School of Mining Engineering

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

CUT-OFF GRADE OPTIMISATION AT AD'DUWAYHI MINE IN SAUDI ARABIA

Mohammed Suliman

(Student Number: 1561775)

Supervisor: Dr. Clinton Birch

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DECLARATION

As the undersigned here, I declare that this thesis and its contents are my own original work, except where citations are made. It is being submitted for the degree of of Master of Science in Engineering at the University of the Witwatersrand, Johannesburg. This work has never been submitted before to any other university for an academic degree or examination.

___________________________ 05_May 2025_

Signature of Candidate Date

ABSTRACT

In terms of the gold mine in Saudi Ariba about 83 % of gold are being mined out through open pit mining methods. In a mining operation, the grade-tonnage distribution of the deposit indicates that not all the material inside the open pit can be processed. Given this grade variability, it is critical to identify ore and waste correctly. In 2018, Saudia Ariba developed a new economic vision focus for developing the gold mining sectors. Therefore, to guarantee achieving the expectations from gold mining industry in terms of techno-economic demands, cut-off grade optimisation is the key. The economic requirement aims towards the maximisation of the return on investment, while techno-economic sustainability aims towards the maximisation of resource recovery. Over the last six decades, tremendous progress has been observed in cut-off grade optimisation. In this study, cut-off grade optimisation was used to maximise the value of Ad‘Duwayhi mining operation based on a finite resource. The traditional approach to cut-off grade calculation was compared with the optimise approach of a cut-off grade, and in so doing, there was an improvement in cut-off grade leading to improved gold produced by 8%. The results of the calculation were modelled in the comparison showing 8% improvement in the net present value. This means that for a given set of design, production and geological parameters, the classification of ore and waste is what ultimately determines the value of a mining operation.

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To my friend Hassan Mukhtar, who started dreaming with me but left in the middle of the journey. May Allah accept him

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AIC: All-in Cost

AISC: All-in sustainable costs

Au g: Gold Content per Grammes

Au oz: Gold Content per ounces

BCOG: Break-even cut-off e grad

Capex: Capital expenditure

C: Maximum concentrator capacity in terms of tons per year.

c: Concentrating cost in terms of $ per ton of material milled.

CF: Cash flow

CIL: Carbon-In-Leach

DCF: Discounted Cash Flow

EBIT : Earnings before interest and tax per US$.

F: Opportunity cost in US$

f: Fixed costs over the production period per year

g/t: Grammes per tonnes

g_{o}: *Optimum cut-off Grade Grammes per tonnes*

GSR: Gross sales revenue in US$

HL: Heap Leach Facility

i: Discount rate in percentage

LoM: Life-of-mine

LP: Linear Problems

M: Maximum mining capacity in terms of tons per year.

m: Mining cost in terms of $ per ton of material mined.

MCF: Mine Call Factor

MRF: Mine Recovery Factor

NPV: Net Present Value

NCF: Net Cash Flow

OpeX: Operating Expenditure

Oz: Ounces

P: Profit In US$

PV: Present Value In US$,

q: Rate of extraction tonnes per annual

Q: The size of the remaining resource

Q_m*: Amount of material mined tonnes per annual*

Q_c*: Amount of ore sent to the mill tonnes per annual*

Q_r*: Ounces Refined.*

S: Selling Price In US$ Per Ounces

Rev: Revenue per US$

R: Maximum refinery capacity in terms of ounces per year.

r: refinery cost in terms of $ per unit product.

T: Time Period Per Years

TFC: Total Fixed Costs

TR: Total Revenue

TUC: Total Unit Cost

TVC: Total Variable Costs

WGC: World Gold Council

UR: Unit Revenue per US$

US$: US Dollar

y: Recovery per percentage %

1 INTRODUCTION

1.1 Research Background and Context

The gold mining sector is important to the economy of Saudi Arabia. It brings more diversity to the economy and offers a range of benefits to the local community through community development programmes. Previously, the contribution of this sector to the economy was limited.

However, in 2018, Saudi Arabia launched a new vision aimed at diversifying the country's revenues not to be reliant on just oil and focusing on developing the mining industry. With the new strategy, there is more pressure on the gold mining sector to increase exploration, development of new resources, and improve mining efficiency to contribute to the economy at the levels expected.

Ma'aden Gold and Base Metals (MGBM) requires improving the efficiency and profitability of current mining operations. The company operates seven mines located in the middle and east of the country. Currently, Ad'Duwayhi is the company's largest and most recent open pit mine at MGBM

Ad'Duwayhi mine located approximately 450km southwest of Riyadh city, capital of Saudi Arabia, 450km northeast of Jeddah, and 125km southeast of the town of Zalim. Access is via sealed highway from Jeddah or Riyadh to 120km from the site, and then via unsealed service road (Xstract Mining, Consultants, 2015). The mine location is indicated in Figure 1.1.

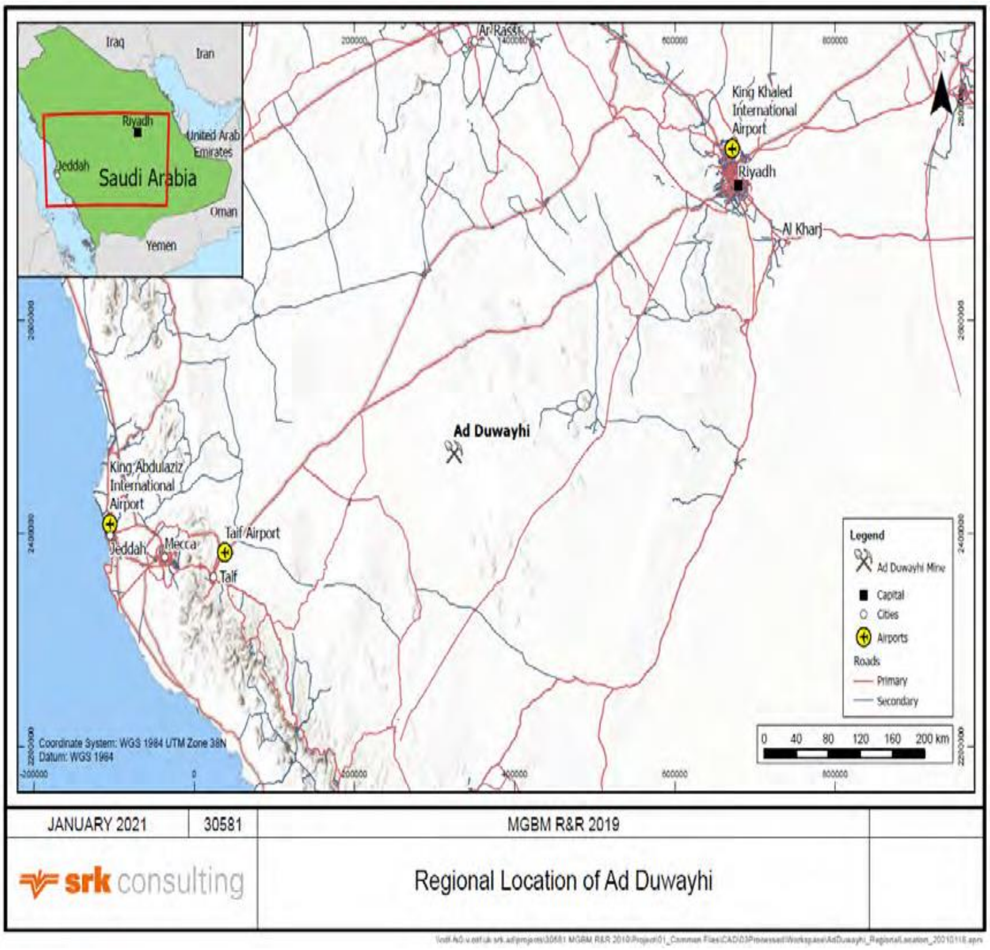

Figure 1.1 Location of Ad'Duwayhi Mine (SRK Consulting, 2017)

1.2 Problem Statement

Ad'Duwayhi mine started producing gold in 2015, and since then there has been a tremendous demand for the gold it produces to cover the shortages caused by other mines within the Ma'aden group. As a result, production plans have focused on mining high-grade material. The current cut-off grade policy being used in the mine has created a huge waste backlog and declining trends in average ore grades.

Waste stripping backlog and declining trends in average ore grades makes mining profitability more critical. The aim of this research is to optimise the cut-off grade of Ad'Duwayhi mining operations and investigate the possibility of optimising the feed grades to improve the net present value (NPV) of the mine.

1.3 Justification Cut-off Grade Optimisation at Ad'Duwayhi Mine

Osanloo (2015) mentioned that optimal open pit mining operation seeks to maximise net present value NPV of cash flow generated from yearly production. The revenue obtainable from the extraction of the mineral is linked to the grade.

Determining the optimum cut-off grade over the life of mine has been one of the major problems of many gold mines. Cut-off grade is a determining factor of operational NPV.

Minnitt (2004) stressed on this point by stating that “The NPV criterion is consistently quoted as the principal determinant of economic value in mining operations and the relationship between cut-off grade and NPV provides a means by which cut-off grades can be optimised”.

In any production process of a mining operation, there are bottlenecks which link with opportunity costs. Processing material with low grade in a production process that has bottlenecks result in lower metal production. Lower metal production results in lower revenue which may lead to loss in an opportunity for the operation, Mugwagwa (2017). Therefore, determining an optimum cut-off grade that maximises the NPV of an operation in the presence of bottlenecks is paramount. In case of Ad'Duwayhi the mill is the bottleneck.

This report seeks to examine various cut-off grade optimisation approaches to identify which one will be the most beneficial to Ad'Duwayhi mine. The optimum cut-off grade policy specifies the yearly feed grades and pit mining progression that correspond with the waste stripping required to expose the ore.

1.4 Research Objectives

This research aims to optimise the cut-off grade of existing mining operations. This aim will be achieved through the following objectives:

a) Conduct a thorough literature review on cut-off grade optimisation.

b) Testing various cut-off grade optimisation approaches to identify. which one will be the most beneficial to Ad'Duwayhi mine.

1.5 Research Methods

The study will focus on optimising cut-off grade at Ad'Duwayhi mining operation. The methodology which will be used for this report will be qualitative and quantitative.

1.5.1 Qualitative research

This would be identifying the various cut-off grade optimisation approaches and identifying the most suitable ones for Ad’Duwayhi mine. This will be done by using qualitative analysis, the qualitative will identify cut-off grade approach which would be applicable for the mine.

1.5.2 Quantitative research

Ad’Duwayhi mine data will be used to calculate optimum cut-off grade and comparing the results of the most suitable cut-off grade optimisation approaches to identify the one that gives the best results as measure using the NPV.

1.6 Sources of Data

This research will apply cut-off grade optimisation at Ad'Duwayhi gold mine. The information for qualitative study will be obtained through journal articles. For the quantitative study the data will be obtained through the mine data. The block mode to be used is called “30581_ADDUW_MOD_ reg.mdl” that was last that was last updated in September 2020. The December 2020 survey month-end topography has been used to deplete the model.

The research will be based on Ad'Duwayhi 2018 pit designs. Other information will be used, including commodity price, geotechnical, mining, and metallurgical parameters obtained from the Ma'aden Strategic Planning Assumptions.

1.7 Criteria for Validation

The data that is used during this research will be provided by the company. The data is managed within a data management system, namely, Geovia Surpac and Whittle 4X®. The data stored in system undergoes a validation process. All data transfer will be electronical, the main data which is from the Ad'Duwayhi mine is periodically audited by an internationally accredited company SRK.

1.8 Structure of the Research Report

The research report will be structured into six chapters. The first chapter will present the research background, problem statement, research justification, and research objective. The second chapter will review some of the literature on cut-off grade and related parameters from sufficient literature on cut-off grade optimisation.

The third chapter will describe the methodology used in this research, break-even and Lane approach will be briefly discussed. The preparation of the required input data will also be highlighted. The fourth chapter will provide a brief of the Ad'Duwayhi mining operations concentrating on location, geology, mining, processing, and economic parameters.

The results obtained from break-even and Lane approach will be presented in the fifth chapter. In addition, the analysis of the results based on the research objective will be discussed. The sixth chapter will provide conclusions and recommendations by summarising the research, showing the findings, and giving suggestions for future work.

1.9 Scientific Contributions and Industrial Significance of the Research

The main contribution of this research is the optimisation of cut-off grades to maximize the NPV of mining operations at the Ad'Duwayhi mine. The key scientific contributions are:

- By demonstrating the importance of optimising cut-off grades, the industry can maximize NPV in mining operations. This research highlights how dynamic cut-off grades, compared to traditional break-even approaches, lead to significant improvements in NPV.
- The research emphasises the role of various economic and technical parameters in determining the optimal cut-off grade. This understanding helps in making informed decisions that enhance the profitability of mining operations
- The research applies Lane's approach to the Ad'Duwayhi mine, showcasing its practical utility. By incorporating opportunity costs and the time value of money, Lane's approach provides a comprehensive framework for cut-off grade optimisation.
- The research provides a framework for cut-off grade optimisation that can be applied to various mining operations, enhancing both scientific understanding and industrial practices. It underscores the importance of integrating economic, technical, and operational factors to achieve sustainable and profitable mining operations

2 LITERATURE REVIEW

2.1 Chapter Overview

Since the 1960s, numerous publications have appeared in the literature considering the cut-off grade optimisation in the mining industry. This section reviews the definition of cut-off grade as given by various authors and shows the significance of cut-off grade in open-pit mining operation.

The section will highlight the different types of cut-off grade to clarify their usage and provides a brief on costs used in cut-off grade calculation as well as the importance of determining the optimum cut-off grade. A brief review on the cut-off grade optimisation and the approaches for the optimisation purposes will be discussed.

2.2 Cut-off Grade Definition

Several authors defined cut-off grade differently, but all agreed that it is the grade is used to distinguish between ore and waste. Lane (1988) (cited in University of the Witwatersrand, 2019) set a classic definition off cut-off grade when he states that cut-off grade is the amount of metal content in the mineralised material that defines what is considered to be ore versus what is considered to be waste.

Dagdelen (2008) brings in the scheduling stage to the definition of cut-off grade by mentioning that it is the grade that is used to distinguish between ore and waste during scheduling process. Rendu (2014) argues that cut-off grade definition should not be limited to distinguishing between ore and waste and scheduling. He provides a broad definition by mentioning that a cut-off grade can define the minimum amount of the product of interest that one metric tonne of ore material must contain before this material is sent to the processing plant. It further defines what ore is considered economical to extract.

Rendu (2014) further indicates that this definition is also used to decide which ore material that should be stockpiled for future processing from ore material should be processed immediately, this is because processing material with a higher grade immediately can generate higher revenue and improve cash flow, while lower-grade material can be stockpiled and processed later when it becomes economically viable.

While these authors provide acceptable definition to the cut-off grade considering the scheduling process, Storrar (1977), cited in (Mugwagwa, 2017) considers the financial aspects by defining it as the minimum value at which ore can be mined and processed without profit or loss.

Ahmadi & Shahabi (2018) bring together these points and define cut-off grade as a technical and economic scale which is determined by various parameters such as geological features, grade distribution, financial parameters and technical factors of operations.

Biswas (2020) pointed out that, besides factors like cost, and revenue , there are other factors influencing the definition of the cut-off grade, known as modifying factors.

Modifying factors encompass a range of technical, economic, financial, and social parameters that influence the conversion of mineral resources into mineral reserves. These factors include mining methods, processing techniques, commodity prices, operating costs, funding availability, environmental regulations, and community relations. The relationship between modifying factors and the definition of cut-off grade is fundamental in mining operations (Mugwagwa, 2017).

The determination of the cut-off grade is directly influenced by these modifying factors. For instance, higher commodity prices may lower the cut-off grade, making more material economically viable, while higher operating costs may increase it. Efficient mining and processing techniques can lower the cut-off grade by enabling the economic processing of lower-grade material. Additionally, strong financial backing can sustain a lower cut-off grade, whereas limited funding may necessitate a higher one to ensure profitability. Social and environmental considerations, such as stricter regulations, may also impact the cut-off grade to minimize waste and environmental impact.

Definition of cut-off grade is particularly important and sensitive in open pit mining where large volumes of material in different grade classes are handled.

2.3 Cut-off Grade in Open Pit Mines.

Understanding the nature of the operation system in open pit mining is important for successful selection of cut-off grade. Figure 2.1 shows the layout of an open pit mine system containing three main components, namely: mining, processing plant, and refinery. The mine produces ore and waste material. The ore product is transported to the processing plant and waste material to the waste dump. At the processing plant, ore material is either stockpiled or processed immediately. The refinery receives the processed ore from the plant and transforms it into the final product specification (Asad, et al., 2016).

Cut-off grade, along with other factors, provide measure and advice of material flow from one point of the mining system to the next. Cut-off grade defines (ore or waste) material that can be extracted from the mine, the quantity and average of the ore material to be processed or stored and the quantity of metal to be refined (Asad, et al., 2016). Figure 2.1 below shows a layout of an open-pit mining system.

