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Shabangu, Thabo

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EXPLORING THE APPLICATION OF PROCEDURAL KNOWLEDGE AND CONCEPTUAL KNOWLEDGE IN GEOMETRY: A CASE OF SCHOOLS IN BUSHBUCKRIDGE IN THE MPUMALANGA PROVINCE, SOUTH AFRICA

Thabo Shabangu

2519462

Supervisor: Dr B. Mofolo-Mbokane

A dissertation

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University of Witwatersrand, Johannesburg

In fulfilment of the requirements for the Master of Education degree by dissertation in Mathematics.

March 2024

DECLARATIONS

I, **Thabo Shabangu**, affirm that the dissertation titled "**Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa**" is my original work. I have appropriately cited and referenced any information that I have quoted to give credit to the sources. I affirm that this dissertation has not been presented for the award of a degree at any other university besides the University of Witwatersrand, Johannesburg, South Africa.

Student: Thabo Shabangu

Signature: 

Date: 15 March 2024

ABSTRACT

The study focuses on the application of procedural and conceptual knowledge in the teaching and learning of geometry. It explores grade 6 and grade 9 learners' proficiency in both types of knowledge through a geometry test. The research also investigates the impact of teaching procedural and conceptual knowledge in learning geometry through semi-structured interviews. Moreover, it explores how grade 6 educators impart both procedural and conceptual knowledge to their learners through educators' observation during geometry lessons. Participants in the study include educators and learners. The theoretical framework for this research is rooted in Van Hiele and Concrete Pictorial Abstract (CPA) theories. The research design for this study utilises a mixed-method approach, incorporating both qualitative and quantitative research. Various data collection methods are employed, including the observation of educators (n=5) during geometry lessons, semi-structured face-to-face interviews with learners (n=10) and educators (n=5), and the administration of a test to learners (n=429).

The research involves 5 primary schools with grade 6 classes and 2 high schools with grade 9 classes. The performance of grade 6 and grade 9 learners is evaluated using the grade 6 assessment to measure their procedural and conceptual knowledge. The study compares the knowledge performances of grade 9 learners with those of grade 6 learners and examines the growth of this knowledge over time. Some notable findings suggest that educators predominantly employ traditional methods in teaching. The results reveal that learners in both grade 6 and grade 9 performed poorly in the assessment of procedural knowledge and conceptual knowledge in geometry. Additionally, the findings indicate that learners generally exhibit a positive attitude towards geometry, displaying a stronger inclination towards learning through conceptual knowledge, which involves using concrete objects, visuals, and technology instead of procedural knowledge or traditional teaching approaches. These findings suggest that the underperformance is attributed to learners being taught with traditional methods and rote memorisation in geometry. The recommendations are that educators adopt teaching methods that actively involve learners, such as utilising tangible objects, technological tools, and real-life examples to facilitate the connection of concepts with their personal experiences.

DEDICATIONS

This dissertation is dedicated to my Mother, Sibongile Nontokozo Lekhuleni-Shabangu. Shortly after registering for this degree, within four months of starting my studies, she fell seriously ill. At that time, I was facing difficulties with my research proposal while also dealing with her illness, which was overwhelming for me. I doubted your recovery and felt close to giving up my studies due to the intense pressure it added to my research responsibilities. Thanks to my Brother Bongani Mafutha Shabangu's encouragement and my mother's resilience, I could persevere and complete my studies. This dissertation is dedicated to you, my mother, whose nurturing and guidance have profoundly shaped my life. I love you dearly and am grateful for all that you have done.

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LIST OF ACRONYMS

2D – Two dimensional

3D – Three dimensional

ATP – Annual Teaching Plan

CK – Conceptual Knowledge

CPA – Concrete Pictorial Abstract

DBE – Department of Basic Education

FET – Further Education Training

L1 – Learner 1

L2 – Learner 2

L3 – Learner 3

MCQ – Multiple Choice Questions

NCS – National Curriculum Statement

PK – Procedural Knowledge

Qual – Qualitative

Quan – Quantitative

RQ – Research Question

TIMSS – Trends in International Mathematics and Science study

KEYWORDS

Analysis, 2D shapes, 3D shapes, Area, Concrete Pictorial Abstract, Conceptual knowledge, Geometry, Measurement, Perimeter, Procedural knowledge, Van Hiele, Visualisation, Volume.

DEFINITION OF CONCEPTS

Procedural Knowledge - as per Johnson, Schneider, and Star (2015), individuals with PK possess expertise in a specific procedure but may not have a comprehensive grasp of the underlying concepts or broader field.

Conceptual Knowledge - is defined as a type of understanding that involves the ability to make connections and relationships. Skemp (1976) describes relational understanding as the capacity to grasp the step-by-step procedures and the underlying reasons behind them.

Intermediate Phase – refers to learners studying from grade 4 to grade 6.

Senior Phase – refers to learners studying from grade 7 to grade 9.

Educators - the educators mentioned in the study are currently teaching mathematics to learners in grade 6 (intermediate phase).

Learners - the learners included in this study are learners in Grade 6 and Grade 9 (2023) from specifically chosen schools in the Bushbuckridge District.

Grade 6 - is the exit grade in the intermediate phase.

Grade 9 - is the exit grade in the senior phase.

Geometry – is a mathematical discipline concerned with the quantification, characteristics, and interconnections of points, lines, angles, surfaces, lines, solids, and angles. (Clapham, Nicholson, and Nicholson, 2014).

Mathematics - is the practice of using symbols to describe relationships between numbers, geometric ideas, and visual representations (DoE, 2011).

Van Hiele - The Van Hiele theory elucidates the process through which learners acquire knowledge in the field of geometry. The theory proposes five distinct stages of geometric cognition, namely visualisation, analysis, abstraction, formal deduction, and rigour. Every level employs its unique language and symbols. Learners progress through the levels gradually and sequentially. The hierarchical structure facilitates their ability to attain enhanced comprehension and outcomes (Vojkuvkova, 2012).

Concrete Pictorial Abstract (CPA) - is an instructional approach that incorporates the three stages of learning described in Brunner's theory: enactive, iconic, and symbolic (Leong, Ho & Chang, 2015, Chang et al., 2017). Brunner's Theory posits that the development of CK is facilitated through direct and interactive engagement with tangible objects. The transmission of this knowledge is predominantly achieved through visual depiction, referred to as iconic representation. Ultimately, the gained expertise and mental pictures are subsequently transformed into abstract representations through symbolic notation (Yurniwati, 2021).

CHAPTER 1: STUDY BACKGROUND

1.1. Introduction and Background of the study

The low mathematical learning outcomes in South Africa have raised concerns among scholars, particularly in the area of geometry (Ginsburg & Lee, 2009; Mamali, 2015; Ngirishi & Bansilal, 2019). According to the Trends in International Mathematics and Scientific Study (TIMSS) (2019), it was found that only four out of ten South African learners demonstrated a basic understanding of mathematics. In 2003, only one out of ten learners demonstrated these skills, but there has been an improvement since then (DBE, 2019). While this may not be considered positive news, it does show progress from the situation in 2003. Furthermore, Zamisa (2019) argues that there has been a consistent lack of proficiency among learners in the field of geometry when compared to other areas of mathematics. Many secondary school learners find geometry to be a challenging topic in mathematics (Zamisa, 2019). Learners facing challenges in secondary mathematics, particularly Geometry, might not have established a solid mathematical foundation during their primary school education. According to Alagao (2022), learners struggle to thoroughly understand geometric concepts, problem-solving skills, and logical reasoning. The challenges learners face can also be attributed to their inability to effectively apply their conceptual knowledge (CK) and procedural knowledge (PK) in geometry.

Furthermore, as asserted by Katrin, van den Ham, and Heinze (2018), the acquisition of mathematical knowledge is a progressive process that relies on the foundation of previously acquired knowledge. Learners must possess a strong mathematical basis encompassing both PK and CK. Reflecting on my educational journey as an educator, I find that there are specific gaps in my recollection of mathematical concepts and principles that I encountered throughout my academic progression, spanning from foundational to tertiary levels. This might be attributed to my limited capacity to fully grasp the concepts of PK and CK in the context of academic pursuits throughout my educational experience. The partial responsibility for this can be attributed to the educators' inefficiency in effectively conveying PK and CK during instructional sessions. According to Johnson, Schneider, and Star (2015), individuals who exhibit PK have proficiency in a particular procedure, although they may not necessarily show a deep understanding of the underlying concepts or broader domain.

The upshot of this phenomenon is that learners may employ specific algorithms in mathematics even without a comprehensive understanding of the foundational concepts upon which such algorithms are built. Skemp (1976) referred to this concept as instrumental understanding and defined it as the acquisition of rules without a deep comprehension of the underlying reasoning. According to Skemp (1976), those who possess the ability to recall and effectively implement an algorithm demonstrate an instrumental understanding of the method. Skemp (1976) characterises relational understanding as the ability to comprehend both the procedural aspects and the underlying rationale. This suggests that learners who have acquired a strong relational understanding will possess the ability to identify appropriate methods, communicate their reasoning for selecting them, and establish meaningful links between various aspects of mathematical knowledge.

Geometry poses challenges for preschool and early primary learners, encompassing the study of 2D and 3D shapes and the exploration of various measurements such as perimeter, area, volume, and capacity. 2D shapes refer to flat shapes and polygons that possess only two dimensions. Several examples of two-dimensional shapes include squares, triangles, quadrilaterals, circles, pentagons, hexagons, heptagons, octagons, nonagons, and decagons. Furthermore, it is possible to conceptualise 2D forms as the external boundaries of 3D objects (Suggate, Davis, & Goulding, 2010). The geometric characteristics inherent in shapes facilitate the process of describing and categorising them. Shapes can exhibit many types of relationships between their sides. These relationships can be categorised as parallel, perpendicular, or neither. Additionally, shapes can possess specific symmetrical properties, such as line symmetry or rotational symmetry, or lack any symmetrical attributes. Furthermore, shapes can be classified as either congruent or comparable based on their corresponding sides and angles. According to Cooper (1986), a 3D shape is characterised as a solid object with three distinct dimensions, namely length, breadth, and height (or thickness). Furthermore, other entities such as buildings, individuals, animals, structures, models, and other objects can be identified as tangible manifestations within the physical realm. Practically everything in the world can be characterized by three dimensions.

I support Brunner's theory that the relationship between the transmission of PK and CK can be established by implementing the Concrete Pictorial Abstract (CPA) approach in the classroom environment. The instructional approach known as CPA integrates three stages of learning as outlined in Brunner's theory: enactive, iconic, and symbolic (Chang, Lee, & Koay, 2017).

Brunner's theory posits that CK is developed by active engagement and manipulation of tangible objects, leading to the formation of visual representations (iconic). Subsequently, these experiences and images are further abstracted through the utilisation of symbolic notation (Soleh, 2021). When employing the CPA method to teach geometry figures, educators can utilise a tangible model that presents learners with real-world features they can connect with. The utilisation of real-world items in the instructional process fosters the development of learners' CK, in contrast to the mere presentation and memorisation of abstract shapes.

When teaching the topic of prisms in three-dimensional shapes using visual aids such as videos or images, it is beneficial to provide learners with relatable examples. By presenting a freezer as an object, learners can establish a connection between the freezer and a rectangular prism (see Figure 1.1). This approach enhances their understanding compared to solely presenting a diagram of a rectangular prism without any reference to their existing knowledge. The identification of a rectangular prism can be extended by examining its surfaces, which reveal that a rectangular prism is composed of six two-dimensional faces in the shape of rectangles. This process enhances the formation of abstract ideas, as the individual has a pre-existing cognitive representation of a rectangular prism, enabling them to effectively identify and label other tangible models or items resembling a rectangular prism. Figure 1.1 illustrates how tangible objects can be utilised by an educator to establish relationships with three-dimensional shapes that are addressed within the context of a geometry curriculum.

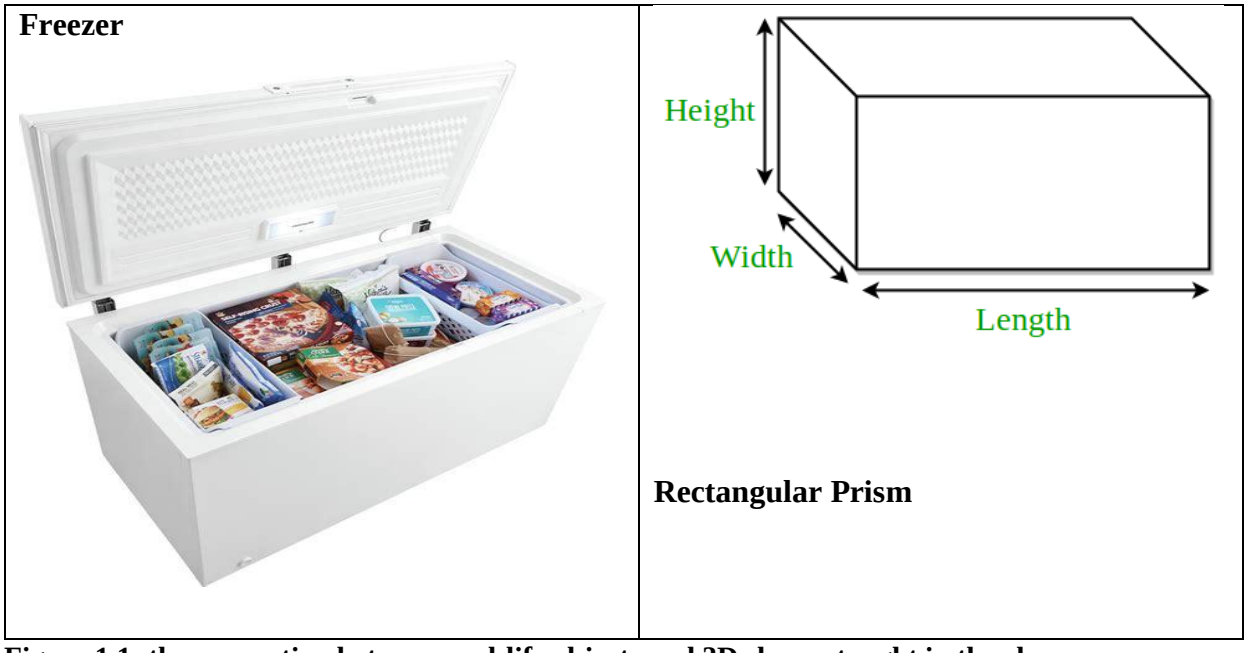


Figure 1.1: the connection between real-life objects and 3D shapes taught in the classroom.

Throughout the process of teaching measurement, an educator has the opportunity to enhance the learners' PK by establishing mathematical associations using the identical rectangular prism represented in three-dimensional formats. Merely presenting learners with the formulas for calculating capacity and volume is inadequate in terms of training them on these topics. Therefore, similar to a rectangular prism, an educator may use a freezer as a pedagogical tool to demonstrate the correlation between the calculation of the freezer's capacity and volume. Learners need to understand that the freezer's capacity refers to its overall storage capability, while the volume pertains to the quantity of space occupied by the products contained within the freezer. As mentioned earlier, the application of PK and CK abilities relies heavily on problem-solving skills. Consequently, learners must be presented with problem-solving scenarios to enhance their proficiency in both areas of knowledge.

Hurrell (2021) critically examines the impact of PK and CK on education, drawing upon current research and scholarly literature in the field. Hurrell (2021) asserts the importance of considering both PK and CK to advance our educational practices. He further asserts that PK and CK exhibit a complementary relationship and offer reciprocal advantages. Moreover, in most instances, CK must be prioritized. Suppose an educator is delivering a lesson on the concept of the perimeter; they must first start by introducing the perimeter and providing a definition that characterizes it as the total distance surrounding a certain shape. The educator may proceed to illustrate the concept of perimeter by employing tangible geometric figures. For example, the educator may employ the dimensions of the classroom, a window, or a door as visual tools to illustrate the concept of perimeter to the learners. Through this process, the learners will have achieved a high level of proficiency in the concept known as the CK of perimeter. Subsequently, learners are instructed in the pedagogical competence of perimeter calculations, whereby they can employ the formula for determining the perimeter.

PK and CK exhibit a dichotomous relationship, wherein their knowledge is interdependent. Consequently, the understanding of the CK of the perimeter will play a crucial role in the acquisition of PK related to the perimeter. The likelihood of learners achieving success is higher when they commence their educational journey by acquiring CK and subsequently progress to PK. One potential drawback of transitioning from a procedural approach to a conceptual approach is that learners may have difficulties in advancing their understanding of core knowledge. According to Hurrell (2021), there is a common misconception among educators regarding the intricate nature of both PK and CK, where both superficial and in-depth

analyses are possible. Hence, it is imperative to emphasize both PK and CK when providing teaching to learners in geometry. While acknowledging the importance of both forms of knowledge, it is argued that teaching CK should take precedence due to its foundational role in facilitating the acquisition of PK and CK skills (Hurrell, 2021). This study affords equal consideration to both PK and CK.

Furthermore, a comprehensive understanding of geometry in mathematics is essential for a thorough appreciation of the natural world and its environment (March & Steadman, 2020). Geometry is seen in both natural and artificial forms, as it is expressed through innovative materials and structures that are utilized and occupied daily. Moreover, geometry is observed in natural entities such as human beings, animals, and geographical features. Consequently, it is imperative for learners to not merely grasp geometry as a set of mathematical operations, but also to appreciate the underlying principles of geometry and its interconnectedness with the surrounding environment. This occurrence ignited a curiosity in examining the responses of learners toward the PK and CK evaluation about their experiences within the classroom. The acquisition of strong foundational mathematical abilities is of utmost importance for learners, starting from the foundation phase. Equally significant is the provision of effective and efficient mathematics instruction during the Intermediate Phase. The Intermediate Phase is widely recognized as a fundamental stage for the development of mathematical abilities in preparation for the Senior Phase. Consequently, the Intermediate Phase plays a crucial role in enhancing the learner's mathematical capabilities.

Based on the numerous literature and studies mentioned above that centre on geometry, it is evident that there is a significant focus on this subject. However, there is currently no research study that specifically examines the CK and PK in geometry among grade 6 and grade 9 learners, compares their knowledge, and tracks the progress of grade 9 from their grade 6. As already mentioned by Katrin et al. (2018), it is emphasised that the development of mathematical knowledge is a gradual process built upon existing knowledge. Therefore, it was significant to conduct the research due to the comparison of the two groups.

1.2. Rationale

Mathematics is a fundamental discipline that is pursued for many objectives. It has played a crucial role in advancing scientific understanding and technological capabilities (Satiti, Alfatah, & Umardiyah, 2021). The ability of learners to apply mathematical concepts in many real-world situations is an essential component of achieving proficiency in mathematics (Satiti et al., 2021). Mathematics helps people think clearly and solve difficulties in many areas of life. It helps people see patterns, extrapolate from their experiences, be creative, and understand other cultures (Hasibuan, Saragih, & Army, 2019). This suggests that mathematics education may improve logical thinking, observational, and analytical skills about the environment. Consequently, learners must possess a solid grounding in mathematics from the onset of their primary schooling. The investigation of this phenomenon holds significant importance due to the recognition of geometry as a mathematical discipline wherein learners commonly encounter difficulties. According to Tachie (2020), educators' classroom challenges were due to a lack of mathematical knowledge and geometry instruction. Educators' attitudes towards mathematics and Euclidean geometry changed. So their lack of knowledge and application has led to textbook-guided chalk-and-talk. Moreover, geometry plays a fundamental role in numerous professional domains, encompassing architecture, scientific research, medical practice, the construction sector, engineering, and the transportation business, among other professions.

The study was initiated due to the underachievement of learners in geometry, as previously mentioned in the introduction and shown by the TIMSS (2019) report, which revealed that 60% of South African learners are facing difficulties in Mathematics, with geometry being one of the specific areas of struggle. The study conducted by Mouton, Louw, and Strydom (2013) highlights the findings of researchers in the field of education at the secondary level. These researchers have consistently pointed out that the teaching practices of senior and primary school educators are responsible for the deficiency in mathematics abilities among learners pursuing higher education and beyond. To reduce the tendency to place blame on previous grades educators and to prepare learners for the demands of the senior phase adequately, Further Education Training (FET) phase, and higher education, it is imperative that the learners have a strong foundation in mathematics throughout their early years of schooling. Consequently, individuals undergoing education may acquire a firm foundation upon which they can construct a prosperous professional trajectory and make valuable contributions to the economic advancement of their nation. Therefore, it is imperative to conduct a thorough

investigation to tackle the mathematical difficulties learners face, given that mathematics holds significant importance in academic achievement, further education institutions, professional environments, and everyday situations.

According to the Curriculum Assessment Policy Statement (CAPS), the subject of mathematics is taught and assessed across various phases. The South African educational system is divided into four distinct phases: the Foundation Phase, which encompasses Grade R to Grade 3; the Intermediate Phase, which spans Grade 4 to Grade 6; the Senior Phase, which covers Grade 7 to Grade 9; and the Further Education Training (FET) phase. At the primary school level, the curriculum area of geometry, space, and measurement carries a weightage of 20% for grade 1, 25% for grade 2, and 27% for grade 3. Geometry holds a weightage of 30% in the Intermediate Phase, spanning from Grade 4 to Grade 6. Geometry and Measurement comprise 35% of the curriculum for grade 7 and grade 8 in the Senior Phase, whereas they account for 40% of the curriculum for grade 9.

Moreover, FET Analytic Geometry, Euclidean Geometry, and Measurement are encompassed within paper 2 for all FET grades. The FET phase consists of two papers, namely paper 1 and paper 2, which encompass a range of topics. In grade 10, the paper 2 is allocated 45 marks out of a total of 100 marks. In the eleventh grade, the paper 2 is assigned 80 marks out of a total of 150 marks. Lastly, in the grade 12, the paper 2 is valued at 90 marks out of a total of 150 marks. This study primarily examined the geometric concepts of the Grade 6 Intermediate Phase, explicitly focusing on measurements, shapes, and spaces. These topics hold significant importance, constituting 30% of the overall content area weight as outlined in the CAPS. This information is crucial as it demonstrates the ongoing nature of the topic. Learners must fully grasp the concepts in every grade to ensure a seamless advancement. This research investigates the CK and PK of mathematics learners in Grade 6 and Grade 9, focusing on the subject covered in the Intermediate Phase. The research offers an identical Intermediate Phase mathematics assessment to learners in both Grade 6 and Grade 9, and subsequently analyses and compares their respective outcomes.

In line with the above, the rationale for selecting both Grade 6 and Grade 9 stems from their designation as the exit grades for the Intermediate Phase and the Senior Phase, respectively. Grade 6 and Grade 9 geometric performances were assessed to evaluate learners' preparedness for the subsequent entering grades, namely Grade 7 in the Senior Phase and Grade 10 in the

Further Education and Training (FET) Phase, respectively. Moreover, the study focused on two high schools and their corresponding feeder primary schools to examine and track the pre-existing geometry CK and PK of grade 9 learners, as acquired during their time in primary school. The identical examination administered to learners in grade 6 is similarly administered to learners in grade 9. This assessment is conducted to evaluate the extent to which Grade 9 learners have acquired a complete understanding of both PK and CK in the field of geometry. As outlined, the Grade 6 curriculum allocates 30% of its content to the subjects of geometry and measurement, while geometry alone constitutes 40% of the Grade 9 curriculum. Again, this assertion aligns with the findings of Katrin et al. (2018), who suggest that mathematics is a discipline characterised by a cumulative nature, wherein the acquisition of knowledge and skills is contingent upon prior learning experiences. This implies that the geometry learned in the intermediate phase builds upon the geometry studied in the foundation phase. The comparison between the two grades is based on tracking the knowledge progression of Grade 9 learners from their Grade 6 level and comparing it with the current Grade 6 learners. This comparison assumes that Grade 9 learners have had more time to develop their CK and PK.

This study centres on the application of the Van Hiele theory[1] stages of geometric understanding as a theoretical framework. The Van Hiele levels of knowledge theory centre around five distinct levels of understanding, including visualisation, analysis, abstraction, deduction, and rigour. The intermediate phase of education emphasises the initial two levels of Van Hiele, namely visualisation and analysis. These levels encompass all the evaluation criteria outlined in the National Curriculum Statement (NCS) as specified by the Department of Education (2015). In the grade 6 curriculum, learners are introduced to the concept of visualisation and explore the properties of two-dimensional (2D) shapes. This encompasses a variety of shapes, both regular and irregular polygons, as well as an examination of their characteristics and angles. Additionally, learners delve into the properties of three-dimensional (3D) shapes, covering a diverse range of objects, their features, and the methods for measuring perimeter, area, and volume. An example of Van Hiele level 1 includes naming regular and irregular polygons, rectangular prisms, cubes, pyramids, and tetrahedrons. In Level 2, learners are expected to demonstrate their ability to discern and articulate the shared characteristics and distinguishing features among various geometric shapes, specifically rectangles, parallelograms, tetrahedrons, and other pyramids. According to the findings of Alex and

[1] This theory is further explored in great detail in the theoretical framework design.

Mammen (2012), Level 1 of the visualisation process involves the identification and labelling of figures. On the other hand, Level 2 of the process, known as analysis, involves the description of the attributes of shapes.

The resolution of numerous geometric problems necessitates the ability to mentally visualise the forms involved, which sometimes poses a challenge for learners when they attempt to construct three-dimensional figures. In a study conducted by Chaudhary (2019), it was found that learners encounter difficulties in visualising geometrical figures, analysing geometrical figures, and understanding the qualities of geometrical concepts. During the transition to the senior phase, the Van Hiele theory emphasises the initial three levels of cognitive development: visualisation, analysis, and abstraction. It is expected that learners possess proficiency in all three levels. In the transition to the FET phase, learners hold the anticipation that they will acquire the ability to engage with all five levels of geometric comprehension as outlined by Van Hiele. According to Masilo (2018), it has been observed that learners are unable to progress to a higher level without first completing the preceding levels. Consequently, it may be deduced that to get Level 2 proficiency, a learner must first attain Level 1 proficiency. Hence, the initial two levels imparted in the intermediate phase are crucial for equipping learners with the requisite skills and knowledge required to attain greater levels of proficiency anticipated in the Further Education and Training (FET) phase. The Van Hiele level is an important factor in the development of learners' geometric knowledge. Therefore, the rationale for employing this theoretical framework is clarified.

In the same vein, Presmeg (2020) contends that the acquisition of a particular Van Hiele level of knowledge cannot be achieved through rote learning or the application of routine algorithms (PK) in the absence of understanding (CK). In essence, the acquisition of knowledge, skills, or information without understanding cannot be deemed as the attainment of a specific level of knowledge. According to Hacıeminoğlu (2021), the enhancement of learning cannot be achieved alone through memorisation or the application of regular algorithms, unless accompanied by a genuine understanding of the subject matter. This implies that learners at a lower level cannot rely solely on rote methods to acquire advanced geometric knowledge. To reach a higher level of geometric understanding, they must first grasp all the required conceptual foundations. Nevertheless, the process of learning is characterised by discontinuity. To clarify, it may be observed that once a learner attains a specific level of proficiency, they tend to remain at that level for a given duration, akin to a process of maturation (Suwito,

Yuwono, Parta, Irawati, & Oktavianingtyas, 2016). Before the completion of the maturation process, it is not efficacious to compel a learner to operate at an elevated level or to administer instruction at an advanced level. According to Suwito et al. (2016), individuals at Level 3 of learning may not immediately grasp the concepts presented in Level 4 geometric presentations. Therefore, this challenge may be connected to insufficient primary school education in the areas of PK and CK. Geometry knowledge encompasses both PK and CK. CK in the field of geometry refers to the understanding and awareness of the interconnectedness among various geometric notions (Habila Elisha, Matawal, Kwalat, 2017). In other words, CK refers to the capacity to characterise and depict the connections among geometric concepts precisely. In contrast, within the field of geometry, PK refers to the cognitive capacity to recall and effectively use recognised methodologies to solve geometric problems. According to a study conducted by Habila Elisha et al. (2017) hence, the establishment of a solid groundwork in both PK and CK during primary education serves as a commendable basis for achieving proficiency throughout all Van Hiele levels, which is essential for attaining success at the secondary school level. Thus, the primary motivation for this study, as described in the existing literature, is the observation that learners have insufficient CK and PK in the field of geometry. Additionally, it has been noted that learners are not progressing adequately in terms of their Van Hiele level of understanding in geometry. Therefore, the investigation of the fundamental issues in geometry teaching and learning in a different context, such as the significance of this study, arose from the intriguing findings and debates surrounding geometry education.

1.3. Purpose of the study

The purpose of this research is to address the prevalent obstacles in geometry mathematics education, specifically in the context of South Africa. The focus on South Africa is primarily due to the widespread issue of poor performance in geometry. The purpose is to contribute to the resolution of the issues related to inadequate geometric performance in schools. The findings of this study, with their potential to be applied to improve the teaching and lessons of Geometry, offer a ray of hope for the future of geometry education. Moreover, the purpose of this study was to examine potential influences that could facilitate the improvement of geometry instruction and learning within the field of mathematics. Conducting an exploration into the phenomena is of utmost importance to generate innovative answers to the issues encountered in the field of geometry. Furthermore, the research seeks to formulate a

comprehensive strategy aimed at enhancing the proficiency of both educators and learners in the domain of geometric mathematics.

1.4. Aim of the study

The main aims of this study are to:

- Investigate how intermediate-phase geometry educators **impart** procedural knowledge and conceptual knowledge while teaching learners.
- Investigates how learners from intermediate and senior phase exit grades **respond** to procedural knowledge and conceptual knowledge assessment geometry in a mathematics classroom.
- Examine the **impact** of teaching conceptual knowledge and procedural knowledge on the learning of geometry.

1.5. Problem statement

The problem in geometry teaching stems from poor academic achievements in mathematics which is pervasive in South Africa, causing considerable concern among all stakeholders. The field of geometry, specifically, is a subject of concern within the context of South Africa, as there exists significant apprehension regarding the subpar academic performance of learners (Ngirishi & Bansilal, 2019). Ngirishi and Bansilal (2019) conducted a study examining the level of geometric concept knowledge among high school learners. The findings of the study revealed that the learners' academic performance was mostly focused on their ability to reason and demonstrate their understanding of the subject matter. This observation suggests that learners may have lacked a strong CK foundation in geometry during their primary education. According to a recent study conducted by Juman, Mathavan, Ambegedara, and Udagedara (2022), it was observed that learners encountered difficulties in grasping the fundamental concepts and theorems of geometry. As per the observations made by educators, a significant proportion of learners exhibit a lack of enthusiasm towards the acquisition of knowledge in the field of geometry. The application of geometrical symbols posed challenges for the learners. The prevailing approach employed by a significant number of educators involves the utilisation of traditional instructional methods for teaching geometry.

Moreover, mathematics educators in the professional field must possess a thorough understanding of geometry, encompassing both conceptual and procedural aspects (Yurniwati

& Soleh, 2019). This implies that educators must possess a high level of competence in their subject matter to transmit their expertise to their learners effectively. Moreover, this suggests that educators who do not possess proficiency in either PK or CK will struggle to adequately convey PK and CK in the context of geometry, leading to subpar academic performance among learners in the subject of geometry and mathematics. Therefore, this study investigated the methods employed by educators to convey PK and CK in the context of geometry. This study aimed to examine the extent to which educators effectively impart PK and CK.

The solving of geometric problems often poses a significant challenge to those studying geometry, as evidenced by the works of Mahdayani (2016) and Sulistiowati, Herman, and Jupri (2019). Learners often encounter difficulties in solving geometry problems due to several factors, including a lack of prior knowledge of the subject, limited experience with the topic, a lack of confidence in their problem-solving abilities, and insufficient knowledge of the appropriate mathematical procedures to apply (Haviger & Vojkůvková, 2014; Sulistiowati et al., 2019). In addition to acknowledging these limitations while addressing geometric problems, assessing the degree to which learners have developed Van Hiele's geometric thinking is crucial. This study employs the Van Hiele theory as a theoretical framework to assess the level of geometric thinking the learners achieve. As per Masilo's (2018) findings, multiple studies indicated that learners' comprehension of geometry remains at level 1 throughout the Further Education and Training (FET) phase, despite the expectation that it should reach level 5. The primary objective of this study was to use Van Hiele's framework to examine the understanding of geometrical thinking, explicitly focusing on the initial two levels of visualisation and analysis that align with the intermediate phase. The study aimed to investigate the learners' responses to PK and CK assessment at the Visualisation and Analysis stages of Van Hiele's model in the context of geometry. This research endeavour is ground-breaking in its use of a custom assessment tool for Grade 6 geometry. The tool is therefore applied to both Grade 6 and Grade 9 learners to compare their geometric PK and CK. This investigation focuses on the levels of Visualisation and Analysis within the Van Hiele framework.

Not all aspects of studying geometry proceed without difficulties. Even though the teaching of geometry is considered one of the most crucial elements in improving learners' spatial abilities (Rohendi, Septian, & Sutarno, 2018). Furthermore, it is widely believed that geometry is one of the most challenging topics, and a significant number of learners harbour negative attitudes

toward it (Juman, Mathavan, Ambegedara, & Udagedara, 2022). The acquisition of geometric knowledge poses significant difficulties for a considerable number of learners, as evidenced by studies conducted by Rohendi, Septian, and Sutarno (2018) and Juman et al. (2022). Visualisation is a crucial skill for tackling many geometrical obstacles; nevertheless, learners frequently encounter difficulties while attempting to construct three-dimensional shapes within an educational setting. Based on a study conducted by Chaudhary (2019), it has been shown that learners encounter difficulties in the visualisation, analysis, and comprehension of geometrical figures and concepts. The learners did not demonstrate active engagement in the classroom activities. There appears to be a noticeable absence of passion among the learners in the classroom.

The inability of mathematics educators to utilise the existing resources was attributed to a shortage of essential materials (Chaudhary, 2019). In the educational context, it was observed that educators predominantly employed traditional teaching methodologies, resulting in limited participation from learners in classroom discussions. Furthermore, educators were found to possess inadequate preparation for their instructional responsibilities. This suggests that learners' proficiency in both PK and CK is deficient, and there is a significant correlation between these two types of knowledge. Numerous studies have demonstrated a significant relationship between learners' PK and CK, indicating that a strong presence of one is indicative of a substantial presence of the other (Surif, Ibrahim & Mokhtar, 2012; Laily, Sunardi, Yuliati & Sari, 2020). Consequently, to enhance learners' comprehension and proficiency in resolving geometric quandaries, learners must attain proficiency in both PK and CK (Laily et al., 2020). Hence, the present study examined the imparting of PK and CK methods employed by educators within the context of geometry lessons.

Currently, educators predominantly employ textual materials in typical classroom environments to instruct learners in the subject of Geometry. Therefore, the geometric concepts exhibit a lack of connection to the tangible world and the experiences of individuals engaged in learning. According to a study conducted by Yuliyanto, Turmudi, Agustin, Putri, and Muqodas (2019), standard instructional approaches are less likely to fully explore the potential and knowledge of learners. The efficacy of learning activities is diminished when learners are directed to engage in rote memorization of knowledge (Jelatu, Sariyasa & Ardana, 2018). Therefore, it is imperative to prioritise both PK and CK when instructing geometry topics in the classroom, to promote a comprehensive understanding and practical application among

learners. According to Mamali (2015), geometry holds significant importance within the realm of mathematics education at both the primary and secondary school levels. As a mathematics educator who specialises in the Intermediate Phase, I have noticed that when the teaching process is solely focused on the educator, learners tend to lose interest and disconnect. In the field of geometry, learners exhibit a preference for active engagement in the learning process, as opposed to assuming a passive role as mere recipients of knowledge. This inclination towards participatory learning allows learners to actively participate and contribute to their own educational experiences. This approach enables learners to actively develop and internalise geometric concepts and methods. This study aimed to examine the application of PK and CK in the field of mathematical geometry. The study emphasised the importance of assigning equal significance to both types of knowledge. While the two forms of knowledge may appear to be dichotomous, they are interrelated and so both are necessary. While both CK and PK are crucial, having a strong foundation in CK is imperative to advance in PK.

Based on the research conducted by Elmagustilla (2020), which examined the relationship between learners' level of interest in learning and their performance in geometry, it was found that individuals with a heightened interest in learning demonstrated a greater likelihood of achieving favourable outcomes in terms of mathematical representation. Learners who possess less motivation to engage in academic pursuits may have greater difficulties in attaining the objectives outlined by the indicators of geometric representation aptitude. The primary objectives of incorporating geometry into the curriculum of educational institutions are to foster learners' self-confidence in their mathematical abilities, enhance their problem-solving skills, facilitate their progression in the field of mathematics, and cultivate their capacity for logical reasoning within the realm of mathematics (Solihatun et al., 2019). Hence, learners must exhibit a strong inclination towards the acquisition of geometric knowledge, as this will enable them to develop self-assurance in their mathematical abilities, enhance their problem-solving capabilities, and progress in their mathematical thinking. Based on a study conducted by Solihatun et al. (2019), a significant proportion of learners demonstrate satisfactory performance on the geometry exam administered by the educator. Nevertheless, the learners had challenges when attempting to articulate and calculate the area and perimeter of geometric shapes such as rectangles, triangles, and squares. Hence, one can infer the absence of perimeter and circumference knowledge in the context of squares, rectangles, and triangles. In consideration of this, the research also examined how learners respond to the assessment of PK and CK in the field of geometry.

Moreover, a comprehensive examination conducted by Adolphus (2011), Mamali (2015), and Tachie (2020) revealed that a significant proportion of educators exhibit deficiencies in both their grasp of mathematical concepts and their pedagogical content knowledge, hence impeding their ability to effectively instruct learners in the domain of geometry. Furthermore, even when presented with comparable instances, the learners possess such inadequate mathematical fundamentals that they are incapable of resolving the problems. The attitudes of learners towards education exhibit a pronounced inclination towards negativity. They exhibit a lack of receptiveness towards novel ideas or alternative approaches. Moreover, educators exhibit a certain degree of reluctance to invest effort due to a deficiency in passion. This study investigated the impact of PK and CK in the field of geometric mathematics, specifically examining how the application of PK and CK in primary school education establishes a strong foundation for secondary school mathematics.

1.6. Research questions

The following three **primary questions** guide the research:

1. How do educators in the intermediate phase **impart** procedural knowledge and conceptual knowledge when teaching geometry?
2. How do learners from intermediate and senior phase exit grades **respond** to procedural knowledge and conceptual knowledge assessment in geometry?
3. What is the **impact** of teaching conceptual knowledge and procedural knowledge on the learning of geometry?

1.7. Significance and focus of the study

This study is the first to focus on CK and PK in geometry within the Bushbuckridge area, specifically in the Mkhuhlu circuit. Furthermore, the study's large sample size is noteworthy as it encompasses both educators and learners, providing insights into CK and PK in geometry from multiple perspectives. The use of mixed methods, that includes observations, semi-structured interviews, and learner assessment, fills a gap left by previous studies. PK and CK both have significant importance in the realm of geometry in mathematics. Exploring the application of PK and CK in geometry is the focus of this study.

The study was conducted in the Mkhuhlu circuit, located in the Mpumalanga Province. This circuit falls under the jurisdiction of the Bushbuckridge Local Municipality, situated within the

Bohlabela district. Most residents in this area live in rural settings, with varying levels of education. Some individuals have low literacy levels, while others possess a higher education background. Essential basic services like water, electricity, sanitation, and healthcare are inadequately provided. Many rural communities face significant poverty, relying on agriculture for sustenance, with high rates of unemployment. Additionally, educational institutions in this circuit grapple with deteriorating physical infrastructure and a shortage of academic resources, including computer laboratories, scientific laboratories, and libraries. It is uncertain how a particular setting or context can impact an individual's capacity to apply PK and CK in geometry.

Nonetheless, Visser, Juan, and Feza (2015) indicated that a school's educational outcomes are affected by the presence or lack of essential resources, with a positive association observed between increased resources and enhanced results. Evidence supporting this claim is provided by Spaull (2013) and Van der Berg (2008), who posit that there is evidence supporting the notion that learners' academic achievements are influenced by the socioeconomic status of the school they attend. Furthermore, Spaull (2013) indicates that economists have identified a correlation between socioeconomic level and academic performance. Consequently, educational institutions and communities characterised by limited resources and a disadvantaged socioeconomic status may potentially contribute to the subpar academic achievement of learners in mathematics. Therefore, the evidence emphasises the significance of context in determining the learners' performance in Mathematics. It is not fully known how learners in the context in which this study is conducted are performing.

1.8. Outline of chapters

Chapter 1 sets the stage for this study. The chapter provides a problem statement, rationale, research aims, objectives, and research questions.

Chapter 2 explores the connection between literature and the topic at hand.

Chapter 3 explores the theoretical framework, specifically focusing on the PK and CK, Van Hiele theory, and Concrete Pictorial Abstract.

Chapter 4 provides a comprehensive overview of the research design, sampling, methodologies, data collection, and data analysis used in this study. The study thoroughly describes the instrument's validity and reliability, as well as the relevant ethical considerations.

Chapter 5, the results obtained and an analysis of the data are presented. This study presents qualitative and quantitative results, utilising mixed methods research approaches.

Chapter 6 presents a thorough data analysis, discussing its implications and providing interpretations.

Chapter 7, delves into the recommendations and conclusions of the study.

1.9. Summary and conclusions

This chapter presented an introductory overview and provided the necessary background information for the study. The review examined different practices for teaching geometry, specifically emphasising PK and CK. The chapter presented the problem statement, rationale, research aims and objectives, research questions, significance, and focus of the study. It also outlined the utilisation of a mixed-methods approach, which involved using both qualitative and quantitative approaches. The composition of the participant pool was described, and the establishment of both a comparison group and an experimental group was delineated. The theoretical frameworks were elucidated. The study established clear definitions for the key concepts to ensure the study had the required content. The subsequent chapter presents an exhaustive literature review of the study.

CHAPTER 2: LITERATURE REVIEW

2.1. Introduction

This chapter examines the literature review in relation to the study's research questions. The focus is on the teaching methods, the process of learners learning Geometry, and the impact of both Procedural Knowledge (PK) and Conceptual Knowledge (CK) approaches on learning. Section 2.2 delves into the imparting of PK and CK in the field of geometry. It explores the methods of teaching and learning 2D and 3D shapes, as well as measurement. Moreover, it examines the utilisation of technology in the application of these concepts. The examination also includes a look at the mathematical register and language. Section 2.3 focuses on the assessment of learners' responses to PK and CK in geometry. Section 2.4 examines the effects of teaching and learning PK and CK in geometry.

2.2. Procedural knowledge and conceptual knowledge

Over time, the distinction between mathematics PK and CK has generated considerable debate and controversy (Davis, 2013). Inquiries regarding the process of mathematics learning, specifically the methods employed in teaching, rely on conjecture on the relative importance of different forms of knowledge and the optimal equilibrium between them. Mathematics has facilitated substantial discussions on both PK and CK due to its well-organized and distinct material (Lefevre & Hiebert, 1986). In addition, Lefevre and Hiebert (1986) contend that there has been intense controversy around the comparative significance of instructing learners' abilities against their comprehension. McClellan and Dewey (1985) advocated for enhanced comprehension and proposed a mathematics curriculum that aimed to elevate learners' understanding beyond the existing level of instruction.

Hiebert and Lefevre (1986) have conducted a comprehensive examination of the philosophical and historical roots of the terms "PK" and "CK" to strengthen their notion. Drawing on Hiebert and Lefevre (1986), they additionally posit that the contemporary differentiation between PK and CK originates from a philosophical basis that links the concepts of "knowing how" and "knowing that" (Ryle, 1949; Scheffler, 1965). Ryle (1949) argues that it is impossible to simplify "knowing how" to "knowing that". In other words, intelligence does not require the step-by-step application of theory and practice. Intelligent conduct, as stated by Skemp (1987), involves thoughtful consideration during the execution of tasks. Conversely, Scheffler defines "Knowing that" as the understanding of propositions or facts. Ferguson-Hessler and de Jong

(1996) propose that knowledge type and quality should be regarded as distinct notions to provide a more accurate description of knowledge.

Thorndike (1922) advocated for the significance of skill development and presented a thorough strategy for efficiently instructing such skills. Brownell (1935) strongly advocated against prioritising the cultivation of specific aptitudes and instead advocated for giving greater importance to understanding. Gagne's (1977) research primarily focused on skill learning, while Bruner (1960) argued for the importance of understanding. Throughout the course, other scholars have provided their perspectives on the subject matter, including Jones (1970) and Shulman (1970). Currently, Hierbert and Lefevre (1986) observe that cognitive psychologists and mathematics educators are re-evaluating the concepts of PK and CK within the field of mathematical education. In addition, occasionally, words and phrases that have not been utilised recently may be introduced into a conversation. Resnick (1982) examines the topics of syntax and semantics, while Gallistel and Gelman (1798) distinguish between PK and declarative knowledge. Although the database is quite large, it encompasses a wide range of terms that are employed to categorise numerous unique forms of knowledge. Barody and Ginsburg (1986) differentiate between meaningful knowledge and mechanical knowledge, whereas Van Lehn (1990) differentiates between schematic knowledge and teleological knowledge. Nevertheless, irrespective of the diverse nomenclature and terminology, the differentiation among the different classifications of knowledge remains mostly unchanged, just like it has been throughout history.

