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Radicals and anti-radicals of near-rings and matrix near-rings

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Abstract

In this thesis the ideal structure of near-rings and the associated matrix near-rings is studied. Firstly, two aspects of near-rings are investigated, these are: the decomposition of a near-ring; and the nilpotency of the Jacobson s-radical, via the nil-rigid series. Secondly, the interplay of ideals of a near-ring and the ideals of the associated matrix near-ring is investigated.

The socle ideal (an anti-radical) of a near-ring R is characterized as an intersection of annihilators of R-groups of a special type, called type-$\mathcal{K}$. That is, the socle ideal is an **annihilator ideal**.

The R-groups of type-$\mathcal{K}$ are shown to separate the Jacobson 0-radical and the Jacobson s-radical, if there is at least one R-group of type-$\mathcal{K}$ contained in some type-0 R-group.

Alternating chains of R-groups of type-0 and type-$\mathcal{K}$ are defined and will be called **Ω-chains**. It is then proved that the maximal length of the Ω-chains give a lower bound on the nil-rigid length of the near-ring. Consequently, one can directly determine that the nil-rigid length of the matrix near-ring, $\mathbb{M}_n(R)$, is equal or bigger than the nil-rigid length of R from the structure of any one of the finite faithful R-groups of R.

For an ideal I being a Jacobson type radical, socle ideal or s-socle ideal of a near-ring, open-questions on the relationship between I^+, I^* and the matrix near-ring ideals of the same type are answered.

A systematic theory is developed that connects each annihilator ideal I of R to its corresponding matrix near-ring ideal, $\mathcal{I}$. This is made possible by the use of Action 2 of the matrix near-ring on the $\mathbb{M}_n(R)$-groups. Specific conditions are given which ensure that $I^+ \subseteq \mathcal{I}$ and $\mathcal{I} \subseteq I^*$. Examples illustrate cases in which no such relationships exist. Necessary and sufficient conditions are given for $I^+ \subseteq \mathcal{I} \subseteq I^*$ to hold.

This thesis thus illustrates the relationship between the structure of a faithful R-group and the ideal structure of the associated matrix near-rings.