Note that t is the discrete variable and the time elapsed is t.T, where T is the model sampling interval.

The ball mill simulation dynamics including the solids feedrate regulator are second order with deadtime and the above model order was chosen accordingly.

On-line Robust Dynamic Model Parameter Estimation: Write above model in regressive form:

$$p(t) = \Phi^T(t-1).\hat{\theta}(t-1)$$

where

$$\Phi^T(t-1) = [-p(t-1) \ -p(t-2) \\ l^*(t-d) \ l^*(t-d-1) \ 1]$$

$$\hat{\theta}^T(t-1) = [a_1 \ a_2 \ b_0 \ b_1 \ c]$$

Assume no deterministic disturbances and no noise then the digital pre-filtering of the measurements in Φ (section "Coping with Deterministic Disturbances and Plant Noise" on page 24) is not required.

θ is calculated using the recursive least squares (RLS) parameter estimator. Knowing that the estimator will execute at least ten times before the parameters are used for optimization control action it seems a good idea to use covariance resetting (section "Estimation of Model Parameters" on page 20) to keep the estimator sensitive. In adaptive control it is unlikely that the parameter estimation will be executed more often than the control actions and the problem with using covariance resetting is to know when to reset the matrix. For the optimizer case the matrix can be reset just after a control action. The estimator can then converge rapidly to the new parameters before they are used in the next control action. Also, this co-ordination between the estimation and control is attractive because, as long as the estimator is given time to converge, the covariance matrix will be small and hence the parameter variance is small when the parameters are used in the control calculation.

Initially the RLS estimator was applied without a relative deadzone (section "Coping with Modelling Error within the Bandwidth of Interest" on page 29) but it did not always converge. This is expected because the simulated P, l^* relationship is non-linear (fourth order polynomial) and the model is linear. Applying the RLS with the relative deadzone solved this problem of model mismatch. The deadzone design is done by simulation. Using the design parameters the deadzone width is decreased to improve parameter estimates but ensure convergence. The deadzone time constant σ_0 is chosen slower than the disturbances. The deadzone only changes after the settling time of the disturbance.

Steady State Model Extraction: Setting $q^{-1} = 1$ in equation (1) gives the linear steady state model:

$$(1 + a_1 + a_2)P = (b_0 + b_1)l^* + c$$

Gradient Calculation: From the above steady state model the gradient is:

$$\frac{\partial P}{\partial l^*} = \frac{(b_0 + b_1)}{(1 + a_1 + a_2)}$$

It is the approximate gradient calculated by the optimizer. The actual steady state simulated mill gradient that this should approximate for a certain l^* and d_s is given in appendix C.

Plant Moves: The new l^* is calculated using the steepest descent gradient search to find the maximum of P.

$$l^*(k+1) = l^*(k) + \mu\frac{\partial P}{\partial l^*}$$

where k is an integer multiple of T_{opt} the optimization update time. T_{opt} is at least ten times greater then T (estimator sampling time).

The stepsize μ and T_{opt} are convergence parameters. T_{opt} is chosen depending on the disturbance bandwidth and the estimator convergence rate. The stepsize μ sets the rate of convergence to the optimum, making μ too large causes instability.

4.5.3 SIMULATION

The block structure in Figure 7 on page 45 is implemented in PASCAL, the program listing is given in appendix D. A fourth order Runge-Kutta method is used to solve the ball mill differential equation as well as to implement the integrator in the solids feedrate regulator. The plant simulation variables and plant constants are given in a file in appendix E and the optimizing regulator constants in a file in appendix F.

4.6 SIMULATION RESULTS

The results are given in Figure 10 on page 57. Only the variables that show the optimizing regulator performance are given. These are:

l (t)	The lower level regulator setpoint that the optimizer calculates
p(t)	The mill product which is the performance criterion that defines the control objective and is maximized.

gradient	The steady state gradient used to determine the next control action
$f(t)$	The mill fractional filling. This is an internal mill state variable

Starting from initial conditions set to zero the optimizer takes approximately 9 plant input moves to reach the desired operating point. Note that the mill time constants depend on the size of the mill and thus the time scale is relative, and for this simulation the units are chosen as minutes. From time 0 to 25 the gradient approximation is bad due to the parameters still being estimated and the linear model changing rapidly. At time 30 the gradient is stable and accurate and the optimizer control actions can be initiated. The optimization update time T_{opt} is every 30 minutes and this is based on the time taken for the estimator to converge after a disturbance. The covariance matrix is reset every 30 minutes, giving the estimator 30 plant samples to converge before the parameters are used in the next control action.

At time 400 a disturbance is introduced that changes the desired operating point from P_{opt} to P'_{opt} and then at time 700 it is changed back to P_{opt}. The magnitude of the disturbance changes F_{opt} by 5 percent and P_{opt} by 20 percent (details of how the simulation model parameters are changed is given in appendix C). The new optimum is found in approximately 8 steps.A disturbance in the other direction is then introduced at time 700. It takes approximately 6 control actions to return to the original desired operating point P_{opt}. While the model is changing rapidly the model parameters are not accurate and the gradient can be sensitive to numerical problems if both the gradient numerator and denominator approach zero. In the unlikely event that the optimizer uses a gradient before it is stable it is advisable to limit the maximum possible gradient magnitude. This is a "safety net" to reduce the effect of a spurious large and incorrect gradient on the controller trajectory.