Hierbert and Lefevre (1986) present three instances illustrating the disparities between current discussions on PK and CK compared to previous ones. They define these disparities as skill and comprehension. Previously, abilities and understandings were seen as instructional objectives in earlier literature and examined in the context of advocating for instructional programmes. This suggests that previously, the emphasis was primarily on educational outcomes and promoting educational initiatives, rather than prioritising knowledge and skills as educational goals. According to Hierbert and Lefervre (1986), the issue at hand is whether classroom education should prioritise abilities, understandings, or a combination of both. The implementation of educational programmes has served as the context for understanding the importance of different abilities. Contemporary literature often focuses on the transmission of knowledge and the connections between different forms of knowledge.

The current focus on the connections between conceptualizations and operationalization is the second characteristic that sets apart studies of PK and CK from previous ones (Hierbert & Lefevre, 1986). Hierbert and Lefervre (1986) argue that historically, the two forms of knowledge have been viewed as separate and in competition for the educator's focus rather than living together harmoniously. This implies that although PK was prioritised in the classroom, CK was given less focus. Hierbert and Lefervre (1986) have noted that there has been a lack of focus on the relationships between the two forms of knowledge. Presently, there is a growing fascination with the correlation between conceptions and procedures. Furthermore, Hierbert and Lefervre (1986) contend that the two categories of information under consideration are perceived as separate, yet they share significant connections that are advantageous to both entities. While PK and CK possess distinct characteristics, they are not mutually exclusive and can be utilized in conjunction to tackle the challenges encountered by learners learning mathematics.

According to Hierbert and Lefervre (1986), a notable difference between past and present discussions is that in the past, the distinction between PK and CK primarily revolved around mathematics learning in formal school settings. However, recent discussions have expanded the scope to include preschool mathematics learning in informal settings. Considering the information presented, the importance of utilising PK and CK should not be undervalued. Consequently, it is vital to incorporate these concepts in the initial stages of a learner's education. Mathematical proficiency is commonly recognised as a significant benefit for youngsters when they begin school (Brownell, 1941; McLaughlin, 1935). Recent comprehensive research has been undertaken in this field to thoroughly examine these skills. Hierbert and Lefervre (1986) observe that the differentiation between PK and CK is equally significant in understanding the acquisition of informal mathematics as it is for formal mathematics. In other words, it implies that learners in a mathematics classroom can acquire knowledge and get instruction on ideas and procedures at the same time, by engaging in informal activities that lay the foundation for more structured activities. "Preschool children develop specific procedures or skills, as well as concepts, understandings, or intuitions related to mathematics" (Hierbert & Lefervre; 1986, p.2). This suggests that for PK and CK to be fully comprehended in mathematics, they must be introduced to a child's mathematical education at an early stage.

The study incorporates PK and CK as the teaching knowledge for this research. This is because the main focus of the study was to investigate the teaching and learning of both types of knowledge. While previous literature indicates debates on the importance of each type of knowledge, this study argues for giving equal importance to both. The distinction between "knowing that" and "knowing how" is evident in this study. "Knowing that" refers to knowing facts, which can be achieved by teaching conceptual knowledge in geometry class. On the other hand, "knowing how" involves following specific procedures or steps. In the context of teaching geometry, this entails teaching content such as measurement. Educators should first impart the CK of this content, such as teaching the concept of the area by connecting it to the real-life experiences of the learners and using visual and tangible objects. Subsequently, the teaching of PK and skills, such as the calculation of area, can be introduced to the learners. This study advocates for the sequential teaching of CK, followed by PK, emphasising the importance of establishing a connection between the two.

2.2.1. Application of procedural and conceptual knowledge in mathematics

Star (2013) conducted comprehensive research on PK and CK in mathematics, focusing primarily on the areas of fractions, counting, and addition of one, two, and three numbers. As per the authors cited by Star (2013), it is commonly recognised that PK and CK have a positive correlation and are acquired simultaneously rather than independently (Rittle-Johnson & Siegler, 1998; Hierbert & Lefevre, 1986). Star (2013) explores the unresolved question of the sequence in which children acquire different types of knowledge and how one type of knowledge impacts the development of another. Ivic states that Piaget categorised knowledge into ‘practical knowledge’ (savoir-faire) and 'CK', whereas Vygotsky separated between ‘manifest content’ (facts and figures) and; instrumental knowledge’ (skills, procedures, and methods) (Haapasalo & Kadijevich, 2000). Furthermore, as stated by Skemp (1979), possessing knowledge, skill, and ability are three discrete yet interrelated notions. A surplus of task-solving algorithms characterises PK, while CK is characterised by a scarcity of algorithms but a prevalence of interpersonal relationships (Hierbert & Lefevre, 1986).

According to Star (2015), a learner with PK is proficient in a specific method or technique, but may not have a deep understanding of the underlying principles or broader subject area. Star (2015) provided an illustration where a learner could do the operation of adding fractions but lacked a deep understanding of the underlying notion of fractions. Likewise, a learner may

possess the ability to perform addition with multi-digit numbers without comprehending the concept of place value. Star (2015) further elucidates that numerous mathematician's harbour scepticism regarding the existence of such comprehension of PK. Some people feel that learners can only memorise the process of adding multi-digit numbers if they lack CK of place value. In this particular scenario, learners are prone to making errors when adding numbers due to their dependence on memorisation rather than understanding the principles of addition and place value. Star (2013) argues that learners can only commit the processes for adding fractions to memory if they possess no prior CK. Star (2013) suggests that broadening one's viewpoint on PK can contribute to advancing theories related to the development of both PK and CK. A learner's verbal expressions or omissions serve as a reliable indicator of their degree of understanding. The proficiency level of a learner can be ascertained based on their actions or lack thereof. By comprehending learners' PK, we can explore alternative approaches to evaluating learners' mathematical proficiency (Star, 2013).

In their study, Kelly, Chappell, and Killpatrick (2003) examined the impact of a concept-based or procedure-based learning environment on learners' proficiency in calculus, specifically their PK and CK. Based on the study, every educator presented different approaches to instructing diverse mathematical subjects. Educators in procedure-based environments stated that their focus was on conveying knowledge of precise methodologies, strategies, and algorithms to their learners. The concept-oriented educator emphasised that her teaching primarily focused on three key aspects: generating ideas, solving problems, and communicating mathematical concepts. The study on midterm exam performance indicates that learners who were exposed to a conceptual curriculum outperformed their peers who followed a procedure-based approach. However, in the final assessment, learners who were exposed to a curriculum that emphasised conceptual understanding rather than procedural learning did not outperform their peers who focused on procedural learning. In contrast, the replication study found that learners in a concept-based classroom fared better than learners in a procedure-based classroom on both the midterm and final tests. The study findings suggest that CK enhances the learning of PK skills.

Similarly, the results suggest that a learner's capacity to perform tasks can be effectively instructed through repeated demonstrations or by establishing a conceptual foundation and subsequently building upon that knowledge (Kelly Et al. 2007). Consequently, this study employs both PK and CK to examine the impact of teaching and gaining PK and CK in a

classroom environment. Furthermore, the study examines how educators at an intermediate level effectively transfer both PK and CK to their learners during geometry teaching. The objective of this study is to gain a deeper understanding of how learners demonstrate their proficiency in geometry through assessments of their PK and CK. This study focuses on PK. PK, a practical form of knowledge involving mathematical calculations, is utilised to teach learners in a geometry class the procedures for calculating area, volume, and perimeter. PK is sometimes employed by educators as a pedagogical approach, wherein learners are instructed through the process of rote memorisation. In this study, CK is utilised as a theory for teaching and learning geometric concepts. This study argues that learners need to acquire CK of certain geometry content before they can be taught the PK of calculating that content.

2.3. The imparting of procedural and conceptual knowledge in geometry

Geometry, a mathematical discipline, focuses on examining and analysing various forms and figures, along with their inherent characteristics (Soleh, 2021). According to Soleh (2021), Geometry holds significant importance in the realm of primary school mathematics as it provides a plethora of visual representations that aid in the comprehension and application of algebra, statistics, and arithmetic. Mathematics, as a discipline, primarily emphasises the development of PK and CK (Hiebert, Miller, & Berk, 2017). Consequently, learners must exhibit a thorough understanding of the concepts they acquire using regular and persistent practice. According to Nadzeri, Chew, Musa, Noor, Hassan, and Ismail (2022), for learners to fully comprehend the subject matter and achieve their maximum potential in the educational setting, it is imperative for educators to meticulously design activities and utilise a diverse array of instructional approaches. Furthermore, according to Nandzeri et al. (2022), there exists a subset of learners who encounter difficulties in understanding the content delivered during mathematics instructional sessions. Nadzeri et al. (2022) argue that their aptitude may influence the academic performance of learners on their assessments to grasp specific mathematical concepts, particularly those of geometry.

According to the findings of Habila Elisha et al. (2017), it is argued that children initially acquire CK through parental explanations or by encountering the inherent constraints within their surroundings. Subsequently, they progress to develop PK through the process of engaging in repetitive practice. This highlights the notion that CK is strengthened when learners can make connections to real-life situations, ultimately resulting in an easier mastery of PK.

Proponents of procedure-first perspectives contend that children initially acquire PK through curiosity and subsequently develop CK about it (Marley & Carbonneau, 2014; Carbonneau & Marley, 2015). Zakaria, Maat, Adnan, and Yaakob (2010) underscored the significance of comprehending mathematical methods and their associated concepts in the process of learning mathematics. Despite the inherent dichotomy between CK and PK, it is essential to recognise that these two types of knowledge are intricately interconnected and rely on each other for a comprehensive understanding. The notion posits that learners ought to be allowed to investigate and cultivate these types of connections as an integral component of their mathematical instruction. The significance of this matter cannot be overstated, as the attainment of mathematical proficiency necessitates a comprehensive understanding of both the underlying principles and the operational techniques within the discipline (Habila Elisha et al., 2017).

Moreover, geometry plays a vital role in our understanding and admiration of the surrounding universe (Zuya & Kwalat, 2015). Therefore, a thorough understanding of PK and CK is essential for mastering geometry. As defined by Habila Elisha et al. (2017), CK in geometry refers to a deep understanding of the connections between different geometric concepts. Put simply, it refers to the capacity to precisely articulate and depict geometric concepts and their interrelationships. On the other hand, PK in geometry pertains to the act of remembering and utilising systematic methods to solve geometric issues (Zuya, 2017). Furthermore, other difficulties can arise when studying geometry at the primary school level. According to a study conducted by Yuniwati and Soleh (2019, p.4), these several challenges were identified;

> Current issues in learning geometry in elementary school include (1) the role of spatial reasoning and its connection to school mathematics in general and school geometry in particular; (2) the action of drawing in the construction of geometric meaning; (3) extending primary school geometry from its typical passive emphasis on vocabulary (naming and sorting shapes by properties) to a more active meaning-making orientation to geometry (including composing/ decomposing, classifying, mapping and orienting, comparing and mentally manipulating two and three-dimensional figures)(Yurniwati & Soleh, 2019, p.4).

Geometry knowledge acquisition can be broadly classified into two main categories: the conceptual part, also known as CK, and the procedural part, also known as PK (Hamdi, 2018). Teaching these two categories necessitates employing distinct methodologies. The abstract components must be converted into sensory experience through visualisation, namely by utilising graphical elements (Hamdi, 2018). Nonetheless, Geometry in the intermediate phase of primary school mathematics encompasses the study of properties related to 2D and 3D

shapes, as well as concepts such as capacity, volume, geometric patterns, symmetry, perimeter, area, measurement, position, movement, and transformation.

This study focused on the geometry content, precisely the attributes of 2D and 3D shapes, as well as their measurement in terms of area, perimeter, and volume. Contrary to the literature mentioned earlier, which primarily focuses on sub-topics within geometry, this study integrates and explores these topics collectively. The research question in the initial research topic aimed to thoroughly investigate how the attributes of 2D and 3D shapes, measurement, technological approach, mathematical register, and language are utilised in the instruction of PK and CK in the field of geometry. This application aims to enhance understanding of the coverage of each subtopic in the geometry curriculum by offering concise and thorough summaries for each.

2.3.1. Studies on properties of 2D and 3D shapes

Instruction on 2D shapes should incorporate materials that establish a link between learners' existing knowledge and the subject matter. Pedagogical approaches and instructional resources should foster learners' ability to apply and comprehend new knowledge effectively. To effectively apply knowledge, educators must employ examples and questions/problems that enhance the acquisition of both PK and CK. As 3D shapes are composed of 2D shape faces, educators must employ instructional materials that facilitate learners in establishing connections between their existing knowledge and the subject matter. By employing solid shapes as a pedagogical instrument, instructional approaches and resources ought to facilitate learners in the application and comprehension of new knowledge. Educators can enhance the application of knowledge by incorporating relevant examples and thought-provoking questions or problems that foster the development of PK and CK. References can be drawn from real-life settings and objects.

In a study by Ismail, Abdullah, Syuhada, and Noh (2020), grade 4 learners encounter difficulties when learning the 'Shape and Space' topic. The study identified six specific challenges faced by these learners. The learners struggled with; recognising different types of 2D and 3D shapes, drawing the 2D and 3D shapes accurately, counting the edges of shapes, identifying the types of straight and curved lines, and the number of surfaces a shape has. Moreover, they encounter difficulties in drawing the net of a 3D shape. Lastly, they struggle with combining basic shapes to form a net. The disparity between this study and the study

conducted by Ismail et al. (2020) solely focused on sampling the learner population, whereas this study aimed to investigate both learners and educators to examine how educators impart knowledge in the field of geometry. The study employed semi-structured interviews and a test as the primary methods for data collection. In contrast, this study emphasised the observation of educators during their teaching sessions. In addition, the study solely focused on the subject of 2D and 3D shapes, while this study encompasses measurements as well.

In a study conducted by Fujita, Kondo, Kumakura, Kunimune, and Jones (2020), the researchers examined the spatial reasoning abilities of learners in grades 4 to 9 when dealing with 2D representations of 3D geometrical shapes. The study revealed that relying solely on spatial visualisation or property-based spatial analytic reasoning is insufficient for solving problems involving multiple reasoning steps. Instead, the researchers found that a combination of these two skills, along with domain-specific knowledge, is necessary to overcome the perceptual appearance of the given diagram. The study by Kondo et al. (2020) primarily utilised surveys to gather learner data. However, I believe that this approach was insufficient for evaluating the learners. In contrast, the present study focused on employing a test as the primary assessment method. Based on a study conducted by Zawani (2014) and Ping and Hua (2016), grade 4 learners cannot still identify the characteristics of 3D shapes. This issue may have emerged due to educators' incompetence in employing effective tactics and approaches during education on this subject. Based on a comprehensive analysis of the literature, there are similarities and differences in the instructional approaches used for the subjects 'space' and 'shape.' The study emphasises the learners' comprehension of the issue rather than rote memorisation (Ping & Hua, 2016).

Ping and Hua (2016) assert that self-discovery is one of the ways employed, wherein learners are required to uncover educational content autonomously. In this scenario, learners should be allowed to engage in autonomous work on geometric concepts. At the same time, educators are expected to provide assistance and guidance to learners when they encounter difficulties, to facilitate a thorough understanding and connection to the fundamental principles of geometry. Furthermore, the research conducted by Ping and Hua (2016) reveals the cause behind the difficulty learners' face in comprehending the features of 3D forms. As a result, many educators resort to traditional methods of memorisation. The traditional method is considered to be time-saving and aids in focusing on more crucial matters. Nevertheless, this approach reduces learners' enthusiasm for mathematics, rendering it unengaging. This suggests that the educators

are solely focusing on teaching geometry, emphasising PK abilities while neglecting CK skills. Moreover, it is plausible that this phenomenon is attributed to the pressure exerted on educators to complete the curriculum.

In their study, Jones and Tzekaki (2016) conducted a qualitative analysis that utilised literature review analysis as a data collection method. In contrast, this study focuses on three previously mentioned methods (assessment, observations and interviews). The findings by Jones and Tzekaki (2016) indicated that educators, upon witnessing a substantial number of examples, systematically endeavoured to improve their learners' understanding by presenting them with impactful challenges and thought-provoking questions. Engaging in activities such as playing games, using toys, or running software programmes that include spatial thinking in the classroom setting enhances a range of spatial skills and spatial reasoning abilities (Jones & Tzekani, 2016). Engaging in both play and study simultaneously can enhance problem-solving skills and promote self-directed learning in children, enabling them to intuitively comprehend mathematical concepts (Ramli, Maat, Khalid, 2020). This implies that incorporating play into the learning process might offer youngsters the chance to explore novel concepts, broaden their understanding, and foster the development of innovative ideas. Moreover, learners can employ the 'gamification' approach to review and reinforce their understanding of the distinctive attributes of 3D, in contrast to Sutiarso, Coesamin, and Nurhanurawati (2018), who advocate for the 'scaffolding' strategy, where learners acquire skills at a certain level and then progress to a higher level under the guidance of the educator. The learners will initially endeavour to achieve that level alone. If learners are unable to attain that degree, the educator should furnish similar yet less complex inquiries to facilitate independent comprehension of the subject matter (Hua & Ping, 2016). Engaging in more advanced tasks can assist learners in cultivating both PK and CK abilities.

The above literature discussions relate to this study as it focused on similar topics, exploring learners' CK and PK in geometry. The literature, exemplified by Ismail et al. (2020), demonstrates that the study explores both 2D and 3D shapes by examining their edges, identifying different types of lines, and counting the number of surfaces each shape possesses. This aspect is also evident in the current study where the lessons observations of the teaching of PK and PK in geometry are explored in this research, whereas some literature indicates that data collection was done through interviews, while this research focuses on observation of educators teaching the knowledge.

2.3.2. Studies on measurement

The principles underlying measurement knowledge assert the importance of adopting a practical approach when teaching measurement. The instruction of these concepts should adopt a more pragmatic approach, wherein learners engage in hands-on activities and perform measurements to facilitate their understanding and application. Measurement encompasses the process of determining dimensions, subsequently enabling the computation of perimeter, volume, and area variables. These calculations facilitate the acquisition of PK skills, while CK can only be acquired through practical measurement and calculations. By engaging in practical measurements, learners gain a conceptual understanding of dimensions, such as perimeter, volume, or area, and the various measuring units used in different measuring tools. Within the realm of mathematics education, students are introduced to the concept of measurement by means of facilitating their perception and application of dimensions necessary for measurement.

In a study conducted by Yurniwati and Soleh (2019) on the PK and CK of educators, it was observed that their CK competency surpassed their PK. This suggests prospective educators possess a CK rather than a PK in which a specific algorithmic ability is taught. In addition, the prospective primary educators in this study (Yurniwati & Soleh, 2019) demonstrated inadequate PK and CK as evidenced by their subpar performance in geometric PK. They appear to possess CK of the formulas used to calculate area and volume. Their low PK score is attributed to a deficiency in mathematical concepts and measuring procedures (Yurniwati & Soleh, 2019). The main distinction between the current study and the study conducted by Yurniwati and Soleh (2019) is that their study specifically examined the PK and CK of educators. In contrast, this study focuses on both learners and educators because teaching and learning involve both parties. Moreover, they only sampled educators and collected data through a diagnostic test and observation, whereas this study also added interviews to the data collection. Hence, it is essential to understand the phenomenon from the perspectives of both learners and educators.

However, a study conducted by Ayvaz, Gündüz, and Bozkus (2017) found that learners in certain teaching schools struggle to understand the concept of diagonally concerning spatial dimensions. According to Safrina and Darmawan (2016), primary school educators struggle

to understand concepts such as measuring angles, defining perpendicular lines, and using the proper methods to draw straight lines. Conversely, a recent study conducted by Idrus, Rahim, and Zulnaidi (2022) reveals that teaching learners concepts can result in the formation of practical and flexible strategies to enhance primary school learners' understanding of measuring areas in the context of mathematics instruction, as well as their ability to retain knowledge. Idrus et al. (2022) argue that establishing a direct connection between CK and area measurements will enhance the interactive and iterative growth of learners' mathematical abilities. An effective approach appears to be teaching primary school learners the theory of area measurement rather than solely focusing on problem-solving techniques. This approach aims to improve their problem-solving skills and deepen their CK (Idrus et al., 2022).

A study conducted by Machaba (2016) discovered that learners' definitions of area and perimeter were wrong. This indicated a deficiency in their previous understanding of these concepts and a lack of CK about perimeter as a measure of distance. In addition, Machaba (2016) found that learners had a deficient CK of the area as a 2D surface, as evidenced by their understanding of the area as the product of length and width. This suggests that learners have utilised their acquired PK of the region formula to describe the given region. Dickson (1989, p. 79) defines area as the measure of the surface of a region and perimeter as the distance around the region. This suggests that learners were only instructed in PK using standard teaching methods without sufficient emphasis on CK through active learning and inquiry-based learning[2]. The difference between this study and Machaba (2016) is that Machaba (2016) solely examined the PK and CK of Area and perimeter among Grade 10 learners. Masilo (2018) contends that, from a pragmatic perspective, education is most efficient when an educator is present to encourage learners' inquiry-based learning and is perceived as a knowledgeable agent who can validate the veracity of learners' knowledge creations. Active learner participation enhances the engagement and effectiveness of learning, reducing the likelihood of disengagement and increasing the understanding of the practical use of knowledge. The measurement in this study focuses on the concepts of area, perimeter, and volume. It involves observing educators teaching learners PK and CK the concepts of area, perimeter, and volume. Whereas Machaba's (2016) research, examined the learners' PK and CK of area and perimeter. In addition to area and perimeter, this study also includes the concepts of volume, where

[2] Inquiry-based learning (IBL) refers to an instructional approach where learners assume accountability for their own education. Learners engage in the process of posing inquiries, gathering and scrutinising evidence, creating solutions, making judgements, providing justifications for their conclusions, and taking action (Friesen, 2014).

calculation and descriptions are taught. This application is designed to evaluate the learners' knowledge and skills in measurement.

2.3.3. Technological approach

Educators exercise discretion in determining how they incorporate the utilisation of technology within their educational settings. Educators have the option to integrate technology into their teaching methods, whether it be during whole class sessions or small group interactions (Hamilton, 2022). The instructional use involves utilising visuals through a document camera to display videos, demonstrating the functionality of a website, exemplifying the use of a digital recorder or e-reader, conducting Skype sessions, and playing podcasts (Hamilton, 2022). Considering this, technology can be utilised in geometry class to visually represent real-life structures of geometric shapes, allowing learners to establish connections with the geometric concepts taught in the classroom. When utilising technology for teaching, it is crucial to emphasise that CK can be enhanced by encouraging active student participation. This allows learners to delve into ideas, connect with their existing understanding, and foster the development and sharing of knowledge. Hamilton (2022) suggests that a technological approach should be employed to actively acquire knowledge by utilising methods such as active learning, discovery learning, problem-based learning, posing essential questions, employing scientific methods, and utilising virtual manipulatives. By engaging in diverse and fulfilling learning experiences, learners will solidify their CK and PK, allowing them to internalise and fully comprehend the information they have acquired.

The existing literature reveals a prevailing study trend that supports using many strategies to enhance learners' understanding of CK in the context of fundamental geometry. Modern technological breakthroughs enable social interaction among learners by offering a collaborative computing environment that allows them to engage in complex mathematical discourse (Prieto, Juanena, & Star, 2014; Kaur, 2015). Furthermore, research has revealed that the use of digitally enhanced instructional platforms facilitates the development of a heightened understanding of the hierarchical connections among different shapes (Bussi & Baccaglini-Frank, 2015; Ng & Chan, 2019). Sharma (2019) acknowledged the importance of technology in promoting a deep grasp of CK and its various forms and representations. However, for technology to be effective in this regard, it is crucial to have well-structured educational preparation.

Ozcakir, Konca, and Arikan (2019) conducted a study in which they employed digital activities to teach geometry. Digital technologies can enhance the accessibility of mathematical topics by facilitating the investigation and manipulation of practical representations. According to Ozcakir et al. (2019), digital activities provide learners with the chance to engage in experimental mathematics. This implies that learners, when given opportunities to engage and acquire knowledge with technology in the field of mathematics, have the potential to enhance their PK and CK capacities. Similar to other implements, the educator plays a crucial role in improving digital educational content by understanding classroom situations, making decisions, modifying plans if necessary, identifying relevant examples, and facilitating communication to promote learners' comprehension and the emergence of personal viewpoints (Biza, 2011). Regarding this matter, educators could initiate the process by examining the pre-existing knowledge of their learners and thereafter employ real-life instances to elucidate the principles being taught in a classroom setting, which the learners can easily relate to. When instructing learners about geometric shapes, such as books, the classroom, rulers, and so on, can be employed as tangible examples. Ozcakir et al. (2019) emphasise the significance of educators' choices regarding the utilisation and integration of digital learning materials. The primary focus of the study conducted by Ozcakir et al. (2019) was to investigate the impact of digital learning tools on the acquisition of knowledge about geometric shapes in early childhood education. However, it is essential to note that this study does not directly examine the effect of technology on geometry as it cannot predict whether all educators involved in the study will utilise technology. Instead, the study aimed to explore the influence of technology in cases where it is utilised.

The findings of Zulnaidi and Zamri's (2017) study suggest that learners who utilise GeoGebra in their mathematics education exhibit a higher level of PK and CK in mathematics compared to those who rely on traditional teaching techniques. Both experimental groups indicate that PK has a significant role in mediating mathematical achievements (Zulnaidi & Zamri, 2017). Similarly, findings from a further study conducted by Jelatu et al. (2018) using GeoGebra[3] demonstrate that learners' understanding of geometry topics may improve more effectively

[3] Geogebra is a versatile software that facilitates the exploration and instruction of geometry, algebra, statistics, and calculus. It is suitable for learners ranging from elementary school to university level. Both in the classroom and at home, it promotes mathematical experimentation and exploration through engaging, problem-centred teaching methods. The source cited is Majerek (2014).

when they are exposed to the GeoGebra-assisted REACT technique, as opposed to traditional teaching methods. This study focused on observing how educators teach PK and CK in the context of geometry. However, it is essential to note that the specific nature of the geometry class taught is determined by each educator. As a result, the teaching methods, strategies, and resources will vary among educators, as they come from different schools with diverse backgrounds and apply different teaching approaches. Therefore, the study does not mandate that educators should implement a technological approach in their lessons, as it is based on overt observation. However, as mentioned earlier in the literature, the use of technology in teaching CK and PK is crucial. Thus, it is essential to examine how this phenomenon is utilised in the teaching and learning geometry, as will be the case in this study.

2.3.4. Mathematical register and language

Mathematics is a discipline that utilises a combination of natural language, symbolism, models, and visual displays to express ideas. It is considered multi-semiotic due to its use of multiple modes of communication. (O'Halloran, 1998, 2015). When learners engage in problem-solving, they utilise various resources and transition between oral and written communication. Learners must establish connections between these three semiotic systems and demonstrate proficiency in comprehending and utilising highly technical, dense, and specific language (Wilkinson, 2015). The study of mathematics relies heavily on language, encompassing both spoken and written forms, and cannot be considered a non-verbal discipline. Halliday's (1975) original formulation implicitly recognised that the language of mathematics encompassed both technical terminology (vocabulary specific to the discipline) and general academic vocabulary. Furthermore, the mathematics register encompasses techniques for structuring communication, such as methods for presenting arguments in the context of mathematical dialogues (Cuevas, 1984). During their education, learners are obligated, and occasionally motivated and assisted, to acquire the skills of verbalising, comprehending, and composing mathematical concepts using the specialised language of the field. The significance of language extends beyond the realm of mathematical education and instruction. For learners to achieve academic success, they must showcase their knowledge in various educational settings, including high-stakes tests. These tests typically involve evaluating learners' understanding through the use of challenging texts, such as intricate word problems (Cheuk, Daro, & Daro, 2018; Martiniello, 2009).

Multiple research has examined the role of language in facilitating the development of CK regarding geometric shapes among learners with limited language proficiency. Their investigations utilised mathematical terminology to establish the definitions of geometry ideas (Friedman, Kazerouni, Lax, & Weisdorf, 2011; Akarsu & Yilmaz, 2015; Kobiela & Lehrer, 2015). Alternatively, other research has examined how learners utilise verbal communication to acquire knowledge about geometric forms (Roth & Gardener, 2012; Mushin, Gardner, & Munro, 2013). The study conducted by Mushin et al. (2013) examined how language and non-verbal behaviour might be utilised to showcase comprehension of geometric shape principles. By employing Conversation Analysis, the researchers examined the data and concluded that learners' difficulty in completing a geometry exercise may be attributed to their limited language comprehension (Mushin et al., 2013). According to Sharma (2019), learner underperformance is primarily attributed to their failure to comprehend and adhere to the educator's instructions, rather than the inherent difficulty of the work. Roth and Gardener (2012) discovered that learners acquire knowledge about geometric forms through engaging in discussions within their regular mathematics classrooms.

A study by Sharma (2019) examines the process of developing comprehension of shapes and their representations in multilingual primary school classrooms. A systematic literature review indicates that multilingual learners utilise their multilingual skills to enhance their comprehension of mathematical concepts while actively engaging in mathematical activities (Adler & Ronda, 2015; Moschkovich, 2015; Sharma, 2019). The argument put forth is that viewing language as a resource emphasises the learner's decision in selecting language during mathematical tasks. The utilisation of several languages in educational settings also brings to light the challenges associated with teaching dilemmas (Sharma, 2019), power dynamics and dominance (Parra & Trinick, 2018), and the comprehension of mathematical concepts (Planas, 2014), as well as its impact on a learner's self-perception and identity.

The importance of selecting appropriate language, recognising power dynamics, and ensuring accessibility to geometric concepts in multilingual classrooms is frequently disregarded (Sharma, 2019). Sharma (2019) conducted a study that examined the literature on the development of conceptual understanding of shapes and their representations among learners aged 5-13 in multilingual primary schools. The study found that existing research does not adequately address the challenges that arise in multilingual contexts when teaching geometry concepts. This study does not directly explore the impacts of language on teaching and

learning. However, it is crucial to acknowledge the potential impact of language and mathematical register on the teaching and learning of geometry. Although not the primary focus of this study, the multilingual nature of the schools involved may provide insight into this aspect.

2.4. Learners' response to procedural and conceptual knowledge assessment in geometry

In their study, Susilawati, Suryadi, and Dahlan (2017) highlight the importance of geometry teaching in cultivating a cognitive framework that enables individuals to perceive and comprehend the interrelationships and characteristics of geometric spaces. They emphasise the significance of providing learners with opportunities and tools to observe, study, and explore geometrical concepts in real-world settings before expecting learners to formally apply their acquired knowledge in the classroom. Murni, Sariyasa, and Ardana (2017) found that traditional methods of teaching geometry, such as using paper and pencil, often result in learners unable to accurately depict geometric concepts. There is a potential drawback that this may not be beneficial to learners in their acquisition of geometry ideas (Jelatu et al., 2018). Alex and Mammen (2018) found that learners excelled in understanding ideas related to lines, fared moderately well with circles, and struggled the most with language associated with quadrilaterals and triangles. Although the majority of learners had a satisfactory understanding of geometric language, their performance improved when provided with visual illustrations of terms rather than verbal explanations. Implementation of Van Hiele's geometric thinking raised worries about the limited ability of most learners to merely perform at the Visualisation stage. An important goal of mathematics education in South Africa is to teach learners about spatial ideas and their connections (Department of Education, 2019). According to Van Hiele, geometry plays a crucial role in the development of logical thinking, which is a vital aspect of understanding mathematics (Alex & Mammen, 2018).

Van Hiele asserts that educators play a substantial role in both the acquisition and teaching of knowledge (Alex & Mammen, 2018). Enhancing mathematical understanding can be achieved more efficiently when educators possess a deep understanding of children's learning processes (Swafford, Jones, & Thornton, 1997). A study conducted by Dhlamini, Chuene, Masha, and Kibirige (2019) which focused on the Grade Nine geometry spatial mathematical reasoning in the South African annual national assessment uncovered several key findings: 1) There was a

presence of unrecorded mixed spatial geometry mathematical reasoning; 2) The majority of learners remembered and manipulated irrelevant mixed spatial geometry mathematical reasoning; 3) Some learners exhibited fragmented mixed spatial geometry mathematical reasoning, which they incorrectly applied to the given context; and 4) A small number of learners effectively utilised their prior knowledge to appropriately blend spatial geometry mathematical reasoning. This can be attributed to learners possessing only procedural skills in geometry and solving problems by rote memorization. Consequently, learners who rely on memory are prone to just utilising their information for a specific evaluation after engaging in repetitive exercises but are likely to lose it unless it is reinforced or revisited. On the other hand, learners who comprehend the subject matter are less likely to forget its content.

The study conducted by Dhlamini et al. (2019) focused exclusively on spatial mathematical reasoning in Grade nine. It was conducted in the Capricorn district of the Limpopo province. This research, on the other hand, is being conducted in the Bushbuckridge region. Thus, the background context of Dlamini et al. (2019) varies from this study, highlighting the significance of conducting this research as the first study in Bushbuckridge. How learners in this study respond to questions that require PK and those that require CK will be explored, with the main focus on Geogebra, as used by educators (where applicable), as in the studies mentioned above. This study examines how learners in grade 6 and grade 9 respond to the Van Hiele level of understanding in terms of their ability to visualise and analyse. This investigation aimed to determine whether learners are progressing in terms of their Van Hiele level. The literature by Ngirishi and Bansisal (2019) indicates that learners are not progressing beyond the visualisation level, despite the expectation for them to reach a higher level. Their study, which focused on FET phase learners, found that most learners were operating at the visualisation and analysis stages of the van Hiele levels of geometric thinking.

2.4.1. Response to 2D and 3D shapes

The curriculum (DoE, 2011) emphasises the importance of learners being able to identify, visualise, analyse, and name various 2D and 3D shapes in their surroundings and in geometric scenarios. This includes a focus on regular and irregular polygons, triangles, as well as understanding the similarities and differences between rectangles and parallelograms. Exploring the attributes of shapes Discuss, categorise, and analyse 2D shapes based on their number of sides, lengths of sides, sizes of angles, and naming of angles. When it comes to 3D

objects, learners should be able to describe the characteristics of 3D objects. Where they can describe, sort, and compare 3D objects based on the number and shape of their faces. Be able to identify and analyse the number of vertices and the number of edges and build 3D models using folding nets.

Bernabeu, Gagatsis, Moreno, Salvador, and Llinares (2019) conducted a study on the comprehension of 2D shapes among nine-year-old learners. The findings indicate that the ability to identify essential characteristics of polygons and recognise shared characteristics among a group of polygons is contingent upon the specific attribute being studied. Furthermore, how learners reason with these qualities is influenced by the distinction between discursive and non-discursive registers. The first outcome can be elucidated by the varied progression of dimensional deconstruction. The second finding suggests that the ability to explain the connection between attributes is enough to create a figure with certain distinct features. This implies a hierarchical relationship between verbal declarative knowledge[4] and perceptual apprehension[5] in this age group (Bernabeu et al., 2019). This implies how the educator imparts knowledge while teaching polygons or geometry in general, the selection of words is, therefore, significant for learners in their development of PK and CK understanding in geometry.

The study conducted by Yilmaz and Kaçar (2016) examines the effects of utilising Vee diagrams and mind maps in teaching the concept of "polygons" to seventh-grade learners in primary education. The study specifically evaluates the influence of these teaching methods on learner accomplishment and the long-term retention of knowledge. The findings of this study indicate that the utilisation of mind mapping and vee diagramming techniques surpasses traditional teaching methods in terms of effectiveness and significantly impacts long-term retention. This indicates that employing diverse instructional approaches and educational resources is imperative to facilitate learners' comprehension of the subject matter and enhance their performance. Yilmaz and Kaçar (2016) propose the utilisation of meaningful learning

[4] The initial stage of learning involves acquiring linguistic knowledge. This form of learning is alternatively referred to as declarative knowledge. Verbs such as "define" and "describe" can be used to measure declarative knowledge. The verb examples above are utilised in the "comprehension" and "application" categories of Bloom's Taxonomy of Learning. Declarative knowledge is classified as a lower level of learning in Bloom's Taxonomy, as it just involves recalling previously acquired knowledge and reflects the acquisition of superficial knowledge. Despite being classified as low, it is necessary across all subject areas as learners can acquire advanced information by first acquiring the fundamental knowledge in this stage (Mofidul & Manjone, 2006).

[5] Perceptual apprehension is the process of straightforwardly recognising and comprehending visual representations, forms, or ideas in order to generate novel understanding (Orozco, 2021).

aids in maths classes to provide comprehensive preliminary information. Therefore, I believe that utilising a range of instructional materials, integrating relevant resources, and employing diverse teaching methodologies would effectively support learners in improving their CK of geometry.

In another study conducted by Pavlovičováa and Švecová (2015), the researchers examined the discovery activities of grade 3 and grade 7 learners in a primary school setting. The findings indicated that grade 3 learners were capable of locating an adequate number of cube nets, although not all of them were accurate. Pavlovičová and Švecová (2015) emphasised the significance of further advancing the understanding of similarity and identity ideas in future research. Additionally, the study demonstrates that utilising Polydron significantly aided learners in their inquiry skills, with motivation playing a substantial part in facilitating this activity. The study found that learners were engaged in a problem-solving activity related to geometry while working as designers, following a discovery process. Pavlovičováa and Švecová (2015) have observed that mind mapping enhances the creativity of young children and facilitates the comprehension of children's cognition. Similar to the Pavlovičováa and Švecová (2015) study, an analysis of the work and solutions of grade 7 learners revealed that the overall test score was 73.54%, which was deemed satisfactory. The learners were more acquainted with the assignment that focused on manipulating and analysing the cube nets. From a cognitive perspective on mental rotation, learners found it more effortless to manage a cube's sides than the edges or vertices. Despite the initial deficiency in handling abilities, the utilisation of Polydron enabled learners to independently devise strategies for resolving assigned tasks. The learners were content with their work and provided favourable feedback regarding the completed activities (Pavlovičováa and Švecová, 2015).

The study by Bernabeu, Moreno, and Llinares (2021) aimed to investigate the comprehension of third-grade primary school pupils about the notion of polygons and their interrelationships. The results indicate that learners' comprehension of the notion of polygons is impacted by their capacity to identify and adapt the pertinent characteristics that are considered in the definition of a polygon. The findings suggest that understanding the concept of a polygon becomes increasingly complicated over time. The discovery of a correlation between the ability to recognise a figure as a polygon and the capacity to change a non-example of a polygon into an actual polygon provides evidence of increasing complexity. Furthermore, the capacity to identify a polygon as an instance of a category depends on the attribute that describes the group

of polygons (Bernabeu et al. 2021). In a study by Hanid, Said, Yahaya, and Abdullah (2022), researchers combined Augmented Reality technology with Computational Thinking to enhance the teaching of Geometry topics. The findings indicate that utilising Augmented Reality applications with Computational Thinking as a teaching approach has a beneficial impact on learners' development of Computational Thinking, Visualisation Skills, and accomplishment in the topic of Geometry. Therefore, it is evident that implementing diverse teaching approaches and technological teaching strategies, rather than relying solely on traditional teaching methods, can significantly improve the performance of learners in a geometry classroom.

This study sought to investigate how learners respond to the PK and CK concerning 2D and 3D shapes. The literature above revealed that mind mapping and Vee diagramming have been shown to enhance learners' creativity and facilitate their comprehension of cognitive processes. Additionally, the literature explains that Polydron greatly enhanced learners' inquiry skills, with motivation playing a significant role in facilitating this activity. However, as previously elucidated in the study, the nature of the geometry class in terms of teaching and learning is influenced by the educator rather than the researcher. Therefore, the choice of resources and teaching methods depends on the educator as the participant in the study. While polyhedron is primarily utilised in the foundation phase, educators in the intermediate phase are expected to employ solids like folding nets to teach learners the connection between 2D and 3D shapes.

2.5. The impact of teaching and learning procedural knowledge and conceptual knowledge in geometry

Conceptual and procedural knowledge are fundamental in mathematics, and both significantly impact the learning and teaching of geometry. Kilpatrick et al. (2001) emphasised the importance of procedural knowledge in supporting conceptual knowledge. It is crucial to connect these two types of knowledge to develop a deep understanding of mathematics (Hiebert & Lefevre, 1986). In addition, the simultaneous development of these two types of knowledge has a beneficial impact on mathematical competence (Rittle-Johnson & Schneider, 2015). The support of algorithms enhances conceptual understanding by providing helpful building blocks for clarifying concepts. On the other hand, students can develop algorithms by grasping the underlying concepts (Aydın, 2018). Conceptual and procedural knowledge are frequently discussed due to their perceived interconnectedness (Rittle-Johnson & Schneider, 2015).

Therefore, providing learners with opportunities to engage these two forms of knowledge is imperative. This is crucial as it allows them to develop a diverse range of perspectives on mathematics, enhancing their problem-solving abilities and facilitating connections between different mathematical concepts. Ultimately, this approach leads to improved performance in mathematics.

The conceptual understanding of mathematics is strongly linked to academic achievement. Exploring CK is crucial for learners to acquire PK (Lauritzen, 2012). Learners who perform well on conceptual tasks also tend to excel in procedural tasks. Numerous studies have pointed out that a deficiency in CK can result in a range of difficulties. Many learners struggle with understanding concepts because they may not have a strong foundation of CK (Tekin-Sitrava, 2017). This suggests that teaching CK can enhance learners' understanding of both CK and PK. However, if conceptual knowledge is lacking, learners may struggle to perform at their full potential. In their study, Khoule, Bonsu, and Houari (2017) investigated the correlations between mathematics anxiety and CK understanding. The study compared the levels of mathematics anxiety in learners who were taught using a CK approach to those who were taught using a PK approach.

The investigation unveiled a statistically significant association between anxiety related to mathematics and the educational approach employed. The CK strategy had a more pronounced impact on lowering mathematics fear compared to the PK approach. The CK technique (Khoule et al., 2017) was found to be beneficial in reducing mathematical anxiety levels in the CK group. Despite receiving training that was similar to more traditional methods of teaching mathematics, the PK group did not experience any change in their levels of mathematics anxiety. Based on this deviation, it was concluded that the PK groups settled on an average. Khoule et al. (2017) found that procedural or traditional teaching did not enhance learners' comfort with mathematics. The research suggests that it had a negative impact. The results support Skemp's (1971) idea that procedurally teaching rote memorisation could cause learners to feel anxious about mathematics. The research conducted by Khoule et al. (2017) demonstrates that children exhibit improved performance in mathematics when they are stimulated to engage in critical thinking rather than relying solely on rote memorisation of procedures and rules. When learners are taught utilising CK and active techniques, they get the confidence to effectively engage with new mathematical subjects. Khoule Et al. (2017) stress

the importance of learners understanding the practical applications of the mathematics they are studying in their prerequisite classes.

However, a recent study conducted by Nahdi and Jatisunda (2020) has revealed that advanced learners of CK are capable of confronting unfamiliar difficulties. In contrast to Khoule Et al. (2017), who focus on teaching learners to apply the CK approach, Nahdi, and Jatisunda (2020) further emphasise that nurturing the growth of learners' PK and CK will have a positive impact on their ability to comprehend new ideas. Research suggests that learners' ability to comprehend a mathematical subject is restricted when they do not possess both PK and CK. Hence, the connection between PK and CK comprehension is vital. Following the work of Nahdi and Jatisunda (2020), this study examines the use of PK and CK in mathematical geometry. It advocates for the equal significance of both PK and CK. In addition, Al-Mutawah, Thomas, Mahmoud, and Fateel (2019) conducted a study on the examination of CK, PK, and Problem-Solving Skills in the field of Mathematics. The outcomes of the calculations for area and circumference in the field of geometry were unsatisfactory. The majority of learners lacked knowledge of the distinction between area and circumference. The learners exhibited a significant deficiency in CK and demonstrated only average proficiency in problem-solving. Al-Mutawah et al. (2019) propose that educators prioritise the development of CK skills over the acquisition of PK skills necessary for doing calculations. This can be achieved by transitioning from the traditional approach to learning, which focuses on the role of the educator, to a more interactive and learner-centred approach known as contextual learning (Jazuli, Setyosari & Sulthon, 2017).

In a study conducted by Laily, Sunardi, Yuliati, Mamah, and Sari (2020), the objective was to analyse the CK and PK of learners in problem-solving involving polyhedral shapes. The study also aimed to explore the correlation between CK and PK concerning problem-solving abilities. The study's findings indicate that learners at the visualisation level demonstrate partial mastery of CK, but none of them exhibit proficiency in PK. On the other hand, learners at the analysis level demonstrate partial mastery of both CK and PK. Lastly, learners at the deduction level demonstrate complete mastery of both CK and PK. The test results demonstrated a positive correlation between learners' level of geometrical thinking and their CK and PK. Specifically, higher geometrical thinking levels were associated with stronger CK and PK (Laily et al. 2020). The study demonstrates a correlation between higher-order geometry thinking and a potent combination of CK and PK. However, achieving a higher degree of geometrical thinking can

only be accomplished through the successful use of CK and PK in teaching and learning geometry.

Moreover, a study conducted by Hock, Suraya, and Salim (2021) aimed to determine the impact of utilising the dynamic software Google SketchUp (GSU), as opposed to not using any software, on the application of van Hiele's theory and traditional teaching strategies in enhancing primary school learners' CK and PK in geometry. The utilisation of van Hiele's phases of learning and van Hiele's levels of thinking, along with the dynamic tool Google SketchUp, in the teaching and learning process yielded several noteworthy results when compared to traditional teaching methods which resulted in the learners' enhancement of their understanding of both the CK and PK aspects of geometry, while the educator and learners engaged in productive and enjoyable interaction. Educators enhanced learning settings and broadened the range of learning opportunities by creating instructional materials to disseminate educational knowledge. Consequently, learners enhance their learning experience by actively interacting with the module and partaking in group discussions to successfully attain the learning objective. These factors serve to inspire and encourage pupils to actively engage in the learning process (Hock et Al. 2021). This implies a beneficial effect when teaching and learning incorporate the application of both CK and PK skills.

