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Mathematical knowledge for teaching fractions and related dilemmas: A case study of a Grade 7 teacher.

Sharon Govender

A research report submitted to the Faculty of Humanities, University of the Witwatersrand, Johannesburg, in partial fulfillment of the requirements for the degree of Master of Education.

Johannesburg, 2008

DECLARATION

I declare that this research report is my own, unaided work. It is being submitted for the Degree of Master of Education in the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination in any other University.

_____

(Signature of candidate)

1st day of February 2008

ABSTRACT

This study investigates what and how mathematics (for teaching) is constituted in classroom practice. Specifically mathematical knowledge for teaching fractions in Grade 7. One teacher was studied to gain insight into the mathematical problem- solving the teacher does and the dilemmas he faces as he goes about his work.

The analysis of the data show that the mathematical problem-solving that this particular teacher engaged in can be classified as demonstrating, encouraging and working with learner ideas. He appealed to mathematics (rules & empirical), experience (everyday) and the curriculum (tests and exams) to fix meaning. The mathematical problem solving and appeals he made threw up dilemmas of representing the content, competing goals and student thinking. This aided in providing a description of what mathematics for teaching is in this practice.

The report concludes with a discussion of what teachers need to know or study in order to become better mathematics teachers and where do they find these courses to accommodate their need to improve as mathematics teachers.

Keywords:

Fractions
Dilemmas
Mathematical problem solving for teaching
Mathematics
Mathematics for teaching
Pedagogic content knowledge
Teaching

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In loving memory of my friend and colleague

Sibusiso

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CHAPTER ONE

INTRODUCTION

1.1 Aim and Rationale

> "Over the past 20 years, two refrains have echoed through the discourse about teachers' knowledge of mathematics: The first is that many primary teachers' mathematical knowledge is weak. And the second is that the mathematical knowledge needed for teaching is different from that needed by mathematicians. Clearly, these two refrains are importantly related, both central to the problem of developing teachers' knowledge for teaching. Still, efforts to develop teachers' mathematical knowledge continue to lack a helpful theoretical or empirical basis for what to work on, and how to connect it to the work that teachers do".
>
> (Ball, Bass and Hill, 2004, 51)

The facts stated by Ball, Bass and Hill (2004) clearly emphasize that teaching mathematics is not a simple and straightforward activity. Being able to teach mathematics successfully requires more than knowing mathematics. It entails knowing how to apply the mathematical knowledge so that learners are able to understand it. So how then do teachers apply their mathematical knowledge so that their learners can make sense of it? What must teachers know and do to make mathematical knowledge accessible for their learners?

Researchers such as Kazima and Adler (2006), Ball, Bass & Hill (2004), Segall (2004), Brodie (2001), Ball and Bass (2000), Marks (1992) and Even, Tirosh, Markovits (1996), have examined, as well as analysed teachers' mathematical knowledge in order to understand and make better sense of the relationship that exists between what teachers know, how they came to know it and how they go about teaching it (Segall, 2004, 491).

There is a great need to develop teachers' mathematical knowledge but it can only be done if there is a deeper understanding of what needs to be developed. An increasing agreement that there is a specialsed mathematical knowledge that teachers need to know

and know how to use is not enough. Understanding and describing this specialised knowledge should now be in focus. There is however, a problem of what to work on and how to connect it to the work teachers do. This raises the important question of how we find out what this specialsed knowledge for teaching mathematics is?

While many discussions and numerous studies have been completed in understanding this specialsed knowledge, it remains vague. Not enough is known empirically about this specialised knowledge that is required to teach mathematics. It is for this reason that I choose to study mathematics for teaching within a particular context.

Another reason for completing this study is that teacher educators, curriculum developers, government as well as teachers have voiced their concern about teacher knowledge in South Africa. Over the past few years, the meaning of successful mathematics learning and teaching has experienced a revolution in South Africa. In the past, teachers focused on teaching procedures and computational skills. Now the new curriculum demands that teachers help learners develop skills that focus both on conceptual and procedural understanding. No longer do we teach mathematics to be able to obtain correct answers and procedures, but instead we now teach so that learners will be able to develop a deeper understanding of mathematics. This requires a specialised kind of knowledge and again raises the question, what is this specialised knowledge in practice? Hopefully this study will aid in contributing to a better understanding and description of this specialised knowledge.

1.1.1 Personal dilemmas

From a personal point of view, I have embarked on this study with the intention of understanding how to manage the dilemmas/problems I experience as a mathematics teacher with regards to teacher knowledge. I have taught Mathematics to primary school learners for the past ten years. My qualifications as a primary school mathematics teacher include a two-year methodology course on how to teach mathematics. I did not do Mathematicsl during my initial years of studying as it was not a requirement and

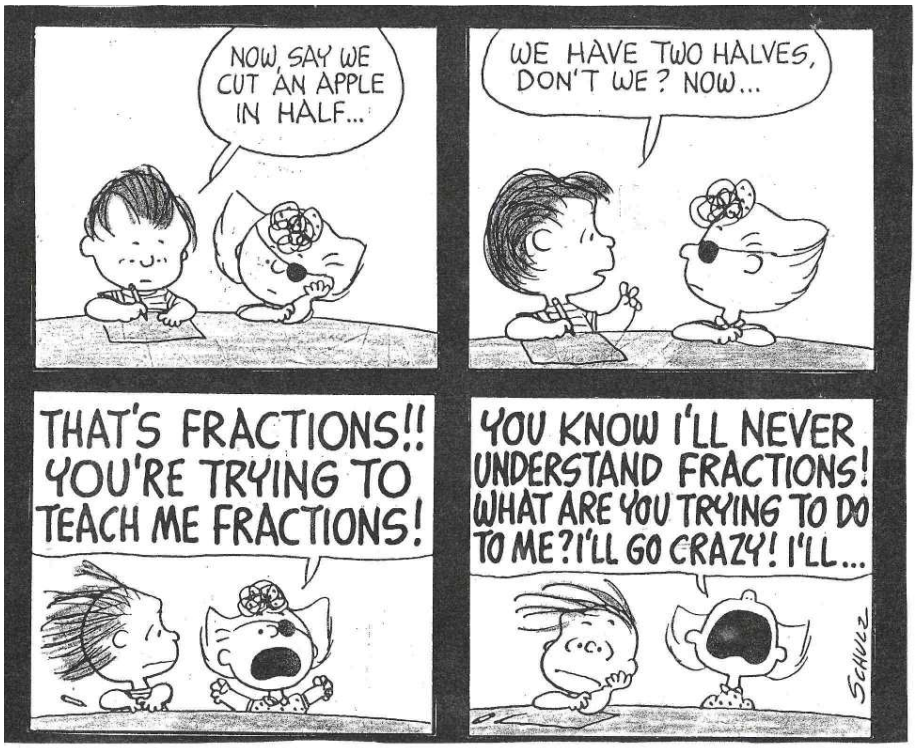
Mathematics was the last subject I wanted to enroll for because of the awful experiences I had when learning it at school. After five years of teaching the different learning areas (i.e. Mathematics, Literacy, Science & Technology, Life Orientation), I found Mathematics to be the most interesting. It was then that I enrolled to do my Honours and later my Masters in Education with a focus on Mathematics. The experience was invaluable. I had decided that after I obtain my Masters degree in Mathematics Education, I will enroll to do Mathematicsl at a university level since it would help my teaching. However, after much deliberation and all the reading I have done on Mathematics for teaching, I am no longer convinced that this will help me become a better mathematics teacher. I am aware that I do not know enough mathematics, but also know that just doing a Mathematics course or more courses in pedagogy is not sufficient. My dilemma is that I want to continue to study in order to improve as a mathematics teacher, so what is it that I must study? Do I study more Mathematics or do I study more pedagogy? This is my dilemma around choices. It is by no means the only dilemma I experience in teaching.

As suggested earlier, the mathematics curriculum requires that teachers teach for both procedural and conceptual understanding. This has proved to be a dilemma for me, particularly when teaching fractions. When teaching fractions I find myself rushing through the work wanting the learners to just follow the procedures in order to obtain correct answers, knowing full well that they do not really understand the underlying concepts. I am faced with the dilemma of teaching fractions for both procedural and conceptual understanding. Teaching for conceptual understanding takes a fair amount of time, learners need time to process what is taking place and make sense of the concepts. It also requires that I engage with the learners on a deeper level mathematically. Learners often get frustrated and lose interest, resulting in me providing them with procedures to follow. Teaching for procedural understanding requires less time and learners can do the mathematics by merely following given procedures. Do I focus on whether they understand the mathematics (why) or whether they can do the mathematics (how)? How do I teach both procedurally and conceptually without compromising either conceptual understanding and so some meaning or procedural skill? This is a real dilemma for me as

a mathematics teacher. Therefore I have chosen the topic of fractions to help me explore how to manage the dilemmas I experience and also to shed more light on what must be known and understood about the mathematical work teachers do, in order to manage this dilemma.

1.2 Why fractions was chosen to contextualise this study.

Fractions is a well-researched topic and findings shows that the teaching of fractions is a problem for any primary school teacher. As mentioned above, it certainly is a problem for me.



The cartoon above speaks a million words of how learners and teachers react towards the learning and teaching of fractions. Learners often give up because fractions are so difficult to understand and teachers usually give up because fractions are so difficult to make understandable.

Is the teaching and learning of fractions and rational numbers an important and necessary part of our mathematics curriculum? This question has plagued my colleagues and I for a number of years. In understanding the importance of fractions and rational numbers in our number system and their complicated nature, I have come to the conclusion that mathematics teachers are threatened by this topic. Research suggest that there are two reasons in particular why teachers may feel this way about teaching fractions: the multifaceted nature of fractions and teachers understanding i.e. their perception and conception of ideas related to the fractions. While teachers may feel insecure about teaching fractions, they cannot ignore the importance of it in both mathematics and everyday life.

Fractions form an important part of our curriculum. It is introduced formally to learners from as early as Grade 1 and forms the basis of what lies ahead in understanding the concepts related to rational numbers in later years. Learners also encounter fractions informally in their everyday lives, for example, sharing out sweets amongst friends or when baking a cake. Although fractions have been an important component of the curriculum for a long time and much research has been done on it, teachers still have difficulty teaching it and learners still experience difficulty understanding fraction concepts.

1.3 Statement of the Problem and Critical questions

As elaborated in Chapter Two, the teaching of fractions requires both procedural skill and conceptual understanding in order to teach it completely and successfully. Teaching fractions is not simple and straightforward and requires specialised knowledge. Therefore a focused study of the teaching of specifically the addition of fractions in Grade 7 was undertaken, with the following critical questions being asked:

1. What mathematical problem solving does this teacher do when teaching addition of fractions in his Grade 7 class?
2. What knowledge resources (appeals) does he call on as he goes about this work?

3. What teaching dilemmas are reflected in this practice?
4. How might this practice, and its teaching dilemmas be explained?

These composite questions and how they are answered will illuminate the mathematical work done by the teacher and also what is constituted as mathematics for teaching in this classroom practice. Ball, Bass and Hill (2004) advocate that finding a helpful theoretical and empirical base for what to work on and how to connect it to the work that teachers do may aid in the effort to develop teachers' mathematical knowledge. Being made aware of and understanding the dilemmas teachers experience as they go about their work may provide the theoretical and empirical base that Ball, Bass and Hill (2004) speak about.

1.4 Summary

In this chapter, I have highlighted the concerns associated with mathematics for teaching, as well as provided my rationale for embarking on this study. I have also engaged in a discussion on the importance of using fractions in this study and how understanding dilemmas teachers experience in their practice can help provide a base for understanding what needs to be developed with regards to teachers' mathematical knowledge. The next chapter focuses on the relevant literature related to the teaching and learning of fractions, dilemmas, as well as the theoretical and analytical framework that underpins this study.

Chapter Two

Literature Review and Theoretical Framework

In this chapter, I will review selections of research literature that relate to the nature and teaching of fractions, the knowledge teachers draw on when teaching fractions as well as tasks for teaching fractions and teaching dilemmas.

I begin with a review of research on the nature and teaching of fractions, as this will illuminate the kinds of problem solving a teacher might be faced with, as well as the knowledge resources he might call on in order to teach fractions.

2.1 The teaching and learning of fractions

'Fractions are among the most complex mathematical concepts that children encounter in their years in primary school' (Charalambous & Pitta-Pantazi, 2005). This statement generates many questions within the realm of mathematics education. Some of these questions are: Why is the teaching and learning of fractions so complex? What can mathematics teachers and teacher educators do in order to 'uncomplicate' the complex mathematical concepts associated with the teaching and learning of fractions? What do mathematics teachers know and do in order to successfully teach fractions?

These are not new questions with regards to the teaching and learning of fractions. Much research has been done on this topic and the findings have been beneficial to teaching and research in this particular field. However, some of these questions remain unanswered, and the difficulties associated with the teaching and learning of fractions persist.

It has been suggested that there are two reasons why children may find it difficult to develop a deep understanding of fractions: (1) the nature of fractions and (2) the way in which fractions are taught (Charalambous & Pitta-Pantazi, 2005).

2.2 The nature of fractions

Charalambous & Pitta-Pantazi (2005, p.233) unequivocally state that, 'To date there is a consensus among researchers that one of the predominant factors contributing to the complexities of teaching and learning fractions lies in the fact that fractions comprise a multifaceted construct'. Behr et al (1992, p. 296) indicate, '…when fractions and rational numbers as applied to real-world problems are looked at from a pedagogical point of view, they take on numerous 'personalities'.

These statements give us insight into why fractions have been so problematic for both teachers to teach and students to learn. It offers to explain that fractions are complex and cannot be viewed simplistically.

Kieren (1976, 1988), Behr, Lesh, Post and Silver (1983), Vergnaud (1983), Freudenthal (1983), and Mack (2001) are a few researchers who have attempted to explain the nature of fractions. Kieren (1976) distinguished four mathematical subconstructs of fractions: measure, ratio, quotient and operator. The notion of the part-whole relationship encourages the development of these four subconstructs and is embedded in all of them. It is for this reason Kieren did not consider it a separate subconstruct (Baturo, 2004). Behr, Lesh, Post and Silver (1983) contested the idea that the part-whole could not form a separate subconstruct and differentiated part-whole, ratio, quotient, operator, and measure as mathematical subconstructs of fractions.

Since fractions have a variety of mathematical meanings there is a large range of ways in which symbols such as $\frac{2}{4}$ might be interpreted. Mamede, Nunes & Bryant (2005, p.282) offer us a clear description of fraction in part-whole and quotient situations. For example, in the part-whole situation, the denominator suggests the number of parts into which a whole has been divided, while the numerator suggests the number of parts taken.

> 'So, $\frac{2}{4}$ in a part- whole situation would mean that a whole – for example- a chocolate was divided into four equal parts, and two were taken' (p.282).

Figure 1.1 shows two parts of a whole chocolate that has been divided into four equal parts, which can be represented by the fraction $\frac{2}{4}$.

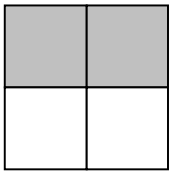


Fig: 2.1: A part –whole representation of the fraction $\frac{2}{4}$.

In the quotient situation, the denominator suggests the number of recipients and the numerator suggest the number of items being shared. In the case of $\frac{2}{4}$, it means that 2 items (chocolates for example) were divided among four people. Furthermore, this construct can be separated into two subconstructs, partitive and quotitive. This means that the fraction $\frac{2}{4}$ represents the division and also the amount that each recipient receives, regardless of how the chocolates were cut. Dividing two chocolates equally amongst four children involves partitive or partitioning while quotitive division represents the part each child receives, even if the chocolates was only cut in half.

Behr et al (1992) explain that,

> The operator concept of rational numbers suggest that the rational number $\frac{3}{4}$ is thought of as a function applied to some number, object or set. As such we can think of an application of the numerator quantity to the object, followed by the denominator quantity applied to this result, or vice versa. The basic notion is that the natural number causes an extension of the quantity, while the denominator causes a contraction…(p.314).

For example, look at the pictures in figure 2.2. To produce the picture of the smaller box, the dimensions of the picture of the larger box were operated on by a factor of $\frac{3}{4}$.

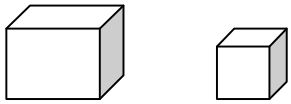


Fig 2.2: Using the operator $\frac{3}{4}$ to shrink the dimensions of a picture.

Central to the measure construct is the concept of unit. When children work with tasks involving number lines (finding points on the number line), they must use the measure construct (Behr et al, 1992). The following is an example where children could use the measure construct, giving each child a piece of string that is one meter long and asking them to cut another length of string $\frac{1}{4}$ metre long. When children are given the opportunity to divide string, for example, into identical lengths of various equally sized parts, they gain experience identifying equivalent fractions. For example, they are able to identify that $\frac{2}{4}$ unit of string is equivalent to $\frac{4}{8}$ unit of string.

The fifth and final construct of fractions is ratio. The ratio construct is different from the other constructs in that the numerator and denominator do not necessarily refer to the same quantity. Addition, unlike for the other fraction constructs, is also defined differently for ratio numbers**.** For example, $\frac{2}{3}+\frac{3}{4}=\frac{2+3}{3+4}$ when $\frac{2}{3}$ and $\frac{3}{4}$ are ratios = $\frac{5}{7}$.

When $\frac{2}{3}$ and $\frac{3}{4}$ is not a ratio, addition is done as follows: $\frac{2}{3}+\frac{3}{4}=\frac{17}{12}$ (Behr et al, 1992)

Connecting the part-whole subconstruct with the process of partitioning provided an opportunity for Behr, Lesh, Post and Silver (1983) to develop a theoretical model which links the different subconstructs of fractions to the basic operations of fractions, fraction equivalence and problem solving (Charalambous & Pitta-Pantazi, 2005). For Mack (2001), 'partitioning' encompasses both the part-whole & quotient subconstructs, while according to Charalambous & Pitta-Pantazi (2005), the model developed by Behr, Lesh, Post and Silver (1983), suggest that in order to develop an understanding of ratio,

operator, quotient and measure, it is imperative to consider the combination of the part-whole subconstruct of rational numbers and the process of partitioning. In order to develop an understanding of the multiplication and addition of fractions, the use of the operator and measure subcontructs would be beneficial. In order to solve problems regarding fractions, understanding part-whole/partitioning, ratio, operator, quotient and measure is needed.

From the literature it is evident that central to the part-whole fraction subconstruct is the notion of partitioning a whole, 'whatever its representation, into a number of equal parts and composing and recomposing (i.e., unitizing and reunitising) the equal parts of the initial whole' (Baturo, 2004, p. 96). Kieren (cited in Baturo, 2004) suggests that in view of the fact that partitioning experiences are so important to the development of rational numbers, students should be afforded numerous opportunities to partition diverse fraction models in different ways in order to develop an understanding of the representation of fractions. Contrary to this, Kerslake (1986) as well as Mamede, Nunes & Bryant (2005) argue that the part-whole model is not the easiest situation for learning fractional representations, in fact Kerslake suggests that instead of promoting the development of the more general idea of a fraction, it inhibits it. Mamede, Nunes & Bryant (2005) in their study reported that there is no supporting evidence that the part-whole fraction subconstruct is the most suitable one for teaching fractions since children performed better in problems presented in quotient situations. Research suggests that in 'traditional' classrooms, part-whole situations are used to introduce the concept of fractions instead of quotient situations and that maybe it is time to rethink which is the most appropriate situation to introduce the concept of fractions. It is evident that these subconstructs make choosing appropriate tasks to represent and teach fractions very complex.

Some mathematics teachers struggle to find the correct representations for teaching fractions because of its multifaceted nature. They often resort to using representations that comprise of regularly shaped objects that are divided into equal parts or they use the number line (Verschaffel, 2006). Hannula (2003) in his study with 5^{th} and 7^{th} grade Finnish students found that the students were unable to use the number line to make sense

of what the appropriate whole was. Instead of being helpful, the number line posed to be problematic. Similarly Charalambous & Pitta-Pantazi (2005) report in their research that the number line is a difficult model for students to use in order to understand the concepts related to fractions and suggest that teachers first focus on establishing an understanding of other notions before embarking on using the number line as a model in their teaching. Kerslake (1986), in her discussion of the different models of fractions children are familiar with, noted that the difficulty children have with fractions arises because of their limited view of a fraction. This in turn generates problems when trying to make sense of the addition or of placing a fraction on a number line. Both Verschaffel et al (2006) and Hannula (2003) draw attention to the fact that since fractions comprise of multiple subconstructs, the way that they are taught and represented in mathematic classrooms at schools should consist of multiple representations that encourage and promote both the procedural and conceptual understanding of fractions. This however, is not evident in such classrooms and thus prompts enquiry into how fractions are being taught in mathematics classrooms.

The focus of this study is not to find solutions or answers to how teachers should teach fractions or how students construct fraction understanding (Mamede et al, 2005, Herman et al, 2004, Pirie, et al, 1994,) but rather, as noted in the previous chapter, it focuses on one teacher and how he teaches fractions specifically, the problem solving he faces, knowledge resources he draws on and the dilemmas reflected in his practice. As noted, despite a substantial amount of research done in order to establish and understand why the teaching and learning of fractions is so difficult, the difficulty persists and remains a constant challenge for teachers. An in-depth study of what happens in practice could offer some explanation and illuminate why the problems related to the teaching and learning of fractions persist, particularly within the South African context. It also hopes that in studying the knowledge for teaching in this practice, more light will be shed on the problem in order to help make a difference with regards to the specialised knowledge needed for teaching mathematics.

2.3 How fractions are being taught

Philippou & Christou (1994) argue that because of the complex nature of fractions, the teaching of fractions should receive more attention and it should be taught in a more meaningful way. However, they maintain that teachers continue to teach fractions procedurally instead of helping students develop a deeper conceptual understanding of them.

Philippou & Christou (1994) in their study of elementary teachers' conceptual and procedural knowledge of fractions conclude that the teachers they worked with portrayed a lack of conceptual understanding of fractions. While they were able to calculate correctly and displayed appropriate knowledge of symbols and algorithms associated with fractions, they had difficulty explaining why particular procedures were used. This highlighted their lack of understanding of conceptual knowledge with regards to the concept of the operations of fractions and the influence it had on how they taught fractions. The subjects involved in this particular study carried out and focused on the procedures to produce correct answers. This indicated that these preservice teachers failed to make a connection between their procedural and conceptual understanding of fractions. Philippou & Christou (1994) dichotomized procedural and conceptual knowledge and argued that the preservice teachers were unable to complete certain problems successfully because of their conceptual knowledge of fractions.

Ma (1999) makes a similar kind of argument, however she does not talk about procedural or conceptual knowledge as separate entities. Instead she refers to 'knowledge packages' where procedural and conceptual knowledge need to be connected. It must be noted that Ma (1999) did not only study fractions, she makes this claim about other topics as well. In her comparative study of teacher's understanding of fundamental mathematics in China and the United States she points out that 43% of U.S teachers were able to complete a calculation on the topic of division by fractions successfully, but none of these teachers were able to offer a mathematically sound explanation for their

calculations. In contrast to this, the Chinese teachers were able to represent the operation and correctly explain its meaning because of their concrete and extensive knowledge of the topic. Ma (1999) alludes to the fact that having an 'all-inclusive' understanding of a topic results in pedagogically powerful representations of that topic.

Charalambous & Pitta-Pantazi (2005), in their study which attempts to provide empirical validity of the theoretical model of fractions, which link the five interpretations of fractions to the operations of fractions and problem solving, suggest that in order for students to become proficient in the operations of fractions, they must master the five interpretations of fractions. This has many implications for the teaching of fractions. Firstly, it proposes that items used when teaching fractions require both procedural and conceptual understanding of the operations of fractions. Secondly it draws attention to the fact that teachers need to develop a supporting framework that would help students develop a deeper understanding of the different 'personalities' of fractions. This in turn could improve students' performances with regards to tasks involving the operations of fractions. Charalambous & Pitta-Pantazi (2005) concur with Philippou & Christou (1994) and other researchers, that when fractions are being taught, there must be an emphasis on the conceptual understanding of it. The teaching of algorithms to perform operations on fractions in order to achieve correct answers must be substituted by the teaching of fractions for conceptual understanding and knowledge. This however, by no means, suggests that teachers must ignore 'procedural learning', but rather as Ma (1999) indicates, teachers with profound understanding of mathematics understand the role of "procedural learning" in conjunction with "conceptual understanding" in the learning of mathematics. This purports that teachers should possess more than just a procedural knowledge of the operations of fractions.

"The real mathematical thinking going on in a classroom, in fact, depends heavily on the teacher's understanding of mathematics" (Ma, 1999, p.153). Fennema and Franke (1992) state, "the evidence is beginning to accumulate to support the idea that when a teacher has a conceptual understanding of mathematics, it influences classroom instruction in a positive way" (p.151). Both of these statements point towards the importance of the kind of understanding that teachers possess regarding the mathematics they teach. What

teachers know and understand, influences their students' thinking and how they teach. It suggests that if teachers are unable to make sense or meaning of the mathematics they teach for themselves, attempting to construct meaning for their students in order to understand fractions could prove to be futile. It also intimates that being able to identify and recognize errors made by students requires that teachers fully understand the subject they teach in order to make valid judgments (Philippou & Christou, 1994). What then should teachers know in order to teach the operations of fractions so that their students develop a profound understanding of it? All of these aspects relate to teacher knowledge, which forms an integral part of teaching.

As teachers embark on the process of teaching fractions they are bound to experience certain dilemmas. Stein et al (2000), highlights a dilemma experienced by one of the teachers in their study. This dilemma experienced by the teacher is closely related to what has been discussed thus far in the literature. Ron, a mathematics teacher, struggles to find a balance between teaching efficient procedures and encouraging students' development of conceptual understanding when teaching fractions, decimals and percentages. The literature clearly points to fact that in order to become proficient in the operations of fractions, students must be exposed to tasks that promote both procedural and conceptual knowledge and the understanding of it. Because of the complex nature of fractions, teachers are faced with the dilemma of teaching algorithms to perform operations on fractions in order to achieve correct answers versus teaching fractions, using more complex, less structured tasks, so that their students are able to develop a deeper understanding of the nature of fractions. If they opt for the latter, they risk the chance of working with students who become anxious because of the uncertainty associated with the tasks (Stein et al, 2000). If they choose the former, their students will have a superficial understanding of the concept of fractions. As mentioned in the previous chapter, I have experienced a similar dilemma when teaching fractions. How do I ensure that my learners know both procedures and concepts without compromising either? What do I need to know in order to promote both conceptual and procedural understanding of fractions? How do I as a teacher manage these dilemmas I experience?

These are not simple questions with straightforward answers, but rather require an in-depth analysis of what takes place in practice when fractions are taught.

From the above discussion, it is evident that the teaching of fractions is not simple and straightforward. Its implication for this study is that learners are bound to experience problems as they grapple with the conceptual and procedural understanding of fractions. This requires that the teacher help learners unpack and understand the mathematics. It will entail some kind of mathematical problems solving by the teacher and he will have to draw on knowledge resources in order to legitimate the mathematics being taught. In other words, in order to make explicit or point out what is valued. A lot of what has been discussed thus far in the literature prescribes what teachers should know and do to teach fractions successfully. As noted earlier, they must possess both conceptual and procedural knowledge and understanding of fractions. They should be able to offer mathematically sound explanations. They are required to choose appropriate representations to teach the concept. They have got to be able to identify and recognize errors made by students and make valid judgments etc. The literature offers an extensive list of what the teacher must know and do when teaching fractions, yet as argued, difficulties persist. My aim in this study is to focus on what the teacher actually does. In other words, I hope to describe what takes place in reality in this particular classroom as the teacher teaches fractions. It will be interesting to see what the teacher does as he teaches fractions. Does he use representations, how does he use them and what does he call on to explain the concepts? Are the explanations he offers mathematically sound? What dilemmas does he experience and what mathematical problem solving does he do? How does he deal with student thinking? This, I hope will provide an opportunity to map out what the teacher does, to what is prescribed by the literature in order to come to an understanding of what and how mathematics for teaching is constituted in this classroom practice.

2.4 Teacher Knowledge and Problem solving

Teacher knowledge comprises more than just subject matter (content) knowledge. It is the blending of both subject matter knowledge and pedagogical knowledge, which Shulman (1986) has termed pedagogical content knowledge.

Shulman (1986) explained that pedagogical content knowledge includes:

> '…the most useful forms of representations of those ideas, the most powerful analogies, illustrations, examples, explanations, and demonstrations- in a word, the ways of representing and formulating the subject that make it comprehensible to others…..It also includes an understanding of what makes the learning of specific topics easy or difficult: the conceptions and preconceptions that students of different ages and backgrounds bring with them to the learning of those most frequently taught topics and lessons….(p.9)

From Shulman, it is understood that pedagogical content knowledge is what describes and evaluates teaching. Ball (2004) suggest it is PCK that helps distinguish teachers from subject area experts. The distinction is not so much in the quality or quantity of the subject matter knowledge, but in how that knowledge is organized and used. Marks (1992) in his study on fifth grade teachers' about their teaching of equivalence of fractions, clearly shows that it is difficult to pin down the concept of pedagogical content knowledge theoretically, however, it signifies a type of knowledge that is specialsed, unique and central to the work done by teachers. This knowledge does not usually form part of non-teaching subject matter experts or teachers who know little of that subject. It is apparent that teachers are required to 'transform' their subject matter knowledge in order to teach.

