

**PRE-SERVICE TEACHER LEARNING AND PRACTICE FOR
MATHEMATICAL LITERACY**

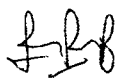
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of Doctor of Philosophy

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DECLARATION

I declare that this thesis is my own work, except as indicated in the acknowledgements. It is being submitted for the degree of Doctor of Philosophy at the University of the Witwatersrand, Johannesburg. It has not been submitted before for any degree or examination at any other institution.



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Signed

30th January, 2014

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Date

DEDICATION

This work is dedicated to my wife, Martha, and son, Garry. You are special to me.

ABSTRACT

This study explores the nature of pre-service Mathematical Literacy teachers' problem solving with a focus on intra-mathematics and extra-mathematics connections, across two years (2011-2012). The pre-service teachers were enrolled into a new three-year Bachelor of Education course, Concepts and literacy in mathematics (CLM), at a large urban University in South Africa. The CLM course aimed specifically at developing the teachers' fundamental mathematical knowledge as well as contextual knowledge, which were believed to be key components in ML teaching. The fact that the course offered a new approach to professional teacher development in ML (pre-service), contrasting the old model (in-service) reported in ML-related literature in South Africa, where qualified teachers from other subjects were re-skilled, coupled with the need to grow the pool of qualified ML teachers, provided a rationale for conducting this study. Data relating to the pre-service teachers' responses to assessment tasks within the course, and their school practicum periods focusing on classroom mathematical working, combined with pedagogical orientations, was collected. PISA's (OECD, 2010, 2013) dimensions of the mathematisation process provided the theoretical framework while Graven and Venkat's (2007a) pedagogic agendas were used to make sense of the pedagogic orientations in practice. The results relating to both learning and practice suggest that the teachers' knowledge relating to model formulation, an aspect of extra-mathematics connections, was weak across the two years. Nevertheless, improvements in ways in which the dimensions of the mathematisation process occurred were noted across the two years, with localised errors. In terms of pedagogic agendas foregrounded by the teachers in ML classrooms, results indicate that agenda 2 (content and context driven) and agenda 3 (mainly content driven) featured more than agenda 1 (context driven) which supports the rhetoric in the ML curriculum. Two implications to teacher training have been noted; first the need for a focus on correctly translating quantities from problem situations into mathematical models, and secondly, the need for promotion of provision of solution procedures with pedagogic links. This study offers two key contributions namely; extending knowledge relating to pre-service ML teacher training, and extending theory for understanding steps in problem solving to incorporate aspects of pedagogy.

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LIST OF ACRONYMS/ABBREVIATIONS

ML:	Mathematical Literacy
FET:	Further Education and Training
GET:	General Education and Training
NCS:	National Curriculum Statement
SAG:	Subject Assessment Guidelines
LPG:	Learning Programme Guidelines
CLM:	Concepts and Literacy in Mathematics
TE:	Teaching Experience
QL:	Quantitative Literacy
CAPS:	Curriculum and Assessment Policy Statement
AMESA:	Association for Mathematics Education of South Africa
PUFM:	Profound Understanding of Fundamental Mathematics
OECD:	Organisation for Economic Co-operation and Development
CDE:	Centre for Development and Enterprise
ACE:	Advanced Certificate in Education

CHAPTER ONE: INTRODUCTION AND CONTEXTUALISATION

1.1 Introduction

The South African government has in the past two decades embarked on education reforms with the view to redress inequalities relating to teaching and learning caused by the legacy of the apartheid education, which was characterised by provision of differentiated curricula based on racial divide (Department of Education, 2003). Within the context of these broad reforms, subject-specific curricula have been reviewed with an aim to improve access and quality among others. Driven by issues relating to access combined with life demands, the South African government also introduced Mathematical Literacy (ML) as a new subject in 2006 within the post-compulsory Further Education and Training (FET) band (Grades 10-12). The introduction of ML was aimed at catering for the large fraction of learners (about 40%) who dropped Mathematics entirely after grade 9 on one hand (Perry, 2004) and redressing concerns relating to difficulties in dealing with situations containing quantitative information, among the South African adult population on the other (Department of Education, 2003). This meant that after 2006, ML was offered in schools as an alternative subject to school mathematics at FET level. The implication was that learners at grade 10 took at least one subject aimed at developing some understandings relating to mathematics. Although ML was introduced in 2006 as a separate subject, some aspects linked to the idea of using mathematics in engaging with world situations had been incorporated previously within Curriculum 2005 (C2005) (Department of Education, 2002). The National Curriculum Statement (NCS) for ML at the time when this study commenced provided the following definition for ML (Department of Education, 2003):

Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop ability and confidence to think numerically and spatially in order to interpret and critically analyze everyday situations and to solve problems (p.9).

This definition suggests that an understanding of both mathematics content and world situations is important in developing competences needed for problem solving. According to the NCS for ML, learners will draw from General Education and Training (GET) (Grades 1-9) mathematics content when engaging with problem situations. Although the ML curriculum

has been reviewed further recently (Department of Basic Education, 2011a), the ML conception has not fundamentally changed.

The introduction of ML as a subject had two immediate implications: it increased the number of learners enrolling into mathematically orientated subjects after grade 9, and it increased the need for qualified teachers to implement ML in the schools. This study was focused on the latter implication, and was set within the context of a new University course which aimed at developing ML knowledge and pedagogic practice among pre-service teachers. Professional teacher development has been conceptualized in a range of different ways in literature. However there is a general understanding that professional development concerns systematic activities aimed at developing an individual's knowledge for teaching, and transforming their knowledge into practice for the benefit of their students' growth (Avalos, 2011; OECD, 2009). More details about the new University course which provided the study context have been provided in chapter 3.

Within the international literature, the concept of mathematical literacy, which is generally based on the notion of quantitative skills needed for adult life and active citizenship, continues to gain relevance in many current academic debates in both developing and developed countries. The imperatives driving the push for quantitative skills among the population are strongly linked to global societal dynamics which have over the years tended to move from the industrial society to the information society (Kaiser & Willander, 2005). Unlike the industrial society, the information society is characterized by analyzing and interpreting various forms of information, many of which require utilizing mathematically based knowledge. The implication is that learners need to develop their capacities to use mathematics in making sense of a diverse range of world contexts in flexible ways, an aspect which has gained global attention over the last decade or so (OECD, 2003; Steen, 2001). For instance, international comparative studies such as the *Programme for International Student Assessment* (PISA) study do not use curricular-bound knowledge to test learners, but they are instead aimed at assessing learners' ability to put mathematical knowledge and skills into functional use by giving learners tasks situated in some contexts (OECD, 2004, 2009). In the United States of America, a similar conception of ML known as Quantitative Literacy (QL) was introduced due to "the need for mathematically literate students who can function in a technology driven society plus the demonstrated lack of success of the current mathematics curriculum" (Burrill, 1990, p.50). Other conceptions like numeracy, mathematical modelling,

etc, have also been used in literature, and broadly concern the development of competences needed to engage with situations containing quantitative information. Although some variations in meaning across these conceptions have been observed, a common thread is that the different notions embrace the ability of an individual to competently manage the quantitative problem situations of everyday life. Details about some of the international conceptions of ML have been provided in chapter 2.

The global focus on the utilitarian value of mathematics emanates from the fact that “citizens in every country are increasingly confronted with a myriad of situations which require quantitative, spatial, probabilistic and relational reasoning” (de Lange, 2006, p.4). Policy debates, political issues, and personal decisions involve judgments about claims based upon quantitative evidence whose analysis and understanding require some knowledge regarding numerical arguments (Manaster, 2001). With the increasing demands for skills needed for active participation in the modern world and the need for workers to understand the meaning of their calculations in the context of work (Gainsburg, 2005), the functional aspect of mathematics appears to be more relevant than before. Steen (2001, p.1) observes that “the world of the twenty-first century is awash with numbers” and this puts huge demands on the individual to have “a predisposition to look at the world with mathematical eyes”. This functionality is critical for individuals to successfully survive in the modern information-based economy and society. The daily needs for the modern society coupled with the pervasiveness of computer technology, print and electronic media have led to the world being flooded with numbers and numerically-based arguments (Brombacher, 2007). Gainsburg (2005) agrees with Steen and Brombacher by arguing that;

It is hard to dispute that the world has become increasingly mathematized. Today, people are surrounded by numbers, charts, graphs and symbols as never before. More and more aspects of our daily lives are controlled by mathematical models, statistics, and computer programs (p.2-3).

Citizens are therefore challenged to make sense of these numbers and related numerical arguments in order to effectively participate in everyday issues that directly or indirectly affect them both at the more basic and more advanced levels.

In the South African context where this study was conducted, ML is not structurally viewed as a mathematical competence (de Lange, 2003) or as a desirable by-product of school mathematics (Gardiner, 2004), but as an alternative option to school mathematics (Venkat &

Graven, 2008). The conception of ML in South Africa (definition given above) has some parallels with other related international conceptions specifically in its emphasis on mathematical knowledge put into functional use in a range of diverse contexts in varied, reflective and insight-based ways (OECD, 2006). There are also contrasts in the sense that some international conceptions emphasize mathematical competences broadly in addition to making sense of contexts using mathematics. As noted above, after grade 9, learners have to make a choice on whether to proceed with school mathematics or take ML at grade 10. The teachers encourage learners to make specific choices informed by grade 9 school mathematics performance where high performers are encouraged to proceed with mathematics and low achievers take ML. This means that ML classrooms consist of learners whose mathematics content understandings are broadly weak. This implies the need for qualified teachers whose understandings of both mathematics content and contexts are developed.

The South African literature around professional teacher development in ML is dominated by studies around 're-skilling' programmes in the form of Advanced Certificate in Education (ACE) courses (Bansilal, 2012; Bansilal, Goba, Webb, James, & Khuzwayo, 2012; Mbekwa, 2007; Nel, 2012). At the core of these endeavours is the understanding that as a short term response to increased numbers of learners enrolling into ML, teachers who were specialists in other disciplines (i.e. Core mathematics, Geography, History etc) could be retrained to teach ML (Mbekwa, 2006). This study reports on an alternative model of professional teacher development focused on pre-service ML teachers who enrolled into a four-year pre-service B.Ed University programme and were tracked across the first two years (2011-2012) of a three-year elective professional development course called 'Concepts and Literacy in Mathematics' (CLM). The course was focused on both primary and ML teachers' preparation. It (course) was comprised of mathematics problem-solving in contexts (focusing on both primary mathematics and ML groups) and ML teaching components (focusing on ML students only). By locating my study within this course, my intention was to gain in-depth insight relating to ways in which the pre-service ML teachers engaged with contextualised mathematics tasks within the context of CLM course assessments and pedagogic practice (during teaching experiences). The specific focus was to explore the nature of the pre-service teachers' working relating to both intra-mathematical and content/contexts connections within written solutions to assessments tasks in the course and across teaching episodes informed by instructional tasks in practice. In terms of learning, this

study falls under Wenger's (1998, p.53) notion of knowledge acquisition, where learning is conceptualised in terms of 'meaning making' through experience. The students in this study were engaged with a range of different mathematical and contextual tasks, and this involved what Wenger calls 'negotiation of meanings'. This learning is understood in relation to knowledge growth in the course and snapshots of how this knowledge plays out in practice.

To understand the constitution of ML in South Africa better, I now give an overview description of the specific context within which the study was conducted.

1.2 The study context and study focus

The study was located in a new University course, called Concepts and Literacy in Mathematics (CLM), and was targeted at pre-service Bachelor of Education (B.Ed) students at a large urban University in South Africa. The CLM course had a dual focus and twin aims; developing pre-service senior primary mathematics teachers and secondary ML teachers. The fact that ML draws from GET (elementary) mathematics, provided a rationale for the needs of the two groups overlapping. The key difference relates to their foci. ML has an overtly utilitarian focus (Davis, 2003) whereas primary mathematics is concerned with laying foundations for mathematical progression and/or for applications in other disciplines that are mathematically based. Further, in ML, skills for making inferences based on "estimates and approximations on incomplete or sometimes inaccurate data" are frequently required (Manaster, 2001, p.68) whereas in primary mathematics, reasoning and justifications on why certain procedures work play a central role. Thus the course was aimed at providing primary mathematics teachers with 'profound understanding of fundamental mathematics' (PUFM) (Ma, 1999) which would allow them to teach competently and with confidence. At the same time, ML teachers needed some competences which would allow them to successfully engage with situations using a range of basic mathematical ideas (Department of Education, 2008). In agreement with the ML specifications, the CLM course included engagement with situations as one of the course goals. As a teaching preparation course, the CLM course focused not only on the 'doing' of the basic mathematics or mathematics in situations, but also had course aims suggesting tasks asking the students to link their problem solving with teaching practice. Further, in addition to a focus on contextualized tasks within the CLM course, the course aims appeared to point, in specific terms, towards developing an understanding of mathematics concepts, an aspect not mentioned across the ML goals. The implication is that the CLM course, in wanting to deal with preparation of primary teachers

and ML teachers, did not neatly map onto ML curriculum goals and its orientation on functional use of mathematics.

Enrolment into the CLM course was based on obtaining a pass mark in ‘Mathematical Routes’ – a course which was offered at first year and was aimed at consolidating the pre-service teachers’ fundamental mathematical knowledge. CLM, in 2011, was a new course and consisted of the first cohort. Therefore, research was required to investigate the nature of students’ development within the course. This study was thus aimed at investigating and reporting on ML teacher development relating to both mathematical and contextual working and the ways in which ML pedagogic practice was linked to a course which also aimed at developing understandings of fundamental mathematical ideas. Within the context of evidence of tensions between foregrounding content understandings versus foregrounding context understandings in ML teaching (Graven & Venkat, 2007a), the study also explored the ways in which ML teachers approached teaching in ML classrooms. It was also important to provide critical reflection on how best this course could be run to ensure that course aims were achieved. Such reflection would help in improving the structure of similar courses in terms of their foci, aims, course content, course enactment, and nature of assessment tasks.

This study was specifically focused on the pre-service ML teachers group within the CLM course during 2011-2012. In order to understand the performance of this group prior to participating in the CLM course, I provide official results from Matric and mathematical routes for the four study participants (sampling technique has been provided in chapter 4), as these provided useful background information in this study.

Name of participant	Matric ML scores (%)	Mathematical routes scores 2010 (%)
Lindiwe	74	52
Mark	86	75
Jabu	76	56
Lebo	73	64

The results show lower mathematical routes (first year university course) scores compared to the ML Matric scores. Further, for the three students achieving below 80% in ML Matric, there is a broad range of ‘Mathematical Routes’ scores in the context of a narrow range of ML Matric scores. Given that this study did not focus on mathematical routes course, an in-depth analysis of these scores was beyond the scope of the current study. However, as noted earlier, the participants’ background information provided insight with regards to their prior

knowledge. While the CLM course had a dual focus, the other group (senior primary mathematics specialists) was not part of this study. This was deliberately done, and aimed at maintaining the study foci on ML. The study therefore aimed at tracking the development of a sample of ML teachers in terms of both knowledge (their own ability to solve, and communicate their solution/reasoning across a range of mathematical and contextual problems) and practice (their teaching of ML in schools) over a two-year period (2011-2012). Although the CLM was a 3-year long course, this study tracked the students across the first two years in the course. This was a pragmatic choice, given that the available resources for conducting this research could not allow for a three-year period of data collection. The context of the study was linked to both the need to expand the pool of teachers qualified to teach ML and also the introduction of the CLM course with its dual foci and twin aims. Chapter 3 provides more details about the CLM course focusing on course objectives, course enactment and course assessment. This study focused on an in- depth qualitative analysis of the development of four pre-service teachers in this course aiming to offer ML as a subject in their subsequent teaching. This analysis also provided some evaluation of this course.

Driven by ML emphasis on contexts combined with the study focus relating to exploring pre-service teachers' connections within mathematics as well as across contexts and mathematics, PISA's (OECD, 2010) notion of mathematisation process has been used as a theoretical framework. It is important to note that the PISA's version of mathematisation process draws from Freudenthal's idea of realistic mathematics education (RME) (Freudenthal, 1973). RME is based on Freudenthal's interpretation of mathematics as a human activity, deeply embedded in 'real' situations (Freudenthal, 1973, 1991). In RME terms, the real situations can include real world contexts or mathematical contexts where learners can experience the problem presented as relevant and real (van den Heuvel-Panhuizen, 2000, 2001). However, engagement with contexts in RME is aimed at achieving mathematical understandings. I therefore found the PISA's theoretical tools useful as they provided handles for thinking about ways in which students worked through contextualized tasks with a focus on understanding the contexts themselves, a central feature of ML in South Africa. Furthermore, some researchers have noted overlaps between PISA mathematisation process and what is described as the mathematical modeling cycle (de Lange, 2006; Kaiser & Willander, 2005). More details about the theoretical lenses underpinning this study have been provided later in Chapter 2. A focus on problem solving processes resonates with my personal research interests which have been buoyed by a study of applied mathematics at Masters degree level,

where the notion of mathematical modeling featured centrally, in recent years. I now provide a brief account of my personal background and how this background links with the need for the broader human population to be numerate, a key feature which underlies ML as a fundamental subject in South Africa.

1.3 Personal background and motivation

My research interest in professional teacher development was initially driven by the need for learners in Malawian schools to be taught by well qualified and competent mathematics teachers. However, my experience teaching mathematics at school level and interacting with learners allowed me to understand how learners were grappling with understanding mathematical concepts often presented in abstract forms. Although some topics within the mathematics curriculum in Malawi were followed by a section focused on application (word problems), problems listed under these sections were often focused on developing mathematical understandings with no connections with real-world situations. Given that this was not a problem at the time, studying applied mathematics at Masters degree level allowed me to appreciate the functionality of mathematical ideas in the real world. Although this view (learning mathematics followed by application) appears to differ with South African conception of ML whose emphasis is on engagement with contextualized tasks, overlaps exist in terms of utilizing mathematical ideas to make sense of problem situations. Further, the mathematical tools used to analyze problem situations at Masters degree level were more complex, and engaging with these abstract concepts gave me the platform to begin to think about how basic mathematical ideas could be utilized to empower citizens so that they could make sense of problem situations often encountered in their everyday lives. This interest in ways in which the mathematical skills needed for life could be developed necessitated a shift from studying for a Masters degree in applied mathematics to a doctoral degree in mathematics education focusing on professional teacher development in ML. Addressing the need for teachers who are numerate and competent to teach ML across the schools in South Africa overlaps with professional teacher development needs within the Malawian context.

The teacher shortage in ML in South African schools constituted the main motivation for conducting research specifically relating to pre-service teacher learning and practice within the context of a new CLM course. Evidence from South African literature has shown that although professional teacher development in the form of in-service teacher trainings have been conducted across the nation, some schools still do not have adequate numbers of

qualified ML teachers (Bansilal, 2012; Mbekwa, 2006, 2007). These studies have reported on in-service training which focused on re-skilling teachers who were qualified in either school mathematics or other non-mathematics subjects. This study however, focused on pre-service ML teacher development with special emphasis on the pre-service teachers' mathematical working (intra-mathematical and mathematics content-context connections). The study was linked to the need for more skilled teachers to teach ML in the schools in the context of evidence of poor learner performance in mathematics and problems with teachers' mathematical knowledge (Reddy, Van der Berg, Janse van Rensburg, & Taylor, 2012).

Globally the idea of focusing on functional aspects of mathematics in an information technology-driven society, where citizens are required to make informed judgements and decisions from quantifiable data, is gaining more relevance. International conceptions like quantitative literacy (Steen, 2001), numeracy (Coben, 2004; de Lange, 2006) and mathematical literacy (Jablonka, 2003; Pugalee, 1999), for instance, emphasize utilitarian value of mathematics. Common to these conceptions is the idea that competences needed to engage with situations are often not been developed within school mathematics learning. The implication for ML implementation in South Africa therefore relates to the need for developing teachers' ML-focused knowledge that would enable them to teach ML effectively in the schools. Given the shortfall in ML teacher numbers, it is thus imperative for a country like South Africa to channel more resources towards ML teacher development if members of the general public are to be active participants and self-managers.

1.4 Definition of terms

The following are operational definitions of terms used, in this study and the literature-based sources from which they are drawn:

- *Situation*: “the situation is the part of the student’s world in which the tasks are placed, [and] it is located at a certain distance from the students. ...the closest situation is the student’s personal life; next is school life, work life and leisure, followed by the local community and society as encountered in daily life” (OECD, 2006, p.81).
- *Context*: “the context of an item is its specific setting within a situation” (OECD, 2006, p.81).

- *Contextual task*: this refers to the scenarios from which students can produce mathematical activity (Department of Education, 2008; Steen, Turner, & Burkhardt, 2007).
- *Consumer of mathematics*: this refers to an individual who is able to receive (or understand) and use information from others (Skovsmose & Valero, 2005). The Learning Programme Guidelines (LPG) for ML in South Africa describes such an individual as a 'self-managing person', someone who is able to understand bank statements, read maps, follow timetables, understand house plans, among others (Department of Education, 2008).
- *Citizenship perspective*: a view which concerns the understanding of information and using this information to make everyday decisions (Skovsmose & Valero, 2005). This view combines understanding and using information involving numerical components at both personal and work place (Department of Education, 2008).
- *Critical perspective*: this view is concerned with using mathematics to *critique* issues of equity and social justice in a society (Frankenstein, 1990; Skovsmose, 2000). In this study, the degree to which curriculum specifications, conceptions, as well as the nature of tasks suggest critical orientation has been explored.
- *Extra-mathematical and intra-mathematical tasks*: this study adopts the PISA's classification of mathematical tasks as follows; tasks are considered to be intra-mathematical if they "refer only to mathematical objects, symbols or structures, and make no reference to matters outside the mathematical world" and are classified as extra-mathematical if "they refer to real-world objects" (OECD, 2006, p.81).
- *Familiar and unfamiliar contexts*: The curriculum statement for ML in South Africa views contexts which learners have seen and engaged with before as familiar and those which are new as unfamiliar (Department of Education, 2003).
- *Real world tasks*: tasks drawn from broader world situations and exhibit some form of concreteness for the problem solver (Department of Education, 2003; van den Heuvel-Panhuizen, 2001).
- *Mathematically-focused tasks*: tasks situated in some context whose focus is on achieving mathematics understanding, in intra-mathematical sense.
- *Mathematics orientation and contextual orientation*: mathematics orientation refers to the teaching approach where mathematics learning or understanding is foregrounded.

If instead, the understanding of context is foregrounded, this approach is said to exhibit contextual orientation (Graven & Venkat, 2007a).

- *Intra-mathematical connections*: connections within and across mathematical strands often characterised by use of symbols and/or mathematical language.
- *Mathematics and context connections*: connections between contexts and mathematical models or statements.
- *Mathematization*: The term mathematization refers to the process concerned with “transforming a problem defined in the real world to a strictly mathematical form ... or interpreting or evaluating a mathematical outcome or a mathematical model in relation to the original problem” (OECD, 2010, p.18).
- *Numeracy*: the state of being “competent, confident, and comfortable with one’s judgements on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context (Coben, et al., 2003, p.10).
- *Coherence*: this study adopts Hall and colleagues’ view that looks at coherence in relation to ‘episodes within solution protocols’ (Hall, Kibler, Wenger, & Truxaw, 1989, p.244). In this study, the solution protocols were divided into three episodes namely, model formulation, intra-mathematical working, and interpretive aspect. And I have used the term coherence largely within ‘intra-mathematical working episode’, as the other two episodes were absent in many solution protocols.

1.5 Rationale for the study

This study was driven by two aspects: first, the need for more qualified ML teachers given increased numbers of learners enrolling into ML in schools; second, to understand the mathematical working of the pre-service ML teachers enrolled into a new University course which sought to develop the teachers’ mathematics content and context understandings. The study was focused on exploring the ML pre-service teachers’ intra-mathematical and mathematics-contexts connections within a pre-service professional development course utilizing a model which contrasted from the in-service teacher development models that have tended to be reported in literature. Within the context of increasing societal demands for better quantitative understanding amongst all citizens, and concerns about content knowledge and shortages of qualified ML teachers, a large urban university developed an undergraduate

B.Ed course (CLM). The CLM course targeted prospective students opting to teach ML while in-service programmes focused on teachers already qualified in other disciplines.

The NCS claims that students in the past have failed to demonstrate expected levels of numeracy and that this prompted the government to introduce ML as a subject (Department of Education, 2003). The ML curriculum further notes that the subject could potentially contribute towards the transformation of South African schooling and society by contributing to the development of individuals' ability to participate fully and critically as citizens. The curricular aim is that future citizens will not only be able to understand and interpret quantitative data, but also become active participants in debating crucial issues that affect their lives. In this way the ML curriculum suggests the adoption of a critical orientation.

Professional teacher education and specifically ML teacher development is one of the priority areas of the National Department of Education in South Africa. The in-service training courses conducted across the country, which were aimed at re-skilling teachers to competently teach ML in the schools support this claim. Some studies have reported on Advanced Certificate in Education (ACE) courses in ML conducted in South Africa since 2006 (Bansilal, 2012; Brown & Schäfer, 2006; Vilakazi & Bansilal, 2012). Since the ACE courses were in-service in nature, the participants were drawn from either the pool of mathematics teachers or other non-mathematical disciplines (Mbekwa, 2007; Webb, Bansilal, James, Khuzwayo, & Goba, 2011). Given the limited pool of mathematics teachers in South Africa (CDE, 2004; Department of Education, 2005), this meant that mathematics teaching was also affected. The findings further raise two major serious concerns relating to ML teaching. First, the report, based on a localised study in Kwazulu-Natal (KZN) province compiled by Bansilal and colleagues (2012), revealed that some of the teachers who successfully completed ML re-skilling courses were not teaching ML in the schools, and that others who failed the course were found teaching ML in some schools. Second, another small scale study done by Mbekwa (2007) indicated that some non-re-skilled mathematics teachers were teaching ML despite evidence showing that mathematics teachers often lack the capacity to both connect their mathematics to real contexts and struggle to see the internal connections between mathematical concepts (Brombacher, 2003; Manaster, 2001; Steen, 2001). Although the idea of well-connected mathematics content understandings has been highlighted in literature as one of the key components of ML (Brown & Schäfer, 2006; Vilakazi & Bansilal, 2012; Webb, et al., 2011), the issue of the ML teachers' intra-

mathematical connections and flexibility has not been dealt with in detail. The need to address these problems was part of the rationale for the CLM course.

Concerns relating to demonstration of some level of numeracy have also been reported in international literature with recommendations towards an investment in professional development aimed at developing teachers' intra-mathematical and mathematics-contexts connections. Drawing from the need for the adult population in the United States to have citizenship skills, Steen (2001) notes;

Unfortunately, despite years of study and life experience in an environment immersed in data, many educated adults remain functionally innumerate. Most U.S. students leave high school with quantitative skills far below what they need to live well in today's society... (p.1).

Steen's concern appears to point towards the need for school leavers to demonstrate some level of competence to deal with everyday challenges, an aspect which overlaps with the rationale for introducing ML in South Africa. Further, parallels can be drawn from Steen's observations noted above with the National Research Council's findings in USA, which suggest that the nature of traditional mathematics instruction which is often formal, less intuitive and abstract, "leads to students who do well in standardized tests and low order skills, but are generally ineffective as teaching strategies for long-term learning, for higher order thinking and for versatile problem solving skills in everyday life" (NRC, 1989, p.57). The above statements suggest a 'citizenship' agenda of schooling, where learning is viewed as preparation for adult life. Brombacher's (2007) observations drawn from a South African context, echo similar sentiments;

Traditional mathematics programmes do not teach people to manage their finances; to understand the impact of hire-purchase agreements on the disposable income; to recognize that there are no 'free' cellular telephones; and to realize that there are no "hot" and "cold" numbers in the national lottery (p.2-3).

The above evidence, which also appears to contain 'citizenship orientation' features, suggests that ML needs to be taught by competent ML teachers – those with strong understandings of both intra-mathematical and mathematics-real world connections. In this view, using non re-skilled school mathematics teachers who reportedly failed ML re-skilling courses to teach ML might not achieve the desired outcomes. There is also a need to grow the pool of ML teachers that can competently and effectively teach the subject in the schools across South

Africa, given the increased numbers of learners enrolled into ML each year. Due to the significant and increasing numbers of learners opting to take ML in FET, an intervention to ensure the availability of qualified ML teachers in the schools is needed (Vithal & Bishop, 2006). The report by the Department of Basic Education (2013) indicates how the numbers of learners writing the ML examinations at Matriculation level have increased over the past four years as shown in table 1.1.

Year	2008	2009	2010	2011	2012
Number of Matriculants	263 464	277 677	280 836	275 380	291 341

Table 1.1: ML learners who wrote Matric (2008-2012) (Department of Education, 2013)

The table shows an upward trend in terms of ML learners writing Matric examinations thereby suggesting an increase in numbers of learners enrolling into ML at grade 10. There is evidence pointing towards poor performance among learners especially on items which require multi-step procedures within intra-mathematical working. According to North (2013), learner performance in ML at Matric appears to be generally high (85%), a percentage based on learners obtaining a mark of 30% and above. He further argues that learners could obtain a pass mark in ML by only taking Paper 1 which is a ‘skills paper’ focusing on assessing questions at level 1 and level 2 of the taxonomy (see Department of Education, 2008 for ML assessment taxonomy). However, there are fewer learners obtaining a mark of 50% and above suggesting that learners struggle to engage with Paper 2 questions which require employment of multi-step procedures. Developing cohorts of ML teachers with competence in handling and teaching more complex ML problems would therefore help to ensure adequate capacity to teach ML in schools.

In summary, this study distinguishes itself from other studies in the following ways. First, it is about pre-service ML teacher learning and practice within a newly introduced three-year course (CLM), a shift from the in-service training programmes reported in South African literature. Secondly, it is located within a course which has dual foci (Senior Primary mathematics group and ML group – prepared to teach at FET level) and twin aims (developing students’ mathematics content and context understandings) - this is what makes the study interesting.

1.6 Statement of the problem

The main purpose of this study was to investigate development in knowledge and practice of a sample of pre-service teachers within the new CLM course. Given that the study was located in a professional development course (CLM) and given the need for tracking the pre-service teachers in the course and in practice, the problem has been divided into three components: exploring the link between CLM course aims and general ML aims, the pre-service teachers learning in the CLM course, and the pre-service teachers' practice. The three components are linked through the idea of connected learning, related to both mathematics and contexts understandings, across the CLM course and school practice.

In relation to linking the aims of the CLM course against those of ML, course documents and ML policy documents respectively, have been analysed. In doing so, I have been guided by the first research question given below. Related results and discussions have been presented in chapters two and three.

1. How did the aims of the CLM course fit with the aims of the ML curriculum in terms of content knowledge and pedagogic practice? How did course materials and course assessment tasks link with ML aims? What were the overlaps and contrasts?

In terms of exploring the pre-service teachers' learning in the CLM course, the study has focused on the pre-service teachers' ways of solving a range of mathematics in context problems given across course assessment. Particular attention has been given to intra-mathematical and contexts/content connections. The second component has been guided by the second research question given below. Results and discussions relating to the second research question have been presented in chapter five.

2. In relation to course tasks and learning, what did a sample of pre-service ML teachers' performance in assessment tasks indicate about their understandings of both mathematics and ML? How did this develop over a two-year period?

The pre-service ML teachers' practice during school experience periods was investigated with a focus on the nature of instructional tasks, classroom mathematical working (related to contexts/content and intra-mathematical connections), and pedagogic agendas foregrounded within teaching. Exploring the third component has been guided by the third research

question given below. Results and discussions relating to the third research question have been presented in chapter six.

3. In relation to practice, how did a sample of pre-service teachers on teaching experience use instructional and assessment tasks in ML lessons?¹

- *What was the nature of the instructional tasks used in school experience ML lessons?*
- *In relation to content/context and the need for well connected intra-mathematical and mathematics-real world understandings, what was the nature of mathematical working within their ML lessons? How did the pre-service ML teachers interpret and communicate their solution strategies during their ML teaching?*
- *How did their ML teaching informed by the selected tasks relate to the 'spectrum of pedagogical agendas' identified by Graven & Venkat (2007)?*

The need for conducting such an investigation therefore appeared to be timely and justified given the shortage of qualified ML teachers in the schools and evidence of poor performance on more complex ML tasks among learners in the schools.

1.7 Limitations of the study

As noted already, the CLM course focused on developing problem solving skills among both senior primary mathematics and ML students. Since the study focus was on professional development of ML teachers only, I was careful when drawing conclusions, bearing in mind that primary mathematics teachers were not part of the study. Such feedback was therefore exploratory and partial. To gain a full understanding of the course dynamics, research is needed focusing on the primary mathematics teachers.

Additionally, this study engaged in depth with four ML student teachers enrolled into the CLM course, and showed willingness to participate in the study. Using such a small sample puts limitations regarding generalizations of results of this study to a larger group. Rather, the results provided in-depth insights on ML student teachers' development in knowledge and how such knowledge links with their practice. According to Gay & Airasian (1996), generalisations in qualitative studies are minimal and sometimes non-existent because the

¹ The original version of this question (third research question) was about teachers' design and use of ML instructional tasks, but it was shifted to 'use' based on empirical data relating to pre-service teachers being told which tasks to cover by their supervising teachers during teaching experiences in schools.

choice of participants is sometimes purposive and small in size. However, in-depth insights were gained as a result of lengthy and intensive engagement with the participants.

1.8 The structure of the thesis

I now present the brief outline of each chapter in this research report.

1.8.1 Chapter one

In this chapter, the reader is introduced to the problem relating to the need for people to be mathematically literate, an essential aspect of citizenship. In order to realise a society comprised of numerate consumers of information and critical citizens, the chapter focuses on the role of teachers and provides a rationale for the need to address the shortage of ML teachers in schools. Empirical evidence has shown that although professional teacher development, in the form of in-service programmes (ACE courses), have been conducted across the nation, some re-skilled teachers are not teaching the subject thereby exacerbating the problem of ML teacher unavailability in the schools. The introduction of the CLM course focused on both intra-mathematical and contextual understandings aimed at addressing the ML teacher shortage in schools and developing the teachers' knowledge essential for problem solving in intra and extra-mathematics tasks. Locating the study within a new professional development course targeted at pre-service ML teachers, was premised on the view that this model could add to the general pool of ML teachers in the country.

1.8.2 Chapter two

In chapter 2, I provide a review of literature related to the focus of this study. I do this by first discussing South African perspectives on Mathematical Literacy including tensions which have been identified in the ML curriculum, advocacy for ML teachers, ML implementation and professional teacher development issues. Different notions of ML documented in international literature including overlaps and contrasts between ML in South Africa and related international conceptions have also been discussed. This literature has provided some insight which has helped me to make sense of the data and also to link the study findings with international literature, especially scholarly articles with similar foci.

Since the study focuses on exploring the nature of pre-service ML teachers' problem solving within the course and during school practice, literature related to mathematical working including the disruptions associated with pedagogic problem solving were drawn on. Due to

limited literature base in South Africa related to problem solving especially within the context of ML, some literature with similar foci drawn from mathematics has been reviewed. Lastly, I provide theoretical lenses for analysing the empirical data generated in this study. PISA's version of the mathematisation process has been found to be useful in this regard, as it focuses on making sense of students' step-by-step process of problem solving. Given that PISA's notion of mathematisation process draws from the broader idea of RME, the rationale for adopting PISA's version has been provided.

1.8.3 Chapter three

This chapter focuses on the context of the study (CLM course), specifically on course aims, course outcomes, course enactment and course assessment. Given that the course was comprised of three sub-courses (CLM1, CLM2, and Method 2), from where the study data was collected, information relating to these sub-courses has been provided. Issues of enactment and assessment especially focusing on the nature of tasks which were made available for the students to engage with have also been discussed. An understanding of the research context allows the reader to not only follow the line of argument in this study but also to check the authenticity of the generated data and related analyses.

1.8.4 Chapter four

In Chapter 4, I describe and explain the methodology used in the study. Given that the study was qualitative in nature, a case study design has been adopted and related details have been provided. I also focus on data collection methods, including when and where the data was collected. The chapter also provides information about sampling. Given that the sample size had to be narrowed down, the rationale for doing so has been detailed. The interpretive approach of data analysis has been adopted in order to understand the teachers' ways of problem solving. Lastly, the ways in which the study addressed ethical issues with a view to protect the study participants have also been highlighted.

1.8.5 Chapter five

This chapter focuses on analysis of the students' responses to course assessment tasks. I draw from the PISA mathematisation process to analyse step-by-step solution procedures provided by the students. Focusing on four participants, their individual accounts have been provided with a focus on how they engaged with course assessment tasks. Since data collection spanned across two years (2011- 2012), analysis of data collected in 2011 is followed by data

generated in 2012 with a view to explore whether there was qualitative improvement in terms of their mathematical working. Lastly, I have drawn from the individual accounts to discuss key results in this chapter.

1.8.6 Chapter six

In Chapter 6, I present the findings and analysis of data relating to pre-service teacher practice. This relates to the students' mathematical working within the context of ML teaching and learning. Although the main aim was to explore the students' problem solving in ML classrooms, an additional focus on how the pre-service teachers communicated their solution procedures also featured. A similar framework used in the analysis of solutions to assessment tasks has been utilised with additional components focusing on more mathematically oriented tasks. Due to limited lessons observed and video-recorded, results within practice provided snapshots of classroom working of the pre-service ML teacher sample.

1.8.7 Chapter seven

This chapter focuses on comparing mathematical working in the course with the teachers' mathematical working in classrooms, focussing on both intra-mathematical and extra-mathematical connections. This comparison leads to some recommendations and implications of the results for ML teacher development. Lastly, possible contributions to knowledge have been highlighted. Further, areas of possible extension of this study have been proposed.

CHAPTER TWO: LITERATURE REVIEW AND CONCEPTUAL FRAMEWORK

2.1 Introduction

Given that this study was aimed at exploring the ML pre-service teachers' development in a new professional development course, this chapter focuses on literature relating to four main aspects. First, I discuss ML in the South African context as documented in the ML policy documents. This includes a discussion of ML definition, aims, tensions within the ML curriculum, the nature of assessment and advocated practices for ML teachers. Reviewing literature relating to ML, documented within policy statements allowed me to make sense of data collected within the CLM course (related to course aims, objectives, and assessment) and in practice (related to ML teaching). Second, I review literature around implementation of ML in South Africa with a focus on learner experiences, problem solving (intra-mathematics and mathematics-contexts connections), and professional teacher development. A review of literature on implementation of ML offered insights regarding the current situation with respect to ML teaching and learning in schools and of course ML teacher training. From these insights, issues for ML teacher development and practice in schools were understood and these led into thinking about some critical lenses for investigating the CLM course that aims at developing understanding of both mathematical content and ML on one hand and the development of pre-service ML teachers' practices in the schools on the other. Third, a discussion around international conceptions of ML and related commentaries, overlaps and contrasts between ML in South Africa and international literature relating to notions of mathematical literacy has been presented. Literature on international conceptions of ML provided useful information on the current arguments from different international researchers who have engaged with related conceptions of ML. From these arguments, links between the findings of this study and results from international studies have been established. Lastly, the chapter focuses on problem solving models and how they were utilised in literature. Models focusing on mathematical working relating to both intra-mathematical and extra-mathematical tasks have been provided with the view to understand theoretical affordances which exist in literature in relation to this study. This literature review therefore provided an understanding of the kinds of competences which the pre-service teachers needed to acquire for ML teaching. The chapter concludes with a discussion relating to the conceptual framework underpinning the study.

2.2 Mathematical Literacy in South Africa

In order to redress some of the damages related to teaching and learning caused by the legacy of apartheid education, South Africa has in the past two decades attempted to reform the education system in a number of different ways. One of the products of these reforms was the introduction of ML as a fundamental subject in schools, a decision driven by evidence related to very low levels of numeracy among the adult population (Department of Education, 2003). However, the introduction of ML led to challenges concerning teacher preparation and subject implementation in schools as more learners opted to do ML at grade 10 since 2006 (Bansilal, et al., 2012). I now present details relating to the ML conception in South Africa, implementation of ML in schools, and issues around professional teacher development. I include detail on tensions and contradictions within the different parts of various curriculum documents.

2.2.1 Conceptions of Mathematical Literacy in South Africa

ML as a fundamental subject was introduced as part of the wave of curriculum reform in 2006 by the South African government with a specific focus on the FET phase (Grade 10-12). Learners from 2006 took either Mathematics or ML as subjects, but not both, at grade 10, implying that every learner has to take one subject that develops some mathematical competences. According to the National Curriculum Statement (NCS) for ML, introducing ML as a subject was aimed at developing competences and skills of learners that would enable them to understand and critically analyze information presented in quantitative forms in both immediate and future everyday lives (Department of Education, 2003). The CAPS document for ML which revised the 2003 ML curriculum concurred with the NCS and noted that these competences and skills “allow individuals to make sense of, participate in and contribute to the twenty-first century world - a world characterised by numbers, numerically based arguments and data represented and misrepresented in a number of different ways” (Department of Basic Education, 2011c, p.8). In order to have these competences developed, CAPS highlights the need for exposure to both mathematical content and real-life contexts - with mathematical content needed for analysing contexts, and contexts determining the content to be utilised. The introduction of ML also sought to deal with a situation where learners who did not do well mathematically in the junior secondary phase usually stopped studying Mathematics (Department of Education, 2003). According to Christiansen (2006, p.10), the number of learners leaving grade 12 every year without Mathematics, before 2006,

was as high as 200 000. In view of this, it was envisaged that the majority of these learners would be studying ML in the FET phase (Mbeka, 2006).

The conception of ML in the South African context largely emphasises the use of real-life situations in both familiar and unfamiliar contexts. The rhetoric suggests the engagement of real-life contexts involving mathematical features aiming at understanding the contexts themselves. The Association for Mathematics Education of South Africa (AMESA), in their response to the initial curriculum formulation, describe ML as “the ability to read, write, and engage with information and realistic situations that are numerical in nature and are mathematical in structure” (Brombacher, 2003, p.4). The AMESA description implies coverage of skills ranging from reading texts, understanding and translating the texts into generalizable forms as well as making sense of numerical forms of information. This conception of ML is linked to the definition given by the NCS which states that:

Mathematical Literacy provides learners with an awareness and understanding of the role that mathematics plays in the modern world. Mathematical Literacy is a subject driven by life-related applications of mathematics. It enables learners to develop ability and confidence to think numerically and spatially in order to interpret and critically analyze everyday situations and to solve problems (Department of Education, 2003, p. 9).

Explicitly clear from both AMESA and NCS definitions is the notion of ML as a subject whose content ought to be derived from the needs of the society in which it is located. This conception suggests that ML focuses on advancing two agendas. First, a citizenship agenda appears to underpin the first statement where ‘understanding of the role of mathematics’ features. Second, there is a sense pointing towards pushing for critical agendas in ML within the last part of the definition. Given that the CLM course was also focused on contextualised problem solving, some tasks which the students engaged with in the course and in practice contained real data drawn from the South African context, which forms the broader society for the learners. The two definitions suggest that the ML curriculum needs to emphasize development of functional understanding of Mathematics among learners. Such a link between ML and ways of life indicates that ML could also be viewed as a social practice within some particular social setting (Frith & Prince, 2006).

The rhetoric of the NCS (Department of Education, 2003) suggests that it is the contexts that drive the selection of mathematical content in ML rather than the mathematics per se. Use of contextualized tasks is emphasized to primarily equip learners with strategies that would assist them in solving everyday and work-related problems that require making sense of quantitative information. Graven and Venkat (2007b) observe that when learners engage with real-life situations, the learning of mathematics can be supported and promoted. Furthermore, they argue that by using mathematics to make sense of situations or contexts, the NCS attempts to emphasize the dialectical relationship that needs to be upheld all the time between mathematics and contexts. For instance the NCS notes some examples of the situations that confront society on a daily basis like hire purchase, mortgage bonds, investments, ability to read maps and time tables, and using medication appropriately (Department of Education, 2003, p.9). It (NCS) further states that a self managing person must be able to understand these situations and be able to solve related problems in both familiar and unfamiliar contexts, using mathematics. The implication is that a functional understanding of mathematics is key to successful engagement with situations. These specifications have however given rise to the content-context tension resulting into dilemma among ML teachers in terms of whether more emphasis should be given to “context-specific problem solving using mathematics, or to the mathematics involved in solving contextual problems”, a concern expressed by Venkat and Graven (2006b, p.20). This is in sharp contrast with school mathematics where use of formal rules and procedures is largely emphasized when solving problems, which often times are very abstract.

2.2.2 Purpose of Mathematical Literacy in South Africa

The NCS (Department of Education, 2003) claims that many South African citizens come from a disadvantaged academic past, characterized by poor quality or lack of education. Evidence reveals that access to Mathematics was denied to the vast majority of black people before 1994 (Vithal & Volmink, 2005). This suggests that the type of education that was offered to the majority of South Africans then, did not prepare the citizens well for real-life challenges that required quantitative reasoning. Although opportunities to study Mathematics were afforded to every learner in the post-apartheid era, evidence has shown that many learners stopped studying Mathematics after grade 9, thus contributing to a perpetuation of high levels of innumeracy in the adult population (Department of Education, 2003). A study report by Howie and Plomp (2002) shows that, of those learners who took Mathematics until

grade 12, the majority demonstrated a very low level of mathematical literacy and were therefore unable to understand and reason with quantitative data. Furthermore, by 2006, one-fifth of South African secondary schools did not offer mathematics beyond grade 10 (Vithal & Bishop, 2006). Further, some 40% of the students took no mathematics at all in the grades 10-12 (CDE, 2004; Perry, 2004). Out of those Senior Certificate candidates in South Africa who took mathematics, less than 5% have historically achieved success in the subject on the higher grade (CDE, 2004). The implication was that the majority of learners were leaving high school without a strong mathematical component, making it difficult for them to engage with problem situations containing quantitative information in their lives.

The introduction of ML was therefore aimed at redressing the numeracy gap among the population which was created by these challenges. In so doing, it was envisaged that critical citizenship skills would be developed among learners which would empower them to confidently solve problems related to both current and future life challenges demanding numerical skills. One of the benefits of being mathematically literate relates to the ability to make informed choices within the context of evidence where people are exploited due to biased reporting (Department of Education, 2008). With global changes technologically or otherwise, effective participation in any society is dependent on confidence in using Mathematics to interpret the world. Against this background, the South African government introduced ML, and further argues that the subject has the potential to provide learners who could have opted not to take mathematics at grade 10, with access to the kind of skills that are crucial for meaningful participation in the modern world (Bowie & Frith, 2006; Frith & Prince, 2006). Given the majority of learners enrolling for ML at grade 10 combined with the limited pool of ML teachers in the schools, ML-focused teacher training was essential to ensure smooth implementation of the ML curriculum. The CLM course was partly introduced to address this need.

2.2.3 Learning Outcomes in Mathematical Literacy

The National Curriculum Statement (NCS) was introduced in 2006 as a national response to the new South African Constitution which was adopted in 1996. Among other issues raised in the preamble, the Constitution aimed at addressing issues of social injustice by healing the divisions that prevented some sections of the society from accessing fundamental human rights during the apartheid regime (Act 108 of 1996, cited in Department of Education,

2003). Since education remains a fundamental human right, the NCS was aimed at redressing the educational imbalances that characterized the past regime by affording equal educational opportunities for all citizens (ibid).

ML, which is conceptualized as a ‘fundamental subject’ in the NCS focuses on developing skills among learners that will make them “highly numerate consumers of mathematics” in the future (Department of Education, 2003, p.9). Although the rhetoric within the ML curriculum appears to combine both consumer and critical perspectives, some critiques have observed that the ‘consumer of mathematics’ view works against the ‘critical perspective’ (Christiansen, 2006). The consumer component of the curriculum implies that future citizens will be expected to have a disposition to understand and analyse a wide range of life situations using their mathematical knowledge. For these skills to be developed, the NCS states that the teaching and learning of ML should “provide learners with opportunities to engage with real-life problems in different contexts, and so as to consolidate and extend their basic mathematical skills” (Department of Education, 2003, p.9). The implication is that the teaching and learning of ML need to emphasise understandings of both contexts and basic mathematics (GET mathematics). The development of both functional and basic mathematics understandings among learners in ML provided a rationale for locating this study within the professional teacher development course (CLM). As noted already, the CLM course aimed at developing pre-service ML teachers’ knowledge in both foundation mathematics and contexts.

In order to achieve the desired competences and skills required for citizenship, the NCS (ibid, p.10) for ML provides a set of critical and developmental outcomes that all learners are envisaged to attain, as follows:

- use mathematical process skills to identify, pose and solve problems creatively and critically;
- work collaboratively in teams and groups to enhance mathematical understanding;
- organise, interpret and manage authentic activities in substantial mathematical ways that demonstrate responsibility and sensitivity to personal and broader societal concerns;
- collect, analyse and organise quantitative data to evaluate and critique conclusions;
- communicate appropriately by using descriptions in words, graphs, symbols, tables and diagrams;

- use mathematical literacy in a critical and effective manner to ensure that science and technology are applied responsibly to the environment and to the health of others;
- demonstrate that knowledge of mathematics assists in understanding the interrelatedness of systems and how they affect each other;
- be prepared to use a variety of individual and co-operative strategies in learning mathematics;
- engage responsibly with quantitative arguments relating to local, national and global issues;
- be sensitive to the aesthetic value of mathematics;
- explore the importance of mathematical literacy for career opportunities;
- realise that mathematical literacy contributes to entrepreneurial success.

These outcomes broadly focus on achieving citizenship and critical agendas. However, a few outcomes appear to point towards realization of mathematics goals. Enhancing mathematics understandings (2nd outcome) and being aware of the aesthetic value of Mathematics (10th outcome) for example suggest a mathematical orientation. Further, these outcomes suggest the importance of both mathematics and context understandings in becoming a self-managing and participating citizen. One of the foci of the CLM course was to develop the ML pre-service teachers' knowledge related to both mathematics and contexts. An understanding of Mathematics and contexts in ML is key to analysing and critiquing quantitative arguments, a central aspect of critical citizenship. The outcomes further suggest that competences related to communicating information (in words, graphs, symbols, tables and diagrams) are central especially within the context of ML problem solving. The need for communicating information using different representations also resonated with the aims of the professional teacher development course (CLM) due to this need within the context of ML practice.

In terms of pre-requisite knowledge for learners to cope with ML, the NCS (Department of Education, 2003) and Learning Programme Guidelines (LPG) (Department of Education, 2008) specify that mathematics content learnt in the GET phase (Grade 7-9) should be used as a basis from which to proceed to the demands of ML in the FET band (Grade 10-12). The LPG specifically mentions "insight into dealing with mathematical and contextualised problems" as a relevant skill needed in ML (ibid, p.10). Within the context of evidence that shows that many learners find Mathematics challenging and struggle to pass standardised examinations in the GET phase (CDE, 2004), the NCS suggests that learners would proceed

to FET level having insight and a good mastery of fundamental mathematical content. However, research studies done in South Africa have revealed that low-performing and failing students in mathematics at grade 9 are strongly encouraged to take ML at grade 10 (Venkat & Graven, 2008). This does not only contradict the policy expectations, but also puts pressure on ML teachers to ensure that the mathematical gap is well bridged within the context of ML learning. Thus the performance evidence suggests that the expectation that learners come into the FET band with sound mathematical content knowledge is an overstatement. The pedagogic implication therefore points to the need for qualified ML teachers whose mathematical and contextual understandings are well connected, an aspect which links to the CLM course rationale.

2.2.4 Progression in Mathematical Literacy

The NCS defines progression broadly as “the process of developing more advanced and complex knowledge and skills” (Department of Education, 2003, p.3). While acknowledging that progression is not markedly evident especially in the Assessment Standards, the NCS argues that “complexity of the situation to be addressed in context, through using the mathematical knowledge and ways of thought available to the learner, is where the extent of the progression needs to be ensured” (p.38). This is also re-emphasised in both LPG and CAPS by noting that increasingly complex situations, deeper and broader understanding of knowledge, attitudes and values need to be achieved in each of Grades 10, 11 and 12. In other words, the broad Learning Outcomes in terms of mathematical content remain the same across the FET band, but the changes are in terms of degree of complexity of contexts (from more familiar to less familiar) and mathematics (one step to multi-step methods) involved.

2.2.5 Tensions related to content and contexts in Mathematical Literacy

A central argument in the ML curriculum and related policy documents associated with the 2003 curriculum namely; the Teacher Guide, LPG, and SAG, is that ML is primarily concerned with preparation of learners for the quantitative demands of everyday life. There is agreement across these policy documents that mathematical content and contexts play an important role in developing citizens who are mathematically literate. However, the integration of both content and contexts has been specified in a range of different ways in these documents, thereby resulting into tensions. Some critiques of the curriculum (Brombacher, 2003; Christiansen, 2007; Venkat, 2010) have observed that the policy

specifications appear to veer ML towards a more mathematical agenda in which mathematical development wrestles with the need for development of the life-related, quantitative data interpretative abilities that dominate the rhetoric. The implication is that the aims of ML could be viewed in terms of promoting either contexts or mathematical content, or both mathematical content and contexts. I now provide a brief discussion of these three orientations as presented in the policy documents.

The context orientation

Critical engagement with contexts drawn from life experiences is central in ML curriculum goals rhetoric for teaching and learning. The LPG for ML specifies that “the emphasis in learning should be on enabling learners to develop mathematical knowledge while dealing with issues, rather than on applying mathematics after learning the basics” (Department of Education, 2008, p.8) as the recommended approach in ML teaching. The justification lies in the fact that the subject (ML) is “rooted in the lives of the learners” (p.42). According to the NCS, the contexts selected for use in an ML classroom need to have some mathematical features and should be “meaningful to the learner” (p.38). This is also echoed in LPG by arguing that teachers of ML need to choose meaningful contexts that learners from different socio-economic backgrounds can access. The two statements suggest that teachers should know the personal experiences of the learners together with their aspirations for the tasks to be meaningful and appealing to every individual learner, something which Julie (2006) describes as difficult to implement. The NCS also states that teachers should be “aware of local contexts which could be more suited to the experiences of the learner” (p.38). Again, the local contexts which might be relevant to the individual learner experiences are too many to be included within ML lessons given the time constraints. However, a balance in terms of selecting or designing contexts that are generally defined at the ‘intersection’ of most learners in the classroom would be desirable.

Within the context of assessment, the NCS notes that assessment “should be contextually based, that is, based in real-life contexts and use real-life data” (Department of Education, 2005, p.7). Although the CLM course where this study was located had some focus on mathematics content, a wide range of contextualised tasks were also given to the pre-service teachers to engage with. The CLM course assessment also comprised some pedagogically-focused tasks which encouraged teachers to attach context to mathematical statements. Thus, the CLM course adopted a contextual frame in a number of assessments.

The mathematics content orientation

Although contexts drawn from learners' everyday real-life experiences become central in ML teaching and learning, there is evidence of curriculum content specifications pulling towards the mathematics content direction in the policy documents. For example, the NCS notes that "ML will result in the ability to understand mathematical terminology" (Department of Education, 2003, p.9), suggesting that mathematics goals would be achieved. The Curriculum Statement also acknowledges that learners do have a set of negative attitudes toward Mathematics called '*mathsphobia*', and warns ML teachers that their main challenge is to "endeavour to win learners to mathematics", through "real-life contexts which lend themselves to mathematical ways of thought" (p.43). In one of the specifications of Assessment Standards, the NCS indicates that learners will be working with complex formulae like the quadratic formulae, $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. Although there is need for ML teachers and learners to use formulae in problem solving, situations in life leading to the need to use quadratic roots formula are rare and do not necessarily resonate with the citizenship perspective adopted by ML. Despite the ML curriculum rhetoric foregrounding real-life contexts, critiques have noted that the content specification seems to largely borrow the structure of school mathematics curriculum (Christiansen, 2006).

Furthermore, the NCS states that ML skills will be useful at workplace among other areas of human life. However, it argues that for an individual to "benefit from specialised training for the workplace, a flexible understanding of mathematical principles is often necessary" (Department of Education, 2003, p.9). The implication for learners is that, they need to have a good mastery of mathematical content for them to be fully prepared for workplace challenges. The LPG also notes that ML as a subject aims at "providing access to mathematics through context" (p.9), implying that engagement with contexts is a vehicle through which mathematical goals need to be achieved. According to the Teacher Guide, learners of ML among other things need to "learn how to perform basic arithmetical operations and work with relationships between arithmetical operations" (Department of Education, 2006, p.1). This relates to doing mathematical computations using operations like addition, subtraction, multiplication and division - aspects linked to intra-mathematics connections. Developing students' skills relating to intra-mathematics connections was one of the central features of the CLM course. Although skills relating to working with arithmetic

operations are critical at the level of translation (setting up of procedures), focusing on working with operations in the absence of contexts appears to push ML learning towards mathematics content frame. In its summary, the LPG argues that ML aims to develop four important abilities, one of which is “the ability to communicate mathematically” (p.8). However, communication in mathematics and understanding mathematics terminology can be viewed as movement within the terrain of mathematical language. Furthermore, the NCS states that ML will enable learners to become “sensitive to the aesthetic value of mathematics” (p.10). This suggests that learners would develop an appreciation of the beauty of mathematics or view mathematics as a collection of concepts and ideas that can be learnt for their own sake, a sharp contrast with the observed contextual frame adopted by the ML curriculum.

The mathematical content and context orientation

Evidence suggesting that both mathematical competences and contextual understandings need to be developed in a dialectical manner in an ML classroom has also been documented within the ML curriculum. The NCS in particular states that “algebraic manipulations skills should be developed as needed in solving contextual problems” (p.12), suggesting a balance between context and content understandings. The Subject Assessment Guidelines (SAG) for ML comes out clearly on the relationship between the content and the contexts in developing ML skills. It notes that:

Learners must be exposed to both mathematical content and real-life contexts to develop competencies. On the one hand, mathematical content is needed to make sense of real life contexts; on the other hand, contexts determine the content that is needed (p.7).

This means that the focus should be on using contexts in ML and utilising GET mathematics knowledge to analyse situations. It also suggests that where learners’ knowledge appears to indicate gaps relating to mathematics content understandings, a second chance to learn mathematics needs to be afforded to the learners. This focus implies the development of students’ skills relating to both intra-mathematics and content-context connections, a key feature within ML problem solving. In terms of ML teaching and assessment, the SAG emphasises the need for both content and contexts to be treated together as follows:

When teaching and assessing Mathematical Literacy, teachers should avoid teaching and assessing mathematical content in the absence of context. ...That is, avoid teaching and assessing contexts without being deliberate about the mathematical content (p.7).

This means that competence in ML depends on combining an understanding of real-life contexts with mathematical content. It (SAG) also appeals to the teachers that tasks selected or designed for assessment need to “provide learners with the opportunity to demonstrate both competence with mathematical content and the ability to make sense of real-life, everyday meaningful problems” (p.8). This implies competences relating to connections within mathematics (intra-mathematical) and across mathematics and contexts (translation). According to the LPG, ML learning needs to emphasise “enabling learners to develop mathematical knowledge while dealing with issues, rather than on applying mathematics after learning the basics” (p.8). Developing learners’ knowledge related to both mathematics content and contexts concurrently has also been echoed in the Teachers’ Guide where teachers are asked to balance both mathematical and contextual agendas;

The challenge for you as the teacher is to use situations or contexts to reveal the underlying mathematics while simultaneously using the mathematics to make sense of the situations or contexts, and in so doing develop in your students the habits or attributes of a mathematically literate person (p.4).

The Teachers’ Guide therefore suggests that it is the engagement with contexts using mathematics content combined with the ability to use mathematics to make sense of the contexts, which leads to competences consistent with a mathematically literate individual in a society. The content-context frame therefore implies that competence with mathematical principles and contextual understanding need to be developed together within ML teaching and learning.

Given the above, two aspects relating to the curriculum specifications are noted. First, it seems unclear whether ML should focus on extending understandings of mathematics content, developing contextual understandings or both. Second, it appears that the curriculum specifications are asking for a balance of all the three orientations. These tensions featuring in the ML curriculum and related policy documents have also been observed at the level of ML implementation in South African schools. Venkat (2007) notes that instead of advancing the citizenship agenda alone through engagement with real life situations containing quantitative

features, some teachers tend to foreground a more mathematical agenda. While some teachers see the need for mathematical and citizenship skills to be developed together (striking a balance) as the Teacher Guide and SAG suggest, a study by Venkat (2010) on a litter project shows a teacher giving more time to learners to understand the situation using mathematics. She observed that ML pupils working on a litter project were “not centrally engaged with making sense of the mathematics per se in their activity; but, primarily, with making sense of the litter project situation” using mathematics (ibid, p.66).

However, empirical evidence drawn from a small scale study suggests that striking a balance between content and contexts is hard to achieve in a classroom (Frith, et al., 2010). Their study focused on their own practices relating to offering a quantitative literacy course to both humanities and law students at an urban University in South Africa. They observed that striking a balance between mathematics content and contexts was a huge challenge, as they realised that they spent more time discussing the contexts at the expense of mathematical content, as noted:

...we became increasingly concerned that the mathematical content was being eclipsed in favour of the context ... I'm kind of wanting to stop and say, "Hey now this is what we've done. All of these questions let's go and look at them. This is percentage increase, or percentage change, these are percentage points" (p.264).

This suggests that successful engagement with contexts containing some mathematics features remains a challenging process for teachers, and therefore implies the need for developing the pre-service ML teachers' knowledge relating to problem solving focusing on balancing their understandings of both mathematics and contexts. Given that the content-context tensions have been observed in practice, analytically, the content/context-driven agenda spectrum proposed by Graven and Venkat (2007a) detailed later in this chapter, provides an important framework to explore pre-service ML teachers' practice during school experience.

2.2.6 What is advocated for Mathematical Literacy teachers in South Africa?

Advocacy in terms of Mathematical Literacy teaching

As noted, there is emphasis in the ML curriculum to engage with contexts in ways that promote situational understanding. In other words the learning of mathematics should not be

in isolation but rather be embedded in contexts. Thus engagement with contexts is advocated as a key driver in ML learning. The ML curriculum also suggests that ML teachers need mastery of the mathematical content, familiarity with different contexts, and skills to effectively link mathematics and contexts. The importance of a good mastery of fundamental mathematics in coherent ways in ML cannot be overemphasised. Evidence has shown that quantitative reasoning, a common feature in ML cannot be achieved if mathematical content knowledge of an individual is fragmented (Gainsburg, 2008). There is need for a deep understanding of what mathematical symbols and procedures mean, and why certain procedures or algorithms work in general, one of the key aspects of the CLM course. This is what Ma (1999) calls profound understanding of fundamental mathematics (PUFM) and refers to “an understanding of the terrain of fundamental mathematics that is deep, broad, and thorough” (p.120). She argues in her book “Knowing and Teaching elementary Mathematics: teacher’s understanding of fundamental mathematics in China and the United States” that PUFM is about conceptual understanding (knowledge of how and why it makes sense), algorithmic competence (justifying algorithm verbally and symbolically), and knowledge about relationships between different topics. However, some studies relating to the specific demands of everyday adult practices reveal that most occupations involve only a low level of mathematical content particularly eighth to ninth grade mathematics (Gainsburg, 2005). Cohering with this view, the ML curriculum is explicit that its focus is predominantly on the use of GET mathematics rather than extending into higher grade mathematical ideas (Department of Education, 2003).

Engaging students in profound understanding of mathematics content which provides the basis for the development of basic skills in quantitative reasoning is thus very critical in terms of preparing them to successfully participate in a “data drenched society” (Steen, 2001, p.2). Similar emphases have been made in the Subject Assessment Guidelines (SAG) of ML with arguments that learners must have a good grasp of both basic mathematical content and real-life contexts to develop competencies needed in life and at workplace (Department of Education, 2008). Even assessment should always allow learners to use mathematical content to solve problems that are contextually based (ibid). This places demands on ML teachers to design and/or select assessment tasks that provide learners with the opportunity to demonstrate both competence with mathematical content and the ability to make sense of varied situations encountered in their everyday lives. This means that profound understanding of mathematics content is as crucial as functional understanding of contexts in ML. Given

debates about how mathematical agendas are fundamentally different from utilitarian agendas (Brombacher, 2007; Davis, 2003), the CLM model to develop knowledge and practice of both primary mathematics and ML teachers together, therefore seemed to be interesting to follow.

Advocacy in terms of assessment in Mathematical Literacy

The idea of assessment in ML, just like in any subject areas is central in terms of documenting students' performance for the purposes of monitoring learning as well as progression and certification. According to SAG (Department of Education, 2008), assessment in ML can either be informal (for improving learning) or formal (for progression or certification purposes). Both forms of assessment seek to measure the extent to which learners are able to make sense of scenarios based on realistic, familiar and unfamiliar real-life contexts by drawing on both mathematical and non-mathematical techniques and/or considerations. To this end, the teachers are warned not to assess the mathematics in the absence of contexts, but allow the contexts to dictate the mathematics to be used (Department of Education, 2008). The SAG for ML (ibid) for instance, indicate that the tasks chosen or designed for assessment in ML need to allow students to demonstrate their understandings in both mathematics content and contexts as follows:

Teachers need to design assessment tasks that provide learners with the opportunity to demonstrate both competence in mathematical content and the ability to use a variety of both mathematical and non-mathematical techniques and/or considerations to make sense of real-life, everyday, meaningful problems (p. 8)

The subject focus on assessing both mathematics and related translations (between contexts and mathematical models and/or solutions) implies that students need to demonstrate abilities in both intra-mathematical and extra-mathematical working. Further, this focus supports the content/context orientation relating to teaching and learning observed in the ML curriculum. Such a focus on both intra-mathematical and extra-mathematical working across assessment tasks suggests theoretical underpinnings related to modelling (Blum & Ferri, 2009; Kaiser, 2007; Maaß & Gurlitt, 2011) or mathematisation processes (OECD, 2006, 2013).

Within the context of problem solving, the Subject Assessment Guidelines (SAG) for ML (Department of Education, 2008, p.17) emphasises use of given formulas, as follows;

Grade 10 We know this when the learner is able to:	Grade 11 We know this when the learner is able to:
10.1.1 Solve problems in various contexts, including financial contexts, by estimating and calculating accurately using mental, written and calculator methods where appropriate, inclusive of: • working with simple formulae e.g. $A = P(1+i)^n$ • using the relationships between arithmetical operations (including the distributive property) to simplify calculations where possible. (NOTE:	11.1.1 In a variety of contexts, find ways to explore and analyse situations that are numerically based, by: • estimating efficiently: • working with formulae by hand and with a calculator, e.g. $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ • showing awareness of the significance of digits:

The idea of using given formulas appears to be linked to the consumer orientation and implies that students need not derive or retrieve formulas when engaging with problem situations. This view is re-emphasised in the revised ML curriculum (CAPS) (Department of Basic Education, 2011a) in the following ways;

In Grade 10, primary focus is on working with 2-dimensional shapes and calculations of perimeter and area of such shapes. In Grades 11 and 12, focus shifts to include 3-Dimensional shapes, with calculations of perimeter, area and volume extended accordingly. All formulae for calculations involving perimeter, area, surface area and volume will be provided in assessments. Note that in all formulae learners are expected to work with the approximate value of pi (π) of 3,142 (p.69).

