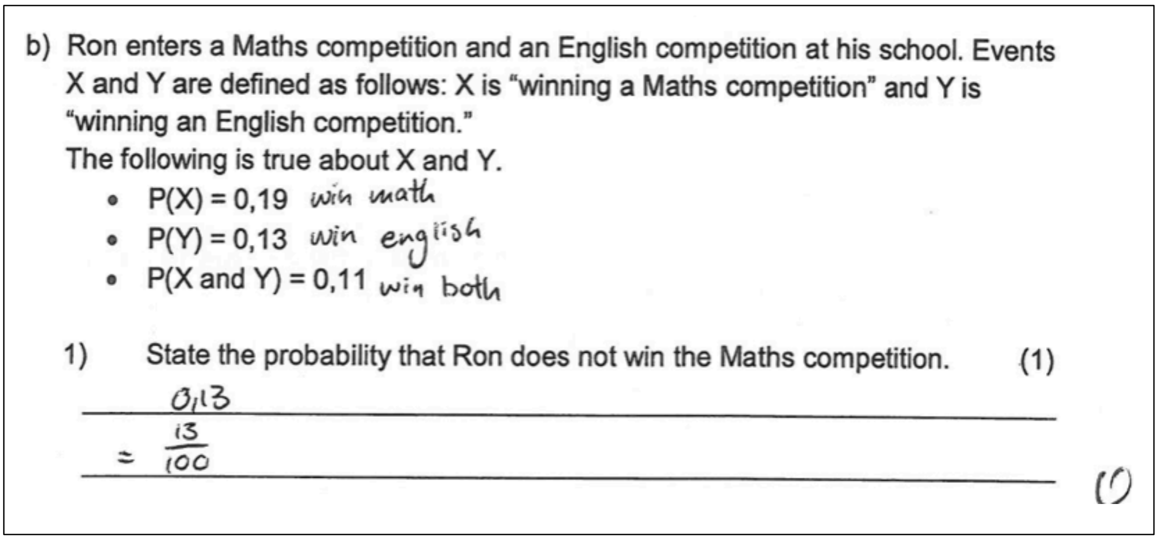
- the probability of winning the Maths competition is 0.19.
- the probability of winning an English competition is 0.13, and
- the probability of winning both competitions is 0.11.

The events were inclusive, but not necessarily dependent. To answer the question, learners were expected to use the relationship between an event and its complement in the rule,

$$\mathrm{P(not\,X) = 1 - P(X)}.$$

However, there was evidence that some learners did not understand the meaning of " not X" as illustrated by the extract in Figure 4.11.

Figure 4. 11 Extract of work by learner C17 showing a procedural error involving single events


b) Ron enters a Maths competition and an English competition at his school. Events X and Y are defined as follows: X is "winning a Maths competition" and Y is "winning an English competition."
The following is true about X and Y.

- P(X) = 0,19 win math
- P(Y) = 0,13 win english
- P(X and Y) = 0,11 win both

1) State the probability that Ron does not win the Maths competition. (1)

0,13
= 13/100

(0)

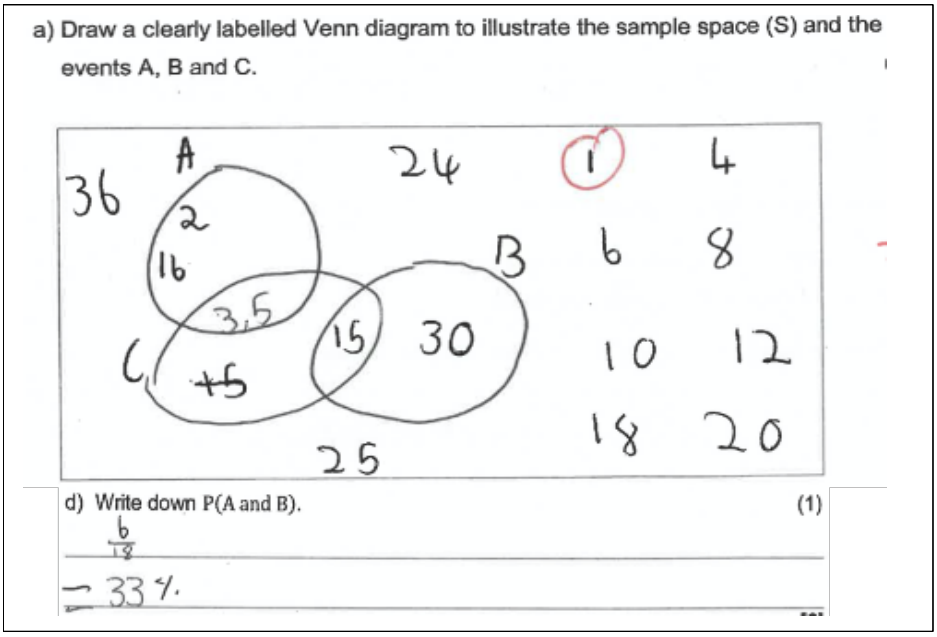
The learner, exemplified in Figure 4.11, interpreted "not X" to mean Y. The thinking that was applied in answering this question was that the probability of not winning the Maths competition is equal to the probability of winning the English competition, as if to say, "if you don't win the Maths competition, then you must win (or you must have won) the English competition." This thinking process is incorrect because it does not take into account that there is a possibility of winning neither competitions.

4.8.2.3 Learners' procedural errors when solving problems involving two events (P3 errors)

Procedural error P3 involved calculation of probability of the intersection of two events. In the pre-test, the number of P3 errors (sum = 10) made by the treatment group represented 37 percent of all procedural errors (sum = 27). There were no P3 errors in the post-test for the treatment group. For the control group, the number of P3 errors in the pre-test (sum = 14) represented 33.3 percent of all the procedural errors (sum = 42). There were also no P3 errors for this group in the post-test. Table 4.9 and Table 4.10 show these results.

In order to compute the probability of the combined event (A∩B), it was important for learners to understand that (A∩B) means "both A and B". This means that outcomes of (A∩B) are all the outcomes that appear in both event A and event B. Some learners misinterpreted (A∩B) to mean "outcomes in A" combined with "outcomes in B". This constituted a misconception which led to a procedural error. In Figure 4.12, the learner added the number of outcomes in A to the number of outcomes in B because he did not understand what (A and B) meant.

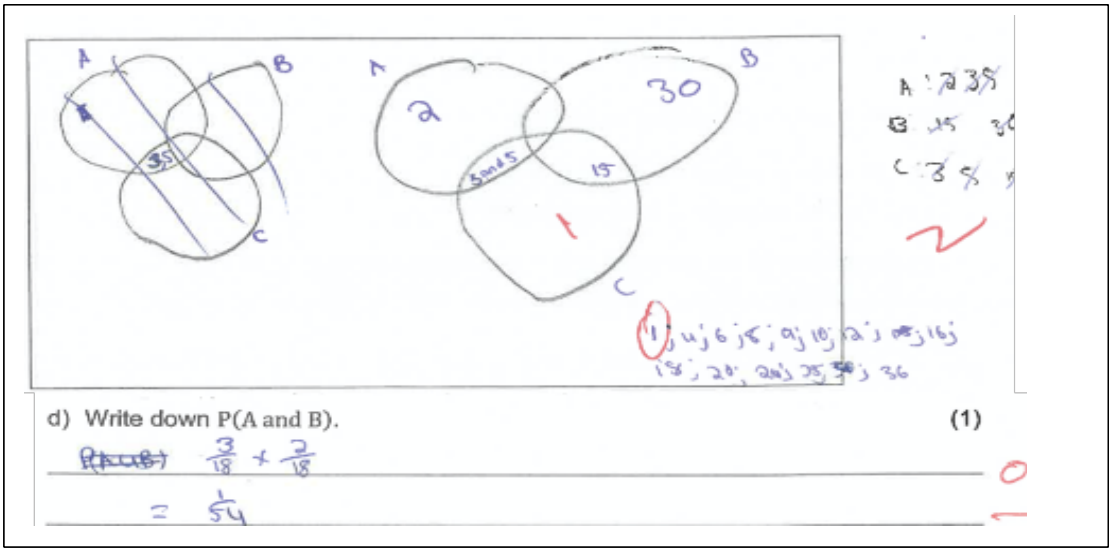
Figure 4. 12 Extract of work by learner T2 showing a procedural error involving intersection of two events



Working from this learner's Venn diagram, the correct answer for P (A∩B) would have been $\frac{0}{18}$ instead of $\frac{6}{18}$. The thinking behind the answer $\frac{6}{18}$ resulted from the learner's misinterpretation of the statement P (A∩B) to mean "all outcomes in A and all outcomes in B" instead of "all common outcomes in A and B".

In other similar cases, instead of adding the number of outcomes in A to the number of outcomes in B, some learners applied the rule P(A∩B) = P(A). P(B) as shown in Figure 4.13. This error suggests that learners thought that event A and event B are independent. Using conditional probability, if A and B are independent, then P(A and B) can be expressed as P(A) . P(B/A). However, since A and B are independent, then P(B/A) = P(B), hence the conclusion that P(A and B) = P(A) . P(B). Figure 4.13 exemplifies this error. According to this learner, there are 3 outcomes in event A and 2 outcomes in event B, and to compute the P(A and B), the learner multiplies $\frac{3}{18}$ by $\frac{2}{18}$.

Figure 4. 13 Extract of work by learner T13 showing a procedural error involving intersection of two events



Procedural errors that students made in the pre-test and post-test indicate that Grade 10 students have difficulties interpreting union of two events, intersection of two events, inclusive events and complementary events. Students' reasoning in tasks involving these concepts was influenced by their lack of understanding of these concepts, and this led to their wrong application of the probability rules for P(A or B) and P(not A).

4.8.2.4 Incidence of procedural errors

Table 4.10 shows the incidence of procedural errors by the treatment group in both the pre-test and post-test. Table 4.11 shows the incidence of procedural errors by the control group in both the pre-test and post-test.

Table 4. 10 Incidence of procedural errors by treatment in the pre-test and post-test

Procedural errors (P) – Treatment Group

		Pre-test		Post-test	
Type	Error description	Freq	%P	Freq	%P
P1	Misunderstanding of P(A or B) rule	9	*33.3*	20	*64.5*
P2	Misunderstanding of P (single event)	1	*3.7*	6	*19.4*
P3	Misunderstanding of P(A and B) rule	10	*37.0*	0	*0.0*
P4	Misunderstanding of inclusive events	5	*18.5*	5	*16.1*
P5	Misunderstanding of complement of combined	2	*7.4*	0	*0.0*
	Total	27	*46.6**	31	*70.5**

Total No. of errors by this group [Pre-test = 58; Post-test = 44]

Total number of errors as a percentage of all the errors for the particular test by group

Table 4. 11 Incidence of procedural errors by control groups in the pre-test and post-test

Procedural errors (P) – Control Group

		Pre-test		Post-test	
Type	Error description	Freq	%P	Freq	%P
P1	Misunderstanding of P(A or B) rule	15	*35.7*	35	*66.0*
P2	Misunderstanding of P (single event)	2	*4.8*	12	*22.6*
P3	Misunderstanding of P(A and B) rule	14	*33.3*	0	*0.0*
P4	Misunderstanding of inclusive events	0	*0.0*	6	*11.3*
P5	Misunderstanding of complement of combined events	11	*26.2*	0	*0.0*
	Total	42	*39.3**	53	*60.9**

Total No. of errors by this group [Pre-test = 107; Post-test = 87]

Total number of errors as a percentage of all the errors for the particular test by group

4.8.3 Conceptual errors (C) made by learners

Some questions in the pre-test and post-test required an understanding of the following probability concepts: biased events, mutually exclusive events, complementary events and inclusive events. Other concepts that were tested were the notion of probability as a measure of magnitude ranging from 0 to 1. Students were also expected to differentiate between actual outcomes of an event, number of outcomes in an event and probability of an event. Conceptual errors made by learners were identified and categorised as follows:

- C1 errors : conceptual errors caused by students' misconception of bias.
- C2 errors: conceptual errors resulting from the representativeness heuristic.
- C3 errors: conceptual errors arising from students' misconception of mutually exclusive events.

- C4 errors: conceptual errors arising from students' misconception of complementary events.
- C5 errors: conceptual errors arising from students' confusion between "outcomes" and "number of outcomes" of an event.
- C6 errors: conceptual errors arising from students' confusion between "outcomes" and "probability of outcomes".
- C7 errors: conceptual errors arising from students' misconception of probability measure.
- C8 errors: conceptual errors arising from students' misconception of inclusive events.

The categories of conceptual errors are explained in the following subsections.

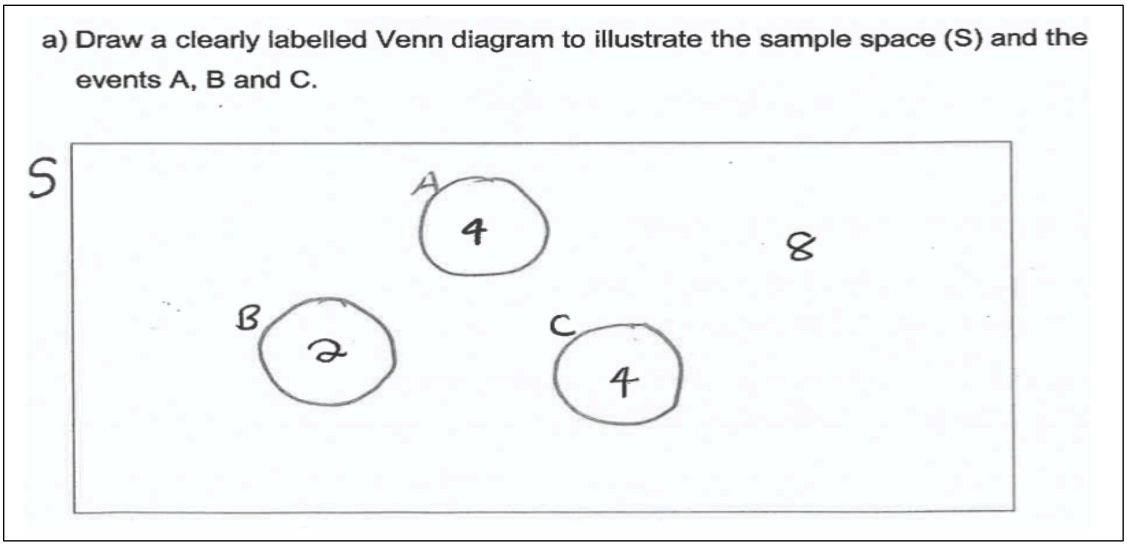
4.8.3.1 Conceptual errors caused by the confusion between outcomes, number of outcomes and probability of events

Some errors students made resulted from the confusion and failure to differentiate between some probability terms, an indication that learner conceptual understanding of these terms was not properly developed. Cases where such confusion was observed included failure to differentiate between:

- actual outcomes and number of outcomes (error C5),
- actual outcomes and probability of outcomes (error C6)
- actual events and probability of events (error C6)

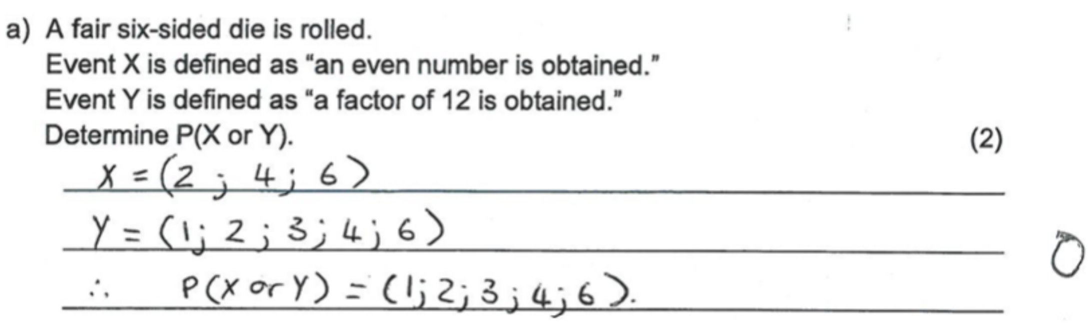
In the pre-test and post-test, the treatment group made 16.7 percent and 7.7 percent of these errors respectively. The corresponding figures for the control were 41.9 percent in the pre-test and 3.4 percent in the post-test. Figure 4.14 illustrates an example of students' errors which resulted from students' lack of understanding of the difference between actual outcomes and the number of outcomes.

Figure 4. 14 Extract of work by learner C5 showing a conceptual error involving outcomes and number of outcomes in an event



The question required the actual outcomes of events A, B and C to be placed in their proper sections of the Venn diagram. The student's response in Figure 4.14, however, shows that the learner wrote the number of outcomes instead of listing the actual outcomes. The event S was given as S = {1;2;3;4;5;6;8;9;10;12;15;16;18;20;24;25;30;36}, and A, B and C were supposed to be worked out to be A = {2;3;5}, B = {15;30} and C = {1;3;5;15}. Instead of listing the outcomes in the correct subsets on the Venn diagram, the learner wrote in the number of outcomes. She made further errors in determining the number of outcomes in event A, but it was clear that she did not understand that she was supposed to use the actual outcomes to answer the question. Further variants of errors in this category involved incidences where learners were asked to calculate the probability of an event, but they listed the outcomes of the event instead as shown in Figure 4.15.

Figure 4. 15 Extract of learner's answer showing confusion between probability of an event and outcomes of an event

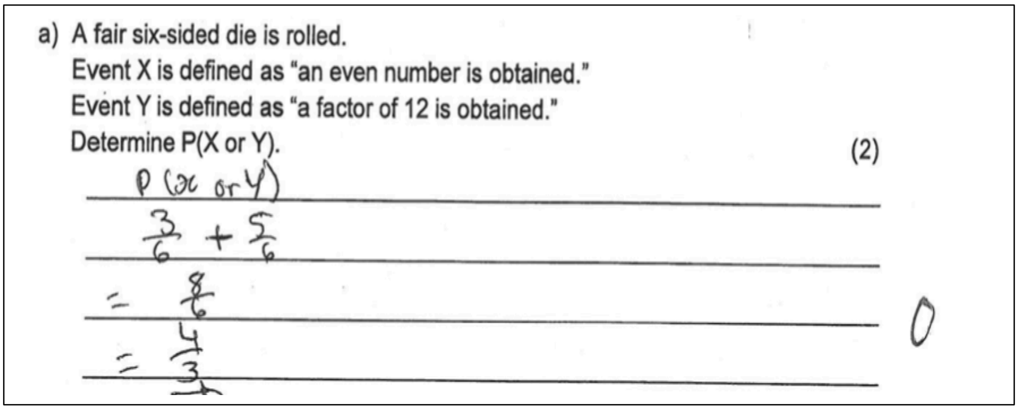


Such errors happen when students do not have an accurate understanding of the concepts that are being tested or when they believe that there is no difference between the concepts. As seen from Figure 4.15, either the student has a misunderstood meaning and nature of "probability of an event" or she believes there is no difference between P(X or Y) and (X or Y). Unfortunately, such misconceptions lead to errors.

4.8.3.2 Conceptual errors involving magnitude of probability values

There was a high incidence of errors where learners gave probability measures as values greater than 1 (error C7). The respondents who chose probability rules made this error when they did not consider the type of events they were dealing with. Figure 4.16 shows an example of error C7.

Figure 4. 16 Extract of work by learner showing a conceptual error involving probability values


a) A fair six-sided die is rolled.
Event X is defined as "an even number is obtained."
Event Y is defined as "a factor of 12 is obtained."
Determine P(X or Y). (2)

P(x or Y)
$\frac{3}{6} + \frac{5}{6}$
$= \frac{8}{6}$
$= \frac{4}{3}$
0

The error in Figure 4.16 shows that the conceptualisation of probability as a measure of chance from 0 to 1 was not well developed. This error was prevalent in questions where event X and event Y were inclusive, but learners treated them as mutually exclusive when calculating P(X or Y). As a result, the probability of (X and Y) was not used in the rule $P(X \cup Y) = P(X) + P(Y) - P(X \cap Y)$. This led to an answer greater than 1.

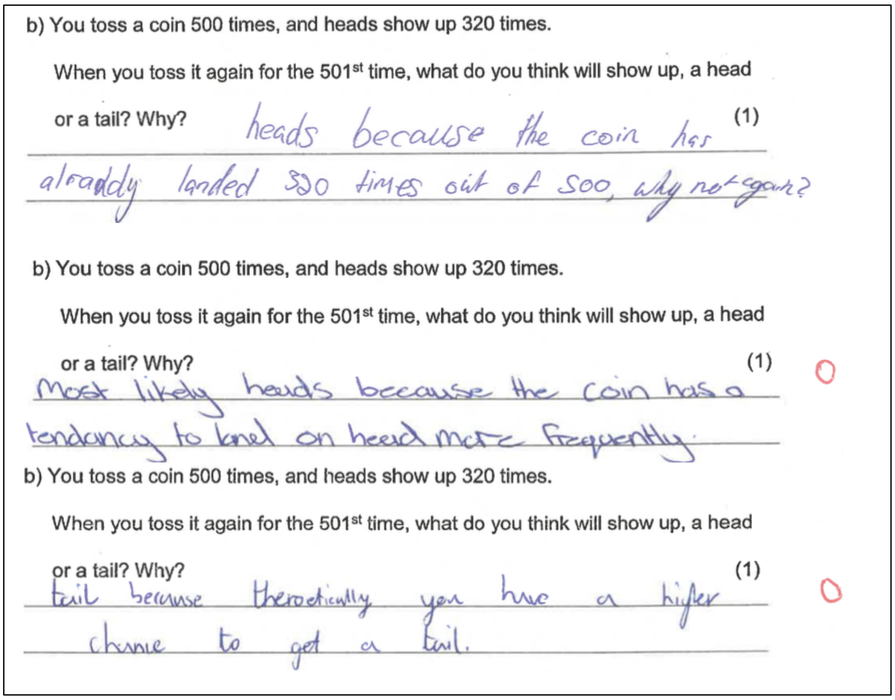
The number of C7 errors by the treatment group constituted 30.8 percent of all conceptual errors in the post-test compared to the corresponding post-test percentage of 20.7 percent for the control group.

4.8.3.3 Conceptual errors caused by the use of a representativeness heuristic

Another most frequent error resulted from the use of a representativeness heuristic (error C2). According to Tversky and Kahneman (1974), a misconception of chance can result from a representativeness heuristic when people judge probability situations based on their evaluation of the degree to which one event resembles another.

In the pre-test Item 1b, a scenario was provided where a coin is tossed 500 times and heads show up 320 times. The question asked what students thought would be the outcome if the coin was tossed again for the 501st time. Some of the responses are shown in Figure 4.17. The first learner's response, viz; "heads because the coin has already landed 320 times out of 500, why not again?", shows that the learner is influenced by his belief that chance is "a self-correcting measure" (Hokor, et al., 2022). According to Hokor, et al., (2022), this view is held by a student who believes that heads will be next simply because they have occurred more than tails previously. This misconception is closely related to the positive recency effect which is a "belief that the previous outcome will repeat itself in the next trials" (Hokor, et al., 2022, p. 3). The flip side of this misconception is the negative recency effect bias which is the belief that the previously obtained occurrence will not be next.

Figure 4. 17 Learners' responses when they use a representativeness heuristic


b) You toss a coin 500 times, and heads show up 320 times.

When you toss it again for the 501st time, what do you think will show up, a head or a tail? Why? (1)

heads because the coin has alraddy landed 320 times out of 500, why not again?

b) You toss a coin 500 times, and heads show up 320 times.

When you toss it again for the 501st time, what do you think will show up, a head or a tail? Why? (1)

Most likely heads because the coin has a tendency to land on head more frequently.

b) You toss a coin 500 times, and heads show up 320 times.

When you toss it again for the 501st time, what do you think will show up, a head or a tail? Why? (1)

tail because therotically you have a hiher chance to get a tail.

From the treatment group 55.5 percent of all their conceptual errors in the pre-test were caused by the representativeness heuristic bias. The corresponding frequency for the control group in the pre-test was 27.9 percent (Table 4.11). The majority of the learners (55.6 percent) thought that the next time the coin is tossed, a head would show up since it had a higher probability of occurring than the tail. Only 5.6 percent of all the students thought that the tail would occur next since they had had many heads already.

4.8.3.4 Incidence of conceptual errors in the pre-test and post-test

Table 4.11 shows the prevalence of conceptual errors by the treatment group and control group in the pre-test and post-test. There was a significant number of learners who failed to answer questions correctly as a result of their lack of adequate conceptual understanding of some of these probability concepts. These types of errors accounted for 33.5% of all the errors that were made by the combined groups. In both the pre-test and the post-test, the frequency of these errors was 30.5% for the treatment group and 36.5% for the control group.

The most frequent errors for the treatment group in the pre-test were C1, C2 and C4 errors (respective frequency rates: 22.2 percent; 55.5 percent; 16.7 percent). In the post-test, the most frequent errors for the treatment group were C4, C7 and C 8 errors (respective frequency rates: 46.2 percent; 30.8 percent; 15.4 percent).

For the control group, on the other hand, the most frequent of these errors in the pre-test were C2, C4 and C6 errors (respective frequency rates: 27.9 percent; 18.6 percent; 32.6 percent). In the post-test, the most frequent errors were C4, C7 and C8 errors (respective frequency rates: 41.4 percent; 20.7 percent; 34.5 percent). Table 4.12 shows these results. These findings indicate that C2, C4, C7 and C8 errors were most problematic for both groups.

Table 4. 12 Incidence of conceptual errors by treatment and control groups in the pre-test and post-test

Conceptual errors (C) – Treatment group					
		Pre-test *58 errors*		**Post-test** *44 errors*	
Type	Error description	Freq	%C	Freq	%C
C1	Understanding of bias	4	*22.2*	0	*0*
C2	Impact of the representativeness heuristic	10	*55.5*	0	*0*
C3	Understanding of mutually exclusive events	1	*5.6*	0	*0*
C4	Understanding of complementary events	3	*16.7*	6	*46.2*
C5	Confusion between outcomes and number of outcomes	0	*0*	0	*0*
C6	Confusion between outcomes/events and probability of outcomes/events	0	*0*	1	*7.7*
C7	Gives probability > 1	0	*0*	4	*30.8*
C8	Understanding of inclusive events	0	*0*	2	*15.4*
	Total	18	*31**	13	*29.5**

Conceptual errors (C) – Control group					
		Pre-test *107 errors*		**Post-test** *87 errors*	
Type	Error description	Freq	%C	Freq	%C
C1	Understanding of bias	0	*0*	0	*0*
C2	Impact of the representativeness heuristic	12	*27.9*	0	*0*
C3	Understanding of mutually exclusive events	2	*4.65*	0	*0*
C4	Understanding of complementary events	8	*18.6*	12	*41.4*
C5	Confusion between outcomes and number of outcomes	4	*9.30*	0	*0*
C6	Confusion between outcomes/events and probability of outcomes/events	14	*32.6*	1	*3.4*
C7	Gives probability > 1	3	*7.0*	6	*20.7*
C8	Understanding of inclusive events	0	*0*	10	*34.5*
	Total	43	*40**	29	*33**

* *Total number of errors as a percentage of all the errors for the particular test by group*

4.9 Prevalence of learners' errors when solving probability problems

A total of 58 learner errors were identified from the treatment group (n = 14) in the pre-test, while 107 were observed from the control group (n = 22). This translated to approximately 4 errors per learner from each group. In the post-test, the total number of learner errors identified was 44 from the treatment group and 87 from the control group. This translated to an average of approximately 3 learner errors per learner in each group.

Table 4.13 shows the prevalence of factual errors (F) that were identified in the pre-test and post-test by both the treatment and control groups. Table 4.14 shows the prevalence of procedural errors (P) that were identified in the pre-test and post-test by both the treatment and control groups. Table 4.15 shows the prevalence of conceptual errors (C) that were identified in the pre-test and post-test by both the treatment and control groups.

Table 4. 13 Number of factual errors (F) identified in the pre-test and post-test from the treatment and control groups

	Number of factual errors (F) in the pre-test	Number of factual errors (F) in the post-test	Total
Treatment group (n = 14)	13	0	13
Control group (n = 22)	22	5	27
Total	35	5	40

Table 4. 14 Number of procedural errors (P)identified in the pre-test and post-test from the treatment and control groups

	Number of procedural errors (P) in the pre-test	Number of procedural errors (P) in the post-test	Total
Treatment group (n = 14)	27	31	58
Control group (n = 22)	42	53	95
Total	69	84	153

Table 4. 15 Number of conceptual errors (C) identified in the pre-test and post-test from the treatment and control groups

	Number of conceptual errors (C) identified in the pre-test	Number of conceptual errors (C) identified in the post-test	Total
Treatment group (n = 14)	18	13	31
Control group (n = 22)	43	29	72
Total	61	42	103

The total number of pre-test errors for both groups combined was 165 (treatment group = 58; control group = 107). In the post-test, the combined number of errors was 131 (treatment group = 44; control group = 87). The total number of errors that respondents in the treatment group made in the pre-test accounted for 35,2 percent of all the pre-test errors, while the corresponding percentage for the control group was 64,8 percent. In the post-test, the percentage of errors made by the treatment group and the control group were 33,6 percent and 66,4 percent respectively. The performance of the treatment group improved slightly in the post-test judging from the decrease in the percentage of errors from 35,2 percent to 33,6 percent. On the other hand, there was no evidence to conclude that the control group improved in the post-test since they contributed more error percentages than the treatment group. Table 4.16 shows a summary of the total number of errors each group made in the pre-test and post-test.

Table 4. 16 Total number of errors each group made in the pre-test and post-test

	Number of Pre-test errors	Number of Post-test errors	Total
Treatment group (n = 14)	58 (35.2%)	44 (33.6%)	102 (34.5%)
Control group (n = 22)	107 (64.8%)	87 (66.4%)	194 (65.5%)
Total	165 (100%)	131 (100%)	296 (100%)

4.10 Comparison between number of pre-test and post-test errors per group

There was a similar general pattern in the number of errors that both the treatment and control groups made in the pre-test and post-test. For both groups, the prevalence of factual errors decreased in the post-test compared to the pre-test. The number of factual errors made by the treatment group in the post-test was 0, compared to 13 in the pre-test. The procedural errors in the post-test for the treatment group increased by a small amount from 27 in the pre-test to 31 in the post-test. Conceptual errors decreased from 18 in the pre-test to 13 in the post-test. This pattern was also observed in the control groups. The factual errors by the control group decreased significantly from 22 to 5, while the procedural errors increased from 42 in the pre-test to 53 in the post-test. The control group conceptual errors also decreased from 43 in the pre-test to 29 in the post-test. Figure 4.18 and Figure 4.19 show this information.

Figure 4. 18 Error patterns between the pre-test and post-test (Treatment group).

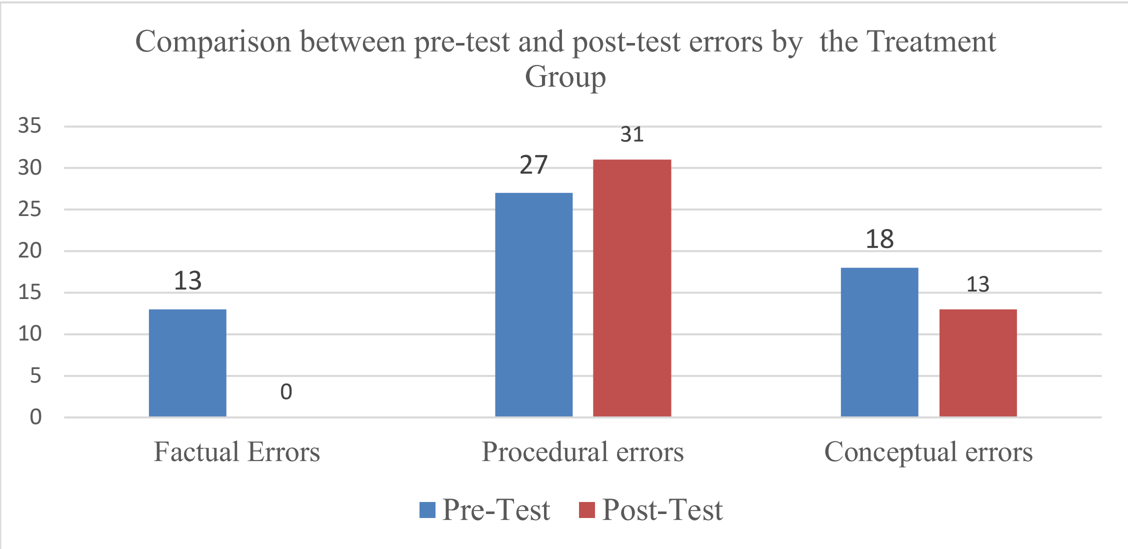
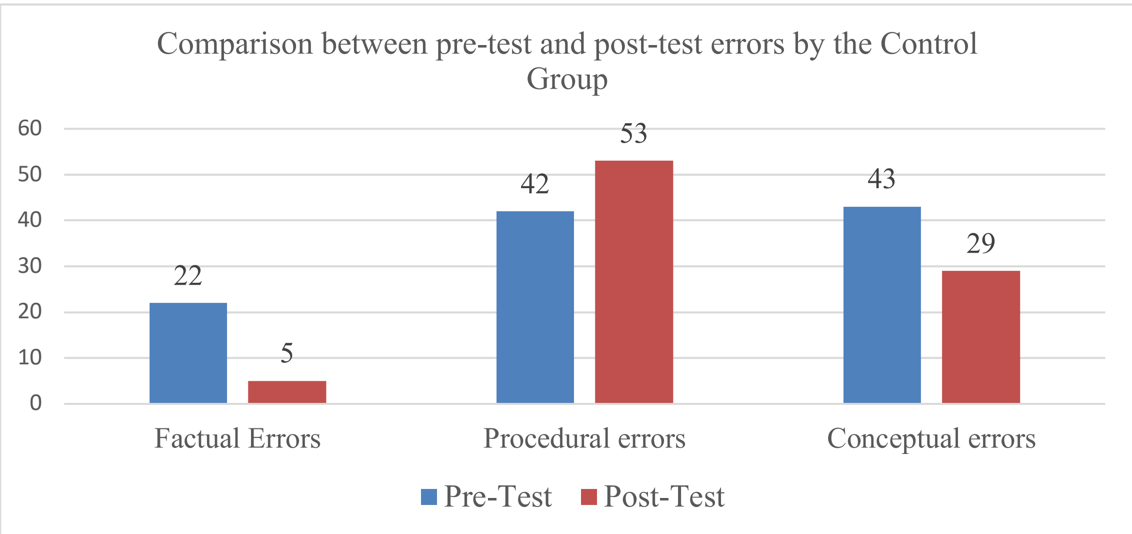


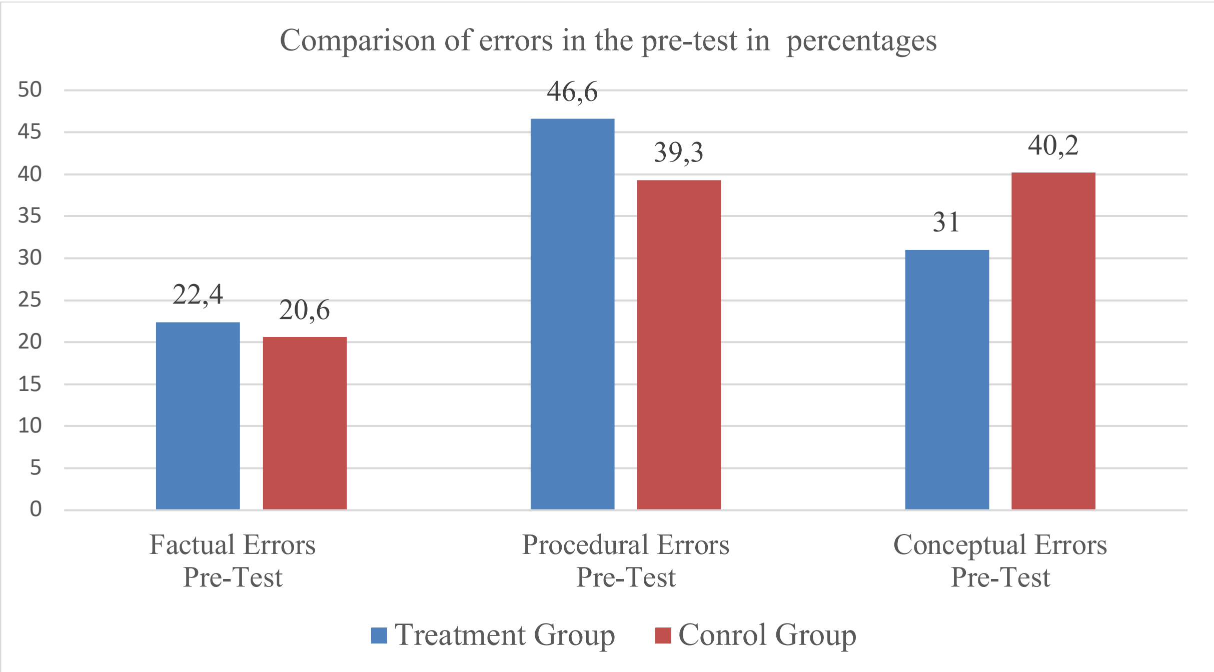
Figure 4. 19 Error patterns between the pre-test and post-test (Control group).