This means that the success of what is taught and how it is taught depends on how teachers use their subject matter knowledge, pedagogical knowledge and their understanding of topics that are difficult or interesting for students and how to represent the content in a useful way (Ball & Bass, 2004). PCK is therefore more than just subject matter knowledge (SMK). It is knowledge that teachers must have in order to teach the subject knowledge successfully and effectively. They must be able to present the content in a way that is comprehensible for students. Their knowledge and understanding of

different ideas, representations, methods etc are important if they want to teach for understanding. They must understand and know why certain concepts are either difficult or easy for students and what would be the best way to teach it. The blending of teacher's subject matter knowledge, pedagogical content knowledge and curricular knowledge must allow for the knowledge to be transformed so that it can be taught. In order for the transformation to take place successfully, so that mathematics is taught well, Ball, Bass and Hill (2004) suggest that teachers 'unpack' or 'decompress' the mathematics they know and have been taught. This implies that teachers must have a good understanding of their subject (mathematical) matter knowledge as well as knowing how to use this mathematical knowledge when teaching (Adler, 2005)

Since the inception of the notion of pedagogical content knowledge, several researchers (Kazima & Adler, 2006, Ball, Bass & Hill, 2004, Segall, 2004, Brodie, 2001, Ball & Bass, 2000, Swafford, et al, 1997, Even, Tirosh, Markovits, 1996, Marks 1992, Lampert, 1985) have examined and analyzed teachers' mathematical knowledge within a specific topic area in order to understand and make better sense of it. These researchers have attempted to understand the relationship that exists between what teachers know, how they came to know it and how they go about teaching it (Segall, 2004). However, some researchers have brought to attention that the notion of pedagogical content knowledge is not simple and straightforward to understand, but instead brings with it a host of complexities that make it difficult to identify and understand within a classroom context. Marks (1992), in his attempt to describe PCK discovered that it was very difficult to do so since it is derived from other types of knowledge (student understanding, subject matter, instructional processes and instructional media). Determining where one kind of knowledge ends and the other begins is very difficult. Marks (1992), therefore suggests that PCK cannot be viewed or assumed as being knowledge that grows out of general pedagogy as well as subject matter. Ma (1999), similarly to Marks, shows that knowledge that is required for teaching mathematics must take into account the relationship between the student and the mathematics as well as the rationale for learning mathematics. Ma (1999) takes it a step further and extends our understanding of mathematical knowledge as part of pedagogical content knowledge (Brodie, 2001). Brodie (2001, p.88) best

describes the nature and development of what Ma calls 'profound understanding of fundamental mathematics':

> "…mathematics teachers understand mathematics for teaching in deeply structural ways. They have 'knowledge packages'(p.113) which represent webs of mathematical concepts and their relationships in ways in which they are best learned by children. A key feature of these 'knowledge packages' are 'conceptual knots' (p.115) which tie together meanings of concepts, symbolic representations, algorithms and their rationales, and procedural and conceptual knowledge"

Ball, Bass and Hill (2004) hold a similar view to Marks (1992) and Ma (1999). Unlike Shulman, they acknowledge that the boundary between SMK and PCK is blurred. They do not make a clear distinction between SMK and PCK and have instead coined the term, "Mathematics for Teaching" (MfT) in order to describe and understand what takes place in practice. In other words, they argue that the categories related to teacher knowledge are not discrete and that they are unclear and therefore a way forward would be to view what takes place in classrooms as mathematical problem solving. Ball and Bass (2000) argue that in order to understand what teachers know, do and learn in the classroom, we must observe classroom practice. As noted, a teacher must be able to 'decompress' the mathematical knowledge they have acquired in order to teach mathematics (Ball & Bass, 2000). They make a clear distinction between knowledge that teachers require in anticipation of practice and knowledge that teachers need in practice. Drawing on both SMK and PCK Ball, Bass and Hill (2004, p59) have suggested types of mathematical problem solving that mathematics teachers do as they go about their work

- Design mathematically accurate explanations that are comprehensible and useful for students;
- Use mathematically appropriate and comprehensible definitions;
- Represent ideas carefully, mapping between a physical or graphical model, the symbolic notation, and the operation or process;
- Interpret and make mathematical and pedagogical judgments about students' questions, solutions, problems, and insights (both predictable and unpredictable);

- Be able to respond both productively to students' mathematical questions and curiosities;
- Make judgments about that mathematical quality of instructional materials and modify as necessary;
- Be able to pose good mathematical questions and problems that are productive for students' learning;
- Assess students' mathematics learning and take the next steps.

When teaching fractions the teacher may find himself engaging in some of the activities mentioned above. For fractions, it would mean that the teacher is able to provide mathematically sound explanations of the different constructs of fractions. He would also have to provide representations of the content that afford learners the opportunity to develop both procedural and conceptual understanding of fractions in their different forms, bearing in mind the cognitive demands of the tasks. It proposes that the teacher has insight into what his learners may be thinking as they engage with different activities i.e. learners will bring to the class their everyday knowledge about fractions that influence their thinking and understanding. The teacher will have to identify, distinguish and mediate between everyday knowledge and scientific knowledge in order to make pedagogical and mathematical judgments. The nature of fractions is complex and this requires that the teacher is able to modify tasks according to the learners' needs without necessarily changing the cognitive demand of the task. This is by no means a complete list of the mathematical problem solving that the teacher in this study could be engaged in. The mathematical problem solving will be identified as the teacher goes about doing his work of teaching.

Adler (2005), drawing from Ball et al (2004) regards the work of mathematics teaching as a particular kind of mathematical problem solving. Adler & Pillay (2007) make it clear that this 'mathematical problem solving' or 'mathematical work' is not synonymous with work someone does when working on or solving a mathematics problem. Instead, they describe it as the work done by mathematics teachers that involves particular kinds of problem solving that has mathematical entailments**.** Teachers are confronted with problems as they go about teaching. The solving of these problems is considered as the

'mathematical problem solving' or 'mathematical work' done by the teacher in the classroom. When teachers are involved in mathematical problem solving, it requires that they call on certain knowledge resources in order to fix meaning or legitimate meaning. Adler et al (2005) provides three knowledge domains that teachers might appeal to in order to legitimate knowledge for learners in mathematics classrooms. The knowledge domains are mathematics, experience and curriculum (see analytical framework for an explanation on each one). In a similar study on functions by Adler & Pillay (2007), it was found that the teacher explained by using empirical examples and calling on what examinations were rather than the nature of functions. It would be interesting to see whether the findings of the study done by Adler & Pillay (2007) correlate with the findings of my study and how the outcome might be explained.

2.5 Teaching Dilemmas and Teacher Knowledge

While the use of Ball, Bass and Hill's (2004) mathematical problem solving categories help describe what takes place in classroom practice, particularly the mathematical work the teacher does, it does not help in addressing certain issues that the literature is starting to throw up. At a personal level dilemmas make more sense and the way Adler (2001) used teaching dilemmas is useful. Therefore, I am going to draw on and explore the issue of teaching dilemmas. I would like to see to what extent dilemmas can be identified by looking at what mathematical problem solving the teacher does and the resources he draws on and how and whether these dilemmas are resolved in relation to what the teacher says. This will provide some insight into what and how mathematics (for teaching) is used and so constituted in this classroom practice.

Adler (2001) in her earlier work brings to our attention the notion of teaching dilemmas experienced in multilingual mathematics classrooms. Similar to Ball and Bass, her interest lies in accessing and understanding teachers' tacit and articulated knowledge for teaching mathematics. Adler (2001) states that while classrooms are complex sites of practice, knowing, understanding and identifying what takes place during this practice aids teachers to reflect and act on their practice. She uses the notion of teaching

dilemmas to capture and open up 'teachers knowledge of the elusive, complex and dialectical nature of teaching and learning mathematics in multilingual classrooms' (2001, p.1). Although Adler's study focused on the dilemmas of code-switching, mediation and transparency experienced in multilingual mathematics classrooms, the idea of identifying dilemmas provides me with 'explanatory tools and analytic devices' with respect to what takes place in a Grade 7 mathematics classroom when fractions are being taught (Adler, 2001, p.1).

The purpose of using dilemmas is that it bridges the abstract ideas of theory and the on-the-ground realities of practice (Adler, 2001, p.1). Adler uses both Lampert (1985) and Berlak and Berlak (1981) amongst others, to show that 'the notion of teaching dilemmas as at once practical, personal and contextual becomes a useful tool through which to capture the complexities and contradictions in teaching' (Adler, 2001, p.56). This means that identifying, acknowledging and understanding teaching dilemmas helps teachers make sense of what is taking place during the process of teaching and learning in practice. Lampert (1985) states that when teachers try to solve pedagogical problems that they encounter daily in the classroom, what actually happens is that they ultimately experience 'practical dilemmas'. These dilemmas are often difficult to address and it is imperative that teachers find a way to manage them instead of ignoring and pretending that they do not exist. It will be interesting to note what the teacher in this study does to manage the dilemmas he faces in particular, what problem solving he does and what resources he calls on as he goes about this work.

As noted, Adler's (2001) study reveals three dilemmas experienced by mathematics teachers in a multilingual classroom. Looking at the dilemma of code switching, Adler (2001) reveals that the teacher is faced with the dilemma of developing English vs. the dilemma of developing meaning (p.68). For Adler, acknowledging this dilemma meant that the teacher was able to understand what was taking place in the classroom with regards to code switching and at the same time act on it, in order to improve her practice. It also provided an understanding of the teacher's mathematical and pedagogical

knowledge required for teaching mathematics in a particular context. In other words, dilemmas help explain what teachers know and what they need to do.

With regards to my study, I hope to gain insight into the teaching dilemmas by studying what mathematical problem solving the teacher engages with and what knowledge resources he draws on to legitimate meaning for his learners. In other words, I hope to see the dilemmas in terms of the problem solving the teacher does and the resources he calls on with a focus on the literature around fractions. Identifying dilemmas and how the teacher manages these, will in turn illuminate his mathematical work.

2.6 Cognitive Demands of Tasks and Associated Dilemmas

I now turn my attention to what forms an important part of this research and that is the tasks used to teach fractions. It so happens that in this study the teacher presents the addition of fractions through a complex task.

There has been a substantial amount of research done on tasks since some of the reform practices in mathematics have been introduced through new kinds of tasks. The teacher in this study does this so it is important for me to examine the task in relation to what problem solving and dilemmas are thrown up.

For the purpose of this study, I will use Stein et al's (2000) understanding of mathematical tasks and how they are enacted within the classroom context.
While tasks per se are not the focus of this study, it is useful to look at Stein et al's work on tasks since it helps identify the kinds of dilemmas teachers' might face. Stein et al distinguish between selection, set-up and implementation of tasks. While they do not talk about tasks in relation to specific topics like fractions however, they offer some understanding of things to think about with regards to dilemmas teachers experience when selecting, setting up and implementing tasks in any topic. Since the teaching of fractions is laden with complexities because of its nature, the tasks that teachers use to represent the content may be an ideal tool to reveal what mathematical problem solving is

taking place, what resources are called on and what dilemmas are experienced. In other words, these tasks are not only central to learning, but they are also important artifacts of practice that could illuminate the mathematical problem solving, resources called on and dilemmas.

Stein et al (2000) suggest that different tasks make different demands on thinking. There is a certain level and kind of thinking required of students in order to successfully engage, complete and solve a task and this is what Stein et al (2000) refer to as cognitive demands. They claim that 'the level and kind of thinking in which students engage determines what they will learn' (2000, p. 13). In other words, teachers hold the key to what learners learn. Teachers either provide tasks that require minimal thinking and reasoning or they can supply their learners with tasks where they are actively engaged cognitively. But in order to provide tasks that are appropriate for a particular lesson, teachers must be able to match the tasks according to their goal for student learning. This means that if a teacher requires that learners become fluent in certain procedure then tasks that involve memorization may be appropriate or if the goal is to develop a deeper understanding of concepts then tasks that involve doing mathematics will be more suitable (Stein et al, 2000).

Stein et al (2000) examines the cognitive demand placed on learners in different kinds of mathematical instructional tasks. They refer to two different levels of cognitive demands of mathematical tasks. They are lower level cognitive demand (procedural) and higher-level cognitive demands (conceptual). Memorization and procedures without connections are considered lower level cognitive demands while procedures with connections and doing mathematics are considered as higher-level cognitive demands.

With regards to the teaching of fractions, finding the correct and appropriate representations can be very difficult for teachers. Often teachers resort to choosing tasks that require procedural applications since their own understanding of fractions are limited and insufficient. A study done by Linchevsky and Vinner (1989) show that in-service and preservice teachers involved in the study, revealed misconceptions and confusions

when the canonical whole was replaced by another whole for fractions of continuous quantities. The teachers' visual representations of fractions were incomplete, unsatisfactory and insufficient to form a complete concept of fractions. Sanchez and Llinares (1992), in their study found that the way fractions are presented, formulated and represented in order to make it understandable to students depends largely on elementary teachers' understanding of fractions (subject matter). Similarly Ball (1993) makes us aware of the dilemma of representing the content that she experienced during her teaching of negative numbers to Grade 3 learners. She reveals that what teachers know about mathematics will determine what tasks they chose in order to teach the content successfully. She explains that representing the content to teach negative numbers and other topics is extremely difficult. From the above and in accordance with previous studies (Charalambous & Pitta-Pantazi, 2005, Philippou & Christou 1994, Marks, 1992 etc) an assumption can be made that teachers will choose or implement tasks that match their subject matter, curriculum and pedagogical knowledge of the topic.

2.7 Positing Potential Dilemmas

As noted, the teaching of fractions is difficult and fraught with dilemmas. It is often seen as being abstract and difficult to visualize. In order for it to be taught successfully, it must be clearly represented. This requires that teachers select appropriate tasks to help them represent the concepts successfully. The selection of good instructional representation and knowing how to use it is of utmost importance and work together to ensure a clear understanding of concepts taught. As Ball (1993) suggests, teachers must have an adequate understanding and knowledge of the concept first before they are able to 'unpack' it so that their learners understand it and can access it. They must have an understanding of how their learners may best 'see' the concepts and predict learners' difficulties so that they know how to best represent the mathematics to them. Teachers are required to use more than one representation since no single representation can fully capture all the ideas embodied in the concept of fractions (Ball, 1993). Selecting and implementing these representations poses problems for teachers. What they select is often difficult to teach because of the abstract nature of fractions. While teachers have

goals and intentions of setting up tasks conceptually, the representations that they use can be problematic, so pushing them to teach procedurally instead. This creates a set of dilemmas for the teacher in that the focus is no longer on teaching both procedures and concepts, but rather on only teaching procedures.

There are three interrelated dilemmas and they all stem from what I have called competing goals. It is described in the literature as the procedural and conceptual work involved when teaching fractions. The first dilemma is related to the task selected, set up and implemented to represent the content. Linked to this is what the learners do as they engage with the task. So it is possible, in fact I will hypothesize, that the teacher will face some dilemmas around representing the task, he will face some dilemmas around what the learners do with the task and thirdly and most centrally he will face dilemmas around the procedural and conceptual work of fractions.

Ball, Bass and Hill (2004) argue that teachers will confront problems as they go about their work and that these problems are mathematical in nature. The teaching of fractions will no doubt throw up a number of problems for the teacher. From the literature and particularly from my own experience, teaching fractions both conceptually and procedurally is a major and a legitimate problem. Ball, Bass and Hill (2004) argue that problem solving is mathematical, so is this dilemma of competing goals mathematical? I would argue that the dilemma is mathematical. How the teacher manages this dilemma affects the learners' mathematical knowledge and their understanding of the concepts related to fractions. If the teacher has a deeper understanding of the interconnection between concepts and procedures in terms of proficiency, the dilemma might be managed in different ways. Managing the dilemma is about understanding the relationship between mathematical concepts and procedures and the knowledge resources called on to legitimate meaning for the learners.

As can be seen, the problem solving suggested by Ball, Bass and Hill is usefully extended by the notion of dilemmas.

While Ball, Bass and Hill's (2004) mathematical problem solving categories enable me to identify the mathematical work the teacher confronts and does when teaching, dilemmas help me think about the pedagogical/mathematical problems he faces and how he manages them as he engages with the mathematical problem solving. This, I posit will provide greater insight into what takes place in this particular practice. It offers to explain more than just what mathematical problem solving the teacher does. Instead the problem solving describes what the teacher does when teaching while the dilemmas describe why he does certain mathematical problem solving as he attempts to address and manage the dilemmas. In other words, the dilemmas help express what the tensions are and will help get to what mathematics is needed to teach fractions and thus provides a description of mathematics for teaching is in this practice.

In this chapter I have reviewed literature and shown that both the nature and the way that fractions are taught is problematic and that although much research has been done around this particular topic, the problems still persist. This brings into question teacher knowledge and what they should know in order to teach fractions successfully. I have attempted to address this by looking at research in terms of what teachers actually do rather than what teachers have to do. There is an assumption within the study that some of what the teacher might do is the function of some of the dilemmas he might experience.

I now turn my attention to how I am going to see teacher knowledge, problem solving, knowledge resources and dilemmas in practice. Firstly, I will not be able to 'see' the dilemmas and therefore I am going to have to infer them. Secondly, to see all of the above-mentioned in practice, I have to develop a gaze. This gaze will help me see things related to the problem.

2.8 Theoretical Framework and Analytical Framework

In researching the mathematical work of teaching, as in other fields of research, a theoretical framework is significant for several reasons. Fundamentally it provides a lens

through which the data is interpreted or analyzed. This in turn enables the 'audience' to know or understand where the researcher is coming from. (Fetherston, 1998).

This is a study of the mathematical work of teaching, focused on the kind of mathematical problem-solving a teacher does as he goes about his work, and the knowledge domains and resources he calls on to do this work, specifically as he selects, sets up and implements mathematical tasks for teaching fractions in his Grade 7 class. In addition to describing this mathematical work, the study seeks to explain this work, relating it to possible teaching dilemmas and the experiences of the teacher.

Where and how is the mathematical work of teaching to be 'seen'? While a teacher's work is not reducible only to their classroom activity, it is central to their work. This study is focused on classroom practice, and so is a study of pedagogy. With this in mind and in line with QUANTUM[1], I have turned to Bernstein's theory of pedagogic discourse as it provides a broad social lens with which to 'see into' pedagogic practice, and is a key component of the mathematical work of the teacher.

Bernstein provided a rather direct description of pedagogy: it is the conversion of knowledge (intellectual, practical, expressive or local knowledge) into pedagogic communication (Bernstein, 1996). In the case of mathematics and the teaching of fractions (as in this study), the teacher's task is to take knowledge of fractions (what they are conceptually, how they can be applied, operations related to fractions and the related rules and so on, together with how this knowledge appears in curriculum documents, texts for teaching and so on) and (re)present these to his learners so that they are understandable or learnable. For Bernstein, pedagogic communication is inherently and intensely social and so, by no means straightforward. In addition to what others have called the transposition of knowledge, pedagogic communication is shaped by:

- How teachers (and the curriculum more broadly) think about learners;

[1] As noted, QUANTUM - a larger study on mathematics for teaching, is elaborated in various research papers. See Adler & Pillay (2007), Pillay (2006), Adler et al (2005), Kazima & Adler (2006)

- The knowledge resources teachers can and do appeal to;
- How they (and the curriculum) interpret mathematics;
- The ways learners respond to mathematical tasks they are given.

But how is pedagogic discourse, and particularly the teachers' mathematical work to be 'seen'? Bernstein posits the notion of a 'pedagogic device' (1996, p. 39) as the central concept and tool in understanding pedagogic practice. He describes the pedagogic device as a set of three inter-related hierarchical rules: distributive, recontextualisation and evaluative (Bernstein, 1996).

The distributive rules control the distribution of different forms of knowledge between social groups, and originates in the recontextualisation rules. The recontextualisation rules 'regulate the formation of specific pedagogic discourse' and originate from the evaluative rules. The evaluative rules determine what counts as valid acquisition of instructional (curricular content) and regulative (social conduct, character and manner) texts (Singh, 2002, p. 573). As Adler, Davis, Kazima, Parker and Webb (2005) have argued, " the distribution of knowledge and the rules for transformation of knowledge into pedagogical communication is condensed in evaluation". Evaluation, as it is at work in pedagogic practice, attempts to control the transmission or acquisition of the available potential meaning. The evaluative rules construct the pedagogic practice by providing the criteria to be transmitted and acquired (Pillay, 2006). Davis et al (2003) argue that the possibilities for meaning are condensed in and through moments of evaluation, and that in mathematics classrooms, evaluation is not a once off thing, but rather happens over a certain period of time.

It follows from this, that the workings of the pedagogic device, and so too the mathematical work of teaching can be made visible through a study of the what, over time, comes to count as valid mathematics in pedagogic practice.

It provides a way of understanding how teachers legitimate or fix meaning when knowledge is distributed and transformed (recontextualised) within the classroom context.

Within a classroom, as in any other pedagogic practice, teachers provide an accepted standard of what it is they want their learners to know (Bernstein, 1996). In providing these criteria, teachers have to comply with recognized rules, standards and traditions of the pedagogic discourse. Consequently teachers are forced to form
sound opinions, sensible decisions or reliable guesses (legitimate meaning).

The transmission of the criteria to learners depends on two rules of acquisition which Bernstein calls recognition and realization rules. While the recognition rule requires that learners recognize/accept the validity or truth of what he/she is expected to know or do**,** realization rule determines how meanings are put together and made public (Bernstein, 1996). An understanding of the recognition rule translates into the realization rule. This means that the learner is able to recognize what it is they are meant to know or do and produce a legitimate text required by the teacher. Recognition and realization rules work in tandem and not in isolation from each other. It is important that learners acquire both rules since recognition rules on their own are insufficient. It is important to note that while Bernstein focuses on the acquirer (learner) and how they come to understand the realisation and recognition rule, this study is concerned with the possibilities set up by the teacher and pedagogic discourse for the learner to acquire these rules. In the context of this study, Bernstein helps to think about what learners are to recognise as mathematics and how do they do this.

Through what is known as the evaluative moments in pedagogic practice, I hope to capture what is taking place in the classroom by 'seeing' the knowledge and experience the teacher draws on as he goes about his work of teaching fractions. In other words, it provides insight into what is actually happening and is going to help me see:

- What problem solving the teacher is doing;
- What is the content of these events;
- What kind of fraction work is demonstrated, valued and validated and how this is done;
- What is happening to the nature of the task.

- And eventually identify what kind of dilemmas the teacher is facing and what is being called on in order to address the dilemmas.

Ultimately an understanding of Bernstein's evaluative rules and principles help us 'see' what the mathematics for teaching is, how this is a function of problem solving, the dilemmas faced by the teacher, and so provides an overall orientation to the study.

The next section focuses on the analytical framework, which will help aid in identifying, and observing what has been highlighted thus far.

2.9 Analytical Framework

The analytical framework used in this study is adapted from the research done by Adler & Pillay (2007), which focuses on mathematics for teaching functions. The data is chunked into units of analysis. Each unit is called an evaluative event or episode. An evaluative event or episode is defined by the moves the teacher makes to legitimate meaning or to fix meaning(Adler & Pillay, in press). An evaluative event is considered to start when the teacher announces the object (that which is to be learnt) and ends when he fixes or legitimates meaning. The end of one event is the start of another.

Upon the identification of an evaluative event I established whether the task used to represent the object requires a high (procedures with connection and doing mathematics) or low (memorization and procedures without connections) level cognitive demand and whether it encourages procedural or conceptual understanding of the object (Stein et al, 2001). The focus of this study is on knowledge resources and what comes to be legitimated as mathematics. Turning to Ball et al (2004) eight types of problem solving (refer to the literature review) that mathematics teachers do, as they go about their work, provide an opportunity to explain what knowledge resources are called on and what comes to be legitimated as mathematics. I have adapted and added to Ball et al's (2004) eight categories as follows: explaining, questioning, demonstrating, representing, restructuring, defining, encouraging, working with learner ideas-identifying errors (mathematical calculations and practical errors). It must be noted that while encouraging

is not per se mathematical I have included it as part of the mathematical problem solving that the teacher does. Kilpatrick (2001, p. 116) refers to productive disposition as 'habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy'. For Kilpatrick (2001) confidence in mathematics goes together with skills. Learners must have confidence in themselves and in the mathematics they are learning. The teacher in this study attempts to encourage the learners through a supportive authoritative role in order to build their confidence in themselves and the mathematics. He often appeals to them to do the mathematics for him or encourages them by letting them know that he believes that they are capable of doing the mathematics. This is discussed in greater detail in the chapters that follow. I am aware that the encouraging he does may not be mathematical per se ,however coupled with what Kilpatrick describes as productive disposition and the work the teacher does there is a certain element of mathematical work involved and therefore I have included it as one of the mathematical problem solving categories.

As discussed previously, when teachers fix or legitimate meaning for their learners they call on certain knowledge resources. In the study by Adler & Pillay (in presss) three knowledge domains that the teachers in that study appealed to legitimate meaning of function ideas were identified. They are described as:

Mathematics (M) - Principles of mathematics: reasoning, defining, representations etc.

Sub categories formed from the principal of mathematics are:

1) **empirica**l (through observation you can make sense of it)
2) **definitions** (the teacher's attempt to define notions based on mathematical definitions and rules in mathematics)
3) **rules** (conventions in mathematics)

Experience (E)- The teacher draws on his personal and **professional** experience. The teacher draws on the experience of his learners (relates notions to the **everyday**).

Curriculum (C)- The teacher legitimates knowledge by making appeals to **textbooks**, **tests and exams**.

As mentioned above, these knowledge domains come from Pillay's (2006) study. It is important to note that the empirical domains (different teacher working differently, different topic) of my study differ from Pillay's and thus these knowledge domains can only offer to be a useful starting point for my study. The teacher in Pillay's (2006) study appealed to the learners' everyday lives. In doing this he used a metaphor of the relationship that exists between a mother and father to explain functions. Relating functions to the everyday lives of the learners suggested that if they understood what happened in everyday life they would understand what it is in mathematics. The teacher also appealed to how the mathematics worked and what the learners would need to do in the examinations. In identifying appeals and the knowledge resources drawn on by the teacher, Pillay (2006) related it to the kind of work/problem solving done by the teacher. With regards to my study, these domains may or may not be the same, they may be elaborated on and how these domains may feature will be future illuminated in terms of the dilemmas.

In the analytical framework I have included some of the dilemmas that might come up. In relation to the mathematical problem solving, it can be seen that there are certainly dilemmas around representation and working with learner ideas. It will be interesting to see the relationship that exists between knowledge resources the teacher calls on and the problem solving he does in order to address and manage the dilemmas he faces.

In summing up, I have put up a framework, the analytical language is going to help me proceed with this study, but these come from different studies. I am hoping to take the research further, so I am expecting that it will be elaborated on and that there will be something more.

The following is a model of the theoretical framework that underpins the study

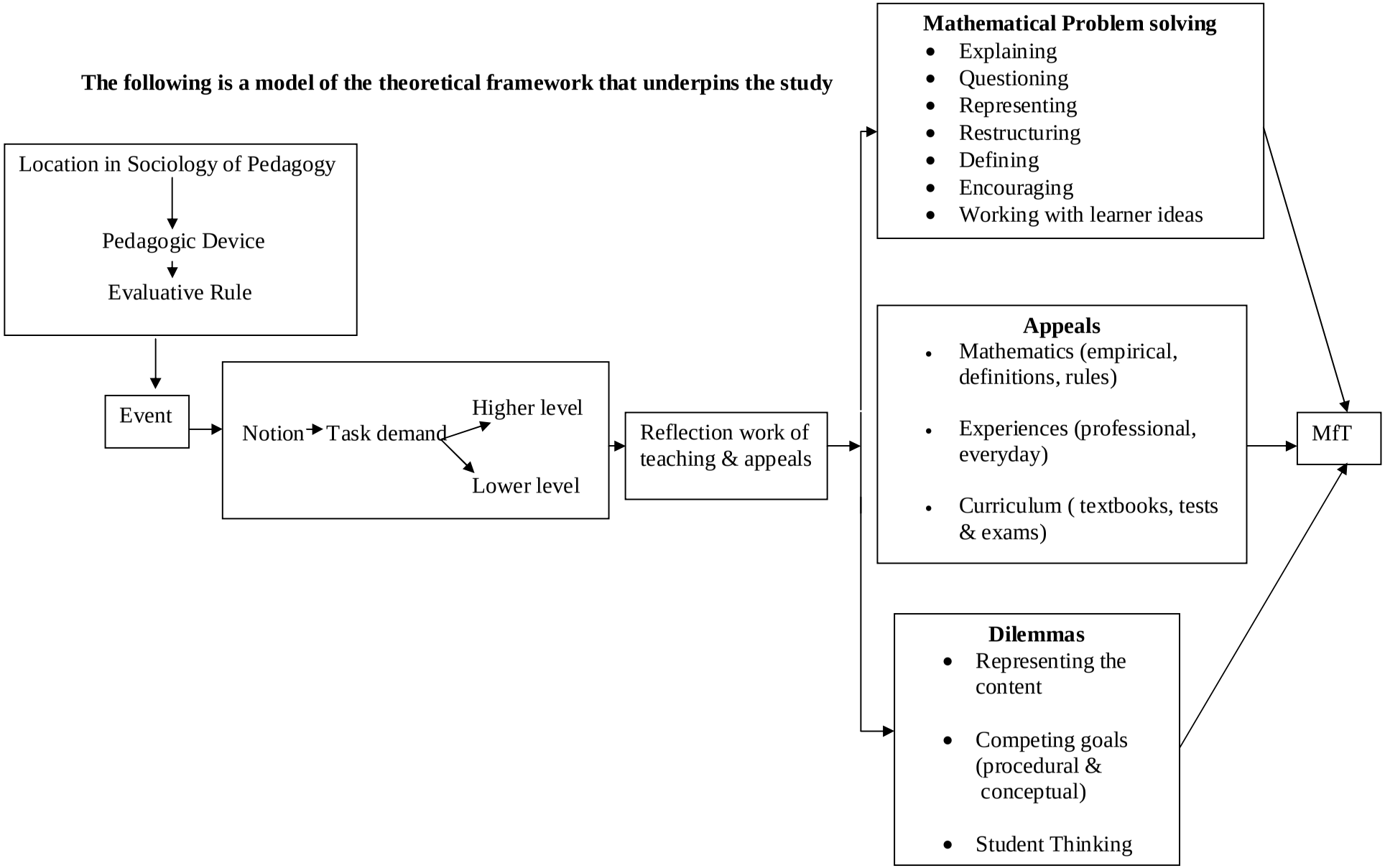


Fig.2.3: A conceptual Map of the Theoretical Framework

2.10 Summary

In this chapter I have outlined the theoretical and analytical framework of the study. I have reviewed the literature pertaining to the nature and teaching and learning of fractions, teacher knowledge pertaining to conceptual and procedural knowledge of fractions as well as tasks used to teach fractions, mathematical problem solving and teaching dilemmas.