Given that everyday situations are often messy and untidy (Brombacher, 2007; Steen, 2001), skills relating to retrieving and/or deriving formulas would appear to be useful. This justifies the CLM course focus on developing the pre-service teachers' skills relating to retrieving formulas. The implication is that in addition to knowing how to use already given formulas, ML teachers need to know how to retrieve and/or derive formulas appropriate for a range of different problem situations.

Implementation of Mathematical Literacy in SA: What do we know?

In this section, I focus on aspects of implementation relating to ML teaching and learning, issues of assessment and professional teacher development, as these are strongly linked with the study focus. At the level of teaching and learning including assessment, the focus was on the degree to which the tasks utilized for teaching and assessment aligned with the ML curriculum specifications and whether opportunities for both intra-mathematics and mathematics-context connections were provided. In terms of professional development, my interest was related to documentation of the kinds of teacher training programmes which

were rolled out since the introduction of ML in 2006, with a view to develop ML skills among either already qualified teachers or pre-service teachers. As already noted, this study involved analyzing ML pre-service teachers' responses to assessment tasks given in the course as well as making sense of their mathematical working within the context of ML classrooms, with a focus on lesson episodes. Reviewing literature around these three aspects allowed me to understand the teachers' ways of problem solving in both the course and in practice.

Mathematical Literacy teaching and learning

Evidence from studies in South Africa suggests that ML is perceived as a subject for low attainers by the larger community (Graven & Venkat, 2007b; Mbekwa, 2007). Even teachers who were supposed to offer ML in schools as an open choice subject for learners, have reportedly being biased towards mathematics as opposed to ML when advising learners on what subjects to take at grade 10 (Graven & Venkat, 2007b). Low performers and failing learners in mathematics at the end of GET phase (Grade 9) are strongly advised to take ML whilst those passing mathematics are advised to take mathematics (Graven & Venkat, 2008). This suggests that rather than viewing mathematics and ML as different, some perceive mathematics and ML in a hierarchical relation with mathematics subsuming ML. The danger of this view is that ML could be treated as a low status subject.

Given the context of learners' weak elementary mathematics understandings, teachers (especially at Grade 10) are challenged to employ teaching and learning methods that seek to motivate learners and transform them into active participants in the ML classrooms. Furthermore, the previous section emphasises that teachers for ML need to have an understanding of both contexts and elementary mathematics in order for them to cope with the classroom demands of ML teaching - given that many learners' performance in mathematics at grade 9 was weak. The importance of mathematics content knowledge is also emphasised by Christiansen (2007) in the following terms:

A teacher of Mathematical Literacy would have to know enough mathematics and enough about applications of mathematics, misuses of mathematics, and effects of using mathematics to further learners' awareness and understanding of the role that mathematics plays in the modern world, help them develop the ability and confidence to interpret and critically analyze social, political and practical situations using mathematical skills transferred from one context to another (p.101).

Christiansen's view appears to suggest 'a learn mathematics and then apply perspective' combined with a citizenship frame (knowing about 'misuses of mathematics'), as key skills that need to be developed among ML teachers. However, this perspective contradicts some ML curriculum specifications asking that contexts need to dictate the mathematics in the curriculum. The need for ML teachers to have functional understandings and a strong mathematical content knowledge justifies the focus on foundational mathematics within the CLM course. Even the course coverage (detailed in chapter 3) in terms of the topics appeared to focus on consolidating the pre-service teachers' understanding of foundation mathematics.

A study done in South Africa by Venkat and Graven (2008) has shown that use of meaningful learning contexts which are driven by real-life everyday experiences and are focused on exploring and understanding, resulted in learners' positive classroom experiences. Furthermore, they found that such classroom tasks stimulated discursive opportunities in ways that promoted learners' participation and understanding (Graven & Venkat, 2007b). Similar results have shown that students who did poorly in traditional mathematics were reportedly showing improvements in their performance if they were given opportunities to engage with contextual problems in the form of 'project works' (Vithal, 2006). Performance improvement observed in Frith's study suggests in some ways positive experiences among learners towards ML learning.

A review of literature shows limited evidence relating to exemplification of instructional tasks used for ML teaching purposes in South Africa. Some of the tasks which teachers have used for instructional purposes in ML have been provided in table 2.1. Given that this study also focused on the pre-service teachers' engagement with instructional tasks during teaching experiences, these tasks provided an understanding of the nature of tasks described in literature relating to ML teaching.

Summary of instructional task	Source
1. Learners, in groups, are asked to consider litter around the school; where litter went, how it got there, whether any recycling occurred, and who to ask and how they might find out answers to these questions if they were not sure. The learners were further asked to devise a data collection sheet for recording data for a corridor in the school that was assigned to their group, conduct an interview with the person/people who would have relevant information and follow-up with them on data collection, to collate this data, do appropriate calculations needed to represent their findings, and then to compare and contrast the findings for each corridor in order to build up a	Venkat, 2010

combined picture of the litter situation across the school.	
2. A handout with a newspaper article about the effect of rain on dams. Article talked about rising levels of water in the Vaal dam. The handout also contained information about water levels in other South African dams in the current week and in the same week in the previous year. The questions asked following this information were as follows; • If 250 000 litres of water are flowing through the vaal dam floodgates in one second, how many litres will flow through in one minute? • By how much percents has the level of water in Hluhluwe dropped since last year? • Explain how important the Vaal dam is to Johannesburg residents?	Graven & Venkat, 2008
3. Numbers of learners in the classroom are counted (i.e. 11 girls and 12 boys). This data is then used to calculate the percentage of girls in the classroom.	Venkat, 2007

Table 2.1: Examples of instructional tasks within ML teaching

The three examples, which were drawn from small scale studies, describe extra-mathematical tasks that were utilised across different ML lessons. The questions in each of the examples appear to focus on making sense of situations using mathematics. In this way the content/context framing advocated in the policy documents for ML appears to feature (Venkat, 2007). However, empirical evidence has shown that ways how teachers tend to mediate such tasks within the context of teaching differ. A range of different teaching orientations adopted by teachers in ML classrooms has been captured in a continuum called a spectrum of pedagogic agendas by Graven and Venkat (2007a). The agendas were related to interpretations of the tensions within ML teaching which have also been observed in the policy documents for ML. The agendas suggest the presence of a range of useful connections which teachers make during teaching. Central to problem solving are two connections namely, context-content and intra-mathematical connections, both of which appear to feature across the agendas. One of the main foci of this study was to explore these connections across the pre-service teachers' lesson episodes within practice. Having talked about ML implementation with regards to teaching and learning, I now discuss issues relating to ML assessment.

Assessment in Mathematical Literacy

Given the study focus, attention in this section has been paid to the nature of ML assessment at two levels - namely at professional teacher development and at school level. Across the teacher development programmes reported in literature, especially the ML re-skilling courses there is some sense pointing towards developing ML teachers' knowledge relating to both mathematics and contextual understandings (Bansilal, Mkhwanazi, & Mahlabela, 2012;

Brown & Schäfer, 2006; Hechter, 2011; Vilakazi & Bansilal, 2012). However these studies appear to largely report on the teachers' performance across extra-mathematical tasks only, an aspect which appears to be in agreement with the nature of exemplification at the level of ML instruction in schools, and broadly with the contextual frame within ML policy documents. Little evidence relating to the nature of tasks used has been provided in literature. Given that consolidating the teachers' knowledge relating to foundation mathematics understandings was one of the ML re-skilling programmes aims (Brown & Schäfer, 2006; Vilakazi & Bansilal, 2012), a focus on contextualised tasks suggests that engagement with these kinds of tasks allows access into the teachers' mathematics content understandings through intra-mathematical connections. Some of the contextualised assessment tasks drawn purposively from literature at the level of ML teacher development (ACE) have been exemplified in table 2.2.

Type of assessment	Assessment tasks	Source
Examination	1. The formula that is used to calculate the transfer duty, payable by a new home owner, is as follows: • For a purchase price of R0-R500 000, the transfer duty is 0%. • For a purchase price of R500 001 to R1 000 000, the transfer duty is 5% on the value above R500 000. • For a purchase price of R1 000 001 and above, the transfer duty is R25 000 + 8% of the value above R1 000 000. a). Calculate the transfer duty payable on a house that is valued at R895 000. b). My friend paid transfer duty of R45 280 on the house that she bought. How much did her house cost?	Bansilal et al, 2012
	2. In March 2003, my new car cost R55 000. What would I have expected to pay for a new car of the same make in March 2005, if increases were in line with inflation? The figures for the monthly inflation rates were provided in a graph containing all the monthly figures over a period of 4 years.	Bansilal, 2011
Assignment	3. Task focused on designing a pattern for a block of tiles and then replicating this block to cover the entire floor was given as an assignment. Dimensions of the hall (21m by 15m) Dimensions of a block (3m by 3m) Dimensions of the tiles (300mm by 300mm) For assessment, they were asked to draw the block they had designed (in a square outline provided), to count or calculate the number of each colour of tile in their block, and then to calculate the number of each colour of tile that would be needed to tile the community hall floor.	Brown & Schafer, 2006

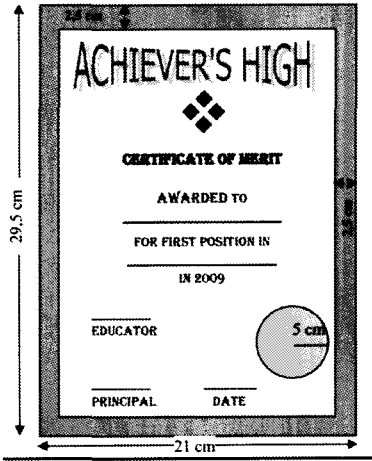
Project	4. Research task For this project you will work on your own and do your own survey. In this chapter you have seen two examples of the kind of survey you should do. All the steps that you have to do to complete a survey are covered in the units in this chapter. Suggested example of a survey: Purpose: To find out about young people's attitudes towards living in South Africa. Questions: • Age, gender, grade at school, etc. • How positive do they feel about their future in South Africa? • How important do they think it is to vote in an election? • Do they think South Africa has a lot to offer young people? • How proud are they to be South African? • How important is it to them that South African sportsmen and women should win in international competitions (such as the Olympics)?	Frith & prince, 2006
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Table 2.2: Examples of assessment tasks within teacher training courses

Three levels of assessment have been highlighted in the table namely examination, assignment and project - all of which appear to support the citizenship framing. Similar assessments, save projects, were carried out in the CLM course. Despite the contextual nature overlapping across the tasks, the tasks show contrasts in terms of the amount of work required when engaging with these tasks. The examination tasks appear to be less demanding than the other two tasks (assignment and project) in terms of time required completing them. Furthermore, unlike the examination and the assignment tasks where numerical data was given, the students needed to collect quantitative information and use it in the solving process in the project task. These examples suggest that skills related to both extra-mathematical (shifting between context and mathematics content) and intra-mathematical (calculations, computations etc) connections would be at play during the problem solving process. Since part of the study focus was to explore the nature of these connections, and how they interact, across the pre-service teachers' mathematical working, these examples provided a useful way of thinking about the kinds of tasks within professional teacher development where these connections feature.

Despite more contextualised tasks featuring at the level of exemplification across ML research studies, a review of the nature of assessment problems in Matriculation examinations, reveals that a combination of mathematically focused tasks and extra-

mathematical tasks are considered (Department of Basic Education, 2008, 2009, 2012). Table 2.3 provides some of the assessment tasks drawn purposively from ML examination past papers between 2008, when the first ML Matric examination was administered, and 2012. These examinations questions allowed me to gain some understanding in terms of the kinds of skills which are examined at Matric in relation to the competences emphasized in the CLM course on one hand and within school experience on the other.

Assessment task	Source
1. a) Decrease 500 kg by 12%. b) Calculate $R450 - R32,40 \times 10$	NSC Exam: ML paper 1 (Department of Basic Education, 2008)
2.  Ms James, an educator at Achiever's High, is responsible for preparing the prize-giving certificates for the annual academic awards day. The certificate is rectangular in shape with a uniform 2,5 cm shaded border, as shown in the diagram below. The outside measurement of the certificate is 21 cm by 29,5 cm. A gold or silver circle with a radius of 5 cm indicating the performance level of the learner is placed on the certificate. a) Write down the length of the diameter of the circle. b) Write down the length of the unshaded part of the certificate. c) Calculate the area of the circle. Use the formula: $\text{Area} = \pi r^2$, where $\pi = 3,14$ and r = radius. d) Calculate the perimeter of the outside of the certificate. Use the formula: $\text{Perimeter} = 2(l + b)$, where l = length and b = breadth. e) Determine the area of the certificate. Use the formula: $\text{Area} = \text{length} \times \text{breadth}$	NSC Exam: ML paper 1 (Department of Basic Education, 2009)
3. Peggy is the owner of the Tasty Sandwich Company. Her weekly expenses are: • Rent R520,00 • Water and electricity R390,00 • Wages 25% of the total weekly expenses • Other R140,00 The cost of the ingredients and packaging is R4,00 per sandwich. a) Calculate her total weekly expenses. b) Write down a formula that Peggy could use to calculate her total costs (in rand) per week for producing x number of sandwiches in the	NSC Exam: ML paper 1 (Department of Basic Education, 2012)

form: Total costs (in rand) per week = ... c) Peggy's total costs for making sandwiches in one week amounted to R2 400. How many sandwiches were made?	
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Table 2.3: Examples from ML Matric past papers

Task one, which is not located in any context (beyond mass), appears to be mathematically focused. This suggests that learners' understanding of mathematics content can be examined in the absence of contexts, contradicting the SAG (Department of Education, 2008) specification on assessment. The use of units such as kilograms (kg), South African currency (Rand), does not necessarily affect the problem solving process and the result, as learners could successfully engage with these tasks without being aware of the units, suggesting a mathematics content frame observed within the ML curriculum. The second task exemplifies cases where the problem is embedded in some context but the focus largely remains mathematical. Although reference is made to the features on the certificate, the questions appear to be concerned with the mathematical calculations and computations, and not necessarily the understanding of the context. This suggests a mathematics content frame observed in the ML curriculum. This is what Usiskin (2001), and FitzSimons & Wedege (2004) call '*artificial word problems*' and '*pseudo-contextualisation*' respectively. While Usiskin (ibid) warns against the use of such tasks 'masquerading as reality in the mathematics classroom', FitzSimons and Wedege argue that tasks of this nature 'fail to prepare learners for participation in the varied discourses of the workplace'. Unlike the first two examples, the third task appears to focus on understanding the business context. The context appears to dictate the mathematics to be used, and the mathematics content is used to make sense of the context. This suggests a contextual frame observed in the curriculum. Overall then, the mix of orientations discussed in the curriculum documents, also appear to play out in ML assessment.

2.2.7 The status of professional teacher development in Mathematical Literacy

Since ML was introduced in 2006 as a school subject in South Africa, professional development programmes for ML do not have long histories like other school subjects. The large majority of ML teachers in the schools received an in-service training through re-skilling programmes (i.e. ACE), which were conducted throughout all the nine provinces in South Africa from 2006 (Bansilal, 2012; Frith & Prince, 2006; Hechter, 2011). This was the case because there was no route to professionally develop new pre-service teachers to teach

ML. Empirical evidence from small scale studies conducted in Western Cape Education Department (WCED) has revealed that teachers who were qualified to teach subjects like mathematics, history, and geography were invited to teach ML at the beginning of 2006 (Mbekwa, 2006, 2007). According to Mbekwa, these non-mathematics specialists were those that showed interest in teaching ML despite their specialization in non-mathematics subjects. The ACE courses comprised modules focusing on mathematics content and contextual understandings (Bansilal, 2012; Mbekwa, 2006), components which also constituted the main features in the CLM course. While some of these teachers were re-skilled first before they could be deployed to schools to teach ML, evidence has shown that other teachers, especially mathematics specialists, did not see the need to be re-skilled (Mbekwa, 2007). This implies the view that teaching and learning methodologies can be directly transferred from mathematics to ML, a claim that AMESA has disputed (Brombacher, 2003). Although similar courses have been conducted in all provinces across the country since 2005 an ongoing shortage of ML teachers has been reported (Bansilal, et al., 2012; Webb, et al., 2011).

Within the context of teacher shortage in ML, some teachers who either dropped out or failed the ACE courses were reportedly found teaching in the schools (Bansilal, et al., 2012). The main reason why the teachers were dropping out or failed was because they could not cope with the mathematical demands (ibid), given their qualifications in non-mathematics subjects. The implication is that some of these teachers were teaching despite having notable knowledge gaps relating to mathematics content, a key feature in ML. Weak mathematics content understandings has also been observed among pre-service ML teachers enrolled into the professional development course where this study was located. In a study conducted by Winter and Venkat (2013), students with this focal cohort were given a test focused on contextualised tasks and analysis of written protocols showed disruptions at the level of intra-mathematical working.

Conceptions of ML in international literature

Conceptions of ML in international literature have been documented in a range of different ways. In this section, I discuss definitions and commentaries related to quantitative literacy, numeracy and mathematical literacy². I am aware of the existence of related conceptions like

² In this study, Mathematical Literacy (ML) is used to refer to a subject in South Africa whereas mathematical literacy (with lowercases) broadly refers to mathematics competences.

mathematical modelling (Blum & Ferri, 2009; Kaiser, 2007; Kaiser & Maaß, 2006) and realistic mathematics education (RME) (Freudenthal, 1973; Gravemeijer, 2004; van den Heuvel-Panhuizen, 2001). However, emphases across these conceptions appear to be similar to those of numeracy and some notions of mathematical literacy (Jablonka, 2003), but with additional focus on mathematics learning including the understanding of models needing advanced mathematics knowledge. A review of these international conceptions has provided insight on current international debates about the ways in which people can be supported to deal with quantitative aspects of situations in their personal lives or at workplaces. It has also provided useful information and illuminated ways in which ML in South Africa can be taught for understanding.

2.2.8 Quantitative literacy

Evidence showing that school mathematics has not led to quantitative literacy makes a convincing case that competences relating to dealing with quantitative information is important in the modern world (Steen, 2001). Studies by Hughes-Hallett (2001) and Steen (2001) suggest that an individual's quantitative literacy cannot be improved by learning more school mathematics. Hughes-Hallett (2003) notes that there are "many examples of students with sophisticated mathematics course work in their backgrounds who possess minimal quantitative literacy, as well as many examples of students with remarkable levels of quantitative literacy but little formal mathematics" (p.94). So, what is quantitative literacy and how can quantitative skills be attained?

Steen (2001), while admitting that there is little agreement on exactly what quantitative literacy (QL) is, provides the following summary:

Quantitative literacy is the capacity to deal effectively with the quantitative aspects of life (p.6). ...Quantitative literacy empowers people by giving them tools to think for themselves, to ask intelligent questions of experts, and to confront authority (p.2).

This conception suggests that QL adopts both consumer and critical perspectives in that it focuses on engaging with quantitative aspects of life combined with taking a critical stance on issues and using related results to 'confront authority'. Steen further argues that citizens who are quantitatively literate need to have a "predisposition to look at the world through mathematical eyes, to see the benefits (and risks) of thinking quantitatively about commonplace issues, and to approach complex problems with confidence in the value of

careful reasoning” (p.5). The idea of searching for benefits and risks within the context of problem solving concurs with Steen’s conception of QL (given above) in ‘critical orientation sense’. Another definition is given by the International Life Skills Survey (ILSS, 2000) cited in Steen (2001), who define quantitative literacy as;

An aggregate of skills, knowledge, beliefs, habits of mind, communication capabilities, dispositions, and problem solving skills that people need in order to engage effectively in quantitative situations arising in life and work.

Besides the ILSS’s definition suggesting a more consumer stance, it appears to agree with Steen’s conception in terms of focusing on individual skills needed to deal with quantitative situations. Within the context of South Africa, Frith and Prince, (2009), in their ‘framework for understanding the quantitative literacy demands of higher education’ define quantitative literacy in the following terms;

“Quantitative literacy is the ability to manage situations or solve problems in practice, and involves responding to quantitative (mathematical and statistical) information that may be presented verbally, visually, in tabular or symbolic form. It requires the activation of enabling knowledge and behaviours and can be observed when it is expressed in the form of a communication, in written, oral or visual mode.”

In addition to consumer elements in this definition, there are also aspects pointing towards taking some critical stance to quantitative information (i.e. responding to information). Overall, the above definitions suggest that quantitative literacy skills could be attained through engagement with contexts, a component which appears to agree with the contextual frame observed in the ML curriculum in South Africa. The implication is that QL needs to be taught in contexts to all school learners as it is a life skill (Hughes-Hallett, 2001), a view which contradicts the current ML structure in South Africa where ML is offered predominantly to failing learners at grade 9.

In conclusion, it appears QL is similar with South African ML although different names have been used in literature. Further, components of both consumer and critical orientations seem to feature across these two conceptions.

2.2.9 Numeracy

The notion of numeracy is conceptualised in a variety of ways ranging from defining numeracy as a set of basic mathematical skills which must be attained at the end of foundation schooling to developing some skills needed to deal with problem situations presented in quantitative terms. Numeracy has its roots in the Crowther report on mathematics education in England and Wales (Cockcroft, 1982). The report proposed reform relating to the ways in which mathematics needed to be taught to learners in schools based on the mathematical demands of adult life. In this report numeracy is defined in the following terms:

an understanding of the scientific approach to the study of phenomena - observation, hypothesis, experiment, verification and the need in the modern world to think quantitatively (cited in Cockcroft, 1982, p.11).

This definition is broad in that it conceptualises numeracy in terms of scientific approaches to phenomena which often are laboratory based and implies a mathematical orientation where the understanding of mathematics seem to feature centrally. The British Department for Education and Employment (DfEE, 1998) provides a more refined definition of numeracy by including competences related to ways in which information is gathered, analysed and presented. In this report, numeracy is defined as follows:

Numeracy means knowing about numbers and number operations. More than this, it requires an ability and inclination to solve numerical problems. ... It also demands familiarity with the ways in which numerical information is gathered by counting and measuring, and is presented in graphs, charts and tables (p.5).

This conception appears to focus on understanding numbers through solving ‘numerical problems’, a key feature of mathematics orientation. However, in addition to understanding numbers, the definition suggests that an individual needs to be confident about ways in which raw data is collected and processed, implying a focus on solving world situations. The utilization of data involves sense making and insight which appear to be central in everyday life situations particularly during the process of making judgments, one of the key attributes of a self managing individual as claimed by the ML curriculum (Department of Education,

2003). The importance of sense-making skills and constructing quality judgments from real-life situations is further elaborated by Coben (2004) as follows:

[Being] numerate means to be competent, confident, and comfortable with one's judgments on whether to use mathematics in a particular situation and if so, what mathematics to use, how to do it, what degree of accuracy is appropriate, and what the answer means in relation to the context (p. 10).

The above description of 'being numerate' by Coben highlights the importance of having insight and interpretation skills in solving problems. Coben's conception suggests a critical view of numeracy where judgments about engagement with everyday situations using mathematics, combined with interpretation of solutions are given primacy. It also implies the need for a predisposition to know exactly which mathematical tools work for particular cases, suggesting that contexts would dictate the mathematics content.

Numeracy is viewed as the ability to use mathematics at a level necessary to function in a specific cultural context (Coben, et al., 2003; FitzSimons & Wedege, 2007). Numeracy can also be understood as "a willingness to engage effectively with quantitative information in simple settings" (Gardiner, 2004, p.14). Gardiner's focus on 'simple settings' contradicts Steen's view who argues that "mathematical literacy focuses on sophisticated uses of elementary mathematics" (Steen, et al., 2007, p.289). Further, the idea of bridging bridges between mathematics and the real world resonates with this study focus relating to exploring extra-mathematical and intra-mathematical connections across the pre-service teachers' mathematical working. Use of situations is fundamental in ensuring the development of numerate skills in an individual. This is why Coben and colleagues (Coben, et al., 2003), reiterate that a functional understanding of mathematical concepts can be achieved through engagement with contexts. These contexts in numeracy according to Coben (2004) could be either authentic or artificial. The use of authentic contexts has also been emphasized within the ML curriculum rhetoric although assessment appears to contradict this rhetoric (Department of Education, 2008; Department of Basic Education, 2011).

Although evidence relating to understanding of numbers has been noted in some definitions of numeracy, a focus on contexts appears to have parallels with the notion of QL. In other words, the two conceptions of ML (Numeracy and QL) suggest the presence of the citizenship orientation.

2.2.10 Mathematical literacy

The mathematical literacy domain, in PISA's terms, is concerned with students' capacities to deal with a variety of situations using mathematics (OECD, 2006). Sense making involving real-world problems is the means to this end. However, PISA argues that successful engagement with problem situations requires some competences in elementary mathematics. Thus an understanding of both mathematics content and contexts is very vital in the development of mathematical literacy. A mathematically literate person makes use of these skills in both trivial situations and "less structured contexts, where the directions are not so clear, and where the student must make decisions about what knowledge may be relevant and how it might usefully be applied" (ibid, p.72). The idea of decision making in terms of what mathematical tools are needed combined with the degree of accuracy when engaging with contexts overlaps with Coben's conception of numeracy, within the critical framing. PISA (ibid) conceptualises mathematical literacy as follows:

Mathematical literacy is an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgements and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen (p.72)

Unlike quantitative literacy conception, the PISA definition puts much emphasis on the utilitarian value of mathematical knowledge by defining "mathematical literacy more strongly in the direction of application" of mathematics and engagement with real-world contexts from which mathematical competences and functional understandings need to be developed (Kaiser & Willander, 2005, p.49). Although the name (mathematical literacy) is similar to the South African conception, the rhetoric in the ML curriculum in South Africa specifies that it the contexts which dictate the mathematics to be used, a view which contrasts with the 'application' orientation. It is argued that the individual's capacity to make sense of situations would be achieved by allowing learners to engage with a variety of real-world contexts (Edge, 2001; OECD, 2006). Emphases relating to engagement with contexts with the view to develop quantitative skills have been noted across the South African conception of ML, quantitative literacy and numeracy – with numeracy conception often using the term 'numeracy skills' rather than 'quantitative skills'. The focus on situations implies that mathematical literacy can only be understood in relation to its context (location) and specificity of cultures (Burkhardt, 2006; de Lange, 2003; Edge, 2001).

The PISA conception of mathematical literacy is similar to de Lange's definition (see de Lange, 2006) in that both conceptions broadly emphasise mathematical knowledge put into functional use and covers mathematical competences in the broadest sense. De Lange has been involved in PISA activities in various capacities ranging from being a member to being chairperson of the group. In his article 'mathematical literacy for living from OECD-PISA perspective' (ibid), he notes that the necessary condition for contextual understandings is conceptual understanding of mathematical principles. He reiterates the need for a coherent understanding of "mathematical terminology, facts, and procedures in performing certain operations, carrying out certain methods, and so forth" when engaging with a problem in context (p.16). The need for functional understandings of mathematics has also been highlighted within the other conceptions (QL, ML and numeracy). The implication is that competence with mathematical language is key to functional understandings. Given a context, learners need to recognise the quantitative features embedded within the context and choose mathematical methods and representations that best simplify and solve the problem (de Lange, 1990). Developing the teachers' skills relating to the procedures involved in problem solving was one of the key features of the CLM course enactment.

De Lange is critical about confining the term 'literacy' to indicate basic or low-level functionality only. He argues that, since being literate in mathematics "means different things according to the needs of the particular community", literacy should be viewed as a "continuous, multidimensional spectrum ranging from aspects of basic functionality to high-level mastery" (ibid, p.16). He further refers to basic mathematical literacy as a level of skills expected of all learners below the age of 15, independent of their role in society, and advanced mathematical literacy as the level of skills expected of students above the age 15 as they begin to think about or identify their future careers, and how they fit into their communities of practice. In South Africa, mathematics is compulsory until grade 9, where many are expected to be 15 years old. However, evidence from literature suggests that levels of mathematical literacy among these learners remain very low (Brombacher, 2007; Howie & Plomp, 2002).

Another definition of mathematical literacy which appears to be similar to PISA conception is given by Eva Jablonka (2003). In her article '*Mathematical Literacy*', she documents what should constitute mathematical literacy in a more general sense. Her rationale for writing the

article appears to be similar with the purpose of ML introduction in South Africa. She draws from the evidence within the context of the society need for individuals to have “the ability to deal with numbers and to interpret quantitative information” (p.76). Although she doesn’t give a precise definition of mathematical literacy, she presents a number of perspectives on the constitution of mathematical literacy namely; mathematical literacy for developing human capital, mathematical literacy for cultural identity, mathematical literacy for social change, mathematical literacy for environmental awareness, and mathematical literacy for evaluating mathematics. It is argued that the varied conceptions are a result of differences in the cultures and contexts of the stakeholders who promote it. From these perspectives, she argues that the notion of mathematical literacy as a tool for evaluating mathematics strongly reflects her view more than any other perspectives. This means that mathematics needs to be learnt first followed by using this knowledge in solving problems, suggesting ‘a learn mathematics and then apply’ view. She describes mathematical literacy as potentially encompassing the following terms:

It may be seen as the ability to use basic computational and geometrical skills in everyday contexts, as the knowledge and understanding of fundamental mathematical notions, as the ability to develop sophisticated mathematical models, or as the capacity for understanding and evaluating another’s use of numbers and mathematical models (p.76).

This description suggests that mathematical literacy should ensure the acquisition of the following skills:

1. Problem solving involving problems encountered in everyday life.
2. Connected understandings of fundamental mathematical ideas.
3. Knowledge of ‘advanced’ mathematical ideas to develop and solve sophisticated mathematical models.
4. Critical analysis of the way how other people use numbers and mathematical models.

Thus according to Jablonka, the domain of mathematical literacy should aim at advancing learners’ understanding and knowledge of mathematical principles, both elementary and advanced, that will enable them among other things to develop and analyse models. In addition to viewing mathematical literacy in the direction of application of mathematics, this description appears to incorporate both citizenship and critical agendas. In contrast, the South

African conception focuses on elementary mathematics (GET level) which in Jablonka's terms relates to 'basic computational and geometric skills'. Although Jablonka's conception is biased towards the knowledge and understanding of mathematics, she acknowledges the need for mathematical literacy to be conceptualised in functional terms as applicable to the situations in which this knowledge is to be applied. In fact it is about the functional aspect of mathematical knowledge - that is about individual skills to use mathematical knowledge in a practical and functional way. She notes:

Any attempt at defining 'mathematical literacy' faces the problem that it cannot be conceptualized exclusively in terms of mathematical knowledge, because it is about individual's capacity to use and apply this knowledge (Jablonka, 2003, p.78).

The cultural differences that exist from one context to another justify the varied ways in which mathematical literacy is conceived. It is indisputable to say that 'everyday life' in a developed or industrialised cultural context is different from that of developing or third-world contexts.

Pugalee (1999) argues in his article '*Constructing a Model of Mathematical Literacy*' that doing mathematics and preparing individuals for higher mathematics is a primary goal of mathematical literacy. He notes that the model must embody the five processes through which students obtain and use their mathematical knowledge namely valuing mathematics, becoming confident in one's ability to do mathematics, becoming problem solvers, communicating mathematically, and reasoning mathematically. These processes appear to foreground mathematics understanding and therefore point towards more mathematics orientations. The focus on mathematics principles is re-emphasised by arguing that:

The model should describe those processes that are central to an individual's capacity to "do mathematics" using the tools of today's society, and it should possess a foundation that allows an individual to adapt to future advances in technology and in mathematical knowledge (p.19)

This suggests that doing mathematics for understanding should form the core of mathematical literacy teaching, a focus which contradicts the rhetoric in South African ML curriculum. However, Coben's idea of numeracy and Jablonka's notion of mathematical literacy appear to overlap with Pugalee's model in that they both refer to mathematical literacy as not

restricted to the ability to solve problem situations but also involve broader understanding of mathematics.

In terms of the approach that needs to be emphasised in ML teaching, Pugalee contends that authentic classroom tasks are critical in developing the level of mathematical understandings that enables learners to competently make sense of everyday situations. By allowing learners to engage with authentic tasks, learners have “full access to the school curriculum and ultimately participate in the adult world” (p.19). Again, by arguing that engagement with authentic tasks would provide access to school curriculum, Pugalee’s conception suggests a push towards some mathematics goals. Furthermore, Pugalee observes that for learners to be equipped with skills needed in life, mathematics should be embedded in tasks that are ‘real’ to them and opportunities for learners to solve the problems using multiple approaches should be provided. The idea of utilising authentic tasks within the context of ML teaching in South Africa has been described as not feasible due to the diversity of learners within ML classrooms (Julie, 2006).

2.3 Overlaps and contrasts between ML in South Africa and international literature

Here, I focus on overlaps and contrasts in relation to contexts and content, as these feature centrally in this study.

2.3.1 Contexts in mathematical literacy

One of the key features in terms of overlaps in both South Africa conceptions of ML and international conceptions is the view of ML as an individual's capacity to reason with quantitative forms of data at some competent level (Edge, 2001). This implies the need for ML to focus on affording students opportunities to engage with real-world situations, an aspect which features across all conceptions.

With regards to contrasts, other international conceptions appear to emphasise mathematically focused tasks in addition to the citizenship oriented situations. For instance, Jablonka’s (2003) and PISA’s (OECD, 2006) notions of mathematical literacy maintain the view that students’ skills relating to working with advanced models need to be equally developed.

While the South African ML curriculum (Department of Education, 2003) claims that realistic and authentic situations need to be used within ML classrooms, this aspect is contradicted at the level of assessment. However, other conceptions maintain a view that contexts need not to be necessarily real but should allow students to imagine the situations (Gravemeijer, 1994a; Jablonka, 2003). Given the multiple orientations observed in ML curriculum, some rhetoric in this curriculum places contexts first and argues that these contexts need to dictate the kinds of mathematics tools to be employed within the problem-solving process, with a focus on understanding the contexts themselves. Other conceptions like Numeracy (PIAAC, 2009; Steen, 1990) and mathematical literacy (Jablonka, 2003) appear to suggest a more application-oriented approach where the learning of mathematics precedes engagement with contexts, in which case the contexts seem to be focused on the particular mathematics principles learnt. In this view, contexts (not necessarily authentic) are used to achieve some mathematics understandings (de Lange, 2003; Gravemeijer, 1994a; Pugalee, 1999).

2.3.2 Content in mathematical literacy

All the conceptions highlight the critical role played by functional understandings of mathematics in making sense of situations (Hughes-Hallett, 2001; Manaster, 2001). The emphasis is largely on the utilitarian dimension of mathematics by focusing on its usage in analysing contexts containing quantitative features. These notions appear to focus not only on simple mathematical computations but also using the mathematical results to take a point of view or an informed decision (Kramarski & Mizrahi, 2004), an aspect of a critical orientation. This implies that being mathematically literate is more than having the ability to use the mathematics; it requires insight, a component which according to Hughes-Hallett (2001) makes the conceptions hard to teach. Furthermore, these conceptions appear to agree that an understanding of elementary mathematics is necessary for developing mathematics literacy (Gainsburg, 2005; Gardiner, 2004).

In terms of contrasts at the level of content, the South African conception of ML emphasises and draws from foundational mathematics (GET mathematics) (Department of Education, 2003) whereas some international notions like mathematical literacy (Jablonka, 2003) and numeracy (Coben, 2004) suggest that both foundational and advanced mathematics are useful in developing life skills. The differences relating to the specified 'mathematics content' appear to be informed by specific contexts where a particular conception is adopted. The

need for recognition of the importance of connections between different mathematics topics and an understanding of sophisticated mathematics including calculus has been emphasised in some international conceptions (de Lange, 2003), an aspect that is absent in other definitions like South African ML and quantitative literacy.

2.3.3 Summary of overlaps and contrasts

Although ML is offered in South Africa schools as a subject, only a fraction of the learners take the subject at grade 10. Given that learners who perform well in school mathematics at grade 9 are encouraged to proceed with mathematics and cannot take ML, the development of citizenship skills is therefore targeted at a small group of learners, despite initial concerns about general levels of innumeracy among South African citizenry (Department of Education 2003). The international conceptions however focus on developing ML-related skills among the majority of learners, as these conceptions are components of mathematics learning, which is afforded to many school going children. In terms of progression, South African ML (Department of Education, 2003) emphasises progression at the level of contexts which are made available for the students to deal with. The international conceptions appear to achieve progression at the level of both contexts and mathematics content, as reference is sometimes made to learning ‘sophisticated mathematics’ (de Lange, 2003). Furthermore, the nature and range of curricula content across the conceptions suggest that both mathematics and mathematics-context connections, an aspect which was part of this study focus, need to be developed among the students. Since contexts feature centrally across the conceptions, pre-service teacher engagement with the contexts in the CLM course allowed students to translate contextual information into mathematics statements (mathematics-context connections) and solve the mathematics models using mathematical language (intra-mathematical connections), aspects linked to the idea of mathematisation (OECD, 2003, 2006). The specificity of these conceptions in terms of ‘cultures’ implies that ML should aim at addressing the needs of particular societies, as it is about peoples’ ways of dealing with everyday issues, which present themselves differently across different societies (Brown & Schäfer, 2006; de Lange, 2003; Jablonka, 2003).

2.4 Theoretical framework - PISA mathematisation process

In problematising this study, I highlighted the need for developing ML teachers’ knowledge relating to both mathematics and contextual understandings. Within the context of exploring

these understandings, the specific focus has been on the teachers' working relating to both intra-mathematics and mathematics-content connections across solutions to tasks within the CLM course and within practice. In view of this focus, reference has been made to studies whose results tend to suggest that besides inadequate numbers of ML teachers across schools in South Africa; their understandings of both fundamental mathematics in connected ways and more complex content-context problem solving, appear weak.

As noted already, this study was guided by the following specific empirical questions; 1) in relation to course tasks and learning, what does a sample of pre-service ML teachers' performance in assessment tasks indicate about their understandings of both mathematics and ML? How does this develop over a two-year period?, and 2) in relation to practice, how does a sample of pre-service teachers on teaching experience work through instructional tasks within ML lessons? The ML emphasis on situations combined with the study focus relating to exploring intra-mathematical and context-content connections across pre-service ML teachers' ways of problem solving, resonated with PISA's notion of mathematisation process³ (OECD, 2010, 2013) a theoretical tool which has provided lenses for exploring step by step processes related to the teachers' problem solving. PISA's version of the mathematisation process draws from Freudenthal's idea of realistic mathematics education (RME) (Freudenthal, 1973). The notion of RME is based on Freudenthal's interpretation of mathematics as a human activity deeply embedded in 'real' situations (Freudenthal, 1973, 1991). Some research studies done in South Africa suggest that RME could be useful in terms of understanding contextual teaching and learning broadly (Barnes & Venter, 2008), and specifically explaining gains made by low attainers in mathematics (Barnes, 2005). The focus on contexts in ML coupled with the fact that ML classrooms generally comprise low attainers in mathematics (Venkat, 2007), suggests that RME work is relevant in ML. Although RME emphasises the idea of 'society relevance' of mathematics, contexts are not necessarily utilised aiming at developing citizenship skills as is the case with the rhetoric within the South African conception of ML, but rather to promote the understanding of mathematical concepts and principles. I therefore found PISA's theoretical tools useful as they provided handles for thinking about ways in which students worked through contextualized tasks with

³ Initially the study proposed that Social theory of learning (Wenger, 1998) and Realistic mathematics education (RME) (Freudenthal, 1973) would provide theoretical lenses for the study. However, due to absence of clearly defined community of practice in the study and RME's focus on mathematics learning combined with lack of emphasis on the interpretive aspect of problem solving, PISA's mathematisation process was selected for this purpose.

a focus on understanding the contexts themselves, a central feature of ML in South Africa. As noted in chapter one, this study adopts Wenger's (1998) conception of learning which is described in terms of 'meaning making'. Problem solving relating to contextualised tasks involved a great deal of meaning constructions as the problem solver enacts the solution procedures. In this way, Wenger's notion of 'meaning making' provides a broad-level complement to PISA's mathematisation cycle.

PISA defines mathematisation as a process which students use to "solve real world problems by shifting between real-world and mathematical world contexts" (OECD, 2009, p.20). Since mathematical literacy concerns individuals' capacities in making sense of real-world problems, both formal and informal strategies are employed in the solution processes (OECD, 2006). While formal strategies employ mathematical language and formal algorithms, informal strategies often make use of non-formal routines (de Lange, 1990). Within the mathematisation process, both formal and informal strategies feature, aspects which also characterised the students' engagement with tasks in this study. Related conceptions of mathematisation such as the modeling cycle have been provided in literature (Blum & Ferri, 2009; Maaß & Gurlitt, 2011; Perrenet & Zwaneveld, 2012; Wake, 2011), and broadly refer to the problem-solving process utilised by learners as they engage with mathematics in contexts tasks. Common to both conceptions of mathematisation and mathematical modelling is the contextual base (emphasis on engaging with contexts). However, within the PISA conception, the focus is on understanding both mathematics and contexts whereas in modelling, mathematical learning constitutes the major focus. Additionally, the broad contexts which need to be explored in PISA are specified and are based on the notion of "distance from the students" (OECD, 2006, p.81) while in modelling, contexts are mentioned at a more generic level. The PISA mathematisation process concerns two major processes; namely translation processes, and solution processes as shown in the figure 2.1.

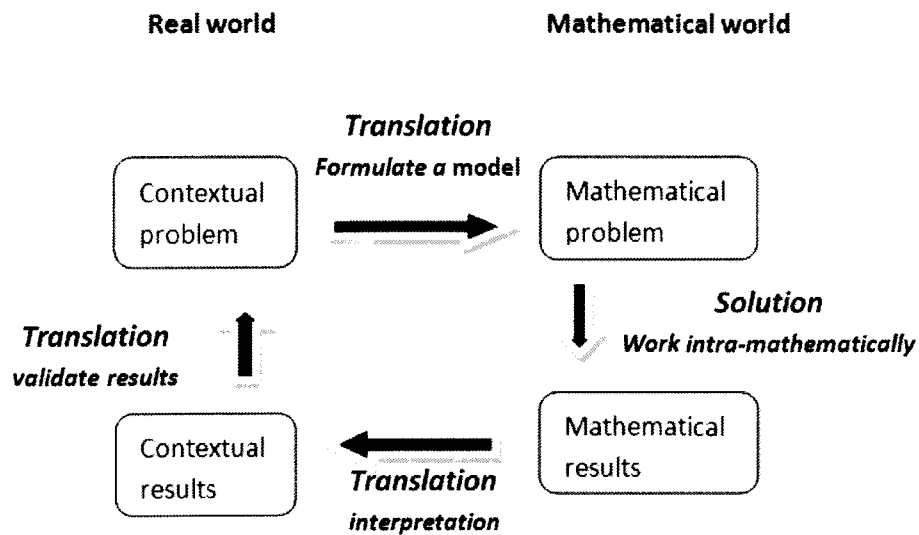


Figure 2.1: Mathematisation cycle (OECD, 2010; 2013)

The translation process

The translation process involves aspects of mathematical working that are at play within the real world context or those associated with the interface between the real world and the mathematical world. The aspects that exemplify translation processes specifically involve:

1. Model formulation

This involves organizing reality according to mathematical concepts and identifying the relevant mathematics involved (OECD, 2010). It also involves gradually trimming away the reality by establishing the relationships between the language of the problem and the symbolic and formal language, leading into a mathematical model amenable to mathematical treatment (OECD, 2006).

Evidence from literature suggests that contextualised tasks can be classified based on the information they contain. Li (1990) for example observes that in some problems, information which is not necessary for the solution of the problem is included, and he calls the extra information, ‘superfluous’. He further argues that incorrect solutions within problem solving are often as a result of inability to formulate a mathematical model due to failure to engage with superfluous information. He also notes that “superfluous information creates a greater cognitive demand” (p.1). In contrast Li (ibid) calls ‘relevant sets’ the contexts where mathematical working utilizes all the given information. This implies that correct model formulation is influenced by contextual understandings across cases where either superfluous

information or relevant sets feature. In addition to solving tasks containing relevant sets, the participants in this study also engaged with tasks containing superfluous information.

At the level of model formulation, some written protocols (solutions) include a listing of the quantities from the context, which Hall and colleagues (1989) call ‘annotation’. They further argue that there are three ways in which annotation features in written students’ protocols namely; (1) problem elements (student recording elements of the problem context); (2) retrieval of formulas (remembering and writing down memorized formulas that seem relevant); and (3) diagram (student draws a picture of the problem situation). ML-related literature in South Africa at the level of professional teacher development suggests that these ways feature across students problem solving. However some errors related to incorrect choice of formulas (Bansilal et al., 2012), incorrect choice of operation(s) or broadly inability to set up equation(s) (Vale, Murray, & Brown, 2012) have been observed at this level. The errors indicate gaps within the teachers’ understandings and imply the need for developing ML teachers’ skills around ‘annotation’. The CLM focus on problem solving processes incorporated attention to this gap.

Furthermore, the literature base is replete with studies which suggest that challenges associated with model formulation are also attributable to difficulty with the language in which the problem is presented (Bernardo, 1999; Kaur, 1997; Koedinger & Nathan, 2004). Related to the language presented in the contexts is the ability to comprehend what in Koedinger & Nathan’s (2004) terms is called ‘external problem representations’ which include arithmetic operations and symbolic algebraic language. According to Bernardo (1999), “the most basic difficulty students have in solving word problems lies in the ability to understand the mathematical problem structure that is embedded in the problem text” (p.149). His study results suggest that often an operation error is associated with the model formulation process and further argues that this error “is most likely a result of a gross inability to parse the meaning of the problem text” (p.155). This result also agrees with other findings focusing on analysing students’ written problem-solving protocols (Borromeo Ferri, 2007; Hall, et al., 1989). They argue that an operation error is a manifestation of conceptual disruption and note that these kinds of errors exist as a result of inclusion of a quantity which is inappropriate for the problem or excludes a quantity that is a critical requirement. This implies the need for a focus on model formulation when teaching problem solving, an aspect which featured within the CLM course.

2. Interpretation

Following the intra-mathematical ‘solution’ process, which I come to later, is interpretation of numerical results. This involves translating a mathematical result into the language of the problem context (OECD, 2010). Interpretation has been described as an important aspect of problem solving involving contextualized tasks (De Corte & Verschaffel, 1989; Sepeng & Webb, 2012). Pedagogically, this means that learners need to be aware that final mathematical results within the context of problem solving are not only informed by the intra-mathematical working but also the original contexts in which the problems are located. Students need to understand the mathematical solution and relate the solution with the given context, an aspect which may require deep understanding of the context. Evidence from studies focusing at the level of both school learners (Greer, 1993; Sepeng & Webb, 2012; Verschaffel, De Corte, & Lasure, 1994) and pre-service teachers (Kaiser & Maaß, 2006) has revealed some difficulties associated with the interpretive aspect when engaging with contextualised mathematics tasks. Sepeng and Webb (2012) for instance focused on South African learners and found out that learners tended to suspend realistic considerations during problem-solving, an aspect which improved after some intervention. Although the study by Sepeng and Webb was located within the context of school mathematics focusing on grade 9 learners, the kinds of tasks which students in ML engage with draw from mathematical knowledge at this level.

3. Validation

Validation refers to evaluating the reasonableness of a mathematical solution in the context of a real-world problem including identifying the limitations of the solution (OECD, 2006). Here, learners ask questions like; what does my solution (contextual result) mean in relation to the given situation? Does my explanation make sense at all in this context (OECD 2010)? To answer these questions, an understanding of both the context and the mathematical solution is needed.

The solution process

The solution process is concerned with aspects of mathematical work that take place substantially within the mathematical world (OECD, 2010). At this level the solver employs mathematical concepts, procedures, facts, and tools to obtain mathematical results. It involves; manipulating mathematical models, formulas or equations; transformation, computation or checking and justifying results within the mathematics domain (OECD,

2006). In addition to competences relating to the translation process, the ability to analyze information or problem situations using mathematics is also key in ML (Department of Education, 2003). Given the context, students need a predisposition to select appropriate mathematical tools to aid them to solve the problems, using a range of different solution pathways (Mousoulides, Sriraman, Pittalis, & Christou, 2007). Hall and colleagues (1989) propose two stages which can be used to interpret and understand students' written mathematics problem solving protocols. They argue that the protocol is "divided into a sequence of coherent problem-solving episodes, and then each episode is scored individually with respect to its content, correctness, and function in the overall sequence" (p.244). According to Hall and colleagues (*ibid*), errors in problem solving can be classified into two broad classes, namely conceptual and manipulation errors. They argue that conceptual errors include errors of commission (incorrect quantities introduced during an episode) and errors of omission (overlooked quantities) whereas manipulation errors include algebraic (i.e. transforming an equation), variable (i.e. incorrect meanings attached to variables), and arithmetic errors (i.e. incorrect arithmetic operations). Intra-mathematical errors have also been noted across local studies in ML especially relating to incorrect calculations (Vale, et al., 2012; Winter & Venkat, 2013). The implication is that students need to develop their functional understandings of mathematics.

Since the study focuses on intra-mathematics and mathematics-context connections, I found the PISA's conception of mathematisation useful due to the inter-connections between real world and mathematical contexts, a key feature in the broad ML rhetoric. This conception was also well linked with the aims and purpose of the CLM course where the study was located. Furthermore, components of the mathematisation process were observable across the students problem solving, in the course and in practice, and this provided a rationale for its adoption in this study.

Whilst in some ways this framework artificially splits up problem solving into these three aspects of modeling cycle, it isn't that I am treating them separately in this study, it is just an analytical device to be able to look at different aspects of the modeling cycle. Relating to data analysis, I have followed through the students' intra-mathematical work and interpretation even if the formulated models are incorrect. I acknowledge the possibility that these incorrect models could be easier in relation to the original problem. However, stopping analysis at this point would mean ignoring the intra-mathematical working that is enacted. Given my interest

in understanding students' intra-mathematical working, I noted errors at the model formulation level, but followed through with whatever intra-mathematical working followed, rather than 'double-penalising' this working by omitting it.

2.4.1 Using mathematisation process to understand problem solving

The notion of mathematisation and/or similar conceptions has been extensively used across international studies. Several researchers have utilised the modelling cycle to make sense of students' problem solving (Galbraith & Stillman, 2006; Kaiser, 2007; Stillman & Brown, 2012). Their findings suggest that modellers (especially beginners) appear to have difficulties with model formulation, an aspect of translation within the PISA mathematisation process. Common to these studies is the focus on understanding step by step mathematical working relating to contextualised problems.

Within the context of ML in South Africa, the PISA mathematisation process has been used by Vilakazi & Bansilal (2012), whose aim was to explore in-service teachers' performance across ML tasks specifically those that demanded employment of algebra. The teachers participated in an ACE programme aimed at reskilling the teachers for ML teaching. The analysis of the contextualised tasks' responses allowed them to gain access into the teachers' understanding of mathematics and contexts, aspects which are key to being mathematically literate. Their findings suggest that a profound understanding of basic algebra was key to solving contextual tasks. Furthermore, they observed that those who had been teaching school mathematics performed better in the tasks than their non-mathematics counterparts—suggesting that competence in mathematics is important in ML. Vale and colleagues (2012) analysed responses to tasks selected from ML examination with a view to explore sources of students errors in terms of whether they are a product of insufficient mathematical literacy or lack of English language proficiency. Utilizing the PISA mathematisation process, their findings suggest that errors occurred at the level of translation, an aspect which they attributed to lack of English proficiency. Furthermore, Brown and Schäfer (2006) conducted a study involving a similar group and context (ACE programme) using a mathematical modelling approach. Their findings revealed that “teachers with weaker mathematical skills took considerably longer to master the contexts and skills developed in the [programme] activities” (p.51). The findings in these studies point towards the need for deep and connected understandings of both mathematics and contexts, as key components needed to successfully engage with ML tasks. The empirical evidence therefore provides justification for a focus on

coherence and connections in mathematical and contextual understandings for ML teachers, an aspect which supports the CLM aims, content and tasks.

In this study, two main empirical data sets have been analysed. The first relates to students' work on CLM assessment tasks and the second concerns lesson episodes informed by some tasks within teaching practice. Working with the data in this study has revealed that theoretical tools provided by the mathematisation process were found to be useful in terms of understanding the steps within the problem solving cycle. However, these lenses did not allow me to make sense of the kinds of connections within the students' problem solving especially within teaching experiences. In view of this, analytical frameworks have been identified and discussed below, aiming to deal with the nuances in the data. An explanation related to how the PISA mathematisation process has been modified to cater for emerging issues from the analysis has been provided later in this chapter. Details showing ways in which constructs have been operationalized have also been provided. This study adopts Martin and colleagues' (Martin, Cohen, & Champion, 2013, p.6) view who refer to operationalization as "development of measurable representations of concepts and/or dimensions of concepts". They further argue that operationalization concerns "the process of putting the concepts of interest into operation or of operating on those concepts in order to measure them, both individually and/or in relation to other concepts". While some of the indicators have been adopted from literature, others are a result of grounded analysis in this study.

2.4.2 Analytic frameworks

This study focused on exploring shifts (if they exist) in knowledge related to the ways in which teachers engaged with tasks, especially in the context of problem solving in the course across two years (2011-2012) and understanding the teachers' practice during teaching experiences. Regarding shifts, the PISA mathematisation process appears to be limited in terms of classifying assessment tasks according to cognitive demands, an aspect which seemed to provide a window for understanding ways in which performance played out across tasks placed at different cognitive levels. The idea of PISA competency clusters was therefore utilized to classify tasks due to the overlapping nature of the CLM assessment tasks and the PISA assessment items. Relating to practice, the empirical data in this study has shown that teachers in some cases utilized 'intra-mathematical tasks' within teaching where connections to world situations featured. In order to make sense of these kinds of instructional tasks,

components of Polya's problem solving model have been used, due to its focus on both intra-mathematics and extra-mathematics tasks. Further, this study has utilized the spectrum of pedagogic agendas in order to explore teaching orientations, which were advanced during teaching experience. I now provide details about these analytic theoretical lenses.

PISA competency clusters (OECD, 2006)

Within the context of ML in South African, the Subject Assessment Guidelines (SAG) for ML (Department of Education, 2008, p.8) offers a taxonomy which is used by teachers when assessing ML learners. It is claimed that the taxonomy was adapted from PISA competency clusters (OECD, 2003). The taxonomy in ML consists of four levels as follows;

Level 1: Knowing

Level 2: Applying routine procedures in familiar contexts

Level 3: Applying multistep procedures in a variety of contexts

Level 4: Reasoning and reflecting

This taxonomy, which lacks detail in terms of providing clear indicators for each level, has been criticized by ML curriculum critics. Venkat and colleagues (2009) note that there is "under-description of the term reasoning" within the taxonomy (p.47). Making reference to everyday life from which ML problem situations need to be selected, they further argue that "reasoning in everyday life involves making sense of a situation by scanning possibilities and deciding on those that fit the question or the argument best" (p.48). They also observed that despite claims suggesting that problem solving is central in ML, this aspect appears to feature only at level 3 and 4 of the taxonomy. In terms of progression across the levels of the taxonomy, Venkat and colleagues argue that the related descriptions "indicate a mathematically-based progression" (p.48), contrasting emphases within ML policy specifications which put progression at the level of contexts – "complexity of the situations" (Department of Education, 2003, p.38). Since reasoning, which is separated from doing (enacting a procedure) in the taxonomy, is understood to be a key feature in ML as well as in the CLM course, I did not find this taxonomy useful in this study. Instead, PISA competency clusters have been used to classify assessment tasks into cognitive demand levels.

In order to explore shifts in terms of students' performance, assessment tasks have been classified according to PISA cognitive demands levels. It is important to note that the 'shift' dimension has not been explored within teaching practice due to the small number of lessons

observed and video-recorded (see more detail in methodology chapter). However, a snapshot of results within practice has provided me with further insight into the students' shifts in knowledge within the course where more detailed data feature. I draw from PISA's (OECD, 2009) and de Lange's (2006) notions of competency clusters to talk about these cognitive demand levels. The competency clusters were developed within the context of analyzing PISA mathematical literacy assessment tasks, whose nature is similar in many ways to the CLM course assessment tasks. The key differences relate to specific contexts (target groups) within which both PISA and CLM assessments are being administered, and partly due to the nature of tasks. PISA's assessment is focused on 15 year-old school learners, whereas CLM course assessment is located within a University teacher professional development. Since the professional development course also focuses on developing the teachers' knowledge related to teaching, the CLM course assessment also included pedagogically linked tasks (requiring linking problem-solving with aspects of teaching and learning); these kinds of tasks are not present in PISA assessment. The way in which I have dealt with such pedagogically linked tasks has been explained in the next sections.

PISA's interest is on comparing learner performance in mathematics in context tasks across member countries with reference to questions' cognitive demand levels. PISA's focus is on performance in relation to mathematical competences at a national level as well as at an individual student level in a particular country. However, this study focused on comparing individual performance of students participating in a University course with an aim of understanding their growth in knowledge related to mathematics content, mathematics in context, and how these two linked with ML teaching practice. The study focus had some parallels with PISA's focus in terms of concentrating on performance as well as mathematical competences used to solve problems. The extent to which students performed across assessment tasks within competency clusters over the two years provided entry points into some understandings related to the students' knowledge development of ML (OECD, 2009). The contextual base in both PISA assessment and CLM course assessment selected for this analysis provided a rationale for drawing tools from PISA. According to PISA there are three competency clusters which provide a description of cognitive processes that students use during problem solving, and these are referred to as the *reproduction cluster (level 1)*, *connection cluster (level 2)*, and *reflection cluster (level 3)*. Ways in which these levels were operationalized into indicators in this study have been presented in table 2.4. Some of the

indicators have been adopted from the PISA framework, and others were grounded from data analysis in this study.

Cognitive demand levels	
Descriptors	Indicators (we know this when the task demands)
Reproduction level	• answering basic questions [level of GET maths] i.e. how many...?, how much...?, What...?, Calculate...? (OECD, 2006) • justifying standard [level of GET maths] quantitative processes or computational processes. (OECD, 2006) • expressing oneself in writing about simple mathematical matters [level of GET maths], such as citing computations and their results, usually not in more than one way. (OECD, 2006) • decoding, encoding familiar, practised standard representations [based on GET maths] of well known mathematical objects. (OECD, 2006) • reading information from tables and diagrams, and using this information in basic calculations. (grounded analysis) • problem-solving involving one-step method. (grounded analysis)
Connections level	• understanding and handling mathematical concepts in contexts that are slightly different from those in which they were first introduced [in CLM course] or have subsequently been practised. (OECD, 2006) • understanding and expressing oneself in writing about explaining matters that include relationships and insight. (OECD, 2006) • translating reality into mathematical structures in contexts that are different from what students have seen or engaged with before in the course. (OECD, 2006) • problem-solving in which connections are made between different mathematical areas and modes of representations (tables, graphs, words, pictures, aspects of teaching and learning). (adapted from OECD, 2006) • selecting formula or theorem to be utilized in problem-solving in contexts. (grounded analysis) • creating real life stories which represent given mathematical statements/model. (grounded analysis) • problem solving involving multi-step methods. (grounded analysis)
Reflection level	• distinguishing between definitions, theorems, conjectures, hypotheses and reflecting upon these distinctions. (OECD, 2006) • understanding and handling mathematical concepts in contexts that are new and complex. (OECD, 2006) • distinguishing between proving and proofs and broader forms of argument and reasoning. Assessing and constructing chains of mathematical arguments of different types. (OECD, 2006) • explaining computations and their results (usually in more than one way), to explaining matters that include complex relationships. (OECD, 2006) • translating reality into mathematical structures in contexts that are largely different from what students had seen or engaged with before in the course. (OECD, 2006) • Reflecting through analysing, offering a critique, and engaging in more complex communication about models and modelling. (OECD, 2006)

Table 2.4: Operationalizing cognitive demand levels

The competency clusters were found to be useful in this study in terms of classifying the assessment tasks according to cognitive demand levels. This allowed for exploration of the pre-service teachers' shifts in performance across tasks placed at different cognitive demand levels, an aspect which points towards growth in problem solving – a key feature in ML. The limitation of this framework in this study relates to its focus on classifying tasks, and not on responses to these tasks. Empirically, there were instances where a task demanded a one-step procedure (and was classified as reproduction level task), but varying solution strategies showing different 'connections of representations' including different number of steps were utilized by students, as illustrated below.

Task	Student's solution 1	Student's solution 2
A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day a) If the Company has one six-man team on the job, how long would it take to complete the contract?	3000m; 6 men, 20m/day $3000 \div 20 = 150$ days	Assuming they work 27 days of the month as the 4 days are Sundays. We then formulate $27 \times 20m = 540m$ first month. Therefore, we can take 540 and divide it by 3000m to get an answer of months and days $3000m \div 540m = 5.56$ months. As we know we can't use 5,56 months. We can say $(27 \times 20) \times 5 = 2700m$, 300m short. Therefore $300m \div 20m = 15$ days. It would take 5 months 15 days.
John and Jane both currently earn R10 000 per month. John performs badly in this job so is demoted and will earn 9% less from next month onwards. How much will he earn?	$R10\ 000 \times \frac{91}{100} =$ R9100	$A = P(1 - \%)$ $A = 10000(1 - 9\%)$ $A = 10000(1 - 0,09)$ $A = 10000 \times 0,91 = 9100$ $A = 9100$

While some students used short one-step methods (solution 1), others employed multi-step methods where connections featured (solution 2), to solve the same problems. For instance, solution 1 in both cases appears to be more efficient than solution 2. However, solution 2 seems to be pedagogically useful, as procedure steps and explanations are provided. Due to the focus at the level of tasks in the classification, this study has classified both tasks as reproduction, as they appear to be 'standard' (practiced within CLM course and GET mathematics). For instance the table shows some responses exhibiting some aspect of connection (in terms of PISA task classification) through definitions and explanations (first

task), as well as retrieving and using some formula, in ways that make sense of the contexts (second task).

Spectrum of pedagogic agendas (Graven & Venkat, 2007)

In an attempt to understand the intra-mathematical and mathematics-contexts connections within the context of ML teaching, exploring pedagogic agendas and how these agendas interact became another interesting aspect. Empirical evidence in South Africa has shown variations in ML teaching agendas observable in ML classrooms which can be linked to either the presence or absence of connections within teaching/learning-based problem solving (Graven & Venkat, 2007a; Venkat, 2010). Analyzing ML from an instruction perspective, Graven & Venkat (2007a, 2008) propose a spectrum of pedagogic agendas based on empirical data from a range of interpretations of content/context link in ML teaching. They further state that the curriculum rhetoric suggests that ML should sit on the first two (left side) agendas which are predominantly contextual with an aim of exploring contexts that learners need to interact and engage with, in their everyday lives and to use mathematics to achieve this goal. These agendas therefore provided entry points into an understanding relating to how these orientations interact within ML teaching informed by instructional tasks. A summary of Graven & Venkat's spectrum of pedagogical agendas has been given in table 2.6.

Context driven (by learner needs) (1)	Content & context driven (2)	Mainly content driven (3)	Content driven (4)
Driving agenda: To explore contexts that learners need in their current everyday and future lives.	Driving agenda: To explore a context so as to deepen maths understanding and to deepen understanding of that context.	Driving agenda: To learn mathematics and then to apply it to various contexts.	Driving agenda: To give learners a 2 nd chance to learn the GET mathematics.
Pedagogic demands: Involves finding and increased discussions of contexts learners and country need for critical participation in variety of situations that call for quantitative	Pedagogic demands: Involves selecting and discussion of contexts (not necessarily realistic) that work to unpack GET math-context relationship	Pedagogic demands: Involves selecting contexts (contrived or more real) that GET math can be applied to	Pedagogic demands: Involves revision of GET math without pedagogic change except slowing down the pace.

disposition			
Issues arising: Progression of math usually sacrificed while authenticity of context is maintained to meet ‘learner needs’. Agenda does not accommodate summative assessments.	Issues arising: Authenticity of context and progression of maths must be balanced. Summative assessments struggle to align and deal with math progression hence gaps (on performance) are likely to occur with continuous assessment.	Issues arising: Authenticity of context is often sacrificed to meet math goals. Math progression can be developed as in math curriculum. Summative assessments are more familiar and performance is more aligned to continuous assessment.	Issues arising: Contexts are not particularly present. Math progression can be developed as in math curriculum with emphasis on increased levels of content abstraction. Traditional summative assessments are similar to continuous assessment hence little discrepancy between the two.

Table 2.5: Spectrum of pedagogical agendas (Graven & Venkat, 2007)

As already highlighted (see chapter one), the CLM course consisted of fundamental mathematics content and contextual-problem solving. The implication is that problem solving in the CLM course was linked to all the agendas across the spectrum. The course started with tasks which afforded pre-service teachers an opportunity to consolidate their content knowledge (agenda 4) before they engaged with contexts in a manner that provided links with the other three agendas (agendas 1, 2, and 3). These agendas provided some useful handles in this study in terms of understanding the nature of link between mathematical content and contexts within ML classrooms.

2.4.3 Conceptual framework for the study

Informed by emphases in ML curriculum relating to contextual understandings and the study focus in terms of exploring intra-mathematics and extra-mathematics connections, the study has focused on extra-mathematics assessment tasks within the CLM course. Although intra-mathematics tasks also featured across the assessment, analysis of the pre-service teachers’ working relating to extra-mathematics assessment tasks sufficiently provided a window for understanding the students’ intra-mathematics connections – with analysis of a sample of intra-mathematical tasks indicating no overall differences in nature and level of mathematical working in either intra-mathematical and extra-mathematical tasks. Due to an aspect relating to intra-mathematical connections within contextualized problem solving, a focus on analysis of intra-mathematical tasks may point to similar results.

The CLM course assessment tasks contained some complexities that could not be analyzed by the theoretical tools provided by the PISA mathematisation cycle. I present two cases which pointed towards the need to modify the PISA framework. I then motivate for modification of the framework in order to capture these complexities in the analysis.

1. Some assessment tasks drawn from the course focused on attaching real world stories to given mathematical statement. Two examples have been illustrated.

- a) *Create a story problem for $4,5 \div 0,75$.*
- b) *Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.*

These two tasks exemplify scenarios where mathematical statements were provided in the context and ‘creation of stories’ representing these statements were sought. In terms of PISA mathematization process this aspect can be linked to ‘reverse’ translation (at the level of model formulation). This reverse translation is described as pedagogically useful in mathematics education – providing situations that help learners to make sense of the mathematics. This is linked to the idea of *specialised content knowledge* for teaching (see Ball, Thames, & Phelps, 2008). Given mathematical weaknesses in ML school population, it would appear that this skill is important within ML teaching.