4.11 Prevalence of pre-test errors: Treatment group vs. Control group

The treatment group had a total of 58 errors in the pre-test (13 factual errors; 27 procedural errors; 18 conceptual errors). Thus, for this group, factual errors constituted 22.4 percent, procedural errors counted 46.6 percent and conceptual errors 31.0 percent. On the other hand, the control group made a total of 107 errors in the pre-test (22 factual errors; 42 procedural errors; 43 conceptual errors). Control group factual errors represented 20.6 percent of the errors, while procedural and conceptual errors represented 39.3 percent and 40.2 percent respectively. Figure 4.20 compares the percentage frequency of errors between the treatment group and the control group in the pre-test.

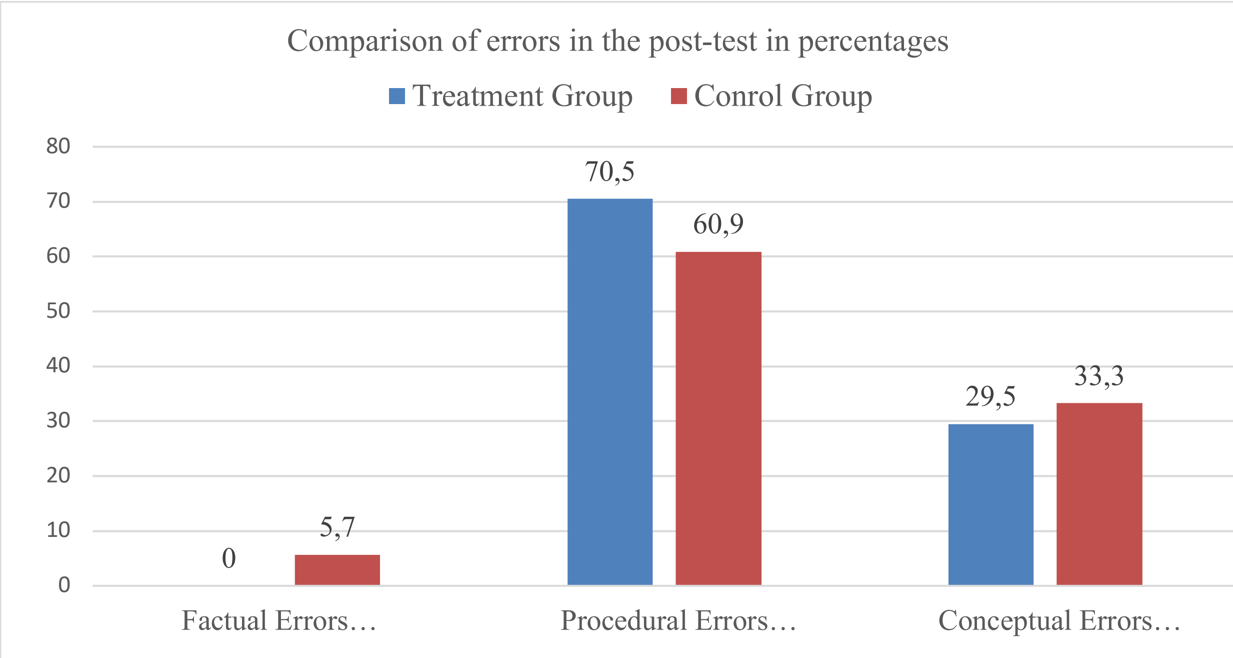
Figure 4. 20 Comparison of errors in the pre-test (each error type as a % of all errors)



4.12 Prevalence of post-test errors: Treatment group vs. Control group

In the post-test, the treatment group made 44 errors (0 factual errors; 31 procedural errors; 13 conceptual errors). Each of the error types were expressed as a percentage of all the errors. Procedural errors were, therefore, 70.5 percent of all the errors, while conceptual errors were 29.5 percent. The control group, on the other hand made in total 87 errors (5 factual errors; 53 procedural errors; 29 conceptual errors). Expressed as percentages of all the errors, 5.7 percent were factual, 60.9 percent were procedural and 33.3 percent were conceptual errors. Figure 4.21 compares the percentage errors made in the post-test by the treatment and control groups.

Figure 4. 21 Comparison of errors in the post-test (each error type as a % of all errors)



It can be concluded from the preceding discussion that learners experienced different levels of difficulty in the questions that were posed. There were cases where correct methods were used leading to correct results being reached. However, there were also cases where correct methods were used, but incorrect results were reached. With regards to questions that were not attempted, this study found that only one learner did not attempt a question. This was not surprising because adequate time was allocated to complete the tasks. It was also found that students made conceptual, procedural and factual errors in their work. There were basic Mathematics concepts such as factors, multiples and primes which were forgotten by some respondents. These concepts are necessary in probability problem solving. This study suggests that activities should be planned in Grade 10 Mathematics lessons to assist learners with these. Performance in probability is likely to improve if adequate time is allocated in class to acquire probability knowledge. This study will also recommend that the probability topic be taught early in the year to afford learners adequate time to grasp the concepts. In the next section, quantitative analysis focuses on the research questions to determine the impact of GeoGebra on specific learning outcomes.

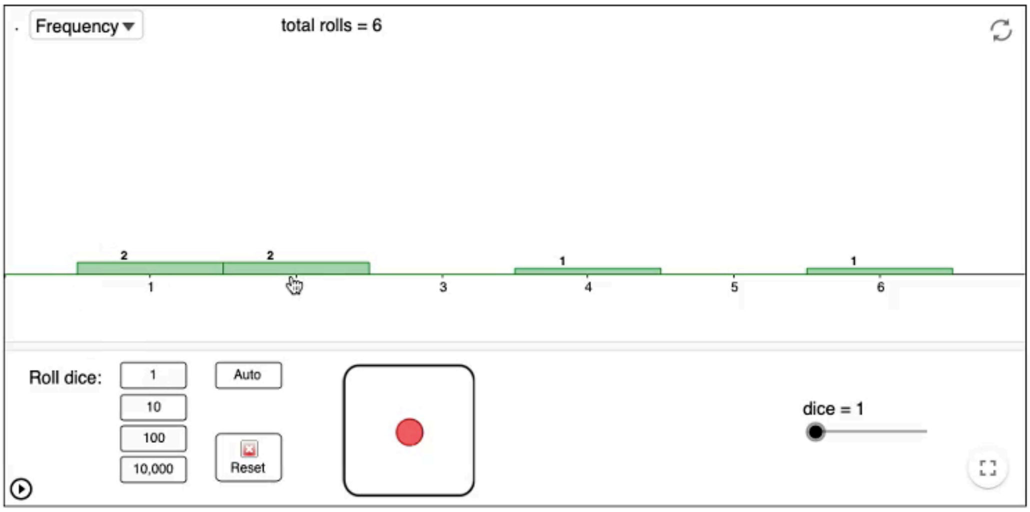
4.13 The impact of GeoGebra intervention: Quantitative analysis

The second research sub-question for this study was developed to address the impact of GeoGebra on the learning of various probability concepts. In particular, the impact of GeoGebra intervention on the prevalence of Grade 10 learners' errors and misconceptions in probability problem solving was sought. Both quantitative and qualitative results were used to answer the research question. After the pre-test was administered to the treatment and control groups, four probability intervention lessons were planned and taught to the treatment group using GeoGebra. Intervention for the control group, on the other hand, was in the form of standard instruction where no ICT application was used.

The affordances of GeoGebra were discussed in the literature review (see section 2.10). They included opportunities presented by the computer algebra systems (CAS) and dynamic geometry software (DGS) functionalities of the tool which allow learners to visualise both algebraic and geometric representations of the same phenomenon simultaneously. GeoGebra further provides a rich learning experience for the students by providing opportunities for them to interact with the graphs that are created.

The current study was centred around GeoGebra and its affordances in creating opportunities for students to learn probability concepts. The intervention was designed to provide opportunities for learners to improve their problem solving and reasoning abilities. It was also the goal of the study to enhance students' conceptual understanding by providing opportunities for them to discover relationships. The GeoGebra artefact was useful in achieving these goals. For example, understanding the relationship between relative frequency and theoretical probability was enhanced when the GeoGebra simulation of rolling a die was used. In theory, when a die is rolled, it is expected that $P(1) = P(2) = P(3) = P(4) = P(5) = P(6) = \frac{1}{6}$. However, in an experiment, the results could be different. The DGS functionality of GeoGebra displayed bar charts to compare the probabilities of getting each outcome in the event $\{1,2,3,4,5,6\}$. After rolling the die a certain number of times, the relative frequency of each outcome was captured in a spreadsheet. Students were able to see the relationship between the relative frequencies and the number of times the die was rolled. In particular, they were able to conclude that the relative frequency of each outcome equals the theoretical probability if the number of times the die is rolled is very large. Learners were also able to observe that the heights of the bar charts became equal as the number of times the die was rolled increased. Figure 4.22 – Figure 4.27 show a sequence of representations that the GeoGebra artefact created to help students understand the relationship between relative frequency and theoretical probability. The Figures show the representations when the die was rolled 6 times (Figure 4.22), 300 times (Figure 4.23), 1200 times (Figure 4.24), 6000 times (Figure 4.25), 30 000 times (Figure 4.26) and 100 000 times (Figure 4.27). The relative frequency of getting each outcome stabilised to 0.17 as the number of times the die was rolled increased.

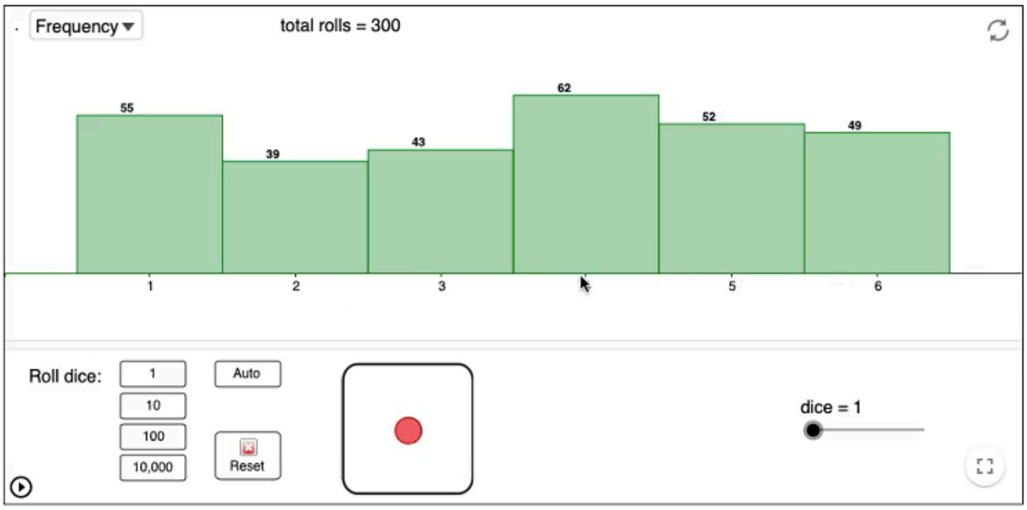
Figure 4. 22 GeoGebra representation showing the relative frequency of outcomes when a die was rolled 6 times



Interpretation: Number of times the die was rolled = 6

Possible Outcome (x)	1	2	3	4	5	6
Relative frequency of x	0.33	0.33	0.00	0.17	0.00	0.17
Theoretical probability of x	0.17	0.17	0.17	0.17	0.17	0.17

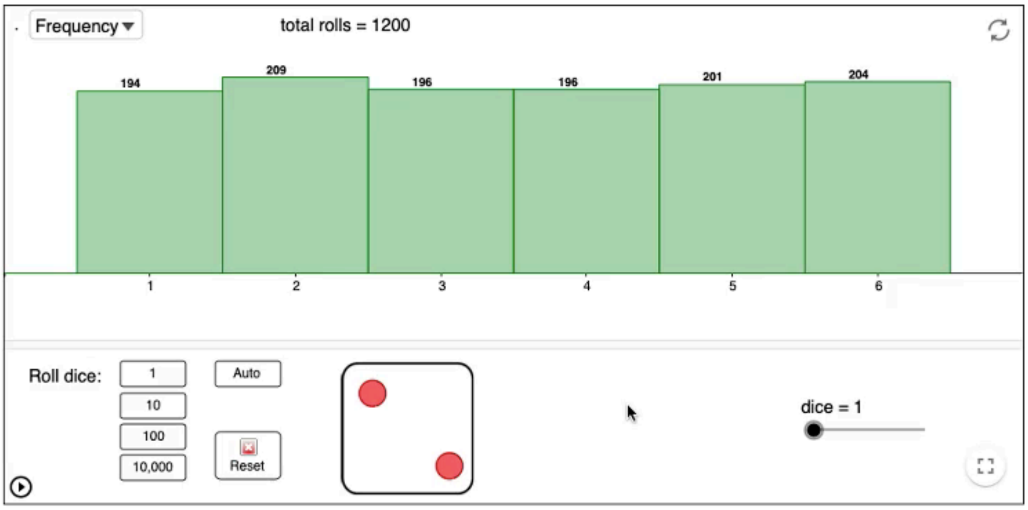
Figure 4. 23 GeoGebra representation showing the relative frequency of outcomes when a die was rolled 300 times



Interpretation: Number of times the die was rolled = 300

Possible Outcome (x)	1	2	3	4	5	6
Relative frequency of x	0.18	0.13	0.14	0.21	0.17	0.16
Theoretical probability of	0.17	0.17	0.17	0.17	0.17	0.17

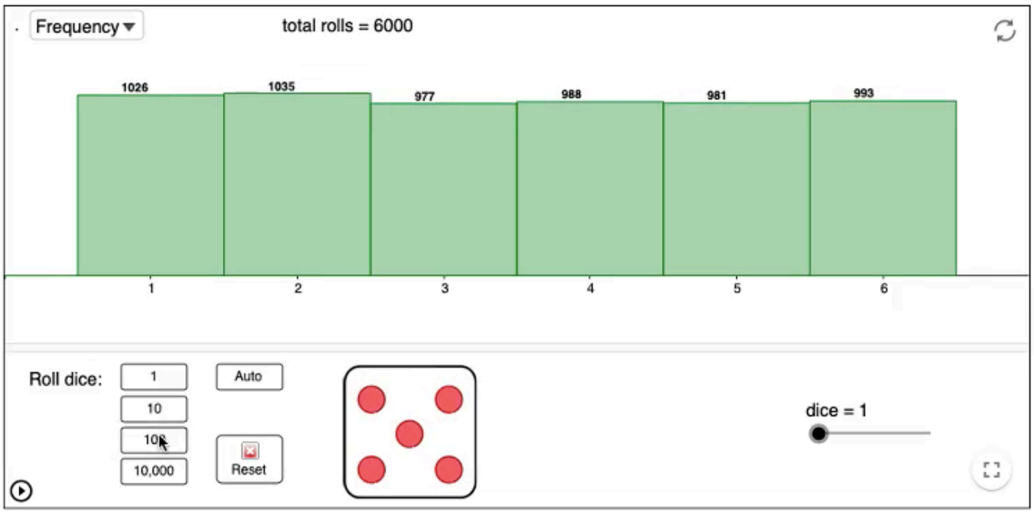
Figure 4. 24 GeoGebra representation showing the relative frequency of outcomes when a die was rolled 1200 times



Interpretation: Number of times the die was rolled = 1200

Possible Outcome (x)	1	2	3	4	5	6
Relative frequency of x	0.16	0.17	0.16	0.16	0.17	0.17
Theoretical probability of x	0.17	0.17	0.17	0.17	0.17	0.17

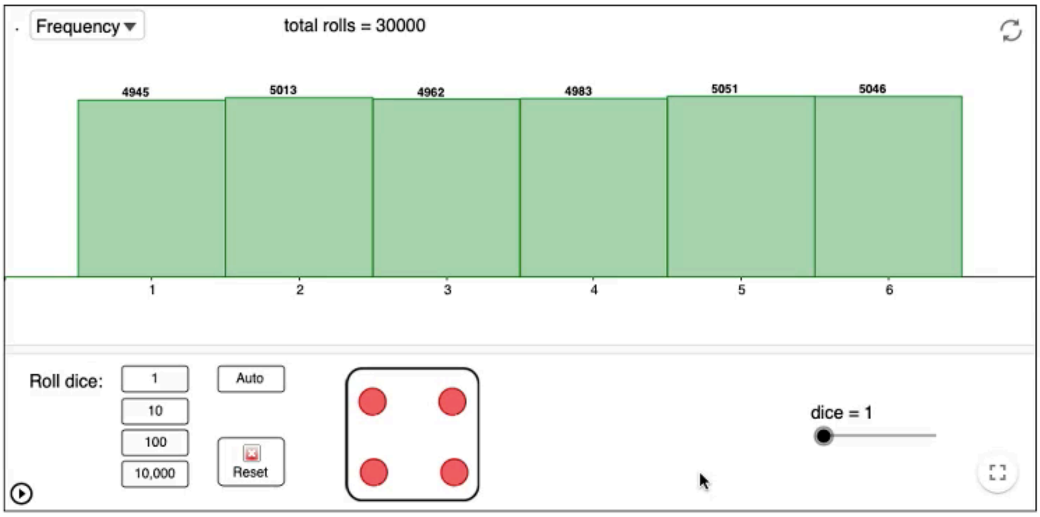
Figure 4. 25 GeoGebra representation showing the relative frequency of outcomes when a die was rolled 6000 times



Interpretation: Number of times the die was rolled = 6000

Possible Outcome (x)	1	2	3	4	5	6
Relative frequency of x	0.17	0.17	0.16	0.16	0.16	0.17
Theoretical probability of x	0.17	0.17	0.17	0.17	0.17	0.17

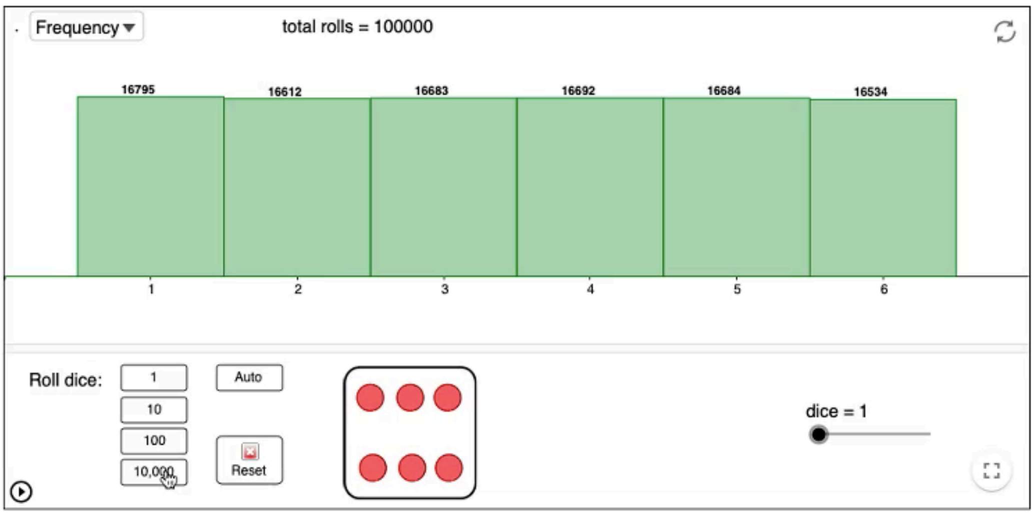
Figure 4. 26 GeoGebra representation showing the relative frequency of outcomes when a die was rolled 30000 times



Interpretation: Number of times the die was rolled = 30 000

Possible Outcome (x)	1	2	3	4	5	6
Relative frequency of x	0.16	0.17	0.17	0.17	0.17	0.17
Theoretical probability of x	0.17	0.17	0.17	0.17	0.17	0.17

Figure 4. 27 GeoGebra representation showing the relative frequency of outcomes when a die was rolled 100 000 times



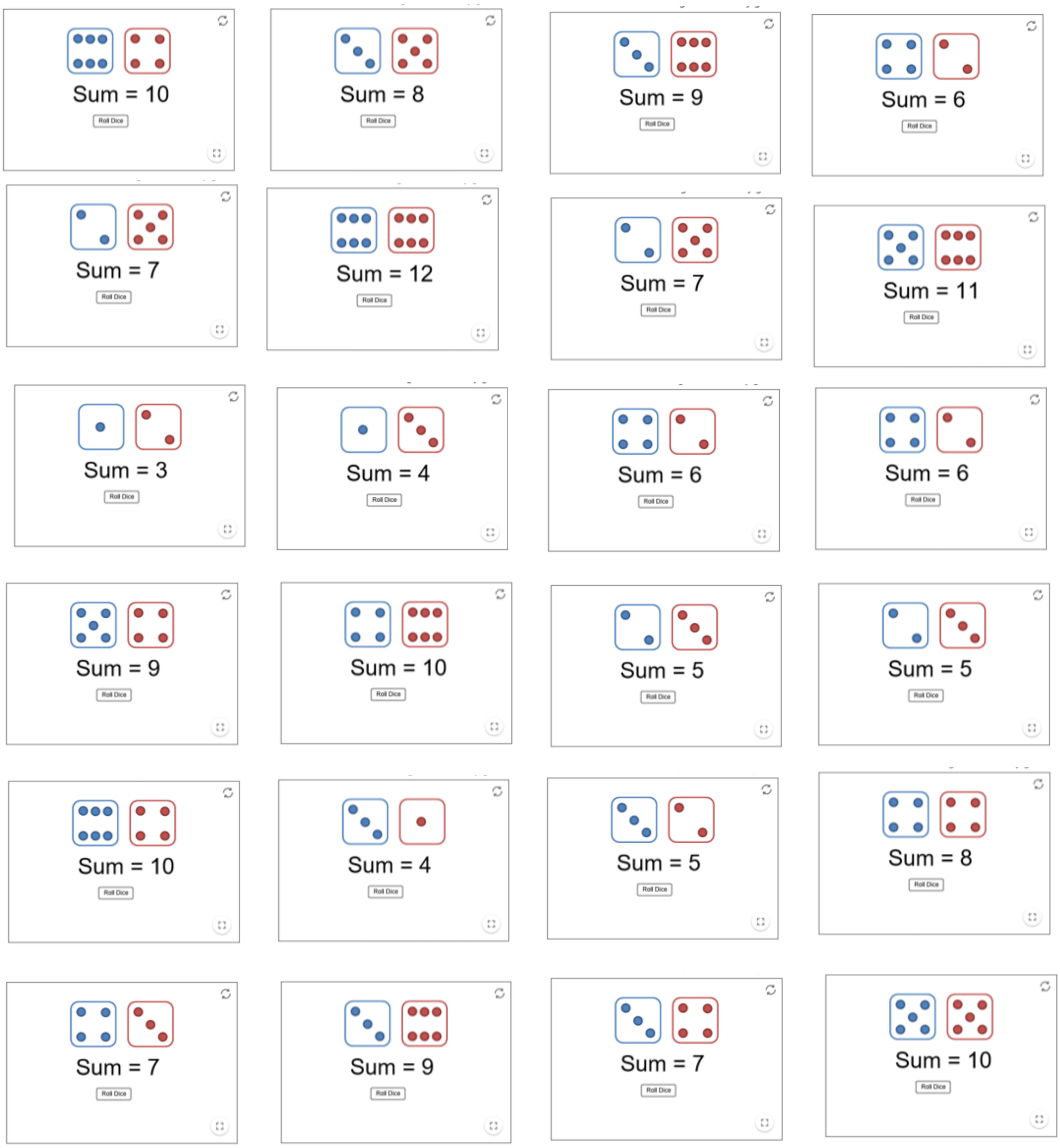
Interpretation: Number of times the die was rolled = 100 000

Possible Outcome (x)	1	2	3	4	5	6
Relative frequency of x	0.17	0.17	0.17	0.17	0.17	0.17
Theoretical probability of x	0.17	0.17	0.17	0.17	0.17	0.17

In another activity, learners were given the following information:

Two dice are rolled. Define A and B as follows: A = {Sum of the numbers appearing on the dice is 7}, B = {At least one of the dice shows a 2}. Students were asked to use a GeoGebra simulation (accessed through the link: https://ggbm.at/J9NyWRa5) to complete the activity. After rolling the dice, students were expected to identify outcomes that fell in A and B. They summarised the results in a table and used them to draw a Venn diagram to illustrate the outcomes. One student's outcomes were captured as screenshots showing the exact numbers from each die. Figure 4.28 shows the outcomes of a GeoGebra simulation of rolling two dice and adding the numbers that showed.

Figure 4. 28 Outcomes obtained by a learner after rolling two dice 24 times using GeoGebra



From this experiment the student was able to identify outcomes for A and B as:

A = {(2;5); (2;5); (4;3); (3;4)}, B = {(4;2); (2;5); (2;5); (1;2); (4;2); (4;2); (2;3); (2;3); (3;2)}.

Different learners had different outcomes as was expected. This information was then represented as a Venn diagram. Students could show the actual outcomes, the number of outcomes or the probability of the outcomes in the regions of the Venn diagram. Thus, GeoGebra created opportunities for students to improve their problem-solving abilities.

In this section, the impact of GeoGebra intervention is analysed by interrogating the quantitative results that were obtained. The results were tested for significance and effect size using JASP (Version 0.14) computer software. I also applied some principles of learning effectiveness in the analysis. According to Moody & Sindre (2003), learning effectiveness can be measured by considering the extent to which the learning activity contributed towards one's learning, and how effective the learning intervention was compared to other learning activities. The dependent variables that the analysis focused on included learners' achievement in tests, difficulty levels of test items, use of methods in problem solving and error frequency in learners' written work. In addition, students' levels of engagement, participation and interaction were identified as dependent variables. For this study, therefore, the following variables were considered as indicators of GeoGebra effectiveness in learning probability:

- success rate as indicated by learner achievement in tests,
- level of difficulty of test items,
- frequency of errors made by learners in tests.
- learner levels of engagement, participation and interaction.

The observed success rate in the pre-test before GeoGebra intervention was compared with the observed success rate in the post-test after intervention. Similarly, difficulty levels and error frequencies before and after intervention were observed and compared to determine the effectiveness of the intervention. Measures were taken to eliminate the effect of external factors other than the independent variables. Independent variables and external factors were discussed in detail in section 3.8. This was addressed by applying some principles of difference-in-differences (DiD) model. A difference-in-differences analysis is used to estimate the impact of an intervention on a programme. Goodman-Bacon (2018) describes a difference-in-differences estimate as "the difference between the change in outcomes before and after a treatment in a treatment versus control group" (p. 1). Outcomes of two groups, the treatment group and the control group, are observed at a particular period before the intervention is introduced. The treatment group is then exposed to the intervention while the control group does not receive it. The outcomes are then observed again after the intervention and the difference-in-differences estimator is calculated using the formula:

$$\text{DiD} = \left(\bar{y}_s = \text{Treatment},\ \text{t} = \text{After} - \bar{y}_s = \text{Treatment}, \text{t} = \text{Before}\right) - \left(\bar{y}_s = Control,\ t = After - \bar{y}_s = Control,\ t = Before\right)$$

where y is the outcome variable, the bar represents the average value (averaged over individuals, typically indexed by i), the group is indexed by s and t is time (Fredriksson & de Oliveira , 2019, p.521).

4.13.1 Impact of GeoGebra intervention on participants' success rate in tests

The pre-test and post-test scores that respondents obtained were converted into percentages. Appendix E6 shows the raw pre-test and post-test scores for the treatment and control groups.

The mean score was calculated to determine the average success rate for each group. Correct responses (or answers) indicated the level of understanding of the content being tested. The pre-test results were analysed for both groups before they received intervention. The success rate was calculated using the scores of correct responses that students gave. The average success rates for the treatment group in the pre-test and in the post-test were 49 percent and 52 percent respectively, while the respective rates for the control group were 45 percent and 44 percent. Table 4.17 shows the success rates for the treatment group and the control group in the pre-test and post-test.

Table 4. 17 Success rate in the pre-test and post-test by the treatment and control groups

	Pre-test – success rate as a %	Post-test – success rate as a %	Difference
Treatment Group	49	52	+3
Control Group	45	44	-1
Difference	+4	+8	+4

Comparison between Pre-test results and Post-test results for each group

There was evidence that students in the treatment group achieved a higher success rate in the post-test than they did in the pre-test. Their post-test average was 3 percent higher than their pre-test success rate.

These results were tested using the Paired Samples t-test (Appendix 7) and it was found that the participants in the treatment group increased their scores by 2,643 (SE:7.103) after receiving GeoGebra intervention. However, this increase was not statistically significant ($t(13) = 0.372$, $p = .716$). Cohen's d = .099 also showed that the effect size was small, suggesting that the GeoGebra intervention had a small effect on learners' achievement scores. It can be concluded that even though the treatment group improved their scores after the GeoGebra intervention, the mean difference between the scores was marginal and not statistically significant.

On the other hand, the control group achieved 1 percent less in the post-test than they did in the pre-test. The paired samples t-test (Appendix E8) indicated that the group's performance decreased on average by 1.273 (SE:4.122). The paired samples t-test also showed that the difference in means between the pre-test and post-test was not statistically significant ($t(21) = 0.309$, $p = .761$), and Cohen's d value of .066 indicated that the interaction had a small effect size. It can be concluded that performance by in the post-test by the control group did not differ significantly from their performance in the pre-test following the intervention that they received.

Generally, the treatment group got better results than the control group although these differences were not statistically significant. The practical implication of these results is that teacher intervention over a period of four days had a small effect on learners' achievements in written tests. The errors and misconceptions which were detected in the pre-test were, more or less, the same in the post-test for both the treatment and the control. The treatment group had received GeoGebra intervention in four lessons lasting between 30 and 40 minutes, while the control group had received theirs through standard teaching practice without ICT use. The intervention was given four months after the pre-test, and nine months after the topic was initially taught.

Comparison between Treatment group performance and Control group performance in the pre- and post-tests

Although the treatment group achieved marginally higher scores in the pre-test (median = 50) than the control group (median = 44.5), a Mann-Whitney U test (Appendix E9) indicated that the mean difference in success rates was not statistically significant ($U = 134$, $p = .524$).

A Mann-Whitney U test (Appendix E10) on the post-test results indicated that the treatment group had a higher success rate (median = 56, mean = 51.929) than the control group (median = 44, mean = 44.091). However, this difference was not statistically significant ($U = 121.5$, $p = .294$).

The observed success rates by the treatment group (n = 14) and the control group (n = 22) in the pre-test and post-test indicate that the treatment group had a slightly higher success rate than the control group in both the pre-test and the post-test. However, these differences were both not statistically significant.

The difference-in-differences (DiD) estimate of the success rates for the two groups between the pre-test scores and the post-test scores was found to be equal to 4. The reported results about the differences in performance between pre-test and post-test results by each group, and between the two groups indicate that the GeoGebra intervention had a positive effect on learners' achievements although these gains were not statistically significant. It can, therefore, be concluded that the use of GeoGebra software to teach probability can potentially lead to probability knowledge gains. In particular, improvements in students' performance in tests can be realised through GeoGebra-supported learning over a reasonable period of time. Further studies with more participants, increased test questions and extended intervention times are recommended to confirm these findings.

4.13.2 Impact of GeoGebra on perceived levels of difficulty in test items

The difficulty levels of the items in the pre-test and post-test were discussed in the previous sections of this chapter (section 4.2.1). The difficulty index was calculated using the following formula: *difficulty index = (number of correct responses)÷(number of learners).*

Test items were classified easy, moderate or difficult. The following guidelines were used:

- the question was Easy if difficulty index ≥ 0.85
- the question was Moderate if $0.51 \leq difficulty\ index\ \leq 0.84$
- the question was Difficult if difficulty index ≤ 0.50

The discrimination indices of the test items were also calculated to get an idea of how well the items discriminated between high scoring students and low scoring students. Each group (treatment group and control group) was divided into two halves, top achievers and bottom achievers. The calculation of the discrimination index was explained in section 4.2.1. A positive discrimination index indicated that students who got higher scores overall got the correct answer more that did students who got lower scores. On the other hand, a negative discrimination index indicated that there were more students in the bottom half who got correct answers than did students who got higher overall scores.

This section reports on the possible impact of GeoGebra intervention in learners' understanding of probability problems. The pre-test and post-test covered the same probability content although different contexts were used in the questions. Pre-test items and post-test items which assessed the same concepts were paired together under the same assessment objective for purposes of analysis. The assessment objectives and the test items were explained in section 4.2. The specific assessment objectives are summarised below.

- *Assessment Objective 1*: *Learners demonstrate knowledge and understanding of probability concepts.* Item (1) in the pre-test and item (5) in the post-test assessed learners' understanding of probability concepts. The change between the difficulty indices of these items was measured.
- *Assessment Objective 2*: *Learners demonstrate the ability to interpret probability models to compare the relative frequency of events with the theoretical probability*. Item (2) in the pre-test was paired against item (1) and item (2) in the post-test. These questions covered experimental probability involving probability of single events, probability of combined events and probability of combined inclusive events using a Venn diagram or the rule P(A or B) = P(A) + P(B) – P(A and B).
- *Assessment Objective 3*: *Learners demonstrate the ability to analyse a problem and select a suitable strategy to solve it.* Item (3) in the pre-test and item (3) in the post-test were problem solving questions which involved representation using a Venn diagram. These items were therefore compared to each other. Also, item (4) in the pre-test was compared with item (4) in the post-test. These questions both required students to interpret a Venn diagram in order to determine the probability of combined inclusive events.

The levels of difficulty of items in the pre-test were compared to the levels of difficulty of similar items in the post-test. Any observed changes were further tested for significance using paired samples t-test.

The impact of GeoGebra intervention on difficulty levels : treatment group

Table 4.18 shows the difficulty index and the discrimination coefficient of pre-test items and post-test items for the treatment group (see also Appendix E.3).

Table 4. 18 Pre-test and Post-test item difficulty index and discrimination coefficient for the Treatment group

GeoGebra effect on difficulty level (treatment group)				
Question/ Item	Difficulty index (Pre-Test)	Discrimination co-efficient (Pre-Test)	Difficulty index (Post-Test)	Discrimination co-efficient (Post-Test)
1	0.07	0.14	0.50	0.14
2	0.21	0.21	0.25	0.36
3	0.21	0.43	0.64	0.43
4	0.14	0.29	0.29	0.43

The results (Table 4.18) show that for all items, difficulty indices increased indicating that the participants found the post-test questions easier than the pre-test questions. However, the questions in the post-test still proved to be difficult as shown by the observed indices that were calculated.

An independent samples T-Test (Appendix E11) indicated that there was a statistically significant reduction in the level of difficulty of questions in the post-test when compared to that of the pre-test ($t(3) = 2.644$, $p = .039$). Cohen's d value of magnitude 1.322 indicates that the effect size was very large. The mean difficulty index for the post-test items was 0.420 compared to the mean difficulty index for the pre-test items which was 0.158. It can be concluded at 95% level of confidence that GeoGebra intervention had a positive effect in reducing the levels of difficulty of the items. The practical implication of this result is that students found questions in the post-test less difficult than matching questions in the pre-test.