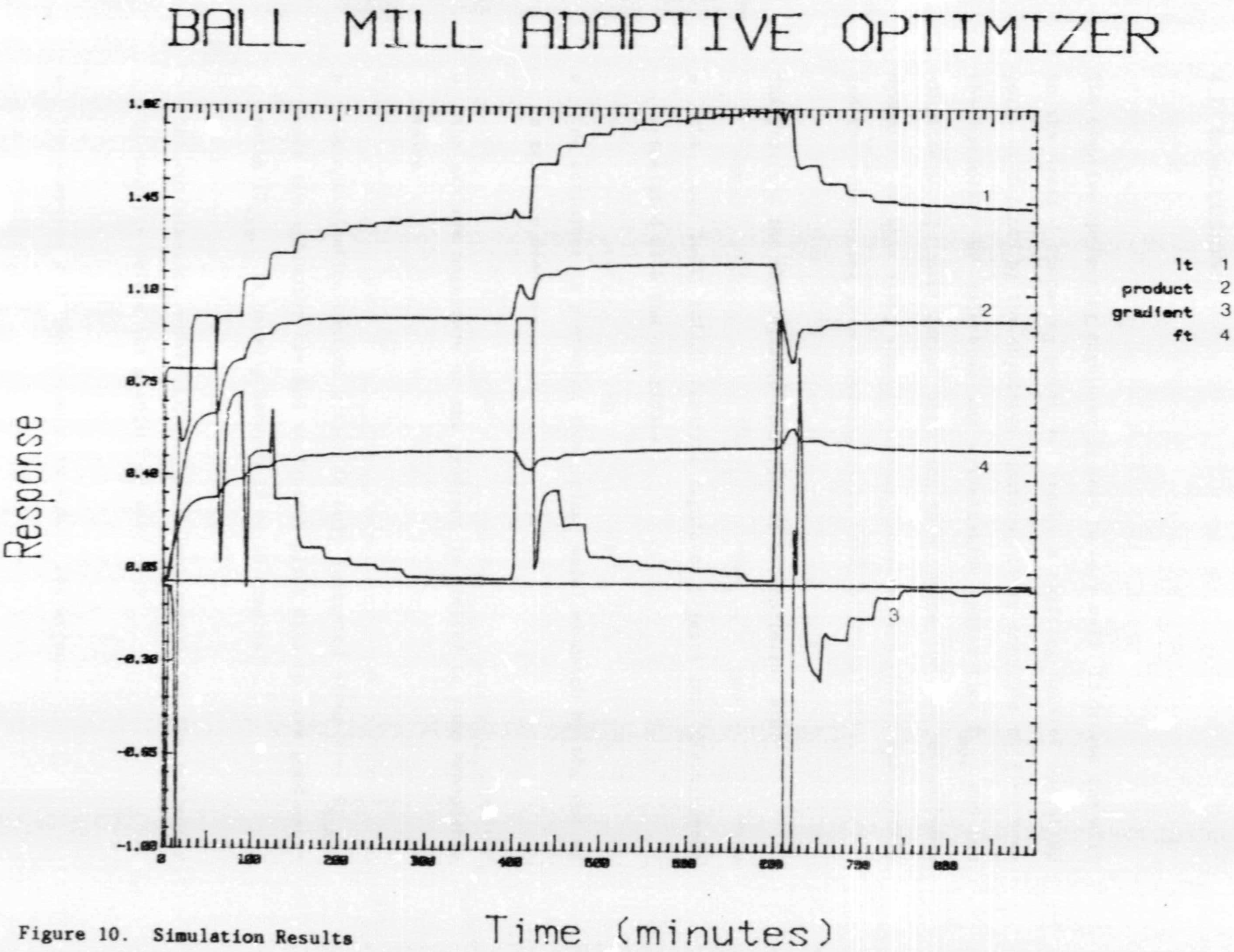
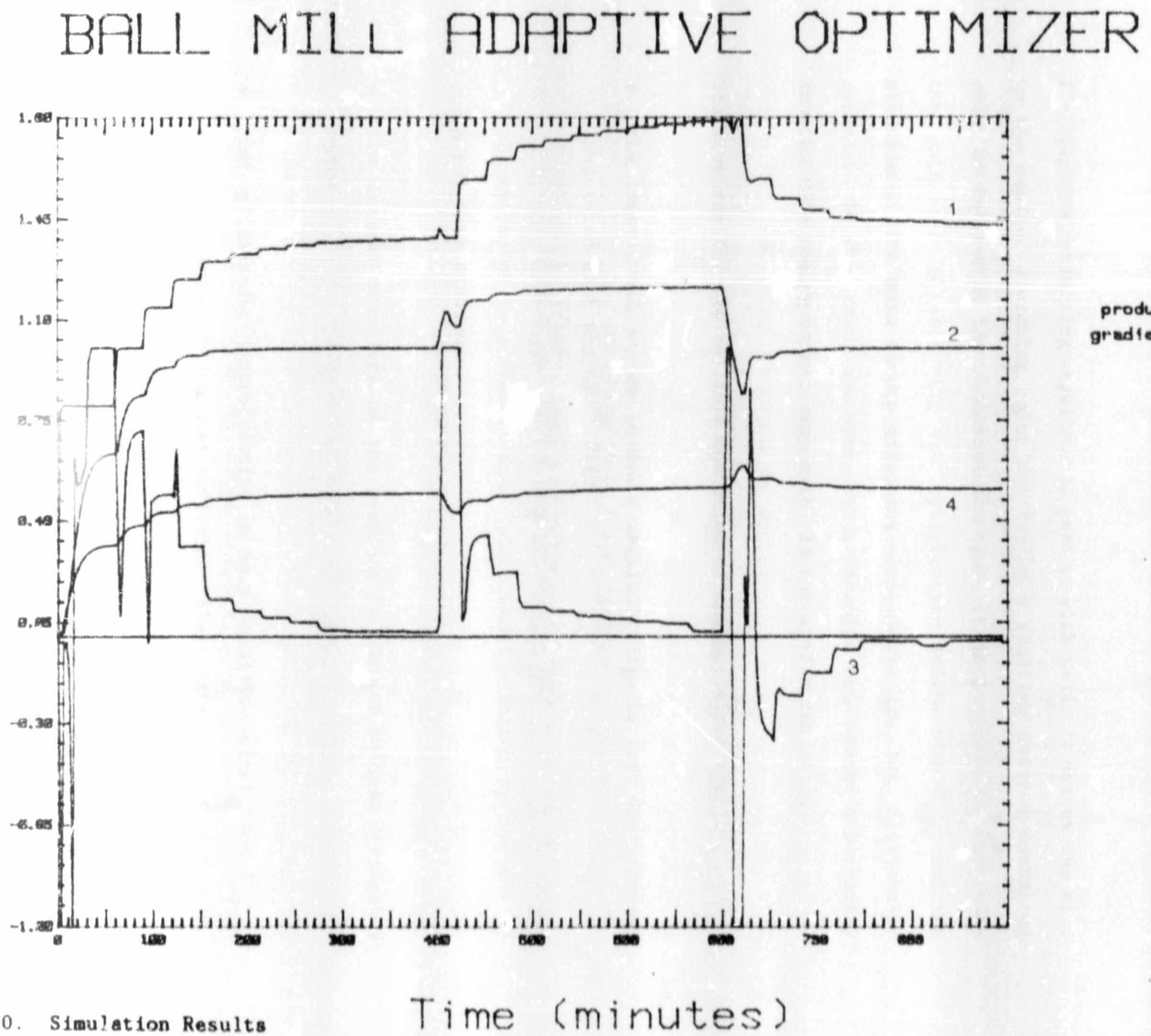