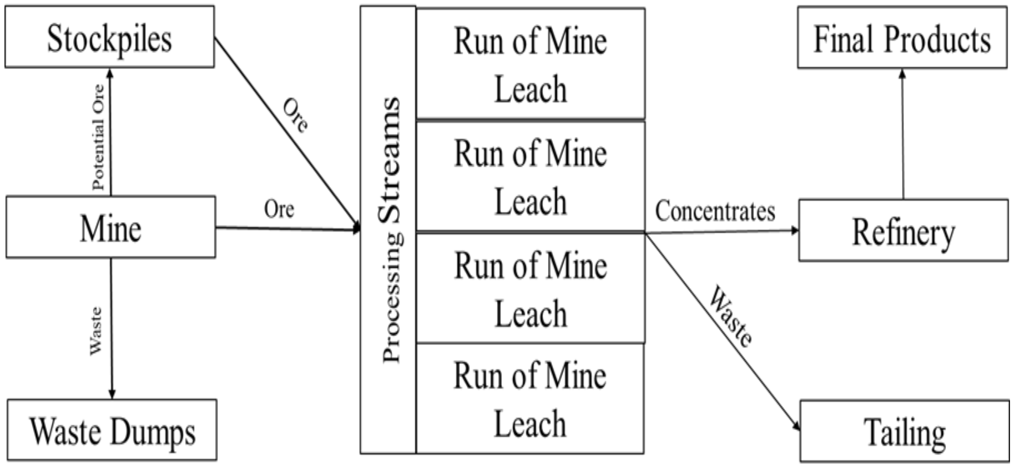

Figure 2.1: Layout of an open-pit mining system (Asad, et al., 2016)

The need to carefully decide the value of the cut-off grade further emphasizes the complexity of the process. Whether during pit optimisation or operational processes, it is important to determine the optimum cut-off grade that can assist to maintain the mine profitably, maximise the use of available resources, and consider all the requirements of the mine operation stakeholders.

2.4 Types of Cut-off Grade

Different types of cut-off grades have been defined according to the level of operations and according to the purpose on open pit mine. Mugwagwa (2017) explains that in an open pit mine, cut-off grade can be classified into two groups, namely external cut-off grade and internal cut-off according to the implementation stage during the life of mine.

The external cut-off grade is used during the optimisation stages and pit design to determine the overall size and shape of the open pit by identifying which blocks of material are economically viable to mine. Once the pit limits are established, the internal cut-off grade is applied to manage the material within these limits, ensuring that only the most valuable ore is processed.

Pasieka and Sotirow (1985) cited in (Kalitenge, 2021) pointed out that various cut-off grades are used in operational and strategic planning shown in Figure 2.2 (attached in Appendix D) . It is crucial to highlight that ore-waste demarcation is only used for operational purposes according to short-term goals. On the other hand Strategic planning, focuses on the whole life of mine project value and long-term objectives.

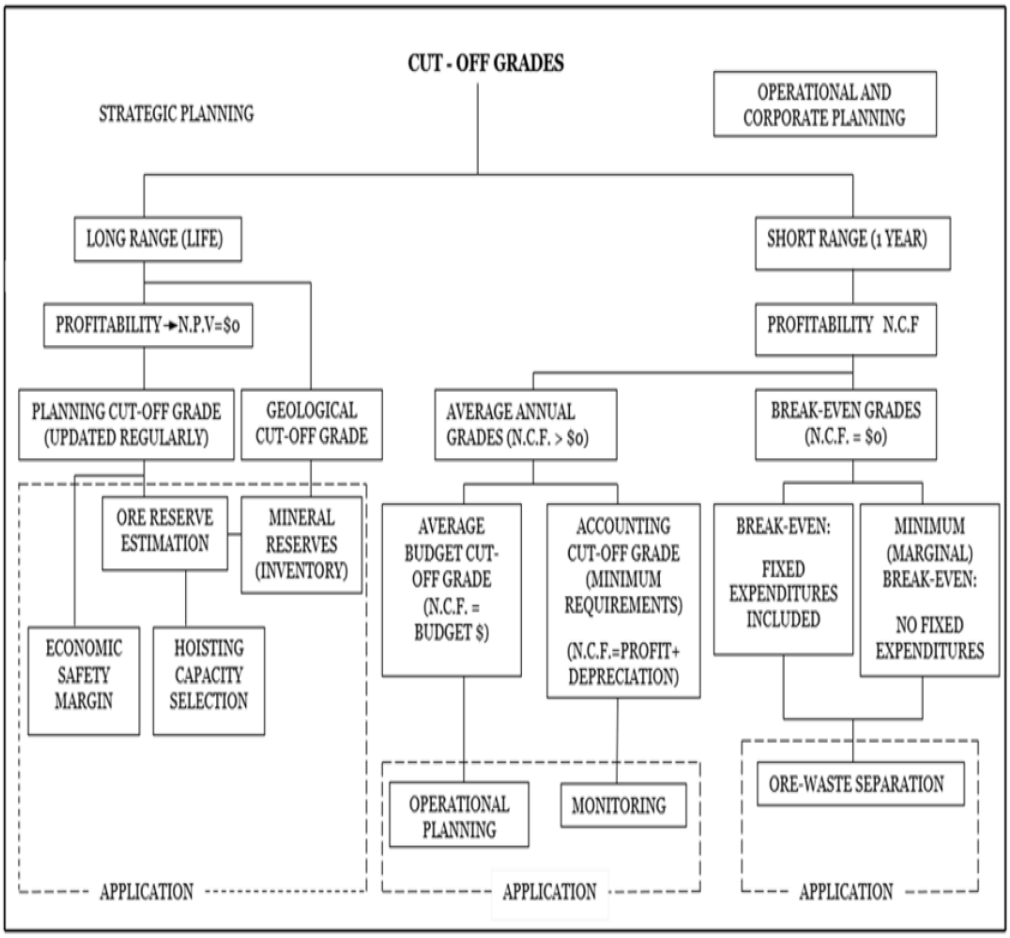

Figure 2.2: Cut-off grades and their respective purposes (Kalitenge, 2021)

Figure 2.3 illustrates different cut-off grades and where they stand in the planning cycle. Given that the success of each planning activity is linked to a specific cut-off grade, it is critical to determine which stage of the planning cycle is being followed to apply the appropriate cut-off grade. The formulae used in the calculation of several cut-off grades are given in Appendix A.

2.4.1 Break-even cut-off grade

The Queen’s University (2019) defines the break-even grade as the grade that the revenue covers all expenses. The University of the Witwatersrand (2019) extended the definition by mentioning that grade below which mining is unprofitable. Rendu (2014) defined as the grade at which the costs of producing a product equal the revenue obtained from the product.

The break-even grade used during pit optimisation to determine the pit limit by defining the ore and waste block within the orebody model. The block with grade above the break-even considers as ore otherwise is waste (Asad, et al., 2016).

2.4.2 Marginal cut-off grade

According to University of the Witwatersrand (2019), marginal cut-off grade is defined as the minimum original head grade of ore material brought for further processing, from which the net revenues after sales of the product will cover the marginal or direct cost of mining, treatment and marketing the product within a given time frame.

Marginal cut-off grade, also known as the milling cut-off grade, applies when there is excess milling capacity in open-pit operations. Once the ultimate pit limit is defined, this cut-off grade helps determine whether a block should be mined and sent to the processing or to the waste dump (Kalitenge, 2021).

2.4.3 Geological cut-off grade

According to Mugwagwa (2017), geological cut-off grade is utilised for generating grade frequency distributions, which helps in distinguishing mineralised material from waste and plot grade-tonnage curves for the mineralized material. Kalitenge (2021) explained that Some mining companies use it to separate mineralized material from waste and to estimate mineral reserves that from a basis for determining ore reserves.

2.4.4 Planning cut-off grade

Mugwagwa (2017) states that the planning cut-off grade is used for strategic planning as well as long-term studies like feasibility studies. Kalitenge (2021) extends more by highlighting that the NPV of the mining operations must cover financial cost. Planning cut-off grade calculation considers that each tonnes with this grade must generate revenue sufficient to cover all capital, operating, and financial costs, plus taxes and royalties that will be paid from the operation during the entire mine life.

2.4.5 Accounting cut-off grade

According to Kalitenge (2021), this is the annual cut-off grade needed to generate sufficient revenue to cover all operating costs, depreciation of fixed assets, and the minimum required profit for a specified capacity. Mugwagwa (2017) notes that depreciation, being a non-cash cost, can negatively impact the operational cut-off grade strategy, ore reserves assessment, and the mine life.

2.5 Cost Related to Cut of Grade Calculation.

Kalitenge (2021) explained that a range of cut-off grades are designed to achieve a specific objective. What distinguishes these cut-off grades are the cost components during its determination. Therefore, it is important to develop a clear understanding of the nature and types of costs that should go into different cut-off grades calculations.

Mugwagwa (2017) highlighted that mining operation involves operating costs, where different costs are incurred from total rock movement (ore and waste) to the final product produced. To be able to accurately derive cut-off grades, it is important to categorise and properly allocate costs to all important stages of the operation.

Hall (2014) explained that once the pit limit has been defined, then the variable mining costs will be incurred for all the rock mined within the definitive pit limit, no matter the grade.

An ultimate decision must be made on which mining block is to be sent to the waste dump or for further processing. Ore material that is economically capable of covering the entire operating costs shall be hauled to the process stream, while all other material shall be sent to the waste dump (Asad, et al., 2016).

According to Dassault Systèmes Geovia (2016), when conducting pit optimisation, it is necessary to calculate the expected costs associated with mining, processing, rehabilitation, and selling. These costs can be expressed as mining cost per tonne, processing cost per tonne, rehabilitation cost per tonne, or selling cost per unit of product produced.

Mining costs contain various elements such as labour, equipment, fuel, maintenance, surveying, geological services, grade control, drill and blast operations, loading and haulage, as well as assaying and sampling. These costs are typically referred to as operating cost per mined tonne.

Processing costs, as described by Dassault Systèmes Geovia (2016), include crushing and grinding, maintenance, mill services, stockpile management, and treatment plant reagents. They are commonly expressed as cost per milled tonne.

Additionally, rehabilitation costs encompass expenses related to waste dump rehabilitation, measures to address acid mine drainage, and revegetation. These costs are typically expressed as cost per milled tonne.

Dassault Systèmes Geovia (2016) explains that selling costs can be expressed as cost per unit of product produced, which includes transportation, insurance, marketing, refining, and smelting expenses.

Furthermore, as emphasised by Dassault Systèmes Geovia (2016), time costs, also known as general and administration costs, include expenses such as accommodation, administration salaries, communications, legal services, safety, and training. These costs need to be allocated among the block values, with the allocation depending on whether the operation is limited.

Dassault Systèmes Geovia (2016) explains that If production is limited by processing capacity, general and administration costs are included in the milling tonnes. This means that when the processing plant is the bottleneck, these costs are distributed over the processed ore, ensuring they are accurately reflected in the cost per milled tonne. Conversely, if the operation is limited by other factors such as mining rate or market demand, the allocation of these costs might differ, potentially spreading over the total mined tonnes or another relevant metric. This approach helps capture the financial impact of these expenses on the overall cost structure of the mining operation

Kalitenge (2021) highlighted that Operating costs are a category of expenses incurred during the operation of a mine when handling rock, ore, and product. These costs are classified into two groups: fixed costs and variable costs.

Fixed costs are expenses that remain constant over a given period, regardless of the level of activity. They are expressed in US$ per unit of time, such as per year or per month. These costs must be paid irrespective of the production capacity or level of activity. examples of fixed costs include equipment depreciation, administration, salaries, marketing, interest expenses, business licenses, insurance, and permit fees (Kalitenge,2021).

In contrast, variable costs fluctuate in direct proportion to the level of activity. They are usually expressed in US$ per unit of production. Such as tonnes mined, tonnes processed, ounces produced, meters drilled, or kilograms of explosives consumed. Each unit handled incurs a corresponding unit cost, meaning that higher production capacity results in higher variable costs. Examples of variable costs include drilling, blasting, loading, crushing and grinding, flotation, filtering and shipping, smelting and refining. (Kalitenge , 2021).

Rendu (2014) highlighted that Capital Expenses (CapEx) refer to costs incurred during pre-stripping, plant construction, and the construction of permanent facilities. These costs are not used to establish the cut-off grade determination. Including CapEx may raise cut-off grade values, resulting in reduced reserves and making the operation uneconomical. Therefore, these costs only influence the cut-off grade during the feasibility study.

Sustaining CapEx, refers to the capital required to maintain and sustain the existing operations These costs need to be included in cut-off grade determination by adding them to an appropriate cost category. (Kalitenge , 2021)

Meredith (2021) explained that It’s important to understand the costs involved in cut-off grade calculations and how these costs relate to different mining activities, and how they are recorded. Errors in cost estimation are relatively frequent, and incorrect cost allocations could mean that incorrect mining decisions are made.

Meredith (2021) pointed out that, prior to 2013, nearly every mining operation classified operating costs differently. To react on this, Leading gold producers, through the World Gold Council (WGC), developed a more representative cost metric for reporting, known as All-In Sustainable Costs (AISC) and All-In Cost (AIC).

The WGC has defined the cost elements reported under these two cost metrics, thereby, they have provided consistency into the costs associated with producing one ounce of gold. Table 1 shows the costs associated with AISC and AIC.

Table1: World Gold Council classification of costs (Meredith, 2021)

Cost Category	Source	US $ / gold ounces sold
On-site mining and processing costs (on a sales basis)	Income Statement	(a)
On-site general and administrative costs	Income Statement	(b)
Royalties and production taxes	Income Statement	(c)
Realised gains and losses on hedges of operating costs	Income Statement	(d)
Community costs related to current operations	Income Statement	(e)
Permitting costs related to current operations	Income Statement	(f)
3rd party smelting, refining and transport costs	Income Statement	(g)
Non-cash remuneration (site-based)	Income Statement	(h)
Stockpile, leach pad and product inventory write-downs	Income Statement	(i)
Operational Stripping Costs	Income Statement	(j)
By-product and co-product credits (*Note: will be a credit*)	Income Statement	(k)
Sub-total (Adjusted operating costs)		(l) = (a)+(b)+(c)+(d)+(e)+(f)+(g)+(h)+(i)+(j)+(k)
Corporate or regional general and administrative costs, including share-based remuneration (sustaining)	Income Statement	(m)
Reclamation & remediation – accretion & amortisation (operating sites)	Income Statement	(n)
Exploration and study costs (sustaining)	Income Statement	(o)
Capital exploration (sustaining)	Cash Flow	(p)
Capitalised stripping & underground mine development (sustaining)	Cash Flow	(q)
Sustaining capital expenditure	Cash Flow	(r)
Sustaining leases	Cash Flow	(s)
All-in Sustaining Costs		(t) = (l)+(m)+(n)+(o)+(p)+(q)+(r)+(s)
Growth and development costs <u>not</u> related to current operations	Income Statement	(u)
Community costs <u>not</u> related to current operations	Income Statement	(v)
Permitting costs <u>not</u> related to current operations	Income Statement	(w)
Reclamation and remediation costs <u>not</u> related to current operations	Income Statement	(x)
Exploration and study costs (non-sustaining)	Income Statement	(y)
Capital exploration (non-sustaining)	Cash Flow	(z)
Capitalised stripping & underground mine development (non-sustaining)	Cash Flow	(aa)
Non-sustaining capital expenditure	Cash Flow	(bb)
Non-sustaining leases	Cash Flow	(cc)
All-in Costs		= (t)+(u)+(v)+(w)+(x)+(y)+(z)+(aa)+(bb)+(cc)

Meredith (2021) said that AIC covers all expenses, including non-mining costs, whereas AISC covers all costs associated with mining activities excluding non-mining costs.

2.6 Challenges Associated with Determining the Optimum Cut-Off Grade

Biwas et al. (2023) outlined several challenges and limitations associated with determining the optimum cut-off grade such as complexity of deposits that may require more detailed procedures for evaluating optimal cut-off grades due to the variability in grade-tonnage distribution.

In addition, Biwas et al. (2023) highlighted analytical method limitations by pointing out the subjectivity and heuristic nature of some methods which may not provide a clear, objective solution for determining the optimal cut-off grade.

Biwas et al. (2023) explained the challenges associated with determining the optimum cut-off grade by emphasising that the dynamic nature of mining operations, including changes in metal prices, processing capabilities, and market demand. The volatility of commodity prices and economic factors can introduce uncertainty into the determination of the optimal cut-off grade.

Biwas et al. (2023) point out other challenges associated with determining the optimum cut-off grade is consideration of environmental and social aspects as well as incorporating additional cost components such as stockpiling options.

2.7 The Cut-off Grade Dilemma

There are several stakeholders for each mining operation, and their interests may differ from those of the company's operating policies. Given that the cut-off grade policy has a strong connection with stakeholder requirement, the interests of all stakeholders should be weighed carefully in the process of determining cut-off grades. The main stakeholders affecting the selection of cut-off grade are shown in Figure 2.3.

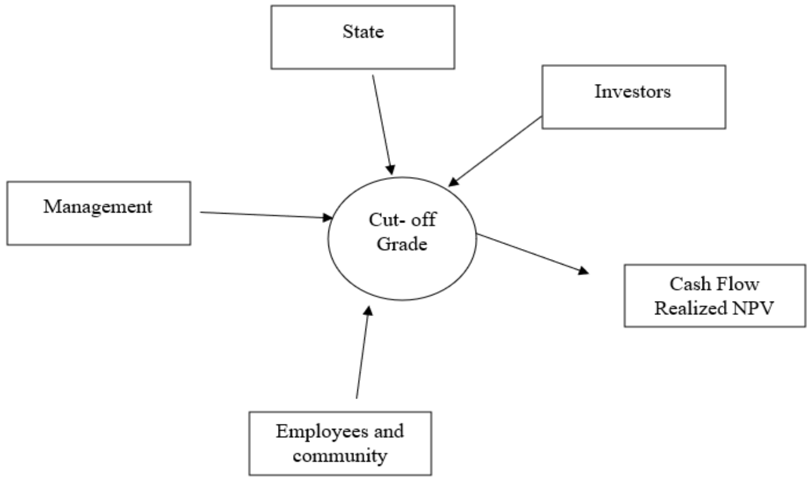

Figure 2.3: *Stakeholders in the cut-off grade determination* (Mugwagwa, 2017)

As it can be observed in the figure above, different stakeholder intersected with the selection of cut- off grade policy. The main stakeholders can be classified in four groups : investors, mine management, state, mine employees and the community.