The impact of GeoGebra intervention on difficulty levels : control group

Table 4. 19 Pre-test and Post-test item difficulty index and discrimination coefficient for the control group

GeoGebra effect on difficulty level (control group)				
Question	Difficulty index (Pre-Test)	Discrimination co-efficient (Pre-Test)	Difficulty index (Post-Test)	Discrimination co-efficient (Post-Test)
1	0.00	0.00	0.27	0.18
2	0.18	0.18	0.025	0.00
3	0.73	0.73	0.68	0.55
4	0.81	0.45	0.36	0.36

Table 4.18 shows the difficulty index and the discrimination coefficient of pre-test items and

post-test items for the control group. The control group answered the same questions as the treatment group and, therefore, the grouping of the items was the same for both groups.

Table 4.19 shows that item (1) and item (2) were difficult for the control group in both the pre-test and the post-test. The difficulty indices for item (3) show that the question was easy in the pre-test (difficulty index = .73), but moderate in the post-test (difficulty index = .68). Item (4), however, was easy in the pre-test (difficulty index = .81), but difficult in the post-test (difficulty index = .36) for the control group. Overall, the difficulty indices for the control group indicate that participants in this group found the post-test more difficult than the pre-test. This was not the case with the treatment group who found the post-test less difficult than the pre-test.

It is not known what the difficulty indices would have looked like for the control group if the GeoGebra intervention was also given to them. These results, however, suggest that to a great extent, learning probability through GeoGebra technology has the potential to increase knowledge gains in learners.

A paired Samples T-Test (Appendix E12) indicated that there was no statistically significant difference in the difficulty indices of the pre-test questions and the post-test questions for the control group ($t(3) = 0.648$, $p = .718$).. This was further evidence that the post-test was more difficult for this group. Cohen's d value of 0.324 indicates that the effect size was small.

4.13.3 GeoGebra impact on prevalence of errors

The prevalence of errors in learners' work in the pre-test and post-test is shown in the tables 4.20 and 4.21 below.

Table 4. 20 Frequency of errors made by learners in the pre-test

	Pre-Test			
	Factual Errors	Procedural Errors	Conceptual Errors	Total
Treatment	13	27	18	58
Control	22	42	43	107
Total	35	69	61	165

Table 4. 21 Frequency of errors made by learners in the post-test

	Post-Test			
	Factual Errors	Procedural Errors	Conceptual Errors	Total
Treatment	0	42	13	55
Control	5	53	29	87
Total	5	95	42	142

The errors that learners made in the pre-test and post-test were coded as factual (F), procedural (P) and conceptual (C). In the pre-test, a total of 58 errors from the treatment group (n = 14) and 107 errors from the control group (n = 22) were identified. In the post-test, 55 errors were made by the treatment group and 87 by the control group. Table 4.20 shows the frequency of errors made by each group in the pre-test and Table 4.21 shows the frequency of errors made by each group in the post-test.

After the pre-test was administered to both the treatment group and the control group, the frequency of errors made by students was determined. The treatment group then received GeoGebra intervention before the post-test was given. The control group did not receive GeoGebra intervention but was taught the same content again using standard instructional practices. A two-way ANOVA (Appendix E 13) was used to measure the interaction between Group (treatment or control group), Test-Type (pre-test or post-test) and intervention (GeoGebra intervention or standard instructional practice). The purpose of this test was to determine whether these variables had an effect on each other and on the prevalence of errors.

It was found that there were no significant main effects between Group ($F(1,\ 8) = 1.779$, $p = .219$, $\omega^2 = 0.071$) and Test-Type ($F(1,\ 8) = 0.143$, $p = .715$, $\omega^2 = 0.000$). This suggests that the errors students made were not influenced by whether a student was in the treatment or control group. The fact that a group of learners received some GeoGebra intervention or not did not influence the prevalence of errors. These results were not surprising. According to Brodie (2014), errors in Mathematics are misconceptions that learners apply. These misconceptions are not easy to remove because they are considered as truth by the learners. Despite any form of intervention, including GeoGebra intervention, learners will always make such errors in Mathematics. This explains why the occurrence of errors in the treatment group was not significantly different from the occurrence of errors in the control group. The objective of teaching should, therefore, never be about completely removing errors in students' work. Instead, errors that students make should be used to plan for further teaching and to understand the needs that learners have.

4.14 Summary: Research sub-question (1)

The effectiveness of GeoGebra intervention in the teaching and learning of probability was analysed under three factors: participants' success rate, difficulty levels of items in the pre-test and post-test, and error incidence in written tests. The results indicated the following findings:

GeoGebra effect on success rate:

- The treatment group (n = 14) had a marginally higher success rate in both the pre-test and the post-test than the control group (n = 22).
- The treatment group (n = 14) had a marginally higher success rate in the post-test than they had in the pre-test.
- The control group (n = 22) had a marginally lower success rate in the post-test than they had in the pre-test.
- The GeoGebra intervention did not disadvantage the treatment group. If anything, it could be the reason for the marginal increases in the success rates that were observed.

GeoGebra effect on difficulty index:

- The observed difficulty indices for the control group indicated that participants in this group found the post-test more difficult than the pre-test. Students in the control group found questions in the post-test more difficult than questions in the pre-test.
- The observed difficulty indices for the treatment group indicated that GeoGebra intervention had a positive and statistically significant effect on reducing the levels of difficulty of the items for the treatment group. Students in the treatment group found questions in the post-test less difficult than questions in the pre-test.

GeoGebra effect on reducing errors in tests:

- The difference in the prevalence of errors made by the treatment group was not statistically significant compared to the prevalence of errors made by the control group.

4.15 Summary

In this chapter, quantitative data were interrogated to try and answer the main research question and research sub-question (1). The study sought to investigate how GeoGebra can be used to support the learning of various probability concepts. It also sought to determine how GeoGebra can be used to address Grade 10 learners' errors and misconceptions in probability problem solving. The effectiveness of GeoGebra intervention was analysed under three variable factors, viz: participants' success rate, difficulty levels of items, and error incidence in written tests.

Among these dependent variables, GeoGebra intervention was found to be significantly effective in reducing difficulty levels of test items. This implies that when learners in the treatment group were answering post-test questions, they made better sense of the given situation more than they did in the pre-test. It also means that learner levels of confidence in the post-test were higher than in the pre-test since the items were easier in the post-test than in the pre-test.

The post-test mean score for the treatment group was 3 percent higher than the average score in the pre-test. On the other hand, the average score in the post-test for the control group was 1 percent less than in the pre-test. Both these differences were, however, not statistically significant ($p = .716$ for the treatment group; $p = .761$ for the control group). It can be argued, based on these non-significant differences, that the different forms of interventions that were given to both groups had the same impact on the performance of the two groups since none yielded significant gains in scores.

GeoGebra intervention did not significantly help in reducing the number of errors learners in the treatment group made. Neither did the intervention received by the control group yield significant reduction in the number of errors made. However, the fact that there was a positive gain in the post-test mean score for the treatment group is important because it shows that GeoGebra intervention was not counterproductive. Future research (with a larger group of participants and an increased number of question items) is recommended to determine the impact of GeoGebra on learners' performance scores and error reduction.

Bearing in mind that this research was more qualitative than quantitative, it is important to point out that qualitative data analysis identified more variables which were significantly influenced by GeoGebra intervention. The next chapter presents qualitative data analysis.

CHAPTER FIVE

QUALITATIVE DATA ANALYSIS

5.0 Introduction

In this chapter, qualitative data are analysed to determine ways in which GeoGebra can be used to overcome specific challenges that Grade 10 learners have when solving probability problems. The analysis further aims to explicate how the use of GeoGebra supported the learning of various probability concepts and addressed learners' errors and misconceptions in probability problem solving. Data analysis was undertaken using coding and the five stages of the Framework approach which are familiarisation, thematic framework, indexing and sorting, data summary and display, and mapping and interpretation (Hackett & Strickland, 2018). Framework approach was discussed in section 3.7.1. The interpretive paradigm which follows a mixed methods methodology within a constructivist perspective was adopted. Themes that emerged from the analysis were interpreted to understand the impact of GeoGebra on learning outcomes. The analysis revealed the following themes:

1) Motivation and language/communicative registers: Learners' conceptualisation of chance as a motivating factor for learning probability
2) Learning opportunities: Opportunities for the development of mathematical competencies
3) Problem-solving strategies: GeoGebra impact on learners' strategies for solving probability problems
4) Active learning: GeoGebra intervention and learner engagement, interaction and participation.

The names used to identify the learners in this study are not their real names in order to protect their confidentiality. Learners with suffix T are learners from the Treatment group, while learners with a suffix C are from the control group. In the interviews, full pseudonyms for the students who participated are used. These are Nikita, Kholwani, Mavusana, Miles, Sanders, and Shareen.

5.1 Theme 1: Motivation and Language/Communicative registers

One of the themes that came up from coding analysis was motivation. In the context of this analysis, motivation will be defined as the reason for learners to learn probability. Since learners' success in solving probability problems was affected to a large extent by their understanding of probability terms and concepts, it was necessary to categorise this theme as language/communicative registers. I will address this theme by discussing one particular factor that militates against learning probability, viz: the misunderstood word as a barrier to understanding probability. This factor was identified from learners' responses in the interviews.

5.1.1 Misunderstood concepts as barriers to understanding probability

Learners were asked whether they agreed with the claim that if a coin is tossed 8000 times and it lands on the heads 8000 times, then that coin is biased. Some of the responses are shown in vignette 1.

Vignette 1 Learners' conceptualisation of bias.

Learner T7: No. Coins always have heads and tails. It was just luck.

Learner T9: No. The coin isn't biased as it has half chance on landing on either heads or tails and by chance it happened to always land on heads.

Researcher*:* The question says: If a coin is flipped 8000 times and it lands on heads 8000 times then that coin must be biased. Do you agree with this statement or not? And you left it unanswered. Why?

Nikita*:* Because I don't understand why a coin can be biased. It's a coin, how can it be biased?

Researcher: Oh, so what you don't understand is that the coin is biased?

Nikita: Yeah, that bias part confused me.

Researcher*:* What's your understanding of bias?

Nikita*:* I understand that it favours one thing over another. Sir, but how can a coin be biased?

The notion of a biased coin was confusing to some learners, judging from the responses they gave (see Vignette 1). Although learner T7 and learner T9 responded correctly to the posed question, it was, nonetheless, clear that they dismissed the notion of a biased coin completely. They attributed the observed outcomes only to chance or luck and ruled out the possibility of bias. Their responses were correct and made sense, but the fact that they did not give bias any consideration suggests that the respondents did not understand how bias applied to such experiments. Thus, in their view, it is not possible that a coin can be biased. In the interview with learner Nikita, it was clear that the concept of bias was confusing. Learner Nikita understood bias as a subjective decision that only a human can take. In other words, according to Nikita's reasoning, it only makes sense when a person has a biased view about something, but it is confusing to imagine a biased coin. This was clear in her question, "Sir, but how can a coin be biased?" The word bias confused Nikita to the extent that she left the question unanswered. When she was asked why she did not attempt the question, part of Nikita's response was, "… that bias part confused me."

Nikita knew the meaning of the term bias, but not in the context of probability. When the term was used in the context of probability, it was misunderstood. According to Sondergaard (2020, p. 80), "not understanding the words one encounters when studying causes the inability to think and thus creates confusion." Sondergaard (2020) further notes that a misunderstood word is a word that is not understood or is understood wrongly. This leads to a variety of negative manifestations on the part of the learner. For example, the learner may feel blank, nervous, tired and not there. The learner may also take long to comprehend what is being studied. Thus, the motivation to learn can be negatively affected when learners do not understand the vocabulary. This confirms the need for probability concepts to be thoroughly discussed in lessons in order to avoid such confusion.

Students also admitted that language factors made it difficult to learn probability. This is so because some terms that are used in probability are technical and difficult to understand by students. This view was reflected in Shareen's response when she said, "you have to know English basically. It's the issue of language and understanding" (Vignette 2, [13] Shareen). Kholwani pointed out that the language used in probability was difficult to understand when he said, " It isn't easy to handle because even though probability is always used, nobody uses the language in everyday life. In everyday use, even though probability is used every day, it [language] is a bit difficult" ([14] Kholwani, Vignette 2]. Learner Shareen and learner Kholwani spoke for many students who find difficulty in understanding mathematical language, particularly the probability language. Mathematics teachers need to be aware of this factor so that they can try and eliminate the language difficulty factor in their teaching and assessment. They can do so by adopting alternative strategies that are appropriate for learners in terms of language levels.

5.1.2 Learners' conceptualisation of chance as a motivating factor in learning probability

Students were also asked what they understood by the term probability. They expressed themselves in different, but related ways. Some of the responses are given in Vignette 2. The aim of this question was to establish whether students had a conceptual understanding of the concept of probability. The question was also intended to determine whether respondents understood the application of probability in real life contexts. The responses students gave indicated that students understood the importance of studying probability. This study argues that when learners understand why probability should be studied, they become motivated to learn it. One of the aims of learning Mathematics according to the CAPS (DBE, 2011b) document states that: "mathematical modeling is an important focal point of the curriculum.

Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible"(p.8).

Vignette 2 Learners' understanding of chance and randomness and its application in real life contexts

[1] Miles: Okay probability … yeah, I think it's like … I'll just say fluctuating. … Like fluctuating a coin.

[2] Kholwani: Probability has a lot to do with the number of chances something has compared to something else. … like when my parents are coming home, who has a higher chance of coming home first between my dad and my mom.

[3]Sanders: Probability … I would say is all about statistics and numbers. Like how many times you would get a head or tail when a coin is tossed.

[4] Shareen: I guess probability is about , you know, chances or possibilities of something happening.

[5] Mavusana: Personally, I would say probability is basically about chance. The chance of certain things happening and the chance of them not happening.

[6] Nikita: It's like the likelihood of something happening. Like what are the chances?

[7] Mavusana: I think probability is applied in literally every single aspect of our lives and in every decision we make, and in everything we do.

[8] Shareen: But we go through it …like maybe the chances of you getting to a mall and choosing a restaurant. There is Spur, there is McDonalds and so on. You could go into Spur or into McDonalds. That's also probability because you could choose one out of the many shops in the mall that you went to.

[9] Sanders: You can use it in population in a certain country or in the whole world. You can use it like to determine how many people are in a country for a particular period of time.

[10] Miles: Okay, like decreasing numbers in birth rates, … mortality … we just have to control that. We have to look at like infrastructure. And we have to measure our population.

[11] Researcher: What do you say about probability? Do you find it easy?

[12] Nikita: It really depends on the question. There are some questions that are easy but others are difficult. But I think like probability is easy to apply … in real life. It's easy to apply in real life but then in Maths questions it's too much.

[13] Shareen: So what you have to figure out is, ... you have to know English basically. It's the issue of language and understanding.

[14] Kholwani: That is actually one of the things that I don't like about it, the language. It isn't easy to handle because even though probability is always used, but nobody uses the language in everyday life. People who are experts in it use it and it's really used in like mathematics and in technical areas like in technology and engineering. But aside from that, in everyday use, even though probability is used every day, it is a bit difficult … it's not user friendly.

It was evident in this study that students had a fair idea of what probability is. The analysis also indicated that participants created their own meanings of what they understood probability to be based on the GeoGebra stimulus that was given. This result confirms the theory of semiotic mediation (Vygotsky, 1978) which postulates that the use of an artefact to mediate learning potentially leads to signs sprouting. With the guidance of the teacher, these signs can evolve into mathematical meaning. Thus, some participants defined probability in terms of what they experienced or observed during the lessons in class. An example of this is seen in Miles and

Kholwani's responses.

Miles used the term "fluctuation" to describe probability. During the GeoGebra simulation of rolling dice and tossing coins, the outcomes were visualised as animated bar graphs which kept moving up and down as the outcomes took various values. Miles understood this phenomenon as fluctuation and used this terminology to define probability as seen in his response: "Okay probability … yeah, I think it's like … I'll just say fluctuating … Like fluctuating a coin" ([1] Miles, Vignette 2]. Thus, GeoGebra simulations produced visualisations that helped Miles to understand the meaning of probability.

Kholwani (see [2]Kholwani, Vignette2) understood probability as a comparison between two different possible outcomes. This conception was also influenced by what students observed during GeoGebra experiments. For example, in the simulation of rolling a die and tossing a coin many times, different outcomes such as 1, 2, 3, 4, 5, 6, Head or Tail were obtained. This appealed to some respondents as an experiment to compare outcomes, thereby influencing their definition of probability. Learner Kholwani, for example, stated that "probability has a lot to do with the number of chances something has compared to something else" ([2]Kholwani, Vignette2). Kholwani supported his idea of probability as a measure of comparison using an analogy of parents arriving home from work when he stated, "like when my parents are coming home, who has a higher chance of coming home first between my dad and my mom" ([2]Kholwani, Vignette 2). The same reasoning was evident in learner Sander's definition of probability. Sanders understood probability in terms of comparisons and stated that it has to do with comparing the number of heads obtained against the number of tails when a coin is tossed ([3] Sanders, Vignette 2]. This suggests that GeoGebra simulations of tossing a coin influenced students' understanding of the concept of probability. This is supported by Vygotsky's (1978) notion of signs which was discussed in Chapter Two. According to Vygotsky (1978), signs are constructions which develop when a learner uses a tool to accomplish a task. These signs can develop into mathematical meanings.

Probability was also understood as a measure of chance by some respondents. This was reflected in the responses by [4] Shareen, [5] Mavusana and [6] Nikita (Vignette 2). Their responses also indicate that students' understanding of probability is influenced by their experience of learning probability. As learners engage in probability related class activities, their understanding of probability is enhanced and they can end up developing their own personal definitions of what probability is.

The analysis of learners' responses also revealed that students were able to fit probability into real life contexts. They were asked for their opinion on how probability was applicable in real life. All the respondents believed that probability was applicable in decision making in everyday life contexts. For example, Mavusana said, "I think probability is applied in literally every single aspect of our lives and in every decision we make, and in everything we do." ([7] Mavusana ,Vignette 2). Other students gave specific areas in real life where probability is used. Shareen stated that probability can be applied in social settings to make social and individual decisions ([8] Shareen). Other responses indicated that students knew that probability can be applied in business decision settings, social decision settings, health settings and environmental settings to mention a few. These results are encouraging and prove that the learning of probability can easily be related to reality in everyday life settings. These results are supported by the aims of CAPS (DBE, 2011b), the theory of the Real Mathematics Education (RME) (van den Heuvel-Panhuizen & Drijvers, 2014) and the principles of the National Council of Teachers of Mathematics (NCTM, 2000) which encourage the teaching of Mathematics from rich and realistic situations. Students who know the importance of acquiring probability knowledge are likely to be motivated to learn it.

Probability, as has already been stated in the literature review, is perceived as a difficult topic to teach and to learn. Teaching it from real life contexts will help simplify it for students. The challenging nature of probability was admitted by some students in the study. Nikita's response ([12] Nikita, Vignette 2) revealed that probability becomes difficult to learn when teachers teach it from isolation. Nikita's answer suggests that teaching probability in context will simplify it for students. Nikita alluded to this when she stated that probability is challenging in Mathematics, but simple to apply in real life. She was referring to her observation that in Mathematics activities, particularly in tests and examinations, probability is more challenging than the probability that is experienced in real life situations. She differentiated between the way probability is handled in class and the way it is experienced in real life. The reason that probability questions in Mathematics assessments are perceived as difficult could be the abstract manner in which probability problems are presented.

Another theme that emerged from coding analysis involved opportunities for instruction in developing mathematical competencies. This theme is discussed in the next section.

5.2 Theme 2: Learning opportunities for developing mathematical competencies

According to Niss & Højgaard (2019, p.12) mathematical competence is "someone's insightful readiness to act appropriately in response to all kinds of mathematical challenges pertaining to given situations." For students to successfully deal with specific challenges in Mathematics, Niss & Højgaard (2019) identify eight competencies that they should develop. These competencies are mathematical thinking, problem solving, modelling, aids and tools, symbols and formalism, reasoning, representation and communication. This study sought to identify ways in which GeoGebra could be used to address learners' errors and misconceptions in probability problem solving. The study also sought to determine how GeoGebra could be used to address specific challenges learners encounter in probability problem solving. An analysis of students' interview responses revealed some opportunities that GeoGebra use created for the development of some of these competencies. Factors that favorably affect the acquisition of probability knowledge were also evident. Learners' views on the educational benefits of GeoGebra intervention were analysed to identify factors that stimulate the development of competencies for learning probability. Learners' responses indicated that the development of competencies was stimulated by time allocation, artefact usefulness and learning environment.

5.2.1 Time allocation: Opportunities for resolving errors and misconceptions

Interview responses by learners indicated that the use of GeoGebra created opportunities for learners to attend to their errors and misconceptions in several specific ways. Vignette 3 shows a summary of interview responses which showed that the use of GeoGebra created adequate time for learners to pay attention to their errors and misconceptions.

Vignette 3 Learner responses showing ways in which the use of GeoGebra created time for learners to focus on their errors and misconceptions.

[1]Shareen: I think I don't know any other way. Maybe I don't know … even our calculator would take so long. I feel like it was very fast … like we were actually able to calculate a die maybe after 10 times of flipping and all that. You could have never done that with a calculator. You actually even need a real die to even experiment what would happen and what would go through.

[2]Shareen: So GeoGebra actually gave us you know that chance … it's actually fast and, you know, you don't have to waste time thinking that okay if I roll the die, but they said there is one die, … you know. It just gives you the things you need to know immediately and now it's up to you and your understanding.

[3]Kholwani: Overall, the lessons were not that long. It didn't take too much time. So it's not time consuming. … We were able to do like … we were able to go to numbers like 800 flips, 30 000 rolls, in order to get the answer and that did not take time at all. It just took you less than five seconds. So it was not time consuming

[4]Mavusana: It definitely gave us a lot of time to work with the numbers. It gave us time to work with unrealistically high numbers. It's a tool that makes us do things that would otherwise be very impossible such as being able to roll a die 30 million times

Time pressure and time constraints are widely given as reasons why teachers fail to meet all their teaching goals. Leong & Chick (2011) note that as a result of lack of adequate teaching time, teachers face challenges in their effort to employ innovative instructional practices. Leong & Chick (2011) also state that teachers' experience of time pressure is a "significant obstacle in the carrying out of novel instructional approaches in the classrooms" (p. 4). They argue that because of the many competing goals of teaching, time-related tensions that evolve from there can bring about complex challenges for the teacher and compromise their instructional choices. Teachers do not only teach, but have other areas to worry about, such as a syllabus that they must complete before examinations, and students that they should assist to learn mathematical reasoning. All these goals require adequate time, but time is not always enough. The failure by Mathematics teachers to use novel instructional practices in their classrooms leads to learners losing out on opportunities to maximise their understanding of the mathematical concepts that ought to be learnt. For example, teachers who perceive time pressure are not likely to probe students' reasoning to understand why students make the errors that they make.

The results of this study showed that the use of GeoGebra created more time for students to learn the actual concepts that are involved in probability. Students were able to use raw data that was generated through the use of GeoGebra to solve problems in probability contexts. The large amount of data that they used in classroom activities was generated quickly. This left enough time for students to interact with the data that was produced, and to learn probability concepts. Learners used the generated data in various ways. First, they used data to construct probability outcomes tables. Second, raw data was used to learn about the relationship between relative frequency and theoretical probability. Third, learners used raw data to draw Venn diagrams based on the outcomes. Learner Shareen attested to the opportunity of GeoGebra to afford learners more learning time when she said, "I feel it was very fast." She also expressed how students' learning experiences benefitted from the efficiency of GeoGebra in achieving the desired learning goals when she said, "GeoGebra actually gave us … that chance … it's actually fast and, …, you don't have to waste time thinking that okay …. It just gives you the things you need to know immediately and now it's up to you and your understanding." The same view was echoed by learner Kholwani and learner Mavusana (Vignette 3, [3] and [4]) who concurred that within a short period of time, the use of GeoGebra was able to produce data from a large number of coin flips. Learner Mavusana referred to these numbers as "unrealistically high numbers" because the numbers were obtained from large amounts of coin flips and dice rolls. "We were able to go to numbers like 800 flips, 30 000 rolls in order to get the answer," said Kholwani. Mavusana admitted that "It's a tool that makes us do things that would otherwise be very impossible, such as being able to roll a die 30 million times." Mavusana understood the importance of analysing data that are generated in probability experiments. He suggested in his response the importance of spending more time in analysing data than in collecting data. The importance of analysing data is supported by Theis & Larose (2013) and Podworny (2016). According to Theis & Larose (2013), data analysis deepens mathematical understanding. Furthermore, data analysis deepens reasoning (Podworny, 2016). These findings are supported by literature on educational practices and learning outcomes. For example, Batanero et al., (2016) argue that information technology allows data collection, data storage, data representation and data organisation to be performed fast and accurately.

5.2.2 Usefulness: Learners' views on the usefulness of GeoGebra in learning probability

Interview responses and observation data revealed that the use of GeoGebra in probability lessons had a positive effect on learners' conceptual development, understanding and knowledge acquisition. Vignette 4 shows learner responses on the impact of GeoGebra-assisted instruction on their probability knowledge acquisition and conceptual understanding.

There was evidence that the use of GeoGebra created opportunities for learners to develop deeper conceptual understanding of probability concepts. Learner Shareen observed that the views that were shared during class discussions were stimulated by the integration of GeoGebra in the lessons. When she was asked to comment on the usefulness of GeoGebra in learning probability, Shareen stated that it "gives people the chance to voice out their own opinions" (see Shareen[7], Vignette 4). During whole-class discussions, it was observed that learners were free to share their thoughts. The discussions that ensued were easily triggered by the GeoGebra activities that learners participated in. Thus, the use of GeoGebra potentially enhanced learners' understanding. In Shareen's own words, learners did not experience difficulties during the lessons because of the support that GeoGebra offered. Learner Kholwani also mentioned that the lessons were eye opening and that they provided light-bulb moments for him when he said, "For me the lessons opened me up … they showed me that no matter how many times you flip a coin, there is a chance that both sides of the coin have an equal chance… It definitely lit a light bulb because everyone was using their mind and thinking to understand" ([5] Kholwani, Vignette 4). This indicates that the use of GeoGebra made a cognitive impact on learners' understanding of some probability concepts. Some respondents evaluated the usefulness of GeoGebra by comparing learning outcomes before and after GeoGebra intervention. Mavusana [2], Sanders [3] and Miles [4] (see Vignette 4) indicated that the concept of relative frequency was initially confusing. However, following the GeoGebra activities, the respondents reported a positive gain in their levels of understanding. Mavusana explained that as a result of GeoGebra, his understanding of relative frequency improved. He pointed out that "due to GeoGebra, you could calculate these big numbers and [could] see how the relative frequency relates with small numbers and the large numbers." Mavusana's response suggested that the use of GeoGebra produced large data which was used to make sense of the concept of relative frequency. Miles attested to the fact that GeoGebra simplified learning when he stated, "at first, I thought it was going to be difficult … but once I was in it, I discovered that okay, I have to work out like this." (Miles [4], Vignette 4). The fact that learners were able to figure out how things were supposed to be done indicates that GeoGebra use enhanced learners' cognitive processes and led to better understanding of probability and its related concepts.

Vignette 4 Learner responses on the impact of GeoGebra on their probability knowledge acquisition, conceptual development and understanding.

[1]Shareen: Even the way people [learners] did not ask questions … like the number of questions that the learners ask actually show that there is maybe a bit of confusion. But the way people[learners] were answering, even giving their own examples, shows that the people [learners] were actually understanding the lesson, The way they could relate maybe to the world and to people… it showed that the lesson was in the right path and they could understand.

[2]**Mavusana**: I think, basically being able to see the relative frequency. Like being able to calculate it. Because due to GeoGebra, you could calculate these big numbers and can [could] see how the relative frequency relates with small numbers and the large numbers ..

[3]Sanders: I think it was …. I don't remember the name, but the thing that we did after tossing the coin. … Yes, relative frequency. I didn't understand it initially but you made it clear for me to understand it.

[4]Miles: At first I thought it was going to be difficult because sometimes like probability … I thought that it's not used in reality, but since you have explained it now I understand…

[5]Kholwani: For me the lessons opened me up … like they showed me that, for example with coins, no matter how many times you flip a coin, there is a chance that both sides of the coin have an equal chance. … Everyone had their own opinions. It definitely lit a light bulb because everyone was using their mind and thinking to understand. And it was fun and we were learning. I learnt something definitely. And other people's opinions were considered.

[6]Miles: I thought it was going to be difficult because sometimes like probability …. I thought it's not used in reality, but since you have explained it now I do understand why we should use probability in our lives. … I thought it was going to be difficult but once I was in it I discovered that okay, I have to work out like this.

[7]Shareen: It actually gives people the chance to voice out their own opinions. As you saw, they used examples from the real world and crime … it was his way of understanding that concept. So the way he might have explained it might have actually helped someone else to understand it better than they already did. So it actually helped us to interact and share some ideas and help each other in the process to understand.

[8]Mavusana: It helped us in a sense that we all had our theories. We all thought certain things and the use of GeoGebra easily showed us like in real time the unrealistic possibilities that we could not possibly do in the classroom setting … being able to roll many times and just proved everyone's theory correct or wrong.

Learning probability through GeoGebra also impacted on learners' expectations in a positive way. Appleton-Knapp & Krentler (2006) argue that student satisfaction about an educational experience can be understood by considering student expectations. Students enter any learning programme with expectations of their own, and at the end of the programme, may recall what their expectations were when they started and use that information to express their level of satisfaction with the learning programme. Some of these expectations revolve around students' expectations of what the course will cover, whether the course will be difficult and whether the course will be interesting. Learner expectations are, therefore, part of their learning and can be understood when attention is paid to how learners experienced learning. Learner Miles, for example, stated that he expected the probability lessons to be difficult, but when he was working hands-on with the GeoGebra tool, everything became clear and he understood how he could go about it. It was also observed in this study that learners came to class with their own conceptions and misconceptions about probability issues. Learner Mavusana confirmed this notion when he stated that the use of GeoGebra helped learners to validate or refute their preconceived theories about probability content. He stated, "It helped us in a sense that we all had our theories. We all thought certain things and the use of GeoGebra easily showed us like in real time the unrealistic possibilities that we could not possibly do in the classroom setting … being able to roll many times … and just proved everyone's theory correct or wrong."

5.2.3 Learning environment: Creation of an inclusive learning environment

Participants in this study were learners of mixed learning abilities. Their different learning needs were considered when selecting pedagogical practices and learning activities. Whereas some learners could understand concepts from simple teacher explanation and exemplification, others found it difficult. Thus, a balance needed to be struck to accommodate the needs of every learner. Respondents were asked to comment on how GeoGebra activities addressed their needs. Vignette 5 shows some learners' responses on the effectiveness of GeoGebra in creating inclusive classrooms.

Shareen's response showed that GeoGebra lessons were effective in addressing individual learners' needs. Shareen said that nothing was difficult to understand, adding that all learners could carry out given instructions because they were clear and easy to follow. Thus, according to learner Shareen, GeoGebra activities created a learning environment where all learners were able to accomplish assigned tasks. "Whether you are a faster learner or a slow learner, they [GeoGebra activities] accommodate both of you", said Shareen.

Vignette 5 Learner responses on the role of GeoGebra in creating an inclusive classroom.

[1]Shareen: I didn't see anything difficult from my own point of view. I think the lessons were straight forward. Whether you are a fast learner or a slow learner they could accommodate both of you…. It was a straightforward lesson.
[2]Kholwani: The numbers were there … in the app. We just clicked and we got the numbers. … It did not take time at all. It just took you less than five seconds.

Learner Kholwani also commented on how GeoGebra activities simplified work for everyone. He stated that data was generated within the shortest possible time by just clicking on the right place. Learner Shareen summed up this idea as follows: "It just gives you the things you need to know immediately and now it's up to you and your understanding." (Shareen [2], Vignette 3). The use of GeoGebra, thus, simplified learners' work and gave the learners an opportunity to interact with the data at their own levels of understanding. Starcic (2010) argues that "educational technology and information communication technology play an important role in creating an effective and adaptable learning environment, especially when teaching pupils with special educational needs and inclusive classrooms" (p. 26). Some of these special educational needs include physical needs, communicational needs, emotional needs and cognitive needs (Starcic, 2010). An effective pedagogical approach is capable of accommodating special educational needs. Respondents in this study admitted that the use of GeoGebra to acquire probability knowledge benefitted them.

Furthermore, learners' responses indicated that the use of GeoGebra to complete activities was fun and interesting. Vignette 6 shows learners' responses on how GeoGebra lessons were interesting and fun.

Vignette 6 Learner responses showing the impact of GeoGebra on their learning experiences.

[1]Sanders: One comment I have is that this thing is quite good. And secondly, it's the best.
[2]Mavusana: I would say for one I really enjoyed the GeoGebra lessons and I feel like it's an app that's necessary.
[3]Shareen: … actually I would like to advise that kids especially from Grade 10 get this app, because … Grade 10 I've noticed personally, is the foundation of all the things that we will learn as we go until we get to Grade 12.
[4]Kholwani: … I really did enjoy the lesson. It was fruitful and I learnt a lot that can be applied. It just opened my mind regarding probability and it just helped me think quickly in a sense.

An analysis of students' responses and reactions revealed that students were satisfied with the GeoGebra-assisted lessons. Participants described the GeoGebra lessons as "quite good", "enjoyable", "fruitful" and "mind opening" (see [1] Sanders, [2] Mavusana and [4] Kholwani, Vignette 6). Appleton-Knapp & Krentler (2006) argue that students' satisfaction with educational courses is influenced by how interesting or not interesting they are perceived to be. Factors that influence student satisfaction can be categorised under personal factors and institutional factors (Appleton-Knapp & Krentler, 2006). According to Appleton-Knapp & Krentler (2006) personal factors are related to the student and include age, gender, temperament and preferred learning style. Institutional factors, on the other hand, are related to educational experiences. These include the teacher's teaching style, quality of teaching, quality of feedback from the teacher, class size, students' interaction with the teacher, students' interaction with each other, students' participation and the ease with which information can be obtained (Appleton-Knapp & Krentler,2006).

There was evidence that the use of GeoGebra simultaneously provided fun for the participants and supported probability knowledge acquisition. The class activities that were planned were adaptable to practical or experimental approaches. Probability experiments using coins, dice, spinners and playing cards provide a learning environment that resembles a gaming situation. Student Mavusana hinted on the value of game situations in developing probability concepts. When he was asked to share about an experience in class or outside class which helped him gain an understanding of some aspect of probability, Mavusana said, "I would say one situation was through rolling a dice. For example, just playing games or anything in class with friends … Probability plays a big role." Mavusana was able to associate playing games with chance. The dynamic animation features of GeoGebra created fascinating visualisations which enhanced learners' cognitive engagement. Students began to understand why their initial thinking patterns were flawed after they watched the simulations using GeoGebra. According to Ramani, Rowe, Eason & Leech (2015), practical number-related activities that children participate in at home or at school have a potential to develop students' numerical proficiency. The meanings that are drawn from practical probability activities are deep because learners get involved in those activities. Studies show that play and everyday informal activities, such as household chores and shopping, support the development of skills and mathematical understanding. Ramani et al.(2014) investigated children's engagement in practical activities at home and the impact they had on their mathematical outcomes. They found that children's engagement in these activities influenced foundational numerical knowledge development. These results support the findings of the current study which indicate that the use of GeoGebra provides opportunities for enhanced learning. However, limited teaching time might pose a possible threat to the effective use of practical approaches. Modelling probability situations using GeoGebra simulations uses up a bit of time. According to the Grade 10 annual teaching plans (ATPs), only two teaching weeks are allocated for the teaching and learning of the probability topic. I argue in this study that the teaching time allocated to probability may not be adequate to accommodate innovative and practical approaches. There is need for probability teaching time to be reviewed to accommodate such approaches.