The literature highlights that teachers often posses only a procedural understanding of fractions. Because of this the tasks they choose often involve only procedural work. Dilemmas surrounding the work of teaching fractions come about as teachers represent the content, work with learner ideas and engage with the procedural and conceptual. It will be interesting if these are similar concerns for the teacher involved in this study. The following chapter involves a discussion on the methodology that influences this research.

Chapter Three

Methodology

This chapter deals with the research methods that I have adopted for this study as well as the data collection techniques that I have employed. It also addresses the sample used for the study and the ethical issues that I have considered.

3.1 Methodological Approach

> 'A research paradigm is a network of coherent ideas about the nature of the world and of the function of researchers which, adhered to by a group of researchers, conditions the patterns of their thinking and underpins their research actions.'
>
> (Bassey, 2003, p. 42).

This statement offers to explain that in making sense of the world researchers have different beliefs about the nature of reality. Bassey (2003) proposes that there exists two particular research paradigms. They are the positivist research paradigm and the interpretive research paradigm.

While positivists believe there is a reality 'out there' in the world that exists, whether it is observed or not and irrespective of who observes, the interpretive researcher cannot accept this idea since, for them, reality is seen as the construct of the human mind. They deem people perceive and so make sense of the world in ways, which are often similar, but not essentially the same. Therefore, there can be different understandings of what is real. Rather than reality being 'out there', it is the observers who are 'out there' (Bassey, 2003). Similarly Cohen et al (2002, p.22) agree that, 'The interpretive paradigm, in contrast to its normative counterpart, is characterized by a concern for the individual'. With this brief explanation of the interpretive research paradigm, I will now discuss it in relation to my study.

My research is classroom-based and its main purpose is to gain an understanding of what and how mathematics (for teaching) is constituted in classroom practice from both the researcher and teacher's perspective. What takes place in the classroom is dependent on the teacher and his pedagogical content knowledge, how he responds to the mathematics, the learner's actions and reactions toward the task, these are all interrelated, interdependent and open to interpretation. It is for this reason that a positivist paradigm would be inappropriate. Instead an in-depth, qualitative and interpretive research paradigm is most suitable.

For this study the most appropriate research method would be a case study. Bell (1987) describes a case study as being "an opportunity for one aspect of a problem to be studied in some depth within a limited timescale". Cohen et al (2002, p.182) defines a case study as portraying "…what it is like to be in a particular situation, to catch the close up reality and 'thick-description' of participants' lived experiences of, thoughts about and feelings for a situation." Similarly Opie (2004.p.74) suggests that a case study "can be viewed as an in-depth study of interactions of a single instance in an enclosed system." Stake (1995) advocates, in qualitative case study, we seek greater understanding of the case. We want to appreciate the uniqueness and complexity of its embeddedness and interaction with its contexts. Mousley (2003) states a 'Case study allows researchers to capture evidence of and synthesize dimensions of teaching theory as well as practice…., it provided a methodological approach for describing components of classroom interaction, but also allowed inquiry into the origins of these elements, the meanings that they seem to hold for the subjects of research, and the ways teachers and children interact with tools and traditions of mathematics education' (p.339).

The above statements reveal that there is uniqueness about case study research in that it aids the understanding of complexities in particular contexts (Bassey, 2003). With regards to my research questions and an understanding of case study research, it would be prudent to suggest that a case study is most beneficial. First, using a case study as a research method provides hypotheses that might be difficult to obtain in other contexts. It provides an opportunity to work in a particular context i.e. the classroom. Secondly the

use of a case study may provide unique situations within the classroom context, which can be used to develop and test hypothesis. Thirdly, the case study may provide new perceptions, help adapt and alter pre-existing conditions, or point out gaps in mathematical knowledge. Fourthly, the case study may demonstrate how a theoretical framework can be exhibited in a concrete example (Sax, 1968). Taking the above mentioned into consideration and using case study as a research method, I hope to gain insights into the kinds of knowledge that a teacher draws on as he teaches fractions, as well as the teaching dilemmas reflected in this practice and how the knowledge resources and teaching dilemmas be explained.

It must be noted that while the use of a case study as a research method may be beneficial, there also exists limitations and difficulties. Stakes (2003) clearly puts forward the limitations of using qualitative research with regards to case studies. He states, 'Qualitative inquiry is subjective. New puzzles are produced more frequently than solutions to old ones. Its contributions to disciplined science are slow and tendentious. The results pay off little in the advancement of social practice. The ethical risks are substantial. And the cost in time and money is high, very high'. (p. 45). While this is true of qualitative research, it must not be forgotten that it is the subjectivity that enables understanding of lived experiences in a complex social world.

3.2 Data Collection Strategies

As already suggested, data was drawn from a case study of a Grade 7 mathematics teacher as he goes about teaching the topic of fractions. In order to obtain this data, qualitative methods of observation and semi structured interviewing were used to ascertain what mathematical problem solving the teacher did, the knowledge resources he called on and the dilemmas he faced as he taught fractions. While the use of in-depth interviews are crucial, it did not afford me the opportunity to observe what was actually happening in practice. Ball and Cohen (1999) and Ball and Bass (2000) argue that if we want to see and understand teacher knowledge production and teacher learning, we must look at what is actually happening in the classroom during teaching.

Stake (1995, p.62) states, “During observation, the qualitative case researcher keeps a good record of events to provide a relatively incontestable description for further analysis and ultimate reporting. He or she lets the occasion tell its story, the situation, the problem, resolution or irresolution of the problem” Using observational research helped me identify and interpret what was actually happening during the teaching process and the implementation of the task and aided in the analysis of data, together with the interviews. It also assisted me in the understanding and establishment of how the teacher implemented the task, what dilemmas he experienced and what mathematical problems solving he did. While using observational research proved to be time consuming, it allowed me to gain direct insight into what was happening during the teaching process.

According to Opie (2004), there are two roles a researcher can take on: participatory or non- participatory. A non- participant role is where the researcher does not communicate with the subjects when collecting the data. This role is often associated with structured observation. A participatory role can take up one of three broad forms: observer as participant, participant as observer and complete participant. In the light of my research, I took on a non-participatory role. During the data collection process, I was present in the class, only observing what was happening. There was no interaction or interference from me with either the children or teacher during the lessons.

As mentioned above, information was collected in the form of observational and interview data. Video recordings, field notes and interviews were looked at through an analytical framework (refer to literature review). An analytical tool has been developed, based on this analytical framework and it will be discussed in the next chapter as the data is presented.

There were two forms of observational data that I used. Firstly, there was a videotape of all the lessons and its transcriptions. The video recordings played a vital role in the data collection strategy. Even though I observed all the lessons, it was impossible to capture everything that was happening in the classroom. It is for this reason that I opted to video

record the lessons. I was able to watch the recordings repeatedly in order to make sense of what was taking place in the classroom and 'see' what I had failed to observe during the actual lessons. The video recordings helped provide additional information such as gestures and body language, which helped understand what was being communicated, by both the teacher and the learners. Each lesson was transcribed[1] verbatim. The purpose of the transcription was to provide a full account or record of what was taking place in the classroom. It was also transcribed so that a systematic and comprehensive analysis could be done in terms of the problem solving done by the teacher. Providing a transcript of each lesson allowed me to chunk the data accordingly. Each lesson also exhibited the time intervals between episodes. This is important in that the time intervals help denote what took place and when.

Secondly, I made my own field notes during the lessons since it was important to note the different events taking place in the classroom. It proved to be beneficial as it helped me recall details that I did not have time to write down during the observation (Fraenkel & Wallen, 1990).

The observational research data collection strategies were supplemented by interview data. Information was collected from interviews conducted with the teacher only. I completed three interviews with him. Each of the interviews were semi- structured and tape-recorded. The first one was done prior to the teaching of fractions and focused more on why he selected the task he did. The second interview was done a week after he started the topic and focused on the problems he confronted when teaching fractions. The final interview was conducted at the end of all the lessons and focused on bringing together what took place over the ten lessons. I have included the third interview as an example (see appendix A). The information collected from these interviews helped shed light on some of the dilemmas experienced by the teacher as well as providing the opportunity to gain further insight into why the teacher did things the way he did.

[1] The full transcript of each lesson is bound and kept separately.

Using interviews as an instrument provided vital information in establishing and understanding what was taking place in the classroom and why, from the teacher's perspective. In other words, it afforded me the opportunity to delve into issues that needed clarification from the teacher's perspective as well as to investigate particular issues in depth. Opie (2004) suggests that the purpose of an interview, unlike a questionnaire, enables one to answer the question 'why'. With regards to my research questions, using interviews enabled me to find out why the teacher selected or ignored certain tasks when teaching fractions. This meant that while the interviews enabled me to understand what the teacher was looking for when selecting a task and how he planned on implementing the selected task, it also provided answers to why he chose the certain tasks and why he planned on implementing them in a particular manner. As suggested by Oppenheim (cited in Opie, 2000, p.111) an interview allows 'the respondents to say what they think and to do so with great richness and spontaneity'. This meant that the teacher was able to freely express himself verbally without having to write down answers which could have been a hindrance, especially since English is not his first language.

The interviews were semi structured which meant they were not solely controlled by the questions I planned, but rather that these questions provided opportunities for me to enquire and expand on what the respondent i.e. the teacher, said (Opie, 2004). It was flexible and aided in allowing the teacher to provide explanations and reasons for why things were done in the way they were done. The semi structured questions also helped prevent straying from the topic or from collecting data that was not relevant to the research project. For ease in transcription and interpretation, all three interviews conducted were tape-recorded and transcribed[2]. No field notes were taken during the interviews as I felt that it would have been overwhelming for the teacher to have the interviews tape-recorded and me taking field notes. I also did not want to lose focus of the interview by taking notes and since I am a novice researcher, there was a great chance of this happening. I did however make notes after each interview.

[2] The full transcript of each interview is bound and kept separately.

3.3 Piloting

The process of piloting enables researchers to ensure the reliability and validity of the instruments used. It suggests that if the researcher is going to use a test instrument, they would want to make sure that it is going to test what they want it to test. There is a process involved. With regards to my research project, I did not follow the process of piloting that ensures validity and reliability, instead, in order to make sure that I was systematic, rigorous and trustworthy, there was an on going process of looking at how I was doing things.

Piloting the instruments in this particular research project was difficult. The only instrument that could possibly have been piloted were the interviews. However, since the interviews were semi-structured and so specific to the teacher, it was difficult to pilot. Instead, I opted to develop the interview instrument. The interview instrument was developed in conversation with my supervisor. This was done so that she could examine the clarity of the interview questions, whether they were comprehensible and since they were semi structured that they covered the relevant information.

All that I could do was to imagine the kind of responses I could get from the teacher. I mentally rehearsed them as suggested by Stake (1999) and knowing that they were semi-structured interviews, I focused on sharpening my listening skills and probing for answers, reasons and explanations.

The analytical tool was developed in order to be used to examine the data. What this tool is and how it was used to do the analysis is elaborated in Chapter Four. As will be shown and discussed, the tool was changed as the data was engaged. The empirical evidence also started to shape what needed to come into the tool. Therefore, piloting the analytical tool was problematic.

3.4 The Sample

The research was carried out at a multilingual, multicultural primary school situated in the south of Johannesburg. It is a co –educational school where English is the language of teaching and learning. However, for many learners this is not their mother tongue. It is a well-resourced government school compared to most in South Africa.

The subject of this study is a Grade 7 mathematics teacher and 36 Grade 7 learners. Mr. T[3] is an opportunistic sample, known to me. I teach at the school and this gave me easy access to the teacher. Mr. T was also selected because of his qualifications and teaching experience. Since the purpose of this study was to gain insight into how mathematics (for teaching) is constituted in classroom practice, qualification and experience was of vital importance. Mr. T has a four-year higher education teaching diploma and has been teaching for 23 years. He taught different subjects and learning areas during the past 23 years. However, over the past 10 years he has been teaching mathematics in primary school. He taught mathematics to both Grade 5 and Grade 6 and for the past five years has been teaching Grade 7 mathematics. He thus fulfils both criteria of experience and qualification. Mr. T is an Afrikaans male whose main language is Afrikaans, nevertheless he teaches in English.

Mr. T is the only Grade 7 mathematics teacher at the school. He teaches five Grade 7 classes every day on a seven-day cycle. He collaborates on a regular basis with the Grade 6 mathematics teacher so as to keep up with what needs to be done in the school with regards to the mathematics curriculum. He is well informed and knowledgeable about the mathematics curriculum and has attended the required training with regards to the Revised National Curriculum Statement as well as professional development workshops for mathematics. His classroom is well resourced with mathematics equipment (unit fractions, geoboards, etc), teaching aids, textbooks as well as a whiteboard and overhead projector.

[3] For the purpose of this study I will use the pseudonym Mr T to refer to the teacher.

Mr. T works closely with local high schools in order to prepare the learners for high school mathematics. The following extract taken from the second interview done with Mr. T reveals his concern about his learners, their parents, and mathematics in his classroom and beyond its four walls.

Please note that Mr. T's first language is Afrikaans and he is teaching in English. I have not corrected the errors typical of an Afrikaans speaker and the text is verbatim.

SG: You work closely with all the high schools, right, School A[4] and School B[5]

Mr. T: Not School B. Not the boys' school, the girls' school. And I ask them, listen what do you expect me to do next year and lot of parents will nail me on the algebra. They'll say to me, you not doing the algebra at School C[6]. Why not? Our kids go to high school and battle. The high school say to me you don't do it. Do things your kids need to know for next year. So I take a Grade 8 book and I go through that and I see more or less, ok, this links up with that, this links up with that…..

Mr. T displays a genuine concern for his learners and what they need in order to excel in mathematics. He is actively involved in the schooling community in order to stay up to date with what is happening. This mathematical knowledge and insight as well as his involvement and dedication to the learning area and learners make him an ideal candidate for my sample.

The past three years have hailed success for Mr. T with regards to his mathematics-teaching career. For three consecutive years he has managed to help produce excellent results with regards to mathematics at his school and region. Mr. T entered both the Grade 6 and 7 learners in what is known as the HH High School [7]Mathematics Challenge.

[4] This is a pseudonym used for the school in order to protect its identity.
[5] This is a pseudonym used for the school in order to protect its identity.
[6] This is a pseudonym used for the school in order to protect its identity. Refers to the school he is teaching at.
[7] This a pseudonym used to protect the identity of the school that runs the competition.

The following extract from the first interview done with Mr. T, further explains the competition and his success:

SG: You recently won an award. Tell me about the award and what it was for.

Mr. T: The award was basically HH High School award, um, we entered it last year and we did very well there. Our school came second there and I came second and our learner came second and we decided to enter the whole Grade 6 and Grade 7 group this year to give more kids an opportunity, to expose the children to that type of competition. You know it was basically arranged, like I said by HH High school, um, and I entered my grade 6's and 7's and I 'm very proud to say that there are 3000 learners that enter that competition, 16 schools and our kids are very smart, I mean, it's not their teachers, it's them as well. And we had 20 kids in the last hundred. We had the top Grade 6 learner out of the 16 schools. We had the top Grade 7 learner out of the 16 schools, which makes me the top maths teacher in the region and I'm planning next year to take all four of those awards, top grade 6, top grade 7, top teacher and top school and yet again I still came second.

SG: Well done and thank you for your time.

(Interview 1)

This year (2007) Mr. T managed once again to help produce the top Grade 7 learner and was awarded the prize for the top mathematics teacher in the region (second time). Unfortunately the school did not manage to win the top school or top Grade 6 learner. However, Mr. T remains optimistic and determined to achieve all four goals next year!

These successes highlight my choice of Mr. T as my sample. Not only is he recognized within his school, but he has also been acknowledged at a regional level. He is actively involved in the development of mathematics education at all levels.

3.5 Ethical considerations

Bassey (2003, p.74) discusses the notion of research ethics under three headings: respect for democracy, respect for truth and respect for persons. I will offer a brief description of each heading and further explain the ethical process embarked on with regards to my research project.

3.5.1 Respect for democracy

Bassey's (2003) main focus regarding respect for democracy relates to the freedom researchers have i.e. 'the freedom to investigate and to ask questions, the freedom to give and receive information, the freedom to express ideas and to critise the ideas of others; and the freedom to publish findings' (p.74). But with this freedom comes certain responsibilities. Researchers are expected to have respect for the truth and for people. If these two are achieved, researchers have the freedom to do things that won't jeopardize themselves or their careers.

With regards to my research project, there were several factors that pointed to respect for democracy. For example, the University of the Witwatersrand expected me to obtain permission from the Gauteng Department of Education (GDE) in order to conduct the research in one of their primary schools. I thus obtained permission from the GDE (see appendix B). I also obtained permission from the principal of the school, the teacher who was involved in the study as well as the learners and their parents. All participants involved in the study (i.e. the principal, the teacher, the learners and their parents) were required to complete a consent form. Letters were sent out informing them of the purpose of the research, ensuring them that the learners, school, principal and teacher will remain anonymous and that they could decide whether they wanted to be a part of this research project. The letter also explained the data collection strategy and how it involved the learners and teacher. It offered an explanation of how the research findings would be used and confirmed the teacher's, learners and parents approval of the use of the transcripts and video-recordings by myself, in publication and by other researchers (see appendix C). Furthermore, I also obtained ethics clearance from the University of the Witwatersrand (See appendix D). The ethical issues addressed above may also form part of respect for truth and respect for persons.

3.5.2 Respect for truth

Being truthful in data collection, analysis and the reporting of findings are expected from researchers. In other words, researchers owe it to both their subjects and themselves to be honest. They must have integrity and not intentionally or unintentionally deceive others

or themselves. It is here where trustworthiness plays an important role. I will address the issue of trustworthiness later in this chapter.

Before conducting the research I took into account what would happen or what I would do if the information gathered from the research was potentially harmful in that it was neither flattering to the school nor the teacher. In order to resolve this dilemma, I made it clear to the principal and teacher from the onset that the aim of my research was not to find fault with the teacher or his teaching strategy, but rather to learn from him.

3.5.3 Respect for persons

Researchers must take cognisance of the fact that when collecting data, the data initially belongs to their subject/s and therefore they owe it to them to treat them as fellow human beings, with dignity and privacy (Bassey, 2003).

There were several ethical issues that arose during the research project. The teacher involved in the study is a colleague of mine. I was unsure of how our professional relationship would be influenced by this study, so before I conducted the research I explained clearly to my colleague what was expected of him as the teacher and me as the researcher, making sure that he was not in anyway threatened, intimidated by me, or insecure about the research. I did this by conversing with him as well as writing an official letter to him.

Time played a vital role during the data collection, especially with regards to the interviews and classroom observation. The teacher involved in the research had a very busy extracurricular timetable and time was a restraining factor. As part of my ethical considerations, we negotiated times that would suit both of us in order to ensure that the research did not become a burden for him. I also ensured that our appointments were diarised and that any cancellations were made in advance by both of us and rescheduled for another time.

With regards to classroom observation, I was very aware of the time the teacher had in order to achieve the different outcomes set out by the Revised National Curriculum Statement. I therefore ensured that being in the class caused minimal distraction and disruption for the teacher and learners.

Patti Lather (in Opie, 2004, p.29) introduces the concept of 'rape research'. It is described as when researchers enter the research field, get all the necessary information they require and forget to return to the field in order to thank the participants for their contribution towards the study. I was very wary of this happening, since I am on the staff of this particular school. I have kept the teacher updated on a regular basis and plan on providing my findings to the teacher once the research report is completed.

It must be noted that respect for democracy, respect for truth and respect for persons appear to be inter-related. It must also be noted that these ethical values clash and may cause problems for the researcher. In order to prevent these clashes, the above-mentioned ethical processes were put in place.

3.6 Rigour in my research

Sikes (in Opie 2004 p. 17) states, " It is on the match between methodology and procedures and research focus/topic/questions that the credibility of any findings, conclusions and claims depend, so the importance of getting it right cannot be overemphasized." This means that in order for my research findings to be credible ,I had to ensure that the methodology and procedures I chose were best suited to my research topic and questions. The general aim of the research is to describe what comes to be mathematics for teaching in this Grade 7 classroom and associated dilemmas and why this is so? What does this tell us about mathematics used in teaching? In order to investigate these broad questions, a focused study of the teaching of fractions in Grade 7 was undertaken with the following critical questions:

1. What mathematical problem-solving does this teacher do when teaching fractions in his Grade 7 class?

2. What knowledge resources (appeals) does he call on as he goes about this work?
3. What teaching dilemmas are reflected in this practice?
4. How might this practice, and its teaching dilemmas be explained?

Bassey (2003, p.75) explains, "reliability is the extent to which a research fact or finding can be repeated, given the same circumstances, and validity is the extent to which a research fact or finding is what it is claimed to be". While both validity and reliability are vital concepts in quantitative research, many researchers believe they are not so with regards to qualitative research. In fact, they argue that it is problematic to use validity and reliably in qualitative research since it views subjectivity as primary. Since validity and reliability are inappropriate for qualitative research, researchers (Guba & Lincoln, 1983) use the concept of trustworthiness (credibility, transferability, dependability and conformability) instead. Interestingly Silverman (2001) disagrees with this and shows how qualitative research can be seen as credible through reliability and validity. I will use Silverman's notion of credibility to show the reliability and validity of my research.

Kirk and Miller (cited in Silverman 2001, p.226) argue that :

> 'Qualitative researchers can no longer afford to beg the issue of reliability. While the forte of field research will always lie in its capability to sort out the validity of propositions, it results will (reasonably) go ignored minus attention to reliability. For reliability to be calculated, it is incumbent on scientific investigators to document his or her procedures.'

The main instrument in qualitative research is the researcher. The complicating issue with this regard is that researchers are human beings and often have different theoretical orientations, therefore making it difficult for the instrument to be repeatable. However, researchers can aim to be consistent by using the criteria and concept of indicators in a consistent and principled manner. This enables different readers of the text/transcripts to witness similar events occurring which help eliminate idiosyncratic interpretations.

The statement above made by Kirk and Miller infers that in order for qualitative research to be highly reliable, qualitative researchers must ensure that what they record when using observational research is as concrete as possible. This means that ' verbatim accounts of what people say, for example, rather than researchers' reconstructions of the

general sense of what a person said, which would allow researchers' personal perspective to influence the reporting (Clive Seale cited in Silverman,2001, p. 227)'.

I would like to note that when conducting this research I acted responsibly and documented all my findings. All video recordings, interviews and audio-recordings were transcribed verbatim in order to strengthen the reliability of the interpretation of the transcripts. The observation and transcripts of the lessons were verified through interviews with the teacher. My supervisor helped with the checking of the interpretation of the transcripts and verified that the codes were applied uniformly. These transcripts are bound, kept separately and available for scrutiny at any time.

As is discussed in detail in the next chapter, the lessons were chunked according to time intervals and the analysis of the data was done coherently, consistently and systematically. As suggested by Silverman (2001), the systematic, consistent and concise recording and analysis of the concrete evidence increases the reliability of the research. This is what I aimed for when doing this research. The analytical tool that was developed from the framework was used to help analyze the data. The categories derived from the use of this tool were used in a standardized way. I provided a description for each category (mathematical problem solving done by the teacher and the appeals made) so as to ensure that any researcher/person would be able to categorise the data in the same way. My supervisor provided accountability in ensuring that the categories were standardized and the data analyzed accordingly. Allowing another researcher to analyze the data according to an agreed set of categories, ensures that any differences are examined and ironed out. Silverman (2001) refers to this as 'inter-rater reliability' and this proved to be very useful in ensuring the reliability of this research.

'Ultimately all methods of data collection are analyzed 'quantitatively', in so far as the act of analysis is an interpretation, and therefore of necessity a selective rendering. Whether the data collected are quantifiable or qualitative, the issue of the warrant for their interferences must be confronted. (Silverman, 2001, p.233)

Silverman (2001) points out that it is important that researchers both qualitative and quantitative, have a 'warrant for their inferences' and that their work is valid. Instead of using triangulation and members' validation, he suggests that in order to validate quantitative research the following methods must be taken into account:

Analytic induction

The constant comparative method

Deviant-case analysis

Comprehensive data treatment

Using appropriate tabulations

Analytical induction depends on a model of how social life works as well as a set of concepts specific to that model (Silverman, 2001). My primary concern with regards to this research is to describe what comes to be mathematics for teaching in this Grade 7 classroom and associated dilemmas. The interpretations of the data is shaped by both an 'involved' theoretical framework, which guarantees contact with established theory, as well as my experience and knowledge of the classroom which ensures contact with the established practices (Brodie, 1994). This relationship that exists between the practical and the theoretical knowledge aid in the establishment of the validity of the research.

The constant comparative method involves 'simply inspecting and comparing all the data fragments that arise in a single case (Glaser & Strauss in Silverman 2001, p.239)'. This comparison was made possible by ensuring that the data I collected was assembled in an analyzable form. The whole-data set from the video recordings was transcribed and chunked into time –intervals. I started by analyzing relatively small chunks of the data. From this I generated and modified a set of categories in conjunction with the theory. I tested out emerging hypothesis by steadily expanding the amount of data. The data was placed into the appropriate categories and used until it was accounted for. This ensured that there was a constant to and fro movement between the different parts of the data. No data was left 'uninspected' or 'unanalyzed' and Silverman (2001) refers to this as comprehensive data treatment.

The data could be placed into the different categories because of the use of appropriate tabulations. As mentioned before, these categories were derived partly from the theoretically defined concepts. While the data collected in the study was qualitative, I employed and incorporated certain quantitative methodology in order to analyze the data and gain insight into the bigger picture. I used a simple counting technique to record how many times the teacher used certain mathematical problem solving and how many times he made certain appeals. This allowed me to survey all the data ensuring that nothing was overlooked. It also enables the readers to survey the data and get an idea of the data as a whole.

Sampling decisions are important with regards to generalisability. Purposive sampling as suggested by Silverman (2001) allows us to select a case for the reason that it exemplifies that which we are interested in. This does not mean the approval of any case chosen, but rather that purposive sampling insists on exercising judgment and thinking critically about accessibility and limitations of the sample researchers are interested in. As I have explained earlier, the reasons for choosing this particular teacher were opportunistic, he was also chosen because of his excellent credentials with regard to his experience and mathematical knowledge and since according to Silverman (2001) generalisability in qualitative research relates to the extent to which the explanations can help illuminate other similar situations, I believe large claims can be made about the analysis of this study.

3.7 Summary

In this chapter I have drawn attention to the research methods employed in this study. The piloting process was discussed, as well as the ethical issues that were considered when embarking on the project. The chapter also addresses issues of credibility with regards to reliability, validity and generalisability.

The way the analysis was done is elaborated in the next chapter where I provide a description of and the indicators of the data collected. I engage in a discussion guided by

the analytical framework of the quantitative analysis of the first four lessons. This in turn leads to providing answers to the first two critical questions of this study i.e. the mathematical work/problem solving the teacher confronts as he goes about teaching fractions to a Grade 7 class and the appeals he makes in order to legitimate meaning for his learners and following that an engagement with questions three and four.

Chapter Four

Analysis and Interpretation of Data

In this chapter I begin with a description of the lessons with particular focus on the mathematical and cognitive demands of the tasks used in lessons observed. This will be followed by a detailed analysis of the lessons to illuminate the problem solving and appeals made by the teacher and the apparent dilemmas he faces.

4.1 Overview of lessons

Mr. T taught a total of 10 lessons related to the topic of fractions (addition and subtraction). I observed all 10 lessons taught to one of the four Grade 7 classes. The lessons were all approximately 50 minutes long and commenced on the 26 July 2006 and ended on the 11 August 2006. The focus of this study is on the first four lessons. I provide a brief description of all ten lessons (see appendix E) since they help locate the four lessons. Lessons five to ten followed the pattern of lessons three and four.

The classroom was organized in 'columns'. The 36 learners were arranged in 3 'columns', as in the diagram below, with the potential for collaborative or group work.

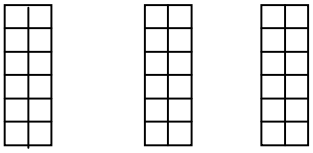


Fig. 4.1: Seating arrangements in classroom

Although learners sat together in 'groups' they worked quietly and independently with little or no communication between them. Mr. T had a white board, which he used frequently for written communication with the learners. The white board was prominent in Mr. T's teaching since he frequently and consistently used it to demonstrate the procedures involved when teaching fractions. During the 10 lessons, Mr. T worked on the addition and subtraction of common fractions, adding and subtracting mixed numbers,

converting mixed numbers to improper fractions and improper fractions to mixed numbers when adding and subtracting fractions.

The first two lessons were dedicated to confirming that when using a fraction wall, certain fractions with different denominators can be combined and add up to one whole. The third lesson focused on finding the lowest common denominator without using the multiples of the denominators as well as changing mixed numbers to improper fractions and vice versa. The fourth and fifth lesson involved correcting the exercise from lesson three. Lesson five also included the introduction to addition and subtraction of mixed numbers. Lesson six was a continuation from lesson five and involved the learners completing the exercise on the addition and subtraction of mixed numbers. During lesson seven Mr. T recapped what was done in lessons one through five. It also included the addition and subtraction of mixed numbers with both operations in one problem. Lesson eight and nine dealt with the corrections of the exercise in lesson seven. The learners were also introduced to their project. Lesson ten was a revision lesson on everything that was covered regarding the addition and subtraction of common fractions.