Manandha, Pant, and Dawadi (2022), observed that CK levels were substantially lower compared to PK levels. The researchers also established a notable correlation and positive relationship between CK and PK. Put simply, learners thrive in PK while CK relies on PK. According to Manandha et al. (2022), educators should prioritise the augmentation of their CK. Furthermore, research has demonstrated that learners' learning of PK and skills, retention of formulas, and problem-solving aptitude were all impacted and facilitated by textbooks, educators, and learning activities. Additional research indicates that a significant proportion of learners perceive mathematics as more difficult than other academic disciplines. The researchers (Manandha et al., 2022) found it difficult to comprehend a scenario where mathematics would have practical applications. Consequently, learners' insufficient CK might be attributed to their extensive time spent in PK. If learners are exclusively instructed through rote repetition, they would not encounter any challenges in mathematics if CK is given equal emphasis as PK.

Within the same context, the study discovered that learners devote a greater amount of time to acquiring PK skills compared to understanding the essential concepts (Manandha et al., 2022). In addition, they did not participate in mathematically-oriented critical discussions either within or outside the classroom. Based on this situation, it seems that learners are focused primarily on achieving a passing grade rather than truly understanding the content. Therefore, the majority of learners prioritise the memorization of information through repetition instead of CK preparation (Manandha et al., 2022). It might be inferred from this that educators persist in using laborious and demanding traditional teaching approaches. Sariyasa (2017) states that learners face difficulties in effectively representing geometrical concepts through traditional instructional approaches. Furthermore, the discovery supports Skemp's (1971) assertion that the employment of procedural teaching methods for memorization can lead to the development of mathematics anxiety in learners. Consequently, learners experience adverse effects when just PK is emphasised, whereas they experience positive effects when both PK and CK are strengthened.

This study examines the influence of teaching PK and CK on the learning of geometry. It focuses on the challenges encountered by both educators and learners, the perspectives of learners and educators on geometry, and the teaching and learning methods employed in the teaching of PK and CK in geometry. According to the literature mentioned above, learners experience anxiety, and the CK strategy has a stronger effect in reducing fear of mathematics compared to the PK approach. Furthermore, the aforementioned studies suggest that learners demonstrate enhanced proficiency in mathematics when they are encouraged to actively participate in critical thinking rather than solely relying on the memorization of procedures and rules. Therefore, this study supports the use of both CK and PK, with an emphasis on teaching CK before PK.

2.6. Summary and conclusions

The literature in this Chapter revolved around prior research that specifically examined the teaching and learning of geometry. This literature chapter reveals that learners encounter numerous obstacles in the field of geometry, particularly in comprehending the characteristics of two-dimensional and three-dimensional shapes, as well as spatial reasoning in both two-dimensional and three-dimensional shapes. The challenges faced by learners are connected to the utilisation of traditional teaching techniques in the field of geometry by educators. The

discussion focused on the reliance on PK and CK, as well as the competence and application of educators' knowledge. Reports indicate that digital technologies possess the capacity to augment learning in the field of geometry. Utilising real-life objects and creating examples that relate to everyday situations can significantly improve the effectiveness of learning in geometry. The next chapter will centre on the examination of the theoretical framework.

CHAPTER 3: THEORETICAL FRAMEWORK

3.1 introduction

The foundation of this study is rooted in the Van Hiele theory and the Concrete Pictorial Abstract (CPA) Theory. Since the study focuses on Procedural knowledge (PK) and Conceptual knowledge (CK) as part of the research topics, the framework includes a discussion of PK and CK using the Van Hiele and CPA theory. This is because the research questions guide the discussion of the two levels of knowledge. The research topic employed the CPA theory to investigate how educators may effectively utilise the CPA theory to teach both PK and CK in geometry. The researcher explores the application of the van Hiele level theory in geometry assessment, as well as its role in facilitating learning and teaching specifically focusing on visualisation and analysis. Therefore, the following section will explore the theoretical frameworks in detail.

3.2. Concrete Pictorial Abstract (CPA) approach

The connection between imparting PK and CK can be established by utilising the Concrete Pictorial Abstract (CPA) theory in the classroom (Africa, Borboran, Guilleno, Mendiol, Portento, Rodriguez & Conde, 2020). CPA is an instructional approach that integrates the three stages of learning outlined in Brunner's theory: enactive, iconic, and symbolic (Leong, Ho & Chang, 2015; Chang et al., 2017). According to Brunner's Theory, CK is developed by direct and interactive engagement with tangible objects. This knowledge is primarily conveyed through visual representation, known as iconic representation. Finally, the acquired experience and visual imagery are further conceptualised and expressed through symbolic notation (Yurniwati, 2021).

> 1) The development Of CK involves the use of teaching aids and visual representations of objects, which require mathematical reasoning.
> 2) Developing PK abilities involves transitioning from the visual aids phase to the symbolic step, which requires making mathematical connections and effective communication.
> 3) Applying problem-solving skills is necessary to put both PK and CK into practice (Soleh & Yurniwati, 2021).

This approach (CPA) in CK involves the use of concrete objects and visuals to enhance mathematical reasoning during teaching and learning. In addition, the acquisition of PK is further developed following the acquisition of CK. Subsequently, algorithms are taught and connected to CK to establish a geometric connection. This aligns with this research, which supports the notion of teaching CK before introducing PK and establishing connections

between the two. Lastly, the application of problem-solving requires the utilisation of both theoretical knowledge (CK) and practical knowledge (PK) to find a solution. The CPA learning strategy is a method that gradually reduces the reliance on multiple senses during teaching, aiming to engage more than one sensory organ. It is also grounded in approaches and perspectives prioritising conceptual learning over memorisation (Yildirim & Karabulut, 2023). Therefore, it is crucial for learners to thoroughly comprehend and grasp geometric concepts before instructing specific procedures, to prevent confusion and promote a deeper understanding of geometry rather than relying on memorisation. Essentially, the present study suggests that the teaching of CK in geometry can be improved by using the CPA theory. Figure 3.1 demonstrates the application of the CPA approach in teaching PK and CK in the classroom.

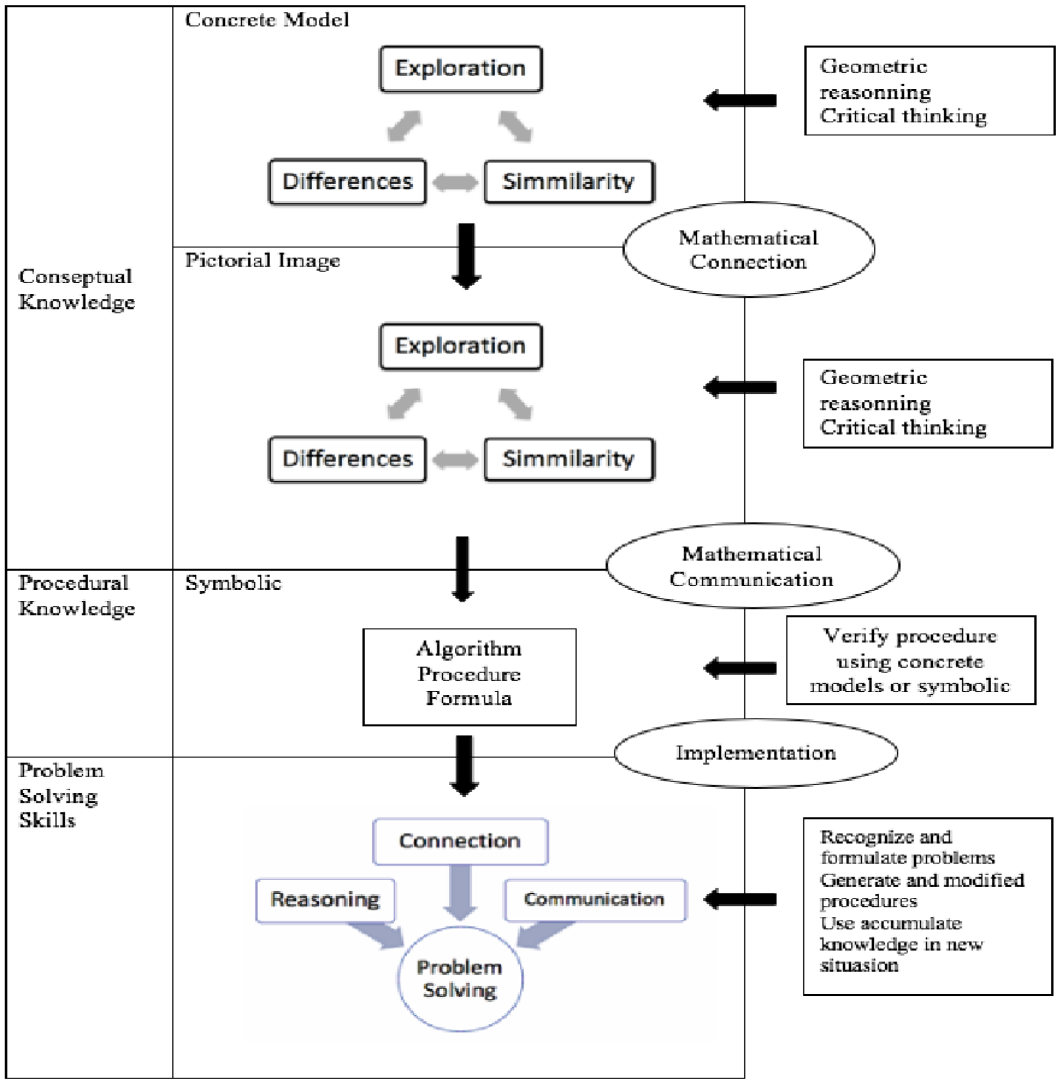


Figure 3.1: The CPA approach is used to teach both procedural and conceptual knowledge in a classroom setting (Yurniwati 2021, p.5).

1. Enactive: Manipulation of concrete objects / an experience. Conduct reasoning when observing congress models of geometry. Find differences and similarities

> between geometry models to form CK, including definitions, and building properties.
> 2. Iconic: Perceptual images of forms of Mathematical ideas. Facilitation of making connections of iconic types and mathematical concepts.
> 3. Symbolic: abstract symbol. Facilitation of communication-based on making connections to abstract mathematical symbols (Yurniwati, 2021, p.4).

The CPA method integrates learning through the physical manipulation of tangible objects, the visual representation of concrete object manipulation, and the use of abstract notation for problem-solving (Putri, 2015). Manipulative objects, such as cakes, measuring tools, or other items, can be used in the process of learning and teaching. To facilitate effective geometry learning, educators must establish a classroom setting that encourages learners to actively engage with tangible objects and practical exercises. By doing so, learners can acquire both PK and CK. This approach goes beyond traditional teaching methods that rely solely on verbal instruction and knowledge transmission. Instead, learners are encouraged to actively participate in the learning process through experimental learning. Picture interpretation refers to the cognitive capacity to generate, comprehend, and analyse visual representations, such as graphs. Abstract notation pertains to the use of symbolic representations, such as numbers or characters, which are employed and understood for problem-solving (Putri, 2007). To familiarise learners with the concept of measurement, an activity can be conducted where they actively engage in measuring length (distance) using tangible objects.

Learners can be provided with measuring equipment to measure various distances throughout the classroom and record their measurements. Learners can ascertain the proportions of the surfaces within the classroom, as well as the dimensions of the doors and windows. Utilising tangible items for learning purposes will enhance learners' understanding of Area and Perimeter. Instead of commencing with a formulaic explanation of area and perimeter, learners initially focus on comprehending the fundamental CK that underlies these mathematical concepts. Similarly, learners can develop both PK and CK by knowing that the area of a door frame can be determined by multiplying its height and width. This knowledge enables them to accurately compute the appropriate door size that will fit the door frame. The perimeter refers to the total distance encompassing the entire outer boundary of the door.

In the context of CPA theory, learners are instructed to use concrete objects, either improvised solids or real-life objects. This approach facilitates the development of geometric reasoning and critical skills. Additionally, the utilisation of technological resources, such as screen

displays of pictorial images, aids in the acquisition of conceptual knowledge by learners. In the subsequent phase, procedural knowledge skills can be acquired by employing concrete objects that learners have previously engaged with. In this stage, learners may be assigned the task of calculating specific metrics, such as the area or perimeter of said concrete objects. Consequently, the development of both CK and PK becomes interconnected. The acquisition of geometric knowledge is closely associated with the development of problem-solving skills. In this particularly crucial stage, learners are required to engage in practical application, enabling them to identify and articulate problems and generate and adopt appropriate procedures. Here, learners apply the recently acquired knowledge in novel contexts.

3.3. Van Hiele theory

This study is based on the Van Hiele (1986) model established by Dina van Hiele-Geldof and her spouse, Pierre Marie van Hiele. This theory highlights the existence of five distinct stages of geometric thinking that learners progress through as they transition from simply recognising a shape to being able to construct geometric proofs. Van Hiele, as referenced by Mason (2003), states that a learner's progression through each level is contingent upon the instruction provided, which is organised into five phases of learning. Paja (2005) further stated that the model elucidates the process by which comprehension of a new subject can evolve and confirms that learners progress sequentially through five levels of geometry learning. Vojkuvkova (2012) elaborates on the Van Hiele model in the following citation.

> The van Hiele theory describes how young people learn geometry. It postulates five levels of geometric thinking which are labelled visualisation, analysis, abstraction, formal deduction, and rigor. Each level uses its language and symbols. Learners or learners pass through the levels "step by step". This hierarchical order helps them to achieve better understanding and results (Vojkuvkova, 2012 p.1).

The above extract implies that Van Hiele provides a framework for how learners should approach the study of geometry. He establishes a hierarchical structure consisting of five distinct stages, each tailored to learners' specific needs and abilities at different grade levels, thereby ensuring optimal learning and teaching methods. The learning process comprises five stages, during which learners spend a specific amount of time acquiring knowledge, mastering skills, and advancing in terms of both level and content. As a result, they gradually progress to the next higher level. Van Hiele's geometric theory, introduced in the 1950s, posits that learners

progress through five distinct stages of understanding logically and sequentially. The following levels are:

- ***Visualisation*** refers to the ability to identify and label geometric figures, including 2D shapes like regular or irregular polygons and 3D objects such as prisms and pyramids.
- ***Analysis*** entails delineating the characteristics of shapes, such as their differences and similarities, the quantity and shapes of their faces, the number of vertices, and the number of edges.
- Informal Deduction, which involves categorising and making generalisations based on qualities.
- Deduction, which uses axioms and definitions to produce proofs.
- Rigour, which requires working with different geometrical systems (Alex & Mammen, 2012).

This study solely focuses on the utilisation of geometrical reasoning throughout the visualisation and analysis phases. The research primarily focuses on the first two stages, as the intermediate phase geometry depends on them. The initial two Van Hiele stages encompass all the evaluation prerequisites for the intermediate phase as outlined by the RNCS (Department of Education, 2019). Hence, the exit level outcomes for learners in the intermediate phase of the curriculum in South Africa may be associated with the expectations of Van Hiele level two (Masilo, 2018).

Level of visualisation: Learners at this level are instructed in geometry using visual representation. According to van Hiele (1986), Figures are evaluated based on their external appearance. A youngster can differentiate between a rectangle and a square based on their distinct shapes. At this level, the main objective is to instruct learners in identifying and manipulating shapes and other geometric elements, such as lines, angles, and grids, based on their visual characteristics. Siyepu (2005) states that learners typically acquire the ability to identify figures only based on their appearance and then compare them to a recognised template. Furthermore, a learner can discern, categorise, contrast, and manipulate geometric shapes (such as parallel lines, triangles, circles, and angles) based on their visual attributes. Thus, it is currently unattainable to comprehend the characteristics of a figure. Hence, decisions are determined by the learners' perception rather than their knowledge or thoughts. Consequently, at this point, a shape is classified as a square, cube, or rectangle solely based on its external characteristics (Siyepu, 2005). Coady and Pegg (1995) assert that there are a

minimum of three classes that exist at this level. During the initial session, learners are assessed based on their ability to establish cognitive connections between geometric forms and familiar objects. For instance, learners are expected to recognise that a cube may be likened to a box or a dice, a rectangle is essentially an elongated square, and parallel lines can be compared to a door (Coady & Pegg, 1995, p.90). Individuals are categorised into the second group when they can identify specific characteristics (as opposed to general qualities) of a shape, such as its sharpness, pointedness, flatness, and corners. However, they struggle to establish the required connections to comprehend the shape as a complete entity. When learners are capable of focusing their attention on only one specific geometric characteristic, they are classified into the third and most basic category (Coady & Pegg, 1995).

At the analysis stage, learners can break down shapes into their components and analyse their characteristics. However, they may encounter difficulty in establishing significant connections between various geometric forms (Mason, 2009). Ngishiri (2019) states that learners at the analytical level possess the ability to identify a square as a geometric shape characterised by four angles of equal measure and four sides of equal length. Every diagonal of a square divides itself into two equal and perpendicular parts. Nevertheless, individuals at this proficiency level may possess an inadequate understanding of the connections between various form features (Ngishiri, 2019). As per Siyepu's (2005) analysis, the process involves thoroughly assessing and scrutinising each constituent or characteristic of an object to comprehend its essence. Individuals perceive shapes as amalgamations of characteristics. They can perceive and recognise the attributes of geometric shapes, but they struggle to comprehend the connections between these attributes (Mason, 2003). Siyepu (2005) contends that the traits are seen as distinct entities independent of one another. An equilateral triangle is characterised by three equal angles, three axes of symmetry, and three equal sides. However, it is important to note that the presence of one of these features does not always imply the presence of the others. As per the RNCS guidelines, learners in the intermediate phase are expected to achieve a level 2 performance (DoE, 2002).

At the Informal Deduction level, learners understand the correlations between traits and geometric shapes (Masilo, 2018). They construct meaningful definitions. They can support their reasoning with clear and concise explanations. According to Mayberry (1983, p.59), we know logical implications and class inclusions at this point. For example, it is presumed that in an equilateral triangle, the angles of the triangle are also equal. Moreover, the relationships

between the figures are understood. An equilateral triangle can be considered an isosceles triangle since all the features of an isosceles triangle are present in an equilateral triangle (Siyepu, 2005). An equilateral triangle is a polygon with three sides that are all of equal length. Additionally, the number three is considered to be a minimum of two. According to this definition, every equilateral triangle is also an isosceles triangle. However, not every isosceles triangle is equilateral. According to the RNCS (DoE, 2002), Senior Phase learners must have achieved a minimum of van Hiele level 3. According to Siyepu (2005), learners at this level are capable of making basic deductions, but they do not fully understand the significance and purpose of formal deductions. Consequently, learners at this stage cannot solve geometry problems that necessitate a series of logical deductions.

Van Hiele's framework distinguishes five distinct stages of knowledge in geometry. Nevertheless, the study primarily focuses on the **initial two levels**, as the detailed explanation has been provided only for the first three levels. The rationale behind this approach is that the study examines the progression from the intermediate phase to the senior phase by comparing learners in grades 6 and 9. Although grade 9 learners are expected to have a level 3 proficiency according to the Van Hiele level of competence, the test specifically assesses the first two levels. The study compares their level 2 proficiency with that of the grade 6s. Acquiring lower-level thinking abilities is a prerequisite for learners to advance to informal deduction, which involves visualisation and analysis. Masilo (2018) states that the difficulty in comprehending geometry, known as cognitive load, arises when learners are taught advanced concepts before obtaining the necessary information and comprehension at a more basic level. Masilo (2018) states that multiple reports indicate that FET learners' understanding of geometry is at level 1, even throughout the FET phase, when they should be at level 5. Consequently, grade 9 is the starting point for the senior phase, during which learners are required to attain proficiency in the initial three levels of Van Hiele's cognitive development framework. During the FET phase, learners are required to compete in all five levels of knowledge. Therefore, the learner needs to possess the required understanding and knowledge at levels 1 and 2 (intermediate phase) and level 3 (senior phase) before progressing to the deduction level and rigour (levels 4 and 5), which are introduced in the FET phase.

The van Hiele theory is employed as a theoretical framework to analyse how educators in the intermediate phase convey PK and CK while teaching geometry. This study examines the performance of learners on assessments measuring PK and CK in the field of geometry,

specifically at level 2 of the Van Hiele knowledge hierarchy. The study employs a test designed based on the initial two levels of the van Hiele standard to assess learners' geometric understanding progression. Furthermore, Van Hiele directs the investigation toward discerning the impacts of PK and CK teaching. These implications arise from either the effective or ineffective utilisation of the van Hiele theoretical level of knowledge in the classroom.

Researchers from South Africa have undertaken several studies focused on Van Hiele's knowledge. A study by Mbusi (2019) examined the application the Van Hiele model to rectify learners' erroneous beliefs on transformation geometry throughout the foundational stage of education. Mbusi and Luneta (2021) conducted a study that examined the erroneous reasoning of pre-service educators in geometric translations using Van Hiele's phase-based instructional design. Similarly, some studies specifically concentrate on the senior phase, such as Kekana's (2016) research, which centres on the utilisation of Geo-gebra in the field of transformation geometry. Ngirishi and Bansilals' (2019) study on high school learners' understanding of geometric concepts is an illustration of the numerous investigations that have examined Van Hiele's Theory in the FET phase. Alex and Mammen (2015) discuss similar ideas in their publication titled "Geometry Lessons at the Senior Secondary Level Using van Hiele's Theory-Based Teaching." Bediako (2021) examined the outcomes of integrating technology into the Euclidean geometry curriculum for 11th-grade learners.

Moreover, further investigation has been conducted, such as the study by Alex and Mammen (2018), which examined learners' understanding of geometric terminology from the standpoint of Van Hiele's theory. Machisi (2021) examined the reflections of Grade 11 learners on their experiences in Euclidean Geometry. Machisi and Feza (2021) have chosen to concentrate on the Van Hiele Teaching method and its impact on the geometric proof skills of 11th-grade learners. However, this research aims to utilise the Van Hiele theory as a theoretical framework for the application of PK and CK in the intermediate phase of a geometry lesson.

3.4. Summary and conclusions

This chapter addressed the PK and CK frameworks and their impact on the study. The application of PK and CK in mathematics was further discussed. Further discussion revolved around the application of the Concrete Pictorial abstract (CPA) theory framework as a teaching lens in the classroom. The van Hiele theory was examined regarding its potential application

as a guiding framework in the geometry classroom. The discussion focused on the five Van Hiele levels of geometry, however, it was clarified that the study only focuses on the first two levels, which involve visualisation and analysis. This choice aligns with the content taught and learned in intermediate geometry.

CHAPTER 4: METHODOLOGY

4.1. Introduction

This chapter offers a thorough examination of the research design, and methodology utilised in this study. The discussion begins by explaining the research design and giving a thorough overview of the methodological approach used in this study. The chapter provides a more detailed explanation of the paradigmatic approach. In addition, the sampling and data collection methods are also discussed. The discussion covers important details about quantitative and qualitative methodologies for data analysis. Subsequently, the discussion turns to the reliability, validity, and ethical considerations of this matter. As a result, the chapter includes a discussion of the limitations and a brief overview of the data collection method.

4.2. Research Design

The research design functions as a dependable foundation for carrying out research (Sileyew, 2019). Sileyew (2019) emphasises the importance of selecting a research approach as it directly affects the effectiveness of data collection. However, it is essential to note that this decision is just one of several challenging choices that need to be taken throughout the study design phase. Commonly employed in the majority of studies is either qualitative or quantitative research. Currently, the mixed methods technique has gained significant appeal. Research design is a systematic and coherent methodology for acquiring necessary information (Patel & Patel, 2019). Qualitative research is centred on the qualitative approach, as suggested by its name. The analysis of human conduct is well-suited for this approach.

This study engages the methodology for observing several aspects of an individual's nonverbal communication, including body language, attitude, perspectives, emotions, and so on (Patel & Patel, 2019). Quantitative research uses statistical methods to investigate phenomena through the collection and analysis of numerical data. Kutscher and Parey (2020) argue that mixed methods research aims to enhance the understanding of a subject by combining qualitative and quantitative research methodologies. This involves incorporating qualitative and quantitative perspectives, as well as utilising diverse approaches for data collection, analysis, and inference. The integration of these methods leads to stronger and more persuasive validation of research findings. In addition, Creswell (1999) argues that this research approach allows a researcher to gain a nuanced understanding of complex events, while simultaneously offering quantitative explanations using visual representations, such as figures, charts, and basic statistical analysis.

Creswell and Poth (2016) contend that the researcher's epistemological and ontological perspective influences the design of the study. Therefore, the research design(s) need to be by both the research paradigm and the subject of investigation (Maxwell, 2010; Creswell & Poth, 2016).

This study employed a mixed methods approach. Creswell (2016) states that mixed methodologies in research involve the integration of qualitative and quantitative research methods. More so, a standard research study would typically involve conducting a survey to ascertain the opinions of participants regarding a certain issue. This would be followed by conducting in-depth interviews to gain insights into individual perspectives (Creswell, 2016). This study used a mixed methods approach to examine the utilisation of Procedural Knowledge (PK) and Conceptual Knowledge (CK) in the context of mathematics geometry. It aimed to investigate the impact of PK and CK on the teaching and learning of geometry in the intermediate phase. Although mixed-methods research incorporates both quantitative and qualitative data, the methods employed to collect and analyse this data may vary across different studies. Researchers have the option to employ a parallel, linear, or iterative approach (Kimmons, 2022). Consequently, the process of acquiring, analysing, and evaluating data for this research followed parallel mixed-methods designs. Kimmons (2022) defines parallel mixed-methods designs as the concurrent collection of quantitative and qualitative data for a single study.

In parallel mixed-methods each process in this design is undertaken independently from the others. An illustration of this would involve the concurrent implementation of surveys and focus groups, followed by the analysis of the outcomes from each method, and the subsequent presentation of the findings in a comparison and contrasting fashion (Kimmons, 2022). Both scenarios involve a data collecting and analysis process that includes distinct phases happening at the same time. The succeeding phase merges qualitative and quantitative findings to compare and evaluate them. This study utilised a parallel mixed method, wherein data was acquired and analysed through a qualitative method involving observations of geometry teaching and learning classes and semi-structured interviews. Conversely, a quantitative approach was utilised to administer a geometry test to assess learners' performance. Afterward, the results obtained from both qualitative and quantitative methods are merged, compared, and evaluated. To provide a clearer explanation of the parallel mixed methods technique, the study specifically utilises a convergent parallel mixed methods design. The convergent parallel design is a

frequently employed and highly accepted approach in the field of mixed methods research (Nataliya, Ivankova, Creswell & Plano Clark, 2016). The researcher employs a combination of quantitative and qualitative methodologies in this study to enhance the comprehensiveness and validity of their understanding of a certain occurrence (Nataliya et al., 2016). This implies that the objective of this method is to offer a comprehensive and dependable understanding of a phenomenon of interest by utilising both qualitative and quantitative methodologies.

To achieve optimal outcomes, it is recommended that researchers employ this approach, wherein they collect both forms of data on the same phenomenon simultaneously. By combining and analysing the distinct results, such as through comparison and contrast, researchers can make well-validated conclusions (Creswell & Plano Clark, 2011). The researcher employing this methodology initially utilises both quantitative and qualitative methodologies simultaneously (Quan + Qual), and thereafter endeavours to integrate the two sets of results by combining them (Nataliya et al., 2016). Figure 4.1 depicts the convergent parallel mixed techniques design.

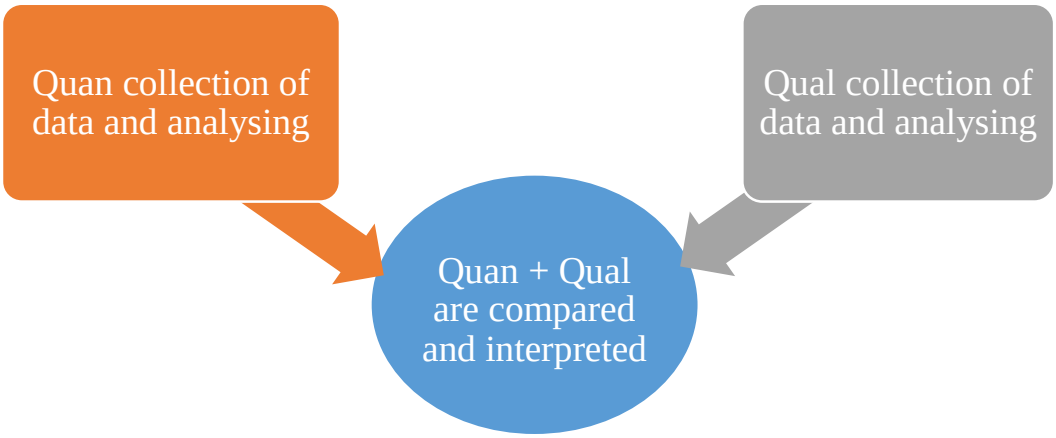


Figure 4.1: Convergent parallel mixed methods design, derived from Nataliya et al.'s work (2016, p.319).

In light of the above, the research approach employed is a convergent parallel mixed methods design, which involves the simultaneous collection of qualitative and quantitative data. Firstly, qualitative methods are employed to examine how educators convey PK and CK in the field of geometry, in response to research question (RQ) 1. The research employed a quantitative method to gather data for RQ2, which aimed to explore the learners' responses when they were assessed on their PK and CK. To address research question 3 and examine the impact of both

PK and CK on geometry teaching and learning, a qualitative methodology is utilised. Therefore, there is a convergence of the two sets (Quan and Qual) of outcomes, which is therefore analysed and compared.

4.3. Paradigm approach

Sefotho (2019) emphasises that paradigms have become a crucial term in the field of social science. While Kuhn (1970, 1974) introduced the term "paradigm," it has since been subject to several interpretations (Cram and Mertens, 2016). This worldview can be characterised by an epistemological position, a collection of widely accepted beliefs among scholars, or outstanding research. In educational research, the term "paradigm" is used to refer to a researcher's worldview or philosophical perspective on the universe (Mackenzie & Knipe, 2006). A paradigm is a framework of fundamental assumptions or beliefs about reality that shape a specific worldview. It encompasses core beliefs about the nature of reality (ontology), the relationship between the knower and the known (epistemology), and the methods used to acquire knowledge (Pietersen & Maree, 2016, p.52). At the core of every paradigm lies a fundamental assumption or a fundamental concept regarding the nature of reality, such as the human experience, the acquisition of knowledge, and the methodology of research. According to Lincoln and Guba (1994, p.15), paradigms can be described as a portrayal of our overall perspective on the world. Our actions in the world and our pursuit of knowledge are dependent on the paradigms we hold, as expressed by the saying "as we think, so we act." The paradigm functions as a lens through which reality is comprehended or as the fundamental principles by which reality is structured (Pietersen & Maree, 2016).

Pragmatism was the guiding paradigm for this research. Pragmatism has gained popularity among researchers of mixed methods, as evidenced by the works of Maxcy (2003), Johnson and Onwuegbuzie (2004), Morgan (2007), and Feilzer (2010). Contrary to common beliefs regarding the characteristics of knowledge, it is primarily focused on resolving tangible issues in the actual world (Feilzer, 2010, p8). The literature on mixed-methods studies frequently cites pragmatism as a fundamental philosophical premise (Tashakkori & Teddlie, 2003; Maxcy, 2003; Biesta & Burbules, 2003; Creswell, 2014; Johnson & Onwuegbuzie, 2004; Mitchell, 2018; Shah, Shah & Khaskhelly, 2019). Johnson and Onwuegbuzie (2007) agree that pragmatism is a sophisticated philosophical framework that combines quantitative and qualitative methodologies. According to Creswell (2014), pragmatism is a philosophical

approach that embraces several methods of collecting and evaluating knowledge. According to Maarouf (2019), pragmatism prioritises the effectiveness of a solution above all else. Primarily, the subject being discussed here is the pragmatic perspective on truth. Goldkuhl (2012) has categorised pragmatism into three unique components to elucidate the intricate relationship between the philosophy of action and knowledge (Maarouf, 2019, p.5).

> First, "functional pragmatism" or "knowledge for action" which means that the purpose of scientific knowledge is to improve action and make a practical difference. Second, the "referential pragmatism" or "knowledge about action" which implies that pragmatism describes the world in an action-oriented way. Third, the "methodological pragmatism" or "knowledge through action" which asserts that knowledge is created through action, or action is our source to knowledge about the world (Maarouf, 2019, p.5).

Based on the information provided, the research employed the pragmatism paradigm to improve action in mathematics and geometric performances, aiming to achieve a practical and quantifiable impact. Moreover, mathematical principles are crucial in our everyday existence; virtually all global technologies have advanced and depended on mathematics for progress, and mathematics has played a significant part in finding practical solutions to worldwide issues. Practicing pragmatism involves emphasising the importance of practical effectiveness. In the context of mathematical teaching, this entails experimenting with different teaching ways and implementing numerous algorithms to determine the most effective methods for learners. Aligned with the philosophy of pragmatism, which posits that knowledge is derived from action, this study actively investigated the concepts of PK and CK within the classroom setting. The researcher, seeking "knowledge for action," addresses the initial study question by examining *observations* that analyse how educators in the intermediate phase impart PK and CK when teaching geometry. The study utilised a *test* to investigate the second question, incorporating both PK and CK geometric questions, to ascertain the most effective method for applying both types of knowledge in the classroom. The research investigated the impact of teaching and learning PK and CK in geometry, specifically focusing on the third research question. Where *Semi-structured interviews* are applied as the lens of action serving as a means to understand the present configuration of the geometry classroom. Pragmatism is crucial in determining the most effective methods for teaching and learning geometry in both PK and CK settings, as it allows for the measurement of objective outcomes.

4.4. Sampling

The process of selecting a subset of individuals from a larger population to make inferences about the entire group is referred to as sampling (Bhardwaj, 2019). Sampling might be considered a nearly instinctive human action, where a component is observed to gather data about the entire entity (Corbetta, 2003). Maree and Pietersen (2016) classify sampling techniques into two main categories: probability approaches and non-probability approaches. Methods based on probability theory and random selection are referred to as "probability approaches," whereas those that do not utilise these principles are known as "non-probability methods." Therefore, only probability sampling methods fulfil the necessary criteria to employ the probability theory to make valid generalisations about the population (Maree & Pietersen, 2016). The study employs both probability and non-probability approaches to collect data, with a specific focus on random sampling, purposeful sampling, and convenience sampling as a means of information gathering. A simple random sample is a subset of a larger population that is selected in a completely random manner. (Thomas, 2020). When employing this sampling strategy, each member of the population has an equal probability of being selected. Purposive sampling is a research method that involves identifying and selecting potential participants or places based on their ability to provide insights into the research topic or primary phenomenon (Creswell, 2013). The convenience sampling method is used when population items are selected based on their easy and simple accessibility (Maree & Pietersen, 2016). Table 4.1 provides a concise overview of the data collection and sampling methods.

Table 4.1: A concise overview of the research design approaches for data collected and sampling

	Research question	**Method design**	**Sampling Method**	**Participants**	**Data Collection instrument**
Mixed methods	RQ1	Qualitative design	Purposive sampling	5 Grade 6 mathematics educators	Observations
	RQ2	Quantitative design	Purposive sampling (secondary schools) Purposive sampling (primary schools)	7 schools (2 secondary and 5 primary school classes)	Test
			Purposive sampling	5 Grade 6 classes (experimental group)	

			Random Sampling	2 Grade 9 classes (control group)	
	RQ3	Qualitative design	Convenience sampling	10 learners (Grade 6)	Semi-structured interviews
			Purposive sampling	5 educators (Grade 6)	

Table 4.1 provides a clear overview of the research design approaches used to collect data in the study. To provide more details on the utilisation of the research design for this research; the experimental group consisted of the grade 6 class, with a primary emphasis on grade 6 subject matter. The present study specifically focused on conducting classroom observations and interviews with grade 6 educators, as well as interviews with grade 6 learners. Grade 9 learners were exclusively utilised for assessment purposes, which was therefore compared to the assessment of grade 6 learners. The current study used purposive sampling to choose Grade 6 learners to assess their geometric knowledge. The motivation for selecting this class was that it has also been chosen for educator observations. Therefore, the study examined the approaches employed by educators in teaching geometry to their learners, as well as the performance of the same learners in geometry. The process of selecting grade 9 classes was carried out via random sampling. The selection of grade 9 classes through random sampling was determined by the prerequisite for all grade 9 learners to take mathematics as per DoE, as well as their prior completion of mathematics at grade 6. Therefore, grade 9 classes were chosen randomly.

For the qualitative aspect of the study, the researcher initiated a process by utilising purposive sampling to carefully select a group of *five educators*. The educators' selection was predicated upon their first-hand expertise teaching mathematics at the grade 6 level. The two secondary schools were chosen using a purposive sampling method. This method was initially selected because these schools receive learners from the five primary schools included in the study as their feeder schools. This was followed by a purposive sampling of the 5 primary schools. These primary schools were chosen because they serve as the feeder schools for the two selected secondary schools. For the qualitative section of the research, one grade 6 mathematics educator was chosen per school from each of the five primary schools. Furthermore, a convenience sampling method was employed to choose *ten learners* from the experimental group (grade 6). Moreover, the interviews involved the deliberate selection of the same educators who were chosen for observation and were also included in the interviews. For

learner interviews, this study exclusively collected data from a demographic (grade 6 class) that was easily reachable and available to participate. The researcher requested that the educator conveniently select two learners, one male and one female, to participate in the study for interview purposes.

For the quantitative aspect of the study, the researcher employed random sampling to select seven classes from two different grades, resulting in two distinct groups. The experimental group comprised five classes from the Intermediate Phase (Grade 6), while the control group comprised two classes from the Senior Phase (Grade 9). The decision to choose Grade 6 learners as the experimental group was based on the researcher administering the assessment to the learners only after the educator had taught the geometry topic and applied intervention strategies to assist the learners in acquiring the PK and CK in geometry. Grade 9 learners were used as a control group because they have already been taught the geometry topic in their maths classes from Grade 6 until their current grade. During the data collection process, specifically in the third term, they were also focusing on the subject of geometry. Their CK and PK progress is monitored and compared to that of the experimental group over some time.

Furthermore, the objective of these two classes is to ascertain the extent to which factors and mathematical geometry scores are correlated. The purpose of this was to include learners from both intermediate and senior phases to evaluate the enduring effects of PK and CK on learners' performance through the administration of a grade 6 test. The assessment selects seven schools through a random process, specifically five primary schools and two secondary schools.

4.5. Data collection methods

Creswell (2016) emphasised the importance of researchers evaluating the different data sources at their disposal carefully and assessing each alternative to identify the data sources that most effectively address the research questions or hypotheses. Qualitative research encompasses a broader spectrum of data compared to quantitative research (Creswell, 2016). Qualitative data can be obtained from diverse sources, such as in-depth interviews, field notes, public and private archives, as well as recordings encompassing sounds, images, and videotapes. Quantitative research has consistently maintained a stable collection of many types of information over time. Researchers collect quantitative data by using instruments that evaluate individual performance or attitudes (Creswell, 2006). In addition, they gather quantitative data

from many sources such as questionnaires, attendance logs, and reviews of previous performance.

The study utilised ***observation*** (see Appendix A), and employed semi-structured interviews as a form of data collection for qualitative designs. Chilisa and Kawulich (2015) classify observations into two main categories. On the one hand, participant observation involves actively participating in the study environment as both an observer and a participant. On the other hand, direct observation simply comprises observing without interacting with the subjects or objects being studied in the research setting (Chilisa & Kawulich, 2015). Hence, the study employed non-participatory direct observation, wherein the researcher refrained from any engagement with the participants. Instead, the researcher solely observed educators teaching geometry to learners in grade 6 classroom settings for ***RQ1,*** focusing specifically on how the educators were imparting PK and CK in geometry.

In a qualitative study, research interviews are commonly acknowledged as the primary and widely employed method of data collection (Qu & Dumay, 2011). This study aimed to investigate the impact of PK and CK on the learning and teaching of geometry. To address research question 3 ***(RQ3)*** The research utilised ***interviews*** (see appendix B), specifically semi-structured interviews, which involved a predetermined set of prompts for open-ended discussion and questions that arose during the interaction between the interviewer and the participants (DiCiccoBloom & Crabtree, 2006). Furthermore, the researcher employed a voice recorder to capture all the interview sessions with the participants, which were subsequently transcribed. The study commenced with semi-structured interviews, enabling me, as the researcher, to elicit additional data from the participants and foster unrestrained expression, unhampered by a predetermined set of inquiries. Therefore, this was presented as a method to delve further into the experiences of learners and educators in a geometry classroom.

To address research question 2 **(*RQ2*)**, the researcher administered a ***test*** (see Appendix C) to the learners. As the researcher, I prepared the test and memo, which were subsequently sent to the intermediate Mathematics subject advisor for validation before being distributed to the learners. This test evaluated the learners' grades based on their proficiency in both PK and CK in the field of geometry. More so, the participants were interviewed in both English and Xitsonga languages. This was done since Mathematics is commonly taught in both languages in Bushbuckridge schools, and the educators frequently move between the two languages

during class, as observed. During the interviews, when learners were facing difficulties with English conversation, questioning, and comprehension, I had to incorporate the Xitsonga language to facilitate their understanding of the questions. The participants' responses demonstrated that they exhibited improved comprehension and responsiveness when I code-switched to the home language (Xitsonga).

4.6. Data analysis

Babbie and Mouton (2004) argue that data should be processed and examined per the study's objectives to ensure the precision of the collected information, as well as its relevance to the research topic and research questions. Rossman and Marshall (2014) define data analysis as the process of organising and attributing significance to a collection of apparently arbitrary data points. The process of theorising about wide classes of knowledge involves obtaining overarching claims, however, it is not a linear process (Schwandt, 2007). Therefore, one may contend that data analysis involves comprehending the data produced through data collection methods and necessitates the application of logical reasoning to do the analysis. Creswell and Plano Clark (2007) emphasise that the traditional way of analysing data in mixed methods studies involves reviewing qualitative data in a qualitative manner and quantitative data in a quantitative manner.

Onwuegbuzie and Teddlie (2003) proposed that researchers should follow seven specific procedures when assessing quantitative and qualitative data using a mixed methods approach: The seven main processes involved in data analysis are: "(a) data reduction, (b) data visualisation, (c) data transformation, (d) data correlation, (e) data consolidation, (f) data comparison, and (g) data integration". In this study, quantitative research was conducted using descriptive analysis. The data was compared using the statistical package for the social sciences (SPSS). Descriptive statistics were used to calculate frequencies, measures of central tendencies (averages), and measures of dispersion (standard deviations, mean, coefficient of variation, minimum and maximum). These calculations were used to compare the scores of the control group and the experimental group in terms of their PK and CK, as well as their performance in 2D Shapes, 3D shapes, Measurement, and Van Hiele's level at the visualisation and analysis level.

The different types of charts serve specific purposes in data visualisation. Bar charts are ideal for representing discrete data, while histograms are better suited for continuous data. Pie charts are effective for illustrating proportions, and box plots are valuable for displaying the distribution of values for multiple variables in a single chart, including their range and medians (Kafle, Price, Cohen, & Kanan, 2018). The bar chart provides a clear visualisation of the highest and lowest values, facilitating easy identification. Furthermore, the bar chart allows for a straightforward comparison of the data. A histogram offers a concise summary of the frequency distribution derived from the collected data. The visual representation of proportions is readily apparent in a pie chart. A box plot is a graphical representation that summarises the distribution of data by displaying the minimum value, first quartile, median, third quartile, maximum value, and any outliers that may be present (Gopal, 2021). Consequently, the study has presented findings in different formats, including bar charts, histograms, pie charts, line graphs, and box plots as needed. The content of learners' responses sampled from the assessments underwent further analysis.

Regarding the qualitative parts of the study, the researcher employed thematic analysis to investigate the educators' observations. Thematic analysis refers to the process of identifying and interpreting recurring themes or patterns in qualitative data. This practice allows for the identification, interpretation, and reporting of commonalities in a dataset. It efficiently organises your data and offers a comprehensive analysis. Thematic analysis conducts a more thorough analysis and explores a wider array of aspects of the subject being investigated (Boyatzis, 1998). This analysis aimed to interpret and analyse how educators teach PK and CK in the context of geometry. In this research, I analysed and classified the themes associated with the study components described before. Consequently, the acquired data was analysed to identify themes or patterns. This enables the researcher to comprehend the impact of PK and CK on geometry learning and teaching, as well as address concerns from the literature that may be classified based on their similarities and differences. A thematic analysis was utilised to examine the effect of PK and CK in the field of geometry.