Sinding and Larsen (1995), cited in (Mugwagwa, 2017) explain that mine employees may adopt lower than optimal cut-off grades in order to extend the life of the mine for longer term employment, however, maximising the life of mine (LOM), may lead to a minimum accepted profitability percentage and higher risk exposure.

Maximising the NPV is the main drive for the investor, where the relationship between cut-off grade and NPV provides a means by which cut-off grades can be optimised (Minnitt, 2004). A high cut-off grade enhances short-term profitability and maximises the project's NPV, benefiting the investor (Mugwagwa, 2017). Although this strategy may be advantageous in the short term, it could lead to a shorter mining operation. This could result in lower long-term economic benefits due to reduced employment and decreased support for employees and the local communities.

Mugwagwa (2017) explained that the state intended to maximise direct revenue, which is the combination of income tax and mineral resource royalties, in such cases a cut-off grade policy which maximises the revenue is preferred.

2.8 Cut-off Grade Optimisation

The policy adopted by a mining operation regarding cut-off grades ultimately determines its profitability and the mine life. It is therefore important to determine the optimum cut-off grade to avoid the negative impact of either overestimating or underestimating.

Hall (2014) explained that during pit optimisation, a higher cut-off grade can lead to value destruction since valuable material can be left as waste, while a lower cut-off grade can destroy the value by adding unprofitable material as shown in Figure 2.4

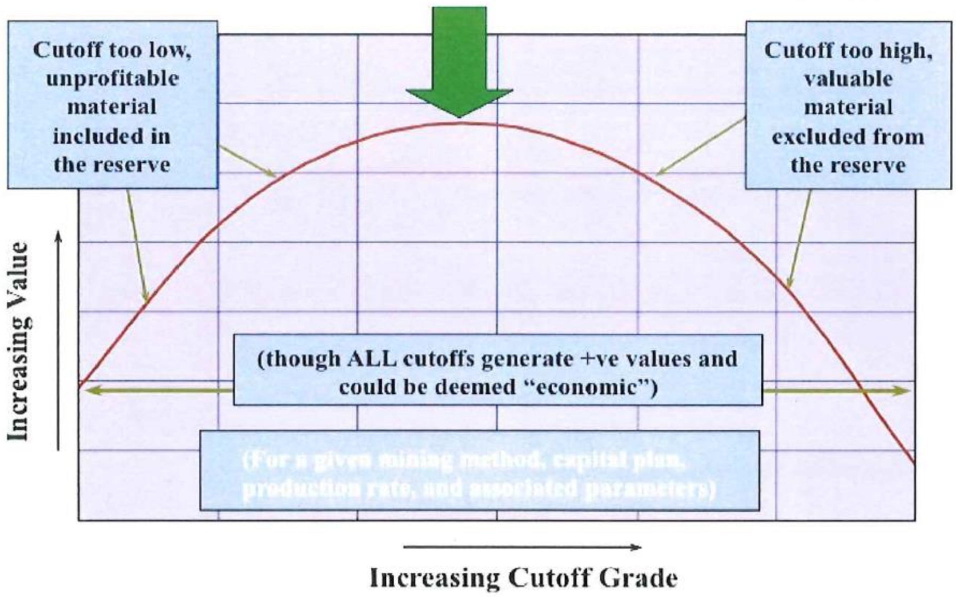

Figure 2.4: Change in value with change in cut-off grade. (University of the Witwatersrand, 2019)

Once the pit limit is defined, low cut-off grade may result in the processing of materials that do not yield high profits especially at the beginning of the life of the mine. This therefore lowers the value of the mining company, while high cut-off grade can reduce the life of the mine.

Mugwagwa (2017) highlighted that once the pit limit has been defined, cut-off grade optimisation is conducted on the material within the pit to determine a strategy that maximises profits and ensures efficient operation. The optimisation of cut-off grade considers factors that significantly impact revenue, such as the dynamic nature of the cut-off grade, the time value of money, and opportunity cost.

More recently, Whittle (2020) argued that cut-off grade optimisation must include other factors involved in an entire mine planning process. These factors include mine scheduling, designs, stockpile, and recovery.

Hall (2014) points out that, for an effective operation, it is important to incorporate all the key parameters simultaneously at their optimal levels. According to Hall (2014), the production rate creates the third dimension, which differs from the two-dimensional approach that considers the cut-off grade and value. This technique produces a three-dimensional surface with a peak, known as a hill of value, as illustrated in Figure 2.5

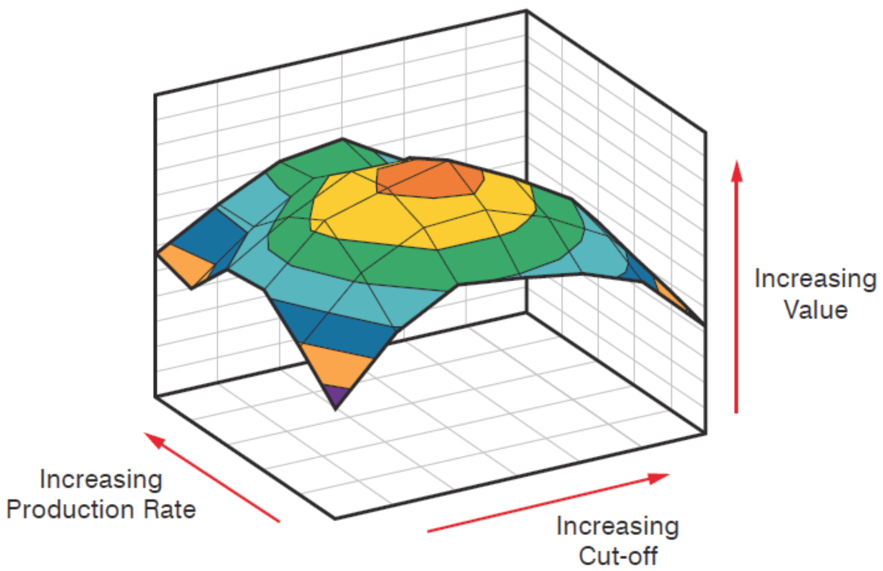

Figure 2.5: A typical underground hill of value versus cut-off and production targets (Hall, 2023).

Figure 2.6 illustrates the relationship between cut-off grade, production rate and the NPV of a mining project. The figure shows a curve that peaks at the optimal cut-off grade, where the NPV is maximised. This peak represents the point at which the balance between costs and revenues is optimised, ensuring the highest profitability for the mining operation. By adjusting the cut-off grade to this optimal point, mining companies can significantly improve their financial performance. (Hall, 2023).

With the implementation of cut-off grade optimisation, Rendu (2014) pointed out that since the cut-off grade expresses the profitability and the life of the mine, a high cut-off grade can grow short-term profitability and NPV but shorten the mine life. Birch (2022) also expanded on this idea, emphasising that cut-off grade optimisation can drive decision makers to prioritise short-term gains over long-term considerations.

Cut-off grade optimisation can achieve higher financial returns in a shorter time. However, this can lead to the waste of low-grade materal that could be processed later at the end of mine life when the higher-grade ore is depleted, especially if there is no option to stockpile it for later processing . Therefore, incorporating the stockpile in cut-off grade optimisation is recommended (Rendu 2014) .

The optimised cut-off grade initially starts at values higher than the break-even grade. This means that ore with marginally economic grades can be processed later when the higher-grade ore is depleted. This material, with grades ranging from the optimal grade to the break-even grade, can be stockpiled for future processing.

Whittle (2020) pointed out that when there is an option to stockpile low-grade material for later processing, the negative consequences of raising the cut-off grade are reduced. If stockpiling is implemented, the overall life of the mine will not be affected since all the material above the marginal cut-off grade will eventually be processed.

2.9 Overview of Cut-Off Grade Approaches and Optimisation.

Several techniques and approaches have been developed to optimise the cut-off grade. Biwas et al. (2023) classified them into two categories: conventional and non-conventional methods. The conventional methods are further divided into four major sub-categories: analytical modelling, integer programming, linear programming, and nonlinear programming. Non-conventional methods, on the other hand, fall under the broad classification of evolutionary algorithms.

Biwas et al. (2023) point out that the conventional methods for determining the optimum cut-off grade in mining operations involve various analytical and mathematical approaches.

2.9.1 Analytical approaches

These methods involve the use of mathematical and analytical models to optimize the cut-off grade based on economic and technical considerations. It provides a theoretical framework for determining the optimum cut-off grade.

Biwas et al. (2023) mentioned that these methods have been used for a long time in the mining industry. However, some of these methods are subjective in nature, leading to potential biases in determining the cut-off grade. In addition, the derivation and application of these methods can be heuristic, meaning they rely on experience and intuition rather than rigorous analysis.

Linear Programming

These methods provide a mathematically rigorous approach to optimising cut-off grades based on defined constraints and objectives. However, it is important to recall that these models can become complex, especially when considering multiple variables and constraints, leading to challenges in practical implementation (Biwas, et al., 2023).

Nonlinear programming

Biwas et al. (2023) explained that these methods can be applied to optimize the cut-off grade based on various factors such as mineral prices, production costs, and resource recovery. Biwas et al. (2023) adds that while nonlinear programming offers significant advantages in modelling complex systems and finding optimal solutions, it also presents challenges related to computational complexity, convergence, and algorithm selection.

Integer programming

These methods are designed to optimise cut-off grades based on multiple metal deposits. while integer programming offers the advantage of finding optimal solutions for discrete decision problems, it also presents challenges related to computational complexity, convergence, and intractability for certain problem instances (Biwas, et al., 2023).

Biwas et al. (2023) explains that the conventional methods for determining cut-off grades offer established and theoretical approaches but may suffer from subjectivity, heuristic nature, limitations in addressing variable costs, and computational complexity. Each method has its strengths and weaknesses, and the choice of approach depends on the specific mining context and the factors that need to be considered in the determination of the optimum cut-off grade. Table 2 provides overview of different methods, highlighting their key characteristics, advantages, and disadvantages.

Table 2 Overview of different analytical approaches, key characteristics, advantages, and disadvantages.

Approach	Description	Advantages	Disadvantages
Linear Programming	Uses linear equations to model the optimisation problem.	Simple to implement, efficient for large problems.	Limited to linear relationships, may not capture complex scenarios.
Nonlinear Programming	Models the optimisation problem using nonlinear equations.	Can handle more complex relationships and constraints.	Computationally intensive, may require sophisticated algorithms and software.
Integer Programming	Uses integer variables in the optimisation model.	Suitable for discrete decision variables, can model complex constraints.	Computationally challenging, especially for large-scale problems.
Dynamic Programming	Breaks down the problem into simpler subproblems and solves them recursively.	Can handle complex, multi-stage decision processes.	Requires significant computational resources, difficult to implement.
Genetic Algorithms	Uses evolutionary techniques to find optimal solutions.	Can handle complex, nonlinear, and multi-objective problems.	Requires careful tuning of parameters, may not always find the global optimum.

2.9.2 Non-conventional approaches

Non-conventional approaches for cut-off grade optimisation offer innovative methods for addressing the complexities of real-world mining operations. These approaches often involve heuristic, trial-and-error-based, or nature-inspired techniques to find optimal cut-off grade. Methods such as genetic algorithms, particle swarm optimisation, Imperialist competitive algorithm, simulated annealing, hybrid methods and machine learning techniques have received a lot of attention in recent years (Biwas, et al., 2023).

Non-conventional approaches offer heuristic, nature-inspired, and global search capabilities for cut-off grade optimisation in mining, but they also present challenges related to computational intensity, convergence, algorithm selection, and theoretical foundation (Biwas, et al., 2023).

Githiria and Musingwini (2018) provide other classification for comparing various cut-off grade approaches in mine planning that improve value generation based on NPV. They determined that the various approaches might be classified into the following main categories:

- Break-even cut-off grade approach.
- Heuristic cut-off grade approach; and
- Lane's optimum cut-off grade approach.

2.10 Break-Even Cut-Off Approach

Traditionally, the determination of cut-off grade used break-even approach. This approach calculated cut-off grade by comparing costs and benefits of the operation.

The Break-even approach provides a basic understanding into the calculation of cut-off grade, it can be used during pit optimisation to determine the pit limit, but during the operation, this approach is unable to deliver the optimum value of cut-off grade.

Pettingell (2017) explains this by mentioning that break-even approach does not maximise the value generated by the operation since it only considers price and costs and ignores other factors such as variability in the ore body and operational capacities. Moreover, Pettingell (2017) demonstrates that neglecting such factors can lead to lower than optimum cut-off grades that have a negative impact on mine efficiency and profitability.

2.11 Heuristic Cut-off Grade Approach

There are several approaches, including swarm intelligence and genetic algorithms. Usually employed for addressing decision problems quickly and where exact accuracy is not crucial (Githiria & Musingwini, 2019).

Heuristic approaches aim to optimise the (NPV). In the early phases of a project, cut-off grade optimisation typically results in higher cut-off grades compared to those calculated using the break-even approach. This increase is attributed to the need to recover the initial capital expenditure. As the project progresses, the cut-off grades tend to decrease, eventually aligning with or even falling below those determined by the break-even approach (Githiria & Musingwini, 2018; Birch, 2022).

Dagdelen (1992), as cited in Birch (2022), points out that these approaches are particularly effective for managing complex, multi-variable optimisation problems and offer more flexible and adaptive solutions. However, they may not yield the optimal cut-off grade because they do not account for capacity constraints and opportunity costs.

2.12 Lane's Optimum Cut-off Grade Approach

As the opposite of the break-even approach, Lane’s approach presented the cut-off grade as a dynamic value that must be optimised to maximise the value generated by the mining operation. In addition, Pettingell (2017) explains that to maximise the profit from the resource, the opportunity cost and time value of money concept should be considered in cut-off grade calculation.

Githiria & Musingwini (2019) highlighted that since Lane published his pioneering work in 1964 and subsequent textbook in 1988 in the field of determining optimum cut-off grade for a mining operation, his approach has been the most common one in the literature.

Githiria & Musingwini (2018) points out that, in his approach, Lane divided the mining operation into three compounds namely mine, concentrator and refinery. He observed that cut-off grade is not only a function of the grade distribution but is also constrained by the capacities of the various interdependent components.

Githiria & Musingwini (2018) extend more by mentioning that, each of these three components has its own limiting capacity and associated costs, therefore, their own optimum cut-off grade. In addition, Lane explained that the whole operation has a set of fixed costs, which do not necessarily relate to any one stage, or the level of production. Figure 2.8 presents mining operation compounds and material flow as shown in Lane's approach.

Material		**Maximum (capacity)**	**Unit (costs)**
	↓		
Ore	**MINE (M)**	**M**	**m**
	↓		
Concentrare	**CONCENTRATOR (C)**	**C**	**c**
	↓		
Product	**REFINERY (R)**	**R**	**r**
	↓		
Market		**Other factors: Fixed costs f, selling price s, recovery y**	

Figure 2.6: Mining operation components and material flow on Lane's Model (Githiria & Musingwini, 2018)

Figure 2.8 demonstrates how ore flows through these mining operation components on Lane's approach and highlights other related costs, such as fixed costs, as well as factors involved in the calculation, including selling price and recovery.

Pettingell (2017) explains time value of money concept in Lane's approach by saying that, processing low grade ore today may reduce the potential future value of high-grade ore if processed later. Therefore, by prioritising mining high cut-off grade ore early in the life of the mine enables the mill to process higher value material earlier.

Mugwagwa (2017) explained in every operation, there exists a bottleneck; one stage or another in the operation process may have limited capacity.

The size of a mining operation is determined by the quantity of the deposit and the extraction rate, which are linked to the operational bottleneck. When an operation encounters a bottleneck, which is a constraint in capacity, opportunity costs are incurred

Mugwagwa (2017) extends more by mentioning that Lane’s approach incorporates geostatistical techniques to approach the grade distribution within the deposit accurately. By accounting for the variability and uncertainty of ore grades, it provides a more realistic assessment of the economic potential and risk associated with different cut-off grades.

Minint (2004) explained that the NPV criteria is frequently cited as the main indicator of economic value in mining operations. In this regard, Lane’s approach provides this optimisation criteria where it seeks to maximise the NPV or profit of the mining operation. By considering the time value of money, it considers the cash flows associated with mining and processing activities over the life of the mine. This optimisation criterion helps identify the cut-off grade that maximises the long-term profitability of the operation.

Biwas et al. (2023) supports Lane’s approach by highlighting that Lane employs an incremental approach, evaluating the economic viability of different sections or blocks within the deposit. This allows for more accurate determination of the cut-off grade by considering the spatial distribution of ore grades and associated costs.

The Lane's approach allows for iterative refinement of the cut-off grade. Mining companies can update and adjust the parameters as new information becomes available, such as changes in metal prices, operational costs, or geological understanding. This iterative process helps ensure that the cut-off grade remains optimized throughout the life of the mine.

The categorisation of the mining operation and the inclusion of the opportunity cost and time value of money developed by Lane makes his approach receive the most attention for the determination of the optimum cut-off grade.

Despite its comprehensive structure and widespread acceptance, several researchers have identified notable limitations in Lane's approach. Dagdelen (2008) argues that Lane's approach has a significant shortcoming because it does not consider stockpile material and the blending process. Stockpiling allows for the storage of ore that may not be immediately processed, helping manage fluctuations in ore grade and ensuring a steady supply to the processing plant. Without accounting for stockpiles, Lane's model may fail to optimise the cut-off grade effectively.

Dagdelen (2008) explains that blending different grades of ore is crucial for maintaining consistent feed quality and processing efficiency. By ignoring blending, Lane's model might not accurately reflect the true economic potential of the ore, resulting in variations in feed quality and processing efficiency.