These kinds of tasks were aimed at providing the students with skills linked with some understanding of the inter-relationship between world contexts and mathematical models, a key aspect related to the idea of translation in mathematisation. This contrasted with the majority of tasks which were situated in extra-mathematical contexts. Given that this is an aspect of translation, a reverse arrow from the ‘mathematical problem’ to the ‘contextual problem’ has been included in the cycle and labeled ‘story creation’. Although these problems were not contextualized (in PISA’s sense), exploring the nature of connections between the mathematical statements and the world contexts were interesting to explore in this study as it linked with ML teaching competences.

2. Some tasks involved considerations related to pedagogy. One of such tasks which specifically demanded connecting problem solving results with pedagogy has been exemplified:

This is the sign in a lift at an office block.

**THIS LIFT CAN CARRY UP
TO 12 PEOPLE**

- a) In a morning rush, 265 people want to go up the lift. How many times must it go up?*
- b) What are the possible errors associated with the mathematical answer which learners can make when answering this question? Why?*

In this example, connections are sought between the mathematics answer and possible learner errors. Just like the above tasks (mathematics statements or models) which demanded attachment of world situations, these kinds of tasks appeared to be pedagogically useful as they related to the idea of teaching and learning. The pedagogic aspect also featured across intra-mathematics tasks within the course assessment, but due to this study focus on both intra-mathematical and context-content connections, these intra-mathematics assessment tasks have not been included in this analysis. As noted already, a focus on extra-mathematical tasks appeared to provide access into the teachers' understanding mathematics. Further, some teachers' solution procedures contained explanations and detail, which could be linked to the idea of 'unpacking', a central feature of pedagogic content knowledge (Hill, Ball, & Schilling, 2008).

Although the pedagogic link appeared to be an aspect of translation, it did not feed back into the original problem context, but rather provided inter-connection aspects of teaching and learning. In this way, the 'pedagogy' domain could not be treated as a subset of the existing domains. Therefore, in this study, an arrow from mathematical results, to 'pedagogy', in the mathematisation process has been introduced. As already noted, provision of explanations and details within the procedure have also been linked to pedagogy. Since this study was located within a professional development course, exploring the nature of this inter-connection provided a snapshot in terms of understanding the teachers' skills relating to teaching and learning. The fact that the two cases are part of the study focus relating to the nature of mathematical work, modifying the PISA mathematisation cycle was justified. I now present the refined framework that incorporates the above cases.

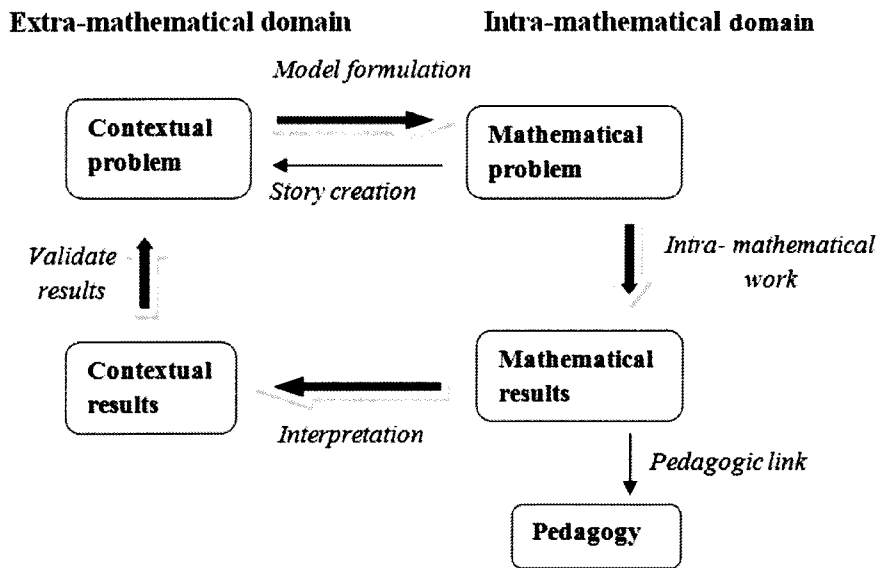


Figure 2.2: Conceptual framework for the study

The study adopts PISA conceptions of model formulation, intra-mathematical work, interpretation of results, and validation of results – presented in section 2.4. These categories according to PISA are intertwined. One of the key features of this framework is that it conceptualizes any working prior to setting up a model as model formulation. This implies that processes like addition, subtraction, division, and multiplication of quantities, including substitution of quantities into a formula, done in the service of formulating a mathematical model, can be understood as model formulation. Hall and colleagues (Hall, et al., 1989) refer to this working as annotation. The LEMA European project (Cabassut, 2013) which adopts PISA mathematisation cycle conceptualizes model formulation in similar ways. This means that the intra-mathematical work is concerned with manipulation of formulated models leading to obtaining mathematical results. I provide an exemplification of how I have coded my data in relation to these above constructs.

Task	Solution	Model formulation	Intra-math work
Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years. Calculate the amount he will receive at the end of the period from Bank B.	$A = P(1 + i)^n$ $i = 7,2\% = 0,072 \div 4 = 0,018; n = 2 \times 4 = 8$ $A = 6000(1 + 0,018)^8 = R6920,44$	$A = P(1 + i)^n$ $i = 7,2\% = 0,072 \div 4 = 0,018; n = 2 \times 4 = 8$ $A = 6000(1 + 0,018)^8$	$A = 6000(1 + 0,018)^8 = R6920,44$

In the context of intra-mathematics tasks, the model formulation step is referred to as devising a plan (Lam, Seng, Hoong, Jaguthsing, & Guan, 2011; Pólya, 1973; Rott, 2012). In my empirical data, if I see intra-mathematical working at this level, literature would use that language (devising a plan), but I will be using model formulation.

As noted above, ‘story creation’ and ‘pedagogic links’ are variants of translation process. While the former aims at connecting mathematical statements with situations, the latter is concerned with connecting aspects of problem solving with teaching and learning in schools. However, both story creation and pedagogic links are pedagogically useful in the sense that they both appeal to specialized knowledge which is “unique to teaching” (Ball, et al., 2008, p.400). Exploring the nature of these ‘variants’ in the CLM course was useful as the students’ learning in this course was followed by teaching experiences which allowed them to practice their learning.

2.5 Operationalizing constructs of the conceptual framework

Following the descriptions above relating to the conceptual framework which underpins this study, I provide indicators for the related constructs. Indicators for analyzing both sets of data (course tasks’ responses and lesson episodes) have been provided in table 2.7.

Mathematisation process	
Descriptors	Indicators (we know this when data contain evidence relating to)
Model formulation	• identifying and selecting the mathematical aspects of a problem context. (OECD, 2013) • recognising mathematical structure (including regularities, relationships, and patterns) in problem situations. (OECD, 2013) • representing a situation mathematically, using appropriate variables, symbols, tables, diagrams, and standard models. (OECD, 2013) • explaining the relationships between the context-specific language of a problem and the symbolic and formal language needed to represent it mathematically. (OECD, 2013) • retrieving or selecting formula. (grounded analysis) • substituting contextual information into a formula. (grounded analysis)
Story creation	• identifying the relationships between the symbolic and formal language of the mathematics statement and the context-specific language. (grounded analysis) • <i>representing</i> the mathematical statement in terms of some world story. (grounded analysis)
Intra-mathematical	• applying mathematical facts, rules, algorithms, and structures when finding solutions. (OECD, 2013)

work	• manipulating numbers, graphical and statistical data and information, algebraic expressions and equations, and geometric representations. (OECD, 2013) • making mathematical diagrams, graphs, and constructions. (OECD, 2013) • using and switching between different representations in the process of finding solutions. (OECD, 2013) • making generalisations based on the results of applying mathematical procedures to find solutions. (OECD, 2013) • reflecting on mathematical arguments, explaining and justifying steps in mathematical procedures (adapted from OECD, 2013)
Interpretation/validation	• interpreting a mathematical result in terms of the problem context. (OECD, 2013) • evaluating the reasonableness of a mathematical solution in the context of a real-world problem. (OECD, 2013) • explaining why a mathematical result or conclusion does, or does not make sense given the context of a problem. (OECD, 2013)
Pedagogic link (applicable within course assessment only)	• providing explanations and/or detail within mathematical working. (grounded analysis) • relating problem solving with possible learner errors or difficulties associated with similar working. (grounded analysis)

Table 2.6: Operationalising the mathematisation process

The rationale for presenting the indicators for analyzing course tasks and classroom practice tasks in one table relates to the overlapping nature of data, as the focus in both cases was on exploring the nature of mathematical working.

2.6 Chapter summary

In this chapter, a discussion around ML in South Africa relating to the rationale for its introduction, conceptions, advocacy issues as specified in the curriculum and related policy documents, issues of implementation focusing on teaching and learning as well as professional teacher development, has been provided.

Introduced in 2006 in South Africa as a fundamental subject, ML was aimed at addressing the innumeracy gap among the population. Driven by the citizenship perspective, the subject focuses on engagement with contexts in order to develop competences needed in solving everyday problematic situations and prepare the learners for future lives (i.e. workplace). Within the citizenship view, specifications relating to both consumer and critical orientations have been noted. Three agendas appear to dominate the curriculum rhetoric namely, contextual agenda, mathematics agenda, and mathematics-context agenda, and these have had an impact in ways how the subject is being implemented in schools. Rather than focusing on

contexts with the view to develop the citizenship skills among learners, some teachers have reportedly engaged learners within the mathematics domain where the learning of mathematics is foregrounded.

Although South Africa was the first country in the world to introduce ML as a subject in schools, evidence suggests that related international conceptions exist namely; quantitative literacy, numeracy, mathematical literacy, realistic mathematics education (RME), and mathematical modeling. The contextual base appears to provide some overlap across the conceptions whereas the focus on mathematics learning in RME, numeracy etc, contrast themselves with the South African conception of ML.

Given that the study was focused on understanding the nature of mathematical working, a review of problem solving models has been provided. These problem solving models have informed a conceptual framework which has been adopted for data analysis in this study.

CHAPTER THREE: THE STUDY CONTEXT - CONCEPTS AND LITERACY IN MATHEMATICS (CLM) COURSE

3.1 Introduction

The aim of this chapter is to provide details about the specific context of the study – a new professional teacher development course-Concepts and Literacy in Mathematics (CLM). I present the rationale underlying the introduction of the CLM course and descriptions relating to general course outcomes and proposed assessment criteria. Since the CLM was introduced as an umbrella course, information about the sub-courses under CLM is also provided. This is followed by detailed discussions around each of these sub-courses with a focus on outcomes, enactment and assessment. Within these descriptions, commentary relating to the degree to which intra-mathematical and mathematics-context connections feature has been provided.

3.2 Overview of the CLM course

The CLM course was a new course introduced in the second year B.Ed program at a large urban University in 2011 and consisted of fundamental mathematics content and problem solving in context. The course, which was a result of a curriculum review in 2010 sought to address problems identified in the structure of the previous B.Ed curriculum and to respond to contextual professional challenges. According to course documents (Academic devt doc, 2010), the course was aimed at building both content and contextual knowledge needed for Foundation Phase (Grades 0-3) and Senior Primary (Grades 4-7) mathematics teaching as well as FET (Grades 10-12) ML teaching. It also attempted to address the insufficiency of ML teachers in schools without drawing from the pool of those already qualifying to become secondary school mathematics teachers as had been the case previously. Thus the course was comprised of two major B.Ed groups, namely Senior primary mathematics and ML pre-service teachers. The CLM course sought to provide students with an advanced perspective and well-connected conceptual understandings of key General Education and Training (GET) mathematical ideas, alongside developing a deep functional understanding at this level and fostering an ability to apply these concepts to real world problems. The CLM course goals were to be realised through the achievement of a set of intended outcomes and assessment criteria, both of which have been presented below.

3.2.1 Course intended outcomes

Given below are the general intended outcomes and assessment criteria for the CLM course

according to official course documents (Academic devt doc, 2010).

Course intended outcomes	Assessment criteria
• Identify and solve problems in which responses display that responsible decisions using critical and creative thinking have been made • Work effectively with others as a member of the team, group, organisation and community • Collect, organise, analyse and critically evaluate information • Communicate effectively using visual, mathematical and/or language skills in the modes of oral and/or written presentations • Demonstrate an understanding of the world as a set of related systems by recognising that problem solving contexts do not exist in isolation.	• Apply the mathematical principles learned in the course to both routine and non-routine tasks • Use investigative approaches to solve problems with and without technology • Apply mathematical concepts to solve contextual problems • Communicate own mathematical thinking using a range of mathematical representations

Table 3.1: CLM course outcomes and assessment criteria

The intended outcomes suggest a critical orientation where evaluation of information presented in problem situations should inform decision making. Further, the outcomes suggest that developing skills relating to communicating information through a range of representations is central to being a critical individual. The critical view has also been observed within the curriculum specifications of ML, suggesting that the CLM course intended outcomes overlapped with the aims of ML.

The assessment criteria on the other hand suggest a more mathematical focus where mathematics learning is followed by application of mathematical ideas within problem solving. Further, the assessment criteria indicate that both intra-mathematics and extra-mathematics tasks would feature, an aspect which supports the course focus on developing both mathematics and contextual understandings. In relation to ML, these criteria agree with some elements of the subject specifications (in terms of its mathematics orientation) and its assessment (as evidenced in the Matric examinations). However, the main emphasis within the ML curriculum relates to developing both consumer and critical skills of learners, a focus which appears to contrast with the CLM assessment criteria.

3.2.2 Introducing the CLM sub-courses

As noted already, the CLM was a three-year long course and was designed and structured with a dual focus. However, this study focused on ML students only. Given that the study

was longitudinal, the teachers were tracked through three sub-courses (CLM 1, CLM 2, and Method 2) with related aims and foci across the two years (2011-2012). The rationale for tracking the students for two years within a three-year long course was pragmatic and based on PhD fellowship timeframes. Collecting data for two consecutive years provided an understanding of the pre-service teachers' ways of solving problems. The CLM 1 and CLM 2 were one-year long, content-based sub-courses and offered in 2011 (B.Ed second year) and 2012 (B.Ed third year) respectively. A one-year long methodology sub-course (Method 2) which specifically aimed at developing pedagogic skills for teaching ML was also offered in 2012. The commonalities between CLM 1 and CLM 2 were in terms of their foci. Both sub-courses emphasized foundation mathematics and mathematics in contexts understandings. The sub-courses' enactment adopted an approach where the students engaged with worksheets containing both pure mathematics and mathematics in contexts tasks, using foundation mathematics concepts. However, the two sub-courses were different in the sense that the CLM 2 course built on concepts developed in CLM 1. Both sub-courses used the same textbook by Billstein and colleagues (2010) titled '*A Problem Solving Approach to Mathematics for Elementary School Teachers*', with CLM 1 focusing on the first part of the textbook and CLM 2 concentrating on the second part. Thus topics which were not covered in CLM 1 were covered in CLM 2. However, two topics namely, 'Sets' and 'Logic', were not covered at all across the sub-courses which focused on content and contextualized problem solving (CLM 1, CLM 2 and CLM 3) – with lack of direct connections between topics in either the ML and primary mathematics. Whilst the emphasis in this textbook is on primary mathematics, several examples and exercises involve mathematical problem-solving in context. The contextual nature of the tasks from this book therefore provided links with the aims, purpose, content, and mathematical level dealt with in ML. CLM 1 and CLM 2 were taught by staff in the Mathematics Education Division at the University. I observed all of the teaching sessions of these 2 sub-courses as a support tutor.

The Method 2 sub-course on the other hand followed a ML textbook series (Grade 10 and 11) titled '*Focus on Mathematical Literacy*' (Bowie, Frith, & Prince, 2006; Bowie, Frith, & Prince, 2007). The focus in this textbook series is on contextualized problem solving. The students were assigned topics from the textbook series to teach during lecture sessions under my supervision. The teaching was followed by some discussions around mathematical work focusing on both intra-mathematical and mathematics-context connections, and ways of communicating solutions in relation to the learners at a particular grade level. While some

topics from the ML textbook series were strongly linked to the CLM 1 and/or CLM 2 sub-course topics like probability, percentages, etc, some concepts covered in Method 2 were not covered in the CLM 1 and CLM 2 sub-courses like solving quadratic equations in the context of acceleration/moving objects, trigonometry in the contexts of house plans, etc. The focus on quadratic equations in this textbook series contradicts the critical and consumer orientations in the ML rhetoric while aligning with ML curriculum specifications. The structure of the B.Ed CLM course across three years can therefore be understood as follows:

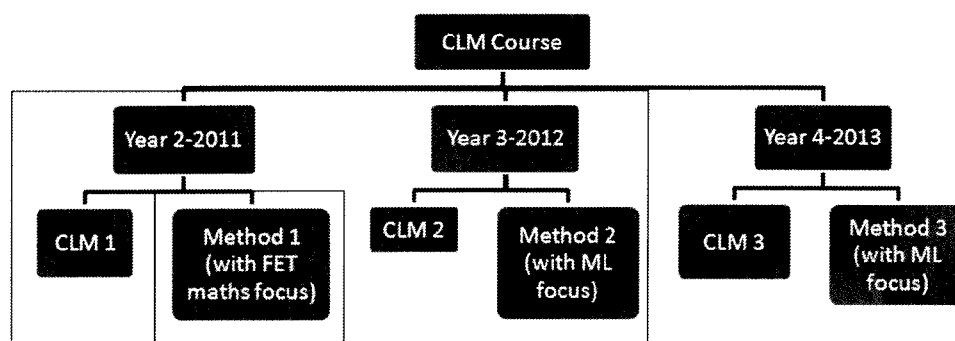


Figure 3.1: Route map for the ML students

The Method 1 sub-course was compulsory for both secondary mathematics and ML student-teachers, with a sole focus on methodology for mathematics teaching at FET level (Grade 10-12). Its course outcomes emphasized development of competences relating to teaching school mathematics. This means that Method 1 adopted a mathematical orientation, a focus which was different from ML focus. Due to this focus in Method 1, analysis in this study did not include data from this sub-course.

Since CLM 3 and Method 3 were offered after the two years of data collection, analysis relating to these sub-courses was beyond the scope of this study. This means that data for this study analysis were selected from CLM 1, CLM 2 and Method 2 sub-courses. Details about each of these three sub-courses (CLM 1, CLM 2 and Method 2), focusing on their aims and emphases have been given later in this chapter.

Since ML was offered as a sub-major subject, the ML student-teachers had to enroll for two method courses each year, one for their major teaching subject and another for their sub-major. Due to the diversity in preferences, the ML teachers enrolled for different major teaching subjects. Knowledge about the major teaching subjects was useful in this study as it provided background information on the study sample. The students' major teaching subjects have been given in the table 3.3. Identities used for participants are pseudonyms.

Teacher Name	Major teaching subject
Lindiwe	Technology
Mark	History
Jabu	Isizulu
Lebo	Technology

Table 3.2: Participants major teaching subjects

The table shows that the students' major teaching subjects ranged from sciences, social sciences to languages. I now provide descriptions of each of the three sub-courses focusing on course outcomes, course aims, course enactment, topics' coverage and course assessment.

3.3 The CLM 1 sub-course

CLM 1 was a content-based sub-course which was offered to second year Primary mathematics and ML B.Ed students with four contact hours per week and ran across both semesters in 2011. This sub-course was focused on developing students' knowledge related to both mathematics content and mathematics in context (extra-mathematical tasks). As already noted, enrollment into the course was based on obtaining a pass mark in *mathematical routes*, a first year course which aimed at developing all B.Ed students' numeracy skills. CLM 1 was taught by a lecturer who was also the course coordinator. I was involved in this sub-course as a tutor, and my role was to assist the students work through problems during lecture sessions, and help the lecturer to mark assignments, quizzes, and tests. My involvement in the sub-course activities allowed for understandings relating to the ways in which the student teachers worked through problems which later helped in understanding the teachers work during the preliminary data analysis phase.

The total number of students enrolled into the sub-course was 23, and comprised of 9 ML, 2 foundation phase, and 12 Senior Primary specialists. The numbers registered for CLM 1 showed that there were more students with a focus at the primary school level than those specializing to teach ML at FET level. Of the 9 ML students, 4 participated in the study. Although concerns relating to inadequate numbers of qualified ML teachers in the schools have been observed in literature, it is important to note that this study focused on a small cohort which will not make significant difference in practice. As noted earlier, the sub-course utilized a textbook by Billstein and colleagues (2010). The emphasis within the CLM 1 sub-course and within the textbook was on understanding intra-mathematical and mathematics-context connections. The sub-course and textbook structures indicated instances of departure

from the rhetoric of the ML curriculum, whilst emphasizing the need for coherence, connection and flexibility within and across mathematical representations of a problem, and between mathematical representations and the problem context. The need for both intra-mathematical and mathematics-context connections and coherence has been noted as important within ML teaching (Department of Education, 2003; Gardiner, 2004; Venkat, 2010).

According to the CLM 1 course outline (Academic devt doc, 2011) the course emphasized the following outcomes;

- Not only knowing how to do a mathematical procedure, but knowing why that mathematical procedure works.
- Mathematical practices: exploration, conjecture and justification/explanation/proof
- Mathematical representations: choosing, using and being able to convert between different representations (verbal, symbolic, graphical, manipulative)
- Understanding mathematical thinking different from your own
- Talking mathematically: being able to communicate your mathematical thinking clearly and coherently using appropriate representations

The sub-course outcomes suggest a mathematical orientation since they do not make explicit reference to a focus on world situations. The emphasis on mathematical procedure, mathematical proof, conjecture, and mathematical thinking locates the sub-course outcomes within the school mathematics domain. Prior evidence of gaps in content knowledge indicates the need for ML teachers to develop their understandings of mathematical ideas in these ways. The need for individual competences to use mathematics knowledge in a practical and functional way in everyday life is supported in related international notions of ML (Jablonka, 2003; Steen, 2001).

The sub-course's coverage was focused on elementary mathematics and mathematics in context knowledge development and included the following specific topics;

Semester 1 topics	Semester 2 topics
Place value	Adding and subtracting fractions
Addition, subtraction, multiplication, division	Multiplying fractions
Negative numbers	Fraction stories
Factors, multiples	Decimal representation of fractions
Divisibility / division algorithm	Ratio and proportion

Prime numbers	Fractions in the classroom
Greatest Common Divisor and Lowest Common Multiple	Percentage
Introduction to fractions (biscuit fractions and drawings)	Financial Mathematics as application of percentage
Relative growth and proportional reasoning	
Equivalent fractions, different notions of fraction (partition)	

Table 3.3: CLM 1 topics coverage

Although the sub-course outcomes suggest a push towards some mathematical agendas, the sub-course topics seem to point towards a balance between pure mathematics and mathematics in context (Department of Education, 2003). Semester 1 topics suggest a more mathematical orientation where mathematical understandings could be developed. In contrast, semester 2 topics engaged with world situations, an aspect which was supported by inclusion of a wide range of contextualized tasks across assessment in this semester. However, there was some specific reference within semester 1 to engaging with contextual problem solving. Connecting the idea of fractions with situations within the context of ‘sharing biscuits’ in semester 1 for instance provided evidence of a contextual frame, as this is linked to problem solving at some personal level (Brombacher, 2007) and relates to the citizenship agenda (Department of Education, 2003).

At the enactment level, semester 1 lecture sessions adopted a more mathematical orientation (intra-mathematical connections) focusing on strengthening the students’ fundamental mathematical knowledge. The lecturer’s demonstrations on how to work through problems were followed by students’ engagement with worksheets which were prepared by the course lecturer. The mathematics orientation emphasized in semester 1 was closely linked with the selection and/or design of the assessment tasks in this semester which also focused more on mathematical tasks than mathematics in context problems. The majority of the assessment tasks which were made available for students to engage with in semester 1, selected purposively here for illustrative purposes, involved intra-mathematical working:

CLM 1-Example 1

Explore and then make a conjecture about what happens when you add an even number and an odd number

Provide a proof of your conjectures. (Note: the proof can be an algebraic proof or it can be a "Smartie" proof)

CLM 1-Example 2

The greatest common divisor of two numbers is 3. The lowest common multiple of the same two numbers is 399. If one of the numbers is 21, what is the other number?

The first two examples show mathematically focused tasks, where no reference is made to objects in the real world. PISA (OECD, 2006) call these kinds of problems 'intra-mathematical tasks' and argue that such tasks are aimed at developing mathematical understandings among students. Given the focus of the course, the pre-service ML teachers needed to engage with these kinds of tasks in order to deepen their understanding of foundation mathematics-a key component in problem solving.

CLM 1-Example3

Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.

The third task exemplifies scenarios where skills relating to connections between mathematics and contexts are played out in new ways. Rather than moving from the contextual world to the mathematical world, as is the case in mathematisation and modelling (Blum & Ferri, 2009; Kaiser & Schwarz, 2006; OECD, 2006), this task asks students to identify a real-life situation or a scenario where the calculation given in the question would be needed, requiring the 'story creation' that I elaborated in the last chapter. Although these examples seem to be different from tasks in ML related literature, and also appear to contradict the ML curriculum rhetoric in citizenship orientation sense (Department of Education, 2003), the related skills remain firmly linked to the need for both primary and ML teachers to have strong connections, especially at the level of translation (OECD, 2010). The implication is that engagement with such problems may also further help to develop the students interpretive skills which are central in making sense of numerical solutions obtained from working with contextual tasks.

There was no examination in semester 1, but a summative assessment in the form of a test was given to the students. The nature of the tasks given in this test was similar in many ways with the tasks given in other course assessments (assignments, short quizzes) in this semester. The three problems given above, selected from the CLM 1 course sessions, exemplify both mathematically focused tasks (examples 1 and 2) in the sense that the tasks were not embedded in some real world context and extra-mathematical tasks (example 3).

Lecture sessions in semester 2 adopted a combination approach with pure mathematics problems mixed with mathematics in context problems, with emphasis on both intra-mathematical and mathematics-context connections. In contrast with semester 1, assessment tasks in this semester included several contextualised tasks. The contextualised tasks have been included in the set of tasks which have been analysed in this study, a choice informed by the study's theoretical lenses. Thus despite the content-driven organisation (Graven & Venkat, 2007a) of the course outcomes and topics, the course enactment and assessment incorporated links to contexts as shown below:

CLM 1-Example 4

1. I spend $\frac{1}{2}$ of my salary on rent and $\frac{1}{5}$ of what I have left on groceries. What fraction of my salary is left for the rest of my expenses?

Although this task is extra-mathematical, in PISA's terms, it remains mathematically focused and therefore seems to support the mathematical orientation, as individuals are unlikely to engage with such kinds of tasks in their everyday lives. Rather than a focus on understanding the context, this task appears to examine mathematics competences.

CLM 1-Example 5

John and Jane both currently earn R10 000 per month.

- a) John performs badly in this job so is demoted and will earn 9% less from next month onwards. How much will he earn?
- b) Last month Jane was actually earning less than R10 000 and she received a raise of 9% which brought her salary up to R10 000. What was she earning last month?
- c) Are the amounts John will earn and the amount Jane earned last month different? Explain why this is so.

Like example 4 above, this task makes reference to some real world objects such as salary and percentage increase/decrease. Although this task supports the contextual orientation observed in the ML curriculum, it contradicts claims of task authenticity (use of real data), given the unlikelihood of 9% increase or reduction in salary.

CLM 1-Example 6

You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?

Example 6 appears to include real data and like examples 4 and 5 supports the contextual frame.

In summary, the three examples appear to be extra-mathematical as they all refer to real world objects. However, whilst these tasks are situated in some context, some are still largely mathematical in focus (Jablonka, 2003; Julie, 2006). Example 4 for instance appears to focus on developing competences relating to working with fractions, rather than understanding the given contexts. The three examples therefore appear to fit in well with the ML curriculum specifications in terms of situating tasks in some contexts.

3.4 The CLM 2 sub-course

The CLM 2 sub-course, which was also content-based, was offered at third year to the same group of students who had completed the CLM 1 component at second year. Just like CLM 1, the CLM 2 sub-course ran for two semesters with four contact hours per week in 2012. The focus of CLM 2 was to further develop the students' knowledge necessary to teach both Primary mathematics and Secondary ML. CLM 2 was taught by a tutor who was also the course coordinator. The sub-course recommended the same textbook used in CLM 1 by Billstein and colleagues (2010), with CLM 2 focusing on topics not covered in CLM 1. As noted already, the book contained examples and exercises in both pure mathematics and mathematics in contexts.

The CLM 2 course outline emphasized the same outcomes which the CLM 1 focused on. Again there were no outcomes with specific focus on situations or problem solving. This means that at the level of planning, the CLM 1 and CLM 2 content sub-courses maintained the same emphasis across the two years despite differences on topics' coverage. Thus the

CLM 2 sub-course outcomes adopted a more content-driven organization (Graven & Venkat, 2007b). The topics which were covered in CLM 2 (Academic devt doc, 2012a) included the following:

Semester 1 topics	Semester 2 topics
Basic Notions (Geometry)	Linear Measure
Polygons (van Hiele puzzle)	Areas of polygons and circles
More about angles	The Pythagoras theorem, distance formula and equation of a circle
Geometry in three dimensions	Surface areas
Congruence through constructions	Volume mass and temperature
Other congruence properties	Displaying data
Similar triangles and similar figures	Measures of central tendency and variation
Lines and equations in a Cartesian coordinate system	Abuses of statistics
Translations and Rotations	Designing experiments and collecting data
Reflections and Glide reflections	How probabilities are determined
Size transformations	Multistage experiments with tree diagrams and geometric probabilities
Symmetries	
Tessellations	

Table 3.4: CLM 2 topics coverage

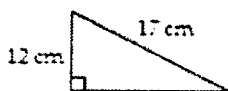
The list of topics for CLM 2 suggests a push towards some mathematical orientation especially in semester 1 where the focus was largely on Geometry (i.e. angles, geometrical constructions and proofs related to congruency and similarity). Semester 2 topics pointed towards a balance between intra-mathematics and extra-mathematics tasks as evidenced in topics related to Pythagoras theorem, equation of a circle, and Statistics (i.e. abuses of Statistics) respectively. Inclusion of a focus on ‘abuses of Statistics’ supports the critical orientation of ML rhetoric.

At the enactment level, semester 1 lecture sessions emphasized more pure mathematics (intra-mathematical connections), specifically relating to finding angle values, geometrical constructions, mathematical generalizations, and proofs related to congruency and similarity of different geometrical figures. The lecture approach in CLM 2 was identical with that of CLM 1 in that the tutor demonstrated how to solve problems followed by students working through problems on a worksheet. Many of the tasks used for assessment in this semester

were intra-mathematical thus supporting the sub-course outcomes and enactment as shown in the following examples, again selected purposively for illustrative purposes.

CLM 2-Example1

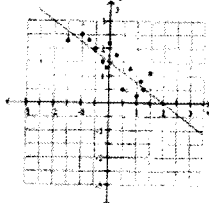
Find the missing length in the following right triangle. Round off to the nearest whole number



- A) 145 cm
- B) 72 cm
- C) 14 cm
- D) 12 cm

CLM 2-Example 2

Determine the equation of the trend line shown in the scatterplot.



- A) $y = \frac{-3}{4}x + 2$
- B) $y = \frac{-3}{4}x + \frac{1}{2}$
- ☒ C) $y = \frac{3}{4}x + \frac{3}{2}$
- D) $y = \frac{-3}{4}x + \frac{3}{2}$

The examples above focus on developing intra-mathematical understandings as no reference to real world objects features.

The second semester combined a focus on mathematics and contextual tasks. The approach adopted at an enactment level was similar to that of semester one. Some of the assessment tasks reflecting topics in semester two have been provided below and relate to 'surface areas' and 'displaying data' respectively.

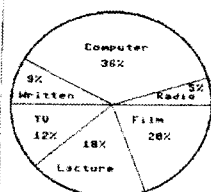
CLM 2-Example 3

Find the surface area of a right regular hexagonal pyramid with sides 3 cm and slant height 6 cm. Round your answer to the nearest hundredth.

- A) 77.39 cm² B) 108.00 cm² C) 65.69 cm² D) 70.15 cm²

CLM 2-Example 4

In a school survey, students showed these preferences for instructional materials



About how many students would you expect to prefer radio in a school of 250 students?

- A) About 13 students
B) About 90 students
C) About 5 students
D) About 45 students

Example 3 suggests that mathematics understandings would be sought. To engage with this task, a student required some knowledge of the 'right hexagonal pyramid' shape as this information was not provided in the problem, before setting up a procedure which in this case involved retrieval of the surface area formula. The fourth example, which was situated in statistical context, demanded some skills relating to reading information followed by using this information to obtain a problem solution.

It is also important to note that the CLM 2 largely adopted multiple choice assessment method. Given the study relating to exploring the nature of the students' mathematical working relating to both intra-mathematical and content-context connections, multiple choice responses did not provide sufficient information in this regard. Although multiple choice responses could potentially point to some common misconceptions, analysis of this data in the absence of solution protocols would lead to speculative claims.

3.5 ML method sub-course (Method 2)

The Method 2 sub-course was introduced in 2012 and was aimed at developing ML pedagogic skills among the teachers. I taught the sub-course, which ran for two semesters and had 2 contact hours per week. The ML teachers had enrolled into a Method 1 course, with FET mathematics teaching focus the previous year. Given that the aims of the two subjects (Mathematics and ML) are different, the ML method course was structured to specifically meet the needs of the ML teachers.

The overarching approach adopted in this course was to have students work through problem situations found in ML textbook series, titled '*Focus on Mathematical Literacy*' (Grade 10 and 11), authored by Bowie and colleagues (2006). This textbook series had been developed in line with the South African Curriculum – National Curriculum Statement (NCS) for ML (Department of Education, 2003). The choices of contexts for lecture discussions were also guided by the Curriculum Assessment Policy Statements (CAPS) document (Department of Basic Education, 2011c). Although the ML textbook series was not CAPS compliant, it was selected because of its ML focus based on the NCS which was phased out at the end of 2011. To prepare the students for the CAPS document, which was introduced in grade 10 at the beginning of 2012, the Method 2 course therefore emphasised linking the topics from the ML textbook series with 'basic skills topics' and 'applications topics' as per CAPS topics organisation. Specifically, given a task, students were required to identify basic skills and the context of the problem situation in terms of the CAPS 'application topics'. In addition, the student presenting the lesson would give homework to be completed by all students before the next lecture session. According to the Method 2 course outline (Academic devt doc, 2012b), this course emphasised achieving the following aims:

- Interrogating the textbook series in terms of its interpretation of the ML curriculum.
- Providing links between the textbook series and the CAPS document for ML in terms of content outline and teaching approach.
- Reflecting on the nature of Mathematical Literacy as communicated by the textbook series in terms of ML conception and what the textbook series said about being a mathematically literate person?
- Preparing students to teach ML at high school level - students to engage largely with contexts using mathematics content.
- Developing students' skills in interpreting and communicating solutions, especially those in numerical form.

The Method 2 sub-course aims, linked well with the CLM course and ML, and therefore figured centrally in this study. There was a strong push towards strengthening the students' pedagogical skills in terms of how to teach specific ML topics. The course aims also suggest that within the context of ML teaching, engagement with mathematics content was not for its own sake but to support the teaching of contexts. Listed in the table below are topics which were covered in the Method 2 sub-course.

Semester 1 topics (Grade 10 textbook)	Semester 2 topics (Grade 11 textbook)
What is Mathematical Literacy?, International conceptions of Mathematical Literacy	Revision: Recap on Grade 10 concepts
Making sense of the Basic Skills description table of the CAPS document	Risky behaviour: alcohol use, tobacco use
Making numbers work: census, water resources, design, finances	Eating and growing: normal weight and height, getting enough food, body mass index
Really useful numbers: AIDS, measurement	Sport: predicting winners, running and proportion, designing a soccer field, motor-car racing
Ratios, percentage and percentage change: crime, scale and measurement, tax	Probability: dice games, contingency tables
Mostly money matters: savings and loans, budgets; Tables and Charts: AIDS, substance abuse, census	Maps and plans: maps and directions, finding bearings, building plans, areas and volumes of houses
Summarizing Numerical information: census, transport, health services	Planning a school: cost of building a school, decorating walls, space-filling designs
Measuring probability: lotto, gambling; Surveys in practice	More money matters: exchanging money, earnings and taxes, inflation
Tabling, Graphing and discussing relationships: exchange rates; Graphs reading the story: AIDS, malaria, food prices	Running a small business: making choices, fixed and variable costs, number of employees
Graphs – Straight lines: housing, Telkom charges	
Graphs that aren't straight: transport, savings, volume, area, water tariffs	

Table 3.5: Method 2 topics coverage

The Method 2 topics coverage fitted with the aims of ML – emphasising engagement with contexts. The majority of topics from the table included specific connections with situations and problem solving. Whilst the semester 1 topics (from grade 10 textbook) drew in some ways from situations at a personal level, the semester 2 topics (from grade 11 textbook) allowed for students to engage with contexts drawn from workplace and/or community level. This supports the ML curriculum's idea which puts progression at the level of contexts' complexity and not necessarily in terms of familiarity at the level of mathematics content as follows:

There are some topics in which the focus in Grade 10 is on contexts relating to the personal lives of learners and/or household issues (e.g. personal finance → cell-phone accounts; household budget), in Grade 11 on contexts relating to the workplace and/or business environment (e.g. business finance → payslips; taxation), and in Grade 12 on contexts relating to scenarios encompassing wider social and political contexts incorporating national and global issues (e.g. exchange rates and inflation) (Department of Basic Education, 2011, p.12).

Similar emphasis can be found in 2003 version of ML curriculum. The Method 2 course adopted an approach which allowed the students to present a lesson each based on the topics from the ML textbook series. The first semester allowed learners to engage with grade 10 textbook while semester 2 learning was focused on grade 11 textbook. Within the 2 hour sessions available per week, one student would use the first 40 minutes to present a lesson while the rest of the students were treated as learners. The teaching was then followed by a class discussion. The whole class discussion which was facilitated by the researcher was focused on the strengths and weaknesses of the lesson presented, and how the weaknesses could be improved. The discussions allowed for feedback on mathematical working in terms of contextual discussion, translating context into mathematics, communication of solution strategies, and interpretations of numerical solutions. Although the researcher was facilitating the Method 2 sessions, more time was given to the students to give feedback to each other and ask questions. In this way there were more opportunities in terms of student-to-student interaction and learning. Thus the tutor could come in to answer students' questions, support or probe some responses to ensure both students' conceptual and procedural understanding.

The Method 2 course assessment involved examining the students' problem solving skills in context as well as how such contexts could be used for teaching in an ML classroom. There were 2 assignments, 2 quizzes, and 1 test in each semester and one examination at the end of the course. It is important to note at this point that there was a mismatch between course enactment and assessment in that Method 2 assessment appeared to tie in with CLM 1 focus (doing ML tasks) more than Method 2 focus (teaching ML tasks). Although I taught the Method 2 sub-course, the course structure including ways in which the course would be assessed was designed by an external ML consultant. An exemplification of the tasks which students engaged with in Method 2 is given below.

Method 2-Example1

Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month.

- i. Determine Nadia's new salary per month.
- ii. Who received the greater percentage increase? Show your working.

Method 2-Example 2

One of your learners in a ML classroom wants to buy a cell phone with internet. The learner has seen the advertisement (see below) and needs advice from you on choosing a better deal. Help the learner and justify your thinking.

Web Exclusive Deals
Better than a bouquet of flowers and a box of chocolates

Nokia X2-01
1GB data bundle
1000 Free SMS
R999.00

Samsung Galaxy Mini
1GB data bundle
1000 Free SMS
R999.00

* 1GB data bundle up to 1GB Free SMS per month. Volume and Conditions apply, which can be found in the advertisement. Offers are valid until 31 March 2012, or until stocks last. We do our best to ensure that the descriptions, prices & pictures are correct. However, changes & errors do sometimes occur for which Virgin Mobile cannot be held liable. ©2012

Source: Cell phone promotion downloaded on 5th May 2012 from <https://www.virginmobile.co.za/Store/?gclid=CL3DmP7lva4CFSgntAodtxxvFg>

Method 2-Example 3

Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years.

- i. Identify the basic skills topic(s) and application topic(s) knowledge according to CAPS document for ML needed to solve this problem situation.
- ii. Calculate the amount he will receive at the end of the period from Bank A.
- iii. Now calculate the amount he will receive at the end of the period from Bank B.

The three tasks exemplify the kinds of situations which learners are likely to deal with which demand them to utilize both mathematical and non-mathematical considerations, a dominant feature of citizenship experiences. Example 1 was concerned with engaging with a context relating to finance involving the idea of percentage. In this case skills related to making comparison informed by some mathematical calculations were examined. The other two

examples comprised some pedagogic aspects in the sense that the problems specifically related to pedagogic demands of the classroom (example 2) and notions of ‘basic skills topics’ and ‘applications topics’ in the ML curriculum (example 3i). These assessment tasks were aimed at measuring progress in knowledge development related to mathematics content and contextual understandings as well as their thinking about the tasks in relation to ML practice.

3.6 Chapter summary

In this chapter, the context within which this study was located has been presented. In this case the CLM course which comprised of sub-courses namely CLM 1, CLM 2 and Method 2, provided this contextual field. Driven by the need for adequate numbers of qualified ML teachers in the schools, the course was introduced in 2010 with a view to be offered across three years (2nd, 3rd, and 4th years), with this study focusing on the 2nd and 3rd years only. The course was aimed at developing mathematical and contextual understandings related to foundation mathematics teachers and ML teachers. Enrolment into the course at second year was based on pass mark in a first year course, Mathematical Routes, which had a numeracy focus.

At enactment level, both CLM 1 and CLM 2 utilised an approach where students engaged with problems on some worksheet, prepared by the course coordinators. The two sub-courses had similar course outcomes despite different foci on topics’ coverage. Given the dual foci, there was an attempt across the two courses to balance the teaching and learning approaches in order to meet the needs of both groups. The Method 2 sub-course on the other hand was targeted at CLM pre-service teachers only and was pedagogically focused, combining problem solving and ways in which problem situations could be utilised in ML teaching. The course followed the ML textbook series (grades 10-12), and assessment across these three sub-courses was focused on both intra-mathematical and extra-mathematical tasks. Furthermore, pedagogically focused tasks also featured especially in CLM 1 and Method 2 sub-courses.

CHAPTER FOUR: METHODOLOGY

4.1 Introduction

In this chapter, I provide a discussion of, and justification for the research methodology that underpinned data collection and data analysis methods for this study. I also give a detailed description of the specific research techniques used in the identification of participants and the ways in which ethical issues were managed. This chapter also presents the details of how concerns about reliability and validity were addressed.

The main purpose of this study was to explore pre-service ML teachers' development in knowledge and practice in the CLM course with specific focus on intra-mathematics and context understandings, and ML pedagogic practice. It involved tracking the pre-service teachers' learning within a new professional teacher development course at a large urban University and exploring how their learning in the course linked with their practices in schools during the two three-week long teaching experiences (TE) (one TE per year) across two years (2011-2012). This implied collecting data relating to the teachers' mathematical and contextual learning as well as their classroom practices over an extended period of time. Thus the research study can be understood as longitudinal and falls into the domain of developmental research. According to Cohen & Manion (1994), developmental research studies are concerned with providing descriptions about relationships between the present state of phenomena and to account for changes occurring within the phenomena under study as a function of time. This study therefore has endeavoured to provide a developmental account in knowledge and snapshots of practice of the pre-service ML teachers across two years.

To address the problem in this study, I employed multiple data collection strategies namely documentation, interviews, non-participant observations and video-recording of the pre-service teachers' practice in the schools during teaching experiences. Multiple data sources ensured that I had overlapping data for each phenomenon (learning and practice) of interest, a component that added rigor to the study. A discussion of these data collection strategies has been provided later in this chapter.

This study adopted an interpretive method which utilized qualitative approaches and methods to explore pre-service ML teachers' development in knowledge and practice. Cohen and

colleagues (2000) posit that interpretive methods are primarily concerned with meaning construction and tend to give descriptive analyses of the phenomena with a particular focus on in-depth understandings of the subjective world of human experience through the mental process of interpretation. Since this study was focused on giving meaning to pre-service teachers' learning and practice in terms of how they engaged with a range of different tasks both within the course and school classrooms, the exploratory methodological approach was justified.

4.2 Research design

To ably talk about the pre-service teachers' learning and practice, I chose qualitative methods utilising an exploratory case study design. Qualitative inquiry assumes that "there are multiple, changing realities, and that different individuals have their own unique constructions of reality" (Merriam, 2002, p.25). In this study, I was interested in the individual teachers' ways of problem solving, focusing on solutions to assessment tasks in the course and mathematical working including communications of solutions in practice. The individual accounts gave me insights on personal development in knowledge and practice. Furthermore, I was also interested in the common patterns across these individual interpretations, and how these were linked with common experiences on the course. The common patterns which emerged from the teachers' accounts provided me with some insights about course development. The idea of multiple realities in qualitative research justifies the use of approaches that put researchers "closer to reality than if an instrument with predefined items had been interjected between the researcher and the phenomenon being studied" (ibid, p.25).

The qualitative case study approach affords researchers opportunities to explore and describe a phenomenon in context using multiple data sources. It seeks meanings and in-depth understandings of a bounded system, supports inductive investigative strategies and generates rich and thick descriptions of the phenomena (Merriam, 1998). The case study approach ensures that the issue is not explored through one lens, but rather a variety of lenses which allow for multiple facets of the phenomenon to be revealed and understood (Baxter & Jack, 2008).

According to Yin (1994), a case study design can be used whenever "an empirical study must examine a contemporary phenomenon in its real-life context, especially when the boundaries

between phenomenon and context are not clearly evident” (p.13). Harling (2002) concurs with Yin in his conceptualization of a case study, and defined it as a “holistic inquiry that investigates a contemporary phenomenon within its natural setting” (p.1). There were two phenomena of interest in this study namely pre-service ML teachers’ learning within the CLM course and pre-service ML teachers’ classroom practices. The natural settings were the newly introduced pre-service professional development course (CLM), and the schools that the pre-service teachers were assigned to for their school experiences. In this case, understanding the phenomena (pre-service teachers’ learning and practice) could not be divorced from the factors characterising the natural settings (CLM course and school environment). The nature of the study therefore could not isolate the specific focus of the study (learning and practice) from the context where learning and practice were located. Thus the notion of case study research based on these qualities was found to be congruent with the aim and focus of this research.

The choice of case study design was based on the nature of study (exploring a unique story), and the fact that I wanted to explore both the phenomena and the context. According to Yin (2009), case studies are the most appropriate strategy whenever contextual conditions are relevant to the phenomena under study. Part of the focus of the study was to give a commentary on how the CLM course learning linked with school practice. In this way, I was able to establish how contextual conditions were related to practice. The contemporary nature of the phenomena, teachers’ development within a newly introduced course, could be fully understood using a case study. Neale and colleagues (2006) contend that case studies are also a preferred strategy when investigating a unique or interesting story with the purpose of seeking meaning and in-depth understanding of the social phenomena within a bounded system. Such investigations can result in a generation of richly descriptive results which provide deeper insights (Merriam, 1998). The qualitative case study inquiry was also appropriate for this study in the sense that it allowed me to talk about issues of interest emerging from the data. In this way, the choice of the case study design for this study was justified.

4.2.1 Defining a case and unit of analysis

According to Yin (1998), a unit of analysis is understood in terms of the basic definition of the case. This means that defining a case implies defining a unit of analysis. The importance of defining a unit of analysis in a case study research lies in its affordances in terms of

limiting the boundaries of a particular study (Yin, 2009). The study approach employed in this research was considered to be a holistic single case inquiry with embedded sub-cases. The case in this study was defined as a group of pre-service ML teachers and sub-cases were individual students within the sample. Consequently, the unit of analysis was defined as the pre-service teachers' development in learning and practice. Thus analysis was focused on the teachers' understandings of mathematical content and context within the CLM course and how such understandings linked with their classroom practices during teaching experience. My interest was on individual analyses - that is within case analysis of individuals and cross case analysis in terms of overlaps and contrasts. It was a holistic approach because it involved collection of in-depth and detailed data and involved multiple sources of data that provided a wide array of information needed to provide deeper insights. According to Baxter & Jack (2008), a powerful engagement with data that involves data analysis within subunits separately (within case analysis), and across all of the subunits (cross-case analysis), can result into deeper insights. This is done based on an understanding that analysis of every single subunit is one piece of a 'puzzle' which contributes to the researcher's understanding of the whole phenomena (unit) (ibid).

4.2.2 Sampling of pre-service teachers

The participants for this study comprised initially *eight* pre-service ML pre-service teachers who were studying for a B.Ed degree at a large urban University in Johannesburg, South Africa. These pre-service teachers were enrolled into a new second year CLM course, which was introduced at the beginning of 2011 academic year. To enrol into CLM course, students needed to have passed 'Mathematical Routes', a first year mathematics course which aimed at developing numerical understandings among all B.Ed students. The whole population size was made up of *nine* ML pre-service teachers. Since the whole population was already small, I decided to invite all the nine specialists in the course. Eight out of the nine, indicated their willingness to participate in the study following informal discussion and the distribution of information letters. Written consent was later given after understanding the purpose and aims of the study in a manner free of any coercion. This meant that almost the whole population which included high, average and low performing students initially participated in the study. I started with a large number (eight participants) to ensure continuity of the study in the event that some participants decided to drop-out at any time of the data collection phase (two

years). Further, I wanted to take care of unanticipated issues which might have affected the participants in terms of not teaching ML in the schools in the course of data collection.

Sampling of research participants was aimed at finding teacher specialists who were willing to have their development in the course in terms of learning and practice, tracked for a period of two years (2011-2012). Collecting data from the eight participants, a number very close to the whole population, added strength to the findings of this study. Due to my interest in understanding individual participants' development, it was imperative for me to find out their major subjects and whether they sat and passed an ML or mathematics examination at Matric. The background information about the two numerical subjects passed at Matric was relevant in this study because competence in ML teaching depended hugely on both mastery of mathematical content and familiarity with contexts. This data therefore provided insight into the nature of numerical knowledge that the pre-service teachers had before enrolling into the CLM course (see table 4.1). As noted in chapter one, pseudonyms have been used to identify the teachers in this study.

Teacher Name	Major subject	Sub-major subject	Passed ML at Matric	Passed Mathematics at Matric
Lindiwe	Technology	ML	X	
Mark	History	ML	X	
Jabu	Isizulu	ML	X	
Lebo	Technology	ML	X	
Linda	English	ML	X	
Thabo	Technology	ML		X
Karabo	Natural science	ML		X
Glein	Technology	ML		X

Table 4.1: Study information of participants

Of the eight ML specialists, 5 passed ML and 3 passed school Mathematics at Matric. However, their main subjects were different ranging from natural sciences to languages. As highlighted already in the introduction chapter, ML could only be chosen as a sub-major within the B.Ed programme structure. Further, none of the participants had school mathematics as a major due to the programme structure not allowing students to combine mathematics and ML. The sample therefore provided insight into the nature of mathematical knowledge, context, and ML teaching of a wide spectrum of students in relation to main subject expertise.

4.2.3 Gaining access into the schools for data collection

‘Experience is a great teacher’. Gaining access to the schools was not as easy as I initially thought. Since pre-service teachers did not maintain schools for their teaching experiences across the two years of data collection, it meant going to new schools each time for data collection related to the teachers’ practice. For each of the data collection phases, I made sure that schools were visited to personally explain the purpose of the study and to ask for permission from school Principals to conduct research at their schools. These visits were also aimed at seeking consent from responsible teachers (students’ supervising teachers) to conduct research in their classrooms. This was normally done a week before the pre-service teachers reported for their teaching experience at the schools. Despite the warm welcome received from the schools, there were differing experiences encountered at some of the schools that resulted into the researcher collecting partial or no data at all. These experiences are noted in the following section.

4.2.4 Narrowing down the sample size

As already noted, starting with 8 participants in the study was aimed at ensuring continuity of the study in situations where some participants decide to withdraw. One of the participants in this study (Glein), withdrew at the beginning of the 2012 academic year. This withdrawal meant that the researcher had to complete the data collection phase (2011-2012) with the remaining 7 participants. Furthermore, there were occurrences during data collection phase which prevented me from collecting data relating to practice for some participants – some of which are:

- School principal not allowing collection of data from school, where reasons for doing so were not provided
- Participant teaching school Mathematics instead of ML during teaching experience
- Participants not allowed to teach ML in the school, as the school Principal cited a University rule specifying that second year students could not teach FET class
- Participant not reporting for teaching experience

In order to provide a full picture of the nature of data collected during the two years (2011-2012), I present a summary of the data sources for each participant in table 4.2.

Students' participants	CLM Course		Teaching Experience (TE)							
	Assessment		Field notes		Video data		Post-lesson interviews			
	2011	2012	F1	F2	V1	V2	IF1	IV1	IF2	IV2
Lindiwe	✓	✓	✓	✓	✓	✓	✓	✓	x	✓
Mark	✓	✓	✓	✓	✓	✓	✓	x	✓	✓
Jabu	✓	✓	✓	✓	✓	✓	✓	x	✓	✓
Lebo	✓	✓	✓	✓	✓	✓	✓	✓	✓	✓
Linda	✓	✓	✓	X	x	x	✓	x	x	x
Thabo	✓	✓	✓	X	x	x	✓	x	x	x
Karabo	✓	✓	✓	X	✓	x	x	✓	x	x
Glein	✓	x	✓	X	✓	x	x	✓	x	x

Table 4.2: Summary of data sources

The table shows details about data collection relating to course assessment, field notes of initial classroom observation (F1, F2), video recording of second classroom observation (V1, V2) and post-lesson interviews (IF1, IV1, IF2, IV2). Since data collection spanned two years (1 TE in 2011 and 1 TE in 2012), 4 lessons (2 lessons across every TE) for each participant needed to either be observed or vide-recorded. The first observed lesson (F1) was given in September 2011 followed by a video recording session (V1), within the same TE. The two lessons (F1 and V1) were not necessarily consecutive due to the fact that the study was not focused on how students teach particular topics, but rather concerned with mathematical working within the instructional space across a range of different tasks. The lessons (i.e. F1) were to be followed by post-lesson interviews (IF1), suggesting that in cases where F1 did not occur, IF1 could not feature. However, in some instances, F1 could feature but IF1 could not, especially when the student teacher had to go to another class immediately after the lesson.

It must be understood that I had no control whatsoever over the students' choices of schools. As a result the participants chose different schools for their TEs across the three phases of data collection. This kind of student placement had two implications. First, it was not feasible to arrange with the students to observe or video-record more than two lessons during a single TE. Second, it involved a lot of travelling since some students were placed far apart from each other. The travelling aspect provided a rationale in terms of why some post-lesson interviews were not conducted, despite the lesson been taught.

Given the evidence from the challenges regarding gaining access into the schools combined with discontinuity of data collected relating to some participants, the sample size was therefore narrowed down to four (Mark, Lindiwe, Jabu, and Lebo). A focus on the four student teachers allowed me to provide a detailed account of their performance across the CLM course and practice, with a view to gain a longitudinal development in relation to the course.

4.3 How data was collected

As already highlighted, this study focused on the ML students only. Assessment tasks in the form of assignments, short quizzes, tests and semester-end examinations were given to the pre-service teachers as a way of monitoring their development and progress in the sub-courses. Both groups of students (Primary and Secondary) who enrolled for CLM 1 and CLM 2 content courses were given the same assessment tasks to work on, across the four semesters in 2011 and 2012. Since the main focus of the CLM 1, CLM 2, and Method 2 was to build the students' knowledge related to mathematics content, mathematics in context, and pedagogic practice, assessment were also focused on testing students' abilities to engage with tasks from both mathematics content and context domains. Tasks which were mathematically focused but demanded linking with aspects of learning were also included in the assessment and especially so in Method 2.

Within the CLM course, data collection focused on assessment tasks and students' marked scripts. The marked scripts were then photocopied before they were handed back to the students, in the case of course assignments, quizzes, and tests. The examinations questions and students' marked scripts were also copied and put in a departmental repository for safekeeping. In practice, data collection focused on instructional tasks and related teaching episodes including interviews relating to the taught lessons. Given my involvement as a tutor in the CLM course which promoted use of contextualized tasks, I acknowledge possibilities of power relations where students would behave in particular ways to please me during data collection of practicum data. However, the fact that non-contextualized instructional tasks were also utilized in practice, suggests that supervising teachers in schools influenced the selection of these tasks more than the CLM course. An overview presentation of how data was collected from the CLM course and school practice has been provided in figure 4.1.

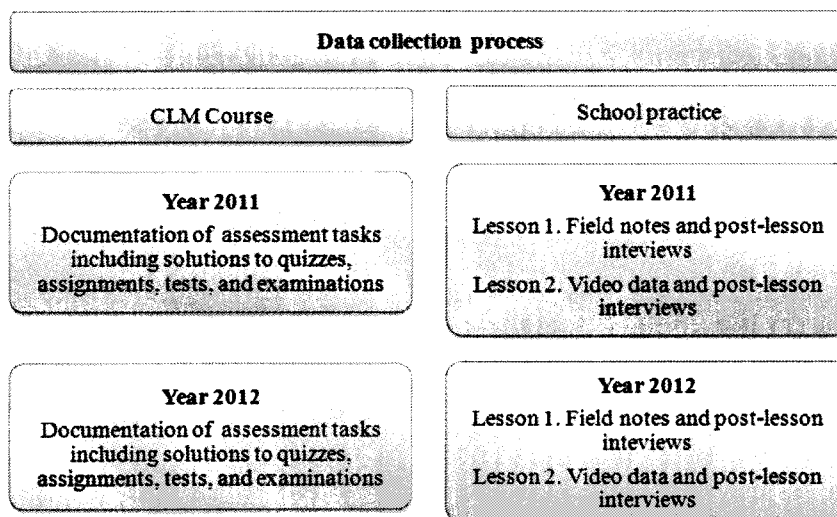


Figure 4.1: Data collection overview

One of the major strengths of a qualitative case study research is the use of multiple data sources, a strategy which also enhances data credibility (Patton, 2002; Yin, 2009). In order to generate valuable data which helped me answer the research questions posed in this study, multiple data collection instruments were used. Due to the qualitative nature of the study design, the data collection instruments included documentation, observation schedule, semi-structured interview schedule, video recorder, and audio recorder. This was aimed at collecting data which would be converted into text and then analysed using qualitative methods. The pieces of data which came from the multiple sources added valuable insights into the nuances of learning and practice as the various strands of data were braided together to promote a greater understanding of the case (Baxter & Jack, 2008). I now explain the techniques I employed to collect data for this study.

4.3.1 Document analysis

Documentation was used to collect data in documentary forms. The documents that were analysed were ML policy documents (National Curriculum Statement, Teachers Guide, Learning Programme Guidelines and Subject Assessment Guidelines) and CLM course development materials including course outlines. Tasks used across the CLM assessment together with related solutions were also collected. Informed by the ML emphasis on contexts (South African conception) and the PISA mathematisation process, collection of course data mainly focused on contextualised tasks. A total of 64 question items including related solutions selected from the CLM course assessments have been analysed. This selection was

also driven by this study focus on extra-mathematics and intra-mathematics connections combined with evidence from South African literature which suggests that a focus on contextualised tasks' responses allows for access into the students' understandings relating to extra-mathematics and intra-mathematics connections (Brown & Schäfer, 2006; Vilakazi & Bansilal, 2012). However, this study did not focus on the teachers' performance in terms of marks as this dimension was not found to be useful in relation to the study focus which was concerned with qualitatively exploring intra-mathematical and mathematics-context connections within the teachers' mathematical working.

My primary interest in reviewing these documents was to analyse how they dealt with mathematics content and contexts, key components in the teaching and learning of ML. The data obtained assisted me to answer the first question that sought to link the aims of ML and those of the new CLM course, with specific focus on mathematics content and context engagement. Furthermore, a focus on course assessment tasks allowed me to understand the ways in which the students' understandings of content and contexts in the context-oriented environment of ML fed through into their teaching practice in terms of design/selection and presentations of ML tasks. In this way, the extent to which pre-service ML teachers engaged with the tasks has been analysed and documented. Data on classroom tasks helped me to answer research question three. Here the spectrum of agendas (Graven & Venkat, 2007a), provided me with useful tools to analyse the nature of link between content and contexts, while the PISA mathematisation process helped me to unpack the nature of dealing with mathematical concepts in contexts. A detailed discussion of these frameworks has been provided in chapter two.

4.3.2 Observation and videotaping

All the CLM course lectures were observed and field notes taken from episodes pertinent to the study focus. A 'pertinent episode' in this case was a situation where the ML students were involved in activities related to either solving mathematical tasks or contextual tasks. The field notes were specifically focused on the depth of content coverage and the nature of tasks and mathematical working. In this way, I was able to gain insights into what and how pre-service teachers were learning. This enabled me to understand how resources and practices on the course linked with ML aims. It also provided me with an understanding of how practices in the CLM course aligned with or differed from teaching practices advocated for ML teaching.

Additionally, selected lesson presentations were observed during the student teachers' teaching experience across the two years to explore whether their ML knowledge and understanding related to ML teaching. Lessons were observed in four phases. The first and second phases were done alongside teaching experiences during the B.Ed second year (first and second semester) of the pre-service teachers' studies and the third and fourth phases were done alongside teaching experiences during the B.Ed third year (first and second semester) of studies. Following teachers on teaching experience for two years allowed for a longitudinal focus on the nature of their teaching, which provided a snapshot in terms of their developments in the course.

In both cases, the observations adopted a non-participant approach. The data obtained through observing course lectures and classroom practices, helped me to answer part of the second research question and part of the third research question. Research questions two and three sought interpretations on the kinds of meanings the pre-service teachers were able to make in the CLM course and ways in which tasks were used in the classroom respectively.

Given that one of the foci of this study was to explore the mathematical working of the pre-service teachers during ML teaching including ways in which mathematical solution strategies were communicated to the learners, videotaping of lessons was ideal. Videotaping ensured that I captured even the small details of classroom practices which could easily be taken for granted or forgotten and yet became very useful during analysis. Furthermore, I wanted to capture teacher talk and explanations linked to the tasks and representations or demonstrations that this talk referred to. Since it was not possible to capture the nature of link between explanations about tasks and related representations using an audio recorder, the use of video was justified. I decided to use observation schedules during my first school visits and video recorder during my second school visits of each teaching experience. Both the observation schedule and video recording were focused on the nature of tasks designed/selection for teaching and the nature of mathematical working including the ways how the teachers communicated their solution strategies in the classroom. Video-recording the second lesson was chosen in order to give ML teachers ample time to settle down as some of the participants indicated 'informally' that video-recording lessons before the teachers had settled down was somehow intimidating.

4.3.3 Semi-structured interviews

The purpose of interviews is to gain insight, understandings, and perspectives of the interviewee's own experiences or knowledge on certain issues (Denzin & Lincoln, 2005). In this study, post-lesson interviews were conducted in order to understand the teachers' rationales relating to instructional decisions from the preparation stage to lesson implementation stage. Although the questions were generic across the four teachers, differences existed as individual teachers taught different lessons and used different tasks.

The participants in this study were interviewed in the form of semi-structured interviews. A semi-structured interview schedule⁴ (see Appendix C) helped me to ask focused questions during one-on-one interviews with the participants. The responses were captured using an audio recorder, which allowed for more thorough and repeated examination of interviewees' answers during analysis (Heritage, 1984). Audio-recording interviews also assisted me to present what was said with greater accuracy (Silverman, 2006).

Since data collection spanned two academic years (four semesters), individual interviews were conducted in two phases (after observed lesson and after video lesson) in each semester. The interviews were focused on reflections relating to presented lessons in the schools including justifications of solution strategies. Interview question items relating to justification of particular teaching approaches sought to explore the teachers' understanding of their own practice especially the kinds of pedagogic decisions which are made within the context of ML teaching. The teaching rationales were then linked with pedagogic agendas with the aim of gaining access into ways in which the context/content tension was addressed by the teachers. This was appropriate because teachers needed to utilize instructional tasks which complemented ML teaching and learning. The interview data obtained helped me to answer part of the third research question (reflections of lessons presented including justifications of decisions made). Thus data from interviews provided me with a way of talking about the teachers' learning and practice in depth.

Another methodological issue deserving attention is trustworthiness of research findings. The next section attempts to explain how issues of accountability were addressed in this study.

⁴ It must be noted that due to changing the theoretical lenses, (a decision which was taken after data collection was completed), combined with evidence from empirical data in this study indicating that pre-service ML teachers were given instructional tasks for their teaching, only one question relating to teaching orientations was found to be useful within the analysis.

4.4 Dealing with validity and reliability

Just like in a positivist research, measures to ensure validity and reliability in a qualitative research are critical in ensuring credible findings. Thus the great concern of any research is whether valid and reliable knowledge has been generated in an ethical manner (Merriam, 1998). Both “producers and consumers of research want to be assured that the findings of an investigation are to be believed and trusted” (Merriam, 2002, p.23). The implication is that research studies need to be conducted in a rigorous and systematic manner. But what guarantee can be given to consumers of research with regards to trustworthiness of the findings? The answer to this question lies in the extent to which some accounting for validity and reliability in research report has been clearly articulated (Merriam, 1998). The whole research process including presentation of results, insights, and conclusions must ring true to the readers. In this report, a detailed account of how this study was conducted combined with justifications of key decisions made has been provided. A discussion of how validity and reliability concerns were addressed in this research study follows.

4.4.1 Validity

Validation of findings in this research study occurred throughout the steps in the research process from the management of raw data to interpretations of results (Creswell, 2009). But what is meant by the term validity in qualitative research? According to Hammersley (1990) validity is interpreted as “the extent to which an account accurately represents the social phenomena to which it refers” (p.57). Validity is about ensuring that the study findings are accurate from the standpoint of the researcher, the participant, or the readers of an account (Creswell & Miller, 2000). The notion of validity can also be understood as “how congruent are one’s findings with reality” (Merriam, 2002, p.25). Drawing from Creswell (2009) and Merriam (1998), this study adopted the following strategies to ensure validity of its results.

Triangulation of data sources

In this study, attention was on triangulation using multiple data collection techniques, as this was congruent with the study aims (Denzin & Lincoln, 2008). A discussion of these techniques has been presented in section 4.3 of this chapter and included interviews, observations, video recording, and document analysis. There was an overlap of data sources for each phenomena of interest to ensure multiplicity of evidence for any claims made about phenomena. One of the major strengths of collecting data using multiple methods is that

“what someone tells you can be checked against what you observe in the field visit or what you read or see in documents relevant to the investigation” (Merriam, 2002, p.25). One of my interests in this study was to explore ways in which the CLM course results related with practice results

Member checks

Member checks or responded validation involves taking data and tentative interpretations back to the participants and asking if claims and codes or categories emerging from the data are plausible (Creswell, 2009; Merriam, 1998). In this study, the researcher was engaging participants in member-checks starting from the preliminary stages of data processing, identification of codes or categories, development of themes and analysis of results. In this way the researcher was able to ask the participants to comment on the raw data, codes, themes and interpretations of the data (Merriam, 2002). This was possible because by the time data collection and analysis were done, the participants were still doing their B.Ed degree program at the same institution where the researcher was based. As noted already, I was tutoring this group of students even after the data collection was completed. Due to this convenience in terms of meeting the participants, I was able to chat to them about this research with no difficulty.