The further steps of data analysis for this investigation are illustrated in Figure 4.2. The diagram depicts the process of knowledge transmission and tracking from feeder primary schools to high schools. The data for geometry assessment has been obtained from two high schools that are also served by the same five primary schools. These primary schools were included in the study through learner geometry assessment, interviews with learners and educators, and lesson

observations. As part of the ethical considerations, the study assigned pseudonyms to the schools, learners, and educators who were participating. Upon analysis, the participants' pseudonyms were designated as follows: school 1 and school 2 represent the two high schools where the geometry assessment was conducted. The schools labelled A-E represent the 5 primary schools that are participating. The educators involved in both observations and interviews are consistent across the 5 primary schools. These educators are identified as educators A-E. Lastly, there are a total of ten learners participating in the interviews, with two learners from each participating primary school. The learners taught by Educator A, are named Learners A1 and A2, those taught by Educator B are named Learners B1 and B2, those taught by Educator C are named Learners C1 and C2, those taught by Educator D are named Learner D1 and D2, and those taught by educator E are named learner E1 and E2. These pseudonyms are employed to substitute the actual names of the participants in the study's analysis. The selection of these pseudonyms was done meticulously to ensure clarity and facilitate the identification of the participants throughout the study. Figure 4.2 illustrates the pseudonym employed in the data analysis.

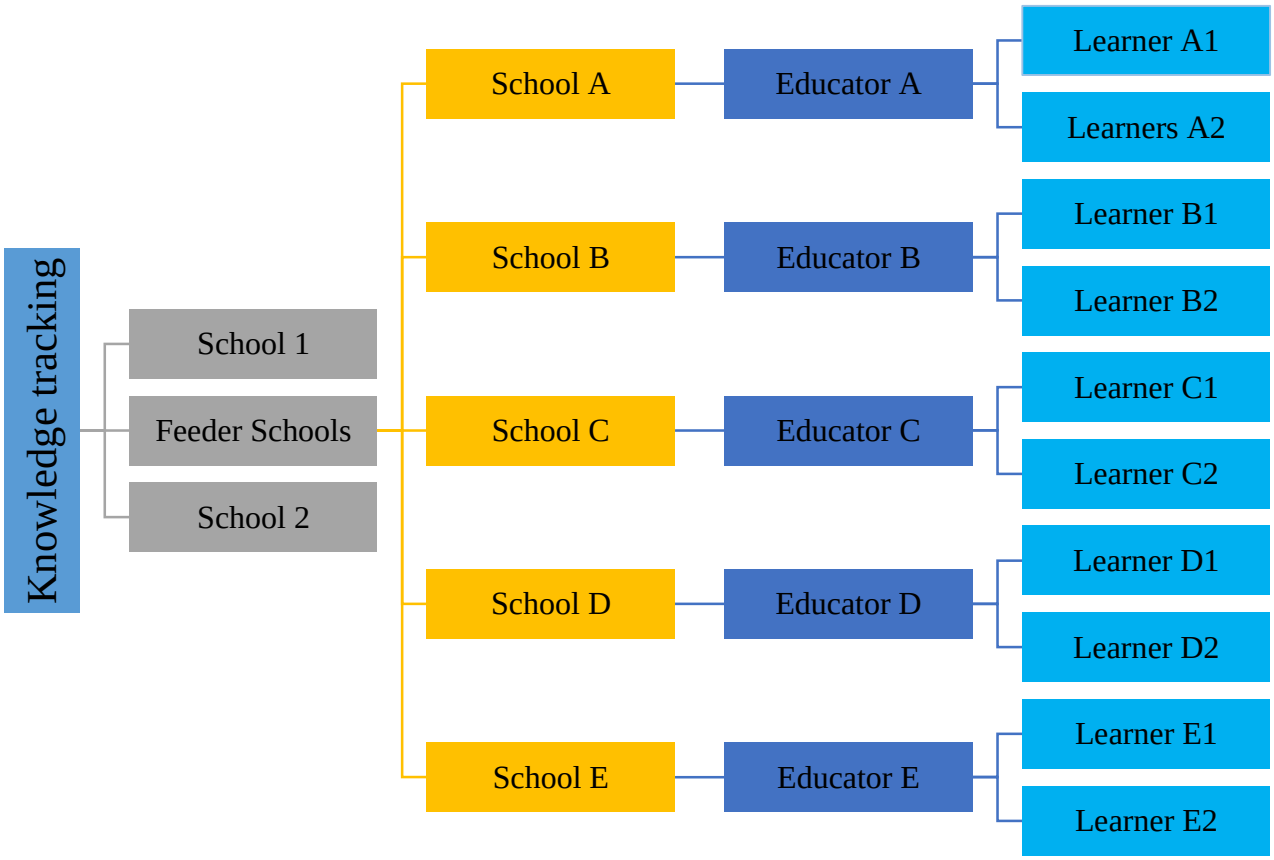


Figure 4.2: the pseudonym employed in the data analysis

4.7. Validity, Reliability, and Trustworthiness

DeCuir-Gunby (2008) emphasises the importance of addressing validity, reliability, and trustworthiness while undertaking mixed-methods research. Validity and reliability concerns in mixed methods studies should be addressed at both the quantitative and qualitative levels. To ensure the validity and reliability of mixed methods research, it is crucial to assess its truthfulness, applicability, consistency, and impartiality (Sale & Brazil, 2004). When examining the validity of qualitative research, concepts such as trustworthiness, dependability, conformability, and comprehension are commonly employed (Kvale, 1996; Maxwell, 1996). Validity, in this particular context, refers to the dependability and accuracy of conclusions made based on collected data (Eisenhart & Howe, 1990, p. 644). Quantitative research, in contrast, prioritises construct validity, encompassing factors such as truth value, credibility, consistency, and dependability (DeCuir-Gunby, 2008). Messick (1995) argues that the significance of test results and a comprehensive examination of data supporting scoring interpretations and their potential repercussions are crucial elements of validity. Construct validity is a framework that considers the practical application, meaning, and interpretation of test results.

Considering the above debate, the researcher opted to incorporate the following elements into the study to enhance the credibility of the qualitative aspect:

- Firstly, the participants are provided with a copy of the transcribed data to confirm the transcription's accuracy.
- Secondly, the study employed well-established research procedures that have already demonstrated their effectiveness in PK/CK studies.
- As a third step, the researcher collected data from schools acquainted with, primarily to ascertain their geographical placements. The objective is to acquaint the researcher with the ambience of the institution.

The validity of the quantitative data in the research is ensured through the following considerations:

- To enhance the accuracy and reliability of the research findings, the researcher acquainted himself with the scales commonly employed in quantitative research.
- Consequently, it is advantageous for the researcher to have a thorough understanding of the appropriate methods for assessing the reliability and validity of scales.

Furthermore, Shenton (2004) emphasised the significance of regular interactions between researchers and supervisors and the evaluation of data by peers in the academic community. According to Shenton (2004), this enhances the researcher's viewpoint by exposing them to a variety of perspectives. It is imperative to maintain a receptive mindset when studying and interpreting the findings of this study.

4.8. Ethical Consideration

Incorporating ethical principles and codes of behaviour into the process of gathering, examining, presenting, and disseminating data on study participants is an essential element of ethical research (Gallagher, 2005, p.8). These principles specifically address matters of personal privacy and dignity, the right of participants to revoke consent, privacy and anonymity, and safeguarding against harm (Qu & Dumay, 2011). Given this, the researcher took into account the subsequent ethical considerations while conducting his research:

- Before starting, the researcher obtained approval from the Ethical Committee in Education of the Faculty of Humanities at the University of Witwatersrand.
- The application was also submitted to the Mpumalanga Department of Education, which granted authorisation to carry out the study in their different schools.
- Consent was sought and obtained from all the schools involved.
- Each participant was provided with permission letters and consent forms.
- The learners who took part in the activity were given assent forms, while consent forms and permission letters were delivered to the parents.
- All participants were informed that their participation is optional and that they have the option to quit at any moment if they no longer choose to continue.
- The risk associated with participating in this study was minimal, as the participants were not subjected to any potential harm or dangers.
- The confidentiality of the research participants' names and the identification of the school was strictly maintained in all academic writing of the study.
- The study ensures the preservation of individual privacy in all published and written data outcomes.
- The research data is securely stored in a computer protected by a password, with exclusive access granted only to me. The data will be deleted within 3-5 years after the project is completed.

- This study utilises overt observation, where participants are conscious of being observed and are informed that it is for research purposes.

4.9. Study limitations

As anticipated, every study probably encounters limitations that are beyond the control of the researchers, and this study is no exception. Various limitations and constraints were encountered while implementing the data collection methods. The intended participants comprised 6 grade 6 educators and 6 grade 6 classes. However, due to ongoing renovations and construction at one school, only 5 educators and 5 classes were included in the study. This decision was made to prioritise the safety of the participants. While observing, it was occasionally challenging to coordinate with all of our educators to schedule observations for a specific teaching topic due to conflicting timetables. Consequently, I could only observe one educator during conflicting times. Only one lesson was observed for each geometry topic. Obtaining interviews with certain educators proved challenging due to their prior personal obligations, which required us to reschedule occasionally. It is important to note that the data collection occurred during the third term, which caused concern as time was elapsing and the Department of Education prohibits data collection during the fourth term. Nevertheless, we successfully conducted the interviews.

4.10. Summary and conclusions

This chapter has examined the research design employed in this study, which utilised a mixed methods approach. The concept of pragmatism was examined within the framework of the paradigm approach. The discussion involved the utilisation of both probability and non-probability sampling methods to select two high schools (specifically, two grade 9 classes) and five primary schools (grade 6 classes). Additionally, a total of ten learners and five educators were chosen to participate in interviews. The data collection methods for qualitative and quantitative research involved educators conducting educator observations, conducting semi-structured interviews with learners and educators and having geometry assessments discussed. The discussion covered the application of descriptive analysis and thematic analysis in both qualitative and quantitative aspects of the study. The topics of validity, reliability, and trustworthiness were deliberated upon. The discussion revolved around the maintenance of ethical considerations during the process of data collection.

CHAPTER 5: DATA ANALYSIS (RESULTS PRESENTATION)

5.1. Introduction

This chapter presents both the qualitative and quantitative data. The analysis is a response to **research question 1**, which asks how educators impart Procedural Knowledge (PK) and Conceptual Knowledge (CK) in geometry. The data collection process consisted of observing teaching lessons in five schools, explicitly focusing on **five grade 6 educators** teaching geometry content. The analysis approach utilised a thematic analysis method, whereby themes were identified and observed during the lesson observations. The subsequent analysis focuses on **research question 2**, which investigated how learners respond to the assessment of PK and CK in geometry. The research question was addressed by collecting quantitative data through administering assessments to participants from 7 schools. The participants included **5 Grade 6 classes** and **2 Grade 9 classes**. The Grade 6 class served as the *experimental group*, consisting of **271 participants**, while the Grade 9 class served as the *control group*, comprising **157 participants**. The analysis adopts a descriptive methodology, employing central tendency and variation measures to succinctly depict the data. **Research question 3** to be analysed is "What is the impact of applying PK and CK in the field of geometry?" The data collection process in this study involved conducting semi-structured interviews with 15 participants, consisting of **5 educators** and **10 learners**. The research question was analysed using thematic analysis.

In this study, grade 9 classes are used as the control group, with *School 1* and *School 2* being assigned as the codes for these classes. The experimental group consists of Grade 6 classes from *School A*, *School B*, *School C, School D, and School E*, each of which has been assigned a specific code. The geometry test participants were learners from both the grade 9 and grade 6 classes. The educator from *School A* is labelled as Educator A, from *School B* as *Educator B,* from *School C* as *Educator C*, from *School D* as *Educator D*, and from *School E* as *Educator E*. These participants are involved in both the lesson observation and the semi-structured interviews. Since the study focuses explicitly on grade 6 content, only educators who teach grade 6 were included as participants. The semi-structured interviews were conducted exclusively with grade 6 learners, who served as the experimental group, while grade 9 learners served as the control group. Two learners were interviewed from each school. The learners from *School A*, instructed by *Educator A*, were labelled as *Learner A1* and *Learner A2*. The learners from *School B*, instructed by *Educator B*, were labelled as *Learner B1* and *Learner B2*. The learners from *School C,* instructed by *Educator C,* were labelled as *Learner C1* and

Learner C2. The learners from *School D*, instructed by *Educator D*, were designated as *Learner D1* and *Learner D2* respectively. Similarly, the learners from *School E*, taught by *Educator E*, were labelled as *Learner E1* and *Learner E2*.

5.2. The imparting of procedural and conceptual knowledge in geometry

The **first research question** "How do educators in the intermediate phase **impart** procedural knowledge and conceptual knowledge when teaching geometry?" Investigated the teaching methods used by grade 6 intermediate educators in mathematics, specifically in the discipline of geometry. The data for this investigation was acquired through lesson observations. The observations were conducted on 5 grade 6 educators from 5 different primary schools. The lesson observation focused on concepts related to 2D shapes, 3D shapes, Area, Perimeter, and Volume. The subsequent data analysis and result presentation are consequently centred on these observed lessons.

In grade 6, learners should be taught Geometry using Van Hiele's level of visualization and analysis. They are expected to have reached the competency level of analysis. When teaching Geometry, educators are required to provide learners with resources such as geometric figures or similar things. These resources serve as examples for learners to relate to real-life objects. Meanwhile, learners at the visualization level should possess the ability to identify and manipulate shapes (such as squares and triangles) and other geometric arrangements (such as angles and lines) only simply on their visual appearance. Furthermore, the educator is supposed to ask questions and encourage learners to generate potential responses based on their existing knowledge. The educator thereafter presents a novel concept and facilitates learner engagement with it.

When teaching at level 2 (analysis) of the Van Hiele model, the educator is supposed to provide explanations based on the evidence collected during the probing process. The learners are required to analyse the forms by considering their attributes and the relationships between their aspects. This involves participating in brainstorming activities, drawing on past knowledge, and exploring new concepts. Similarly, the educator provides learners with a systematic approach and a set of inquiries to assist them in determining the characteristics of various geometric forms. Participants actively participate in tasks that result in the discovery of hidden relationships. Within the CPA framework, learners have the opportunity to

participate in activities that encourage discovery learning and incorporate learning through physical manipulation of tangible things, graphic depiction of actual object manipulation, and the use of abstract notation for problem-solving. Therefore, educators are expected to instruct geometry using tangible objects and visuals, incorporating real-life illustrations and experiences to facilitate effective teaching and learning. This approach also caters to the learning requirements of individuals who have a preference for visual learning. The variables observed in the application of PK and CK are the facilitation of pre-existing knowledge, acquisition of new knowledge, and integration of new knowledge.

5.2.1. 2D shapes teaching and learning

When conducting lessons on 2D objects, educators should utilise resources that connect the learners' existing knowledge with the current subject matter. When conveying new knowledge, teaching methods and materials should encourage learners to apply and comprehend the new knowledge in a meaningful way. The educator must use examples and questions/problems that enhance the acquisition of both PK and CK to apply knowledge effectively.

5.2.1.1. Grade 6 Prior Knowledge Facilitation

When teaching 2D shapes, educators attempted to activate prior knowledge by introducing the lessons in a way that connected to what the learners already knew. However, some educators failed to make a clear connection between the current topic and the learners' prior knowledge. I made the following observation while facilitating learners' prior knowledge of 2D shapes with *Educator A*. To introduce the concept of circles, the educator initiated the lesson by asking the learners what we refer to as a flat, round shape. The learners correctly identified the shape as a circle. However, the educator failed to introduce the topic of the lesson and explain its purpose to the learners, connecting it to their prior knowledge. On the other hand, *educator B* As a part of their pre-knowledge, the educator assigned a task to the learners. The task involved drawing various shapes such as rectangles, squares, parallelograms, triangles, and pentagons. Additionally, the learners were asked to identify the number of parallel lines, equal sides, right angles, and equal angles in each shape.

During the lesson with *Educator C*, she began by asking the learners to define polygons. The learners had some difficulty answering, so she proceeded to draw several shapes on the chalkboard and asked the learners to identify which ones were polygons. The learners correctly

identified the shape as a square, which is a type of polygon. They also recognised that a square has four sides that are all equal in length. The learner did not mention that it consists of four right angles. The educator asked the learners if triangles are considered polygons, but the learners had difficulty determining whether triangles are indeed polygons. The educator referred back to the definition of polygons and explained to the learners that polygons are shapes with straight sides that are closed. However, the learners still faced difficulties, so the educator provided additional explanations about polygons and switched to the learners' home language to ensure their understanding. When teaching about angles, *Educator D* began by showing a poster that displayed various types of polygon, ranging from triangles to octagons. Then, *Educator D* engaged the learners by asking them to identify and name each shape. The learners successfully named the shapes. The educator effectively connected the learners' prior knowledge to the topic of angles by highlighting that angles can be identified in 2D shapes. The observation on prior knowledge reveals that while educators do enquire about learners' prior knowledge, they are not effectively connecting it to the new topic.

5.2.1.2. Teaching New Knowledge in Grade 6

When introducing the topic of circles, *Educator A* began by providing a clear definition of what a circle is. The educator draws a circle and places a dot in the centre of it. An example of the distance around a circle shape is called the circumference. A line segment that passes through the centre of a circle, from one point on the circumference to the opposite point on the circumference, is called a diameter. The term "radius" is defined as half of the diameter. When an object has a single radius, it is referred to as a radius. However, when there are two or more radii, they are called radii. Learners and educators often lack essential teaching and learning resources, such as physical campuses and rulers, which are necessary to draw circles within a specific radius. The educator demonstrated to the learners how to draw a radius on the chalkboard, but the learners did not have any learning materials. Learners demonstrate a keen interest in helping the educator by drawing on the chalkboard. The *Educator B* drew a trapezium and asked the learners to identify the shape. The learners incorrectly stated that it was a parallelogram. The educator described the characteristics of the trapezium shape, including an illustration of the parallel lines present in a trapezium, the number of sides that are equal or unequal, and the number of angles that may be found in a trapezium. The educator utilised chalkboards, learners' textbooks, and chalk to teach the concepts of 2D shapes.

Educator C explains the concept of 2D shapes and defines them as shapes that possess both width and length. Nevertheless, this definition lacks clarity and can mislead learners, as there exist other 2D shapes that do not possess length and breadth. Additionally, it classifies 2-dimensional forms as polygons, defining them as shapes that possess two or more sides. She presents two types of triangles: those with three equal sides and those with two equal sides. Furthermore, it is inaccurate to state that polygons can be categorised as 2D shapes when the relationship between the two is the opposite: 2D shapes can be classed as polygons. Furthermore, she introduces quadrilaterals and clarifies that "quad" refers to the number four, indicating that a quadrilateral has four sides. In addition, learners were asked to identify types of quadrilaterals. The learner listed the following: rhombus, trapezium, parallelogram, kite, square, and rectangle. Instructed learners to identify the types of angles included in the quadrilateral. Learners provided their responses as acute, right, and obtuse angles. This lesson explores the shared characteristics of parallelograms and rectangles, specifically their possession of two pairs of opposite sides that are equal in length. A rhombus and a square both have equal side lengths. However, a rhombus has two obtuse angles and two acute angles, whereas a square has four right angles. A kite possesses two pairs of congruent sides. The educator employs a traditional teaching approach by relying on a textbook. The majority of the concepts are taught through reading directly from the textbook. She incorrectly distinguished between regular and irregular shapes. She defines irregular shape as a shape that is not straight, and regular as a shape that is not straight, which is considered improper. This indicates a deficiency in the educator's content knowledge.

Educator D's lesson observations included the introduction of many types of angle, such as right, acute, obtuse, straight, reflex, and revolution angles. The educator provided definitions for each of these angles. Utilising a poster as a visual aid to illustrate various classifications of angles. The educator illustrates the angles but fails to provide a comprehensive explanation of the underlying concepts. The learners showed a lack of engagement with the class due to the educator's reliance on traditional teaching methods, which involved describing concepts without actively integrating the learners in the process of discovery learning. The educator has just elucidated by providing definitions from the textbook and fails to offer practical illustrations from real-life scenarios. The observations indicate that many educators are still using traditional teaching methods. Additionally, some educators lack subject-specific content knowledge, and they rarely incorporate real-life objects that learners can easily relate to.

5.2.1.3. Grade 6 Learners' Knowledge Application

Educator A's lesson observations reveal that the learners encountered difficulty in distinguishing between the radius and diameter when applying their knowledge. The learners were unable to demonstrate that the radius is equal to half of the diameter. The educator reiterated that the diameter is twice the length of the radius. This indicates that the learners' CK was still deficient. During the observation of *Educator C*, it was evident that the learners were able to successfully identify and differentiate between quadrilaterals and triangles. However, they faced challenges when it came to identifying irregular and regular shapes, as well as correctly naming certain quadrilaterals. The learners encountered difficulties when attempting to draw 2D shapes with unequal sides. In *Educator D*'s session on knowledge application, the educator enquired about the angles that can be identified in the given polygons. Only a small number of learners showed a willingness to respond. The educator provided examples of angles that may be observed through the classroom windows, and the learners correctly identified them as right angles. Observations on knowledge application reveal that learners faced difficulties in applying their knowledge. As a result, educators had to employ repetitive explanations and practice to reinforce the learning process.

5.2.2. 3D shapes Teaching and Learning

Similar to 2D shapes when teaching about 3D shapes, educators should use materials that help learners relate their prior knowledge to the current topic, as 3D shapes are composed of 2D shape faces. When imparting new knowledge, it is important for teaching techniques and materials to promote learners' application and comprehension of the new knowledge in a meaningful manner, utilising solid shapes as a tool. To effectively apply knowledge, the educator should utilise instances and questions/problems that facilitate the acquisition of PK and CK. This can be achieved by including real-life settings and objects as references. The following observations were made on 3D shapes.

5.2.2.1 Grade 6 Prior Knowledge Facilitation

During the lesson on 3D shapes, *Educator E* began by showing the learners pictures of various shapes such as squares, circles, triangles, hexagons, pentagons, octagons, rectangles, and ovals. The learners were then asked to identify and name each shape. The learners began to provide answers. The educator asked them whether they were 2D or 3D shapes. When learners are asked why a certain shape is considered 2D, they often respond by saying that it is because the

shape is curved. However, others argue that it is because the shape has only one face. When the educator asked about the description of 3D shapes, the learners provided various answers. Some mentioned that 3D shapes have multiple faces, while others pointed out that you can observe angles and edges on these shapes. The educator provided scaffolding by asking the learners what the term is for the point where two points intersect. The answer provided by the learners was that it is a vertex. The educator demonstrated to the learners the concept of 2D shapes by using cut-out shapes. She explained that in a 2D shape, two lines meet at a vertex. Moreover, referring to solid shapes, the educator showed the learners that in a 3D shape, three lines meet at a vertex. The educator further explained the difference between 2D shapes and 3D shapes. The educator explained that 2D shapes, such as squares or circles, have two dimensions: length and breadth. These shapes are flat and can only be measured in terms of length and width. On the other hand, 3D shapes, like cubes or spheres, have three dimensions: length, height, and width. These shapes are not flat but rather have depth or thickness. They can be measured in terms of their length, height, and width. So, while 2D shapes are limited to two dimensions, 3D shapes have an additional dimension, giving them more depth and volume. The educator used real-life objects to illustrate concepts during their teaching. The educator utilized prior knowledge of 2D shapes to help facilitate understanding and make connections to 3D shapes.

Educator B began the lesson by instructing the learners to draw various 2D shapes, such as rectangles, squares, triangles, and pentagons, on the chalkboard. The educator explained that 3D shapes can be created by combining 2D shapes. The educator began by introducing the learners to the concept of a rectangular prism. "I will demonstrate how 2D shapes combine to create a 3D shape. I would like to point out that the faces of 3D shapes are composed of 2D faces." He demonstrated to the learners that a rectangular prism is composed of six rectangles. The lesson for *Educator A* began with the educator asking the learners to identify all the shapes they knew. The learners began to identify and name different shapes, including both 2D and 3D shapes. The educator asked the learners about the number of sides of the 2D shapes, and the learners provided various answers. The educator asked why these shapes are referred to as 2D shapes, but the learners had difficulty providing an answer. However, the educator's introduction of 3D shapes did not effectively establish a connection between the previously taught 2D shapes and the new concept.

To facilitate learners' prior knowledge, *Educator C*, the educator, began the lesson by drawing

2D shapes and enquiring about the names of these shapes from the learners. The educator failed to effectively connect the prior knowledge to the current topic being discussed. She just made revisions on 2D shapes and moved straight to 3D shapes without explaining or making connections to the learners. Workbooks were utilised as a teaching aid. *Educator D* started the lesson by introducing that today's lesson is 3d shapes mainly focussing on the vertices and edges, and no prior knowledge was observed. During *Educator E*'s second lesson observation on teaching 3D shapes, the educator instructed the learners to review the concepts of 3D shapes covered in the previous lesson. The learners responded that 3D shapes have many faces, while others mentioned that 3D shapes also have vertices. The educator assigned the learners a small activity in which they had to determine the number of faces, edges, and vertices on a 3D shape. To make this task more engaging, the educator provided tangible 3D shapes for which the models were created by the learners themselves as part of their homework.

5.2.2.2. Teaching New Knowledge in Grade 6

Educator A began the lesson by explaining that a 3D shape is so named because it possesses length, height, and width. She presented the concepts of prism and pyramids. The educator introduced the prism as such since it possesses identical bases. The educator used a 3D rectangular prism to demonstrate to the learners that a rectangular prism is named as such because it is composed of rectangular faces that form the entire shape. The educator demonstrated to the learners that a pyramid has an apex, which is a sharp point where the edges meet. She compared a cube to ice cubes, highlighting their relevance. While instructing on the characteristics of a pentagonal prism, she displayed the shape's base and inquired about its name. The learners promptly identified it as a pentagon. She inquired with the learners about the name of the 3D shape they were having difficulty with. She then informed them that it is called a pentagonal prism due to its base. By presenting a hexagonal prism and a heptagonal prism and inquiring about the bases, the educator enabled the learners to provide accurate responses. While discussing the qualities of a square pyramid, the educator clarifies that its square base is what classifies it as a square pyramid. A pentagonal pyramid is a geometric solid with a base that consists of five sides and a pointed apex at the top. While discussing the characteristics of a triangular pyramid, she emphasized that it is commonly referred to as a tetrahedron.

The lesson for *Educator B* was characterised by a high level of engagement between the educator and learners, resulting in a lesson that focused on discovery learning. The educator

instructs the learners to sketch a cube on the board. The learner accurately depicted the cube and successfully labelled the vertex, edge, and face on the 3D shape. However, they had difficulties with the spelling. The concept of teaching vertex involves instructing and demonstrating to learners that a vertex is the corner of the book. The educator instructs learners to utilize their textbook as a rectangular prism. The learners were given the task of drawing a net of a rectangular prism on the board and then counting the number of faces on the net. The learner mistakenly drew a cube instead of a rectangular prism.

Educator C began the lesson by instructing the learners to turn to a specific page in their workbooks that displayed 3D objects. She drew various 2D shapes and asked learners to identify the objects and describe them. Learners responded as a 2d shape. She presented learners with 3D objects and inquired about the different 3D objects featured in the workbook. The educator began explaining to the learners how to identify a 3D object based on its base. The educator began questioning the learners about the 3D shapes shown in the workbook. The educator explains to the learners that a cube is a special prism, characterised by its flat faces and six identical faces. Prism - all faces are flat with two identical ends. The educator informs the learners that the triangular pyramid is also known as a tetrahedron and holds a unique status among pyramids. The educator fails to explain the naming of specific shapes as special pyramids or prisms.

The lesson on 3D shapes taught by *Educator D* began by examining the vertices and edges. The educator explained the concept of 3-dimensional shapes, which involve length, width, and height. Let's dive into the topic of Pyramids and prisms. According to his explanation, Prism has two identical ends. The educator posed a question about the number of sides in a pentagon, and the learners promptly responded with the correct answer of 5. The cube is a special type of prism that was explained by the educator. The educator presents learners with a three-dimensional cube. Inform them that it has 6 faces. The educator does not provide any information about what the faces of the cube are, he encourages learners to explore and discover it on their own. However, learners can correctly identify them as squares. While discussing the qualities of pyramids, the educator explains the concept of a pyramid but does not provide a detailed explanation of why it is referred to as a square-based pyramid. During the lesson, the educator focused on demonstrating the attributes of rectangular prisms by using visual examples. The educators showed the learners a 3D Pentagonal prism when the learners were asked about the 3D object, they identified it as a pentagon. However, the educator corrected

them by stating that it was a pentagonal prism. However, the educator did not explain as to why it is called a pentagonal prism. The use of nets in teaching allows learners to visualise how 3D shapes are formed and understand their structure. Figure 5.1 below illustrates the 2D 3D geometric solids with folding nets.

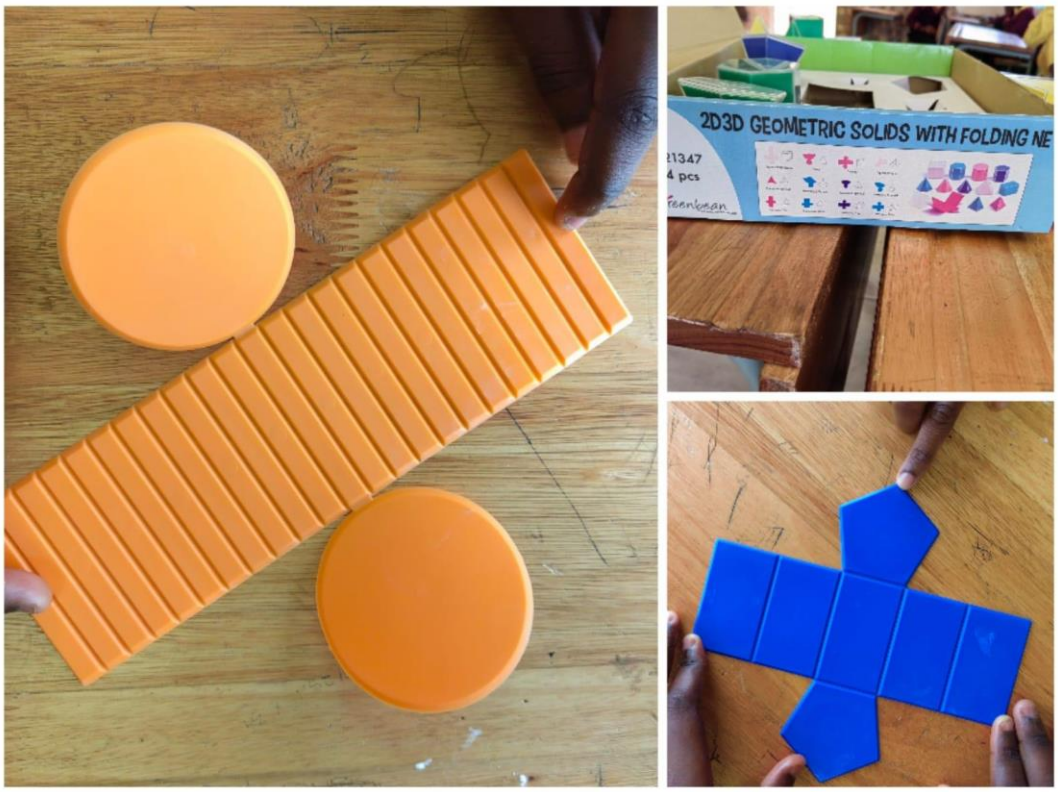


Figure 5.1: Transformation of 2D folding nets into 3D geometric solids in grade 6 (*School D*)

The picture above depicts the 3D geometric solids with folding nets that *Educator D* used to effectively teach learners about 3D shapes. The image on the left displays a foldable net of a cylinder. The educator demonstrated to the learners how a cylinder can be created using two circles and a rectangle. The educator demonstrated to the learners that a pentagonal prism is created by combining two pentagons and five rectangles, as shown in the bottom right frame. The picture in the top right corner features a box filled with a variety of 3D geometric solids, which can be utilized for teaching about 3D shapes.

For her lesson on 3D shapes, *Educator E* brought the learners to the mathematics lab. The video she played showcased a wide range of 3D shapes, accompanied by detailed descriptions and a narrated voiceover. The educator would play the video, pausing it at intervals to engage the learner in discussions and pose questions about the shapes, before continuing to play. The

educator would select a learner to identify a 2D shape within the 3D shape shown on the visuals. Furthermore, she encouraged learners to observe and identify the point where two faces meet, known as an edge. She asked the learner about the point where two or more straight lines meet. The learner correctly answered that it is called a vertex. Another learner was chosen to demonstrate the vertex to the rest of the learners. The educator would ask the learner to identify objects on the screen, such as a vertex or a cylinder. The learner would then be instructed to physically point to their answer on the screen. The educator requested that the learners demonstrate a three-dimensional object in the classroom. One of the learners confidently pointed to a cupboard. The video effectively demonstrated various 3D shapes, including solid shapes and real-life objects. Take, for instance, The Great Pyramid of Giza in Egypt. The video effectively demonstrates various 3D shapes, such as balls and books, and connects them to real-life objects that learners encounter in the classroom. The lesson showcased a remarkable level of engagement and active participation from the learners.

5.2.2.3. Grade 6 Learners' Knowledge Application

For *educator A,* a few examples promoted CK. The learners were only shown 3D objects and their properties, but there were no examples given to show the relevance of these objects in their everyday lives. *Educator B*'s lesson focused on introducing the learners to the concept of a rectangular prism. Demonstrating the formation of a 3D shape using 2D shapes. Additionally, the learners can understand that the faces of 3D shapes are composed of 2D faces. However, learners found it challenging to distinguish between a square-based pyramid and a triangular pyramid. The lesson fostered active participation and interaction between the educator and learners, promoting a discovery-based approach to learning. During the lesson, *Educator C* focused on discussing the 3D objects displayed on the page. However, it did not provide any concrete examples or real-life connections to help the learners apply their new knowledge. Simply learners memorising the concept of 3D shapes. There was no connection to real-life problems. The assessment required learners to apply both PK and CK. However, the teaching approach focused solely on promoting PK and did not effectively foster CK.

The lesson of *Educator D* involved showcasing different 3D solid geometric folding nets to the learners. He also provided them with the names of the 3D shapes, along with information about their number of faces, edges, and vertices. The educator had to teach objects at their disposal, but they primarily relied on the traditional method of imparting geometry knowledge. This approach involved the educator doing most of the talking, which unfortunately led to boredom

among the learners. As a result, some learners began to misbehave and engage in conversations with their friends during class. The educator primarily relied on questions and problems that focused on PK. The lesson for *Educator E* involved teaching with a screen projector. The content included videos showcasing real-life objects. The engagement in the class fostered a deep understanding of both CK and PK. The 3D shapes were described in great detail, and real-life geometric examples were seamlessly connected to them. The assessment highlighted the importance of PK and CK. The learners were tasked with understanding the concept of 3D shapes and their relevance to real-life situations, as well as their various attributes.

5.2.3. Teaching and Learning of Measurement

The study focuses on teaching the concepts of Area, Perimeter, and Volume in measurement. The analysis includes observing how prior knowledge is facilitated, new knowledge is taught, and knowledge is applied in the context of Area, Perimeter, and Volume. When teaching prior knowledge, the educator should ask questions and encourage learners to provide answers based on their existing knowledge. Similarly, when introducing new knowledge and promoting knowledge application, the educator should present a new concept and allow learners to engage with it. Additionally, the educator should provide learners with a method and relevant questions to help them understand how PK and CK can be applied to measurement concepts.

5.2.3.1. Prior knowledge Facilitation Grade 6

Educator A began the lesson by sketching rectangles and squares on the board. Then asked the learners to identify the shapes. The learners provided their answers. The Educator recorded the dimensions of the shapes and inquired about their measurements, to which the learners provided their responses. The educator connected the learners' past knowledge of 2D shapes with the concepts of Perimeter and Area. The educator utilized traditional teaching tools such as a chalkboard and chalk to write and illustrate concepts. *For educator B,* the lesson lacked the incorporation of prior knowledge and instead began by introducing the topic of measurement. *Educator C* started the lesson by providing definitions of the concepts of Area and Perimeter. Participants were not provided with a chance to contribute based on their existing knowledge. The lesson did not connect with any previous knowledge. Teaching tools like workbooks, chalks, and chalkboards were commonly used in the classroom. The lesson for *Educator D* began by providing definitions of Area, Perimeter, and Volume, but it did not

effectively tap into the learners' existing knowledge. No utilization of pre-existing knowledge was observed. *Educator E* started the lesson by engaging the learners in a discussion about their understanding of the concepts of Area, Perimeter, and Volume. The learners made an effort to answer, although they faced some difficulties. The educator provided support and introduced the topic of Area, Perimeter, and Volume. The observation of the absence of prior knowledge utilization in measurement lessons indicates that educators are failing to effectively incorporate learners' existing knowledge while teaching measurement topics.

5.2.3.2. New Knowledge Teaching in Grade 6

The teaching of new knowledge began with *Educator A* drawing a rectangle measuring 3 cm in length and 2 cm in width. The learners were asked about the length and width. The educator effectively conveyed the importance of learning how to calculate the Area and Perimeter. The educator clearly explained the Area formula and gave a concise definition of what an Area is. The educator provided the formula$A = LxB$, but no examples were covered and commenced with the calculation algorithms for determining the Area of a square with dimensions of 3cm by 2cm, resulting in a total of 6cm squared. However, the educator failed to explain the presence of the square in the centimetre. Invited learners to participate in a role-playing activity where they can demonstrate their skills in calculating the Area of both a rectangle and a square. The educator explained that Perimeter is the distance around a 2D shape, however, did not make a practical example of it in real life. The formula for calculating the Perimeter is P = l + b + l + b, the educator explained. By applying this formula to the given measurements of 3cm, 2cm, 3cm, and 2cm, we find that the Perimeter is equal to 10cm. The educator provided a clear explanation to learners about calculating Perimeter, emphasising that there are no square units involved since it involves calculating the sum of all four sides. However, this explanation is wrong; the exclusion of square units in the calculation of Area is not due to the consideration of the four sides in the Perimeter. *Educator A* primarily focused on teaching the calculations involved in determining Volume. The educator explained to the learners how Volume is measured in cubic units but did not delve into the reasons behind this measurement.

When teaching the Volume, *Educator B* emphasised the relationship between Volume and capacity. He provided a clear explanation of how Volume is calculated: by multiplying the length, breadth, and height. The learners lacked discipline, engaging in activities such as eating and chatting with friends during class, showing an apparent lack of focus. The educator presented learners with a 3D shape made up of cubes and instructed them to determine the

number of cubes for their length, width, and height. The learners diligently counted the cubes and provided the educator with the answers. After assigning the learners the task of calculating the Volume, they completed the calculations. However, the learners seemed to be wholly engrossed in their activities during class. It appeared that only a few of them were actively engaged and focused. The educator struggled with managing the classroom effectively.

In the same vein, when introducing the concept of Area and Perimeter, *Educator B* began by engaging the learners in a discussion about the definition of Area. The concept of Area was explained as the surface of an object. The educator clearly explained how Area is measured in squares, using examples of square centimetres and square meters. However, the educator failed to provide learners with an explanation of the origin of the squares. The educator asked learners to sketch a rectangle with dimensions of 5cm by 3cm. Participants actively engaged in the activity under the educator's guidance and completed it. In addition, learners were tasked with calculating the Perimeter and Area of that shape. The Perimeter of the rectangle was calculated correctly using the formula P = 5cm+3cm+5cm+3cm = 16cm. The Area of the square is A = 5cmx3cm = 15cm squares. The educator instructed the learners to draw a square with a side measurement of 6cm. Next, learners were tasked with calculating both the Area and Perimeter of the square. The classroom was equipped with floor tiles. The educator requested that the class determine the length of the room using tiles, while another learner focused on calculating the width of the tiles. The educator assigned the learners a task to measure and calculate the Area of the classroom.

Educator C began the lesson by explaining the concept of Perimeter as the distance around a 2D shape. The educator illustrated how to calculate the Perimeter using rectangles and measurements. She provided examples where she gave the learners specific measurements of 10cm for length and 5cm for width. Learners successfully solved for the Perimeter. She explained that an Area is a measure of the space enclosed by a Perimeter. During the explanation of calculating the Area, she provided an example and mentioned that the answer was in square centimetre squares. However, she did not elaborate on why the Area is measured in this unit. The educator began the lesson on Volume by directing the learners to their workbooks. She provided an example of calculating a Volume to help illustrate the concept. The educator failed to adequately demonstrate the correct method for calculating Volume, instead presenting the learners with inaccurate calculations. She presented learners with visual representations from different angles. She illustrated that the Volume was calculated by adding

together the Areas of each side of the rectangular prism, resulting in an incorrect method of calculating the Volume and getting the wrong value.

Educator D began the lesson by inquiring about the concept of Perimeter from the learners. Learners made an effort to respond but encountered some difficulties. The educator explains to the learners that the Perimeter of a 2D shape is the distance around it. To help them understand, he provides real-life examples, like using a trundle wheel to measure the Perimeter of the school. The educator proceeded to explain the calculations for finding the Perimeter, demonstrating the formula. $P = L + L + B + B$. However, the lesson involved the educator lecturing rather than actively involving the learners in the lesson. Consequently, learners become disinterested and begin to cause disruptions. The educator introduced the area concept by explaining it as a space within a 2D shape. The formula for calculating Area, A=lxB, was introduced and explained. The concept of Area being measured in square units was demonstrated through a calculation example, where the answer was given in square centimetres. However, the reason behind this unit of measurement was not provided. Here is an example made for the learners: When purchasing tiles for a classroom, it is important to measure the length and breadth of the space. By calculating the Area, we can determine the exact number of tiles required. Moving to Volume, the educator explained that Volume refers to the space contained within a three-dimensional shape. The educator began the lesson by engaging the learners in a discussion about the concept of Volume. The learners remained silent, so he proceeded to provide them with the definition. The educator introduces the formula as $V = lxbxh$ moreover, mentions that it is measured in cubic units, but does not provide an explanation for this.

Educator E's lesson of teaching Perimeter began by prompting the learners to define Perimeter. The definitions were provided by the learners. The educator emphasises that the Perimeter is only calculated for 2D shapes. She provides a clear explanation of the concept of finding the total distance around a 2D shape. She simplifies it for learners by emphasising the importance of adding all the measurements around the shape to find the total. Learners begin to incorporate new knowledge and arrive at the solution. The educator effectively guided the learners in identifying the missing measurements of the sides of a rectangle with a given single width and length. The learners successfully demonstrated their understanding by using equal opposite lines. Both learners and educators began working on developing the formula for calculating the

Perimeter. The educator began the lesson on the concept of Area by asking the learners to provide their definitions, which they did. The educator began sketching the surface of the classroom. She clearly states that Area focused on two sides of a 2d shape. Therefore, the formula allows for calculating the length and breadth of a 2D shape. The educator and the learners collaborated to solve a calculation involving Area, with the answer provided in square centimetres.

In line with the above, the educator then inquired about the reason for squaring it. The learners faced some difficulties, but the educator clarified that the square also represents the units being multiplied. When transitioning to the topic of teaching Volume, the educator inquired about the definition of Volume. The explanation was a bit challenging, but she supported the learners as they discussed 3D shapes and their Volume. The educator used a visual aid to demonstrate how to calculate the Volume of the box. The box was placed in a rectangular prism, and the educator requested a learner to come up to the board and illustrate the 3D shape. She explained the differences between 2D and 3D shapes to the learners using demonstrations. The learners were taught about the three sides of an object, namely length, width, and height. They were shown that the formula for calculating the Volume is $V = L\ x\ B\ x\ H$. The educator instructed the learners to estimate the measurements of the rectangular prism independently. However, the educator made a mistake by assigning identical measurements to the height and breadth, indicating a failure to distinguish between the two dimensions of the Volume.

5.2.3.3. Grade 6 Learners Knowledge Application

When it comes to applying knowledge *Educator A* utilised a technique in which learners were required to repeatedly sing the formula to achieve mastery. However, this method did not effectively foster CK. The educator failed to connect the content to real-life problems. The assessment primarily focuses on PK, as it includes questions that require learners to apply algorithms to calculate area, volume, and perimeter. The learners exhibited a positive response to the calculations using volume. However, the concept of Volume was not adequately elucidated in a manner that fosters a deep understanding of volumes. Regarding *educator B*, the learners had a positive response to the application of knowledge. The educator effectively linked the measurement concept to real-life situations inside the classroom setting. Despite the crowded nature of the class, the educator faced challenges with class management. Some learners were not fully engaged in the lesson. However, those who were attentive responded enthusiastically. On the other hand, *Educator C* defined Perimeter, Area, and Volume, but

failed to offer a conceptual explanation to the learners on the purpose and rationale for calculating perimeter or area. The concepts of volume were not adequately explained, as the focus was primarily on calculations. The lesson lacked practical application to real-world problems. However, the Assessment did enhance learners' PK and CK.

Educator D provided practical examples, like using a trundle wheel to measure the perimeter of the school. Learners were having difficulty applying their newly acquired knowledge to problem-solving tasks. The educator successfully connected the new content to real-life situations, although there was limited learner engagement in the learning and teaching process. The assessment evaluated both PK and CK, however, learners encountered difficulties in demonstrating their performance. Lesson for *Educator E* The lesson on teaching measurement successfully incorporated real-life situations, fostering active participation between learners and the educator. This approach not only enhanced CK but also developed PK. The learners showed a positive response to the assessment in the class.