Githiria and Musingwini (2019) observe that Lane’s approach is unable to determine the optimum cut-off grade for polymetallic deposits. This limitation arises because Lane's model primarily focuses on single-metal deposits and does not account for the complexities associated with multiple metals.

Biswas (2020) notes that in the event of an aggressive increase in the selling price, Lane’s approach does not provide a clear selection of the optimum cut-off grade. This can lead to indecisiveness and suboptimal economic outcome. Despite these shortcomings the comprehensive structure of the Lane’s approach is becoming a reason for large-scale acceptance of the mining operation.

Lane’s approach has been modified in several studies to address its deficiencies and incorporate different mine scenarios. These modifications have positively impacted the accuracy of determining the optimum cut-off grade. Table 2 shows some of the studies addressing several shortcomings and suggest various scenarios to enhance the accuracy of Lane’s approach. Githiria & Musingwini (2018) pointed out that these studies succeed to improve Lane's approach accuracy, as they address the most techno-economic variables of the mining operation.

Table 3: Example of studies addressing some shortcomings and suggestions for some improvements in Lane's approach.

Research	Summary of improvement
(Asad, 2005)	Asad developed a cut-off grade optimisation algorithm based on Lane's work, but with a stockpiling option for open-pit mining operations with two economic minerals (Githiria & Musingwini, 2019)
(King, 2011)	Look to consider the operational and administrative costs into Lane's Model (Githiria & Musingwini, 2019)
(Maleki, 2016.)	Propose a new model consider the blending process based on Lane's work.
(Zarshenas, Y & Saeedi, G 2017)	Used Lane's algorithm and demonstrated that with an increase of dilution there was a decrease in average grade and consequently an increase in cut-off grade.

Githiria & Musingwini (2018) highlighted that some of the studies are now incorporated into some of the existing mine planning and optimisation software such as Maptek Evolution®, OptiPit®, and Geovia Whittle 4X®.

Ad'Duwayhi mine has experienced a period of high-grading and reduced waste stripping, leading to a reduction in the grade of the remaining ore and an increase in the cost of waste stripping backlog. This situation necessitates cut-off grade optimisation to address these issues.

To address the problem, various approaches for cut-off grade optimisation were considered. Lane's approach was selected as it provides a comprehensive framework that balances the capacities of mining, milling, and processing stages to maximise NPV. As well as it defines three balancing cut-off grades to balance the capacities of each pair of stages. This dynamic nature allows for adjustments as the mining operation progresses, making it suitable for varying conditions.

Minnitt (2004) supports that by explaining that Lane’s approach is widely recognised as the most suitable and consistent approach for representing the sequential cash flows associated with extracting mineral reserves from a finite resource.

With the rapid depletion of high-grade ore, Ad'Duwayhi mine is left with low-grade ore. This situation encourages the adoption of Lane's approach, as it incorporates dynamic programming techniques to ensure an optimum cut-off grade is selected at all points in time. Pettingell (2017) supports this by explaining that Lane's approach provides an elegant and straightforward method to optimise cut-off grades in mining operations, particularly in scenarios where ore grades are low.

2.13 Chapter Summary

This chapter reviews the basic concepts and definitions of cut-off grades. highlighting the importance of optimising the cut-off grade and link with the NPV. Further, it reviews the different cut-off grade optimisation approaches in the literature.

The break-even cut-off grade approch has been widely used in many operations to discriminate between ore and waste. The cut-off grade established by this approch ensures every ton of ore mined pays for itself. It is commonly used during pit optimisation to determine the pit limit, but during the operation, this approach is unable to deliver the optimum value of cut-off grade. To address this limitation, Lane's approach has been selected as an optimisation approch to verify the opportunity to achieve the optimum cut-off grade.

Chapter 3 will present how optimum cut-off grade is determined in the Ad'Duwayhi gold mine. These approaches are then compared to the resultant cash flows.

3 METHODOLOGIES

3.1 Chapter Overview

This chapter describes the methodology that has been used for the research. The cut-off grade optimisation approaches selected are presented in this chapter. The break-even cut-off grade calculated to show the impact of pursuing the break-even cut-off grade policy and Lane's approach applied as optimisation approaches which attempts to maximise the NPV.

3.2 Break-even Analysis

Pettingell (2017) explained that break-even cut-off grade is where the costs of producing a sealable product are equal to the revenue earned from that product.

Birch (2016a) mentioned that mining companies calculate cut-off grades to determine what proportion of a mineral deposit can be extracted profitably. The common method is break-even, where costs and benefits of the operation are compared.

Birch (2022) presents the break-even cut-off grade formula as follows:

At break-even cut-off grade:

$$units/t = ((TFC \div X) + UVC) \ / \ (\mathrm{y} * S) \ \ldots\ldots\ldots\ldots\ldots\ldots \ (3.1)$$

Where: $\mathrm{units/t}$ is the break-even cut-off grade in units per tonnes,

TFC = Total fixed costs,

UVC = Unit variable cost,

X = Units of production.

y = Mine recovery factor in %, and

S = Metal price in US$ per unit.

But:

$$TR = TFC + TVC \text{ .... } (3.2)$$

where: TR = Total revenue in US$,

TFC = Total fixed costs, and

TVC = Total variable costs.

But:

$$TR = UR * X$$

And:

$$TVC = UVC * X$$

Where: UR = Unit revenue,

UVC = Unit variable cost, and

X = Units of production.

Therefore:

$$UR * X = TFC + UVC * X$$

Dividing both sides by X:

$$UR = (TFC \div X) + UVC$$

But:

$$UR = units/t * \mathrm{y} * \mathrm{S}$$

Where: $\mathrm{units/t}$ is the break-even grade in units per tonnes,

y = Mine recovery factor in %, and

S = Metal price is US$ per unit.

Therefore, at break-even cut-off grade.

$$units/t = ((TFC \div X) + UVC) \ / \ (\mathrm{y} * \mathrm{S})$$

The cost included in the calculation are based on a break-even principle, where the revenues earned are equal to the costs of producing. In this aspect, Asad (2016) stated that once an ultimate pit limit is defined, all mining blocks within the ultimate pit limit will be mined whether ore or waste, so the subsequent step involves determining if these blocks are suitable for processing plant or should be send to the waste dumps.

Asad (2016) explained that at this stage, a mining block that can cover the Concentrating, refining and sales costs shall be sent to the Concentrating Plant, otherwise, it shall be hauled to the waste dumps. Figure 3.1 demonstrates the break-even points for unit total and unit variable costs.

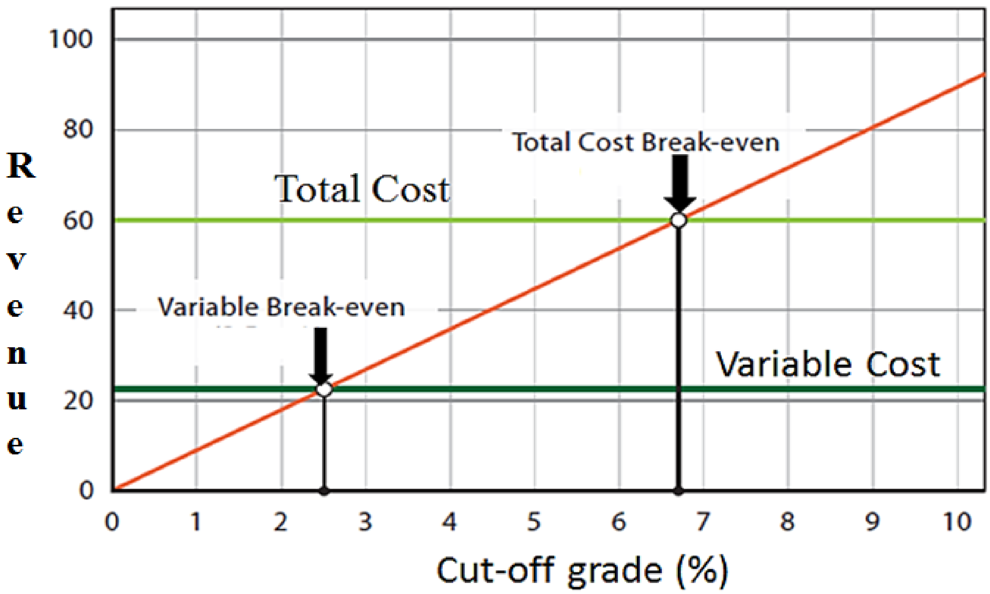

Figure 3.1: Break-even points for unit total and unit variable costs (Mugwagwa, 2017)

Figure 3.1 illustrates the relationship between total costs, variable costs, and revenue at different levels of production. The break-even point is where total revenue equals total costs, meaning there is no profit or loss. This point is crucial for understanding the minimum production level required to cover all costs (Mugwagwa, 2017).

This research is based on the Ad'Duwayhi 2018 pit designs. Therefore, calculating the overall break-even cut-off grade is not required at this stage. Since the pit limit already determines which material is economically viable to mine. Instead, it is required to calculate the break-even cut-off grade based on the processing cost (Gholamnejad, 2008).

Kalitenge (2021) highlighted that some of the noticeable features of the break-even cut-off approach include neglecting the grade distribution of the mineralistion and the capacity of the mining production system's components. Kalitenge (2021) explains that one of the serious consequences that mines can face by relying on the method that does not consider the geology of the orebody and capacities of different components of the mining system is the failure to establish the optimum cut-off grades that hinders the strategy to maximise the NPV of the operations. Furthermore, while the approach can be used to predict the profitability of the operations, there is no guarantee if the desired profit will be achieved.

3.3 Lane's Model Analysis

Determining the optimal cut-off grade based on a defined resource and the capacities that limit the extraction and production of that resource is a challenge in the mining industry (Gholamnejad, 2008).

To determine the optimum cut-off grades for a deposit, it is necessary to identify the optimal exploitation approach that serves the strategy of maximising the present value of an operation based on a finite resource and then connects cash flows to cut-off grades. In this regard, Lane proposed that value is a function of the size of the remaining resource and the rate of extraction (Pettingell, 2017).

Pettingell (2017) explained that the present value of the mining operation at any time is a function of the mine's life, the quantity of the remaining resource, and the rate of extraction.

Present value can be express as follows:

$$\mathrm{PV} = (\mathrm{T} + \mathrm{t}, \mathrm{Q} - \mathrm{q}) \qquad (3.3)$$

Where: PV = Present value in US$,

T = Life of the mine per years,

Q = Size of the remaining resource in tonnes, and

q = Rate of extraction tonnes per year.

t = Specific time period.

Differentiating PV with respect to Q and T,

$$\partial PV / \partial Q = P - t/q\ (\delta v - \partial Pv / \partial T) \ \ldots\ldots\ldots\ldots\ldots \ (3.4)$$

Where: P = The cash flow arising from one unit of resource,

t = Time it takes to process that unit, and

t/q = The tonnage is given by, which represents the time required to process one unit of mineralized material.

Another concept defined in Lane’s approach is the opportunity cost of mining at a specific extraction rate. It can be rewritten as F, and the formula that maximizes the present value of a mining operation is given by.

$$F = (iPV + (\partial PV / \partial T), \ \ldots\ldots\ldots\ldots\ldots \ (3.5)$$

Where: F = Opportunity cost per US$

i = Discount rate

Discount rate is multiplied by PV to reflect the decrease in value of the resource resulting from extraction, which is subtracted by the first derivative of the value PV with respect to time T.

Pettingell (2017) explained thet to conncet relates cash flows to cut-off grades, the formula for the cash flow arising from one unit of mineralized material can express as follows:

Pettingell (2017) explained that to connect related cash flows to cut-off grades, the formula for the cash flow arising from one unit of mineralised material can be expressed as follows

$$P = (S\text{-}r)\ (g\ y\ Q_c) - (mQ_m) - (cQ_c) - (fT) \ldots\ldots\ldots \ (3.6)$$

where: *P* = Total cash flow,

S = Commodity price,

r = Refining, transportation, of the saleable product,

g = Average grade (g/ton) of material sent for concentrating facilities,

y = Recovery of product from ore as a percentage,

Q_c = Quantity of ore sent for concentrating facilities,

m = Mining cost,

Q_m = Quantity of material mined (both ore and waste),

c = Concentrated variable cost,

f = Fixed cost per annum, and

T = Mine life in years.

The above formula determines the cash flow generated from concentrating one unit of mineralised material considering various costs and recovery rates.

Limiting economic cut-off grades

Lane's approach determines the optimum cut-off grade by considering both the limiting economic and the balancing cut-off grade. According to Githiria & Musingwini, (2018) the three limiting economic cut-off grades are the mining limiting cut-off grade (g_m), concentrator limiting cut-off grade (g_c), and refinery limiting cut-off grade (g_r). They are based on the capacities of the individual mining compound.

Kalitenge (2021) explained that the mining system throughput is limited by each stage's capacity. The mining rate, which determines the amount of material extracted from the mine, is influenced by factors such as the size of the truck fleet, the number of shovels, cycle time and the drilling equipment. In the concentrating stage, the throughput is constrained by either the ore handling facilities or the concentrating plant. Additionally, the refinery stage faces restrictions due to the sales contract or the limiting capacity of the refinery or smelter.

Each limited capacity has its own calculation considering the unit costs and the specified capacity for that system, these calculations also account for the opportunity cost of not mining the remaining of the deposit due to the limiting capacities of the mine, concentrating facilities, and the refinery (Pettingell, 2017). Therefore, depending on which area of the system is limiting, the time '*T* ' can be express as follows, If the mining capacity limits the system, the time '*T*' can be expressed as follows

$$T = Q_m / M \text{ ......} \quad (3.7)$$

Where: T = Mine life in years,

Q_m = Quantity of tonnes mined, and

M = Maximum annual capacity for the mine.

If the concentrator's capacity limits the system, the time '*T*' can be expressed as follows

$$T = Q_c / C \text{ ......} \quad (3.8)$$

Where: T = Mine life in years,

Q_c = Quantity of tons milled, and

C = Maximum annual capacity for the Concentrating plant.

If the refinery capacity is the limited the system, the time '*T*' can be expressed as follows,

$$T = Q_r / R \text{ ......} \quad (3.9)$$

Where: T = Mine life in years,

Qr = Quantity of ounces refined in each period, and

R = Maximum annual capacity of the refinery (market).

Kalitenge (2021) mentioned that the opportunity cost should be distributed per tonne of material mined, per tonne of ore concentrated, or per ounce of metal refined, depending on which component is limiting the mining operation.

Therefore, mine limiting economic cut-off grade can be expressed as;

$$g_m = c / (S - r)\, y \text{ ......} \quad (3.10)$$

Where: g_m = Mining limiting grade (grammes of product per tonne of ore),

c = Concentrating variable cost,

S = Price of a unit of product,

r = Refinery costs per unit of product sold, and

y = Recovery of product from ore as a percentage.

The formula for determining the mine limiting economic cut-off grade indicates that the mineralised material should be classified as ore since its retains its implicit value.

The Concentrate limiting economic cut-off grade can be expressed as.

$$g_c = (c + ((F+f)\ /C)\ /\ (S - r)\ y \ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \ (3.11)$$

where: g_c = Concentrated limiting grade (grammes of product per tonne of ore),

c = Concentrating variable cost,

f = Fixed cost per annum,

F = Opportunity cost,

C = Concentrating capacity,

S = Price of a unit of product,

r = Refinery costs per unit of product, and

y = Recovery of product from ore as a percentage.

Pettingell (2017) stated that for the Concentrating limiting equation, the opportunity cost indicates the additional time required to process the ore. This results in larger cut-offs, which are considerable in the early years of production, and smaller cut-offs as the opportunity cost declines with the resource. This realization exemplifies dynamic cut-off theory, in which cut-off grades change during the life of a mine to maximise the value of the material being mined.

Kalitenge (2021) States that the refinery limiting cut-off grade (g_r) can be expressed as follows.

$$g_r = c / ((S - r - ((f + F) / R)) * y \quad (3.12)$$

where g_r = Refinery limiting grade (grammes of product per tonne of ore),

c = Concentrating variable cost,

S = Price of a unit of product,

r = Refinery costs per unit of product sold,

y = Recovery of product from ore as a percentage,

f = Fixed cost per annum,

F = Opportunity cost, and

R= Refinery capacity per annum.

Pettingell (2017) explained that the refinery limiting equation incorporates the present value in the form of an opportunity cost, which along with the fixed costs is distributed according to the limiting capacity. As a result, cut-off grades decrease as opportunity cost declines due to resource depletion.

Three limited cut-off grades definned in Lane's approach, Githiria & Musingwini (2018) point out that these limiting cut-off grades change with changes in economic and technical parameters.

Balancing cut-off grades

The three balancing cut-off grades are the mine and concentrator balancing cut-off grade (g_{mc}), refinery and concentrator balancing cut-off grade (g_{rc}), and mining and refinery balancing cut-off grade (g_{mr}). They are based on if more than one compound limits the operation (Dadi and Sattarvand, 2016).

If both the mine and concentrate facility are limiting than cut-off grade g_{mc} is the grade that satisfies the equation

$$Q_m / M = Q_c / C$$

Where: Q_m = Quantity of tonnes mined,

M = Maximum annual capacity for the mine,

Q_c = Quantity of tons milled, and

C = Maximum annual capacity for the concentrating plant.

In case the concentrate facility and the refinery are the limiting factors then g_{cr} is the grade that satisfies,

$$\mathrm{Qc} / \mathrm{C} = \mathrm{Qr} / \mathrm{R}$$

Where: Q_c = Quantity of tons milled

C = Maximum annual capacity for the concentrating plant.

Q_r = Quantity of ounces refined in each period and

R = Maximum annual capacity for the refinery (market).

In case the mine and the refinery are both limited, then g_{mr} must satisfy

$$\mathrm{Qm} / \mathrm{M} = \mathrm{Qr} / \mathrm{R}$$

Where: Q_m = Quantity of tonnes mined,

M = Maximum annual capacity for the mine,

Q_r = Quantity of ounces refined in each period, and

R = Maximum annual capacity for the refinery (market).

