

https://dspace.org/

Theoretical and practical aspects of variography: in particular, estimation and modelling of semi-variograms over areas of limited and clustered or widely spaced data in a two-dimensional South African gold mining context.

Morgan, Craig John

2012-01-30
http://hdl.handle.net/10539/11193

Downloaded from a DSpace repository. https://dspace.org/

LIST OF FIGURES

Page

Figure 2.1 A typical bounded semi-variogram function, modified from Armstrong (1998 :26). 17

Figure 2.2 Some common behaviours of semi-variograms near the origin. From left to right: parabolic behaviour; linear behaviour; discontinuity at the origin; pure nugget effect. Figure 2.2 is taken from Journel & Huijbregts (1978 : 39). . 19

Figure 2.3 A typical semi-variogram function showing a nugget effect, range and sill parameter. Figure 2.3 is modified from Armstrong (1998 : 26). 21

Figure 2.4 The basic shape of the spherical semi-variogram model. . . . 22

Figure 2.5 The basic shape of the exponential semi-variogram model. . 23

Figure 2.6 The basic shape of the Gaussian semi-variogram model. . . . 24

Figure 2.7 An example of an experimental semi-variogram calculated for various distance lags of the vector *h* along the 90 degree azimuth (i.e. in an east-west direction). 26

Figure 2.8 An example of a semi-variogram cloud taken from Wackernagel (1995 : 31). 28

Figure 2.9 A spherical semi-variogram model fitted to an experimental Cr concentration semi-variogram. This figure is taken from Goovaerts (1997 : 100). 29

Figure 2.10 The point and regularized semi-variograms defined over a particular spatial region. Notice that for the larger modulus values of the vector *h*, the two semi-variograms simply differ in value by $\bar{\gamma}(v,v)$. This diagram has been taken from Journel & Huijbregts (1978 : 78). 36

Figure 3.1 The kriging prediction surface obtained using (a) the nugget-free semi-variogram model and (b) the nugget-inclusive semi-variogram model. 40

Figure 3.2 The estimated grade distribution of the 16 minable blocks, assuming that the semi-variogram is accurately estimated and modelled. 41

Figure 3.3 The estimated grade distribution of the 16 minable blocks, assuming that the semi-variogram's nugget effect is now too highly estimated. 41

Figure 3.4 An experimental reef width semi-variogram from an actual South African gold mine. 43

Figure 3.5 Two similar semi-variogram models fitted to the experimental reef width semi-variogram in Figure 3.4, one having (a) no nugget effect, and one having (b) a large nugget effect. 43

Figure 3.6 An acceptable spherical semi-variogram model fitted to the experimental reef width semi-variogram in Figure 3.4. 44

Figure 3.7 An example of a drilling programme conducted over the virgin ground area of a gold mine. 48

Figure 4.1 Two blocks of support v separated by a distance equal to the point semi-variogram range plus v (i.e. the distance between the block centres). Notice that any point within the one block will always be at least the point range distance from any point within the other block. Consequently, all the points within the one block are uncorrelated with all the points in the other block. Hence the two blocks are also uncorrelated. 51

Figure 4.2 The relationship between the point and regularized semi-variogram as discussed in Proof 4.1. This figure is taken directly from Rendu (1978 : 25). 53

Figure 4.3 Semi-variograms over support sizes w and W ($w < W$), illustrating the relationship between the support size of the regionalized variable and the regularized semi-variogram parameters. 55

Figure 4.4 Spatial location of the point support data used in Section 4.2. 57

Figure 4.5 The nested relative isotropic three-structure spherical point support semi-variogram fitted to the experimental semi-variogram of over 260,000 chip samples. The data is of gold content at various locations, and is taken from an actual operational gold mine in South Africa. 58

Figure 4.6 The nested relative isotropic three-structure spherical block support semi-variogram model discussed above, fitted to the experimental semi-variogram calculated from a 20m x 20m data regularization. 59