In summary, over 10 lessons involving fractions, with the first four focused on the addition of fractions, including mixed numbers, provided me the opportunity to hone into the mathematical problem solving the teacher did, the appeals he made and the dilemmas he faced in this context.

4.2 Task focus and demands

I will now provide an analysis of the two tasks used in the first four lessons. What follows is the worksheet given to learners. I have divided it into task 1 and task 2. Task 1 was focused on during the first two lessons and task 2 was focused on in the next two lessons. The tasks will be analyzed according to their cognitive demands and the relevant literature on fractions.

Task 1 (my naming)



COMMON FRACTIONS
ADDITION AND SUBTRACTIONS

DISCOVER....DISCOVER.....DISCOVER.....DISCOVER.....

Adding and subtracting common fractions is easy if you understand the simple basic rules:

- **If the denominator is the same just add or subtract to find answer to sum.**
- **If the denominator is not the same find LCD then work out the answer.**

DISCUSSION AND CONCLUSION

But before we can do the work we need to travel on the road of discovery.

- **WHEN TO USE A COMMON DENOMINATOR**
- **HOW TO PROVE BY USING A FRACTION DIAGRAM/GRID**

IF YOU ADD FRACTIONS WHERE THE DENOMINATORS ARE DIFFERENT THE ANSWER WILL BE ONE WHOLE.

E.g. $\frac{1}{2}+\frac{1}{4}+\frac{1}{4}=1$ **NOW LET US PROVE IT!!!!**

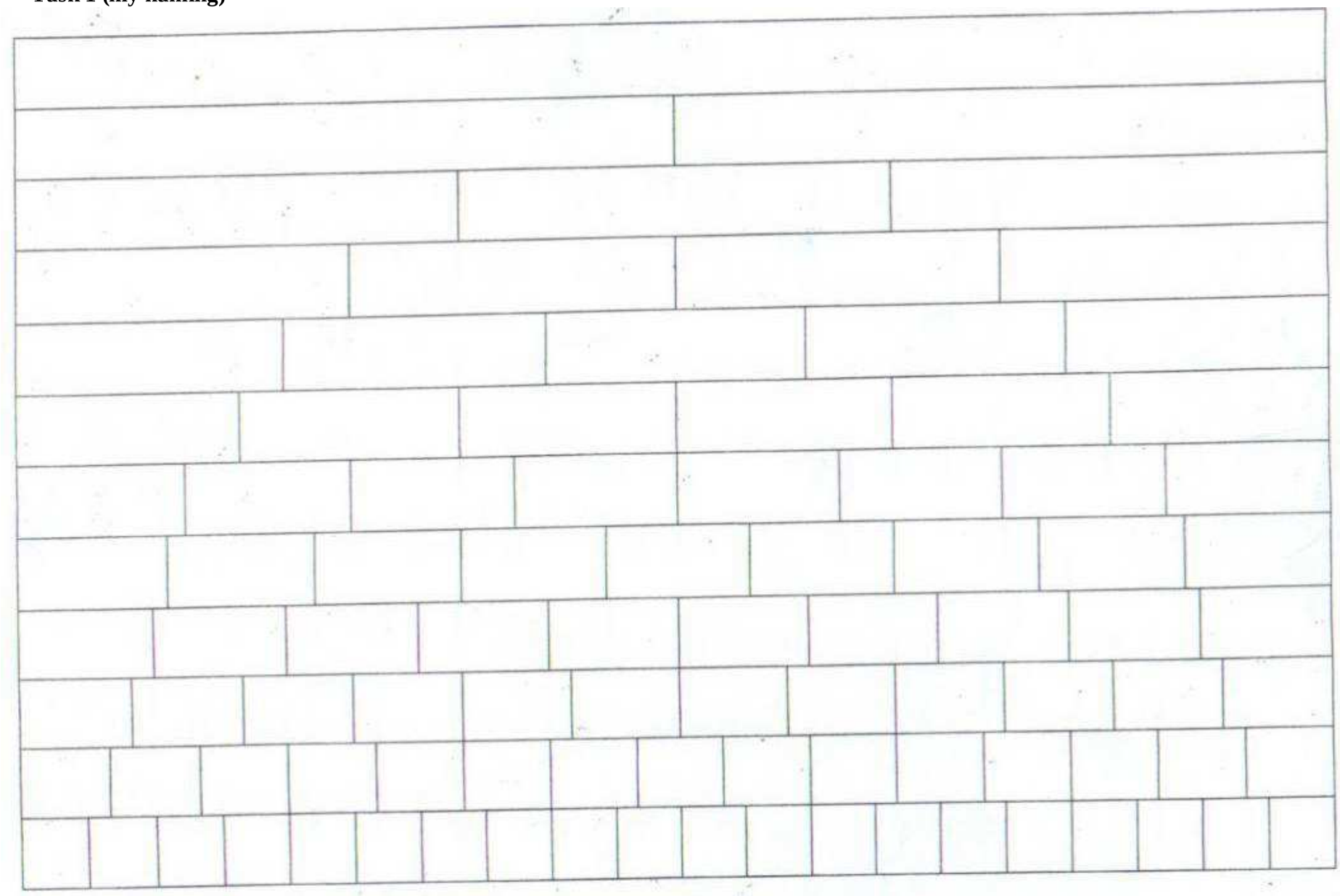
Instructions

- **Draw the diagram/grid at home.**
- **Fill in the different fractions.**
- **Cut into different size fractions.**
- **Make sure it is neatly done.**
- **Make up 10 number sentences to prove the above.**
- **Remember it is for marks, you have two periods to complete the project.**

ASSESSMENT RUBRIC/TABLE

COMMON FRACTIONS: ADDITION	
1. FOLLOWING INSTRUCTIONS	5
2. COMPLETE DIAGRAM/GRID	5
3. 10 NUMBER SENTENCES/ANSWERS	20
TOTAL	30

Task 1 (my naming)



Task 2 (my naming)

Find the lowest common denominator and then add the following fractions. The first one has been done for you.

a. $\frac{3}{8} + \frac{5}{6} = 1\frac{5}{24}$

$= \frac{3 \times 3}{8 \times 3} + \frac{5 \times 4}{6 \times 4}$

$= \frac{9}{24} + \frac{20}{24}$

$= \frac{29}{24} = 1\frac{5}{24}$

$M_8 = \{ 8; 16; 24; 32; \ldots \}$

$M_6 = \{ 6; 12; 18; 24; \ldots \}$

LCD = 24

b. $\frac{11}{12} + \frac{1}{15} =$

c. $\frac{7}{9} + \frac{5}{8} =$

d. $\frac{5}{18} + \frac{2}{9} =$

e. $\frac{11}{5} + \frac{5}{6} =$

f. $\frac{5}{12} + \frac{3}{8} =$

g. $\frac{4}{9} + \frac{5}{15} =$

Find the difference between the following fractions: Remember to find the lowest common denominator before subtraction.

a. $\frac{11}{20}$ and $\frac{2}{15} =$ _____

b. $\frac{9}{16}$ and $\frac{3}{20} =$ _____

c. $\frac{19}{20}$ and $\frac{7}{24} =$ _____

d. $\frac{20}{21}$ and $\frac{5}{14} =$

Task 1 was given to the learners with the intention of allowing them to engage with the concept of fractions. During the initial interview, Mr. T provided the following explanation for why he chose this particular task to introduce the addition and subtraction of fractions:

> Mr. T: Because I felt in the past by just going straight to the addition and subtraction the children don't understand why an $\frac{1}{8}$ and a $\frac{1}{8}$ plus whatever can make one whole. Why the denominators are not the same and if they are different they have to look for the LCD. I find it easier to work with a fraction board, which is actually grade 5 and grade 6 work, before I go to the grade 7 work. Just to make them understand it better and I find that they find it quite interesting, it's also a little bit of a discover, that's the main thing. They have to discover that by using different equal parts you can actually work out a sum total.
>
> (Interview 1)

The teacher did not merely want to give the learners a large number of sums in order to practice addition. Instead he designed the task with an in-built self-checking mechanism. This in-built self checking focused the learners' attention on the addition of fractions while it provided them with the opportunity to check that their answers were correct and added up to one whole. This is a notable task in that it provides opportunity for the learners to learn about unit fractions and wholes and it can be carried out in different ways. The task allows for the consolidation of these concepts through no particular set of procedures, and lends itself to learners being able to add fractions and get a sense of meaning for how fractions add up to one whole. It can therefore be considered "doing" mathematics (Stein et al 2000). Built into the task is the possibility for learners to add the unit fraction in a range of ways. For example, in this strip the learner had repeated tenths seven times and therefore could have added seven tenths.

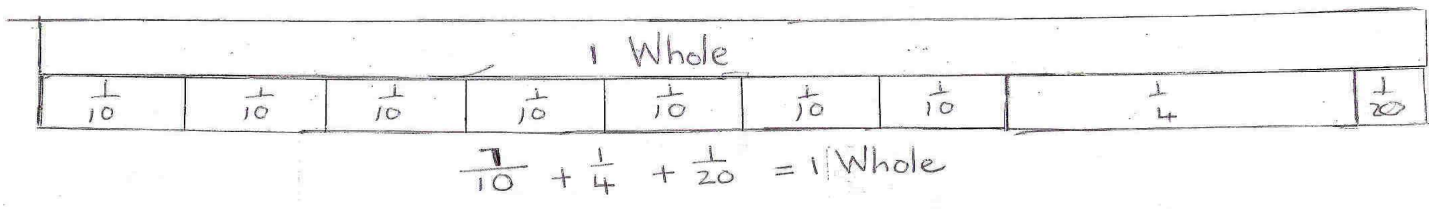


Fig. 4.2: Extract from learner 's book.

Or, learners could have used each unit fraction once only in order to get to one whole as shown in the example below:

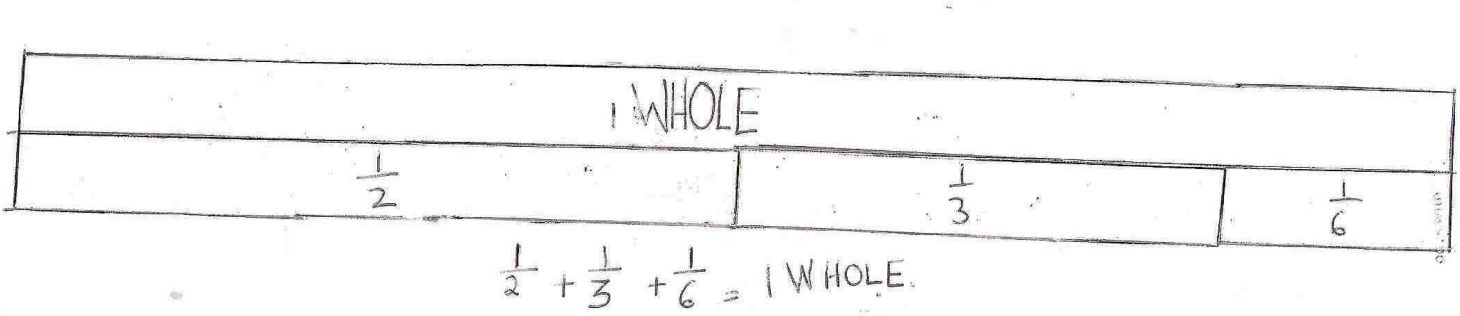


Fig 4.3: Extract from learner's book

The task allowed for the consolidation of concepts, like equivalent fractions. As you will see, the teacher gave directions on how to add unit fractions using lowest common denominators and expected the additions to be done that way. An alternative approach would have been to leave this open. In this sense, Mr. T reduced the task to a set of procedures that is followed when adding fractions and so shaped and constrained the potential meaning of the task. This will be discussed later in this chapter and is not uncommon with regards to the implementation of tasks (see Stein et al, 2000).

There are other possibilities that lie in this task. It is an open task in that learners can put the different fraction pieces together in which ever way they want, as long as they get to one whole. This in turn factors into what they need to add to get a whole and shifts the task between doing mathematics and carrying out procedures. As I will be showing later, there is more potential in this task than is realized by the teacher. While there are many aspects of fractions that can be explored with regards to this task, Mr. T focuses on the self-check feature of the task. Given the way that Mr. T deals with the task, it raises the question of how he then copes with both the procedural and conceptual demands of the task as these emerge and as his learners work on the task.

Task 2 involved procedures with connections. The task was set up so that learners could engage more with the procedural aspect related to fractions and also so that they are able to make the connection between the procedural and conceptual. Part two of the task

allows the learners to make a connection between adding fractions with different denominators and finding the lowest common denominator. While the task requires that learners follow a general procedure, it cannot be done mindlessly (Stein et al, 2000).

The literature clearly points to the fact that in order for learners to gain an understanding of fractions, they must engage both conceptually and procedurally with the topic. However, research has shown that teachers experience difficulty finding the balance in teaching fractions both procedurally and conceptually because of their nature. Teachers place emphasis on the procedural aspects. It will be interesting to observe whether this is true for Mr. T and if he faces similar challenges. As argued in the literature, teachers' knowledge and understanding of fractions determines how they teach fractions and facilitate their learner's procedural and conceptual understanding. Using appropriate representations, working with student ideas in order to move them on mathematically, offering mathematically sound explanations, engaging with and responding productively to student questions, using mathematically appropriate and comprehensible definitions (Ball et al, 2004) will affect how Mr. T manages the procedural and conceptual demands of the task as these emerge while his learners work on the task.

4.3 Working with the procedural and conceptual in teaching fractions

The task allows for the development of both conceptual and procedural knowledge and I will now discuss these two concepts in the light of Stein et al's (2000) description of the cognitive demands placed on learners through tasks. While Stein et al (2000) considers what happens to the task during the set up and implementation phase, this is not my focus. Stein et al's (2000) focus is on tasks and how teachers sustain or reduce demands. These task demands shift from the set up to implementation and vice versa. In terms of teaching fractions, research explains that there exists a tension between the set-up that may be conceptual and the implementation that may be procedural or vice versa. In other words, there might well be a problem with the intention to set up a task conceptually but it is reduced to procedurally teaching. Thus the link between the set-up and the implementation of a task is not straightforward.

As this research involves how a teacher works with a particular task as well as his learners, it is interesting to look at in terms of Stein et al's work. My focus, however, is more on what problems the teacher confronts, how he deals with these and the dilemmas he faces. Stein et al (2000) are concerned with tasks for all mathematics, I am however, only focused and concerned with fractions.

In light of the above mentioned task, the literature on fractions and the criteria set by Stein et al (2000), tasks that are procedural in nature may place lower cognitive demands on learners while tasks that engage in conceptual understanding may be of a high cognitive demand. While this may be true, there exists the possibility that certain procedural work may demand a higher level of thinking. Stein et al (2000) refer to procedures with connections as well as procedures without connections. They classify procedures with connections as placing a high cognitive demand on learners. They use the word 'procedural' and it fits into both high and low cognitive demand tasks. Therefore, it cannot be unequivocally stated that tasks, which require procedural work, make low cognitive demands on learners but rather that it depends on what the procedures are doing. The task used by Mr. T may appear to be procedural in nature however, going beyond its surface and superficial nature, there is evidence that it requires a high level of thinking. The task presents both procedural and conceptual goals. It allows the learners to work with and develop an understanding that fractions are part of a whole and at the same time show that when adding fractions there are certain procedures that need to be followed. The task demands engagement with the concepts involved in fractions (addition, multiples, LCD) and stimulates purposeful connections to meaning or relevant mathematical ideas, which lead to a different set of opportunities for student thinking (Stein et al, 2000). Thus both conceptual and procedural understanding of fractions can be promoted and developed.

As mentioned previously, there exists a tension between the procedural and conceptual teaching of fractions. High demand tasks that are more conceptual in nature, the complex nature of fractions and teachers' understanding of the topic, throw up dilemmas around procedural and conceptual work, the representation of the content and working with

student ideas. Teachers might intend to teach and develop certain concepts by using specific tasks, however, what they might intend is not always what happens. The tasks are sometimes inappropriate to develop the concepts, or they are difficult and require mediation from the teacher. The teacher may not be able to provide efficient mediation since they themselves find it difficult to re/present the content in an alternative way because of its complex nature. Their conceptual and procedural knowledge of fractions influence how they teach it. They might be able to complete calculations on the topic of fractions successfully, but not be able to offer a mathematically sound explanation for their calculations; or they are able to represent the operation and correctly explain its meaning because of their concrete and extensive knowledge of the topic. If the former is true, it is inevitable that the teacher will be faced with the dilemma of teaching algorithms to perform operations on fractions in order to achieve correct answers versus teaching for conceptual understanding and knowledge. This in turn, determines the representations the teacher will use. Reducing the task to simple algorithms, devoid of any high level thinking versus using the more complex, less structured task, so that their students are able to develop a deeper understanding of the nature of fractions then becomes a dilemma for teachers. Linked to this, is the dilemma of working with students' ideas. Using more open-ended tasks requires that the teacher has a deep understanding of fractions so as to move the learners on mathematically. If the teacher does not possess the necessary knowledge and skills, it leads to them working with student ideas at a superficial level. How they engage with their learners mathematically will ultimately determine the mathematical learning that takes place. Do teachers once again teach procedurally at the expense of conceptual understanding or do they teach conceptually, knowing that they themselves are unable to facilitate the necessary mediation required for a deeper understanding of fractions? There exists a relationship between the task selected to teach fractions, how it is implemented, the teacher's knowledge of the topic, and the dilemmas that the teacher faces when he works with the learners and the task.

4.4 Description of data and indicators

What I present next is an extract and commentary of lesson 1 where Mr. T encounters difficulties working both procedurally and conceptually with the task. I also present a table that illustrates how the data was classified and recorded for this extract. There are two main reasons for the selection of this extract. Firstly, it shows the criteria for the data analysis i.e., the concepts and the indicators. It will explain what mathematical problem solving he does and the appeals he makes to legitimate meaning for the learners. Secondly, it provides an example of when the teacher is faced with the dilemma of working both procedurally and conceptually with the task.

The table on the next page illustrates how the data was classified and recorded. As initially discussed in Chapter Two, the classroom transcripts were chunked into evaluative events (see appendix F). Each event was marked by the start of a new idea, what I refer to as a notion or sub notion. In this particular extract, indeed in the whole lesson, there was one notion with two sub notions at play, and all of these were around the addition of fractions. The first column represents the events, the second column represents the timing of the events, the third column signifies the notion, and the fourth column represents sub notions respectively. From this, each event is analyzed according to the cognitive demands of the task, the problem solving the teacher engages in and the appeals he makes. It must be noted that in practice, all these categories are inter-related. Separating them analytically enables 'seeing' what is happening with new eyes.

Events	Timing	Concept			Task Demands				Problem Solving/ Mathematical Work								Appeals					
					Lower Level		Higher Level									Working with learner ideas		Maths			Experience		Curriculum
		Notion		Sub-Notion	Memorization	Procedures Without Connections	Procedures With Connections	Doing mathematics	Explanations	Questioning	Demonstrations	Representing	Restructuring	Defining	Encouraging	Mathematical Calculation Errors	Practical' Errors	Empirical	Definitions	Rules	Professional	Experience	tests & exams
Lesson 1 (50 Min)																						
4	31:53-36:19	Adding fractions with different denominators	4.1	Finding the LCD using multiples		✓			✓						✓	✓		✓		✓			
	36:20 - 37:56		4.2	Using fraction strips to prove one whole & finding the LCD		✓					✓	✓	✓		✓	✓		✓		✓			
	37:57 - 39:49		4.3			✓					✓	✓	✓			✓		✓		✓			
	39:50-45:13		4.4	Using fractions strips to prove one whole & finding the LCD				✓	✓	✓	✓	✓	✓		✓	✓		✓		✓			
5	45:14-46:36	Adding fractions with different denominators	5.1	Using fractions strips to prove one whole		✓			✓					✓					✓	✓			

Table 1: Event 4 from Lesson 1

I am going to show what an event looks like. An event starts when the teacher introduces something new. In the extract that follows for example, it is allowing the learners to work with the unit fractions on the floor. Bringing in this new representation is the start of a new event.

I will proceed with the analysis by presenting some transcript. I will emphasis aspects of the transcript that I am going to discuss. This emphasis will be indicated by either underlining or bold words. Excerpts of the transcript will be referred to in order to show how I have identified the problem solving and how I have identified the appeals. After the transcript I will make some comment on what is going on in the lesson because the transcript is not always illuminating of what was happening. I will contextualize the transcript and what will follow is analysis of the problem solving and appeals.

The following extract was identified as event 4 of lesson 1. I have already explained the lessons. However, it is important to note what has happened prior to when this extract begins. Up until event 4, the teacher explained the task to the learners. He did a few examples on the board of how to find the lowest common denominators. The learners were than required to complete the 10 sums they made up from their charts. Mr. T walked around the classroom to assist the learners individually. A few learners encountered problems while engaging with the task. These problems will be discussed in detail later on. Event 4 commences when Mr. T invites a learner who is having difficulty finding the lowest common denominator for one of his sums to work with the unit fractions on the floor.

The extract from the transcript of lesson 1 starts at 36:20 and lasts for 9 minutes and 7 seconds.

Between the time interval 36:20 to 37:56 I have identified the sub-notions as *using unit fractions to confirm one whole* and *finding the lowest common denominator using multiples.* This is event 4.1.

36:20

Mr. T: You still battling with that one (learner cannot find the LCD in order to complete he sum).

L: Yes sir

Mr. T: What's it, three, five, four. Bring me your book. Come here *(moves to the front of the class where unit fractions are*). Put your book on the floor. Sit down on the floor. Now take, let's do this. Put this (*unit fraction*) on the floor. Grade 7, young man would you like to leave? Put this on the floor. Just imagine that's a whole (*2 half unit fractions*). Take your combination now that you've got. Ok, and take it from here (*the table*) and put it over there (*the floor*) and see if it will work out. I'm going to call you up guys, one by one just now (*addresses whole class*) and I want to see the ones you battling with. Sir is going to ask you to come and build it on the floor here and lets just see if you've gone over. Remember my pieces were cut equally. So let's just see. Here's the other piece

(*speaks to learners*) Ken[8] stay seated please. (*Learner works on the floor, while teacher watches*).

Mr. T: **Ok now, if you look at that *(refers to the unit fractions on the floor that the learner has put together)*. Can I tell you what, your answer must work out. You just got to find the lowest common denominator. Go and find it. I bet you'll be able to do it.** Ok, who else did I see was battling with this sum? Whom of you just said to me you were battling?

L: (*few learners raise their hands*)

Comment

Table 1 shows that the demand of the task was identified as lower level cognitive demand that focused on procedures without connections.

This is evident in that when the learner struggles to find the lowest common denominator, the teacher instructs him to use the unit fraction provided to see whether his answer will work out. The learner puts the unit fractions together to form one whole. He acknowledges that his sum should add up to one whole and goes back to his desk to find the lowest common denominator in order to complete the sum. When completing the sum, he is involved in merely generating multiples from memory and in order to find the lowest common denominator he must follow the procedures set out by the teacher in the beginning of the lesson. Thus the task was considered as a lower level cognitive demand task involving procedures without connections.

The mathematical work or problem solving that Mr. T engaged in, I classified as demonstrating, representing, restructuring, encouraging and identifying errors (mathematical calculations-learner cannot find the LCD). Mr. T uses the unit fractions to **represent** the learner's sum (visually & graphically). He represents the sum of the unit fractions as a whole. He also **demonstrates** how to add the unit fractions to get one whole. He restructures (so as to get the learner back on the task) the task in that he moves the learner from finding the lowest common denominator to allowing him to work with the unit fractions so as to establish that the sum must work out to one whole. He then

[8] Pseudonym to protect the identity of the learner

moves him back onto the task that requires him to find the answer one by finding the lowest common denominator and adding the unit fractions. Mr. T attempts to make a link between the conceptual and the procedural as he restructures the task. Identifying the mathematical error (learner cannot find the lowest common denominator) made by the learner is evident in that Mr. T immediately recognizes that the learner is unable to calculate the sum and as a result instructs him to use the unit fractions so as to confirm that the answer must work out and to encourage the learner that he can perform the task. He also **encourages** the learner by saying to the learner that he bets he will be able to do the sum.

Analysis of the appeals

In Mr. T's attempt to fix meaning, I classified his appeals to be mathematics. As discussed in Chapter Two there are three ways in which appeals to mathematics can be distinguished i.e. principled, rule based and empirical. As is seen in this extract (refer to the bold print), the appeals are mathematical and they are rule based. The teacher explains to the learner that since the fractions units add up to one whole, the calculations must also add up to one whole, he further explains that in finding the lowest common denominator the sum will add up to one whole. This indicates, that Mr. T is appealing to a mathematical rule as well as the empirical (a concrete example- the unit fractions must add up to one whole). In encouraging the learner he appeals to mathematical rules in that he suggests that he knows this learner will be able to find the lowest common denominator and show that the unit fractions add up to one whole because his answer must work out.

Transcript

Between the times interval 37:57 to 39:49 the sub notions are similar to the previous ones i.e. *using unit fractions to prove one whole* and *finding the lowest common denominator using multiples*. This is event 4.2.

37:57

Mr. T: Come here Joe[9]. Bring me your book as well.

L: (*learner comes to the front of the classroom*)

Mr. T: Before you do that, take these pieces (*pieces on the floor used by the previous learner*) and put it back (*on the table*). Just leave the two halves. Give me your book. Put those other pieces back here (*on the table*). No, no, leave the halves. I want…(*inaudible*) because I haven't got a whole. So I'm going to use that as a whole ok. Now take the one that you battling with. Take the pieces off here (*table*) and put it there (*floor*). Ok, so see if your sum will work out.

L: (*asks for certain strip, inaudible*)

Mr. T: Sorry young man. You need a (*inaudible*). There isn't one. Is that what your sum is?

Mr. T: (*Marks Joe's book while he works on the floor*).

Mr. T: (*addresses previous learner*). Right it's just somewhere with your lowest common denominator. You didn't do something right (*refers to the sum in his book*). And that makes you feel like you can do it *(referring to the fact that the strips add up to one whole so the sum must work out*). Am I right? You smarter than what you thought you were.

Mr. T: (*comes back to help Ken*) Just push your puzzle together (*referring to unit fractions*). Look there. Can you see the gap there?

L: Yes sir

Mr. T: Maybe if you have (*inaudible*) it will work out. Ok right. Go try it again. Just try it again for sir. See if you can do it and you'll come back here again.

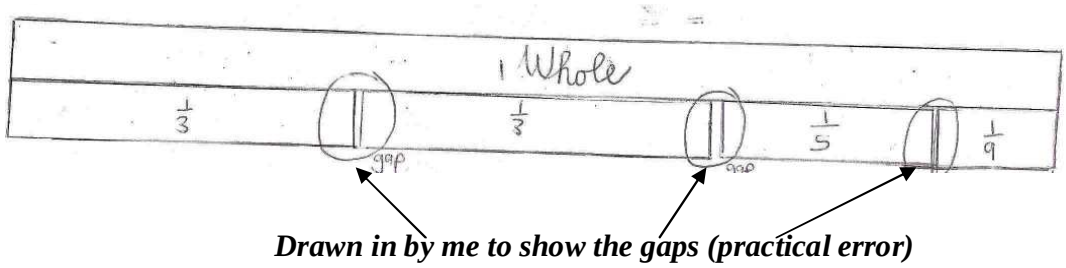
Mr. T: Ok, who else? (*addresses entire class).* Come Tom[10]. Bring me your book.

Comment

The teacher continues to engage with the learners individually. The task is once again classified and remains as a low-level cognitive demand task with the focus on procedures without connections. In other words, it encouraged and focused on procedural learning and there is no connection to concepts since the teacher provided the steps for the learners

[9] Pseudonym to protect the identity of the learner
[10] Pseudonym to protect the identity of the learner

to follow. This came to be a sub-notion in that the teacher has the opportunity to engage with a different learner with regards to the same task at hand. The learner in this instance experienced difficulty in that his sum would not add up to one whole. Unlike the previous learner, this learner's combination of unit fractions was incorrect in that it did not add up to a whole. This is the example from his book:



Drawn in by me to show the gaps (practical error)

Fig. 4.4 : Extract from Joe's book

Mr. T once again gets the learner to use the unit fractions to show him that the sum must add up to one whole, however in this instance Mr. T did not have the unit fraction ($\frac{1}{9}$) so the learner used $\frac{1}{8}$ instead. Mr. T points out the gap to him and suggests that if they had $\frac{1}{9}$ it may work out. He does not note or refer to the 'gaps' in the learner's book as shown in Figure 4.4. He recommends that the learner tries again to work out the sum and come back later to use the unit fractions if he is unsuccessful. It is interesting to note that the sum was as follows: $\frac{1}{3} + \frac{1}{3} + \frac{1}{5} + \frac{1}{9} =$, this does not add up to one whole.

It appears that Mr. T sent the learner to work out the sum without realising that the sum will not work out to one whole. Mr. T did not identify the practical error or the mathematical error made by the learner. He assumed that the learner was struggling to find the lowest common denominator and therefore could not get to one whole. When the learner used the unit fractions provided by Mr. T, it was difficult for him to tell that the unit fractions he used would not add up to one because he did not have the unit

fraction ($\frac{1}{9}$) to represent his sum. Unfortunately time ran out and this was not addressed again.

The problem solving that Mr. T was engaged with in this event was very similar to the previous one. He **demonstrated, represented, restructured** and **identified a mathematical error (unable to find the LCD)** made by the learner. He demonstrated and represented the content using the unit fractions. However, the unit fractions did not work for him in this case in that he did not have the $\frac{1}{9}$ to show the learner that his sum would not work out. If he was able to represent the sum using the appropriate unit fractions, Mr. T may have realised that it was the practical error (gaps) that led to the mathematical error (not combining the correct unit fractions resulting in the sum not adding up to one whole). Instead he allowed the learner to go back and try the sum again.

Mr. T again appeals to the rules of mathematics and the empirical as he points out the gap when using the unit fractions and proposes that if he had the appropriate unit fraction ($\frac{1}{9}$), it may work out to one whole.