Pettingell (2017) explains that the balancing ratio is observed when the proportion of mineralised material, recoverable mineral per unit of mineralised material, and recoverable mineral per unit of ore above the corresponding grade are applied to the cumulative grade distribution curve, graphical representation of g_{mc} is shown in Figure 3.2.

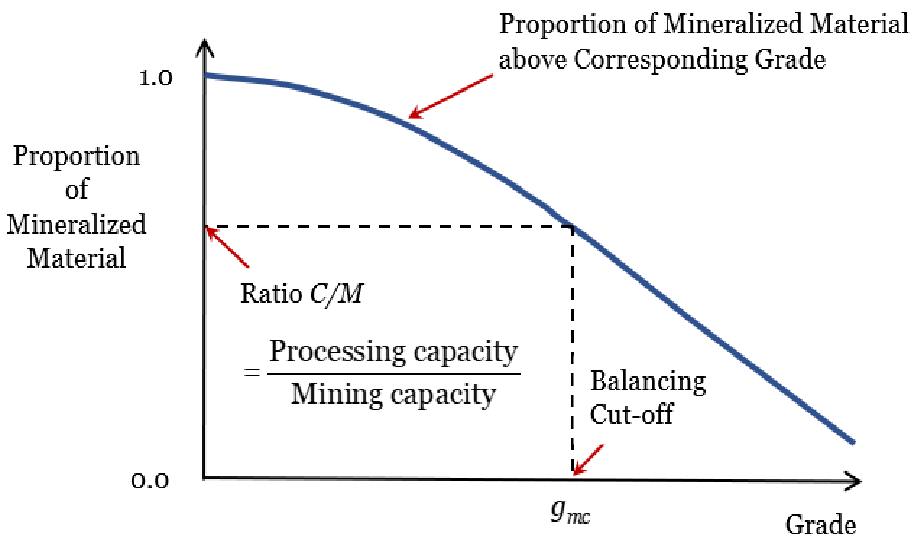

Figure 3.2 Graphical representation of balancing cut-off when the mine and the mill are limiting (Kalitenge, 2021).

As can obseved in figure above at this cut-off grade, both the mining and concentrating stages simultaneously operate at their maximum capacities.

Kalitenge (2021) highlighted that at specific point of the graph where the ratio of recoverable minerals to the total tonnes of mined material equals the ratio of refined material to tonnes of mined material, graphical representation of g_{mr} is shown in Figure 3.3.

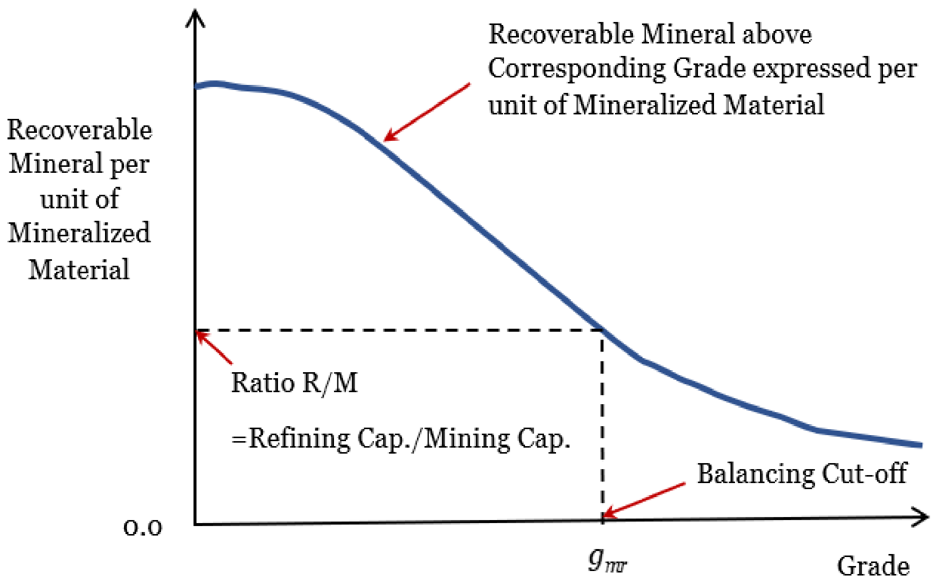

Figure 3.3 Graphical representation of balancing cut-off when the mine and the refinery are limiting (Kalitenge, 2021).

There is a point along the resulting line at which the ratio of recoverable mineral content to cumulative tonnage equals the ratio refinery and concentrate capacity. At this point, the two stages are said to be simultaneously operating at their maximum capacities, and the grade required to achieve this is called mill-refinery balancing cut-off grade and is denoted as g_{rc}.

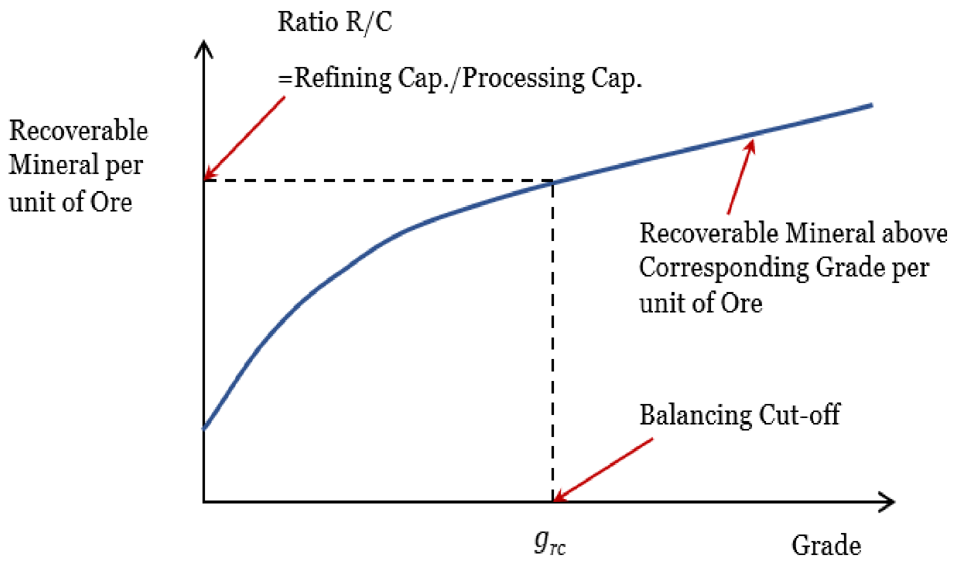

Figure 3.4 Graphical representation of balancing cut-off when the mill and the refinery are limiting (Kalitenge, 2021).

Therefore, to determine the effective optimum cut-off, six potential cut-off grades need to be evaluated: three break-even cut-off grades based on limiting capacity, and three balancing cut-offs that depend on which capacities are limiting the mining system.

Effective optimum cut-off grade

The optimum cut-off grade is selected from the limiting and balancing cut-off grades, Pettingell (2017) explained that since the cut-off grade corresponds to the present value of the mine and with the limiting and balancing cut-offs, the optimum cut-off grade can be determined by maximising the rate of change of present value.

In case the system is limited by the mine, the maximum change in the present values can be expressed by

$$V_m = (S - r)\, xyg_w - xc - m - ((f + F) / M), \ldots\ldots\ldots\ (3.13)$$

Where: V_m = Maximin change in the present value in case mine is limited the system in US$

c = Concentrate variable cost,

S = Price of a unit of product,

r = Refinery costs per unit of product sold,

y = Recovery of product from ore as a percentage,

f = Fixed cost per annum,

F = Opportunity cost,

R= Marketing capacity per annum,

M = Maximum annual capacity for the mine,

m = Mining cost,

x = Proportion of mineralized material classified as ore, and

g_w = Weighted average grade of the ore grammes per tonnes.

In case the system is limited by the Concentrating unit, the maximum change in the present values can be expressed by

$$Vc = (S - r)\, xyg_w - x\,(c + ((f + F) / C) - m \ldots\ldots\ldots\ (3.14)$$

Where: V_c = the maximin change in the present value in case Concentrate is limited the system in US$,

c = Concentrating cost,

S = Price of a unit of product,

r = Marketing costs per unit of product sold,

y = Recovery of product from ore as a percentage,

f = Fixed cost per annum,

F = Opportunity cost,

g_w = Weighted average grade of the ore grammes per tonnes,

x = The proportion of ore proceed, and

C = Maximum annual capacity for the Concentrating plant.

In case the system is limited by the Refinery, the maximum change in the present values can be expressed by

$$V_r = ((S - r - (f + F) / C))\, xyg_w - xc - m, \ldots\ldots\ldots\ldots \quad (3.15)$$

Where V_r = the maximin change in the present value in case refinery is limited the system.

c = Concentrating variable cost,

S = Price of a unit of product,

r = Marketing costs per unit of product sold,

y = Recovery of product from ore as a percentage,

f = Fixed cost per annum,

F = Opportunity cost,

x = The proportion of ore refined, and

C = Maximum annual capacity for the Concentrating plant.

Plotting V as a function cut-off grade, indicates that the graph is concave with a single maximum. The effective optimum cut-off grade becomes the peak point along the feasible v_e curve. V_e represents the maximin change in the present value in case refinery is limited the system. Figure 3.5 shows the increment in present value versus cut-off grade for three stages.

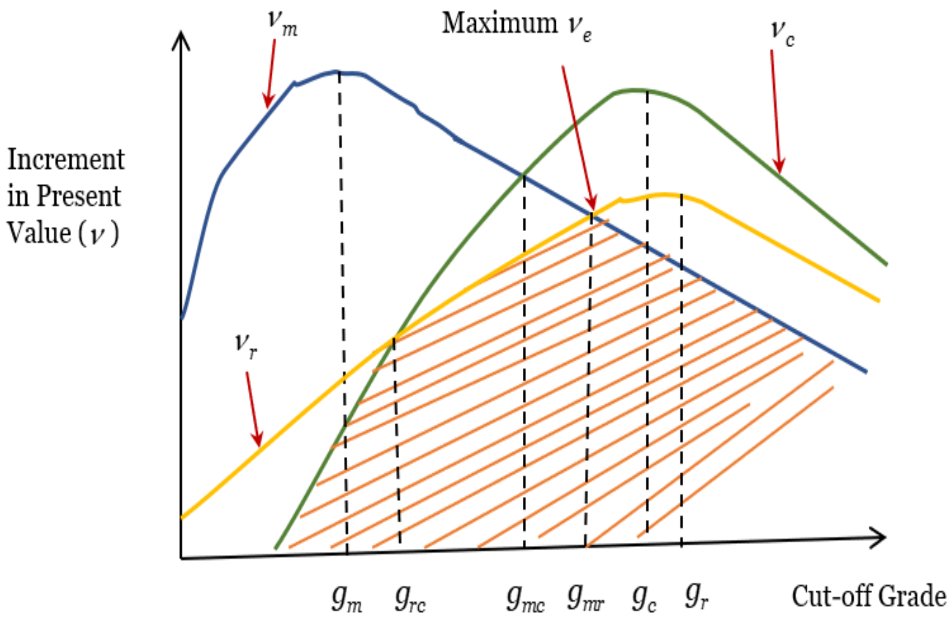

Figure 3.5 Increment in present value versus cut-off grade for three stages (Kalitenge, 2021)

The optimum cut-off grade of the six cut-offs is found by first finding the effective cut-off grade of each pair of the three mining components in balance, Then, three effective optimums cut-off grades for components limiting the operation in pair and the optimum cutoff grade (g_G) are determined (Kalitenge, 2021).

Asad and Topal (2011) explain that optimum cut-off grade may never be less than the limited mine cut-off grade since it is the lowest (break-even) cut-off grade, and it never exceeds a limited cut-off process, since this may schedule some of the valuable ore to the waste dumps.

Gholamnejad (2008) explained that the local optimums for each pair of operations are obtained using the following rules;

When the mine and the Concentrating facility are limiting

$$G_{mc} = g_m, \text{ if } g_{mc} < g_m$$

$$= g_c, \text{ if } g_{mc} > g_c$$

$$= g_{mc}, \text{ otherwise.}$$

In Case the mine and market are limiting,

$$G_{mr} = g_{m,\ if}\ g_{mr} < g_m$$
$$= g_{r,\ if}\ g_{mr} > g_r$$
$$= g_{mr,}\ \text{otherwise.}$$

When the mill and refinery are limiting,

$$G_{cr} = g_{r,\ if}\ g_{cr} < g_r$$
$$= g_{c,}\ \text{if}\ g_{cr} > g_r$$
$$= g_{cr,}\ \text{otherwise.}$$

Pettingell (2017) explained more the effective optimum grade occurs at the Concentrating limiting cut-off $g_{c,}$ or the middle value between the limiting and balancing cut-offs

Kalitenge (2021) points out that Lane admits that the peaks for the various incremental present value curves are easily identified. However, the exact intersections of the feasible region can be difficult to determine graphically and a more robust method such as the golden search method must be applied.

3.4 The Cash Flow Model

This initial research aimed to test cut-off grades calculation using different optimisation approaches and compare the resultant cash flows to the base case (break-even cut-off grade). The model developed for this purpose includes the following core components:

- Production,
- Revenue,
- Costs, and
- Cash Flow.

Production

The production rate for this research refers to the ounces production rate for the mill. Data obtained from mine annual reports and the ore flow is used to determine the ratio of ore milled, which comes directly from the mine.

The model is populated with the assumption that production will continue until the mineral resource above the cut-off grade is depleted. This includes the Measured and Indicated resource blocks, while the Inferred resource blocks are not included.

The metal produced in the cash flow model is determined using Equation

$$Au(oz) = (\mathrm{g} * \mathrm{t} * \mathrm{y}) * 31.1, \qquad (3.16)$$

Where: $Au(oz)$ = Metal produced in troy ounces

t = The milled tonnes,

y = Mine recovery factor is a %, and

g = Average mining grade,

Revenue

The gold price used for this study is for 2020 and the research states the metal price and resultant revenue in US$. The formula to determine the gross sales revenue is as shown in Equation (3.16).

$$GSR = (Au(oz) * p) \qquad (3.17)$$

Where: GSR = Gross sales revenue in US$

$Au(oz)$ = Metal recovered in troy ounces,

P = Metal price in US$/oz.

Costs

Birch (2022) mentioned that the basic structure of the cut-off grade calculation considers both fixed and variable costs. However, fixed costs are divided by predicted production to calculate unit fixed costs. These unit fixed costs are then added to the variable costs, resulting in the total unit cost.

For this research, the projected costs obtained from the mine are not separated into core fixed and variable components. Instead, the costs are presented as unit Operating Expenditure (OpEx) costs in US$ per tonne (US$/tonne). In addition, the Ad'Duwayhi mine is a mature operation where the initial Capital Expenditure (CapEx) is considered recouped, so capital costs were excluded from the model. Sustaining capex was consider as part form fix cost.

Earnings before interest and tax

Earnings before interest and tax (EBIT) is the gross sales revenue minus the total costs (excluding the Tax and income Tax), as shown in Equation (3.18)

$$EBIT = (GSR - TC) \qquad (3.18)$$

Where: GSR = Gross sales revenue,

TC = Total costs in US$, and

EBIT= Earnings before interest and tax per US$.

Cash flow

The cash flow (CF) is calculated for each year of production. The formula is as follows:

$$CF = (EBIT - it), \qquad (3.19)$$

Where: CF = Cash flow in US$

$EBIT$ = Earnings before interest and tax,and

it = income tax in US$.

Discounted cash flow

The present value of expected future cash flows is determined using a discount rate, to calculate the Discounted cash flow (DCF) formula is ;

$$DCF = (CF_1/(1+i)) + (CF_2/(1+i^2)) + (CF_n/(1+i^n)) \qquad (3.20)$$

Where: DCF is the discounted cash flow (US$),

CF_1 = Cash flow for the first year,

CF_2 = Cash flow for the second year,

CF_n = Cash flow for additional years, and

i = Discount rate per percentage (%).

Net present value

The NPV is the difference between the present value of cash inflows and the present value of cash outflows over a specific period, discounted at a particular rate. The formula for determining the NPV is as follows:

$$\mathrm{NPV} = \Sigma\ (\mathrm{CFt} \ /\ (1 + \mathrm{i})\ \hat{}\,\mathrm{t}) \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (3.21)$$

where CFt = represents the expected cash flow in period t,

i = Discount rate per percentage (%),

t = Time period.

The model used for this research has been adopted from the cash flow model presented in Birch (Birch, 2022).

3.5 Chapter Summary

This chapter has presented some cut-off grade optimisation approaches and how they work. It also presents the cash flow optimisation model, which has been developed for this research. The following chapters present the mine data used in this research and then describe how this model identifies the optimal approach.

4 AD‘DUWAYHI MINING OPERATIONS OVERVIEW

4.1 Chapter Overview

This chapter provides a general description of the Ad'Duwayhi mining operations. A brief description of the regional and local geology is given. The local stratigraphy and the Ad'Duwayhi orebodies are shown. The mining and processing operations overview is highlighted to provide better understanding of the bottlenecks in the system. The cost and production figures are provided as well as the measured and indicated resources estimation.

4.2 Ad‘Duwayhi Mine Data

Ad’Duwayhi is open pit mine located in the eastern part of Saudi Arabia. The mining right for mineral extraction at Ad‘Duwayhi mine is owned by Ma’aden Gold and Base Metals Company (MGBM). The site consists of an open pit mine, run of mine pad (ROM), crusher, carbon-in-leach (CIL) processing plant and refining facility, waste rock dumps and tailings storage facility as well as ancillary buildings and maintenance facilities. The site also comprises an assay laboratory and metallurgical lab (WAI, 2018).