Figure 10. Simulation Results



10. Simulation Results

4.7 CONCLUSIONS

The adaptive optimizing regulator is seen to work well. It drives the mill to the desired operating point and tracks a shifting desired operating point as defined by the objective function. It is possible to fine tune the optimizer by selecting the optimization update time T_{opt} and the stepsize μ. The main factor affecting this choice are the disturbance dynamics. In this study the disturbance dynamics were chosen arbitrarily and the fine tuning is not meaningful in terms of a real mill.

The provisos for the reliable operation of the optimizer are:

- the lower level solids feedrate regulator rejects fast disturbances associated with the classifier
- the estimator is protected from plant model mismatch by the use of a relative deadzone
- the maximum magnitude of the gradient is limited to some reasonable value
- the optimization control action is only initiated after time T_{start} once the estimator has had sufficient plant data to give good estimates (T_{start} is usually between 20 to 50 samples)
- the optimization update time T_{opt} is long enough for the estimator to converge before the gradient is used
- the stepsize μ is small enough to ensure convergence

The assumption that there is no process noise or no deterministic disturbances can easily be dropped if the signal processing of the measurements is done carefully (section "Coping with Deterministic Disturbances and Plant Noise" on page 24).

Alternative methods to covariance resetting, namely forgetting factor and constant trace RLS estimation, failed to give improved performance. It seems that covariance resetting, as used in the optimizer, is less susceptible to the condition that the plant input has to be persistently exciting for estimation. Another consideration is that the design parameters for covariance resetting are more easily chosen than for the other methods.

5.0 APPLICATION OF ADAPTIVE OPTIMIZING REGULATOR TO AN AUTOGENOUS GRINDING CIRCUIT

5.1 BACKGROUND

5.1.1 MILLING

The South African gold-mining industry grinds 100 Mt of ore per year to a fineness that permits a high percentage of the gold to be extracted by the cyanide-leaching process. At the conservative figure of 30 kW hours of electrical energy consumed per ton of material smaller than 75µm produced, the electrical energy alone amounts to a cost of over 70 million rand per annum.

The mining operation constitutes up to 90 percent of the total gold mining capital and energy costs. A breakdown of the remaining cost, that associated with extraction, shows the milling stage to be the significant contributor. Also the milling stage is often the bottleneck in the extraction process that prevents the reduction of the costly surface ore inventory. Due to the importance of the milling operation in the gold mining and recovery process the more control and flexibility that can be achieved the better. By changing the long term mill throughput the rate of mining must be altered and by changing the mill product size distribution the downstream process is affected. In fact one of the ways to determine the economics of the whole process is to control the milling stage.

The introduction of autogenous run-of-mine (ROM) tumbling mills has been one of the most important developments in South African Milling practice during recent decades. With the hard quartzite ore it is highly desirable to eliminate the crushing stage and mill the run-of-mine ore directly. A typical autogenous grinding circuit is given in Figure 11 on page 62. It has three components; the autogenous mill, the sump with pump and the hydrocyclone classifier. The major difference is, instead of using ferrous grinding media as in ball and rod mills, it is the larger rocks that do the grinding. The control of a typical autogenous mill involves the manipulation of fresh rock feedrate, mill water addition, sump water addition and pump speed to obtain desired mill operation. The desired mill operation is gauged from mill measurements such as mill power draft, mill weight, pulp flowrate and sump level.

5.1.2 AUTOGENOUS MILL CONTROL

The potential economic advantages of autogenous milling have been partially offset by the difficulties encountered in achieving consistent product quality and flexibility of milling circuit operation. These difficulties can very often be traced to the inadequacy of the automatic control strategies presently in use. Hulbert and Barker (1985) have this to say:

> Milling is an operation for which there is not yet a generally accepted method of control for the achievement of optimum results. One of the reasons for this is the complicated array of states required at any particular time, and the limited number of measurements available for determination of those states.