Transcript

The next time interval, 39:50 to 45:13 is a continuation of the previous sub notion. This is event 4.3.

39:50

L: (*Tom comes to the front of the classroom and sits on the floor. Hands his book to the teacher*)

Mr. T: Put those pieces back on the board. Just leave the halves there. Put the other pieces back on the table for me please.

L: (*puts unit fractions on the table. Leaves two halves on the floor*)

Mr. T: Take the one you battling with. Put it together.

L: (p*uts unit fractions together*)

40:48
Announcements over intercom

Leaner needs certain unit fractions but not available. Teacher continues to work with him, but intercom is on and very difficult to hear conversation. Teacher tries using unit fractions from a pie chart.

42: 41
Mr. T: They don't have twenties, ah. My plan is not working. **Three twenties will be how many tenths? Think carefully.**

L: *No response.*

Mr. T: **If I had to replace that (*refers to the* $\frac{1}{20}+\frac{1}{20}+\frac{1}{20}$ *unit fractions*) with tenths what would it be?**

L: *No response*

Mr. T: **Those three twenties. If I take a ten here (*give learner a tenth).***

L: *Places tenth in position.*

Mr. T: **We've got, ok it won't work. Take it off. So we've got to find an alternative for that.** What I though was, if we have ninths, just put that on there (*pie graph*). I'll see something now. What else do you need?

L: One eighth

Mr. T: One eighth, uh, sometimes they don't give you all the puzzles that you need so they can't work. Ag, it frustrates me! I can't do this one. Oh eighths, here's an eighth. You only need one?

L: Yes sir.

Mr. T: You see what sir was trying to do now is. I thought, I don't think a ninth will fit in there. And I don't have the. Ok, here you are (*hands learner pieces*). Put it in there. Let's see what you going to be short of.

L: *Learner takes pieces*

Mr. T: Put it on here. Put it on this. Make believe this is going to be your whole (*pie chart*)

43:46
Bell rings

Mr. T: Okay, now put it on there. Let's fill the puzzle. Okay so you still need, what else do you need? Two fifths. No you've got fifths.

L: Thirds

Mr. T: Thirds, you need a third. This is most frustrating. **But I actually thought that we could use this. Okay, mmm, okay so we can't use it. We can't use it on this side either. So we need a ninth there. It doesn't look like your sum is going to work out now can you see, because already battling um, to put the pieces together?** Okay, so it's not the end of the world. Um, you did it over here (*referring to work in learner's book*). What did you find? Ah Grade 7 don't talk please. Um, okay, where, where, which one was it? Which one was it?

L: Points to sum (*unit fractions*)

Mr. T: A third, a fifth a fifth. It seems to fit in there. Okay, I do believe that if we had a ninth over there it would work out. Okay, so just go and try, your lowest common denominator what did you find?

L: I couldn't find it. Had to carry on?

Mr. T: Okay but you must carry on. That's why you didn't find it. Super, okay we know where your problem was. Okay?

L: Yes sir

Comment

This is what appears in this learner's book:

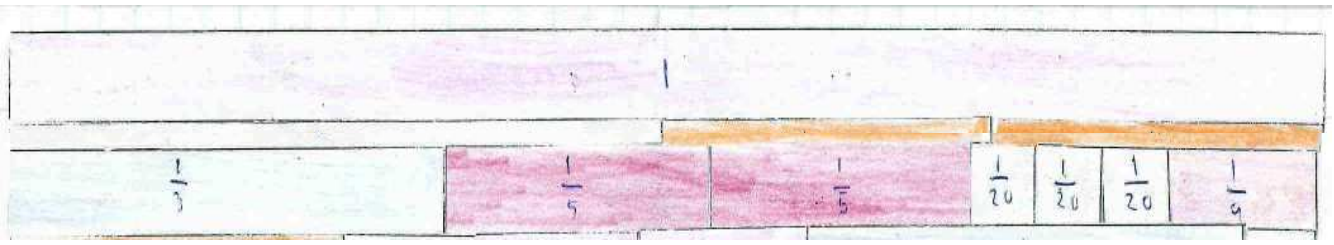


Fig: 4.5: Extract from Tom's book

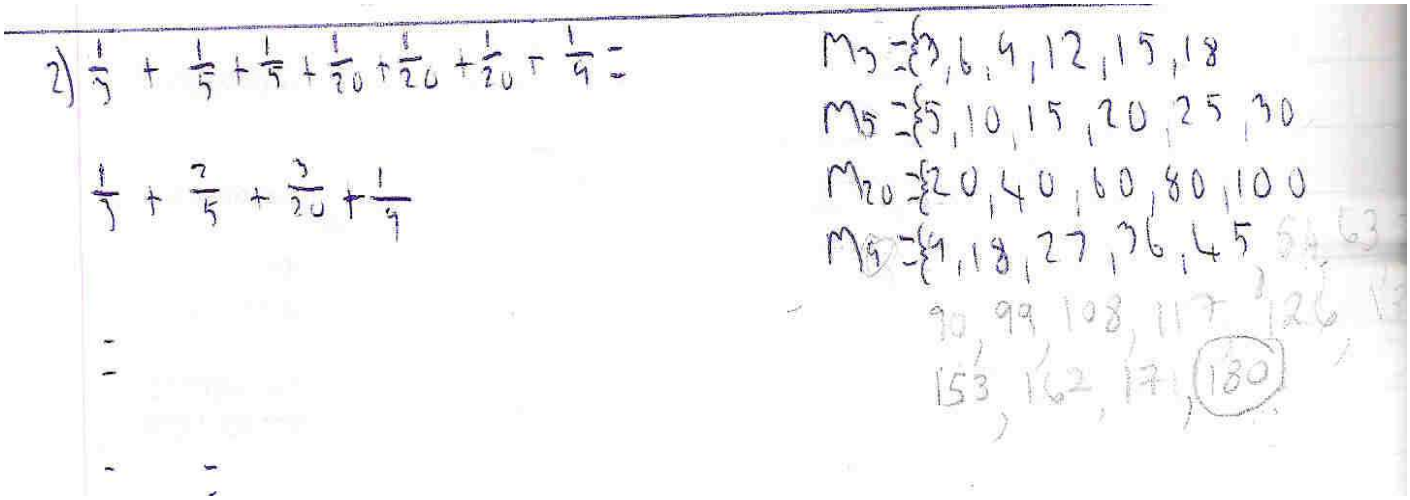


Fig. 4.6: Extract from Tom's book

Here the teacher engages with a different learner and in doing so, restructures the task to doing mathematics as he problem solves and makes appeal. The learner in this particular sub notion is having difficulty finding the lowest common denominator. He uses the fractions strips provided by Mr. T to confirm that the sum must work out to one whole.

Once again there is no ninth unit fraction so it is impossible for the learner to check whether his sum will add up to one whole.

Mr. T attempts to use different unit fractions to represent the ninth and thus moves the task into doing mathematics. It is evident that the task demands are changing, as up until now all that was required of the learner was to follow procedures. At this point in the lesson, Mr. T and the learner face a problem and Mr. T attempts to work more mathematically. He asks questions that force the learner and himself into thinking more mathematically about the problem instead of merely following procedures as before (refer to bold print). Mr. T tries to find an alternative representation for the sum, but fails. When he and the learner are unable to solve it using the unit fractions, he turns to a different form of representation and uses the unit fractions of a pie chart in order to show the learner that it will add up to one whole. He runs into problems again because once more he does not have all the unit fractions. Eventually Mr. T refers to the learner's book

and suggests that the unit fractions that the learner used will add up to one since they fitted perfectly in his book (refer to figure 4.6). He also suggests that the learner's problem was adding fractions with large denominators and not practical errors.

Referring to figure 4.6, it is clear that the learner found the lowest common denominator (i.e. 180) and circled it, but was unable to complete the sum. However, it is interesting to note that $\frac{1}{3} + \frac{1}{5} + \frac{1}{5} + \frac{1}{5} + \frac{1}{20} + \frac{1}{20} + \frac{1}{20}$, does not add up to one whole (i.e. $\frac{180}{180}$). It adds up to $\frac{179}{180}$, which is very close to one whole. When $\frac{1}{3} + \frac{1}{5} + \frac{1}{5} + \frac{1}{20} + \frac{1}{20}$ is added we get $\frac{159}{180}$. The unit fraction piece that will fit in most neatly to get close to $\frac{180}{180}$ is in fact $\frac{1}{9}$ because what is actually missing is $\frac{21}{180}$ or $\frac{7}{60}$, which is almost $\frac{1}{9}$. Neither the teacher nor the learner was able to pick this up. In fact I only discovered this close to the end of the analysis when my supervisor pointed it out to me. Perhaps if the learner completed the sum he would have realized the error. Visually it appears as though the unit fractions fit together and add up to one whole. However, there is a bigger problem presented here and I will return to it when I engage in a discussion regarding the structuring and representation of the task in the chapters to follow.

Mr. T was involved in seven of the eight mathematical problem-solving categories (i.e. **explaining, encouraging, identifying mathematical calculation errors, restructuring, representing, demonstrating, questioning**). As with the previous events, appeals here are classified as Mr. T appealing to mathematical rules and the empirical.

The event, including the notions and sub notions, focused on thus far, involves the teacher and the work he did with individual learners. They do not illustrate all the possible problem solving and appeals e.g. defining (problem solving) and curriculum (appeals) are not classified. In order to illustrate the notion of defining, I will make use of the extract that involves work done by the teacher with the whole class.

The next extract was taken from lesson 3 and is event 3.1. It is approximately a minute and a half into the lesson. Mr. T recaps what has taken place the past two lessons and then proceeds to explain and demonstrate how to change a mixed number into an improper fraction. In doing so he requires that the learners define what an improper fraction and mixed number is.

Transcript

01:38 – 03:28

Mr. T: Okay boys and girls, we took a fraction board. We cut it up. We stuck it back together again. You worked out a couple of sums and you proved that if I put certain fractions together, look at me I'm here. Um that it works out to one whole. Now we going to go a little bit further. I'm going to show you two ways of getting to the lowest common denominator without using multiples of what I'm working out. Short and sweet. Quickly you can do it, but step by step. Now like I said to the other classes and I said to you yesterday, **it's like getting a play station game, right. I'm putting it into the play station and I start playing. It's a brand new one I just discovered of the shelf. I'm putting it on and I'm playing with it, but I'm battling because I've got to discover how to get to how to win the game or whatever the case may be. The same with maths okay. The same with this work.** There is shorter methods of doing this but at this stage I think if you take the long road and the long method like I'm going to show you and why I'm doing things that you'll be able to get the sum right. And then at the end you can say, okay I know a shorter method. I worked out the answer quicker. That's great. You can do that. But at the moment boys and girls let's just follow. There's two ways of doing things. I'm going to put it on the board. I'm going to show you. I'm going to ask you going to copy it down. I'm going to give you your worksheet and I am expecting you guys to do it like that okay. And look at your examples. Say, this is the example; I don't understand how sir got that. Call me and I'll come and help you again. But at this stage if you just listen to what I'm doing on the board, you can do it. Okay, and its easy sums that sir is giving you.

03:29 – 05:30

Mr. T: Okay, we are now going to mixed number, improper fractions. **Whom of you can explain to me what an improper fraction is? What is an improper fraction? You told me a fraction is an equal part of a whole. What is an improper fraction?**

L: **An improper fraction is when the denominator is smaller than the numerator.**

Mr. T: Numerator is smaller[11]. Give me an example.

L: Like uh, thirteen over eleven.

Mr. T: Thirteen over eleven (*writes* $\frac{13}{11}$ *on the board*). So it depends on, somebody took my pens (*battles to write on board with his pens and mumbles under his breath*). That's an improper fraction. Now I'd like to change, what is a mixed number? What is a mixed number then? Ja.

L: When there is a whole number and a fraction.

Mr. T**: A whole number and a fraction. Good answer. In other words I can change that (*Referring to fraction on board)* into a mixed number.**

L: Yes sir (*respond together*)

Mr. T: What will my answer then be?

L: One whole

Mr. T: One whole (*repeats after learners*)

L: The fraction mustn't be (t*eacher interrupts*)

Mr. T: No, no give me the answer to this. You telling me (*learners calls out answer*)

L: Two elevenths.

Mr. T: Two elevenths (*repeats after learner*). Okay. That's exactly what I'm looking for. So that over there (*writes* $1\frac{2}{11}$ *on the board*) boys and girls is known as a mixed number, okay.

L: Mixed number (*that repeat after the teacher*)

Mr. T: And you know that now from grade six and that is over there ($\frac{13}{11}$ *and writes improper fraction next to it) is an improper fraction.*

L: Improper fraction (*they repeat after the teacher*)

[11] I am aware of what the teacher said here and that it could just be a mistake on his part.

Comment

Before I attempt to explain how I recognized defining as part of the problem solving done by the teacher, I must make mention of the appeal Mr. T makes with regards to the learners' everyday lives. He likens the procedures of completing and being competent in adding fractions to playing a play station game (see bold and underlined print). This indicates to me that he appeals to experience. In this particular event, he equates the activity of doing mathematics to having fun. He communicates the message that fractions are meaningful because it is like playing a game, rather than it is meaningful because of what fractions are and what they represent. It appears as though he is trying to say that mathematics, as an activity, should be fun. In this extract, he does not relate a mathematical idea to an everyday concept, like the work done by Adler & Pillay (2007), which reports on how the teacher uses an everyday concept (marriage) to explain the mathematical idea of an equation. Mr. T makes different everyday appeals. He appeals to the learners' everyday experiences, but at different levels, one is at a level of experience and disposition (learners should not be struggling with fractions, but rather they should be having fun) and the other is at the level of concepts (in lesson 1 he associates the fraction wall with a brick wall and how the wall cannot go into the neighbours yard). Both of these are linked to the learners' everyday knowledge and experience, but since this is not a study of everyday practices, I will refer to both of these as everyday experience[12].

This extract from lesson 3, illustrates the sub notion changing a mixed number to an improper fraction. In order to explain to the learners the procedures and rules to follow when changing a mixed number into an improper fraction, Mr. T questions them by asking them firstly to define what an improper fraction is and secondly, what a mixed number is. In getting the learners to define these two concepts, it is evident that Mr. T

[12] I am aware that one is about how (how you do mathematics is like playing a play station game) and the other is about what (that a fraction and a fraction wall is like a brick layer).

appeals to mathematics (see bold print). He particularly makes an appeal to definitions and rules.

None of the extracts drawn on thus far describe appeals to the curriculum (e.g. test and exams) made by Mr. T. When Mr. T makes an appeal to tests or exams he informs his learners that they must follow certain procedures in order to obtain marks. He also stresses the importance of what will be required by the learners in preparation for the exams or tests, for example he constantly reminds them that they must correct the sums they have done so that when they are studying for the exams or test they are able to study the correct methods and procedures. Mr. T clearly emphasizes the set of rules the learners need to follow as described by the curriculum. For example in lesson 4 he states the following:

> *Mr. T: That's why I keep on telling you maths is a study subject boys and girls. When I give you something to do at home, go over it like you study. Remember the stuff you were taught in class. Just keep on going over it. And now look at this, you sitting here thinking I could have done that, oh that was easy, this was easy…*
>
> (Lesson 4)

The next section involves a composite table of the problem solving and appeals made by Mr. T in the first four lessons. This is done lesson-by-lesson and then taking all the lessons into account.

4. 5 A Quantitative Analysis

In order to make overall sense of the task demands, the mathematical problem solving done and the appeals made by Mr. T, it was useful to quantify the data. This helped depict the presence, absence and frequency and as such provided a view of lesson 1 to 4 of the task demands, problem solving and appeals. It helps to show up dominant practices and by linking the problem solving and the appeals, it starts to reflect what

dilemmas the teacher faces. This was done by tallying occurrences of the task demands, problem solving and appeals made.

The following table quantifies, per lesson, the task demands, the problem solving that Mr. T engaged with and ultimately the appeals made by him in order to legitimate meaning for his learners. The last column of the table displays the average of the percentage of the occurrence of each item. I offer an explanation with regards to the values that appear in the percentage occurred column later in this chapter.

		Total Occurrence				**% Occurred**				Averages
Lesson Number		1	2	3	4	1	2	3	4	
Notions (including sub-notions)		14	17	7	5					
Task demands										
Memorization		0	0	0	0	0	0	0	0	0
Procedure without connections		13	17	7	5	93	100	100	100	98
Procedures with connections		0	0	0	0	0	0	0	0	0
Doing mathematics		2	0	0	0	14	0	0	0	4
Problem Solving										
Explanations		5	5	4	2	36	29	57	40	41
Questioning		1	0	0	0	7	0	0	0	2
Demonstrating		9	7	4	0	64	41	57	0	41
Representing		4	6	0	0	29	35	0	0	16
Restructuring		3	2	0	0	21	12	0	0	8
Defining		1	0	1	0	7	0	14	0	5
Encouraging		8	7	1	4	57	41	14	80	48
Working with Student Ideas	Mathematic Calculations	6	5	1	2	43	29	14	40	32
	Practical errors	7	12	0	0	50	71	0	0	30
Appeals										
Mathematics	Empirical	7	4	0	0	50	24	0	0	19
	Definitions	1	0	1	0	7	0	14	0	5
	Rules	13	16	7	5	93	94	100	100	97
Experience	Profession	0	0	0	0	0	0	0	0	0
	Everyday	2	2	3	1	14	12	43	20	22
Curriculum	Test/Exams	1	6	1	1	7	35	14	20	19

Table 2 : Quantitative Results per Lesson

The next table quantifies the task demands, problem solving and appeals made by Mr. T, taking into account all four lessons. It illuminates the overall practice. From the table below it is important to note that within an event, the categories do not necessarily occur exclusively. For example, when legitimating meaning for his learners, Mr. T could appeal to more than one category. The table indicates that there is a total of 43 events, inclusive of notions and sub-notions. The total occurrences signify the frequency that each of the categories could be identified. The percentage occurred simply denotes a percentage of the total occurrence out of the total of 43.

		Total Occurrence	**% Occurred**
Notions (including sub-notions)		43	
Task demands			
Memorization		0	0
Procedure without connections		42	98
Procedures with connections		0	0
Doing mathematics		2	5
Problem Solving			
Explaining		16	37
Questioning		1	2
Demonstrating		20	47
Representing		10	23
Restructuring		5	12
Defining		2	5
Encouraging		20	47
Working with student Ideas:	Mathematic Calculations	14	33
	Practical errors	19	44
Appeals			
Mathematics	Empirical	11	26
	Definitions	2	5
	Rules	41	95
Experience	Profession	0	0
	Everyday	8	19
Curriculum	Test/Exams	9	21

Table 3 : Composite Quantitative Results

The next step involved in this study would be to revisit and answer the critical questions that underpin the study in order to provide an analysis of the data. The critical questions are as follows:

1. What mathematical problem solving does this teacher do when teaching fractions in his Grade 7 class?
2. What knowledge resources (appeals) does he call on as he goes about this work?
3. What teaching dilemmas are reflected in this practice?
4. How might this practice, and its teaching dilemmas be explained?

4.6. Identifying the mathematical problem solving Mr. T confronts in his teaching

In this section I will discuss the mathematical problem solving or mathematical work done by Mr. T as he teaches the additions of fractions to his Grade 7 learners. Interestingly Pillay[13] (2006), who I have mentioned before, did not find these same observations. In his study of the mathematical work done by teachers, the kind of mathematical problem solving carried out by the teacher was seen as a function of pedagogy. He draws attention to a similar study done by Kazima and Adler (2006) on the mathematical knowledge for teaching probability in a South African classroom and explains that certain tasks/activities will elicit specific mathematical problem solving for the teacher. In Pillay's study he describes the teacher's pedagogy as 'traditional' and suggests that this 'limited' the problem solving that took place. Taking the study done by Pillay (2006) and the one done by Kazima and Adler (2006) into consideration, it is useful to think about what the teacher involved in my study does and in what way. I will look at whether there are any similarities or differences between the studies.

From Table 3 it can be observed that Mr. T is involved in the full range of mathematical problem solving identified in the literature. However, some are carried out at a greater

[13] A study investigating mathematics for teaching; the kind of mathematical problem solving a teacher does as he goes about teaching the topic of Functions.

degree compared to others. For example, it appears as though he spends more time encouraging his learners than he does posing mathematical questions and problems. My study, similar to Kazima and Adler (2006) shows that while defining, explaining, representing and questioning are present, they are not central to Mr. T's lessons. This is in contrast to the study done by Pillay (2006), where it was reported that these were central features of the teacher's lessons. An important difference to note between the two studies is the way learners were engaged. While the teacher in Pillay's study took on a more 'traditional' approach and did not use an extended or complex task to teach functions, Mr. T did. Thus, Mr. T had to deal with what the learners produced and engage their thinking. This was done largely at the level of mathematical rules and so correcting errors made by the learners. While the task was set up to be a doing mathematics task, Table 3 depicts that it declined almost immediately to a lower level task that involved procedures without connections. This certainly influenced the mathematical problem solving that Mr. T undertook during his lessons.

I will now take a closer look at the mathematical problem solving done by Mr. T and attempt to explain, using extracts from different lessons, the mathematical work of teaching that Mr. T confronts.

It is expedient to use some of the extracts previously discussed as these have already been described and contextualized.

4.6.1 Explaining and demonstrating

Table 3 reveals that the average for the categories **explaining and demonstrating** are 37% and 47% respectively. The following extracts will elucidate what the notion of explaining and demonstrating involve.

5:35

Mr. T: Now boys and girls, I've got two examples ($\frac{3}{4} + \frac{5}{8} =$ and $\frac{3}{4} + \frac{5}{15} =$) on the board like I said. I hope you can see this pen, (*mumbles under his breath about the pen*). If you can't see at the back boys and girls, you are welcome to move to the front. Right the first one boys and girls, I look at my denominator. I want to add,

you can see it's addition. I want to add $\frac{3}{4}$ and $\frac{5}{8}$. Okay, but I see my denominators is not

L: The same (*respond together*)

Mr. T: Not the same. What do you want me to do then?

L: You have to figure out the lowest common multiple.

Mr. T: Figure out the lowest common multiple without saying multiples of four equals four, eight, twelve, and sixteen. We going to leave that bit of the sum out now. We just going to… (*Inaudible*). There is a easy way to do that now. All you have to say is, can four go into itself (*refers to sum on the board*)

L: Yes (*respond together*)

Mr. T: Yes. Can four go into eight?

L: Yes (*respond together*)

Mr. T: Can eight go into itself?

L: Yes (*respond together*)

Mr. T: Can eight go into four?

L: No. Yes (*some answer yes, some no*)

Mr. T: So which one of those two will I use then as the lowest common denominator?

L: Four (*one learner responds*)

L: Four, eight (*respond together with different answers*).

Teacher ignores the incorrect answer and carries on.

Mr. T: Okay, equals (*draws line and writes eight as the denominator*). Okay boys and girls, what I'm saying to you is change (*walks to his desk to find a pen*). Uh change, boys and girls, to same de-no-minator (*circles the number 8 and writes ' change to same denominator' on the board next to the sum*). Look I'm writing over here because I want you to see it. Change that and it will be?

L: Eight (*respond together*).

Mr. T: That will be class?

L: Eight (*respond together*)

Mr. T: Guys do me a favour. Just speak up so that we can all hear you.

L: Yes sir (*respond together*)

Mr. T: Eight. So now I'm saying four goes into eight how many times?

L: Twice (*respond together*)

Mr. T: I say times two (*writes it on the board*). Eight goes in itself?

L: Once (*respond together*).

Mr. T: Times one. What I do at the bottom I must do at the?

L: Top (*respond together*)

Mr. T: Okay, times two (*writes it next to the numerator*). Sorry bout that *(makes a mistake)* and times one.

Leaner brings pens from the other teacher.

Mr. T: Right okay. Now we say four times two is eight, eight times one is eight. Is that clear? You understand that?

L: Yes (*respond together*).

Mr. T: Boys and girls you understand where I got that from?

L: Yes.

Mr. T: Now, three times two is?

L: Six (*respond together*)

Mr. T: Six (*repeats after learners*). Plus boys and girls?

L: Five (*respond together*)

Mr. T: Where did we get the five from?

L: Five times one is five (*respond together*)

Mr. T: Five time one equals (*repeats after learners*). Now?

L: Eleven (*respond together*)

Mr. T: Hang on, hang on before you shout out the answer. Eight, the denominator stays the same there boys and girls. Can you see that? (*Writes eight as the denominator*)

L: Yes sir (*respond together*)

Mr. T: Six plus five is?

L: Eleven (*respond together*)

Mr. T: Okay (*writes eleven as the numerator*). *Reprimands a learner*.

Mr. T: What do we have here boys and girls. We got a ?

L &Mr. T: Improper fraction (*respond together*)

Mr. T: I just want to write it over here so you can see. (*writes the words improper fractions next to the fraction*). Now what must I do with that? I must change it to a?

L: Mixed number (*respond together*)

Mr. T: Mixed number (*repeats after learners*). What will my answer be?

L: (*individual learner):* One whole sir and two over eight.

Mr. T: One whole, are you sure? And, Yes?

L: (*Shout out answer together)*

Mr. T: One whole and?

L: Three over eight

Mr. T: 1 whole and $\frac{3}{8}$ boys and girls. Now can I simplify that? (*learner gives teacher another pen*). Thank you very much young lady. Can I simplify that?

L: No response form the learners

Mr. T: Guys I'm asking you a question. Can I simplify that? We've done it before.

L: No sir (*respond together*)

Mr. T: So my final answer would be 1 whole and $\frac{3}{8}$ okay (writes it on the board).

That's one way of doing it. The first way is find the lowest common denominator and you work out your multiples okay. The second one is all I've to say is can that denominator go into that and can that denominator go into that (refers to sum on the board) and that will be my lowest common denominator. The third one, you ,look on the board boys and girls and you'll see three quarters plus five fifteens….. (*explains how to do the sum*)

(Lesson 3, time interval 5:35)

To teach the learners how to add the different unit fractions Mr. T engages in explaining to them how to find the lowest common denominator.

The extract reveals Mr. T explaining how to add different unit fractions using three different methods. Prior to this extract he recaps how to change a mixed number to an improper fraction and vice versa. This extract starts with Mr. T explaining that to find the lowest common denominator, the learners must find the lowest common multiples of the denominators of the fractions they are adding. The second method he explains that learners need to look at the denominators 4 and 8 and figure out whether they are divisible by 8. The third method, shows them how to simplify a fraction and then add it to another fraction by multiplying the denominators in order to find the lowest common denominators. He does not offer a full explanation of the first method since the learners have already completed a number of sums using it. The following shows the two sums he used to explain the two methods:

Method 2

$$\frac{3(x2)}{4} + \frac{5(x1)}{8}$$

$$= \frac{6+5}{8}$$

$$= \frac{11}{8}$$

$$= 1\frac{3}{8}$$

Method 3

$$\frac{3}{4} + \frac{5}{15}$$

$$= \frac{3(x3)}{4} + \frac{1(x4)}{3} \text{ (simplified)}$$

$$= \frac{9+4}{12}$$

$$= \frac{13}{12}$$

$$= 1\frac{1}{12}$$

Mr. T's explanations of the different procedures to follow when adding fractions with different denominators are all procedural in nature and linked to mathematical rules. This was largely a result of the decline of the task used**.** He does not offer to explain why or when certain procedures are used. Referring to the underlined text from the extract, it appears that Mr. T in his explanations, attempts to provide some mathematical rationale. However, he poses questions to the learners that illicit responses of how to do the sum rather than why they are following certain procedures. His questions are of a procedural nature in that the answers he requires follow a pattern. He meticulously writes down all the steps involved when following the procedures and explains clearly how they must be done.

4.6.2 Demonstrating

While explaining involved an attempt to provide some mathematical rational for adding fractions and verbalizing the procedures of the algorithms (Ma, 1999), demonstrating involved showing or displaying the steps of the computation or procedures and /or verbalizing procedures For example:

3:29

Mr. T: Okay, we are now going to mixed number, improper fractions. Whom of you can explain to me what an improper fraction is? What is an improper fraction? You told me a fraction is an equal part of a whole. What is an improper fraction?

L : An improper fraction is when the denominator is smaller than the numerator.

Mr. T: Numerator is smaller. Give me an example.

L: Like uh, thirteen over eleven.

Mr. T: $\frac{13}{11}$ (*writes it on the board*). So it depends on, somebody took my pens (*battles to write on board with his pens and mumbles under his breathe*). That's an improper fraction. Now I'd like to change, what is a mixed number? What is a mixed number then? Ja.

L: When there is a whole number and a fraction.

Mr. T: A whole number and a fraction. Good answer. In other words I can change that (*referring to fraction on board*) into a mixed number.

L: Yes sir (*respond together*)

Mr. T: What will my answer then be?

L: One whole

Mr. T: One whole (*repeats after learners*)

L : The fraction mustn't be (*teacher interrupts*)

Mr. T: No, no give me the answer to this. You telling me (*learners calls out answer*)

L: Two elevenths.

Mr. T: Two elevenths (repeats after learner). Okay. That's exactly what I'm looking for. So that over there (*writes 1* $\frac{2}{11}$ *on the board)* boys and girls is known as a mixed number, okay.

L: Mixed number (*learners repeat after the teacher*)

Mr. T: And you know that now from grade six and that is over there (*referring to* $\frac{13}{11}$ *and writes improper fraction next to it*) is an improper fraction.

L: Improper fraction (*they repeat after the teacher*)

Asks one of the learners to get him pens from another teacher because his are not working.

Mr. T: Okay can you boys and girls, improper fraction, mixed number (*refers to board*). So you understand that concept. How do I get from one to another. How do I test my answer? I just say 1 x 11 + 2 give me $\frac{13}{11}$ okay. Am I right?

L: Yes sir (respond together)

Mr. T: Is that what you were taught in Grade five and Grade six. Okay.