The Mineral Resources were estimated utilising block models with block dimensions of 10 x 10 x 5 m for the Measured and Indicated blocks, (SRK 2021). The Mineral Resources estimation are shown in Table 4 below.

Table 4: Ad‘Duwayhi Mineral Resource estimation (SRK, 2020).

Resource Classification	Tonnes Mt	AU_PPM g/t	Au t.Koz
Measured	1.66	2.12	110
Indicated	12.5	1.13	193
Measured and Indicated	14.1	1.41	303

The simulated ore grade distribution was lognormal with a low-grade skew throughout the mineralised body. The grade tonnage data serves as the sole grade tonnage curve for the entire life of the mine as shown in Figure 4.1.

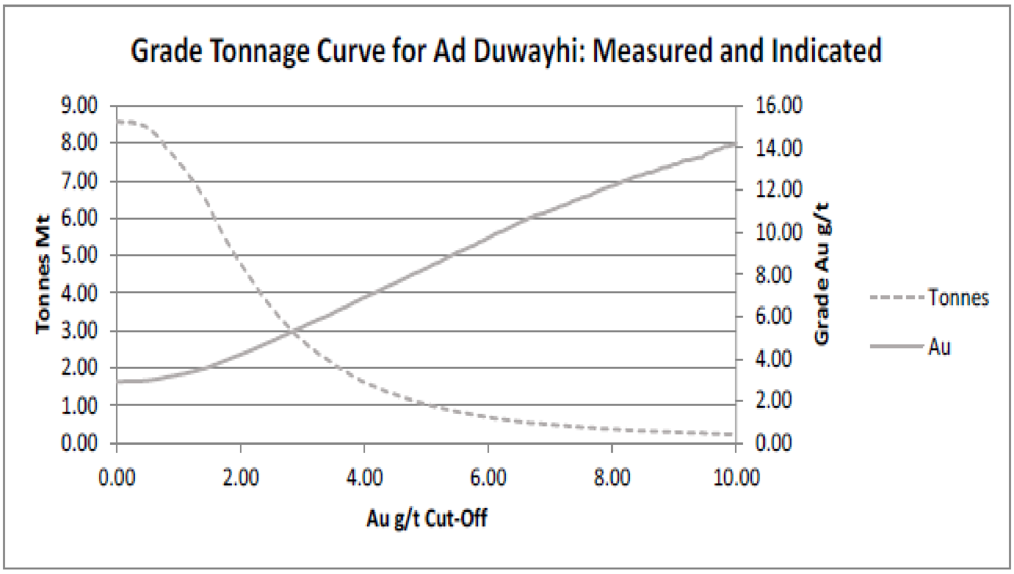

Figure 4.1: Grade Tonnage Curve: Measured and Indicated Open Pit Material at various Au cut-off's (SRK ,2020)

The figure shows the data set includes the tons of material, the average grade of the material and the gram of gold contained in each grade category. Grade tonnage data for the grade tonnage curves used in this research is presented in Appendix A.

The cost and production data are obtained from the company' annual Mineral Resource and Reserve reports.The annual mined production is 16 Mt per year with mine recovery factor 5 %. The Concentrating plant capacity is 2.3 Mt milled per annual with recovery of 92%. The mining cost is 1.13 US$ per tonnes mined and Concentrating cost is set on 15.3 US$ per proecess tonnes. The admin cost is 3.2US$ per tonnes Concentrated while 0.04 US$ per ounces produced is covering the selling cost.

4.3 Geology

4.3.1 Regional geology

Snowden Mining Consultants (2011) state that Ad'Duwayhi is situated within Pre-Cambrian rocks of the Arabian Shield, part of the Arabian-Nubian Shield (ANS). The Pre- Cambrian rocks of the ANS are now exposed either side of the Red Sea, in western Saudi Arabia and Yemen to the east (Arabian Shield) and in Egypt, Sudan, Eritrea, Somalia and Ethiopia to the west (Nubian Shield). Figure 4.2 shows the tectonic setting of Saudi Arabia

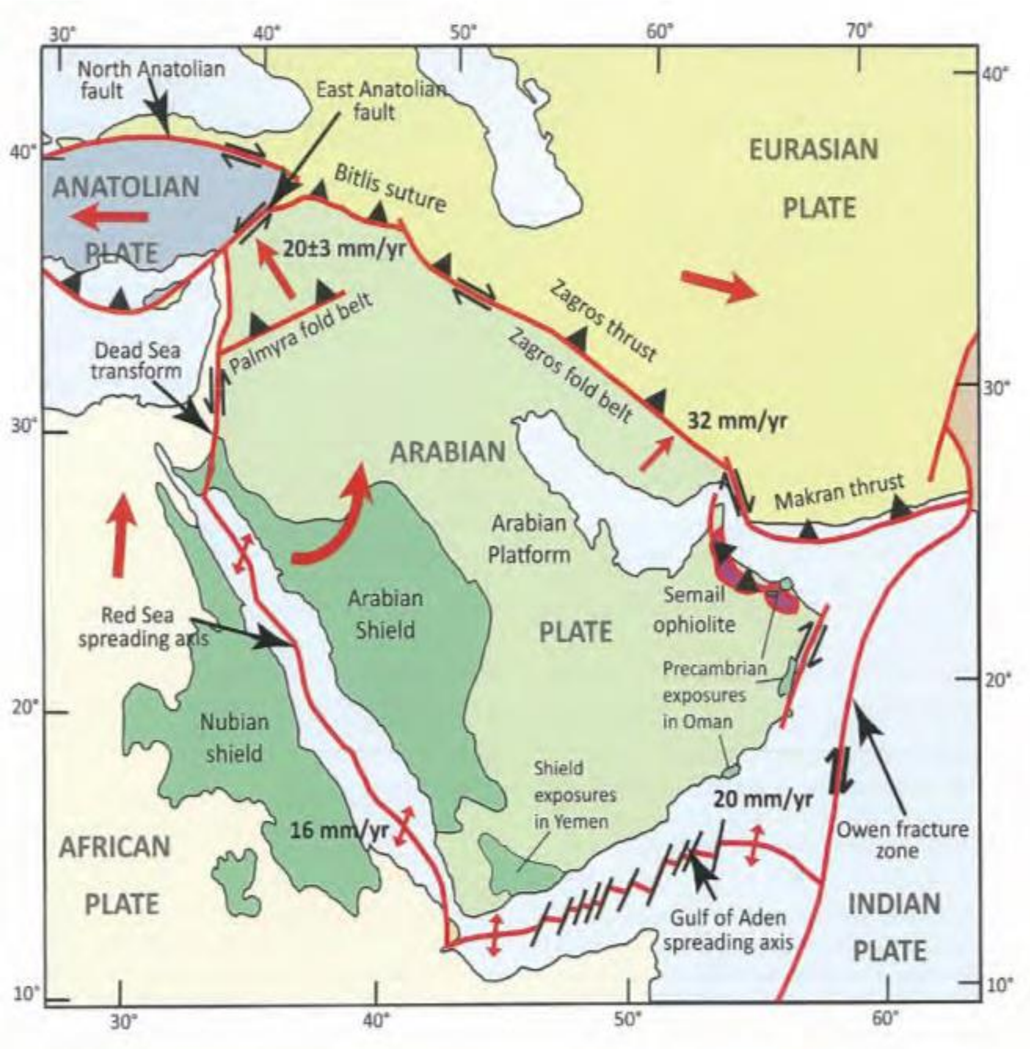

Figure 4.2: Tectonic Setting of Pre-Cambrian Rocks of the Arabian Shield (Snowden, 2011).

4.3.2 Mineralisation style

The main zone of gold mineralisation at Ad'Duwayhi is located and structurally controlled within a major thrust fault. This thrust zone is 2-10m in thickness and contains heavily deformed and shattered catacaustic rocks and well-developed quartz veins. Moving away from the main thrust zone, high-grade veins can be found which often show evidence of faulted contacts and which may have exploited existing structures (SRK , 2019).

Gold is often associated with pyrite and tetrahedrite which forms 3-5% by volume. Minor amounts of galena, chalcopyrite and sphalerite are also present. Gold is most often found in the form of fine-grained free gold not associated with any other mineral assemblage. However, some can be found as gold-telluride in high grade veins or silver-telluride with a trace amount of gold (Doebrich, et al., 2004). A molybdenum mineralizing event has been identified associated with low gold grades (1-3g/t Au), while higher grade molybdenum is found within the granite at Ad'Duwayhi (WAI , 2017).

The mineralized zones have low sulphide and base metal content. The gold to silver ratio is approximately 6:1. Gold shows only moderate correlation with silver and no significant correlation with other metals apart from lead.

The main vein averages five metres wide and strikes along 50º and dips at about 30º to the southeast. This vein is contained within a lower grade halo vein averaging about 30m wide. There is also a vertical granite vein which crosscuts roughly the centre of the pit with a width averaging 60m, and a flat lying lower grade vein which sits on the hanging wall side of the main veins in the southern section of the pit (WAI, 2018).

4.4 Mining

The mine consists of a single open pit with a total of 12 predominately 20m hight final benches. The final pit design has a length of 1,500m, a maximum width of 580m, striking at 50°. The maximum depth of the pit design is 240m, with overall slopes of 50° in the fresh rock (SRK , 2019).

Mining operation is carried out by conventional diesel-powered equipment, a combination of track mounted hydraulic shovels and rubber tyre rigid frame dump trucks. Drill and blast operations use mobile tracked drill rigs. The support fleet comprises secondary excavators (rock breaker), tracked dozers, graders, and a water truck to assist mining operations (WAI, 2017).

Most mining activities are conducted by the mining contractors at Ad'Duwayhi mine. All the mining equipment is supplied and operated by the contractor. Table 5 below shows the main equipment at the mine.

Table 5: The Primary mining equipment in the mine (WAI, 2017)

Equipment	**Type**	**Number**
Hydraulic Excavator	CAT 6020	2
Hydraulic Excavator	CAT 6015	2
Hydraulic Excavator	Komatsu 1250	2
Haul Truck	CAT 777	9
Haul Truck	CAT775	15
Blast hole Drill Rig	Atlas Copco T35	2
Blast hole Drill Rig	Sandvik DP1100	3
Blast hole Drill Rig	Sandvik DX700	5
Dozer	CAT D9	4
Grader	CAT	2

The waste material from the pit is hauled to the designated waste dump. Ore material is hauled to the graded stockpiles or ROM pad where the material is either loaded directly into the grizzly at the primary crusher or stockpiled for future processing.

4.5 Mineral Processing

The Ad'Duwayhi Concentrating plant utilises gravity and CIL technology to process 2.0 Mtpa of ore to produce ore product. Ore is essentially non-refractory, with high cyanidation recoveries, generally more than 95%. A simplified processing flow diagram is shown in Figure 4.3(attached in Appendix D).

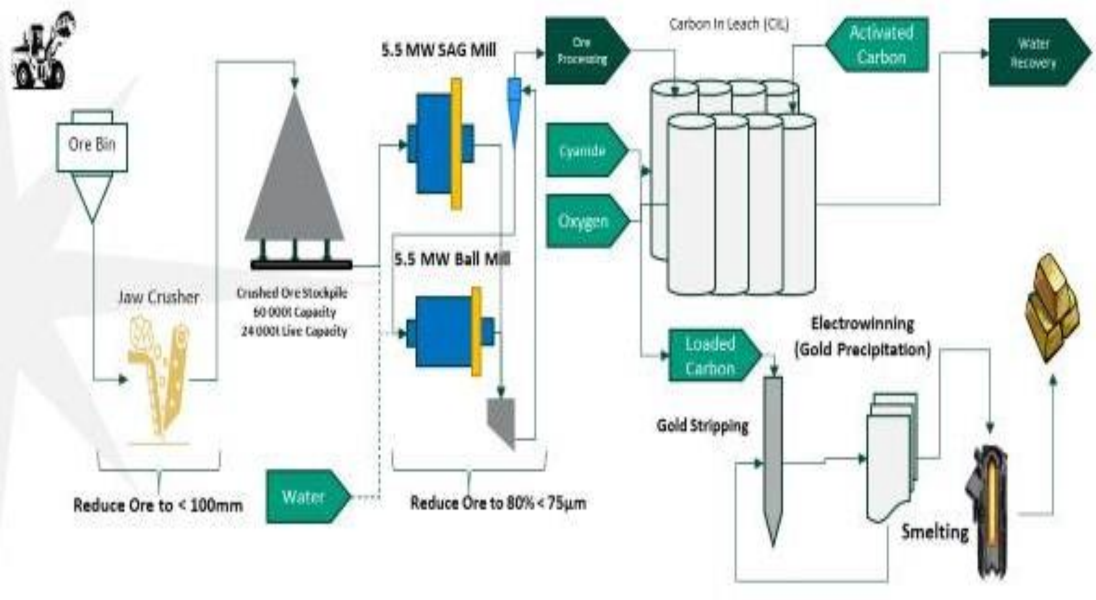

Figure 4.3: Simplified processing flow diagram (SRK, 2019)

Figure 4.3 illustrates the simplified processing flow diagram for the Ad Duwayhi gold Processing Plant. The process starts with ore handling, where ore is transported, crushed, and milled. The ore undergoes gravity concentration and intensive leaching to recover gold. The slurry is then processed in leach tanks and the CIL circuit, where gold is adsorbed onto activated carbon. The loaded carbon is stripped of gold in the elution circuit, and the gold is recovered through electrowinning and smelting. Finally, the tailings are thickened, filtered, and transported to a storage facility

Gold is the main product, with silver produced as a by-product through precipitation. The plant is designed to treat 180 KT of feed ore per month.

While the plant unit has capacity constraints based on various physical quantities or rates, the total plant capacity is typically limited by a single bottleneck, which is a capacity of 2000 KT per annum. This bottleneck is primarily in the leaching circuit, measured in terms of mill feed tonnes

4.6 Chapter Summary

This chapter introduced Ad'Duwayhi gold mine where the research is conducted, detailing the geology of the area and the ore bodies. The mine infrastructure has been described. The mine and mill capacities have been defined, as well as identifying the bottlenecks in the operation. Financial and production estimates for the research have been determined.

Measured and Indicated Resources have been tabulated, with cost and production figures obtained from the 2021 Mineral Resource and Reserve reports for the mine. The following chapter will utilise the data from the developed model to determine the best approach for optimising the cut-off grade.

5 RESULTS

5.1 Chapter Overview

This chapter outlines the calculation of cut-off grades using various optimisation approaches and compares the resultant cash flows. It also details the development of a model that describes how the tonnes and average mining grades, determined by each cut-off grade, will impact the overall cash flow

5.2 Model Assumptions and Limitations

Economic assumptions such as prices, costs, and production capacity used in this model were obtained from the annual mineral resource and reserve reports for the Ad'Duwayhi mine. For simplicity, price and cost were kept the same throughout the research.

The model is suited for an open pit gold mine that has one source of ore, one processing method, and one refinery. A defined resource and an ultimate pit limit had been determined. Capital costs were excluded from the model when calculating the NPV.

The parameters include the assumed gold price, mining, processing, selling and time costs, annual fixed costs. It was considered plant recovery on 95% with mine dilution at 5% and the refinery assumed to be a 100%. Table 6 below shows the Model Assumptions and Limitations

Table 6: The Assumptions Input (SRK, 2019)

Assumptions	**Unit**	**Number**
Price	(US$/g)	$ 41.80
Mining cost	($/t mined)	$ 3,20
Concentrating cost	($/t milled)	$ 21.20
Admin costs	($/t milled)	$ 1.67
Selling cost	($/g product sold)	$1.51
Discount rate.	%	8%
Recovery	%	95%
Fix cost	(Millon US$/year)	$ 24

The assumptions used in the calculations include the mine operating 365 days a year with a mining capacity of 16 million tonnes per year. The processing capacity is up to 2.4 million tonnes per year, and the refinery capacity is 1 million ounces per year. These capacities were carefully selected to ensure that neither mining nor refining would limit production. The unit mining cost is US$3.20 per tonne, and this cost is assumed to be the same for both ore and waste

5.3 Identifying the Optimal Approach

Break-even approach

The break-even grade is considered the base case, and it would be calculated using the break-even grade formula (from Equation 3.1). The main input is the price in US$ per gram of gold, the sales cost in US$ per grams of metal, processing cost in US$ per tonnes milled, the mining cost in US$ per tonnes mined, the recovery in percentage %, and the profit is expressed in US$ per gram of metal.

This research uses the 2018 pit designs for the Ad'Duwayhi mine. Therefore, there is no need to determine the overall cut-off grade at this point. Instead, it is only necessary to calculate the cut-off grade based on the capacity of the processing plant, which is the bottleneck in the production process at Ad'Duwayhi.

Lane’s approach

The objective is to maximise NPV by selecting the optimum cut-off grade. However, calculating the optimal cut-off grade using NPV requires an iterative mathematical method. Lane’s approach presents the opportunity cost associated to mining operation at a particular cut-off affects the overall NPV. This means that an opportunity cost needs to be included when calculating the cut-off grade.

Opportunity cost refers to the potential benefits that are forfeited when one option is chosen over another. In Lane's approach, opportunity cost involves the trade-off between mining ore of different grades and the associated costs. It is calculated as the rate of change in present value over time minus the discounted present value. Opportunity cost is expressed as F in the following equation

$$F = (\mu\ PV- (dPV / d\ T) \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.1)$$

Where: F = Opportunity cost

μ = Any estimated value for PV is valid if the resulting terminal value is zero (or a specified value).