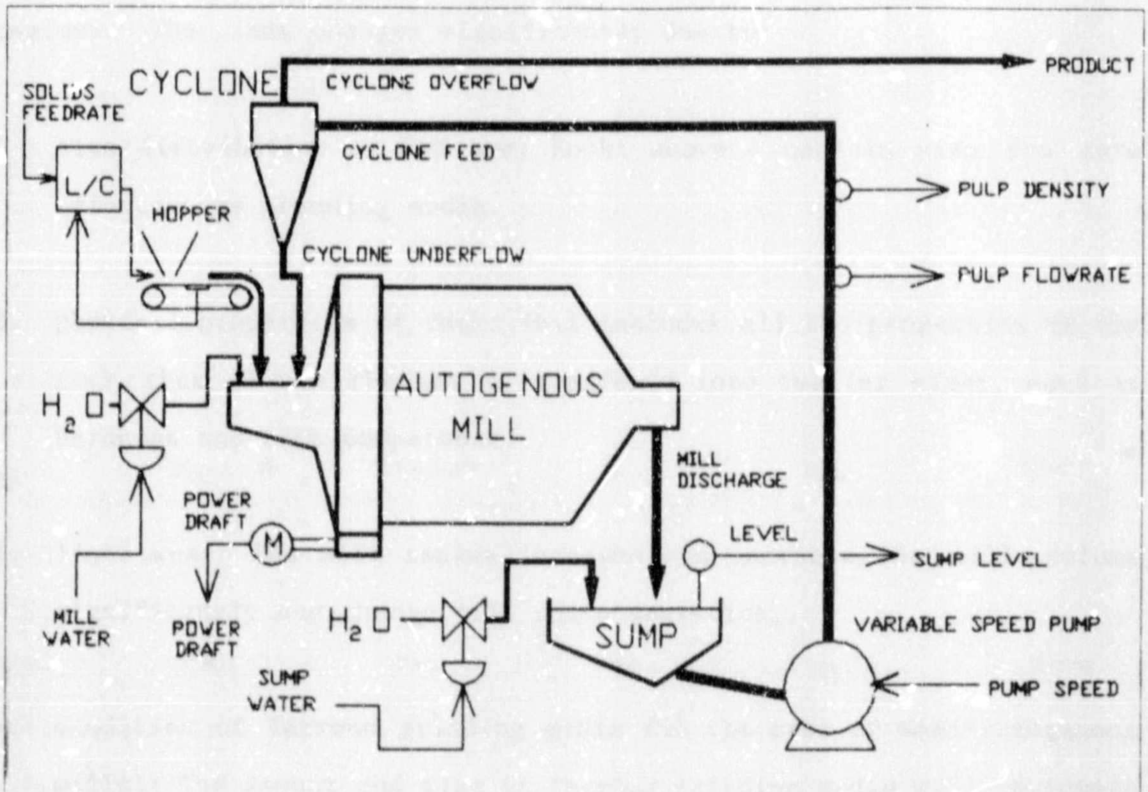


Figure 11. Typical Autogenous Grinding Circuit

Existing control strategies (Lynch, 1977, Williamson, 1975 and Flook, 1975) are taken directly from methods used in ball mill control. They have not gone much beyond the simple objective of maintaining certain process variables at pre-specified setpoints through the use of single loop PID controllers or multi-variable controllers. Present-day standard practice is to additionally use a separate power peak-seeking controller that uses heuristic logic to manipulate only the fresh rock feed rate. This type of control has been successfully applied to ball mills. The presence of ferrous grinding media makes the ball mill performance far less sensitive to disturbances and much easier to control.

The autogenous mill is a prime example of a plant that is impossible to model physically due to the large variety of physical mechanisms and complex interactions. An empirical modelling approach is accurate for certain mill conditions, but is not accurate under typically changing mill regimes. The plant changes significantly due to:

1. size distribution of ROM ore: Rocks above a certain size are more effective as grinding media.

2. physical properties of rock: This includes all the properties of the rock that change the way it fragments into smaller sizes, such as hardness and rock competence.

3. liner wear: The mill radius increase can increase the mill volume significantly and change mill characteristics.

4. condition of ferrous grinding media (in the case of semi-autogenous mills): The amount and size of ferrous grinding media will obviously affect the mill operation.

5. degree of pulp hold-up: If the interstices between the larger rocks are full this can cushion the impact breakage and reduce the grinding efficiency.

6. load volume: The amount of material inside the mill, this can change the dominant physical process inside the mill eg. from attrition to abrasion. It also changes the effectiveness of the epicyclic gear effect (Lynch, 1977), too high a load causes mill clogging.

What makes matters worse is that the first five disturbances can not be measured reliably. The most acute disturbance is the large size variations in the ore received from underground. Heavy blasting at depth in narrow stopes frequently means that ore delivered contains a high proportion of fine material and a dearth of material larger than 100mm.

As a general rule in control the less you know about the plant the less you can expect from the controller. It is not surprising that only limited success has been achieved with existing controllers. One must accept performance well below average, under certain conditions, so the controller has a big enough margin to cope with extreme conditions and is robust. A further complicating factor that makes this a pathological case is that the desired or optimum operating point is very close to an unstable region (Duckworth and Lynch, 1982).