L: Yes sir.

(Lesson 3, time interval 3:29)

This extract reveals that Mr. T demonstrates to the learners how to change a mixed number to an improper fraction and vice versa. He merely asks the children to define a mixed number and improper fraction and demonstrates to them how to convert them. Mr. T does not engage in a discussion with regards to why it is necessary to convert mixed numbers to improper fractions or vice versa. Neither does he offer to explain the mathematical link (they have the same value, but are represented differently) between improper fractions and mixed numbers, or what they mean mathematically.

While the following extract reveals several things about the task, the mathematical work done and the appeals made by the teacher, I want to draw attention especially to the mathematical work of demonstrating exhibited by Mr. T. I have underlined the text in order to draw your attention to the specific area of the extract in question.

Here Mr. T explains to the learners what he is going to do on the board and what is required from the learners. The extract reveals what Mr. T's intentions were, i.e. to show (demonstrate) to the learners how to use two different methods to find the lowest common denominator. When Mr. T says that he is going to '**show'** the learners how to do the sum, this is what he literally does. When he refers to ' ...**the long road** …' and '**why I'm doing things that you'll be able to get the sum right…'**. He is referring to the procedures involved and following them step by step in order to get the correct answer. He encourages and ensures them to just follow the procedures and they will be able to do it. He makes no reference to explaining mathematically why the two different methods can be used, what makes them different, when they can be used etc. There are no mathematical explanations just a demonstration of 'how'.

01:38

Mr. T: Okay boys and girls, we took a fraction board. We cut it up. We stuck it back together again. You worked out a couple of sums and you proved that if I put certain fractions together, look at me I'm here. Um that it works out to one whole. Now we going to go a little bit further. I'm going to show you two ways of getting to the lowest common denominator without using multiples of what I'm working out. Short and sweet. Quickly you can do it, but step by step. Now like I said to the other classes and I said to you yesterday, it's like getting a play station game, right. I'm putting it into the play station and I start playing. It's a brand new one I just discovered off the shelf. I'm putting it on and I'm playing with it, but I'm battling because I've got to discover how to get to how to win the

> game or whatever the case may be. The same with maths okay. The same with this work. There is shorter methods of doing this, but at this stage I think if you take the long road and the long method like I'm going to **show** you and why I'm doing things that you'll be able to get the sum right. And then at the end you can say, okay I know a shorter method. I worked out the answer quicker. That's great. You can do that. But at the moment boys and girls let's just follow. There's two ways of doing things. I'm going to put it on the board. I'm going to **show** you. I'm going to ask you going to copy it down. I'm going to give you your worksheet and I am expecting you guys to do it like that okay. And look at your examples. Say, this is the example; I don't understand how sir got that. Call me and I'll come and help you again. But at this stage if you just listen to what I'm doing on the board, you can do it. Okay, and its easy sums that sir is giving you.
>
> (Lesson3, time interval, 01:38)

Demonstrating formed a large portion of the mathematical problem solving done by Mr. T and was influenced by the nature of the task.

4.6.3 Working with student ideas and encouraging learners

There exists a definite link between the mathematical work done by Mr. T regarding working with student ideas (mathematical calculations and practical errors) and encouraging learners. Encouraging, similar to demonstrating, is ranked as the highest (47%) problem solving Mr. T confronts, while working with student's ideas also makes up a substantial amount of the problem solving he engages in.

The following extract depicts the mathematical work of encouraging and working with student ideas.

Prior to this extract, while Mr. T was marking the learners' books, he discovered that many of them did not cut out the unit fractions accurately and so he wanted them to see their mistakes. Mr. T got different learners to come to the board and use his unit fractions to check whether their answers would add up to one whole.

The following is the example from the learner's book that Mr. is referring to in the extract:

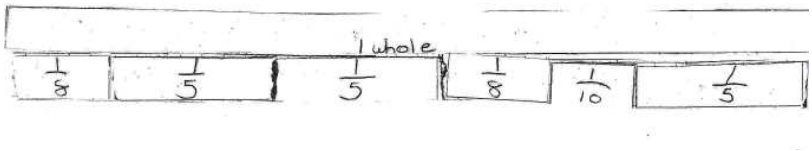


Fig 4.7: Extract from learner's book

37:55

Mr. T: Uh, I want you to look on the board. Please don't have your own conversations Grade 7's. I'm trying to, I'm just going to see whether we can solve a few problems and see where you guys, the area where you guys battled with. (*teacher sticks unit fractions on the board)*

Mr. T: (*addresses individual learner once done sticking the unit fractions*) And now look here. Okay, what do you think was your problem? Tell me.

L: *Response is inaudible*

Mr. T: It was too short. Okay. So you left out pieces. Guys I'm not going to do the sums physically. I'm just wanting you to come to the board to show you where you've gone wrong. I mean now, look at his now, thank you.

Learner goes back to his seat

Mr. T: And then we can see whether we can solve. If you have, he cut out his pieces now and he pasted his pieces back into his book. And this is all the way (*refers to unit fractions on the board*) am I right young man?

L: Yes sir

Mr. T: And but look at the gap. So that's one of the reasons why his sum did not work out. It's a simple, the start of his project already when he cut up his fraction wall and he stuck them back together and sort of mixing them up. He has already made mistakes. It's not a serious problem. Don't worry. You not going to be, uh, get into trouble for it. I just wanted to show you what silly mistakes we can make in order to prove or either we rush through the work.

Mr. T: (*refers to next learner*) Your problem would now be (*looks at book with learner*). Okay, let's put it together and see what happens.

Mr. T: (*addresses entire class*) Who else sitting here think they've got a problem. They in the same boat.

L: *Put up their hands*

Mr. T: Before you put up your hands. Who of you sitting here think you made the same problem as Paul[14]? Where your pieces, you cut your pieces out, stick them back into your book. It look to you like it's fitting but it's not. Just put your hands up and then you'll see. Put up your hands high.

L: Quite *a few learners put their hands up.*
Mr. T: Look at that. Can you see that? (*refers to gaps*) And you think it's a serious mistake or is it just a silly mistake from your part?

L: Silly mistake (*Respond together*)

Mr. T: Silly mistake. If I give you a fraction board again and I say to you go cut it out and put it back together again. You think you'd get all the answers right next time round.

L: Yes sir (*respond together*)

Mr. T: And you will be able to prove to me that if the denominators are all the same and you stick it back properly underneath the whole and your fraction wall, your answer will be one whole. Would you be able to do that?

L: Yes sir (*respond together*)

Mr. T: Are you sure?

L: Yes sir (*respond together*)

(Lesson 2, time interval 37:55)

When working with learners' errors, both practical and mathematical calculations, Mr. T often refers to the procedures that should be followed in order to obtain the correct answer. He then follows that up with a word of encouragement. For example, in the above extract (refer to underlined text), Mr. T identifies that a learner did not cut out the unit fractions accurately so his sum was not adding up to one whole. Upon identifying the error, Mr. T explains to him why his sum will not add up to one whole, i.e. because he did not follow the instructions/procedures carefully. Thereafter, Mr. T encouraged the learner, 'It's not a serious problem. Don't worry. You not going to be, uh, get into

[14] Pseudonym to protect the identity of the learner.

trouble for it'. While Mr. T does pay some attention to the mathematics and what was done incorrectly, he is more concerned about how the learners feel and finds the need to consistently encourage them.

In the extract below from lesson 1, Mr. T addresses the whole class. He identifies that the learners are struggling to find the lowest common denominator. He encourages them not to give up and ensures them that they will find the lowest common denominator. He also encourages them by reassuring them that they can do the mathematics. It is clear that when Mr. T deals with student thinking, he encourages them and moves them on emotionally, whether he also does this mathematically is in question.

35:49

Mr. T: (*addresses whole class while marking a learner's book*) Some of your combinations are brilliant. Just shows me that you can do maths. And it does work. And some of your combinations are right but you not finding the right lowest common denominator. Somewhere along the line you are slipping. So just keep on going back to that. Because I'm telling you, you will discover the right one. It's very interesting I must say.

(Lesson 1, time interval 35:49)

4.6.4 Representing, questioning, restructuring and defining

Very little mathematical problem solving is done with regards to representing, questioning, restructuring and defining by Mr. T. He initially represents the fractions empirically using unit fractions and later represents the fractions symbolically (using number). He constantly works between the empirical and symbolic in order to address learners' errors and misunderstandings and also in some instances attempts to restructure the task in order to promote understanding. His representations are limited to the unit fractions, symbols and what I describe as unsuitable explanations related to the learners' everyday lives. They are unsuitable in that they do not carry mathematical meaning**.** The questions posed by Mr. T are more encouraging than mathematical. For example:

Mr. T: Look at that. Can you see that? (*refers to gaps*) And you think it's a serious mistake or is it just a silly mistake from your part?

L: Silly mistake (*Respond together*)

Mr. T: Silly mistake. If I give you a fraction board again and I say to you go cut it out and put it back together again. You think you'd get all the answers right next time round.

L: Yes sir (*respond together*)

Mr. T: And you will be able to prove to me that if the denominators are all the same and you stick it back properly underneath the whole and your fraction wall, your answer will be one whole. Would you be able to do that?

L: Yes sir (*respond together*)

Mr. T: Are you sure?

L: Yes sir (*respond together*)

(Lesson 2, time interval 37:55)

4.7 The knowledge resources (appeals) and experiences that Mr. T calls on as he goes about this work

As initially discussed in Chapter Two, **mathematics** (empirical, definitions, rules), **experience** (professional, everyday) and **curriculum** (textbooks, examinations/tests) were identified as the categories of appeals. These knowledge domains were what Adler and Pillay (in press) found in their study, and these provide a starting point for me. I am now going to elaborate on my findings. These appeals were drawn on by Mr. T to legitimate meaning for his learners. Table 2 (Quantitative Results per Lesson) reveals that Mr. T either made appeals to a single category or to different combinations of the three categories in his attempt to fix meaning. Thus, not all categories may have been appealed to in each event. The next section clarifies the knowledge resources and experiences that Mr. T draws on as he engages in the different mathematical problem solving while teaching.

4.7.1 Mathematics

The statistics in Table 3 (Composite Quantitative Results) clearly illuminate that Mr. T often appealed to rules in mathematics in order to fix meaning for his learners. The appeals he makes can be associated and closely linked to the task used. While the task was set up as a higher-level cognitive demand task, it declined to a lower level procedures without connections task, resulting in Mr. T engaging in the mathematical work of demonstrating and explaining. Therefore, many of the appeals made by Mr. T were to mathematical rules. He also called on empirical observations from time to time to help learners make sense of the procedures and concepts. The following extract highlights the mathematical work he

engages in as well as the appeals he makes, particularly to mathematical rules. For the purpose of drawing your attention to the specific area in question, I have underlined the text.

03:23

Mr. T: (*interrupts learner*) What have we noticed already? That?

L: The lowest common denominator is twelve

Mr. T: (*repeats what learners have said*) That the lowest common denominator is?

L: Twelve

Mr. T: We've already picked that up. So, circle that boys and girls (*circles 12*). This is simple work Grade 7's. Ok**.** It's knowing your times tables. That's what lowest common denominator, lowest common multiple means**.** (*Writes the following on the board : LCD*) equals lowest common denominator. Sorry, I'm scratching on the board now. Right that's twelve (*writes: LCD = 12*). That (*draws line*). Twelve (*writes 12 as denominator*). Now we say can four go into twelve?

L: Yes

Mr. T: How many times?

L: Three times

Mr. T: Times 3. Times 3 there. (*Writes X3 next to denominators that are 4*). Can six go into twelve?

L: Yes.

Mr. T: How many times:

L: Twice

Mr. T: Times two (*writes X2 next to denominators which are 6*), times two, times two. And what we do at the bottom, we must do at the top *(writes x2 and X3 next to numerators respectively*). Guys it's step by step. Just follow. Three, times three, times two, times two. Okay. One times three is? (*Writes each step as he explains*)

L: (*respond together*) three

Mr. T: One times three is?

L: (*respond together*) three

Mr. T: One times two is?

L: (*respond together)* two

Mr. T: One times two is?

L: (*respond together*) two

Mr. T: One times two is?

L: (*respond together*) two

Mr. T: Equals twelve over twelve. If you add up three plus three equals?

L: Six

Mr. T: Plus two equals?

L: Eight

Mr. T: Plus two equals

L: Twelve

Mr. T: That is?

L: 1 whole

Mr. T: Do you see that.

L: Yes sir.

Mr. T: I proved that part (*points to unit fractions that shows problem*) of the sum by following, by doing it step by step by step. There's no short cuts guys. The minute you take a short cut you are going to battle. Right, you need to do that (*points to example showing multiple on the board)* and you need to do this (*points to working out of sum*) for me. Will you be able to do that?

(Lesson 1, time interval 03:23)

In this extract, in Mr. T's attempts to explain and demonstrate how to find the lowest common denominator, he appeals to the mathematical rules of knowing the times table. Mr. T explains to the learners that, '*It's knowing your times tables. That's what lowest common denominator, lowest common multiple means'*. The mathematical problem solving he engages in requires that he calls on this particular mathematical rule. He later refers to the representation of the sum in the form of unit fractions, to show the learners empirically that the sum must add up to one whole. So, Mr. T emphasises that following the correct procedures (based on the rule of finding the lowest common multiple and following each step) and having the correct unit fractions will ultimately produce the correct answer. It is also noted that Mr. T encourages the learners by stating that the work is simple and makes a personal and an emotional appeal to the learners to do the work for him. Again it is noted with interest that there is not an appeal of a mathematical kind.

4.7.2 Experience

When engaging in the procedural work, Mr. T appeals to mathematical rules. However, in his attempt to make sense of the conceptual, he appeals to the everyday lives of the learners and their experiences. Consider the following extract from lesson 1, I have underlined the text to assist in drawing attention to the pertinent aspects in the discussion that follows:

30:31

L: Can one of … (*inaudible*)

Mr. T: No, no. Remember we working, we want to discover whether it's one whole. We don't want to go into a mixed number. Okay, so. Can you see why, look at your fraction wall. Come I show you something. Look there (*points to learner's fraction wall in book*.). Look how you've done it. Can you see that? *(gaps between unit fractions*) Now look at mine on the board. Look here. Mine fits in properly here. Can you see that? And yours are not. The minute I do this and I (*takes* $\frac{1}{5}$ *and replaces it with existing* $\frac{1}{6}$) take the $\frac{1}{5}$ and I take this $\frac{1}{6}$ off here. I put that in here *(the unit fraction sticks out and does not make one whole)* . Look what's going to happen now. Can you see there? And it's now going to become more than a whole. Okay, and that's what we don't want. So now you got to go back to your fraction board and have to redo that bit so that you find out a piece that fits in here John[15]. Ok, you got to put that back in here. Now the wall looks. If you building a wall for a house. John, if you're building a wall. John, look at me. I'm building a wall at home. So now I'm gonna, this is my neighbour's yard (*uses unit fractions on board to illustrate*) so I'm gonna put my wall like that *(the unit fractions put together are greater than one whole)* . Never mind it's going over into my neighbour's yard. Do you think he is going to be happy with that?

L: No sir.

Mr. T: No, he's not. His gonna tell you take that away and put it back on. There's lots of examples I can use to say to you it's got to be equal.

(Lesson 1, time interval 30:31)

This extract illustrates two things. Firstly, Mr. T appeals to the empirical to show the learner that the sum must add up to one whole. He shows the learner that if the correct pieces are placed accurately (i.e. without any gaps between them), the sum must add up to one whole. He demonstrates that when an incorrect combination of unit fractions are used, the sum will not add up to one whole, but instead the answer will be a mixed number. Secondly, in Mr. T's attempt to explain conceptually the importance of the correct combination of unit fractions in order to get one whole, he appeals to the everyday experience of building a wall. Learners may comprehend and understand that you cannot build a wall that leads into your neighbour's yard, but what does this mean mathematically when adding unit fractions that add up to one whole? Mr. T's appeal to

[15] Pseudonym to protect the identity of the learner.

something visual and emotional can make an impact on the learners, but it is not exactly clear how this might carry a mathematical meaning that is sufficient to help solve the problem.

When dealing with procedural work Mr. T calls on mathematical rules and the empirical to legitimate meaning. When working with the conceptual and trying to make meaning of it, he calls on the everyday. It is this disparity that severs the mathematical connection between the procedural and the conceptual. As mentioned in the literature review, there exists an important connection between the procedural and conceptual understanding of fractions because of its complex nature and thus a good understanding of both will enable mathematically meaningful explanations and representations of concepts and procedures. The following extract illustrates another way in which Mr. T calls on the everyday to legitimate meaning for his learners:

27:18

Mr. T: Imagine boys and girls that you are playing a play station game. Ok, the first time you get it. How do you feel? That you are going to battle to play it. Am I right?

L: (*respond together*) Yes sir.

Mr. T: And then, you discover, and the more you discover to play the game, the easier it is becoming and the more fun it's becoming. That's exactly what you doing now. You are going to find, uh, what's that, short cuts in the game. In this game too, that you playing now. You are going to discover that if you add a $\frac{1}{3}$ plus and a $\frac{1}{3}$ and a $\frac{1}{6}$ and a $\frac{1}{6}$ and a $\frac{1}{5}$ and a $\frac{1}{5}$ together, that it might not work out….

(Lesson 1, time interval 27:18)

Mr. T wants to make learning mathematics fun so he relates it to the learners' everyday experience of playing a play station game. He likens the mastery of the skill of playing the game to that of following the procedures in order to complete the sum successfully and stresses that just like playing the game is fun, so is doing mathematics.

4.7.3 Curriculum

When Mr. T engages in the mathematical work of encouraging and/or working with learner ideas(refer to the chunking of lessons 1-4, Appendix E) , he sometimes appeals to the curriculum in the form of tests and examinations. He encourages them by saying that the mathematics is easy and they can do well in the exams if they follow the steps exactly like they did in their books. Thus again, moving them on emotionally, and not appealing to the experience of success. When marking their books, he frequently comments on the number of marks they will obtain for getting the sum correct and following the instructions and procedures. The following extract from lesson 4 helps shed some light on the appeals he makes regarding the curriculum.

27:04

Mr. T: That's why I keep on telling you maths is a study subject boys and girls. When I give you something to do at home, go over it like you study. Remember the stuff you were taught in class. Just keep on going over it. And now look at this, you sitting here thinking I could have done that, oh that was easy, this was easy…

(Lesson 4, time interval 27:04)

From the discussion regarding the problem solving and appeals Mr. T makes, it seems apparent that despite a potentially interesting and mathematically demanding task, he can sustain his practice without pushing mathematical thinking, either his own or his learners. This in turn, questions Ball and Bass's (2002) specialised mathematical problems teachers solve as they go about their work of teaching. In the final chapter I will discuss these problem solving categories and show how they have helped or hindered my research.

4.8 Summary

In summing up, Mr. T selects, sets up and implements a conceptual task to teach the addition of fractions. From the literature we are made aware that the teaching of fractions is difficult because of the tension between the conceptual and the procedural. When engaging with the conceptual representation, he does certain mathematical problem

solving. The problem solving centers mostly around the empirical, metaphorical and the examples he uses about the everyday. Mathematics for teaching in this practice can be described as predominantly demonstrating, encouraging, working with learner ideas and appeals to mathematics as a set of rules. As Mr. T goes about his work of mathematical problem solving, he faces certain dilemmas. These dilemmas are thrown up as a result of the problem solving and appeals he makes.

In the next chapter I will provide a discussion related to the teaching dilemmas that are reflected in Mr. T's practice and how this practice and its teaching dilemmas might be explained.

Chapter Five

Discussion

The discussion that follows is concerned with answering the last two critical questions, and they are:

- What teaching dilemmas are reflected in this practice?
- How might this practice, and its teaching dilemmas be explained?

5.1 What teaching dilemmas are reflected in this practice?

As discussed previously, dilemmas are thrown up in Mr. T's practice as a result of the mathematical problem solving and appeals he engages with. There are three main teaching dilemmas that are reflected in Mr. T's practice:

- Representing the content
- Competing goals (procedural versus conceptual)
- Student Thinking

It must be noted that these were anticipated dilemmas and are not necessarily reflective of what the teacher experienced as dilemmas. They are rather what was apparent to me, the observer, and I refer to these as what he faced. I will discuss these dilemmas from the perspective of the teacher as well as from the literature and what I have observed. I hope that this will help me reflect if the teacher is to move his practice on in ways that align with the new curriculum goals, what he will need to know and be able to do, mathematically as well as pedagogically (or what MfT will he need) to acquire and where this might be possible for him.

5.1.1 Representing the content (Cognitive demand vs. Encouragement)

In order for Mr. T to present the topic of fractions, he had to represent it in some way. In this case, it came as a task. The task was potentially mathematically demanding. Its

representation, however was predominantly procedural. The task was open and unstructured, i.e. the learners labeled the unit fractions, cut them out, stuck them together randomly so as to get one whole. With this came the following problems: the learners made both practical and mathematical errors. Some learners did not cut the unit fractions out accurately or they left gaps between the unit fractions resulting in their answers not adding up to one whole. Others did not use a variety of unit fractions and this defeated the purpose of finding the lowest common denominator. The mathematical errors occurred when they were unable to find the lowest common denominators because of the combinations they chose. Leaving the task open ended and unstructured, with few or no restrictions resulted ultimately into the teacher telling the learners to "follow these steps". This in turn led to a decline into a lower level cognitive demand task and practical and mathematical problems. For example the problem mentioned earlier, where the visual overrode the mathematical (refer to Fig: 4.5: Extract from Tom's book).

Neither Mr. T nor the learner were able to tell that the sum would not add up to one whole because the unit fractions appeared to fit together and make a whole. It was difficult to spot the error visually and this caused a set of problems for both the teacher and the learner. Perhaps the teacher could have excluded certain unit fractions e.g. ninths, placed restrictions on what unit fractions were used or put up some of the combinations for the learners to practice the addition of fractions. This brings us to the fact that without direction from the teacher it could lead to a whole lot of misconceptions. Mr. T found himself engaging in the mathematical work of encouraging his learners, and appealing to mathematical rules and experience in order to help them arrive at the correct answer and feel secure about their ability to do mathematics. Ensuring that his learners felt as though they could do mathematics and that mathematics is a 'fun' subject was of paramount importance to Mr. T. This is indicated in the third interview carried out with him:

> Mr. T: ……..I wanted them to discover that if you take that fraction board and that wall and you cut it apart and you put the pieces together again, you can find out if different pieces make one whole. Okay, and that you can actually play, make a game of it and make maths fun. Um, and if they discover that making has got to do with discovery. If they can discover um, that you just take a plain simple thing like that fraction board and

> make it fun and put it together again and work out different sums and see where um, um, how important the denominator is in that whole…(mumbles something that is inaudible). I can't think now. Basically it's just for them to discover. You know to see that maths can be a fun subject and anything is possible in maths. If you really try you can do it……
>
> (Interview 3)

While Mr. T wanted his learners to be able to 'discover' that different unit fractions add up to one whole and that in order to do this successfully they must know how to find the lowest common denominator, he also wanted to make sure that the mathematics was fun for his learners.

If the task was closed or more structured with certain restriction (for example, learners were only allowed to use each unit fraction once only, or only certain combinations of unit fractions were allowed), it might have maintained its cognitive demand. This would have urged them to look more closely at the visual organization of the unit fractions and allowed them to think and generate mathematical ideas of when to use a common denominator. It may not have eliminated all the practical errors, but there is a possibility that it would have allowed the teacher to engage more in the mathematical work of explaining, working with the learners' ideas and defining at a conceptual level, instead of demonstrating, encouraging learners and appealing to mathematical rules in order to correct and address practical and mathematical errors..

Ma (1999, p.83) states that '…in order to have a pedagogically powerful representation for a topic, a teacher should first have a comprehensive understanding of it'. This entails being able to determine how to use tasks appropriately, foreseeing possible difficulties, responses and errors from the learners. By doing this the teacher could make decisions regarding the best way to represent the mathematics to them. While Mr. T wanted learners to engage with the mathematics, he also wanted them to see mathematics as fun and to encourage them. In engaging with the mathematical problem solving of restructuring and scaling the task so that the learners could understand the mathematics and have fun doing it, a dilemma is reflected. How does he do this simultaneously?

How does he maintain the cognitive demand of the task and still provide opportunities for learners to feel secure about their mathematical abilities and have fun when doing mathematics? If he opts for tasks that are more challenging in nature, does he risk the chance of working with students who become anxious because of the uncertainty associated with the tasks (Stein et al, 2000), or does he use tasks that will encourage and make mathematics 'fun' and risk having learners' who possess a superficial understanding of the concept of fractions? Mr. T does not articulate this as a dilemma, but he emphasizes the need for his learners to be happy, 'They must have fun when doing mathematics.' It is because of this that the dilemma is thrown up. I have named this dilemma **cognitive demands vs. encouragement** and will discuss later in this chapter why this dilemma may be reflected in Mr. T's practice.

5.1.2 Competing goals (procedural vs. conceptual)

The literature unequivocally states that the teaching of fractions is difficult because of its complex nature. Being able to teach it successfully requires a comprehensive understanding of the nature and different subconstructs of fractions as well as a sound pedagogical knowledge. This involves teaching fractions both procedurally and conceptually and this is not an easy task for teachers. Studies have shown that teachers often resort to choosing tasks that require procedural applications or they lower task demands and argue this as a function of their own limited understanding of fractions.

As discussed earlier, the task chosen by Mr. T was potentially interesting and mathematically demanding. However, in his representation of the task he taught procedurally. Teaching procedurally involved Mr. T largely engaging in the mathematical work of demonstrating. When doing this, he appealed mainly to the empirical and mathematical rules. When attempting to explain conceptually, Mr. T appealed to the experience of the learners' everyday lives. Ma (1999, p. 82) explains,

> '…the "real world" cannot produce mathematical content by itself. Without a solid knowledge of what to represent, no matter how rich one's knowledge of students' lives,

> no matter how much one is motivated to connect mathematics with students' lives, one still cannot produce a conceptually correct representation'

Evidence (refer to previously mentioned extracts) from my research shows that what is mentioned above is true of Mr. T. Mr. T, in his interviews, explains that he wants mathematics to be fun and the learners to enjoy it. He attempts to make a connection between their everyday lives and the mathematics in order to make this possible. However, in his attempt to explain that if the incorrect unit fractions are placed together, it would not add up to one whole is not helpful in connecting the mathematics to the everyday. His appeals made to the everyday lives of learners (building a wall and playing a play station game) did not produce a conceptually correct representation since it was unclear how this might carry a mathematical meaning that is sufficient to help solve the problem experienced by the learners.

As discussed earlier, there lies a gap between the procedural and the conceptual teaching of fractions in Mr. T's classroom. Mr. T's mathematical work was reduced to demonstrating since the task declined to procedures without connections. This threw up a dilemma for the teacher in that the focus was no longer on teaching both procedures and concepts, but rather on only teaching procedures. Mr. T does not recognize this dilemma as procedural vs. conceptual. Instead, for him, the dilemma is one of time. The following extract from interview 3 done with Mr. T illuminates his concerns with regards to the amount of time he has to teach.

> Mr. T: Um, I found that there was a problem and I think that, uh, I don't get enough time to, like I said, I just don't get enough time to spend with the children that really, really need my help. You got to get through the work. You got to rush the kids. And by looking at the videos I would have loved to spend more time with a lot of kids because when you were filming it we were looking at some of the kids and, the camera was on them, I could see they were desperately crying saying 'please sir we need more time with this ' but we just didn't have the time. The problem, um, um, maybe they should, I don't know. I don't know how to solve the problem. I just think that the kids are not getting enough um, I can't give them enough to make them understand, um how easy it is to do the work. You see for me it's easy. I've done it a million times but it's because I'm doing it over and over it's easy. Just imagine what we'll be able to do if we could teach them over and over the same thing and eventually they will pick up and the difficult things will just fall into place. Because it's the easy things

> that they must understand before they can go to the next level. I can see now with the work I'm doing now. Um, and we doing to the power of square roots. If I had more time they would understand it like this (clicks his finger). But there isn't.
>
> (Interview 3)

From this extract we see that time is a proxy for the problem Mr. T faces. He is pressured by the curriculum, tests and examinations, so time is a major concern for him. He articulates clearly that he does not know what to do with regards to this problem. If he had more time he sounds convinced that the learners would achieve more. Teaching for conceptual understanding requires time. Learners have to make sense of the concepts and engage with the mathematics at a deeper level. Mr. T clearly feels that he does not have this time at his disposal and therefore sees it as a problem. He has to move on to other topics whether his learners understand the mathematics or not.

The literature states that in order for fractions to be meaningful for learners they must be exposed to both the procedural and the conceptual work surrounding it. So, if Mr. T wants his learners to be both procedurally fluent and possess a conceptual understanding of fractions, as suggested by the literature, how does he do this? How does he ensure that since the procedural is "easier" and less time consuming to teach, it does not dominate, resulting in the conceptual being ignored? How does he avoid using representations that force him to teach procedurally? This is a dilemma for him. Does he teach algorithms to perform operations on fractions in order to achieve correct answers or does he teach fractions, using more complex tasks, so that his learners are able to develop a deeper understanding of the nature of fractions?

5.1.3 Working with learner ideas (Scientific vs. Everyday and Conceptual vs. Procedural)

The task that Mr. T used, was intended to connect the unit fractions to the symbolic representation of fractions (e.g. $\frac{1}{8}$), and to connect the symbolic operation of addition with the process of physically finding unit fractions that add up to one whole. The

learners had to think about how they would go about the activity i.e. what unit fractions go together to get one whole, how they would add the different unit fractions to get to one whole etc. The way the task was intended meant that they could not apply the procedures mindlessly. However, Mr. T modeled/demonstrated 'how the learners should do it', what procedures to follow etc. This resulted in learners being unable to make connections to the conceptual meaning associated with the task (Stein et al, 2000). They focused on following the procedures. Mr. T's attempt to bring the learners back to focusing on the conceptual part of the task, as well as following the procedures correctly, threw up a dilemma for him.