5.3 Theme 3: GeoGebra impact on problem-solving strategies

Problem solving questions in the pre-test and post-test required learners to figure out effective strategies to solve the problems. Some of these questions could be answered using various strategies such as drawing appropriate Venn diagrams and using probability rules. The importance of representing mathematical information in various forms is well documented (e.g. Niemi, 1996; Suh & Moyer, 2007). Suh & Moyer (2007) state that representational fluency is the ability by learners to create and interpret representations such as pictures, graphs and symbols to solve problems. Lesh's (2007) translation model (Suh & Moyer, 2007) advances that mathematical ideas must be presented in different modes in order for students to grasp the concepts. This section presents a qualitative analysis of the impact of GeoGebra on learners' choice and adoption of problem-solving strategies.

5.3.1 Students' choice and use of a Venn diagram as a strategy to solve problems

Venn diagrams could be used in the pre-test and post-test as a strategy to solve probability problems. It was found that learners used Venn diagrams as a strategy whenever the preceding part (or parts) of the question elicited for a Venn diagram to be drawn. The results also showed that the majority of learners (but not all the learners) used Venn diagrams whenever they were asked to use them.

In the pre-test, item # 3(b) (see frame below) instructed participants to draw a Venn diagram to illustrate the given information.

Pre-test Item # 3
In a group of 40 learners the following information is true:

- *7 learners are left -handed (L)*
- *18 learners play soccer (S)*
- *4 learners play soccer and are left-handed*
- *All 40 learners are either right-handed or left-handed (not both)*

a) How many learners in the group are right-handed and do NOT play soccer?

b) Draw a Venn diagram to represent the above information.

c) Determine the probability that one learner selected at random is left-handed or plays soccer.

The question after that (item # 3(c)) did not specify the strategy to be used to answer it. Students could use their Venn diagrams or adopt any other correct strategy of their choice to answer the question. The majority of the students (57 percent of the students in the treatment group) preferred to answer item # 3(c) using the Venn diagram from item # 3(b). The remaining 43 percent drew their Venn diagrams, which were either correctly or incorrectly drawn, but preferred to use probability rules to answer the questions. Thus the choice to use a Venn diagram as a strategy to answer item # 3(c) was influenced by the context of the preceding question.

An equivalent question was asked in the post-test (see frame below) describing a different situation.

Post-test Item # 3
During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows"

- *28 learners said they preferred boerewors rolls and koeksisters (B)*
- *48 said they preferred kota with atchar, chips and Russian sausages (K)*
- *x said they preferred both B and K, and*
- *20 said they preferred neither.*

a) Draw a labelled Venn diagram to illustrate this information.
b) Use the Venn diagram to calculate the probability that a Grade 10 learner selected at random from the group prefers B or K.
c) Use the Venn diagram to state the probability that a Grade 10 learner selected at random prefers neither B nor K.

In this question, students were asked to represent the given information in a Venn diagram. The ensuing questions (item # 3 (b) and item # 3 (c)) specified that the Venn diagram in item # 3 (a) should be used. It was observed that the majority of the students (64 percent of the students in the treatment group) used their Venn diagrams to answer the given questions as required.

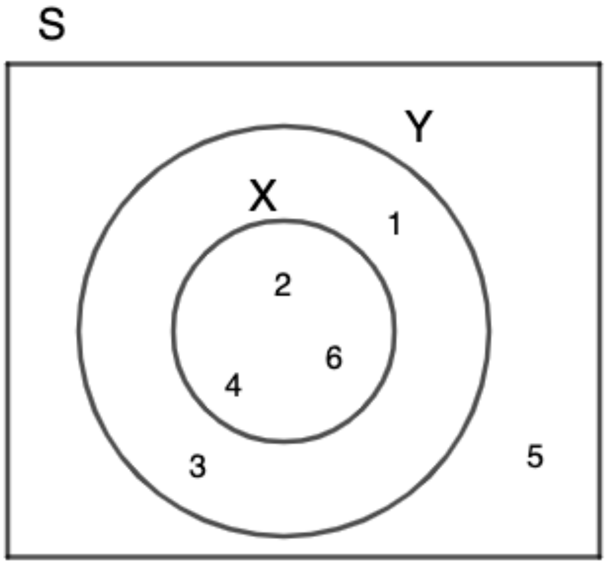
The remaining 36 percent drew the required Venn diagram but did not use it to answer the questions as directed in the question. They preferred, instead, to use the probability rules for P(A or B) and P(A and B). It can be concluded from the foregoing analysis that the choice of a strategy to answer the posed questions was influenced by the structure of the question. For example, the support that learners received through scaffolding assisted them to adopt a strategy that worked to answer that particular question. After drawing a Venn diagram, it is likely that learners will use it to answer ensuing questions.

This study also found that the majority of the respondents did not choose the Venn diagram strategy in a question that did not mention or suggest a Venn diagram. Some questions did not suggest a strategy for learners to use and respondents had to decide on a strategy that worked. For example, in the following question (see frame below), either a Venn diagram or the probability rule, P(X or Y) = P(X) + P(Y) – P(X and Y) could be used successfully.

Post-test item # 2 *A fair six-sided die is rolled.* *Event X is defined as "an even number is obtained"* *Event Y is defined as "a factor of 12 is obtained"* *Determine P(X or Y).*

Using a Venn diagram, after entering the correct outcomes in their set, the learners would only need to read the answers from the diagram (see Method 1 below).

Method 1: *P(X or Y)* $= \frac{n(X \text{ or } Y)}{n(S)} = \frac{5}{6}$ (from Ven diagram).



Another alternative and effective strategy was the use of the probability rule (see method 2 below).

Method 2: P(X or Y) = P(X) + P(Y) – P(X and Y).

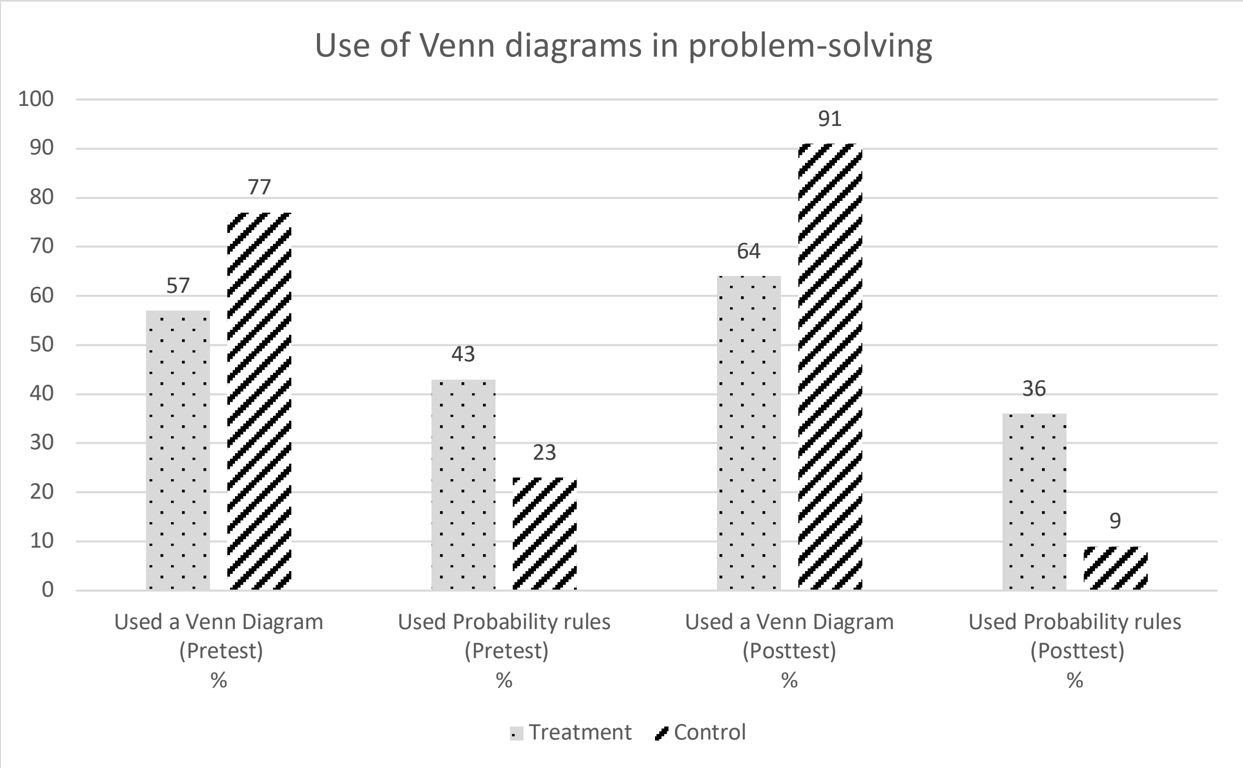
$$= \frac{3}{6} + \frac{5}{6} - \frac{3}{6}$$

$$= \frac{5}{6}$$

As already stated, most students chose to use the probability rule instead of the Ven diagram to answer this question. However, the rule was incorrectly applied in some instances where students tended to treat X and Y as mutually exclusive events. As a result, they ended up with P(X or Y) = P(X) + P(Y) = 133.3% as their answer. Creating a Venn diagram for such a question could have possibly assisted these students to reach the correct answer.

The extent to which learners used Venn diagram strategies was determined. Figure 5.1 gives a summary of the results.

Figure 5. 1 Use of Venn diagrams to solve probability problems



The results indicate that in the pre-test, the majority of the students in both groups (57 percent in the treatment group and 77 percent in the control group) used Venn diagrams even though the question did not instruct them to use a Venn diagram. In the post-test, the results revealed that the majority of the students in both the treatment group (64 percent) and the control group (91 percent) used Venn diagrams to answer the questions. These results suggest that students found Venn diagrams more favourable to use in solving these problems than probability rules. However, as already observed, the use of a Venn diagram to answer these questions was influenced by the fact that a Venn diagram in a preceding question was required. When no Venn diagram was mentioned in the question, the majority of the students decided that the rule was the best option. The percentage number of learners who used Venn diagrams increased in the post-test for both groups (from 57 percent to 64 percent for the treatment group; from 77 percent to 91 percent for the control group).

The use of Venn diagrams to represent information and solve probability problems improved students' conceptual understanding of probability. Some learners (e.g. Nikita) understood the advantage of using Venn diagrams. In the pre-test, Nikita drew Venn diagrams as a strategy to answer some questions even though Venn diagrams were not a requirement. Nikita was asked why she used Venn diagrams and her response is shown in Vignette 7.

Vignette 7 Learners' responses on the use of Venn diagram methods to solve problems.

[1]Researcher: I notice that when you were trying to answer these questions, you tried to come up with a strategy. You decided to draw a Venn diagram for (d) and (e). How did you hope that drawing the Venn diagram was going to help you?

[2]Nikita: I just think a Venn diagram , like … it just makes the information easier to read. … Instead of reading it a couple of times, I draw it out so that I can just see the information and just see exactly where everything falls in the Venn diagram.

[3]Researcher: You said it's mutually exclusive.

[4]Nikita: Yes. I don't think I actually wrote the correct answer.

[5]Researcher: So you are saying mutually exclusive? Do you think it's related to your diagram?

[6]Nikita: I think … I think so.

[7]Researcher: Okay. Why?

[8]Nikita: Because it's included. Oh, it's not.

[9] Researcher: Is it included or not?

[10] Nikita: In the circles [or not?]

[11] Researcher: What is included there? Like, in your diagram what is included?

[12] Nikita: I think the numbers that are inside the two circles.

[13] Researcher: Yeah. Okay. Inclusive means there is something common.

[14] Nikita: What does that mean?

Learner Nikita identified three ways in which her use of Venn diagrams improved her understanding of probability. Her ideas are captured in the interview response in Vignette 7.

Nikita stated that drawing a Venn diagram benefitted her in three ways, which are: 1) it makes it easy to read the information, 2) it makes it easy to visualise the information, and 3) it makes it easy to see where the information fits in the Venn diagram. This is evidence that the Venn diagram assisted Nikita to make sense of the posed problems. This evidence is in line with the notion of representational fluency as an indicator of mathematical conceptual understanding (Niemi ,1996 ; Suh and Moyer, 2007). Representational fluency emphasises creation and interpretation of information in different forms including pictures, graphs and symbols to enhance conceptual understanding.

This study mainly focused on the impact of GeoGebra on modelling probability situations. Learners in the treatment group used Venn diagrams to represent information obtained from GeoGebra activities. The use of GeoGebra was intended to enhance students' understanding of Venn diagram methods. For example, two dice were rolled using a GeoGebra simulation of rolling dice. Event A and event B were defined as follows: A = {sum of numbers on the dice is 7} and B = {at least one of the dice shows a 2}. For this question, learners were supposed to represent the outcomes on a Venn diagram. In a conversation between the researcher and learner Cebisile (see Vignette 8), it was clear that learner Cebisile understood the relationship between events A, B and S (the sample space). However, she was not sure how she could present her results on a Venn diagram in a feasible and efficient way.

Vignette 8 Use of GeoGebra to probe students' understanding of Venn diagram methods in probability problem solving

[1]Cebisile: Sir. I was wondering for question (f) of the experimental probability, when we draw the Venn diagram, do we have to put all the other outcomes that don't fit in A and/or B outside the circles. For example (3;6). It doesn't have a two or add up to 7. Do we write these outside the circles? Because if we do, then I'm going to have to write like 20.

[2]Researcher: You can just write the probabilities instead of the actual outcomes to avoid writing too much.

[3]Cebisile: … For A they ask for the possible outcomes to get 7. If I got (4;3) three times, do I write it three times or just once? But then when I put in the Venn diagram it will be three times? Because my A has four possible outcomes, but P(A) is $\frac{7}{36}$ because some were repeated.

Cebisile's reasoning indicated that she was engaged and understood the concepts that were involved in this activity. The practical challenges that she encountered, such as writing in too much information, showed that she was engaged. Her responses suggested that she had alternative options in her mind, an indication that the use of a Venn diagram supported her mathematical understanding of the concepts she was learning. Learner Nikita's responses (Vignette 7) also confirmed that Venn diagram methods were useful for enhancing understanding. Initially, Nikita did not have a good understanding of the concept of "mutually exclusive" and "inclusive." By using a Venn diagram, she was able to gain a better understanding of these concepts. These results are in line with one of the aims of the Curriculum and Assessment Policy Statement (CAPS), (DBE, 2011b) which state that Mathematics teaching must seek to produce learners who are able to "communicate effectively using visual, symbolic and/or language skills in various modes" (p.5).

5.4 Theme 4: Promoting active learning through GeoGebra mediation

The Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011b) stipulates that the teaching and learning of Mathematics should be guided by the principles of active and critical learning. This study was conducted within a constructivist perspective and was informed by the theory of semiotic mediation. In order to determine the level of engagement of students during the GeoGebra-assisted lessons, interview and observation data sources were analysed. Observation sources included video recordings of parts of the lessons, photographs and field notes. The analysis of these data revealed that learners' levels of engagement in classroom activities was enhanced by the GeoGebra activities. Also, the level of interaction between learners was high during these activities. It was further observed that the rate of participation among learners was high. I present the evidence of these findings in the following sub-sections.

5.4.1 The impact of GeoGebra on enhancing learner engagement in classroom activities

Mango (2015) defines student engagement as "the extent to which [students] take part in educationally effective practices (p. 53). According to Mango, student engagement can be categorised as behavioral, cognitive and emotional engagement. Behavioral engagement has to do with students' physical and active involvement in learning activities (Mango, 2015). On the other hand, "emotional engagement is thought to occur when students have a positive attitude and enjoy what they are doing; while cognitive engagement is thought to take place when students invest into learning in a focused, self-regulating and strategic way" (Mango, 2015, p. 53). The integration of GeoGebra in learning probability provided an opportunity for learners to engage actively in the planned activities. These findings are supported by studies which show that student engagement can increase when technologies are integrated in classroom activities. For example, Diemer, Fernandez & Streepey (2012) argue that "positive learning outcomes are likely to accompany use of iPads within university classrooms if the device effectively increases the level of student engagement" (p.22). In a study by Diemer et al., (2012) on students' perceptions of classroom engagement and learning using iPads, it was found that students' level of engagement while using iPads correlated positively with their learning outcomes. In another study to examine the ways in which students perceived the influence of using iPads on students' learning and students' engagement, Mango (2015) found that students' learning engagement was positively influenced by the use of the technology. It was observed that participants' active engagement and interest in classroom activities were stimulated by the GeoGebra activities. At the close of one GeoGebra lesson, one learner commented, "this was fun."

Participants were asked about an incident, activity or experience in class (or outside class) which helped them to understand some aspect of probability. Learner Mavusana responded as follows: "I would say one situation was through rolling a dice … just playing games or anything in class with friends, like monopoly. Probability plays a big role and you get to see how it really works there…" Mavusana's response sheds some light on how students learn through play. During the GeoGebra activities some students were fascinated by the animations that were visualised. For example, in a GeoGebra simulation of tossing a coin many times, the results were visualised as bars indicating the frequencies of the outcomes. The frequency bars were visualised as upward and downward motion. The varying heights of the bars produced an animation which was not only effective in describing the phenomenon of relative frequency but was also exciting to watch. As students watched this motion, they began to predict and talk among themselves about what would eventually happen. The animations managed to create a game situation, and because this was happening in the context of a probability lesson, students restricted their views and arguments on probability. In all the GeoGebra lessons that were conducted in this study, students' engagement was very high. They had worksheets to complete during the experiments. Since the experiments could be paused, students had adequate time to interrogate their data and draw conclusions from them. Some students were observed using their calculators to complete some tasks. The Venn diagrams that were drawn and the conclusions that were drawn show that students were engaged cognitively.

The use of GeoGebra also enabled students to engage with each other and with the researcher in a productive manner. This provided students with the necessary support that they needed to complete the tasks.

5.4.2 Impact of GeoGebra on learner interaction with each other and with the teacher

Student interaction with the researcher and with classmates was observed during the lessons and in the interview. The researcher facilitated these lessons and allowed students to interact with each other and with him as they worked through probability problems using GeoGebra as a supporting tool. It was observed during the lessons that GeoGebra-assisted lessons allowed students to share ideas among themselves. The different views that students held about some aspects of probability were freely expressed during the lessons. All the lessons were characterised by useful conversations among students. This is what one of the students said about the level of interaction that was facilitated by GeoGebra-assisted lessons (Vignette 9).

Vignette 9 The impact of GeoGebra on the level of interaction among learners

[1]Shareen: It actually gives people the chance to voice out their own opinions. As you saw, they used examples from the real world and crime … it was his way of understanding that concept. So the way he might have explained it might have actually helped someone else to understand it better than they already did. So it actually helped us to interact and share some ideas and help each other in the process to understand.

The response by Nikita was evidence that the use of GeoGebra provoked discussions and interactions that helped validate or dismiss students' preconceived ideas about some aspects of probability. These findings were consistent with findings from earlier studies which examined student levels of engagement during technology-based educational practices. For example, Bahati, Fors, Hansen, Nouri & Mukuma (2019) examined students' learning with formative e-assessment strategies and found that the quality of the level of engagement among students was very high.

Vygotsky's (1978) constructivist perspective describes the zone of proximal development (ZPD) and states that with the help of an adult or a more knowledgeable other, a student can be guided through a task using scaffolding techniques. Scaffolding creates a supportive environment for the students and enables them to complete the task successfully. Interview scripts and field notes were analysed in this study to determine the extent to which students had adequate support for them to successfully learn probability using GeoGebra. The findings revealed that students received adequate support from the researcher to learn probability. The researcher worked one-on-one with some students and applied scaffolding techniques to guide them in achieving their learning goals. The following conversation is an example of the support that helped Nikita to gain a better understanding of a learning situation.

Vignette 10 Evidence of learner support – interaction between learners and the researcher

[1]Researcher: What about this one? X means boys in Grade 10 who play soccer and Y means girls in Grade 10 who play soccer. Are they [X and Y] mutually exclusive?
[2]Nikita: I think they are mutually exclusive because they have nothing in common, boys and girls.
[3]Researcher: But do you think X and Y are complementary?
[4]Nikita: Yes they should be complementary because they will give you all the Grade 10s. No they won't. No wait. What? …. Uhm. …[silent for a long time]
[5]Researcher: Think about it like this. If I take all the Grade 10 boys who play soccer and all the Grade 10 girls who play soccer and I put them together does that give us all the Grade 10s in the school?
[6]Nikita: No because not necessarily all the boys play soccer and not necessarily all the girls play soccer.
[7] Sanders: I didn't understand it initially, but you made it clear for me to understand it.

After this type of assistance from the researcher, some students showed that they benefitted from the support and made comments such as, "I didn't understand it initially, but you made it clear for me to understand. An analysis of the learning environment that was created and the help that the teacher gave to students on a one-on-one basis revealed that students were adequately supported to use GeoGebra for probability knowledge acquisition. It was also evident from the analysis that students did not find GeoGebra difficult to use. They completed their homework tasks using the GeoGebra application, and no one reported any technical challenges.

Teachers are capable of influencing students' learning outcomes in many ways. for example, according to Domino (2009), teachers can influence students' learning outcomes in the way they teach mathematics and in the way they help students. Domino (2009) found that students' understanding of mathematics were particularly influenced by the way the teacher planned and delivered lessons that were fun and interesting, planned and delivered lessons which supported students' active engagement, planned and delivered content that was related to real life, paced his lessons, helped students and showed enthusiasm about mathematics. The CAPS document stipulates the following two specific aims of teaching mathematics: "mathematical modeling is an important focal point of the curriculum. Real life problems should be incorporated into all sections whenever appropriate. Examples used should be realistic and not contrived. Contextual problems should include issues relating to health, social, economic, cultural, scientific, political and environmental issues whenever possible*"* and "to promote accessibility of Mathematical content to all learners. It could be achieved by catering for learners with different needs (DBE, 2011b, p.8)

There are several implications for teaching in these specific aims including the role that mathematics teachers play in influencing students' learning outcomes. It is the responsibility of the teacher to ensure that the content that is learnt in class is related to real life contexts and that the teaching strategies that are used are inclusive. This study took these aims into consideration when developing the GeoGebra strategies that were used in class. The levels of learner engagement and interaction that prevailed were a product of the planning that was done to foster the CAPS principles on teaching and learning mathematics. Learners reported satisfaction with these lessons when they said that they found the GeoGebra lessons interesting and fun and that opportunities were created for them to be interact with each other in an active learning environment.

5.5 Summary of analysis

The main research question of this study sought to find ways in which GeoGebra can be used to overcome specific challenges that Grade 10 learners have when solving probability problems. The following themes emerged from the analysis of qualitative data:

Theme 1: Motivation to learn: There are several barriers to learning Mathematics. A misunderstood probability concept can cause confusion for learners, resulting in failure to answer questions comprehensively. Also, learners who struggle to understand a mathematical term will inevitably experience a reduced level of motivation to answer the affected question. The analysis of learners' interview responses showed that learners are motivated to learn probability if they understand what probability entails and how it applies to real life settings. The results showed that respondents had a fair understanding of the concept of chance. This understanding becomes motivation for the learning of the probability topic.

Theme 2: Opportunities for the development of mathematical competencies. The use of GeoGebra provided several opportunities for learners to develop their mathematical competencies. First, since all data were generated through GeoGebra simulations, learners did not have to spend too much time on that aspect. This time was used to interact with the data and to make sense out of it. Learners attested to the usefulness of GeoGebra in this regard. Second, respondents confirmed some affordances of GeoGebra which were useful for their understanding. Some spoke about how GeoGebra helped them to gain deeper insight into the situation under study, which led to deeper understanding. For example, learners' preconceived ideas were either validated or refuted through the use of GeoGebra simulations. Third, the use of GeoGebra created a learning environment which was favourable for skills and competencies development. The learning environment that was created was inclusive in the sense that it accommodated learners of different abilities.

Theme 3: Problem solving. There was evidence that the use of GeoGebra impacted on learners' choice and use of problem-solving strategies. The decision to use Venn diagram strategies was mainly influenced by the way in which the question was structured. For example, scaffolding supported learners' decisions by suggesting a strategy that could be used to answer questions.

Theme 4: Active learning. The use of GeoGebra supported learner engagement in classroom activities. Learners were observed working actively to complete tasks. Engagement was enhanced because learners were expected to capture as much information as possible from the GeoGebra generated data. This included capturing and summarising the results from the GeoGebra experiments. Since learners were working as a collective, interaction was observed to be high as well. Learners were free to share their views, and as they did so, their preconceived ideas were either validated or refuted. Students said that GeoGebra-assisted lessons allowed them to interact with each other and to share ideas freely.

5.6 Summary

In this chapter, qualitative data analysis was presented. The results showed that the use of GeoGebra supported learning of probability concepts. Also, students' challenges in probability were addressed through the use of GeoGebra. Students indicated in the interviews that GeoGebra was useful for acquiring and understanding probability concepts. They also expressed their satisfaction with the level of interaction and engagement that were created as a result of the tool.

CHAPTER SIX

DISCUSSION

6.0 Introduction

The purpose of this study was to explore the impact of the use of GeoGebra intervention on Grade 10 learners' understanding of probability. This chapter presents a discussion of major findings in relation to the errors and misconceptions learners make when solving probability problems and the impact of GeoGebra intervention on learners' understanding of probability. The chapter concludes with a discussion of the pedagogical framework which was developed for teaching probability using ICT mediation. The key findings are discussed to help answer the research questions of the study.

6.1 Key findings

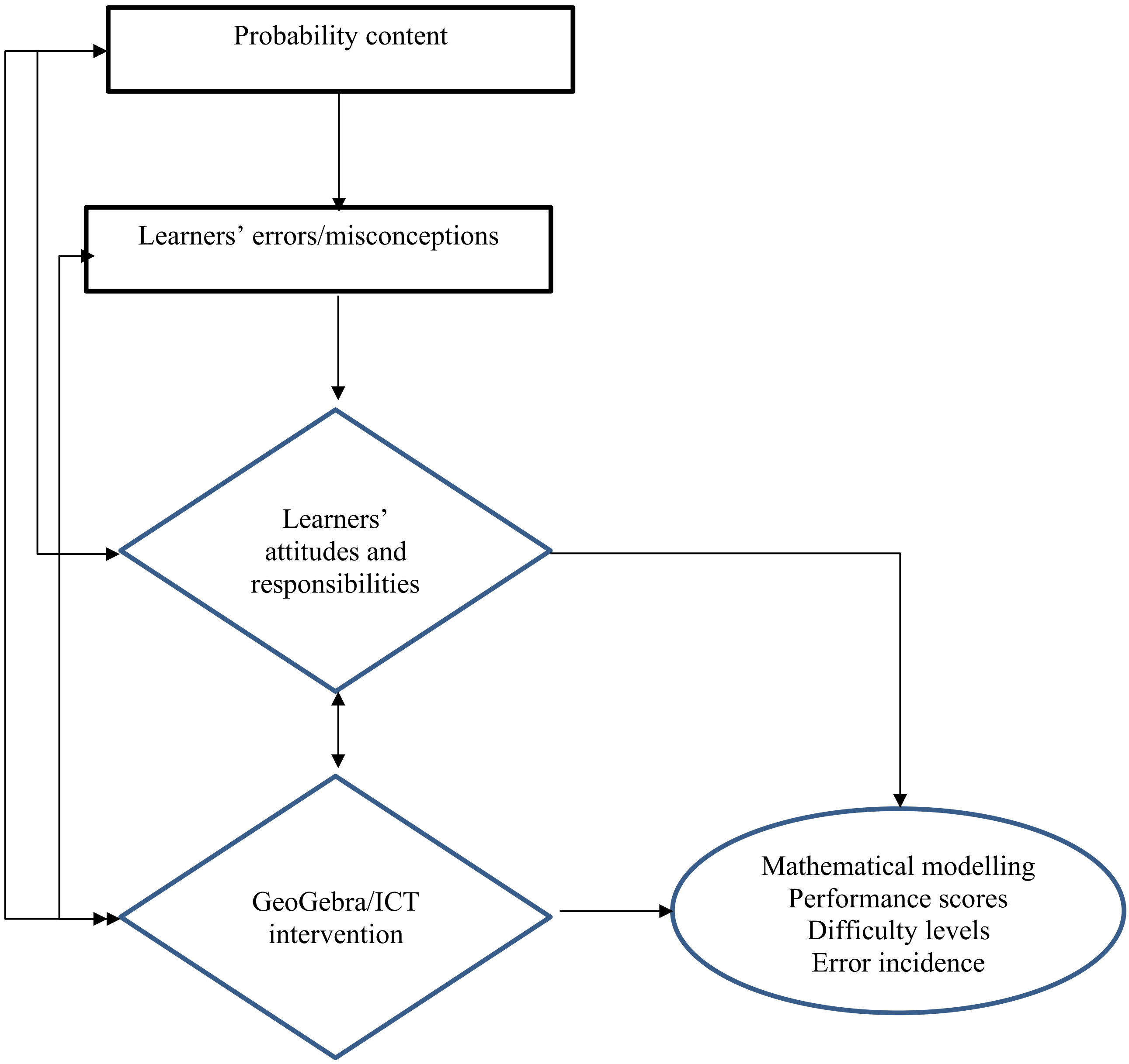
The value of GeoGebra in addressing learners' errors and misconceptions was found to be multi-dimensional. It comprised six factors or themes which contributed towards the creation of a learning environment where learners' misconceptions were addressed. These are: (a) the persistence of students' errors in spite of ICT mediation, (b) students' useful attitudes and responsibilities for overcoming learning barriers using ICTs , (c) difficulty levels in written tests as indicators of the impact of ICTs mediation, (d) mathematical modelling as a critical indicator of conceptualisation, (e) active learning of concepts as an indicator of the student-centredness of ICTs, and (f) performance scores which still remain a challenge in probability tests. All of these factors contribute towards a creation of an environment where GeoGebra can be used to address misconceptions faced by students when solving probability problems.

6.2 Interpretation of the findings

Each of these themes stood out as major contributors towards finding ways in which GeoGebra use can influence probability learning. The themes interacted with each other to create an active learning environment where learners' misconcepions were addressed. Figure 6.1 shows a conceptual framework of the interaction among these themes.

The active learning environment is created by the interaction between and among the elements that are shown in the diagram. For example, as students acquire probability knowledge, errors may be made which require decisions to be taken. Learners on their part need useful attitudes to handle challenges that relate to learning. Both learners and the teacher need to agree on approaches and methods of learning. GeoGebra intervention as a teaching practice is a decision taken by the teacher and accepted by the learner, which serves as a means for achieving the learning objectives. Each theme is discussed in detail in the following sub-sections.

Figure 6. 1 A learning environment where learners' misconceptions were addressed



6.2.1 The persistence of students' errors in spite of ICT mediation

The results of this study showed that students made factual, procedural and conceptual errors in the pre-test as well as in the post-test. Although the number of errors made by the treatment group dropped from 58 in the pre-test to 44 in the post-test, the Paired Samples independent T-Test showed that this change was not statistically significant. Similarly, the study found that success rates in the post-test as measured by the average of the students' scores did not significantly improve for the treatment group. The average scores improved by only 3% in the post-test for the treatment group. Both these sets of results were inconsistent with existing research which found that the use of ICTs increased test results (e.g. Carr, 2012; Shieh, 2012). Carr (2012) investigated the effect of an iPad on students' achievement in Mathematics, while Shieh (2012) examined the impact of Technology-Enabled Active Learning (TEAL) on students' performance. Both studies reported significant gains in achievement. Although the findings of the current study were unexpected in relation to improving test results and reducing the incidence of errors, they were not surprising if viewed from a constructivist lens. Constructivism advances that errors are a result of misconceptions that students construct as they make sense of the world around them. These errors are not easy to remove through instruction. Instead, they require teachers to guide learners to restructure those conceptions so that they fit in their current domain. This might explain why performance in the post-test was not different from performance in the pre-test.

According to Brodie (2014) a shift in teachers' thinking about learner errors is needed for effective teaching. Teachers should begin to ask themselves about the relationship between learners' errors and their (teachers') own pedagogical content knowledge (PCK). Brodie (2014) suggests that student errors should be interpreted in terms of "the underlying mathematics rather than in relation to learners' abilities and attitudes" (p. 222). The findings of the current study were, therefore, not surprising because the occurrence or non-occurrence of errors in students' work did not necessarily reflect students' abilities or attitudes. Despite the intervention they received, errors still occurred in their written post-test.

These results have pedagogical implications for Mathematics teaching, one of which is to harness the learning opportunities that learners' errors and misconceptions present. This requires re-thinking assessments to prioritise measuring mathematical understanding. Traditional tests are sometimes criticised for focusing on mathematical skills and procedures only. Rosli et al., (2013) argue that traditional tests have no ability to accurately measure students' mathematical understanding. According to Rosli et al. (2013) performance rubrics should be used to check students' work because rubrics are capable of measuring student's understanding of mathematical concepts. In traditional tests, which are marked using a memorandum, focus is placed on the procedures that students follow to arrive at answers. Procedures that are judged as incorrect do not get rewarded. This study suggests that instead of rejecting students' errors, educators should find a way to assess the mathematical reasoning behind all the errors made. In other words, assessments should be designed to accommodate even those unexpected responses that students may give in their written work. A performance rubric is a useful tool for this purpose.

6.2.2 Students' useful attitudes and responsibilities for overcoming learning barriers

The results of this study indicated that motivation was a critical factor in learning probability. Students need a reason for them to learn Mathematics, particularly probability. This reason should go beyond just getting good marks to realizing the connection between probability and the real world. It is generally admitted that probability is a challenging topic because of its abstract and complex nature. The Department of Basic Education (DBE, 2015, 2016) examination reports show that candidates confuse probability concepts, leading to low examination results. To address such challenges, Sarwadi & Shahrill (2014) emphasise the importance of properly teaching the topic. They argue that a systematic approach that allows concepts to hierarchically build on each other is critical. When students continue to get low marks or fail to make conceptual sense of the content they are learning, they may become frustrated and give up. However, learners who understand the importance of learning probability will most likely want to learn it despite its challenging nature. This was the reason for establishing participants' appreciation of the role probability plays in their everyday lives. The study findings showed that students' useful attitudes for learning probability were motivated by their understanding of the place of probability in everyday life. The National Council of Teachers of Mathematics (NCTM, 2000) gives guidelines about the need for the curriculum to help students see connections between Mathematics and the real world.

Motivation is affected when learning barriers are present. Students' responses in the interviews showed that misunderstood words or concepts became potential barriers to understanding. Sondergaard (2020) argues that misunderstood words create confusion for learners. Learner Nikita experienced this phenomenon when the concept of bias was used in a question. On the basis of these results, it can be concluded that motivation is enhanced when the connection between Mathematics and the real world is understood. Also, students are motivated when they understand the concepts that are used in the questions or lesson. This explains the importance of teaching probability for conceptual understanding. GeoGebra integration was found to be effective in supporting conceptual understanding. This will be discussed in section 6.2.4.

6.2.3 Difficulty levels in written tests as indicators of the impact of ICTs mediation

Difficulty indices for the treatment group indicated that the post-test was less difficult than the pre-test. In contrast, the control group found the post-test more difficult than the pre-test. Based on the fact that difficulty indices measure the number of students who got correct answers, it can be stated that the number of students who got correct answers increased in the treatment group after GeoGebra intervention. This further explains that the intervention was effective because the increase in the number of learners who answered questions correctly was found to be statistically significant.

This improvement can be attributed to the way GeoGebra intervention helped in building learners' confidence and general understanding. Learner Kholwani, for example, said that GeoGebra lessons were eye opening when he said, "The lessons opened me up. It definitely lit a light bulb. It just helped me think quickly in a sense." Learner Miles also explained how his confidence grew when he said, "I thought it was going to be difficult, but once I was in it, I discovered that okay, I have to work [it] out like this." Learner Sanders also shared that GeoGebra helped improve his understanding. He said, "I didn't understand it initially, but you made it clear for me to understand it."