The improved control and optimization of autogenous milling is receiving increasing attention (Duckworth and Lynch, 1982, Flook and Plasket, 1983, Pauw and Co-workers, 1985). This is due, not only to the unsatisfactory existing control, but also to more sophisticated instrumentation (Mokken, 1986), inexpensive powerful on-line process control computers (eg. PROSCON) and new control theory (Herbst and Rajamani, 1982). There is a strong, economically motivated need to bring together these recently available resources to improve autogenous ROM milling.

5.2 A NEW LOOK AT AUTOGENOUS MILL CONTROL OBJECTIVES

Duckworth and Lynch (1982) point out that one of the main problems in developing control strategies for ROM milling circuits relates to the conceptual difficulty of designing the control system to meet the control objectives. The common control objective stated by most workers, for example, Duckworth and Lynch (1982), Pauw and Co-workers (1985) and Lynch (1977), is to operate the circuit at the maximum throughput tonnage that will allow the desired product particle size to be maintained. Occasionally an alternative stated objective is to produce the finest possible product size for a given throughput. Various control schemes are then synthesized, but invariably these only address the real control objectives indirectly. For example, control of cyclone feed flow conditions emphasizes the optimal use of the classifier, but neglects the resulting effect on grinding efficiency in the mill, whilst power peak-seeking control emphasizes optimal grinding without taking into account the separation process. The peak-seeking control functions independently of the classifier control and vice versa causing considerable adverse interaction. Optimization of the whole circuit is not achieved.

Recent studies into the internal mechanisms and physical processes taking place in autogenous mills (Stanley, 1974, 1977 and Flook, 1977) provide clear new indications as to how milling circuits should be controlled. Strong motivation for the objective of maximizing mill power draft is given by Stanley(1974):

> Results of tests on a typical autogenous mill (St. Helena) provide a triumphant vindication of the "maximum power" control philosophy, and add further support that the "constant feedrate" and "constant mill power" bases of operation and the relatively low load levels commonly used in American practice are wasteful of capital and expensive in operation. An examination of the results brings to light the apparently anomalous fact that in contrast to conventional mills, the harder a ROM mill is pushed (up to the power peak), the finer will be the product. This of course, is due to the fact that, up to the power peak, the power increases faster than the feedrate, and so the energy input per unit mass increases.

Stanley (1977) highlights the fact that there are two major aspects to the autogenous milling problem, namely the control of the load of larger rock of grinding media size in the mill and the control of the mill pulp loading. The power drawn by the mill is a good indicator of grinding media load, but it is also affected by changes in pulp loading. A good automatic control system must treat the problem as a multi-variable one in order to handle the interaction between these two aspects.

Power drawn by the mill however, is only a partial performance indicator. The transfer of power to the pulp and consequently the amount of breakage within the pulp, is determined by mill pulp loading. Optimum transfer of energy could be indicated by other measurements such as mill rattle, discharge pulp temperature and discharge fineness. An experienced mill operator uses these measurements to empirically gauge the mill operation.

Economically significant unmeasureable disturbance variables to be dealt with by a good control scheme include changes in feed-size distribution, grinding media competence, grinding media hardness, the amount and condition of ferrous grinding media in the mill (in the case of semi-autogenous milling) and the degree of wear of the mill liner (Flook, 1977).

In view of the above factors we see the problem of autogenous ROM milling circuit control as a direct multi-input optimization problem. The three usual control inputs of interest are fresh rock feedrate, flowrate of water to the mill and flowrate of water to the sump. Depending on the values of the disturbance variables at a point in time, a unique combination of these control inputs must be determined to optimize the circuit economic performance. In the following work mill power is taken as the economic criterion to be optimized. Further research needs to be done to investigate the dual-criterion optimization of mill power and the distribution of that power, indicated by, for example, the discharge pulp temperature.

5.3 APPLICABILITY OF ADAPTIVE OPTIMIZING REGULATOR

Hulbert and Barker(1985) propose a wet-milling circuit multi-variable controller designed using the Inverse Nyquist Array technique. They state that optimization consists of the selection of set points at which the process operates most efficiently according to an **off-line** evaluation of its performance. The adaptive optimizer does not remove the need for setpoint following controllers. These remain an essential part of the first-level control scheme. However, in the case of the autogenous mill the plant is changing reasonably fast, and so will the desired regulator setpoints, making it essential that the performance evaluation is done **on-line** and is adaptive.

The adaptive optimizing regulator proposed in chapters 2 and 3 seems particularly suited to achieving the control objectives discussed above. The features of the adaptive optimizing regulator that make it an attractive solution to the autogenous milling control problem are:

1. The mill has an easily measured instantaneous objective, namely mill power draft.

2. It is an integrated approach and is inherently multi-variable. The high level control objective can be reconciled with the control algorithm.

3. It can track a shifting optimum reasonably fast. The on-line model parameter estimator will change the parameters to fit a changing plant.

4. It is not very sensitive to process noise. The regulator processes the signals to prevent adverse effects of noise and the model identification has good statistical properties.

5. It provides sufficient flexibility for the handling of constraints and control objectives which change from time to time.

6. Its computer processing requirements are easily met and it is easily implemented, without capital cost, on a typical existing ROM autogenous mill.

7. It provides a structured framework, that can easily be systematically extended, to tak advantage of new on-line instrumentation, in the event that it becomes available.

If the adaptive optimizer proves to be successful it will form the foundation for an even higher level of control. An envisaged advanced hierarchical controller could make use of the flexibility of the adaptive optimizing regulator. It could change the control objective depending on downstream process measurements, current market prices. ore inventory and ore reserves. By using a heuristic rule based expert system it could select the most economical control objective. Examples of possible objectives are: minimize energy or maximize throughput or maximize fineness of product for a given throughput. The objective may even be a multicriterion optimization. Provided that the mill objective can be calculated from plant measurements and has a single extremum within the boundaries of operation, then the adaptive optimizer can be used.