5.2.4. Observation summary for teaching geometry.

The observation of prior knowledge in the instruction of 2D shapes indicates that although educators do prompt learners to share their existing knowledge, they are not adequately establishing a connection between this prior knowledge and the new subject matter. The findings pertaining to the instruction of 2D shapes suggest that a significant number of educators continue to employ traditional teaching approaches. Furthermore, certain educators exhibit a deficiency in subject-specific content knowledge, and they seldom integrate tangible objects from real-world contexts that learners can readily connect with. The analysis of knowledge application in the context of 2D shapes indicates that learners encountered challenges when attempting to apply their acquired knowledge. Consequently, educators were compelled to utilise repetitive explanations and facilitate practice sessions in order to strengthen the learning process. In the evaluation of measurement instruction, it was noted that there is a lack of application of prior knowledge in measurement lessons, suggesting that educators are not effectively integrating learners' existing knowledge when teaching measurement topics. The educators place significant emphasis on instructing the concepts of measurements through explanations rather than actively engaging learners through practical exercises or providing visual representations of concept objects or pictures. This approach does

not allow learners to establish connections with real-life experiences and enhance their conceptual understanding.

5.3. Learners' response to procedural and conceptual knowledge assessment in geometry (quantitative part of the study)

This section examines the data gathered from assessments conducted on learners in five primary schools and two high schools, to address study **research question 2**: "How do learners respond to PK and CK assessment in geometry?" The analysis adopts a descriptive approach. The discussion begins by examining the specific performances of schools in 2D shapes, 3D shapes, and measurements involving grade 6 (experimental group) and grade 9 (control group). We then delve into the overall geometry performance of learners in both primary and secondary schools. They involve the grade 6 (experimental group) and grade 9 (control group).

5.3.1. Grade 6 and 9 Learners response to 2D shape assessment

This section focuses on the learners' performance in assessing 2D shapes. The learners were assigned a task (see Appendix C) that focused on the characteristics of 2D shapes. This included identifying and labelling the shapes, determining if they were regular or irregular, labelling the angles inside the shapes, and describing the sides of the shapes. The 2D shape comprised a triangle, hexagons, and a quadrilateral. The work consisted of a 12-mark assessment, with each mark granted for specific criteria. One mark was given for correctly naming the 2D shape, another for determining if it was regular or irregular, one for recognising the type of angle displayed, and one for describing the number of sides of the shapes (See Figure 5.5). The performance summary for 2D shape in the control group is presented in Table 5.3.

Learners performance 2D shape (Control group with Grade 9)

		Frequency	Percent	Cumulative Percent
Marks obtained	0	2	1,3	1,3
	1	1	,6	1,9
	2	3	1,9	3,8
	3	8	5,1	8,9
	4	10	6,4	15,3
	5	18	11,5	26,8
	6	26	16,6	43,3
	7	41	26,1	69,4
	8	26	16,6	86,0
	9	10	6,4	92,4
	10	6	3,8	96,2
	11	6	3,8	100,0
	Total	157	100,0	

Table 5.3: Learners' performance in the control group regarding the topic of 2D shapes

The table displays the performance of learners in 2D shapes for the control group. The task was graded out of 12 marks. In the table, we observe a range of marks from 0 to 11, with the maximum achievable mark being 12. This indicates that there are learners who scored between 0 and 11 marks. The frequency distribution reveals that 2 learners attained the minimum score of 0, a solitary learner obtained a score of 1, 3 learners achieved a score of 2, 8 learners attained a score of 3, only 10 learners achieved a score of 4, 18 learners who attained a score of 5, 26 learners who obtained a score of 6, and the highest frequency of 41 learners who achieved a score of 7. 10 learners scored 9, 6 learners who scored 10, and an additional 6 learners who scored 11. The cumulative percentage indicates that 43.3% of learners achieved a mark below the average in 2D shape, while 56.7% fared above average. The results indicate that the performance of learners in the control group in properties of 2D shapes is above average. The following table presents a concise overview of the experimental group's performance concerning the attributes of 2D shapes.

Learners' performances 2D shape (an experimental group with Grade 6)

		Frequency	Percent	Cumulative Percent
Mark obtained	0	7	2,6	2,6
	1	3	1,1	3,7
	2	12	4,4	8,1
	3	12	4,4	12,5
	4	18	6,6	19,2
	5	30	11,1	30,3
	6	50	18,5	48,7
	7	42	15,5	64,2
	8	51	18,8	83,0
	9	29	10,7	93,7
	10	12	4,4	98,2
	11	4	1,5	99,6
	12	1	,4	100,0
	Total	271	100,0	

Table 5.4: Learners' performance in the experimental group regarding the topic of 2D shapes

Table 5.4 presents learners' performance in the field of 2D shape for the control group. Within the table, we may perceive a spectrum of scores spanning from 0 to 12. This suggests that there are learners who obtained scores ranging from 0 to 12 marks. The frequency distribution shows that 7 learners scored the minimum of 0, 3 learners who scored 1, 12 learners who scored 2, 12 learners who scored 3, only 18 learners who scored 4, 30 learners who scored 5, 50 learners who scored 6, 42 learners who scored 7, and the highest frequency of 51 learners who scored 8. A total of 29 learners achieved a score of 9, while 12 learners achieved a score of 10, and 4 learners achieved a score of 11. There was only one learner who obtained a perfect score of 12. The cumulative percentage reveals that 30.3% of learners scored below the average in 2D shape, while 69.7% performed above average level. The results suggest that the learners in the control group have achieved a performance level in characteristics of 2D shapes that are significantly higher than expected. The comparison of 2D shape performances between the control and experimental groups indicates that learners in the experimental group (grade 6) earned a significantly higher percentage of 69.7% over the average, compared to the control group (grade 9) who achieved an average percentage of 56.7%. The figure below illustrates the activity involving a 2D shape that was given to the learners. It provides a glimpse into how the learners attempted to answer questions on the attributes of 2D shapes.

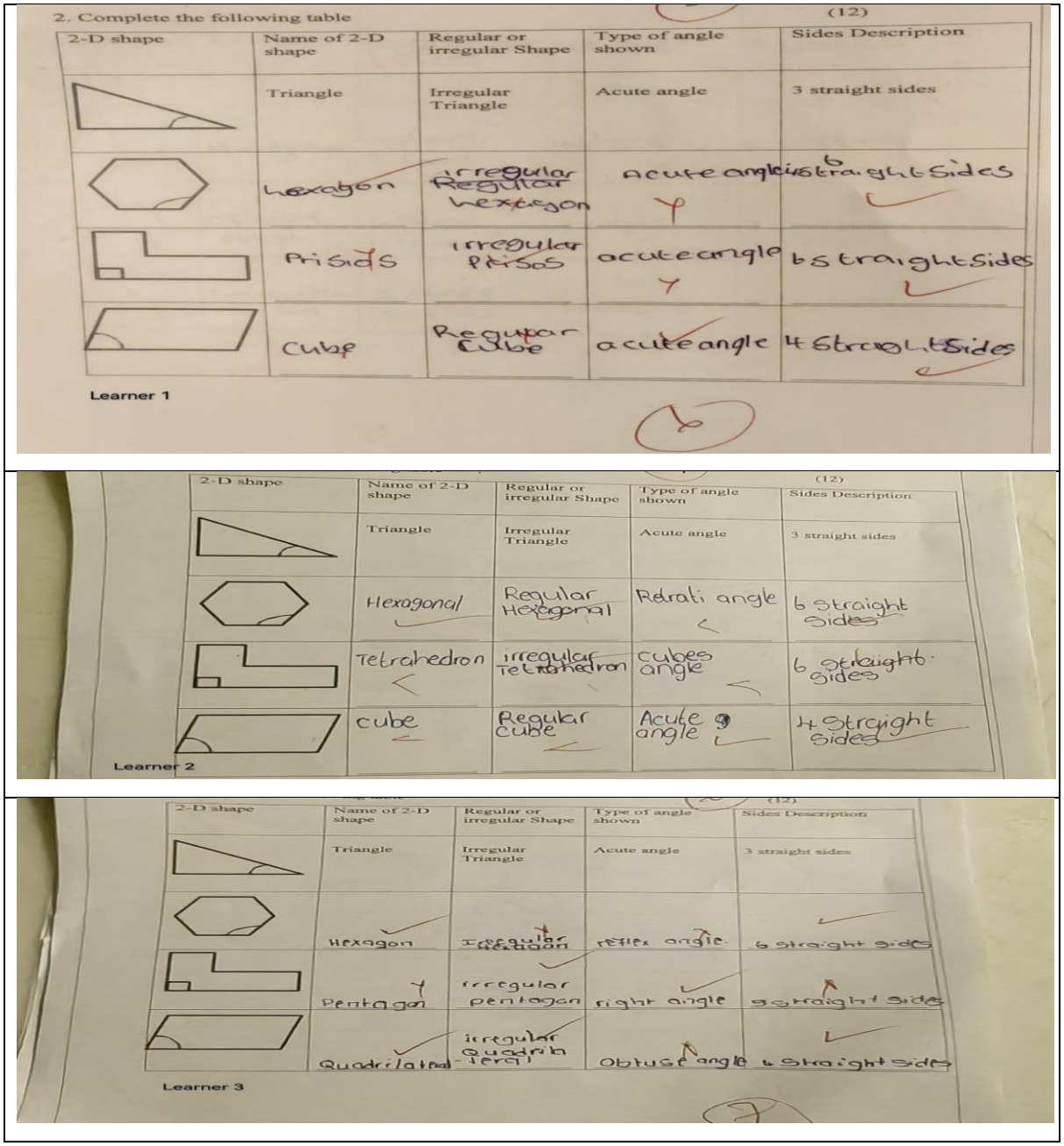


Figure 5.5: Properties of 2D shapes learners' activity

The provided figure depicts the activity with 2D shapes that were presented to the learners. The image is sourced from three learners' scripts as part of their assessment. The learners were presented with a triangle as an illustrative example, along with the missing answers for that triangle. They were then required to complete the missing details for the remaining shapes. Looking at 2nd and 3rd shapes both of them are hexagons. However, you can see that the learner correctly named the 2nd shape as a hexagon. However, the learners mostly failed to identify the third shape as a hexagon since it is an irregular hexagon, which is familiar to learners. This indicates that the learners lack a CK of a hexagon. Instead, they have just memorised its visual appearance without grasping the defining characteristics of a hexagon as a closed two-

dimensional form with six sides. Regarding the final shape, it is evident that the learners erroneously recognised the 2D shape as a cube; however, learner 3 (L3) named it correctly as a quadrilateral. The learners' most frequently seen incorrect responses were recognising the shape as a square or rhombus. This also indicates that the learners' comprehension of quadrilateral concepts is deficient, as they were unable to recognise the shape as a parallelogram, which is a type of quadrilateral.

Examining the area where learners had to determine whether the shape was irregular, it was evident that many learners encountered difficulties with this question. Upon further examination of the two hexagons, Learner 1 (L1) and L3 determined that both shapes were irregular, whereas Learner 2 (L2) correctly identified the shapes as regular and irregular, respectively. The learner struggles to distinguish between regular and irregular shapes, potentially resulting in guessing the mark for the irregular hexagon. The learner's knowledge of angles appears to be quite limited. It seems that L1 copied the acute angle from the examples and filled in the missing spaces. The learners performed admirably when describing the sides of each shape, which was a consistent trend among them. The learners demonstrate a strong proficiency in counting the number of sides based on the shape, indicating their competence in procedural skills. Although the learner correctly identified the irregular hexagon sides as six, if they had a strong CK of hexagons, they would have readily identified the shape as a hexagon due to its composition of six closed sides. The following section examines the proficiency of learners in the attributes of 3D shapes.

5.3.2. Grade 6 and 9 learners' response to 3D shape assessment

This section explicitly examines the learners' performances in assessing 3D shapes. The learners were given a task (see Appendix C) that specifically emphasised the attributes of three-dimensional shapes. This involved the process of recognising and categorising the three-dimensional figures, providing a detailed account of their characteristics, and determining the number of edges and vertices they possess. The 3D shape consisted of a cube, hexagonal prism, hexagonal pyramid, triangular prism, and triangular prism. The task involved a 21-mark assessment, where each mark was awarded based on specific criteria. A single mark was awarded for accurately identifying the 2D form, another mark for identifying the faces and the 2D shape that composes the 3D shape, one for recognising the number of edges, and one for

characterising the number of vertices of the shapes (see Figure 5.8). The following table analyses the learners' performance in the properties of 3D shapes.

3D shapes learner performance – Grade 9 and 6		
	Control group	Experimental group
N	157	271
Mean	5,13	6,41
Median	4	5
Mode	2	1
Std. Deviation	3,951	5,198
Variance	15,612	27,021
Skewness	0,81	0,791
Kurtosis	-0,184	-0,358
Range	16	20
Minimum	0	0
Maximum	16	20
Q1	2	2
Q3	8	10
IQR	. 8	8

Table 5.5: Descriptive statistics and distribution of data for 3D shapes assessment

Based on the mean values of the control group with grade 9 (5.13) and the experimental group with grade 9 (6.41), it can be observed that both groups of learners have performed below the average mark. The control group (3,951) and experimental group (5,198) have low standard deviations, indicating that the data is tightly grouped around their respective means. The variance (15,612) for control is larger than the mean; this tells us that data is more spread out from the mean and each, and so is the variance (27,021) of the experimental group to its mean. The mode for the control group is 2 and 1 for the experimental group. Examining the distribution of data for the control group, we observe a difference of 1.13 between the mean and the median, indicating a slight rightward spread. Similarly, in the experimental group, we observe a mean difference of 1.41, suggesting a slight rightward spread. The control group and experimental group both exhibit positive Skewness values (0.81 and 0.791, respectively), indicating a slight positive skew in the data distribution for both the experimental and control groups as they both exhibit a right tail. Meanwhile, the kurtosis negative value (-0,184) of the control group indicates that the data follows a platykurtic distribution. For the experimental

group, the kurtosis (-0,358) indicates that the data also follows a platykurtic distribution. The probability scores for the control group in this data set exhibit a wide range, with the minimum score obtained being 0 and the maximum score being 16. Furthermore, the data set for the experimental group shows a wide range of 20, with the lowest score obtained being 0 and the highest score being 21. The range of values for the control group in this sample is between 2 and 8, indicating an interquartile range of 6. Regarding the experimental group, the range of values for the control group in this sample falls between 2 and 10, with an interquartile range of 8 between the upper and lower quartiles. The box plot figures for both the control group and the experimental group provide insights into the distribution of the data.

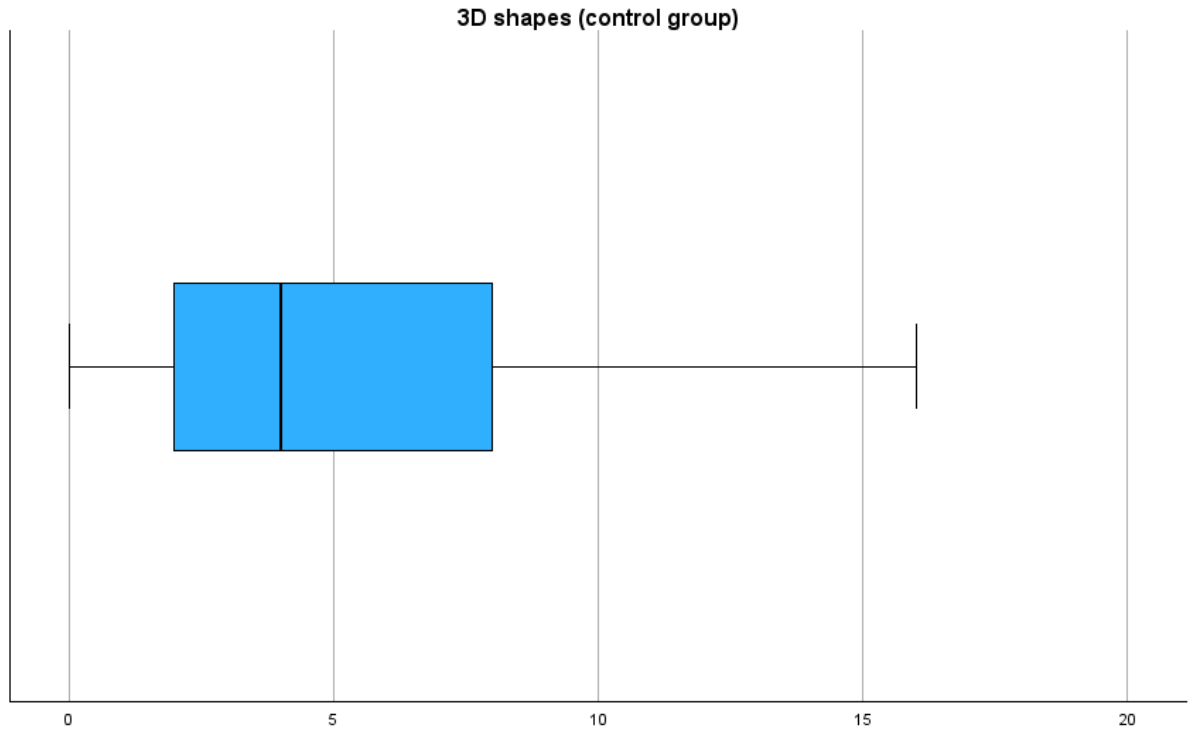


Figure 5.6: Boxplot of the control group (grade 9) spreads data in 3D shapes

The box plot (Figure 5.6) for the control group provides evidence of data Skewness, as there is a larger dispersion of data to the right of the mean compared to the dispersion on the left side of the mean. The Box plot verifies that the minimum value is 0 and the greatest value is 16. The Q3 data indicates that 75% of the learners scored below 8 marks. This data collection does not contain any outliers. The data values are more widely dispersed at the higher end of the mean, indicating a positive skew in the data. Although the high scores acquired were distributed towards the right of the mean, the box plot at the third quartile (Q3) reveals that the learners

scored below the average. The boxplot figure provides an analysis of the data spread within the experimental group.

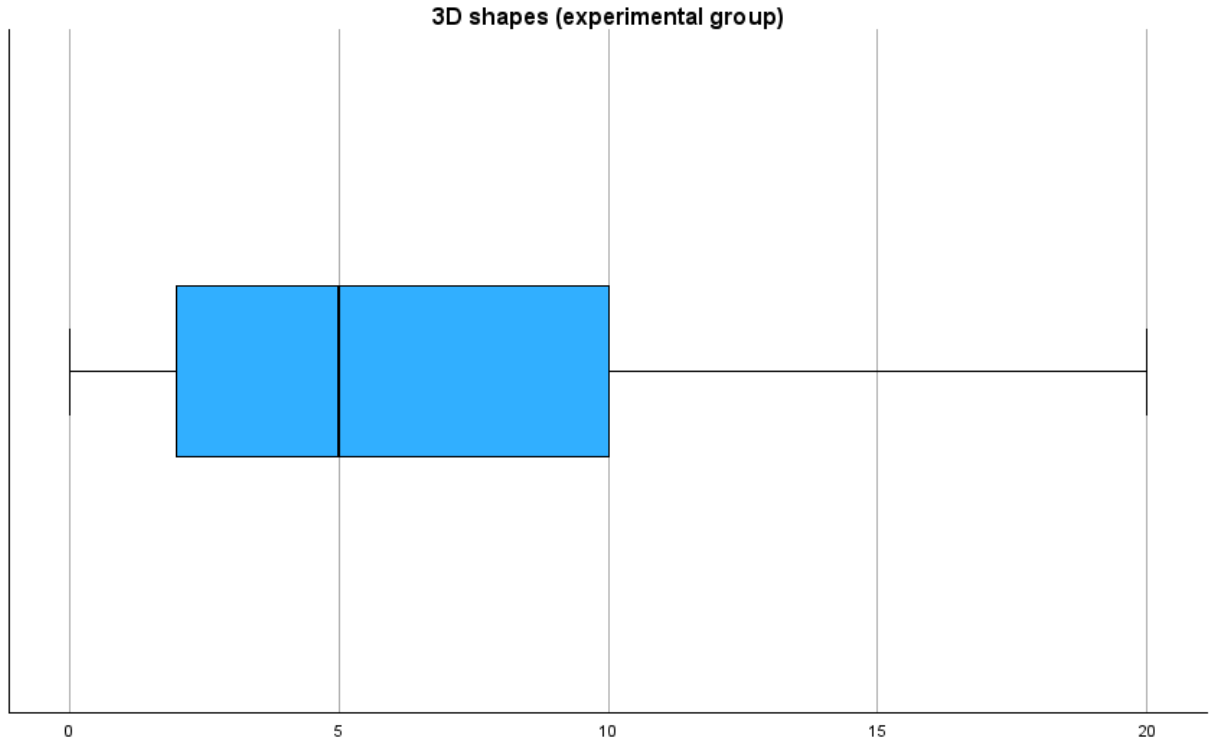
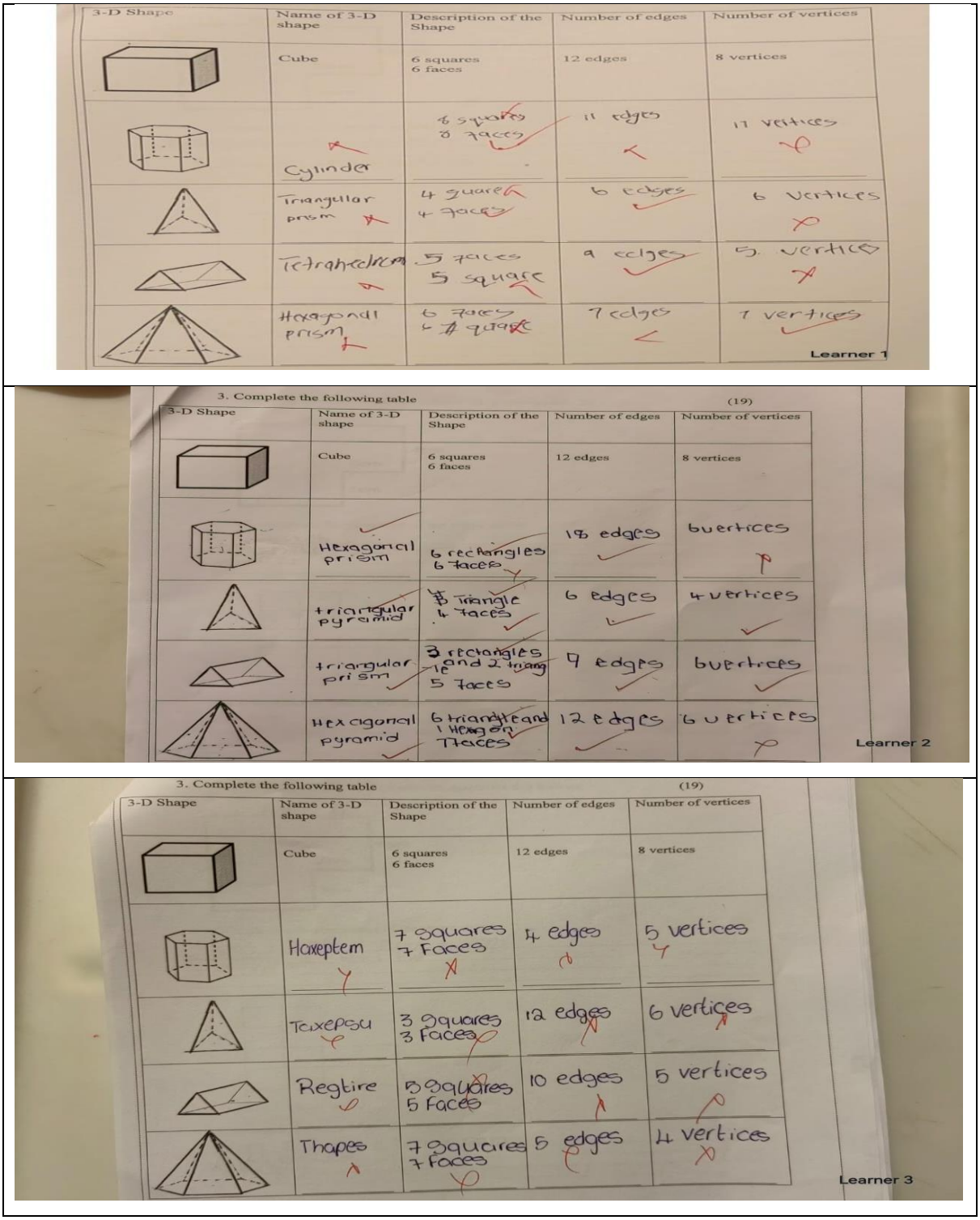


Figure 5.7: Boxplot of experimental group (grade 6) spread data in 3D shapes

The box plot (Figure 5.7) for the experimental group indicates data Skewness since there is a greater spread of data to the right of the mean compared to the spread on the left side of the mean. The Box plot confirms that the minimum value is 0 and the maximum value is 20. According to the Q3 data, 75% of the learners achieved a score of less than 10 marks. The data collection is free from any outliers. Furthermore, the data values exhibit a greater degree of dispersion towards the upper end of the mean, suggesting a positive skew in the data. While the majority of the high scores were above the mean, the box plot at the third quartile (Q3) indicates that the learners scored below the average. Despite the experimental group scores being below average and identical to the control group, it is worth noting that the third quartile (10) of the experimental group suggests that they outperformed the control group, which had a third quartile of 8. The following figure depicts the task assigned to the learners, which involves 3D shapes activity. It offers insights into how the learners approached answering questions about the attributes of 3D shapes.



3-D Shape	Name of 3-D shape	Description of the Shape	Number of edges	Number of vertices
	Cube	6 squares 6 faces	12 edges	8 vertices
	Cylinder	8 squares 8 faces	11 edges	17 vertices
	Triangular prism	4 square 4 faces	6 edges	6 vertices
	Tetrahedron	5 faces 5 square	9 edges	5 vertices
	Hexagonal prism	6 faces 4 square	7 edges	7 vertices

Learner 1

3. Complete the following table (19)

3-D Shape	Name of 3-D shape	Description of the Shape	Number of edges	Number of vertices
	Cube	6 squares 6 faces	12 edges	8 vertices
	Hexagonal prism	6 rectangles 6 faces	18 edges	6 vertices
	triangular pyramid	3 triangle 4 faces	6 edges	4 vertices
	triangular prism	3 rectangles and 2 triang 5 faces	9 edges	6 vertices
	Hexagonal pyramid	6 triangles and 1 Hexagon 7 faces	12 edges	6 vertices

Learner 2

3. Complete the following table (19)

3-D Shape	Name of 3-D shape	Description of the Shape	Number of edges	Number of vertices
	Cube	6 squares 6 faces	12 edges	8 vertices
	Hexeptem	7 squares 7 Faces	14 edges	5 vertices
	Texepsu	3 squares 3 Faces	12 edges	6 vertices
	Regtire	5 squares 5 Faces	10 edges	5 vertices
	Thapes	7 squares 7 Faces	5 edges	4 vertices

Learner 3

Figure 5.8: Properties of 3D shapes learners activity

The provided figure depicts the three-dimensional shapes activity of a learner, which was included as part of the geometry assessment given to the learners. The assessment consisted of evaluating the characteristics of three-dimensional shapes. The task consisted of identifying the 3D shapes portrayed, providing a description of each shape, and stating the number of edges and vertices in each shape. The learners were given an example of a cube and they had to

complete the other shapes with missing answers. The activity included hexagonal prism, triangular prism, tetrahedron, and hexagonal prism. Upon examining the learner's response about the second shape, a hexagonal prism, it was determined that the L1 mistakenly recognized it as a cylinder. The potential error the learner may have made here is visualising the hexagon as a cylinder due to their comparable visual characteristics. This indicates that the learner's CK of the hexagonal prism is deficient, as they are unable to distinguish between the hexagonal prism and cylinder. The L1 misidentified the 3rd shape, which is a tetrahedron, as a triangular prism. Upon observing the fourth shape, which happens to be a triangular prism, the learner made the erroneous identification of calling it a tetrahedron. The learner had difficulty distinguishing between a triangular prism and a tetrahedron. Furthermore, upon examining the most last 3D shape, a hexagonal pyramid, the L1 misidentified it as a hexagonal prism. L2 performed exceptionally well in naming 3D, whereas L3 performed significantly poorly. This indicates that learners face difficulty in distinguishing between prisms and pyramids, and this was a prevalent observation among the majority of learners.

The description of the 3D shapes focuses on identifying the 2D shapes that compose each shape and determining the total number of faces. The L1 correctly identified the 8 faces of the second 3D shape, which is a hexagonal prism. However, did not recognise the 2 hexagons and mistakenly identified the rectangles as squares. This indicates that learners who have a limited understanding of 2D shapes may encounter difficulties when it comes to comprehending 3D shapes, as a solid grasp of 2D shapes is necessary for understanding 3D shapes. The L1 accurately described the tetrahedron as having 4 faces but failed to describe that the faces are made up of 4 triangles. L1 accurately identified the 5 faces of the triangular prism but did not recognise the presence of the 3 rectangles. Moreover, the L1 did not recognise that the hexagonal pyramid is composed of 7 faces, which include 6 triangles and 1 hexagon. When it comes to the edges L1 failed to show that the hexagonal prism consisted of 18 edges instead identified 11 edges.

L1 correctly identified the edges of both tetrahedron and triangular prism, respectively as 6 edges and 9 edges. L1 also failed to identify the edges of the hexagonal prism, and the learner identified the edges as 7 instead of 12. This tells us that the learners lack the procedural skills for calculating the edges. The learner erroneously calculated the number of vertices of the hexagonal prism as 17 instead of the correct number, which is 12. L1 incorrectly recognised 6 vertices instead of the correct number of 4 on the tetrahedron. The learner erroneously

recognised the number of vertices of the triangular prism as 5 instead of the correct value of 6. L1 successfully identified 7 vertices on the hexagonal prism, specifically those of the hexagonal pyramid. L2 performed well in the description of 3D shapes, number of edges, and number of vertices. In comparison, L3 struggled in all aspects. When examining the properties of 3D shapes, it becomes apparent that the proficiency of learners varies from one school to another. For example, you find that in School A, the learners excel at naming 3D shapes, while you find that in *School B*, their skills in this area are weaker. Conversely, you find that *School A* struggles with describing the 3D shapes, whereas School B excels in this aspect. These patterns are consistent across all seven schools and their respective concepts.

5.3.3. Grade 6 and 9 Learners response to Measurement assessment

In this section, we examine the performance of the learners' assessment in Measurement content related to perimeter, area, and volume calculations. The task (see Appendix C) had a maximum score of 7, with 3 marks allocated for area calculation, 3 marks for perimeter calculation, and 2 marks for volume calculation (see Figure 5.17). The table below provides a summary of the performance of the learners in the control group in the field of Measurement.

Learners performance in Measurement (Control group with grade 9)

		Frequency	Percent	Cumulative Percent
Mark obtained	0	107	68,2	68,2
	2	37	23,6	91,7
	3	1	,6	92,4
	4	12	7,6	100,0
	Total	157	100,0	

Table 5.6: Summary of the experimental group's performance on the topic of Measurement

Table 5.6 presents a summary of the performance of the learners in the measurement aspect of the control group. The table indicates that learners in this section showed a significantly poor level of performance. The frequency data reveals that 107 learners obtained a score of 0 in the assessment, accounting for a cumulative proportion of 68.2. The report indicates that no learner achieved a score of 1. The frequency data indicates that 37 learners obtained a score of two marks, which corresponds to a percentage of 23.6. Just one learner achieved a score of 3. Consequently, 145 learners did not meet the average score in the assessment, indicating that 92.4% of the participants performed below average. There were only 12 learners who achieved a score of 4, corresponding to a percentage of 7.6. The results indicate that there were no learners who achieved a score between 5 and 7 marks. This indicates that all the learners

achieved scores lower than 4 out of the 7 marks. The below pie chart shows the distribution of data and learners' performance in the measurement of the control group.

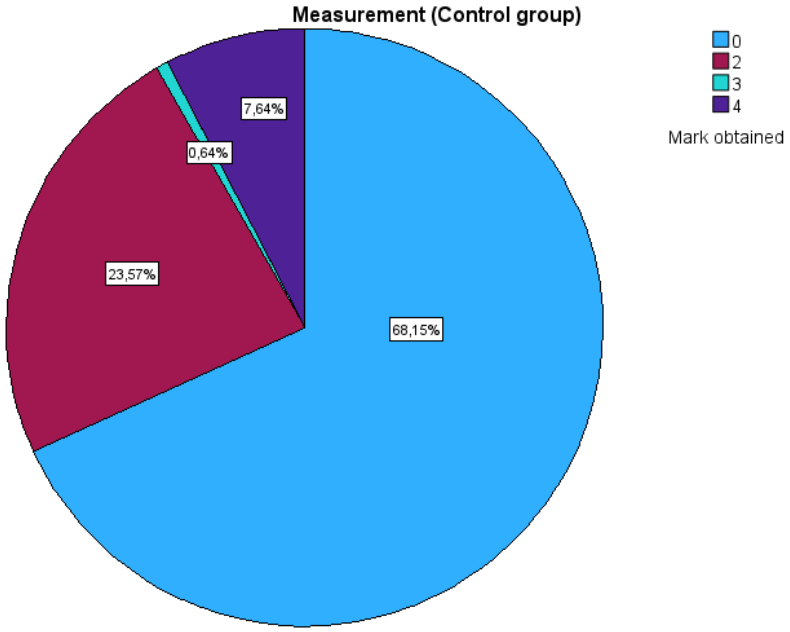


Figure 5.9: Distribution of data in the measurement for the control group (grade 9)

The pie chart above validates the data distribution for measurement in the control group. The colours of the pie chart represent the proportion of learners who achieved a specific score, as represented by the corresponding area occupied with the total percentage. The larger the percentage, the greater the corresponding area in the pie chart. The area percentage colour corresponds to the colour of the mark received by the learners. The pie chart indicates that the learner's scores were limited to 0, 2, 3, and 4. The pie charts indicate that 68.15% of individuals attained a score of 0, 23.57% attained a score of 2, 0.64% attained a score of 3, and 7.64% attained a score of 4. The following table illustrates the experimental group learners' performances in the measurement.

Learner performances in measurement (experimental group with grade 6)

		Frequency	Percent	Cumulative Percent
Mark obtained	0	176	64,9	64,9
	2	84	31,0	95,9
	4	11	4,1	100,0
	Total	271	100,0	

Table 5.7: Summary of the experimental group's performance on the topic of Measurement

Table 5.7 provides an overview of the performance of the learners in the measurement aspect of the experimental group. The table shows that learners in this section for the experimental group have demonstrated a significantly lower level of performance. According to the frequency data, a total of 176 learners received a score of 0 in the assessment, making up a cumulative proportion of 64.9. According to the report, no learner received a score of 1. According to the frequency data, 84 learners achieved a score of two marks, which accounts for 31% of the total. None of the learners scored a 3. As a result, a total of 260 learners fell short of the average score in the assessment, revealing that a staggering 95.9% of the participants performed below par. Out of all the learners, only 11 managed to achieve a score of 4, which translates to a percentage of 4.1. According to the results, none of the learners scored between 5 and 7 marks. All the learners achieved scores lower than 4 out of the 7 marks. The pie chart below further illustrates the distribution of data and learners' performance in the measurement of the experimental group.

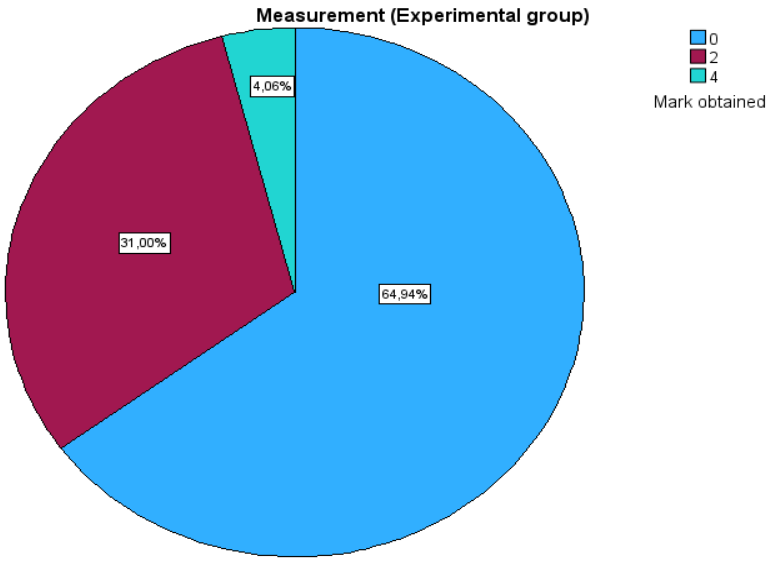
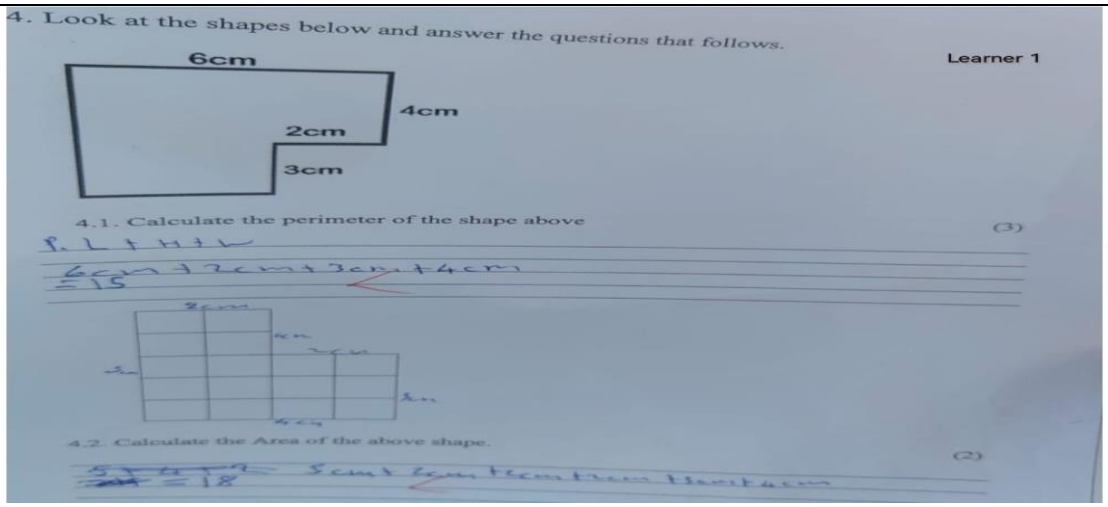
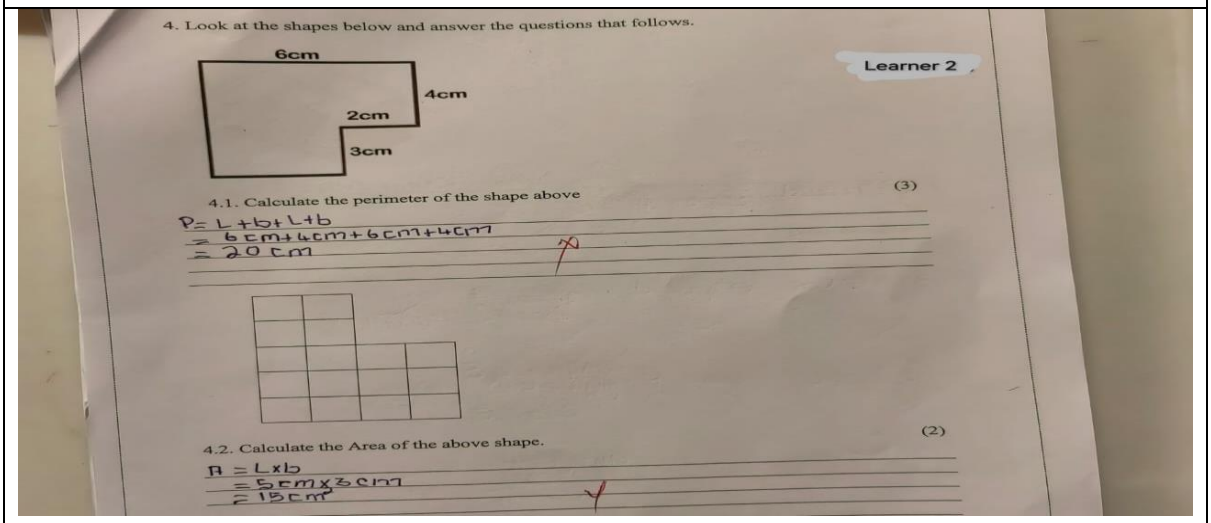


Figure 5.10: The distribution of data in the measurement for the experimental group

The pie chart above confirms the distribution of data for measurement in the experimental group. Based on the pie chart, it can be observed that the learner's scores were only 0, 2, and 4. Based on the pie charts, it can be observed that the majority of individuals, 64.94%, achieved a score of 0. Additionally, 31% of individuals attained a score of 2, while a smaller percentage, 4.1%, achieved a score of 4. Both the results from the control group and experimental group indicate subpar performance among the learners in the measurement. The performance of the experimental group and the control group is noticeably lacking. Based on the findings, 7.6%

of learners from the control group achieved an average pass of 4 marks. Similarly, a low percentage of 4.1 in the experimental group attained the average mark of 4 marks. Based on the results, it can be observed that the performance of the control group is 3.5% higher compared to the experimental group. The figure illustrates the assigned task to the learners, which entailed a measurement activity. It provides valuable perspectives on the learners' attempts at answering questions related to Measurement content.





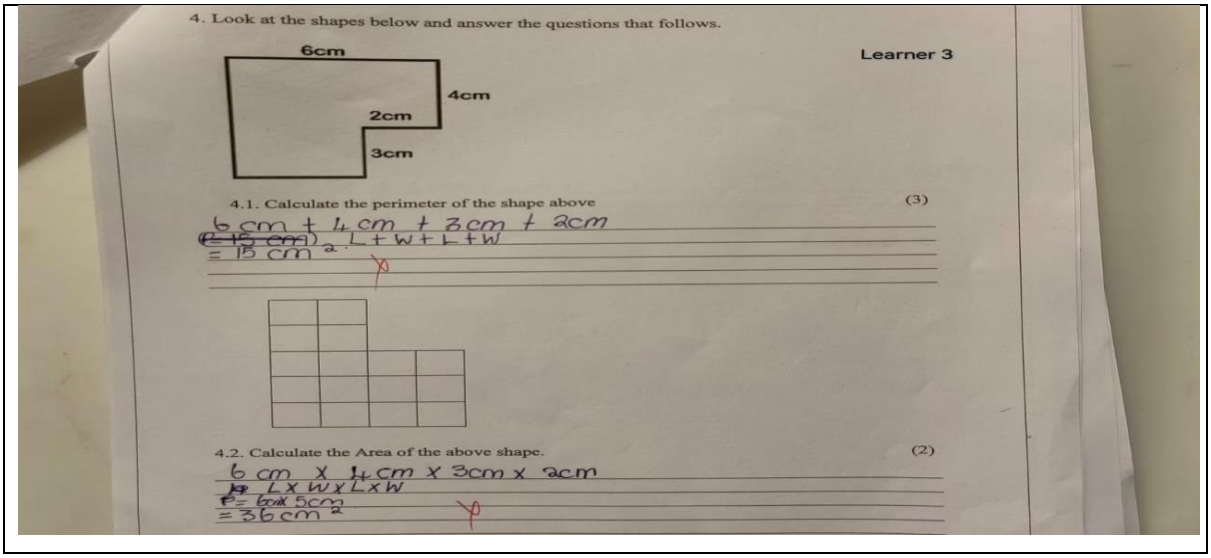


Figure 5.11: Properties of 3D shapes learner activity

Figure 5.11 illustrates several activities that were involved in the measurement process. The activity was graded out of a maximum of 7 marks. 3 marks were allocated for calculating the perimeter, 2 marks for calculating the area, and 2 marks for calculating the volume. However, Figure 5.11 solely addresses the concepts of area and perimeter. An illustration of a volume-related question can be found in Figure 5.17. Looking into the calculation of the perimeter, all the learners were unsuccessful in utilising the concept of equal opposite lines to calculate the perimeter. By using the length of the given sides, they could have determined the length of the missing opposite sides. This was prevalent among the majority of learners. When it comes to calculating the area. The L1, instead, calculated the perimeter of the shape. The learner is struggling to distinguish between the concepts of area and perimeter. It would have been beneficial for them to first grasp the fundamentals of area and perimeter. The concept of area involves measuring the size of a surface, typically in terms of squares. In contrast, the perimeter is the complete distance around a 2D shape. While L2 and L3 were unable to fully apply the formulas for calculating the area, they could have first divided the shape into a square and a rectangle. By applying the formulas to calculate the area of both shapes and adding the results, they would have obtained the overall area of the shape. Alternatively, they could calculate the area of the shape by determining the number of squares inside it. The learners have a limited CK of Area and Perimeter. This section examines the performance of learners in their van Hiele level of geometry thinking.

5.3.4. Grade 6 and 9 Learners response to van Hiele level

This section analyses the learners' responses concerning their level of geometry thinking according to the van Hiele model. The assessment included numerous multiple-choice questions (MCQ). However, for our analysis, we will focus on two specific questions from this section: one question about the visualisation level and another question on the analysis level. Each MCQ was assigned only one mark. The subsequent phase involves analysing the visualisation competence of learners in both the control group and the experimental group.

5.3.4.1. Visualisation level

The visualisation task required the learners to identify the 2D shape represented by a drawn pentagonal polygon. They were presented with four answer options, of which only one is correct. The table below provides a summary of the performance of the learners in the visualisation level of the control group.

Visualisation level (Control group with grade 9)

		Frequency	Percent
Mark obtained	0	69	43,9
	1	88	56,1
	Total	157	100,0

Table 5.8: Summary of the control group's performance on the visualisation level

The table displays the performance of the learners in the control group with their level of visualisation in geometry thinking. The table indicates that 69 learners scored zero in the visualisation level of pentagons, accounting for 43.9% of the total learners. The table indicates that 88 learners achieved a full score, accounting for 56.1% of the total number of learners. The data indicates that, in the control group, the majority of learners have an above-average level of visualisation when it comes to hexagons, with an average of 56.1%. The table below analyses the performance of the experimental group in terms of their level of visualisation.