Of particular note was the interaction between difficulty levels and performance levels in tests. Results showed that although more students in the treatment group answered questions correctly in the post-test than in the pre-test, the average performance score did not improve significantly. There was an improvement in performance scores by an average of 3% only, which was not statistically significant. This should be expected because to calculate the average performance in tests, each learners' score is used. Some of these scores could be outliers with the potential to either inflate or deflate the mean score.

This study also found that the number of errors made in the post-test did not drop significantly after intervention. Based on these results, it can be concluded that the levels of difficulty of test items, test performance scores and number of errors in students' work are not always directly proportional. The findings of this study, as already stated, differed from existing research (e.g. Burns et al., 2006; Carr, 2012). Burns et al., (2006) found that difficulty levels positively influenced students' performance in tests. The possible cause for this difference was not clear, but it might be that the participants in such studies tend to over-rely on the ICT tool that is used for intervention to the extent that their own reasoning functions are limited.

Notwithstanding, existing study findings validate the results of this study by suggesting that there are various other factors that affect performance in tests (e.g. Ashcraft & Moore, 2009; Burns et al., 2006). Ashcraft & Moore (2009) found that anxiety is another factor which potentially hinders a significant gain in performance scores. Burns et al.,(2006) also argue that very difficult instructional material can lead to student frustration, while very easy material can lead to boredom. They argue that an appropriate level of challenge, where questions are neither too difficult nor too easy, will keep students appropriately motivated in Mathematics instruction and assessment. It is, therefore, possible that whereas GeoGebra intervention reduced difficulty levels of test items, other factors which hindered significant improvement in students' performance scores were present. Mathematics teachers should be aware of unexpected factors in their planning.

6.2.4 Mathematical modelling as a critical indicator of conceptualisation

Problem solving questions in this study were answered using Venn diagrams or probability rules. A Venn diagram was correct if all events, their outcomes, number of outcomes or probabilities were correctly identified and represented in the model. It was found that the incidence of correct Venn diagrams leading to correct results (Category A) improved significantly by 100 percent for the treatment group after the intervention. The results also showed that the incidence of correct Venn diagrams followed by incorrect results (Category B) dropped by 56 percent in the post-test for the treatment group. For the control group, the number of cases in Category A was 8 in the pre-test and 9 in the post-test. In Category B, the number of cases increased from 7 in the pre-test to 10 in the post-test. Both these increases were not statistically significant.

A correct Venn diagram drawn was an indication that students were able to translate probability knowledge from one mode into another. According to Lesh's translation model (Suh & Moyer, 2007), deeper mathematical understanding is developed when students are able to translate information into different forms such as pictures, diagrams and symbols. Interview responses showed that students' representational fluency was supported by GeoGebra use through its dynamic geometry software (DGS) functionality. This functionality allowed students to view visual and numerical feedback from the artefact. Learner Shareen stated that "looking at the video" helped them understand better than "actually reading a question or maybe a textbook." This shows that Shareen was aware of other forms of presenting information. The "videos" she was referring to were the GeoGebra simulations which created graphs alongside the generated data. Students interacted with these data and created mathematical meaning. They attributed their understanding to GeoGebra and stated that "it was eye opening", "it lit a light bulb", "it was fun", "it was practical", "it was immediate" and it "challenged their opinions."

These findings are consistent with existing research (e.g. Lee & Hollebrands, 2006). In their study, Lee & Hollebrands (2006) used a java applet to investigate how the features of the technology tool supported, or did not support, students' problem solving. They found that most features of the technology tool supported students' problem solving. The dynamic features of the applet included animation and motion, while feedback features included graphs and numerical results. In particular, the dynamic feedback feature present in the applet allowed students to move an object from one location to another, thus helping them to achieve their goals using different strategies. In the current study, the GeoGebra tool helped learners in similar ways using its feedback features. Graphs and numerical data were visualised simultaneously, allowing students to interact with both to see patterns and create meaning. This is supported by the theory of semiotic mediation (TSM) (Vygotsky, 1978) which posits that as students work with technological tools, signs are produced which lead to creation of mathematical meaning.

The results of this study also showed that some students used, or tried to use, the probability rule P(A or B) = P(A) + P(B) – P (A and B) instead of the Venn diagram. The explanation for this might be that the Venn diagram was not mandatory in some of these questions. Students were free to choose any strategy to use. It was found that the probability rule was not correctly used by the majority of the students because the relationship between event A and event B was not taken into account. The use of the rule or formula required students to take into account whether these events were inclusive or exclusive in order to reach the correct result. Many students who did not understand this aspect just substituted the given numbers into the formula and ended up getting probability values that were greater than 100%. It was clear in such cases that there was no full understanding of the conditions under which the formula can be used. On the basis of these findings, it can be said that students perceived Venn diagrams to be difficult to use in probability problem solving. This conclusion is supported by Uesaka, Manalo & Ichikawa (2007) who argue that students' lack of confidence in using diagrams, and their belief that diagrams are difficult, constitute some of the factors that hinder the use of diagrams.

Nevertheless, some students who used Venn diagrams to solve problems gave convincing reasons why diagrams helped to simplify the work for them. Student Nikita reasoned that the use of Venn diagram methods increased her understanding of the question. Nikita said, "I just think that a Venn diagram … just makes the information easier to read. Instead of reading it a couple of times, I try to draw it out so that I can just see the information and just see exactly where everything falls in the Venn diagram." Thus, Nikita applauded Venn diagram strategies for their ability to simplify information and create a better understanding of the situation.

6.2.5 Active learning of concepts as an indicator of the student-centredness of ICTs

Participants expressed satisfaction with the level of active learning that was stimulated by the GeoGebra learning environment. They referenced the levels of engagement, interaction and participation in classroom activities as indicators of active learning. The CAPS (DBE, 2011b) guidelines stipulate that active and critical learning should be the goal of Mathematics teaching. According to Mango (2015), engaged learners are not only actively engaged in learning activities, but are also focused and self-regulating. Engaged learners also show a positive attitude which allows them to have fun doing what they are doing. The results of this study showed that students' engagement in classroom activities was high. The level of engagement was attributed to GeoGebra usage. These findings are consistent with existing research which shows that the effective use of a technological device potentially leads to positive learning outcomes (e.g. Diemer et al., 2012).

Students' interview responses also attested to the GeoGebra potential to promote collaboration among students. Participants pointed out that through discussion, "other people's opinions were considered" and they were given a chance "to voice out their own opinions." This interaction took place in a collaborative manner as conceptualised by constructivist perspectives and the theory of semiotic mediation (Vygotsky, 1978). The interaction among students also promoted conceptual understanding and addressed errors and misconceptions that were present.

On the basis of these findings, it can be argued that GeoGebra use promoted active learning. A student-centred learning environment was created in which students took responsibility of their learning. They did this by engaging in classroom discussions which were stimulated by the feedback they received from GeoGebra. This interaction was characterised by collaboration among students which led to validation or refutal of preconceptions and opinions that existed prior to the intervention. Thus, the GeoGebra learning environment that was created addressed students' specific challenges in probability and provided opportunities for developing their conceptual understanding.

Students' personal factors, such as their attitudes towards probability and the GeoGebra tool also played a significant role in bringing about positive learning outcomes. Students showed interest in the work that was done and in the methods that were used. This was important because it made them cooperate with the teacher and increase their participation. The whole learning experience was satisfying as a result.

6.2.6 Performance scores still remain a challenge in probability tests

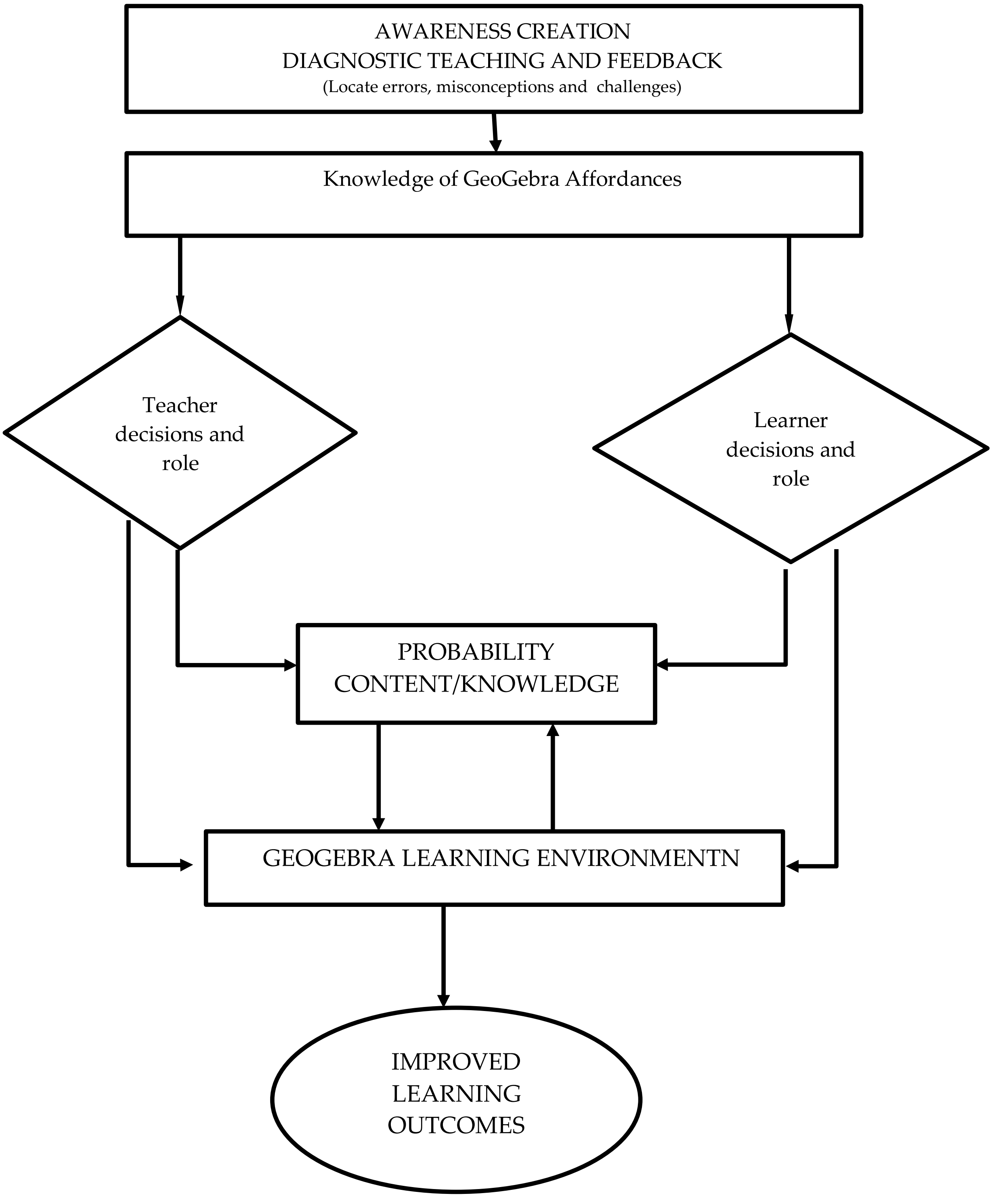
The results of this study showed that GeoGebra intervention did not impact significantly on students' performance in tests. The average score improved in the post-test for the treatment group by only 3%. However, the Paired Samples independent T-Test showed that this improvement was not significant. This study also found that the number of errors made by students in their written work did not drop significantly after GeoGebra intervention contrary to expectation. Constructivist theories advance that knowledge is created by learners during interaction in a social setting. The way knowledge is created is the same way errors are created. Brodie (2014) posits that errors are conceptual structures which are developed in the mind when students are learning. This helps to explain why errors might be difficult to eradicate because as far as learners are concerned, these are not errors, but knowledge. Adequate planning is required to address learners' errors. It is not surprising, therefore, that students continued to make errors after intervention was given. As a result, performance in the post-test, particularly from the treatment group, did not improve significantly. The results further suggest that there are other factors that influence students' performance in tests which should be researched by the teacher in order to plan adequately.

6.3 A pedagogical framework for teaching and learning probability using ICT: Research Sub-Question 3

The findings of this study suggest a pedagogical framework that can be recommended for teaching probability using GeoGebra. Figure 6.2 shows the pedagogical framework that was developed based on the results of this study.

The study found that students made factual, procedural and conceptual errors in their work. It also found that GeoGebra intervention had an impact on several learning outcomes including creating a learning environment where learners' levels of engagement, interaction and participation were enhanced and supported probability learning. GeoGebra intervention also supported learners' ability to use effective methods to solve probability problems. Through the use of GeoGebra-supported activities, students found questions in the post-test less challenging than questions in the pre-test. The pedagogical framework for teaching probability using GeoGebra was derived from the discussion of data that were collected by the researcher in the classroom where he was a teacher. The framework attempts to explain how probability can be taught using GeoGebra. Literature shows that there are several pedagogical frameworks that have been suggested by researchers for Mathematics lessons. For example, Chua & Wu (2005) suggest a pedagogical framework for technology-based Mathematics lessons which consists of

Figure 6. 2 Pedagogical framework for GeoGebra-supported intervention



four components, which are: exploring, conjecturing, verifying, and generalising (p. 389). Their framework emphasises mathematical thinking (conjecturing, verifying, generalisation), investigative learning (exploring), own knowledge creation (conjecturing) and mathematical knowledge creation (generalisation). This framework states that students solve mathematical problems by exploring, conjecturing and verifying knowledge. If they discover that their conjecture is incorrect, they can go back and go through the whole process again; and so, learning Mathematics is a process of going back and moving forward. Adler & Ronda (2015) argue for a framework which starts with the realisation of the object of learning, and is characterised by exemplification, explanatory talk and student participation within a sociocultural context. The sociocultural context involves the use of tools in mediating learning. On the same subject, Sellar & Cormack (2007) mention that pedagogy is a triad relationship between the teacher, the student and knowledge. According to Sellar & Cormack (2007), pedagogy "involves challenging students to make meaning of their worlds through language and other sign systems, as a result of the teacher disorganising their current understandings and helping them to produce new ones" (p. 3). These pedagogical approaches share the same view that students learn effectively in a constructivist learning environment where the teacher employs teaching methods which support students' creation of knowledge through active engagement and interaction.

The proposed pedagogical framework (Figure 6.2) in this study is a product of an interacting relationship between the teacher, the student, the probability content/knowledge and the GeoGebra tool. These interact and create a GeoGebra-learning environment which potentially leads to improved learning outcomes. The components of the framework were derived from the analysis of the data. These include, 1) creating awareness, 2) teacher decisions , 3) learner decisions, and 4) learning environment. These four components are not linear but interact with each other to bring about the desired learning outcomes. The learning environment that is created is influenced by the nature of interactions between the teacher, the student and the content. The framework is explained under the following subheadings:

- creating an awareness of knowledge gaps to support decision making,
- teacher and student decisions
- probability content/knowledge
- intentional integration of technology into teaching
- improved learning outcomes

6.3.1 Creating an awareness of knowledge gaps to support decision making

Quantitative and qualitative results of this study informed the pedagogical framework that was developed for teaching probability using GeoGebra. Being aware of the existing students' knowledge gaps was considered to be one of the important components of the framework. During intervention, the researcher and the learners benefitted from knowing the gaps that existed in learners' probability knowledge. For the researcher, this knowledge provided useful information for planning the intervention lessons. It also assisted me in choosing appropriate learning activities and teaching techniques. For the students, their awareness provided some form of feedback which helped them make their own decisions about the required actions. Students who are aware of their challenges in specific areas of probability are likely to become more willing and motivated to participate in planned interventions. In this study, for example, students were highly motivated during intervention.

Based on the fact that their errors were pointed out to them prior to intervention, it can be concluded that students' levels of motivation were influenced by their awareness of existing gaps. In other words, feedback that students received after the pre-test helped them understood that they had knowledge gaps. This convinced them to adjust their attitudes towards the planned intervention because they believed it was going to help fix their challenges and improve their test scores. Thus, students' performance expectancy about GeoGebra was enhanced by their awareness and understanding of the errors and misconceptions that they held. According to the Unified Theory of Acceptance and Use of Technology (UTAUT) model (Venkatesh, et al., 2003), people's use of technology is influenced by their belief that it will improve their performance. Students were best prepared for the intervention. They were also diligent in completing the assigned activities.

The relationship between the researcher and the students also played an important role in making the lessons a success. Students had faith in the researcher and trusted that his methods would work, so they learned from the researcher. The researcher also learnt from the students about how to teach probability in a GeoGebra-supported learning environment. The professional relationship between the researcher and the learners is, therefore, important in pedagogical practices.

6.3.2 Teacher and students' decisions

The purpose of studying students' errors was to get an idea of the nature of intervention students needed. The findings were used to plan for appropriate intervention. Decisions were made about the type of learning activities that suited the learners. As a teacher, the researcher also made decisions about the appropriate teaching and learning strategies to effectively address students' challenges in probability. Teacher decisions regarding the allocation of resources, such as time, were also made. The type of learning activities that were presented to the learners were carefully selected to allow active learning. For example, the researcher selected learning activities that landed themselves to practical activity or experimentation. These activities were not too easy to avoid learners getting bored. Neither were the activities too difficult because that would create anxiety and unnecessary perceived difficulty levels in the work.

After the pre-test was marked, feedback was given to students specifically focusing on the errors made in the pre-test. After this feedback, learners received intervention before they wrote the post-test. The treatment group received their intervention using GeoGebra technology. Since learners were made aware of their errors, they were able to accept the planned intervention as a remedial measure that would address their challenges. Thus, students also made important decisions which were driven by their desire to improve their performance outcomes. Based on the analysis of the interview responses, students' attitudes and motivation to learn probability using GeoGebra technology were shaped by their knowledge that they needed remediation to overcome their challenges. Decision making by both the teacher and the learner is, therefore, seen as a necessary component in a GeoGebra-supported educational programme.

6.3.3 Probability content/knowledge

The pedagogical framework that this study developed drew from Adler & Ronda's (2015) Mathematics discourse in instruction (MDI). According to Adler & Ronda (2015), one of the variables that describe Mathematics teaching is the object of learning, or the goal of the lesson. MDI was discussed in the literature review under section 2.3.1. The proposed framework recognises the importance of the probability content that learners should be taught and why it should be taught. The Curriculum and Assessment Policy Statement (CAPS) (DBE, 2011a) specifies the probability content that should be taught to Grade 10 learners. The subject assessment guidelines provided in the CAPS Mathematics syllabus also specify learning and assessment objectives which should be used to direct probability teaching. In developing the pedagogical framework for using GeoGebra in probability lessons, these standard guidelines as spelt out in the CAPS document were followed. Probability content and knowledge is a central component of the framework because it addresses epistemological issues about the nature of probability knowledge and what it entails. Mathematics teachers and students should be made aware of the value of probability knowledge in real life so that they become motivated to acquire it. Students' interview responses showed that learners had a fair understanding of probability and how it applies to real life.

6.3.4 Intentional integration of technology into teaching

The decision to integrate GeoGebra into learning should be intentional. The teacher's pedagogical content knowledge and his/her understanding of the affordances of GeoGebra should be motivating factors for him/her to use the artefact. This knowledge includes the potential for GeoGebra to support the creation of knowledge through students' interaction with each other and with the tool. It also includes the knowledge of the sociocultural context within which knowledge is created through the process of manipulating signs and symbols that begin to emerge as a result of the interaction between students and students, students and teacher, as well as students and the technology tool. This further implies that the successful integration of GeoGebra requires an understanding of the theories that explain knowledge creation through technology tools. The constructivist theory of semiotic mediation (TSM) (Vygotsky, 1978) which underpins this study gave the researcher some useful guidelines regarding the organisation of the GeoGebra-supported lessons. It is important that decisions that help in creating a constructivist learning environment are made by the teacher in the teaching and learning of probability using GeoGebra.

6.3.5 Improved learning outcomes

The final framework component which was identified from data analysis related to the learning outcomes. In particular, it is hoped that following this teaching plan potentially leads to increased levels of learner engagement, interaction and participation. The GeoGebra learning environment supported active learning. Students' preconceived ideas, or theories in their own words, were tested and either validated or refuted when new knowledge emerged as a result of the GeoGebra-supported probability lessons. Due to the level of student engagement in the learning activities, and the level of interaction among students, there was perceived satisfaction and perceived usefulness of GeoGebra-supported lessons. This suggests that for a pedagogical framework for teaching probability using GeoGebra, a learning environment that supports engagement, interaction and participation must be created. For example, the teacher could select learning activities that land themselves to practical activities or experimentation. These activities must not be too easy for the learners because they may get bored, or too difficult because that might create anxiety and unnecessary perceived difficulty levels in the work.

Furthermore, the suggested framework will potentially see improvements in various learning outcomes. For example, it is the goal of every teaching intervention to improve learners' performance in written assessments, use of correct methods and conceptual understanding.

6.4 Summary

In this chapter, findings were discussed to answer the research question of the study. The discussion revealed that students continue to make errors in spite of planned intervention. Constructivist theories view errors and misconceptions as some form of knowledge that learners construct in their minds during the process of learning. While these errors and misconceptions might have a negative influence on certain learning outcomes, such as performance, the same are rich sources of pedagogical content in the sense that they are able to inform teachers' pedagogical decisions for improved learning outcomes. Teachers who are aware of the nature and source of their students' misconceptions are well equipped and prepared to manage students' misconceptions because they can utilise them to plan effective learning activities.

Whereas there is no guarantee that GeoGebra intervention will eliminate or significantly reduce the incidence of errors in students' work, it can be concluded, based on the findings of this study, that GeoGebra-supported lessons have the potential to reduce difficulty levels in written tests. In addition, the use of GeoGebra was found to be effective in increasing the incidence of the use of correct methods in written tests. Students' performance also improved marginally by 3%, although this improvement was statistically non-significant.

The effectiveness of GeoGebra was supported by students' learning experiences and their perceptions. The high levels of engagement, participation and interaction were a product of students' perceived usefulness of the GeoGebra artefact, students' perceived satisfaction about GeoGebra and students' attitudes. Learners also stated that they enjoyed utilising GeoGebra technology to acquire probability knowledge.

A pedagogical framework that supports high levels of engagement and interaction is needed to resolve learner errors and misconceptions in probability. The use of GeoGebra in this study provided an interface where teacher roles, learner roles and the probability curriculum converged to make sense of the object of learning. In this kind of environment, students were able to either validate or refute their preconceived ideas. They were also able to discover new knowledge as a result. It is argued in this study that the best way to address an error that is caused by a certain misconception is to plan activities that have the potential to directly challenge the misconception. Planning such activities is not always easy for teachers, especially as a result of time limitations. It might also take long for the learner to realise that his or her ideas need to be adjusted to fit into the new knowledge, but it is important to realise that learning probability is going to be exciting and effective when learners get the opportunity to engage in practical activities such as experiments.

In the final chapter, I present the summary of results, the conclusion and recommendations for future consideration, limitations of my study.

CHAPTER SEVEN

SUMMARY OF RESULTS, CONCLUSION AND RECOMMENDATIONS

7.0 Introduction

This chapter provides the summary, conclusions and recommendations derived from the investigation of the impact of GeoGebra on learners' understanding of probability. The study aimed to achieve three objectives. The first objective of the study was to identify the errors and misconceptions that Grade 10 learners make when solving probability problems. The second objective of the study was to explore how the use of GeoGebra can support the learning of probability concepts. The third objective was to explore how GeoGebra can be used to address Grade 10 learners' errors and misconceptions in probability problem solving.

The study answered four research questions:

- In what ways can GeoGebra be used to overcome errors and misconceptions that Grade 10 learners have when solving probability problems?
- What errors and misconceptions do Grade 10 learners make when modelling and solving probability problems?
- How can the use of GeoGebra support the learning of various probability concepts? In particular, how can GeoGebra be used to address Grade 10 learners' errors and misconceptions in probability problem solving?
- What pedagogical framework can be suggested for ICT mediations in resolving learner errors and misconceptions in probability?

The study was conducted at one Johannesburg high school in Gauteng, South Africa. The respondents were Grade 10 learners who were selected using purposive and convenient sampling. The students were assigned to the treatment group and control group. The study employed a mixed-methods quasi-experimental design. Quantitative data were collected through a pre-test and post-test, while qualitative data were collected mainly through participant observation and semi-structured individual interviews. Before the post-test was written, participants received intervention using GeoGebra, while the control group received their intervention through standard instructional practice without any technology.

7.1 Summary of Results

The findings of the study are summarised according to the research questions from the first chapter.

7.1.1 Secondary Research Question 1

What errors and misconceptions do Grade 10 learners make when modelling and solving probability problems?

Students made factual, procedural and conceptual errors in both the pre-test and post-test. Factual errors resulted when students failed to recall and classify multiples of a number, factors of a number and prime numbers. This led to them listing numbers in wrong sets or excluding numbers from their sets, leading to wrong answers. Procedural errors were made when students used wrong procedures as a result of misunderstanding events. Students misunderstood procedures for probabilities of union of events, single events, intersection of events, inclusive and complementary events. Conceptual errors were caused by a number of factors including the following: misunderstanding of bias, representativeness heuristics, misunderstanding of types of events, confusion between outcomes, number of outcomes and probability of outcomes.

The total number of errors made by the treatment group (n = 14) before GeoGebra intervention was 58. After intervention, the total number of errors dropped to 55 in the post-test. For the control group (n = 22), the total number of errors also dropped from 107 in the pre-test to 87 in the post-test. There was no evidence to suggest that students' errors were caused by the type of intervention students received. On the basis of these results, it can be concluded that GeoGebra intervention was not effective in reducing the number of errors students made when solving probability problems. Errors in students' work persisted in spite of the intervention.

7.1.2 Secondary Research Question 2

How can the use of GeoGebra support the learning of various probability concepts? In particular, how can GeoGebra be used to address Grade 10 learners' errors and misconceptions in probability problem solving?

Students encountered specific challenges when they were solving probability problems. In the pre-test, they had problems understanding probability concepts such as bias and types of probability events. This caused them to misinterpret the posed questions and to misapply methods. Misunderstood words or mathematical concepts are potential learning barriers and require immediate attention. The learning of probability concepts was supported by the use of GeoGebra which created opportunities for active learning. An analysis of interview and observation data showed that students' engagement, interaction and participation in classroom activities were high when GeoGebra was used. Through the use of GeoGebra, large volumes of raw data were generated. Students summarised and analysed these data to answer specific questions. As a result, they were kept busy and engaged throughout the lessons. Students also interacted with each other in various ways. Their diverse views and preconceptions were challenged or validated through group discussion. Some students indicated that GeoGebra helped them to learn. They were satisfied with the technology and they enjoyed the lessons. Some of them said that they had fun using the artefact. On the basis of these results, it was concluded that GeoGebra addressed specific challenges faced by learners through a student-centred learning environment that stimulated discussion. As students engaged in discussions, conceptual understanding was enhanced.

Another challenge experienced by students was based on the belief that the probability topic is difficult. Some students admitted that probability was difficult, especially in tests. Others indicated that before intervention, they thought that everything was going to be difficult. The pre-conceived belief that probability is difficult was imported to the probability lessons, creating potential learning barriers for students. However, after GeoGebra intervention, students admitted that the work was not difficult. They attributed this to the GeoGebra artefact which, they said, was easy to use. Students' positive attitudes were observed. Their motivation and willingness to use the artefact was driven by their desire to acquire deeper understanding of probability concepts.

7.1.3 What is the impact of GeoGebra on students' use of methods in problem solving?

The use of methods to solve probability problems was another focus of this study. The adopted framework for error analysis as suggested by Herholdt & Sapire (2014) includes an analysis of students' use of methods. The use of methods was categorised for ease of analysis as follows:

- Category A: learners drew correct Venn diagrams and used them to reach correct results
- Category B: learners drew correct Venn diagrams, but were not able to use them to reach correct results
- Category C: learners drew incorrect Venn diagrams, but interpreted them correctly to reach the results
- Category D: learners drew incorrect Venn diagrams and interpreted them incorrectly to arrive at a result

For the treatment group, the frequency of Category A improved from 5 in the pre-test to 10 in the post-test, while the frequency for Category B dropped from 9 in the pre-test to 4 in the post-test. These numbers were encouraging because they suggested that the number of cases in which correct Venn diagrams were drawn and correct answers obtained increased by 100 percentage. Whereas there was an improvement in Category A by the treatment group, there was a drop in Category B by the same group. This indicated that the number of cases where students failed to correctly interpret Venn diagrams dropped, which was an encouraging result because they implied that interpretation of Venn diagrams improved.

On the basis of these results, it was concluded that the intervention using GeoGebra was effective in improving correct use of methods. The dynamic geometric software (DGS) functionality of GeoGebra was cited as the possible reason for this improvement. The visual feedback that students received from the artefact allowed them (students) adequate time to interpret the numerical data that was produced, and to represent it in different forms.

7.1.4 What is the impact of GeoGebra on the difficulty of test items?

Findings on the difficulty levels of test items showed that for the treatment group, the post-test questions were less difficult than the pre-test questions. This difference was tested using Paired Samples T-Test and was found to be statistically significant ($t(3) = 2.644$, $p = .039$). In contrast, for the control group the post-test was more difficult than the pre-test. However, this difference was not statistically significant. The results suggest that the intervention given to the treatment group was effective in reducing difficulty levels of test questions. Difficulty levels measure the number of learners who answered the questions correctly. In the light of this, it can be concluded that the number of students who answered questions correctly increased significantly in the post-test for the treatment group. This outcome can be attributed to the student-centredness of GeoGebra activities which allowed students to take responsibility of their learning. Students' levels of confidence increased during GeoGebra lessons where they were actively and critically involved. Students admitted that before intervention, they thought the lessons were going to be difficult. However, after intervention, students said that they understood the work.

7.1.5 What is the impact of GeoGebra on performance outcomes?

The results of this study indicated that GeoGebra was not effective in improving performance scores and reducing the incidence of errors students made when solving probability problems. Although there was an average percentage score improvement of 3% in the post-test by the treatment group, this difference was not statistically significant (Paired Samples T-Test, $t(13) = 0.372$, $p = .716$). There was also an average percentage decrease of 1% in the post-test by the control group, although this difference was not statistically significant. On the basis of the above findings, it can be argued that there are other factors besides question difficulty which can potentially influence students' performance in tests. These factors need to be researched to help explain why students' test scores may not significantly improve when test items are not difficult. Also, an improved use of correct methods is expected to yield improved test scores. However, in this study the results showed that despite an improvement in modelling probability situations using Venn diagrams, students' scores in tests did not significantly improve. Further research, with more test items and more participants, is required to verify these findings.

7.2 Conclusions

Based on the findings, the following conclusions were drawn:

Conclusion 1: Harnessing learning opportunities behind students' errors

There was no significant evidence that the incidence of errors in students' work declined due to GeoGebra intervention. Students continued to make errors in their written work even after receiving intervention. Errors and misconceptions are conceptual constructs that learners create when learning. As such, it is not easy to eradicate them. Therefore, Mathematics teachers should seek to understand the sources of these errors and to find pedagogical ways of harnessing the underlying learning opportunities that are there. The objective of teaching Mathematics should always be about developing conceptual understanding and improving strategies for solving problems.

Conclusion 2: Emphasis on performance scores should not undermine the impact of ICT potentialities.

The results of this study showed that GeoGebra was effective in improving students' conceptual understanding through active learning. The results also showed that strategies for solving probability problems improved after intervention. However, the mean performance scores did not improve significantly despite intervention. Whereas performance scores are important, they cannot be used as indicators of mathematical understanding. An ICT tool which supports conceptual development and improves problem solving strategies is adequately effective.

Conclusion 3: GeoGebra use creates a student-centred learning environment which supports active learning.

The pedagogical framework suggested in this study includes four elements which interact to produce the desired outcome: the learner, the teacher, probability content (object of learning) and the mediating tool (GeoGebra). Based on the observations of what worked well in this study, it was found that both the teacher and the learner should be intentional in using GeoGebra to teach and learn. On its own, GeoGebra, like any other ICT tool, has no ability to bring about any change. A deliberate decision to use GeoGebra in probability teaching and learning was a critical starting point for harnessing its potentialities. The framework started with giving feedback to students about the errors they made in the pre-test. Students who are aware of their existing knowledge gaps are more likely to be motivated to use the technology that the teacher introduces in the lesson. The teacher's role included planning activities which allowed students to learn through participation, interaction and engagement. GeoGebra use provoked productive discussions which saw students sharing their views. This helped many of them to gain better understanding of the phenomenon under scrutiny. The learning environment that was created through GeoGebra created opportunities for students to evaluate their preconceived ideas.

Conclusion 4: Affordances of GeoGebra in influencing students' attitudes and learning
Although it was not the aim of this study to measure the attitudes of students towards the use of GeoGebra, it was found that the use of GeoGebra led to increased students' perceptions about the usefulness of GeoGebra. Students were satisfied with the technological tool and showed a positive attitude towards its use in learning probability. They also had fun learning probability using the tool.

Contribution of this study
This study adds to already existing knowledge of ICT-driven approaches in Mathematics teaching and learning. It suggests a pedagogical framework for teaching probability using GeoGebra and emphasises the far-reaching effect of the teacher's knowledge of the affordances of ICT artefacts in teaching and learning. Such knowledge makes ICT integration intentional and helps in creating a conducive learning environment in which the learner and the artefact interact in the activity of mathematical knowledge acquisition. The deployment of GeoGebra in learning probability can enhance the quality of learner engagement in planned activities, which leads to deeper understanding of probability concepts.

7.3 Recommendations

Students make errors while modelling and solving probability problems. Some of these errors are caused by students' misconceptions and require teachers to harness them to improve their pedagogical practices. This study showed that GeoGebra is an effective tool in addressing the errors and misconceptions that learners have in probability. Based on the findings of this study, the following recommendations are made:

1. Curriculum planners and authorities should revise the Mathematics syllabus to reduce the number of topics that are taught. Fewer topics could help create sufficient time for technological integration. Currently, teachers' attempts to deliver technology-based

practice are frustrated by the demands of the curriculum. They are expected to complete a long syllabus in a short space of time and at the same time meet the assessment requirements. This leaves teachers with no choice, but to adopt poor teaching strategies.

2. Education authorities should invest extensively in teacher development, especially in teachers' ability and confidence to use technology. In-service training in technology integration should be provided to boost teachers' technological pedagogical content knowledge. In particular, teachers should be aware of the affordances and constraints of several ICT tools that are available for use in Mathematics lessons.
3. Through ongoing teacher development programmes and workshops, teachers should be reminded about important theories of learning. A clear understanding of the implications of these theories for teaching and learning could equip them to plan meaningful pedagogical practices.
4. Mathematics classrooms should be well equipped with technologies such as computers, data projectors and audio speakers to support mathematics learning.
5. Education planners should re-think assessment to emphasise mathematical understanding over and above performance scores. Currently, there is too much importance attached to obtaining good marks in examinations. Many students get the good marks but may not fully understand the Mathematics involved. Examining students for mathematical understanding should go beyond using marking memoranda to allocate marks to include assessment rubrics that are carefully planned to assess full understanding. Students who show mathematical understanding are students who have been well taught. This study argues that teaching well involves technological integration in a student-centred learning environment. Thus, by putting more emphasis on mathematical understanding, teachers will be encouraged to involve technology in their teaching.

7.4 Limitations

Some challenges that were encountered in this study pose limitations to the transferability and generalisability of these findings to other sites. Curriculum issues that were encountered in this study and the teaching plans that were followed may not be similar to curriculum issues and teaching plans in other schools. The teaching plan for Mathematics at the research site needed to be adjusted to accommodate all the processes of this study. Probability teaching was planned towards the end of the year, and that did not leave adequate time for probability to be taught using technology due to the pressure that is there towards the end of each year in South African schools. According to the South African annual teaching plans (ATPs), the probability topic is the last topic to be taught in the year for all grades. Although teachers are able to adjust the teaching plan and teach probability at their preferred time, the rigidity of the assessment programme does not encourage that. Teachers, therefore, take the easy route out and teach the topic at the end according to the curriculum plan. Textbooks and examination papers also bring probability content at the end. This reality, coupled with the accepted reality that probability is a new and difficult topic in the curriculum, compromises the effective teaching of the topic.