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5.4 SIMULATION

Now that the theory and algorithms have been proposed (chapters 2 and 3) and their suitability to the autogenous mill control problem has been discussed, the next stage is to test the adaptive optimizer under simulation. The simulation will test the global stability of the optimizer, it also provides an invaluable tool for selecting design parameters. The versatility, generality and speed of a simulation make it possible to test the optimizer under a wide variety of scenarios.

The autogenous mill is simulated using a fourth-order Runge-Kutta method to calculate the states.The program (listing in Appendix G) is written in PASCAL to make it readable and easily modifiable. Simulations were done on an HP 9000 series 300 computer operating under UNIX and NMODE environments. The variables that need to be experimented with are read from the following files:

init_(file number)	contains the initial mill states (given in Appendix H)
const_(file number)	contains the grinding circuit constants (given in Appendix I)
setup_(file number)	contains the simulation setup and controller variables (given in appendix J)

where the file number is any integer between 0 and 9.

Contained in the PASCAL include file model.p (given in Appendix K) is the autogenous mill model, sump model and hydrocyclone model. Output calculations are done in the include file output_calc.p (given in Appendix L). Controllers, the estimator and optimizer are included in the main source listing (Appendix G). Simulation results are written to the

various files in directory plot_data_(file number), where the file number corresponds to the file number in setup_(file number). Within the plot_data_(file number) directory is the group_dir which contains all the variables that one wants to display on a single graph. The results are plotted using STARBASE graphics routines (Hewlett Packard Software) in the PASCAL program plot.p (given in appendix M). The above file structure gives flexibility and helps to keep track of the different simulation runs.

5.4.1 DETAILS OF AUTOGENOUS MILL MODEL

The model used to simulate the behaviour of a general autogenous grinding circuit (in Figure 11 on page 62) was developed in an M.Sc.(Eng) project by Kramersh (1987). There are three main functional components: the mill, the sump and the hydrocyclone. Each component is modelled separately and then interlinked in a modular modelling approach.

The essentially mechanistic mill model is based on principles given by Hinde (1977). It is targeted towards a control based environment. The number of internal states is limited to those that are significant for controller performance evaluation and output measurement calculation. Mill load is split into three size fractions: grinding media, fines and particles satisfying the required grind size. The physical mechanisms describing comminution included in the model are chipping, abrasion, attrition and impact breakage. Mill power draft changes dynamically with changes in feed rate, feed coarseness, mill water addition, sump water addition and pulp flow rate. Effects of mill overload, pulp hold-up, pulp density, interstices overfill and load slipping are included in the model.

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The hydrocyclone model is based on Plitt's (1980) equations. In Kramersh's model, because of the limited number of rock size fractions, statistical techniques are used to calculate the probability of a particle reporting to the underflow rather than the overflow. The hydrocyclone dynamics are instantaneous relative to the mill dynamics and it can be treated as a static element. The sump dynamics are described as a single perfect mixer with negligible size reduction occurring in its volume.

The simulation is not meant to model a particular mill but rather to illustrate important autogenous mill characteristics. Its qualitative validity and accuracy was verified by comparing the simulation with Stanley's paper on the behaviour of an autogenous mill. Limitations of the simulation include: the step changes in the breakage rates used for mill overload conditions as well as the linear transformations used to relate power consumption to load coarseness.

5.4.2 DETAILS OF ADAPTIVE OPTIMIZING REGULATOR

Problem Formulation: In order to simplify explanation and highlight the basic principles, we treat here a simplified implementation. The implementation is the simplest possible working solution that is easily extended to include additional mill measurements and additional mill inputs. For this initial study these additional variables only complicate the issue and make it difficult to choose design parameters and evaluate the regulators performance. A schematic of the autogenous grinding circuit incorporating the adaptive optimizing regulator is given in Figure 12 on page 72.