Mr. T was faced with working with his learners ideas on two levels. The first level involved identifying practical errors that the learners made, while the second focused on the mathematical calculation errors the learners performed.

5.1.4 Identifying practical errors (Scientific concepts vs. Everyday concepts)

The learners did not follow the instructions of the task carefully, they cut out the unit fractions incorrectly i.e. they were too short and/or they stuck them down leaving gaps between them or the combinations of the unit fractions were greater than 1 whole. This created what I have called practical errors. Because the combinations of unit fractions were incorrect and/or there were gaps, the sums did not add up to one whole.
In the first interview with Mr. T, he offers an explanation of how he planned to deal with these practical errors:

Mr. T : I have, funnily enough I already experienced my first problem today with another class with exactly the same thing. They are putting the fraction board pieces together and it is not equaling a whole. They haven't, it's a simple thing, you can see it, because they a $\frac{1}{16}$ $+\frac{1}{8}$ $+\frac{1}{4}$ $+\frac{1}{4}$ $+\frac{1}{2}$ + whatever will equal one whole, I've experienced that already and **the only way to now solve the problem is to sit with them and say here's another fraction board, look at it, that's equal to that, that's equal to that, that's why it's called equivalent fractions.** I've had a few of them; it's not a lot of them. It's a little thing like equivalent, equal fractions that by just cutting out the things wrong they find their whole is like this and their

two halves are shorter. Their half and half is short because they cut it out wrong. Then they are already puzzled because now it doesn't make one whole and they are looking at it and they haven't even started sums yet. It's fine to then say to them, OK guys now do it again. Just look at it carefully, just put your pieces together, put your puzzle, I always say to them it's like a puzzle you are building now, if one of the pieces of the puzzle is gone, the puzzle don't look right. When you finished putting your pieces together, you can actually put a frame around it, and it's got to fit perfect into that frame, if there's a gap in the frame it means one of your puzzle pieces are wrong, one of your section pieces are wrong

(Interview 1)

From the explanation it is evident that Mr. T calls on the empirical (see bold print text) and the everyday experiences (see underlined text) of his learners to address the errors.

I now offer an example of how Mr. T addressed the practical errors.

The following is an example of a learner who experienced a practical error:

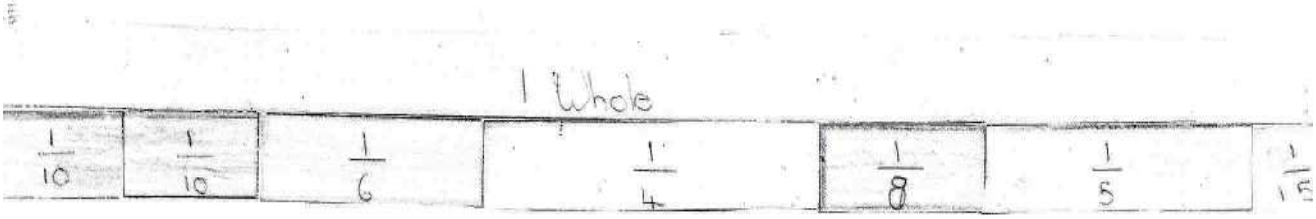


Fig 5.1: Extract from learner's book

In Mr. T's effort to correct the learner, he offers the following explanation:

Mr. T: If you building a wall for a house. John, if you're building a wall. John, look at me. I'm building a wall at home. So now I'm going to, this is my neighbour's yard (uses unit fractions on board to illustrate)
So I'm going to put my wall like that (the unit fractions put together are greater than one whole, similar to illustration above). Never mind it's going over into my neighbour's yard. Do you think he is going to be happy with that?

(Lesson 1, time interval 30:31)

Here Mr. T is trying to show the learner that the unit fractions he used will not add up to one whole. He represents the learner's symbolic representation ($\frac{1}{10}+\frac{1}{10}+\frac{1}{6}+\frac{1}{4}+\frac{1}{8}+\frac{1}{5}+\frac{1}{15}=$) by using the empirical (unit fractions) to show him that the sum will be greater

than one. He attempts to restructure the task by moving the learner from the procedural work of completing the algorithm, to visually understanding the problem. In his attempt to explain conceptually why the unit fractions will not add up to one, he appeals to the everyday experience of building a wall. Mr. T tries to make a connection between the procedural work done by the learner and the conceptual meaning associated with the task. He is unsuccessful in doing this because his representation of the conceptual (appeal to the everyday) does not lend itself to helping the learner understand the mathematical concept.

The following extract is another example of Mr. T's attempt to work with a learner who has made a practical error:

> Mr. T: I gave you a grid which has already been done for you and all you had to fill in was the missing link and then you had to cut it up and stick it back together using different fractions. And now we discovering it's not that easy. In the first place you have to know your times tables, okay. You have to know that it's got to fit, the pieces have got to fit in properly before the sum works out. It's like a puzzle as well. Ross[16], if one of the pieces of your puzzle is gone, is your puzzle working out?
>
> (Lesson 1, time interval 27:18)

He appeals to the mathematical rule of knowing the times table in order to find the lowest common denominator. He then explains that the unit fractions are just like a puzzle, if one of the puzzle pieces is missing, the puzzle will be incomplete. Here is another instance of where Mr. T uses the learner's everyday experience of building a puzzle in order to legitimate meaning and does not provide a mathematical explanation for why the unit fractions will not add up to one whole if they are not the correct combination or stuck down incorrectly with gaps in-between them.

The next extract reveals Mr. T's attempt to explain mathematically why the sum will not work out if the unit fractions are not cut out equally:

> Mr. T: Fractions are equal parts of a whole, okay. The minute you don't cut your pieces equally boys and girls, it won't work, okay because, what's the word I'm looking for? Um, never mind, I can't think of it now but I'm

[16] Pseudonym to protect the identity of the learner.

> going to do, these pieces on the floor and the table are equal. So what you are going to do now is, you going to come and sit here and just take your sum that you are battling with and you are going to put the pieces together just to see whether you've done it correctly okay. And if you haven't then it's not going to work out. It's going to work out to more than a whole and that's what we'll do later. Okay. Thank you Grade seven. We'll carry on with this when you come to me tomorrow.
>
> (Lesson 1, time interval 45:14)

Mr. T tries to explain mathematically why the unit fractions must be cut out correctly so that the sum can work out to one whole. He experiences difficulty doing this and so abandons the idea and refers to the empirical to help the learners make sense of their practical errors.

In other attempts to correct learners' practical errors, Mr. T appealed to the empirical in order to explain why the sums will not add up to one whole. He does not explain mathematically why the symbolic representation would be different from the visual representation.

5.1.5 Mathematical calculation errors (Conceptual vs. Procedural)

In his attempt to correct the mathematical calculation errors made by the learners (these included not being able to find the LCD, not choosing the correct multiple as the LCD, adding incorrectly, using the incorrect LCD), Mr. T sometimes attempted to work with the empirical in order to show the learners where they made a mistake and to bridge the learners procedural and conceptual understanding. This threw up dilemmas for him and the following extract highlights his plight in linking procedural and conceptual knowledge.

39:50

L: (*Tom comes to the front of the classroom and sits on the floor. Hands his book to the teacher*)

Mr. T: Put those pieces back on the board. Just leave the halves there. Put the other pieces back on the table for me please.

L: (*puts pieces on the table. Leavers two halves on the floor*)

Mr. T: Take the one you battling with. Put it together.

L: (p*uts strips together*)

40:48
Announcements over intercom

Leaner needs certain strips but not available. Teacher continues to work with him but intercom is on and very difficult to hear conversation. Teacher tries using fraction pieces form a pie chart.

42: 41

Mr. T: They don't have twenties, ah. My plan is not working. $\frac{3}{20}$ **will be how many tenths ? Think carefully.**

L: *No response.*

Mr. T: **If I had to replace that (*refers to the* $\frac{1}{20}+\frac{1}{20}+\frac{1}{20}$ *unit fractions*) with tenths what would it be?**

L: *No response*

Mr. T: **Those** $\frac{3}{20}$**. If I, take a** $\frac{1}{10}$ **here (*gives learner a tenth).***

L: *Places* $\frac{1}{10}$ *in position.*

Mr. T: **We've got, ok it won't work. Take it off. So we've got to find an alternative for that.** What I though was, if we have ninths, just put that on there (*pie graph*). I'll see something now. What else do you need?

L: One eighth

Mr. T: One eighth, uh, sometimes they don't give you all the puzzles that you need so they can't work. Ag, it frustrates me! I can't do this one. Oh eighths, here's an eight. You only need one?

L: Yes sir.

Mr. T: You see what sir was trying to do now is. I thought, I don't think a ninth

will fit in there. And I don't have the? Ok, here you are (*hands learner pieces*). Put it in there. Let's see what you going to be short of.

L: *Learner takes pieces*

Mr. T: Put it on here. Put it on this. Make believe this is going to be your whole (*pie chart*)

43:46
Bell rings

Mr. T: Okay, now put it on there. Let's fill the puzzle. Okay so you still need, what else do you need? Two fifths. No you've got fifths.

L: Thirds

Mr. T: Thirds, you need a third. This is most frustrating. **But I actually thought that we could use this. Okay, mmm, okay so we can't use it. We can't use it on this side either. So we need a ninth there. It doesn't look like your sum is going to work out now can you see, because already battling um, to put the pieces together?** Okay, so it's not the end of the world. Um, you did it over here (*referring to work in learner's book*). What did you find? Ah Grade 7 don't talk please. Um, okay, where, where, which one was it? Which one was it?

L: Points to sum (*unit fractions*)

Mr. T: A third, a fifth a fifth. It seems to fit in there. Okay, I do believe that if we had a ninth over there it would work out. Okay, so just go and try, your lowest common denominator what did you find?

L: I couldn't find it. Had to carry on?

Mr. T: Okay but you must carry on. That's why you didn't find it. Super, okay we know where your problem was. Okay.

L: Yes sir

(Lesson 1, time interval 39:50)

Firstly Mr. T tries to represent the learner's symbolic representation empirically by using the unit fractions. It appears that in doing this, he hopes the learner gains an understanding of why his sum must work out to one whole. He restructures the task in that he moves the learner from merely working with procedures, to engaging with the concept of whether the unit fraction chosen will add up to one whole and what other fractions (equivalent fractions) could be used. He is unable to do this because he does

not have the appropriate unit fractions. He then attempts to use a different form of representation, still using unit fractions, except these are now in the form of a pie chart. Once again he runs into trouble because he does not have the correct unit fractions to represent the sum. Eventually, Mr. T refers to the learner's book and uses the unit fractions in the learner's book to point out that the sum must add up to one whole. An interesting issue appears here which does not even surface for the teacher. To the human eye it appears as though the combination of unit fractions used by the learner will add up to one whole, but as discussed previously, in Chapter Four, it does not. The teacher suggests that if they had a ninth (see bold print) the sum would work. However, a ninth is too small and so the sum does not add up to one whole. As mentioned, this is very difficult to notice with the human eye in the learner's book and perhaps if the teacher had the correct unit fractions they would have been able to identify this error. The teacher is unaware of the misconception and does not mediate between the practical error and the mathematical calculation. If the learner completed the sum and saw that it did not add up to one whole, it could have thrown up a different dilemma of the conceptual versus the procedural for the teacher. The teacher resorts back to the procedural when he urges and encourages the learner to go back and find the lowest common denominator and follow the rules. This will help him find the answer and it must work out to one whole because the visual representation in the learner's book appears to add up to one whole. Engaging with the mathematical work of restructuring and representing, threw up a dilemma for Mr. T of the conceptual versus the procedural knowledge.

In an attempt to understand and explain the mathematical work done by Mr. T when working with this particular learner, I posed the following question during our third and final interview, and got the following response:

SG: Do you think that using the unit fractions aided the learner's understanding of the concept when you were teaching?

Mr. T: Yes, I think I did as you so rightfully noticed. I think the problem is that I should have had, like I said early on, if I only had the right material. The right hands on material, those kids would have understood it perfectly. But he knew what he was

doing but he couldn't figure it, he couldn't get the right piece. One of the pieces were missing and because I just didn't have it and I think if in that if I can get more equipment like that with it standing all over my classroom and kids can actually go to it and say oh lets see if we can solve the problem. They will solve the problem. Even a computer, if I had a computer in my class.

(interview 3)

Not being satisfied with this explanation, and after much probing for answers that would give me an understanding of Mr. T's mathematical knowledge regarding fractions, I posed the following question:

SG: Okay, what I was thinking was that because the unit fractions were inadequate in representing the concept of finding the lowest common denominator- we didn't have enough. This might have impacted on the possibilities for learners to understand. What do you think?

Mr. T: I think the school must get more maths equipment. More of those type of things that kids can. You know what, I've learned that if kids, if they can be hands on to a lot of things, like in science, if they can fiddle around with things it can make them understand things better. It's anything in life, if you do it, if you do it yourself and you can build on those things, you understand things. It's like a puzzle. If I put a puzzle together and I know how to do it or this video games the kids are playing. Once they understand the basic things where they actually got their hands on the things, on the equipment they understand things better. That's what I think.

(Interview 3)

From the interview and discussion, I concluded that Mr. T continuously went back to stating that the 'equipment' (referring to the unit fractions) he used would have worked better if he had the correct and different unit fractions. He did not refer to the mathematics in order to justify his statements, but rather felt strongly about having sufficient mathematic equipment so that the children could experience what it is like to work with the 'equipment' in order to understand. Mr. T. did not recognize that the dilemma was the procedural versus the conceptual knowledge, since for him not having the correct equipment led to the problem.

From the discussion on both the practical and mathematical calculation errors, it appears as though Mr. T is faced with the dilemma of mediating between the conceptual and the procedural as well as mediating between scientific concepts and everyday concepts. The dilemma of mediating between the procedural and the conceptual when working with learner ideas is not a new dilemma, but is rather connected to what I have referred to as competing goals and is an extension of the procedural versus the conceptual. I assumed that as Mr. T worked with learner ideas different dilemmas would be thrown up. Evidently this is not so and the dilemma of competing goals manifested itself as practical and mathematical problems.

Not being able to mediate between scientific concepts and everyday concepts was a real dilemma for Mr. T. According to Vygotsky, when working with topics like fractions, it is possible to move from the abstract to the concrete. In other words, we can teach from the scientific to the everyday. We do not always teach from the concrete to the abstract. Moving from the abstract to the concrete is also a way of filling out and understanding a concept. This is what Mr. T attempts to do. However, the concrete (everyday) is not the 'right kind' of concrete. Choosing appropriate everyday concepts to relate to the scientific concepts or moving between the concrete and the abstract becomes a real difficulty and throws up dilemmas of mediation and the relationship between scientific concepts and everyday concepts. What has been discussed thus far regarding the mediation between scientific concepts and everyday concepts goes beyond the scope of this research. I have not used any of the above mentioned in this research project and am only aware of it from my own studies. Had I known this at the beginning of the study it might have been something interesting to explore. I will return to this later in the chapter when I engage in a discussion on the emphasis and relevance placed by the new curriculum on mediating between scientific concepts and everyday concepts and how it challenges teachers to bring in the everyday when teaching.

Mr. T faced the dilemma of mediating between the conceptual and the procedural knowledge and mediating between scientific concepts and everyday concepts because of how the task was represented. Perhaps if the task had been less open, more structured

and not had so much emphasis placed on it being fun, Mr. T would have had less issues to deal with. Once again this is not a new dilemma, but rather an extension of the dilemma of representation (cognitive demand vs. encouragement). The dilemmas faced when working with learner ideas was a result of how the task was represented and/or competing goals.

When Mr. T worked with learner ideas, he appealed to the everyday, the empirical and mathematical rules. It must be noted that during lessons 1 and 2, when working with learner ideas, he made appeals to the everyday, empirical and mathematical rules (refer to chunking of lessons 1-4, Appendix F). During lessons 3 and 4, while he made some appeals to the everyday, he appealed mostly to mathematical rules and made no appeals to the empirical. A possible explanation for this is the total decline of the task from a doing mathematics task to a task involving procedures without connections. While lessons 1 and 2 required intensive work by Mr. T in order to correct both practical and mathematical errors made by learners when completing the task, lessons 3 and 4 involved Mr. T working mostly procedurally to correct mathematical errors.

I will now engage in a discussion on how this practice and its teaching dilemmas may be explained.

5.2 How might this practice, and its teaching dilemmas be explained?

Teaching dilemmas make explicit the tensions inherent in teaching (Adler, 2001). With regards to Mr. T's practice, the teaching dilemmas reflected in his practice are illuminated by the problem –solving or mathematical work he does and the appeals he makes. They also aid in providing a description for what constitutes mathematics for teaching (MfT) in this practice.

Ball, Bass and Hill (2004) report that knowing mathematics is not sufficient to teach successfully. Knowing mathematics for teaching is fundamental to both teaching and learning mathematics. Knowing mathematics for teaching involves teachers having a

comprehensive understanding and pedagogy of the topics they teach. Teachers are obliged to 'unpack' or 'decompress' the mathematics they know so that their learners are able to make sense of it. Fractions especially is a difficult and complicated topic to engage with. Its multifaceted nature makes it difficult to understand and requires a special kind of knowledge to 'unpack' and 'decompress' so that learners can understand and make sense of it both procedurally and conceptually.

Mr. T, in his attempt to 'unpack' the mathematical knowledge of fractions he possesses, reflects certain dilemmas. These dilemmas may have occurred for several reasons and I will try to explain why.

There runs a common thread through the dilemmas in that they all revolve around the conceptual and procedural work done by the teacher. This is not a new occurrence and confirms what is said in the literature about the conceptual and procedural work around the teaching of fractions.

The dilemmas may be because of the influence of the new curriculum (C2005) and/or Mr. T's subject as well as pedagogical knowledge of the topic of fractions. Unfortunately, during our interviews I did not address the impact of the new curriculum and how its implementation has influenced his teaching. In hindsight I realise my error and therefore can only speculate that the new curriculum has had much influence on the task he chose, the mathematical work/problem solving he does and the appeals he makes.

Curriculum 2005 states that the purpose of teaching and learning mathematics must aim to develop 'the necessary confidenc**e** and competence to deal with any mathematical situation without being hindered by a fear of Mathematic' as well as ' a love for Mathematics' (RNCS, 2001, p. 91).

From observing Mr. T's lessons and engaging with him during the interviews, it is clear that he wants his learners to be confident when they are in his classroom learning

mathematics and he wants them to love Mathematics. The following extract explains how he feels about teaching mathematics:

Mr. T: **You know to see that maths can be a fun subject and anything is possible in maths. If you really try you can do it.** You know, and by taking that fraction board, I think that the kids have a, uh, uh, what's the word I'm looking for? If we had the right equipment, we could have fraction boards like that lying around and children that can't imagine that, that you can put things together and make it and make something of it. **So that the kids can just see it's not difficult to do maths. It's a simple subject. The thing is we've got to think anything is possible**. I don't know whether, it's a difficult thing to ask because I discovered **that I do that it makes the children excited, it makes the children want to be at maths. I don't know whether when you were here saw that a lot of kids got excited about it.**

(Interview 3)

This extract shows us that Mr. T would really love for his learners to see mathematics as being fun and at the same time build up their confidence so that they can do mathematics. This in turn will foster a love for mathematics. From the analysis of the mathematical work he does, it reveals that he constantly encourages the learners. This highlights the fact that he sees the mathematical work of encouraging as a vital aspect of teaching and learning mathematics. While this may be in accordance with the curriculum, developing a deeper understanding of fractions is also important. Ma (1999) states that while encouraging learners is necessary, it is not sufficient to promote significant mathematical learning. As teachers encourage learners, they must also be able to support mathematical learning and inquiry. Stein et al (2000) suggest that in order for teachers to help their learners obtain and/or maintain a high level order of thinking, tasks must be structured to elicit this. As explained in the analysis, Mr. T intended to use the task to teach for conceptual understanding. However, in his representation of the content, he reduced the task to procedural work and shifted the focus to encouraging learners to achieve correct answers to build up their confidence when doing mathematics. This led to him appealing to mathematical rules and the everyday to legitimate meaning for his learners. The tension arose here and caused a dilemma of cognitive demand versus encouragement.

Literature regarding the teaching of fractions is laden with examples of teachers who are able to perform calculations on the topic of fractions, but are unable to offer mathematically sound explanations for their calculations. Their understanding of the topic influences how they work with their learners' ideas and how they move them on mathematically. Mr. T, in his attempt to move his learners on mathematically experienced problems. Concurring with findings recorded in Ma's (2000) study, although Mr. T was concerned with teaching for conceptual understanding, teaching fractions for him meant following a set of procedures step-by step to arrive at answers.

5.3 Summary

In this chapter I have focused on questions 3 and 4 i.e. what teaching dilemmas are reflected in this practice and how might this practice, and its teaching dilemmas be explained? Representing the content in the form of a task revealed that mathematics for teaching in Mr. T's classroom constitutes predominately the mathematical work/problem solving of demonstrating, encouraging and working with learners ideas. He legitimated meaning for his learners by appealing to mathematics (empirical, mathematical rules), experience (everyday) and the curriculum (test, exams). Engaging with these mathematical problem solving and appeals reflected dilemmas of representing the content, competing goals and working with learner ideas in this practice. I also explain that the dilemmas faced by Mr. T could be a result of
the demands placed on teachers by the new curriculum.

In the concluding chapter I reflect back on the study, its theoretical underpinnings and its results.

Chapter six

Conclusion

6.1 Introduction

This study was concerned with investigating what and how mathematics (for teaching) is constituted in classroom practice?
Why is this so? What are the implications of this for the improvement of teaching and learning mathematics?

The questions that guided this study were:

1. What mathematical problem solving does this teacher do when teaching fractions in his Grade 7 class?
2. What knowledge resources (appeals) does he call on as he goes about this work?
3. What teaching dilemmas are reflected in this practice?
4. How might this practice, and its teaching dilemmas be explained

The use of an extended task was critical in this study as it aided in unveiling the mathematical work/problem-solving the teacher does, as well as the appeals he makes. It also helped reflect the dilemmas in this practice.

The topic of fractions was chosen since it is problematic for both teachers and learners and despite the substantial amount of research done on this topic, difficulties associated with the teaching and learning of fractions still persist.

The theoretical lens that informed this study was drawn form the QUANTUM project. It is located in the sociology of pedagogy and Bernstein's (1990) notion of the pedagogic device provided an orientation that enabled me to study the detail of this pedagogic practice. It helped me select evaluative events as a unit and see how they were grounded through appeals, which in turn was linked to the mathematical problem solving.

The study is influenced by the work on tasks done by Stein et al (2000) and the work on teaching dilemmas done by Adler (2001). Stein et al's (2000) work on understanding the cognitive demands of tasks helped categorise the task used by Mr. T. Ball, Bass and Hill's (2004), eight types of mathematical problem solving that teachers do as they go about their teaching, aided in identifying the categories that were used to described the mathematical work of the teacher. Since these categories were not adequate in describing what took place in this particular practice, it was necessary to add to the categories. The categories that were used to identify the appeals Mr. T made in order to legitimate meaning were taken from work done by Adler & Pillay's (2007) and provided a starting point for the analysis. Once the data was chunked into evaluative events it was clear that the mathematical work/problem solving that made up Mr. T's practice were predominantly: demonstrating, encouraging and working with learners' ideas. While he did engage in the other mathematical work/problem solving, they were not the focus of his teaching practice. He appealed to mathematics (rules & empirical), experience (everyday) and the curriculum (tests and exams) to fix meaning. The mathematical problem solving and appeals he made threw up certain dilemmas. The dilemmas were representing the content (cognitive demand vs. encouragement), competing goals (procedural vs. conceptual) and student thinking (scientific vs. everyday and conceptual vs. procedural). This aided in providing a description of what mathematics for teaching is in this practice.

6.2 Personal Coda

Referring to the my personal dilemma expressed in Chapter One regarding what I need to study in order to improve as a mathematics teacher, I have come to conclude that I am no different from Mr. T. When I reflect on my own practice and that of Mr. T's I empathize

with what I have observed, which is why it has been so difficult to do this study. I know that I need to learn more about fractions. Completing a course in higher level Mathematics may extend my mathematical knowledge and help me feel more confident in my mathematical capability. Attending a course that is pedagogically focused i.e. taking pedagogical artifacts like tasks that deal with fractions and engaging with them will most definitely benefit me and I will learn a great deal from it. A third option would be to do an in-depth study of fractions, what fractions are, the different subconstructs of fractions, examining various kinds of tasks, how these tasks work, the kind of fractions they represent and do not represent, what research has been done on fractions, studying more and observing other teachers teaching fractions and observing the kind of problems they experience etc. Of these three options I would ague that while each one of them may benefit me personally, not all of them will help with my professional development. For me to improve how I teach fractions, I will have to choose a combination of the three options. Does a course like this exist anywhere in teacher development or training? I know that courses focusing on either higher-level Mathematics or pedagogy are readily available at universities and teacher training colleges. A combination of the pedagogy and mathematics is advanced. Where would I find a course that addresses the issues related to the teaching of fractions so that I can improve my ability to teach fractions? After doing this research project I am still not sure that I would be able to select, set up and implement a task that would include, focus and develop both the conceptual and procedural understanding of fractions. Where would I learn to do this? This did not happen while completing my BEd Honours or Masters. It certainly is not going to happen if I complete my PhD. So my question changes from, "What do I study to become a better mathematics teacher?" to, "Where do I find the appropriated courses to learn more about how to teach mathematics successfully?"

6.3 Reflections

After 10 days of observing Mr. T I was distressed and convinced that the data I had collected was of no use. All that I saw was that the teacher was doing the same thing over and over again. It was only when stepping back and adjusting my gaze, with the

help of the analytical framework, i.e. mathematical work/problem solving by Ball, bass & Hill (2004), appeals by Adler & Davis (2005), Dilemmas by Adler (2001) and tasks by Stein et al (2000) that I was able to see beyond the superficial and common sense.

Using the mathematical problem solving categories defined by Ball et al (2004) proved to be most useful in that it provided a description of what was taking place in the classroom. The categories assisted me in seeing that while the teacher did not engage successfully with the conceptual understanding of fractions, there was most definitely an attempt on his part to do so. This was really helpful with regards to working with learner ideas. It showed how the teacher worked with learner ideas and also showed that working with learners' ideas formed an important part of this teacher's job. It explained so much of what this research is trying make sense of i.e. what constitutes mathematics for teaching in this practice.

Using the task as a tool revealed that tasks are not straightforward. Stein et all (2000) and related literature report the strains and stresses connected to working with tasks that involve both the procedural and conceptual. This was no different for Mr. T. The appeals helped me see how difficult it was for him to provide mathematical grounds for a lot of what he was doing, while the dilemmas revealed the demands placed on him by the new curriculum.

So mapping the analytical framework onto the classroom practice helped me see what I could not see before. It provided me with a reading of the classroom practice. I am by no means suggesting that this is the only reading for this classroom practice, but rather that it provided a particular reading which was very illuminating.

While all of the above is true, not all was so simple and straightforward. I conducted the research as rigorously as I could. The mathematical work/problem solving categories were on one level very helpful and yet on another, they were problematic. The categories have specific meanings that are not clear in Ball, Bass and Hill's (2004) writing. For example they describe the work of explaining as being able to, 'design mathematically

accurate explanation that are comprehensible and useful for students' (Ball, Bass & Hill, 2004, p.59). What they see as the work of explaining and what could count as explaining in different classrooms are not necessarily the same. For example, it was very difficult to determine whether Mr. T's attempt to explain how to add fractions with different denominators was in fact what they prescribe as being an explanation. The difficulty in categorising whether his explanations fitted the prescription arose when he demonstrated how the different unit fractions added up to one whole without any mathematical rationale. Another instance is his 'explanation' of how to follow the procedures in order to complete the algorithms. While they were mathematically accurate and both comprehensive and useful (they understood the procedures and it enabled them to complete the algorithm successfully) for the learners, they lacked a mathematical rationale. This convinced me that the work he was involved in was more of demonstrating than it was explaining. I therefore generated a category called demonstrating. With the help of work done by Ma (1999, p.47), I redefined explaining as being able to offer a mathematical rationale for the procedures and ' verbalizing the procedures of the algorithm'. Demonstrating meant, displaying the steps of computation or the unit fractions and/or verbalizing the procedures of the algorithm' (Ma, 1999, p.7). This meant that when Mr. T was engaging in the mathematical work of explaining, he could also be demonstrating. However, if he were demonstrating, there would be no explaining involved. I conducted the analysis as rigorously as I could, however I grappled with this dilemma of explaining versus demonstrating right up until the end, and I know that much work still needs to go into understanding it better and formulating categories that will best describe the mathematical work.

The mathematical work/problem solving of encouraging was also very difficult to fit into any of the categories formulated by Ball, Bass and Hill (2004). As mentioned previously, since Mr. T did a significant amount of encouraging it was necessary to include it as one of the problem solving categories, yet it is not per se mathematical.

For Ball, Bass and Hill (2004) mathematics for teaching is about defining, explaining, representing, questioning, working with learner ideas and restructuring tasks. If teachers,

for example, are asked whether they work with learners' ideas, I am sure that many would respond in the affirmative. The question then to ask is, is it in line with Ball, Bass and Hill's (2004) category of working with learners' ideas? If not, what does this mean? Does Ball, Bass and Hill's (2004) guide for looking at MfT work for any pedagogy? There is a problem with these problem -solving categories. They are tasks of teaching and this is what teachers do, but what makes it so?

From this particular study, it is evident that while the teacher engages in all of the mathematical work/problem solving, it is done at different degrees with certain categories dominating and does not always fit precisely into the stipulated categories. So what is prescribed by Ball, Bass and Hill's (2004) will not hold true for any pedagogy and may be more of a hindrance than a help.