Another limitation of the study was that some of the observed changes in the learning outcomes might have been influenced by other factors other than GeoGebra intervention. This possibility was minimised in a number of ways. First, the control group (n = 22) was introduced into the study to help evaluate the effect of GeoGebra by comparing the results of the two groups before and after the interventions. Second, a difference-in-differences estimate was used to evaluate the impact of GeoGebra on performance scores. Third, various sources were used to triangulate the results. These sources included interviews, observation, tests, video recordings and photographs.

7.5 Suggestions for future research

The main purpose of this study was to determine the impact of GeoGebra on Grade 10 students' understanding of probability at one particular school in Gauteng. The effect of GeoGebra that was discussed in this study was, therefore, observed on a small group of students (n = 14) in Grade 10. Further research is recommended to determine how learners in other grades respond to the use of GeoGebra in acquiring probability knowledge.

The quantitative result analysis showed that the improvement in test scores after the intervention was only 3%. However, this increase was not statistically significant. This marginal improvement in scores was rather unanticipated given that a significant reduction in difficulty levels and an increased use of correct methods were observed after GeoGebra intervention. This is likely due to the small number of test items that were asked in both the pre-test and the post-test. Increasing the number of items in the test and increasing the number of participants in the study is likely to alter this outcome.

7.6 Final word

The teaching and learning of probability cannot be left to chance. A deliberate effort by all stake holders in education from curriculum developers to curriculum implementers must be made in South Africa to ensure that ICTs are integrated into the teaching and learning of the topic. A successful integration of ICTs in probability teaching and learning, however, requires teachers' content knowledge (CK) of probability and their technological pedagogical content knowledge (TPACK) to be up to date. There is need for teacher training programmes to not only focus on equipping Mathematics teachers with basic computer skills, but to develop their confidence and competence in using ICT technologies in teaching Mathematics.

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Appendices

Appendix A: Consent Letters

A1 : Information sheet to the principal

Wits School of Education
University of the Witwatersrand
Private Bag 3
Wits
2050

The Principal and SGB
………..
26 September 2018

Dear Sir/Madam

RE: PERMISSION TO CONDUCT RESEARCH AT YOUR SCHOOL

My name is Innocent Moyo. I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing a research study entitled: "*The impact of ICT mediation on Grade 10 learners' understanding of probability: The case of GeoGebra in one Gauteng school*"

I am inviting your school to participate in this research.

My research seeks to identify the errors and misconceptions Grade 10 learners have when solving probability problems and to explore the impact of GeoGebra software in remedying such misconceptions. The students will be given a pen-and-paper pre-test on the topic. The test will be marked using a memo and an error analysis will be carried out. A smaller number of learners will then be asked to participate in a semi-structured interview in order to obtain a deeper understanding of their misconceptions. These interviews will be audio recorded for those participants who will have given me permission to do so. This will be followed by a teaching intervention which will be mediated using GeoGebra to simulate several probability situations. A minimum of one week and a maximum of two weeks will be used for these lessons. The lessons will be video recorded after consent from the learners and their parents has been sought and granted. The audio and video recordings will be transcribed for data analysis. Field notes will also be taken during these activities to try and obtain an understanding of how learners interact with the content of probability, the GeoGebra artefact and with each other. The final phase of the study will be a pen-and-paper post-test which will be analysed using error analysis.

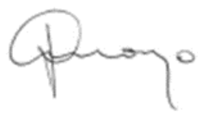
Participation by learners is completely voluntary and they can withdraw their participation at any stage of the study without any penalty. There are no foreseeable risks in participating in this study. The study will not disadvantage the participants in any way. There is a potential benefit for participants in that their understanding of probability concepts may be enhanced. The participants will not be paid for this study.

The names of the research participants and the identity of the school will be kept confidential at all times and in all academic writing about the study. No names will be used on the tests to identify the learners. Your individual privacy will be maintained in all published and written data resulting from the study. All research data will be destroyed between 3-5 years after completion of the project.

The reason why I have chosen your school is because of its convenient access for me as an educator and as a researcher. The learners at your school are confident in using technology to enhance their learning and have technological devices that can use GeoGebra.

Please let me know if you require any further information. I look forward to your response as soon as is convenient.

Yours sincerely,



SIGNATURE:

NAME: INNOCENT MOYO

EMAIL ADDRESSES: innoetho@gmail.com ; 1773068@students.wits.ac.za

CELLPHONE NUMBERS: 078 594 2264

NAME OF SUPERVISOR: Dr Judah Makonye

EMAIL ADDRESS: Judah.Makonye@wits.ac.za

A2: Information sheet parents

Wits School of Education
University of the Witwatersrand
Private Bag 3
Wits
2050

26 September 2018

Dear Parent

<u>RE: REQUEST FOR PERMISSION FOR YOUR CHILD TO TAKE PART IN A RESEARCH STUDY</u>

My name is Innocent Moyo and I am a PhD student in the School of Education at the University of the Witwatersrand. I am kindly asking for your permission for your child to participate in my research study entitled: "T*he impact of ICT mediation on Grade 10 learners' understanding of probability: the case of GeoGebra in one Gauteng school"*

The reason why I have chosen your child's class is that in Grade 10, the probability topic is covered in the syllabus and learners find the concepts associated with it confusing. I want to investigate the errors and misconceptions that learners have when solving probability problems and explore the impact that GeoGebra software has on the understanding of the topic.

My investigation involves identifying the errors and misconceptions Grade 10 learners have when solving probability problems and exploring the impact of GeoGebra software in remedying such misconceptions. The participating students will be given a pre-test on Grade 10 probability concepts. The test will be marked using a memo and an analysis of the type of errors made will be carried out. A smaller number of consenting learners will then be asked to participate in a semi-structured interview which will help me get a deeper understanding of their misconceptions. These interviews will be audio recorded with the permission of the participating learners and their parents. This will be followed by probability lessons which will be facilitated by me using GeoGebra to help participants to visualise the probability situations. A minimum of one week and a maximum of two weeks will be used for these lessons. The lessons will be video recorded with the permission of the participating learners and their parents. These lessons will then be followed by another test and the errors made will be analysed and the results used to determine the impact that GeoGebra had on the learning of probability.

If you give permission for your child to participate in the study, I need your child's help with participating in the following stages of my research: writing the pre-test, participating in the interview, participating in the lessons with GeoGebra and writing the post-test. I also need your permission for your child to be audio recorded during the interview and video recorded during the lessons using GeoGebra.
Your child will not be disadvantaged in any way. S/he will be reassured that s/he can withdraw her/his permission at any time during this project without any penalty. There are no foreseeable risks in participating and your child will not be paid for this study.

Your child's name and identity will be kept confidential at all times and in all academic writing about the study. His/her individual privacy will be maintained in all published and written data resulting from the study.

All research data will be destroyed between 3-5 years after completion of the project.

Please let me know if you require any further information.

Thank you very much for your help.

Yours sincerely,

SIGNATURE:

NAME: INNOCENT MOYO

EMAIL ADDRESSES: innoetho@gmail.com ; 1773068@students.wits.ac.za

CELLPHONE NUMBERS: 078 594 2264

NAME OF SUPERVISOR: Dr Judah Makonye

EMAIL ADDRESS: Judah.Makonye@wits.ac.za

Parent Consent Form

Please fill in the reply slip below indicating your willingness to allow your child to participate in the research study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

I, ______________________________ the parent of ______________________________

Permission for my child to participate in the pre-test **Circle one**
Agree that my child may write a pre-test for this study. YES/NO

Permission to participate in the post-test **Circle one**
Agree that my child may write a post-test for this study. YES/NO

Informed Consent
I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- s/he does not have to answer every question and can withdraw from the study at any time.
- s/he can ask not to be audiotaped, photographed and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ………………………………………

Parent Consent Form

Please fill in the reply slip below indicating your willingness to allow your child to participate in the research study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

I, ______________________________ the parent of ______________________________

Permission for my child to be interviewed	**Circle one**
Agree that my child may be interviewed for this study.	YES/NO

Informed Consent
I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- s/he does not have to answer every question and can withdraw from the study at any time.
- s/he can ask not to be audiotaped, photographed and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ………………………………………

Parent Consent Form

Please fill in the reply slip below indicating your willingness to allow your child to participate in the research study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

I, ______________________________ the parent of ______________________________

Permission for my child to be observed in class	**Circle one**
Agree that my child may be observed in class for this study.	YES/NO

Informed Consent
I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- s/he does not have to answer every question and can withdraw from the study at any time.
- s/he can ask not to be audiotaped, photographed and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ……………………………………….

Parent Consent Form

Please fill in the reply slip below indicating your willingness to allow your child to participate in the research study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

I, ______________________________ the parent of ______________________________

Permission for my child to be audiotaped	**Circle one**
Agree that my child may be audiotaped during the interview for this study.	YES/NO
I know that the audiotapes will be used for this project only.	YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- s/he does not have to answer every question and can withdraw from the study at any time.
- s/he can ask not to be audiotaped, photographed and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: …………………………….… Date: ……………………………………….

Parent Consent Form

Please fill in the reply slip below indicating your willingness to allow your child to participate in the research study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

I, ______________________________ the parent of ______________________________

Permission for my child to be videotaped	**Circle one**
Agree that my child may be videotaped in class.	YES/NO
I know that the videotapes will be used for this project only.	YES/NO

Informed Consent

I understand that:

- my child's name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- s/he does not have to answer every question and can withdraw from the study at any time.
- s/he can ask not to be audiotaped, photographed and/or videotaped.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: …………………………….… Date: ……………………………………….

A3: information sheet for learners

Wits School of Education
University of the Witwatersrand
Private Bag 3
Wits
2050

Dainfern College
P O Box 3199
Dainfern
2055

26 September 2018

Dear Learner: ……………………………………………………

My name is Innocent Moyo and I am a PhD student in the School of Education at the University of the Witwatersrand.

I am doing a research study entitled "*The impact of ICT mediation on Grade 10 learners' understanding of probability: The case of GeoGebra in one Gauteng school*". I am inviting you to participate in this research.

My investigation involves identifying the errors and misconceptions Grade 10 learners have when solving probability problems and exploring the impact of GeoGebra software in remedying such misconceptions. The participating students will be given a pre-test on Grade 10 probability concepts. The test will be marked using a memo and an analysis of the type of errors made will be carried out. A smaller number of consenting learners will then be asked to participate in a semi-structured interview which will help me get a deeper understanding of their misconceptions. These interviews will be audio recorded with the permission of the participating learners. This will be followed by probability lessons which will be facilitated by me using GeoGebra to help participants to visualise the probability situations. A minimum of one week and a maximum of two weeks will be used for these lessons. The lessons will be video recorded with the permission of the participating learners and their parents. These lessons will be followed by another test and the errors made will be analysed and the results used to determine the impact that GeoGebra had on the learning of probability.

Your participation in this research is purely voluntary. You can withdraw from the study at any time without any penalty. I do not foresee any risks in you participating in the study. Rather, your participation may potentially benefit you by enhancing your understanding of probability concepts through the use of GeoGebra. Please note that the pre-test and post-test are not tests for marks.

Your real name will not be used throughout the research, but I will make up a name that I will use to refer to your data in order to protect your privacy and confidentiality. No information about you will be disclosed when reporting the findings of the study. I will store all data collected from you in a safe place, and between 3 – 5 years after I have completed my research, all this information will be destroyed.

Your parents have also been given an information sheet and consent form, but at the end of the day it is your decision to join us in this study.

If you accept my invitation, I need your help with participating in the following stages of my research: writing the pre-test, participating in the interview, participating in the lessons with GeoGebra and writing the post-test. I also need your permission to be audio recorded during the interview and video recorded during the lessons using GeoGebra.

I look forward to working with you!

Please feel free to contact me if you have any questions.

Thank you

SIGNATURE:

NAME: Innocent Moyo

EMAIL ADDRESSES: innoetho@gmail.com; 1773068@students.wits.ac.za

CELLPHONE NUMBER: 078 594 2264

NAME OF SUPERVISOR: Dr Judah Makonye

EMAIL ADDRESS: Judah.Makonye@wits.ac.za

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

My name is: ____

Permission to participate in the pre-test	**Circle one**
I agree to write a pre-test for this study.	YES/NO
Permission to participate in the post-test	**Circle one**
I agree to write a post-test for this study.	YES/NO

Informed Consent
I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ………………………………..… Date: ……………………………………….

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

My name is: ____

Permission to be questionnaire

I agree to fill in a questionnaire for this study YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ……………………………………….

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

My name is: ____

Permission to be interviewed

I would like to be interviewed for this study. YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I can stop the interview at any time and don't have to answer all the questions asked.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ……………………………………….

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

My name is: ____

Permission to observe you in class

I agree to be observed in class. YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ……………………………………….

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

My name is: ____

Permission to be audiotaped

I agree to be audiotaped during the interview or observation lesson	YES/NO
I know that the audiotapes will be used for this project only	YES/NO

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign: ……………………………..… Date: ……………………………………….

Learner Consent Form

Please fill in the reply slip below if you agree to participate in my study called:

THE IMPACT OF ICT MEDIATION ON GRADE 10 LEARNERS' UNDERSTANDING OF PROBABILITY: THE CASE OF GEOGEBRA IN ONE GAUTENG SCHOOL

My name is: ____

Permission to be videotaped

I agree to be videotaped in class.	YES/NO
I know that the videotapes will be used for this project only.	YES/NO

Informed Consent

I understand that:

- my name and information will be kept confidential and safe and that my name and the name of my school will not be revealed.
- I do not have to answer every question and can withdraw from the study at any time.
- I can ask not to be audiotaped, photographed and/or videotape
- all the data collected during this study will be destroyed within 3-5 years after completion of my project.

Sign____________________________ Date__________________________

Appendix B: Ethics clearance

B1 University of Witwatersrand

Wits School of Education

WITS UNIVERSITY

27 St Andrews Road, Parktown, Johannesburg, 2193 Private Bag 3, Wits 2050, South Africa. Tel: +27 11 717-3064 Fax: +27 11 717-3100 E-mail: enquiries@educ.wits.ac.za Website: www.wits.ac.za

25 October 2018

Student Number: 1773068

Protocol Number: 2018ECE022D

Dear Innocent Moyo

Application for Ethics Clearance: Doctor of Philosophy

Thank you very much for your ethics application. The Ethics Committee in Education of the Faculty of Humanities, acting on behalf of the Senate has considered your application for ethics clearance for your proposal entitled:

The impact of ICT mediation on Grade 10 learners' understanding of probability: The case of GeoGebra in one Gauteng school.

The committee recently met and I am pleased to inform you that **clearance was granted**. However, there were a few small issues which the committee would appreciate you attending to before embarking on your research.

The following comments were made:

- State that the video must not be made public and once used, for the study, is wiped.
- The number of consent forms needing signatures, for participants and their parents, may annoy the participants. Maybe consider combining some of these forms so that participants only need to sign 2 or 3 at the most.
- It is not clear how and when the questionnaires will be used in the study, please clarify.

Please use the above protocol number in all correspondence to the relevant research parties (schools, parents, learners etc.) and include it in your research report or project on the title page.

The Protocol Number above should be submitted to the Graduate Studies in Education Committee upon submission of your final research report.

All the best with your research project.

Yours sincerely,

Wits School of Education

011 717-3416

Cc Supervisor: Dr Judah Makonye

Appendix C: Interview questions and transcripts

C1 Interview questions used to develop the interview protocol

1. In your own words, would you please describe what probability is or what it entails?
2. Do you think that probability knowledge is applicable in real life?
3. How confident are you in probability? Rate yourself from 0 to 10, 0 being poor and 10 being excellent..
4. What do you like about probability?
5. What do you dislike about probability?
6. Can you describe any experience in class that helped you gain a better understanding of any concept in probability?
7. Can you describe any experience in class that prevented you from gaining a better understanding of any probability concept?
8. Can you describe any experience in class or outside class that helped you solve probability problems?
9. Can you describe any experiences in class or outside class which prevented you from solving probability problems?
10. What do you think were good things about these lessons? What do you think made them good?
11. What main things did you learn in this lesson? What do you think helped you learn them?
12. What things do you think were difficult in this lesson? What made them difficult? What helped you with them?
13. In what ways did the use of GeoGebra help or not help? What made it helpful or not helpful?
14. What do you think could have been done better in this lesson? How do you think that would have helped you?
15. How would you evaluate the contribution of GeoGebra intervention to your personal experience?
16. What knowledge and skills do you think you acquired or developed through the use of GeoGebra?
17. Do you have any comments or anything you would like to say?

C2 Matrix showing the link between research questions and the interview questions

	Background information	Primary research question	RSQ 1	RSQ 2	RSQ 3
Interview Q 1	x				
Interview Q 2	x				
Interview Q 3	x		x	x	
Interview Q 4	x		x	x	
Interview Q 5	x		x	x	
Interview Q 6	x	x	x	x	
Interview Q 7	x	x	x	x	
Interview Q 8	x	x	x	x	
Interview Q 9	x	x	x	x	
Interview Q 10		x	x	x	
Interview Q 11		x	x	x	
Interview Q 12		x	x	x	
Interview Q 13		x	x	x	x
Interview Q 14		x	x	x	x
Interview Q 15			x	x	
Interview Q 16			x	x	
Interview Q 17			x	x	

C3 Interview transcript NN

1	Tr: Tell me did you attend probability lessons in the beginning of the year?
2	NN: Yes I did?
3	Tr: What do you say about probability? Did you find it easy?
4	NN: It's ok ... it's aahh .. it really depends on the question.
5	Tr: So you think it depends on the question?
6	NN: Yeah.
7	Tr: What do you mean it all depends on the question?
8	NN: I mean there are some questions that are easy, but others are difficult. but I think like probability is easy to apply.
9	Tr: To apply where?
10	NN: In real life. It's easy to apply in real life but then … in Maths questions it's too much.
11	Tr: So, you are saying probability is applicable in real life?
12	NN: Yeah.
13	Tr: I agree with you. But what does it entail though? What are your own thoughts for on what probability entails?
14	NN: For me it's like the likelihood of something happening.
15	Tr: Ok.
16	NN: It's like the likelihood of something happening. Like what are the chances.
17	Tr: Okay. Yeah, that's true. That's true. You spoke about some aspects of probability that are challenging and you said it depends on the question. Do you remember one section or one aspect of probability that you find challenging? … any concepts in probability that you find challenging? or difficult?
18	NN: I can't remember.
19	Tr: Okay, that's fine.
20	Tr: I want to look at what you did here in this paper.
21	NN: (… sounds anxious)
22	Tr: You actually did very well.
23	NN: Okay [... sounds relieved]
24	Tr: You really did well. There are just these few questions that you did not get right. My effort is to try and understand what you were thinking. I want to see if I can think like you. I don't really want to correct you, but I just want to think Like you. For instance, why did you not respond to this question? I wrote here "why did you not provide an answer for question 1 (a)? The question says "If a coin is flipped 8000 times and it lands on heads 8000 times then that coin must be biased. Do you agree with this statement or not?" And you left it unanswered. Why?
25	NN: [Laughing]
26	Tr: Do you have any reason why you could not answer this question?
27	NN: Because I don't understand why a coin can be biased. It's a coin. How can it be biased?
28	Tr: Oh, so what you don't understand is that the coin is biased?
29	NN: Yeah, that bias part confused me.
30	Tr: what's your understanding of bias?
31	NN: I understand that it favours one thing over another. Sir, but how can a coin be biased?
32	Tr: So, in this case we are saying we flip this coin 8000 times and every time it is showing us heads.
33	NN: Yeah, but then the coin is not biased it's just … luck. [laughing]
34	Tr: Oh, so it's just luck?

35	NN: Yeah.
36	Tr: So, are you saying it is possible that you can throw a coin 8000 times and it can lend on one face? Is there a chance like that?
37	NN: There is a chance. It might be small but there is a chance.
38	Tr: What about a dice or a die? Can it be biased?
39	NN: I don't think so.
40	Tr: Okay, that is fine. So, tell me, if you believe that it coin cannot be biased what do you think it can be? Do you a term for that?
41	NN: [silence]
42	Tr: It's not biased, so what do you think it is?
43	NN: [silence]
44	Tr: Do you know the opposite of bias maybe?
45	NN: No sir. I don't know the opposite of bias.
46	Tr: It's fair.
47	NN: Oh! Okay. Okay.
48	Tr: So, you are right. Bias means it favours one over the other. Fair means it gives the heads and tails equal chances.
49	NN: Equal chance … yes.
50	Tr: I agree that most of the coins or all of the coins that we use are not biased. But do you believe that you can toss a coin 8000 times and you get a thousand heads?
51	NN: That wouldn't be fair for the tail really.
52	Tr: But is it possible?
53	NN: It is possible. It's chance, so anything is possible.
54	Tr: I agree with you. Then you tried this question here.
55	NN: Tjo!
56	Tr: You toss a coin 500 times and heads show up 320 times. When you toss it again, do you think it will lend on a head or on a tail? And you said Head because the probability of getting it is higher.
57	NN: Wow! What was I saying?
58	Tr: Do you want to change that?
59	NN: A coin is tossed 500 times and heads show up 320 times …. Hmmm … [thinking]
60	Tr: Can you explain what you were thinking or what you think? Or would you stick to your idea that because the head shows many times then it has to be the head again in the next toss?
61	NN: No... that was silly … that's just a silly answer.
62	Tr: So, what do you think is the answer?
63	NN: Sir, I feel like …uhm … wait what's the question? When you toss it again…?
64	Tr: Yeah, like you tossed it 500 times and you get the hands showing 320 times. So, if you toss the coin one more time what will happen?
65	NN: Oh. Actually, now that I'm looking at it properly, I feel like sir it's a coin again. and a coin is fair. So, it could be heads or it could be tail. I don't think that the past … I don't think that the past outcome should affect what will happen next. I don't know what I was thinking.
66	Tr: Okay, so you think the next time we toss it again …
67	NN: It really won't depend on what happened.
68	Tr: So, it could be either a head or a tail and not necessarily a head? But you gave a nice answer though.
69	NN: No, that is not a nice answer.
70	[both laughing]

71	Tr: Okay, what about (d) and (e) and (f). My interest in these is that I noticed that when you were trying to answer these questions you tried to come up with a strategy. You decided to draw a Venn diagram. You drew a Venn diagram for (d) and (e) and for this one. How did you hope that drawing the Venn diagrams was going to help you?
72	NN: I just think that a Venn diagram like… it just makes the information easier to read.
73	Tr: Ok.
74	NN: Instead of reading it a couple of times … I try to draw it out so that I can just see the information and just see exactly where everything falls in the Venn diagram.
75	Tr: Okay.
76	Tr: Here you said it's mutually inclusive.
77	NN: Yes, I don't think I actually wrote the correct answer.
78	Tr: So, you are saying mutually inclusive. Do you think it's related to your diagram?
79	NH: I think …. Uhm. I think so.
80	Tr: Okay. Why?
81	NN: Because it's included or it's not.
82	Tr: Included or it's not?
83	NN: In the circles or not.
84	Tr: What is included there? Like in your diagram, what is included?
85	NN: I think the numbers that are inside the two circles.
86	Tr: Yeah, okay. Inclusive means there's something common.
87	NN: What does that mean?
88	Tr: If you look at your 2 circles there, they do share something. So that is the meaning of inclusive.
89	NN: Oh, okay.
90	Tr: The way you drew it there and the conclusion you have here is fine because that shows that it's inclusive and you said it's inclusive.
91	NN: Right. Yes.
92	Tr: But the question was asking whether the events are mutually exclusive or not. And I see you are saying they are not mutually exclusive but inclusive which is correct.
93	NN: Yeah.
94	Tr: Initially I was asking about the strategy. I noticed that your Venn diagram matches your answer which is okay. I'm happy with that. Then here I see you drew only one circle there and the question was asking again whether X and Y are mutually exclusive. But, however, you decided to draw only one circle and not two and you said they are mutually exclusive. So, what do you think mutually exclusive means?
95	NN: Like they don't have anything in common.
96	Tr: Okay, that's correct. I thought you were going to draw two circles that don't have anything in common.
97	NN: Yeah, I should have.
98	Tr: I like the fact that you realise you may need to draw Venn diagrams to answer some of these questions. That's a very good strategy.
99	NN: Yeah.
100	Tr: Do you think maybe that these statements were confusing? Like when you say X represents learners in grade 10 that are in the swimming team and Y represents learners in grade 10 who are in the debating team. Do you think X and Y are exclusive or inclusive? Did you find that confusing?
101	NN: I don't think so.
102	Tr: Is it clear?
103	NN: Yeah, I think … I think it's clear

104	Tr: Okay. And this one here says X is red cars driven by women and Y is white cars driven by women. You said these too a mutually exclusive and you gave the reason as one cannot drive a car that is both red and white.
105	NN: But one can
106	Tr: But at the same time?
107	NN: Oh, not at the same time. No.
108	Tr: How do you differentiate between complementary events and mutually exclusive events? Do you think they are different?
109	NN: Okay, I don't know. What is that?
110	Tr: Do you want me to help you with that?
111	NN: Yes. Yes please.
112	Tr: Mutually exclusive means they do not share anything in common. …. [tr explains the difference] …. complementary means (1) they should be mutually exclusive and (2) they should be exhaustive [tr explains in detail]
113	Tr: What about this one? X means "boys in grade 10 or play soccer" and Y means "girls in grade 10 or play soccer" Are they mutually exclusive?
114	NN: I think they are mutually exclusive. Because they have nothing in common boys and girls.
115	Tr: But do you think X and Y are complementary?
116	NN: Yes, they should be complementary because they will give you all the Grade 10s. […] No, they won't. No, wait. What? Uhm.
117	Tr: Think about it like this if I take all the Grade 10 boys who play soccer and all the grade 10 girls who play soccer and I put them together does that give us all the grade tens in the school?
118	NN: No, because not necessarily all the boys play soccer and not necessarily all the girls play soccer.
119	Tr: What about 12 c? You drew a Venn diagram, and everything is correct. We want to find the probability that one learner selected at random is left-handed or plays soccer.
120	NN: Okay. Let me look at it again. Then isn't it …. Uhm….
121	Tr: You said 3 + 4. Where is 3 + 4 coming from?
122	NN: Hmm. I think what I'm seeing here, Mmm ...from the ... hmmm …The learners that are left-handed … the learners that are left-handed and play soccer. that's why I see a 4 there.
123	Tr: Is it this 3 and that 4?
124	NN: Yeah.
125	Tr: Okay. But it says left-handed or play soccer. So, what do you think about that 14 according to your diagram?
126	NN: I should have included it. …. So, they can be right-handed as long as they play soccer.
127	Tr: Yes. What do you think?
128	NN: They can be right-handed as long as they play soccer.
129	Tr: Exactly.
130	NN: So, I should have added 14.
131	Tr: Yes. But if the question said they are left-handed and play soccer what would you have?
132	NN: I would just use four.

C4 Interview transcript SH

1	Tr: Alright Sh.. , can you describe in your own words what probability entails, what it involves or what it is all about?
2	Sh: Ok. Eish, probability … You just asked one of those questions that are just there, but you can't explain. I guess probability is about, you know, chances or possibilities of something happening.
3	Tr: Okay. Do you think the knowledge of probability is applicable in real life in real life? Do you think we need probability in real life?
4	Sh: I think in real life basically … can I say maybe we go through probability every day and maybe we just don't notice or see that this is probability. But we go through it, like maybe the chances of you getting to a mall and choosing a restaurant … there is Spur, there is McDonalds and so on. You could go into Spur or into McDonalds. That's also a probability because you could choose one out of the many shops in the mall that you went to.
5	Tr: Good. That's a brilliant answer. We use probability every time. Do you like probability though?
6	Sh: I like probability although it's challenging. I love the challenges it comes with. The fact that it gets you to think deeper. Even though sometimes the answers is really simple and you think deep for no reason, but I feel like that challenge that you go through actually helps you, you know. Sometimes it starts as probability but, you know, it ends up helping you with other things and that could be in life problems in decision-making.
7	Tr: Ok so you know when we are learning probability it's all about understanding, especially in class. I want you to (if you can remember anything), any experience in class maybe … it could be an activity that took place in class which actually helped you to gain a better understanding of a particular concept regarding probability. Do you remember such an experience or an activity?

8	Sh: Yeah, yeah. I guess probability …. I'm not too sure what I can choose. But like, I don't know, it is just one of those topics where I go to class and the teacher explains. You just have to understand … there is no way around probability. For example a coin, you can't really say there is three sides to a coin. It's always gonna be two sides. Like a pack of cards, they can either tell you, you are picking a Joker, you know. The thing with probability is the thing they give you (like a die) it can never change. They can never say a die that has eight sides. So what you have to figure out is you have to know English basically. It's the issue of language and understanding.
9	Tr: Okay. Yeah, that's deep. That's good. So what do you think. We talk about methods of learning. We used GeoGebra to carry out some experiments. Do you think there were any good things about those lessons?
10	Sh: Yeah.
11	Tr: What do you think those are and why do you think those were good?
12	Sh: I think …. for some people, isn't it we are different learners you know. So for visual learners who get to see that okay, the die here, the chance of getting a one while you are looking at the video, it actually helps them understand deeper than actually reading a question or maybe a textbook. You know it's different, so … that chance to just get to see that okay the die was rolled maybe two times or maybe two dice were rolled here and the probability and stuff you know, so .. it depends I guess on the type of learner that you are. I guess for visual … I would say for visual learners, it is very advisable that we use GeoGebra.
13	Tr: Ok. So it's about visualising things.
14	Sh: It accommodates many learners.
15	Tr: Anything maybe that you think could have been done better in those lessons? Or anything that was difficult because of the use of GeoGebra?

16	Sh: I didn't see anything difficult from my own point of view. I think the lesson was straight forward. Whether you are a fast learner or a slow learner it could accommodate both of you. So it was just … it was a straight forward lesson. Nothing was missing. Even the way people did not ask questions … the questions … like the number of questions that the learners ask actually show that there is maybe a bit confusion. But the way people were answering even giving their own examples shows that people were actually understanding the lesson. The way they could relate maybe to the world and to people … it showed that the lesson was in the right path and they could understand.
17	Tr: So talking about the relating to the world and relating to people, do you think using GeoGebra to learn gives you as learners a chance to interact with one another?
18	Sh: [….]
19	Tr: So during those lessons, do you think people were talking to each other or interacting with each other?
20	Sh: Yeah … it actually gives people the chance to voice out their own opinion as you see … you see as people … as you saw, they used examples from the real world and crime … it was his way of understanding that concept. So the way he might have explained it might have actually helped someone else to understand it better than the already did. So it actually helped us to interact and share some ideas and help each other in the process to understand.
21	Tr: Okay. Good. We were using GeoGebra to generate numbers because in statistics and probability we need to be working with numbers. Do you think we could have may be obtained those numbers in another way? Do you think using GeoGebra was a waste of effort?
22	Sh: No. It was definitely not a waste of effort. I think I don't know of any other way. Maybe I don’t know … even our calculator would take so long. I feel like, it was very fast like we were actually able to calculate a die maybe after 10 times of flipping and all that. You could have never done that with a calculator. You actually even need a real die to even experiment what would happen and what would go through. So GeoGebra actually gave us you know that chance … it’s actually fast and you know

	you don’t have to waste time thinking that okay if I roll the die, but they said there is one die, … you know. It’s just gives you the things you need to know immediately and now it’s up you and your understanding.
23	Tr: That's nice. Do you have any comments or anything you would like to stay in closing?
24	Sh: In closing I’d like to say … actually I would like to advise that kids especially from Grade 10 get this app, … because Grade 10 I've noticed personally, is the foundation of all the things that we will learn as we go until we get to grade 12. It would actually be better if we were able to have this app and have access to this app and … you know … use it in our favour may be in class when we don't understand and the teacher can further explain and assist and maybe the friends can also explain and assist and stuff like that.
25	Tr: Thank you so much. And the good thing is about the app is that it is not only for probability. You can use it in algebra, you can use it in functions, you can use it for geometry … It was particularly designed for maths and it's free
26	Sh: It’s free?
27	Tr: Yes, it’s free. You can download it anytime on your phone, or on any device and you can use it offline. You won't need any data or internet. … Thank you so much for your insight. That was so important.
28	Sh: Thank you.

C5 Interview transcript MV

1	Tr: Thank you Mv… I want you to tell me in your own words what you think probability is all about? What does probability entail?
2	Mv: Personally, I'd say probability is basically about chance … the chance of certain things happening and the chance of them not happening … along those lines.
3	Tr: Okay. And do you think in real life the knowledge of probability is applicable? Where do we apply probability in real life?
4	Mv: I think probability is applied in literally every single aspect of our lives and in every decision we make and in everything we do, we have to always check okay there is a probability of something happening … if I do this or if I do that.
5	Tr: Do you like it? Do you like probability?
6	Mv: I think everyone has to like it because there is no choice. If you like it or you don't it's a part of life. Yes I do like it.
7	Tr: Okay. Would you think of an experience inside class or outside class where you were learning probability and maybe something helped you to gain an understanding of something? Can you think of an activity or an experience in class or outside of class that helped you to understand a certain aspect of probability in a better way?
8	Mv: I'd say one situation is through rolling a dice. For example, just playing games or anything in class with friends like monopoly and everything … probability plays a big role and you get to see how it really works there and yeah …
9	Tr: Okay. Good. To acquire knowledge, like probability knowledge, especially in classroom settings, we need to come up with or to think about methods of learning. I want you to tell me … you can refer to the GeoGebra lessons that we had… are there any good things about those learn lessons that you can remember which you can share with me?

10	Mv: I'll say one pretty interesting thing is we get to realise that the more dice you roll, the more …the more dice you roll it's pretty much the same as the less you roll in the sense that if you roll 10 times you basically get the same result as when you roll 1000 times.
11	Tr: Okay. Alright. And in terms of … because in probability you are working with numbers … we need numbers to talk about probability. I know that chance can be expressed in many other ways, like we can use words like "almost", "probably" etc, but we can put that as a measure using numbers. So we were generating numbers using GeoGebra. Do you think there could be a better of getting the numbers that we used?
12	Mv: Oh you mean like you could check around the world and gaining …
13	Tr: So you are saying like checking around the world ..
14	Mv: Yes, and other people's experiments. But otherwise I feel like GeoGebra probably the most practical thing to do.
15	Tr: Okay. So you think it was effective?
16	Mv: Yes, definitely.
17	Tr: Okay. Did it allow you time, like do you think you got enough time to analyse your numbers? Like I said, in statistics and probability you get numbers and then you analyse the numbers. And, like you said earlier on, you then make decisions using the numbers. Was there enough time to analyse the numbers?
18	Mv: It definitely gave us a lot of time to work with the numbers. It gave us time to work with unrealistically high numbers. It's a tool that makes us do things that would otherwise be very impossible such as being able to roll a die like 30 million times.

19	Tr: Are there any skills that you think you developed or you gained from the use of GeoGebra? Mv: I think, basically being able to see the relative frequency like being able to calculate it. Because due to GeoGebra you could calculate these big numbers and can see how the relative frequency relates with the small numbers and the large number of dice rolled or flipping of the coin. So I would say through GeoGebra I could basically get a one hundred percent factual answer.
20	Tr: Okay. How do you think the use of GeoGebra in our lessons helped the class to interact with each other?
21	Mv: Okay, it helped us in a sense that we all had our own theories. We all thought certain things and the use of GeoGebra easily showed us like in real time the unrealistic possibilities that we could not possibly do in the classroom setting … being able to roll many times and just proved everyone's theory correct or wrong.
22	Tr: Okay. So would you say there was enough interaction among the students?
23	Mv: Yes. Through GeoGebra we were able to go through the simulation of our theories and see what's actually probable.
24	Tr: What about the benefits of using GeoGebra? I know that we have five sense, I just want you to think about your senses. How do you think GeoGebra helped you to apply your senses?
25	Mv: Okay. Personally I would say the only thing you probably saw or used was the sense of sight. So I guess one negative is that a person doesn't really get to feel it, but it's more of a simulation that happens in the [face?]. Usually some people believe by… by actually rolling the dice. Because someone could say, I can roll the die in a certain way and get whatever number I actually want. They can have their own strategies … while GeoGebra does everything fairly in a way. So, that's where it kind of makes it unfair because you only get the sense of sight. Other people have other senses they can use to manipulate the game and get heads always.
26	Tr: Okay. That's good. Do you have any comments or any questions?