Reflectivity

According to Merriam (2002), reflectivity concerns “critical self-reflection by the researcher regarding assumptions, worldview, biases, theoretical orientation, and relationship to the study that may affect the investigation” (p.31). Keeping reports of major decisions made during the entire research process, which were later fed into analysis ensured truthfulness of results in this study. The quality of qualitative research is often judged by the level of commentary by the researcher about “how their interpretation of the findings is shaped by their background, such as their gender, culture, history, and socio-economic origin” (Creswell, 2009, p.192). In this study reflectivity has been achieved through a continuous reference to the researcher’s background and experience, values and assumptions that might have affected data collection and analysis. The fact that the researcher was engaged in data collection and analysis over a period of two years allowed for more reflectivity.

4.4.2 Reliability

Reliability refers to the “degree of consistency with which instances are assigned to the same category by different observers or by the same observer on different occasions” (Hammersley, 1990, p.67). The term reliability has been closely associated with the traditional sense of ‘*replicability*’ within the positivism paradigm. A more relevant term in qualitative research suggested by Lincoln and Guba (1985) is ‘*dependability*’ of results - the notion that puts more emphasis on consistency of the research process as opposed to demanding an outsider to replicate the study and get the same results. This shifts the focus from “whether findings will be found again [to] whether the results are consistent with the data collected” (Merriam, 1998, p.206). But how did this research study ensure consistency in its approaches in exploring development in knowledge and practice? In ensuring consistency and dependability of results in this study, the researcher employed Gibbs (2007) procedures namely;

- Checking transcripts to make sure that they do not contain obvious mistakes made during transcription. Since I transcribed some of the interview and video data, some obvious mistakes were eliminated at this stage.
- Making sure that there was no drift in the definition of categories during analysis. I had an audit trail which contained a detailed description on how data was collected, how categories were derived, and how decisions were made throughout the inquiry (Guba & Lincoln, 1981). This ensured flexibility in this study in terms of defining and redefining codes or categories, and use these to analyse data.
- Triangulation in terms of using multiple methods of data collection in this study strengthened both validity and reliability (Merriam, 1998).

4.5 Analysis of data

The study adopted an interpretive perspective of data analysis, as this was consistent with the study aims focusing on exploring the pre-service teachers’ mathematical working. Due to large volumes of data from different sources which have been highlighted earlier, I decided to do data collection and analysis concurrently. This approach assisted me to check whether my instruments needed to be sharpened before the next phase of data collection across the two years, and this was done while continuously making reference to the research questions. As already highlighted, this research study adopted a case study design with embedded sub-

cases. According to Yin (2009), the preferred method of analysis for multiple case studies is 'cross-case synthesis'. This technique treats each individual case separately before aggregating findings or looking for patterns across the cases.

Drawing from Miles and Huberman (1994) this study adopted a two-phase qualitative data analysis design. According to Miles and Huberman, this design is suitable for within and cross-case analyses of multiple case study research. The strategy involves the following phases;

1. Data reduction
2. Conclusion drawing and verification

I now explain each of these phases and provide a link with this research study process.

4.5.1 Data Reduction

Data reduction entails; simplifying, summarising, coding, and identification of categories and themes (Miles & Huberman, 1994). In this study, data reduction was aimed at ensuring that data were reduced to manageable size before the final analysis. The interview and video data were first transcribed verbatim and the transcribed data were stored as soft copies in the computer. Field and documentation notes were also typed and kept in soft form.

The sub-cases in this study were the individual teachers' accounts related to their learning and practice. I was interested in understanding the nature of meanings constructed by each pre-service teacher in the CLM course as well as understanding their practices in terms of how tasks were implemented in ML classrooms. To make sense of data belonging to individual teachers, I created tables for each teacher that displayed data from the individual cases according to the identified categories or codes (Yin, 2009). The categories, which were tentative at this level, were either grounded or theory/literature – driven. Each category in the tables had corresponding evidence extracted from the interview and video transcript as well as field notes. The choice of the non-grounded categories was informed by both theory and literature whereas the grounded ones were issues of interest emergent from the data which were part of this study focus. Repeating this process for every individual case, I came up with word tables that were slightly different especially in grounded categories. Looking across the word tables of individual cases for patterns, I maintained the recurrent categories and

discarded the others to proceed with analysis. The different pieces of individual cases were then combined in one word table for easy comparison.

4.5.2 Drawing conclusions and verification

This is the interpretation phase where meanings and insights were constructed that provided answers to the research questions. The recurrent categories were turned into themes and relationships between these themes were probed for insights. This phase involved referring back to the raw data at all times to make sure that the interpretations were consistent with the study findings.

4.6 Ethical Considerations

This study ensured that all participants were given details of the proposed study and then invited to participate in the study. Opportunities to give their informed consent before participating in this study were also afforded. Research details included providing access to their marked scripts, classroom observations, voice-recording interviews and video-recordings of selected school lesson presentations. The rationale for seeking informed consent was to protect and respect the right of self-determination of the participants (Cohen, et al., 2000). The right to self-determination also included the right to refuse to take part or to withdraw once the research had begun. Permission to observe lectures was also sought from the course coordinator and all students – including students that did not participate in the study. Consent forms were given to all sampled participants to fill in a manner free of coercion. These consent forms included information on aims and purpose of study, likely publication of findings as well as issues of confidentiality and benefits, risks, dangers if any, involved as a consequence of their participation in the research study. Since this study was on professional teacher development, a research clearance to undertake the research study from the University was applied for and granted before data collection started. The application among other things included explanation of the aims, purpose and benefits of doing the research and how the results would help to inform the professional teacher development in ML. Additionally, the head of the institution where the CLM course was implemented was informed about the study. Clearance from the Gauteng Department of Education (GDE) was also obtained to conduct research from the schools that teachers were assigned to for their teaching experiences throughout the two years of data collection. The schools were not predetermined as they were informed by the choice of the pre-service teachers during their

teaching experiences. I therefore made it clear in the GDE application that the choice of schools was to be made by the pre-service teachers and consequently I was cleared to conduct research in any of the schools in Johannesburg, following student allocations to these schools. Before conducting research at the schools, consent was sought from the School Principals, supervising teachers and learners from the class where data collection would be done.

4.7 Chapter summary

In this chapter, I have provided a discussion about the methodology which underpinned the study. A case study design was adopted with the view to gain an in-depth understanding of the phenomena (Yin, 1994). Driven by the longitudinal study approach where data was collected over an extended period of time, data collection methods included; lesson observations, video data, semi-structured interview data, and document analysis. The focus across these data sets was on the students' mathematical working (i.e. extra-mathematics and intra-mathematics connections). Data analysis adopted an interpretive approach focusing on the individual cases followed by the general overview of all the participants. The next chapter concerns analysing the CLM course data focusing on individual case accounts relating to problem solving.

CHAPTER FIVE: PRE-SERVICE TEACHER LEARNING FOR MATHEMATICAL LITERACY

5.1 Introduction

In this chapter, I present results and discussion focused on responses to ‘mathematics in context’ assessment tasks. The data, which was qualitative in nature, was generated in the context of the CLM course using the methodology and data collection techniques described in chapter four. The organization of the chapter focuses on three components. First, an overview of course tasks, categorized on the basis of openings for aspects of the mathematisation process, and based on competency clusters across 2011 and 2012 has been provided. This is followed by an in-depth analysis of each pre-service teacher’s working in relation to tasks across different competency clusters, and then to aspects of mathematisation process, using selected tasks to illustrate the points. Lastly, I provide a quantitative summary of working across competency clusters and mathematisation process, in order to evaluate development⁵, if any, in relation to elements of problem solving.

A total of 64 assessment question items were classified using PISA competency clusters in this analysis (see Appendix A). These tasks represented a selection of tasks from the three CLM sub-courses CLM1, CLM 2, and Method 2, across 2011 and 2012. The tasks’ selection depended on tasks that were embedded in some world contexts (extra-mathematical but not necessarily authentic), and were amenable to mathematics treatment. The results of this classification indicated that the assessment tasks belonged to either reproduction level or connections level. Of the 64 items, 24 tasks were classified as reproduction level tasks and 40 as connections level items. This meant that assessment tasks at the level of reflection did not feature across the assessment tasks in this analysis.

In summarising pre-service teachers’ working quantitatively, I have considered the frequencies of instances where model formulation, intra-mathematical work and interpretation either featured or did not feature. The focus was on the three elements of the mathematisation process due to the fact that these featured across most tasks. A consideration of the frequencies was aimed at exploring whether pre-service teachers’ working has shown improvements in terms of how they engaged with the tasks across the two years. As noted

⁵ Initially, my interest was to explore development in relation to the pre-service teachers’ performance across the CLM course assessment tasks. However, a focus on different mathematical topics combined with different situations within these assessments made this difficult to do.

earlier, 64 question items were considered, and these items were the same across the four teachers. Of the 64 items, 26 were selected from 2011 assessment and these have been classified (using PISA competency clusters) into reproduction (10 items) and connections (16 items) levels. The remaining items (38) were selected from 2012 assessment and comprised 14 reproduction and 24 connections level items. The fact that it was not feasible to exemplify all the responses to these questions items provided a useful rationale for looking at the frequencies.

At model formulation level, the focus was on whether the translation was correct, incorrect or 'did not feature'. 'Did not feature' (written as 'no feature' in some cases) responses refer to those cases where pre-service teachers' working did not show the translation aspect or the question item did not demand this. In relation to intra-mathematical working, the focus was on whether pre-service teachers' working is coherent, incoherent, or 'did not feature'. The concentration on coherence was consistent with the broad study focus relating to exploring the nature of the teachers' mathematical working. In some cases, the intra-mathematical working was not shown although this aspect could have featured (no feature). These were instances where only the interpretive aspect was provided or the solutions were not given at all. Furthermore, at the interpretation level, whether this aspect was correct, incorrect and 'did not feature' have been examined. At this level, instances where the interpretive aspect was consistent with the mathematical results and the problem situations even if the mathematical results were incorrect, have been coded as correct interpretation. This was the case because the analysis has shown that broadly incorrect mathematical results were a result of incorrect formulations.

5.2 Engagement with assessment tasks within the CLM course

Data in the form of solutions to assessment tasks collected in the first year of the study (2011) were analyzed separately from those collected in the second year of the study (2012). The rationale was to explore and document shifts (if any) in terms of the pre-service teachers' development in knowledge related to both intra-mathematical and content-context connections across tasks at both reproduction and connections levels within the two years (2011-2012).

Engagement with the CLM course assessment tasks involved the pre-service teachers' activation of elementary mathematics knowledge, an aspect which was consistent with the

ML curriculum specifications. Evidence from the South African literature base (Vale, et al., 2012; Vilakazi & Bansilal, 2012) suggests that notions of ‘cognitive demands levels’ and ‘mathematisation process’ are useful within the context of understanding students’ mathematical working by focusing on responses to assessment tasks. While Vilakazi and Bansilal paid attention to students’ performance across examination scripts within the context of an ML re-skilling programme (ACE), Vale and colleagues focused on analyzing FET college students’ errors associated with aspects of mathematisation process (translation and solution processes) as they worked through problems selected from ML Matric examination items. In this study, the mathematisation process was found to be useful due to the overlapping nature of the tasks with those used in the above studies.

The pre-service teachers’ responses to tasks in this study indicate that *validation* of mathematical answers was only foregrounded in a few instances. This observation provided a rationale for combining *interpretation* and *validation* in this analysis since both descriptors are concerned with relating mathematical answers with the problem context. They differ in that interpretation situates the mathematical answer in the problem context whereas validation aims at “evaluating mathematical outcomes and their reasonableness in the context of a real-world-based problem” (OECD, 2013, p.26) or whether the solution is “appropriate” (Blum & Ferri, 2009, p.48) within the problem context. An overview of the competences which form a central feature of analysis in this chapter against tasks’ classification based on cognitive demand levels across 2011 and 2012 is provided in table 5.1.

Elements of mathematisation	Reproduction tasks 2011 (10 items)	Connections tasks 2011 (16 items)	Reproduction tasks 2012 (14 items)	Connections tasks 2012 (24 items)
Model formulation	✓	✓	✓	✓
Story creation	X	✓	x	X
Intra-mathematical work	✓	✓	✓	✓
Interpretation/ Validation	✓	✓	✓	✓
Pedagogic links	✓	✓	✓	✓

Table 5.1: Competencies examined across tasks in 2011 and 2012

This table shows occurrences relating to translation (model formulation, story creation, interpretation), and intra-mathematical working, and how they featured across responses to assessment tasks at both cognitive demand levels. This supports an earlier argument (see chapter four) that a focus on contextualized assessment tasks within the course may provide access to students’ understandings related to both intra-mathematical and mathematics-

context connections. I now present the results for each of the four pre-service teachers; Lindiwe, Mark, Jabu, and Lebo (pseudonyms). Since my intention was to explore the pre-service teachers' ways of problem solving, I have focused on both the 'presences' and the 'absences' (with emphasis on the presences) within the pre-service teachers' responses to the assessment tasks.

Within the individual participants' accounts, I have presented exemplifications of a selection of tasks and related responses in tables. The examples were selected purposively and were informed by the need to provide evidence relating to claims regarding the categories drawn from the mathematisation process. The responses to tasks which were not included in the tables were similar with the selected tasks. It is important to note that although the same categories were maintained across the four participants, the evidence supporting similar claims could be drawn from either the same tasks (with different responses) or different tasks.

5.3 Lindiwe's mathematical working

5.3.1 Mathematisation of tasks across 2011 academic year

Table 5.2 shows examples relating to Lindiwe's problem solving in 2011. The discussion that follows makes reference to the examples in this table.

Task	Lindiwe's solution
Reproduction level tasks	
Rp1: A person has $29\frac{1}{2}$ metres of material available to make doll's dresses. Each dress requires $\frac{3}{4}$ metre of material. a) How many dresses can be made?	$29\frac{1}{2} \div \frac{3}{4} = \frac{59}{2} \times \frac{4}{3} = \frac{236}{6}$ $= \frac{118}{3} = 39\frac{1}{3}$
b) How much material will be left over	$29\frac{1}{2} - 39\frac{1}{3} = \frac{59}{2} - \frac{118}{3} = \frac{177}{6} - \frac{236}{6}$ $= \frac{59}{6} = 9\frac{5}{6}$
Rp2: I have 8,2m of material. I need 0,4m of material to make doll's dress. a) How many complete dresses can I make from the material?	8,2m material $8,2m \div 0,4m$ $= 20 \text{ dresses}$
b) How much material will I have left over?	$\frac{1}{2} - 0,4 = 0,1m$
Rp3: I have $\frac{3}{4}$ litres of milk in the fridge. I drink $\frac{1}{3}$ of it. How much milk (in litres) do I have left?	$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ $= 0.42 \text{ litres}$
Rp4: A recipe for a full pot of stew requires that I use $\frac{4}{5}$ of a cup of beef stock. I only want to make $\frac{1}{2}$ of a pot of stew. How much beef stock do I need?	$\frac{1}{2} \times \frac{4}{5}$ $= \frac{2}{5}$

Rp5: The price of a shirt is reduced from R350 to R280. By what percentage has the price of the shirt been reduced?	Percentage decrease $= \frac{\text{initial value} - \text{final value}}{\text{initial value}}$ $= \frac{R350 - R280}{R350} \times 100 = 20\%$
Rp6: John and Jane both currently earn R10 000 per month. a) John performs badly in this job so is demoted and will earn 9% less from next month onwards. How much will he earn?	$R10000 (1 - 9\%)$ $= R9100$
b) Last month Jane was actually earning less than R10 000 and she received a raise of 9% which brought her salary up to R10 000. What was she earning last month?	$R10000 = P(1 + 9\%)$ $P = R10000 \div (1 + 9\%)$ $P = 9174,31$
Connections level tasks	
Cn1: A factory A manufactures candles One worker can make 60 candles in a day. Factory B makes glass candle holders. One worker can make 18 glass candle holders in a day. The factory owners decide to collaborate and so want to make the same number of glass holders as candles each day. What is the smallest number of candle-makers factory A can employ and the smallest number of holder-makers factory B can employ so that they can do this?	The smallest number that makers of factory A can employ is 3 people as 3 people will be able to make 180 candles. Makers of factory B can employ the smallest amount of 10 people so they can make the same amount of glass candle holders a day.
Cn2: Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a picture to explain how you got your solution.	 Anna gives Buhle half. The [shaded] part is the half given to Buhle. Buhle's half now cut into 3 equal pieces, she gives Anna 1 of her 3 pieces
Cn4: Create a story problem for $4,5 \div 0,75$.	I have 4,5 metres of ribbon. I want to cut 0,75 metres long strips. How many strips can I cut?
Cn5: Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.	Jane has borrowed R2 from Sue in 3 days. How much debt is Jane in now with Sue? Jane owes Sue R6.
Cn6: Lynn says it will take her $\frac{1}{2}$ of a day to mark all the assignments. Mark says it will take him $\frac{1}{4}$ of a day to mark all assignments. If they work together to mark the assignments, how quickly will they be able to mark the assignments? (you can assume they each keep up the same pace as they would working alone).	$\text{Lynn } \frac{1}{2} \text{ of day} = \left(\frac{1}{2} \times 24\right) = 12 \text{ hours}$ $\text{Mark } \frac{1}{4} \text{ of day} = \left(\frac{1}{4} \times 24\right) = 6 \text{ hours}$ $\text{Together} = 18 \text{ hours} \div 2$ $= \text{it will take them 9 hours}$
Cn8: I have 150 exams to mark. I mark $\frac{1}{2}$ of them. I persuade a friend to mark $\frac{1}{3}$ of what I have left. How many do I have left to mark?	$150 \times \frac{1}{2} = 75 ; 75 \times \frac{1}{3} = 25$ $150 - 75 - 25 = 50. \text{ Left to mark}$
Cn9: Buhle invested money at a bank that paid 8% annual	$P = ? ; A = R4118,36 ; r = 8\% (\div 4) =$

interest compounded quarterly. If she had R4118,36 in her account at the end of 4 years, what was her initial investment	$2\% ; n=4\text{years} (x4)=16$ $A = P(1 + i)^n$ $R4118,36 = ? (1 + 2\%)^{16}$ $P = R4118,36 \div (1 + 2\%)^{16}$ $P=R3000$									
Cn11: a) In order to make strawberry milkshake the instructions tell me I must mix $\frac{2}{3}$ of a cup of milk with $\frac{1}{5}$ of a cup of strawberry syrup. If I want to make 10 cups of milkshake, how many cups of milk and how many cups of syrup will I need?	<table border="1"><tr><td>Milk</td><td>2/3</td><td>7,69</td></tr><tr><td>St. syrup</td><td>1/5</td><td>2,31</td></tr><tr><td>milkshake</td><td>13/15</td><td>10</td></tr></table> $\frac{2}{3} \times 11,54 = 7,69$ cups milk $\frac{1}{5} \times 11,54 = 2,31$ cup syrup $\frac{13}{15} \times 11,54 = 10$	Milk	2/3	7,69	St. syrup	1/5	2,31	milkshake	13/15	10
Milk	2/3	7,69								
St. syrup	1/5	2,31								
milkshake	13/15	10								
Cn12: At Pizzaz, the pizza with a 10cm radius costs R30. The pizza with a 15cm radius costs R45. Which is the better deal or is there no difference? Explain fully and clearly why you say so.	There is a difference a 10cm radius to a 15cm radius has a 5cm difference meaning 15cm is bigger than 10cm. the better deal is the 15cm radius pizza is better as the pizza is bigger									
Cn13: You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?	$R85000(1 - 10\%)^3 = R61965$									
Cn14: I spend $\frac{1}{2}$ of my salary on rent and $\frac{1}{5}$ of what I have left on groceries. What fraction of my salary is left for the rest of my expenses?	$\frac{1}{2} - \frac{1}{5} = \frac{5}{10} - \frac{2}{10}$ $= \frac{3}{10}$									

Table 5.2: Lindiwe's responses to assessment tasks in 2011

Model formulation

Results at the level of model formulation indicate that both correct and incorrect formulations featured across Lindiwe's solutions. In cases where correct formulations were observed, Lindiwe's solutions were characterized by correct identification and selection of measurable quantities, followed by correct choices of operation(s). There were also instances where formulae were used in setting up procedures, a step which was often followed by substitution of the contextual quantities. However, incorrect formulations have been observed in some cases. Some of the instances where correct formulations featured, included Rp1a, Rp4, Rp5, Rp6a, Rp6b, Cn8, Cn9, and Cn13.

Across these instances, variations in terms of formulations have been noted, and these included; use of formula, use of diagram, use of table, and direct formulations. In cases where formulas were used, the results suggest that selecting an appropriate formula for the situation

was key. Choosing an appropriate mathematical formula is linked to mathematics content understandings in connected ways (Stillman, 2012; Vale, et al., 2012). In some instances (see Cn9), the translation was preceded by the listing of the selected quantities, referred to as annotation by Hall and colleagues (1989). Annotation was found to be useful in ML problem solving as it provided an understanding relating to ways in which the students' thinking processes were played out during the setting up of procedures. The contexts where formulae were used across the examples appear to be associated with finance (see Rp5, Rp6, Cn9, and Cn13), suggesting that Lindiwe was familiar with these kinds of contexts. Use of diagram (i.e. Cn2) and use of a table (i.e. Cn11a) were also utilized by Lindiwe to set up procedures, although these techniques were observed in fewer cases. Further, direct formulations, where selected quantities from the contexts were directly transformed into a mathematical statement with the help of operation(s), were also noted across Lindiwe's working (i.e. Rp1a, Rp4, and Cn8).

Despite evidence of correct formulations across Lindiwe's working in 2011, some incorrect formulations were also noted. The errors relating to model formulation were interesting to explore in this study since ML emphasizes engagement with contextualized tasks, and correct translation of contextual information becomes central to successful problem-solving. The examples (Rp1b, Rp2b, Rp3, Cn6, and Cn14) provided evidence of scenarios where Lindiwe had difficulties setting up mathematical procedures. While quantities were incorrectly selected in Rp1b, the results in Rp2b, Rp3, Cn6, and Cn14 suggest that correct quantities were identified and selected, but these were followed by incorrect operations. As noted earlier, comprehension of contextual language is key to selecting the correct mathematical operations (Bernardo, 1999; Koedinger & Nathan, 2004). These operations (i.e. subtraction) were linked to the language in the context which suggests the 'take away' notion, especially in responses relating to tasks Rp1b, Rp3 and Cn14. The choices of operations across these examples suggest difficulties relating to linking contextual language and mathematical language. Evidence from ML literature in South Africa points towards difficulties relating to translating contexts into mathematical language within the context of problem solving (Vale, et al., 2012). Lindiwe's understanding therefore resonates with everyday use of the ideas of 'left over' (Rp1b), 'drinking' (Rp3), and 'spending' (Cn14) which are linked to 'taking away', and thus suggests subtraction. Lindiwe's model formulation exhibits errors at the level of both reproduction and connections level tasks.

Story creation

As already noted, the assessment tasks which demanded attachment of contexts to mathematics models/statements only featured in the CLM 1 content course in 2011. Lindiwe's engagement with these tasks (Cn4 and Cn5) suggests that she had no difficulties identifying suitable world stories for the models. The mathematical meanings within the statements were maintained in the stories. The choices of the quantities within the stories (i.e. 4,5m of ribbon and borrowing R2) support the view that these were personal level-focused tasks. As already noted, engagement with these kinds of tasks allowed for the pre-service ML teachers' development of content-context and intra-connection skills.

Intra-mathematical work

Lindiwe's solutions in most instances show that she was strong in terms of intra-mathematical working across tasks in 2011. The setting up of procedures was often followed by correct execution of these procedures in terms of logic, calculations, and manipulation of the mathematical statements as illustrated in examples Rp5, Rp6a, Rp6b, Cn2, Cn8, Cn9, and Cn11a. The examples show some coherent vertical working from the formulated models. In some cases, where solving equations was involved (Rp6b, Cn9), Lindiwe was able to logically work out the problems, an aspect which is central to ML problem solving and was linked to intra-mathematical connections (Vilakazi & Bansilal, 2012). Lindiwe's working also suggests that summarizing contextual information into a table could be useful in terms of solving problems. In task Cn11a for instance, her vertical working started in the table followed by the selection of some results to complete the enactment of the procedure. Further, task Cn2 exemplifies Lindiwe providing a diagrammatically correct representation of the answer. However, this quantity was not given in numerical terms as demanded in the problem context. In this example, a procedure accompanied by correct explanations was provided but this procedure was not brought to its mathematical conclusion. Failure to complete a procedure might suggest breakdown in enacting multi-step methods (Hall, et al., 1989), a central feature of connections level tasks.

Lindiwe's intra-mathematical working also indicated that coherent working was achieved even in cases where models were incorrectly formulated. In these cases, her working logically followed from formulated models as shown in tasks Rp3, Cn6, and Cn14. The coherent working achieved across these examples suggests that incorrect mathematical results were not necessarily a result of incoherent intra-mathematical working, but rather a consequence of incorrectly formulated models. This suggests that competencies relating to

model formulation, an aspect of ‘extra-mathematics knowledge’, are important to develop her ML-related skills.

One case (Rp1b), was observed across Lindiwe’s intra-mathematical working where incoherent working featured.

Rp1b	$29\frac{1}{2} - 39\frac{1}{3} = \frac{59}{2} - \frac{118}{3} = \frac{177}{6} - \frac{236}{6} = \frac{59}{6} = 9\frac{5}{6}$
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The solution to task Rp1b, which was preceded by incorrect model formulation, shows that a bigger rational number was subtracted from a smaller rational number. Given that coherent working was achieved in many cases across 2011, obtaining a negative answer here was more likely. However, Lindiwe’s working suggests that the negative sign has been dropped at the level of the final mathematical result. This may be to a realization that a positive number would be more suitable for the answer and that negative numbers would not make sense in the context.

Interpretation and validation of mathematical answers

Across 2011, Lindiwe’s responses to tasks show that she was able to interpret the mathematical answers to reflect the context. It should be understood that in some cases the tasks did not specifically ask for the interpretive aspect. Some of the examples relating to how mathematical answers were interpreted are illustrated in tasks Rp2a, Rp3, Cn6, and Cn8. The interpretive aspect, where it featured across 2011, was consistent with the given contexts. The examples referred to above indicate that the interpretive aspect was preceded by some intra-mathematical working. In this way, the understanding of both the mathematical result obtained and the context informed the nature of translation. However, evidence from Lindiwe’s working also shows that translation in the form of interpretation was given in situations where intra-mathematical working was not provided (see Cn1 and Cn12). Lindiwe’s response to Cn1 shows that the interpretation was correct and consistent with the problem situation. This explanation suggests that it was informed by some intra-mathematical working which was not provided. The other case (Cn12) exemplifies a situation where Lindiwe did not appropriately engage with the context mathematically. The response in this example suggests that she struggled with formulation of an appropriate model whose mathematical result could have informed the correct interpretive aspect. Further, Lindiwe did not seem to pay attention to the interpretation aspect in Rp1a where the answer was given as $39\frac{1}{3}$. Since the question demanded the answer in terms of ‘number of dresses’, the final result

would be a whole number (i.e. 39). However, a similar task (see Rp2a), whose mathematical answer was 20.5 suggests that the interpretive aspect was considered. It is important to note that Rp1a was included in an assessment at the beginning of semester 1 in 2011 whereas Rp2 was part of an assessment which was given to the pre-service teachers at the end of this semester. This suggests a shift to successful interpretation in this period.

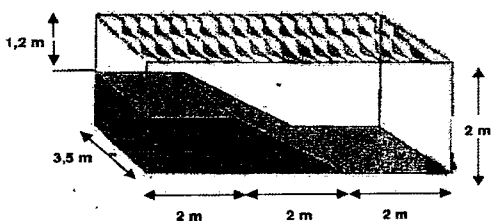
Pedagogic links

In few cases, Lindiwe included details and explanations within procedures in ways that provided links with pedagogy. Task Cn2 and Cn12 have a feature of this link. However, most procedures show aspects of less unpacking (Hill, et al., 2008), suggesting a weak pedagogic link.

5.3.2 Mathematisation of tasks across 2012 academic year

Presented in table 5.3 are examples of Lindiwe's responses to assessment tasks in 2012 which have been used for reference purposes in the following discussion.

Task	Lindiwe's solution
Reproduction level tasks	
Rp7: A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day. a) If the Company has one six-man team on the job, how long would it take to complete the contract?	Assuming they work 27 days of the month as the 4 days are Sundays. We then formulate $27 \times 20m = 540m$ first month. Therefore, we can take 540 and divide it by 3000m to get an answer of months and days $3000m \div 540m = 5.56$ months. As we know we can't use 5,56 months. We can say $(27 \times 20) \times 5 = 2700m$, 300m short. Therefore $300m \div 20m = 15$ days. It would take 5 months 15 days.
b) (i) How many teams must they put on the job if they have to get the contract finished in one day	$\begin{array}{r} 6 \times 20 \\ x : 3000 \\ 6 \times 3000 = 18000 \\ 20 \times x = 20x \\ \frac{18000}{20} = \frac{20x}{20} \\ 900 = x \text{ workers for 1 day} \end{array}$
Rp8: Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month.	$R6075 \times 3,5\% = R212,625$ Therefore $R6075 + R212,625 = R6287,625$ Nadia's new salary

a) Determine Nadia's new salary per month.	
b) Who received the greater percentage increase? Show your working	Sekuru: $\frac{R259,86}{R8000} \times 100 = 3,2\%$ Nadia: Got 3,5% increase. Therefore Nadia got the greater percentage
Rp13: The scale on a map is 1:35 000. If the distance between two towns on the map is 2,75cm, determine the actual distance between the towns in kilometres.	Scale 1:35000 $2,75 \times 35000 = 96250cm \approx 9,625km$
Connections level tasks	
Cn16: Nombuso went to a supermarket on Saturday 10th March, 2012. She wanted to buy chicken portions for a family of three. She found out that a 2 kg packet of mixed portions cost R31.99 and a 5 kg packet of the same type cost R89.99. Which one is a better deal in terms of money saving? Show all your working.	Family of 3 R31,99; R89,99; 2kg:R31,99; 5kg: R89,99 Therefore $(R89,99 \times 2) \div 5 = R35,996$ $(R31,99 \times 5) \div 2 = R79,975$ Would be better to buy 5kg as its cheaper than buying 2kg
Cn18: Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years.	Simple Interest $= P \times i \times n = R6000 \times 7,2\% \times 2 \text{ years} = R864$ Therefore $R6000 + R864 = R6864$ Will be received
a). Calculate the amount he will receive at the end of the period from Bank A	
b). Now calculate the amount he will receive at the end of the period from Bank B.	Compound Interest $= P(1 + i)^n = R6000(1 + 5,4\%)^2$ $= R6679,46$
Cn19: Jane and Tom plan to install a sloping pool in their back garden. A sketch of the pool is shown below. The length of the pool is 6 m and its width is 3,5m. The depth of the water in the shallow end is 1,2m and 2m deep in the deep end.	$Volume = l \times b \times h$ $= 4m \times 3,5m \times 0,8m$ $= 11,2m^3$
	
a). Calculate the volume of the raised cemented portion at the shallow end of the pool.	
b). Hence, determine the volume of water, in litres, required to fill the pool to the top. (NOTE: 1 000 litres = 1 m ³ .)	$V = l \times b \times h$ $= 6m \times 3,5m \times 2m = 42m^3$ $42m^3 \times 1000l = 42000 l \text{ per } m^3$
c). Jane and Tom are planning to put up a security fence, one metre away from the edges of the pool.	$Area = l \times b = 6m \times 3,5m = 21m^2$ Therefore 1 meter away

The fence will be right around the pool. Determine how many metres of fencing Jane and Tom would need to buy.	$(6 + 1) 7m \times (3,5 + 1)4,5m = 31,5 \text{ meters}$. Need to buy 32 meters of fencing
Cn20: Volume of Sound Model is given by; $L = 10.\log\left(\frac{I}{10^{-12}}\right)$. Here the volume L is measured in decibels (db) and I is the intensity in watts per square meter (W/m^2). a). An alarm has an intensity of $5.8 \times 10^{-9} W/m^2$. How loud is the alarm in decibels?	$10.\log\left(\frac{I}{10^{-12}}\right)$ $= 10.\log\left(\frac{1}{5.8 \times 10^{-9}}\right)$ $= 8.24 \text{ db}$
Cn25: This is the sign in a lift at an office block. THIS LIFT CAN CARRY UP TO 12 PEOPLE a) In a morning rush, 265 people want to go up the lift. How many times must it go up?	$265 \div 12 = 22,1$ The lift can go up 23 times as if 12 people can fit in then the 264 person will go up the lift on their own.
b) What are the possible errors associated with the mathematical answer which learners can make when answering this question? Why?	Learners can assume that since the answer is 22,1 that means the lift can go up that many of times forgetting that a lift cannot go 22,1. It will have to go 23 times to include the last individual
Cn29: The diagram below (not drawn to scale) is a plan of Sandile's flat which they are planning to redecorate. a) All the ceilings are to be painted with 2 coats of white paint. Each litre of paint will cover 10 m^2 of ceiling. How much paint will she need to paint the ceilings?	$5m \times 2m \times 3m \times 1,5m \times 4m \times 4,5m \times 2,5m = 2025m^2$ Therefore $2025m^2 \div 10m^2 = 202,50$ $202,50 \times 2 \text{ coats}$ $405l \text{ of paints is needed}$

Table 5.3: Lindiwe's responses to assessment tasks in 2012

Model formulation

As in 2011, the 2012 results across reproduction and connections level tasks show both correct and incorrect formulations. In cases which involved Lindiwe choosing formulas, three phenomena were noted. First, correct choices of formula were followed by substitution involving correct contextual quantities (Rp8b, Cn18a). Second, correct choices of formulas

were followed by substitution involving incorrect quantities (Cn18b, Cn19a, and Cn19b). In Cn18b, Lindiwe's formulation suggests that she chose the correct formula for compound interest and appeared to struggle with translating the contextual information where interest was charged quarterly across two years, into the formula. However, this points to a possibility of a 'slip' as engagement with a similar task (see Cn9, table 5.2) in 2011 was correct. Incorrect selection of quantities in Cn19a and Cn19b appeared to be a result of failure to understand the different shapes in the given figure. Third, the choice of inappropriate formulae followed by substitution of incorrect quantities into these formulae (Cn19c and Cn29a). In Cn19c for instance, selecting an area formula instead of perimeter formula combined with the translation of length and width of the pool to length and width of fencing, were incorrect. Unlike studies which have shown that errors associated with model formulation are largely a consequence of language comprehension difficulties (Vale et al, 2012), Lindiwe's responses especially to Cn19 and Cn29a suggest that inability to relate shapes of figures with corresponding formulas could affect the accuracy of solutions. These results mean that Lindiwe's formulation of models was weak at connections level tasks in 2012 especially within the area and perimeter topic areas.

Further, cases where formulae were provided in the question, followed by incorrect substitution, were noted. In Cn20a, the problem solver was required to identify and select contextual quantities needed to set up the procedure using a given formula. It involved substituting the intensity 'I' for a quantity given in the context. In this case Lindiwe's working suggests that instead of substituting 'I' for the value given in the contexts she resorted to replacing the denominator 10^{-12} by the intensity $I = 5.8 \times 10^{-9}$. Using given formulas within problem solving has been emphasised in ML curriculum.

Another interesting aspect within Lindiwe's working was the utilization of informal strategies in engaging with tasks. In Rp7a, for instance, her solution shows that the number of working days in a week was clearly defined (i.e. 27 days in a month).

Rp7: A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day. a) If the Company has one six-man team on the job, how long would it take to complete the contract?	Assuming they work 27 days of the month as the 4 days are Sundays. We then formulate $27 \times 20m = 540m$ first month. Therefore, we can take 540 and divide it by 3000m to get an answer of months and days $3000m \div 540m = 5.56$ months. As we know we can't use 5,56 months. We can say $(27 \times 20) \times 5 = 2700m$, 300m short. Therefore $300m \div$
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	20m = 15 days. It would take 5 months 15 days.
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This definition seemed to have formed the basis for her argument leading to a correct answer. Use of informal ways of problem solving has been noted as one of the useful means of engaging with contexts within the ML curriculum (Department of Education, 2003), a view which supports the citizenship orientation, in the sense that no ‘pencil’ and ‘paper’ are often used to solve everyday situational problems. However, Lindiwe’s working relating to this task (Rp7a) shows that reference is made to dividing a smaller number by a bigger number, (*‘we can take 540 and divide it by 3000’*), although the subsequent written mathematical statement was correctly presented. Despite this disruption, she was able to provide an explanation for her steps, which is pedagogically useful. Further, in one of the responses to 2011 assessment tasks (see Rp1b), a similar disruption was observed involving subtracting a bigger number from a smaller number in her mathematical statement, at model formulation level, a scenario which resulted in obtaining a positive answer. Pedagogically, these kinds of errors have the potential of disrupting meaningful learning.

Intra-mathematical work

Across 2012, Lindiwe’s vertical working suggests that she was able to employ appropriate mathematical tools to correctly work out the problems. Instances relating to her intra-mathematical working leading into correct mathematical results have been exemplified in table 5.3 (see Rp7a Rp7b, Rp8a, Rp8b, Rp13, Cn16, Cn17a, and Cn25). In order to obtain a correct mathematical answer, formulating a correct model is key within the context of problem solving (Stillman, 2012). The examples which have been referred to above show cases where intra-mathematical working was preceded by correct formulations. The results suggest that her working was logical and coherent with respect to formulated mathematical models.

Despite instances where Lindiwe’s responses were both mathematically and contextually correct, incorrect answers have also been observed in 2012 (see Cn18b, Cn19a, Cn19b, Cn20a, and Cn29a). Evidence from the examples show that Lindiwe’s intra-mathematical working was coherent and consistent with the formulated models. Given that the vertical working logically followed from the models, it suggests that Lindiwe’s difficulty was at the level of formulation (translation) and was also located within connections level tasks. While in some cases the errors occur at substitution level (Cn18b, Cn19a, Cn20a, Cn29a) (cases

where formulas are used), other instances (see Cn19b) suggest that Lindiwe misunderstood the problem, an aspect which may have contributed to non-completion of the procedure. In this example (Cn19b), Lindiwe was able to calculate volume of the whole figure, but could not recognize that the space occupied by the ‘raised cemented portion’ would not be filled with water. These results agree with findings relating to her intra-mathematical working in 2011 and therefore imply the need to emphasize the model formulation aspect of the mathematisation process when teaching problem solving.

Interpretation and validation of mathematical answers

Mathematical answers across 2012 were largely interpreted and validated in the context of situations (Rp7a, Rp8a, Rp8b, Cn16, and Cn19c). Further, the interpretation featured even in situations where the mathematical answers were incorrect in relation to the original context (see Cn19 and Cn29). Across these cases, the interpretive aspect was consistent with the obtained mathematical answers and the original problem situations.

There were instances where Lindiwe’s interpretations were incorrect and not consistent with the mathematical working. For example, both the mathematical working and the mathematical results in Cn16 were correct but these were followed by an error within the interpretation. The results also suggest that both incorrect interpretations and correct interpretations preceded by incorrect mathematical working were related to tasks at connections level. Unlike in 2011 where the featuring of this aspect was not preceded by any mathematical working in some cases, the interpretations in 2012 were often drawn from the obtained mathematical results. It is also important to note that Lindiwe could not provide solutions to some of the tasks, provided in appendix A.

Pedagogical links

Like in 2011, two forms of pedagogic links were noted in 2012. First, provision of explanations and detailed steps within procedures (i.e. Rp7a and Rp7bi). Second, connecting mathematical result with some aspects of teaching and learning (i.e. Cn25b). Across these examples, Lindiwe was able to demonstrate skills relating to pedagogy. However, her working in other instances suggest weak pedagogic links.

5.3.3 Quantitative summary of Lindiwe's mathematical working

As noted earlier, this study looked at occurrences of aspects of mathematisation process in the form of frequencies to explore shifts relating to problem solving competences. The table showing the frequencies and percentages of these occurrences across Lindiwe's working is given below (table 5.4). The frequencies have been converted into percentages in order to compare Lindiwe's performance across both cognitive levels of tasks over the two years.

			2011 academic year (26 question items; 10 reproduction and 16 connections level items)		2012 academic year (38 question items; 14 reproduction and 24 connections level items)	
Elements of mathematisation	Cognitive levels	Performance	Frequency 2011	Percentage (%) 2011	Frequency 2012	Percentage (%) 2012
Model formulation	Reproduction level	Correct	7 /10	70	7/14	50
		Incorrect	2/10	20	3/14	21
		No feature	1/10	10	4/14	29
	Connections level	Correct	9/16	56	9/24	38
		Incorrect	4/16	25	7/24	29
		No feature	3/16	19	8/24	33
Intra-mathematical working	Reproduction level	Coherent	9/10	90	10/14	71
		Incoherent	1/10	10	0/14	0
		No feature	0/10	0	4/14	29
	Connections level	Coherent	13/16	81	16/24	67
		Incoherent	0/16	0	0/24	0
		No feature	3/16	19	8/24	33
Interpretation	Reproduction level	Correct	7/10	70	11/14	79
		Incorrect	0/10	0	0/14	0
		No feature	3/10	30	3/14	21
	Connections level	Correct	10/16	63	15/24	63
		Incorrect	0/16	0	2/24	8
		No feature	6/16	37	7/24	29

Table 5.4: Frequency table showing Lindiwe's performance across tasks

Model formulation

The table shows that Lindiwe was able to correctly translate more question items in 2011 than in 2012 at both reproduction and connections levels. While 70% of the responses to reproduction items were correct in 2011, only 50% in 2012 were correct at this level. At connections level, 56% formulations were correct in 2011 as opposed to 38% in 2012 at the

same level. As already noted at the level of evidence, incorrect formulations featured more at connections level than at reproduction level, especially in 2012. As noted earlier, errors associated with formulations concern choosing operations and substituting contextual quantities into formulas. However, the errors at this level were not linked with any specific strands or topics, but rather featured across the whole range of different tasks. Given different topics, this suggests ‘localised’ translation competence in 2011 topics, and weaker translation competence in 2012 topics. Literature relating to mathematics problem solving is replete with evidence suggesting occurrence of errors at model formulation level (Clarkson, 1991a; Hall, et al., 1989; Maat & Zakaria, 2010). Related findings can be found in Vale and colleagues (Vale, et al., 2012) paper within ML problem solving, although their results suggest that more errors are attributed to inaccurate mathematics calculations (intra-mathematical working) than model formulation. The implication is that Lindiwe’s competences relating to model formulation show no improvement across the two years (2011-2012).

Intra-mathematical working

At intra-mathematical level, more coherent working has been observed in 2011 than in 2012 across both reproduction (from 90% in 2011 to 71% in 2012) and connections (from 81% in 2011 to 67% in 2012) levels. Largely, these are cases where formulations were either correct or incorrect, an aspect which was consistent with the results at model formulation level. As noted already, coherence appeared to feature even in cases where formulations were incorrect, as the vertical working followed logically from the formulated models – thus contradicting findings by Vale and colleagues (2012). Although some of these incorrect models could be easier than the correct ones, Lindiwe was able to follow through the procedures. The results also suggest that if formulations were correct in these cases, coherence could still be achieved and correct mathematical results could be obtained. This implies that Lindiwe’s competence relating to intra-mathematical working was relatively strong, despite reductions in percentages from 2011 to 2012. However, the results have indicated that incorrect mathematical results were a result of incorrect formulations and not necessarily due to disruptions within the vertical working.

Interpretation of mathematical answers

Results have shown that the interpretive aspect was broadly stable across 2011 and 2012. In 2012, 79% of the interpretations at reproduction level were correct, as opposed to 70% in 2011. At connections level, 63% of the interpretations were correct across the two years.

Again, my interest was in whether the interpretive aspect was consistent with both the mathematical result and the problem context even if the preceding mathematical result was incorrect, in relation to the original problem context. These results imply that Lindiwe's competence relating to interpretation of mathematical results showed no marked improvement across the two years. Furthermore, the results indicate that the strength of this aspect was not specific to particular strands or topics although finance contexts were more prevalent.

The summaries therefore indicate some stable performance across reproduction and connections tasks in all the three PISA categories – a result which agrees with the qualitative analysis.

5.4 Mark's mathematical work

5.4.1 Mathematisation of tasks across 2011 academic year

Examples for Mark which have been referred to within the discussion that follows are detailed in table 5.5.

Task	Mark's solution
Reproduction level tasks	
Rp1: A person has $29\frac{1}{2}$ metres of material available to make doll's dresses. Each dress requires $\frac{3}{4}$ metre of material. a) How many dresses can be made?	$\frac{3}{4} \times 29\frac{1}{2} = 22,125$ Therefore 22 dresses can be made
b) How much material will be left over	$22 \div 0,75 = 29,3$ $29,3 - 29 = 0,3 \times 0,75 = 0,25$ Therefore 0,25 m will be left over
Rp2: I have 8,2m of material. I need 0,4m of material to make doll's dress. a) How many complete dresses can I make from the material?	$8,2m \div 0,4 = 20,5$ Therefore 20 complete dresses
b) How much material will I have left over?	$0,5 \times 0,4 = 0,2m$ Of material left over
Rp3: I have $\frac{3}{4}$ litres of milk in the fridge. I drink $\frac{1}{3}$ of it. How much milk (in litres) do I have left?	$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ litres of milk Therefore 416,67 ml = 0,42 litres
Rp4: A recipe for a full pot of stew requires that I use $\frac{4}{5}$ of a cup of beef stock. I only want to make $\frac{1}{2}$ of a pot of stew. How much beef stock do I need?	$\frac{4}{5} \div 2 = \frac{2}{5}$ of a cup of beef stock
Rp5: The price of a shirt is reduced from R350 to R280. By what percentage has the price of the shirt been reduced?	R350 to R280 % decrease = $\frac{\text{initial} - \text{final}}{\text{initial}} \times 100$

	$= \frac{350-280}{350} \times 100$, 20% decrease
Rp6: John and Jane both currently earn R10 000 per month. a) John performs badly in this job so is demoted and will earn 9% less from next month onwards. How much will he earn?	9% of 10000=R900 10000-900=R9100 Therefore he will now earn R9100
Connections level tasks	
Cn1: A factory A manufactures candles One worker can make 60 candles in a day. Factory B makes glass candle holders. One worker can make 18 glass candle holders in a day. The factory owners decide to collaborate and so want to make the same number of glass holders as candles each day. What is the smallest number of candle-makers factory A can employ and the smallest number of holder-makers factory B can employ so that they can do this?	LCM (60; 18) 18; 36; 54; 72; 90; 108; 126; 144; 162; 180 60; 120; 180 Therefore A must employ 3 candle-makers. Factory B must employ 10 holder-makers.
Cn2: Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a picture to explain how you got your solution.	 Therefore Rashad got $\frac{1}{6}$ of the chocolate bar
Cn4: Create a story problem for $4,5 \div 0,75$.	You have 4,5 km of road. There is a stop street every 0,75 km. How many stop streets will there be?
Cn5: Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.	Rory is playing at the Ned bank Golf challenge. On the first 3 holes he shoots a score of 2 under par. What is his score at the end of the first three?
Cn6: Lynn says it will take her $\frac{1}{2}$ of a day to mark all the assignments. Mark says it will take him $\frac{1}{4}$ of a day to mark all assignments. If they work together to mark the assignments, how quickly will they be able to mark the assignments? (you can assume they each keep up the same pace as they would working alone.)	12 hours for Lynn 6 hours for Mark Therefore $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0,125$ of a day. Therefore $0,125 \times 24 = 3$ hours if they work together
Cn8: I have 150 exams to mark. I mark $\frac{1}{2}$ of them. I persuade a friend to mark $\frac{1}{3}$ of what I have left. How many do I have left to mark?	$\frac{1}{2} \times 150 = 75$ $\frac{1}{3} \times 75 = 25$ 75-25=50, You have to mark 50
Cn9: Buhle invested money at a bank that paid 8% annual interest compounded quarterly. If she had R4118,36 in her account at the end of 4 years, what was her initial investment	$A = P(1 + i)^n$; A=4118,36 $P=?$; $= \frac{8\%}{4}$; $n = 4 \times 4$ $4118,36 = P(1 + 0,02)^{16}$ $4118,36 = P(1,02)^{16}$

	$4118,36 = P(1,372785705)$ $\frac{4118,36}{1,372785705} = P$ Therefore $P = R3000,00$. Therefore she initially invested R3000,00
Cn10: My daughter wants to paint her bedroom pink. I mixed 3 tins of red paint with 5 tins of white paint and she says the pink it makes is perfect. I figure we need about 12 tins of paint to paint her bedroom. a) If I add 2 tins of white paint and 2 tins of red paint to the perfect pink mix will it be too red, too white or still perfect?	$\frac{3red}{3} : \frac{5white}{3} ; 1:1,666$ $\frac{5red}{5} : \frac{7white}{5} ; 1:1,4$ Therefore the mixture will have less white in it, making the mixture too red
Cn12: At Pizzaz, the pizza with a 10cm radius costs R30. The pizza with a 15cm radius costs R45. Which is the better deal or is there no difference? Explain fully and clearly why you say so.	$10cm=30; 15cm=45$ $\frac{10}{10} : \frac{30}{10} = R3/cm$ $\frac{15}{15} : \frac{45}{15} = R3/cm$ There is no difference in price because both the 10cm and 15cm pizzas cost R3 per cm. thus no matter how large the pizza will be provided it costs R3 per cm, there will be no difference in price.
Cn13: You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?	$85000; 10\%/year; 3 \text{ years}$ $A = P(1 - i)^n$ $= 85000(1 - 0,1)^3$ $= R61965 \text{ at the end of 3 years}$
Cn14: I spend $\frac{1}{2}$ of my salary on rent and $\frac{1}{5}$ of what I have left on groceries. What fraction of my salary is left for the rest of my expenses?	$\frac{1}{2} \times \frac{1}{5}$ $= \frac{1}{10} \text{ is left}$

Table 5.5: Mark's responses to assessment tasks in 2011

Model formulation

Mark's competence related to model formulation in 2011 across both reproduction and connections level tasks, appears to be generally strong (see Rp2a, Rp2b, Rp4, Rp5, Rp6a, Cn8, Cn9, Cn13). Like Lindiwe, formulae were used across both reproduction and connections level question items to set up procedures. Although formulae are generally provided in ML assessments (Department of Basic Education, 2013), Mark's responses indicate that he was able to select the appropriate formulae (see Rp5, Cn9, and Cn13), as these were not provided within the CLM assessment. Like Lindiwe, Mark's results suggest competent selection of the formulae over a wide range of different contexts. In instances

where Mark either utilized formulas or employed ‘direct’ formulations, the results show that contextual quantities were listed before they are substituted into the formulae, an aspect referred to as annotation (Hall, et al., 1989). At the level of professional teacher development, listing quantities within model formulation appears to provide a window into the students’ understandings of the problem contexts, in relation to the questions posed, and suggest a useful methodology for teaching ML (Department of Education, 2003).

The results also indicate some instances of incorrect model formulations in other cases (see Rp1a, Rp1b, Rp3, Cn6, and Cn12). These examples indicate that Mark correctly identified and selected contextual quantities but this was followed by incorrect choice of operations, across both reproduction and connections level question items. These errors occurred within the context of working with fractions. Like Lindiwe, Mark appeared to relate the phrase ‘I drink $\frac{1}{3}$ of it’ (i.e. Rp3), with the idea of ‘taking away’ in everyday language, suggesting subtraction. However, Mark was able to formulate a model for task Cn14 (although this formulation was incomplete) unlike Lindiwe who treated this task like Rp3. Regarding task Cn12, Mark’s formulation suggests that his reasoning was somewhat restricted to the basic arithmetic operations (+, −, ÷, ×) which in this case appears to be less useful, an aspect which also characterized Lindiwe’s engagement with this task. As already noted, errors relating to choosing operations are linked to difficulties in terms of establishing the interrelationship between contextual or everyday language and mathematical language, a key component within contextualized mathematics problem solving (Stillman, 2012).

Story creation

With regards to story creation, Mark’s responses indicate that world stories were correctly chosen and were consistent with the meanings contained in the original mathematical models or statements. Two tasks involving ‘story creation’ have been exemplified in table 5.5 (see Cn4 and Cn5). The responses provided suggest that Mark was able to make sense of the quantities included in the mathematical statements and how these quantities were interrelated. In addition, the examples indicate Mark’s familiarity with the chosen contexts provided in the mathematics statements. Like Lindiwe, Mark’s stories related to situations affecting the individual person. The ability to draw stories from a range of different situations suggests familiarity and confidence with contexts, one of the key skills needed within ML teaching and problem solving.

Intra-mathematical work

Mark's responses characterized by coherent working preceded by correct model formulation, are exemplified in table 5.5 (see Rp2a, Rp2b, Rp4, Rp5, Rp6a, Cn1, Cn2, Cn8, and Cn13). The responses in 2011 appear to exhibit coherent intra-mathematical working across both reproduction and connections level tasks. This meant that the model formulation step was largely followed by logical vertical working where mathematical language and symbols were employed (Freudenthal, 1991; van den Heuvel-Panhuizen, 2001, 2003). The results also provided evidence of horizontal mathematisation approaches to problem solving (Gravemeijer, 1994b). Task Cn1 for instance exemplifies a solution procedure relating to finding the lowest common multiple (LCM) involving listing of numbers in the form of sequences whose common differences were 18 and 60 respectively. Given that this strategy did not feature within problem solving in the CLM course, Mark's working suggests some level of understanding of the problem context. Visual representation was another feature within Mark's intra-mathematical working. Task Cn2 provides evidence relating to Mark's ability in terms of solving a problem using a diagram, although it does not show the reasoning process used to get the result. Lindiwe's engagement with this task showed that details relating to reasoning process were provided but she could not provide a mathematical conclusion, an aspect which was addressed by Mark. Showing reasoning process within solution procedures is pedagogically useful especially within the context of professional teacher development. Further, in some cases, Mark's working shows that he was able to work with formulas. Competence across different ways of representing solutions, points towards Mark's strength in knowledge relating to intra-mathematical connections (Mousoulides, et al., 2007).

Coherent problem solving was also been noted in 2011 across cases where incorrect formulations featured (see Rp1a, Rp3, and Cn6). Across these examples, coherent vertical working was achieved although the mathematical answers were incorrect. As in Lindiwe's case, the incorrect mathematical results appeared to be a result of incorrect model formulation, especially at the level of choosing operations. These results overlap with Lindiwe's working and therefore suggest the need for both students' knowledge development relating to translating contextual language into mathematical language, with a focus on choosing operations.

Interpretation and validation of mathematical answers

Mark's mathematical working shows that he was able to translate the mathematical answers in relation to the contexts across both reproduction and connections level tasks. In many instances across his responses, interpretation competences were apparent (Rp1a, Rp1b, Rp2a, Rp2b, Rp3, Rp6a, Cn2, and Cn10a). This interpretation included introduction of units to mathematical results (Rp3), and dealing with fractional results (Rp1a, Rp1b, and Rp2a) among others. These examples show that at both reproduction and connections levels of tasks, the interpretive aspect was consistent with both the mathematical results and the problem situations, even across cases where the mathematical results were incorrect (see Rp1a, Rp1b, and Rp3). Unlike Lindiwe, whose engagement with task Cn2 did not provide a numerical quantity to summarize her visual solution representation, Mark appears to be able to give this numerical value within the interpretive aspect.

Pedagogic links

Mark appears to provide solutions showing less pedagogic explanations in most cases (Rp1a, Rp5, Rp6a, Cn2, and Cn13). Task Cn2 for example indicate a correct solution with less detail in terms of his reasoning process, employed to get to the result, unlike Lindiwe who provided step-by-step explanations.

Cn2: Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a picture to explain how you got your solution.	 Therefore Rashad got $\frac{1}{6}$ of the chocolate bar
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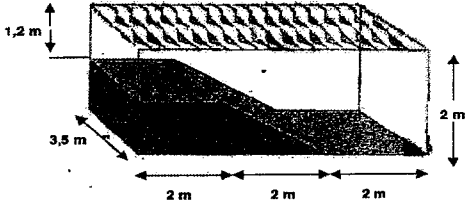
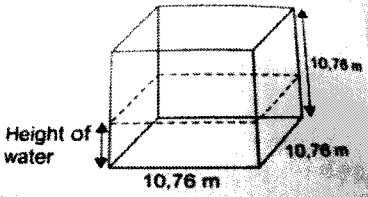
This exemplifies mathematical working involving less 'unpacking' of mathematical concepts (Hill, et al., 2008).

5.4.2 Mathematisation of tasks across 2012 academic year

Table 5.6 provides a selection of examples relating to Mark's problem-solving in 2012.

Task	Mark's solution
Reproduction level tasks	
Rp7: A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day a) If the Company has one six-man team on the job, how long would it take to complete the contract?	3000m; 6 men, 20m/day $3000 \div 20 = 150$ days

b) How many teams must they put on the job if they have to get the contract finished i) in one day	1 team=150 days ? = 1 day $3000 \div 150 = 20$ Therefore you will need 150 teams																		
Rp8: Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month. a) Determine Nadia's new salary per month.	$103,5\% \times R6075 = R6287,63$ per month																		
Rp12: The table below shows two sets of ML method test scores. <table><tr><td>Test A</td><td>50</td><td>70</td><td>50</td><td>50</td><td>60</td><td>80</td><td>100</td><td>100</td></tr><tr><td>Test B</td><td>40</td><td>62</td><td>64</td><td>72</td><td>70</td><td>68</td><td>68</td><td>68</td></tr></table> Calculate the mean for each set of the test scores.	Test A	50	70	50	50	60	80	100	100	Test B	40	62	64	72	70	68	68	68	Test A $\frac{50+70+50+50+60+80+100+100}{8} = 70$ Test B $\frac{40+62+64+72+70+68+68+68}{8} = 64$
Test A	50	70	50	50	60	80	100	100											
Test B	40	62	64	72	70	68	68	68											
Rp14: The only sports offered at Burg High School are soccer and netball. The principal loves soccer so he allocates the sports budget so that for every R2 spent on netball, R3 will be spent on soccer. b) If the school gets R8000 to spend on sport, how much will be allocated to netball?	R8000 total $\frac{8000}{5} = 1600$ $1600 \times 2 = 32000$ Therefore R32000 will be allocated to netball																		
Rp15: If we start with a principal of P Rands then the amount A in an account after t years, with an annual interest rate r compounded continuously, is given by: $A = Pe^{rt}$. If R5000 is deposited and earn $4\frac{1}{4}\%$ compounded continuously then how much will be accumulated at the end of a 3 year period?	$P=5000, r = \frac{4,25}{100}, t=3$ $A = Pe^{rt}$ $A = 5000e^{0,0425 \times 3}$ $A = R5679,92$ at the end of 3 years																		
Connections level tasks																			
Cn15: One of your learners in a Mathematical Literacy classroom wants to buy a cell phone with internet. The learner has seen the advertisement (see attached page) and needs an advice from you on choosing a better deal. Help the learner and justify your thinking. Nokia X2-01 Valued at R1549, starter pack included <u>TopUp 99 Phone Contract</u> Get R100 airtime p.m. for calls and data. 1000 free SMSs p.m. R99 P.M. x 24 MP3 player, FM radio, OVI store, Push E-mail and instant messaging, web browsing Terms and conditions apply Web Exclusive Deals  Deals are valid until 31 March, 2012 Samsung Galaxy Mini Valued at R2299, starter pack included <u>TopUp 199 Phone Contract</u> Get R200 airtime p.m. for calls and data. 1000 free SMSs p.m. R199 P.M. x 24 HSDPA 7.2 - Wi-Fi Android OS 2.2 Froyo. Push E-mail Terms and conditions apply	Nokia = R1549; $99 \times 24 = 2376$ Galaxy Mini=R2299; $199 \times 24 = 4776$ Mp3, instant messaging, 1000 sms, wi-fi. I would suggest the Nokia. The deal is cheaper. Teenagers don't phone much thus the 1000 sms's is good. The phone has mp3 player which will appeal to a teenager, the phone also looks good. Thus I recommend the Nokia X2-01.																		
Cn17: A loaf of bread is a regular purchase for many	$A = P(1 - i)^n$																		

families. If a loaf of bread costs R7.24 and that the cost of the loaf has risen by the average inflation rate of 4,5% in the last 20 years. Find how much a loaf of bread would cost 20 years ago.	$= 7,24(1 - 0,045)^{20}$ $= R2,88$ Therefore, 20 years ago a loaf of bread would have cost R2,88
Cn18: Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years. b): Now calculate the amount he will receive at the end of the period from Bank B.	$A = P(1 + i)^n$ $i = 7,2\% = 0,072 \div 4 = 0,018; n = 2 \times 4 = 8$ $A = 6000(1 + 0,018)^8$ $= R6920,44$
Cn19: Jane and Tom plan to install a sloping pool in their back garden. A sketch of the pool is shown below. The length of the pool is 6 m and its width is 3,5 m. The depth of the water in the shallow end is 1,2 m and 2 m deep in the deep end.  a). Calculate the volume of the raised cemented portion at the shallow end of the pool.	Volume of the rectangular prism $= l \times b \times h$ $= 2 \times 3,5 \times 0,8$ $= 5,6 \text{ m}^3$ Volume of triangular prism $= \frac{1}{2} b \times h \times w$ $= 1 \times 0,8 \times 3,5$ $= 2,8 \text{ m}^3$ Therefore, $5,6 + 2,8 = 8,4 \text{ m}^3$
c). Jane and Tom are planning to put up a security fence, one metre away from the edges of the pool. The fence will be right around the pool. Determine how many metres of fencing Jane and Tom would need to buy.	Perimeter of pool + area for fencing $= 2(3,5 + 1 + 1) + 2(6 + 1 + 1)$ $= 27 \text{ m of fencing will be needed}$
Cn24: The figure below shows a cube-shaped tank. The tank contains 500 kilolitres of water, what is the height of the water in the tank? [$1 \text{ m}^3 = 1 \text{ kl}$] 	$10,76 \times 10,76 \times 10,76 = 1245,77 \text{ m}^3 = 1245,77 \text{ kl}$ Therefore, $\frac{500}{1245,77} \times 100 = 40,14\%$ Therefore, 40,14% of 10,76 $= 4,32$ Therefore, $10,76 \times 10,76 \times 4,32 = 500$. Therefore the height of the water is 4,32m
Cn25: This is the sign in a lift at an office block. THIS LIFT CAN CARRY UP TO 12 PEOPLE a) In a morning rush, 265 people want to go up the lift. How many times must it go up?	$\frac{265}{12} = 22,083$ $22 \times 12 = 264$ $265 - 264 = 1$ Therefore the lift must go up 23 times
b) What are the possible errors associated with the mathematical answer which learners can make when	People will assume that the 0,083 does not mean anything and will therefore

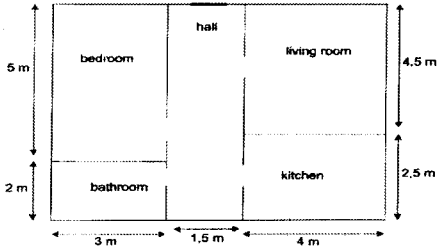
answering this question? Why?	exclude that person from the findings. Also people may assume that because it is only one person, they may squeeze into the 22 nd journey
Cn29: The diagram below (not drawn to scale) is a plan of Sandile's flat which they are planning to redecorate.  a) All the ceilings are to be painted with 2 coats of white paint. Each litre of paint will cover 10 m² of ceiling. How much paint will she need to paint the ceilings?	$7 \times 8,5 = 59,5m^2 \times 2 = 119m^2$ Therefore $119 \div 10 = 11,9$ She will need 12 litres of paint
b) Is your answer an exact, underestimation or overestimation? Give a reason for your argument.	Overestimation. 11 litres will not quite cover the whole amount thus you will buy more than what you need to get the job done, so you will have 0,1 litres left over, therefore it is an overestimation

Table 5.6: Mark's responses to assessment tasks in 2012

Model formulation

At both reproduction and connections level tasks in 2012, Mark appears to demonstrate competence and confidence in terms of setting up procedures, illustrated in Rp8a, Rp12, Rp15, Cn19a, and Cn24. Like in 2011, Mark employed formulation approaches involving direct translation and formulae in 2012. Task Rp8 for instance was concerned with percentage increase which was similar to task Rp6 given in 2011 (table 5.5), but ways in which the translations played out across the two tasks appear to be different. Translation in task Rp8 suggests a deeper understanding of the idea of percentage increase as 3,55% is added to 100%, a formulation which offers a more direct approach of obtaining the mathematical result.

Furthermore, Mark's formulation relating to tasks Cn19a, Cn19c, Cn24, and Cn29 indicates understandings of solving problems involving area and perimeter. In task Cn19, Mark was able to recognize that the cemented portion was comprised of both rectangular and triangular prisms before appropriate formulae were chosen and used in the procedure set up. The related formulation therefore suggests that identification of these two shapes involved some

construction. Engagement with Cn24 at model formulation level involved using the idea of percentage in order to set up a procedure for finding the height of water in the tank. Lindiwe, in contrast, was unable to correctly formulate models relating to area and perimeter. As in 2011, Mark included annotations in his problem solving in 2012 (Hall, et al., 1989). Although the idea of including annotations featured in CLM course, as already noted, Lindiwe's working did not specifically show this aspect. The results therefore suggest that Mark's knowledge at the level of model formulation was relatively strong in 2012.

Mark was unable, in some cases, to set up procedures in 2012 especially at connections level, although the errors at this level only featured in a few cases (Cn17 and 18b). While contextual quantities and formulae were correctly selected in Cn17, an incorrect quantity (i.e. 7.2% instead of 5.4%) was selected in 18b. However, in both cases, further disruptions have been noted at the level of substitution and these appear to be 'slips' as working with similar tasks in 2011 showed that Mark was able to successfully engage with these kinds of tasks at model formulation level. Task Cn17 was similar to 'car depreciation task' (Cn13, table 5.5) and Cn18 was similar to 'investment task' (Cn9, table 5.5). These slips were associated with substituting quantities into either depreciation or compound interest formulas. Unlike in 2011 where incorrect formulation were also observed at the level of reproductions tasks and involved choosing operations, these results suggest that Mark's skills relating to model formulation were stronger in 2012.