Ball, Bass and Hill's (2004) problem solving categories was put up as a framework and during the initial stages of the research it made sense to use it, but as the study progressed it got complicated and difficult to work with. As mentioned before, while the problem solving categories aided in revealing what took place in Mr. T's practice, it posed a problem in that the boundaries of the categories were not clear. For lack of a better word, they were 'iffy', their meanings were not clear and could not be relied on. This indicates to me that it does not relate to different pedagogy. Since it is not possible to use these categories in any classroom, it is insufficient in helping us understand MfT in different classrooms.

Not only have I used the mathematical work/problem solving categories, but this study was also driven by a pedagogy discourse condensed in evaluation. This methodology I believe, was a useful and successful way of analyzing the data for following reasons: Chunking the data helped me account for everything that took place in the classroom during the lessons. Nothing was left to chance. It also made working with the insurmountable data easier. After collecting the data, observing the lessons and transcribing them, I felt incredibly insecure and overwhelmed. It was only when I started chunking the data that I was able to make some sense of it. Determining the cognitive demand of the task, identifying a notion, categorizing the mathematical work or

problem/solving done by the teacher and the appeals he made to legitimate meaning were all very insightful, but not without problems or concerns.

The appeals used to legitimate meaning were also a cause for concern. They were vague and while they aided in understanding and showed how Mr. T fixed meaning for his learners, they were not clear in meaning or intention. Similar to my experience with the problem solving categories, I sometimes experienced difficulty categorising the appeals made by the teacher. For example, when Mr. T engaged in the mathematical work of encouraging his learners, he called on the everyday. This initially seemed easy to categories as his appeals related to the learners everyday experiences, however, once the analysis progressed, I realized that the new curriculum had an even greater effect on his attempt to legitimate meaning for his learners. The knowledge domain related to the curriculum and explained by Adler & Davis does not provide a clear description of legitimating meaning through the new curriculum. These knowledge domains may have to be revisited in order to make them clearer and more relevant to the pedagogy.

As I reflect back on the process involved in completing this study, I realize how much I have learnt. While this has been a long and challenging road to travel it has been worthwhile. As a developing researcher, I look back and question the framework, tools and orientation I have adopted for this study. They have shaped what I have come to see, and while they explain many aspects that this study aimed to investigate, it must be noted that the framework is still in its developmental stages and not yet stable. I used the categories from Ball, Bass and Hill's (2004) and the knowledge domain from Adler & Pillay (2007) the best way possible. Building on and enhancing the work already done by these researchers could prove most beneficial for understanding mathematics for teaching in different practices involving different pedagogy.

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Appendix A
Example of Interview Schedules

Interview 3 with Mr. T

Present: Mr. T
Sharon Govender

Question:	**Thank you for making time again to talk with me. Just a reminder about the focus of my research and the wider project it connects with. We are trying to understand better the mathematical work teachers do as they prepare and then teach- what do teachers of mathematics need to know mathematically and in relation to learners and learning so to teach well. So the questions I want to explore with you all relate to learning more about this from you, your experience and your practice. So what I want to know is what was your goal in lesson one? You know you gave them the fraction board, what did you want to teach and what did you want them to learn?**
Question:	**From observing your lessons I concluded that the main aim was to get learners to add and subtract fractions that have different denominators. You used a fraction board/wall, got the learners to make up ten different sums by sticking different fraction pieces together in order to get one whole. They had to show all their calculations of how they added the fractions up. In order to do this they needed to find the LCD. So your focus was to find the LCD. Why did you choose this**

	particular method, the fraction wall, to teach this particular concept, finding the LCD?
Question:	**What was the purpose of the fraction wall?**
Question:	**So you got to watch the DVDs?**
Question:	**When you think back did this work as you hoped, using the fraction board, did it work the way you wanted it to work? Did it work well?**
Question:	**How do you think this helped them learn about LCD's?**
Question:	**What problems or concerns, if there were any, arose for you?**
Question:	**Mathematically what problems did you encounter?**
Question:	**So close to the end of lesson one of the learners, I think it was Tom[17], he struggled to show that his unit fractions added up to one whole. Do you remember that?**
Question:	**He was sitting here in the front and um, he tried to get it to one whole but he couldn't because he did not have the correct unit fractions. You tried to help him but you also ran into difficulty. And it seemed to me this was because you didn't have the correct unit fractions.**
Question:	**Can you remember?**
Question:	**Do you think that using the unit fractions aided the learners understanding of the concept when you were teaching?**
Question:	**How do you think the unit fractions helped them understand lowest common denominators?**
Question:	**Okay um, do you think it posed any problems – unit fractions? To you think it**

[17] Pseudonym to protect the identity of the learner

	was a problem?
Question:	**Okay, what I was thinking was that because the unit fractions were inadequate in representing the concept of the lowest common denominator- we didn't have enough. This might have impacted on the possibilities for learners to understand. What do you think?**
Question:	**So you don't think Tom found it difficult because they didn't have the proper unit fractions?**
Question:	**Okay, now have you ever come across or used other ways that you could have used to present the concept of finding the lowest common denominator in order to add or subtract fractions?**
Question:	**Have always used a fraction board?**
Question:	**It's very interesting because usually they use it to teach equivalent fractions. You are using it to teach the addition and subtraction of fraction?**
Question:	**Do you think you selected what was appropriate and did it aid you teaching the concept of fractions? Did you find it a success?**

Question:	**I would like us to shift the now attention to some of the learners so you could reflect with me on the kind of work we have to do to help them especially when they are having difficulties. For example in lesson three, Ted[18], I don't know if you remember this. He struggled with one of the sums. He couldn't find the lowest common denominator. Eventually you had to show him how to get to it. Do you remember that?**

[18] Pseudonym to protect the identity of the learner.

Question:	**When you asked him questions regarding the examples you gave them on the board, he was able to articulate that you simplified it and found the multiples in order to find the lowest common denominator. However, when he completed his own sum and explained how he got the lowest common denominator he was unable to do that. Can you elaborate on this?**
Question:	**He could tell you exactly what you did, this and that and the multiples and how to get to it but when he did his own he just couldn't do the maths. He was able to say what you did in the example but he couldn't do it in his book. Now Mr. E, as a teacher are there are other ways that you could help him?**
Question:	**Mathematically to find the lowest common denominator, can you think of any other ways that you would use besides using the unit fractions or besides what you did with, showing them the multiples, because you went through them. You went, what are the multiples of four and you said two and four. You said 'what are the multiples of eight?', and you went through all, I can't remember what you said. And you went and you said circle it. Is there any other way that you could think of that you would teach that?**
Question:	**Ok Mr., that's it. Thank you for your time**

Appendix B

Letter of approval to conduct study issued by the Gauteng Department of Education

UMnyango WezeMfundo
Department of Education

Lefapha la Thuto
Departement van Onderwys

Date:	18 July 2006
Name of Researcher:	Bennett Sharon
Address of Researcher:	186 Floreston Road
	Mondeor
	2091
Telephone Number:	(011) 6805454
Fax Number:	(011) 4330692
Research Topic:	The dilemmas teachers' experience when teaching fractions: A case study of a Grade 7 teacher
Number and type of schools:	1 Primary School
District/s/HO	Johannesburg South

Re: Approval in Respect of Request to Conduct Research

This letter serves to indicate that approval is hereby granted to the above-mentioned researcher to proceed with research in respect of the study indicated above. The onus rests with the researcher to negotiate appropriate and relevant time schedules with the school/s and/or offices involved to conduct the research. A separate copy of this letter must be presented to both the School (both Principal and SGB) and the District/Head Office Senior Manager confirming that permission has been granted for the research to be conducted.

Permission has been granted to proceed with the above study subject to the conditions listed below being met, and may be withdrawn should any of these conditions be flouted:

1. ***The District/Head Office Senior Manager/s concerned must be presented with a copy of this letter that would indicate that the said researcher/s has/have been granted permission from the Gauteng Department of Education to conduct the research study.***
2. ***The District/Head Office Senior Manager/s must be approached separately, and in writing, for permission to involve District/Head Office Officials in the project.***
3. ***A copy of this letter must be forwarded to the school principal and the chairperson of the School Governing Body (SGB) that would indicate that the researcher/s have been granted permission from the Gauteng Department of Education to conduct the research study.***

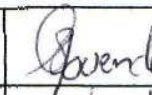
Office of the Senior Manager – Strategic Policy Research & Development
Room 525, 111 Commissioner Street, Johannesburg, 2001 P.O.Box 7710, Johannesburg, 2000
Tel: (011) 355-0488 Fax: (011) 355-0286

4. *A letter / document that outlines the purpose of the research and the anticipated outcomes of such research must be made available to the principals, SGBs and District/Head Office Senior Managers of the schools and districts/offices concerned, respectively.*
5. *The Researcher will make every effort obtain the goodwill and co-operation of all the GDE officials, principals, chairpersons of the SGBs, teachers and learners involved. Persons who offer their co-operation will not receive additional remuneration from the Department while those that opt not to participate will not be penalised in any way.*
6. *Research may only be conducted after school hours so that the normal school programme is not interrupted. The Principal (if at a school) and/or Senior Manager (if at a district/head office) must be consulted about an appropriate time when the researcher/s may carry out their research at the sites that they manage.*
7. *Research may only commence from the second week of February and must be concluded before the beginning of the last quarter of the academic year.*
8. *Items 6 and 7 will not apply to any research effort being undertaken on behalf of the GDE. Such research will have been commissioned and be paid for by the Gauteng Department of Education.*
9. *It is the researcher's responsibility to obtain written parental consent of all learners that are expected to participate in the study.*
10. *The researcher is responsible for supplying and utilising his/her own research resources, such as stationery, photocopies, transport, faxes and telephones and should not depend on the goodwill of the institutions and/or the offices visited for supplying such resources.*
11. *The names of the GDE officials, schools, principals, parents, teachers and learners that participate in the study may not appear in the research report without the written consent of each of these individuals and/or organisations.*
12. *On completion of the study the researcher must supply the Senior Manager: Strategic Policy Development, Management & Research Coordination with one Hard Cover bound and one Ring bound copy of the final, approved research report. The researcher would also provide the said manager with an electronic copy of the research abstract/summary and/or annotation.*
13. *The researcher may be expected to provide short presentations on the purpose, findings and recommendations of his/her research to both GDE officials and the schools concerned.*
14. *Should the researcher have been involved with research at a school and/or a district/head office level, the Senior Manager concerned must also be supplied with a brief summary of the purpose, findings and recommendations of the research study.*

The Gauteng Department of Education wishes you well in this important undertaking and looks forward to examining the findings of your research study.

Kind regards

ALBERT CHANEE
ACTING DIVISIONAL MANAGER: OFSTED

The contents of this letter has been read and understood by the researcher.	
Signature of Researcher:	
Date:	20/07/2006

Appendix C
Letters Seeking Permission

Letter to Principal

Dear Mr. Shutte

As you are aware I am currently completing my Masters of Education in Mathematics Educations at the University of the Witwatersrand and am required to conduct a research project.

For the purpose of this study I require a grade 7 Mathematics educator as well as a group of grade seven learners who will participate in my study. The study aims to investigate the dilemmas/ problems experienced by grade 7 mathematics educators when teaching fractions.

With your consent, and permission from the GDE, governing body and parents, I would like Mr. Esterhuisen and Grade 7E to be participants in the study. Mr. Esterhuisen has informally indicated that he would be a willing to participate in the project.

I plan to observe lessons that are dedicated to the teaching of fractions. I plan to videotape these lessons as well as to have access to copies of some of the materials produced by the learners during these lessons. Lessons will continue as normal and as scheduled, with my presence in the back of the classroom. I also plan on conducting approximately 3 interviews with Mr. Esterhuisen. I will interview him before he introduces the concept of fractions and after he has taught it. Another interview will be conducted during the interim period of completing the unit. I will be collecting data for approximately 2 cycles (i.e. 7 days per cycle, each lesson is an hour long). This works out to be approximately 14 hours in the classroom. The interviews will be about 45 minutes each. The total time required of the educator is approximately 17 hours.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports by making use of a pseudonym to refer to the school, teacher and learners. Video extracts, where anonymity cannot be provided, will only be used with consent from the learners and parents/guardians. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years.
Please note that if consent is not granted I will respect your decision. In addition, if at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Thank you for your support.

Yours sincerely
Sharon Govender

CONSENT FORM (PRINCIPAL):

I, _____ (please print full name,) the principal of Mondeor Primary School give consent to the following:

1. Allowing Mrs. Bennett to conduct her research at Mondeor Primary School.

   YES [ ] NO [ ] please tick

2. Videotaping of lessons on mathematics fractions in which an educator of the school might appear as part of the videotext.

   YES [ ] NO [ ] please tick

3. Copies made of classwork, homework or assessment that learners might produce as part of these lessons.

   YES [ ] NO [ ] please tick

4. Tape recoding of interviews with researcher.

   YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Letter to Parents/Guardian

Dear Parents/Guardians

I am currently completing my Masters of Education in Mathematics Educations at the University of the Witwatersrand in Johannesburg. As part of my study, I am investigating the dilemmas/ problems experienced by grade 7 mathematics educators when teaching fractions. This letter is to request your consent for your child/ward to participate in the above mentioned research project.

In this phase of the project the focus will be on classroom teaching of fractions in grade 7. I plan to observe lessons that are dedicated to the teaching of fractions. I plan to videotape these lessons as well as to have access to copies of some of the materials produced by your child/ward during these lessons. Since you are the parent/guardian of the learners in these classes, I ask for your consent to allow your child/ward to appear as part of the videotext and where necessary to have access to copies of materials that your child/ward might produce. Lessons will continue as normal and as scheduled, with my presence in the back of the classroom.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports by making use of a pseudonym to refer to the school, teacher and learners. Video extracts, where anonymity cannot be provided, will only be used with your and your child/wards' consent. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years. I will be collecting data for approximately 2 cycles (i.e. 7 days per cycle, each lesson is an hour long). This works out to be approximately 14 hours in the classroom. The total time required of the learners is approximately 14 hours.

Please note that if consent is not granted I will respect your decision. Therefore your child/ward together with any other children not participating in the project will be seated on one side of the classroom and will not be videotaped. Furthermore, any text that your child/ward might produce will not be used in the project. In addition, if at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Please complete the form attached and return it to Mrs. Bennett at your earliest convenience. I will be happy to answer any questions or queries that you might have.

Looking forward to hearing from you. Thank you for your support.

Yours sincerely
Ms. Govender

CONSENT FORM (PARENTS/GUARDIANS):

I, _____ (please print full name,)
parent/guardian of _____,
give consent to the following:

1. Videotaping of lessons on mathematics fractions in which my child/ward might appear as part of the videotext.

YES [ ] NO [ ] please tick

2. Copies made of classwork, homework or assessment that my child/ward might produce as part of these lessons.

YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Letter to the teacher

Dear Mr. T

As you are aware I am currently completing my Masters of Education in Mathematics Educations at the University of the Witwatersrand and am required to conduct a research project.

For the purpose of this study I require a grade 7 Mathematics educator as well as a group of grade seven learners who will participate in my study. The study aims to investigate the dilemmas/ problems experienced by grade 7 mathematics educators when teaching fractions.

With your consent, and permission from Mr. Shutte, the governing body, the GDE and parents, I would like you and Grade 7E to be participants in the study.

In this phase of the project the focus will be on classroom teaching of fractions in grade 7, as well as interviews with you at three points. The interviews will be conducted before you introduce the concept of fractions and after you have taught it. Another interview will be conducted during the interim period of completing the unit. I will be collecting data for approximately 2 cycles (i.e. 7 days per cycle, each lesson is an hour long). This works out to be approximately 14 hours in the classroom. The interviews will be about 45 minutes each. The total time required of you is approximately 17 hours.

I plan to observe lessons that are dedicated to the teaching of fractions. I plan to videotape these lessons as well as to have access to copies of some of the materials produced by your learners during these lessons. Since you are the teacher of these learners in these classes, I ask for your consent to allow me where necessary to have access to copies of materials that your learners might produce. Lessons will continue as normal and as scheduled, with my presence in the back of the classroom.

All data collected will only be used for research purposes. There is a possibility that the research could be reported at appropriate conferences or in relevant journals. I assure you that anonymity and confidentiality will be protected in all written and verbal reports by making use of a pseudonym to refer to the school, teacher and learners. Video extracts, where anonymity cannot be provided, will only be used with your and your learners' consent. Upon completion of the project, all data collected will be archived and securely stored at the University of the Witwatersrand for a maximum period of five years. The findings of my study will be communicated with you, if you so desire, upon completion of my study. Please note that the results of the study will not be shared with the school's principal.

Please note that if consent is not granted I will respect your decision. In addition, if at any point you wish to withdraw your consent, you may do so without any penalty or prejudice.

Please complete the form attached and return it to Mrs. Bennett at your earliest convenience. I will be happy to answer any questions or queries that you might have.

Looking forward to hearing from you. Thank you for your support.

Yours sincerely
Ms. Govender

CONSENT FORM (TEACHER):

I, _____ (please print full name), a grade 7 teacher, give consent to the following:

2. Videotaping of lessons on mathematics fractions in which I might appear as part of the videotext.

YES [ ] NO [ ] please tick

3. Copies made of classwork, homework or assessment that my learners might produce as part of these lessons.

YES [ ] NO [ ] please tick

4. Tape recoding of interviews with researcher.

YES [ ] NO [ ] please tick

Signed: _____

Date: _____

Appendix D

Ethics Clearance issued by the University of the Witwatersrand

Appendix D

Ethics Clearance issued by the University of the Witwatersrand

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG
Division of the Deputy Registrar (Research & Academic)

MEMORANDUM

TO **Mrs S Bennett**
School of Education
Sent by e-mail –adlerj@educ.wits.ac.za
C/O Prof J Adler

jillian.adler@wits.ac.za

FROM: Ms Anisa Keshav
Secretary: Human Research Ethics committee (Non-Medical)
Tel: 717 1234 Fax 339 5708 e-mail:Keshava@research.wits.ac.za

DATE: 18 July 2006

Ref: **R14/49/1**

Protocol: H060709: The Dilemmas/Problems Teachers' Experience when Teaching Fractions: A Case Study of Grade 7 Teacher

The above protocol was considered by members the Human Research Ethics Committee (Non-Medical). This application cannot be approved until the following amendments/correction have been made.

- Written consent from the Department of Education must be submitted
- A child assent form is needed
- The information sheets need to indicate how much time the researcher would like of participants
- The information sheet to the instructor should explicitly indicate that results of the study will not be shared with the schools Principal

Please let me have the amendments as soon as possible as protocols on which no action has been taken will be removed from the agenda without approval after two months.

Cc

NB: Please highlight the changes you submit





31 January 2008

To Whom It May Concern:

Dear Sir/Madam

This letter is to confirm that I Sharon Govender (student number: 9309109G) upon applying for Ethical clearance from the University of Witwatersrand received the following reply (see attached document). I made the necessary changes as suggested by the ethics committee and resubmitted my application. My supervisor was notified via e-mail that it had been accepted and that I was allowed to continue with the research project.

Yours sincerely

Sharon Govender

Confirmation

I confirm the above information. I am Ms Govender's supervisor and she proceeded having done the necessary refinements to the ethics procedures as requesed.

I did not keep the email copy – assuming this was held in the University and sent to the student. Ms Govender has not been successful in her attempts to reach the University Research Office, and obtain a copy.

Yours truly

Jill Adler

Professor Jill Adler, Chair of Mathematics Education

Director: Marang Centre

Wits School of Education

Appendix E
Overview of lessons Observed

DATE	DURATION	TOPIC	MAIN IDEAS DISCUSSED (CONCEPTS, SKILLS, FORMULAE	COMMENT
Wed 26/07/06	1 hour	Introduction – addition of common fractions	Finding the LCD using multiples	Mr. T showed learners how to find the LCD using multiples. Learners' work independently to prove mathematically how different fractions add up to one whole. Mr. T discovered a few practical problems: • Learners did not cut out and paste in fractions strips accurately making it difficult to complete the sums. • They were also unable to find the **L**CD. • They did not mix the fractions pieces to get different sums. • When putting the fraction pieces together they do not make up a whole but when they work it out mathematically it works out to be a whole. Related doing the mathematics to playing a play station game. At first it is difficult to play but the more you play the game the more you discover about it and the better you get. Mr. T stated that doing these

				sums are exactly the same. Used the example of a garden wall and how it is inappropriate for you to build a wall that ends in your neighbour's yard. Used this to try to explain to learners their mistake with the unit fractions. Got a few learners to come to the front of the class and use the available unit fractions to show how the different pieces add up to one whole. Tried using a different representation (correct unit fractions not available so used a pie graph instead). Ran into difficulties because could not represent it. Ran out of time because lesson ended.
Thurs 27/07/06	1 hour	Addition of common fractions	Finding the LCD using multiples	Explained and discussed the practical problems the learners where experiencing. Explained and demonstrated (on the board) if they chose incorrect combinations it would not add up to one whole. Learners continued with previous task. Mr. T walked around aiding the different learners by explaining and demonstrating how to find the LCD and identifying their practical errors.
Fri 28/07/06	1 hour	Addition & subtraction of common fractions without using	Finding the LCD without using multiples	Recapped what was expected of the learners from the previous task. Referred again to the play station game and how it is similar to doing mathematics. Mr. T explained and demonstrated on the board the two different ways of finding the LCD without using multiples. Showed

		multiples		how to convert from a mixed number to an improper fraction and vice versa. Learners completed a worksheet related to explanation.
Mon 31/07/06	1 hour	Addition & subtraction of common fractions without using multiples	Finding the LCD without using multiples	12 learners completed and explained the procedures related to the previous worksheet on the board while the others marked their work. Mr. T went through all the problems on the board doing the necessary corrections with the help of the children. Mr. T refers to mathematics as a study subject. He implied that doing procedures repeatedly would help. Motivates the learners by saying that they can do mathematics, gives an example that if they can work with money they can do mathematics.
Tue 1/08/06	1 hour	Addition & subtraction of common fractions without using multiples	Adding and subtracting mixed numbers	Continued with the corrections from the previous lesson. Insisted on children doing the corrections so that when they study they know the different procedures. Walked around the class helping individual learners with their corrections. Do two examples on the board of how to add and subtract mixed numbers. Uses the learners to help him by asking them the procedures. Insists that they use the method that requires them to change a mixed number to an improper fraction. Learners copy the examples exactly they way Mr. T wrote it on the board. Learners completed the worksheet in their books.

Wed 2/08/06	1 hour	Addition & subtraction of common fractions without using multiples	Adding and subtracting mixed numbers	Mr. T explained the progression of the lesson. He called out the correct answers form the previous worksheet so that the learners could mark their works. Once learners were able to do their corrections successfully they were allowed to move onto the next worksheet.
Thurs 3/08/06	1 hour	Addition & subtraction of common fractions	Finding LCD (with & without multiples) Adding & subtracting mixed numbers (changing a mixed number to a improper fraction and vice versa) Adding and subtracting mixed numbers with both operations in a single problem	Mr. T recapped lessons 1-5 by doing examples on the board. Explained and demonstrated on the board how to work with problems that have both addition and subtraction in a single problem. Learners completed a worksheet related to this.
Fri	1 hour	Addition &	:	Introduced the project and explained what had to be done. The project

4/08/06		subtraction of common fractions	Finding LCD (with & without multiples) Adding & subtracting mixed numbers (changing a mixed number to a improper fraction and vice versa) Adding and subtracting mixed numbers with both operations in a single problem	tested all the concepts that were taught the past two weeks. Learners were only able to start the project once their corrections for the previous worksheet were done.
Mon 7/08/06	1 hour	Addition & subtraction of common fractions	Finding LCD (with & without multiples) Adding & subtracting mixed numbers (changing a mixed number to a improper fraction and vice versa)	Learners continued working on their corrections and the project.

			Adding and subtracting mixed numbers with both operations in a single problem	
Tue 8/08/06	1 hour	Addition & subtraction of common fractions	Finding LCD (with & without multiples) Adding & subtracting mixed numbers (changing a mixed number to a improper fraction and vice versa) Adding and subtracting mixed numbers with both operations in a single problem	Recapped what was done in Grade 6 and the past two weeks. Did an example on the board on how to add mixed numbers. Referred to the importance of symbols in mathematics (= sign). Referred to a marriage to explain when to decide which number can be used as the lowest common denominator when not using multiples. Learners completed the project. Mr. T was concerned that the learners did not know their times table so they struggled with finding the LCD.

Appendix F

Categorizing and Chunking of Lessons 1-4 into Evaluative Events

Events	Timing	Concept			Task Demands				Problem Solving/ Mathematical Work									Appeals					
					Lower Level		Higher Level									Working with learner ideas		Maths			Experience		Curriculum
		Notion	Sub-Notion		Memorization	P Without C	P With C	Doing mathematics	Explanations	Questioning	Demonstrations	Representing	Restructuring	Defining	Encouraging	Mathematical Calculation Errors	Practical' Errors	Empirical	Definitions	Rules	Professional	Experience	tests & exams
Lesson 1 (50 Min)																							
1	00:36-01:41	Adding fractions with different denominators	1.1	Using fractions strips to prove one whole				✓	✓								✓						
2	01:46 - 06:29	Adding fractions with different denominators	2.1	Finding the LCD using multiples		✓			✓	✓	✓				✓			✓		✓			
	06:30-07:00		2.2			✓									✓					✓			
	07:01-22:18		2.3			✓				✓	✓				✓	✓	✓			✓			✓
3	22:19-23:04:	Adding fractions with different denominators	3.1	Using fractions strips to prove one whole		✓											✓			✓			

	23:05 - 27:17		3.2	Finding the LCD using multiples		✓			✓						✓	✓	✓			✓			
	27:18- 29:37		3.3	Finding the LCD using multiples		✓			✓								✓	✓		✓		✓	
	29:38- 30:30		3.4	Finding the LCD using multiples		✓			✓											✓			
	30:31- 31:52		3.5	Fraction Wall & Finding LCD		✓			✓	✓	✓	✓					✓	✓		✓		✓	
4	31:53- 36:19	Adding fractions with different denominators	4.1	Finding the LCD using multiples		✓			✓						✓	✓		✓		✓			
	36:20 - 37:56		4.2	Using unit fractions to prove one whole & finding the LCD		✓					✓	✓	✓		✓	✓		✓		✓			
	37:57 - 39:49		4.3			✓					✓	✓	✓			✓		✓		✓			
	39:50- 45:13		4.4	Using fractions strips to prove one whole & finding the LCD				✓	✓	✓	✓	✓	✓		✓	✓		✓		✓			
5	45:14- 46:36	Adding fractions with different denominators	5.1	Using fractions strips to prove one whole		✓			✓					✓					✓	✓			

Lesson 2 (50 min)																							
1	00:00-03:49	Adding fractions with different denominators	1.1	Using fractions strips to prove one whole		✓			✓	✓	✓	✓					✓	✓		✓			✓
	03:50-07:06		1.2	Finding LCD & intro to improper fractions		✓			✓	✓	✓	✓					✓			✓			
2	07:07-10:18	Adding fractions with different denominators		Finding the LCD using multiples		✓			✓		✓	✓								✓		✓	
3	10:19-12:00	Adding fractions with different denominators	3.1	Finding the LCD using multiples		✓									✓								✓
	12:01-15:13		3.2			✓			✓											✓			
	15:14-17:15		3.3			✓									✓		✓			✓			✓
4	17:16-22:32	Adding fractions with different denominators	4.1	Using fractions strips to prove one whole		✓						✓					✓	✓		✓			
5	22:33-26:19	Adding fractions with different denominators	5.1	Finding the LCD using multiples		✓										✓				✓			
	26:20-29:47		5.2			✓									✓	✓	✓			✓			✓
	29:48-30:35		5.3			✓										✓				✓			
	30:36-32:32		5.4			✓									✓	✓	✓			✓			

6	32:33-35:28	Adding fractions with different denominators	6.1	Finding the LCD using multiples		✓									✓	✓	✓			✓		✓	✓
7	35:29-39:09	Adding fractions with different denominators	7.1	Using fractions strips to prove one whole		✓			✓	✓	✓	✓	✓				✓	✓		✓			
	39:10-41:54		7.2			✓				✓					✓		✓			✓			
	41:55-42:36		7.3			✓				✓							✓			✓			
	42:38-43:47		7.4			✓			✓						✓		✓			✓			✓
	43:48-47:57		7.5	Finding the LCD &using strips to prove one whole		✓			✓		✓	✓	✓				✓	✓		✓			

Lesson 3 (50 min)																							
1	01:38-03:28	Adding fractions with different denominators	1.1	Finding the LCD using multiples		✓			✓											✓		✓	
	03:30-05:34		1.2	Changing a mixed no. to an improper fraction		✓			✓	✓	✓			✓					✓	✓		✓	
2	05:35-09:27	Adding fractions with different denominators	2.1	Finding the LCD not using multiples & changing a mixed no. to an improper fraction		✓			✓	✓	✓									✓			
	09:28-24:15		2.2	Simplifying fractions & multiplying denominators to find LCD		✓			✓	✓	✓									✓			
3	24:16 - 29:30	Adding & subtracting fractions with different denominators	3.1	Finding LCD's using different methods		✓									✓					✓		✓	✓
	29:31-31:45		3.2			✓			✓											✓			
	31:46-42:46		3.3			✓				✓						✓				✓			

Lesson 4 (50 min)																								
1	00:00-03:00	Adding & subtracting fractions with different denominator	1.1	Finding the LCD		✓			✓												✓			
2	03:00-23:49	Adding & subtracting fractions with different denominator	2.1	Finding the LCD		✓									✓	✓					✓			
3	23:50 - 24:30	Adding & subtracting fractions with different denominator	3.1			✓									✓						✓			✓
4	24:31-41:33	Adding & subtracting fractions with different denominator	4.1	Finding the LCD		✓			✓	✓					✓	✓					✓		✓	
5	41:34-42-40	Adding & subtracting fractions with different denominator	5.1	Finding the LCD		✓									✓						✓			