Visualisation level (Experimental group with grade 6)

		Frequency	Percent
Mark obtained	0	116	42,8
	1	155	57,2
	Total	271	100,0

Table 5.9: Summary of the experimental group's performance at the visualisation level

The table presents the performance of the learners in the experimental group regarding their proficiency in visualising geometric concepts. The table shows that 116 learners, constituting

42.8% of the total, achieved a score of zero in the visualisation level of pentagons. The table shows that 155 learners obtained a full score, representing 57.2% of the total learners. According to the data, the experimental group shows that most learners have a higher-than-average ability to visualise hexagons, with an average of 57.2%. Both the control and experimental groups exhibited above-average performance in terms of their level of visualisation. Nevertheless, the experimental group outperformed the control group by a marginal 1.1%. The figure illustrates a learner's attempt to respond to a pentagon at the visualisation level.

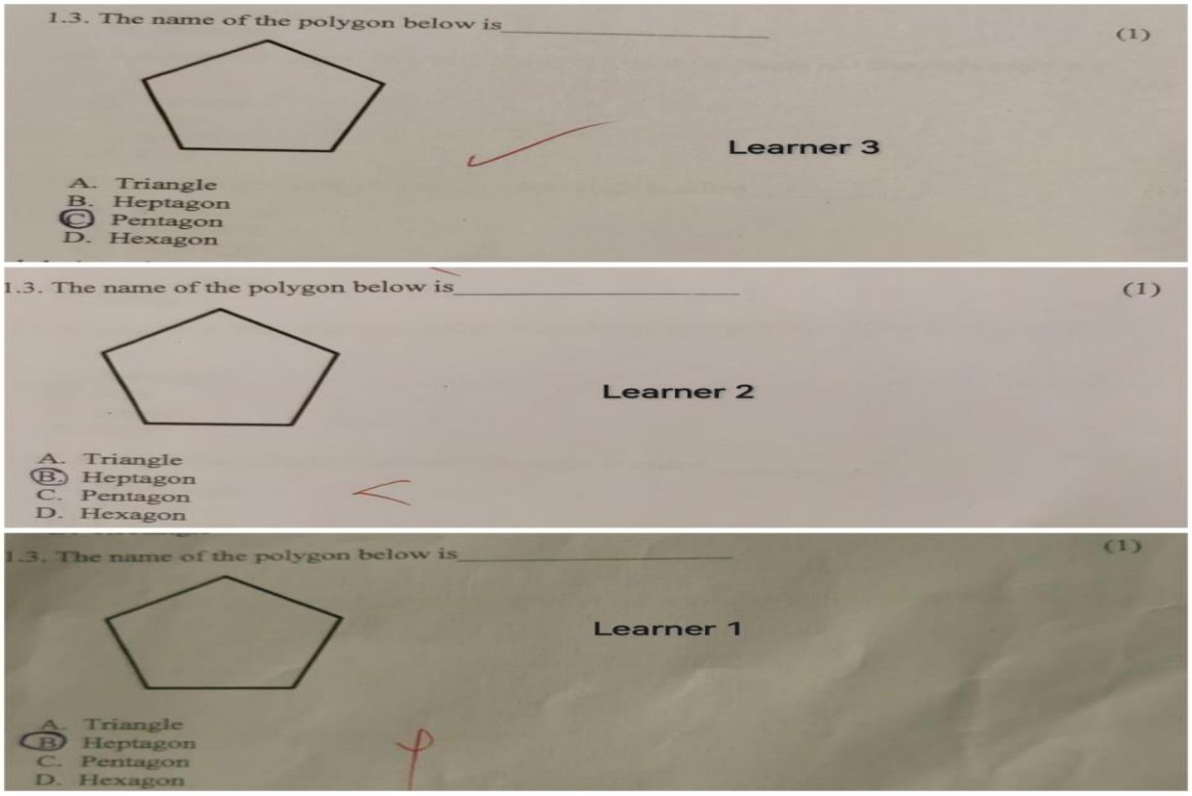


Figure 5.12: Pentagon 2D shape visualization level learners' activity

Figure 5.12 displays the MCQ that was included in the geometry assessment for the learners who were sampled. The task required the learner to identify a two-dimensional shape with five sides, known as a pentagon. The shape in question is composed of four potential options, with only one being the correct answer. The available options are as follows: triangle, heptagon, pentagon, and hexagon. The L1 and L2 incorrectly identified the shape as a heptagon instead of a pentagon. This indicates that the learners were unable to visualise the Pentagon accurately. This may be because the pentagon and heptagon have similar appearances but differ in the number of sides. As a result, the learner mistakenly visualised the pentagon as a heptagon. In

the following section, we will analyse the learner's performance in the domain of geometric thinking at the analysis level.

5.3.4.2. Analysis level

At the geometric thinking of analysis level, the task necessitated learners to engage in analyses of the properties of a square. They were given four answer choices, with only one being accurate. The following table presents a concise overview of the performance of the learners in the analysis level of the control group.

Analysis level (Control group with grade 9)

		Frequency	Percent
Mark obtained	0	94	59,9
	1	63	40,1
	Total	157	100,0

Table 5.10: Summary of the control group's performance at analysis level

The table presents the performance of the learners in the control group about their level of analytical thinking in geometry. The table shows that 94 learners, which represents 59.9% of the total learners, obtained a score of zero in the visualisation level of pentagons. The table shows that 63 learners attained a full score, representing 40.1% of the total learners. The data reveals that 40.1% of learners in the control group demonstrated a below-average level of analysis when analysing a square. The following table examines the performance of the experimental group about their level of analysis.

Analysis level (Experimental group with grade 6)

		Frequency	Percent
Mark obtained	0	127	46,9
	1	144	53,1
	Total	271	100,0

Table 5.10: Summary of the experimental group's performance at the analysis level

The table displays the performance of the learners in the experimental group about their proficiency in the analytical level of geometric thinking. The table shows that 127 learners, constituting 46.9% of the total, achieved a score of zero in the analysis level of properties of squares. The data presented in the table indicates that 144 learners achieved a full score, accounting for 53.1% of the total number of learners. Based on the data, the experimental group demonstrates that the majority of learners possess an above-average aptitude for analysing the square, with an average score of 53.1%. The results indicate that the learners in the control group performed below the average, whereas the experimental group demonstrated above-average performance in their ability to analyse a square. According to the data, the

experimental group achieved better results than the control group, with a margin of 13%. Figure 5.13 depicts a learner's effort to analyse the properties of a square.

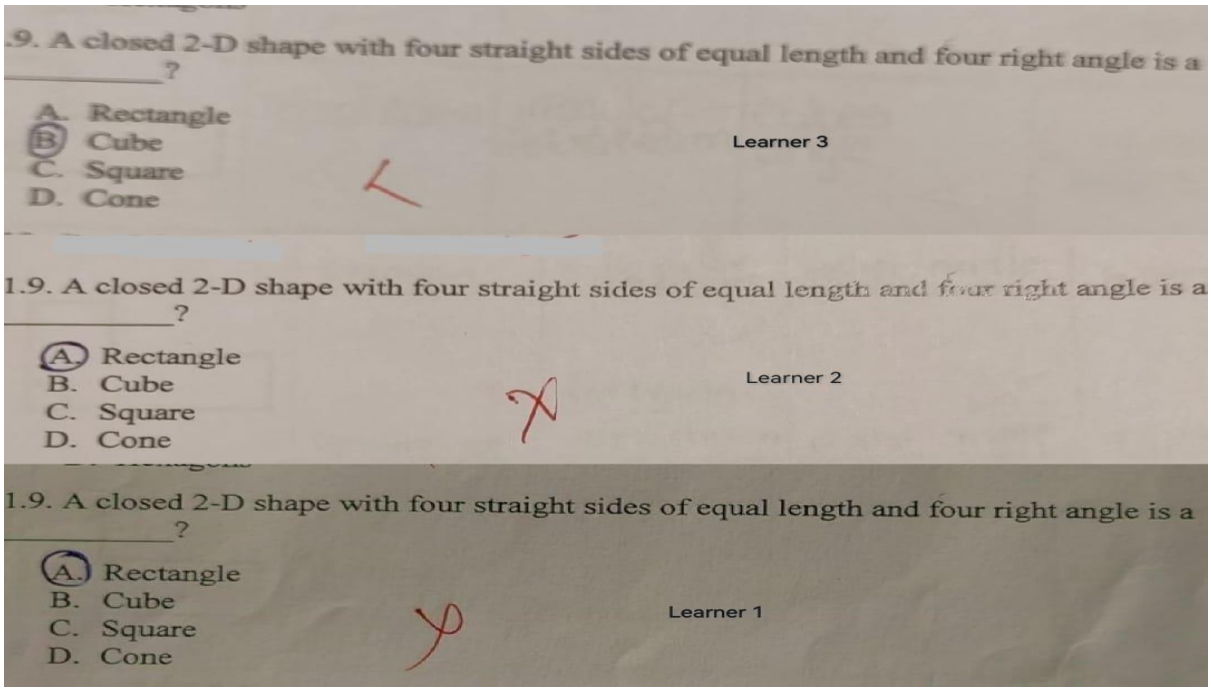


Figure 5.13: Square 2D shape analysis level learner activity

Figure 5.13 presents the MCQ related to the properties of a square. These questions were part of the geometry assessment administered to a selected group of learners. The learner was provided with a description of a square, which is defined as a two-dimensional shape with four sides of equal length and four right angles, forming a closed shape. The question consists of four possible options, of which only one is the correct answer. The available choices include the following geometric shapes: rectangle, cube, square, and cone. L1 and L2 erroneously classified the shape described as a rectangle, whereas L3 described it as a cube instead of a square. This suggests that the learners could not accurately analyse the properties of a square. The possible reason for this might be that both squares and rectangles are types of quadrilaterals that possess four right angles and four sides. However, squares have the additional property of having all sides equal in length, whereas rectangles have opposite sides that are equal and parallel. Consequently, the learner erroneously classified the square as a rectangle. This suggests that the learner's geometric thinking has not yet reached an analytical level. The following section specifically examines how learners respond to CK and PK.

5.3.5. Grade 6 and 9 Learners response to Conceptual and Procedural Knowledge

The comprehensive approach to mathematics teaching and learning encompasses both CK and PK across all topics. In this section, we focus on the learners' responses regarding their understanding and application of CK and PK in geometry, specifically emphasising volume. Within this section, we select a question about the PK of volume and another related to the CK of volume. The CK question required a description and definition of volume in a multiple-choice format. However, the PK question required a calculation of the volume. The CK question had a weightage of one mark, while the PK question had a weightage of two marks for calculating the volume. The line graph figure below depicts the summarised performances of the learners in the control group for CK and PK of the volume.

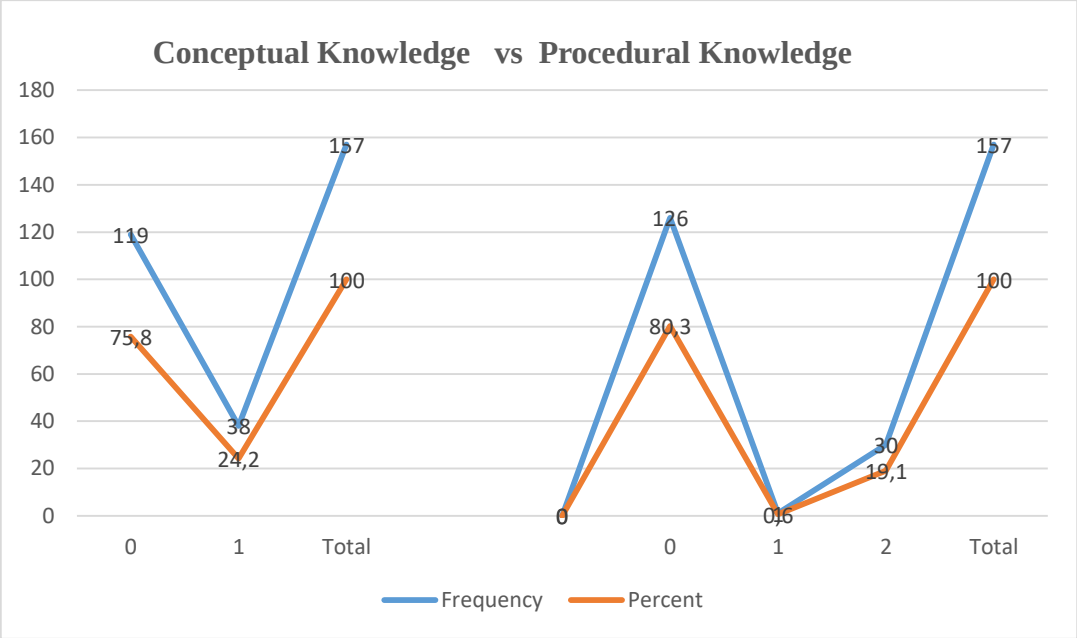


Figure 5.14: Control Group (grade 9) learners' conceptual and procedural knowledge

Figure 5.14 depicts the control group's performance in terms of CK and PK volume. The left line graph displays the performance of CK learners in terms of volume, while the right line graph illustrates the performance of PK learners in terms of volume. The frequency data indicates that a total of 157 learners took part in both the CK and PK activities. The CK question had a total mark of 1. The frequency data reveals that 119 learners received a score of zero, accounting for 75.8% of the total. The frequency analysis for the CK also indicates that 38 learners achieved a full score, accounting for 24.2% of the total. This shows that only 24.2% of the learners achieved satisfactory performance on the CK volume question, while 75.8% performed poorly. Looking at the learners' performances in the PK question, the question

consisted of a total two mark allocation of 2. The frequency data indicates that 126 learners received a score of zero, accounting for 80.3% of the total. Additionally, one learner, representing 0.16%, scored 1. The frequency data reveals that 30 learners, constituting 19.1%, achieved a total score of 2. The data indicates that the performance of the control group's learners in PK and CK is significantly low. The line graph figure below depicts the summarised performances of the learners in the experimental group for CK and PK of the volume.

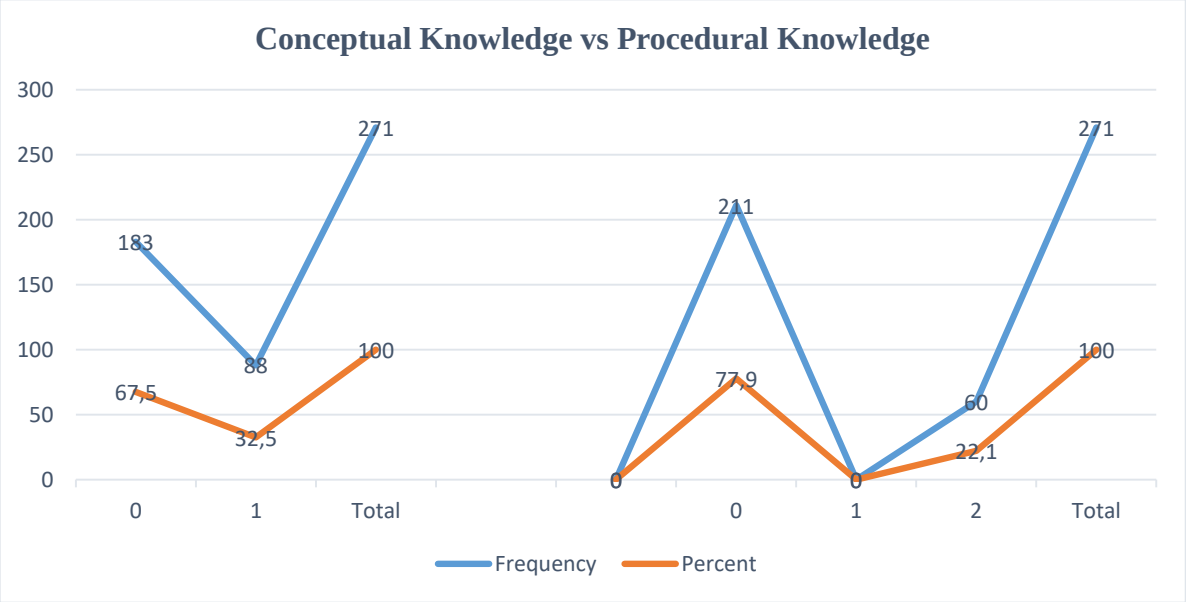


Figure 5.15: Experimental Group (grade 6) learners' conceptual and procedural knowledge

Figure 5.15 illustrates the experimental group's performance in regard to the volume of CK and PK. The left line graph depicts the performance of CK learners regarding the volume, while the right line graph portrays the performance of PK learners in terms of volume. According to the frequency data, there were a combined total of 271 participants in both the CK and PK activities. The CK question had a maximum score of 1. According to the frequency data, 183 learners obtained a zero score, representing 67.5% of the total. The frequency analysis of the CK reveals that 88 learners attained a perfect score, constituting 32.5% of the overall total. These findings suggest that a mere 32.5% of the learners achieved a satisfactory level of performance on the CK volume question, while a significant 67.5% of the learners exhibited poor performance. Upon reviewing the learners' performances in the PK question, the question had a total mark allocation of 2. According to the frequency data, 211 learners obtained a zero score, representing 77.9% of the total. In addition, no learners, accounting for 0% of the total, achieved a score of 1. In addition, the frequency data indicates that 60 learners, accounting for

22.1% of the total, obtained a score of 2. The data demonstrates that the performance of the learners in the experimental group is notably deficient in both PK and CK.

When comparing the performance of learners in the control and experimental groups, both groups performed significantly poorly in terms of the volume of CK and PK. Upon examining the CK performances, it is evident that only 24.2% of the learners achieved a satisfactory level, in contrast to 32.5% of learners in the experimental group. This suggests that although both groups performed poorly, the experimental group demonstrated a slightly better performance with an 8.3% in volume. The researchers analysed the PK of the volume and found that the control group achieved a result of 19.1%, while the experimental group achieved a higher 22.1%. These findings indicate that the experimental group outperformed the control group by a margin of 3%. The results of both groups suggest that the learners performed slightly better in CK than in PK. This figure examines how a learner approached, answering the CK question about volume.

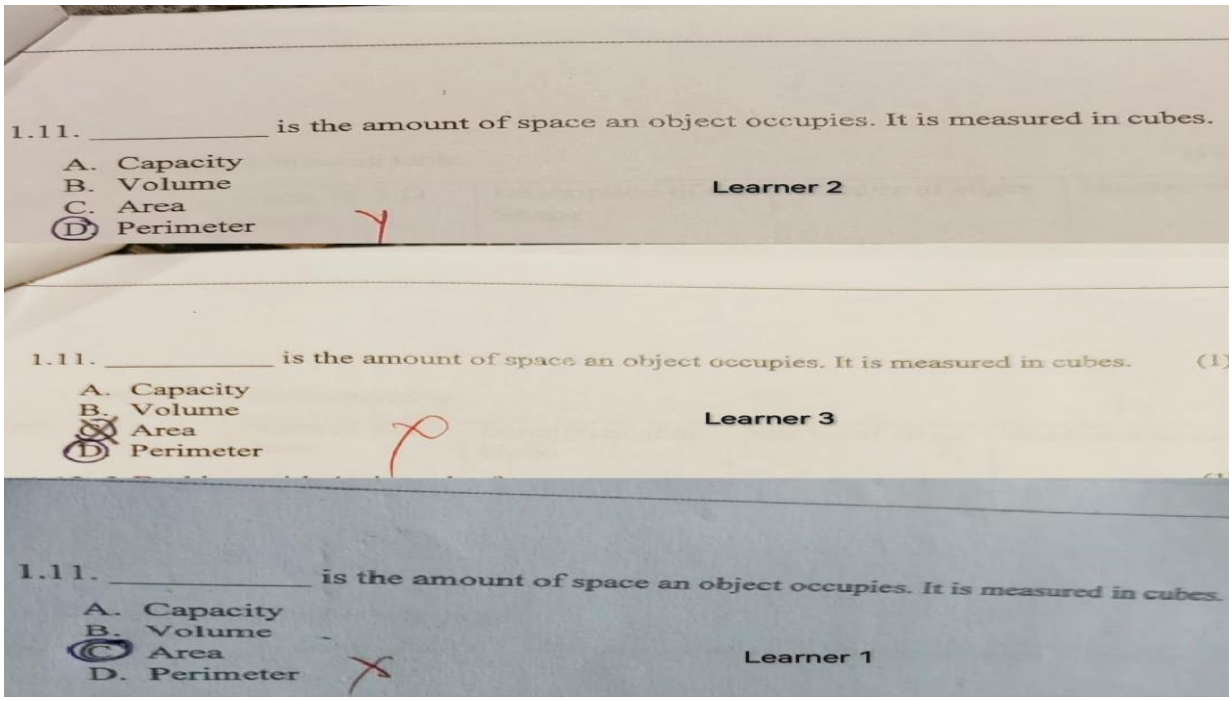


Figure 5.16: Volume Conceptual Knowledge Learners' activity

Figure 5.16 displays the effort put forth by a learner during an activity related to volume CK. The question was a multiple-choice question where learners had to select one correct answer from four options. The available options were capacity, volume, area, and perimeter. The question provided a clear definition of volume as the measure of space occupied by an object.

Moreover, that "it is measured in cubes." L1 responded and mistakenly identified the given definition as the Area. Whereas L2 and L3 incorrectly identified the description as one of the perimeters. It is somewhat challenging to comprehend the learner's process to arrive at that answer, given that area is typically defined as the "size of the surface of a shape and is measured in squares." There seems to be a misconception here where the learner has connected the idea of "the size of the surface of a shape" with "the amount of space an object occupies." These concepts are somewhat similar, and the learner has also made a connection between squares and cubes without distinguishing between the two. It appears that the learner's CK in geometry is weak. Hence, the learner struggled to differentiate between the concepts of area and volume.

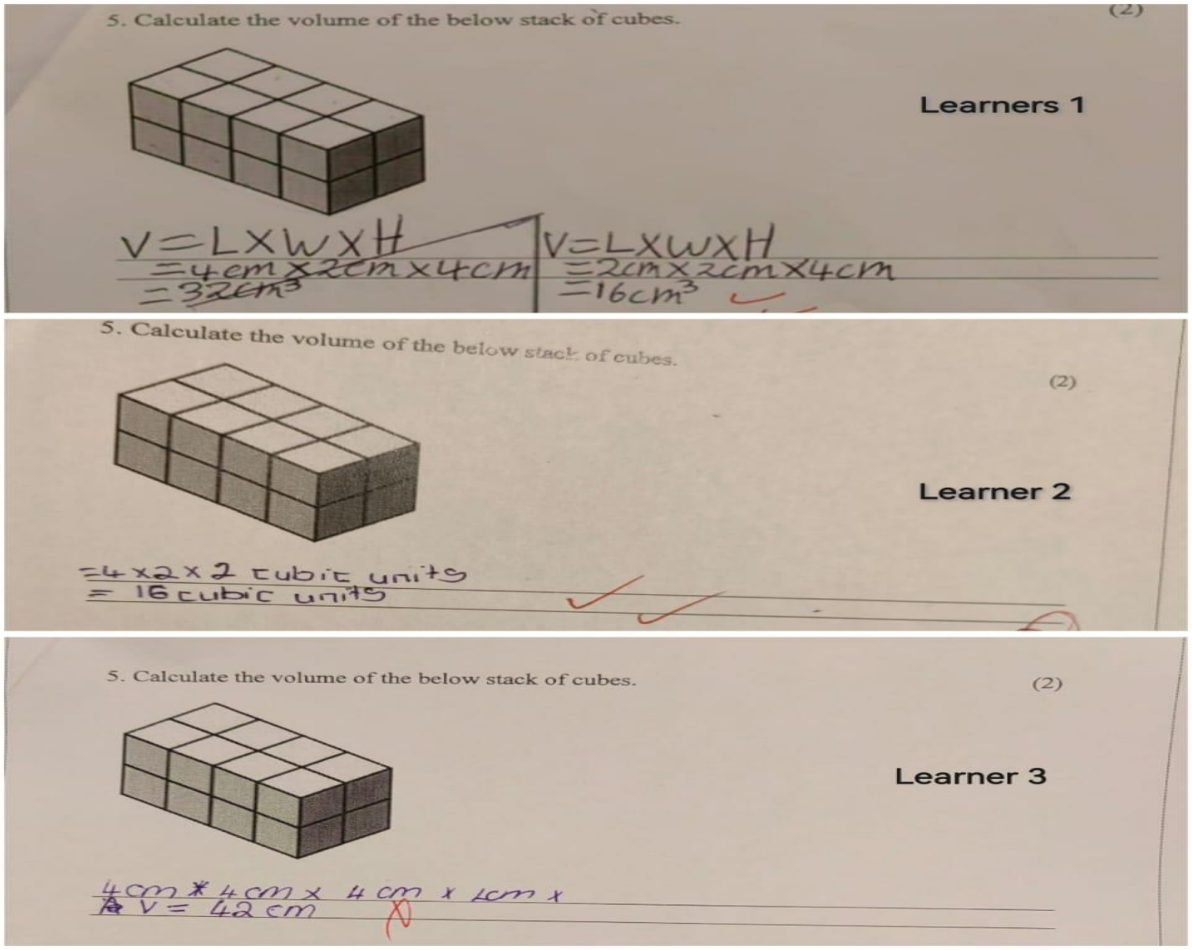


Figure 5.17: Volume Procedural Knowledge Learners' activity

Figure 5.17 displays the PK learner's activity, precisely their volume calculation. The activity required the learner to complete a 2-mark task, which involved calculating the volume of a rectangular prism. The sides of the unit were not provided with numerical dimensions, but the learner could easily count the cubes. There are three different ways in which the learner can arrive at the final answer to this question. The learner has multiple options for calculating the

total number of cubes. They can count the top cubes and perform multiplication (8x2) or by repeated addition (8+8). Alternatively, they can count the cubes from the width side and multiply them by the length of the cubes (4x4) or use repeated addition (4+4+4+4). The learner could also use the volume formula $V = lxwxh$ (4x2x2) to find the correct answer, which is 16 cubic units. The L1 and L2 utilised the volume formula in this instance. The learners demonstrated accuracy in applying the formula, but there was a mismatch in applying the dimension to the formula with L1. The learner mistakenly confused the length and height dimensions, but accurately applied the width dimension. Despite this, the learner was still able to reach the final answer and received full marks. The volume calculation did not specify the unit, but the learner consistently used cm for all dimensions and arrived at an answer of $16cm3$. However, L3 struggled with the application of the formula and algorithms of volume. L1 and L2 demonstrate a PK for calculating volume, but there may be errors due to insufficient CK. Although this learner demonstrated strong performance in the PK of the volume, a notable number of learners from both the control and experimental groups struggled with this aspect.

5.3.6. Overview of schools performances in Geometry

The results of geometry were obtained from schools (n=7) and learners (n=429) involving the grade 6 (experimental group) and grade 9 (control group). The school achievement is discussed using descriptive statistics.

School Code	N	Geometry performance %
School A	45	32
School B	79	34
School C	43	43
School D	37	34
School E	67	35
School 1	75	33
School 2	83	33

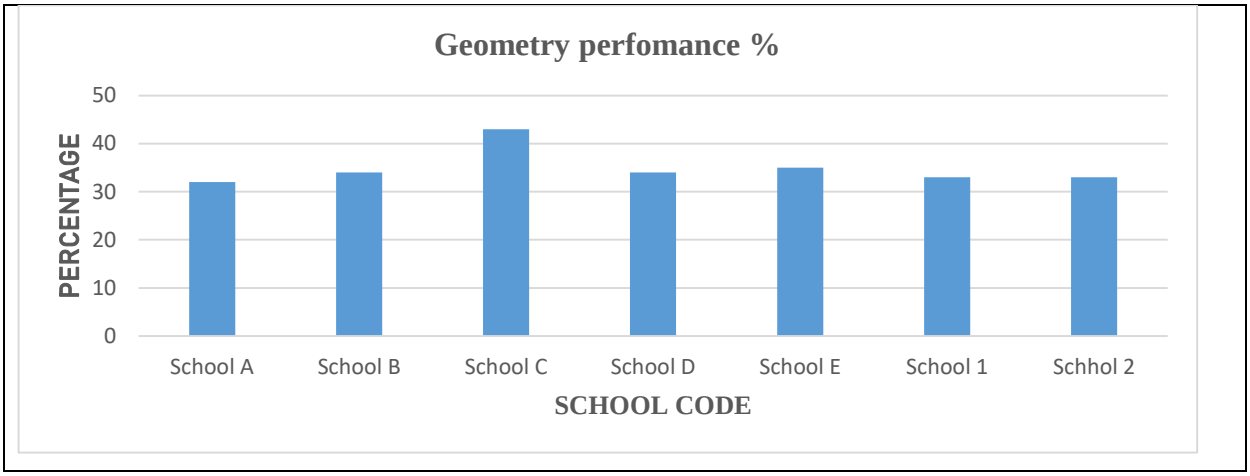


Figure 5.2.: Summary of schools' performance in geometry

Figure 5.2 presents an overview of the performance of learners from different schools. It includes the grade 6 learners from 5 primary schools and the two grade 9 classes from two secondary schools. The data reveals the following results: School A had an achievement rate of 32% with a sample size of 45, School B achieved 34% with a sample size of 79, School C achieved 43% with a sample size of 43, School D achieved 34% with a sample size of 37, School E achieved 35% with a sample size of 67, School 1 achieved 33% with a sample size of 75, and School 2 also achieved 33% with a sample size of 83. Based on the results, it can be concluded that the performance of all the learners in the 5 primary schools is below average. Among the schools, school A had the lowest performance at 32%, while school C had the highest performance at 43%. Schools B, D, and E had similar performances, ranging from 34% to 35%. The results of the two high schools indicate that both schools achieved a pass rate of 33%, which is below average performance. Based on the results, it is evident that three primary schools have slightly outperformed the two secondary schools in terms of their performance in geometry. It appears that the performance of grade 6 learners was slightly better than that of grade 9. Overall, the performances of all the schools are quite low. Table 5.1 shows the overall comparison between the control group and the experimental group.

The values related to the descriptive statistics for the two groups in the geometry assessment are summarised in Table 5.2. The control group comprised 158 participants, while the experimental group had 271 participants. The test score for geometry was 52 marks in total. During the intermediate phase, the pass percentage is 40%. This implies that learners need to score at least 21 marks out of 52 to pass the test.

Table 5.1: Descriptive statistics and distribution of data for geometry assessment

Geometry learner's performances in Grades 9 and 6		
	Control group	Experimental group
N	157	271
Mean	17,38	18,45
Median	16	16,54
Mode	16	16
Std. Deviation	6,389	8,699
Coefficient Variance	36,76	47,15
Skewness	0,593	0,686
Kurtosis	1,197	-0,212
Range	40	41
Minimum	0	2
Maximum	40	43
IQ1	13	12,01
IQ3	21	24,03
IQR	8	12,02

Based on the mean values of the control group (17.38) and experimental group (18.45), it can be observed that both groups of learners have performed below the moderate requirement for pass marks. The control group (6,389) and experimental group (8,699) have low standard deviations, indicating that the data is tightly grouped around their respective means. The mode for both groups is 16. Examining the distribution of data for the control group, we observe a difference of 1.38 between the mean and the median, indicating a slight rightward spread. Similarly, we observe a mean difference of 1.91 in the experimental group, suggesting a slight rightward spread. The control group and experimental group both exhibit positive Skewness values (0.593 and 0.686, respectively), indicating a slight positive skew in the data distribution for the experimental group as it shows a right tail, and the control group data are symmetrically distributed around the mean. Meanwhile, the kurtosis positive value (1,197) of the control group indicates that the data follows a Leptokurtic distribution. For the experimental group, the kurtosis negative score of -0.212 demonstrates that the data follows a platykurtic distribution. The probability scores for the control group in this data set exhibit a wide range, with the minimum score obtained being 0 and the maximum score being 40. Furthermore, the data set for the experimental group shows a wide range of 41, with the lowest score obtained being two and the highest score being 43. The range of values for the control group in this sample is between 13 and 21, indicating an interquartile range of 8. Regarding the experimental

group, the range of values for the control group in this sample falls between 12.01 and 24.03, with an interquartile range of 12.02 between the upper and lower quartiles. Exploring below are histograms (figure 5.3 and Figure 5.4) and tables (table 5.2 and Table 5.3) that illustrate the data spread.

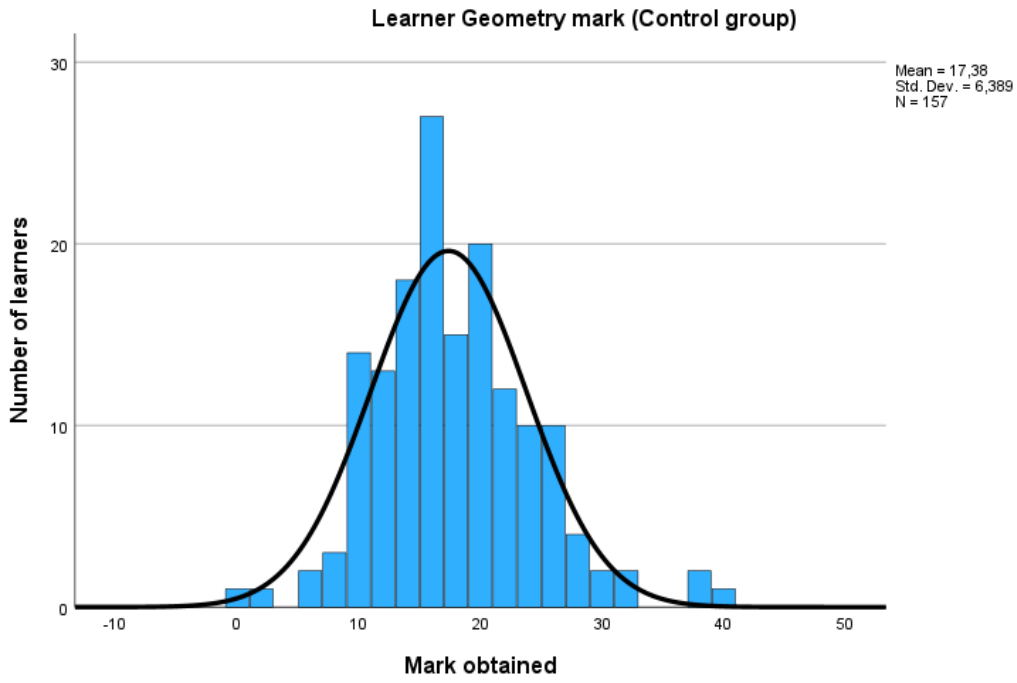


Figure 5.3: Histogram for marks obtained in geometry for the control group (grade 9)

The histogram visually displays the geometric outcomes from the control group's experiments. Most learners scored marks within the interquartile 1 (13) and interquartile 3 (21). The analysis shows that the data is evenly distributed around the mean, indicating a symmetrical distribution.

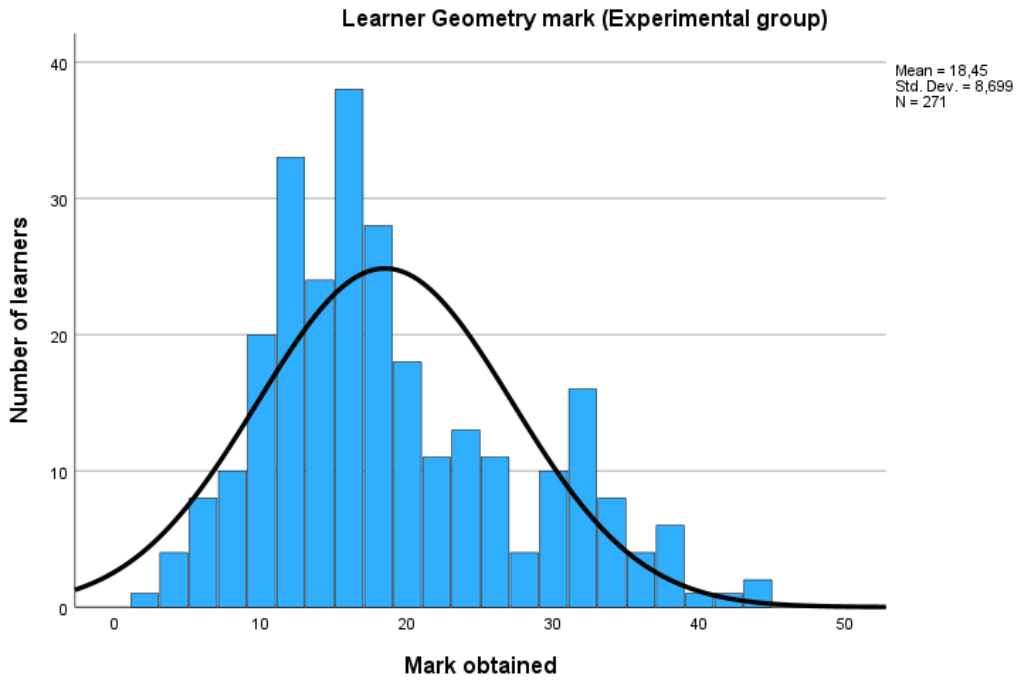


Figure 5.4: Histogram for marks obtained in geometry for the experimental group (grade 6)

The histogram offers a visual representation of the results of the experimental geometry. Most of the learners achieved scores within the range of the first quartile (12.01) and the third quartile (24.03). Based on the data provided, the data exhibits a longer tail on the right side, indicating a right skew.

Table 5.2: Dispersed data on learners' performance (grades 9 and 6)

Geometry learner's performance (Control group with Grade 9)

		Frequency	Percent	Cumulative Percent
No. of learners	0	1	,6	,6
	2	1	,6	1,3
	6	2	1,3	2,5
	8	3	1,9	4,5
	9	6	3,8	8,3
	10	8	5,1	13,4
	11	5	3,2	16,6
	12	8	5,1	21,7
	13	6	3,8	25,5
	14	12	7,6	33,1
	15	13	8,3	41,4
	16	14	8,9	50,3
	17	5	3,2	53,5
	18	10	6,4	59,9
	19	14	8,9	68,8
	20	6	3,8	72,6
	21	8	5,1	77,7
	22	4	2,5	80,3
	23	7	4,5	84,7
	24	3	1,9	86,6
	25	6	3,8	90,4
	26	4	2,5	93,0
	27	2	1,3	94,3
	28	2	1,3	95,5
	29	2	1,3	96,8
	32	2	1,3	98,1
	37	2	1,3	99,4
	40	1	,6	100,0
	Total	157	100,0	

Geometry learner performance (experimental group with Grade 6)

		Frequency	Percent	Cumulative Percent
No. of learner	2	1	,4	,4
	4	4	1,5	1,8
	5	4	1,5	3,3
	6	4	1,5	4,8
	7	4	1,5	6,3
	8	6	2,2	8,5
	9	9	3,3	11,8
	10	11	4,1	15,9
	11	16	5,9	21,8
	12	17	6,3	28,0
	13	18	6,6	34,7
	14	6	2,2	36,9
	15	12	4,4	41,3
	16	26	9,6	50,9
	17	13	4,8	55,7
	18	15	5,5	61,3
	19	8	3,0	64,2
	20	10	3,7	67,9
	21	6	2,2	70,1
	22	5	1,8	72,0
	23	3	1,1	73,1
	24	10	3,7	76,8
	25	7	2,6	79,3
	26	4	1,5	80,8
	27	1	,4	81,2
	28	3	1,1	82,3
	29	6	2,2	84,5
	30	4	1,5	86,0
	31	8	3,0	88,9
	32	8	3,0	91,9
	33	3	1,1	93,0
	34	5	1,8	94,8
	35	2	,7	95,6
	36	2	,7	96,3
	37	4	1,5	97,8
	38	2	,7	98,5
	39	1	,4	98,9
	42	1	,4	99,3
	43	2	,7	100,0
	Total	271	100,0	

The table illustrates the distribution of learners' marks for both the control and experimental groups on an exam with a total of 52 marks. Based on the analysis of the data, it is evident that there is only one learner who scored the lowest mark of 0, while another learner achieved the highest mark of 40 out of 52. As mentioned earlier, the expected pass mark for this test is 21. However, the cumulative percentage reveals that a significant number of learners, precisely 72.6%, did not pass the test. Based on the numbers, there appears to be a higher frequency of individuals who failed the assessment (n=114) compared to those who passed (n=43). This suggests that the cumulative percentage of individuals who passed is 27.5%. Furthermore, the frequency reveals that 28 individuals attained a score between 40% - 49%, indicating a moderate level of achievement for learners and a cumulative percentage of 17.8. 10 learners achieved an average percentage between 50% and 59%, resulting in a cumulative percentage of 6.4. 4 learners achieved a substantial percentage between 60% and 69%, making up a cumulative percentage of 2.5. Additionally, 1 learner achieved a meritorious percentage between 70% and 79%, resulting in a cumulative percentage of 0.6.

Analysing the data, it is clear that one learner received the lowest score of 2, while two received the highest score of 43 out of 52. On the other hand, it is worth noting that a considerable proportion of learners, precisely 67.9%, did not achieve a passing score on the test. According to the data, it seems that more individuals did not pass the assessment (n=184) compared to those who did pass (n=87). It can be inferred that the overall pass rate is 32.1% based on the given evidence. Furthermore, the data shows that 31 learners achieved a score between 40% and 49%, suggesting a moderate level of achievement for learners. This accounts for a cumulative percentage of 17.8. The data shows that 26 learners had an average percentage between 50% and 59%, resulting in a cumulative percentage 9.6. A total of 18 learners achieved a substantial percentage of between 60% and 69%, contributing to a cumulative percentage of 6.6. Moreover, 9 learners achieved a meritorious percentage between 70% and 79%, resulting in a cumulative percentage of 0.6. The data indicates that we have 3 learners who have achieved exceptional scores, ranging from 80% to 100%, resulting in a cumulative percentage of 1.1%. The analysis of data comparing the performance in geometry between the control group and experimental group indicates that a higher percentage of learners, precisely 32.1%, performed well in the experimental group compared to the control group, which had a performance rate of 27.5%. The results indicate that the experimental group had a higher percentage of individuals who scored between 50% and 100% compared to the control group.

5.4. The impact of teaching and learning procedural knowledge and conceptual knowledge in geometry

This section examines the data gathered using a qualitative approach, specifically through interviews, to address the third research question: "What are the effects of teaching and learning CK and PK in geometry?" The data collection process encompassed five primary schools, with one educator and two learners being interviewed per school, resulting in five educator interviews and ten learner interviews. Firstly, we examine the interviews conducted with the educators below. This section comprises interviews conducted individually with the five participating educators and 10 participating learners. The interviews were held separately, with each educator/learner, in a one-on-one session. The following section centres on the initial theme 'Geometry teaching and learning conceptualisation'.

5.4.1. Geometry teaching and learning conceptualisation

In this theme, we examine how learners and educators conceptualise teaching and learning in geometry. The theme through interview questions explores the perspectives of learners and educators on the significance of geometry in real life and as a subject taught and learned in the classroom. Learners and educators must comprehend the purpose of learning and teaching geometry in the classroom. They should not merely perceive it as a subject for learners to pass but rather as a body of knowledge to be acquired and applied in real-life situations. The question asked during interviews was, "Can you outline the importance of teaching Geometry at school? How do you think teaching geometry benefits the children"? When questioned about the significance of teaching Geometry and the potential benefits it can offer to children, educators expressed the following opinions;

> *It can help kids outside of school, and it can also help them choose careers. For example, I think that engineers use measurements and some of the things they learn in class when they're working (Educator A).*
> *Teaching Geometry in class is crucial as it equips learners with the skills to calculate the area of a house and assist their parents with tile fittings, among other practical applications. Also, some of them will pursue careers in construction as they grow up (Educator B).*
> *Hmm, Geometry teaching can be enhanced through the use of measurements. This will significantly benefit the learners by helping them apply their knowledge to real-life situations. It will also prepare them for future career decisions, especially for those interested in pursuing careers in construction (Educator C).*

However, the question asked to learners was, "Do you understand the importance of learning geometry? Explain". When asked about the significance of geometry and its practical applications, learners expressed their opinions as follows:

> *Yes, I do. It's fun. Also, geometry is fun because it helps us better understand the shapes around us. It's also fun because we can connect and compare it to things we see daily outside of school. When I think of angles, I also picture them at home and outside of school (learner A2).*
> *Indeed, I do. Geometry provides valuable knowledge about mathematics, allowing us to explore patterns and shapes. By studying geometry, we develop the ability to identify and classify different shapes and determine the number of lines of symmetry they possess (learner B1).*
> *I find it helpful because it helps me learn about different shapes, and I truly enjoy it. I may not have all the answers at the moment, but I have faith that things will improve with time. Also, it holds importance as it will contribute to the advancement of my studies (learner C1).*

Learner A1 had this to say "Yes. I know what volume, area, perimeter, and all that stuff mean." *Learner B2* responded by saying "No, I don't understand." On the other hand, learner *C2* Yes, I understand how important it is to study geometry because it teaches us how to figure out numbers and shapes. *Learner D1* responded by stating, "Yes, but not everything she understands." Upon further inquiry, she mentioned that she takes time to learn.

The quotes from the educators demonstrate a clear understanding of the significance of learning geometry in the classroom. They recognise its importance in shaping learners' future careers and its practical application in the real world. However, regarding the learners, while some grasp the importance of geometry, the quotes demonstrate that other learners fail to comprehend its significance and are unable to connect their learning to the real world.