Figure 5.1 A spherical semi-variogram model fitted closely to its experimental semi-variogram. 62

Figure 5.2 A spherical semi-variogram model arbitrarily fitted to its experimental semi-variogram. 63

Figure 5.3 The five identified 120m x 120m blocks within the area of underground chip sampling (highlighted by the green mesh). 64

Figure 5.4 Each 120m x 120m block in Figure 5.3 (shown here in orange) has the following six adjacent 20m x 40m blocks (shown in blue). 64

Figure 5.5 Point support data surrounding a pillar of unmined ground. Blue indicates low value, while red indicates high value. Continuity in higher grade value is visible from left to right across the pillar. 66

Figure 5.6 The directional experimental semi-variogram of the data in Figure 5.5. The longest range of influence occurs along the 90 degree azimuth (i.e. horizontally). 67

Figure 5.7 An omni-directional semi-variogram model fitted to the experimental semi-variogram in Figure 5.6. 68

Figure 5.8 An anisotropic semi-variogram model fitted to the longest and shortest range directions of the experimental semi-variogram in Figure 5.6. 68

Figure 5.9 Three separate data configurations, showing a point to be estimated (in red) and five available samples from which the estimate can be calculated. 70

Figure 5.10 The experimental semi-variogram over the area of concern. 71

Figure 5.11 Model A – a one parameter spherical semi-variogram model fitted to the experimental semi-variogram in Figure 5.10. Model B – a three parameter spherical semi-variogram model fitted to the experimental semi-variogram in Figure 5.10. 72

Figure 5.12 (a) Experimental semi-variogram with a sill apparently below the dotted sample variance. (b) Experimental semi-variogram with a sill well above the dotted sample variance. 75

Figure 5.13 (a) A semi-variogram model that appears to fit the experimental semi-variogram and not consider the dotted sample variance. (b) However, the semi-variogram model does actually have a sill equal to the sample variance. It just reaches the sill at an arbitrarily large distance lag. 77

Figure 5.14 An example of a hole effect semi-variogram model with a nugget effect of c_0 and a sill of $c + c_0$. The figure is taken from Clark & Harper (2000 : 221). 78

Figure 5.15 An experimental semi-variogram that may benefit from a hole effect semi-variogram model fit. 79

Figure 5.16 A semi-variogram model fitted with a sill not equal to the sample variance, but rather equal to the apparent sill of the experimental semi-variogram. This is because the sample variance is deemed to be less reliable than the experimental sill (see Section 5.3). 79

Figure 6.1 An example of a second-order or intrinsically stationary zone (red line) that is adequately represented by a reasonable amount of data (black triangles). All statistics, including the experimental semi-variogram, can therefore be calculated with a great deal of confidence. 81

Figure 6.2 An example of a second-order or intrinsically stationary zone in which mining activities has just begun. The data is limited and usually clustered in a certain area. In this case, the limited, clustered data covers approximately 5% of the total zonal area. 82

Figure 6.3 An example of a second-order or intrinsically stationary zone which has been sampled on a wide scale. Note that the data can be reasonably plentiful or limited dependent on the number of wide scale samples taken. 82

Figure 6.4 The average estimated sill value as a proportion of the actual zonal sill value for Zone A, calculated over various data coverage percentages. The information is taken from Table 6.2. 89

Figure 6.5 The average estimated sill value as a proportion of the actual zonal sill value for Zone B, calculated over various data coverage percentages. The information is taken from Table 6.3. 93

Figure 6.6 The average estimated sill value as a proportion of the actual zonal sill value for Zone C, calculated over various data coverage percentages. The information is taken from Table 6.4. 96

Figure 6.7 The average estimated sill value of accumulation as a proportion of the true zonal sill value, shown over the number of samples contained in the various data clusters. . . 97

Figure 6.8 The average estimated sill value of channel width as a proportion of the true zonal sill value, shown over the number of samples contained in the various data clusters. . . 98