Intra-mathematical work

In 2012, Mark's skills related to intra-mathematical working were strong with attention to detail when enacting procedures. The procedures were also logical and coherent as illustrated in Rp7a, Rp7bi, Rp8a, Rp12, Cn18b, Cn19a, Cn24, and Cn29a. Mark was able to utilize multi-step methods, especially at connections level, across the examples (OECD, 2006, 2013). Within responses to tasks Cn19 and Cn24, Mark appears to demonstrate an understanding relating to three dimensional objects. His understanding especially related to task Cn19 was evident in the ways in which he used the idea of construction (volume of raised cemented portion), an aspect which was hidden in the problem situation. In task Cn24, Mark used the idea of percentage to calculate the height of the water in the tank. In doing this, he ensured that the units were the same before working out the percentage, suggesting his understanding of the problem. While the strategy used in task Cn19 was emphasized

within the CLM course, the vertical procedure utilized in task Cn24 appears to be a product of Mark's own problem solving.

Example Cn17 exemplifies coherent working within situations where models were incorrectly formulated. This feature was also a characteristic of Lindiwe's problem solving across 2011 and 2012 as well as Mark's mathematical working in 2011. This example provided information suggesting that coherence was achieved across some tasks despite evidence of incorrect mathematics models constraining the correctness of the mathematical answers. This means that Mark would have obtained some correct mathematical results if the formulations had been correct given the appropriateness with which he engaged with these problems intra-mathematically.

Interpretation and validation of mathematical answers

In terms of the interpretive aspect, Mark's responses indicate that this aspect of the translation process was strong and consistent with both the mathematical answers and the features of the contexts across 2012. Examples of solutions, where the interpretive aspect featured across 2012, included Rp7a, Rp7bi, Rp14b, Cn17, Cn24, Cn29a, and Cn29b. Like his own working in 2011, and Lindiwe's case, the interpretive aspect featured correctly in situations where answers were incorrect (i.e. Cn17). However, the interpretive aspect in Cn29b was generic and did not reflect Sandile's flat in terms of considering areas covered by elected walls. Although the response to Cn29b show that Mark was able to figure out that the answer was an overestimation, the commentary supporting this decision ignored the realistic consideration where the ceiling did not cover certain areas (i.e. where walls demarcating rooms are elected). This consideration would suggest that the answer was indeed an overestimation but with a wider margin than the 0.1 litre mentioned in Mark's explanation.

Pedagogical links

Task Cn25 provided in table 5.6 shows an example of tasks which demanded linking problem solving results with ways in which learners think in terms of translating these answers. Unlike Lindiwe whose explanation focused on learners and their ways of reasoning around translating a mathematical result, Mark's answer did not make specific reference to learners but problem solvers in broad sense. However the two responses (from Mark and Lindiwe) point towards interpretation, which incorporates realistic considerations, being in conflict with basic mathematics rules relating to 'rounding up' or 'rounding down'. Further, more

‘efficient’ or ‘direct’ working has been noted across Mark’s working, linking it to the idea of ‘compression’ that is valued in mathematics. In contrast Lindiwe’s working included explanations, an aspect linked to ‘unpacking’, which is an important pedagogic skill (Hill, et al., 2008).

5.4.3 Quantitative summary of Mark’s mathematical working

Mark’s mathematical working is summarized in table 5.7. Like in Lindiwe’s case, the focus was on model formulation, intra-mathematical work, and interpretation. The rationale for doing so has already been highlighted in earlier sections in this chapter.

			2011 academic year (26 question items; 10 reproduction and 16 connections level items)		2012 academic year (38 question items; 14 reproduction and 24 connections level items)	
Elements of mathematisation	Cognitive levels	Performance	Frequency 2011	Percentage (%) 2011	Frequency 2012	Percentage (%) 2012
Model formulation	Reproduction level	Correct	5/10	50	11/14	79
		Incorrect	4/10	40	2/14	14
		No feature	1/10	10	1/14	7
	Connections level	Correct	10/16	62	19/24	79
		Incorrect	3/16	19	1/24	4
		No feature	3/16	19	4/24	17
Intra-mathematical working	Reproduction level	Coherent	9/10	90	13/14	93
		Incoherent	0	0	0	0
		No feature	1/10	10	1/14	7
	Connections level	Coherent	13/16	81	20/24	83
		Incoherent	0	0	0	0
		No feature	3/16	19	4/24	17
Interpretation	Reproduction level	Correct	8/10	80	11/14	79
		Incorrect	1/10	10	1/14	7
		No feature	1/10	10	2/14	14
	Connections level	Correct	12/16	75	23/24	96
		Incorrect	1/16	6	0	0
		No feature	3/16	19	1/24	4

Table 5.7: Frequency table showing Mark’s performance across tasks

Model formulation

Results from the table indicate that there were more instances where model formulation was correct in 2012 than 2011, at both reproduction and connections level items. For instance, at reproduction level items, 79% of the formulations were correct in 2012 compared to 50% in 2011. Across connections level items, the results indicate that only 62% of the formulations were correct in 2011 compared to 79% in 2012. Thus more correct formulations were observed at reproduction level. Furthermore, the results show a reduction in the number of incorrect formulation across both reproduction (from 40% in 2011 to 14% in 2012) and connections items (from 19% in 2011 to 4% in 2012) – suggesting more reduction at reproduction level than connections level. Given that there were more connections level items in 2012 which appeared to require multi-step methods, Mark's responses suggest growth in performance in terms of how models were formulated across the two years.

Intra-mathematical working

Mark's competences relating to the solution process suggest that his intra-mathematical working slightly improved across 2011 and 2012. As already noted, by coherence, I refer to adherence to logic and mathematical rules within the context of doing calculations, manipulation of symbols, leading to obtaining a mathematical result. The table shows that 93 % of the solution procedures in 2012 as opposed to 90% in 2011 achieved coherence at reproduction level items. Mark's performance at connections level tasks indicate that 83% of the solution procedures in 2012 compared to 81% in 2011 exhibited coherence, again suggesting a small shift. There were no cases of incoherent vertical working across the two years as coherence featured even in cases where mathematical models were incorrectly formulated, as noted at the level of evidence. Unlike the case of Lindiwe, where no improvement in performance was observed at this level, Mark's intra-mathematical working shows a small improvement in 2012.

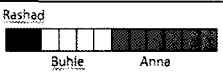
Interpretation of mathematical answers

Overall, Mark's skills relating to interpretation of mathematical results improved at connections level tasks and stabilised at reproduction level. The results indicate that across reproduction items, 79% of the responses in 2012 as opposed to 80% in 2011 included some form of interpretation. However, of the responses relating to connections level items, 96% in 2012 had a feature of the interpretive aspect compared to 75% in 2011. Although an upward

trend in percentages has not been observed across reproduction level items from 2011 to 2012 (i.e. from 80% to 79%), the large improvement (from 75% to 96%) at connections level suggests that Mark's skills had broadly developed further.

5.5 Jabu's mathematical work

5.5.1 Mathematisation of tasks across 2011 academic year

Task	Jabu's solution
Reproduction level tasks	
Rp1: A person has $29\frac{1}{2}$ metres of material available to make doll's dresses. Each dress requires $\frac{3}{4}$ metre of material a) How many dresses can be made?	$29\frac{1}{2} \div \frac{3}{4} = 29\frac{1}{2} \times \frac{4}{3} = \frac{59}{2} \times \frac{4}{3} = \frac{236}{6}$ $= 39\frac{1}{3}$ 39 dresses
b) How much material will be left over	$\frac{3}{4} - \frac{1}{3} = \frac{2}{12}$ $39\frac{1}{3} - 39 \text{ dresses} = \frac{1}{3}$
Rp2: I have 8,2m of material. I need 0,4m of material to make doll's dress. a) How many complete dresses can I make from the material?	$8.2m \div 0,4 = 20,5$ You can make 20 complete dresses
b) How much material will I have left over?	$\frac{1}{2} \text{ of } 0,4 = \frac{1}{2} \times 0,4 = 0,2$
Rp3: I have $\frac{3}{4}$ litres of milk in the fridge. I drink $\frac{1}{3}$ of it. How much milk (in litres) do I have left?	$\frac{3}{4} - \frac{1}{3} = \frac{2}{12}$
Rp4: A recipe for a full pot of stew requires that I use $\frac{4}{5}$ of a cup of beef stock. I only want to make $\frac{1}{2}$ of a pot of stew. How much beef stock do I need?	$\frac{1}{2} \text{ of } \frac{4}{5} = \frac{1}{2} \times \frac{4}{5} = \frac{2}{5}$
Rp5: The price of a shirt is reduced from R350 to R280. By what percentage has the price of the shirt been reduced?	$\frac{280}{350} \times 100 = 80\%$ it has been reduced by 80%
Rp6: John and Jane both currently earn R10 000 per month. a) John performs badly in this job so is demoted and will earn 9% less from next month onwards. How much will he earn?	$\frac{91}{100} \times 10000 = R9100$
Connections level tasks	
Cn2: Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a picture to explain how you got your solution.	 Anna had chocolate with 12 pieces, and she gave half $\frac{1}{2}$ to Buhle of which is 6 pieces of the total 12 pieces. Buhle gave Rashad her friend two pieces of which is a third $\frac{1}{3}$ of the half and $\frac{1}{6}$ of the total chocolate.

Cn4: Create a story problem for $4,5 \div 0,75$.	My car has 4,5 litres of petrol and for every kilometer it consumes 0,75 l, how many kilometers will it take me to run out of petrol.
Cn5: Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.	If I have a bank balance of 0.00 and my mother promises to send me money on my account, if I go to the bank to check if the money is in yet, and every time I check my account the bank charged me R2 which become a negative balance because I do not have any money in the bank/bank overdraft. If I check my balance 3 times my bank overdraft will be -R6 because I have been charged by the 3 times R2 which can be expressed as $3 \times (-2) = -6$
Cn6: Lynn says it will take her $\frac{1}{2}$ of a day to mark all the assignments. Mark says it will take him $\frac{1}{4}$ of a day to mark all assignments. If they work together to mark the assignments, how quickly will they be able to mark the assignments? (you can assume they each keep up the same pace as they would working alone)	$\frac{1}{2}$ of 12 hours = 6 hours $\frac{1}{4}$ of 12 hours = 3 hours $\frac{6}{12} - \frac{3}{12} = \frac{9}{12}$
Cn8: I have 150 exams to mark. I mark $\frac{1}{2}$ of them. I persuade a friend to mark $\frac{1}{3}$ of what I have left. How many do I have left to mark?	$\frac{150}{2} = 75$ $= \frac{1}{3} \times 75 = 25$ $= 75 - 25 = 50$ left to mark
Cn9: Buhle invested money at a bank that paid 8% annual interest compounded quarterly. If she had R4118,36 in her account at the end of 4 years, what was her initial investment	$8\% = \frac{8}{100} = 0,08$ $\frac{0,08}{16} = 0,005$ Therefore $\frac{R4118,36}{0,005} = 823672$
Cn10: My daughter wants to paint her bedroom pink. I mixed 3 tins of red paint with 5 tins of white paint and she says the pink it makes is perfect. I figure we need about 12 tins of paint to paint her bedroom. a) If I add 2 tins of white paint and 2 tins of red paint to the perfect pink mix will it be too red, too white or still perfect?	It would be too white
Cn13: You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?	$\frac{10}{100} \times 85000 = 8500$ $85000 - 8500 = R76500$ year 1 $\frac{10}{100} \times R76500 = 7650$ $\frac{90}{100} \times 76500 = 68850$ year 2

	$\frac{90}{100} \times R68850 = R61965$ year 3. The value of the car after 3 years will be R61965
Cn14: I spend $\frac{1}{2}$ of my salary on rent and $\frac{1}{5}$ of what I have left on groceries. What fraction of my salary is left for the rest of my expenses?	$\frac{1}{2} - \frac{1}{5} = \frac{5}{10} - \frac{2}{10} = \frac{3}{10}$

Table 5.8: Jabu's responses to assessment tasks in 2011

Model formulation

Instances where correct formulations featured across Jabu's working in 2011 regarding both reproduction and connections level tasks included, Rp1a, Rp2a, Rp2b, Rp4, Cn2, Cn8, and Cn13. Like Lindiwe and Mark, Jabu was, in many instances, able to correctly translate contextual information into some models. However, Jabu's formulation relating to task Cn13 appears to be different from the other two cases. In this example, Jabu focused on employing an iterative technique where model formulation occurred at three levels based on the number of years. Across Lindiwe and Mark's working, a depreciation formula was utilized to solve this problem, and only involved fewer steps. Although both approaches seem to be correct, Jabu's strategy appears to be pedagogically useful, as it provided details regarding the steps in which the actual value of the car at the end of each year was reflected, across three years. This is in sharp contrast with the strategy that utilized the formula as only the value of the car at the end of the three years was given.

Some incorrect formulations also featured in 2011 (Rp1b, Rp3, Cn6, Cn9, Cn14). Tasks Rp3 and Cn14 were also translated in similar ways by Lindiwe and Mark, despite similar tasks featuring within CLM course enactment, with subtraction used instead of a multiplication operation. Like the others, Jabu was unable to formulate a correct model for task Cn6. The examples show that Jabu had problems working with fractions especially at the level of both model formulation and simplification. Errors relating to simplifying the fractions have been discussed at intra-mathematical level later in this section. Furthermore, although Jabu appears to struggle to set up a procedure for task Cn9, results have indicated that Lindiwe and Mark were able to choose a compound interest formula for this task. Given that an iterative technique was employed in a similar case involving depreciation (see Cn13), Jabu's working suggests that his understanding of this technique was disconnected, as both tasks were part of the same assessment.

Story creation

Two tasks involving attaching stories to mathematical models (statements) are exemplified in table 5.8 (i.e. Cn4 and Cn5). These examples indicate that Jabu had no problems engaging with tasks involving creation of stories. The situations from which Jabu's stories were drawn overlapped with Mark and Lindiwe's stories in that they appear to affect individuals at some personal level. The variations in choices of contexts, implies diversity in terms of the familiarity of situations across the individual students. Another similarity relates to the nature of the stories 'created' as they seem to contain the exact quantitative information given in the models, rather than the 'messier' superfluous information that Steen (2001) describes as common in everyday life.

Intra-mathematical work

Some examples showing Jabu's coherent vertical working have been provided in table 5.8 (see Rp1a, Rp2a, Rp2b, Rp6, Cn2, Cn8, and Cn13). These examples show that Jabu was able to correctly engage with task Rp2b which involved working with fractions, although he failed to solve a similar task (see Rp1b). The fact that Rp1b was assessed prior to Rp2a may provide an explanation for this; suggesting interim learning. Diagrammatic representation of solutions was another feature across Jabu's examples (e.g. Cn2). In this example, Jabu was able to connect mathematical example to a particular chocolate bar with 12 pieces, and used the answer to derive the fractional answer. Relatively, he provided more 'pedagogic' explanations of steps than Mark, and also provided a numerical quantity as an answer unlike Lindiwe where her solution was not brought to its mathematical conclusion. In terms of task Cn13, Jabu's iterative technique indicates that the two mathematical statements within the first step were combined in the subsequent steps where 10% was subtracted from 100% to obtain some percentage (90%) representing the new value, an aspect which can also be observed within his working relating to task Rp6a. This implies confidence in terms of working flexibly with percentage increase or decrease.

With Lindiwe and Mark, I highlighted instances where coherent working was preceded by incorrect model formulations, with incorrect mathematical results obtained. Within Jabu's working, despite incorrect model formulation, incoherent working was also observed (Rp3, Cn6, and Cn14). Examples show Jabu's inability to engage with situations at an intra-mathematical level, especially involving fractions. His working highlights disruptions at the level of model formulation as well as within calculations. Jabu was also unable to solve

problems involving subtraction of fractions within cases where denominators were either different (Rp3, Cn14) or the same (Cn6). This implies gaps in mathematical understandings relating to working with fractions. As already noted across Lindiwe and Mark, coherent working featured in these cases although similar errors occurred at the level of model formulation.

Interpretation and validation of mathematical answers

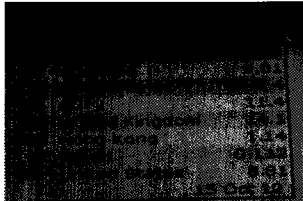
For Lindiwe and Mark, the interpretive aspect appears to be largely preceded by some mathematical results. Across these instances, the mathematical results and the problem situations informed the nature of interpretations. Some cases across Jabu's working overlap with these kinds of solution presentations. However, other cases show that the interpretive aspect was provided without any mathematical working (see Cn10). His response in this example suggests that it was not informed by any mathematical working as the given option was proposed in the question. This may suggest failure to engage with the problem mathematically.

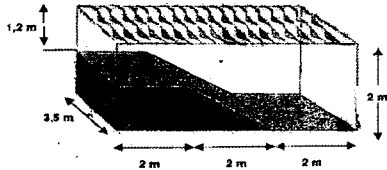
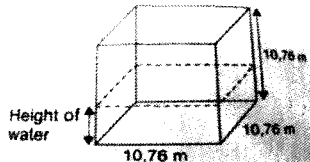
Pedagogic links

Despite more cases exhibiting less or no explanations relating to Jabu's reasoning leading to mathematical results, some procedures indicate that explanations (i.e. Cn2) and detailed working (i.e. Cn13) were provided. His working in Cn2 for instance, was similar to Lindiwe in that the diagrammatic representation of the solution was accompanied by some 'unpacking', contrasting Mark's working in this task.

5.5.2 Mathematisation of tasks across 2012 academic year

Task	Jabu's solution
Reproduction level tasks	
Rp7: A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day a) If the Company has one six-man team on the job, how long would it take to complete the contract?	$\frac{3000}{20} = 150$ It would take them 150 days to complete the contract
Rp8: Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month. a) Determine Nadia's new salary per month.	$\frac{103,5}{100} \times R6075$ $= R6287,63$

b) Who received the greater percentage increase? Show your working	$\frac{R259,86}{R8000} \times 100 = 3,2\%$ Nadia got a greater percentage increase because she got 3,5% increase while Sekuru got about 3,2% increase																		
Rp9: A table showing exchange rates as of 15 th September 2012 is given below.	$R50000 \times 0,112 = 5600 \text{ Yen}$																		
 Kabelo is travelling to Japan within the next five days for a business trip, how much Japanese Yen can he buy if he has R50,000.00?																			
Rp12: The table below shows two sets of ML method test scores.	Test A mean $\frac{50+70+50+50+60+80+100+100}{8} \quad \text{Mean} = 70$ Test B mean $\frac{40+62+64+72+70+68+68+68}{8} \quad \text{Mean} = 64$																		
<table border="1"><tr><td>Test A</td><td>50</td><td>70</td><td>50</td><td>50</td><td>60</td><td>80</td><td>100</td><td>100</td></tr><tr><td>Test B</td><td>40</td><td>62</td><td>64</td><td>72</td><td>70</td><td>68</td><td>68</td><td>68</td></tr></table> Calculate the mean for each set of the test scores.	Test A	50	70	50	50	60	80	100	100	Test B	40	62	64	72	70	68	68	68	
Test A	50	70	50	50	60	80	100	100											
Test B	40	62	64	72	70	68	68	68											
Rp14: The only sports offered at Burg High School are soccer and netball. The principal loves soccer so he allocates the sports budget so that for every R2 spent on netball, R3 will be spent on soccer. b) If the school gets R8000 to spend on sport, how much will be allocated to netball?	$R8000 \div 5 = R1600$ $R1600 \times 2 = R3200$ R3200 will be allocated to netball																		
Connections level tasks																			
Cn16: Nombuso went to a supermarket on Saturday 10th March, 2012. She wanted to buy chicken portions for a family of three. She found out that a 2 kg packet of mixed portions cost R31.99 and a 5 kg packet of the same type cost R89.99. Which one is a better deal in terms of money saving. Show all your working.	$R31.99 \div 2 = R15.995$ $R89.99 \div 5 = R17.998$ The 2kg will be the cheaper because each kg is cheaper compared to the 5kg																		
Cn18: Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years. a). Calculate the amount he will receive at the end of the period from Bank A	$I = P \times R \times T$ $A = R6000 \times 0,072 \times 2$ $= R864 + R6000$ $A = R6864$																		

b). Now calculate the amount he will receive at the end of the period from Bank B.	$R6000(1 + \frac{0,072}{2})^8 = R7962,13$
Cn19: Jane and Tom plan to install a sloping pool in their back garden. A sketch of the pool is shown below. The length of the pool is 6 m and its width is 3,5 m. The depth of the water in the shallow end is 1,2 m and 2 m deep in the deep end.  a). Calculate the volume of the raised cemented portion at the shallow end of the pool.	Surface area = $l \times b \times h$ $= 2m + 2m = 4m$ Therefore $2m - 1,2m$ $= 0,8m$ Raised cemented portion $= 4 \times 3,5 \times 0,8$ $= 11,2 m$
Cn20: Volume of Sound Model is given by; $L = 10 \cdot \log\left(\frac{I}{10^{-12}}\right)$. Here the volume L is measured in decibels (db) and I is the intensity in watts per square meter (W/m^2). a). An alarm has an intensity of $5.8 \times 10^{-9} W/m^2$. How loud is the alarm in decibels?	$L = 10 \cdot \log\left(\frac{5,8 \times 10^{-9}}{10^{-12}}\right)$ $= 10 \cdot \log\left(\frac{5,8}{10^{-3}}\right)$ $= 37,63$
Cn22: A lift at an office block can only carry 12 people. In a morning rush, 51 people want to go up the lift. How many times must it go up? Show your working.	$\frac{51}{12} = 4,25$ Therefore $4 \times 12 = 48$ $51 - 48 = 3$ Therefore $4 + 1 = 5$. The lift will go up 5 times, 4 times with 12 people and 1 time with 3 people
Cn23: Of the 112 learners in Grade 10 at Greenside High School, three-quarters ($\frac{3}{4}$) have pets. One-sixth ($\frac{1}{6}$) of those with pets have cats. Use a model or picture to find the number of learners who have other kinds of pets.	$\frac{3}{4} \text{ of } 112 = \frac{3}{4} \times 112 = 84$ Therefore $\frac{84}{6} = 14$ So 14 of them have cats as pets. Therefore $84 - 14 = 70$ 70 learners have other kinds of pets.
Cn24: The figure below shows a cube-shaped tank. The tank contains 500 kilolitres of water, what is the height of the water in the tank? [$1 m^3 = 1kl$] 	$10,76 \times 10,76 \times 10,76 = 1245,76 m^3$ Therefore $1245,76 m^3 - 500 m^3$ $= 745,76 m^3$ Therefore $500 m^3 = 500000 m$
Cn25: This is the sign in a lift at an office block.	$\frac{265}{12} = 22,08$

THIS LIFT CAN CARRY UP TO 12 a) In a morning rush, 265 people want to go up the lift. How many times must it go up?	Therefore $22 \times 12 = 264 = 23$ The lift will go up 23 times, 22 times with 12 people and one with 1 person. Because $\frac{264}{12} = 22 + 1$ trip. Its 23
b) What are the possible errors associated with the mathematical answer which learners can make when answering this question? Why?	Learners can say $\frac{265}{12} = 22,08$ and leave the answer as a decimal, forgetting there can't be broken trip or less than 1 trip because learners may forget that there cannot be a half or quarter of a trip, its either a trip or not
Cn27: If we assume that each time a woman has a baby (one at a time), the probability that the baby will be a boy is the same as the probability that the baby will be a girl, what is the probability that a woman who has three children will have; a) Three girls	$\frac{1}{8}$ because there are 8 possible outcomes and three girls is only 1 outcome so it is $\frac{1}{8}$ because we have 2^3 outcomes BBB; BBG; BGB; BGG GGG; GBB; GBG; GGB

Table 5.9: Jabu's responses to assessment tasks in 2012

Model formulation

Some examples showing Jabu's correct model formulation in 2012 have been provided in table 5.9 (see Rp8a, Rp14b, Cn20a, and Cn27a). Jabu's ways of model formulation in 2012 appears to be similar to Lindiwe and Mark's formulations. However, contrasts have been noted, especially in terms of how Jabu formulated his models relating to tasks Rp8 and Cn27a. While Jabu appears to add 3,5% to 100% in order to obtain a percentage representing the new salary in Rp8, Lindiwe's results indicate that she first worked out 3,5% of R6075 followed by adding this result to R6075 to obtain the new salary; suggesting pedagogic links. Since this kind of formulation was also observed across Jabu's working related to similar tasks (see Rp6, table 5.8) in 2011, this suggests understanding of percentage increase (or decrease) in connected ways. Further, Jabu was able to provide a list of possible outcomes in task Cn27a before probability was calculated, although this was not preceded by a tree diagram as in Mark's case. Listing of possible outcomes in probabilities appears to be useful within the context of learning problem solving for teaching purposes, as these kinds of skills allow for school learners' understanding of ways in which contextual information was linked with mathematical models (Ball, et al., 2008).

Incorrect formulations also featured in 2012 (see Rp9, Cn18b, and Cn19a). These examples show that Jabu was unable to choose an appropriate operation (i.e. Rp9) in order to convert South African Rand into Japanese Yen. Currency conversion has been described as one of the key skills in ML in a task that can be related to consumer orientation (Department of Education, 2003). He also appears to struggle in terms of selecting some of the quantities including substituting these quantities in formula (Cn18b). In this example 7,2% was selected instead of 5,4%. Although this could be understood as a slip, related substitution suggests that an error was involved. Errors relating to both choices of operations and substituting quantities into formulae were also observed in Lindiwe and Mark's work, and suggest gaps relating to extra-mathematical knowledge (Borromeo Ferri, 2007). Jabu's working relating to task Cn19 indicates that he was unable to provide a written statement relating to the formula. He appears unsure whether the formula for surface area or volume would be useful. Furthermore, the results show that the introduced units in this example (Cn19) were not consistent with either surface area or volume, an aspect which has also been highlighted later in this section (at the level of intra-mathematical work). This implies that Jabu was unable to work with retrieved or derived formulae, supporting an earlier observation relating to his working in 2011.

Intra-mathematical work

At intra-mathematical level in 2012, Jabu successfully engaged with some tasks in coherent ways. These were cases where vertical working was preceded by correct model formulations (i.e. Rp8a, Rp8b, Rp14b Cn16, Cn22, and Cn23). These examples show that Jabu approached some of the tasks in similar ways to Lindiwe and Mark. However, some strategies were interesting as they did not feature in both Lindiwe and Mark's work. In task Cn16 for instance, Jabu utilized the unitization method which provided a useful basis for comparison. Furthermore, solving task Cn22 involved the breakdown of the mathematical result before the interpretive aspect featured, suggesting a deep understanding of the problem context. His working took into account realistic considerations as application of mathematical rules would push for the rounding down of the mathematical result to 4 since 0,3 is closer to zero. Although Jabu obtained a correct mathematical answer in task Cn23, he appears not to address the question in terms of the solution method proposed within the task. Rather than using a picture, as the task demanded, his strategy was algebraic than Mark and Lindiwe's, despite successfully engaging with a similar task (see Cn2, table 5.8) in 2011.

Cases characterized by incoherent working in 2012 have been exemplified in table 5.9 (i.e. Cn18a, Cn19a, and Cn24). In Cn18a, the solution shows that after a simple interest formula was chosen, the letter ‘A’ which appears to represent ‘amount’ was introduced to replace ‘I’ in the formula. This was followed by some illogical use of equal sign (i.e. $R6000 \times 0,072 \times 2 \neq R864 + R6000$), although the mathematical result was eventually correct. Furthermore, the other two examples (Cn19, Cn24) show errors relating to some breakdown in terms of units. While task Cn19 indicates that the units ‘metres’ were introduced within the context of working with volume, task Cn24 shows that a measure of volume (m^3) was converted to a measure of length (m). The errors relating to both mathematical syntax and use of units suggest gaps in mathematical understandings.

Interpretation and validation of mathematical answers

Jabu’s responses appear to show interpretive aspects across many examples in 2012 as illustrated in Rp7a, Rp8b, Rp14b, Cn16, and Cn22. The examples indicate that Jabu was able to provide interpretations for the mathematical results obtained across 2012. The interpretive aspect appears to be similar to both Lindiwe and Mark’s work. However, in task Cn22, Jabu provided a more detailed explanation where he paid attention to realistic considerations with respect to the problem context. Although the task was mathematically focused, these kinds of interpretation skills form an important part of ML problem solving and resonate with the citizenship perspective.

Pedagogical links

Like in 2011, and in the other students’ working, Jabu’s responses in 2012 showed that pedagogic explanations and more detailed working featured in a few cases. Instances where pedagogic links were noted include Cn25b and Cn27. In Cn25b, the mathematical result was translated in relation to possible learner errors while in Cn27; explanations relating to Jabu’s working were given. Further, unlike Lindiwe’s working in Rp7, which included explanations, Jabu’s working in this task was similar to Mark’s, with no unpacking.

5.5.3 Quantitative summary of Jabu’s mathematical working

Table 5.10 provides a summary of Jabu’s mathematical working in terms of frequencies relating to occurrences of model formulation, intra-mathematical working, and interpretive aspect.

			2011 academic year (26 question items; 10 reproduction and 16 connections level items)		2012 academic year (38 question items; 14 reproduction and 24 connections level items)	
Elements of mathematisation	Cognitive levels	Performance	Frequency 2011	Percentage (%) 2011	Frequency 2012	Percentage (%) 2012
Model formulation	Reproduction level	Correct	8/10	80	10/14	72
		Incorrect	2/10	20	3/14	21
		No feature	0	0	1/14	7
	Connections level	Correct	4/16	25	13/24	54
		Incorrect	7/16	44	5/24	21
		No feature	5/16	31	6/24	25
Intra-mathematical working	Reproduction level	Coherent	7/10	70	12/14	86
		Incoherent	2/10	20	0	0
		No feature	1/10	10	2/14	14
	Connections level	Coherent	6/16	38	18/24	75
		Incoherent	3/16	18	1/24	4
		No feature	7/16	44	5/24	21
Interpretation	Reproduction level	Correct	5/10	50	12/14	86
		Incorrect	0	0	0	0
		No feature	5/10	50	2/14	14
	Connections level	Correct	7/16	44	19/24	79
		Incorrect	1/16	6	0	0
		No feature	8/16	50	5/24	21

Table 5.10: Frequency table showing Jabu's performance across tasks

Model formulation

Although the results show a reduction in terms of the percentage of correct formulations at the level of reproduction tasks (from 80% in 2011 to 72% in 2012), a marked increase in performance was noted at connections level items (25% in 2011 to 54% in 2012). A focus on incorrect formulations indicate that there was no improvement at the reproduction level across the two years as 21% of the formulations were incorrect in 2012 compared to 20% in 2011. However, improvement was observed at connections level items as the percentage of incorrect formulations reduced (44% in 2011 and 21% in 2012) across the years. Overall, the results suggest a slight improvement in terms of Jabu's skills relating to formulating models across the two years (2011-2012).

Intra-mathematical working

The results seem to suggest that Jabu's skills relating to calculations, computations, simplifications, etc were more coherent in 2012 than in 2011. At reproduction level, 86% of the solution procedures were coherent in 2012 compared to 70% in 2011. A large improvement has been noted at the level of connections question items, where 75% of the solutions exhibited coherence in 2012 as opposed to 38% in 2011. These results support the earlier observations where more responses in 2011 exhibited more incoherence than in 2012. This suggests that Jabu's competences relating to coherent intra-mathematical working improved, with a larger shift observed at connections level.

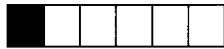
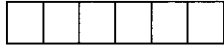
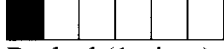
Interpretation and validation of mathematical answers

More responses had a feature of the interpretive aspect at both reproduction and connections level in 2012 than in 2011. For instance, 86% of the solutions in 2012 included some form of interpretation as opposed to 50% in 2011 at reproduction level items. At connections level tasks, 79% of the answers were interpreted in 2012 compared to 44% in 2011. Furthermore, there seem to be a reduction in terms of cases where this aspect did not feature at all. As already noted, these were cases where the solution procedure was terminated before an interpretive aspect was provided. These cases reduced from 50% at both reproduction and connections levels in 2011 to 14% at reproduction level and 21% at connection level in 2012. This implies growth in terms of Jabu's skills relating to paying attention to interpretation.

5.6 Lebo's mathematical work

5.6.1 Mathematisation of tasks across 2011 academic year

Task	Lebo's solution
Reproduction level tasks	
Rp1: A person has $29\frac{1}{2}$ metres of material available to make doll's dresses. Each dress requires $\frac{3}{4}$ metre of material a). How many dresses can be made?	$29\frac{1}{2} \div \frac{3}{4} = \frac{59}{2} \times \frac{4}{3} = \frac{118}{3} = 39\frac{1}{3}$ Therefore 39,333.... Therefore 39 dresses
b). How much material will be left over	$29\frac{1}{2} - 39\frac{1}{3} = \frac{59}{2} - \frac{118}{3} = \frac{59}{6}$ Therefore 9.83 material
Rp2: I have 8,2m of material. I need 0,4m of material to make doll's dress. a) How many complete dresses can I make from the material?	$8,2 \div 0,4 = 20,5$ 20 dresses because can't have 0,5 of a dress
b) How much material will I have left over?	$20,5 - 20 = 0,5$

	Therefore 0,5m of material will be left over
Rp4: A recipe for a full pot of stew requires that I use $\frac{4}{5}$ of a cup of beef stock. I only want to make $\frac{1}{2}$ of a pot of stew. How much beef stock do I need?	$\frac{4}{5} \times \frac{1}{2} = \frac{2}{5} = 0,4$ of beef stock. Therefore $\frac{1}{2}$ pot will need $\frac{2}{5}$ beef stock
Rp5: The price of a shirt is reduced from R350 to R280. By what percentage has the price of the shirt been reduced?	R350 to R280=R70 reduction cost. But $R350 \times 80\% = R280$. Therefore 80% is the reduction percentage
Rp6: John and Jane both currently earn R10 000 per month. a) John performs badly in this job so is demoted and will earn 9% less from next month onwards. How much will he earn?	$A = P(1 - \%)$ $A = 10000(1 - 9\%)$ $A = 10000(1 - 0,09)$ $A = 10000 \times 0,91 = 9100$ $A = R9100$
Connections level tasks	
Cn2: Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a picture to explain how you got your solution.	Buhle (3 pieces)  Anna  Whole is 6 pieces  Rashad (1 piece) Therefore of the 6 pieces Rashad got 1 piece, this means $\frac{1}{6}$ - the whole is six pieces
Cn4: Create a story problem for $4,5 \div 0,75$.	I have 4,5 litres of petrol to get from home to school. If 0,75 litres is what I have left when I got to school, how much petrol did I use that day?
Cn5: Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.	Olwethu has R0. She would love to buy these sweets that cost R6. Lutho offers to borrow her R2 and the other friends. Olwethu owes three people R2 each. How much does she owe everyone together? $3 \times (-R2) = -R6$. This means she is in debt by R6. If you owe something it is (negative) because you do not have that thing.
Cn6: Lynn says it will take her $\frac{1}{2}$ of a day to mark all the assignments. Mark says it will take him $\frac{1}{4}$ of a day to mark all assignments. If they work together to mark the assignments, how quickly will they be able to mark the assignments?	Lynn $\frac{1}{2}$ day Mark $\frac{1}{4}$ day $\frac{1}{2} \times 4 + \frac{1}{4} \times 2 = \frac{4}{8} + \frac{2}{8} = \frac{6}{8}$ together of a day
Cn9: Buhle invested money at a bank that	$A = P(1 + i)^n$

paid 8% annual interest compounded quarterly. If she had R4118,36 in her account at the end of 4 years, what was her initial investment	$R4118,36 = P(1 + \frac{8}{100} \times \frac{1}{4})^{4 \times 4}$ $R4118,36 = P(1,373)$ $R4118,36 \div 1,373 = P$ Therefore $P = R3000,00$
Cn11: My daughter wants to paint her bedroom pink. I mixed 3 tins of red paint with 5 tins of white paint and she says the pink it makes is perfect. I figure we need about 12 tins of paint to paint her bedroom. a) If I add 2 tins of white paint and 2 tins of red paint to the perfect pink mix will it be too red, too white or still perfect?	Too red
Cn13: You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?	$A = P(1 - in)$ $R85000 = P(1 - \frac{10}{100} \times 3)$ $\frac{R85000}{0,7} = P = R36428,57$ Therefore R36 428,57

Table 5.11: Lebo's responses to assessment tasks in 2011

Model formulation

With regard to correct identification and selection of quantities from the contexts, and setting up procedures broadly, Lebo's responses indicate several similarities with Lindiwe, Mark, and Jabu, across both reproduction and connections level tasks. Some examples are provided in table 5.11 and include Rp1a, Rp2a, Rp4, Rp6a, Cn2, and Cn9. Regarding task Rp6a, Lebo correctly selected formula followed by correct substitution, in ways very similar to Lindiwe's working. Despite these differences, the same reasoning seems to underlie both strategies. Furthermore, Lebo correctly represented her formulation diagrammatically, within her working relating to task Cn2, in ways that overlapped with the other three students. Her drawings were also accompanied by explanations suggesting that she understood and could communicate the steps in the formulation. Unlike Jabu, Lebo correctly chose formulae in some cases which allowed her to correctly set up procedures (i.e. Cn9).

In some cases, Lebo's model formulation exhibited some disruptions as illustrated in Rp1b, Rp2b, Cn6, and Cn13. Across these examples, incorrect selection of contextual quantities and operations especially across tasks Rp1b, Rp2b, and Cn6, featured, as in Lindiwe and Jabu. Her response to Rp1b for instance shows that a bigger number was subtracted from a smaller number in which case a positive result was obtained. Lindiwe had engaged with this task in

similar ways (see table 5.2). Some of the quantities appear to be selected from the preceding steps (Rp1b, Rp2b) of the problem solving process. Sometimes information was used within Lebo's working, whose source was not clear and was not explained (i.e. quantities 4 and 2 in Cn6). Although an incorrect choice of formula featured in task Cn13, a similar case (Cn9, table 5.11) indicate that she was able to select an appropriate formula. Across most of these examples, the kinds of disruptions, noted, relate to working with fractions (see Rp1b, Rp2b, Cn6), a result similar to Lindiwe, Mark, and Jabu.

Story creation

Two examples are provided in table 5.11 to illustrate Lebo's ability relating to 'story creation' given a mathematical model (see Cn4 and Cn5). Lebo's response to Cn4 suggests that she was unable to choose a context which correctly represented the given model. Rather than creating a story which depicted the idea of division, Lebo chose a story which suggested a procedure leading to subtraction of the two quantities, which was incorrect. The second example exhibited an idea of borrowing from friends, especially when an individual has no money (R0.00). Thus borrowing R2.00 from three people led to a debt amounting to R6 (i.e. she had -R6), which was correct. It is interesting to note that Lebo's stories appear to have been drawn from similar contexts to Jabu with similar formulations (see table 5.8). However, unlike Jabu who provided correct stories in both examples, Lebo's response to Cn4 suggests that her skills relating to attaching contexts to mathematics statements needed further development.

Intra-mathematical work

Examples showing coherent intra-mathematical working within situations where the model formulation was correct, included, Rp1a, Rp2a, Rp4, Rp6a, Cn2, and Rp9. Lebo's intra-mathematical working was generally similar to the other three cases. Lebo's working, like Lindiwe and Mark, indicates that she was able to manipulate equations (i.e. Cn9) unlike Jabu whose working relating to this task suggest the contrary. Further, Lebo appears to confidently use diagrammatic representation to solve problems (i.e. Cn2). The explanations which accompanied her drawings and the connections made with the numerical quantities suggest understanding of her solution strategy. Although Lebo's working at the level of model formulation suggests that she was unable to set up procedures in some instances where fractions were involved, her working in this example implies the ability to use varied solution procedures within problem solving.

Like the others, coherence across Lebo's vertical working was also achieved in situations where models were incorrectly formulated (i.e. Rp2b). Further, evidence has shown that in some instances incoherence across intra-mathematics working featured (see Rp1b and Cn6). In task Rp1b, Lebo appears to subtract a bigger number from a smaller number, in which case a negative number would be expected. However, the negative sign was dropped at the level of the mathematical result. This suggests that Lebo may have understood that the task demanded a positive result but was unable to set up a procedure leading to this result. As noted already, Lindiwe engaged with this task in similar ways. Task Cn6 response also featured a mathematical breakdown as the statement that followed the first equal sign ($\frac{4}{8} + \frac{2}{8}$) illogically followed from the first statement ($\frac{1}{2} \times 4 + \frac{1}{4} \times 2$). This implies gaps in mathematical understandings especially in the context of solving problems involving fractions. Similar incoherent vertical working was also observed in Jabu's working involving fractions.

Interpretation and validation of mathematical answers

Lebo's interpretive aspect across 2011 featured in most procedures across both reproduction and connections level tasks, exemplified in Rp1a, Rp2a, Rp4, Rp5, Cn2, and Cn11a. Lebo's interpretation of mathematical results overlaps with the other students in that the translation was consistent with the mathematical results and the problem situations, even in situations where incorrect mathematical results were obtained. However, unlike Lindiwe who could not provide an interpretive aspect in Rp1a (see table 5.2), Lebo was able to do so.

Furthermore, interpretation was also provided in cases where the mathematical working was not provided (i.e. Cn11). Similar working was observed across Jabu (see Cn10, table 5.6) and Lindiwe (see Cn1, table 5.2), although different tasks were involved in these instances.

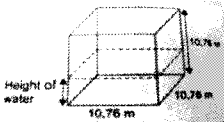
Pedagogic links

Like Lindiwe, Mark and Jabu, most procedures across Lebo's working exhibited less or no explanations, an aspect linked to ML teaching and learning. Few instances where some explanations and detailed working were provided include Rp6, Cn2, and Cn9. Similar working was observed in Lindiwe's working and less so in Mark and Jabu's solutions.

5.6.2 Mathematisation of tasks across 2012 academic year

Task	Lebo's solution
Reproduction level tasks	
Rp7: A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day b) How many teams must they put on the job if they have to get the contract finished i) in one day	$6 : 20$ $x : 3000$ $6 \times 3000 = 18000$ $20 \times x = 20x$ $\frac{18000}{20} = \frac{20x}{20}$ $900 = x$ Therefore they will need 900 workers to finish the job in one day.
Rp8: Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month. a) Determine Nadia's new salary per month.	$R6075 \text{ of } 3,5\%$ $= R212,63 + R6075$ $= R6287,63$
b) Who received the greater percentage increase? Show your working	$R6287,63$ (Nadia) Nadia increase by 3,5% $R8000 + R259,86 = R8259,86$ (Sekuru) $= 3,24\%$ Sekuru increase Therefore Nadia gets a greater % increase
Rp14: The only sports offered at Burg High School are soccer and netball. The principal loves soccer so he allocates the sports budget so that for every R2 spent on netball, R3 will be spent on soccer. a) If R450 is allocated to soccer, how much will be allocated to netball?	$R2 : R3$ $R450 : x$ $R3 : R2$ $= \frac{900}{3} = 300$ Therefore R300 will be allocated to netball
b) If the school gets R8000 to spend on sport, how much will be allocated to netball?	$R8000 : x$ $R3 : R2$ $= \frac{16000}{3} = R5333,33$ (soccer) Therefore $R8000 - R5333,33 = R2666,67$
Connections level tasks	
Cn16: Nombuso went to a supermarket on Saturday 10th March, 2012. She wanted to buy chicken portions for a family of three. She found out that a 2 kg packet of mixed portions cost R31.99 and a 5 kg packet of the same type cost R89.99. Which one is a better deal in terms of money saving? Show all your working.	$2\text{kg} = R31,99$; $1\text{kg} = R15,99$; $3\text{ kg} = R47,98$; $4\text{kg} = R63,98$; $5\text{kg} = R89,99$. So if she buys 2(2kg), $R63,98 + 1\text{kg } R15,99 = R79,97$ Which is still cheaper than the 5kg R89,99. Therefore she will save if she buys option 1. She will save R10,02
Cn17: A loaf of bread is a regular purchase for	$1) R7,24 \times 4,5\% = 0,3258$

many families. If a loaf of bread costs R7.24 and that the cost of the loaf has risen by the average inflation rate of 4,5% in the last 20 years. Find how much a loaf of bread would cost 20 years ago.	$R7,24 - 0,3258 = R6,91$ 2) $R7,24 \times 4,5\% \times 20 = R6,52$
Cn18: Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years. a). Calculate the amount he will receive at the end of the period from Bank A	$P=R6000$; $r=7,2\%$; $n=2$ yrs $I = P \times r \times n$ $I = R6000 \times 7,2\% \times 2$ $I = R864$ $A = R6000 + R864$ $= R6864$
b). Now calculate the amount he will receive at the end of the period from Bank B.	$A = P(1 + \frac{r}{100})^n$ $A = R6000(1 + \frac{5,4}{100})^2 = R6665,496$ Therefore $= R6665,50$
Cn19: Jane and Tom plan to install a sloping pool in their back garden. A sketch of the pool is shown below. The length of the pool is 6 m and its width is 3,5 m. The depth of the water in the shallow end is 1,2 m and 2 m deep in the deep end.	$V = l \times b \times h$ $V = 2m \times 3,5m \times 1,2m$ $V = 8,4 \text{ m}^3$
a). Calculate the volume of the raised cemented portion at the shallow end of the pool.	
c). Jane and Tom are planning to put up a security fence, one metre away from the edges of the pool. The fence will be right around the pool. Determine how many metres of fencing Jane and Tom would need to buy.	$A = l \times b$ $A = 6 \times 2 = 6m^2$ Therefore $\approx 6 m^2$
Cn20: Volume of Sound Model is given by; $L = 10.\log(\frac{I}{10^{-12}})$. Here the volume L is measured in decibels (db) and I is the intensity in watts per square meter (W/m^2). b). Anna can scream at 56 db and Billy can yell at 48 db. How many more times intense is Anna's scream than Billy's yell?	$48db = 10.\log(i \div 10^{-12})$ $48db - 10.\log = (i \div 10^{-12})$ $48db - 10.\log \times 10^{-12} = i$ $168 = i$ $56db = 10.\log(i \div 10^{-12})$ $56db - 10.\log = (i \div 10^{-12})$ $56db - 10.\log \times 10^{-12} = i$ $176 = i$ $176-168=8$. Anna screams 8 times more than Billy

Cn22: A lift at an office block can only carry 12 people. In a morning rush, 51 people want to go up the lift. How many times must it go up? Show your working.	12 people= 1 lift to go up 51 people = x $\begin{array}{r} 12:1 \\ \times \\ 51:x \\ \hline \end{array}$ $= \frac{12x}{12} = \frac{51}{12}$ $x = 4,25$ We cannot have a lift that goes up comma 25 times. So we can say it would go up 5 times
Cn23: Of the 112 learners in Grade 10 at Greenside High School, three-quarters ($\frac{3}{4}$) have pets. One-sixth ($\frac{1}{6}$) of those with pets have cats. Use a model or picture to find the number of learners who have other kinds of pets.	$\frac{3}{4} \times 112 = 84$ have pets $\frac{1}{6} \times 84 = 14$ have cats $112 - 84 = 28$ have other kinds of pets $84 \div 112 \times 100 = 75$ $(14 \div 84 \times 100 = 16,6667)$ $28 \div 112 \times 100 = 25$
Cn24: The figure below shows a cube-shaped tank. The tank contains 500 kilolitres of water, what is the height of the water in the tank? [1 m³=1kl] 	$A = 4S$ 500 $V = 4S \times h$ $500 = 4(10,76) \times h$ $500 \div 4 = 10,76 \times h$ $500 \div 4 \div 10,76 = h$ $11,617 = h$ $11,72 \approx h$ $V = 4S \times h$ $V = 4(10,76) \times 11,617 = 499,99568$ Rounded off = 500m³ Therefore 500m³ = 500kl
Cn25: This is the sign in a lift at an office block. THIS LIFT CAN CARRY UP TO 12 a) In a morning rush, 265 people want to go up the lift. How many times must it go up?	$\frac{265}{12} = 22,083$ The lift will go up 23 times to fetch all the people
b) What are the possible errors associated with the mathematical answer which learners can make when answering this question? Why?	Learners might think that the lift will go up and down, so they will count the coming down as the times the lift will move. Learners might not see “how many times must it go up?” The lift has to go up with the people who are left (22,083), we cannot round off, because they are still people to go up. Even

	though they are less than 12 of 1 person, the lift will still have to go up again.
Cn29: The diagram below (not drawn to scale) is a plan of Sandile's flat which they are planning to redecorate. a) All the ceilings are to be painted with 2 coats of white paint. Each litre of paint will cover 10 m^2 of ceiling. How much paint will she need to paint the ceilings?	$A = l \times b$ $= 7 \times 8,5$ $= 59,5 \text{ m}^2$ $1l = 10 \text{ m}^2$ $\frac{59,5}{10} = 5,95 \times 2 \text{ coats}$ $= 11,9l$ Therefore, she will need 11,9 litres to paint the ceilings. Therefore 12 litres will be fine

Table 5.12: Lebo's responses to assessment tasks in 2012

Model formulation

Examples of correct formulations in 2012 across both reproduction and connections level tasks are provided in table 5.12 and include Rp7b, Rp14a, Cn16, Cn18a, and Cn22. Lebo did not seem to formulate models differently from the other students in 2012. However, the examples suggest that the idea of proportionality was utilized successfully in cases where other students employed alternative translation techniques. For instance in task Cn22 the results have shown that Lindiwe, Mark, and Jabu, simply divided the quantity 51 by 12 without formulating equations.

Disruptions relating to setting up procedures continued to feature in 2012, as exemplified in Rp14b, Cn17, Cn18b, Cn19a, Cn19c, and Cn24. These examples indicate that there were more errors occurring at connections level tasks than at reproduction level tasks in 2012, suggesting that Lebo was unable to engage with tasks involving multi-step methods. Incorrect selection of formulae particularly relating to area and/or perimeter and volume of 3 dimensional shapes (i.e. Cn19a, Cn19c, and Cn24) were noted across the same tasks for Lindiwe, Mark and Jabu. Although formulae are often provided in ML assessment (Department of Education, 2008), knowledge relating to choosing appropriate formulae is needed at the level of the teacher. Given that choosing formulae was a key feature of CLM course at the course enactment stage, choosing appropriate formulae was part of this assessment, as formulae were not provided within problem situations.

Intra-mathematical work

Examples showing Lebo's coherent intra-mathematics working include Rp7bi, Cn16, Cn18a, and Cn29a. As noted earlier, Lebo's working across tasks was similar to the other students. However, some contrasts were noted. Task Rp7b shows that the idea of proportionality was employed to set up the procedure leading to an equation. In this example, despite accurate working, Lebo does not conclude the procedure in terms of providing the number of teams to be put on the job. Failure to complete solution procedures, especially at interpretation level, were also noted within Lindiwe's working, and is linked to gaps relating to extra-mathematics knowledge (Borromeo Ferri, 2007; Verschaffel, et al., 1994).

Incoherent vertical working across Lebo's responses to assessment tasks which featured in 2011 appears to continue characterizing some responses in 2012 (see Rp8a, Cn20b, and Cn23). Example Rp8a shows a response where Lebo attempted to combine two steps into a single step thereby creating a scenario in which the left hand side of the equation did not equal the right hand side. This incoherence appears to be at the level of logic. Although these kinds of illogical presentation of solutions did not often lead to incorrect mathematical results, the accuracy of intra-mathematical working was constrained. Regarding task Cn23, the response appears to show some working towards calculating percentages, which was not part of the focus in this question. Again, this shows that the question was not properly understood (Kaur, 1997; Koedinger & Nathan, 2004). The other example (Cn20) presents a situation where algebraic rules relating to solving equations were flouted. Although Lebo showed confidence within the contexts of solving similar tasks (involving equation), she was unable to apply the same algebraic rules in this example, a characteristic of manipulation errors (Hall, et al., 1989). This suggests difficulties relating to applying mathematical rules in novel situations, as this task involved numbers in standard forms, an aspect which did not feature in the CLM course. Other participants successfully engaged with this task.

Interpretation and validation of mathematical answers

Examples showing Lebo's interpretive aspect in 2012 included Rp7b, Rp8b, Rp14a, Cn16, Cn20, Cn22, Cn25a, and Cn29. The responses show that the mathematical results were interpreted in similar ways to the other students, featuring comparisons of answers in order to take a view point in some cases (i.e. Rp8b, Cn16) and provision of realistic considerations in others (i.e. Cn22, Cn25a).

Pedagogical links

Explanations and step-by-step procedures were noted across Lebo's working (i.e. Rp7b, Cn16, Cn18, Cn22, and Cn25b). However, in some cases, incorrect explanations were observed – suggesting weak pedagogic links. In Cn25 for example, Lebo's intra-mathematical working was coherent, but linking the mathematical result with learners' possible ways of reasoning appeared to be vague. She made reference to the idea of 'rounding off' and argued that the answer did not need to be treated this way, thus contradicting her own sense making of the mathematical result. In contrast, the other participants were able to successfully engage with this task. Further, Lebo's working in Rp7b provided more detail, like Lindiwe's, an aspect which could not be observed across Mark and Jabu's working, relating to the same task.

5.6.3 Quantitative summary of Lebo's mathematical working

Table 5.13 summarises Lebo's mathematical working across two years (2011-2012) in terms of frequencies relating to occurrences of model formulation, intra-mathematical working, and interpretive aspect.

			2011 academic year (26 question items; 10 reproduction and 16 connections level items)		2012 academic year (38 question items; 14 reproduction and 24 connections level items)	
Elements of mathematisation	Cognitive levels	Performance	Frequency 2011	Percentage (%) 2011	Frequency 2012	Percentage (%) 2012
Model formulation	Reproduction level	Correct	5/10	50	11/14	79
		Incorrect	3/10	30	2/14	14
		No feature	2/10	20	1/14	7
	Connections level	Correct	3/16	19	12/24	50
		Incorrect	5/16	31	6/24	25
		No feature	8/16	50	6/24	25
Intra-mathematical working	Reproduction level	Coherent	5/10	50	13/14	93
		Incoherent	3/10	30	0	0
		No feature	2/10	20	1/14	7
	Connections level	Coherent	6/16	38	15/24	62
		Incoherent	1/16	6	4/24	17
		No feature	9/16	56	5/24	21
Interpretation	Reproduction	Correct	8/10	80	13/14	93

	level	Incorrect	1/10	10	0	0
		No feature	1/10	10	1/14	7
	Connections level	Correct	9/16	56	19/24	79
		Incorrect	0	0	2/24	8
		No feature	7/16	44	3/24	13

Table 5.13: Frequency table showing Lebo's performance across tasks

Model formulation

Results show an upward trend in terms of occurrences of correct formulations at both reproduction tasks (from 50% in 2011 to 79 % in 2012) and connections tasks (from 19% in 2011 to 50% in 2012). Further, occurrences relating to incorrect formulations declined over the two years at both cognitive demand levels. Although occurrences of correct formulations at connections level tasks remained less than those at reproduction level, overall results suggest that there was an improvement in this aspect in 2012.

Intra-mathematical working

At the level of intra-mathematical working, occurrences of coherent working appear to have almost doubled for reproduction level tasks (from 50% in 2011 to 93% in 2012). The improvement was also noted at connections level tasks, although the shift (in occurrences) was lower than that at reproduction level across the same period (from 38% in 2011 to 62% in 2012). Again this means that there was an improvement in terms of achieving coherence across Lebo's working.

Interpretation and validation of mathematical answers

Like the other aspects (model formulation and intra-mathematical working), a marked increase in occurrences relating to interpretation was noted. At reproduction level, 93% of the responses had a feature of interpretation in 2012 compared to 80% in 2011. In terms of connections tasks' responses, 79% had a feature of interpretation in 2012 compared to 56% in 2011. This suggests that Lebo's consideration of the interpretive aspect improved across the two years.

5.7. Chapter summary

5.7.1 The nature of the teachers' translation processes

Problem situations in ML are often situated in some context. To solve these kinds of problems by utilizing mathematics ideas, a number of translation sub-processes are played out. Informed by the mathematisation process, and some grounded focus, I focused my analysis in this chapter on model formulation, story creation, interpretation or validation, and pedagogic link, with a view to explore ways in which these sub-processes played out across the students' mathematical working.

Model formulation

The pre-service ML teachers' working relating to translating contextual information into mathematical models across 2011 and 2012 was characterised by both correct and incorrect formulations, a feature which played out at both reproduction and connections levels of the assessment tasks. One interesting finding concerns the idea of annotations (Hall, et al., 1989) which featured in some of the students' working (i.e. Lindiwe, Mark, and Lebo) and less so across Jabu's solutions. Annotation provided information about how the contextual quantities were selected for model formulation, an aspect which appeared to be pedagogically useful as it allows learners to see the relationship between the contextual features and the model. Although Jabu did not include annotation within his protocols across the two years, there was no evidence suggesting that the absence of 'annotation' in his protocols impacted his mathematical working in any way. Further, this study also sought to explore whether the absence of annotation fed into Jabu's pedagogic practice.

Another feature of model formulation was related to selecting arithmetic operations to establish the inter-relationships between the identified contextual quantities. Across the four participants, picking correct operations to represent the quantitative features of the contexts remained a challenging exercise across the two years (2011-2012), although some marked increase in occurrences of instances where correct operations featured in 2012 was noted. One common error was concerned with formulating models involving fractions, with results suggesting difficulties relating to translating everyday language into mathematical statements. This concurs with Bernardo's (1999) results indicating that "the most basic difficulty students have in solving word problems lies in the ability to understand the mathematical problem structure that is embedded in the problem text" (p.149), a component which is key to

selecting operations. Operation errors are also linked to ‘conception disruptions’ (Hall, et al., 1989). Overall, the skill related to choosing operations in a bid to set up the mathematical model was generally stronger in 2012 than in 2011, an aspect which provided a pointer towards the students’ skills development. Further growth in this respect (selecting correct operations) was observed specifically across Mark, Jabu, and Lebo’s working and less so in Lindiwe’s working.

Another aspect of model formulation adopted in the pre-service teachers written protocols concerned the choice or selection of formulas followed by substitution (Hall, et al., 1989). Results have shown that retrieval of formulas was a success within financial contexts and less so in cases where area and perimeter of 3-dimensional shapes were involved. Only Mark seemed to confidently engage with the area and perimeter situations at this level. In other cases the choice of correct formulas was followed by incorrect substitution. Besides situations involving area and perimeter, incorrect substitutions were also noted across some financial contexts despite selecting the correct formula. For instance the case where compounded interest was charged more than once in one year (i.e. quarterly, see Cn18b), all the participants gave an incorrect substitution. The pre-service teachers’ grapple with substitution suggests disruptions relating to their understanding of the link between the contextual quantities and the variables given in the formulas. This supports empirical claims indicating that “the interface between the real world problem and the mathematical model ... presents difficulties for students” (Crouch & Haines, 2004, p.198).

Story creation

A related competence to model formulation was the story creation, an aspect of mathematisation which has emerged from grounded analysis in this study. This aspect was concerned with the reverse process of model formulation, where a mathematical model or statement was given and demanded the problem solver to identify a story which best represented the model. The ‘story creation’ skill was found to be useful in ML given the need for the pre-service ML teachers to exhibit some deep and connected understandings of both mathematics content and situations within the context of ML learning and subsequently in practice. The results show that the pre-service teachers had no difficulty identifying the stories for the given models, except one case (Lebo) where one of the stories provided a weak link with the related model. Unlike the high school students in Koedinger & Nathan’s (2004) study, who showed difficulties with comprehending symbolic algebra representation, the pre-

service teachers in this study showed more developed algebra-language (or symbolic arithmetic language) comprehension skills. Further, the ‘created’ stories across the four participants were at the personal level (OECD, 2006), and this provided links to citizenship orientation. Another interesting finding regarding story creation competence relates to the way the pre-service teachers introduced the ‘units’ in order to ensure that the quantities exhibited some meaning relevant to the selected contexts. Although these kinds of problems were given in 2011 only, the ways how the pre-service teachers interpreted the mathematical answers (mathematics content-context connections) across 2012 might give some pointers into the pre-service teachers’ growth in terms of inter-connecting symbolic mathematical language and contexts.

Interpretation and validation

The study results have shown that the pre-service teachers’ interpretive aspect did not feature in most of the protocols especially in 2011. This suggests that students had difficulties interpreting answers (Greer, 1993; Kaiser & Maaß, 2006; Sepeng & Webb, 2012; Verschaffel, et al., 1994). In some cases the tasks had either explicitly or implicitly implied that the mathematical answers needed to be located within the context of the problems, the interpretive aspect could have featured. In 2012, most of the protocols included translation in the form of the interpretive aspect and/or validation. Not only did the inclusion of the interpretive aspect improve in 2012, but also the accuracy of the translation was better than the ones given in 2011, across all the participants. The phrase ‘better quality’ refers to scenarios in which the translations were more consistent with the mathematical answers as well as the original problem situations.

In other cases, the results indicated that realistic considerations were ignored. This finding agrees with other empirical studies (Greer, 1993; Sepeng & Webb, 2012; Verschaffel, et al., 1994) although these studies have a focus on children, with overlaps at the level of tasks. The nature of tasks in terms of cognitive demands in this study was similar in many ways with the kinds of tasks used in the above studies. The idea of interpretation involving authentic considerations was one of the central features of ML related competences. The ML subject specifications suggest an emphasis relating to translating mathematical answers in the context of the problem, some of which require the problem solver to evoke realistic reasoning (Department of Education, 2003). The CAPS for ML refers to the interpretive aspect as ‘interpretation and communication’, and is a feature of the ‘basic skills topics’ (Department

of Basic Education, 2011b). The results have not showed differential results for specific mathematics topics or strands.

Pedagogic links

Literature relating to teacher development broadly suggests the need for teachers to develop their skills relating to dealing with learner problems about their learning (Carpenter, Fennema, Peterson, & Carey, 1988; Kramarski, 2009). This study results indicate that the pre-service teachers included explanations and detailed solution procedures in a few cases – features linked to pedagogy. Across these cases, the teachers’ pedagogically-linked skills appear to be strong, although weak problem-solving/pedagogy connection was noted in Lebo’s working. Most procedures exhibited less unpacking of concepts, an aspect which can be described as ‘compression’ of solutions. Both ‘unpacked’ and ‘compressed’ solution procedures are useful within the context of professional development, as these skills are needed in practice.

5.7.2 The nature of the teachers’ intra-mathematical working

In addition to competences relating to the translation process, the ability to analyze information or problem situations using mathematics is also key in ML (Department of Education, 2003). Given the context, students need a predisposition to select appropriate mathematical tools to aid them to solve the problems. Drawing from Hall et al (1989) and some grounded sense making, I focused my analysis within the solution process on three sub-processes namely; coherence of procedures, incomplete solution procedures and incorrect mathematical answers. Discussing these three aspects allowed me to understand the nature of pre-service teachers’ intra-mathematical working, a feature which complemented the understanding of translation processes, both of which help to address one of the research questions relating to the nature of students’ mathematical working.

Coherence of procedures

I focus on what Hall et al, (ibid) call conceptual coherence, which is noticed in the protocols when a ‘student is exhibiting the same conceptualisation of the problem’ (p.245). The intra-mathematical working especially in 2011 was characterised by both coherent and incoherent procedures across the four participants. Coherent procedures were more prevalent at reproduction level tasks than at the connections level questions. Broadly, the results have

shown that the pre-service teachers gave coherent protocols even in cases where the intra-mathematical working was preceded by incorrect model formulation. Incoherent working has also been noted especially within the mathematical working of Lindiwe and Lebo. In relation to mathematics topics, errors were observed in situations involving fractions and area/perimeter of 3-dimensional shapes. Coherent procedures featured more in 2012 than in 2011, suggesting that the pre-service teachers' competences relating to producing procedures that cohere had improved further in 2012.

Incomplete procedures (pre-service teachers' protocols)

Incomplete procedures refer to situations where the final answer was not included in the solution procedure. In most cases where the pre-service teachers' procedures exhibited incompleteness, the first part of the procedures was often correct and coherent. This was then followed by an abrupt end in procedures, suggesting a view that the mathematical answer had finally been arrived at. This means that the pre-service teachers either misunderstood the questions' demands or did not know how to proceed and when to terminate the procedure. These kinds of responses were more prevalent at connections level tasks than at reproduction level tasks, suggesting difficulties with multi-step methods. Incomplete procedures were more prevalent in Lindiwe and Lebo's responses.

In other cases, especially in 2011, mathematical answers were provided without being preceded by some mathematical working, an aspect which did not feature in 2012. It may be that the teachers used separate sheets of paper to solve problems, whose final answers were then transferred onto the answer sheets which were eventually submitted for marking. Related to providing answers without showing procedures were cases where the interpretation featured without any related mathematical working. Again this could be related to instances where the procedure was written on a separate sheet as noted above, but this time only the interpretive aspect was transferred onto the answer sheet. Evidence of these kinds of working has been noted across Lindiwe, Jabu, and Lebo's working.

Incorrect mathematical answers

Overall, the results have shown that the incorrect answers provided by the pre-service teachers, especially at reproduction level tasks in 2011, were not necessarily the function of incoherent intra-mathematical working. Errors were often made at the level of model formulation especially during choices relating to arithmetic operations and substitution of

contextual quantities into formulas (Winter & Venkat, 2013). The results in 2012 showed that fewer incorrect answers were obtained as a result of breakdowns related to model formulation. This analysis has also revealed that, in addition to errors committed at model formulation level, some incorrect answers particularly in 2012 were as a result of incoherent procedures (manipulation errors) especially at connections level tasks (Hall et al, 1989). Incorrect mathematics answers and incoherent intra-mathematics working were observed across Lindiwe, Jabu, and Lebo's working more than Mark's working. Since these incorrect answers were more prevalent in 2011, this implies that the pre-service teachers' competences related to both the model formulation and the intra-mathematical sub-process may have developed further. Although the study finding related to coherence of procedures is comparative (2011 against 2012), some parallels can be drawn with the findings in Hall et al (1989), who found that "analyses of errors encountered by students when attempting solutions suggest that conceptual errors of omission and commission are both more prevalent and more damaging than manipulative errors in algebra or arithmetic" (p.269).

CHAPTER SIX: PRESENTATION AND DISCUSSION OF FINDINGS: PRE-SERVICE ML TEACHERS' PRACTICE

6.1 Introduction

In this chapter, findings from the ML pre-service teachers' practice data sources namely; observed lessons, video lessons, and post-lesson interviews, were analyzed and discussed. Due to the qualitative nature of the generated data, an interpretive approach to data analysis was adopted. The purpose was primarily to answer the third research question which related to exploring the nature of mathematical working within teaching episodes informed by the tasks utilized within lessons. Furthermore, post-lesson interviews were analysed with a focus on the rationales for advancing particular pedagogic goals within ML teaching and learning. The ways in which the pedagogic goals played out and interacted allowed for an understanding of connections and disconnections across the pre-service teachers' mathematical working within practice. Figure 6.1 presents a framework which has been used as a guide for data analysis and discussion of results in this chapter.