5.4.2. Challenges in teaching and learning geometry

Learners often encounter obstacles when learning geometry, just as educators teaching geometry also face numerous challenges in delivering the content to the learners. It is crucial to address these challenges to ensure ongoing effectiveness in teaching and learning. The question asked to educators was, "What challenges do you come across when teaching geometry?" When educators were questioned about the difficulties they encounter while teaching Geometry and how they handle them, they responded as follows:

> *I often encounter the challenge of children struggling with accurate calculations. To address this, I provide them with additional examples and exercises to help them improve (educator A).*
> *It can be quite a challenge when learners are presented with a triangle that lacks certain measurements and they only want to measure when the length is already given. I explained to them that it is necessary to measure the length in cases where the length is not provided. Also, there seems to be some confusion among learners when it comes to reflex and acute angles.*

> *However, I always explain to them that every angle has a line of rotation. The line needs to rotate beyond 180 degrees to indicate that the angle is outside. On the other hand, an acute angle does not exceed 180 degrees and is considered to be inside. The challenge revolves around acute and reflex angles (Educator B).*
> *My learners struggle to differentiate between 2D and 3D objects; they are unaware that 2D objects are shapes and also encounter difficulties when trying to calculate volume (educator C).*

When asked about their thoughts on improving teaching and learning in Geometry, *Educator C* provided the following response:

> *Um, I believe that the repetition of teaching about 3D prisms, specifically rectangular prisms, is necessary because learners often struggle with counting the vertices and edges of these objects, as well as calculating their volumes.*
> *The main issue I'm facing is that the children are struggling with understanding and identifying 3D shapes. They are having difficulty calculating the vertices and faces of these objects (educator D).*
> *Teaching Geometry can be quite challenging, especially when it comes to explaining complex concepts to learners. I often find myself searching for ways to make the subject more relatable and understandable for them. Additionally, the fact that some learners struggle with drawing can make it even more difficult, as visual representations are often necessary to gauge their level of comprehension. Another hurdle I face is helping learners grasp and apply the various formulas involved in Geometry (Educator E)*

Based on the quotes provided, it is evident that educators are encountering a range of difficulties when teaching geometry. These challenges primarily revolve around learners struggling to understand the content. Specifically, learners face challenges with calculations, angles, 3D shapes, and grasping new concepts in geometry. However, the question asked to learners was "Do you experience problems when learning geometry? If so/yes/ what kind of problems? What do you think can be done to address these problems"? When learners were asked about their struggles in learning geometry, they had this to say:

> *While some of my classmates struggle with Geometry and lack focus, I can confidently say that I have a solid understanding of this topic. I don't face any difficulties (Learner B1)*
> *I struggle with calculations, particularly when it comes to understanding how to calculate area, volume, and perimeter (Learner D1).*
> *Yes, sometimes I do, but our teacher ensures our understanding by providing clear explanations and realistic examples (Learner D2).*
> *I don't have any issues at all. However, there are situations when I struggle with understanding shapes (Learner E1).*
> *I do have difficulties; I struggle to understand the vertices. I usually see them. Counting without visual aid is a challenge for me (Educator E2).*

The responses from other learners, which are not quoted here, indicated that they do not face any difficulties in geometry. Upon examining the quotes provided above by learners, it is evident that they are facing difficulties with calculations, shapes, and vertices. These are

precisely the areas that educators have identified as the most challenging for learners in geometry. In the following section, we will explore the learning and teaching resources that learners and educators find valuable when it comes to teaching and learning geometry.

5.4.3. Learning and teaching material resources in geometry

When it comes to teaching and learning geometry, there are undoubtedly effective teaching resources and methods that cater to both learners and educators. Educators should utilize teaching resources that facilitate a better understanding of geometry for learners. The question asked to educators was "What material/ or resources do you find necessary to teach Geometry topics?" When educators were interviewed and asked about how they utilise the teaching and learning resources, this is what they had to say;

> *I think workbooks, rulers, pencils pictures, and even magazines can help (Educators A).*

When asked about the materials used to help struggling learners, *Educator A* responded as follows:

> *You provide them with visual aids, such as pictures or tangible objects, to facilitate their learning process, as some learners learn most effectively through sight, touch, and visual perception (Educator A).*
> *As I previously emphasised, solid shapes are extremely helpful. At times, I instruct the learners to create and bring the shapes, and at other times, I construct the shapes myself. (Educator B).*

Educator B provided the following information on the methods used to support learners who are experiencing difficulties in understanding geometry:

> *Uh, alright, so occasionally the learners struggle with distinguishing between 2D and 3D. For example, I could illustrate the concept using a brick as a visual object or tool. The face of the brick represents the 2D aspect, while the entire brick represents the 3D aspect. This is something that they struggle to grasp. One of the challenges we face is the large number of learners at our school, making it difficult to provide individualised assistance.*
> *The materials I use are the likes of dusters, windows, books, and bottles (Educator C).*
> *Yeah, I use the one that we currently have in our class, because they are very visual and it's simple for them to learn from what they see every day (educator D).*
> *Textbooks, charts, and chalk are preferred because they provide an easily understandable visual representation. I also utilise Google and YouTube to search for relevant examples (Educator E).*

However, the question asked to learners was "What kinds of learning and teaching tools does your teacher use to teach you geometry in the classroom? Do you think these tools and methods are effective in helping you understand geometry better and what other effective tools/methods do you think teachers can also adopt or use?)". During interviews with the learners, they were

asked about the learning and teaching materials used by their educator. Here is how they responded:

> *Um. Shapes such as squares, pyramids, square pyramids, triangles, and tetrahedrons (Learner A1)*

When *Learner A1* was asked about what the educator does with these objects, what do the objects look like? The learner provided the following response;

> *The teacher holds them and explains them by identifying them. The teacher would point out different shapes, such as a pyramid and a tetrahedron, which appeared to be made of wood.*
> *Uh, like uh...Various small-sized shapes, including 2D shapes and 3D shapes. The teacher effectively incorporates solid shapes into their lessons to help us understand the concepts of 2D and 3D shapes (Learner A2).*

When queried about their preferred learning and teaching methods, as well as their difficulties in geometry, *Learner A2* responded as follows:

> *I think it would be better if they gave us a visual presentation of the angles, like if they would show us exactly what an acute angle looks like and not see it as a drawing (learner A2).*
> *To help us understand what we are learning, the teacher not only uses our workbooks but also provides examples such as visual presentations, formulas, and calculations to check for understanding (Learner B1).*
> *During the lesson on 3D, our teacher shows the similarities between the board duster and the shape of a rectangle by using the duster itself. Our teacher uses real-life matters to help us understand things better (Learner C1).*
> *The teacher utilises various tools and objects in the classroom, including dusters, bottles, tables, chairs, and the chalkboard. Our teacher uses materials like 3D shapes like boxes in class for us to understand better. (Learner C2).*
> *The teacher teaches us by providing many examples and including visual shapes that allow us to both see and interact with different shapes (Learner D1).*
> *Hmm, umm. Sometimes, there are boards, chalk, and a box (Learner D2).*
> *She uses the classroom or 3D and 2D objects like papers or books to make examples of rectangles and squares. She also uses containers or our lunch boxes to make examples. Our teacher also uses the maths lab to show us the visual presentations of 3D and 2D (Learner E1).*
> *She uses methods, and takes us to the lab, shows us the objects (learner E2)*

Learner E2, when queried about how to tackle the challenges encountered, replied as follows:

> *I believe it would be helpful to me to be often taken to the mathematics laboratory to count the vertices of various objects, as I perceive this activity to be useful for my learning.*

The quotes provided by educators and learners highlight the effective use of real-life objects and solids in teaching geometry. This approach helps learners grasp the concepts more easily. The quotes indicate that educators heavily depend on textbooks, chalkboards, chalk, and visual aids. The learners have expressed a preference for visual presentations that connect their

lessons with real-life objects. The following section delves into the preferences of learners when it comes to conceptual or procedural learning.

5.4.4. Learning Conceptually or Procedurally

When instructing geometry in the classroom, educators can choose to teach learners either through procedural methods, conceptual methods, or a combination of both. This section focuses on analysing the preferences of learners when it comes to acquiring knowledge in the field of geometry within a classroom setting. The question asked to learners was, "Do you find it easier to learn geometry by memorising or by understanding the concepts first?" The learners provided the following feedback during their interviews regarding their preferred methods of learning in the field of geometry:

> *I find that my learning process becomes much easier when I understand the underlying concepts and calculations. When these are explained to me, I can understand them more quickly and effectively. (Learner A1)*
> *I find it more effective to grasp the concepts rather than relying on memorisation. Memorising tends to lead to forgetting, and it can be frustrating when teachers ask about something we've already forgotten the following year (Learner A2).*
> *I prefer to learn through understanding as I believe it is more valuable to understand concepts rather than simply memorising them (Learner B!).*
> *When given a shape's name and a description of its sides, I can grasp it immediately, so understanding is easier for me than memorization (Learner B2).*
> *I appreciate the importance of understanding concepts in my learning process. Memorisation alone tends to lead to confusion, whereas understanding allows me to independently discover solutions (Learner D2).*
> *I believe that my learning process is more effective when I strive to understand the material rather than relying solely on memorization. In my experience, I tend to forget information that I have simply memorised without truly grasping its meaning. For instance, we are currently in term 3, and our teacher introduced a concept from term 2. If I had memorised it, I would have likely forgotten it by now. So, just to clarify, I want to make sure I understand correctly (Learner E1).*

Considering the statements made by the learner above. The learners interviewed expressed a preference for initially learning geometry conceptually, by understanding the concepts, rather than relying on procedural memorization. The learners mentioned that understanding concepts more quickly and effectively is important to them. They find that learning conceptually helps them to independently find solutions while relying solely on procedural learning can lead to confusion. This section examines how learners establish connections between the concepts of geometry and their everyday experiences.

5.4.5. Learners' application of geometry in real-life scenarios.

It is crucial for learners to not merely acquire knowledge of geometry to pass exams. Learners must comprehend the geometry concepts they are studying and possess the ability to apply them in their daily lives. This facilitates a deeper understanding of these concepts and fosters an appreciation for the significance of geometry education. The question asked to the learners was "Do you think the things you learn in geometry help you or anyone in real life, explain how?" When questioned about the practical applications of Geometry in everyday life, learners were asked to reflect on how the concepts they learn in this subject can be beneficial to themselves and others. Learners had this to say:

> *I sometimes see it. Nonetheless, I cannot relate to it or comprehend how to apply it in the real world (Learner A1).*
> *I believe it is beneficial because it enables me to assist those who are unable to comprehend geometry; thus, we both possess the same body of knowledge (Learner A2).*
> *The practical application of this knowledge became evident when our teacher led us outside to measure the tiles, as the school was undergoing construction at the time. We utilised a measuring wheel to accurately determine the Volume, ensuring that they purchased the appropriate amount of tiles and avoided any unnecessary excess or shortage (Learner B1).*

When asked about how the learning of geometry assists them or anyone in real life, *Learner B2*, *Learner C1*, and *Learner D2* struggled to answer the question. This suggests that they have difficulty relating what they learn in geometry to real-life situations.

> *It is useful for calculating the number of lines in houses and areas, as well as perimeters, no matter where we are. It also assists me in understanding how to calculate the perimeter of the hose or shops (Learner C2).*
> *It is beneficial to me as I can not only help myself but also share my knowledge with others. Also, the skills we learn in school can be applied at home (Learner D1).*
> *They are quite helpful to me. Mathematics is essential in various aspects of our daily lives, such as calculating tables and other objects at home (Learner E1).*
> *I believe they are beneficial as they assist us in our personal growth and equip us with the necessary knowledge and skills for the future. It's important to have a clear understanding of the vertices or faces of an object when they are being discussed (Learner E2).*

Based on the aforementioned interviews conducted with learners, it is evident that the majority of learners are encountering difficulties in establishing connections between the geometry concepts taught in class. However, learners do acknowledge the practical relevance of geometry in everyday life situations. Only a small number of learners can establish a connection between geometry and real-life situations. In the following section, we explore the educators' familiarity with technology and how they may be incorporating it into the geometry classroom.

5.4.6. Application of Technology in Geometry

In this section, we examine the educators' level of proficiency with technology and how they are integrating it into the geometry classroom. The application of technology in the geometry class is crucial as it can enhance the engagement of auditory and visual learners. The question asked was "Does your school have technological facilities/materials that you think can assist in teaching geometry? If yes, how often do you use the facilities/materials? If not, if you had them do you think these materials can play a role in improving learning and teaching in geometry? Would you be able to use the technologies materials/facilities without any challenges if provided to the school? When queried about the presence of technological facilities or materials in their schools that could aid in teaching Geometry, as well as their ability to utilise such resources, the educators responded as follows:

> *No, not much. There are certain resources that I require, but unfortunately, I do not possess them. Also, I lack the necessary technological proficiency to utilise them (Educator A).*
> *Yes, we possess the necessary materials; however, their utilisation is currently hindered due to ongoing construction, resulting in the disconnection of the cables. I am proficient in using them, as the process is straightforward. Also, we have access to projectors and smart TVs. We simply log in and access any topic that is relevant for the day (Educator B).*
> *Unfortunately, we do not have any technological facilities available. If they could provide me with some guidance on how to use the facilities or materials, I believe I would be able to make use of them (Educator C).*
> *Yes, our school does offer a wide range of technological materials. I haven't used them much, as I find it easier to explain concepts in a classroom. I tend to struggle with technology, so I prefer to avoid relying on it (Educator D).*
> *Yes we do, we have a lab around. We use the facility once or twice a week (Educator E).*

Based on the interview data, it is evident that out of the 5 schools that participated, only 3 of them have been confirmed by the educators to have technological facilities. The remaining 2 schools, however, lack such facilities and materials. Out of all the educators teaching geometry, only one of them utilised technological facilities. Although one educator mentioned being well-versed in technology, they expressed frustration at not being able to utilise it due to ongoing construction at their school. The other educators confirmed that while they have access to technology facilities, they tend to struggle with using technology. As a result, they prefer to avoid relying on it in the classroom. Two other educators mentioned that they were not familiar with technology, but they expressed their willingness to learn if they were given the necessary resources and training. One learner at the school, who has access to technology facilities, expressed a preference for being taken to the mathematics laboratory when facing learning difficulties. This learner finds it more effective to visually comprehend the content by observing shapes. The next section focuses on the learners' attitude towards geometry.

5.4.7. Learner's Attitude towards Geometry

A positive disposition towards geometry is crucial for learners' academic progress. Learners who possess a pessimistic attitude towards the subject are likely to underperform, as a positive attitude fosters learners' enthusiasm for the learning process. The question asked was "How do you like/feel about learning geometry in math class?" When probed about their attitudes towards learning Geometry in the classroom, learners had the following to articulate:

> *I feel happy because I understand what my teacher is teaching me in geometry (Learner A1).*
> *Umm, I like learning Geometry and I feel like every year they could teach us Geometry all over again (learner A2).*
> *Um, okay. I feel great. We gain a great deal of knowledge. We primarily learn about measuring houses and homes (Learner B1).*
> *I feel content, as it assists my focus and brings a sense of calm to my mind. I feel like I have a solid foundation on everything, which will serve as evidence of my learning (Learner C1).*
> *I feel really good because I get to express myself and get to enjoy what I learn in class (learner D2).*

Shown above are quotes from learners regarding their sentiments toward learning geometry. All of them conveyed a sense of enthusiasm and joy in the process of studying geometry. This indicates that the learners possess a favourable disposition towards geometry.

5.5. Summary and conclusions.

The primary focus of this chapter was to analyse the lesson observations of grade 6 educators teaching Geometry. The observed topics included the teaching of 2D shapes, 3D shapes, and measurement. The themes observed in this study were prior knowledge facilitation, teaching new knowledge, and application of the learners' knowledge. The analysis conducted a detailed examination of the geometry assessment administered to the learners. The analysis centred on summarising the schools' performance in geometry by utilising descriptive statistics and examining the distribution of data for this subject. Furthermore, the examined content included the learners' responses to assessments on 2D shapes, 3D shapes, and measurements. Furthermore, their responses to the Van Hiele level in terms of visualisation and analysis and their understanding of CK and PK were analysed. The final segment examined comprises the structured interviews conducted with the learners and educators. The analysis encompassed the following identified themes: the conceptualisation of geometry teaching and learning, challenges encountered in teaching and learning geometry, the learning and teaching resources in geometry, the learning of conceptual and/or procedural learning, the application of geometry in real-life scenarios by learners, the utilisation of technology in geometry, and learners' attitudes towards geometry. In the following chapter, data analysis is discussed.

CHAPTER 6: DISCUSSION (RESULTS INTERPRETATION)

6.1. Introduction

This chapter provides an interpretation and discussion of the analysed data, following the data presentation in the previous chapter. The study addresses three primary research questions: 1) how do educators in the intermediate phase **impart** Procedural Knowledge (PK) and Conceptual Knowledge (CK) in geometry? 2) How do learners in the intermediate and senior phases **respond** to assessments of PK and CK in geometry? 3) What is the **impact** of teaching CK and PK in learning geometry? Therefore, this chapter examines the findings from the analysis of educators' observations, learners' assessments, and interviews with educators and learners to address the three research questions. The next section addresses the first research question.

6.2. The imparting of procedural and conceptual knowledge in geometry (Research Question 1)

The discussion begins by addressing the first research question and the discussion of the analysed data, which includes observations of educators teaching about 2D shapes, 3D shapes, and measurement. The subsequent section centres on the discourse surrounding the teaching and learning of 2D geometric figures.

6.2.1. 2D shapes teaching and learning

Although the results, as indicated in 5.2.1, show that educators do help learners recall their prior knowledge when teaching 2D shapes, they are using traditional teaching methods that do not enhance learners' CK. This approach fails to tap into learners' everyday experiences and real-life objects to explore their prior knowledge. Following the moment when learners encountered difficulty in determining whether triangles are polygons. Bernabeu et al. (2019) discovered that the way learners think about these attributes is affected by the differentiation between discursive and non-discursive registers. Research indicates that possessing the capacity to elucidate the relationship between characteristics is sufficient to generate a figure with specific distinguishing traits. During my observation, the educator encountered difficulties with learners' understanding of the definition of polygons. Consequently, the educator resorted to using the learners' home language to ensure comprehension. The findings of this study were consistent with the research conducted by Alex and Mammen (2018), which highlighted the challenges learners faced in understanding the language used in relation to quadrilaterals and

triangles. Bernabeu et al. (2019) suggest that there is a hierarchical relationship between verbal declarative knowledge and perceptual apprehension in such learners. This indicates that the choice of words used by the educator during the teaching of 2D shapes was crucial for the learners' progress and comprehension of both PK and CK.

When introducing new concepts about 2D shapes, it has been observed that many educators still rely on traditional teaching methods. Moreover, certain educators possess insufficient expertise in subject-specific content, and they infrequently integrate tangible objects from everyday life that learners can readily connect with. This is in contrast to the CPA theory. CPA theory posits that CK is acquired through active and interactive involvement with physical objects (Leong et al. 2015, Chang et al., 2017). This suggests that without the active and interactive use of physical objects in teaching 2D shapes, learners will persist in having limited CK and encounter difficulties in comprehending 2D shapes. More so, the findings are consistent with Tachie's (2019) research, which highlighted that educators lacked both mathematical subject content knowledge and pedagogical content knowledge to teach geometry effectively. The observation revealed that certain educators, relying solely on textbook definitions, fail to provide practical examples from real-life situations when teaching 2D shapes. In line with the findings of Murni et al. (2017), it has been observed that traditional approaches to teaching geometry, such as utilising paper and pencil, frequently lead to learners' inability to accurately represent geometric concepts. More so, Khoule et al. (2017) discovered that using procedural or traditional teaching methods did not improve learners' level of comfort with mathematics.

Observations on the application of knowledge for 2D shapes indicate that learners encountered challenges in applying their acquired knowledge. The study findings indicate that learners encountered difficulties comprehending concepts related to quadrilaterals, and triangles, distinguishing irregular and regular shapes, and accurately labelling specific quadrilaterals. Consequently, educators had to utilise repetitive explanations and practice to strengthen the learning process. The learners faced challenges when trying to sketch 2D shapes with sides of different lengths. Consistent with the research conducted by Ismail et al. (2020), it was discovered that learners encounter challenges in identifying various categories of 2D shapes and face difficulties in accurately drawing these shapes. Therefore, the lesson observations of educators teaching 2D shapes from this study indicate that learners face difficulties applying 2D shapes. This may be attributed to educators' use of traditional teaching methods for 2D

shapes. According to Jelatu et al. (2018), it has been suggested that traditional teaching methods may not be advantageous for learners when it comes to acquiring geometry concepts. The following section will address the discussions and interpretation of the observed results regarding the teaching and learning of 3D shapes.

6.2.2. 3D Shapes teaching and learning

The findings from the study on the facilitation of prior knowledge suggest that most educators effectively facilitated learners' prior knowledge by establishing connections between 3D shapes and 2D shapes. The educators connected tangible objects from everyday life to the lesson to enhance the learners' existing knowledge. By establishing a connection between the concepts of 2D shapes and 3D shapes, learners develop a solid foundation of CK of 3D shapes. This foundation facilitates their exploration of the topic in greater depth. Based on the research conducted by Haviger and Vojkůvková (2014) and Sulistiowati et al. (2019), learners frequently face challenges when attempting to solve geometry problems. These difficulties can be attributed to various factors, including a deficiency in their pre-existing knowledge of the subject matter (Haviger & Vojkůvková, 2014; Sulistiowati et al., 2019). Therefore, it is crucial to take into account the learners' existing knowledge when introducing the concept of 3D shapes to enhance their understanding. Star (2013) contends that learners can only memorise the processes for concepts if they do not have any prior CK. This implies that prior to instructing learners on the PK of the concepts, it is crucial to prioritise the CK first.

Educators utilise various teaching tools when teaching new knowledge about 3D shapes. The observations revealed that some educators effectively engaged learners by employing tools like 3D geometric solid folding nets and instructional videos accessed through the math lab. However, other educators relied on traditional methods such as chalkboards and textbooks to convey the knowledge, although some of their lessons still managed to captivate the learners. Utilising instructional materials such as tangible objects and videos is crucial for enhancing learners' understanding. When teaching about three-dimensional shapes, it is essential for learners to visually perceive the actual three-dimensional aspects of the shape, rather than relying solely on textbooks or chalkboard drawings. This approach will greatly improve learners' comprehension of three-dimensional shapes. Consequently, Ping and Hua (2016) elucidate the reason for the challenges learners encounter in understanding the characteristics of 3D shapes, leading many educators to rely on traditional methods of rote learning.

Furthermore, a study conducted by Ismail et al. (2020) revealed that learners face challenges when learning to draw the net of a 3D shape. Specifically, they encounter difficulties in combining basic shapes to form a net. As illustrated in Figure 5.1, *Educator D* utilised geometric 3D solids to teach the concept of folding nets in class. This was essential for the learners to understand how 2D shapes can be transformed into 3D shapes through the process of folding nets. Consequently, learners may encounter difficulties when merging the nets if they lack comprehension of the relationship between 2D and 3D shapes.

It is evident that certain educators predominantly relied on traditional methods to teach geometry, indicating a lack of innovation in knowledge application. *Educator D*'s teaching approach involved a significant amount of lecturing, resulting in a lack of engagement from the learners. Consequently, certain learners started to exhibit disruptive behaviour and engage in conversations with their peers during class. These findings align with the study conducted by Khoule et al. (2017), which concluded that using procedural or traditional teaching methods did not improve learners' level of comfort with mathematics. Consequently, learners lose interest when they are not actively engaged in the class, particularly if the lesson is focused on the educator rather than the learners themselves. Therefore, the results of Zulnaidi and Zamri's (2017) study indicate that learners who incorporate technology resources in their geometry class demonstrate greater proficiency in both PK and CK in mathematics, in comparison to those who depend on traditional teaching methods. The effectiveness of the lesson in *Educator E*'s Math lab was evident as the learners were actively engaged and responsive. The use of videos and visuals proved to be beneficial, especially for those who learn better through visual aids. The following section centres on the discussions and interpretations of the analysis of the observation in the teaching of measurement.

6.2.3. Teaching and Learning of Measurement

The observations on measurement indicate that educators are not making connections between learners' prior knowledge and the teaching of measurement concepts. The lack of utilisation of prior knowledge in measurement lessons suggests that educators are not effectively integrating learners' existing knowledge into the teaching of measurement topics. In contrast to the findings of Katrin et al. (2018), who propose that mathematics is a subject characterised by a cumulative nature, meaning that acquiring knowledge and skills depends on previous learning experiences. Therefore, educators must activate learners' prior knowledge before introducing a new topic to

enhance and cultivate their new knowledge. Furthermore, a study by Dhlamini et al. (2019) revealed that a limited number of learners could effectively apply their existing knowledge to integrate spatial geometry mathematical reasoning. This can be attributed to learners possessing solely procedural skills in geometry and solving problems through rote memorization, as they are taught.

The study's findings on instructing learners in new knowledge indicate that educators often fail to establish links between the topic of measurements and real-life examples. The results show that when teaching the area, volume, and perimeter educators explain the definitions of the concepts without making a connection to their real life. With *Educator B* the educator delivered a concise explanation to learners regarding the computation of perimeter while teaching the relevant concepts. However, the incorrect explanation given to learners was that the perimeter does not involve square units because it encompasses calculating all four sides. Nevertheless, this explanation is incorrect. The omission of the square unit in the perimeter calculation is not attributed to the inclusion of the four sides in the perimeter. The educator should have demonstrated to learners that square units are derived from calculating the products of two lengths. This demonstrates that the educator possesses insufficient content knowledge in the field of measurement. This aligns with a study by Safrina and Darmawan (2016), which discovered that primary school educators face difficulties comprehending concepts such as angle measurement, defining perpendicular lines, and employing appropriate techniques for drawing straight lines.

On the other hand, *Educator D* elucidated to the learners that the Perimeter of a two-dimensional shape refers to the total distance around it. To facilitate understanding, he offered practical illustrations, such as employing a trundle wheel to measure the Perimeter of the school. This aligns with the findings of Idrus et al. (2022), who contend that establishing a direct correlation between CK and measurements will facilitate the interactive and iterative development of learners' mathematical proficiency. Furthermore, educators who merely provide definitions without offering real-life examples in the classroom demonstrate a lack of creativity and instead rely on traditional approaches to teaching. A study conducted by Makgakga (2023) found that lesson-study using custom-made videos effectively enhances the teaching of surface area and volume concepts for rectangular and cylindrical prisms, leading to improved learner outcomes. This indicates that educators can effectively enhance learners' knowledge by employing creativity and technology in geometry lessons.

The outcomes of the observation related to the application of knowledge *Educator A* employed a pedagogical approach wherein learners were compelled to sing the formula repeatedly to attain mastery. Nevertheless, this approach failed to adequately promote the development of CK. According to Jelatu et al. (2018), the effectiveness of learning activities decreases when learners are instructed to simply memorise knowledge without understanding it. This suggests that when educators employ repetitive teaching methods and passive knowledge transmission, learners may not fully comprehend or internalise the content. Consistent with the findings of Yurniwati and Soleh (2019), this study suggests that educators have a strong CK regarding the formulas used for calculating area and volume. However, their low PK score can be attributed to a lack of understanding of mathematical concepts and measuring procedures. These findings indicate that educators relied on rote memorisation to teach the formulas for calculating area and volume, while their understanding of the concepts of measurement was lacking. The following section presents the outcomes of the analysed assessment of the learners and provides an interpretation of the findings.

6.3. Learners' response to procedural and conceptual knowledge assessment in geometry (Research Question 2)

This section of the discussion addresses the second research question and presents the analysed data. The data includes an evaluation of the learners' performance in the field of geometry. The discussion centres on evaluating learners' performances of 2D shapes, 3D shapes, and measurements such as area, perimeter, and volume. Furthermore, the discussion and analysis centre on learners' proficiency concerning the Van Hiele level (visualisation and analysis) and delve deeper into their conceptual and procedural abilities. An analysis of data comparing the overall performance in geometry between grade 6 and grade 9 learners reveals that a greater proportion of grade 6 learners, specifically 32.1%, achieved high performance compared to grade 9 learners, who had a performance rate of 27.5%. The findings suggest that grade 6 learners had a greater proportion of individuals who obtained scores ranging from 50% to 100% compared to grade 9 learners. This indicates that the grade 9 learners CK did not exhibit any progress or improvement over some time. In the following section, we will focus specifically on the discussion and interpretation of the performance of grade 6 and grade 9 learners in assessing 2D shapes.

6.3.1. Learners performance in 2D shapes

When examining the performance of grade 6 and grade 9 learners in 2D shape, it is evident that grade 6 learners achieved a significantly higher percentage of 69.7% above the average, while grade 9 learners attained an average percentage of 56.7%. The study results indicate that although grades 6 and 9 performed above average, grade 6 outperformed grade 9. This suggests that learners are taught through procedural methods, leading them to learn the content, prepare for exams, and subsequently forget it. Since the content was familiar to them from grade 6 and they had studied geometry consistently up to grade 9, higher performance was anticipated. This study aligns with the research conducted by Fujita et al. (2020), which focused on spatial reasoning skills related to 2D representations of 3D geometrical shapes in learners from grades 4 to 9. The findings indicated that while some progress is observed across the grades, specific issues show limited improvement from Grades 6 to 9. Many Grade 9 learners also faced difficulties. Moreover, this is in line with a study conducted by Bernabeu et al. (2021), where their research showed a progressive improvement in primary learners' understanding of 2D shape categories. In the following section, we will focus specifically on the discussion and interpretation of the performance of grade 6 and grade 9 learners with the assessment of 3D shapes.

6.3.2. Learners performance in 3D shapes

Comparing the mean values of the control group (5.13) and experimental group (6.41), it is evident that both groups of learners scored below the average. Grade 9 learners averaged 24.4% on 3D shapes, while grade 6 learners averaged 30.5%. Both percentages indicate that the grade's performance was significantly low and below average. Although both groups' performances were significantly low, the grade 6 group outperformed the grade 9 group slightly. This study contradicts the findings of Pavlovičováa and Švecová (2015), who reported that the overall test score for 3D shapes was 73.54%, considered satisfactory. They found that learners in their study found it easier to handle the sides of 3D shapes compared to the edges or vertices. However, this study aligns with Ismail et al. (2020), who found that learners struggled with 3D shapes and counting their edges. Similarly, in this study, learners faced challenges counting the edges of 2D shapes. The study aligns with previous research by Zawani (2014) and Ping and Hua (2016), indicating that learners continue to struggle with identifying the characteristics of 3D shapes. In the following section, we will focus specifically on the

discussion and interpretation of the performance of grade 6 and grade 9 learners with the assessment of 3D measurement.

6.3.3. Learners Performance in Measurement

The results from both grade 9 and grade 6 show below-average performance among the learners in the subject of measurement. There is a noticeable lack in the performance of grade 6 and 9 learners. According to the results, it is clear that just 7.6% of grade 6 learners managed to attain an average pass. In the same way, only a small percentage of 4.1 in grade 9 achieved the average. The results indicate that grade 6 outperformed grade 9 by 3.5%. Although these findings are concerning, they indicate that learners face significant challenges in calculating area, volume, and perimeter. This study's findings align with Machaba (2016), indicating that learners struggle with understanding area as a 2D surface, as shown by their interpretation of area as the result of length and width. As a result, learners do not have a clear understanding of the concept of area. Furthermore, learners were found to have a limited grasp of the concept of 'perimeter'. All the learners struggled to provide an accurate definition of perimeter, according to Machaba (2016). This study highlights that learners faced challenges when calculating area and perimeter, possibly due to a lack of understanding of the concepts of Area and Volume.

This study is in line with Solihatun et al. (2019), who discovered that learners faced difficulties in expressing and calculating the area and perimeter of geometric shapes like rectangles, triangles, and squares. In line with Abadi and Amiri's (2022) study, this research reveals that primary school learners face difficulties when solving problems related to the perimeter and area of two-dimensional shapes. In addition, this study contrasts with Makgakga's (2023) research, which found a significant improvement in understanding surface area and volume concepts for rectangular and cylindrical prisms. In Makgakga's study, technology played a crucial role in improving outcomes by creating an interactive and interesting learning environment in the classroom, focusing on teaching both CK and PK in contrast to traditional teaching methods. It is suggested that the use of technology played a crucial role in improving learners' CK and PK in the measurement topic. In the following section, we will focus specifically on the discussion and interpretation of the performance of grade 6 and grade 9 learners about their performance in terms of Van Hiele level (visualisation and analysis).

6.3.4. Performance of learners at the Van Hiele level

The study results indicate that in grade 9, the majority of learners demonstrate an above-average level of visualisation skills with hexagons, with an average of 56.1%. Conversely, data indicates that in grade 6, the majority of learners' exhibit above-average skills in visualising hexagons, with an average of 57.2%. The grade 9 and grade 6 learners demonstrated above-average visualisation skills, scoring nearly the same percentage. The results indicate that both groups of learners' performances regarding the Van Hiele visualisation level are adequate. 40.1% of grade 9 learners showed subpar analysis skills when analysing a square. The data shows that most grade 6 learners can analyse squares, with an average score of 53.1%. The results show that grade 9 learners performed below average, while grade 6 learners demonstrated above-average skills in analysing a square. The grade 9 learners are currently performing at a visualisation level but have not reached the analysis level. They are expected to be at the Informal Deduction stage, which involves categorising and making generalisations based on qualities.

This study aligns with Masilo's (2018) research, indicating that despite expectations for learners' comprehension of geometry to reach level 5 during the Further Education and Training (FET) phase, it often remains at level 1. This study highlights that grade 9 learners are currently at Van Hiele level 1, while senior phase learners are expected to be at level 3. This may be due to educators teaching learners using PK and rote learning methods without ensuring they truly understand the concepts. Presmeg (2020) argues that reaching a specific Van Hiele level of knowledge requires more than just memorisation or following set procedures without comprehension. The study by Machizi and Feza (2021) demonstrated the potential to assist underprepared learners in upper secondary school geometry classes to improve and excel in mastering geometric proofs at level 5. The study demonstrated that the Van Hiele theory-based instructional model had a more significant positive effect on learners' geometric proof skills compared to the traditional teaching method. In the following section, we will focus specifically on the discussion and interpretation of the performance of grade 6 and grade 9 learners concerning their performance in terms of CK and PK.

6.3.5. Learners' performance in conceptual and procedural knowledge

When comparing the performance of grade 9 and grade 6 learners, the results indicate that both groups performed poorly in terms of the volume of CK and PK. Analysis of the CK

performances shows that 24.2% of grade 9 learners reached a satisfactory level, while 32.5% of grade 9 learners achieved the same. Both groups performed poorly, but the grade 6 group showed slightly better performance, with an 8.3% increase in volume. The PK of the volume was analysed, showing that grade 9 achieved a 19.1% result, while grade 6 achieved a slightly higher result of 22.1%. The results show that the grade 6 learners performed better than the control group by a 3% margin. Both groups' results show that the learners performed slightly better in CK than in PK. This finding is consistent with previous research in geometry, such as Machaba (2016), which revealed that learners lacked CK of area as a 2D surface, as demonstrated by their belief that area is the result of multiplying length and width. In this study, learners faced challenges related to volume in both CK and PK. According to Al-Mutawah et al. (2019), the results of the area and circumference calculations in geometry were inadequate. Additionally, a study by Abadi and Amiri (2022) revealed that learners faced challenges primarily due to language difficulties and a lack of adequate CK. Zulnaidi and Zamri's (2017) study indicates that learners who use GeoGebra in their maths education demonstrate higher levels of PK and CK in maths than those who use traditional teaching methods.

Tan-Sasisman and Aksu (2016) found that learners' error patterns were shaped by their pre-existing CK and PK schemas established earlier. Tan-Sasisman and Aksu (2016) found that learners need both the knowledge to perform measurements and an understanding of the purpose and significance of measurements to fully comprehend the concept of measurement. Khoule et al. (2017) discovered that both PK and traditional teaching methods did not improve learners' mathematical comfort, indicating a detrimental effect. The CK teaching method, as described by Khoule et al. in 2017, effectively decreased mathematical anxiety. A study by Nahdi and Jatisunda (2019) found that advanced learners of CK can effectively deal with unfamiliar challenges. Whereas a study by Laily et al. (2020) shows that learners at the visualisation level have some understanding of CK, but none of them show expertise in PK. The study results align closely with those mentioned here. Learners faced challenges with both CK and PK, although in other studies learners struggled with only one type of knowledge. Furthermore, multiple studies suggest that learners who struggle with CK are also likely to struggle with PK. In this study, we recommended incorporating both CK and PK in teaching, emphasising presenting CK before PK to facilitate learning for learners. Past studies further indicate that geometry lessons structured with technology are more effective than traditional teaching methods. The following section presents the outcomes of the analysed interview of the learners and educators through the interpretation and discussion of the findings.

6.4. The impact of teaching procedural knowledge and conceptual knowledge in learning of geometry (Research Question 3)

This section discusses and interprets the viewpoints of educators and learners regarding geometry, focusing on the impact of teaching PK and CK. The section focused on the perspectives of educators and learners regarding the significance of geometry as a subject in both academic settings and practical applications, as well as their attitudes toward the subject. We examine the teaching materials favoured by educators and the materials that learners are exposed to. We explore the obstacles encountered in the teaching and learning of geometry. We examine whether learners prefer being taught geometry through CK or PK. The following section focused on the conceptualisation of geometry by learners and educators.

6.4.1. Learners and educators Geometry conceptualisation

The results indicate that educators have a firm grasp of the importance of teaching geometry in the classroom. They understand how it can impact learners' future careers and its relevance in real-world scenarios. On the one hand, some learners understand the importance of geometry, while on the other hand, the results show that other learners struggle to see its relevance and apply it to real-life situations. The study findings align with Mamiala, Mji, and Simelane-Mnisi (2019) research, which revealed a significant lack of interest and understanding of Geometry among learners. The results show that most learners are struggling to make connections between the geometry concepts taught in class. Learners recognise the practical importance of geometry in everyday life situations. Only a marginal of learners can relate geometry to real-world scenarios. This aligns with Rohendi et al.'s (2018) and Juman et al. (2022) study, indicating that many learners face significant challenges in acquiring geometric knowledge. Satiti et al. (2021) argued that learners' capacity to utilise mathematical concepts in various real-world scenarios is crucial to attaining proficiency in mathematics. The results indicate that learners expressed enthusiasm and joy while learning geometry, indicating a positive attitude toward the subject. This study contradicts Mamali's (2015) and Mamiala et al.'s (2019) studies, which show learners have a negative attitude towards geometry. The following section discusses the challenges encountered by learners and educators in geometry.

6.4.2. Challenges encountered in geometry

The results indicate that educators face various challenges when teaching geometry. The main challenges involve learners having difficulty grasping the content. Learners encounter difficulties with calculations, angles, 3D shapes, and understanding new concepts in geometry. This study aligns with Ismail, Abdullah, Syuhada, and Noh's (2020) research, which concluded that learning difficulties in Mathematics lead to learners facing challenges with numbers, formulas, calculations, concepts, and related shapes due to the requirement of patience and persistence. A study conducted by Juman et al. (2022) found that both learners and educators perceive geometry as a challenging topic within mathematics. The results of this study indicate that some learners reported no difficulties in geometry based on their responses. However, many learners are experiencing challenges with calculations, shapes, and vertices. The challenges identified by the learners are the same areas that educators have pinpointed as the most difficult for learners in geometry. This discovery aligns with research by Mamiala et al. (2020), indicating that difficulties in learning Geometry are a factor in learners' underperformance. The study revealed a significant lack of interest and understanding among learners in the subject of geometry. Geometry is often perceived as a challenging mathematical concept due to its complexity and difficulty in comprehension. The study (Mamiala et al., 2020) argued that despite concerns about learners' interest, most still consider Mathematics important and recognise its relevance. The findings align with the results of my study, indicating that despite facing specific challenges in geometry, learners still value the subject and demonstrate interest in learning. The following section discusses the learning and teaching resources in geometry that educators utilise and learners find beneficial.

6.4.3. Learning and teaching material resources in geometry

The educators and learners emphasised the successful utilisation of real-life objects and solids in teaching geometry. This method assists learners in understanding the concepts more effectively. The results show that educators rely significantly on textbooks, chalkboards, chalk, and pictures. The learners have clearly shown a preference for visual presentations that link their lessons to real-life objects. The study by Ismail et al. (2020) highlights that challenges in understanding the 'Shape and Space' topic stem from educators' reliance on traditional teaching methods. This includes an educator-centred approach during the facilitation process and limited technology integration in the classroom, often relying solely on textbooks for teaching. The

study by Tachie (2020) reveals that educators often talk and write during teaching without providing explanations for learners to grasp concepts.

From the results on technology use, it is clear that out of the 5 participating schools, only 3 have been verified by educators to possess technological facilities. However, the other 2 schools do not have those facilities and materials. Of all the geometry educators, only one used technological resources. Despite being knowledgeable in technology, one educator expressed frustration about being unable to use it because of ongoing construction at their school. The other educators acknowledged that, despite having access to technology facilities, they find it challenging to use technology. Consequently, they choose to refrain from depending on it in the classroom. Two educators admitted their lack of familiarity with technology but stated their readiness to learn if provided with the required resources and training. A learner at the school with access to technology resources preferred going to the mathematics laboratory when encountering learning challenges. This learner prefers to visually understand the content by looking at shapes. The results indicate that educators are not utilising technological resources to teach geometry. Research has shown that utilising digitally enhanced instructional platforms helps enhance the understanding of hierarchical connections among different shapes (Bussi & Baccaglini-Frank, 2015; Ng & Chan, 2019). Sharma (2019) recognised the significance of technology in enhancing the understanding of CK and its different forms and representations. A study by Naidoo (2018) clearly shows the critical role of technology-enhanced learning in supporting learners to comprehend the dynamic relationship between area and perimeter. The following section discusses the learners' preferred method of teaching, whether it is CK or PK.

6.4.4. Preference for teaching methods based on conceptual or procedural knowledge among learners.

Findings from The learners interviewed indicated a preference for starting with a CK of geometry, focusing on grasping the concepts rather than memorising procedures. The learners emphasised the significance of grasping concepts rapidly and efficiently. The learners emphasised that learning conceptually enables them to independently discover solutions, whereas depending only on procedural learning may result in confusion. This study supports Skemp's (1971) claim that using procedural teaching methods for memorisation can contribute to the development of mathematics anxiety in learners. Presmeg (2020) argues that reaching a specific Van Hiele level of knowledge requires more than just memorisation or following set

procedures without comprehension. Manandha et al. (2022) noted that CK levels were significantly lower than PK levels. Manandha et al. (2022) stress the significance of educators improving their CK. Mutawah et al. (2019) suggest that educators should focus on developing CK skills rather than emphasising the acquisition of PK skills needed for calculations. Nahdi and Jatisunda (2019) assert that developing learners' PK and CK will enhance their capacity to understand novel concepts. This study emphasises the equal importance of PK and CK; however, it suggests that CK should be taught before PK. The following section consolidates and integrates the discussion of the results from the three research questions.

6.5. Consolidation and conclusion of results discussion

The study revealed that educators predominantly rely on traditional teaching methods, such as textbooks, writing on the board, and excessive educator-centred verbal communication. Learners have expressed a preference for learning geometry using concrete objects that are linked to their real-life experiences. The educators do not utilise technology to teach geometry in their schools. However, learners who have been exposed to technology have shown enthusiasm and a preference for learning geometry with technological assistance. The learners' geometry performances were subpar, yet they still showed enthusiasm for learning the subject. This indicates that the poor performances are attributed to learners being instructed using traditional methods and rote memorisation in geometry. Learners have stated a preference for conceptual teaching, emphasising that it allows them to independently discover solutions instead of relying solely on procedural learning, which can lead to confusion.

The comparison of the performances of grade 6 and grade 9 learners shows that the grade 9 learners performed slightly worse than the grade 6 learners. This assessment was conducted during term three, in line with their Annual Teaching Programme (ATP), indicating that they were studying the topic of geometry at that time. This indicates that the grade 9 CK did not progress from grade 6 to grade 9 due to learning geometry through rote memorisation rather than understanding the concepts. Therefore, they scored poorly on the grade 6 assessment, which focused on the visualisation and analysis level of the van Hiele model of geometry understanding. By grade 9, they are expected to perform at the informal deduction level. The learners are currently at the visualisation level due to being instructed through rote memorisation and procedural teaching, hindering the development of CK and advancement to

higher Van Hiele levels. The following chapter will focus on the recommendations and conclusions drawn from the study.

CHAPTER 7: RECOMMENDATIONS AND CONCLUSIONS

7.1. Introduction

This chapter provides recommendations based on the results and findings of the three research questions: 1) how do educators in the intermediate phase impart PK and CK in geometry? 2) How do learners in the intermediate and senior phases respond to assessments of PK and CK in geometry? 3) What is the impact of teaching CK and PK in learning geometry? The chapter further focuses on the study's limitations, Recommendations for future research, and concluding remarks. The following section will address the study's recommendations.