Limiting cut-off grade

Ad'Duwayhi operation is constrained by the concentrating stage of the system it is assumed that the mine and the refinery will never be a bottleneck the operation, so balancing cut-off grades do not need to be calculated. The calculations for determining the limiting economic cut-off grades

$$g_p = ((cp + (f + F) / Cp)) / ((S - r) * y\ p)), \quad \ldots\ldots\ldots\ldots\ldots(5.2)$$

where: g_p = Concentrating Limiting cut- off grade p = Refers to the specific process being analyzed. This means that each process p, will have its own unit costs, capacities and recovery used to determine the limiting cut-off for that process.

Quantity of ore, waste and average grade

Once the limiting economic grade g_p for the process is calculated the quantity of ore T_O, the quantity of waste T_W and the average grade of ore g is determined as a function of g_p

$$TO_{(g_p)} = \sum_{n \geq n^*} T_n \qquad \text{.. (5.3)}$$

Where TO $_{(gp)}$ = Quantity of ore in tonnes ,

g_p = limiting economic grade grammes per tonnes, and,

n = Specific period

$$TW_{(g_p)} = \sum_{n < n^*} T_n \qquad \text{.. (5.4)}$$

Where Tw $_{(gp)}$ = Quantity of waste in tonnes ,

g_p = limiting economic grade grammes per tonnes, and

n = Specific period

The average grade for specific period calculated by

$$\bar{g}_{(g_p)}\Big| = \left(\sum_n \geq n^* T_n \left(\frac{g(n)+g(n+1)}{2} \right) \right) / \; TO_{(g_p)} \text{ , ... (5.5)}$$

where g $_{(gp)}$ = Refers to the individual grade categories that make up the grade tonnage curve,

T = Tonnes of material within those grade categories,

n* = The grade category selected as cut-off,

TO_{gp}. = The sum of all tonnes in each grade category above cut-off is denoted, and

TW_{gp} = Sum of all tons within each grade category below cut-off.

Quantity mined, concentrated and refined

The quantity of material mined, Qm, the quantity of ore concentrated, Qc and the quantity of ounces refined, Qr, are computed as a function of the cut-off grade as follows:

$$Qm = \left[\sum_{p} Qc_{p}\right] \qquad (5.6)$$

Where Qm = Quantity of material mined per tonne

Qc = Quantity of ore processed, and

Considering all annual concentrate quantities are within the maximum capacity.

The quantity refined can be expressed as

$$Qr = \sum_{p} Qc_{p} * \bar{g}_{(g_p)} * y_{p} \qquad (5.7)$$

where Qr = The quantity refined for the entire mine is equal to the sum quantity of ore processed multiplied by the product of the average grade of ore sent process and the average mineral recovery.

Qc_p = Quantity of ore concentrated,

$g_{(gp)}$ = product of the average grade of ore sent to consecrate, and

y_p = The average mineral recovery.

The formula for calculating cut-off policy for lane optimisation approach including annual Q_m, Q_r , Q_c and profit per year, is provided in Appendix B

Ad'Duwayhi optimum cut-off grade

The break-even grade for Ad'Duwayhi mine is calculated to be 0.70 g/t. The resultant 292,12 k.oz produced, and the tonnes processed are estimated to be 13.8 million. The cut-off grade presented in Table 7 below.

Table 7: Optimisation results based on break-even approach

Year	Mining Tonnage (Mt)	Ore Mined (Mt)	Process (Mt)	Process grade (g/t)	K.Oz
1	19.00	1.70	1.70	0.70	36,34
2	18.30	2.10	2.10	0.70	44,90
3	16.20	2.40	2.10	0.70	44,90
4	13.80	3.10	2.20	0.70	51,31
5	10.70	2.60	2.20	0.70	50,03
6	8.80	1.90	1.90	0.70	40,62
7			1.20	0.70	24,02
Total	**92.00**	**13.80**	**13.80**	**0.70**	**292,12**

A total of 92.00 Mt of mining tonnage was recorded, with 13.80 Mt of ore mined and processed. The process grade remained consistent at 0.70 g/t, resulting in 292.12 K.Oz produced over seven years. However, the ore processed fluctuated between 1.70 Mt and 3.10 Mt annually, indicating a reduction in available ore that could fully utilize the processing capacity.

By optimising the cut-off grade through Lane’s approach, the ounces produced increases by 8.0 % (292.12 Koz to 306,51Koz). Optimised cut-off grade using Lane’s approach presented in Table 8 below.

Table 8: Optimisation results based on Lane’s approach.

Year	Mining Tonnage (Mt)	Ore Mined (Mt)	Process (Mt)	Process grade g/t	KOz
1	19.00	1.72	1.70	0.70	41.53
2	17.30	2.11	2.10	0.71	70.55
3	15.20	2.50	2.20	1.10	64.14
4	13.78	2.27	2.20	0.90	57.72
5	12.50	2.22	2.30	0.80	43.98
6	1100	1.30	1.50	0.60	28.58
Total	**70.00**	**12.05**	**12.05**	**0.80**	**306,51**

The results of the research indicate that both optimisation approaches suffered a shortage of material to fully utilise the plant capacity. This shortage was a consequence of limited ore material resulting from the backlog in waste stripping.

5.4 Cash Flow Model

The simple cash flow model developed used the (AISC) as the main cost input. However, royalties and income tax payable were not included in the model. This exclusion was due to limitations in data collection, as comprehensive and accurate data on royalties and tax obligations were not available at the time of the model's development.

The discount rate used to calculate the DCF, and the resulting NPV is crucial for determining the cut-off grade. It is defined as the interest rate used to find the present value of future cash flows, and it is usually calculated using the weighted average cost of capital.

To calculate the NPV, the below formula has been used

$$\mathrm{NPV} = \Sigma\ (\mathrm{CFt}\ /\ (1 + \mathrm{r})\ \hat{}\ \mathrm{t})\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(5.8)$$

Where: CF_t = the expected cash flow in period t,

i = Discount rate (8%).

t = Time period.

Cash flow model resultant from break-even cut-off grade approach presented in Table 9 below.

Table 9: Cash flow mode resultant break-even cut-off grade approach

Cash Flow								
	Claender Year	2020	2021	2022	2023	2024	2025	2026
	Project Year	0	1	2	3	4	5	6
	Units							
Production								
Remaining Tonnes	Mt	19,00	18,30	16,20	13,80	10,70	8,80	
Ore mined	Mt	1,7	2,10	2,40	3,10	2,60	1,90	
Mill Tonnes	Mt	1,70	2,10	2,10	2,40	2,34	1,90	1,20
Mill Grade	g/t	0,70	0,70	0,70	0,70	0,70	0,70	0,70
Recovery	%	95	95	95	95	95	95	95
Gold Produced	oz	36,34	44,90	44,90	51,31	50,03	40,62	24,02
Revenu								
Gold Price	$/oz	1550	1550	1550	1550	1550	1550	1550
Gross Sales Revenue	US$ millons	56 329	69 597	69 597	79 533	77 549	62 963	37 236
cost								
Adjusted Operating Cost	US$/t	(19,20)	(18,20)	(17,53)	(16,20)	(14,90)	(13,32)	(15,70)
Total cost	US$ millons	(53 656)	(61 503)	(66 222)	(78 152)	(70 502)	(52 518)	(4 881)
EBIT								
Earnings Before Interest and Tax	US$ millions	2 673	8 094	3 375	1 381	7 047	10 445	32 355
Capital Expenditure and Tax Shield								
Sustaining Cost	US$ millons	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00
Net Sales Revenue After Capex	US$ millons		3 015	3 184	1 169	8 819	10 278	34 780
Royalties								
Royalties	US$ millons	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00
Income Tax								
Income Tax Payable	US$ millons	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00
Cash Flow								
Acutal Cash flow after tax	US$ millons	$0,00	$8 094	$3 375	$1 381	$7 047	$10 445	$32 355
Discounted Cash flow	US$ millons	8,00%	7 494	3 125	1 279	6 525	9 671	29 958
NPV	US$ millons	$33,03						

The basics of break-even analysis is only considering price and cost, however in practice, the deposit grades are varied. These grades quality fluctuations affect the ounces produced and consequently the revenue of the operation and achievement of the predicted NPV.

Cut-off grade optimisation was developed through Lane’s approach resulting in increasing the average grade by 8 %. Table 10 identifies how resulting tonnes and average mining grades from Lane’s approach will affect a common cash flow model.

Table 10: Cash flow mode resultant Lane’s approach

Cash Flow								
	Claender Year	2020	2021	2022	2023	2024	2025	2026
	Project Year	0	1	2	3	4	5	6
	Units							
Production								
Remaining Tonnes	Mt	19.00	17,30	15,20	13,78	12,50	10,00	
Ore mined	Mt	1,70	2,10	2,50	2,70	2,30	1,30	
Mill Tonnes	Mt	1,70	2,10	2,20	2.10	1,80	1,30	
Mill Grade	g/t	0,80	1,10	1.00	0,90	0,80	0,72	
Recovery	%	95,00	95,00	95,00	95,00	95,00	95,00	
Gold Produced	oz	41 537	70 552	64 138	57 724	43 980	28 587	
Revenu								
Gold Price	$/oz	1550	1550	1550	1550	1550	1550	
Gross Sales Revenue	US$ millons	64 385	109 360	99 417	89 476	68 171	41 895	
cost								
Adjusted Operating Cost	US$/t	(19,20)	(19,20)	(19,20)	(19,20)	(19,20)	(19,20)	
Total cost	US$ millons	(56 276)	(64 252)	(70 517)	(62 769)	(61 608)	(38 890)	(354 311)
EBIT								
Earnings Before Interest and Tax	US$ millons	8 109	45 108	28 900	26 707	6 563	3 005	118 393
Capital Expenditure and Tax Shield								
Sustaining Cost	US$ millons	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00	
Net Sales Revenue After Capex	US$ millons		45 108	28 900	26 707	6 563	3 005	
Royalties								
Royalties	US$ millons	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00	
Income Tax								
Income Tax Payable	US$ millons	$0,00	$0,00	$0,00	$0,00	$0,00	$0,00	
Cash Flow								
Acutal Cash flow after tax	US$ millons	$0,00	$45 108	$28 900	$26 707	$6 563	$3 005	
Discounted Cash flow	US$ millons	8,00%	45 108	26 760	24 729	6 077	2 783	
NPV	US$ millons	$36,60						

The dynamic feed grade policy, implemented through Lane's approach, has contributed to improving the NPV of the operation. However, it’s important to recall that the ore mined tonnes is decreased.

For this cash flow model, it is important to note that the model is expanded to include as many years as required by the mine being evaluated. It is crucial that the financial model accommodates the entire Mineral reserve. If this is not the case, the possibility of incorrect cut-off grades being determined because not all the possible revenue generated for the corresponding cut-off grade is considered (Birch, 2022).

Comparing the resultant cash flows to the base case (break-even cut-off grade). Table 11 shows Ad'Duwayhi mine break-even cut-off grade vs Lane optimised cut-off grade.

Table 11: Ad'Duwayhi mine break-even cut-off grade vs Lane optimised cut-off grade.

Value	Unit	Break-even	Lane	Differences
Cut-off Grade	g/t	0.70	0.81	+ 12.9.6%
Mining Tonnes	(Mt)	83	74	- 12.0%
Ore Mined Tonnes	(Mt)	13.8	12.05	- 14.5%
Process Tonnes	(Mt)	13.8	12.6	- 13.7%
Process grade	g/t	0.76	0.84	+14.6%
Ounces produced	KOz	292,12	306,51	+5.1%
Gross Sales Revenue	US$ millions	$452,503	$472,253	+4%
cost	US$ millions	($387,434)	($ 354,434)	-9.3%
NPV @ 8 % Discount	US$ millions	$35.03	$39.60	+8%
LOM	Year	7	6	+16.7%

5.5 Chapter Summary

This chapter of the research experimented with the optimisation of the cut-off grade. The optimal approach that would enhance the ounces produced and maximise the NPV was used. This approach improved the NPV by approximately 3 US$ million. In this chapter, the focus was on utilizing the module introduced in Chapter 3, applying various inputs to the model to document the resulting outputs.

6 ANALYSES OF RESULTS

6.1 Chapter Overview

The chapter gives the analysis and comparison of results obtained through the different approaches to calculate optimum cut-off grade. This comparison includes tonnes processed, life of mine and NPV.

6.2 Comparison of Tonnes Processed from the Approaches

The cut-off grade policy derived from the break-even approach resulted in higher tonnes processed compared to Lane’s approach. This is due to the nature of the break-even approach, which sets the cut-off grade based on maintaining a balance between cost and price. This allows for the processing of lower-grade ore, leading to a higher overall volume of ore processed. In contrast, Lane’s approach focuses on prioritising higher-grade ore, resulting in fewer tonnes processed as it quickly depletes the high-grade resources.

6.3 Comparison of Life of Mine from the Optimisation Approaches

The cut-off grade policy derived from the break-even approach resulted in a longer LOM of 7 years compared to Lane’s approach, which had a LOM of 6 years. This is mainly because the break-even approach focuses on maintaining a balance between cost and price, ensuring that the operation remains profitable over a longer period. By setting a cut-off grade that considers only the immediate economic factors, the break-even approach allows for the processing of lower-grade ore, which extends the mine's life.

In contrast, Lane’s approach incorporates a dynamic cut-off grade policy that optimises for higher short-term profitability by prioritising the processing of higher-grade ore. it often results in a shorter LOM as the higher-grade ore is depleted more quickly.

6.4 Comparison of NPV from the Approaches

Comparing the NPV of the two approaches confirms that using dynamic cut-off grades in Lane’s approach can improve NPV about 8% over that of the break-even approach. The cut- off grade optimisation gives the highest NPV by increasing the ounces produced.

Optimise cut-off grade policy that improves the NPV depends on availability of hight-grade material the mine, especially for a mine with a short life. However massive stripping at the beginning of a mine's life has negative impact on that because of increased cash outflows generated by early stripping.

6.5 Chapter Summary

This chapter has discussed the results of testing cut-off grades calculated using different optimisation approaches and compare the resultant cash flows using the model's developed for this research.

It has been shown in the research that optimising cut-off grade through lane approach led to maximising the NPV, increasing the ounces produced, reducing the mine life and ore mined tonnes. The following chapter is the conclusion and makes recommendations for further research.

7 Conclusions and Recommendations

7.1 Conclusions

This research aims to optimise the cut-off grade at Ad'Duwayhi mining operation. Test cut-off grades calculated using different optimisation approaches and compare the resultant cash flows.

The results show that the NPV for Lane's approach was, on average, 8% higher than the break-even approach. This suggests that using a dynamic feed grade can increase the NPV.

The results also demonstrate that for Lane’s approach ore mined quantity was less by 14.5% compared with the break-even. This indicates that some ore material may be sent to the waste dump . It was demonstrated through analysis that. cut-off grade policy determined by the break-even approach results in a 7-year LOM while the cut of grade policy based on Lane’s approach have a 6-year LOM.

The hypothetical scenario in this research did not consider stockpiling. However, this research suggests that incorporating cut-off grade optimisation with a stockpiling policy can improve the NPV and minimise the negative effects of raising the cut-off grade and the possibility of wasting ore material.

The hypotheses of this research also did not consider the geological uncertainty of the deposits, the research rather relied on a fixed grade-tonnage distribution; however, the nature of the orebody needs to be considered in cut-off grade optimisation since is influence of availability of ore material in specific time.

There is an opportunity to enhance the NPV of Ad'Duwayhi mining by optimising the cut-off grade. The results show that, cut-off grade optimisation can improve the NPV by 8% compared to break even approach.

7.2 Recommendations

Monitoring the mining rate balance in the earlier years of the life of the mine is important. Developing a production plan that focuses on high grade without considering the waste stripping may lead to creation of waste backlog and declining of remaining material grade, which negatively impact in mine's profitability.

As price and cost play a vital role in determining the cut-off grade, it is important to recall that, with the difficulties in confidently predicting the long-term trend in gold prices due to the current high economic and political instability, it is necessary to update the long-term cut-off grade on an annual basis and adjust the ore reserves based on the new cut-off grade.

The cut-off grade optimisation must not be treated in isolation. Optimising other factors such as stockpile, blending, recovery, and push-back design also needs to be considered and done simultaneously. It is important to note that to get the best possible value for the operation, we need to optimise the key parameters simultaneously.

The work done in this research report on optimal cut-off policies is adapted for Ad'Duwayhi mine. For implementation in other operation sites, it requires big data and more time. However, to get a better assessment for cut-off grade optimisation, further research needs to be conducted in different operation sites and conditions.

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APPENDIX A .Cut-off grade calculation formulae

Planning cut-off grade = ((m + c + (f + F + X) / C) / (s – r) y))

Budget cut-off grade = (((m + c + ((f + P + c) / C) / (s– r) y))

Accounting cut-off grade = (((m + C+ ((f + P + X + c) / C) / (s – r) y))

Marginal cut−off grade = ((s + h) / ((s – r) y)

Where :

m = mining variable cost

f = fixed cost

F = opportunity cost

X = ongoing capital expenditure

C = concentrates capacity

P = minimum profit or net cash flow

s = price

r = variable refinery cost

c: Concentrating cost

y = yield (recovery)

r = rehandle cost from stockpile

APPENDIX B. Simulated Grade Tonnage Data

Grade tonnage data for the grade tonnage curves used in this research is presented in Tables A1 below. The data set includes the tons of material, the average grade of the material and the gram of gold contained in each grade category.