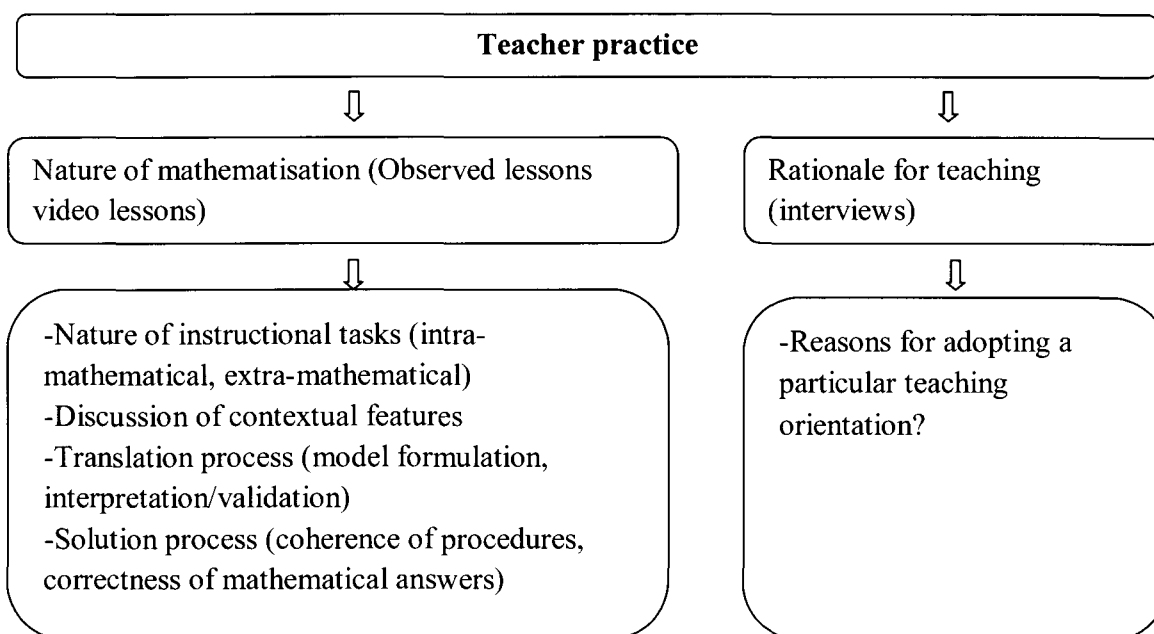


Figure 6.1: Instructional practice analysis

As in the last chapter, results and analysis of each of the four teachers have been presented independently as the practice data were specifically linked to lessons taught by individual teachers. The analysis of each of the cases was then followed by an overview analysis and discussion of all four participants looking for emerging patterns across them.

6.1.1 Why intra-mathematical tasks were included within analysis of teachers' practices

In Chapter five, I argued for choosing only assessment tasks which referred to real world objects (extra-mathematical) either explicitly or implicitly. The key rationale was related to the ML curriculum emphasis on extra-mathematical tasks combined with the CLM course focus on contextualized problem solving, as well as the fact that intra-mathematical working could be seen in these tasks. However, data collected in practicum sessions, including broader ML classroom based studies in South Africa, indicate that teachers utilize both intra-mathematical and extra-mathematical tasks in lessons. In addition to a focus on contextualized tasks in practice, analysis of lesson episodes relating to intra-mathematical tasks were included in this chapter. My interest in exploring the nature of orientations to both content and contextual working provided the rationale. The table showing the classification of instructional tasks, as intra or extra-mathematical, utilized by all the four teachers has been included in Appendix B. This classification provided entry points into understanding the kinds of tasks used in ML classrooms across 2011 and 2012, as well as linking these tasks with pedagogic goals foregrounded in classrooms.

It should be noted that the PISA mathematisation process focuses more on problem solving involving extra-mathematical tasks as opposed to engagement with intra-mathematical tasks. However, within the context of ML teaching, it has been observed that teachers included real world exemplifications of mathematics ideas, within their teaching episodes using intra-mathematical tasks. Teaching relating to intra-mathematical tasks included devising a plan (translating quantities from an intra-mathematical context to a mathematical model), intra-mathematical working (manipulating the model), and checking results (relating the mathematical answer with the intra-mathematical context). Further, the mathematisation process included a focus on the concept of 'solution process' that is concerned with intra-mathematical working, an aspect which provided a window for making sense of the teachers' vertical mathematisation. In this study the following links have been noted within the teachers' practices;

- The teaching of some intra-mathematical tasks was accompanied by real life exemplifications of mathematical ideas involving tasks' objects in either the lesson introduction or the course of explaining the problem solving procedure. This idea was also pushed (though much more specifically) in the CLM course where the teachers were asked to create stories which represent given mathematical models.

- The teaching of intra-mathematical tasks was immediately followed by occasional contextualized tasks with related foci.
- Some intra-mathematical tasks required both translation (model formulation and interpretation) and vertical working-key features of the mathematisation process.

These kinds of connections between mathematics content ideas and problem contexts (either mathematical or real world situations) within the context of ML teaching and learning provided a rationale for including both intra-mathematical and extra-mathematical tasks for analysis in this chapter.

6.1.2 Practice for Mathematical Literacy

Within practice, the study focused on sixteen lessons in total, presented by the four pre-service teachers across 2011 and 2012. This was comprised of two lessons for each teacher (one observed and the other video-recorded) in each of the two years, given the fact that these teachers were given only three weeks to teach ML in each academic year, during practicum periods. The average length of lesson periods across schools where data was collected was 40 minutes (ranging from 35 minutes to 45 minutes). Furthermore, most lessons were delayed due to learners arriving late from other classrooms and in some cases because of prolonged teacher meetings. There is also evidence pointing towards broad problems relating to learners' weak mathematical understandings (Venkat & Graven, 2006a, 2007), an aspect which potentially affects pace in ML classrooms. The reader should be reminded that enrolling into ML at grade 10 was largely dependent on low achievement in school mathematics in grade 9. The other factor was related to issues of interest. Chatting to school ML teachers during data collection, I got some sense indicating that while some high performing learners in mathematics chose to do ML because of their future career aspirations, these cases were few. Having learners with weak mathematics understandings in ML classrooms can therefore be related to teachers using fewer tasks for teaching as more time is spent on explaining mathematical concepts. As already noted, the study aimed at exploring growth in knowledge among pre-service ML teachers. However, due to a relatively small set of data in practice, it was hard to make claims relating to development based on this data set. Rather, snapshots at the level of practice have provided useful information in terms of understanding the teachers' knowledge development in the CLM course where more detailed data at the level of mathematical and contextual development has been analysed.

Since my focus was on teachers' mathematical working, the emphasis was on teaching episodes informed by the tasks utilized in the lessons. Due to this focus, only teaching episodes which were characterized by whole class discussion where the teacher either did some problem solving or provided commentary related to solutions given by learners were analyzed. This means that work done by learners where the solutions were neither presented to the whole class nor attracted comment from the teacher for the benefit of the whole class, has not been considered in this analysis. As noted in the methodology chapter, the video camera was focused at the teacher in front of the class and field notes only targeted at what the teacher said or did within the context of whole class teaching and learning.

In relation to the post-lesson interviews, the semi-structured questions which were used, informed the initial categories which were used for analysis. According to Denzin and Lincoln (2005), the purpose of interviews is to gain insight, understandings, and perspectives of the interviewee's own experiences or knowledge on certain issues. In this study, post-lesson interviews were conducted in order to understand the teachers' rationales relating to instructional decisions in terms of pedagogical agendas advanced in the classroom. Questions asked during the interviews were linked to the specific lessons taught but at the same time maintained the general foci on the nature of instructional tasks and related pedagogic approaches. The question items for the interview (see Appendix C) were specifically focused on one major aspect as indicated in italics below. This focus during post-lesson interviews was consistent with the study purpose – exploring connections across the pre-service teachers' working.

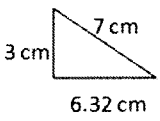
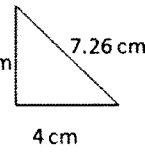
- *Justification of teaching approach:* Why did you approach your lesson the way you did (i.e. from context to formal mathematics or operating within mathematics throughout or operating within the situation itself throughout)?

Selected excerpts are used to exemplify each teacher's responses to the above question within each teacher's accounts presented later in this chapter. I now present results for each of the four participants. Within the accounts of the individual participants, results from observation data are presented followed by results from video data for each of the years 2011 and 2012. This presentation provides a chronological narrative of events. Results from interviews were also presented, followed by a chapter summary at the end.

6.2 Lindiwe's practice

Lindiwe utilized 7 tasks within whole class discussions during her teaching across 2011 and 2012. Of the 7 tasks, four tasks appear to be intra-mathematical and three are extra-mathematical. Table 6.1 provides summaries of the lessons observed and video-recorded and information about the nature of tasks utilized within these lessons.

Lesson	Summary of Lindiwe's lessons
Lindiwe's lessons in 2011	
Lesson 1 (observation)	The following examples were used in the lesson; LW1: Sketch a graph of the function $y = -x^2$ LW2: Solve the following quadratic equation $2x^2 - 5x - 3 = 0$ In the lesson preamble the teacher asked learners to differentiate between linear and quadratic equations. A hand was raised and the teacher invited the learner whose hand was raised to write her answer on the white board. Two equations were presented on the board by the learner as follows; $\text{Linear} = y = mx + c$ $Q = ax^2 + bx + c = 0$ The teacher accepted these answers as correct. The teacher then noted that graphs were useful in real life. Her reference to graphs appeared to be linked to the first example (LW1) which focused on sketching a quadratic 'graph'. She mentioned Newspaper articles and survey reports as some of the situations where the idea of graphing features, at a very generic level. Solving task LW1 involved drawing a table of values with the teacher deciding on the range of values ($x \in [-4, 4]$) and number of columns, without providing a rationale. Corresponding y-values for $x = -4$ and $x = -3$ were filled in the table by the teacher and learners were told to complete the table followed by sketching graph. The teacher then moved around the class supervising the learners' work. In terms of task LW2, the teacher started by writing the quadratic formula on the board followed by listing down of quantities from the mathematics context, while referring to her lesson notes. The quantities were then substituted into the quadratic formula by the teacher with some errors. After being interrupted by a learner, the substitution was corrected. The teacher continued referring to her lesson notes while enacting the solution procedure, which led to correct mathematical results. After these two examples, similar tasks were given to the learners as a class exercise.
Lesson 2 (Video)	The lesson utilized the following tasks; LW3: The sum of two numbers is 56 and the difference between the numbers is 22. Find the two numbers. LW4: The cost of the theatre tickets for 4 adults and 3 children is £47.50. The cost for 2 adults and 6 children is £44. How much does each adult and

	child ticket cost? The lesson started with the teacher reading the first task (LW3) from a slide. Another slide was displayed showing explanations relating to selecting information from the problems to set up equations. Learners were involved in suggesting what the two simultaneous equations should be. After the learners' feedback, the teacher displayed the third slide showing the following two equations. $a + b = 56$ $a - b = 22$ The teacher did not explain the need for 2 equations. The third slide was accompanied by explanations focusing on identification of problem quantities and how these fed into the model (equations). The next slide showed the solution method. A similar approach was used for the second example (task LW4). Unlike the first example, task LW4 included quantities which referred to world objects like 'theatre tickets' and related costs. In this example the solution method was preceded by these equations; $4a + 3c = 47.50$ $2a + 6c = 44$ Errors were noted at the level of enacting the procedure especially relating to the second example (LW4).
Lindiwe's lessons in 2012	
Lesson 1 (observation)	The lesson focused on these tasks; LW5: Show whether the following triangle is right angled a)  b)  The lesson started with the teacher asking learners what they had learnt in the previous lesson. One learner's response suggested that the notion of Pythagoras theorem was discussed and applied in solving problems involving right-angled triangles. The teacher then asks learners to think about how the idea of Pythagoras theorem would be used to show whether a triangle was right-angled. There was silence in the classroom. Reacting to the learners' silence, the teacher demonstrated on the board how the problem could be answered. Pythagoras theorem was introduced and written on the board by the teacher as; $Hyp^2 = S^2 + S^2$ where 'Hyp' and 'S' were stated as hypotenuse and side respectively. The problem quantities (dimensions of triangle) were substituted in the equation above, starting with task LW5a. After simplification, using a calculator, the numerical values from both sides of the equal sign were compared and Lindiwe stated that the triangle was right-angled. Task LW5b was solved in

		similar ways but learners were more involved in solving this task than in the first task. Similar tasks were given towards the end of the lesson as class exercise.												
Lesson (Video)	2	Two tasks are used in this lesson; LW6: Assume that each time a woman has a baby; she has 50% chance of having a boy and 50% chance of having a girl. a) if a woman has two children, draw a tree diagram to show all the possible outcomes in terms of the gender of the two children b) if a woman has two children, what is the probability that both her children being boys. LW7: A travel agent plans trips for tourists from Chicago to Miami. He gives them three ways to get from town to town: airplane, bus, train. Once the tourists arrive, there are two ways to get to the hotel: hotel van or taxi. The cost of each type of transportation is given in the table below. <table><tr><td>Transportation</td><td>Cost</td></tr><tr><td>Airplane</td><td>\$350</td></tr><tr><td>Bus</td><td>\$150</td></tr><tr><td>Train</td><td>\$225</td></tr><tr><td>Hotel van</td><td>\$60</td></tr><tr><td>Taxi</td><td>\$40</td></tr></table> a) Draw a tree diagram to illustrate the possible choices for the tourists. Determine the cost for each outcome. b) what is the probability that a person’s trip cost less than \$300 c) what is the probability that a person’s trip costs more than \$350 The lesson started with a discussion of the contextual features relating to task LW6. Learners were asked to provide their reasoning around the chances of giving birth to a male or a female baby, of which the learners answered in a chorus 50-50. The teacher then instructed the learners to be in two groups of four, within a class size of 8, where they could engage with the task and present their results after 7 minutes. The results from the two groups (Team Literacy and Team Pirate) were presented with similar errors, but these errors were accepted by the teacher as correct. The errors occurred when learners were drawing the tree diagrams, a solution method proposed in the question. The teacher’s discussion relating to features within the second task (LW7) focused broadly on modes of transportation and later narrowed down to features provided in the given contexts. Unlike the group work utilized in the first lesson episode, the second lesson episode adopted a lecture method. The teacher dominated the class discussion relating to providing solutions for the task, with reference to the class notes. Like in the first episode, her working in this task involved some errors. These errors occurred when	Transportation	Cost	Airplane	\$350	Bus	\$150	Train	\$225	Hotel van	\$60	Taxi	\$40
Transportation	Cost													
Airplane	\$350													
Bus	\$150													
Train	\$225													
Hotel van	\$60													
Taxi	\$40													

	selecting quantities from previous steps within the procedure. Towards the end of the lesson, some tasks with a similar focus were given as class exercise.
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Table 6.1: Lindiwe's lesson episodes

Since the study focused on the nature of mathematical working within practice, I draw from the categories identified and discussed in the conceptual framework (chapter 3) in order to make sense of this data. A discussion focused on lesson episodes is provided below.

6.2.1 Analysis of Lindiwe's 2011 teaching experience

Translation process: model formulation and devising solution plan

Across the 4 tasks used in 2011, the translation process at the level of model formulation (or devising a plan) took the form of choosing the range of values (for graphing) (LW1), or choosing formulae (theorem) (LW2) including selecting information from the mathematical context to be substituted into the formulae (theorem), and formulating equations (LW3 and LW4).

Solving LW1 instance involved transforming functional representations from an equation, to tabular and then graphical forms. The range of values ($x \in [-4, 4]$) was decided by the teacher without providing a rationale for this selection. This was followed by the drawing of a table of values where all x -values were filled in as shown below.

x	-4	-3	-2	-1	0	1	2	3	4
$-x^2$									

The table of values shows that the numbers were arranged in 'standard' ascending order, suggesting that Lindiwe knew how to use tables of values and correctly represent this information, though not in ways that communicated rationales for her decisions.

Regarding task LW2, Lindiwe introduced the quadratic formula on the board as;

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

which was followed by identification of the quantities from the problem (quadratic equation).

While referring to her lesson notes, she identified the values for a , b , and c , as follows;

- coefficient of x^2 as a , $a = 2$

- coefficient of x as b , $b = -5$
- the number before the equal sign as c , $c = -3$

This working was followed by her emphasis on learners listing the contextual quantities before these were substituted into the formula, although without providing a rationale for doing so. Without involving learners, the quantities were substituted into the formula by Lindiwe as follows;

$$x = \frac{-5 \pm \sqrt{-5^2 - 4(2)(-3)}}{2(2)}$$

Lindiwe's substitution is incorrect in terms of translating the value of b which was negative. Errors relating to substituting contextual quantities in formulae were also noted within her working in the CLM course. This incorrect substitution was spotted by one of the learners in the classroom who proposed that 'brackets' be introduced where -5 had been written in the formula. At this stage, the teacher did not seem to realize what had gone wrong with her substitution until the learner was invited to do the corrections on the board. After the correction, the equation was written as;

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

Lindiwe's incorrect substitution coupled by failure to recognise her own mistake when interrupted by the learner suggests a gap in her mathematical understandings related to substitution in cases where negative numbers were involved. Thus Lindiwe's skill relating to model formulation appears to be strong within tabular translation and less so within substituting negative values into formula. Although Lindiwe's competences relating to selecting both formulae and contextual quantities appear to be strong, rationales for her decisions within ML teaching – key to learners' understanding of the problem solving process was absent. More generally, Lindiwe's supervision of learners' work in the classroom relating to LW1 provided a classroom opportunity for learners to have personalised feedback on their problem solving.

The examples used in the video lesson focused on the notion of simultaneous equations. Lindiwe's translations relating to task LW3 involved identification of variables to represent the contextual (mathematics) quantities – in this case the two unknown numbers.

Lindiwe: the following says [points at the problem on a projector slide]; The sum of two numbers is 56 and the difference between the numbers is 22. Find the two

numbers. Does anyone have an idea about how you would find the two numbers?'

[...]

Learner: you can say $x + y = 56$

Lindiwe: $x + y = ?$

Learner: 56

Lindiwe: ok. and then?

Learner: you can say $56 - 22 = x - y$

[...]

Lindiwe: We will call the numbers a and b to actually work it out. ... So we will still call the numbers a and b. [reads on from the slide: We can use the given information to write a pair of simultaneous equations in terms of a and b]. So what we do is [shows $a + b = 56$ on a slide] a plus b equals 56, that will be your first equation. Alright so you can copy this down and write it as your first equation. Put number one, first equation. You don't have to copy everything down, just copy a plus b first equation. Ok then your second one will be [shows $a - b = 22$ on a slide] a minus b equals 22, that's your second equation.

The excerpt shows a correct formulation of the pair of equations although Lindiwe provided no rationale for choosing a and b as the feedback from learners suggests that x and y were proposed as the unknowns representing the two numbers. There was also an emphasis relating to formulating two equations (simultaneous equations) ignoring the second equation proposed by one of the learners ($56 - 22 = x - y$) which included the two numerical quantities given in the context. This suggests that Lindiwe was unable to move from her own formulation to consider learner offers.

In contrast, the choice of the unknowns was justified in task LW4:

Lindiwe: Now what we will do is, we will actually say a plus c, why we would say a plus c, because now we don't know. [...] We will substitute this a, but we just say because we need variables, we will say a and c alright. So now what we do here is we say 4a because there are 4 adults plus 3c because there are 3 children equals 47 rand 50, well not rand, pounds, and then we will say for

this one, what will we say for this one?[refers to the second statement of the problem]

Learner: 2a plus 6c

Lindiwe: 2a plus 6c [shows slide] so what they did here, the question is [reads from slide]. How much does each adult and child ticket cost? So let's call the cost of an adult's ticket a and the cost of a child's ticket c. We can write $4a + 3c = 47.50$ (equation 1); $2a + 6c = 44$ (equation 2). Please write that down. So now, 4a, where do you think 4a comes from, the lady at the back

Learner: The adults number of tickets

Lindiwe: The adults number of tickets, which we don't actually know how much the adult tickets cost right? And the 3c we don't actually know how much the children tickets cost. So what we, they have only told us that together that amount gives us 47.50. So now we need to figure out how much did actually the ticket cost for an adult and how much did the ticket actually cost for those three children. Is it clear now?

The excerpt above shows inter-connections between the contextual features and choices of variables for the mathematics model. The emphasis in the excerpt; '*So now what we do here is we say 4a because there are 4 adults plus 3c because there are 3 children*', suggests Lindiwe's confidence relating to translation. Learner feedback appears to point towards some understanding in terms of formulating simultaneous equations. Furthermore, the second example shows continuity from the first example on the basis of simultaneous equations, not on context. This contradicts claims made in the ML curriculum in South Africa which suggest that continuity at the level of context features centrally. The tasks utilized by Lindiwe in 2011 also shows that no formulas were provided (i.e. quadratic formula) within the given problems, suggesting that learners needed to retrieve formulas during problem solving. While the CLM course emphasized deciding on appropriate formula within problem solving, the ML curriculum specifications suggest that formulas would be given within problems. As in LW3, her working in this example shows a jump into formulating a mathematical model where mathematical language was used. In terms of Graven and Venkat's (2007) pedagogic agendas, this suggests that a mathematics content frame dominates overall.

Solution process: intra-mathematical work

The graphing task (LW1) was concerned with completion of table of values followed by the actual graph sketching, a part which was done by the individual learners, providing less detail relating to intra-mathematical working. In this example, the teacher completed two values accurately. Since completing these values involved calculations, this was an aspect of intra-mathematical working. However, given that much of the intra-mathematical working in this task was completed by the learners, it fell outside the scope of this analysis.

In relation to task LW2, Lindiwe's intra-mathematical work involved simplifying

$$x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$$

to obtain the final answer

$$x = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5}{4} \pm \frac{7}{4}$$

The final answer was not preceded by explanation relating to how the numbers were simplified as the teacher referred to the lesson notes before writing on the board. While there was lack of explanation within the intra-mathematical work in this task, there appeared to be mathematical coherence across the operational steps leading to the correct mathematical answer.

Lindiwe's intra-mathematical working across the video lesson in 2011 indicates numerous attempts to explain the steps in the solution procedures. Engaging with task LW3 for instance shows that procedure steps were explained. Her move from the formulated simultaneous equations ($a + b = 56$; $a - b = 22$) follows;

Lindiwe: Ok! Adding these equations it gives 2a equals 78. And now I am going to show you how it got to here. So what you do is because we want to get actually a and b to actually give us 56 for the first one and a minus b which is 22, we actually want to work out 56, how we actually get to the answer which is 2a equals 78. So what you do is, ok in your brackets we actually take, so it will be 22 plus b, close bracket plus b equals 56 [writes $(22 + b) + b = 56$] because now we try to actually work out what a is, that's why we are substituting with 22 plus b. so alright from this step does anybody know what we actually do here? Anybody with an idea of what we should do here?

Learner: We times what the brackets are.

- Lindiwe: So what would that be? Because we have to get 22 to 56 so what we do is because of the two b's they will give us? $2b$, because $b + b$ gives $2b$, ok, and then because we want to get this [points at 22] to the other side, there will be a minus in front of 22 and it will be 56 minus 22, equals 34 [writes $2b = 56 - 22 = 34$]. Now because we want b to be on its own, yes [recognizes a hand from a learner]*
- Learner: Divide 2 by 34*
- Lindiwe: 2 by 34 and this side?*
- Learner: $2b$ by 2*
- Lindiwe: Then what's my answer?*
- Learner: 17*

Within the context of explaining the procedure steps, the excerpt shows that Lindiwe started by adding the two equations suggesting that she was utilizing elimination method. However this method was abandoned prematurely as she drew the learners' attention to another method (substitution) without providing a rationale for doing so. The substitution method was successfully employed until the correct answer was obtained. However, an error in her explanation was noted relating to dividing two numbers. Dividing 34 by 2 in the written statement was accompanied by Lindiwe's explanation relating to dividing 2 by 34. Although the error originated from the learner, Lindiwe was echoing the same error suggesting that she did not notice that what the learner said would result in a fraction (less than 1) and not a whole number as indicated in the written statement. The presence of slips and errors combined with premature termination of one solution method in favour of another within practice was interesting as these may disrupt learners' understandings.

Lindiwe's intra-mathematical working relating to task LW4 was accompanied by explanations. After the model was formulated ($4a + 3c = 47.50$; $2a + 6c = 44$), Lindiwe employed substitution method, as follows;

- Lindiwe: So it will be $2a$ equals 44 minus $6c$ [writes $2a = 44 - 6c$]. Please follow with me. Now what they did they divided 2 so we divide that by 2 [divides 44 by 2], that by 2 [divides $6c$ by 2]. And then it would actually give me [writes $2a = 22 - 3c$]. Now moving from that we can actually say that a equals 22 minus $3c$ [refers to lesson notes and writes $a = 22 - 3c$]. Now we need to*

work out $4a$ plus $3c$, remember that's the first one they gave us. So that was $4a$ plus $3c$ equals 47 comma 50 [writes $4a + 3c = 47.50$]. Now working with that, because we want actually to work out what 4 was. We take 4 bracket, substitute a , which was 22 minus $3c$ equals 47. 50 [writes $4(22 - 3c) = 47.50$; the term $3c$ is left out]. Now if I have to substitute, what would then my sum be? The ladies at the back, give me my answers.

Learner: 88 minus $12c$ equals 47.50

Lindiwe: 88 minus $12c$ equals 47.50 [writes $88 - 12c = 47.50$]. What would I have to do? I need $12c$ on its own.

Learner: Take 88 over

Lindiwe: So it would be

Learner: 47.50 minus 88

The excerpt (first part) shows that after dividing the equation $2a = 44 - 6c$ by 2 Lindiwe writes $2a = 22 - 3c$ which was later written as $a = 22 - 3c$ after lesson notes were referred to. Another error noted in the excerpt was related to incorrect selection of information from the previous step in the procedure. After obtaining $a = 22 - 3c$ from one of the equations, Lindiwe attempted to substitute this result into $4a + 3c = 47.50$ in order to find the value of c . However, this substitution went wrong as $4a = 47.50$ was selected leaving out a term $3c$. The resultant substitution became $4(22 - 3c) = 47.50$. This omission resulted in an incorrect mathematical answer, an aspect which was not noticed by both learners and the teacher. In several of these tasks, Lindiwe's teaching indicates gaps in her ability to produce logical deductive steps in working.

Translation process: interpretation and checking results

Results related to the first two tasks (LW1 and LW2) utilised in lesson 1 show that interpretation of mathematical answers in relation to the intra-mathematical contexts did not feature within the solutions. Of the two examples which focused on simultaneous equations, an interpretive aspect only features in one case, where interpretation is with respect to the intra-mathematical situation.

Lindiwe: So now, so I have showed you another way of doing it [slide shows $2a = 78$; $a = 39$]. You have actually worked the whole thing out. As you can follow [shows a slide written: Substituting $a = 39$ into the first equation gives

39 + b = 56 . Subtracting 39 from both sides: = 17. So the two numbers are 39 and 17]. We can check these solutions by substituting them into the second equation a - b = 22. But we are not gonna do that alright [slide shows 39-17=22].

After obtaining the mathematical result in LW4, Lindiwe attempted to address the question by providing the conclusion; '*so the two numbers are 39 and 17*'. She further noted that the obtained answer could also be substituted back in the original equations which were formulated as a result of translating the original context. Although she indicated to learners that she could not demonstrate how the checking of results was done, her next slide showed this aspect.

Another form of interpretation is concerned with inclusion of units. Task LW4 for instance was situated in a theatre ticket sales context where the idea of currency became very critical. In this case, the key question was concerned with finding the unit cost of an adult's ticket and a child's ticket. This implies that the final answer needed to have units in order to ensure its location within the context. However, Lindiwe's mathematical working related to this task shows that the units (£) were not introduced at the end of the solution procedure thereby delinking the answer from the original context. This suggests a lack of sense of context during Lindiwe's problem solving in her ML teaching, reinforcing the content-drive agenda.

6.2.2 Analysis of Lindiwe's 2012 teaching experience

Translation process: model formulation

Lindiwe had no difficulty with model formulation related to tasks utilised within the observed lesson in 2012. Pythagoras theorem was identified as a useful tool in determining whether the given triangles were right-angled. The theorem was given as:

$$Hyp^2 = S^2 + S^2$$

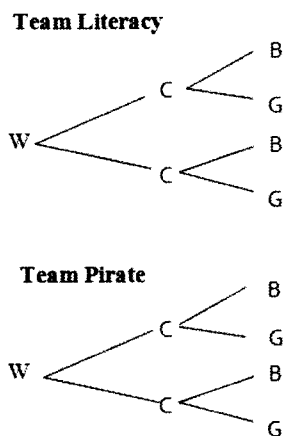
And substituting values from task LW5a resulted in a mathematical statement given as:

$$7cm^2 = 6,32cm^2 + 3cm^2$$

Mathematically, her use of Pythagoras theorem suggests that both shorter sides were the same. However, the substitution indicates awareness that the 'S's referred to two shorter sides. While the statement could be read as consisting of numbers with units (cm^2), it appears that her intention was to write $(7\text{ cm})^2 = (6,32\text{ cm})^2 + (3\text{ cm})^2$, something which became

evident in the next step (see sub-section below). Similar working relating to LW5b was adopted by the teacher.

Across the video lesson in 2012, the idea of tree diagrams within the context of solving probabilities featured centrally. Task LW6 was translated by learners who were assigned into two groups whose names were Team Literacy and Team Pirate. The answers provided by the two groups are provided below:



The learner who presented on behalf of team literacy provided this explanation:

Team Literacy: this is our tree diagram [learner refers to drawing on board]. There is 50% chance of a boy and 50% chance of a girl, right? So, W stand for woman, C stand for chance, B stand for boy and G stand for girl. Now probability, the probability of two children, both boys, there are two boys, one, two [learner refers to 2 B's in the tree diagram]. So our answer is 2 over 4 [learner writes $P(\text{both boys}) = \frac{2}{4}$ on board while referring to the notes].

Lindiwe: is that correct?

Class: yes

Lindiwe: that's correct. Another group! present your solution.

[...]

Lindiwe: attention please! You know what, both solutions are correct

The excerpt shows that Lindiwe accepted the learners' translation as correct, despite errors in the learner's explanation relating to the way in which the boys and girls were represented on

the tree diagram. This suggests that the phenomena were treated as mutually exclusive, rather than independent events.

Despite accepting the incorrect formulation of the drawing in task LW6, Lindiwe provided a correct tree diagram for task LW7 which was similar to task LW6. The correct formulation in LW7 was achieved through constant reference to the lesson notes across the episode. At the level of formulation, Lindiwe's working suggests lack of confidence in her ability to work with tree diagrams in classroom setting, although she produced correct model in prior independent working.

Solution process: intra-mathematical work

Similar solution procedures were adopted in tasks LW4 and LW5. After translating information from the mathematics context into Pythagoras theorem, the numbers on both sides of the equal sign were squared, simplified and compared. The two intra-mathematical steps relating to task LW3, provide a sense of how Lindiwe engaged with these tasks.

$$7cm^2 = 6,32cm^2 + 3cm^2$$

$$49cm = 48,94cm$$

The first mathematical statement shown above followed immediately after Pythagoras theorem was introduced, meaning that the numbers had just been translated from the intra-mathematical context (triangle) into the theorem. The teacher then concluded that the triangle was not right-angled because the two numbers were different. She noted that unless the two numbers were equal, the triangle could not be right-angled. This conclusion suggests a conceptual understanding of the idea of Pythagoras theorem, in that her talk communicated her awareness of when the theorem holds, rather than just how to use it.

As in 2011, Lindiwe's teaching in 2012 using tasks focused on probabilities (LW6 and LW7) revealed further disruptions. After learners had engaged with task LW6 in two groups, they both gave $P(\text{both boys}) = \frac{2}{4}$ as an answer which was a correct translation from the incorrect tree diagram both groups had drawn - since there were 2 B's out of the four possible outcomes. The learners' answers then attracted a comment from the teacher noting that the answer was correct and further noted that the answer could also be written as a percentage, she equated $\frac{2}{4}$ to 25%:

$$P(\text{both boys}) = \frac{2}{4} = 25\%$$

Although $\frac{2}{4} \neq 25\%$, Lindiwe's percentage answer is mathematically correct in the context of the original question. This suggests that she had some knowledge of the correct answer but she was unable to make links between the result provided by the learners and her answer, an aspect of weak intra-mathematical connections, and possibly relating to lack of confidence in the classroom setting. Furthermore, the teacher's affirmation of the learners solutions as correct and suggesting that $\frac{2}{4} = 25\%$ implied gaps in her mathematical knowledge related to using tree diagrams as a strategy for solving probability problems as well as linking fractions with percentages. It should be noted that similar errors were also noted within the context of engaging with LW7 which focused on calculating probabilities relating to using public transport.

Translation process: interpretation/validation

Tasks LW5a and LW5b required some form of interpretation in relation to the intra-mathematical problem context (right-angled triangles), a component which was accurately dealt with by Lindiwe in her working. Since these tasks were focused on determining whether the triangles were right-angled, the intra-mathematical working needed to inform a commentary relating to the nature of the triangles. Furthermore, although tasks LW5a and LW5b contained units, disruptions relating to the use of these units observed within the intra-mathematical work did not affect the conclusion, as this aspect was informed by numerical quantities. In these examples, Lindiwe obtained correct results after taking the squares. However, the correct units (cm^2) were dropped in favour of cm without providing a rationale for doing so.

The interpretive aspect was absent within the tasks which focused on calculating probabilities. The solution procedures in these tasks were terminated after the probabilities were calculated.

6.2.3 Orientation to ML teaching

Across the two years, Lindiwe introduced her lessons in two ways. First, in cases where intra-mathematics tasks were utilized, there was a jump from the intra-mathematical context to formulating mathematical models. Only in one case (LW1) did Lindiwe attempt to provide some connection between the mathematical idea (graph) and situations where this idea was useful – though at a more generic level. Second, in instances where extra-mathematics tasks

were utilized, very brief discussions of contextual features were included in the lesson preamble before moving into a step where mathematics models were formulated. In both cases, Lindiwe involved learners in translating contextual quantities into some models, with errors in some cases. This means that there was little time spent on discussing the context in favour of model formulation. At the intra-mathematical level, like the model formulation stage, Lindiwe involved learners at the level of answering questions within problem solving. However, her working exhibited disruptions in many cases, despite continuous reference to her lesson notes. These errors were associated with her inability to produce logical deductive steps in her intra-mathematical working (Hall, et al., 1989), an aspect of intra-mathematical connections (Mhlolo, Venkat, & Schäfer, 2012; Venkat & Adler, 2012). Similar errors were also noted and highlighted within the CLM course assessment tasks' analysis (Chapter five). Further, her working and explanation at this level were not related to features of the problem situations, suggesting a more mathematical focus. Regarding interpretation of mathematics answers, this aspect did not feature in either her written working or her explanations in many cases. There were also instances where units were involved and these were dropped at the end of the problem solving process. Drawn from Lindiwe's mathematisation within practice, her working adopted a strongly mathematical orientation (Graven & Venkat, 2007a).

6.2.4 Lindiwe's justification of her teaching approach: interviews

Results from interviews across 2011 and 2012 have revealed that rationales for teaching can be grouped under two headings:

- I teach how I was taught
- Approach adopted from the supervising teacher

These two categories have been briefly discussed and evidence from the data has been provided below.

I teach how I was taught

Despite the theories of teaching and learning learnt during professional development focused on ML teaching, there is a sense suggesting that some teaching were being influenced by Lindiwe's high school teachers.

Researcher: Your approach was confined within the domain of mathematics, and there were limited or no connections with learners' lives, why?

Lindiwe: That's how I was taught. So I assumed that's the way you should do it.

Interestingly, the lesson which preceded this interview was presented after the teacher had enrolled for a method course at the University where theories related to ML pedagogy were covered. Despite enrolling into the method course, Lindiwe's response suggests that she was resisting the teaching and learning strategies emphasized in the CLM course which specifically addressed pedagogic issues within the context of ML teaching and learning. This excerpt was taken from a post-lesson interview which followed Lindiwe's teaching where quadratic equation was utilized. The fact that the CLM course did not cover any materials related to solving quadratic equations prior to this lesson may provide a rationale for Lindiwe adopting the teaching style of her high school teacher. However, it also points to a broader lack of transfer of course advocated pedagogies.

Approach adopted from the supervising teacher

Another dilemma faced by pre-service teachers relates to whether they should divert from the pedagogic orientations often advanced by the supervising teachers during their teaching experience. This constituted another constraint where pre-service teachers were bound to adopt teaching strategies from their mentors given a short period in which they had to teach in the schools. Lindiwe's response to a question aimed at understanding her rationale for adopting a more mathematical agenda despite using a contextualized task appears to suggest that she was following the teaching tradition of her supervising teacher.

Researcher: Your approach was confined within the domain of mathematics, why?

Lindiwe: Because she [supervising teacher] just started with them at the beginning of the week, or last Thursday, I don't know but it hasn't been long when they have done this, so this is where they are at, at the moment, so I thought that if I can recap what they have...because next week I think they only have up to Wednesday then they start exams.

The above extract also suggests that Lindiwe's teaching approach was aimed at preparing learners for oncoming examinations. Changing the pedagogic approach would therefore imply that learners would approach the question items in the examination differently from the preferred approach of the supervising teacher.

6.3 Mark's practice

Seven tasks were utilized across lessons that Mark taught in 2011 and 2012. Details about these tasks and summaries of corresponding lesson episodes are presented in table 6.2.

Lesson	Summary										
Mark's lessons in 2011											
Lesson 1 (observation)	Two tasks were utilized in the lesson; MK1: Task based on activity focused on drawing the classroom on A3 size paper to scale. The scale was given as 1:30. The dimensions of the class and A3 paper were 720cm X 720cm and 42cm X 30cm respectively. MK2: Task based on map reading. Johannesburg central map (see appendix D) was used as reference for the activity. a) Give the degrees, latitude and longitude of the points marked A, B, and C. b) What is the closest road to 26°11'30"S and 28°02'30"E? c) Using the scale, write down the length and breadth of Joubert Park? The teacher started the lesson by making reference to the previous lesson where learners had measured and recorded the dimensions of the classroom. After this announcement, a worksheet drawn from the learners' textbook was distributed in class. The worksheet showed that the scale (1:30) was specified within the task. The teacher then reminded learners of the classroom measurements which were taken by the learners as follows; <table> <tr> <td>Classroom:</td><td>720cm X 720 cm</td></tr> <tr> <td>A3 size paper:</td><td>42cm X 30 cm</td></tr> <tr> <td>Desks:</td><td>54cm X 42cm</td></tr> </table> Using the above measurements together with the scale, Mark calculates the dimensions of classroom and desks on paper as follows; <table> <tr> <td><u>Class</u></td><td><u>Desks</u></td></tr> <tr> <td>$720 \div 30 = 24$</td><td>$54 \div 30 = 1.8$</td></tr> </table> After the calculations, learners were told to go into groups of three where their focus was to draw the classroom on an A3 size paper containing 30 desks. The teacher emphasized drawing the classroom to scale. Whilst the learners were busy working in groups, the teacher was seen walking around the class checking the learners' working and continuing to emphasize that the drawing must be to scale. The second period focused on map reading (task MK2). The first part of the task engagement was characterized by discussion of contextual features with a focus on defining terminologies like scale of a map, longitudes, and	Classroom:	720cm X 720 cm	A3 size paper:	42cm X 30 cm	Desks:	54cm X 42cm	<u>Class</u>	<u>Desks</u>	$720 \div 30 = 24$	$54 \div 30 = 1.8$
Classroom:	720cm X 720 cm										
A3 size paper:	42cm X 30 cm										
Desks:	54cm X 42cm										
<u>Class</u>	<u>Desks</u>										
$720 \div 30 = 24$	$54 \div 30 = 1.8$										

	latitudes. Reading coordinates in this activity involved addition (and/or subtraction) of 'seconds' to/from gridlines in order to locate unmarked points on the map (details have been provided in later sections). Within the context of engaging with this task, some contextual inaccuracies were observed within the teacher's explanations.
Lesson 2 (Video)	The following tasks were utilized within the video lesson; MK3: Mrs Sibayi charged R2 500 per month for the rental of a flat she owned in East London in 2002. She raised the rent every year by the same percentage as inflation. The inflation for the next three years was approximately 7% in 2003, 3,5% in 2004 and 3,8% in 2005. Estimate the monthly rental in 2005. (R2 873,83) MK4: People often get an annual salary increase that is similar to the inflation rate. A man earns R4 200 per month after his annual increase, which was the same as the inflation rate of 5,5%. What did he earn per month during the previous year? (R3 981) MK5: You read in the newspaper that the inflation rate is decreasing. Which of the following statements is or are true in this case? - Prices are not going up. - Prices are going up more slowly than before. - Prices are going down. - Prices are going up faster than before. In the lesson preamble, Mark provided an overview discussion around 'personal finance' including definitions of key terminologies which he noted that they were often encountered within the context of solving personal finance-related problems. No reference was made to specific tasks within this discussion. <i>Mark: Alright. So we're just going to keep talking about that. Alright, we're going to deal with various different ideas surrounding the whole term of finance. Right, we're going to talk about banking, we're going to talk about things like that. And in the end I've got an activity for you that you need to use the simple interest formula and some common sense, ok, because it's not all just formulas, alright – that's not what Maths is about. Alright! So personal finance, these are basically just some of the key words. Right, you've got interest – not interest as in you are keen to do it – right, interest in terms of Maths, alright, they mean different things. Ok! you've got your withdrawals, your interest rate, your stop order, your statement, your transactions, your deposits, your debit order and your bank fees. Ok! So now what we're going to do is we're just going to discuss each of them individually.</i> This generic discussion was followed by engagement with each of the tasks

	given above (MK3, MK4, and MK5). Mark's working, including communicating procedures relating to MK3 and MK5 appeared to be accurate. However, some errors within both explanations and written statements were observed within the context of engaging with MK4.
Mark's lessons in 2012	
Lesson 1 (observation)	One task was used in this lesson; MK6: Task was based on mathematising the Nelson Mandela bridge. Facts about the Nelson Mandela bridge -It is about 284m long cable stayed bridge in Johannesburg which crosses over 40 railway lines which links Braamfontein and the north of Johannesburg to Newtown in the south. -It has a 66m north-back span, 176m main span and a 42m south-back span -The north pylon is 77m high, and the south pylon is 42m high -The width of the bridge is 15m -The pylons are concrete-filled steel tubes. Approximately 4000m³ of concrete and 100 tons of structural steel was used for the bridge construction, with around 500 tons of reinforcing steel cast into the concretes. a) Estimate the area covered by the bridge b) Estimate the maximum number of cars that could be parked on the bridge, supposing the whole bridge was used as a parking area. Assume that the average car has a length of 4,5 m and a width of 1,8 m. (Hint: think about the area covered by one car, and work from there) This task was focused on calculating the area covered by the Nelson Mandela Bridge. This task was drawn from the ML textbook under the topic 'structures and designs' (Dickson, 2005). The teacher began with a contextual discussion broadly focusing on different types of bridges. Arch bridges, Suspension bridges, and Cable-stayed bridges were mentioned as some of the types of bridges and related examples were given drawing from both local and international contexts. The broad discussion of bridges was followed by a specific example of one cable-stayed bridge which was the Nelson Mandela Bridge. An iPad was then connected to a projector and used in the lesson to show learners different pictures of the Mandela bridge. The teacher used the device to search for more information on internet relating to history of the Mandela Bridge, and its ability to attract tourists. Engagement with the task involved retrieval of formula for finding the 'surface area' of the rectangular-shaped bridge and selection of information from the context to set up the procedure, leading to some mathematical results.
Lesson 2 (Video)	The lesson was focused on understanding formulas related to volume and surface area of prisms MK7: Volumes and surface area of shapes

Volume and external surface area of prisms

		VOLUME	TOTAL EXTERNAL AREA (closed top and bottom)
1.	Cylinder: Base $\odot$ 	Area of Base $\times$ Height $\odot$ $Vol = \pi r^2 H$	Perimeter $\times H + 2 \text{ Area}$ $\odot$ $= 2\pi rH + 2\pi r^2$
2.	Rectangular Prism: 	Area Base $\times$ Height $\square$ $= l \times b \times H$	
3.	Cube: 'Square' 	Area Base $\times$ Height $\square$ $= side \times side \times side$ $= s^3$	
4.	Prism with Base: Triangle 	Area Base $\times$ Height $\triangle$ $= \frac{1}{2} b \times h$	

The teacher started by distributing a worksheet containing information about prisms and some formulas among the learners. He then provided a discussion of some real world contexts where the idea of surface area was useful. A wall-painting situation which was described as similar to a situation discussed in the previous lesson was used by the teacher in this regard. He noted that the total surface area of the wall needed to be painted often inform the quantity of paint to be bought.

Mark: So if you work out the area, [...] if you work out how much, so the example that we used on Friday was if we have our classroom, what was the example I used with our classroom?

Learner: I think it was paint or something.

Mark: It was paint, ok. So if we have our classroom, this is our classroom, we want to paint all the walls in our classroom. We want to paint them blue. Ok, now we need to know how much blue paint we need in order to paint all the walls in our classroom. So what do we do? We take the length multiply it by the width, ok? And that's how, and then we get our total surface area. So how much paint will cover the walls, just the walls, will be the total surface area of the classroom, ok?

Mark's explanations included familiarizing learners with notions relating to two dimensional (2D) and three dimensional (3D) shapes with a specific focus on a triangle followed by a discussion around the difference between a pyramid and a prism.

Mark: [...] So to work out the surface area of a triangle, just a 2D triangle is half base times height. Now we need to work out how to work out the total external area of a 3D triangle, so a triangular prism, or a pyramid. Here's a question. What's the difference between a pyramid and a triangular prism?

	<i>Learner: Sir, isn't the pyramid the length is shorter than the – what was the other one?</i> <i>Mark: A triangular prism. So both have triangles in them but I want to know what the difference is. Is there a difference between triangular prisms and prisms? [...] Ok, the key difference between a pyramid and a prism, or a triangular pyramid and a triangular prism, ok, is that in a pyramid all the points meet at a single vertex. At a single point they all meet at the top. So think about the pyramids at Giza, ok, in Egypt, right, they all go up and they all meet at the top. A triangular prism, ok, they meet at an edge. So the two triangles they meet at an edge which connects, that's the idea of a prism. Alright!</i> This discussion was followed by Mark's explanations and mathematical working showing learners how to deal with situations involving different shapes where a combination of formulas was required.
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Table 6.2: Mark's lesson episodes

6.3.1 Analysis of Mark's 2011 teaching experience

Translation process: model formulation

Mark's translations in 2011 included explanations relating to how quantities were identified and selected from the contexts. Engagement with task MK1, involved selecting quantities relating to the classroom dimensions which learners had found during the previous lesson - they measured length and width of classroom using a tape-measure in groups. In the observed lesson, Mark reminded learners how the measurements were taken and referred them to the scale which was provided on the worksheet.

As in MK1, translation relating to task MK2 was focused on identifying and selecting gridlines from the given map (Johannesburg Central map):

Mark: There are 60 seconds in a minute. Now if you want to find A, then you have to look at the lines [latitude and longitude] that meet at A. Another important point when referencing, you always put north or south first. So if you are referencing something in the northern hemisphere, you start with the 'north' coordinates and then 'east' or 'west' coordinates, separated by a semi colon. If you do it the other way round you get it wrong. Similarly in the southern hemisphere you start with the 'south' coordinates. So looking at point A, there is no coordinate so we need to find

coordinates for that. So looking at the latitude line at the bottom, you got 28 degrees, 2 minutes 30 seconds E and 28 degrees, 3 minutes 0 second E [Teacher refers to longitude lines and 'E' refers to East of 'Prime Meridian', a longitude line that sits at 0 degrees]

The excerpt shows that Mark made reference to longitude and latitude lines on the map as useful handles within the context of locating particular points on a map. Since referencing using coordinates follows a particular pattern, a way of presenting coordinates in writing was explained to the learners before engaging with the task. Learners were thus warned that referencing in the northern hemisphere was different from referencing in the southern hemisphere. According to Mark, locating a point in the Southern hemisphere, starts with a latitude line (line running from east to west) followed by a longitude line (line running from south to north). Latitude lines are located either south or north of Equator (latitude line sitting at 0°) whereas longitude lines are located either east or west of Prime Meridian (longitude line sitting at 0°). This means that Mark's explanations relating to referencing were broadly accurate. However, the last line of the excerpt shows that he made reference to gridlines running from south to north as latitudes rather than longitudes. This resulted in providing readings for longitude lines first – contradicting his earlier remarks that referencing in the southern hemisphere begins with 'S'.

Across the video lesson (lesson 2), the results have shown that Mark was able to correctly identify and select quantities to set up the solution procedures in some cases. Task MK3 involved selecting percentages for each of the three years 2003, 2004, and 2005 and the rental charge for 2002.

Mark: So if we look at the first question, alright. ... Now step by step we need to work through this, right. It's given you your interest or interest rates. Right! So, what are we going to do?

[...]

Learner: Then you times it by 3,8% [other learners prompted him with 3,8%]

Mark: 3,8%? 3,8%?

Learner: Yes Sir

[...]

Mark: Are you sure? Have you read the question properly?

Learner: Yes

Mark: We'll read it again. Right! What year is the inflation 3.8%?

Learners: 2005

Mark: In 2005. What year is the question asking you to work out?

Learner: 2005

Mark: 2005, right. [...] If you look at it she charges R2 500 per month for the rental of a flat she owned in East London in?

Learners: 2002

Mark: In 2002. Alright! She raises the rent every year by the same percentage as inflation. So in 2003 she raises her percentage by?

Learner: 7%

Mark: 7%. Right! So in 2004 it's 3,5%, in 2005 it's 3,8%. So think again. Are we right when we say this? [points to the initially proposed 3,8%]

Learners: No

Mark: Why not?

Learner: Because we have to work out all the increases

Mark: Exactly. We have to work out the increase per year in order to get to your final increase in 2005. Right! So, what are we going to do?

Learner: 2 500 times 7%

Mark: 2 500 x 7%?

In the excerpt, learners select an incorrect percentage before the teacher shows them that the choice of 3,8% was incorrect to start with. Although the question focused on obtaining an increase in 2005, Mark noted the need for a focus on an increase in 2003, 2004 and then 2005, in succession. This means that the solution procedure needed to be multi-step where results from the first step needed to be used in the second step and so on. Task MK4 was dealt with in similar ways where correct quantities were selected for the procedure.

In contrast, setting up a procedure relating to task MK4 was incorrect, despite selecting the correct quantities from the context.

Mark: [...] We will do it together. Right. After his increase he earns R4 200. It's asking us to work out how much he earned before the increase. How are we going to do that? [...] How do we work that out? What was his increase?

Learner: R200

Mark: No, no, not in Rands. What was the rate of his increase? What was his interest rate, his inflation rate?

Learner: 5,5%

Mark: 5,5%. Right! So we've got 2 values, right [writes: R4 200; 5,5%]. Now we need to use the R4 200 and the 5,5% in order to work out what his amount of, what his salary was before his increase. So how are we going to do that?

Learner: Times it by 5,5%. 4 200 times 5,5 percent. Then you subtract them.

Learners: Its 231

Mark: [writes = R231] Alright. Now what do you get?

Learner: That R231 and the R4 200

Mark: [writes: = R4 200 – R231]

Learners: [call out 3 969]

Mark: [writes: = R3 969,00] Alright. Is that the correct way to do it? Does everyone agree with Glen? [name of learner]

Learners: Yes

Mark: Alright. Let's see if Glen is right. This is the correct answer. That is the correct answer [refers to R3969.00]. I don't know. I apologize; I must have had a finger filled last night. Apologies, right! So let's just ignore this [refers to the correct answer given at the end of the question]

The excerpt shows that correct quantities (i.e. R4 200 and 5,5%) were selected from the context. However, the way in which Mark dealt with the percentage (multiplying R4200 by 5.5% and subtract result from R4200), a strategy proposed by the learner and eventually adopted by Mark was incorrect. While the mathematical answer to the question was provided at the end of the problem (i.e. R3981, MK4), Mark struggled in terms of setting up a procedure which could result in the given answer. Similar errors were also observed across his problem solving relating to assessment tasks in the CLM course (see Chapter five). This implies gaps in his ability to engage with contexts demanding calculating an original number or amount before a percentage increase was implemented.

Solution process: intra-mathematical work

The task which focused on translating the classroom context into a drawing, involved some calculations at the level where measurements relating to the classroom and desks were being converted into dimensions for the drawing using a scale (1:30). While the measurements of

the classroom (720cm X 720cm), desks (42cm X 30cm), A3 size paper (42cm X 30cm) were decided upon in the previous lesson, the scale was given in the original problem from the worksheet.

Class

$$720/30=24\text{cm}$$

$$720/30=24\text{cm}$$

Desks

$$54/30=1.8\text{cm}$$

$$42/30=1.4\text{cm}$$

Mark's working suggests that the dimensions of the classroom and desks on paper needed to be 24cm X 24cm and 1.8cm X 1.4cm respectively. Learners were instructed by the teacher to go into groups of three and use the information above to complete the activity. With the inclusion of the desks in their drawing, the learners struggled to complete the activity. Although the teacher was moving around the classroom supervising the activity, more time was needed for the activity than the 35 minutes which was available for the lesson period. Furthermore the inclusion of desks (which were not fixed) in the drawing implied that learners needed to measure distances between the desks. Thus the time factor was very crucial in ensuring that the activity was successfully completed.

Regarding the 'map reading' context, the intra-mathematical work involved addition/subtraction and division in order to locate the points on the map.

Mark: [...] What do you think this line [line that passes through point A, which is between the two longitude lines] will be?

Learner: Each two blocks is 30 seconds so that the difference between the two given coordinates

Mark: That's right. The degrees aren't changing; the minutes and the seconds are changing. So there is 30 seconds difference to the next longitude line. So each block represents 15 seconds. So this line is gonna be 28 degrees, 2 minutes, 45 seconds East. Now let's do the one on the vertical. So 26 degrees, 12 minutes 45 seconds, is that correct?

Learner: no sir

Mark: no. why do you disagree with me?

Learner: because ah it should be 26 degrees, 11 minutes 45 seconds.

Mark: Sure? Did anyone see the mistake i made? [makes correction]. So it is 26 degrees, 11 minutes 45 seconds South. Now work out the answer on your own.

Learner: 26 degrees, 11 minutes, 45 seconds South; 28 degrees, 2 minutes 30 seconds East.

Mark: do you agree with her?

Learners: yes

Mark: now let's look at B. Yes [points at a learner]

The results above suggest that after noting the difference between two lines (latitude or longitude), the result, 30 seconds (30'') appears to have been divided by 2 to obtain 15 seconds (15''). However, the idea of proportionality could be very useful in these kinds of situations as it provides a general strategy for calculating a line between two gridlines. The calculated difference was then added to (or subtracted) from a nearest gridline reading in order to find the reading of the midpoint line. Further, Mark's working suggests that the coordinate for point 'A' which was given as $26^{\circ} 11' 45'' S; 28^{\circ} 2' 30'' E$ agreed with his explanation that referencing in the southern hemisphere (where Johannesburg is located) needed to start with South, and then East, since Johannesburg is to the East of Prime Meridian.

Within the explanations relating to locating points on a map using coordinates, errors were noted.

Mark: Now let's look at B. Yes [points at a learner]

Learner: I have no idea what's going on.

Mark: That's right, no problem with that. Now I am trying to get to my girl friend's house at point B. I don't know what the street name and number is. So let's find out what the coordinates are.

Learner: Do the minutes increase when you ah! ah! going down and when you are going up, does it decrease?

Mark: It depends with the map, but yes generally. In this case yes it increases as you go down and decreases as you go up.

Learner: It also increases as you go to the right.

Mark: Yes! Now let's try to get coordinates for B

Learner: 26 degrees, 12 minutes, 15 seconds South; 28 degrees, 2 minutes 45 seconds East.

Mark: correct

The excerpt suggests that coordinates can be very useful in contexts where the physical address of a particular point is not known. Although it is difficult to locate such points in practice, proportional reasoning could be useful in such cases. In terms of whether coordinates readings increase when you go down, Mark's response '*but yes generally*' suggests that he has limited knowledge relating to the pattern of latitude and longitude lines. His feedback was true within the context of the problem but not in general terms. While latitude lines increase as you go down in the southern hemisphere, they increase as you go up in the northern hemisphere. Since the problem context (Johannesburg Central map) was drawn from the southern hemisphere, the former becomes true but not in general sense. These errors appear to be linked to lack of broader understanding relating to working with maps.

In terms of the video lesson, Mark's intra-mathematical working is characterized by aspects of solution coherence even in cases where difficulties relating to setting up procedures featured. The mathematical working relating to MK4 shows that coherence was achieved across the steps within the procedure.

Mark: So if we look at the first question, alright. Ok, let's pay attention. [...] Alright! Now step by step we need to work through this, right. It's given you your interest or interest rates, right? So, what are we going to do?

Learner: 2 500 times 7%

Mark: 2 500 x 7%?

Learner: Which is equal to 175

Mark: Equals 175? Then you go two thousand – that's supposed to be a 5, plus 175 correct? [writes: 2500+175=]

Learners: Yes

Mark: Gives you?

Learner: 2 675

Mark: 2 675 [writes: $2500 \times 7\% = R175 = R2500 + R175 = R2675$]. Right! Now what do we do? Now we times this [refers to R2675]. Ok, what are we timesing by?

Learner: 3.5

Mark: But what are we timesing by 3.5%?

Learner: R2 675

Mark: Why?

Learner: Because that is the rate in 2003.

In this excerpt, Mark demonstrates his confidence relating to engaging with contexts involving multi-step methods. Despite his ability to enact the procedure, some errors relating to mathematical syntax were noted where the equal sign was incorrectly used in the written mathematical statement. Although these disruptions do not impact negatively on the mathematical results, they have the potential to affect learners' understandings of problem solving.

Mark's coherent intra-mathematical working was also evident within the context of solving task MK4. As noted already, the term coherence is used within the context of intra-mathematical working (Hall, et al., 1989).

Mark: [...] Now we need to use the R4 200 and the 5,5% in order to work out what his amount of, what his salary was before his increase. So how are we going to do that?

Learner: Times it by 5,5%. 4 200 times 5,5 percent. Then you subtract them.

Learners: Its 231

Mark: [writes = R231] Alright. Now what do you get?

Learner: That R231 and the R4 200

Mark: [writes: = R4 200 – R231]

Learners: [call out 3 969]

Mark: [writes: = R3 969,00] Alright. Is that the correct way to do it? Does everyone agree with Glen? [name of learner]

Learners: Yes

Mark: Alright. Let's see if Glen is right. This is the correct answer. That is the correct answer [refers to R3969.00]. I don't know. I apologize; I must have had a finger filled last night. Apologies, right! So let's just ignore this [refers to the correct answer given at the end of the question]

In this example, the error occurred at the level of model formulation. Mark's working indicates that there was logical presentation of the procedure until the final result was obtained. However, this result was incorrect with respect to the given context. Failure to consolidate the answer provided in the question and the result obtained within this working,

prompted Mark to apologize to the learners. By apologizing to the learners the teacher's gesture may suggest that he did not verify the answer provided in the question before the task was introduced in the classroom. It also implies that Mark's understanding regarding solving such problems was weak. In this case, the incorrect result appears to be a consequence of failure to formulate a correct model. Cases where coherent intra-mathematical working were achieved across instances preceded by incorrect model formulation were also observed across his engagement with assessment tasks within the CLM course.

Translation process: interpretation/validation

The tasks relating to classroom drawing and map reading (MK1 and MK2) involved constant reference to the original contexts. In terms of the drawing task, the focus was on drawing a classroom on A3 paper using some given scale to represent the classroom. Features like desks, distance between desks, dimensions of the drawing in relation to the actual classroom, featured centrally in this task. There was little intra-mathematical working focusing on calculating the dimensions of the drawing. Relating to the map reading context, Mark's explanations were focused on the map features as opposed to the related mathematical working throughout the problem solving process. Use of gridlines (longitude and latitude lines) featured centrally in this task, where these were used to find coordinates of points on the map. The nature of calculations (which involved proportional reasoning) was aimed at obtaining gridlines which were located between 'marked' longitude and latitude lines on the map. Across the video lesson, a similar focus was adopted where Mark appears to advance the understanding of rent payments versus inflation (MK3) and salary increase (MK4). In this way, the primacy was given to the understanding of the context, suggesting a link to pedagogic agenda 2 of the spectrum (Graven & Venkat, 2007a), but with errors.

6.3.2 Analysis of Mark's 2012 teaching experience

Translation process: model formulation

Although the original question in task MK6 was related to estimating the area covered by the Nelson Mandela Bridge, Mark's working shows that the focus was on calculating the surface area of the Bridge supported by each of the two Pylons (north and south) as surface area formula (surface area = $l \times b$) was proposed with a view to be used in the procedure. A simplified cross-section of the bridge has been given in figure 6.2.

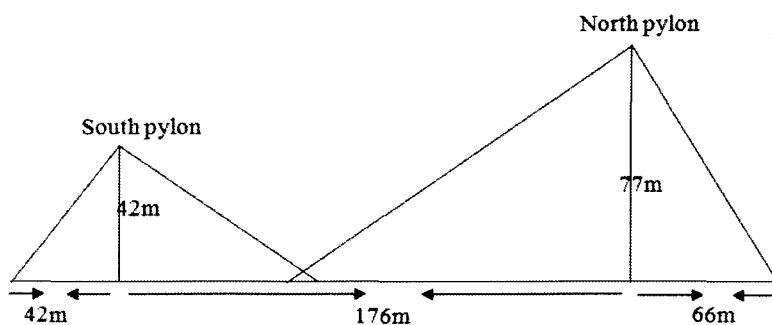


Figure 6.2: A cross-section of Nelson Mandela Bridge

In his working, he noted that the north pylon is bigger than the south pylon and that the ‘middle part was shared equally’. Equally sharing of the middle part seemed to mean that the north and south pylons supported equal distances of the bridge, an aspect which was reflected in his working (see working below).

$$176m \div 2 = 88m$$

To calculate the area supported by the north pylon, the teacher writes

$$(88 + 66) \times 15m = ?$$

To calculate the area supported by the south pylon, the teacher writes

$$(88 + 42) \times 15m = ?$$

Although Mark correctly observed that the north pylon was bigger than the south pylon, dividing the middle part equally was inaccurate. This means that the context was misunderstood and therefore affected the setting up of procedure. A closer look at the Bridge shows that the north pylon supports a larger surface area (see figure 6.2). Since the heights of the two pylons were provided, using proportional reasoning could provide some good approximation of the surface areas supported by south and north pylons. Mark’s failure to correctly set up a procedure resulted in an incorrect answer, implying the need for skills relating to setting up of procedures.

Relating to task MK7 where the focus was on deriving surface area formulas, the whole problem appears to be located within the translation process, at the level of model formulation. It involved selecting information from the mathematical context as well as choosing area formulas for different faces of the given shapes. Although formula derivation implies working with variables (abstract objects), Mark attempted to start from working with concrete objects by introducing numbers for each side of the shapes. These numbers were

used to calculate the total surface area of the shapes. The use of numbers was then replaced by variables leading to some abstract formula.

Mark: [...] That's the key thing that we're looking at now, ok, is the cube, all lengths of the cube are the same, everything is equal. So what we're going to do is just to make it a little bit easier for you to view it, to get it, to grasp it, to catch the gist, is I'm gonna measure this and say that this is 2. [writes 2 on one side of the cube] Right! So the length of one side is 2.

Learner: Ok

Mark: Ok. How do you work out the, how do you work out the area for one face?

Learner: Length times breadth

Mark: Ok, length times breadth. Ok, it's the same as everything else. So its length times breadth. So in this case it would be

Learner: 2 times 2

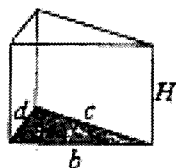
Mark: 2 times 2. Ok. How many faces are there?

Learners: 6

Rather than directly working with variables, numbers were utilised first suggesting that learners would understand the abstract formulation better with the help of numbers, as learners at this level (Grade 11) were already familiar with calculating areas using numbers. Using numbers in terms of finding the surface area of the cube provided leverage for learners to understand the abstract formulation. The use of numbers in this case was therefore in the service of the abstract formulation of formulae.

Relating to formulation of a formula for the prism with triangular base, results indicate that Mark made an attempt to explain the formulation. Further, errors associated with mathematical terms and assumptions in terms of the type of the triangle forming the base of the prism were noted. The assumption error appears to have influenced the whole formulation, an aspect which was not noticed by either the teacher or learners. I include the prism before the excerpt for reference purposes.

Prism with Base: Triangle



Mark: The rectangle. But I want to differentiate because these, the c is a rectangle as well, so I don't want to say there are three rectangles, because this rectangle, the coloured in rectangle [with side d] and the rectangles of c are different sizes, so you can't just work it out together. Ok? What is, to work out the area of a triangle, area of a triangle equals what?

Learners: Half base times height.

Mark: Half base times? Ok, quite right. Half base times height. This is to work out the area. So now, how many triangles are there?

Learner: 2

Mark: So you multiply that by 2. Now, how many rectangles are there?

Learner: 1

Mark: How do you work out the area of a rectangle?

Learner: Length times breadth.

Mark: Length times breadth. How many are there?

Learner: 3

Mark: Well, ok we've got to work. This is why I'm working out c now, I've written there c , ok. So then you've got to multiply that by 2 because it's 2, ok. Now what about for d ? So you've got, uh, you've got d as well, ok. Length times breadth times what?

Learner: Number 1

Mark: Ok, that's it: length times breadth times 1, ok, because it's the only shape of that kind in the shape. [writes: $\text{Area triangle} = (\frac{1}{2}b \times h) \times 2$; $c = (l \times b) \times 2$; $d = (l \times b)$]. So these are all the different options that we have to work out. That's how we have to work it out. [...] Ok, we need to look at this and say right, now through here what we've done here [points to the overhead] we're able to work out the surface area. Remember, we don't need to know the volume at the moment. Ok, volume involves the height. Surface area involves the amount of paint needed to paint the total exterior, exterior surface of the shape. Right, you just, you add it, you add them together, alright? That's all

you have to do. So you [adds terms together: $\text{Area triangle} = (\frac{1}{2}b \times h) \times 2 + c = (l \times b) \times 2 + d = (l \times b)$]

Although the prism had b, c, d, and H as side lengths, the excerpt suggests that Mark used some of these variables (b, c and d) to represent faces. The introduction of letters 'h' for height and 'b' for breadth within area formulae for triangle and rectangles respectively was not accompanied by explanation in terms of how these differ with the variables 'H' and 'b' given in the problem. Further, Mark's formulation assumed that the base triangle was isosceles (two sides equal), an understanding which eventually led to multiplying the area of one face (marked by c) by 2. Since the focus was on deriving a generalized surface area formula (with each side distinct from the other two), Mark's assumption may point towards weak mathematical understandings relating to setting up procedures. There was also a 'mathematics syntax' disruption at the level of relating the areas of the individual faces (a, b, and c), as the equal sign was incorrectly used, despite selecting the correct operation (addition).