Figure 6.9 Mining activity, indicated by the black sample points, moving from the well sampled red zone into the blue zone. Until the amount of mining activity in the blue zone has increased, the data will be limited and clustered in this zone. 101

Figure 6.10 A closer look at the zonal boundary between the red and blue zones in Figure 6.9. The exact boundary change does not occur instantaneously at the geozone line, but rather it is a gradual change from one zonal area to another. The area between the purple dotted lines can therefore be thought of as a combination of the red and blue zones. 102

Figure 6.11 The average estimated nugget value as a proportion of the actual zonal nugget value, calculated over various data coverage percentages. The information is taken from Table 6.6. 104

Figure 6.12 The average estimated nugget value as a proportion of the actual zonal nugget value, calculated over various data coverage percentages. The information is taken from Table 6.7. 108

Figure 6.13 The 907 sample points available in the zone identified for mining. 112

Figure 6.14 The calculated directional experimental semi-variogram for gold accumulation. It can be seen that the 135 degree and 45 degree pair define the long and short range of the semi-variogram respectively. 113

Figure 6.15 The calculated directional experimental semi-variogram for channel width. It can again be seen that the 135 degree and 45 degree pair define the long and short range of the semi-variogram respectively. 113

Figure 6.16 The nearby, well sampled zone (indicated by a star) of similar variability characteristics. 114

Figure 6.17 The directional experimental gold accumulation semi-variogram of the nearby well sampled zone. 115

Figure 6.18 The directional experimental channel width semi-variogram of the nearby well sampled zone. 115

Figure 6.19 The accumulation and channel width semi-variograms based on 5 randomly selected clusters of point data from the well sampled zone. 118

Figure 6.20 The estimated accumulation semi-variogram from Figure 6.14 (solid lines) compared to the actual accumulation semi-variogram of that zone (ornamented lines). The under-estimation of the sill, nugget and range parameters is clearly apparent. 119

Figure 6.21 The estimated channel width semi-variogram from Figure 6.15 (solid lines) compared to the actual channel width semi-variogram of that zone (ornamented lines). The under-estimation of the sill and nugget parameters is again apparent. 119

Figure 6.22 The adjusted accumulation semi-variogram (solid lines) compared to the actual accumulation semi-variogram of that zone (ornamented lines). Except for the range parameter, the sill and nugget parameters are well estimated. 120

Figure 6.23 After the range parameter is also adjusted, the adjusted accumulation semi-variogram (solid lines) compares much better to the actual accumulation semi-variogram of that zone (ornamented lines) than the semi-variogram in Figure 6.20. 121

Figure 6.24 The adjusted channel width semi-variogram (solid lines) compared to the actual channel width semi-variogram of that zone (ornamented lines). The fit is much better than that in Figure 6.21. 121

Figure 7.1 A q-q plot of the effect estimates in Table 7.2, showing the main effect C lying way-off the normal distribution probability line. 130

Figure 7.2 A scatter plot, taken from Table 7.1, illustrating the main and interaction effects on sill bias. 130

Figure 7.3 A q-q plot of the effect estimates in Table 7.5, showing the main effect G and the interaction effect G × V lying way-off the normal distribution probability line. 136

Figure 7.4 A scatter plot, taken from Table 7.4, illustrating the main and interaction effects on nugget effect bias. 136

Figure 7.5 A q-q plot of the effect estimates in Table 7.8, showing the main effect C lying way-off the normal distribution probability line. 141

Figure 7.6 A scatter plot, taken from Table 7.7, illustrating the main and interaction effects on range parameter bias. 141

Figure I.1 A quadratic trend surface fits the data well, and shows that higher cmg/t values are expected in the west, with values decreasing towards the east. Hence, there is significant trend within the data. 154

Figure I.2 The residuals show no trend across the project area. Hence, the mean residual value over the project area can now be considered a constant. 155