7.2. Recommendations for the study

The study aimed to explore how geometry educators teach PK and CK to learners during the intermediate phase. Examines the responses of intermediate and senior phase exit grade learners' responses to Procedural Knowledge (PK) and Conceptual Knowledge (CK) assessments in geometry within a mathematics classroom. Explore how CK and PK impact geometry teaching and learning. The aims were successfully achieved. Thus, the study's objectives were to address the common challenges in geometry mathematics education, particularly in South Africa, and to help improve geometric performance in schools. Enhancing the teaching and lessons of Geometry and developing a strategy to improve the skills of both educators and learners in geometric mathematics.

Recommendations are formulated according to the study's objectives and aims. Results indicated that teaching and learning in geometry often rely on traditional methods. Recommendations that educators transition to teaching styles that actively engage learners. The teaching should adopt a more pragmatic and interactive approach, allowing learners to actively engage in the entire learning process, thereby facilitating the acquisition of conceptual knowledge through practical application. Geometry should be taught using solids and real-life objects to help learners connect the concepts to their own experiences. Educators struggle with technology skills, and some schools lack necessary technology resources. It is recommended that the department provide schools with technological equipment and resources to support educators in teaching geometry. Moreover, organising training workshops for educators who may not be technologically competent would be beneficial. According to the learners, they strongly prefer learning with technology, which is evident by their excitement when visiting the maths lab. Learners in grades 6 and 9 struggled with the geometry assessment, likely due

to the use of traditional teaching methods and educators relying on rote memorisation teaching. It is recommended to teach CK and PK effectively in geometry. Educators should focus on teaching CK skills before moving on to PK. This approach will improve learners' CK and PK skills and help them reach the expected Van Hiele level of understanding for their grades. Other concerning factors have been observed. Educators have expressed their concerns about overcrowding, stating that it makes it challenging to address all the learners' needs due to the large number of learners. It is recommended that the DBE take action to address overcrowding in schools. The following section will address the study's limitations.

7.3. Limitations of the study

Only learners in grades 6 and 9 and educators in grades 6 from selected schools in the Mkhuhlu Circuit were included in the study; therefore, it is not possible to draw generalised conclusions. The study conditions in the Mkhuhlu Circuit may differ from those of schools in other regions of South Africa. Hence, the findings and conclusions are primarily related to specific schools within the Mkhuhlu circuit. The study examined specific aspects of geometry, including 2D shapes, 3D shapes, and measurements, through selected questions. It did not cover the entire scope of geometry, which limits the generalisability of the study's results. Due to the large amount of data collected and the word count limitation, the study focused solely on pentagons and squares when examining Van Hiele levels. This limited scope prevents generalisation to the entire geometry topic. The study specifically analysed the volume CK and PK, while drawing general conclusions on other aspects. The study concentrated solely on descriptive statistics due to extensive data collection and did not include inferential statistics. The study did not examine factors such as sex, social and economic background, age, and educational background, which also added to the limitations of the study. The following section will focus on the study's recommendations for future research.

7.4. Recommendations for Future Research

The limitations of the study of the study suggest that the samples were collected from schools in a specific region only. It is recommended to conduct the study in different regions of the country. Further research should explore aspects of geometry not addressed in this study. Moreover, a study should be conducted to assess learners' understanding of all geometry components according to the Van Hiele levels. It is recommended to include inferential statistics and also to investigate factors like gender, social and economic background, age, and

educational history, which were limitations of the study. The following section will focus on the finalisation of the study.

7.5. Concluding remarks

Firstly, the study has provided me with an opportunity to explore much. Throughout the writing process, I have gained extensive knowledge about the topic of mathematics geometry and the associated theories. Additionally, I have gained valuable insights from both learners and educators during the data collection process. I will utilise the positive findings and address the negative ones in my implementation.

The study examined three research questions. The first question investigated how educators teach PK and CK in geometry. The results indicated that educators are utilising traditional methods to teach this knowledge. The second research question aimed to investigate the performance of learners in PK and CK assessment tasks in geometry. The results indicated that the students of grades 6 and 9 performed poorly in geometry assessments. The third research question examined the impact of teaching and learning PK and CK in geometry. The results showed that learners had a positive attitude towards geometry, with a greater interest in learning through CK, concrete objects, visuals, and technology rather than PK or traditional teaching methods. The study's findings were consistent with some of the literature, while others were in contrast to it.

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9. APPENDICES

Appendix A

Observation schedule

The following observation schedule aims to gather data on how educators in the intermediate phase impart procedural knowledge in the classroom.

Observation focus variable: Van Hiele visualisation level

Variables themes		Observation results(Data)
Teacher activity	Learners' activity	
Learners have access to resources in the form of geometric figures (or other comparable forms). The educator poses a question and allows learners to produce possible answers based on their prior knowledge. The educator delivers a new concept and allows learners to interact with it.	The learner recognises and manipulates figures (e.g., squares, triangles) and other geometric configurations (e.g., angles, lines,) based on the look of the forms.	

Observation focus variable: Van Hiele Analysis level

Variables themes		Observation results(Data)
Teacher activity	Learners' activity	
The teacher explains based on the proof gathered during the probing process. The teacher gives students a method and some questions that will help them figure out the properties of different geometric shapes.	Learners study the shapes based on their qualities and interactions between properties (brainstorming using prior knowledge and working with new ideas). Learners engage in activities that lead to the discovery of hidden connections.	

Observation focus variable: Pre-knowledge (Conceptual Knowledge and Procedural knowledge)

Variables themes		Observation results(Data)
Teacher activity	Learners' activity	
How did the educator facilitate the student's recall of prior knowledge? How prior knowledge was made applicable to the current topic? What resources were utilized? How were the resources utilized?	How did the students react to the teacher's method of recalling prior knowledge? Was the students' prior knowledge relevant? Is it possible to apply or connect the students' prior knowledge to the new topic?	

Observation focus variable: New knowledge (Conceptual Knowledge and procedural knowledge)

Variables themes		Observation results(Data)
Teacher activity	Learners' activity	
How did the educator introduce the new topic at hand and concepts? What resources/materials were utilized to deliver the new content? What teaching approach did the educator employ?	Did the learners participate in the discovery of new information? Were they reliant on the educator? How did the learners respond to the educational approach and teaching resources?	

Observation focus variable: knowledge application (Conceptual Knowledge and procedural knowledge)

Variables themes		Observation results(Data)
Teacher activity	Learners' activity	
What examples and questions/problems did the educator utilize to apply new procedural and conceptual knowledge?	Did the learners understand and grab the new knowledge? Did they successfully apply new knowledge in problem-solving?	

Observation focus variable: conclusion (Conceptual Knowledge and procedural knowledge)

Variables themes		Observation results(Data)
Teacher activity	Learners' activity	
How did the educator link the whole content with real life problems? How was assessment planned to evaluate learners' procedural knowledge and conceptual knowledge?	How did learners get involved in discovering answers to real-life problems? How was the response or performance of the pupils on the assessment?	

APPENDIX B

MATHEMATICS

GEOMETRY ASSESMENT

GRADE:

SCHOOL CODE: _____

LEARNER CODE: _____

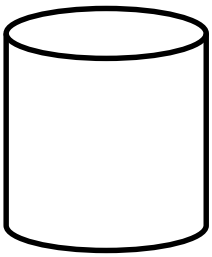
MARKS: 50 **DURATION: 2 HOURS**

INSTRUCTIONS

- **Read all the instructions carefully.**
- **Answer all the questions.**
- **Write neatly and legibly.**

1. Circle the letter of the correct answer.

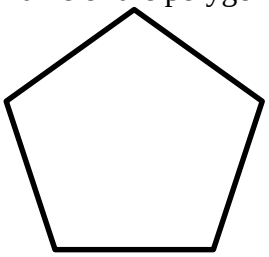
1.1. The following 3-d shape is a_____ (1)



A. Cube
B. Rectangular Prism
C. Square based pyramid
D. Cylinder

1.2. A 2-D shape with 7 sides is called_____? (1)
A. Hexagon
B. Heptagon
C. Pentagon
D. Rectangle

1.3. The name of the polygon below is_____ (1)



A. Triangle
B. Heptagon
C. Pentagon
D. Hexagon

1.4. A total distance around the 2-D shape is called_____? (1)

A. Area
B. Volume
C. Perimeter
D. Cube

1.5. The name of the polygon below is _____? (1)



A. Quadrilateral
B. parallelogram
C. Rectangle
D. Square

1.6. The size of the surface of a shape and is measured in squares _____? (1)

A. Perimeter
B. Area
C. Squares
D. Volume

1.7. A 3D shape with one square face and four triangular faces_____? (1)

A. Triangular pyramid
B. Triangular prism
C. Tetrahedron
D. Square based pyramid

1.8. Shapes with one base and have triangular faces which meet at one vertex (point) are called _____? (1)

A. Pyramids
B. Prisms
C. Quadrilaterals
D. Hexagons

1.9. A closed 2-D shape with four straight sides of equal length and four right angle is a _____? (1)

A. Rectangle
B. Cube
C. Square
D. Cone

1.10. An angle that is bigger than a straight angle is called_____? (1)

A. Reflex angle
B. Revolution
C. Right angle
D. Obtuse Angle

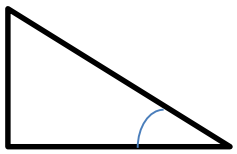
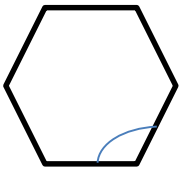
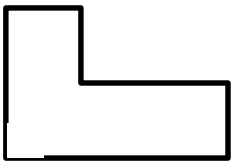
1.11. _____ is the amount of space an object occupies. It is measured in cubes. (1)

A. Capacity
B. Volume
C. Area
D. Perimeter

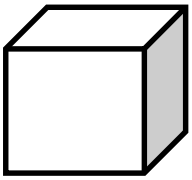
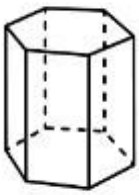
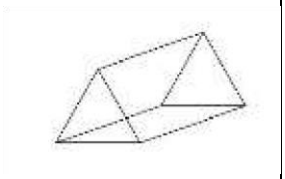
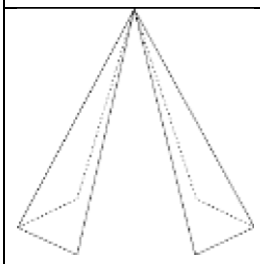
1.12. 3-D object with 4 triangular faces (1)

A. Hexagonal prism
B. Cube
C. Triangular Prism
D. Tetrahedron

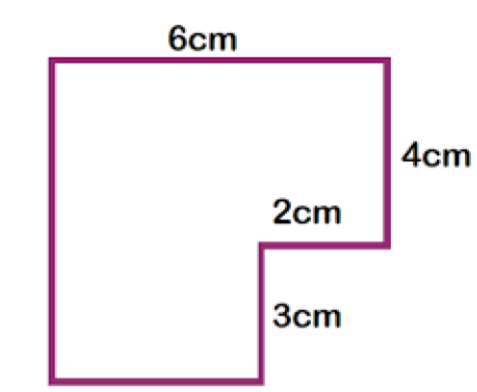
2. Complete the following table (12)

2-D shape	Name of 2-D shape	Regular or irregular Shape	Type of angle shown	Sides Description
	Triangle	Irregular Triangle	Acute angle	3 straight sides
				
				
				

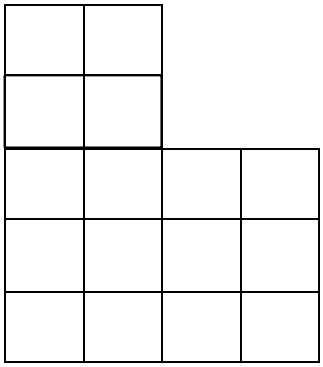
3. Complete the following table (19)

3-D Shape	Name of 3-D shape	Description of the Shape	Number of edges	Number of vertices
	Cube	6 squares 6 faces	12 edges	8 vertices
	_____	_____	_____	_____
	_____	_____	_____	_____
	_____	_____	_____	_____
	_____	_____	_____	_____

4. Look at the shapes below and answer the questions that follows.

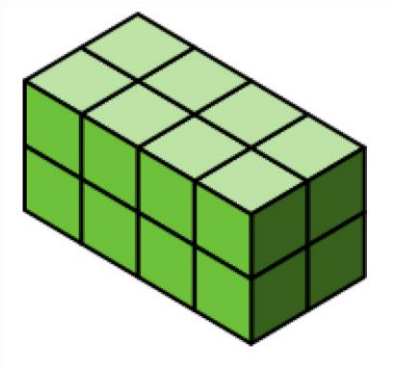


4.1. Calculate the perimeter of the shape above (3)



Calculate the Area of the above shape. (2)

5. Calculate the volume of the below stack of cubes. (2)



Total marks: 50

Appendix c

Interview Questions for Learners and Teachers

The following interview questions address research question 3

This interview question aims to gather information about the impact of teaching and learning conceptual knowledge and procedural knowledge in geometry.

For learners

1. Do you understand the importance of learning geometry?
2. How do you like/feel about learning geometry in math class?
3. What kinds of learning and teaching tools does your teacher use to teach you geometry in the classroom? (Teaching methods/ teaching styles. Probe "Do you think these tools & methods are effective in helping you understand geometry better & what other effective tools/methods do you think teachers can also adopt or use?)
4. Do you find it easier to learn geometry by memorizing or by understanding the concepts, explain?
5. Do you think the things you learn in geometry help you or anyone in real life, explain how.
6. What additional help would assist you to understand geometry better or more?
7. How does your teacher motivate you to learn geometry and make it easier for you?
8. Do you experience problems when learning geometry? If so/yes/ what kind of problems? Probe "What do you think can be done to address this problem"

For Educators

1. Can you outline the importance of teaching Geometry at school? “how do you think teaching geometry benefits the children”
2. What challenges do you come across when teaching geometry?
3. How do you assist struggling learners in geometry?
4. What material/ or resources do you find necessary to teach Geometry topics?
5. Do you use any of the mentioned materials?
6. How and what was the outcome of your last lesson?
7. How can you rate your teaching and your student’s performance in Geometry last year and this year (poor, average, good)?
8. What is the main cause of such performance?
9. What improvements would you like to make for better teaching and learning in Geometry? “Any kind of assistance you need to improve…”
10. Does your school have technological facilities/materials that you think can assist in teaching geometry?
11. **If yes,** how often do you use the facilities/materials? **If not**, so if you had them do you think these materials can play a role in improving learning and teaching in geometry?
12. Would you be able to use the technologies materials/facilities without any challenges if provided to the school?

Appendix D

UNIVERSITY OF THE WITWATERSRAND, JOHANNESBURG

Research Office

HUMAN RESEARCH ETHICS COMMITTEE (NON-MEDICAL)
R14/49 Shabangu

CLEARANCE CERTIFICATE	**PROTOCOL NUMBER: H23/06/41**
PROJECT TITLE	Exploring the application of procedural knowledge and conceptual knowledge in geometry: a case of schools in Bushbuckridge in the Mpumalanga province, South Africa
INVESTIGATOR(S)	Mr T Shabangu
SCHOOL/DEPARTMENT	Wits School of Education/
DATE CONSIDERED	23 June 2023
DECISION OF THE COMMITTEE	Approved Risk Level: Minimal
EXPIRY DATE	30 July 2026

DATE 22 August 2023 **CHAIRPERSON** _____
(Professor J Watermeyer)

cc: Supervisor : Dr B Mofolo-Mbokane

DECLARATION OF INVESTIGATOR(S)

To be completed in duplicate and **A SIGNED COPY** returned to the Secretary electronically. Unreported changes to the application may invalidate the clearance given by the HREC (Non-Medical)

I/We fully understand the conditions under which I am/we are authorized to carry out the abovementioned research and I/we guarantee to ensure compliance with these conditions. Should any departure be contemplated from the research procedure as approved I/we undertake to submit an amendment of the protocol to the Committee. **I/we agree to completion of a regular progress report. For Minimal and Low Risk studies, this is due annually on 31 December. For Medium and High Risk studies, this is due twice annually on 30 June and 31 December.**

_____ _____/_____/_____
Signature **Date**

PLEASE QUOTE THE PROTOCOL NUMBER ON ALL ENQUIRIES

Appendix E

education
MPUMALANGA PROVINCE
REPUBLIC OF SOUTH AFRICA

Ikhamanga Building, Government Boulevard, Riverside Park, Mpumalanga Province
Private Bag X11341, Mbombela, 1200.
Tel: 013 766 5552/5115, Toll Free Line: 0800 203 116

Litiko le Temfundvo, Umnyango we Fundo | Departement van Onderwys | Ndzawulo ya Dyondzo

Enq: Thabo Shabangu
University of the Witwatersrand
Tel: 079 1234 361
Email: 2519462@students.wits.ac.za

RE: "EXPLORING THE APPLICATION OF PROCEDURAL KNOWLEDGE AND CONCEPTUAL KNOWLEDGE IN GEOMETRY: A CASE OF SCHOOLS IN BUSHBUCKRIDGE IN THE MPUMALANGA PROVINCE, SOUTH AFRICA"

Your application to conduct research study was received and is therefore acknowledged. The title of your research project reads: **"Exploring the application of procedural knowledge and conceptual knowledge in geometry: a case of schools in Bushbuckridge in the Mpumalanga province, South Africa".**

I trust that the aims and the objectives of the study will benefit the whole department especially the beneficiaries. Your request is approved subject to you observing the provisions of the departmental research policy which is available in the department website. You are requested to adhere to your university's research ethics as spelt out in your research ethics.

In terms of the research policy, data or any research activity can be conducted after school hours as per appointment with affected participants and COVID -19 regulations to observed. You are also requested to share your findings with the relevant sections of the department so that we may consider implementing your findings if that will be in the best interest of the department. To this effect, your final approved research report (both soft and hard copy) should be submitted to the department so that your recommendations could be implemented. You may be required to prepare a presentation and present at the departments' annual research dialogue.

For more information kindly liaise with the department's research unit @ 013 766 5015/5124 0r c.maphanga@mpuedu.gov.za

The department wishes you well in this important project and pledges to give you the necessary support you may need.

Moyane

MRS LH MOYANE
HEAD: EDUCATION

02 / 08 / 2023
DATE

MPUMALANGA
THE PLACE OF THE RISING SUN

Appendix F

Mpumalanga Department of Education Permission request

From: Thabo Shabangu

Master of Education Student

University of the Witwatersrand

2519462@students.wits.ac.za

079 1234 361

To: Department of Education Mpumalanga

Research Ethics section

Head of Department

SUBJECT: RESEARCH ETHICS APPLICATION

My name is Thabo Shabangu. I am a Master of education student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr. Batseba Mofolo-Mbokane. I am conducting a research study examining the application of conceptual knowledge and procedural knowledge in geometry teaching and learning in Mathematics. The study title is Exploring the Application of procedural knowledge and conceptual knowledge in Geometry: a case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

The purpose of this communication is to formally request permission to conduct research in multiple schools under the jurisdiction of the Mpumalanga Department of Education. The research is situated specifically within the Mkhuhlu circuit, which falls under the Bohlabela region. Upon receiving approval from the Department of Education, I will seek permission from the principals of the schools where my research will be conducted. Furthermore, I will request consent from the educators, parents, and learners who will be participating in my study.

My research involves classroom observation in geometry classroom mathematics, a geometry assessment of the learners, and face-to-face structured interviews with the learners. The observations will take part during the mathematics periods. I will use direct observation where I will observe without engaging with the learners or the educator. I will be making written notes on my observations. Structured face-to-face interviews will take part after classes between me and selected learners from the schools under the Mkhuhlu circuit. The geometry observation aims to gather data about how educators in the intermediate phase teach both procedural and conceptual knowledge in the classroom. The main focus is to look at the ways that conceptual and procedural knowledge about geometry is taught, with a

particular emphasis on pre-existing knowledge, newly acquired knowledge, and knowledge application. Structured face-to-face interviews will take part after classes between me and selected learners and educators from your schools.

The interviews will take place on the school premises and the venue within the school will be provided by the participating and consenting teachers. The assessment will be given to the learners during mathematics classes for grade 6 as the content of the test will be in line with the Grade 6 ATP for term 3. And the test for grade 9 learners will be given after school. The test will last for one hour. Because I will use the same test to evaluate learners in grades 6 and 9, for consistency the test will be designed by me rather than an educator. As a result, it will be a comparative test. The purpose of the test is to investigate how learners from intermediate and senior phase exit grades respond to procedural knowledge and conceptual knowledge assessment in geometry in a mathematics classroom

Given the issues surrounding poor geometric performance in mathematics in South Africa, the purpose of this research is to contribute to the resolution of the increased challenges in mathematics encountered in schools by exploring the applications of procedural knowledge and conceptual knowledge in geometry mathematics. The findings of this study could be used to enhance the learning and teaching of Geometry.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks for participating in this study. The participants will not be paid for this study.

With your permission, I would like to observe educators' geometry lessons. Furthermore, I would like to ask permission to give learners a geometry test and I would also like to audio record the interviews with the learners. The names of the research participants and the identity of the school will be kept confidential at all times and in all academic writings about the study. Individual privacy will be maintained in all published and written data results from the study. All research data will be kept in a password-protected computer to which only I will have access and be destroyed between 3-5 years after completion of the project.
This research study will be written up as a dissertation. The dissertation will be available online for public access. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterward about this research study, feel free to contact me or my supervisor at the details listed below. If you have any concerns or complaints about the ethical

procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), by telephone at +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,

Thabo Shabangu

_____

Signature

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 079 1234 361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 011 7171 000

Appendix G

Permission information sheet (Principal)

LETTER TO THE PRINCIPAL

Dear School Principal

My name is Thabo Shabangu. I am a Masters student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr. Batseba Mofolo-Mbokane. I am conducting a research study examining the application of conceptual knowledge and procedural knowledge in geometry teaching and learning in Mathematics. The study title is; Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

I would like to invite your school to participate in this research. My research involves classroom observation in geometry classroom mathematics, a geometry assessment for the learners, and face-to-face structured interviews with the learners. I have completed the application process and obtained approval from both the Mpumalanga Department of Education and the Wits Human Research Ethics Committee. Upon receiving approval from you, I will seek permission and consent from the educators, consent from the parents of the learners, and assent from the learners who will be participating in my study.

The observations will take part during the mathematics periods. I will use direct observation where I will observe without engaging with the learners or the teacher. I will be making written notes on my observations. The geometry observation aims to gather data about how educators in the intermediate phase teach both procedural and conceptual knowledge in the classroom. The main focus of this study is to look at the ways that conceptual and procedural knowledge about geometry is taught, with a particular emphasis on pre-existing knowledge, newly acquired knowledge, and knowledge application. Structured face-to-face interviews will take part after classes between me and selected learners and educators from your school. The venue of the interviews within the school will be provided by the educators. The assessment for grade 6 will be given to the learners during the mathematics period and this assessment will be in alignment with the ATP. On the other hand assessment for grade 9 will take place after school. The test will last for one hour. Because I will use the same test to evaluate learners in grades 6 and 9, for consistency the test will be designed by me rather than an educator. As a result, it will be a comparative test.

Given the issues surrounding poor geometric performance in mathematics in South Africa, the purpose of this research is to contribute to the resolution of the increased challenges in mathematics encountered in schools by exploring the applications of procedural knowledge and conceptual knowledge in geometry mathematics. The findings of this study could be used to enhance the learning and teaching of Geometry.

The research participants will not be advantaged or disadvantaged in any way. They will be reassured that they can withdraw their permission at any time during this project without any penalty. There are no foreseeable risks for participating in this study. The participants will not be paid for this study.

With your permission, I would like to observe educators' geometry lessons. Furthermore, I would like to ask permission to give learners a geometry test and I would also like to audio record the interviews with the learners. The names of the research participants and the identity of the school will be kept confidential at all times and in all academic writings about the study. Individual privacy will be maintained in all published and written data results from the study. All research data will be kept in a password-protected computer to which only I will have access and be destroyed between 3-5 years after completion of the project.

This research study will be written up as a dissertation. The dissertation will be available online for public access. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterward about this research study, feel free to contact me or my supervisor at the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), by telephone at +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

_____
Signature

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix H

Teacher observation Participant Information Sheet (Grade 6)

Dear Teacher

My name is Thabo Shabangu. I am a Masters student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr Batseba Mofolo-Mbokane. I am conducting a research study about examining the application of conceptual knowledge and procedural knowledge on geometry teaching and learning in Mathematics. The study title is; Exploring the Application of Procedural Knowledge and conceptual knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

Your Mathematics geometry classroom has been selected to participate in the classroom observation study. Given the issues surrounding poor geometric performance in mathematics in South Africa, the purpose of this research is to contribute to the resolution of the increased challenges in mathematics encountered in schools by exploring the applications of procedural knowledge and conceptual knowledge in geometry mathematics. The findings of this study could be used to enhance the learning and teaching of Geometry.

My research involves classroom observation in geometry classroom mathematics, a geometry assessment for the learners and face-to-face structured interviews with the learners and educators. I have completed the application process and obtained approval from both the Mpumalanga Department of Education, the Wits Human Research Ethics Committee and the principal of the school. Upon receiving approval from you, I will seek permission and consent from the parents of the learners, and assent to learners who will be participating in my study.

The observations will take part during the mathematics periods. I will use direct observation where I will observe without engaging with the learners or the teacher. I will be making written notes on my observations. The geometry observation aims to gather data about how educators in the intermediate phase teach both procedural and conceptual skills in the classroom. The main focus of this study is to look at the ways that conceptual and procedural knowledge about geometry is taught, with a particular emphasis on pre-existing knowledge, newly acquired knowledge, and knowledge application. Structured face-to-face interviews will take part after classes between me and selected the learners from your school. Moreover, structured face-to-face interviews will take place between me and you at your preferred schedule. The assessment will be given to the learners after school.

With your permission, I would like to observe your geometry lessons. Furthermore, I would like to ask permission to give learners a geometry test and I would also like to audio record the interviews with the learners. The names of the research participants and the identity of the school will be kept confidential at all times and in all academic writings about the study. Individual privacy will be maintained in all published and written data results from the study. All research data will be kept in a password-protected computer in which only I will have access to and be destroyed between 3-5 years after completion of the project.

If you decide to take part in the research study, it should be because you want to volunteer. You do not have to take part. You can stop being in the study at any time. You will not get any direct benefits if you choose to join the research study. You will not lose any services, benefits or rights you would normally have if you decide not to join. Taking part in the research study will not cost you anything. You will not be paid for being in this research study. There are no foreseeable risk for participating in this study.

This research study will be written up as a dissertation. The dissertation will be available online for public access. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor on the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix I

Teacher Consent Form

Title of project: Exploring the application of procedural knowledge and conceptual knowledge in geometry: a case of schools in Bushbuckridge in the Mpumalanga province, South Africa.

Name of researcher: Shabangu Thabo

I, ……., agree to participate in this research project.
Educator's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I understand that I can volunteer to take part in the study	YES	NO
I agree with classroom observation	YES	NO
I agree that my learners can be interviewed	YES	NO
I agree to be interviewed	YES	NO
I agree for learners to write a test	YES	NO
I agree that my participation or that of my learners will remain confidential (my name will not be used by the researcher in their research dissertation.	YES	NO

…… (Teacher's name)
…… (Teacher's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking consent)
…… (Date)

Appendix J

Learner Assessment Permission Information Sheet (Grade 9)

Dear Teacher

My name is Thabo Shabangu. I am a Masters student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr. Batseba Mofolo-Mbokane. I am conducting a research study examining the application of conceptual knowledge and procedural knowledge in geometry teaching and learning in Mathematics. The study title is; Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

We want to extend an invitation to your Mathematics geometry classroom to participate in our classroom observation study. Given the issues surrounding poor geometric performance in mathematics in South Africa, this research aims to contribute to the resolution of the increased challenges in mathematics encountered in schools by exploring the applications of procedural knowledge and conceptual knowledge in geometry mathematics. The findings of this study could be used to enhance the learning and teaching of Geometry.

My research involves a geometry assessment for the learners. I have completed the application process and obtained approval from the Mpumalanga Department of Education, the Wits Human Research Ethics Committee, and the school principal. Upon receiving approval from you, I will seek permission and consent of parents of the learners, and learners who will be participating in my study. The assessment will be given to the learners after school. The test will last for one hour. Because I will use the same test to evaluate learners in grades 6 and 9, for consistency the test will be designed by me rather than an educator. As a result, it will be a comparative test, where the grade 9 geometry performances is compared to the one of the grade 6. May I kindly ask if it would be possible to administer the assessment during the mathematics period? The purpose of the test is to investigate how learners from intermediate and senior phase exit grades respond to procedural knowledge and conceptual knowledge assessment in geometry in a mathematics classroom.

With your permission, I would like to ask permission to give learners a geometry test. The names of the research participants and the identity of the school will be kept confidential at all times and in all academic writings about the study. Individual privacy will be maintained in all published and written data results from the study. All research data will be kept in a password-protected computer to which only I will have access and be destroyed between 3-5 years after completion of the project.

If you decide that the learners take part in the research study, it should be because they want to volunteer. They do not have to take part. They can stop being in the study at any time. Learners will not get any direct benefits if they choose to join the research study. Learners will not lose any services, benefits, or rights they would normally have if they decide not to join. Taking part in the research study will not cost you or the learners anything. You or the learners will not be paid for being in this research study. There is no foreseeable risk for participating in this study.

This research study will be written up as a dissertation. The dissertation will be available online for public access. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterward about this research study, feel free to contact me or my supervisor at the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), by telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu
Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix K

Parent Information Sheet (Test)

Dear Parent

My name is Thabo Shabangu. I am a Masters student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr. Batseba Mofolo-Mbokane. I am conducting a research study examining the application of conceptual knowledge and procedural knowledge in geometry teaching and learning in Mathematics. The study title is; Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

A Mathematics class of your child has been selected to participate in a mathematics geometry assessment. Given the issues surrounding poor geometric performance in mathematics in South Africa, this research aims to contribute to the resolution of the increased challenges in mathematics encountered in schools by exploring the applications of procedural knowledge and conceptual knowledge in geometry mathematics. The findings of this study could be used to enhance the learning and teaching of Geometry.

Learners will write a geometry assessment for my research. The test will be taken in their class at his/her school after school. The test will last for one hour. Because I will use the same test to evaluate learners in grades 6 and 9, for consistency the test will be designed by me rather than an educator. As a result, it will be a comparative test, where the grade 9 geometry procedural and conceptual knowledge performances is compared to the one of the grade 6. Teachers of grade 6 can use the test for grades if they want to, either as a formal or informal task. However, teachers of grade 9 cannot use the test for grades because the test is mostly based on grade 6 content.

All the written test scripts will be noted by an anonymous designator and confidentiality. When I share the results of the research study, I will not include the name of your child or anything else that could identify them. For analysis only codes will be used will used in the place of the names of the participants. The data will be used to improve teaching and learning in geometry.

With your permission, I request your authorization to administer a geometry assessment to your child. I have obtained permission and approval from both the educator and principal. If you consent that your child takes part in the research study, it should be because it is voluntary. Your child is not obliged to take part. He/she can stop being in the study at any time. Your child will not get any direct benefits if you choose that he/she join the research study. He/she will not lose any services, benefits, or rights she/he would normally have if you decide not to join. Taking part in the research study will not cost you anything. Your child will not be paid for being in this research study. There are no foreseeable risks for participating in this study.

This research study will be written up as a dissertation. The dissertation will be available online for public access. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor at the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), by telephone at +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix L

Parent Consent Form

Title of project: Exploring the application of procedural knowledge and conceptual knowledge in geometry: a case of schools in Bushbuckridge in the Mpumalanga province, South Africa.

Name of researcher: Shabangu Thabo

I, ……., agree that my child can participate in this research project.
Parent's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I volunteer that my child take part in the study	YES	NO
I agree that my child write a geometry mathematics test	YES	NO
I agree that the results from the script of my child can be used for analysis and reporting.	YES	NO
I agree that the participation of my child will remain anonymous (his/her name will not be used by the researcher in their research report).	YES	NO

…… (Parent's name)
…… (Learner's name)
…… (Parent's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking assent)
…… (Date)

Appendix M

Learner Assessment Participant Information Sheet

Dear Learner

Hi, I am Thabo Shabangu. I am studying for a higher degree at a university called the University of the Witwatersrand in Johannesburg. The person who is in charge of me is called Dr. Batseba Mofolo-Mbokane. I am doing a study to see how people learn and teach geometry in math. I want to know how people use their ideas and skills to understand and solve problems in geometry. The study is about how students in schools in Bushbuckridge, South Africa use different types of knowledge in geometry. The two types of knowledge are procedural knowledge and conceptual knowledge. The study wants to understand how these types of knowledge are used in geometry by students in this area.

Your Math class is invited to take part in a test about shapes and angles. This research aims to help solve the problems with geometry math in South Africa. It will look at how procedural knowledge and conceptual knowledge can be used to improve math education in schools. The results of this study could help make learning and teaching Geometry better.

May I please have your permission to give you a geometry test? I asked your teacher and principal, and they said its okay. You will write a test about shapes and measurements for my study. The test will happen in your class, at your school, during your mathematics period or at the time as arranged by your teacher. The test will go on for one hour. I will make the test for students in grades 6 and 9 myself, so that it is the same for both grades. This way, the test will be consistent. So, it will be a test where things are compared.

The test scripts will be marked with a secret code and kept private. When I tell others about the research study, I won't mention your name or anything else that could reveal who you are. Only codes will be used for analysis instead of using the names of the participants.

If you choose to join the research study, it should be because you want to. You don't need to participate if you feel like not. You can leave the study whenever you want. If you decide to join the research study, you won't receive any immediate advantages. If you choose not to join, you won't miss out on any services, benefits, or rights that you would usually have. Participating in the research study won't require you to pay anything. You won't receive any money for participating in this research study. There are minimal risks that we can see for joining this study.

This research study will be written as a long essay called a dissertation. The dissertation will be online for anyone to read. If you want a short version of this long paper, I can send it to you. Just let me know!

If you have any questions while or after reading about this research study, feel free to contact me or my supervisor using the contact information provided below. If you have any worries or problems about how this research study is being done, you can get in touch with the University Human Research Ethics Committee (Non-Medical). You can call them at +27(0) 11 717 1408 or send an email to hrecnon-medical@wits.ac.za.

Yours sincerely, Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix N

Learner Assent Form

Title of project: Exploring the application of procedural knowledge and conceptual knowledge in geometry: a case of schools in Bushbuckridge in the Mpumalanga province, South Africa.

Name of researcher: Shabangu Thabo

I, …….., agree to participate in this research project.
Learner's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I volunteer to take part in the study	YES	NO
I agree to writing a geometry mathematics test	YES	NO
I agree that the results from my script of can be used for analysis and reporting.	YES	NO
I agree that my participation will remain anonymous (my name will not be used by the researcher in their research report).	YES	NO

…… (Learner's name)
…… (Learner's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking assent)
…… (Date)

Appendix O

Learner Observation Participant Information Sheet

Dear Learner

I am Thabo Shabangu. I am studying for a higher degree at a university called the University of the Witwatersrand in Johannesburg. The person who is in charge of me is called Dr. Batseba Mofolo-Mbokane. I am doing a study to see how people learn and teach geometry in math. I want to know how people use their ideas and skills to understand and solve problems in geometry. The study is called "Learning about Geometry in Schools in Bushbuckridge, South Africa." It looks at how students use different types of knowledge, like procedural knowledge and conceptual knowledge, when learning about shapes and measurements.

Your Math class is invited to take part in a study where people will watch and learn from your class. This research aims to help solve the problems with geometry math in South Africa. We want to understand how using different types of knowledge, like knowing how to do something and understanding the concepts behind it, can make math easier for students. The results of this study could help make learning and teaching Geometry better.

The observations will happen during math class. I will watch and pay attention to the learners and the teacher without talking or interacting with them. I will write down what I see instead of using any recordings. The geometry observation is about collecting information on how teachers in the middle grades teach both the step-by-step methods and the big ideas in geometry. This study also wants to see how people are taught about shapes and measurements in geometry. They will focus on what people already know, what they learn, and how they use that knowledge. We will keep all the observations private and not share them with anyone. When I tell others about the research study, I won't mention your name or anything else that could show who you are. The information will be used to make teaching and learning about shapes better.

If you choose to join the research study, it should be because you want to offer your help willingly. You don't have to join in if you feel like you don’t want. You can stop being in the study at any time. If you decide to join the research study, you won't receive any immediate advantages. If you choose not to join, you won't miss out on any services, benefits, or rights that you would usually have. Participating in the research study won't require you to pay anything. You won't receive any money for participating in this research study. There are no risks that we can see for joining this study. This research study will be written as a long essay called a dissertation. The dissertation will be on the internet for anyone to read. If you want a short version of this long paper, I can send it to you. Just let me know!

If you have any questions while or after reading about this research study, you can contact me or my supervisor using the information provided below. Feel free to reach out! If you have any worries or problems about how this research study is being done, you can get in touch with the University Human Research Ethics Committee (Non-Medical). You can call them at +27(0) 11 717 1408 or send them an email at hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix P

Learners Assent Form for observation

Title of project: Exploring the application of procedural knowledge and conceptual knowledge in geometry: a case of schools in Bushbuckridge in the Mpumalanga province, South Africa.

Name of researcher: Shabangu Thabo

I, ... , agree to participate in this research project.
Learner's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I volunteer to take part in the study	YES	NO
I agree that I should be observed	YES	NO
I agree that my participation will remain confidential (my name will not be used by the researcher in their research report).	YES	NO
	YES	NO

I understand that I can withdraw at any point

…… (Learners name
…… (Learner's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking assent)
…… (Date)

Appendix Q

Parent/guardian permission Information Sheet (PIS) for interviews

Dear Parent or Guardian

My name is Thabo Shabangu. I am a Masters student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr. Batseba Mofolo-Mbokane. I am conducting a research study examining the application of conceptual knowledge and procedural knowledge in geometry teaching and learning in Mathematics. The study title is Exploring the Application of procedural knowledge and conceptual knowledge in Geometry: a case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

I am inviting your child to take part in an interview. My research involves face-to-face semi-structured interviews. If you decide that your child will take part, his/her participation in this research study will last about 30 minutes. The interview will take place at his/her school after all the school lessons have been concluded. The venue of the interview within the school will be confirmed by his/her teacher.

With your permission, I request your approval to interview your child. I have obtained permission and approval from both the educator and principal. With your permission, I would like to audio record the interview. This data will be stored in a password-protected computer for about 3- 5 years and deleted after the project is completed. Only the researcher will have access to the data. The interview will be confidential. When I share the results of the research study, I will not include the name of the learners or anything else that could identify them.

If you decide that your child take part in the research study, it should be because it is voluntary. The child is not obliged to take part. All learners for whom we have parental consent will be asked if they wish to participate and only those who agree will participate in the interviews. Moreover, the learners can stop participating in the study at any time. They do not have to answer any questions if they do not want to. Learners will not get any direct benefits if they choose to join the research study. Learners will not lose any services, benefits, or rights they would normally have if they decide not to join. Taking part in the research study will not cost the learner anything. The learner will not be paid for being in this research study. There are no foreseeable risks for participating in this study.

This research study will be written up as a dissertation. The dissertation will be available online. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterward about this research study, feel free to contact me or my supervisor at the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), by telephone +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix R

Parental Consent Form

Title of project: Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

Name of researcher: Shabangu Thabo

I, ..., agree that my child can participate in this research project.
The parent's or guardian's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I understand that my child can volunteer to take part in the study	YES	NO
I agree that the interview may be audio recorded	YES	NO
I agree that direct quotations from my interview may be used by the researcher in their research report/ manuscript/book chapter	YES	NO
I agree that my child's participation will remain confidential and anonymous (my name will not be used by the researcher in their research report).	YES	NO
I understand that my child can withdraw at any point of the interview	YES	NO

…… (Learners name)
…… (Parent's or guardian's name)
…… (Parent's or guardian's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking consent)
…… (Date)

Appendix S

Learner permission Information Sheet (PIS) for interviews

Dear learner

I am called Thabo Shabangu. I am studying for a higher degree at a university called the University of the Witwatersrand in Johannesburg. The person who is in charge of me is called Dr. Batseba Mofolo-Mbokane. I am doing a study to see how we use what we know and how we do things in geometry teaching and learning in math. The study is called "Learning about Geometry in Schools in Bushbuckridge, South Africa." It looks at how students use different types of knowledge to understand Geometry.

I want to ask you some questions in an interview. I am doing a study where I talk to learners in person and flexibly ask those questions. If you choose to join, your involvement in this research study will take around 30 minutes. The interview will happen at your school after all the classes are finished. Your teacher will tell you where the interview will take place.

Can I please record the interview with your permission? This information will be kept on a computer that is protected by a password. It will be stored for around 3 years or deleted once the project is finished. Only the person doing the research will be able to see the information. The interview will be private and secret. When I tell others about the findings from the research, I won't mention your names or anything else that could reveal who you are.

If you choose to join the research study, it should be your decision and not something you are forced to do. You don't have to participate in this study. Also, you have the option to stop being part of the study whenever you want. You don't have to respond to any questions if you don't want to. If you decide to join the research study, you won't receive any immediate advantages. If you choose not to join, you won't miss out on any services, benefits, or rights that you would usually have. Participating in the research study won't require you to pay anything. You won't receive any money for participating in this research study. There are no risks that we can see for taking part in this study.

This research study will be written as a long essay called a dissertation. The dissertation can be found on the internet. If you want a short version of this long paper, I can send it to you. Just let me know!

If you have any questions while or after reading about this research study, you can contact me or my supervisor using the information provided below. If you have any worries or problems about the right and wrong ways of doing this research study, you can get in touch with the University Human Research Ethics Committee (Non-Medical). You can call them at +27(0) 11 717 1408 or send them an email at hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361

Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix T

Learners Assent Form

Title of project: Exploring the Application of Procedural Knowledge and conceptual knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

Name of researcher: Shabangu Thabo

I, ..., agree to participate in this research project.
Learner's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I volunteer to take part in the study	YES	NO
I agree that the interview may be audio-recorded	YES	NO
I agree that direct quotations from my interview may be used by the researcher	YES	NO
I agree that my participation will remain anonymous (my name will not be used by the researcher in their research report).	YES	NO
I understand that I can withdraw at any point	YES	NO

…… (Learners name
…… (Learner's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking assent)
…… (Date)

Appendix U

Parent Information Sheet (Learner Observation)

Dear Parent

My name is Thabo Shabangu. I am a Masters student at the University of the Witwatersrand, Johannesburg. My supervisor is Dr. Batseba Mofolo-Mbokane. I am conducting a research study examining the application of conceptual knowledge and procedural knowledge in geometry teaching and learning in Mathematics. The study title is; Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

Your child is invited to participate in the classroom observation study in a mathematics class. Given the issues surrounding poor geometric performance in mathematics in South Africa, this research aims to contribute to the resolution of the increased challenges in mathematics encountered in schools by exploring the applications of procedural knowledge and conceptual knowledge in geometry mathematics. The findings of this study could be used to enhance the learning and teaching of Geometry.

The observations will take part during the mathematics periods. I will use direct observation where I will observe without engaging with the learners or the teacher. I will be making written notes on my observations, there will be no recordings. The geometry observation aims to gather data about how educators in the intermediate phase teach both procedural and conceptual knowledge in the classroom. The main focus of this study will be on how van Hiele's levels of learning geometry are used in the classroom. This study also wants to look at the ways that conceptual and procedural knowledge about geometry is taught, with a particular emphasis on pre-existing knowledge, newly acquired knowledge, and knowledge application.

All the observations will be noted with confidentiality. When I share the results of the research study, I will not include the name of your child or anything else that could identify them. The data will further be used to improve teaching and learning in geometry.

If you decide that your child take part in the research study, it should be because it is voluntary. Your child is not obliged to take part. He/she can stop being in the study at any time. Your child will not get any direct benefits if you decide that he/she chooses to join the research study. He/she will not lose any services, benefits, or rights you would normally have if you decide not to join. Taking part in the research study will not cost you anything. She/he will not be paid for being in this research study. There are no foreseeable risks for participating in this study. This research study will be written up as a dissertation. The dissertation will be available online for public access. If you would like to receive a summary of this dissertation, I will be happy to send it to you.

If you have any questions during or afterwards about this research study, feel free to contact me or my supervisor at the details listed below. If you have any concerns or complaints about the ethical procedures of this research study, you are welcome to contact the University Human Research Ethics Committee (Non-Medical), by telephone at +27(0) 11 717 1408, email hrecnon-medical@wits.ac.za.

Yours sincerely,
Thabo Shabangu

Researcher:
Thabo Shabangu, 2519462@students.wits.ac.za, 0791234361
Supervisor:
Dr Batseba Mofolo-Mbokane, Batseba.mbokane@wits.ac.za, 0117171000

Appendix V

Parent Consent Form

Title of project: Exploring the Application of Procedural Knowledge and Conceptual Knowledge in Geometry: A Case of Schools in Bushbuckridge in the Mpumalanga Province, South Africa.

Name of researcher: Shabangu Thabo

I, ..., agree that my child can participate in this research project.
Parent's name

I agree to the following:

(Please circle the relevant options below)

The research study was explained to me. I understand what this study is about.	YES	NO
I volunteer that my child take part in the study	YES	NO
I agree that my child should be observed	YES	NO
I agree that the participation of my child will remain anonymous and confidential (his/her name will not be used by the researcher in their research report).	YES	NO
I understand that my child can withdraw at any point	YES	NO

…… (Parent's name)
…… (Learner's name)
…… (Parent's signature)
…… (Date)

…… (Signature)
…… (Name of researcher/person seeking assent)
…… (Date)