Mark's working also shows that the area formulas for the individual faces were combined in one statement. In this way, a specific formula needed to deal with specific cases of shapes was derived. Despite the emphasis in ML pushing towards application of formulas when engaging with situations, there was need for learners (especially within the context of calculating surface areas of 3-dimensional shapes) to understand how such pieces of formulae could be combined in order to solve problems. The CLM course also focused on some problem contexts where formulae were not provided – implying that students needed to decide on the choice of formulae to be used within the related solution procedures.

Translation process: interpretation/validation

Across the two lessons, the interpretive aspect featured. In the episode relating to task MK6, the translation was provided where Mark observed that the north pylon supported a larger surface area than the southern pylon, and therefore the north pylon needed to be stronger. On the other hand, task MK7 was not necessarily concerned with finding numerical answers but deriving the surface area formulae for the given shapes. Due to this focus, the task was concerned with translation at the level of model formulation, as noted already, and therefore did not require some interpretive aspect in the sense of 'interpretation of mathematical results' or 'checking results' since intra-mathematical working did not feature.

6.3.3 Orientation to ML teaching

Mark utilized seven tasks across 2011 and 2012. Six of the seven tasks were extra-mathematical. Across these extra-mathematical tasks, Mark was able to spend considerable time in the lesson preamble providing discussions of the contexts. While in some cases (MK1, MK2, and MK6), the discussions were specific to the given contexts, some generic discussion were also provided (relating to MK3, MK4, and MK5). Within examples where generic discussions were provided in the lesson preamble, further connections which appeared to be specific to the contexts seemed to feature within the problem solving process. Regarding the intra-mathematical task, there was a reference to the ‘painting’ situation as an exemplification of the idea of area, still at a more generic level. A case where some discussion at an overview level featured within the lesson introduction was also noted within Lindiwe’s working. In formulating mathematics models, Mark involved learners, like Lindiwe.

Within the solution process, Mark’s working, which involved learners, continued to show connections with the context features. Words like inflation, prices, salary increase, etc were often referred to, at this level. There were also instances where Mark emphasized correct intra-mathematical working among learners. Engaging with MK3 for example, shows that learners suggested an incorrect solution method, which Mark corrected later. Mark was also able to pay attention to interpretation of solutions especially across the extra-mathematics contexts. This implies that Mark’s teaching adopted a teaching orientation that balanced the understanding of mathematics and contexts, but with errors at the level of formulation and mathematics syntax.

6.3.4 Mark’s justification of his teaching approach: interviews

Mark’s rationale for adopting a particular pedagogic orientation can be classified into two namely;

- Understanding the context
- Dealing with misconceptions

These two components have been briefly discussed below and evidence from the data has been provided to support the claims.

Understanding the context

The interview results appear to indicate that Mark's teaching was focused on finishing the activity which was introduced in the previous lesson where measurements for the classroom were taken by the learners.

Researcher: Your teaching was focused on drawing the classroom on paper using some scale, why?

Mark: So on Tuesday, we did the measuring of the classroom and we did the measuring of the desks, so all those measurements had been made before I came into the class today. So the learners came into class and started the activity because they had already done the prep for the lesson so there was no reason for me to go back and say, lets measure it again because they had already done it and then after recap and say ok as we measured on Tuesday the classroom is 720cm by 720cm and the desks are 54cm by 40cm, things like that. So I did have to recap but I didn't have to re-measure.

Since the lesson was focused on completing some activity, Mark appears to indicate the importance of recapping to ensure that there was common understanding among the learners before dealing with the last part of the activity. The measurements of the classroom and the desks did not appear to constitute the focus but were aimed at developing learners' understanding of the drawing context.

Dealing with misconceptions

Another rationale related to Mark's pedagogic orientation was concerned with assisting learners in terms of dealing with their misconceptions. Where learners didn't understand a concept due to misconceptions, Mark suggests that a second chance should be given to the learners to learn about the same concept with a focus on their misconceptions.

Researcher: Your lesson was focused on translating information from the context followed by intra-mathematical working, why that approach?

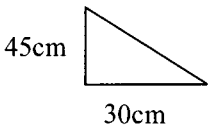
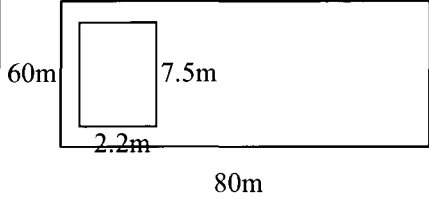
Mark: ah basically the fundamental reason why I chose that was because this is revision. So specifically what I wanted to do, was I wanted to jump straight into it because they have already learnt it, I didn't want to take them all the way back to the beginning because I thought they would get bored so I tried to make sure that it was enough contextualization in the sense that the question was contextualized, so it was in the real world, but I didn't want to go into too

much into the context and I would rather want to focus on the misconceptions that they were having and I wanted to solve their misconceptions, that was my main goal.

The excerpt suggests that in cases where the task was situated in a real-world context, discussing the features of the context would make learners become bored, a statement which appears to be in sharp contrast with emphases within ML which indicate that learners need to familiarize themselves with the contextual features before engaging with it. Furthermore the excerpt shows that another agenda in Mark's lesson was to deal with learners' misconceptions despite lack of this focus within his teaching. The lesson which preceded this interview progressed without any emphases on specific concepts, an aspect which could have provided a window into gaps within the learners' understandings.

6.4 Jabu's practice

A total of nine tasks were utilized in Jabu's lessons across 2011 and 2012. The nature of these tasks including summaries of corresponding lesson episodes are provided in table 6.3.

Lesson	Summary
Jabu's lessons in 2011	
Lesson 1 (observation)	The following tasks were utilised in this lesson; JB1: Find the area of the triangle  JB2: A hall is to be constructed with tiled floor. The tiles cost R7/m². If the hall, 80m by 60m, is to have a stage 7.5m by 2.5m, calculate the floor area excluding the area occupied by the stage.  This lesson utilized two tasks focusing on calculating the area of two dimensional (2D) shapes. In the lesson preamble, the teacher briefly discussed, in general terms, the usefulness of the notion of area in real life. Learners were involved and provided examples of situations where the idea of area was found to be useful, as follows:

	• Putting carpet in the room • Painting the wall • Buying curtains, tells you how much curtain material you need These examples were accepted by Jabu, who noted that if there was lack of understanding relating to area, 'one would buy less or more materials'. The discussion around ways in which the idea of area was used in real life was then followed by whole class discussion relating to solving task JB1. Learners were asked to provide area formula for the triangle, followed by translating the contextual (intra-mathematical) information in the formula. $A = \frac{1}{2} \times \text{base} \times h$ $A = \frac{1}{2} \times 30 \times 45$ The formulated model was simplified with the help of calculator to obtain a numerical answer. A similar approach was employed in the second example (task JB2), where the formula for finding the area of the rectangle was selected and used. Some quantitative information in this example (i.e. cost of tiles) was not included in the procedure set up. $A = l \times b$ $\text{Area of Hall} = 60m \times 80m = 4800m^2$ $\text{Area of stage} = 7.5m \times 2.2m = 16.5m^2$ $\text{Area} = 4800m^2 - 16.5m^2 = 4783.5m^2$ As in JB1The model was then simplified using calculator to obtain a mathematical answer. Tasks with similar foci were given as a class exercise after the examples.
Lesson 2 (Video)	The following tasks were utilized in this lesson; JB3: If we have 14 girls and 5 boys in this class, how do we go about calculating the percentage of girls to boys? JB4: Karen earns R77 560 and is given a 3% wage increase. And Darren earns R75 420 and is given an increase of 3.5%. a) Who received the larger increase in Rand terms? b) Who earns more after the increase? This lesson was delayed by 7 minutes because the teacher was attending staff meeting. The introduction included a discussion around the meaning of percentage and connections between the mathematical idea of percentage with ways in which percentages were used in real life. <i>Jabu: Now, if we talk of percentage we're talking of something like this [writes %]. So what comes to mind? What comes to mind if you're talking of a percentage? Yes!</i> <i>Learner: A mark out of 100.</i>

	<i>Jabu: A mark out of 100. That's good. Yes!</i> <i>Learner: A portion of something</i> <i>Jabu: A portion of something. Ok. Now if you talk about a percentage, I mean, it is all over us, it is around us, we see it every day. A percentage, it is something that is marked out of 100. The denominator is always 100 because it is a whole. Do you understand?</i> <i>Learners: Yes Sir</i> <i>Jabu: Now we see that every day in our daily lives. They always talk about percentages in newspapers; percentage, if you go to the shop something that's got VAT. [writes: 14%]. Now VAT is always marked 14% of the price that you are buying.</i> The explanations were followed by problem solving focusing on percentage of girls and boys in the classroom (JB3), and wage increase (JB4). Two solution methods relating to the second example (JB4) were provided, both of which resulted in the same mathematical answer. Similar tasks with different numerical values were given as a class exercise after the examples.
Jabu's lessons in 2012	
Lesson 1 (observation)	These tasks were utilised in this lesson; JB5: According to the 1996 census there are 1,8 million Tsonga speaking and 9 million Zulu-speaking people in South Africa. Determine the ratio of Tsonga-speaking to Zulu-speaking people. Write the ratio in its simplest form. JB6: The ratio of the distance a motorist travelled to the distance a cyclist travelled is 40:3. How far did the motorist travel if the cyclist travelled 21 km? JB7: A piece of wood is cut in the ratio 2:5. If the shorter piece is 56 cm long, how long was the whole piece of wood before cutting? The lesson introduction included a discussion around the meaning of ratio and situations in real life where the idea of ratio was useful. In his discussion, Jabu emphasized that 'ratio is concerned with two related quantities'. In terms of real world examples, Jabu provided a situation relating to running a marathon where 'one person takes 15 hours and the other takes 25 hours to complete'. He concluded that the ratio relating to running times between the two persons was 15:25. Within the discussion around ratio, a related terminology referred to as 'rate' was also introduced by Jabu and he noted that it was concerned with 'unrelated quantities'. Exemplifications relating to 'rate' included, 'a car travelling at 100km/h' and 'a snake growing at 2mm/week'. This discussion was followed by problem solving involving three examples (JB5, JB6, and JB7) where Jabu demonstrated how to engage with ratio-focused tasks within the whole class

		discussions.
Lesson (Video)	2	This lesson was focused on these tasks; JB8: I borrow R5 000 at 5% over 1 year simple interest. Calculate interest. JB9: If I had my P, which is my principal, the principal amount is R10 000. The rate, 7 percent, and this must be paid over 5 years. Find interest. This lesson was delayed by 10 minutes, due to staff meeting. Jabu started the lesson with a discussion around ways in which the mathematical ideas related to ‘interest’ were useful in the real world. He also made an attempt to motivate learners towards ML learning in a more general sense. <i>Jabu: Something that, something that hopefully you see every day and even talk about which is interest. Now, remember that you talk about things and you learn about things that are all around you. We are putting things into context. Maths literacy is a study of maths that makes sense. All the maths that we do here is maths that is very central to society. It’s the maths that people use in their daily lives. Now, if I may ask what do you, what do you understand by the term ‘interest’? If somebody says, um, ‘I have to pay interest’? Yes?</i> <i>Learner: You pay more than you ask</i> <i>Jabu: Ok, you have to pay more than you ask for. Good answer. What do you others think? Just think. You have to be able to think. It has to be, it has to come from? Yes?</i> <i>Learner: Extra amounts that you have to pay.</i> <i>Jabu: Extra amounts that you have to pay, ok.</i> Further, some errors relating to misunderstanding of some terminology often used within the context of solving problems involving interest were noted in Jabu’s explanations. The discussion around ‘interest’ was followed by problem solving (Tasks JB8 and JB9). In both cases, simple interest formula was used to set up procedures. Calculators were also used for calculations.

Table 6.3: Jabu’s lesson episodes

6.4.1 Analysis of Jabu’s 2011 teaching experience

Translation process: model formulation

Like Lindiwe and Mark, discussion in the lesson preamble was generic, and not focused on specific tasks. At the level of model formulation, results indicate that engagement with tasks JB1 and JB2 involved identification of formulae followed by substitution. In this lesson, Jabu

was able to translate information from the contexts. The area formula for a triangle (JB1) was identified as

$$A = \frac{1}{2} \times \text{base} \times \text{height}$$

followed by substitution

$$A = \frac{1}{2} \times 30 \times 45$$

Engaging with task JB2 was similar to his working relating to JB1 except that some information provided in JB2 relating to ‘cost of tiles’ was not utilised in the formulation as the question did not need this. By correctly selecting contextual information where some data was left out, Jabu’s working suggests that he was able to understand the problem context.

In terms of video lesson, similar (generic) connections between the mathematical idea (percentage) and the real world were made within the lesson preamble.

Jabu: Now we see that every day in our daily lives. They always talk about percentages in newspapers; percentage, if you go to the shop something that’s got VAT. [writes: 14%]. Now VAT is always marked 14% of the price that you are buying. If you are buying cold drink and the cold drink costs, how much does a 2 litre cost?

Learner: R10

Jabu: R15. [writes: R15,00] Now the VAT it would be 14 of the R15 that you are buying the 2 litre for. You understand that?

Learner: Sir, what is the VAT, Sir?

Another learner: Value Added Tax.

Jabu: Value Added Tax. So the government takes a certain percentage on other foods. Like, um, for example if you, if you buy cold drink and the cold drink is R15, the government will take 14% of the R15 for tax. They are taxing you. Ok?

The extract shows that in addition to discussing percentage broadly, Jabu utilized a context involving the buying of a ‘cold drink’ whose price included Value Added Tax (VAT) given as 14%. The example (cost of 2 litre cold drink) was a scenario which appears to be familiar among most learners. However, Jabu’s explanations indicate that his knowledge relating to how VAT was calculated was weak. His explanation did not take into consideration the fact that the cost of cold drink (R15) was inclusive of VAT. This meant that the actual cost of the

cold drink (VAT excluded) was less than R15, contrasting what Jabu had suggested. This error suggests gaps in his ability to work with problems, involving finding an original number before it was increased by a certain percentage. A similar error was also noted within Mark's practice, at the level of model formulation. The extract also suggests that although VAT appears to be a commonly used notion within buying and selling contexts, the mathematics involved could be misunderstood. This implies the need for explaining or discussing contextual features of situations used for ML classrooms, an aspect which was also emphasized in the CLM course.

Further, formulating a model for task JB3 involved identifying the numbers of the boys and girls in the classroom through head count before a procedure for calculating percentage was set up. This approach made the context more relevant, accessible, and authentic as classroom data was used.

Jabu: Ok let's do a percentage here in class. We are going to do a percentage of boys to girls. How many girls do you have? Girls, raise your hands! [counts the girls and writes 12]

Learners: 13.

Jabu: There are 13? Now, how many boys? [counts the boys and writes 4]

Learner: 5, there are 5, Sir

Jabu: [erases the 4 and writes 5 boys; and changes 13 girls to 14 girls]. Now, ok we are going to work on the original question. Now, if we have 14 girls and 5 boys, how do we go about calculating the percentage of girls to boys? Any ideas? Yes! Remember a percentage is always out of 100. Yes!

Learner: Sir, don't you add them together and then you get the final amount?

Jabu: Ok, we add it together and so what will be this?

Learners: 19

Jabu: 19. Ok. So 19 is going to be our denominator, right?

Learners: Yes

Jabu: Ok, so we are going to say 14 over 19 [writes: $14/19 \times 100 =$]

The extract shows some mathematical working being grounded within the context of understanding the proportion of boys and that of girls with respect to the total classroom size. However Jabu used the phrase 'calculating the percentage of girls to boys' in the original task and within his explanations as the main focus in the task, a statement which appears to

be misleading in terms of mathematical meaning it portrayed. Although the task was designed in the classroom (quantities generated through head count), there was need for choosing and using the appropriate phrases as one of the foci of ML was to develop understandings of both mathematics content and contexts, a key feature of mathematics-context frame in ML (Department of Education, 2003). Formulating question items for ML was also one of the CLM course focus, especially the Method 2 sub-course.

Solution process: intra-mathematical working

Results relating to Jabu's solutions in 2011 show both correct working and errors. One of the instances where correct intra-mathematical working featured was in JB1, whose solution was given as:

$$A = \frac{1}{2} \times 30 \times 45 = 675$$

The calculations appear to be accurate. Although a calculator was used to calculate the answer, Jabu was able to present the solution procedure in written form coherently. However, the nature of the task combined with Jabu's working, suggest that the focus in this example was mathematical. Engaging with task JB2 involved the recognition of the figure and selection of an appropriate formula. At this point, Jabu appears to take control as he told learners that the hall had a rectangular shape and began to demonstrate to the learners how to solve the problem.

$$\text{Area of Hall} = 60m \times 80m = 4800m^2$$

$$\text{Area of stage} = 7.5m \times 2.2m = 16.5m^2$$

$$\text{Area} = 4800m^2 - 16.5m^2 = 4783.5m^2$$

In this working, which was accompanied by explanations, Jabu coherently presented the solution. The explanations were focused on how to move across the steps in the procedure. Although Jabu had included information about the cost of the tiles in the initial question, this information was never used in the solution. Further Jabu's working show that he was not confused by this additional information.

The lesson episode relating to task JB4a indicates that two solution strategies were offered, both of which were correct.

Solution strategy 1

Jabu: [...] Ok we're going to first calculate what Karen received. So how do you go, do you get...?

Learner: 3 over 100...

Jabu: 3 over 100 [writes: $3/100$]. Times?

Learner: R77 560

Jabu: R77 560 [writes: $3/100 \times R77\ 560 =$]. What do you get?

Learner: You get R2 326.8

Jabu: What's that?

Learner: R2 326.8

Jabu: Two thousand? [writes: R2 326,8]. That's the cents, ok! [points to the ,8]. Now this is what the increase was. This is 3% of R77 560. Ok.

Solution strategy 2

Jabu: [...] Now another way to do this. Because you are increasing a percent, what you can do is. Listen, because you are adding 3% on top of this amount [points to R77 560]. What you can do is what is 103 percent of [writes: $103/100 \times R77\ 560$]. So we are adding the percentage, ok. This will give you 103% of this [points to R77 560]. And then after that you are going to minus the initial amount from what he got as an increase. Do you understand?

Learners: Yes

Jabu: You are adding the 3% on top. So what is 103% of 77 560?

Learner: R79 886.80

Jabu: [writes: R79 886,8] Now this is 103%. So now you are going to subtract the 100% and see what increase he got. What was the initial amount? [writes: $R79\ 886,8 - R77\ 560 =$] What do we have as an answer? It should be the same as what we got.

Learner: It's R2 326.8

Jabu: [writes: R2 326,8]. Is it the same amount that you got?

Learners: Yes

The two strategies involved learners in terms of answering questions and doing calculations. The teacher's working was also accompanied by explanations which was accurate. Since the two strategies had the same focus (calculating the actual wage increase), Jabu emphasized that the answers in both solution strategies needed to be the same. Empirical evidence within

mathematics education tends to strongly recommend provision of multiple ways of solving the same problem due to variations in learning styles among learners (Bingolbali, 2011; Leikin & Levav-Waynberg, 2008; Schoenfeld, 1983). The South African literature relating to ML implementation suggests a similar need within ML teaching and learning (Vithal, 2006).

Despite accurate intra-mathematical working across the video lessons, some errors were noted in one of the episodes.

Jabu: Ok, so we are going to say 14 over 19 [writes: $14/19 \times 100 =$]. What do we get? [class is silent]. Make calculations. I know you can do maths.

Learner: 78.68

Jabu: 78 point? [writes: 78,]

Learner: 68

Jabu: [writes: 78,7%]. Automatically you can see if you have the girls it's 78.7, so what will be the boys? So because a percentage is always over a 100, the percentage of boys will be the difference between from 100 minus 78.7. Do we agree?

Learner: Yes

The excerpt shows that 78.68% was offered by one of the learners as the answer which was eventually affirmed by the teacher as correct. However, the correct answer is 73.68%. Since the error was preceded by correct model formulation, it may suggest failure by the teacher to make sense of the answer before it was accepted. Acceptance of an incorrect answer may either be a slip or due to lack of preparation prior to the lesson presentation. Furthermore, the excerpt shows that after the learner had proposed 78.68%, the teacher writes 78.7% suggesting that the answer has been corrected to one decimal place, a step which was not explained and its rationale not provided.

Translation process: interpretation/validation

The results show that Jabu paid little attention to units when solving problems in some cases where this aspect needed to feature. Units in most cases locate the solution within the contexts where the problem is situated, in both intra- mathematical and extra-mathematical cases. For instance, the solution to JB1 focusing on calculating the area of the triangle was given by the teacher as;

$$A = \frac{1}{2} \times 30 \times 45 = 675$$

Although the calculations in this solution were accurate, the dropping of units (cm^2) suggests that the solution was incomplete in relation to the original intra-mathematical context. While the problem was not embedded in some world context, including units within the solution procedure identifies the solution with the original problem context.

Unlike the other lesson (utilizing JB2), the video lesson episode shows that units were included within the problem solving process. For instance the explanations in the lesson episode relating to task JB4 appears to sit within the context where the contextual language, (i.e. Darren, Karen, and wage increase) continued to feature centrally throughout the solution process. By locating the discussion in the context in this example, the mathematical answer did not constitute the main goal of the solution process, but rather, the aim was to understand the context in terms of who received the larger increase - with mathematical working forming part of this understanding.

Jabu: Now who's earning more after the increase? Is it Karen or Darren?

Learners: Karen

Jabu: Hey?

Learners: Karen

Jabu: [speaks to one learner]. Who's earning more? Can you see who's earning more?

Learner: Yes, Sir

Jabu: Who?

Learner: It's Karen, Sir

Jabu: Karen. Good. Now after the increase Karen is earning more. Even though Darren got 3.5% increase and he got 3% increase, but then he [Karen] still earns more. Do you see?

The excerpt shows Jabu's emphasis in terms of translating the mathematical answer to establish its connection with the context. By keeping the conversation close to the context and interpreting the mathematical answer in the context of the problem, Jabu's pedagogical approach in this episode suggests a context orientation.

6.4.2 Analysis of Jabu's 2012 teaching experience

Translation process: model formulation

The discussion within the introduction across the two lessons in 2012 adopted a similar approach to the other lessons in 2011, where teacher talk in the lesson preamble lacked specificity to the utilized tasks.

Jabu: Something that, something that hopefully you see every day and even talk about which is interest. Now, remember that you talk about things and you learn about things that are all around you. We are putting things into context. Maths literacy is a study of maths that makes sense. All the maths that we do here is maths that is very central to society. It's the maths that people use in their daily lives. Now, if I may ask what do you, what do you understand by the term 'interest'? If somebody says, um, 'I have to pay interest'? Yes?

Learner: You pay more than you ask

Jabu: Ok, you have to pay more than you ask for. Good answer. What do you others think? Just think. You have to be able to think. It has to be, it has to come from? Yes?

Learner: Extra amounts that you have to pay.

Jabu: Extra amounts that you have to pay, ok.

There is an emphasis in the extract to push ML teaching and learning towards a focus on the kinds of mathematics that make sense in the society. The extract also suggests that Jabu's knowledge relating to pedagogic requirements of ML which emphasise structuring lessons around contextual themes from which the underlying mathematics can emerge (Bowie & Frith, 2006), despite evidence from problem solving relating to intra-mathematical tasks (JB1) which points towards a more mathematical frame.

Furthermore, some errors relating to misunderstanding of some terminology often used within in the context of problem-solving involving interest were noted in Jabu's explanations.

Jabu: Now if you borrow money you can either accumulate monthly so that means every month they are going to add on the money that you have borrowed on the money that you owe them each and every month, ok. So if it's 10% they are going to add 10% each and every month. There's also a yearly, so that

means the interest accumulates every year. So if you borrow R10 000 over 5 years, each and every year they are going to add that certain percentage, ok. There's also something that you call half-yearly. Half-yearly means the interest will accumulate every 6 months because we have 12 months in a year, you divide that by 2 [writes: $\frac{12}{2} = 6$]. So that's 6 months. So that means in a year it will accumulate twice, do we understand?

Learners: Yes

Jabu: From January to June, June to December. Ok, and then we have semi-annual, when we say semi-annual, that means it will accumulate 4 times a year. So you take your 12 and divide it by 4, that means it will accumulate 4 times a year, that's semi-annual. Understand?

The excerpt shows Jabu's attempts to explain different interest periods. However, he noted that if the interest accumulated semi-annually, then it meant four times a year, which was incorrect. He emphasized this point by exemplifying it with dividing 12 (months) by 4, suggesting that it was not a slip. The implication in terms of model formulation is that mathematical models resulting from this kind of translation would be inaccurate.

In situations where the formula was required to solve the problems (JB8 and JB9), Jabu attempted to explain the variables in the formula and how they related to the contextual quantities.

Jabu: [...] Ok, I borrow R5 000 at 5% over 1 year simple interest. Ok, let's go. We are using the formula [writes: $P \times R \times T$]. What is our principal amount?

Learner: R5 000

Jabu: It's R5 000, so our P will be R5 000 [writes: $P = R5\ 000$]. That's the principal amount. Ok? Now, what will be our rate? [writes: $Rate =$]. Raise your hand. Somebody else in this class!

Learner: It will be 5%.

Jabu: Now 5% simply means it's 5 out of 100. Ok. Because a percentage is out of 100. So 5%. That's what it actually means 5 is out of 100. [writes: $Rate = \frac{5}{100} = 0,05$]

Learners: Yes

Jabu: Good. Then the Time? [writes: $T =$]

Learner: 1 year

Jabu: It's 1 year [writes: $T=1$ year]. Ok. Now we are going to use the formula that we have. [...] What are we going to do?

[...]

Learner: You have to say R5 000 and times 0,05 and

Jabu: [writes: $5\,000 \times 0,05$]

Learner: times the 1 year

Jabu: Times the 1 year [writes: $R5\,000 \times 0,05 \times 1 =$]

The extract above shows how learners were taken through the translation process where contextual quantities were identified and used in the formula. Unlike the disruptions which were noted in other cases, Jabu's working in this example suggests that he was able to both formulate models and communicate the procedure relating to this formulation to learners.

Solution process: intra-mathematical work

Jabu's teaching and intra-mathematical working in 2012 has revealed some weaknesses in these aspects. His solution to JB5 was provided as follows:

<u>Tsonga</u>	:	<u>Zulu</u>
1,800,000	:	9,000,000
18	:	90
1.8	:	9

In this example, Jabu's working appears to exhibit some coherence although the solution suggests that he was unable to simplify the ratio to its simplest form as demanded by the question. The solution suggests that Jabu failed to choose the common factor(s) for 18 and 90 which could lead into 1:5. This implies weak mathematical understandings relating to identifying common factors of numbers.

The other two tasks (JB6 and JB7) in this lesson were given to learners to engage with in a form of a classroom exercise. After the learners had worked through the problems, some learners were invited to present their solutions to the whole class. Two solutions to task JB6 which were given by learners are provided below to illustrate variations in their strategies.

Learner 1

$$= 40:3$$

$$\text{Therefore } 3 = 21\text{km}$$

$$40 \div 3 = 13,33 = 21\text{km} \times 13,33 = 280\text{km}$$

Learner 2

$$M = 40$$

$$C = 3$$

$$C = 21 \div 3 = 7 = 40 \times 7 = 280$$

The solutions presented by the two learners indicate that two slightly different approaches were used despite obtaining the same answer in both cases. The teacher accepted both solutions without asking the learners to explain their procedures. Further, there was no teacher commentary despite these variations, and some disruptions relating to mathematical syntax (i.e. involving the use of equal sign) within the two solution procedures. Jabu's inability to comment on the learners' working especially relating to mathematical syntax and dropping of units may be due to the fact that this working resonated with his own problem solving, and therefore did not notice these gaps.

Translation process: interpretation/validation

Despite cases where the interpretive aspect featured correctly (i.e. JB8 and JB9), the absence of this aspect was also noted in other cases across Jabu's working. The absence of the teacher's commentary related to dropping of the units in task JB6 supports earlier observations linked to gaps in Jabu's knowledge concerning the interpretive aspect.

Further, the solution to JB5 indicates that Jabu's focus was on simply writing the ratio in its simplest form while ignoring the problem context. By leaving the answer as 1.8:9, with a fractional part and yet the numbers represented people, Jabu's working suggests a mathematically driven goal where the context becomes a mere vehicle (Graven & Venkat, 2007b). Drawing from Jabu's own mathematical working relating to CLM course assessment tasks' (see Chapter five), his interpretive aspect appears to be weak.

6.4.3 Orientation to ML teaching

As with Lindiwe and Mark, Jabu's working shows that his lessons included some discussion of broad contexts especially in the lesson preamble at an overview level where connections between mathematical ideas underlying the given contexts and world situations were established. Despite the fact that more extra-mathematical tasks (8 out of 9) were utilized, features relating to these tasks were not discussed, contrasting the ML curriculum specifications, which emphasize familiarizing learners with the contextual features before

engagement with these tasks (Department of Education, 2003). Further, Jabu's working shows that he was able in many cases to correctly identify and select contextual quantities for models. His formulations were often accompanied by explanations focused on learners' conceptualization of this mathematisation steps. Despite showing confidence in formulating models, some error associated with explaining the idea of Value Added Tax (VAT) and related calculations was noted, an aspect which was not noted by both the teacher and the learners. Like in the model formulation stage, Jabu's intra-mathematical working shows that he was able to communicate his solutions to the learners, although some disruptions were also observed at this stage. Further, in one of the cases, Jabu was able to offer two accurate solution methods to a single problem. Cases where learners were allowed to solve problems and present their solutions to the whole class were also observed. The implication is that Jabu was aware of differences in learning styles in the classroom. In terms of interpretation, Jabu's working in some cases was characterized by dropping of units, an aspect which was also observed across his problem solving in CLM course. Dropping of units suggests weak interpretive aspect. Jabu's teaching may therefore suggest adopting both mathematics and contextual orientations.

6.4.4 Jabu's justification of his teaching approaches: interviews

There was one main justification for a pedagogic approach used in Jabu's lessons – to ensure that the learners were motivated to learn.

Researcher: In terms of your teaching approach, you made sure that learners got involved in the lesson where the focus was on mathematics understandings, why?

Jabu: personally I believe that it is important that when you introduce a lesson even when you teach it you don't lose your learners, you make sure that they are still with you. And it's important that learners you use them to teach them, so you use what they know to get to what they do not know. So from using the learner-centered approach, I think it also helped because if one of them, one of their peers goes and do the problem on the chalkboard they actually see that it's not something difficult that they cannot do, but something they can do, it motivates them.

There was reference made to the idea of using what learners already know to facilitate the understanding of new knowledge. This according to Jabu could be achieved through

involving learners sharing their solution procedures with the whole class by writing solutions on the board. However, for learners to get involved at this level, an understanding of the problem context appears to be vitally important. Within his teaching, the contexts in which the problems were located were not discussed thus implying that learners were already familiar with the situations, a view which contradicts the ML curriculum specifications relating to the need for discussing contexts before learners engage with them. When this was further probed, there appeared to be a contradiction in Jabu's feedback.

Researcher: In your lesson you utilised more than one context, but no discussion of the contexts was provided, do you think learners were familiar with the contexts?

Jabu: what I realized was, the, especially the first one [JB5] it would have been better if I had discussed what census is. Why do we have to count people and all that, and I could have also used the fact that it happened early this year, they were counting people and all that but I can see that it was one weakness of the lesson because I didn't use something which is around them to actually explain the context. And ah I took for granted the whole thing, then in this question I actually focused on the content instead of the context.

There was realisation by Jabu relating to the need for discussing contexts before problem solving. Jabu's response appears to attach importance to contextual discussion especially related to similar situations which learners were familiar with, although this was not done in the lesson. The post-lesson reflection indicates that Jabu acknowledged the weakness of his lesson and appears to suggest ways of improving on the weaknesses in the next lessons – by focusing on the context. Furthermore, the interview results show that Jabu was knowledgeable about what was expected of his teaching in ML (aiming at contextual understandings), despite his pedagogic orientations suggesting the contrary.

6.5 Lebo's practice

Eight tasks were utilized within Lebo's lessons across 2011 and 2012. Of the eight tasks, four were situated in some real world contexts and the other four were intra-mathematical, as shown in table 6.4.

Lesson	Summary
Jabu's lessons in 2011	
Lesson 1	Two tasks focused on contingency tables were used in this lesson;

(observation)	LB1: Imagine a hotel that has a large casino attached to it. The management knows that the more hotel guests there are that gamble, the more money they are likely to make. They want to know how many of their guests gamble at their casino and whether male or female guests are more likely to gamble. They keep careful records about the gender and gambling habits of the next 500 guests visiting the hotel. They find that 247 guests are male and 253 are female. Of these guests, 145 gamble and 355 don't. Of the men, 87 gamble and 160 don't. Altogether, 58 women gamble and 195 don't. Summarise this data by completing the following table. <table><tr><td></td><td>Gamble</td><td>Don't gamble</td><td>Total</td></tr><tr><td>Male</td><td></td><td>160</td><td></td></tr><tr><td>Female</td><td>58</td><td></td><td>253</td></tr><tr><td>Total</td><td></td><td>355</td><td>500</td></tr></table> LB2: I want to find out how many learners are absent every week in my class. I also want to know if these learners are male or female. A record of absenteeism for the next week is kept. I have 35 learners in my class. There are 20 males and 15 females in the class. Of the learners 25 are never absent and 10 are always absent. Of the males 6 are always absent and 4 females are always absent. Summarise this information in a contingency table. In this lesson, Lebo provided a discussion focusing on the features of the specific tasks, and this aspect preceded any mathematical working. Specifically Lebo's explanations were concerned with gambling especially in task LB1. She noted that gambling often occurs in casinos where winners receive monetary prizes. She also observed that only people who are 18 years and above are legally allowed to gamble. A similar discussion relating to LB2 was also provided focusing specifically on learner absenteeism. Lebo made reference to her ML class, linking it to situations where attendance registers were taken. In terms of the nature of mathematical working, the first task (LB1) involved completing a contingency table which was given in the context with some quantities already filled in the boxes. The teacher together with the learners, were involved in selecting contextual quantities needed to fill in particular boxes. The second task (LB2) was concerned with deciding on the nature of the contingency table to be drawn, in terms of number of rows and columns, before identifying and selecting contextual quantities to fill in the table. In this way, there was progression on the basis of contingency tables. One task was given as a class exercise towards the end of the lesson.		Gamble	Don't gamble	Total	Male		160		Female	58		253	Total		355	500
	Gamble	Don't gamble	Total														
Male		160															
Female	58		253														
Total		355	500														
Lesson 2 (Video)	The lesson utilised the following two tasks; LB3: An insurance company divides its clients into two age groups, under 30 and over 30. In a particular year, 120 of the 500 clients were under 30. In that year 150 clients, of whom 50 were under 30, made claims. a) draw a contingency table to summarise this data																

- b) find the probability that a randomly selected client:
 i) is under 30 ii) has made a claim iii) is under 30 and has not made a claim

LB4: The South African National Spatial Biodiversity Assessment 2004 investigated the status of ecosystems across South Africa. The purpose was to plan for what needed to be done to protect our natural environment, plants and animals. Ecosystems were classified according to type of vegetation and status. The status of an ecosystem was classified as:

- least threatened if the ecosystem was still mostly intact.
- vulnerable if the ecosystem was reasonably intact but nearing the point beyond which it could no longer function properly.
- endangered if it had lost significant amounts of its natural habitat.
- critically endangered if it had very little habitat left and species associated with it were being lost.

	least threatened	vulnerable	endangered	critically endangered	Total
Albany thicket	16	1	1	0	18
Desert	16	0	1	0	17
Forest	7	0	0	5	12
Fynbos	67	12	29	14	122
Grassland	33	28	18	1	80
Nama-Karoo	14	1	0	0	15
savannah	59	21	7	0	87
Succulent Karoo	65	2	1	0	68
Wetland	12	5	1	1	19
Total	289	70	58	21	438

- a) what percentage of ecosystems in South Africa are critically endangered?
 b) what percentage of ecosystems in South Africa are either endangered or critically endangered?
 c) what percentage of the critically endangered ecosystems is fynbos?

This lesson was focused on calculating probabilities and percentages within the context of contingency tables. A discussion around specific features of the contexts was not provided, as Lebo's lesson preamble only focused on recapping the ideas introduced in the previous lesson relating to contingency tables.

Lebo: What were the three steps that we wrote down yesterday for drawing a contingency table?

Learner: Organizing the information into groups.

	<i>Lebo: Organizing the information into groups. Let's just wait for everyone to settle down. [...] Ok, you said the first one is to organize the information into groups. And the second one?</i> <i>[...]</i> <i>Learner: Ok, decide on rows and columns and the information that goes there.</i> <i>Lebo: Ja. And then the last point?</i> <i>Learners: They must have totals.</i> <i>Lebo: I beg your pardon?</i> <i>Learner: They must have totals.</i> <i>Lebo: Must have what?</i> <i>Learner: Totals</i> <i>Lebo: Oh, must have totals column on the thing. Ja?</i> While the first task (LB3) involved deciding on the nature of the contingency table in terms of rows and columns before translating the contextual quantities into the table, the second task (LB4) focused on calculating percentages using the information already presented in the contingency table. Disruptions within the mathematical working were observed in both cases.
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Jabu's lessons in 2012

Lesson 1 (observation)	The lesson utilised the following tasks;												
	LB5: Complete the table for the following formula												
	$m = \frac{2}{n}$												
	<table border="1"><tr><td>n</td><td>-1</td><td>0</td><td>$\frac{1}{2}$</td><td>2</td><td>4</td></tr><tr><td>m</td><td></td><td></td><td></td><td></td><td></td></tr></table>	n	-1	0	$\frac{1}{2}$	2	4	m					
	n	-1	0	$\frac{1}{2}$	2	4							
m													
LB6: Find the roots of the equation $2x^2 - 6x + 1 = 0$													
LB7: Use the following formula to find y in each case, $y = A\left(\frac{100+r}{100}\right)^x$:													
	i) $A = 200$; $r = 20$ and $x = 7$												
	ii) $A = 12\,200$; $r = 9$ and $x = 120$												
	The lesson was focused on three intra-mathematical tasks. These tasks which were utilized within the whole class discussion were concerned with completing the table of an inverse relationship of m and n, given the values of n (LB5), finding the zeroes (roots) of a quadratic equation (LB6), and substitution (LB7). These intra-mathematical contexts were not preceded by any explanations involving exemplifications as was the case with the other pre-service teachers. Solving task LB6 involved using a quadratic formula followed by computations which involved the learners at the level of doing calculations using their calculators. For the other two tasks (LB5 and LB7)												

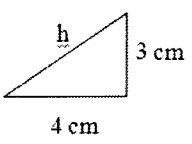
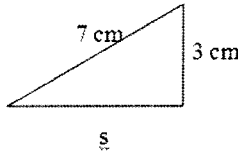
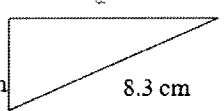
	Lebo instructed the learners to use calculators to find the answers. Tasks with similar foci were given as class exercise after working out the examples.
Lesson 2 (Video)	One task was utilised in this lesson; LB8: Find the length of the unknown side in each triangle (answers rounded off to one decimal place). a)  b)  c)  This lesson was concerned with calculating unknown sides in right-angled triangles. Within the lesson preamble, Lebo attempted to check learners' understanding relating to Pythagoras theorem and for which type of triangles the theorem was applicable. This was followed by the introduction of Pythagoras theorem which was used to calculate the third side of the triangles. <i>Lebo: Can anyone tell me anything that you know or heard about Pythagoras? What was his theorem?</i> <i>Learner: $a^2 + b^2 = c^2$</i> <i>[...]</i> <i>Lebo: So does it work for all triangles?</i> <i>Learner: No, isosceles</i> <i>Learner: No, it only works for</i> <i>Lebo: Isosceles triangles?</i> <i>Learner: Right angled triangles</i> <i>Lebo: It works for right angled triangles and we know that the side opposite to the 90° or the right-angle is the hypotenuse. So whenever you are given a triangle. When Tristan [name of learner] was saying that [draws right-angled triangle marked a, b, c] So Tristan was saying that to be able to find this side c [points to c and writes a question mark next to c] which we know is the hypotenuse.</i> Within her working relating to using Pythagoras theorem, some errors were observed.

Table 6.4: Jabu's lesson episodes

6.5.1 Analysis of Lebo's 2011 teaching experience

Translation process: model formulation

The results relating to observation lesson indicate that the translation process at the level of model formulation took two forms. The first form involved organizing contextual information into some contingency table where this table was given in the problem context (LB1), and the second form was related to the situation where the contingency table had to be decided. The contingency table for LB1 was completed on the board where learners were allowed to contribute. This step was accompanied by Lebo's explanations relating to deciding on the number of rows and columns of the table and how the information from the context needed to be translated into the table. The organization of the contextual information resulted in the following table:

	Gamble	Don't gamble	Total
Male	87	160	247
Female	58	195	253
Total	147	355	500

The table shows that the quantities were correctly translated into appropriate boxes despite the information presented in a disorganized manner in the context. This suggests that Lebo understood the context before engaging with it. Regarding task LB2, the contingency table was decided by the teacher followed by translation of the contextual information as follows:

	Present	Absent	Total
Males		6	20
Females		4	15
Total	25	10	35

The decision relating to the number of rows and columns together with the translation appears to be accurate. Further, her working was accompanied by explanations focusing on how the number of rows and columns were decided as well as how to fill in the table.

The video lesson also focused on working with contingency tables, suggesting some form of progression on the basis of this aspect and not on contexts. Engaging with the insurance

context (LB3), Lebo's working shows that learners were reminded that engagement with the current task was similar to problem solving in the previous lesson.

Lebo: So the first question says you must draw a contingency table to summarise the information that you've been given. So the first step in doing that we have to group the information. Organise the information into groups. So how would you go about doing that? Remember the other lesson we had I gave you the contingency table and you had to fill in the words. Now you're drawing contingency tables yourselves with the information that I just gave you. Yes! [acknowledges a learner]

Learner: I said the left hand side I said under...

Lebo: Ok, just let's talk about rows and columns. How many columns would we have?

Learner: 4

Lebo: [draws a table with 4 columns on the board] Like that. And how many...

Learner: 4 rows

Lebo: [adds 4 rows to the table] And then someone else. What else do you do? Do you remember the other day I gave you this and then with the information and then you just had to fill in the missing words. It's similar to that but it's just that now you're dividing your own, you're making your own contingency table. Yesterday we said to be able to do that you have to organize your information into groups and then we have to write the information into totals etc. So what's really difficult?

Learner: can you put in clients who were over 30 and under 30.

Lebo: [writes over 30 and under 30 in the first column, second and third rows] Like that?

Learner: Ja

The excerpt suggests that drawing a contingency table started with deciding on the number of columns and rows before appropriate headings for the rows and columns were written. In the excerpt, Lebo appears to involve learners during table drawing, something learners were able to do. The completed contingency table without contextual quantities filled in, was correct and was given as:

	Made claims	Made no claims	Total
Over 30			
Under 30			
Total			

The drawing of the table was followed by filling in the boxes using the quantities provided in the context. Lebo's explanations provided a step by step procedure in terms of how the quantities were translated from the extra-mathematical context.

Lebo: [...] Now we need to put the numbers in the blocks. What do we already know? Let's start with everything that we already know. We know that there were 500 people given, so write that down. [writes: 500 in the last column, last row] And then what else do we know?

Learner: Those who were under 30. There's 120, ja.

Learner: That are 30. That are under 30.

Lebo: Uh huh, there are 120

Learner: In total

Lebo: [writes: 120 in the last column, third row] What else do we know?

Learner: There are 150 people that made claims.

Lebo: That made claims

Learner: Ja

Lebo: [writes: 150 in the second column, last row]

With continued learner involvement, the quantities were translated from the context into the table correctly as shown below.

	Made claims	Made no claims	Total
Over 30			
Under 30	50		120
Total	150		500

The drawing of the table and the filling in of the contextual quantities in the appropriate boxes combined with related explanations suggest Lebo's competence with contingency tables. Another level where model formulation featured within the two extra-mathematical tasks was where numbers were selected from the contingency table in order to set up a

procedure for calculating probabilities (LB3) and percentages (LB4). For instance, the completed table relating to LB3 was given as:

	Made claims	Made no claims	Total
Over 30	100	280	380
Under 30	50	70	120
Total	150	350	500

While some learners seemed able to identify the relevant contextual information needed to answer questions, some errors involving selection of numbers from the table which was proposed by the learner and accepted by the teacher were noted. Answers, $120/500$, simplified to $6/25$ to question LB3bi and $150/500$ simplified to $3/10$ to question LB3bii were obtained, and these were correct. However, an answer to question LB3biii given as $70/350$ which was later simplified to $1/5$ was offered by the learner and affirmed by the teacher as correct. Rather than using 500 as the denominator, 350 was selected to represent the total number of clients.

Lebo: Under 30 and has not made a claim.

Learner: 70 over 350

Lebo: Why over 350?

Learner: Because it has?

*Lebo: [looks at the table] So you're working under 30 and has not made claims
[points to 70 in the table and writes: $70/350$]*

Learner: A fifth

Lebo: [writes: $= 1/5$]

The teacher's question, *why over 350?*, suggests that she had enough time to detect and engage with the error. A similar error was also noted within problem solving relating to task LB4 in this episode. This suggests gaps in Lebo's ability to identify and select quantities from tables correctly for mathematical models, although she was able to translate contextual quantities into the contingency tables earlier.

Solution process: intra-mathematical work

The intra-mathematical working relating to the task focusing on completing the contingency table largely involved subtraction or addition. After translating the contextual quantities into the table in task LB2, some boxes were left without being filled as follows;

	Present	Absent	Total
Males		6	20
Females		4	15
Total	25	10	35

In order to fill in the remaining boxes the teacher noted that certain numbers needed to be identified from the table that should subtract. For instance, to fill the second row, second column Lebo observed that 6 needed to be subtracted from 20. Proceeding in this manner filling in the third row, second column, the completed table was given as:

	Present	Absent	Total
Males	14	6	20
Females	11	4	15
Total	25	10	35

The table shows that the boxes were correctly filled and that Lebo's working appears to be secure.

The idea relating to subtracting some numbers within the context of filling in the boxes in the contingency tables featured across the video lesson in similar ways (i.e. LB3). However some errors within explanations were observed within Lebo's intra-mathematical working in this task. The following extract made reference to this table.

	Made claims	Made no claims	Total
Over 30			
Under 30	50		120
Total	150		500

Lebo: So now we need to work out because you already know that the total of the people that made claims is 150. So obviously 150 minus the 50 what's missing?

Learners: 100

Lebo: 100 [writes: 100 in the second column, second row]. Of the under 30, 50 made claims, how many didn't make claims because there were 120 altogether?

Learners: 70

Lebo: [writes: 70 in the third column, third row]. Weren't we told how many people were over 30?

Learner: No.

Lebo: [re-reads the question]

Learner: Ma'am if there's 120 that made claims...

Lebo: [points to 120 on the table] Oh ja, ja, ja, ja. 120 minus 500?

Learners: [speaks very softly] 380

Lebo: What's the answer?

Learners: 380

Lebo: [writes: 380 in the last column, second row]

In this extract, Lebo appears to lead the learners into some calculations leading into filling in the empty boxes in the table. However, Lebo referred to subtracting 120 from 500 as '120 minus 500', in her explanation, where a positive answer, 380, was obtained. This type of error characterized Lebo's intra-mathematical working where subtraction was involved, as similar errors were also noted within the context of solving assessment tasks in the CLM course. This suggests that Lebo understood subtraction of numbers as commutative, which is incorrect.

Translation process: interpretation/validation

The tasks utilized across 2011 did not specifically demand for the interpretive aspect. However, solving examples where the focus was on drawing contingency tables involved continuous reference to the features and language of the context, an aspect of number-context connections, which is a useful skill in ML.

6.5.2 Analysis of Lebo's 2012 teaching experience

Translation process: model formulation

In cases where formulae were needed to solve some problems, Lebo was able to choose or retrieve appropriate formulae to set up procedures. For instance, a quadratic formula was selected in order to solve an intra-mathematical task LB6 as:

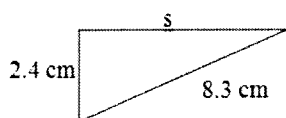
$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

This choice, which was correct, was followed by identification of quantities from the equation and substitution. While referring to the lesson notes, Lebo identified the variables as follows: $a = 2, b = -6, c = 1$. Upon substituting these quantities in the formula, she wrote:

$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 2 \times 1}}{2 \times 2}$$

This substitution was accurate despite dropping the equal sign in the written statement. A similar task was utilized by Lindiwe in one of her lessons where the value of 'b' was also negative. However, Lindiwe's working indicated that this substitution was incorrect until the learner corrected the error.

Within the video lesson, the model formulation involved identifying Pythagoras theorem followed by translating (substituting) the lengths of the triangles into the theorem as follows:



Lebo: Adam [name of learner], the last one please.

Learner: Ok, I haven't done it, Ma'am, but I'll do it now.

Lebo: Ok

Learner: c squared equals

Lebo: [writes: $c^2 =$] Please, please, please, Tristan [name of learner]

Learner: a squared plus b, ja, equals a squared plus b squared.

Lebo: [writes: $c^2 = a^2 + b^2$]

Learner: And then you say c squared equals 2 point 4 squared minus 8 point 3 squared.

Lebo: [writes: $c^2 = 2,4^2 - 8,3^2$]

The excerpt shows how information from the intra-mathematical context (triangle) was translated into Pythagoras theorem with learners' involvement. However, the substitution was incorrect as 'c' was treated as the opposite side of the triangle within Lebo's formulation and yet it was identified as the hypotenuse side when the theorem was introduced in the lesson preamble.

Lebo: Can anyone tell me anything that you know or heard about Pythagoras? What was his theorem?

Learner: $a^2 + b^2 = c^2$

Lebo: Ok, what does that mean [waves her hands]

[...]

Learner: hypotenuse of a triangle.

Lebo: So but why, why do we work with these theorems?

Learner: Because we don't know the other side of that angle and this?

Lebo: So does it work for all triangles?

Learner: No, isosceles

Lebo: Isosceles triangles?

Learner: Right angled triangles

Lebo: It works for right angled triangles and we know that the side opposite to the 90° , or the right-angle is the hypotenuse.

Furthermore the introduction of minus sign was not justified in this formulation. Again by showing that a bigger number was been subtracted from a smaller number in the written statement suggests that Lebo was not able to make sense of the learner's answer, an aspect linked to weak mathematics understandings. A case where a bigger number was subtracted from a smaller number resulting into a 'positive' result was also observed within her working relating to contingency tables in 2011.

Solution process: intra-mathematical work

Despite Lebo's problem solving exhibiting coherent intra-mathematical working in some cases, other instances continued to be characterized by disruptions. The solution to task LB6 for instance shows that rather than obtaining two roots for the quadratic equation, only one answer was provided.

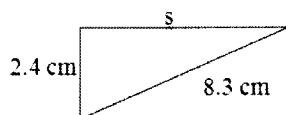
$$\frac{-(-6) \pm \sqrt{(-6)^2 - 4 \times 2 \times 1}}{2 \times 2}$$

$$\frac{-(-6) + \sqrt{36 - 8}}{4} = \frac{6 + \sqrt{28}}{4}$$

The intra-mathematical working shows that in addition to dropping the equal sign, the negative part of the solution had been dropped without providing a rationale for doing so. However, given the intra-mathematical nature of the task, there was a need for two roots

(both positive and negative). Although in some cases a positive solution is most preferred, these cases are often informed by the context where the solution makes sense (i.e. distance or length). Lebo's solution therefore suggests failure to understand the mathematics context.

In terms of the video lesson, Lebo's working across the sub-questions was accurate in most cases leading to correct mathematical answers, with learners' involvement. However, some disruptions related to 'mathematics syntax' were noted as Lebo was solving for the unknown variable 's' represented by 'c' in her working.



Lebo: [writes: $c^2 = a^2 + b^2$]

Learner: And then you say 'c' squared equals 2.4 squared minus 8.3 squared.

Lebo: [writes: $c^2 = 2,4^2 - 8,3^2$; writes $c^2 =$]

Learner: Um, minus 63. It can't, it can't be.

Lebo: That's because you minused the smaller one to the bigger one [circles $2,4^2$ and $8,3^2$]

Learner: Ja. The answer was 63

Lebo: [writes: 63] Point?

Learner: Point 13. And then square that.

Lebo: [writes: $\sqrt{c^2} = \sqrt{63.13}$; Then writes $c =$ and looks at the learner]

Learner: Um... And that equals 7 point 9.

Lebo: [writes: $c = 7,9$ cm]

In the excerpt, Lebo appears to accept the learner's feedback relating to an incorrect translation ($c^2 = 2,4^2 - 8,3^2$). Although one would expect a negative answer from this formulation, the excerpt shows that a positive answer was obtained ($2,4^2 - 8,3^2 = 63.13$) – an issue which was picked up by one of the learners but not fully addressed by the teacher. By not addressing this error, coupled with other similar errors already highlighted in this lesson, Lebo's mathematical understandings could be understood as insecure.

Translation process: interpretation/validation

Tasks in 2012 generally had a focus on intra-mathematics tasks. These tasks did not demand an interpretive aspect, especially the tasks utilized in the observation lesson.

On the other hand, Lebo's working relating to task LB8 suggests that Lebo was pushing towards obtaining a positive answer (and ignoring the negative one) from the quadratic equation which followed from Pythagoras theorem. This may suggest that Lebo was unable to interpret the mathematical solution with respect to the intra-mathematical context.

6.5.3 Orientation to ML teaching

Like in the lessons given by the other pre-service teachers, Lebo's teaching was preceded by some discussion of contexts at a more generic level especially for extra-mathematical tasks. Thus there was no discussion focused on features relating to specific tasks. Rather, the focus was on understandings relating to how mathematics models were formulated followed by intra-mathematical working where mathematical language was used. Engagement with the intra-mathematical tasks was not preceded by any connections with world situations, a feature which characterized the other pre-service teachers' practice. Although this was not necessary, it may suggest limited understandings of the relationship between the mathematical models and world contexts. These connections featured within the CLM course.

Although Lebo has demonstrated confidence and competences in engaging with some tasks, errors were also observed. Some of these errors relate to incorrect selection of quantities from the contingency tables with the view to calculate probabilities and percentages. There were also cases where a bigger number was subtracted from the smaller number resulting in a positive number. Although the results have shown that some errors did not negatively impact on the answer, meaningful learning may have been disrupted. Further, Lebo's working suggests that the interpretive aspect did not feature after the mathematical solutions were obtained. Despite utilizing a range of different tasks for her lessons, Lebo's pedagogic approaches suggest a focus on achieving mathematical goals.

6.5.4 Lebo's justification of her teaching approach: interviews

Results from interviews with Lebo suggest three rationales for adopting a particular pedagogic approach within her teaching across 2011 and 2012, namely;

- Approach adopted from supervising teacher
- Approach driven by on-coming examinations
- Approach informed by the nature of task

A brief discussion of the rationales and presentation of related excerpts follow.

Approach adopted from supervising teacher

Evidence from the results indicates that some pedagogic approaches were driven by teaching orientations often adopted by the supervising teacher. Observing lessons taught by the supervising teachers appears to have allowed the students on teaching experience to appreciate and understand the culture of the school relating to teaching before they could handle their own classes confidently.

Researcher: Your lesson appeared to focus on striking a balance between contextual and mathematics understandings, why did you approach the lesson this way?

Lebo: That approach came from the previous class, like classes that we had altogether, like some of them I have been observing him teaching and some of them I have been teaching myself. So from that we were like learning, so especially the gambling one he did it yah yesterday he taught and the way he explained the gambling concept and numbers everything made it easier for me to extract the numbers and everything like that.

By adopting the supervisor's way of teaching, the excerpt suggests that teaching became easier than if new approaches were implemented, especially in the context of teaching where similar tasks were utilized. Her confident engagement with tasks focusing on gambling (and contingency tables) at the level of formulation suggests her understanding of the supervising teacher's explanations. Although there is a sense of the student on teaching experience learning from experienced teachers, observing lessons may limit the novice teachers in terms of employing their innovative ways during practice. Thus often times after lesson observations, the novice teacher tends to teach in ways similar to the supervising teacher's pedagogy.

Approach driven by on-coming examinations

The misalignment that sometimes exists between examinations and the rhetoric related to ML teaching and learning appears to create a dilemma in terms of pedagogic focus. Although the rhetoric in the ML curriculum suggests teaching using contexts, some question items in Matric examinations focus primarily on mathematics content understandings (Department of Basic Education, 2008, 2009). This implies the need for teachers to emphasize not only on contextualized tasks but also on content focused tasks, aspects which also featured within the CLM course.

Researcher: Your lesson approach was more orientated towards mathematics, such as demonstrating how to substitute values in a quadratic formula. Why did you choose that approach?

Lebo: The approach, well firstly because yesterday in the other lesson we were focusing more on other parts of, like more mathematical literate type of questions where we focused on bank statements and those type of things. We substitute from those type of things, like the example which we have just done now other questions which was yah that. So I felt that since it is in the textbook and something they need to know in case it comes out for the exams so I thought even though it was, I was not supposed to be too mathematical and stating it but then there was no other way for me.

The excerpt highlights another tension related to ML teaching, where Lebo's understanding in terms of utilizing contextualized tasks was contradicted by textbook specifications. The need for learners to develop a deep and connected understanding of mathematics content may be used as a rationale for including content-based tasks in the textbooks, and that such tasks should not be ignored.

Approach informed by the nature of task

Within the context of mathematics problem solving, there were instances where the pedagogic approaches to tasks were often informed by the nature of the tasks themselves. Although this claim could be true for most mathematically focused tasks, it was less so for contextualized tasks. In contrast with intra-mathematics tasks, problem solving involving extra-mathematical tasks may have a focus on mathematics, contextual, or both contextual and mathematical understandings.

Researcher: Your lesson approach was mainly focused on mathematics understanding, why did you choose that approach?

Lebo: I didn't understand it in another way where I can say no rather than thinking about this formula being like this, you can think of it being bananas for instance, I couldn't come up with anything so that's why I thought that may be if I just state it as it is then it would be easier for them to understand but then which wasn't because now I am coming with a lot of numbers and telling them this is that and that and that so yah it was a bit tricky but then I think by the end they got something

It is important to note that this interview was conducted after a lesson which focused on finding the roots of a quadratic equation. Lebo's response suggests that she found it difficult to relate her discourse with the real world objects or situations. Although the CLM course had a focus on linking mathematical models with real life stories, none of the CLM tasks were quadratic.

6.6 Chapter summary

6.6.1 The nature of tasks utilized in practice

The results have shown that both extra-mathematical and intra-mathematical tasks were utilized by the pre-service teachers within their practice across 2011 and 2012. Like in the course assessment, some tasks utilized in practice were mathematically focused despite being situated in a context. While Mark's extra-mathematical tasks appeared to be more textual with lots of information needed to be decoded before procedures were set up, others were less textual and required minimal effort to comprehend. In some cases, the need for skills needed to selecting formulae was noted as these were not given or suggested in the problem. This means that in addition to knowing how to utilize a particular formula, the teachers needed to be able to select the appropriate formulae for particular problem contexts. For instance problems related to finding roots of a quadratic equation, calculating the third side of a right-angled triangle, featured across school practice and required some competences relating to selecting and using formulae. The kinds of problems which the students engaged with within the CLM course also required the students to select formulae (see chapter five). Thus selection of formulae within problem solving was part of the CLM course focus.

6.6.2 The nature of connections within lesson preamble

The pre-service teachers appear to start their lessons in similar ways in the sense that they all attempted to connect mathematics ideas underlying the problems world situations where these ideas would be useful, in the lesson preamble. These content/context connections which took the form of explanations did not focus on specific problem contexts in many cases but rather they were more generic. While these connections were observed within the context of solving intra-mathematical tasks, they also preceded some extra-mathematical tasks' teaching. Within problem solving involving intra-mathematical tasks, these kinds of connections appear to focus on motivating learners in order to let them appreciate the usefulness of the mathematics content in life even if the tasks seemed to be mathematically focused. At the level of extra-mathematical tasks' teaching, explanations were focused on features (i.e. definitions and clarification of terms) of general situations. In some cases the teachers' explanations were characterized by errors relating to both lack of familiarity with the contextual features and gaps within the teachers' mathematical understandings. Regarding gaps in mathematics understandings, the results have shown instances where exemplifications were provided within the explanations in the lesson preamble, but these were accompanied by incorrect mathematical working (i.e. Jabu's explanations relating to VAT). Further, the study results have shown that these generic explanations located within the lesson preamble did not necessarily help in terms of understanding the problem situations utilized in these lessons.

6.6.3 Strong mathematical working

In this report, strong mathematical working refers to coherent problem solving accompanied by correct pedagogic explanations. PISA's (OECD, 2010) components of the mathematisation process were used to provide a sense of key steps within problem solving. The explanation component was an addition through grounded sense making of problem solving in practice. In this study this component, where it featured, had among other things been characterized by the teachers' insightful feedback relating to the learners' answers, procedures and questions.

The results have shown that some teaching across 2011 and 2012 exhibited coherent mathematical working. These were instances characterized by; 1) correct identification and selection of quantities from the context (either intra-mathematical or extra-mathematical), 2) correct selection of operations, and 3) logical presentation of the solution procedure. In other

cases some correct formulae were selected followed by accurate substitution of the contextual information. After the solution plan was set up, mathematical tools were correctly used to obtain the mathematical answers. Where applicable, the mathematical solutions were interpreted within the problem context. Jabu's working for instance relating to 'wage increase' task (JB4) provides evidence of some coherent working. His working also provided an alternative solution strategy to the same task thereby providing learners with different perspectives of engaging with the problem. Due to the nature of the context (practice), Jabu's problem solving was also accompanied by correct explanations focusing on the three key aspects of mathematisation process.

6.6.4 Weak mathematical working

Despite some lesson episodes exhibiting strong mathematical working, the quality of other solution procedures was constrained by disruptions. These disruptions were observed across the mathematisation process at the level of model formulation, vertical working and interpretation/validation. At the level of model formulation, where most errors were observed, the results have shown that the teachers were unable to correctly select contextual quantities, operations, and in some cases incorrectly substituted contextual information into some formulae. Incorrect translation of quantities from real models (tree diagrams and contingency tables), were noted as some of the examples of these errors. There was also evidence pointing towards difficulties relating to setting up procedures. Two similar instances (Mark and Jabu) were observed in the data where a solution plan related to working with percentages could not be accurately decided. Similar findings were reported within the context of problem solving involving contextualized tasks (Clarkson, 1991b; Maat & Zakaria, 2010). However, within the context of ML in South Africa, a study by Vale and colleagues (Vale, et al., 2012) has shown that more errors in problem solving were attributed to inaccurate mathematics calculations and incorrect mathematics models.

Regarding intra-mathematical working, incorrect use of equal sign was observed across the pre-service teachers, suggesting existence of gaps in knowledge relating to mathematics syntax. There were also examples of situations where the teachers accepted incorrect answers or feedback from learners within problem solving, suggesting failure by the teachers to make sense of these learners' answers/feedback. In other instances, inaccurate explanations specifically related to key contextual features were provided. For instance, Mark's definition of a map as 'a scale' and Jabu's statement that semi-annual means 'four times a year' were

incorrect. As already highlighted, incorrect explanations relating to contextual features may suggest lack of familiarity with the contexts.

As noted above, analysis of the errors has shown that there were more translation (model formulation) errors than calculation errors, thus supporting the course tasks' findings (Chapter five). The errors committed at the level of model formulation did not necessarily inform errors occurring at the level of intra-mathematical working. Thus translation errors and vertical working errors appear to be independent of each other. In most cases model formulation errors resulted in incorrect answers even if the formulations were followed by correct intra-mathematical working. While some errors committed at intra-mathematical working level resulted in incorrect answers, others (related to mathematical syntax) did not negatively affect the mathematics results.

6.6.5 The nature of pedagogic orientations

Interviews

Analysis of the interview data was focused on one aspect namely; justification of particular pedagogic orientations across teaching. By adopting grounded analysis, the results have revealed two themes relating to either teachers or learners. While some teaching approaches were adopted from qualified teachers, others were targeted at allowing the learners to maximize their learning.

1. Teaching approach adopted from other qualified teachers

Since student teachers were assigned mentors (supervising teachers) during teaching experiences, they were allowed to observe the mentors' lessons before they could independently teach their own lessons. Evidence from the interview data has revealed that the student teachers often adopted their mentors' ways of teaching. Although the teaching orientations appear to contradict the ML teaching approaches specified in the curriculum and emphasized in the CLM course, the student teachers continued to teach the mentor's way. Besides teaching the mentor's way, there was also evidence indicating that some student teachers preferred to teach the same way they were taught at high school, in some cases.

2. Approach with learner focus

Evidence from the interview data suggests that some pedagogic approaches were more focused on the learners. Three aspects were noted under this learner focused pedagogy. They relate to; understanding the context, dealing with misconceptions, and delivering a motivating lesson.

Although the analysis of the lesson episodes have not shown teaching orientations characterized by ‘context driven’, interview results indicate that certain pedagogic moves were targeted at allowing learners to specifically understand the contexts. This relates closely to the pedagogical agenda sitting to the far left of the spectrum (Graven & Venkat, 2007a). Another rationale for a learner-focused pedagogic approach was concerned with dealing with learner misconceptions. Such lessons took the form of revision where the intention was primarily to engage with the learners’ misunderstandings relating to specific concepts (relating to either mathematics or contexts) in order to counter their misconceptions. Delivering motivating lessons was another rationale for certain pedagogic approaches adopted by some teachers. The results have shown that motivating lessons were aimed at actively involving the learners (learner-centered) throughout the lesson especially through activities with clear instructions and adequate supervision.

Which pedagogic Agendas are foregrounded within practice?

The study results have shown that the teaching orientations across the episodes relate to the last three agendas on the spectrum identified by Graven and Venkat (2007a) namely; content and context driven, mainly content driven, and content driven.

Lessons relating to the ‘content and context driven agenda’ utilized contextualized tasks (not necessarily realistic) and appear to focus on developing learners’ understandings of mathematics content and context. A lesson given by Mark on map reading for instance was more focused on understanding the context of map reading and related mathematics (proportional reasoning), and therefore provided a good example of a ‘content and context driven’ lesson. Pedagogic orientations pointing towards ‘mainly content driven agenda’ were also noted where contextualized tasks were used but the related mathematical working appears to advance mathematically-focused goals. Lindiwe’s lesson which utilized a ‘theatre ticket sales’ context exemplifies ‘mainly content driven agenda’ where content/context connections did not feature beyond the model formulation level. This implies that the context

was selected so that mathematics could be applied to. The absence of content/context connections including the interpretive aspect at the level of intra-mathematical working supports this claim. Furthermore, tasks which did not refer to any real world objects (non-contextualized) were also utilized across different teaching episodes. Although engagement with some of these tasks was preceded by a discussion around ways in which understanding mathematical concepts could be useful in life, the pedagogic agendas remain ‘content driven’. Lessons around ‘solving quadratic equations’ and ‘finding the side lengths of triangles’ taught by Lindiwe and Lebo respectively provided useful examples of the ‘content driven’ agenda.