Figure II.1 The location of the 5 mined-out pillars with respect to the pillar being estimated. 158

Figure III.1 A q-q plot of the effect estimates, showing the main effect F lying way-off the normal distribution probability line. 165

Figure IV.1 The spatial coverage of the chosen real-life gold mining data set. It can be seen that the data is numerous and covers the entire zonal area. 170

Figure IV.2 A cluster of data selected from the full data set in Figure IV.1. This cluster is then used to calculate a semi-variogram, of which the parameters are then compared to the semi-variogram calculated from the all the data in Figure IV.1. 170

Figure IV.3 The estimated semi-variogram model using all the data in Figure IV.1. However, the data is slightly manipulated so as to have both a high average grade and grade variability. . . . 172

Figure IV.4 An estimated semi-variogram model calculated from a high cluster size of modified data (modified to have both a high average grade and grade variability). 173

Figure IV.5 Another estimated semi-variogram model calculated from a high cluster size of modified data (modified to have both a high average grade and grade variability). 173

Figure IV.6 An estimated semi-variogram model calculated from a low cluster size of modified data (modified to have both a high average grade and grade variability). 174

Figure IV.7 Another estimated semi-variogram model calculated from a low cluster size of modified data (modified to have both a high average grade and grade variability). 174

Figure IV.8 The estimated semi-variogram model using all the data in Figure IV.1. However, the data is slightly manipulated so as to have a high average grade but low grade variability. 176

Figure IV.9 An estimated semi-variogram model calculated from a high cluster size of modified data (modified to have a high average grade and a low grade variability). 177

Figure IV.10 Another estimated semi-variogram model calculated from a high cluster size of modified data (modified to have a high average grade and a low grade variability). 178

Figure IV.11 An estimated semi-variogram model calculated from a low cluster size of modified data (modified to have a high average grade and a low grade variability). 178

Figure IV.12 Another estimated semi-variogram model calculated from a low cluster size of modified data (modified to have a high average grade and a low grade variability). 179

Figure IV.13 The estimated semi-variogram model using all the data in Figure IV.1. However, the data is slightly manipulated so as to have a low average grade but high grade variability. 181

Figure IV.14 An estimated semi-variogram model calculated from a high cluster size of modified data (modified to have a low average grade and a high grade variability). 182

Figure IV.15 Another estimated semi-variogram model calculated from a high cluster size of modified data (modified to have a low average grade and a high grade variability). 182

Figure IV.16 An estimated semi-variogram model calculated from a low cluster size of modified data (modified to have a low average grade and a high grade variability). 183

Figure IV.17 Another estimated semi-variogram model calculated from a low cluster size of modified data (modified to have a low average grade and a high grade variability). 183

Figure IV.18 The estimated semi-variogram model using all the data in Figure IV.1. However, the data is slightly manipulated so as to have both a low average grade and low grade variability. 185

Figure IV.19 An estimated semi-variogram model calculated from a high cluster size of modified data (modified to have both a low average grade and low grade variability). 186

Figure IV.20 Another estimated semi-variogram model calculated from a high cluster size of modified data (modified to have both a low average grade and low grade variability). 187

Figure IV.21 An estimated semi-variogram model calculated from a low cluster size of modified data (modified to have both a low average grade and low grade variability). 187

Figure IV.22 Another estimated semi-variogram model calculated from a low cluster size of modified data (modified to have both a low average grade and low grade variability). 188

Figure V.1 The probability density function of $X \sim LN(2,\sigma^2)$ for σ = 0.25, 0.5, 0.75 and 1.00. This figure has been taken from Hogg & Tanis (1997 : 223) and shows the various shapes of the two-parameter lognormal distribution. It also illustrates that σ^2 is a good measure of spread of the lognormal distribution. 194

Figure V.2 A normal probability plot of lognormally distributed data. Two extreme outliers (on the right) are apparent from this plot. 203