27	Mv: I would say for one I really enjoyed the GeoGebra lessons and I feel like it's an app that’s necessary.
28	Tr: Thank you so much Mv.
29	Mv: You are welcome.

C6 Interview transcript KV

1	Tr: So Kv .., can you describe for me in your own words what probability is all about?
2	Kv: In my own words, probability has a lot to do with the with the number of chances something has compared to something else. Like how many chances does it have in occurring compared to something else.
3	Tr: Okay so its chances, comparisons and so on? Yes, you are right. What about in real life, do you think the knowledge of probability is applicable? Do you think we use it in real life?
4	Kv: Yes we do use it in real life. For example we use it to compare … for example, like when my parents are coming home who was a higher of coming home between my dad and my mom.
5	Tr: Okay. So that you can fix the area that needs to be fixed.
6	Kv: Exactly. If my dad is coming first I have to yeah … fixed something related to him.
7	Tr: And what about in business, … in the business world? Do you think we need probability?
8	Kv: Yes we do because in order to like … when it comes to business … I think of when it comes to making decisions and actually getting the best out of the decisions you have to consider probability. Let's say you are entering a market … like, let's say you are starting a new business I look at the … like … the higher chance I have of succeeding in one certain market compared to another. For example, there is a higher chance of success in the engineering industry compared to another field.
9	Tr: So you can use it approximate the success rates in different fields of or occupation?
10	Kv: Yes.
11	Tr: Yes, I agree with you. But do you like probability? And what do you like about it if you do?

12	Kv: I do like it because it gives you a clear road … you can use probability to make the right decisions.
13	Tr: Okay, for decisions?
14	Kv: Especially for decisions for everyday life, comparing things so that you can get the best chance out of anything really in life. For example, when I go to university I'll have to decide which course I want to have and basically I have to compare the different courses.
15	Tr: Okay, that's good. But is there any particular thing you don't like about probability? It doesn't have to be there but can you think of anything?
16	Kv: It can be complicated. I don't like it when applying it I guess. There are so many factors to consider which does make it a little bit complicated because it's not only just about coins.
17	Tr: Okay. Do you think the language that is used in probability is easy to handle?
18	Kv: It's not easy to handle. That is actually one of the things that I don't like about it, the language. It isn't easy to handle because even though probability is always used, but nobody uses the language in everyday life. People who are experts in it use it and it's really used in like mathematics and in technical areas like in technology and engineering. But aside from that, in everyday use, even though probability is used every day, it is a bit difficult … it's not user friendly.
20	Tr: Okay. That's good. What about the lessons now, the GeoGebra lessons? Do you think there were any good things about the lessons especially where we were using GeoGebra? Anything you can think of from those lessons that you think were good?
21	Kv: Yes, there were lots of good things. For me the lessons opened me up .. like they showed me that, for example with coins, no matter how many times you flip a coin, there is a chance that both the sides of the coin have an equal chance. I guess you can use that knowledge to apply in real life as well.
22	Tr: So in life for instance, as human beings we need to interact. Do you think the use of GeoGebra in a lesson helps learners or people to interact with each other?

23	Kv: It definitely does.
24	Tr: Did you notice that in our lessons?
25	Kv: Yes everyone had their own opinions. It definitely lit a light bulb because everyone was using their mind and thinking trying to understand and it was fun and we were learning something. I learnt something definitely … and other people's opinions were considered. The thing about probability and the use of the app is that it's factual, it's right there. So all your questions are answered immediately.
26	Tr: Okay. What about in terms of time (talking about efficiency), how do you think GeoGebra addresses that problem?
27	Kv: In terms of time … it answers .. uhm… in terms of time…
28	Tr: Yeah. You can refer back to the lessons. Do you think you noticed anything about whether time was managed in a certain way? It could be a good way or a bad way. Anything you can remember about the time factor?
29	Kv: Overall, the lessons were not that long. It didn't take too much time …. so it's not time consuming.
30	Tr: Remember we were using many numbers. Where did we get those numbers from?
31	Kv: We got those numbers from … the numbers were there, or present in the app. We just clicked and we got the numbers.
32	Tr: Without the app, how would we get those numbers?
33	Kv: We probably have to do it practically or manually. We were able to do like … we were able to go numbers like 800 flips, 30 000 rolls, in order to get the answer and that did not take time at all. It just took you less than five seconds. So it was it was not time consuming.
34	Tr: Okay. Good. Then we used the rest of the time to talk about those numbers.
35	Kv: Yes exactly.
36	Tr: Thank you so much. Do you have any comments or anything in closing that you would want to say?

37	Kv: Not much but I really did enjoy the lesson. It was fruitful and I learnt a lot that can be applied. It just opened my mind regarding probability and it just helped me think quickly in a sense.
38	Tr: Okay. Thank you Kv.
39	Kv: Thank you sir.

C7 Interview transcript EL

1	**Tr**	So, what I want to know is …
2	**El**	Yes sir
3	**Tr**	… they are asking here for question two for you to draw a Venn diagram. And the Venn diagram should show your outcomes for A, B and C, and you put a 1 in the intersection of a and C or K. But what outcomes go inside A?
4	**El**	2, 3, 5
5	**Tr**	And what are they called?
6	**El**	Prime numbers.
7	**Tr**	So, what mistake did you make?
8	**El**	I put 1
9	**Tr**	And what is wrong with that?
10	**El**	1 is not a prime number.
11	**Tr**	Ok. Alright. Good.
12	**Tr**	So, do you think you know where 1 should have gone?
13	**El**	I think I should have put 1 just for C.
14	**Tr**	Ok, 1 just for C?
15	**El**	Yes sir
16	**Tr**	Ok, right. I agree with you. And then your answers for the next question, like B, what does that symbol mean?
17	**El**	Probability of A
18	**Tr**	And what is your answer?
19	**El**	Uhm ...
20	**Tr**	You said 1,2,3,5. Where are those numbers coming from?
21	**El**	From S
22	**Tr**	Ok, yeah. They are there in S … but the question says probability of A.
23	**El**	Oh…
24	**Tr**	The numbers 1,2, 3, 5 … where are they coming from?
25	**El**	A
26	**Tr**	Coming from A?

27	**El**	Yes sir
28	**Tr**	Ok yeah. That is correct. That is the correct. Set A.
29	**El**	Yes. Yes.
30	**Tr**	But then you wrote the actual numbers 1,2, 3, 5.
31	**El**	Oooh…
32	**Tr**	Do you remember what we measure when we measure probability?
33	**El**	Like numbers divided by the like …. I should have just said like … like 3 / the numbers of… (not audible)
34	**Tr**	Right.
35	**El**	So maybe 5 /
36	**Tr**	So, in your case how many numbers are in A?
37	**El**	3÷ 5
38	**Tr**	Where is the five coming from?
39	**El**	All the numbers in the Venn diagram
40	**Tr**	I see six numbers. You wrote 6 numbers.
41	**El**	Yes, but 1 should be here.
42	**Tr**	Okay. So, 1 should be there.
43	**El**	Yes
44	**Tr**	So, is it / 5 or / 6?
45	**El**	/ 6
46	**El**	÷ 6?
47	**Tr**	But do we only have the six numbers in the Venn diagram?
48	**El**	Silence….
49	**Tr**	Is it the only numbers that we have?
50	**El**	No, maybe these numbers let me see.
51	**El**	This one's right?
52	**Tr**	Yes.
53	**El**	So, if you count …...
54	**El**	They will be 18
55	**Tr**	okay they'll be 18?
56	**El**	Yes
57	**Tr**	Inside the whole set S there should be 18 numbers.

58	**El**	Yes
59	**Tr**	And so …. but inside A there is three numbers.
60	**El**	Yes
61	**Tr**	So, there is 3 out of 18.
62	**El**	Yes sir.
63	**Tr**	Ok so 3 out of 18. What is the maximum value that you can ever get for probability? Remember probability is a measure.
64	**El**	100
65	**Tr**	You mean 100%? and hundred percent means 1.
66	**El**	Yes.
67	**Tr**	What about the smallest or the least value for probability?
68	**El**	0
69	**Tr**	Ok 0 correct.
70	**Tr**	So, do you understand the mistake that you made there?
71	**El**	Yes, I do now.
72	**Tr**	And you see that this is the same mistake here?
73	**El**	Yes
74	**Tr**	So, if you were to correct that 1 and put it where it belongs, how many numbers will be there in A?
75	**El**	There will be three.
76	**Tr**	Okay. 3
77	**Tr**	But how many numbers are in S?
78	**El**	18.
79	**Tr**	Then tell me, what is the meaning of A or B?
80	**El**	It means like it's contained in both of them.
81	**Tr**	Ok
82	**El**	So, what did you do here? In your case I can see that you are giving me a summary of the numbers in A. You are giving me 1, 2, 3, 5, 15 and 30. These are the numbers I suppose which are in A and in B?
83	**El**	Yes
84	**Tr**	So, if you understand part (b) your answer here was going to be …?

85	**El**	Probability of A + probability of B - probability of A + probability of B.
86	**Tr**	Minus?
87	**El**	Probability of A + probability of B minus probability of A intersection B.
88	**Tr**	Yes. Correct. That is the formula. I'm happy you remember it. But just remember probability is a measure between 0 and 1 you can express it as a percentage if you want.
89	**El**	I think I get confused by the factors and stuff ..
90	**Tr**	Okay. Yeah, factors and the related stuff. Thank you El
91	**El**	Thank you, sir.

C8 Interview transcript S

1	Tr: So tell me S, in your own words how would you describe probability?
2	S: Probability in my own words I would say is all about statistics and numbers. Like what we did in our lessons … like how many times you would get a head or a tail when a coin is tossed.
3	Tr: Okay so you think it's about numbers
4	S: Yes numbers and statistics
5	Tr: Do you think that the knowledge of probability is applicable in real life?
6	S: Yes I think it's applicable because ….you can use it in population in a certain country or in the whole world. You can use it like to determine how many people are in a country for a particular period of time.
7	Tr: So tell me if you were to rate yourself from 0 to 10 where 0 is poor and 10 is excellent how confident do you think you are in probability?
8	S: I would say 8 out of 10.
9	Tr: Okay. Do you have something in particular that you like about probability?
10	S: Yeah. I like the way it gives you the amount like the way it gives you how the certain amount is calculated … It gives you the correct answer. It doesn't give you a wrong answer.
11	Tr: Oh, so you like the accuracy of the answers that you get from probability?
12	S: Yes
13	Tr: Is there anything that you don't like about probability?
14	S: No. There is nothing I don't like about probability.
15	Tr: Right. If you think back in class or during any lesson, would you think of an experience which helped you to gain a better understanding of a particular concept in probability?

16	S: Yeah, I think it was one time when we were doing simple interest. There was a lot of thinking and I did not understand it and the teacher didn't explain it the way I understood it. So I asked my parents at home because they understand the sums and so I got a better understanding.
17	Tr: Ok. So it was about a method or how it was taught to you?
18	S: Yeah. It wasn't easy.
19	Tr: And in terms of probability … I'm trying to see if you can think of something that happened in class which helped you to understand something in probability … a method or an activity that happened in class which you think helped you to understand something in probability.
20	S: It wasn't in Maths. It was in Geography.
21	Tr: It was in Geography? Okay, what do you remember?
22	S: Population density … it was about determining the statistics of babies born in a year in a country and the number of deaths … likes of life expectancy … so it made me … I wouldn't say easier, … it was more good for me to go to Maths and just do it because I saw that this it was similar to Maths
23	Tr: Ok. So it's similar to Maths?
24	S: Yeah.
25	Tr: Ok. So what about the GeoGebra lessons that we had? Do you think there were any good things?
26	S: Yeah, there were good things. I think it was [….]. I don't remember the name, but the thing that we did after tossing the coin. I think total number or …. I don't remember the name.
27	Tr: Relative frequency?
28	S: Yes, relative frequency. I didn't understand it initially but you made it clear for me to understand it.
29	Tr: We were using GeoGebra to carry out some experiments. Do you think that was effective for you?
30	S: Could you please repeat the question?

31	Tr: Ok remember we were using GeoGebra to roll dice and to toss coins instead of actually taking the actual coin or dice. We were using a GeoGebra software. Do you think that was an effective approach?
32	S: It made our lives easier. Because you cannot toss a coin three thousand times. So you can toss a coin maybe 10 times, but to toss a coin 3000 times you get tired and annoyed.
33	Tr: So tell me, after tossing the coin many times we got many numbers.
34	S: Yes, we got many numbers.
35	Tr: And what did we do with the numbers? How did you process those numbers?
36	S: We kept on counting and counting and counting and we got the total numbers that were huge … they were big numbers because we kept on increasing in tens and in hundreds and in thousands . That's why we got large numbers because we kept on adding and the numbers kept on increasing.
37	Tr: So do you think GeoGebra can be used to save time, and how do you think it can be used in probability lessons?
38	S: Yeah because in class there are no types of things like smart boards and things like that. So, we need these things to take over because … for example if you are doing an example that you need a solution to you don't need to toss a coin for like 20 times because there is a software that can do it for you up to thousands and thousands of times. So I think they should be (introduced) in schools so that the lives of learners can be easier for them instead of tossing a coin many times .. for the software to take over and do what you are supposed to do.
39	Tr: So that means you actually let the software generate the numbers for you and then you can do the analysis. I agree with you there.
40	S: Yes.
41	Tr: Do you have any particular skills or knowledge that you think you could develop or acquire through the use of GeoGebra?
42	S: Knowledge and skills?
43	Tr: Yes.

44	S: Firstly, the knowledge is that I learnt something, and something going into my mind will stay. And the skills are that I know how to use it now. I know how to use it and I know when to use it.
45	Tr: Okay. Yeah that is important. Because you can look at your situation and select what aspect of GeoGebra you can use.
46	Tr: Do you have any comments or any questions that you would want to ask?
47	S: One comment I have is that this thing is quite good. And secondly, it's the best. Yeah, that's what I can say for now.
48	Tr: Thank you S… I appreciate your input.

C9 Interview transcript M

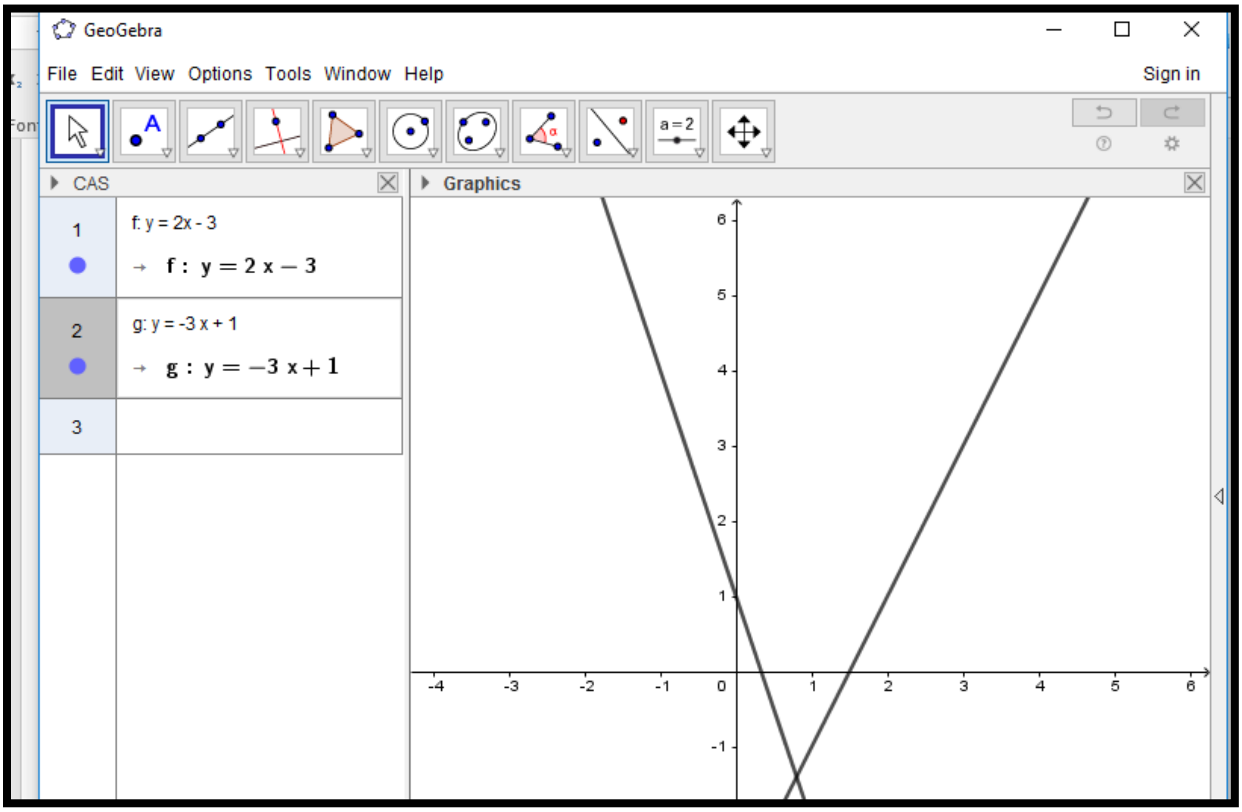
1	Tr: So M … tell me, in your own words, can you describe what probability means to you?
2	M: Means … Ok probability … yeah I think it's like [pause] I'll just say fluctuating. Okay can I use a coin?
3	Tr: Yeah. Anything.
4	M: Ok. For example, like fluctuating a coin like 50%
5	Tr: Yeah, like what does it involve? What does probability entail?
6	M: To me it involves population … it could be a coin
7	Tr: What in terms of population?
8	M: Mmm…
9	Tr: Not necessarily something that happened in the lesson. You can even try and tell me what is involved … or what is probability?
10	M: Mmm okay … it's just like …. in simple terms … it's just like playing with the number that you actually are given … extending a number if I'm not mistaken.
11	Tr: Ok so tell me, do you think we need to learn probability? Like in real life do you think there is an application for probability?
12	M: Yes, yes I do.
13	Tr: Can you think of one area in real life where we can use probability? I know you said population. Can you elaborate?
14	M: Ok like decreasing numbers in birth rates … mortality …. we just have to control that. We have to look at like infrastructure … and we have to measure our population.
15	Tr: So if you were to rate yourself from 0 to 10 where zero is poor and 10 is excellent how confident are you with probability?
16	M: I'd say 50%.
17	Tr: Do you have something that you like about probability?
18	M: Yeah playing with numbers
19	Tr: Playing with numbers? Anything you don't like?

20	M: Probably given hard questions or tricky questions.
21	Tr: Ok, let's link it to this one. Do you remember in class any experience that helped you gain a better understanding of a concept in probability?
22	M: Yeah, yeah.
23	Tr: Can you describe it?
24	M: Yes I think I can … I was using a dice. From what I was actually learning it was like 50- 50 or 60 - 40 or the dice from one to six.
25	Tr: Ok. So it is the use of the dice that helped you understand something?
26	M: Yes, yes, yes.
27	Tr: Ok. Do you remember one thing that you understood by using a die?
28	M: A die you can actually roll it … you can roll it like any how and it will give you like a random number. If it was to me and I want actually 2 to come out and I roll my dice and 2 doesn't pop up, I can mad or I get like this thing.
29	Tr: Okay so you understand randomness?
30	M: Yes, that anything is possible and can happen anytime.
31	Tr: I want you to think about the GeoGebra lessons that you had. Do you think there were any good things in those GeoGebra lessons?
32	M: Yes
33	Tr: What do you think were those good things and what do you think maybe made them good?
34	M: Ok for example if I have a coin physically, and I wanted to spin it 3000 times I wouldn't have enough time to do that in class. It's time wasting. But if I have an app which is GeoGebra, it can assist me if I want to spin it three thousand times I can just automatically click on it and it will just [not audible] up.
35	Tr: So you are saying this app can help you generate …
36	M: Yes large numbers.
37	Tr: Anything you think maybe was difficult in those GeoGebra lessons? Anything that you can think of?

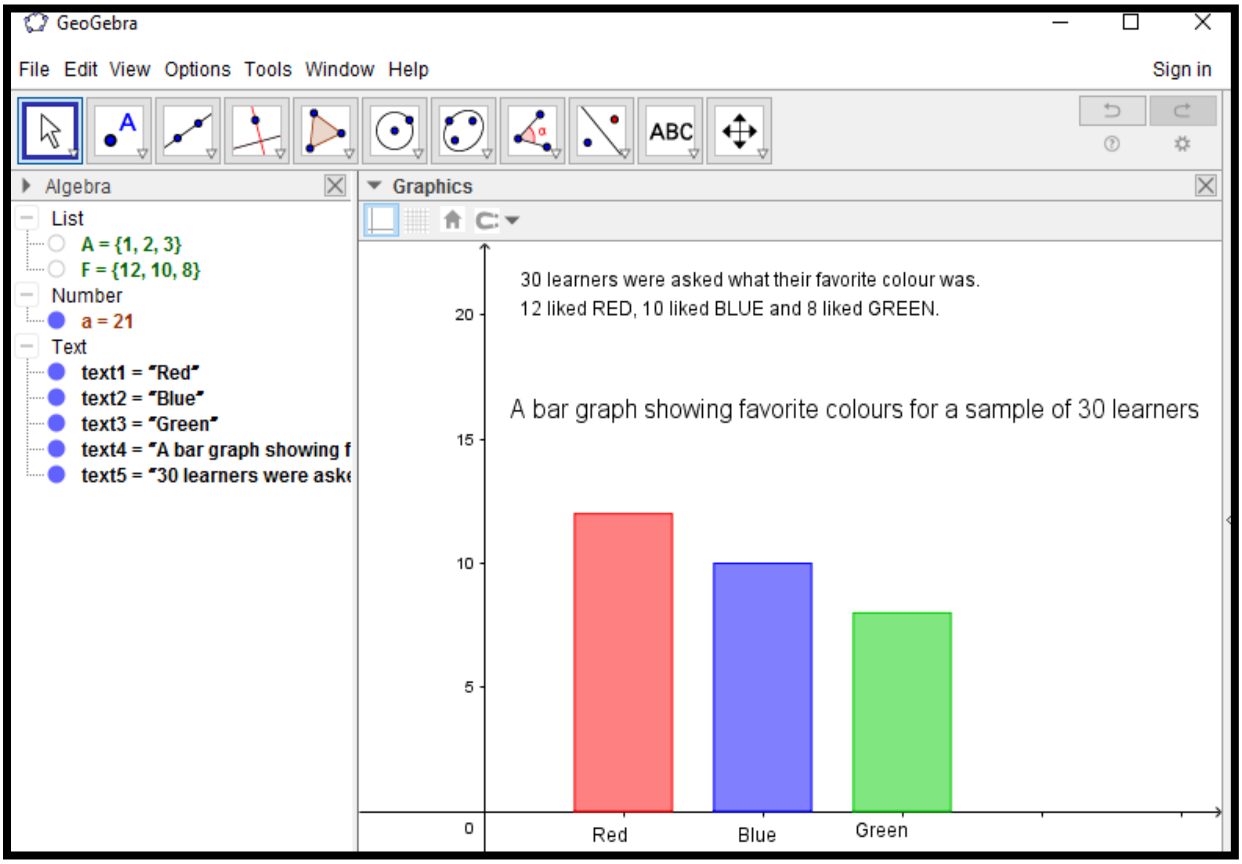
38	M: Nothing was difficult.
39	Tr: Do you think it was easy operating or working with the app, the GeoGebra app?
40	M: It is easy to operate and access and it's easy to use because you just press your numbers or you just click … everything just works smoothly. The system actually works for you
41	Tr: The system actually generates the numbers for you?
42	M: Yes. Especially large numbers.
43	Tr: Right. And how did you feel when you were working with the GeoGebra app and the probability problems?
44	M: At first I thought that it was going to be difficult because sometimes like probability … I thought that it's not used in reality, but since you have explained it now I do understand why we should use probability in our lives. So, yeah …
45	Tr: Okay. So initially you thought it was going to be to be difficult?
46	M: Yes, I thought it was going to be difficult but once I was in it I discovered that Ok, I have to work out like this.
47	Tr: Okay. That's good.do you have any comments. Anything you would like to say?
48	M: How did that app get like into [……]
49	Tr: Into? It's a Maths app. It was designed particularly for Maths and it's free. You can download it and use it to work out algebra, trigonometry, geometry and so on.
50	M: Because us teenagers need this.
51	Tr: But do you think there are certain skills that you need in order to use this app?
52	M: I think you have to just learn the basics. … you must just master the basics and get more knowledge before you go into the system.
53	Tr: That's a good observation.

Appendix D: GeoGebra affordances

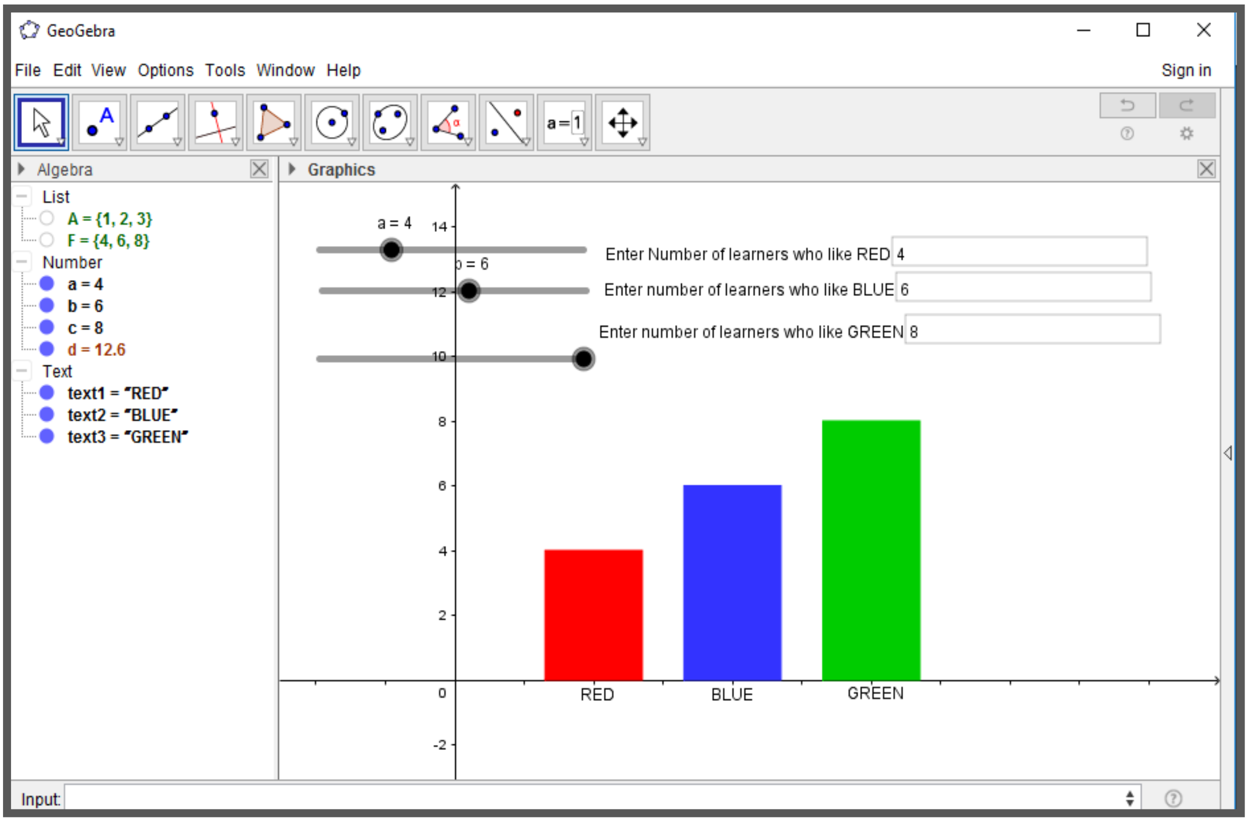
D1 A GeoGebra screen showing its CAS and DGS functionalities



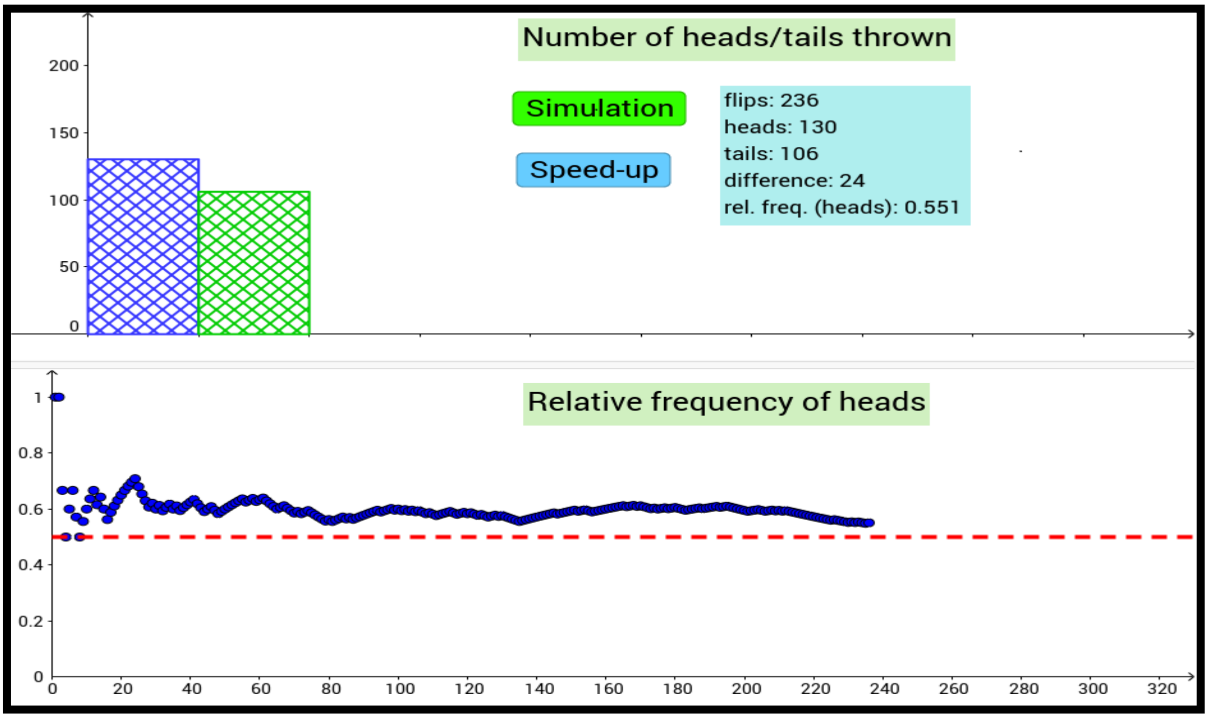
D2 A GeoGebra screen showing a visualisation of colour preferences by a sample of 30 learners.



D3 A GeoGebra screen showing the bar graph when parameters are adjusted using sliders



D4 A simulation of flipping a coin and recording the relative frequency of heads (Zoltán Fehér)



Appendix E: Quantitative data findings – Error analysis

E. 1 Pre-test item by item raw scores for both groups

Pre-test raw marks for each item
(Treatment Group, n = 14)

ITEM #	1	2	3	4	Total
OUT OF	(8)	(6)	(3)	(2)	(19)
T1	6	1	2	0	9
T2	5	2	3	0	10
T3	3	2	0	2	7
T4	5	0	1	0	6
T5	5	5	2	0	12
T6	6	3	3	2	14
T7	2	5	3	0	10
T8	3	0	0	2	5
T9	6	4	0	0	10
T10	6	2	0	2	10
T11	4	1	1	2	8
T12	7	5	1	0	13
T13	5	4	0	0	9
T14	4	2	2	2	10
% Mean	60%	43%	43%	43%	50%

Pre-test raw marks for each item
(Control Group, n = 22)

ITEM #	1	2	3	4	Total
OUT OF	(8)	(6)	(3)	(2)	(19)
C1	6	1	3	2	12
C2	2	0	1	2	5
C3	3	6	3	0	12
C4	6	0	0	0	6
C5	5	0	0	0	5
C6	6	2	3	2	13
C7	5	3	1	0	9
C8	6	0	3	2	11
C9	4	0	1	2	7
C10	3	1	0	0	4
C11	6	5	1	0	12
C12	5	0	1	0	6
C13	6	2	3	2	13
C14	6	2	3	0	11
C15	4	0	1	0	5
C16	6	1	1	0	8
C17	6	2	0	0	8
C18	5	3	0	2	10
C19	5	3	3	2	13
C20	3	2	3	2	10
C21	3	1	1	0	5
C22	3	1	1	0	5
% Mean	59%	27%	50%	41%	45%

E.2 Post-test item by item raw scores for both groups

Post-test raw marks for each item
(Treatment Group, n = 14)

ITEM #	1	2	3	4	5	Total
OUT OF	(2)	(4)	(3)	(5)	(2)	(16)
T1	0	0	0	1	2	3
T2	0	1	3	3	2	9
T3	0	0	3	2	2	7
T4	0	0	3	3	0	6
T5	0	1	3	5	1	10
T6	2	3	3	1	0	9
T7	2	4	3	5	2	16
T8	0	0	3	5	1	9
T9	2	4	3	3	2	14
T10	2	1	0	0	0	3
T11	0	0	1	3	0	4
T12	0	4	1	3	2	10
T13	0	1	1	1	1	4
T14	0	1	3	3	2	9
% Mean	29%	36%	71%	54%	61%	50%

Post-test raw marks for each item
(Control Group, n = 22)

ITEM #	1	2	3	4	5	Total
OUT OF	(2)	(4)	(3)	(5)	(2)	(16)
C1	2	3	3	5	0	13
C2	0	3	3	3	0	9
C3	0	0	3	5	2	10
C4	0	0	3	3	0	6
C5	0	1	3	1	0	5
C6	0	1	3	3	2	9
C7	0	0	3	5	1	9
C8	0	0	3	1	1	5
C9	0	1	3	3	0	7
C10	0	0	1	5	0	6
C11	0	0	1	1	1	3
C12	0	1	3	3	0	7
C13	0	1	3	5	0	9
C14	0	1	3	3	1	8
C15	0	0	1	5	0	6
C16	0	0	3	5	2	10
C17	0	0	3	1	1	5
C18	0	0	1	5	2	8
C19	0	0	1	0	2	3
C20	0	3	3	3	0	9
C21	0	1	1	1	2	5
C22	2	0	1	3	0	6
% Mean	9%	18%	79%	63%	39%	45%

E. 3 Pre-test and Post-test : Level of difficulty of each item

Pre-test difficulty level		*Post-test difficulty level*	
Treatment group	**Control group**	**Treatment group**	**Control group**
Item 1: **Difficult** **(p = 0.07)** Probability concepts. Identification with no reasons.	Item 1: **Difficult** **(p = 0.00)** Probability concepts. Identification with no reasons.	**Item 2: Difficult** **(p = 0.21)** Experimental probability problem involving single and combined events of inclusive events (No method suggested. Students could use a Venn diagram or definitions). Given: Probabilities of events.	Item 2: **Difficult** **(p = 0.00)** Experimental probability problem involving single and combined events of inclusive events (No method suggested. Students could use a Venn diagram or definitions). Given: Probabilities of events.
Item 4: **Difficult** **(p = 0.14)** Interpret Venn diagram to determine probability of combined inclusive events	Item 2 : **Difficult** **(p = 0.18)** Experimental probability problem of single and combined events using a Venn diagram. Given: Outcomes of events.	Item 1: **Difficult** **(p = 0.29)** Probability of combined inclusive events	Item 1: **Difficult** **(p = 0.05)** Probability of combined inclusive events
Item 2 : **Difficult** **(p = 0.21)** Experimental probability problem of single and combined events using a Venn diagram. Given: Outcomes of events.	Item 3: **Easy** **(p = 0,73)** Problem solving – representation using Venn diagram	**Item 4: Difficult** **(p = 0.29)** Interpret Venn diagram to determine probability of combined inclusive events	Item 5: **Difficult** **(p = 0.27)** Probability concepts. Giving reasons why events are mutually exclusive, complementary of inclusive
Item 3: **Difficult** **(p = 0.21)** Problem solving – representation using Venn diagram	Item 4: **Easy** **(p = 0.81)** Interpret Venn diagram to determine probability of combined inclusive events	Item 5: **Difficult** **(p = 0.50)** Probability concepts. Giving reasons why events are mutually exclusive, complementary of inclusive (61% success rate)	Item 4: **Difficult** **(p = 0.36)** Interpret Venn diagram to determine probability of combined inclusive events
		Item 3: **Moderate** **(p = 0.64)** Problem solving – representation using Venn diagram	Item 3:**Moderate** **(p = 0.68)** Problem solving – representation using Venn diagram

E.4 Student's use of methods in the pre-test.