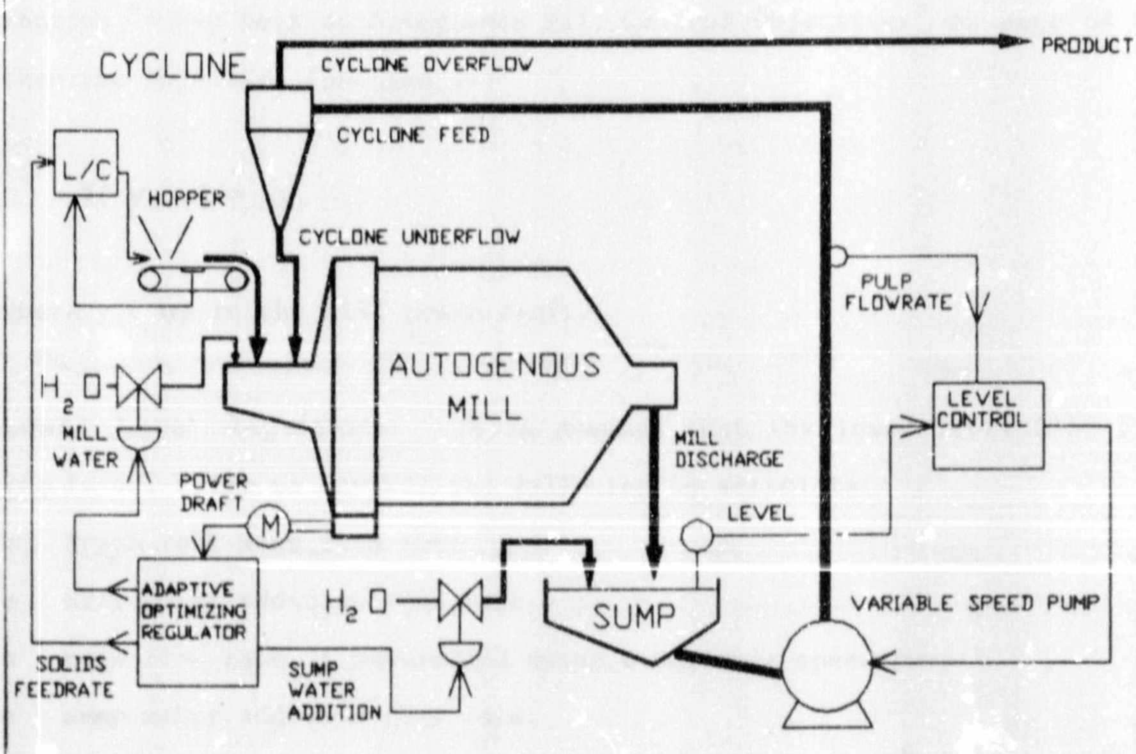

Figure 12. Autogenous Grinding Circuit incorporating an Adaptive Optimizing Regulator

The static optimization problem (defined in "Problem Simplification" on page 7) is:

$$\min_{\mathbf{u}} \ \Psi(\, \mathbf{y}(\mathbf{u}), \mathbf{u})$$

The control vector is:

$$\mathbf{u} = [u_1 \ u_2]^T$$

where $u_1 \equiv$ fresh rock feed rate and $u_2 \equiv$ mill water addition, an obvious additional input would be sump water addition.

Assume we choose to control the plant to achieve maximum power (see section "A New Look at Autogenous Mill Control Objectives" on page 64) then the objective function is:

$$\Psi(\mathbf{y}, \mathbf{u}) = -y_1$$

where $y_1(\mathbf{u})$ is the mill power draft.

Lower Level Regulators: It is assumed that the lower level SISO PI regulators exist to control the following to setpoints:

- fresh rock mass feed rate
- mill water addition flow rate
- pulp flow rate (controlled using a variable speed pump)
- sump water addition flow rate

The dynamics of these regulators are much faster than the mill dynamics and are ignored. A weakly tuned proportional controller adjusts the pulp flow rate setpoint to keep the sump level approximately constant. A tightly tuned sump level controller would prevent the sump from performing its function as a buffer between the mill and the pump. It is understood that a multi-variable regulator, having setpoints such as pulp density and sump level and using control inputs such as pulp flow rate and sump water addition, is preferable. The optimizer then determines the setpoints for the grinding circuit trajectory towards the optimum without violating constraints on, for example, pulp density and sump level. Also, the multi-variable regulator will keep the mill in a feasible operating region between optimizer moves. For this case, where the constraints are not simply constraints on the control vector $\mathbf{u}$ but are some function of $\mathbf{u}$, $g_i(\mathbf{u})$, the constrained optimization algorithm (se section "On-line Optimization" on page 33) requires that $g_i(\mathbf{u})$ are known. Because of the nature of the mill (large physical disturbances) these constraint functions would have to be identified on-line, this could be done in an identical way to that in which the objective $\Psi(\mathbf{u})$ is identified.

When running a simulation the mill operating environment can be carefully controlled so that constraints do not become active and the simpler unconstrained optimization algorithm can be used. As a starting point and in order to solve more fundamental issues the simulations in this report assume the mill is inside the allowed operating region and the optimization is constrained only by limits on the control vector $\mathbf{u}$. It is envisaged that the constrained case is simply a matter of extending the ideas for objective function model identification to constraint function identification and applying the constrained optimization linear program (section "Constrained Optimization" on page 36).

Dynamic Model: It is assumed that a second-order ARMA model will provide an adequate representation of the dynamic behaviour of the mill power. This can be justified on the grounds that, physically, certain effects contributing to power have slow dynamics compared with others. Also, from an optimization point of view, the search directions are determined using first derivatives only and a higher order model is therefore not justified. Simulations using a non-linear Hammerstein model (section "Choice of Model" on page 16) were performed, but criteria for comparison were difficult to establish and the advantage over a linear model was not clear, so the added complication was avoided. The predicted power is assumed to be given by the linear model:

$$\begin{aligned} y_1(t) = -a_1 y_1(t-1) - a_2 y_1(t-2) + b_{11} u_1(t-d) + b_{21} u_1(t-d-1) \\ + b_{12} u_2(t-d) + b_{22} u(t-d-1) + c \quad \ldots\ldots\ldots\ldots (1) \end{aligned}$$

where a_i and b_{ij} are the dynamic parameters and c is the dc value, all of which are estimated by the RLS estimator. Expressed compactly in regressive form the above model becomes:

$$y_1(t) = \phi(t-1)^T\hat{\theta}(t-1) \quad \ldots\ldots\ldots\ldots (2)$$

where

$$\phi(t-1)^T = [-y_1(t-1) \quad -y_1(t-2) \quad u_1(t-d) \quad u_1(t-d-1) \quad u_2(t-d) \quad u_2(t-d-1) \quad 1]$$

$$\hat{\theta}(t-1)^T = [a_1 \; a_2 \; b_{11} \; b_{21} \; b_{12} \; b_{22} \; c]$$

Parameter Estimation: A RLS estimator is used to estimate the seven model parameters. It is modified to allow continuous tracking of a changing plant; three different algorithms were tested, namely covariance resetting, exponential data weighting and the constant trace algorithm (described in section "Estimation of Model Parameters" on page 20).