Au Plan	Tonnes	Au Plan	gram
0.0 -> 0.1	20,127,766	0	0
0.1 -> 0.2	1,161,766	0.146	169,618
0.2 -> 0.3	867,655	0.247	214,311
0.3 -> 0.4	749,230	0.349	261,481
0.4 -> 0.5	723,551	0.45	325,598
0.5 -> 0.6	1,995,274	0.55	393,617
0.6 -> 0.7	696,476	0.649	452,013
0.7 -> 0.8	660,017	0.747	493,033
0.8 -> 0.9	563,123	0.849	478,091
0.9 -> 1.0	486,962	0.949	462,127
1.0 -> 1.1	454,490	1.049	476,760
1.1 -> 1.2	402,406	1.15	462,767
1.2 -> 1.3	429,824	1.25	537,280
1.3 -> 1.4	370,288	1.35	499,889
1.4 -> 1.5	387,503	1.449	561,492
1.5 -> 1.6	340,072	1.55	527,112
1.6 -> 1.7	318,040	1.649	524,448
1.7 -> 1.8	312,291	1.75	546,509
1.8 -> 1.9	275,943	1.85	510,495
1.9 -> 2.0	270,277	1.947	526,229
2.0 -> 3.0	275,943	2.00	4,904,383
3.0 -> 4.0	966,526	3.448	3,332,582
4.0 -> 5.0	550,626	4.447	2,448,634
5.0 -> 6.0	349,780	5.476	1,915,395
6.0 -> 7.0	209,951	6.46	1,356,283

7.0 -> 8.0	160,382	7.453	1,195,327
8.0 -> 9.0	108,460	8.463	917,897
9.0 -> 10.0	84,978	9.476	805,252
10.0 -> 11.0	66,858	10.501	702,076
11.0 -> 12.0	57,871	11.438	661,928
12.0 -> 13.0	42,801	12.479	534,114
13.0 -> 14.0	27,750	13.541	375,763
14.0 -> 15.0	26,398	14.438	381,134
15.0 -> 16.0	25,058	15.538	389,351
16.0 -> 17.0	19,382	16.525	320,288
17.0 -> 18.0	16706	17.434	291,252
18.0 -> 19.0	8684	18.441	160,142
	34,591,108	**0.61**	**29,114,671**

APPENDIX C. Lane Approach calculation per Year

Calculated complete cut-off policy for lane optimisation approach including annual Q_{m}, Q_{r} , Q_{c} and profit.

Year	g_m	g_c	Q_m	Q_c	Q_r	P
1	0,70	1,60	16,000,000	1,700,000	31,15	47 481
2	0,71	1,23	16,000,000	2,100,000	48,54	69 406
3	1,10	1,10	16,000,000	2,200,000	73,91	112 651
4	0,90	1.10	16,000,000	2,200,000	60,48	92 169
5	0,80	0.90	14,000,000	2,200,000	56,20	85 652
6	0,60	0,80	10,000,000	2,200,000	57,20	41 895

Year 1

Mining limiting economic cut-off grade (g_m) as Equation (3.10)

$$g_m = 26.2/ (40.7 - 1.51)*0.95 = 0.70 \text{ g/t}$$

The Concentrate limiting economic cut-off grade (g_c) as Equation (3.11)

$$g_c = (26.2 + (33 /1.7)) / (40.7 - 1.5)*0.95 = 1.60 \text{ g/t}$$

The quantities (*Qm*, *Qc* and *Qr*) and total profits as a function of cut-off grade under different constraints are given in Appendex A. and Equations (5.6 , 5.7 , 5,88 and 3,6 Respectively)

Qm = 16,000,000 mined tonnes

Qc = 1,700,000 mill tonnes

$$P = (40.7 - 1.5)*31.15 - 16*3.1 - ((26.2+24.2/1,7) - (16 -2.2) = \text{US\$ } 47{,}481$$

Year 2

Mining limiting economic cut-off grade (g_m) as Equation (3.10)

$$g_m = 21.2/ (40.7 - 1.51)*0.95 = 0.71 \text{ g/t}$$

The Concentrate limiting economic cut-off grade (g_c) as Equation (3.11)

$$g_c = (21.2 + (33 /2.1)) / (40.7 - 1.5)*0.95 =1.23 \text{ g/t}$$

The quantities (*Qm*, *Qc* and *Qr*) and total profits as a function of cut-off grade under different constraints are given in Appendex A. and Equations (5.6 , 5.7 , 5,88 and 3,6 Respectively)

Q_m = 16,000,000 mined tonnes

Q_c = 2,100,000 mill tonnes

$$P = (40.7 - 1.5)* 48{,}54 - 16*3.1 - ((21.2+24.2/2.1) - (16 -2.1) = US\$ 69{,}406$$

Year 3

Mining limiting economic cut-off grade (g_m) as Equation (3.10)

$$g_m = 21.2/ (40.7 - 1.51) *0.95 = 1.10 \text{ g/t}$$

The Concentrate limiting economic cut-off grade (g_c) as Equation (3.11)

$$g_c = (22.4 + (33 /2.2)) / (40.7 - 1.5)*0.95 =1.10 \text{ g/t}$$

The quantities (*Qm*, *Qc* and *Qr*) and total profits as a function of cut-off grade under different constraints are given in Appendex A. and Equations (5.6 , 5.7 , 5,88 and 3,6 Respectively)

Qm = 16,000,000 mined tonnes

Qc = 2,200,000 mill tonnes

$$P = (40.7 - 1.5)* 73{,}91 - 16*3.1 - ((21.2+24.2/2.2) - (16 -2.2) = US\$ 112{,}651$$

Year 4

Mining limiting economic cut-off grade (g_m) as Equation (3.10)

$$g_m = 21.2/ (40.7 - 1.51)*0.95 = 0.90 \text{ g/t}$$

The Concentrate limiting economic cut-off grade (g_c) as Equation (3.11)

$$g_c = (21.2 + (23 /2.2)) / (40.7 - 1.5)*0.95 =1.10 \text{ g/t}$$

The quantities (*Qm*, *Qc* and *Qr*) and total profits as a function of cut-off grade under different constraints are given in Appendex A and Equations (5.6 , 5.7 , 5,88 and 3,6 Respectively)

Qm = 16,000,000 mined tonnes . Equation (5.3)

Qc = 2,200,000 mill tonnes

$\mathrm{P} = (40.7 - 1.5)*60.48 - 16*3.1 - ((21.2+24.2/2.2) - (16 -2.2)$
$= \mathrm{US\$}\ 92\ ,\ 169$

Year 5

Mining limiting economic cut-off grade (g_m) as Equation (3.10)

$g_m = 21.2/ (40.7 - 1.51)*0.95 = 0.80\ \mathrm{g/t}$

The Concentrate limiting economic cut-off grade (g_c) as Equation (3.11)

$g_c = (21.2 + (21 /2.2)) / (40.7 - 1.5)*0.95 =0.90\ \mathrm{g/t}$

The quantities (*Qm*, *Qc* and *Qr*) and total profits as a function of cut-off grade under different constraints are given in Appendex A and Equations (5.6 , 5.7 , 5,88 and 3,6 Respectively)

Qm = 14,000,000 mined tonnes

Qc = 1,700,000 mill tonnes

$\mathrm{P} = (40.7 - 1.5)*56.20 - 14*3.1 - (21.2+24.2/1.40) - (16 -2.2)$
$= \mathrm{US\$}\ 85{,}562$

Year 6

Mining limiting economic cut-off grade (g_m) as Equation (3.10)

$g_m = 21.2/ (35.6- 1.51)*0.95 = 0.60\ \mathrm{g/t}$

The Concentrate limiting economic cut-off grade (g_c) as Equation (3.1)

$g_c = (21.2 + (27 /1.7)) / (35.6 - 1.5)*0.95 =0.90\ \mathrm{g/t}$

The quantities (*Qm*, *Qc* and *Qr*) and total profits as a function of cut-off grade under different constraints are given in Appendex A. and Equations (5.6 , 5.7 , 5,88 and 3,6 Respectively)

Qm = 10,000,000 mined tonnes

Qc = 2,200,000 mill tonnes

$\mathrm{P} = (40.7 - 1.5)*57{,}20 - 10*3.1 - ((21.2+24.2/2.2) - (16 - 2.2)$

$= \mathrm{US\$}\ 41{,}895.$

APPENDIX D. REVISED TABLS AND FIGURES

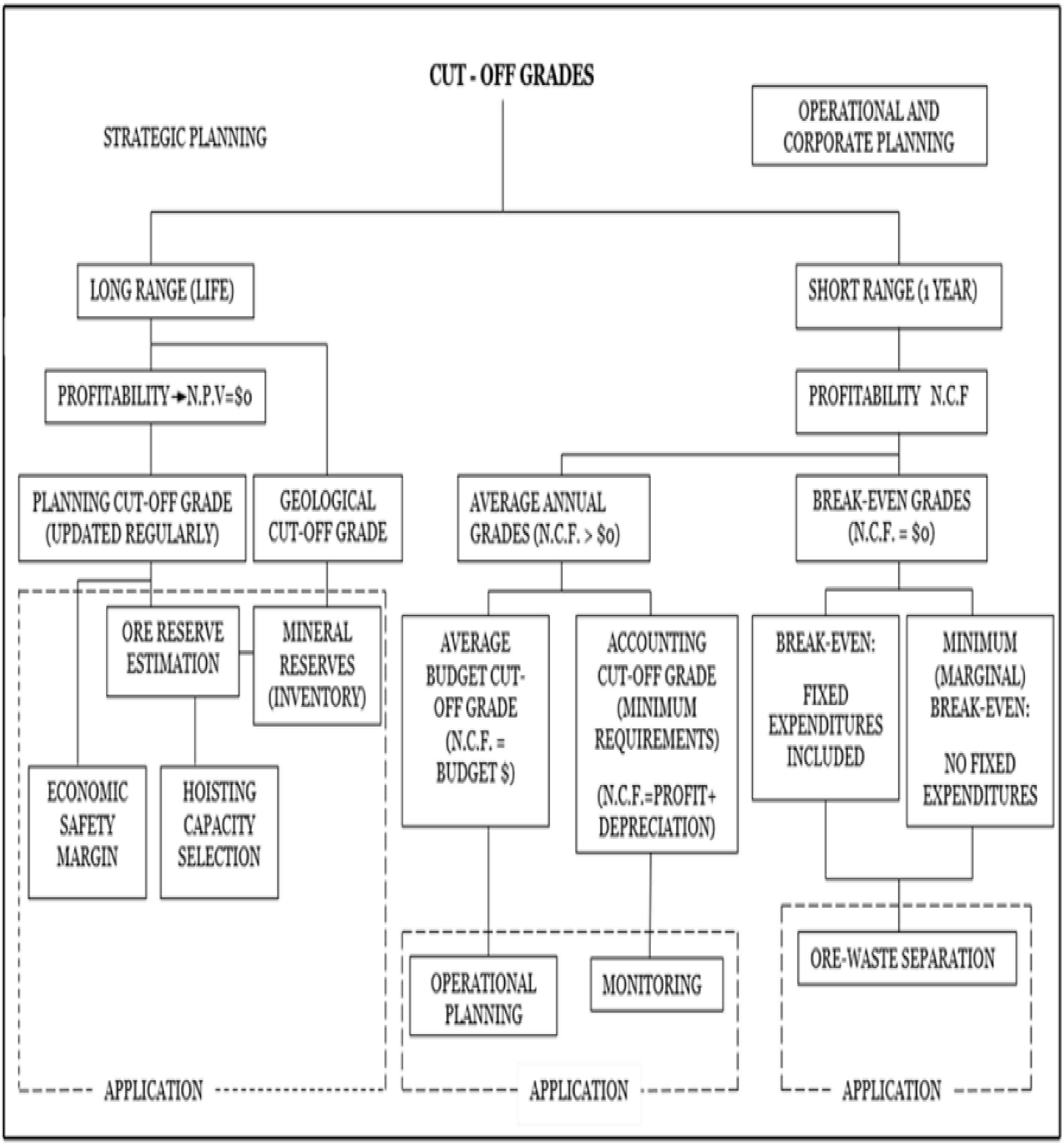

Figure D.1: Cut-off grades and their respective purposes (Kalitenge, 2021)

TableD.1: World Gold Council classification of costs (Meredith, 2021)

Cost Category	Source	US $ / gold ounces sold
On-site mining and processing costs (on a sales basis)	Income Statement	(a)
On-site general and administrative costs	Income Statement	(b)
Royalties and production taxes	Income Statement	(c)
Realised gains and losses on hedges of operating costs	Income Statement	(d)
Community costs related to current operations	Income Statement	(e)
Permitting costs related to current operations	Income Statement	(f)
3rd party smelting, refining and transport costs	Income Statement	(g)
Non-cash remuneration (site-based)	Income Statement	(h)
Stockpile, leach pad and product inventory write-downs	Income Statement	(i)
Operational Stripping Costs	Income Statement	(j)
By-product and co-product credits (*Note: will be a credit)*	Income Statement	(k)
Sub-total (Adjusted operating costs)		(l) = (a)+(b)+(c)+(d)+(e)+(f)+(g)+(h)+(i)+(j)+(k)
Corporate or regional general and administrative costs, including share-based remuneration (sustaining)	Income Statement	(m)
Reclamation & remediation – accretion & amortisation (operating sites)	Income Statement	(n)
Exploration and study costs (sustaining)	Income Statement	(o)
Capital exploration (sustaining)	Cash Flow	(p)
Capitalised stripping & underground mine development (sustaining)	Cash Flow	(q)
Sustaining capital expenditure	Cash Flow	(r)
Sustaining leases	Cash Flow	(s)
All-in Sustaining Costs		(t) = (l)+(m)+(n)+(o)+(p)+(q)+(r)+(s)
Growth and development costs not related to current operations	Income Statement	(u)
Community costs not related to current operations	Income Statement	(v)
Permitting costs not related to current operations	Income Statement	(w)
Reclamation and remediation costs not related to current operations	Income Statement	(x)
Exploration and study costs (non-sustaining)	Income Statement	(y)
Capital exploration (non-sustaining)	Cash Flow	(z)
Capitalised stripping & underground mine development (non-sustaining)	Cash Flow	(aa)
Non-sustaining capital expenditure	Cash Flow	(bb)
Non-sustaining leases	Cash Flow	(cc)
All-in Costs		= (t)+(u)+(v)+(w)+(x)+(y)+(z)+(aa)+(bb)+(cc)

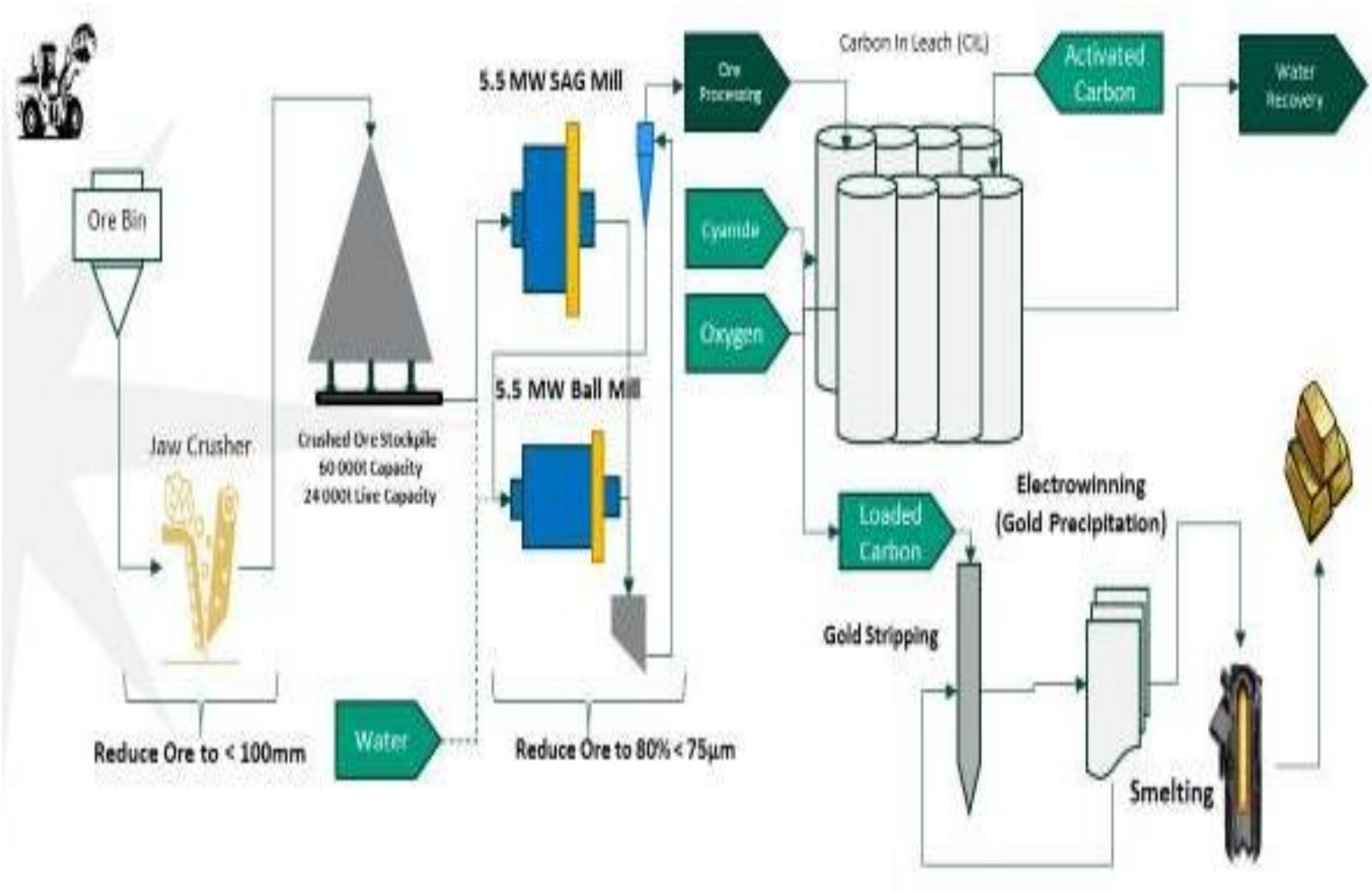

Figure D.2: Simplified processing flow diagram (SRK, 2019)