Correct Venn diagram, correct follow through method = A Incorrect Venn diagram, correct follow through method = C										Correct Venn diagram, incorrect follow-through method = B I incorrect Venn diagram, incorrect follow though method = D								
Pre-Test : Treatment group											Pre-test: Control group							
	Student	ITEM 2				ITEM 3				Student	ITEM 2				ITEM 3			
		A	B	C	D	A	B	C	D		A	B	C	D	A	B	C	D
1	T1				1				1	C1				1	1			
2	T2		1				1			C2				1			1	
3	T3				1		1			C3	1					1		
4	T4				1				1	C4				1				1
5	T5	1					1			C5				1				1
6	T6		1			1				C6		1			1			
7	T7	1					1			C7		1						1
8	T8				1			1		C8				1	1			
9	T9		1						1	C9				1			1	
10	T1				1			1		C10				1				1
11	T1				1			1		C11	1							1
12	T1	1							1	C12				1				1
13	T1		1						1	C13			1		1			
14	T1		1			1				C14		1				1		
15										C15				1				1
16										C16				1				1
17										C17		1						1
18										C18		1					1	
19										C19				1	1			
20										C20				1	1			
21										C21				1				1
22										C22				1				1
Total		3	5	0	6	2	4	3	5		2	5	1	14	6	2	3	11
Average %		21	36	0	43	14	29	21	36		9	23	4	64	27	9	14	50

Incidence of each category in the pre-test (Item 2 and Item 3)

Category	A	B	C	D
Treatment (n = 14)	5	9	3	11
Control (n = 22)	8	7	4	25

E.5 Student's use of methods in the post-test.

Correct Venn diagram, correct follow through method = A Incorrect Venn diagram, correct follow through method = C											Correct Venn diagram, incorrect follow-through method = B I incorrect Venn diagram, incorrect follow though method = D							
Post-Test : Treatment group											Post-test: Control group							
		ITEM 1				ITEM 2					ITEM 1				ITEM 2			
	Student	A	B	C	D	A	B	C	D	Student	A	B	C	D	A	B	C	D
1	T1				1				1	C1	1				1			
2	T2				1		1			C2	1				1			
3	T3	1				1				C3				1	1			
4	T4				1		1			C4				1		1		
5	T5				1	1				C5				1		1		
6	T6	1					1			C6				1		1		
7	T7	1				1				C7				1	1			
8	T8				1	1				C8				1		1		
9	T9	1				1				C9				1		1		
10	T1				1				1	C1				1				1
11	T1				1				1	C1				1				1
12	T1	1							1	C1				1		1		
13	T1				1				1	C1				1	1			
14	T1				1		1			C1				1		1		
15										C1				1				1
16										C1				1	1			
17										C1				1		1		
18										C1				1			1	
19										C1				1				1
20										C2	1					1		
21										C2				1				1
22										C2				1		1		
Total		5	0	0	9	5	4	0	5		3	0	0	19	6	10	1	5
Average %		36	0	0	64	36	28	0	36		14	0	0	86	27	45	5	23

Incidence of each category in the post-test

Category	A	B	C	D
Treatment (n = 14)	10	4	0	14
Control (n = 22)	9	10	1	5

E. 6 Scores obtained in the pre-test and post-test by students

Pre-Test (Out of 19)						Post-test (Out of 16)					
Treatment Group			Control Group			Treatment Group			Control Group		
	Score	%		Score	%		Score	%		Score	%
T1	9	47	C1	12	63	T1	3	19	C1	13	81
T2	10	53	C2	5	26	T2	9	56	C2	9	56
T3	7	37	C3	12	63	T3	10	63	C3	10	63
T4	6	32	C4	6	32	T4	6	38	C4	6	38
T5	12	63	C5	5	26	T5	10	63	C5	5	31
T6	14	74	C6	13	68	T6	9	56	C6	9	56
T7	8	42	C7	9	47	T7	16	100	C7	9	56
T8	5	26	C8	11	58	T8	9	56	C8	5	31
T9	10	53	C9	7	37	T9	14	88	C9	6	38
T10	10	53	C10	4	21	T10	3	19	C10	7	44
T11	8	42	C11	12	63	T11	4	25	C11	4	25
T12	13	68	C12	6	32	T12	10	63	C12	7	44
T13	9	47	C13	13	68	T13	4	25	C13	9	56
T14	10	53	C14	11	58	T14	9	56	C14	8	50
			C15	5	26				C15	2	13
			C16	8	42				C16	10	63
			C17	8	42				C17	5	31
			C18	10	53				C18	8	50
			C19	13	68				C19	3	19
			C20	10	53				C20	9	56
			C21	5	26				C21	5	31
			C22	5	26				C22	6	38
	Av.	Av.%		Av.	Av.%		Av.	Av.%		Av.	Av.%
	9	**49**		**9**	**45**		**8**	**52**		**7**	**44**

E.7 Paired Samples T-Test for Treatment group pre-test and post-test scores

Results

Paired Samples T-Test

Paired Samples T-Test

Measure 1		Measure 2	t	df	p	Mean Difference	SE Difference	95% CI for Mean Difference Lower	95% CI for Mean Difference Upper	Cohen's d	95% CI for Cohen's d Lower	95% CI for Cohen's d Upper
Treatment Pre-test Scores	-	Treatment Post-test Scores	−0.372	13	0.716	−2.643	7.103	−17.989	12.703	−0.099	−0.623	0.428

Note. Student's t-test.

Assumption Checks

Test of Normality (Shapiro-Wilk)

			W	p
Treatment Pre-test Scores	-	Treatment Post-test Scores	0.951	0.570

Note. Significant results suggest a deviation from normality.

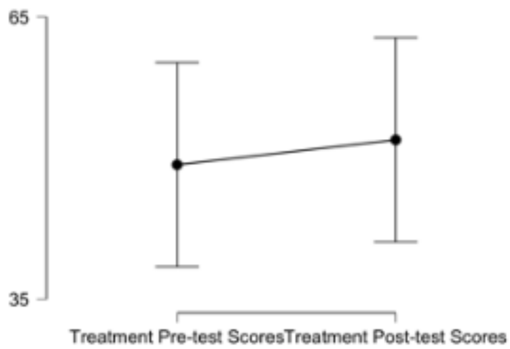
Descriptives

Descriptives

	N	Mean	SD	SE
Treatment Pre-test Scores	14	49.286	13.338	3.565
Treatment Post-test Scores	14	51.929	24.587	6.571

Descriptives Plots

Treatment Pre-test Scores - Treatment Post-test Scores



E.8 Paired Samples T-Test for Control group pre-test and post-test scores

Results

Paired Samples T-Test

Paired Samples T-Test

Measure 1		Measure 2	Test	Statistic	df	p	Location Parameter	SE Difference	95% CI for Location Parameter		Effect Size
									Lower	Upper	
Control Pre-test Scores	-	Control Post-Test Scores	Student	0.309	21	0.761	1.273	4.122	−7.299	9.844	0.066
			Wilcoxon	114.500		0.986	−1.400e −5		−8.500	10.500	−0.009

Note. For the Student t-test, effect size is given by Cohen's *d*. For the Wilcoxon test, effect size is given by the matched rank biserial correlation.
Note. For the Student t-test, location parameter is given by mean difference *d*. For the Wilcoxon test, effect size is given by the Hodges-Lehmann estimate.

Assumption Checks

Test of Normality (Shapiro-Wilk)

			W	p
Control Pre-test Scores	-	Control Post-Test Scores	0.951	0.336

Note. Significant results suggest a deviation from normality.

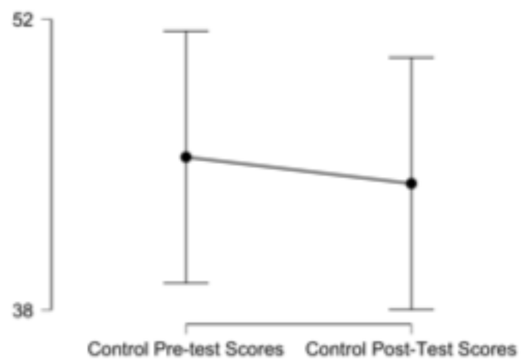
Descriptives

Descriptives

	N	Mean	SD	SE
Control Pre-test Scores	22	45.364	16.658	3.551
Control Post-Test Scores	22	44.091	16.350	3.486

Descriptives Plots

Control Pre-test Scores - Control Post-Test Scores



E.9 Independent Samples T-Test for Treatment group vs Control group pre-test scores

Results

Independent Samples T-Test

Independent Samples T-Test

	Test	Statistic	df	p	Location Parameter	SE Difference	Effect Size
Pre-test Score	Student	−0.741	34.000	0.464	−3.922	5.290	−0.253
	Welch	−0.779	32.060	0.441	−3.922	5.032	−0.260
	Mann-Whitney	134.000		0.524	−5.000		−0.130

Note. For the Student t-test and Welch t-test, effect size is given by Cohen's d. For the Mann-Whitney test, effect size is given by the rank biserial correlation.

Note. For the Student t-test and Welch t-test, location parameter is given by mean difference. For the Mann-Whitney test, location parameter is given by the Hodges-Lehmann estimate.

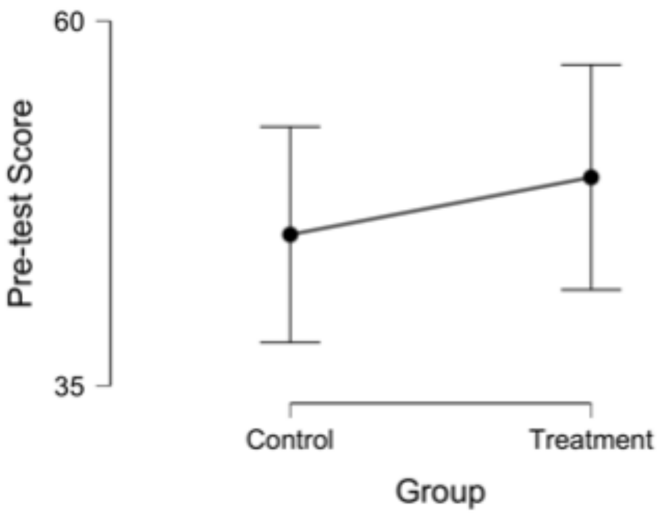
Descriptives

Group Descriptives

	Group	N	Mean	SD	SE
Pre-test Score	Control	22	45.364	16.658	3.551
	Treatment	14	49.286	13.338	3.565

Descriptives Plots

Pre-test Score



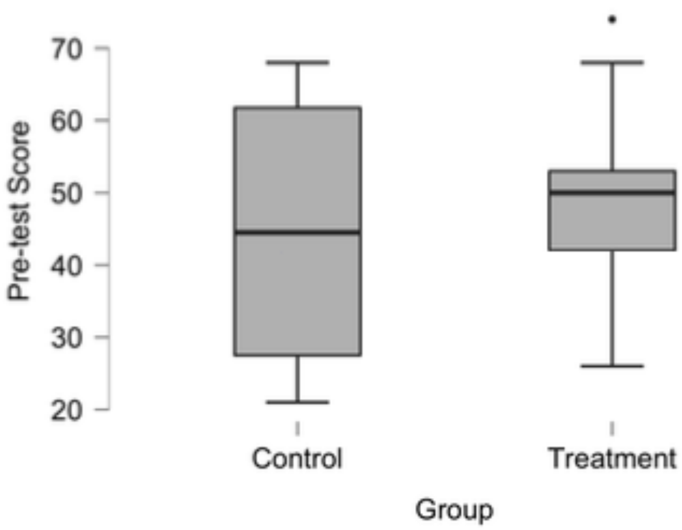
Descriptive Statistics

Descriptive Statistics

	Pre-test Score	
	Control	Treatment
Valid	22	14
Missing	0	0
Mean	45.364	49.286
Median	44.500	50.000
Std. Deviation	16.658	13.338
Kurtosis	−1.620	−0.193
Std. Error of Kurtosis	0.953	1.154
Shapiro-Wilk	0.891	0.974
P-value of Shapiro-Wilk	0.019	0.928
Minimum	21.000	26.000
Maximum	68.000	74.000

Boxplots

Pre-test Score



E.10 Independent Samples T-Test for Treatment group vs Control group post-test scores

Results

Independent Samples T-Test

Independent Samples T-Test

	Test	Statistic	df	p	Location Parameter	SE Difference	Effect Size
Post-test Score	Student	−1.152	34.000	0.257	−7.838	6.806	−0.394
	Welch	−1.054	20.348	0.304	−7.838	7.439	−0.375
	Mann-Whitney	121.500		0.294	−7.000		−0.211

Note. For the Student t-test and Welch t-test, effect size is given by Cohen's d. For the Mann-Whitney test, effect size is given by the rank biserial correlation.

Note. For the Student t-test and Welch t-test, location parameter is given by mean difference. For the Mann-Whitney test, location parameter is given by the Hodges-Lehmann estimate.

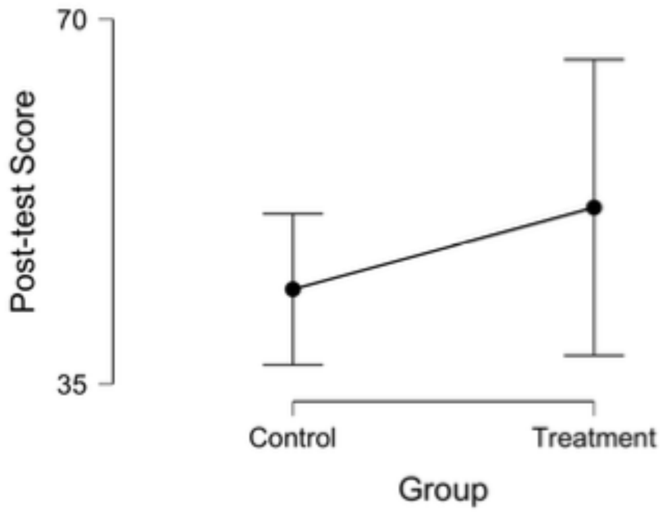
Descriptives

Group Descriptives

	Group	N	Mean	SD	SE
Post-test Score	Control	22	44.091	16.350	3.486
	Treatment	14	51.929	24.587	6.571

Descriptives Plots

Post-test Score



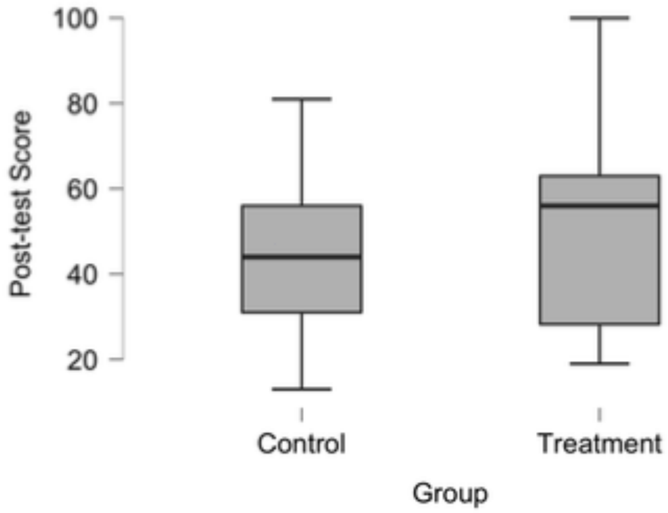
Descriptive Statistics

Descriptive Statistics

	Post-test Score	
	Control	Treatment
Valid	22	14
Missing	0	0
Mean	44.091	51.929
Median	44.000	56.000
Std. Deviation	16.350	24.587
Kurtosis	−0.110	−0.318
Std. Error of Kurtosis	0.953	1.154
Shapiro-Wilk	0.971	0.913
P-value of Shapiro-Wilk	0.743	0.176
Minimum	13.000	19.000
Maximum	81.000	100.000

Boxplots

Post-test Score



E.11 Paired Samples T-Test results for the effect of GeoGebra on the difficulty index of questions for the Treatment group

Results

Paired Samples T-Test

Paired Samples T-Test

Measure 1		Measure 2	t	df	p	Cohen's d
Difficulty Index (Pre-Test) Treatment Group	-	Difficulty Index (Post -Test) Treatment Group)	−2.644	3	0.039	−1.322

Note. For all tests, the alternative hypothesis specifies that Difficulty Index (Pre-Test) Treatment Group is less than Difficulty Index (Post -Test) Treatment Group).
Note. Student's t-test.

Assumption Checks

Test of Normality (Shapiro-Wilk)

			W	p
Difficulty Index (Pre-Test) Treatment Group	-	Difficulty Index (Post -Test) Treatment Group)	0.837	0.186

Note. Significant results suggest a deviation from normality.

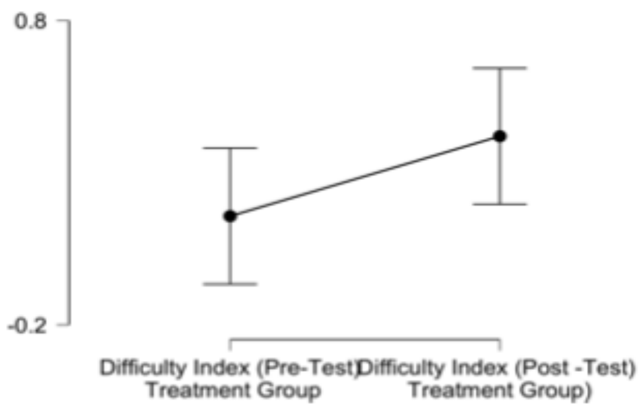
Descriptives

Descriptives

	N	Mean	SD	SE
Difficulty Index (Pre-Test) Treatment Group	4	0.158	0.067	0.034
Difficulty Index (Post -Test) Treatment Group)	4	0.420	0.183	0.092

Descriptives Plots

Difficulty Index (Pre-Test) Treatment Group - Difficulty Index (Post -Test) Treatment Group)



E.12 Paired Samples T-Test results for the effect of GeoGebra on the difficulty index of questions for the Control group

Results

Paired Samples T-Test

Paired Samples T-Test

Measure 1		Measure 2	t	df	p	Cohen's d
Diffuclty index Control Group (Pre-Test)	-	Difficulty index Control Group (Post-Test)	0.648	3	0.718	0.324

Note. For all tests, the alternative hypothesis specifies that Diffuclty index Control Group (Pre-Test) is less than Difficulty index Control Group (Post-Test).
Note. Student's t-test.

Assumption Checks

Test of Normality (Shapiro-Wilk)

			W	p
Diffuclty index Control Group (Pre-Test)	-	Difficulty index Control Group (Post-Test)	0.991	0.962

Note. Significant results suggest a deviation from normality.

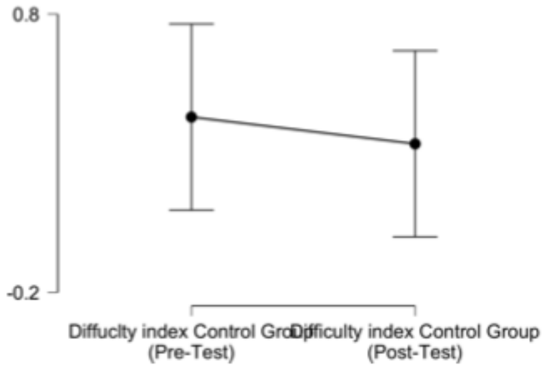
Descriptives

Descriptives

	N	Mean	SD	SE
Diffuclty index Control Group (Pre-Test)	4	0.430	0.401	0.200
Difficulty index Control Group (Post-Test)	4	0.334	0.271	0.135

Descriptives Plots

Diffuclty index Control Group (Pre-Test) - Difficulty index Control Group (Post-Test)



E. 13 GeoGebra effect on error frequency between groups and tests

Results

ANOVA

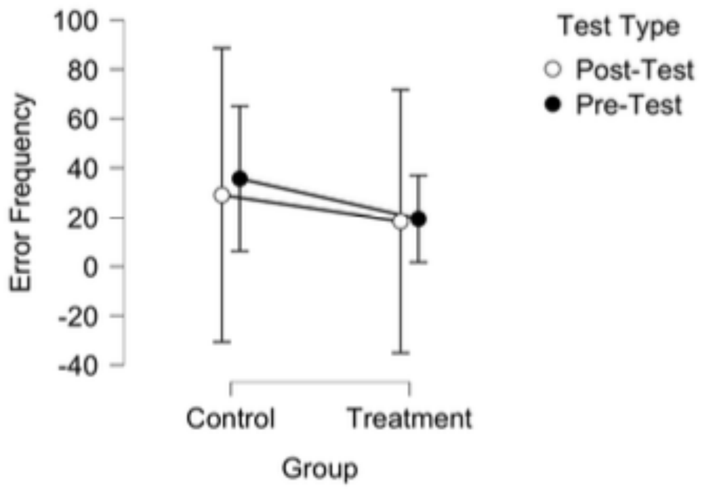
ANOVA - Error Frequency

Cases	Sum of Squares	df	Mean Square	F	p	ω^2
Group	546.750	1	546.750	1.779	0.219	0.071
Test Type	44.083	1	44.083	0.143	0.715	0.000
Group ∗ Test Type	24.083	1	24.083	0.078	0.787	0.000
Residuals	2458.000	8	307.250			

Note. Type III Sum of Squares

Descriptives

Descriptives plots



Appendix F: Worksheets for classroom activities with GeoGebra.

F.1 Lesson 1 Basic probability concepts.

Watch the following video https://www.youtube.com/embed/uzkc-qNVoOk and answer the following questions:

1. Column A gives a description or definition of the probability concepts that you encountered in the video. For each definition/description, write down the concept (s) that the definition refers to. Choose from the following concepts. You can repeat the concepts and one description can have more than one concepts.

Experiment; Possibilities; Fair; Biased; Trial, Outcomes; Random Event; Sample Space; probability; Mutually Exclusive.

Column A	Column B
When a die is rolled, it can land on a 1 or 2 or 3 or 4 or 5 or 6. Which term is used to describe these numbers?	
A coin is tossed a large number of times in an experiment. What term is used to describe each toss?	
The term used to explain that a coin is equally likely to land on a head or a tail.	
The term used to explain that there are 6 outcomes that can be obtained when a die is rolled.	
What term is used to describe the process of tossing a coin and recording the results?	
The term used to refer to the measure of chance of a coin landing on a head.	
The term used to explain that events cannot happen simultaneously.	

2. In an experiment, a fair sided coin is tossed once. Write down the sample space:
3. In an experiment, a fair six-sided dice with faces numbered 1, 2, 3, 4, 5 and 6 is rolled. Write down the sample space for this experiment.
4. In an experiment, two fair six-sided dice with faces numbered 1, 2, 3, 4, 5 and 6 are rolled. The sample space for this experiment is “the sum of the numbers rolled.” Write down the sample space for this experiment.

F.2 Lesson 2 Probability models to compare theoretical and experimental probability using a coin simulation.

Click on the following link https://ggbm.at/LZbwMZtJ to use a simulation for tossing a coin.

1. If you toss a coin 100 times, how many heads and how many tails do you expect to get? Why?
2. Toss a single coin up to 100 times and record the number of heads and the number of tails that are obtained after 10, 20, 30 etc. tosses. Write your results in the table below.

Number of coins	Flips	Number of Heads	Number of Tails	Relative frequency (Head)	Relative frequency (Tail)
1	30				
1	40				
1	50				
1	60				
1	70				
1	80				
1	90				
1	100				
1					
1					
1					
1					
1					
1					

What do you notice about relative frequencies and theoretical probabilities of obtaining heads and tails?

F.3 Lesson 3 Probability models to compare theoretical and experimental probability using a dice simulation.

Click on the following link https://ggbm.at/UsoH4eNl to use a simulation of rolling a dice.

1. When you roll a die once, how many times do you expect to get each of the numbers 1, 2, 3, 4, 5 or 6? Why?

2. Experimental Probability: Roll a single die, many times, say 1000 times, and record the number of 1s, 2s, 3s, 4s, 5s, and 6s that are obtained. Write your results in the table below.

Number of dice	Trial	n(1)	n(2)	n(3)	n(4)	n(5)	n(6)
1	100 rolls						
1	200 rolls						
1	800 rolls						
Increase the number of rolls significantly (n large)							

3. Theoretical Probability: A single die is rolled 1000 times. What is the theoretical probability of obtaining each of the numbers: 1, 2, 3, 4, 5 and 6?

4. What do you notice about the relative frequencies and the theoretical probabilities of obtaining the outcomes 1,2,3,4,5, and 6?

5. Experimental Probability: Roll 2 dice simultaneously and add up the numbers that show up for each trial. Complete the following table.

No. of rolls	The **sum** of the numbers obtained when 2 dice are rolled 10 times at the same time. For each trial, write down the number of times each sum appeared.											
			3	4	5	6	7	8	9	10	11	12

6. Theoretical Probability: Draw an outcomes table for the sum of the numbers when two dice are rolled.

7. Comment of the experimental and theoretical probabilities.

F.4 Lesson 4 Probability models and Venn diagrams using a dice simulation.

Two dice are rolled. Define A and B as follows: Event A: Sum of the numbers appearing on the dice is 7. Event B: At least one of the dice shows a 2
Click on the following link https://ggbm.at/J9NyWRa5 to use a simulation for rolling dice and adding the numbers that show up.

a) Roll the dice up to 36 times and record the outcomes in the table below. Record the blue dice first, followed by the red dice. For example, if the blue dice shows a 1 and the red dice shows a 3, write (1;3) in the appropriate box as the outcome. These numbers may repeat as you keep on rolling the dice. Make sure you leave enough space for any possible repetitions.

		Blue Dice					
Red Dice		1	2	3	4	5	6
	1						
	2						
	3						
	4						
	5						
	6						

b) Write down A, the set of all possible outcomes that give a sum of 7. (Use the experiment above)

c) Write down B, the set of all possible outcomes that contain at least a 2. (Use the experiment above)

d) Are there any outcomes from your experiment that fall into both A and B? Write them down.

e) Draw a Venn diagram to illustrate the information you obtained from your experiment.

f) Complete the table below for the outcomes you would expect to get theoretically.

g) Write down A, the ordered pair of outcomes that add up to 7. Use the table above.

h) Write down B, the ordered pair of outcomes that have at least a 2. Use the table above.

i) Draw a Venn diagram to illustrate the above information.

j) What do you notice about the results obtained experimentally and theoretically?

k) Calculate the probability of $A \cap B$.

l) Calculate the probability of $A \cup B$.

Appendix G: Pre-Test and Post-Test Items

G.1 Pre-test

Item 1

a) When a coin is flipped 8 000 times and it lands on heads 8000 times, then that coin must be biased.

Do you agree with the statement above? Why? (1)

b) You toss a coin 500 times, and heads show up 320 times.
When you toss it again for the 501st time, what do you think will show up, a head or a tail? Why? (1)

c) You toss 2 fair coins at the same time just once and record the outcomes for all the coins.

i) Write down all the possible outcomes for this experiment using H and T for heads and tails respectively. (1)

ii) What is the probability that either one of the outcomes HH or TT will occur? (1)

d) In a school, there are 40 Grade 10 learners. It is given that 25 of these learners participate in swimming, 10 participate in debating, 5 participate in both swimming and debating and 10 participate in neither swimming nor debating.

Events X and Y are defined as follows:

- X: The learners in Grade 10 in the swimming team.
- Y: The learners in Grade 10 in the debating team.

State, whether events X and Y are mutually exclusive or not. (1)

e) A group of parents attended a swimming gala at a certain school to support their children.16 of them were women. Each woman drove either a red car or a white car. Events X and Y are defined as follows:

- X: Red cars which are driven by women.
- Y: White cars which are driven by women.

State whether or not events X and Y are mutually exclusive. (1)

f) Events X and Y are defined as follows:

- X: Boys in Grade 10 who play soccer.
- Y: Girls in Grade 10 who play soccer.

State whether or not events X and Y are complementary. (1)

g) The sample space S is such that $S = \{1; 2; 3; 4; 5; 6; 7; 8\}$.

Event X and event Y are defined as follows:

- $X = \{1; 3; 4; 8\}$
- $Y = \{2; 3; 5; 6; 7; 8\}$

State whether or not events X and Y are complementary. (1)

[8]

Item 2

Two dice are rolled at the same time. The numbers showing on the dice are multiplied by each other to get the sample space, S.
S = {1;2;3;4;5;6;8;9;10;12;15;16;18;20;24;25;30;36}

- Define event A as follows: A = {outcomes that are prime numbers}
- Define event B as follows: B = {outcomes that are multiples of 15}
- Define event C as follows: C = {outcomes that are factors of 15}

a) Draw a clearly labelled Venn diagram to illustrate the sample space (S) and the events A, B and C. (3)

b) Write down P (A). (1)

c) Write down P(A or B). (1)

d) Write down P(A and B). (1)

[6]

Item 3

In a group of 40 learners the following information is TRUE:

- 7 learners are left-handed
- 18 learners play soccer
- 4 learners play soccer and are left-handed
- All 40 learners are either right-handed or left-handed (not both)

Let L be the set of all left-handed people and S be the set of all learners who play soccer.

a) How many learners in the group are right-handed and do NOT play soccer? (1)

b) Draw a Venn diagram to represent the above information. (2)

[3]

Item 4

[Refer to item 3 above] Determine the probability that one learner selected at random is left-handed or plays soccer. (2)

[2]

G.2 Post-test

Item 1

a) A fair six-sided die is rolled.

Event X is defined as "an even number is obtained."

Event Y is defined as "a factor of 12 is obtained."

Determine P(X or Y). **[2]**

Item 2

b) Ron enters a Maths and an English competitions at his school. Events X and Y are defined as follows: X is "winning a Maths competition" and Y is "winning an English competition."

The following is true about X and Y.

- P(X) = 0,19
- P(Y) = 0,13
- P(X and Y) = 0,11

i. State the probability that Ron does not win the Maths competition. (1)

ii. Calculate the probability that Ron does not win any competition. (3)

[4]

Item 3

During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows:

- 28 learners said they preferred B, "Boerewors rolls and koeksisters"
- 48 said they preferred K, "Kota with atchar, chips and Russian sausages"
- x said they preferred both B and K
- 20 said they preferred neither.

a) Draw a labelled Venn diagram to illustrate this information **[3]**

Item 4

[Refer to item 3 above to answer this question]

b) Use your Venn diagram to calculate the probability that a Grade 10 learner selected at random prefers B or K. (4)

c) Use your Venn diagram to calculate the probability that a Grade 10 learner selected at random prefers neither B nor K. (1)

[5]

Item 5

[Refer to item 3 above to answer this question]

d) State, with a reason, whether B and K are mutually exclusive, complementary or inclusive. [2]

G.3. Pre-test and post- test items for specific learning outcome 1

Pre-test items (Item 1)
a) When a coin is flipped 8 000 times and it lands on heads 8000 times, then that coin must be biased. Do you agree with the statement above? Why?
b) You toss a coin 500 times, and heads show up 320 times. When you toss it again for the 501st time, what do you think will show up, a head or a tail? Why?
c) You toss 2 fair coins at the same time just once and record the outcomes for all the coins. i) Write down all the possible outcomes for this experiment using H and T for heads and tails respectively. ii) What is the probability that either one of the outcomes HH or TT will occur?
d) In a school, there are 40 Grade 10 learners. It is given that 25 of these learners participate in swimming, 10 participate in debating, 5 participate in both swimming and debating and 10 participate in neither swimming nor debating. Events X and Y are defined as follows: X: the learners in Grade 10 in the swimming team. Y: the learners in Grade 10 in the debating team. State, whether events X and Y are mutually exclusive or not.
e) A group of parents attended a swimming gala at a certain school to support their children.16 of them were women. Each woman drove either a red car or a white car. Events X and Y are defined as follows: X: Red cars which are driven by women; Y: White cars which are driven by women. State whether or not events X and Y are mutually exclusive.
f) Events X and Y are defined as follows: X: Boys in Grade 10 who play soccer; Y: Girls in Grade 10 who play soccer. State whether or not events X and Y are complementary.
g) The sample space S is such that $S = \{1; 2; 3; 4; 5; 6; 7; 8\}$. Event X and event Y are defined as follows: $X = \{1; 3; 4; 8\}$; $Y = \{2; 3; 5; 6; 7; 8\}$ State whether or not events X and Y are complementary.

Post-test item (Item 5)
During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows: 28 learners said they preferred B, “Boerewors rolls and koeksisters”, 48 said they preferred K, “Kota with atchar, chips and Russian sausages”, x said they preferred both B and K, 20 said they preferred neither. State, with a reason, whether B and K are mutually exclusive, complementary or inclusive.

G.4 : Pre-test and post- test items for specific learning outcome 2

Pre-test item (Item 2)
Two dice are rolled at the same time. The numbers showing on the dice are multiplied by each other to get the sample space, S. S = {1;2;3;4;5;6;8;9;10;12;15;16;18;20;24;25;30;36} Define event A as follows: A = {outcomes that are prime numbers}. Define event B as follows: B = {outcomes that are multiples of 15}. Define event C as follows: C = {outcomes that are factors of 15}. e) Draw a clearly labelled Venn diagram to illustrate the sample space (S) and the events A, B and C. f) Write down P (A). g) Write down P(A or B). h) Write down P(A and B).
Post-test item (Item 1)
A fair six-sided die is rolled. Event X is defined as "an even number is obtained." Event Y is defined as "a factor of 12 is obtained." Determine P(X or Y).
Post-test item (Item 2)
Ron enters a Maths and English competitions at his school. Events X and Y are defined as follows: X is "winning a Maths competition" and Y is "winning an English competition." The following is true about X and Y. P(X) = 0,19; P(Y) = 0,13; P(X and Y) = 0,11 iii. State the probability that Ron does not win the Maths competition. iv. Calculate the probability that Ron does not win any competition.

G. 5 : Pre-test and post-test items for specific learning outcome 3

Pre-test item (Item 3)
In a group of 40 learners the following information is TRUE: 7 learners are left-handed, 18 learners play soccer, 4 learners play soccer and are left-handed, all 40 learners are either right-handed or left-handed (not both). Let L be the set of all left-handed people and S be the set of all learners who play soccer. a) How many learners in the group are right-handed and do NOT play soccer? b) Draw a Venn diagram to represent the above information.
Post-test item (Item 3)
During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows: 28 learners said they preferred B, “Boerewors rolls and koeksisters”, 48 said they preferred K, “Kota with atchar, chips and Russian sausages”, x said they preferred both B and K, 20 said they preferred neither. Draw a labelled Venn diagram to illustrate this information.

Pre-test item (Item 4)
In a group of 40 learners the following information is TRUE: 7 learners are left-handed, 18 learners play soccer, 4 learners play soccer and are left-handed, all 40 learners are either right-handed or left-handed (not both). Let L be the set of all left-handed people and S be the set of all learners who play soccer. Determine the probability that one learner selected at random is left-handed or plays soccer.

Post-test item (Item 4)
During the 2019 heritage week, 80 Grade 10 pupils in a particular school were asked which food they preferred. The results were as follows: 28 learners said they preferred B, “Boerewors rolls and koeksisters”, 48 said they preferred K, “Kota with atchar, chips and Russian sausages”, x said they preferred both B and K, 20 said they preferred neither. a) Use your Venn diagram to calculate the probability that a Grade 10 learner selected at random prefers B or K. b) Use your Venn diagram to calculate the probability that a Grade 10 learner selected at random prefers neither B nor K.