Since no high frequency noise nor deterministic disturbances were introduced in the simulation the digital pre-filtering (described in "Coping with Deterministic Disturbances and Plant Noise" on page 24) of the model variables is not necessary.

It was found that a relative deadzone (described in section "Coping with Modelling Error within the Bandwidth of Interest" on page 29) is required to prevent estimator divergence due to plant model mismatch. The deadzone design parameters were chosen using rough guidelines and then checked and adjusted by simulation. The parameter ε_0 is chosen as of the order of the noise level in the system, ε_1 - ε_3 are chosen as weighting factors to account for the relative plant model mismatch expected from each variable. The relative deadzone time constant parameter σ_0 is chosen slower than the disturbance dynamics and also slow enough to give the estimator time to converge before the deadzone width changes. The deadzone width scaling constant β is selected by simulation because of the lack of a more systematic approach. The idea is to reduce β to improve parameter estimates while ensuring the estimator still converges.

Steady State Model Extraction: Assuming the variables in the above objective function model do not change with time, or equivalently setting $q^{-1}=1$ if the model is written in q^{-1} operator notation, gives the steady state model: (this procedure is described in "Extracting the Steady-State Model from the Dynamic Model" on page 32)

$$y_1(1 + a_1 + a_2) = u_1(b_{11} + b_{21}) + u_2(b_{12} + b_{22}) + c \quad .... (3)$$

Note that this is a linear model that will give accurate gradients as long as the model is a good linear approximation to the non-linear objective characteristic at the operating point. The gradient is only valid for the operating point at which the parameters have converged.

Gradient Calculation and Plant Moves Ψ can be minimized by means of a simple gradient search algorithm:

$$\mathbf{u}(k+1) = \mathbf{u}(k) - \mu \nabla_{\mathbf{u}} \Psi|_k \quad .... (4)$$

where $\nabla_{\mathbf{u}}$ denotes the gradient with respect to $\mathbf{u}$. This algorithm is executed regularly at times $t = nT_0$, n being a whole number. An analytic expression for the gradient in terms of the model parameters is obtained by applying the chain rule for differentiation.

$$\nabla_{\mathbf{u}} \Psi(\mathbf{y}, \mathbf{u}) \,|_k = \frac{\partial \Psi(\mathbf{y},\mathbf{u})}{\partial \mathbf{u}}\bigg|_k + \frac{\partial \Psi(\mathbf{u}, \mathbf{y})}{\partial \mathbf{y}}\bigg|_k \left(\frac{\partial \mathbf{y}}{\partial \mathbf{u}}\right)_k \quad ...(5)$$

In the present example the objective function is $\Psi(\mathbf{y}, \mathbf{u}) = -y_1$ and the gradient at k is:

$$\nabla_{\mathbf{u}}\Psi|_k = -\left[\begin{array}{cc} \frac{\partial y_1}{\partial u_1} & \frac{\partial y_1}{\partial u_2} \end{array}\right]_k^T$$

From the steady state model (equation (3)) we can calculate the gradient from the parameters:

$$\nabla_{\mathbf{u}}\Psi|_k = -\left[\begin{array}{cc} \frac{b_{11} + b_{21}}{1 + a_1 + a_2} & \frac{b_{12} + b_{22}}{1 + a_1 + a_2} \end{array}\right]_k^T$$

Equation (4) can now be used as the control law that drives the plant towards the objective characteristic minimum.

The control algorithm discussed above is implemented in the PASCAL program given in appendix G.

5.5 RESULTS OF SIMULATION

There are two many aspects and associated design parameters to do exhaustive tests. Ten simulation runs were performed and a representative simulation was chosen for documentation. The simulated economically significant disturbance is limited to that caused by a change in the size distribution of the fresh rock conveyed into the mill. The reason for this is that size distribution is the major significant disturbance and the other disturbances have intricate and difficult to understand effects. However, the optimizer should have no problem in coping with all persistent disturbances.

Two simulations are presented, the first (Figure 13 on page 80) shows how the optimizer tracks an **increase** in feed coarseness and the second (Figure 14 on page 81) shows how the optimizer tracks a **decrease** in feed coarseness. Apart from the disturbances, the simulation setup variables given in appendix J are identical for both runs. Only the variables relevant to optimizer performance are shown on the graphs. The simulations were run for 800 minutes (approximately 13 hours). The speed of the mill dynamics vary widely depending on mill size and rock breakage rates and so the time scale is relative. The mill measurement sampling rate is 30 seconds and the estimator uses every sample. This sampling rate is fast enough to capture all the important system modes. The optimization is performed after every 50 samples, corresponding to an update time of 25 minutes; this is ample time for the estimator to converge to new parameters after an input move. Between input moves there is very little spectral content in the input signal and it is necessary to add a 5 percent perturbation or excitation signal (bandlimited Gaussian noise) to ensure persistently exciting input signals for the estimator. The covariance matrix is reset just after an input move so the estimator has 50 samples to converge again before the model parameters are used in the next move. The initial solids feed rate and initial mill water addition are chosen to keep the mill in an unconstrained operating region. On a real plant this would correspond to only switching in the optimizer once the mill is in the region of the desired operating point. The optimizer is only allowed to make plant moves after the estimator has converged and the model is accurate. Looking at the error between the predicted and measured power and also at the model parameter variance an estimation start time of 50 minutes was selected. Maximum and minimum gradients were limited, this is purely a safety net in case of numerical problems when dividing small numbers. In this simulation the gradient never needs to be limited.

Important model and estimator variables dumped to a file at strategic times are given in appendix N. The small estimation error and small