Despite the teaching orientations sitting on the last three agendas of the spectrum, the results indicate that the third and fourth agendas dominated the lesson episodes taught across 2011 and 2012. Since the ML curriculum supported by empirical results suggest that ML teaching need to sit on the first and the second agendas (DoE, 2003; Graven & Venkat, 2007b), the study results show that the teachers’ knowledge related to practice was still developing.

CHAPTER SEVEN: CONCLUSIONS

7.1 Introduction

This study was concerned with exploring the pre-service ML teachers' development in knowledge within the context of a new professional development course (CLM), at a major urban University in South Africa. The specific focus was twofold. First, I explored the ways in which the pre-service teachers engaged with contextualized assessment tasks in the course, zeroing on both extra-mathematics and intra-mathematics connections within the teachers' mathematical working. Second, the teachers were followed during their teaching practicum experiences with a view to explore their classroom mathematical working relating to instructional tasks and how their working was communicated to the learners. Understanding orientations to ML teaching was also part of this focus within the teachers' practices. This study focus was premised on the view that both extra-mathematics and intra-mathematics connections were central to both problem solving and ML teaching. Concerns relating to weak problem solving competences among ML teachers provided the rationale for locating this study within pre-service teachers' professional development course. Unlike the in-service teacher training model (i.e. ACE) reported in ML-related literature (Bansilal, 2012; Bansilal, Goba, et al., 2012; Mbekwa, 2006), the CLM course offered an alternative 'pre-service' route of ML teacher development, and studying the students' knowledge growth in the new course was worthwhile.

7.2 Pre-service teachers' growth in knowledge

As already noted, given the small data set relating to the pre-service teachers' practice, the focus in this study was on exploring growth in knowledge development in relation to the CLM course where detailed data featured.

7.2.1 The nature of extra-mathematical connections across course work and ML practice

Skills relating to translating contextual features into mathematical models across the four teachers appear to be generally weak. The weakness was observed within the CLM course as well as in practice. Within the course, despite evidence pointing towards accurate identification and selection of contextual information in many cases, the choice of mathematics operations, remained problematic, especially at connections level tasks. Some of the instances where inaccurate choices of operations were observed involved working with

fractions. These disruptions were linked to inability to decode the language used in the problem contexts (i.e. everyday language) into mathematical language (Clarkson, 1991b; Maat & Zakaria, 2010; Vale, et al., 2012). Substituting contextual quantities into formulae was another problematic feature across the pre-service teachers' problem solving. In most cases, this step was preceded by correct choices of formulae, appropriate for the problem situations. Extra-mathematical connections also featured in written responses through 'annotations', an aspect which was concerned with listing of contextual information before the mathematical model was formulated (Hall, et al., 1989). The idea of annotation was interesting in this study because it provided an understanding relating to the identification and selection of quantities from the contexts. The absence of annotation has been associated with a 'jump' into the mathematical world, an approach which has been described as a move towards a focus on the mathematical aspect of the problem solving process. Further, the interpretive aspect across the teachers' responses was consistent with the mathematical solutions in many cases – even in situations where the mathematical results were incorrect.

In order to understand improvement in performance relating to extra-mathematics connections, this study looked at frequencies focusing on occurrences of correct and inaccurate model formulations as well as correct and incorrect interpretations, across 2011 and 2012. Despite the presence of isolated cases relating to inaccurate model formulations, noted within solution protocols in this study, improvements in overall performance relating to this aspect were noted in Mark, Jabu and Lebo's working. Lindiwe's working, however, showed no improvement at the level of model formulation.

Further, more occurrences in 2012 relating to instances where the interpretive aspect featured, were also observed in the course, with results suggesting some improvement in this year compared to 2011, across the four participants. However, at the level of individual students, results have shown that the interpretive aspect did not feature, even in situations where the problem context demanded so (i.e. Jabu's working). In terms of growth relating to extra-mathematics connections in the course, the results have shown that improvements occurred, although some inaccuracies were still evident in 2012. This implies that the pre-service teachers' knowledge developed further across the two years in relation to this aspect.

Extra-mathematical connections within practice were also observed, although variations were noted relating to how this aspect featured. Many lessons, particularly those utilizing intra-mathematical tasks, included some generic discussions in the lesson preamble linking the

mathematical ideas in the tasks (i.e. ratio, interest, percentages, etc) with related functionality in the world. In contrast, teaching using extra-mathematics tasks involved a discussion of contextual features often within the lesson introduction at an overview level. Lack of focus on specific contextual features within discussions provided an overlap between lessons utilizing extra-mathematics and intra-mathematics tasks. Further, it provided a point of dissonance with the CLM course, where ‘story creation’ tasks asked for a direct and specific linking of a mathematical situation to an extra-mathematical situation. The pedagogic discussions within instances where intra-mathematical tasks were utilized were often very brief, with more time spent within the model formulation step before engaging with the problems intra-mathematically. This suggests a mathematics content orientation. However, although cases were noted where teachers rushed through contextual discussions, Mark saw the need to spend considerable time familiarizing learners with the contextual features especially within contexts where extra-mathematical tasks were utilized, despite these discussions being generic. Relating to translation of contextual quantities at the level of model formulation, some errors similar to those observed within course assessment-based problem solving (i.e. choosing incorrect operations, incorrect substitution) were also noted in practice, despite teachers’ reference to lesson notes within teaching (i.e. Lindiwe, Lebo). In terms of interpreting mathematical results, omissions of the interpretive aspect and incoherent use of units, were noted in some lessons. This suggests that extra-mathematical connections, particularly model formulation, is a point of weakness across course knowledge and classroom practice, pointing to the need for greater emphasis on this feature within ML teacher development.

7.2.2 The nature of intra-mathematical connections across course work and ML practice

Overall, the pre-service teachers’ skills relating to intra-mathematical working were relatively secure, and improving across the two years, as they were able to correctly calculate, compute and/or manipulate mathematical statements in many cases. The cases which were preceded by incorrect model formulations also exhibited coherent working, in the sense of logically enacting procedures in intra-mathematical sense. One interesting finding in this study was that inaccurate vertical working was not generally a consequence of incorrect model formulations. Rather, translation errors at the level of model formulation were independent of procedure enactment errors (where these featured). These results contrast findings in ML literature which suggest that errors within ML problem solving are largely attributed to

inaccurate mathematical calculations or manipulation of mathematical statements (Vale, et al., 2012). This implies the need for emphases on model formulation within ML problem solving.

Within practice, accuracy relating to vertical working was also achieved in many instances. The results across the four teachers have also shown that attempts were made in many instances to explain steps within the solution procedures. Unlike the vertical working in the course which suggested a mathematical focus due to limited or no context-mathematics connections within solution protocols, ML teaching combined vertical working with explanations, linking this working to features of the contexts in many cases especially across lessons utilizing extra-mathematical tasks. Within these cases the teachers' pedagogic approaches appeared to adopt either contextual or content-context orientations (Graven & Venkat, 2007a). Teaching using intra-mathematical tasks was often mathematically focused, suggesting that intra-mathematical steps within practice adopted some content driven orientation (ibid).

Within the CLM course, errors were observed across the students problem solving. These gaps were not ascribable to topics, but to specific tasks within topics. Some of the tasks where disruptions featured were linked to percentages, fractions, and area/perimeter of 3-dimensional shapes. While errors within 'fractions' contexts were related to inability to deal with scenarios involving different 'wholes', disruptions relating to area/perimeter were linked to difficulties in terms of retrieving and/or deriving formulae appropriate for the given shapes. The results have shown that all the students struggled to deal with many tasks involving fractions. Further, Lindiwe, Jabu, and Lebo's working indicate that they struggled to engage with area/perimeter tasks. Findings relating to incoherent vertical working have been reported in literature (Bernardo, 1999; Clarkson, 1991b; Maat & Zakaria, 2010). Relating to classroom practice, the results have shown errors at the level of talk and problems with flexible working with procedures (i.e. Lindiwe, Jabu). Further, incorrect use of equal sign was observed across the four pre-service teachers, both within the course and in practice, suggesting existence of gaps in knowledge relating to mathematics syntax. While some errors committed at intra-mathematical working level resulted in incorrect answers, others (related to mathematical syntax) did not negatively affect the mathematics results. Although errors relating to mathematics syntax did not often lead to incorrect mathematics results, the accuracy of intra-mathematical working across these cases was constrained.

The results show that there is a link between what happened in practice and the CLM course, in that contextualized tasks which were largely emphasized in the course were utilized in many lessons which were observed. The solution strategies to problem situations which were adopted in the course were also utilized in practice. However, the fact that supervising teachers in schools influenced the selection of tasks and sometimes solution methods used in the classrooms may suggest that these mentor teachers had more control than the influence of the course. This implies existence of tensions between what was taught in the course and what was selected to be used for instructional purposes in schools. Further, in terms of considering learning in terms of developing in meaning-making, the results indicate that this aspect generally improved among the four students across the two years, with exceptions of disruptions in some instances.

7.3 Recommendations and implications for ML teacher development

The study results concerning gaps within the teachers' knowledge relating to contexts-mathematical connections suggest the need for an emphasis on this aspect in teacher development programmes. Further, I have argued that disruptions relating to setting up procedures in problem solving focusing on contextualized tasks are linked to language problems (Koedinger & Nathan, 2004). Weak mathematics understandings among in-service ML teachers have also been reported in ML literature (Brown & Schäfer, 2006; Vilakazi & Bansilal, 2012), and suggests the importance of a focus on developing the teachers' deep and connected understandings of mathematics.

Another implication at the level of pedagogy concerns the need for more practical experience. Drawing from the generic extra-mathematics connections which the teachers were making in the lesson pre-ambles, especially when engaging with intra-mathematics tasks, the emphasis in the ML method sub-course needs to be on utilizing both extra-mathematical and intra-mathematical tasks. Given this study results, I make three recommendations relating to ML teacher training:

- Emphasis within ML teacher training programmes should be on translation at the level of model formulation, an aspect which was weak across the teachers' working. Extra-mathematics connections appear to feature centrally within ML problem solving due to its citizenship orientation.

- Given that this study was located within professional teacher development, I found ‘pedagogic links’ to be an important aspect within problem solving in the course. Despite observed weaknesses relating to model formulation, the pedagogic links, in many cases allowed the students to be able to work from context to mathematics and from mathematics to context within practice.
- The ML Method course needs to focus more on providing explanations which are specific to the instructional tasks. The generic explanations within practice, which were observed in this study, appear to potentially disrupt meaningful learning.

7.4 Contribution to research

One possible contribution of this research relates to exploring step-by-step analysis of pre-service ML teachers’ problem solving processes across two years, with the view to understand the teachers’ growth in knowledge. This provided a window for understanding the sources of disruptions across the mathematisation cycle in ML problem solving, an aspect which was not addressed in ML literature. Literature in both school mathematics (Bernardo, 1999; Hall, et al., 1989) and South African ML (Vale, et al., 2012) relating to errors within problem solving revealed that more disruptions in problem solving occur at the level of intra-mathematical/vertical working. However, this study results have shown more translation errors at the level of model formulation. Further, this focus has provided an understanding of the nature of intra-mathematical and extra-mathematical connections, key aspects of ML problem solving. Disruptions at the level of model formulation suggest that competences relating to extra-mathematical connections were weak. Since problem solving is at the heart of ML, an understanding of possible sources of errors within this process may help in ensuring that ML professional development courses with similar structure and focus emphasise these aspects.

Another possible contribution concerns extending the PISA mathematisation process to a framework which can be used to understand mathematical working in ML classrooms, where both intra-mathematical and extra-mathematical tasks are utilised. This is useful given the context and mathematics content orientations advanced within ML teaching and learning (Graven & Venkat, 2007a).

7.5 Limitations

Given that the CLM course, which provided an empirical field for this study, had dual foci and twin aims, combined with the fact that the focus was on ML students only, the study findings are therefore exploratory and partial. To gain a full understanding of the course dynamics, further research is needed focused on the primary mathematics students. Further, the study has noted a mismatch between assessment and course work within the ML-focused sub-course (Method 2), in that whilst the course enactment emphasised ways of ML teaching, the assessment was focused on examining problem solving skills. Analysis of practice data has therefore provided insight relating to their ML pedagogic skills, an aspect which could have also been explored within the analysis of course data.

Additionally, this study engaged in depth with four ML specialists who enrolled into the CLM course, and showed willingness to participate in the study. Given such a small sample, the study results could not be generalised to a larger population. Rather, the generalisability of the findings is only limited to the studied cases. However, the results provided in-depth insights on ML student teachers development in knowledge and how such knowledge linked with their practice. As noted by Gay and Airasian (1996), generalisations in qualitative studies are minimal and sometimes non-existent because the choice of participants is sometimes purposive and small in size. However, in-depth insights were gained as a result of lengthy and intensive engagement with the participants.

7.6 Future research

Two possibilities relating to future research can be drawn from this study.

- (a) Extending this study to a larger population of pre-service ML teachers to explore the nature of extra-mathematical and intra-mathematical connections. This focus would help to establish global problematic areas within the problem solving process.
- (b) Exploring the relationship between pedagogic orientations advanced by the pre-service teachers in practice and the learners' understandings to determine the extent to which these orientations allow for development of learners' competences relating to problem solving.

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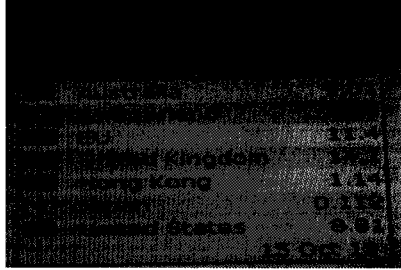
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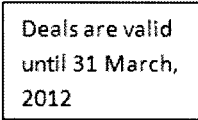
APPENDIX A: CLASSIFICATION OF ASSESSMENT TASKS

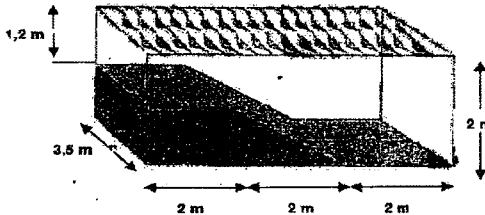
Task code	Reproduction (Rp) level tasks
2011 academic year	
Rp1	A person has $29\frac{1}{2}$ metres of material available to make doll's dresses. Each dress requires $\frac{3}{4}$ metre of material. a) How many dresses can be made? b) How much material will be left over
Rp2	I have 8,2m of material. I need 0,4m of material to make doll's dress. a) How many complete dresses can I make from the material? b) How much material will I have left over?
Rp3	I have $\frac{3}{4}$ litres of milk in the fridge. I drink $\frac{1}{3}$ of it. How much milk (in litres) do I have left?
Rp4	A recipe for a full pot of stew requires that I use $\frac{4}{5}$ of a cup of beef stock. I only want to make $\frac{1}{2}$ of a pot of stew. How much beef stock do I need?
Rp5	The price of a shirt is reduced from R350 to R280. By what percentage has the price of the shirt been reduced?
Rp6	John and Jane both currently earn R10 000 per month. John performs badly in this job so is demoted and will earn 9% less from next month onwards. a) How much will he earn? b) Last month Jane was actually earning less than R10 000 and she received a raise of 9% which brought her salary up to R10 000. What was she earning last month? c) Are the amounts John will earn and the amount Jane earned last month different? Explain why this is so.
2012 academic year	
Rp7	A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day a) If the Company has one six-man team on the job, how long would it take to complete the contract? b) How many teams must they put on the job if they have to get the contract finished i) in one day ii) in six days c) How many teams should they use if they have to get the contract finished within 20 days? (they do not split up their teams of workers).
Rp8	Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month. a) Determine Nadia's new salary per month.

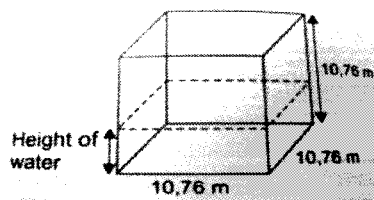
	b) Who received the greater percentage increase? Show your working
Rp9	A table showing exchange rates as of 15th September 2012 is given below.  Kabelo is travelling to Japan within the next five days for a business trip, how much Japanese Yen can he buy if he has R50,000.00?
Rp10	The figure below shows change in commodity prices in the past 10 months.  Calculate the percentage increase for the items listed in the table above within this 10-month period.
Rp11	Paul reads in the newspaper that a recent study in the United States has revealed that 37% of the people in America are overweight and that 22% are obese. Furthermore, 15,5% of all teenagers in America are obese. This gets him thinking about the situation at his school and in the rest of South Africa and how it compares with the situation in America. To determine your weight status according to the classification given below, you have to determine your Body Mass Index (BMI) using the following formula: $= \frac{\text{weight in kg}}{(\text{height in m})^2}$

	The BMI is then used to classify someone as follows: <table border="1"> <thead> <tr> <th>BMI</th><th>Classification</th></tr> </thead> <tbody> <tr> <td>< 18.5</td><td>underweight</td></tr> <tr> <td>≥ 18.5 and <25</td><td>Normal weight</td></tr> <tr> <td>≥ 25 and <30</td><td>overweight</td></tr> <tr> <td>≥ 30</td><td>obese</td></tr> </tbody> </table> i. Calculate Paul's BMI if he weighs 85 kg and his height is 1,75 m ii. Using the table above based on BMI, how would you classify Paul's BMI?	BMI	Classification	< 18.5	underweight	≥ 18.5 and <25	Normal weight	≥ 25 and <30	overweight	≥ 30	obese								
BMI	Classification																		
< 18.5	underweight																		
≥ 18.5 and <25	Normal weight																		
≥ 25 and <30	overweight																		
≥ 30	obese																		
Rp12	The table below shows two sets of ML method test scores. <table border="1"> <thead> <tr> <th>Test A</th><th>Test B</th></tr> </thead> <tbody> <tr><td>50</td><td>40</td></tr> <tr><td>70</td><td>62</td></tr> <tr><td>50</td><td>64</td></tr> <tr><td>50</td><td>72</td></tr> <tr><td>60</td><td>70</td></tr> <tr><td>80</td><td>68</td></tr> <tr><td>100</td><td>68</td></tr> <tr><td>100</td><td>68</td></tr> </tbody> </table> Calculate the mean for each set of the test scores.	Test A	Test B	50	40	70	62	50	64	50	72	60	70	80	68	100	68	100	68
Test A	Test B																		
50	40																		
70	62																		
50	64																		
50	72																		
60	70																		
80	68																		
100	68																		
100	68																		
Rp13	The scale on a map is 1:35 000. If the distance between two towns on the map is 2,75cm, determine the actual distance between the towns in kilometres.																		
Rp14	The only sports offered at Burg High School are soccer and netball. The principal loves soccer so he allocates the sports budget so that for every R2 spent on netball, R3 will be spent on soccer. a) If R450 is allocated to soccer, how much will be allocated to netball? b) If the school gets R8000 to spend on sport, how much will be allocated to netball?																		
Rp15	If we start with a principal of P Rands then the amount A in an account after t years, with an annual interest rate r compounded continuously, is given by: $A = Pe^{rt}$.																		

	If R5000 is deposited and earn $4\frac{1}{4}\%$ compounded continuously then how much will be accumulated at the end of a 3 year period?
Connections (Cn) level tasks	
	2011 academic year
Cn1	A factory A manufactures candles One worker can make 60 candles in a day. Factory B makes glass candle holders. One worker can make 18 glass candle holders in a day. The factory owners decide to collaborate and so want to make the same number of glass holders as candles each day. What is the smallest number of candle-makers factory A can employ and the smallest number of holder-makers factory B can employ so that they can do this?
Cn2	Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a picture to explain how you got your solution.
Cn3	Kara spent $\frac{1}{2}$ of her pocket money on Saturday and $\frac{1}{3}$ of what had left on Sunday. Can this situation be modeled as $\frac{1}{2} - \frac{1}{3}$? Explain why or why not.
Cn4	Create a story problem for $4,5 \div 0,75$.
Cn5	Use a real-life context to explain why it makes sense to say that the product of a positive and negative number is negative. Use an example like $3 \times (-2) = -6$ to illustrate it.
Cn6	Lynn says it will take her $\frac{1}{2}$ of a day to mark all the assignments. Mark says it will take him $\frac{1}{4}$ of a day to mark all assignments. If they work together to mark the assignments, how quickly will they be able to mark the assignments? (you can assume they each keep up the same pace as they would working alone.)
Cn7	I have $\frac{2}{3}$ of a pizza left over. My friend eats $\frac{1}{2}$ of the pizza. What fraction of the pizza do I have left? Does this scenario give you $\frac{2}{3} - \frac{1}{2}$. If yes, explain why. If not, change the wording of the problem so that it does.
Cn8	I have 150 exams to mark. I mark $\frac{1}{2}$ of them. I persuade a friend to mark $\frac{1}{3}$ of what I have left. How many do I have left to mark?
Cn9	Buhle invested money at a bank that paid 8% annual interest compounded quarterly. If she had R41 18,36 in her account at the end of 4 years, what was her initial investment
Cn10	My daughter wants to paint her bedroom pink. I mixed 3 tins of red paint with 5 tins of white paint and she says the pink it makes is perfect. I figure we need about 12 tins of paint to paint her bedroom. a) If I add 2 tins of white paint and 2 tins of red paint to the perfect pink mix will it be too red, too white or still perfect?

	b) How many tins of red paint and white paint must I add to the original perfect pink mix to make 12 tins worth of perfect pink mix?
Cn11	a) In order to make strawberry milkshake the instructions tell me I must mix $\frac{2}{3}$ of a cup of milk with $\frac{1}{5}$ of a cup of strawberry syrup. If I want to make 10 cups of milkshake, how many cups of milk and how many cups of syrup will I need? b) If you mixed $\frac{2}{3}$ of a cup of milk with $\frac{1}{4}$ of a cup of strawberry syrup would you have a milkshake that is stronger (i.e. more strawberry) or weaker than the original milkshake in (a). Explain your answer.
Cn12	At Pizzaz, the pizza with a 10cm radius costs R30. The pizza with a 15cm radius costs R45. Which is the better deal or is there no difference? Explain fully and clearly why you say so.
Cn13	You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?
Cn14	I spend $\frac{1}{2}$ of my salary on rent and $\frac{1}{5}$ of what I have left on groceries. What fraction of my salary is left for the rest of my expenses?
2012 academic year	
Cn15	One of your learners in a Mathematical Literacy classroom wants to buy a cell phone with internet. The learner has seen the advertisement (see attached page) and needs an advice from you on choosing a better deal. Help the learner and justify your thinking. Nokia X2-01 Valued at R1549, starter pack included <u>TopUp 99 Phone Contract</u> Get R100 airtime p.m. for calls and data. 1000 free SMSs p.m. R99 P.M. x 24 MP3 player, FM radio, OVI store, Push E-mail and instant messaging, web browsing Terms and conditions apply Web Exclusive Deals   Samsung Galaxy Mini Valued at R2299, starter pack included <u>TopUp 199 Phone Contract</u> Get R200 airtime p.m. for calls and data. 1000 free SMSs p.m. R199 P.M. x 24 HSDPA 7.2 + WI-FI Android OS 2.2 Froyo, Push E-mail Terms and conditions apply
Cn16	Nombuso went to a supermarket on Saturday 10th March, 2012. She wanted to buy chicken portions for a family of

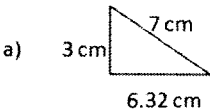
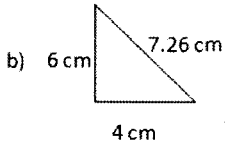
	three. She found out that a 2 kg packet of mixed portions cost R31.99 and a 5 kg packet of the same type cost R89.99. Which one is a better deal in terms of money saving. Show all your working.
Cn17	A loaf of bread is a regular purchase for many families. If a loaf of bread costs R7.24 and that the cost of the loaf has risen by the average inflation rate of 4,5% in the last 20 years. Find how much a loaf of bread would cost 20 years ago.
Cn18	Bank A offers an interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years. a) Calculate the amount he will receive at the end of the period from Bank A b) Now calculate the amount he will receive at the end of the period from Bank B. c) At which bank should he invest and why?
Cn19	Jane and Tom plan to install a sloping pool in their back garden. A sketch of the pool is shown below. The length of the pool is 6 m and its width is 3,5 m. The depth of the water in the shallow end is 1,2 m and 2 m deep in the deep end.  a) Calculate the volume of the raised cemented portion at the shallow end of the pool. b) Hence, determine the volume of water, in litres, required to fill the pool to the top. (NOTE: 1 000 litres = 1 m³.) c) Jane and Tom are planning to put up a security fence, one metre away from the edges of the pool. The fence will be right around the pool. Determine how many metres of fencing Jane and Tom would need to buy.
Cn20	Volume of Sound Model is given by; $L = 10 \cdot \log \left(\frac{I}{10^{-12}} \right)$. Here the volume L is measured in decibels (db) and I is the intensity in watts per square meter (W/m^2). a) An alarm has an intensity of $5,8 \times 10^{-9} W/m^2$. How loud is the alarm in decibels? b) Anna can scream at 56 db and Billy can yell at 48 db. How many more times intense is Anna's scream than Billy's yell?
Cn21	Sketch a scatter plot of the data given below and find out if the data have a positive correlation, a negative correlation, or relatively no correlation.

	<table><tr><td>x</td><td>1</td><td>2</td><td>3</td><td>3</td><td>5</td><td>5</td><td>6</td><td>7</td><td>8</td><td>8</td></tr><tr><td>y</td><td>2</td><td>4</td><td>5</td><td>6</td><td>3</td><td>4</td><td>8</td><td>7</td><td>7</td><td>8</td></tr></table>	x	1	2	3	3	5	5	6	7	8	8	y	2	4	5	6	3	4	8	7	7	8
x	1	2	3	3	5	5	6	7	8	8													
y	2	4	5	6	3	4	8	7	7	8													
Cn22	A lift at an office block can only carry 12 people. In a morning rush, 51 people want to go up the lift. How many times must it go up? Show your working.																						
Cn23	Of the 112 learners in Grade 10 at Greenside High School, three-quarters (3/4) have pets. One-sixth (1/6) of those with pets have cats. Use a model or picture to find the number of learners who have other kinds of pets.																						
Cn24	The figure below shows a cube-shaped tank. The tank contains 500 kilolitres of water, what is the height of the water in the tank? [1 m³=1kl] 																						
Cn25	This is the sign in a lift at an office block. THIS LIFT CAN CARRY UP TO 12 PEOPLE a) In a morning rush, 265 people want to go up the lift. How many times must it go up? b) What are the possible errors associated with the mathematical answer which learners can make when answering this question? Why?																						
Cn26	The table below shows the weight values for 250 boys given in the form of frequencies. <table><tr><th>Weight (kg)</th><th>Number of boys</th></tr><tr><td>62</td><td>4</td></tr><tr><td>63</td><td>20</td></tr><tr><td>64</td><td>53</td></tr><tr><td>65</td><td>98</td></tr><tr><td>66</td><td>49</td></tr><tr><td>67</td><td>20</td></tr><tr><td>68</td><td>6</td></tr></table>	Weight (kg)	Number of boys	62	4	63	20	64	53	65	98	66	49	67	20	68	6						
Weight (kg)	Number of boys																						
62	4																						
63	20																						
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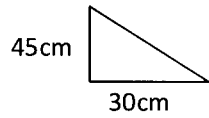
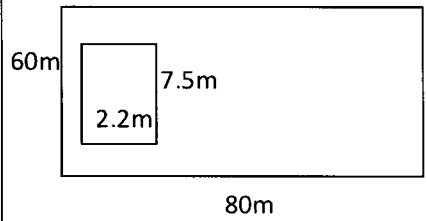
	a) Draw a frequency table showing the percentage frequencies and cumulative percentage frequencies. b) Draw a graph of the cumulative percentage frequencies, labeling clearly both the horizontal and vertical axes.																		
Cn27	If we assume that each time a woman has a baby (one at a time), the probability that the baby will be a boy is the same as the probability that the baby will be a girl, what is the probability that a woman who has three children will have; a) Three girls b) Two boys and one girl (in any order).																		
Cn28	Calculate the standard deviation for each set of the test scores. <table border="1"> <thead> <tr> <th>Test A</th><th>Test B</th></tr> </thead> <tbody> <tr><td>50</td><td>40</td></tr> <tr><td>70</td><td>62</td></tr> <tr><td>50</td><td>64</td></tr> <tr><td>50</td><td>72</td></tr> <tr><td>60</td><td>70</td></tr> <tr><td>80</td><td>68</td></tr> <tr><td>100</td><td>68</td></tr> <tr><td>100</td><td>68</td></tr> </tbody> </table> Compare performance in the two tests. In which test was performance better and why?	Test A	Test B	50	40	70	62	50	64	50	72	60	70	80	68	100	68	100	68
Test A	Test B																		
50	40																		
70	62																		
50	64																		
50	72																		
60	70																		
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100	68																		
100	68																		
Cn29	The diagram below (not drawn to scale) is a plan of Sandile's flat which they are planning to redecorate.																		

	a) All the ceilings are to be painted with 2 coats of white paint. Each litre of paint will cover 10 m^2 of ceiling. How much paint will she need to paint the ceilings? b) Is your answer an exact, underestimation or overestimation? Give a reason for your argument.
Cn30	The figure below shows front view of a round house plan.  Calculate the length of the roof beams, using only the dimensions that are labelled on the diagram. Show your calculations.

APPENDIX B: INSTRUCTION TASKS UTILIZED BY PRE-SERVICE TEACHERS ACROSS 2011 AND 2012

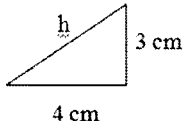
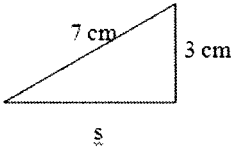
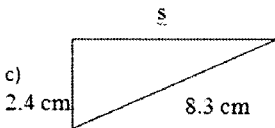
Instructional tasks used by the four teachers		Nature of task											
Tasks used by Lindiwe (LW)													
Observation lesson 2011	LW1: Sketch a graph of the function $y = -x^2$	Intra-mathematical task											
	LW2: Solve the following quadratic equation $2x^2 - 5x - 3 = 0$	Intra-mathematical task											
Video lesson 2011	LW3: The sum of two numbers is 56 and the difference between the numbers is 22. Find the two numbers.	Intra-mathematical task											
	LW4: The cost of the theatre tickets for 4 adults and 3 children is £47.50. The cost for 2 adults and 6 children is £44. How much does each adult and child ticket cost?	Extra-mathematical task											
Observation lesson 2012	LW5: Show whether the following triangle is right angled a)  b) 	Intra-mathematical task											
Video lesson 2012	LW6: Assume that each time a woman has a baby; she has 50% chance of having a boy and 50% chance of having a girl. a) if a woman has two children, draw a tree diagram to show all the possible outcomes in terms of the gender of the two children b) if a woman has two children, what is the probability that both her children being boys.	Extra-mathematical task											
	LW7: A travel agent plans trips for tourists from Chicago to Miami. He gives them three ways to get from town to town: airplane, bus, train. Once the tourists arrive, there are two ways to get to the hotel: hotel van or taxi. The cost of each type of transportation is given in the table below. <table border="1" data-bbox="497 1069 945 1287"><tr><td>Transportation</td><td>Cost</td></tr><tr><td>Airplane</td><td>\$350</td></tr><tr><td>Bus</td><td>\$150</td></tr><tr><td>Train</td><td>\$225</td></tr><tr><td>Hotel van</td><td>\$60</td></tr><tr><td>Taxi</td><td>\$40</td></tr></table> a) Draw a tree diagram to illustrate the possible choices for the tourists. Determine the cost for each outcome.	Transportation	Cost	Airplane	\$350	Bus	\$150	Train	\$225	Hotel van	\$60	Taxi	\$40
Transportation	Cost												
Airplane	\$350												
Bus	\$150												
Train	\$225												
Hotel van	\$60												
Taxi	\$40												

	b) If these six outcomes are chosen equally by tourists, what is the probability that a randomly selected tourist travel in a bus? c) What is the probability that a person's trip cost less than \$300 d) what is the probability that a person's trip costs more than \$350	
Tasks used by Mark (MK)		
Observation lesson 2011	MK1: Task based on scale drawing. Learners are instructed to draw the class on A4 paper to scale. The dimensions of the class and A4 paper are 720cm X 720cm and 42cm X 30cm respectively.	Extra-mathematical task
	MK2: Task based on map reading. Johannesburg central map is used as reference for the activity. a) Give the degrees, latitude and longitude of the points marked A, B, and C. b) What is the closest road to 26°11'30"S and 28°02'30"E? c) Using the scale, write down the length and breadth of Joubert Park?	Extra-mathematical task
Video lesson 2011	MK3: Mrs. Sibayi charged R2 500 per month for the rental of a flat she owned in East London in 2002. She raised the rent every year by the same percentage as inflation. The inflation for the next three years was approximately 7% in 2003, 3,5% in 2004 and 3,8% in 2005. Estimate the monthly rental in 2005. (R2 873,83)	Extra-mathematical task
	MK4: People often get an annual salary increase that is similar to the inflation rate. A man earns R4 200 per month after his annual increase, which was the same as the inflation rate of 5,5%. What did he earn per month during the previous year? (R3 981)	Extra-mathematical task
	MK5: You read in the newspaper that the inflation rate is decreasing. Which of the following statements is or are true in this case? (a) Prices are not going up. (b) Prices are going up more slowly than before. (c) Prices are going down. (d) Prices are going up faster than before.	Extra-mathematical task
Observation lesson 2012	MK6: Task was based on mathematising the Nelson Mandela bridge. The width of the deck of the bridge is 15 m. a) Estimate the area covered by the bridge b) Estimate the maximum number of cars that could be parked on the bridge, supposing the whole bridge was used as a parking area. Assume that the	Extra-mathematical task

	average car has a length of 4,5 m and a width of 1,8 m. (Hint: think about the area covered by one car, and work from there)																					
Video lesson 2012	MK7 Volume and external surface area of prisms <table><thead><tr><th></th><th></th><th>VOLUME</th><th>TOTAL EXTERNAL AREA (closed top and bottom)</th></tr></thead><tbody><tr><td>1.</td><td>Cylinder: Base  </td><td>Area of Base $\times$ Height  $Vol = \pi r^2 H$</td><td>Perimeter $\times H + 2 \text{ Area}$  $= 2\pi rH + 2\pi r^2$</td></tr><tr><td>2.</td><td>Rectangular Prism: </td><td>Area Base $\times$ Height  $= l \times b \times H$</td><td></td></tr><tr><td>3.</td><td>Cube: (Square) </td><td>Area Base $\times$ Height  $= side \times side \times side$ $= side^3$</td><td></td></tr><tr><td>4.</td><td>Prism with Base: Triangle </td><td>Area Base $\times$ Height  $= \frac{1}{2}bh \times H$</td><td></td></tr></tbody></table>			VOLUME	TOTAL EXTERNAL AREA (closed top and bottom)	1.	Cylinder: Base  	Area of Base $\times$ Height  $Vol = \pi r^2 H$	Perimeter $\times H + 2 \text{ Area}$  $= 2\pi rH + 2\pi r^2$	2.	Rectangular Prism: 	Area Base $\times$ Height  $= l \times b \times H$		3.	Cube: (Square) 	Area Base $\times$ Height  $= side \times side \times side$ $= side^3$		4.	Prism with Base: Triangle 	Area Base $\times$ Height  $= \frac{1}{2}bh \times H$		Intra-mathematical task
		VOLUME	TOTAL EXTERNAL AREA (closed top and bottom)																			
1.	Cylinder: Base  	Area of Base $\times$ Height  $Vol = \pi r^2 H$	Perimeter $\times H + 2 \text{ Area}$  $= 2\pi rH + 2\pi r^2$																			
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3.	Cube: (Square) 	Area Base $\times$ Height  $= side \times side \times side$ $= side^3$																				
4.	Prism with Base: Triangle 	Area Base $\times$ Height  $= \frac{1}{2}bh \times H$																				
Tasks used by Jabu (JB)																						
Observation lesson 2011	JB1: Find the area of the triangle 	Intra-mathematical task																				
	JB2: A hall is to be constructed with tiled floor. The tiles cost R7/m². If the hall, 80m by 60m, is to have a stage 7.5m by 2.5m, calculate the floor area excluding the area occupied by the stage. 	Extra-mathematical task																				

Video lesson 2011	JB3: If we have 14 girls and 5 boys in this class, how do we go about calculating the percentage of girls to boys?	Extra-mathematical task																
	JB4: Karen earns R77 560 and is given a 3% wage increase. And Darren earns R75 420 and is given an increase of 3.5%. a) Who received the larger increase in Rand terms? b) Who earns more after the increase?	Extra-mathematical task																
Observation lesson 2012	JB5: According to the 1996 census there are 1,8 million Tsonga speaking and 9 million Zulu-speaking people in South Africa. Determine the ratio of Tsonga-speaking to Zulu-speaking people. Write the ratio in its simplest form	Extra-mathematical task																
	JB6: The ratio of the distance a motorist travelled to the distance a cyclist travelled is 40:3. How far did the motorist travel if the cyclist travelled 21 km?	Extra-mathematical task																
	JB7: A piece of wood is cut in the ratio 2:5. If the shorter piece is 56 cm long, how long was the whole piece of wood before cutting?	Extra-mathematical task																
Video lesson 2012	JB8: I borrow R5 000 at 5% over 1 year simple interest. Calculate interest	Extra-mathematical task																
	JB9: If I had my P, which is my principal, the principal amount is 10 000. The rate, 7 percent, and this must be paid over 5 years. Find interest.	Extra-mathematical task																
Tasks used by Lebo (LB)																		
Observation lesson 2011	LB1: Imagine a hotel that has a large casino attached to it. The management knows that the more hotel guests there are that gamble, the more money they are likely to make. They want to know how many of their guests gamble at their casino and whether male or female guests are more likely to gamble. They keep careful records about the gender and gambling habits of the next 500 guests visiting the hotel. They find that 247 guests are male and 253 don't. Of the men, 87 gamble and 160 don't. Altogether, 58 women gamble and 195 don't. Summarise this data by completing the following table. <table><tr><td></td><td>Gamble</td><td>Don't gamble</td><td>Total</td></tr><tr><td>Male</td><td></td><td>160</td><td></td></tr><tr><td>Female</td><td>58</td><td></td><td>253</td></tr><tr><td>Total</td><td></td><td>355</td><td>500</td></tr></table>		Gamble	Don't gamble	Total	Male		160		Female	58		253	Total		355	500	Extra-mathematical task
		Gamble	Don't gamble	Total														
Male		160																
Female	58		253															
Total		355	500															
	LB2: I want to find out how many learners are absent every week in my class. I also want to know if these learners are male or female. A record of absenteeism for	Extra-mathematical task																

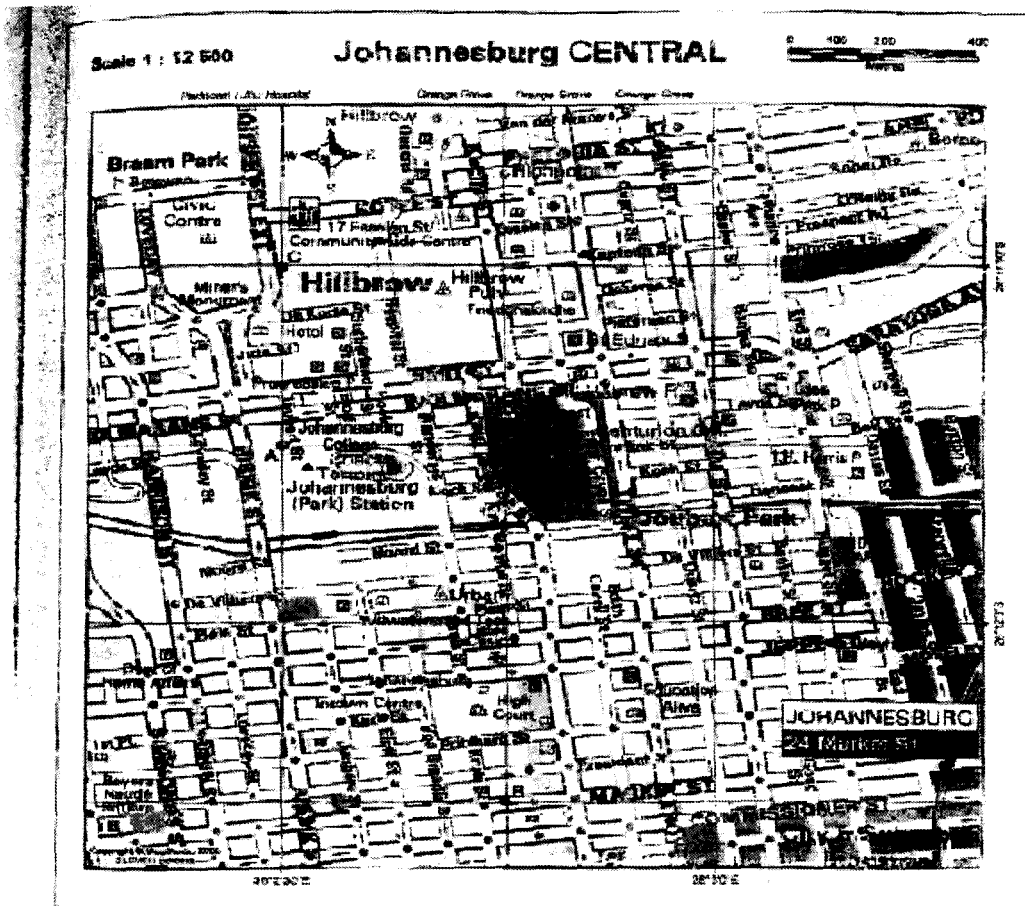
	the next week is kept. I have 35 learners in my class. There are 20 males and 15 females in the class. Of the learners 25 are never absent and 10 are always absent. Of the males 6 are always absent and 4 females are always absent. Summarise this information in a contingency table.																																																												
Video lesson 2011	LB3: An insurance company divides its clients into two age groups, under 30 and over 30. In a particular year, 120 of the 500 clients were under 30. In that year 150 clients, of whom 50 were under 30, made claims. a). draw a contingency table to summarise this data b). find the probability that a randomly selected client: i) is under 30 ii) has made a claim iii) is under 30 and has not made a claim	Extra-mathematical task																																																											
	LB4: The South African National Spatial Biodiversity Assessment 2004 investigated the status of ecosystems across South Africa. The purpose was to plan for what needed to be done to protect our natural environment, plants and animals. Ecosystems were classified according to type of vegetation and status. The status of an ecosystem was classified as: • least threatened if the ecosystem was still mostly intact. • vulnerable if the ecosystem was reasonably intact but nearing the point beyond which it could no longer function properly. • endangered if it had lost significant amounts of its natural habitat. • critically endangered if it had very little habitat left and species associated with it were being lost. <table><tr><td></td><td>least threatened</td><td>vulnerable</td><td>endangered</td><td>critically endangered</td><td>Total</td></tr><tr><td>Albany thicket</td><td>16</td><td>1</td><td>1</td><td>0</td><td>18</td></tr><tr><td>Desert</td><td>16</td><td>0</td><td>1</td><td>0</td><td>17</td></tr><tr><td>Forest</td><td>7</td><td>0</td><td>0</td><td>5</td><td>12</td></tr><tr><td>Fynbos</td><td>67</td><td>12</td><td>29</td><td>14</td><td>122</td></tr><tr><td>Grassland</td><td>33</td><td>28</td><td>18</td><td>1</td><td>80</td></tr><tr><td>Nama-Karoo</td><td>14</td><td>1</td><td>0</td><td>0</td><td>15</td></tr><tr><td>Savannah</td><td>59</td><td>21</td><td>7</td><td>0</td><td>87</td></tr><tr><td>Succulent Karoo</td><td>65</td><td>2</td><td>1</td><td>0</td><td>68</td></tr><tr><td>Wetland</td><td>12</td><td>5</td><td>1</td><td>1</td><td>19</td></tr></table>		least threatened	vulnerable	endangered	critically endangered	Total	Albany thicket	16	1	1	0	18	Desert	16	0	1	0	17	Forest	7	0	0	5	12	Fynbos	67	12	29	14	122	Grassland	33	28	18	1	80	Nama-Karoo	14	1	0	0	15	Savannah	59	21	7	0	87	Succulent Karoo	65	2	1	0	68	Wetland	12	5	1	1	19
	least threatened	vulnerable	endangered	critically endangered	Total																																																								
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	<table><tr><td>Total</td><td>289</td><td>70</td><td>58</td><td>21</td><td>438</td></tr></table>	Total	289	70	58	21	438							
Total	289	70	58	21	438									
	a) what percentage of ecosystems in South Africa are critically endangered? b) what percentage of ecosystems in South Africa are either endangered or critically endangered? c) what percentage of the critically endangered ecosystems is fynbos?													
Observation lesson 2012	LB5: Complete the table for the following formula $m = \frac{2}{n}$ <table><tr><td>n</td><td>-1</td><td>0</td><td>1/2</td><td>2</td><td>4</td></tr><tr><td>m</td><td></td><td></td><td></td><td></td><td></td></tr></table> LB6: Find the roots of the equation $2x^2 - 6x + 1 = 0$ LB7: Use the following formula to find y in each case, $y = A(\frac{100+r}{100})^x$: a) A = 200; r = 20 and x = 7 b) A = 12 200; r = 9 and x = 120	n	-1	0	1/2	2	4	m						Intra-mathematical task Intra-mathematical task Intra-mathematical task
n	-1	0	1/2	2	4									
m														
Video lesson 2012	LB8: Find the length of the unknown side in each triangle (answers rounded off to one decimal place). a)  b)  c) 	Intra-mathematical tasks												

APPENDIX C: POST-LESSON INTERVIEW SCHEDULE FOR PRE-SERVICE ML TEACHERS

1. What do you think were the strengths and weaknesses of your lesson? If you are to teach the lesson again would you do it differently?
2. With reference to your classroom tasks, why did you choose those tasks to support your ML teaching?
3. Do you discuss the design/selection of your tasks with your supervising teacher or other pre-service ML teachers at your school? If so, do you find this useful and how?
4. Did you feel that the tasks achieve their intended purpose in the lesson? If so, how?
5. What documents/textbooks/other sources do you use to inform your selection of classroom tasks?
6. What are some of the difficulties you experience in designing/selecting tasks for your ML lesson?
7. Why did you approach your lesson the way you did (i.e. from context to formal mathematics or operating within the situation itself throughout)?
8. Do you think the approach you took helped learners to understand the context(s)? If so how?
9. Do you think the approach you took helped learners to understand the mathematical content? If so how?
10. Any other comments related to your lesson?

APPENDIX D: MAP OF JOHANNESBURG CENTRAL



1. Give the degrees, latitude and longitude of the points marked A, B and C.
2. What is the closest road to 26°11'30"S and 28°02'30"E?
3. Using the scale, write down the length and breadth of Joubert Park.

APPENDIX E: CODING OF TASKS

Task	Task solution	Indicators		
		Model formulation	Intra-mathematical working	Interpretive aspect
Tasks from CLM course				
Rp1: A person has $29\frac{1}{2}$ metres of material available to make doll's dresses. Each dress requires $\frac{3}{4}$ metre of material. How many dresses can be made?	$\frac{3}{4} \times 29\frac{1}{2}$ = 22,125 Therefore 22 dresses can be made	$\frac{3}{4} \times 29\frac{1}{2}$	$\frac{3}{4} \times 29\frac{1}{2}$ = 22,125	Therefore 22 dresses can be made
Rp2: I spend $\frac{1}{2}$ of my salary on rent and $\frac{1}{5}$ of what I have left on groceries. What fraction of my salary is left for the rest of my expenses?	$\frac{1}{2} \times \frac{1}{5}$ = $\frac{1}{10}$ is left	$\frac{1}{2} \times \frac{1}{5}$	$\frac{1}{2} \times \frac{1}{5}$ = $\frac{1}{10}$	$\frac{1}{10}$ is left
Rp3: I have $\frac{3}{4}$ litres of milk in the fridge. I drink $\frac{1}{3}$ of it. How much milk (in litres) do I have left?	$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$ litres of milk Therefore 416,67 ml =0,42 litres	$\frac{3}{4} - \frac{1}{3}$	$\frac{3}{4} - \frac{1}{3} = \frac{9}{12} - \frac{4}{12} = \frac{5}{12}$	$\frac{5}{12}$ litres of milk Therefore 416,67 ml =0,42 litres
Rp5: The price of a shirt is reduced from R350 to R280. By what percentage has the price of the shirt been reduced?	R350 to R280 % decrease= $\frac{\text{initial}-\text{final}}{\text{initial}} \times 100$ $= \frac{350-280}{350} \times 100$ 20% decrease	R350 to R280 % decrease= $\frac{\text{initial}-\text{final}}{\text{initial}} \times 100$ $= \frac{350-280}{350} \times 100$	$\frac{350-280}{350} \times 100$ 20%	20% decrease
Rp6: John and Jane both currently earn R10 000 per month. a) John performs badly in	9% of 10000=R900 10000-900=R9100	9% of 10000	9% of 10000=R900 10000-900=R9100	Therefore he will now earn R9100

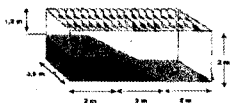
this job so is demoted and will earn 9% less from next month onwards. How much will he earn?	Therefore he will now earn R9100			
b) Last month Jane was actually earning less than R10 000 and she received a raise of 9% which brought her salary up to R10 000. What was she earning last month?	Initial=? %=9% of ? Final=10000 $9174,32(1+0,09)$ =R10000,01 Therefore she was earning R9174,32 last month	Initial=? %=9% of ? Final=10000 $9174,32(1+0,09)$	$9174,32(1+0,09)$ =R10000,01	Therefore she was earning R9175,32 last month
Rp8: Nadia is getting a 3,5% increase in salary and Sekuru is getting an increase in salary of R259,86 more per month. Nadia earns R6 075 per month and Sekuru earns R8 000 per month. a) Determine Nadia's new salary per month.	$103,5\% \times R6075 =$ $R6287,63$ per month	$103,5\% \times R6075$	$103,5\% \times R6075$ = R6287,63	R6287,63 per month
b) Who received the greater percentage increase? Show your working	$103,5\% \times R7740,14 =$ $R8011,04$ Therefore Nadia earns a greater percentage increase	$103,5\% \times R7740,14$	$103,5\% \times R7740,14 =$ $R8011,04$	Therefore Nadia earns a greater percentage increase
Rp9: A company has a contract to put up 3 000 metres of fencing around a golf course. A team of six workers can complete 20 metres of fencing in one day a) If the Company has one six-man team on the job,	3000m; 6 men, 20m/day $3000 \div 20 = 150$ days	3000m; 6 men, 20m/day $3000 \div 20$	$3000 \div 20 = 150$	150 days

how long would it take to complete the contract?																						
b) How many teams must they put on the job if they have to get the contract finished, in one day?	1 team=150 days ? = 1 day $3000 \div 150 = 20$ Therefore you will need 150 teams	1 team=150 days ? = 1 day $3000 \div 150$	$3000 \div 150 = 20$	Therefore you will need 150 teams																		
Rp12: The table below shows two sets of ML method test scores. <table border="1"><thead><tr><th>Test A</th><th>Test B</th></tr></thead><tbody><tr><td>50</td><td>40</td></tr><tr><td>70</td><td>62</td></tr><tr><td>50</td><td>64</td></tr><tr><td>50</td><td>72</td></tr><tr><td>60</td><td>70</td></tr><tr><td>80</td><td>68</td></tr><tr><td>100</td><td>68</td></tr><tr><td>100</td><td>68</td></tr></tbody></table> Calculate the mean for each set of the test scores.	Test A	Test B	50	40	70	62	50	64	50	72	60	70	80	68	100	68	100	68	Test A $\frac{50+70+50+50+60+80+100+100}{8} = 70$ Test B $\frac{40+62+64+72+70+68+68+68}{8} = 64$	Test A $\frac{50+70+50+50+60+80+100+100}{8} = 70$ Test B $\frac{40+62+64+72+70+68+68+68}{8} = 64$		
Test A	Test B																					
50	40																					
70	62																					
50	64																					
50	72																					
60	70																					
80	68																					
100	68																					
100	68																					
Rp13: The scale on a map is 1:35 000. If the distance between two towns on the map is 2,75cm, determine the actual distance between the towns in kilometres.	1:35000 2,75: ? $2,75 \times 35000 = 96250cm \div 100000 = 0,9625 km$	1:35000 2,75: ? $2,75 \times 35000$	$2,75 \times 35000 = 96250cm \div 100000 = 0,9625 km$																			
Rp14: The only sports offered at Burg High School are soccer and netball. The principal loves soccer so he allocates the sports budget	Budget; 2net: 3 soccer R450 to soccer $450 + \left(\frac{2}{3} \times 450\right) = total$	Budget; 2net: 3 soccer R450 to soccer $450 + \left(\frac{2}{3} \times 450\right) = total$	$450+300=750$	Therefore netball receives R300																		

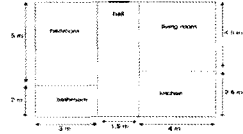
so that for every R2 spent on netball, R3 will be spent on soccer. a) If R450 is allocated to soccer, how much will be allocated to netball?	450+300=750 Therefore netball receives R300			
b) If the school gets R8000 to spend on sport, how much will be allocated to netball?	R8000 total $\frac{8000}{5} = 1600$ $1600 \times 2 = 32000$ Therefore R32000 will be allocated to netball	R8000 total $\frac{8000}{5}$	$\frac{8000}{5} = 1600$ $1600 \times 2 = 32000$	Therefore R32000 will be allocated to netball
Rp15: If we start with a principal of P Rands then the amount A in an account after t years, with an annual interest rate r compounded continuously, is given by: $A = Pe^{rt}$. If R5000 is deposited and earn $4\frac{1}{4}\%$ compounded continuously then how much will be accumulated at the end of a 3 year period?	$P=5000, r = \frac{4,25}{100}, t=3$ $A = Pe^{rt}$ $A = 5000e^{0,0425 \times 3}$ $A = R5679,92$ at the end of 3 years	$P=5000, r = \frac{4,25}{100}, t=3$ $A = Pe^{rt}$ $A = 5000e^{0,0425 \times 3}$	$5000e^{0,0425 \times 3}$ $= R5679,92$	R5679,92 at the end of 3 years
Cn2: Anna gave $\frac{1}{2}$ of her chocolate bar to Buhle. Buhle gave $\frac{1}{3}$ of the chocolate she got from Anna to Rashad. What fraction of the chocolate bar did Rashad get? Use a	 Therefore Rashad got $\frac{1}{6}$ of the chocolate bar			Therefore Rashad got $\frac{1}{6}$ of the chocolate bar

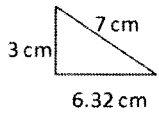
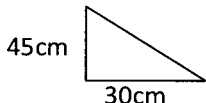
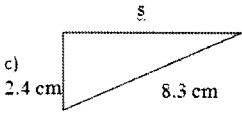
picture to explain how you got your solution.				
Cn6: Lynn says it will take her $\frac{1}{2}$ of a day to mark all the assignments. Mark says it will take him $\frac{1}{4}$ of a day to mark all assignments. If they work together to mark the assignments, how quickly will they be able to mark the assignments? (you can assume they each keep up the same pace as they would working alone.)	12 hours for Lynn 6 hours for Mark Therefore $\frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $= 0,125$ of a day Therefore $0,125 \times 24 = 3$ hours if they work together	12 hours for Lynn 6 hours for Mark Therefore $\frac{1}{2} \times \frac{1}{4}$	$\frac{1}{2} \times \frac{1}{4} = \frac{1}{8} = 0,125$ $0,125 \times 24 = 3$	0,125 of a day 3 hours if they work together
Cn8: I have 150 exams to mark. I mark $\frac{1}{2}$ of them. I persuade a friend to mark $\frac{1}{3}$ of what I have left. How many do I have left to mark?	$\frac{1}{2} \times 150 = 75$ $\frac{1}{3} \times 75 = 25$ 75-25=50 You have to mark 50	$\frac{1}{2} \times 150$ $\frac{1}{3} \times 75$	$\frac{1}{2} \times 150 = 75$ $\frac{1}{3} \times 75 = 25$ 75-25=50	You have to mark 50
Cn9: Buhle invested money at a bank that paid 8% annual interest compounded quarterly. If she had R4118,36 in her account at the end of 4 years, what was her initial investment	$A = P(1 + i)^n$ A=4118,36 P=? $i = \frac{8\%}{4}$ $n = 4 \times 4$ $4118,36 = P(1 + 0,02)^{16}$ $4118,36 = P(1,02)^{16}$ $4118,36 = P(1,372785705)$ $\frac{4118,36}{1,372785705} = P$ Therefore P=R3000,00 Therefore she initially invested R3000,00	$A = P(1 + i)^n$ A=4118,36 P=? $i = \frac{8\%}{4}$ $n = 4 \times 4$ $4118,36 = P(1 + 0,02)^{16}$	$4118,36 = P(1 + 0,02)^{16}$ $4118,36 = P(1,02)^{16}$ $4118,36 = P(1,372785705)$ $\frac{4118,36}{1,372785705} = P$ P=R3000,00	Therefore she initially invested R3000,00

Cn13: You buy a car for R85 000. If each year the value of the car depreciates by 10% of its value the previous year, what will its value be at the end of 3 years?	85000; 10%/year; 3 years $A = P(1 - i)^n$ $= 85000(1 - 0,1)^3$ $= R61965$ at the end of 3 years	85000; 10%/year; 3 years $A = P(1 - i)^n$ $= 85000(1 - 0,1)^3$	$85000(1 - 0,1)^3 = R61965$	R61965 at the end of 3 years
Cn16: Nombuso went to a supermarket on Saturday 10th March, 2012. She wanted to buy chicken portions for a family of three. She found out that a 2 kg packet of mixed portions cost R31.99 and a 5 kg packet of the same type cost R89.99. Which one is a better deal in terms of money saving. Show all your working.	2kg = 31,99 5kg=89,99 $\frac{31,99}{2} = R15,99$ per kg $\frac{89,99}{5} = R17,99$ per kg Therefore It would be cheaper to buy the 2kg chicken as it is R2,00 cheaper per kg than the 5kg chicken. She should buy 2x2kg chickens in order for it to be a good deal. However 3 people won't need more than 2kg of chicken.	2kg = 31,99 5kg=89,99 $\frac{31,99}{2}$ $\frac{89,99}{5}$	$\frac{31,99}{2} = R15,99$ $\frac{89,99}{5} = R17,99$	R15.99 per kg 17,99 per kg It would be cheaper to buy the 2kg chicken as it is R2,00 cheaper per kg than the 5kg chicken. She should buy 2x2kg chickens in order for it to be a good deal. However 3 people won't need more than 2kg of chicken.
Cn17: A loaf of bread is a regular purchase for many families. If a loaf of bread costs R7.24 and that the cost of the loaf has risen by the average inflation rate of 4,5% in the last 20 years. Find how much a loaf of bread would cost 20 years ago.	$A = P(1 - i)^n$ $= 7,24(1 - 0,045)^{20}$ $= R2,88$ Therefore, 20 years ago a loaf of bread would have cost R2,88	$A = P(1 - i)^n$ $= 7,24(1 - 0,045)^{20}$	$7,24(1 - 0,045)^{20} = R2,88$	Therefore, 20 years ago a loaf of bread would have cost R2,88
Cn18: Bank A offers an				

interest of 7,2% per annum simple interest. Bank B offers an interest of 5,4% per annum compounded quarterly. Mr Mazibuko wants to invest R6 000,00 for 2 years. a). Calculate the amount he will receive at the end of the period from Bank A	$7,2\% = 0,072$ $A = P(1 + i \cdot n)^n$ $= R6000(1 + 0,072 \times 2)$ $= R6864,00$	$7,2\% = 0,072$ $A = PP(1 + i \cdot n)^n$ $= R6000(1 + 0,072 \times 2)$	$R6000(1 + 0,072 \times 2)$ $= R6864,00$	
b). Now calculate the amount he will receive at the end of the period from Bank B.	$A = P(1 + i)^n$ $i = 7,2 = 0,072 \div 6 = 0,018$ $n = 2 \times 4 = 8$ $A = 6000(1 + 0,018)^8$ $= R6920,44$	$A = P(1 + i)^n$ $i = 7,2 = 0,072 \div 6 = 0,018$ $n = 2 \times 4 = 8$ $A = 6000(1 + 0,018)^8$	$A = 6000(1 + 0,018)^8$ $= R6920,44$	
Cn19: Jane and Tom plan to install a sloping pool in their back garden. A sketch of the pool is shown below. The length of the pool is 6 m and its width is 3,5 m. The depth of the water in the shallow end is 1,2 m and 2 m deep in the deep end.  a). Calculate the volume of the raised cemented	Volume of the rectangular prism $= l \times b \times h$ $= 2 \times 3,5 \times 0,8$ $= 5,6 \text{ m}^3$ Volume of triangular prism $= \frac{1}{2} b \times h \times w$ $= 0,5 \times 0,8 \times 3,5$ $= 1,4 \text{ m}^3$ Therefore, $5,6 + 1,4 = 7 \text{ m}^3$	Volume of the rectangular prism $= l \times b \times h$ $= 2 \times 3,5 \times 0,8$ Volume of triangular prism $= \frac{1}{2} b \times h \times w$ $= 0,5 \times 0,8 \times 3,5$	$2 \times 3,5 \times 0,8$ $= 5,6 \text{ m}^3$... $0,5 \times 0,8 \times 3,5$ $= 1,4 \text{ m}^3$... Therefore, $5,6 + 1,4 = 7 \text{ m}^3$	

portion at the shallow end of the pool.				
b). Hence, determine the volume of water, in litres, required to fill the pool to the top. (NOTE: 1 000 litres = 1 m³.)	Total volume $= l \times b \times h$ $= 6 \times 3,5 \times 2 = 42 \text{ m}^3$ Therefore volume of the concrete area $= 42 - 7 = 35 \text{ m}^3$ Therefore $35 \times 1000 = 35000$ litres of water is required.	Total volume $= l \times b \times h$ $= 6 \times 3,5 \times 2$	$6 \times 3,5 \times 2 = 42 \text{ m}^3$ Therefore volume of the concrete area $= 42 - 7 = 35 \text{ m}^3$ Therefore $35 \times 1000 = 35000$ litres	35000 litres of water is required.
c). Jane and Tom are planning to put up a security fence, one metre away from the edges of the pool. The fence will be right around the pool. Determine how many metres of fencing Jane and Tom would need to buy.	Perimeter of pool + area for fencing $= 2(3,5 + 1 + 1) + 2(6 + 1 + 1) = 27 \text{ m}$ of fencing will be needed	Perimeter of pool + area for fencing $= 2(3,5 + 1 + 1) + 2(6 + 1 + 1)$	$2(3,5 + 1 + 1) + 2(6 + 1 + 1) = 27 \text{ m}$	27m of fencing will be needed
Cn25: This is the sign in a lift at an office block. THIS LIFT CAN CARRY UP TO 12 PEOPLE In a morning rush, 265 people want to go up the lift. How many times must it go up?	$265/12 = 22,083$ $22 \times 12 = 264$ $265 - 264 = 1$ Therefore the lift must go up 23 times	$265/12$	$265/12 = 22,083$ $22 \times 12 = 264$ $265 - 264 = 1$	Therefore the lift must go up 23 times
Cn29: The diagram below (not drawn to scale) is a plan of Sandile's flat	$7 \times 8,5 = 59,5 \text{ m}^2 \times 2 = 119 \text{ m}^2$	$7 \times 8,5$	$7 \times 8,5 = 59,5 \text{ m}^2 \times 2 = 119 \text{ m}^2$	She will need 12 litres of paint

which they are planning to redecorate.  a) All the ceilings are to be painted with 2 coats of white paint. Each litre of paint will cover 10 m² of ceiling. How much paint will she need to paint the ceilings? b) Is your answer an exact, underestimation or overestimation? Give a reason for your argument.	Therefore $119/10=11,9$ She will need 12 litres of paint		Therefore $119/10=11,9$	
	Overestimation. 11 litres will not quite cover the whole amount thus you will buy more than what you need to get the job done, so you will have 0,1 litres left over, therefore it is an overestimation			Overestimation. 11 litres will not quite cover the whole amount thus you will buy more than what you need to get the job done, so you will have 0,1 litres left over, therefore it is an overestimation
Tasks from practice (for pedagogic explanations, refer to chapter 6)				
LW2: Solve the following quadratic equation $2x^2 - 5x - 3 = 0$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2; b = -5; c = -3$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$ $x = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5}{4} \pm \frac{7}{4}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2; b = -5; c = -3$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$	$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ $a = 2; b = -5; c = -3$ $x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(-3)}}{2(2)}$ $x = \frac{5 \pm \sqrt{25 + 24}}{4} = \frac{5}{4} \pm \frac{7}{4}$	
LW5: Show whether the following triangle is right angled	$Hyp^2 = S^2 + S^2$ $7cm^2 = 6,32cm^2 + 3cm^2$ $49cm = 48,94cm$	$Hyp^2 = S^2 + S^2$ $7cm^2 = 6,32cm^2 + 3cm^2$	$7cm^2 = 6,32cm^2 + 3cm^2$ $49cm = 48,94cm$	Since the two numbers are different, the triangle is not right angled

	Since the two numbers are different, the triangle is not right angled			
MK4: People often get an annual salary increase that is similar to the inflation rate. A man earns R4 200 per month after his annual increase, which was the same as the inflation rate of 5,5%. What did he earn per month during the previous year? (R3 981)	$R4200 \times 5,5\% = R231$ $R4200 - R231 = R3969$	$R4200 \times 5,5\%$	$R4200 \times 5,5\% = R231$	
JB1: Find the area of the triangle 	$A = \frac{1}{2} \times \text{base} \times \text{height}$ $A = \frac{1}{2} \times 30 \times 45 = 675$	$A = \frac{1}{2} \times \text{base} \times \text{height}$ $A = \frac{1}{2} \times 30 \times 45$	$A = \frac{1}{2} \times 30 \times 45 = 675$	
LB8: Find the length of the unknown side in the triangle (answers rounded off to one decimal place). 	$c^2 = a^2 + b^2$ $c^2 = 2,4^2 + 8,3^2 = 63.13$ $\sqrt{c^2} = \sqrt{63.13}$ $c = 7,9 \text{ cm}$	$c^2 = a^2 + b^2$ $c^2 = 2,4^2 + 8,3^2$	$c^2 = 2,4^2 + 8,3^2 = 63.13$ $\sqrt{c^2} = \sqrt{63.13}$ $c = 7,9 \text{ cm}$	